Computer Algebra Independent Integration Tests

Summer 2023 edition

1-Algebraic-functions/1.1-Binomial-products/1.1.3-General/27-1.1.3.4-e-x-^m-a+b-x^n-^p-c+d-x^n-^q

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [1081]. This is test number [27].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (1081)	0.00 (0)
Mathematica	100.00 (1081)	0.00 (0)
Maple	84.27 (911)	15.73 (170)
Fricas	75.21 (813)	24.79 (268)
Giac	52.82 (571)	47.18 (510)
Mupad	49.12 (531)	50.88 (550)
Sympy	37.56 (406)	62.44 (675)
Maxima	36.17 (391)	63.83 (690)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

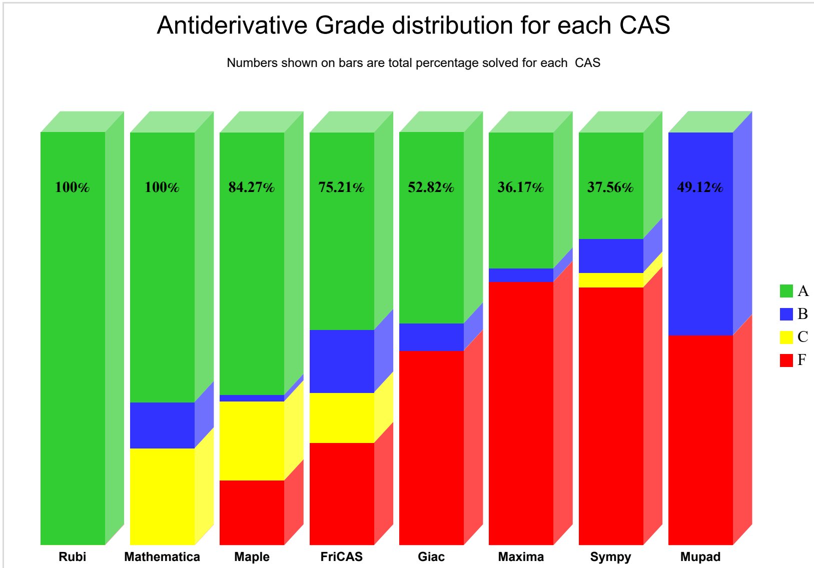
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

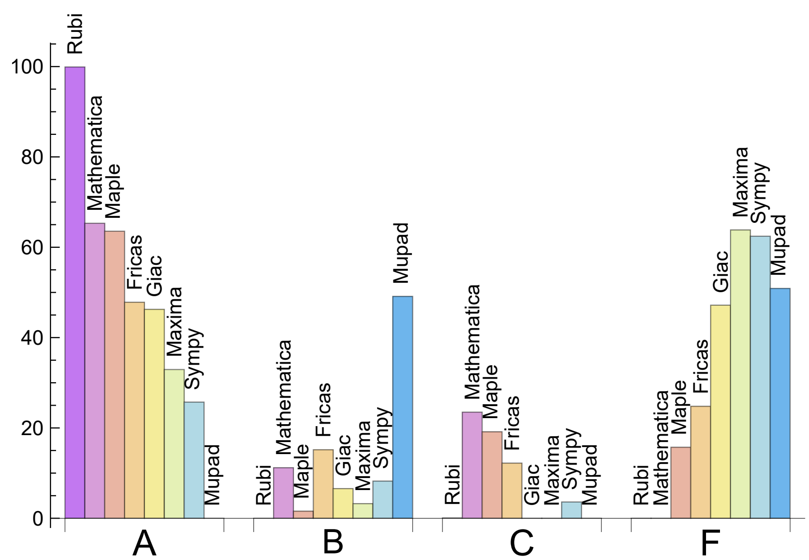
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.907	0.000	0.093	0.000
Mathematica	65.310	11.193	23.497	0.000
Maple	63.552	1.573	19.149	15.726
Fricas	47.826	15.171	12.211	24.792
Giac	46.253	6.568	0.000	47.179
Maxima	32.932	3.238	0.000	63.830
Sympy	25.717	8.233	3.608	62.442
Mupad	0.000	49.121	0.000	50.879

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	170	100.00	0.00	0.00
Fricas	268	39.93	57.46	2.61
Giac	510	95.88	0.20	3.92
Mupad	550	0.00	100.00	0.00
Sympy	675	79.70	18.22	2.07
Maxima	690	89.28	0.00	10.72

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.15
Maxima	0.25
Giac	0.35
Fricas	0.85
Mathematica	3.54
Maple	5.15
Mupad	8.09
Sympy	11.53

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	119.81	1.27	97.00	1.02
Mathematica	143.86	1.22	112.00	0.95
Giac	154.59	1.30	116.00	1.05
Sympy	204.11	1.93	114.00	1.18
Rubi	208.52	1.00	115.00	1.00
Maple	274.80	1.84	133.00	0.96
Mupad	439.68	2.41	117.00	1.12
Fricas	608.80	3.52	222.00	2.04

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

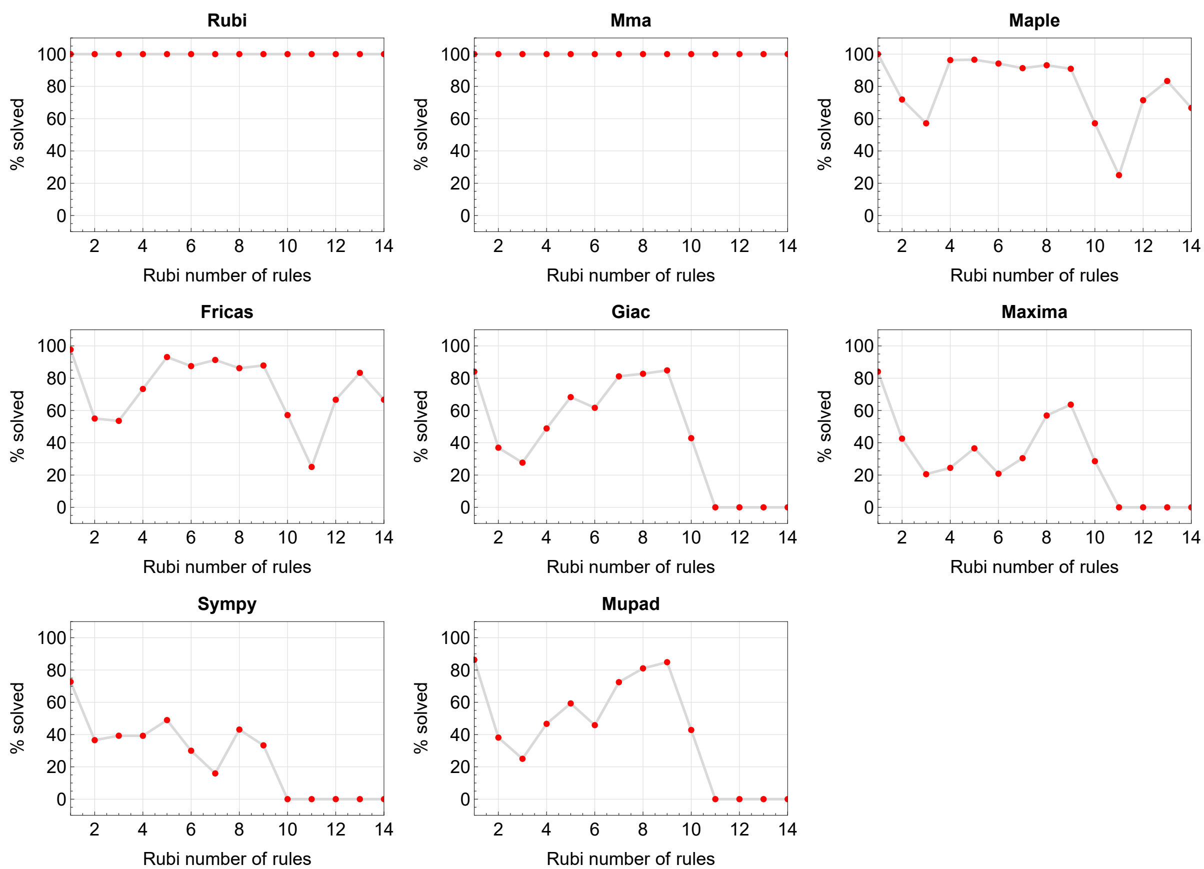


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

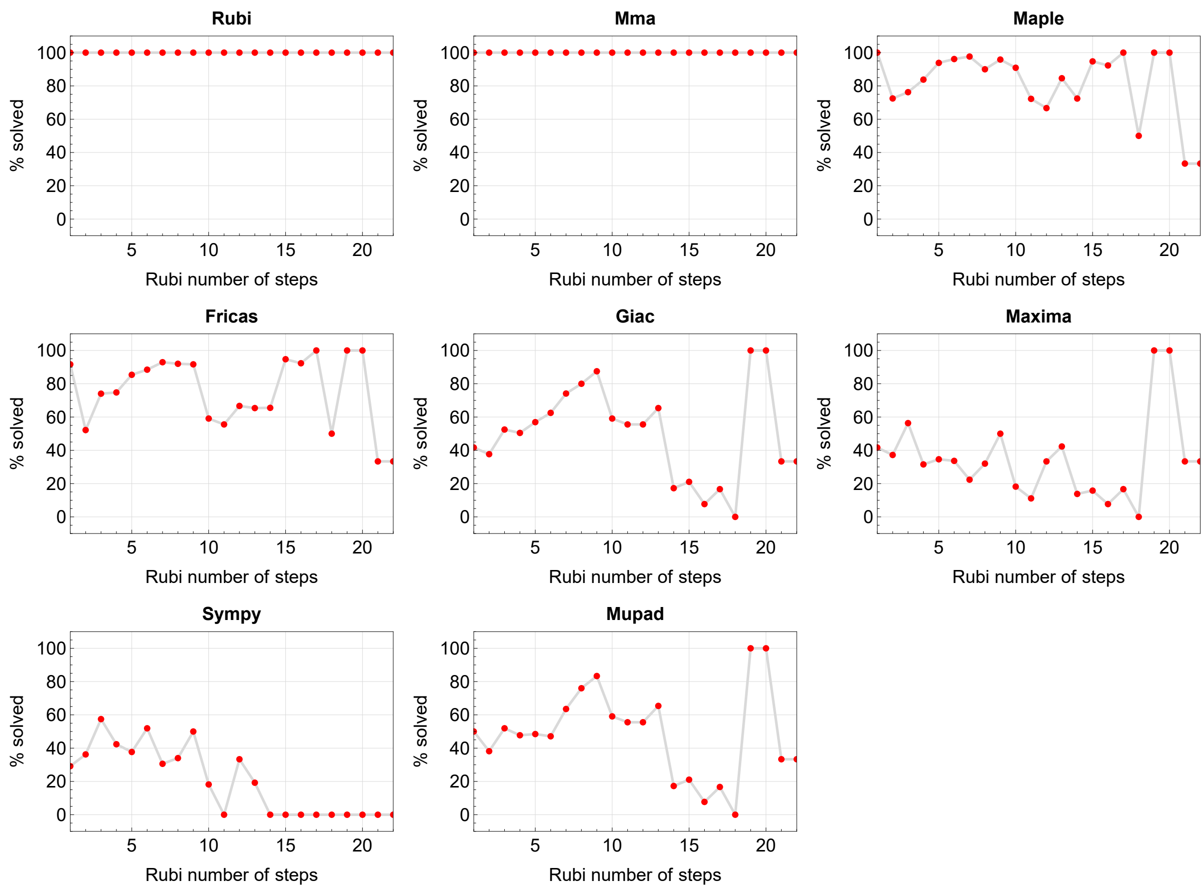


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

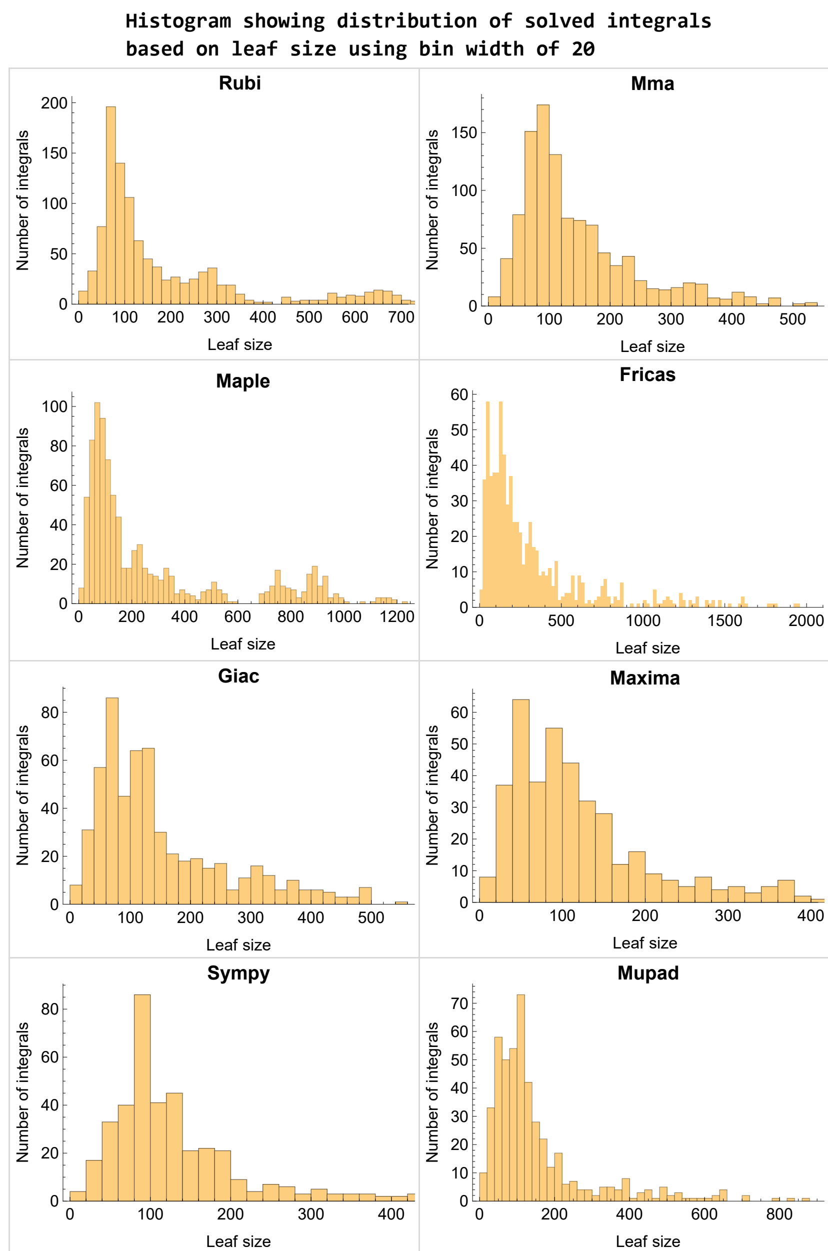


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

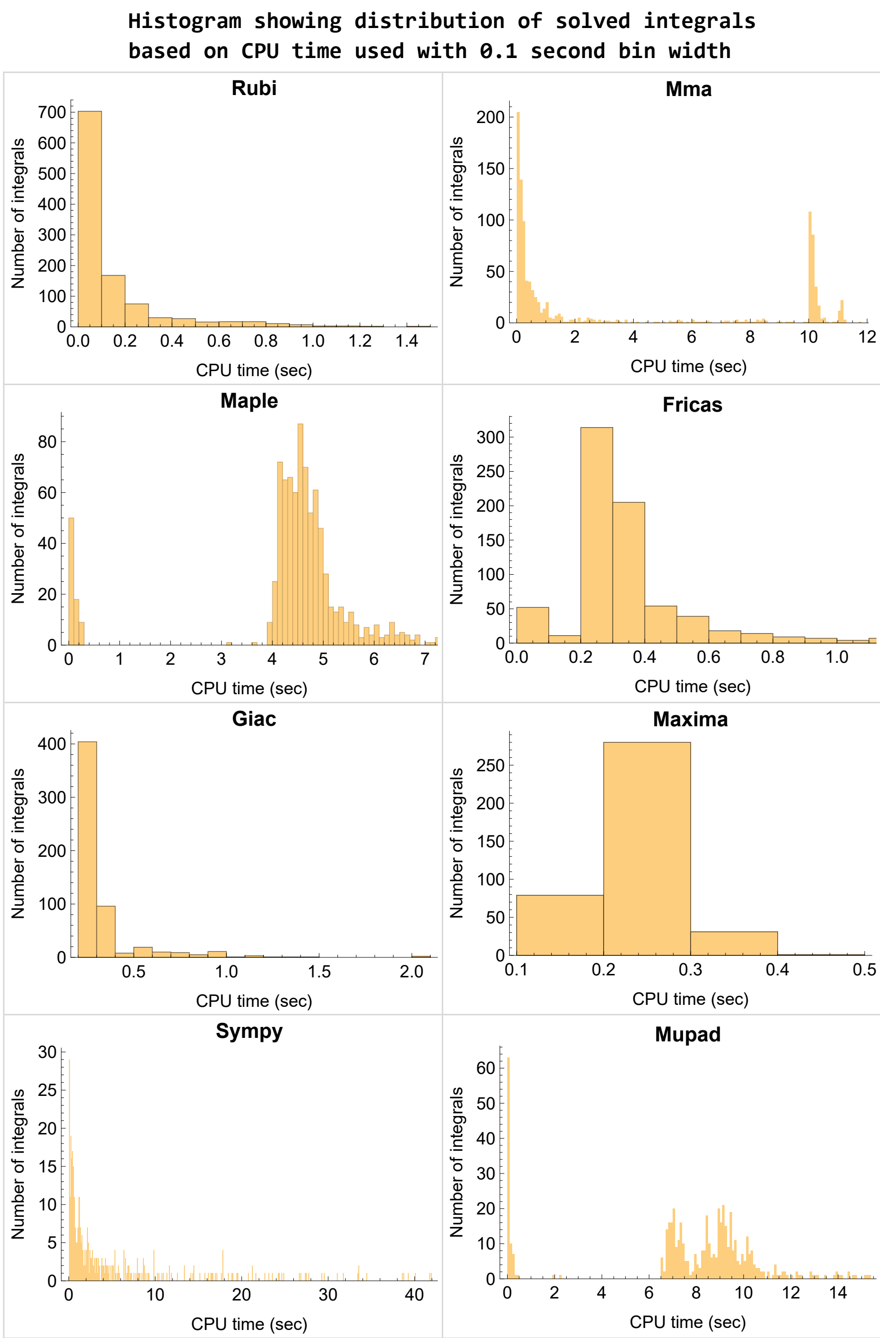


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

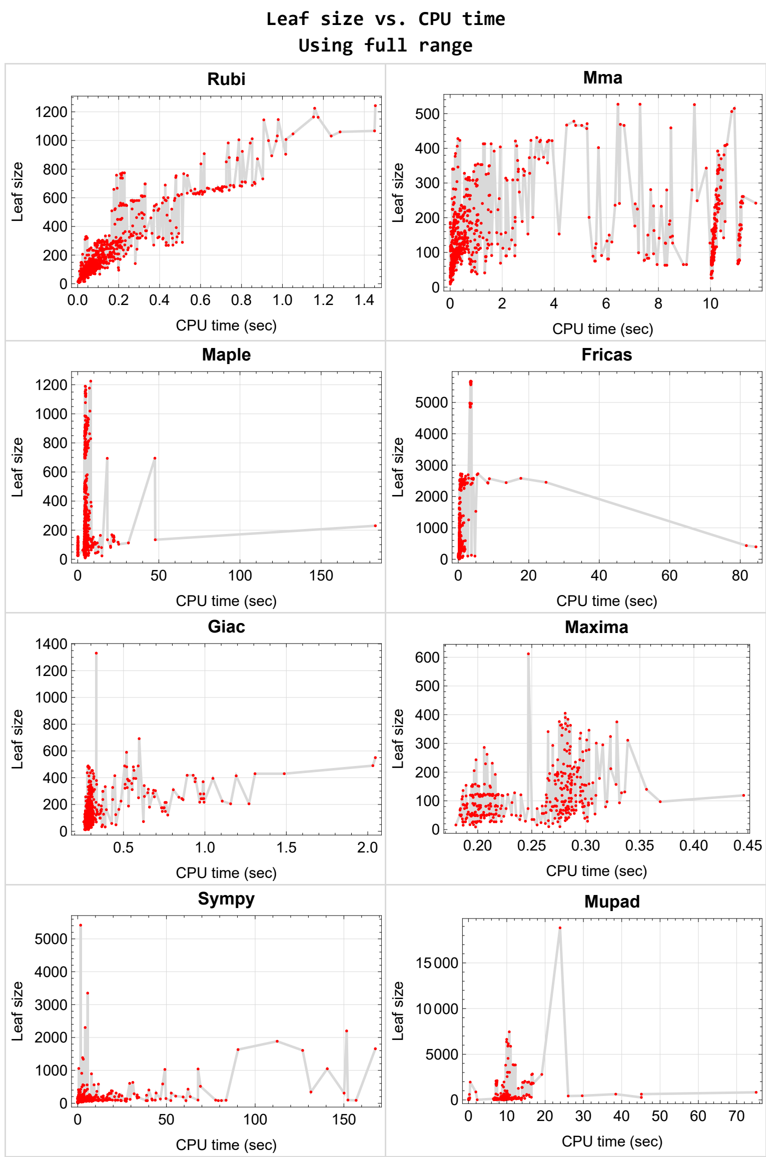


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {795, 797, 799, 800, 819, 821}

Mathematica {267, 268, 269, 279, 280, 322, 323, 324, 338, 339, 340, 341, 363, 365, 367, 373, 375, 377, 385, 387, 393, 395, 397, 421, 437, 438, 439, 440, 441, 455, 456, 457, 458, 459, 465, 467, 469, 475, 477, 479, 485, 487, 489, 495, 497, 499, 513, 515, 581, 582, 584, 585, 634, 636, 637, 672, 673, 674, 675, 676, 709, 710, 711, 712, 713, 743, 745, 746, 796, 798, 800, 815, 817, 818, 834, 835, 836, 837, 866, 883, 905, 926, 994, 1002, 1003, 1011, 1012}

Maple {264, 265, 266, 267, 268, 269, 275, 276, 277, 278, 279, 280, 289, 290, 291, 292, 293, 294, 302, 303, 304, 305, 306, 307, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 363, 364, 365, 366, 367, 373, 374, 375, 376, 377, 383, 384, 385, 386, 387, 393, 394, 395, 396, 397, 405, 406, 407, 408, 409, 410, 418, 419, 420, 422, 423, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 449, 450, 451, 452, 453, 454, 456, 457, 458, 459, 465, 466, 467, 468, 469, 475, 476, 477, 478, 479, 485, 486, 487, 488, 489, 495, 496, 497, 498, 499, 634, 637, 795, 796, 797, 798, 799, 800, 815, 816, 817, 818, 819, 820, 821}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

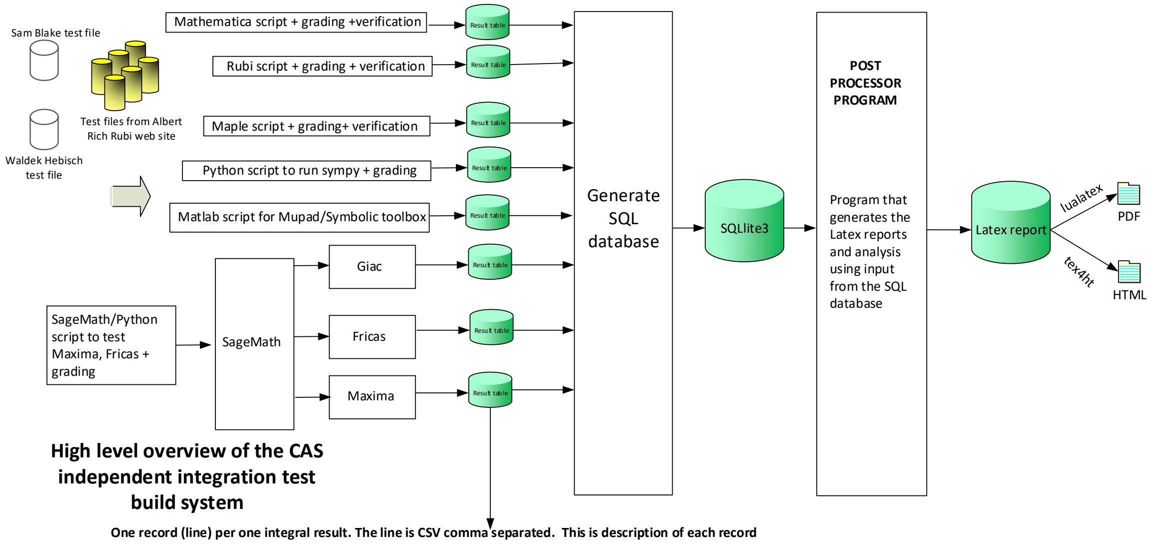
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 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621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081 }

B grade { }

C grade { 455 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 196, 197, 198, 199, 200, 201, 213, 214, 215, 216, 217, 218, 228, 229, 230, 231, 232, 233, 244, 245, 246, 247, 248, 259, 260, 261, 262, 263, 270, 271, 272, 273, 274, 278, 282, 283, 284, 285, 286, 287, 288, 295, 296, 297, 298, 299, 300, 301, 308, 309, 310, 311, 312, 313, 314, 321, 325, 326, 327, 328, 329, 330, 331, 358, 359, 360, 361, 362, 364, 368, 369, 370, 371, 372, 378, 379, 380, 381, 382, 383, 384, 388, 389, 390, 391, 392, 398, 399, 400, 401, 402, 403, 404, 411, 412, 413, 414, 415, 416, 417, 424, 425, 426, 427, 428, 429, 430, 442, 443, 444, 445, 446, 447, 448, 460, 461, 462, 463, 464, 470, 471, 472, 473, 474, 480, 481, 482, 483, 484, 490, 491, 492, 493, 494, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 516, 519, 522, 525, 527, 530, 533, 535, 538, 541, 543, 546, 549, 551, 554, 557, 559, 562, 565, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 583, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 620, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 658, 659, 660, 661, 662, 663, 664, 677, 678, 679, 680, 681, 682, 683, 693, 696, 697, 698, 699, 700, 701, 714, 715, 716, 717, 718, 719, 720, 728, 729, 732, 733, 734, 735, 736, 737, 744, 747, 748, 749, 750, 751, 752, 753, 763, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 801, 802, 803, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 841, 842, 843, 844, 846, 847, 848, 849, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 903, 904, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1008, 1009, 1010, 1011, 1012, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1054, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078 }

B grade { 30, 54, 267, 268, 269, 279, 280, 322, 323, 324, 338, 339, 340, 341, 363, 365, 366, 367, 373, 374, 375, 376, 377, 385, 386, 387, 393, 394, 395, 396, 397, 437, 438, 439, 440, 441, 456, 457, 458, 459, 465, 466, 467, 468, 469, 475, 476, 477, 478, 479, 485, 486, 487, 488, 489, 495, 496, 497, 498, 499, 513, 514, 515, 672, 673, 674, 675, 676, 691, 692, 694, 695, 709, 710, 711, 712, 713, 727, 730, 731, 743, 745, 746, 761, 762, 764, 765, 766, 804, 845, 850, 866, 867, 868, 869, 880, 881, 882, 883, 884, 885, 886, 905, 906, 907, 924, 925, 926, 927, 928, 1004, 1005, 1006, 1007, 1013, 1014, 1015, 1016, 1051, 1052, 1053 }

C grade { 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 219, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 264, 265, 266, 275, 276, 277, 281, 289, 290, 291, 292, 293, 294, 302, 303, 304, 305, 306, 307, 315, 316, 317, 318, 319, 320, 332, 333, 334, 335, 336, 337, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 405, 406, 407, 408, 409, 410, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 449, 450, 451, 452, 453, 454, 455, 517, 518, 520, 521, 523, 524, 526, 528, 529, 531, 532, 534, 536, 537, 539, 540, 542, 544, 545, 547, 548, 550, 552, 553, 555, 556, 558, 560, 561, 563, 564, 566, 581, 582, 584, 585, 600, 601, 602, 603, 604, 618, 619, 621, 622, 634, 635, 636, 637, 652, 653, 654, 655, 656, 657, 665, 666, 667, 668, 669, 670, 671, 684, 685, 686, 687, 688, 689, 690, 702, 703, 704, 705, 706, 707, 708, 721, 722, 723, 724, 725, 726, 738, 739, 740, 741, 742, 754, 755, 756, 757, 758, 759, 760, 795, 796, 797, 798, 799, 800, 815, 816, 817, 818, 819, 820, 821, 834, 835, 836, 837, 838, 839, 840, 897, 898, 899, 900, 901, 902, 918, 919, 920, 921, 922, 923, 1024, 1055, 1079, 1080, 1081 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 57, 60, 63, 64, 65, 66, 67, 68, 69, 70, 72, 75, 78, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 126, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 270, 271, 272, 273, 274, 282, 283, 284, 285, 286, 287, 288, 295, 296, 297, 298, 299, 300, 301, 308, 309, 310, 311, 312, 313, 314, 325, 326, 327, 328, 329, 330, 331, 358, 359, 360, 361, 362, 368, 369, 370, 371, 372, 378, 379, 380, 381, 382, 388, 389, 390, 391, 392, 398, 399, 400, 401, 402, 403, 404, 411, 412, 413, 414, 415, 416, 417, 421, 424, 425, 426, 427, 428, 429, 430, 442, 443, 444, 445, 446, 447, 448, 455, 460, 461, 462, 463, 464, 470, 471, 472, 473, 474, 480, 481, 482, 483, 484, 490, 491, 492, 493, 494, 516, 519, 522, 525, 527, 530, 533, 535, 538, 541, 543, 546, 549, 551, 554, 557, 559, 562, 565, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 638, 639, 640, 641, 642, 643, 646, 647, 648, 649, 650, 651, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 1002, 1003, 1004, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1015, 1016, 1017, 1023, 1029, 1036, 1039, 1040, 1041, 1042, 1043, 1044, 1046, 1047, 1048, 1049, 1050, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1078, 1079, 1080 }

B grade { 30, 54, 124, 125, 644, 645, 1005, 1014, 1024, 1045, 1051, 1052, 1053, 1054, 1055, 1065, 1081 }

C grade { 56, 58, 59, 61, 62, 71, 73, 74, 76, 77, 79, 80, 95, 96, 97, 98, 99, 100, 101, 102, 264, 265, 266, 267, 268, 269, 275, 276, 277, 278, 279, 280, 281, 289, 290, 291, 292, 293, 294, 302, 303, 304, 305, 306, 307, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 363, 364, 365, 366, 367, 373, 374, 375, 376, 377, 383, 384, 385, 386, 387, 393, 394, 395, 396, 397, 405, 406, 407, 408, 409, 410, 418, 419, 420, 422, 423, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 449, 450, 451, 452, 453, 454, 456, 457, 458, 459, 465, 466, 467, 468, 469, 475, 476, 477, 478, 479, 485, 486, 487, 488, 489, 495, 496, 497, 498, 499, 517, 518, 520, 521, 523, 524, 526, 528, 529, 531, 532, 534, 536, 537, 539, 540, 542, 544, 545, 547, 548, 550, 552, 553, 555, 556, 558, 560, 561, 563, 564, 566, 634, 637, 795, 796, 797, 798, 799, 800, 815, 816, 817, 818, 819, 820, 821, 834, 835, 836, 837, 838, 839, 840 }

F normal fail { 127, 128, 129, 130, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 581, 582, 583, 584, 585, 600, 601, 602, 603, 604, 618, 619, 620, 621, 622, 635, 636, 652, 653, 654, 655, 656, 657, 672, 673, 674, 675, 676, 691, 692, 693, 694, 695, 709, 710, 711, 712, 713, 727, 728, 729, 730, 731, 743, 744, 745, 746, 761, 762, 763, 764, 765, 766, 801, 802, 803, 804, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 863, 864, 865, 866, 867, 868, 869, 880, 881, 882, 883, 884, 885, 886, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1018, 1019, 1020, 1021, 1022, 1025, 1026, 1027, 1028, 1030, 1031, 1032, 1033, 1034, 1035, 1037, 1038, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 95, 96, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 158, 161, 163, 166, 169, 171, 174, 177, 179, 180, 181, 182, 183, 184, 196, 197, 198, 199, 200, 201, 213, 214, 215, 216, 217, 218, 228, 229, 230, 231, 232, 233, 244, 245, 246, 247, 248, 259, 260, 261, 262, 263, 270, 271, 272, 273, 274, 282, 283, 284, 285, 286, 287, 288, 295, 296, 297, 298, 299, 300, 301, 308, 309, 310, 311, 312, 313, 314, 325, 326, 327, 328, 329, 330, 331, 358, 359, 360, 361, 362, 368, 369, 370, 371, 372, 378, 379, 380, 381, 382, 390, 398, 399, 400, 401, 402, 403, 404, 411, 412, 413, 414, 415, 416, 417, 424, 425, 426, 427, 428, 429, 430, 442, 443, 444, 445, 446, 447, 448, 460, 461, 462, 464, 470, 471, 472, 473, 474, 481, 483, 484, 506, 509, 516, 519, 522, 525, 527, 530, 533, 535, 538, 541, 543, 546, 549, 551, 554, 557, 559, 562, 565, 567, 568, 569, 570, 572, 573, 574, 575, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 605, 606, 607, 608, 609, 611, 612, 623, 624, 625, 626, 627, 628, 629, 630, 634, 637, 638, 639, 640, 642, 643, 644, 658, 659, 660, 661, 662, 663, 664, 665, 667, 682, 683, 696, 697, 698, 699, 700, 701, 702, 703, 714, 715, 719, 720, 721, 722, 751, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 787, 788, 789, 790, 791, 792, 793, 794, 805, 806, 807, 808, 809, 810, 811, 814, 822, 824, 826, 827, 828, 829, 833, 853, 854, 855, 856, 857, 858, 859, 862, 871, 873, 874, 875, 879, 887, 888, 889, 890, 891, 892, 893, 896, 909, 911, 912, 913, 917, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1015, 1016, 1017, 1023, 1024, 1029, 1039, 1040, 1041, 1042, 1043, 1045, 1046, 1047, 1048, 1049, 1056, 1057, 1058, 1059, 1064, 1066, 1067, 1068, 1069, 1071, 1072, 1073, 1074, 1075, 1078, 1079, 1080, 1081 }

B grade { 30, 54, 93, 94, 97, 98, 99, 100, 101, 102, 103, 104, 124, 125, 126, 156, 157, 159, 160, 162, 164, 165, 167, 168, 170, 172, 173, 175, 176, 178, 267, 268, 269, 276, 278, 279, 280, 317, 321, 322, 323, 324, 338, 339, 340, 341, 350, 351, 352, 353, 354, 355, 356, 357, 388, 389, 391, 392, 437, 438, 439, 441, 456, 457, 458, 459, 463, 480, 482, 490, 491, 492, 493, 494, 507, 508, 571, 576, 577, 596, 614, 615, 616, 617, 620, 631, 632, 633, 641, 645, 646, 647, 648, 649, 650, 651, 666, 677, 678, 679, 680, 681, 684, 685, 686, 716, 717, 718, 732, 733, 734, 735, 736, 737, 738, 739, 747, 748, 749, 750, 752, 753, 754, 755, 812, 813, 823, 825, 830, 831, 832, 860, 861, 870, 872, 876, 877, 878, 894, 895, 908, 910, 914, 915, 916, 949, 975, 1014, 1036, 1044, 1050, 1051, 1052, 1053, 1054, 1055, 1060, 1061, 1062, 1063, 1065, 1070, 1076, 1077 }

C grade { 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 219, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 264, 265, 266, 275, 277, 281, 289, 290, 291, 292, 293, 294, 302, 303, 304, 306, 307, 315, 316, 318, 319, 320, 332, 333, 334, 335, 336, 337, 342, 343, 344, 345, 346, 347, 348, 349, 405, 406, 407, 408, 409, 410, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 449, 450, 451, 452, 453, 454, 455, 526, 550, 555, 556, 558, 560, 561, 563, 564, 566, 610, 613, 779, 780, 781, 782, 783, 784, 785, 786 }

F normal fail { 127, 128, 129, 130, 500, 501, 502, 503, 504, 505, 510, 511, 512, 513, 514, 515, 517, 518, 520, 521, 523, 524, 528, 529, 531, 532, 534, 536, 537, 539, 540, 542, 544, 545, 547, 548, 552, 553, 618, 619, 621, 622, 635, 636, 652, 653, 654, 655, 656, 657, 796, 801, 804, 816, 819, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 867, 880, 884, 899, 900, 906, 927, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1018, 1019, 1020, 1021, 1022, 1025, 1026, 1027, 1028, 1030, 1031, 1032, 1033, 1034, 1035, 1037, 1038 }

F(-1) timedout fail { 305, 363, 364, 365, 373, 374, 375, 376, 377, 383, 384, 385, 386, 387, 395, 396, 397, 440, 465, 466, 467, 468, 469, 475, 476, 477, 478, 479, 485, 486, 487, 488, 489, 495, 496, 497, 498, 499, 578, 579, 580, 581, 582, 583, 584, 585, 597, 598, 599, 600, 601, 602, 603, 604, 668, 669, 670, 671, 672, 673, 674, 675, 676, 687, 688, 689, 690, 691, 692, 693, 694, 695, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 723, 724, 725, 726, 727, 728, 729, 730, 731, 740, 741, 742, 743, 744, 745, 746, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 795, 797, 798, 799, 800, 802, 803, 815, 817, 818, 820, 821, 834, 835, 836, 837, 838, 839, 840, 864, 865, 868, 869, 881, 882, 883, 885, 886, 897, 898, 901, 902, 903, 905, 907, 918, 919, 920, 921, 922, 923, 924, 925, 926, 928 }

F(-2) exception fail { 366, 367, 393, 394, 863, 866, 904 }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 196, 197, 198, 199, 200, 201, 213, 214, 215, 216, 228, 229, 230, 231, 244, 245, 246, 247, 248, 259, 260, 261, 270, 271, 272, 282, 283, 284, 285, 295, 296, 297, 298, 308, 309, 310, 311, 325, 326, 327, 328, 398, 399, 400, 401, 411, 412, 413, 414, 424, 425, 426, 427, 442, 443, 444, 445, 525, 567, 568, 569, 570, 586, 587, 588, 589, 605, 606, 607, 608, 609, 623, 624, 625, 626, 638, 639, 640, 641, 642, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 964, 965, 966, 967, 968, 969, 970, 975, 976, 977, 978, 979, 980, 981, 982, 983, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1015, 1016, 1017, 1023, 1024, 1029, 1036, 1039, 1040, 1041, 1042, 1043, 1045, 1046, 1047, 1048, 1049, 1055, 1056, 1057, 1058, 1064, 1078, 1079, 1080, 1081 }

B grade { 30, 54, 184, 217, 218, 232, 233, 929, 930, 943, 944, 945, 959, 960, 961, 962, 963, 971, 972, 973, 974, 984, 985, 1014, 1044, 1050, 1051, 1052, 1053, 1054, 1059, 1060, 1061, 1062, 1063 }

C grade { }

F normal fail { 127, 128, 129, 130, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 219, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 262, 263, 264, 265, 266, 267, 268, 269, 273, 274, 275, 276, 277, 278, 279, 280, 281, 286, 287, 288, 289, 290, 291, 292, 293, 294, 299, 300, 301, 302, 303, 304, 305, 306, 307, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 361, 362, 363, 364, 365, 366, 367, 371, 372, 373, 374, 375, 376, 377, 381, 382, 383, 384, 385, 386, 387, 391, 392, 393, 394, 395, 396, 397, 402, 403, 404, 405, 406, 407, 408, 409, 410, 415, 416, 417, 418, 419, 420, 421, 422, 423, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 463, 464, 465, 466, 467, 468, 469, 473, 474, 475, 476, 477, 478, 479, 483, 484, 485, 486, 487, 488, 489, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 788, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1018, 1019, 1020, 1021, 1022, 1025, 1026, 1027, 1028, 1030, 1031, 1032, 1033, 1034, 1035, 1037, 1038, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077 }

F(-1) timedout fail { }

F(-2) exception fail { 358, 359, 360, 368, 369, 370, 378, 379, 380, 388, 389, 390, 460, 461, 462, 470, 471, 472, 480, 481, 482, 490, 491, 492, 506, 507, 508, 509, 658, 659, 660, 661, 677, 678, 679, 680, 696, 697, 698, 714, 715, 716, 717, 718, 732, 733, 734, 735, 747, 748, 749, 750, 751, 787, 789, 805, 806, 807, 822, 823, 824, 825, 853, 854, 855, 870, 871, 872, 887, 888, 889, 908, 909, 910 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 196, 197, 198, 199, 200, 201, 213, 214, 215, 216, 217, 218, 228, 229, 230, 231, 232, 233, 244, 245, 246, 247, 248, 259, 260, 261, 262, 263, 270, 271, 272, 273, 274, 282, 283, 284, 285, 286, 287, 288, 295, 296, 297, 298, 299, 300, 301, 308, 309, 310, 311, 312, 313, 314, 325, 326, 327, 328, 329, 330, 331, 358, 359, 360, 361, 362, 368, 369, 370, 371, 372, 378, 379, 380, 381, 382, 388, 389, 390, 391, 392, 398, 399, 400, 401, 402, 403, 404, 411, 412, 413, 414, 415, 416, 417, 424, 425, 426, 427, 428, 429, 430, 442, 443, 444, 445, 446, 447, 448, 460, 461, 462, 463, 464, 470, 471, 472, 473, 474, 480, 481, 482, 483, 484, 490, 491, 492, 493, 494, 506, 507, 519, 525, 543, 546, 549, 551, 554, 559, 562, 567, 568, 569, 570, 571, 572, 573, 586, 587, 588, 589, 590, 591, 592, 605, 606, 607, 608, 609, 610, 611, 623, 624, 625, 626, 627, 628, 638, 639, 640, 641, 642, 643, 644, 658, 659, 660, 661, 662, 663, 664, 677, 678, 681, 682, 683, 696, 698, 699, 700, 701, 714, 715, 716, 717, 718, 719, 720, 732, 733, 734, 735, 736, 737, 747, 748, 749, 752, 753, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 789, 791, 793, 805, 806, 807, 808, 809, 812, 813, 822, 823, 824, 825, 826, 827, 853, 854, 855, 856, 857, 860, 870, 871, 872, 873, 874, 887, 888, 889, 890, 891, 894, 895, 908, 909, 910, 911, 912, 929, 930, 931, 937, 938, 939, 940, 942, 943, 944, 945, 946, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 968, 969, 971, 972, 973, 974, 975, 976, 980, 981, 982, 984, 985, 1002, 1003, 1009, 1010, 1011, 1012, 1013, 1015, 1016, 1017, 1023, 1051, 1056, 1057, 1058, 1060, 1061, 1062, 1064, 1078 }

B grade { 27, 30, 54, 124, 125, 126, 508, 509, 516, 527, 530, 535, 538, 557, 679, 680, 697, 750, 751, 792, 794, 814, 828, 829, 830, 831, 832, 833, 858, 859, 875, 877, 896, 913, 914, 915, 916, 917, 932, 933, 934, 935, 936, 941, 947, 948, 949, 950, 951, 952, 965, 966, 967, 970, 977, 978, 979, 983, 1004, 1005, 1007, 1008, 1014, 1024, 1052, 1053, 1054, 1055, 1079, 1080, 1081 }

C grade { }

F normal fail { 127, 128, 129, 130, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 219, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 264, 265, 266, 267, 268, 269, 275, 276, 277, 278, 279, 280, 281, 289, 290, 291, 292, 293, 294, 302, 303, 304, 305, 306, 307, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 363, 364, 365, 366, 367, 373, 374, 375, 376, 377, 383, 384, 385, 386, 387, 393, 394, 395, 396, 397, 405, 406, 407, 408, 409, 410, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 465, 466, 467, 468, 469, 475, 476, 477, 478, 479, 485, 486, 487, 488, 489, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 510, 511, 512, 513, 514, 515, 517, 518, 520, 521, 523, 524, 526, 528, 529, 531, 532, 534, 536, 537, 539, 540, 542, 544, 545, 547, 548, 550, 552, 553, 555, 556, 558, 560, 561, 563, 564, 566, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 629, 630, 631, 632, 633, 634, 635, 636, 637, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 738, 739, 740, 741, 742, 743, 744, 745, 746, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 815, 816, 817, 818, 819, 820, 821, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 861, 862, 863, 864, 865, 866, 867, 868, 869, 876, 878, 879, 880, 881, 882, 883, 884, 885, 886, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 998, 999, 1000, 1001, 1018, 1019, 1020, 1021, 1022, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1059, 1063, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077 }

F(-1) timedout fail { 1006 }

F(-2) exception fail { 350, 351, 352, 353, 354, 355, 356, 357, 522, 533, 541, 565, 788, 790, 810, 811, 892, 893, 996, 997 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 196, 197, 198, 199, 200, 201, 213, 214, 215, 216, 217, 218, 228, 229, 230, 231, 232, 233, 244, 245, 246, 247, 248, 259, 260, 261, 262, 263, 270, 271, 272, 273, 274, 276, 281, 282, 283, 284, 285, 286, 287, 288, 295, 296, 297, 298, 299, 300, 301, 308, 309, 310, 311, 312, 313, 314, 317, 325, 326, 327, 328, 329, 330, 331, 358, 359, 360, 361, 362, 368, 369, 370, 371, 372, 378, 379, 380, 381, 382, 388, 389, 390, 391, 392, 398, 399, 400, 401, 402, 403, 404, 411, 412, 413, 414, 415, 416, 417, 424, 425, 426, 427, 428, 429, 430, 442, 443, 444, 445, 446, 447, 448, 460, 461, 462, 463, 464, 470, 471, 472, 473, 474, 480, 481, 482, 483, 484, 490, 491, 492, 493, 494, 506, 507, 508, 509, 557, 562, 565, 567, 568, 569, 570, 571, 572, 573, 586, 587, 588, 589, 590, 591, 592, 605, 606, 607, 608, 609, 610, 611, 623, 624, 625, 626, 627, 628, 638, 639, 640, 641, 642, 643, 644, 658, 659, 660, 661, 662, 663, 664, 677, 678, 679, 680, 681, 682, 683, 696, 697, 698, 699, 700, 701, 714, 715, 716, 717, 718, 719, 720, 732, 733, 734, 735, 736, 737, 747, 748, 749, 750, 751, 752, 753, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 789, 791, 793, 805, 806, 807, 808, 809, 822, 823, 824, 825, 826, 827, 853, 854, 855, 856, 857, 870, 871, 872, 873, 874, 887, 888, 889, 890, 891, 908, 909, 910, 911, 912, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 959, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 1002, 1003, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1014, 1015, 1016, 1017, 1023, 1024, 1029, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1078, 1079, 1080, 1081 }

C grade { }

F normal fail { }

F(-1) timedout fail { 127, 128, 129, 130, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 219, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 264, 265, 266, 267, 268, 269, 275, 277, 278, 279, 280, 289, 290, 291, 292, 293, 294, 302, 303, 304, 305, 306, 307, 315, 316, 318, 319, 320, 321, 322, 323, 324, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 363, 364, 365, 366, 367, 373, 374, 375, 376, 377, 383, 384, 385, 386, 387, 393, 394, 395, 396, 397, 405, 406, 407, 408, 409, 410, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 465, 466, 467, 468, 469, 475, 476, 477, 478, 479, 485, 486, 487, 488, 489, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 558, 559, 560, 561, 563, 564, 566, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 629, 630, 631, 632, 633, 634, 635, 636, 637, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 738, 739, 740, 741, 742, 743, 744, 745, 746, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 788, 790, 792, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 943, 944, 957, 958, 960, 961, 972, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1004, 1013, 1018, 1019, 1020, 1021, 1022, 1025, 1026, 1027, 1028, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 112, 114, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 182, 183, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 199, 200, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 247, 249, 250, 251, 252, 254, 255, 256, 257, 258, 259, 260, 261, 262, 270, 271, 272, 273, 282, 283, 284, 285, 286, 295, 296, 297, 298, 299, 308, 309, 310, 311, 312, 325, 326, 327, 328, 329, 358, 359, 360, 368, 369, 370, 380, 381, 390, 391, 522, 525, 543, 549, 554, 557, 787, 789, 807, 808, 855, 856, 889, 890, 930, 931, 932, 933, 934, 935, 936, 941, 942, 943, 947, 948, 949, 950, 951, 952, 958, 962, 963, 964, 965, 966, 967, 969, 970, 971, 972, 973, 975, 976, 977, 978, 979, 982, 985, 1005, 1023, 1056, 1057, 1058, 1064 }

B grade { 27, 30, 110, 113, 124, 125, 126, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 179, 180, 181, 184, 196, 197, 198, 201, 218, 232, 244, 245, 246, 248, 361, 371, 516, 519, 527, 530, 533, 535, 538, 541, 546, 770, 791, 929, 937, 938, 939, 940, 944, 945, 946, 953, 954, 955, 956, 957, 959, 960, 961, 968, 974, 980, 981, 983, 984, 1024, 1029, 1039, 1040, 1041, 1045, 1046, 1047, 1051, 1052, 1053, 1054, 1055, 1059, 1060, 1061, 1062, 1063, 1078, 1079 }

C grade { 127, 128, 500, 501, 502, 503, 504, 517, 518, 520, 521, 523, 524, 526, 528, 529, 531, 532, 534, 536, 537, 539, 540, 542, 544, 545, 547, 548, 550, 553, 555, 556, 558, 841, 842, 843, 848, 849, 1081 }

F normal fail { 263, 264, 265, 266, 267, 268, 269, 274, 275, 276, 277, 278, 279, 280, 281, 287, 288, 289, 290, 291, 292, 293, 294, 300, 301, 302, 303, 304, 305, 306, 307, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 330, 331, 334, 335, 336, 337, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 362, 363, 364, 365, 366, 367, 372, 373, 374, 375, 376, 377, 378, 379, 382, 383, 384, 385, 386, 387, 388, 389, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 405, 406, 407, 408, 409, 413, 414, 415, 419, 420, 421, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 472, 473, 475, 476, 477, 478, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 788, 790, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 844, 845, 847, 850, 853, 854, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 911, 912, 915, 916, 917, 919, 920, 922, 923, 924, 925, 926, 927, 928, 1002, 1003, 1004, 1006, 1007, 1008, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1025, 1026, 1027, 1028, 1030, 1065, 1068, 1069, 1070, 1074, 1075, 1076 }

F(-1) timedout fail { 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 107, 108, 109, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 129, 130, 163, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 253, 332, 333, 338, 404, 410, 411, 412, 416, 417, 418, 422, 423, 470, 471, 474, 479, 505, 551, 552, 559, 560, 561, 562, 563, 564, 565, 566, 708, 767, 768, 769, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 822, 823, 846, 851, 852, 870, 871, 908, 909, 910, 913, 914, 918, 921, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1009, 1010, 1066, 1067, 1071, 1072, 1073, 1077, 1080 }

F(-2) exception fail { 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1042, 1043, 1044, 1048, 1049, 1050 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	27	29	29	28
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.88	0.88	0.85
time (sec)	N/A	0.023	0.009	0.213	0.211	0.238	0.017	0.267	0.046

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	27	29	29	28
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.88	0.88	0.85
time (sec)	N/A	0.013	0.007	0.238	0.199	0.270	0.017	0.277	6.760

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	24	26	26	25
N.S.	1	1.00	1.00	0.89	0.86	0.86	0.93	0.93	0.89
time (sec)	N/A	0.009	0.007	0.247	0.259	0.227	0.017	0.276	0.037

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	27	28	25	27	28	26
N.S.	1	1.00	1.00	0.93	0.97	0.86	0.93	0.97	0.90
time (sec)	N/A	0.015	0.011	0.169	0.222	0.234	0.050	0.275	6.712

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	30	27	29	26	29	28
N.S.	1	1.00	1.00	0.97	0.87	0.94	0.84	0.94	0.90
time (sec)	N/A	0.012	0.012	0.047	0.217	0.234	0.045	0.270	0.042

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	24	24	28	24	23	24
N.S.	1	1.00	1.00	0.86	0.86	1.00	0.86	0.82	0.86
time (sec)	N/A	0.014	0.010	0.044	0.199	0.234	0.051	0.276	6.700

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	28	30	26	40	25
N.S.	1	1.00	1.00	0.90	0.97	1.03	0.90	1.38	0.86
time (sec)	N/A	0.015	0.013	0.051	0.200	0.224	0.108	0.267	0.045

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	32	28	29	29	31	31	29
N.S.	1	1.00	1.03	0.90	0.94	0.94	1.00	1.00	0.94
time (sec)	N/A	0.012	0.011	0.055	0.202	0.226	0.129	0.275	0.038

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	30	25	27	29	29	29	28
N.S.	1	1.00	1.07	0.89	0.96	1.04	1.04	1.04	1.00
time (sec)	N/A	0.011	0.013	0.047	0.199	0.310	0.144	0.268	6.716

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	31	26	30	31	29	37	29
N.S.	1	1.00	1.07	0.90	1.03	1.07	1.00	1.28	1.00
time (sec)	N/A	0.015	0.018	0.039	0.212	0.250	0.282	0.271	0.056

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	51	52	51	51	54	53	51
N.S.	1	1.00	1.21	1.24	1.21	1.21	1.29	1.26	1.21
time (sec)	N/A	0.049	0.017	4.142	0.194	0.257	0.027	0.262	6.784

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	51	51	54	53	51
N.S.	1	1.00	1.00	0.95	0.93	0.93	0.98	0.96	0.93
time (sec)	N/A	0.023	0.008	4.156	0.205	0.349	0.022	0.264	0.049

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	51	50	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.02	1.00	0.96
time (sec)	N/A	0.016	0.009	4.016	0.219	0.232	0.023	0.279	0.044

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	51	50	52	49	53	52	49
N.S.	1	1.00	1.11	1.09	1.13	1.07	1.15	1.13	1.07
time (sec)	N/A	0.027	0.017	3.926	0.197	0.255	0.063	0.280	0.041

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	52	51	53	49	52	50
N.S.	1	1.00	1.00	0.98	0.96	1.00	0.92	0.98	0.94
time (sec)	N/A	0.019	0.021	4.227	0.206	0.237	0.060	0.263	0.049

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	53	49	48	48
N.S.	1	1.00	1.00	0.98	0.96	1.06	0.98	0.96	0.96
time (sec)	N/A	0.020	0.019	4.076	0.200	0.240	0.062	0.298	0.050

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	49	49	52	54	51	69	49
N.S.	1	1.00	0.96	0.96	1.02	1.06	1.00	1.35	0.96
time (sec)	N/A	0.031	0.026	4.187	0.215	0.246	0.150	0.294	0.046

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	51	50	53	53	53	54	52
N.S.	1	1.00	0.96	0.94	1.00	1.00	1.00	1.02	0.98
time (sec)	N/A	0.020	0.019	4.256	0.217	0.236	0.161	0.278	0.050

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	46	51	53	53	51	50
N.S.	1	1.00	1.00	0.92	1.02	1.06	1.06	1.02	1.00
time (sec)	N/A	0.019	0.021	3.930	0.231	0.233	0.187	0.286	6.752

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	46	54	55	51	70	52
N.S.	1	1.00	1.00	0.90	1.06	1.08	1.00	1.37	1.02
time (sec)	N/A	0.035	0.022	4.093	0.213	0.240	0.402	0.256	0.056

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	54	48	54	53	58	56	53
N.S.	1	1.00	1.02	0.91	1.02	1.00	1.09	1.06	1.00
time (sec)	N/A	0.020	0.019	4.089	0.195	0.236	0.495	0.267	0.051

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	45	51	53	54	53	50
N.S.	1	1.00	1.00	0.90	1.02	1.06	1.08	1.06	1.00
time (sec)	N/A	0.020	0.024	4.106	0.208	0.260	0.524	0.263	0.048

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	121	119	119	136	125	107
N.S.	1	1.00	1.00	1.03	1.02	1.02	1.16	1.07	0.91
time (sec)	N/A	0.073	0.025	4.190	0.205	0.248	0.029	0.276	0.052

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	107	121	119	119	136	125	107
N.S.	1	1.00	1.13	1.27	1.25	1.25	1.43	1.32	1.13
time (sec)	N/A	0.200	0.026	4.179	0.211	0.257	0.031	0.274	6.691

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	121	119	119	134	125	107
N.S.	1	1.00	1.00	1.03	1.02	1.02	1.15	1.07	0.91
time (sec)	N/A	0.049	0.020	4.034	0.202	0.269	0.029	0.266	0.041

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	121	119	119	136	125	107
N.S.	1	1.00	1.00	1.03	1.02	1.02	1.16	1.07	0.91
time (sec)	N/A	0.046	0.019	4.309	0.195	0.250	0.038	0.276	0.043

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	107	121	119	119	138	125	107
N.S.	1	1.00	1.60	1.81	1.78	1.78	2.06	1.87	1.60
time (sec)	N/A	0.108	0.026	4.010	0.204	0.240	0.031	0.279	0.041

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	121	119	119	136	125	107
N.S.	1	1.00	1.00	1.03	1.02	1.02	1.16	1.07	0.91
time (sec)	N/A	0.045	0.017	4.036	0.213	0.330	0.030	0.266	0.042

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	120	119	119	133	124	107
N.S.	1	1.00	1.00	1.03	1.02	1.02	1.14	1.06	0.91
time (sec)	N/A	0.044	0.018	4.315	0.222	0.255	0.030	0.274	0.041

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	107	121	119	119	136	125	107
N.S.	1	1.00	2.55	2.88	2.83	2.83	3.24	2.98	2.55
time (sec)	N/A	0.052	0.026	4.072	0.213	0.242	0.032	0.269	0.045

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	120	119	119	134	124	106
N.S.	1	1.00	1.00	1.03	1.02	1.02	1.15	1.06	0.91
time (sec)	N/A	0.067	0.017	4.031	0.195	0.239	0.029	0.264	0.044

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	109	116	115	115	128	120	103
N.S.	1	1.00	1.00	1.06	1.06	1.06	1.17	1.10	0.94
time (sec)	N/A	0.039	0.017	4.028	0.202	0.239	0.029	0.281	0.048

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	113	119	120	117	134	124	105
N.S.	1	1.00	1.28	1.35	1.36	1.33	1.52	1.41	1.19
time (sec)	N/A	0.045	0.032	3.945	0.193	0.242	0.105	0.279	0.046

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	112	121	118	121	129	124	106
N.S.	1	1.00	1.00	1.08	1.05	1.08	1.15	1.11	0.95
time (sec)	N/A	0.044	0.036	4.115	0.217	0.244	0.106	0.272	0.047

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	112	120	116	121	128	119	104
N.S.	1	1.00	1.00	1.07	1.04	1.08	1.14	1.06	0.93
time (sec)	N/A	0.049	0.035	4.191	0.202	0.239	0.110	0.283	0.043

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	115	121	120	123	133	143	105
N.S.	1	1.00	1.02	1.07	1.06	1.09	1.18	1.27	0.93
time (sec)	N/A	0.085	0.046	4.105	0.206	0.241	0.194	0.266	0.053

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	115	122	121	121	133	127	109
N.S.	1	1.00	1.02	1.08	1.07	1.07	1.18	1.12	0.96
time (sec)	N/A	0.069	0.039	4.160	0.202	0.238	0.209	0.266	0.044

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	113	119	120	121	133	124	108
N.S.	1	1.00	1.00	1.05	1.06	1.07	1.18	1.10	0.96
time (sec)	N/A	0.047	0.038	4.416	0.207	0.245	0.222	0.274	0.048

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	106	117	122	123	131	148	113
N.S.	1	1.00	0.93	1.03	1.07	1.08	1.15	1.30	0.99
time (sec)	N/A	0.081	0.053	4.029	0.198	0.256	0.508	0.282	0.050

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	110	117	121	121	129	127	113
N.S.	1	1.00	1.00	1.06	1.10	1.10	1.17	1.15	1.03
time (sec)	N/A	0.054	0.039	4.129	0.240	0.249	0.595	0.284	6.754

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	113	114	120	121	133	124	111
N.S.	1	1.00	1.00	1.01	1.06	1.07	1.18	1.10	0.98
time (sec)	N/A	0.052	0.039	4.125	0.195	0.253	0.616	0.273	0.048

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	106	111	123	123	129	150	118
N.S.	1	1.00	0.93	0.97	1.08	1.08	1.13	1.32	1.04
time (sec)	N/A	0.084	0.058	4.143	0.211	0.239	1.530	0.275	0.054

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	118	111	122	121	131	127	118
N.S.	1	1.00	1.03	0.97	1.06	1.05	1.14	1.10	1.03
time (sec)	N/A	0.054	0.025	4.114	0.202	0.257	8.427	0.262	6.594

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	109	108	120	121	131	124	116
N.S.	1	1.00	1.00	0.99	1.10	1.11	1.20	1.14	1.06
time (sec)	N/A	0.050	0.042	4.134	0.205	0.256	27.720	0.269	0.069

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	118	106	123	123	129	149	122
N.S.	1	1.00	1.04	0.93	1.08	1.08	1.13	1.31	1.07
time (sec)	N/A	0.071	0.041	4.336	0.214	0.305	46.573	0.273	0.064

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	117	107	122	121	0	128	123
N.S.	1	1.00	1.02	0.93	1.06	1.05	0.00	1.11	1.07
time (sec)	N/A	0.044	0.040	3.967	0.212	0.241	0.000	0.308	6.562

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	110	102	119	121	0	123	120
N.S.	1	1.00	1.00	0.93	1.08	1.10	0.00	1.12	1.09
time (sec)	N/A	0.049	0.044	4.130	0.203	0.242	0.000	0.271	6.549

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	116	102	123	123	0	145	121
N.S.	1	1.00	1.03	0.90	1.09	1.09	0.00	1.28	1.07
time (sec)	N/A	0.068	0.059	4.331	0.219	0.233	0.000	0.291	6.511

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	118	104	122	121	0	128	121
N.S.	1	1.00	1.03	0.90	1.06	1.05	0.00	1.11	1.05
time (sec)	N/A	0.041	0.037	4.344	0.228	0.236	0.000	0.281	6.588

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	110	101	119	121	0	125	119
N.S.	1	1.00	1.00	0.92	1.08	1.10	0.00	1.14	1.08
time (sec)	N/A	0.044	0.047	4.132	0.202	0.267	0.000	0.273	0.080

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	121	102	123	123	0	136	121
N.S.	1	1.00	1.33	1.12	1.35	1.35	0.00	1.49	1.33
time (sec)	N/A	0.043	0.040	4.117	0.209	0.234	0.000	0.288	0.096

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	119	104	121	121	0	127	119
N.S.	1	1.00	1.05	0.92	1.07	1.07	0.00	1.12	1.05
time (sec)	N/A	0.045	0.033	4.116	0.219	0.238	0.000	0.305	6.598

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	121	104	121	121	0	127	121
N.S.	1	1.00	1.03	0.89	1.03	1.03	0.00	1.09	1.03
time (sec)	N/A	0.042	0.034	3.965	0.214	0.241	0.000	0.278	6.744

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	118	104	121	121	0	127	122
N.S.	1	1.00	2.46	2.17	2.52	2.52	0.00	2.65	2.54
time (sec)	N/A	0.022	0.032	4.135	0.218	0.236	0.000	0.282	6.773

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	104	121	121	0	127	121
N.S.	1	1.00	1.00	0.89	1.03	1.03	0.00	1.09	1.03
time (sec)	N/A	0.055	0.046	4.126	0.211	0.232	0.000	0.291	0.065

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	171	84	182	167	114	217	164
N.S.	1	1.00	0.93	0.46	0.99	0.91	0.62	1.19	0.90
time (sec)	N/A	0.109	0.106	4.008	0.295	0.252	0.315	0.282	0.280

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	47	49	50	51	46	52	52
N.S.	1	1.00	0.87	0.91	0.93	0.94	0.85	0.96	0.96
time (sec)	N/A	0.047	0.026	4.220	0.206	0.239	0.252	0.289	0.079

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	154	65	157	162	114	207	144
N.S.	1	1.00	0.92	0.39	0.94	0.97	0.68	1.24	0.86
time (sec)	N/A	0.091	0.080	4.019	0.296	0.257	0.259	0.289	7.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	152	60	154	145	87	186	162
N.S.	1	1.00	0.94	0.37	0.95	0.90	0.54	1.15	1.00
time (sec)	N/A	0.079	0.081	4.277	0.300	0.367	0.274	0.292	7.055

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	31	32	31	30	27	32	31
N.S.	1	1.00	0.89	0.91	0.89	0.86	0.77	0.91	0.89
time (sec)	N/A	0.034	0.014	4.363	0.213	0.240	0.219	0.388	0.067

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	152	45	131	382	92	161	126
N.S.	1	1.00	1.01	0.30	0.87	2.55	0.61	1.07	0.84
time (sec)	N/A	0.073	0.045	4.024	0.336	0.256	0.221	0.395	7.002

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	129	42	128	369	71	133	123
N.S.	1	1.00	0.89	0.29	0.88	2.54	0.49	0.92	0.85
time (sec)	N/A	0.054	0.059	4.170	0.333	0.297	0.245	0.357	6.995

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	35	32	26	34	36
N.S.	1	1.00	1.00	0.97	1.03	0.94	0.76	1.00	1.06
time (sec)	N/A	0.025	0.015	4.155	0.239	0.251	0.606	0.367	0.115

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	134	114	140	372	90	155	126
N.S.	1	1.00	0.91	0.78	0.95	2.53	0.61	1.05	0.86
time (sec)	N/A	0.071	0.086	4.179	0.278	0.258	0.266	0.361	6.964

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	135	113	140	411	73	161	126
N.S.	1	1.00	0.91	0.76	0.94	2.76	0.49	1.08	0.85
time (sec)	N/A	0.063	0.101	4.201	0.356	0.265	0.274	0.335	0.251

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	49	46	48	47	41	69	46
N.S.	1	1.00	0.98	0.92	0.96	0.94	0.82	1.38	0.92
time (sec)	N/A	0.038	0.024	3.999	0.220	0.271	0.621	0.333	0.118

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	154	130	147	158	112	197	178
N.S.	1	1.00	0.93	0.79	0.89	0.96	0.68	1.19	1.08
time (sec)	N/A	0.090	0.113	4.203	0.295	0.251	0.288	0.348	6.803

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	154	130	148	176	99	176	145
N.S.	1	1.00	0.92	0.77	0.88	1.05	0.59	1.05	0.86
time (sec)	N/A	0.089	0.124	4.201	0.283	0.260	0.329	0.308	6.725

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	70	64	70	73	61	99	70
N.S.	1	1.00	1.01	0.93	1.01	1.06	0.88	1.43	1.01
time (sec)	N/A	0.054	0.030	4.143	0.201	0.232	0.678	0.322	0.128

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	173	150	178	180	139	216	161
N.S.	1	1.00	0.94	0.82	0.97	0.98	0.76	1.17	0.88
time (sec)	N/A	0.100	0.137	4.198	0.313	0.368	0.354	0.331	6.813

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	203	114	218	271	156	244	209
N.S.	1	1.00	0.87	0.49	0.94	1.16	0.67	1.05	0.90
time (sec)	N/A	0.111	0.134	4.171	0.289	0.328	0.575	0.289	6.798

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	72	76	82	121	82	106	86
N.S.	1	1.00	0.88	0.93	1.00	1.48	1.00	1.29	1.05
time (sec)	N/A	0.070	0.071	4.258	0.199	0.240	0.577	0.287	0.089

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	185	94	192	257	151	236	179
N.S.	1	1.00	0.86	0.44	0.89	1.20	0.70	1.10	0.83
time (sec)	N/A	0.095	0.128	4.034	0.289	0.260	0.619	0.296	6.834

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	181	87	187	240	126	211	193
N.S.	1	1.00	0.85	0.41	0.88	1.13	0.59	0.99	0.91
time (sec)	N/A	0.090	0.125	4.194	0.284	0.267	0.528	0.301	6.866

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	50	57	60	81	56	91	62
N.S.	1	1.00	0.83	0.95	1.00	1.35	0.93	1.52	1.03
time (sec)	N/A	0.043	0.039	4.145	0.197	0.248	0.520	0.305	0.085

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	165	71	162	578	126	189	158
N.S.	1	1.00	0.84	0.36	0.83	2.95	0.64	0.96	0.81
time (sec)	N/A	0.081	0.120	4.210	0.287	0.280	0.545	0.311	6.832

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	160	65	157	573	102	166	150
N.S.	1	1.00	0.84	0.34	0.83	3.02	0.54	0.87	0.79
time (sec)	N/A	0.076	0.130	4.198	0.328	0.253	0.439	0.294	6.807

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	38	40	44	36	65	37
N.S.	1	1.00	1.00	0.93	0.98	1.07	0.88	1.59	0.90
time (sec)	N/A	0.030	0.015	3.990	0.206	0.254	0.299	0.291	6.609

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	146	67	160	548	117	186	145
N.S.	1	1.00	0.85	0.39	0.94	3.20	0.68	1.09	0.85
time (sec)	N/A	0.064	0.096	4.205	0.286	0.265	0.373	0.269	6.770

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	145	65	158	537	97	160	143
N.S.	1	1.00	0.86	0.38	0.93	3.18	0.57	0.95	0.85
time (sec)	N/A	0.071	0.091	4.192	0.272	0.256	0.331	0.286	6.814

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	46	48	51	70	46	61	47
N.S.	1	1.00	0.90	0.94	1.00	1.37	0.90	1.20	0.92
time (sec)	N/A	0.039	0.031	4.008	0.204	0.230	0.292	0.277	0.148

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	196	164	139	166	570	122	180	156
N.S.	1	1.01	0.84	0.71	0.85	2.92	0.63	0.92	0.80
time (sec)	N/A	0.078	0.129	4.369	0.283	0.270	0.382	0.279	6.896

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	163	138	172	618	109	188	159
N.S.	1	1.00	0.83	0.70	0.88	3.15	0.56	0.96	0.81
time (sec)	N/A	0.074	0.130	4.036	0.275	0.272	0.402	0.279	0.259

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	64	76	76	118	70	80	78
N.S.	1	1.00	0.84	1.00	1.00	1.55	0.92	1.05	1.03
time (sec)	N/A	0.061	0.050	4.305	0.190	0.253	0.709	0.277	6.848

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	185	155	186	259	153	231	209
N.S.	1	1.00	0.86	0.72	0.87	1.20	0.71	1.07	0.97
time (sec)	N/A	0.099	0.147	4.443	0.274	0.259	0.424	0.278	7.034

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	183	154	186	277	138	206	176
N.S.	1	1.00	0.85	0.72	0.87	1.29	0.64	0.96	0.82
time (sec)	N/A	0.095	0.145	4.211	0.278	0.258	0.464	0.283	6.986

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	85	96	106	154	100	149	100
N.S.	1	1.00	0.88	0.99	1.09	1.59	1.03	1.54	1.03
time (sec)	N/A	0.077	0.101	4.180	0.185	0.319	0.803	0.281	0.152

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	94	100	115	179	112	131	117
N.S.	1	1.00	0.88	0.93	1.07	1.67	1.05	1.22	1.09
time (sec)	N/A	0.108	0.069	4.234	0.186	0.297	1.703	0.293	0.105

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	92	76	94	142	94	93	94
N.S.	1	1.00	1.05	0.86	1.07	1.61	1.07	1.06	1.07
time (sec)	N/A	0.065	0.041	4.016	0.232	0.329	1.404	0.286	6.831

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	64	57	72	89	70	61	70
N.S.	1	1.00	0.97	0.86	1.09	1.35	1.06	0.92	1.06
time (sec)	N/A	0.045	0.026	4.168	0.211	0.248	1.077	0.285	6.830

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	30	29	42	42	42	28	44
N.S.	1	1.00	0.94	0.91	1.31	1.31	1.31	0.88	1.38
time (sec)	N/A	0.016	0.017	3.976	0.214	0.255	0.441	0.277	6.791

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	59	61	77	119	75	74	71
N.S.	1	1.00	0.87	0.90	1.13	1.75	1.10	1.09	1.04
time (sec)	N/A	0.043	0.049	4.186	0.203	0.268	0.396	0.288	0.169

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	87	98	109	197	107	136	107
N.S.	1	1.00	0.86	0.97	1.08	1.95	1.06	1.35	1.06
time (sec)	N/A	0.090	0.059	4.175	0.216	0.264	0.836	0.289	6.905

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	108	123	136	229	133	131	130
N.S.	1	1.00	0.89	1.01	1.11	1.88	1.09	1.07	1.07
time (sec)	N/A	0.097	0.085	4.180	0.186	0.262	0.911	0.282	0.159

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	216	116	228	364	192	259	213
N.S.	1	1.00	0.88	0.47	0.93	1.48	0.78	1.05	0.87
time (sec)	N/A	0.114	0.191	4.372	0.279	0.252	9.878	0.295	7.081

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	210	109	223	347	163	234	227
N.S.	1	1.00	0.86	0.45	0.91	1.42	0.67	0.96	0.93
time (sec)	N/A	0.122	0.160	4.188	0.269	0.290	1.388	0.292	0.337

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	194	90	196	792	162	210	187
N.S.	1	1.00	0.87	0.41	0.88	3.57	0.73	0.95	0.84
time (sec)	N/A	0.101	0.166	4.151	0.279	0.292	7.931	0.288	6.995

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	188	85	191	789	141	187	183
N.S.	1	1.00	0.85	0.39	0.87	3.59	0.64	0.85	0.83
time (sec)	N/A	0.093	0.164	4.444	0.272	0.296	1.219	0.284	0.280

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	181	85	195	756	155	206	175
N.S.	1	1.00	0.90	0.42	0.97	3.76	0.77	1.02	0.87
time (sec)	N/A	0.079	0.176	4.040	0.267	0.299	5.366	0.276	6.965

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	178	83	193	743	136	187	173
N.S.	1	1.00	0.89	0.42	0.97	3.73	0.68	0.94	0.87
time (sec)	N/A	0.088	0.158	4.266	0.274	0.292	0.779	0.281	6.972

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	178	86	195	752	153	207	175
N.S.	1	1.00	0.89	0.43	0.97	3.74	0.76	1.03	0.87
time (sec)	N/A	0.082	0.135	4.292	0.281	0.369	0.524	0.292	0.268

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	175	84	192	743	133	180	173
N.S.	1	1.00	0.89	0.43	0.97	3.77	0.68	0.91	0.88
time (sec)	N/A	0.073	0.132	4.108	0.273	0.307	0.421	0.291	0.263

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	193	159	199	776	162	204	185
N.S.	1	1.00	0.85	0.70	0.88	3.42	0.71	0.90	0.81
time (sec)	N/A	0.092	0.163	4.261	0.274	0.294	0.482	0.307	7.014

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	189	158	201	812	143	209	188
N.S.	1	1.00	0.83	0.70	0.89	3.58	0.63	0.92	0.83
time (sec)	N/A	0.099	0.154	4.089	0.292	0.270	0.498	0.288	6.871

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	214	175	221	366	189	254	240
N.S.	1	1.00	0.87	0.71	0.90	1.49	0.77	1.03	0.98
time (sec)	N/A	0.121	0.193	4.273	0.283	0.274	0.568	0.283	6.862

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	210	174	221	384	173	229	207
N.S.	1	1.00	0.85	0.71	0.90	1.56	0.70	0.93	0.84
time (sec)	N/A	0.107	0.218	4.134	0.285	0.273	0.587	0.282	6.826

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	66	65	68	72	0	70	68
N.S.	1	1.00	0.94	0.93	0.97	1.03	0.00	1.00	0.97
time (sec)	N/A	0.053	0.036	4.154	0.217	0.368	0.000	0.288	7.044

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	242	228	324	273	0	311	1751
N.S.	1	1.00	0.80	0.76	1.08	0.91	0.00	1.03	5.82
time (sec)	N/A	0.233	0.153	4.212	0.323	0.545	0.000	0.287	16.266

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	238	225	349	228	0	308	873
N.S.	1	1.00	0.80	0.76	1.18	0.77	0.00	1.04	2.95
time (sec)	N/A	0.187	0.120	4.524	0.281	0.291	0.000	0.287	1.926

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	43	43	49	42	144	51	51
N.S.	1	1.00	0.81	0.81	0.92	0.79	2.72	0.96	0.96
time (sec)	N/A	0.034	0.022	4.160	0.198	0.301	5.027	0.284	7.081

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	224	207	289	244	0	286	1364
N.S.	1	1.00	0.78	0.72	1.00	0.85	0.00	0.99	4.74
time (sec)	N/A	0.109	0.085	4.291	0.286	0.271	0.000	0.301	14.053

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	224	207	317	199	342	278	1265
N.S.	1	1.00	0.78	0.72	1.10	0.69	1.19	0.97	4.39
time (sec)	N/A	0.105	0.087	4.571	0.294	0.269	131.427	0.294	13.021

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	31	32	41	31	138	51	602
N.S.	1	1.00	0.69	0.71	0.91	0.69	3.07	1.13	13.38
time (sec)	N/A	0.023	0.019	4.160	0.203	0.268	0.726	0.298	0.253

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	224	207	265	201	515	290	982
N.S.	1	1.00	0.78	0.72	0.92	0.70	1.79	1.01	3.41
time (sec)	N/A	0.115	0.099	4.168	0.285	0.280	69.176	0.290	10.072

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	224	207	293	254	0	278	1364
N.S.	1	1.00	0.78	0.72	1.02	0.88	0.00	0.97	4.74
time (sec)	N/A	0.105	0.103	4.238	0.269	0.315	0.000	0.305	13.944

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	54	55	61	54	0	71	58
N.S.	1	1.00	0.87	0.89	0.98	0.87	0.00	1.15	0.94
time (sec)	N/A	0.045	0.031	4.138	0.198	0.483	0.000	0.297	7.310

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	244	228	300	238	0	305	716
N.S.	1	1.00	0.82	0.76	1.00	0.80	0.00	1.02	2.39
time (sec)	N/A	0.198	0.141	4.206	0.283	0.304	0.000	0.294	8.432

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	259	228	328	301	0	309	1829
N.S.	1	1.00	0.86	0.76	1.09	1.00	0.00	1.03	6.08
time (sec)	N/A	0.201	0.169	4.454	0.278	0.778	0.000	0.315	16.873

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	88	82	87	99	0	111	87
N.S.	1	1.00	1.01	0.94	1.00	1.14	0.00	1.28	1.00
time (sec)	N/A	0.072	0.044	4.310	0.186	1.343	0.000	0.290	7.682

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	282	248	341	305	0	328	1734
N.S.	1	1.00	0.89	0.78	1.07	0.96	0.00	1.03	5.45
time (sec)	N/A	0.291	0.213	4.377	0.265	0.786	0.000	0.295	16.468

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	282	248	369	356	0	336	1860
N.S.	1	1.00	0.88	0.77	1.15	1.11	0.00	1.05	5.79
time (sec)	N/A	0.332	0.209	4.377	0.281	0.336	0.000	0.289	16.539

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	115	114	117	127	0	165	118
N.S.	1	1.00	0.97	0.96	0.98	1.07	0.00	1.39	0.99
time (sec)	N/A	0.093	0.060	4.175	0.189	3.729	0.000	0.286	7.641

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	352	304	279	376	332	0	377	1814
N.S.	1	1.00	0.86	0.79	1.07	0.94	0.00	1.07	5.15
time (sec)	N/A	0.363	0.261	4.365	0.275	0.404	0.000	0.290	16.927

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	137	1077	205	851	5418	1331	559
N.S.	1	1.00	0.93	7.28	1.39	5.75	36.61	8.99	3.78
time (sec)	N/A	0.088	0.558	4.737	0.197	0.274	1.651	0.333	7.625

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	66	261	91	215	1057	332	177
N.S.	1	1.00	0.93	3.68	1.28	3.03	14.89	4.68	2.49
time (sec)	N/A	0.032	0.146	4.073	0.195	0.261	0.602	0.283	6.906

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	42	53	53	92	410	143	95
N.S.	1	1.00	0.93	1.18	1.18	2.04	9.11	3.18	2.11
time (sec)	N/A	0.023	0.070	0.085	0.190	0.270	0.399	0.270	6.839

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	55	0	0	0	187	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	2.83	0.00	0.00
time (sec)	N/A	0.025	0.134	0.000	0.000	0.000	7.587	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	80	0	0	0	1049	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	11.28	0.00	0.00
time (sec)	N/A	0.029	0.224	0.000	0.000	0.000	140.649	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	80	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.028	0.383	0.000	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	112	112	86	0	0	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.048	0.363	0.000	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	32	46	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.011	0.033	0.271	0.198	0.240	0.976	0.301	0.055

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	32	46	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.011	0.032	0.281	0.189	0.243	0.630	0.274	7.001

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	32	46	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.012	0.028	0.253	0.203	0.243	0.434	0.266	0.045

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	30	46	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.77	1.18	0.74	0.79
time (sec)	N/A	0.015	0.033	0.244	0.186	0.243	0.583	0.278	0.045

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	35	28	27	29	44	29	31
N.S.	1	1.00	0.95	0.76	0.73	0.78	1.19	0.78	0.84
time (sec)	N/A	0.011	0.030	0.259	0.196	0.249	0.266	0.303	6.976

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	35	30	27	29	44	29	31
N.S.	1	1.00	0.95	0.81	0.73	0.78	1.19	0.78	0.84
time (sec)	N/A	0.011	0.037	0.082	0.186	0.258	0.361	0.273	7.024

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	34	30	27	28	46	29	31
N.S.	1	1.00	0.87	0.77	0.69	0.72	1.18	0.74	0.79
time (sec)	N/A	0.011	0.039	0.086	0.186	0.249	0.393	0.271	0.044

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	35	30	27	29	42	29	30
N.S.	1	1.00	0.95	0.81	0.73	0.78	1.14	0.78	0.81
time (sec)	N/A	0.015	0.037	0.084	0.215	0.266	0.479	0.270	0.043

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	59	52	51	56	80	53	51
N.S.	1	1.00	0.94	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.021	0.052	4.195	0.185	0.274	1.577	0.273	6.979

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	60	52	51	56	80	53	51
N.S.	1	1.00	0.95	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.031	0.059	4.140	0.183	0.263	1.131	0.277	0.051

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	60	52	51	56	80	53	51
N.S.	1	1.00	0.95	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.027	0.057	4.055	0.210	0.244	0.768	0.293	0.050

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	59	52	51	54	80	53	51
N.S.	1	1.00	0.94	0.83	0.81	0.86	1.27	0.84	0.81
time (sec)	N/A	0.023	0.059	4.169	0.196	0.245	0.853	0.263	0.055

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	59	52	51	53	78	53	51
N.S.	1	1.00	0.97	0.85	0.84	0.87	1.28	0.87	0.84
time (sec)	N/A	0.034	0.053	4.156	0.195	0.253	0.518	0.275	0.047

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	59	54	51	53	78	53	51
N.S.	1	1.00	0.97	0.89	0.84	0.87	1.28	0.87	0.84
time (sec)	N/A	0.021	0.074	4.194	0.193	0.252	0.620	0.280	0.048

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	59	54	51	53	80	53	51
N.S.	1	1.00	0.94	0.86	0.81	0.84	1.27	0.84	0.81
time (sec)	N/A	0.022	0.060	4.217	0.204	0.258	0.722	0.275	0.050

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	59	54	51	53	76	53	51
N.S.	1	1.00	0.97	0.89	0.84	0.87	1.25	0.87	0.84
time (sec)	N/A	0.022	0.068	4.335	0.199	0.263	0.881	0.279	0.052

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	81	76	73	78	114	77	69
N.S.	1	1.00	0.95	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.042	0.070	4.533	0.184	0.425	2.504	0.286	6.939

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	91	76	73	78	114	77	69
N.S.	1	1.00	1.07	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.030	0.077	4.309	0.188	0.262	1.859	0.281	0.030

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	81	76	73	78	114	77	69
N.S.	1	1.00	0.95	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.030	0.087	4.157	0.202	0.281	1.352	0.272	0.033

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	80	76	73	76	114	77	69
N.S.	1	1.00	0.94	0.89	0.86	0.89	1.34	0.91	0.81
time (sec)	N/A	0.028	0.072	4.222	0.255	0.257	1.271	0.280	0.032

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	80	76	73	75	112	77	69
N.S.	1	1.00	0.96	0.92	0.88	0.90	1.35	0.93	0.83
time (sec)	N/A	0.028	0.071	4.225	0.247	0.251	1.011	0.261	0.031

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	78	73	75	112	77	69
N.S.	1	1.00	1.00	0.94	0.88	0.90	1.35	0.93	0.83
time (sec)	N/A	0.029	0.094	4.225	0.210	0.301	1.108	0.271	0.034

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	77	78	73	75	112	77	69
N.S.	1	1.00	0.91	0.92	0.86	0.88	1.32	0.91	0.81
time (sec)	N/A	0.029	0.069	4.229	0.226	0.278	1.184	0.281	0.036

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	78	78	73	75	110	77	69
N.S.	1	1.00	0.94	0.94	0.88	0.90	1.33	0.93	0.83
time (sec)	N/A	0.034	0.088	4.242	0.221	0.266	1.409	0.275	0.035

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	67	55	58	143	428	64	111
N.S.	1	1.00	0.92	0.75	0.79	1.96	5.86	0.88	1.52
time (sec)	N/A	0.036	0.108	4.281	0.298	0.286	62.164	0.270	7.023

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	172	206	295	1289	605	289	1933
N.S.	1	1.00	0.60	0.72	1.02	4.48	2.10	1.00	6.71
time (sec)	N/A	0.432	0.297	4.361	0.316	0.296	29.283	0.300	7.334

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	153	190	212	1603	581	260	1640
N.S.	1	1.00	0.57	0.70	0.79	5.94	2.15	0.96	6.07
time (sec)	N/A	0.465	0.304	4.317	0.323	0.309	11.674	0.628	7.262

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	40	39	108	381	39	93
N.S.	1	1.00	1.00	0.75	0.74	2.04	7.19	0.74	1.75
time (sec)	N/A	0.024	0.084	4.447	0.299	0.283	4.678	0.272	7.051

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	152	190	278	1245	558	280	1915
N.S.	1	1.00	0.57	0.71	1.04	4.65	2.08	1.04	7.15
time (sec)	N/A	0.432	0.319	4.172	0.303	0.296	5.311	0.288	7.344

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	156	191	212	1636	561	257	1700
N.S.	1	1.00	0.58	0.71	0.79	6.10	2.09	0.96	6.34
time (sec)	N/A	0.443	0.360	4.172	0.287	0.276	9.206	0.690	7.292

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	40	39	120	371	39	102
N.S.	1	1.00	1.00	0.75	0.74	2.26	7.00	0.74	1.92
time (sec)	N/A	0.025	0.082	4.108	0.275	0.336	19.499	0.268	0.110

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	158	191	278	1290	586	280	2023
N.S.	1	1.00	0.59	0.71	1.03	4.78	2.17	1.04	7.49
time (sec)	N/A	0.371	0.304	4.327	0.280	0.341	47.882	0.284	7.383

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	77	65	68	222	0	68	116
N.S.	1	1.00	0.81	0.68	0.72	2.34	0.00	0.72	1.22
time (sec)	N/A	0.039	0.158	4.144	0.277	0.301	0.000	0.292	7.081

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	312	181	216	311	1426	1658	313	1884
N.S.	1	1.00	0.58	0.69	1.00	4.57	5.31	1.00	6.04
time (sec)	N/A	0.415	0.990	4.223	0.339	0.281	167.741	0.295	7.347

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	169	213	235	1773	1885	288	1578
N.S.	1	1.00	0.58	0.74	0.81	6.13	6.52	1.00	5.46
time (sec)	N/A	0.482	0.815	3.997	0.290	0.289	112.409	0.649	7.291

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	71	61	61	190	1042	63	115
N.S.	1	1.00	1.00	0.86	0.86	2.68	14.68	0.89	1.62
time (sec)	N/A	0.029	0.142	4.143	0.276	0.265	67.820	0.288	7.020

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	168	213	301	1417	1632	302	1922
N.S.	1	1.00	0.58	0.74	1.04	4.90	5.65	1.04	6.65
time (sec)	N/A	0.511	0.752	4.271	0.310	0.323	90.341	0.299	7.321

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	184	216	240	1788	2200	284	1757
N.S.	1	1.00	0.58	0.68	0.75	5.62	6.92	0.89	5.53
time (sec)	N/A	0.463	0.841	4.504	0.298	0.373	151.557	0.681	7.170

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	97	79	66	67	232	0	66	139
N.S.	1	1.01	0.82	0.69	0.70	2.42	0.00	0.69	1.45
time (sec)	N/A	0.048	0.161	4.042	0.287	0.429	0.000	0.277	6.797

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	188	217	312	1463	0	313	2080
N.S.	1	1.00	0.59	0.68	0.98	4.60	0.00	0.98	6.54
time (sec)	N/A	0.417	0.774	4.282	0.301	0.427	0.000	0.292	7.183

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	92	81	96	314	0	84	133
N.S.	1	1.00	0.88	0.78	0.92	3.02	0.00	0.81	1.28
time (sec)	N/A	0.043	0.236	4.384	0.294	0.388	0.000	0.279	7.018

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	327	192	234	341	1618	0	328	1944
N.S.	1	1.00	0.59	0.72	1.04	4.95	0.00	1.00	5.94
time (sec)	N/A	0.398	1.076	4.357	0.285	0.398	0.000	0.291	7.392

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	327	193	235	271	1959	0	314	1672
N.S.	1	1.00	0.59	0.72	0.83	5.99	0.00	0.96	5.11
time (sec)	N/A	0.485	1.048	4.186	0.297	0.429	0.000	0.652	7.347

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	92	82	96	313	0	84	136
N.S.	1	1.00	0.88	0.79	0.92	3.01	0.00	0.81	1.31
time (sec)	N/A	0.048	0.224	4.195	0.280	0.288	0.000	0.276	7.158

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	189	233	336	1588	0	322	1952
N.S.	1	1.00	0.59	0.73	1.05	4.95	0.00	1.00	6.08
time (sec)	N/A	0.391	1.021	4.182	0.295	0.418	0.000	0.295	0.453

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	208	236	273	1934	0	306	1786
N.S.	1	1.00	0.59	0.67	0.78	5.51	0.00	0.87	5.09
time (sec)	N/A	0.493	1.032	4.124	0.282	0.366	0.000	0.685	7.377

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	130	102	86	100	347	0	88	163
N.S.	1	1.01	0.79	0.67	0.78	2.69	0.00	0.68	1.26
time (sec)	N/A	0.063	0.230	4.221	0.296	0.308	0.000	0.270	7.179

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	209	237	346	1608	0	334	2109
N.S.	1	1.00	0.60	0.68	0.99	4.58	0.00	0.95	6.01
time (sec)	N/A	0.459	1.068	4.391	0.303	0.284	0.000	0.300	7.474

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	75	67	118	99	219	104	154
N.S.	1	1.00	0.73	0.65	1.15	0.96	2.13	1.01	1.50
time (sec)	N/A	0.063	0.091	4.478	0.214	0.344	0.395	0.286	7.063

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	56	49	84	75	168	73	114
N.S.	1	1.00	0.77	0.67	1.15	1.03	2.30	1.00	1.56
time (sec)	N/A	0.042	0.068	4.087	0.201	0.250	0.298	0.265	7.056

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	34	31	49	50	117	44	44
N.S.	1	1.00	0.74	0.67	1.07	1.09	2.54	0.96	0.96
time (sec)	N/A	0.028	0.045	4.110	0.198	0.255	0.213	0.271	6.976

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	61	50	67	125	87	61	80
N.S.	1	1.00	0.95	0.78	1.05	1.95	1.36	0.95	1.25
time (sec)	N/A	0.038	0.111	4.106	0.279	0.429	4.654	0.289	7.148

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	65	56	107	143	134	68	76
N.S.	1	1.00	0.77	0.67	1.27	1.70	1.60	0.81	0.90
time (sec)	N/A	0.048	0.152	4.359	0.300	0.280	11.828	0.264	7.361

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	78	65	158	172	160	120	93
N.S.	1	1.00	0.89	0.74	1.80	1.95	1.82	1.36	1.06
time (sec)	N/A	0.051	0.206	4.216	0.290	0.284	33.568	0.278	7.596

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	89	349	0	91	83	0	0
N.S.	1	1.00	0.29	1.15	0.00	0.30	0.27	0.00	0.00
time (sec)	N/A	0.143	5.486	4.421	0.000	0.104	1.170	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	75	325	0	67	82	0	0
N.S.	1	1.00	0.28	1.21	0.00	0.25	0.31	0.00	0.00
time (sec)	N/A	0.085	5.575	4.387	0.000	0.084	1.022	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	81	317	0	57	85	0	0
N.S.	1	1.00	0.30	1.18	0.00	0.21	0.32	0.00	0.00
time (sec)	N/A	0.092	6.082	4.411	0.000	0.090	1.216	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	80	328	0	64	94	0	0
N.S.	1	1.00	0.29	1.21	0.00	0.24	0.35	0.00	0.00
time (sec)	N/A	0.084	10.093	4.469	0.000	0.088	1.343	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	80	352	0	89	97	0	0
N.S.	1	1.00	0.26	1.15	0.00	0.29	0.32	0.00	0.00
time (sec)	N/A	0.103	10.093	4.557	0.000	0.083	1.532	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	581	581	91	503	0	102	83	0	0
N.S.	1	1.00	0.16	0.87	0.00	0.18	0.14	0.00	0.00
time (sec)	N/A	0.299	5.816	4.638	0.000	0.082	1.182	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	548	548	75	479	0	76	83	0	0
N.S.	1	1.00	0.14	0.87	0.00	0.14	0.15	0.00	0.00
time (sec)	N/A	0.215	5.555	4.607	0.000	0.081	1.103	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	545	545	81	469	0	64	85	0	0
N.S.	1	1.00	0.15	0.86	0.00	0.12	0.16	0.00	0.00
time (sec)	N/A	0.203	6.051	4.352	0.000	0.088	1.249	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	546	546	80	480	0	70	92	0	0
N.S.	1	1.00	0.15	0.88	0.00	0.13	0.17	0.00	0.00
time (sec)	N/A	0.239	10.090	4.249	0.000	0.092	1.366	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	581	581	80	504	0	97	97	0	0
N.S.	1	1.00	0.14	0.87	0.00	0.17	0.17	0.00	0.00
time (sec)	N/A	0.261	10.091	4.332	0.000	0.090	1.441	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	614	614	80	530	0	123	97	0	0
N.S.	1	1.00	0.13	0.86	0.00	0.20	0.16	0.00	0.00
time (sec)	N/A	0.314	10.097	4.489	0.000	0.078	1.651	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	80	68	118	124	267	104	206
N.S.	1	1.00	0.78	0.66	1.15	1.20	2.59	1.01	2.00
time (sec)	N/A	0.065	0.100	4.310	0.223	0.271	0.584	0.269	6.990

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	56	49	84	99	216	73	211
N.S.	1	1.00	0.77	0.67	1.15	1.36	2.96	1.00	2.89
time (sec)	N/A	0.043	0.074	4.338	0.210	0.245	0.424	0.268	6.833

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	34	31	49	73	165	44	150
N.S.	1	1.00	0.74	0.67	1.07	1.59	3.59	0.96	3.26
time (sec)	N/A	0.032	0.045	4.183	0.198	0.245	0.322	0.274	6.914

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	85	66	80	172	109	80	131
N.S.	1	1.00	1.05	0.81	0.99	2.12	1.35	0.99	1.62
time (sec)	N/A	0.039	0.147	4.204	0.284	0.261	10.221	0.267	6.906

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	81	79	134	169	223	103	111
N.S.	1	1.00	0.74	0.72	1.22	1.54	2.03	0.94	1.01
time (sec)	N/A	0.065	0.163	4.396	0.297	0.265	14.415	0.268	7.368

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	81	78	171	191	243	131	110
N.S.	1	1.00	0.70	0.68	1.49	1.66	2.11	1.14	0.96
time (sec)	N/A	0.065	0.255	4.402	0.266	0.361	38.798	0.283	7.560

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	93	373	0	115	172	0	0
N.S.	1	1.00	0.28	1.11	0.00	0.34	0.51	0.00	0.00
time (sec)	N/A	0.154	7.544	4.575	0.000	0.082	1.970	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	77	349	0	91	170	0	0
N.S.	1	1.00	0.26	1.17	0.00	0.30	0.57	0.00	0.00
time (sec)	N/A	0.097	7.453	4.601	0.000	0.082	1.700	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	83	334	0	80	172	0	0
N.S.	1	1.00	0.28	1.13	0.00	0.27	0.58	0.00	0.00
time (sec)	N/A	0.109	7.579	4.252	0.000	0.103	2.061	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	82	329	0	67	184	0	0
N.S.	1	1.00	0.28	1.11	0.00	0.23	0.62	0.00	0.00
time (sec)	N/A	0.106	10.081	4.488	0.000	0.099	2.198	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	302	82	351	0	89	196	0	0
N.S.	1	1.00	0.27	1.16	0.00	0.29	0.65	0.00	0.00
time (sec)	N/A	0.120	10.080	4.521	0.000	0.076	2.543	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	614	614	96	527	0	126	172	0	0
N.S.	1	1.00	0.16	0.86	0.00	0.21	0.28	0.00	0.00
time (sec)	N/A	0.310	7.812	4.514	0.000	0.077	2.075	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	581	581	78	503	0	100	172	0	0
N.S.	1	1.00	0.13	0.87	0.00	0.17	0.30	0.00	0.00
time (sec)	N/A	0.268	7.466	4.290	0.000	0.078	1.871	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	573	573	83	486	0	87	173	0	0
N.S.	1	1.00	0.14	0.85	0.00	0.15	0.30	0.00	0.00
time (sec)	N/A	0.271	7.629	4.435	0.000	0.167	2.170	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	578	578	85	482	0	77	182	0	0
N.S.	1	1.00	0.15	0.83	0.00	0.13	0.31	0.00	0.00
time (sec)	N/A	0.274	10.079	4.488	0.000	0.117	2.236	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	576	576	82	503	0	95	194	0	0
N.S.	1	1.00	0.14	0.87	0.00	0.16	0.34	0.00	0.00
time (sec)	N/A	0.274	10.086	4.502	0.000	0.113	2.440	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	608	608	82	529	0	123	199	0	0
N.S.	1	1.00	0.13	0.87	0.00	0.20	0.33	0.00	0.00
time (sec)	N/A	0.316	10.084	4.549	0.000	0.079	2.735	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	80	68	118	76	175	101	104
N.S.	1	1.00	0.78	0.66	1.15	0.74	1.70	0.98	1.01
time (sec)	N/A	0.067	0.082	4.234	0.201	0.242	0.397	0.266	7.119

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	56	49	83	52	124	70	52
N.S.	1	1.00	0.77	0.67	1.14	0.71	1.70	0.96	0.71
time (sec)	N/A	0.054	0.063	4.093	0.201	0.272	0.304	0.267	7.033

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	33	30	48	29	75	38	29
N.S.	1	1.00	0.72	0.65	1.04	0.63	1.63	0.83	0.63
time (sec)	N/A	0.028	0.043	4.367	0.240	0.256	0.196	0.264	7.014

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	37	54	105	71	40	57
N.S.	1	1.00	1.00	0.77	1.12	2.19	1.48	0.83	1.19
time (sec)	N/A	0.026	0.077	4.476	0.309	0.262	1.719	0.274	7.167

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	47	109	126	80	62	67
N.S.	1	1.00	1.00	0.81	1.88	2.17	1.38	1.07	1.16
time (sec)	N/A	0.041	0.101	4.140	0.286	0.249	5.757	0.276	7.304

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	78	67	178	173	163	121	95
N.S.	1	1.00	0.87	0.74	1.98	1.92	1.81	1.34	1.06
time (sec)	N/A	0.062	0.208	4.162	0.265	0.266	14.461	0.266	7.387

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	89	325	0	67	80	0	0
N.S.	1	1.00	0.33	1.20	0.00	0.25	0.30	0.00	0.00
time (sec)	N/A	0.097	10.085	4.197	0.000	0.090	1.232	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	74	309	0	42	78	0	0
N.S.	1	1.00	0.31	1.29	0.00	0.18	0.33	0.00	0.00
time (sec)	N/A	0.056	10.039	4.158	0.000	0.073	0.938	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	78	311	0	50	82	0	0
N.S.	1	1.00	0.32	1.28	0.00	0.21	0.34	0.00	0.00
time (sec)	N/A	0.058	10.036	4.317	0.000	0.073	1.038	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	78	329	0	62	90	0	0
N.S.	1	1.00	0.28	1.20	0.00	0.23	0.33	0.00	0.00
time (sec)	N/A	0.085	10.038	4.394	0.000	0.082	1.215	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	548	548	91	479	0	78	80	0	0
N.S.	1	1.00	0.17	0.87	0.00	0.14	0.15	0.00	0.00
time (sec)	N/A	0.232	10.083	4.255	0.000	0.083	1.293	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	517	517	75	463	0	51	80	0	0
N.S.	1	1.00	0.15	0.90	0.00	0.10	0.15	0.00	0.00
time (sec)	N/A	0.181	10.058	4.347	0.000	0.084	1.174	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	509	509	77	463	0	54	82	0	0
N.S.	1	1.00	0.15	0.91	0.00	0.11	0.16	0.00	0.00
time (sec)	N/A	0.167	10.035	4.211	0.000	0.078	0.993	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	550	550	78	481	0	70	88	0	0
N.S.	1	1.00	0.14	0.87	0.00	0.13	0.16	0.00	0.00
time (sec)	N/A	0.236	10.039	4.393	0.000	0.082	1.189	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	581	581	78	504	0	97	94	0	0
N.S.	1	1.00	0.13	0.87	0.00	0.17	0.16	0.00	0.00
time (sec)	N/A	0.289	10.042	4.452	0.000	0.084	1.327	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	77	68	116	88	175	114	152
N.S.	1	1.00	0.75	0.66	1.13	0.85	1.70	1.11	1.48
time (sec)	N/A	0.066	0.092	4.403	0.201	0.246	0.465	0.280	7.280

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	55	49	81	63	124	77	60
N.S.	1	1.00	0.75	0.67	1.11	0.86	1.70	1.05	0.82
time (sec)	N/A	0.054	0.074	4.335	0.208	0.253	0.339	0.276	7.243

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	33	30	47	41	75	38	33
N.S.	1	1.00	0.72	0.65	1.02	0.89	1.63	0.83	0.72
time (sec)	N/A	0.029	0.056	4.106	0.203	0.241	0.236	0.270	7.190

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	51	70	170	78	53	65
N.S.	1	1.00	1.00	0.88	1.21	2.93	1.34	0.91	1.12
time (sec)	N/A	0.036	0.113	4.266	0.282	0.255	3.864	0.278	7.310

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	77	74	144	233	264	99	131
N.S.	1	1.00	0.90	0.86	1.67	2.71	3.07	1.15	1.52
time (sec)	N/A	0.055	0.208	4.322	0.280	0.269	23.526	0.280	7.470

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	100	94	215	289	192	137	167
N.S.	1	1.00	0.85	0.80	1.82	2.45	1.63	1.16	1.42
time (sec)	N/A	0.079	0.249	4.352	0.292	0.273	59.398	0.295	7.690

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	103	406	0	122	80	0	0
N.S.	1	1.00	0.34	1.35	0.00	0.41	0.27	0.00	0.00
time (sec)	N/A	0.117	10.098	4.813	0.000	0.076	8.748	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	78	348	0	95	80	0	0
N.S.	1	1.00	0.29	1.29	0.00	0.35	0.30	0.00	0.00
time (sec)	N/A	0.096	10.075	4.679	0.000	0.085	4.060	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	73	336	0	83	78	0	0
N.S.	1	1.00	0.29	1.34	0.00	0.33	0.31	0.00	0.00
time (sec)	N/A	0.065	10.044	4.305	0.000	0.115	2.794	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	86	350	0	102	82	0	0
N.S.	1	1.00	0.32	1.29	0.00	0.38	0.30	0.00	0.00
time (sec)	N/A	0.092	10.041	4.674	0.000	0.076	8.289	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	72	386	0	119	90	0	0
N.S.	1	1.00	0.24	1.27	0.00	0.39	0.30	0.00	0.00
time (sec)	N/A	0.121	10.040	4.837	0.000	0.081	24.213	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	547	547	79	504	0	104	80	0	0
N.S.	1	1.00	0.14	0.92	0.00	0.19	0.15	0.00	0.00
time (sec)	N/A	0.214	10.077	5.056	0.000	0.080	4.844	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	524	524	71	490	0	94	80	0	0
N.S.	1	1.00	0.14	0.94	0.00	0.18	0.15	0.00	0.00
time (sec)	N/A	0.183	10.069	4.184	0.000	0.082	2.801	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	548	548	72	504	0	106	82	0	0
N.S.	1	1.00	0.13	0.92	0.00	0.19	0.15	0.00	0.00
time (sec)	N/A	0.226	10.038	4.951	0.000	0.079	6.359	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	580	580	72	540	0	127	88	0	0
N.S.	1	1.00	0.12	0.93	0.00	0.22	0.15	0.00	0.00
time (sec)	N/A	0.279	10.039	5.362	0.000	0.080	16.465	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	611	611	72	573	0	156	94	0	0
N.S.	1	1.00	0.12	0.94	0.00	0.26	0.15	0.00	0.00
time (sec)	N/A	0.327	10.039	5.225	0.000	0.092	43.675	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	73	70	116	98	338	104	145
N.S.	1	1.00	0.71	0.68	1.13	0.95	3.28	1.01	1.41
time (sec)	N/A	0.064	0.094	4.388	0.204	0.303	0.588	0.294	7.294

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	56	49	84	75	240	63	60
N.S.	1	1.00	0.77	0.67	1.15	1.03	3.29	0.86	0.82
time (sec)	N/A	0.049	0.073	4.371	0.190	0.312	0.437	0.302	7.215

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	33	30	49	52	144	32	33
N.S.	1	1.00	0.72	0.65	1.07	1.13	3.13	0.70	0.72
time (sec)	N/A	0.030	0.054	4.207	0.203	0.316	0.342	0.284	6.956

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	70	70	81	243	100	67	80
N.S.	1	1.00	0.91	0.91	1.05	3.16	1.30	0.87	1.04
time (sec)	N/A	0.038	0.157	4.234	0.273	0.295	6.744	0.302	7.033

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	99	94	170	351	1608	101	198
N.S.	1	1.00	0.88	0.83	1.50	3.11	14.23	0.89	1.75
time (sec)	N/A	0.067	0.198	4.268	0.271	0.333	126.770	0.307	7.232

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	108	397	0	153	80	0	0
N.S.	1	1.00	0.36	1.33	0.00	0.51	0.27	0.00	0.00
time (sec)	N/A	0.120	10.126	5.359	0.000	0.091	60.996	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	99	372	0	141	80	0	0
N.S.	1	1.00	0.35	1.31	0.00	0.50	0.28	0.00	0.00
time (sec)	N/A	0.094	10.104	4.434	0.000	0.086	38.552	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	103	364	0	145	78	0	0
N.S.	1	1.00	0.36	1.29	0.00	0.51	0.28	0.00	0.00
time (sec)	N/A	0.084	10.057	4.383	0.000	0.132	24.335	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	116	389	0	163	82	0	0
N.S.	1	1.00	0.39	1.30	0.00	0.54	0.27	0.00	0.00
time (sec)	N/A	0.104	10.070	5.541	0.000	0.085	81.003	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	83	425	0	178	0	0	0
N.S.	1	1.00	0.25	1.27	0.00	0.53	0.00	0.00	0.00
time (sec)	N/A	0.135	10.047	5.575	0.000	0.083	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	577	577	109	555	0	162	80	0	0
N.S.	1	1.00	0.19	0.96	0.00	0.28	0.14	0.00	0.00
time (sec)	N/A	0.271	10.109	5.561	0.000	0.081	79.235	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	559	559	92	528	0	154	80	0	0
N.S.	1	1.00	0.16	0.94	0.00	0.28	0.14	0.00	0.00
time (sec)	N/A	0.222	10.098	4.462	0.000	0.086	39.258	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	563	563	81	520	0	154	80	0	0
N.S.	1	1.00	0.14	0.92	0.00	0.27	0.14	0.00	0.00
time (sec)	N/A	0.209	10.077	4.276	0.000	0.108	24.604	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	578	578	86	545	0	167	82	0	0
N.S.	1	1.00	0.15	0.94	0.00	0.29	0.14	0.00	0.00
time (sec)	N/A	0.271	10.043	5.219	0.000	0.082	52.467	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	610	610	83	581	0	186	88	0	0
N.S.	1	1.00	0.14	0.95	0.00	0.30	0.14	0.00	0.00
time (sec)	N/A	0.318	10.044	5.878	0.000	0.084	156.888	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	78	64	69	156	95	82	109
N.S.	1	1.00	0.80	0.66	0.71	1.61	0.98	0.85	1.12
time (sec)	N/A	0.084	0.104	8.331	0.283	0.310	8.164	0.291	8.910

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	66	54	53	129	78	64	88
N.S.	1	1.00	0.87	0.71	0.70	1.70	1.03	0.84	1.16
time (sec)	N/A	0.047	0.086	4.535	0.275	0.290	3.407	0.294	8.534

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	54	44	42	110	61	44	71
N.S.	1	1.00	0.95	0.77	0.74	1.93	1.07	0.77	1.25
time (sec)	N/A	0.037	0.063	4.342	0.284	0.256	1.543	0.294	8.159

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	59	45	0	147	78	50	93
N.S.	1	1.00	0.91	0.69	0.00	2.26	1.20	0.77	1.43
time (sec)	N/A	0.052	0.067	4.699	0.000	0.258	2.584	0.307	9.019

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	66	0	194	0	72	113
N.S.	1	1.00	1.00	0.75	0.00	2.20	0.00	0.82	1.28
time (sec)	N/A	0.083	0.177	4.468	0.000	0.275	0.000	0.298	9.332

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	689	689	133	867	0	2442	0	0	0
N.S.	1	1.00	0.19	1.26	0.00	3.54	0.00	0.00	0.00
time (sec)	N/A	0.428	6.024	4.849	0.000	2.121	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	659	659	63	848	0	2202	0	0	0
N.S.	1	1.00	0.10	1.29	0.00	3.34	0.00	0.00	0.00
time (sec)	N/A	0.176	8.321	4.206	0.000	0.842	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	697	697	136	868	0	2253	0	0	0
N.S.	1	1.00	0.20	1.25	0.00	3.23	0.00	0.00	0.00
time (sec)	N/A	0.330	11.083	5.007	0.000	0.529	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	66	66	236	713	0	2387	0	0	0
N.S.	1	1.00	3.58	10.80	0.00	36.17	0.00	0.00	0.00
time (sec)	N/A	0.038	6.336	4.743	0.000	1.872	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	64	64	165	696	0	2240	0	0	0
N.S.	1	1.00	2.58	10.88	0.00	35.00	0.00	0.00	0.00
time (sec)	N/A	0.020	10.157	4.652	0.000	0.513	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	66	66	244	716	0	2361	0	0	0
N.S.	1	1.00	3.70	10.85	0.00	35.77	0.00	0.00	0.00
time (sec)	N/A	0.038	11.148	4.639	0.000	0.987	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	65	54	53	129	78	64	88
N.S.	1	1.00	0.83	0.69	0.68	1.65	1.00	0.82	1.13
time (sec)	N/A	0.055	0.095	4.389	0.286	0.257	8.734	0.290	9.021

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	56	44	43	112	63	49	71
N.S.	1	1.00	0.95	0.75	0.73	1.90	1.07	0.83	1.20
time (sec)	N/A	0.033	0.069	4.432	0.301	0.255	4.321	0.285	9.312

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	30	29	87	48	29	56
N.S.	1	1.00	1.00	0.75	0.72	2.18	1.20	0.72	1.40
time (sec)	N/A	0.027	0.049	4.145	0.302	0.244	2.832	0.297	8.422

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	59	45	0	148	80	53	94
N.S.	1	1.00	0.91	0.69	0.00	2.28	1.23	0.82	1.45
time (sec)	N/A	0.048	0.074	4.239	0.000	0.251	3.454	0.287	9.204

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	66	0	194	0	72	112
N.S.	1	1.00	1.00	0.75	0.00	2.20	0.00	0.82	1.27
time (sec)	N/A	0.058	0.138	4.455	0.000	0.264	0.000	0.282	9.496

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	667	667	67	848	0	2268	0	0	0
N.S.	1	1.00	0.10	1.27	0.00	3.40	0.00	0.00	0.00
time (sec)	N/A	0.193	10.041	4.379	0.000	1.643	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	206	206	67	416	0	2289	0	0	453
N.S.	1	1.00	0.33	2.02	0.00	11.11	0.00	0.00	2.20
time (sec)	N/A	0.024	10.040	4.350	0.000	0.673	0.000	0.000	29.695

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	697	697	136	868	0	2293	0	0	0
N.S.	1	1.00	0.20	1.25	0.00	3.29	0.00	0.00	0.00
time (sec)	N/A	0.329	11.089	4.936	0.000	0.697	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	66	66	67	696	0	2300	0	0	0
N.S.	1	1.00	1.02	10.55	0.00	34.85	0.00	0.00	0.00
time (sec)	N/A	0.037	10.039	4.362	0.000	0.712	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	64	64	165	416	0	2347	0	0	0
N.S.	1	1.00	2.58	6.50	0.00	36.67	0.00	0.00	0.00
time (sec)	N/A	0.020	10.052	4.329	0.000	0.717	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	66	66	243	716	0	2381	0	0	0
N.S.	1	1.00	3.68	10.85	0.00	36.08	0.00	0.00	0.00
time (sec)	N/A	0.040	11.180	5.039	0.000	1.416	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	28	164	0	1116	0	0	653
N.S.	1	1.00	0.22	1.29	0.00	8.79	0.00	0.00	5.14
time (sec)	N/A	0.016	0.017	13.970	0.000	0.393	0.000	0.000	7.157

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	82	70	96	169	109	100	118
N.S.	1	1.00	0.74	0.63	0.86	1.52	0.98	0.90	1.06
time (sec)	N/A	0.079	0.131	3.654	0.264	0.290	18.481	0.279	7.226

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	71	58	82	147	92	83	98
N.S.	1	1.00	0.79	0.64	0.91	1.63	1.02	0.92	1.09
time (sec)	N/A	0.067	0.104	4.274	0.263	0.343	8.083	0.288	7.284

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	59	47	66	121	75	65	78
N.S.	1	1.00	0.86	0.68	0.96	1.75	1.09	0.94	1.13
time (sec)	N/A	0.041	0.088	4.428	0.273	0.296	3.742	0.282	7.326

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	47	38	56	101	56	43	59
N.S.	1	1.00	0.94	0.76	1.12	2.02	1.12	0.86	1.18
time (sec)	N/A	0.035	0.064	4.341	0.260	0.295	1.895	0.274	7.291

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	53	38	0	138	71	48	125
N.S.	1	1.00	0.91	0.66	0.00	2.38	1.22	0.83	2.16
time (sec)	N/A	0.040	0.071	4.547	0.000	0.313	2.769	0.304	8.425

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	81	65	0	186	0	73	69
N.S.	1	1.00	1.00	0.80	0.00	2.30	0.00	0.90	0.85
time (sec)	N/A	0.055	0.157	4.299	0.000	0.268	0.000	0.296	7.408

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	95	77	0	188	0	100	83
N.S.	1	1.00	0.89	0.72	0.00	1.76	0.00	0.93	0.78
time (sec)	N/A	0.101	0.216	4.448	0.000	0.293	0.000	0.304	7.574

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	648	648	150	884	0	2442	0	0	0
N.S.	1	1.00	0.23	1.36	0.00	3.77	0.00	0.00	0.00
time (sec)	N/A	0.668	6.109	4.912	0.000	13.579	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	624	624	130	867	0	2428	0	0	0
N.S.	1	1.00	0.21	1.39	0.00	3.89	0.00	0.00	0.00
time (sec)	N/A	0.609	6.211	4.837	0.000	3.951	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	601	601	63	848	0	2194	0	0	0
N.S.	1	1.00	0.10	1.41	0.00	3.65	0.00	0.00	0.00
time (sec)	N/A	0.419	8.252	4.204	0.000	0.764	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	632	632	137	868	0	2219	0	0	0
N.S.	1	1.00	0.22	1.37	0.00	3.51	0.00	0.00	0.00
time (sec)	N/A	0.584	11.098	5.114	0.000	0.364	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	654	654	153	882	0	2401	0	0	0
N.S.	1	1.00	0.23	1.35	0.00	3.67	0.00	0.00	0.00
time (sec)	N/A	0.696	11.102	4.789	0.000	0.744	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	678	678	164	895	0	2436	0	0	0
N.S.	1	1.00	0.24	1.32	0.00	3.59	0.00	0.00	0.00
time (sec)	N/A	0.706	10.109	5.028	0.000	1.919	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	93	80	110	191	143	117	135
N.S.	1	1.00	0.72	0.62	0.85	1.47	1.10	0.90	1.04
time (sec)	N/A	0.084	0.146	4.398	0.264	0.272	49.715	0.292	7.461

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	82	69	96	169	122	100	115
N.S.	1	1.00	0.75	0.63	0.88	1.55	1.12	0.92	1.06
time (sec)	N/A	0.072	0.133	4.456	0.267	0.274	27.488	0.280	7.424

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	71	58	82	147	95	83	95
N.S.	1	1.00	0.81	0.66	0.93	1.67	1.08	0.94	1.08
time (sec)	N/A	0.049	0.102	4.355	0.260	0.273	13.341	0.291	7.441

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	59	47	68	121	71	65	75
N.S.	1	1.00	0.88	0.70	1.01	1.81	1.06	0.97	1.12
time (sec)	N/A	0.039	0.093	4.163	0.267	0.283	6.337	0.274	7.429

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	52	0	152	92	61	89
N.S.	1	1.00	1.00	0.71	0.00	2.08	1.26	0.84	1.22
time (sec)	N/A	0.053	0.081	4.406	0.000	0.273	4.392	0.282	9.971

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	78	59	0	186	0	64	56
N.S.	1	1.00	1.00	0.76	0.00	2.38	0.00	0.82	0.72
time (sec)	N/A	0.050	0.143	4.571	0.000	0.298	0.000	0.287	7.726

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	95	77	0	218	0	101	87
N.S.	1	1.00	0.91	0.74	0.00	2.10	0.00	0.97	0.84
time (sec)	N/A	0.068	0.198	4.737	0.000	0.289	0.000	0.289	7.907

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	669	669	163	895	0	2453	0	0	0
N.S.	1	1.00	0.24	1.34	0.00	3.67	0.00	0.00	0.00
time (sec)	N/A	0.706	7.829	4.752	0.000	24.896	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	645	645	150	884	0	2442	0	0	0
N.S.	1	1.00	0.23	1.37	0.00	3.79	0.00	0.00	0.00
time (sec)	N/A	0.706	7.595	4.894	0.000	8.320	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	627	627	127	864	0	2368	0	0	0
N.S.	1	1.00	0.20	1.38	0.00	3.78	0.00	0.00	0.00
time (sec)	N/A	0.593	9.980	4.846	0.000	1.810	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	626	626	137	859	0	0	0	0	0
N.S.	1	1.00	0.22	1.37	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.576	10.087	4.894	0.000	0.000	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	651	651	154	882	0	2369	0	0	0
N.S.	1	1.00	0.24	1.35	0.00	3.64	0.00	0.00	0.00
time (sec)	N/A	0.703	10.105	4.956	0.000	0.559	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	675	675	167	895	0	2442	0	0	0
N.S.	1	1.00	0.25	1.33	0.00	3.62	0.00	0.00	0.00
time (sec)	N/A	0.726	10.105	4.931	0.000	1.513	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	69	58	82	146	92	82	98
N.S.	1	1.00	0.77	0.64	0.91	1.62	1.02	0.91	1.09
time (sec)	N/A	0.060	0.114	4.500	0.271	0.308	16.603	0.311	7.472

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	58	47	66	121	76	65	78
N.S.	1	1.00	0.82	0.66	0.93	1.70	1.07	0.92	1.10
time (sec)	N/A	0.053	0.092	4.447	0.268	0.301	8.639	0.280	7.439

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	49	38	56	103	60	48	60
N.S.	1	1.00	0.94	0.73	1.08	1.98	1.15	0.92	1.15
time (sec)	N/A	0.032	0.058	4.545	0.261	0.278	4.868	0.279	7.489

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	24	42	78	42	27	45
N.S.	1	1.00	1.00	0.73	1.27	2.36	1.27	0.82	1.36
time (sec)	N/A	0.023	0.045	4.566	0.277	0.281	3.126	0.287	7.501

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	51	38	0	139	75	54	47
N.S.	1	1.00	0.88	0.66	0.00	2.40	1.29	0.93	0.81
time (sec)	N/A	0.036	0.065	4.286	0.000	0.310	3.294	0.292	7.309

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	81	64	0	184	0	73	73
N.S.	1	1.00	1.00	0.79	0.00	2.27	0.00	0.90	0.90
time (sec)	N/A	0.057	0.140	4.484	0.000	0.288	0.000	0.276	7.587

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	95	77	0	217	0	101	94
N.S.	1	1.00	0.89	0.72	0.00	2.03	0.00	0.94	0.88
time (sec)	N/A	0.073	0.218	4.328	0.000	0.287	0.000	0.292	7.673

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	630	630	130	867	0	2428	0	0	0
N.S.	1	1.00	0.21	1.38	0.00	3.85	0.00	0.00	0.00
time (sec)	N/A	0.573	10.094	4.873	0.000	8.429	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	601	601	67	848	0	2254	0	0	0
N.S.	1	1.00	0.11	1.41	0.00	3.75	0.00	0.00	0.00
time (sec)	N/A	0.441	10.051	4.449	0.000	1.632	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	141	141	67	416	0	2285	0	0	272
N.S.	1	1.00	0.48	2.95	0.00	16.21	0.00	0.00	1.93
time (sec)	N/A	0.281	10.039	4.231	0.000	0.540	0.000	0.000	45.124

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	632	632	137	868	0	2259	0	0	0
N.S.	1	1.00	0.22	1.37	0.00	3.57	0.00	0.00	0.00
time (sec)	N/A	0.535	11.086	4.888	0.000	0.461	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	654	654	152	882	0	2403	0	0	0
N.S.	1	1.00	0.23	1.35	0.00	3.67	0.00	0.00	0.00
time (sec)	N/A	0.693	11.111	4.932	0.000	1.108	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	678	678	167	895	0	2436	0	0	0
N.S.	1	1.00	0.25	1.32	0.00	3.59	0.00	0.00	0.00
time (sec)	N/A	0.716	10.116	5.023	0.000	2.621	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	66	66	67	696	0	2284	0	0	0
N.S.	1	1.00	1.02	10.55	0.00	34.61	0.00	0.00	0.00
time (sec)	N/A	0.043	10.045	4.361	0.000	0.562	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	64	64	166	416	0	2319	0	0	0
N.S.	1	1.00	2.59	6.50	0.00	36.23	0.00	0.00	0.00
time (sec)	N/A	0.020	10.160	4.225	0.000	0.594	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	66	66	242	716	0	2373	0	0	0
N.S.	1	1.00	3.67	10.85	0.00	35.95	0.00	0.00	0.00
time (sec)	N/A	0.040	11.236	4.899	0.000	0.991	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	66	66	261	732	0	2417	0	0	0
N.S.	1	1.00	3.95	11.09	0.00	36.62	0.00	0.00	0.00
time (sec)	N/A	0.039	11.219	4.916	0.000	2.512	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	70	65	82	189	90	82	95
N.S.	1	1.00	0.78	0.72	0.91	2.10	1.00	0.91	1.06
time (sec)	N/A	0.074	0.133	4.276	0.272	0.296	21.226	0.274	8.726

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	59	54	68	161	75	58	75
N.S.	1	1.00	0.83	0.76	0.96	2.27	1.06	0.82	1.06
time (sec)	N/A	0.058	0.105	4.474	0.277	0.288	11.697	0.428	7.924

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	49	48	56	149	56	47	60
N.S.	1	1.00	0.94	0.92	1.08	2.87	1.08	0.90	1.15
time (sec)	N/A	0.037	0.085	4.697	0.289	0.350	7.903	0.371	7.886

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	52	48	58	147	60	48	63
N.S.	1	1.00	0.95	0.87	1.05	2.67	1.09	0.87	1.15
time (sec)	N/A	0.036	0.088	4.255	0.277	0.268	6.507	0.454	7.886

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	69	71	0	213	97	68	68
N.S.	1	1.00	0.91	0.93	0.00	2.80	1.28	0.89	0.89
time (sec)	N/A	0.047	0.124	4.429	0.000	0.267	4.402	0.272	7.941

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	85	76	0	272	0	100	88
N.S.	1	1.00	0.85	0.76	0.00	2.72	0.00	1.00	0.88
time (sec)	N/A	0.064	0.210	4.512	0.000	0.280	0.000	0.276	8.061

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	100	88	0	303	0	118	112
N.S.	1	1.00	0.78	0.69	0.00	2.37	0.00	0.92	0.88
time (sec)	N/A	0.094	0.283	4.524	0.000	0.287	0.000	0.275	8.292

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	629	629	127	869	0	2384	0	0	0
N.S.	1	1.00	0.20	1.38	0.00	3.79	0.00	0.00	0.00
time (sec)	N/A	0.565	8.544	4.323	0.000	3.572	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	635	635	126	878	0	2525	0	0	0
N.S.	1	1.00	0.20	1.38	0.00	3.98	0.00	0.00	0.00
time (sec)	N/A	0.586	8.277	4.360	0.000	0.446	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	632	632	124	875	0	2525	0	0	0
N.S.	1	1.00	0.20	1.38	0.00	4.00	0.00	0.00	0.00
time (sec)	N/A	0.594	10.081	4.550	0.000	0.431	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	653	653	140	890	0	2379	0	0	0
N.S.	1	1.00	0.21	1.36	0.00	3.64	0.00	0.00	0.00
time (sec)	N/A	0.740	11.100	5.422	0.000	0.643	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	675	675	153	911	0	2529	0	0	0
N.S.	1	1.00	0.23	1.35	0.00	3.75	0.00	0.00	0.00
time (sec)	N/A	0.763	11.100	5.480	0.000	1.743	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	699	699	167	930	0	2564	0	0	0
N.S.	1	1.00	0.24	1.33	0.00	3.67	0.00	0.00	0.00
time (sec)	N/A	0.866	10.128	5.477	0.000	4.259	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	66	66	233	724	0	2535	0	0	0
N.S.	1	1.00	3.53	10.97	0.00	38.41	0.00	0.00	0.00
time (sec)	N/A	0.042	8.110	4.236	0.000	0.782	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	64	64	230	721	0	2483	0	0	0
N.S.	1	1.00	3.59	11.27	0.00	38.80	0.00	0.00	0.00
time (sec)	N/A	0.022	10.158	4.408	0.000	0.915	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	66	66	248	736	0	2495	0	0	0
N.S.	1	1.00	3.76	11.15	0.00	37.80	0.00	0.00	0.00
time (sec)	N/A	0.044	11.191	5.197	0.000	1.590	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	66	66	261	757	0	2543	0	0	0
N.S.	1	1.00	3.95	11.47	0.00	38.53	0.00	0.00	0.00
time (sec)	N/A	0.043	11.261	5.288	0.000	4.221	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	737	737	80	977	0	4847	0	0	0
N.S.	1	1.00	0.11	1.33	0.00	6.58	0.00	0.00	0.00
time (sec)	N/A	0.218	10.099	5.023	0.000	3.426	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	757	757	80	924	0	4855	0	0	0
N.S.	1	1.00	0.11	1.22	0.00	6.41	0.00	0.00	0.00
time (sec)	N/A	0.214	10.091	6.279	0.000	3.367	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	774	774	87	926	0	4963	0	0	0
N.S.	1	1.00	0.11	1.20	0.00	6.41	0.00	0.00	0.00
time (sec)	N/A	0.228	10.078	4.971	0.000	3.735	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	768	768	90	983	0	4981	0	0	0
N.S.	1	1.00	0.12	1.28	0.00	6.49	0.00	0.00	0.00
time (sec)	N/A	0.214	10.079	5.086	0.000	3.280	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	738	738	80	977	0	4931	0	0	0
N.S.	1	1.00	0.11	1.32	0.00	6.68	0.00	0.00	0.00
time (sec)	N/A	0.210	10.104	5.290	0.000	3.389	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	758	758	80	924	0	4953	0	0	0
N.S.	1	1.00	0.11	1.22	0.00	6.53	0.00	0.00	0.00
time (sec)	N/A	0.190	10.097	5.004	0.000	3.376	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	774	774	89	926	0	4867	0	0	0
N.S.	1	1.00	0.11	1.20	0.00	6.29	0.00	0.00	0.00
time (sec)	N/A	0.218	10.072	4.742	0.000	3.391	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	768	768	89	983	0	4875	0	0	0
N.S.	1	1.00	0.12	1.28	0.00	6.35	0.00	0.00	0.00
time (sec)	N/A	0.209	10.073	4.769	0.000	3.414	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	318	318	83	538	0	5563	0	0	0
N.S.	1	1.00	0.26	1.69	0.00	17.49	0.00	0.00	0.00
time (sec)	N/A	0.046	10.099	4.832	0.000	3.556	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	324	324	83	509	0	5587	0	0	0
N.S.	1	1.00	0.26	1.57	0.00	17.24	0.00	0.00	0.00
time (sec)	N/A	0.041	10.083	4.688	0.000	3.499	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	328	328	85	510	0	5667	0	0	0
N.S.	1	1.00	0.26	1.55	0.00	17.28	0.00	0.00	0.00
time (sec)	N/A	0.039	10.085	4.855	0.000	3.686	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	330	330	87	541	0	5679	0	0	0
N.S.	1	1.00	0.26	1.64	0.00	17.21	0.00	0.00	0.00
time (sec)	N/A	0.042	10.077	4.751	0.000	3.547	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	310	310	83	538	0	5631	0	0	0
N.S.	1	1.00	0.27	1.74	0.00	18.16	0.00	0.00	0.00
time (sec)	N/A	0.036	10.115	4.720	0.000	3.430	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	316	316	83	509	0	5667	0	0	0
N.S.	1	1.00	0.26	1.61	0.00	17.93	0.00	0.00	0.00
time (sec)	N/A	0.043	10.105	4.990	0.000	3.441	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	320	320	84	510	0	5599	0	0	0
N.S.	1	1.00	0.26	1.59	0.00	17.50	0.00	0.00	0.00
time (sec)	N/A	0.047	10.091	4.851	0.000	3.569	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	322	322	86	541	0	5599	0	0	0
N.S.	1	1.00	0.27	1.68	0.00	17.39	0.00	0.00	0.00
time (sec)	N/A	0.043	10.102	4.966	0.000	3.598	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	121	124	0	280	173	139	176
N.S.	1	1.00	0.97	0.99	0.00	2.24	1.38	1.11	1.41
time (sec)	N/A	0.103	0.390	5.612	0.000	0.277	8.957	0.288	10.527

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	88	81	0	195	128	96	136
N.S.	1	1.00	0.95	0.87	0.00	2.10	1.38	1.03	1.46
time (sec)	N/A	0.060	0.274	4.370	0.000	0.275	4.212	0.280	10.421

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	62	0	156	95	66	82
N.S.	1	1.00	1.00	0.89	0.00	2.23	1.36	0.94	1.17
time (sec)	N/A	0.047	0.126	4.381	0.000	0.274	2.410	0.294	10.443

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	81	98	0	383	165	79	114
N.S.	1	1.00	0.95	1.15	0.00	4.51	1.94	0.93	1.34
time (sec)	N/A	0.051	0.156	4.501	0.000	0.311	4.288	0.299	12.462

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	107	96	0	513	0	107	137
N.S.	1	1.00	0.93	0.83	0.00	4.46	0.00	0.93	1.19
time (sec)	N/A	0.091	0.427	4.582	0.000	0.301	0.000	0.269	9.444

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	64	64	241	741	0	0	0	0	0
N.S.	1	1.00	3.77	11.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.042	6.502	5.088	0.000	0.000	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	64	64	65	857	0	0	0	0	0
N.S.	1	1.00	1.02	13.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	9.079	4.435	0.000	0.000	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	59	59	161	705	0	0	0	0	0
N.S.	1	1.00	2.73	11.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.019	10.172	4.279	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	62	62	139	892	0	0	0	0	0
N.S.	1	1.00	2.24	14.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.048	10.113	5.236	0.000	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	64	64	335	740	0	0	0	0	0
N.S.	1	1.00	5.23	11.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.040	10.268	5.353	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	143	149	0	410	201	193	330
N.S.	1	1.00	0.93	0.97	0.00	2.66	1.31	1.25	2.14
time (sec)	N/A	0.141	0.472	4.396	0.000	0.276	33.412	0.296	10.386

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	111	113	0	297	153	151	215
N.S.	1	1.00	0.92	0.94	0.00	2.48	1.28	1.26	1.79
time (sec)	N/A	0.086	0.306	4.680	0.000	0.278	16.104	0.298	10.355

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	85	94	0	204	124	113	143
N.S.	1	1.00	0.89	0.98	0.00	2.12	1.29	1.18	1.49
time (sec)	N/A	0.084	0.274	4.416	0.000	0.281	7.951	0.287	10.179

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	106	104	0	486	189	112	155
N.S.	1	1.00	1.02	1.00	0.00	4.67	1.82	1.08	1.49
time (sec)	N/A	0.090	0.337	4.589	0.000	0.313	6.570	0.292	12.332

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	108	118	0	538	0	121	167
N.S.	1	1.00	0.93	1.02	0.00	4.64	0.00	1.04	1.44
time (sec)	N/A	0.109	0.395	4.429	0.000	0.305	0.000	0.297	13.457

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	65	65	280	800	0	0	0	0	0
N.S.	1	1.00	4.31	12.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.038	8.318	5.245	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	65	65	149	921	0	0	0	0	0
N.S.	1	1.00	2.29	14.17	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.029	10.163	5.570	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	60	60	351	769	0	0	0	0	0
N.S.	1	1.00	5.85	12.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.020	10.326	5.171	0.000	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	63	63	148	920	0	0	0	0	0
N.S.	1	1.00	2.35	14.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.041	10.133	5.699	0.000	0.000	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	65	65	343	771	0	0	0	0	0
N.S.	1	1.00	5.28	11.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.040	10.353	5.255	0.000	0.000	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	91	79	0	289	0	106	121
N.S.	1	1.00	0.88	0.76	0.00	2.78	0.00	1.02	1.16
time (sec)	N/A	0.083	0.309	4.337	0.000	0.351	0.000	0.266	9.690

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	75	61	0	205	0	64	86
N.S.	1	1.00	1.01	0.82	0.00	2.77	0.00	0.86	1.16
time (sec)	N/A	0.045	0.153	4.532	0.000	0.343	0.000	0.266	9.409

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	39	0	130	88	40	70
N.S.	1	1.00	1.00	0.76	0.00	2.55	1.73	0.78	1.37
time (sec)	N/A	0.044	0.072	4.673	0.000	0.292	3.930	0.281	10.230

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	82	66	0	431	119	71	114
N.S.	1	1.00	0.96	0.78	0.00	5.07	1.40	0.84	1.34
time (sec)	N/A	0.061	0.208	4.361	0.000	0.329	5.103	0.277	11.765

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	109	92	0	565	0	104	142
N.S.	1	1.00	0.93	0.79	0.00	4.83	0.00	0.89	1.21
time (sec)	N/A	0.085	0.399	4.606	0.000	0.347	0.000	0.269	12.828

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	64	64	65	719	0	0	0	0	0
N.S.	1	1.00	1.02	11.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.041	10.041	4.285	0.000	0.000	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	64	64	65	429	0	0	0	0	0
N.S.	1	1.00	1.02	6.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.028	10.036	4.262	0.000	0.000	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	59	59	161	429	0	0	0	0	0
N.S.	1	1.00	2.73	7.27	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.019	10.060	4.420	0.000	0.000	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	62	62	141	890	0	0	0	0	0
N.S.	1	1.00	2.27	14.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.046	10.115	4.929	0.000	0.000	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	64	64	339	738	0	0	0	0	0
N.S.	1	1.00	5.30	11.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.043	10.296	4.879	0.000	0.000	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	111	100	0	440	0	103	115
N.S.	1	1.00	1.04	0.93	0.00	4.11	0.00	0.96	1.07
time (sec)	N/A	0.107	0.486	4.540	0.000	0.391	0.000	0.273	10.806

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	80	88	0	326	0	78	94
N.S.	1	1.00	0.98	1.07	0.00	3.98	0.00	0.95	1.15
time (sec)	N/A	0.062	0.264	4.351	0.000	0.335	0.000	0.288	10.269

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	76	83	0	236	121	73	89
N.S.	1	1.00	0.99	1.08	0.00	3.06	1.57	0.95	1.16
time (sec)	N/A	0.045	0.168	4.202	0.000	0.349	8.689	0.283	10.220

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	110	103	0	790	155	111	139
N.S.	1	1.00	0.96	0.90	0.00	6.93	1.36	0.97	1.22
time (sec)	N/A	0.095	0.601	4.914	0.000	0.380	6.873	0.285	12.983

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	142	144	0	1120	0	173	597
N.S.	1	1.00	0.90	0.91	0.00	7.09	0.00	1.09	3.78
time (sec)	N/A	0.173	0.678	4.563	0.000	0.454	0.000	0.295	15.206

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	67	67	231	749	0	0	0	0	0
N.S.	1	1.00	3.45	11.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.039	10.192	4.454	0.000	0.000	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	67	67	142	907	0	0	0	0	0
N.S.	1	1.00	2.12	13.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	10.126	4.429	0.000	0.000	0.000	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	62	62	338	753	0	0	0	0	0
N.S.	1	1.00	5.45	12.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.020	10.323	4.261	0.000	0.000	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	65	65	193	952	0	0	0	0	0
N.S.	1	1.00	2.97	14.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.044	10.213	5.861	0.000	0.000	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	67	67	408	798	0	0	0	0	0
N.S.	1	1.00	6.09	11.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.043	10.576	6.049	0.000	0.000	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	91	98	107	219	0	110	127
N.S.	1	1.00	0.78	0.84	0.91	1.87	0.00	0.94	1.09
time (sec)	N/A	0.068	0.244	4.407	0.272	0.344	0.000	0.276	8.317

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	79	79	91	191	0	93	107
N.S.	1	1.00	0.77	0.77	0.89	1.87	0.00	0.91	1.05
time (sec)	N/A	0.057	0.163	4.669	0.297	0.305	0.000	0.285	8.279

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	69	62	79	165	0	69	87
N.S.	1	1.00	0.84	0.76	0.96	2.01	0.00	0.84	1.06
time (sec)	N/A	0.045	0.136	4.876	0.287	0.466	0.000	0.269	8.223

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	61	49	66	149	0	53	72
N.S.	1	1.00	0.95	0.77	1.03	2.33	0.00	0.83	1.12
time (sec)	N/A	0.042	0.107	4.243	0.281	0.297	0.000	0.276	8.118

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	83	78	0	226	0	79	76
N.S.	1	1.00	0.94	0.89	0.00	2.57	0.00	0.90	0.86
time (sec)	N/A	0.055	0.146	4.525	0.000	0.314	0.000	0.290	8.252

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	97	98	0	278	0	113	117
N.S.	1	1.00	0.78	0.79	0.00	2.24	0.00	0.91	0.94
time (sec)	N/A	0.076	0.291	4.629	0.000	0.333	0.000	0.287	8.433

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	112	116	0	310	0	105	154
N.S.	1	1.00	0.68	0.71	0.00	1.89	0.00	0.64	0.94
time (sec)	N/A	0.107	0.373	4.637	0.000	0.328	0.000	0.280	8.702

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	663	663	176	897	0	2568	0	0	0
N.S.	1	1.00	0.27	1.35	0.00	3.87	0.00	0.00	0.00
time (sec)	N/A	0.685	7.094	5.543	0.000	8.825	0.000	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	641	641	167	877	0	2397	0	0	0
N.S.	1	1.00	0.26	1.37	0.00	3.74	0.00	0.00	0.00
time (sec)	N/A	0.598	10.153	4.566	0.000	1.735	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	644	644	164	883	0	2496	0	0	0
N.S.	1	1.00	0.25	1.37	0.00	3.88	0.00	0.00	0.00
time (sec)	N/A	0.620	10.105	4.488	0.000	0.402	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	665	665	179	898	0	2390	0	0	0
N.S.	1	1.00	0.27	1.35	0.00	3.59	0.00	0.00	0.00
time (sec)	N/A	0.640	11.126	5.668	0.000	0.500	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	687	687	199	919	0	2549	0	0	0
N.S.	1	1.00	0.29	1.34	0.00	3.71	0.00	0.00	0.00
time (sec)	N/A	0.853	10.154	5.649	0.000	1.434	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	711	711	209	938	0	2582	0	0	0
N.S.	1	1.00	0.29	1.32	0.00	3.63	0.00	0.00	0.00
time (sec)	N/A	0.818	10.160	5.512	0.000	2.939	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	103	109	119	239	0	127	147
N.S.	1	1.00	0.77	0.81	0.89	1.78	0.00	0.95	1.10
time (sec)	N/A	0.101	0.223	4.776	0.295	0.305	0.000	0.281	8.316

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	93	98	107	219	0	111	127
N.S.	1	1.00	0.78	0.82	0.90	1.84	0.00	0.93	1.07
time (sec)	N/A	0.070	0.194	4.640	0.305	0.322	0.000	0.271	8.266

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	81	79	93	192	0	93	107
N.S.	1	1.00	0.84	0.81	0.96	1.98	0.00	0.96	1.10
time (sec)	N/A	0.054	0.167	4.414	0.331	0.306	0.000	0.277	8.045

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	72	63	79	162	0	69	87
N.S.	1	1.00	0.94	0.82	1.03	2.10	0.00	0.90	1.13
time (sec)	N/A	0.047	0.130	4.488	0.318	0.304	0.000	0.282	8.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	85	64	0	220	0	70	101
N.S.	1	1.00	1.00	0.75	0.00	2.59	0.00	0.82	1.19
time (sec)	N/A	0.062	0.133	4.531	0.000	0.315	0.000	0.272	8.754

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	97	99	0	280	0	114	110
N.S.	1	1.00	0.80	0.82	0.00	2.31	0.00	0.94	0.91
time (sec)	N/A	0.080	0.264	4.642	0.000	0.409	0.000	0.284	8.215

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	112	118	0	310	0	129	151
N.S.	1	1.00	0.70	0.73	0.00	1.93	0.00	0.80	0.94
time (sec)	N/A	0.120	0.339	4.654	0.000	0.362	0.000	0.287	8.588

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	681	681	191	921	0	2580	0	0	0
N.S.	1	1.00	0.28	1.35	0.00	3.79	0.00	0.00	0.00
time (sec)	N/A	0.801	8.442	5.508	0.000	17.752	0.000	0.000	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	657	657	176	897	0	2568	0	0	0
N.S.	1	1.00	0.27	1.37	0.00	3.91	0.00	0.00	0.00
time (sec)	N/A	0.662	10.176	5.523	0.000	3.788	0.000	0.000	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	638	638	141	874	0	2335	0	0	0
N.S.	1	1.00	0.22	1.37	0.00	3.66	0.00	0.00	0.00
time (sec)	N/A	0.612	10.194	4.489	0.000	0.994	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	522	522	242	483	0	68	0	0	0
N.S.	1	1.00	0.46	0.93	0.00	0.13	0.00	0.00	0.00
time (sec)	N/A	0.266	11.739	5.352	0.000	0.101	0.000	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	684	684	199	919	0	2549	0	0	0
N.S.	1	1.00	0.29	1.34	0.00	3.73	0.00	0.00	0.00
time (sec)	N/A	0.793	10.153	5.615	0.000	0.896	0.000	0.000	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	708	708	212	938	0	2582	0	0	0
N.S.	1	1.00	0.30	1.32	0.00	3.65	0.00	0.00	0.00
time (sec)	N/A	0.862	10.169	5.837	0.000	1.946	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	81	79	93	195	0	93	107
N.S.	1	1.00	0.85	0.83	0.98	2.05	0.00	0.98	1.13
time (sec)	N/A	0.055	0.186	4.575	0.302	0.280	0.000	0.269	8.046

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	70	63	79	167	0	69	87
N.S.	1	1.00	0.84	0.76	0.95	2.01	0.00	0.83	1.05
time (sec)	N/A	0.048	0.127	4.393	0.297	0.286	0.000	0.263	8.043

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	63	50	67	155	0	58	72
N.S.	1	1.00	0.98	0.78	1.05	2.42	0.00	0.91	1.12
time (sec)	N/A	0.035	0.119	4.352	0.291	0.302	0.000	0.282	7.963

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	64	52	72	153	0	59	75
N.S.	1	1.00	0.96	0.78	1.07	2.28	0.00	0.88	1.12
time (sec)	N/A	0.038	0.092	4.246	0.284	0.276	0.000	0.277	7.921

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	83	78	0	226	0	79	80
N.S.	1	1.00	0.94	0.89	0.00	2.57	0.00	0.90	0.91
time (sec)	N/A	0.063	0.144	4.512	0.000	0.294	0.000	0.280	7.999

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	97	99	0	280	0	114	117
N.S.	1	1.00	0.78	0.80	0.00	2.26	0.00	0.92	0.94
time (sec)	N/A	0.083	0.285	4.639	0.000	0.308	0.000	0.276	8.119

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	112	118	0	310	0	128	155
N.S.	1	1.00	0.68	0.72	0.00	1.89	0.00	0.78	0.95
time (sec)	N/A	0.111	0.359	4.642	0.000	0.307	0.000	0.273	8.434

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	641	641	167	877	0	2397	0	0	0
N.S.	1	1.00	0.26	1.37	0.00	3.74	0.00	0.00	0.00
time (sec)	N/A	0.620	10.167	4.691	0.000	3.665	0.000	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	647	647	166	886	0	2538	0	0	0
N.S.	1	1.00	0.26	1.37	0.00	3.92	0.00	0.00	0.00
time (sec)	N/A	0.604	10.115	4.886	0.000	0.452	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	644	644	164	883	0	2540	0	0	0
N.S.	1	1.00	0.25	1.37	0.00	3.94	0.00	0.00	0.00
time (sec)	N/A	0.599	10.105	4.318	0.000	0.567	0.000	0.000	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	665	665	180	898	0	2391	0	0	0
N.S.	1	1.00	0.27	1.35	0.00	3.60	0.00	0.00	0.00
time (sec)	N/A	0.676	11.132	5.487	0.000	0.677	0.000	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	687	687	196	919	0	2549	0	0	0
N.S.	1	1.00	0.29	1.34	0.00	3.71	0.00	0.00	0.00
time (sec)	N/A	0.745	10.163	5.596	0.000	2.066	0.000	0.000	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	711	711	212	938	0	2582	0	0	0
N.S.	1	1.00	0.30	1.32	0.00	3.63	0.00	0.00	0.00
time (sec)	N/A	0.803	10.165	5.797	0.000	4.286	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	66	66	239	723	0	2425	0	0	0
N.S.	1	1.00	3.62	10.95	0.00	36.74	0.00	0.00	0.00
time (sec)	N/A	0.038	10.327	4.505	0.000	0.718	0.000	0.000	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	66	66	237	732	0	2548	0	0	0
N.S.	1	1.00	3.59	11.09	0.00	38.61	0.00	0.00	0.00
time (sec)	N/A	0.044	10.216	4.582	0.000	0.899	0.000	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	64	64	237	729	0	2498	0	0	0
N.S.	1	1.00	3.70	11.39	0.00	39.03	0.00	0.00	0.00
time (sec)	N/A	0.021	10.199	4.481	0.000	0.874	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	66	66	266	744	0	0	0	0	0
N.S.	1	1.00	4.03	11.27	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.039	10.209	5.565	0.000	0.000	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	66	66	279	765	0	2561	0	0	0
N.S.	1	1.00	4.23	11.59	0.00	38.80	0.00	0.00	0.00
time (sec)	N/A	0.039	10.217	5.559	0.000	4.252	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	99	75	98	233	0	88	111
N.S.	1	1.00	1.04	0.79	1.03	2.45	0.00	0.93	1.17
time (sec)	N/A	0.053	0.184	4.582	0.286	0.313	0.000	0.294	8.308

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	71	61	81	223	0	67	94
N.S.	1	1.00	0.86	0.73	0.98	2.69	0.00	0.81	1.13
time (sec)	N/A	0.046	0.177	4.769	0.283	0.337	0.000	0.279	8.362

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	73	61	83	223	0	76	96
N.S.	1	1.00	0.86	0.72	0.98	2.62	0.00	0.89	1.13
time (sec)	N/A	0.046	0.152	4.381	0.291	0.352	0.000	0.296	8.278

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	73	62	85	219	0	72	97
N.S.	1	1.00	0.83	0.70	0.97	2.49	0.00	0.82	1.10
time (sec)	N/A	0.049	0.153	4.274	0.302	0.315	0.000	0.288	8.252

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	93	92	0	316	0	93	101
N.S.	1	1.00	0.88	0.87	0.00	2.98	0.00	0.88	0.95
time (sec)	N/A	0.082	0.202	4.554	0.000	0.353	0.000	0.289	8.347

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	109	113	0	368	0	129	133
N.S.	1	1.00	0.76	0.79	0.00	2.57	0.00	0.90	0.93
time (sec)	N/A	0.089	0.352	4.701	0.000	0.316	0.000	0.299	8.478

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	123	135	0	398	0	149	171
N.S.	1	1.00	0.66	0.73	0.00	2.15	0.00	0.81	0.92
time (sec)	N/A	0.120	0.431	4.497	0.000	0.328	0.000	0.293	8.821

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	668	668	168	910	0	2681	0	0	0
N.S.	1	1.00	0.25	1.36	0.00	4.01	0.00	0.00	0.00
time (sec)	N/A	0.633	10.125	4.591	0.000	0.596	0.000	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	671	671	169	910	0	2723	0	0	0
N.S.	1	1.00	0.25	1.36	0.00	4.06	0.00	0.00	0.00
time (sec)	N/A	0.685	10.120	4.619	0.000	0.650	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	665	665	167	904	0	2684	0	0	0
N.S.	1	1.00	0.25	1.36	0.00	4.04	0.00	0.00	0.00
time (sec)	N/A	0.695	10.119	4.528	0.000	0.762	0.000	0.000	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	686	686	180	920	0	2534	0	0	0
N.S.	1	1.00	0.26	1.34	0.00	3.69	0.00	0.00	0.00
time (sec)	N/A	0.761	11.154	6.327	0.000	0.909	0.000	0.000	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	708	708	198	943	0	2692	0	0	0
N.S.	1	1.00	0.28	1.33	0.00	3.80	0.00	0.00	0.00
time (sec)	N/A	0.819	10.163	6.309	0.000	2.796	0.000	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	732	732	210	962	0	2725	0	0	0
N.S.	1	1.00	0.29	1.31	0.00	3.72	0.00	0.00	0.00
time (sec)	N/A	0.904	10.185	6.671	0.000	5.575	0.000	0.000	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	256	66	189	339	0	89	0	0	0
N.S.	1	0.26	0.74	1.32	0.00	0.35	0.00	0.00	0.00
time (sec)	N/A	0.051	10.562	4.401	0.000	0.102	0.000	0.000	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	66	66	242	754	0	2713	0	0	0
N.S.	1	1.00	3.67	11.42	0.00	41.11	0.00	0.00	0.00
time (sec)	N/A	0.061	10.291	4.422	0.000	1.067	0.000	0.000	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	64	64	253	748	0	2640	0	0	0
N.S.	1	1.00	3.95	11.69	0.00	41.25	0.00	0.00	0.00
time (sec)	N/A	0.021	10.265	4.546	0.000	0.975	0.000	0.000	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	66	66	259	764	0	2650	0	0	0
N.S.	1	1.00	3.92	11.58	0.00	40.15	0.00	0.00	0.00
time (sec)	N/A	0.058	10.246	6.045	0.000	2.074	0.000	0.000	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	66	66	283	787	0	2698	0	0	0
N.S.	1	1.00	4.29	11.92	0.00	40.88	0.00	0.00	0.00
time (sec)	N/A	0.048	10.242	6.209	0.000	5.324	0.000	0.000	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	126	129	0	469	0	136	202
N.S.	1	1.00	0.78	0.80	0.00	2.91	0.00	0.84	1.25
time (sec)	N/A	0.170	0.434	4.801	0.000	0.314	0.000	0.290	10.881

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	98	98	0	334	0	102	152
N.S.	1	1.00	0.72	0.72	0.00	2.46	0.00	0.75	1.12
time (sec)	N/A	0.090	0.291	4.666	0.000	0.407	0.000	0.278	10.135

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	65	0	255	0	79	125
N.S.	1	1.00	1.00	0.81	0.00	3.19	0.00	0.99	1.56
time (sec)	N/A	0.061	0.229	4.457	0.000	0.351	0.000	0.273	9.669

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	111	120	0	856	0	114	182
N.S.	1	1.00	0.92	0.99	0.00	7.07	0.00	0.94	1.50
time (sec)	N/A	0.092	0.630	4.442	0.000	0.393	0.000	0.277	12.814

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	132	151	0	870	0	183	438
N.S.	1	1.00	0.82	0.94	0.00	5.40	0.00	1.14	2.72
time (sec)	N/A	0.158	0.781	4.761	0.000	0.397	0.000	0.286	14.409

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	64	64	235	748	0	0	0	0	0
N.S.	1	1.00	3.67	11.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.035	10.218	4.743	0.000	0.000	0.000	0.000	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	64	64	153	908	0	0	0	0	0
N.S.	1	1.00	2.39	14.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.030	10.109	4.557	0.000	0.000	0.000	0.000	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	59	59	232	753	0	0	0	0	0
N.S.	1	1.00	3.93	12.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.020	10.226	4.395	0.000	0.000	0.000	0.000	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	62	62	172	920	0	0	0	0	0
N.S.	1	1.00	2.77	14.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.045	10.151	6.387	0.000	0.000	0.000	0.000	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	64	64	338	766	0	0	0	0	0
N.S.	1	1.00	5.28	11.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.034	10.323	6.576	0.000	0.000	0.000	0.000	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	162	169	0	443	0	211	331
N.S.	1	1.00	0.86	0.89	0.00	2.34	0.00	1.12	1.75
time (sec)	N/A	0.175	0.562	4.651	0.000	0.384	0.000	0.287	12.037

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	125	133	0	314	0	173	229
N.S.	1	1.00	0.77	0.82	0.00	1.93	0.00	1.06	1.40
time (sec)	N/A	0.099	0.361	4.725	0.000	0.322	0.000	0.276	11.577

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	94	108	0	234	0	122	170
N.S.	1	1.00	1.00	1.15	0.00	2.49	0.00	1.30	1.81
time (sec)	N/A	0.056	0.311	4.259	0.000	0.315	0.000	0.274	11.373

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	122	140	0	686	0	155	214
N.S.	1	1.00	0.93	1.07	0.00	5.24	0.00	1.18	1.63
time (sec)	N/A	0.107	0.524	4.657	0.000	0.423	0.000	0.278	13.944

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	154	165	0	838	0	216	531
N.S.	1	1.00	0.91	0.97	0.00	4.93	0.00	1.27	3.12
time (sec)	N/A	0.207	0.929	4.725	0.000	0.351	0.000	0.286	15.795

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	65	65	338	808	0	0	0	0	0
N.S.	1	1.00	5.20	12.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.044	10.466	6.478	0.000	0.000	0.000	0.000	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	65	65	177	955	0	0	0	0	0
N.S.	1	1.00	2.72	14.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.041	10.199	4.767	0.000	0.000	0.000	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	60	60	339	801	0	0	0	0	0
N.S.	1	1.00	5.65	13.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.020	10.323	4.568	0.000	0.000	0.000	0.000	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	63	63	190	970	0	0	0	0	0
N.S.	1	1.00	3.02	15.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.051	10.253	6.316	0.000	0.000	0.000	0.000	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	65	65	370	815	0	0	0	0	0
N.S.	1	1.00	5.69	12.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.037	10.408	6.813	0.000	0.000	0.000	0.000	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	130	132	0	475	0	134	160
N.S.	1	1.00	1.06	1.07	0.00	3.86	0.00	1.09	1.30
time (sec)	N/A	0.112	0.616	4.600	0.000	0.288	0.000	0.287	11.608

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	100	83	0	348	0	116	111
N.S.	1	1.00	1.01	0.84	0.00	3.52	0.00	1.17	1.12
time (sec)	N/A	0.074	0.286	4.638	0.000	0.320	0.000	0.276	11.036

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	86	90	0	302	0	93	104
N.S.	1	1.00	0.99	1.03	0.00	3.47	0.00	1.07	1.20
time (sec)	N/A	0.049	0.260	4.440	0.000	0.269	0.000	0.279	10.663

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	124	146	0	862	0	139	162
N.S.	1	1.00	0.94	1.11	0.00	6.53	0.00	1.05	1.23
time (sec)	N/A	0.109	0.549	4.571	0.000	0.341	0.000	0.271	14.692

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	163	191	0	1236	0	257	355
N.S.	1	1.00	0.88	1.03	0.00	6.68	0.00	1.39	1.92
time (sec)	N/A	0.176	1.019	4.803	0.000	0.405	0.000	0.268	15.994

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	64	64	238	764	0	0	0	0	0
N.S.	1	1.00	3.72	11.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.044	10.225	4.511	0.000	0.000	0.000	0.000	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	64	64	172	923	0	0	0	0	0
N.S.	1	1.00	2.69	14.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.028	10.175	4.560	0.000	0.000	0.000	0.000	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	59	59	392	769	0	0	0	0	0
N.S.	1	1.00	6.64	13.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.028	10.277	4.666	0.000	0.000	0.000	0.000	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	62	62	226	963	0	0	0	0	0
N.S.	1	1.00	3.65	15.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.037	10.263	6.403	0.000	0.000	0.000	0.000	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	64	64	411	809	0	0	0	0	0
N.S.	1	1.00	6.42	12.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.039	10.634	6.312	0.000	0.000	0.000	0.000	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	149	133	146	0	746	0	195	367
N.S.	1	0.99	0.89	0.97	0.00	4.97	0.00	1.30	2.45
time (sec)	N/A	0.135	0.662	4.394	0.000	0.352	0.000	0.272	12.269

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	110	124	0	630	0	181	247
N.S.	1	1.00	0.82	0.93	0.00	4.70	0.00	1.35	1.84
time (sec)	N/A	0.096	0.473	4.446	0.000	0.376	0.000	0.278	11.733

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	101	86	0	450	0	153	199
N.S.	1	1.00	0.94	0.80	0.00	4.17	0.00	1.42	1.84
time (sec)	N/A	0.062	0.376	4.479	0.000	0.334	0.000	0.266	11.305

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	157	199	0	1819	0	226	288
N.S.	1	1.00	0.91	1.16	0.00	10.58	0.00	1.31	1.67
time (sec)	N/A	0.183	1.179	4.550	0.000	0.602	0.000	0.270	16.413

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	223	250	0	2384	0	367	18847
N.S.	1	1.00	0.93	1.04	0.00	9.89	0.00	1.52	78.20
time (sec)	N/A	0.289	1.464	4.892	0.000	0.722	0.000	0.288	23.902

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	67	67	381	787	0	0	0	0	0
N.S.	1	1.00	5.69	11.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.038	10.323	4.435	0.000	0.000	0.000	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	67	67	216	986	0	0	0	0	0
N.S.	1	1.00	3.22	14.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.029	10.283	4.383	0.000	0.000	0.000	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	62	62	381	830	0	0	0	0	0
N.S.	1	1.00	6.15	13.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.020	10.518	4.412	0.000	0.000	0.000	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	65	65	308	1019	0	0	0	0	0
N.S.	1	1.00	4.74	15.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.040	10.421	7.528	0.000	0.000	0.000	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	67	67	515	863	0	0	0	0	0
N.S.	1	1.00	7.69	12.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.037	10.917	7.337	0.000	0.000	0.000	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	134	113	0	0	0	379	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	2.83	0.00	0.00
time (sec)	N/A	0.065	0.956	0.000	0.000	0.000	18.960	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	132	111	0	0	0	246	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	1.86	0.00	0.00
time (sec)	N/A	0.062	0.340	0.000	0.000	0.000	6.887	0.000	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	110	0	0	0	121	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.92	0.00	0.00
time (sec)	N/A	0.050	0.228	0.000	0.000	0.000	2.224	0.000	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	110	0	0	0	117	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.89	0.00	0.00
time (sec)	N/A	0.067	0.263	0.000	0.000	0.000	1.946	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	133	133	113	0	0	0	117	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.88	0.00	0.00
time (sec)	N/A	0.053	0.496	0.000	0.000	0.000	29.539	0.000	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	133	133	113	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.073	0.998	0.000	0.000	0.000	0.000	0.000	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	88	0	0	256	0	104	283
N.S.	1	1.00	1.00	0.00	0.00	2.91	0.00	1.18	3.22
time (sec)	N/A	0.063	1.382	0.000	0.000	0.300	0.000	0.310	13.121

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	194	0	54	49
N.S.	1	1.00	1.00	0.00	0.00	4.04	0.00	1.12	1.02
time (sec)	N/A	0.046	0.833	0.000	0.000	0.280	0.000	0.287	8.873

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	204	0	89	136
N.S.	1	1.00	1.00	0.00	0.00	4.25	0.00	1.85	2.83
time (sec)	N/A	0.037	0.862	0.000	0.000	0.289	0.000	0.277	11.449

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	91	0	0	278	0	413	481
N.S.	1	1.00	1.00	0.00	0.00	3.05	0.00	4.54	5.29
time (sec)	N/A	0.053	1.594	0.000	0.000	0.317	0.000	0.288	15.120

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	90	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.067	2.485	0.000	0.000	0.000	0.000	0.000	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	90	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.073	2.142	0.000	0.000	0.000	0.000	0.000	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	90	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.047	2.162	0.000	0.000	0.000	0.000	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	170	0	0	0	0	0	0
N.S.	1	1.00	2.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.041	2.398	0.000	0.000	0.000	0.000	0.000	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	189	0	0	0	0	0	0
N.S.	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.082	2.496	0.000	0.000	0.000	0.000	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	365	0	0	0	0	0	0
N.S.	1	1.00	4.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.068	2.592	0.000	0.000	0.000	0.000	0.000	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	123	138	0	295	298	327	0
N.S.	1	1.00	0.76	0.86	0.00	1.83	1.85	2.03	0.00
time (sec)	N/A	0.082	1.077	5.648	0.000	0.568	18.801	0.434	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	324	112	777	0	0	97	0	0
N.S.	1	1.00	0.35	2.40	0.00	0.00	0.30	0.00	0.00
time (sec)	N/A	0.252	10.179	5.045	0.000	0.000	21.289	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	581	581	94	1140	0	0	97	0	0
N.S.	1	1.00	0.16	1.96	0.00	0.00	0.17	0.00	0.00
time (sec)	N/A	0.472	10.116	4.587	0.000	0.000	6.920	0.000	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	96	109	0	221	253	177	0
N.S.	1	1.00	0.79	0.90	0.00	1.83	2.09	1.46	0.00
time (sec)	N/A	0.066	0.408	4.769	0.000	0.578	1.552	0.324	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	93	744	0	0	97	0	0
N.S.	1	1.00	0.33	2.60	0.00	0.00	0.34	0.00	0.00
time (sec)	N/A	0.170	10.078	4.981	0.000	0.000	2.645	0.000	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	580	580	98	1123	0	0	100	0	0
N.S.	1	1.00	0.17	1.94	0.00	0.00	0.17	0.00	0.00
time (sec)	N/A	0.432	10.064	4.832	0.000	0.000	3.521	0.000	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	82	100	0	207	160	0	0
N.S.	1	1.00	0.69	0.85	0.00	1.75	1.36	0.00	0.00
time (sec)	N/A	0.066	0.387	4.599	0.000	0.567	6.392	0.000	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	97	743	0	0	100	0	0
N.S.	1	1.00	0.34	2.63	0.00	0.00	0.35	0.00	0.00
time (sec)	N/A	0.188	10.061	4.699	0.000	0.000	12.581	0.000	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	564	564	81	1127	0	0	97	0	0
N.S.	1	1.00	0.14	2.00	0.00	0.00	0.17	0.00	0.00
time (sec)	N/A	0.413	10.100	4.984	0.000	0.000	9.852	0.000	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	75	84	81	180	131	109	0
N.S.	1	1.00	0.95	1.06	1.03	2.28	1.66	1.38	0.00
time (sec)	N/A	0.033	0.449	4.460	0.299	0.362	26.869	0.298	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	80	745	0	76	97	0	0
N.S.	1	1.00	0.30	2.77	0.00	0.28	0.36	0.00	0.00
time (sec)	N/A	0.156	10.096	4.516	0.000	0.077	67.776	0.000	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	144	162	0	355	634	488	0
N.S.	1	1.00	0.72	0.81	0.00	1.77	3.15	2.43	0.00
time (sec)	N/A	0.106	0.588	4.620	0.000	0.570	31.069	0.507	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	364	116	801	0	0	199	0	0
N.S.	1	1.00	0.32	2.20	0.00	0.00	0.55	0.00	0.00
time (sec)	N/A	0.252	10.188	4.532	0.000	0.000	63.406	0.000	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	621	621	96	1164	0	0	199	0	0
N.S.	1	1.00	0.15	1.87	0.00	0.00	0.32	0.00	0.00
time (sec)	N/A	0.489	10.142	4.992	0.000	0.000	20.756	0.000	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	121	134	0	273	546	415	0
N.S.	1	1.00	0.75	0.83	0.00	1.70	3.39	2.58	0.00
time (sec)	N/A	0.082	0.526	4.634	0.000	0.686	3.441	0.447	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	324	96	768	0	0	199	0	0
N.S.	1	1.00	0.30	2.37	0.00	0.00	0.61	0.00	0.00
time (sec)	N/A	0.187	10.086	4.795	0.000	0.000	7.280	0.000	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	614	614	84	1140	0	0	202	0	0
N.S.	1	1.00	0.14	1.86	0.00	0.00	0.33	0.00	0.00
time (sec)	N/A	0.494	10.078	5.050	0.000	0.000	9.875	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	101	117	0	255	289	0	0
N.S.	1	1.00	0.66	0.77	0.00	1.68	1.90	0.00	0.00
time (sec)	N/A	0.077	0.563	4.563	0.000	0.559	17.816	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	85	759	0	0	202	0	0
N.S.	1	1.00	0.27	2.42	0.00	0.00	0.64	0.00	0.00
time (sec)	N/A	0.208	10.082	4.771	0.000	0.000	23.154	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	165	186	0	409	1028	692	0
N.S.	1	1.00	0.68	0.77	0.00	1.70	4.27	2.87	0.00
time (sec)	N/A	0.132	0.736	4.982	0.000	0.587	49.127	0.596	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	404	404	116	825	0	0	308	0	0
N.S.	1	1.00	0.29	2.04	0.00	0.00	0.76	0.00	0.00
time (sec)	N/A	0.268	10.232	4.886	0.000	0.000	150.121	0.000	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	661	661	99	1188	0	0	308	0	0
N.S.	1	1.00	0.15	1.80	0.00	0.00	0.47	0.00	0.00
time (sec)	N/A	0.572	10.159	4.815	0.000	0.000	52.726	0.000	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	141	158	0	323	896	590	0
N.S.	1	1.00	0.70	0.79	0.00	1.61	4.46	2.94	0.00
time (sec)	N/A	0.093	0.643	4.463	0.000	0.569	7.754	0.519	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	364	84	792	0	0	308	0	0
N.S.	1	1.00	0.23	2.18	0.00	0.00	0.85	0.00	0.00
time (sec)	N/A	0.232	10.090	4.844	0.000	0.000	17.853	0.000	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	650	650	87	1166	0	0	311	0	0
N.S.	1	1.00	0.13	1.79	0.00	0.00	0.48	0.00	0.00
time (sec)	N/A	0.553	10.060	4.939	0.000	0.000	21.625	0.000	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	127	143	0	309	403	0	0
N.S.	1	1.00	0.68	0.76	0.00	1.64	2.14	0.00	0.00
time (sec)	N/A	0.104	0.669	4.586	0.000	0.578	40.093	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	352	88	786	0	0	311	0	0
N.S.	1	1.00	0.25	2.23	0.00	0.00	0.88	0.00	0.00
time (sec)	N/A	0.223	10.055	4.862	0.000	0.000	44.170	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	100	114	0	245	194	146	0
N.S.	1	1.00	0.83	0.94	0.00	2.02	1.60	1.21	0.00
time (sec)	N/A	0.062	0.554	4.685	0.000	0.549	13.369	0.339	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	98	753	0	0	94	0	0
N.S.	1	1.00	0.34	2.63	0.00	0.00	0.33	0.00	0.00
time (sec)	N/A	0.175	10.130	4.900	0.000	0.000	15.700	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	543	543	80	1124	0	0	94	0	0
N.S.	1	1.00	0.15	2.07	0.00	0.00	0.17	0.00	0.00
time (sec)	N/A	0.375	10.101	4.809	0.000	0.000	5.781	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	79	94	0	184	151	94	0
N.S.	1	1.00	0.95	1.13	0.00	2.22	1.82	1.13	0.00
time (sec)	N/A	0.048	0.797	4.622	0.000	0.544	1.508	0.325	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	80	728	0	0	94	0	0
N.S.	1	1.00	0.32	2.92	0.00	0.00	0.38	0.00	0.00
time (sec)	N/A	0.171	10.069	4.704	0.000	0.000	2.035	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	542	542	83	1119	0	0	97	0	0
N.S.	1	1.00	0.15	2.06	0.00	0.00	0.18	0.00	0.00
time (sec)	N/A	0.374	10.049	4.960	0.000	0.000	2.407	0.000	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	66	87	0	183	60	106	0
N.S.	1	1.00	0.88	1.16	0.00	2.44	0.80	1.41	0.00
time (sec)	N/A	0.047	0.424	4.623	0.000	0.379	5.117	0.312	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	82	740	0	58	97	0	0
N.S.	1	1.00	0.33	3.01	0.00	0.24	0.39	0.00	0.00
time (sec)	N/A	0.162	10.053	4.939	0.000	0.080	15.173	0.000	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	93	129	0	307	0	132	0
N.S.	1	1.00	0.78	1.08	0.00	2.56	0.00	1.10	0.00
time (sec)	N/A	0.065	0.836	4.693	0.000	0.562	0.000	0.338	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	87	787	0	0	0	0	0
N.S.	1	1.00	0.30	2.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.172	10.131	5.912	0.000	0.000	0.000	0.000	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	553	553	77	1154	0	0	94	0	0
N.S.	1	1.00	0.14	2.09	0.00	0.00	0.17	0.00	0.00
time (sec)	N/A	0.384	10.134	4.591	0.000	0.000	77.731	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	81	92	0	234	95	93	0
N.S.	1	1.00	0.95	1.08	0.00	2.75	1.12	1.09	0.00
time (sec)	N/A	0.055	0.738	4.282	0.000	0.391	19.572	0.315	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	79	754	0	92	94	0	0
N.S.	1	1.00	0.31	2.92	0.00	0.36	0.36	0.00	0.00
time (sec)	N/A	0.141	10.070	4.788	0.000	0.089	22.605	0.000	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	585	585	77	1177	0	108	97	0	0
N.S.	1	1.00	0.13	2.01	0.00	0.18	0.17	0.00	0.00
time (sec)	N/A	0.427	10.061	7.205	0.000	0.107	41.997	0.000	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	44	39	0	57	90	98	70
N.S.	1	1.00	0.66	0.58	0.00	0.85	1.34	1.46	1.04
time (sec)	N/A	0.022	0.971	4.482	0.000	0.276	83.591	0.357	8.856

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	95	784	0	117	97	0	0
N.S.	1	1.00	0.34	2.77	0.00	0.41	0.34	0.00	0.00
time (sec)	N/A	0.163	10.069	6.015	0.000	0.086	152.274	0.000	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	100	157	0	345	0	126	0
N.S.	1	1.00	0.88	1.38	0.00	3.03	0.00	1.11	0.00
time (sec)	N/A	0.056	1.014	4.418	0.000	0.388	0.000	0.348	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	108	809	0	170	0	0	0
N.S.	1	1.00	0.36	2.71	0.00	0.57	0.00	0.00	0.00
time (sec)	N/A	0.174	10.164	4.603	0.000	0.088	0.000	0.000	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	596	596	86	1190	0	169	0	0	0
N.S.	1	1.00	0.14	2.00	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.475	10.131	4.627	0.000	0.086	0.000	0.000	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	44	39	0	59	0	72	73
N.S.	1	1.00	0.56	0.49	0.00	0.75	0.00	0.91	0.92
time (sec)	N/A	0.023	0.818	4.457	0.000	0.253	0.000	0.314	8.580

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	107	785	0	150	0	0	0
N.S.	1	1.00	0.36	2.64	0.00	0.51	0.00	0.00	0.00
time (sec)	N/A	0.174	10.117	4.543	0.000	0.091	0.000	0.000	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	624	624	85	1225	0	167	0	0	0
N.S.	1	1.00	0.14	1.96	0.00	0.27	0.00	0.00	0.00
time (sec)	N/A	0.508	10.066	8.026	0.000	0.093	0.000	0.000	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	67	62	0	93	0	0	115
N.S.	1	1.00	0.64	0.60	0.00	0.89	0.00	0.00	1.11
time (sec)	N/A	0.038	0.755	3.103	0.000	0.274	0.000	0.000	8.758

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	121	829	0	178	0	0	0
N.S.	1	1.00	0.38	2.59	0.00	0.56	0.00	0.00	0.00
time (sec)	N/A	0.211	10.096	7.982	0.000	0.088	0.000	0.000	0.000

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	202	147	183	186	0	215	240
N.S.	1	1.00	0.92	0.67	0.83	0.85	0.00	0.98	1.09
time (sec)	N/A	0.172	0.405	6.640	0.287	0.270	0.000	0.762	9.041

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	210	136	155	174	0	176	219
N.S.	1	1.00	1.21	0.78	0.89	1.00	0.00	1.01	1.26
time (sec)	N/A	0.137	0.277	5.047	0.287	0.256	0.000	0.738	8.983

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	188	125	153	157	0	174	200
N.S.	1	1.00	1.09	0.73	0.89	0.91	0.00	1.01	1.16
time (sec)	N/A	0.107	0.260	4.596	0.282	0.265	0.000	0.747	8.902

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	169	126	139	144	0	148	194
N.S.	1	1.00	1.13	0.84	0.93	0.96	0.00	0.99	1.29
time (sec)	N/A	0.084	0.228	4.734	0.283	0.254	0.000	0.738	8.662

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	235	182	0	634	0	217	345
N.S.	1	1.00	1.10	0.85	0.00	2.96	0.00	1.01	1.61
time (sec)	N/A	0.123	0.430	4.943	0.000	0.264	0.000	0.995	8.984

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	282	222	0	321	0	243	455
N.S.	1	1.00	1.05	0.83	0.00	1.20	0.00	0.91	1.70
time (sec)	N/A	0.208	0.622	4.858	0.000	0.281	0.000	0.984	9.291

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	316	254	0	345	0	279	490
N.S.	1	1.00	1.12	0.90	0.00	1.22	0.00	0.99	1.73
time (sec)	N/A	0.192	0.687	4.703	0.000	0.277	0.000	0.972	9.452

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	327	273	0	362	0	0	0
N.S.	1	1.00	1.22	1.02	0.00	1.35	0.00	0.00	0.00
time (sec)	N/A	0.193	1.469	7.070	0.000	0.269	0.000	0.000	0.000

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	293	243	0	338	0	0	0
N.S.	1	1.00	1.26	1.04	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.125	1.041	4.746	0.000	0.267	0.000	0.000	0.000

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	265	219	0	313	0	0	0
N.S.	1	1.00	1.32	1.09	0.00	1.56	0.00	0.00	0.00
time (sec)	N/A	0.054	0.703	4.680	0.000	0.265	0.000	0.000	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	190	150	0	395	0	0	0
N.S.	1	1.00	1.22	0.96	0.00	2.53	0.00	0.00	0.00
time (sec)	N/A	0.058	0.523	4.797	0.000	84.462	0.000	0.000	0.000

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	215	149	0	0	0	0	0
N.S.	1	1.00	1.17	0.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.111	0.564	4.689	0.000	0.000	0.000	0.000	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	206	160	0	0	0	0	0
N.S.	1	1.00	0.98	0.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.182	0.671	4.877	0.000	0.000	0.000	0.000	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	219	171	0	0	0	0	0
N.S.	1	1.00	0.92	0.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.245	0.761	4.932	0.000	0.000	0.000	0.000	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	521	521	234	0	0	0	0	0	0
N.S.	1	1.00	0.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.459	7.814	0.000	0.000	0.000	0.000	0.000	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	494	494	225	0	0	0	0	0	0
N.S.	1	1.00	0.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.315	7.192	0.000	0.000	0.000	0.000	0.000	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	416	416	431	0	0	0	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.182	3.327	0.000	0.000	0.000	0.000	0.000	0.000

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	496	496	231	0	0	0	0	0	0
N.S.	1	1.00	0.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.312	11.163	0.000	0.000	0.000	0.000	0.000	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	523	523	243	0	0	0	0	0	0
N.S.	1	1.00	0.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.397	11.190	0.000	0.000	0.000	0.000	0.000	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	202	148	183	209	0	215	261
N.S.	1	1.00	0.91	0.66	0.82	0.94	0.00	0.96	1.17
time (sec)	N/A	0.176	0.464	4.637	0.292	0.390	0.000	0.755	8.721

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	210	136	155	197	0	176	206
N.S.	1	1.00	1.19	0.77	0.88	1.11	0.00	0.99	1.16
time (sec)	N/A	0.130	0.332	4.791	0.299	0.331	0.000	0.750	8.763

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	188	128	155	181	0	175	221
N.S.	1	1.00	1.07	0.73	0.89	1.03	0.00	1.00	1.26
time (sec)	N/A	0.112	0.306	5.127	0.299	0.344	0.000	0.734	8.661

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	170	126	140	167	0	148	186
N.S.	1	1.00	1.11	0.82	0.92	1.09	0.00	0.97	1.22
time (sec)	N/A	0.092	0.283	4.594	0.286	0.355	0.000	0.757	8.661

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	236	182	0	530	0	217	369
N.S.	1	1.00	1.10	0.85	0.00	2.48	0.00	1.01	1.72
time (sec)	N/A	0.125	0.431	4.674	0.000	0.365	0.000	0.968	8.975

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	282	221	0	612	0	243	490
N.S.	1	1.00	1.05	0.82	0.00	2.28	0.00	0.90	1.82
time (sec)	N/A	0.187	0.659	4.694	0.000	0.353	0.000	0.977	9.488

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	316	254	0	660	0	279	513
N.S.	1	1.00	1.11	0.89	0.00	2.32	0.00	0.98	1.81
time (sec)	N/A	0.193	0.734	4.737	0.000	0.368	0.000	0.993	9.401

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	325	271	0	701	0	0	0
N.S.	1	1.00	1.23	1.03	0.00	2.66	0.00	0.00	0.00
time (sec)	N/A	0.158	1.381	4.960	0.000	0.342	0.000	0.000	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	291	241	0	653	0	0	0
N.S.	1	1.00	1.27	1.05	0.00	2.85	0.00	0.00	0.00
time (sec)	N/A	0.089	1.012	4.885	0.000	0.303	0.000	0.000	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	264	219	0	611	0	0	0
N.S.	1	1.00	1.32	1.10	0.00	3.06	0.00	0.00	0.00
time (sec)	N/A	0.045	0.720	4.534	0.000	0.318	0.000	0.000	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	195	156	0	434	0	0	0
N.S.	1	1.00	1.24	0.99	0.00	2.76	0.00	0.00	0.00
time (sec)	N/A	0.073	0.523	4.750	0.000	81.744	0.000	0.000	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	216	150	0	0	0	0	0
N.S.	1	1.00	1.19	0.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.131	0.568	4.876	0.000	0.000	0.000	0.000	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	206	159	0	0	0	0	0
N.S.	1	1.00	0.99	0.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.163	0.667	4.828	0.000	0.000	0.000	0.000	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	219	174	0	0	0	0	0
N.S.	1	1.00	0.93	0.74	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.223	0.738	5.066	0.000	0.000	0.000	0.000	0.000

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	512	512	147	0	0	0	0	0	0
N.S.	1	1.00	0.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.392	8.541	0.000	0.000	0.000	0.000	0.000	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	485	485	127	0	0	0	0	0	0
N.S.	1	1.00	0.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.300	8.267	0.000	0.000	0.000	0.000	0.000	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	457	457	63	0	0	0	0	0	0
N.S.	1	1.00	0.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.216	10.049	0.000	0.000	0.000	0.000	0.000	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	483	483	136	0	0	0	0	0	0
N.S.	1	1.00	0.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.325	11.108	0.000	0.000	0.000	0.000	0.000	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	512	512	148	0	0	0	0	0	0
N.S.	1	1.00	0.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.372	11.093	0.000	0.000	0.000	0.000	0.000	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	145	109	119	118	0	134	133
N.S.	1	1.00	1.14	0.86	0.94	0.93	0.00	1.06	1.05
time (sec)	N/A	0.065	0.233	9.742	0.278	0.318	0.000	0.307	8.427

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	139	102	119	137	0	127	133
N.S.	1	1.00	1.09	0.80	0.93	1.07	0.00	0.99	1.04
time (sec)	N/A	0.063	0.205	10.106	0.446	0.308	0.000	0.294	8.362

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	127	109	97	106	0	98	111
N.S.	1	1.00	1.31	1.12	1.00	1.09	0.00	1.01	1.14
time (sec)	N/A	0.056	0.161	9.163	0.369	0.324	0.000	0.293	8.330

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	126	95	97	125	0	98	111
N.S.	1	1.00	1.29	0.97	0.99	1.28	0.00	1.00	1.13
time (sec)	N/A	0.045	0.157	9.077	0.285	0.334	0.000	0.304	8.355

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	104	80	86	90	0	87	100
N.S.	1	1.00	1.27	0.98	1.05	1.10	0.00	1.06	1.22
time (sec)	N/A	0.039	0.012	4.968	0.279	0.277	0.000	0.288	8.517

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	185	145	0	410	0	149	256
N.S.	1	1.00	1.35	1.06	0.00	2.99	0.00	1.09	1.87
time (sec)	N/A	0.073	0.272	4.933	0.000	1.002	0.000	0.314	8.408

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	202	211	0	187	0	163	382
N.S.	1	1.00	1.29	1.34	0.00	1.19	0.00	1.04	2.43
time (sec)	N/A	0.075	0.385	6.521	0.000	0.289	0.000	0.302	8.478

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	220	233	0	201	0	0	0
N.S.	1	1.00	1.43	1.51	0.00	1.31	0.00	0.00	0.00
time (sec)	N/A	0.049	0.682	6.395	0.000	0.259	0.000	0.000	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	205	179	0	452	0	0	0
N.S.	1	1.00	1.52	1.33	0.00	3.35	0.00	0.00	0.00
time (sec)	N/A	0.021	0.491	4.671	0.000	1.045	0.000	0.000	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	114	95	0	253	0	0	0
N.S.	1	1.00	1.30	1.08	0.00	2.88	0.00	0.00	0.00
time (sec)	N/A	0.009	0.019	4.628	0.000	1.711	0.000	0.000	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	141	126	0	307	0	0	0
N.S.	1	1.00	1.34	1.20	0.00	2.92	0.00	0.00	0.00
time (sec)	N/A	0.029	0.407	21.995	0.000	1.687	0.000	0.000	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	142	129	0	283	0	0	0
N.S.	1	1.00	1.15	1.04	0.00	2.28	0.00	0.00	0.00
time (sec)	N/A	0.053	0.438	22.078	0.000	1.758	0.000	0.000	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	154	135	0	320	0	0	0
N.S.	1	1.00	1.09	0.96	0.00	2.27	0.00	0.00	0.00
time (sec)	N/A	0.083	0.458	22.177	0.000	1.696	0.000	0.000	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	271	271	40	0	0	0	0	0	0
N.S.	1	1.00	0.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.155	10.033	0.000	0.000	0.000	0.000	0.000	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	254	254	26	0	0	0	0	0	0
N.S.	1	1.00	0.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.093	10.025	0.000	0.000	0.000	0.000	0.000	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	233	233	283	0	0	373	0	0	0
N.S.	1	1.00	1.21	0.00	0.00	1.60	0.00	0.00	0.00
time (sec)	N/A	0.078	0.124	0.000	0.000	1.702	0.000	0.000	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	270	270	67	0	0	0	0	0	0
N.S.	1	1.00	0.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.117	11.054	0.000	0.000	0.000	0.000	0.000	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	289	289	76	0	0	0	0	0	0
N.S.	1	1.00	0.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.175	11.056	0.000	0.000	0.000	0.000	0.000	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	137	100	119	142	0	127	135
N.S.	1	1.00	1.10	0.80	0.95	1.14	0.00	1.02	1.08
time (sec)	N/A	0.061	0.215	9.498	0.309	0.272	0.000	0.378	8.404

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	127	109	97	114	0	98	113
N.S.	1	1.00	1.30	1.11	0.99	1.16	0.00	1.00	1.15
time (sec)	N/A	0.051	0.178	9.509	0.279	0.270	0.000	0.357	8.435

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	126	95	97	130	0	98	113
N.S.	1	1.00	1.33	1.00	1.02	1.37	0.00	1.03	1.19
time (sec)	N/A	0.064	0.137	8.606	0.320	0.258	0.000	0.355	8.416

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	102	78	86	98	0	87	102
N.S.	1	1.00	1.23	0.94	1.04	1.18	0.00	1.05	1.23
time (sec)	N/A	0.042	0.122	4.615	0.303	0.293	0.000	0.328	8.558

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	185	145	0	182	0	149	344
N.S.	1	1.00	1.35	1.06	0.00	1.33	0.00	1.09	2.51
time (sec)	N/A	0.068	0.252	4.514	0.000	0.260	0.000	0.363	8.488

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	202	211	0	195	0	163	368
N.S.	1	1.00	1.28	1.34	0.00	1.23	0.00	1.03	2.33
time (sec)	N/A	0.072	0.330	7.280	0.000	0.278	0.000	0.365	8.557

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	222	234	0	232	0	0	0
N.S.	1	1.00	1.39	1.46	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.062	0.734	6.537	0.000	0.259	0.000	0.000	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	206	179	0	197	0	0	0
N.S.	1	1.00	1.48	1.29	0.00	1.42	0.00	0.00	0.00
time (sec)	N/A	0.024	0.487	4.668	0.000	0.267	0.000	0.000	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	114	96	0	283	0	0	0
N.S.	1	1.00	1.30	1.09	0.00	3.22	0.00	0.00	0.00
time (sec)	N/A	0.011	0.367	4.891	0.000	1.467	0.000	0.000	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	143	121	0	272	0	0	0
N.S.	1	1.00	1.39	1.17	0.00	2.64	0.00	0.00	0.00
time (sec)	N/A	0.020	0.397	22.118	0.000	1.425	0.000	0.000	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	141	127	0	312	0	0	0
N.S.	1	1.00	1.14	1.02	0.00	2.52	0.00	0.00	0.00
time (sec)	N/A	0.051	0.455	22.479	0.000	1.410	0.000	0.000	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	291	291	115	694	0	356	0	0	0
N.S.	1	1.00	0.40	2.38	0.00	1.22	0.00	0.00	0.00
time (sec)	N/A	0.132	10.129	18.211	0.000	1.723	0.000	0.000	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	294	294	26	0	0	0	0	0	0
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.141	10.063	0.000	0.000	0.000	0.000	0.000	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	293	293	111	0	0	0	0	0	0
N.S.	1	1.00	0.38	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.130	10.131	0.000	0.000	0.000	0.000	0.000	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	294	294	120	695	0	396	0	0	0
N.S.	1	1.00	0.41	2.36	0.00	1.35	0.00	0.00	0.00
time (sec)	N/A	0.124	11.134	47.562	0.000	1.729	0.000	0.000	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	142	138	128	140	0	136	148
N.S.	1	1.00	1.01	0.98	0.91	0.99	0.00	0.96	1.05
time (sec)	N/A	0.077	0.346	8.934	0.286	0.299	0.000	0.302	8.538

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	137	133	119	159	0	120	139
N.S.	1	1.00	1.05	1.02	0.92	1.22	0.00	0.92	1.07
time (sec)	N/A	0.074	0.277	8.617	0.290	0.301	0.000	0.296	8.487

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	132	127	108	130	0	109	128
N.S.	1	1.00	1.15	1.10	0.94	1.13	0.00	0.95	1.11
time (sec)	N/A	0.069	0.261	8.368	0.272	0.329	0.000	0.299	8.475

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	126	122	97	148	0	98	117
N.S.	1	1.00	1.26	1.22	0.97	1.48	0.00	0.98	1.17
time (sec)	N/A	0.061	0.240	8.470	0.291	0.335	0.000	0.294	8.434

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	127	122	97	125	0	98	117
N.S.	1	1.00	1.27	1.22	0.97	1.25	0.00	0.98	1.17
time (sec)	N/A	0.047	0.208	8.416	0.278	0.298	0.000	0.310	8.445

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	198	210	0	226	0	160	253
N.S.	1	1.00	1.29	1.36	0.00	1.47	0.00	1.04	1.64
time (sec)	N/A	0.081	0.412	6.857	0.000	0.329	0.000	0.309	8.559

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	209	267	0	238	0	181	399
N.S.	1	1.00	1.19	1.53	0.00	1.36	0.00	1.03	2.28
time (sec)	N/A	0.077	0.515	6.549	0.000	0.347	0.000	0.288	8.603

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	227	292	0	271	0	0	0
N.S.	1	1.00	1.30	1.68	0.00	1.56	0.00	0.00	0.00
time (sec)	N/A	0.069	0.911	6.647	0.000	0.325	0.000	0.000	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	220	219	0	239	0	0	0
N.S.	1	1.00	1.44	1.43	0.00	1.56	0.00	0.00	0.00
time (sec)	N/A	0.036	0.668	5.545	0.000	0.334	0.000	0.000	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	139	141	0	318	0	0	0
N.S.	1	1.00	1.31	1.33	0.00	3.00	0.00	0.00	0.00
time (sec)	N/A	0.022	0.498	7.633	0.000	1.906	0.000	0.000	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	140	141	0	288	0	0	0
N.S.	1	1.00	1.32	1.33	0.00	2.72	0.00	0.00	0.00
time (sec)	N/A	0.016	0.481	6.833	0.000	1.775	0.000	0.000	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	148	156	0	340	0	0	0
N.S.	1	1.00	1.19	1.26	0.00	2.74	0.00	0.00	0.00
time (sec)	N/A	0.055	0.561	21.601	0.000	1.678	0.000	0.000	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	152	161	0	316	0	0	0
N.S.	1	1.00	1.06	1.12	0.00	2.19	0.00	0.00	0.00
time (sec)	N/A	0.098	0.582	22.168	0.000	1.694	0.000	0.000	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	157	168	0	351	0	0	0
N.S.	1	1.00	0.97	1.04	0.00	2.17	0.00	0.00	0.00
time (sec)	N/A	0.109	0.589	21.044	0.000	1.706	0.000	0.000	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	292	292	71	0	0	0	0	0	0
N.S.	1	1.00	0.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.172	10.084	0.000	0.000	0.000	0.000	0.000	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	274	274	66	0	0	0	0	0	0
N.S.	1	1.00	0.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.129	10.072	0.000	0.000	0.000	0.000	0.000	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	274	274	66	0	0	0	0	0	0
N.S.	1	1.00	0.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.145	10.071	0.000	0.000	0.000	0.000	0.000	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	274	274	45	0	0	0	0	0	0
N.S.	1	1.00	0.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.132	10.044	0.000	0.000	0.000	0.000	0.000	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	292	292	76	0	0	0	0	0	0
N.S.	1	1.00	0.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.144	11.080	0.000	0.000	0.000	0.000	0.000	0.000

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	308	308	79	0	0	0	0	0	0
N.S.	1	1.00	0.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.205	11.103	0.000	0.000	0.000	0.000	0.000	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	308	283	0	325	0	379	442
N.S.	1	1.00	1.17	1.07	0.00	1.23	0.00	1.44	1.67
time (sec)	N/A	0.261	0.870	6.038	0.000	0.333	0.000	0.334	9.202

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	265	235	0	282	0	320	336
N.S.	1	1.00	1.20	1.07	0.00	1.28	0.00	1.45	1.53
time (sec)	N/A	0.176	0.626	4.731	0.000	0.395	0.000	0.299	8.894

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	226	205	0	222	0	276	298
N.S.	1	1.00	1.22	1.10	0.00	1.19	0.00	1.48	1.60
time (sec)	N/A	0.138	0.381	4.819	0.000	0.278	0.000	0.301	8.543

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	205	221	0	206	0	223	249
N.S.	1	1.00	1.29	1.39	0.00	1.30	0.00	1.40	1.57
time (sec)	N/A	0.114	0.284	4.711	0.000	0.277	0.000	0.304	8.457

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	312	258	0	276	0	311	1607
N.S.	1	1.00	1.27	1.05	0.00	1.12	0.00	1.26	6.53
time (sec)	N/A	0.146	0.603	4.830	0.000	0.276	0.000	0.553	8.824

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	351	357	0	429	0	351	1917
N.S.	1	1.00	1.03	1.05	0.00	1.26	0.00	1.03	5.64
time (sec)	N/A	0.260	0.971	5.234	0.000	0.344	0.000	0.510	14.134

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	413	428	0	472	0	455	2767
N.S.	1	1.00	1.12	1.16	0.00	1.28	0.00	1.23	7.48
time (sec)	N/A	0.331	1.275	5.110	0.000	1.120	0.000	0.564	16.499

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	527	445	0	494	0	0	0
N.S.	1	1.00	1.57	1.32	0.00	1.47	0.00	0.00	0.00
time (sec)	N/A	0.270	6.446	6.375	0.000	1.147	0.000	0.000	0.000

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	467	393	0	452	0	0	0
N.S.	1	1.00	1.69	1.42	0.00	1.64	0.00	0.00	0.00
time (sec)	N/A	0.160	4.475	4.861	0.000	0.335	0.000	0.000	0.000

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	423	338	0	330	0	0	0
N.S.	1	1.00	1.81	1.44	0.00	1.41	0.00	0.00	0.00
time (sec)	N/A	0.055	3.791	4.628	0.000	0.276	0.000	0.000	0.000

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	309	219	0	0	0	0	0
N.S.	1	1.00	1.84	1.30	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.057	2.405	4.765	0.000	0.000	0.000	0.000	0.000

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	333	245	0	0	0	0	0
N.S.	1	1.00	1.63	1.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.118	2.615	4.811	0.000	0.000	0.000	0.000	0.000

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	373	266	0	0	0	0	0
N.S.	1	1.00	1.45	1.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.210	3.043	4.839	0.000	0.000	0.000	0.000	0.000

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	419	306	0	0	0	0	0
N.S.	1	1.00	1.32	0.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.299	3.441	4.915	0.000	0.000	0.000	0.000	0.000

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	281	0	0	0	0	0	0
N.S.	1	1.00	4.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.035	7.703	0.000	0.000	0.000	0.000	0.000	0.000

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	240	0	0	0	0	0	0
N.S.	1	1.00	3.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.036	7.109	0.000	0.000	0.000	0.000	0.000	0.000

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	160	0	0	0	0	0	0
N.S.	1	1.00	2.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.019	0.040	0.000	0.000	0.000	0.000	0.000	0.000

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	327	0	0	0	0	0	0
N.S.	1	1.00	5.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.038	10.269	0.000	0.000	0.000	0.000	0.000	0.000

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	289	0	0	0	0	0	0
N.S.	1	1.00	4.52	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.039	10.383	0.000	0.000	0.000	0.000	0.000	0.000

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	310	272	0	455	0	409	490
N.S.	1	1.00	1.17	1.02	0.00	1.71	0.00	1.54	1.84
time (sec)	N/A	0.200	1.038	4.729	0.000	0.521	0.000	0.318	9.470

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	266	235	0	398	0	350	385
N.S.	1	1.00	1.19	1.05	0.00	1.78	0.00	1.57	1.73
time (sec)	N/A	0.175	0.623	4.756	0.000	0.486	0.000	0.335	9.303

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	228	206	0	353	0	306	302
N.S.	1	1.00	1.21	1.10	0.00	1.88	0.00	1.63	1.61
time (sec)	N/A	0.127	0.491	4.722	0.000	0.453	0.000	0.301	9.492

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	206	222	0	323	0	259	238
N.S.	1	1.00	1.27	1.37	0.00	1.99	0.00	1.60	1.47
time (sec)	N/A	0.105	0.287	4.721	0.000	0.519	0.000	0.307	9.317

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	310	258	0	425	0	341	1963
N.S.	1	1.00	1.27	1.05	0.00	1.73	0.00	1.39	8.01
time (sec)	N/A	0.153	0.663	4.627	0.000	0.538	0.000	0.627	9.273

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	347	352	292	0	1030	0	395	1908
N.S.	1	1.00	1.01	0.84	0.00	2.97	0.00	1.14	5.50
time (sec)	N/A	0.261	0.964	4.904	0.000	0.626	0.000	0.563	14.445

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	413	333	0	1151	0	488	2788
N.S.	1	1.00	1.12	0.90	0.00	3.11	0.00	1.32	7.54
time (sec)	N/A	0.336	1.326	5.029	0.000	1.356	0.000	0.590	19.141

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	527	356	0	1164	0	0	0
N.S.	1	1.00	1.58	1.07	0.00	3.49	0.00	0.00	0.00
time (sec)	N/A	0.263	7.293	5.320	0.000	1.589	0.000	0.000	0.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	466	391	0	1091	0	0	0
N.S.	1	1.00	1.71	1.44	0.00	4.01	0.00	0.00	0.00
time (sec)	N/A	0.128	5.055	4.818	0.000	0.600	0.000	0.000	0.000

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	423	339	0	469	0	0	0
N.S.	1	1.00	1.82	1.45	0.00	2.01	0.00	0.00	0.00
time (sec)	N/A	0.048	0.371	4.775	0.000	0.500	0.000	0.000	0.000

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	309	222	0	0	0	0	0
N.S.	1	1.00	1.83	1.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.063	2.591	5.075	0.000	0.000	0.000	0.000	0.000

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	334	245	0	0	0	0	0
N.S.	1	1.00	1.62	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.117	2.861	4.846	0.000	0.000	0.000	0.000	0.000

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	374	275	0	0	0	0	0
N.S.	1	1.00	1.46	1.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.214	3.252	4.855	0.000	0.000	0.000	0.000	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	422	307	0	0	0	0	0
N.S.	1	1.00	1.32	0.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.305	3.916	4.917	0.000	0.000	0.000	0.000	0.000

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	181	0	0	0	0	0	0
N.S.	1	1.00	2.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.039	8.420	0.000	0.000	0.000	0.000	0.000	0.000

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	141	0	0	0	0	0	0
N.S.	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.040	8.028	0.000	0.000	0.000	0.000	0.000	0.000

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	10.036	0.000	0.000	0.000	0.000	0.000	0.000

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	138	0	0	0	0	0	0
N.S.	1	1.00	2.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.039	10.107	0.000	0.000	0.000	0.000	0.000	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	181	0	0	0	0	0	0
N.S.	1	1.00	2.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.039	10.161	0.000	0.000	0.000	0.000	0.000	0.000

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	308	260	0	369	0	394	477
N.S.	1	1.00	1.23	1.04	0.00	1.47	0.00	1.57	1.90
time (sec)	N/A	0.248	0.878	4.888	0.000	0.289	0.000	0.317	8.912

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	260	228	0	298	0	348	348
N.S.	1	1.00	1.23	1.08	0.00	1.41	0.00	1.65	1.65
time (sec)	N/A	0.162	0.545	4.895	0.000	0.289	0.000	0.300	8.912

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	221	203	0	246	0	297	304
N.S.	1	1.00	1.18	1.09	0.00	1.32	0.00	1.59	1.63
time (sec)	N/A	0.153	0.411	5.010	0.000	0.288	0.000	0.315	8.555

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	333	291	0	320	0	357	796
N.S.	1	1.00	1.28	1.11	0.00	1.23	0.00	1.37	3.05
time (sec)	N/A	0.199	0.905	4.773	0.000	0.288	0.000	0.551	9.915

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	399	353	364	0	383	0	394	2047
N.S.	1	1.00	0.88	0.91	0.00	0.96	0.00	0.99	5.13
time (sec)	N/A	0.326	1.175	5.042	0.000	0.428	0.000	0.576	14.720

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	440	440	413	454	0	503	0	481	2841
N.S.	1	1.00	0.94	1.03	0.00	1.14	0.00	1.09	6.46
time (sec)	N/A	0.451	1.573	5.024	0.000	1.166	0.000	0.561	16.695

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	526	472	0	550	0	0	0
N.S.	1	1.00	1.57	1.41	0.00	1.65	0.00	0.00	0.00
time (sec)	N/A	0.315	9.383	5.109	0.000	1.183	0.000	0.000	0.000

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	469	385	0	396	0	0	0
N.S.	1	1.00	1.69	1.39	0.00	1.43	0.00	0.00	0.00
time (sec)	N/A	0.167	6.538	5.190	0.000	0.406	0.000	0.000	0.000

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	457	334	0	0	0	0	0
N.S.	1	1.00	1.80	1.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.166	5.247	4.946	0.000	0.000	0.000	0.000	0.000

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	328	243	0	0	0	0	0
N.S.	1	1.00	1.63	1.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.131	3.056	5.025	0.000	0.000	0.000	0.000	0.000

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	369	259	0	0	0	0	0
N.S.	1	1.00	1.48	1.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.214	3.407	4.845	0.000	0.000	0.000	0.000	0.000

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	419	294	0	0	0	0	0
N.S.	1	1.00	1.32	0.92	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.333	3.731	5.048	0.000	0.000	0.000	0.000	0.000

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	392	392	478	336	0	0	0	0	0
N.S.	1	1.00	1.22	0.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.481	4.753	5.061	0.000	0.000	0.000	0.000	0.000

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	343	0	0	0	0	0	0
N.S.	1	1.00	5.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.044	9.839	0.000	0.000	0.000	0.000	0.000	0.000

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	280	0	0	0	0	0	0
N.S.	1	1.00	4.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.041	9.288	0.000	0.000	0.000	0.000	0.000	0.000

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	346	0	0	0	0	0	0
N.S.	1	1.00	5.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.021	0.299	0.000	0.000	0.000	0.000	0.000	0.000

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	341	0	0	0	0	0	0
N.S.	1	1.00	5.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.040	10.359	0.000	0.000	0.000	0.000	0.000	0.000

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	286	0	0	0	0	0	0
N.S.	1	1.00	4.40	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.042	10.384	0.000	0.000	0.000	0.000	0.000	0.000

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	307	275	0	1004	0	454	438
N.S.	1	1.00	1.06	0.95	0.00	3.46	0.00	1.57	1.51
time (sec)	N/A	0.202	1.008	4.803	0.000	0.298	0.000	0.307	8.955

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	263	234	0	873	0	371	339
N.S.	1	1.00	1.08	0.96	0.00	3.58	0.00	1.52	1.39
time (sec)	N/A	0.192	0.657	4.731	0.000	0.314	0.000	0.299	8.986

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	231	204	0	768	0	313	267
N.S.	1	1.00	1.14	1.00	0.00	3.78	0.00	1.54	1.32
time (sec)	N/A	0.156	0.515	5.059	0.000	0.320	0.000	0.299	9.034

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	203	186	0	667	0	257	219
N.S.	1	1.00	1.21	1.11	0.00	3.97	0.00	1.53	1.30
time (sec)	N/A	0.108	0.285	4.852	0.000	0.272	0.000	0.307	8.935

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	162	152	0	592	0	226	208
N.S.	1	1.00	1.12	1.05	0.00	4.08	0.00	1.56	1.43
time (sec)	N/A	0.092	0.172	4.500	0.000	0.265	0.000	0.286	8.849

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	309	248	0	628	0	326	702
N.S.	1	1.00	1.27	1.02	0.00	2.57	0.00	1.34	2.88
time (sec)	N/A	0.138	0.678	4.619	0.000	0.280	0.000	0.523	10.226

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	353	284	0	837	0	378	1929
N.S.	1	1.00	1.19	0.96	0.00	2.83	0.00	1.28	6.52
time (sec)	N/A	0.213	1.221	4.839	0.000	0.335	0.000	0.530	15.388

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	466	311	0	826	0	0	0
N.S.	1	1.00	1.71	1.14	0.00	3.03	0.00	0.00	0.00
time (sec)	N/A	0.123	4.821	4.969	0.000	0.342	0.000	0.000	0.000

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	423	286	0	761	0	0	0
N.S.	1	1.00	1.82	1.23	0.00	3.27	0.00	0.00	0.00
time (sec)	N/A	0.055	3.104	4.639	0.000	0.281	0.000	0.000	0.000

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	255	171	0	0	0	0	0
N.S.	1	1.00	1.72	1.16	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.016	0.074	4.543	0.000	0.000	0.000	0.000	0.000

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	314	216	0	0	0	0	0
N.S.	1	1.00	1.78	1.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.058	2.136	4.859	0.000	0.000	0.000	0.000	0.000

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	340	231	0	0	0	0	0
N.S.	1	1.00	1.59	1.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.158	2.622	4.986	0.000	0.000	0.000	0.000	0.000

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	374	270	0	0	0	0	0
N.S.	1	1.00	1.43	1.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.205	3.586	4.983	0.000	0.000	0.000	0.000	0.000

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	144	0	0	0	0	0	0
N.S.	1	1.00	2.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.036	8.461	0.000	0.000	0.000	0.000	0.000	0.000

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.042	7.949	0.000	0.000	0.000	0.000	0.000	0.000

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.027	10.033	0.000	0.000	0.000	0.000	0.000	0.000

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	141	0	0	0	0	0	0
N.S.	1	1.00	2.27	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.043	10.107	0.000	0.000	0.000	0.000	0.000	0.000

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	183	0	0	0	0	0	0
N.S.	1	1.00	2.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.047	10.176	0.000	0.000	0.000	0.000	0.000	0.000

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	264	234	0	1322	0	372	331
N.S.	1	1.00	1.10	0.97	0.00	5.49	0.00	1.54	1.37
time (sec)	N/A	0.178	0.777	4.773	0.000	0.346	0.000	0.297	8.968

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	231	202	0	1156	0	312	292
N.S.	1	1.00	1.15	1.00	0.00	5.75	0.00	1.55	1.45
time (sec)	N/A	0.147	0.558	4.935	0.000	0.318	0.000	0.322	8.671

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	202	186	0	1060	0	253	232
N.S.	1	1.00	1.22	1.13	0.00	6.42	0.00	1.53	1.41
time (sec)	N/A	0.129	0.277	5.042	0.000	0.318	0.000	0.294	8.796

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	162	152	0	927	0	221	213
N.S.	1	1.00	1.12	1.05	0.00	6.39	0.00	1.52	1.47
time (sec)	N/A	0.090	0.183	4.503	0.000	0.330	0.000	0.296	9.165

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	308	249	0	472	0	321	1413
N.S.	1	1.00	1.26	1.02	0.00	1.93	0.00	1.31	5.77
time (sec)	N/A	0.142	0.754	4.650	0.000	0.320	0.000	0.509	9.151

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	355	297	0	562	0	377	1959
N.S.	1	1.00	1.19	0.99	0.00	1.88	0.00	1.26	6.55
time (sec)	N/A	0.218	1.065	4.902	0.000	0.793	0.000	0.536	15.747

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	471	329	0	558	0	0	0
N.S.	1	1.00	1.69	1.18	0.00	2.00	0.00	0.00	0.00
time (sec)	N/A	0.145	5.260	4.955	0.000	0.654	0.000	0.000	0.000

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	423	287	0	530	0	0	0
N.S.	1	1.00	1.81	1.23	0.00	2.26	0.00	0.00	0.00
time (sec)	N/A	0.073	3.470	4.662	0.000	0.276	0.000	0.000	0.000

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	255	169	0	0	0	0	0
N.S.	1	1.00	1.71	1.13	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.019	1.886	4.540	0.000	0.000	0.000	0.000	0.000

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	308	211	0	0	0	0	0
N.S.	1	1.00	1.78	1.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.061	2.186	4.761	0.000	0.000	0.000	0.000	0.000

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	340	231	0	0	0	0	0
N.S.	1	1.00	1.58	1.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.124	2.805	4.884	0.000	0.000	0.000	0.000	0.000

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	249	0	0	0	0	0	0
N.S.	1	1.00	3.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.042	9.487	0.000	0.000	0.000	0.000	0.000	0.000

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.038	8.959	0.000	0.000	0.000	0.000	0.000	0.000

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	161	0	0	0	0	0	0
N.S.	1	1.00	2.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.020	0.059	0.000	0.000	0.000	0.000	0.000	0.000

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	338	0	0	0	0	0	0
N.S.	1	1.00	5.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.038	10.276	0.000	0.000	0.000	0.000	0.000	0.000

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	347	349	314	0	1300	0	431	564
N.S.	1	1.00	1.01	0.90	0.00	3.75	0.00	1.24	1.63
time (sec)	N/A	0.287	1.492	4.796	0.000	0.363	0.000	0.315	9.133

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	298	266	0	1141	0	372	493
N.S.	1	1.00	1.18	1.05	0.00	4.51	0.00	1.47	1.95
time (sec)	N/A	0.205	0.971	4.716	0.000	0.359	0.000	0.295	9.480

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	254	233	0	1004	0	325	449
N.S.	1	1.00	1.25	1.15	0.00	4.95	0.00	1.60	2.21
time (sec)	N/A	0.166	0.751	4.687	0.000	0.339	0.000	0.307	9.625

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	227	224	0	872	0	301	412
N.S.	1	1.00	1.30	1.29	0.00	5.01	0.00	1.73	2.37
time (sec)	N/A	0.112	0.556	4.503	0.000	0.344	0.000	0.299	9.516

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	220	197	0	262	0	285	389
N.S.	1	1.00	1.32	1.18	0.00	1.57	0.00	1.71	2.33
time (sec)	N/A	0.102	0.372	4.639	0.000	0.315	0.000	0.304	9.271

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	350	304	0	975	0	389	3804
N.S.	1	1.00	1.29	1.12	0.00	3.60	0.00	1.44	14.04
time (sec)	N/A	0.230	1.509	4.738	0.000	0.344	0.000	0.525	9.707

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	392	349	0	1386	0	481	5875
N.S.	1	1.00	1.10	0.98	0.00	3.88	0.00	1.35	16.46
time (sec)	N/A	0.275	1.690	4.998	0.000	0.817	0.000	0.520	11.004

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	506	494	0	1329	0	0	0
N.S.	1	1.00	1.57	1.53	0.00	4.13	0.00	0.00	0.00
time (sec)	N/A	0.293	10.815	5.414	0.000	0.803	0.000	0.000	0.000

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	466	343	0	1127	0	0	0
N.S.	1	1.00	1.79	1.32	0.00	4.33	0.00	0.00	0.00
time (sec)	N/A	0.116	6.681	4.772	0.000	0.309	0.000	0.000	0.000

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	322	217	0	0	0	0	0
N.S.	1	1.00	1.87	1.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.072	2.836	4.569	0.000	0.000	0.000	0.000	0.000

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	328	243	0	0	0	0	0
N.S.	1	1.00	1.83	1.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.037	0.548	4.579	0.000	0.000	0.000	0.000	0.000

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	358	257	0	0	0	0	0
N.S.	1	1.00	1.56	1.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.119	3.739	4.764	0.000	0.000	0.000	0.000	0.000

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	402	302	0	0	0	0	0
N.S.	1	1.00	1.40	1.05	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.190	5.697	4.818	0.000	0.000	0.000	0.000	0.000

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	459	356	0	0	0	0	0
N.S.	1	1.00	1.31	1.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.306	8.478	4.893	0.000	0.000	0.000	0.000	0.000

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	194	0	0	0	0	0	0
N.S.	1	1.00	2.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.039	10.233	0.000	0.000	0.000	0.000	0.000	0.000

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	144	0	0	0	0	0	0
N.S.	1	1.00	2.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.039	10.129	0.000	0.000	0.000	0.000	0.000	0.000

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	129	0	0	0	0	0	0
N.S.	1	1.00	1.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.038	10.098	0.000	0.000	0.000	0.000	0.000	0.000

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	141	0	0	0	0	0	0
N.S.	1	1.00	2.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.029	10.127	0.000	0.000	0.000	0.000	0.000	0.000

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	193	0	0	0	0	0	0
N.S.	1	1.00	2.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.040	10.210	0.000	0.000	0.000	0.000	0.000	0.000

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	264	0	0	0	0	0	0
N.S.	1	1.00	3.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.039	10.317	0.000	0.000	0.000	0.000	0.000	0.000

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	82	78	84	100	0	88	88
N.S.	1	1.00	0.91	0.87	0.93	1.11	0.00	0.98	0.98
time (sec)	N/A	0.062	0.054	4.564	0.188	2.693	0.000	0.282	10.040

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	66	65	68	72	0	70	68
N.S.	1	1.00	0.94	0.93	0.97	1.03	0.00	1.00	0.97
time (sec)	N/A	0.047	0.039	4.534	0.184	1.033	0.000	0.284	9.908

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	43	43	49	42	0	51	51
N.S.	1	1.00	0.81	0.81	0.92	0.79	0.00	0.96	0.96
time (sec)	N/A	0.035	0.027	4.515	0.235	0.480	0.000	0.280	9.505

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	31	32	41	31	138	51	1012
N.S.	1	1.00	0.69	0.71	0.91	0.69	3.07	1.13	22.49
time (sec)	N/A	0.022	0.023	4.522	0.190	0.263	0.908	0.271	9.511

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	54	55	61	54	0	73	58
N.S.	1	1.00	0.87	0.89	0.98	0.87	0.00	1.18	0.94
time (sec)	N/A	0.044	0.036	4.531	0.191	1.153	0.000	0.275	10.198

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	88	82	87	99	0	112	87
N.S.	1	1.00	1.01	0.94	1.00	1.14	0.00	1.29	1.00
time (sec)	N/A	0.066	0.043	4.823	0.187	4.728	0.000	0.276	11.381

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	104	98	100	576	0	112	532
N.S.	1	1.00	0.93	0.88	0.89	5.14	0.00	1.00	4.75
time (sec)	N/A	0.198	0.164	4.588	0.281	1.494	0.000	0.284	10.768

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	82	81	80	416	0	80	518
N.S.	1	1.00	0.89	0.88	0.87	4.52	0.00	0.87	5.63
time (sec)	N/A	0.080	0.146	4.701	0.298	0.468	0.000	0.298	10.545

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	66	60	59	325	0	59	379
N.S.	1	1.00	0.84	0.76	0.75	4.11	0.00	0.75	4.80
time (sec)	N/A	0.046	0.042	4.801	0.263	0.303	0.000	0.298	10.433

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	66	60	59	325	0	59	399
N.S.	1	1.00	0.84	0.76	0.75	4.11	0.00	0.75	5.05
time (sec)	N/A	0.046	0.050	4.529	0.284	0.303	0.000	0.277	10.411

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	169	81	80	432	0	80	354
N.S.	1	1.00	1.84	0.88	0.87	4.70	0.00	0.87	3.85
time (sec)	N/A	0.088	0.225	4.581	0.269	0.531	0.000	0.303	10.189

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	193	101	101	592	0	103	535
N.S.	1	1.00	1.72	0.90	0.90	5.29	0.00	0.92	4.78
time (sec)	N/A	0.145	0.213	4.589	0.274	1.989	0.000	0.278	9.925

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	457	457	377	234	375	1196	0	469	6361
N.S.	1	1.00	0.82	0.51	0.82	2.62	0.00	1.03	13.92
time (sec)	N/A	0.316	0.192	4.559	0.329	0.405	0.000	0.288	9.940

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	340	226	363	1427	0	453	2553
N.S.	1	1.00	0.76	0.50	0.81	3.18	0.00	1.01	5.69
time (sec)	N/A	0.207	0.096	4.563	0.286	0.314	0.000	0.310	10.089

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	340	218	361	1067	0	437	5889
N.S.	1	1.00	0.76	0.49	0.80	2.38	0.00	0.97	13.12
time (sec)	N/A	0.212	0.098	4.546	0.283	0.276	0.000	0.297	10.268

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	340	218	363	1331	0	477	6633
N.S.	1	1.00	0.76	0.49	0.81	2.96	0.00	1.06	14.77
time (sec)	N/A	0.194	0.134	4.558	0.277	0.273	0.000	0.290	9.985

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	340	226	365	1171	0	437	6153
N.S.	1	1.00	0.76	0.50	0.81	2.61	0.00	0.97	13.70
time (sec)	N/A	0.196	0.119	4.444	0.278	0.355	0.000	0.278	10.389

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	460	460	385	237	384	1461	0	488	5962
N.S.	1	1.00	0.84	0.52	0.83	3.18	0.00	1.06	12.96
time (sec)	N/A	0.357	0.206	4.683	0.283	0.499	0.000	0.283	10.357

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	462	462	406	241	390	1255	0	472	7459
N.S.	1	1.00	0.88	0.52	0.84	2.72	0.00	1.02	16.15
time (sec)	N/A	0.287	0.273	4.584	0.281	2.937	0.000	0.289	10.652

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	479	479	428	261	405	1526	0	483	4547
N.S.	1	1.00	0.89	0.54	0.85	3.19	0.00	1.01	9.49
time (sec)	N/A	0.436	0.298	4.996	0.281	4.974	0.000	0.280	10.327

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	88	84	0	195	122	96	87
N.S.	1	1.00	0.95	0.90	0.00	2.10	1.31	1.03	0.94
time (sec)	N/A	0.063	0.204	6.012	0.000	0.269	5.091	0.276	9.049

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	142	124	0	714	0	0	0
N.S.	1	1.00	1.18	1.03	0.00	5.95	0.00	0.00	0.00
time (sec)	N/A	0.115	1.262	5.291	0.000	0.309	0.000	0.000	0.000

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	69	62	0	156	92	66	54
N.S.	1	1.00	0.99	0.89	0.00	2.23	1.31	0.94	0.77
time (sec)	N/A	0.061	0.128	4.862	0.000	0.247	2.907	0.276	9.043

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	107	77	0	612	0	0	0
N.S.	1	1.00	1.18	0.85	0.00	6.73	0.00	0.00	0.00
time (sec)	N/A	0.054	0.399	5.108	0.000	0.274	0.000	0.000	0.000

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	81	105	0	383	158	79	199
N.S.	1	1.00	0.95	1.24	0.00	4.51	1.86	0.93	2.34
time (sec)	N/A	0.059	0.170	4.867	0.000	0.273	4.431	0.269	9.236

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	96	69	0	281	0	121	0
N.S.	1	1.00	1.26	0.91	0.00	3.70	0.00	1.59	0.00
time (sec)	N/A	0.067	0.451	5.447	0.000	0.264	0.000	0.772	0.000

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	107	96	0	513	0	107	269
N.S.	1	1.00	0.93	0.83	0.00	4.46	0.00	0.93	2.34
time (sec)	N/A	0.097	0.416	4.960	0.000	0.266	0.000	0.263	9.582

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	118	92	0	329	0	225	0
N.S.	1	1.00	1.07	0.84	0.00	2.99	0.00	2.05	0.00
time (sec)	N/A	0.133	0.670	5.678	0.000	0.284	0.000	1.106	0.000

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	857	1067	141	332	0	0	0	0	0
N.S.	1	1.25	0.16	0.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.450	10.142	7.130	0.000	0.000	0.000	0.000	0.000

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	700	904	241	303	0	0	0	0	0
N.S.	1	1.29	0.34	0.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.016	10.437	6.054	0.000	0.000	0.000	0.000	0.000

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	786	1012	65	299	0	0	0	0	0
N.S.	1	1.29	0.08	0.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.852	10.039	4.677	0.000	0.000	0.000	0.000	0.000

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	679	881	161	273	0	0	0	0	0
N.S.	1	1.30	0.24	0.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.740	10.173	4.929	0.000	0.000	0.000	0.000	0.000

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	809	1031	138	318	0	0	0	0	0
N.S.	1	1.27	0.17	0.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.238	10.098	7.401	0.000	0.000	0.000	0.000	0.000

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	703	893	333	294	0	0	0	0	0
N.S.	1	1.27	0.47	0.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.948	10.286	6.798	0.000	0.000	0.000	0.000	0.000

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	70	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.083	11.079	0.000	0.000	0.000	0.000	0.000	0.000

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	70	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.077	11.065	0.000	0.000	0.000	0.000	0.000	0.000

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	68	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.051	11.047	0.000	0.000	0.000	0.000	0.000	0.000

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	143	0	0	0	0	0	0
N.S.	1	1.00	2.07	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.084	11.148	0.000	0.000	0.000	0.000	0.000	0.000

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	91	91	0	289	0	106	102
N.S.	1	1.00	0.88	0.88	0.00	2.78	0.00	1.02	0.98
time (sec)	N/A	0.095	0.293	5.060	0.000	0.312	0.000	0.277	9.271

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	73	72	0	205	0	64	58
N.S.	1	1.00	0.99	0.97	0.00	2.77	0.00	0.86	0.78
time (sec)	N/A	0.046	0.167	4.917	0.000	0.307	0.000	0.268	9.144

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	39	0	130	87	40	40
N.S.	1	1.00	1.00	0.76	0.00	2.55	1.71	0.78	0.78
time (sec)	N/A	0.034	0.070	4.959	0.000	0.325	5.884	0.284	9.046

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	80	78	0	431	114	71	652
N.S.	1	1.00	0.94	0.92	0.00	5.07	1.34	0.84	7.67
time (sec)	N/A	0.053	0.191	4.896	0.000	0.329	5.308	0.282	9.198

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	109	92	0	565	0	104	396
N.S.	1	1.00	0.93	0.79	0.00	4.83	0.00	0.89	3.38
time (sec)	N/A	0.087	0.439	4.979	0.000	0.363	0.000	0.266	9.649

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	136	152	0	739	0	0	0
N.S.	1	1.00	1.11	1.24	0.00	6.01	0.00	0.00	0.00
time (sec)	N/A	0.105	0.946	5.620	0.000	0.338	0.000	0.000	0.000

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	108	85	0	632	0	0	0
N.S.	1	1.00	1.19	0.93	0.00	6.95	0.00	0.00	0.00
time (sec)	N/A	0.060	0.530	5.210	0.000	0.312	0.000	0.000	0.000

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	74	42	0	245	0	72	0
N.S.	1	1.00	1.37	0.78	0.00	4.54	0.00	1.33	0.00
time (sec)	N/A	0.036	0.789	5.126	0.000	0.300	0.000	0.278	0.000

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	100	80	0	332	0	116	0
N.S.	1	1.00	1.25	1.00	0.00	4.15	0.00	1.45	0.00
time (sec)	N/A	0.076	0.571	5.349	0.000	0.300	0.000	0.310	0.000

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	121	103	0	418	0	205	0
N.S.	1	1.00	1.05	0.90	0.00	3.63	0.00	1.78	0.00
time (sec)	N/A	0.122	1.084	6.302	0.000	0.309	0.000	1.158	0.000

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	872	872	249	298	0	0	0	0	0
N.S.	1	1.00	0.29	0.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.878	10.346	6.114	0.000	0.000	0.000	0.000	0.000

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	638	837	65	265	0	0	0	0	0
N.S.	1	1.31	0.10	0.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.602	10.056	4.722	0.000	0.000	0.000	0.000	0.000

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	638	742	161	191	0	0	0	0	0
N.S.	1	1.16	0.25	0.30	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.479	10.076	4.687	0.000	0.000	0.000	0.000	0.000

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	677	864	337	288	0	0	0	0	0
N.S.	1	1.28	0.50	0.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.780	10.288	6.319	0.000	0.000	0.000	0.000	0.000

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	804	982	65	292	0	0	0	0	0
N.S.	1	1.22	0.08	0.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.843	10.055	4.760	0.000	0.000	0.000	0.000	0.000

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	656	756	65	191	0	0	0	0	0
N.S.	1	1.15	0.10	0.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.536	10.055	4.798	0.000	0.000	0.000	0.000	0.000

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	833	1007	141	310	0	0	0	0	0
N.S.	1	1.21	0.17	0.37	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.017	10.121	6.489	0.000	0.000	0.000	0.000	0.000

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	175	171	0	622	0	180	186
N.S.	1	1.00	1.00	0.98	0.00	3.55	0.00	1.03	1.06
time (sec)	N/A	0.136	0.679	5.152	0.000	0.623	0.000	0.295	9.455

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	130	133	0	475	0	134	144
N.S.	1	1.00	1.06	1.08	0.00	3.86	0.00	1.09	1.17
time (sec)	N/A	0.097	0.492	5.176	0.000	0.642	0.000	0.297	9.834

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	100	83	0	348	0	116	95
N.S.	1	1.00	1.01	0.84	0.00	3.52	0.00	1.17	0.96
time (sec)	N/A	0.064	0.283	5.224	0.000	0.911	0.000	0.277	9.690

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	86	90	0	302	0	93	84
N.S.	1	1.00	0.99	1.03	0.00	3.47	0.00	1.07	0.97
time (sec)	N/A	0.049	0.281	4.886	0.000	0.353	0.000	0.278	9.537

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	124	146	0	862	0	139	3017
N.S.	1	1.00	0.94	1.11	0.00	6.53	0.00	1.05	22.86
time (sec)	N/A	0.094	0.529	5.007	0.000	0.538	0.000	0.281	10.723

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	163	191	0	1236	0	257	3822
N.S.	1	1.00	0.88	1.03	0.00	6.68	0.00	1.39	20.66
time (sec)	N/A	0.171	0.975	5.134	0.000	0.543	0.000	0.308	11.896

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	189	164	0	1386	0	333	0
N.S.	1	1.00	0.99	0.86	0.00	7.26	0.00	1.74	0.00
time (sec)	N/A	0.225	2.768	6.404	0.000	1.448	0.000	0.388	0.000

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	153	117	0	1077	0	298	0
N.S.	1	1.00	1.09	0.83	0.00	7.64	0.00	2.11	0.00
time (sec)	N/A	0.109	1.957	5.705	0.000	0.522	0.000	0.373	0.000

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	112	81	0	426	0	244	0
N.S.	1	1.00	1.20	0.87	0.00	4.58	0.00	2.62	0.00
time (sec)	N/A	0.070	1.161	5.564	0.000	0.448	0.000	0.858	0.000

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	124	90	0	467	0	237	0
N.S.	1	1.00	1.19	0.87	0.00	4.49	0.00	2.28	0.00
time (sec)	N/A	0.071	1.454	5.847	0.000	0.661	0.000	0.318	0.000

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	157	112	0	612	0	418	0
N.S.	1	1.00	1.05	0.75	0.00	4.11	0.00	2.81	0.00
time (sec)	N/A	0.135	1.500	6.038	0.000	0.720	0.000	0.891	0.000

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	201	134	0	760	0	395	0
N.S.	1	1.00	0.97	0.64	0.00	3.65	0.00	1.90	0.00
time (sec)	N/A	0.236	2.497	6.751	0.000	0.737	0.000	0.942	0.000

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	996	996	253	337	0	0	0	0	0
N.S.	1	1.00	0.25	0.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.970	10.297	5.311	0.000	0.000	0.000	0.000	0.000

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	908	908	238	324	0	0	0	0	0
N.S.	1	1.00	0.26	0.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.618	10.229	5.204	0.000	0.000	0.000	0.000	0.000

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	983	983	392	333	0	0	0	0	0
N.S.	1	1.00	0.40	0.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.735	10.276	4.959	0.000	0.000	0.000	0.000	0.000

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	1046	1046	408	364	0	0	0	0	0
N.S.	1	1.00	0.39	0.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.052	10.549	8.392	0.000	0.000	0.000	0.000	0.000

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1146	1146	162	353	0	0	0	0	0
N.S.	1	1.00	0.14	0.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.979	10.391	5.024	0.000	0.000	0.000	0.000	0.000

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1144	1144	172	359	0	0	0	0	0
N.S.	1	1.00	0.15	0.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.909	10.175	4.980	0.000	0.000	0.000	0.000	0.000

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1225	1225	226	392	0	0	0	0	0
N.S.	1	1.00	0.18	0.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.157	10.283	8.135	0.000	0.000	0.000	0.000	0.000

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	200	194	164	0	0	0	182	0	0
N.S.	1	0.97	0.82	0.00	0.00	0.00	0.91	0.00	0.00
time (sec)	N/A	0.161	11.133	0.000	0.000	0.000	6.383	0.000	0.000

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	123	115	110	0	0	0	117	0	0
N.S.	1	0.93	0.89	0.00	0.00	0.00	0.95	0.00	0.00
time (sec)	N/A	0.047	2.272	0.000	0.000	0.000	2.194	0.000	0.000

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	66	0	0	0	56	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.82	0.00	0.00
time (sec)	N/A	0.013	0.785	0.000	0.000	0.000	0.601	0.000	0.000

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	125	0	0	0	0	0	0
N.S.	1	1.00	1.54	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.041	5.606	0.000	0.000	0.000	0.000	0.000	0.000

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	179	0	0	0	0	0	0
N.S.	1	1.00	2.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.040	11.181	0.000	0.000	0.000	0.000	0.000	0.000

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	77	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.040	11.109	0.000	0.000	0.000	0.000	0.000	0.000

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	198	198	167	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.125	11.186	0.000	0.000	0.000	0.000	0.000	0.000

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	132	113	0	0	0	117	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.89	0.00	0.00
time (sec)	N/A	0.042	5.577	0.000	0.000	0.000	17.871	0.000	0.000

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	69	0	0	0	56	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.79	0.00	0.00
time (sec)	N/A	0.016	1.934	0.000	0.000	0.000	0.734	0.000	0.000

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	169	0	0	0	0	0	0
N.S.	1	1.00	2.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.042	11.188	0.000	0.000	0.000	0.000	0.000	0.000

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	77	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.053	11.112	0.000	0.000	0.000	0.000	0.000	0.000

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	77	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.045	11.113	0.000	0.000	0.000	0.000	0.000	0.000

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	91	91	0	288	0	106	103
N.S.	1	1.00	0.88	0.88	0.00	2.77	0.00	1.02	0.99
time (sec)	N/A	0.076	0.292	7.314	0.000	0.527	0.000	0.292	9.193

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	73	72	0	205	0	64	58
N.S.	1	1.00	0.99	0.97	0.00	2.77	0.00	0.86	0.78
time (sec)	N/A	0.044	0.155	5.396	0.000	0.434	0.000	0.286	9.096

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	39	0	130	87	40	40
N.S.	1	1.00	1.00	0.76	0.00	2.55	1.71	0.78	0.78
time (sec)	N/A	0.034	0.073	5.112	0.000	0.320	8.628	0.298	9.120

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	80	78	0	431	114	71	652
N.S.	1	1.00	0.94	0.92	0.00	5.07	1.34	0.84	7.67
time (sec)	N/A	0.047	0.193	5.272	0.000	0.305	6.994	0.299	9.297

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	109	92	0	565	0	104	396
N.S.	1	1.00	0.93	0.79	0.00	4.83	0.00	0.89	3.38
time (sec)	N/A	0.075	0.432	5.389	0.000	0.330	0.000	0.288	9.853

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	136	152	0	739	0	237	0
N.S.	1	1.00	1.11	1.24	0.00	6.01	0.00	1.93	0.00
time (sec)	N/A	0.091	1.410	9.611	0.000	0.825	0.000	0.434	0.000

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	108	85	0	632	0	156	0
N.S.	1	1.00	1.19	0.93	0.00	6.95	0.00	1.71	0.00
time (sec)	N/A	0.061	0.697	7.217	0.000	0.608	0.000	0.303	0.000

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	74	42	0	245	0	72	0
N.S.	1	1.00	1.37	0.78	0.00	4.54	0.00	1.33	0.00
time (sec)	N/A	0.029	0.859	6.881	0.000	0.733	0.000	0.300	0.000

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	100	80	0	332	0	0	0
N.S.	1	1.00	1.25	1.00	0.00	4.15	0.00	0.00	0.00
time (sec)	N/A	0.056	0.720	8.124	0.000	0.372	0.000	0.000	0.000

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	121	103	0	416	0	0	0
N.S.	1	1.00	1.05	0.90	0.00	3.62	0.00	0.00	0.00
time (sec)	N/A	0.109	1.490	10.953	0.000	0.389	0.000	0.000	0.000

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	10.049	0.000	0.000	0.000	0.000	0.000	0.000

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.047	10.044	0.000	0.000	0.000	0.000	0.000	0.000

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.038	10.049	0.000	0.000	0.000	0.000	0.000	0.000

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	161	0	0	0	0	0	0
N.S.	1	1.00	2.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.016	10.189	0.000	0.000	0.000	0.000	0.000	0.000

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	141	0	0	0	0	0	0
N.S.	1	1.00	2.27	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.038	10.122	0.000	0.000	0.000	0.000	0.000	0.000

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	141	0	0	0	0	0	0
N.S.	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.063	10.122	0.000	0.000	0.000	0.000	0.000	0.000

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	141	0	0	0	0	0	0
N.S.	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.061	10.134	0.000	0.000	0.000	0.000	0.000	0.000

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	130	133	0	475	0	134	144
N.S.	1	1.00	1.06	1.08	0.00	3.86	0.00	1.09	1.17
time (sec)	N/A	0.097	0.535	5.356	0.000	0.641	0.000	0.276	9.466

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	100	83	0	348	0	116	95
N.S.	1	1.00	1.01	0.84	0.00	3.52	0.00	1.17	0.96
time (sec)	N/A	0.056	0.276	5.325	0.000	0.378	0.000	0.279	9.795

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	86	90	0	302	0	93	84
N.S.	1	1.00	0.99	1.03	0.00	3.47	0.00	1.07	0.97
time (sec)	N/A	0.047	0.251	5.159	0.000	0.411	0.000	0.273	9.241

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	124	146	0	862	0	139	3025
N.S.	1	1.00	0.94	1.11	0.00	6.53	0.00	1.05	22.92
time (sec)	N/A	0.087	0.521	5.352	0.000	0.480	0.000	0.281	10.306

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	163	189	0	1236	0	257	3860
N.S.	1	1.00	0.88	1.02	0.00	6.68	0.00	1.39	20.86
time (sec)	N/A	0.161	1.007	5.446	0.000	0.478	0.000	0.284	11.632

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	153	117	0	1077	0	343	0
N.S.	1	1.00	1.09	0.83	0.00	7.64	0.00	2.43	0.00
time (sec)	N/A	0.109	2.987	10.579	0.000	0.848	0.000	0.296	0.000

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	112	81	0	426	0	0	0
N.S.	1	1.00	1.20	0.87	0.00	4.58	0.00	0.00	0.00
time (sec)	N/A	0.057	1.433	9.337	0.000	0.361	0.000	0.000	0.000

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	124	90	0	467	0	237	0
N.S.	1	1.00	1.19	0.87	0.00	4.49	0.00	2.28	0.00
time (sec)	N/A	0.060	1.599	9.441	0.000	0.506	0.000	0.283	0.000

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	157	112	0	612	0	0	0
N.S.	1	1.00	1.05	0.75	0.00	4.11	0.00	0.00	0.00
time (sec)	N/A	0.127	1.789	12.056	0.000	0.515	0.000	0.000	0.000

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	201	134	0	760	0	0	0
N.S.	1	1.00	0.97	0.64	0.00	3.65	0.00	0.00	0.00
time (sec)	N/A	0.200	3.176	18.346	0.000	0.686	0.000	0.000	0.000

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	169	0	0	0	0	0	0
N.S.	1	1.00	2.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.035	10.201	0.000	0.000	0.000	0.000	0.000	0.000

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	168	0	0	0	0	0	0
N.S.	1	1.00	2.62	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.051	10.193	0.000	0.000	0.000	0.000	0.000	0.000

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	172	0	0	0	0	0	0
N.S.	1	1.00	2.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.036	10.175	0.000	0.000	0.000	0.000	0.000	0.000

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	329	0	0	0	0	0	0
N.S.	1	1.00	5.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.017	10.297	0.000	0.000	0.000	0.000	0.000	0.000

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	226	0	0	0	0	0	0
N.S.	1	1.00	3.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.034	10.297	0.000	0.000	0.000	0.000	0.000	0.000

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	226	0	0	0	0	0	0
N.S.	1	1.00	3.53	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.056	10.281	0.000	0.000	0.000	0.000	0.000	0.000

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	225	0	0	0	0	0	0
N.S.	1	1.00	3.52	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.056	10.288	0.000	0.000	0.000	0.000	0.000	0.000

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	91	91	0	288	0	106	103
N.S.	1	1.00	0.88	0.88	0.00	2.77	0.00	1.02	0.99
time (sec)	N/A	0.065	0.287	8.891	0.000	0.402	0.000	0.274	9.154

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	73	72	0	205	0	64	58
N.S.	1	1.00	0.99	0.97	0.00	2.77	0.00	0.86	0.78
time (sec)	N/A	0.043	0.153	5.986	0.000	0.422	0.000	0.280	9.110

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	39	0	130	87	40	40
N.S.	1	1.00	1.00	0.76	0.00	2.55	1.71	0.78	0.78
time (sec)	N/A	0.029	0.072	5.898	0.000	0.452	10.988	0.274	9.144

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	80	78	0	431	114	71	652
N.S.	1	1.00	0.94	0.92	0.00	5.07	1.34	0.84	7.67
time (sec)	N/A	0.045	0.190	6.177	0.000	0.492	9.149	0.269	9.356

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	109	92	0	565	0	104	396
N.S.	1	1.00	0.93	0.79	0.00	4.83	0.00	0.89	3.38
time (sec)	N/A	0.072	0.429	6.098	0.000	0.877	0.000	0.314	10.114

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	136	152	0	739	0	0	0
N.S.	1	1.00	1.11	1.24	0.00	6.01	0.00	0.00	0.00
time (sec)	N/A	0.085	2.361	22.368	0.000	0.381	0.000	0.000	0.000

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	108	85	0	632	0	0	0
N.S.	1	1.00	1.19	0.93	0.00	6.95	0.00	0.00	0.00
time (sec)	N/A	0.054	1.093	12.914	0.000	0.395	0.000	0.000	0.000

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	74	42	0	245	0	72	0
N.S.	1	1.00	1.37	0.78	0.00	4.54	0.00	1.33	0.00
time (sec)	N/A	0.031	0.959	10.886	0.000	0.445	0.000	0.288	0.000

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	100	80	0	332	0	116	0
N.S.	1	1.00	1.25	1.00	0.00	4.15	0.00	1.45	0.00
time (sec)	N/A	0.052	0.878	14.678	0.000	0.505	0.000	0.283	0.000

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	121	103	0	416	0	205	0
N.S.	1	1.00	1.05	0.90	0.00	3.62	0.00	1.78	0.00
time (sec)	N/A	0.096	2.498	25.258	0.000	0.521	0.000	1.271	0.000

Problem 897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	851	851	65	0	0	0	0	0	0
N.S.	1	1.00	0.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.726	10.063	0.000	0.000	0.000	0.000	0.000	0.000

Problem 898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	754	754	65	0	0	0	0	0	0
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.481	10.058	0.000	0.000	0.000	0.000	0.000	0.000

Problem 899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	878	878	141	0	0	0	0	0	0
N.S.	1	1.00	0.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.780	10.157	0.000	0.000	0.000	0.000	0.000	0.000

Problem 900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1005	1005	65	0	0	0	0	0	0
N.S.	1	1.00	0.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.787	10.069	0.000	0.000	0.000	0.000	0.000	0.000

Problem 901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	768	768	65	0	0	0	0	0	0
N.S.	1	1.00	0.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.518	10.062	0.000	0.000	0.000	0.000	0.000	0.000

Problem 902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1032	1032	141	0	0	0	0	0	0
N.S.	1	1.00	0.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.975	10.125	0.000	0.000	0.000	0.000	0.000	0.000

Problem 903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.032	10.070	0.000	0.000	0.000	0.000	0.000	0.000

Problem 904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.030	10.062	0.000	0.000	0.000	0.000	0.000	0.000

Problem 905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	161	0	0	0	0	0	0
N.S.	1	1.00	2.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.017	10.198	0.000	0.000	0.000	0.000	0.000	0.000

Problem 906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	141	0	0	0	0	0	0
N.S.	1	1.00	2.27	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.037	10.133	0.000	0.000	0.000	0.000	0.000	0.000

Problem 907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	141	0	0	0	0	0	0
N.S.	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.034	10.145	0.000	0.000	0.000	0.000	0.000	0.000

Problem 908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	130	133	0	475	0	134	144
N.S.	1	1.00	1.06	1.08	0.00	3.86	0.00	1.09	1.17
time (sec)	N/A	0.095	0.536	5.954	0.000	0.278	0.000	0.274	9.483

Problem 909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	100	83	0	348	0	116	95
N.S.	1	1.00	1.01	0.84	0.00	3.52	0.00	1.17	0.96
time (sec)	N/A	0.055	0.273	5.837	0.000	0.324	0.000	0.273	9.294

Problem 910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	86	90	0	302	0	93	84
N.S.	1	1.00	0.99	1.03	0.00	3.47	0.00	1.07	0.97
time (sec)	N/A	0.045	0.251	5.819	0.000	0.413	0.000	0.265	9.137

Problem 911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	124	146	0	862	0	139	3017
N.S.	1	1.00	0.94	1.11	0.00	6.53	0.00	1.05	22.86
time (sec)	N/A	0.083	0.527	5.923	0.000	0.451	0.000	0.270	10.155

Problem 912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	163	191	0	1236	0	257	3832
N.S.	1	1.00	0.88	1.03	0.00	6.68	0.00	1.39	20.71
time (sec)	N/A	0.152	1.023	6.152	0.000	0.522	0.000	0.270	12.251

Problem 913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	153	117	0	1077	0	298	0
N.S.	1	1.00	1.09	0.83	0.00	7.64	0.00	2.11	0.00
time (sec)	N/A	0.102	4.186	24.895	0.000	1.194	0.000	0.374	0.000

Problem 914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	112	81	0	426	0	244	0
N.S.	1	1.00	1.20	0.87	0.00	4.58	0.00	2.62	0.00
time (sec)	N/A	0.057	2.026	20.282	0.000	0.353	0.000	0.993	0.000

Problem 915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	124	90	0	467	0	237	0
N.S.	1	1.00	1.19	0.87	0.00	4.49	0.00	2.28	0.00
time (sec)	N/A	0.062	1.825	20.214	0.000	0.394	0.000	0.373	0.000

Problem 916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	157	112	0	612	0	418	0
N.S.	1	1.00	1.05	0.75	0.00	4.11	0.00	2.81	0.00
time (sec)	N/A	0.119	2.131	31.252	0.000	0.554	0.000	0.928	0.000

Problem 917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	201	134	0	760	0	395	0
N.S.	1	1.00	0.97	0.64	0.00	3.65	0.00	1.90	0.00
time (sec)	N/A	0.193	5.337	47.789	0.000	0.661	0.000	1.049	0.000

Problem 918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	924	924	159	0	0	0	0	0	0
N.S.	1	1.00	0.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.804	10.164	0.000	0.000	0.000	0.000	0.000	0.000

Problem 919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	999	999	169	0	0	0	0	0	0
N.S.	1	1.00	0.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.932	10.178	0.000	0.000	0.000	0.000	0.000	0.000

Problem 920	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1060	1060	225	0	0	0	0	0	0
N.S.	1	1.00	0.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.282	10.316	0.000	0.000	0.000	0.000	0.000	0.000

Problem 921	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1164	1164	159	0	0	0	0	0	0
N.S.	1	1.00	0.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.152	10.171	0.000	0.000	0.000	0.000	0.000	0.000

Problem 922	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1162	1162	169	0	0	0	0	0	0
N.S.	1	1.00	0.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.174	10.198	0.000	0.000	0.000	0.000	0.000	0.000

Problem 923	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1243	1243	226	0	0	0	0	0	0
N.S.	1	1.00	0.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.454	10.310	0.000	0.000	0.000	0.000	0.000	0.000

Problem 924	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	170	0	0	0	0	0	0
N.S.	1	1.00	2.66	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.037	10.217	0.000	0.000	0.000	0.000	0.000	0.000

Problem 925	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	170	0	0	0	0	0	0
N.S.	1	1.00	2.66	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	10.186	0.000	0.000	0.000	0.000	0.000	0.000

Problem 926	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	328	0	0	0	0	0	0
N.S.	1	1.00	5.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.017	10.296	0.000	0.000	0.000	0.000	0.000	0.000

Problem 927	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	226	0	0	0	0	0	0
N.S.	1	1.00	3.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.034	10.330	0.000	0.000	0.000	0.000	0.000	0.000

Problem 928	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	226	0	0	0	0	0	0
N.S.	1	1.00	3.53	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.034	10.320	0.000	0.000	0.000	0.000	0.000	0.000

Problem 929	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	118	116	243	242	226	143	134
N.S.	1	1.00	0.96	0.94	1.98	1.97	1.84	1.16	1.09
time (sec)	N/A	0.063	0.644	0.093	0.294	0.432	24.897	0.273	10.008

Problem 930	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	97	91	159	191	144	105	93
N.S.	1	1.00	1.08	1.01	1.77	2.12	1.60	1.17	1.03
time (sec)	N/A	0.042	0.171	0.075	0.303	0.435	17.849	0.280	9.655

Problem 931	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	80	77	108	155	107	87	68
N.S.	1	1.00	0.95	0.92	1.29	1.85	1.27	1.04	0.81
time (sec)	N/A	0.036	0.261	0.083	0.287	0.453	17.144	0.328	9.611

Problem 932	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	82	84	67	166	85	163	57
N.S.	1	1.00	1.39	1.42	1.14	2.81	1.44	2.76	0.97
time (sec)	N/A	0.029	0.205	0.082	0.289	0.307	4.533	0.471	9.554

Problem 933	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	47	48	49	60	73	250	91
N.S.	1	1.00	1.02	1.04	1.07	1.30	1.59	5.43	1.98
time (sec)	N/A	0.024	0.149	0.069	0.208	0.356	1.178	0.592	9.221

Problem 934	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	69	70	84	85	109	310	126
N.S.	1	1.00	0.93	0.95	1.14	1.15	1.47	4.19	1.70
time (sec)	N/A	0.036	0.199	0.087	0.203	0.284	1.211	0.806	9.431

Problem 935	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	93	94	118	109	143	370	168
N.S.	1	1.00	0.89	0.90	1.13	1.05	1.38	3.56	1.62
time (sec)	N/A	0.050	0.225	0.104	0.202	0.326	1.293	0.989	9.704

Problem 936	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	113	118	152	133	177	430	210
N.S.	1	1.00	0.84	0.88	1.13	0.99	1.32	3.21	1.57
time (sec)	N/A	0.066	0.255	0.141	0.211	0.362	1.382	1.487	10.113

Problem 937	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	108	113	158	131	1386	175	117
N.S.	1	1.00	0.72	0.75	1.05	0.87	9.24	1.17	0.78
time (sec)	N/A	0.046	0.102	0.095	0.198	0.303	2.757	0.275	9.161

Problem 938	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	88	89	124	107	910	140	97
N.S.	1	1.00	0.75	0.76	1.06	0.91	7.78	1.20	0.83
time (sec)	N/A	0.039	0.084	0.081	0.200	0.286	2.252	0.269	8.914

Problem 939	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	64	65	90	82	422	105	77
N.S.	1	1.00	0.76	0.77	1.07	0.98	5.02	1.25	0.92
time (sec)	N/A	0.030	0.066	0.068	0.197	0.285	1.654	0.270	9.047

Problem 940	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	42	43	55	57	119	72	54
N.S.	1	1.00	0.79	0.81	1.04	1.08	2.25	1.36	1.02
time (sec)	N/A	0.016	0.049	0.060	0.217	0.277	1.230	0.623	8.905

Problem 941	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	68	83	75	156	107	116	80
N.S.	1	1.00	1.03	1.26	1.14	2.36	1.62	1.76	1.21
time (sec)	N/A	0.022	0.128	0.059	0.282	0.359	1.427	0.267	9.183

Problem 942	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	75	93	133	164	107	76	97
N.S.	1	1.00	0.88	1.09	1.56	1.93	1.26	0.89	1.14
time (sec)	N/A	0.029	0.200	0.095	0.295	0.426	2.024	0.285	9.635

Problem 943	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	91	102	193	194	144	130	0
N.S.	1	1.00	1.00	1.12	2.12	2.13	1.58	1.43	0.00
time (sec)	N/A	0.031	0.253	0.085	0.286	0.371	3.287	0.291	0.000

Problem 944	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	124	127	277	244	226	153	0
N.S.	1	1.00	1.01	1.03	2.25	1.98	1.84	1.24	0.00
time (sec)	N/A	0.051	0.281	0.096	0.285	0.401	7.141	0.297	0.000

Problem 945	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	123	116	240	243	253	144	130
N.S.	1	1.00	1.00	0.94	1.95	1.98	2.06	1.17	1.06
time (sec)	N/A	0.058	0.255	0.085	0.297	0.691	33.529	0.287	10.272

Problem 946	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	98	96	171	203	216	121	105
N.S.	1	1.00	0.85	0.83	1.49	1.77	1.88	1.05	0.91
time (sec)	N/A	0.054	0.486	0.089	0.287	0.311	55.222	0.332	10.183

Problem 947	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	103	99	134	195	187	225	95
N.S.	1	1.00	0.94	0.90	1.22	1.77	1.70	2.05	0.86
time (sec)	N/A	0.048	0.349	0.093	0.286	0.291	16.511	0.485	10.131

Problem 948	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	91	108	80	213	114	254	72
N.S.	1	1.00	1.20	1.42	1.05	2.80	1.50	3.34	0.95
time (sec)	N/A	0.033	0.338	0.096	0.283	0.315	9.839	0.840	10.370

Problem 949	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	49	48	49	84	189	370	122
N.S.	1	1.00	1.07	1.04	1.07	1.83	4.11	8.04	2.65
time (sec)	N/A	0.025	0.235	0.088	0.197	0.310	3.388	0.939	10.106

Problem 950	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	71	70	84	109	258	430	164
N.S.	1	1.00	0.96	0.95	1.14	1.47	3.49	5.81	2.22
time (sec)	N/A	0.035	0.291	0.111	0.217	0.300	3.547	1.307	10.253

Problem 951	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	94	94	118	134	326	490	206
N.S.	1	1.00	0.90	0.90	1.13	1.29	3.13	4.71	1.98
time (sec)	N/A	0.049	0.343	0.165	0.196	0.354	3.767	2.031	10.679

Problem 952	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	115	118	152	157	393	550	248
N.S.	1	1.00	0.86	0.88	1.13	1.17	2.93	4.10	1.85
time (sec)	N/A	0.062	0.409	0.263	0.200	0.398	3.927	2.047	11.330

Problem 953	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	110	115	158	155	3351	175	137
N.S.	1	1.00	0.73	0.77	1.05	1.03	22.34	1.17	0.91
time (sec)	N/A	0.046	0.113	0.103	0.206	0.304	5.607	0.275	9.059

Problem 954	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	89	91	124	132	2304	140	118
N.S.	1	1.00	0.76	0.78	1.06	1.13	19.69	1.20	1.01
time (sec)	N/A	0.036	0.095	0.085	0.206	0.289	4.205	0.274	9.021

Problem 955	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	66	67	90	106	1340	105	97
N.S.	1	1.00	0.79	0.80	1.07	1.26	15.95	1.25	1.15
time (sec)	N/A	0.026	0.075	0.078	0.212	0.297	3.147	0.268	8.964

Problem 956	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	44	45	55	80	498	72	77
N.S.	1	1.00	0.83	0.85	1.04	1.51	9.40	1.36	1.45
time (sec)	N/A	0.020	0.052	0.067	0.211	0.299	2.366	0.273	8.993

Problem 957	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	81	99	91	203	184	140	0
N.S.	1	1.00	0.94	1.15	1.06	2.36	2.14	1.63	0.00
time (sec)	N/A	0.034	0.178	0.068	0.306	0.305	2.181	0.279	0.000

Problem 958	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	105	147	163	190	202	115	0
N.S.	1	1.00	0.87	1.21	1.35	1.57	1.67	0.95	0.00
time (sec)	N/A	0.038	0.204	0.108	0.306	0.336	3.240	0.293	0.000

Problem 959	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	92	115	207	216	216	145	78
N.S.	1	1.00	0.82	1.03	1.85	1.93	1.93	1.29	0.70
time (sec)	N/A	0.041	0.298	0.102	0.293	0.323	5.378	0.299	10.150

Problem 960	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	117	127	275	246	253	173	0
N.S.	1	1.00	0.95	1.03	2.24	2.00	2.06	1.41	0.00
time (sec)	N/A	0.042	0.388	0.109	0.300	0.298	10.454	0.305	0.000

Problem 961	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	150	151	354	298	287	214	0
N.S.	1	1.00	0.94	0.95	2.23	1.87	1.81	1.35	0.00
time (sec)	N/A	0.059	0.354	0.116	0.278	0.335	31.778	0.304	0.000

Problem 962	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	105	99	178	192	150	111	99
N.S.	1	1.00	1.17	1.10	1.98	2.13	1.67	1.23	1.10
time (sec)	N/A	0.044	0.434	0.083	0.285	0.332	14.385	0.283	9.850

Problem 963	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	80	82	109	146	66	79	59
N.S.	1	1.00	1.36	1.39	1.85	2.47	1.12	1.34	1.00
time (sec)	N/A	0.028	0.101	0.073	0.285	0.304	14.104	0.273	9.557

Problem 964	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	76	69	54	130	71	66	35
N.S.	1	1.00	1.77	1.60	1.26	3.02	1.65	1.53	0.81
time (sec)	N/A	0.022	0.108	0.075	0.279	0.320	2.951	0.290	9.299

Problem 965	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	39	44	48	39	65	124	35
N.S.	1	1.00	0.91	1.02	1.12	0.91	1.51	2.88	0.81
time (sec)	N/A	0.024	0.125	0.064	0.206	0.309	1.054	0.452	9.004

Problem 966	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	60	67	83	62	100	180	58
N.S.	1	1.00	0.83	0.93	1.15	0.86	1.39	2.50	0.81
time (sec)	N/A	0.036	0.159	0.071	0.222	0.312	1.056	0.660	9.044

Problem 967	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	93	91	118	86	133	236	102
N.S.	1	1.00	0.92	0.90	1.17	0.85	1.32	2.34	1.01
time (sec)	N/A	0.049	0.194	0.080	0.194	0.302	1.181	0.866	9.091

Problem 968	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	56	62	85	59	338	99	53
N.S.	1	1.00	0.68	0.76	1.04	0.72	4.12	1.21	0.65
time (sec)	N/A	0.023	0.075	0.066	0.197	0.283	1.464	0.320	9.703

Problem 969	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	34	39	49	36	70	66	67
N.S.	1	1.00	0.67	0.76	0.96	0.71	1.37	1.29	1.31
time (sec)	N/A	0.015	0.049	0.058	0.197	0.287	1.077	0.280	9.422

Problem 970	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	71	73	58	131	39	80	65
N.S.	1	1.00	1.51	1.55	1.23	2.79	0.83	1.70	1.38
time (sec)	N/A	0.018	0.076	0.058	0.282	0.289	1.156	0.273	9.460

Problem 971	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	80	93	121	144	66	64	94
N.S.	1	1.00	1.31	1.52	1.98	2.36	1.08	1.05	1.54
time (sec)	N/A	0.023	0.127	0.084	0.263	0.290	2.104	0.289	10.069

Problem 972	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	101	112	200	201	150	125	0
N.S.	1	1.00	1.09	1.20	2.15	2.16	1.61	1.34	0.00
time (sec)	N/A	0.033	0.244	0.088	0.277	0.406	4.095	0.294	0.000

Problem 973	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	159	140	215	304	177	144	134
N.S.	1	1.00	1.35	1.19	1.82	2.58	1.50	1.22	1.14
time (sec)	N/A	0.057	0.548	0.114	0.294	0.430	34.409	0.290	10.336

Problem 974	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	139	115	144	249	264	105	90
N.S.	1	1.00	1.62	1.34	1.67	2.90	3.07	1.22	1.05
time (sec)	N/A	0.039	0.328	0.103	0.277	0.510	16.943	0.274	9.820

Problem 975	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	75	75	69	200	75	69	54
N.S.	1	1.00	1.44	1.44	1.33	3.85	1.44	1.33	1.04
time (sec)	N/A	0.026	0.132	0.066	0.289	0.456	4.103	0.272	9.467

Problem 976	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	36	46	46	46	68	66	46
N.S.	1	1.00	0.86	1.10	1.10	1.10	1.62	1.57	1.10
time (sec)	N/A	0.025	0.111	0.091	0.224	0.314	0.479	0.313	8.939

Problem 977	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	60	69	81	73	85	188	66
N.S.	1	1.00	0.88	1.01	1.19	1.07	1.25	2.76	0.97
time (sec)	N/A	0.037	0.167	0.095	0.200	0.288	2.250	0.531	8.977

Problem 978	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	81	94	116	98	114	303	91
N.S.	1	1.00	0.81	0.94	1.16	0.98	1.14	3.03	0.91
time (sec)	N/A	0.048	0.213	0.105	0.202	0.286	2.646	0.696	9.204

Problem 979	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	104	118	151	121	146	414	154
N.S.	1	1.00	0.83	0.94	1.20	0.96	1.16	3.29	1.22
time (sec)	N/A	0.062	0.243	0.116	0.194	0.302	3.056	1.192	9.346

Problem 980	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	80	91	128	95	561	146	79
N.S.	1	1.00	0.72	0.82	1.15	0.86	5.05	1.32	0.71
time (sec)	N/A	0.032	0.096	0.110	0.234	0.289	2.920	0.282	10.138

Problem 981	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	57	66	90	70	267	106	81
N.S.	1	1.00	0.72	0.84	1.14	0.89	3.38	1.34	1.03
time (sec)	N/A	0.019	0.076	0.091	0.190	0.307	2.613	0.272	9.695

Problem 982	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	33	43	53	47	65	62	38
N.S.	1	1.00	0.73	0.96	1.18	1.04	1.44	1.38	0.84
time (sec)	N/A	0.020	0.050	0.086	0.193	0.279	2.456	0.273	9.212

Problem 983	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	71	79	80	195	206	108	60
N.S.	1	1.00	1.20	1.34	1.36	3.31	3.49	1.83	1.02
time (sec)	N/A	0.021	0.107	0.063	0.270	0.257	3.840	0.272	9.449

Problem 984	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	91	127	162	248	262	107	0
N.S.	1	1.00	0.99	1.38	1.76	2.70	2.85	1.16	0.00
time (sec)	N/A	0.032	0.251	0.104	0.278	0.279	6.588	0.307	0.000

Problem 985	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	113	155	243	314	180	148	0
N.S.	1	1.00	0.92	1.26	1.98	2.55	1.46	1.20	0.00
time (sec)	N/A	0.047	0.269	0.119	0.274	0.279	11.330	0.309	0.000

Problem 986	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	101	115	0	0	0	0	0	0
N.S.	1	0.96	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.080	0.335	0.000	0.000	0.000	0.000	0.000	0.000

Problem 987	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	106	0	0	0	0	0	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.058	0.267	0.000	0.000	0.000	0.000	0.000	0.000

Problem 988	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	100	100	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.064	0.258	0.000	0.000	0.000	0.000	0.000	0.000

Problem 989	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	106	0	0	0	0	0	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.064	0.244	0.000	0.000	0.000	0.000	0.000	0.000

Problem 990	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	100	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.048	0.251	0.000	0.000	0.000	0.000	0.000	0.000

Problem 991	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	104	0	0	0	0	0	0
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.055	0.091	0.000	0.000	0.000	0.000	0.000	0.000

Problem 992	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	97	97	95	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.051	0.234	0.000	0.000	0.000	0.000	0.000	0.000

Problem 993	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	82	106	0	0	0	0	0	0
N.S.	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.042	0.263	0.000	0.000	0.000	0.000	0.000	0.000

Problem 994	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	110	0	0	0	0	0	0
N.S.	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.043	0.102	0.000	0.000	0.000	0.000	0.000	0.000

Problem 995	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	106	0	0	0	0	0	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.058	0.351	0.000	0.000	0.000	0.000	0.000	0.000

Problem 996	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	111	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.080	0.489	0.000	0.000	0.000	0.000	0.000	0.000

Problem 997	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	111	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.082	0.486	0.000	0.000	0.000	0.000	0.000	0.000

Problem 998	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	111	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.079	0.427	0.000	0.000	0.000	0.000	0.000	0.000

Problem 999	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	111	0	0	0	0	0	0
N.S.	1	1.00	1.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.081	0.459	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1000	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	111	0	0	0	0	0	0
N.S.	1	1.00	1.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.058	0.535	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1001	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	111	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.077	0.527	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1002	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	99	75	57	62	0	162	831
N.S.	1	1.00	0.73	0.56	0.42	0.46	0.00	1.20	6.16
time (sec)	N/A	0.040	7.236	4.579	0.195	0.309	0.000	0.288	75.080

Problem 1003	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	87	65	47	57	0	127	632
N.S.	1	1.00	0.84	0.62	0.45	0.55	0.00	1.22	6.08
time (sec)	N/A	0.028	1.366	4.599	0.184	0.283	0.000	0.286	45.134

Problem 1004	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	404	52	37	52	0	92	0
N.S.	1	1.00	5.53	0.71	0.51	0.71	0.00	1.26	0.00
time (sec)	N/A	0.021	1.927	4.592	0.189	0.277	0.000	0.286	0.000

Problem 1005	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	264	72	26	46	61	57	41
N.S.	1	1.00	7.14	1.95	0.70	1.24	1.65	1.54	1.11
time (sec)	N/A	0.013	1.492	4.593	0.187	0.284	0.650	0.277	9.381

Problem 1006	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	184	47	27	55	0	0	129
N.S.	1	1.00	2.75	0.70	0.40	0.82	0.00	0.00	1.93
time (sec)	N/A	0.019	1.192	4.574	0.274	0.274	0.000	0.000	10.549

Problem 1007	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	421	23	10	30	0	48	31
N.S.	1	1.00	13.58	0.74	0.32	0.97	0.00	1.55	1.00
time (sec)	N/A	0.006	2.507	4.574	0.276	0.262	0.000	0.299	9.136

Problem 1008	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	36	28	21	37	0	90	43
N.S.	1	1.00	0.57	0.44	0.33	0.59	0.00	1.43	0.68
time (sec)	N/A	0.014	10.024	4.825	0.263	0.271	0.000	0.284	9.296

Problem 1009	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	41	33	31	44	0	111	55
N.S.	1	1.00	0.44	0.35	0.33	0.47	0.00	1.18	0.59
time (sec)	N/A	0.019	10.050	4.608	0.265	0.262	0.000	0.310	9.416

Problem 1010	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	46	38	41	49	0	132	67
N.S.	1	1.00	0.37	0.30	0.33	0.39	0.00	1.06	0.54
time (sec)	N/A	0.028	10.078	4.696	0.281	0.250	0.000	0.319	9.363

Problem 1011	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	88	65	47	57	0	76	632
N.S.	1	1.00	0.85	0.62	0.45	0.55	0.00	0.73	6.08
time (sec)	N/A	0.029	1.359	4.820	0.194	0.258	0.000	0.282	38.441

Problem 1012	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	75	55	37	52	0	59	429
N.S.	1	1.00	1.03	0.75	0.51	0.71	0.00	0.81	5.88
time (sec)	N/A	0.022	1.366	4.748	0.186	0.263	0.000	0.299	26.035

Problem 1013	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	265	41	24	46	0	39	0
N.S.	1	1.00	7.57	1.17	0.69	1.31	0.00	1.11	0.00
time (sec)	N/A	0.014	1.521	4.609	0.195	0.258	0.000	0.276	0.000

Problem 1014	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	38	40	16	27	0	20	6
N.S.	1	1.00	4.75	5.00	2.00	3.38	0.00	2.50	0.75
time (sec)	N/A	0.007	1.034	4.587	0.180	0.259	0.000	0.273	9.438

Problem 1015	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	146	20	10	25	0	25	19
N.S.	1	1.00	5.03	0.69	0.34	0.86	0.00	0.86	0.66
time (sec)	N/A	0.006	1.130	4.576	0.266	0.260	0.000	0.290	9.677

Problem 1016	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	407	25	21	34	0	48	33
N.S.	1	1.00	6.46	0.40	0.33	0.54	0.00	0.76	0.52
time (sec)	N/A	0.014	2.557	4.592	0.269	0.271	0.000	0.281	9.681

Problem 1017	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	41	30	31	39	0	69	43
N.S.	1	1.00	0.44	0.32	0.33	0.41	0.00	0.73	0.46
time (sec)	N/A	0.022	1.327	4.588	0.281	0.319	0.000	0.304	9.710

Problem 1018	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	80	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.030	0.088	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1019	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	70	72	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.021	0.066	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1020	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.019	0.060	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1021	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	73	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.045	0.082	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1022	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	78	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.027	0.069	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1023	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	15	11	10	16	11
N.S.	1	1.00	1.00	0.80	1.00	0.73	0.67	1.07	0.73
time (sec)	N/A	0.010	0.009	4.601	0.186	0.315	0.050	0.280	0.084

Problem 1024	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	110	46	36	40	36	48	31
N.S.	1	1.00	5.00	2.09	1.64	1.82	1.64	2.18	1.41
time (sec)	N/A	0.009	0.272	12.094	0.283	0.307	2.174	0.312	9.198

Problem 1025	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	114	114	88	0	0	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.042	0.295	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1026	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	78	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.027	0.127	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1027	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	78	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.021	0.113	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1028	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	64	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.016	0.033	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1029	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	58	58	69	58	348	0	162
N.S.	1	1.00	0.92	0.92	1.10	0.92	5.52	0.00	2.57
time (sec)	N/A	0.045	0.091	4.883	0.282	0.333	1.335	0.000	9.984

Problem 1030	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	74	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.026	0.122	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1031	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	77	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.027	0.127	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1032	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	175	175	141	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.196	0.437	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1033	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	142	135	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.152	0.218	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1034	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	143	134	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.126	0.217	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1035	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	108	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.088	0.137	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1036	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	107	100	151	224	0	0	0
N.S.	1	1.00	1.06	0.99	1.50	2.22	0.00	0.00	0.00
time (sec)	N/A	0.075	0.216	5.128	0.201	0.303	0.000	0.000	0.000

Problem 1037	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	142	133	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.142	0.238	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1038	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	145	145	136	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.139	0.231	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1039	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	134	188	231	177	347	0	0
N.S.	1	1.00	1.03	1.45	1.78	1.36	2.67	0.00	0.00
time (sec)	N/A	0.094	0.197	5.406	0.213	0.518	3.833	0.000	0.000

Problem 1040	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	87	118	150	108	221	0	0
N.S.	1	1.00	0.97	1.31	1.67	1.20	2.46	0.00	0.00
time (sec)	N/A	0.061	0.137	4.932	0.195	0.446	2.132	0.000	0.000

Problem 1041	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	50	65	83	56	116	0	0
N.S.	1	1.00	0.83	1.08	1.38	0.93	1.93	0.00	0.00
time (sec)	N/A	0.037	0.081	4.688	0.202	0.264	1.258	0.000	0.000

Problem 1042	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	44	59	60	45	0	0	0
N.S.	1	1.00	0.81	1.09	1.11	0.83	0.00	0.00	0.00
time (sec)	N/A	0.041	0.085	5.064	0.205	0.276	0.000	0.000	0.000

Problem 1043	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	107	121	120	0	0	0
N.S.	1	1.00	1.00	1.43	1.61	1.60	0.00	0.00	0.00
time (sec)	N/A	0.043	0.134	5.375	0.203	0.288	0.000	0.000	0.000

Problem 1044	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	100	157	243	267	0	0	0
N.S.	1	1.00	0.95	1.50	2.31	2.54	0.00	0.00	0.00
time (sec)	N/A	0.063	0.191	6.238	0.198	0.308	0.000	0.000	0.000

Problem 1045	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	185	342	286	230	428	0	0
N.S.	1	1.00	1.17	2.16	1.81	1.46	2.71	0.00	0.00
time (sec)	N/A	0.106	0.242	5.720	0.206	0.296	5.482	0.000	0.000

Problem 1046	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	125	157	192	146	277	0	0
N.S.	1	1.00	1.06	1.33	1.63	1.24	2.35	0.00	0.00
time (sec)	N/A	0.076	0.169	4.905	0.216	0.275	3.199	0.000	0.000

Problem 1047	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	76	91	112	82	150	0	0
N.S.	1	1.00	0.88	1.06	1.30	0.95	1.74	0.00	0.00
time (sec)	N/A	0.051	0.112	4.820	0.196	0.276	1.859	0.000	0.000

Problem 1048	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	66	78	81	74	0	0	0
N.S.	1	1.00	0.93	1.10	1.14	1.04	0.00	0.00	0.00
time (sec)	N/A	0.048	0.119	5.027	0.216	0.283	0.000	0.000	0.000

Problem 1049	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	93	125	147	166	0	0	0
N.S.	1	1.00	0.98	1.32	1.55	1.75	0.00	0.00	0.00
time (sec)	N/A	0.060	0.166	5.405	0.212	0.293	0.000	0.000	0.000

Problem 1050	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	94	169	262	301	0	0	0
N.S.	1	1.00	0.78	1.41	2.18	2.51	0.00	0.00	0.00
time (sec)	N/A	0.094	0.277	6.234	0.209	0.278	0.000	0.000	0.000

Problem 1051	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	172	155	154	154	175	13	154
N.S.	1	1.00	12.29	11.07	11.00	11.00	12.50	0.93	11.00
time (sec)	N/A	0.002	0.006	4.543	0.189	0.266	0.043	0.286	0.169

Problem 1052	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	182	157	156	156	182	156	156
N.S.	1	1.00	11.38	9.81	9.75	9.75	11.38	9.75	9.75
time (sec)	N/A	0.034	0.007	4.741	0.195	0.270	0.044	0.285	9.152

Problem 1053	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	186	157	156	156	185	156	156
N.S.	1	1.00	11.62	9.81	9.75	9.75	11.56	9.75	9.75
time (sec)	N/A	0.029	0.006	4.659	0.195	0.254	0.045	0.292	9.182

Problem 1054	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	230	229	189	360	189	229
N.S.	1	1.00	1.00	10.95	10.90	9.00	17.14	9.00	10.90
time (sec)	N/A	0.011	0.061	183.423	0.207	0.267	10.799	0.364	9.550

Problem 1055	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	107	50	16	32	39	70	25
N.S.	1	1.00	8.23	3.85	1.23	2.46	3.00	5.38	1.92
time (sec)	N/A	0.011	0.256	12.568	0.255	0.278	2.239	0.315	9.163

Problem 1056	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	9	9	9	9	10	8	11	8
N.S.	1	1.12	1.12	1.12	1.12	1.25	1.00	1.38	1.00
time (sec)	N/A	0.005	0.007	4.615	0.198	0.247	0.051	0.266	9.027

Problem 1057	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	17	13	12	18	13
N.S.	1	1.00	1.00	0.93	1.13	0.87	0.80	1.20	0.87
time (sec)	N/A	0.013	0.009	4.603	0.193	0.256	0.080	0.274	9.150

Problem 1058	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	17	13	12	15	13
N.S.	1	1.00	1.00	0.93	1.13	0.87	0.80	1.00	0.87
time (sec)	N/A	0.012	0.010	4.626	0.196	0.256	0.087	0.278	9.100

Problem 1059	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	19	17	47	17	29	0	15
N.S.	1	1.00	1.27	1.13	3.13	1.13	1.93	0.00	1.00
time (sec)	N/A	0.012	0.044	4.691	0.194	0.377	0.282	0.000	8.696

Problem 1060	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	81	81	87	13	12
N.S.	1	1.00	1.00	0.93	5.79	5.79	6.21	0.93	0.86
time (sec)	N/A	0.001	0.024	4.626	0.218	0.487	0.428	0.261	11.279

Problem 1061	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	81	81	87	15	14
N.S.	1	1.00	1.00	0.94	5.06	5.06	5.44	0.94	0.88
time (sec)	N/A	0.010	0.034	4.648	0.207	0.389	0.656	0.272	2.276

Problem 1062	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	81	81	87	15	14
N.S.	1	1.00	1.00	0.94	5.06	5.06	5.44	0.94	0.88
time (sec)	N/A	0.009	0.041	4.817	0.212	0.690	0.895	0.283	13.805

Problem 1063	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	23	612	105	129	0	105
N.S.	1	1.00	1.00	1.10	29.14	5.00	6.14	0.00	5.00
time (sec)	N/A	0.010	0.107	14.915	0.247	0.293	41.767	0.000	9.389

Problem 1064	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	46	69	53	46	58	56	37
N.S.	1	1.00	0.88	1.33	1.02	0.88	1.12	1.08	0.71
time (sec)	N/A	0.019	0.086	5.315	0.274	0.273	27.375	0.267	9.000

Problem 1065	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	142	143	0	757	0	0	0
N.S.	1	1.00	1.53	1.54	0.00	8.14	0.00	0.00	0.00
time (sec)	N/A	0.059	10.505	0.154	0.000	0.419	0.000	0.000	0.000

Problem 1066	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	252	252	223	0	0	607	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	2.41	0.00	0.00	0.00
time (sec)	N/A	0.164	1.836	0.000	0.000	0.318	0.000	0.000	0.000

Problem 1067	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	199	199	178	0	0	469	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	2.36	0.00	0.00	0.00
time (sec)	N/A	0.121	1.032	0.000	0.000	0.309	0.000	0.000	0.000

Problem 1068	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	146	146	141	0	0	359	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	2.46	0.00	0.00	0.00
time (sec)	N/A	0.089	0.568	0.000	0.000	0.304	0.000	0.000	0.000

Problem 1069	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	123	0	0	281	0	0	0
N.S.	1	1.00	1.38	0.00	0.00	3.16	0.00	0.00	0.00
time (sec)	N/A	0.059	0.338	0.000	0.000	0.290	0.000	0.000	0.000

Problem 1070	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	122	0	0	408	0	0	0
N.S.	1	1.00	1.34	0.00	0.00	4.48	0.00	0.00	0.00
time (sec)	N/A	0.058	1.056	0.000	0.000	0.350	0.000	0.000	0.000

Problem 1071	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	57	0	0	135	0	0	0
N.S.	1	1.00	0.60	0.00	0.00	1.42	0.00	0.00	0.00
time (sec)	N/A	0.047	0.932	0.000	0.000	0.422	0.000	0.000	0.000

Problem 1072	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	358	358	274	0	0	771	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	2.15	0.00	0.00	0.00
time (sec)	N/A	0.275	2.657	0.000	0.000	0.366	0.000	0.000	0.000

Problem 1073	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	291	291	241	0	0	607	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	2.09	0.00	0.00	0.00
time (sec)	N/A	0.211	1.238	0.000	0.000	0.322	0.000	0.000	0.000

Problem 1074	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	221	221	191	0	0	471	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	2.13	0.00	0.00	0.00
time (sec)	N/A	0.158	0.643	0.000	0.000	0.295	0.000	0.000	0.000

Problem 1075	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	150	150	157	0	0	361	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	2.41	0.00	0.00	0.00
time (sec)	N/A	0.111	0.425	0.000	0.000	0.299	0.000	0.000	0.000

Problem 1076	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	133	133	185	0	0	540	0	0	0
N.S.	1	1.00	1.39	0.00	0.00	4.06	0.00	0.00	0.00
time (sec)	N/A	0.100	0.736	0.000	0.000	0.392	0.000	0.000	0.000

Problem 1077	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	147	147	217	0	0	769	0	0	0
N.S.	1	1.00	1.48	0.00	0.00	5.23	0.00	0.00	0.00
time (sec)	N/A	0.092	1.567	0.000	0.000	0.490	0.000	0.000	0.000

Problem 1078	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	29	25	46	35	22
N.S.	1	1.00	1.00	1.05	1.45	1.25	2.30	1.75	1.10
time (sec)	N/A	0.003	0.052	4.806	0.244	0.436	0.788	0.290	9.116

Problem 1079	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	97	26	35	32	80	52	47
N.S.	1	1.00	3.59	0.96	1.30	1.19	2.96	1.93	1.74
time (sec)	N/A	0.006	0.129	5.363	0.250	0.519	26.646	0.284	8.968

Problem 1080	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	97	26	35	32	0	56	47
N.S.	1	1.00	3.59	0.96	1.30	1.19	0.00	2.07	1.74
time (sec)	N/A	0.007	0.130	6.586	0.252	0.274	0.000	0.279	8.916

Problem 1081	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	101	60	39	35	116	66	54
N.S.	1	1.00	3.74	2.22	1.44	1.30	4.30	2.44	2.00
time (sec)	N/A	0.009	0.197	6.683	0.271	0.252	17.653	0.315	9.078

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [622] had the largest ratio of [.5908999999999999981]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	18	0.111
2	A	2	1	1.00	16	0.062
3	A	2	1	1.00	15	0.067
4	A	3	2	1.00	18	0.111
5	A	2	1	1.00	18	0.056
6	A	2	1	1.00	18	0.056
7	A	3	2	1.00	18	0.111
8	A	2	1	1.00	18	0.056
9	A	2	1	1.00	18	0.056
10	A	3	2	1.00	18	0.111
11	A	3	2	1.00	20	0.100
12	A	2	1	1.00	18	0.056
13	A	2	1	1.00	17	0.059
14	A	4	3	1.00	20	0.150
15	A	2	1	1.00	20	0.050
16	A	2	1	1.00	20	0.050
17	A	3	2	1.00	20	0.100
18	A	2	1	1.00	20	0.050
19	A	2	1	1.00	20	0.050
20	A	3	2	1.00	20	0.100
21	A	2	1	1.00	20	0.050
22	A	2	1	1.00	20	0.050
23	A	2	1	1.00	20	0.050
24	A	3	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	2	1	1.00	20	0.050
26	A	2	1	1.00	20	0.050
27	A	3	2	1.00	20	0.100
28	A	2	1	1.00	20	0.050
29	A	2	1	1.00	20	0.050
30	A	3	2	1.00	20	0.100
31	A	2	1	1.00	18	0.056
32	A	2	1	1.00	17	0.059
33	A	4	3	1.00	20	0.150
34	A	2	1	1.00	20	0.050
35	A	2	1	1.00	20	0.050
36	A	3	2	1.00	20	0.100
37	A	2	1	1.00	20	0.050
38	A	2	1	1.00	20	0.050
39	A	3	2	1.00	20	0.100
40	A	2	1	1.00	20	0.050
41	A	2	1	1.00	20	0.050
42	A	3	2	1.00	20	0.100
43	A	2	1	1.00	20	0.050
44	A	2	1	1.00	20	0.050
45	A	3	2	1.00	20	0.100
46	A	2	1	1.00	20	0.050
47	A	2	1	1.00	20	0.050
48	A	3	2	1.00	20	0.100
49	A	2	1	1.00	20	0.050
50	A	2	1	1.00	20	0.050
51	A	4	3	1.00	20	0.150
52	A	2	1	1.00	20	0.050
53	A	2	1	1.00	20	0.050
54	A	3	3	1.00	20	0.150
55	A	2	1	1.00	20	0.050
56	A	9	8	1.00	20	0.400
57	A	3	2	1.00	20	0.100
58	A	8	8	1.00	20	0.400
59	A	8	8	1.00	20	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	3	2	1.00	20	0.100
61	A	7	7	1.00	18	0.389
62	A	7	7	1.00	17	0.412
63	A	3	2	1.00	20	0.100
64	A	7	7	1.00	20	0.350
65	A	7	7	1.00	20	0.350
66	A	3	2	1.00	20	0.100
67	A	8	8	1.00	20	0.400
68	A	8	8	1.00	20	0.400
69	A	3	2	1.00	20	0.100
70	A	9	8	1.00	20	0.400
71	A	9	8	1.00	20	0.400
72	A	3	2	1.00	20	0.100
73	A	9	8	1.00	20	0.400
74	A	9	8	1.00	20	0.400
75	A	3	2	1.00	20	0.100
76	A	8	8	1.00	20	0.400
77	A	8	8	1.00	20	0.400
78	A	3	2	1.00	20	0.100
79	A	7	7	1.00	18	0.389
80	A	7	7	1.00	17	0.412
81	A	3	2	1.00	20	0.100
82	A	8	8	1.01	20	0.400
83	A	8	8	1.00	20	0.400
84	A	3	2	1.00	20	0.100
85	A	9	8	1.00	20	0.400
86	A	9	8	1.00	20	0.400
87	A	3	2	1.00	20	0.100
88	A	3	2	1.00	20	0.100
89	A	3	2	1.00	20	0.100
90	A	3	2	1.00	20	0.100
91	A	2	2	1.00	20	0.100
92	A	3	2	1.00	20	0.100
93	A	3	2	1.00	20	0.100
94	A	3	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	10	9	1.00	20	0.450
96	A	10	9	1.00	20	0.450
97	A	9	9	1.00	20	0.450
98	A	9	9	1.00	20	0.450
99	A	8	8	1.00	20	0.400
100	A	8	8	1.00	20	0.400
101	A	8	8	1.00	18	0.444
102	A	8	8	1.00	17	0.471
103	A	9	9	1.00	20	0.450
104	A	9	9	1.00	20	0.450
105	A	10	9	1.00	20	0.450
106	A	10	9	1.00	20	0.450
107	A	3	2	1.00	22	0.091
108	A	15	8	1.00	22	0.364
109	A	14	8	1.00	22	0.364
110	A	3	2	1.00	22	0.091
111	A	13	7	1.00	22	0.318
112	A	13	7	1.00	22	0.318
113	A	4	3	1.00	22	0.136
114	A	13	7	1.00	20	0.350
115	A	13	7	1.00	19	0.368
116	A	3	2	1.00	22	0.091
117	A	15	8	1.00	22	0.364
118	A	14	8	1.00	22	0.364
119	A	3	2	1.00	22	0.091
120	A	16	9	1.00	22	0.409
121	A	15	9	1.00	22	0.409
122	A	3	2	1.00	22	0.091
123	A	17	9	1.00	22	0.409
124	A	2	1	1.00	20	0.050
125	A	2	1	1.00	20	0.050
126	A	2	1	1.00	18	0.056
127	A	2	2	1.00	20	0.100
128	A	2	2	1.00	20	0.100
129	A	2	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	3	2	1.00	24	0.083
131	A	2	1	1.00	20	0.050
132	A	2	1	1.00	20	0.050
133	A	2	1	1.00	20	0.050
134	A	2	1	1.00	20	0.050
135	A	2	1	1.00	20	0.050
136	A	2	1	1.00	20	0.050
137	A	2	1	1.00	20	0.050
138	A	2	1	1.00	20	0.050
139	A	2	1	1.00	22	0.045
140	A	2	1	1.00	22	0.045
141	A	2	1	1.00	22	0.045
142	A	2	1	1.00	22	0.045
143	A	2	1	1.00	22	0.045
144	A	2	1	1.00	22	0.045
145	A	2	1	1.00	22	0.045
146	A	2	1	1.00	22	0.045
147	A	2	1	1.00	22	0.045
148	A	2	1	1.00	22	0.045
149	A	2	1	1.00	22	0.045
150	A	2	1	1.00	22	0.045
151	A	2	1	1.00	22	0.045
152	A	2	1	1.00	22	0.045
153	A	2	1	1.00	22	0.045
154	A	2	1	1.00	22	0.045
155	A	5	5	1.00	22	0.227
156	A	13	9	1.00	22	0.409
157	A	12	8	1.00	22	0.364
158	A	4	4	1.00	22	0.182
159	A	12	8	1.00	22	0.364
160	A	12	8	1.00	22	0.364
161	A	4	4	1.00	22	0.182
162	A	12	8	1.00	22	0.364
163	A	5	5	1.00	22	0.227
164	A	13	9	1.00	22	0.409

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	12	8	1.00	22	0.364
166	A	4	4	1.00	22	0.182
167	A	12	8	1.00	22	0.364
168	A	13	9	1.00	22	0.409
169	A	5	5	1.01	22	0.227
170	A	13	9	1.00	22	0.409
171	A	5	5	1.00	22	0.227
172	A	13	9	1.00	22	0.409
173	A	13	9	1.00	22	0.409
174	A	5	5	1.00	22	0.227
175	A	13	9	1.00	22	0.409
176	A	14	10	1.00	22	0.454
177	A	6	6	1.01	22	0.273
178	A	14	10	1.00	22	0.454
179	A	3	2	1.00	22	0.091
180	A	3	2	1.00	22	0.091
181	A	3	2	1.00	22	0.091
182	A	5	5	1.00	22	0.227
183	A	5	5	1.00	22	0.227
184	A	5	5	1.00	22	0.227
185	A	4	4	1.00	22	0.182
186	A	3	3	1.00	19	0.158
187	A	3	3	1.00	22	0.136
188	A	3	3	1.00	22	0.136
189	A	4	4	1.00	22	0.182
190	A	6	6	1.00	22	0.273
191	A	5	5	1.00	20	0.250
192	A	5	5	1.00	22	0.227
193	A	5	5	1.00	22	0.227
194	A	6	6	1.00	22	0.273
195	A	7	6	1.00	22	0.273
196	A	3	2	1.00	22	0.091
197	A	3	2	1.00	22	0.091
198	A	3	2	1.00	22	0.091
199	A	6	5	1.00	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	6	5	1.00	22	0.227
201	A	6	6	1.00	22	0.273
202	A	5	4	1.00	22	0.182
203	A	4	3	1.00	19	0.158
204	A	4	3	1.00	22	0.136
205	A	4	4	1.00	22	0.182
206	A	4	3	1.00	22	0.136
207	A	7	6	1.00	22	0.273
208	A	6	5	1.00	20	0.250
209	A	6	5	1.00	22	0.227
210	A	6	6	1.00	22	0.273
211	A	6	5	1.00	22	0.227
212	A	7	6	1.00	22	0.273
213	A	3	2	1.00	22	0.091
214	A	3	2	1.00	22	0.091
215	A	3	2	1.00	22	0.091
216	A	4	4	1.00	22	0.182
217	A	4	4	1.00	22	0.182
218	A	5	5	1.00	22	0.227
219	A	3	3	1.00	22	0.136
220	A	2	2	1.00	19	0.105
221	A	2	2	1.00	22	0.091
222	A	3	3	1.00	22	0.136
223	A	5	5	1.00	22	0.227
224	A	4	4	1.00	20	0.200
225	A	4	4	1.00	22	0.182
226	A	5	5	1.00	22	0.227
227	A	6	5	1.00	22	0.227
228	A	3	2	1.00	22	0.091
229	A	3	2	1.00	22	0.091
230	A	3	2	1.00	22	0.091
231	A	4	4	1.00	22	0.182
232	A	5	5	1.00	22	0.227
233	A	6	6	1.00	22	0.273
234	A	4	4	1.00	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	3	3	1.00	22	0.136
236	A	2	2	1.00	19	0.105
237	A	3	3	1.00	22	0.136
238	A	4	4	1.00	22	0.182
239	A	5	5	1.00	22	0.227
240	A	4	4	1.00	20	0.200
241	A	5	5	1.00	22	0.227
242	A	6	6	1.00	22	0.273
243	A	7	6	1.00	22	0.273
244	A	3	2	1.00	22	0.091
245	A	3	2	1.00	22	0.091
246	A	3	2	1.00	22	0.091
247	A	5	5	1.00	22	0.227
248	A	6	5	1.00	22	0.227
249	A	4	3	1.00	22	0.136
250	A	3	3	1.00	22	0.136
251	A	3	3	1.00	19	0.158
252	A	4	3	1.00	22	0.136
253	A	5	4	1.00	22	0.182
254	A	6	5	1.00	22	0.227
255	A	5	5	1.00	22	0.227
256	A	5	5	1.00	20	0.250
257	A	6	5	1.00	22	0.227
258	A	7	6	1.00	22	0.273
259	A	6	5	1.00	26	0.192
260	A	5	5	1.00	26	0.192
261	A	4	4	1.00	26	0.154
262	A	6	5	1.00	26	0.192
263	A	7	6	1.00	26	0.231
264	A	7	6	1.00	26	0.231
265	A	5	5	1.00	24	0.208
266	A	7	6	1.00	26	0.231
267	A	2	2	1.00	26	0.077
268	A	2	2	1.00	23	0.087
269	A	2	2	1.00	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
270	A	5	4	1.00	26	0.154
271	A	4	4	1.00	26	0.154
272	A	3	3	1.00	26	0.115
273	A	6	5	1.00	26	0.192
274	A	7	6	1.00	26	0.231
275	A	5	5	1.00	26	0.192
276	A	1	1	1.00	24	0.042
277	A	7	6	1.00	26	0.231
278	A	2	2	1.00	26	0.077
279	A	2	2	1.00	23	0.087
280	A	2	2	1.00	26	0.077
281	A	1	1	1.00	22	0.045
282	A	6	5	1.00	27	0.185
283	A	6	5	1.00	27	0.185
284	A	5	5	1.00	27	0.185
285	A	4	4	1.00	27	0.148
286	A	6	5	1.00	27	0.185
287	A	7	6	1.00	27	0.222
288	A	8	7	1.00	27	0.259
289	A	15	13	1.00	27	0.482
290	A	14	12	1.00	27	0.444
291	A	12	11	1.00	25	0.440
292	A	14	12	1.00	27	0.444
293	A	15	13	1.00	27	0.482
294	A	16	13	1.00	27	0.482
295	A	7	5	1.00	27	0.185
296	A	7	5	1.00	27	0.185
297	A	6	5	1.00	27	0.185
298	A	5	4	1.00	27	0.148
299	A	7	6	1.00	27	0.222
300	A	7	6	1.00	27	0.222
301	A	8	7	1.00	27	0.259
302	A	16	13	1.00	27	0.482
303	A	15	13	1.00	27	0.482
304	A	14	12	1.00	25	0.480

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
305	A	14	12	1.00	27	0.444
306	A	15	13	1.00	27	0.482
307	A	16	13	1.00	27	0.482
308	A	5	4	1.00	27	0.148
309	A	5	4	1.00	27	0.148
310	A	4	4	1.00	27	0.148
311	A	3	3	1.00	27	0.111
312	A	6	5	1.00	27	0.185
313	A	7	6	1.00	27	0.222
314	A	8	7	1.00	27	0.259
315	A	14	12	1.00	27	0.444
316	A	12	11	1.00	27	0.407
317	A	8	7	1.00	25	0.280
318	A	14	12	1.00	27	0.444
319	A	15	13	1.00	27	0.482
320	A	16	13	1.00	27	0.482
321	A	2	2	1.00	27	0.074
322	A	2	2	1.00	24	0.083
323	A	2	2	1.00	27	0.074
324	A	2	2	1.00	27	0.074
325	A	7	5	1.00	27	0.185
326	A	5	4	1.00	27	0.148
327	A	4	4	1.00	27	0.148
328	A	4	4	1.00	27	0.148
329	A	7	6	1.00	27	0.222
330	A	8	7	1.00	27	0.259
331	A	9	8	1.00	27	0.296
332	A	14	12	1.00	27	0.444
333	A	14	12	1.00	27	0.444
334	A	14	12	1.00	25	0.480
335	A	15	13	1.00	27	0.482
336	A	16	13	1.00	27	0.482
337	A	17	13	1.00	27	0.482
338	A	2	2	1.00	27	0.074
339	A	2	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
340	A	2	2	1.00	27	0.074
341	A	2	2	1.00	27	0.074
342	A	5	5	1.00	33	0.152
343	A	5	5	1.00	35	0.143
344	A	5	5	1.00	35	0.143
345	A	5	5	1.00	37	0.135
346	A	5	5	1.00	33	0.152
347	A	5	5	1.00	35	0.143
348	A	5	5	1.00	36	0.139
349	A	5	5	1.00	36	0.139
350	A	1	1	1.00	33	0.030
351	A	1	1	1.00	35	0.029
352	A	1	1	1.00	35	0.029
353	A	1	1	1.00	37	0.027
354	A	1	1	1.00	33	0.030
355	A	1	1	1.00	35	0.029
356	A	1	1	1.00	36	0.028
357	A	1	1	1.00	36	0.028
358	A	6	5	1.00	24	0.208
359	A	5	5	1.00	24	0.208
360	A	4	4	1.00	24	0.167
361	A	6	4	1.00	24	0.167
362	A	7	5	1.00	24	0.208
363	A	2	2	1.00	24	0.083
364	A	2	2	1.00	22	0.091
365	A	2	2	1.00	21	0.095
366	A	2	2	1.00	24	0.083
367	A	2	2	1.00	24	0.083
368	A	7	5	1.00	24	0.208
369	A	6	5	1.00	24	0.208
370	A	5	4	1.00	24	0.167
371	A	7	5	1.00	24	0.208
372	A	7	5	1.00	24	0.208
373	A	2	2	1.00	24	0.083
374	A	2	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
375	A	2	2	1.00	21	0.095
376	A	2	2	1.00	24	0.083
377	A	2	2	1.00	24	0.083
378	A	5	4	1.00	24	0.167
379	A	4	4	1.00	24	0.167
380	A	3	3	1.00	24	0.125
381	A	6	4	1.00	24	0.167
382	A	7	5	1.00	24	0.208
383	A	2	2	1.00	24	0.083
384	A	2	2	1.00	22	0.091
385	A	2	2	1.00	21	0.095
386	A	2	2	1.00	24	0.083
387	A	2	2	1.00	24	0.083
388	A	5	4	1.00	24	0.167
389	A	4	4	1.00	24	0.167
390	A	4	4	1.00	24	0.167
391	A	7	5	1.00	24	0.208
392	A	8	6	1.00	24	0.250
393	A	2	2	1.00	24	0.083
394	A	2	2	1.00	22	0.091
395	A	2	2	1.00	21	0.095
396	A	2	2	1.00	24	0.083
397	A	2	2	1.00	24	0.083
398	A	6	6	1.00	27	0.222
399	A	6	6	1.00	27	0.222
400	A	5	5	1.00	27	0.185
401	A	4	4	1.00	27	0.148
402	A	7	6	1.00	27	0.222
403	A	8	7	1.00	27	0.259
404	A	9	7	1.00	27	0.259
405	A	15	13	1.00	27	0.482
406	A	14	12	1.00	27	0.444
407	A	14	12	1.00	25	0.480
408	A	15	13	1.00	27	0.482
409	A	16	13	1.00	27	0.482

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
410	A	17	13	1.00	27	0.482
411	A	7	7	1.00	27	0.259
412	A	7	6	1.00	27	0.222
413	A	6	5	1.00	27	0.185
414	A	5	5	1.00	27	0.185
415	A	7	6	1.00	27	0.222
416	A	8	7	1.00	27	0.259
417	A	9	7	1.00	27	0.259
418	A	16	14	1.00	27	0.518
419	A	15	13	1.00	27	0.482
420	A	14	12	1.00	25	0.480
421	A	6	6	1.00	27	0.222
422	A	16	13	1.00	27	0.482
423	A	17	13	1.00	27	0.482
424	A	5	5	1.00	27	0.185
425	A	5	5	1.00	27	0.185
426	A	4	4	1.00	27	0.148
427	A	4	4	1.00	27	0.148
428	A	7	6	1.00	27	0.222
429	A	8	7	1.00	27	0.259
430	A	9	7	1.00	27	0.259
431	A	14	12	1.00	27	0.444
432	A	14	12	1.00	27	0.444
433	A	14	12	1.00	25	0.480
434	A	15	13	1.00	27	0.482
435	A	16	13	1.00	27	0.482
436	A	17	13	1.00	27	0.482
437	A	2	2	1.00	27	0.074
438	A	2	2	1.00	27	0.074
439	A	2	2	1.00	24	0.083
440	A	2	2	1.00	27	0.074
441	A	2	2	1.00	27	0.074
442	A	5	5	1.00	27	0.185
443	A	5	5	1.00	27	0.185
444	A	5	5	1.00	27	0.185

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
445	A	5	5	1.00	27	0.185
446	A	8	7	1.00	27	0.259
447	A	9	8	1.00	27	0.296
448	A	10	8	1.00	27	0.296
449	A	15	13	1.00	27	0.482
450	A	15	13	1.00	27	0.482
451	A	15	13	1.00	25	0.520
452	A	16	14	1.00	27	0.518
453	A	17	14	1.00	27	0.518
454	A	18	14	1.00	27	0.518
455	C	2	2	0.26	27	0.074
456	A	2	2	1.00	27	0.074
457	A	2	2	1.00	24	0.083
458	A	2	2	1.00	27	0.074
459	A	2	2	1.00	27	0.074
460	A	6	6	1.00	24	0.250
461	A	5	5	1.00	24	0.208
462	A	4	4	1.00	24	0.167
463	A	7	5	1.00	24	0.208
464	A	8	6	1.00	24	0.250
465	A	2	2	1.00	24	0.083
466	A	2	2	1.00	22	0.091
467	A	2	2	1.00	21	0.095
468	A	2	2	1.00	24	0.083
469	A	2	2	1.00	24	0.083
470	A	7	6	1.00	24	0.250
471	A	6	5	1.00	24	0.208
472	A	5	5	1.00	24	0.208
473	A	7	5	1.00	24	0.208
474	A	8	6	1.00	24	0.250
475	A	2	2	1.00	24	0.083
476	A	2	2	1.00	22	0.091
477	A	2	2	1.00	21	0.095
478	A	2	2	1.00	24	0.083
479	A	2	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
480	A	5	5	1.00	24	0.208
481	A	4	4	1.00	24	0.167
482	A	4	4	1.00	24	0.167
483	A	7	5	1.00	24	0.208
484	A	8	6	1.00	24	0.250
485	A	2	2	1.00	24	0.083
486	A	2	2	1.00	22	0.091
487	A	2	2	1.00	21	0.095
488	A	2	2	1.00	24	0.083
489	A	2	2	1.00	24	0.083
490	A	5	5	0.99	24	0.208
491	A	5	5	1.00	24	0.208
492	A	5	5	1.00	24	0.208
493	A	8	6	1.00	24	0.250
494	A	9	7	1.00	24	0.292
495	A	2	2	1.00	24	0.083
496	A	2	2	1.00	22	0.091
497	A	2	2	1.00	21	0.095
498	A	2	2	1.00	24	0.083
499	A	2	2	1.00	24	0.083
500	A	3	3	1.00	24	0.125
501	A	3	3	1.00	24	0.125
502	A	3	3	1.00	24	0.125
503	A	3	3	1.00	24	0.125
504	A	3	3	1.00	24	0.125
505	A	3	3	1.00	24	0.125
506	A	5	5	1.00	26	0.192
507	A	4	4	1.00	26	0.154
508	A	3	3	1.00	26	0.115
509	A	4	4	1.00	26	0.154
510	A	3	2	1.00	26	0.077
511	A	3	2	1.00	26	0.077
512	A	3	2	1.00	24	0.083
513	A	3	2	1.00	23	0.087
514	A	3	2	1.00	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
515	A	3	2	1.00	26	0.077
516	A	7	7	1.00	26	0.269
517	A	5	5	1.00	26	0.192
518	A	6	6	1.00	26	0.231
519	A	6	6	1.00	26	0.231
520	A	4	4	1.00	26	0.154
521	A	6	6	1.00	26	0.231
522	A	6	6	1.00	26	0.231
523	A	4	4	1.00	26	0.154
524	A	6	6	1.00	24	0.250
525	A	6	6	1.00	24	0.250
526	A	4	4	1.00	24	0.167
527	A	8	7	1.00	26	0.269
528	A	6	5	1.00	26	0.192
529	A	7	6	1.00	26	0.231
530	A	7	6	1.00	26	0.231
531	A	5	4	1.00	26	0.154
532	A	7	6	1.00	26	0.231
533	A	7	6	1.00	26	0.231
534	A	5	4	1.00	26	0.154
535	A	9	7	1.00	26	0.269
536	A	7	5	1.00	26	0.192
537	A	8	6	1.00	26	0.231
538	A	8	6	1.00	26	0.231
539	A	6	4	1.00	26	0.154
540	A	8	6	1.00	26	0.231
541	A	8	6	1.00	26	0.231
542	A	6	4	1.00	26	0.154
543	A	6	6	1.00	26	0.231
544	A	4	4	1.00	26	0.154
545	A	5	5	1.00	26	0.192
546	A	5	5	1.00	26	0.192
547	A	3	3	1.00	26	0.115
548	A	5	5	1.00	26	0.192
549	A	5	5	1.00	26	0.192

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
550	A	3	3	1.00	26	0.115
551	A	6	6	1.00	26	0.231
552	A	4	4	1.00	26	0.154
553	A	5	5	1.00	26	0.192
554	A	5	5	1.00	26	0.192
555	A	3	3	1.00	26	0.115
556	A	6	6	1.00	26	0.231
557	A	2	2	1.00	26	0.077
558	A	4	4	1.00	26	0.154
559	A	6	6	1.00	26	0.231
560	A	4	4	1.00	26	0.154
561	A	6	6	1.00	26	0.231
562	A	2	2	1.00	26	0.077
563	A	4	4	1.00	26	0.154
564	A	7	6	1.00	26	0.231
565	A	3	3	1.00	26	0.115
566	A	5	4	1.00	26	0.154
567	A	8	7	1.00	28	0.250
568	A	8	7	1.00	28	0.250
569	A	7	7	1.00	28	0.250
570	A	6	6	1.00	28	0.214
571	A	10	6	1.00	28	0.214
572	A	13	8	1.00	28	0.286
573	A	12	8	1.00	28	0.286
574	A	6	5	1.00	28	0.179
575	A	5	4	1.00	28	0.143
576	A	3	3	1.00	26	0.115
577	A	3	3	1.00	28	0.107
578	A	4	4	1.00	28	0.143
579	A	5	4	1.00	28	0.143
580	A	6	4	1.00	28	0.143
581	A	22	14	1.00	28	0.500
582	A	21	13	1.00	28	0.464
583	A	14	8	1.00	25	0.320
584	A	21	13	1.00	28	0.464

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
585	A	22	14	1.00	28	0.500
586	A	8	7	1.00	28	0.250
587	A	8	7	1.00	28	0.250
588	A	7	7	1.00	28	0.250
589	A	6	6	1.00	28	0.214
590	A	10	6	1.00	28	0.214
591	A	13	8	1.00	28	0.286
592	A	12	8	1.00	28	0.286
593	A	5	5	1.00	28	0.179
594	A	4	4	1.00	28	0.143
595	A	3	3	1.00	25	0.120
596	A	3	3	1.00	28	0.107
597	A	4	4	1.00	28	0.143
598	A	5	4	1.00	28	0.143
599	A	6	4	1.00	28	0.143
600	A	14	13	1.00	28	0.464
601	A	13	12	1.00	28	0.429
602	A	11	11	1.00	26	0.423
603	A	13	12	1.00	28	0.429
604	A	14	13	1.00	28	0.464
605	A	7	6	1.00	22	0.273
606	A	7	6	1.00	22	0.273
607	A	7	6	1.00	22	0.273
608	A	6	6	1.00	22	0.273
609	A	5	5	1.00	22	0.227
610	A	10	7	1.00	22	0.318
611	A	11	8	1.00	22	0.364
612	A	4	4	1.00	22	0.182
613	A	3	3	1.00	22	0.136
614	A	1	1	1.00	19	0.053
615	A	3	3	1.00	22	0.136
616	A	4	4	1.00	22	0.182
617	A	5	4	1.00	22	0.182
618	A	12	12	1.00	22	0.546
619	A	10	10	1.00	22	0.454

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
620	A	8	8	1.00	20	0.400
621	A	12	11	1.00	22	0.500
622	A	14	13	1.00	22	0.591
623	A	7	6	1.00	22	0.273
624	A	7	6	1.00	22	0.273
625	A	6	6	1.00	22	0.273
626	A	5	5	1.00	22	0.227
627	A	10	7	1.00	22	0.318
628	A	11	8	1.00	22	0.364
629	A	5	4	1.00	22	0.182
630	A	3	3	1.00	22	0.136
631	A	1	1	1.00	20	0.050
632	A	2	2	1.00	22	0.091
633	A	4	4	1.00	22	0.182
634	A	15	9	1.00	22	0.409
635	A	18	11	1.00	22	0.500
636	A	16	10	1.00	19	0.526
637	A	16	10	1.00	22	0.454
638	A	11	9	1.00	22	0.409
639	A	11	9	1.00	22	0.409
640	A	9	8	1.00	22	0.364
641	A	6	6	1.00	22	0.273
642	A	6	6	1.00	22	0.273
643	A	11	8	1.00	22	0.364
644	A	13	9	1.00	22	0.409
645	A	5	5	1.00	22	0.227
646	A	4	4	1.00	22	0.182
647	A	2	2	1.00	22	0.091
648	A	2	2	1.00	19	0.105
649	A	4	4	1.00	22	0.182
650	A	5	4	1.00	22	0.182
651	A	6	4	1.00	22	0.182
652	A	13	12	1.00	22	0.546
653	A	12	11	1.00	22	0.500
654	A	12	11	1.00	22	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
655	A	11	11	1.00	20	0.550
656	A	13	12	1.00	22	0.546
657	A	14	12	1.00	22	0.546
658	A	8	7	1.00	24	0.292
659	A	8	7	1.00	24	0.292
660	A	7	7	1.00	24	0.292
661	A	6	6	1.00	24	0.250
662	A	10	7	1.00	24	0.292
663	A	13	9	1.00	24	0.375
664	A	12	9	1.00	24	0.375
665	A	6	5	1.00	24	0.208
666	A	5	4	1.00	24	0.167
667	A	3	3	1.00	22	0.136
668	A	3	3	1.00	24	0.125
669	A	4	4	1.00	24	0.167
670	A	5	4	1.00	24	0.167
671	A	6	4	1.00	24	0.167
672	A	2	2	1.00	24	0.083
673	A	2	2	1.00	24	0.083
674	A	2	2	1.00	21	0.095
675	A	2	2	1.00	24	0.083
676	A	2	2	1.00	24	0.083
677	A	8	7	1.00	24	0.292
678	A	8	7	1.00	24	0.292
679	A	7	7	1.00	24	0.292
680	A	6	6	1.00	24	0.250
681	A	10	7	1.00	24	0.292
682	A	13	9	1.00	24	0.375
683	A	12	9	1.00	24	0.375
684	A	5	5	1.00	24	0.208
685	A	4	4	1.00	24	0.167
686	A	3	3	1.00	21	0.143
687	A	3	3	1.00	24	0.125
688	A	4	4	1.00	24	0.167
689	A	5	4	1.00	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
690	A	6	4	1.00	24	0.167
691	A	2	2	1.00	24	0.083
692	A	2	2	1.00	24	0.083
693	A	2	2	1.00	22	0.091
694	A	2	2	1.00	24	0.083
695	A	2	2	1.00	24	0.083
696	A	9	7	1.00	24	0.292
697	A	8	7	1.00	24	0.292
698	A	7	6	1.00	24	0.250
699	A	11	8	1.00	24	0.333
700	A	15	9	1.00	24	0.375
701	A	14	10	1.00	24	0.417
702	A	6	5	1.00	24	0.208
703	A	5	4	1.00	22	0.182
704	A	5	4	1.00	24	0.167
705	A	4	4	1.00	24	0.167
706	A	5	4	1.00	24	0.167
707	A	6	4	1.00	24	0.167
708	A	7	4	1.00	24	0.167
709	A	2	2	1.00	24	0.083
710	A	2	2	1.00	24	0.083
711	A	2	2	1.00	21	0.095
712	A	2	2	1.00	24	0.083
713	A	2	2	1.00	24	0.083
714	A	7	6	1.00	24	0.250
715	A	7	6	1.00	24	0.250
716	A	7	6	1.00	24	0.250
717	A	6	6	1.00	24	0.250
718	A	5	5	1.00	24	0.208
719	A	10	7	1.00	24	0.292
720	A	11	8	1.00	24	0.333
721	A	4	4	1.00	24	0.167
722	A	3	3	1.00	24	0.125
723	A	1	1	1.00	21	0.048
724	A	3	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
725	A	4	4	1.00	24	0.167
726	A	5	4	1.00	24	0.167
727	A	2	2	1.00	24	0.083
728	A	2	2	1.00	24	0.083
729	A	2	2	1.00	22	0.091
730	A	2	2	1.00	24	0.083
731	A	2	2	1.00	24	0.083
732	A	7	6	1.00	24	0.250
733	A	7	6	1.00	24	0.250
734	A	6	6	1.00	24	0.250
735	A	5	5	1.00	24	0.208
736	A	10	7	1.00	24	0.292
737	A	11	8	1.00	24	0.333
738	A	5	4	1.00	24	0.167
739	A	3	3	1.00	24	0.125
740	A	1	1	1.00	22	0.045
741	A	3	3	1.00	24	0.125
742	A	4	4	1.00	24	0.167
743	A	2	2	1.00	24	0.083
744	A	2	2	1.00	24	0.083
745	A	2	2	1.00	21	0.095
746	A	2	2	1.00	24	0.083
747	A	11	7	1.00	24	0.292
748	A	9	7	1.00	24	0.292
749	A	7	6	1.00	24	0.250
750	A	6	6	1.00	24	0.250
751	A	6	6	1.00	24	0.250
752	A	11	8	1.00	24	0.333
753	A	13	9	1.00	24	0.375
754	A	5	5	1.00	24	0.208
755	A	4	4	1.00	24	0.167
756	A	3	3	1.00	24	0.125
757	A	2	2	1.00	21	0.095
758	A	4	4	1.00	24	0.167
759	A	5	4	1.00	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
760	A	6	4	1.00	24	0.167
761	A	2	2	1.00	24	0.083
762	A	2	2	1.00	24	0.083
763	A	2	2	1.00	24	0.083
764	A	2	2	1.00	22	0.091
765	A	2	2	1.00	24	0.083
766	A	2	2	1.00	24	0.083
767	A	3	2	1.00	22	0.091
768	A	3	2	1.00	22	0.091
769	A	3	2	1.00	22	0.091
770	A	4	3	1.00	22	0.136
771	A	3	2	1.00	22	0.091
772	A	3	2	1.00	22	0.091
773	A	6	5	1.00	22	0.227
774	A	5	4	1.00	22	0.182
775	A	4	3	1.00	22	0.136
776	A	4	3	1.00	20	0.150
777	A	5	4	1.00	22	0.182
778	A	6	5	1.00	22	0.227
779	A	20	8	1.00	22	0.364
780	A	19	7	1.00	22	0.318
781	A	19	7	1.00	22	0.318
782	A	19	7	1.00	22	0.318
783	A	19	7	1.00	19	0.368
784	A	21	8	1.00	22	0.364
785	A	20	8	1.00	22	0.364
786	A	22	9	1.00	22	0.409
787	A	5	5	1.00	24	0.208
788	A	7	7	1.00	24	0.292
789	A	4	4	1.00	24	0.167
790	A	6	6	1.00	22	0.273
791	A	6	4	1.00	24	0.167
792	A	5	5	1.00	24	0.208
793	A	7	5	1.00	24	0.208
794	A	6	6	1.00	24	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
795	A	13	8	1.25	24	0.333
796	A	10	6	1.29	24	0.250
797	A	11	7	1.29	24	0.292
798	A	9	5	1.30	21	0.238
799	A	13	8	1.27	24	0.333
800	A	10	6	1.27	24	0.250
801	A	3	3	1.00	28	0.107
802	A	3	3	1.00	28	0.107
803	A	3	3	1.00	28	0.107
804	A	3	3	1.00	28	0.107
805	A	5	4	1.00	24	0.167
806	A	4	4	1.00	24	0.167
807	A	3	3	1.00	24	0.125
808	A	6	4	1.00	24	0.167
809	A	7	5	1.00	24	0.208
810	A	7	7	1.00	24	0.292
811	A	6	6	1.00	24	0.250
812	A	3	3	1.00	22	0.136
813	A	5	5	1.00	24	0.208
814	A	6	6	1.00	24	0.250
815	A	10	6	1.00	24	0.250
816	A	9	5	1.31	24	0.208
817	A	7	4	1.16	21	0.190
818	A	10	6	1.28	24	0.250
819	A	11	7	1.22	24	0.292
820	A	7	4	1.15	24	0.167
821	A	13	8	1.21	24	0.333
822	A	5	5	1.00	24	0.208
823	A	5	5	1.00	24	0.208
824	A	4	4	1.00	24	0.167
825	A	4	4	1.00	24	0.167
826	A	7	5	1.00	24	0.208
827	A	8	6	1.00	24	0.250
828	A	8	8	1.00	24	0.333
829	A	7	7	1.00	24	0.292

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
830	A	5	5	1.00	24	0.208
831	A	4	4	1.00	22	0.182
832	A	6	6	1.00	24	0.250
833	A	7	6	1.00	24	0.250
834	A	10	6	1.00	24	0.250
835	A	10	6	1.00	24	0.250
836	A	10	6	1.00	21	0.286
837	A	11	7	1.00	24	0.292
838	A	13	8	1.00	24	0.333
839	A	13	8	1.00	24	0.333
840	A	14	9	1.00	24	0.375
841	A	4	4	0.97	26	0.154
842	A	3	3	0.93	24	0.125
843	A	2	2	1.00	17	0.118
844	A	2	2	1.00	26	0.077
845	A	2	2	1.00	26	0.077
846	A	2	2	1.00	26	0.077
847	A	4	4	1.00	26	0.154
848	A	3	3	1.00	24	0.125
849	A	2	2	1.00	17	0.118
850	A	2	2	1.00	26	0.077
851	A	2	2	1.00	26	0.077
852	A	2	2	1.00	26	0.077
853	A	5	4	1.00	24	0.167
854	A	4	4	1.00	24	0.167
855	A	3	3	1.00	24	0.125
856	A	6	4	1.00	24	0.167
857	A	7	5	1.00	24	0.208
858	A	7	7	1.00	24	0.292
859	A	6	6	1.00	24	0.250
860	A	3	3	1.00	24	0.125
861	A	5	5	1.00	24	0.208
862	A	6	6	1.00	24	0.250
863	A	2	2	1.00	24	0.083
864	A	3	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
865	A	3	3	1.00	22	0.136
866	A	2	2	1.00	21	0.095
867	A	2	2	1.00	24	0.083
868	A	3	3	1.00	24	0.125
869	A	3	3	1.00	24	0.125
870	A	5	5	1.00	24	0.208
871	A	4	4	1.00	24	0.167
872	A	4	4	1.00	24	0.167
873	A	7	5	1.00	24	0.208
874	A	8	6	1.00	24	0.250
875	A	7	7	1.00	24	0.292
876	A	5	5	1.00	24	0.208
877	A	4	4	1.00	24	0.167
878	A	6	6	1.00	24	0.250
879	A	7	6	1.00	24	0.250
880	A	2	2	1.00	24	0.083
881	A	3	3	1.00	24	0.125
882	A	3	3	1.00	22	0.136
883	A	2	2	1.00	21	0.095
884	A	2	2	1.00	24	0.083
885	A	3	3	1.00	24	0.125
886	A	3	3	1.00	24	0.125
887	A	5	4	1.00	24	0.167
888	A	4	4	1.00	24	0.167
889	A	3	3	1.00	24	0.125
890	A	6	4	1.00	24	0.167
891	A	7	5	1.00	24	0.208
892	A	7	7	1.00	24	0.292
893	A	6	6	1.00	24	0.250
894	A	3	3	1.00	24	0.125
895	A	5	5	1.00	24	0.208
896	A	6	6	1.00	24	0.250
897	A	10	6	1.00	24	0.250
898	A	8	5	1.00	22	0.227
899	A	11	7	1.00	24	0.292

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
900	A	12	8	1.00	24	0.333
901	A	8	5	1.00	24	0.208
902	A	14	9	1.00	24	0.375
903	A	2	2	1.00	24	0.083
904	A	2	2	1.00	24	0.083
905	A	2	2	1.00	21	0.095
906	A	2	2	1.00	24	0.083
907	A	2	2	1.00	24	0.083
908	A	5	5	1.00	24	0.208
909	A	4	4	1.00	24	0.167
910	A	4	4	1.00	24	0.167
911	A	7	5	1.00	24	0.208
912	A	8	6	1.00	24	0.250
913	A	7	7	1.00	24	0.292
914	A	5	5	1.00	24	0.208
915	A	4	4	1.00	24	0.167
916	A	6	6	1.00	24	0.250
917	A	7	6	1.00	24	0.250
918	A	11	7	1.00	24	0.292
919	A	11	7	1.00	22	0.318
920	A	12	8	1.00	24	0.333
921	A	14	9	1.00	24	0.375
922	A	14	9	1.00	24	0.375
923	A	15	10	1.00	24	0.417
924	A	2	2	1.00	24	0.083
925	A	2	2	1.00	24	0.083
926	A	2	2	1.00	21	0.095
927	A	2	2	1.00	24	0.083
928	A	2	2	1.00	24	0.083
929	A	6	6	1.00	22	0.273
930	A	5	5	1.00	22	0.227
931	A	5	5	1.00	20	0.250
932	A	5	5	1.00	22	0.227
933	A	3	2	1.00	22	0.091
934	A	3	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
935	A	3	2	1.00	22	0.091
936	A	3	2	1.00	22	0.091
937	A	5	3	1.00	22	0.136
938	A	4	3	1.00	22	0.136
939	A	3	3	1.00	22	0.136
940	A	2	2	1.00	22	0.091
941	A	5	5	1.00	22	0.227
942	A	5	5	1.00	19	0.263
943	A	5	5	1.00	22	0.227
944	A	6	6	1.00	22	0.273
945	A	6	5	1.00	22	0.227
946	A	6	6	1.00	22	0.273
947	A	6	5	1.00	20	0.250
948	A	6	5	1.00	22	0.227
949	A	3	2	1.00	22	0.091
950	A	3	2	1.00	22	0.091
951	A	3	2	1.00	22	0.091
952	A	3	2	1.00	22	0.091
953	A	5	3	1.00	22	0.136
954	A	4	3	1.00	22	0.136
955	A	3	3	1.00	22	0.136
956	A	2	2	1.00	22	0.091
957	A	6	5	1.00	22	0.227
958	A	6	6	1.00	22	0.273
959	A	6	5	1.00	19	0.263
960	A	6	5	1.00	22	0.227
961	A	7	6	1.00	22	0.273
962	A	5	5	1.00	22	0.227
963	A	4	4	1.00	20	0.200
964	A	4	4	1.00	22	0.182
965	A	3	2	1.00	22	0.091
966	A	3	2	1.00	22	0.091
967	A	3	2	1.00	22	0.091
968	A	3	3	1.00	22	0.136
969	A	2	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
970	A	4	4	1.00	19	0.210
971	A	4	4	1.00	22	0.182
972	A	5	5	1.00	22	0.227
973	A	6	6	1.00	22	0.273
974	A	5	5	1.00	20	0.250
975	A	4	4	1.00	22	0.182
976	A	3	2	1.00	22	0.091
977	A	3	2	1.00	22	0.091
978	A	3	2	1.00	22	0.091
979	A	3	2	1.00	22	0.091
980	A	4	4	1.00	22	0.182
981	A	3	3	1.00	22	0.136
982	A	3	3	1.00	19	0.158
983	A	4	4	1.00	22	0.182
984	A	5	5	1.00	22	0.227
985	A	6	6	1.00	22	0.273
986	A	4	3	0.96	24	0.125
987	A	4	3	1.00	22	0.136
988	A	3	3	1.00	22	0.136
989	A	4	3	1.00	22	0.136
990	A	3	3	1.00	20	0.150
991	A	4	3	1.00	19	0.158
992	A	3	3	1.00	22	0.136
993	A	4	3	1.00	22	0.136
994	A	3	3	1.00	22	0.136
995	A	4	3	1.00	22	0.136
996	A	4	3	1.00	26	0.115
997	A	4	3	1.00	26	0.115
998	A	4	3	1.00	26	0.115
999	A	4	3	1.00	26	0.115
1000	A	4	3	1.00	26	0.115
1001	A	4	3	1.00	26	0.115
1002	A	6	4	1.00	28	0.143
1003	A	5	4	1.00	28	0.143
1004	A	4	4	1.00	28	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1005	A	3	3	1.00	28	0.107
1006	A	4	4	1.00	28	0.143
1007	A	1	1	1.00	28	0.036
1008	A	2	2	1.00	28	0.071
1009	A	3	2	1.00	28	0.071
1010	A	4	2	1.00	28	0.071
1011	A	5	3	1.00	28	0.107
1012	A	4	3	1.00	28	0.107
1013	A	3	3	1.00	28	0.107
1014	A	2	2	1.00	28	0.071
1015	A	1	1	1.00	28	0.036
1016	A	2	2	1.00	28	0.071
1017	A	3	2	1.00	28	0.071
1018	A	3	3	1.00	24	0.125
1019	A	3	3	1.00	22	0.136
1020	A	3	3	1.00	21	0.143
1021	A	4	4	1.00	24	0.167
1022	A	3	3	1.00	24	0.125
1023	A	3	2	1.00	18	0.111
1024	A	1	1	1.00	33	0.030
1025	A	3	2	1.00	24	0.083
1026	A	3	2	1.00	22	0.091
1027	A	3	2	1.00	20	0.100
1028	A	3	2	1.00	19	0.105
1029	A	3	2	1.00	22	0.091
1030	A	3	2	1.00	22	0.091
1031	A	3	2	1.00	22	0.091
1032	A	5	3	1.00	24	0.125
1033	A	5	3	1.00	22	0.136
1034	A	5	3	1.00	20	0.150
1035	A	4	3	1.00	19	0.158
1036	A	3	2	1.00	22	0.091
1037	A	5	3	1.00	22	0.136
1038	A	5	3	1.00	22	0.136
1039	A	3	2	1.00	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1040	A	3	2	1.00	26	0.077
1041	A	3	2	1.00	24	0.083
1042	A	3	2	1.00	26	0.077
1043	A	3	2	1.00	26	0.077
1044	A	3	2	1.00	26	0.077
1045	A	3	2	1.00	26	0.077
1046	A	3	2	1.00	26	0.077
1047	A	3	2	1.00	24	0.083
1048	A	3	2	1.00	26	0.077
1049	A	3	2	1.00	26	0.077
1050	A	3	2	1.00	26	0.077
1051	A	1	1	1.00	17	0.059
1052	A	2	2	1.00	21	0.095
1053	A	2	2	1.00	21	0.095
1054	A	2	2	1.00	25	0.080
1055	A	1	1	1.00	31	0.032
1056	A	2	1	1.12	17	0.059
1057	A	3	2	1.00	21	0.095
1058	A	3	2	1.00	21	0.095
1059	A	3	2	1.00	21	0.095
1060	A	1	1	1.00	17	0.059
1061	A	2	2	1.00	21	0.095
1062	A	2	2	1.00	21	0.095
1063	A	2	2	1.00	25	0.080
1064	A	5	5	1.00	22	0.227
1065	A	8	8	1.00	26	0.308
1066	A	8	6	1.00	30	0.200
1067	A	7	6	1.00	30	0.200
1068	A	6	6	1.00	30	0.200
1069	A	5	5	1.00	30	0.167
1070	A	5	5	1.00	30	0.167
1071	A	3	3	1.00	30	0.100
1072	A	9	7	1.00	30	0.233
1073	A	8	7	1.00	30	0.233
1074	A	7	7	1.00	30	0.233

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1075	A	6	6	1.00	30	0.200
1076	A	6	6	1.00	30	0.200
1077	A	6	6	1.00	30	0.200
1078	A	1	1	1.00	17	0.059
1079	A	1	1	1.00	27	0.037
1080	A	1	1	1.00	27	0.037
1081	A	1	1	1.00	27	0.037

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^2(a+bx^3)\left(A+Bx^3\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 33

$$\int x^2\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{1}{3}aAx^3+\frac{1}{6}(Ab+aB)x^6+\frac{1}{9}bBx^9$$

```
[Out] 1/3*a*A*x^3+1/6*(A*b+B*a)*x^6+1/9*b*B*x^9
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {455, 45}

$$\int x^2\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{1}{6}x^6(aB+Ab)+\frac{1}{3}aAx^3+\frac{1}{9}bBx^9$$

```
[In] Int[x^2*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (a*A*x^3)/3 + ((A*b + a*B)*x^6)/6 + (b*B*x^9)/9
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int (a+bx)(A+Bx)\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(aA+(Ab+aB)x+bBx^2\right)\,dx, x, x^3\right)\\ &= \frac{1}{3}aAx^3+\frac{1}{6}(Ab+aB)x^6+\frac{1}{9}bBx^9\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x^2\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{1}{3}aAx^3+\frac{1}{6}(Ab+aB)x^6+\frac{1}{9}bBx^9$$

```
[In] Integrate[x^2*(a + b*x^3)*(A + B*x^3),x]
[Out] (a*A*x^3)/3 + ((A*b + a*B)*x^6)/6 + (b*B*x^9)/9
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{aA\,x^3}{3}+\frac{(Ab+Ba)x^6}{6}+\frac{bB\,x^9}{9}$	28
norman	$\frac{bB\,x^9}{9}+\left(\frac{Ab}{6}+\frac{Ba}{6}\right)x^6+\frac{aA\,x^3}{3}$	29
gosper	$\frac{1}{9}bB\,x^9+\frac{1}{6}x^6Ab+\frac{1}{6}x^6Ba+\frac{1}{3}aA\,x^3$	30
risch	$\frac{1}{9}bB\,x^9+\frac{1}{6}x^6Ab+\frac{1}{6}x^6Ba+\frac{1}{3}aA\,x^3$	30
parallelrisch	$\frac{1}{9}bB\,x^9+\frac{1}{6}x^6Ab+\frac{1}{6}x^6Ba+\frac{1}{3}aA\,x^3$	30

```
[In] int(x^2*(b*x^3+a)*(B*x^3+A),x,method=_RETURNVERBOSE)
[Out] 1/3*a*A*x^3+1/6*(A*b+B*a)*x^6+1/9*b*B*x^9
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x^2(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{1}{9}\,Bbx^9+\frac{1}{6}\,(Ba+Ab)x^6+\frac{1}{3}\,Aax^3$$

```
[In] integrate(x^2*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
[Out] 1/9*B*b*x^9 + 1/6*(B*a + A*b)*x^6 + 1/3*A*a*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x^2(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{Aax^3}{3}+\frac{Bbx^9}{9}+x^6\left(\frac{Ab}{6}+\frac{Ba}{6}\right)$$

```
[In] integrate(x**2*(b*x**3+a)*(B*x**3+A),x)
[Out] A*a*x**3/3 + B*b*x**9/9 + x**6*(A*b/6 + B*a/6)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x^2(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{1}{9}\,Bbx^9+\frac{1}{6}\,(Ba+Ab)x^6+\frac{1}{3}\,Aax^3$$

```
[In] integrate(x^2*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
[Out] 1/9*B*b*x^9 + 1/6*(B*a + A*b)*x^6 + 1/3*A*a*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x^2(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{1}{9}\,Bbx^9+\frac{1}{6}\,Bax^6+\frac{1}{6}\,Abx^6+\frac{1}{3}\,Aax^3$$

```
[In] integrate(x^2*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
[Out] 1/9*B*b*x^9 + 1/6*B*a*x^6 + 1/6*A*b*x^6 + 1/3*A*a*x^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int x^2(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{B\,b\,x^9}{9}+\left(\frac{A\,b}{6}+\frac{B\,a}{6}\right)\,x^6+\frac{A\,a\,x^3}{3}$$

```
[In] int(x^2*(A + B*x^3)*(a + b*x^3),x)
[Out] x^6*((A*b)/6 + (B*a)/6) + (A*a*x^3)/3 + (B*b*x^9)/9
```

3.2 $\int x(a+bx^3)(A+Bx^3)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 33

$$\int x(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{1}{2}aAx^2+\frac{1}{5}(Ab+aB)x^5+\frac{1}{8}bBx^8$$

```
[Out] 1/2*a*A*x^2+1/5*(A*b+B*a)*x^5+1/8*b*B*x^8
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {459}

$$\int x(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{1}{5}x^5(aB+Ab)+\frac{1}{2}aAx^2+\frac{1}{8}bBx^8$$

```
[In] Int[x*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (a*A*x^2)/2 + ((A*b + a*B)*x^5)/5 + (b*B*x^8)/8
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int\left(aAx+(Ab+aB)x^4+bBx^7\right)\,dx\\&=\frac{1}{2}aAx^2+\frac{1}{5}(Ab+aB)x^5+\frac{1}{8}bBx^8\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x(a+bx^3)(A+Bx^3)\,dx=\frac{1}{2}aAx^2+\frac{1}{5}(Ab+aB)x^5+\frac{1}{8}bBx^8$$

```
[In] Integrate[x*(a + b*x^3)*(A + B*x^3),x]
[Out] (a*A*x^2)/2 + ((A*b + a*B)*x^5)/5 + (b*B*x^8)/8
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{aA\,x^2}{2}+\frac{(Ab+Ba)x^5}{5}+\frac{bB\,x^8}{8}$	28
norman	$\frac{bB\,x^8}{8}+\left(\frac{Ab}{5}+\frac{Ba}{5}\right)x^5+\frac{aA\,x^2}{2}$	29
gosper	$\frac{1}{8}bB\,x^8+\frac{1}{5}x^5Ab+\frac{1}{5}aB\,x^5+\frac{1}{2}aA\,x^2$	30
risch	$\frac{1}{8}bB\,x^8+\frac{1}{5}x^5Ab+\frac{1}{5}aB\,x^5+\frac{1}{2}aA\,x^2$	30
parallelrisch	$\frac{1}{8}bB\,x^8+\frac{1}{5}x^5Ab+\frac{1}{5}aB\,x^5+\frac{1}{2}aA\,x^2$	30

```
[In] int(x*(b*x^3+a)*(B*x^3+A),x,method=_RETURNVERBOSE)
[Out] 1/2*a*A*x^2+1/5*(A*b+B*a)*x^5+1/8*b*B*x^8
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x(a+bx^3)(A+Bx^3)\,dx=\frac{1}{8}\,Bbx^8+\frac{1}{5}\,(Ba+Ab)x^5+\frac{1}{2}\,Aax^2$$

```
[In] integrate(x*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
[Out] 1/8*B*b*x^8 + 1/5*(B*a + A*b)*x^5 + 1/2*A*a*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{Aax^2}{2}+\frac{Bbx^8}{8}+x^5\left(\frac{Ab}{5}+\frac{Ba}{5}\right)$$

```
[In] integrate(x*(b*x**3+a)*(B*x**3+A),x)
[Out] A*a*x**2/2 + B*b*x**8/8 + x**5*(A*b/5 + B*a/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{1}{8}\,Bbx^8+\frac{1}{5}\,(Ba+Ab)x^5+\frac{1}{2}\,Aax^2$$

```
[In] integrate(x*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
[Out] 1/8*B*b*x^8 + 1/5*(B*a + A*b)*x^5 + 1/2*A*a*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{1}{8}\,Bbx^8+\frac{1}{5}\,Bax^5+\frac{1}{5}\,Abx^5+\frac{1}{2}\,Aax^2$$

```
[In] integrate(x*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
[Out] 1/8*B*b*x^8 + 1/5*B*a*x^5 + 1/5*A*b*x^5 + 1/2*A*a*x^2
```

Mupad [B] (verification not implemented)

Time = 6.76 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int x(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{B\,b\,x^8}{8}+\left(\frac{A\,b}{5}+\frac{B\,a}{5}\right)\,x^5+\frac{A\,a\,x^2}{2}$$

```
[In] int(x*(A + B*x^3)*(a + b*x^3),x)
[Out] x^5*((A*b)/5 + (B*a)/5) + (A*a*x^2)/2 + (B*b*x^8)/8
```

3.3 $\int (a+bx^3)(A+Bx^3)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int (a+bx^3)(A+Bx^3)\,dx = aAx+\frac{1}{4}(Ab+aB)x^4+\frac{1}{7}bBx^7$$

```
[Out] a*A*x+1/4*(A*b+B*a)*x^4+1/7*b*B*x^7
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {380}

$$\int (a+bx^3)(A+Bx^3)\,dx = \frac{1}{4}x^4(aB+Ab)+aAx+\frac{1}{7}bBx^7$$

```
[In] Int[(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] a*A*x + ((A*b + a*B)*x^4)/4 + (b*B*x^7)/7
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int (aA+(Ab+aB)x^3+bBx^6)\,dx\\ &= aAx+\frac{1}{4}(Ab+aB)x^4+\frac{1}{7}bBx^7\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (a+bx^3)(A+Bx^3)\,dx = aAx+\frac{1}{4}(Ab+aB)x^4+\frac{1}{7}bBx^7$$

```
[In] Integrate[(a + b*x^3)*(A + B*x^3),x]
[Out] a*A*x + ((A*b + a*B)*x^4)/4 + (b*B*x^7)/7
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$aAx+\frac{(Ab+Ba)x^4}{4}+\frac{bB\,x^7}{7}$	25
norman	$\frac{bB\,x^7}{7}+\left(\frac{Ab}{4}+\frac{Ba}{4}\right)x^4+aAx$	26
gosper	$\frac{1}{7}bB\,x^7+\frac{1}{4}x^4Ab+\frac{1}{4}Ba\,x^4+aAx$	27
risch	$\frac{1}{7}bB\,x^7+\frac{1}{4}x^4Ab+\frac{1}{4}Ba\,x^4+aAx$	27
parallelrisch	$\frac{1}{7}bB\,x^7+\frac{1}{4}x^4Ab+\frac{1}{4}Ba\,x^4+aAx$	27

```
[In] int((b*x^3+a)*(B*x^3+A),x,method=_RETURNVERBOSE)
[Out] a*A*x+1/4*(A*b+B*a)*x^4+1/7*b*B*x^7
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int (a+bx^3)(A+Bx^3)\,dx = \frac{1}{7}\,Bbx^7+\frac{1}{4}\,(Ba+Ab)x^4+Aax$$

```
[In] integrate((b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
[Out] 1/7*B*b*x^7 + 1/4*(B*a + A*b)*x^4 + A*a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int (a+bx^3)\left(A+Bx^3\right)\,dx = Aax + \frac{Bbx^7}{7} + x^4\left(\frac{Ab}{4} + \frac{Ba}{4}\right)$$

```
[In] integrate((b*x**3+a)*(B*x**3+A),x)
[Out] A*a*x + B*b*x**7/7 + x**4*(A*b/4 + B*a/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int (a+bx^3)\left(A+Bx^3\right)\,dx = \frac{1}{7}\,Bbx^7 + \frac{1}{4}\,(Ba+Ab)x^4 + Aax$$

```
[In] integrate((b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
[Out] 1/7*B*b*x^7 + 1/4*(B*a + A*b)*x^4 + A*a*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int (a+bx^3)\left(A+Bx^3\right)\,dx = \frac{1}{7}\,Bbx^7 + \frac{1}{4}\,Bax^4 + \frac{1}{4}\,Abx^4 + Aax$$

```
[In] integrate((b*x^3+a)*(B*x^3+A),x, algorithm="giac")
[Out] 1/7*B*b*x^7 + 1/4*B*a*x^4 + 1/4*A*b*x^4 + A*a*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int (a+bx^3)\left(A+Bx^3\right)\,dx = \frac{B\,b\,x^7}{7} + \left(\frac{A\,b}{4} + \frac{B\,a}{4}\right)\,x^4 + A\,a\,x$$

```
[In] int((A + B*x^3)*(a + b*x^3),x)
[Out] x^4*((A*b)/4 + (B*a)/4) + A*a*x + (B*b*x^7)/7
```

3.4 $\int \frac{(a+bx^3)(A+Bx^3)}{x} dx$

Optimal result

Integrand size = 18, antiderivative size = 29

$$\int \frac{(a+bx^3)(A+Bx^3)}{x} dx = \frac{1}{3}(Ab+aB)x^3 + \frac{1}{6}bBx^6 + aA\log(x)$$

```
[Out] 1/3*(A*b+B*a)*x^3+1/6*b*B*x^6+a*A*ln(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {457, 77}

$$\int \frac{(a+bx^3)(A+Bx^3)}{x} dx = \frac{1}{3}x^3(aB+Ab) + aA\log(x) + \frac{1}{6}bBx^6$$

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x,x]
```

```
[Out] ((A*b + a*B)*x^3)/3 + (b*B*x^6)/6 + a*A*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)(A+Bx)}{x}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(Ab+aB+\frac{aA}{x}+bBx\right)\,dx, x, x^3\right)\\ &= \frac{1}{3}(Ab+aB)x^3+\frac{1}{6}bBx^6+aA\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)(A+Bx^3)}{x}\,dx=\frac{1}{3}(Ab+aB)x^3+\frac{1}{6}bBx^6+aA\log(x)$$

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x,x]
[Out] ((A*b + a*B)*x^3)/3 + (b*B*x^6)/6 + a*A*Log[x]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

method	result	size
norman	$\left(\frac{Ab}{3}+\frac{Ba}{3}\right)x^3+\frac{bB\,x^6}{6}+aA\ln\left(x\right)$	27
default	$\frac{bB\,x^6}{6}+\frac{Ab\,x^3}{3}+\frac{Ba\,x^3}{3}+aA\ln\left(x\right)$	28
parallelrisch	$\frac{bB\,x^6}{6}+\frac{Ab\,x^3}{3}+\frac{Ba\,x^3}{3}+aA\ln\left(x\right)$	28
risch	$\frac{bB\,x^6}{6}+\frac{Ab\,x^3}{3}+\frac{Ba\,x^3}{3}+\frac{b\,A^2}{6B}+\frac{Aa}{3}+\frac{B\,a^2}{6b}+aA\ln\left(x\right)$	50

```
[In] int((b*x^3+a)*(B*x^3+A)/x,x,method=_RETURNVERBOSE)
[Out] (1/3*A*b+1/3*B*a)*x^3+1/6*b*B*x^6+a*A*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)(A+Bx^3)}{x}\,dx = \frac{1}{6}\,Bbx^6 + \frac{1}{3}\,(Ba+Ab)x^3 + Aa\log\left(x\right)$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x,x, algorithm="fricas")
[Out] 1/6*B*b*x^6 + 1/3*(B*a + A*b)*x^3 + A*a*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)(A+Bx^3)}{x}\,dx = Aa\log\left(x\right) + \frac{Bbx^6}{6} + x^3\left(\frac{Ab}{3} + \frac{Ba}{3}\right)$$

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x,x)
[Out] A*a*log(x) + B*b*x**6/6 + x**3*(A*b/3 + B*a/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)(A+Bx^3)}{x}\,dx = \frac{1}{6}\,Bbx^6 + \frac{1}{3}\,(Ba+Ab)x^3 + \frac{1}{3}\,Aa\log\left(x^3\right)$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x,x, algorithm="maxima")
[Out] 1/6*B*b*x^6 + 1/3*(B*a + A*b)*x^3 + 1/3*A*a*log(x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)(A+Bx^3)}{x}\,dx = \frac{1}{6}\,Bbx^6 + \frac{1}{3}\,Bax^3 + \frac{1}{3}\,Abx^3 + Aa\log\left(|x|\right)$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x,x, algorithm="giac")
[Out] 1/6*B*b*x^6 + 1/3*B*a*x^3 + 1/3*A*b*x^3 + A*a*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 6.71 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)(A+Bx^3)}{x}\,dx = x^3\left(\frac{A\,b}{3}+\frac{B\,a}{3}\right)+\frac{B\,b\,x^6}{6}+A\,a\,\ln\left(x\right)$$

```
[In] int(((A + B*x^3)*(a + b*x^3))/x,x)
[Out] x^3*((A*b)/3 + (B*a)/3) + (B*b*x^6)/6 + A*a*log(x)
```

3.5 $\int \frac{(a+bx^3)(A+Bx^3)}{x^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^2} dx = -\frac{aA}{x} + \frac{1}{2}(Ab+aB)x^2 + \frac{1}{5}bBx^5$$

```
[Out] -a*A/x+1/2*(A*b+B*a)*x^2+1/5*b*B*x^5
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {459}

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^2} dx = \frac{1}{2}x^2(aB+Ab) - \frac{aA}{x} + \frac{1}{5}bBx^5$$

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^2,x]
```

```
[Out] -((a*A)/x) + ((A*b + a*B)*x^2)/2 + (b*B*x^5)/5
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{aA}{x^2} + (Ab+aB)x + bBx^4\right) dx \\ &= -\frac{aA}{x} + \frac{1}{2}(Ab+aB)x^2 + \frac{1}{5}bBx^5 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^2}\,dx=-\frac{aA}{x}+\frac{1}{2}(Ab+aB)x^2+\frac{1}{5}bBx^5$$

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^2,x]
[Out] -((a*A)/x) + ((A*b + a*B)*x^2)/2 + (b*B*x^5)/5
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{bB\,x^5}{5}+\frac{Ab\,x^2}{2}+\frac{Ba\,x^2}{2}-\frac{aA}{x}$	30
norman	$\frac{\frac{bB\,x^6}{5}+\left(\frac{Ab}{2}+\frac{Ba}{2}\right)x^3-Aa}{x}$	30
risch	$\frac{bB\,x^5}{5}+\frac{Ab\,x^2}{2}+\frac{Ba\,x^2}{2}-\frac{aA}{x}$	30
gosper	$-\frac{-2bB\,x^6-5Ab\,x^3-5Ba\,x^3+10Aa}{10x}$	32
parallelrisch	$\frac{2bB\,x^6+5Ab\,x^3+5Ba\,x^3-10Aa}{10x}$	32

```
[In] int((b*x^3+a)*(B*x^3+A)/x^2,x,method=_RETURNVERBOSE)
[Out] 1/5*b*B*x^5+1/2*A*b*x^2+1/2*B*a*x^2-a*A/x
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^2}\,dx=\frac{2\,Bbx^6+5\,(Ba+Ab)x^3-10\,Aa}{10\,x}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^2,x, algorithm="fricas")
[Out] 1/10*(2*B*b*x^6 + 5*(B*a + A*b)*x^3 - 10*A*a)/x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^2}\,dx = -\frac{Aa}{x} + \frac{Bbx^5}{5} + x^2\left(\frac{Ab}{2} + \frac{Ba}{2}\right)$$

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**2,x)
[Out] -A*a/x + B*b*x**5/5 + x**2*(A*b/2 + B*a/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^2}\,dx = \frac{1}{5}\,Bbx^5 + \frac{1}{2}\,(Ba+Ab)x^2 - \frac{Aa}{x}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^2,x, algorithm="maxima")
[Out] 1/5*B*b*x^5 + 1/2*(B*a + A*b)*x^2 - A*a/x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^2}\,dx = \frac{1}{5}\,Bbx^5 + \frac{1}{2}\,Bax^2 + \frac{1}{2}\,Abx^2 - \frac{Aa}{x}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^2,x, algorithm="giac")
[Out] 1/5*B*b*x^5 + 1/2*B*a*x^2 + 1/2*A*b*x^2 - A*a/x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^2}\,dx = x^2\left(\frac{A\,b}{2} + \frac{B\,a}{2}\right) - \frac{A\,a}{x} + \frac{B\,b\,x^5}{5}$$

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^2,x)
[Out] x^2*((A*b)/2 + (B*a)/2) - (A*a)/x + (B*b*x^5)/5
```

3.6 $\int \frac{(a+bx^3)(A+Bx^3)}{x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 28

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^3}\,dx = -\frac{aA}{2x^2}+(Ab+aB)x+\frac{1}{4}bBx^4$$

```
[Out] -1/2*a*A/x^2+(A*b+B*a)*x+1/4*b*B*x^4
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {459}

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^3}\,dx = x(aB+Ab)-\frac{aA}{2x^2}+\frac{1}{4}bBx^4$$

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^3,x]
```

```
[Out] -1/2*(a*A)/x^2 + (A*b + a*B)*x + (b*B*x^4)/4
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(Ab\left(1+\frac{aB}{Ab}\right)+\frac{aA}{x^3}+bBx^3\right)\,dx \\ &= -\frac{aA}{2x^2}+(Ab+aB)x+\frac{1}{4}bBx^4\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^3}\,dx = -\frac{aA}{2x^2} + (Ab+aB)x + \frac{1}{4}bBx^4$$

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^3,x]
[Out] -1/2*(a*A)/x^2 + (A*b + a*B)*x + (b*B*x^4)/4
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{bB\,x^4}{4}+Abx+Bax-\frac{aA}{2x^2}$	24
risch	$\frac{bB\,x^4}{4}+Abx+Bax-\frac{aA}{2x^2}$	24
norman	$\frac{\frac{bB\,x^6}{4}+(Ab+Ba)x^3-\frac{Aa}{2}}{x^2}$	28
parallelrisch	$\frac{bB\,x^6+4Ab\,x^3+4Ba\,x^3-2Aa}{4x^2}$	31
gosper	$-\frac{-bB\,x^6-4Ab\,x^3-4Ba\,x^3+2Aa}{4x^2}$	32

```
[In] int((b*x^3+a)*(B*x^3+A)/x^3,x,method=_RETURNVERBOSE)
[Out] 1/4*b*B*x^4+A*b*x+B*a*x-1/2*a*A/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^3}\,dx = \frac{Bbx^6 + 4\,(Ba+Ab)x^3 - 2\,Aa}{4\,x^2}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^3,x, algorithm="fricas")
[Out] 1/4*(B*b*x^6 + 4*(B*a + A*b)*x^3 - 2*A*a)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^3}\,dx = -\frac{Aa}{2x^2} + \frac{Bbx^4}{4} + x(Ab+Ba)$$

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**3,x)
[Out] -A*a/(2*x**2) + B*b*x**4/4 + x*(A*b + B*a)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^3}\,dx = \frac{1}{4}\,Bbx^4 + (Ba+Ab)x - \frac{Aa}{2\,x^2}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^3,x, algorithm="maxima")
[Out] 1/4*B*b*x^4 + (B*a + A*b)*x - 1/2*A*a/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^3}\,dx = \frac{1}{4}\,Bbx^4 + Bax + Abx - \frac{Aa}{2\,x^2}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^3,x, algorithm="giac")
[Out] 1/4*B*b*x^4 + B*a*x + A*b*x - 1/2*A*a/x^2
```

Mupad [B] (verification not implemented)

Time = 6.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^3}\,dx = x\,(A\,b+B\,a) - \frac{A\,a}{2\,x^2} + \frac{B\,b\,x^4}{4}$$

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^3,x)
[Out] x*(A*b + B*a) - (A*a)/(2*x^2) + (B*b*x^4)/4
```

3.7 $\int \frac{(a+bx^3)(A+Bx^3)}{x^4} dx$

Optimal result

Integrand size = 18, antiderivative size = 29

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^4} dx = -\frac{aA}{3x^3} + \frac{1}{3}bBx^3 + (Ab+aB)\log(x)$$

```
[Out] -1/3*a*A/x^3+1/3*b*B*x^3+(A*b+B*a)*ln(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {457, 77}

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^4} dx = \log(x)(aB+Ab) - \frac{aA}{3x^3} + \frac{1}{3}bBx^3$$

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^4,x]
```

```
[Out] -1/3*(a*A)/x^3 + (b*B*x^3)/3 + (A*b + a*B)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)(A+Bx)}{x^2}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(bB+\frac{aA}{x^2}+\frac{Ab+aB}{x}\right)\,dx, x, x^3\right)\\ &= -\frac{aA}{3x^3}+\frac{1}{3}bBx^3+(Ab+aB)\log(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^4}\,dx = -\frac{aA}{3x^3}+\frac{1}{3}bBx^3+(Ab+aB)\log(x)$$

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^4,x]
[Out] -1/3*(a*A)/x^3 + (b*B*x^3)/3 + (A*b + a*B)*Log[x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{aA}{3x^3}+\frac{bB\,x^3}{3}+(Ab+Ba)\ln(x)$	26
risch	$-\frac{aA}{3x^3}+\frac{bB\,x^3}{3}+A\ln(x)\,b+aB\ln(x)$	26
norman	$\frac{-\frac{Aa}{3}+\frac{bB\,x^6}{3}}{x^3}+(Ab+Ba)\ln(x)$	28
parallelrisch	$\frac{bB\,x^6+3A\ln(x)x^3b+3B\ln(x)x^3a-Aa}{3x^3}$	35

```
[In] int((b*x^3+a)*(B*x^3+A)/x^4,x,method=_RETURNVERBOSE)
[Out] -1/3*a*A/x^3+1/3*b*B*x^3+(A*b+B*a)*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^4}\,dx = \frac{Bbx^6+3\,(Ba+Ab)x^3\log(x)-Aa}{3\,x^3}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^4,x, algorithm="fricas")
[Out] 1/3*(B*b*x^6 + 3*(B*a + A*b)*x^3*log(x) - A*a)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^4}\,dx = -\frac{Aa}{3x^3}+\frac{Bbx^3}{3}+(Ab+Ba)\log(x)$$

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**4,x)
[Out] -A*a/(3*x**3) + B*b*x**3/3 + (A*b + B*a)*log(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^4}\,dx = \frac{1}{3}\,Bbx^3+\frac{1}{3}\,(Ba+Ab)\log\left(x^3\right)-\frac{Aa}{3\,x^3}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^4,x, algorithm="maxima")
[Out] 1/3*B*b*x^3 + 1/3*(B*a + A*b)*log(x^3) - 1/3*A*a/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^4}\,dx = \frac{1}{3}\,Bbx^3+(Ba+Ab)\log(|x|)-\frac{Bax^3+Abx^3+Aa}{3\,x^3}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^4,x, algorithm="giac")
[Out] 1/3*B*b*x^3 + (B*a + A*b)*log(abs(x)) - 1/3*(B*a*x^3 + A*b*x^3 + A*a)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^4}\,dx = \ln(x)\,(A\,b+B\,a) - \frac{A\,a}{3\,x^3} + \frac{B\,b\,x^3}{3}$$

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^4,x)
[Out] log(x)*(A*b + B*a) - (A*a)/(3*x^3) + (B*b*x^3)/3
```

3.8 $\int \frac{(a+bx^3)(A+Bx^3)}{x^5} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^5} \, dx = -\frac{aA}{4x^4}-\frac{Ab+aB}{x}+\frac{1}{2}bBx^2$$

```
[Out] -1/4*a*A/x^4+(-A*b-B*a)/x+1/2*b*B*x^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {459}

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^5} \, dx = -\frac{aB+Ab}{x}-\frac{aA}{4x^4}+\frac{1}{2}bBx^2$$

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^5,x]
```

```
[Out] -1/4*(a*A)/x^4 - (A*b + a*B)/x + (b*B*x^2)/2
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{aA}{x^5}+\frac{Ab+aB}{x^2}+bBx\right) \, dx \\ &= -\frac{aA}{4x^4}-\frac{Ab+aB}{x}+\frac{1}{2}bBx^2\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^5}\,dx = -\frac{aA}{4x^4}+\frac{-Ab-aB}{x}+\frac{1}{2}bBx^2$$

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^5,x]
```

```
[Out] -1/4*(a*A)/x^4 + (-(A*b) - a*B)/x + (b*B*x^2)/2
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{bB\,x^2}{2}-\frac{Ab+Ba}{x}-\frac{aA}{4x^4}$	28
norman	$\frac{\frac{bB\,x^6}{2}+(-Ab-Ba)x^3-\frac{Aa}{4}}{x^4}$	30
gosper	$-\frac{-2bB\,x^6+4Ab\,x^3+4Ba\,x^3+Aa}{4x^4}$	31
risch	$\frac{bB\,x^2}{2}+\frac{(-Ab-Ba)x^3-\frac{Aa}{4}}{x^4}$	31
parallelrisch	$-\frac{-2bB\,x^6+4Ab\,x^3+4Ba\,x^3+Aa}{4x^4}$	31

```
[In] int((b*x^3+a)*(B*x^3+A)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b*B*x^2-(A*b+B*a)/x-1/4*a*A/x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^5}\,dx = \frac{2\,Bbx^6-4\,(Ba+Ab)x^3-Aa}{4\,x^4}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(2*B*b*x^6 - 4*(B*a + A*b)*x^3 - A*a)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^5}\,dx = \frac{Bbx^2}{2} + \frac{-Aa + x^3(-4Ab - 4Ba)}{4x^4}$$

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**5,x)
[Out] B*b*x**2/2 + (-A*a + x**3*(-4*A*b - 4*B*a))/(4*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^5}\,dx = \frac{1}{2}\,Bbx^2 - \frac{4\,(Ba+Ab)x^3 + Aa}{4\,x^4}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^5,x, algorithm="maxima")
[Out] 1/2*B*b*x^2 - 1/4*(4*(B*a + A*b)*x^3 + A*a)/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^5}\,dx = \frac{1}{2}\,Bbx^2 - \frac{4\,Bax^3 + 4\,Abx^3 + Aa}{4\,x^4}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^5,x, algorithm="giac")
[Out] 1/2*B*b*x^2 - 1/4*(4*B*a*x^3 + 4*A*b*x^3 + A*a)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^5}\,dx = \frac{B\,b\,x^2}{2} - \frac{(A\,b + B\,a)\,x^3 + \frac{A\,a}{4}}{x^4}$$

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^5,x)
[Out] (B*b*x^2)/2 - ((A*a)/4 + x^3*(A*b + B*a))/x^4
```

3.9 $\int \frac{(a+bx^3)(A+Bx^3)}{x^6} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 28

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^6} \, dx = -\frac{aA}{5x^5} - \frac{Ab+aB}{2x^2} + bBx$$

```
[Out] -1/5*a*A/x^5+1/2*(-A*b-B*a)/x^2+b*B*x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{459\}$

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^6} \, dx = -\frac{aB+Ab}{2x^2} - \frac{aA}{5x^5} + bBx$$

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^6,x]
[Out] -1/5*(a*A)/x^5 - (A*b + a*B)/(2*x^2) + b*B*x
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(bB + \frac{aA}{x^6} + \frac{Ab+aB}{x^3}\right) dx \\ &= -\frac{aA}{5x^5} - \frac{Ab+aB}{2x^2} + bBx \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^6}\,dx = -\frac{aA}{5x^5}+\frac{-Ab-aB}{2x^2}+bBx$$

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^6,x]
[Out] -1/5*(a*A)/x^5 + (-(A*b) - a*B)/(2*x^2) + b*B*x
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$bBx-\frac{Ab+Ba}{2x^2}-\frac{aA}{5x^5}$	25
risch	$bBx+\frac{\left(-\frac{Ab}{2}-\frac{Ba}{2}\right)x^3-\frac{Aa}{5}}{x^5}$	28
norman	$\frac{bB\,x^6+\left(-\frac{Ab}{2}-\frac{Ba}{2}\right)x^3-\frac{Aa}{5}}{x^5}$	29
gosper	$-\frac{-10bB\,x^6+5Ab\,x^3+5Ba\,x^3+2Aa}{10x^5}$	32
parallelrisch	$-\frac{-10bB\,x^6+5Ab\,x^3+5Ba\,x^3+2Aa}{10x^5}$	32

```
[In] int((b*x^3+a)*(B*x^3+A)/x^6,x,method=_RETURNVERBOSE)
[Out] b*B*x-1/2*(A*b+B*a)/x^2-1/5*a*A/x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^6}\,dx = \frac{10\,Bbx^6-5\,(Ba+Ab)x^3-2\,Aa}{10\,x^5}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^6,x, algorithm="fricas")
[Out] 1/10*(10*B*b*x^6 - 5*(B*a + A*b)*x^3 - 2*A*a)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^6}\,dx = Bbx + \frac{-2Aa + x^3(-5Ab - 5Ba)}{10x^5}$$

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**6,x)
[Out] B*b*x + (-2*A*a + x**3*(-5*A*b - 5*B*a))/(10*x**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^6}\,dx = Bbx - \frac{5\,(Ba+Ab)x^3 + 2\,Aa}{10\,x^5}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^6,x, algorithm="maxima")
[Out] B*b*x - 1/10*(5*(B*a + A*b)*x^3 + 2*A*a)/x^5
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^6}\,dx = Bbx - \frac{5\,Bax^3 + 5\,Abx^3 + 2\,Aa}{10\,x^5}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^6,x, algorithm="giac")
[Out] B*b*x - 1/10*(5*B*a*x^3 + 5*A*b*x^3 + 2*A*a)/x^5
```

Mupad [B] (verification not implemented)

Time = 6.72 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^6}\,dx = B\,b\,x - \frac{\left(\frac{A\,b}{2} + \frac{B\,a}{2}\right)\,x^3 + \frac{A\,a}{5}}{x^5}$$

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^6,x)
[Out] B*b*x - ((A*a)/5 + x^3*((A*b)/2 + (B*a)/2))/x^5
```

3.10 $\int \frac{(a+bx^3)(A+Bx^3)}{x^7}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 29

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^7}\,dx = -\frac{aA}{6x^6} - \frac{Ab+aB}{3x^3} + bB\log(x)$$

```
[Out] -1/6*a*A/x^6+1/3*(-A*b-B*a)/x^3+b*B*ln(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {457, 77}

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^7}\,dx = -\frac{aB+Ab}{3x^3} - \frac{aA}{6x^6} + bB\log(x)$$

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^7,x]
```

```
[Out] -1/6*(a*A)/x^6 - (A*b + a*B)/(3*x^3) + b*B*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)(A+Bx)}{x^3}\,dx, x, x^3\right) \\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{aA}{x^3}+\frac{Ab+aB}{x^2}+\frac{bB}{x}\right)\,dx, x, x^3\right) \\ &= -\frac{aA}{6x^6}-\frac{Ab+aB}{3x^3}+bB\log(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^7}\,dx = -\frac{aA}{6x^6}+\frac{-Ab-aB}{3x^3}+bB\log(x)$$

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^7,x]
[Out] -1/6*(a*A)/x^6 + (-(A*b) - a*B)/(3*x^3) + b*B*Log[x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

method	result	size
default	$bB\ln(x)-\frac{aA}{6x^6}-\frac{Ab+Ba}{3x^3}$	26
norman	$\frac{\left(-\frac{Ab}{3}-\frac{Ba}{3}\right)x^3-\frac{Aa}{6}}{x^6}+bB\ln(x)$	29
risch	$\frac{\left(-\frac{Ab}{3}-\frac{Ba}{3}\right)x^3-\frac{Aa}{6}}{x^6}+bB\ln(x)$	29
parallelrisch	$-\frac{-6Bb\ln(x)x^6+2Ab\,x^3+2Ba\,x^3+Aa}{6x^6}$	33

```
[In] int((b*x^3+a)*(B*x^3+A)/x^7,x,method=_RETURNVERBOSE)
[Out] b*B*ln(x)-1/6*a*A/x^6-1/3*(A*b+B*a)/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^7}\,dx = \frac{6\,Bbx^6\log(x) - 2\,(Ba+Ab)x^3 - Aa}{6\,x^6}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^7,x, algorithm="fricas")
[Out] 1/6*(6*B*b*x^6*log(x) - 2*(B*a + A*b)*x^3 - A*a)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^7}\,dx = Bb\log(x) + \frac{-Aa + x^3(-2Ab - 2Ba)}{6x^6}$$

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**7,x)
[Out] B*b*log(x) + (-A*a + x**3*(-2*A*b - 2*B*a))/(6*x**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^7}\,dx = \frac{1}{3}\,Bb\log\left(x^3\right) - \frac{2\,(Ba+Ab)x^3 + Aa}{6\,x^6}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^7,x, algorithm="maxima")
[Out] 1/3*B*b*log(x^3) - 1/6*(2*(B*a + A*b)*x^3 + A*a)/x^6
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^7}\,dx = Bb\log(|x|) - \frac{3\,Bbx^6 + 2\,Bax^3 + 2\,Abx^3 + Aa}{6\,x^6}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^7,x, algorithm="giac")
[Out] B*b*log(abs(x)) - 1/6*(3*B*b*x^6 + 2*B*a*x^3 + 2*A*b*x^3 + A*a)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^7}\,dx = B\,b\,\ln(x) - \frac{\left(\frac{A\,b}{3}+\frac{B\,a}{3}\right)x^3+\frac{A\,a}{6}}{x^6}$$

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^7,x)
[Out] B*b*log(x) - ((A*a)/6 + x^3*((A*b)/3 + (B*a)/3))/x^6
```

3.11 $\int x^2(a+bx^3)^2 (A+Bx^3)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 42

$$\int x^2\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{(Ab-aB)\left(a+bx^3\right)^3}{9b^2}+\frac{B\left(a+bx^3\right)^4}{12b^2}$$

```
[Out] 1/9*(A*b-B*a)*(b*x^3+a)^3/b^2+1/12*B*(b*x^3+a)^4/b^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {455, 45}

$$\int x^2\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{\left(a+bx^3\right)^3(Ab-aB)}{9b^2}+\frac{B\left(a+bx^3\right)^4}{12b^2}$$

```
[In] Int[x^2*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] ((A*b - a*B)*(a + b*x^3)^3)/(9*b^2) + (B*(a + b*x^3)^4)/(12*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int (a+bx)^2(A+Bx)\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{(Ab-aB)(a+bx)^2}{b}+\frac{B(a+bx)^3}{b}\right)\,dx, x, x^3\right)\\ &= \frac{(Ab-aB)\left(a+bx^3\right)^3}{9b^2}+\frac{B(a+bx^3)^4}{12b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int x^2\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx = \frac{1}{36}x^3\left(12a^2A+6a(2Ab+aB)x^3+4b(Ab+2aB)x^6+3b^2Bx^9\right)$$

```
[In] Integrate[x^2*(a + b*x^3)^2*(A + B*x^3),x]
[Out] (x^3*(12*a^2*A + 6*a*(2*A*b + a*B)*x^3 + 4*b*(A*b + 2*a*B)*x^6 + 3*b^2*B*x^
9))/36
```

Maple [A] (verified)

Time = 4.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24

method	result	size
default	$\frac{B\,b^2x^{12}}{12}+\frac{(b^2A+2abB)x^9}{9}+\frac{(2abA+a^2B)x^6}{6}+\frac{a^2A\,x^3}{3}$	52
norman	$\frac{B\,b^2x^{12}}{12}+\left(\frac{1}{9}b^2A+\frac{2}{9}abB\right)x^9+\left(\frac{1}{3}abA+\frac{1}{6}a^2B\right)x^6+\frac{a^2A\,x^3}{3}$	52
gosper	$\frac{1}{12}B\,b^2x^{12}+\frac{1}{9}x^9b^2A+\frac{2}{9}x^9abB+\frac{1}{3}x^6abA+\frac{1}{6}x^6a^2B+\frac{1}{3}a^2A\,x^3$	54
risch	$\frac{1}{12}B\,b^2x^{12}+\frac{1}{9}x^9b^2A+\frac{2}{9}x^9abB+\frac{1}{3}x^6abA+\frac{1}{6}x^6a^2B+\frac{1}{3}a^2A\,x^3$	54
parallelrisch	$\frac{1}{12}B\,b^2x^{12}+\frac{1}{9}x^9b^2A+\frac{2}{9}x^9abB+\frac{1}{3}x^6abA+\frac{1}{6}x^6a^2B+\frac{1}{3}a^2A\,x^3$	54

```
[In] int(x^2*(b*x^3+a)^2*(B*x^3+A),x,method=_RETURNVERBOSE)
[Out] 1/12*B*b^2*x^12+1/9*(A*b^2+2*B*a*b)*x^9+1/6*(2*A*a*b+B*a^2)*x^6+1/3*a^2*A*x
^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int x^2(a+bx^3)^2(A+Bx^3)\,dx = \frac{1}{12}Bb^2x^{12}+\frac{1}{9}(2Bab+Ab^2)x^9+\frac{1}{6}(Ba^2+2Aab)x^6+\frac{1}{3}Aa^2x^3$$

```
[In] integrate(x^2*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
[Out] 1/12*B*b^2*x^12 + 1/9*(2*B*a*b + A*b^2)*x^9 + 1/6*(B*a^2 + 2*A*a*b)*x^6 + 1
/3*A*a^2*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.29

$$\int x^2(a+bx^3)^2(A+Bx^3)\,dx = \frac{Aa^2x^3}{3}+\frac{Bb^2x^{12}}{12}+x^9\left(\frac{Ab^2}{9}+\frac{2Bab}{9}\right)+x^6\left(\frac{Aab}{3}+\frac{Ba^2}{6}\right)$$

```
[In] integrate(x**2*(b*x**3+a)**2*(B*x**3+A),x)
[Out] A*a**2*x**3/3 + B*b**2*x**12/12 + x**9*(A*b**2/9 + 2*B*a*b/9) + x**6*(A*a*b
/3 + B*a**2/6)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int x^2(a+bx^3)^2(A+Bx^3)\,dx = \frac{1}{12}Bb^2x^{12}+\frac{1}{9}(2Bab+Ab^2)x^9+\frac{1}{6}(Ba^2+2Aab)x^6+\frac{1}{3}Aa^2x^3$$

```
[In] integrate(x^2*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
[Out] 1/12*B*b^2*x^12 + 1/9*(2*B*a*b + A*b^2)*x^9 + 1/6*(B*a^2 + 2*A*a*b)*x^6 + 1
/3*A*a^2*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int x^2(a+bx^3)^2(A+Bx^3)\,dx = \frac{1}{12}Bb^2x^{12} + \frac{2}{9}Babx^9 + \frac{1}{9}Ab^2x^9 + \frac{1}{6}Ba^2x^6 + \frac{1}{3}Aabx^6 + \frac{1}{3}Aa^2x^3$$

```
[In] integrate(x^2*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/12*B*b^2*x^12 + 2/9*B*a*b*x^9 + 1/9*A*b^2*x^9 + 1/6*B*a^2*x^6 + 1/3*A*a*b
*x^6 + 1/3*A*a^2*x^3
```

Mupad [B] (verification not implemented)

Time = 6.78 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int x^2(a+bx^3)^2(A+Bx^3)\,dx = x^6\left(\frac{B\,a^2}{6}+\frac{A\,b\,a}{3}\right)+x^9\left(\frac{A\,b^2}{9}+\frac{2\,B\,a\,b}{9}\right)+\frac{A\,a^2\,x^3}{3}+\frac{B\,b^2\,x^{12}}{12}$$

```
[In] int(x^2*(A + B*x^3)*(a + b*x^3)^2,x)
```

```
[Out] x^6*((B*a^2)/6 + (A*a*b)/3) + x^9*((A*b^2)/9 + (2*B*a*b)/9) + (A*a^2*x^3)/3
 + (B*b^2*x^12)/12
```

3.12 $\int x(a+bx^3)^2\left(A+Bx^3\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 55

$$\int x\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{1}{2}a^2Ax^2+\frac{1}{5}a(2Ab+aB)x^5+\frac{1}{8}b(Ab+2aB)x^8+\frac{1}{11}b^2Bx^{11}$$

```
[Out] 1/2*a^2*A*x^2+1/5*a*(2*A*b+B*a)*x^5+1/8*b*(A*b+2*B*a)*x^8+1/11*b^2*B*x^11
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {459}

$$\int x\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{1}{2}a^2Ax^2+\frac{1}{8}bx^8(2aB+Ab)+\frac{1}{5}ax^5(aB+2Ab)+\frac{1}{11}b^2Bx^{11}$$

```
[In] Int[x*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (a^2*A*x^2)/2 + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^8)/8 + (b^2*B*
x^11)/11
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int\left(a^2Ax+a(2Ab+aB)x^4+b(Ab+2aB)x^7+b^2Bx^{10}\right)\,dx\\&=\frac{1}{2}a^2Ax^2+\frac{1}{5}a(2Ab+aB)x^5+\frac{1}{8}b(Ab+2aB)x^8+\frac{1}{11}b^2Bx^{11}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int x(a+bx^3)^2 (A+Bx^3)\, dx = \frac{1}{2}a^2Ax^2+\frac{1}{5}a(2Ab+aB)x^5+\frac{1}{8}b(Ab+2aB)x^8+\frac{1}{11}b^2Bx^{11}$$

```
[In] Integrate[x*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (a^2*A*x^2)/2 + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^8)/8 + (b^2*B*
x^11)/11
```

Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{b^2 B x^{11}}{11}+\frac{(b^2 A+2abB)x^8}{8}+\frac{(2abA+a^2 B)x^5}{5}+\frac{a^2 A x^2}{2}$	52
norman	$\frac{b^2 B x^{11}}{11}+\left(\frac{1}{8}b^2 A+\frac{1}{4}abB\right)x^8+\left(\frac{2}{5}abA+\frac{1}{5}a^2 B\right)x^5+\frac{a^2 A x^2}{2}$	52
gosper	$\frac{1}{11}b^2 B x^{11}+\frac{1}{8}x^8 b^2 A+\frac{1}{4}x^8 abB+\frac{2}{5}x^5 abA+\frac{1}{5}a^2 B x^5+\frac{1}{2}a^2 A x^2$	54
risch	$\frac{1}{11}b^2 B x^{11}+\frac{1}{8}x^8 b^2 A+\frac{1}{4}x^8 abB+\frac{2}{5}x^5 abA+\frac{1}{5}a^2 B x^5+\frac{1}{2}a^2 A x^2$	54
parallelrisch	$\frac{1}{11}b^2 B x^{11}+\frac{1}{8}x^8 b^2 A+\frac{1}{4}x^8 abB+\frac{2}{5}x^5 abA+\frac{1}{5}a^2 B x^5+\frac{1}{2}a^2 A x^2$	54

```
[In] int(x*(b*x^3+a)^2*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*b^2*B*x^11+1/8*(A*b^2+2*B*a*b)*x^8+1/5*(2*A*a*b+B*a^2)*x^5+1/2*a^2*A*x
^2
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x(a+bx^3)^2 (A+Bx^3)\, dx = \frac{1}{11}Bb^2x^{11}+\frac{1}{8}(2Bab+Ab^2)x^8+\frac{1}{5}(Ba^2+2Aab)x^5+\frac{1}{2}Aa^2x^2$$

```
[In] integrate(x*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/11*B*b^2*x^11 + 1/8*(2*B*a*b + A*b^2)*x^8 + 1/5*(B*a^2 + 2*A*a*b)*x^5 + 1
/2*A*a^2*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int x(a+bx^3)^2\left(A+Bx^3\right)\,dx=\frac{Aa^2x^2}{2}+\frac{Bb^2x^{11}}{11}+x^8\left(\frac{Ab^2}{8}+\frac{Bab}{4}\right)+x^5\cdot\left(\frac{2Aab}{5}+\frac{Ba^2}{5}\right)$$

```
[In] integrate(x*(b*x**3+a)**2*(B*x**3+A),x)
```

```
[Out] A*a**2*x**2/2 + B*b**2*x**11/11 + x**8*(A*b**2/8 + B*a*b/4) + x**5*(2*A*a*b
/5 + B*a**2/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x(a+bx^3)^2\left(A+Bx^3\right)\,dx=\frac{1}{11}\,Bb^2x^{11}+\frac{1}{8}\,\left(2\,Bab+Ab^2\right)x^8+\frac{1}{5}\,\left(Ba^2+2\,Aab\right)x^5+\frac{1}{2}\,Aa^2x^2$$

```
[In] integrate(x*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/11*B*b^2*x^11 + 1/8*(2*B*a*b + A*b^2)*x^8 + 1/5*(B*a^2 + 2*A*a*b)*x^5 + 1
/2*A*a^2*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int x(a+bx^3)^2\left(A+Bx^3\right)\,dx=\frac{1}{11}\,Bb^2x^{11}+\frac{1}{4}\,Babx^8+\frac{1}{8}\,Ab^2x^8+\frac{1}{5}\,Ba^2x^5+\frac{2}{5}\,Aabx^5+\frac{1}{2}\,Aa^2x^2$$

```
[In] integrate(x*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/11*B*b^2*x^11 + 1/4*B*a*b*x^8 + 1/8*A*b^2*x^8 + 1/5*B*a^2*x^5 + 2/5*A*a*b
*x^5 + 1/2*A*a^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=x^5\left(\frac{B\,a^2}{5}+\frac{2\,A\,b\,a}{5}\right)+x^8\left(\frac{A\,b^2}{8}+\frac{B\,a\,b}{4}\right)+\frac{A\,a^2\,x^2}{2}+\frac{B\,b^2\,x^{11}}{11}$$

```
[In] int(x*(A + B*x^3)*(a + b*x^3)^2,x)
```

```
[Out] x^5*((B*a^2)/5 + (2*A*a*b)/5) + x^8*((A*b^2)/8 + (B*a*b)/4) + (A*a^2*x^2)/2
 + (B*b^2*x^11)/11
```

3.13 $\int (a+bx^3)^2 (A+Bx^3)\, dx$

Optimal result

Integrand size = 17, antiderivative size = 50

$$\int (a+bx^3)^2 (A+Bx^3)\, dx = a^2Ax + \frac{1}{4}a(2Ab+aB)x^4 + \frac{1}{7}b(Ab+2aB)x^7 + \frac{1}{10}b^2Bx^{10}$$

```
[Out] a^2*A*x+1/4*a*(2*A*b+B*a)*x^4+1/7*b*(A*b+2*B*a)*x^7+1/10*b^2*B*x^10
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {380}

$$\int (a+bx^3)^2 (A+Bx^3)\, dx = a^2Ax + \frac{1}{7}bx^7(2aB+Ab) + \frac{1}{4}ax^4(aB+2Ab) + \frac{1}{10}b^2Bx^{10}$$

```
[In] Int[(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] a^2*A*x + (a*(2*A*b + a*B)*x^4)/4 + (b*(A*b + 2*a*B)*x^7)/7 + (b^2*B*x^10)/
10
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^2A + a(2Ab+aB)x^3 + b(Ab+2aB)x^6 + b^2Bx^9\right) dx \\ &= a^2Ax + \frac{1}{4}a(2Ab+aB)x^4 + \frac{1}{7}b(Ab+2aB)x^7 + \frac{1}{10}b^2Bx^{10}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (a+bx^3)^2 (A+Bx^3)\, dx = a^2Ax + \frac{1}{4}a(2Ab+aB)x^4 + \frac{1}{7}b(Ab+2aB)x^7 + \frac{1}{10}b^2Bx^{10}$$

```
[In] Integrate[(a + b*x^3)^2*(A + B*x^3),x]
[Out] a^2*A*x + (a*(2*A*b + a*B)*x^4)/4 + (b*(A*b + 2*a*B)*x^7)/7 + (b^2*B*x^10)/
10
```

Maple [A] (verified)

Time = 4.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{b^2Bx^{10}}{10}+\frac{(b^2A+2abB)x^7}{7}+\frac{(2abA+a^2B)x^4}{4}+a^2Ax$	49
norman	$\frac{b^2Bx^{10}}{10}+\left(\frac{1}{7}b^2A+\frac{2}{7}abB\right)x^7+\left(\frac{1}{2}abA+\frac{1}{4}a^2B\right)x^4+a^2Ax$	49
gosper	$\frac{1}{10}b^2Bx^{10}+\frac{1}{7}x^7b^2A+\frac{2}{7}x^7abB+\frac{1}{2}x^4abA+\frac{1}{4}a^2Bx^4+a^2Ax$	51
risch	$\frac{1}{10}b^2Bx^{10}+\frac{1}{7}x^7b^2A+\frac{2}{7}x^7abB+\frac{1}{2}x^4abA+\frac{1}{4}a^2Bx^4+a^2Ax$	51
parallelrisch	$\frac{1}{10}b^2Bx^{10}+\frac{1}{7}x^7b^2A+\frac{2}{7}x^7abB+\frac{1}{2}x^4abA+\frac{1}{4}a^2Bx^4+a^2Ax$	51

```
[In] int((b*x^3+a)^2*(B*x^3+A),x,method=_RETURNVERBOSE)
[Out] 1/10*b^2*B*x^10+1/7*(A*b^2+2*B*a*b)*x^7+1/4*(2*A*a*b+B*a^2)*x^4+a^2*A*x
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (a+bx^3)^2 (A+Bx^3)\, dx = \frac{1}{10}Bb^2x^{10} + \frac{1}{7}(2Bab+Ab^2)x^7 + \frac{1}{4}(Ba^2+2Aab)x^4 + Aa^2x$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
[Out] 1/10*B*b^2*x^10 + 1/7*(2*B*a*b + A*b^2)*x^7 + 1/4*(B*a^2 + 2*A*a*b)*x^4 + A
*a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int (a + bx^3)^2 (A + Bx^3) \, dx = Aa^2x + \frac{Bb^2x^{10}}{10} + x^7\left(\frac{Ab^2}{7} + \frac{2Bab}{7}\right) + x^4\left(\frac{Aab}{2} + \frac{Ba^2}{4}\right)$$

```
[In] integrate((b*x**3+a)**2*(B*x**3+A),x)
```

```
[Out] A*a**2*x + B*b**2*x**10/10 + x**7*(A*b**2/7 + 2*B*a*b/7) + x**4*(A*a*b/2 +
B*a**2/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (a + bx^3)^2 (A + Bx^3) \, dx = \frac{1}{10} Bb^2x^{10} + \frac{1}{7} (2 Bab + Ab^2)x^7 + \frac{1}{4} (Ba^2 + 2 Aab)x^4 + Aa^2x$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/10*B*b^2*x^10 + 1/7*(2*B*a*b + A*b^2)*x^7 + 1/4*(B*a^2 + 2*A*a*b)*x^4 + A
*a^2*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (a + bx^3)^2 (A + Bx^3) \, dx = \frac{1}{10} Bb^2x^{10} + \frac{2}{7} Babx^7 + \frac{1}{7} Ab^2x^7 + \frac{1}{4} Ba^2x^4 + \frac{1}{2} Aabx^4 + Aa^2x$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/10*B*b^2*x^10 + 2/7*B*a*b*x^7 + 1/7*A*b^2*x^7 + 1/4*B*a^2*x^4 + 1/2*A*a*b
*x^4 + A*a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^3\right)^2 \left(A+Bx^3\right)\,dx = x^4\left(\frac{B\,a^2}{4}+\frac{A\,b\,a}{2}\right)+x^7\left(\frac{A\,b^2}{7}+\frac{2\,B\,a\,b}{7}\right)+\frac{B\,b^2\,x^{10}}{10}+A\,a^2\,x$$

```
[In] int((A + B*x^3)*(a + b*x^3)^2,x)
```

```
[Out] x^4*((B*a^2)/4 + (A*a*b)/2) + x^7*((A*b^2)/7 + (2*B*a*b)/7) + (B*b^2*x^10)/
10 + A*a^2*x
```

3.14 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x} dx$

Optimal result

Integrand size = 20, antiderivative size = 46

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x} dx = \frac{2}{3}aAbx^3 + \frac{1}{6}Ab^2x^6 + \frac{B(a+bx^3)^3}{9b} + a^2A\log(x)$$

```
[Out] 2/3*a*A*b*x^3+1/6*A*b^2*x^6+1/9*B*(b*x^3+a)^3/b+a^2*A*ln(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {457, 81, 45}

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x} dx = a^2A\log(x) + \frac{2}{3}aAbx^3 + \frac{B(a+bx^3)^3}{9b} + \frac{1}{6}Ab^2x^6$$

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x,x]
```

```
[Out] (2*a*A*b*x^3)/3 + (A*b^2*x^6)/6 + (B*(a + b*x^3)^3)/(9*b) + a^2*A*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^2(A+Bx)}{x}\,dx, x, x^3\right) \\
&= \frac{B(a+bx^3)^3}{9b} + \frac{1}{3}A\text{Subst}\left(\int \frac{(a+bx)^2}{x}\,dx, x, x^3\right) \\
&= \frac{B(a+bx^3)^3}{9b} + \frac{1}{3}A\text{Subst}\left(\int \left(2ab+\frac{a^2}{x}+b^2x\right)\,dx, x, x^3\right) \\
&= \frac{2}{3}aAbx^3 + \frac{1}{6}Ab^2x^6 + \frac{B(a+bx^3)^3}{9b} + a^2A\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x}\,dx = \frac{1}{3}a(2Ab+aB)x^3 + \frac{1}{6}b(Ab+2aB)x^6 + \frac{1}{9}b^2Bx^9 + a^2A\log(x)$$

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x,x]
[Out] (a*(2*A*b + a*B)*x^3)/3 + (b*(A*b + 2*a*B)*x^6)/6 + (b^2*B*x^9)/9 + a^2*A*L
og[x]
```

Maple [A] (verified)

Time = 3.93 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

method	result	size
norman	$\left(\frac{1}{6}b^{2}A+\frac{1}{3}abB\right)x^{6}+\left(\frac{2}{3}abA+\frac{1}{3}a^{2}B\right)x^{3}+\frac{b^{2}Bx^{9}}{9}+a^{2}A\ln\left(x\right)$	50
default	$\frac{b^{2}Bx^{9}}{9}+\frac{Ab^{2}x^{6}}{6}+\frac{Babx^{6}}{3}+\frac{2aAbx^{3}}{3}+\frac{a^{2}Bx^{3}}{3}+a^{2}A\ln\left(x\right)$	52
risch	$\frac{b^{2}Bx^{9}}{9}+\frac{Ab^{2}x^{6}}{6}+\frac{Babx^{6}}{3}+\frac{2aAbx^{3}}{3}+\frac{a^{2}Bx^{3}}{3}+a^{2}A\ln\left(x\right)$	52
parallelrisch	$\frac{b^{2}Bx^{9}}{9}+\frac{Ab^{2}x^{6}}{6}+\frac{Babx^{6}}{3}+\frac{2aAbx^{3}}{3}+\frac{a^{2}Bx^{3}}{3}+a^{2}A\ln\left(x\right)$	52

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x,x,method=_RETURNVERBOSE)
[Out] (1/6*b^2*A+1/3*a*b*B)*x^6+(2/3*a*b*A+1/3*a^2*B)*x^3+1/9*b^2*B*x^9+a^2*A*ln(
x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int\frac{(a+bx^{3})^{2}(A+Bx^{3})}{x}\,dx=\frac{1}{9}Bb^{2}x^{9}+\frac{1}{6}\left(2Bab+Ab^{2}\right)x^{6}+\frac{1}{3}\left(Ba^{2}+2Aab\right)x^{3}+Aa^{2}\log\left(x\right)$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x,x, algorithm="fricas")
[Out] 1/9*B*b^2*x^9 + 1/6*(2*B*a*b + A*b^2)*x^6 + 1/3*(B*a^2 + 2*A*a*b)*x^3 + A*a
^2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15

$$\int\frac{(a+bx^{3})^{2}(A+Bx^{3})}{x}\,dx=Aa^{2}\log\left(x\right)+\frac{Bb^{2}x^{9}}{9}+x^{6}\left(\frac{Ab^{2}}{6}+\frac{Bab}{3}\right)+x^{3}\cdot\left(\frac{2Aab}{3}+\frac{Ba^{2}}{3}\right)$$

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x,x)
[Out] A*a**2*log(x) + B*b**2*x**9/9 + x**6*(A*b**2/6 + B*a*b/3) + x**3*(2*A*a*b/3
 + B*a**2/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x}\,dx = \frac{1}{9}\,Bb^2x^9 + \frac{1}{6}\,(2\,Bab + Ab^2)x^6 + \frac{1}{3}\,(Ba^2 + 2\,Aab)x^3 + \frac{1}{3}\,Aa^2\log\left(x^3\right)$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x,x, algorithm="maxima")
```

```
[Out] 1/9*B*b^2*x^9 + 1/6*(2*B*a*b + A*b^2)*x^6 + 1/3*(B*a^2 + 2*A*a*b)*x^3 + 1/3
*A*a^2*log(x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x}\,dx = \frac{1}{9}\,Bb^2x^9 + \frac{1}{3}\,Babx^6 + \frac{1}{6}\,Ab^2x^6 + \frac{1}{3}\,Ba^2x^3 + \frac{2}{3}\,Aabx^3 + Aa^2\log\left(|x|\right)$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x,x, algorithm="giac")
```

```
[Out] 1/9*B*b^2*x^9 + 1/3*B*a*b*x^6 + 1/6*A*b^2*x^6 + 1/3*B*a^2*x^3 + 2/3*A*a*b*x
^3 + A*a^2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x}\,dx = x^3\left(\frac{B\,a^2}{3}+\frac{2\,A\,b\,a}{3}\right)+x^6\left(\frac{A\,b^2}{6}+\frac{B\,a\,b}{3}\right)+\frac{B\,b^2\,x^9}{9}+A\,a^2\,\ln\left(x\right)$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x,x)
```

```
[Out] x^3*((B*a^2)/3 + (2*A*a*b)/3) + x^6*((A*b^2)/6 + (B*a*b)/3) + (B*b^2*x^9)/9
 + A*a^2*log(x)
```

3.15 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^2}\,dx = -\frac{a^2A}{x}+\frac{1}{2}a(2Ab+aB)x^2+\frac{1}{5}b(Ab+2aB)x^5+\frac{1}{8}b^2Bx^8$$

```
[Out] -a^2*A/x+1/2*a*(2*A*b+B*a)*x^2+1/5*b*(A*b+2*B*a)*x^5+1/8*b^2*B*x^8
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^2}\,dx = -\frac{a^2A}{x}+\frac{1}{5}bx^5(2aB+Ab)+\frac{1}{2}ax^2(aB+2Ab)+\frac{1}{8}b^2Bx^8$$

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^2,x]
```

```
[Out] -((a^2*A)/x) + (a*(2*A*b + a*B)*x^2)/2 + (b*(A*b + 2*a*B)*x^5)/5 + (b^2*B*x
^8)/8
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a^2A}{x^2} + a(2Ab+aB)x + b(Ab+2aB)x^4 + b^2Bx^7\right) dx$$
$$= -\frac{a^2A}{x} + \frac{1}{2}a(2Ab+aB)x^2 + \frac{1}{5}b(Ab+2aB)x^5 + \frac{1}{8}b^2Bx^8$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^2} dx = -\frac{a^2A}{x} + \frac{1}{2}a(2Ab+aB)x^2 + \frac{1}{5}b(Ab+2aB)x^5 + \frac{1}{8}b^2Bx^8$$

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^2,x]
[Out] -((a^2*A)/x) + (a*(2*A*b + a*B)*x^2)/2 + (b*(A*b + 2*a*B)*x^5)/5 + (b^2*B*x
^8)/8
```

Maple [A] (verified)

Time = 4.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

method	result	size
norman	$\frac{\frac{b^2Bx^9}{8}+\left(\frac{1}{5}b^2A+\frac{2}{5}abB\right)x^6+\left(abA+\frac{1}{2}a^2B\right)x^3-a^2A}{x}$	52
default	$\frac{b^2Bx^8}{8}+\frac{Ab^2x^5}{5}+\frac{2Babx^5}{5}+aAbx^2+\frac{a^2Bx^2}{2}-\frac{a^2A}{x}$	53
risch	$\frac{b^2Bx^8}{8}+\frac{Ab^2x^5}{5}+\frac{2Babx^5}{5}+aAbx^2+\frac{a^2Bx^2}{2}-\frac{a^2A}{x}$	53
gosper	$-\frac{-5b^2Bx^9-8Ab^2x^6-16Babx^6-40aAbx^3-20a^2Bx^3+40a^2A}{40x}$	56
parallelrisch	$\frac{5b^2Bx^9+8Ab^2x^6+16Babx^6+40aAbx^3+20a^2Bx^3-40a^2A}{40x}$	56

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^2,x,method=_RETURNVERBOSE)
[Out] 1/x*(1/8*b^2*B*x^9+(1/5*b^2*A+2/5*a*b*B)*x^6+(a*b*A+1/2*a^2*B)*x^3-a^2*A)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^2} \, dx = \frac{5 \, Bb^2x^9 + 8 \, (2 \, Bab + Ab^2)x^6 + 20 \, (Ba^2 + 2 \, Aab)x^3 - 40 \, Aa^2}{40 \, x}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/40*(5*B*b^2*x^9 + 8*(2*B*a*b + A*b^2)*x^6 + 20*(B*a^2 + 2*A*a*b)*x^3 - 40
*A*a^2)/x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^2} \, dx = -\frac{Aa^2}{x} + \frac{Bb^2x^8}{8} + x^5 \left(\frac{Ab^2}{5} + \frac{2Bab}{5}\right) + x^2 \left(Aab + \frac{Ba^2}{2}\right)$$

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**2,x)
```

```
[Out] -A*a**2/x + B*b**2*x**8/8 + x**5*(A*b**2/5 + 2*B*a*b/5) + x**2*(A*a*b + B*a
**2/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^2} \, dx = \frac{1}{8} \, Bb^2x^8 + \frac{1}{5} \, (2 \, Bab + Ab^2)x^5 + \frac{1}{2} \, (Ba^2 + 2 \, Aab)x^2 - \frac{Aa^2}{x}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/8*B*b^2*x^8 + 1/5*(2*B*a*b + A*b^2)*x^5 + 1/2*(B*a^2 + 2*A*a*b)*x^2 - A*a
^2/x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^2}\,dx = \frac{1}{8}\,Bb^2x^8 + \frac{2}{5}\,Babx^5 + \frac{1}{5}\,Ab^2x^5 + \frac{1}{2}\,Ba^2x^2 + Aabx^2 - \frac{Aa^2}{x}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/8*B*b^2*x^8 + 2/5*B*a*b*x^5 + 1/5*A*b^2*x^5 + 1/2*B*a^2*x^2 + A*a*b*x^2 -
 A*a^2/x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^2}\,dx = x^2\left(\frac{B\,a^2}{2} + A\,b\,a\right) + x^5\left(\frac{A\,b^2}{5} + \frac{2\,B\,a\,b}{5}\right) - \frac{A\,a^2}{x} + \frac{B\,b^2\,x^8}{8}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^2,x)
```

```
[Out] x^2*((B*a^2)/2 + A*a*b) + x^5*((A*b^2)/5 + (2*B*a*b)/5) - (A*a^2)/x + (B*b^
2*x^8)/8
```

3.16 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^3} dx = -\frac{a^2A}{2x^2} + a(2Ab+aB)x + \frac{1}{4}b(Ab+2aB)x^4 + \frac{1}{7}b^2Bx^7$$

```
[Out] -1/2*a^2*A/x^2+a*(2*A*b+B*a)*x+1/4*b*(A*b+2*B*a)*x^4+1/7*b^2*B*x^7
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^3} dx = -\frac{a^2A}{2x^2} + \frac{1}{4}bx^4(2aB+Ab) + ax(aB+2Ab) + \frac{1}{7}b^2Bx^7$$

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^3,x]
```

```
[Out] -1/2*(a^2*A)/x^2 + a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*x^4)/4 + (b^2*B*x^7
)/7
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a(2Ab+aB)+\frac{a^2A}{x^3}+b(Ab+2aB)x^3+b^2Bx^6\right)\,dx\\ &= -\frac{a^2A}{2x^2}+a(2Ab+aB)x+\frac{1}{4}b(Ab+2aB)x^4+\frac{1}{7}b^2Bx^7\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2\,(A+Bx^3)}{x^3}\,dx = -\frac{a^2A}{2x^2}+a(2Ab+aB)x+\frac{1}{4}b(Ab+2aB)x^4+\frac{1}{7}b^2Bx^7$$

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^3,x]
[Out] -1/2*(a^2*A)/x^2 + a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*x^4)/4 + (b^2*B*x^7
)/7
```

Maple [A] (verified)

Time = 4.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{b^2Bx^7}{7}+\frac{Ab^2x^4}{4}+\frac{Babx^4}{2}+2aAbx+a^2Bx-\frac{a^2A}{2x^2}$	49
risch	$\frac{b^2Bx^7}{7}+\frac{Ab^2x^4}{4}+\frac{Babx^4}{2}+2aAbx+a^2Bx-\frac{a^2A}{2x^2}$	49
norman	$\frac{\frac{b^2Bx^9}{7}+\left(\frac{1}{4}b^2A+\frac{1}{2}abB\right)x^6+\left(2abA+a^2B\right)x^3-\frac{a^2A}{2}}{x^2}$	52
gosper	$-\frac{-4b^2Bx^9-7Ab^2x^6-14Babx^6-56aAbx^3-28a^2Bx^3+14a^2A}{28x^2}$	56
parallelrisch	$\frac{4b^2Bx^9+7Ab^2x^6+14Babx^6+56aAbx^3+28a^2Bx^3-14a^2A}{28x^2}$	56

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^3,x,method=_RETURNVERBOSE)
[Out] 1/7*b^2*B*x^7+1/4*A*b^2*x^4+1/2*B*a*b*x^4+2*a*A*b*x+a^2*B*x-1/2*a^2*A/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^3} \, dx = \frac{4 \, Bb^2x^9 + 7 \, (2 \, Bab + Ab^2)x^6 + 28 \, (Ba^2 + 2 \, Aab)x^3 - 14 \, Aa^2}{28 \, x^2}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/28*(4*B*b^2*x^9 + 7*(2*B*a*b + A*b^2)*x^6 + 28*(B*a^2 + 2*A*a*b)*x^3 - 14
*A*a^2)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^3} \, dx = -\frac{Aa^2}{2x^2} + \frac{Bb^2x^7}{7} + x^4 \left(\frac{Ab^2}{4} + \frac{Bab}{2} \right) + x \left(2Aab + Ba^2 \right)$$

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**3,x)
```

```
[Out] -A*a**2/(2*x**2) + B*b**2*x**7/7 + x**4*(A*b**2/4 + B*a*b/2) + x*(2*A*a*b +
 B*a**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^3} \, dx = \frac{1}{7} \, Bb^2x^7 + \frac{1}{4} \, (2 \, Bab + Ab^2)x^4 + \left(Ba^2 + 2 \, Aab \right)x - \frac{Aa^2}{2 \, x^2}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/7*B*b^2*x^7 + 1/4*(2*B*a*b + A*b^2)*x^4 + (B*a^2 + 2*A*a*b)*x - 1/2*A*a^2
/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^3} dx = \frac{1}{7} Bb^2x^7 + \frac{1}{2} Babx^4 + \frac{1}{4} Ab^2x^4 + Ba^2x + 2 Aabx - \frac{Aa^2}{2 x^2}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/7*B*b^2*x^7 + 1/2*B*a*b*x^4 + 1/4*A*b^2*x^4 + B*a^2*x + 2*A*a*b*x - 1/2*A
*a^2/x^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^3} dx = x^4 \left(\frac{A b^2}{4} + \frac{B a b}{2}\right) + x \left(B a^2 + 2 A b a\right) - \frac{A a^2}{2 x^2} + \frac{B b^2 x^7}{7}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^3,x)
```

```
[Out] x^4*((A*b^2)/4 + (B*a*b)/2) + x*(B*a^2 + 2*A*a*b) - (A*a^2)/(2*x^2) + (B*b^
2*x^7)/7
```

3.17 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 51

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^4}\,dx = -\frac{a^2A}{3x^3}+\frac{1}{3}b(Ab+2aB)x^3+\frac{1}{6}b^2Bx^6+a(2Ab+aB)\log(x)$$

```
[Out] -1/3*a^2*A/x^3+1/3*b*(A*b+2*B*a)*x^3+1/6*b^2*B*x^6+a*(2*A*b+B*a)*ln(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 77}

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^4}\,dx = -\frac{a^2A}{3x^3}+\frac{1}{3}bx^3(2aB+Ab)+a\log(x)(aB+2Ab)+\frac{1}{6}b^2Bx^6$$

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^4,x]
```

```
[Out] -1/3*(a^2*A)/x^3 + (b*(A*b + 2*a*B)*x^3)/3 + (b^2*B*x^6)/6 + a*(2*A*b + a*B
)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^2(A+Bx)}{x^2}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(b(Ab+2aB)+\frac{a^2A}{x^2}+\frac{a(2Ab+aB)}{x}+b^2Bx\right)\,dx, x, x^3\right)\\ &= -\frac{a^2A}{3x^3}+\frac{1}{3}b(Ab+2aB)x^3+\frac{1}{6}b^2Bx^6+a(2Ab+aB)\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^4}\,dx = \frac{1}{6}\left(-\frac{2a^2A}{x^3}+2b(Ab+2aB)x^3+b^2Bx^6+6a(2Ab+aB)\log(x)\right)$$

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^4,x]
[Out] ((-2*a^2*A)/x^3 + 2*b*(A*b + 2*a*B)*x^3 + b^2*B*x^6 + 6*a*(2*A*b + a*B)*Log
[x])/6
```

Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{b^2Bx^6}{6}+\frac{Ab^2x^3}{3}+\frac{2Babx^3}{3}+a(2Ab+Ba)\ln(x)-\frac{a^2A}{3x^3}$	49
norman	$\frac{\left(\frac{1}{3}b^2A+\frac{2}{3}abB\right)x^6-\frac{a^2A}{3}+\frac{b^2Bx^9}{6}}{x^3}+(2abA+a^2B)\ln(x)$	52
parallelrisch	$\frac{b^2Bx^9+2Ab^2x^6+4Babx^6+12A\ln(x)x^3ab+6B\ln(x)x^3a^2-2a^2A}{6x^3}$	59
risch	$\frac{b^2Bx^6}{6}+\frac{Ab^2x^3}{3}+\frac{2Babx^3}{3}+\frac{A^2b^2}{6B}+\frac{2abA}{3}+\frac{2a^2B}{3}-\frac{a^2A}{3x^3}+2A\ln(x)ab+a^2B\ln(x)$	73

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^4,x,method=_RETURNVERBOSE)
[Out] 1/6*b^2*B*x^6+1/3*A*b^2*x^3+2/3*B*a*b*x^3+a*(2*A*b+B*a)*ln(x)-1/3*a^2*A/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^4} \, dx = \frac{Bb^2x^9 + 2\,(2\,Bab + Ab^2)x^6 + 6\,(Ba^2 + 2\,Aab)x^3 \log(x) - 2\,Aa^2}{6\,x^3}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^4,x, algorithm="fricas")
[Out] 1/6*(B*b^2*x^9 + 2*(2*B*a*b + A*b^2)*x^6 + 6*(B*a^2 + 2*A*a*b)*x^3*log(x) -
 2*A*a^2)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^4} \, dx = -\frac{Aa^2}{3x^3} + \frac{Bb^2x^6}{6} + a(2Ab+Ba)\log(x) + x^3\left(\frac{Ab^2}{3} + \frac{2Bab}{3}\right)$$

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**4,x)
[Out] -A*a**2/(3*x**3) + B*b**2*x**6/6 + a*(2*A*b + B*a)*log(x) + x**3*(A*b**2/3
+ 2*B*a*b/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^4} \, dx = \frac{1}{6}\,Bb^2x^6 + \frac{1}{3}\,(2\,Bab + Ab^2)x^3 + \frac{1}{3}\,(Ba^2 + 2\,Aab)\log(x^3) - \frac{Aa^2}{3\,x^3}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^4,x, algorithm="maxima")
[Out] 1/6*B*b^2*x^6 + 1/3*(2*B*a*b + A*b^2)*x^3 + 1/3*(B*a^2 + 2*A*a*b)*log(x^3)
- 1/3*A*a^2/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^4}\,dx = \frac{1}{6}Bb^2x^6 + \frac{2}{3}Babx^3 + \frac{1}{3}Ab^2x^3 + (Ba^2 + 2Aab)\log(|x|) - \frac{Ba^2x^3 + 2Aabx^3 + Aa^2}{3x^3}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/6*B*b^2*x^6 + 2/3*B*a*b*x^3 + 1/3*A*b^2*x^3 + (B*a^2 + 2*A*a*b)*log(abs(x
)) - 1/3*(B*a^2*x^3 + 2*A*a*b*x^3 + A*a^2)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^4}\,dx = x^3\left(\frac{A\,b^2}{3} + \frac{2\,B\,a\,b}{3}\right) + \ln(x)\,\left(B\,a^2 + 2\,A\,b\,a\right) - \frac{A\,a^2}{3\,x^3} + \frac{B\,b^2\,x^6}{6}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^4,x)
```

```
[Out] x^3*((A*b^2)/3 + (2*B*a*b)/3) + log(x)*(B*a^2 + 2*A*a*b) - (A*a^2)/(3*x^3)
+ (B*b^2*x^6)/6
```

3.18 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^5} dx$

Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^5} dx = -\frac{a^2A}{4x^4} - \frac{a(2Ab+aB)}{x} + \frac{1}{2}b(Ab+2aB)x^2 + \frac{1}{5}b^2Bx^5$$

```
[Out] -1/4*a^2*A/x^4-a*(2*A*b+B*a)/x+1/2*b*(A*b+2*B*a)*x^2+1/5*b^2*B*x^5
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^5} dx = -\frac{a^2A}{4x^4} + \frac{1}{2}bx^2(2aB+Ab) - \frac{a(aB+2Ab)}{x} + \frac{1}{5}b^2Bx^5$$

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^5,x]
```

```
[Out] -1/4*(a^2*A)/x^4 - (a*(2*A*b + a*B))/x + (b*(A*b + 2*a*B)*x^2)/2 + (b^2*B*x
^5)/5
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{a^2A}{x^5}+\frac{a(2Ab+aB)}{x^2}+b(Ab+2aB)x+b^2Bx^4\right)\,dx\\ &= -\frac{a^2A}{4x^4}-\frac{a(2Ab+aB)}{x}+\frac{1}{2}b(Ab+2aB)x^2+\frac{1}{5}b^2Bx^5\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2\,(A+Bx^3)}{x^5}\,dx = \frac{-5a^2A-20a(2Ab+aB)x^3+10b(Ab+2aB)x^6+4b^2Bx^9}{20x^4}$$

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^5,x]
[Out] (-5*a^2*A - 20*a*(2*A*b + a*B)*x^3 + 10*b*(A*b + 2*a*B)*x^6 + 4*b^2*B*x^9)/
(20*x^4)
```

Maple [A] (verified)

Time = 4.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{b^2Bx^5}{5}+\frac{Ab^2x^2}{2}+Bab\,x^2-\frac{a(2Ab+Ba)}{x}-\frac{a^2A}{4x^4}$	50
norman	$\frac{\frac{b^2Bx^9}{5}+\left(\frac{1}{2}b^2A+abB\right)x^6+\left(-2abA-a^2B\right)x^3-\frac{a^2A}{4}}{x^4}$	52
risch	$\frac{b^2Bx^5}{5}+\frac{Ab^2x^2}{2}+Bab\,x^2+\frac{\left(-2abA-a^2B\right)x^3-\frac{a^2A}{4}}{x^4}$	54
gosper	$-\frac{-4b^2Bx^9-10Ab^2x^6-20Babx^6+40aAbx^3+20a^2Bx^3+5a^2A}{20x^4}$	56
parallelrisch	$\frac{4b^2Bx^9+10Ab^2x^6+20Babx^6-40aAbx^3-20a^2Bx^3-5a^2A}{20x^4}$	56

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^5,x,method=_RETURNVERBOSE)
[Out] 1/5*b^2*B*x^5+1/2*A*b^2*x^2+B*a*b*x^2-a*(2*A*b+B*a)/x-1/4*a^2*A/x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^5} \, dx = \frac{4 \, Bb^2x^9 + 10 \, (2 \, Bab + Ab^2)x^6 - 20 \, (Ba^2 + 2 \, Aab)x^3 - 5 \, Aa^2}{20 \, x^4}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^5,x, algorithm="fricas")
[Out] 1/20*(4*B*b^2*x^9 + 10*(2*B*a*b + A*b^2)*x^6 - 20*(B*a^2 + 2*A*a*b)*x^3 - 5
*A*a^2)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^5} \, dx = \frac{Bb^2x^5}{5} + x^2\left(\frac{Ab^2}{2} + Bab\right) + \frac{-Aa^2 + x^3(-8Aab - 4Ba^2)}{4x^4}$$

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**5,x)
[Out] B*b**2*x**5/5 + x**2*(A*b**2/2 + B*a*b) + (-A*a**2 + x**3*(-8*A*a*b - 4*B*a
**2))/(4*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^5} \, dx = \frac{1}{5} \, Bb^2x^5 + \frac{1}{2} \, (2 \, Bab + Ab^2)x^2 - \frac{4 \, (Ba^2 + 2 \, Aab)x^3 + Aa^2}{4 \, x^4}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^5,x, algorithm="maxima")
[Out] 1/5*B*b^2*x^5 + 1/2*(2*B*a*b + A*b^2)*x^2 - 1/4*(4*(B*a^2 + 2*A*a*b)*x^3 +
A*a^2)/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^5} dx = \frac{1}{5} Bb^2x^5 + Babx^2 + \frac{1}{2} Ab^2x^2 - \frac{4 Ba^2x^3 + 8 Aabx^3 + Aa^2}{4 x^4}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^5,x, algorithm="giac")
```

```
[Out] 1/5*B*b^2*x^5 + B*a*b*x^2 + 1/2*A*b^2*x^2 - 1/4*(4*B*a^2*x^3 + 8*A*a*b*x^3
+ A*a^2)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^5} dx = x^2 \left(\frac{A b^2}{2} + B a b\right) - \frac{x^3 (B a^2 + 2 A b a) + \frac{A a^2}{4}}{x^4} + \frac{B b^2 x^5}{5}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^5,x)
```

```
[Out] x^2*((A*b^2)/2 + B*a*b) - (x^3*(B*a^2 + 2*A*a*b) + (A*a^2)/4)/x^4 + (B*b^2*
x^5)/5
```

3.19 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^6}\,dx = -\frac{a^2A}{5x^5} - \frac{a(2Ab+aB)}{2x^2} + b(Ab+2aB)x + \frac{1}{4}b^2Bx^4$$

```
[Out] -1/5*a^2*A/x^5-1/2*a*(2*A*b+B*a)/x^2+b*(A*b+2*B*a)*x+1/4*b^2*B*x^4
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^6}\,dx = -\frac{a^2A}{5x^5} - \frac{a(aB+2Ab)}{2x^2} + bx(2aB+Ab) + \frac{1}{4}b^2Bx^4$$

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^6,x]
```

```
[Out] -1/5*(a^2*A)/x^5 - (a*(2*A*b + a*B))/(2*x^2) + b*(A*b + 2*a*B)*x + (b^2*B*x
^4)/4
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(b(Ab+2aB)+\frac{a^2A}{x^6}+\frac{a(2Ab+aB)}{x^3}+b^2Bx^3\right)\,dx\\ &= -\frac{a^2A}{5x^5}-\frac{a(2Ab+aB)}{2x^2}+b(Ab+2aB)x+\frac{1}{4}b^2Bx^4\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2\,(A+Bx^3)}{x^6}\,dx = -\frac{a^2A}{5x^5}-\frac{a(2Ab+aB)}{2x^2}+b(Ab+2aB)x+\frac{1}{4}b^2Bx^4$$

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^6,x]
[Out] -1/5*(a^2*A)/x^5 - (a*(2*A*b + a*B))/(2*x^2) + b*(A*b + 2*a*B)*x + (b^2*B*x
^4)/4
```

Maple [A] (verified)

Time = 3.93 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{b^2Bx^4}{4}+Ab^2x+2Babx-\frac{a(2Ab+Ba)}{2x^2}-\frac{a^2A}{5x^5}$	46
risch	$\frac{b^2Bx^4}{4}+Ab^2x+2Babx+\frac{\left(-abA-\frac{1}{2}a^2B\right)x^3-\frac{a^2A}{5}}{x^5}$	50
norman	$\frac{\frac{b^2Bx^9}{4}+\left(b^2A+2abB\right)x^6+\left(-abA-\frac{1}{2}a^2B\right)x^3-\frac{a^2A}{5}}{x^5}$	52
gosper	$-\frac{-5b^2Bx^9-20Ab^2x^6-40Babx^6+20aAbx^3+10a^2Bx^3+4a^2A}{20x^5}$	56
parallelrisch	$\frac{5b^2Bx^9+20Ab^2x^6+40Babx^6-20aAbx^3-10a^2Bx^3-4a^2A}{20x^5}$	56

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^6,x,method=_RETURNVERBOSE)
[Out] 1/4*b^2*B*x^4+A*b^2*x+2*B*a*b*x-1/2*a*(2*A*b+B*a)/x^2-1/5*a^2*A/x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^6} \, dx = \frac{5 \, Bb^2x^9 + 20 \, (2 \, Bab + Ab^2)x^6 - 10 \, (Ba^2 + 2 \, Aab)x^3 - 4 \, Aa^2}{20 \, x^5}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^6,x, algorithm="fricas")
```

```
[Out] 1/20*(5*B*b^2*x^9 + 20*(2*B*a*b + A*b^2)*x^6 - 10*(B*a^2 + 2*A*a*b)*x^3 - 4
*A*a^2)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^6} \, dx = \frac{Bb^2x^4}{4} + x(Ab^2 + 2Bab) + \frac{-2Aa^2 + x^3(-10Aab - 5Ba^2)}{10x^5}$$

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**6,x)
```

```
[Out] B*b**2*x**4/4 + x*(A*b**2 + 2*B*a*b) + (-2*A*a**2 + x**3*(-10*A*a*b - 5*B*a
**2))/(10*x**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^6} \, dx = \frac{1}{4} \, Bb^2x^4 + \left(2 \, Bab + Ab^2\right)x - \frac{5 \, (Ba^2 + 2 \, Aab)x^3 + 2 \, Aa^2}{10 \, x^5}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^6,x, algorithm="maxima")
```

```
[Out] 1/4*B*b^2*x^4 + (2*B*a*b + A*b^2)*x - 1/10*(5*(B*a^2 + 2*A*a*b)*x^3 + 2*A*a
^2)/x^5
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^6} \, dx = \frac{1}{4} Bb^2x^4 + 2\,Babx + Ab^2x - \frac{5\,Ba^2x^3 + 10\,Aabx^3 + 2\,Aa^2}{10\,x^5}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^6,x, algorithm="giac")
```

```
[Out] 1/4*B*b^2*x^4 + 2*B*a*b*x + A*b^2*x - 1/10*(5*B*a^2*x^3 + 10*A*a*b*x^3 + 2*
A*a^2)/x^5
```

Mupad [B] (verification not implemented)

Time = 6.75 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^6} \, dx = x\left(A\,b^2 + 2\,B\,a\,b\right) - \frac{x^3\left(\frac{B\,a^2}{2} + A\,b\,a\right) + \frac{A\,a^2}{5}}{x^5} + \frac{B\,b^2\,x^4}{4}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^6,x)
```

```
[Out] x*(A*b^2 + 2*B*a*b) - (x^3*((B*a^2)/2 + A*a*b) + (A*a^2)/5)/x^5 + (B*b^2*x^
4)/4
```

3.20 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^7} dx$

Optimal result

Integrand size = 20, antiderivative size = 51

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^7} dx = -\frac{a^2A}{6x^6} - \frac{a(2Ab+aB)}{3x^3} + \frac{1}{3}b^2Bx^3 + b(Ab+2aB)\log(x)$$

```
[Out] -1/6*a^2*A/x^6-1/3*a*(2*A*b+B*a)/x^3+1/3*b^2*B*x^3+b*(A*b+2*B*a)*ln(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 77}

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^7} dx = -\frac{a^2A}{6x^6} - \frac{a(aB+2Ab)}{3x^3} + b\log(x)(2aB+Ab) + \frac{1}{3}b^2Bx^3$$

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^7,x]
```

```
[Out] -1/6*(a^2*A)/x^6 - (a*(2*A*b + a*B))/(3*x^3) + (b^2*B*x^3)/3 + b*(A*b + 2*a
*B)*Log[x]
```

<u>Rule 77</u>

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^2(A+Bx)}{x^3}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(b^2B + \frac{a^2A}{x^3} + \frac{a(2Ab+aB)}{x^2} + \frac{b(Ab+2aB)}{x}\right)\,dx, x, x^3\right)\\ &= -\frac{a^2A}{6x^6} - \frac{a(2Ab+aB)}{3x^3} + \frac{1}{3}b^2Bx^3 + b(Ab+2aB)\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^7}\,dx = \frac{1}{6}\left(-\frac{4aAb}{x^3} + 2b^2Bx^3 - \frac{a^2(A+2Bx^3)}{x^6} + 6b(Ab+2aB)\log(x)\right)$$

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^7,x]
[Out] ((-4*a*A*b)/x^3 + 2*b^2*B*x^3 - (a^2*(A + 2*B*x^3))/x^6 + 6*b*(A*b + 2*a*B)
*Log[x])/6
```

Maple [A] (verified)

Time = 4.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{a^2A}{6x^6} - \frac{a(2Ab+Ba)}{3x^3} + \frac{b^2Bx^3}{3} + b(Ab+2Ba)\ln(x)$	46
norman	$\frac{\left(-\frac{2}{3}abA - \frac{1}{3}a^2B\right)x^3 - \frac{a^2A}{6} + \frac{b^2Bx^9}{3}}{x^6} + (b^2A + 2abB)\ln(x)$	52
risch	$\frac{b^2Bx^3}{3} + \frac{\left(-\frac{2}{3}abA - \frac{1}{3}a^2B\right)x^3 - \frac{a^2A}{6}}{x^6} + A\ln(x)\,b^2 + 2B\ln(x)\,ab$	52
parallelrisch	$\frac{2b^2Bx^9 + 6A\ln(x)x^6b^2 + 12B\ln(x)x^6ab - 4aAbx^3 - 2a^2Bx^3 - a^2A}{6x^6}$	60

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^7,x,method=_RETURNVERBOSE)
[Out] -1/6*a^2*A/x^6-1/3*a*(2*A*b+B*a)/x^3+1/3*b^2*B*x^3+b*(A*b+2*B*a)*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^7} \, dx = \frac{2\, Bb^2x^9 + 6\,(2\, Bab + Ab^2)x^6 \log{(x)} - 2\,(Ba^2 + 2\, Aab)x^3 - Aa^2}{6\, x^6}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^7,x, algorithm="fricas")
[Out] 1/6*(2*B*b^2*x^9 + 6*(2*B*a*b + A*b^2)*x^6*log(x) - 2*(B*a^2 + 2*A*a*b)*x^3
 - A*a^2)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^7} \, dx = \frac{Bb^2x^3}{3} + b(Ab + 2Ba) \log{(x)} + \frac{-Aa^2 + x^3(-4Aab - 2Ba^2)}{6x^6}$$

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**7,x)
[Out] B*b**2*x**3/3 + b*(A*b + 2*B*a)*log(x) + (-A*a**2 + x**3*(-4*A*a*b - 2*B*a*
*2))/(6*x**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^7} \, dx = \frac{1}{3}\, Bb^2x^3 + \frac{1}{3}\, \left(2\, Bab + Ab^2\right) \log{\left(x^3\right)} - \frac{2\,(Ba^2 + 2\, Aab)x^3 + Aa^2}{6\, x^6}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^7,x, algorithm="maxima")
[Out] 1/3*B*b^2*x^3 + 1/3*(2*B*a*b + A*b^2)*log(x^3) - 1/6*(2*(B*a^2 + 2*A*a*b)*x
^3 + A*a^2)/x^6
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^7}\,dx = \frac{1}{3}Bb^2x^3 + \left(2Bab + Ab^2\right)\log\left(|x|\right) - \frac{6Babx^6 + 3Ab^2x^6 + 2Ba^2x^3 + 4Aabx^3 + Aa^2}{6x^6}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^7,x, algorithm="giac")
```

```
[Out] 1/3*B*b^2*x^3 + (2*B*a*b + A*b^2)*log(abs(x)) - 1/6*(6*B*a*b*x^6 + 3*A*b^2*
x^6 + 2*B*a^2*x^3 + 4*A*a*b*x^3 + A*a^2)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^7}\,dx = \ln(x)\,\left(A\,b^2 + 2\,B\,a\,b\right) - \frac{x^3\left(\frac{B\,a^2}{3} + \frac{2\,A\,b\,a}{3}\right) + \frac{A\,a^2}{6}}{x^6} + \frac{B\,b^2\,x^3}{3}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^7,x)
```

```
[Out] log(x)*(A*b^2 + 2*B*a*b) - (x^3*((B*a^2)/3 + (2*A*a*b)/3) + (A*a^2)/6)/x^6
+ (B*b^2*x^3)/3
```

3.21 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^8} dx$

Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^8} dx = -\frac{a^2A}{7x^7} - \frac{a(2Ab+aB)}{4x^4} - \frac{b(Ab+2aB)}{x} + \frac{1}{2}b^2Bx^2$$

```
[Out] -1/7*a^2*A/x^7-1/4*a*(2*A*b+B*a)/x^4-b*(A*b+2*B*a)/x+1/2*b^2*B*x^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^8} dx = -\frac{a^2A}{7x^7} - \frac{a(aB+2Ab)}{4x^4} - \frac{b(2aB+Ab)}{x} + \frac{1}{2}b^2Bx^2$$

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^8,x]
```

```
[Out] -1/7*(a^2*A)/x^7 - (a*(2*A*b + a*B))/(4*x^4) - (b*(A*b + 2*a*B))/x + (b^2*B
*x^2)/2
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{a^2 A}{x^8} + \frac{a(2Ab + aB)}{x^5} + \frac{b(Ab + 2aB)}{x^2} + b^2 Bx\right) dx \\ &= -\frac{a^2 A}{7x^7} - \frac{a(2Ab + aB)}{4x^4} - \frac{b(Ab + 2aB)}{x} + \frac{1}{2} b^2 Bx^2 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{(a + bx^3)^2 (A + Bx^3)}{x^8} dx = -\frac{-14b^2 x^6(-2A + Bx^3) + 14abx^3(A + 4Bx^3) + a^2(4A + 7Bx^3)}{28x^7}$$

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^8,x]
[Out] -1/28*(-14*b^2*x^6*(-2*A + B*x^3) + 14*a*b*x^3*(A + 4*B*x^3) + a^2*(4*A + 7
*B*x^3))/x^7
```

Maple [A] (verified)

Time = 4.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{a^2 A}{7x^7} - \frac{a(2Ab+Ba)}{4x^4} - \frac{b(Ab+2Ba)}{x} + \frac{b^2 B x^2}{2}$	48
norman	$\frac{\frac{b^2 B x^9}{2} + (-b^2 A - 2abB)x^6 + (-\frac{1}{2}abA - \frac{1}{4}a^2 B)x^3 - \frac{a^2 A}{7}}{x^7}$	53
risch	$\frac{b^2 B x^2}{2} + \frac{(-b^2 A - 2abB)x^6 + (-\frac{1}{2}abA - \frac{1}{4}a^2 B)x^3 - \frac{a^2 A}{7}}{x^7}$	54
gosper	$-\frac{-14b^2 B x^9 + 28A b^2 x^6 + 56Bab x^6 + 14aAb x^3 + 7a^2 B x^3 + 4a^2 A}{28x^7}$	56
parallelrisch	$-\frac{-14b^2 B x^9 + 28A b^2 x^6 + 56Bab x^6 + 14aAb x^3 + 7a^2 B x^3 + 4a^2 A}{28x^7}$	56

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^8,x,method=_RETURNVERBOSE)
[Out] -1/7*a^2*A/x^7-1/4*a*(2*A*b+B*a)/x^4-b*(A*b+2*B*a)/x+1/2*b^2*B*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^8}\,dx = \frac{14\,Bb^2x^9 - 28\,(2\,Bab + Ab^2)x^6 - 7\,(Ba^2 + 2\,Aab)x^3 - 4\,Aa^2}{28\,x^7}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^8,x, algorithm="fricas")
[Out] 1/28*(14*B*b^2*x^9 - 28*(2*B*a*b + A*b^2)*x^6 - 7*(B*a^2 + 2*A*a*b)*x^3 - 4
*A*a^2)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^8}\,dx = \frac{Bb^2x^2}{2} + \frac{-4Aa^2 + x^6(-28Ab^2 - 56Bab) + x^3(-14Aab - 7Ba^2)}{28x^7}$$

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**8,x)
[Out] B*b**2*x**2/2 + (-4*A*a**2 + x**6*(-28*A*b**2 - 56*B*a*b) + x**3*(-14*A*a*b
 - 7*B*a**2))/(28*x**7)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^8}\,dx = \frac{1}{2}\,Bb^2x^2 - \frac{28\,(2\,Bab + Ab^2)x^6 + 7\,(Ba^2 + 2\,Aab)x^3 + 4\,Aa^2}{28\,x^7}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^8,x, algorithm="maxima")
[Out] 1/2*B*b^2*x^2 - 1/28*(28*(2*B*a*b + A*b^2)*x^6 + 7*(B*a^2 + 2*A*a*b)*x^3 +
4*A*a^2)/x^7
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^8}\,dx = \frac{1}{2}Bb^2x^2 - \frac{56\,Babx^6 + 28\,Ab^2x^6 + 7\,Ba^2x^3 + 14\,Aabx^3 + 4\,Aa^2}{28\,x^7}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^8,x, algorithm="giac")
```

```
[Out] 1/2*B*b^2*x^2 - 1/28*(56*B*a*b*x^6 + 28*A*b^2*x^6 + 7*B*a^2*x^3 + 14*A*a*b*
x^3 + 4*A*a^2)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^8}\,dx = \frac{B\,b^2\,x^2}{2} - \frac{x^3\,\left(\frac{B\,a^2}{4}+\frac{A\,b\,a}{2}\right) + x^6\,\left(A\,b^2+2\,B\,a\,b\right) + \frac{A\,a^2}{7}}{x^7}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^8,x)
```

```
[Out] (B*b^2*x^2)/2 - (x^3*((B*a^2)/4 + (A*a*b)/2) + x^6*(A*b^2 + 2*B*a*b) + (A*a
^2)/7)/x^7
```

3.22 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^9}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^9}\,dx = -\frac{a^2A}{8x^8}-\frac{a(2Ab+aB)}{5x^5}-\frac{b(Ab+2aB)}{2x^2}+b^2Bx$$

```
[Out] -1/8*a^2*A/x^8-1/5*a*(2*A*b+B*a)/x^5-1/2*b*(A*b+2*B*a)/x^2+b^2*B*x
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^9}\,dx = -\frac{a^2A}{8x^8}-\frac{a(aB+2Ab)}{5x^5}-\frac{b(2aB+Ab)}{2x^2}+b^2Bx$$

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^9,x]
```

```
[Out] -1/8*(a^2*A)/x^8 - (a*(2*A*b + a*B))/(5*x^5) - (b*(A*b + 2*a*B))/(2*x^2) +
b^2*B*x
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(b^2B + \frac{a^2A}{x^9} + \frac{a(2Ab+aB)}{x^6} + \frac{b(Ab+2aB)}{x^3}\right)dx\\ &= -\frac{a^2A}{8x^8} - \frac{a(2Ab+aB)}{5x^5} - \frac{b(Ab+2aB)}{2x^2} + b^2Bx\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^9}\,dx = -\frac{a^2A}{8x^8} - \frac{a(2Ab+aB)}{5x^5} - \frac{b(Ab+2aB)}{2x^2} + b^2Bx$$

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^9,x]
[Out] -1/8*(a^2*A)/x^8 - (a*(2*A*b + a*B))/(5*x^5) - (b*(A*b + 2*a*B))/(2*x^2) +
b^2*B*x
```

Maple [A] (verified)

Time = 4.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{a^2A}{8x^8}-\frac{a(2Ab+Ba)}{5x^5}-\frac{b(Ab+2Ba)}{2x^2}+b^2Bx$	45
risch	$b^2Bx+\frac{\left(-\frac{1}{2}b^2A-abB\right)x^6+\left(-\frac{2}{5}abA-\frac{1}{5}a^2B\right)x^3-\frac{a^2A}{8}}{x^8}$	51
norman	$\frac{b^2B\,x^9+\left(-\frac{1}{2}b^2A-abB\right)x^6+\left(-\frac{2}{5}abA-\frac{1}{5}a^2B\right)x^3-\frac{a^2A}{8}}{x^8}$	52
gosper	$-\frac{-40b^2B\,x^9+20A\,b^2x^6+40Bab\,x^6+16aAb\,x^3+8a^2B\,x^3+5a^2A}{40x^8}$	56
parallelrisch	$-\frac{-40b^2B\,x^9+20A\,b^2x^6+40Bab\,x^6+16aAb\,x^3+8a^2B\,x^3+5a^2A}{40x^8}$	56

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^9,x,method=_RETURNVERBOSE)
[Out] -1/8*a^2*A/x^8-1/5*a*(2*A*b+B*a)/x^5-1/2*b*(A*b+2*B*a)/x^2+b^2*B*x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^9}\,dx = \frac{40\,Bb^2x^9-20\,(2\,Bab+Ab^2)x^6-8\,(Ba^2+2\,Aab)x^3-5\,Aa^2}{40\,x^8}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^9,x, algorithm="fricas")
```

```
[Out] 1/40*(40*B*b^2*x^9 - 20*(2*B*a*b + A*b^2)*x^6 - 8*(B*a^2 + 2*A*a*b)*x^3 - 5
*A*a^2)/x^8
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^9}\,dx = Bb^2x+\frac{-5Aa^2+x^6(-20Ab^2-40Bab)+x^3(-16Aab-8Ba^2)}{40x^8}$$

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**9,x)
```

```
[Out] B*b**2*x + (-5*A*a**2 + x**6*(-20*A*b**2 - 40*B*a*b) + x**3*(-16*A*a*b - 8*
B*a**2))/(40*x**8)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^9}\,dx = Bb^2x-\frac{20\,(2\,Bab+Ab^2)x^6+8\,(Ba^2+2\,Aab)x^3+5\,Aa^2}{40\,x^8}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^9,x, algorithm="maxima")
```

```
[Out] B*b^2*x - 1/40*(20*(2*B*a*b + A*b^2)*x^6 + 8*(B*a^2 + 2*A*a*b)*x^3 + 5*A*a^
2)/x^8
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^9} \, dx = Bb^2x - \frac{40 \, Babx^6 + 20 \, Ab^2x^6 + 8 \, Ba^2x^3 + 16 \, Aabx^3 + 5 \, Aa^2}{40 \, x^8}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^9,x, algorithm="giac")
```

```
[Out] B*b^2*x - 1/40*(40*B*a*b*x^6 + 20*A*b^2*x^6 + 8*B*a^2*x^3 + 16*A*a*b*x^3 +
5*A*a^2)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^9} \, dx = B \, b^2 \, x - \frac{x^3 \left(\frac{B \, a^2}{5} + \frac{2 \, A \, b \, a}{5}\right) + x^6 \left(\frac{A \, b^2}{2} + B \, a \, b\right) + \frac{A \, a^2}{8}}{x^8}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^9,x)
```

```
[Out] B*b^2*x - (x^3*((B*a^2)/5 + (2*A*a*b)/5) + x^6*((A*b^2)/2 + B*a*b) + (A*a^2
)/8)/x^8
```

3.23 $\int x^9(a+bx^3)^5 (A+Bx^3)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int x^9 \left(a+bx^3\right)^5 \left(A+Bx^3\right)\,dx = \frac{1}{10}a^5Ax^{10}+\frac{1}{13}a^4(5Ab+aB)x^{13}+\frac{5}{16}a^3b(2Ab+aB)x^{16} +\frac{10}{19}a^2b^2(Ab+aB)x^{19}+\frac{5}{22}ab^3(Ab+2aB)x^{22} +\frac{1}{25}b^4(Ab+5aB)x^{25}+\frac{1}{28}b^5Bx^{28}$$

```
[Out] 1/10*a^5*A*x^10+1/13*a^4*(5*A*b+B*a)*x^13+5/16*a^3*b*(2*A*b+B*a)*x^16+10/19
*a^2*b^2*(A*b+B*a)*x^19+5/22*a*b^3*(A*b+2*B*a)*x^22+1/25*b^4*(A*b+5*B*a)*x^
25+1/28*b^5*B*x^28
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int x^9 \left(a+bx^3\right)^5 \left(A+Bx^3\right)\,dx = \frac{1}{10}a^5Ax^{10}+\frac{1}{13}a^4x^{13}(aB+5Ab)+\frac{5}{16}a^3bx^{16}(aB+2Ab) +\frac{10}{19}a^2b^2x^{19}(aB+Ab)+\frac{1}{25}b^4x^{25}(5aB+Ab) +\frac{5}{22}ab^3x^{22}(2aB+Ab)+\frac{1}{28}b^5Bx^{28}$$

```
[In] Int[x^9*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^10)/10 + (a^4*(5*A*b + a*B)*x^13)/13 + (5*a^3*b*(2*A*b + a*B)*x^16
)/16 + (10*a^2*b^2*(A*b + a*B)*x^19)/19 + (5*a*b^3*(A*b + 2*a*B)*x^22)/22 +
 (b^4*(A*b + 5*a*B)*x^25)/25 + (b^5*B*x^28)/28
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^5Ax^9 + a^4(5Ab + aB)x^{12} + 5a^3b(2Ab + aB)x^{15} + 10a^2b^2(Ab + aB)x^{18} \right. \\ &\qquad \left. + 5ab^3(Ab + 2aB)x^{21} + b^4(Ab + 5aB)x^{24} + b^5Bx^{27}\right) dx \\ &= \frac{1}{10}a^5Ax^{10} + \frac{1}{13}a^4(5Ab + aB)x^{13} + \frac{5}{16}a^3b(2Ab + aB)x^{16} + \frac{10}{19}a^2b^2(Ab + aB)x^{19} \\ &\quad + \frac{5}{22}ab^3(Ab + 2aB)x^{22} + \frac{1}{25}b^4(Ab + 5aB)x^{25} + \frac{1}{28}b^5Bx^{28}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^9\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx &= \frac{1}{10}a^5Ax^{10} + \frac{1}{13}a^4(5Ab + aB)x^{13} + \frac{5}{16}a^3b(2Ab + aB)x^{16} \\ &\quad + \frac{10}{19}a^2b^2(Ab + aB)x^{19} + \frac{5}{22}ab^3(Ab + 2aB)x^{22} \\ &\quad + \frac{1}{25}b^4(Ab + 5aB)x^{25} + \frac{1}{28}b^5Bx^{28}\end{aligned}$$

```
[In] Integrate[x^9*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^10)/10 + (a^4*(5*A*b + a*B)*x^13)/13 + (5*a^3*b*(2*A*b + a*B)*x^16
)/16 + (10*a^2*b^2*(A*b + a*B)*x^19)/19 + (5*a*b^3*(A*b + 2*a*B)*x^22)/22 +
 (b^4*(A*b + 5*a*B)*x^25)/25 + (b^5*B*x^28)/28
```

Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

method	result
norman	$\frac{a^5 A x^{10}}{10}+\left(\frac{5}{13}a^4 bA+\frac{1}{13}a^5 B\right)x^{13}+\left(\frac{5}{8}a^3 b^2 A+\frac{5}{16}a^4 bB\right)x^{16}+\left(\frac{10}{19}a^2 b^3 A+\frac{10}{19}a^3 b^2 B\right)x^{19}+\left(\frac{5}{22}\right.$
default	$\frac{b^5 B x^{28}}{28}+\frac{(b^5 A+5a b^4 B)x^{25}}{25}+\frac{(5a b^4 A+10a^2 b^3 B)x^{22}}{22}+\frac{(10a^2 b^3 A+10a^3 b^2 B)x^{19}}{19}+\frac{(10a^3 b^2 A+5a^4 bB)x^{16}}{16}+\frac{(5a^4 b}{}$
gosper	$\frac{1}{10}a^5 A\,x^{10}+\frac{5}{13}x^{13}a^4 bA+\frac{1}{13}x^{13}a^5 B+\frac{5}{8}x^{16}a^3 b^2 A+\frac{5}{16}x^{16}a^4 bB+\frac{10}{19}x^{19}a^2 b^3 A+\frac{10}{19}x^{19}a^3 b^2 B$
risch	$\frac{1}{10}a^5 A\,x^{10}+\frac{5}{13}x^{13}a^4 bA+\frac{1}{13}x^{13}a^5 B+\frac{5}{8}x^{16}a^3 b^2 A+\frac{5}{16}x^{16}a^4 bB+\frac{10}{19}x^{19}a^2 b^3 A+\frac{10}{19}x^{19}a^3 b^2 B$
parallelrisch	$\frac{1}{10}a^5 A\,x^{10}+\frac{5}{13}x^{13}a^4 bA+\frac{1}{13}x^{13}a^5 B+\frac{5}{8}x^{16}a^3 b^2 A+\frac{5}{16}x^{16}a^4 bB+\frac{10}{19}x^{19}a^2 b^3 A+\frac{10}{19}x^{19}a^3 b^2 B$

```
[In] int(x^9*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*a^5*A*x^10+(5/13*a^4*b*A+1/13*a^5*B)*x^13+(5/8*a^3*b^2*A+5/16*a^4*b*B)
*x^16+(10/19*a^2*b^3*A+10/19*a^3*b^2*B)*x^19+(5/22*a*b^4*A+5/11*a^2*b^3*B)*
x^22+(1/25*b^5*A+1/5*a*b^4*B)*x^25+1/28*b^5*B*x^28
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^9\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=\frac{1}{28}\,Bb^5x^{28}+\frac{1}{25}\,(5\,Bab^4+Ab^5)x^{25}+\frac{5}{22}\,(2\,Ba^2b^3+Aab^4)x^{22}+\frac{10}{19}\,(Ba^3b^2+Aa^2b^3)x^{19}+\frac{5}{16}\,(Ba^4b+2\,Aa^3b^2)x^{16}+\frac{1}{10}\,Aa^5x^{10}+\frac{1}{13}\,(Ba^5+5\,Aa^4b)x^{13}$$

```
[In] integrate(x^9*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/28*B*b^5*x^28 + 1/25*(5*B*a*b^4 + A*b^5)*x^25 + 5/22*(2*B*a^2*b^3 + A*a*b
^4)*x^22 + 10/19*(B*a^3*b^2 + A*a^2*b^3)*x^19 + 5/16*(B*a^4*b + 2*A*a^3*b^2
)*x^16 + 1/10*A*a^5*x^10 + 1/13*(B*a^5 + 5*A*a^4*b)*x^13
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.16

$$\int x^9\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=\frac{Aa^5x^{10}}{10}+\frac{Bb^5x^{28}}{28}+x^{25}\left(\frac{Ab^5}{25}+\frac{Bab^4}{5}\right)+x^{22}\cdot\left(\frac{5Aab^4}{22}+\frac{5Ba^2b^3}{11}\right)+x^{19}\cdot\left(\frac{10Aa^2b^3}{19}+\frac{10Ba^3b^2}{19}\right)+x^{16}\cdot\left(\frac{5Aa^3b^2}{8}+\frac{5Ba^4b}{16}\right)+x^{13}\cdot\left(\frac{5Aa^4b}{13}+\frac{Ba^5}{13}\right)$$

```
[In] integrate(x**9*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**10/10 + B*b**5*x**28/28 + x**25*(A*b**5/25 + B*a*b**4/5) + x**22*
(5*A*a*b**4/22 + 5*B*a**2*b**3/11) + x**19*(10*A*a**2*b**3/19 + 10*B*a**3*b
**2/19) + x**16*(5*A*a**3*b**2/8 + 5*B*a**4*b/16) + x**13*(5*A*a**4*b/13 +
B*a**5/13)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^9(a+bx^3)^5(A+Bx^3)\,dx = \frac{1}{28}Bb^5x^{28}+\frac{1}{25}(5Bab^4+Ab^5)x^{25}+\frac{5}{22}(2Ba^2b^3+Aab^4)x^{22} + \frac{10}{19}(Ba^3b^2+Aa^2b^3)x^{19}+\frac{5}{16}(Ba^4b+2Aa^3b^2)x^{16} + \frac{1}{10}Aa^5x^{10}+\frac{1}{13}(Ba^5+5Aa^4b)x^{13}$$

```
[In] integrate(x^9*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/28*B*b^5*x^28 + 1/25*(5*B*a*b^4 + A*b^5)*x^25 + 5/22*(2*B*a^2*b^3 + A*a*b
^4)*x^22 + 10/19*(B*a^3*b^2 + A*a^2*b^3)*x^19 + 5/16*(B*a^4*b + 2*A*a^3*b^2
)*x^16 + 1/10*A*a^5*x^10 + 1/13*(B*a^5 + 5*A*a^4*b)*x^13
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07

$$\int x^9(a+bx^3)^5(A+Bx^3)\,dx = \frac{1}{28}Bb^5x^{28}+\frac{1}{5}Bab^4x^{25}+\frac{1}{25}Ab^5x^{25}+\frac{5}{11}Ba^2b^3x^{22} + \frac{5}{22}Aab^4x^{22}+\frac{10}{19}Ba^3b^2x^{19}+\frac{10}{19}Aa^2b^3x^{19}+\frac{5}{16}Ba^4bx^{16} + \frac{5}{8}Aa^3b^2x^{16}+\frac{1}{13}Ba^5x^{13}+\frac{5}{13}Aa^4bx^{13}+\frac{1}{10}Aa^5x^{10}$$

```
[In] integrate(x^9*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/28*B*b^5*x^28 + 1/5*B*a*b^4*x^25 + 1/25*A*b^5*x^25 + 5/11*B*a^2*b^3*x^22
+ 5/22*A*a*b^4*x^22 + 10/19*B*a^3*b^2*x^19 + 10/19*A*a^2*b^3*x^19 + 5/16*B*
a^4*b*x^16 + 5/8*A*a^3*b^2*x^16 + 1/13*B*a^5*x^13 + 5/13*A*a^4*b*x^13 + 1/1
0*A*a^5*x^10
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int x^9 (a+bx^3)^5 \left(A+Bx^3\right) \, dx = x^{13} \left(\frac{B\,a^5}{13}+\frac{5\,A\,b\,a^4}{13}\right)+x^{25} \left(\frac{A\,b^5}{25}+\frac{B\,a\,b^4}{5}\right) + \frac{A\,a^5\,x^{10}}{10}+\frac{B\,b^5\,x^{28}}{28}+\frac{10\,a^2\,b^2\,x^{19}\,(A\,b+B\,a)}{19} + \frac{5\,a^3\,b\,x^{16}\,(2\,A\,b+B\,a)}{16}+\frac{5\,a\,b^3\,x^{22}\,(A\,b+2\,B\,a)}{22}$$

```
[In] int(x^9*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^13*((B*a^5)/13 + (5*A*a^4*b)/13) + x^25*((A*b^5)/25 + (B*a*b^4)/5) + (A*a
^5*x^10)/10 + (B*b^5*x^28)/28 + (10*a^2*b^2*x^19*(A*b + B*a))/19 + (5*a^3*b
*x^16*(2*A*b + B*a))/16 + (5*a*b^3*x^22*(A*b + 2*B*a))/22
```

3.24 $\int x^8(a+bx^3)^5\left(A+Bx^3\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 95

$$\int x^8\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=\frac{a^2(Ab-aB)\left(a+bx^3\right)^6}{18b^4}-\frac{a(2Ab-3aB)\left(a+bx^3\right)^7}{21b^4}+\frac{(Ab-3aB)\left(a+bx^3\right)^8}{24b^4}+\frac{B\left(a+bx^3\right)^9}{27b^4}$$

```
[Out] 1/18*a^2*(A*b-B*a)*(b*x^3+a)^6/b^4-1/21*a*(2*A*b-3*B*a)*(b*x^3+a)^7/b^4+1/2
4*(A*b-3*B*a)*(b*x^3+a)^8/b^4+1/27*B*(b*x^3+a)^9/b^4
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 77}

$$\int x^8\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=\frac{a^2\left(a+bx^3\right)^6(Ab-aB)}{18b^4}+\frac{\left(a+bx^3\right)^8(Ab-3aB)}{24b^4}-\frac{a\left(a+bx^3\right)^7(2Ab-3aB)}{21b^4}+\frac{B\left(a+bx^3\right)^9}{27b^4}$$

```
[In] Int[x^8*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^2*(A*b - a*B)*(a + b*x^3)^6)/(18*b^4) - (a*(2*A*b - 3*a*B)*(a + b*x^3)^7
)/(21*b^4) + ((A*b - 3*a*B)*(a + b*x^3)^8)/(24*b^4) + (B*(a + b*x^3)^9)/(27
*b^4)
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
```

```
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int x^2(a+bx)^5(A+Bx)\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int\left(-\frac{a^2(-Ab+aB)(a+bx)^5}{b^3}+\frac{a(-2Ab+3aB)(a+bx)^6}{b^3}+\frac{(Ab-3aB)(a+bx)^7}{b^3}+\frac{B(a+bx)^8}{b^3}\right)dx,x,x^3\right)\\ &= \frac{a^2(Ab-aB)\left(a+bx^3\right)^6}{18b^4}-\frac{a(2Ab-3aB)\left(a+bx^3\right)^7}{21b^4}+\frac{(Ab-3aB)\left(a+bx^3\right)^8}{24b^4}+\frac{B\left(a+bx^3\right)^9}{27b^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13

$$\int x^8\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx = \frac{x^9(168a^5A+126a^4(5Ab+aB)x^3+504a^3b(2Ab+aB)x^6+840a^2b^2(Ab+aB)x^9+360ab^3(Ab+2aB)x^{12}}{1512}$$

```
[In] Integrate[x^8*(a + b*x^3)^5*(A + B*x^3),x]
[Out] (x^9*(168*a^5*A + 126*a^4*(5*A*b + a*B)*x^3 + 504*a^3*b*(2*A*b + a*B)*x^6 +
 840*a^2*b^2*(A*b + a*B)*x^9 + 360*a*b^3*(A*b + 2*a*B)*x^12 + 63*b^4*(A*b +
 5*a*B)*x^15 + 56*b^5*B*x^18))/1512
```

Maple [A] (verified)

Time = 4.18 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.27

method	result
norman	$\frac{b^5 B x^{27}}{27}+\frac{a^5 A x^9}{9}+\left(\frac{5}{12}a^4bA+\frac{1}{12}a^5B\right)x^{12}+\left(\frac{2}{3}a^3b^2A+\frac{1}{3}a^4bB\right)x^{15}+\left(\frac{5}{9}a^2b^3A+\frac{5}{9}a^3b^2B\right)x^{18}+$
default	$\frac{b^5 B x^{27}}{27}+\frac{(b^5A+5a\,b^4B)x^{24}}{24}+\frac{(5a\,b^4A+10a^2b^3B)x^{21}}{21}+\frac{(10a^2b^3A+10a^3b^2B)x^{18}}{18}+\frac{(10a^3b^2A+5a^4bB)x^{15}}{15}+$
gosper	$\frac{1}{27}b^5B\,x^{27}+\frac{1}{9}a^5A\,x^9+\frac{5}{12}x^{12}a^4bA+\frac{1}{12}x^{12}a^5B+\frac{2}{3}x^{15}a^3b^2A+\frac{1}{3}x^{15}a^4bB+\frac{5}{9}x^{18}a^2b^3A+\frac{5}{9}x^{18}$
risch	$\frac{1}{27}b^5B\,x^{27}+\frac{1}{9}a^5A\,x^9+\frac{5}{12}x^{12}a^4bA+\frac{1}{12}x^{12}a^5B+\frac{2}{3}x^{15}a^3b^2A+\frac{1}{3}x^{15}a^4bB+\frac{5}{9}x^{18}a^2b^3A+\frac{5}{9}x^{18}$
parallelrisch	$\frac{1}{27}b^5B\,x^{27}+\frac{1}{9}a^5A\,x^9+\frac{5}{12}x^{12}a^4bA+\frac{1}{12}x^{12}a^5B+\frac{2}{3}x^{15}a^3b^2A+\frac{1}{3}x^{15}a^4bB+\frac{5}{9}x^{18}a^2b^3A+\frac{5}{9}x^{18}$

```
[In] int(x^8*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/27*b^5*B*x^27+1/9*a^5*A*x^9+(5/12*a^4*b*A+1/12*a^5*B)*x^12+(2/3*a^3*b^2*A
+1/3*a^4*b*B)*x^15+(5/9*a^2*b^3*A+5/9*a^3*b^2*B)*x^18+(5/21*a*b^4*A+10/21*a
^2*b^3*B)*x^21+(1/24*b^5*A+5/24*a*b^4*B)*x^24
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.25

$$\int x^8(a+bx^3)^5(A+Bx^3)\,dx=\frac{1}{27}Bb^5x^{27}+\frac{1}{24}(5Bab^4+Ab^5)x^{24}+\frac{5}{21}(2Ba^2b^3+Aab^4)x^{21}+\frac{5}{9}(Ba^3b^2+Aa^2b^3)x^{18}+\frac{1}{3}(Ba^4b+2Aa^3b^2)x^{15}+\frac{1}{9}Aa^5x^9+\frac{1}{12}(Ba^5+5Aa^4b)x^{12}$$

```
[In] integrate(x^8*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/27*B*b^5*x^27 + 1/24*(5*B*a*b^4 + A*b^5)*x^24 + 5/21*(2*B*a^2*b^3 + A*a*b
^4)*x^21 + 5/9*(B*a^3*b^2 + A*a^2*b^3)*x^18 + 1/3*(B*a^4*b + 2*A*a^3*b^2)*x
^15 + 1/9*A*a^5*x^9 + 1/12*(B*a^5 + 5*A*a^4*b)*x^12
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.43

$$\int x^8(a+bx^3)^5\left(A+Bx^3\right)\,dx=\frac{Aa^5x^9}{9}+\frac{Bb^5x^{27}}{27}+x^{24}\left(\frac{Ab^5}{24}+\frac{5Bab^4}{24}\right)+x^{21}\cdot\left(\frac{5Aab^4}{21}+\frac{10Ba^2b^3}{21}\right)+x^{18}\cdot\left(\frac{5Aa^2b^3}{9}+\frac{5Ba^3b^2}{9}\right)+x^{15}\cdot\left(\frac{2Aa^3b^2}{3}+\frac{Ba^4b}{3}\right)+x^{12}\cdot\left(\frac{5Aa^4b}{12}+\frac{Ba^5}{12}\right)$$

```
[In] integrate(x**8*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**9/9 + B*b**5*x**27/27 + x**24*(A*b**5/24 + 5*B*a*b**4/24) + x**21
*(5*A*a*b**4/21 + 10*B*a**2*b**3/21) + x**18*(5*A*a**2*b**3/9 + 5*B*a**3*b*
*2/9) + x**15*(2*A*a**3*b**2/3 + B*a**4*b/3) + x**12*(5*A*a**4*b/12 + B*a**
5/12)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.25

$$\int x^8(a+bx^3)^5\left(A+Bx^3\right)\,dx=\frac{1}{27}\,Bb^5x^{27}+\frac{1}{24}\,\left(5\,Bab^4+Ab^5\right)x^{24}+\frac{5}{21}\,\left(2\,Ba^2b^3+Aab^4\right)x^{21}+\frac{5}{9}\,\left(Ba^3b^2+Aa^2b^3\right)x^{18}+\frac{1}{3}\,\left(Ba^4b+2\,Aa^3b^2\right)x^{15}+\frac{1}{9}\,Aa^5x^9+\frac{1}{12}\,\left(Ba^5+5\,Aa^4b\right)x^{12}$$

```
[In] integrate(x^8*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/27*B*b^5*x^27 + 1/24*(5*B*a*b^4 + A*b^5)*x^24 + 5/21*(2*B*a^2*b^3 + A*a*b
^4)*x^21 + 5/9*(B*a^3*b^2 + A*a^2*b^3)*x^18 + 1/3*(B*a^4*b + 2*A*a^3*b^2)*x
^15 + 1/9*A*a^5*x^9 + 1/12*(B*a^5 + 5*A*a^4*b)*x^12
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.32

$$\int x^8(a+bx^3)^5(A+Bx^3)\,dx = \frac{1}{27}Bb^5x^{27} + \frac{5}{24}Bab^4x^{24} + \frac{1}{24}Ab^5x^{24} + \frac{10}{21}Ba^2b^3x^{21} + \frac{5}{21}Aab^4x^{21} + \frac{5}{9}Ba^3b^2x^{18} + \frac{5}{9}Aa^2b^3x^{18} + \frac{1}{3}Ba^4bx^{15} + \frac{2}{3}Aa^3b^2x^{15} + \frac{1}{12}Ba^5x^{12} + \frac{5}{12}Aa^4bx^{12} + \frac{1}{9}Aa^5x^9$$

```
[In] integrate(x^8*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/27*B*b^5*x^27 + 5/24*B*a*b^4*x^24 + 1/24*A*b^5*x^24 + 10/21*B*a^2*b^3*x^2
1 + 5/21*A*a*b^4*x^21 + 5/9*B*a^3*b^2*x^18 + 5/9*A*a^2*b^3*x^18 + 1/3*B*a^4
*b*x^15 + 2/3*A*a^3*b^2*x^15 + 1/12*B*a^5*x^12 + 5/12*A*a^4*b*x^12 + 1/9*A*
a^5*x^9
```

Mupad [B] (verification not implemented)

Time = 6.69 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13

$$\int x^8(a+bx^3)^5(A+Bx^3)\,dx = x^{12}\left(\frac{B\,a^5}{12} + \frac{5\,A\,b\,a^4}{12}\right) + x^{24}\left(\frac{A\,b^5}{24} + \frac{5\,B\,a\,b^4}{24}\right) + \frac{A\,a^5\,x^9}{9} + \frac{B\,b^5\,x^{27}}{27} + \frac{5\,a^2\,b^2\,x^{18}\,(A\,b+B\,a)}{9} + \frac{a^3\,b\,x^{15}\,(2\,A\,b+B\,a)}{3} + \frac{5\,a\,b^3\,x^{21}\,(A\,b+2\,B\,a)}{21}$$

```
[In] int(x^8*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^12*((B*a^5)/12 + (5*A*a^4*b)/12) + x^24*((A*b^5)/24 + (5*B*a*b^4)/24) + (
A*a^5*x^9)/9 + (B*b^5*x^27)/27 + (5*a^2*b^2*x^18*(A*b + B*a))/9 + (a^3*b*x^
15*(2*A*b + B*a))/3 + (5*a*b^3*x^21*(A*b + 2*B*a))/21
```

3.25 $\int x^7(a+bx^3)^5\,(A+Bx^3)\;dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int x^7\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=\frac{1}{8}a^5Ax^8+\frac{1}{11}a^4(5Ab+aB)x^{11}+\frac{5}{14}a^3b(2Ab+aB)x^{14}+\frac{10}{17}a^2b^2(Ab+aB)x^{17}+\frac{1}{4}ab^3(Ab+2aB)x^{20}+\frac{1}{23}b^4(Ab+5aB)x^{23}+\frac{1}{26}b^5Bx^{26}$$

```
[Out] 1/8*a^5*A*x^8+1/11*a^4*(5*A*b+B*a)*x^11+5/14*a^3*b*(2*A*b+B*a)*x^14+10/17*a
^2*b^2*(A*b+B*a)*x^17+1/4*a*b^3*(A*b+2*B*a)*x^20+1/23*b^4*(A*b+5*B*a)*x^23+
1/26*b^5*B*x^26
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int x^7\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=\frac{1}{8}a^5Ax^8+\frac{1}{11}a^4x^{11}(aB+5Ab)+\frac{5}{14}a^3bx^{14}(aB+2Ab)+\frac{10}{17}a^2b^2x^{17}(aB+Ab)+\frac{1}{23}b^4x^{23}(5aB+Ab)+\frac{1}{4}ab^3x^{20}(2aB+Ab)+\frac{1}{26}b^5Bx^{26}$$

```
[In] Int[x^7*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^8)/8 + (a^4*(5*A*b + a*B)*x^11)/11 + (5*a^3*b*(2*A*b + a*B)*x^14)/
14 + (10*a^2*b^2*(A*b + a*B)*x^17)/17 + (a*b^3*(A*b + 2*a*B)*x^20)/4 + (b^4
*(A*b + 5*a*B)*x^23)/23 + (b^5*B*x^26)/26
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^5Ax^7 + a^4(5Ab + aB)x^{10} + 5a^3b(2Ab + aB)x^{13} + 10a^2b^2(Ab + aB)x^{16} \right. \\ &\qquad \left. + 5ab^3(Ab + 2aB)x^{19} + b^4(Ab + 5aB)x^{22} + b^5Bx^{25}\right) dx \\ &= \frac{1}{8}a^5Ax^8 + \frac{1}{11}a^4(5Ab + aB)x^{11} + \frac{5}{14}a^3b(2Ab + aB)x^{14} + \frac{10}{17}a^2b^2(Ab + aB)x^{17} \\ &\quad + \frac{1}{4}ab^3(Ab + 2aB)x^{20} + \frac{1}{23}b^4(Ab + 5aB)x^{23} + \frac{1}{26}b^5Bx^{26}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^7\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx &= \frac{1}{8}a^5Ax^8 + \frac{1}{11}a^4(5Ab + aB)x^{11} + \frac{5}{14}a^3b(2Ab + aB)x^{14} \\ &\quad + \frac{10}{17}a^2b^2(Ab + aB)x^{17} + \frac{1}{4}ab^3(Ab + 2aB)x^{20} \\ &\quad + \frac{1}{23}b^4(Ab + 5aB)x^{23} + \frac{1}{26}b^5Bx^{26}\end{aligned}$$

```
[In] Integrate[x^7*(a + b*x^3)^5*(A + B*x^3),x]
[Out] (a^5*A*x^8)/8 + (a^4*(5*A*b + a*B)*x^11)/11 + (5*a^3*b*(2*A*b + a*B)*x^14)/
14 + (10*a^2*b^2*(A*b + a*B)*x^17)/17 + (a*b^3*(A*b + 2*a*B)*x^20)/4 + (b^4
*(A*b + 5*a*B)*x^23)/23 + (b^5*B*x^26)/26
```

Maple [A] (verified)

Time = 4.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

method	result
norman	$\frac{a^5 A x^8}{8}+\left(\frac{5}{11}a^4 bA+\frac{1}{11}a^5 B\right)x^{11}+\left(\frac{5}{7}a^3 b^2 A+\frac{5}{14}a^4 bB\right)x^{14}+\left(\frac{10}{17}a^2 b^3 A+\frac{10}{17}a^3 b^2 B\right)x^{17}+\left(\frac{1}{4}a\right.$
default	$\frac{b^5 B x^{26}}{26}+\frac{(b^5 A+5a b^4 B)x^{23}}{23}+\frac{(5a b^4 A+10a^2 b^3 B)x^{20}}{20}+\frac{(10a^2 b^3 A+10a^3 b^2 B)x^{17}}{17}+\frac{(10a^3 b^2 A+5a^4 bB)x^{14}}{14}+\frac{(5a^4 b}{}$
gosper	$\frac{1}{8}a^5 A x^8+\frac{5}{11}x^{11}a^4 bA+\frac{1}{11}x^{11}a^5 B+\frac{5}{7}x^{14}a^3 b^2 A+\frac{5}{14}x^{14}a^4 bB+\frac{10}{17}x^{17}a^2 b^3 A+\frac{10}{17}x^{17}a^3 b^2 B+$
risch	$\frac{1}{8}a^5 A x^8+\frac{5}{11}x^{11}a^4 bA+\frac{1}{11}x^{11}a^5 B+\frac{5}{7}x^{14}a^3 b^2 A+\frac{5}{14}x^{14}a^4 bB+\frac{10}{17}x^{17}a^2 b^3 A+\frac{10}{17}x^{17}a^3 b^2 B+$
parallelrisch	$\frac{1}{8}a^5 A x^8+\frac{5}{11}x^{11}a^4 bA+\frac{1}{11}x^{11}a^5 B+\frac{5}{7}x^{14}a^3 b^2 A+\frac{5}{14}x^{14}a^4 bB+\frac{10}{17}x^{17}a^2 b^3 A+\frac{10}{17}x^{17}a^3 b^2 B+$

```
[In] int(x^7*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*a^5*A*x^8+(5/11*a^4*b*A+1/11*a^5*B)*x^11+(5/7*a^3*b^2*A+5/14*a^4*b*B)*x
^14+(10/17*a^2*b^3*A+10/17*a^3*b^2*B)*x^17+(1/4*a*b^4*A+1/2*a^2*b^3*B)*x^20
+(1/23*b^5*A+5/23*a*b^4*B)*x^23+1/26*b^5*B*x^26
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^7\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=\frac{1}{26}\,Bb^5x^{26}+\frac{1}{23}\,(5\,Bab^4+Ab^5)x^{23}+\frac{1}{4}\,(2\,Ba^2b^3+Aab^4)x^{20}+\frac{10}{17}\,(Ba^3b^2+Aa^2b^3)x^{17}+\frac{5}{14}\,(Ba^4b+2\,Aa^3b^2)x^{14}+\frac{1}{8}\,Aa^5x^8+\frac{1}{11}\,(Ba^5+5\,Aa^4b)x^{11}$$

```
[In] integrate(x^7*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/26*B*b^5*x^26 + 1/23*(5*B*a*b^4 + A*b^5)*x^23 + 1/4*(2*B*a^2*b^3 + A*a*b^
4)*x^20 + 10/17*(B*a^3*b^2 + A*a^2*b^3)*x^17 + 5/14*(B*a^4*b + 2*A*a^3*b^2)
*x^14 + 1/8*A*a^5*x^8 + 1/11*(B*a^5 + 5*A*a^4*b)*x^11
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.15

$$\int x^7\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=\frac{Aa^5x^8}{8}+\frac{Bb^5x^{26}}{26}+x^{23}\left(\frac{Ab^5}{23}+\frac{5Bab^4}{23}\right)+x^{20}\left(\frac{Aab^4}{4}+\frac{Ba^2b^3}{2}\right)+x^{17}\cdot\left(\frac{10Aa^2b^3}{17}+\frac{10Ba^3b^2}{17}\right)+x^{14}\cdot\left(\frac{5Aa^3b^2}{7}+\frac{5Ba^4b}{14}\right)+x^{11}\cdot\left(\frac{5Aa^4b}{11}+\frac{Ba^5}{11}\right)$$

```
[In] integrate(x**7*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**8/8 + B*b**5*x**26/26 + x**23*(A*b**5/23 + 5*B*a*b**4/23) + x**20
*(A*a*b**4/4 + B*a**2*b**3/2) + x**17*(10*A*a**2*b**3/17 + 10*B*a**3*b**2/1
7) + x**14*(5*A*a**3*b**2/7 + 5*B*a**4*b/14) + x**11*(5*A*a**4*b/11 + B*a**
5/11)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^7 \left(a+bx^3\right)^5 \left(A+Bx^3\right)\,dx = \frac{1}{26}\,Bb^5x^{26} + \frac{1}{23}\,\left(5\,Bab^4 + Ab^5\right)x^{23} + \frac{1}{4}\,\left(2\,Ba^2b^3 + Aab^4\right)x^{20} + \frac{10}{17}\,\left(Ba^3b^2 + Aa^2b^3\right)x^{17} + \frac{5}{14}\,\left(Ba^4b + 2\,Aa^3b^2\right)x^{14} + \frac{1}{8}\,Aa^5x^8 + \frac{1}{11}\,\left(Ba^5 + 5\,Aa^4b\right)x^{11}$$

```
[In] integrate(x^7*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/26*B*b^5*x^26 + 1/23*(5*B*a*b^4 + A*b^5)*x^23 + 1/4*(2*B*a^2*b^3 + A*a*b^
4)*x^20 + 10/17*(B*a^3*b^2 + A*a^2*b^3)*x^17 + 5/14*(B*a^4*b + 2*A*a^3*b^2)
*x^14 + 1/8*A*a^5*x^8 + 1/11*(B*a^5 + 5*A*a^4*b)*x^11
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07

$$\int x^7 \left(a+bx^3\right)^5 \left(A+Bx^3\right)\,dx = \frac{1}{26}\,Bb^5x^{26} + \frac{5}{23}\,Bab^4x^{23} + \frac{1}{23}\,Ab^5x^{23} + \frac{1}{2}\,Ba^2b^3x^{20} + \frac{1}{4}\,Aab^4x^{20} + \frac{10}{17}\,Ba^3b^2x^{17} + \frac{10}{17}\,Aa^2b^3x^{17} + \frac{5}{14}\,Ba^4bx^{14} + \frac{5}{7}\,Aa^3b^2x^{14} + \frac{1}{11}\,Ba^5x^{11} + \frac{5}{11}\,Aa^4bx^{11} + \frac{1}{8}\,Aa^5x^8$$

```
[In] integrate(x^7*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/26*B*b^5*x^26 + 5/23*B*a*b^4*x^23 + 1/23*A*b^5*x^23 + 1/2*B*a^2*b^3*x^20
+ 1/4*A*a*b^4*x^20 + 10/17*B*a^3*b^2*x^17 + 10/17*A*a^2*b^3*x^17 + 5/14*B*a
^4*b*x^14 + 5/7*A*a^3*b^2*x^14 + 1/11*B*a^5*x^11 + 5/11*A*a^4*b*x^11 + 1/8*
A*a^5*x^8
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int x^7\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=x^{11}\left(\frac{B\,a^5}{11}+\frac{5\,A\,b\,a^4}{11}\right)+x^{23}\left(\frac{A\,b^5}{23}+\frac{5\,B\,a\,b^4}{23}\right)+\frac{A\,a^5\,x^8}{8}+\frac{B\,b^5\,x^{26}}{26}+\frac{10\,a^2\,b^2\,x^{17}\,(A\,b+B\,a)}{17}+\frac{5\,a^3\,b\,x^{14}\,(2\,A\,b+B\,a)}{14}+\frac{a\,b^3\,x^{20}\,(A\,b+2\,B\,a)}{4}$$

```
[In] int(x^7*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^11*((B*a^5)/11 + (5*A*a^4*b)/11) + x^23*((A*b^5)/23 + (5*B*a*b^4)/23) + (
A*a^5*x^8)/8 + (B*b^5*x^26)/26 + (10*a^2*b^2*x^17*(A*b + B*a))/17 + (5*a^3*
b*x^14*(2*A*b + B*a))/14 + (a*b^3*x^20*(A*b + 2*B*a))/4
```

3.26 $\int x^6(a+bx^3)^5 (A+Bx^3) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int x^6 \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx = \frac{1}{7}a^5Ax^7 + \frac{1}{10}a^4(5Ab+aB)x^{10} + \frac{5}{13}a^3b(2Ab+aB)x^{13} + \frac{5}{8}a^2b^2(Ab+aB)x^{16} + \frac{5}{19}ab^3(Ab+2aB)x^{19} + \frac{1}{22}b^4(Ab+5aB)x^{22} + \frac{1}{25}b^5Bx^{25}$$

```
[Out] 1/7*a^5*A*x^7+1/10*a^4*(5*A*b+B*a)*x^10+5/13*a^3*b*(2*A*b+B*a)*x^13+5/8*a^2
*b^2*(A*b+B*a)*x^16+5/19*a*b^3*(A*b+2*B*a)*x^19+1/22*b^4*(A*b+5*B*a)*x^22+1
/25*b^5*B*x^25
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int x^6 \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx = \frac{1}{7}a^5Ax^7 + \frac{1}{10}a^4x^{10}(aB+5Ab) + \frac{5}{13}a^3bx^{13}(aB+2Ab) + \frac{5}{8}a^2b^2x^{16}(aB+Ab) + \frac{1}{22}b^4x^{22}(5aB+Ab) + \frac{5}{19}ab^3x^{19}(2aB+Ab) + \frac{1}{25}b^5Bx^{25}$$

```
[In] Int[x^6*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^7)/7 + (a^4*(5*A*b + a*B)*x^10)/10 + (5*a^3*b*(2*A*b + a*B)*x^13)/
13 + (5*a^2*b^2*(A*b + a*B)*x^16)/8 + (5*a*b^3*(A*b + 2*a*B)*x^19)/19 + (b^
4*(A*b + 5*a*B)*x^22)/22 + (b^5*B*x^25)/25
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^5Ax^6 + a^4(5Ab+aB)x^9 + 5a^3b(2Ab+aB)x^{12} + 10a^2b^2(Ab+aB)x^{15}\right.\\ &\qquad \left.+ 5ab^3(Ab+2aB)x^{18} + b^4(Ab+5aB)x^{21} + b^5Bx^{24}\right)\,dx\\ &= \frac{1}{7}a^5Ax^7 + \frac{1}{10}a^4(5Ab+aB)x^{10} + \frac{5}{13}a^3b(2Ab+aB)x^{13} + \frac{5}{8}a^2b^2(Ab+aB)x^{16}\\ &\quad + \frac{5}{19}ab^3(Ab+2aB)x^{19} + \frac{1}{22}b^4(Ab+5aB)x^{22} + \frac{1}{25}b^5Bx^{25}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^6\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx &= \frac{1}{7}a^5Ax^7 + \frac{1}{10}a^4(5Ab+aB)x^{10} + \frac{5}{13}a^3b(2Ab+aB)x^{13}\\ &\quad + \frac{5}{8}a^2b^2(Ab+aB)x^{16} + \frac{5}{19}ab^3(Ab+2aB)x^{19}\\ &\quad + \frac{1}{22}b^4(Ab+5aB)x^{22} + \frac{1}{25}b^5Bx^{25}\end{aligned}$$

```
[In] Integrate[x^6*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^7)/7 + (a^4*(5*A*b + a*B)*x^10)/10 + (5*a^3*b*(2*A*b + a*B)*x^13)/
13 + (5*a^2*b^2*(A*b + a*B)*x^16)/8 + (5*a*b^3*(A*b + 2*a*B)*x^19)/19 + (b^
4*(A*b + 5*a*B)*x^22)/22 + (b^5*B*x^25)/25
```

Maple [A] (verified)

Time = 4.31 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

method	result
norman	$\frac{a^5 A x^7}{7}+\left(\frac{1}{2}a^4bA+\frac{1}{10}a^5B\right)x^{10}+\left(\frac{10}{13}a^3b^2A+\frac{5}{13}a^4bB\right)x^{13}+\left(\frac{5}{8}a^2b^3A+\frac{5}{8}a^3b^2B\right)x^{16}+\left(\frac{5}{19}a\,b^4\right.$
default	$\frac{b^5 B\,x^{25}}{25}+\frac{(b^5A+5a\,b^4B)x^{22}}{22}+\frac{(5a\,b^4A+10a^2b^3B)x^{19}}{19}+\frac{(10a^2b^3A+10a^3b^2B)x^{16}}{16}+\frac{(10a^3b^2A+5a^4bB)x^{13}}{13}+\frac{(5a^4bA}{}$
gosper	$\frac{1}{7}a^5Ax^7+\frac{1}{2}x^{10}a^4bA+\frac{1}{10}x^{10}a^5B+\frac{10}{13}x^{13}a^3b^2A+\frac{5}{13}x^{13}a^4bB+\frac{5}{8}x^{16}a^2b^3A+\frac{5}{8}x^{16}a^3b^2B+\frac{5}{19}x$
risch	$\frac{1}{7}a^5Ax^7+\frac{1}{2}x^{10}a^4bA+\frac{1}{10}x^{10}a^5B+\frac{10}{13}x^{13}a^3b^2A+\frac{5}{13}x^{13}a^4bB+\frac{5}{8}x^{16}a^2b^3A+\frac{5}{8}x^{16}a^3b^2B+\frac{5}{19}x$
parallelrisch	$\frac{1}{7}a^5Ax^7+\frac{1}{2}x^{10}a^4bA+\frac{1}{10}x^{10}a^5B+\frac{10}{13}x^{13}a^3b^2A+\frac{5}{13}x^{13}a^4bB+\frac{5}{8}x^{16}a^2b^3A+\frac{5}{8}x^{16}a^3b^2B+\frac{5}{19}x$

```
[In] int(x^6*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*a^5*A*x^7+(1/2*a^4*b*A+1/10*a^5*B)*x^10+(10/13*a^3*b^2*A+5/13*a^4*b*B)*
x^13+(5/8*a^2*b^3*A+5/8*a^3*b^2*B)*x^16+(5/19*a*b^4*A+10/19*a^2*b^3*B)*x^19
+(1/22*b^5*A+5/22*a*b^4*B)*x^22+1/25*b^5*B*x^25
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^6(a+bx^3)^5(A+Bx^3)\,dx=\frac{1}{25}Bb^5x^{25}+\frac{1}{22}(5Bab^4+Ab^5)x^{22}+\frac{5}{19}(2Ba^2b^3+Aab^4)x^{19}+\frac{5}{8}(Ba^3b^2+Aa^2b^3)x^{16}+\frac{5}{13}(Ba^4b+2Aa^3b^2)x^{13}+\frac{1}{7}Aa^5x^7+\frac{1}{10}(Ba^5+5Aa^4b)x^{10}$$

```
[In] integrate(x^6*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/25*B*b^5*x^25 + 1/22*(5*B*a*b^4 + A*b^5)*x^22 + 5/19*(2*B*a^2*b^3 + A*a*b
^4)*x^19 + 5/8*(B*a^3*b^2 + A*a^2*b^3)*x^16 + 5/13*(B*a^4*b + 2*A*a^3*b^2)*
x^13 + 1/7*A*a^5*x^7 + 1/10*(B*a^5 + 5*A*a^4*b)*x^10
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.16

$$\int x^6(a+bx^3)^5(A+Bx^3)\,dx=\frac{Aa^5x^7}{7}+\frac{Bb^5x^{25}}{25}+x^{22}\left(\frac{Ab^5}{22}+\frac{5Bab^4}{22}\right)+x^{19}\cdot\left(\frac{5Aab^4}{19}+\frac{10Ba^2b^3}{19}\right)+x^{16}\cdot\left(\frac{5Aa^2b^3}{8}+\frac{5Ba^3b^2}{8}\right)+x^{13}\cdot\left(\frac{10Aa^3b^2}{13}+\frac{5Ba^4b}{13}\right)+x^{10}\left(\frac{Aa^4b}{2}+\frac{Ba^5}{10}\right)$$

```
[In] integrate(x**6*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**7/7 + B*b**5*x**25/25 + x**22*(A*b**5/22 + 5*B*a*b**4/22) + x**19
*(5*A*a*b**4/19 + 10*B*a**2*b**3/19) + x**16*(5*A*a**2*b**3/8 + 5*B*a**3*b*
*2/8) + x**13*(10*A*a**3*b**2/13 + 5*B*a**4*b/13) + x**10*(A*a**4*b/2 + B*a
**5/10)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^6(a+bx^3)^5 (A+Bx^3)\ dx = \frac{1}{25}\,Bb^5x^{25}+\frac{1}{22}\,(5\,Bab^4+Ab^5)x^{22}+\frac{5}{19}\,(2\,Ba^2b^3+Aab^4)x^{19} + \frac{5}{8}\,(Ba^3b^2+Aa^2b^3)x^{16}+\frac{5}{13}\,(Ba^4b+2\,Aa^3b^2)x^{13} + \frac{1}{7}\,Aa^5x^7+\frac{1}{10}\,(Ba^5+5\,Aa^4b)x^{10}$$

```
[In] integrate(x^6*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/25*B*b^5*x^25 + 1/22*(5*B*a*b^4 + A*b^5)*x^22 + 5/19*(2*B*a^2*b^3 + A*a*b
^4)*x^19 + 5/8*(B*a^3*b^2 + A*a^2*b^3)*x^16 + 5/13*(B*a^4*b + 2*A*a^3*b^2)*
x^13 + 1/7*A*a^5*x^7 + 1/10*(B*a^5 + 5*A*a^4*b)*x^10
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07

$$\int x^6(a+bx^3)^5 (A+Bx^3)\ dx = \frac{1}{25}\,Bb^5x^{25}+\frac{5}{22}\,Bab^4x^{22}+\frac{1}{22}\,Ab^5x^{22}+\frac{10}{19}\,Ba^2b^3x^{19} + \frac{5}{19}\,Aab^4x^{19}+\frac{5}{8}\,Ba^3b^2x^{16}+\frac{5}{8}\,Aa^2b^3x^{16}+\frac{5}{13}\,Ba^4bx^{13} + \frac{10}{13}\,Aa^3b^2x^{13}+\frac{1}{10}\,Ba^5x^{10}+\frac{1}{2}\,Aa^4bx^{10}+\frac{1}{7}\,Aa^5x^7$$

```
[In] integrate(x^6*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/25*B*b^5*x^25 + 5/22*B*a*b^4*x^22 + 1/22*A*b^5*x^22 + 10/19*B*a^2*b^3*x^1
9 + 5/19*A*a*b^4*x^19 + 5/8*B*a^3*b^2*x^16 + 5/8*A*a^2*b^3*x^16 + 5/13*B*a^
4*b*x^13 + 10/13*A*a^3*b^2*x^13 + 1/10*B*a^5*x^10 + 1/2*A*a^4*b*x^10 + 1/7*
A*a^5*x^7
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int x^6 \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx = x^{10} \left(\frac{B\,a^5}{10}+\frac{A\,b\,a^4}{2}\right)+x^{22} \left(\frac{A\,b^5}{22}+\frac{5\,B\,a\,b^4}{22}\right) + \frac{A\,a^5\,x^7}{7}+\frac{B\,b^5\,x^{25}}{25}+\frac{5\,a^2\,b^2\,x^{16}\,(A\,b+B\,a)}{8} + \frac{5\,a^3\,b\,x^{13}\,(2\,A\,b+B\,a)}{13}+\frac{5\,a\,b^3\,x^{19}\,(A\,b+2\,B\,a)}{19}$$

```
[In] int(x^6*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^10*((B*a^5)/10 + (A*a^4*b)/2) + x^22*((A*b^5)/22 + (5*B*a*b^4)/22) + (A*a
^5*x^7)/7 + (B*b^5*x^25)/25 + (5*a^2*b^2*x^16*(A*b + B*a))/8 + (5*a^3*b*x^1
3*(2*A*b + B*a))/13 + (5*a*b^3*x^19*(A*b + 2*B*a))/19
```

3.27 $\int x^5(a+bx^3)^5 (A+Bx^3)\, dx$

Optimal result

Integrand size = 20, antiderivative size = 67

$$\int x^5(a+bx^3)^5 (A+Bx^3)\, dx = -\frac{a(Ab-aB)(a+bx^3)^6}{18b^3} + \frac{(Ab-2aB)(a+bx^3)^7}{21b^3} + \frac{B(a+bx^3)^8}{24b^3}$$

```
[Out] -1/18*a*(A*b-B*a)*(b*x^3+a)^6/b^3+1/21*(A*b-2*B*a)*(b*x^3+a)^7/b^3+1/24*B*(
b*x^3+a)^8/b^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 77}

$$\int x^5(a+bx^3)^5 (A+Bx^3)\, dx = \frac{(a+bx^3)^7 (Ab-2aB)}{21b^3} - \frac{a(a+bx^3)^6 (Ab-aB)}{18b^3} + \frac{B(a+bx^3)^8}{24b^3}$$

```
[In] Int[x^5*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] -1/18*(a*(A*b - a*B)*(a + b*x^3)^6)/b^3 + ((A*b - 2*a*B)*(a + b*x^3)^7)/(21
*b^3) + (B*(a + b*x^3)^8)/(24*b^3)
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
```

```
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int x(a+bx)^5(A+Bx)\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)(a+bx)^5}{b^2}+\frac{(Ab-2aB)(a+bx)^6}{b^2}+\frac{B(a+bx)^7}{b^2}\right)\,dx, x, x^3\right)\\ &= -\frac{a(Ab-aB)\left(a+bx^3\right)^6}{18b^3}+\frac{(Ab-2aB)\left(a+bx^3\right)^7}{21b^3}+\frac{B\left(a+bx^3\right)^8}{24b^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.60

$$\int x^5\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx = \frac{1}{504}x^6\left(84a^5A+56a^4(5Ab+aB)x^3+210a^3b(2Ab+aB)x^6 + 336a^2b^2(Ab+aB)x^9+140ab^3(Ab+2aB)x^{12} + 24b^4(Ab+5aB)x^{15}+21b^5Bx^{18}\right)$$

```
[In] Integrate[x^5*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (x^6*(84*a^5*A + 56*a^4*(5*A*b + a*B)*x^3 + 210*a^3*b*(2*A*b + a*B)*x^6 + 3
36*a^2*b^2*(A*b + a*B)*x^9 + 140*a*b^3*(A*b + 2*a*B)*x^12 + 24*b^4*(A*b + 5
*a*B)*x^15 + 21*b^5*B*x^18))/504
```

Maple [A] (verified)

Time = 4.01 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.81

method	result
norman	$\frac{a^5 A x^6}{6}+\left(\frac{5}{9}a^4bA+\frac{1}{9}a^5B\right)x^9+\left(\frac{5}{6}a^3b^2A+\frac{5}{12}a^4bB\right)x^{12}+\left(\frac{2}{3}a^2b^3A+\frac{2}{3}a^3b^2B\right)x^{15}+\left(\frac{5}{18}a\,b^4A\right.$
default	$\frac{b^5 B x^{24}}{24}+\frac{(b^5A+5a\,b^4B)x^{21}}{21}+\frac{(5a\,b^4A+10a^2b^3B)x^{18}}{18}+\frac{(10a^2b^3A+10a^3b^2B)x^{15}}{15}+\frac{(10a^3b^2A+5a^4bB)x^{12}}{12}+\frac{(5a^4b}{}$
gosper	$\frac{1}{6}a^5Ax^6+\frac{5}{9}x^9a^4bA+\frac{1}{9}x^9a^5B+\frac{5}{6}x^{12}a^3b^2A+\frac{5}{12}x^{12}a^4bB+\frac{2}{3}x^{15}a^2b^3A+\frac{2}{3}x^{15}a^3b^2B+\frac{5}{18}x^{18}$
risch	$\frac{1}{6}a^5Ax^6+\frac{5}{9}x^9a^4bA+\frac{1}{9}x^9a^5B+\frac{5}{6}x^{12}a^3b^2A+\frac{5}{12}x^{12}a^4bB+\frac{2}{3}x^{15}a^2b^3A+\frac{2}{3}x^{15}a^3b^2B+\frac{5}{18}x^{18}$
parallelrisch	$\frac{1}{6}a^5Ax^6+\frac{5}{9}x^9a^4bA+\frac{1}{9}x^9a^5B+\frac{5}{6}x^{12}a^3b^2A+\frac{5}{12}x^{12}a^4bB+\frac{2}{3}x^{15}a^2b^3A+\frac{2}{3}x^{15}a^3b^2B+\frac{5}{18}x^{18}$

```
[In] int(x^5*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a^5*A*x^6+(5/9*a^4*b*A+1/9*a^5*B)*x^9+(5/6*a^3*b^2*A+5/12*a^4*b*B)*x^12
+(2/3*a^2*b^3*A+2/3*a^3*b^2*B)*x^15+(5/18*a*b^4*A+5/9*a^2*b^3*B)*x^18+(1/21
*b^5*A+5/21*a*b^4*B)*x^21+1/24*b^5*B*x^24
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.78

$$\int x^5\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=\frac{1}{24}Bb^5x^{24}+\frac{1}{21}\left(5Bab^4+Ab^5\right)x^{21}+\frac{5}{18}\left(2Ba^2b^3+Aab^4\right)x^{18}+\frac{2}{3}\left(Ba^3b^2+Aa^2b^3\right)x^{15}+\frac{5}{12}\left(Ba^4b+2Aa^3b^2\right)x^{12}+\frac{1}{6}Aa^5x^6+\frac{1}{9}\left(Ba^5+5Aa^4b\right)x^9$$

```
[In] integrate(x^5*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/24*B*b^5*x^24 + 1/21*(5*B*a*b^4 + A*b^5)*x^21 + 5/18*(2*B*a^2*b^3 + A*a*b
^4)*x^18 + 2/3*(B*a^3*b^2 + A*a^2*b^3)*x^15 + 5/12*(B*a^4*b + 2*A*a^3*b^2)*
x^12 + 1/6*A*a^5*x^6 + 1/9*(B*a^5 + 5*A*a^4*b)*x^9
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(60) = 120.

Time = 0.03 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.06

$$\int x^5(a+bx^3)^5(A+Bx^3)\,dx = \frac{Aa^5x^6}{6} + \frac{Bb^5x^{24}}{24} + x^{21}\left(\frac{Ab^5}{21} + \frac{5Bab^4}{21}\right) + x^{18} \cdot \left(\frac{5Aab^4}{18} + \frac{5Ba^2b^3}{9}\right) + x^{15} \cdot \left(\frac{2Aa^2b^3}{3} + \frac{2Ba^3b^2}{3}\right) + x^{12} \cdot \left(\frac{5Aa^3b^2}{6} + \frac{5Ba^4b}{12}\right) + x^9 \cdot \left(\frac{5Aa^4b}{9} + \frac{Ba^5}{9}\right)$$

```
[In] integrate(x**5*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**6/6 + B*b**5*x**24/24 + x**21*(A*b**5/21 + 5*B*a*b**4/21) + x**18
*(5*A*a*b**4/18 + 5*B*a**2*b**3/9) + x**15*(2*A*a**2*b**3/3 + 2*B*a**3*b**2
/3) + x**12*(5*A*a**3*b**2/6 + 5*B*a**4*b/12) + x**9*(5*A*a**4*b/9 + B*a**5
/9)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.78

$$\int x^5(a+bx^3)^5(A+Bx^3)\,dx = \frac{1}{24}Bb^5x^{24} + \frac{1}{21}\left(5\,Bab^4 + Ab^5\right)x^{21} + \frac{5}{18}\left(2\,Ba^2b^3 + Aab^4\right)x^{18} + \frac{2}{3}\left(Ba^3b^2 + Aa^2b^3\right)x^{15} + \frac{5}{12}\left(Ba^4b + 2\,Aa^3b^2\right)x^{12} + \frac{1}{6}Aa^5x^6 + \frac{1}{9}\left(Ba^5 + 5\,Aa^4b\right)x^9$$

```
[In] integrate(x^5*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/24*B*b^5*x^24 + 1/21*(5*B*a*b^4 + A*b^5)*x^21 + 5/18*(2*B*a^2*b^3 + A*a*b
^4)*x^18 + 2/3*(B*a^3*b^2 + A*a^2*b^3)*x^15 + 5/12*(B*a^4*b + 2*A*a^3*b^2)*
x^12 + 1/6*A*a^5*x^6 + 1/9*(B*a^5 + 5*A*a^4*b)*x^9
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(62) = 124$.

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.87

$$\int x^5(a+bx^3)^5\left(A+Bx^3\right)\,dx = \frac{1}{24}\,Bb^5x^{24} + \frac{5}{21}\,Bab^4x^{21} + \frac{1}{21}\,Ab^5x^{21} + \frac{5}{9}\,Ba^2b^3x^{18} + \frac{5}{18}\,Aab^4x^{18} + \frac{2}{3}\,Ba^3b^2x^{15} + \frac{2}{3}\,Aa^2b^3x^{15} + \frac{5}{12}\,Ba^4bx^{12} + \frac{5}{6}\,Aa^3b^2x^{12} + \frac{1}{9}\,Ba^5x^9 + \frac{5}{9}\,Aa^4bx^9 + \frac{1}{6}\,Aa^5x^6$$

```
[In] integrate(x^5*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/24*B*b^5*x^24 + 5/21*B*a*b^4*x^21 + 1/21*A*b^5*x^21 + 5/9*B*a^2*b^3*x^18
+ 5/18*A*a*b^4*x^18 + 2/3*B*a^3*b^2*x^15 + 2/3*A*a^2*b^3*x^15 + 5/12*B*a^4*
b*x^12 + 5/6*A*a^3*b^2*x^12 + 1/9*B*a^5*x^9 + 5/9*A*a^4*b*x^9 + 1/6*A*a^5*x
^6
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.60

$$\int x^5(a+bx^3)^5\left(A+Bx^3\right)\,dx = x^9\left(\frac{B\,a^5}{9}+\frac{5\,A\,b\,a^4}{9}\right) + x^{21}\left(\frac{A\,b^5}{21}+\frac{5\,B\,a\,b^4}{21}\right) + \frac{A\,a^5\,x^6}{6} + \frac{B\,b^5\,x^{24}}{24} + \frac{2\,a^2\,b^2\,x^{15}\,(A\,b+B\,a)}{3} + \frac{5\,a^3\,b\,x^{12}\,(2\,A\,b+B\,a)}{12} + \frac{5\,a\,b^3\,x^{18}\,(A\,b+2\,B\,a)}{18}$$

```
[In] int(x^5*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^9*((B*a^5)/9 + (5*A*a^4*b)/9) + x^21*((A*b^5)/21 + (5*B*a*b^4)/21) + (A*a
^5*x^6)/6 + (B*b^5*x^24)/24 + (2*a^2*b^2*x^15*(A*b + B*a))/3 + (5*a^3*b*x^1
2*(2*A*b + B*a))/12 + (5*a*b^3*x^18*(A*b + 2*B*a))/18
```

3.28 $\int x^4(a+bx^3)^5(A+Bx^3)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int x^4(a+bx^3)^5(A+Bx^3)\,dx = \frac{1}{5}a^5Ax^5+\frac{1}{8}a^4(5Ab+aB)x^8+\frac{5}{11}a^3b(2Ab+aB)x^{11} + \frac{5}{7}a^2b^2(Ab+aB)x^{14}+\frac{5}{17}ab^3(Ab+2aB)x^{17} + \frac{1}{20}b^4(Ab+5aB)x^{20}+\frac{1}{23}b^5Bx^{23}$$

```
[Out] 1/5*a^5*A*x^5+1/8*a^4*(5*A*b+B*a)*x^8+5/11*a^3*b*(2*A*b+B*a)*x^11+5/7*a^2*b
^2*(A*b+B*a)*x^14+5/17*a*b^3*(A*b+2*B*a)*x^17+1/20*b^4*(A*b+5*B*a)*x^20+1/2
3*b^5*B*x^23
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int x^4(a+bx^3)^5(A+Bx^3)\,dx = \frac{1}{5}a^5Ax^5+\frac{1}{8}a^4x^8(aB+5Ab)+\frac{5}{11}a^3bx^{11}(aB+2Ab) + \frac{5}{7}a^2b^2x^{14}(aB+Ab)+\frac{1}{20}b^4x^{20}(5aB+Ab) + \frac{5}{17}ab^3x^{17}(2aB+Ab)+\frac{1}{23}b^5Bx^{23}$$

```
[In] Int[x^4*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^5)/5 + (a^4*(5*A*b + a*B)*x^8)/8 + (5*a^3*b*(2*A*b + a*B)*x^11)/11
 + (5*a^2*b^2*(A*b + a*B)*x^14)/7 + (5*a*b^3*(A*b + 2*a*B)*x^17)/17 + (b^4*
(A*b + 5*a*B)*x^20)/20 + (b^5*B*x^23)/23
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^5Ax^4 + a^4(5Ab+aB)x^7 + 5a^3b(2Ab+aB)x^{10} + 10a^2b^2(Ab+aB)x^{13} \right. \\ &\qquad \left. + 5ab^3(Ab+2aB)x^{16} + b^4(Ab+5aB)x^{19} + b^5Bx^{22}\right)\,dx \\ &= \frac{1}{5}a^5Ax^5 + \frac{1}{8}a^4(5Ab+aB)x^8 + \frac{5}{11}a^3b(2Ab+aB)x^{11} + \frac{5}{7}a^2b^2(Ab+aB)x^{14} \\ &\quad + \frac{5}{17}ab^3(Ab+2aB)x^{17} + \frac{1}{20}b^4(Ab+5aB)x^{20} + \frac{1}{23}b^5Bx^{23}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^4\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx &= \frac{1}{5}a^5Ax^5 + \frac{1}{8}a^4(5Ab+aB)x^8 + \frac{5}{11}a^3b(2Ab+aB)x^{11} \\ &\quad + \frac{5}{7}a^2b^2(Ab+aB)x^{14} + \frac{5}{17}ab^3(Ab+2aB)x^{17} \\ &\quad + \frac{1}{20}b^4(Ab+5aB)x^{20} + \frac{1}{23}b^5Bx^{23}\end{aligned}$$

```
[In] Integrate[x^4*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^5)/5 + (a^4*(5*A*b + a*B)*x^8)/8 + (5*a^3*b*(2*A*b + a*B)*x^11)/11
 + (5*a^2*b^2*(A*b + a*B)*x^14)/7 + (5*a*b^3*(A*b + 2*a*B)*x^17)/17 + (b^4*
(A*b + 5*a*B)*x^20)/20 + (b^5*B*x^23)/23
```

Maple [A] (verified)

Time = 4.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

method	result
norman	$\frac{a^5 A x^5}{5} + \left(\frac{5}{8}a^4bA + \frac{1}{8}a^5B\right)x^8 + \left(\frac{10}{11}a^3b^2A + \frac{5}{11}a^4bB\right)x^{11} + \left(\frac{5}{7}a^2b^3A + \frac{5}{7}a^3b^2B\right)x^{14} + \left(\frac{5}{17}a\,b^4A\right.$
default	$\frac{b^5 B\,x^{23}}{23} + \frac{(b^5A+5a\,b^4B)x^{20}}{20} + \frac{(5a\,b^4A+10a^2b^3B)x^{17}}{17} + \frac{(10a^2b^3A+10a^3b^2B)x^{14}}{14} + \frac{(10a^3b^2A+5a^4bB)x^{11}}{11} + \frac{(5a^4bA}{}$
gosper	$\frac{1}{5}a^5A\,x^5 + \frac{5}{8}x^8a^4bA + \frac{1}{8}x^8a^5B + \frac{10}{11}x^{11}a^3b^2A + \frac{5}{11}x^{11}a^4bB + \frac{5}{7}x^{14}a^2b^3A + \frac{5}{7}x^{14}a^3b^2B + \frac{5}{17}x^{17}$
risch	$\frac{1}{5}a^5A\,x^5 + \frac{5}{8}x^8a^4bA + \frac{1}{8}x^8a^5B + \frac{10}{11}x^{11}a^3b^2A + \frac{5}{11}x^{11}a^4bB + \frac{5}{7}x^{14}a^2b^3A + \frac{5}{7}x^{14}a^3b^2B + \frac{5}{17}x^{17}$
parallelrisch	$\frac{1}{5}a^5A\,x^5 + \frac{5}{8}x^8a^4bA + \frac{1}{8}x^8a^5B + \frac{10}{11}x^{11}a^3b^2A + \frac{5}{11}x^{11}a^4bB + \frac{5}{7}x^{14}a^2b^3A + \frac{5}{7}x^{14}a^3b^2B + \frac{5}{17}x^{17}$

```
[In] int(x^4*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a^5*A*x^5+(5/8*a^4*b*A+1/8*a^5*B)*x^8+(10/11*a^3*b^2*A+5/11*a^4*b*B)*x^
11+(5/7*a^2*b^3*A+5/7*a^3*b^2*B)*x^14+(5/17*a*b^4*A+10/17*a^2*b^3*B)*x^17+(
1/20*b^5*A+1/4*a*b^4*B)*x^20+1/23*b^5*B*x^23
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^4\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx = \frac{1}{23}\,Bb^5x^{23} + \frac{1}{20}\,\left(5\,Bab^4 + Ab^5\right)x^{20} + \frac{5}{17}\,\left(2\,Ba^2b^3 + Aab^4\right)x^{17} + \frac{5}{7}\,\left(Ba^3b^2 + Aa^2b^3\right)x^{14} + \frac{5}{11}\,\left(Ba^4b + 2\,Aa^3b^2\right)x^{11} + \frac{1}{5}\,Aa^5x^5 + \frac{1}{8}\,\left(Ba^5 + 5\,Aa^4b\right)x^8$$

```
[In] integrate(x^4*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/23*B*b^5*x^23 + 1/20*(5*B*a*b^4 + A*b^5)*x^20 + 5/17*(2*B*a^2*b^3 + A*a*b
^4)*x^17 + 5/7*(B*a^3*b^2 + A*a^2*b^3)*x^14 + 5/11*(B*a^4*b + 2*A*a^3*b^2)*
x^11 + 1/5*A*a^5*x^5 + 1/8*(B*a^5 + 5*A*a^4*b)*x^8
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.16

$$\int x^4\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx = \frac{Aa^5x^5}{5} + \frac{Bb^5x^{23}}{23} + x^{20}\left(\frac{Ab^5}{20} + \frac{Bab^4}{4}\right) + x^{17}\cdot\left(\frac{5Aab^4}{17} + \frac{10Ba^2b^3}{17}\right) + x^{14}\cdot\left(\frac{5Aa^2b^3}{7} + \frac{5Ba^3b^2}{7}\right) + x^{11}\cdot\left(\frac{10Aa^3b^2}{11} + \frac{5Ba^4b}{11}\right) + x^8\cdot\left(\frac{5Aa^4b}{8} + \frac{Ba^5}{8}\right)$$

```
[In] integrate(x**4*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**5/5 + B*b**5*x**23/23 + x**20*(A*b**5/20 + B*a*b**4/4) + x**17*(5
*A*a*b**4/17 + 10*B*a**2*b**3/17) + x**14*(5*A*a**2*b**3/7 + 5*B*a**3*b**2/
7) + x**11*(10*A*a**3*b**2/11 + 5*B*a**4*b/11) + x**8*(5*A*a**4*b/8 + B*a**
5/8)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^4 \left(a+bx^3\right)^5 \left(A+Bx^3\right)\,dx = \frac{1}{23}\,Bb^5x^{23} + \frac{1}{20}\left(5\,Bab^4 + Ab^5\right)x^{20} + \frac{5}{17}\left(2\,Ba^2b^3 + Aab^4\right)x^{17} + \frac{5}{7}\left(Ba^3b^2 + Aa^2b^3\right)x^{14} + \frac{5}{11}\left(Ba^4b + 2\,Aa^3b^2\right)x^{11} + \frac{1}{5}\,Aa^5x^5 + \frac{1}{8}\left(Ba^5 + 5\,Aa^4b\right)x^8$$

```
[In] integrate(x^4*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/23*B*b^5*x^23 + 1/20*(5*B*a*b^4 + A*b^5)*x^20 + 5/17*(2*B*a^2*b^3 + A*a*b
^4)*x^17 + 5/7*(B*a^3*b^2 + A*a^2*b^3)*x^14 + 5/11*(B*a^4*b + 2*A*a^3*b^2)*
x^11 + 1/5*A*a^5*x^5 + 1/8*(B*a^5 + 5*A*a^4*b)*x^8
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07

$$\int x^4 \left(a+bx^3\right)^5 \left(A+Bx^3\right)\,dx = \frac{1}{23}\,Bb^5x^{23} + \frac{1}{4}\,Bab^4x^{20} + \frac{1}{20}\,Ab^5x^{20} + \frac{10}{17}\,Ba^2b^3x^{17} + \frac{5}{17}\,Aab^4x^{17} + \frac{5}{7}\,Ba^3b^2x^{14} + \frac{5}{7}\,Aa^2b^3x^{14} + \frac{5}{11}\,Ba^4bx^{11} + \frac{10}{11}\,Aa^3b^2x^{11} + \frac{1}{8}\,Ba^5x^8 + \frac{5}{8}\,Aa^4bx^8 + \frac{1}{5}\,Aa^5x^5$$

```
[In] integrate(x^4*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/23*B*b^5*x^23 + 1/4*B*a*b^4*x^20 + 1/20*A*b^5*x^20 + 10/17*B*a^2*b^3*x^17
 + 5/17*A*a*b^4*x^17 + 5/7*B*a^3*b^2*x^14 + 5/7*A*a^2*b^3*x^14 + 5/11*B*a^4
*b*x^11 + 10/11*A*a^3*b^2*x^11 + 1/8*B*a^5*x^8 + 5/8*A*a^4*b*x^8 + 1/5*A*a^
5*x^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int x^4 (a+bx^3)^5 (A+Bx^3)\,dx = x^8\left(\frac{B\,a^5}{8}+\frac{5\,A\,b\,a^4}{8}\right)+x^{20}\left(\frac{A\,b^5}{20}+\frac{B\,a\,b^4}{4}\right) + \frac{A\,a^5\,x^5}{5}+\frac{B\,b^5\,x^{23}}{23}+\frac{5\,a^2\,b^2\,x^{14}\,(A\,b+B\,a)}{7} + \frac{5\,a^3\,b\,x^{11}\,(2\,A\,b+B\,a)}{11}+\frac{5\,a\,b^3\,x^{17}\,(A\,b+2\,B\,a)}{17}$$

```
[In] int(x^4*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^8*((B*a^5)/8 + (5*A*a^4*b)/8) + x^20*((A*b^5)/20 + (B*a*b^4)/4) + (A*a^5*
x^5)/5 + (B*b^5*x^23)/23 + (5*a^2*b^2*x^14*(A*b + B*a))/7 + (5*a^3*b*x^11*(
2*A*b + B*a))/11 + (5*a*b^3*x^17*(A*b + 2*B*a))/17
```

3.29 $\int x^3(a+bx^3)^5 (A+Bx^3)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int x^3(a+bx^3)^5 (A+Bx^3)\,dx = \frac{1}{4}a^5Ax^4+\frac{1}{7}a^4(5Ab+aB)x^7+\frac{1}{2}a^3b(2Ab+aB)x^{10} + \frac{10}{13}a^2b^2(Ab+aB)x^{13}+\frac{5}{16}ab^3(Ab+2aB)x^{16} + \frac{1}{19}b^4(Ab+5aB)x^{19}+\frac{1}{22}b^5Bx^{22}$$

```
[Out] 1/4*a^5*A*x^4+1/7*a^4*(5*A*b+B*a)*x^7+1/2*a^3*b*(2*A*b+B*a)*x^10+10/13*a^2*
b^2*(A*b+B*a)*x^13+5/16*a*b^3*(A*b+2*B*a)*x^16+1/19*b^4*(A*b+5*B*a)*x^19+1/
22*b^5*B*x^22
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int x^3(a+bx^3)^5 (A+Bx^3)\,dx = \frac{1}{4}a^5Ax^4+\frac{1}{7}a^4x^7(aB+5Ab)+\frac{1}{2}a^3bx^{10}(aB+2Ab) + \frac{10}{13}a^2b^2x^{13}(aB+Ab)+\frac{1}{19}b^4x^{19}(5aB+Ab) + \frac{5}{16}ab^3x^{16}(2aB+Ab)+\frac{1}{22}b^5Bx^{22}$$

```
[In] Int[x^3*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^4)/4 + (a^4*(5*A*b + a*B)*x^7)/7 + (a^3*b*(2*A*b + a*B)*x^10)/2 +
(10*a^2*b^2*(A*b + a*B)*x^13)/13 + (5*a*b^3*(A*b + 2*a*B)*x^16)/16 + (b^4*(
A*b + 5*a*B)*x^19)/19 + (b^5*B*x^22)/22
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^5Ax^3+a^4(5Ab+aB)x^6+5a^3b(2Ab+aB)x^9+10a^2b^2(Ab+aB)x^{12}\right.\\&\qquad\left.+5ab^3(Ab+2aB)x^{15}+b^4(Ab+5aB)x^{18}+b^5Bx^{21}\right)\,dx\\&=\frac{1}{4}a^5Ax^4+\frac{1}{7}a^4(5Ab+aB)x^7+\frac{1}{2}a^3b(2Ab+aB)x^{10}+\frac{10}{13}a^2b^2(Ab+aB)x^{13}\\&\quad+\frac{5}{16}ab^3(Ab+2aB)x^{16}+\frac{1}{19}b^4(Ab+5aB)x^{19}+\frac{1}{22}b^5Bx^{22}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^3\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx &= \frac{1}{4}a^5Ax^4+\frac{1}{7}a^4(5Ab+aB)x^7+\frac{1}{2}a^3b(2Ab+aB)x^{10}\\&\quad+\frac{10}{13}a^2b^2(Ab+aB)x^{13}+\frac{5}{16}ab^3(Ab+2aB)x^{16}\\&\quad+\frac{1}{19}b^4(Ab+5aB)x^{19}+\frac{1}{22}b^5Bx^{22}\end{aligned}$$

```
[In] Integrate[x^3*(a + b*x^3)^5*(A + B*x^3),x]
[Out] (a^5*A*x^4)/4 + (a^4*(5*A*b + a*B)*x^7)/7 + (a^3*b*(2*A*b + a*B)*x^10)/2 +
(10*a^2*b^2*(A*b + a*B)*x^13)/13 + (5*a*b^3*(A*b + 2*a*B)*x^16)/16 + (b^4*(
A*b + 5*a*B)*x^19)/19 + (b^5*B*x^22)/22
```

Maple [A] (verified)

Time = 4.32 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03

method	result
norman	$\frac{a^5 A x^4}{4}+\left(\frac{5}{7}a^4bA+\frac{1}{7}a^5B\right)x^7+\left(a^3b^2A+\frac{1}{2}a^4bB\right)x^{10}+\left(\frac{10}{13}a^2b^3A+\frac{10}{13}a^3b^2B\right)x^{13}+\left(\frac{5}{16}a\,b^4A\right.$
default	$\frac{b^5 B x^{22}}{22}+\frac{(b^5A+5a\,b^4B)x^{19}}{19}+\frac{(5a\,b^4A+10a^2b^3B)x^{16}}{16}+\frac{(10a^2b^3A+10a^3b^2B)x^{13}}{13}+\frac{(10a^3b^2A+5a^4bB)x^{10}}{10}+\frac{(5a^4b}{}$
gosper	$\frac{1}{4}a^5A\,x^4+\frac{5}{7}x^7a^4bA+\frac{1}{7}x^7a^5B+x^{10}a^3b^2A+\frac{1}{2}x^{10}a^4bB+\frac{10}{13}x^{13}a^2b^3A+\frac{10}{13}x^{13}a^3b^2B+\frac{5}{16}x^{16}$
risch	$\frac{1}{4}a^5A\,x^4+\frac{5}{7}x^7a^4bA+\frac{1}{7}x^7a^5B+x^{10}a^3b^2A+\frac{1}{2}x^{10}a^4bB+\frac{10}{13}x^{13}a^2b^3A+\frac{10}{13}x^{13}a^3b^2B+\frac{5}{16}x^{16}$
parallelrisch	$\frac{1}{4}a^5A\,x^4+\frac{5}{7}x^7a^4bA+\frac{1}{7}x^7a^5B+x^{10}a^3b^2A+\frac{1}{2}x^{10}a^4bB+\frac{10}{13}x^{13}a^2b^3A+\frac{10}{13}x^{13}a^3b^2B+\frac{5}{16}x^{16}$

```
[In] int(x^3*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^5*A*x^4+(5/7*a^4*b*A+1/7*a^5*B)*x^7+(a^3*b^2*A+1/2*a^4*b*B)*x^10+(10/
13*a^2*b^3*A+10/13*a^3*b^2*B)*x^13+(5/16*a*b^4*A+5/8*a^2*b^3*B)*x^16+(1/19*
b^5*A+5/19*a*b^4*B)*x^19+1/22*b^5*B*x^22
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^3\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=\frac{1}{22}\,Bb^5x^{22}+\frac{1}{19}\left(5\,Bab^4+Ab^5\right)x^{19}+\frac{5}{16}\left(2\,Ba^2b^3+Aab^4\right)x^{16}+\frac{10}{13}\left(Ba^3b^2+Aa^2b^3\right)x^{13}+\frac{1}{2}\left(Ba^4b+2\,Aa^3b^2\right)x^{10}+\frac{1}{4}\,Aa^5x^4+\frac{1}{7}\left(Ba^5+5\,Aa^4b\right)x^7$$

```
[In] integrate(x^3*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/22*B*b^5*x^22 + 1/19*(5*B*a*b^4 + A*b^5)*x^19 + 5/16*(2*B*a^2*b^3 + A*a*b
^4)*x^16 + 10/13*(B*a^3*b^2 + A*a^2*b^3)*x^13 + 1/2*(B*a^4*b + 2*A*a^3*b^2)
*x^10 + 1/4*A*a^5*x^4 + 1/7*(B*a^5 + 5*A*a^4*b)*x^7
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.14

$$\int x^3\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=\frac{Aa^5x^4}{4}+\frac{Bb^5x^{22}}{22}+x^{19}\left(\frac{Ab^5}{19}+\frac{5Bab^4}{19}\right)+x^{16}\cdot\left(\frac{5Aab^4}{16}+\frac{5Ba^2b^3}{8}\right)+x^{13}\cdot\left(\frac{10Aa^2b^3}{13}+\frac{10Ba^3b^2}{13}\right)+x^{10}\left(Aa^3b^2+\frac{Ba^4b}{2}\right)+x^7\cdot\left(\frac{5Aa^4b}{7}+\frac{Ba^5}{7}\right)$$

```
[In] integrate(x**3*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**4/4 + B*b**5*x**22/22 + x**19*(A*b**5/19 + 5*B*a*b**4/19) + x**16
*(5*A*a*b**4/16 + 5*B*a**2*b**3/8) + x**13*(10*A*a**2*b**3/13 + 10*B*a**3*b
**2/13) + x**10*(A*a**3*b**2 + B*a**4*b/2) + x**7*(5*A*a**4*b/7 + B*a**5/7)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^3(a+bx^3)^5(A+Bx^3)\,dx = \frac{1}{22}Bb^5x^{22} + \frac{1}{19}(5Bab^4 + Ab^5)x^{19} + \frac{5}{16}(2Ba^2b^3 + Aab^4)x^{16} + \frac{10}{13}(Ba^3b^2 + Aa^2b^3)x^{13} + \frac{1}{2}(Ba^4b + 2Aa^3b^2)x^{10} + \frac{1}{4}Aa^5x^4 + \frac{1}{7}(Ba^5 + 5Aa^4b)x^7$$

```
[In] integrate(x^3*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/22*B*b^5*x^22 + 1/19*(5*B*a*b^4 + A*b^5)*x^19 + 5/16*(2*B*a^2*b^3 + A*a*b
^4)*x^16 + 10/13*(B*a^3*b^2 + A*a^2*b^3)*x^13 + 1/2*(B*a^4*b + 2*A*a^3*b^2)
*x^10 + 1/4*A*a^5*x^4 + 1/7*(B*a^5 + 5*A*a^4*b)*x^7
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

$$\int x^3(a+bx^3)^5(A+Bx^3)\,dx = \frac{1}{22}Bb^5x^{22} + \frac{5}{19}Bab^4x^{19} + \frac{1}{19}Ab^5x^{19} + \frac{5}{8}Ba^2b^3x^{16} + \frac{5}{16}Aab^4x^{16} + \frac{10}{13}Ba^3b^2x^{13} + \frac{10}{13}Aa^2b^3x^{13} + \frac{1}{2}Ba^4bx^{10} + Aa^3b^2x^{10} + \frac{1}{7}Ba^5x^7 + \frac{5}{7}Aa^4bx^7 + \frac{1}{4}Aa^5x^4$$

```
[In] integrate(x^3*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/22*B*b^5*x^22 + 5/19*B*a*b^4*x^19 + 1/19*A*b^5*x^19 + 5/8*B*a^2*b^3*x^16
+ 5/16*A*a*b^4*x^16 + 10/13*B*a^3*b^2*x^13 + 10/13*A*a^2*b^3*x^13 + 1/2*B*a
^4*b*x^10 + A*a^3*b^2*x^10 + 1/7*B*a^5*x^7 + 5/7*A*a^4*b*x^7 + 1/4*A*a^5*x^
4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int x^3 (a + bx^3)^5 (A + Bx^3) \, dx = x^7 \left(\frac{B a^5}{7} + \frac{5 A b a^4}{7}\right) + x^{19} \left(\frac{A b^5}{19} + \frac{5 B a b^4}{19}\right) + \frac{A a^5 x^4}{4} + \frac{B b^5 x^{22}}{22} + \frac{10 a^2 b^2 x^{13} (A b + B a)}{13} + \frac{a^3 b x^{10} (2 A b + B a)}{2} + \frac{5 a b^3 x^{16} (A b + 2 B a)}{16}$$

```
[In] int(x^3*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^7*((B*a^5)/7 + (5*A*a^4*b)/7) + x^19*((A*b^5)/19 + (5*B*a*b^4)/19) + (A*a
^5*x^4)/4 + (B*b^5*x^22)/22 + (10*a^2*b^2*x^13*(A*b + B*a))/13 + (a^3*b*x^1
0*(2*A*b + B*a))/2 + (5*a*b^3*x^16*(A*b + 2*B*a))/16
```

3.30 $\int x^2(a+bx^3)^5\left(A+Bx^3\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 42

$$\int x^2\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=\frac{(Ab-aB)\left(a+bx^3\right)^6}{18b^2}+\frac{B\left(a+bx^3\right)^7}{21b^2}$$

```
[Out] 1/18*(A*b-B*a)*(b*x^3+a)^6/b^2+1/21*B*(b*x^3+a)^7/b^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {455, 45}

$$\int x^2\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=\frac{\left(a+bx^3\right)^6(Ab-aB)}{18b^2}+\frac{B\left(a+bx^3\right)^7}{21b^2}$$

```
[In] Int[x^2*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] ((A*b - a*B)*(a + b*x^3)^6)/(18*b^2) + (B*(a + b*x^3)^7)/(21*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int (a+bx)^5(A+Bx)\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{(Ab-aB)(a+bx)^5}{b}+\frac{B(a+bx)^6}{b}\right)\,dx, x, x^3\right)\\ &= \frac{(Ab-aB)\left(a+bx^3\right)^6}{18b^2}+\frac{B\left(a+bx^3\right)^7}{21b^2}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 107 vs. 2(42) = 84.

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.55

$$\int x^2\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx = \frac{1}{126}x^3\left(42a^5A+21a^4(5Ab+aB)x^3+70a^3b(2Ab+aB)x^6 + 105a^2b^2(Ab+aB)x^9+42ab^3(Ab+2aB)x^{12} + 7b^4(Ab+5aB)x^{15}+6b^5Bx^{18}\right)$$

```
[In] Integrate[x^2*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (x^3*(42*a^5*A + 21*a^4*(5*A*b + a*B)*x^3 + 70*a^3*b*(2*A*b + a*B)*x^6 + 10
5*a^2*b^2*(A*b + a*B)*x^9 + 42*a*b^3*(A*b + 2*a*B)*x^12 + 7*b^4*(A*b + 5*a*
B)*x^15 + 6*b^5*B*x^18))/126
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(38) = 76.

Time = 4.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.88

method	result
norman	$\frac{a^5Ax^3}{3}+\left(\frac{5}{6}a^4bA+\frac{1}{6}a^5B\right)x^6+\left(\frac{10}{9}a^3b^2A+\frac{5}{9}a^4bB\right)x^9+\left(\frac{5}{6}a^2b^3A+\frac{5}{6}a^3b^2B\right)x^{12}+\left(\frac{1}{3}a\,b^4A\right.$
default	$\frac{b^5Bx^{21}}{21}+\frac{(b^5A+5ab^4B)x^{18}}{18}+\frac{(5ab^4A+10a^2b^3B)x^{15}}{15}+\frac{(10a^2b^3A+10a^3b^2B)x^{12}}{12}+\frac{(10a^3b^2A+5a^4bB)x^9}{9}+\frac{(5a^4b}{}$
gosper	$\frac{1}{3}a^5Ax^3+\frac{5}{6}x^6a^4bA+\frac{1}{6}x^6a^5B+\frac{10}{9}x^9a^3b^2A+\frac{5}{9}x^9a^4bB+\frac{5}{6}x^{12}a^2b^3A+\frac{5}{6}x^{12}a^3b^2B+\frac{1}{3}x^{15}a$
risch	$\frac{1}{3}a^5Ax^3+\frac{5}{6}x^6a^4bA+\frac{1}{6}x^6a^5B+\frac{10}{9}x^9a^3b^2A+\frac{5}{9}x^9a^4bB+\frac{5}{6}x^{12}a^2b^3A+\frac{5}{6}x^{12}a^3b^2B+\frac{1}{3}x^{15}a$
parallelrisch	$\frac{1}{3}a^5Ax^3+\frac{5}{6}x^6a^4bA+\frac{1}{6}x^6a^5B+\frac{10}{9}x^9a^3b^2A+\frac{5}{9}x^9a^4bB+\frac{5}{6}x^{12}a^2b^3A+\frac{5}{6}x^{12}a^3b^2B+\frac{1}{3}x^{15}a$

```
[In] int(x^2*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^5*A*x^3+(5/6*a^4*b*A+1/6*a^5*B)*x^6+(10/9*a^3*b^2*A+5/9*a^4*b*B)*x^9+
(5/6*a^2*b^3*A+5/6*a^3*b^2*B)*x^12+(1/3*a*b^4*A+2/3*a^2*b^3*B)*x^15+(1/18*b
^5*A+5/18*a*b^4*B)*x^18+1/21*b^5*B*x^21
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(38) = 76$.

Time = 0.24 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.83

$$\int x^2(a+bx^3)^5(A+Bx^3)\,dx = \frac{1}{21}Bb^5x^{21} + \frac{1}{18}(5\,Bab^4 + Ab^5)x^{18} + \frac{1}{3}(2\,Ba^2b^3 + Aab^4)x^{15} + \frac{5}{6}(Ba^3b^2 + Aa^2b^3)x^{12} + \frac{5}{9}(Ba^4b + 2\,Aa^3b^2)x^9 + \frac{1}{3}Aa^5x^3 + \frac{1}{6}(Ba^5 + 5\,Aa^4b)x^6$$

```
[In] integrate(x^2*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/21*B*b^5*x^21 + 1/18*(5*B*a*b^4 + A*b^5)*x^18 + 1/3*(2*B*a^2*b^3 + A*a*b^
4)*x^15 + 5/6*(B*a^3*b^2 + A*a^2*b^3)*x^12 + 5/9*(B*a^4*b + 2*A*a^3*b^2)*x^
9 + 1/3*A*a^5*x^3 + 1/6*(B*a^5 + 5*A*a^4*b)*x^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(36) = 72$.

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.24

$$\int x^2(a+bx^3)^5(A+Bx^3)\,dx = \frac{Aa^5x^3}{3} + \frac{Bb^5x^{21}}{21} + x^{18}\left(\frac{Ab^5}{18} + \frac{5Bab^4}{18}\right) + x^{15}\left(\frac{Aab^4}{3} + \frac{2Ba^2b^3}{3}\right) + x^{12}\cdot\left(\frac{5Aa^2b^3}{6} + \frac{5Ba^3b^2}{6}\right) + x^9\cdot\left(\frac{10Aa^3b^2}{9} + \frac{5Ba^4b}{9}\right) + x^6\cdot\left(\frac{5Aa^4b}{6} + \frac{Ba^5}{6}\right)$$

```
[In] integrate(x**2*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**3/3 + B*b**5*x**21/21 + x**18*(A*b**5/18 + 5*B*a*b**4/18) + x**15
*(A*a*b**4/3 + 2*B*a**2*b**3/3) + x**12*(5*A*a**2*b**3/6 + 5*B*a**3*b**2/6)
 + x**9*(10*A*a**3*b**2/9 + 5*B*a**4*b/9) + x**6*(5*A*a**4*b/6 + B*a**5/6)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(38) = 76$.

Time = 0.21 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.83

$$\int x^2(a+bx^3)^5(A+Bx^3)\,dx = \frac{1}{21}Bb^5x^{21} + \frac{1}{18}(5Bab^4 + Ab^5)x^{18} + \frac{1}{3}(2Ba^2b^3 + Aab^4)x^{15} + \frac{5}{6}(Ba^3b^2 + Aa^2b^3)x^{12} + \frac{5}{9}(Ba^4b + 2Aa^3b^2)x^9 + \frac{1}{3}Aa^5x^3 + \frac{1}{6}(Ba^5 + 5Aa^4b)x^6$$

```
[In] integrate(x^2*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/21*B*b^5*x^21 + 1/18*(5*B*a*b^4 + A*b^5)*x^18 + 1/3*(2*B*a^2*b^3 + A*a*b^
4)*x^15 + 5/6*(B*a^3*b^2 + A*a^2*b^3)*x^12 + 5/9*(B*a^4*b + 2*A*a^3*b^2)*x^
9 + 1/3*A*a^5*x^3 + 1/6*(B*a^5 + 5*A*a^4*b)*x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(38) = 76$.

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.98

$$\int x^2(a+bx^3)^5(A+Bx^3)\,dx = \frac{1}{21}Bb^5x^{21} + \frac{5}{18}Bab^4x^{18} + \frac{1}{18}Ab^5x^{18} + \frac{2}{3}Ba^2b^3x^{15} + \frac{1}{3}Aab^4x^{15} + \frac{5}{6}Ba^3b^2x^{12} + \frac{5}{6}Aa^2b^3x^{12} + \frac{5}{9}Ba^4bx^9 + \frac{10}{9}Aa^3b^2x^9 + \frac{1}{6}Ba^5x^6 + \frac{5}{6}Aa^4bx^6 + \frac{1}{3}Aa^5x^3$$

```
[In] integrate(x^2*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/21*B*b^5*x^21 + 5/18*B*a*b^4*x^18 + 1/18*A*b^5*x^18 + 2/3*B*a^2*b^3*x^15
+ 1/3*A*a*b^4*x^15 + 5/6*B*a^3*b^2*x^12 + 5/6*A*a^2*b^3*x^12 + 5/9*B*a^4*b*
x^9 + 10/9*A*a^3*b^2*x^9 + 1/6*B*a^5*x^6 + 5/6*A*a^4*b*x^6 + 1/3*A*a^5*x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.55

$$\int x^2(a+bx^3)^5(A+Bx^3)\,dx = x^6\left(\frac{B\,a^5}{6} + \frac{5\,A\,b\,a^4}{6}\right) + x^{18}\left(\frac{A\,b^5}{18} + \frac{5\,B\,a\,b^4}{18}\right) + \frac{A\,a^5\,x^3}{3} + \frac{B\,b^5\,x^{21}}{21} + \frac{5\,a^2\,b^2\,x^{12}\,(A\,b + B\,a)}{6} + \frac{5\,a^3\,b\,x^9\,(2\,A\,b + B\,a)}{9} + \frac{a\,b^3\,x^{15}\,(A\,b + 2\,B\,a)}{3}$$

```
[In] int(x^2*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^6*((B*a^5)/6 + (5*A*a^4*b)/6) + x^18*((A*b^5)/18 + (5*B*a*b^4)/18) + (A*a
^5*x^3)/3 + (B*b^5*x^21)/21 + (5*a^2*b^2*x^12*(A*b + B*a))/6 + (5*a^3*b*x^9
*(2*A*b + B*a))/9 + (a*b^3*x^15*(A*b + 2*B*a))/3
```

3.31 $\int x(a+bx^3)^5 (A+Bx^3)\, dx$

Optimal result

Integrand size = 18, antiderivative size = 117

$$\int x(a+bx^3)^5 (A+Bx^3)\, dx = \frac{1}{2}a^5Ax^2 + \frac{1}{5}a^4(5Ab+aB)x^5 + \frac{5}{8}a^3b(2Ab+aB)x^8 + \frac{10}{11}a^2b^2(Ab+aB)x^{11} + \frac{5}{14}ab^3(Ab+2aB)x^{14} + \frac{1}{17}b^4(Ab+5aB)x^{17} + \frac{1}{20}b^5Bx^{20}$$

```
[Out] 1/2*a^5*A*x^2+1/5*a^4*(5*A*b+B*a)*x^5+5/8*a^3*b*(2*A*b+B*a)*x^8+10/11*a^2*b
^2*(A*b+B*a)*x^11+5/14*a*b^3*(A*b+2*B*a)*x^14+1/17*b^4*(A*b+5*B*a)*x^17+1/2
0*b^5*B*x^20
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {459}

$$\int x(a+bx^3)^5 (A+Bx^3)\, dx = \frac{1}{2}a^5Ax^2 + \frac{1}{5}a^4x^5(aB+5Ab) + \frac{5}{8}a^3bx^8(aB+2Ab) + \frac{10}{11}a^2b^2x^{11}(aB+Ab) + \frac{1}{17}b^4x^{17}(5aB+Ab) + \frac{5}{14}ab^3x^{14}(2aB+Ab) + \frac{1}{20}b^5Bx^{20}$$

```
[In] Int[x*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^2)/2 + (a^4*(5*A*b + a*B)*x^5)/5 + (5*a^3*b*(2*A*b + a*B)*x^8)/8 +
 (10*a^2*b^2*(A*b + a*B)*x^11)/11 + (5*a*b^3*(A*b + 2*a*B)*x^14)/14 + (b^4*
(A*b + 5*a*B)*x^17)/17 + (b^5*B*x^20)/20
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^5Ax + a^4(5Ab+aB)x^4 + 5a^3b(2Ab+aB)x^7 + 10a^2b^2(Ab+aB)x^{10} \right. \\ &\qquad \left. + 5ab^3(Ab+2aB)x^{13} + b^4(Ab+5aB)x^{16} + b^5Bx^{19}\right)\,dx \\ &= \frac{1}{2}a^5Ax^2 + \frac{1}{5}a^4(5Ab+aB)x^5 + \frac{5}{8}a^3b(2Ab+aB)x^8 + \frac{10}{11}a^2b^2(Ab+aB)x^{11} \\ &\quad + \frac{5}{14}ab^3(Ab+2aB)x^{14} + \frac{1}{17}b^4(Ab+5aB)x^{17} + \frac{1}{20}b^5Bx^{20}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x(a+bx^3)^5\,(A+Bx^3)\;dx &= \frac{1}{2}a^5Ax^2 + \frac{1}{5}a^4(5Ab+aB)x^5 + \frac{5}{8}a^3b(2Ab+aB)x^8 \\ &\quad + \frac{10}{11}a^2b^2(Ab+aB)x^{11} + \frac{5}{14}ab^3(Ab+2aB)x^{14} \\ &\quad + \frac{1}{17}b^4(Ab+5aB)x^{17} + \frac{1}{20}b^5Bx^{20}\end{aligned}$$

```
[In] Integrate[x*(a + b*x^3)^5*(A + B*x^3),x]
[Out] (a^5*A*x^2)/2 + (a^4*(5*A*b + a*B)*x^5)/5 + (5*a^3*b*(2*A*b + a*B)*x^8)/8 +
 (10*a^2*b^2*(A*b + a*B)*x^11)/11 + (5*a*b^3*(A*b + 2*a*B)*x^14)/14 + (b^4*
(A*b + 5*a*B)*x^17)/17 + (b^5*B*x^20)/20
```

Maple [A] (verified)

Time = 4.03 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03

method	result
norman	$\frac{a^5Ax^2}{2}+\left(a^4bA+\frac{1}{5}a^5B\right)x^5+\left(\frac{5}{4}a^3b^2A+\frac{5}{8}a^4bB\right)x^8+\left(\frac{10}{11}a^2b^3A+\frac{10}{11}a^3b^2B\right)x^{11}+\left(\frac{5}{14}ab^4A\right.$
default	$\frac{b^5Bx^{20}}{20}+\frac{(b^5A+5ab^4B)x^{17}}{17}+\frac{(5ab^4A+10a^2b^3B)x^{14}}{14}+\frac{(10a^2b^3A+10a^3b^2B)x^{11}}{11}+\frac{(10a^3b^2A+5a^4bB)x^8}{8}+$
gosper	$\frac{1}{2}a^5Ax^2+x^5a^4bA+\frac{1}{5}x^5a^5B+\frac{5}{4}x^8a^3b^2A+\frac{5}{8}x^8a^4bB+\frac{10}{11}x^{11}a^2b^3A+\frac{10}{11}x^{11}a^3b^2B+\frac{5}{14}x^{14}a$
risch	$\frac{1}{2}a^5Ax^2+x^5a^4bA+\frac{1}{5}x^5a^5B+\frac{5}{4}x^8a^3b^2A+\frac{5}{8}x^8a^4bB+\frac{10}{11}x^{11}a^2b^3A+\frac{10}{11}x^{11}a^3b^2B+\frac{5}{14}x^{14}a$
parallelrisch	$\frac{1}{2}a^5Ax^2+x^5a^4bA+\frac{1}{5}x^5a^5B+\frac{5}{4}x^8a^3b^2A+\frac{5}{8}x^8a^4bB+\frac{10}{11}x^{11}a^2b^3A+\frac{10}{11}x^{11}a^3b^2B+\frac{5}{14}x^{14}a$

```
[In] int(x*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^5*A*x^2+(a^4*b*A+1/5*a^5*B)*x^5+(5/4*a^3*b^2*A+5/8*a^4*b*B)*x^8+(10/1
1*a^2*b^3*A+10/11*a^3*b^2*B)*x^11+(5/14*a*b^4*A+5/7*a^2*b^3*B)*x^14+(1/17*b
^5*A+5/17*a*b^4*B)*x^17+1/20*b^5*B*x^20
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=\frac{1}{20}\,Bb^5x^{20}+\frac{1}{17}\,(5\,Bab^4+Ab^5)x^{17}+\frac{5}{14}\,(2\,Ba^2b^3+Aab^4)x^{14}+\frac{10}{11}\,(Ba^3b^2+Aa^2b^3)x^{11}+\frac{5}{8}\,(Ba^4b+2\,Aa^3b^2)x^8+\frac{1}{2}\,Aa^5x^2+\frac{1}{5}\,(Ba^5+5\,Aa^4b)x^5$$

```
[In] integrate(x*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/20*B*b^5*x^20 + 1/17*(5*B*a*b^4 + A*b^5)*x^17 + 5/14*(2*B*a^2*b^3 + A*a*b
^4)*x^14 + 10/11*(B*a^3*b^2 + A*a^2*b^3)*x^11 + 5/8*(B*a^4*b + 2*A*a^3*b^2)
*x^8 + 1/2*A*a^5*x^2 + 1/5*(B*a^5 + 5*A*a^4*b)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.15

$$\int x\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=\frac{Aa^5x^2}{2}+\frac{Bb^5x^{20}}{20}+x^{17}\left(\frac{Ab^5}{17}+\frac{5Bab^4}{17}\right)+x^{14}\cdot\left(\frac{5Aab^4}{14}+\frac{5Ba^2b^3}{7}\right)+x^{11}\cdot\left(\frac{10Aa^2b^3}{11}+\frac{10Ba^3b^2}{11}\right)+x^8\cdot\left(\frac{5Aa^3b^2}{4}+\frac{5Ba^4b}{8}\right)+x^5\left(Aa^4b+\frac{Ba^5}{5}\right)$$

```
[In] integrate(x*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**2/2 + B*b**5*x**20/20 + x**17*(A*b**5/17 + 5*B*a*b**4/17) + x**14
*(5*A*a*b**4/14 + 5*B*a**2*b**3/7) + x**11*(10*A*a**2*b**3/11 + 10*B*a**3*b
**2/11) + x**8*(5*A*a**3*b**2/4 + 5*B*a**4*b/8) + x**5*(A*a**4*b + B*a**5/5
)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x(a+bx^3)^5 (A+Bx^3)\ dx = \frac{1}{20}Bb^5x^{20} + \frac{1}{17}(5Bab^4 + Ab^5)x^{17} + \frac{5}{14}(2Ba^2b^3 + Aab^4)x^{14} + \frac{10}{11}(Ba^3b^2 + Aa^2b^3)x^{11} + \frac{5}{8}(Ba^4b + 2Aa^3b^2)x^8 + \frac{1}{2}Aa^5x^2 + \frac{1}{5}(Ba^5 + 5Aa^4b)x^5$$

```
[In] integrate(x*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/20*B*b^5*x^20 + 1/17*(5*B*a*b^4 + A*b^5)*x^17 + 5/14*(2*B*a^2*b^3 + A*a*b
^4)*x^14 + 10/11*(B*a^3*b^2 + A*a^2*b^3)*x^11 + 5/8*(B*a^4*b + 2*A*a^3*b^2)
*x^8 + 1/2*A*a^5*x^2 + 1/5*(B*a^5 + 5*A*a^4*b)*x^5
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

$$\int x(a+bx^3)^5 (A+Bx^3)\ dx = \frac{1}{20}Bb^5x^{20} + \frac{5}{17}Bab^4x^{17} + \frac{1}{17}Ab^5x^{17} + \frac{5}{7}Ba^2b^3x^{14} + \frac{5}{14}Aab^4x^{14} + \frac{10}{11}Ba^3b^2x^{11} + \frac{10}{11}Aa^2b^3x^{11} + \frac{5}{8}Ba^4bx^8 + \frac{5}{4}Aa^3b^2x^8 + \frac{1}{5}Ba^5x^5 + Aa^4bx^5 + \frac{1}{2}Aa^5x^2$$

```
[In] integrate(x*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/20*B*b^5*x^20 + 5/17*B*a*b^4*x^17 + 1/17*A*b^5*x^17 + 5/7*B*a^2*b^3*x^14
+ 5/14*A*a*b^4*x^14 + 10/11*B*a^3*b^2*x^11 + 10/11*A*a^2*b^3*x^11 + 5/8*B*a
^4*b*x^8 + 5/4*A*a^3*b^2*x^8 + 1/5*B*a^5*x^5 + A*a^4*b*x^5 + 1/2*A*a^5*x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int x(a+bx^3)^5\,(A+Bx^3)\ dx = x^5\left(\frac{B\,a^5}{5}+A\,b\,a^4\right)+x^{17}\left(\frac{A\,b^5}{17}+\frac{5\,B\,a\,b^4}{17}\right)+\frac{A\,a^5\,x^2}{2}+\frac{B\,b^5\,x^{20}}{20}+\frac{10\,a^2\,b^2\,x^{11}\,(A\,b+B\,a)}{11}+\frac{5\,a^3\,b\,x^8\,(2\,A\,b+B\,a)}{8}+\frac{5\,a\,b^3\,x^{14}\,(A\,b+2\,B\,a)}{14}$$

```
[In] int(x*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^5*((B*a^5)/5 + A*a^4*b) + x^17*((A*b^5)/17 + (5*B*a*b^4)/17) + (A*a^5*x^2
)/2 + (B*b^5*x^20)/20 + (10*a^2*b^2*x^11*(A*b + B*a))/11 + (5*a^3*b*x^8*(2*
A*b + B*a))/8 + (5*a*b^3*x^14*(A*b + 2*B*a))/14
```

3.32 $\int (a+bx^3)^5 (A+Bx^3)\, dx$

Optimal result

Integrand size = 17, antiderivative size = 109

$$\int (a+bx^3)^5 (A+Bx^3)\, dx = a^5Ax + \frac{1}{4}a^4(5Ab+aB)x^4 + \frac{5}{7}a^3b(2Ab+aB)x^7 + a^2b^2(Ab+aB)x^{10} + \frac{5}{13}ab^3(Ab+2aB)x^{13} + \frac{1}{16}b^4(Ab+5aB)x^{16} + \frac{1}{19}b^5Bx^{19}$$

```
[Out] a^5*A*x+1/4*a^4*(5*A*b+B*a)*x^4+5/7*a^3*b*(2*A*b+B*a)*x^7+a^2*b^2*(A*b+B*a)
*x^10+5/13*a*b^3*(A*b+2*B*a)*x^13+1/16*b^4*(A*b+5*B*a)*x^16+1/19*b^5*B*x^19
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {380}

$$\int (a+bx^3)^5 (A+Bx^3)\, dx = a^5Ax + \frac{1}{4}a^4x^4(aB+5Ab) + \frac{5}{7}a^3bx^7(aB+2Ab) + a^2b^2x^{10}(aB+Ab) + \frac{1}{16}b^4x^{16}(5aB+Ab) + \frac{5}{13}ab^3x^{13}(2aB+Ab) + \frac{1}{19}b^5Bx^{19}$$

```
[In] Int[(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] a^5*A*x + (a^4*(5*A*b + a*B)*x^4)/4 + (5*a^3*b*(2*A*b + a*B)*x^7)/7 + a^2*b
^2*(A*b + a*B)*x^10 + (5*a*b^3*(A*b + 2*a*B)*x^13)/13 + (b^4*(A*b + 5*a*B)*
x^16)/16 + (b^5*B*x^19)/19
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
```

```
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^5A + a^4(5Ab + aB)x^3 + 5a^3b(2Ab + aB)x^6 + 10a^2b^2(Ab + aB)x^9 \right.\\ &\qquad \left. + 5ab^3(Ab + 2aB)x^{12} + b^4(Ab + 5aB)x^{15} + b^5Bx^{18}\right)\,dx\\ &= a^5Ax + \frac{1}{4}a^4(5Ab + aB)x^4 + \frac{5}{7}a^3b(2Ab + aB)x^7 + a^2b^2(Ab + aB)x^{10}\\ &\quad + \frac{5}{13}ab^3(Ab + 2aB)x^{13} + \frac{1}{16}b^4(Ab + 5aB)x^{16} + \frac{1}{19}b^5Bx^{19}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00

$$\int \left(a + bx^3\right)^5 \left(A + Bx^3\right)\,dx = a^5Ax + \frac{1}{4}a^4(5Ab + aB)x^4 + \frac{5}{7}a^3b(2Ab + aB)x^7 + a^2b^2(Ab + aB)x^{10} + \frac{5}{13}ab^3(Ab + 2aB)x^{13} + \frac{1}{16}b^4(Ab + 5aB)x^{16} + \frac{1}{19}b^5Bx^{19}$$

```
[In] Integrate[(a + b*x^3)^5*(A + B*x^3),x]
[Out] a^5*A*x + (a^4*(5*A*b + a*B)*x^4)/4 + (5*a^3*b*(2*A*b + a*B)*x^7)/7 + a^2*b
^2*(A*b + a*B)*x^10 + (5*a*b^3*(A*b + 2*a*B)*x^13)/13 + (b^4*(A*b + 5*a*B)*
x^16)/16 + (b^5*B*x^19)/19
```

Maple [A] (verified)

Time = 4.03 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.06

method	result
norman	$a^5Ax + \left(\frac{5}{4}a^4bA + \frac{1}{4}a^5B\right)x^4 + \left(\frac{10}{7}a^3b^2A + \frac{5}{7}a^4bB\right)x^7 + \left(a^2b^3A + a^3b^2B\right)x^{10} + \left(\frac{5}{13}a\,b^4A + \right.$
gosper	$a^5Ax + \frac{5}{4}x^4a^4bA + \frac{1}{4}x^4a^5B + \frac{10}{7}x^7a^3b^2A + \frac{5}{7}x^7a^4bB + A\,a^2b^3x^{10} + B\,a^3b^2x^{10} + \frac{5}{13}x^{13}a\,b^4A$
default	$\frac{b^5Bx^{19}}{19} + \frac{(b^5A+5a\,b^4B)x^{16}}{16} + \frac{(5a\,b^4A+10a^2b^3B)x^{13}}{13} + \frac{(10a^2b^3A+10a^3b^2B)x^{10}}{10} + \frac{(10a^3b^2A+5a^4bB)x^7}{7} + $
risch	$a^5Ax + \frac{5}{4}x^4a^4bA + \frac{1}{4}x^4a^5B + \frac{10}{7}x^7a^3b^2A + \frac{5}{7}x^7a^4bB + A\,a^2b^3x^{10} + B\,a^3b^2x^{10} + \frac{5}{13}x^{13}a\,b^4A$
parallelrisch	$a^5Ax + \frac{5}{4}x^4a^4bA + \frac{1}{4}x^4a^5B + \frac{10}{7}x^7a^3b^2A + \frac{5}{7}x^7a^4bB + A\,a^2b^3x^{10} + B\,a^3b^2x^{10} + \frac{5}{13}x^{13}a\,b^4A$

```
[In] int((b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] a^5*A*x+(5/4*a^4*b*A+1/4*a^5*B)*x^4+(10/7*a^3*b^2*A+5/7*a^4*b*B)*x^7+(A*a^2
*b^3+B*a^3*b^2)*x^10+(5/13*a*b^4*A+10/13*a^2*b^3*B)*x^13+(1/16*b^5*A+5/16*a
*b^4*B)*x^16+1/19*b^5*B*x^19
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06

$$\int \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx = \frac{1}{19} Bb^5x^{19} + \frac{1}{16} \left(5 Bab^4 + Ab^5\right)x^{16} + \frac{5}{13} \left(2 Ba^2b^3 + Aab^4\right)x^{13} + \left(Ba^3b^2 + Aa^2b^3\right)x^{10} + \frac{5}{7} \left(Ba^4b + 2 Aa^3b^2\right)x^7 + Aa^5x + \frac{1}{4} \left(Ba^5 + 5 Aa^4b\right)x^4$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/19*B*b^5*x^19 + 1/16*(5*B*a*b^4 + A*b^5)*x^16 + 5/13*(2*B*a^2*b^3 + A*a*b
^4)*x^13 + (B*a^3*b^2 + A*a^2*b^3)*x^10 + 5/7*(B*a^4*b + 2*A*a^3*b^2)*x^7 +
 A*a^5*x + 1/4*(B*a^5 + 5*A*a^4*b)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.17

$$\int \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx = Aa^5x + \frac{Bb^5x^{19}}{19} + x^{16}\left(\frac{Ab^5}{16} + \frac{5Bab^4}{16}\right) + x^{13} \cdot \left(\frac{5Aab^4}{13} + \frac{10Ba^2b^3}{13}\right) + x^{10}\left(Aa^2b^3 + Ba^3b^2\right) + x^7 \cdot \left(\frac{10Aa^3b^2}{7} + \frac{5Ba^4b}{7}\right) + x^4 \cdot \left(\frac{5Aa^4b}{4} + \frac{Ba^5}{4}\right)$$

```
[In] integrate((b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x + B*b**5*x**19/19 + x**16*(A*b**5/16 + 5*B*a*b**4/16) + x**13*(5*A
*a*b**4/13 + 10*B*a**2*b**3/13) + x**10*(A*a**2*b**3 + B*a**3*b**2) + x**7*
(10*A*a**3*b**2/7 + 5*B*a**4*b/7) + x**4*(5*A*a**4*b/4 + B*a**5/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06

$$\int (a+bx^3)^5 (A+Bx^3)\,dx = \frac{1}{19}Bb^5x^{19} + \frac{1}{16}(5\,Bab^4 + Ab^5)x^{16} + \frac{5}{13}(2\,Ba^2b^3 + Aab^4)x^{13} + (Ba^3b^2 + Aa^2b^3)x^{10} + \frac{5}{7}(Ba^4b + 2\,Aa^3b^2)x^7 + Aa^5x + \frac{1}{4}(Ba^5 + 5\,Aa^4b)x^4$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/19*B*b^5*x^19 + 1/16*(5*B*a*b^4 + A*b^5)*x^16 + 5/13*(2*B*a^2*b^3 + A*a*b
^4)*x^13 + (B*a^3*b^2 + A*a^2*b^3)*x^10 + 5/7*(B*a^4*b + 2*A*a^3*b^2)*x^7 +
 A*a^5*x + 1/4*(B*a^5 + 5*A*a^4*b)*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.10

$$\int (a+bx^3)^5 (A+Bx^3)\,dx = \frac{1}{19}Bb^5x^{19} + \frac{5}{16}Bab^4x^{16} + \frac{1}{16}Ab^5x^{16} + \frac{10}{13}Ba^2b^3x^{13} + \frac{5}{13}Aab^4x^{13} + Ba^3b^2x^{10} + Aa^2b^3x^{10} + \frac{5}{7}Ba^4bx^7 + \frac{10}{7}Aa^3b^2x^7 + \frac{1}{4}Ba^5x^4 + \frac{5}{4}Aa^4bx^4 + Aa^5x$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/19*B*b^5*x^19 + 5/16*B*a*b^4*x^16 + 1/16*A*b^5*x^16 + 10/13*B*a^2*b^3*x^1
3 + 5/13*A*a*b^4*x^13 + B*a^3*b^2*x^10 + A*a^2*b^3*x^10 + 5/7*B*a^4*b*x^7 +
 10/7*A*a^3*b^2*x^7 + 1/4*B*a^5*x^4 + 5/4*A*a^4*b*x^4 + A*a^5*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int (a+bx^3)^5 (A+Bx^3)\,dx = x^4\left(\frac{B\,a^5}{4} + \frac{5\,A\,b\,a^4}{4}\right) + x^{16}\left(\frac{A\,b^5}{16} + \frac{5\,B\,a\,b^4}{16}\right) + \frac{B\,b^5\,x^{19}}{19} + A\,a^5\,x + a^2\,b^2\,x^{10}\,(A\,b + B\,a) + \frac{5\,a^3\,b\,x^7\,(2\,A\,b + B\,a)}{7} + \frac{5\,a\,b^3\,x^{13}\,(A\,b + 2\,B\,a)}{13}$$

```
[In] int((A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^4*((B*a^5)/4 + (5*A*a^4*b)/4) + x^16*((A*b^5)/16 + (5*B*a*b^4)/16) + (B*b
^5*x^19)/19 + A*a^5*x + a^2*b^2*x^10*(A*b + B*a) + (5*a^3*b*x^7*(2*A*b + B*
a))/7 + (5*a*b^3*x^13*(A*b + 2*B*a))/13
```

3.33 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 88

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x} \, dx = \frac{5}{3}a^4Abx^3 + \frac{5}{3}a^3Ab^2x^6 + \frac{10}{9}a^2Ab^3x^9 + \frac{5}{12}aAb^4x^{12} + \frac{1}{15}Ab^5x^{15} + \frac{B(a+bx^3)^6}{18b} + a^5A\log(x)$$

```
[Out] 5/3*a^4*A*b*x^3+5/3*a^3*A*b^2*x^6+10/9*a^2*A*b^3*x^9+5/12*a*A*b^4*x^12+1/15
*A*b^5*x^15+1/18*B*(b*x^3+a)^6/b+a^5*A*ln(x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {457, 81, 45}

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x} \, dx = a^5A\log(x) + \frac{5}{3}a^4Abx^3 + \frac{5}{3}a^3Ab^2x^6 + \frac{10}{9}a^2Ab^3x^9 + \frac{5}{12}aAb^4x^{12} + \frac{B(a+bx^3)^6}{18b} + \frac{1}{15}Ab^5x^{15}$$

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x,x]
```

```
[Out] (5*a^4*A*b*x^3)/3 + (5*a^3*A*b^2*x^6)/3 + (10*a^2*A*b^3*x^9)/9 + (5*a*A*b^4
*x^12)/12 + (A*b^5*x^15)/15 + (B*(a + b*x^3)^6)/(18*b) + a^5*A*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x}\,dx, x, x^3\right)\\
&= \frac{B(a+bx^3)^6}{18b} + \frac{1}{3}A\text{Subst}\left(\int \frac{(a+bx)^5}{x}\,dx, x, x^3\right)\\
&= \frac{B(a+bx^3)^6}{18b} + \frac{1}{3}A\text{Subst}\left(\int \left(5a^4b+\frac{a^5}{x}+10a^3b^2x+10a^2b^3x^2+5ab^4x^3+b^5x^4\right)dx, x, x^3\right)\\
&= \frac{5}{3}a^4Abx^3+\frac{5}{3}a^3Ab^2x^6+\frac{10}{9}a^2Ab^3x^9+\frac{5}{12}aAb^4x^{12}+\frac{1}{15}Ab^5x^{15}+\frac{B(a+bx^3)^6}{18b}+a^5A\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.28

$$\begin{aligned}
\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x}\,dx &= \frac{1}{3}a^4(5Ab+aB)x^3 + \frac{5}{6}a^3b(2Ab+aB)x^6\\
&\quad + \frac{10}{9}a^2b^2(Ab+aB)x^9 + \frac{5}{12}ab^3(Ab+2aB)x^{12}\\
&\quad + \frac{1}{15}b^4(Ab+5aB)x^{15} + \frac{1}{18}b^5Bx^{18} + a^5A\log(x)
\end{aligned}$$

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x,x]
[Out] (a^4*(5*A*b + a*B)*x^3)/3 + (5*a^3*b*(2*A*b + a*B)*x^6)/6 + (10*a^2*b^2*(A*
b + a*B)*x^9)/9 + (5*a*b^3*(A*b + 2*a*B)*x^12)/12 + (b^4*(A*b + 5*a*B)*x^15
)/15 + (b^5*B*x^18)/18 + a^5*A*Log[x]
```

Maple [A] (verified)

Time = 3.94 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.35

method	result
norman	$\left(\frac{1}{15}b^{5}A+\frac{1}{3}a\,b^{4}B\right)x^{15}+\left(\frac{5}{12}a\,b^{4}A+\frac{5}{6}a^{2}b^{3}B\right)x^{12}+\left(\frac{10}{9}a^{2}b^{3}A+\frac{10}{9}a^{3}b^{2}B\right)x^{9}+\left(\frac{5}{3}a^{3}b^{2}A+\frac{5}{6}a\right.$
default	$\frac{b^{5}B\,x^{18}}{18}+\frac{A\,b^{5}x^{15}}{15}+\frac{Ba\,b^{4}x^{15}}{3}+\frac{5aA\,b^{4}x^{12}}{12}+\frac{5B\,a^{2}b^{3}x^{12}}{6}+\frac{10a^{2}A\,b^{3}x^{9}}{9}+\frac{10B\,a^{3}b^{2}x^{9}}{9}+\frac{5a^{3}A\,b^{2}x^{6}}{3}+\frac{5B\,a^{4}b}{6}$
risch	$\frac{b^{5}B\,x^{18}}{18}+\frac{A\,b^{5}x^{15}}{15}+\frac{Ba\,b^{4}x^{15}}{3}+\frac{5aA\,b^{4}x^{12}}{12}+\frac{5B\,a^{2}b^{3}x^{12}}{6}+\frac{10a^{2}A\,b^{3}x^{9}}{9}+\frac{10B\,a^{3}b^{2}x^{9}}{9}+\frac{5a^{3}A\,b^{2}x^{6}}{3}+\frac{5B\,a^{4}b}{6}$
parallelrisch	$\frac{b^{5}B\,x^{18}}{18}+\frac{A\,b^{5}x^{15}}{15}+\frac{Ba\,b^{4}x^{15}}{3}+\frac{5aA\,b^{4}x^{12}}{12}+\frac{5B\,a^{2}b^{3}x^{12}}{6}+\frac{10a^{2}A\,b^{3}x^{9}}{9}+\frac{10B\,a^{3}b^{2}x^{9}}{9}+\frac{5a^{3}A\,b^{2}x^{6}}{3}+\frac{5B\,a^{4}b}{6}$

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (1/15*b^5*A+1/3*a*b^4*B)*x^15+(5/12*a*b^4*A+5/6*a^2*b^3*B)*x^12+(10/9*a^2*b
^3*A+10/9*a^3*b^2*B)*x^9+(5/3*a^3*b^2*A+5/6*a^4*b*B)*x^6+(5/3*a^4*b*A+1/3*a
^5*B)*x^3+1/18*b^5*B*x^18+a^5*A*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x}\,dx = \frac{1}{18}\,Bb^5x^{18}+\frac{1}{15}\,\left(5\,Bab^4+Ab^5\right)x^{15}+\frac{5}{12}\,\left(2\,Ba^2b^3+Aab^4\right)x^{12}+\frac{10}{9}\,\left(Ba^3b^2+Aa^2b^3\right)x^9+\frac{5}{6}\,\left(Ba^4b+2\,Aa^3b^2\right)x^6+Aa^5\log\left(x\right)+\frac{1}{3}\,\left(Ba^5+5\,Aa^4b\right)x^3$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x,x, algorithm="fricas")
```

```
[Out] 1/18*B*b^5*x^18 + 1/15*(5*B*a*b^4 + A*b^5)*x^15 + 5/12*(2*B*a^2*b^3 + A*a*b
^4)*x^12 + 10/9*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 5/6*(B*a^4*b + 2*A*a^3*b^2)*x
^6 + A*a^5*log(x) + 1/3*(B*a^5 + 5*A*a^4*b)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x}\,dx = Aa^5\log\left(x\right)+\frac{Bb^5x^{18}}{18}+x^{15}\left(\frac{Ab^5}{15}+\frac{Bab^4}{3}\right)+x^{12}\cdot\left(\frac{5Aab^4}{12}+\frac{5Ba^2b^3}{6}\right)+x^9\cdot\left(\frac{10Aa^2b^3}{9}+\frac{10Ba^3b^2}{9}\right)+x^6\cdot\left(\frac{5Aa^3b^2}{3}+\frac{5Ba^4b}{6}\right)+x^3\cdot\left(\frac{5Aa^4b}{3}+\frac{Ba^5}{3}\right)$$

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x,x)
```

```
[Out] A*a**5*log(x) + B*b**5*x**18/18 + x**15*(A*b**5/15 + B*a*b**4/3) + x**12*(5
*A*a*b**4/12 + 5*B*a**2*b**3/6) + x**9*(10*A*a**2*b**3/9 + 10*B*a**3*b**2/9
) + x**6*(5*A*a**3*b**2/3 + 5*B*a**4*b/6) + x**3*(5*A*a**4*b/3 + B*a**5/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x} dx = \frac{1}{18} Bb^5x^{18} + \frac{1}{15} (5 Bab^4 + Ab^5)x^{15} + \frac{5}{12} (2 Ba^2b^3 + Aab^4)x^{12} + \frac{10}{9} (Ba^3b^2 + Aa^2b^3)x^9 + \frac{5}{6} (Ba^4b + 2 Aa^3b^2)x^6 + \frac{1}{3} Aa^5 \log (x^3) + \frac{1}{3} (Ba^5 + 5 Aa^4b)x^3$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x,x, algorithm="maxima")
```

```
[Out] 1/18*B*b^5*x^18 + 1/15*(5*B*a*b^4 + A*b^5)*x^15 + 5/12*(2*B*a^2*b^3 + A*a*b
^4)*x^12 + 10/9*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 5/6*(B*a^4*b + 2*A*a^3*b^2)*x
^6 + 1/3*A*a^5*log(x^3) + 1/3*(B*a^5 + 5*A*a^4*b)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.41

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x} dx = \frac{1}{18} Bb^5x^{18} + \frac{1}{3} Bab^4x^{15} + \frac{1}{15} Ab^5x^{15} + \frac{5}{6} Ba^2b^3x^{12} + \frac{5}{12} Aab^4x^{12} + \frac{10}{9} Ba^3b^2x^9 + \frac{10}{9} Aa^2b^3x^9 + \frac{5}{6} Ba^4bx^6 + \frac{5}{3} Aa^3b^2x^6 + \frac{1}{3} Ba^5x^3 + \frac{5}{3} Aa^4bx^3 + Aa^5 \log (|x|)$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x,x, algorithm="giac")
```

```
[Out] 1/18*B*b^5*x^18 + 1/3*B*a*b^4*x^15 + 1/15*A*b^5*x^15 + 5/6*B*a^2*b^3*x^12 +
 5/12*A*a*b^4*x^12 + 10/9*B*a^3*b^2*x^9 + 10/9*A*a^2*b^3*x^9 + 5/6*B*a^4*b*
x^6 + 5/3*A*a^3*b^2*x^6 + 1/3*B*a^5*x^3 + 5/3*A*a^4*b*x^3 + A*a^5*log(abs(x
))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x} \, dx = x^3 \left(\frac{B\,a^5}{3}+\frac{5\,A\,b\,a^4}{3}\right)+x^{15} \left(\frac{A\,b^5}{15}+\frac{B\,a\,b^4}{3}\right) + \frac{B\,b^5\,x^{18}}{18}+A\,a^5\,\ln\left(x\right)+\frac{10\,a^2\,b^2\,x^9\,\left(A\,b+B\,a\right)}{9} + \frac{5\,a^3\,b\,x^6\,\left(2\,A\,b+B\,a\right)}{6}+\frac{5\,a\,b^3\,x^{12}\,\left(A\,b+2\,B\,a\right)}{12}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x,x)
```

```
[Out] x^3*((B*a^5)/3 + (5*A*a^4*b)/3) + x^15*((A*b^5)/15 + (B*a*b^4)/3) + (B*b^5*
x^18)/18 + A*a^5*log(x) + (10*a^2*b^2*x^9*(A*b + B*a))/9 + (5*a^3*b*x^6*(2*
A*b + B*a))/6 + (5*a*b^3*x^12*(A*b + 2*B*a))/12
```

3.34 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 112

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^2} dx = -\frac{a^5A}{x} + \frac{1}{2}a^4(5Ab+aB)x^2 + a^3b(2Ab+aB)x^5 + \frac{5}{4}a^2b^2(Ab+aB)x^8 + \frac{5}{11}ab^3(Ab+2aB)x^{11} + \frac{1}{14}b^4(Ab+5aB)x^{14} + \frac{1}{17}b^5Bx^{17}$$

```
[Out] -a^5*A/x+1/2*a^4*(5*A*b+B*a)*x^2+a^3*b*(2*A*b+B*a)*x^5+5/4*a^2*b^2*(A*b+B*a
)*x^8+5/11*a*b^3*(A*b+2*B*a)*x^11+1/14*b^4*(A*b+5*B*a)*x^14+1/17*b^5*B*x^17
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^2} dx = -\frac{a^5A}{x} + \frac{1}{2}a^4x^2(aB+5Ab) + a^3bx^5(aB+2Ab) + \frac{5}{4}a^2b^2x^8(aB+Ab) + \frac{1}{14}b^4x^{14}(5aB+Ab) + \frac{5}{11}ab^3x^{11}(2aB+Ab) + \frac{1}{17}b^5Bx^{17}$$

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^2,x]
```

```
[Out] -((a^5*A)/x) + (a^4*(5*A*b + a*B)*x^2)/2 + a^3*b*(2*A*b + a*B)*x^5 + (5*a^2
*b^2*(A*b + a*B)*x^8)/4 + (5*a*b^3*(A*b + 2*a*B)*x^11)/11 + (b^4*(A*b + 5*a
*B)*x^14)/14 + (b^5*B*x^17)/17
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a^5 A}{x^2} + a^4(5Ab + aB)x + 5a^3b(2Ab + aB)x^4 + 10a^2b^2(Ab + aB)x^7 + 5ab^3(Ab + 2aB)x^{10} + b^4(Ab + 5aB)x^{13} + b^5Bx^{16}\right) dx$$

$$= -\frac{a^5 A}{x} + \frac{1}{2}a^4(5Ab + aB)x^2 + a^3b(2Ab + aB)x^5 + \frac{5}{4}a^2b^2(Ab + aB)x^8 + \frac{5}{11}ab^3(Ab + 2aB)x^{11} + \frac{1}{14}b^4(Ab + 5aB)x^{14} + \frac{1}{17}b^5Bx^{17}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$\int \frac{(a + bx^3)^5 (A + Bx^3)}{x^2} dx = -\frac{a^5 A}{x} + \frac{1}{2}a^4(5Ab + aB)x^2 + a^3b(2Ab + aB)x^5 + \frac{5}{4}a^2b^2(Ab + aB)x^8 + \frac{5}{11}ab^3(Ab + 2aB)x^{11} + \frac{1}{14}b^4(Ab + 5aB)x^{14} + \frac{1}{17}b^5Bx^{17}$$

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^2,x]
```

```
[Out] -((a^5*A)/x) + (a^4*(5*A*b + a*B)*x^2)/2 + a^3*b*(2*A*b + a*B)*x^5 + (5*a^2
*b^2*(A*b + a*B)*x^8)/4 + (5*a*b^3*(A*b + 2*a*B)*x^11)/11 + (b^4*(A*b + 5*a
*B)*x^14)/14 + (b^5*B*x^17)/17
```

Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.08

method	result
norman	$\frac{-a^5A+(\frac{5}{2}a^4bA+\frac{1}{2}a^5B)x^3+(2a^3b^2A+a^4bB)x^6+(\frac{5}{4}a^2b^3A+\frac{5}{4}a^3b^2B)x^9+(\frac{5}{11}ab^4A+\frac{10}{11}a^2b^3B)x^{12}+(\frac{1}{14}b^5A+\frac{5}{14}ab^4B)x^{15}+\frac{b^5}{}}{x}$
default	$\frac{b^5Bx^{17}}{17}+\frac{Ab^5x^{14}}{14}+\frac{5Bab^4x^{14}}{14}+\frac{5Aab^4x^{11}}{11}+\frac{10Ba^2b^3x^{11}}{11}+\frac{5Aa^2b^3x^8}{4}+\frac{5Ba^3b^2x^8}{4}+2a^3Ab^2x^5+Ba^4b$
risch	$\frac{b^5Bx^{17}}{17}+\frac{Ab^5x^{14}}{14}+\frac{5Bab^4x^{14}}{14}+\frac{5Aab^4x^{11}}{11}+\frac{10Ba^2b^3x^{11}}{11}+\frac{5Aa^2b^3x^8}{4}+\frac{5Ba^3b^2x^8}{4}+2a^3Ab^2x^5+Ba^4b$
gosper	$-\frac{-308b^5Bx^{18}-374Ab^5x^{15}-1870Bab^4x^{15}-2380aAb^4x^{12}-4760Ba^2b^3x^{12}-6545a^2Ab^3x^9-6545Ba^3b^2x^9-10472a^3Ab^2x^6-}{5236x}$
parallelrisch	$\frac{308b^5Bx^{18}+374Ab^5x^{15}+1870Bab^4x^{15}+2380aAb^4x^{12}+4760Ba^2b^3x^{12}+6545a^2Ab^3x^9+6545Ba^3b^2x^9+10472a^3Ab^2x^6+5236}{5236x}$

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/x*(-a^5*A+(5/2*a^4*b*A+1/2*a^5*B)*x^3+(2*A*a^3*b^2+B*a^4*b)*x^6+(5/4*a^2*
b^3*A+5/4*a^3*b^2*B)*x^9+(5/11*a*b^4*A+10/11*a^2*b^3*B)*x^12+(1/14*b^5*A+5/
14*a*b^4*B)*x^15+1/17*b^5*B*x^18)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^2}\,dx = \frac{308\,Bb^5x^{18}+374\,(5\,Bab^4+Ab^5)x^{15}+2380\,(2\,Ba^2b^3+Aab^4)x^{12}+6545\,(Ba^3b^2+Aa^2b^3)x^9+5236\,(Ba^4b}{5236\,x}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/5236*(308*B*b^5*x^18 + 374*(5*B*a*b^4 + A*b^5)*x^15 + 2380*(2*B*a^2*b^3 +
 A*a*b^4)*x^12 + 6545*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 5236*(B*a^4*b + 2*A*a^3
*b^2)*x^6 - 5236*A*a^5 + 2618*(B*a^5 + 5*A*a^4*b)*x^3)/x
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^2}\,dx = -\frac{Aa^5}{x}+\frac{Bb^5x^{17}}{17}+x^{14}\left(\frac{Ab^5}{14}+\frac{5Bab^4}{14}\right)+x^{11}\cdot\left(\frac{5Aab^4}{11}+\frac{10Ba^2b^3}{11}\right)+x^8\cdot\left(\frac{5Aa^2b^3}{4}+\frac{5Ba^3b^2}{4}\right)+x^5\cdot\left(2Aa^3b^2+Ba^4b\right)+x^2\cdot\left(\frac{5Aa^4b}{2}+\frac{Ba^5}{2}\right)$$

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**2,x)
```

```
[Out] -A*a**5/x + B*b**5*x**17/17 + x**14*(A*b**5/14 + 5*B*a*b**4/14) + x**11*(5*
A*a*b**4/11 + 10*B*a**2*b**3/11) + x**8*(5*A*a**2*b**3/4 + 5*B*a**3*b**2/4)
 + x**5*(2*A*a**3*b**2 + B*a**4*b) + x**2*(5*A*a**4*b/2 + B*a**5/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^2} dx = \frac{1}{17} Bb^5x^{17} + \frac{1}{14} (5 Bab^4 + Ab^5)x^{14} + \frac{5}{11} (2 Ba^2b^3 + Aab^4)x^{11} + \frac{5}{4} (Ba^3b^2 + Aa^2b^3)x^8 + (Ba^4b + 2 Aa^3b^2)x^5 - \frac{Aa^5}{x} + \frac{1}{2} (Ba^5 + 5 Aa^4b)x^2$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/17*B*b^5*x^17 + 1/14*(5*B*a*b^4 + A*b^5)*x^14 + 5/11*(2*B*a^2*b^3 + A*a*b
^4)*x^11 + 5/4*(B*a^3*b^2 + A*a^2*b^3)*x^8 + (B*a^4*b + 2*A*a^3*b^2)*x^5 -
A*a^5/x + 1/2*(B*a^5 + 5*A*a^4*b)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^2} dx = \frac{1}{17} Bb^5x^{17} + \frac{5}{14} Bab^4x^{14} + \frac{1}{14} Ab^5x^{14} + \frac{10}{11} Ba^2b^3x^{11} + \frac{5}{11} Aab^4x^{11} + \frac{5}{4} Ba^3b^2x^8 + \frac{5}{4} Aa^2b^3x^8 + Ba^4bx^5 + 2 Aa^3b^2x^5 + \frac{1}{2} Ba^5x^2 + \frac{5}{2} Aa^4bx^2 - \frac{Aa^5}{x}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/17*B*b^5*x^17 + 5/14*B*a*b^4*x^14 + 1/14*A*b^5*x^14 + 10/11*B*a^2*b^3*x^1
1 + 5/11*A*a*b^4*x^11 + 5/4*B*a^3*b^2*x^8 + 5/4*A*a^2*b^3*x^8 + B*a^4*b*x^5
 + 2*A*a^3*b^2*x^5 + 1/2*B*a^5*x^2 + 5/2*A*a^4*b*x^2 - A*a^5/x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^2} \, dx = x^2 \left(\frac{B a^5}{2} + \frac{5 A b a^4}{2}\right) + x^{14} \left(\frac{A b^5}{14} + \frac{5 B a b^4}{14}\right) - \frac{A a^5}{x} + \frac{B b^5 x^{17}}{17} + \frac{5 a^2 b^2 x^8 (A b + B a)}{4} + a^3 b x^5 (2 A b + B a) + \frac{5 a b^3 x^{11} (A b + 2 B a)}{11}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^2,x)
```

```
[Out] x^2*((B*a^5)/2 + (5*A*a^4*b)/2) + x^14*((A*b^5)/14 + (5*B*a*b^4)/14) - (A*a
^5)/x + (B*b^5*x^17)/17 + (5*a^2*b^2*x^8*(A*b + B*a))/4 + a^3*b*x^5*(2*A*b
+ B*a) + (5*a*b^3*x^11*(A*b + 2*B*a))/11
```

3.35 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 112

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^3}\,dx = -\frac{a^5A}{2x^2}+a^4(5Ab+aB)x+\frac{5}{4}a^3b(2Ab+aB)x^4+\frac{10}{7}a^2b^2(Ab+aB)x^7+\frac{1}{2}ab^3(Ab+2aB)x^{10}+\frac{1}{13}b^4(Ab+5aB)x^{13}+\frac{1}{16}b^5Bx^{16}$$

```
[Out] -1/2*a^5*A/x^2+a^4*(5*A*b+B*a)*x+5/4*a^3*b*(2*A*b+B*a)*x^4+10/7*a^2*b^2*(A*
b+B*a)*x^7+1/2*a*b^3*(A*b+2*B*a)*x^10+1/13*b^4*(A*b+5*B*a)*x^13+1/16*b^5*B*
x^16
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^3}\,dx = -\frac{a^5A}{2x^2}+a^4x(aB+5Ab)+\frac{5}{4}a^3bx^4(aB+2Ab)+\frac{10}{7}a^2b^2x^7(aB+Ab)+\frac{1}{13}b^4x^{13}(5aB+Ab)+\frac{1}{2}ab^3x^{10}(2aB+Ab)+\frac{1}{16}b^5Bx^{16}$$

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^3,x]
```

```
[Out] -1/2*(a^5*A)/x^2 + a^4*(5*A*b + a*B)*x + (5*a^3*b*(2*A*b + a*B)*x^4)/4 + (1
0*a^2*b^2*(A*b + a*B)*x^7)/7 + (a*b^3*(A*b + 2*a*B)*x^10)/2 + (b^4*(A*b + 5
*a*B)*x^13)/13 + (b^5*B*x^16)/16
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^4(5Ab+aB)+\frac{a^5A}{x^3}+5a^3b(2Ab+aB)x^3+10a^2b^2(Ab+aB)x^6\right.\\&\qquad\left.+5ab^3(Ab+2aB)x^9+b^4(Ab+5aB)x^{12}+b^5Bx^{15}\right)dx\\&=-\frac{a^5A}{2x^2}+a^4(5Ab+aB)x+\frac{5}{4}a^3b(2Ab+aB)x^4+\frac{10}{7}a^2b^2(Ab+aB)x^7\\&\quad+\frac{1}{2}ab^3(Ab+2aB)x^{10}+\frac{1}{13}b^4(Ab+5aB)x^{13}+\frac{1}{16}b^5Bx^{16}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{(a+bx^3)^5(A+Bx^3)}{x^3}\,dx &= -\frac{a^5A}{2x^2}+a^4(5Ab+aB)x+\frac{5}{4}a^3b(2Ab+aB)x^4\\&\quad+\frac{10}{7}a^2b^2(Ab+aB)x^7+\frac{1}{2}ab^3(Ab+2aB)x^{10}\\&\quad+\frac{1}{13}b^4(Ab+5aB)x^{13}+\frac{1}{16}b^5Bx^{16}\end{aligned}$$

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^3,x]
```


```
[Out] -1/2*(a^5*A)/x^2 + a^4*(5*A*b + a*B)*x + (5*a^3*b*(2*A*b + a*B)*x^4)/4 + (1
0*a^2*b^2*(A*b + a*B)*x^7)/7 + (a*b^3*(A*b + 2*a*B)*x^10)/2 + (b^4*(A*b + 5
*a*B)*x^13)/13 + (b^5*B*x^16)/16
```

Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.07

method	result
default	$\frac{b^{5} B \,x^{16}}{16}+\frac{A \,b^{5} x^{13}}{13}+\frac{5 B a \,b^{4} x^{13}}{13}+\frac{A a \,b^{4} x^{10}}{2}+B \,a^{2} b^{3} x^{10}+\frac{10 a^{2} A \,b^{3} x^{7}}{7}+\frac{10 B \,a^{3} b^{2} x^{7}}{7}+\frac{5 a^{3} A \,b^{2} x^{4}}{2}+\frac{5 B \,a^{4} b}{4}$
norman	$\frac{-\frac{a^{5} A}{2}+\left(5 a^{4} b A+a^{5} B\right) x^{3}+\left(\frac{5}{2} a^{3} b^{2} A+\frac{5}{4} a^{4} b B\right) x^{6}+\left(\frac{10}{7} a^{2} b^{3} A+\frac{10}{7} a^{3} b^{2} B\right) x^{9}+\left(\frac{1}{2} a \,b^{4} A+a^{2} b^{3} B\right) x^{12}+\left(\frac{1}{13} b^{5} A+\frac{5}{13} a \,b^{4} B\right) x^{15}+b}{x^{2}}$
risch	$\frac{b^{5} B \,x^{16}}{16}+\frac{A \,b^{5} x^{13}}{13}+\frac{5 B a \,b^{4} x^{13}}{13}+\frac{A a \,b^{4} x^{10}}{2}+B \,a^{2} b^{3} x^{10}+\frac{10 a^{2} A \,b^{3} x^{7}}{7}+\frac{10 B \,a^{3} b^{2} x^{7}}{7}+\frac{5 a^{3} A \,b^{2} x^{4}}{2}+\frac{5 B \,a^{4} b}{4}$
gosper	$-\frac{-91 b^{5} B \,x^{18}-112 A \,b^{5} x^{15}-560 B a \,b^{4} x^{15}-728 a A \,b^{4} x^{12}-1456 B \,a^{2} b^{3} x^{12}-2080 a^{2} A \,b^{3} x^{9}-2080 B \,a^{3} b^{2} x^{9}-3640 a^{3} A \,b^{2} x^{6}-18}{1456 x^{2}}$
parallelrisch	$\frac{91 b^{5} B \,x^{18}+112 A \,b^{5} x^{15}+560 B a \,b^{4} x^{15}+728 a A \,b^{4} x^{12}+1456 B \,a^{2} b^{3} x^{12}+2080 a^{2} A \,b^{3} x^{9}+2080 B \,a^{3} b^{2} x^{9}+3640 a^{3} A \,b^{2} x^{6}+1820 B}{1456 x^{2}}$

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*b^5*B*x^16+1/13*A*b^5*x^13+5/13*B*a*b^4*x^13+1/2*A*a*b^4*x^10+B*a^2*b^
3*x^10+10/7*a^2*A*b^3*x^7+10/7*B*a^3*b^2*x^7+5/2*a^3*A*b^2*x^4+5/4*B*a^4*b*
x^4+5*a^4*A*b*x+a^5*B*x-1/2*a^5*A/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^3} \, dx = \frac{91 \, Bb^5x^{18} + 112 \, (5 \, Bab^4 + Ab^5)x^{15} + 728 \, (2 \, Ba^2b^3 + Aab^4)x^{12} + 2080 \, (Ba^3b^2 + Aa^2b^3)x^9 + 1820 \, (Ba^4b +}{1456 \, x^2}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/1456*(91*B*b^5*x^18 + 112*(5*B*a*b^4 + A*b^5)*x^15 + 728*(2*B*a^2*b^3 + A
*a*b^4)*x^12 + 2080*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 1820*(B*a^4*b + 2*A*a^3*b
^2)*x^6 - 728*A*a^5 + 1456*(B*a^5 + 5*A*a^4*b)*x^3)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^3} \, dx = -\frac{Aa^5}{2x^2} + \frac{Bb^5x^{16}}{16} + x^{13}\left(\frac{Ab^5}{13} + \frac{5Bab^4}{13}\right) + x^{10}\left(\frac{Aab^4}{2} + Ba^2b^3\right) + x^7 \cdot \left(\frac{10Aa^2b^3}{7} + \frac{10Ba^3b^2}{7}\right) + x^4 \cdot \left(\frac{5Aa^3b^2}{2} + \frac{5Ba^4b}{4}\right) + x\left(5Aa^4b + Ba^5\right)$$

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**3,x)
```

```
[Out] -A*a**5/(2*x**2) + B*b**5*x**16/16 + x**13*(A*b**5/13 + 5*B*a*b**4/13) + x*
*10*(A*a*b**4/2 + B*a**2*b**3) + x**7*(10*A*a**2*b**3/7 + 10*B*a**3*b**2/7)
 + x**4*(5*A*a**3*b**2/2 + 5*B*a**4*b/4) + x*(5*A*a**4*b + B*a**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^3} dx = \frac{1}{16} Bb^5x^{16} + \frac{1}{13} (5 Bab^4 + Ab^5)x^{13} + \frac{1}{2} (2 Ba^2b^3 + Aab^4)x^{10} + \frac{10}{7} (Ba^3b^2 + Aa^2b^3)x^7 + \frac{5}{4} (Ba^4b + 2 Aa^3b^2)x^4 - \frac{Aa^5}{2 x^2} + (Ba^5 + 5 Aa^4b)x$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/16*B*b^5*x^16 + 1/13*(5*B*a*b^4 + A*b^5)*x^13 + 1/2*(2*B*a^2*b^3 + A*a*b^
4)*x^10 + 10/7*(B*a^3*b^2 + A*a^2*b^3)*x^7 + 5/4*(B*a^4*b + 2*A*a^3*b^2)*x^
4 - 1/2*A*a^5/x^2 + (B*a^5 + 5*A*a^4*b)*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^3} dx = \frac{1}{16} Bb^5x^{16} + \frac{5}{13} Bab^4x^{13} + \frac{1}{13} Ab^5x^{13} + Ba^2b^3x^{10} + \frac{1}{2} Aab^4x^{10} + \frac{10}{7} Ba^3b^2x^7 + \frac{10}{7} Aa^2b^3x^7 + \frac{5}{4} Ba^4bx^4 + \frac{5}{2} Aa^3b^2x^4 + Ba^5x + 5 Aa^4bx - \frac{Aa^5}{2 x^2}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*B*b^5*x^16 + 5/13*B*a*b^4*x^13 + 1/13*A*b^5*x^13 + B*a^2*b^3*x^10 + 1/
2*A*a*b^4*x^10 + 10/7*B*a^3*b^2*x^7 + 10/7*A*a^2*b^3*x^7 + 5/4*B*a^4*b*x^4
+ 5/2*A*a^3*b^2*x^4 + B*a^5*x + 5*A*a^4*b*x - 1/2*A*a^5/x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x^3}\,dx = x\,\left(B\,a^5+5\,A\,b\,a^4\right)+x^{13}\,\left(\frac{A\,b^5}{13}+\frac{5\,B\,a\,b^4}{13}\right)-\frac{A\,a^5}{2\,x^2}+\frac{B\,b^5\,x^{16}}{16}+\frac{10\,a^2\,b^2\,x^7\,(A\,b+B\,a)}{7}+\frac{5\,a^3\,b\,x^4\,(2\,A\,b+B\,a)}{4}+\frac{a\,b^3\,x^{10}\,(A\,b+2\,B\,a)}{2}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^3,x)
```

```
[Out] x*(B*a^5 + 5*A*a^4*b) + x^13*((A*b^5)/13 + (5*B*a*b^4)/13) - (A*a^5)/(2*x^2
) + (B*b^5*x^16)/16 + (10*a^2*b^2*x^7*(A*b + B*a))/7 + (5*a^3*b*x^4*(2*A*b
+ B*a))/4 + (a*b^3*x^10*(A*b + 2*B*a))/2
```

3.36 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^4} dx$

Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^4} dx = -\frac{a^5A}{3x^3} + \frac{5}{3}a^3b(2Ab+aB)x^3 + \frac{5}{3}a^2b^2(Ab+aB)x^6 + \frac{5}{9}ab^3(Ab+2aB)x^9 + \frac{1}{12}b^4(Ab+5aB)x^{12} + \frac{1}{15}b^5Bx^{15} + a^4(5Ab+aB)\log(x)$$

```
[Out] -1/3*a^5*A/x^3+5/3*a^3*b*(2*A*b+B*a)*x^3+5/3*a^2*b^2*(A*b+B*a)*x^6+5/9*a*b^
3*(A*b+2*B*a)*x^9+1/12*b^4*(A*b+5*B*a)*x^12+1/15*b^5*B*x^15+a^4*(5*A*b+B*a)
*ln(x)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 77}

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^4} dx = -\frac{a^5A}{3x^3} + a^4\log(x)(aB+5Ab) + \frac{5}{3}a^3bx^3(aB+2Ab) + \frac{5}{3}a^2b^2x^6(aB+Ab) + \frac{1}{12}b^4x^{12}(5aB+Ab) + \frac{5}{9}ab^3x^9(2aB+Ab) + \frac{1}{15}b^5Bx^{15}$$

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^4,x]
```

```
[Out] -1/3*(a^5*A)/x^3 + (5*a^3*b*(2*A*b + a*B)*x^3)/3 + (5*a^2*b^2*(A*b + a*B)*x
^6)/3 + (5*a*b^3*(A*b + 2*a*B)*x^9)/9 + (b^4*(A*b + 5*a*B)*x^12)/12 + (b^5*
B*x^15)/15 + a^4*(5*A*b + a*B)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^2}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(5a^3b(2Ab+aB)+\frac{a^5A}{x^2}+\frac{a^4(5Ab+aB)}{x}+10a^2b^2(Ab+aB)x\right.\right.\\ &\qquad\left.\left.+5ab^3(Ab+2aB)x^2+b^4(Ab+5aB)x^3+b^5Bx^4\right)\,dx,x,x^3\right)\\ &= -\frac{a^5A}{3x^3}+\frac{5}{3}a^3b(2Ab+aB)x^3+\frac{5}{3}a^2b^2(Ab+aB)x^6+\frac{5}{9}ab^3(Ab+2aB)x^9\\ &\quad+\frac{1}{12}b^4(Ab+5aB)x^{12}+\frac{1}{15}b^5Bx^{15}+a^4(5Ab+aB)\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x^4}\,dx &= -\frac{a^5A}{3x^3}+\frac{5}{3}a^3b(2Ab+aB)x^3+\frac{5}{3}a^2b^2(Ab+aB)x^6\\ &\quad+\frac{5}{9}ab^3(Ab+2aB)x^9+\frac{1}{12}b^4(Ab+5aB)x^{12}\\ &\quad+\frac{1}{15}b^5Bx^{15}+\left(5a^4Ab+a^5B\right)\log(x)\end{aligned}$$

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^4,x]
```

```
[Out] -1/3*(a^5*A)/x^3 + (5*a^3*b*(2*A*b + a*B)*x^3)/3 + (5*a^2*b^2*(A*b + a*B)*x
^6)/3 + (5*a*b^3*(A*b + 2*a*B)*x^9)/9 + (b^4*(A*b + 5*a*B)*x^12)/12 + (b^5*
B*x^15)/15 + (5*a^4*A*b + a^5*B)*Log[x]
```

Maple [A] (verified)

Time = 4.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

method	result
default	$\frac{b^5 B x^{15}}{15}+\frac{A b^5 x^{12}}{12}+\frac{5 B a b^4 x^{12}}{12}+\frac{5 A a b^4 x^9}{9}+\frac{10 B a^2 b^3 x^9}{9}+\frac{5 a^2 A b^3 x^6}{3}+\frac{5 B a^3 b^2 x^6}{3}+\frac{10 a^3 A b^2 x^3}{3}+\frac{5 B a^4 b x^3}{3}$
norman	$\frac{\left(\frac{1}{12} b^5 A+\frac{5}{12} a b^4 B\right) x^{15}+\left(\frac{5}{9} a b^4 A+\frac{10}{9} a^2 b^3 B\right) x^{12}+\left(\frac{5}{3} a^2 b^3 A+\frac{5}{3} a^3 b^2 B\right) x^9+\left(\frac{10}{3} a^3 b^2 A+\frac{5}{3} a^4 b B\right) x^6-\frac{a^5 A}{3}+\frac{b^5 B x^{18}}{15}}{x^3}+(5 a^4 b A$
risch	$\frac{b^5 B x^{15}}{15}+\frac{A b^5 x^{12}}{12}+\frac{5 B a b^4 x^{12}}{12}+\frac{5 A a b^4 x^9}{9}+\frac{10 B a^2 b^3 x^9}{9}+\frac{5 a^2 A b^3 x^6}{3}+\frac{5 B a^3 b^2 x^6}{3}+\frac{10 a^3 A b^2 x^3}{3}+\frac{5 B a^4 b x^3}{3}$
parallelrisch	$\frac{12 b^5 B x^{18}+15 A b^5 x^{15}+75 B a b^4 x^{15}+100 a A b^4 x^{12}+200 B a^2 b^3 x^{12}+300 a^2 A b^3 x^9+300 B a^3 b^2 x^9+600 a^3 A b^2 x^6+300 B a^4 b x^6+90}{180 x^3}$

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*b^5*B*x^15+1/12*A*b^5*x^12+5/12*B*a*b^4*x^12+5/9*A*a*b^4*x^9+10/9*B*a^
2*b^3*x^9+5/3*a^2*A*b^3*x^6+5/3*B*a^3*b^2*x^6+10/3*a^3*A*b^2*x^3+5/3*B*a^4*
b*x^3+a^4*(5*A*b+B*a)*ln(x)-1/3*a^5*A/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^4} dx = \frac{12 Bb^5x^{18} + 15 (5 Bab^4 + Ab^5)x^{15} + 100 (2 Ba^2b^3 + Aab^4)x^{12} + 300 (Ba^3b^2 + Aa^2b^3)x^9 + 300 (Ba^4b + 2 A}{180 x^3}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^4,x, algorithm="fricas")
```

```
[Out] 1/180*(12*B*b^5*x^18 + 15*(5*B*a*b^4 + A*b^5)*x^15 + 100*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 300*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 300*(B*a^4*b + 2*A*a^3*b^2)*
x^6 - 60*A*a^5 + 180*(B*a^5 + 5*A*a^4*b)*x^3*log(x))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^4}\,dx = -\frac{Aa^5}{3x^3} + \frac{Bb^5x^{15}}{15} + a^4\cdot(5Ab+Ba)\log(x) + x^{12}\left(\frac{Ab^5}{12}+\frac{5Bab^4}{12}\right) + x^9\cdot\left(\frac{5Aab^4}{9}+\frac{10Ba^2b^3}{9}\right) + x^6 \cdot\left(\frac{5Aa^2b^3}{3}+\frac{5Ba^3b^2}{3}\right) + x^3\cdot\left(\frac{10Aa^3b^2}{3}+\frac{5Ba^4b}{3}\right)$$

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**4,x)
```

```
[Out] -A*a**5/(3*x**3) + B*b**5*x**15/15 + a**4*(5*A*b + B*a)*log(x) + x**12*(A*b
**5/12 + 5*B*a*b**4/12) + x**9*(5*A*a*b**4/9 + 10*B*a**2*b**3/9) + x**6*(5*
A*a**2*b**3/3 + 5*B*a**3*b**2/3) + x**3*(10*A*a**3*b**2/3 + 5*B*a**4*b/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^4}\,dx = \frac{1}{15}Bb^5x^{15} + \frac{1}{12}\left(5\,Bab^4+Ab^5\right)x^{12} + \frac{5}{9}\left(2\,Ba^2b^3+Aab^4\right)x^9 + \frac{5}{3}\left(Ba^3b^2+Aa^2b^3\right)x^6 + \frac{5}{3}\left(Ba^4b+2\,Aa^3b^2\right)x^3 - \frac{Aa^5}{3\,x^3} + \frac{1}{3}\left(Ba^5+5\,Aa^4b\right)\log\left(x^3\right)$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/15*B*b^5*x^15 + 1/12*(5*B*a*b^4 + A*b^5)*x^12 + 5/9*(2*B*a^2*b^3 + A*a*b^
4)*x^9 + 5/3*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 5/3*(B*a^4*b + 2*A*a^3*b^2)*x^3
- 1/3*A*a^5/x^3 + 1/3*(B*a^5 + 5*A*a^4*b)*log(x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^4}\,dx = \frac{1}{15}Bb^5x^{15} + \frac{5}{12}Bab^4x^{12} + \frac{1}{12}Ab^5x^{12} + \frac{10}{9}Ba^2b^3x^9 + \frac{5}{9}Aab^4x^9 + \frac{5}{3}Ba^3b^2x^6 + \frac{5}{3}Aa^2b^3x^6 + \frac{5}{3}Ba^4bx^3 + \frac{10}{3}Aa^3b^2x^3 + \left(Ba^5+5\,Aa^4b\right)\log\left(|x|\right) - \frac{Ba^5x^3+5\,Aa^4bx^3+Aa^5}{3\,x^3}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/15*B*b^5*x^15 + 5/12*B*a*b^4*x^12 + 1/12*A*b^5*x^12 + 10/9*B*a^2*b^3*x^9
+ 5/9*A*a*b^4*x^9 + 5/3*B*a^3*b^2*x^6 + 5/3*A*a^2*b^3*x^6 + 5/3*B*a^4*b*x^3
 + 10/3*A*a^3*b^2*x^3 + (B*a^5 + 5*A*a^4*b)*log(abs(x)) - 1/3*(B*a^5*x^3 +
5*A*a^4*b*x^3 + A*a^5)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^4} dx = x^{12} \left(\frac{A b^5}{12} + \frac{5 B a b^4}{12}\right) + \ln(x) \left(B a^5 + 5 A b a^4\right) - \frac{A a^5}{3 x^3} + \frac{B b^5 x^{15}}{15} + \frac{5 a^2 b^2 x^6 (A b + B a)}{3} + \frac{5 a^3 b x^3 (2 A b + B a)}{3} + \frac{5 a b^3 x^9 (A b + 2 B a)}{9}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^4,x)
```

```
[Out] x^12*((A*b^5)/12 + (5*B*a*b^4)/12) + log(x)*(B*a^5 + 5*A*a^4*b) - (A*a^5)/(
3*x^3) + (B*b^5*x^15)/15 + (5*a^2*b^2*x^6*(A*b + B*a))/3 + (5*a^3*b*x^3*(2*
A*b + B*a))/3 + (5*a*b^3*x^9*(A*b + 2*B*a))/9
```

3.37 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^5} dx$

Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^5} dx = -\frac{a^5A}{4x^4} - \frac{a^4(5Ab+aB)}{x} + \frac{5}{2}a^3b(2Ab+aB)x^2 + 2a^2b^2(Ab+aB)x^5 + \frac{5}{8}ab^3(Ab+2aB)x^8 + \frac{1}{11}b^4(Ab+5aB)x^{11} + \frac{1}{14}b^5Bx^{14}$$

```
[Out] -1/4*a^5*A/x^4-a^4*(5*A*b+B*a)/x+5/2*a^3*b*(2*A*b+B*a)*x^2+2*a^2*b^2*(A*b+B
*a)*x^5+5/8*a*b^3*(A*b+2*B*a)*x^8+1/11*b^4*(A*b+5*B*a)*x^11+1/14*b^5*B*x^14
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^5} dx = -\frac{a^5A}{4x^4} - \frac{a^4(aB+5Ab)}{x} + \frac{5}{2}a^3bx^2(aB+2Ab) + 2a^2b^2x^5(aB+Ab) + \frac{1}{11}b^4x^{11}(5aB+Ab) + \frac{5}{8}ab^3x^8(2aB+Ab) + \frac{1}{14}b^5Bx^{14}$$

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^5,x]
```

```
[Out] -1/4*(a^5*A)/x^4 - (a^4*(5*A*b + a*B))/x + (5*a^3*b*(2*A*b + a*B)*x^2)/2 +
2*a^2*b^2*(A*b + a*B)*x^5 + (5*a*b^3*(A*b + 2*a*B)*x^8)/8 + (b^4*(A*b + 5*a
*B)*x^11)/11 + (b^5*B*x^14)/14
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
```

```
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a^5A}{x^5}+\frac{a^4(5Ab+aB)}{x^2}+5a^3b(2Ab+aB)x+10a^2b^2(Ab+aB)x^4 + 5ab^3(Ab+2aB)x^7+b^4(Ab+5aB)x^{10}+b^5Bx^{13}\right)dx$$

$$= -\frac{a^5A}{4x^4}-\frac{a^4(5Ab+aB)}{x}+\frac{5}{2}a^3b(2Ab+aB)x^2+2a^2b^2(Ab+aB)x^5 + \frac{5}{8}ab^3(Ab+2aB)x^8+\frac{1}{11}b^4(Ab+5aB)x^{11}+\frac{1}{14}b^5Bx^{14}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^5}\,dx = -\frac{a^5A}{4x^4}+\frac{-5a^4Ab-a^5B}{x}+\frac{5}{2}a^3b(2Ab+aB)x^2 + 2a^2b^2(Ab+aB)x^5+\frac{5}{8}ab^3(Ab+2aB)x^8 + \frac{1}{11}b^4(Ab+5aB)x^{11}+\frac{1}{14}b^5Bx^{14}$$

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^5,x]
[Out] -1/4*(a^5*A)/x^4 + (-5*a^4*A*b - a^5*B)/x + (5*a^3*b*(2*A*b + a*B)*x^2)/2 +
 2*a^2*b^2*(A*b + a*B)*x^5 + (5*a*b^3*(A*b + 2*a*B)*x^8)/8 + (b^4*(A*b + 5*
a*B)*x^11)/11 + (b^5*B*x^14)/14
```

Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.08

method	result
norman	$\frac{-\frac{a^5 A}{4}+(-5a^4bA-a^5B)x^3+(5a^3b^2A+\frac{5}{2}a^4bB)x^6+(2a^2b^3A+2a^3b^2B)x^9+(\frac{5}{8}a\,b^4A+\frac{5}{4}a^2b^3B)x^{12}+(\frac{1}{11}b^5A+\frac{5}{11}a\,b^4B)x^{15}+b}{x^4}$
default	$\frac{b^5 B\,x^{14}}{14}+\frac{A\,b^5x^{11}}{11}+\frac{5Ba\,b^4x^{11}}{11}+\frac{5aA\,b^4x^8}{8}+\frac{5B\,a^2b^3x^8}{4}+2A\,a^2b^3x^5+2B\,a^3b^2x^5+5a^3A\,b^2x^2+\frac{5B}{}$
risch	$\frac{b^5 B\,x^{14}}{14}+\frac{A\,b^5x^{11}}{11}+\frac{5Ba\,b^4x^{11}}{11}+\frac{5aA\,b^4x^8}{8}+\frac{5B\,a^2b^3x^8}{4}+2A\,a^2b^3x^5+2B\,a^3b^2x^5+5a^3A\,b^2x^2+\frac{5B}{}$
gosper	$-\frac{-44b^5B\,x^{18}-56A\,b^5x^{15}-280Ba\,b^4x^{15}-385aA\,b^4x^{12}-770B\,a^2b^3x^{12}-1232a^2A\,b^3x^9-1232B\,a^3b^2x^9-3080a^3A\,b^2x^6-1540}{616x^4}$
parallelrisch	$\frac{44b^5B\,x^{18}+56A\,b^5x^{15}+280Ba\,b^4x^{15}+385aA\,b^4x^{12}+770B\,a^2b^3x^{12}+1232a^2A\,b^3x^9+1232B\,a^3b^2x^9+3080a^3A\,b^2x^6+1540B\,a^4}{616x^4}$

[In] int((b*x^3+a)^5*(B*x^3+A)/x^5,x,method=_RETURNVERBOSE)

[Out] 1/x^4*(-1/4*a^5*A+(-5*A*a^4*b-B*a^5)*x^3+(5*a^3*b^2*A+5/2*a^4*b*B)*x^6+(2*A *a^2*b^3+2*B*a^3*b^2)*x^9+(5/8*a*b^4*A+5/4*a^2*b^3*B)*x^12+(1/11*b^5*A+5/11 *a*b^4*B)*x^15+1/14*b^5*B*x^18)

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^5}\,dx = \frac{44\,Bb^5x^{18}+56\,(5\,Bab^4+Ab^5)x^{15}+385\,(2\,Ba^2b^3+Aab^4)x^{12}+1232\,(Ba^3b^2+Aa^2b^3)x^9+1540\,(Ba^4b+}{616\,x^4}$$

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^5,x, algorithm="fricas")

[Out] 1/616*(44*B*b^5*x^18 + 56*(5*B*a*b^4 + A*b^5)*x^15 + 385*(2*B*a^2*b^3 + A*a *b^4)*x^12 + 1232*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 1540*(B*a^4*b + 2*A*a^3*b^2)*x^6 - 154*A*a^5 - 616*(B*a^5 + 5*A*a^4*b)*x^3)/x^4

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^5}\,dx = \frac{Bb^5x^{14}}{14}+x^{11}\left(\frac{Ab^5}{11}+\frac{5Bab^4}{11}\right)+x^8\cdot\left(\frac{5Aab^4}{8}+\frac{5Ba^2b^3}{4}\right)+x^5\cdot\left(2Aa^2b^3+2Ba^3b^2\right)+x^2\cdot\left(5Aa^3b^2+\frac{5Ba^4b}{2}\right)+\frac{-Aa^5+x^3(-20Aa^4b-4Ba^5)}{4x^4}$$

[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**5,x)

```
[Out] B*b**5*x**14/14 + x**11*(A*b**5/11 + 5*B*a*b**4/11) + x**8*(5*A*a*b**4/8 +
5*B*a**2*b**3/4) + x**5*(2*A*a**2*b**3 + 2*B*a**3*b**2) + x**2*(5*A*a**3*b*
*2 + 5*B*a**4*b/2) + (-A*a**5 + x**3*(-20*A*a**4*b - 4*B*a**5))/(4*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^5}\,dx = \frac{1}{14}Bb^5x^{14} + \frac{1}{11}\left(5Bab^4 + Ab^5\right)x^{11} + \frac{5}{8}\left(2Ba^2b^3 + Aab^4\right)x^8 + 2\left(Ba^3b^2 + Aa^2b^3\right)x^5 + \frac{5}{2}\left(Ba^4b + 2Aa^3b^2\right)x^2 - \frac{Aa^5 + 4(Ba^5 + 5Aa^4b)x^3}{4x^4}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^5,x, algorithm="maxima")
```

```
[Out] 1/14*B*b^5*x^14 + 1/11*(5*B*a*b^4 + A*b^5)*x^11 + 5/8*(2*B*a^2*b^3 + A*a*b^
4)*x^8 + 2*(B*a^3*b^2 + A*a^2*b^3)*x^5 + 5/2*(B*a^4*b + 2*A*a^3*b^2)*x^2 -
1/4*(A*a^5 + 4*(B*a^5 + 5*A*a^4*b)*x^3)/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^5}\,dx = \frac{1}{14}Bb^5x^{14} + \frac{5}{11}Bab^4x^{11} + \frac{1}{11}Ab^5x^{11} + \frac{5}{4}Ba^2b^3x^8 + \frac{5}{8}Aab^4x^8 + 2Ba^3b^2x^5 + 2Aa^2b^3x^5 + \frac{5}{2}Ba^4bx^2 + 5Aa^3b^2x^2 - \frac{4Ba^5x^3 + 20Aa^4bx^3 + Aa^5}{4x^4}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^5,x, algorithm="giac")
```

```
[Out] 1/14*B*b^5*x^14 + 5/11*B*a*b^4*x^11 + 1/11*A*b^5*x^11 + 5/4*B*a^2*b^3*x^8 +
 5/8*A*a*b^4*x^8 + 2*B*a^3*b^2*x^5 + 2*A*a^2*b^3*x^5 + 5/2*B*a^4*b*x^2 + 5*
A*a^3*b^2*x^2 - 1/4*(4*B*a^5*x^3 + 20*A*a^4*b*x^3 + A*a^5)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^5} \, dx = x^{11} \left(\frac{A b^5}{11} + \frac{5 B a b^4}{11}\right) - \frac{\frac{A a^5}{4} + x^3 (B a^5 + 5 A b a^4)}{x^4} + \frac{B b^5 x^{14}}{14} + 2 a^2 b^2 x^5 (A b + B a) + \frac{5 a^3 b x^2 (2 A b + B a)}{2} + \frac{5 a b^3 x^8 (A b + 2 B a)}{8}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^5,x)
```

```
[Out] x^11*((A*b^5)/11 + (5*B*a*b^4)/11) - ((A*a^5)/4 + x^3*(B*a^5 + 5*A*a^4*b))/
x^4 + (B*b^5*x^14)/14 + 2*a^2*b^2*x^5*(A*b + B*a) + (5*a^3*b*x^2*(2*A*b + B
*a))/2 + (5*a*b^3*x^8*(A*b + 2*B*a))/8
```

3.38 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^6}\,dx = -\frac{a^5A}{5x^5}-\frac{a^4(5Ab+aB)}{2x^2}+5a^3b(2Ab+aB)x+\frac{5}{2}a^2b^2(Ab+aB)x^4+\frac{5}{7}ab^3(Ab+2aB)x^7+\frac{1}{10}b^4(Ab+5aB)x^{10}+\frac{1}{13}b^5Bx^{13}$$

```
[Out] -1/5*a^5*A/x^5-1/2*a^4*(5*A*b+B*a)/x^2+5*a^3*b*(2*A*b+B*a)*x+5/2*a^2*b^2*(A
*b+B*a)*x^4+5/7*a*b^3*(A*b+2*B*a)*x^7+1/10*b^4*(A*b+5*B*a)*x^10+1/13*b^5*B*
x^13
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^6}\,dx = -\frac{a^5A}{5x^5}-\frac{a^4(aB+5Ab)}{2x^2}+5a^3bx(aB+2Ab)+\frac{5}{2}a^2b^2x^4(aB+Ab)+\frac{1}{10}b^4x^{10}(5aB+Ab)+\frac{5}{7}ab^3x^7(2aB+Ab)+\frac{1}{13}b^5Bx^{13}$$

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^6,x]
```

```
[Out] -1/5*(a^5*A)/x^5 - (a^4*(5*A*b + a*B))/(2*x^2) + 5*a^3*b*(2*A*b + a*B)*x +
(5*a^2*b^2*(A*b + a*B)*x^4)/2 + (5*a*b^3*(A*b + 2*a*B)*x^7)/7 + (b^4*(A*b +
 5*a*B)*x^10)/10 + (b^5*B*x^13)/13
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(5a^3b(2Ab+aB) + \frac{a^5A}{x^6} + \frac{a^4(5Ab+aB)}{x^3} + 10a^2b^2(Ab+aB)x^3 + 5ab^3(Ab+2aB)x^6 + b^4(Ab+5aB)x^9 + b^5Bx^{12}\right) dx$$

$$= -\frac{a^5A}{5x^5} - \frac{a^4(5Ab+aB)}{2x^2} + 5a^3b(2Ab+aB)x + \frac{5}{2}a^2b^2(Ab+aB)x^4 + \frac{5}{7}ab^3(Ab+2aB)x^7 + \frac{1}{10}b^4(Ab+5aB)x^{10} + \frac{1}{13}b^5Bx^{13}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^6} dx = -\frac{a^5A}{5x^5} - \frac{a^4(5Ab+aB)}{2x^2} + 5a^3b(2Ab+aB)x + \frac{5}{2}a^2b^2(Ab+aB)x^4 + \frac{5}{7}ab^3(Ab+2aB)x^7 + \frac{1}{10}b^4(Ab+5aB)x^{10} + \frac{1}{13}b^5Bx^{13}$$

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^6,x]
[Out] -1/5*(a^5*A)/x^5 - (a^4*(5*A*b + a*B))/(2*x^2) + 5*a^3*b*(2*A*b + a*B)*x +
(5*a^2*b^2*(A*b + a*B)*x^4)/2 + (5*a*b^3*(A*b + 2*a*B)*x^7)/7 + (b^4*(A*b +
 5*a*B)*x^10)/10 + (b^5*B*x^13)/13
```

Maple [A] (verified)

Time = 4.42 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.05

method	result
default	$\frac{b^5 B x^{13}}{13}+\frac{A b^5 x^{10}}{10}+\frac{B a b^4 x^{10}}{2}+\frac{5 A a b^4 x^7}{7}+\frac{10 B a^2 b^3 x^7}{7}+\frac{5 a^2 A b^3 x^4}{2}+\frac{5 B a^3 b^2 x^4}{2}+10 a^3 b^2 A x+5 a^4 b B x$
norman	$\frac{-\frac{a^5 A}{5}+\left(-\frac{5}{2} a^4 b A-\frac{1}{2} a^5 B\right) x^3+\left(10 a^3 b^2 A+5 a^4 b B\right) x^6+\left(\frac{5}{2} a^2 b^3 A+\frac{5}{2} a^3 b^2 B\right) x^9+\left(\frac{5}{7} a b^4 A+\frac{10}{7} a^2 b^3 B\right) x^{12}+\left(\frac{1}{10} b^5 A+\frac{1}{2} a b^4 B\right) x^{15}+}{x^5}$
risch	$\frac{b^5 B x^{13}}{13}+\frac{A b^5 x^{10}}{10}+\frac{B a b^4 x^{10}}{2}+\frac{5 A a b^4 x^7}{7}+\frac{10 B a^2 b^3 x^7}{7}+\frac{5 a^2 A b^3 x^4}{2}+\frac{5 B a^3 b^2 x^4}{2}+10 a^3 b^2 A x+5 a^4 b B x$
gosper	$-\frac{-70 b^5 B x^{18}-91 A b^5 x^{15}-455 B a b^4 x^{15}-650 a A b^4 x^{12}-1300 B a^2 b^3 x^{12}-2275 a^2 A b^3 x^9-2275 B a^3 b^2 x^9-9100 a^3 A b^2 x^6-4550 B}{910 x^5}$
parallelrisch	$\frac{70 b^5 B x^{18}+91 A b^5 x^{15}+455 B a b^4 x^{15}+650 a A b^4 x^{12}+1300 B a^2 b^3 x^{12}+2275 a^2 A b^3 x^9+2275 B a^3 b^2 x^9+9100 a^3 A b^2 x^6+4550 B a^4 b}{910 x^5}$

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b^5*B*x^13+1/10*A*b^5*x^10+1/2*B*a*b^4*x^10+5/7*A*a*b^4*x^7+10/7*B*a^2
*b^3*x^7+5/2*a^2*A*b^3*x^4+5/2*B*a^3*b^2*x^4+10*a^3*b^2*A*x+5*a^4*b*B*x-1/2
*a^4*(5*A*b+B*a)/x^2-1/5*a^5*A/x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^6} \, dx = \frac{70 \, B b^5 x^{18} + 91 \, (5 \, B a b^4 + A b^5) x^{15} + 650 \, (2 \, B a^2 b^3 + A a b^4) x^{12} + 2275 \, (B a^3 b^2 + A a^2 b^3) x^9 + 4550 \, (B a^4 b + 2}{910 \, x^5}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^6,x, algorithm="fricas")
```

```
[Out] 1/910*(70*B*b^5*x^18 + 91*(5*B*a*b^4 + A*b^5)*x^15 + 650*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 2275*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 4550*(B*a^4*b + 2*A*a^3*b^2
)*x^6 - 182*A*a^5 - 455*(B*a^5 + 5*A*a^4*b)*x^3)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^6} \, dx = \frac{Bb^5x^{13}}{13} + x^{10}\left(\frac{Ab^5}{10} + \frac{Bab^4}{2}\right) + x^7 \cdot \left(\frac{5Aab^4}{7} + \frac{10Ba^2b^3}{7}\right) + x^4 \cdot \left(\frac{5Aa^2b^3}{2} + \frac{5Ba^3b^2}{2}\right) + x\left(10Aa^3b^2 + 5Ba^4b\right) + \frac{-2Aa^5 + x^3(-25Aa^4b - 5Ba^5)}{10x^5}$$

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**6,x)
```

```
[Out] B*b**5*x**13/13 + x**10*(A*b**5/10 + B*a*b**4/2) + x**7*(5*A*a*b**4/7 + 10*
B*a**2*b**3/7) + x**4*(5*A*a**2*b**3/2 + 5*B*a**3*b**2/2) + x*(10*A*a**3*b*
*2 + 5*B*a**4*b) + (-2*A*a**5 + x**3*(-25*A*a**4*b - 5*B*a**5))/(10*x**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^6}\,dx = \frac{1}{13}\,Bb^5x^{13} + \frac{1}{10}\left(5\,Bab^4 + Ab^5\right)x^{10} + \frac{5}{7}\left(2\,Ba^2b^3 + Aab^4\right)x^7 + \frac{5}{2}\left(Ba^3b^2 + Aa^2b^3\right)x^4 + 5\left(Ba^4b + 2\,Aa^3b^2\right)x - \frac{2\,Aa^5 + 5\,(Ba^5 + 5\,Aa^4b)x^3}{10\,x^5}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^6,x, algorithm="maxima")
```

```
[Out] 1/13*B*b^5*x^13 + 1/10*(5*B*a*b^4 + A*b^5)*x^10 + 5/7*(2*B*a^2*b^3 + A*a*b^
4)*x^7 + 5/2*(B*a^3*b^2 + A*a^2*b^3)*x^4 + 5*(B*a^4*b + 2*A*a^3*b^2)*x - 1/
10*(2*A*a^5 + 5*(B*a^5 + 5*A*a^4*b)*x^3)/x^5
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^6}\,dx = \frac{1}{13}\,Bb^5x^{13} + \frac{1}{2}\,Bab^4x^{10} + \frac{1}{10}\,Ab^5x^{10} + \frac{10}{7}\,Ba^2b^3x^7 + \frac{5}{7}\,Aab^4x^7 + \frac{5}{2}\,Ba^3b^2x^4 + \frac{5}{2}\,Aa^2b^3x^4 + 5\,Ba^4bx + 10\,Aa^3b^2x - \frac{5\,Ba^5x^3 + 25\,Aa^4bx^3 + 2\,Aa^5}{10\,x^5}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^6,x, algorithm="giac")
```

```
[Out] 1/13*B*b^5*x^13 + 1/2*B*a*b^4*x^10 + 1/10*A*b^5*x^10 + 10/7*B*a^2*b^3*x^7 +
 5/7*A*a*b^4*x^7 + 5/2*B*a^3*b^2*x^4 + 5/2*A*a^2*b^3*x^4 + 5*B*a^4*b*x + 10
*A*a^3*b^2*x - 1/10*(5*B*a^5*x^3 + 25*A*a^4*b*x^3 + 2*A*a^5)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^6}\,dx = x^{10}\left(\frac{A\,b^5}{10}+\frac{B\,a\,b^4}{2}\right)-\frac{\frac{A\,a^5}{5}+x^3\left(\frac{B\,a^5}{2}+\frac{5\,A\,b\,a^4}{2}\right)}{x^5}+\frac{B\,b^5\,x^{13}}{13}+\frac{5\,a^2\,b^2\,x^4\,(A\,b+B\,a)}{2}+5\,a^3\,b\,x\,(2\,A\,b+B\,a)+\frac{5\,a\,b^3\,x^7\,(A\,b+2\,B\,a)}{7}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^6,x)
```

```
[Out] x^10*((A*b^5)/10 + (B*a*b^4)/2) - ((A*a^5)/5 + x^3*((B*a^5)/2 + (5*A*a^4*b)
/2))/x^5 + (B*b^5*x^13)/13 + (5*a^2*b^2*x^4*(A*b + B*a))/2 + 5*a^3*b*x*(2*A
*b + B*a) + (5*a*b^3*x^7*(A*b + 2*B*a))/7
```

3.39 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^7}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 114

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^7}\,dx = -\frac{a^5A}{6x^6}-\frac{a^4(5Ab+aB)}{3x^3}+\frac{10}{3}a^2b^2(Ab+aB)x^3+\frac{5}{6}ab^3(Ab+2aB)x^6+\frac{1}{9}b^4(Ab+5aB)x^9+\frac{1}{12}b^5Bx^{12}+5a^3b(2Ab+aB)\log(x)$$

```
[Out] -1/6*a^5*A/x^6-1/3*a^4*(5*A*b+B*a)/x^3+10/3*a^2*b^2*(A*b+B*a)*x^3+5/6*a*b^3
*(A*b+2*B*a)*x^6+1/9*b^4*(A*b+5*B*a)*x^9+1/12*b^5*B*x^12+5*a^3*b*(2*A*b+B*a
)*ln(x)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 77}

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^7}\,dx = -\frac{a^5A}{6x^6}-\frac{a^4(aB+5Ab)}{3x^3}+5a^3b\log(x)(aB+2Ab)+\frac{10}{3}a^2b^2x^3(aB+Ab)+\frac{1}{9}b^4x^9(5aB+Ab)+\frac{5}{6}ab^3x^6(2aB+Ab)+\frac{1}{12}b^5Bx^{12}$$

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^7,x]
```

```
[Out] -1/6*(a^5*A)/x^6 - (a^4*(5*A*b + a*B))/(3*x^3) + (10*a^2*b^2*(A*b + a*B)*x^
3)/3 + (5*a*b^3*(A*b + 2*a*B)*x^6)/6 + (b^4*(A*b + 5*a*B)*x^9)/9 + (b^5*B*x
^12)/12 + 5*a^3*b*(2*A*b + a*B)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^3}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(10a^2b^2(Ab+aB)+\frac{a^5A}{x^3}+\frac{a^4(5Ab+aB)}{x^2}+\frac{5a^3b(2Ab+aB)}{x}\right.\right.\\
&\qquad \left.\left. +5ab^3(Ab+2aB)x+b^4(Ab+5aB)x^2+b^5Bx^3\right)\,dx, x, x^3\right)\\
&= -\frac{a^5A}{6x^6}-\frac{a^4(5Ab+aB)}{3x^3}+\frac{10}{3}a^2b^2(Ab+aB)x^3+\frac{5}{6}ab^3(Ab+2aB)x^6\\
&\quad +\frac{1}{9}b^4(Ab+5aB)x^9+\frac{1}{12}b^5Bx^{12}+5a^3b(2Ab+aB)\log(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.93

$$
\begin{aligned}
\int \frac{(a+bx^3)^5(A+Bx^3)}{x^7}\,dx &= \frac{1}{36}\left(-\frac{6a^5A}{x^6}-\frac{12a^4(5Ab+aB)}{x^3}+120a^2b^2(Ab+aB)x^3\right.\\
&\quad +30ab^3(Ab+2aB)x^6+4b^4(Ab+5aB)x^9+3b^5Bx^{12}\\
&\quad \left.+180a^3b(2Ab+aB)\log(x)\right)
\end{aligned}
$$

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^7,x]
```

```
[Out] ((-6*a^5*A)/x^6 - (12*a^4*(5*A*b + a*B))/x^3 + 120*a^2*b^2*(A*b + a*B)*x^3
+ 30*a*b^3*(A*b + 2*a*B)*x^6 + 4*b^4*(A*b + 5*a*B)*x^9 + 3*b^5*B*x^12 + 180
*a^3*b*(2*A*b + a*B)*Log[x])/36
```

Maple [A] (verified)

Time = 4.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.03

method	result
default	$\frac{b^5 B x^{12}}{12}+\frac{A b^5 x^9}{9}+\frac{5 B a b^4 x^9}{9}+\frac{5 A a b^4 x^6}{6}+\frac{5 B a^2 b^3 x^6}{3}+\frac{10 a^2 A b^3 x^3}{3}+\frac{10 B a^3 b^2 x^3}{3}+5 a^3 b(2 A b+B a) \ln$
norman	$\frac{\left(\frac{1}{9} b^5 A+\frac{5}{9} a b^4 B\right) x^{15}+\left(\frac{5}{6} a b^4 A+\frac{5}{3} a^2 b^3 B\right) x^{12}+\left(\frac{10}{3} a^2 b^3 A+\frac{10}{3} a^3 b^2 B\right) x^9+\left(-\frac{5}{3} a^4 b A-\frac{1}{3} a^5 B\right) x^3-\frac{a^5 A}{6}+\frac{b^5 B x^{18}}{12}}{x^6}+(10 a^3 b^2$
risch	$\frac{b^5 B x^{12}}{12}+\frac{A b^5 x^9}{9}+\frac{5 B a b^4 x^9}{9}+\frac{5 A a b^4 x^6}{6}+\frac{5 B a^2 b^3 x^6}{3}+\frac{10 a^2 A b^3 x^3}{3}+\frac{10 B a^3 b^2 x^3}{3}+\frac{\left(-\frac{5}{3} a^4 b A-\frac{1}{3} a^5 B\right) x^3-\frac{a^5}{6}}{x^6}$
parallelrisch	$\frac{3 b^5 B x^{18}+4 A b^5 x^{15}+20 B a b^4 x^{15}+30 a A b^4 x^{12}+60 B a^2 b^3 x^{12}+120 a^2 A b^3 x^9+120 B a^3 b^2 x^9+360 A \ln (x) x^6 a^3 b^2+180 B \ln (x) x^6}{36 x^6}$

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*b^5*B*x^12+1/9*A*b^5*x^9+5/9*B*a*b^4*x^9+5/6*A*a*b^4*x^6+5/3*B*a^2*b^3
*x^6+10/3*a^2*A*b^3*x^3+10/3*B*a^3*b^2*x^3+5*a^3*b*(2*A*b+B*a)*ln(x)-1/6*a^
5*A/x^6-1/3*a^4*(5*A*b+B*a)/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^7}\,dx = \frac{3\,Bb^5x^{18}+4\,(5\,Bab^4+Ab^5)x^{15}+30\,(2\,Ba^2b^3+Aab^4)x^{12}+120\,(Ba^3b^2+Aa^2b^3)x^9+180\,(Ba^4b+2\,Aa^3}{36\,x^6}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^7,x, algorithm="fricas")
```

```
[Out] 1/36*(3*B*b^5*x^18 + 4*(5*B*a*b^4 + A*b^5)*x^15 + 30*(2*B*a^2*b^3 + A*a*b^4
)*x^12 + 120*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 180*(B*a^4*b + 2*A*a^3*b^2)*x^6*
log(x) - 6*A*a^5 - 12*(B*a^5 + 5*A*a^4*b)*x^3)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^7} dx = \frac{Bb^5x^{12}}{12} + 5a^3b(2Ab+Ba)\log(x) + x^9\left(\frac{Ab^5}{9}+\frac{5Bab^4}{9}\right) + x^6\cdot\left(\frac{5Aab^4}{6}+\frac{5Ba^2b^3}{3}\right) + x^3 \cdot\left(\frac{10Aa^2b^3}{3}+\frac{10Ba^3b^2}{3}\right) + \frac{-Aa^5+x^3(-10Aa^4b-2Ba^5)}{6x^6}$$

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**7,x)
```

```
[Out] B*b**5*x**12/12 + 5*a**3*b*(2*A*b + B*a)*log(x) + x**9*(A*b**5/9 + 5*B*a*b*
*4/9) + x**6*(5*A*a*b**4/6 + 5*B*a**2*b**3/3) + x**3*(10*A*a**2*b**3/3 + 10
*B*a**3*b**2/3) + (-A*a**5 + x**3*(-10*A*a**4*b - 2*B*a**5))/(6*x**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^7} dx = \frac{1}{12}Bb^5x^{12} + \frac{1}{9}\left(5Bab^4+Ab^5\right)x^9 + \frac{5}{6}\left(2Ba^2b^3+Aab^4\right)x^6 + \frac{10}{3}\left(Ba^3b^2+Aa^2b^3\right)x^3 + \frac{5}{3}\left(Ba^4b+2Aa^3b^2\right)\log\left(x^3\right) - \frac{Aa^5+2\left(Ba^5+5Aa^4b\right)x^3}{6x^6}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^7,x, algorithm="maxima")
```

```
[Out] 1/12*B*b^5*x^12 + 1/9*(5*B*a*b^4 + A*b^5)*x^9 + 5/6*(2*B*a^2*b^3 + A*a*b^4)
*x^6 + 10/3*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 5/3*(B*a^4*b + 2*A*a^3*b^2)*log(x
^3) - 1/6*(A*a^5 + 2*(B*a^5 + 5*A*a^4*b)*x^3)/x^6
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^7} dx = \frac{1}{12}Bb^5x^{12} + \frac{5}{9}Bab^4x^9 + \frac{1}{9}Ab^5x^9 + \frac{5}{3}Ba^2b^3x^6 + \frac{5}{6}Aab^4x^6 + \frac{10}{3}Ba^3b^2x^3 + \frac{10}{3}Aa^2b^3x^3 + 5\left(Ba^4b+2Aa^3b^2\right)\log(|x|) - \frac{15Ba^4bx^6+30Aa^3b^2x^6+2Ba^5x^3+10Aa^4bx^3+Aa^5}{6x^6}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^7,x, algorithm="giac")
```

```
[Out] 1/12*B*b^5*x^12 + 5/9*B*a*b^4*x^9 + 1/9*A*b^5*x^9 + 5/3*B*a^2*b^3*x^6 + 5/6
*A*a*b^4*x^6 + 10/3*B*a^3*b^2*x^3 + 10/3*A*a^2*b^3*x^3 + 5*(B*a^4*b + 2*A*a
^3*b^2)*log(abs(x)) - 1/6*(15*B*a^4*b*x^6 + 30*A*a^3*b^2*x^6 + 2*B*a^5*x^3
+ 10*A*a^4*b*x^3 + A*a^5)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x^7}\,dx = \ln(x)\,\left(5\,B\,a^4\,b+10\,A\,a^3\,b^2\right)-\frac{\frac{A\,a^5}{6}+x^3\,\left(\frac{B\,a^5}{3}+\frac{5\,A\,b\,a^4}{3}\right)}{x^6}+x^9\,\left(\frac{A\,b^5}{9}+\frac{5\,B\,a\,b^4}{9}\right)+\frac{B\,b^5\,x^{12}}{12}+\frac{10\,a^2\,b^2\,x^3\,(A\,b+B\,a)}{3}+\frac{5\,a\,b^3\,x^6\,(A\,b+2\,B\,a)}{6}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^7,x)
```

```
[Out] log(x)*(10*A*a^3*b^2 + 5*B*a^4*b) - ((A*a^5)/6 + x^3*((B*a^5)/3 + (5*A*a^4*
b)/3))/x^6 + x^9*((A*b^5)/9 + (5*B*a*b^4)/9) + (B*b^5*x^12)/12 + (10*a^2*b^
2*x^3*(A*b + B*a))/3 + (5*a*b^3*x^6*(A*b + 2*B*a))/6
```

3.40 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^8}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 110

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^8}\,dx = -\frac{a^5A}{7x^7}-\frac{a^4(5Ab+aB)}{4x^4}-\frac{5a^3b(2Ab+aB)}{x}+5a^2b^2(Ab+aB)x^2 + ab^3(Ab+2aB)x^5+\frac{1}{8}b^4(Ab+5aB)x^8+\frac{1}{11}b^5Bx^{11}$$

```
[Out] -1/7*a^5*A/x^7-1/4*a^4*(5*A*b+B*a)/x^4-5*a^3*b*(2*A*b+B*a)/x+5*a^2*b^2*(A*b
+B*a)*x^2+a*b^3*(A*b+2*B*a)*x^5+1/8*b^4*(A*b+5*B*a)*x^8+1/11*b^5*B*x^11
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^8}\,dx = -\frac{a^5A}{7x^7}-\frac{a^4(aB+5Ab)}{4x^4}-\frac{5a^3b(aB+2Ab)}{x}+5a^2b^2x^2(aB+Ab) + \frac{1}{8}b^4x^8(5aB+Ab)+ab^3x^5(2aB+Ab)+\frac{1}{11}b^5Bx^{11}$$

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^8,x]
```

```
[Out] -1/7*(a^5*A)/x^7 - (a^4*(5*A*b + a*B))/(4*x^4) - (5*a^3*b*(2*A*b + a*B))/x
+ 5*a^2*b^2*(A*b + a*B)*x^2 + a*b^3*(A*b + 2*a*B)*x^5 + (b^4*(A*b + 5*a*B)*
x^8)/8 + (b^5*B*x^11)/11
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
```

```
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{a^5 A}{x^8}+\frac{a^4(5Ab+aB)}{x^5}+\frac{5a^3b(2Ab+aB)}{x^2}+10a^2b^2(Ab+aB)x \right.\\ &\quad \left. +5ab^3(Ab+2aB)x^4+b^4(Ab+5aB)x^7+b^5Bx^{10}\right) dx\\ &= -\frac{a^5 A}{7x^7}-\frac{a^4(5Ab+aB)}{4x^4}-\frac{5a^3b(2Ab+aB)}{x}+5a^2b^2(Ab+aB)x^2\\ &\quad +ab^3(Ab+2aB)x^5+\frac{1}{8}b^4(Ab+5aB)x^8+\frac{1}{11}b^5Bx^{11}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x^8}\,dx &= -\frac{a^5 A}{7x^7}-\frac{a^4(5Ab+aB)}{4x^4}-\frac{5a^3b(2Ab+aB)}{x}+5a^2b^2(Ab+aB)x^2\\ &\quad +ab^3(Ab+2aB)x^5+\frac{1}{8}b^4(Ab+5aB)x^8+\frac{1}{11}b^5Bx^{11}\end{aligned}$$

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^8,x]
[Out] -1/7*(a^5*A)/x^7 - (a^4*(5*A*b + a*B))/(4*x^4) - (5*a^3*b*(2*A*b + a*B))/x
+ 5*a^2*b^2*(A*b + a*B)*x^2 + a*b^3*(A*b + 2*a*B)*x^5 + (b^4*(A*b + 5*a*B)*
x^8)/8 + (b^5*B*x^11)/11
```

Maple [A] (verified)

Time = 4.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.06

method	result
default	$\frac{b^5 B x^{11}}{11}+\frac{A b^5 x^8}{8}+\frac{5Ba b^4 x^8}{8}+Aa b^4 x^5+2B a^2 b^3 x^5+5A a^2 b^3 x^2+5B a^3 b^2 x^2-\frac{a^5 A}{7x^7}-\frac{5a^3 b(2Ab+}{x}$
norman	$\frac{-\frac{a^5 A}{7}+\left(-\frac{5}{4}a^4 bA-\frac{1}{4}a^5 B\right)x^3+\left(-10a^3b^2A-5a^4bB\right)x^6+\left(5a^2b^3A+5a^3b^2B\right)x^9+\left(a b^4 A+2a^2b^3B\right)x^{12}+\left(\frac{1}{8}b^5A+\frac{5}{8}a b^4 B\right)x^{15}+}{x^7}$
risch	$\frac{b^5 B x^{11}}{11}+\frac{A b^5 x^8}{8}+\frac{5Ba b^4 x^8}{8}+Aa b^4 x^5+2B a^2 b^3 x^5+5A a^2 b^3 x^2+5B a^3 b^2 x^2+\frac{(-10a^3b^2A-5a^4bB}{}$
gosper	$-\frac{-56b^5 B x^{18}-77A b^5 x^{15}-385Ba b^4 x^{15}-616aA b^4 x^{12}-1232B a^2 b^3 x^{12}-3080a^2 A b^3 x^9-3080B a^3 b^2 x^9+6160a^3 A b^2 x^6+308}{616x^7}$
parallelrisch	$\frac{56b^5 B x^{18}+77A b^5 x^{15}+385Ba b^4 x^{15}+616aA b^4 x^{12}+1232B a^2 b^3 x^{12}+3080a^2 A b^3 x^9+3080B a^3 b^2 x^9-6160a^3 A b^2 x^6-3080B a}{616x^7}$

[In] int((b*x^3+a)^5*(B*x^3+A)/x^8,x,method=_RETURNVERBOSE)

[Out] 1/11*b^5*B*x^11+1/8*A*b^5*x^8+5/8*B*a*b^4*x^8+A*a*b^4*x^5+2*B*a^2*b^3*x^5+5
*A*a^2*b^3*x^2+5*B*a^3*b^2*x^2-1/7*a^5*A/x^7-5*a^3*b*(2*A*b+B*a)/x-1/4*a^4*
(5*A*b+B*a)/x^4

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^8} dx = \frac{56 Bb^5x^{18} + 77 (5 Bab^4 + Ab^5)x^{15} + 616 (2 Ba^2b^3 + Aab^4)x^{12} + 3080 (Ba^3b^2 + Aa^2b^3)x^9 - 3080 (Ba^4b + 2}{616 x^7}$$

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^8,x, algorithm="fricas")

[Out] 1/616*(56*B*b^5*x^18 + 77*(5*B*a*b^4 + A*b^5)*x^15 + 616*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 3080*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 3080*(B*a^4*b + 2*A*a^3*b^2
)*x^6 - 88*A*a^5 - 154*(B*a^5 + 5*A*a^4*b)*x^3)/x^7

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^8} dx = \frac{Bb^5x^{11}}{11} + x^8\left(\frac{Ab^5}{8} + \frac{5Bab^4}{8}\right) + x^5\left(Aab^4 + 2Ba^2b^3\right) + x^2 \cdot \left(5Aa^2b^3 + 5Ba^3b^2\right) + \frac{-4Aa^5 + x^6(-280Aa^3b^2 - 140Ba^4b) + x^3(-35Aa^4b - 7Ba^5)}{28x^7}$$

[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**8,x)

[Out] B*b**5*x**11/11 + x**8*(A*b**5/8 + 5*B*a*b**4/8) + x**5*(A*a*b**4 + 2*B*a**
2*b**3) + x**2*(5*A*a**2*b**3 + 5*B*a**3*b**2) + (-4*A*a**5 + x**6*(-280*A*
a**3*b**2 - 140*B*a**4*b) + x**3*(-35*A*a**4*b - 7*B*a**5))/(28*x**7)

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^8} dx = \frac{1}{11} Bb^5x^{11} + \frac{1}{8} (5 Bab^4 + Ab^5)x^8 + (2 Ba^2b^3 + Aab^4)x^5 + 5 (Ba^3b^2 + Aa^2b^3)x^2 - \frac{140 (Ba^4b + 2 Aa^3b^2)x^6 + 4 Aa^5 + 7 (Ba^5 + 5 Aa^4b)x^3}{28 x^7}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^8,x, algorithm="maxima")
```

```
[Out] 1/11*B*b^5*x^11 + 1/8*(5*B*a*b^4 + A*b^5)*x^8 + (2*B*a^2*b^3 + A*a*b^4)*x^5
 + 5*(B*a^3*b^2 + A*a^2*b^3)*x^2 - 1/28*(140*(B*a^4*b + 2*A*a^3*b^2)*x^6 +
4*A*a^5 + 7*(B*a^5 + 5*A*a^4*b)*x^3)/x^7
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^8} dx = \frac{1}{11} Bb^5x^{11} + \frac{5}{8} Bab^4x^8 + \frac{1}{8} Ab^5x^8 + 2 Ba^2b^3x^5 + Aab^4x^5 + 5 Ba^3b^2x^2 + 5 Aa^2b^3x^2 - \frac{140 Ba^4bx^6 + 280 Aa^3b^2x^6 + 7 Ba^5x^3 + 35 Aa^4bx^3 + 4 Aa^5}{28 x^7}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^8,x, algorithm="giac")
```

```
[Out] 1/11*B*b^5*x^11 + 5/8*B*a*b^4*x^8 + 1/8*A*b^5*x^8 + 2*B*a^2*b^3*x^5 + A*a*b
^4*x^5 + 5*B*a^3*b^2*x^2 + 5*A*a^2*b^3*x^2 - 1/28*(140*B*a^4*b*x^6 + 280*A*
a^3*b^2*x^6 + 7*B*a^5*x^3 + 35*A*a^4*b*x^3 + 4*A*a^5)/x^7
```

Mupad [B] (verification not implemented)

Time = 6.75 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^8} dx = x^8 \left(\frac{A b^5}{8} + \frac{5 B a b^4}{8}\right) - \frac{\frac{A a^5}{7} + x^6 (5 B a^4 b + 10 A a^3 b^2) + x^3 \left(\frac{B a^5}{4} + \frac{5 A b a^4}{4}\right)}{x^7} + \frac{B b^5 x^{11}}{11} + 5 a^2 b^2 x^2 (A b + B a) + a b^3 x^5 (A b + 2 B a)$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^8,x)
```

```
[Out] x^8*((A*b^5)/8 + (5*B*a*b^4)/8) - ((A*a^5)/7 + x^6*(10*A*a^3*b^2 + 5*B*a^4*
b) + x^3*((B*a^5)/4 + (5*A*a^4*b)/4))/x^7 + (B*b^5*x^11)/11 + 5*a^2*b^2*x^2
*(A*b + B*a) + a*b^3*x^5*(A*b + 2*B*a)
```

3.41 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^9}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x^9}\,dx = -\frac{a^5A}{8x^8}-\frac{a^4(5Ab+aB)}{5x^5}-\frac{5a^3b(2Ab+aB)}{2x^2}+10a^2b^2(Ab+aB)x+\frac{5}{4}ab^3(Ab+2aB)x^4+\frac{1}{7}b^4(Ab+5aB)x^7+\frac{1}{10}b^5Bx^{10}$$

```
[Out] -1/8*a^5*A/x^8-1/5*a^4*(5*A*b+B*a)/x^5-5/2*a^3*b*(2*A*b+B*a)/x^2+10*a^2*b^2
*(A*b+B*a)*x+5/4*a*b^3*(A*b+2*B*a)*x^4+1/7*b^4*(A*b+5*B*a)*x^7+1/10*b^5*B*x
^10
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x^9}\,dx = -\frac{a^5A}{8x^8}-\frac{a^4(aB+5Ab)}{5x^5}-\frac{5a^3b(aB+2Ab)}{2x^2}+10a^2b^2x(aB+Ab)+\frac{1}{7}b^4x^7(5aB+Ab)+\frac{5}{4}ab^3x^4(2aB+Ab)+\frac{1}{10}b^5Bx^{10}$$

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^9,x]
```

```
[Out] -1/8*(a^5*A)/x^8 - (a^4*(5*A*b + a*B))/(5*x^5) - (5*a^3*b*(2*A*b + a*B))/(2
*x^2) + 10*a^2*b^2*(A*b + a*B)*x + (5*a*b^3*(A*b + 2*a*B)*x^4)/4 + (b^4*(A*
b + 5*a*B)*x^7)/7 + (b^5*B*x^10)/10
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(10a^2b^2(Ab+aB) + \frac{a^5A}{x^9} + \frac{a^4(5Ab+aB)}{x^6} + \frac{5a^3b(2Ab+aB)}{x^3} + 5ab^3(Ab+2aB)x^3 + b^4(Ab+5aB)x^6 + b^5Bx^9\right) dx$$

$$= -\frac{a^5A}{8x^8} - \frac{a^4(5Ab+aB)}{5x^5} - \frac{5a^3b(2Ab+aB)}{2x^2} + 10a^2b^2(Ab+aB)x + \frac{5}{4}ab^3(Ab+2aB)x^4 + \frac{1}{7}b^4(Ab+5aB)x^7 + \frac{1}{10}b^5Bx^{10}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^9}\,dx = -\frac{a^5A}{8x^8} - \frac{a^4(5Ab+aB)}{5x^5} - \frac{5a^3b(2Ab+aB)}{2x^2} + 10a^2b^2(Ab+aB)x + \frac{5}{4}ab^3(Ab+2aB)x^4 + \frac{1}{7}b^4(Ab+5aB)x^7 + \frac{1}{10}b^5Bx^{10}$$

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^9,x]
[Out] -1/8*(a^5*A)/x^8 - (a^4*(5*A*b + a*B))/(5*x^5) - (5*a^3*b*(2*A*b + a*B))/(2
*x^2) + 10*a^2*b^2*(A*b + a*B)*x + (5*a*b^3*(A*b + 2*a*B)*x^4)/4 + (b^4*(A*
b + 5*a*B)*x^7)/7 + (b^5*B*x^10)/10
```

Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.01

method	result
default	$\frac{b^5 B x^{10}}{10}+\frac{A b^5 x^7}{7}+\frac{5 B a b^4 x^7}{7}+\frac{5 a A b^4 x^4}{4}+\frac{5 B a^2 b^3 x^4}{2}+10 A a^2 b^3 x+10 B a^3 b^2 x-\frac{a^5 A}{8 x^8}-\frac{5 a^3 b(2 A b+B a}{2 x^2}$
norman	$\frac{-\frac{a^5 A}{8}+\left(-a^4 b A-\frac{1}{5} a^5 B\right) x^3+\left(-5 a^3 b^2 A-\frac{5}{2} a^4 b B\right) x^6+\left(10 a^2 b^3 A+10 a^3 b^2 B\right) x^9+\left(\frac{5}{4} a b^4 A+\frac{5}{2} a^2 b^3 B\right) x^{12}+\left(\frac{1}{7} b^5 A+\frac{5}{7} a b^4 B\right) x^{15}}{x^8}$
risch	$\frac{b^5 B x^{10}}{10}+\frac{A b^5 x^7}{7}+\frac{5 B a b^4 x^7}{7}+\frac{5 a A b^4 x^4}{4}+\frac{5 B a^2 b^3 x^4}{2}+10 A a^2 b^3 x+10 B a^3 b^2 x+\frac{\left(-5 a^3 b^2 A-\frac{5}{2} a^4 b B\right) x^6}{}$
gosper	$-\frac{-28 b^5 B x^{18}-40 A b^5 x^{15}-200 B a b^4 x^{15}-350 a A b^4 x^{12}-700 B a^2 b^3 x^{12}-2800 a^2 A b^3 x^9-2800 B a^3 b^2 x^9+1400 a^3 A b^2 x^6+700 B}{280 x^8}$
parallelrisch	$\frac{28 b^5 B x^{18}+40 A b^5 x^{15}+200 B a b^4 x^{15}+350 a A b^4 x^{12}+700 B a^2 b^3 x^{12}+2800 a^2 A b^3 x^9+2800 B a^3 b^2 x^9-1400 a^3 A b^2 x^6-700 B a^4 b}{280 x^8}$

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*b^5*B*x^10+1/7*A*b^5*x^7+5/7*B*a*b^4*x^7+5/4*a*A*b^4*x^4+5/2*B*a^2*b^3
*x^4+10*A*a^2*b^3*x+10*B*a^3*b^2*x-1/8*a^5*A/x^8-5/2*a^3*b*(2*A*b+B*a)/x^2-
1/5*a^4*(5*A*b+B*a)/x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \frac{\left(a+bx^3\right)^5 \left(A+Bx^3\right)}{x^9}\,dx = \frac{28\,Bb^5x^{18}+40\,(5\,Bab^4+Ab^5)x^{15}+350\,(2\,Ba^2b^3+Aab^4)x^{12}+2800\,(Ba^3b^2+Aa^2b^3)x^9-700\,(Ba^4b+2}{280\,x^8}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^9,x, algorithm="fricas")
```

```
[Out] 1/280*(28*B*b^5*x^18 + 40*(5*B*a*b^4 + A*b^5)*x^15 + 350*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 2800*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 700*(B*a^4*b + 2*A*a^3*b^2)
*x^6 - 35*A*a^5 - 56*(B*a^5 + 5*A*a^4*b)*x^3)/x^8
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.18

$$\int \frac{\left(a+bx^3\right)^5 \left(A+Bx^3\right)}{x^9}\,dx = \frac{Bb^5x^{10}}{10}+x^7\left(\frac{Ab^5}{7}+\frac{5Bab^4}{7}\right)+x^4\cdot\left(\frac{5Aab^4}{4}+\frac{5Ba^2b^3}{2}\right)+x\left(10Aa^2b^3+10Ba^3b^2\right) + \frac{-5Aa^5+x^6(-200Aa^3b^2-100Ba^4b)+x^3(-40Aa^4b-8Ba^5)}{40x^8}$$

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**9,x)
```

```
[Out] B*b**5*x**10/10 + x**7*(A*b**5/7 + 5*B*a*b**4/7) + x**4*(5*A*a*b**4/4 + 5*B
*a**2*b**3/2) + x*(10*A*a**2*b**3 + 10*B*a**3*b**2) + (-5*A*a**5 + x**6*(-2
00*A*a**3*b**2 - 100*B*a**4*b) + x**3*(-40*A*a**4*b - 8*B*a**5))/(40*x**8)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^9}\,dx = \frac{1}{10}\,Bb^5x^{10} + \frac{1}{7}\left(5\,Bab^4 + Ab^5\right)x^7 + \frac{5}{4}\left(2\,Ba^2b^3 + Aab^4\right)x^4 + 10\left(Ba^3b^2 + Aa^2b^3\right)x - \frac{100\,(Ba^4b + 2\,Aa^3b^2)x^6 + 5\,Aa^5 + 8\,(Ba^5 + 5\,Aa^4b)x^3}{40\,x^8}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^9,x, algorithm="maxima")
```

```
[Out] 1/10*B*b^5*x^10 + 1/7*(5*B*a*b^4 + A*b^5)*x^7 + 5/4*(2*B*a^2*b^3 + A*a*b^4)
*x^4 + 10*(B*a^3*b^2 + A*a^2*b^3)*x - 1/40*(100*(B*a^4*b + 2*A*a^3*b^2)*x^6
 + 5*A*a^5 + 8*(B*a^5 + 5*A*a^4*b)*x^3)/x^8
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^9}\,dx = \frac{1}{10}\,Bb^5x^{10} + \frac{5}{7}\,Bab^4x^7 + \frac{1}{7}\,Ab^5x^7 + \frac{5}{2}\,Ba^2b^3x^4 + \frac{5}{4}\,Aab^4x^4 + 10\,Ba^3b^2x + 10\,Aa^2b^3x - \frac{100\,Ba^4bx^6 + 200\,Aa^3b^2x^6 + 8\,Ba^5x^3 + 40\,Aa^4bx^3 + 5\,Aa^5}{40\,x^8}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^9,x, algorithm="giac")
```

```
[Out] 1/10*B*b^5*x^10 + 5/7*B*a*b^4*x^7 + 1/7*A*b^5*x^7 + 5/2*B*a^2*b^3*x^4 + 5/4
*A*a*b^4*x^4 + 10*B*a^3*b^2*x + 10*A*a^2*b^3*x - 1/40*(100*B*a^4*b*x^6 + 20
0*A*a^3*b^2*x^6 + 8*B*a^5*x^3 + 40*A*a^4*b*x^3 + 5*A*a^5)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^9}\,dx = x^7\left(\frac{A\,b^5}{7}+\frac{5\,B\,a\,b^4}{7}\right) - \frac{\frac{A\,a^5}{8}+x^6\left(\frac{5\,B\,a^4\,b}{2}+5\,A\,a^3\,b^2\right)+x^3\left(\frac{B\,a^5}{5}+A\,b\,a^4\right)}{x^8} + \frac{B\,b^5\,x^{10}}{10}+10\,a^2\,b^2\,x\,(A\,b+B\,a)+\frac{5\,a\,b^3\,x^4\,(A\,b+2\,B\,a)}{4}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^9,x)
```

```
[Out] x^7*((A*b^5)/7 + (5*B*a*b^4)/7) - ((A*a^5)/8 + x^6*(5*A*a^3*b^2 + (5*B*a^4*
b)/2) + x^3*((B*a^5)/5 + A*a^4*b))/x^8 + (B*b^5*x^10)/10 + 10*a^2*b^2*x*(A*
b + B*a) + (5*a*b^3*x^4*(A*b + 2*B*a))/4
```

3.42 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{10}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 114

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{10}}\,dx = -\frac{a^5A}{9x^9}-\frac{a^4(5Ab+aB)}{6x^6}-\frac{5a^3b(2Ab+aB)}{3x^3}+\frac{5}{3}ab^3(Ab+2aB)x^3 + \frac{1}{6}b^4(Ab+5aB)x^6+\frac{1}{9}b^5Bx^9+10a^2b^2(Ab+aB)\log(x)$$

```
[Out] -1/9*a^5*A/x^9-1/6*a^4*(5*A*b+B*a)/x^6-5/3*a^3*b*(2*A*b+B*a)/x^3+5/3*a*b^3*
(A*b+2*B*a)*x^3+1/6*b^4*(A*b+5*B*a)*x^6+1/9*b^5*B*x^9+10*a^2*b^2*(A*b+B*a)*
ln(x)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 77}

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{10}}\,dx = -\frac{a^5A}{9x^9}-\frac{a^4(aB+5Ab)}{6x^6}-\frac{5a^3b(aB+2Ab)}{3x^3} + 10a^2b^2\log(x)(aB+Ab)+\frac{1}{6}b^4x^6(5aB+Ab) + \frac{5}{3}ab^3x^3(2aB+Ab)+\frac{1}{9}b^5Bx^9$$

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^10,x]
```

```
[Out] -1/9*(a^5*A)/x^9 - (a^4*(5*A*b + a*B))/(6*x^6) - (5*a^3*b*(2*A*b + a*B))/(3
*x^3) + (5*a*b^3*(A*b + 2*a*B)*x^3)/3 + (b^4*(A*b + 5*a*B)*x^6)/6 + (b^5*B*
x^9)/9 + 10*a^2*b^2*(A*b + a*B)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^4}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(5ab^3(Ab+2aB)+\frac{a^5A}{x^4}+\frac{a^4(5Ab+aB)}{x^3}+\frac{5a^3b(2Ab+aB)}{x^2}\right.\right.\\
&\qquad \left.\left.+\frac{10a^2b^2(Ab+aB)}{x}+b^4(Ab+5aB)x+b^5Bx^2\right)\,dx, x, x^3\right)\\
&= -\frac{a^5A}{9x^9}-\frac{a^4(5Ab+aB)}{6x^6}-\frac{5a^3b(2Ab+aB)}{3x^3}+\frac{5}{3}ab^3(Ab+2aB)x^3\\
&\quad +\frac{1}{6}b^4(Ab+5aB)x^6+\frac{1}{9}b^5Bx^9+10a^2b^2(Ab+aB)\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.93

$$\begin{aligned}
\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x^{10}}\,dx &= \frac{1}{18}\left(-\frac{2a^5A}{x^9}-\frac{3a^4(5Ab+aB)}{x^6}-\frac{30a^3b(2Ab+aB)}{x^3}\right.\\
&\quad +30ab^3(Ab+2aB)x^3+3b^4(Ab+5aB)x^6+2b^5Bx^9\\
&\quad \left.+180a^2b^2(Ab+aB)\log(x)\right)
\end{aligned}$$

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^10,x]
```

```
[Out] ((-2*a^5*A)/x^9 - (3*a^4*(5*A*b + a*B))/x^6 - (30*a^3*b*(2*A*b + a*B))/x^3
+ 30*a*b^3*(A*b + 2*a*B)*x^3 + 3*b^4*(A*b + 5*a*B)*x^6 + 2*b^5*B*x^9 + 180*
a^2*b^2*(A*b + a*B)*Log[x])/18
```

Maple [A] (verified)

Time = 4.14 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

method	result
default	$\frac{b^5 B x^9}{9}+\frac{A b^5 x^6}{6}+\frac{5 B a b^4 x^6}{6}+\frac{5 A a b^4 x^3}{3}+\frac{10 B a^2 b^3 x^3}{3}+10 a^2 b^2 (A b+B a) \ln(x)-\frac{a^4 (5 A b+B a)}{6 x^6}-\frac{5 a^3 b(2}{3}$
norman	$\frac{(\frac{1}{6} b^5 A+\frac{5}{6} a b^4 B) x^{15}+(\frac{5}{3} a b^4 A+\frac{10}{3} a^2 b^3 B) x^{12}+(-\frac{10}{3} a^3 b^2 A-\frac{5}{3} a^4 b B) x^6+(-\frac{5}{6} a^4 b A-\frac{1}{6} a^5 B) x^3-\frac{a^5 A}{9}+\frac{b^5 B x^{18}}{9}}{x^9}+(10 a^2 b^3 A$
risch	$\frac{b^5 B x^9}{9}+\frac{A b^5 x^6}{6}+\frac{5 B a b^4 x^6}{6}+\frac{5 A a b^4 x^3}{3}+\frac{10 B a^2 b^3 x^3}{3}+\frac{(-\frac{10}{3} a^3 b^2 A-\frac{5}{3} a^4 b B) x^6+(-\frac{5}{6} a^4 b A-\frac{1}{6} a^5 B) x^3-\frac{a^5 A}{9}}{x^9}+10$
parallelrisch	$\frac{2 b^5 B x^{18}+3 A b^5 x^{15}+15 B a b^4 x^{15}+30 a A b^4 x^{12}+60 B a^2 b^3 x^{12}+180 A \ln(x) x^9 a^2 b^3+180 B \ln(x) x^9 a^3 b^2-60 a^3 A b^2 x^6-30 B a^4 b x^6-}{18 x^9}$

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*b^5*B*x^9+1/6*A*b^5*x^6+5/6*B*a*b^4*x^6+5/3*A*a*b^4*x^3+10/3*B*a^2*b^3*
x^3+10*a^2*b^2*(A*b+B*a)*ln(x)-1/6*a^4*(5*A*b+B*a)/x^6-5/3*a^3*b*(2*A*b+B*a
)/x^3-1/9*a^5*A/x^9
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{10}}\,dx$$
$$= \frac{2\,Bb^5x^{18}+3\,(5\,Bab^4+Ab^5)x^{15}+30\,(2\,Ba^2b^3+Aab^4)x^{12}+180\,(Ba^3b^2+Aa^2b^3)x^9\log(x)-30\,(Ba^4b+2}{18\,x^9}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^10,x, algorithm="fricas")
```

```
[Out] 1/18*(2*B*b^5*x^18 + 3*(5*B*a*b^4 + A*b^5)*x^15 + 30*(2*B*a^2*b^3 + A*a*b^4
)*x^12 + 180*(B*a^3*b^2 + A*a^2*b^3)*x^9*log(x) - 30*(B*a^4*b + 2*A*a^3*b^2
)*x^6 - 2*A*a^5 - 3*(B*a^5 + 5*A*a^4*b)*x^3)/x^9
```

Sympy [A] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.13

$$\begin{aligned}\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{10}}\,dx = {} & \frac{Bb^5x^9}{9}+10a^2b^2(Ab+Ba)\log(x) \\ & +x^6\left(\frac{Ab^5}{6}+\frac{5Bab^4}{6}\right)+x^3\cdot\left(\frac{5Aab^4}{3}+\frac{10Ba^2b^3}{3}\right) \\ & +\frac{-2Aa^5+x^6(-60Aa^3b^2-30Ba^4b)+x^3(-15Aa^4b-3Ba^5)}{18x^9}\end{aligned}$$

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**10,x)
```

```
[Out] B*b**5*x**9/9 + 10*a**2*b**2*(A*b + B*a)*log(x) + x**6*(A*b**5/6 + 5*B*a*b*
*4/6) + x**3*(5*A*a*b**4/3 + 10*B*a**2*b**3/3) + (-2*A*a**5 + x**6*(-60*A*a
**3*b**2 - 30*B*a**4*b) + x**3*(-15*A*a**4*b - 3*B*a**5))/(18*x**9)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{10}}\,dx = \frac{1}{9}\,Bb^5x^9 + \frac{1}{6}\,\left(5\,Bab^4 + Ab^5\right)x^6 + \frac{5}{3}\,\left(2\,Ba^2b^3 + Aab^4\right)x^3 + \frac{10}{3}\,\left(Ba^3b^2 + Aa^2b^3\right)\log\left(x^3\right) - \frac{30\,(Ba^4b + 2\,Aa^3b^2)x^6 + 2\,Aa^5 + 3\,(Ba^5 + 5\,Aa^4b)x^3}{18\,x^9}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^10,x, algorithm="maxima")
```

```
[Out] 1/9*B*b^5*x^9 + 1/6*(5*B*a*b^4 + A*b^5)*x^6 + 5/3*(2*B*a^2*b^3 + A*a*b^4)*x
^3 + 10/3*(B*a^3*b^2 + A*a^2*b^3)*log(x^3) - 1/18*(30*(B*a^4*b + 2*A*a^3*b^
2)*x^6 + 2*A*a^5 + 3*(B*a^5 + 5*A*a^4*b)*x^3)/x^9
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{10}}\,dx = \frac{1}{9}\,Bb^5x^9 + \frac{5}{6}\,Bab^4x^6 + \frac{1}{6}\,Ab^5x^6 + \frac{10}{3}\,Ba^2b^3x^3 + \frac{5}{3}\,Aab^4x^3 + 10\,\left(Ba^3b^2 + Aa^2b^3\right)\log\left(|x|\right) - \frac{110\,Ba^3b^2x^9 + 110\,Aa^2b^3x^9 + 30\,Ba^4bx^6 + 60\,Aa^3b^2x^6 + 3\,Ba^5x^3 + 15\,Aa^4bx^3 + 2\,Aa^5}{18\,x^9}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^10,x, algorithm="giac")
```

```
[Out] 1/9*B*b^5*x^9 + 5/6*B*a*b^4*x^6 + 1/6*A*b^5*x^6 + 10/3*B*a^2*b^3*x^3 + 5/3*
A*a*b^4*x^3 + 10*(B*a^3*b^2 + A*a^2*b^3)*log(abs(x)) - 1/18*(110*B*a^3*b^2*
x^9 + 110*A*a^2*b^3*x^9 + 30*B*a^4*b*x^6 + 60*A*a^3*b^2*x^6 + 3*B*a^5*x^3 +
 15*A*a^4*b*x^3 + 2*A*a^5)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{10}} \, dx = x^6 \left(\frac{A b^5}{6} + \frac{5 B a b^4}{6}\right) - \frac{\frac{A a^5}{9} + x^6 \left(\frac{5 B a^4 b}{3} + \frac{10 A a^3 b^2}{3}\right) + x^3 \left(\frac{B a^5}{6} + \frac{5 A b a^4}{6}\right)}{x^9} + \ln(x) \left(10 B a^3 b^2 + 10 A a^2 b^3\right) + \frac{B b^5 x^9}{9} + \frac{5 a b^3 x^3 (A b + 2 B a)}{3}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^10,x)
```

```
[Out] x^6*((A*b^5)/6 + (5*B*a*b^4)/6) - ((A*a^5)/9 + x^6*((10*A*a^3*b^2)/3 + (5*B
*a^4*b)/3) + x^3*((B*a^5)/6 + (5*A*a^4*b)/6))/x^9 + log(x)*(10*A*a^2*b^3 +
10*B*a^3*b^2) + (B*b^5*x^9)/9 + (5*a*b^3*x^3*(A*b + 2*B*a))/3
```

3.43 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{11}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 115

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{11}}\,dx = -\frac{a^5A}{10x^{10}} - \frac{a^4(5Ab+aB)}{7x^7} - \frac{5a^3b(2Ab+aB)}{4x^4} - \frac{10a^2b^2(Ab+aB)}{x} + \frac{5}{2}ab^3(Ab+2aB)x^2 + \frac{1}{5}b^4(Ab+5aB)x^5 + \frac{1}{8}b^5Bx^8$$

```
[Out] -1/10*a^5*A/x^10-1/7*a^4*(5*A*b+B*a)/x^7-5/4*a^3*b*(2*A*b+B*a)/x^4-10*a^2*b
^2*(A*b+B*a)/x+5/2*a*b^3*(A*b+2*B*a)*x^2+1/5*b^4*(A*b+5*B*a)*x^5+1/8*b^5*B*
x^8
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{11}}\,dx = -\frac{a^5A}{10x^{10}} - \frac{a^4(aB+5Ab)}{7x^7} - \frac{5a^3b(aB+2Ab)}{4x^4} - \frac{10a^2b^2(aB+Ab)}{x} + \frac{1}{5}b^4x^5(5aB+Ab) + \frac{5}{2}ab^3x^2(2aB+Ab) + \frac{1}{8}b^5Bx^8$$

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^11,x]
```

```
[Out] -1/10*(a^5*A)/x^10 - (a^4*(5*A*b + a*B))/(7*x^7) - (5*a^3*b*(2*A*b + a*B))/
(4*x^4) - (10*a^2*b^2*(A*b + a*B))/x + (5*a*b^3*(A*b + 2*a*B)*x^2)/2 + (b^4
*(A*b + 5*a*B)*x^5)/5 + (b^5*B*x^8)/8
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{a^5A}{x^{11}} + \frac{a^4(5Ab+aB)}{x^8} + \frac{5a^3b(2Ab+aB)}{x^5} + \frac{10a^2b^2(Ab+aB)}{x^2} \right. \\ &\qquad \left. + 5ab^3(Ab+2aB)x + b^4(Ab+5aB)x^4 + b^5Bx^7\right) dx \\ &= -\frac{a^5A}{10x^{10}} - \frac{a^4(5Ab+aB)}{7x^7} - \frac{5a^3b(2Ab+aB)}{4x^4} - \frac{10a^2b^2(Ab+aB)}{x} \\ &\quad + \frac{5}{2}ab^3(Ab+2aB)x^2 + \frac{1}{5}b^4(Ab+5aB)x^5 + \frac{1}{8}b^5Bx^8\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{11}} dx$$
$$= \frac{1400a^2b^3x^9(-2A+Bx^3) + 140ab^4x^{12}(5A+2Bx^3) - 700a^3b^2x^6(A+4Bx^3) + 7b^5x^{15}(8A+5Bx^3) - 50a^4b}{280x^{10}}$$

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^11,x]
```

```
[Out] (1400*a^2*b^3*x^9*(-2*A + B*x^3) + 140*a*b^4*x^12*(5*A + 2*B*x^3) - 700*a^3
*b^2*x^6*(A + 4*B*x^3) + 7*b^5*x^15*(8*A + 5*B*x^3) - 50*a^4*b*x^3*(4*A + 7
*B*x^3) - 4*a^5*(7*A + 10*B*x^3))/(280*x^10)
```

Maple [A] (verified)

Time = 4.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

method	result
default	$\frac{b^5 B x^8}{8}+\frac{A b^5 x^5}{5}+B a b^4 x^5+\frac{5 A a b^4 x^2}{2}+5 B a^2 b^3 x^2-\frac{a^4(5 A b+B a)}{7 x^7}-\frac{a^5 A}{10 x^{10}}-\frac{10 a^2 b^2(A b+B a)}{x}-\frac{5 a^3 b(2}{4}$
norman	$\frac{-\frac{a^5 A}{10}+(-\frac{5}{7} a^4 b A-\frac{1}{7} a^5 B) x^3+(-\frac{5}{2} a^3 b^2 A-\frac{5}{4} a^4 b B) x^6+(-10 a^2 b^3 A-10 a^3 b^2 B) x^9+(\frac{5}{2} a b^4 A+5 a^2 b^3 B) x^{12}+(\frac{1}{5} b^5 A+a b^4 B) x^1}{x^{10}}$
risch	$\frac{b^5 B x^8}{8}+\frac{A b^5 x^5}{5}+B a b^4 x^5+\frac{5 A a b^4 x^2}{2}+5 B a^2 b^3 x^2+\frac{(-10 a^2 b^3 A-10 a^3 b^2 B) x^9+(-\frac{5}{2} a^3 b^2 A-\frac{5}{4} a^4 b B) x^6+(-}{x^{10}}$
gosper	$-\frac{-35 b^5 B x^{18}-56 A b^5 x^{15}-280 B a b^4 x^{15}-700 a A b^4 x^{12}-1400 B a^2 b^3 x^{12}+2800 a^2 A b^3 x^9+2800 B a^3 b^2 x^9+700 a^3 A b^2 x^6+350 B}{280 x^{10}}$
parallelrisch	$\frac{35 b^5 B x^{18}+56 A b^5 x^{15}+280 B a b^4 x^{15}+700 a A b^4 x^{12}+1400 B a^2 b^3 x^{12}-2800 a^2 A b^3 x^9-2800 B a^3 b^2 x^9-700 a^3 A b^2 x^6-350 B a^4 b}{280 x^{10}}$

[In] int((b*x^3+a)^5*(B*x^3+A)/x^11,x,method=_RETURNVERBOSE)

[Out] 1/8*b^5*B*x^8+1/5*A*b^5*x^5+B*a*b^4*x^5+5/2*A*a*b^4*x^2+5*B*a^2*b^3*x^2-1/7*a^4*(5*A*b+B*a)/x^7-1/10*a^5*A/x^10-10*a^2*b^2*(A*b+B*a)/x-5/4*a^3*b*(2*A*b+B*a)/x^4

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{11}}\,dx = \frac{35\,Bb^5x^{18}+56\,(5\,Bab^4+Ab^5)x^{15}+700\,(2\,Ba^2b^3+Aab^4)x^{12}-2800\,(Ba^3b^2+Aa^2b^3)x^9-350\,(Ba^4b+2}{280\,x^{10}}$$

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^11,x, algorithm="fricas")

[Out] 1/280*(35*B*b^5*x^18 + 56*(5*B*a*b^4 + A*b^5)*x^15 + 700*(2*B*a^2*b^3 + A*a*b^4)*x^12 - 2800*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 350*(B*a^4*b + 2*A*a^3*b^2)*x^6 - 28*A*a^5 - 40*(B*a^5 + 5*A*a^4*b)*x^3)/x^10

Sympy [A] (verification not implemented)

Time = 8.43 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{11}}\,dx = \frac{Bb^5x^8}{8} + x^5\left(\frac{Ab^5}{5} + Bab^4\right) + x^2\cdot\left(\frac{5Aab^4}{2} + 5Ba^2b^3\right) + \frac{-14Aa^5 + x^9(-1400Aa^2b^3 - 1400Ba^3b^2) + x^6(-350Aa^3b^2 - 175Ba^4b) + x^3(-100Aa^4b - 20Ba^5)}{140x^{10}}$$

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**11,x)
```

```
[Out] B*b**5*x**8/8 + x**5*(A*b**5/5 + B*a*b**4) + x**2*(5*A*a*b**4/2 + 5*B*a**2*
b**3) + (-14*A*a**5 + x**9*(-1400*A*a**2*b**3 - 1400*B*a**3*b**2) + x**6*(-
350*A*a**3*b**2 - 175*B*a**4*b) + x**3*(-100*A*a**4*b - 20*B*a**5))/(140*x*
*10)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{11}}\,dx = \frac{1}{8}\,Bb^5x^8 + \frac{1}{5}\,(5\,Bab^4 + Ab^5)x^5 + \frac{5}{2}\,(2\,Ba^2b^3 + Aab^4)x^2 - \frac{1400\,(Ba^3b^2 + Aa^2b^3)x^9 + 175\,(Ba^4b + 2\,Aa^3b^2)x^6 + 14\,Aa^5 + 20\,(Ba^5 + 5\,Aa^4b)x^3}{140\,x^{10}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^11,x, algorithm="maxima")
```

```
[Out] 1/8*B*b^5*x^8 + 1/5*(5*B*a*b^4 + A*b^5)*x^5 + 5/2*(2*B*a^2*b^3 + A*a*b^4)*x
^2 - 1/140*(1400*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 175*(B*a^4*b + 2*A*a^3*b^2)*
x^6 + 14*A*a^5 + 20*(B*a^5 + 5*A*a^4*b)*x^3)/x^10
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{11}}\,dx = \frac{1}{8}\,Bb^5x^8 + Bab^4x^5 + \frac{1}{5}\,Ab^5x^5 + 5\,Ba^2b^3x^2 + \frac{5}{2}\,Aab^4x^2 - \frac{1400\,Ba^3b^2x^9 + 1400\,Aa^2b^3x^9 + 175\,Ba^4bx^6 + 350\,Aa^3b^2x^6 + 20\,Ba^5x^3 + 100\,Aa^4bx^3 + 14\,Aa^5}{140\,x^{10}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^11,x, algorithm="giac")
```

```
[Out] 1/8*B*b^5*x^8 + B*a*b^4*x^5 + 1/5*A*b^5*x^5 + 5*B*a^2*b^3*x^2 + 5/2*A*a*b^4
*x^2 - 1/140*(1400*B*a^3*b^2*x^9 + 1400*A*a^2*b^3*x^9 + 175*B*a^4*b*x^6 + 3
50*A*a^3*b^2*x^6 + 20*B*a^5*x^3 + 100*A*a^4*b*x^3 + 14*A*a^5)/x^10
```

Mupad [B] (verification not implemented)

Time = 6.59 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{11}}\,dx = x^5 \left(\frac{A\,b^5}{5}+B\,a\,b^4\right) - \frac{\frac{A\,a^5}{10}+x^6\left(\frac{5\,B\,a^4\,b}{4}+\frac{5\,A\,a^3\,b^2}{2}\right)+x^3\left(\frac{B\,a^5}{7}+\frac{5\,A\,b\,a^4}{7}\right)+x^9\,(10\,B\,a^3\,b^2+10\,A\,a^2\,b^3)}{x^{10}} + \frac{B\,b^5\,x^8}{8}+\frac{5\,a\,b^3\,x^2\,(A\,b+2\,B\,a)}{2}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^11,x)
```

```
[Out] x^5*((A*b^5)/5 + B*a*b^4) - ((A*a^5)/10 + x^6*((5*A*a^3*b^2)/2 + (5*B*a^4*b
)/4) + x^3*((B*a^5)/7 + (5*A*a^4*b)/7) + x^9*(10*A*a^2*b^3 + 10*B*a^3*b^2))
/x^10 + (B*b^5*x^8)/8 + (5*a*b^3*x^2*(A*b + 2*B*a))/2
```

3.44 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{12}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 109

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{12}} \, dx = -\frac{a^5A}{11x^{11}} - \frac{a^4(5Ab+aB)}{8x^8} - \frac{a^3b(2Ab+aB)}{x^5} - \frac{5a^2b^2(Ab+aB)}{x^2} + 5ab^3(Ab+2aB)x + \frac{1}{4}b^4(Ab+5aB)x^4 + \frac{1}{7}b^5Bx^7$$

```
[Out] -1/11*a^5*A/x^11-1/8*a^4*(5*A*b+B*a)/x^8-a^3*b*(2*A*b+B*a)/x^5-5*a^2*b^2*(A
*b+B*a)/x^2+5*a*b^3*(A*b+2*B*a)*x+1/4*b^4*(A*b+5*B*a)*x^4+1/7*b^5*B*x^7
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{12}} \, dx = -\frac{a^5A}{11x^{11}} - \frac{a^4(aB+5Ab)}{8x^8} - \frac{a^3b(aB+2Ab)}{x^5} - \frac{5a^2b^2(aB+Ab)}{x^2} + \frac{1}{4}b^4x^4(5aB+Ab) + 5ab^3x(2aB+Ab) + \frac{1}{7}b^5Bx^7$$

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^12,x]
```

```
[Out] -1/11*(a^5*A)/x^11 - (a^4*(5*A*b + a*B))/(8*x^8) - (a^3*b*(2*A*b + a*B))/x^
5 - (5*a^2*b^2*(A*b + a*B))/x^2 + 5*a*b^3*(A*b + 2*a*B)*x + (b^4*(A*b + 5*a
*B)*x^4)/4 + (b^5*B*x^7)/7
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
```

```
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(5ab^3(Ab+2aB)+\frac{a^5A}{x^{12}}+\frac{a^4(5Ab+aB)}{x^9}+\frac{5a^3b(2Ab+aB)}{x^6}\right.\\ &\quad\left.+\frac{10a^2b^2(Ab+aB)}{x^3}+b^4(Ab+5aB)x^3+b^5Bx^6\right)dx\\ &= -\frac{a^5A}{11x^{11}}-\frac{a^4(5Ab+aB)}{8x^8}-\frac{a^3b(2Ab+aB)}{x^5}-\frac{5a^2b^2(Ab+aB)}{x^2}\\ &\quad+5ab^3(Ab+2aB)x+\frac{1}{4}b^4(Ab+5aB)x^4+\frac{1}{7}b^5Bx^7\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{12}}\,dx = -\frac{a^5A}{11x^{11}}-\frac{a^4(5Ab+aB)}{8x^8}-\frac{a^3b(2Ab+aB)}{x^5}-\frac{5a^2b^2(Ab+aB)}{x^2}+5ab^3(Ab+2aB)x+\frac{1}{4}b^4(Ab+5aB)x^4+\frac{1}{7}b^5Bx^7$$

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^12,x]
[Out] -1/11*(a^5*A)/x^11 - (a^4*(5*A*b + a*B))/(8*x^8) - (a^3*b*(2*A*b + a*B))/x^
5 - (5*a^2*b^2*(A*b + a*B))/x^2 + 5*a*b^3*(A*b + 2*a*B)*x + (b^4*(A*b + 5*a
*B)*x^4)/4 + (b^5*B*x^7)/7
```

Maple [A] (verified)

Time = 4.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99

method	result
default	$\frac{b^5Bx^7}{7}+\frac{Ab^5x^4}{4}+\frac{5Bab^4x^4}{4}+5Aab^4x+10Ba^2b^3x-\frac{a^4(5Ab+Ba)}{8x^8}-\frac{a^5A}{11x^{11}}-\frac{5a^2b^2(Ab+Ba)}{x^2}-\frac{a^3b(2A}{x}$
risch	$\frac{b^5Bx^7}{7}+\frac{Ab^5x^4}{4}+\frac{5Bab^4x^4}{4}+5Aab^4x+10Ba^2b^3x+\frac{(-5a^2b^3A-5a^3b^2B)x^9+(-2a^3b^2A-a^4bB)x^6+(-\frac{5}{8}a^4}{x^{11}}$
norman	$\frac{-\frac{a^5A}{11}+(-\frac{5}{8}a^4bA-\frac{1}{8}a^5B)x^3+(-2a^3b^2A-a^4bB)x^6+(-5a^2b^3A-5a^3b^2B)x^9+(5ab^4A+10a^2b^3B)x^{12}+(\frac{1}{4}b^5A+\frac{5}{4}ab^4B)x^{15}+}{x^{11}}$
gosper	$-\frac{-88b^5Bx^{18}-154Ab^5x^{15}-770Bab^4x^{15}-3080aAb^4x^{12}-6160Ba^2b^3x^{12}+3080a^2Ab^3x^9+3080Ba^3b^2x^9+1232a^3Ab^2x^6+6}{616x^{11}}$
parallelrisch	$\frac{88b^5Bx^{18}+154Ab^5x^{15}+770Bab^4x^{15}+3080aAb^4x^{12}+6160Ba^2b^3x^{12}-3080a^2Ab^3x^9-3080Ba^3b^2x^9-1232a^3Ab^2x^6-616B}{616x^{11}}$

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^12,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*b^5*B*x^7+1/4*A*b^5*x^4+5/4*B*a*b^4*x^4+5*A*a*b^4*x+10*B*a^2*b^3*x-1/8*
a^4*(5*A*b+B*a)/x^8-1/11*a^5*A/x^11-5*a^2*b^2*(A*b+B*a)/x^2-a^3*b*(2*A*b+B*
a)/x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{12}}\,dx = \frac{88\,Bb^5x^{18}+154\,(5\,Bab^4+Ab^5)x^{15}+3080\,(2\,Ba^2b^3+Aab^4)x^{12}-3080\,(Ba^3b^2+Aa^2b^3)x^9-616\,(Ba^4b+}{616\,x^{11}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^12,x, algorithm="fricas")
```

```
[Out] 1/616*(88*B*b^5*x^18 + 154*(5*B*a*b^4 + A*b^5)*x^15 + 3080*(2*B*a^2*b^3 + A
*a*b^4)*x^12 - 3080*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 616*(B*a^4*b + 2*A*a^3*b^
2)*x^6 - 56*A*a^5 - 77*(B*a^5 + 5*A*a^4*b)*x^3)/x^11
```

Sympy [A] (verification not implemented)

Time = 27.72 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{12}}\,dx = \frac{Bb^5x^7}{7}+x^4\left(\frac{Ab^5}{4}+\frac{5Bab^4}{4}\right)+x\left(5Aab^4+10Ba^2b^3\right) + \frac{-8Aa^5+x^9(-440Aa^2b^3-440Ba^3b^2)+x^6(-176Aa^3b^2-88Ba^4b)+x^3(-55Aa^4b-11Ba^5)}{88x^{11}}$$

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**12,x)
```

```
[Out] B*b**5*x**7/7 + x**4*(A*b**5/4 + 5*B*a*b**4/4) + x*(5*A*a*b**4 + 10*B*a**2*
b**3) + (-8*A*a**5 + x**9*(-440*A*a**2*b**3 - 440*B*a**3*b**2) + x**6*(-176
*A*a**3*b**2 - 88*B*a**4*b) + x**3*(-55*A*a**4*b - 11*B*a**5))/(88*x**11)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{12}}\,dx = \frac{1}{7}Bb^5x^7 + \frac{1}{4}\left(5\,Bab^4 + Ab^5\right)x^4 + 5\left(2\,Ba^2b^3 + Aab^4\right)x - \frac{440\,(Ba^3b^2 + Aa^2b^3)x^9 + 88\,(Ba^4b + 2\,Aa^3b^2)x^6 + 8\,Aa^5 + 11\,(Ba^5 + 5\,Aa^4b)x^3}{88\,x^{11}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^12,x, algorithm="maxima")
```

```
[Out] 1/7*B*b^5*x^7 + 1/4*(5*B*a*b^4 + A*b^5)*x^4 + 5*(2*B*a^2*b^3 + A*a*b^4)*x -
 1/88*(440*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 88*(B*a^4*b + 2*A*a^3*b^2)*x^6 + 8
*A*a^5 + 11*(B*a^5 + 5*A*a^4*b)*x^3)/x^11
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{12}}\,dx = \frac{1}{7}Bb^5x^7 + \frac{5}{4}Bab^4x^4 + \frac{1}{4}Ab^5x^4 + 10\,Ba^2b^3x + 5\,Aab^4x - \frac{440\,Ba^3b^2x^9 + 440\,Aa^2b^3x^9 + 88\,Ba^4bx^6 + 176\,Aa^3b^2x^6 + 11\,Ba^5x^3 + 55\,Aa^4bx^3 + 8\,Aa^5}{88\,x^{11}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^12,x, algorithm="giac")
```

```
[Out] 1/7*B*b^5*x^7 + 5/4*B*a*b^4*x^4 + 1/4*A*b^5*x^4 + 10*B*a^2*b^3*x + 5*A*a*b^
4*x - 1/88*(440*B*a^3*b^2*x^9 + 440*A*a^2*b^3*x^9 + 88*B*a^4*b*x^6 + 176*A*
a^3*b^2*x^6 + 11*B*a^5*x^3 + 55*A*a^4*b*x^3 + 8*A*a^5)/x^11
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{12}}\,dx = x^4\left(\frac{A\,b^5}{4} + \frac{5\,B\,a\,b^4}{4}\right) - \frac{\frac{A\,a^5}{11} + x^6\,(B\,a^4\,b + 2\,A\,a^3\,b^2) + x^3\left(\frac{B\,a^5}{8} + \frac{5\,A\,b\,a^4}{8}\right) + x^9\,(5\,B\,a^3\,b^2 + 5\,A\,a^2\,b^3)}{x^{11}} + \frac{B\,b^5\,x^7}{7} + 5\,a\,b^3\,x\,(A\,b + 2\,B\,a)$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^12,x)
```

```
[Out] x^4*((A*b^5)/4 + (5*B*a*b^4)/4) - ((A*a^5)/11 + x^6*(2*A*a^3*b^2 + B*a^4*b)
 + x^3*((B*a^5)/8 + (5*A*a^4*b)/8) + x^9*(5*A*a^2*b^3 + 5*B*a^3*b^2))/x^11
+ (B*b^5*x^7)/7 + 5*a*b^3*x*(A*b + 2*B*a)
```

3.45 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{13}} dx$

Optimal result

Integrand size = 20, antiderivative size = 114

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{13}} dx = -\frac{a^5A}{12x^{12}} - \frac{a^4(5Ab+aB)}{9x^9} - \frac{5a^3b(2Ab+aB)}{6x^6} - \frac{10a^2b^2(Ab+aB)}{3x^3} + \frac{1}{3}b^4(Ab+5aB)x^3 + \frac{1}{6}b^5Bx^6 + 5ab^3(Ab + 2aB)\log(x)$$

```
[Out] -1/12*a^5*A/x^12-1/9*a^4*(5*A*b+B*a)/x^9-5/6*a^3*b*(2*A*b+B*a)/x^6-10/3*a^2
*b^2*(A*b+B*a)/x^3+1/3*b^4*(A*b+5*B*a)*x^3+1/6*b^5*B*x^6+5*a*b^3*(A*b+2*B*a
)*ln(x)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 77}

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{13}} dx = -\frac{a^5A}{12x^{12}} - \frac{a^4(aB+5Ab)}{9x^9} - \frac{5a^3b(aB+2Ab)}{6x^6} - \frac{10a^2b^2(aB+Ab)}{3x^3} + \frac{1}{3}b^4x^3(5aB+Ab) + 5ab^3\log(x)(2aB + Ab) + \frac{1}{6}b^5Bx^6$$

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^13,x]
```

```
[Out] -1/12*(a^5*A)/x^12 - (a^4*(5*A*b + a*B))/(9*x^9) - (5*a^3*b*(2*A*b + a*B))/
(6*x^6) - (10*a^2*b^2*(A*b + a*B))/(3*x^3) + (b^4*(A*b + 5*a*B)*x^3)/3 + (b
^5*B*x^6)/6 + 5*a*b^3*(A*b + 2*a*B)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^5}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(b^4(Ab+5aB)+\frac{a^5A}{x^5}+\frac{a^4(5Ab+aB)}{x^4}+\frac{5a^3b(2Ab+aB)}{x^3}\right.\right.\\
&\qquad\left.\left.+\frac{10a^2b^2(Ab+aB)}{x^2}+\frac{5ab^3(Ab+2aB)}{x}+b^5Bx\right)\,dx, x, x^3\right)\\
&= -\frac{a^5A}{12x^{12}}-\frac{a^4(5Ab+aB)}{9x^9}-\frac{5a^3b(2Ab+aB)}{6x^6}-\frac{10a^2b^2(Ab+aB)}{3x^3}\\
&\quad+\frac{1}{3}b^4(Ab+5aB)x^3+\frac{1}{6}b^5Bx^6+5ab^3(Ab+2aB)\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{13}}\,dx = -\frac{120a^2Ab^3x^9-60ab^4Bx^{15}-6b^5x^{15}(2A+Bx^3)+60a^3b^2x^6(A+2Bx^3)+10a^4bx^3(2A+3Bx^3)+a^5(3A}{36x^{12}}$$

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^13,x]
```

```
[Out] -1/36*(120*a^2*A*b^3*x^9 - 60*a*b^4*B*x^15 - 6*b^5*x^15*(2*A + B*x^3) + 60*
a^3*b^2*x^6*(A + 2*B*x^3) + 10*a^4*b*x^3*(2*A + 3*B*x^3) + a^5*(3*A + 4*B*x
^3) - 180*a*b^3*(A*b + 2*a*B)*x^12*Log[x])/x^12
```

Maple [A] (verified)

Time = 4.34 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.93

method	result
default	$\frac{b^5 B x^6}{6}+\frac{A b^5 x^3}{3}+\frac{5 B a b^4 x^3}{3}+5 a b^3 (A b+2 B a) \ln (x)-\frac{5 a^3 b (2 A b+B a)}{6 x^6}-\frac{a^5 A}{12 x^{12}}-\frac{10 a^2 b^2 (A b+B a)}{3 x^3}-$
norman	$\frac{\left(\frac{1}{3} b^5 A+\frac{5}{3} a b^4 B\right) x^{15}+\left(-\frac{10}{3} a^2 b^3 A-\frac{10}{3} a^3 b^2 B\right) x^9+\left(-\frac{5}{3} a^3 b^2 A-\frac{5}{6} a^4 b B\right) x^6+\left(-\frac{5}{9} a^4 b A-\frac{1}{9} a^5 B\right) x^3-\frac{a^5 A}{12}+\frac{b^5 B x^{18}}{6}}{x^{12}}+(5 a b^4$
parallelrisch	$\frac{6 b^5 B x^{18}+12 A b^5 x^{15}+60 B a b^4 x^{15}+180 A \ln (x) x^{12} a b^4+360 B \ln (x) x^{12} a^2 b^3-120 a^2 A b^3 x^9-120 B a^3 b^2 x^9-60 a^3 A b^2 x^6-30 B a}{36 x^{12}}$
risch	$\frac{b^5 B x^6}{6}+\frac{A b^5 x^3}{3}+\frac{5 B a b^4 x^3}{3}+\frac{b^5 A^2}{6 B}+\frac{5 a b^4 A}{3}+\frac{25 a^2 b^3 B}{6}+\frac{\left(-\frac{10}{3} a^2 b^3 A-\frac{10}{3} a^3 b^2 B\right) x^9+\left(-\frac{5}{3} a^3 b^2 A-\frac{5}{6} a^4 b B\right) x^6+}{x^{12}}$

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^13,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*b^5*B*x^6+1/3*A*b^5*x^3+5/3*B*a*b^4*x^3+5*a*b^3*(A*b+2*B*a)*ln(x)-5/6*a
^3*b*(2*A*b+B*a)/x^6-1/12*a^5*A/x^12-10/3*a^2*b^2*(A*b+B*a)/x^3-1/9*a^4*(5*
A*b+B*a)/x^9
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{13}} dx$$
$$= \frac{6 Bb^5x^{18} + 12 (5 Bab^4 + Ab^5)x^{15} + 180 (2 Ba^2b^3 + Aab^4)x^{12} \log (x) - 120 (Ba^3b^2 + Aa^2b^3)x^9 - 30 (Ba^4b}{36 x^{12}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^13,x, algorithm="fricas")
```

```
[Out] 1/36*(6*B*b^5*x^18 + 12*(5*B*a*b^4 + A*b^5)*x^15 + 180*(2*B*a^2*b^3 + A*a*b
^4)*x^12*log(x) - 120*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 30*(B*a^4*b + 2*A*a^3*b
^2)*x^6 - 3*A*a^5 - 4*(B*a^5 + 5*A*a^4*b)*x^3)/x^12
```

Sympy [A] (verification not implemented)

Time = 46.57 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{13}} dx = \frac{Bb^5x^6}{6} + 5ab^3(Ab+2Ba) \log (x) + x^3 \left(\frac{Ab^5}{3} + \frac{5Bab^4}{3}\right)$$
$$+ \frac{-3Aa^5 + x^9(-120Aa^2b^3 - 120Ba^3b^2) + x^6(-60Aa^3b^2 - 30Ba^4b) + x^3(-20Aa^4b - 4Ba^5)}{36x^{12}}$$

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**13,x)
```

```
[Out] B*b**5*x**6/6 + 5*a*b**3*(A*b + 2*B*a)*log(x) + x**3*(A*b**5/3 + 5*B*a*b**4
/3) + (-3*A*a**5 + x**9*(-120*A*a**2*b**3 - 120*B*a**3*b**2) + x**6*(-60*A*
a**3*b**2 - 30*B*a**4*b) + x**3*(-20*A*a**4*b - 4*B*a**5))/(36*x**12)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{13}}\,dx = \frac{1}{6}\,Bb^5x^6 + \frac{1}{3}\,(5\,Bab^4 + Ab^5)x^3 + \frac{5}{3}\,\left(2\,Ba^2b^3 + Aab^4\right)\log\left(x^3\right) - \frac{120\,(Ba^3b^2 + Aa^2b^3)x^9 + 30\,(Ba^4b + 2\,Aa^3b^2)x^6 + 3\,Aa^5 + 4\,(Ba^5 + 5\,Aa^4b)x^3}{36\,x^{12}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^13,x, algorithm="maxima")
```

```
[Out] 1/6*B*b^5*x^6 + 1/3*(5*B*a*b^4 + A*b^5)*x^3 + 5/3*(2*B*a^2*b^3 + A*a*b^4)*l
og(x^3) - 1/36*(120*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 30*(B*a^4*b + 2*A*a^3*b^2
)*x^6 + 3*A*a^5 + 4*(B*a^5 + 5*A*a^4*b)*x^3)/x^12
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.31

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{13}}\,dx = \frac{1}{6}\,Bb^5x^6 + \frac{5}{3}\,Bab^4x^3 + \frac{1}{3}\,Ab^5x^3 + 5\,\left(2\,Ba^2b^3 + Aab^4\right)\log\left(|x|\right) - \frac{250\,Ba^2b^3x^{12} + 125\,Aab^4x^{12} + 120\,Ba^3b^2x^9 + 120\,Aa^2b^3x^9 + 30\,Ba^4bx^6 + 60\,Aa^3b^2x^6 + 4\,Ba^5x^3 + 20\,Aa}{36\,x^{12}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^13,x, algorithm="giac")
```

```
[Out] 1/6*B*b^5*x^6 + 5/3*B*a*b^4*x^3 + 1/3*A*b^5*x^3 + 5*(2*B*a^2*b^3 + A*a*b^4)
*log(abs(x)) - 1/36*(250*B*a^2*b^3*x^12 + 125*A*a*b^4*x^12 + 120*B*a^3*b^2*
x^9 + 120*A*a^2*b^3*x^9 + 30*B*a^4*b*x^6 + 60*A*a^3*b^2*x^6 + 4*B*a^5*x^3 +
 20*A*a^4*b*x^3 + 3*A*a^5)/x^12
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{13}} \, dx$$
$$= \ln(x) \left(10 B a^2 b^3 + 5 A a b^4\right) - \frac{\frac{A a^5}{12} + x^6 \left(\frac{5 B a^4 b}{6} + \frac{5 A a^3 b^2}{3}\right) + x^3 \left(\frac{B a^5}{9} + \frac{5 A b a^4}{9}\right) + x^9 \left(\frac{10 B a^3 b^2}{3} + \frac{10 A a^2 b^3}{3}\right)}{x^{12}} + x^3 \left(\frac{A b^5}{3} + \frac{5 B a b^4}{3}\right) + \frac{B b^5 x^6}{6}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^13,x)
```

```
[Out] log(x)*(10*B*a^2*b^3 + 5*A*a*b^4) - ((A*a^5)/12 + x^6*((5*A*a^3*b^2)/3 + (5
*B*a^4*b)/6) + x^3*((B*a^5)/9 + (5*A*a^4*b)/9) + x^9*((10*A*a^2*b^3)/3 + (1
0*B*a^3*b^2)/3))/x^12 + x^3*((A*b^5)/3 + (5*B*a*b^4)/3) + (B*b^5*x^6)/6
```

3.46 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{14}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 115

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{14}}\,dx = -\frac{a^5A}{13x^{13}}-\frac{a^4(5Ab+aB)}{10x^{10}}-\frac{5a^3b(2Ab+aB)}{7x^7}-\frac{5a^2b^2(Ab+aB)}{2x^4}-\frac{5ab^3(Ab+2aB)}{x}+\frac{1}{2}b^4(Ab+5aB)x^2+\frac{1}{5}b^5Bx^5$$

```
[Out] -1/13*a^5*A/x^13-1/10*a^4*(5*A*b+B*a)/x^10-5/7*a^3*b*(2*A*b+B*a)/x^7-5/2*a^
2*b^2*(A*b+B*a)/x^4-5*a*b^3*(A*b+2*B*a)/x+1/2*b^4*(A*b+5*B*a)*x^2+1/5*b^5*B
*x^5
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{14}}\,dx = -\frac{a^5A}{13x^{13}}-\frac{a^4(aB+5Ab)}{10x^{10}}-\frac{5a^3b(aB+2Ab)}{7x^7}-\frac{5a^2b^2(aB+Ab)}{2x^4}+\frac{1}{2}b^4x^2(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{x}+\frac{1}{5}b^5Bx^5$$

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^14,x]
```

```
[Out] -1/13*(a^5*A)/x^13 - (a^4*(5*A*b + a*B))/(10*x^10) - (5*a^3*b*(2*A*b + a*B)
)/(7*x^7) - (5*a^2*b^2*(A*b + a*B))/(2*x^4) - (5*a*b^3*(A*b + 2*a*B))/x + (
b^4*(A*b + 5*a*B)*x^2)/2 + (b^5*B*x^5)/5
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a^5A}{x^{14}} + \frac{a^4(5Ab+aB)}{x^{11}} + \frac{5a^3b(2Ab+aB)}{x^8} + \frac{10a^2b^2(Ab+aB)}{x^5} + \frac{5ab^3(Ab+2aB)}{x^2} + b^4(Ab+5aB)x + b^5Bx^4\right) dx$$

$$= -\frac{a^5A}{13x^{13}} - \frac{a^4(5Ab+aB)}{10x^{10}} - \frac{5a^3b(2Ab+aB)}{7x^7} - \frac{5a^2b^2(Ab+aB)}{2x^4} - \frac{5ab^3(Ab+2aB)}{x} + \frac{1}{2}b^4(Ab+5aB)x^2 + \frac{1}{5}b^5Bx^5$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{14}}\,dx = -\frac{-2275ab^4x^{12}(-2A+Bx^3) - 91b^5x^{15}(5A+2Bx^3) + 2275a^2b^3x^9(A+4Bx^3) + 325a^3b^2x^6(4A+7Bx^3)}{910x^{13}}$$

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^14,x]
```

```
[Out] -1/910*(-2275*a*b^4*x^12*(-2*A + B*x^3) - 91*b^5*x^15*(5*A + 2*B*x^3) + 227
5*a^2*b^3*x^9*(A + 4*B*x^3) + 325*a^3*b^2*x^6*(4*A + 7*B*x^3) + 65*a^4*b*x^
3*(7*A + 10*B*x^3) + a^5*(70*A + 91*B*x^3))/x^13
```

Maple [A] (verified)

Time = 3.97 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.93

method	result
default	$\frac{b^5 B x^5}{5}+\frac{A b^5 x^2}{2}+\frac{5 B a b^4 x^2}{2}-\frac{5 a^3 b(2 A b+B a)}{7 x^7}-\frac{a^4(5 A b+B a)}{10 x^{10}}-\frac{a^5 A}{13 x^{13}}-\frac{5 a b^3(A b+2 B a)}{x}-\frac{5 a^2 b^2(A b+B a)}{2 x^4}$
norman	$\frac{-\frac{a^5 A}{13}+(-\frac{1}{2} a^4 b A-\frac{1}{10} a^5 B) x^3+(-\frac{10}{7} a^3 b^2 A-\frac{5}{7} a^4 b B) x^6+(-\frac{5}{2} a^2 b^3 A-\frac{5}{2} a^3 b^2 B) x^9+(-5 a b^4 A-10 a^2 b^3 B) x^{12}+(\frac{1}{2} b^5 A+\frac{5}{2} a b^4 B)}{x^{13}}$
risch	$\frac{b^5 B x^5}{5}+\frac{A b^5 x^2}{2}+\frac{5 B a b^4 x^2}{2}+\frac{(-5 a b^4 A-10 a^2 b^3 B) x^{12}+(-\frac{5}{2} a^2 b^3 A-\frac{5}{2} a^3 b^2 B) x^9+(-\frac{10}{7} a^3 b^2 A-\frac{5}{7} a^4 b B) x^6+(-\frac{1}{2} a^4 b A-}{x^{13}}$
gosper	$-\frac{-182 b^5 B x^{18}-455 A b^5 x^{15}-2275 B a b^4 x^{15}+4550 a A b^4 x^{12}+9100 B a^2 b^3 x^{12}+2275 a^2 A b^3 x^9+2275 B a^3 b^2 x^9+1300 a^3 A b^2 x^6+6}{910 x^{13}}$
parallelrisch	$\frac{182 b^5 B x^{18}+455 A b^5 x^{15}+2275 B a b^4 x^{15}-4550 a A b^4 x^{12}-9100 B a^2 b^3 x^{12}-2275 a^2 A b^3 x^9-2275 B a^3 b^2 x^9-1300 a^3 A b^2 x^6-650 B}{910 x^{13}}$

[In] int((b*x^3+a)^5*(B*x^3+A)/x^14,x,method=_RETURNVERBOSE)

[Out] 1/5*b^5*B*x^5+1/2*A*b^5*x^2+5/2*B*a*b^4*x^2-5/7*a^3*b*(2*A*b+B*a)/x^7-1/10*a^4*(5*A*b+B*a)/x^10-1/13*a^5*A/x^13-5*a*b^3*(A*b+2*B*a)/x-5/2*a^2*b^2*(A*b+B*a)/x^4

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{14}} dx = \frac{182 Bb^5x^{18}+455(5Bab^4+Ab^5)x^{15}-4550(2Ba^2b^3+Aab^4)x^{12}-2275(Ba^3b^2+Aa^2b^3)x^9-650(Ba^4b+}{910x^{13}}$$

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^14,x, algorithm="fricas")

[Out] 1/910*(182*B*b^5*x^18 + 455*(5*B*a*b^4 + A*b^5)*x^15 - 4550*(2*B*a^2*b^3 + A*a*b^4)*x^12 - 2275*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 650*(B*a^4*b + 2*A*a^3*b^2)*x^6 - 70*A*a^5 - 91*(B*a^5 + 5*A*a^4*b)*x^3)/x^13

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{14}} dx = \text{Timed out}$$

[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**14,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{14}} \, dx = \frac{1}{5} Bb^5x^5 + \frac{1}{2} (5 Bab^4 + Ab^5)x^2 - \frac{4550 (2 Ba^2b^3 + Aab^4)x^{12} + 2275 (Ba^3b^2 + Aa^2b^3)x^9 + 650 (Ba^4b + 2 Aa^3b^2)x^6 + 70 Aa^5 + 91 (Ba^5 + \cdots}{910 x^{13}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^14,x, algorithm="maxima")
```

```
[Out] 1/5*B*b^5*x^5 + 1/2*(5*B*a*b^4 + A*b^5)*x^2 - 1/910*(4550*(2*B*a^2*b^3 + A*
a*b^4)*x^12 + 2275*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 650*(B*a^4*b + 2*A*a^3*b^2
)*x^6 + 70*A*a^5 + 91*(B*a^5 + 5*A*a^4*b)*x^3)/x^13
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{14}} \, dx = \frac{1}{5} Bb^5x^5 + \frac{5}{2} Bab^4x^2 + \frac{1}{2} Ab^5x^2 - \frac{9100 Ba^2b^3x^{12} + 4550 Aab^4x^{12} + 2275 Ba^3b^2x^9 + 2275 Aa^2b^3x^9 + 650 Ba^4bx^6 + 1300 Aa^3b^2x^6 + 91 Ba\cdots}{910 x^{13}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^14,x, algorithm="giac")
```

```
[Out] 1/5*B*b^5*x^5 + 5/2*B*a*b^4*x^2 + 1/2*A*b^5*x^2 - 1/910*(9100*B*a^2*b^3*x^1
2 + 4550*A*a*b^4*x^12 + 2275*B*a^3*b^2*x^9 + 2275*A*a^2*b^3*x^9 + 650*B*a^4
*b*x^6 + 1300*A*a^3*b^2*x^6 + 91*B*a^5*x^3 + 455*A*a^4*b*x^3 + 70*A*a^5)/x^
13
```

Mupad [B] (verification not implemented)

Time = 6.56 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{14}} \, dx = x^2 \left(\frac{A\,b^5}{2} + \frac{5\,B\,a\,b^4}{2}\right) - \frac{\frac{A\,a^5}{13} + x^{12}\,(10\,B\,a^2\,b^3 + 5\,A\,a\,b^4) + x^6\left(\frac{5\,B\,a^4\,b}{7} + \frac{10\,A\,a^3\,b^2}{7}\right) + x^3\left(\frac{B\,a^5}{10} + \frac{A\,b\,a^4}{2}\right) + x^9\left(\frac{5\,B\,a^3\,b^2}{2} + \frac{5\,A\,a^2\,b^3}{2}\right)}{x^{13}} + \frac{B\,b^5\,x^5}{5}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^14,x)
```

```
[Out] x^2*((A*b^5)/2 + (5*B*a*b^4)/2) - ((A*a^5)/13 + x^12*(10*B*a^2*b^3 + 5*A*a*
b^4) + x^6*((10*A*a^3*b^2)/7 + (5*B*a^4*b)/7) + x^3*((B*a^5)/10 + (A*a^4*b)
/2) + x^9*((5*A*a^2*b^3)/2 + (5*B*a^3*b^2)/2))/x^13 + (B*b^5*x^5)/5
```

3.47 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{15}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 110

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{15}}\,dx = -\frac{a^5A}{14x^{14}}-\frac{a^4(5Ab+aB)}{11x^{11}}-\frac{5a^3b(2Ab+aB)}{8x^8}-\frac{2a^2b^2(Ab+aB)}{x^5}-\frac{5ab^3(Ab+2aB)}{2x^2}+b^4(Ab+5aB)x+\frac{1}{4}b^5Bx^4$$

```
[Out] -1/14*a^5*A/x^14-1/11*a^4*(5*A*b+B*a)/x^11-5/8*a^3*b*(2*A*b+B*a)/x^8-2*a^2*
b^2*(A*b+B*a)/x^5-5/2*a*b^3*(A*b+2*B*a)/x^2+b^4*(A*b+5*B*a)*x+1/4*b^5*B*x^4
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{15}}\,dx = -\frac{a^5A}{14x^{14}}-\frac{a^4(aB+5Ab)}{11x^{11}}-\frac{5a^3b(aB+2Ab)}{8x^8}-\frac{2a^2b^2(aB+Ab)}{x^5}+b^4x(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{2x^2}+\frac{1}{4}b^5Bx^4$$

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^15,x]
```

```
[Out] -1/14*(a^5*A)/x^14 - (a^4*(5*A*b + a*B))/(11*x^11) - (5*a^3*b*(2*A*b + a*B)
)/(8*x^8) - (2*a^2*b^2*(A*b + a*B))/x^5 - (5*a*b^3*(A*b + 2*a*B))/(2*x^2) +
 b^4*(A*b + 5*a*B)*x + (b^5*B*x^4)/4
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(b^4(Ab+5aB) + \frac{a^5A}{x^{15}} + \frac{a^4(5Ab+aB)}{x^{12}} + \frac{5a^3b(2Ab+aB)}{x^9} + \frac{10a^2b^2(Ab+aB)}{x^6} + \frac{5ab^3(Ab+2aB)}{x^3} + b^5Bx^3\right) dx$$

$$= -\frac{a^5A}{14x^{14}} - \frac{a^4(5Ab+aB)}{11x^{11}} - \frac{5a^3b(2Ab+aB)}{8x^8} - \frac{2a^2b^2(Ab+aB)}{x^5} - \frac{5ab^3(Ab+2aB)}{2x^2} + b^4(Ab+5aB)x + \frac{1}{4}b^5Bx^4$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x^{15}}\,dx = -\frac{a^5A}{14x^{14}} - \frac{a^4(5Ab+aB)}{11x^{11}} - \frac{5a^3b(2Ab+aB)}{8x^8} - \frac{2a^2b^2(Ab+aB)}{x^5} - \frac{5ab^3(Ab+2aB)}{2x^2} + b^4(Ab+5aB)x + \frac{1}{4}b^5Bx^4$$

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^15,x]
```

```
[Out] -1/14*(a^5*A)/x^14 - (a^4*(5*A*b + a*B))/(11*x^11) - (5*a^3*b*(2*A*b + a*B)
)/(8*x^8) - (2*a^2*b^2*(A*b + a*B))/x^5 - (5*a*b^3*(A*b + 2*a*B))/(2*x^2) +
 b^4*(A*b + 5*a*B)*x + (b^5*B*x^4)/4
```

Maple [A] (verified)

Time = 4.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.93

method	result
default	$\frac{b^5 B x^4}{4}+A b^5 x+5 B a b^4 x-\frac{a^5 A}{14 x^{14}}-\frac{5 a^3 b(2 A b+B a)}{8 x^8}-\frac{a^4(5 A b+B a)}{11 x^{11}}-\frac{5 a b^3(A b+2 B a)}{2 x^2}-\frac{2 a^2 b^2(A b+B a)}{x^5}$
risch	$\frac{b^5 B x^4}{4}+A b^5 x+5 B a b^4 x+\frac{\left(-\frac{5}{2} a b^4 A-5 a^2 b^3 B\right) x^{12}+\left(-2 a^2 b^3 A-2 a^3 b^2 B\right) x^9+\left(-\frac{5}{4} a^3 b^2 A-\frac{5}{8} a^4 b B\right) x^6+\left(-\frac{5}{11} a^4 b A\right.}{x^{14}}$
norman	$\frac{-\frac{a^5 A}{14}+\left(-\frac{5}{11} a^4 b A-\frac{1}{11} a^5 B\right) x^3+\left(-\frac{5}{4} a^3 b^2 A-\frac{5}{8} a^4 b B\right) x^6+\left(-2 a^2 b^3 A-2 a^3 b^2 B\right) x^9+\left(-\frac{5}{2} a b^4 A-5 a^2 b^3 B\right) x^{12}+\left(b^5 A+5 a b^4 B\right) x}{x^{14}}$
gosper	$-\frac{-154 b^5 B x^{18}-616 A b^5 x^{15}-3080 B a b^4 x^{15}+1540 a A b^4 x^{12}+3080 B a^2 b^3 x^{12}+1232 a^2 A b^3 x^9+1232 B a^3 b^2 x^9+770 a^3 A b^2 x^6+}{616 x^{14}}$
parallelrisch	$\frac{154 b^5 B x^{18}+616 A b^5 x^{15}+3080 B a b^4 x^{15}-1540 a A b^4 x^{12}-3080 B a^2 b^3 x^{12}-1232 a^2 A b^3 x^9-1232 B a^3 b^2 x^9-770 a^3 A b^2 x^6-385 B}{616 x^{14}}$

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^15,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b^5*B*x^4+A*b^5*x+5*B*a*b^4*x-1/14*a^5*A/x^14-5/8*a^3*b*(2*A*b+B*a)/x^8
-1/11*a^4*(5*A*b+B*a)/x^11-5/2*a*b^3*(A*b+2*B*a)/x^2-2*a^2*b^2*(A*b+B*a)/x^
5
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{15}}\,dx = \frac{154\,Bb^5x^{18}+616\,(5\,Bab^4+Ab^5)x^{15}-1540\,(2\,Ba^2b^3+Aab^4)x^{12}-1232\,(Ba^3b^2+Aa^2b^3)x^9-385\,(Ba^4b}{616\,x^{14}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^15,x, algorithm="fricas")
```

```
[Out] 1/616*(154*B*b^5*x^18 + 616*(5*B*a*b^4 + A*b^5)*x^15 - 1540*(2*B*a^2*b^3 +
A*a*b^4)*x^12 - 1232*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 385*(B*a^4*b + 2*A*a^3*b
^2)*x^6 - 44*A*a^5 - 56*(B*a^5 + 5*A*a^4*b)*x^3)/x^14
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{15}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**15,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{15}}\,dx = \frac{1}{4}\,Bb^5x^4 + \left(5\,Bab^4 + Ab^5\right)x - \frac{1540\,(2\,Ba^2b^3 + Aab^4)x^{12} + 1232\,(Ba^3b^2 + Aa^2b^3)x^9 + 385\,(Ba^4b + 2\,Aa^3b^2)x^6 + 44\,Aa^5 + 56\,(Ba^5 + 5}{616\,x^{14}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^15,x, algorithm="maxima")
```

```
[Out] 1/4*B*b^5*x^4 + (5*B*a*b^4 + A*b^5)*x - 1/616*(1540*(2*B*a^2*b^3 + A*a*b^4)
*x^12 + 1232*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 385*(B*a^4*b + 2*A*a^3*b^2)*x^6
+ 44*A*a^5 + 56*(B*a^5 + 5*A*a^4*b)*x^3)/x^14
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{15}}\,dx = \frac{1}{4}\,Bb^5x^4 + 5\,Bab^4x + Ab^5x - \frac{3080\,Ba^2b^3x^{12} + 1540\,Aab^4x^{12} + 1232\,Ba^3b^2x^9 + 1232\,Aa^2b^3x^9 + 385\,Ba^4bx^6 + 770\,Aa^3b^2x^6 + 56\,Ba^5x^3}{616\,x^{14}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^15,x, algorithm="giac")
```

```
[Out] 1/4*B*b^5*x^4 + 5*B*a*b^4*x + A*b^5*x - 1/616*(3080*B*a^2*b^3*x^12 + 1540*A
*a*b^4*x^12 + 1232*B*a^3*b^2*x^9 + 1232*A*a^2*b^3*x^9 + 385*B*a^4*b*x^6 + 7
70*A*a^3*b^2*x^6 + 56*B*a^5*x^3 + 280*A*a^4*b*x^3 + 44*A*a^5)/x^14
```

Mupad [B] (verification not implemented)

Time = 6.55 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{15}}\,dx = x\left(A\,b^5 + 5\,B\,a\,b^4\right) - \frac{\frac{A\,a^5}{14} + x^{12}\left(5\,B\,a^2\,b^3 + \frac{5\,A\,a\,b^4}{2}\right) + x^6\left(\frac{5\,B\,a^4\,b}{8} + \frac{5\,A\,a^3\,b^2}{4}\right) + x^3\left(\frac{B\,a^5}{11} + \frac{5\,A\,b\,a^4}{11}\right) + x^9\,\left(2\,B\,a^3\,b^2 + 2\,A\,a^2\,b^3\right)}{x^{14}} + \frac{B\,b^5\,x^4}{4}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^15,x)
```

```
[Out] x*(A*b^5 + 5*B*a*b^4) - ((A*a^5)/14 + x^12*(5*B*a^2*b^3 + (5*A*a*b^4)/2) +
x^6*((5*A*a^3*b^2)/4 + (5*B*a^4*b)/8) + x^3*((B*a^5)/11 + (5*A*a^4*b)/11) +
 x^9*(2*A*a^2*b^3 + 2*B*a^3*b^2))/x^14 + (B*b^5*x^4)/4
```

3.48 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{16}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{16}}\,dx = -\frac{a^5A}{15x^{15}}-\frac{a^4(5Ab+aB)}{12x^{12}}-\frac{5a^3b(2Ab+aB)}{9x^9}-\frac{5a^2b^2(Ab+aB)}{3x^6}-\frac{5ab^3(Ab+2aB)}{3x^3}+\frac{1}{3}b^5Bx^3+b^4(Ab+5aB)\log(x)$$

```
[Out] -1/15*a^5*A/x^15-1/12*a^4*(5*A*b+B*a)/x^12-5/9*a^3*b*(2*A*b+B*a)/x^9-5/3*a^
2*b^2*(A*b+B*a)/x^6-5/3*a*b^3*(A*b+2*B*a)/x^3+1/3*b^5*B*x^3+b^4*(A*b+5*B*a)
*ln(x)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 77}

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{16}}\,dx = -\frac{a^5A}{15x^{15}}-\frac{a^4(aB+5Ab)}{12x^{12}}-\frac{5a^3b(aB+2Ab)}{9x^9}-\frac{5a^2b^2(aB+Ab)}{3x^6}+b^4\log(x)(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{3x^3}+\frac{1}{3}b^5Bx^3$$

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^16,x]
```

```
[Out] -1/15*(a^5*A)/x^15 - (a^4*(5*A*b + a*B))/(12*x^12) - (5*a^3*b*(2*A*b + a*B)
)/(9*x^9) - (5*a^2*b^2*(A*b + a*B))/(3*x^6) - (5*a*b^3*(A*b + 2*a*B))/(3*x^
3) + (b^5*B*x^3)/3 + b^4*(A*b + 5*a*B)*Log[x]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^6}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(b^5B+\frac{a^5A}{x^6}+\frac{a^4(5Ab+aB)}{x^5}+\frac{5a^3b(2Ab+aB)}{x^4}+\frac{10a^2b^2(Ab+aB)}{x^3}+\frac{5ab^3(Ab+2aB)}{x^2}+\frac{b^4(Ab+5aB)}{x}\right)dx, x, x^3\right)\\
&= -\frac{a^5A}{15x^{15}}-\frac{a^4(5Ab+aB)}{12x^{12}}-\frac{5a^3b(2Ab+aB)}{9x^9}-\frac{5a^2b^2(Ab+aB)}{3x^6}-\frac{5ab^3(Ab+2aB)}{3x^3}+\frac{1}{3}b^5Bx^3+b^4(Ab+5aB)\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{16}}\,dx = -\frac{300aAb^4x^{12}-60b^5Bx^{18}+300a^2b^3x^9(A+2Bx^3)+100a^3b^2x^6(2A+3Bx^3)+25a^4bx^3(3A+4Bx^3)+3}{180x^{15}} + b^4(Ab+5aB)\log(x)$$

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^16,x]
```

```
[Out] -1/180*(300*a*A*b^4*x^12 - 60*b^5*B*x^18 + 300*a^2*b^3*x^9*(A + 2*B*x^3) +
100*a^3*b^2*x^6*(2*A + 3*B*x^3) + 25*a^4*b*x^3*(3*A + 4*B*x^3) + 3*a^5*(4*A
 + 5*B*x^3))/x^15 + b^4*(A*b + 5*a*B)*Log[x]
```

Maple [A] (verified)

Time = 4.33 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.90

method	result
default	$-\frac{a^5A}{15x^{15}}-\frac{a^4(5Ab+Ba)}{12x^{12}}-\frac{5a^3b(2Ab+Ba)}{9x^9}-\frac{5a^2b^2(Ab+Ba)}{3x^6}-\frac{5a\,b^3(Ab+2Ba)}{3x^3}+\frac{b^5B\,x^3}{3}+b^4(Ab+5Ba)\ln($
norman	$\frac{(-\frac{5}{3}a\,b^4A-\frac{10}{3}a^2b^3B)x^{12}+(-\frac{5}{3}a^2b^3A-\frac{5}{3}a^3b^2B)x^9+(-\frac{10}{9}a^3b^2A-\frac{5}{9}a^4bB)x^6+(-\frac{5}{12}a^4bA-\frac{1}{12}a^5B)x^3-\frac{a^5A}{15}+\frac{b^5B\,x^{18}}{3}}{x^{15}}+(b^5$
risch	$\frac{b^5B\,x^3}{3}+\frac{-\frac{a^5A}{15}+(-\frac{5}{12}a^4bA-\frac{1}{12}a^5B)x^3+(-\frac{10}{9}a^3b^2A-\frac{5}{9}a^4bB)x^6+(-\frac{5}{3}a^2b^3A-\frac{5}{3}a^3b^2B)x^9+(-\frac{5}{3}a\,b^4A-\frac{10}{3}a^2b^3B)x^{12}}{x^{15}}+$
parallelrisch	$\frac{60b^5B\,x^{18}+180A\ln(x)x^{15}b^5+900B\ln(x)x^{15}a\,b^4-300aA\,b^4x^{12}-600B\,a^2b^3x^{12}-300a^2A\,b^3x^9-300B\,a^3b^2x^9-200a^3A\,b^2x^6-100}{180x^{15}}$

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^16,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*a^5*A/x^15-1/12*a^4*(5*A*b+B*a)/x^12-5/9*a^3*b*(2*A*b+B*a)/x^9-5/3*a^
2*b^2*(A*b+B*a)/x^6-5/3*a*b^3*(A*b+2*B*a)/x^3+1/3*b^5*B*x^3+b^4*(A*b+5*B*a)
*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{16}}\,dx = \frac{60\,Bb^5x^{18}+180\,(5\,Bab^4+Ab^5)x^{15}\log{(x)}-300\,(2\,Ba^2b^3+Aab^4)x^{12}-300\,(Ba^3b^2+Aa^2b^3)x^9-100\,(Ba}{180\,x^{15}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^16,x, algorithm="fricas")
```

```
[Out] 1/180*(60*B*b^5*x^18 + 180*(5*B*a*b^4 + A*b^5)*x^15*log(x) - 300*(2*B*a^2*b
^3 + A*a*b^4)*x^12 - 300*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 100*(B*a^4*b + 2*A*a
^3*b^2)*x^6 - 12*A*a^5 - 15*(B*a^5 + 5*A*a^4*b)*x^3)/x^15
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{16}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**16,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{16}}\,dx = \frac{1}{3}Bb^5x^3 + \frac{1}{3}\left(5Bab^4 + Ab^5\right)\log\left(x^3\right) - \frac{300\,(2\,Ba^2b^3 + Aab^4)x^{12} + 300\,(Ba^3b^2 + Aa^2b^3)x^9 + 100\,(Ba^4b + 2\,Aa^3b^2)x^6 + 12\,Aa^5 + 15\,(Ba^5 + 5\,A}{180\,x^{15}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^16,x, algorithm="maxima")
```

```
[Out] 1/3*B*b^5*x^3 + 1/3*(5*B*a*b^4 + A*b^5)*log(x^3) - 1/180*(300*(2*B*a^2*b^3
+ A*a*b^4)*x^12 + 300*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 100*(B*a^4*b + 2*A*a^3*
b^2)*x^6 + 12*A*a^5 + 15*(B*a^5 + 5*A*a^4*b)*x^3)/x^15
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{16}}\,dx = \frac{1}{3}Bb^5x^3 + \left(5Bab^4 + Ab^5\right)\log\left(|x|\right) - \frac{685\,Bab^4x^{15} + 137\,Ab^5x^{15} + 600\,Ba^2b^3x^{12} + 300\,Aab^4x^{12} + 300\,Ba^3b^2x^9 + 300\,Aa^2b^3x^9 + 100\,Ba^4bx^6}{180\,x^{15}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^16,x, algorithm="giac")
```

```
[Out] 1/3*B*b^5*x^3 + (5*B*a*b^4 + A*b^5)*log(abs(x)) - 1/180*(685*B*a*b^4*x^15 +
 137*A*b^5*x^15 + 600*B*a^2*b^3*x^12 + 300*A*a*b^4*x^12 + 300*B*a^3*b^2*x^9
 + 300*A*a^2*b^3*x^9 + 100*B*a^4*b*x^6 + 200*A*a^3*b^2*x^6 + 15*B*a^5*x^3 +
 75*A*a^4*b*x^3 + 12*A*a^5)/x^15
```

Mupad [B] (verification not implemented)

Time = 6.51 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{16}}\,dx = \ln(x)\,\left(A\,b^5 + 5\,B\,a\,b^4\right) - \frac{\frac{A\,a^5}{15} + x^{12}\left(\frac{10\,B\,a^2\,b^3}{3} + \frac{5\,A\,a\,b^4}{3}\right) + x^6\left(\frac{5\,B\,a^4\,b}{9} + \frac{10\,A\,a^3\,b^2}{9}\right) + x^3\left(\frac{B\,a^5}{12} + \frac{5\,A\,b\,a^4}{12}\right) + x^9\left(\frac{5\,B\,a^3\,b^2}{3} + \frac{5\,A\,a^2\,b^3}{3}\right)}{x^{15}} + \frac{B\,b^5\,x^3}{3}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^16,x)
```

```
[Out] log(x)*(A*b^5 + 5*B*a*b^4) - ((A*a^5)/15 + x^12*((10*B*a^2*b^3)/3 + (5*A*a*
b^4)/3) + x^6*((10*A*a^3*b^2)/9 + (5*B*a^4*b)/9) + x^3*((B*a^5)/12 + (5*A*a
^4*b)/12) + x^9*((5*A*a^2*b^3)/3 + (5*B*a^3*b^2)/3))/x^15 + (B*b^5*x^3)/3
```

3.49 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{17}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 115

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{17}}\,dx = -\frac{a^5A}{16x^{16}}-\frac{a^4(5Ab+aB)}{13x^{13}}-\frac{a^3b(2Ab+aB)}{2x^{10}}-\frac{10a^2b^2(Ab+aB)}{7x^7}-\frac{5ab^3(Ab+2aB)}{4x^4}-\frac{b^4(Ab+5aB)}{x}+\frac{1}{2}b^5Bx^2$$

```
[Out] -1/16*a^5*A/x^16-1/13*a^4*(5*A*b+B*a)/x^13-1/2*a^3*b*(2*A*b+B*a)/x^10-10/7*
a^2*b^2*(A*b+B*a)/x^7-5/4*a*b^3*(A*b+2*B*a)/x^4-b^4*(A*b+5*B*a)/x+1/2*b^5*B
*x^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{17}}\,dx = -\frac{a^5A}{16x^{16}}-\frac{a^4(aB+5Ab)}{13x^{13}}-\frac{a^3b(aB+2Ab)}{2x^{10}}-\frac{10a^2b^2(aB+Ab)}{7x^7}-\frac{b^4(5aB+Ab)}{x}-\frac{5ab^3(2aB+Ab)}{4x^4}+\frac{1}{2}b^5Bx^2$$

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^17,x]
```

```
[Out] -1/16*(a^5*A)/x^16 - (a^4*(5*A*b + a*B))/(13*x^13) - (a^3*b*(2*A*b + a*B))/
(2*x^10) - (10*a^2*b^2*(A*b + a*B))/(7*x^7) - (5*a*b^3*(A*b + 2*a*B))/(4*x^
4) - (b^4*(A*b + 5*a*B))/x + (b^5*B*x^2)/2
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a^5A}{x^{17}} + \frac{a^4(5Ab+aB)}{x^{14}} + \frac{5a^3b(2Ab+aB)}{x^{11}} + \frac{10a^2b^2(Ab+aB)}{x^8} + \frac{5ab^3(Ab+2aB)}{x^5} + \frac{b^4(Ab+5aB)}{x^2} + b^5Bx\right) dx$$

$$= -\frac{a^5A}{16x^{16}} - \frac{a^4(5Ab+aB)}{13x^{13}} - \frac{a^3b(2Ab+aB)}{2x^{10}} - \frac{10a^2b^2(Ab+aB)}{7x^7} - \frac{5ab^3(Ab+2aB)}{4x^4} - \frac{b^4(Ab+5aB)}{x} + \frac{1}{2}b^5Bx^2$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{17}} \, dx = -\frac{-728b^5x^{15}(-2A+Bx^3) + 1820ab^4x^{12}(A+4Bx^3) + 520a^2b^3x^9(4A+7Bx^3) + 208a^3b^2x^6(7A+10Bx^3)}{1456x^{16}}$$

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^17,x]
```

```
[Out] -1/1456*(-728*b^5*x^15*(-2*A + B*x^3) + 1820*a*b^4*x^12*(A + 4*B*x^3) + 520
*a^2*b^3*x^9*(4*A + 7*B*x^3) + 208*a^3*b^2*x^6*(7*A + 10*B*x^3) + 56*a^4*b*
x^3*(10*A + 13*B*x^3) + 7*a^5*(13*A + 16*B*x^3))/x^16
```

Maple [A] (verified)

Time = 4.34 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

method	result
default	$-\frac{a^5 A}{16x^{16}}-\frac{a^4(5Ab+Ba)}{13x^{13}}-\frac{a^3b(2Ab+Ba)}{2x^{10}}-\frac{10a^2b^2(Ab+Ba)}{7x^7}-\frac{5a\,b^3(Ab+2Ba)}{4x^4}-\frac{b^4(Ab+5Ba)}{x}+\frac{b^5B\,x^2}{2}$
norman	$\frac{-\frac{a^5A}{16}+(-\frac{5}{13}a^4bA-\frac{1}{13}a^5B)x^3+(-a^3b^2A-\frac{1}{2}a^4bB)x^6+(-\frac{10}{7}a^2b^3A-\frac{10}{7}a^3b^2B)x^9+(-\frac{5}{4}a\,b^4A-\frac{5}{2}a^2b^3B)x^{12}+(-b^5A-5a\,b^4}{x^{16}}$
risch	$\frac{b^5B\,x^2}{2}+\frac{-\frac{a^5A}{16}+(-\frac{5}{13}a^4bA-\frac{1}{13}a^5B)x^3+(-a^3b^2A-\frac{1}{2}a^4bB)x^6+(-\frac{10}{7}a^2b^3A-\frac{10}{7}a^3b^2B)x^9+(-\frac{5}{4}a\,b^4A-\frac{5}{2}a^2b^3B)x^{12}+(-}{x^{16}}$
gosper	$-\frac{-728b^5B\,x^{18}+1456A\,b^5x^{15}+7280Ba\,b^4x^{15}+1820aA\,b^4x^{12}+3640B\,a^2b^3x^{12}+2080a^2A\,b^3x^9+2080B\,a^3b^2x^9+1456a^3A\,b^2x^6}{1456x^{16}}$
parallelrisch	$-\frac{-728b^5B\,x^{18}+1456A\,b^5x^{15}+7280Ba\,b^4x^{15}+1820aA\,b^4x^{12}+3640B\,a^2b^3x^{12}+2080a^2A\,b^3x^9+2080B\,a^3b^2x^9+1456a^3A\,b^2x^6}{1456x^{16}}$

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^17,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*a^5*A/x^16-1/13*a^4*(5*A*b+B*a)/x^13-1/2*a^3*b*(2*A*b+B*a)/x^10-10/7*
a^2*b^2*(A*b+B*a)/x^7-5/4*a*b^3*(A*b+2*B*a)/x^4-b^4*(A*b+5*B*a)/x+1/2*b^5*B
*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{17}}\,dx = \frac{728\,Bb^5x^{18}-1456\,(5\,Bab^4+Ab^5)x^{15}-1820\,(2\,Ba^2b^3+Aab^4)x^{12}-2080\,(Ba^3b^2+Aa^2b^3)x^9-728\,(Ba^4}{1456\,x^{16}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^17,x, algorithm="fricas")
```

```
[Out] 1/1456*(728*B*b^5*x^18 - 1456*(5*B*a*b^4 + A*b^5)*x^15 - 1820*(2*B*a^2*b^3
+ A*a*b^4)*x^12 - 2080*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 728*(B*a^4*b + 2*A*a^3
*b^2)*x^6 - 91*A*a^5 - 112*(B*a^5 + 5*A*a^4*b)*x^3)/x^16
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{17}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**17,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{17}} \, dx = \frac{1}{2} Bb^5x^2 - \frac{1456 (5 Bab^4 + Ab^5)x^{15} + 1820 (2 Ba^2b^3 + Aab^4)x^{12} + 2080 (Ba^3b^2 + Aa^2b^3)x^9 + 728 (Ba^4b + 2 Aa^3b^2)x}{1456 x^{16}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^17,x, algorithm="maxima")
```

```
[Out] 1/2*B*b^5*x^2 - 1/1456*(1456*(5*B*a*b^4 + A*b^5)*x^15 + 1820*(2*B*a^2*b^3 +
 A*a*b^4)*x^12 + 2080*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 728*(B*a^4*b + 2*A*a^3*
b^2)*x^6 + 91*A*a^5 + 112*(B*a^5 + 5*A*a^4*b)*x^3)/x^16
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{17}} \, dx = \frac{1}{2} Bb^5x^2 - \frac{7280 Bab^4x^{15} + 1456 Ab^5x^{15} + 3640 Ba^2b^3x^{12} + 1820 Aab^4x^{12} + 2080 Ba^3b^2x^9 + 2080 Aa^2b^3x^9 + 728 Ba}{1456 x^{16}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^17,x, algorithm="giac")
```

```
[Out] 1/2*B*b^5*x^2 - 1/1456*(7280*B*a*b^4*x^15 + 1456*A*b^5*x^15 + 3640*B*a^2*b^
3*x^12 + 1820*A*a*b^4*x^12 + 2080*B*a^3*b^2*x^9 + 2080*A*a^2*b^3*x^9 + 728*
B*a^4*b*x^6 + 1456*A*a^3*b^2*x^6 + 112*B*a^5*x^3 + 560*A*a^4*b*x^3 + 91*A*a
^5)/x^16
```

Mupad [B] (verification not implemented)

Time = 6.59 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{17}} \, dx = \frac{B\,b^5\,x^2}{2} - \frac{\frac{A\,a^5}{16} + x^6\left(\frac{B\,a^4\,b}{2} + A\,a^3\,b^2\right) + x^{12}\left(\frac{5\,B\,a^2\,b^3}{2} + \frac{5\,A\,a\,b^4}{4}\right) + x^3\left(\frac{B\,a^5}{13} + \frac{5\,A\,b\,a^4}{13}\right) + x^{15}\left(A\,b^5 + 5\,B\,a\,b^4\right) + x^9\left(\frac{1}{}\right.}{x^{16}}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^17,x)
```

```
[Out] (B*b^5*x^2)/2 - ((A*a^5)/16 + x^6*(A*a^3*b^2 + (B*a^4*b)/2) + x^12*((5*B*a^
2*b^3)/2 + (5*A*a*b^4)/4) + x^3*((B*a^5)/13 + (5*A*a^4*b)/13) + x^15*(A*b^5
 + 5*B*a*b^4) + x^9*((10*A*a^2*b^3)/7 + (10*B*a^3*b^2)/7))/x^16
```

3.50 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{18}} dx$

Optimal result

Integrand size = 20, antiderivative size = 110

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{18}} dx = -\frac{a^5 A}{17x^{17}} - \frac{a^4(5Ab+aB)}{14x^{14}} - \frac{5a^3b(2Ab+aB)}{11x^{11}} - \frac{5a^2b^2(Ab+aB)}{4x^8} - \frac{ab^3(Ab+2aB)}{x^5} - \frac{b^4(Ab+5aB)}{2x^2} + b^5 Bx$$

```
[Out] -1/17*a^5*A/x^17-1/14*a^4*(5*A*b+B*a)/x^14-5/11*a^3*b*(2*A*b+B*a)/x^11-5/4*
a^2*b^2*(A*b+B*a)/x^8-a*b^3*(A*b+2*B*a)/x^5-1/2*b^4*(A*b+5*B*a)/x^2+b^5*B*x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{18}} dx = -\frac{a^5 A}{17x^{17}} - \frac{a^4(aB+5Ab)}{14x^{14}} - \frac{5a^3b(aB+2Ab)}{11x^{11}} - \frac{5a^2b^2(aB+Ab)}{4x^8} - \frac{b^4(5aB+Ab)}{2x^2} - \frac{ab^3(2aB+Ab)}{x^5} + b^5 Bx$$

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^18,x]
```

```
[Out] -1/17*(a^5*A)/x^17 - (a^4*(5*A*b + a*B))/(14*x^14) - (5*a^3*b*(2*A*b + a*B)
)/(11*x^11) - (5*a^2*b^2*(A*b + a*B))/(4*x^8) - (a*b^3*(A*b + 2*a*B))/x^5 -
 (b^4*(A*b + 5*a*B))/(2*x^2) + b^5*B*x
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(b^5B + \frac{a^5A}{x^{18}} + \frac{a^4(5Ab+aB)}{x^{15}} + \frac{5a^3b(2Ab+aB)}{x^{12}} + \frac{10a^2b^2(Ab+aB)}{x^9} + \frac{5ab^3(Ab+2aB)}{x^6} + \frac{b^4(Ab+5aB)}{x^3}\right) dx$$

$$= -\frac{a^5A}{17x^{17}} - \frac{a^4(5Ab+aB)}{14x^{14}} - \frac{5a^3b(2Ab+aB)}{11x^{11}} - \frac{5a^2b^2(Ab+aB)}{4x^8} - \frac{ab^3(Ab+2aB)}{x^5} - \frac{b^4(Ab+5aB)}{2x^2} + b^5Bx$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{18}}\,dx = -\frac{a^5A}{17x^{17}} - \frac{a^4(5Ab+aB)}{14x^{14}} - \frac{5a^3b(2Ab+aB)}{11x^{11}} - \frac{5a^2b^2(Ab+aB)}{4x^8} - \frac{ab^3(Ab+2aB)}{x^5} - \frac{b^4(Ab+5aB)}{2x^2} + b^5Bx$$

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^18,x]
[Out] -1/17*(a^5*A)/x^17 - (a^4*(5*A*b + a*B))/(14*x^14) - (5*a^3*b*(2*A*b + a*B)
)/(11*x^11) - (5*a^2*b^2*(A*b + a*B))/(4*x^8) - (a*b^3*(A*b + 2*a*B))/x^5 -
 (b^4*(A*b + 5*a*B))/(2*x^2) + b^5*B*x
```

Maple [A] (verified)

Time = 4.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.92

method	result
default	$-\frac{a^5A}{17x^{17}}-\frac{a^4(5Ab+Ba)}{14x^{14}}-\frac{5a^3b(2Ab+Ba)}{11x^{11}}-\frac{5a^2b^2(Ab+Ba)}{4x^8}-\frac{a\,b^3(Ab+2Ba)}{x^5}-\frac{b^4(Ab+5Ba)}{2x^2}+b^5Bx$
risch	$b^5Bx+\frac{-\frac{a^5A}{17}+(-\frac{5}{14}a^4bA-\frac{1}{14}a^5B)x^3+(-\frac{10}{11}a^3b^2A-\frac{5}{11}a^4bB)x^6+(-\frac{5}{4}a^2b^3A-\frac{5}{4}a^3b^2B)x^9+(-a\,b^4A-2a^2b^3B)x^{12}+(-}{x^{17}}$
norman	$\frac{-\frac{a^5A}{17}+(-\frac{5}{14}a^4bA-\frac{1}{14}a^5B)x^3+(-\frac{10}{11}a^3b^2A-\frac{5}{11}a^4bB)x^6+(-\frac{5}{4}a^2b^3A-\frac{5}{4}a^3b^2B)x^9+(-a\,b^4A-2a^2b^3B)x^{12}+(-\frac{1}{2}b^5A-\frac{5}{2}a\,b}{x^{17}}$
gosper	$-\frac{-5236b^5B\,x^{18}+2618A\,b^5x^{15}+13090Ba\,b^4x^{15}+5236aA\,b^4x^{12}+10472B\,a^2b^3x^{12}+6545a^2A\,b^3x^9+6545B\,a^3b^2x^9+4760a^3A\,b}{5236x^{17}}$
parallelrisch	$-\frac{-5236b^5B\,x^{18}+2618A\,b^5x^{15}+13090Ba\,b^4x^{15}+5236aA\,b^4x^{12}+10472B\,a^2b^3x^{12}+6545a^2A\,b^3x^9+6545B\,a^3b^2x^9+4760a^3A\,b}{5236x^{17}}$

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^18,x,method=_RETURNVERBOSE)
[Out] -1/17*a^5*A/x^17-1/14*a^4*(5*A*b+B*a)/x^14-5/11*a^3*b*(2*A*b+B*a)/x^11-5/4*
a^2*b^2*(A*b+B*a)/x^8-a*b^3*(A*b+2*B*a)/x^5-1/2*b^4*(A*b+5*B*a)/x^2+b^5*B*x
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x^{18}}\,dx = \frac{5236\,Bb^5x^{18}-2618\,(5\,Bab^4+Ab^5)x^{15}-5236\,(2\,Ba^2b^3+Aab^4)x^{12}-6545\,(Ba^3b^2+Aa^2b^3)x^9-2380\,(B}{5236\,x^{17}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^18,x, algorithm="fricas")
[Out] 1/5236*(5236*B*b^5*x^18 - 2618*(5*B*a*b^4 + A*b^5)*x^15 - 5236*(2*B*a^2*b^3
 + A*a*b^4)*x^12 - 6545*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 2380*(B*a^4*b + 2*A*a
^3*b^2)*x^6 - 308*A*a^5 - 374*(B*a^5 + 5*A*a^4*b)*x^3)/x^17
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x^{18}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**18,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{18}}\,dx = Bb^5x - \frac{2618\,(5\,Bab^4+Ab^5)x^{15}+5236\,(2\,Ba^2b^3+Aab^4)x^{12}+6545\,(Ba^3b^2+Aa^2b^3)x^9+2380\,(Ba^4b+2\,Aa^3b^2)}{5236\,x^{17}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^18,x, algorithm="maxima")
```

```
[Out] B*b^5*x - 1/5236*(2618*(5*B*a*b^4 + A*b^5)*x^15 + 5236*(2*B*a^2*b^3 + A*a*b
^4)*x^12 + 6545*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 2380*(B*a^4*b + 2*A*a^3*b^2)*
x^6 + 308*A*a^5 + 374*(B*a^5 + 5*A*a^4*b)*x^3)/x^17
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{18}}\,dx = Bb^5x - \frac{13090\,Bab^4x^{15}+2618\,Ab^5x^{15}+10472\,Ba^2b^3x^{12}+5236\,Aab^4x^{12}+6545\,Ba^3b^2x^9+6545\,Aa^2b^3x^9+2380}{5236\,x^{17}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^18,x, algorithm="giac")
```

```
[Out] B*b^5*x - 1/5236*(13090*B*a*b^4*x^15 + 2618*A*b^5*x^15 + 10472*B*a^2*b^3*x^
12 + 5236*A*a*b^4*x^12 + 6545*B*a^3*b^2*x^9 + 6545*A*a^2*b^3*x^9 + 2380*B*a
^4*b*x^6 + 4760*A*a^3*b^2*x^6 + 374*B*a^5*x^3 + 1870*A*a^4*b*x^3 + 308*A*a^
5)/x^17
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{18}}\,dx = B\,b^5\,x - \frac{\frac{A\,a^5}{17}+x^{12}\,(2\,B\,a^2\,b^3+A\,a\,b^4)+x^6\left(\frac{5\,B\,a^4\,b}{11}+\frac{10\,A\,a^3\,b^2}{11}\right)+x^3\left(\frac{B\,a^5}{14}+\frac{5\,A\,b\,a^4}{14}\right)+x^{15}\left(\frac{A\,b^5}{2}+\frac{5\,B\,a\,b^4}{2}\right)+x^9\,(}{x^{17}}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^18,x)
```

```
[Out] B*b^5*x - ((A*a^5)/17 + x^12*(2*B*a^2*b^3 + A*a*b^4) + x^6*((10*A*a^3*b^2)/
11 + (5*B*a^4*b)/11) + x^3*((B*a^5)/14 + (5*A*a^4*b)/14) + x^15*((A*b^5)/2
+ (5*B*a*b^4)/2) + x^9*((5*A*a^2*b^3)/4 + (5*B*a^3*b^2)/4))/x^17
```

3.51 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{19}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 91

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{19}}\,dx = -\frac{a^5B}{15x^{15}} - \frac{5a^4bB}{12x^{12}} - \frac{10a^3b^2B}{9x^9} - \frac{5a^2b^3B}{3x^6} - \frac{5ab^4B}{3x^3} - \frac{A(a+bx^3)^6}{18ax^{18}} + b^5B\log(x)$$

```
[Out] -1/15*a^5*B/x^15-5/12*a^4*b*B/x^12-10/9*a^3*b^2*B/x^9-5/3*a^2*b^3*B/x^6-5/3
*a*b^4*B/x^3-1/18*A*(b*x^3+a)^6/a/x^18+b^5*B*ln(x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {457, 79, 45}

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{19}}\,dx = -\frac{a^5B}{15x^{15}} - \frac{5a^4bB}{12x^{12}} - \frac{10a^3b^2B}{9x^9} - \frac{5a^2b^3B}{3x^6} - \frac{A(a+bx^3)^6}{18ax^{18}} - \frac{5ab^4B}{3x^3} + b^5B\log(x)$$

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^19,x]
```

```
[Out] -1/15*(a^5*B)/x^15 - (5*a^4*b*B)/(12*x^12) - (10*a^3*b^2*B)/(9*x^9) - (5*a^
2*b^3*B)/(3*x^6) - (5*a*b^4*B)/(3*x^3) - (A*(a + b*x^3)^6)/(18*a*x^18) + b^
5*B*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^7}\,dx,x,x^3\right)\\
&= -\frac{A(a+bx^3)^6}{18ax^{18}}+\frac{1}{3}B\text{Subst}\left(\int \frac{(a+bx)^5}{x^6}\,dx,x,x^3\right)\\
&= -\frac{A(a+bx^3)^6}{18ax^{18}}+\frac{1}{3}B\text{Subst}\left(\int \left(\frac{a^5}{x^6}+\frac{5a^4b}{x^5}+\frac{10a^3b^2}{x^4}+\frac{10a^2b^3}{x^3}+\frac{5ab^4}{x^2}+\frac{b^5}{x}\right)\,dx,x,x^3\right)\\
&= -\frac{a^5B}{15x^{15}}-\frac{5a^4bB}{12x^{12}}-\frac{10a^3b^2B}{9x^9}-\frac{5a^2b^3B}{3x^6}-\frac{5ab^4B}{3x^3}-\frac{A(a+bx^3)^6}{18ax^{18}}+b^5B\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x^{19}}\,dx = -\frac{60Ab^5x^{15}+150ab^4x^{12}(A+2Bx^3)+100a^2b^3x^9(2A+3Bx^3)+50a^3b^2x^6(3A+4Bx^3)+15a^4bx^3(4A+5}{180x^{18}}$$

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^19,x]
```

[Out] -1/180*(60*A*b^5*x^15 + 150*a*b^4*x^12*(A + 2*B*x^3) + 100*a^2*b^3*x^9*(2*A + 3*B*x^3) + 50*a^3*b^2*x^6*(3*A + 4*B*x^3) + 15*a^4*b*x^3*(4*A + 5*B*x^3) + 2*a^5*(5*A + 6*B*x^3) - 180*b^5*B*x^18*Log[x])/x^18

Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.12

method	result
default	$b^5 B \ln(x) - \frac{5a\, b^3(Ab+2Ba)}{6x^6} - \frac{5a^3 b(2Ab+Ba)}{12x^{12}} - \frac{a^4(5Ab+Ba)}{15x^{15}} - \frac{a^5 A}{18x^{18}} - \frac{b^4(Ab+5Ba)}{3x^3} - \frac{10a^2 b^2(Ab+Ba)}{9x^9}$
norman	$\frac{(-\frac{1}{3}b^5 A-\frac{5}{3}a\, b^4 B)x^{15}+(-\frac{5}{6}a\, b^4 A-\frac{5}{3}a^2 b^3 B)x^{12}+(-\frac{10}{9}a^2 b^3 A-\frac{10}{9}a^3 b^2 B)x^9+(-\frac{5}{6}a^3 b^2 A-\frac{5}{12}a^4 bB)x^6+(-\frac{1}{3}a^4 bA-\frac{1}{15}a^5 B)x}{x^{18}}$
risch	$\frac{(-\frac{1}{3}b^5 A-\frac{5}{3}a\, b^4 B)x^{15}+(-\frac{5}{6}a\, b^4 A-\frac{5}{3}a^2 b^3 B)x^{12}+(-\frac{10}{9}a^2 b^3 A-\frac{10}{9}a^3 b^2 B)x^9+(-\frac{5}{6}a^3 b^2 A-\frac{5}{12}a^4 bB)x^6+(-\frac{1}{3}a^4 bA-\frac{1}{15}a^5 B)x}{x^{18}}$
parallelrisch	$-\frac{-180b^5 B\ln(x)x^{18}+60A\, b^5 x^{15}+300Ba\, b^4 x^{15}+150aA\, b^4 x^{12}+300B\, a^2 b^3 x^{12}+200a^2 A\, b^3 x^9+200B\, a^3 b^2 x^9+150a^3 A\, b^2 x^6+75}{180x^{18}}$

[In] int((b*x^3+a)^5*(B*x^3+A)/x^19,x,method=_RETURNVERBOSE)

[Out] b^5*B*ln(x)-5/6*a*b^3*(A*b+2*B*a)/x^6-5/12*a^3*b*(2*A*b+B*a)/x^12-1/15*a^4*(5*A*b+B*a)/x^15-1/18*a^5*A/x^18-1/3*b^4*(A*b+5*B*a)/x^3-10/9*a^2*b^2*(A*b+B*a)/x^9

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{19}}\, dx = \frac{180\, Bb^5 x^{18}\log(x) - 60\,(5\, Bab^4 + Ab^5)x^{15} - 150\,(2\, Ba^2 b^3 + Aab^4)x^{12} - 200\,(Ba^3 b^2 + Aa^2 b^3)x^9 - 75\,(Ba}{180\, x^{18}}$$

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^19,x, algorithm="fricas")

[Out] 1/180*(180*B*b^5*x^18*log(x) - 60*(5*B*a*b^4 + A*b^5)*x^15 - 150*(2*B*a^2*b^3 + A*a*b^4)*x^12 - 200*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 75*(B*a^4*b + 2*A*a^3*b^2)*x^6 - 10*A*a^5 - 12*(B*a^5 + 5*A*a^4*b)*x^3)/x^18

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{19}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**19,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{19}}\,dx = \frac{1}{3}\,Bb^5\log\left(x^3\right) - \frac{60\,(5\,Bab^4 + Ab^5)x^{15} + 150\,(2\,Ba^2b^3 + Aab^4)x^{12} + 200\,(Ba^3b^2 + Aa^2b^3)x^9 + 75\,(Ba^4b + 2\,Aa^3b^2)x^6 + 10}{180\,x^{18}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^19,x, algorithm="maxima")
```

```
[Out] 1/3*B*b^5*log(x^3) - 1/180*(60*(5*B*a*b^4 + A*b^5)*x^15 + 150*(2*B*a^2*b^3
+ A*a*b^4)*x^12 + 200*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 75*(B*a^4*b + 2*A*a^3*b
^2)*x^6 + 10*A*a^5 + 12*(B*a^5 + 5*A*a^4*b)*x^3)/x^18
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.49

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{19}}\,dx = Bb^5\log\left(|x|\right) - \frac{147\,Bb^5x^{18} + 300\,Bab^4x^{15} + 60\,Ab^5x^{15} + 300\,Ba^2b^3x^{12} + 150\,Aab^4x^{12} + 200\,Ba^3b^2x^9 + 200\,Aa^2b^3x^9 + 7}{180\,x^{18}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^19,x, algorithm="giac")
```

```
[Out] B*b^5*log(abs(x)) - 1/180*(147*B*b^5*x^18 + 300*B*a*b^4*x^15 + 60*A*b^5*x^1
5 + 300*B*a^2*b^3*x^12 + 150*A*a*b^4*x^12 + 200*B*a^3*b^2*x^9 + 200*A*a^2*b
^3*x^9 + 75*B*a^4*b*x^6 + 150*A*a^3*b^2*x^6 + 12*B*a^5*x^3 + 60*A*a^4*b*x^3
 + 10*A*a^5)/x^18
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{19}} \, dx = B\, b^5 \ln(x) - \frac{\frac{A\,a^5}{18} + x^{12}\left(\frac{5\,B\,a^2\,b^3}{3} + \frac{5\,A\,a\,b^4}{6}\right) + x^6\left(\frac{5\,B\,a^4\,b}{12} + \frac{5\,A\,a^3\,b^2}{6}\right) + x^3\left(\frac{B\,a^5}{15} + \frac{A\,b\,a^4}{3}\right) + x^{15}\left(\frac{A\,b^5}{3} + \frac{5\,B\,a\,b^4}{3}\right) + x^9\left(\right.}{x^{18}}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^19,x)
```

```
[Out] B*b^5*log(x) - ((A*a^5)/18 + x^12*((5*B*a^2*b^3)/3 + (5*A*a*b^4)/6) + x^6*(
(5*A*a^3*b^2)/6 + (5*B*a^4*b)/12) + x^3*((B*a^5)/15 + (A*a^4*b)/3) + x^15*(
(A*b^5)/3 + (5*B*a*b^4)/3) + x^9*((10*A*a^2*b^3)/9 + (10*B*a^3*b^2)/9))/x^1
8
```

3.52 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{20}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{20}}\,dx = -\frac{a^5A}{19x^{19}}-\frac{a^4(5Ab+aB)}{16x^{16}}-\frac{5a^3b(2Ab+aB)}{13x^{13}}-\frac{a^2b^2(Ab+aB)}{x^{10}}-\frac{5ab^3(Ab+2aB)}{7x^7}-\frac{b^4(Ab+5aB)}{4x^4}-\frac{b^5B}{x}$$

```
[Out] -1/19*a^5*A/x^19-1/16*a^4*(5*A*b+B*a)/x^16-5/13*a^3*b*(2*A*b+B*a)/x^13-a^2*
b^2*(A*b+B*a)/x^10-5/7*a*b^3*(A*b+2*B*a)/x^7-1/4*b^4*(A*b+5*B*a)/x^4-b^5*B/
x
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{20}}\,dx = -\frac{a^5A}{19x^{19}}-\frac{a^4(aB+5Ab)}{16x^{16}}-\frac{5a^3b(aB+2Ab)}{13x^{13}}-\frac{a^2b^2(aB+Ab)}{x^{10}}-\frac{b^4(5aB+Ab)}{4x^4}-\frac{5ab^3(2aB+Ab)}{7x^7}-\frac{b^5B}{x}$$

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^20,x]
```

```
[Out] -1/19*(a^5*A)/x^19 - (a^4*(5*A*b + a*B))/(16*x^16) - (5*a^3*b*(2*A*b + a*B)
)/(13*x^13) - (a^2*b^2*(A*b + a*B))/x^10 - (5*a*b^3*(A*b + 2*a*B))/(7*x^7)
- (b^4*(A*b + 5*a*B))/(4*x^4) - (b^5*B)/x
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a^5 A}{x^{20}} + \frac{a^4(5Ab + aB)}{x^{17}} + \frac{5a^3b(2Ab + aB)}{x^{14}} + \frac{10a^2b^2(Ab + aB)}{x^{11}} + \frac{5ab^3(Ab + 2aB)}{x^8} + \frac{b^4(Ab + 5aB)}{x^5} + \frac{b^5B}{x^2}\right) dx$$

$$= -\frac{a^5A}{19x^{19}} - \frac{a^4(5Ab + aB)}{16x^{16}} - \frac{5a^3b(2Ab + aB)}{13x^{13}} - \frac{a^2b^2(Ab + aB)}{x^{10}} - \frac{5ab^3(Ab + 2aB)}{7x^7} - \frac{b^4(Ab + 5aB)}{4x^4} - \frac{b^5B}{x}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.05

$$\int \frac{(a + bx^3)^5 (A + Bx^3)}{x^{20}} dx = -\frac{6916b^5x^{15}(A + 4Bx^3) + 4940ab^4x^{12}(4A + 7Bx^3) + 3952a^2b^3x^9(7A + 10Bx^3) + 2128a^3b^2x^6(10A + 13B}{27664x^{19}}$$

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^20,x]
```

```
[Out] -1/27664*(6916*b^5*x^15*(A + 4*B*x^3) + 4940*a*b^4*x^12*(4*A + 7*B*x^3) + 3
952*a^2*b^3*x^9*(7*A + 10*B*x^3) + 2128*a^3*b^2*x^6*(10*A + 13*B*x^3) + 665
*a^4*b*x^3*(13*A + 16*B*x^3) + 91*a^5*(16*A + 19*B*x^3))/x^19
```

Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.92

method	result
default	$-\frac{a^5 A}{19x^{19}}-\frac{a^4(5Ab+Ba)}{16x^{16}}-\frac{5a^3b(2Ab+Ba)}{13x^{13}}-\frac{a^2b^2(Ab+Ba)}{x^{10}}-\frac{5a\,b^3(Ab+2Ba)}{7x^7}-\frac{b^4(Ab+5Ba)}{4x^4}-\frac{b^5B}{x}$
norman	$\frac{-\frac{a^5A}{19}+(-\frac{5}{16}a^4bA-\frac{1}{16}a^5B)x^3+(-\frac{10}{13}a^3b^2A-\frac{5}{13}a^4bB)x^6+(-a^2b^3A-a^3b^2B)x^9+(-\frac{5}{7}a\,b^4A-\frac{10}{7}a^2b^3B)x^{12}+(-\frac{1}{4}b^5A-\frac{5}{4}a\,b^4B}{x^{19}}$
risch	$\frac{-\frac{a^5A}{19}+(-\frac{5}{16}a^4bA-\frac{1}{16}a^5B)x^3+(-\frac{10}{13}a^3b^2A-\frac{5}{13}a^4bB)x^6+(-a^2b^3A-a^3b^2B)x^9+(-\frac{5}{7}a\,b^4A-\frac{10}{7}a^2b^3B)x^{12}+(-\frac{1}{4}b^5A-\frac{5}{4}a\,b^4B}{x^{19}}$
gosper	$-\frac{27664b^5B\,x^{18}+6916A\,b^5x^{15}+34580Ba\,b^4x^{15}+19760aA\,b^4x^{12}+39520B\,a^2b^3x^{12}+27664a^2A\,b^3x^9+27664B\,a^3b^2x^9+21280a^3A}{27664x^{19}}$
parallelrisch	$-\frac{27664b^5B\,x^{18}+6916A\,b^5x^{15}+34580Ba\,b^4x^{15}+19760aA\,b^4x^{12}+39520B\,a^2b^3x^{12}+27664a^2A\,b^3x^9+27664B\,a^3b^2x^9+21280a^3A}{27664x^{19}}$

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^20,x,method=_RETURNVERBOSE)
```

```
[Out] -1/19*a^5*A/x^19-1/16*a^4*(5*A*b+B*a)/x^16-5/13*a^3*b*(2*A*b+B*a)/x^13-a^2*
b^2*(A*b+B*a)/x^10-5/7*a*b^3*(A*b+2*B*a)/x^7-1/4*b^4*(A*b+5*B*a)/x^4-b^5*B/
x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{20}}\,dx = -\frac{27664\,Bb^5x^{18}+6916\,(5\,Bab^4+Ab^5)x^{15}+19760\,(2\,Ba^2b^3+Aab^4)x^{12}+27664\,(Ba^3b^2+Aa^2b^3)x^9+106}{27664\,x^{19}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^20,x, algorithm="fricas")
```

```
[Out] -1/27664*(27664*B*b^5*x^18 + 6916*(5*B*a*b^4 + A*b^5)*x^15 + 19760*(2*B*a^2
*b^3 + A*a*b^4)*x^12 + 27664*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 10640*(B*a^4*b +
 2*A*a^3*b^2)*x^6 + 1456*A*a^5 + 1729*(B*a^5 + 5*A*a^4*b)*x^3)/x^19
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{20}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**20,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{20}} dx = -\frac{27664 Bb^5x^{18} + 6916 (5 Bab^4 + Ab^5)x^{15} + 19760 (2 Ba^2b^3 + Aab^4)x^{12} + 27664 (Ba^3b^2 + Aa^2b^3)x^9 + 10\ldots}{27664 x^{19}}$$

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^20,x, algorithm="maxima")

[Out] -1/27664*(27664*B*b^5*x^18 + 6916*(5*B*a*b^4 + A*b^5)*x^15 + 19760*(2*B*a^2*b^3 + A*a*b^4)*x^12 + 27664*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 10640*(B*a^4*b + 2*A*a^3*b^2)*x^6 + 1456*A*a^5 + 1729*(B*a^5 + 5*A*a^4*b)*x^3)/x^19

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{20}} dx = -\frac{27664 Bb^5x^{18} + 34580 Bab^4x^{15} + 6916 Ab^5x^{15} + 39520 Ba^2b^3x^{12} + 19760 Aab^4x^{12} + 27664 Ba^3b^2x^9 + 2\ldots}{27664 x^{19}}$$

[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^20,x, algorithm="giac")

[Out] -1/27664*(27664*B*b^5*x^18 + 34580*B*a*b^4*x^15 + 6916*A*b^5*x^15 + 39520*B*a^2*b^3*x^12 + 19760*A*a*b^4*x^12 + 27664*B*a^3*b^2*x^9 + 27664*A*a^2*b^3*x^9 + 10640*B*a^4*b*x^6 + 21280*A*a^3*b^2*x^6 + 1729*B*a^5*x^3 + 8645*A*a^4*b*x^3 + 1456*A*a^5)/x^19

Mupad [B] (verification not implemented)

Time = 6.60 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{20}} dx = -\frac{\frac{A a^5}{19} + x^{12} \left(\frac{10 B a^2 b^3}{7} + \frac{5 A a b^4}{7}\right) + x^6 \left(\frac{5 B a^4 b}{13} + \frac{10 A a^3 b^2}{13}\right) + x^3 \left(\frac{B a^5}{16} + \frac{5 A b a^4}{16}\right) + x^{15} \left(\frac{A b^5}{4} + \frac{5 B a b^4}{4}\right) + x^9\ldots}{x^{19}}$$

[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^20,x)

[Out] -((A*a^5)/19 + x^12*((10*B*a^2*b^3)/7 + (5*A*a*b^4)/7) + x^6*((10*A*a^3*b^2)/13 + (5*B*a^4*b)/13) + x^3*((B*a^5)/16 + (5*A*a^4*b)/16) + x^15*((A*b^5)/4 + (5*B*a*b^4)/4) + x^9*(A*a^2*b^3 + B*a^3*b^2) + B*b^5*x^18)/x^19

3.53 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{21}} dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{21}} dx = -\frac{a^5A}{20x^{20}} - \frac{a^4(5Ab+aB)}{17x^{17}} - \frac{5a^3b(2Ab+aB)}{14x^{14}} - \frac{10a^2b^2(Ab+aB)}{11x^{11}} - \frac{5ab^3(Ab+2aB)}{8x^8} - \frac{b^4(Ab+5aB)}{5x^5} - \frac{b^5B}{2x^2}$$

```
[Out] -1/20*a^5*A/x^20-1/17*a^4*(5*A*b+B*a)/x^17-5/14*a^3*b*(2*A*b+B*a)/x^14-10/1
1*a^2*b^2*(A*b+B*a)/x^11-5/8*a*b^3*(A*b+2*B*a)/x^8-1/5*b^4*(A*b+5*B*a)/x^5-
1/2*b^5*B/x^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{21}} dx = -\frac{a^5A}{20x^{20}} - \frac{a^4(aB+5Ab)}{17x^{17}} - \frac{5a^3b(aB+2Ab)}{14x^{14}} - \frac{10a^2b^2(aB+Ab)}{11x^{11}} - \frac{b^4(5aB+Ab)}{5x^5} - \frac{5ab^3(2aB+Ab)}{8x^8} - \frac{b^5B}{2x^2}$$

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^21,x]
```

```
[Out] -1/20*(a^5*A)/x^20 - (a^4*(5*A*b + a*B))/(17*x^17) - (5*a^3*b*(2*A*b + a*B)
)/(14*x^14) - (10*a^2*b^2*(A*b + a*B))/(11*x^11) - (5*a*b^3*(A*b + 2*a*B))/
(8*x^8) - (b^4*(A*b + 5*a*B))/(5*x^5) - (b^5*B)/(2*x^2)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a^5A}{x^{21}} + \frac{a^4(5Ab+aB)}{x^{18}} + \frac{5a^3b(2Ab+aB)}{x^{15}} + \frac{10a^2b^2(Ab+aB)}{x^{12}} + \frac{5ab^3(Ab+2aB)}{x^9} + \frac{b^4(Ab+5aB)}{x^6} + \frac{b^5B}{x^3}\right) dx$$

$$= -\frac{a^5A}{20x^{20}} - \frac{a^4(5Ab+aB)}{17x^{17}} - \frac{5a^3b(2Ab+aB)}{14x^{14}} - \frac{10a^2b^2(Ab+aB)}{11x^{11}} - \frac{5ab^3(Ab+2aB)}{8x^8} - \frac{b^4(Ab+5aB)}{5x^5} - \frac{b^5B}{2x^2}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{21}} dx = -\frac{5236b^5x^{15}(2A+5Bx^3)+6545ab^4x^{12}(5A+8Bx^3)+5950a^2b^3x^9(8A+11Bx^3)+3400a^3b^2x^6(11A+14}{52360x^{20}}$$

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^21,x]
```

```
[Out] -1/52360*(5236*b^5*x^15*(2*A + 5*B*x^3) + 6545*a*b^4*x^12*(5*A + 8*B*x^3) +
 5950*a^2*b^3*x^9*(8*A + 11*B*x^3) + 3400*a^3*b^2*x^6*(11*A + 14*B*x^3) + 1
100*a^4*b*x^3*(14*A + 17*B*x^3) + 154*a^5*(17*A + 20*B*x^3))/x^20
```

Maple [A] (verified)

Time = 3.96 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a^5A}{20x^{20}}-\frac{a^4(5Ab+Ba)}{17x^{17}}-\frac{5a^3b(2Ab+Ba)}{14x^{14}}-\frac{10a^2b^2(Ab+Ba)}{11x^{11}}-\frac{5a\,b^3(Ab+2Ba)}{8x^8}-\frac{b^4(Ab+5Ba)}{5x^5}-\frac{b^5B}{2x^2}$
norman	$\frac{-\frac{a^5A}{20}+(-\frac{5}{17}a^4bA-\frac{1}{17}a^5B)x^3+(-\frac{5}{7}a^3b^2A-\frac{5}{14}a^4bB)x^6+(-\frac{10}{11}a^2b^3A-\frac{10}{11}a^3b^2B)x^9+(-\frac{5}{8}a\,b^4A-\frac{5}{4}a^2b^3B)x^{12}+(-\frac{1}{5}b^5A-a\,b^4}{x^{20}}$
risch	$\frac{-\frac{a^5A}{20}+(-\frac{5}{17}a^4bA-\frac{1}{17}a^5B)x^3+(-\frac{5}{7}a^3b^2A-\frac{5}{14}a^4bB)x^6+(-\frac{10}{11}a^2b^3A-\frac{10}{11}a^3b^2B)x^9+(-\frac{5}{8}a\,b^4A-\frac{5}{4}a^2b^3B)x^{12}+(-\frac{1}{5}b^5A-a\,b^4}{x^{20}}$
gosper	$-\frac{26180b^5B\,x^{18}+10472A\,b^5x^{15}+52360Ba\,b^4x^{15}+32725aA\,b^4x^{12}+65450B\,a^2b^3x^{12}+47600a^2A\,b^3x^9+47600B\,a^3b^2x^9+37400a^3}{52360x^{20}}$
parallelrisch	$-\frac{26180b^5B\,x^{18}+10472A\,b^5x^{15}+52360Ba\,b^4x^{15}+32725aA\,b^4x^{12}+65450B\,a^2b^3x^{12}+47600a^2A\,b^3x^9+47600B\,a^3b^2x^9+37400a^3}{52360x^{20}}$

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^21,x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*a^5*A/x^20-1/17*a^4*(5*A*b+B*a)/x^17-5/14*a^3*b*(2*A*b+B*a)/x^14-10/1
1*a^2*b^2*(A*b+B*a)/x^11-5/8*a*b^3*(A*b+2*B*a)/x^8-1/5*b^4*(A*b+5*B*a)/x^5-
1/2*b^5*B/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x^{21}}\,dx = -\frac{26180\,Bb^5x^{18}+10472\,(5\,Bab^4+Ab^5)x^{15}+32725\,(2\,Ba^2b^3+Aab^4)x^{12}+47600\,(Ba^3b^2+Aa^2b^3)x^9+18}{52360\,x^{20}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^21,x, algorithm="fricas")
```

```
[Out] -1/52360*(26180*B*b^5*x^18 + 10472*(5*B*a*b^4 + A*b^5)*x^15 + 32725*(2*B*a^
2*b^3 + A*a*b^4)*x^12 + 47600*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 18700*(B*a^4*b
+ 2*A*a^3*b^2)*x^6 + 2618*A*a^5 + 3080*(B*a^5 + 5*A*a^4*b)*x^3)/x^20
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^5\,(A+Bx^3)}{x^{21}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**21,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{21}}\,dx = -\frac{26180\,Bb^5x^{18}+10472\,(5\,Bab^4+Ab^5)x^{15}+32725\,(2\,Ba^2b^3+Aab^4)x^{12}+47600\,(Ba^3b^2+Aa^2b^3)x^9+1}{52360\,x^{20}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^21,x, algorithm="maxima")
```

```
[Out] -1/52360*(26180*B*b^5*x^18 + 10472*(5*B*a*b^4 + A*b^5)*x^15 + 32725*(2*B*a^
2*b^3 + A*a*b^4)*x^12 + 47600*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 18700*(B*a^4*b
+ 2*A*a^3*b^2)*x^6 + 2618*A*a^5 + 3080*(B*a^5 + 5*A*a^4*b)*x^3)/x^20
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{21}}\,dx = -\frac{26180\,Bb^5x^{18}+52360\,Bab^4x^{15}+10472\,Ab^5x^{15}+65450\,Ba^2b^3x^{12}+32725\,Aab^4x^{12}+47600\,Ba^3b^2x^9+}{52360\,x^{20}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^21,x, algorithm="giac")
```

```
[Out] -1/52360*(26180*B*b^5*x^18 + 52360*B*a*b^4*x^15 + 10472*A*b^5*x^15 + 65450*
B*a^2*b^3*x^12 + 32725*A*a*b^4*x^12 + 47600*B*a^3*b^2*x^9 + 47600*A*a^2*b^3
*x^9 + 18700*B*a^4*b*x^6 + 37400*A*a^3*b^2*x^6 + 3080*B*a^5*x^3 + 15400*A*a
^4*b*x^3 + 2618*A*a^5)/x^20
```

Mupad [B] (verification not implemented)

Time = 6.74 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{21}}\,dx = -\frac{\frac{A\,a^5}{20}+x^{12}\left(\frac{5\,B\,a^2\,b^3}{4}+\frac{5\,A\,a\,b^4}{8}\right)+x^6\left(\frac{5\,B\,a^4\,b}{14}+\frac{5\,A\,a^3\,b^2}{7}\right)+x^3\left(\frac{B\,a^5}{17}+\frac{5\,A\,b\,a^4}{17}\right)+x^{15}\left(\frac{A\,b^5}{5}+B\,a\,b^4\right)+x^9\,(}{x^{20}}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^21,x)
```

```
[Out] -((A*a^5)/20 + x^12*((5*B*a^2*b^3)/4 + (5*A*a*b^4)/8) + x^6*((5*A*a^3*b^2)/
7 + (5*B*a^4*b)/14) + x^3*((B*a^5)/17 + (5*A*a^4*b)/17) + x^15*((A*b^5)/5 +
 B*a*b^4) + x^9*((10*A*a^2*b^3)/11 + (10*B*a^3*b^2)/11) + (B*b^5*x^18)/2)/x
^20
```

3.54 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{22}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 48

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{22}}\,dx = -\frac{A(a+bx^3)^6}{21ax^{21}} + \frac{(Ab-7aB)(a+bx^3)^6}{126a^2x^{18}}$$

```
[Out] -1/21*A*(b*x^3+a)^6/a/x^21+1/126*(A*b-7*B*a)*(b*x^3+a)^6/a^2/x^18
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {457, 79, 37}

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{22}}\,dx = \frac{(a+bx^3)^6(Ab-7aB)}{126a^2x^{18}} - \frac{A(a+bx^3)^6}{21ax^{21}}$$

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^22,x]
```

```
[Out] -1/21*(A*(a + b*x^3)^6)/(a*x^21) + ((A*b - 7*a*B)*(a + b*x^3)^6)/(126*a^2*x
^18)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^8}\,dx, x, x^3\right)\\
&= -\frac{A(a+bx^3)^6}{21ax^{21}} + \frac{(-Ab+7aB)\text{Subst}\left(\int \frac{(a+bx)^5}{x^7}\,dx, x, x^3\right)}{21a}\\
&= -\frac{A(a+bx^3)^6}{21ax^{21}} + \frac{(Ab-7aB)\left(a+bx^3\right)^6}{126a^2x^{18}}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 118 vs. 2(48) = 96.

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.46

$$\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{22}}\,dx = -\frac{21b^5x^{15}(A+2Bx^3)+35ab^4x^{12}(2A+3Bx^3)+35a^2b^3x^9(3A+4Bx^3)+21a^3b^2x^6(4A+5Bx^3)+7a^4bx^3(}{126x^{21}}$$

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^22,x]
[Out] -1/126*(21*b^5*x^15*(A + 2*B*x^3) + 35*a*b^4*x^12*(2*A + 3*B*x^3) + 35*a^2*
b^3*x^9*(3*A + 4*B*x^3) + 21*a^3*b^2*x^6*(4*A + 5*B*x^3) + 7*a^4*b*x^3*(5*A
 + 6*B*x^3) + a^5*(6*A + 7*B*x^3))/x^21
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(44) = 88$.

Time = 4.14 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.17

method	result
default	$-\frac{b^4(Ab+5Ba)}{6x^6}-\frac{5a^2b^2(Ab+Ba)}{6x^{12}}-\frac{a^5A}{21x^{21}}-\frac{a^3b(2Ab+Ba)}{3x^{15}}-\frac{a^4(5Ab+Ba)}{18x^{18}}-\frac{b^5B}{3x^3}-\frac{5a\,b^3(Ab+2Ba)}{9x^9}$
norman	$\frac{-\frac{a^5A}{21}+(-\frac{5}{18}a^4bA-\frac{1}{18}a^5B)x^3+(-\frac{2}{3}a^3b^2A-\frac{1}{3}a^4bB)x^6+(-\frac{5}{6}a^2b^3A-\frac{5}{6}a^3b^2B)x^9+(-\frac{5}{9}a\,b^4A-\frac{10}{9}a^2b^3B)x^{12}+(-\frac{1}{6}b^5A-\frac{5}{6}a}{x^{21}}$
risch	$\frac{-\frac{a^5A}{21}+(-\frac{5}{18}a^4bA-\frac{1}{18}a^5B)x^3+(-\frac{2}{3}a^3b^2A-\frac{1}{3}a^4bB)x^6+(-\frac{5}{6}a^2b^3A-\frac{5}{6}a^3b^2B)x^9+(-\frac{5}{9}a\,b^4A-\frac{10}{9}a^2b^3B)x^{12}+(-\frac{1}{6}b^5A-\frac{5}{6}a}{x^{21}}$
gosper	$-\frac{42b^5B\,x^{18}+21A\,b^5x^{15}+105Ba\,b^4x^{15}+70aA\,b^4x^{12}+140B\,a^2b^3x^{12}+105a^2A\,b^3x^9+105B\,a^3b^2x^9+84a^3A\,b^2x^6+42B\,a^4b\,x^6+}{126x^{21}}$
parallelrisch	$-\frac{42b^5B\,x^{18}+21A\,b^5x^{15}+105Ba\,b^4x^{15}+70aA\,b^4x^{12}+140B\,a^2b^3x^{12}+105a^2A\,b^3x^9+105B\,a^3b^2x^9+84a^3A\,b^2x^6+42B\,a^4b\,x^6+}{126x^{21}}$

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^22,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*b^4*(A*b+5*B*a)/x^6-5/6*a^2*b^2*(A*b+B*a)/x^12-1/21*a^5*A/x^21-1/3*a^3
*b*(2*A*b+B*a)/x^15-1/18*a^4*(5*A*b+B*a)/x^18-1/3*b^5*B/x^3-5/9*a*b^3*(A*b+
2*B*a)/x^9
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(45) = 90$.

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.52

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{22}}\,dx = -\frac{42\,Bb^5x^{18}+21\,(5\,Bab^4+Ab^5)x^{15}+70\,(2\,Ba^2b^3+Aab^4)x^{12}+105\,(Ba^3b^2+Aa^2b^3)x^9+42\,(Ba^4b+2\,A}{126\,x^{21}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^22,x, algorithm="fricas")
```

```
[Out] -1/126*(42*B*b^5*x^18 + 21*(5*B*a*b^4 + A*b^5)*x^15 + 70*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 105*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 42*(B*a^4*b + 2*A*a^3*b^2)*x
^6 + 6*A*a^5 + 7*(B*a^5 + 5*A*a^4*b)*x^3)/x^21
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{22}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**22,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(45) = 90.

Time = 0.22 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.52

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{22}}\,dx = -\frac{42\,Bb^5x^{18} + 21\,(5\,Bab^4 + Ab^5)x^{15} + 70\,(2\,Ba^2b^3 + Aab^4)x^{12} + 105\,(Ba^3b^2 + Aa^2b^3)x^9 + 42\,(Ba^4b + 2\,A}{126\,x^{21}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^22,x, algorithm="maxima")
[Out] -1/126*(42*B*b^5*x^18 + 21*(5*B*a*b^4 + A*b^5)*x^15 + 70*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 105*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 42*(B*a^4*b + 2*A*a^3*b^2)*x
^6 + 6*A*a^5 + 7*(B*a^5 + 5*A*a^4*b)*x^3)/x^21
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(45) = 90.

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.65

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{22}}\,dx = -\frac{42\,Bb^5x^{18} + 105\,Bab^4x^{15} + 21\,Ab^5x^{15} + 140\,Ba^2b^3x^{12} + 70\,Aab^4x^{12} + 105\,Ba^3b^2x^9 + 105\,Aa^2b^3x^9 + 42\,I}{126\,x^{21}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^22,x, algorithm="giac")
[Out] -1/126*(42*B*b^5*x^18 + 105*B*a*b^4*x^15 + 21*A*b^5*x^15 + 140*B*a^2*b^3*x^
12 + 70*A*a*b^4*x^12 + 105*B*a^3*b^2*x^9 + 105*A*a^2*b^3*x^9 + 42*B*a^4*b*x
^6 + 84*A*a^3*b^2*x^6 + 7*B*a^5*x^3 + 35*A*a^4*b*x^3 + 6*A*a^5)/x^21
```

Mupad [B] (verification not implemented)

Time = 6.77 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.54

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{22}} \, dx = -\frac{\frac{A a^5}{21} + x^6 \left(\frac{B a^4 b}{3} + \frac{2 A a^3 b^2}{3}\right) + x^{12} \left(\frac{10 B a^2 b^3}{9} + \frac{5 A a b^4}{9}\right) + x^3 \left(\frac{B a^5}{18} + \frac{5 A b a^4}{18}\right) + x^{15} \left(\frac{A b^5}{6} + \frac{5 B a b^4}{6}\right) + x^9 \left(\right.}{x^{21}}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^22,x)
```

```
[Out] -((A*a^5)/21 + x^6*((2*A*a^3*b^2)/3 + (B*a^4*b)/3) + x^12*((10*B*a^2*b^3)/9
 + (5*A*a*b^4)/9) + x^3*((B*a^5)/18 + (5*A*a^4*b)/18) + x^15*((A*b^5)/6 + (
5*B*a*b^4)/6) + x^9*((5*A*a^2*b^3)/6 + (5*B*a^3*b^2)/6) + (B*b^5*x^18)/3)/x
^21
```

3.55 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{23}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{23}}\,dx = -\frac{a^5A}{22x^{22}} - \frac{a^4(5Ab+aB)}{19x^{19}} - \frac{5a^3b(2Ab+aB)}{16x^{16}} - \frac{10a^2b^2(Ab+aB)}{13x^{13}} - \frac{ab^3(Ab+2aB)}{2x^{10}} - \frac{b^4(Ab+5aB)}{7x^7} - \frac{b^5B}{4x^4}$$

```
[Out] -1/22*a^5*A/x^22-1/19*a^4*(5*A*b+B*a)/x^19-5/16*a^3*b*(2*A*b+B*a)/x^16-10/1
3*a^2*b^2*(A*b+B*a)/x^13-1/2*a*b^3*(A*b+2*B*a)/x^10-1/7*b^4*(A*b+5*B*a)/x^7
-1/4*b^5*B/x^4
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{23}}\,dx = -\frac{a^5A}{22x^{22}} - \frac{a^4(aB+5Ab)}{19x^{19}} - \frac{5a^3b(aB+2Ab)}{16x^{16}} - \frac{10a^2b^2(aB+Ab)}{13x^{13}} - \frac{b^4(5aB+Ab)}{7x^7} - \frac{ab^3(2aB+Ab)}{2x^{10}} - \frac{b^5B}{4x^4}$$

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^23,x]
```

```
[Out] -1/22*(a^5*A)/x^22 - (a^4*(5*A*b + a*B))/(19*x^19) - (5*a^3*b*(2*A*b + a*B)
)/(16*x^16) - (10*a^2*b^2*(A*b + a*B))/(13*x^13) - (a*b^3*(A*b + 2*a*B))/(2
*x^10) - (b^4*(A*b + 5*a*B))/(7*x^7) - (b^5*B)/(4*x^4)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a^5 A}{x^{23}} + \frac{a^4(5Ab+aB)}{x^{20}} + \frac{5a^3b(2Ab+aB)}{x^{17}} + \frac{10a^2b^2(Ab+aB)}{x^{14}} + \frac{5ab^3(Ab+2aB)}{x^{11}} + \frac{b^4(Ab+5aB)}{x^8} + \frac{b^5 B}{x^5}\right) dx$$

$$= -\frac{a^5 A}{22x^{22}} - \frac{a^4(5Ab+aB)}{19x^{19}} - \frac{5a^3b(2Ab+aB)}{16x^{16}} - \frac{10a^2b^2(Ab+aB)}{13x^{13}} - \frac{ab^3(Ab+2aB)}{2x^{10}} - \frac{b^4(Ab+5aB)}{7x^7} - \frac{b^5 B}{4x^4}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{23}} dx = -\frac{a^5 A}{22x^{22}} - \frac{a^4(5Ab+aB)}{19x^{19}} - \frac{5a^3b(2Ab+aB)}{16x^{16}} - \frac{10a^2b^2(Ab+aB)}{13x^{13}} - \frac{ab^3(Ab+2aB)}{2x^{10}} - \frac{b^4(Ab+5aB)}{7x^7} - \frac{b^5 B}{4x^4}$$

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^23,x]
```

```
[Out] -1/22*(a^5*A)/x^22 - (a^4*(5*A*b + a*B))/(19*x^19) - (5*a^3*b*(2*A*b + a*B)
)/(16*x^16) - (10*a^2*b^2*(A*b + a*B))/(13*x^13) - (a*b^3*(A*b + 2*a*B))/(2
*x^10) - (b^4*(A*b + 5*a*B))/(7*x^7) - (b^5*B)/(4*x^4)
```

Maple [A] (verified)

Time = 4.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a^5A}{22x^{22}}-\frac{a^4(5Ab+Ba)}{19x^{19}}-\frac{5a^3b(2Ab+Ba)}{16x^{16}}-\frac{10a^2b^2(Ab+Ba)}{13x^{13}}-\frac{a\,b^3(Ab+2Ba)}{2x^{10}}-\frac{b^4(Ab+5Ba)}{7x^7}-\frac{b^5B}{4x^4}$
norman	$\frac{-\frac{a^5A}{22}+(-\frac{5}{19}a^4bA-\frac{1}{19}a^5B)x^3+(-\frac{5}{8}a^3b^2A-\frac{5}{16}a^4bB)x^6+(-\frac{10}{13}a^2b^3A-\frac{10}{13}a^3b^2B)x^9+(-\frac{1}{2}a\,b^4A-a^2b^3B)x^{12}+(-\frac{1}{7}b^5A-\frac{5}{7}a\,b^4}{x^{22}}$
risch	$\frac{-\frac{a^5A}{22}+(-\frac{5}{19}a^4bA-\frac{1}{19}a^5B)x^3+(-\frac{5}{8}a^3b^2A-\frac{5}{16}a^4bB)x^6+(-\frac{10}{13}a^2b^3A-\frac{10}{13}a^3b^2B)x^9+(-\frac{1}{2}a\,b^4A-a^2b^3B)x^{12}+(-\frac{1}{7}b^5A-\frac{5}{7}a\,b^4}{x^{22}}$
gosper	$-\frac{76076b^5B\,x^{18}+43472A\,b^5x^{15}+217360Ba\,b^4x^{15}+152152aA\,b^4x^{12}+304304B\,a^2b^3x^{12}+234080a^2A\,b^3x^9+234080B\,a^3b^2x^9+19}{304304x^{22}}$
parallelrisch	$-\frac{76076b^5B\,x^{18}+43472A\,b^5x^{15}+217360Ba\,b^4x^{15}+152152aA\,b^4x^{12}+304304B\,a^2b^3x^{12}+234080a^2A\,b^3x^9+234080B\,a^3b^2x^9+19}{304304x^{22}}$

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^23,x,method=_RETURNVERBOSE)
```

```
[Out] -1/22*a^5*A/x^22-1/19*a^4*(5*A*b+B*a)/x^19-5/16*a^3*b*(2*A*b+B*a)/x^16-10/1
3*a^2*b^2*(A*b+B*a)/x^13-1/2*a*b^3*(A*b+2*B*a)/x^10-1/7*b^4*(A*b+5*B*a)/x^7
-1/4*b^5*B/x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{23}}\,dx = -\frac{76076\,Bb^5x^{18}+43472\,(5\,Bab^4+Ab^5)x^{15}+152152\,(2\,Ba^2b^3+Aab^4)x^{12}+234080\,(Ba^3b^2+Aa^2b^3)x^9+}{304304\,x^{22}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^23,x, algorithm="fricas")
```

```
[Out] -1/304304*(76076*B*b^5*x^18 + 43472*(5*B*a*b^4 + A*b^5)*x^15 + 152152*(2*B*
a^2*b^3 + A*a*b^4)*x^12 + 234080*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 95095*(B*a^4
*b + 2*A*a^3*b^2)*x^6 + 13832*A*a^5 + 16016*(B*a^5 + 5*A*a^4*b)*x^3)/x^22
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{23}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**23,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{23}}\,dx = -\frac{76076\,Bb^5x^{18} + 43472\,(5\,Bab^4 + Ab^5)x^{15} + 152152\,(2\,Ba^2b^3 + Aab^4)x^{12} + 234080\,(Ba^3b^2 + Aa^2b^3)x^9 + \cdots}{304304\,x^{22}}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^23,x, algorithm="maxima")
```

```
[Out] -1/304304*(76076*B*b^5*x^18 + 43472*(5*B*a*b^4 + A*b^5)*x^15 + 152152*(2*B*
a^2*b^3 + A*a*b^4)*x^12 + 234080*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 95095*(B*a^4
*b + 2*A*a^3*b^2)*x^6 + 13832*A*a^5 + 16016*(B*a^5 + 5*A*a^4*b)*x^3)/x^22
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{23}}\,dx = -\frac{76076\,Bb^5x^{18} + 217360\,Bab^4x^{15} + 43472\,Ab^5x^{15} + 304304\,Ba^2b^3x^{12} + 152152\,Aab^4x^{12} + 234080\,Ba^3b^2 \cdots}{30430\cdots}$$

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^23,x, algorithm="giac")
```

```
[Out] -1/304304*(76076*B*b^5*x^18 + 217360*B*a*b^4*x^15 + 43472*A*b^5*x^15 + 3043
04*B*a^2*b^3*x^12 + 152152*A*a*b^4*x^12 + 234080*B*a^3*b^2*x^9 + 234080*A*a
^2*b^3*x^9 + 95095*B*a^4*b*x^6 + 190190*A*a^3*b^2*x^6 + 16016*B*a^5*x^3 + 8
0080*A*a^4*b*x^3 + 13832*A*a^5)/x^22
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{23}}\,dx = -\frac{\frac{A\,a^5}{22} + x^{12}\left(B\,a^2\,b^3 + \frac{A\,a\,b^4}{2}\right) + x^6\left(\frac{5\,B\,a^4\,b}{16} + \frac{5\,A\,a^3\,b^2}{8}\right) + x^3\left(\frac{B\,a^5}{19} + \frac{5\,A\,b\,a^4}{19}\right) + x^{15}\left(\frac{A\,b^5}{7} + \frac{5\,B\,a\,b^4}{7}\right) + x^9\left(\cdots\right.}{x^{22}}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^23,x)
```

```
[Out] -((A*a^5)/22 + x^12*(B*a^2*b^3 + (A*a*b^4)/2) + x^6*((5*A*a^3*b^2)/8 + (5*B
*a^4*b)/16) + x^3*((B*a^5)/19 + (5*A*a^4*b)/19) + x^15*((A*b^5)/7 + (5*B*a*
b^4)/7) + x^9*((10*A*a^2*b^3)/13 + (10*B*a^3*b^2)/13) + (B*b^5*x^18)/4)/x^2
2
```

3.56 $\int \frac{x^6(A+Bx^3)}{a+bx^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 183

$$\int \frac{x^6(A+Bx^3)}{a+bx^3} \, dx = -\frac{a(Ab-aB)x}{b^3}+\frac{(Ab-aB)x^4}{4b^2}+\frac{Bx^7}{7b} - \frac{a^{4/3}(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{10/3}} + \frac{a^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{10/3}} - \frac{a^{4/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{10/3}}$$

```
[Out] -a*(A*b-B*a)*x/b^3+1/4*(A*b-B*a)*x^4/b^2+1/7*B*x^7/b+1/3*a^(4/3)*(A*b-B*a)*
ln(a^(1/3)+b^(1/3)*x)/b^(10/3)-1/6*a^(4/3)*(A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/b^(10/3)-1/3*a^(4/3)*(A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^
(1/3)*x)/a^(1/3)*3^(1/2))/b^(10/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used

= {470, 308, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^6(A+Bx^3)}{a+bx^3}\,dx = -\frac{a^{4/3}(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{10/3}} - \frac{a^{4/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{10/3}} + \frac{a^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{10/3}} - \frac{ax(Ab-aB)}{b^3} + \frac{x^4(Ab-aB)}{4b^2} + \frac{Bx^7}{7b}$$

```
[In] Int[(x^6*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] -((a*(A*b - a*B)*x)/b^3) + ((A*b - a*B)*x^4)/(4*b^2) + (B*x^7)/(7*b) - (a^(
4/3)*(A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3
]*b^(10/3)) + (a^(4/3)*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(10/3)) -
 (a^(4/3)*(A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^
(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{Bx^7}{7b} - \frac{(-7Ab+7aB)\int \frac{x^6}{a+bx^3}\,dx}{7b} \\
&= \frac{Bx^7}{7b} - \frac{(-7Ab+7aB)\int \left(-\frac{a}{b^2}+\frac{x^3}{b}+\frac{a^2}{b^2(a+bx^3)}\right)dx}{7b} \\
&= -\frac{a(Ab-aB)x}{b^3} + \frac{(Ab-aB)x^4}{4b^2} + \frac{Bx^7}{7b} + \frac{\left(a^2(Ab-aB)\right)\int \frac{1}{a+bx^3}\,dx}{b^3} \\
&= -\frac{a(Ab-aB)x}{b^3} + \frac{(Ab-aB)x^4}{4b^2} + \frac{Bx^7}{7b} + \frac{\left(a^{4/3}(Ab-aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^3} \\
&\quad + \frac{\left(a^{4/3}(Ab-aB)\right)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3b^3}
\end{aligned}$$

$$= -\frac{a(Ab-aB)x}{b^3} + \frac{(Ab-aB)x^4}{4b^2} + \frac{Bx^7}{7b} + \frac{a^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{10/3}} - \frac{\left(a^{4/3}(Ab-aB)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^{10/3}} + \frac{\left(a^{5/3}(Ab-aB)\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2b^3}$$

$$= -\frac{a(Ab-aB)x}{b^3} + \frac{(Ab-aB)x^4}{4b^2} + \frac{Bx^7}{7b} + \frac{a^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{10/3}} - \frac{a^{4/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{10/3}} + \frac{\left(a^{4/3}(Ab-aB)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{b^{10/3}}$$

$$= -\frac{a(Ab-aB)x}{b^3} + \frac{(Ab-aB)x^4}{4b^2} + \frac{Bx^7}{7b} - \frac{a^{4/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{10/3}} + \frac{a^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{10/3}} - \frac{a^{4/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{10/3}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.93

$$\int \frac{x^6(A+Bx^3)}{a+bx^3}\,dx$$

$$= \frac{84a\sqrt[3]{b}(-Ab+aB)x + 21b^{4/3}(Ab-aB)x^4 + 12b^{7/3}Bx^7 + 28\sqrt{3}a^{4/3}(-Ab+aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 28a}{84b^{10/3}}$$

```
[In] Integrate[(x^6*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (84*a*b^(1/3)*(-(A*b) + a*B)*x + 21*b^(4/3)*(A*b - a*B)*x^4 + 12*b^(7/3)*B*
x^7 + 28*Sqrt[3]*a^(4/3)*(-(A*b) + a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/
Sqrt[3]] - 28*a^(4/3)*(-(A*b) + a*B)*Log[a^(1/3) + b^(1/3)*x] + 14*a^(4/3)*
(-(A*b) + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(84*b^(10/3)
)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.01 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.46

method	result
risch	$\frac{B\,x^{7}}{7b}+\frac{A\,x^{4}}{4b}-\frac{Ba\,x^{4}}{4b^{2}}-\frac{aAx}{b^{2}}+\frac{a^{2}Bx}{b^{3}}+\frac{a^{2}\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{(Ab-Ba)\ln\left(x-\_R\right)}{\_R^{2}}\right)}{3b^{4}}$
default	$-\frac{-\frac{1}{7}b^{2}B\,x^{7}-\frac{1}{4}A\,b^{2}x^{4}+\frac{1}{4}Bab\,x^{4}+aAbx-a^{2}Bx}{b^{3}}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a^{2}(}{b^{3}}$

[In] int(x^6*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] 1/7*B*x^7/b+1/4/b*A*x^4-1/4/b^2*B*a*x^4-1/b^2*a*A*x+1/b^3*a^2*B*x+1/3/b^4*a^2*sum((A*b-B*a)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.91

$$\int \frac{x^6(A+Bx^3)}{a+bx^3}\,dx = \frac{12\,Bb^2x^7-21\,(Bab-Ab^2)x^4-28\,\sqrt{3}(Ba^2-Aab)\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3\,a}\right)+14\,(Ba^2-Aab)\left(\frac{a}{b}\right)^{\frac{1}{3}}}{84\,b^3}$$

[In] integrate(x^6*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")

[Out] 1/84*(12*B*b^2*x^7 - 21*(B*a*b - A*b^2)*x^4 - 28*sqrt(3)*(B*a^2 - A*a*b)*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 14*(B*a^2 - A*a*b)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 28*(B*a^2 - A*a*b)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 84*(B*a^2 - A*a*b)*x)/b^3

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.62

$$\int \frac{x^6(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^7}{7b} + x^4\left(\frac{A}{4b} - \frac{Ba}{4b^2}\right) + x\left(-\frac{Aa}{b^2} + \frac{Ba^2}{b^3}\right) + \operatorname{RootSum}\left(27t^3b^{10} - A^3a^4b^3 + 3A^2Ba^5b^2 - 3AB^2a^6b + B^3a^7, \left(t \mapsto t\log\left(-\frac{3tb^3}{-Aab+Ba^2} + x\right)\right)\right)$$

```
[In] integrate(x**6*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] B*x**7/(7*b) + x**4*(A/(4*b) - B*a/(4*b**2)) + x*(-A*a/b**2 + B*a**2/b**3)
+ RootSum(27*_t**3*b**10 - A**3*a**4*b**3 + 3*A**2*B*a**5*b**2 - 3*A*B**2*a
**6*b + B**3*a**7, Lambda(_t, _t*log(-3*_t*b**3/(-A*a*b + B*a**2) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.99

$$\int \frac{x^6(A+Bx^3)}{a+bx^3}\,dx = \frac{4\,Bb^2x^7 - 7\,(Bab - Ab^2)x^4 + 28\,(Ba^2 - Aab)x}{28\,b^3} - \frac{\sqrt{3}(Ba^3 - Aa^2b)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(Ba^3 - Aa^2b)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(Ba^3 - Aa^2b)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/28*(4*B*b^2*x^7 - 7*(B*a*b - A*b^2)*x^4 + 28*(B*a^2 - A*a*b)*x)/b^3 - 1/3
*sqrt(3)*(B*a^3 - A*a^2*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/
3))/(b^4*(a/b)^(2/3)) + 1/6*(B*a^3 - A*a^2*b)*log(x^2 - x*(a/b)^(1/3) + (a/
b)^(2/3))/(b^4*(a/b)^(2/3)) - 1/3*(B*a^3 - A*a^2*b)*log(x + (a/b)^(1/3))/(b
^4*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.19

$$\int \frac{x^6(A+Bx^3)}{a+bx^3}\,dx = -\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba^2-\left(-ab^2\right)^{\frac{1}{3}}Aab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^4} - \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}Ba^2-\left(-ab^2\right)^{\frac{1}{3}}Aab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^4} + \frac{\left(Ba^3b^4-Aa^2b^5\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^7} + \frac{4Bb^6x^7-7Bab^5x^4+7Ab^6x^4+28Ba^2b^4x-28Aab^5x}{28b^7}$$

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(1/3)*B*a^2 - (-a*b^2)^(1/3)*A*a*b)*arctan(1/3*sqrt(
3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 1/6*((-a*b^2)^(1/3)*B*a^2 - (-a
*b^2)^(1/3)*A*a*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 + 1/3*(B*a^
3*b^4 - A*a^2*b^5)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^7) + 1/28*(
4*B*b^6*x^7 - 7*B*a*b^5*x^4 + 7*A*b^6*x^4 + 28*B*a^2*b^4*x - 28*A*a*b^5*x)/
b^7
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.90

$$\int \frac{x^6(A+Bx^3)}{a+bx^3}\,dx = x^4\left(\frac{A}{4b}-\frac{Ba}{4b^2}\right)+\frac{Bx^7}{7b} + \frac{a^{4/3}\ln\left(b^{1/3}x+a^{1/3}\right)\left(Ab-Ba\right)}{3b^{10/3}} - \frac{ax\left(\frac{A}{b}-\frac{Ba}{b^2}\right)}{b} - \frac{a^{4/3}\ln\left(a^{1/3}-2b^{1/3}x+\sqrt{3}a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(Ab-Ba\right)}{3b^{10/3}} + \frac{a^{4/3}\ln\left(2b^{1/3}x-a^{1/3}+\sqrt{3}a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(Ab-Ba\right)}{3b^{10/3}}$$

```
[In] int((x^6*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] x^4*(A/(4*b) - (B*a)/(4*b^2)) + (B*x^7)/(7*b) + (a^(4/3)*log(b^(1/3)*x + a^
(1/3))*(A*b - B*a))/(3*b^(10/3)) - (a*x*(A/b - (B*a)/b^2))/b - (a^(4/3)*log
(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b -
B*a))/(3*b^(10/3)) + (a^(4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3
))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/(3*b^(10/3))
```

3.57 $\int \frac{x^5(A+Bx^3)}{a+bx^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 54

$$\int \frac{x^5(A+Bx^3)}{a+bx^3} dx = \frac{(Ab-aB)x^3}{3b^2} + \frac{Bx^6}{6b} - \frac{a(Ab-aB)\log(a+bx^3)}{3b^3}$$

```
[Out] 1/3*(A*b-B*a)*x^3/b^2+1/6*B*x^6/b-1/3*a*(A*b-B*a)*ln(b*x^3+a)/b^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 78}

$$\int \frac{x^5(A+Bx^3)}{a+bx^3} dx = -\frac{a(Ab-aB)\log(a+bx^3)}{3b^3} + \frac{x^3(Ab-aB)}{3b^2} + \frac{Bx^6}{6b}$$

```
[In] Int[(x^5*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] ((A*b - a*B)*x^3)/(3*b^2) + (B*x^6)/(6*b) - (a*(A*b - a*B)*Log[a + b*x^3])/
(3*b^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x(A+Bx)}{a+bx}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{Ab-aB}{b^2}+\frac{Bx}{b}+\frac{a(-Ab+aB)}{b^2(a+bx)}\right)\,dx, x, x^3\right)\\ &= \frac{(Ab-aB)x^3}{3b^2}+\frac{Bx^6}{6b}-\frac{a(Ab-aB)\log\left(a+bx^3\right)}{3b^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{x^5(A+Bx^3)}{a+bx^3}\,dx = \frac{bx^3(2Ab-2aB+bBx^3)+2a(-Ab+aB)\log\left(a+bx^3\right)}{6b^3}$$

```
[In] Integrate[(x^5*(A + B*x^3))/(a + b*x^3),x]
[Out] (b*x^3*(2*A*b - 2*a*B + b*B*x^3) + 2*a*(-(A*b) + a*B)*Log[a + b*x^3])/(6*b^
3)
```

Maple [A] (verified)

Time = 4.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

method	result	size
norman	$\frac{(Ab-Ba)x^3}{3b^2}+\frac{B\,x^6}{6b}-\frac{a(Ab-Ba)\ln\left(b\,x^3+a\right)}{3b^3}$	49
default	$\frac{\frac{1}{2}bB\,x^6+Ab\,x^3-Ba\,x^3}{3b^2}-\frac{a(Ab-Ba)\ln\left(b\,x^3+a\right)}{3b^3}$	50
parallelrisch	$-\frac{-b^2B\,x^6-2A\,b^2x^3+2Bab\,x^3+2A\ln\left(b\,x^3+a\right)ab-2B\ln\left(b\,x^3+a\right)a^2}{6b^3}$	60
risch	$\frac{B\,x^6}{6b}+\frac{A\,x^3}{3b}-\frac{Ba\,x^3}{3b^2}+\frac{A^2}{6bB}-\frac{aA}{3b^2}+\frac{a^2B}{6b^3}-\frac{a\ln\left(b\,x^3+a\right)A}{3b^2}+\frac{a^2\ln\left(b\,x^3+a\right)B}{3b^3}$	89

```
[In] int(x^5*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
[Out] 1/3*(A*b-B*a)*x^3/b^2+1/6*B*x^6/b-1/3*a*(A*b-B*a)*ln(b*x^3+a)/b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{x^5(A+Bx^3)}{a+bx^3}\,dx = \frac{Bb^2x^6 - 2\,(Bab - Ab^2)x^3 + 2\,(Ba^2 - Aab)\log\left(bx^3 + a\right)}{6\,b^3}$$

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
[Out] 1/6*(B*b^2*x^6 - 2*(B*a*b - A*b^2)*x^3 + 2*(B*a^2 - A*a*b)*log(b*x^3 + a))/
b^3
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{x^5(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^6}{6b} + \frac{a(-Ab+Ba)\log\left(a+bx^3\right)}{3b^3} + x^3\left(\frac{A}{3b} - \frac{Ba}{3b^2}\right)$$

```
[In] integrate(x**5*(B*x**3+A)/(b*x**3+a),x)
[Out] B*x**6/(6*b) + a*(-A*b + B*a)*log(a + b*x**3)/(3*b**3) + x**3*(A/(3*b) - B*
a/(3*b**2))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{x^5(A+Bx^3)}{a+bx^3}\,dx = \frac{Bbx^6 - 2\,(Ba - Ab)x^3}{6\,b^2} + \frac{(Ba^2 - Aab)\log\left(bx^3 + a\right)}{3\,b^3}$$

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
[Out] 1/6*(B*b*x^6 - 2*(B*a - A*b)*x^3)/b^2 + 1/3*(B*a^2 - A*a*b)*log(b*x^3 + a)/
b^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \frac{x^5(A+Bx^3)}{a+bx^3}\,dx = \frac{Bbx^6-2\,Bax^3+2\,Abx^3}{6\,b^2}+\frac{(Ba^2-Aab)\log\left(|bx^3+a|\right)}{3\,b^3}$$

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*(B*b*x^6 - 2*B*a*x^3 + 2*A*b*x^3)/b^2 + 1/3*(B*a^2 - A*a*b)*log(abs(b*x
^3 + a))/b^3
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \frac{x^5(A+Bx^3)}{a+bx^3}\,dx = x^3\left(\frac{A}{3\,b}-\frac{B\,a}{3\,b^2}\right)+\frac{\ln\left(b\,x^3+a\right)\,\left(B\,a^2-A\,a\,b\right)}{3\,b^3}+\frac{B\,x^6}{6\,b}$$

```
[In] int((x^5*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] x^3*(A/(3*b) - (B*a)/(3*b^2)) + (log(a + b*x^3)*(B*a^2 - A*a*b))/(3*b^3) +
(B*x^6)/(6*b)
```

3.58 $\int \frac{x^4(A+Bx^3)}{a+bx^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 167

$$\int \frac{x^4(A+Bx^3)}{a+bx^3}\,dx = \frac{(Ab-aB)x^2}{2b^2}+\frac{Bx^5}{5b}+\frac{a^{2/3}(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{8/3}} + \frac{a^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}} - \frac{a^{2/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{8/3}}$$

```
[Out] 1/2*(A*b-B*a)*x^2/b^2+1/5*B*x^5/b+1/3*a^(2/3)*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*
x)/b^(8/3)-1/6*a^(2/3)*(A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/
b^(8/3)+1/3*a^(2/3)*(A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1
/2))/b^(8/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {470, 327, 298, 31, 648, 631, 210, 642}

$$\int \frac{x^4(A+Bx^3)}{a+bx^3}\,dx = \frac{a^{2/3}(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{8/3}} - \frac{a^{2/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{8/3}} + \frac{a^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}} + \frac{x^2(Ab-aB)}{2b^2} + \frac{Bx^5}{5b}$$

```
[In] Int[(x^4*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] ((A*b - a*B)*x^2)/(2*b^2) + (B*x^5)/(5*b) + (a^(2/3)*(A*b - a*B)*ArcTan[(a^
(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(8/3)) + (a^(2/3)*(A*b
- a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(8/3)) - (a^(2/3)*(A*b - a*B)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{Bx^5}{5b} - \frac{(-5Ab+5aB)\int \frac{x^4}{a+bx^3}\,dx}{5b} \\
&= \frac{(Ab-aB)x^2}{2b^2} + \frac{Bx^5}{5b} - \frac{(a(Ab-aB))\int \frac{x}{a+bx^3}\,dx}{b^2} \\
&= \frac{(Ab-aB)x^2}{2b^2} + \frac{Bx^5}{5b} + \frac{\left(a^{2/3}(Ab-aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{7/3}} \\
&\quad - \frac{\left(a^{2/3}(Ab-aB)\right)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3b^{7/3}}
\end{aligned}
$$

$$= \frac{(Ab-aB)x^2}{2b^2} + \frac{Bx^5}{5b} + \frac{a^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}} - \frac{\left(a^{2/3}(Ab-aB)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{6b^{8/3}} - \frac{(a(Ab-aB))\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{2b^{7/3}}$$

$$= \frac{(Ab-aB)x^2}{2b^2} + \frac{Bx^5}{5b} + \frac{a^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}} - \frac{a^{2/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{8/3}} - \frac{\left(a^{2/3}(Ab-aB)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{b^{8/3}}$$

$$= \frac{(Ab-aB)x^2}{2b^2} + \frac{Bx^5}{5b} + \frac{a^{2/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{8/3}} + \frac{a^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}} - \frac{a^{2/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{8/3}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.92

$$\int \frac{x^4(A+Bx^3)}{a+bx^3}dx$$

$$= \frac{15b^{2/3}(Ab-aB)x^2 + 6b^{5/3}Bx^5 - 10\sqrt{3}a^{2/3}(-Ab+aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 10a^{2/3}(-Ab+aB)\log\left(\sqrt[3]{a}\right.}{30b^{8/3}}$$

```
[In] Integrate[(x^4*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (15*b^(2/3)*(A*b - a*B)*x^2 + 6*b^(5/3)*B*x^5 - 10*Sqrt[3]*a^(2/3)*(-(A*b)
+ a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 10*a^(2/3)*(-(A*b) + a
*B)*Log[a^(1/3) + b^(1/3)*x] + 5*a^(2/3)*(-(A*b) + a*B)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(30*b^(8/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.39

method	result	size
risch	$\frac{B x^5}{5b}+\frac{A x^2}{2b}-\frac{B a x^2}{2b^2}+\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(-Ab+Ba)\ln\left(x-\_R\right)}{\_R}\right)}{3b^3}$	65
default	$\frac{\frac{bB\,x^5}{5}+\frac{(Ab-Ba)x^2}{2}}{b^2}-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)a(Ab-Ba)}{b^2}$	131

[In] int(x^4*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] 1/5*B*x^5/b+1/2/b*A*x^2-1/2/b^2*B*a*x^2+1/3/b^3*a*sum((-A*b+B*a)/_R*ln(x-_R
),_R=RootOf(_Z^3*b+a))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.97

$$\int \frac{x^4(A+Bx^3)}{a+bx^3}\,dx$$
$$=\frac{6\,Bbx^5-15\,(Ba-Ab)x^2+10\,\sqrt{3}(Ba-Ab)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bx\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}a}{3\,a}\right)+5\,(Ba-Ab)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(a\right.}{30\,b^2}$$

[In] integrate(x^4*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")

[Out] 1/30*(6*B*b*x^5 - 15*(B*a - A*b)*x^2 + 10*sqrt(3)*(B*a - A*b)*(a^2/b^2)^(1/
3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3) - sqrt(3)*a)/a) + 5*(B*a - A*b
)*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a*(a^2/b^2)^(1/3)) - 10
*(B*a - A*b)*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/b^2

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.68

$$\int \frac{x^4(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^5}{5b} + x^2\left(\frac{A}{2b} - \frac{Ba}{2b^2}\right) + \text{RootSum}\left(27t^3b^8 - A^3a^2b^3 + 3A^2Ba^3b^2 - 3AB^2a^4b + B^3a^5, \left(t \mapsto t\log\left(\frac{9t^2b^5}{A^2ab^2 - 2ABa^2b + B^2a^3} + \right.\right.\right.$$

```
[In] integrate(x**4*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] B*x**5/(5*b) + x**2*(A/(2*b) - B*a/(2*b**2)) + RootSum(27*_t**3*b**8 - A**3
*a**2*b**3 + 3*A**2*B*a**3*b**2 - 3*A*B**2*a**4*b + B**3*a**5, Lambda(_t, _
t*log(9*_t**2*b**5/(A**2*a*b**2 - 2*A*B*a**2*b + B**2*a**3) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.94

$$\int \frac{x^4(A+Bx^3)}{a+bx^3}\,dx = \frac{\sqrt{3}(Ba^2 - Aab)\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{2Bbx^5 - 5(Ba - Ab)x^2}{10b^2} + \frac{(Ba^2 - Aab)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(Ba^2 - Aab)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(B*a^2 - A*a*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1
/3))/(b^3*(a/b)^(1/3)) + 1/10*(2*B*b*x^5 - 5*(B*a - A*b)*x^2)/b^2 + 1/6*(B*
a^2 - A*a*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(1/3)) - 1/3
*(B*a^2 - A*a*b)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.24

$$\int \frac{x^4(A+Bx^3)}{a+bx^3}\,dx = -\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}Ba-\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^4} + \frac{\left(\left(-ab^2\right)^{\frac{2}{3}}Ba-\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^4} - \frac{\left(Ba^2b^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}-Aab^4\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^5} + \frac{2Bb^4x^5-5Bab^3x^2+5Ab^4x^2}{10b^5}$$

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(2/3)*B*a - (-a*b^2)^(2/3)*A*b)*arctan(1/3*sqrt(3)*(
2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 + 1/6*((-a*b^2)^(2/3)*B*a - (-a*b^2)^
(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 - 1/3*(B*a^2*b^3*(-
a/b)^(1/3) - A*a*b^4*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/
(a*b^5) + 1/10*(2*B*b^4*x^5 - 5*B*a*b^3*x^2 + 5*A*b^4*x^2)/b^5
```

Mupad [B] (verification not implemented)

Time = 7.00 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.86

$$\int \frac{x^4(A+Bx^3)}{a+bx^3}\,dx = x^2\left(\frac{A}{2b}-\frac{Ba}{2b^2}\right)+\frac{Bx^5}{5b}+\frac{a^{2/3}\ln\left(b^{1/3}x+a^{1/3}\right)(Ab-Ba)}{3b^{8/3}} + \frac{a^{2/3}\ln\left(a^{1/3}-2b^{1/3}x+\sqrt{3}a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(Ab-Ba)}{3b^{8/3}} - \frac{a^{2/3}\ln\left(2b^{1/3}x-a^{1/3}+\sqrt{3}a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(Ab-Ba)}{3b^{8/3}}$$

```
[In] int((x^4*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] x^2*(A/(2*b) - (B*a)/(2*b^2)) + (B*x^5)/(5*b) + (a^(2/3)*log(b^(1/3)*x + a^
(1/3))*(A*b - B*a))/(3*b^(8/3)) + (a^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/
3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/(3*b^(8/3)) - (a^(2/3)*
log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b
 - B*a))/(3*b^(8/3))
```

3.59 $\int \frac{x^3(A+Bx^3)}{a+bx^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 162

$$\int \frac{x^3(A+Bx^3)}{a+bx^3} dx = \frac{(Ab-aB)x}{b^2} + \frac{Bx^4}{4b} + \frac{\sqrt[3]{a}(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{7/3}} - \frac{\sqrt[3]{a}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{7/3}} + \frac{\sqrt[3]{a}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{7/3}}$$

```
[Out] (A*b-B*a)*x/b^2+1/4*B*x^4/b-1/3*a^(1/3)*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/b^(
7/3)+1/6*a^(1/3)*(A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(7/3
)+1/3*a^(1/3)*(A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b
^(7/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {470, 327, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^3(A+Bx^3)}{a+bx^3}\,dx = \frac{\sqrt[3]{a}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{7/3}} + \frac{\sqrt[3]{a}(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{7/3}} - \frac{\sqrt[3]{a}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{7/3}} + \frac{x(Ab-aB)}{b^2} + \frac{Bx^4}{4b}$$

```
[In] Int[(x^3*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] ((A*b - a*B)*x)/b^2 + (B*x^4)/(4*b) + (a^(1/3)*(A*b - a*B)*ArcTan[(a^(1/3)
- 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(7/3)) - (a^(1/3)*(A*b - a*B)
*Log[a^(1/3) + b^(1/3)*x])/(3*b^(7/3)) + (a^(1/3)*(A*b - a*B)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{Bx^4}{4b} - \frac{(-4Ab+4aB)\int \frac{x^3}{a+bx^3}\,dx}{4b} \\
&= \frac{(Ab-aB)x}{b^2} + \frac{Bx^4}{4b} - \frac{(a(Ab-aB))\int \frac{1}{a+bx^3}\,dx}{b^2} \\
&= \frac{(Ab-aB)x}{b^2} + \frac{Bx^4}{4b} - \frac{\left(\sqrt[3]{a}(Ab-aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^2} \\
&\quad - \frac{\left(\sqrt[3]{a}(Ab-aB)\right)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3b^2}
\end{aligned}$$

$$= \frac{(Ab-aB)x}{b^2} + \frac{Bx^4}{4b} - \frac{\sqrt[3]{a}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{7/3}} + \frac{\left(\sqrt[3]{a}(Ab-aB)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^{7/3}} - \frac{\left(a^{2/3}(Ab-aB)\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2b^2}$$

$$= \frac{(Ab-aB)x}{b^2} + \frac{Bx^4}{4b} - \frac{\sqrt[3]{a}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{7/3}} + \frac{\sqrt[3]{a}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{7/3}} - \frac{\left(\sqrt[3]{a}(Ab-aB)\right)\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{b^{7/3}}$$

$$= \frac{(Ab-aB)x}{b^2} + \frac{Bx^4}{4b} + \frac{\sqrt[3]{a}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{7/3}} - \frac{\sqrt[3]{a}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{7/3}} + \frac{\sqrt[3]{a}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{7/3}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.94

$$\int \frac{x^3(A+Bx^3)}{a+bx^3}\,dx$$

$$= \frac{12\sqrt[3]{b}(Ab-aB)x + 3b^{4/3}Bx^4 - 4\sqrt{3}\sqrt[3]{a}(-Ab+aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 4\sqrt[3]{a}(-Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{12b^{7/3}}$$

```
[In] Integrate[(x^3*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (12*b^(1/3)*(A*b - a*B)*x + 3*b^(4/3)*B*x^4 - 4*Sqrt[3]*a^(1/3)*(-(A*b) + a
*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 4*a^(1/3)*(-(A*b) + a*B)*
Log[a^(1/3) + b^(1/3)*x] - 2*a^(1/3)*(-(A*b) + a*B)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/(12*b^(7/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.37

method	result	size
risch	$\frac{B\,x^4}{4b}+\frac{Ax}{b}-\frac{Bax}{b^2}+\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(-Ab+Ba)\ln\left(x-\_R\right)}{\_R^2}\right)}{3b^3}$	60
default	$\frac{\frac{1}{4}bB\,x^4+Abx-Bax}{b^2}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a(Ab-Ba)}{b^2}$	127

```
[In] int(x^3*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*B*x^4/b+1/b*A*x-1/b^2*B*a*x+1/3/b^3*a*sum((-A*b+B*a)/_R^2*ln(x-_R),_R=R
ootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.90

$$\int \frac{x^3(A+Bx^3)}{a+bx^3}\,dx$$
$$=\frac{3\,Bbx^4-4\,\sqrt{3}(Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3\,a}\right)+2\,(Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\right.\right.}{12\,b^2}$$

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*B*b*x^4 - 4*sqrt(3)*(B*a - A*b)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*
b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) + 2*(B*a - A*b)*(-a/b)^(1/3)*log(x^2 + x*(
-a/b)^(1/3) + (-a/b)^(2/3)) - 4*(B*a - A*b)*(-a/b)^(1/3)*log(x - (-a/b)^(1/
3)) - 12*(B*a - A*b)*x)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.54

$$\int \frac{x^3(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^4}{4b} + x\left(\frac{A}{b}-\frac{Ba}{b^2}\right) + \operatorname{RootSum}\left(27t^3b^7 + A^3ab^3 - 3A^2Ba^2b^2 + 3AB^2a^3b - B^3a^4, \left(t \mapsto t\log\left(\frac{3tb^2}{-Ab+Ba}+x\right)\right)\right)$$

```
[In] integrate(x**3*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] B*x**4/(4*b) + x*(A/b - B*a/b**2) + RootSum(27*_t**3*b**7 + A**3*a*b**3 - 3
*A**2*B*a**2*b**2 + 3*A*B**2*a**3*b - B**3*a**4, Lambda(_t, _t*log(3*_t*b**
2/(-A*b + B*a) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.95

$$\int \frac{x^3(A+Bx^3)}{a+bx^3}\,dx = \frac{Bbx^4 - 4\,(Ba-Ab)x}{4\,b^2} + \frac{\sqrt{3}(Ba^2-Aab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(Ba^2-Aab)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(Ba^2-Aab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^3\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/4*(B*b*x^4 - 4*(B*a - A*b)*x)/b^2 + 1/3*sqrt(3)*(B*a^2 - A*a*b)*arctan(1/
3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(2/3)) - 1/6*(B*a^2 -
 A*a*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) + 1/3*(B*a
^2 - A*a*b)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.15

$$\int \frac{x^3(A+Bx^3)}{a+bx^3}\,dx = \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3} - \frac{\left(Ba^2b^2-Aab^3\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^4} + \frac{Bb^3x^4-4\,Bab^2x+4\,Ab^3x}{4\,b^4}$$

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(2
*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 + 1/6*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(
1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^3 - 1/3*(B*a^2*b^2 - A
*a*b^3)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4) + 1/4*(B*b^3*x^4 -
4*B*a*b^2*x + 4*A*b^3*x)/b^4
```

Mupad [B] (verification not implemented)

Time = 7.05 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00

$$\int \frac{x^3(A+Bx^3)}{a+bx^3}\,dx = x\left(\frac{A}{b}-\frac{B\,a}{b^2}\right)+\frac{B\,x^4}{4\,b}+\frac{(-a)^{1/3}\,\ln\left((-a)^{4/3}+a\,b^{1/3}\,x\right)\,(A\,b-B\,a)}{3\,b^{7/3}} - \frac{(-a)^{1/3}\,\ln\left(2\,a\,b^{1/3}\,x-(-a)^{4/3}-\sqrt{3}\,(-a)^{4/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(A\,b-B\,a)}{3\,b^{7/3}} + \frac{(-a)^{1/3}\,\ln\left(2\,a\,b^{1/3}\,x-(-a)^{4/3}+\sqrt{3}\,(-a)^{4/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(A\,b-B\,a)}{3\,b^{7/3}}$$

```
[In] int((x^3*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] x*(A/b - (B*a)/b^2) + (B*x^4)/(4*b) + ((-a)^(1/3)*log((-a)^(4/3) + a*b^(1/3
)*x)*(A*b - B*a))/(3*b^(7/3)) - ((-a)^(1/3)*log(2*a*b^(1/3)*x - 3^(1/2)*(-a
)^(4/3)*1i - (-a)^(4/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*b^(7/3)) +
((-a)^(1/3)*log(3^(1/2)*(-a)^(4/3)*1i - (-a)^(4/3) + 2*a*b^(1/3)*x)*((3^(1/
2)*1i)/2 - 1/2)*(A*b - B*a))/(3*b^(7/3))
```

3.60 $\int \frac{x^2(A+Bx^3)}{a+bx^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 35

$$\int \frac{x^2(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^3}{3b} + \frac{(Ab-aB)\log(a+bx^3)}{3b^2}$$

```
[Out] 1/3*B*x^3/b+1/3*(A*b-B*a)*ln(b*x^3+a)/b^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {455, 45}

$$\int \frac{x^2(A+Bx^3)}{a+bx^3}\,dx = \frac{(Ab-aB)\log(a+bx^3)}{3b^2} + \frac{Bx^3}{3b}$$

```
[In] Int[(x^2*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (B*x^3)/(3*b) + ((A*b - a*B)*Log[a + b*x^3])/(3*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{a+bx}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{B}{b}+\frac{Ab-aB}{b(a+bx)}\right)\,dx, x, x^3\right)\\ &= \frac{Bx^3}{3b}+\frac{(Ab-aB)\log\left(a+bx^3\right)}{3b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{x^2(A+Bx^3)}{a+bx^3}\,dx = \frac{bBx^3+(Ab-aB)\log\left(a+bx^3\right)}{3b^2}$$

```
[In] Integrate[(x^2*(A + B*x^3))/(a + b*x^3),x]
[Out] (b*B*x^3 + (A*b - a*B)*Log[a + b*x^3])/(3*b^2)
```

Maple [A] (verified)

Time = 4.36 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{B\,x^3}{3b}+\frac{(Ab-Ba)\ln\left(b\,x^3+a\right)}{3b^2}$	32
norman	$\frac{B\,x^3}{3b}+\frac{(Ab-Ba)\ln\left(b\,x^3+a\right)}{3b^2}$	32
parallelrisch	$\frac{bB\,x^3+A\ln\left(b\,x^3+a\right)b-B\ln\left(b\,x^3+a\right)a}{3b^2}$	36
risch	$\frac{B\,x^3}{3b}+\frac{\ln\left(b\,x^3+a\right)A}{3b}-\frac{\ln\left(b\,x^3+a\right)Ba}{3b^2}$	40

```
[In] int(x^2*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
[Out] 1/3*B*x^3/b+1/3*(A*b-B*a)*ln(b*x^3+a)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{x^2(A+Bx^3)}{a+bx^3}\,dx = \frac{Bbx^3-(Ba-Ab)\log\left(bx^3+a\right)}{3\,b^2}$$

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
[Out] 1/3*(B*b*x^3 - (B*a - A*b)*log(b*x^3 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{x^2(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^3}{3b} - \frac{(-Ab+Ba)\log\left(a+bx^3\right)}{3b^2}$$

```
[In] integrate(x**2*(B*x**3+A)/(b*x**3+a),x)
[Out] B*x**3/(3*b) - (-A*b + B*a)*log(a + b*x**3)/(3*b**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{x^2(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^3}{3\,b} - \frac{(Ba-Ab)\log\left(bx^3+a\right)}{3\,b^2}$$

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
[Out] 1/3*B*x^3/b - 1/3*(B*a - A*b)*log(b*x^3 + a)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{x^2(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^3}{3\,b} - \frac{(Ba-Ab)\log\left(|bx^3+a|\right)}{3\,b^2}$$

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
[Out] 1/3*B*x^3/b - 1/3*(B*a - A*b)*log(abs(b*x^3 + a))/b^2
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{x^2(A+Bx^3)}{a+bx^3}\,dx=\frac{B\,x^3}{3\,b}+\frac{\ln\left(b\,x^3+a\right)\,\left(A\,b-B\,a\right)}{3\,b^2}$$

```
[In] int((x^2*(A + B*x^3))/(a + b*x^3),x)
[Out] (B*x^3)/(3*b) + (log(a + b*x^3)*(A*b - B*a))/(3*b^2)
```

3.61 $\int \frac{x(A+Bx^3)}{a+bx^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 150

$$\int \frac{x(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^2}{2b} - \frac{(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{5/3}} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{5/3}} + \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{5/3}}$$

```
[Out] 1/2*B*x^2/b-1/3*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(5/3)+1/6*(A*b-B*
a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/b^(5/3)-1/3*(A*b-B*a)*
arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(1/3)/b^(5/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {470, 298, 31, 648, 631, 210, 642}

$$\int \frac{x(A+Bx^3)}{a+bx^3}\,dx = \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{5/3}} - \frac{(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{5/3}} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{5/3}} + \frac{Bx^2}{2b}$$

```
[In] Int[(x*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (B*x^2)/(2*b) - ((A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/(Sqrt[3]*a^(1/3)*b^(5/3)) - ((A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*
a^(1/3)*b^(5/3)) + ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(6*a^(1/3)*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{Bx^2}{2b} - \frac{(-2Ab+2aB)\int \frac{x}{a+bx^3}\,dx}{2b} \\
&= \frac{Bx^2}{2b} - \frac{(Ab-aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}b^{4/3}} + \frac{(Ab-aB)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3\sqrt[3]{a}b^{4/3}} \\
&= \frac{Bx^2}{2b} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{5/3}} + \frac{(Ab-aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6\sqrt[3]{a}b^{5/3}} \\
&\quad + \frac{(Ab-aB)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2b^{4/3}} \\
&= \frac{Bx^2}{2b} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{5/3}} + \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{5/3}} \\
&\quad + \frac{(Ab-aB)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}b^{5/3}} \\
&= \frac{Bx^2}{2b} - \frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{5/3}} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{5/3}} \\
&\quad + \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{5/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.01

$$\begin{aligned}
\int \frac{x(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^2}{2b} &- \frac{(-Ab+aB)\arctan\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{5/3}} \\
&+ \frac{(-Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{5/3}} \\
&- \frac{(-Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{5/3}}
\end{aligned}$$

```
[In] Integrate[(x*(A + B*x^3))/(a + b*x^3),x]
[Out] (B*x^2)/(2*b) - ((-(A*b) + a*B)*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*a^(1/3)*b^(5/3)) + ((-(A*b) + a*B)*Log[a^(1/3) + b^(1/3)*
x])/(3*a^(1/3)*b^(5/3)) - ((-(A*b) + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/(6*a^(1/3)*b^(5/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.30

method	result	size
risch	$\frac{B x^2}{2b}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^3+a\right)}\frac{(Ab-Ba)\ln\left(x-\_R\right)}{\_R}}{3b^2}$	45
default	$\frac{B x^2}{2b}+\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)(Ab-Ba)}{b}$	113

```
[In] int(x*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
[Out] 1/2*B*x^2/b+1/3/b^2*sum((A*b-B*a)/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 382, normalized size of antiderivative = 2.55

$$\int\frac{x(A+Bx^3)}{a+bx^3}\,dx$$

$$=\left[\frac{3\,Bab^2x^2-3\,\sqrt{\frac{1}{3}}(Ba^2b-Aab^2)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3-ab+3\,\sqrt{\frac{1}{3}}\left(abx+2\,(-ab^2)^{\frac{2}{3}}x^2+(-ab^2)^{\frac{1}{3}}a\right)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}-3\,(-ab^2)^{\frac{2}{3}}\cdots}{bx^3+a}\right.}{6\cdots}\right.$$

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(3*B*a*b^2*x^2 - 3*sqrt(1/3)*(B*a^2*b - A*a*b^2)*sqrt((-a*b^2)^(1/3)/a
)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^
2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - (-a
*b^2)^(2/3)*(B*a - A*b)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3))
+ 2*(-a*b^2)^(2/3)*(B*a - A*b)*log(b*x - (-a*b^2)^(1/3)))/(a*b^3), 1/6*(3*B
*a*b^2*x^2 - 6*sqrt(1/3)*(B*a^2*b - A*a*b^2)*sqrt(-(-a*b^2)^(1/3)/a)*arctan
(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - (-a*b^2)^(
2/3)*(B*a - A*b)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*(-a
*b^2)^(2/3)*(B*a - A*b)*log(b*x - (-a*b^2)^(1/3)))/(a*b^3)]
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.61

$$\int \frac{x(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^2}{2b} + \operatorname{RootSum}\left(27t^3ab^5 + A^3b^3 - 3A^2Bab^2 + 3AB^2a^2b - B^3a^3, \left(t \mapsto t\log\left(\frac{9t^2ab^3}{A^2b^2 - 2ABab + B^2a^2} + x\right)\right)\right)$$

```
[In] integrate(x*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] B*x**2/(2*b) + RootSum(27*_t**3*a*b**5 + A**3*b**3 - 3*A**2*B*a*b**2 + 3*A*
B**2*a**2*b - B**3*a**3, Lambda(_t, _t*log(9*_t**2*a*b**3/(A**2*b**2 - 2*A*
B*a*b + B**2*a**2) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

$$\int \frac{x(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^2}{2\,b} - \frac{\sqrt{3}(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(Ba-Ab)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/2*B*x^2/b - 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3)
)/(a/b)^(1/3))/(b^2*(a/b)^(1/3)) - 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3)
+ (a/b)^(2/3))/(b^2*(a/b)^(1/3)) + 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(b^
2*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.07

$$\int \frac{x(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^2}{2\,b} - \frac{\sqrt{3}(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,\left(-ab^2\right)^{\frac{1}{3}}\,b} + \frac{(Ba-Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,\left(-ab^2\right)^{\frac{1}{3}}\,b} + \frac{\left(Bab\left(-\frac{a}{b}\right)^{\frac{1}{3}}-Ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^2}$$

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/b - 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*b) + 1/6*(B*a - A*b)*log(x^2 + x*(-a/b)^(1
/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*b) + 1/3*(B*a*b*(-a/b)^(1/3) - A*b^2*(-
a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2)
```

Mupad [B] (verification not implemented)

Time = 7.00 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.84

$$\int \frac{x(A+Bx^3)}{a+bx^3}\,dx = \frac{B\,x^2}{2\,b} - \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-B\,a\right)}{3\,a^{1/3}\,b^{5/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{1/3}\,b^{5/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{1/3}\,b^{5/3}}$$

```
[In] int((x*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] (B*x^2)/(2*b) - (log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a^(1/3)*b^(5/3))
- (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(
A*b - B*a))/(3*a^(1/3)*b^(5/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a
^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*a^(1/3)*b^(5/3))
```

3.62 $\int \frac{A+Bx^3}{a+bx^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 145

$$\int \frac{A+Bx^3}{a+bx^3} dx = \frac{Bx}{b} - \frac{(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}} - \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}$$

```
[Out] B*x/b+1/3*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(4/3)-1/6*(A*b-B*a)*ln(
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(4/3)-1/3*(A*b-B*a)*arctan
(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {396, 206, 31, 648, 631, 210, 642}

$$\int \frac{A+Bx^3}{a+bx^3} dx = -\frac{(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}} - \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}} + \frac{Bx}{b}$$

```
[In] Int[(A + B*x^3)/(a + b*x^3),x]
```

```
[Out] (B*x)/b - ((A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*a^(2/3)*b^(4/3)) + ((A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3
)*b^(4/3)) - ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
6*a^(2/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{Bx}{b} - \frac{(-Ab+aB)\int \frac{1}{a+bx^3}\,dx}{b} \\
&= \frac{Bx}{b} + \frac{(Ab-aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}b} + \frac{(Ab-aB)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}b} \\
&= \frac{Bx}{b} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}} - \frac{(Ab-aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}b^{4/3}} \\
&\quad + \frac{(Ab-aB)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{a}b} \\
&= \frac{Bx}{b} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}} - \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}} \\
&\quad + \frac{(Ab-aB)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{2/3}b^{4/3}} \\
&= \frac{Bx}{b} - \frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}} \\
&\quad - \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^3}{a+bx^3}\,dx$$

$$= \frac{6a^{2/3}\sqrt[3]{b}Bx - 2\sqrt{3}(Ab-aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right) - (Ab-aB)\log\left(a^{2/3} - \right.}{6a^{2/3}b^{4/3}}$$

```
[In] Integrate[(A + B*x^3)/(a + b*x^3),x]
[Out] (6*a^(2/3)*b^(1/3)*B*x - 2*Sqrt[3]*(A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^
(1/3))/Sqrt[3]] + 2*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x] - (A*b - a*B)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(4/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.29

method	result	size
risch	$\frac{Bx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(Ab-Ba)\ln\left(x-\_R\right)}{\_R^2}}{3b^2}$	42
default	$\frac{Bx}{b}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)(Ab-Ba)}{b}$	110

```
[In] int((B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] B*x/b+1/3/b^2*sum((A*b-B*a)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 369, normalized size of antiderivative = 2.54

$$\int \frac{A+Bx^3}{a+bx^3}\,dx = \left[\frac{6\,Ba^2bx-3\,\sqrt{\frac{1}{3}}(Ba^2b-Aab^2)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\,(a^2b)^{\frac{1}{3}}ax-a^2+3\,\sqrt{\frac{1}{3}}\left(2\,abx^2+(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)}{6\,a^2b^2}\right.$$

```
[In] integrate((B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(6*B*a^2*b*x - 3*sqrt(1/3)*(B*a^2*b - A*a*b^2)*sqrt(-(a^2*b)^(1/3)/b)*
log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*
b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) + (a^2*b
```

```
)^(2/3)*(B*a - A*b)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*(a
^2*b)^(2/3)*(B*a - A*b)*log(a*b*x + (a^2*b)^(2/3)))/(a^2*b^2), 1/6*(6*B*a^2
*b*x - 6*sqrt(1/3)*(B*a^2*b - A*a*b^2)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/
3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + (a^2*
b)^(2/3)*(B*a - A*b)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*(
a^2*b)^(2/3)*(B*a - A*b)*log(a*b*x + (a^2*b)^(2/3)))/(a^2*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.49

$$\int \frac{A+Bx^3}{a+bx^3}\,dx = \frac{Bx}{b} + \operatorname{RootSum}\left(27t^3a^2b^4 - A^3b^3 + 3A^2Bab^2 - 3AB^2a^2b + B^3a^3, \left(t \mapsto t\log\left(-\frac{3tab}{-Ab+Ba} + x\right)\right)\right)$$

```
[In] integrate((B*x**3+A)/(b*x**3+a),x)
```

```
[Out] B*x/b + RootSum(27*_t**3*a**2*b**4 - A**3*b**3 + 3*A**2*B*a*b**2 - 3*A*B**2
*a**2*b + B**3*a**3, Lambda(_t, _t*log(-3*_t*a*b/(-A*b + B*a) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^3}{a+bx^3}\,dx = \frac{Bx}{b} - \frac{\sqrt{3}(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(Ba-Ab)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] B*x/b - 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b
)^(1/3))/(b^2*(a/b)^(2/3)) + 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b
)^(2/3))/(b^2*(a/b)^(2/3)) - 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(b^2*(a/b
)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^3}{a+bx^3}\,dx = \frac{\sqrt{3}(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}} + \frac{(Ba-Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}} + \frac{Bx}{b} + \frac{(Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab}$$

```
[In] integrate((B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3
))/(-a*b^2)^(2/3) + 1/6*(B*a - A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)
)/(-a*b^2)^(2/3) + B*x/b + 1/3*(B*a - A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^
(1/3)))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 6.99 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^3}{a+bx^3}\,dx = \frac{B\,x}{b} + \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-B\,a\right)}{3\,a^{2/3}\,b^{4/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{2/3}\,b^{4/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{2/3}\,b^{4/3}}$$

```
[In] int((A + B*x^3)/(a + b*x^3),x)
```

```
[Out] (B*x)/b + (log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a^(2/3)*b^(4/3)) - (log
(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b -
B*a))/(3*a^(2/3)*b^(4/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3)
)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/(3*a^(2/3)*b^(4/3))
```

3.63 $\int \frac{A+Bx^3}{x(a+bx^3)} dx$

Optimal result

Integrand size = 20, antiderivative size = 34

$$\int \frac{A+Bx^3}{x(a+bx^3)} dx = \frac{A\log(x)}{a} - \frac{(Ab-aB)\log(a+bx^3)}{3ab}$$

```
[Out] A*ln(x)/a-1/3*(A*b-B*a)*ln(b*x^3+a)/a/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 78}

$$\int \frac{A+Bx^3}{x(a+bx^3)} dx = \frac{A\log(x)}{a} - \frac{(Ab-aB)\log(a+bx^3)}{3ab}$$

```
[In] Int[(A + B*x^3)/(x*(a + b*x^3)),x]
```

```
[Out] (A*Log[x])/a - ((A*b - a*B)*Log[a + b*x^3])/(3*a*b)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x(a+bx)}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{A}{ax}+\frac{-Ab+aB}{a(a+bx)}\right)\,dx, x, x^3\right)\\ &= \frac{A\log(x)}{a}-\frac{(Ab-aB)\log\left(a+bx^3\right)}{3ab}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)}\,dx=\frac{A\log(x)}{a}+\frac{(-Ab+aB)\log\left(a+bx^3\right)}{3ab}$$

```
[In] Integrate[(A + B*x^3)/(x*(a + b*x^3)),x]
[Out] (A*Log[x])/a + ((-(A*b) + a*B)*Log[a + b*x^3])/(3*a*b)
```

Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{A\ln(x)}{a}-\frac{(Ab-Ba)\ln\left(b\,x^3+a\right)}{3ab}$	33
norman	$\frac{A\ln(x)}{a}-\frac{(Ab-Ba)\ln\left(b\,x^3+a\right)}{3ab}$	33
risch	$\frac{A\ln(x)}{a}-\frac{\ln\left(b\,x^3+a\right)A}{3a}+\frac{\ln\left(b\,x^3+a\right)B}{3b}$	37
parallelrisch	$\frac{3A\ln(x)b-A\ln\left(b\,x^3+a\right)b+B\ln\left(b\,x^3+a\right)a}{3ab}$	39

```
[In] int((B*x^3+A)/x/(b*x^3+a),x,method=_RETURNVERBOSE)
[Out] A*ln(x)/a-1/3*(A*b-B*a)*ln(b*x^3+a)/a/b
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^3}{x(a+bx^3)}\,dx = \frac{3\,Ab\log(x)+(Ba-Ab)\log(bx^3+a)}{3\,ab}$$

```
[In] integrate((B*x^3+A)/x/(b*x^3+a),x, algorithm="fricas")
[Out] 1/3*(3*A*b*log(x) + (B*a - A*b)*log(b*x^3 + a))/(a*b)
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{A+Bx^3}{x(a+bx^3)}\,dx = \frac{A\log(x)}{a} + \frac{(-Ab+Ba)\log\left(\frac{a}{b}+x^3\right)}{3ab}$$

```
[In] integrate((B*x**3+A)/x/(b*x**3+a),x)
[Out] A*log(x)/a + (-A*b + B*a)*log(a/b + x**3)/(3*a*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^3}{x(a+bx^3)}\,dx = \frac{A\log(x^3)}{3\,a} + \frac{(Ba-Ab)\log(bx^3+a)}{3\,ab}$$

```
[In] integrate((B*x^3+A)/x/(b*x^3+a),x, algorithm="maxima")
[Out] 1/3*A*log(x^3)/a + 1/3*(B*a - A*b)*log(b*x^3 + a)/(a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^3}{x(a+bx^3)}\,dx = \frac{A\log(|x|)}{a} + \frac{(Ba-Ab)\log(|bx^3+a|)}{3\,ab}$$

```
[In] integrate((B*x^3+A)/x/(b*x^3+a),x, algorithm="giac")
[Out] A*log(abs(x))/a + 1/3*(B*a - A*b)*log(abs(b*x^3 + a))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)}\,dx=\frac{B\,\ln\left(b\,x^3+a\right)}{3\,b}-\frac{A\,\ln\left(b\,x^3+a\right)}{3\,a}+\frac{A\,\ln\left(x\right)}{a}$$

```
[In] int((A + B*x^3)/(x*(a + b*x^3)),x)
[Out] (B*log(a + b*x^3))/(3*b) - (A*log(a + b*x^3))/(3*a) + (A*log(x))/a
```

3.64 $\int \frac{A+Bx^3}{x^2(a+bx^3)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 147

$$\int \frac{A+Bx^3}{x^2(a+bx^3)}\,dx=-\frac{A}{ax}+\frac{(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}b^{2/3}}+\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{2/3}}-\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}b^{2/3}}$$

```
[Out] -A/a/x+1/3*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)/b^(2/3)-1/6*(A*b-B*a)*ln
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(2/3)+1/3*(A*b-B*a)*arcta
n(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/b^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {464, 298, 31, 648, 631, 210, 642}

$$\int \frac{A+Bx^3}{x^2(a+bx^3)}\,dx=\frac{(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}b^{2/3}}-\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}b^{2/3}}+\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{2/3}}-\frac{A}{ax}$$

```
[In] Int[(A + B*x^3)/(x^2*(a + b*x^3)),x]
```

```
[Out] -(A/(a*x)) + ((A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]
)/(Sqrt[3]*a^(4/3)*b^(2/3)) + ((A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(
4/3)*b^(2/3)) - ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(6*a^(4/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{A}{ax} - \frac{(Ab-aB)\int \frac{x}{a+bx^3}\,dx}{a} \\
&= -\frac{A}{ax} + \frac{(Ab-aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{4/3}\sqrt[3]{b}} - \frac{(Ab-aB)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{4/3}\sqrt[3]{b}} \\
&= -\frac{A}{ax} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{2/3}} - \frac{(Ab-aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{4/3}b^{2/3}} \\
&\quad - \frac{(Ab-aB)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a\sqrt[3]{b}} \\
&= -\frac{A}{ax} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{2/3}} - \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}b^{2/3}} \\
&\quad - \frac{(Ab-aB)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{4/3}b^{2/3}} \\
&= -\frac{A}{ax} + \frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}b^{2/3}} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{2/3}} \\
&\quad - \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}b^{2/3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.91

$$
\int \frac{A+Bx^3}{x^2(a+bx^3)}\,dx
$$

$$
= \frac{-6\sqrt[3]{a}Ab^{2/3} + 2\sqrt{3}(Ab-aB)x\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2(Ab-aB)x\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right) - (Ab-aB)x\log\left(a^2\right.}{6a^{4/3}b^{2/3}x}
$$

```
[In] Integrate[(A + B*x^3)/(x^2*(a + b*x^3)),x]
```

```
[Out] (-6*a^(1/3)*A*b^(2/3) + 2*Sqrt[3]*(A*b - a*B)*x*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]] + 2*(A*b - a*B)*x*Log[a^(1/3) + b^(1/3)*x] - (A*b - a*B)*x
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*b^(2/3)*x)
```

Maple [A] (verified)

Time = 4.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.78

method	result
default	$-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)(Ab-Ba)}{a}-\frac{A}{ax}$
risch	$-\frac{A}{ax}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{3}b^{2}a^{4}-A^{3}b^{3}+3A^{2}Ba\,b^{2}-3A\,B^{2}a^{2}b+B^{3}a^{3}\right)}\_R\ln\left(\left(-4\_R^{3}a^{4}b^{2}+3A^{3}b^{3}-9A^{2}Ba\,b^{2}+9A\,B^{2}a^{2}b-3B^{3}a^{3}\right.\right.\right.}{3}$

```
[In] int((B*x^3+A)/x^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)
*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1)))*(A*b-B*a)/a-A/a/x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.53

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)}\,dx = \left[-\frac{6\,Aab^2+3\,\sqrt{\frac{1}{3}}(Ba^2b-Aab^2)x\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3-ab-3\,\sqrt{\frac{1}{3}}\left(abx+2\,(ab^2)^{\frac{2}{3}}x^2-(ab^2)^{\frac{1}{3}}a\right)\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}-3\,(ab^2)^{\frac{2}{3}}x}{bx^3+a}\right)}{6\,a^2b^2x}, -\frac{6\,Aab^2+6\,\sqrt{\frac{1}{3}}(Ba^2b-Aab^2)x\sqrt{\frac{(ab^2)^{\frac{1}{3}}}{a}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(2\,bx-(ab^2)^{\frac{1}{3}}\right)\sqrt{\frac{(ab^2)^{\frac{1}{3}}}{a}}}{b}\right)-(ab^2)^{\frac{2}{3}}\,(Ba-Ab)x\log\left(b\right.}{6\,a^2b^2x}\right.$$

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/6*(6*A*a*b^2 + 3*sqrt(1/3)*(B*a^2*b - A*a*b^2)*x*sqrt(-(a*b^2)^(1/3)/a)
*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^
(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) - (a*b^2)
^(2/3)*(B*a - A*b)*x*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) + 2*(
a*b^2)^(2/3)*(B*a - A*b)*x*log(b*x + (a*b^2)^(1/3)))/(a^2*b^2*x), -1/6*(6*A
*a*b^2 + 6*sqrt(1/3)*(B*a^2*b - A*a*b^2)*x*sqrt((a*b^2)^(1/3)/a)*arctan(-sq
rt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) - (a*b^2)^(2/3)*(B
*a - A*b)*x*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) + 2*(a*b^2)^(2
/3)*(B*a - A*b)*x*log(b*x + (a*b^2)^(1/3)))/(a^2*b^2*x)]
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.61

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)}\,dx = -\frac{A}{ax} + \operatorname{RootSum}\left(27t^3a^4b^2-A^3b^3+3A^2Bab^2-3AB^2a^2b+B^3a^3, \left(t\mapsto t\log\left(\frac{9t^2a^3b}{A^2b^2-2ABab+B^2a^2}+x\right)\right)\right)$$

```
[In] integrate((B*x**3+A)/x**2/(b*x**3+a),x)
```

```
[Out] -A/(a*x) + RootSum(27*_t**3*a**4*b**2 - A**3*b**3 + 3*A**2*B*a*b**2 - 3*A*B
**2*a**2*b + B**3*a**3, Lambda(_t, _t*log(9*_t**2*a**3*b/(A**2*b**2 - 2*A*B
*a*b + B**2*a**2) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)}\,dx = \frac{\sqrt{3}(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,ab\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{A}{ax}$$

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))
/(a*b*(a/b)^(1/3)) + 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))
/(a*b*(a/b)^(1/3)) - 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(a*b*(a/b)^(1/3))
 - A/(a*x)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)}\,dx = \frac{\sqrt{3}(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,\left(-ab^2\right)^{\frac{1}{3}}\,a} - \frac{(Ba-Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,\left(-ab^2\right)^{\frac{1}{3}}\,a} - \frac{\left(Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}-Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2} - \frac{A}{ax}$$

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3
))/((-a*b^2)^(1/3)*a) - 1/6*(B*a - A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(
2/3))/((-a*b^2)^(1/3)*a) - 1/3*(B*a*(-a/b)^(1/3) - A*b*(-a/b)^(1/3))*(-a/b)
^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 - A/(a*x)
```

Mupad [B] (verification not implemented)

Time = 6.96 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx^3}{x^2\,(a+bx^3)}\,dx = \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-B\,a\right)}{3\,a^{4/3}\,b^{2/3}}-\frac{A}{a\,x}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{4/3}\,b^{2/3}}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{4/3}\,b^{2/3}}$$

```
[In] int((A + B*x^3)/(x^2*(a + b*x^3)),x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a^(4/3)*b^(2/3)) - A/(a*x) + (log
(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b -
B*a))/(3*a^(4/3)*b^(2/3)) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3)
)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*a^(4/3)*b^(2/3))
```

3.65 $\int \frac{A+Bx^3}{x^3(a+bx^3)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 149

$$\int \frac{A+Bx^3}{x^3(a+bx^3)}\,dx = -\frac{A}{2ax^2}+\frac{(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}\sqrt[3]{b}} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}} + \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt[3]{b}}$$

```
[Out] -1/2*A/a/x^2-1/3*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(1/3)+1/6*(A*b-B
*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(1/3)+1/3*(A*b-B*a)
*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {464, 206, 31, 648, 631, 210, 642}

$$\int \frac{A+Bx^3}{x^3(a+bx^3)}\,dx = \frac{(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}\sqrt[3]{b}} + \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt[3]{b}} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}} - \frac{A}{2ax^2}$$

```
[In] Int[(A + B*x^3)/(x^3*(a + b*x^3)),x]
```

```
[Out] -1/2*A/(a*x^2) + ((A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(Sqrt[3]*a^(5/3)*b^(1/3)) - ((A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3
*a^(5/3)*b^(1/3)) + ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2])/(6*a^(5/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
```

```
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = -\frac{A}{2ax^2} - \frac{(2Ab-2aB)\int \frac{1}{a+bx^3}\,dx}{2a}$$

$$= -\frac{A}{2ax^2} - \frac{(Ab-aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{5/3}} - \frac{(Ab-aB)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{5/3}}$$

$$= -\frac{A}{2ax^2} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}} - \frac{(Ab-aB)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a^{4/3}} + \frac{(Ab-aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{5/3}\sqrt[3]{b}}$$

$$= -\frac{A}{2ax^2} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}} + \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt[3]{b}} - \frac{(Ab-aB)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{5/3}\sqrt[3]{b}}$$

$$= -\frac{A}{2ax^2} + \frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}\sqrt[3]{b}} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}} + \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt[3]{b}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^3}{x^3(a+bx^3)}\,dx = \frac{-\frac{3a^{2/3}A}{x^2} + \frac{2\sqrt{3}(Ab-aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} + \frac{2(-Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}} + \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}}}{6a^{5/3}}$$

```
[In] Integrate[(A + B*x^3)/(x^3*(a + b*x^3)),x]
[Out] ((-3*a^(2/3)*A)/x^2 + (2*Sqrt[3]*(A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]])/b^(1/3) + (2*(-(A*b) + a*B)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3
) + ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3))/(
6*a^(5/3))
```

Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.76

method	result
default	$\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)(-Ab+Ba)}{a}-\frac{A}{2a\,x^2}$
risch	$-\frac{A}{2a\,x^2}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^3 b\,a^5+A^3b^3-3A^2Ba\,b^2+3A\,B^2a^2b-B^3a^3\right)}\_R\ln\left(\left(-4\_R^3a^5b-3A^3b^3+9A^2Ba\,b^2-9A\,B^2a^2b+3B^3a^3\right)\right.\right.}{3}$

```
[In] int((B*x^3+A)/x^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] (1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x
+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1)))*(-A*b+B*a)/a-1/2*A/a/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 411, normalized size of antiderivative = 2.76

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)}\,dx = \left[-\frac{3\sqrt{\frac{1}{3}}(Ba^2b-Aab^2)x^2\sqrt{\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3+3\left(-a^2b\right)^{\frac{1}{3}}ax-a^2-3\sqrt{\frac{1}{3}}\left(2\,abx^2+\left(-a^2b\right)^{\frac{2}{3}}x+\left(-a^2b\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)+(\cdots}{\cdots}\right.$$

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(1/3)*(B*a^2*b - A*a*b^2)*x^2*sqrt((-a^2*b)^(1/3)/b)*log((2*a*
b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3
)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) + (-a^2*b)^(2/
3)*(B*a - A*b)*x^2*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) - 2*(
-a^2*b)^(2/3)*(B*a - A*b)*x^2*log(a*b*x + (-a^2*b)^(2/3)) + 3*A*a^2*b)/(a^3
*b*x^2), 1/6*(6*sqrt(1/3)*(B*a^2*b - A*a*b^2)*x^2*sqrt(-(-a^2*b)^(1/3)/b)*a
rctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3
)/b)/a^2) - (-a^2*b)^(2/3)*(B*a - A*b)*x^2*log(a*b*x^2 - (-a^2*b)^(2/3)*x -
 (-a^2*b)^(1/3)*a) + 2*(-a^2*b)^(2/3)*(B*a - A*b)*x^2*log(a*b*x + (-a^2*b)^
(2/3)) - 3*A*a^2*b)/(a^3*b*x^2)]
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.49

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)}\,dx = -\frac{A}{2ax^2} + \operatorname{RootSum}\left(27t^3a^5b+A^3b^3-3A^2Bab^2+3AB^2a^2b-B^3a^3,\left(t\mapsto t\log\left(\frac{3ta^2}{-Ab+Ba}+x\right)\right)\right)$$

```
[In] integrate((B*x**3+A)/x**3/(b*x**3+a),x)
```

```
[Out] -A/(2*a*x**2) + RootSum(27*_t**3*a**5*b + A**3*b**3 - 3*A**2*B*a*b**2 + 3*A
*B**2*a**2*b - B**3*a**3, Lambda(_t, _t*log(3*_t*a**2/(-A*b + B*a) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^3}{x^3(a+bx^3)}\,dx = \frac{\sqrt{3}(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{A}{2ax^2}$$

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))
/(a*b*(a/b)^(2/3)) - 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))
/(a*b*(a/b)^(2/3)) + 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(a*b*(a/b)^(2/3))
 - 1/2*A/(a*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx^3}{x^3(a+bx^3)}\,dx = -\frac{(Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^2} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2b} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^2b} - \frac{A}{2ax^2}$$

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*(B*a - A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1/3*sqrt(3)*
((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^
(1/3))/(-a/b)^(1/3))/(a^2*b) + 1/6*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b
)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b) - 1/2*A/(a*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^3}{x^3\,(a+bx^3)}\,dx = -\frac{A}{2\,a\,x^2} - \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-B\,a\right)}{3\,a^{5/3}\,b^{1/3}} + \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{5/3}\,b^{1/3}} - \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{5/3}\,b^{1/3}}$$

```
[In] int((A + B*x^3)/(x^3*(a + b*x^3)),x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*
b - B*a))/(3*a^(5/3)*b^(1/3)) - (log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a
^(5/3)*b^(1/3)) - A/(2*a*x^2) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(
1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/(3*a^(5/3)*b^(1/3))
```

3.66 $\int \frac{A+Bx^3}{x^4(a+bx^3)} dx$

Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)}\,dx=-\frac{A}{3ax^3}-\frac{(Ab-aB)\log(x)}{a^2}+\frac{(Ab-aB)\log\left(a+bx^3\right)}{3a^2}$$

```
[Out] -1/3*A/a/x^3-(A*b-B*a)*ln(x)/a^2+1/3*(A*b-B*a)*ln(b*x^3+a)/a^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 78}

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)}\,dx=\frac{(Ab-aB)\log\left(a+bx^3\right)}{3a^2}-\frac{\log(x)(Ab-aB)}{a^2}-\frac{A}{3ax^3}$$

```
[In] Int[(A + B*x^3)/(x^4*(a + b*x^3)),x]
```

```
[Out] -1/3*A/(a*x^3) - ((A*b - a*B)*Log[x])/a^2 + ((A*b - a*B)*Log[a + b*x^3])/(3
*a^2)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{A}{ax^2}+\frac{-Ab+aB}{a^2x}-\frac{b(-Ab+aB)}{a^2(a+bx)}\right)\,dx, x, x^3\right)\\ &= -\frac{A}{3ax^3}-\frac{(Ab-aB)\log(x)}{a^2}+\frac{(Ab-aB)\log\left(a+bx^3\right)}{3a^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)}\,dx = -\frac{A}{3ax^3}+\frac{(-Ab+aB)\log(x)}{a^2}+\frac{(Ab-aB)\log\left(a+bx^3\right)}{3a^2}$$

```
[In] Integrate[(A + B*x^3)/(x^4*(a + b*x^3)),x]
[Out] -1/3*A/(a*x^3) + ((-(A*b) + a*B)*Log[x])/a^2 + ((A*b - a*B)*Log[a + b*x^3])
/(3*a^2)
```

Maple [A] (verified)

Time = 4.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{A}{3a\,x^3}+\frac{(-Ab+Ba)\ln(x)}{a^2}+\frac{(Ab-Ba)\ln\left(b\,x^3+a\right)}{3a^2}$	46
norman	$-\frac{A}{3a\,x^3}-\frac{(Ab-Ba)\ln(x)}{a^2}+\frac{(Ab-Ba)\ln\left(b\,x^3+a\right)}{3a^2}$	47
parallelrisch	$-\frac{3A\ln(x)x^3b-A\ln\left(b\,x^3+a\right)x^3b-3B\ln(x)x^3a+B\ln\left(b\,x^3+a\right)x^3a+Aa}{3x^3a^2}$	60
risch	$-\frac{A}{3a\,x^3}-\frac{\ln(x)Ab}{a^2}+\frac{B\ln(x)}{a}+\frac{\ln\left(-b\,x^3-a\right)Ab}{3a^2}-\frac{\ln\left(-b\,x^3-a\right)B}{3a}$	62

```
[In] int((B*x^3+A)/x^4/(b*x^3+a),x,method=_RETURNVERBOSE)
[Out] -1/3*A/a/x^3+1/a^2*(-A*b+B*a)*ln(x)+1/3*(A*b-B*a)*ln(b*x^3+a)/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)}\,dx = -\frac{(Ba-Ab)x^3\log\left(bx^3+a\right)-3\,(Ba-Ab)x^3\log\left(x\right)+Aa}{3\,a^2x^3}$$

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a),x, algorithm="fricas")
[Out] -1/3*((B*a - A*b)*x^3*log(b*x^3 + a) - 3*(B*a - A*b)*x^3*log(x) + A*a)/(a^2
*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)}\,dx = -\frac{A}{3ax^3}+\frac{(-Ab+Ba)\log\left(x\right)}{a^2}-\frac{(-Ab+Ba)\log\left(\frac{a}{b}+x^3\right)}{3a^2}$$

```
[In] integrate((B*x**3+A)/x**4/(b*x**3+a),x)
[Out] -A/(3*a*x**3) + (-A*b + B*a)*log(x)/a**2 - (-A*b + B*a)*log(a/b + x**3)/(3*
a**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)}\,dx = -\frac{(Ba-Ab)\log\left(bx^3+a\right)}{3\,a^2}+\frac{(Ba-Ab)\log\left(x^3\right)}{3\,a^2}-\frac{A}{3\,ax^3}$$

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a),x, algorithm="maxima")
[Out] -1/3*(B*a - A*b)*log(b*x^3 + a)/a^2 + 1/3*(B*a - A*b)*log(x^3)/a^2 - 1/3*A/
(a*x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.38

$$\int \frac{A+Bx^3}{x^4\,(a+bx^3)}\,dx = \frac{(Ba-Ab)\log\left(|x|\right)}{a^2} - \frac{(Bab-Ab^2)\log\left(|bx^3+a|\right)}{3\,a^2b} - \frac{Bax^3-Abx^3+Aa}{3\,a^2x^3}$$

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a),x, algorithm="giac")
[Out] (B*a - A*b)*log(abs(x))/a^2 - 1/3*(B*a*b - A*b^2)*log(abs(b*x^3 + a))/(a^2*
b) - 1/3*(B*a*x^3 - A*b*x^3 + A*a)/(a^2*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^3}{x^4\,(a+bx^3)}\,dx = \frac{\ln\left(b\,x^3+a\right)\,\left(A\,b-B\,a\right)}{3\,a^2} - \frac{A}{3\,a\,x^3} - \frac{\ln\left(x\right)\,\left(A\,b-B\,a\right)}{a^2}$$

```
[In] int((A + B*x^3)/(x^4*(a + b*x^3)),x)
[Out] (log(a + b*x^3)*(A*b - B*a))/(3*a^2) - A/(3*a*x^3) - (log(x)*(A*b - B*a))/a
^2
```

3.67 $\int \frac{A+Bx^3}{x^5(a+bx^3)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 165

$$\int \frac{A+Bx^3}{x^5(a+bx^3)}\,dx = -\frac{A}{4ax^4}+\frac{Ab-aB}{a^2x}-\frac{\sqrt[3]{b}(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}} - \frac{\sqrt[3]{b}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}} + \frac{\sqrt[3]{b}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{7/3}}$$

```
[Out] -1/4*A/a/x^4+(A*b-B*a)/a^2/x-1/3*b^(1/3)*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^
(7/3)+1/6*b^(1/3)*(A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(7/
3)-1/3*b^(1/3)*(A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/
a^(7/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {464, 331, 298, 31, 648, 631, 210, 642}

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)}\,dx = -\frac{\sqrt[3]{b}(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}} + \frac{\sqrt[3]{b}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{7/3}} - \frac{\sqrt[3]{b}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}} + \frac{Ab-aB}{a^2x} - \frac{A}{4ax^4}$$

```
[In] Int[(A + B*x^3)/(x^5*(a + b*x^3)),x]
```

```
[Out] -1/4*A/(a*x^4) + (A*b - a*B)/(a^2*x) - (b^(1/3)*(A*b - a*B)*ArcTan[(a^(1/3)
 - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)) - (b^(1/3)*(A*b - a*B
)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(7/3)) + (b^(1/3)*(A*b - a*B)*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{A}{4ax^4}-\frac{(4Ab-4aB)\int \frac{1}{x^2(a+bx^3)}\,dx}{4a}\\
&= -\frac{A}{4ax^4}+\frac{Ab-aB}{a^2x}+\frac{(b(Ab-aB))\int \frac{x}{a+bx^3}\,dx}{a^2}\\
&= -\frac{A}{4ax^4}+\frac{Ab-aB}{a^2x}-\frac{\left(b^{2/3}(Ab-aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{7/3}}\\
&\quad+\frac{\left(b^{2/3}(Ab-aB)\right)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{7/3}}
\end{aligned}
$$

$$= -\frac{A}{4ax^4} + \frac{Ab-aB}{a^2x} - \frac{\sqrt[3]{b}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}} + \frac{\left(\sqrt[3]{b}(Ab-aB)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{7/3}} + \frac{\left(b^{2/3}(Ab-aB)\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a^2}$$

$$= -\frac{A}{4ax^4} + \frac{Ab-aB}{a^2x} - \frac{\sqrt[3]{b}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}} + \frac{\sqrt[3]{b}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{7/3}} + \frac{\left(\sqrt[3]{b}(Ab-aB)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{7/3}}$$

$$= -\frac{A}{4ax^4} + \frac{Ab-aB}{a^2x} - \frac{\sqrt[3]{b}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}} - \frac{\sqrt[3]{b}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}} + \frac{\sqrt[3]{b}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{7/3}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^3}{x^5(a+bx^3)}\,dx$$

$$= \frac{-\frac{3a^{4/3}A}{x^4} + \frac{12\sqrt[3]{a}(Ab-aB)}{x} - 4\sqrt{3}\sqrt[3]{b}(Ab-aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 4\sqrt[3]{b}(-Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right) + 2}{12a^{7/3}}$$

```
[In] Integrate[(A + B*x^3)/(x^5*(a + b*x^3)),x]
```

```
[Out] ((-3*a^(4/3)*A)/x^4 + (12*a^(1/3)*(A*b - a*B))/x - 4*Sqrt[3]*b^(1/3)*(A*b -
 a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 4*b^(1/3)*(-(A*b) + a*B
)*Log[a^(1/3) + b^(1/3)*x] + 2*b^(1/3)*(A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(12*a^(7/3))
```

Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.79

method	result
default	$-\frac{A}{4a\,x^{4}}-\frac{-Ab+Ba}{x\,a^{2}}+\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b(Ab-Ba)}{a^{2}}$
risch	$\frac{\frac{(Ab-Ba)x^{3}}{a^{2}}-\frac{A}{4a}}{x^{4}}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^{7}\_Z^{3}+A^{3}b^{4}-3A^{2}Ba\,b^{3}+3A\,B^{2}a^{2}b^{2}-B^{3}a^{3}b\right)}\_R\ln\left(\left(-4a^{7}\_R^{3}-3A^{3}b^{4}+9A^{2}Ba\,b^{3}-9A\,B^{2}a^{2}b^{2}\right.\right.\right.}{3}$

```
[In] int((B*x^3+A)/x^5/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*A/a/x^4-(-A*b+B*a)/x/a^2+(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(
a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arct
an(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*b*(A*b-B*a)/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^3}{x^5\,(a+bx^3)}\,dx = -\frac{4\sqrt{3}(Ba-Ab)x^4\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-2\,(Ba-Ab)x^4\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(-\frac{b}{a}\right)^{\frac{2}{3}}-\right.}{12\,a^2x^4}$$

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/12*(4*sqrt(3)*(B*a - A*b)*x^4*(-b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(
1/3) + 1/3*sqrt(3)) - 2*(B*a - A*b)*x^4*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)
^(2/3) - a*(-b/a)^(1/3)) + 4*(B*a - A*b)*x^4*(-b/a)^(1/3)*log(b*x + a*(-b/a
)^(2/3)) + 12*(B*a - A*b)*x^3 + 3*A*a)/(a^2*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.68

$$\int \frac{A+Bx^3}{x^5(a+bx^3)}\,dx = \mathrm{RootSum}\left(27t^3a^7+A^3b^4-3A^2Bab^3+3AB^2a^2b^2-B^3a^3b, \left(t\mapsto t\log\left(\frac{9t^2a^5}{A^2b^3-2ABab^2+B^2a^2b}+x\right)\right)\right) + \frac{-Aa+x^3\cdot(4Ab-4Ba)}{4a^2x^4}$$

```
[In] integrate((B*x**3+A)/x**5/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**7 + A**3*b**4 - 3*A**2*B*a*b**3 + 3*A*B**2*a**2*b**2 -
B**3*a**3*b, Lambda(_t, _t*log(9*_t**2*a**5/(A**2*b**3 - 2*A*B*a*b**2 + B**
2*a**2*b) + x))) + (-A*a + x**3*(4*A*b - 4*B*a))/(4*a**2*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^3}{x^5(a+bx^3)}\,dx = -\frac{\sqrt{3}(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{4(Ba-Ab)x^3+Aa}{4a^2x^4}$$

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3)
)/(a^2*(a/b)^(1/3)) - 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)
)/(a^2*(a/b)^(1/3)) + 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(a^2*(a/b)^(1/3)
) - 1/4*(4*(B*a - A*b)*x^3 + A*a)/(a^2*x^4)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx^3}{x^5\,(a+bx^3)}\,dx = \frac{\left(Bab\left(-\frac{a}{b}\right)^{\frac{1}{3}} - Ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3} + \frac{\sqrt{3}\left((-ab^2)^{\frac{2}{3}}\,Ba - (-ab^2)^{\frac{2}{3}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^3b} - \frac{\left((-ab^2)^{\frac{2}{3}}\,Ba - (-ab^2)^{\frac{2}{3}}\,Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^3b} - \frac{4\,Bax^3-4\,Abx^3+Aa}{4\,a^2x^4}$$

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*(B*a*b*(-a/b)^(1/3) - A*b^2*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/
b)^(1/3)))/a^3 + 1/3*sqrt(3)*((-a*b^2)^(2/3)*B*a - (-a*b^2)^(2/3)*A*b)*arct
an(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b) - 1/6*((-a*b^2)^(
2/3)*B*a - (-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^
3*b) - 1/4*(4*B*a*x^3 - 4*A*b*x^3 + A*a)/(a^2*x^4)
```

Mupad [B] (verification not implemented)

Time = 6.80 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx^3}{x^5\,(a+bx^3)}\,dx = \frac{(-b)^{1/3}\,\ln\left(a^{1/3}\,(-b)^{8/3}+b^3\,x\right)\,(A\,b-B\,a)}{3\,a^{7/3}} - \frac{\frac{A}{4\,a}-\frac{x^3\,(A\,b-B\,a)}{a^2}}{x^4} + \frac{(-b)^{1/3}\,\ln\left(a^{1/3}\,(-b)^{8/3}-2\,b^3\,x+\sqrt{3}\,a^{1/3}\,(-b)^{8/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(A\,b-B\,a)}{3\,a^{7/3}} - \frac{(-b)^{1/3}\,\ln\left(2\,b^3\,x-a^{1/3}\,(-b)^{8/3}+\sqrt{3}\,a^{1/3}\,(-b)^{8/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(A\,b-B\,a)}{3\,a^{7/3}}$$

```
[In] int((A + B*x^3)/(x^5*(a + b*x^3)),x)
```

```
[Out] ((-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) + b^3*x)*(A*b - B*a))/(3*a^(7/3)) - (A/(
4*a) - (x^3*(A*b - B*a))/a^2)/x^4 + ((-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) - 2*
b^3*x + 3^(1/2)*a^(1/3)*(-b)^(8/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/
(3*a^(7/3)) - ((-b)^(1/3)*log(2*b^3*x - a^(1/3)*(-b)^(8/3) + 3^(1/2)*a^(1/3
)*(-b)^(8/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*a^(7/3))
```

3.68 $\int \frac{A+Bx^3}{x^6(a+bx^3)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 168

$$\int \frac{A+Bx^3}{x^6(a+bx^3)}\,dx = -\frac{A}{5ax^5}+\frac{Ab-aB}{2a^2x^2}-\frac{b^{2/3}(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}} + \frac{b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}} - \frac{b^{2/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{8/3}}$$

```
[Out] -1/5*A/a/x^5+1/2*(A*b-B*a)/a^2/x^2+1/3*b^(2/3)*(A*b-B*a)*ln(a^(1/3)+b^(1/3)
*x)/a^(8/3)-1/6*b^(2/3)*(A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)
/a^(8/3)-1/3*b^(2/3)*(A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(
1/2))/a^(8/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {464, 331, 206, 31, 648, 631, 210, 642}

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)}\,dx = -\frac{b^{2/3}(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}} - \frac{b^{2/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{8/3}} + \frac{b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}} + \frac{Ab-aB}{2a^2x^2} - \frac{A}{5ax^5}$$

```
[In] Int[(A + B*x^3)/(x^6*(a + b*x^3)),x]
```

```
[Out] -1/5*A/(a*x^5) + (A*b - a*B)/(2*a^2*x^2) - (b^(2/3)*(A*b - a*B)*ArcTan[(a^(
1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(8/3)) + (b^(2/3)*(A*b -
 a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(8/3)) - (b^(2/3)*(A*b - a*B)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{A}{5ax^5} - \frac{(5Ab-5aB)\int \frac{1}{x^3(a+bx^3)}\,dx}{5a} \\
&= -\frac{A}{5ax^5} + \frac{Ab-aB}{2a^2x^2} + \frac{(b(Ab-aB))\int \frac{1}{a+bx^3}\,dx}{a^2} \\
&= -\frac{A}{5ax^5} + \frac{Ab-aB}{2a^2x^2} + \frac{(b(Ab-aB))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{8/3}} + \frac{(b(Ab-aB))\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{8/3}}
\end{aligned}$$

$$= -\frac{A}{5ax^5} + \frac{Ab-aB}{2a^2x^2} + \frac{b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}} - \frac{\left(b^{2/3}(Ab-aB)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{8/3}} + \frac{(b(Ab-aB))\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a^{7/3}}$$

$$= -\frac{A}{5ax^5} + \frac{Ab-aB}{2a^2x^2} + \frac{b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}} - \frac{b^{2/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{8/3}} + \frac{\left(b^{2/3}(Ab-aB)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{8/3}}$$

$$= -\frac{A}{5ax^5} + \frac{Ab-aB}{2a^2x^2} - \frac{b^{2/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}} + \frac{b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}} - \frac{b^{2/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{8/3}}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^3}{x^6(a+bx^3)}\,dx = \frac{-\frac{6a^{5/3}A}{x^5} + \frac{15a^{2/3}(Ab-aB)}{x^2} - 10\sqrt{3}b^{2/3}(Ab-aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 10b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right) + 5}{30a^{8/3}}$$

```
[In] Integrate[(A + B*x^3)/(x^6*(a + b*x^3)),x]
```

```
[Out] ((-6*a^(5/3)*A)/x^5 + (15*a^(2/3)*(A*b - a*B))/x^2 - 10*Sqrt[3]*b^(2/3)*(A*
b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 10*b^(2/3)*(A*b - a*
B)*Log[a^(1/3) + b^(1/3)*x] + 5*b^(2/3)*(-(A*b) + a*B)*Log[a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2])/(30*a^(8/3))
```

Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.77

method	result
default	$-\frac{A}{5a\,x^{5}}-\frac{-Ab+Ba}{2x^{2}a^{2}}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)b(Ab-Ba)}{a^{2}}$
risch	$\frac{\frac{(Ab-Ba)x^{3}}{2a^{2}}-\frac{A}{5a}}{x^{5}}+\frac{\left(\sum_{\_R=\text{RootOf}\left(a^{8}\_Z^{3}-A^{3}b^{5}+3A^{2}Ba\,b^{4}-3A\,B^{2}a^{2}b^{3}+B^{3}a^{3}b^{2}\right)}\_R\ln\left(\left(-4\_R^{3}a^{8}+3A^{3}b^{5}-9A^{2}Ba\,b^{4}+9A\,B^{2}a^{2}\right.\right.\right.}{3}$

[In] int((B*x^3+A)/x^6/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] -1/5*A/a/x^5-1/2*(-A*b+B*a)/x^2/a^2+(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*b*(A*b-B*a)/a^2

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.05

$$\int\frac{A+Bx^{3}}{x^{6}\left(a+bx^{3}\right)}\,dx=-\frac{10\sqrt{3}(Ba-Ab)x^{5}\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}-\sqrt{3}b}{3\,b}\right)-5\,(Ba-Ab)x^{5}\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(b^{2}x^{2}-abx\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}+a\right.}{30\,a^{2}x^{5}}$$

[In] integrate((B*x^3+A)/x^6/(b*x^3+a),x, algorithm="fricas")

[Out] -1/30*(10*sqrt(3)*(B*a - A*b)*x^5*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 5*(B*a - A*b)*x^5*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*b*x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) + 10*(B*a - A*b)*x^5*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)) + 15*(B*a - A*b)*x^3 + 6*A*a)/(a^2*x^5)

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.59

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)}\,dx = \operatorname{RootSum}\left(27t^3a^8-A^3b^5+3A^2Bab^4-3AB^2a^2b^3+B^3a^3b^2,\left(t\mapsto t\log\left(-\frac{3ta^3}{-Ab^2+Bab}+x\right)\right)\right) + \frac{-2Aa+x^3\cdot(5Ab-5Ba)}{10a^2x^5}$$

```
[In] integrate((B*x**3+A)/x**6/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**8 - A**3*b**5 + 3*A**2*B*a*b**4 - 3*A*B**2*a**2*b**3 +
B**3*a**3*b**2, Lambda(_t, _t*log(-3*_t*a**3/(-A*b**2 + B*a*b) + x))) + (-2
*A*a + x**3*(5*A*b - 5*B*a))/(10*a**2*x**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)}\,dx = -\frac{\sqrt{3}(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{5(Ba-Ab)x^3+2Aa}{10a^2x^5}$$

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3)
)/(a^2*(a/b)^(2/3)) + 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)
)/(a^2*(a/b)^(2/3)) - 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(a^2*(a/b)^(2/3)
) - 1/10*(5*(B*a - A*b)*x^3 + 2*A*a)/(a^2*x^5)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)}\,dx = -\frac{\sqrt{3}\left((-ab^2)^{\frac{1}{3}}Ba-(-ab^2)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^3} + \frac{(Bab-Ab^2)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^3} - \frac{\left((-ab^2)^{\frac{1}{3}}Ba-(-ab^2)^{\frac{1}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^3} - \frac{5Bax^3-5Abx^3+2Aa}{10a^2x^5}$$

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(
2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^3 + 1/3*(B*a*b - A*b^2)*(-a/b)^(1/3)*lo
g(abs(x - (-a/b)^(1/3)))/a^3 - 1/6*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b
)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^3 - 1/10*(5*B*a*x^3 - 5*A*b*x^
3 + 2*A*a)/(a^2*x^5)
```

Mupad [B] (verification not implemented)

Time = 6.73 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)}\,dx = \frac{b^{2/3}\ln\left(b^{1/3}x+a^{1/3}\right)(Ab-Ba)}{3a^{8/3}} - \frac{\frac{A}{5a}-\frac{x^3(Ab-Ba)}{2a^2}}{x^5} - \frac{b^{2/3}\ln\left(a^{1/3}-2b^{1/3}x+\sqrt{3}a^{1/3}1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}1\mathrm{i}}{2}\right)(Ab-Ba)}{3a^{8/3}} + \frac{b^{2/3}\ln\left(2b^{1/3}x-a^{1/3}+\sqrt{3}a^{1/3}1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}1\mathrm{i}}{2}\right)(Ab-Ba)}{3a^{8/3}}$$

```
[In] int((A + B*x^3)/(x^6*(a + b*x^3)),x)
```

```
[Out] (b^(2/3)*log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a^(8/3)) - (A/(5*a) - (x^
3*(A*b - B*a))/(2*a^2))/x^5 - (b^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x
 + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*a^(8/3)) + (b^(2/3)*log(
3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B
*a))/(3*a^(8/3))
```

3.69 $\int \frac{A+Bx^3}{x^7(a+bx^3)} dx$

Optimal result

Integrand size = 20, antiderivative size = 69

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)}\,dx=-\frac{A}{6ax^6}+\frac{Ab-aB}{3a^2x^3}+\frac{b(Ab-aB)\log(x)}{a^3}-\frac{b(Ab-aB)\log\left(a+bx^3\right)}{3a^3}$$

```
[Out] -1/6*A/a/x^6+1/3*(A*b-B*a)/a^2/x^3+b*(A*b-B*a)*ln(x)/a^3-1/3*b*(A*b-B*a)*ln
(b*x^3+a)/a^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 78}

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)}\,dx=-\frac{b(Ab-aB)\log\left(a+bx^3\right)}{3a^3}+\frac{b\log(x)(Ab-aB)}{a^3}+\frac{Ab-aB}{3a^2x^3}-\frac{A}{6ax^6}$$

```
[In] Int[(A + B*x^3)/(x^7*(a + b*x^3)),x]
```

```
[Out] -1/6*A/(a*x^6) + (A*b - a*B)/(3*a^2*x^3) + (b*(A*b - a*B)*Log[x])/a^3 - (b*
(A*b - a*B)*Log[a + b*x^3])/(3*a^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{A}{ax^3}+\frac{-Ab+aB}{a^2x^2}-\frac{b(-Ab+aB)}{a^3x}+\frac{b^2(-Ab+aB)}{a^3(a+bx)}\right)dx, x, x^3\right)\\ &= -\frac{A}{6ax^6}+\frac{Ab-aB}{3a^2x^3}+\frac{b(Ab-aB)\log(x)}{a^3}-\frac{b(Ab-aB)\log\left(a+bx^3\right)}{3a^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)}\,dx = \frac{-a(aA-2Abx^3+2aBx^3)+6b(Ab-aB)x^6\log(x)+2b(-Ab+aB)x^6\log\left(a+bx^3\right)}{6a^3x^6}$$

```
[In] Integrate[(A + B*x^3)/(x^7*(a + b*x^3)),x]
```

```
[Out] (-(a*(a*A - 2*A*b*x^3 + 2*a*B*x^3)) + 6*b*(A*b - a*B)*x^6*Log[x] + 2*b*(-(A
*b) + a*B)*x^6*Log[a + b*x^3])/(6*a^3*x^6)
```

Maple [A] (verified)

Time = 4.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{A}{6a\,x^{6}}-\frac{-Ab+Ba}{3x^{3}a^{2}}+\frac{b(Ab-Ba)\ln(x)}{a^{3}}-\frac{b(Ab-Ba)\ln(b\,x^{3}+a)}{3a^{3}}$	64
norman	$\frac{-\frac{A}{6a}+\frac{(Ab-Ba)x^{3}}{3a^{2}}}{x^{6}}+\frac{b(Ab-Ba)\ln(x)}{a^{3}}-\frac{b(Ab-Ba)\ln(b\,x^{3}+a)}{3a^{3}}$	66
risch	$\frac{-\frac{A}{6a}+\frac{(Ab-Ba)x^{3}}{3a^{2}}}{x^{6}}+\frac{b^{2}\ln(x)A}{a^{3}}-\frac{b\ln(x)B}{a^{2}}-\frac{b^{2}\ln(b\,x^{3}+a)A}{3a^{3}}+\frac{b\ln(b\,x^{3}+a)B}{3a^{2}}$	80
parallelrisch	$\frac{6A\ln(x)x^{6}b^{2}-2A\ln(b\,x^{3}+a)x^{6}b^{2}-6B\ln(x)x^{6}ab+2B\ln(b\,x^{3}+a)x^{6}ab+2aAb\,x^{3}-2a^{2}B\,x^{3}-a^{2}A}{6a^{3}x^{6}}$	87

```
[In] int((B*x^3+A)/x^7/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*A/a/x^6-1/3*(-A*b+B*a)/x^3/a^2+b*(A*b-B*a)*ln(x)/a^3-1/3*b*(A*b-B*a)*l
n(b*x^3+a)/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^3}{x^7\,(a+bx^3)}\,dx = \frac{2\,(Bab-Ab^2)x^6\log\left(bx^3+a\right)-6\,(Bab-Ab^2)x^6\log\left(x\right)-2\,(Ba^2-Aab)x^3-Aa^2}{6\,a^3x^6}$$

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*(B*a*b - A*b^2)*x^6*log(b*x^3 + a) - 6*(B*a*b - A*b^2)*x^6*log(x) -
2*(B*a^2 - A*a*b)*x^3 - A*a^2)/(a^3*x^6)
```

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^3}{x^7\,(a+bx^3)}\,dx = \frac{-Aa+x^3\cdot(2Ab-2Ba)}{6a^2x^6}-\frac{b(-Ab+Ba)\log\left(x\right)}{a^3}+\frac{b(-Ab+Ba)\log\left(\frac{a}{b}+x^3\right)}{3a^3}$$

```
[In] integrate((B*x**3+A)/x**7/(b*x**3+a),x)
```

```
[Out] (-A*a + x**3*(2*A*b - 2*B*a))/(6*a**2*x**6) - b*(-A*b + B*a)*log(x)/a**3 +
b*(-A*b + B*a)*log(a/b + x**3)/(3*a**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^3}{x^7(a+bx^3)}\,dx = \frac{(Bab-Ab^2)\log(bx^3+a)}{3\,a^3} - \frac{(Bab-Ab^2)\log(x^3)}{3\,a^3} - \frac{2\,(Ba-Ab)x^3+Aa}{6\,a^2x^6}$$

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*(B*a*b - A*b^2)*log(b*x^3 + a)/a^3 - 1/3*(B*a*b - A*b^2)*log(x^3)/a^3 -
 1/6*(2*(B*a - A*b)*x^3 + A*a)/(a^2*x^6)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.43

$$\int \frac{A+Bx^3}{x^7(a+bx^3)}\,dx = -\frac{(Bab-Ab^2)\log(|x|)}{a^3} + \frac{(Bab^2-Ab^3)\log(|bx^3+a|)}{3\,a^3b} + \frac{3\,Babx^6-3\,Ab^2x^6-2\,Ba^2x^3+2\,Aabx^3-Aa^2}{6\,a^3x^6}$$

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -(B*a*b - A*b^2)*log(abs(x))/a^3 + 1/3*(B*a*b^2 - A*b^3)*log(abs(b*x^3 + a)
)/(a^3*b) + 1/6*(3*B*a*b*x^6 - 3*A*b^2*x^6 - 2*B*a^2*x^3 + 2*A*a*b*x^3 - A*
a^2)/(a^3*x^6)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^3}{x^7(a+bx^3)}\,dx = \frac{\ln(x)\,\left(A\,b^2-B\,a\,b\right)}{a^3} - \frac{\ln\left(b\,x^3+a\right)\,\left(A\,b^2-B\,a\,b\right)}{3\,a^3} - \frac{\frac{A}{6\,a}-\frac{x^3\,(A\,b-B\,a)}{3\,a^2}}{x^6}$$

```
[In] int((A + B*x^3)/(x^7*(a + b*x^3)),x)
```

```
[Out] (log(x)*(A*b^2 - B*a*b))/a^3 - (log(a + b*x^3)*(A*b^2 - B*a*b))/(3*a^3) - (
A/(6*a) - (x^3*(A*b - B*a))/(3*a^2))/x^6
```

3.70 $\int \frac{A+Bx^3}{x^8(a+bx^3)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 184

$$\int \frac{A+Bx^3}{x^8(a+bx^3)}\,dx = -\frac{A}{7ax^7}+\frac{Ab-aB}{4a^2x^4}-\frac{b(Ab-aB)}{a^3x}+\frac{b^{4/3}(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{10/3}} + \frac{b^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}} - \frac{b^{4/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{10/3}}$$

```
[Out] -1/7*A/a/x^7+1/4*(A*b-B*a)/a^2/x^4-b*(A*b-B*a)/a^3/x+1/3*b^(4/3)*(A*b-B*a)*
ln(a^(1/3)+b^(1/3)*x)/a^(10/3)-1/6*b^(4/3)*(A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/a^(10/3)+1/3*b^(4/3)*(A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^
(1/3)*x)/a^(1/3)*3^(1/2))/a^(10/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {464, 331, 298, 31, 648, 631, 210, 642}

$$\int \frac{A+Bx^3}{x^8\,(a+bx^3)}\,dx = \frac{b^{4/3}(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{10/3}} - \frac{b^{4/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{10/3}} + \frac{b^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}} - \frac{b(Ab-aB)}{a^3x} + \frac{Ab-aB}{4a^2x^4} - \frac{A}{7ax^7}$$

```
[In] Int[(A + B*x^3)/(x^8*(a + b*x^3)),x]
```

```
[Out] -1/7*A/(a*x^7) + (A*b - a*B)/(4*a^2*x^4) - (b*(A*b - a*B))/(a^3*x) + (b^(4/
3)*(A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(10/3)) + (b^(4/3)*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(10/3)) - (
b^(4/3)*(A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1
0/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{A}{7ax^7} - \frac{(7Ab-7aB)\int \frac{1}{x^5(a+bx^3)}\,dx}{7a} \\
&= -\frac{A}{7ax^7} + \frac{Ab-aB}{4a^2x^4} + \frac{(b(Ab-aB))\int \frac{1}{x^2(a+bx^3)}\,dx}{a^2} \\
&= -\frac{A}{7ax^7} + \frac{Ab-aB}{4a^2x^4} - \frac{b(Ab-aB)}{a^3x} - \frac{(b^2(Ab-aB))\int \frac{x}{a+bx^3}\,dx}{a^3} \\
&= -\frac{A}{7ax^7} + \frac{Ab-aB}{4a^2x^4} - \frac{b(Ab-aB)}{a^3x} + \frac{\left(b^{5/3}(Ab-aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{10/3}} \\
&\quad - \frac{\left(b^{5/3}(Ab-aB)\right)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{10/3}}
\end{aligned}
$$

$$=-\frac{A}{7ax^7}+\frac{Ab-aB}{4a^2x^4}-\frac{b(Ab-aB)}{a^3x}+\frac{b^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}}-\frac{\left(b^{4/3}(Ab-aB)\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{10/3}}-\frac{\left(b^{5/3}(Ab-aB)\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a^3}$$

$$=-\frac{A}{7ax^7}+\frac{Ab-aB}{4a^2x^4}-\frac{b(Ab-aB)}{a^3x}+\frac{b^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}}-\frac{b^{4/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{10/3}}-\frac{\left(b^{4/3}(Ab-aB)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{10/3}}$$

$$=-\frac{A}{7ax^7}+\frac{Ab-aB}{4a^2x^4}-\frac{b(Ab-aB)}{a^3x}+\frac{b^{4/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{10/3}}+\frac{b^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}}-\frac{b^{4/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{10/3}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.94

$$\int\frac{A+Bx^3}{x^8\left(a+bx^3\right)}\,dx$$

$$=\frac{-\frac{12a^{7/3}A}{x^7}+\frac{21a^{4/3}(Ab-aB)}{x^4}+\frac{84\sqrt[3]{a}b(-Ab+aB)}{x}+28\sqrt{3}b^{4/3}(Ab-aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+28b^{4/3}(Ab-aB)\text{lo}}{84a^{10/3}}$$

```
[In] Integrate[(A + B*x^3)/(x^8*(a + b*x^3)),x]
```

```
[Out] ((-12*a^(7/3)*A)/x^7 + (21*a^(4/3)*(A*b - a*B))/x^4 + (84*a^(1/3)*b*(-(A*b)
 + a*B))/x + 28*Sqrt[3]*b^(4/3)*(A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]] + 28*b^(4/3)*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x] + 14*b^(4/3)
*(-(A*b) + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(84*a^(10/3
))
```

Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.82

method	result
default	$-\frac{A}{7a\,x^{7}}-\frac{-Ab+Ba}{4x^{4}a^{2}}-\frac{b(Ab-Ba)}{a^{3}x}-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^{2}(Ab-Ba)}{a^{3}}$
risch	$\frac{-\frac{b(Ab-Ba)x^{6}}{a^{3}}+\frac{(Ab-Ba)x^{3}}{4a^{2}}-\frac{A}{7a}}{x^{7}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{10}\_Z^{3}-A^{3}b^{7}+3A^{2}Ba\,b^{6}-3A\,B^{2}a^{2}b^{5}+B^{3}a^{3}b^{4}\right)}\_R\ln\left(\left(-4a^{10}\_R^{3}+3A^{3}b^{7}-9A^{2}\right.\right.\right.}{3}$

[In] int((B*x^3+A)/x^8/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] -1/7*A/a/x^7-1/4*(-A*b+B*a)/x^4/a^2-b*(A*b-B*a)/a^3/x-(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*b^2*(A*b-B*a)/a^3

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^{3}}{x^{8}\left(a+bx^{3}\right)}\,dx = \frac{28\sqrt{3}(Bab-Ab^{2})x^{7}\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+14\,(Bab-Ab^{2})x^{7}\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^{2}-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right.\right.}{84\,a^{3}x^{7}}$$

[In] integrate((B*x^3+A)/x^8/(b*x^3+a),x, algorithm="fricas")

[Out] 1/84*(28*sqrt(3)*(B*a*b - A*b^2)*x^7*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3)) + 14*(B*a*b - A*b^2)*x^7*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 28*(B*a*b - A*b^2)*x^7*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3)) + 84*(B*a*b - A*b^2)*x^6 - 21*(B*a^2 - A*a*b)*x^3 - 12*A*a^2)/(a^3*x^7)

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.76

$$\int \frac{A+Bx^3}{x^8(a+bx^3)}\,dx$$
$$=\operatorname{RootSum}\left(27t^3a^{10}-A^3b^7+3A^2Bab^6-3AB^2a^2b^5+B^3a^3b^4,\left(t\mapsto t\log\left(\frac{9t^2a^7}{A^2b^5-2ABab^4+B^2a^2b^3}+x\right.\right.\right.$$
$$+\frac{-4Aa^2+x^6(-28Ab^2+28Bab)+x^3\cdot(7Aab-7Ba^2)}{28a^3x^7}$$

```
[In] integrate((B*x**3+A)/x**8/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**10 - A**3*b**7 + 3*A**2*B*a*b**6 - 3*A*B**2*a**2*b**5 +
 B**3*a**3*b**4, Lambda(_t, _t*log(9*_t**2*a**7/(A**2*b**5 - 2*A*B*a*b**4 +
 B**2*a**2*b**3) + x))) + (-4*A*a**2 + x**6*(-28*A*b**2 + 28*B*a*b) + x**3*
(7*A*a*b - 7*B*a**2))/(28*a**3*x**7)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^3}{x^8(a+bx^3)}\,dx=\frac{\sqrt{3}(Bab-Ab^2)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(Bab-Ab^2)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(Bab-Ab^2)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{28(Bab-Ab^2)x^6-7(Ba^2-Aab)x^3-4Aa^2}{28a^3x^7}$$

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(B*a*b - A*b^2)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1
/3))/(a^3*(a/b)^(1/3)) + 1/6*(B*a*b - A*b^2)*log(x^2 - x*(a/b)^(1/3) + (a/b
)^(2/3))/(a^3*(a/b)^(1/3)) - 1/3*(B*a*b - A*b^2)*log(x + (a/b)^(1/3))/(a^3*
(a/b)^(1/3)) + 1/28*(28*(B*a*b - A*b^2)*x^6 - 7*(B*a^2 - A*a*b)*x^3 - 4*A*a
^2)/(a^3*x^7)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx^3}{x^8\,(a+bx^3)}\,dx = -\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}Ba-\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^4} - \frac{\left(Bab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}-Ab^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^4} + \frac{\left(\left(-ab^2\right)^{\frac{2}{3}}Ba-\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^4} + \frac{28\,Babx^6-28\,Ab^2x^6-7\,Ba^2x^3+7\,Aabx^3-4\,Aa^2}{28\,a^3x^7}$$

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(2/3)*B*a - (-a*b^2)^(2/3)*A*b)*arctan(1/3*sqrt(3)*(
2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^4 - 1/3*(B*a*b^2*(-a/b)^(1/3) - A*b^3*(
-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 + 1/6*((-a*b^2)^(2
/3)*B*a - (-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^4
+ 1/28*(28*B*a*b*x^6 - 28*A*b^2*x^6 - 7*B*a^2*x^3 + 7*A*a*b*x^3 - 4*A*a^2)/
(a^3*x^7)
```

Mupad [B] (verification not implemented)

Time = 6.81 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^3}{x^8\,(a+bx^3)}\,dx = \frac{b^{4/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-B\,a\right)}{3\,a^{10/3}} - \frac{\frac{A}{7\,a}-\frac{x^3\,\left(A\,b-B\,a\right)}{4\,a^2}+\frac{b\,x^6\,\left(A\,b-B\,a\right)}{a^3}}{x^7} + \frac{b^{4/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{10/3}} - \frac{b^{4/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{10/3}}$$

```
[In] int((A + B*x^3)/(x^8*(a + b*x^3)),x)
```

```
[Out] (b^(4/3)*log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a^(10/3)) - (A/(7*a) - (x
^3*(A*b - B*a))/(4*a^2) + (b*x^6*(A*b - B*a))/a^3)/x^7 + (b^(4/3)*log(3^(1/
2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/
(3*a^(10/3)) - (b^(4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3
^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*a^(10/3))
```

3.71 $\int \frac{x^9(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 233

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^2}\,dx = -\frac{a(7Ab-10aB)x}{3b^4}+\frac{(7Ab-10aB)x^4}{12b^3}-\frac{(7Ab-10aB)x^7}{21ab^2} + \frac{(Ab-aB)x^{10}}{3ab\,(a+bx^3)}-\frac{a^{4/3}(7Ab-10aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{13/3}} + \frac{a^{4/3}(7Ab-10aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{13/3}} - \frac{a^{4/3}(7Ab-10aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{13/3}}$$

```
[Out] -1/3*a*(7*A*b-10*B*a)*x/b^4+1/12*(7*A*b-10*B*a)*x^4/b^3-1/21*(7*A*b-10*B*a)
*x^7/a/b^2+1/3*(A*b-B*a)*x^10/a/b/(b*x^3+a)+1/9*a^(4/3)*(7*A*b-10*B*a)*ln(a
^(1/3)+b^(1/3)*x)/b^(13/3)-1/18*a^(4/3)*(7*A*b-10*B*a)*ln(a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/b^(13/3)-1/9*a^(4/3)*(7*A*b-10*B*a)*arctan(1/3*(a^(1/
3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(13/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {468, 308, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^2}\,dx = -\frac{a^{4/3}(7Ab-10aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{13/3}} - \frac{a^{4/3}(7Ab-10aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{13/3}} + \frac{a^{4/3}(7Ab-10aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{13/3}} - \frac{ax(7Ab-10aB)}{3b^4} + \frac{x^4(7Ab-10aB)}{12b^3} - \frac{x^7(7Ab-10aB)}{21ab^2} + \frac{x^{10}(Ab-aB)}{3ab(a+bx^3)}$$

```
[In] Int[(x^9*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -1/3*(a*(7*A*b - 10*a*B)*x)/b^4 + ((7*A*b - 10*a*B)*x^4)/(12*b^3) - ((7*A*b
 - 10*a*B)*x^7)/(21*a*b^2) + ((A*b - a*B)*x^10)/(3*a*b*(a + b*x^3)) - (a^(4
/3)*(7*A*b - 10*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*
Sqrt[3]*b^(13/3)) + (a^(4/3)*(7*A*b - 10*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*
b^(13/3)) - (a^(4/3)*(7*A*b - 10*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(18*b^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(Ab-aB)x^{10}}{3ab\,(a+bx^3)}+\frac{(-7Ab+10aB)\int\frac{x^9}{a+bx^3}\,dx}{3ab}\\
&= \frac{(Ab-aB)x^{10}}{3ab\,(a+bx^3)}+\frac{(-7Ab+10aB)\int\left(\frac{a^2}{b^3}-\frac{ax^3}{b^2}+\frac{x^6}{b}-\frac{a^3}{b^3(a+bx^3)}\right)dx}{3ab}\\
&= -\frac{a(7Ab-10aB)x}{3b^4}+\frac{(7Ab-10aB)x^4}{12b^3}-\frac{(7Ab-10aB)x^7}{21ab^2}\\
&\quad+\frac{(Ab-aB)x^{10}}{3ab\,(a+bx^3)}+\frac{\left(a^2(7Ab-10aB)\right)\int\frac{1}{a+bx^3}\,dx}{3b^4}
\end{aligned}$$

$$= -\frac{a(7Ab-10aB)x}{3b^4} + \frac{(7Ab-10aB)x^4}{12b^3} - \frac{(7Ab-10aB)x^7}{21ab^2} + \frac{(Ab-aB)x^{10}}{3ab(a+bx^3)} + \frac{\left(a^{4/3}(7Ab-10aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9b^4} + \frac{\left(a^{4/3}(7Ab-10aB)\right)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9b^4}$$

$$= -\frac{a(7Ab-10aB)x}{3b^4} + \frac{(7Ab-10aB)x^4}{12b^3} - \frac{(7Ab-10aB)x^7}{21ab^2} + \frac{(Ab-aB)x^{10}}{3ab(a+bx^3)} + \frac{a^{4/3}(7Ab-10aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{13/3}} - \frac{\left(a^{4/3}(7Ab-10aB)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18b^{13/3}} + \frac{\left(a^{5/3}(7Ab-10aB)\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^4}$$

$$= -\frac{a(7Ab-10aB)x}{3b^4} + \frac{(7Ab-10aB)x^4}{12b^3} - \frac{(7Ab-10aB)x^7}{21ab^2} + \frac{(Ab-aB)x^{10}}{3ab(a+bx^3)} + \frac{a^{4/3}(7Ab-10aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{13/3}} - \frac{a^{4/3}(7Ab-10aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{13/3}} + \frac{\left(a^{4/3}(7Ab-10aB)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3b^{13/3}}$$

$$= -\frac{a(7Ab-10aB)x}{3b^4} + \frac{(7Ab-10aB)x^4}{12b^3} - \frac{(7Ab-10aB)x^7}{21ab^2} + \frac{(Ab-aB)x^{10}}{3ab(a+bx^3)} - \frac{a^{4/3}(7Ab-10aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{13/3}} + \frac{a^{4/3}(7Ab-10aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{13/3}} - \frac{a^{4/3}(7Ab-10aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{13/3}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.87

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$= \frac{252a\sqrt[3]{b}(-2Ab+3aB)x+63b^{4/3}(Ab-2aB)x^4+36b^{7/3}Bx^7+\frac{84a^2\sqrt[3]{b}(-Ab+aB)x}{a+bx^3}+28\sqrt{3}a^{4/3}(-7Ab+10a}{}$$

```
[In] Integrate[(x^9*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (252*a*b^(1/3)*(-2*A*b + 3*a*B)*x + 63*b^(4/3)*(A*b - 2*a*B)*x^4 + 36*b^(7/
3)*B*x^7 + (84*a^2*b^(1/3)*(-(A*b) + a*B)*x)/(a + b*x^3) + 28*Sqrt[3]*a^(4/
3)*(-7*A*b + 10*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 28*a^(4/
3)*(-7*A*b + 10*a*B)*Log[a^(1/3) + b^(1/3)*x] + 14*a^(4/3)*(-7*A*b + 10*a*B
)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(252*b^(13/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.17 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.49

method	result
risch	$\frac{B\,x^{7}}{7b^{2}}+\frac{A\,x^{4}}{4b^{2}}-\frac{Ba\,x^{4}}{2b^{3}}-\frac{2aAx}{b^{3}}+\frac{3a^{2}Bx}{b^{4}}+\frac{\left(-\frac{1}{3}a^{2}bA+\frac{1}{3}a^{3}B\right)x}{b^{4}\left(b\,x^{3}+a\right)}+\frac{a^{2}\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{(7Ab-10Ba)\ln\left(x-\_R\right)}{\_R^{2}}\right)}{9b^{5}}$
default	$-\frac{-\frac{1}{7}b^{2}B\,x^{7}-\frac{1}{4}A\,b^{2}x^{4}+\frac{1}{2}Bab\,x^{4}+2aAbx-3a^{2}Bx}{b^{4}}+\frac{a^{2}\left(\frac{\left(-\frac{Ab}{3}+\frac{Ba}{3}\right)x}{b\,x^{3}+a}+\frac{(7Ab-10Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\cdots}{\cdots}\right)}{3}\cdots\right)}{b^{4}}$

[In] int(x^9*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)

[Out] 1/7*B*x^7/b^2+1/4/b^2*A*x^4-1/2/b^3*B*a*x^4-2/b^3*a*A*x+3/b^4*a^2*B*x+(-1/3*a^2*b*A+1/3*a^3*B)*x/b^4/(b*x^3+a)+1/9/b^5*a^2*sum((7*A*b-10*B*a)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.16

$$\int\frac{x^{9}(A+Bx^{3})}{\left(a+bx^{3}\right)^{2}}\,dx$$

$$=\frac{36\,Bb^{3}x^{10}-9\,(10\,Bab^{2}-7\,Ab^{3})x^{7}+63\,(10\,Ba^{2}b-7\,Aab^{2})x^{4}-28\,\sqrt{3}(10\,Ba^{3}-7\,Aa^{2}b+(10\,Ba^{2}b-7\,Aa\cdots}{\cdots}$$

[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] 1/252*(36*B*b^3*x^10 - 9*(10*B*a*b^2 - 7*A*b^3)*x^7 + 63*(10*B*a^2*b - 7*A*a*b^2)*x^4 - 28*sqrt(3)*(10*B*a^3 - 7*A*a^2*b + (10*B*a^2*b - 7*A*a*b^2)*x^3)*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 14*(

```
10*B*a^3 - 7*A*a^2*b + (10*B*a^2*b - 7*A*a*b^2)*x^3)*(a/b)^(1/3)*log(x^2 -
x*(a/b)^(1/3) + (a/b)^(2/3)) - 28*(10*B*a^3 - 7*A*a^2*b + (10*B*a^2*b - 7*A
*a*b^2)*x^3)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 84*(10*B*a^3 - 7*A*a^2*b)*x
)/(b^5*x^3 + a*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.67

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Bx^7}{7b^2} + x^4\left(\frac{A}{4b^2} - \frac{Ba}{2b^3}\right) + x\left(-\frac{2Aa}{b^3} + \frac{3Ba^2}{b^4}\right) + \frac{x(-Aa^2b+Ba^3)}{3ab^4+3b^5x^3} + \mathrm{RootSum}\left(729t^3b^{13} - 343A^3a^4b^3 + 1470A^2Ba^5b^2 - 2100AB^2a^6b + 1000B^3a^7, \left(t \mapsto t\log\left(-\frac{9tb}{-7Aab+\dots}\right.\right.\right.$$

```
[In] integrate(x**9*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x**7/(7*b**2) + x**4*(A/(4*b**2) - B*a/(2*b**3)) + x*(-2*A*a/b**3 + 3*B*a
**2/b**4) + x*(-A*a**2*b + B*a**3)/(3*a*b**4 + 3*b**5*x**3) + RootSum(729*_
t**3*b**13 - 343*A**3*a**4*b**3 + 1470*A**2*B*a**5*b**2 - 2100*A*B**2*a**6*
b + 1000*B**3*a**7, Lambda(_t, _t*log(-9*_t*b**4/(-7*A*a*b + 10*B*a**2) + x
)))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.94

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(Ba^3-Aa^2b)x}{3\,(b^5x^3+ab^4)} + \frac{4\,Bb^2x^7 - 7\,(2\,Bab - Ab^2)x^4 + 28\,(3\,Ba^2 - 2\,Aab)x}{28\,b^4} - \frac{\sqrt{3}(10\,Ba^3 - 7\,Aa^2b)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^5\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(10\,Ba^3 - 7\,Aa^2b)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^5\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(10\,Ba^3 - 7\,Aa^2b)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^5\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(B*a^3 - A*a^2*b)*x/(b^5*x^3 + a*b^4) + 1/28*(4*B*b^2*x^7 - 7*(2*B*a*b
- A*b^2)*x^4 + 28*(3*B*a^2 - 2*A*a*b)*x)/b^4 - 1/9*sqrt(3)*(10*B*a^3 - 7*A*
```

```
a^2*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^5*(a/b)^(2/3)
) + 1/18*(10*B*a^3 - 7*A*a^2*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^5
*(a/b)^(2/3)) - 1/9*(10*B*a^3 - 7*A*a^2*b)*log(x + (a/b)^(1/3))/(b^5*(a/b)^
(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.05

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^2}\,dx = -\frac{\sqrt{3}\left(10\,(-ab^2)^{\frac{1}{3}}\,Ba^2 - 7\,(-ab^2)^{\frac{1}{3}}\,Aab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^5} + \frac{(10\,Ba^3 - 7\,Aa^2b)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^4} - \frac{\left(10\,(-ab^2)^{\frac{1}{3}}\,Ba^2 - 7\,(-ab^2)^{\frac{1}{3}}\,Aab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^5} + \frac{Ba^3x - Aa^2bx}{3\,(bx^3+a)b^4} + \frac{4\,Bb^{12}x^7 - 14\,Bab^{11}x^4 + 7\,Ab^{12}x^4 + 84\,Ba^2b^{10}x - 56\,Aab^{11}x}{28\,b^{14}}$$

```
[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(10*(-a*b^2)^(1/3)*B*a^2 - 7*(-a*b^2)^(1/3)*A*a*b)*arctan(1/3*
sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^5 + 1/9*(10*B*a^3 - 7*A*a^2*b)
*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4) - 1/18*(10*(-a*b^2)^(1/3)*
B*a^2 - 7*(-a*b^2)^(1/3)*A*a*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^
5 + 1/3*(B*a^3*x - A*a^2*b*x)/((b*x^3 + a)*b^4) + 1/28*(4*B*b^12*x^7 - 14*B
*a*b^11*x^4 + 7*A*b^12*x^4 + 84*B*a^2*b^10*x - 56*A*a*b^11*x)/b^14
```

Mupad [B] (verification not implemented)

Time = 6.80 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.90

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^2}\,dx = x^4\left(\frac{A}{4\,b^2}-\frac{B\,a}{2\,b^3}\right)-x\left(\frac{2\,a\,\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b}+\frac{B\,a^2}{b^4}\right)+\frac{B\,x^7}{7\,b^2}+\frac{x\,\left(\frac{B\,a^3}{3}-\frac{A\,a^2\,b}{3}\right)}{b^5\,x^3+a\,b^4}+\frac{a^{4/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(7\,A\,b-10\,B\,a\right)}{9\,b^{13/3}}-\frac{a^{4/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(7\,A\,b-10\,B\,a\right)}{9\,b^{13/3}}+\frac{a^{4/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(7\,A\,b-10\,B\,a\right)}{9\,b^{13/3}}$$

```
[In] int((x^9*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] x^4*(A/(4*b^2) - (B*a)/(2*b^3)) - x*((2*a*(A/b^2 - (2*B*a)/b^3))/b + (B*a^2
)/b^4) + (B*x^7)/(7*b^2) + (x*((B*a^3)/3 - (A*a^2*b)/3))/(a*b^4 + b^5*x^3)
+ (a^(4/3)*log(b^(1/3)*x + a^(1/3))*(7*A*b - 10*B*a))/(9*b^(13/3)) - (a^(4/
3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(
7*A*b - 10*B*a))/(9*b^(13/3)) + (a^(4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)
*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(7*A*b - 10*B*a))/(9*b^(13/3))
```

3.72 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 82

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(Ab-2aB)x^3}{3b^3}+\frac{Bx^6}{6b^2}-\frac{a^2(Ab-aB)}{3b^4(a+bx^3)}-\frac{a(2Ab-3aB)\log(a+bx^3)}{3b^4}$$

```
[Out] 1/3*(A*b-2*B*a)*x^3/b^3+1/6*B*x^6/b^2-1/3*a^2*(A*b-B*a)/b^4/(b*x^3+a)-1/3*a
*(2*A*b-3*B*a)*ln(b*x^3+a)/b^4
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 78}

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2}\,dx = -\frac{a^2(Ab-aB)}{3b^4(a+bx^3)}-\frac{a(2Ab-3aB)\log(a+bx^3)}{3b^4}+\frac{x^3(Ab-2aB)}{3b^3}+\frac{Bx^6}{6b^2}$$

```
[In] Int[(x^8*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((A*b - 2*a*B)*x^3)/(3*b^3) + (B*x^6)/(6*b^2) - (a^2*(A*b - a*B))/(3*b^4*(a
 + b*x^3)) - (a*(2*A*b - 3*a*B)*Log[a + b*x^3])/(3*b^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx)^2}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{Ab-2aB}{b^3}+\frac{Bx}{b^2}-\frac{a^2(-Ab+aB)}{b^3(a+bx)^2}+\frac{a(-2Ab+3aB)}{b^3(a+bx)}\right)\,dx, x, x^3\right)\\ &= \frac{(Ab-2aB)x^3}{3b^3}+\frac{Bx^6}{6b^2}-\frac{a^2(Ab-aB)}{3b^4\left(a+bx^3\right)}-\frac{a(2Ab-3aB)\log\left(a+bx^3\right)}{3b^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88

$$\int \frac{x^8(A+Bx^3)}{\left(a+bx^3\right)^2}\,dx = \frac{2b(Ab-2aB)x^3+b^2Bx^6+\frac{2a^2(-Ab+aB)}{a+bx^3}+2a(-2Ab+3aB)\log\left(a+bx^3\right)}{6b^4}$$

```
[In] Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^2,x]
[Out] (2*b*(A*b - 2*a*B)*x^3 + b^2*B*x^6 + (2*a^2*(-(A*b) + a*B))/(a + b*x^3) + 2
*a*(-2*A*b + 3*a*B)*Log[a + b*x^3])/(6*b^4)
```

Maple [A] (verified)

Time = 4.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

method	result
default	$\frac{(bBx^3+Ab-2Ba)^2}{6b^4B}-\frac{a\left(\frac{(2Ab-3Ba)\ln(bx^3+a)}{b}+\frac{a(Ab-Ba)}{b(bx^3+a)}\right)}{3b^3}$
norman	$\frac{\frac{Bx^9}{6b}-\frac{a(2abA-3a^2B)}{3b^4}+\frac{(2Ab-3Ba)x^6}{6b^2}}{bx^3+a}-\frac{a(2Ab-3Ba)\ln(bx^3+a)}{3b^4}$
parallelrisch	$-\frac{-b^3Bx^9-2x^6b^3A+3Bx^6ab^2+4A\ln(bx^3+a)x^3ab^2-6B\ln(bx^3+a)x^3a^2b+4A\ln(bx^3+a)a^2b-6B\ln(bx^3+a)a^3+4a^2bA-6a}{6b^4(bx^3+a)}$
risch	$\frac{Bx^6}{6b^2}+\frac{Ax^3}{3b^2}-\frac{2Bax^3}{3b^3}+\frac{A^2}{6b^2B}-\frac{2aA}{3b^3}+\frac{2a^2B}{3b^4}-\frac{a^2A}{3b^3(bx^3+a)}+\frac{a^3B}{3b^4(bx^3+a)}-\frac{2a\ln(bx^3+a)A}{3b^3}+\frac{a^2\ln(bx^3+a)}{b^4}$

```
[In] int(x^8*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(B*b*x^3+A*b-2*B*a)^2/b^4/B-1/3*a/b^3*((2*A*b-3*B*a)/b*ln(b*x^3+a)+a*(A
*b-B*a)/b/(b*x^3+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.48

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Bb^3x^9-(3\,Bab^2-2\,Ab^3)x^6+2\,Ba^3-2\,Aa^2b-2\,(2\,Ba^2b-Aab^2)x^3+2\,(3\,Ba^3-2\,Aa^2b+(3\,Ba^2b-2\,A}{6\,(b^5x^3+ab^4)}$$

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(B*b^3*x^9 - (3*B*a*b^2 - 2*A*b^3)*x^6 + 2*B*a^3 - 2*A*a^2*b - 2*(2*B*a
^2*b - A*a*b^2)*x^3 + 2*(3*B*a^3 - 2*A*a^2*b + (3*B*a^2*b - 2*A*a*b^2)*x^3)
*log(b*x^3 + a))/(b^5*x^3 + a*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Bx^6}{6b^2}+\frac{a(-2Ab+3Ba)\log(a+bx^3)}{3b^4}+x^3\left(\frac{A}{3b^2}-\frac{2Ba}{3b^3}\right)+\frac{-Aa^2b+Ba^3}{3ab^4+3b^5x^3}$$

```
[In] integrate(x**8*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x**6/(6*b**2) + a*(-2*A*b + 3*B*a)*log(a + b*x**3)/(3*b**4) + x**3*(A/(3*
b**2) - 2*B*a/(3*b**3)) + (-A*a**2*b + B*a**3)/(3*a*b**4 + 3*b**5*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Ba^3-Aa^2b}{3\,(b^5x^3+ab^4)} + \frac{Bbx^6-2\,(2\,Ba-Ab)x^3}{6\,b^3} + \frac{(3\,Ba^2-2\,Aab)\log(bx^3+a)}{3\,b^4}$$

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(B*a^3 - A*a^2*b)/(b^5*x^3 + a*b^4) + 1/6*(B*b*x^6 - 2*(2*B*a - A*b)*x^
3)/b^3 + 1/3*(3*B*a^2 - 2*A*a*b)*log(b*x^3 + a)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.29

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(3\,Ba^2-2\,Aab)\log(|bx^3+a|)}{3\,b^4} + \frac{Bb^2x^6-4\,Babx^3+2\,Ab^2x^3}{6\,b^4} - \frac{3\,Ba^2bx^3-2\,Aab^2x^3+2\,Ba^3-Aa^2b}{3\,(bx^3+a)b^4}$$

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*B*a^2 - 2*A*a*b)*log(abs(b*x^3 + a))/b^4 + 1/6*(B*b^2*x^6 - 4*B*a*b*
x^3 + 2*A*b^2*x^3)/b^4 - 1/3*(3*B*a^2*b*x^3 - 2*A*a*b^2*x^3 + 2*B*a^3 - A*a
^2*b)/((b*x^3 + a)*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2}\,dx = x^3\left(\frac{A}{3\,b^2}-\frac{2\,B\,a}{3\,b^3}\right) + \frac{\ln(b\,x^3+a)\,\left(3\,B\,a^2-2\,A\,a\,b\right)}{3\,b^4} + \frac{B\,x^6}{6\,b^2} + \frac{B\,a^3-A\,a^2\,b}{3\,b\,\left(b^4\,x^3+a\,b^3\right)}$$

```
[In] int((x^8*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] x^3*(A/(3*b^2) - (2*B*a)/(3*b^3)) + (log(a + b*x^3)*(3*B*a^2 - 2*A*a*b))/(3
*b^4) + (B*x^6)/(6*b^2) + (B*a^3 - A*a^2*b)/(3*b*(a*b^3 + b^4*x^3))
```

3.73 $\int \frac{x^7(A+Bx^3)}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 215

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^2} dx = \frac{(5Ab-8aB)x^2}{6b^3} - \frac{(5Ab-8aB)x^5}{15ab^2} + \frac{(Ab-aB)x^8}{3ab(a+bx^3)} + \frac{a^{2/3}(5Ab-8aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{11/3}} + \frac{a^{2/3}(5Ab-8aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{11/3}} - \frac{a^{2/3}(5Ab-8aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{11/3}}$$

```
[Out] 1/6*(5*A*b-8*B*a)*x^2/b^3-1/15*(5*A*b-8*B*a)*x^5/a/b^2+1/3*(A*b-B*a)*x^8/a/
b/(b*x^3+a)+1/9*a^(2/3)*(5*A*b-8*B*a)*ln(a^(1/3)+b^(1/3)*x)/b^(11/3)-1/18*a
^(2/3)*(5*A*b-8*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(11/3)+1/9
*a^(2/3)*(5*A*b-8*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^
(11/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {468, 308, 298, 31, 648, 631, 210, 642}

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{a^{2/3}(5Ab-8aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{11/3}} - \frac{a^{2/3}(5Ab-8aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{11/3}} + \frac{a^{2/3}(5Ab-8aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{11/3}} + \frac{x^2(5Ab-8aB)}{6b^3} - \frac{x^5(5Ab-8aB)}{15ab^2} + \frac{x^8(Ab-aB)}{3ab\,(a+bx^3)}$$

```
[In] Int[(x^7*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((5*A*b - 8*a*B)*x^2)/(6*b^3) - ((5*A*b - 8*a*B)*x^5)/(15*a*b^2) + ((A*b -
a*B)*x^8)/(3*a*b*(a + b*x^3)) + (a^(2/3)*(5*A*b - 8*a*B)*ArcTan[(a^(1/3) -
2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*b^(11/3)) + (a^(2/3)*(5*A*b - 8
*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(11/3)) - (a^(2/3)*(5*A*b - 8*a*B)*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*b^(11/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{(Ab-aB)x^8}{3ab\,(a+bx^3)} + \frac{(-5Ab+8aB)\int \frac{x^7}{a+bx^3}\,dx}{3ab}$$

$$= \frac{(Ab-aB)x^8}{3ab\,(a+bx^3)} + \frac{(-5Ab+8aB)\int \left(-\frac{ax}{b^2}+\frac{x^4}{b}+\frac{a^2x}{b^2(a+bx^3)}\right)\,dx}{3ab}$$

$$= \frac{(5Ab-8aB)x^2}{6b^3} - \frac{(5Ab-8aB)x^5}{15ab^2} + \frac{(Ab-aB)x^8}{3ab\,(a+bx^3)} - \frac{(a(5Ab-8aB))\int \frac{x}{a+bx^3}\,dx}{3b^3}$$

$$= \frac{(5Ab-8aB)x^2}{6b^3} - \frac{(5Ab-8aB)x^5}{15ab^2} + \frac{(Ab-aB)x^8}{3ab(a+bx^3)} + \frac{\left(a^{2/3}(5Ab-8aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9b^{10/3}} - \frac{\left(a^{2/3}(5Ab-8aB)\right)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9b^{10/3}}$$

$$= \frac{(5Ab-8aB)x^2}{6b^3} - \frac{(5Ab-8aB)x^5}{15ab^2} + \frac{(Ab-aB)x^8}{3ab(a+bx^3)} + \frac{a^{2/3}(5Ab-8aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{11/3}} - \frac{\left(a^{2/3}(5Ab-8aB)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18b^{11/3}} - \frac{(a(5Ab-8aB))\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^{10/3}}$$

$$= \frac{(5Ab-8aB)x^2}{6b^3} - \frac{(5Ab-8aB)x^5}{15ab^2} + \frac{(Ab-aB)x^8}{3ab(a+bx^3)} + \frac{a^{2/3}(5Ab-8aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{11/3}} - \frac{a^{2/3}(5Ab-8aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{11/3}} - \frac{\left(a^{2/3}(5Ab-8aB)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3b^{11/3}}$$

$$= \frac{(5Ab-8aB)x^2}{6b^3} - \frac{(5Ab-8aB)x^5}{15ab^2} + \frac{(Ab-aB)x^8}{3ab(a+bx^3)} + \frac{a^{2/3}(5Ab-8aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{11/3}} + \frac{a^{2/3}(5Ab-8aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{11/3}} - \frac{a^{2/3}(5Ab-8aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{11/3}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.86

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$= \frac{45b^{2/3}(Ab-2aB)x^2 + 18b^{5/3}Bx^5 + \frac{30ab^{2/3}(Ab-aB)x^2}{a+bx^3} - 10\sqrt{3}a^{2/3}(-5Ab+8aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 10a^{2}}{90b^{11/3}}$$

```
[In] Integrate[(x^7*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (45*b^(2/3)*(A*b - 2*a*B)*x^2 + 18*b^(5/3)*B*x^5 + (30*a*b^(2/3)*(A*b - a*B
)*x^2)/(a + b*x^3) - 10*Sqrt[3]*a^(2/3)*(-5*A*b + 8*a*B)*ArcTan[(1 - (2*b^(
```

```
1/3)*x)/a^(1/3))/Sqrt[3]] - 10*a^(2/3)*(-5*A*b + 8*a*B)*Log[a^(1/3) + b^(1/
3)*x] + 5*a^(2/3)*(-5*A*b + 8*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(90*b^(11/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.44

method	result
risch	$\frac{B x^5}{5b^2} + \frac{A x^2}{2b^2} - \frac{Ba x^2}{b^3} + \frac{\left(\frac{1}{3}abA - \frac{1}{3}a^2B\right)x^2}{b^3(b x^3+a)} + \frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)} \frac{(-5Ab+8Ba)\ln\left(x-\_R\right)}{\_R}\right)}{9b^4}$
default	$\frac{\frac{bB x^5}{5}+\frac{x^2(Ab-2Ba)}{2}}{b^3} - \frac{a\left(\frac{\left(-\frac{Ab}{3}+\frac{Ba}{3}\right)x^2}{b x^3+a}+\left(\frac{5Ab}{3}-\frac{8Ba}{3}\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\right)}{b^3}$

```
[In] int(x^7*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*B*x^5/b^2+1/2/b^2*A*x^2-1/b^3*B*a*x^2+(1/3*a*b*A-1/3*a^2*B)*x^2/b^3/(b*
x^3+a)+1/9/b^4*a*sum((-5*A*b+8*B*a)/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.20

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$= \frac{18\,Bb^2x^8 - 9\,(8\,Bab - 5\,Ab^2)x^5 - 15\,(8\,Ba^2 - 5\,Aab)x^2 + 10\,\sqrt{3}((8\,Bab - 5\,Ab^2)x^3 + 8\,Ba^2 - 5\,Aab)\left(\frac{a^2}{b^2}\right)}{}$$

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/90*(18*B*b^2*x^8 - 9*(8*B*a*b - 5*A*b^2)*x^5 - 15*(8*B*a^2 - 5*A*a*b)*x^2
 + 10*sqrt(3)*((8*B*a*b - 5*A*b^2)*x^3 + 8*B*a^2 - 5*A*a*b)*(a^2/b^2)^(1/3)
```

```
*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3) - sqrt(3)*a)/a) + 5*((8*B*a*b -
5*A*b^2)*x^3 + 8*B*a^2 - 5*A*a*b)*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)
^(2/3) + a*(a^2/b^2)^(1/3)) - 10*((8*B*a*b - 5*A*b^2)*x^3 + 8*B*a^2 - 5*A*a
*b)*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/(b^4*x^3 + a*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.70

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Bx^5}{5b^2} + x^2\left(\frac{A}{2b^2} - \frac{Ba}{b^3}\right) + \frac{x^2(Aab-Ba^2)}{3ab^3+3b^4x^3} + \mathrm{RootSum}\left(729t^3b^{11} - 125A^3a^2b^3 + 600A^2Ba^3b^2 - 960AB^2a^4b + 512B^3a^5, \left(t \mapsto t\log\left(\frac{81}{25A^2ab^2 - 80A\right.\right.\right.$$

```
[In] integrate(x**7*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x**5/(5*b**2) + x**2*(A/(2*b**2) - B*a/b**3) + x**2*(A*a*b - B*a**2)/(3*a
*b**3 + 3*b**4*x**3) + RootSum(729*_t**3*b**11 - 125*A**3*a**2*b**3 + 600*A
**2*B*a**3*b**2 - 960*A*B**2*a**4*b + 512*B**3*a**5, Lambda(_t, _t*log(81*_
t**2*b**7/(25*A**2*a*b**2 - 80*A*B*a**2*b + 64*B**2*a**3) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.89

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^2}\,dx = -\frac{(Ba^2-Aab)x^2}{3\,(b^4x^3+ab^3)} + \frac{\sqrt{3}(8\,Ba^2-5\,Aab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{2\,Bbx^5-5\,(2\,Ba-Ab)x^2}{10\,b^3} + \frac{(8\,Ba^2-5\,Aab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(8\,Ba^2-5\,Aab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(B*a^2 - A*a*b)*x^2/(b^4*x^3 + a*b^3) + 1/9*sqrt(3)*(8*B*a^2 - 5*A*a*b
)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^4*(a/b)^(1/3)) + 1
/10*(2*B*b*x^5 - 5*(2*B*a - A*b)*x^2)/b^3 + 1/18*(8*B*a^2 - 5*A*a*b)*log(x^
2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(1/3)) - 1/9*(8*B*a^2 - 5*A*a*b
)*log(x + (a/b)^(1/3))/(b^4*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.10

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^2}\,dx = -\frac{\left(8\,Ba^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,Aab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^3} - \frac{\sqrt{3}\left(8\,(-ab^2)^{\frac{2}{3}}\,Ba-5\,(-ab^2)^{\frac{2}{3}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^5} - \frac{Ba^2x^2-Aabx^2}{3\,(bx^3+a)b^3} + \frac{\left(8\,(-ab^2)^{\frac{2}{3}}\,Ba-5\,(-ab^2)^{\frac{2}{3}}\,Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^5} + \frac{2\,Bb^8x^5-10\,Bab^7x^2+5\,Ab^8x^2}{10\,b^{10}}$$

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(8*B*a^2*(-a/b)^(1/3) - 5*A*a*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x -
 (-a/b)^(1/3)))/(a*b^3) - 1/9*sqrt(3)*(8*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2)^(2
/3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^5 - 1/3*(B
*a^2*x^2 - A*a*b*x^2)/((b*x^3 + a)*b^3) + 1/18*(8*(-a*b^2)^(2/3)*B*a - 5*(-
a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^5 + 1/10*(2*B*
b^8*x^5 - 10*B*a*b^7*x^2 + 5*A*b^8*x^2)/b^10
```

Mupad [B] (verification not implemented)

Time = 6.83 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.83

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^2}\,dx = x^2\left(\frac{A}{2\,b^2}-\frac{B\,a}{b^3}\right)+\frac{B\,x^5}{5\,b^2}-\frac{x^2\left(\frac{B\,a^2}{3}-\frac{A\,a\,b}{3}\right)}{b^4\,x^3+a\,b^3} + \frac{a^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(5\,A\,b-8\,B\,a\right)}{9\,b^{11/3}} + \frac{a^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(5\,A\,b-8\,B\,a\right)}{9\,b^{11/3}} - \frac{a^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(5\,A\,b-8\,B\,a\right)}{9\,b^{11/3}}$$

```
[In] int((x^7*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] x^2*(A/(2*b^2) - (B*a)/b^3) + (B*x^5)/(5*b^2) - (x^2*((B*a^2)/3 - (A*a*b)/3
))/(a*b^3 + b^4*x^3) + (a^(2/3)*log(b^(1/3)*x + a^(1/3))*(5*A*b - 8*B*a))/(
9*b^(11/3)) + (a^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^
(1/2)*1i)/2 - 1/2)*(5*A*b - 8*B*a))/(9*b^(11/3)) - (a^(2/3)*log(3^(1/2)*a^(
1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*A*b - 8*B*a))/(9
*b^(11/3))
```

3.74 $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 213

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(4Ab-7aB)x}{3b^3} - \frac{(4Ab-7aB)x^4}{12ab^2} + \frac{(Ab-aB)x^7}{3ab(a+bx^3)} + \frac{\sqrt[3]{a}(4Ab-7aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{10/3}} - \frac{\sqrt[3]{a}(4Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{10/3}} + \frac{\sqrt[3]{a}(4Ab-7aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{10/3}}$$

```
[Out] 1/3*(4*A*b-7*B*a)*x/b^3-1/12*(4*A*b-7*B*a)*x^4/a/b^2+1/3*(A*b-B*a)*x^7/a/b/
(b*x^3+a)-1/9*a^(1/3)*(4*A*b-7*B*a)*ln(a^(1/3)+b^(1/3)*x)/b^(10/3)+1/18*a^(
1/3)*(4*A*b-7*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(10/3)+1/9*a
^(1/3)*(4*A*b-7*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(1
0/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {468, 308, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{\sqrt[3]{a}(4Ab-7aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{10/3}} + \frac{\sqrt[3]{a}(4Ab-7aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{10/3}} - \frac{\sqrt[3]{a}(4Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{10/3}} + \frac{x(4Ab-7aB)}{3b^3} - \frac{x^4(4Ab-7aB)}{12ab^2} + \frac{x^7(Ab-aB)}{3ab\,(a+bx^3)}$$

```
[In] Int[(x^6*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((4*A*b - 7*a*B)*x)/(3*b^3) - ((4*A*b - 7*a*B)*x^4)/(12*a*b^2) + ((A*b - a*
B)*x^7)/(3*a*b*(a + b*x^3)) + (a^(1/3)*(4*A*b - 7*a*B)*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*b^(10/3)) - (a^(1/3)*(4*A*b - 7*a
*B)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(10/3)) + (a^(1/3)*(4*A*b - 7*a*B)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*b^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{(Ab-aB)x^7}{3ab\,(a+bx^3)} + \frac{(-4Ab+7aB)\int \frac{x^6}{a+bx^3}\,dx}{3ab}$$

$$= \frac{(Ab-aB)x^7}{3ab\,(a+bx^3)} + \frac{(-4Ab+7aB)\int \left(-\frac{a}{b^2}+\frac{x^3}{b}+\frac{a^2}{b^2(a+bx^3)}\right)\,dx}{3ab}$$

$$= \frac{(4Ab-7aB)x}{3b^3} - \frac{(4Ab-7aB)x^4}{12ab^2} + \frac{(Ab-aB)x^7}{3ab\,(a+bx^3)} - \frac{(a(4Ab-7aB))\int \frac{1}{a+bx^3}\,dx}{3b^3}$$

$$= \frac{(4Ab-7aB)x}{3b^3} - \frac{(4Ab-7aB)x^4}{12ab^2} + \frac{(Ab-aB)x^7}{3ab(a+bx^3)} - \frac{\left(\sqrt[3]{a}(4Ab-7aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9b^3} - \frac{\left(\sqrt[3]{a}(4Ab-7aB)\right)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9b^3}$$

$$= \frac{(4Ab-7aB)x}{3b^3} - \frac{(4Ab-7aB)x^4}{12ab^2} + \frac{(Ab-aB)x^7}{3ab(a+bx^3)} - \frac{\sqrt[3]{a}(4Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{10/3}} + \frac{\left(\sqrt[3]{a}(4Ab-7aB)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18b^{10/3}} - \frac{\left(a^{2/3}(4Ab-7aB)\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^3}$$

$$= \frac{(4Ab-7aB)x}{3b^3} - \frac{(4Ab-7aB)x^4}{12ab^2} + \frac{(Ab-aB)x^7}{3ab(a+bx^3)} - \frac{\sqrt[3]{a}(4Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{10/3}} + \frac{\sqrt[3]{a}(4Ab-7aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{10/3}} - \frac{\left(\sqrt[3]{a}(4Ab-7aB)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3b^{10/3}}$$

$$= \frac{(4Ab-7aB)x}{3b^3} - \frac{(4Ab-7aB)x^4}{12ab^2} + \frac{(Ab-aB)x^7}{3ab(a+bx^3)} + \frac{\sqrt[3]{a}(4Ab-7aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{10/3}} - \frac{\sqrt[3]{a}(4Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{10/3}} + \frac{\sqrt[3]{a}(4Ab-7aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{10/3}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.85

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$= \frac{36\sqrt[3]{b}(Ab-2aB)x + 9b^{4/3}Bx^4 + \frac{12a\sqrt[3]{b}(Ab-aB)x}{a+bx^3} - 4\sqrt{3}\sqrt[3]{a}(-4Ab+7aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 4\sqrt[3]{a}(-4A}{36b^{10/3}}$$

```
[In] Integrate[(x^6*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (36*b^(1/3)*(A*b - 2*a*B)*x + 9*b^(4/3)*B*x^4 + (12*a*b^(1/3)*(A*b - a*B)*x
)/(a + b*x^3) - 4*Sqrt[3]*a^(1/3)*(-4*A*b + 7*a*B)*ArcTan[(1 - (2*b^(1/3)*x
)/a^(1/3))/Sqrt[3]] + 4*a^(1/3)*(-4*A*b + 7*a*B)*Log[a^(1/3) + b^(1/3)*x] -
 2*a^(1/3)*(-4*A*b + 7*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])
/(36*b^(10/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.41

method	result
risch	$\frac{B x^4}{4b^2}+\frac{Ax}{b^2}-\frac{2Bax}{b^3}+\frac{\left(\frac{1}{3}abA-\frac{1}{3}a^2B\right)x}{b^3(b x^3+a)}+\frac{a\left(\sum_{\_R=\operatorname{RootOf}\left(b\_Z^3+a\right)}\frac{(-4Ab+7Ba)\ln\left(x-\_R\right)}{\_R^2}\right)}{9b^4}$
default	$\frac{\frac{1}{4}bB x^4+Abx-2Bax}{b^3}-\frac{a\left(\frac{\left(-\frac{Ab}{3}+\frac{Ba}{3}\right)x}{b x^3+a}+\frac{(4Ab-7Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3}\right)}{b^3}$

```
[In] int(x^6*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*B*x^4/b^2+1/b^2*A*x-2/b^3*B*a*x+(1/3*a*b*A-1/3*a^2*B)*x/b^3/(b*x^3+a)+1
/9/b^4*a*sum((-4*A*b+7*B*a)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.13

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{9\,Bb^2x^7 - 9\,(7\,Bab - 4\,Ab^2)x^4 - 4\,\sqrt{3}((7\,Bab - 4\,Ab^2)x^3 + 7\,Ba^2 - 4\,Aab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}} - \sqrt{3}}{3\,a}\right.}{}$$

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(9*B*b^2*x^7 - 9*(7*B*a*b - 4*A*b^2)*x^4 - 4*sqrt(3)*((7*B*a*b - 4*A*b
^2)*x^3 + 7*B*a^2 - 4*A*a*b)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^
(2/3) - sqrt(3)*a)/a) + 2*((7*B*a*b - 4*A*b^2)*x^3 + 7*B*a^2 - 4*A*a*b)*(-a
/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)) - 4*((7*B*a*b - 4*A*b^2)
*x^3 + 7*B*a^2 - 4*A*a*b)*(-a/b)^(1/3)*log(x - (-a/b)^(1/3)) - 12*(7*B*a^2
- 4*A*a*b)*x)/(b^4*x^3 + a*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.59

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Bx^4}{4b^2} + x\left(\frac{A}{b^2} - \frac{2Ba}{b^3}\right) + \frac{x(Aab - Ba^2)}{3ab^3 + 3b^4x^3}$$
$$+\mathrm{RootSum}\left(729t^3b^{10} + 64A^3ab^3 - 336A^2Ba^2b^2 + 588AB^2a^3b - 343B^3a^4, \left(t \mapsto t\log\left(\frac{9tb^3}{-4Ab + 7Ba} + \right.\right.\right.$$

```
[In] integrate(x**6*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x**4/(4*b**2) + x*(A/b**2 - 2*B*a/b**3) + x*(A*a*b - B*a**2)/(3*a*b**3 +
3*b**4*x**3) + RootSum(729*_t**3*b**10 + 64*A**3*a*b**3 - 336*A**2*B*a**2*b
**2 + 588*A*B**2*a**3*b - 343*B**3*a**4, Lambda(_t, _t*log(9*_t*b**3/(-4*A*
b + 7*B*a) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.88

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^2}\,dx = -\frac{(Ba^2-Aab)x}{3\,(b^4x^3+ab^3)} + \frac{Bbx^4-4\,(2\,Ba-Ab)x}{4\,b^3} + \frac{\sqrt{3}(7\,Ba^2-4\,Aab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(7\,Ba^2-4\,Aab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(7\,Ba^2-4\,Aab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(B*a^2 - A*a*b)*x/(b^4*x^3 + a*b^3) + 1/4*(B*b*x^4 - 4*(2*B*a - A*b)*x
)/b^3 + 1/9*sqrt(3)*(7*B*a^2 - 4*A*a*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/
3))/(a/b)^(1/3))/(b^4*(a/b)^(2/3)) - 1/18*(7*B*a^2 - 4*A*a*b)*log(x^2 - x*(
a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(2/3)) + 1/9*(7*B*a^2 - 4*A*a*b)*log(x
 + (a/b)^(1/3))/(b^4*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.99

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{\sqrt{3}\left(7\,(-ab^2)^{\frac{1}{3}}\,Ba-4\,(-ab^2)^{\frac{1}{3}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^4} - \frac{(7\,Ba^2-4\,Aab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^3} + \frac{\left(7\,(-ab^2)^{\frac{1}{3}}\,Ba-4\,(-ab^2)^{\frac{1}{3}}\,Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^4} - \frac{Ba^2x-Aabx}{3\,(bx^3+a)b^3} + \frac{Bb^6x^4-8\,Bab^5x+4\,Ab^6x}{4\,b^8}$$

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(7*(-a*b^2)^(1/3)*B*a - 4*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3
)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 1/9*(7*B*a^2 - 4*A*a*b)*(-a/b)^(
1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^3) + 1/18*(7*(-a*b^2)^(1/3)*B*a - 4*(-
a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 - 1/3*(B*a^2
*x - A*a*b*x)/((b*x^3 + a)*b^3) + 1/4*(B*b^6*x^4 - 8*B*a*b^5*x + 4*A*b^6*x)
/b^8
```

Mupad [B] (verification not implemented)

Time = 6.87 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.91

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$= x\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)-\frac{x\,\left(\frac{B\,a^2}{3}-\frac{A\,a\,b}{3}\right)}{b^4\,x^3+a\,b^3}+\frac{B\,x^4}{4\,b^2}$$
$$+\frac{(-a)^{1/3}\,\ln\left((-a)^{4/3}+a\,b^{1/3}\,x\right)\,(4\,A\,b-7\,B\,a)}{9\,b^{10/3}}$$
$$-\frac{(-a)^{1/3}\,\ln\left((-a)^{4/3}-2\,a\,b^{1/3}\,x+\sqrt{3}\,(-a)^{4/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(4\,A\,b-7\,B\,a)}{9\,b^{10/3}}$$
$$+\frac{(-a)^{1/3}\,\ln\left(2\,a\,b^{1/3}\,x-(-a)^{4/3}+\sqrt{3}\,(-a)^{4/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(4\,A\,b-7\,B\,a)}{9\,b^{10/3}}$$

```
[In] int((x^6*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] x*(A/b^2 - (2*B*a)/b^3) - (x*((B*a^2)/3 - (A*a*b)/3))/(a*b^3 + b^4*x^3) + (
B*x^4)/(4*b^2) + ((-a)^(1/3)*log((-a)^(4/3) + a*b^(1/3)*x)*(4*A*b - 7*B*a))
/(9*b^(10/3)) - ((-a)^(1/3)*log((-a)^(4/3) + 3^(1/2)*(-a)^(4/3)*1i - 2*a*b^
(1/3)*x)*((3^(1/2)*1i)/2 + 1/2)*(4*A*b - 7*B*a))/(9*b^(10/3)) + ((-a)^(1/3)
*log(3^(1/2)*(-a)^(4/3)*1i - (-a)^(4/3) + 2*a*b^(1/3)*x)*((3^(1/2)*1i)/2 -
1/2)*(4*A*b - 7*B*a))/(9*b^(10/3))
```

3.75 $\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2} dx = \frac{Bx^3}{3b^2} + \frac{a(Ab-aB)}{3b^3(a+bx^3)} + \frac{(Ab-2aB)\log(a+bx^3)}{3b^3}$$

```
[Out] 1/3*B*x^3/b^2+1/3*a*(A*b-B*a)/b^3/(b*x^3+a)+1/3*(A*b-2*B*a)*ln(b*x^3+a)/b^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 78}

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2} dx = \frac{a(Ab-aB)}{3b^3(a+bx^3)} + \frac{(Ab-2aB)\log(a+bx^3)}{3b^3} + \frac{Bx^3}{3b^2}$$

```
[In] Int[(x^5*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (B*x^3)/(3*b^2) + (a*(A*b - a*B))/(3*b^3*(a + b*x^3)) + ((A*b - 2*a*B)*Log[
a + b*x^3])/(3*b^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x(A+Bx)}{(a+bx)^2}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{B}{b^2}+\frac{a(-Ab+aB)}{b^2(a+bx)^2}+\frac{Ab-2aB}{b^2(a+bx)}\right)dx, x, x^3\right)\\ &= \frac{Bx^3}{3b^2}+\frac{a(Ab-aB)}{3b^3(a+bx^3)}+\frac{(Ab-2aB)\log(a+bx^3)}{3b^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{bBx^3+\frac{a(Ab-aB)}{a+bx^3}+(Ab-2aB)\log(a+bx^3)}{3b^3}$$

```
[In] Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^2,x]
[Out] (b*B*x^3 + (a*(A*b - a*B))/(a + b*x^3) + (A*b - 2*a*B)*Log[a + b*x^3])/(3*b
^3)
```

Maple [A] (verified)

Time = 4.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

method	result	size
norman	$\frac{\frac{B x^6}{3b}+\frac{a(Ab-2Ba)}{3b^3}}{b x^3+a}+\frac{(Ab-2Ba)\ln(b x^3+a)}{3b^3}$	57
default	$\frac{B x^3}{3b^2}+\frac{\frac{(Ab-2Ba)\ln(b x^3+a)}{b}+\frac{a(Ab-Ba)}{b(b x^3+a)}}{3b^2}$	59
risch	$\frac{B x^3}{3b^2}+\frac{aA}{3b^2(b x^3+a)}-\frac{a^2 B}{3b^3(b x^3+a)}+\frac{\ln(b x^3+a)A}{3b^2}-\frac{2\ln(b x^3+a)Ba}{3b^3}$	74
parallelrisch	$\frac{b^2 B x^6+A\ln(b x^3+a)x^3 b^2-2B\ln(b x^3+a)x^3 ab+A\ln(b x^3+a)ab-2B\ln(b x^3+a)a^2+abA-2a^2 B}{3b^3(b x^3+a)}$	92

```
[In] int(x^5*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/3*B*x^6/b+1/3*a*(A*b-2*B*a)/b^3)/(b*x^3+a)+1/3*(A*b-2*B*a)*ln(b*x^3+a)/b
^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.35

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Bb^2x^6+Babx^3-Ba^2+Aab-((2\,Bab-Ab^2)x^3+2\,Ba^2-Aab)\log(bx^3+a)}{3\,(b^4x^3+ab^3)}$$

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(B*b^2*x^6 + B*a*b*x^3 - B*a^2 + A*a*b - ((2*B*a*b - A*b^2)*x^3 + 2*B*a
^2 - A*a*b)*log(b*x^3 + a))/(b^4*x^3 + a*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Bx^3}{3b^2}+\frac{Aab-Ba^2}{3ab^3+3b^4x^3}-\frac{(-Ab+2Ba)\log(a+bx^3)}{3b^3}$$

```
[In] integrate(x**5*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x**3/(3*b**2) + (A*a*b - B*a**2)/(3*a*b**3 + 3*b**4*x**3) - (-A*b + 2*B*a
)*log(a + b*x**3)/(3*b**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Bx^3}{3\,b^2}-\frac{Ba^2-Aab}{3\,(b^4x^3+ab^3)}-\frac{(2\,Ba-Ab)\log(bx^3+a)}{3\,b^3}$$

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*B*x^3/b^2 - 1/3*(B*a^2 - A*a*b)/(b^4*x^3 + a*b^3) - 1/3*(2*B*a - A*b)*l
og(b*x^3 + a)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.52

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{\frac{(bx^3+a)B}{b^2} + \frac{(2\,Ba-Ab)\log\left(\frac{|bx^3+a|}{(bx^3+a)^2|b|}\right)}{b^2} - \frac{\frac{Ba^2b}{bx^3+a}-\frac{Aab^2}{bx^3+a}}{b^3}}{3\,b}$$

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*((b*x^3 + a)*B/b^2 + (2*B*a - A*b)*log(abs(b*x^3 + a)/((b*x^3 + a)^2*ab
s(b)))/b^2 - (B*a^2*b/(b*x^3 + a) - A*a*b^2/(b*x^3 + a))/b^3)/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.03

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{B\,x^3}{3\,b^2} + \frac{\ln\left(b\,x^3+a\right)\,\left(A\,b-2\,B\,a\right)}{3\,b^3} - \frac{B\,a^2-A\,a\,b}{3\,b\,\left(b^3\,x^3+a\,b^2\right)}$$

```
[In] int((x^5*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (B*x^3)/(3*b^2) + (log(a + b*x^3)*(A*b - 2*B*a))/(3*b^3) - (B*a^2 - A*a*b)/
(3*b*(a*b^2 + b^3*x^3))
```

3.76 $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 196

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^2} dx = -\frac{(2Ab-5aB)x^2}{6ab^2} + \frac{(Ab-aB)x^5}{3ab(a+bx^3)} - \frac{(2Ab-5aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}b^{8/3}} - \frac{(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9\sqrt[3]{a}b^{8/3}} + \frac{(2Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18\sqrt[3]{a}b^{8/3}}$$

```
[Out] -1/6*(2*A*b-5*B*a)*x^2/a/b^2+1/3*(A*b-B*a)*x^5/a/b/(b*x^3+a)-1/9*(2*A*b-5*B
*a)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(8/3)+1/18*(2*A*b-5*B*a)*ln(a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/b^(8/3)-1/9*(2*A*b-5*B*a)*arctan(1/3*(a
^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(1/3)/b^(8/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used

= {468, 327, 298, 31, 648, 631, 210, 642}

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(2Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18\sqrt[3]{a}b^{8/3}} - \frac{(2Ab-5aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}b^{8/3}} - \frac{(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9\sqrt[3]{a}b^{8/3}} - \frac{x^2(2Ab-5aB)}{6ab^2} + \frac{x^5(Ab-aB)}{3ab\,(a+bx^3)}$$

```
[In] Int[(x^4*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -1/6*((2*A*b - 5*a*B)*x^2)/(a*b^2) + ((A*b - a*B)*x^5)/(3*a*b*(a + b*x^3))
- ((2*A*b - 5*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sq
rt[3]*a^(1/3)*b^(8/3)) - ((2*A*b - 5*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(1
/3)*b^(8/3)) + ((2*A*b - 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(18*a^(1/3)*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(Ab-aB)x^5}{3ab\left(a+bx^3\right)}+\frac{(-2Ab+5aB)\int \frac{x^4}{a+bx^3}\,dx}{3ab} \\
&= -\frac{(2Ab-5aB)x^2}{6ab^2}+\frac{(Ab-aB)x^5}{3ab\left(a+bx^3\right)}+\frac{(2Ab-5aB)\int \frac{x}{a+bx^3}\,dx}{3b^2} \\
&= -\frac{(2Ab-5aB)x^2}{6ab^2}+\frac{(Ab-aB)x^5}{3ab\left(a+bx^3\right)}-\frac{(2Ab-5aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9\sqrt[3]{a}b^{7/3}} \\
&\quad +\frac{(2Ab-5aB)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9\sqrt[3]{a}b^{7/3}}
\end{aligned}
$$

$$= -\frac{(2Ab-5aB)x^2}{6ab^2} + \frac{(Ab-aB)x^5}{3ab\,(a+bx^3)} - \frac{(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9\sqrt[3]{a}b^{8/3}} + \frac{(2Ab-5aB)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18\sqrt[3]{a}b^{8/3}} + \frac{(2Ab-5aB)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^{7/3}}$$

$$= -\frac{(2Ab-5aB)x^2}{6ab^2} + \frac{(Ab-aB)x^5}{3ab\,(a+bx^3)} - \frac{(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9\sqrt[3]{a}b^{8/3}} + \frac{(2Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18\sqrt[3]{a}b^{8/3}} + \frac{(2Ab-5aB)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3\sqrt[3]{a}b^{8/3}}$$

$$= -\frac{(2Ab-5aB)x^2}{6ab^2} + \frac{(Ab-aB)x^5}{3ab\,(a+bx^3)} - \frac{(2Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}b^{8/3}} - \frac{(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9\sqrt[3]{a}b^{8/3}} + \frac{(2Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18\sqrt[3]{a}b^{8/3}}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.84

$$\int\frac{x^4(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$= \frac{9b^{2/3}Bx^2 - \frac{6b^{2/3}(Ab-aB)x^2}{a+bx^3} + \frac{2\sqrt{3}(-2Ab+5aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} + \frac{2(-2Ab+5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}} + \frac{(2Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{a}}}{18b^{8/3}}$$

```
[In] Integrate[(x^4*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (9*b^(2/3)*B*x^2 - (6*b^(2/3)*(A*b - a*B)*x^2)/(a + b*x^3) + (2*Sqrt[3]*(-2
*A*b + 5*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (2*(-2
*A*b + 5*a*B)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + ((2*A*b - 5*a*B)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(1/3))/(18*b^(8/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.36

method	result	size
risch	$\frac{B\,x^{2}}{2b^{2}}+\frac{\left(-\frac{Ab}{3}+\frac{Ba}{3}\right)x^{2}}{b^{2}\left(b\,x^{3}+a\right)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{(2Ab-5Ba)\ln\left(x-\_R\right)}{\_R}}{9b^{3}}$	71
default	$\frac{B\,x^{2}}{2b^{2}}+\frac{\frac{\left(-\frac{Ab}{3}+\frac{Ba}{3}\right)x^{2}}{b\,x^{3}+a}+\left(-\frac{5Ba}{3}+\frac{2Ab}{3}\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{2}}$	138

```
[In] int(x^4*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*B*x^2/b^2+(-1/3*A*b+1/3*B*a)*x^2/b^2/(b*x^3+a)+1/9/b^3*sum((2*A*b-5*B*a
)/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 578, normalized size of antiderivative = 2.95

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$= \left[\frac{9\,Bab^3x^5+3\,(5\,Ba^2b^2-2\,Aab^3)x^2-3\,\sqrt{\frac{1}{3}}(5\,Ba^3b-2\,Aa^2b^2+(5\,Ba^2b^2-2\,Aab^3)x^3)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x}{\cdots}\right.}{\cdots}\right.$$

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(9*B*a*b^3*x^5 + 3*(5*B*a^2*b^2 - 2*A*a*b^3)*x^2 - 3*sqrt(1/3)*(5*B*a
^3*b - 2*A*a^2*b^2 + (5*B*a^2*b^2 - 2*A*a*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*
```

```
log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)
^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - ((5*B
*a*b - 2*A*b^2)*x^3 + 5*B*a^2 - 2*A*a*b)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b
^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*((5*B*a*b - 2*A*b^2)*x^3 + 5*B*a^2 - 2*
A*a*b)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a*b^5*x^3 + a^2*b^4), 1/1
8*(9*B*a*b^3*x^5 + 3*(5*B*a^2*b^2 - 2*A*a*b^3)*x^2 - 6*sqrt(1/3)*(5*B*a^3*b
 - 2*A*a^2*b^2 + (5*B*a^2*b^2 - 2*A*a*b^3)*x^3)*sqrt(-(-a*b^2)^(1/3)/a)*arc
tan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - ((5*B*a
*b - 2*A*b^2)*x^3 + 5*B*a^2 - 2*A*a*b)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2
)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*((5*B*a*b - 2*A*b^2)*x^3 + 5*B*a^2 - 2*A*
a*b)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a*b^5*x^3 + a^2*b^4)]
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.64

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Bx^2}{2b^2} + \frac{x^2(-Ab+Ba)}{3ab^2+3b^3x^3} + \text{RootSum}\left(729t^3ab^8 + 8A^3b^3 - 60A^2Bab^2 + 150AB^2a^2b - 125B^3a^3, \left(t \mapsto t\log\left(\frac{81t^2ab^5}{4A^2b^2 - 20ABab + 2\right.\right.\right.$$

```
[In] integrate(x**4*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x**2/(2*b**2) + x**2*(-A*b + B*a)/(3*a*b**2 + 3*b**3*x**3) + RootSum(729*
_t**3*a*b**8 + 8*A**3*b**3 - 60*A**2*B*a*b**2 + 150*A*B**2*a**2*b - 125*B**
3*a**3, Lambda(_t, _t*log(81*_t**2*a*b**5/(4*A**2*b**2 - 20*A*B*a*b + 25*B*
*2*a**2) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.83

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(Ba-Ab)x^2}{3\,(b^3x^3+ab^2)} + \frac{Bx^2}{2\,b^2} - \frac{\sqrt{3}(5\,Ba-2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(5\,Ba-2\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(5\,Ba-2\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(B*a - A*b)*x^2/(b^3*x^3 + a*b^2) + 1/2*B*x^2/b^2 - 1/9*sqrt(3)*(5*B*a
- 2*A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(1/
3)) - 1/18*(5*B*a - 2*A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b
)^(1/3)) + 1/9*(5*B*a - 2*A*b)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.96

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Bx^2}{2\,b^2} - \frac{\sqrt{3}(5\,Ba-2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^2\right)^{\frac{1}{3}}\,b^2} + \frac{(5\,Ba-2\,Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^2\right)^{\frac{1}{3}}\,b^2} + \frac{\left(5\,Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^2} + \frac{Bax^2-Abx^2}{3\,(bx^3+a)b^2}$$

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/b^2 - 1/9*sqrt(3)*(5*B*a - 2*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*b^2) + 1/18*(5*B*a - 2*A*b)*log(x^2
+ x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*b^2) + 1/9*(5*B*a*(-a/b)^(
1/3) - 2*A*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2)
+ 1/3*(B*a*x^2 - A*b*x^2)/((b*x^3 + a)*b^2)
```

Mupad [B] (verification not implemented)

Time = 6.83 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.81

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{B\,x^2}{2\,b^2} - \frac{x^2\,\left(\frac{A\,b}{3}-\frac{B\,a}{3}\right)}{b^3\,x^3+a\,b^2} - \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(2\,A\,b-5\,B\,a\right)}{9\,a^{1/3}\,b^{8/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(2\,A\,b-5\,B\,a\right)}{9\,a^{1/3}\,b^{8/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(2\,A\,b-5\,B\,a\right)}{9\,a^{1/3}\,b^{8/3}}$$

```
[In] int((x^4*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (B*x^2)/(2*b^2) - (x^2*((A*b)/3 - (B*a)/3))/(a*b^2 + b^3*x^3) - (log(b^(1/3
)*x + a^(1/3))*(2*A*b - 5*B*a))/(9*a^(1/3)*b^(8/3)) - (log(3^(1/2)*a^(1/3)*
1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(2*A*b - 5*B*a))/(9*a^(1
/3)*b^(8/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1
i)/2 + 1/2)*(2*A*b - 5*B*a))/(9*a^(1/3)*b^(8/3))
```

3.77 $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 190

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2} dx = -\frac{(Ab-4aB)x}{3ab^2} + \frac{(Ab-aB)x^4}{3ab(a+bx^3)} - \frac{(Ab-4aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{7/3}} + \frac{(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{7/3}} - \frac{(Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{7/3}}$$

```
[Out] -1/3*(A*b-4*B*a)*x/a/b^2+1/3*(A*b-B*a)*x^4/a/b/(b*x^3+a)+1/9*(A*b-4*B*a)*ln
(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(7/3)-1/18*(A*b-4*B*a)*ln(a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(7/3)-1/9*(A*b-4*B*a)*arctan(1/3*(a^(1/3)-2*b
^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(7/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used

= {468, 327, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2}\,dx = -\frac{(Ab-4aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{7/3}} - \frac{(Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{7/3}} + \frac{(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{7/3}} - \frac{x(Ab-4aB)}{3ab^2} + \frac{x^4(Ab-aB)}{3ab\,(a+bx^3)}$$

```
[In] Int[(x^3*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -1/3*((A*b - 4*a*B)*x)/(a*b^2) + ((A*b - a*B)*x^4)/(3*a*b*(a + b*x^3)) - ((
A*b - 4*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*
a^(2/3)*b^(7/3)) + ((A*b - 4*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(2/3)*b^(7
/3)) - ((A*b - 4*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a
^(2/3)*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(Ab-aB)x^4}{3ab\,(a+bx^3)} + \frac{(-Ab+4aB)\int \frac{x^3}{a+bx^3}\,dx}{3ab}\\
&= -\frac{(Ab-4aB)x}{3ab^2} + \frac{(Ab-aB)x^4}{3ab\,(a+bx^3)} + \frac{(Ab-4aB)\int \frac{1}{a+bx^3}\,dx}{3b^2}\\
&= -\frac{(Ab-4aB)x}{3ab^2} + \frac{(Ab-aB)x^4}{3ab\,(a+bx^3)} + \frac{(Ab-4aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{2/3}b^2}\\
&\quad + \frac{(Ab-4aB)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{2/3}b^2}
\end{aligned}$$

$$= -\frac{(Ab-4aB)x}{3ab^2} + \frac{(Ab-aB)x^4}{3ab(a+bx^3)} + \frac{(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{7/3}} - \frac{(Ab-4aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{2/3}b^{7/3}} + \frac{(Ab-4aB)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6\sqrt[3]{a}b^2}$$

$$= -\frac{(Ab-4aB)x}{3ab^2} + \frac{(Ab-aB)x^4}{3ab(a+bx^3)} + \frac{(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{7/3}} - \frac{(Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{7/3}} + \frac{(Ab-4aB)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{2/3}b^{7/3}}$$

$$= -\frac{(Ab-4aB)x}{3ab^2} + \frac{(Ab-aB)x^4}{3ab(a+bx^3)} - \frac{(Ab-4aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{7/3}} + \frac{(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{7/3}} - \frac{(Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{7/3}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.84

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$= \frac{18\sqrt[3]{b}Bx - \frac{6\sqrt[3]{b}(Ab-aB)x}{a+bx^3} + \frac{2\sqrt{3}(-Ab+4aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} + \frac{2(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}} + \frac{(-Ab+4aB)\log\left(a^{2/3}-\sqrt[3]{a}\right.}{a^{2/3}}}{18b^{7/3}}$$

```
[In] Integrate[(x^3*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (18*b^(1/3)*B*x - (6*b^(1/3)*(A*b - a*B)*x)/(a + b*x^3) + (2*Sqrt[3]*(-(A*b
) + 4*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (2*(A*b -
 4*a*B)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + ((-(A*b) + 4*a*B)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3))/(18*b^(7/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{Bx}{b^2}+\frac{\left(-\frac{Ab}{3}+\frac{Ba}{3}\right)x}{b^2(b\,x^3+a)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(Ab-4Ba)\ln\left(x-\_R\right)}{\_R^2}}{9b^3}$	65
default	$\frac{Bx}{b^2}+\frac{\frac{\left(-\frac{Ab}{3}+\frac{Ba}{3}\right)x}{b\,x^3+a}+\frac{(Ab-4Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3}}{b^2}$	133

```
[In] int(x^3*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] B*x/b^2+(-1/3*A*b+1/3*B*a)*x/b^2/(b*x^3+a)+1/9/b^3*sum((A*b-4*B*a)/_R^2*ln(
x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 573, normalized size of antiderivative = 3.02

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$=\left[\frac{18\,Ba^2b^2x^4-3\,\sqrt{\frac{1}{3}}(4\,Ba^3b-Aa^2b^2+(4\,Ba^2b^2-Aab^3)x^3)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\,(a^2b)^{\frac{1}{3}}ax-a^2+3\,\sqrt{\frac{1}{3}}\left(2\,abx^2+\right.}{bx^3+a}\right.}{}\right.$$

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(18*B*a^2*b^2*x^4 - 3*sqrt(1/3)*(4*B*a^3*b - A*a^2*b^2 + (4*B*a^2*b^2
 - A*a*b^3)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*
```

```
x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(
-(a^2*b)^(1/3)/b))/(b*x^3 + a)) + ((4*B*a*b - A*b^2)*x^3 + 4*B*a^2 - A*a*b)
*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*((4*B*a
*b - A*b^2)*x^3 + 4*B*a^2 - A*a*b)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3))
 + 6*(4*B*a^3*b - A*a^2*b^2)*x)/(a^2*b^4*x^3 + a^3*b^3), 1/18*(18*B*a^2*b^2
*x^4 - 6*sqrt(1/3)*(4*B*a^3*b - A*a^2*b^2 + (4*B*a^2*b^2 - A*a*b^3)*x^3)*sq
rt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*
sqrt((a^2*b)^(1/3)/b)/a^2) + ((4*B*a*b - A*b^2)*x^3 + 4*B*a^2 - A*a*b)*(a^2
*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*((4*B*a*b -
A*b^2)*x^3 + 4*B*a^2 - A*a*b)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 6*
(4*B*a^3*b - A*a^2*b^2)*x)/(a^2*b^4*x^3 + a^3*b^3)]
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.54

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Bx}{b^2} + \frac{x(-Ab+Ba)}{3ab^2+3b^3x^3} + \operatorname{RootSum}\left(729t^3a^2b^7 - A^3b^3 + 12A^2Bab^2 - 48AB^2a^2b + 64B^3a^3, \left(t \mapsto t\log\left(-\frac{9tab^2}{-Ab+4Ba}+x\right)\right)\right)$$

```
[In] integrate(x**3*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x/b**2 + x*(-A*b + B*a)/(3*a*b**2 + 3*b**3*x**3) + RootSum(729*_t**3*a**2
*b**7 - A**3*b**3 + 12*A**2*B*a*b**2 - 48*A*B**2*a**2*b + 64*B**3*a**3, Lam
bda(_t, _t*log(-9*_t*a*b**2/(-A*b + 4*B*a) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.83

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(Ba-Ab)x}{3\,(b^3x^3+ab^2)} + \frac{Bx}{b^2} - \frac{\sqrt{3}(4\,Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(4\,Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(4\,Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(B*a - A*b)*x/(b^3*x^3 + a*b^2) + B*x/b^2 - 1/9*sqrt(3)*(4*B*a - A*b)*a
rctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(2/3)) + 1/18
*(4*B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) - 1
/9*(4*B*a - A*b)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.87

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{\sqrt{3}(4\,Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^2\right)^{\frac{2}{3}}\,b} + \frac{(4\,Ba-Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^2\right)^{\frac{2}{3}}\,b} + \frac{Bx}{b^2} + \frac{(4\,Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^2} + \frac{Bax-Abx}{3\,(bx^3+a)b^2}$$

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(4*B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1
/3))/((-a*b^2)^(2/3)*b) + 1/18*(4*B*a - A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a
/b)^(2/3))/((-a*b^2)^(2/3)*b) + B*x/b^2 + 1/9*(4*B*a - A*b)*(-a/b)^(1/3)*lo
g(abs(x - (-a/b)^(1/3)))/(a*b^2) + 1/3*(B*a*x - A*b*x)/((b*x^3 + a)*b^2)
```

Mupad [B] (verification not implemented)

Time = 6.81 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.79

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{B\,x}{b^2} - \frac{x\,\left(\frac{A\,b}{3}-\frac{B\,a}{3}\right)}{b^3\,x^3+a\,b^2} + \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-4\,B\,a\right)}{9\,a^{2/3}\,b^{7/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-4\,B\,a\right)}{9\,a^{2/3}\,b^{7/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-4\,B\,a\right)}{9\,a^{2/3}\,b^{7/3}}$$

```
[In] int((x^3*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (B*x)/b^2 - (x*((A*b)/3 - (B*a)/3))/(a*b^2 + b^3*x^3) + (log(b^(1/3)*x + a^
(1/3))*(A*b - 4*B*a))/(9*a^(2/3)*b^(7/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(
1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - 4*B*a))/(9*a^(2/3)*b^(7/3))
 + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*
(A*b - 4*B*a))/(9*a^(2/3)*b^(7/3))
```

3.78 $\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 41

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2} dx = \frac{-Ab+aB}{3b^2(a+bx^3)} + \frac{B\log(a+bx^3)}{3b^2}$$

```
[Out] 1/3*(-A*b+B*a)/b^2/(b*x^3+a)+1/3*B*ln(b*x^3+a)/b^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {455, 45}

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2} dx = \frac{B\log(a+bx^3)}{3b^2} - \frac{Ab-aB}{3b^2(a+bx^3)}$$

```
[In] Int[(x^2*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -1/3*(A*b - a*B)/(b^2*(a + b*x^3)) + (B*Log[a + b*x^3])/(3*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{(a+bx)^2}\,dx, x, x^3\right) \\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{Ab-aB}{b(a+bx)^2}+\frac{B}{b(a+bx)}\right)\,dx, x, x^3\right) \\ &= -\frac{Ab-aB}{3b^2\left(a+bx^3\right)}+\frac{B\log\left(a+bx^3\right)}{3b^2} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{x^2(A+Bx^3)}{\left(a+bx^3\right)^2}\,dx = \frac{-Ab+aB}{3b^2\left(a+bx^3\right)}+\frac{B\log\left(a+bx^3\right)}{3b^2}$$

```
[In] Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^2,x]
[Out] (-(A*b) + a*B)/(3*b^2*(a + b*x^3)) + (B*Log[a + b*x^3])/(3*b^2)
```

Maple [A] (verified)

Time = 3.99 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{B\ln\left(b\,x^{3}+a\right)}{3b^{2}}-\frac{Ab-Ba}{3b^{2}\left(b\,x^{3}+a\right)}$	38
norman	$\frac{B\ln\left(b\,x^{3}+a\right)}{3b^{2}}-\frac{Ab-Ba}{3b^{2}\left(b\,x^{3}+a\right)}$	38
risch	$\frac{B\ln\left(b\,x^{3}+a\right)}{3b^{2}}-\frac{A}{3b\left(b\,x^{3}+a\right)}+\frac{Ba}{3b^{2}\left(b\,x^{3}+a\right)}$	47
parallelrisch	$-\frac{-B\ln\left(b\,x^{3}+a\right)x^{3}b-B\ln\left(b\,x^{3}+a\right)a+Ab-Ba}{3b^{2}\left(b\,x^{3}+a\right)}$	50

```
[In] int(x^2*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/3*B*ln(b*x^3+a)/b^2-1/3/b^2*(A*b-B*a)/(b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Ba-Ab+(Bbx^3+Ba)\log(bx^3+a)}{3\,(b^3x^3+ab^2)}$$

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
[Out] 1/3*(B*a - A*b + (B*b*x^3 + B*a)*log(b*x^3 + a))/(b^3*x^3 + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{B\log(a+bx^3)}{3b^2} + \frac{-Ab+Ba}{3ab^2+3b^3x^3}$$

```
[In] integrate(x**2*(B*x**3+A)/(b*x**3+a)**2,x)
[Out] B*log(a + b*x**3)/(3*b**2) + (-A*b + B*a)/(3*a*b**2 + 3*b**3*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{Ba-Ab}{3\,(b^3x^3+ab^2)} + \frac{B\log(bx^3+a)}{3\,b^2}$$

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
[Out] 1/3*(B*a - A*b)/(b^3*x^3 + a*b^2) + 1/3*B*log(b*x^3 + a)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.59

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2}\,dx = -\frac{B\left(\frac{\log\left(\frac{|bx^3+a|}{(bx^3+a)^2|b|}\right)}{b} - \frac{a}{(bx^3+a)b}\right)}{3\,b} - \frac{A}{3\,(bx^3+a)b}$$

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*B*(log(abs(b*x^3 + a)/((b*x^3 + a)^2*abs(b)))/b - a/((b*x^3 + a)*b))/b
 - 1/3*A/((b*x^3 + a)*b)
```

Mupad [B] (verification not implemented)

Time = 6.61 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{B\,\ln\left(b\,x^3+a\right)}{3\,b^2} - \frac{A\,b-B\,a}{3\,b^2\,\left(b\,x^3+a\right)}$$

```
[In] int((x^2*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (B*log(a + b*x^3))/(3*b^2) - (A*b - B*a)/(3*b^2*(a + b*x^3))
```

3.79 $\int \frac{x(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 171

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(Ab-aB)x^2}{3ab\,(a+bx^3)} - \frac{(Ab+2aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{5/3}} - \frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{5/3}} + \frac{(Ab+2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{4/3}b^{5/3}}$$

```
[Out] 1/3*(A*b-B*a)*x^2/a/b/(b*x^3+a)-1/9*(A*b+2*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(4/
3)/b^(5/3)+1/18*(A*b+2*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/
3)/b^(5/3)-1/9*(A*b+2*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2)
)/a^(4/3)/b^(5/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used

$= \{468, 298, 31, 648, 631, 210, 642\}$

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^2}\,dx = -\frac{(2aB+Ab)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{5/3}} + \frac{(2aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{4/3}b^{5/3}} - \frac{(2aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{5/3}} + \frac{x^2(Ab-aB)}{3ab\,(a+bx^3)}$$

```
[In] Int[(x*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((A*b - a*B)*x^2)/(3*a*b*(a + b*x^3)) - ((A*b + 2*a*B)*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)*b^(5/3)) - ((A*b + 2*a*B)
*Log[a^(1/3) + b^(1/3)*x])/(9*a^(4/3)*b^(5/3)) + ((A*b + 2*a*B)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(4/3)*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{(Ab-aB)x^2}{3ab\,(a+bx^3)} + \frac{(Ab+2aB)\int \frac{x}{a+bx^3}\,dx}{3ab}$$

$$= \frac{(Ab-aB)x^2}{3ab\,(a+bx^3)} - \frac{(Ab+2aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{4/3}b^{4/3}} + \frac{(Ab+2aB)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{4/3}b^{4/3}}$$

$$= \frac{(Ab-aB)x^2}{3ab\,(a+bx^3)} - \frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{5/3}} + \frac{(Ab+2aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{4/3}b^{5/3}} + \frac{(Ab+2aB)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6ab^{4/3}}$$

$$= \frac{(Ab-aB)x^2}{3ab\,(a+bx^3)} - \frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{5/3}} + \frac{(Ab+2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{4/3}b^{5/3}} + \frac{(Ab+2aB)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{4/3}b^{5/3}}$$

$$= \frac{(Ab-aB)x^2}{3ab\left(a+bx^3\right)} - \frac{(Ab+2aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{5/3}} - \frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{5/3}} + \frac{(Ab+2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{4/3}b^{5/3}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.85

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$= \frac{-\frac{6\sqrt[3]{a}b^{2/3}(-Ab+aB)x^2}{a+bx^3} - 2\sqrt{3}(Ab+2aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right) + (Ab+2aB)}{18a^{4/3}b^{5/3}}$$

```
[In] Integrate[(x*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((-6*a^(1/3)*b^(2/3)*(-(A*b) + a*B)*x^2)/(a + b*x^3) - 2*Sqrt[3]*(A*b + 2*a
*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*(A*b + 2*a*B)*Log[a^(1/
3) + b^(1/3)*x] + (A*b + 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(18*a^(4/3)*b^(5/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.39

method	result	size
risch	$\frac{(Ab-Ba)x^2}{3ab(b\,x^3+a)}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\,\_Z^3+a\right)}\frac{(Ab+2Ba)\ln\left(x-\_R\right)}{\_R}}{9a\,b^2}$	67
default	$\frac{(Ab-Ba)x^2}{3ab(b\,x^3+a)}+\frac{(Ab+2Ba)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab}$	136

```
[In] int(x*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(A*b-B*a)*x^2/a/b/(b*x^3+a)+1/9/a/b^2*sum((A*b+2*B*a)/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 548, normalized size of antiderivative = 3.20

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^2}\,dx$$

$$= \left[-\frac{6\,(Ba^2b^2 - Aab^3)x^2 - 3\,\sqrt{\frac{1}{3}}(2\,Ba^3b + Aa^2b^2 + (2\,Ba^2b^2 + Aab^3)x^3)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3 - ab + 3\,\sqrt{\frac{1}{3}}\left(abx+\right.}{}\right.}{}\right.$$

$$-\frac{6\,(Ba^2b^2 - Aab^3)x^2 - 6\,\sqrt{\frac{1}{3}}(2\,Ba^3b + Aa^2b^2 + (2\,Ba^2b^2 + Aab^3)x^3)\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,bx+(-ab^2)\right.}{b}\right.}{}$$

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/18*(6*(B*a^2*b^2 - A*a*b^3)*x^2 - 3*sqrt(1/3)*(2*B*a^3*b + A*a^2*b^2 +
(2*B*a^2*b^2 + A*a*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b +
3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)
^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - ((2*B*a*b + A*b^2)*x^3 + 2*B
*a^2 + A*a*b)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2
/3)) + 2*((2*B*a*b + A*b^2)*x^3 + 2*B*a^2 + A*a*b)*(-a*b^2)^(2/3)*log(b*x -
 (-a*b^2)^(1/3)))/(a^2*b^4*x^3 + a^3*b^3), -1/18*(6*(B*a^2*b^2 - A*a*b^3)*x
^2 - 6*sqrt(1/3)*(2*B*a^3*b + A*a^2*b^2 + (2*B*a^2*b^2 + A*a*b^3)*x^3)*sqrt
(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2
)^(1/3)/a)/b) - ((2*B*a*b + A*b^2)*x^3 + 2*B*a^2 + A*a*b)*(-a*b^2)^(2/3)*lo
g(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*((2*B*a*b + A*b^2)*x^3
 + 2*B*a^2 + A*a*b)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^4*x^3
+ a^3*b^3)]
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.68

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{x^2(Ab-Ba)}{3a^2b+3ab^2x^3} + \text{RootSum}\left(729t^3a^4b^5+A^3b^3+6A^2Bab^2+12AB^2a^2b+8B^3a^3, \left(t \mapsto t\log\left(\frac{81t^2a^3b^3}{A^2b^2+4ABab+4B^2a^2}+\right.\right.\right.$$

```
[In] integrate(x*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] x**2*(A*b - B*a)/(3*a**2*b + 3*a*b**2*x**3) + RootSum(729*_t**3*a**4*b**5 +
 A**3*b**3 + 6*A**2*B*a*b**2 + 12*A*B**2*a**2*b + 8*B**3*a**3, Lambda(_t, _
t*log(81*_t**2*a**3*b**3/(A**2*b**2 + 4*A*B*a*b + 4*B**2*a**2) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.94

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^2}\,dx = -\frac{(Ba-Ab)x^2}{3\,(ab^2x^3+a^2b)} + \frac{\sqrt{3}(2\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(2\,Ba+Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(2\,Ba+Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(B*a - A*b)*x^2/(a*b^2*x^3 + a^2*b) + 1/9*sqrt(3)*(2*B*a + A*b)*arctan
(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(1/3)) + 1/18*(2
*B*a + A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(1/3)) - 1/
9*(2*B*a + A*b)*log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.09

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{\sqrt{3}(2\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^2\right)^{\frac{1}{3}}\,ab} - \frac{(2\,Ba+Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^2\right)^{\frac{1}{3}}\,ab} - \frac{\left(2\,Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}+Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b} - \frac{Bax^2-Abx^2}{3\,(bx^3+a)ab}$$

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
[Out] 1/9*sqrt(3)*(2*B*a + A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1
/3))/((-a*b^2)^(1/3)*a*b) - 1/18*(2*B*a + A*b)*log(x^2 + x*(-a/b)^(1/3) + (
-a/b)^(2/3))/((-a*b^2)^(1/3)*a*b) - 1/9*(2*B*a*(-a/b)^(1/3) + A*b*(-a/b)^(1
/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b) - 1/3*(B*a*x^2 - A*b*x
^2)/((b*x^3 + a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 6.77 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.85

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{x^2\,(A\,b-B\,a)}{3\,a\,b\,\left(b\,x^3+a\right)} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(A\,b+2\,B\,a)}{9\,a^{4/3}\,b^{5/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(A\,b+2\,B\,a)}{9\,a^{4/3}\,b^{5/3}} - \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(A\,b+2\,B\,a)}{9\,a^{4/3}\,b^{5/3}}$$

```
[In] int((x*(A + B*x^3))/(a + b*x^3)^2,x)
[Out] (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*
b + 2*B*a))/(9*a^(4/3)*b^(5/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a
^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b + 2*B*a))/(9*a^(4/3)*b^(5/3)) - (log(b^
(1/3)*x + a^(1/3))*(A*b + 2*B*a))/(9*a^(4/3)*b^(5/3)) + (x^2*(A*b - B*a))/(
3*a*b*(a + b*x^3))
```

3.80 $\int \frac{A+Bx^3}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 169

$$\int \frac{A+Bx^3}{(a+bx^3)^2}\,dx = \frac{(Ab-aB)x}{3ab(a+bx^3)} - \frac{(2Ab+aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}} + \frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}} - \frac{(2Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}}$$

```
[Out] 1/3*(A*b-B*a)*x/a/b/(b*x^3+a)+1/9*(2*A*b+B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)
/b^(4/3)-1/18*(2*A*b+B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)
/b^(4/3)-1/9*(2*A*b+B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/
a^(5/3)/b^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used

$= \{393, 206, 31, 648, 631, 210, 642\}$

$$\int \frac{A+Bx^3}{(a+bx^3)^2}\,dx = -\frac{(aB+2Ab)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}} - \frac{(aB+2Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}} + \frac{(aB+2Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}} + \frac{x(Ab-aB)}{3ab\,(a+bx^3)}$$

```
[In] Int[(A + B*x^3)/(a + b*x^3)^2,x]
```

```
[Out] ((A*b - a*B)*x)/(3*a*b*(a + b*x^3)) - ((2*A*b + a*B)*ArcTan[(a^(1/3) - 2*b^
(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(4/3)) + ((2*A*b + a*B)*L
og[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(4/3)) - ((2*A*b + a*B)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{(Ab-aB)x}{3ab\,(a+bx^3)} + \frac{(2Ab+aB)\int \frac{1}{a+bx^3}\,dx}{3ab}$$

$$= \frac{(Ab-aB)x}{3ab\,(a+bx^3)} + \frac{(2Ab+aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{5/3}b} + \frac{(2Ab+aB)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}b}$$

$$= \frac{(Ab-aB)x}{3ab\,(a+bx^3)} + \frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}} - \frac{(2Ab+aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{5/3}b^{4/3}} + \frac{(2Ab+aB)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{4/3}b}$$

$$= \frac{(Ab-aB)x}{3ab\,(a+bx^3)} + \frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}} - \frac{(2Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}} + \frac{(2Ab+aB)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{5/3}b^{4/3}}$$

$$=\frac{(Ab-aB)x}{3ab\,(a+bx^3)}-\frac{(2Ab+aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}}+\frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}}-\frac{(2Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.86

$$\int\frac{A+Bx^3}{(a+bx^3)^2}\,dx$$
$$=\frac{-\frac{6a^{2/3}\sqrt[3]{b}(-Ab+aB)x}{a+bx^3}-2\sqrt{3}(2Ab+aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-(2Ab+aB}{18a^{5/3}b^{4/3}}$$

```
[In] Integrate[(A + B*x^3)/(a + b*x^3)^2,x]
```

```
[Out] ((-6*a^(2/3)*b^(1/3)*(-(A*b) + a*B)*x)/(a + b*x^3) - 2*Sqrt[3]*(2*A*b + a*B
)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(2*A*b + a*B)*Log[a^(1/3)
 + b^(1/3)*x] - (2*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(18*a^(5/3)*b^(4/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.38

method	result	size
risch	$\frac{(Ab-Ba)x}{3ab(b\,x^3+a)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(2Ab+Ba)\ln\left(x-\_R\right)}{\_R^2}}{9a\,b^2}$	65
default	$\frac{(Ab-Ba)x}{3ab(b\,x^3+a)}+\frac{(2Ab+Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3ab}$	134

```
[In] int((B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(A*b-B*a)*x/a/b/(b*x^3+a)+1/9/a/b^2*sum((2*A*b+B*a)/_R^2*ln(x-_R),_R=Ro
otOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 537, normalized size of antiderivative = 3.18

$$\int \frac{A+Bx^3}{(a+bx^3)^2}\,dx$$

$$= \left[\frac{3\sqrt{\frac{1}{3}}(Ba^3b+2\,Aa^2b^2+(Ba^2b^2+2\,Aab^3)x^3)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\,(a^2b)^{\frac{1}{3}}ax-a^2+3\,\sqrt{\frac{1}{3}}\left(2\,abx^2+(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right.}{bx^3+a}\right.}{}\right.$$

```
[In] integrate((B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(B*a^3*b + 2*A*a^2*b^2 + (B*a^2*b^2 + 2*A*a*b^3)*x^3)*sq
rt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/
3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/
(b*x^3 + a)) - ((B*a*b + 2*A*b^2)*x^3 + B*a^2 + 2*A*a*b)*(a^2*b)^(2/3)*log(
a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((B*a*b + 2*A*b^2)*x^3 + B
*a^2 + 2*A*a*b)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) - 6*(B*a^3*b - A*a
^2*b^2)*x)/(a^3*b^3*x^3 + a^4*b^2), 1/18*(6*sqrt(1/3)*(B*a^3*b + 2*A*a^2*b^
2 + (B*a^2*b^2 + 2*A*a*b^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*
(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - ((B*a*b + 2
*A*b^2)*x^3 + B*a^2 + 2*A*a*b)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x
+ (a^2*b)^(1/3)*a) + 2*((B*a*b + 2*A*b^2)*x^3 + B*a^2 + 2*A*a*b)*(a^2*b)^(2
/3)*log(a*b*x + (a^2*b)^(2/3)) - 6*(B*a^3*b - A*a^2*b^2)*x)/(a^3*b^3*x^3 +
a^4*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.57

$$\int \frac{A+Bx^3}{(a+bx^3)^2}\,dx = \frac{x(Ab-Ba)}{3a^2b+3ab^2x^3} + \operatorname{RootSum}\left(729t^3a^5b^4-8A^3b^3-12A^2Bab^2-6AB^2a^2b-B^3a^3,\left(t\mapsto t\log\left(\frac{9ta^2b}{2Ab+Ba}+x\right)\right)\right)$$

```
[In] integrate((B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] x*(A*b - B*a)/(3*a**2*b + 3*a*b**2*x**3) + RootSum(729*_t**3*a**5*b**4 - 8*
A**3*b**3 - 12*A**2*B*a*b**2 - 6*A*B**2*a**2*b - B**3*a**3, Lambda(_t, _t*l
og(9*_t*a**2*b/(2*A*b + B*a) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^3}{(a+bx^3)^2}\,dx = -\frac{(Ba-Ab)x}{3\,(ab^2x^3+a^2b)} + \frac{\sqrt{3}(Ba+2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(Ba+2\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(Ba+2\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(B*a - A*b)*x/(a*b^2*x^3 + a^2*b) + 1/9*sqrt(3)*(B*a + 2*A*b)*arctan(1
/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(2/3)) - 1/18*(B*a
 + 2*A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3)) + 1/9*
(B*a + 2*A*b)*log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx^3}{(a+bx^3)^2}\,dx = -\frac{\sqrt{3}(Ba+2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,(-ab^2)^{\frac{2}{3}}\,a} - \frac{(Ba+2\,Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,(-ab^2)^{\frac{2}{3}}\,a} - \frac{(Ba+2\,Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b} - \frac{Bax-Abx}{3\,(bx^3+a)ab}$$

```
[In] integrate((B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(B*a + 2*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/((-a*b^2)^(2/3)*a) - 1/18*(B*a + 2*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/9*(B*a + 2*A*b)*(-a/b)^(1/3)*log(abs(x -
 (-a/b)^(1/3)))/(a^2*b) - 1/3*(B*a*x - A*b*x)/((b*x^3 + a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 6.81 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^3}{(a+bx^3)^2}\,dx = \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(2\,A\,b+B\,a)}{9\,a^{5/3}\,b^{4/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(2\,A\,b+B\,a)}{9\,a^{5/3}\,b^{4/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(2\,A\,b+B\,a)}{9\,a^{5/3}\,b^{4/3}} + \frac{x\,(A\,b-B\,a)}{3\,a\,b\,(b\,x^3+a)}$$

```
[In] int((A + B*x^3)/(a + b*x^3)^2,x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(2*A*b + B*a))/(9*a^(5/3)*b^(4/3)) - (log(3^(1/2)
*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*A*b + B*a))/
(9*a^(5/3)*b^(4/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^
(1/2)*1i)/2 - 1/2)*(2*A*b + B*a))/(9*a^(5/3)*b^(4/3)) + (x*(A*b - B*a))/(3*
a*b*(a + b*x^3))
```

3.81 $\int \frac{A+Bx^3}{x(a+bx^3)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 51

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^2}\,dx=\frac{Ab-aB}{3ab\left(a+bx^3\right)}+\frac{A\log(x)}{a^2}-\frac{A\log\left(a+bx^3\right)}{3a^2}$$

```
[Out] 1/3*(A*b-B*a)/a/b/(b*x^3+a)+A*ln(x)/a^2-1/3*A*ln(b*x^3+a)/a^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 78}

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^2}\,dx=-\frac{A\log\left(a+bx^3\right)}{3a^2}+\frac{A\log(x)}{a^2}+\frac{Ab-aB}{3ab\left(a+bx^3\right)}$$

```
[In] Int[(A + B*x^3)/(x*(a + b*x^3)^2),x]
```

```
[Out] (A*b - a*B)/(3*a*b*(a + b*x^3)) + (A*Log[x])/a^2 - (A*Log[a + b*x^3])/(3*a^
2)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x(a+bx)^2}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{A}{a^2x}+\frac{-Ab+aB}{a(a+bx)^2}-\frac{Ab}{a^2(a+bx)}\right)\,dx, x, x^3\right)\\
&= \frac{Ab-aB}{3ab\,(a+bx^3)}+\frac{A\log(x)}{a^2}-\frac{A\log\left(a+bx^3\right)}{3a^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$
\int \frac{A+Bx^3}{x\left(a+bx^3\right)^2}\,dx = \frac{\frac{a(Ab-aB)}{b(a+bx^3)}+3A\log(x)-A\log\left(a+bx^3\right)}{3a^2}
$$

```
[In] Integrate[(A + B*x^3)/(x*(a + b*x^3)^2),x]
[Out] ((a*(A*b - a*B))/(b*(a + b*x^3)) + 3*A*Log[x] - A*Log[a + b*x^3])/(3*a^2)
```

Maple [A] (verified)

Time = 4.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{A\ln(x)}{a^2}-\frac{A\ln\left(b\,x^3+a\right)-\frac{a(Ab-Ba)}{b\left(b\,x^3+a\right)}}{3a^2}$	48
norman	$-\frac{(Ab-Ba)x^3}{3a^2(b\,x^3+a)}+\frac{A\ln(x)}{a^2}-\frac{A\ln\left(b\,x^3+a\right)}{3a^2}$	48
risch	$\frac{A}{3a(b\,x^3+a)}-\frac{B}{3b(b\,x^3+a)}+\frac{A\ln(x)}{a^2}-\frac{A\ln\left(b\,x^3+a\right)}{3a^2}$	53
parallelrisch	$\frac{3A\ln(x)x^3b-A\ln\left(b\,x^3+a\right)x^3b-Ab\,x^3+Ba\,x^3+3aA\ln(x)-A\ln\left(b\,x^3+a\right)a}{3a^2(b\,x^3+a)}$	71

```
[In] int((B*x^3+A)/x/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
[Out] A*ln(x)/a^2-1/3/a^2*(A*ln(b*x^3+a)-a*(A*b-B*a)/b/(b*x^3+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.37

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^2}\,dx = -\frac{Ba^2 - Aab + \left(Ab^2x^3 + Aab\right)\log\left(bx^3+a\right) - 3\left(Ab^2x^3 + Aab\right)\log\left(x\right)}{3\left(a^2b^2x^3 + a^3b\right)}$$

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^2,x, algorithm="fricas")
[Out] -1/3*(B*a^2 - A*a*b + (A*b^2*x^3 + A*a*b)*log(b*x^3 + a) - 3*(A*b^2*x^3 + A
*a*b)*log(x))/(a^2*b^2*x^3 + a^3*b)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^2}\,dx = \frac{A\log\left(x\right)}{a^2} - \frac{A\log\left(\frac{a}{b}+x^3\right)}{3a^2} + \frac{Ab-Ba}{3a^2b+3ab^2x^3}$$

```
[In] integrate((B*x**3+A)/x/(b*x**3+a)**2,x)
[Out] A*log(x)/a**2 - A*log(a/b + x**3)/(3*a**2) + (A*b - B*a)/(3*a**2*b + 3*a*b*
*2*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^2}\,dx = -\frac{Ba-Ab}{3\left(ab^2x^3+a^2b\right)} - \frac{A\log\left(bx^3+a\right)}{3\,a^2} + \frac{A\log\left(x^3\right)}{3\,a^2}$$

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^2,x, algorithm="maxima")
[Out] -1/3*(B*a - A*b)/(a*b^2*x^3 + a^2*b) - 1/3*A*log(b*x^3 + a)/a^2 + 1/3*A*log
(x^3)/a^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^2}\,dx=-\frac{A\log\left(|bx^3+a|\right)}{3\,a^2}+\frac{A\log\left(|x|\right)}{a^2}+\frac{Ab^2x^3-Ba^2+2\,Aab}{3\,(bx^3+a)a^2b}$$

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*A*log(abs(b*x^3 + a))/a^2 + A*log(abs(x))/a^2 + 1/3*(A*b^2*x^3 - B*a^2
 + 2*A*a*b)/((b*x^3 + a)*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^2}\,dx=\frac{A\,\ln\left(x\right)}{a^2}-\frac{A\,\ln\left(b\,x^3+a\right)}{3\,a^2}+\frac{A\,b-B\,a}{3\,a\,b\,\left(b\,x^3+a\right)}$$

```
[In] int((A + B*x^3)/(x*(a + b*x^3)^2),x)
```

```
[Out] (A*log(x))/a^2 - (A*log(a + b*x^3))/(3*a^2) + (A*b - B*a)/(3*a*b*(a + b*x^3
))
```

3.82 $\int \frac{A+Bx^3}{x^2(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 195

$$\int \frac{A+Bx^3}{x^2(a+bx^3)^2}\,dx = \frac{-4Ab+aB}{3a^2bx}+\frac{Ab-aB}{3abx(a+bx^3)}+\frac{(4Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}b^{2/3}}+\frac{(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}b^{2/3}}-\frac{(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{7/3}b^{2/3}}$$

```
[Out] 1/3*(-4*A*b+B*a)/a^2/b/x+1/3*(A*b-B*a)/a/b/x/(b*x^3+a)+1/9*(4*A*b-B*a)*ln(a
^(1/3)+b^(1/3)*x)/a^(7/3)/b^(2/3)-1/18*(4*A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/
3)*x+b^(2/3)*x^2)/a^(7/3)/b^(2/3)+1/9*(4*A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(
1/3)*x)/a^(1/3)*3^(1/2))/a^(7/3)/b^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used

= {468, 331, 298, 31, 648, 631, 210, 642}

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^2}\,dx = \frac{(4Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}b^{2/3}} - \frac{(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{7/3}b^{2/3}} + \frac{(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}b^{2/3}} - \frac{4Ab-aB}{3a^2bx} + \frac{Ab-aB}{3abx\left(a+bx^3\right)}$$

```
[In] Int[(A + B*x^3)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -1/3*(4*A*b - a*B)/(a^2*b*x) + (A*b - a*B)/(3*a*b*x*(a + b*x^3)) + ((4*A*b
- a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3
)*b^(2/3)) + ((4*A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(7/3)*b^(2/3)) -
 ((4*A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(7/3)
*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{Ab-aB}{3abx\left(a+bx^3\right)}+\frac{(4Ab-aB)\int\frac{1}{x^2(a+bx^3)}\,dx}{3ab}\\
&= -\frac{4Ab-aB}{3a^2bx}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}-\frac{(4Ab-aB)\int\frac{x}{a+bx^3}\,dx}{3a^2}\\
&= -\frac{4Ab-aB}{3a^2bx}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}+\frac{(4Ab-aB)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{7/3}\sqrt[3]{b}}\\
&\quad-\frac{(4Ab-aB)\int\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{7/3}\sqrt[3]{b}}
\end{aligned}
$$

$$= -\frac{4Ab-aB}{3a^2bx}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}+\frac{(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}b^{2/3}} - \frac{(4Ab-aB)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{7/3}b^{2/3}} - \frac{(4Ab-aB)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^2\sqrt[3]{b}}$$

$$= -\frac{4Ab-aB}{3a^2bx}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}+\frac{(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}b^{2/3}} - \frac{(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{7/3}b^{2/3}} - \frac{(4Ab-aB)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{7/3}b^{2/3}}$$

$$= -\frac{4Ab-aB}{3a^2bx}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}+\frac{(4Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}b^{2/3}} + \frac{(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}b^{2/3}} - \frac{(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{7/3}b^{2/3}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.84

$$\int\frac{A+Bx^3}{x^2\left(a+bx^3\right)^2}\,dx$$

$$= \frac{-\frac{18\sqrt[3]{a}A}{x}+\frac{6\sqrt[3]{a}(-Ab+aB)x^2}{a+bx^3}+\frac{2\sqrt{3}(4Ab-aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{2(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}+\frac{(-4Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\right.}{b^{2/3}}}{18a^{7/3}}$$

```
[In] Integrate[(A + B*x^3)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] ((-18*a^(1/3)*A)/x + (6*a^(1/3)*(-(A*b) + a*B)*x^2)/(a + b*x^3) + (2*Sqrt[3
]*(4*A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (2*(
4*A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((-4*A*b + a*B)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(18*a^(7/3))
```

Maple [A] (verified)

Time = 4.37 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.71

method	result
default	$-\frac{A}{a^{2}x}-\frac{\frac{\left(\frac{Ab}{3}-\frac{Ba}{3}\right)x^{2}}{b\,x^{3}+a}+\left(\frac{4Ab}{3}-\frac{Ba}{3}\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^{2}}$
risch	$\frac{-\frac{(4Ab-Ba)x^{3}}{3a^{2}}-\frac{A}{a}}{x(b\,x^{3}+a)}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^{7}b^{2}\_Z^{3}-64A^{3}b^{3}+48A^{2}Ba\,b^{2}-12A\,B^{2}a^{2}b+B^{3}a^{3}\right)}\_R\ln\left(\left(-4\_R^{3}a^{7}b^{2}+192A^{3}b^{3}-144A^{2}Ba\right.\right.\right.}{9}$

```
[In] int((B*x^3+A)/x^2/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -A/a^2/x-1/a^2*((1/3*A*b-1/3*B*a)*x^2/(b*x^3+a)+(4/3*A*b-1/3*B*a)*(-1/3/b/(
a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(
2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 570, normalized size of antiderivative = 2.92

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^2}\,dx$$

$$= \left[-\frac{18\,Aa^2b^2-6\,(Ba^2b^2-4\,Aab^3)x^3+3\,\sqrt{\frac{1}{3}}((Ba^2b^2-4\,Aab^3)x^4+(Ba^3b-4\,Aa^2b^2)x)\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2}{}\right.}{}\right.$$

$$-\frac{18\,Aa^2b^2-6\,(Ba^2b^2-4\,Aab^3)x^3+6\,\sqrt{\frac{1}{3}}((Ba^2b^2-4\,Aab^3)x^4+(Ba^3b-4\,Aa^2b^2)x)\sqrt{\frac{(ab^2)^{\frac{1}{3}}}{a}}\arctan\left(-\right.}{}$$

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/18*(18*A*a^2*b^2 - 6*(B*a^2*b^2 - 4*A*a*b^3)*x^3 + 3*sqrt(1/3)*((B*a^2*
b^2 - 4*A*a*b^3)*x^4 + (B*a^3*b - 4*A*a^2*b^2)*x)*sqrt(-(a*b^2)^(1/3)/a)*lo
g((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/
3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) - ((B*a*b -
4*A*b^2)*x^4 + (B*a^2 - 4*A*a*b)*x)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/
3)*b*x + (a*b^2)^(2/3)) + 2*((B*a*b - 4*A*b^2)*x^4 + (B*a^2 - 4*A*a*b)*x)*(
a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^3*x^4 + a^4*b^2*x), -1/18*(18
*A*a^2*b^2 - 6*(B*a^2*b^2 - 4*A*a*b^3)*x^3 + 6*sqrt(1/3)*((B*a^2*b^2 - 4*A*
a*b^3)*x^4 + (B*a^3*b - 4*A*a^2*b^2)*x)*sqrt((a*b^2)^(1/3)/a)*arctan(-sqrt(
1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) - ((B*a*b - 4*A*b^2)*
x^4 + (B*a^2 - 4*A*a*b)*x)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x +
(a*b^2)^(2/3)) + 2*((B*a*b - 4*A*b^2)*x^4 + (B*a^2 - 4*A*a*b)*x)*(a*b^2)^(2
/3)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^3*x^4 + a^4*b^2*x)]
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.63

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^2}\,dx = \frac{-3Aa+x^3(-4Ab+Ba)}{3a^3x+3a^2bx^4} + \operatorname{RootSum}\left(729t^3a^7b^2-64A^3b^3+48A^2Bab^2-12AB^2a^2b+B^3a^3, \left(t\mapsto t\log\left(\frac{81t^2a^5b}{16A^2b^2-8ABab+B^2a^2}\right.\right.\right.$$

```
[In] integrate((B*x**3+A)/x**2/(b*x**3+a)**2,x)
```

```
[Out] (-3*A*a + x**3*(-4*A*b + B*a))/(3*a**3*x + 3*a**2*b*x**4) + RootSum(729*_t*
*3*a**7*b**2 - 64*A**3*b**3 + 48*A**2*B*a*b**2 - 12*A*B**2*a**2*b + B**3*a*
*3, Lambda(_t, _t*log(81*_t**2*a**5*b/(16*A**2*b**2 - 8*A*B*a*b + B**2*a**2
) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^2}\,dx = \frac{(Ba-4\,Ab)x^3-3\,Aa}{3\left(a^2bx^4+a^3x\right)} + \frac{\sqrt{3}(Ba-4\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(Ba-4\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(Ba-4\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((B*a - 4*A*b)*x^3 - 3*A*a)/(a^2*b*x^4 + a^3*x) + 1/9*sqrt(3)*(B*a - 4*
A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(1/3)
) + 1/18*(B*a - 4*A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^
(1/3)) - 1/9*(B*a - 4*A*b)*log(x + (a/b)^(1/3))/(a^2*b*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^2}\,dx = \frac{\sqrt{3}(Ba-4\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{1}{3}}a^2} - \frac{(Ba-4\,Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{1}{3}}a^2} - \frac{\left(Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}-4\,Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3} + \frac{Bax^3-4\,Abx^3-3\,Aa}{3\left(bx^4+ax\right)a^2}$$

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(B*a - 4*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1
/3))/((-a*b^2)^(1/3)*a^2) - 1/18*(B*a - 4*A*b)*log(x^2 + x*(-a/b)^(1/3) + (
-a/b)^(2/3))/((-a*b^2)^(1/3)*a^2) - 1/9*(B*a*(-a/b)^(1/3) - 4*A*b*(-a/b)^(1
/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^3 + 1/3*(B*a*x^3 - 4*A*b*x^3
 - 3*A*a)/((b*x^4 + a*x)*a^2)
```

Mupad [B] (verification not implemented)

Time = 6.90 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^2}\,dx = \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(4\,A\,b-B\,a\right)}{9\,a^{7/3}\,b^{2/3}} - \frac{\frac{A}{a}+\frac{x^3\,(4\,A\,b-B\,a)}{3\,a^2}}{b\,x^4+a\,x} + \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(4\,A\,b-B\,a\right)}{9\,a^{7/3}\,b^{2/3}} - \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(4\,A\,b-B\,a\right)}{9\,a^{7/3}\,b^{2/3}}$$

```
[In] int((A + B*x^3)/(x^2*(a + b*x^3)^2),x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(4*A*b - B*a))/(9*a^(7/3)*b^(2/3)) - (A/a + (x^3*
(4*A*b - B*a))/(3*a^2))/(a*x + b*x^4) + (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)
*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(4*A*b - B*a))/(9*a^(7/3)*b^(2/3)) - (
log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(4*A
*b - B*a))/(9*a^(7/3)*b^(2/3))
```

3.83 $\int \frac{A+Bx^3}{x^3(a+bx^3)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 196

$$\int \frac{A+Bx^3}{x^3(a+bx^3)^2} dx = \frac{-5Ab+2aB}{6a^2bx^2} + \frac{Ab-aB}{3abx^2(a+bx^3)} + \frac{(5Ab-2aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}\sqrt[3]{b}} - \frac{(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}\sqrt[3]{b}} + \frac{(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}\sqrt[3]{b}}$$

```
[Out] 1/6*(-5*A*b+2*B*a)/a^2/b/x^2+1/3*(A*b-B*a)/a/b/x^2/(b*x^3+a)-1/9*(5*A*b-2*B
*a)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^(1/3)+1/18*(5*A*b-2*B*a)*ln(a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(1/3)+1/9*(5*A*b-2*B*a)*arctan(1/3*(a
^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used

$= \{468, 331, 206, 31, 648, 631, 210, 642\}$

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^2}\,dx = \frac{(5Ab-2aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}\sqrt[3]{b}} + \frac{(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}\sqrt[3]{b}} - \frac{(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}\sqrt[3]{b}} - \frac{5Ab-2aB}{6a^2bx^2} + \frac{Ab-aB}{3abx^2\left(a+bx^3\right)}$$

```
[In] Int[(A + B*x^3)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -1/6*(5*A*b - 2*a*B)/(a^2*b*x^2) + (A*b - a*B)/(3*a*b*x^2*(a + b*x^3)) + ((
5*A*b - 2*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3
]*a^(8/3)*b^(1/3)) - ((5*A*b - 2*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(8/3)*
b^(1/3)) + ((5*A*b - 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])
/(18*a^(8/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{Ab-aB}{3abx^2\,(a+bx^3)}+\frac{(5Ab-2aB)\int\frac{1}{x^3(a+bx^3)}\,dx}{3ab}\\
&= -\frac{5Ab-2aB}{6a^2bx^2}+\frac{Ab-aB}{3abx^2\,(a+bx^3)}-\frac{(5Ab-2aB)\int\frac{1}{a+bx^3}\,dx}{3a^2}\\
&= -\frac{5Ab-2aB}{6a^2bx^2}+\frac{Ab-aB}{3abx^2\,(a+bx^3)}-\frac{(5Ab-2aB)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{8/3}}\\
&\quad-\frac{(5Ab-2aB)\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{8/3}}
\end{aligned}$$

$$= -\frac{5Ab-2aB}{6a^2bx^2} + \frac{Ab-aB}{3abx^2(a+bx^3)} - \frac{(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}\sqrt[3]{b}} - \frac{(5Ab-2aB)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{7/3}} + \frac{(5Ab-2aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{8/3}\sqrt[3]{b}}$$

$$= -\frac{5Ab-2aB}{6a^2bx^2} + \frac{Ab-aB}{3abx^2(a+bx^3)} - \frac{(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}\sqrt[3]{b}} + \frac{(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}\sqrt[3]{b}} - \frac{(5Ab-2aB)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{8/3}\sqrt[3]{b}}$$

$$= -\frac{5Ab-2aB}{6a^2bx^2} + \frac{Ab-aB}{3abx^2(a+bx^3)} + \frac{(5Ab-2aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}\sqrt[3]{b}} - \frac{(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}\sqrt[3]{b}} + \frac{(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}\sqrt[3]{b}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx^3}{x^3(a+bx^3)^2}\,dx$$

$$= \frac{-\frac{9a^{2/3}A}{x^2} + \frac{6a^{2/3}(-Ab+aB)x}{a+bx^3} + \frac{2\sqrt{3}(5Ab-2aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} + \frac{2(-5Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}} + \frac{(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{\cdots}\right.}{\sqrt[3]{b}}}{18a^{8/3}}$$

```
[In] Integrate[(A + B*x^3)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] ((-9*a^(2/3)*A)/x^2 + (6*a^(2/3)*(-(A*b) + a*B)*x)/(a + b*x^3) + (2*Sqrt[3]
*(5*A*b - 2*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (2*
(-5*A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + ((5*A*b - 2*a*B)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3))/(18*a^(8/3))
```

Maple [A] (verified)

Time = 4.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.70

method	result
default	$-\frac{A}{2a^2x^2}-\frac{\frac{\left(\frac{Ab}{3}-\frac{Ba}{3}\right)x}{b\,x^3+a}+\frac{(5Ab-2Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3}}{a^2}$
risch	$\frac{-\frac{(5Ab-2Ba)x^3}{6a^2}-\frac{A}{2a}}{x^2(b\,x^3+a)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^8b\_Z^3+125A^3b^3-150A^2Ba\,b^2+60A\,B^2a^2b-8B^3a^3\right)}\_R\ln\left(\left(-4\_R^3a^8b-375A^3b^3+450A^2B\right.\right.\right.}{9}$

[In] int((B*x^3+A)/x^3/(b*x^3+a)^2,x,method=_RETURNVERBOSE)

[Out] -1/2*A/a^2/x^2-1/a^2*((1/3*A*b-1/3*B*a)*x/(b*x^3+a)+1/3*(5*A*b-2*B*a)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))
)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 618, normalized size of antiderivative = 3.15

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^2}\,dx$$

$$= \left[-\frac{9\,Aa^3b - 3\,(2\,Ba^3b - 5\,Aa^2b^2)x^3 + 3\,\sqrt{\tfrac{1}{3}}((2\,Ba^2b^2 - 5\,Aab^3)x^5 + (2\,Ba^3b - 5\,Aa^2b^2)x^2)\sqrt{\tfrac{(-a^2b)^{\frac{1}{3}}}{b}}\log\Bigg(\right.$$

$$-\frac{9\,Aa^3b - 3\,(2\,Ba^3b - 5\,Aa^2b^2)x^3 - 6\,\sqrt{\tfrac{1}{3}}((2\,Ba^2b^2 - 5\,Aab^3)x^5 + (2\,Ba^3b - 5\,Aa^2b^2)x^2)\sqrt{-\tfrac{(-a^2b)^{\frac{1}{3}}}{b}}\,\mathrm{arct}}{}$$

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/18*(9*A*a^3*b - 3*(2*B*a^3*b - 5*A*a^2*b^2)*x^3 + 3*sqrt(1/3)*((2*B*a^2
*b^2 - 5*A*a*b^3)*x^5 + (2*B*a^3*b - 5*A*a^2*b^2)*x^2)*sqrt((-a^2*b)^(1/3)/
b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (
-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) +
((2*B*a*b - 5*A*b^2)*x^5 + (2*B*a^2 - 5*A*a*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*
x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) - 2*((2*B*a*b - 5*A*b^2)*x^5 + (
2*B*a^2 - 5*A*a*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))/(a^4*b^
2*x^5 + a^5*b*x^2), -1/18*(9*A*a^3*b - 3*(2*B*a^3*b - 5*A*a^2*b^2)*x^3 - 6*
sqrt(1/3)*((2*B*a^2*b^2 - 5*A*a*b^3)*x^5 + (2*B*a^3*b - 5*A*a^2*b^2)*x^2)*s
qrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3
)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) + ((2*B*a*b - 5*A*b^2)*x^5 + (2*B*a^2 - 5
*A*a*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)
*a) - 2*((2*B*a*b - 5*A*b^2)*x^5 + (2*B*a^2 - 5*A*a*b)*x^2)*(-a^2*b)^(2/3)*
log(a*b*x + (-a^2*b)^(2/3)))/(a^4*b^2*x^5 + a^5*b*x^2)]
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.56

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^2}\,dx = \frac{-3Aa+x^3(-5Ab+2Ba)}{6a^3x^2+6a^2bx^5} + \operatorname{RootSum}\left(729t^3a^8b+125A^3b^3-150A^2Bab^2+60AB^2a^2b-8B^3a^3,\left(t\mapsto t\log\left(\frac{9ta^3}{-5Ab+2Ba}+x\right)\right)\right)$$

```
[In] integrate((B*x**3+A)/x**3/(b*x**3+a)**2,x)
```

```
[Out] (-3*A*a + x**3*(-5*A*b + 2*B*a))/(6*a**3*x**2 + 6*a**2*b*x**5) + RootSum(72
9*_t**3*a**8*b + 125*A**3*b**3 - 150*A**2*B*a*b**2 + 60*A*B**2*a**2*b - 8*B
**3*a**3, Lambda(_t, _t*log(9*_t*a**3/(-5*A*b + 2*B*a) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^2}\,dx = \frac{(2\,Ba-5\,Ab)x^3-3\,Aa}{6\left(a^2bx^5+a^3x^2\right)} + \frac{\sqrt{3}(2\,Ba-5\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(2\,Ba-5\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(2\,Ba-5\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*((2*B*a - 5*A*b)*x^3 - 3*A*a)/(a^2*b*x^5 + a^3*x^2) + 1/9*sqrt(3)*(2*B*
a - 5*A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)
^(2/3)) - 1/18*(2*B*a - 5*A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*
b*(a/b)^(2/3)) + 1/9*(2*B*a - 5*A*b)*log(x + (a/b)^(1/3))/(a^2*b*(a/b)^(2/3
))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^2}\,dx = -\frac{(2\,Ba-5\,Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3} + \frac{\sqrt{3}\left(2\,(-ab^2)^{\frac{1}{3}}\,Ba-5\,(-ab^2)^{\frac{1}{3}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b} + \frac{Bax-Abx}{3\,(bx^3+a)a^2} + \frac{\left(2\,(-ab^2)^{\frac{1}{3}}\,Ba-5\,(-ab^2)^{\frac{1}{3}}\,Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3b} - \frac{A}{2\,a^2x^2}$$

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*B*a - 5*A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^3 + 1/9*sqrt
(3)*(2*(-a*b^2)^(1/3)*B*a - 5*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(2*x +
 (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b) + 1/3*(B*a*x - A*b*x)/((b*x^3 + a)*a^2
) + 1/18*(2*(-a*b^2)^(1/3)*B*a - 5*(-a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(
1/3) + (-a/b)^(2/3))/(a^3*b) - 1/2*A/(a^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^2}\,dx = -\frac{\frac{A}{2\,a}+\frac{x^3\,(5\,A\,b-2\,B\,a)}{6\,a^2}}{b\,x^5+a\,x^2} - \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(5\,A\,b-2\,B\,a)}{9\,a^{8/3}\,b^{1/3}} + \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,A\,b-2\,B\,a)}{9\,a^{8/3}\,b^{1/3}} - \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,A\,b-2\,B\,a)}{9\,a^{8/3}\,b^{1/3}}$$

```
[In] int((A + B*x^3)/(x^3*(a + b*x^3)^2),x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*
A*b - 2*B*a))/(9*a^(8/3)*b^(1/3)) - (log(b^(1/3)*x + a^(1/3))*(5*A*b - 2*B*
a))/(9*a^(8/3)*b^(1/3)) - (A/(2*a) + (x^3*(5*A*b - 2*B*a))/(6*a^2))/(a*x^2
+ b*x^5) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2
 - 1/2)*(5*A*b - 2*B*a))/(9*a^(8/3)*b^(1/3))
```

3.84 $\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 76

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^2}\,dx = -\frac{A}{3a^2x^3}-\frac{Ab-aB}{3a^2\left(a+bx^3\right)}-\frac{(2Ab-aB)\log(x)}{a^3}+\frac{(2Ab-aB)\log\left(a+bx^3\right)}{3a^3}$$

```
[Out] -1/3*A/a^2/x^3+1/3*(-A*b+B*a)/a^2/(b*x^3+a)-(2*A*b-B*a)*ln(x)/a^3+1/3*(2*A*
b-B*a)*ln(b*x^3+a)/a^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 78}

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^2}\,dx = \frac{(2Ab-aB)\log\left(a+bx^3\right)}{3a^3}-\frac{\log(x)(2Ab-aB)}{a^3}-\frac{Ab-aB}{3a^2\left(a+bx^3\right)}-\frac{A}{3a^2x^3}$$

```
[In] Int[(A + B*x^3)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] -1/3*A/(a^2*x^3) - (A*b - a*B)/(3*a^2*(a + b*x^3)) - ((2*A*b - a*B)*Log[x])
/a^3 + ((2*A*b - a*B)*Log[a + b*x^3])/(3*a^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
```

```
&& ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)^2}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{A}{a^2x^2}+\frac{-2Ab+aB}{a^3x}-\frac{b(-Ab+aB)}{a^2(a+bx)^2}-\frac{b(-2Ab+aB)}{a^3(a+bx)}\right)\,dx, x, x^3\right)\\ &= -\frac{A}{3a^2x^3}-\frac{Ab-aB}{3a^2\left(a+bx^3\right)}-\frac{(2Ab-aB)\log(x)}{a^3}+\frac{(2Ab-aB)\log\left(a+bx^3\right)}{3a^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^2}\,dx = \frac{-\frac{aA}{x^3}+\frac{a(-Ab+aB)}{a+bx^3}+3(-2Ab+aB)\log(x)+(2Ab-aB)\log\left(a+bx^3\right)}{3a^3}$$

```
[In] Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^2),x]
[Out] (-((a*A)/x^3) + (a*(-(A*b) + a*B))/(a + b*x^3) + 3*(-2*A*b + a*B)*Log[x] +
(2*A*b - a*B)*Log[a + b*x^3])/(3*a^3)
```

Maple [A] (verified)

Time = 4.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

method	result
default	$-\frac{A}{3a^2x^3}+\frac{(-2Ab+Ba)\ln(x)}{a^3}+\frac{b\left(\frac{(2Ab-Ba)\ln\left(b\,x^3+a\right)}{b}-\frac{a(Ab-Ba)}{b\left(b\,x^3+a\right)}\right)}{3a^3}$
norman	$\frac{-\frac{A}{3a}+\frac{b(2Ab-Ba)x^6}{3a^3}}{x^3(b\,x^3+a)}-\frac{(2Ab-Ba)\ln(x)}{a^3}+\frac{(2Ab-Ba)\ln(b\,x^3+a)}{3a^3}$
risch	$\frac{-\frac{(2Ab-Ba)x^3}{3a^2}-\frac{A}{3a}}{x^3(b\,x^3+a)}-\frac{2\ln(x)Ab}{a^3}+\frac{B\ln(x)}{a^2}+\frac{2\ln(-b\,x^3-a)Ab}{3a^3}-\frac{\ln(-b\,x^3-a)B}{3a^2}$
parallelrisch	$-\frac{6A\ln(x)x^6b^2-2A\ln(b\,x^3+a)x^6b^2-3B\ln(x)x^6ab+B\ln(b\,x^3+a)x^6ab-2A\,b^2x^6+B\,x^6ab+6A\ln(x)x^3ab-2A\ln(b\,x^3+a)x^3a}{3a^3x^3(b\,x^3+a)}$

```
[In] int((B*x^3+A)/x^4/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*A/a^2/x^3+(-2*A*b+B*a)/a^3*ln(x)+1/3/a^3*b*((2*A*b-B*a)/b*ln(b*x^3+a)-
a*(A*b-B*a)/b/(b*x^3+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.55

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^2}\,dx = \frac{(Ba^2-2\,Aab)x^3-Aa^2-((Bab-2\,Ab^2)x^6+(Ba^2-2\,Aab)x^3)\log\left(bx^3+a\right)+3\left((Bab-2\,Ab^2)x^6+(B\right.}{3\left(a^3bx^6+a^4x^3\right)}$$

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*((B*a^2 - 2*A*a*b)*x^3 - A*a^2 - ((B*a*b - 2*A*b^2)*x^6 + (B*a^2 - 2*A*
a*b)*x^3)*log(b*x^3 + a) + 3*((B*a*b - 2*A*b^2)*x^6 + (B*a^2 - 2*A*a*b)*x^3
)*log(x))/(a^3*b*x^6 + a^4*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^2}\,dx = \frac{-Aa+x^3(-2Ab+Ba)}{3a^3x^3+3a^2bx^6}+\frac{(-2Ab+Ba)\log\left(x\right)}{a^3} - \frac{(-2Ab+Ba)\log\left(\frac{a}{b}+x^3\right)}{3a^3}$$

```
[In] integrate((B*x**3+A)/x**4/(b*x**3+a)**2,x)
```

```
[Out] (-A*a + x**3*(-2*A*b + B*a))/(3*a**3*x**3 + 3*a**2*b*x**6) + (-2*A*b + B*a)
*log(x)/a**3 - (-2*A*b + B*a)*log(a/b + x**3)/(3*a**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^2}\,dx = \frac{(Ba-2\,Ab)x^3-Aa}{3\left(a^2bx^6+a^3x^3\right)} - \frac{(Ba-2\,Ab)\log\left(bx^3+a\right)}{3\,a^3} + \frac{(Ba-2\,Ab)\log\left(x^3\right)}{3\,a^3}$$

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((B*a - 2*A*b)*x^3 - A*a)/(a^2*b*x^6 + a^3*x^3) - 1/3*(B*a - 2*A*b)*log
(b*x^3 + a)/a^3 + 1/3*(B*a - 2*A*b)*log(x^3)/a^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^2}\,dx = \frac{(Ba-2\,Ab)\log\left(|x|\right)}{a^3} + \frac{Bax^3-2\,Abx^3-Aa}{3\left(bx^6+ax^3\right)a^2} - \frac{\left(Bab-2\,Ab^2\right)\log\left(|bx^3+a|\right)}{3\,a^3b}$$

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] (B*a - 2*A*b)*log(abs(x))/a^3 + 1/3*(B*a*x^3 - 2*A*b*x^3 - A*a)/((b*x^6 + a
*x^3)*a^2) - 1/3*(B*a*b - 2*A*b^2)*log(abs(b*x^3 + a))/(a^3*b)
```

Mupad [B] (verification not implemented)

Time = 6.85 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^2}\,dx = \frac{\ln\left(b\,x^3+a\right)\,\left(2\,A\,b-B\,a\right)}{3\,a^3} - \frac{\frac{A}{3\,a}+\frac{x^3\,\left(2\,A\,b-B\,a\right)}{3\,a^2}}{b\,x^6+a\,x^3} - \frac{\ln\left(x\right)\,\left(2\,A\,b-B\,a\right)}{a^3}$$

```
[In] int((A + B*x^3)/(x^4*(a + b*x^3)^2),x)
```

```
[Out] (log(a + b*x^3)*(2*A*b - B*a))/(3*a^3) - (A/(3*a) + (x^3*(2*A*b - B*a))/(3*
a^2))/(a*x^3 + b*x^6) - (log(x)*(2*A*b - B*a))/a^3
```

3.85 $\int \frac{A+Bx^3}{x^5(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 215

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx=\frac{-7Ab+4aB}{12a^2bx^4}+\frac{7Ab-4aB}{3a^3x}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}-\frac{\sqrt[3]{b}(7Ab-4aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{10/3}}-\frac{\sqrt[3]{b}(7Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{10/3}}+\frac{\sqrt[3]{b}(7Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{10/3}}$$

```
[Out] 1/12*(-7*A*b+4*B*a)/a^2/b/x^4+1/3*(7*A*b-4*B*a)/a^3/x+1/3*(A*b-B*a)/a/b/x^4
/(b*x^3+a)-1/9*b^(1/3)*(7*A*b-4*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)+1/18*b^
(1/3)*(7*A*b-4*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(10/3)-1/9*
b^(1/3)*(7*A*b-4*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(
10/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {468, 331, 298, 31, 648, 631, 210, 642}

$$\int \frac{A+Bx^3}{x^5 (a+bx^3)^2}\,dx = -\frac{\sqrt[3]{b}(7Ab-4aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{10/3}} + \frac{\sqrt[3]{b}(7Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{10/3}} - \frac{\sqrt[3]{b}(7Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{10/3}} + \frac{7Ab-4aB}{3a^3x} - \frac{7Ab-4aB}{12a^2bx^4} + \frac{Ab-aB}{3abx^4(a+bx^3)}$$

```
[In] Int[(A + B*x^3)/(x^5*(a + b*x^3)^2),x]
```

```
[Out] -1/12*(7*A*b - 4*a*B)/(a^2*b*x^4) + (7*A*b - 4*a*B)/(3*a^3*x) + (A*b - a*B)
/(3*a*b*x^4*(a + b*x^3)) - (b^(1/3)*(7*A*b - 4*a*B)*ArcTan[(a^(1/3) - 2*b^(
1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(10/3)) - (b^(1/3)*(7*A*b - 4*a*B)
*Log[a^(1/3) + b^(1/3)*x])/(9*a^(10/3)) + (b^(1/3)*(7*A*b - 4*a*B)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{Ab-aB}{3abx^4\left(a+bx^3\right)}+\frac{(7Ab-4aB)\int\frac{1}{x^5(a+bx^3)}\,dx}{3ab}\\
&= -\frac{7Ab-4aB}{12a^2bx^4}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}-\frac{(7Ab-4aB)\int\frac{1}{x^2(a+bx^3)}\,dx}{3a^2}\\
&= -\frac{7Ab-4aB}{12a^2bx^4}+\frac{7Ab-4aB}{3a^3x}+\frac{Ab-aB}{3abx^4\left(a+bx^3\right)}+\frac{(b(7Ab-4aB))\int\frac{x}{a+bx^3}\,dx}{3a^3}
\end{aligned}
$$

$$= -\frac{7Ab-4aB}{12a^2bx^4} + \frac{7Ab-4aB}{3a^3x} + \frac{Ab-aB}{3abx^4(a+bx^3)} - \frac{\left(b^{2/3}(7Ab-4aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{10/3}} + \frac{\left(b^{2/3}(7Ab-4aB)\right)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{10/3}}$$

$$= -\frac{7Ab-4aB}{12a^2bx^4} + \frac{7Ab-4aB}{3a^3x} + \frac{Ab-aB}{3abx^4(a+bx^3)} - \frac{\sqrt[3]{b}(7Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{10/3}} + \frac{\left(\sqrt[3]{b}(7Ab-4aB)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{10/3}} + \frac{\left(b^{2/3}(7Ab-4aB)\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^3}$$

$$= -\frac{7Ab-4aB}{12a^2bx^4} + \frac{7Ab-4aB}{3a^3x} + \frac{Ab-aB}{3abx^4(a+bx^3)} - \frac{\sqrt[3]{b}(7Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{10/3}} + \frac{\sqrt[3]{b}(7Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{10/3}} + \frac{\left(\sqrt[3]{b}(7Ab-4aB)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{10/3}}$$

$$= -\frac{7Ab-4aB}{12a^2bx^4} + \frac{7Ab-4aB}{3a^3x} + \frac{Ab-aB}{3abx^4(a+bx^3)} - \frac{\sqrt[3]{b}(7Ab-4aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{10/3}} - \frac{\sqrt[3]{b}(7Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{10/3}} + \frac{\sqrt[3]{b}(7Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{10/3}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx^3}{x^5(a+bx^3)^2}\,dx$$

$$= \frac{-\frac{9a^{4/3}A}{x^4} - \frac{36\sqrt[3]{a}(-2Ab+aB)}{x} - \frac{12\sqrt[3]{a}b(-Ab+aB)x^2}{a+bx^3} - 4\sqrt{3}\sqrt[3]{b}(7Ab-4aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 4\sqrt[3]{b}(-7Ab+4a}{36a^{10/3}}$$

```
[In] Integrate[(A + B*x^3)/(x^5*(a + b*x^3)^2),x]
```

```
[Out] ((-9*a^(4/3)*A)/x^4 - (36*a^(1/3)*(-2*A*b + a*B))/x - (12*a^(1/3)*b*(-(A*b)
 + a*B)*x^2)/(a + b*x^3) - 4*Sqrt[3]*b^(1/3)*(7*A*b - 4*a*B)*ArcTan[(1 - (2
*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 4*b^(1/3)*(-7*A*b + 4*a*B)*Log[a^(1/3) + b^
(1/3)*x] + 2*b^(1/3)*(7*A*b - 4*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/(36*a^(10/3))
```

Maple [A] (verified)

Time = 4.44 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.72

method	result
default	$-\frac{A}{4a^2x^4}-\frac{-2Ab+Ba}{x\,a^3}+\frac{b\left(\frac{\left(\frac{Ab}{3}-\frac{Ba}{3}\right)x^2}{b\,x^3+a}+\left(\frac{7Ab}{3}-\frac{4Ba}{3}\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\right)}{a^3}$
risch	$\frac{\frac{b(7Ab-4Ba)x^6}{3a^3}+\frac{(7Ab-4Ba)x^3}{4a^2}-\frac{A}{4a}}{x^4(b\,x^3+a)}+\frac{\left(\sum_{\_R=\text{RootOf}\left(a^{10}\_Z^3+343A^3b^4-588A^2Ba\,b^3+336A\,B^2a^2b^2-64B^3a^3b\right)}\_R\ln\left(\left(-4a^{10}\_R^3-\right.\right.\right.}{9}$

```
[In] int((B*x^3+A)/x^5/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*A/a^2/x^4-(-2*A*b+B*a)/x/a^3+1/a^3*b*((1/3*A*b-1/3*B*a)*x^2/(b*x^3+a)+
(7/3*A*b-4/3*B*a)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*l
n(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/
2)*(2/(a/b)^(1/3)*x-1))))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.20

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx =$$

$$-\frac{12\,(4\,Bab-7\,Ab^2)x^6+9\,(4\,Ba^2-7\,Aab)x^3+9\,Aa^2+4\,\sqrt{3}((4\,Bab-7\,Ab^2)x^7+(4\,Ba^2-7\,Aab)x^4)(\cdots}{}$$

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(12*(4*B*a*b - 7*A*b^2)*x^6 + 9*(4*B*a^2 - 7*A*a*b)*x^3 + 9*A*a^2 + 4
*sqrt(3)*((4*B*a*b - 7*A*b^2)*x^7 + (4*B*a^2 - 7*A*a*b)*x^4)*(-b/a)^(1/3)*a
rctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqrt(3)) - 2*((4*B*a*b - 7*A*b^2)*x^
7 + (4*B*a^2 - 7*A*a*b)*x^4)*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a*
(-b/a)^(1/3)) + 4*((4*B*a*b - 7*A*b^2)*x^7 + (4*B*a^2 - 7*A*a*b)*x^4)*(-b/a
)^(1/3)*log(b*x + a*(-b/a)^(2/3)))/(a^3*b*x^7 + a^4*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.71

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx$$
$$= \operatorname{RootSum}\left(729t^3a^{10} + 343A^3b^4 - 588A^2Bab^3 + 336AB^2a^2b^2 - 64B^3a^3b, \left(t \mapsto t\log\left(\frac{81t^2a^7}{49A^2b^3 - 56ABab^2 \cdots}\right.\right.\right.$$
$$+ \frac{-3Aa^2 + x^6\cdot(28Ab^2 - 16Bab) + x^3\cdot(21Aab - 12Ba^2)}{12a^4x^4 + 12a^3bx^7}$$

```
[In] integrate((B*x**3+A)/x**5/(b*x**3+a)**2,x)
```

```
[Out] RootSum(729*_t**3*a**10 + 343*A**3*b**4 - 588*A**2*B*a*b**3 + 336*A*B**2*a*
*2*b**2 - 64*B**3*a**3*b, Lambda(_t, _t*log(81*_t**2*a**7/(49*A**2*b**3 - 5
6*A*B*a*b**2 + 16*B**2*a**2*b) + x))) + (-3*A*a**2 + x**6*(28*A*b**2 - 16*B
*a*b) + x**3*(21*A*a*b - 12*B*a**2))/(12*a**4*x**4 + 12*a**3*b*x**7)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx = -\frac{4\,(4\,Bab-7\,Ab^2)x^6 + 3\,(4\,Ba^2-7\,Aab)x^3 + 3\,Aa^2}{12\,(a^3bx^7+a^4x^4)} - \frac{\sqrt{3}(4\,Ba-7\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(4\,Ba-7\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(4\,Ba-7\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/12*(4*(4*B*a*b - 7*A*b^2)*x^6 + 3*(4*B*a^2 - 7*A*a*b)*x^3 + 3*A*a^2)/(a^
3*b*x^7 + a^4*x^4) - 1/9*sqrt(3)*(4*B*a - 7*A*b)*arctan(1/3*sqrt(3)*(2*x -
(a/b)^(1/3))/(a/b)^(1/3))/(a^3*(a/b)^(1/3)) - 1/18*(4*B*a - 7*A*b)*log(x^2
- x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*(a/b)^(1/3)) + 1/9*(4*B*a - 7*A*b)*log(
x + (a/b)^(1/3))/(a^3*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx = \frac{\left(4\,Bab\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 7\,Ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^4} + \frac{\sqrt{3}\left(4\left(-ab^2\right)^{\frac{2}{3}}Ba - 7\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^4b} - \frac{Babx^2 - Ab^2x^2}{3\left(bx^3+a\right)a^3} - \frac{\left(4\left(-ab^2\right)^{\frac{2}{3}}Ba - 7\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^4b} - \frac{4\,Bax^3 - 8\,Abx^3 + Aa}{4\,a^3x^4}$$

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*(4*B*a*b*(-a/b)^(1/3) - 7*A*b^2*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x -
(-a/b)^(1/3)))/a^4 + 1/9*sqrt(3)*(4*(-a*b^2)^(2/3)*B*a - 7*(-a*b^2)^(2/3)*A
*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b) - 1/3*(B*
a*b*x^2 - A*b^2*x^2)/((b*x^3 + a)*a^3) - 1/18*(4*(-a*b^2)^(2/3)*B*a - 7*(-a
*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b) - 1/4*(4*
B*a*x^3 - 8*A*b*x^3 + A*a)/(a^3*x^4)
```

Mupad [B] (verification not implemented)

Time = 7.03 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^3}{x^5\,(a+bx^3)^2}\,dx$$
$$=\frac{\frac{x^3\,(7\,A\,b-4\,B\,a)}{4\,a^2}-\frac{A}{4\,a}+\frac{b\,x^6\,(7\,A\,b-4\,B\,a)}{3\,a^3}}{b\,x^7+a\,x^4}+\frac{(-b)^{1/3}\,\ln\left(a^{1/3}\,(-b)^{8/3}+b^3\,x\right)\,(7\,A\,b-4\,B\,a)}{9\,a^{10/3}}$$
$$+\frac{(-b)^{1/3}\,\ln\left(a^{1/3}\,(-b)^{8/3}-2\,b^3\,x+\sqrt{3}\,a^{1/3}\,(-b)^{8/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(7\,A\,b-4\,B\,a)}{9\,a^{10/3}}$$
$$-\frac{(-b)^{1/3}\,\ln\left(2\,b^3\,x-a^{1/3}\,(-b)^{8/3}+\sqrt{3}\,a^{1/3}\,(-b)^{8/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(7\,A\,b-4\,B\,a)}{9\,a^{10/3}}$$

```
[In] int((A + B*x^3)/(x^5*(a + b*x^3)^2),x)
```

```
[Out] ((x^3*(7*A*b - 4*B*a))/(4*a^2) - A/(4*a) + (b*x^6*(7*A*b - 4*B*a))/(3*a^3))
/(a*x^4 + b*x^7) + ((-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) + b^3*x)*(7*A*b - 4*B
*a))/(9*a^(10/3)) + ((-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) - 2*b^3*x + 3^(1/2)*
a^(1/3)*(-b)^(8/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(7*A*b - 4*B*a))/(9*a^(10/3))
 - ((-b)^(1/3)*log(2*b^3*x - a^(1/3)*(-b)^(8/3) + 3^(1/2)*a^(1/3)*(-b)^(8/3
)*1i)*((3^(1/2)*1i)/2 + 1/2)*(7*A*b - 4*B*a))/(9*a^(10/3))
```

3.86 $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 215

$$\begin{aligned}\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^2}\,dx=&\frac{-8Ab+5aB}{15a^2bx^5}+\frac{8Ab-5aB}{6a^3x^2}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}\\&-\frac{b^{2/3}(8Ab-5aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{11/3}}\\&+\frac{b^{2/3}(8Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{11/3}}\\&-\frac{b^{2/3}(8Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{11/3}}\end{aligned}$$

```
[Out] 1/15*(-8*A*b+5*B*a)/a^2/b/x^5+1/6*(8*A*b-5*B*a)/a^3/x^2+1/3*(A*b-B*a)/a/b/x
^5/(b*x^3+a)+1/9*b^(2/3)*(8*A*b-5*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)-1/18*
b^(2/3)*(8*A*b-5*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(11/3)-1/
9*b^(2/3)*(8*A*b-5*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a
^(11/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {468, 331, 206, 31, 648, 631, 210, 642}

$$\int \frac{A+Bx^3}{x^6(a+bx^3)^2}\,dx = -\frac{b^{2/3}(8Ab-5aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{11/3}} - \frac{b^{2/3}(8Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{11/3}} + \frac{b^{2/3}(8Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{11/3}} + \frac{8Ab-5aB}{6a^3x^2} - \frac{8Ab-5aB}{15a^2bx^5} + \frac{Ab-aB}{3abx^5(a+bx^3)}$$

```
[In] Int[(A + B*x^3)/(x^6*(a + b*x^3)^2),x]
```

```
[Out] -1/15*(8*A*b - 5*a*B)/(a^2*b*x^5) + (8*A*b - 5*a*B)/(6*a^3*x^2) + (A*b - a*
B)/(3*a*b*x^5*(a + b*x^3)) - (b^(2/3)*(8*A*b - 5*a*B)*ArcTan[(a^(1/3) - 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(11/3)) + (b^(2/3)*(8*A*b - 5*a*
B)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(11/3)) - (b^(2/3)*(8*A*b - 5*a*B)*Log[a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(11/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{Ab-aB}{3abx^5\left(a+bx^3\right)}+\frac{(8Ab-5aB)\int \frac{1}{x^6(a+bx^3)}\,dx}{3ab} \\
&= -\frac{8Ab-5aB}{15a^2bx^5}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}-\frac{(8Ab-5aB)\int \frac{1}{x^3(a+bx^3)}\,dx}{3a^2} \\
&= -\frac{8Ab-5aB}{15a^2bx^5}+\frac{8Ab-5aB}{6a^3x^2}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}+\frac{(b(8Ab-5aB))\int \frac{1}{a+bx^3}\,dx}{3a^3}
\end{aligned}$$

$$= -\frac{8Ab-5aB}{15a^2bx^5} + \frac{8Ab-5aB}{6a^3x^2} + \frac{Ab-aB}{3abx^5(a+bx^3)} + \frac{(b(8Ab-5aB))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{11/3}} + \frac{(b(8Ab-5aB))\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{11/3}}$$

$$= -\frac{8Ab-5aB}{15a^2bx^5} + \frac{8Ab-5aB}{6a^3x^2} + \frac{Ab-aB}{3abx^5(a+bx^3)} + \frac{b^{2/3}(8Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{11/3}} - \frac{\left(b^{2/3}(8Ab-5aB)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{11/3}} + \frac{(b(8Ab-5aB))\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{10/3}}$$

$$= -\frac{8Ab-5aB}{15a^2bx^5} + \frac{8Ab-5aB}{6a^3x^2} + \frac{Ab-aB}{3abx^5(a+bx^3)} + \frac{b^{2/3}(8Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{11/3}} - \frac{b^{2/3}(8Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{11/3}} + \frac{\left(b^{2/3}(8Ab-5aB)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a^{11/3}}$$

$$= -\frac{8Ab-5aB}{15a^2bx^5} + \frac{8Ab-5aB}{6a^3x^2} + \frac{Ab-aB}{3abx^5(a+bx^3)} - \frac{b^{2/3}(8Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{11/3}} + \frac{b^{2/3}(8Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{11/3}} - \frac{b^{2/3}(8Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{11/3}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^3}{x^6(a+bx^3)^2}\,dx$$

$$= \frac{-\frac{18a^{5/3}A}{x^5} - \frac{45a^{2/3}(-2Ab+aB)}{x^2} - \frac{30a^{2/3}b(-Ab+aB)x}{a+bx^3} - 10\sqrt{3}b^{2/3}(8Ab-5aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 10b^{2/3}(8Ab-5}{90a^{11/3}}$$

```
[In] Integrate[(A + B*x^3)/(x^6*(a + b*x^3)^2),x]
```

```
[Out] ((-18*a^(5/3)*A)/x^5 - (45*a^(2/3)*(-2*A*b + a*B))/x^2 - (30*a^(2/3)*b*(-(A
*b) + a*B)*x)/(a + b*x^3) - 10*Sqrt[3]*b^(2/3)*(8*A*b - 5*a*B)*ArcTan[(1 -
(2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 10*b^(2/3)*(8*A*b - 5*a*B)*Log[a^(1/3) +
b^(1/3)*x] + 5*b^(2/3)*(-8*A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2])/(90*a^(11/3))
```

Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.72

method	result
default	$-\frac{A}{5a^2x^5}-\frac{-2Ab+Ba}{2x^2a^3}+\frac{b\left(\frac{\left(\frac{Ab}{3}-\frac{Ba}{3}\right)x}{b\,x^3+a}+\frac{(8Ab-5Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{3}\right)}{a^3}$
risch	$\frac{\frac{b(8Ab-5Ba)x^6}{6a^3}+\frac{(8Ab-5Ba)x^3}{10a^2}-\frac{A}{5a}}{x^5(b\,x^3+a)}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^{11}\_Z^3-512A^3b^5+960A^2Ba\,b^4-600A\,B^2a^2b^3+125B^3a^3b^2\right)}\_R\ln\left(\left(-4\_R^3a^{11}\right.\right.\right.}{}$

[In] int((B*x^3+A)/x^6/(b*x^3+a)^2,x,method=_RETURNVERBOSE)

[Out] -1/5*A/a^2/x^5-1/2*(-2*A*b+B*a)/x^2/a^3+1/a^3*b*((1/3*A*b-1/3*B*a)*x/(b*x^3
+a)+1/3*(8*A*b-5*B*a)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3
)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^
(1/2)*(2/(a/b)^(1/3)*x-1))))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.29

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^2}\,dx =$$

$$-\frac{15\,(5\,Bab-8\,Ab^2)x^6+9\,(5\,Ba^2-8\,Aab)x^3+18\,Aa^2+10\,\sqrt{3}((5\,Bab-8\,Ab^2)x^8+(5\,Ba^2-8\,Aab)x^5}{}$$

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/90*(15*(5*B*a*b - 8*A*b^2)*x^6 + 9*(5*B*a^2 - 8*A*a*b)*x^3 + 18*A*a^2 +
10*sqrt(3)*((5*B*a*b - 8*A*b^2)*x^8 + (5*B*a^2 - 8*A*a*b)*x^5)*(b^2/a^2)^(1
/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 5*((5*B*a*b
 - 8*A*b^2)*x^8 + (5*B*a^2 - 8*A*a*b)*x^5)*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*
b*x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) + 10*((5*B*a*b - 8*A*b^2)*x^8 +
(5*B*a^2 - 8*A*a*b)*x^5)*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)))/(a^3
*b*x^8 + a^4*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.64

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^2}\,dx$$
$$=\mathrm{RootSum}\left(729t^3a^{11}-512A^3b^5+960A^2Bab^4-600AB^2a^2b^3+125B^3a^3b^2,\left(t\mapsto t\log\left(-\frac{9ta^4}{-8Ab^2+5Bab}\right.\right.\right.$$
$$+\frac{-6Aa^2+x^6\cdot(40Ab^2-25Bab)+x^3\cdot(24Aab-15Ba^2)}{30a^4x^5+30a^3bx^8}$$

```
[In] integrate((B*x**3+A)/x**6/(b*x**3+a)**2,x)
```

```
[Out] RootSum(729*_t**3*a**11 - 512*A**3*b**5 + 960*A**2*B*a*b**4 - 600*A*B**2*a*
*2*b**3 + 125*B**3*a**3*b**2, Lambda(_t, _t*log(-9*_t*a**4/(-8*A*b**2 + 5*B
*a*b) + x))) + (-6*A*a**2 + x**6*(40*A*b**2 - 25*B*a*b) + x**3*(24*A*a*b -
15*B*a**2))/(30*a**4*x**5 + 30*a**3*b*x**8)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^2}\,dx=-\frac{5\,(5\,Bab-8\,Ab^2)x^6+3\,(5\,Ba^2-8\,Aab)x^3+6\,Aa^2}{30\,(a^3bx^8+a^4x^5)}$$
$$-\frac{\sqrt{3}(5\,Ba-8\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{(5\,Ba-8\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$-\frac{(5\,Ba-8\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/30*(5*(5*B*a*b - 8*A*b^2)*x^6 + 3*(5*B*a^2 - 8*A*a*b)*x^3 + 6*A*a^2)/(a^
3*b*x^8 + a^4*x^5) - 1/9*sqrt(3)*(5*B*a - 8*A*b)*arctan(1/3*sqrt(3)*(2*x -
(a/b)^(1/3))/(a/b)^(1/3))/(a^3*(a/b)^(2/3)) + 1/18*(5*B*a - 8*A*b)*log(x^2
- x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*(a/b)^(2/3)) - 1/9*(5*B*a - 8*A*b)*log(
x + (a/b)^(1/3))/(a^3*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^2}\,dx = -\frac{\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}Ba-8\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^4} + \frac{\left(5Bab-8Ab^2\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^4} - \frac{\left(5\left(-ab^2\right)^{\frac{1}{3}}Ba-8\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^4} - \frac{Babx-Ab^2x}{3\left(bx^3+a\right)a^3} - \frac{5Bax^3-10Abx^3+2Aa}{10a^3x^5}$$

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(5*(-a*b^2)^(1/3)*B*a - 8*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(
3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^4 + 1/9*(5*B*a*b - 8*A*b^2)*(-a/b)^
(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 - 1/18*(5*(-a*b^2)^(1/3)*B*a - 8*(-a*b
^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^4 - 1/3*(B*a*b*x
- A*b^2*x)/((b*x^3 + a)*a^3) - 1/10*(5*B*a*x^3 - 10*A*b*x^3 + 2*A*a)/(a^3*x
^5)
```

Mupad [B] (verification not implemented)

Time = 6.99 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^2}\,dx=\frac{\frac{x^3\,(8\,A\,b-5\,B\,a)}{10\,a^2}-\frac{A}{5\,a}+\frac{b\,x^6\,(8\,A\,b-5\,B\,a)}{6\,a^3}}{b\,x^8+a\,x^5}+\frac{b^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(8\,A\,b-5\,B\,a)}{9\,a^{11/3}}-\frac{b^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(8\,A\,b-5\,B\,a)}{9\,a^{11/3}}+\frac{b^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(8\,A\,b-5\,B\,a)}{9\,a^{11/3}}$$

```
[In] int((A + B*x^3)/(x^6*(a + b*x^3)^2),x)
```

```
[Out] ((x^3*(8*A*b - 5*B*a))/(10*a^2) - A/(5*a) + (b*x^6*(8*A*b - 5*B*a))/(6*a^3)
)/(a*x^5 + b*x^8) + (b^(2/3)*log(b^(1/3)*x + a^(1/3))*(8*A*b - 5*B*a))/(9*a
^(11/3)) - (b^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/
2)*1i)/2 + 1/2)*(8*A*b - 5*B*a))/(9*a^(11/3)) + (b^(2/3)*log(3^(1/2)*a^(1/3
)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(8*A*b - 5*B*a))/(9*a^
(11/3))
```

3.87 $\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 97

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^2}\,dx=-\frac{A}{6a^2x^6}+\frac{2Ab-aB}{3a^3x^3}+\frac{b(Ab-aB)}{3a^3\left(a+bx^3\right)}+\frac{b(3Ab-2aB)\log(x)}{a^4}-\frac{b(3Ab-2aB)\log\left(a+bx^3\right)}{3a^4}$$

```
[Out] -1/6*A/a^2/x^6+1/3*(2*A*b-B*a)/a^3/x^3+1/3*b*(A*b-B*a)/a^3/(b*x^3+a)+b*(3*A
*b-2*B*a)*ln(x)/a^4-1/3*b*(3*A*b-2*B*a)*ln(b*x^3+a)/a^4
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 78}

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^2}\,dx=-\frac{b(3Ab-2aB)\log\left(a+bx^3\right)}{3a^4}+\frac{b\log(x)(3Ab-2aB)}{a^4}+\frac{b(Ab-aB)}{3a^3\left(a+bx^3\right)}+\frac{2Ab-aB}{3a^3x^3}-\frac{A}{6a^2x^6}$$

```
[In] Int[(A + B*x^3)/(x^7*(a + b*x^3)^2),x]
```

```
[Out] -1/6*A/(a^2*x^6) + (2*A*b - a*B)/(3*a^3*x^3) + (b*(A*b - a*B))/(3*a^3*(a +
b*x^3)) + (b*(3*A*b - 2*a*B)*Log[x])/a^4 - (b*(3*A*b - 2*a*B)*Log[a + b*x^3
])/(3*a^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)^2}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{A}{a^2x^3}+\frac{-2Ab+aB}{a^3x^2}-\frac{b(-3Ab+2aB)}{a^4x}+\frac{b^2(-Ab+aB)}{a^3(a+bx)^2}+\frac{b^2(-3Ab+2aB)}{a^4(a+bx)}\right)dx, x, x^3\right)\\ &= -\frac{A}{6a^2x^6}+\frac{2Ab-aB}{3a^3x^3}+\frac{b(Ab-aB)}{3a^3(a+bx^3)}+\frac{b(3Ab-2aB)\log(x)}{a^4}-\frac{b(3Ab-2aB)\log(a+bx^3)}{3a^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^3}{x^7(a+bx^3)^2}\,dx = -\frac{\frac{a^2A}{x^6}+\frac{2a(-2Ab+aB)}{x^3}+\frac{2ab(-Ab+aB)}{a+bx^3}-6b(3Ab-2aB)\log(x)+2b(3Ab-2aB)\log(a+bx^3)}{6a^4}$$

```
[In] Integrate[(A + B*x^3)/(x^7*(a + b*x^3)^2),x]
```

```
[Out] -1/6*((a^2*A)/x^6 + (2*a*(-2*A*b + a*B))/x^3 + (2*a*b*(-(A*b) + a*B))/(a +
b*x^3) - 6*b*(3*A*b - 2*a*B)*Log[x] + 2*b*(3*A*b - 2*a*B)*Log[a + b*x^3])/a
^4
```

Maple [A] (verified)

Time = 4.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99

method	result
default	$-\frac{A}{6a^{2}x^{6}}-\frac{-2Ab+Ba}{3x^{3}a^{3}}+\frac{b(3Ab-2Ba)\ln(x)}{a^{4}}-\frac{b^{2}\left(\frac{(3Ab-2Ba)\ln\left(b\,x^{3}+a\right)}{b}-\frac{a(Ab-Ba)}{b\left(b\,x^{3}+a\right)}\right)}{3a^{4}}$
norman	$\frac{-\frac{A}{6a}+\frac{(3Ab-2Ba)x^{3}}{6a^{2}}-\frac{b\left(3b^{2}A-2abB\right)x^{9}}{3a^{4}}}{x^{6}(b\,x^{3}+a)}+\frac{b(3Ab-2Ba)\ln(x)}{a^{4}}-\frac{b(3Ab-2Ba)\ln\left(b\,x^{3}+a\right)}{3a^{4}}$
risch	$\frac{\frac{b(3Ab-2Ba)x^{6}}{3a^{3}}+\frac{(3Ab-2Ba)x^{3}}{6a^{2}}-\frac{A}{6a}}{x^{6}(b\,x^{3}+a)}+\frac{3b^{2}\ln(x)A}{a^{4}}-\frac{2b\ln(x)B}{a^{3}}-\frac{b^{2}\ln\left(b\,x^{3}+a\right)A}{a^{4}}+\frac{2b\ln\left(b\,x^{3}+a\right)B}{3a^{3}}$
parallelrisch	$\frac{18A\ln(x)x^{9}b^{3}-6A\ln\left(b\,x^{3}+a\right)x^{9}b^{3}-12B\ln(x)x^{9}a\,b^{2}+4B\ln\left(b\,x^{3}+a\right)x^{9}a\,b^{2}-6A\,x^{9}b^{3}+4B\,x^{9}a\,b^{2}+18A\ln(x)x^{6}a\,b^{2}-6A\ln(b\,x}{6a^{4}x^{6}(b\,x^{3}+a)}$

```
[In] int((B*x^3+A)/x^7/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*A/a^2/x^6-1/3*(-2*A*b+B*a)/x^3/a^3+b*(3*A*b-2*B*a)*ln(x)/a^4-1/3/a^4*b
^2*((3*A*b-2*B*a)/b*ln(b*x^3+a)-a*(A*b-B*a)/b/(b*x^3+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.59

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^2}\,dx = -\frac{2\left(2\,Ba^2b-3\,Aab^2\right)x^6+Aa^3+\left(2\,Ba^3-3\,Aa^2b\right)x^3-2\left(\left(2\,Bab^2-3\,Ab^3\right)x^9+\left(2\,Ba^2b-3\,Aab^2\right)x^6\right)\log}{6\left(a^4bx^9+a^5x^6\right)}$$

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*(2*B*a^2*b - 3*A*a*b^2)*x^6 + A*a^3 + (2*B*a^3 - 3*A*a^2*b)*x^3 - 2
*((2*B*a*b^2 - 3*A*b^3)*x^9 + (2*B*a^2*b - 3*A*a*b^2)*x^6)*log(b*x^3 + a) +
 6*((2*B*a*b^2 - 3*A*b^3)*x^9 + (2*B*a^2*b - 3*A*a*b^2)*x^6)*log(x))/(a^4*b
*x^9 + a^5*x^6)
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^3}{x^7(a+bx^3)^2}\,dx = \frac{-Aa^2+x^6\cdot(6Ab^2-4Bab)+x^3\cdot(3Aab-2Ba^2)}{6a^4x^6+6a^3bx^9} - \frac{b(-3Ab+2Ba)\log(x)}{a^4} + \frac{b(-3Ab+2Ba)\log\left(\frac{a}{b}+x^3\right)}{3a^4}$$

```
[In] integrate((B*x**3+A)/x**7/(b*x**3+a)**2,x)
```

```
[Out] (-A*a**2 + x**6*(6*A*b**2 - 4*B*a*b) + x**3*(3*A*a*b - 2*B*a**2))/(6*a**4*x
**6 + 6*a**3*b*x**9) - b*(-3*A*b + 2*B*a)*log(x)/a**4 + b*(-3*A*b + 2*B*a)*
log(a/b + x**3)/(3*a**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx^3}{x^7(a+bx^3)^2}\,dx = -\frac{2\,(2\,Bab-3\,Ab^2)x^6+(2\,Ba^2-3\,Aab)x^3+Aa^2}{6\,(a^3bx^9+a^4x^6)} + \frac{(2\,Bab-3\,Ab^2)\log(bx^3+a)}{3\,a^4} - \frac{(2\,Bab-3\,Ab^2)\log(x^3)}{3\,a^4}$$

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(2*B*a*b - 3*A*b^2)*x^6 + (2*B*a^2 - 3*A*a*b)*x^3 + A*a^2)/(a^3*b*x
^9 + a^4*x^6) + 1/3*(2*B*a*b - 3*A*b^2)*log(b*x^3 + a)/a^4 - 1/3*(2*B*a*b -
 3*A*b^2)*log(x^3)/a^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.54

$$\int \frac{A+Bx^3}{x^7(a+bx^3)^2}\,dx = -\frac{(2\,Bab-3\,Ab^2)\log(|x|)}{a^4} + \frac{(2\,Bab^2-3\,Ab^3)\log(|bx^3+a|)}{3\,a^4b} - \frac{2\,Bab^2x^3-3\,Ab^3x^3+3\,Ba^2b-4\,Aab^2}{3\,(bx^3+a)a^4} + \frac{6\,Babx^6-9\,Ab^2x^6-2\,Ba^2x^3+4\,Aabx^3-Aa^2}{6\,a^4x^6}$$

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -(2*B*a*b - 3*A*b^2)*log(abs(x))/a^4 + 1/3*(2*B*a*b^2 - 3*A*b^3)*log(abs(b*
x^3 + a))/(a^4*b) - 1/3*(2*B*a*b^2*x^3 - 3*A*b^3*x^3 + 3*B*a^2*b - 4*A*a*b^
2)/((b*x^3 + a)*a^4) + 1/6*(6*B*a*b*x^6 - 9*A*b^2*x^6 - 2*B*a^2*x^3 + 4*A*a
*b*x^3 - A*a^2)/(a^4*x^6)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^2}\,dx=\frac{\frac{x^3\,\left(3\,A\,b-2\,B\,a\right)}{6\,a^2}-\frac{A}{6\,a}+\frac{b\,x^6\,\left(3\,A\,b-2\,B\,a\right)}{3\,a^3}}{b\,x^9+a\,x^6}-\frac{\ln\left(b\,x^3+a\right)\,\left(3\,A\,b^2-2\,B\,a\,b\right)}{3\,a^4}+\frac{\ln\left(x\right)\,\left(3\,A\,b^2-2\,B\,a\,b\right)}{a^4}$$

```
[In] int((A + B*x^3)/(x^7*(a + b*x^3)^2),x)
```

```
[Out] ((x^3*(3*A*b - 2*B*a))/(6*a^2) - A/(6*a) + (b*x^6*(3*A*b - 2*B*a))/(3*a^3))
/(a*x^6 + b*x^9) - (log(a + b*x^3)*(3*A*b^2 - 2*B*a*b))/(3*a^4) + (log(x)*(
3*A*b^2 - 2*B*a*b))/a^4
```

3.88 $\int \frac{x^{11}(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 107

$$\int \frac{x^{11}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(Ab-3aB)x^3}{3b^4}+\frac{Bx^6}{6b^3}+\frac{a^3(Ab-aB)}{6b^5(a+bx^3)^2}-\frac{a^2(3Ab-4aB)}{3b^5(a+bx^3)}-\frac{a(Ab-2aB)\log(a+bx^3)}{b^5}$$

```
[Out] 1/3*(A*b-3*B*a)*x^3/b^4+1/6*B*x^6/b^3+1/6*a^3*(A*b-B*a)/b^5/(b*x^3+a)^2-1/3
*a^2*(3*A*b-4*B*a)/b^5/(b*x^3+a)-a*(A*b-2*B*a)*ln(b*x^3+a)/b^5
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 78}

$$\int \frac{x^{11}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{a^3(Ab-aB)}{6b^5(a+bx^3)^2}-\frac{a^2(3Ab-4aB)}{3b^5(a+bx^3)}-\frac{a(Ab-2aB)\log(a+bx^3)}{b^5}+\frac{x^3(Ab-3aB)}{3b^4}+\frac{Bx^6}{6b^3}$$

```
[In] Int[(x^11*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - 3*a*B)*x^3)/(3*b^4) + (B*x^6)/(6*b^3) + (a^3*(A*b - a*B))/(6*b^5*(a
 + b*x^3)^2) - (a^2*(3*A*b - 4*a*B))/(3*b^5*(a + b*x^3)) - (a*(A*b - 2*a*B)
*Log[a + b*x^3])/b^5
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^3(A+Bx)}{(a+bx)^3}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{Ab-3aB}{b^4}+\frac{Bx}{b^3}+\frac{a^3(-Ab+aB)}{b^4(a+bx)^3}-\frac{a^2(-3Ab+4aB)}{b^4(a+bx)^2}\right.\right.\\
&\qquad\left.\left.+\frac{3a(-Ab+2aB)}{b^4(a+bx)}\right)\,dx, x, x^3\right)\\
&= \frac{(Ab-3aB)x^3}{3b^4}+\frac{Bx^6}{6b^3}+\frac{a^3(Ab-aB)}{6b^5\left(a+bx^3\right)^2}-\frac{a^2(3Ab-4aB)}{3b^5\left(a+bx^3\right)}-\frac{a(Ab-2aB)\log\left(a+bx^3\right)}{b^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.88

$$
\int \frac{x^{11}(A+Bx^3)}{\left(a+bx^3\right)^3}\,dx = \frac{2b(Ab-3aB)x^3+b^2Bx^6+\frac{a^3(Ab-aB)}{(a+bx^3)^2}+\frac{2a^2(-3Ab+4aB)}{a+bx^3}+6a(-Ab+2aB)\log\left(a+bx^3\right)}{6b^5}
$$

```
[In] Integrate[(x^11*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (2*b*(A*b - 3*a*B)*x^3 + b^2*B*x^6 + (a^3*(A*b - a*B))/(a + b*x^3)^2 + (2*a
^2*(-3*A*b + 4*a*B))/(a + b*x^3) + 6*a*(-(A*b) + 2*a*B)*Log[a + b*x^3])/(6*
b^5)
```

Maple [A] (verified)

Time = 4.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

method	result
norman	$\frac{\frac{B\,x^{12}}{6b}-\frac{a^2\left(3abA-6a^2B\right)}{2b^5}+\frac{(Ab-2Ba)x^9}{3b^2}-\frac{2a\left(abA-2a^2B\right)x^3}{b^4}}{\left(b\,x^3+a\right)^2}-\frac{a(Ab-2Ba)\ln\left(b\,x^3+a\right)}{b^5}$
default	$\frac{\left(bB\,x^3+Ab-3Ba\right)^2}{6b^5B}-\frac{a\left(\frac{(3Ab-6Ba)\ln\left(b\,x^3+a\right)}{b}-\frac{a^2(Ab-Ba)}{2b\left(b\,x^3+a\right)^2}+\frac{a(3Ab-4Ba)}{b\left(b\,x^3+a\right)}\right)}{3b^4}$
risch	$\frac{B\,x^6}{6b^3}+\frac{A\,x^3}{3b^3}-\frac{Ba\,x^3}{b^4}+\frac{A^2}{6b^3B}-\frac{Aa}{b^4}+\frac{3B\,a^2}{2b^5}+\frac{\left(-a^2bA+\frac{4}{3}a^3B\right)x^3-\frac{a^3(5Ab-7Ba)}{6b}}{b^4\left(b\,x^3+a\right)^2}-\frac{a\ln\left(b\,x^3+a\right)A}{b^4}+\frac{2a^2\ln\left(b\,x^3\right.}{b^5}$
parallelrisch	$-\frac{-B\,x^{12}b^4-2A\,x^9b^4+4B\,x^9a\,b^3+6A\ln\left(b\,x^3+a\right)x^6a\,b^3-12B\ln\left(b\,x^3+a\right)x^6a^2b^2+12A\ln\left(b\,x^3+a\right)x^3a^2b^2-24B\ln\left(b\,x^3+a\right)x^3a^3}{6b^5\left(b\,x^3+a\right)^2}$

```
[In] int(x^11*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/6*B/b*x^12-1/2*a^2*(3*A*a*b-6*B*a^2)/b^5+1/3*(A*b-2*B*a)/b^2*x^9-2*a*(A*
a*b-2*B*a^2)/b^4*x^3)/(b*x^3+a)^2-a*(A*b-2*B*a)*ln(b*x^3+a)/b^5
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.67

$$\int \frac{x^{11}(A+Bx^3)}{(a+bx^3)^3}\,dx$$
$$=\frac{Bb^4x^{12}-2\,(2\,Bab^3-Ab^4)x^9-(11\,Ba^2b^2-4\,Aab^3)x^6+7\,Ba^4-5\,Aa^3b+2\,(Ba^3b-2\,Aa^2b^2)x^3+6\,((2\,B}{6\,(b^7x^6+2\,ab^6x^3+a^2b^5)}$$

```
[In] integrate(x^11*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(B*b^4*x^12 - 2*(2*B*a*b^3 - A*b^4)*x^9 - (11*B*a^2*b^2 - 4*A*a*b^3)*x^
6 + 7*B*a^4 - 5*A*a^3*b + 2*(B*a^3*b - 2*A*a^2*b^2)*x^3 + 6*((2*B*a^2*b^2 -
 A*a*b^3)*x^6 + 2*B*a^4 - A*a^3*b + 2*(2*B*a^3*b - A*a^2*b^2)*x^3)*log(b*x^
3 + a))/(b^7*x^6 + 2*a*b^6*x^3 + a^2*b^5)
```

Sympy [A] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.05

$$\int \frac{x^{11}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{Bx^6}{6b^3} + \frac{a(-Ab+2Ba)\log\left(a+bx^3\right)}{b^5} + x^3\left(\frac{A}{3b^3} - \frac{Ba}{b^4}\right) + \frac{-5Aa^3b+7Ba^4+x^3(-6Aa^2b^2+8Ba^3b)}{6a^2b^5+12ab^6x^3+6b^7x^6}$$

```
[In] integrate(x**11*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*x**6/(6*b**3) + a*(-A*b + 2*B*a)*log(a + b*x**3)/b**5 + x**3*(A/(3*b**3)
- B*a/b**4) + (-5*A*a**3*b + 7*B*a**4 + x**3*(-6*A*a**2*b**2 + 8*B*a**3*b))
/(6*a**2*b**5 + 12*a*b**6*x**3 + 6*b**7*x**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.07

$$\int \frac{x^{11}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{7\,Ba^4-5\,Aa^3b+2\,(4\,Ba^3b-3\,Aa^2b^2)x^3}{6\,(b^7x^6+2\,ab^6x^3+a^2b^5)} + \frac{Bbx^6-2\,(3\,Ba-Ab)x^3}{6\,b^4} + \frac{(2\,Ba^2-Aab)\log\left(bx^3+a\right)}{b^5}$$

```
[In] integrate(x^11*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(7*B*a^4 - 5*A*a^3*b + 2*(4*B*a^3*b - 3*A*a^2*b^2)*x^3)/(b^7*x^6 + 2*a*
b^6*x^3 + a^2*b^5) + 1/6*(B*b*x^6 - 2*(3*B*a - A*b)*x^3)/b^4 + (2*B*a^2 - A
*a*b)*log(b*x^3 + a)/b^5
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.22

$$\int \frac{x^{11}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(2\,Ba^2-Aab)\log\left(|bx^3+a|\right)}{b^5} + \frac{Bb^3x^6-6\,Bab^2x^3+2\,Ab^3x^3}{6\,b^6} - \frac{18\,Ba^2b^2x^6-9\,Aab^3x^6+28\,Ba^3bx^3-12\,Aa^2b^2x^3+11\,Ba^4-4\,Aa^3b}{6\,(bx^3+a)^2b^5}$$

```
[In] integrate(x^11*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] (2*B*a^2 - A*a*b)*log(abs(b*x^3 + a))/b^5 + 1/6*(B*b^3*x^6 - 6*B*a*b^2*x^3
+ 2*A*b^3*x^3)/b^6 - 1/6*(18*B*a^2*b^2*x^6 - 9*A*a*b^3*x^6 + 28*B*a^3*b*x^3
 - 12*A*a^2*b^2*x^3 + 11*B*a^4 - 4*A*a^3*b)/((b*x^3 + a)^2*b^5)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.09

$$\int \frac{x^{11}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{\frac{7\,B\,a^4-5\,A\,a^3\,b}{6\,b}+x^3\,\left(\frac{4\,B\,a^3}{3}-A\,a^2\,b\right)}{a^2\,b^4+2\,a\,b^5\,x^3+b^6\,x^6}+x^3\,\left(\frac{A}{3\,b^3}-\frac{B\,a}{b^4}\right)+\frac{\ln\left(b\,x^3+a\right)\,\left(2\,B\,a^2-A\,a\,b\right)}{b^5}+\frac{B\,x^6}{6\,b^3}$$

```
[In] int((x^11*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] ((7*B*a^4 - 5*A*a^3*b)/(6*b) + x^3*((4*B*a^3)/3 - A*a^2*b))/(a^2*b^4 + b^6*
x^6 + 2*a*b^5*x^3) + x^3*(A/(3*b^3) - (B*a)/b^4) + (log(a + b*x^3)*(2*B*a^2
 - A*a*b))/b^5 + (B*x^6)/(6*b^3)
```

3.89 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 88

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3} dx = \frac{Bx^3}{3b^3} - \frac{a^2(Ab-aB)}{6b^4(a+bx^3)^2} + \frac{a(2Ab-3aB)}{3b^4(a+bx^3)} + \frac{(Ab-3aB)\log(a+bx^3)}{3b^4}$$

```
[Out] 1/3*B*x^3/b^3-1/6*a^2*(A*b-B*a)/b^4/(b*x^3+a)^2+1/3*a*(2*A*b-3*B*a)/b^4/(b*
x^3+a)+1/3*(A*b-3*B*a)*ln(b*x^3+a)/b^4
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {457, 78}

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3} dx = -\frac{a^2(Ab-aB)}{6b^4(a+bx^3)^2} + \frac{a(2Ab-3aB)}{3b^4(a+bx^3)} + \frac{(Ab-3aB)\log(a+bx^3)}{3b^4} + \frac{Bx^3}{3b^3}$$

```
[In] Int[(x^8*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (B*x^3)/(3*b^3) - (a^2*(A*b - a*B))/(6*b^4*(a + b*x^3)^2) + (a*(2*A*b - 3*a
*B))/(3*b^4*(a + b*x^3)) + ((A*b - 3*a*B)*Log[a + b*x^3])/(3*b^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx)^3}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{B}{b^3}-\frac{a^2(-Ab+aB)}{b^3(a+bx)^3}+\frac{a(-2Ab+3aB)}{b^3(a+bx)^2}+\frac{Ab-3aB}{b^3(a+bx)}\right)dx, x, x^3\right)\\ &= \frac{Bx^3}{3b^3}-\frac{a^2(Ab-aB)}{6b^4(a+bx^3)^2}+\frac{a(2Ab-3aB)}{3b^4(a+bx^3)}+\frac{(Ab-3aB)\log(a+bx^3)}{3b^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.05

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{Bx^3}{3b^3}+\frac{-a^2Ab+a^3B}{6b^4(a+bx^3)^2}+\frac{2aAb-3a^2B}{3b^4(a+bx^3)}+\frac{(Ab-3aB)\log(a+bx^3)}{3b^4}$$

```
[In] Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^3,x]
[Out] (B*x^3)/(3*b^3) + (-(a^2*A*b) + a^3*B)/(6*b^4*(a + b*x^3)^2) + (2*a*A*b - 3
*a^2*B)/(3*b^4*(a + b*x^3)) + ((A*b - 3*a*B)*Log[a + b*x^3])/(3*b^4)
```

Maple [A] (verified)

Time = 4.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.86

method	result
norman	$\frac{\frac{B x^{9}}{3 b}+\frac{a^{2}(A b-3 B a)}{2 b^{4}}+\frac{2 a(A b-3 B a) x^{3}}{3 b^{3}}}{\left(b x^{3}+a\right)^{2}}+\frac{(A b-3 B a) \ln \left(b x^{3}+a\right)}{3 b^{4}}$
default	$\frac{B x^{3}}{3 b^{3}}+\frac{\frac{(A b-3 B a) \ln \left(b x^{3}+a\right)}{b}-\frac{a^{2}(A b-B a)}{2 b\left(b x^{3}+a\right)^{2}}+\frac{a(2 A b-3 B a)}{b\left(b x^{3}+a\right)}}{3 b^{3}}$
risch	$\frac{B x^{3}}{3 b^{3}}+\frac{\left(\frac{2}{3} a b A-a^{2} B\right) x^{3}+\frac{a^{2}(3 A b-5 B a)}{6 b}}{b^{3}\left(b x^{3}+a\right)^{2}}+\frac{\ln \left(b x^{3}+a\right) A}{3 b^{3}}-\frac{\ln \left(b x^{3}+a\right) B a}{b^{4}}$
parallelrisch	$\frac{2 b^{3} B x^{9}+2 A \ln \left(b x^{3}+a\right) x^{6} b^{3}-6 B \ln \left(b x^{3}+a\right) x^{6} a b^{2}+4 A \ln \left(b x^{3}+a\right) x^{3} a b^{2}-12 B \ln \left(b x^{3}+a\right) x^{3} a^{2} b+4 a A b^{2} x^{3}-12 B a^{2} b x^{3}+2 A}{6 b^{4}\left(b x^{3}+a\right)^{2}}$

[In] int(x^8*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)

[Out] (1/3*B/b*x^9+1/2*a^2*(A*b-3*B*a)/b^4+2/3*a*(A*b-3*B*a)/b^3*x^3)/(b*x^3+a)^2 +1/3*(A*b-3*B*a)*ln(b*x^3+a)/b^4

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.61

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{2\,Bb^3x^9+4\,Bab^2x^6-5\,Ba^3+3\,Aa^2b-4\,(Ba^2b-Aab^2)x^3-2\,((3\,Bab^2-Ab^3)x^6+3\,Ba^3-Aa^2b+2\,(3}{6\,(b^6x^6+2\,ab^5x^3+a^2b^4)}$$

[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")

[Out] 1/6*(2*B*b^3*x^9 + 4*B*a*b^2*x^6 - 5*B*a^3 + 3*A*a^2*b - 4*(B*a^2*b - A*a*b^2)*x^3 - 2*((3*B*a*b^2 - A*b^3)*x^6 + 3*B*a^3 - A*a^2*b + 2*(3*B*a^2*b - A*a*b^2)*x^3)*log(b*x^3 + a))/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)

Sympy [A] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.07

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{Bx^3}{3b^3} + \frac{3Aa^2b-5Ba^3+x^3\cdot(4Aab^2-6Ba^2b)}{6a^2b^4+12ab^5x^3+6b^6x^6} - \frac{(-Ab+3Ba)\log{(a+bx^3)}}{3b^4}$$

[In] integrate(x**8*(B*x**3+A)/(b*x**3+a)**3,x)

[Out] B*x**3/(3*b**3) + (3*A*a**2*b - 5*B*a**3 + x**3*(4*A*a*b**2 - 6*B*a**2*b))/(6*a**2*b**4 + 12*a*b**5*x**3 + 6*b**6*x**6) - (-A*b + 3*B*a)*log(a + b*x**3)/(3*b**4)

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.07

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{Bx^3}{3\,b^3} - \frac{5\,Ba^3 - 3\,Aa^2b + 2\,(3\,Ba^2b - 2\,Aab^2)x^3}{6\,(b^6x^6 + 2\,ab^5x^3 + a^2b^4)} - \frac{(3\,Ba - Ab)\log\left(bx^3 + a\right)}{3\,b^4}$$

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3*B*x^3/b^3 - 1/6*(5*B*a^3 - 3*A*a^2*b + 2*(3*B*a^2*b - 2*A*a*b^2)*x^3)/(
b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4) - 1/3*(3*B*a - A*b)*log(b*x^3 + a)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.06

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{Bx^3}{3\,b^3} - \frac{(3\,Ba - Ab)\log\left(|bx^3 + a|\right)}{3\,b^4} + \frac{9\,Bab^2x^6 - 3\,Ab^3x^6 + 12\,Ba^2bx^3 - 2\,Aab^2x^3 + 4\,Ba^3}{6\,(bx^3 + a)^2b^4}$$

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*B*x^3/b^3 - 1/3*(3*B*a - A*b)*log(abs(b*x^3 + a))/b^4 + 1/6*(9*B*a*b^2*
x^6 - 3*A*b^3*x^6 + 12*B*a^2*b*x^3 - 2*A*a*b^2*x^3 + 4*B*a^3)/((b*x^3 + a)^
2*b^4)
```

Mupad [B] (verification not implemented)

Time = 6.83 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.07

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{B\,x^3}{3\,b^3} - \frac{x^3\,\left(B\,a^2 - \frac{2\,A\,a\,b}{3}\right) + \frac{5\,B\,a^3 - 3\,A\,a^2\,b}{6\,b}}{a^2\,b^3 + 2\,a\,b^4\,x^3 + b^5\,x^6} + \frac{\ln\left(b\,x^3 + a\right)\,\left(A\,b - 3\,B\,a\right)}{3\,b^4}$$

```
[In] int((x^8*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (B*x^3)/(3*b^3) - (x^3*(B*a^2 - (2*A*a*b)/3) + (5*B*a^3 - 3*A*a^2*b)/(6*b))
/(a^2*b^3 + b^5*x^6 + 2*a*b^4*x^3) + (log(a + b*x^3)*(A*b - 3*B*a))/(3*b^4)
```

3.90 $\int \frac{x^5(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 66

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{a(Ab-aB)}{6b^3(a+bx^3)^2}-\frac{Ab-2aB}{3b^3(a+bx^3)}+\frac{B\log(a+bx^3)}{3b^3}$$

```
[Out] 1/6*a*(A*b-B*a)/b^3/(b*x^3+a)^2+1/3*(-A*b+2*B*a)/b^3/(b*x^3+a)+1/3*B*ln(b*x
^3+a)/b^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 78}

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{Ab-2aB}{3b^3(a+bx^3)}+\frac{a(Ab-aB)}{6b^3(a+bx^3)^2}+\frac{B\log(a+bx^3)}{3b^3}$$

```
[In] Int[(x^5*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (a*(A*b - a*B))/(6*b^3*(a + b*x^3)^2) - (A*b - 2*a*B)/(3*b^3*(a + b*x^3)) +
 (B*Log[a + b*x^3])/(3*b^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x(A+Bx)}{(a+bx)^3}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^2(a+bx)^3}+\frac{Ab-2aB}{b^2(a+bx)^2}+\frac{B}{b^2(a+bx)}\right)\,dx, x, x^3\right)\\ &= \frac{a(Ab-aB)}{6b^3\left(a+bx^3\right)^2}-\frac{Ab-2aB}{3b^3\left(a+bx^3\right)}+\frac{B\log\left(a+bx^3\right)}{3b^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int \frac{x^5(A+Bx^3)}{\left(a+bx^3\right)^3}\,dx = \frac{3a^2B-2Ab^2x^3-ab(A-4Bx^3)+2B\left(a+bx^3\right)^2\log\left(a+bx^3\right)}{6b^3\left(a+bx^3\right)^2}$$

```
[In] Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^3,x]
[Out] (3*a^2*B - 2*A*b^2*x^3 - a*b*(A - 4*B*x^3) + 2*B*(a + b*x^3)^2*Log[a + b*x^
3])/(6*b^3*(a + b*x^3)^2)
```

Maple [A] (verified)

Time = 4.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

method	result	size
norman	$\frac{-\frac{a(Ab-3Ba)}{6b^3}-\frac{(Ab-2Ba)x^3}{3b^2}}{\left(b\,x^3+a\right)^2}+\frac{B\ln\left(b\,x^3+a\right)}{3b^3}$	57
risch	$\frac{-\frac{a(Ab-3Ba)}{6b^3}-\frac{(Ab-2Ba)x^3}{3b^2}}{\left(b\,x^3+a\right)^2}+\frac{B\ln\left(b\,x^3+a\right)}{3b^3}$	57
default	$\frac{B\ln\left(b\,x^3+a\right)}{3b^3}+\frac{a(Ab-Ba)}{6b^3\left(b\,x^3+a\right)^2}-\frac{Ab-2Ba}{3b^3\left(b\,x^3+a\right)}$	61
parallelrisch	$-\frac{-2B\ln\left(b\,x^3+a\right)x^6b^2-4B\ln\left(b\,x^3+a\right)x^3ab+2A\,b^2x^3-4Bab\,x^3-2B\ln\left(b\,x^3+a\right)a^2+abA-3a^2B}{6b^3\left(b\,x^3+a\right)^2}$	90

```
[In] int(x^5*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/6*a*(A*b-3*B*a)/b^3-1/3*(A*b-2*B*a)/b^2*x^3)/(b*x^3+a)^2+1/3*B*ln(b*x^3
+a)/b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.35

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{2\,(2\,Bab-Ab^2)x^3+3\,Ba^2-Aab+2\,(Bb^2x^6+2\,Babx^3+Ba^2)\log\left(bx^3+a\right)}{6\,(b^5x^6+2\,ab^4x^3+a^2b^3)}$$

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(2*B*a*b - A*b^2)*x^3 + 3*B*a^2 - A*a*b + 2*(B*b^2*x^6 + 2*B*a*b*x^3
 + B*a^2)*log(b*x^3 + a))/(b^5*x^6 + 2*a*b^4*x^3 + a^2*b^3)
```

Sympy [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{B\log\left(a+bx^3\right)}{3b^3}+\frac{-Aab+3Ba^2+x^3(-2Ab^2+4Bab)}{6a^2b^3+12ab^4x^3+6b^5x^6}$$

```
[In] integrate(x**5*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*log(a + b*x**3)/(3*b**3) + (-A*a*b + 3*B*a**2 + x**3*(-2*A*b**2 + 4*B*a*b
))/(6*a**2*b**3 + 12*a*b**4*x**3 + 6*b**5*x**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{2\,(2\,Bab-Ab^2)x^3+3\,Ba^2-Aab}{6\,(b^5x^6+2\,ab^4x^3+a^2b^3)}+\frac{B\log\left(bx^3+a\right)}{3\,b^3}$$

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(2*B*a*b - A*b^2)*x^3 + 3*B*a^2 - A*a*b)/(b^5*x^6 + 2*a*b^4*x^3 + a^
2*b^3) + 1/3*B*log(b*x^3 + a)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{B\log\left(|bx^3+a|\right)}{3\,b^3} + \frac{2\,(2\,Ba-Ab)x^3+\frac{3\,Ba^2-Aab}{b}}{6\,(bx^3+a)^2b^2}$$

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*B*log(abs(b*x^3 + a))/b^3 + 1/6*(2*(2*B*a - A*b)*x^3 + (3*B*a^2 - A*a*b
)/b)/((b*x^3 + a)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 6.83 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{\frac{3\,B\,a^2-A\,a\,b}{6\,b^3}-\frac{x^3\,(A\,b-2\,B\,a)}{3\,b^2}}{a^2+2\,a\,b\,x^3+b^2\,x^6} + \frac{B\,\ln\left(b\,x^3+a\right)}{3\,b^3}$$

```
[In] int((x^5*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] ((3*B*a^2 - A*a*b)/(6*b^3) - (x^3*(A*b - 2*B*a))/(3*b^2))/(a^2 + b^2*x^6 +
2*a*b*x^3) + (B*log(a + b*x^3))/(3*b^3)
```

3.91 $\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 32

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3} dx = -\frac{(A+Bx^3)^2}{6(Ab-aB)(a+bx^3)^2}$$

```
[Out] -1/6*(B*x^3+A)^2/(A*b-B*a)/(b*x^3+a)^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {455, 37}

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3} dx = -\frac{(A+Bx^3)^2}{6(a+bx^3)^2(Ab-aB)}$$

```
[In] Int[(x^2*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] -1/6*(A + B*x^3)^2/((A*b - a*B)*(a + b*x^3)^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{(a+bx)^3}\,dx, x, x^3\right)\\ &= -\frac{(A+Bx^3)^2}{6(Ab-aB)\,(a+bx^3)^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{Ab+B(a+2bx^3)}{6b^2\,(a+bx^3)^2}$$

```
[In] Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^3,x]
[Out] -1/6*(A*b + B*(a + 2*b*x^3))/(b^2*(a + b*x^3)^2)
```

Maple [A] (verified)

Time = 3.98 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
gosper	$-\frac{2bB\,x^3+Ab+Ba}{6(b\,x^3+a)^2b^2}$	29
parallelrisch	$-\frac{2bB\,x^3+Ab+Ba}{6(b\,x^3+a)^2b^2}$	29
norman	$\frac{-\frac{B\,x^3}{3b}-\frac{Ab+Ba}{6b^2}}{(b\,x^3+a)^2}$	33
risch	$\frac{-\frac{B\,x^3}{3b}-\frac{Ab+Ba}{6b^2}}{(b\,x^3+a)^2}$	33
default	$-\frac{Ab-Ba}{6b^2(b\,x^3+a)^2}-\frac{B}{3b^2(b\,x^3+a)}$	39

```
[In] int(x^2*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
[Out] -1/6*(2*B*b*x^3+A*b+B*a)/(b*x^3+a)^2/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{2\,Bbx^3+Ba+Ab}{6\,(b^4x^6+2\,ab^3x^3+a^2b^2)}$$

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
[Out] -1/6*(2*B*b*x^3 + B*a + A*b)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{-Ab-Ba-2Bbx^3}{6a^2b^2+12ab^3x^3+6b^4x^6}$$

```
[In] integrate(x**2*(B*x**3+A)/(b*x**3+a)**3,x)
[Out] (-A*b - B*a - 2*B*b*x**3)/(6*a**2*b**2 + 12*a*b**3*x**3 + 6*b**4*x**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{2\,Bbx^3+Ba+Ab}{6\,(b^4x^6+2\,ab^3x^3+a^2b^2)}$$

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
[Out] -1/6*(2*B*b*x^3 + B*a + A*b)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{2\,Bbx^3+Ba+Ab}{6\,(bx^3+a)^2b^2}$$

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
[Out] -1/6*(2*B*b*x^3 + B*a + A*b)/((b*x^3 + a)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 6.79 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.38

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{\frac{A\,b+B\,a}{6\,b^2}+\frac{B\,x^3}{3\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}$$

```
[In] int((x^2*(A + B*x^3))/(a + b*x^3)^3,x)
[Out] -((A*b + B*a)/(6*b^2) + (B*x^3)/(3*b))/(a^2 + b^2*x^6 + 2*a*b*x^3)
```

3.92 $\int \frac{A+Bx^3}{x(a+bx^3)^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 68

$$\int \frac{A+Bx^3}{x(a+bx^3)^3} \, dx = \frac{Ab-aB}{6ab(a+bx^3)^2}+\frac{A}{3a^2(a+bx^3)}+\frac{A\log(x)}{a^3}-\frac{A\log(a+bx^3)}{3a^3}$$

```
[Out] 1/6*(A*b-B*a)/a/b/(b*x^3+a)^2+1/3*A/a^2/(b*x^3+a)+A*ln(x)/a^3-1/3*A*ln(b*x^
3+a)/a^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 78}

$$\int \frac{A+Bx^3}{x(a+bx^3)^3} \, dx = -\frac{A\log(a+bx^3)}{3a^3}+\frac{A\log(x)}{a^3}+\frac{A}{3a^2(a+bx^3)}+\frac{Ab-aB}{6ab(a+bx^3)^2}$$

```
[In] Int[(A + B*x^3)/(x*(a + b*x^3)^3),x]
```

```
[Out] (A*b - a*B)/(6*a*b*(a + b*x^3)^2) + A/(3*a^2*(a + b*x^3)) + (A*Log[x])/a^3
- (A*Log[a + b*x^3])/(3*a^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x(a+bx)^3}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{A}{a^3x}+\frac{-Ab+aB}{a(a+bx)^3}-\frac{Ab}{a^2(a+bx)^2}-\frac{Ab}{a^3(a+bx)}\right)dx, x, x^3\right)\\
&= \frac{Ab-aB}{6ab\left(a+bx^3\right)^2}+\frac{A}{3a^2\left(a+bx^3\right)}+\frac{A\log(x)}{a^3}-\frac{A\log\left(a+bx^3\right)}{3a^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^3}\,dx = \frac{\frac{a\left(3aAb-a^2B+2Ab^2x^3\right)}{b\left(a+bx^3\right)^2}+6A\log(x)-2A\log\left(a+bx^3\right)}{6a^3}$$

```
[In] Integrate[(A + B*x^3)/(x*(a + b*x^3)^3),x]
[Out] ((a*(3*a*A*b - a^2*B + 2*A*b^2*x^3))/(b*(a + b*x^3)^2) + 6*A*Log[x] - 2*A*L
og[a + b*x^3])/(6*a^3)
```

Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

method	result
risch	$\frac{\frac{A b\,x^{3}}{3 a^{2}}+\frac{3 A b -B a}{6 a b}}{\left(b\,x^{3}+a \right)^{2}}+\frac{A \ln \left(x \right)}{a^{3}}-\frac{A \ln \left(b\,x^{3}+a \right)}{3 a^{3}}$
default	$\frac{A \ln \left(x \right)}{a^{3}}-\frac{A \ln \left(b\,x^{3}+a \right)-\frac{a^{2} \left(A b -B a \right)}{2 b \left(b\,x^{3}+a \right)^{2}}-\frac{A a}{b\,x^{3}+a}}{3 a^{3}}$
norman	$\frac{-\frac{\left(2 A b -B a \right) x^{3}}{3 a^{2}}-\frac{b \left(3 A b -B a \right) x^{6}}{6 a^{3}}}{\left(b\,x^{3}+a \right)^{2}}+\frac{A \ln \left(x \right)}{a^{3}}-\frac{A \ln \left(b\,x^{3}+a \right)}{3 a^{3}}$
parallelrisch	$\frac{6 A \ln \left(x \right) x^{6} b^{2}-2 A \ln \left(b\,x^{3}+a \right) x^{6} b^{2}-3 A\,b^{2} x^{6}+B\,x^{6} a b +12 A \ln \left(x \right) x^{3} a b -4 A \ln \left(b\,x^{3}+a \right) x^{3} a b -4 a A b\,x^{3}+2 a^{2} B\,x^{3}+6 a^{2} A \ln \left(x \right)-2}{6 a^{3} \left(b\,x^{3}+a \right)^{2}}$

```
[In] int((B*x^3+A)/x/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/3/a^2*A*b*x^3+1/6*(3*A*b-B*a)/a/b)/(b*x^3+a)^2+A*ln(x)/a^3-1/3*A*ln(b*x^
3+a)/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.75

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^3}\,dx$$
$$=\frac{2\,Aab^2x^3-Ba^3+3\,Aa^2b-2\left(Ab^3x^6+2\,Aab^2x^3+Aa^2b\right)\log\left(bx^3+a\right)+6\left(Ab^3x^6+2\,Aab^2x^3+Aa^2b\right)\log}{6\left(a^3b^3x^6+2\,a^4b^2x^3+a^5b\right)}$$

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*A*a*b^2*x^3 - B*a^3 + 3*A*a^2*b - 2*(A*b^3*x^6 + 2*A*a*b^2*x^3 + A*a
^2*b)*log(b*x^3 + a) + 6*(A*b^3*x^6 + 2*A*a*b^2*x^3 + A*a^2*b)*log(x))/(a^3
*b^3*x^6 + 2*a^4*b^2*x^3 + a^5*b)
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^3}\,dx=\frac{A\log\left(x\right)}{a^3}-\frac{A\log\left(\frac{a}{b}+x^3\right)}{3a^3}+\frac{3Aab+2Ab^2x^3-Ba^2}{6a^4b+12a^3b^2x^3+6a^2b^3x^6}$$

```
[In] integrate((B*x**3+A)/x/(b*x**3+a)**3,x)
```

```
[Out] A*log(x)/a**3 - A*log(a/b + x**3)/(3*a**3) + (3*A*a*b + 2*A*b**2*x**3 - B*a
**2)/(6*a**4*b + 12*a**3*b**2*x**3 + 6*a**2*b**3*x**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.13

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^3}\,dx=\frac{2\,Ab^2x^3-Ba^2+3\,Aab}{6\left(a^2b^3x^6+2\,a^3b^2x^3+a^4b\right)}-\frac{A\log\left(bx^3+a\right)}{3\,a^3}+\frac{A\log\left(x^3\right)}{3\,a^3}$$

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*A*b^2*x^3 - B*a^2 + 3*A*a*b)/(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b) -
 1/3*A*log(b*x^3 + a)/a^3 + 1/3*A*log(x^3)/a^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^3}\,dx=-\frac{A\log\left(|bx^3+a|\right)}{3\,a^3}+\frac{A\log\left(|x|\right)}{a^3}+\frac{3\,Ab^3x^6+8\,Aab^2x^3-Ba^3+6\,Aa^2b}{6\left(bx^3+a\right)^2a^3b}$$

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/3*A*log(abs(b*x^3 + a))/a^3 + A*log(abs(x))/a^3 + 1/6*(3*A*b^3*x^6 + 8*A
*a*b^2*x^3 - B*a^3 + 6*A*a^2*b)/((b*x^3 + a)^2*a^3*b)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^3}\,dx=\frac{\frac{3\,A\,b-B\,a}{6\,a\,b}+\frac{A\,b\,x^3}{3\,a^2}}{a^2+2\,a\,b\,x^3+b^2\,x^6}-\frac{A\,\ln\left(b\,x^3+a\right)}{3\,a^3}+\frac{A\,\ln\left(x\right)}{a^3}$$

```
[In] int((A + B*x^3)/(x*(a + b*x^3)^3),x)
```

```
[Out] ((3*A*b - B*a)/(6*a*b) + (A*b*x^3)/(3*a^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) - (
A*log(a + b*x^3))/(3*a^3) + (A*log(x))/a^3
```

3.93 $\int \frac{A+Bx^3}{x^4(a+bx^3)^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 101

$$\int \frac{A+Bx^3}{x^4(a+bx^3)^3}\,dx = -\frac{A}{3a^3x^3}-\frac{Ab-aB}{6a^2(a+bx^3)^2}-\frac{2Ab-aB}{3a^3(a+bx^3)} - \frac{(3Ab-aB)\log(x)}{a^4}+\frac{(3Ab-aB)\log(a+bx^3)}{3a^4}$$

```
[Out] -1/3*A/a^3/x^3+1/6*(-A*b+B*a)/a^2/(b*x^3+a)^2+1/3*(-2*A*b+B*a)/a^3/(b*x^3+a
)-(3*A*b-B*a)*ln(x)/a^4+1/3*(3*A*b-B*a)*ln(b*x^3+a)/a^4
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 78}

$$\int \frac{A+Bx^3}{x^4(a+bx^3)^3}\,dx = \frac{(3Ab-aB)\log(a+bx^3)}{3a^4}-\frac{\log(x)(3Ab-aB)}{a^4} - \frac{2Ab-aB}{3a^3(a+bx^3)}-\frac{A}{3a^3x^3}-\frac{Ab-aB}{6a^2(a+bx^3)^2}$$

```
[In] Int[(A + B*x^3)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] -1/3*A/(a^3*x^3) - (A*b - a*B)/(6*a^2*(a + b*x^3)^2) - (2*A*b - a*B)/(3*a^3
*(a + b*x^3)) - ((3*A*b - a*B)*Log[x])/a^4 + ((3*A*b - a*B)*Log[a + b*x^3])
/(3*a^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)^3}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{A}{a^3x^2}+\frac{-3Ab+aB}{a^4x}-\frac{b(-Ab+aB)}{a^2(a+bx)^3}-\frac{b(-2Ab+aB)}{a^3(a+bx)^2}-\frac{b(-3Ab+aB)}{a^4(a+bx)}\right)dx, x, x^3\right)\\
&= -\frac{A}{3a^3x^3}-\frac{Ab-aB}{6a^2\left(a+bx^3\right)^2}-\frac{2Ab-aB}{3a^3\left(a+bx^3\right)}-\frac{(3Ab-aB)\log(x)}{a^4}+\frac{(3Ab-aB)\log\left(a+bx^3\right)}{3a^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^3}\,dx = \frac{-\frac{2aA}{x^3}+\frac{a^2(-Ab+aB)}{\left(a+bx^3\right)^2}+\frac{2a(-2Ab+aB)}{a+bx^3}+6(-3Ab+aB)\log(x)+2(3Ab-aB)\log\left(a+bx^3\right)}{6a^4}$$

```
[In] Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^3),x]
[Out] ((-2*a*A)/x^3 + (a^2*(-(A*b) + a*B))/(a + b*x^3)^2 + (2*a*(-2*A*b + a*B))/(
a + b*x^3) + 6*(-3*A*b + a*B)*Log[x] + 2*(3*A*b - a*B)*Log[a + b*x^3])/(6*a
^4)
```

Maple [A] (verified)

Time = 4.18 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.97

method	result
norman	$\frac{-\frac{A}{3a}+\frac{2b(3Ab-Ba)x^6}{3a^3}+\frac{b^2(3Ab-Ba)x^9}{2a^4}}{x^3(b x^3+a)^2}-\frac{(3Ab-Ba)\ln(x)}{a^4}+\frac{(3Ab-Ba)\ln(b x^3+a)}{3a^4}$
default	$-\frac{A}{3a^3x^3}+\frac{(-3Ab+Ba)\ln(x)}{a^4}+\frac{b\left(\frac{(3Ab-Ba)\ln(b x^3+a)}{b}-\frac{a^2(Ab-Ba)}{2b(b x^3+a)^2}-\frac{a(2Ab-Ba)}{b(b x^3+a)}\right)}{3a^4}$
risch	$\frac{-\frac{b(3Ab-Ba)x^6}{3a^3}-\frac{(3Ab-Ba)x^3}{2a^2}-\frac{A}{3a}}{x^3(b x^3+a)^2}-\frac{3\ln(x)Ab}{a^4}+\frac{B\ln(x)}{a^3}+\frac{\ln(-b x^3-a)Ab}{a^4}-\frac{\ln(-b x^3-a)B}{3a^3}$
parallelrisch	$-\frac{18A\ln(x)x^9b^3-6A\ln(b x^3+a)x^9b^3-6B\ln(x)x^9a b^2+2B\ln(b x^3+a)x^9a b^2-9A x^9b^3+3B x^9a b^2+36A\ln(x)x^6a b^2-12A\ln(}{}$

```
[In] int((B*x^3+A)/x^4/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/3*A/a+2/3*b*(3*A*b-B*a)/a^3*x^6+1/2*b^2*(3*A*b-B*a)/a^4*x^9)/x^3/(b*x^3
+a)^2-(3*A*b-B*a)*ln(x)/a^4+1/3*(3*A*b-B*a)*ln(b*x^3+a)/a^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(89) = 178$.

Time = 0.26 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.95

$$\int \frac{A+Bx^3}{x^4(a+bx^3)^3}\,dx$$
$$=\frac{2\,(Ba^2b-3\,Aab^2)x^6-2\,Aa^3+3\,(Ba^3-3\,Aa^2b)x^3-2\,((Bab^2-3\,Ab^3)x^9+2\,(Ba^2b-3\,Aab^2)x^6+(Ba^3}{6\,(a^4b^2x^9+2\,a^5}$$

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(B*a^2*b - 3*A*a*b^2)*x^6 - 2*A*a^3 + 3*(B*a^3 - 3*A*a^2*b)*x^3 - 2*
((B*a*b^2 - 3*A*b^3)*x^9 + 2*(B*a^2*b - 3*A*a*b^2)*x^6 + (B*a^3 - 3*A*a^2*b
)*x^3)*log(b*x^3 + a) + 6*((B*a*b^2 - 3*A*b^3)*x^9 + 2*(B*a^2*b - 3*A*a*b^2
)*x^6 + (B*a^3 - 3*A*a^2*b)*x^3)*log(x))/(a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x
^3)
```

Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^3}\,dx = \frac{-2Aa^2+x^6(-6Ab^2+2Bab)+x^3(-9Aab+3Ba^2)}{6a^5x^3+12a^4bx^6+6a^3b^2x^9} + \frac{(-3Ab+Ba)\log{(x)}}{a^4} - \frac{(-3Ab+Ba)\log{\left(\frac{a}{b}+x^3\right)}}{3a^4}$$

```
[In] integrate((B*x**3+A)/x**4/(b*x**3+a)**3,x)
```

```
[Out] (-2*A*a**2 + x**6*(-6*A*b**2 + 2*B*a*b) + x**3*(-9*A*a*b + 3*B*a**2))/(6*a*
*5*x**3 + 12*a**4*b*x**6 + 6*a**3*b**2*x**9) + (-3*A*b + B*a)*log(x)/a**4 -
 (-3*A*b + B*a)*log(a/b + x**3)/(3*a**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^3}\,dx = \frac{2\,(Bab-3\,Ab^2)x^6+3\,(Ba^2-3\,Aab)x^3-2\,Aa^2}{6\,(a^3b^2x^9+2\,a^4bx^6+a^5x^3)} - \frac{(Ba-3\,Ab)\log{(bx^3+a)}}{3\,a^4} + \frac{(Ba-3\,Ab)\log{(x^3)}}{3\,a^4}$$

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(B*a*b - 3*A*b^2)*x^6 + 3*(B*a^2 - 3*A*a*b)*x^3 - 2*A*a^2)/(a^3*b^2*
x^9 + 2*a^4*b*x^6 + a^5*x^3) - 1/3*(B*a - 3*A*b)*log(b*x^3 + a)/a^4 + 1/3*(
B*a - 3*A*b)*log(x^3)/a^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.35

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^3}\,dx = \frac{(Ba-3\,Ab)\log{(|x|)}}{a^4} - \frac{(Bab-3\,Ab^2)\log{(|bx^3+a|)}}{3\,a^4b} + \frac{3\,Bab^2x^6-9\,Ab^3x^6+8\,Ba^2bx^3-22\,Aab^2x^3+6\,Ba^3-14\,Aa^2b}{6\,(bx^3+a)^2a^4} - \frac{Bax^3-3\,Abx^3+Aa}{3\,a^4x^3}$$

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] (B*a - 3*A*b)*log(abs(x))/a^4 - 1/3*(B*a*b - 3*A*b^2)*log(abs(b*x^3 + a))/(
a^4*b) + 1/6*(3*B*a*b^2*x^6 - 9*A*b^3*x^6 + 8*B*a^2*b*x^3 - 22*A*a*b^2*x^3
+ 6*B*a^3 - 14*A*a^2*b)/((b*x^3 + a)^2*a^4) - 1/3*(B*a*x^3 - 3*A*b*x^3 + A*
a)/(a^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 6.90 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^3}\,dx = \frac{\ln\left(b\,x^3+a\right)\,\left(3\,A\,b-B\,a\right)}{3\,a^4} - \frac{\frac{A}{3\,a}+\frac{x^3\,\left(3\,A\,b-B\,a\right)}{2\,a^2}+\frac{b\,x^6\,\left(3\,A\,b-B\,a\right)}{3\,a^3}}{a^2\,x^3+2\,a\,b\,x^6+b^2\,x^9} - \frac{\ln\left(x\right)\,\left(3\,A\,b-B\,a\right)}{a^4}$$

```
[In] int((A + B*x^3)/(x^4*(a + b*x^3)^3),x)
```

```
[Out] (log(a + b*x^3)*(3*A*b - B*a))/(3*a^4) - (A/(3*a) + (x^3*(3*A*b - B*a))/(2*
a^2) + (b*x^6*(3*A*b - B*a))/(3*a^3))/(a^2*x^3 + b^2*x^9 + 2*a*b*x^6) - (lo
g(x)*(3*A*b - B*a))/a^4
```

3.94 $\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 122

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^3}\,dx=-\frac{A}{6a^3x^6}+\frac{3Ab-aB}{3a^4x^3}+\frac{b(Ab-aB)}{6a^3\left(a+bx^3\right)^2}+\frac{b(3Ab-2aB)}{3a^4\left(a+bx^3\right)}+\frac{3b(2Ab-aB)\log(x)}{a^5}-\frac{b(2Ab-aB)\log\left(a+bx^3\right)}{a^5}$$

```
[Out] -1/6*A/a^3/x^6+1/3*(3*A*b-B*a)/a^4/x^3+1/6*b*(A*b-B*a)/a^3/(b*x^3+a)^2+1/3*
b*(3*A*b-2*B*a)/a^4/(b*x^3+a)+3*b*(2*A*b-B*a)*ln(x)/a^5-b*(2*A*b-B*a)*ln(b*
x^3+a)/a^5
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 78}

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^3}\,dx=-\frac{b(2Ab-aB)\log\left(a+bx^3\right)}{a^5}+\frac{3b\log(x)(2Ab-aB)}{a^5}+\frac{b(3Ab-2aB)}{3a^4\left(a+bx^3\right)}+\frac{3Ab-aB}{3a^4x^3}+\frac{b(Ab-aB)}{6a^3\left(a+bx^3\right)^2}-\frac{A}{6a^3x^6}$$

```
[In] Int[(A + B*x^3)/(x^7*(a + b*x^3)^3),x]
```

```
[Out] -1/6*A/(a^3*x^6) + (3*A*b - a*B)/(3*a^4*x^3) + (b*(A*b - a*B))/(6*a^3*(a +
b*x^3)^2) + (b*(3*A*b - 2*a*B))/(3*a^4*(a + b*x^3)) + (3*b*(2*A*b - a*B)*Lo
g[x])/a^5 - (b*(2*A*b - a*B)*Log[a + b*x^3])/a^5
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)^3}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{A}{a^3x^3}+\frac{-3Ab+aB}{a^4x^2}-\frac{3b(-2Ab+aB)}{a^5x}+\frac{b^2(-Ab+aB)}{a^3(a+bx)^3}\right.\right.\\
&\qquad\left.\left.+\frac{b^2(-3Ab+2aB)}{a^4(a+bx)^2}+\frac{3b^2(-2Ab+aB)}{a^5(a+bx)}\right)dx, x, x^3\right)\\
&= -\frac{A}{6a^3x^6}+\frac{3Ab-aB}{3a^4x^3}+\frac{b(Ab-aB)}{6a^3(a+bx^3)^2}+\frac{b(3Ab-2aB)}{3a^4(a+bx^3)}\\
&\quad+\frac{3b(2Ab-aB)\log(x)}{a^5}-\frac{b(2Ab-aB)\log(a+bx^3)}{a^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^3}{x^7(a+bx^3)^3}\,dx = \frac{-\frac{a^2A}{x^6}-\frac{2a(-3Ab+aB)}{x^3}+\frac{a^2b(Ab-aB)}{(a+bx^3)^2}+\frac{2ab(3Ab-2aB)}{a+bx^3}+18b(2Ab-aB)\log(x)+6b(-2Ab+aB)\log(a+bx^3)}{6a^5}$$

```
[In] Integrate[(A + B*x^3)/(x^7*(a + b*x^3)^3),x]
```

```
[Out] (-((a^2*A)/x^6) - (2*a*(-3*A*b + a*B))/x^3 + (a^2*b*(A*b - a*B))/(a + b*x^3
)^2 + (2*a*b*(3*A*b - 2*a*B))/(a + b*x^3) + 18*b*(2*A*b - a*B)*Log[x] + 6*b
*(-2*A*b + a*B)*Log[a + b*x^3])/(6*a^5)
```

Maple [A] (verified)

Time = 4.18 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.01

method	result
default	$-\frac{A}{6a^{3}x^{6}}-\frac{-3Ab+Ba}{3a^{4}x^{3}}+\frac{3b(2Ab-Ba)\ln(x)}{a^{5}}-\frac{b^{2}\left(\frac{(6Ab-3Ba)\ln\left(b\,x^{3}+a\right)}{b}-\frac{a^{2}(Ab-Ba)}{2b\left(b\,x^{3}+a\right)^{2}}-\frac{a(3Ab-2Ba)}{b\left(b\,x^{3}+a\right)}\right)}{3a^{5}}$
norman	$\frac{-\frac{A}{6a}+\frac{(2Ab-Ba)x^{3}}{3a^{2}}-\frac{2b\left(2b^{2}A-abB\right)x^{9}}{a^{4}}-\frac{b^{2}\left(6b^{2}A-3abB\right)x^{12}}{2a^{5}}}{x^{6}(b\,x^{3}+a)^{2}}+\frac{3b(2Ab-Ba)\ln(x)}{a^{5}}-\frac{b(2Ab-Ba)\ln\left(b\,x^{3}+a\right)}{a^{5}}$
risch	$\frac{\frac{b^{2}(2Ab-Ba)x^{9}}{a^{4}}+\frac{3b(2Ab-Ba)x^{6}}{2a^{3}}+\frac{(2Ab-Ba)x^{3}}{3a^{2}}-\frac{A}{6a}}{x^{6}(b\,x^{3}+a)^{2}}+\frac{6b^{2}\ln(x)A}{a^{5}}-\frac{3b\ln(x)B}{a^{4}}-\frac{2b^{2}\ln(b\,x^{3}+a)A}{a^{5}}+\frac{b\ln(b\,x^{3}+a)B}{a^{4}}$
parallelrisch	$\frac{36A\ln(x)x^{12}b^{4}-12A\ln(b\,x^{3}+a)x^{12}b^{4}-18B\ln(x)x^{12}a\,b^{3}+6B\ln(b\,x^{3}+a)x^{12}a\,b^{3}-18A\,x^{12}b^{4}+9B\,x^{12}a\,b^{3}+72A\ln(x)x^{9}a\,b^{3}-24}{}$

[In] int((B*x^3+A)/x^7/(b*x^3+a)^3,x,method=_RETURNVERBOSE)

[Out] -1/6*A/a^3/x^6-1/3*(-3*A*b+B*a)/a^4/x^3+3*b*(2*A*b-B*a)*ln(x)/a^5-1/3/a^5*b^2*((6*A*b-3*B*a)/b*ln(b*x^3+a)-1/2*a^2*(A*b-B*a)/b/(b*x^3+a)^2-a*(3*A*b-2*B*a)/b/(b*x^3+a))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(110) = 220.

Time = 0.26 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.88

$$\int \frac{A+Bx^{3}}{x^{7}\left(a+bx^{3}\right)^{3}}\,dx = -\frac{6\left(Ba^{2}b^{2}-2\,Aab^{3}\right)x^{9}+9\left(Ba^{3}b-2\,Aa^{2}b^{2}\right)x^{6}+Aa^{4}+2\left(Ba^{4}-2\,Aa^{3}b\right)x^{3}-6\left(\left(Bab^{3}-2\,Ab^{4}\right)x^{12}+2\left(\right.\right.}{}$$

[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^3,x, algorithm="fricas")

[Out] -1/6*(6*(B*a^2*b^2 - 2*A*a*b^3)*x^9 + 9*(B*a^3*b - 2*A*a^2*b^2)*x^6 + A*a^4 + 2*(B*a^4 - 2*A*a^3*b)*x^3 - 6*((B*a*b^3 - 2*A*b^4)*x^12 + 2*(B*a^2*b^2 - 2*A*a*b^3)*x^9 + (B*a^3*b - 2*A*a^2*b^2)*x^6)*log(b*x^3 + a) + 18*((B*a*b^3 - 2*A*b^4)*x^12 + 2*(B*a^2*b^2 - 2*A*a*b^3)*x^9 + (B*a^3*b - 2*A*a^2*b^2)*x^6)*log(x))/(a^5*b^2*x^12 + 2*a^6*b*x^9 + a^7*x^6)

Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^3}\,dx = \frac{-Aa^3+x^9\cdot\left(12Ab^3-6Bab^2\right)+x^6\cdot\left(18Aab^2-9Ba^2b\right)+x^3\cdot\left(4Aa^2b-2Ba^3\right)}{6a^6x^6+12a^5bx^9+6a^4b^2x^{12}} - \frac{3b(-2Ab+Ba)\log{(x)}}{a^5} + \frac{b(-2Ab+Ba)\log{\left(\frac{a}{b}+x^3\right)}}{a^5}$$

```
[In] integrate((B*x**3+A)/x**7/(b*x**3+a)**3,x)
```

```
[Out] (-A*a**3 + x**9*(12*A*b**3 - 6*B*a*b**2) + x**6*(18*A*a*b**2 - 9*B*a**2*b)
+ x**3*(4*A*a**2*b - 2*B*a**3))/(6*a**6*x**6 + 12*a**5*b*x**9 + 6*a**4*b**2
*x**12) - 3*b*(-2*A*b + B*a)*log(x)/a**5 + b*(-2*A*b + B*a)*log(a/b + x**3)
/a**5
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^3}\,dx = -\frac{6\left(Bab^2-2\,Ab^3\right)x^9+9\left(Ba^2b-2\,Aab^2\right)x^6+Aa^3+2\left(Ba^3-2\,Aa^2b\right)x^3}{6\left(a^4b^2x^{12}+2\,a^5bx^9+a^6x^6\right)} + \frac{\left(Bab-2\,Ab^2\right)\log\left(bx^3+a\right)}{a^5} - \frac{\left(Bab-2\,Ab^2\right)\log\left(x^3\right)}{a^5}$$

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(6*(B*a*b^2 - 2*A*b^3)*x^9 + 9*(B*a^2*b - 2*A*a*b^2)*x^6 + A*a^3 + 2*(
B*a^3 - 2*A*a^2*b)*x^3)/(a^4*b^2*x^12 + 2*a^5*b*x^9 + a^6*x^6) + (B*a*b - 2
*A*b^2)*log(b*x^3 + a)/a^5 - (B*a*b - 2*A*b^2)*log(x^3)/a^5
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^3}\,dx = -\frac{3\left(Bab-2\,Ab^2\right)\log\left(|x|\right)}{a^5}+\frac{\left(Bab^2-2\,Ab^3\right)\log\left(|bx^3+a|\right)}{a^5b} - \frac{6\,Bab^2x^9-12\,Ab^3x^9+9\,Ba^2bx^6-18\,Aab^2x^6+2\,Ba^3x^3-4\,Aa^2bx^3+Aa^3}{6\left(bx^6+ax^3\right)^2a^4}$$

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -3*(B*a*b - 2*A*b^2)*log(abs(x))/a^5 + (B*a*b^2 - 2*A*b^3)*log(abs(b*x^3 +
a))/(a^5*b) - 1/6*(6*B*a*b^2*x^9 - 12*A*b^3*x^9 + 9*B*a^2*b*x^6 - 18*A*a*b^
2*x^6 + 2*B*a^3*x^3 - 4*A*a^2*b*x^3 + A*a^3)/((b*x^6 + a*x^3)^2*a^4)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^3}\,dx = \frac{\frac{x^3\,\left(2\,A\,b-B\,a\right)}{3\,a^2}-\frac{A}{6\,a}+\frac{b^2\,x^9\,\left(2\,A\,b-B\,a\right)}{a^4}+\frac{3\,b\,x^6\,\left(2\,A\,b-B\,a\right)}{2\,a^3}}{a^2\,x^6+2\,a\,b\,x^9+b^2\,x^{12}} - \frac{\ln\left(b\,x^3+a\right)\,\left(2\,A\,b^2-B\,a\,b\right)}{a^5}+\frac{\ln\left(x\right)\,\left(6\,A\,b^2-3\,B\,a\,b\right)}{a^5}$$

```
[In] int((A + B*x^3)/(x^7*(a + b*x^3)^3),x)
```

```
[Out] ((x^3*(2*A*b - B*a))/(3*a^2) - A/(6*a) + (b^2*x^9*(2*A*b - B*a))/a^4 + (3*b
*x^6*(2*A*b - B*a))/(2*a^3))/(a^2*x^6 + b^2*x^12 + 2*a*b*x^9) - (log(a + b*
x^3)*(2*A*b^2 - B*a*b))/a^5 + (log(x)*(6*A*b^2 - 3*B*a*b))/a^5
```

3.95 $\int \frac{x^{10}(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 246

$$\int \frac{x^{10}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{2(5Ab-11aB)x^2}{9b^4} - \frac{4(5Ab-11aB)x^5}{45ab^3} + \frac{(Ab-aB)x^{11}}{6ab\,(a+bx^3)^2} + \frac{(5Ab-11aB)x^8}{18ab^2\,(a+bx^3)} + \frac{4a^{2/3}(5Ab-11aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}b^{14/3}} + \frac{4a^{2/3}(5Ab-11aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{14/3}} - \frac{2a^{2/3}(5Ab-11aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27b^{14/3}}$$

```
[Out] 2/9*(5*A*b-11*B*a)*x^2/b^4-4/45*(5*A*b-11*B*a)*x^5/a/b^3+1/6*(A*b-B*a)*x^11
/a/b/(b*x^3+a)^2+1/18*(5*A*b-11*B*a)*x^8/a/b^2/(b*x^3+a)+4/27*a^(2/3)*(5*A*
b-11*B*a)*ln(a^(1/3)+b^(1/3)*x)/b^(14/3)-2/27*a^(2/3)*(5*A*b-11*B*a)*ln(a^(
2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(14/3)+4/27*a^(2/3)*(5*A*b-11*B*a)*ar
ctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(14/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {468, 294, 308, 298, 31, 648, 631, 210, 642}

$$\int \frac{x^{10}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{4a^{2/3}(5Ab-11aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}b^{14/3}} - \frac{2a^{2/3}(5Ab-11aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27b^{14/3}} + \frac{4a^{2/3}(5Ab-11aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{14/3}} + \frac{2x^2(5Ab-11aB)}{9b^4} - \frac{4x^5(5Ab-11aB)}{45ab^3} + \frac{x^8(5Ab-11aB)}{18ab^2(a+bx^3)} + \frac{x^{11}(Ab-aB)}{6ab(a+bx^3)^2}$$

```
[In] Int[(x^10*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (2*(5*A*b - 11*a*B)*x^2)/(9*b^4) - (4*(5*A*b - 11*a*B)*x^5)/(45*a*b^3) + ((
A*b - a*B)*x^11)/(6*a*b*(a + b*x^3)^2) + ((5*A*b - 11*a*B)*x^8)/(18*a*b^2*(
a + b*x^3)) + (4*a^(2/3)*(5*A*b - 11*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(S
qrt[3]*a^(1/3))])/(9*Sqrt[3]*b^(14/3)) + (4*a^(2/3)*(5*A*b - 11*a*B)*Log[a^
(1/3) + b^(1/3)*x])/(27*b^(14/3)) - (2*a^(2/3)*(5*A*b - 11*a*B)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*b^(14/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{(Ab-aB)x^{11}}{6ab\left(a+bx^3\right)^2}+\frac{(-5Ab+11aB)\int\frac{x^{10}}{\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^{11}}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab-11aB)x^8}{18ab^2\left(a+bx^3\right)} - \frac{(4(5Ab-11aB))\int \frac{x^7}{a+bx^3}\,dx}{9ab^2}$$

$$= \frac{(Ab-aB)x^{11}}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab-11aB)x^8}{18ab^2\left(a+bx^3\right)} - \frac{(4(5Ab-11aB))\int \left(-\frac{ax}{b^2}+\frac{x^4}{b}+\frac{a^2x}{b^2(a+bx^3)}\right)dx}{9ab^2}$$

$$= \frac{2(5Ab-11aB)x^2}{9b^4} - \frac{4(5Ab-11aB)x^5}{45ab^3} + \frac{(Ab-aB)x^{11}}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab-11aB)x^8}{18ab^2\left(a+bx^3\right)} - \frac{(4a(5Ab-11aB))\int \frac{x}{a+bx^3}\,dx}{9b^4}$$

$$= \frac{2(5Ab-11aB)x^2}{9b^4} - \frac{4(5Ab-11aB)x^5}{45ab^3} + \frac{(Ab-aB)x^{11}}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab-11aB)x^8}{18ab^2\left(a+bx^3\right)} + \frac{\left(4a^{2/3}(5Ab-11aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27b^{13/3}} - \frac{\left(4a^{2/3}(5Ab-11aB)\right)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27b^{13/3}}$$

$$= \frac{2(5Ab-11aB)x^2}{9b^4} - \frac{4(5Ab-11aB)x^5}{45ab^3} + \frac{(Ab-aB)x^{11}}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab-11aB)x^8}{18ab^2\left(a+bx^3\right)} + \frac{4a^{2/3}(5Ab-11aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{14/3}} - \frac{\left(2a^{2/3}(5Ab-11aB)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27b^{14/3}} - \frac{(2a(5Ab-11aB))\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9b^{13/3}}$$

$$= \frac{2(5Ab-11aB)x^2}{9b^4} - \frac{4(5Ab-11aB)x^5}{45ab^3} + \frac{(Ab-aB)x^{11}}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab-11aB)x^8}{18ab^2\left(a+bx^3\right)} + \frac{4a^{2/3}(5Ab-11aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{14/3}} - \frac{2a^{2/3}(5Ab-11aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27b^{14/3}} - \frac{\left(4a^{2/3}(5Ab-11aB)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9b^{14/3}}$$

$$= \frac{2(5Ab-11aB)x^2}{9b^4} - \frac{4(5Ab-11aB)x^5}{45ab^3} + \frac{(Ab-aB)x^{11}}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab-11aB)x^8}{18ab^2\left(a+bx^3\right)} + \frac{4a^{2/3}(5Ab-11aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}b^{14/3}} + \frac{4a^{2/3}(5Ab-11aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{14/3}} - \frac{2a^{2/3}(5Ab-11aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27b^{14/3}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.88

$$\int \frac{x^{10}(A+Bx^3)}{\left(a+bx^3\right)^3}\,dx$$

$$= \frac{135b^{2/3}(Ab-3aB)x^2 + 54b^{5/3}Bx^5 + \frac{45a^2b^{2/3}(-Ab+aB)x^2}{(a+bx^3)^2} + \frac{30ab^{2/3}(7Ab-10aB)x^2}{a+bx^3} - 40\sqrt{3}a^{2/3}(-5Ab+11aB)\,\mathrm{a}}{}$$

```
[In] Integrate[(x^10*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (135*b^(2/3)*(A*b - 3*a*B)*x^2 + 54*b^(5/3)*B*x^5 + (45*a^2*b^(2/3)*(-(A*b)
 + a*B)*x^2)/(a + b*x^3)^2 + (30*a*b^(2/3)*(7*A*b - 10*a*B)*x^2)/(a + b*x^3
) - 40*Sqrt[3]*a^(2/3)*(-5*A*b + 11*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))
/Sqrt[3]] - 40*a^(2/3)*(-5*A*b + 11*a*B)*Log[a^(1/3) + b^(1/3)*x] + 20*a^(2
/3)*(-5*A*b + 11*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(270*
b^(14/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.37 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.47

method	result
risch	$\frac{B\,x^{5}}{5b^{3}}+\frac{A\,x^{2}}{2b^{3}}-\frac{3Ba\,x^{2}}{2b^{4}}+\frac{\left(\frac{7}{9}a\,b^{2}A-\frac{10}{9}a^{2}bB\right)x^{5}+\frac{a^{2}(11Ab-17Ba)x^{2}}{18}}{b^{4}\left(b\,x^{3}+a\right)^{2}}-\frac{4a\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{(5Ab-11Ba)\ln\left(x-\_R\right)}{\_R}\right)}{27b^{5}}$
default	$\frac{\frac{bB\,x^{5}}{5}+\frac{(Ab-3Ba)x^{2}}{2}}{b^{4}}-\frac{a\left(\frac{\left(-\frac{7}{9}b^{2}A+\frac{10}{9}abB\right)x^{5}-\frac{a(11Ab-17Ba)x^{2}}{18}}{\left(b\,x^{3}+a\right)^{2}}+\left(\frac{20Ab}{9}-\frac{44Ba}{9}\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\ \mathrm{ar}}{}\right.\right.}{b^{4}}$

```
[In] int(x^10*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*B*x^5/b^3+1/2/b^3*A*x^2-3/2/b^4*B*a*x^2+((7/9*a*b^2*A-10/9*a^2*b*B)*x^5
+1/18*a^2*(11*A*b-17*B*a)*x^2)/b^4/(b*x^3+a)^2-4/27/b^5*a*sum((5*A*b-11*B*a
)/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.48

$$\int \frac{x^{10}(A+Bx^3)}{\left(a+bx^3\right)^3}\,dx$$

$$=\frac{54\,Bb^{3}x^{11}-27\,(11\,Bab^{2}-5\,Ab^{3})x^{8}-96\,(11\,Ba^{2}b-5\,Aab^{2})x^{5}-60\,(11\,Ba^{3}-5\,Aa^{2}b)x^{2}+40\,\sqrt{3}((11\,Bab}{}$$

```
[In] integrate(x^10*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/270*(54*B*b^3*x^11 - 27*(11*B*a*b^2 - 5*A*b^3)*x^8 - 96*(11*B*a^2*b - 5*A
*a*b^2)*x^5 - 60*(11*B*a^3 - 5*A*a^2*b)*x^2 + 40*sqrt(3)*((11*B*a*b^2 - 5*A
*b^3)*x^6 + 11*B*a^3 - 5*A*a^2*b + 2*(11*B*a^2*b - 5*A*a*b^2)*x^3)*(a^2/b^2
)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3) - sqrt(3)*a)/a) + 20*((11
*B*a*b^2 - 5*A*b^3)*x^6 + 11*B*a^3 - 5*A*a^2*b + 2*(11*B*a^2*b - 5*A*a*b^2)
```

```
*x^3)*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a*(a^2/b^2)^(1/3))
- 40*((11*B*a*b^2 - 5*A*b^3)*x^6 + 11*B*a^3 - 5*A*a^2*b + 2*(11*B*a^2*b - 5
*A*a*b^2)*x^3)*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/(b^6*x^6 + 2*a
*b^5*x^3 + a^2*b^4)
```

Sympy [A] (verification not implemented)

Time = 9.88 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.78

$$\int \frac{x^{10}(A+Bx^3)}{(a+bx^3)^3}\,dx$$
$$= \frac{Bx^5}{5b^3} + x^2\left(\frac{A}{2b^3} - \frac{3Ba}{2b^4}\right) + \frac{x^5\cdot(14Aab^2-20Ba^2b)+x^2\cdot(11Aa^2b-17Ba^3)}{18a^2b^4+36ab^5x^3+18b^6x^6}$$
$$+\text{RootSum}\left(19683t^3b^{14} - 8000A^3a^2b^3 + 52800A^2Ba^3b^2 - 116160AB^2a^4b + 85184B^3a^5, \left(t \mapsto t\log\left(\frac{}{400}\right.\right.\right.$$

```
[In] integrate(x**10*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*x**5/(5*b**3) + x**2*(A/(2*b**3) - 3*B*a/(2*b**4)) + (x**5*(14*A*a*b**2 -
 20*B*a**2*b) + x**2*(11*A*a**2*b - 17*B*a**3))/(18*a**2*b**4 + 36*a*b**5*x
**3 + 18*b**6*x**6) + RootSum(19683*_t**3*b**14 - 8000*A**3*a**2*b**3 + 528
00*A**2*B*a**3*b**2 - 116160*A*B**2*a**4*b + 85184*B**3*a**5, Lambda(_t, _t
*log(729*_t**2*b**9/(400*A**2*a*b**2 - 1760*A*B*a**2*b + 1936*B**2*a**3) +
x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.93

$$\int \frac{x^{10}(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{2\,(10\,Ba^2b-7\,Aab^2)x^5+(17\,Ba^3-11\,Aa^2b)x^2}{18\,(b^6x^6+2\,ab^5x^3+a^2b^4)} + \frac{4\,\sqrt{3}(11\,Ba^2-5\,Aab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{2\,Bbx^5-5\,(3\,Ba-Ab)x^2}{10\,b^4} + \frac{2\,(11\,Ba^2-5\,Aab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{4\,(11\,Ba^2-5\,Aab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate(x^10*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*(2*(10*B*a^2*b - 7*A*a*b^2)*x^5 + (17*B*a^3 - 11*A*a^2*b)*x^2)/(b^6*x
^6 + 2*a*b^5*x^3 + a^2*b^4) + 4/27*sqrt(3)*(11*B*a^2 - 5*A*a*b)*arctan(1/3*
sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^5*(a/b)^(1/3)) + 1/10*(2*B*b*x^
5 - 5*(3*B*a - A*b)*x^2)/b^4 + 2/27*(11*B*a^2 - 5*A*a*b)*log(x^2 - x*(a/b)^
(1/3) + (a/b)^(2/3))/(b^5*(a/b)^(1/3)) - 4/27*(11*B*a^2 - 5*A*a*b)*log(x +
(a/b)^(1/3))/(b^5*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.05

$$\int \frac{x^{10}(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{4\left(11\,Ba^2\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 5\,Aab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^4} - \frac{4\sqrt{3}\left(11\left(-ab^2\right)^{\frac{2}{3}}Ba - 5\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^6} + \frac{2\left(11\left(-ab^2\right)^{\frac{2}{3}}Ba - 5\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,b^6} - \frac{20\,Ba^2bx^5 - 14\,Aab^2x^5 + 17\,Ba^3x^2 - 11\,Aa^2bx^2}{18\left(bx^3+a\right)^2b^4} + \frac{2\,Bb^{12}x^5 - 15\,Bab^{11}x^2 + 5\,Ab^{12}x^2}{10\,b^{15}}$$

```
[In] integrate(x^10*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -4/27*(11*B*a^2*(-a/b)^(1/3) - 5*A*a*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x
 - (-a/b)^(1/3)))/(a*b^4) - 4/27*sqrt(3)*(11*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2
)^(2/3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^6 + 2/
27*(11*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3)
+ (-a/b)^(2/3))/b^6 - 1/18*(20*B*a^2*b*x^5 - 14*A*a*b^2*x^5 + 17*B*a^3*x^2
- 11*A*a^2*b*x^2)/((b*x^3 + a)^2*b^4) + 1/10*(2*B*b^12*x^5 - 15*B*a*b^11*x^
2 + 5*A*b^12*x^2)/b^15
```

Mupad [B] (verification not implemented)

Time = 7.08 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.87

$$\int \frac{x^{10}(A+Bx^3)}{(a+bx^3)^3}\,dx$$
$$= \frac{x^5\left(\frac{7\,A\,a\,b^2}{9}-\frac{10\,B\,a^2\,b}{9}\right)-x^2\left(\frac{17\,B\,a^3}{18}-\frac{11\,A\,a^2\,b}{18}\right)}{a^2\,b^4+2\,a\,b^5\,x^3+b^6\,x^6}+x^2\left(\frac{A}{2\,b^3}-\frac{3\,B\,a}{2\,b^4}\right)$$
$$+\frac{B\,x^5}{5\,b^3}+\frac{4\,a^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(5\,A\,b-11\,B\,a)}{27\,b^{14/3}}$$
$$+\frac{4\,a^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,A\,b-11\,B\,a)}{27\,b^{14/3}}$$
$$-\frac{4\,a^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,A\,b-11\,B\,a)}{27\,b^{14/3}}$$

```
[In] int((x^10*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (x^5*((7*A*a*b^2)/9 - (10*B*a^2*b)/9) - x^2*((17*B*a^3)/18 - (11*A*a^2*b)/1
8))/(a^2*b^4 + b^6*x^6 + 2*a*b^5*x^3) + x^2*(A/(2*b^3) - (3*B*a)/(2*b^4)) +
 (B*x^5)/(5*b^3) + (4*a^(2/3)*log(b^(1/3)*x + a^(1/3))*(5*A*b - 11*B*a))/(2
7*b^(14/3)) + (4*a^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((
3^(1/2)*1i)/2 - 1/2)*(5*A*b - 11*B*a))/(27*b^(14/3)) - (4*a^(2/3)*log(3^(1/
2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*A*b - 11*B
*a))/(27*b^(14/3))
```

3.96 $\int \frac{x^9(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 244

$$\begin{aligned}\int \frac{x^9(A+Bx^3)}{(a+bx^3)^3}\,dx =& \frac{7(2Ab-5aB)x}{9b^4}-\frac{7(2Ab-5aB)x^4}{36ab^3}+\frac{(Ab-aB)x^{10}}{6ab\,(a+bx^3)^2}\\ &+\frac{(2Ab-5aB)x^7}{9ab^2\,(a+bx^3)}+\frac{7\sqrt[3]{a}(2Ab-5aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}b^{13/3}}\\ &-\frac{7\sqrt[3]{a}(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{13/3}}\\ &+\frac{7\sqrt[3]{a}(2Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54b^{13/3}}\end{aligned}$$

```
[Out] 7/9*(2*A*b-5*B*a)*x/b^4-7/36*(2*A*b-5*B*a)*x^4/a/b^3+1/6*(A*b-B*a)*x^10/a/b
/(b*x^3+a)^2+1/9*(2*A*b-5*B*a)*x^7/a/b^2/(b*x^3+a)-7/27*a^(1/3)*(2*A*b-5*B*
a)*ln(a^(1/3)+b^(1/3)*x)/b^(13/3)+7/54*a^(1/3)*(2*A*b-5*B*a)*ln(a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(13/3)+7/27*a^(1/3)*(2*A*b-5*B*a)*arctan(1/3*
(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(13/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {468, 294, 308, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{7\sqrt[3]{a}(2Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54b^{13/3}} + \frac{7\sqrt[3]{a}(2Ab-5aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}b^{13/3}} - \frac{7\sqrt[3]{a}(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{13/3}} + \frac{7x(2Ab-5aB)}{9b^4} - \frac{7x^4(2Ab-5aB)}{36ab^3} + \frac{x^7(2Ab-5aB)}{9ab^2(a+bx^3)} + \frac{x^{10}(Ab-aB)}{6ab(a+bx^3)^2}$$

```
[In] Int[(x^9*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (7*(2*A*b - 5*a*B)*x)/(9*b^4) - (7*(2*A*b - 5*a*B)*x^4)/(36*a*b^3) + ((A*b
- a*B)*x^10)/(6*a*b*(a + b*x^3)^2) + ((2*A*b - 5*a*B)*x^7)/(9*a*b^2*(a + b*
x^3)) + (7*a^(1/3)*(2*A*b - 5*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(9*Sqrt[3]*b^(13/3)) - (7*a^(1/3)*(2*A*b - 5*a*B)*Log[a^(1/3) +
b^(1/3)*x])/(27*b^(13/3)) + (7*a^(1/3)*(2*A*b - 5*a*B)*Log[a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{(Ab - aB)x^{10}}{6ab\,(a + bx^3)^2} + \frac{(-4Ab + 10aB)\int \frac{x^9}{(a+bx^3)^2}\,dx}{6ab}$$

$$=\frac{(Ab-aB)x^{10}}{6ab\left(a+bx^3\right)^2}+\frac{(2Ab-5aB)x^7}{9ab^2\left(a+bx^3\right)}-\frac{(7(2Ab-5aB))\int\frac{x^6}{a+bx^3}\,dx}{9ab^2}$$

$$=\frac{(Ab-aB)x^{10}}{6ab\left(a+bx^3\right)^2}+\frac{(2Ab-5aB)x^7}{9ab^2\left(a+bx^3\right)}-\frac{(7(2Ab-5aB))\int\left(-\frac{a}{b^2}+\frac{x^3}{b}+\frac{a^2}{b^2(a+bx^3)}\right)dx}{9ab^2}$$

$$=\frac{7(2Ab-5aB)x}{9b^4}-\frac{7(2Ab-5aB)x^4}{36ab^3}+\frac{(Ab-aB)x^{10}}{6ab\left(a+bx^3\right)^2}+\frac{(2Ab-5aB)x^7}{9ab^2\left(a+bx^3\right)}-\frac{(7a(2Ab-5aB))\int\frac{1}{a+bx^3}\,dx}{9b^4}$$

$$=\frac{7(2Ab-5aB)x}{9b^4}-\frac{7(2Ab-5aB)x^4}{36ab^3}+\frac{(Ab-aB)x^{10}}{6ab\left(a+bx^3\right)^2}+\frac{(2Ab-5aB)x^7}{9ab^2\left(a+bx^3\right)}-\frac{\left(7\sqrt[3]{a}(2Ab-5aB)\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27b^4}-\frac{\left(7\sqrt[3]{a}(2Ab-5aB)\right)\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27b^4}$$

$$=\frac{7(2Ab-5aB)x}{9b^4}-\frac{7(2Ab-5aB)x^4}{36ab^3}+\frac{(Ab-aB)x^{10}}{6ab\left(a+bx^3\right)^2}+\frac{(2Ab-5aB)x^7}{9ab^2\left(a+bx^3\right)}-\frac{7\sqrt[3]{a}(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{13/3}}+\frac{\left(7\sqrt[3]{a}(2Ab-5aB)\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54b^{13/3}}-\frac{\left(7a^{2/3}(2Ab-5aB)\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18b^4}$$

$$=\frac{7(2Ab-5aB)x}{9b^4}-\frac{7(2Ab-5aB)x^4}{36ab^3}+\frac{(Ab-aB)x^{10}}{6ab\left(a+bx^3\right)^2}+\frac{(2Ab-5aB)x^7}{9ab^2\left(a+bx^3\right)}-\frac{7\sqrt[3]{a}(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{13/3}}+\frac{7\sqrt[3]{a}(2Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54b^{13/3}}-\frac{\left(7\sqrt[3]{a}(2Ab-5aB)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9b^{13/3}}$$

$$= \frac{7(2Ab-5aB)x}{9b^4} - \frac{7(2Ab-5aB)x^4}{36ab^3} + \frac{(Ab-aB)x^{10}}{6ab\,(a+bx^3)^2} + \frac{(2Ab-5aB)x^7}{9ab^2\,(a+bx^3)} + \frac{7\sqrt[3]{a}(2Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}b^{13/3}} - \frac{7\sqrt[3]{a}(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{13/3}} + \frac{7\sqrt[3]{a}(2Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54b^{13/3}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.86

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$= \frac{108\sqrt[3]{b}(Ab-3aB)x + 27b^{4/3}Bx^4 + \frac{18a^2\sqrt[3]{b}(-Ab+aB)x}{(a+bx^3)^2} + \frac{6a\sqrt[3]{b}(13Ab-19aB)x}{a+bx^3} - 28\sqrt{3}\sqrt[3]{a}(-2Ab+5aB)\arctan}{108}$$

```
[In] Integrate[(x^9*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (108*b^(1/3)*(A*b - 3*a*B)*x + 27*b^(4/3)*B*x^4 + (18*a^2*b^(1/3)*(-(A*b) +
 a*B)*x)/(a + b*x^3)^2 + (6*a*b^(1/3)*(13*A*b - 19*a*B)*x)/(a + b*x^3) - 28
*Sqrt[3]*a^(1/3)*(-2*A*b + 5*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]] + 28*a^(1/3)*(-2*A*b + 5*a*B)*Log[a^(1/3) + b^(1/3)*x] - 14*a^(1/3)*(-2*
A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(108*b^(13/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.19 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.45

method	result
risch	$\frac{B\,x^{4}}{4b^{3}}+\frac{Ax}{b^{3}}-\frac{3Bax}{b^{4}}+\frac{\left(\frac{13}{18}a\,b^{2}A-\frac{19}{18}a^{2}bB\right)x^{4}+\frac{a^{2}(5Ab-8Ba)x}{9}}{b^{4}\left(b\,x^{3}+a\right)^{2}}-\frac{7a\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{(2Ab-5Ba)\ln\left(x-\_R\right)}{\_R^{2}}\right)}{27b^{5}}$
default	$\frac{\frac{1}{4}bB\,x^{4}+Abx-3Bax}{b^{4}}-\frac{a\left(\frac{\left(-\frac{13}{18}b^{2}A+\frac{19}{18}abB\right)x^{4}-\frac{a(5Ab-8Ba)x}{9}}{\left(b\,x^{3}+a\right)^{2}}+\frac{7(2Ab-5Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\right.}{}\right.}{9}\right.}{b^{4}}$

[In] int(x^9*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)

[Out] 1/4*B*x^4/b^3+1/b^3*A*x-3/b^4*B*a*x+((13/18*a*b^2*A-19/18*a^2*b*B)*x^4+1/9*a^2*(5*A*b-8*B*a)*x)/b^4/(b*x^3+a)^2-7/27/b^5*a*sum((2*A*b-5*B*a)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.42

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$=\frac{27\,Bb^3x^{10}-54\,(5\,Bab^2-2\,Ab^3)x^7-147\,(5\,Ba^2b-2\,Aab^2)x^4-28\,\sqrt{3}((5\,Bab^2-2\,Ab^3)x^6+5\,Ba^3-2\,A}{}$$

```
[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/108*(27*B*b^3*x^10 - 54*(5*B*a*b^2 - 2*A*b^3)*x^7 - 147*(5*B*a^2*b - 2*A*
a*b^2)*x^4 - 28*sqrt(3)*((5*B*a*b^2 - 2*A*b^3)*x^6 + 5*B*a^3 - 2*A*a^2*b +
2*(5*B*a^2*b - 2*A*a*b^2)*x^3)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b
)^(2/3) - sqrt(3)*a)/a) + 14*((5*B*a*b^2 - 2*A*b^3)*x^6 + 5*B*a^3 - 2*A*a^2
*b + 2*(5*B*a^2*b - 2*A*a*b^2)*x^3)*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) +
 (-a/b)^(2/3)) - 28*((5*B*a*b^2 - 2*A*b^3)*x^6 + 5*B*a^3 - 2*A*a^2*b + 2*(5
*B*a^2*b - 2*A*a*b^2)*x^3)*(-a/b)^(1/3)*log(x - (-a/b)^(1/3)) - 84*(5*B*a^3
 - 2*A*a^2*b)*x)/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)
```

Sympy [A] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.67

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{Bx^4}{4b^3} + x\left(\frac{A}{b^3} - \frac{3Ba}{b^4}\right) + \frac{x^4\cdot(13Aab^2-19Ba^2b) + x(10Aa^2b-16Ba^3)}{18a^2b^4+36ab^5x^3+18b^6x^6}$$
$$+\mathrm{RootSum}\left(19683t^3b^{13} + 2744A^3ab^3 - 20580A^2Ba^2b^2 + 51450AB^2a^3b - 42875B^3a^4, \left(t \mapsto t\log\left(\frac{2}{-14A}\right.\right.\right.$$

```
[In] integrate(x**9*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*x**4/(4*b**3) + x*(A/b**3 - 3*B*a/b**4) + (x**4*(13*A*a*b**2 - 19*B*a**2*
b) + x*(10*A*a**2*b - 16*B*a**3))/(18*a**2*b**4 + 36*a*b**5*x**3 + 18*b**6*
x**6) + RootSum(19683*_t**3*b**13 + 2744*A**3*a*b**3 - 20580*A**2*B*a**2*b*
*2 + 51450*A*B**2*a**3*b - 42875*B**3*a**4, Lambda(_t, _t*log(27*_t*b**4/(-
14*A*b + 35*B*a) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.91

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{(19\,Ba^2b-13\,Aab^2)x^4+2\,(8\,Ba^3-5\,Aa^2b)x}{18\,(b^6x^6+2\,ab^5x^3+a^2b^4)} + \frac{Bbx^4-4\,(3\,Ba-Ab)x}{4\,b^4} + \frac{7\,\sqrt{3}(5\,Ba^2-2\,Aab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^5\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{7\,(5\,Ba^2-2\,Aab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^5\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{7\,(5\,Ba^2-2\,Aab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^5\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*((19*B*a^2*b - 13*A*a*b^2)*x^4 + 2*(8*B*a^3 - 5*A*a^2*b)*x)/(b^6*x^6
+ 2*a*b^5*x^3 + a^2*b^4) + 1/4*(B*b*x^4 - 4*(3*B*a - A*b)*x)/b^4 + 7/27*sqr
t(3)*(5*B*a^2 - 2*A*a*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3)
)/(b^5*(a/b)^(2/3)) - 7/54*(5*B*a^2 - 2*A*a*b)*log(x^2 - x*(a/b)^(1/3) + (a
/b)^(2/3))/(b^5*(a/b)^(2/3)) + 7/27*(5*B*a^2 - 2*A*a*b)*log(x + (a/b)^(1/3)
)/(b^5*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.96

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{7\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}Ba - 2\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^5} - \frac{7\left(5\,Ba^2 - 2\,Aab\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^4} + \frac{7\left(5\left(-ab^2\right)^{\frac{1}{3}}Ba - 2\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^5} - \frac{19\,Ba^2bx^4 - 13\,Aab^2x^4 + 16\,Ba^3x - 10\,Aa^2bx}{18\left(bx^3+a\right)^2b^4} + \frac{Bb^9x^4 - 12\,Bab^8x + 4\,Ab^9x}{4\,b^{12}}$$

```
[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 7/27*sqrt(3)*(5*(-a*b^2)^(1/3)*B*a - 2*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(
3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^5 - 7/27*(5*B*a^2 - 2*A*a*b)*(-a/b)
^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4) + 7/54*(5*(-a*b^2)^(1/3)*B*a - 2*
(-a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^5 - 1/18*(19
*B*a^2*b*x^4 - 13*A*a*b^2*x^4 + 16*B*a^3*x - 10*A*a^2*b*x)/((b*x^3 + a)^2*b
^4) + 1/4*(B*b^9*x^4 - 12*B*a*b^8*x + 4*A*b^9*x)/b^12
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.93

$$\int \frac{x^9(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{x^4\left(\frac{13\,A\,a\,b^2}{18} - \frac{19\,B\,a^2\,b}{18}\right) - x\left(\frac{8\,B\,a^3}{9} - \frac{5\,A\,a^2\,b}{9}\right)}{a^2\,b^4 + 2\,a\,b^5\,x^3 + b^6\,x^6} + x\left(\frac{A}{b^3} - \frac{3\,B\,a}{b^4}\right) + \frac{B\,x^4}{4\,b^3} + \frac{7\,{(-a)}^{1/3}\,\ln\left({(-a)}^{4/3} + a\,b^{1/3}\,x\right)\,\left(2\,A\,b - 5\,B\,a\right)}{27\,b^{13/3}} - \frac{7\,{(-a)}^{1/3}\,\ln\left({(-a)}^{4/3} - 2\,a\,b^{1/3}\,x + \sqrt{3}\,{(-a)}^{4/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(2\,A\,b - 5\,B\,a\right)}{27\,b^{13/3}} + \frac{7\,{(-a)}^{1/3}\,\ln\left(2\,a\,b^{1/3}\,x - {(-a)}^{4/3} + \sqrt{3}\,{(-a)}^{4/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(2\,A\,b - 5\,B\,a\right)}{27\,b^{13/3}}$$

```
[In] int((x^9*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (x^4*((13*A*a*b^2)/18 - (19*B*a^2*b)/18) - x*((8*B*a^3)/9 - (5*A*a^2*b)/9))
/(a^2*b^4 + b^6*x^6 + 2*a*b^5*x^3) + x*(A/b^3 - (3*B*a)/b^4) + (B*x^4)/(4*b
^3) + (7*(-a)^(1/3)*log((-a)^(4/3) + a*b^(1/3)*x)*(2*A*b - 5*B*a))/(27*b^(1
3/3)) - (7*(-a)^(1/3)*log((-a)^(4/3) + 3^(1/2)*(-a)^(4/3)*1i - 2*a*b^(1/3)*
x)*((3^(1/2)*1i)/2 + 1/2)*(2*A*b - 5*B*a))/(27*b^(13/3)) + (7*(-a)^(1/3)*lo
g(3^(1/2)*(-a)^(4/3)*1i - (-a)^(4/3) + 2*a*b^(1/3)*x)*((3^(1/2)*1i)/2 - 1/2
)*(2*A*b - 5*B*a))/(27*b^(13/3))
```

3.97 $\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 222

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3} dx = -\frac{5(Ab-4aB)x^2}{18ab^3} + \frac{(Ab-aB)x^8}{6ab(a+bx^3)^2} + \frac{(Ab-4aB)x^5}{9ab^2(a+bx^3)} - \frac{5(Ab-4aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\sqrt[3]{a}b^{11/3}} - \frac{5(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27\sqrt[3]{a}b^{11/3}} + \frac{5(Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54\sqrt[3]{a}b^{11/3}}$$

```
[Out] -5/18*(A*b-4*B*a)*x^2/a/b^3+1/6*(A*b-B*a)*x^8/a/b/(b*x^3+a)^2+1/9*(A*b-4*B*
a)*x^5/a/b^2/(b*x^3+a)-5/27*(A*b-4*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(11
/3)+5/54*(A*b-4*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/b^(1
1/3)-5/27*(A*b-4*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(
1/3)/b^(11/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used

= {468, 294, 327, 298, 31, 648, 631, 210, 642}

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{5(Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54\sqrt[3]{a}b^{11/3}} - \frac{5(Ab-4aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\sqrt[3]{a}b^{11/3}} - \frac{5(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27\sqrt[3]{a}b^{11/3}} - \frac{5x^2(Ab-4aB)}{18ab^3} + \frac{x^5(Ab-4aB)}{9ab^2(a+bx^3)} + \frac{x^8(Ab-aB)}{6ab(a+bx^3)^2}$$

```
[In] Int[(x^7*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (-5*(A*b - 4*a*B)*x^2)/(18*a*b^3) + ((A*b - a*B)*x^8)/(6*a*b*(a + b*x^3)^2)
 + ((A*b - 4*a*B)*x^5)/(9*a*b^2*(a + b*x^3)) - (5*(A*b - 4*a*B)*ArcTan[(a^(
1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(1/3)*b^(11/3)) - (5*(
A*b - 4*a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(1/3)*b^(11/3)) + (5*(A*b - 4*
a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(1/3)*b^(11/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(Ab-aB)x^8}{6ab\left(a+bx^3\right)^2}+\frac{(-2Ab+8aB)\int \frac{x^7}{\left(a+bx^3\right)^2}\,dx}{6ab} \\
&= \frac{(Ab-aB)x^8}{6ab\left(a+bx^3\right)^2}+\frac{(Ab-4aB)x^5}{9ab^2\left(a+bx^3\right)}-\frac{(5(Ab-4aB))\int \frac{x^4}{a+bx^3}\,dx}{9ab^2} \\
&= -\frac{5(Ab-4aB)x^2}{18ab^3}+\frac{(Ab-aB)x^8}{6ab\left(a+bx^3\right)^2}+\frac{(Ab-4aB)x^5}{9ab^2\left(a+bx^3\right)}+\frac{(5(Ab-4aB))\int \frac{x}{a+bx^3}\,dx}{9b^3}
\end{aligned}$$

$$= -\frac{5(Ab-4aB)x^2}{18ab^3} + \frac{(Ab-aB)x^8}{6ab\,(a+bx^3)^2} + \frac{(Ab-4aB)x^5}{9ab^2\,(a+bx^3)} - \frac{(5(Ab-4aB))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27\sqrt[3]{a}b^{10/3}} + \frac{(5(Ab-4aB))\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27\sqrt[3]{a}b^{10/3}}$$

$$= -\frac{5(Ab-4aB)x^2}{18ab^3} + \frac{(Ab-aB)x^8}{6ab\,(a+bx^3)^2} + \frac{(Ab-4aB)x^5}{9ab^2\,(a+bx^3)} - \frac{5(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27\sqrt[3]{a}b^{11/3}} + \frac{(5(Ab-4aB))\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54\sqrt[3]{a}b^{11/3}} + \frac{(5(Ab-4aB))\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18b^{10/3}}$$

$$= -\frac{5(Ab-4aB)x^2}{18ab^3} + \frac{(Ab-aB)x^8}{6ab\,(a+bx^3)^2} + \frac{(Ab-4aB)x^5}{9ab^2\,(a+bx^3)} - \frac{5(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27\sqrt[3]{a}b^{11/3}} + \frac{5(Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54\sqrt[3]{a}b^{11/3}} + \frac{(5(Ab-4aB))\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9\sqrt[3]{a}b^{11/3}}$$

$$= -\frac{5(Ab-4aB)x^2}{18ab^3} + \frac{(Ab-aB)x^8}{6ab\,(a+bx^3)^2} + \frac{(Ab-4aB)x^5}{9ab^2\,(a+bx^3)} - \frac{5(Ab-4aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\sqrt[3]{a}b^{11/3}} - \frac{5(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27\sqrt[3]{a}b^{11/3}} + \frac{5(Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54\sqrt[3]{a}b^{11/3}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.87

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$= \frac{27b^{2/3}Bx^2 + \frac{9ab^{2/3}(Ab-aB)x^2}{(a+bx^3)^2} - \frac{6b^{2/3}(4Ab-7aB)x^2}{a+bx^3} + \frac{10\sqrt{3}(-Ab+4aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} + \frac{10(-Ab+4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}}{54b^{11/3}}$$

```
[In] Integrate[(x^7*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (27*b^(2/3)*B*x^2 + (9*a*b^(2/3)*(A*b - a*B)*x^2)/(a + b*x^3)^2 - (6*b^(2/3
)*(4*A*b - 7*a*B)*x^2)/(a + b*x^3) + (10*Sqrt[3]*(-(A*b) + 4*a*B)*ArcTan[(1
```

```
 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (10*(-(A*b) + 4*a*B)*Log[a^(1
/3) + b^(1/3)*x])/a^(1/3) + (5*(A*b - 4*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/a^(1/3))/(54*b^(11/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.41

method	result
risch	$\frac{B\,x^{2}}{2b^{3}}+\frac{\left(-\frac{4}{9}b^{2}A+\frac{7}{9}abB\right)x^{5}-\frac{a(5Ab-11Ba)x^{2}}{18}}{b^{3}\left(b\,x^{3}+a\right)^{2}}+\frac{5\left(\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{3}+a\right)}\frac{(Ab-4Ba)\ln\left(x-\_R\right)}{\_R}\right)}{27b^{4}}$
default	$\frac{B\,x^{2}}{2b^{3}}+\frac{\frac{\left(-\frac{4}{9}b^{2}A+\frac{7}{9}abB\right)x^{5}-\frac{a(5Ab-11Ba)x^{2}}{18}}{\left(b\,x^{3}+a\right)^{2}}+\left(\frac{5Ab}{9}-\frac{20Ba}{9}\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{b^{3}}$

```
[In] int(x^7*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*B*x^2/b^3+((-4/9*b^2*A+7/9*a*b*B)*x^5-1/18*a*(5*A*b-11*B*a)*x^2)/b^3/(b
*x^3+a)^2+5/27/b^4*sum((A*b-4*B*a)/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(184) = 368.

Time = 0.29 (sec) , antiderivative size = 792, normalized size of antiderivative = 3.57

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$=\left[\frac{27\,Bab^4x^8+24\,(4\,Ba^2b^3-Aab^4)x^5+15\,(4\,Ba^3b^2-Aa^2b^3)x^2-15\,\sqrt{\frac{1}{3}}((4\,Ba^2b^3-Aab^4)x^6+4\,Ba^4b-\right.$$

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(27*B*a*b^4*x^8 + 24*(4*B*a^2*b^3 - A*a*b^4)*x^5 + 15*(4*B*a^3*b^2 -
A*a^2*b^3)*x^2 - 15*sqrt(1/3)*((4*B*a^2*b^3 - A*a*b^4)*x^6 + 4*B*a^4*b - A*
a^3*b^2 + 2*(4*B*a^3*b^2 - A*a^2*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^
2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)
*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - 5*((4*B*a*b^2
- A*b^3)*x^6 + 4*B*a^3 - A*a^2*b + 2*(4*B*a^2*b - A*a*b^2)*x^3)*(-a*b^2)^(2
/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 10*((4*B*a*b^2 - A
*b^3)*x^6 + 4*B*a^3 - A*a^2*b + 2*(4*B*a^2*b - A*a*b^2)*x^3)*(-a*b^2)^(2/3)
*log(b*x - (-a*b^2)^(1/3)))/(a*b^7*x^6 + 2*a^2*b^6*x^3 + a^3*b^5), 1/54*(27
*B*a*b^4*x^8 + 24*(4*B*a^2*b^3 - A*a*b^4)*x^5 + 15*(4*B*a^3*b^2 - A*a^2*b^3
)*x^2 - 30*sqrt(1/3)*((4*B*a^2*b^3 - A*a*b^4)*x^6 + 4*B*a^4*b - A*a^3*b^2 +
 2*(4*B*a^3*b^2 - A*a^2*b^3)*x^3)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*
(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - 5*((4*B*a*b^2 - A*b^3
)*x^6 + 4*B*a^3 - A*a^2*b + 2*(4*B*a^2*b - A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log
(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 10*((4*B*a*b^2 - A*b^3)*x
^6 + 4*B*a^3 - A*a^2*b + 2*(4*B*a^2*b - A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b*
x - (-a*b^2)^(1/3)))/(a*b^7*x^6 + 2*a^2*b^6*x^3 + a^3*b^5)]
```

Sympy [A] (verification not implemented)

Time = 7.93 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.73

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{Bx^2}{2b^3} + \frac{x^5(-8Ab^2+14Bab)+x^2(-5Aab+11Ba^2)}{18a^2b^3+36ab^4x^3+18b^5x^6}$$
$$+\mathrm{RootSum}\left(19683t^3ab^{11}+125A^3b^3-1500A^2Bab^2+6000AB^2a^2b-8000B^3a^3,\left(t\mapsto t\log\left(\frac{}{25A^2b^2-2}\right.\right.\right.$$

```
[In] integrate(x**7*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*x**2/(2*b**3) + (x**5*(-8*A*b**2 + 14*B*a*b) + x**2*(-5*A*a*b + 11*B*a**2
))/(18*a**2*b**3 + 36*a*b**4*x**3 + 18*b**5*x**6) + RootSum(19683*_t**3*a*b
**11 + 125*A**3*b**3 - 1500*A**2*B*a*b**2 + 6000*A*B**2*a**2*b - 8000*B**3*
a**3, Lambda(_t, _t*log(729*_t**2*a*b**7/(25*A**2*b**2 - 200*A*B*a*b + 400*
B**2*a**2) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.88

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{2\,(7\,Bab-4\,Ab^2)x^5+(11\,Ba^2-5\,Aab)x^2}{18\,(b^5x^6+2\,ab^4x^3+a^2b^3)}+\frac{Bx^2}{2\,b^3} - \frac{5\,\sqrt{3}(4\,Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^4\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{5\,(4\,Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^4\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{5\,(4\,Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^4\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(2*(7*B*a*b - 4*A*b^2)*x^5 + (11*B*a^2 - 5*A*a*b)*x^2)/(b^5*x^6 + 2*a*
b^4*x^3 + a^2*b^3) + 1/2*B*x^2/b^3 - 5/27*sqrt(3)*(4*B*a - A*b)*arctan(1/3*
sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^4*(a/b)^(1/3)) - 5/54*(4*B*a -
A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(1/3)) + 5/27*(4*B*a
 - A*b)*log(x + (a/b)^(1/3))/(b^4*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.95

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{Bx^2}{2\,b^3} - \frac{5\,\sqrt{3}(4\,Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{1}{3}}\,b^3} + \frac{5\,(4\,Ba-Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^2\right)^{\frac{1}{3}}\,b^3} + \frac{5\left(4\,Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}-Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^3} + \frac{14\,Babx^5-8\,Ab^2x^5+11\,Ba^2x^2-5\,Aabx^2}{18\,(bx^3+a)^2b^3}$$

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/b^3 - 5/27*sqrt(3)*(4*B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)
^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*b^3) + 5/54*(4*B*a - A*b)*log(x^2 + x
*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*b^3) + 5/27*(4*B*a*(-a/b)^(1/
3) - A*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^3) + 1/
18*(14*B*a*b*x^5 - 8*A*b^2*x^5 + 11*B*a^2*x^2 - 5*A*a*b*x^2)/((b*x^3 + a)^2
*b^3)
```

Mupad [B] (verification not implemented)

Time = 7.00 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.84

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{x^2\left(\frac{11\,B\,a^2}{18}-\frac{5\,A\,a\,b}{18}\right)-x^5\left(\frac{4\,A\,b^2}{9}-\frac{7\,B\,a\,b}{9}\right)}{a^2\,b^3+2\,a\,b^4\,x^3+b^5\,x^6} + \frac{B\,x^2}{2\,b^3}-\frac{5\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-4\,B\,a\right)}{27\,a^{1/3}\,b^{11/3}} - \frac{5\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-4\,B\,a\right)}{27\,a^{1/3}\,b^{11/3}} + \frac{5\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-4\,B\,a\right)}{27\,a^{1/3}\,b^{11/3}}$$

```
[In] int((x^7*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (x^2*((11*B*a^2)/18 - (5*A*a*b)/18) - x^5*((4*A*b^2)/9 - (7*B*a*b)/9))/(a^2
*b^3 + b^5*x^6 + 2*a*b^4*x^3) + (B*x^2)/(2*b^3) - (5*log(b^(1/3)*x + a^(1/3
))*(A*b - 4*B*a))/(27*a^(1/3)*b^(11/3)) - (5*log(3^(1/2)*a^(1/3)*1i - 2*b^(
1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b - 4*B*a))/(27*a^(1/3)*b^(11/3
)) + (5*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1
/2)*(A*b - 4*B*a))/(27*a^(1/3)*b^(11/3))
```

3.98 $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 220

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^3} dx = -\frac{2(Ab-7aB)x}{9ab^3} + \frac{(Ab-aB)x^7}{6ab(a+bx^3)^2} + \frac{(Ab-7aB)x^4}{18ab^2(a+bx^3)} - \frac{2(Ab-7aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{2/3}b^{10/3}} + \frac{2(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{2/3}b^{10/3}} - \frac{(Ab-7aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{2/3}b^{10/3}}$$

```
[Out] -2/9*(A*b-7*B*a)*x/a/b^3+1/6*(A*b-B*a)*x^7/a/b/(b*x^3+a)^2+1/18*(A*b-7*B*a)
*x^4/a/b^2/(b*x^3+a)+2/27*(A*b-7*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(10/3
)-1/27*(A*b-7*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(10/
3)-2/27*(A*b-7*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/
3)/b^(10/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used

= {468, 294, 327, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{2(Ab-7aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{2/3}b^{10/3}} - \frac{(Ab-7aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{2/3}b^{10/3}} + \frac{2(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{2/3}b^{10/3}} - \frac{2x(Ab-7aB)}{9ab^3} + \frac{x^4(Ab-7aB)}{18ab^2(a+bx^3)} + \frac{x^7(Ab-aB)}{6ab(a+bx^3)^2}$$

```
[In] Int[(x^6*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (-2*(A*b - 7*a*B)*x)/(9*a*b^3) + ((A*b - a*B)*x^7)/(6*a*b*(a + b*x^3)^2) +
((A*b - 7*a*B)*x^4)/(18*a*b^2*(a + b*x^3)) - (2*(A*b - 7*a*B)*ArcTan[(a^(1/
3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(2/3)*b^(10/3)) + (2*(A*
b - 7*a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(2/3)*b^(10/3)) - ((A*b - 7*a*B)
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*a^(2/3)*b^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{(Ab-aB)x^7}{6ab\,(a+bx^3)^2} + \frac{(-Ab+7aB)\int \frac{x^6}{(a+bx^3)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^7}{6ab\,(a+bx^3)^2} + \frac{(Ab-7aB)x^4}{18ab^2\,(a+bx^3)} - \frac{(2(Ab-7aB))\int \frac{x^3}{a+bx^3}\,dx}{9ab^2}$$

$$= -\frac{2(Ab-7aB)x}{9ab^3} + \frac{(Ab-aB)x^7}{6ab\left(a+bx^3\right)^2} + \frac{(Ab-7aB)x^4}{18ab^2\left(a+bx^3\right)} + \frac{(2(Ab-7aB))\int \frac{1}{a+bx^3}\,dx}{9b^3}$$

$$= -\frac{2(Ab-7aB)x}{9ab^3} + \frac{(Ab-aB)x^7}{6ab\left(a+bx^3\right)^2} + \frac{(Ab-7aB)x^4}{18ab^2\left(a+bx^3\right)} + \frac{(2(Ab-7aB))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{2/3}b^3} + \frac{(2(Ab-7aB))\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{2/3}b^3}$$

$$= -\frac{2(Ab-7aB)x}{9ab^3} + \frac{(Ab-aB)x^7}{6ab\left(a+bx^3\right)^2} + \frac{(Ab-7aB)x^4}{18ab^2\left(a+bx^3\right)} + \frac{2(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{2/3}b^{10/3}} - \frac{(Ab-7aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{2/3}b^{10/3}} + \frac{(Ab-7aB)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9\sqrt[3]{a}b^3}$$

$$= -\frac{2(Ab-7aB)x}{9ab^3} + \frac{(Ab-aB)x^7}{6ab\left(a+bx^3\right)^2} + \frac{(Ab-7aB)x^4}{18ab^2\left(a+bx^3\right)} + \frac{2(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{2/3}b^{10/3}} - \frac{(Ab-7aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{2/3}b^{10/3}} + \frac{(2(Ab-7aB))\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{2/3}b^{10/3}}$$

$$= -\frac{2(Ab-7aB)x}{9ab^3} + \frac{(Ab-aB)x^7}{6ab\left(a+bx^3\right)^2} + \frac{(Ab-7aB)x^4}{18ab^2\left(a+bx^3\right)} - \frac{2(Ab-7aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{2/3}b^{10/3}} + \frac{2(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{2/3}b^{10/3}} - \frac{(Ab-7aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{2/3}b^{10/3}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.85

$$\int \frac{x^6(A+Bx^3)}{\left(a+bx^3\right)^3}\,dx$$

$$= \frac{54\sqrt[3]{b}Bx + \frac{9a\sqrt[3]{b}(Ab-aB)x}{\left(a+bx^3\right)^2} - \frac{3\sqrt[3]{b}(7Ab-13aB)x}{a+bx^3} + \frac{4\sqrt{3}(-Ab+7aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} + \frac{4(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}} + \frac{2(}{}}{54b^{10/3}}$$

```
[In] Integrate[(x^6*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (54*b^(1/3)*B*x + (9*a*b^(1/3)*(A*b - a*B)*x)/(a + b*x^3)^2 - (3*b^(1/3)*(7
*A*b - 13*a*B)*x)/(a + b*x^3) + (4*Sqrt[3]*(-(A*b) + 7*a*B)*ArcTan[(1 - (2*
b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (4*(A*b - 7*a*B)*Log[a^(1/3) + b^(1
/3)*x])/a^(2/3) + (2*(-(A*b) + 7*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/a^(2/3))/(54*b^(10/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.44 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.39

method	result
risch	$\frac{Bx}{b^3}+\frac{\left(-\frac{7}{18}b^2A+\frac{13}{18}abB\right)x^4-\frac{a(2Ab-5Ba)x}{9}}{b^3(b\,x^3+a)^2}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(Ab-7Ba)\ln\left(x-\_R\right)}{\_R^2}\right)}{27b^4}$
default	$\frac{Bx}{b^3}+\frac{\frac{\left(-\frac{7}{18}b^2A+\frac{13}{18}abB\right)x^4-\frac{a(2Ab-5Ba)x}{9}}{\left(b\,x^3+a\right)^2}+\frac{2(Ab-7Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{9}}{b^3}$

```
[In] int(x^6*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] B*x/b^3+((-7/18*b^2*A+13/18*a*b*B)*x^4-1/9*a*(2*A*b-5*B*a)*x)/b^3/(b*x^3+a)
^2+2/27/b^4*sum((A*b-7*B*a)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(182) = 364$.

Time = 0.30 (sec) , antiderivative size = 789, normalized size of antiderivative = 3.59

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$= \left[\frac{54\,Ba^2b^3x^7 + 21\,(7\,Ba^3b^2 - Aa^2b^3)x^4 - 6\,\sqrt{\frac{1}{3}}((7\,Ba^2b^3 - Aab^4)x^6 + 7\,Ba^4b - Aa^3b^2 + 2\,(7\,Ba^3b^2 - Aa}{}\right.$$

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(54*B*a^2*b^3*x^7 + 21*(7*B*a^3*b^2 - A*a^2*b^3)*x^4 - 6*sqrt(1/3)*((
7*B*a^2*b^3 - A*a*b^4)*x^6 + 7*B*a^4*b - A*a^3*b^2 + 2*(7*B*a^3*b^2 - A*a^2
*b^3)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^
2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*
b)^(1/3)/b))/(b*x^3 + a)) + 2*((7*B*a*b^2 - A*b^3)*x^6 + 7*B*a^3 - A*a^2*b
+ 2*(7*B*a^2*b - A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x
+ (a^2*b)^(1/3)*a) - 4*((7*B*a*b^2 - A*b^3)*x^6 + 7*B*a^3 - A*a^2*b + 2*(7*
B*a^2*b - A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(7*B*
a^4*b - A*a^3*b^2)*x)/(a^2*b^6*x^6 + 2*a^3*b^5*x^3 + a^4*b^4), 1/54*(54*B*a
^2*b^3*x^7 + 21*(7*B*a^3*b^2 - A*a^2*b^3)*x^4 - 12*sqrt(1/3)*((7*B*a^2*b^3
- A*a*b^4)*x^6 + 7*B*a^4*b - A*a^3*b^2 + 2*(7*B*a^3*b^2 - A*a^2*b^3)*x^3)*s
qrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)
*sqrt((a^2*b)^(1/3)/b)/a^2) + 2*((7*B*a*b^2 - A*b^3)*x^6 + 7*B*a^3 - A*a^2*
b + 2*(7*B*a^2*b - A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*
x + (a^2*b)^(1/3)*a) - 4*((7*B*a*b^2 - A*b^3)*x^6 + 7*B*a^3 - A*a^2*b + 2*(
7*B*a^2*b - A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(7*
B*a^4*b - A*a^3*b^2)*x)/(a^2*b^6*x^6 + 2*a^3*b^5*x^3 + a^4*b^4)]
```

Sympy [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.64

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{Bx}{b^3} + \frac{x^4(-7Ab^2+13Bab)+x(-4Aab+10Ba^2)}{18a^2b^3+36ab^4x^3+18b^5x^6} + \operatorname{RootSum}\left(19683t^3a^2b^{10}-8A^3b^3+168A^2Bab^2-1176AB^2a^2b+2744B^3a^3, \left(t\mapsto t\log\left(-\frac{27tab^3}{-2Ab+14B\right.\right.\right.$$

```
[In] integrate(x**6*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*x/b**3 + (x**4*(-7*A*b**2 + 13*B*a*b) + x*(-4*A*a*b + 10*B*a**2))/(18*a**
2*b**3 + 36*a*b**4*x**3 + 18*b**5*x**6) + RootSum(19683*_t**3*a**2*b**10 -
8*A**3*b**3 + 168*A**2*B*a*b**2 - 1176*A*B**2*a**2*b + 2744*B**3*a**3, Lamb
da(_t, _t*log(-27*_t*a*b**3/(-2*A*b + 14*B*a) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.87

$$\begin{aligned}\int \frac{x^6(A+Bx^3)}{(a+bx^3)^3}\,dx = {} & \frac{(13\,Bab-7\,Ab^2)x^4+2\,(5\,Ba^2-2\,Aab)x}{18\,(b^5x^6+2\,ab^4x^3+a^2b^3)} + \frac{Bx}{b^3} \\ & - \frac{2\sqrt{3}(7\,Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\ & + \frac{(7\,Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} \\ & - \frac{2\,(7\,Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}\end{aligned}$$

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*((13*B*a*b - 7*A*b^2)*x^4 + 2*(5*B*a^2 - 2*A*a*b)*x)/(b^5*x^6 + 2*a*b^
4*x^3 + a^2*b^3) + B*x/b^3 - 2/27*sqrt(3)*(7*B*a - A*b)*arctan(1/3*sqrt(3)*
(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^4*(a/b)^(2/3)) + 1/27*(7*B*a - A*b)*log
(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(2/3)) - 2/27*(7*B*a - A*b)*
log(x + (a/b)^(1/3))/(b^4*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.85

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{2\sqrt{3}(7Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}b^2} + \frac{(7Ba-Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}b^2} + \frac{Bx}{b^3} + \frac{2(7Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27ab^3} + \frac{13Babx^4-7Ab^2x^4+10Ba^2x-4Aabx}{18(bx^3+a)^2b^3}$$

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 2/27*sqrt(3)*(7*B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/((-a*b^2)^(2/3)*b^2) + 1/27*(7*B*a - A*b)*log(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/((-a*b^2)^(2/3)*b^2) + B*x/b^3 + 2/27*(7*B*a - A*b)*(-a/b)^(1
/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^3) + 1/18*(13*B*a*b*x^4 - 7*A*b^2*x^4 +
 10*B*a^2*x - 4*A*a*b*x)/((b*x^3 + a)^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.83

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{B\,x}{b^3} - \frac{x^4\left(\frac{7\,A\,b^2}{18}-\frac{13\,B\,a\,b}{18}\right)-x\left(\frac{5\,B\,a^2}{9}-\frac{2\,A\,a\,b}{9}\right)}{a^2\,b^3+2\,a\,b^4\,x^3+b^5\,x^6} + \frac{2\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(A\,b-7\,B\,a)}{27\,a^{2/3}\,b^{10/3}} - \frac{2\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(A\,b-7\,B\,a)}{27\,a^{2/3}\,b^{10/3}} + \frac{2\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(A\,b-7\,B\,a)}{27\,a^{2/3}\,b^{10/3}}$$

```
[In] int((x^6*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (B*x)/b^3 - (x^4*((7*A*b^2)/18 - (13*B*a*b)/18) - x*((5*B*a^2)/9 - (2*A*a*b
)/9))/(a^2*b^3 + b^5*x^6 + 2*a*b^4*x^3) + (2*log(b^(1/3)*x + a^(1/3))*(A*b
```

```
- 7*B*a))/(27*a^(2/3)*b^(10/3)) - (2*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x +
 a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - 7*B*a))/(27*a^(2/3)*b^(10/3)) + (2*
log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b
 - 7*B*a))/(27*a^(2/3)*b^(10/3))
```

3.99 $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 201

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3} dx = \frac{(Ab-aB)x^5}{6ab(a+bx^3)^2} - \frac{(Ab+5aB)x^2}{18ab^2(a+bx^3)} - \frac{(Ab+5aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{8/3}} - \frac{(Ab+5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{8/3}} + \frac{(Ab+5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{4/3}b^{8/3}}$$

```
[Out] 1/6*(A*b-B*a)*x^5/a/b/(b*x^3+a)^2-1/18*(A*b+5*B*a)*x^2/a/b^2/(b*x^3+a)-1/27
*(A*b+5*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)/b^(8/3)+1/54*(A*b+5*B*a)*ln(a^(2
/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(8/3)-1/27*(A*b+5*B*a)*arctan(
1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/b^(8/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used

$= \{468, 294, 298, 31, 648, 631, 210, 642\}$

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{(5aB+Ab)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{8/3}} + \frac{(5aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{4/3}b^{8/3}} - \frac{(5aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{8/3}} - \frac{x^2(5aB+Ab)}{18ab^2(a+bx^3)} + \frac{x^5(Ab-aB)}{6ab(a+bx^3)^2}$$

```
[In] Int[(x^4*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^5)/(6*a*b*(a + b*x^3)^2) - ((A*b + 5*a*B)*x^2)/(18*a*b^2*(a
+ b*x^3)) - ((A*b + 5*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(9*Sqrt[3]*a^(4/3)*b^(8/3)) - ((A*b + 5*a*B)*Log[a^(1/3) + b^(1/3)*x])/(
27*a^(4/3)*b^(8/3)) + ((A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/(54*a^(4/3)*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(Ab-aB)x^5}{6ab\,(a+bx^3)^2} + \frac{(Ab+5aB)\int \frac{x^4}{(a+bx^3)^2}\,dx}{6ab} \\
&= \frac{(Ab-aB)x^5}{6ab\,(a+bx^3)^2} - \frac{(Ab+5aB)x^2}{18ab^2\,(a+bx^3)} + \frac{(Ab+5aB)\int \frac{x}{a+bx^3}\,dx}{9ab^2} \\
&= \frac{(Ab-aB)x^5}{6ab\,(a+bx^3)^2} - \frac{(Ab+5aB)x^2}{18ab^2\,(a+bx^3)} - \frac{(Ab+5aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{4/3}b^{7/3}} \\
&\quad + \frac{(Ab+5aB)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{4/3}b^{7/3}}
\end{aligned}$$

$$= \frac{(Ab - aB)x^5}{6ab\,(a + bx^3)^2} - \frac{(Ab + 5aB)x^2}{18ab^2\,(a + bx^3)} - \frac{(Ab + 5aB)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{27a^{4/3}b^{8/3}} + \frac{(Ab + 5aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}\,dx}{54a^{4/3}b^{8/3}} + \frac{(Ab + 5aB)\int \frac{1}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}\,dx}{18ab^{7/3}}$$

$$= \frac{(Ab - aB)x^5}{6ab\,(a + bx^3)^2} - \frac{(Ab + 5aB)x^2}{18ab^2\,(a + bx^3)} - \frac{(Ab + 5aB)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{27a^{4/3}b^{8/3}} + \frac{(Ab + 5aB)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{54a^{4/3}b^{8/3}} + \frac{(Ab + 5aB)\text{Subst}\left(\int \frac{1}{-3 - x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{4/3}b^{8/3}}$$

$$= \frac{(Ab - aB)x^5}{6ab\,(a + bx^3)^2} - \frac{(Ab + 5aB)x^2}{18ab^2\,(a + bx^3)} - \frac{(Ab + 5aB)\tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{8/3}} - \frac{(Ab + 5aB)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{27a^{4/3}b^{8/3}} + \frac{(Ab + 5aB)\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{54a^{4/3}b^{8/3}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.90

$$\int \frac{x^4(A + Bx^3)}{(a + bx^3)^3}\,dx$$

$$= \frac{-\frac{9b^{2/3}(Ab - aB)x^2}{(a + bx^3)^2} + \frac{6b^{2/3}(Ab - 4aB)x^2}{a(a + bx^3)} - \frac{2\sqrt{3}(Ab + 5aB)\arctan\left(\frac{1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}} - \frac{2(Ab + 5aB)\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{a^{4/3}} + \frac{(Ab + 5aB)\log\left(a^{2/3}\ldots\right.}{a\ldots}}{54b^{8/3}}$$

```
[In] Integrate[(x^4*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((-9*b^(2/3)*(A*b - a*B)*x^2)/(a + b*x^3)^2 + (6*b^(2/3)*(A*b - 4*a*B)*x^2)
/(a*(a + b*x^3)) - (2*Sqrt[3]*(A*b + 5*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]])/a^(4/3) - (2*(A*b + 5*a*B)*Log[a^(1/3) + b^(1/3)*x])/a^(4/3)
+ ((A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3))/(
54*b^(8/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.42

method	result	size
risch	$\frac{\frac{(Ab-4Ba)x^5}{9ab}-\frac{(Ab+5Ba)x^2}{18b^2}}{(b\,x^3+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(Ab+5Ba)\ln\left(x-\_R\right)}{\_R}}{27a\,b^3}$	85
default	$\frac{\frac{(Ab-4Ba)x^5}{9ab}-\frac{(Ab+5Ba)x^2}{18b^2}}{(b\,x^3+a)^2}+\frac{(Ab+5Ba)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9b^2a}$	154

```
[In] int(x^4*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/9*(A*b-4*B*a)/a/b*x^5-1/18*(A*b+5*B*a)/b^2*x^2)/(b*x^3+a)^2+1/27/a/b^3*s
um((A*b+5*B*a)/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(160) = 320.

Time = 0.30 (sec) , antiderivative size = 756, normalized size of antiderivative = 3.76

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$= \left[-\frac{6\,(4\,Ba^2b^3-Aab^4)x^5+3\,(5\,Ba^3b^2+Aa^2b^3)x^2-3\,\sqrt{\frac{1}{3}}((5\,Ba^2b^3+Aab^4)x^6+5\,Ba^4b+Aa^3b^2+2\,(5\,Ba}{}\right.$$

$$-\frac{6\,(4\,Ba^2b^3-Aab^4)x^5+3\,(5\,Ba^3b^2+Aa^2b^3)x^2-6\,\sqrt{\frac{1}{3}}((5\,Ba^2b^3+Aab^4)x^6+5\,Ba^4b+Aa^3b^2+2\,(5\,Ba}{}$$

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/54*(6*(4*B*a^2*b^3 - A*a*b^4)*x^5 + 3*(5*B*a^3*b^2 + A*a^2*b^3)*x^2 - 3
*sqrt(1/3)*((5*B*a^2*b^3 + A*a*b^4)*x^6 + 5*B*a^4*b + A*a^3*b^2 + 2*(5*B*a^
3*b^2 + A*a^2*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqr
t(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3
)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - ((5*B*a*b^2 + A*b^3)*x^6 + 5*B*a^
3 + A*a^2*b + 2*(5*B*a^2*b + A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a
*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*((5*B*a*b^2 + A*b^3)*x^6 + 5*B*a^3 +
A*a^2*b + 2*(5*B*a^2*b + A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1
/3)))/(a^2*b^6*x^6 + 2*a^3*b^5*x^3 + a^4*b^4), -1/54*(6*(4*B*a^2*b^3 - A*a*
b^4)*x^5 + 3*(5*B*a^3*b^2 + A*a^2*b^3)*x^2 - 6*sqrt(1/3)*((5*B*a^2*b^3 + A*
a*b^4)*x^6 + 5*B*a^4*b + A*a^3*b^2 + 2*(5*B*a^3*b^2 + A*a^2*b^3)*x^3)*sqrt(
-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)
^(1/3)/a)/b) - ((5*B*a*b^2 + A*b^3)*x^6 + 5*B*a^3 + A*a^2*b + 2*(5*B*a^2*b
+ A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^
(2/3)) + 2*((5*B*a*b^2 + A*b^3)*x^6 + 5*B*a^3 + A*a^2*b + 2*(5*B*a^2*b + A*
a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^6*x^6 + 2*a^3*
b^5*x^3 + a^4*b^4)]
```

Sympy [A] (verification not implemented)

Time = 5.37 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.77

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{x^5\cdot(2Ab^2-8Bab)+x^2(-Aab-5Ba^2)}{18a^3b^2+36a^2b^3x^3+18ab^4x^6} + \text{RootSum}\left(19683t^3a^4b^8+A^3b^3+15A^2Bab^2+75AB^2a^2b+125B^3a^3, \left(t\mapsto t\log\left(\frac{729t^2a^3b^5}{A^2b^2+10ABab+2\right.\right.\right.$$

```
[In] integrate(x**4*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] (x**5*(2*A*b**2 - 8*B*a*b) + x**2*(-A*a*b - 5*B*a**2))/(18*a**3*b**2 + 36*a
**2*b**3*x**3 + 18*a*b**4*x**6) + RootSum(19683*_t**3*a**4*b**8 + A**3*b**3
 + 15*A**2*B*a*b**2 + 75*A*B**2*a**2*b + 125*B**3*a**3, Lambda(_t, _t*log(7
29*_t**2*a**3*b**5/(A**2*b**2 + 10*A*B*a*b + 25*B**2*a**2) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.97

$$\begin{aligned}\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3}\,dx = &-\frac{2\,(4\,Bab-Ab^2)x^5+(5\,Ba^2+Aab)x^2}{18\,(ab^4x^6+2\,a^2b^3x^3+a^3b^2)} \\ &+\frac{\sqrt{3}(5\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} \\ &+\frac{(5\,Ba+Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} \\ &-\frac{(5\,Ba+Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\end{aligned}$$

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*(2*(4*B*a*b - A*b^2)*x^5 + (5*B*a^2 + A*a*b)*x^2)/(a*b^4*x^6 + 2*a^2*
b^3*x^3 + a^3*b^2) + 1/27*sqrt(3)*(5*B*a + A*b)*arctan(1/3*sqrt(3)*(2*x - (
a/b)^(1/3))/(a/b)^(1/3))/(a*b^3*(a/b)^(1/3)) + 1/54*(5*B*a + A*b)*log(x^2 -
 x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(1/3)) - 1/27*(5*B*a + A*b)*log(
x + (a/b)^(1/3))/(a*b^3*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.02

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{\sqrt{3}(5\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{1}{3}}ab^2} - \frac{(5\,Ba+Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^2\right)^{\frac{1}{3}}ab^2} - \frac{\left(5\,Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}+Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b^2} - \frac{8\,Babx^5-2\,Ab^2x^5+5\,Ba^2x^2+Aabx^2}{18\,(bx^3+a)^2ab^2}$$

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(5*B*a + A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/((-a*b^2)^(1/3)*a*b^2) - 1/54*(5*B*a + A*b)*log(x^2 + x*(-a/b)^(1/3)
+ (-a/b)^(2/3))/((-a*b^2)^(1/3)*a*b^2) - 1/27*(5*B*a*(-a/b)^(1/3) + A*b*(-a
/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^2) - 1/18*(8*B*a*
b*x^5 - 2*A*b^2*x^5 + 5*B*a^2*x^2 + A*a*b*x^2)/((b*x^3 + a)^2*a*b^2)
```

Mupad [B] (verification not implemented)

Time = 6.97 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.87

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{\frac{x^2\,(A\,b+5\,B\,a)}{18\,b^2}-\frac{x^5\,(A\,b-4\,B\,a)}{9\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6} - \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(A\,b+5\,B\,a)}{27\,a^{4/3}\,b^{8/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(A\,b+5\,B\,a)}{27\,a^{4/3}\,b^{8/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(A\,b+5\,B\,a)}{27\,a^{4/3}\,b^{8/3}}$$

```
[In] int((x^4*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*
b + 5*B*a))/(27*a^(4/3)*b^(8/3)) - (log(b^(1/3)*x + a^(1/3))*(A*b + 5*B*a))
/(27*a^(4/3)*b^(8/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((
3^(1/2)*1i)/2 - 1/2)*(A*b + 5*B*a))/(27*a^(4/3)*b^(8/3)) - ((x^2*(A*b + 5*B
*a))/(18*b^2) - (x^5*(A*b - 4*B*a))/(9*a*b))/(a^2 + b^2*x^6 + 2*a*b*x^3)
```

3.100 $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 199

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(Ab-aB)x^4}{6ab\,(a+bx^3)^2} - \frac{(Ab+2aB)x}{9ab^2\,(a+bx^3)} - \frac{(Ab+2aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{7/3}} + \frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{7/3}} - \frac{(Ab+2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{7/3}}$$

```
[Out] 1/6*(A*b-B*a)*x^4/a/b/(b*x^3+a)^2-1/9*(A*b+2*B*a)*x/a/b^2/(b*x^3+a)+1/27*(A
*b+2*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(7/3)-1/54*(A*b+2*B*a)*ln(a^(2/3)
-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(7/3)-1/27*(A*b+2*B*a)*arctan(1/3
*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(7/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used

$= \{468, 294, 206, 31, 648, 631, 210, 642\}$

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{(2aB+Ab)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{7/3}} - \frac{(2aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{7/3}} + \frac{(2aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{7/3}} - \frac{x(2aB+Ab)}{9ab^2(a+bx^3)} + \frac{x^4(Ab-aB)}{6ab(a+bx^3)^2}$$

```
[In] Int[(x^3*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^4)/(6*a*b*(a + b*x^3)^2) - ((A*b + 2*a*B)*x)/(9*a*b^2*(a + b
*x^3)) - ((A*b + 2*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
(9*Sqrt[3]*a^(5/3)*b^(7/3)) + ((A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*
a^(5/3)*b^(7/3)) - ((A*b + 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2])/(54*a^(5/3)*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(Ab-aB)x^4}{6ab\left(a+bx^3\right)^2}+\frac{(2Ab+4aB)\int \frac{x^3}{\left(a+bx^3\right)^2}\,dx}{6ab}\\
&= \frac{(Ab-aB)x^4}{6ab\left(a+bx^3\right)^2}-\frac{(Ab+2aB)x}{9ab^2\left(a+bx^3\right)}+\frac{(Ab+2aB)\int \frac{1}{a+bx^3}\,dx}{9ab^2}\\
&= \frac{(Ab-aB)x^4}{6ab\left(a+bx^3\right)^2}-\frac{(Ab+2aB)x}{9ab^2\left(a+bx^3\right)}+\frac{(Ab+2aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{5/3}b^2}\\
&\quad+\frac{(Ab+2aB)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{5/3}b^2}
\end{aligned}
$$

$$= \frac{(Ab-aB)x^4}{6ab\left(a+bx^3\right)^2} - \frac{(Ab+2aB)x}{9ab^2\left(a+bx^3\right)} + \frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{7/3}} - \frac{(Ab+2aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{5/3}b^{7/3}} + \frac{(Ab+2aB)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{4/3}b^2}$$

$$= \frac{(Ab-aB)x^4}{6ab\left(a+bx^3\right)^2} - \frac{(Ab+2aB)x}{9ab^2\left(a+bx^3\right)} + \frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{7/3}} - \frac{(Ab+2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{7/3}} + \frac{(Ab+2aB)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{5/3}b^{7/3}}$$

$$= \frac{(Ab-aB)x^4}{6ab\left(a+bx^3\right)^2} - \frac{(Ab+2aB)x}{9ab^2\left(a+bx^3\right)} - \frac{(Ab+2aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{7/3}} + \frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{7/3}} - \frac{(Ab+2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{7/3}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.89

$$\int \frac{x^3(A+Bx^3)}{\left(a+bx^3\right)^3}\,dx$$

$$= \frac{-\frac{9\sqrt[3]{b}(Ab-aB)x}{\left(a+bx^3\right)^2} + \frac{3\sqrt[3]{b}(Ab-7aB)x}{a(a+bx^3)} - \frac{2\sqrt{3}(Ab+2aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}} + \frac{2(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}} - \frac{(Ab+2aB)\log\left(a^{2/3}-\right.}{a^{5/}}}{54b^{7/3}}$$

```
[In] Integrate[(x^3*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((-9*b^(1/3)*(A*b - a*B)*x)/(a + b*x^3)^2 + (3*b^(1/3)*(A*b - 7*a*B)*x)/(a*
(a + b*x^3)) - (2*Sqrt[3]*(A*b + 2*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/
Sqrt[3]])/a^(5/3) + (2*(A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) - ((
A*b + 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(54*b
^(7/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.42

method	result	size
risch	$\frac{\frac{(Ab-7Ba)x^4}{18ab}-\frac{(Ab+2Ba)x}{9b^2}}{(b\,x^3+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(Ab+2Ba)\ln\left(x-\_R\right)}{\_R^2}}{27a\,b^3}$	83
default	$\frac{\frac{(Ab-7Ba)x^4}{18ab}-\frac{(Ab+2Ba)x}{9b^2}}{(b\,x^3+a)^2}+\frac{(Ab+2Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{9b^2a}$	152

```
[In] int(x^3*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/18*(A*b-7*B*a)/a/b*x^4-1/9*(A*b+2*B*a)/b^2*x)/(b*x^3+a)^2+1/27/a/b^3*sum
((A*b+2*B*a)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(158) = 316.

Time = 0.29 (sec) , antiderivative size = 743, normalized size of antiderivative = 3.73

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$= \left[-\frac{3\,(7\,Ba^3b^2 - Aa^2b^3)x^4 - 3\,\sqrt{\frac{1}{3}}((2\,Ba^2b^3 + Aab^4)x^6 + 2\,Ba^4b + Aa^3b^2 + 2\,(2\,Ba^3b^2 + Aa^2b^3)x^3)\sqrt{-\frac{(a^2b}{b}}}{} \right.$$

$$-\frac{3\,(7\,Ba^3b^2 - Aa^2b^3)x^4 - 6\,\sqrt{\frac{1}{3}}((2\,Ba^2b^3 + Aab^4)x^6 + 2\,Ba^4b + Aa^3b^2 + 2\,(2\,Ba^3b^2 + Aa^2b^3)x^3)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}}{}$$

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/54*(3*(7*B*a^3*b^2 - A*a^2*b^3)*x^4 - 3*sqrt(1/3)*((2*B*a^2*b^3 + A*a*b
^4)*x^6 + 2*B*a^4*b + A*a^3*b^2 + 2*(2*B*a^3*b^2 + A*a^2*b^3)*x^3)*sqrt(-(a
^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*
a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3
 + a)) + ((2*B*a*b^2 + A*b^3)*x^6 + 2*B*a^3 + A*a^2*b + 2*(2*B*a^2*b + A*a*
b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) -
2*((2*B*a*b^2 + A*b^3)*x^6 + 2*B*a^3 + A*a^2*b + 2*(2*B*a^2*b + A*a*b^2)*x^
3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 6*(2*B*a^4*b + A*a^3*b^2)*x)/
(a^3*b^5*x^6 + 2*a^4*b^4*x^3 + a^5*b^3), -1/54*(3*(7*B*a^3*b^2 - A*a^2*b^3)
*x^4 - 6*sqrt(1/3)*((2*B*a^2*b^3 + A*a*b^4)*x^6 + 2*B*a^4*b + A*a^3*b^2 + 2
*(2*B*a^3*b^2 + A*a^2*b^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(
a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + ((2*B*a*b^2
+ A*b^3)*x^6 + 2*B*a^3 + A*a^2*b + 2*(2*B*a^2*b + A*a*b^2)*x^3)*(a^2*b)^(2/
3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*((2*B*a*b^2 + A*b^3
)*x^6 + 2*B*a^3 + A*a^2*b + 2*(2*B*a^2*b + A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(
a*b*x + (a^2*b)^(2/3)) + 6*(2*B*a^4*b + A*a^3*b^2)*x)/(a^3*b^5*x^6 + 2*a^4*
b^4*x^3 + a^5*b^3)]
```

Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.68

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{x^4(Ab^2-7Bab)+x(-2Aab-4Ba^2)}{18a^3b^2+36a^2b^3x^3+18ab^4x^6} + \operatorname{RootSum}\left(19683t^3a^5b^7-A^3b^3-6A^2Bab^2-12AB^2a^2b-8B^3a^3, \left(t\mapsto t\log\left(\frac{27ta^2b^2}{Ab+2Ba}+x\right)\right)\right)$$

```
[In] integrate(x**3*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] (x**4*(A*b**2 - 7*B*a*b) + x*(-2*A*a*b - 4*B*a**2))/(18*a**3*b**2 + 36*a**2
*b**3*x**3 + 18*a*b**4*x**6) + RootSum(19683*_t**3*a**5*b**7 - A**3*b**3 -
6*A**2*B*a*b**2 - 12*A*B**2*a**2*b - 8*B**3*a**3, Lambda(_t, _t*log(27*_t*a
**2*b**2/(A*b + 2*B*a) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.97

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{(7\,Bab-Ab^2)x^4+2\,(2\,Ba^2+Aab)x}{18\,(ab^4x^6+2\,a^2b^3x^3+a^3b^2)} + \frac{\sqrt{3}(2\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(2\,Ba+Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(2\,Ba+Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*((7*B*a*b - A*b^2)*x^4 + 2*(2*B*a^2 + A*a*b)*x)/(a*b^4*x^6 + 2*a^2*b^
3*x^3 + a^3*b^2) + 1/27*sqrt(3)*(2*B*a + A*b)*arctan(1/3*sqrt(3)*(2*x - (a/
b)^(1/3))/(a/b)^(1/3))/(a*b^3*(a/b)^(2/3)) - 1/54*(2*B*a + A*b)*log(x^2 - x
*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(2/3)) + 1/27*(2*B*a + A*b)*log(x
+ (a/b)^(1/3))/(a*b^3*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.94

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{\sqrt{3}(2\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{2}{3}}\,ab} - \frac{(2\,Ba+Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{2}{3}}\,ab} - \frac{(2\,Ba+Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b^2} - \frac{7\,Babx^4-Ab^2x^4+4\,Ba^2x+2\,Aabx}{18\,(bx^3+a)^2ab^2}$$

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(2*B*a + A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^
(1/3))/((-a*b^2)^(2/3)*a*b) - 1/54*(2*B*a + A*b)*log(x^2 + x*(-a/b)^(1/3) +
 (-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b) - 1/27*(2*B*a + A*b)*(-a/b)^(1/3)*log(a
bs(x - (-a/b)^(1/3)))/(a^2*b^2) - 1/18*(7*B*a*b*x^4 - A*b^2*x^4 + 4*B*a^2*x
 + 2*A*a*b*x)/((b*x^3 + a)^2*a*b^2)
```

Mupad [B] (verification not implemented)

Time = 6.97 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.87

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b+2\,B\,a\right)}{27\,a^{5/3}\,b^{7/3}} - \frac{\frac{x\,(A\,b+2\,B\,a)}{9\,b^2}-\frac{x^4\,(A\,b-7\,B\,a)}{18\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b+2\,B\,a\right)}{27\,a^{5/3}\,b^{7/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b+2\,B\,a\right)}{27\,a^{5/3}\,b^{7/3}}$$

```
[In] int((x^3*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(A*b + 2*B*a))/(27*a^(5/3)*b^(7/3)) - ((x*(A*b +
2*B*a))/(9*b^2) - (x^4*(A*b - 7*B*a))/(18*a*b))/(a^2 + b^2*x^6 + 2*a*b*x^3)
 - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*
(A*b + 2*B*a))/(27*a^(5/3)*b^(7/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x
 - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b + 2*B*a))/(27*a^(5/3)*b^(7/3))
```

3.101 $\int \frac{x(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 201

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(Ab-aB)x^2}{6ab\,(a+bx^3)^2} + \frac{(2Ab+aB)x^2}{9a^2b\,(a+bx^3)} - \frac{(2Ab+aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{5/3}} - \frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{5/3}} + \frac{(2Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{7/3}b^{5/3}}$$

```
[Out] 1/6*(A*b-B*a)*x^2/a/b/(b*x^3+a)^2+1/9*(2*A*b+B*a)*x^2/a^2/b/(b*x^3+a)-1/27*
(2*A*b+B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(7/3)/b^(5/3)+1/54*(2*A*b+B*a)*ln(a^(2/
3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(7/3)/b^(5/3)-1/27*(2*A*b+B*a)*arctan(1
/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(7/3)/b^(5/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used

$= \{468, 296, 298, 31, 648, 631, 210, 642\}$

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{(aB+2Ab)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{5/3}} + \frac{(aB+2Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{7/3}b^{5/3}} - \frac{(aB+2Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{5/3}} + \frac{x^2(aB+2Ab)}{9a^2b\,(a+bx^3)} + \frac{x^2(Ab-aB)}{6ab\,(a+bx^3)^2}$$

```
[In] Int[(x*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^2)/(6*a*b*(a + b*x^3)^2) + ((2*A*b + a*B)*x^2)/(9*a^2*b*(a +
 b*x^3)) - ((2*A*b + a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]
)/(9*Sqrt[3]*a^(7/3)*b^(5/3)) - ((2*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x])/(2
7*a^(7/3)*b^(5/3)) + ((2*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/(54*a^(7/3)*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(Ab-aB)x^2}{6ab\,(a+bx^3)^2}+\frac{(4Ab+2aB)\int \frac{x}{(a+bx^3)^2}\,dx}{6ab}\\
&= \frac{(Ab-aB)x^2}{6ab\,(a+bx^3)^2}+\frac{(2Ab+aB)x^2}{9a^2b\,(a+bx^3)}+\frac{(2Ab+aB)\int \frac{x}{a+bx^3}\,dx}{9a^2b}\\
&= \frac{(Ab-aB)x^2}{6ab\,(a+bx^3)^2}+\frac{(2Ab+aB)x^2}{9a^2b\,(a+bx^3)}-\frac{(2Ab+aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{7/3}b^{4/3}}\\
&\quad+\frac{(2Ab+aB)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{7/3}b^{4/3}}\end{aligned}$$

$$= \frac{(Ab-aB)x^2}{6ab(a+bx^3)^2} + \frac{(2Ab+aB)x^2}{9a^2b(a+bx^3)} - \frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{5/3}} + \frac{(2Ab+aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{7/3}b^{5/3}} + \frac{(2Ab+aB)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^2b^{4/3}}$$

$$= \frac{(Ab-aB)x^2}{6ab(a+bx^3)^2} + \frac{(2Ab+aB)x^2}{9a^2b(a+bx^3)} - \frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{5/3}} + \frac{(2Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{7/3}b^{5/3}} + \frac{(2Ab+aB)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{7/3}b^{5/3}}$$

$$= \frac{(Ab-aB)x^2}{6ab(a+bx^3)^2} + \frac{(2Ab+aB)x^2}{9a^2b(a+bx^3)} - \frac{(2Ab+aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{5/3}} - \frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{5/3}} + \frac{(2Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{7/3}b^{5/3}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.89

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$= \frac{-\frac{9a^{4/3}b^{2/3}(-Ab+aB)x^2}{(a+bx^3)^2} + \frac{6\sqrt[3]{a}b^{2/3}(2Ab+aB)x^2}{a+bx^3} - 2\sqrt{3}(2Ab+aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2(2Ab+aB)\log\left(\sqrt[3]{a}+\right.}{54a^{7/3}b^{5/3}}$$

```
[In] Integrate[(x*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((-9*a^(4/3)*b^(2/3)*(-(A*b) + a*B)*x^2)/(a + b*x^3)^2 + (6*a^(1/3)*b^(2/3)
*(2*A*b + a*B)*x^2)/(a + b*x^3) - 2*Sqrt[3]*(2*A*b + a*B)*ArcTan[(1 - (2*b^
(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*(2*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x] + (2*
A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*b^(5
/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.43

method	result	size
risch	$\frac{\frac{(2Ab+Ba)x^5}{9a^2}+\frac{(7Ab-Ba)x^2}{18ab}}{(b x^3+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{(2Ab+Ba)\ln(x-\_R)}{\_R}}{27a^2b^2}$	86
default	$\frac{\frac{(2Ab+Ba)x^5}{9a^2}+\frac{(7Ab-Ba)x^2}{18ab}}{(b x^3+a)^2}+\frac{(2Ab+Ba)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^2b}$	155

```
[In] int(x*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/9*(2*A*b+B*a)/a^2*x^5+1/18*(7*A*b-B*a)/a/b*x^2)/(b*x^3+a)^2+1/27/a^2/b^2
*sum((2*A*b+B*a)/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. 2(160) = 320.

Time = 0.37 (sec) , antiderivative size = 752, normalized size of antiderivative = 3.74

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^3}\,dx$$

$$= \left[\frac{6\,(Ba^2b^3+2\,Aab^4)x^5-3\,(Ba^3b^2-7\,Aa^2b^3)x^2+3\,\sqrt{\frac{1}{3}}((Ba^2b^3+2\,Aab^4)x^6+Ba^4b+2\,Aa^3b^2+2\,(Ba^3\right.$$

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(6*(B*a^2*b^3 + 2*A*a*b^4)*x^5 - 3*(B*a^3*b^2 - 7*A*a^2*b^3)*x^2 + 3*
sqrt(1/3)*((B*a^2*b^3 + 2*A*a*b^4)*x^6 + B*a^4*b + 2*A*a^3*b^2 + 2*(B*a^3*b
```

```
^2 + 2*A*a^2*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt
(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)
/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + ((B*a*b^2 + 2*A*b^3)*x^6 + B*a^3 +
 2*A*a^2*b + 2*(B*a^2*b + 2*A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*
b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*((B*a*b^2 + 2*A*b^3)*x^6 + B*a^3 + 2*A
*a^2*b + 2*(B*a^2*b + 2*A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/
3)))/(a^3*b^5*x^6 + 2*a^4*b^4*x^3 + a^5*b^3), 1/54*(6*(B*a^2*b^3 + 2*A*a*b^
4)*x^5 - 3*(B*a^3*b^2 - 7*A*a^2*b^3)*x^2 + 6*sqrt(1/3)*((B*a^2*b^3 + 2*A*a*
b^4)*x^6 + B*a^4*b + 2*A*a^3*b^2 + 2*(B*a^3*b^2 + 2*A*a^2*b^3)*x^3)*sqrt(-(
-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(
1/3)/a)/b) + ((B*a*b^2 + 2*A*b^3)*x^6 + B*a^3 + 2*A*a^2*b + 2*(B*a^2*b + 2*
A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2
/3)) - 2*((B*a*b^2 + 2*A*b^3)*x^6 + B*a^3 + 2*A*a^2*b + 2*(B*a^2*b + 2*A*a*
b^2)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^3*b^5*x^6 + 2*a^4*b^
4*x^3 + a^5*b^3)]
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.76

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{x^5\cdot(4Ab^2+2Bab)+x^2\cdot(7Aab-Ba^2)}{18a^4b+36a^3b^2x^3+18a^2b^3x^6} + \mathrm{RootSum}\left(19683t^3a^7b^5+8A^3b^3+12A^2Bab^2+6AB^2a^2b+B^3a^3,\left(t\mapsto t\log\left(\frac{729t^2a^5b^3}{4A^2b^2+4ABab+B^2a^2}\right.\right.\right.$$

```
[In] integrate(x*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] (x**5*(4*A*b**2 + 2*B*a*b) + x**2*(7*A*a*b - B*a**2))/(18*a**4*b + 36*a**3*
b**2*x**3 + 18*a**2*b**3*x**6) + RootSum(19683*_t**3*a**7*b**5 + 8*A**3*b**
3 + 12*A**2*B*a*b**2 + 6*A*B**2*a**2*b + B**3*a**3, Lambda(_t, _t*log(729*_
t**2*a**5*b**3/(4*A**2*b**2 + 4*A*B*a*b + B**2*a**2) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.97

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{2\,(Bab+2\,Ab^2)x^5-(Ba^2-7\,Aab)x^2}{18\,(a^2b^3x^6+2\,a^3b^2x^3+a^4b)} + \frac{\sqrt{3}(Ba+2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(Ba+2\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(Ba+2\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(2*(B*a*b + 2*A*b^2)*x^5 - (B*a^2 - 7*A*a*b)*x^2)/(a^2*b^3*x^6 + 2*a^3
*b^2*x^3 + a^4*b) + 1/27*sqrt(3)*(B*a + 2*A*b)*arctan(1/3*sqrt(3)*(2*x - (a
/b)^(1/3))/(a/b)^(1/3))/(a^2*b^2*(a/b)^(1/3)) + 1/54*(B*a + 2*A*b)*log(x^2
- x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2*(a/b)^(1/3)) - 1/27*(B*a + 2*A*b)*l
og(x + (a/b)^(1/3))/(a^2*b^2*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.03

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{\sqrt{3}(Ba+2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{1}{3}}a^2b} - \frac{(Ba+2\,Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^2\right)^{\frac{1}{3}}a^2b} - \frac{\left(Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b} + \frac{2\,Babx^5+4\,Ab^2x^5-Ba^2x^2+7\,Aabx^2}{18\,\left(bx^3+a\right)^2a^2b}$$

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(B*a + 2*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/((-a*b^2)^(1/3)*a^2*b) - 1/54*(B*a + 2*A*b)*log(x^2 + x*(-a/b)^(1/3)
+ (-a/b)^(2/3))/((-a*b^2)^(1/3)*a^2*b) - 1/27*(B*a*(-a/b)^(1/3) + 2*A*b*(-a
/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b) + 1/18*(2*B*a*b*
x^5 + 4*A*b^2*x^5 - B*a^2*x^2 + 7*A*a*b*x^2)/((b*x^3 + a)^2*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.87

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{\frac{x^5\,(2\,A\,b+B\,a)}{9\,a^2}+\frac{x^2\,(7\,A\,b-B\,a)}{18\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(2\,A\,b+B\,a)}{27\,a^{7/3}\,b^{5/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(2\,A\,b+B\,a)}{27\,a^{7/3}\,b^{5/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(2\,A\,b+B\,a)}{27\,a^{7/3}\,b^{5/3}}$$

```
[In] int((x*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] ((x^5*(2*A*b + B*a))/(9*a^2) + (x^2*(7*A*b - B*a))/(18*a*b))/(a^2 + b^2*x^6
 + 2*a*b*x^3) - (log(b^(1/3)*x + a^(1/3))*(2*A*b + B*a))/(27*a^(7/3)*b^(5/3
)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2
)*(2*A*b + B*a))/(27*a^(7/3)*b^(5/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)
*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*A*b + B*a))/(27*a^(7/3)*b^(5/3))
```

3.102 $\int \frac{A+Bx^3}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 197

$$\int \frac{A+Bx^3}{(a+bx^3)^3}\,dx = \frac{(Ab-aB)x}{6ab(a+bx^3)^2}+\frac{(5Ab+aB)x}{18a^2b(a+bx^3)} - \frac{(5Ab+aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{4/3}}+\frac{(5Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{4/3}} - \frac{(5Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{4/3}}$$

```
[Out] 1/6*(A*b-B*a)*x/a/b/(b*x^3+a)^2+1/18*(5*A*b+B*a)*x/a^2/b/(b*x^3+a)+1/27*(5*
A*b+B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^(4/3)-1/54*(5*A*b+B*a)*ln(a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(4/3)-1/27*(5*A*b+B*a)*arctan(1/3*
(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {393, 205, 206, 31, 648, 631, 210, 642}

$$\int \frac{A+Bx^3}{(a+bx^3)^3}\,dx = -\frac{(aB+5Ab)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{4/3}} - \frac{(aB+5Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{4/3}} + \frac{(aB+5Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{4/3}} + \frac{x(aB+5Ab)}{18a^2b(a+bx^3)} + \frac{x(Ab-aB)}{6ab(a+bx^3)^2}$$

```
[In] Int[(A + B*x^3)/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x)/(6*a*b*(a + b*x^3)^2) + ((5*A*b + a*B)*x)/(18*a^2*b*(a + b*
x^3)) - ((5*A*b + a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
9*Sqrt[3]*a^(8/3)*b^(4/3)) + ((5*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*a
^(8/3)*b^(4/3)) - ((5*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2])/(54*a^(8/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(Ab-aB)x}{6ab\left(a+bx^3\right)^2}+\frac{(5Ab+aB)\int \frac{1}{\left(a+bx^3\right)^2}\,dx}{6ab}\\
&= \frac{(Ab-aB)x}{6ab\left(a+bx^3\right)^2}+\frac{(5Ab+aB)x}{18a^2b\left(a+bx^3\right)}+\frac{(5Ab+aB)\int \frac{1}{a+bx^3}\,dx}{9a^2b}\\
&= \frac{(Ab-aB)x}{6ab\left(a+bx^3\right)^2}+\frac{(5Ab+aB)x}{18a^2b\left(a+bx^3\right)}+\frac{(5Ab+aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{8/3}b}\\
&\quad+\frac{(5Ab+aB)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{8/3}b}
\end{aligned}$$

$$= \frac{(Ab-aB)x}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab+aB)x}{18a^2b\left(a+bx^3\right)} + \frac{(5Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{4/3}} - \frac{(5Ab+aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{8/3}b^{4/3}} + \frac{(5Ab+aB)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{7/3}b}$$

$$= \frac{(Ab-aB)x}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab+aB)x}{18a^2b\left(a+bx^3\right)} + \frac{(5Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{4/3}} - \frac{(5Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{4/3}} + \frac{(5Ab+aB)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{8/3}b^{4/3}}$$

$$= \frac{(Ab-aB)x}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab+aB)x}{18a^2b\left(a+bx^3\right)} - \frac{(5Ab+aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{4/3}} + \frac{(5Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{4/3}} - \frac{(5Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{4/3}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^3}{\left(a+bx^3\right)^3}\,dx$$

$$= \frac{-\frac{9a^{5/3}\sqrt[3]{b}(-Ab+aB)x}{\left(a+bx^3\right)^2} + \frac{3a^{2/3}\sqrt[3]{b}(5Ab+aB)x}{a+bx^3} - 2\sqrt{3}(5Ab+aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2(5Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right) \ldots}{54a^{8/3}b^{4/3}}$$

```
[In] Integrate[(A + B*x^3)/(a + b*x^3)^3,x]
```

```
[Out] ((-9*a^(5/3)*b^(1/3)*(-(A*b) + a*B)*x)/(a + b*x^3)^2 + (3*a^(2/3)*b^(1/3)*(
5*A*b + a*B)*x)/(a + b*x^3) - 2*Sqrt[3]*(5*A*b + a*B)*ArcTan[(1 - (2*b^(1/3
)*x)/a^(1/3))/Sqrt[3]] + 2*(5*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x] - (5*A*b
+ a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(4/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.43

method	result	size
risch	$\frac{\frac{(5Ab+Ba)x^4}{18a^2}+\frac{(4Ab-Ba)x}{9ab}}{(b\,x^3+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{(5Ab+Ba)\ln(x-\_R)}{\_R^2}}{27b^2a^2}$	84
default	$\frac{\frac{(5Ab+Ba)x^4}{18a^2}+\frac{(4Ab-Ba)x}{9ab}}{(b\,x^3+a)^2}+\frac{(5Ab+Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{9b\,a^2}$	153

```
[In] int((B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/18*(5*A*b+B*a)/a^2*x^4+1/9*(4*A*b-B*a)/a/b*x)/(b*x^3+a)^2+1/27/b^2/a^2*s
um((5*A*b+B*a)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(156) = 312.

Time = 0.31 (sec) , antiderivative size = 743, normalized size of antiderivative = 3.77

$$\int \frac{A+Bx^3}{(a+bx^3)^3}\,dx$$

$$= \left[\frac{3\,(Ba^3b^2+5\,Aa^2b^3)x^4+3\,\sqrt{\tfrac{1}{3}}((Ba^2b^3+5\,Aab^4)x^6+Ba^4b+5\,Aa^3b^2+2\,(Ba^3b^2+5\,Aa^2b^3)x^3)\sqrt{-\frac{(a^2b}{b}}}{}\right.$$

```
[In] integrate((B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(3*(B*a^3*b^2 + 5*A*a^2*b^3)*x^4 + 3*sqrt(1/3)*((B*a^2*b^3 + 5*A*a*b^
4)*x^6 + B*a^4*b + 5*A*a^3*b^2 + 2*(B*a^3*b^2 + 5*A*a^2*b^3)*x^3)*sqrt(-(a^
```

```
2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a
*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3
+ a)) - ((B*a*b^2 + 5*A*b^3)*x^6 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*A*a*b
^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2
*((B*a*b^2 + 5*A*b^3)*x^6 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*A*a*b^2)*x^3
)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) - 6*(B*a^4*b - 4*A*a^3*b^2)*x)/(
a^4*b^4*x^6 + 2*a^5*b^3*x^3 + a^6*b^2), 1/54*(3*(B*a^3*b^2 + 5*A*a^2*b^3)*x
^4 + 6*sqrt(1/3)*((B*a^2*b^3 + 5*A*a*b^4)*x^6 + B*a^4*b + 5*A*a^3*b^2 + 2*(
B*a^3*b^2 + 5*A*a^2*b^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^
2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - ((B*a*b^2 + 5*
A*b^3)*x^6 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*A*a*b^2)*x^3)*(a^2*b)^(2/3)
*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((B*a*b^2 + 5*A*b^3)*
x^6 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*
b*x + (a^2*b)^(2/3)) - 6*(B*a^4*b - 4*A*a^3*b^2)*x)/(a^4*b^4*x^6 + 2*a^5*b^
3*x^3 + a^6*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.68

$$\int \frac{A+Bx^3}{(a+bx^3)^3}\,dx = \frac{x^4\cdot(5Ab^2+Bab)+x(8Aab-2Ba^2)}{18a^4b+36a^3b^2x^3+18a^2b^3x^6} + \operatorname{RootSum}\left(19683t^3a^8b^4-125A^3b^3-75A^2Bab^2-15AB^2a^2b-B^3a^3,\left(t\mapsto t\log\left(\frac{27ta^3b}{5Ab+Ba}+x\right)\right)\right)$$

```
[In] integrate((B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] (x**4*(5*A*b**2 + B*a*b) + x*(8*A*a*b - 2*B*a**2))/(18*a**4*b + 36*a**3*b**
2*x**3 + 18*a**2*b**3*x**6) + RootSum(19683*_t**3*a**8*b**4 - 125*A**3*b**3
 - 75*A**2*B*a*b**2 - 15*A*B**2*a**2*b - B**3*a**3, Lambda(_t, _t*log(27*_t
*a**3*b/(5*A*b + B*a) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^3}{(a+bx^3)^3}\,dx = \frac{(Bab+5\,Ab^2)x^4-2\,(Ba^2-4\,Aab)x}{18\,(a^2b^3x^6+2\,a^3b^2x^3+a^4b)} + \frac{\sqrt{3}(Ba+5\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(Ba+5\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(Ba+5\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*((B*a*b + 5*A*b^2)*x^4 - 2*(B*a^2 - 4*A*a*b)*x)/(a^2*b^3*x^6 + 2*a^3*b
^2*x^3 + a^4*b) + 1/27*sqrt(3)*(B*a + 5*A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b
)^(1/3))/(a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3)) - 1/54*(B*a + 5*A*b)*log(x^2 -
x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2*(a/b)^(2/3)) + 1/27*(B*a + 5*A*b)*log
(x + (a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^3}{(a+bx^3)^3}\,dx = -\frac{\sqrt{3}(Ba+5\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{2}{3}}\,a^2} - \frac{(Ba+5\,Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^2\right)^{\frac{2}{3}}\,a^2} - \frac{(Ba+5\,Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3b} + \frac{Babx^4+5\,Ab^2x^4-2\,Ba^2x+8\,Aabx}{18\,\left(bx^3+a\right)^2a^2b}$$

```
[In] integrate((B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(B*a + 5*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^
(1/3))/((-a*b^2)^(2/3)*a^2) - 1/54*(B*a + 5*A*b)*log(x^2 + x*(-a/b)^(1/3) +
 (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) - 1/27*(B*a + 5*A*b)*(-a/b)^(1/3)*log(a
bs(x - (-a/b)^(1/3)))/(a^3*b) + 1/18*(B*a*b*x^4 + 5*A*b^2*x^4 - 2*B*a^2*x +
 8*A*a*b*x)/((b*x^3 + a)^2*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^3}{(a+bx^3)^3}\,dx = \frac{\frac{x^4\,(5\,A\,b+B\,a)}{18\,a^2}+\frac{x\,(4\,A\,b-B\,a)}{9\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(5\,A\,b+B\,a)}{27\,a^{8/3}\,b^{4/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,A\,b+B\,a)}{27\,a^{8/3}\,b^{4/3}} + \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,A\,b+B\,a)}{27\,a^{8/3}\,b^{4/3}}$$

```
[In] int((A + B*x^3)/(a + b*x^3)^3,x)
```

```
[Out] ((x^4*(5*A*b + B*a))/(18*a^2) + (x*(4*A*b - B*a))/(9*a*b))/(a^2 + b^2*x^6 +
 2*a*b*x^3) + (log(b^(1/3)*x + a^(1/3))*(5*A*b + B*a))/(27*a^(8/3)*b^(4/3))
 - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*
(5*A*b + B*a))/(27*a^(8/3)*b^(4/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x
 - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*A*b + B*a))/(27*a^(8/3)*b^(4/3))
```

3.103 $\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 227

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^3}\,dx = -\frac{2(7Ab-aB)}{9a^3bx}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}+\frac{7Ab-aB}{18a^2bx\left(a+bx^3\right)} + \frac{2(7Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}b^{2/3}}+\frac{2(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}b^{2/3}} - \frac{(7Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{10/3}b^{2/3}}$$

```
[Out] -2/9*(7*A*b-B*a)/a^3/b/x+1/6*(A*b-B*a)/a/b/x/(b*x^3+a)^2+1/18*(7*A*b-B*a)/a
^2/b/x/(b*x^3+a)+2/27*(7*A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)/b^(2/3)-1/
27*(7*A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(10/3)/b^(2/3)+2
/27*(7*A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(10/3)/
b^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used

= {468, 296, 331, 298, 31, 648, 631, 210, 642}

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^3}\,dx = \frac{2(7Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}b^{2/3}} - \frac{(7Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{10/3}b^{2/3}} + \frac{2(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}b^{2/3}} - \frac{2(7Ab-aB)}{9a^3bx} + \frac{7Ab-aB}{18a^2bx\left(a+bx^3\right)} + \frac{Ab-aB}{6abx\left(a+bx^3\right)^2}$$

```
[In] Int[(A + B*x^3)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] (-2*(7*A*b - a*B))/(9*a^3*b*x) + (A*b - a*B)/(6*a*b*x*(a + b*x^3)^2) + (7*A
*b - a*B)/(18*a^2*b*x*(a + b*x^3)) + (2*(7*A*b - a*B)*ArcTan[(a^(1/3) - 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(10/3)*b^(2/3)) + (2*(7*A*b - a*
B)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*b^(2/3)) - ((7*A*b - a*B)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*a^(10/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{Ab-aB}{6abx\,(a+bx^3)^2} + \frac{(7Ab-aB)\int \frac{1}{x^2(a+bx^3)^2}\,dx}{6ab}$$

$$= \frac{Ab-aB}{6abx\,(a+bx^3)^2} + \frac{7Ab-aB}{18a^2bx\,(a+bx^3)} + \frac{(2(7Ab-aB))\int \frac{1}{x^2(a+bx^3)}\,dx}{9a^2b}$$

$$= -\frac{2(7Ab-aB)}{9a^3bx} + \frac{Ab-aB}{6abx\,(a+bx^3)^2} + \frac{7Ab-aB}{18a^2bx\,(a+bx^3)} - \frac{(2(7Ab-aB))\int \frac{x}{a+bx^3}\,dx}{9a^3}$$

$$= -\frac{2(7Ab-aB)}{9a^3bx} + \frac{Ab-aB}{6abx\,(a+bx^3)^2} + \frac{7Ab-aB}{18a^2bx\,(a+bx^3)} + \frac{(2(7Ab-aB))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{10/3}\sqrt[3]{b}} - \frac{(2(7Ab-aB))\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{10/3}\sqrt[3]{b}}$$

$$= -\frac{2(7Ab-aB)}{9a^3bx} + \frac{Ab-aB}{6abx\,(a+bx^3)^2} + \frac{7Ab-aB}{18a^2bx\,(a+bx^3)} + \frac{2(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}b^{2/3}} - \frac{(7Ab-aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{10/3}b^{2/3}} - \frac{(7Ab-aB)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^3\sqrt[3]{b}}$$

$$= -\frac{2(7Ab-aB)}{9a^3bx} + \frac{Ab-aB}{6abx\,(a+bx^3)^2} + \frac{7Ab-aB}{18a^2bx\,(a+bx^3)} + \frac{2(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}b^{2/3}} - \frac{(7Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{10/3}b^{2/3}} - \frac{(2(7Ab-aB))\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{10/3}b^{2/3}}$$

$$= -\frac{2(7Ab-aB)}{9a^3bx} + \frac{Ab-aB}{6abx\,(a+bx^3)^2} + \frac{7Ab-aB}{18a^2bx\,(a+bx^3)} + \frac{2(7Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}b^{2/3}} + \frac{2(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}b^{2/3}} - \frac{(7Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{10/3}b^{2/3}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^3}{x^2\,(a+bx^3)^3}\,dx$$

$$= \frac{-\frac{54\sqrt[3]{a}A}{x} + \frac{9a^{4/3}(-Ab+aB)x^2}{(a+bx^3)^2} + \frac{6\sqrt[3]{a}(-5Ab+2aB)x^2}{a+bx^3} + \frac{4\sqrt{3}(7Ab-aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} + \frac{4(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}} + \frac{2(}{}}{54a^{10/3}}$$

```
[In] Integrate[(A + B*x^3)/(x^2*(a + b*x^3)^3),x]
```

[Out] ((-54*a^(1/3)*A)/x + (9*a^(4/3)*(-(A*b) + a*B)*x^2)/(a + b*x^3)^2 + (6*a^(1/3)*(-5*A*b + 2*a*B)*x^2)/(a + b*x^3) + (4*Sqrt[3]*(7*A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (4*(7*A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + (2*(-7*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(54*a^(10/3))

Maple [A] (verified)

Time = 4.26 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.70

method	result
default	$-\frac{A}{a^3 x}-\frac{\frac{\left(\frac{5}{9}b^2A-\frac{2}{9}abB\right)x^5+\frac{a(13Ab-7Ba)x^2}{18}}{\left(bx^3+a\right)^2}+\left(\frac{14Ab}{9}-\frac{2Ba}{9}\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right.}{3}\right.}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{a^3}$
risch	$\frac{-\frac{2b(7Ab-Ba)x^6}{9a^3}-\frac{7(7Ab-Ba)x^3}{18a^2}-\frac{A}{a}}{x\left(bx^3+a\right)^2}+\frac{2\left(\sum_{\_R=\operatorname{RootOf}\left(a^{10}b^2\_Z^3-343A^3b^3+147A^2Bab^2-21AB^2a^2b+B^3a^3\right)}\_R\ln\left(\left(-4\_R^3a^{10}b^2\right.\right.\right.}{27}$

[In] int((B*x^3+A)/x^2/(b*x^3+a)^3,x,method=_RETURNVERBOSE)

[Out] -A/a^3/x-1/a^3*(((5/9*b^2*A-2/9*a*b*B)*x^5+1/18*a*(13*A*b-7*B*a)*x^2)/(b*x^3+a)^2+(14/9*A*b-2/9*B*a)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(179) = 358$.

Time = 0.29 (sec) , antiderivative size = 776, normalized size of antiderivative = 3.42

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^3}\,dx = \left[\frac{12\,(Ba^2b^3-7\,Aab^4)x^6-54\,Aa^3b^2+21\,(Ba^3b^2-7\,Aa^2b^3)x^3-6\,\sqrt{\tfrac{1}{3}}((Ba^2b^3-7\,Aab^4)x^7+2\,(Ba^3b^2-7}{}\right.$$

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(12*(B*a^2*b^3 - 7*A*a*b^4)*x^6 - 54*A*a^3*b^2 + 21*(B*a^3*b^2 - 7*A*
a^2*b^3)*x^3 - 6*sqrt(1/3)*((B*a^2*b^3 - 7*A*a*b^4)*x^7 + 2*(B*a^3*b^2 - 7*
A*a^2*b^3)*x^4 + (B*a^4*b - 7*A*a^3*b^2)*x)*sqrt(-(a*b^2)^(1/3)/a)*log((2*b
^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*
sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) + 2*((B*a*b^2 - 7*
A*b^3)*x^7 + 2*(B*a^2*b - 7*A*a*b^2)*x^4 + (B*a^3 - 7*A*a^2*b)*x)*(a*b^2)^(
2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 4*((B*a*b^2 - 7*A*b
^3)*x^7 + 2*(B*a^2*b - 7*A*a*b^2)*x^4 + (B*a^3 - 7*A*a^2*b)*x)*(a*b^2)^(2/3
)*log(b*x + (a*b^2)^(1/3)))/(a^4*b^4*x^7 + 2*a^5*b^3*x^4 + a^6*b^2*x), 1/54
*(12*(B*a^2*b^3 - 7*A*a*b^4)*x^6 - 54*A*a^3*b^2 + 21*(B*a^3*b^2 - 7*A*a^2*b
^3)*x^3 - 12*sqrt(1/3)*((B*a^2*b^3 - 7*A*a*b^4)*x^7 + 2*(B*a^3*b^2 - 7*A*a^
2*b^3)*x^4 + (B*a^4*b - 7*A*a^3*b^2)*x)*sqrt((a*b^2)^(1/3)/a)*arctan(-sqrt(
1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) + 2*((B*a*b^2 - 7*A*b
^3)*x^7 + 2*(B*a^2*b - 7*A*a*b^2)*x^4 + (B*a^3 - 7*A*a^2*b)*x)*(a*b^2)^(2/3
)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 4*((B*a*b^2 - 7*A*b^3)
*x^7 + 2*(B*a^2*b - 7*A*a*b^2)*x^4 + (B*a^3 - 7*A*a^2*b)*x)*(a*b^2)^(2/3)*l
og(b*x + (a*b^2)^(1/3)))/(a^4*b^4*x^7 + 2*a^5*b^3*x^4 + a^6*b^2*x)]
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.71

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^3}\,dx = \frac{-18Aa^2+x^6(-28Ab^2+4Bab)+x^3(-49Aab+7Ba^2)}{18a^5x+36a^4bx^4+18a^3b^2x^7} + \text{RootSum}\left(19683t^3a^{10}b^2-2744A^3b^3+1176A^2Bab^2-168AB^2a^2b+8B^3a^3, \left(t\mapsto t\log\left(\frac{729\ldots}{196A^2b^2-56\ldots}\right.\right.\right.$$

```
[In] integrate((B*x**3+A)/x**2/(b*x**3+a)**3,x)
```

```
[Out] (-18*A*a**2 + x**6*(-28*A*b**2 + 4*B*a*b) + x**3*(-49*A*a*b + 7*B*a**2))/(1
8*a**5*x + 36*a**4*b*x**4 + 18*a**3*b**2*x**7) + RootSum(19683*_t**3*a**10*
b**2 - 2744*A**3*b**3 + 1176*A**2*B*a*b**2 - 168*A*B**2*a**2*b + 8*B**3*a**
3, Lambda(_t, _t*log(729*_t**2*a**7*b/(196*A**2*b**2 - 56*A*B*a*b + 4*B**2*
a**2) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^3}\,dx = \frac{4\,(Bab-7\,Ab^2)x^6+7\,(Ba^2-7\,Aab)x^3-18\,Aa^2}{18\,(a^3b^2x^7+2\,a^4bx^4+a^5x)} + \frac{2\,\sqrt{3}(Ba-7\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(Ba-7\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^3b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{2\,(Ba-7\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(4*(B*a*b - 7*A*b^2)*x^6 + 7*(B*a^2 - 7*A*a*b)*x^3 - 18*A*a^2)/(a^3*b^
2*x^7 + 2*a^4*b*x^4 + a^5*x) + 2/27*sqrt(3)*(B*a - 7*A*b)*arctan(1/3*sqrt(3
)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b*(a/b)^(1/3)) + 1/27*(B*a - 7*A*b)
*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(1/3)) - 2/27*(B*a - 7
*A*b)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^3}{x^2(a+bx^3)^3}\,dx = \frac{2\sqrt{3}(Ba-7Ab)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{1}{3}}a^3} - \frac{(Ba-7Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(-ab^2\right)^{\frac{1}{3}}a^3} - \frac{2\left(Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}-7Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^4} - \frac{A}{a^3x} + \frac{4Babx^5-10Ab^2x^5+7Ba^2x^2-13Aabx^2}{18(bx^3+a)^2a^3}$$

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 2/27*sqrt(3)*(B*a - 7*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/((-a*b^2)^(1/3)*a^3) - 1/27*(B*a - 7*A*b)*log(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/((-a*b^2)^(1/3)*a^3) - 2/27*(B*a*(-a/b)^(1/3) - 7*A*b*(-a/b)^
(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 - A/(a^3*x) + 1/18*(4*B*
a*b*x^5 - 10*A*b^2*x^5 + 7*B*a^2*x^2 - 13*A*a*b*x^2)/((b*x^3 + a)^2*a^3)
```

Mupad [B] (verification not implemented)

Time = 7.01 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx^3}{x^2(a+bx^3)^3}\,dx = \frac{2\ln\left(b^{1/3}x+a^{1/3}\right)(7Ab-Ba)}{27a^{10/3}b^{2/3}} - \frac{\frac{A}{a}+\frac{7x^3(7Ab-Ba)}{18a^2}+\frac{2bx^6(7Ab-Ba)}{9a^3}}{a^2x+2abx^4+b^2x^7} + \frac{2\ln\left(a^{1/3}-2b^{1/3}x+\sqrt{3}a^{1/3}1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}1\mathrm{i}}{2}\right)(7Ab-Ba)}{27a^{10/3}b^{2/3}} - \frac{2\ln\left(2b^{1/3}x-a^{1/3}+\sqrt{3}a^{1/3}1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}1\mathrm{i}}{2}\right)(7Ab-Ba)}{27a^{10/3}b^{2/3}}$$

```
[In] int((A + B*x^3)/(x^2*(a + b*x^3)^3),x)
```

```
[Out] (2*log(b^(1/3)*x + a^(1/3))*(7*A*b - B*a))/(27*a^(10/3)*b^(2/3)) - (A/a + (
7*x^3*(7*A*b - B*a))/(18*a^2) + (2*b*x^6*(7*A*b - B*a))/(9*a^3))/(a^2*x + b
^2*x^7 + 2*a*b*x^4) + (2*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((
3^(1/2)*1i)/2 - 1/2)*(7*A*b - B*a))/(27*a^(10/3)*b^(2/3)) - (2*log(3^(1/2)*
a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(7*A*b - B*a))/(
27*a^(10/3)*b^(2/3))
```

3.104 $\int \frac{A+Bx^3}{x^3(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 227

$$\int \frac{A+Bx^3}{x^3(a+bx^3)^3}\,dx = -\frac{5(4Ab-aB)}{18a^3bx^2}+\frac{Ab-aB}{6abx^2(a+bx^3)^2}+\frac{4Ab-aB}{9a^2bx^2(a+bx^3)} + \frac{5(4Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}\sqrt[3]{b}}-\frac{5(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}\sqrt[3]{b}} + \frac{5(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{11/3}\sqrt[3]{b}}$$

```
[Out] -5/18*(4*A*b-B*a)/a^3/b/x^2+1/6*(A*b-B*a)/a/b/x^2/(b*x^3+a)^2+1/9*(4*A*b-B*
a)/a^2/b/x^2/(b*x^3+a)-5/27*(4*A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)/b^(1
/3)+5/54*(4*A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(11/3)/b^(
1/3)+5/27*(4*A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(
11/3)/b^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used

= {468, 296, 331, 206, 31, 648, 631, 210, 642}

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^3}\,dx = \frac{5(4Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}\sqrt[3]{b}} + \frac{5(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{11/3}\sqrt[3]{b}} - \frac{5(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}\sqrt[3]{b}} - \frac{5(4Ab-aB)}{18a^3bx^2} + \frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)} + \frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2}$$

```
[In] Int[(A + B*x^3)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] (-5*(4*A*b - a*B))/(18*a^3*b*x^2) + (A*b - a*B)/(6*a*b*x^2*(a + b*x^3)^2) +
 (4*A*b - a*B)/(9*a^2*b*x^2*(a + b*x^3)) + (5*(4*A*b - a*B)*ArcTan[(a^(1/3)
 - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)*b^(1/3)) - (5*(4*A*
b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*b^(1/3)) + (5*(4*A*b - a*B)
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(11/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2}+\frac{(8Ab-2aB)\int\frac{1}{x^3(a+bx^3)^2}\,dx}{6ab}\\ &= \frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2}+\frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)}+\frac{(5(4Ab-aB))\int\frac{1}{x^3(a+bx^3)}\,dx}{9a^2b}\end{aligned}$$

$$= -\frac{5(4Ab-aB)}{18a^3bx^2} + \frac{Ab-aB}{6abx^2(a+bx^3)^2} + \frac{4Ab-aB}{9a^2bx^2(a+bx^3)} - \frac{(5(4Ab-aB))\int \frac{1}{a+bx^3}\,dx}{9a^3}$$

$$= -\frac{5(4Ab-aB)}{18a^3bx^2} + \frac{Ab-aB}{6abx^2(a+bx^3)^2} + \frac{4Ab-aB}{9a^2bx^2(a+bx^3)} - \frac{(5(4Ab-aB))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{11/3}} - \frac{(5(4Ab-aB))\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{11/3}}$$

$$= -\frac{5(4Ab-aB)}{18a^3bx^2} + \frac{Ab-aB}{6abx^2(a+bx^3)^2} + \frac{4Ab-aB}{9a^2bx^2(a+bx^3)} - \frac{5(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}\sqrt[3]{b}} - \frac{(5(4Ab-aB))\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{10/3}} + \frac{(5(4Ab-aB))\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{11/3}\sqrt[3]{b}}$$

$$= -\frac{5(4Ab-aB)}{18a^3bx^2} + \frac{Ab-aB}{6abx^2(a+bx^3)^2} + \frac{4Ab-aB}{9a^2bx^2(a+bx^3)} - \frac{5(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}\sqrt[3]{b}} + \frac{5(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{11/3}\sqrt[3]{b}} - \frac{(5(4Ab-aB))\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{11/3}\sqrt[3]{b}}$$

$$= -\frac{5(4Ab-aB)}{18a^3bx^2} + \frac{Ab-aB}{6abx^2(a+bx^3)^2} + \frac{4Ab-aB}{9a^2bx^2(a+bx^3)} + \frac{5(4Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}\sqrt[3]{b}} - \frac{5(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}\sqrt[3]{b}} + \frac{5(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{11/3}\sqrt[3]{b}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx^3}{x^3(a+bx^3)^3}\,dx$$

$$= \frac{-\frac{27a^{2/3}A}{x^2} + \frac{9a^{5/3}(-Ab+aB)x}{(a+bx^3)^2} + \frac{3a^{2/3}(-11Ab+5aB)x}{a+bx^3} + \frac{10\sqrt{3}(4Ab-aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} + \frac{10(-4Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}} +}{54a^{11/3}}$$

```
[In] Integrate[(A + B*x^3)/(x^3*(a + b*x^3)^3),x]
```

[Out] ((-27*a^(2/3)*A)/x^2 + (9*a^(5/3)*(-(A*b) + a*B)*x)/(a + b*x^3)^2 + (3*a^(2/3)*(-11*A*b + 5*a*B)*x)/(a + b*x^3) + (10*Sqrt[3]*(4*A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (10*(-4*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + (5*(4*A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3))/(54*a^(11/3))

Maple [A] (verified)

Time = 4.09 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.70

method	result
default	$-\frac{A}{2a^3x^2}-\frac{\frac{\left(\frac{11}{18}b^2A-\frac{5}{18}abB\right)x^4+\frac{a(7Ab-4Ba)x}{9}}{\left(bx^3+a\right)^2}+\frac{5(4Ab-Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{9}}{a^3}$
risch	$\frac{-\frac{5b(4Ab-Ba)x^6}{18a^3}-\frac{4(4Ab-Ba)x^3}{9a^2}-\frac{A}{2a}}{x^2\left(bx^3+a\right)^2}+\frac{5\left(\sum_{\_R=\mathrm{RootOf}\left(a^{11}b\_Z^3+64A^3b^3-48A^2Bab^2+12AB^2a^2b-B^3a^3\right)}\_R\ln\left(\left(-4\_R^3a^{11}b-19\right.\right.\right.}{27}$

[In] int((B*x^3+A)/x^3/(b*x^3+a)^3,x,method=_RETURNVERBOSE)

[Out] -1/2*A/a^3/x^2-1/a^3*(((11/18*b^2*A-5/18*a*b*B)*x^4+1/9*a*(7*A*b-4*B*a)*x)/(b*x^3+a)^2+5/9*(4*A*b-B*a)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. 2(179) = 358.

Time = 0.27 (sec) , antiderivative size = 812, normalized size of antiderivative = 3.58

$$\int \frac{A+Bx^3}{x^3(a+bx^3)^3}\,dx$$

$$= \left[\frac{15\,(Ba^3b^2-4\,Aa^2b^3)x^6-27\,Aa^4b+24\,(Ba^4b-4\,Aa^3b^2)x^3-15\,\sqrt{\frac{1}{3}}((Ba^2b^3-4\,Aab^4)x^8+2\,(Ba^3b^2-4\right.$$

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(15*(B*a^3*b^2 - 4*A*a^2*b^3)*x^6 - 27*A*a^4*b + 24*(B*a^4*b - 4*A*a^
3*b^2)*x^3 - 15*sqrt(1/3)*((B*a^2*b^3 - 4*A*a*b^4)*x^8 + 2*(B*a^3*b^2 - 4*A
*a^2*b^3)*x^5 + (B*a^4*b - 4*A*a^3*b^2)*x^2)*sqrt((-a^2*b)^(1/3)/b)*log((2*
a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2
/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - 5*((B*a*b^
2 - 4*A*b^3)*x^8 + 2*(B*a^2*b - 4*A*a*b^2)*x^5 + (B*a^3 - 4*A*a^2*b)*x^2)*(
-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 10*((B*a
*b^2 - 4*A*b^3)*x^8 + 2*(B*a^2*b - 4*A*a*b^2)*x^5 + (B*a^3 - 4*A*a^2*b)*x^2
)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))/(a^5*b^3*x^8 + 2*a^6*b^2*x^5
+ a^7*b*x^2), 1/54*(15*(B*a^3*b^2 - 4*A*a^2*b^3)*x^6 - 27*A*a^4*b + 24*(B*a
^4*b - 4*A*a^3*b^2)*x^3 + 30*sqrt(1/3)*((B*a^2*b^3 - 4*A*a*b^4)*x^8 + 2*(B*
a^3*b^2 - 4*A*a^2*b^3)*x^5 + (B*a^4*b - 4*A*a^3*b^2)*x^2)*sqrt(-(-a^2*b)^(1
/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2
*b)^(1/3)/b)/a^2) - 5*((B*a*b^2 - 4*A*b^3)*x^8 + 2*(B*a^2*b - 4*A*a*b^2)*x^
5 + (B*a^3 - 4*A*a^2*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x
- (-a^2*b)^(1/3)*a) + 10*((B*a*b^2 - 4*A*b^3)*x^8 + 2*(B*a^2*b - 4*A*a*b^2)
*x^5 + (B*a^3 - 4*A*a^2*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))
/(a^5*b^3*x^8 + 2*a^6*b^2*x^5 + a^7*b*x^2)]
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.63

$$\int \frac{A+Bx^3}{x^3(a+bx^3)^3}\,dx = \frac{-9Aa^2+x^6(-20Ab^2+5Bab)+x^3(-32Aab+8Ba^2)}{18a^5x^2+36a^4bx^5+18a^3b^2x^8} + \mathrm{RootSum}\left(19683t^3a^{11}b+8000A^3b^3-6000A^2Bab^2+1500AB^2a^2b-125B^3a^3,\left(t\mapsto t\log\left(\frac{27ta^4}{-20Ab+5\right.\right.\right.$$

```
[In] integrate((B*x**3+A)/x**3/(b*x**3+a)**3,x)
```

```
[Out] (-9*A*a**2 + x**6*(-20*A*b**2 + 5*B*a*b) + x**3*(-32*A*a*b + 8*B*a**2))/(18
*a**5*x**2 + 36*a**4*b*x**5 + 18*a**3*b**2*x**8) + RootSum(19683*_t**3*a**1
1*b + 8000*A**3*b**3 - 6000*A**2*B*a*b**2 + 1500*A*B**2*a**2*b - 125*B**3*a
**3, Lambda(_t, _t*log(27*_t*a**4/(-20*A*b + 5*B*a) + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^3}{x^3(a+bx^3)^3}\,dx = \frac{5\,(Bab-4\,Ab^2)x^6+8\,(Ba^2-4\,Aab)x^3-9\,Aa^2}{18\,(a^3b^2x^8+2\,a^4bx^5+a^5x^2)} + \frac{5\,\sqrt{3}(Ba-4\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{5\,(Ba-4\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{5\,(Ba-4\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(5*(B*a*b - 4*A*b^2)*x^6 + 8*(B*a^2 - 4*A*a*b)*x^3 - 9*A*a^2)/(a^3*b^2
*x^8 + 2*a^4*b*x^5 + a^5*x^2) + 5/27*sqrt(3)*(B*a - 4*A*b)*arctan(1/3*sqrt(
3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b*(a/b)^(2/3)) - 5/54*(B*a - 4*A*b
)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(2/3)) + 5/27*(B*a -
4*A*b)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^3}\,dx = -\frac{5\,(Ba-4\,Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^4} + \frac{5\,\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}\,Ba-4\left(-ab^2\right)^{\frac{1}{3}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b} + \frac{5\left(\left(-ab^2\right)^{\frac{1}{3}}\,Ba-4\left(-ab^2\right)^{\frac{1}{3}}\,Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^4b} + \frac{5\,Babx^6-20\,Ab^2x^6+8\,Ba^2x^3-32\,Aabx^3-9\,Aa^2}{18\left(bx^4+ax\right)^2a^3}$$

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -5/27*(B*a - 4*A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 + 5/27*sqrt
(3)*((-a*b^2)^(1/3)*B*a - 4*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (
-a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b) + 5/54*((-a*b^2)^(1/3)*B*a - 4*(-a*b^2)^
(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b) + 1/18*(5*B*a*b
*x^6 - 20*A*b^2*x^6 + 8*B*a^2*x^3 - 32*A*a*b*x^3 - 9*A*a^2)/((b*x^4 + a*x)^
2*a^3)
```

Mupad [B] (verification not implemented)

Time = 6.87 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^3}\,dx = -\frac{\frac{A}{2\,a}+\frac{4\,x^3\,(4\,A\,b-B\,a)}{9\,a^2}+\frac{5\,b\,x^6\,(4\,A\,b-B\,a)}{18\,a^3}}{a^2\,x^2+2\,a\,b\,x^5+b^2\,x^8} - \frac{5\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(4\,A\,b-B\,a)}{27\,a^{11/3}\,b^{1/3}} + \frac{5\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(4\,A\,b-B\,a)}{27\,a^{11/3}\,b^{1/3}} - \frac{5\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(4\,A\,b-B\,a)}{27\,a^{11/3}\,b^{1/3}}$$

```
[In] int((A + B*x^3)/(x^3*(a + b*x^3)^3),x)
```

```
[Out] (5*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(
4*A*b - B*a))/(27*a^(11/3)*b^(1/3)) - (5*log(b^(1/3)*x + a^(1/3))*(4*A*b -
```

```
B*a))/(27*a^(11/3)*b^(1/3)) - (A/(2*a) + (4*x^3*(4*A*b - B*a))/(9*a^2) + (5
*b*x^6*(4*A*b - B*a))/(18*a^3))/(a^2*x^2 + b^2*x^8 + 2*a*b*x^5) - (5*log(3^
(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(4*A*b - B
*a))/(27*a^(11/3)*b^(1/3))
```

3.105 $\int \frac{A+Bx^3}{x^5(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 246

$$\int \frac{A+Bx^3}{x^5(a+bx^3)^3}\,dx = -\frac{7(5Ab-2aB)}{36a^3bx^4}+\frac{7(5Ab-2aB)}{9a^4x}+\frac{Ab-aB}{6abx^4(a+bx^3)^2}+\frac{5Ab-2aB}{9a^2bx^4(a+bx^3)}-\frac{7\sqrt[3]{b}(5Ab-2aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{13/3}}-\frac{7\sqrt[3]{b}(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{13/3}}+\frac{7\sqrt[3]{b}(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{13/3}}$$

```
[Out] -7/36*(5*A*b-2*B*a)/a^3/b/x^4+7/9*(5*A*b-2*B*a)/a^4/x+1/6*(A*b-B*a)/a/b/x^4
/(b*x^3+a)^2+1/9*(5*A*b-2*B*a)/a^2/b/x^4/(b*x^3+a)-7/27*b^(1/3)*(5*A*b-2*B*
a)*ln(a^(1/3)+b^(1/3)*x)/a^(13/3)+7/54*b^(1/3)*(5*A*b-2*B*a)*ln(a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(13/3)-7/27*b^(1/3)*(5*A*b-2*B*a)*arctan(1/3*
(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(13/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {468, 296, 331, 298, 31, 648, 631, 210, 642}

$$\int \frac{A+Bx^3}{x^5(a+bx^3)^3}\,dx = -\frac{7\sqrt[3]{b}(5Ab-2aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{13/3}} + \frac{7\sqrt[3]{b}(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{13/3}} - \frac{7\sqrt[3]{b}(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{13/3}} + \frac{7(5Ab-2aB)}{9a^4x} - \frac{7(5Ab-2aB)}{36a^3bx^4} + \frac{5Ab-2aB}{9a^2bx^4(a+bx^3)} + \frac{Ab-aB}{6abx^4(a+bx^3)^2}$$

```
[In] Int[(A + B*x^3)/(x^5*(a + b*x^3)^3),x]
```

```
[Out] (-7*(5*A*b - 2*a*B))/(36*a^3*b*x^4) + (7*(5*A*b - 2*a*B))/(9*a^4*x) + (A*b
 - a*B)/(6*a*b*x^4*(a + b*x^3)^2) + (5*A*b - 2*a*B)/(9*a^2*b*x^4*(a + b*x^3)
) - (7*b^(1/3)*(5*A*b - 2*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1
/3))])/(9*Sqrt[3]*a^(13/3)) - (7*b^(1/3)*(5*A*b - 2*a*B)*Log[a^(1/3) + b^(1
/3)*x])/(27*a^(13/3)) + (7*b^(1/3)*(5*A*b - 2*a*B)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(54*a^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{(10Ab-4aB)\int \frac{1}{x^5(a+bx^3)^2}\,dx}{6ab}$$

$$= \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} + \frac{(7(5Ab-2aB))\int \frac{1}{x^5(a+bx^3)}\,dx}{9a^2b}$$

$$= -\frac{7(5Ab-2aB)}{36a^3bx^4} + \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} - \frac{(7(5Ab-2aB))\int \frac{1}{x^2(a+bx^3)}\,dx}{9a^3}$$

$$= -\frac{7(5Ab-2aB)}{36a^3bx^4} + \frac{7(5Ab-2aB)}{9a^4x} + \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} + \frac{(7b(5Ab-2aB))\int \frac{x}{a+bx^3}\,dx}{9a^4}$$

$$= -\frac{7(5Ab-2aB)}{36a^3bx^4} + \frac{7(5Ab-2aB)}{9a^4x} + \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} - \frac{\left(7b^{2/3}(5Ab-2aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{13/3}} + \frac{\left(7b^{2/3}(5Ab-2aB)\right)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{13/3}}$$

$$= -\frac{7(5Ab-2aB)}{36a^3bx^4} + \frac{7(5Ab-2aB)}{9a^4x} + \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} - \frac{7\sqrt[3]{b}(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{13/3}} + \frac{\left(7\sqrt[3]{b}(5Ab-2aB)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{54a^{13/3}} + \frac{\left(7b^{2/3}(5Ab-2aB)\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^4}$$

$$= -\frac{7(5Ab-2aB)}{36a^3bx^4} + \frac{7(5Ab-2aB)}{9a^4x} + \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} - \frac{7\sqrt[3]{b}(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{13/3}} + \frac{7\sqrt[3]{b}(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{13/3}} + \frac{\left(7\sqrt[3]{b}(5Ab-2aB)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{13/3}}$$

$$= -\frac{7(5Ab-2aB)}{36a^3bx^4} + \frac{7(5Ab-2aB)}{9a^4x} + \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} - \frac{7\sqrt[3]{b}(5Ab-2aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{13/3}} - \frac{7\sqrt[3]{b}(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{13/3}} + \frac{7\sqrt[3]{b}(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{13/3}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx$$

$$= \frac{-\frac{27a^{4/3}A}{x^4} - \frac{108\sqrt[3]{a}(-3Ab+aB)}{x} - \frac{18a^{4/3}b(-Ab+aB)x^2}{\left(a+bx^3\right)^2} - \frac{12\sqrt[3]{a}b(-8Ab+5aB)x^2}{a+bx^3} - 28\sqrt{3}\sqrt[3]{b}(5Ab-2aB)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) \cdots}{108a^{13/3}}$$

```
[In] Integrate[(A + B*x^3)/(x^5*(a + b*x^3)^3),x]
```

```
[Out] ((-27*a^(4/3)*A)/x^4 - (108*a^(1/3)*(-3*A*b + a*B))/x - (18*a^(4/3)*b*(-(A*
b) + a*B)*x^2)/(a + b*x^3)^2 - (12*a^(1/3)*b*(-8*A*b + 5*a*B)*x^2)/(a + b*x
^3) - 28*Sqrt[3]*b^(1/3)*(5*A*b - 2*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))
/Sqrt[3]] + 28*b^(1/3)*(-5*A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x] + 14*b^(1/
3)*(5*A*b - 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(108*a^(
13/3))
```

Maple [A] (verified)

Time = 4.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.71

method	result
default	$-\frac{A}{4a^3x^4}-\frac{-3Ab+Ba}{a^4x}+\frac{b\left(\frac{\left(\frac{8}{9}b^2A-\frac{5}{9}abB\right)x^5+\frac{a(19Ab-13Ba)x^2}{18}}{\left(bx^3+a\right)^2}+\left(\frac{35Ab}{9}-\frac{14Ba}{9}\right)\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\cdots\right.\right.}{a^4}$
risch	$\frac{\frac{7b^2(5Ab-2Ba)x^9}{9a^4}+\frac{49b(5Ab-2Ba)x^6}{36a^3}+\frac{(5Ab-2Ba)x^3}{2a^2}-\frac{A}{4a}}{x^4(bx^3+a)^2}+\frac{7\left(\sum_{\_R=\text{RootOf}\left(a^{13}\_Z^3+125A^3b^4-150A^2Bab^3+60AB^2a^2b^2-8B^3a^3b\right)}\_R\cdots\right.}{\cdots}$

```
[In] int((B*x^3+A)/x^5/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*A/a^3/x^4-(-3*A*b+B*a)/a^4/x+1/a^4*b*(((8/9*b^2*A-5/9*a*b*B)*x^5+1/18*
a*(19*A*b-13*B*a)*x^2)/(b*x^3+a)^2+(35/9*A*b-14/9*B*a)*(-1/3/b/(a/b)^(1/3)*
ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3
^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.49

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx =$$

$$-\frac{84\,(2\,Bab^2-5\,Ab^3)x^9+147\,(2\,Ba^2b-5\,Aab^2)x^6+27\,Aa^3+54\,(2\,Ba^3-5\,Aa^2b)x^3+28\,\sqrt{3}((2\,Bab^2-\cdots}{\cdots}$$

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/108*(84*(2*B*a*b^2 - 5*A*b^3)*x^9 + 147*(2*B*a^2*b - 5*A*a*b^2)*x^6 + 27
*A*a^3 + 54*(2*B*a^3 - 5*A*a^2*b)*x^3 + 28*sqrt(3)*((2*B*a*b^2 - 5*A*b^3)*x
^10 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^7 + (2*B*a^3 - 5*A*a^2*b)*x^4)*(-b/a)^(1/
3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqrt(3)) - 14*((2*B*a*b^2 - 5*A*
b^3)*x^10 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^7 + (2*B*a^3 - 5*A*a^2*b)*x^4)*(-b/
a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a*(-b/a)^(1/3)) + 28*((2*B*a*b^2 -
5*A*b^3)*x^10 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^7 + (2*B*a^3 - 5*A*a^2*b)*x^4)*
```

```
(-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)))/(a^4*b^2*x^10 + 2*a^5*b*x^7 + a^6*x
^4)
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.77

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx$$
$$=\operatorname{RootSum}\left(19683t^3a^{13}+42875A^3b^4-51450A^2Bab^3+20580AB^2a^2b^2-2744B^3a^3b,\left(t\mapsto t\log\left(\frac{}{1225A^2b\cdots}\right.\right.\right.$$
$$+\frac{-9Aa^3+x^9\cdot(140Ab^3-56Bab^2)+x^6\cdot(245Aab^2-98Ba^2b)+x^3\cdot(90Aa^2b-36Ba^3)}{36a^6x^4+72a^5bx^7+36a^4b^2x^{10}}$$

```
[In] integrate((B*x**3+A)/x**5/(b*x**3+a)**3,x)
```

```
[Out] RootSum(19683*_t**3*a**13 + 42875*A**3*b**4 - 51450*A**2*B*a*b**3 + 20580*A
*B**2*a**2*b**2 - 2744*B**3*a**3*b, Lambda(_t, _t*log(729*_t**2*a**9/(1225*
A**2*b**3 - 980*A*B*a*b**2 + 196*B**2*a**2*b) + x))) + (-9*A*a**3 + x**9*(1
40*A*b**3 - 56*B*a*b**2) + x**6*(245*A*a*b**2 - 98*B*a**2*b) + x**3*(90*A*a
**2*b - 36*B*a**3))/(36*a**6*x**4 + 72*a**5*b*x**7 + 36*a**4*b**2*x**10)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx$$
$$=-\frac{28\,(2\,Bab^2-5\,Ab^3)x^9+49\,(2\,Ba^2b-5\,Aab^2)x^6+9\,Aa^3+18\,(2\,Ba^3-5\,Aa^2b)x^3}{36\,(a^4b^2x^{10}+2\,a^5bx^7+a^6x^4)}$$
$$-\frac{7\,\sqrt{3}(2\,Ba-5\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$
$$-\frac{7\,(2\,Ba-5\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^4\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{7\,(2\,Ba-5\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^4\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/36*(28*(2*B*a*b^2 - 5*A*b^3)*x^9 + 49*(2*B*a^2*b - 5*A*a*b^2)*x^6 + 9*A*
a^3 + 18*(2*B*a^3 - 5*A*a^2*b)*x^3)/(a^4*b^2*x^10 + 2*a^5*b*x^7 + a^6*x^4)
```

```
- 7/27*sqrt(3)*(2*B*a - 5*A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)
^(1/3))/(a^4*(a/b)^(1/3)) - 7/54*(2*B*a - 5*A*b)*log(x^2 - x*(a/b)^(1/3) +
(a/b)^(2/3))/(a^4*(a/b)^(1/3)) + 7/27*(2*B*a - 5*A*b)*log(x + (a/b)^(1/3))/
(a^4*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx = \frac{7\left(2\,Bab\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,Ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^5} + \frac{7\sqrt{3}\left(2\left(-ab^2\right)^{\frac{2}{3}}Ba-5\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^5b} - \frac{7\left(2\left(-ab^2\right)^{\frac{2}{3}}Ba-5\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^5b} - \frac{10\,Bab^2x^5-16\,Ab^3x^5+13\,Ba^2bx^2-19\,Aab^2x^2}{18\left(bx^3+a\right)^2a^4} - \frac{4\,Bax^3-12\,Abx^3+Aa}{4\,a^4x^4}$$

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 7/27*(2*B*a*b*(-a/b)^(1/3) - 5*A*b^2*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x -
 (-a/b)^(1/3)))/a^5 + 7/27*sqrt(3)*(2*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2)^(2/3)
*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^5*b) - 7/54*
(2*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3))/(a^5*b) - 1/18*(10*B*a*b^2*x^5 - 16*A*b^3*x^5 + 13*B*a^2*b*x^2
- 19*A*a*b^2*x^2)/((b*x^3 + a)^2*a^4) - 1/4*(4*B*a*x^3 - 12*A*b*x^3 + A*a)/
(a^4*x^4)
```

Mupad [B] (verification not implemented)

Time = 6.86 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx$$
$$=\frac{\frac{x^3\,(5\,A\,b-2\,B\,a)}{2\,a^2}-\frac{A}{4\,a}+\frac{7\,b^2\,x^9\,(5\,A\,b-2\,B\,a)}{9\,a^4}+\frac{49\,b\,x^6\,(5\,A\,b-2\,B\,a)}{36\,a^3}}{a^2\,x^4+2\,a\,b\,x^7+b^2\,x^{10}}$$
$$+\frac{7\,(-b)^{1/3}\,\ln\left(a^{1/3}\,(-b)^{8/3}+b^3\,x\right)\,(5\,A\,b-2\,B\,a)}{27\,a^{13/3}}$$
$$+\frac{7\,(-b)^{1/3}\,\ln\left(a^{1/3}\,(-b)^{8/3}-2\,b^3\,x+\sqrt{3}\,a^{1/3}\,(-b)^{8/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,A\,b-2\,B\,a)}{27\,a^{13/3}}$$
$$-\frac{7\,(-b)^{1/3}\,\ln\left(2\,b^3\,x-a^{1/3}\,(-b)^{8/3}+\sqrt{3}\,a^{1/3}\,(-b)^{8/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,A\,b-2\,B\,a)}{27\,a^{13/3}}$$

```
[In] int((A + B*x^3)/(x^5*(a + b*x^3)^3),x)
```

```
[Out] ((x^3*(5*A*b - 2*B*a))/(2*a^2) - A/(4*a) + (7*b^2*x^9*(5*A*b - 2*B*a))/(9*a
^4) + (49*b*x^6*(5*A*b - 2*B*a))/(36*a^3))/(a^2*x^4 + b^2*x^10 + 2*a*b*x^7)
 + (7*(-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) + b^3*x)*(5*A*b - 2*B*a))/(27*a^(13
/3)) + (7*(-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) - 2*b^3*x + 3^(1/2)*a^(1/3)*(-b
)^(8/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(5*A*b - 2*B*a))/(27*a^(13/3)) - (7*(-b)
^(1/3)*log(2*b^3*x - a^(1/3)*(-b)^(8/3) + 3^(1/2)*a^(1/3)*(-b)^(8/3)*1i)*((
3^(1/2)*1i)/2 + 1/2)*(5*A*b - 2*B*a))/(27*a^(13/3))
```

3.106 $\int \frac{A+Bx^3}{x^6(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 246

$$\int \frac{A+Bx^3}{x^6(a+bx^3)^3}\,dx = -\frac{4(11Ab-5aB)}{45a^3bx^5}+\frac{2(11Ab-5aB)}{9a^4x^2}+\frac{Ab-aB}{6abx^5(a+bx^3)^2}+\frac{11Ab-5aB}{18a^2bx^5(a+bx^3)}-\frac{4b^{2/3}(11Ab-5aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{14/3}}+\frac{4b^{2/3}(11Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{14/3}}-\frac{2b^{2/3}(11Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{14/3}}$$

```
[Out] -4/45*(11*A*b-5*B*a)/a^3/b/x^5+2/9*(11*A*b-5*B*a)/a^4/x^2+1/6*(A*b-B*a)/a/b
/x^5/(b*x^3+a)^2+1/18*(11*A*b-5*B*a)/a^2/b/x^5/(b*x^3+a)+4/27*b^(2/3)*(11*A
*b-5*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(14/3)-2/27*b^(2/3)*(11*A*b-5*B*a)*ln(a^(
2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(14/3)-4/27*b^(2/3)*(11*A*b-5*B*a)*ar
ctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(14/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {468, 296, 331, 206, 31, 648, 631, 210, 642}

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^3}\,dx = -\frac{4b^{2/3}(11Ab-5aB)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{14/3}} - \frac{2b^{2/3}(11Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{14/3}} + \frac{4b^{2/3}(11Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{14/3}} + \frac{2(11Ab-5aB)}{9a^4x^2} - \frac{4(11Ab-5aB)}{45a^3bx^5} + \frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)} + \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2}$$

```
[In] Int[(A + B*x^3)/(x^6*(a + b*x^3)^3),x]
```

```
[Out] (-4*(11*A*b - 5*a*B))/(45*a^3*b*x^5) + (2*(11*A*b - 5*a*B))/(9*a^4*x^2) + (
A*b - a*B)/(6*a*b*x^5*(a + b*x^3)^2) + (11*A*b - 5*a*B)/(18*a^2*b*x^5*(a +
b*x^3)) - (4*b^(2/3)*(11*A*b - 5*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(9*Sqrt[3]*a^(14/3)) + (4*b^(2/3)*(11*A*b - 5*a*B)*Log[a^(1/3
) + b^(1/3)*x])/(27*a^(14/3)) - (2*b^(2/3)*(11*A*b - 5*a*B)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*a^(14/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2} + \frac{(11Ab-5aB)\int \frac{1}{x^6\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2} + \frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)} + \frac{(4(11Ab-5aB))\int \frac{1}{x^6\left(a+bx^3\right)}\,dx}{9a^2b}$$

$$= -\frac{4(11Ab-5aB)}{45a^3bx^5} + \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2} + \frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)} - \frac{(4(11Ab-5aB))\int \frac{1}{x^3\left(a+bx^3\right)}\,dx}{9a^3}$$

$$= -\frac{4(11Ab-5aB)}{45a^3bx^5} + \frac{2(11Ab-5aB)}{9a^4x^2} + \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2} + \frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)} + \frac{(4b(11Ab-5aB))\int \frac{1}{a+bx^3}\,dx}{9a^4}$$

$$= -\frac{4(11Ab-5aB)}{45a^3bx^5} + \frac{2(11Ab-5aB)}{9a^4x^2} + \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2} + \frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)} + \frac{(4b(11Ab-5aB))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{14/3}} + \frac{(4b(11Ab-5aB))\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{14/3}}$$

$$= -\frac{4(11Ab-5aB)}{45a^3bx^5} + \frac{2(11Ab-5aB)}{9a^4x^2} + \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2} + \frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)} + \frac{4b^{2/3}(11Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{14/3}} - \frac{\left(2b^{2/3}(11Ab-5aB)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{27a^{14/3}} + \frac{(2b(11Ab-5aB))\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{13/3}}$$

$$= -\frac{4(11Ab-5aB)}{45a^3bx^5} + \frac{2(11Ab-5aB)}{9a^4x^2} + \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2} + \frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)} + \frac{4b^{2/3}(11Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{14/3}} - \frac{2b^{2/3}(11Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{14/3}} + \frac{\left(4b^{2/3}(11Ab-5aB)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{9a^{14/3}}$$

$$= -\frac{4(11Ab-5aB)}{45a^3bx^5} + \frac{2(11Ab-5aB)}{9a^4x^2} + \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2} + \frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)} - \frac{4b^{2/3}(11Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{14/3}} + \frac{4b^{2/3}(11Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{14/3}} - \frac{2b^{2/3}(11Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{14/3}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^3}\,dx$$

$$= \frac{-\frac{54a^{5/3}A}{x^5} - \frac{135a^{2/3}(-3Ab+aB)}{x^2} - \frac{45a^{5/3}b(-Ab+aB)x}{\left(a+bx^3\right)^2} - \frac{15a^{2/3}b(-17Ab+11aB)x}{a+bx^3} - 40\sqrt{3}b^{2/3}(11Ab-5aB)\arctan\left(\frac{1-\cdots}{\cdots}\right)}{270a\cdots}$$

```
[In] Integrate[(A + B*x^3)/(x^6*(a + b*x^3)^3),x]
```

```
[Out] ((-54*a^(5/3)*A)/x^5 - (135*a^(2/3)*(-3*A*b + a*B))/x^2 - (45*a^(5/3)*b*(-(
A*b) + a*B)*x)/(a + b*x^3)^2 - (15*a^(2/3)*b*(-17*A*b + 11*a*B)*x)/(a + b*x
^3) - 40*Sqrt[3]*b^(2/3)*(11*A*b - 5*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3)
)/Sqrt[3]] + 40*b^(2/3)*(11*A*b - 5*a*B)*Log[a^(1/3) + b^(1/3)*x] + 20*b^(2
/3)*(-11*A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(270*
a^(14/3))
```

Maple [A] (verified)

Time = 4.13 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.71

method	result
default	$-\frac{A}{5a^3x^5}-\frac{-3Ab+Ba}{2x^2a^4}+\frac{b\left(\frac{\left(\frac{17}{18}b^2A-\frac{11}{18}abB\right)x^4+\frac{a(10Ab-7Ba)x}{9}}{\left(bx^3+a\right)^2}+\frac{4(11Ab-5Ba)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan}{}\right.}{9}\right.}{a^4}$
risch	$\frac{\frac{2b^2(11Ab-5Ba)x^9}{9a^4}+\frac{16b(11Ab-5Ba)x^6}{45a^3}+\frac{(11Ab-5Ba)x^3}{10a^2}-\frac{A}{5a}}{x^5(bx^3+a)^2}+\frac{4\left(\sum_{\_R=\text{RootOf}\left(a^{14}\_Z^3-1331A^3b^5+1815A^2Bab^4-825AB^2a^2b^3+125B^3a^3b\right.}\right.}{}$

[In] int((B*x^3+A)/x^6/(b*x^3+a)^3,x,method=_RETURNVERBOSE)

[Out] -1/5*A/a^3/x^5-1/2*(-3*A*b+B*a)/x^2/a^4+1/a^4*b*(((17/18*b^2*A-11/18*a*b*B)*x^4+1/9*a*(10*A*b-7*B*a)*x)/(b*x^3+a)^2+4/9*(11*A*b-5*B*a)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.56

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^3}\,dx =$$

$$-\frac{60\,(5\,Bab^2-11\,Ab^3)x^9+96\,(5\,Ba^2b-11\,Aab^2)x^6+54\,Aa^3+27\,(5\,Ba^3-11\,Aa^2b)x^3+40\,\sqrt{3}((5\,Bab^2}{}$$

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/270*(60*(5*B*a*b^2 - 11*A*b^3)*x^9 + 96*(5*B*a^2*b - 11*A*a*b^2)*x^6 + 5
4*A*a^3 + 27*(5*B*a^3 - 11*A*a^2*b)*x^3 + 40*sqrt(3)*((5*B*a*b^2 - 11*A*b^3
)*x^11 + 2*(5*B*a^2*b - 11*A*a*b^2)*x^8 + (5*B*a^3 - 11*A*a^2*b)*x^5)*(b^2/
a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 20*(
(5*B*a*b^2 - 11*A*b^3)*x^11 + 2*(5*B*a^2*b - 11*A*a*b^2)*x^8 + (5*B*a^3 - 1
1*A*a^2*b)*x^5)*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*b*x*(b^2/a^2)^(1/3) + a^2*(
b^2/a^2)^(2/3)) + 40*((5*B*a*b^2 - 11*A*b^3)*x^11 + 2*(5*B*a^2*b - 11*A*a*b
^2)*x^8 + (5*B*a^3 - 11*A*a^2*b)*x^5)*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)
^(1/3)))/(a^4*b^2*x^11 + 2*a^5*b*x^8 + a^6*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^3}\,dx$$
$$= \mathrm{RootSum}\left(19683t^3a^{14}-85184A^3b^5+116160A^2Bab^4-52800AB^2a^2b^3+8000B^3a^3b^2,\left(t\mapsto t\log\left(-\frac{}{-4}\right.\right.\right.$$
$$+\frac{-18Aa^3+x^9\cdot\left(220Ab^3-100Bab^2\right)+x^6\cdot\left(352Aab^2-160Ba^2b\right)+x^3\cdot\left(99Aa^2b-45Ba^3\right)}{90a^6x^5+180a^5bx^8+90a^4b^2x^{11}}$$

```
[In] integrate((B*x**3+A)/x**6/(b*x**3+a)**3,x)
```

```
[Out] RootSum(19683*_t**3*a**14 - 85184*A**3*b**5 + 116160*A**2*B*a*b**4 - 52800*
A*B**2*a**2*b**3 + 8000*B**3*a**3*b**2, Lambda(_t, _t*log(-27*_t*a**5/(-44*
A*b**2 + 20*B*a*b) + x))) + (-18*A*a**3 + x**9*(220*A*b**3 - 100*B*a*b**2)
+ x**6*(352*A*a*b**2 - 160*B*a**2*b) + x**3*(99*A*a**2*b - 45*B*a**3))/(90*
a**6*x**5 + 180*a**5*b*x**8 + 90*a**4*b**2*x**11)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^3}{x^6 (a+bx^3)^3}\,dx = -\frac{20\,(5\,Bab^2-11\,Ab^3)x^9+32\,(5\,Ba^2b-11\,Aab^2)x^6+18\,Aa^3+9\,(5\,Ba^3-11\,Aa^2b)x^3}{90\,(a^4b^2x^{11}+2\,a^5bx^8+a^6x^5)} - \frac{4\,\sqrt{3}(5\,Ba-11\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{2\,(5\,Ba-11\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{4\,(5\,Ba-11\,Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/90*(20*(5*B*a*b^2 - 11*A*b^3)*x^9 + 32*(5*B*a^2*b - 11*A*a*b^2)*x^6 + 18
*A*a^3 + 9*(5*B*a^3 - 11*A*a^2*b)*x^3)/(a^4*b^2*x^11 + 2*a^5*b*x^8 + a^6*x^
5) - 4/27*sqrt(3)*(5*B*a - 11*A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(
a/b)^(1/3))/(a^4*(a/b)^(2/3)) + 2/27*(5*B*a - 11*A*b)*log(x^2 - x*(a/b)^(1/
3) + (a/b)^(2/3))/(a^4*(a/b)^(2/3)) - 4/27*(5*B*a - 11*A*b)*log(x + (a/b)^(
1/3))/(a^4*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^3}{x^6 (a+bx^3)^3}\,dx = -\frac{4\,\sqrt{3}\left(5\,(-ab^2)^{\frac{1}{3}}\,Ba-11\,(-ab^2)^{\frac{1}{3}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^5} + \frac{4\,(5\,Bab-11\,Ab^2)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^5} - \frac{2\left(5\,(-ab^2)^{\frac{1}{3}}\,Ba-11\,(-ab^2)^{\frac{1}{3}}\,Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^5} - \frac{11\,Bab^2x^4-17\,Ab^3x^4+14\,Ba^2bx-20\,Aab^2x}{18\,(bx^3+a)^2a^4} - \frac{5\,Bax^3-15\,Abx^3+2\,Aa}{10\,a^4x^5}$$

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -4/27*sqrt(3)*(5*(-a*b^2)^(1/3)*B*a - 11*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqr
t(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^5 + 4/27*(5*B*a*b - 11*A*b^2)*(-a
/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^5 - 2/27*(5*(-a*b^2)^(1/3)*B*a - 11*
(-a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^5 - 1/18*(11
*B*a*b^2*x^4 - 17*A*b^3*x^4 + 14*B*a^2*b*x - 20*A*a*b^2*x)/((b*x^3 + a)^2*a
^4) - 1/10*(5*B*a*x^3 - 15*A*b*x^3 + 2*A*a)/(a^4*x^5)
```

Mupad [B] (verification not implemented)

Time = 6.83 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^3}\,dx=\frac{\frac{x^3\,\left(11\,A\,b-5\,B\,a\right)}{10\,a^2}-\frac{A}{5\,a}+\frac{2\,b^2\,x^9\,\left(11\,A\,b-5\,B\,a\right)}{9\,a^4}+\frac{16\,b\,x^6\,\left(11\,A\,b-5\,B\,a\right)}{45\,a^3}}{a^2\,x^5+2\,a\,b\,x^8+b^2\,x^{11}}+\frac{4\,b^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(11\,A\,b-5\,B\,a\right)}{27\,a^{14/3}}-\frac{4\,b^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(11\,A\,b-5\,B\,a\right)}{27\,a^{14/3}}+\frac{4\,b^{2/3}\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(11\,A\,b-5\,B\,a\right)}{27\,a^{14/3}}$$

```
[In] int((A + B*x^3)/(x^6*(a + b*x^3)^3),x)
```

```
[Out] ((x^3*(11*A*b - 5*B*a))/(10*a^2) - A/(5*a) + (2*b^2*x^9*(11*A*b - 5*B*a))/(
9*a^4) + (16*b*x^6*(11*A*b - 5*B*a))/(45*a^3))/(a^2*x^5 + b^2*x^11 + 2*a*b*
x^8) + (4*b^(2/3)*log(b^(1/3)*x + a^(1/3))*(11*A*b - 5*B*a))/(27*a^(14/3))
 - (4*b^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/
2 + 1/2)*(11*A*b - 5*B*a))/(27*a^(14/3)) + (4*b^(2/3)*log(3^(1/2)*a^(1/3)*1
i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(11*A*b - 5*B*a))/(27*a^(
14/3))
```

3.107 $\int \frac{x^8}{(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 70

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)}\,dx = \frac{x^3}{3bd} + \frac{a^2\log(a+bx^3)}{3b^2(bc-ad)} - \frac{c^2\log(c+dx^3)}{3d^2(bc-ad)}$$

```
[Out] 1/3*x^3/b/d+1/3*a^2*ln(b*x^3+a)/b^2/(-a*d+b*c)-1/3*c^2*ln(d*x^3+c)/d^2/(-a*
d+b*c)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 84}

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)}\,dx = \frac{a^2\log(a+bx^3)}{3b^2(bc-ad)} - \frac{c^2\log(c+dx^3)}{3d^2(bc-ad)} + \frac{x^3}{3bd}$$

```
[In] Int[x^8/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] x^3/(3*b*d) + (a^2*Log[a + b*x^3])/(3*b^2*(b*c - a*d)) - (c^2*Log[c + d*x^3
])/(3*d^2*(b*c - a*d))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(a+bx)(c+dx)}\,dx, x, x^3\right) \\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{bd} + \frac{a^2}{b(bc-ad)(a+bx)} + \frac{c^2}{d(-bc+ad)(c+dx)}\right) dx, x, x^3\right) \\
&= \frac{x^3}{3bd} + \frac{a^2\log\left(a+bx^3\right)}{3b^2(bc-ad)} - \frac{c^2\log\left(c+dx^3\right)}{3d^2(bc-ad)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)}\,dx = \frac{a^2d^2\log\left(a+bx^3\right) - b\left(d(-bc+ad)x^3 + bc^2\log\left(c+dx^3\right)\right)}{3b^2d^2(bc-ad)}$$

```
[In] Integrate[x^8/((a + b*x^3)*(c + d*x^3)),x]
[Out] (a^2*d^2*Log[a + b*x^3] - b*(d*(-(b*c) + a*d)*x^3 + b*c^2*Log[c + d*x^3]))/
(3*b^2*d^2*(b*c - a*d))
```

Maple [A] (verified)

Time = 4.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{x^3}{3bd}+\frac{c^2\ln\left(d\,x^3+c\right)}{3(ad-bc)d^2}-\frac{a^2\ln\left(b\,x^3+a\right)}{3(ad-bc)b^2}$	65
norman	$\frac{x^3}{3bd}+\frac{c^2\ln\left(d\,x^3+c\right)}{3(ad-bc)d^2}-\frac{a^2\ln\left(b\,x^3+a\right)}{3(ad-bc)b^2}$	65
risch	$\frac{x^3}{3bd}-\frac{a^2\ln\left(-b\,x^3-a\right)}{3b^2(ad-bc)}+\frac{c^2\ln\left(d\,x^3+c\right)}{3(ad-bc)d^2}$	68
parallelrisch	$-\frac{-x^3ab\,d^2+x^3b^2cd+a^2\ln\left(b\,x^3+a\right)d^2-c^2\ln\left(d\,x^3+c\right)b^2}{3b^2d^2(ad-bc)}$	70

```
[In] int(x^8/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3/b/d+1/3*c^2/(a*d-b*c)/d^2*ln(d*x^3+c)-1/3*a^2/(a*d-b*c)/b^2*ln(b*x^
3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)}\,dx = \frac{a^2d^2\log(bx^3+a) - b^2c^2\log(dx^3+c) + (b^2cd - abd^2)x^3}{3(b^3cd^2 - ab^2d^3)}$$

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*d^2*log(b*x^3 + a) - b^2*c^2*log(d*x^3 + c) + (b^2*c*d - a*b*d^2)*
x^3)/(b^3*c*d^2 - a*b^2*d^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate(x**8/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)}\,dx = \frac{a^2\log(bx^3+a)}{3(b^3c - ab^2d)} - \frac{c^2\log(dx^3+c)}{3(bcd^2 - ad^3)} + \frac{x^3}{3\,bd}$$

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*a^2*log(b*x^3 + a)/(b^3*c - a*b^2*d) - 1/3*c^2*log(d*x^3 + c)/(b*c*d^2
- a*d^3) + 1/3*x^3/(b*d)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)}\,dx = \frac{a^2\log\left(|bx^3+a|\right)}{3\left(b^3c-ab^2d\right)} - \frac{c^2\log\left(|dx^3+c|\right)}{3\left(bcd^2-ad^3\right)} + \frac{x^3}{3\,bd}$$

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*a^2*log(abs(b*x^3 + a))/(b^3*c - a*b^2*d) - 1/3*c^2*log(abs(d*x^3 + c))
/(b*c*d^2 - a*d^3) + 1/3*x^3/(b*d)
```

Mupad [B] (verification not implemented)

Time = 7.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)}\,dx = \frac{a^2\,\ln\left(b\,x^3+a\right)}{3\,b^3\,c-3\,a\,b^2\,d} + \frac{c^2\,\ln\left(d\,x^3+c\right)}{3\,a\,d^3-3\,b\,c\,d^2} + \frac{x^3}{3\,b\,d}$$

```
[In] int(x^8/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] (a^2*log(a + b*x^3))/(3*b^3*c - 3*a*b^2*d) + (c^2*log(c + d*x^3))/(3*a*d^3
- 3*b*c*d^2) + x^3/(3*b*d)
```

3.108 $\int \frac{x^7}{(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 301

$$\int \frac{x^7}{(a+bx^3)(c+dx^3)}\,dx = \frac{x^2}{2bd} - \frac{a^{5/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}(bc-ad)} + \frac{c^{5/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}d^{5/3}(bc-ad)} - \frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}(bc-ad)} + \frac{c^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{5/3}(bc-ad)} + \frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}(bc-ad)} - \frac{c^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6d^{5/3}(bc-ad)}$$

```
[Out] 1/2*x^2/b/d-1/3*a^(5/3)*ln(a^(1/3)+b^(1/3)*x)/b^(5/3)/(-a*d+b*c)+1/3*c^(5/3
)*ln(c^(1/3)+d^(1/3)*x)/d^(5/3)/(-a*d+b*c)+1/6*a^(5/3)*ln(a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/b^(5/3)/(-a*d+b*c)-1/6*c^(5/3)*ln(c^(2/3)-c^(1/3)*d^(
1/3)*x+d^(2/3)*x^2)/d^(5/3)/(-a*d+b*c)-1/3*a^(5/3)*arctan(1/3*(a^(1/3)-2*b^
(1/3)*x)/a^(1/3)*3^(1/2))/b^(5/3)/(-a*d+b*c)*3^(1/2)+1/3*c^(5/3)*arctan(1/3
*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/d^(5/3)/(-a*d+b*c)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {490, 598, 298, 31, 648, 631, 210, 642}

$$\int \frac{x^7}{(a+bx^3)(c+dx^3)}\,dx = -\frac{a^{5/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}(bc-ad)}+\frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}(bc-ad)} - \frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}(bc-ad)}+\frac{c^{5/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}d^{5/3}(bc-ad)} - \frac{c^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6d^{5/3}(bc-ad)} + \frac{c^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{5/3}(bc-ad)}+\frac{x^2}{2bd}$$

```
[In] Int[x^7/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] x^2/(2*b*d) - (a^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*b^(5/3)*(b*c - a*d)) + (c^(5/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqr
t[3]*c^(1/3))])/(Sqrt[3]*d^(5/3)*(b*c - a*d)) - (a^(5/3)*Log[a^(1/3) + b^(1
/3)*x])/(3*b^(5/3)*(b*c - a*d)) + (c^(5/3)*Log[c^(1/3) + d^(1/3)*x])/(3*d^(
5/3)*(b*c - a*d)) + (a^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(6*b^(5/3)*(b*c - a*d)) - (c^(5/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2
/3)*x^2])/(6*d^(5/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^2}{2bd} - \frac{\int \frac{x(2ac+2(bc+ad)x^3)}{(a+bx^3)(c+dx^3)}\,dx}{2bd} \\
&= \frac{x^2}{2bd} - \frac{\int \left(\frac{2a^2dx}{(-bc+ad)(a+bx^3)} + \frac{2bc^2x}{(bc-ad)(c+dx^3)}\right)dx}{2bd} \\
&= \frac{x^2}{2bd} + \frac{a^2\int \frac{x}{a+bx^3}\,dx}{b(bc-ad)} - \frac{c^2\int \frac{x}{c+dx^3}\,dx}{d(bc-ad)}
\end{aligned}
$$

$$= \frac{x^2}{2bd} - \frac{a^{5/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{4/3}(bc-ad)} + \frac{a^{5/3}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3b^{4/3}(bc-ad)} + \frac{c^{5/3}\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3d^{4/3}(bc-ad)} - \frac{c^{5/3}\int \frac{\sqrt[3]{c}+\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{3d^{4/3}(bc-ad)}$$

$$= \frac{x^2}{2bd} - \frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}(bc-ad)} + \frac{c^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{5/3}(bc-ad)} + \frac{a^{5/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^{5/3}(bc-ad)} + \frac{a^2\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2b^{4/3}(bc-ad)} - \frac{c^{5/3}\int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{6d^{5/3}(bc-ad)} - \frac{c^2\int \frac{1}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{2d^{4/3}(bc-ad)}$$

$$= \frac{x^2}{2bd} - \frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}(bc-ad)} + \frac{c^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{5/3}(bc-ad)} + \frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}(bc-ad)} - \frac{c^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6d^{5/3}(bc-ad)} + \frac{a^{5/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{b^{5/3}(bc-ad)} - \frac{c^{5/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{d^{5/3}(bc-ad)}$$

$$= \frac{x^2}{2bd} - \frac{a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}(bc-ad)} + \frac{c^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}d^{5/3}(bc-ad)} - \frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}(bc-ad)} + \frac{c^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{5/3}(bc-ad)} + \frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}(bc-ad)} - \frac{c^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6d^{5/3}(bc-ad)}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.80

$$\int \frac{x^7}{(a+bx^3)(c+dx^3)}\,dx$$

$$= \frac{-\frac{3ax^2}{b} + \frac{3cx^2}{d} - \frac{2\sqrt{3}a^{5/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{5/3}} + \frac{2\sqrt{3}c^{5/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{d^{5/3}} - \frac{2a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{5/3}} + \frac{2c^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{d^{5/3}}}{6bc-6ad}$$

```
[In] Integrate[x^7/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((-3*a*x^2)/b + (3*c*x^2)/d - (2*Sqrt[3]*a^(5/3)*ArcTan[(1 - (2*b^(1/3)*x)/
a^(1/3))/Sqrt[3]])/b^(5/3) + (2*Sqrt[3]*c^(5/3)*ArcTan[(1 - (2*d^(1/3)*x)/c
^(1/3))/Sqrt[3]])/d^(5/3) - (2*a^(5/3)*Log[a^(1/3) + b^(1/3)*x])/b^(5/3) +
(2*c^(5/3)*Log[c^(1/3) + d^(1/3)*x])/d^(5/3) + (a^(5/3)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3) - (c^(5/3)*Log[c^(2/3) - c^(1/3)*d^(1/
3)*x + d^(2/3)*x^2])/d^(5/3))/(6*b*c - 6*a*d)
```

Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.76

method	result
default	$\frac{x^2}{2bd}+\frac{\left(-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)c^2}{(ad-bc)d}-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{(ad-bc}$
risch	$\frac{x^2}{2bd}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\left(a^3d^5-3a^2bc\,d^4+3a\,b^2c^2d^3-b^3c^3d^2\right)\_Z^3+b^3c^5\right)}\_R\ln\left(\left(\left(\left(-a^5b^2c\,d^6+2a^4b^3c^2d^5-2a^3b^4c^3d^4+2a^2b^5c^4d^3-a\,b^6c^5d\right.\right.\right.\right.}{}$

```
[In] int(x^7/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2/b/d+(-1/3/d/(c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/6/d/(c/d)^(1/3)*ln(x^2-
(c/d)^(1/3)*x+(c/d)^(2/3))+1/3*3^(1/2)/d/(c/d)^(1/3)*arctan(1/3*3^(1/2)*(2/
(c/d)^(1/3)*x-1)))*c^2/(a*d-b*c)/d-(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/
6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)
*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*a^2/(a*d-b*c)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.55 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.91

$$\int\frac{x^7}{(a+bx^3)(c+dx^3)}\,dx$$
$$=\frac{2\sqrt{3}ad\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}a}{3a}\right)-2\sqrt{3}bc\left(-\frac{c^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}dx\left(-\frac{c^2}{d^2}\right)^{\frac{1}{3}}+\sqrt{3}c}{3c}\right)+ad\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax\right.}{}$$

```
[In] integrate(x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*a*d*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3
) - sqrt(3)*a)/a) - 2*sqrt(3)*b*c*(-c^2/d^2)^(1/3)*arctan(1/3*(2*sqrt(3)*d*
x*(-c^2/d^2)^(1/3) + sqrt(3)*c)/c) + a*d*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a
^2/b^2)^(2/3) + a*(a^2/b^2)^(1/3)) + b*c*(-c^2/d^2)^(1/3)*log(c*x^2 - d*x*(
-c^2/d^2)^(2/3) - c*(-c^2/d^2)^(1/3)) - 2*a*d*(a^2/b^2)^(1/3)*log(a*x + b*(
a^2/b^2)^(2/3)) - 2*b*c*(-c^2/d^2)^(1/3)*log(c*x + d*(-c^2/d^2)^(2/3)) + 3*
(b*c - a*d)*x^2)/(b^2*c*d - a*b*d^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7}{(a+bx^3)(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate(x**7/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.08

$$\begin{aligned}\int \frac{x^7}{(a+bx^3)(c+dx^3)}\,dx &= \frac{\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,(b^3c-ab^2d)\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\sqrt{3}c^2\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,(bcd^2-ad^3)\left(\frac{c}{d}\right)^{\frac{1}{3}}} \\ &+ \frac{a^2\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^3c\left(\frac{a}{b}\right)^{\frac{1}{3}}-ab^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} - \frac{c^2\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd^2\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^3\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} \\ &- \frac{a^2\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^3c\left(\frac{a}{b}\right)^{\frac{1}{3}}-ab^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} + \frac{c^2\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bcd^2\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^3\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} + \frac{x^2}{2\,bd}\end{aligned}$$

```
[In] integrate(x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^3*c
 - a*b^2*d)*(a/b)^(1/3)) - 1/3*sqrt(3)*c^2*arctan(1/3*sqrt(3)*(2*x - (c/d)^
(1/3))/(c/d)^(1/3))/((b*c*d^2 - a*d^3)*(c/d)^(1/3)) + 1/6*a^2*log(x^2 - x*(
a/b)^(1/3) + (a/b)^(2/3))/(b^3*c*(a/b)^(1/3) - a*b^2*d*(a/b)^(1/3)) - 1/6*c
^2*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c*d^2*(c/d)^(1/3) - a*d^3*(c/d
)^(1/3)) - 1/3*a^2*log(x + (a/b)^(1/3))/(b^3*c*(a/b)^(1/3) - a*b^2*d*(a/b)^
(1/3)) + 1/3*c^2*log(x + (c/d)^(1/3))/(b*c*d^2*(c/d)^(1/3) - a*d^3*(c/d)^(1
/3)) + 1/2*x^2/(b*d)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.03

$$\int \frac{x^7}{(a+bx^3)(c+dx^3)}\,dx = -\frac{a^2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(ab^2c-a^2bd)} + \frac{c^2\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2d-acd^2)} - \frac{(-ab^2)^{\frac{2}{3}}\,a\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^4c-\sqrt{3}ab^3d} + \frac{(-cd^2)^{\frac{2}{3}}\,c\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^3-\sqrt{3}ad^4} + \frac{(-ab^2)^{\frac{2}{3}}\,a\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(b^4c-ab^3d)} - \frac{(-cd^2)^{\frac{2}{3}}\,c\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(bcd^3-ad^4)} + \frac{x^2}{2\,bd}$$

```
[In] integrate(x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*a^2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2*c - a^2*b*d) + 1/3*
c^2*(-c/d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2*d - a*c*d^2) - (-a*b^2)^
(2/3)*a*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*b^4*
c - sqrt(3)*a*b^3*d) + (-c*d^2)^(2/3)*c*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1
/3))/(-c/d)^(1/3))/(sqrt(3)*b*c*d^3 - sqrt(3)*a*d^4) + 1/6*(-a*b^2)^(2/3)*a
*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(b^4*c - a*b^3*d) - 1/6*(-c*d^2)^
(2/3)*c*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c*d^3 - a*d^4) + 1/2*x^
2/(b*d)
```

Mupad [B] (verification not implemented)

Time = 16.27 (sec) , antiderivative size = 1751, normalized size of antiderivative = 5.82

$$\int \frac{x^7}{(a+bx^3)(c+dx^3)}\,dx = \text{Too large to display}$$

```
[In] int(x^7/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(((((27*a^2*b*c^2*d*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + 27*a*b^3*c*d^3
*(a*d + b*c)*(a*d - b*c)^4*(a^5/(b^5*(a*d - b*c)^3))^(2/3))*(a^5/(b^5*(a*d
- b*c)^3))^(1/3))/3 - (9*(a*b^7*c^8 + a^8*c*d^7 - a^2*b^6*c^7*d - a^7*b*c^2
```

*d^6))/(b^2*d^2))*(a^5/(b^5*(a*d - b*c)^3))^(2/3))/9 - (a^4*c^4*x*(a^2*d^2 + b^2*c^2 + a*b*c*d))/(b^2*d^2))*(-a^5/(27*b^8*c^3 - 27*a^3*b^5*d^3 + 81*a^2*b^6*c*d^2 - 81*a*b^7*c^2*d))^(1/3) + log(((((27*a^2*b*c^2*d*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + 27*a*b^3*c*d^3*(a*d + b*c)*(a*d - b*c)^4*(-c^5/(d^5*(a*d - b*c)^3))^(2/3))*(-c^5/(d^5*(a*d - b*c)^3))^(1/3))/3 - (9*(a*b^7*c^8 + a^8*c*d^7 - a^2*b^6*c^7*d - a^7*b*c^2*d^6))/(b^2*d^2))*(-c^5/(d^5*(a*d - b*c)^3))^(2/3))/9 - (a^4*c^4*x*(a^2*d^2 + b^2*c^2 + a*b*c*d))/(b^2*d^2))*(-c^5/(27*a^3*d^8 - 27*b^3*c^3*d^5 + 81*a*b^2*c^2*d^6 - 81*a^2*b*c*d^7))^(1/3) - (log(((3^(1/2)*1i + 1)^2*(((3^(1/2)*1i + 1)*(27*a^2*b*c^2*d*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^4*(a^5/(b^5*(a*d - b*c)^3))^(2/3))/4)*(a^5/(b^5*(a*d - b*c)^3))^(1/3))/6 + (9*(a*b^7*c^8 + a^8*c*d^7 - a^2*b^6*c^7*d - a^7*b*c^2*d^6))/(b^2*d^2))*(a^5/(b^5*(a*d - b*c)^3))^(2/3))/36 + (a^4*c^4*x*(a^2*d^2 + b^2*c^2 + a*b*c*d))/(b^2*d^2))*(-a^5/(27*b^8*c^3 - 27*a^3*b^5*d^3 + 81*a^2*b^6*c*d^2 - 81*a*b^7*c^2*d))^(1/3)*(3^(1/2)*1i + 1))/2 + (log(((3^(1/2)*1i - 1)^2*(((3^(1/2)*1i - 1)*(27*a^2*b*c^2*d*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(a^5/(b^5*(a*d - b*c)^3))^(2/3))/4)*(a^5/(b^5*(a*d - b*c)^3))^(1/3))/6 - (9*(a*b^7*c^8 + a^8*c*d^7 - a^2*b^6*c^7*d - a^7*b*c^2*d^6))/(b^2*d^2))*(a^5/(b^5*(a*d - b*c)^3))^(2/3))/36 - (a^4*c^4*x*(a^2*d^2 + b^2*c^2 + a*b*c*d))/(b^2*d^2))*(-a^5/(27*b^8*c^3 - 27*a^3*b^5*d^3 + 81*a^2*b^6*c*d^2 - 81*a*b^7*c^2*d))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(((3^(1/2)*1i + 1)^2*(((3^(1/2)*1i + 1)*(27*a^2*b*c^2*d*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-c^5/(d^5*(a*d - b*c)^3))^(2/3))/4)*(-c^5/(d^5*(a*d - b*c)^3))^(1/3))/6 + (9*(a*b^7*c^8 + a^8*c*d^7 - a^2*b^6*c^7*d - a^7*b*c^2*d^6))/(b^2*d^2))*(-c^5/(d^5*(a*d - b*c)^3))^(2/3))/36 + (a^4*c^4*x*(a^2*d^2 + b^2*c^2 + a*b*c*d))/(b^2*d^2))*(-c^5/(27*a^3*d^8 - 27*b^3*c^3*d^5 + 81*a*b^2*c^2*d^6 - 81*a^2*b*c*d^7))^(1/3)*(3^(1/2)*1i + 1))/2 + (log(((3^(1/2)*1i - 1)^2*(((3^(1/2)*1i - 1)*(27*a^2*b*c^2*d*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-c^5/(d^5*(a*d - b*c)^3))^(2/3))/4)*(-c^5/(d^5*(a*d - b*c)^3))^(1/3))/6 - (9*(a*b^7*c^8 + a^8*c*d^7 - a^2*b^6*c^7*d - a^7*b*c^2*d^6))/(b^2*d^2))*(-c^5/(d^5*(a*d - b*c)^3))^(2/3))/36 - (a^4*c^4*x*(a^2*d^2 + b^2*c^2 + a*b*c*d))/(b^2*d^2))*(-c^5/(27*a^3*d^8 - 27*b^3*c^3*d^5 + 81*a*b^2*c^2*d^6 - 81*a^2*b*c*d^7))^(1/3)*(3^(1/2)*1i - 1))/2 + x^2/(2*b*d)

3.109 $\int \frac{x^6}{(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 296

$$\int \frac{x^6}{(a+bx^3)(c+dx^3)}\,dx = \frac{x}{bd} - \frac{a^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{4/3}(bc-ad)} + \frac{c^{4/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}d^{4/3}(bc-ad)} + \frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}(bc-ad)} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{4/3}(bc-ad)} - \frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}(bc-ad)} + \frac{c^{4/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6d^{4/3}(bc-ad)}$$

```
[Out] x/b/d+1/3*a^(4/3)*ln(a^(1/3)+b^(1/3)*x)/b^(4/3)/(-a*d+b*c)-1/3*c^(4/3)*ln(c
^(1/3)+d^(1/3)*x)/d^(4/3)/(-a*d+b*c)-1/6*a^(4/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)
*x+b^(2/3)*x^2)/b^(4/3)/(-a*d+b*c)+1/6*c^(4/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x
+d^(2/3)*x^2)/d^(4/3)/(-a*d+b*c)-1/3*a^(4/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*
x)/a^(1/3)*3^(1/2))/b^(4/3)/(-a*d+b*c)*3^(1/2)+1/3*c^(4/3)*arctan(1/3*(c^(1
/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/d^(4/3)/(-a*d+b*c)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {490, 536, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^6}{(a+bx^3)(c+dx^3)}\,dx = -\frac{a^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{4/3}(bc-ad)} - \frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}(bc-ad)} + \frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}(bc-ad)} + \frac{c^{4/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}d^{4/3}(bc-ad)} + \frac{c^{4/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6d^{4/3}(bc-ad)} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{4/3}(bc-ad)} + \frac{x}{bd}$$

```
[In] Int[x^6/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] x/(b*d) - (a^(4/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt
[3]*b^(4/3)*(b*c - a*d)) + (c^(4/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]
*c^(1/3))])/(Sqrt[3]*d^(4/3)*(b*c - a*d)) + (a^(4/3)*Log[a^(1/3) + b^(1/3)*
x])/(3*b^(4/3)*(b*c - a*d)) - (c^(4/3)*Log[c^(1/3) + d^(1/3)*x])/(3*d^(4/3)
*(b*c - a*d)) - (a^(4/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6
*b^(4/3)*(b*c - a*d)) + (c^(4/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*
x^2])/(6*d^(4/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x}{bd} - \frac{\int \frac{ac+(bc+ad)x^3}{(a+bx^3)(c+dx^3)}\,dx}{bd}\\ &= \frac{x}{bd} + \frac{a^2\int \frac{1}{a+bx^3}\,dx}{b(bc-ad)} - \frac{c^2\int \frac{1}{c+dx^3}\,dx}{d(bc-ad)}\end{aligned}$$

$$= \frac{x}{bd} + \frac{a^{4/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b(bc-ad)} + \frac{a^{4/3}\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3b(bc-ad)} - \frac{c^{4/3}\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3d(bc-ad)} - \frac{c^{4/3}\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{3d(bc-ad)}$$

$$= \frac{x}{bd} + \frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}(bc-ad)} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{4/3}(bc-ad)} - \frac{a^{4/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^{4/3}(bc-ad)} + \frac{a^{5/3}\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2b(bc-ad)} + \frac{c^{4/3}\int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{6d^{4/3}(bc-ad)} - \frac{c^{5/3}\int \frac{1}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{2d(bc-ad)}$$

$$= \frac{x}{bd} + \frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}(bc-ad)} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{4/3}(bc-ad)} - \frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}(bc-ad)} + \frac{c^{4/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6d^{4/3}(bc-ad)} + \frac{a^{4/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{b^{4/3}(bc-ad)} - \frac{c^{4/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{d^{4/3}(bc-ad)}$$

$$= \frac{x}{bd} - \frac{a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{4/3}(bc-ad)} + \frac{c^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}d^{4/3}(bc-ad)} + \frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}(bc-ad)} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{4/3}(bc-ad)} - \frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}(bc-ad)} + \frac{c^{4/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6d^{4/3}(bc-ad)}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.80

$$\int \frac{x^6}{(a+bx^3)(c+dx^3)}\,dx = \frac{-\frac{6ax}{b}+\frac{6cx}{d}-\frac{2\sqrt{3}a^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{4/3}}+\frac{2\sqrt{3}c^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{d^{4/3}}+\frac{2a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{4/3}}-\frac{2c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{d^{4/3}}-}{6bc-6ad}$$

```
[In] Integrate[x^6/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((-6*a*x)/b + (6*c*x)/d - (2*Sqrt[3]*a^(4/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]])/b^(4/3) + (2*Sqrt[3]*c^(4/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/
3))/Sqrt[3]])/d^(4/3) + (2*a^(4/3)*Log[a^(1/3) + b^(1/3)*x])/b^(4/3) - (2*c
^(4/3)*Log[c^(1/3) + d^(1/3)*x])/d^(4/3) - (a^(4/3)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/b^(4/3) + (c^(4/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x
 + d^(2/3)*x^2])/d^(4/3))/(6*b*c - 6*a*d)
```

Maple [A] (verified)

Time = 4.52 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.76

method	result
default	$\frac{x}{bd}+\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)c^2}{d(ad-bc)}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\cdots\right.}{b(ad-bc)}$
risch	$\frac{x}{bd}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\left(a^3bd^3-3cd^2a^2b^2+3c^2dab^3-b^4c^3\right)\_Z^3+a^4d^3\right)}\_R\ln\left(\left(-a^5bcd^5-ab^5c^5d\right)x+\left(-a^5bd^6+3d^5cb^2a^4-2d^4c^2b^3a^3-2d\cdots\right.\right.}{3bd}$

```
[In] int(x^6/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] x/b/d+(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(
1/3)*x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(
1/3)*x-1)))/d*c^2/(a*d-b*c)-(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b
```

```
)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(
1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))/b*a^2/(a*d-b*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.77

$$\int \frac{x^6}{(a+bx^3)(c+dx^3)}\,dx = -\frac{2\sqrt{3}ad\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)+2\sqrt{3}bc\left(\frac{c}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}dx\left(\frac{c}{d}\right)^{\frac{2}{3}}-\sqrt{3}c}{3c}\right)-ad\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2+\right.}{}$$

```
[In] integrate(x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*a*d*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - s
qrt(3)*a)/a) + 2*sqrt(3)*b*c*(c/d)^(1/3)*arctan(1/3*(2*sqrt(3)*d*x*(c/d)^(2
/3) - sqrt(3)*c)/c) - a*d*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2
/3)) - b*c*(c/d)^(1/3)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3)) + 2*a*d*(-a/b
)^(1/3)*log(x - (-a/b)^(1/3)) + 2*b*c*(c/d)^(1/3)*log(x + (c/d)^(1/3)) - 6*
(b*c - a*d)*x)/(b^2*c*d - a*b*d^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^3)(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate(x**6/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.18

$$\int \frac{x^6}{(a+bx^3)(c+dx^3)}\,dx = \frac{\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^3c\left(\frac{a}{b}\right)^{\frac{1}{3}}-ab^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\sqrt{3}c^2\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bcd^2\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^3\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} - \frac{a^2\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^3c\left(\frac{a}{b}\right)^{\frac{2}{3}}-ab^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} + \frac{c^2\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad^3\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} + \frac{a^2\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^3c\left(\frac{a}{b}\right)^{\frac{2}{3}}-ab^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} - \frac{c^2\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bcd^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad^3\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} + \frac{x}{bd}$$

```
[In] integrate(x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^3*c
*(a/b)^(1/3) - a*b^2*d*(a/b)^(1/3))*(a/b)^(1/3)) - 1/3*sqrt(3)*c^2*arctan(1
/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c*d^2*(c/d)^(1/3) - a*d^3*(
c/d)^(1/3))*(c/d)^(1/3)) - 1/6*a^2*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(
b^3*c*(a/b)^(2/3) - a*b^2*d*(a/b)^(2/3)) + 1/6*c^2*log(x^2 - x*(c/d)^(1/3)
+ (c/d)^(2/3))/(b*c*d^2*(c/d)^(2/3) - a*d^3*(c/d)^(2/3)) + 1/3*a^2*log(x +
(a/b)^(1/3))/(b^3*c*(a/b)^(2/3) - a*b^2*d*(a/b)^(2/3)) - 1/3*c^2*log(x + (c
/d)^(1/3))/(b*c*d^2*(c/d)^(2/3) - a*d^3*(c/d)^(2/3)) + x/(b*d)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.04

$$\int \frac{x^6}{(a+bx^3)(c+dx^3)}\,dx = -\frac{a^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(ab^2c-a^2bd\right)} + \frac{c^2\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^2d-acd^2\right)} + \frac{\left(-ab^2\right)^{\frac{1}{3}}a\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^3c-\sqrt{3}ab^2d} - \frac{\left(-cd^2\right)^{\frac{1}{3}}c\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^2-\sqrt{3}ad^3} + \frac{\left(-ab^2\right)^{\frac{1}{3}}a\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^3c-ab^2d\right)} - \frac{\left(-cd^2\right)^{\frac{1}{3}}c\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd^2-ad^3\right)} + \frac{x}{bd}$$

```
[In] integrate(x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*a^2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2*c - a^2*b*d) + 1/3*
c^2*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2*d - a*c*d^2) + (-a*b^2)^
(1/3)*a*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*b^3*
c - sqrt(3)*a*b^2*d) - (-c*d^2)^(1/3)*c*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1
/3))/(-c/d)^(1/3))/(sqrt(3)*b*c*d^2 - sqrt(3)*a*d^3) + 1/6*(-a*b^2)^(1/3)*a
*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(b^3*c - a*b^2*d) - 1/6*(-c*d^2)^
(1/3)*c*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c*d^2 - a*d^3) + x/(b*d
)
```

Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 873, normalized size of antiderivative = 2.95

$$\int \frac{x^6}{(a+bx^3)(c+dx^3)}\,dx = \ln\left(a\,x+b^2\,c\left(-\frac{a^4}{b^4\,(a\,d-b\,c)^3}\right)^{1/3} - a\,b\,d\left(-\frac{a^4}{b^4\,(a\,d-b\,c)^3}\right)^{1/3}\right)\left(\frac{a^4}{-27\,a^3\,b^4\,d^3+81\,a^2\,b^5\,c\,d^2-81\,a\,b^6\,c^2\,d+27\,b^7\,c^3}\right)^{1/3}$$

$$+\ln\left(c\,x+a\,d^2\left(\frac{c^4}{d^4\,(a\,d-b\,c)^3}\right)^{1/3}-b\,c\,d\left(\frac{c^4}{d^4\,(a\,d-b\,c)^3}\right)^{1/3}\right)\left(\frac{c^4}{27\,a^3\,d^7-81\,a^2\,b\,c\,d^6+81\,a\,b^2\,c^2\,d^5-\right.$$

```
[In] int(x^6/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(a*x + b^2*c*(-a^4/(b^4*(a*d - b*c)^3))^(1/3) - a*b*d*(-a^4/(b^4*(a*d -
b*c)^3))^(1/3))*(a^4/(27*b^7*c^3 - 27*a^3*b^4*d^3 + 81*a^2*b^5*c*d^2 - 81*a
*b^6*c^2*d))^(1/3) + log(c*x + a*d^2*(c^4/(d^4*(a*d - b*c)^3))^(1/3) - b*c*
d*(c^4/(d^4*(a*d - b*c)^3))^(1/3))*(c^4/(27*a^3*d^7 - 27*b^3*c^3*d^4 + 81*a
*b^2*c^2*d^5 - 81*a^2*b*c*d^6))^(1/3) + x/(b*d) + (log((3*x*(a^2*b^4*c^6 +
a^6*c^2*d^4))/(b*d) - (3*a*c^2*(3^(1/2)*1i - 1)*(-a^4/(b^4*(a*d - b*c)^3))^
(1/3)*(a^5*d^5 - b^5*c^5 + a*b^4*c^4*d - a^4*b*c*d^4))/(2*d))*(a^4/(27*b^7*
c^3 - 27*a^3*b^4*d^3 + 81*a^2*b^5*c*d^2 - 81*a*b^6*c^2*d))^(1/3)*(3^(1/2)*1
i - 1))/2 - (log((3*x*(a^2*b^4*c^6 + a^6*c^2*d^4))/(b*d) + (3*a*c^2*(3^(1/2
)*1i + 1)*(-a^4/(b^4*(a*d - b*c)^3))^(1/3)*(a^5*d^5 - b^5*c^5 + a*b^4*c^4*d
 - a^4*b*c*d^4))/(2*d))*(a^4/(27*b^7*c^3 - 27*a^3*b^4*d^3 + 81*a^2*b^5*c*d^
2 - 81*a*b^6*c^2*d))^(1/3)*(3^(1/2)*1i + 1))/2 + (log((3*x*(a^2*b^4*c^6 + a
^6*c^2*d^4))/(b*d) + (3*a^2*c*(3^(1/2)*1i - 1)*(c^4/(d^4*(a*d - b*c)^3))^(1
/3)*(a^5*d^5 - b^5*c^5 + a*b^4*c^4*d - a^4*b*c*d^4))/(2*b))*(c^4/(27*a^3*d^
7 - 27*b^3*c^3*d^4 + 81*a*b^2*c^2*d^5 - 81*a^2*b*c*d^6))^(1/3)*(3^(1/2)*1i
- 1))/2 - (log((3*x*(a^2*b^4*c^6 + a^6*c^2*d^4))/(b*d) - (3*a^2*c*(3^(1/2)*
1i + 1)*(c^4/(d^4*(a*d - b*c)^3))^(1/3)*(a^5*d^5 - b^5*c^5 + a*b^4*c^4*d -
a^4*b*c*d^4))/(2*b))*(c^4/(27*a^3*d^7 - 27*b^3*c^3*d^4 + 81*a*b^2*c^2*d^5 -
 81*a^2*b*c*d^6))^(1/3)*(3^(1/2)*1i + 1))/2
```

3.110 $\int \frac{x^5}{(a+bx^3)(c+dx^3)} dx$

Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)} dx = -\frac{a\log(a+bx^3)}{3b(bc-ad)} + \frac{c\log(c+dx^3)}{3d(bc-ad)}$$

```
[Out] -1/3*a*ln(b*x^3+a)/b/(-a*d+b*c)+1/3*c*ln(d*x^3+c)/d/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)} dx = \frac{c\log(c+dx^3)}{3d(bc-ad)} - \frac{a\log(a+bx^3)}{3b(bc-ad)}$$

```
[In] Int[x^5/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/3*(a*Log[a + b*x^3])/(b*(b*c - a*d)) + (c*Log[c + d*x^3])/(3*d*(b*c - a*
d))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{(a+bx)(c+dx)}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)}+\frac{c}{(bc-ad)(c+dx)}\right)dx, x, x^3\right)\\ &= -\frac{a\log(a+bx^3)}{3b(bc-ad)}+\frac{c\log(c+dx^3)}{3d(bc-ad)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)}\,dx = -\frac{ad\log(a+bx^3)-bc\log(c+dx^3)}{3b^2cd-3abd^2}$$

```
[In] Integrate[x^5/((a + b*x^3)*(c + d*x^3)),x]
[Out] -((a*d*Log[a + b*x^3] - b*c*Log[c + d*x^3])/(3*b^2*c*d - 3*a*b*d^2))
```

Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

method	result	size
parallelrisch	$\frac{a\ln(b\,x^3+a)d-c\ln(d\,x^3+c)b}{3(ad-bc)bd}$	43
default	$-\frac{c\ln(d\,x^3+c)}{3(ad-bc)d}+\frac{a\ln(b\,x^3+a)}{3(ad-bc)b}$	50
norman	$-\frac{c\ln(d\,x^3+c)}{3(ad-bc)d}+\frac{a\ln(b\,x^3+a)}{3(ad-bc)b}$	50
risch	$-\frac{c\ln(-d\,x^3-c)}{3(ad-bc)d}+\frac{a\ln(b\,x^3+a)}{3(ad-bc)b}$	53

```
[In] int(x^5/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
[Out] 1/3*(a*ln(b*x^3+a)*d-c*ln(d*x^3+c)*b)/(a*d-b*c)/b/d
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)}\,dx = -\frac{ad\log(bx^3+a) - bc\log(dx^3+c)}{3(b^2cd - abd^2)}$$

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/3*(a*d*log(b*x^3 + a) - b*c*log(d*x^3 + c))/(b^2*c*d - a*b*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(39) = 78.

Time = 5.03 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.72

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)}\,dx = \frac{a\log\left(x^3 + \frac{\frac{a^3d^2}{b(ad-bc)} - \frac{2a^2cd}{ad-bc} + \frac{abc^2}{ad-bc} + 2ac}{ad+bc}\right)}{3b(ad-bc)} - \frac{c\log\left(x^3 + \frac{-\frac{a^2cd}{ad-bc} + \frac{2abc^2}{ad-bc} + 2ac - \frac{b^2c^3}{d(ad-bc)}}{ad+bc}\right)}{3d(ad-bc)}$$

```
[In] integrate(x**5/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] a*log(x**3 + (a**3*d**2/(b*(a*d - b*c)) - 2*a**2*c*d/(a*d - b*c) + a*b*c**2
/(a*d - b*c) + 2*a*c)/(a*d + b*c))/(3*b*(a*d - b*c)) - c*log(x**3 + (-a**2*
c*d/(a*d - b*c) + 2*a*b*c**2/(a*d - b*c) + 2*a*c - b**2*c**3/(d*(a*d - b*c)
))/(a*d + b*c))/(3*d*(a*d - b*c))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)}\,dx = -\frac{a\log(bx^3+a)}{3(b^2c - abd)} + \frac{c\log(dx^3+c)}{3(bcd - ad^2)}$$

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*a*log(b*x^3 + a)/(b^2*c - a*b*d) + 1/3*c*log(d*x^3 + c)/(b*c*d - a*d^2
)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)}\,dx = -\frac{a\log\left(|bx^3+a|\right)}{3\left(b^2c-abd\right)} + \frac{c\log\left(|dx^3+c|\right)}{3\left(bcd-ad^2\right)}$$

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
[Out] -1/3*a*log(abs(b*x^3 + a))/(b^2*c - a*b*d) + 1/3*c*log(abs(d*x^3 + c))/(b*c
*d - a*d^2)
```

Mupad [B] (verification not implemented)

Time = 7.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)}\,dx = -\frac{a\,\ln\left(b\,x^3+a\right)}{3\,b^2\,c-3\,a\,b\,d} - \frac{c\,\ln\left(d\,x^3+c\right)}{3\,a\,d^2-3\,b\,c\,d}$$

```
[In] int(x^5/((a + b*x^3)*(c + d*x^3)),x)
[Out] - (a*log(a + b*x^3))/(3*b^2*c - 3*a*b*d) - (c*log(c + d*x^3))/(3*a*d^2 - 3*
b*c*d)
```

3.111 $\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 288

$$
\begin{aligned}
\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx = {} & \frac{a^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{2/3}(bc-ad)} - \frac{c^{2/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}d^{2/3}(bc-ad)} \\
& + \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}(bc-ad)} - \frac{c^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{2/3}(bc-ad)} \\
& - \frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{2/3}(bc-ad)} \\
& + \frac{c^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6d^{2/3}(bc-ad)}
\end{aligned}
$$

```
[Out] 1/3*a^(2/3)*ln(a^(1/3)+b^(1/3)*x)/b^(2/3)/(-a*d+b*c)-1/3*c^(2/3)*ln(c^(1/3)
+d^(1/3)*x)/d^(2/3)/(-a*d+b*c)-1/6*a^(2/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/b^(2/3)/(-a*d+b*c)+1/6*c^(2/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/
3)*x^2)/d^(2/3)/(-a*d+b*c)+1/3*a^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(
1/3)*3^(1/2))/b^(2/3)/(-a*d+b*c)*3^(1/2)-1/3*c^(2/3)*arctan(1/3*(c^(1/3)-2*
d^(1/3)*x)/c^(1/3)*3^(1/2))/d^(2/3)/(-a*d+b*c)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {492, 298, 31, 648, 631, 210, 642}

$$\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx = \frac{a^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{2/3}(bc-ad)} - \frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{2/3}(bc-ad)} + \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}(bc-ad)} - \frac{c^{2/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}d^{2/3}(bc-ad)} + \frac{c^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6d^{2/3}(bc-ad)} - \frac{c^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{2/3}(bc-ad)}$$

```
[In] Int[x^4/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] (a^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(2/3
)*(b*c - a*d)) - (c^(2/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))]
)/(Sqrt[3]*d^(2/3)*(b*c - a*d)) + (a^(2/3)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(
2/3)*(b*c - a*d)) - (c^(2/3)*Log[c^(1/3) + d^(1/3)*x])/(3*d^(2/3)*(b*c - a*
d)) - (a^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(2/3)*(
b*c - a*d)) + (c^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*d
^(2/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = -\frac{a\int \frac{x}{a+bx^3}\,dx}{bc-ad} + \frac{c\int \frac{x}{c+dx^3}\,dx}{bc-ad}$$

$$= \frac{a^{2/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\sqrt[3]{b}(bc-ad)} - \frac{a^{2/3}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3\sqrt[3]{b}(bc-ad)} - \frac{c^{2/3}\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3\sqrt[3]{d}(bc-ad)} + \frac{c^{2/3}\int \frac{\sqrt[3]{c}+\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{3\sqrt[3]{d}(bc-ad)}$$

$$= \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}(bc-ad)} - \frac{c^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{2/3}(bc-ad)} - \frac{a^{2/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^{2/3}(bc-ad)} - \frac{a\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{b}(bc-ad)} + \frac{c^{2/3}\int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{6d^{2/3}(bc-ad)} + \frac{c\int \frac{1}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{2\sqrt[3]{d}(bc-ad)}$$

$$= \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}(bc-ad)} - \frac{c^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{2/3}(bc-ad)} - \frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{2/3}(bc-ad)} + \frac{c^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6d^{2/3}(bc-ad)} - \frac{a^{2/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{b^{2/3}(bc-ad)} + \frac{c^{2/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{d^{2/3}(bc-ad)}$$

$$= \frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{2/3}(bc-ad)} - \frac{c^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}d^{2/3}(bc-ad)} + \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}(bc-ad)} - \frac{c^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{2/3}(bc-ad)} - \frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{2/3}(bc-ad)} + \frac{c^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6d^{2/3}(bc-ad)}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.78

$$\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx = \frac{\frac{2\sqrt{3}a^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} - \frac{2\sqrt{3}c^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{d^{2/3}} + \frac{2a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}} - \frac{2c^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{d^{2/3}} - \frac{a^{2/3}\log\left(a^{2/3}\cdots\right.}{\cdots}}{6bc-6ad}$$

```
[In] Integrate[x^4/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((2*Sqrt[3]*a^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) -
(2*Sqrt[3]*c^(2/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/d^(2/3) + (
2*a^(2/3)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) - (2*c^(2/3)*Log[c^(1/3) + d^(1
/3)*x])/d^(2/3) - (a^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
b^(2/3) + (c^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/d^(2/3))
/(6*b*c - 6*a*d)
```

Maple [A] (verified)

Time = 4.29 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.72

method	result
default	$-\frac{\left(-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)c}{ad-bc}+\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\cdots\right.}{ad-bc}$
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(a^{3}b^{2}d^{3}-3a^{2}b^{3}c\,d^{2}+3a\,b^{4}c^{2}d-b^{5}c^{3}\right)\_Z^{3}+a^{2}\right)}\_R\ln\left(\left(\left(-2a^{3}b^{2}c\,d^{4}+4a^{2}b^{3}c^{2}d^{3}-2a\,b^{4}c^{3}d^{2}\right)\_R^{3}-a^{2}cd-b\,c^{2}a\right)x+(-a\cdots\right.}{3}$

[In] int(x^4/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] -(-1/3/d/(c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/6/d/(c/d)^(1/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3*3^(1/2)/d/(c/d)^(1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1)))*c/(a*d-b*c)+(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*a/(a*d-b*c)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx =$$
$$-\frac{2\sqrt{3}\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}+\sqrt{3}a}{3a}\right)-2\sqrt{3}\left(\frac{c^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}dx\left(\frac{c^2}{d^2}\right)^{\frac{1}{3}}-\sqrt{3}c}{3c}\right)-\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2-\cdots\right.}{}$$

[In] integrate(x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")

[Out] -1/6*(2*sqrt(3)*(-a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a^2/b^2)^(1/3) + sqrt(3)*a)/a) - 2*sqrt(3)*(c^2/d^2)^(1/3)*arctan(1/3*(2*sqrt(3)*d*x*(c^2/d^2)^(1/3) - sqrt(3)*c)/c) - (-a^2/b^2)^(1/3)*log(a*x^2 - b*x*(-a^2/b^2)^(2/3) - a*(-a^2/b^2)^(1/3)) - (c^2/d^2)^(1/3)*log(c*x^2 - d*x*(c^2/d^2)^(2/3) + c*(c^2/d^2)^(1/3)) + 2*(-a^2/b^2)^(1/3)*log(a*x + b*(-a^2/b^2)^(2/3)) + 2*(c^2/d^2)^(1/3)*log(c*x + d*(c^2/d^2)^(2/3)))/(b*c - a*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate(x**4/(b*x**3+a)/(d*x**3+c),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx = &-\frac{\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^2c-abd\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}c\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bcd-ad^2\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} \\ &-\frac{a\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}+\frac{c\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^2\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} \\ &+\frac{a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}-\frac{c\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bcd\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^2\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}\end{aligned}$$

```
[In] integrate(x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^2*c
 - a*b*d)*(a/b)^(1/3)) + 1/3*sqrt(3)*c*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3)
)/(c/d)^(1/3))/((b*c*d - a*d^2)*(c/d)^(1/3)) - 1/6*a*log(x^2 - x*(a/b)^(1/3
) + (a/b)^(2/3))/(b^2*c*(a/b)^(1/3) - a*b*d*(a/b)^(1/3)) + 1/6*c*log(x^2 -
x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c*d*(c/d)^(1/3) - a*d^2*(c/d)^(1/3)) + 1/3*
a*log(x + (a/b)^(1/3))/(b^2*c*(a/b)^(1/3) - a*b*d*(a/b)^(1/3)) - 1/3*c*log(
x + (c/d)^(1/3))/(b*c*d*(c/d)^(1/3) - a*d^2*(c/d)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.99

$$\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx = \frac{a\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(abc-a^2d)} - \frac{c\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2-acd)} + \frac{(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^3c-\sqrt{3}ab^2d} - \frac{(-cd^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^2-\sqrt{3}ad^3} - \frac{(-ab^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(b^3c-ab^2d)} + \frac{(-cd^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(bcd^2-ad^3)}$$

```
[In] integrate(x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*a*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) - 1/3*c*(-c/d
)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) + (-a*b^2)^(2/3)*arctan(
1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*b^3*c - sqrt(3)*a*b
^2*d) - (-c*d^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3)
)/(sqrt(3)*b*c*d^2 - sqrt(3)*a*d^3) - 1/6*(-a*b^2)^(2/3)*log(x^2 + x*(-a/b)
^(1/3) + (-a/b)^(2/3))/(b^3*c - a*b^2*d) + 1/6*(-c*d^2)^(2/3)*log(x^2 + x*(
-c/d)^(1/3) + (-c/d)^(2/3))/(b*c*d^2 - a*d^3)
```

Mupad [B] (verification not implemented)

Time = 14.05 (sec) , antiderivative size = 1364, normalized size of antiderivative = 4.74

$$\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx = \text{Too large to display}$$

```
[In] int(x^4/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(a*x + b^3*c^2*(-a^2/(b^2*(a*d - b*c)^3))^(2/3) + a^2*b*d^2*(-a^2/(b^2*(
a*d - b*c)^3))^(2/3) - 2*a*b^2*c*d*(-a^2/(b^2*(a*d - b*c)^3))^(2/3))*(a^2/(
27*b^5*c^3 - 27*a^3*b^2*d^3 + 81*a^2*b^3*c*d^2 - 81*a*b^4*c^2*d))^(1/3) + l
og(c*x + a^2*d^3*(c^2/(d^2*(a*d - b*c)^3))^(2/3) + b^2*c^2*d*(c^2/(d^2*(a*d
```

```
 - b*c)^3))^(2/3) - 2*a*b*c*d^2*(c^2/(d^2*(a*d - b*c)^3))^(2/3))*(c^2/(27*a
^3*d^5 - 27*b^3*c^3*d^2 + 81*a*b^2*c^2*d^3 - 81*a^2*b*c*d^4))^(1/3) + (log(
((3^(1/2)*1i - 1)^2*(-a^2/(b^2*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i - 1)*(54
*a^2*b^3*c^2*d^3*x*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i - 1)^2*(a*d
+ b*c)*(a*d - b*c)^4*(-a^2/(b^2*(a*d - b*c)^3))^(2/3))/4)*(-a^2/(b^2*(a*d -
 b*c)^3))^(1/3))/6 - 9*a^2*b^4*c^4*d^2 - 9*a^4*b^2*c^2*d^4 + 9*a*b^5*c^5*d
+ 9*a^5*b*c*d^5))/36 + a^2*b*c^2*d*x*(a*d + b*c))*(a^2/(27*b^5*c^3 - 27*a^3
*b^2*d^3 + 81*a^2*b^3*c*d^2 - 81*a*b^4*c^2*d))^(1/3)*(3^(1/2)*1i - 1))/2 -
(log(((3^(1/2)*1i + 1)^2*(-a^2/(b^2*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i + 1
)*(54*a^2*b^3*c^2*d^3*x*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i + 1)^2*
(a*d + b*c)*(a*d - b*c)^4*(-a^2/(b^2*(a*d - b*c)^3))^(2/3))/4)*(-a^2/(b^2*(
a*d - b*c)^3))^(1/3))/6 + 9*a^2*b^4*c^4*d^2 + 9*a^4*b^2*c^2*d^4 - 9*a*b^5*c
^5*d - 9*a^5*b*c*d^5))/36 - a^2*b*c^2*d*x*(a*d + b*c))*(a^2/(27*b^5*c^3 - 2
7*a^3*b^2*d^3 + 81*a^2*b^3*c*d^2 - 81*a*b^4*c^2*d))^(1/3)*(3^(1/2)*1i + 1))
/2 + (log(((3^(1/2)*1i - 1)^2*(c^2/(d^2*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i
 - 1)*(54*a^2*b^3*c^2*d^3*x*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i - 1
)^2*(a*d + b*c)*(a*d - b*c)^4*(c^2/(d^2*(a*d - b*c)^3))^(2/3))/4)*(c^2/(d^2
*(a*d - b*c)^3))^(1/3))/6 - 9*a^2*b^4*c^4*d^2 - 9*a^4*b^2*c^2*d^4 + 9*a*b^5
*c^5*d + 9*a^5*b*c*d^5))/36 + a^2*b*c^2*d*x*(a*d + b*c))*(c^2/(27*a^3*d^5 -
 27*b^3*c^3*d^2 + 81*a*b^2*c^2*d^3 - 81*a^2*b*c*d^4))^(1/3)*(3^(1/2)*1i - 1
))/2 - (log(((3^(1/2)*1i + 1)^2*(c^2/(d^2*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*
1i + 1)*(54*a^2*b^3*c^2*d^3*x*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i +
 1)^2*(a*d + b*c)*(a*d - b*c)^4*(c^2/(d^2*(a*d - b*c)^3))^(2/3))/4)*(c^2/(d
^2*(a*d - b*c)^3))^(1/3))/6 + 9*a^2*b^4*c^4*d^2 + 9*a^4*b^2*c^2*d^4 - 9*a*b
^5*c^5*d - 9*a^5*b*c*d^5))/36 - a^2*b*c^2*d*x*(a*d + b*c))*(c^2/(27*a^3*d^5
 - 27*b^3*c^3*d^2 + 81*a*b^2*c^2*d^3 - 81*a^2*b*c*d^4))^(1/3)*(3^(1/2)*1i +
 1))/2
```

3.112 $\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 288

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx = \frac{\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{b}(bc-ad)} - \frac{\sqrt[3]{c}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{d}(bc-ad)} - \frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{b}(bc-ad)} + \frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{d}(bc-ad)} + \frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{b}(bc-ad)} - \frac{\sqrt[3]{c}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6\sqrt[3]{d}(bc-ad)}$$

```
[Out] -1/3*a^(1/3)*ln(a^(1/3)+b^(1/3)*x)/b^(1/3)/(-a*d+b*c)+1/3*c^(1/3)*ln(c^(1/3
)+d^(1/3)*x)/d^(1/3)/(-a*d+b*c)+1/6*a^(1/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^
(2/3)*x^2)/b^(1/3)/(-a*d+b*c)-1/6*c^(1/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2
/3)*x^2)/d^(1/3)/(-a*d+b*c)+1/3*a^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^
(1/3)*3^(1/2))/b^(1/3)/(-a*d+b*c)*3^(1/2)-1/3*c^(1/3)*arctan(1/3*(c^(1/3)-2
*d^(1/3)*x)/c^(1/3)*3^(1/2))/d^(1/3)/(-a*d+b*c)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {492, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx = \frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{b}(bc-ad)} + \frac{\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{b}(bc-ad)} - \frac{\sqrt[3]{c}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{d}(bc-ad)} - \frac{\sqrt[3]{c}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6\sqrt[3]{d}(bc-ad)} - \frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{b}(bc-ad)} + \frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{d}(bc-ad)}$$

```
[In] Int[x^3/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] (a^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(1/3
)*(b*c - a*d)) - (c^(1/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))]
)/(Sqrt[3]*d^(1/3)*(b*c - a*d)) - (a^(1/3)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(
1/3)*(b*c - a*d)) + (c^(1/3)*Log[c^(1/3) + d^(1/3)*x])/(3*d^(1/3)*(b*c - a*
d)) + (a^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(1/3)*(
b*c - a*d)) - (c^(1/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*d
^(1/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{a\int\frac{1}{a+bx^3}\,dx}{bc-ad}+\frac{c\int\frac{1}{c+dx^3}\,dx}{bc-ad}\\ &= -\frac{\sqrt[3]{a}\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3(bc-ad)}-\frac{\sqrt[3]{a}\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3(bc-ad)}\\ &\quad+\frac{\sqrt[3]{c}\int\frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3(bc-ad)}+\frac{\sqrt[3]{c}\int\frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{3(bc-ad)}\end{aligned}$$

$$= -\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{b}(bc-ad)} + \frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{d}(bc-ad)}$$
$$- \frac{a^{2/3}\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{2(bc-ad)} + \frac{\sqrt[3]{a}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{6\sqrt[3]{b}(bc-ad)}$$
$$+ \frac{c^{2/3}\int \frac{1}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}dx}{2(bc-ad)} - \frac{\sqrt[3]{c}\int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}dx}{6\sqrt[3]{d}(bc-ad)}$$

$$= -\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{b}(bc-ad)} + \frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{d}(bc-ad)}$$
$$+ \frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{b}(bc-ad)} - \frac{\sqrt[3]{c}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6\sqrt[3]{d}(bc-ad)}$$
$$- \frac{\sqrt[3]{a}\text{Subst}\left(\int \frac{1}{-3-x^2}dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}(bc-ad)} + \frac{\sqrt[3]{c}\text{Subst}\left(\int \frac{1}{-3-x^2}dx, x, 1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{\sqrt[3]{d}(bc-ad)}$$

$$= \frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{b}(bc-ad)} - \frac{\sqrt[3]{c}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{d}(bc-ad)}$$
$$- \frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{b}(bc-ad)} + \frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{d}(bc-ad)}$$
$$+ \frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{b}(bc-ad)} - \frac{\sqrt[3]{c}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6\sqrt[3]{d}(bc-ad)}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)}dx$$
$$= \frac{\frac{2\sqrt{3}\sqrt[3]{a}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} - \frac{2\sqrt{3}\sqrt[3]{c}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}} - \frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}} + \frac{2\sqrt[3]{c}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt[3]{d}} + \frac{\sqrt[3]{a}\log\left(a^{2/ \right.}{}}{6bc-6ad}$$

```
[In] Integrate[x^3/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((2*Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) -
(2*Sqrt[3]*c^(1/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/d^(1/3) - (
2*a^(1/3)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + (2*c^(1/3)*Log[c^(1/3) + d^(1
/3)*x])/d^(1/3) + (a^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
b^(1/3) - (c^(1/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/d^(1/3))
/(6*b*c - 6*a*d)
```

Maple [A] (verified)

Time = 4.57 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.72

method	result
default	$-\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)c}{ad-bc}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\mathrm{arct}}{}\right.}{ad-bc}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^3b\,d^3-3c\,d^2a^2b^2+3c^2da\,b^3-b^4c^3\right)\_Z^3-a\right)}\_R\ln\left(\left(\left(a^4b\,d^5-4a^3b^2c\,d^4+6a^2b^3c^2d^3-4a\,b^4c^3d^2+b^5c^4d\right)\_R^3-a^2d^2-b^2\right.\right.\right.}{3}$

```
[In] int(x^3/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*
x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*
x-1)))*c/(a*d-b*c)+(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*l
n(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/
2)*(2/(a/b)^(1/3)*x-1)))*a/(a*d-b*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx = -\frac{2\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)+2\sqrt{3}\left(-\frac{c}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}dx\left(-\frac{c}{d}\right)^{\frac{2}{3}}-\sqrt{3}c}{3c}\right)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\right.\right.}{6\,(bc-}$$

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)
*a)/a) + 2*sqrt(3)*(-c/d)^(1/3)*arctan(1/3*(2*sqrt(3)*d*x*(-c/d)^(2/3) - sq
rt(3)*c)/c) - (a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - (-c/d)^(
1/3)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3)) + 2*(a/b)^(1/3)*log(x + (a/b)
^(1/3)) + 2*(-c/d)^(1/3)*log(x - (-c/d)^(1/3)))/(b*c - a*d)
```

Sympy [A] (verification not implemented)

Time = 131.43 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.19

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx$$
$$= \mathrm{RootSum}\left(t^3\cdot\left(27a^3d^4-81a^2bcd^3+81ab^2c^2d^2-27b^3c^3d\right)+c,\left(t\mapsto t\log\left(x+\frac{162t^4a^4bd^5-648t^4a^3b^2c\right.\right.\right.$$
$$+\mathrm{RootSum}\left(t^3\cdot\left(27a^3bd^3-81a^2b^2cd^2+81ab^3c^2d-27b^4c^3\right)-a,\left(t\mapsto t\log\left(x+\frac{162t^4a^4bd^5-648t^4a^3\right.\right.\right.$$

```
[In] integrate(x**3/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] RootSum(_t**3*(27*a**3*d**4 - 81*a**2*b*c*d**3 + 81*a*b**2*c**2*d**2 - 27*b
**3*c**3*d) + c, Lambda(_t, _t*log(x + (162*_t**4*a**4*b*d**5 - 648*_t**4*a
**3*b**2*c*d**4 + 972*_t**4*a**2*b**3*c**2*d**3 - 648*_t**4*a*b**4*c**3*d**
2 + 162*_t**4*b**5*c**4*d - 3*_t*a**2*d**2 + 6*_t*a*b*c*d - 3*_t*b**2*c**2)
/(a*d + b*c)))) + RootSum(_t**3*(27*a**3*b*d**3 - 81*a**2*b**2*c*d**2 + 81*
a*b**3*c**2*d - 27*b**4*c**3) - a, Lambda(_t, _t*log(x + (162*_t**4*a**4*b*
d**5 - 648*_t**4*a**3*b**2*c*d**4 + 972*_t**4*a**2*b**3*c**2*d**3 - 648*_t*
*4*a*b**4*c**3*d**2 + 162*_t**4*b**5*c**4*d - 3*_t*a**2*d**2 + 6*_t*a*b*c*d
 - 3*_t*b**2*c**2)/(a*d + b*c))))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.10

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx = -\frac{\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^2c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\sqrt{3}c\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bcd\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^2\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} + \frac{a\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-abd\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} - \frac{c\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad^2\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} - \frac{a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^2c\left(\frac{a}{b}\right)^{\frac{2}{3}}-abd\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} + \frac{c\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bcd\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad^2\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}$$

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^2*c*
(a/b)^(1/3) - a*b*d*(a/b)^(1/3))*(a/b)^(1/3)) + 1/3*sqrt(3)*c*arctan(1/3*sq
rt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c*d*(c/d)^(1/3) - a*d^2*(c/d)^(1
/3))*(c/d)^(1/3)) + 1/6*a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*c*(a/
b)^(2/3) - a*b*d*(a/b)^(2/3)) - 1/6*c*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3)
)/(b*c*d*(c/d)^(2/3) - a*d^2*(c/d)^(2/3)) - 1/3*a*log(x + (a/b)^(1/3))/(b^2
*c*(a/b)^(2/3) - a*b*d*(a/b)^(2/3)) + 1/3*c*log(x + (c/d)^(1/3))/(b*c*d*(c/
d)^(2/3) - a*d^2*(c/d)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.97

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx = \frac{a\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3(abc-a^2d)} - \frac{c\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3(bc^2-acd)} - \frac{(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c-\sqrt{3}abd} + \frac{(-cd^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd-\sqrt{3}ad^2} - \frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(b^2c-abd)} + \frac{(-cd^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6(bcd-ad^2)}$$

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*a*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) - 1/3*c*(-c/d
)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) - (-a*b^2)^(1/3)*arctan(
1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*b^2*c - sqrt(3)*a*b
*d) + (-c*d^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/
(sqrt(3)*b*c*d - sqrt(3)*a*d^2) - 1/6*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/
3) + (-a/b)^(2/3))/(b^2*c - a*b*d) + 1/6*(-c*d^2)^(1/3)*log(x^2 + x*(-c/d)^
(1/3) + (-c/d)^(2/3))/(b*c*d - a*d^2)
```

Mupad [B] (verification not implemented)

Time = 13.02 (sec) , antiderivative size = 1265, normalized size of antiderivative = 4.39

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx = \text{Too large to display}$$

```
[In] int(x^3/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(x + a*d*(a/(b*(a*d - b*c)^3))^(1/3) - b*c*(a/(b*(a*d - b*c)^3))^(1/3))*
(-a/(27*b^4*c^3 - 27*a^3*b*d^3 + 81*a^2*b^2*c*d^2 - 81*a*b^3*c^2*d))^(1/3)
+ log(x - a*d*(-c/(d*(a*d - b*c)^3))^(1/3) + b*c*(-c/(d*(a*d - b*c)^3))^(1/
3))*(-c/(27*a^3*d^4 - 27*b^3*c^3*d + 81*a*b^2*c^2*d^2 - 81*a^2*b*c*d^3))^(1
```

```
/3) + (log(((3^(1/2)*1i - 1)*(a/(b*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i - 1)
^2*(81*a*b^3*c*d^3*x*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^(1/2)*1i - 1)*(a*d
+ b*c)*(a*d - b*c)^4*(a/(b*(a*d - b*c)^3))^(1/3))/2)*(a/(b*(a*d - b*c)^3))^
(2/3))/36 + 9*a*b^5*c^4*d^2 + 9*a^4*b^2*c*d^5 - 9*a^2*b^4*c^3*d^3 - 9*a^3*b
^3*c^2*d^4))/6 - 3*a*b^2*c*d^2*x*(a^2*d^2 + b^2*c^2))*(3^(1/2)*1i - 1)*(-a/
(27*b^4*c^3 - 27*a^3*b*d^3 + 81*a^2*b^2*c*d^2 - 81*a*b^3*c^2*d))^(1/3))/2 -
 (log(((3^(1/2)*1i + 1)*(a/(b*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(8
1*a*b^3*c*d^3*x*(a*d - b*c)^4 + (81*a*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*d + b*c
)*(a*d - b*c)^4*(a/(b*(a*d - b*c)^3))^(1/3))/2)*(a/(b*(a*d - b*c)^3))^(2/3)
)/36 + 9*a*b^5*c^4*d^2 + 9*a^4*b^2*c*d^5 - 9*a^2*b^4*c^3*d^3 - 9*a^3*b^3*c^
2*d^4))/6 + 3*a*b^2*c*d^2*x*(a^2*d^2 + b^2*c^2))*(3^(1/2)*1i + 1)*(-a/(27*b
^4*c^3 - 27*a^3*b*d^3 + 81*a^2*b^2*c*d^2 - 81*a*b^3*c^2*d))^(1/3))/2 + (log
(((3^(1/2)*1i - 1)*(-c/(d*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i - 1)^2*(81*a*
b^3*c*d^3*x*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a
*d - b*c)^4*(-c/(d*(a*d - b*c)^3))^(1/3))/2)*(-c/(d*(a*d - b*c)^3))^(2/3))/
36 + 9*a*b^5*c^4*d^2 + 9*a^4*b^2*c*d^5 - 9*a^2*b^4*c^3*d^3 - 9*a^3*b^3*c^2*
d^4))/6 - 3*a*b^2*c*d^2*x*(a^2*d^2 + b^2*c^2))*(3^(1/2)*1i - 1)*(-c/(27*a^3
*d^4 - 27*b^3*c^3*d + 81*a*b^2*c^2*d^2 - 81*a^2*b*c*d^3))^(1/3))/2 - (log((
(3^(1/2)*1i + 1)*(-c/(d*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81*a*b^
3*c*d^3*x*(a*d - b*c)^4 + (81*a*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)*(a*d
 - b*c)^4*(-c/(d*(a*d - b*c)^3))^(1/3))/2)*(-c/(d*(a*d - b*c)^3))^(2/3))/36
 + 9*a*b^5*c^4*d^2 + 9*a^4*b^2*c*d^5 - 9*a^2*b^4*c^3*d^3 - 9*a^3*b^3*c^2*d^
4))/6 + 3*a*b^2*c*d^2*x*(a^2*d^2 + b^2*c^2))*(3^(1/2)*1i + 1)*(-c/(27*a^3*d
^4 - 27*b^3*c^3*d + 81*a*b^2*c^2*d^2 - 81*a^2*b*c*d^3))^(1/3))/2
```

3.113 $\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 45

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx = \frac{\log(a+bx^3)}{3(bc-ad)} - \frac{\log(c+dx^3)}{3(bc-ad)}$$

```
[Out] 1/3*ln(b*x^3+a)/(-a*d+b*c)-1/3*ln(d*x^3+c)/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {455, 36, 31}

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx = \frac{\log(a+bx^3)}{3(bc-ad)} - \frac{\log(c+dx^3)}{3(bc-ad)}$$

```
[In] Int[x^2/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] Log[a + b*x^3]/(3*(b*c - a*d)) - Log[c + d*x^3]/(3*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)}\,dx, x, x^3\right)\\ &= \frac{b\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, x^3\right)}{3(bc-ad)} - \frac{d\text{Subst}\left(\int \frac{1}{c+dx}\,dx, x, x^3\right)}{3(bc-ad)}\\ &= \frac{\log(a+bx^3)}{3(bc-ad)} - \frac{\log(c+dx^3)}{3(bc-ad)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx = \frac{\log(a+bx^3) - \log(c+dx^3)}{3bc-3ad}$$

```
[In] Integrate[x^2/((a + b*x^3)*(c + d*x^3)),x]
[Out] (Log[a + b*x^3] - Log[c + d*x^3])/(3*b*c - 3*a*d)
```

Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.71

method	result	size
parallelrisch	$-\frac{\ln(b\,x^3+a)-\ln(d\,x^3+c)}{3(ad-bc)}$	32
default	$\frac{\ln(d\,x^3+c)}{3ad-3bc}-\frac{\ln(b\,x^3+a)}{3(ad-bc)}$	42
norman	$\frac{\ln(d\,x^3+c)}{3ad-3bc}-\frac{\ln(b\,x^3+a)}{3(ad-bc)}$	42
risch	$\frac{\ln(d\,x^3+c)}{3ad-3bc}-\frac{\ln(-b\,x^3-a)}{3(ad-bc)}$	45

```
[In] int(x^2/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
[Out] -1/3*(ln(b*x^3+a)-ln(d*x^3+c))/(a*d-b*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx = \frac{\log(bx^3+a)-\log(dx^3+c)}{3(bc-ad)}$$

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
[Out] 1/3*(log(b*x^3 + a) - log(d*x^3 + c))/(b*c - a*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(36) = 72.

Time = 0.73 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.07

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx = \frac{\log\left(x^3+\frac{-\frac{a^2d^2}{ad-bc}+\frac{2abcd}{ad-bc}+ad-\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{3(ad-bc)} - \frac{\log\left(x^3+\frac{\frac{a^2d^2}{ad-bc}-\frac{2abcd}{ad-bc}+ad+\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{3(ad-bc)}$$

```
[In] integrate(x**2/(b*x**3+a)/(d*x**3+c),x)
[Out] log(x**3 + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2*c**
2/(a*d - b*c) + b*c)/(2*b*d))/(3*(a*d - b*c)) - log(x**3 + (a**2*d**2/(a*d
- b*c) - 2*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2*b*d)
)/(3*(a*d - b*c))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx = \frac{\log(bx^3+a)}{3(bc-ad)} - \frac{\log(dx^3+c)}{3(bc-ad)}$$

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
[Out] 1/3*log(b*x^3 + a)/(b*c - a*d) - 1/3*log(d*x^3 + c)/(b*c - a*d)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx = \frac{b\log(|bx^3+a|)}{3(b^2c-abd)} - \frac{d\log(|dx^3+c|)}{3(bcd-ad^2)}$$

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b*log(abs(b*x^3 + a))/(b^2*c - a*b*d) - 1/3*d*log(abs(d*x^3 + c))/(b*c*
d - a*d^2)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 602, normalized size of antiderivative = 13.38

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)}\,dx =$$

$$-\frac{\operatorname{atan}\left(\frac{\frac{\left(\frac{x^3\left(36\,c\,b^4\,d^3+36\,a\,b^3\,d^4\right)+\frac{x^3\left(54\,a^2\,b^3\,d^5+108\,a\,b^4\,c\,d^4+54\,b^5\,c^2\,d^3\right)+108\,a\,b^4\,c^2\,d^3+108\,a^2\,b^3\,c\,d^4}{3\,a\,d-3\,b\,c}+36\,a\,b^3\,c\,d^3}{3\,a\,d-3\,b\,c}+6\,b^3\,d^3\,x^3\right)\,1\mathrm{i}}{3\,a\,d-3\,b\,c}-\frac{\left(\frac{x^3\left(36\,c\,b^4\,d\cdots\right.}{\cdots}\right.}{\cdots}}{\frac{\frac{x^3\left(36\,c\,b^4\,d^3+36\,a\,b^3\,d^4\right)+\frac{x^3\left(54\,a^2\,b^3\,d^5+108\,a\,b^4\,c\,d^4+54\,b^5\,c^2\,d^3\right)+108\,a\,b^4\,c^2\,d^3+108\,a^2\,b^3\,c\,d^4}{3\,a\,d-3\,b\,c}+36\,a\,b^3\,c\,d^3}{3\,a\,d-3\,b\,c}+6\,b^3\,d^3\,x^3}{3\,a\,d-3\,b\,c}+\frac{x^3\left(36\,c\,b^4\,d^3\cdots\right.}{\cdots}}\right)\cdots}{3\,a\,d-3\,b\,c}$$

```
[In] int(x^2/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] -(atan(((((x^3*(36*a*b^3*d^4 + 36*b^4*c*d^3) + (x^3*(54*a^2*b^3*d^5 + 54*b^
5*c^2*d^3 + 108*a*b^4*c*d^4) + 108*a*b^4*c^2*d^3 + 108*a^2*b^3*c*d^4)/(3*a*
d - 3*b*c) + 36*a*b^3*c*d^3)/(3*a*d - 3*b*c) + 6*b^3*d^3*x^3)*1i)/(3*a*d -
3*b*c) - (((x^3*(36*a*b^3*d^4 + 36*b^4*c*d^3) - (x^3*(54*a^2*b^3*d^5 + 54*b
^5*c^2*d^3 + 108*a*b^4*c*d^4) + 108*a*b^4*c^2*d^3 + 108*a^2*b^3*c*d^4)/(3*a
*d - 3*b*c) + 36*a*b^3*c*d^3)/(3*a*d - 3*b*c) - 6*b^3*d^3*x^3)*1i)/(3*a*d -
 3*b*c))/(((x^3*(36*a*b^3*d^4 + 36*b^4*c*d^3) + (x^3*(54*a^2*b^3*d^5 + 54*b
^5*c^2*d^3 + 108*a*b^4*c*d^4) + 108*a*b^4*c^2*d^3 + 108*a^2*b^3*c*d^4)/(3*a
*d - 3*b*c) + 36*a*b^3*c*d^3)/(3*a*d - 3*b*c) + 6*b^3*d^3*x^3)/(3*a*d - 3*b
*c) + ((x^3*(36*a*b^3*d^4 + 36*b^4*c*d^3) - (x^3*(54*a^2*b^3*d^5 + 54*b^5*c
^2*d^3 + 108*a*b^4*c*d^4) + 108*a*b^4*c^2*d^3 + 108*a^2*b^3*c*d^4)/(3*a*d -
 3*b*c) + 36*a*b^3*c*d^3)/(3*a*d - 3*b*c) - 6*b^3*d^3*x^3)/(3*a*d - 3*b*c))
)*2i)/(3*a*d - 3*b*c)
```

3.114 $\int \frac{x}{(a+bx^3)(c+dx^3)} dx$

Optimal result

Integrand size = 20, antiderivative size = 288

$$\int \frac{x}{(a+bx^3)(c+dx^3)} dx = -\frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}(bc-ad)} + \frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}(bc-ad)} + \frac{\sqrt[3]{d}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}(bc-ad)} + \frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}(bc-ad)} - \frac{\sqrt[3]{d}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6\sqrt[3]{c}(bc-ad)}$$

```
[Out] -1/3*b^(1/3)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/(-a*d+b*c)+1/3*d^(1/3)*ln(c^(1/3
)+d^(1/3)*x)/c^(1/3)/(-a*d+b*c)+1/6*b^(1/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^
(2/3)*x^2)/a^(1/3)/(-a*d+b*c)-1/6*d^(1/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2
/3)*x^2)/c^(1/3)/(-a*d+b*c)-1/3*b^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^
(1/3)*3^(1/2))/a^(1/3)/(-a*d+b*c)*3^(1/2)+1/3*d^(1/3)*arctan(1/3*(c^(1/3)-2
*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(1/3)/(-a*d+b*c)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {493, 298, 31, 648, 631, 210, 642}

$$\int \frac{x}{(a+bx^3)(c+dx^3)}\,dx = \frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}(bc-ad)} - \frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}(bc-ad)} + \frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)} - \frac{\sqrt[3]{d}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6\sqrt[3]{c}(bc-ad)} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}(bc-ad)} + \frac{\sqrt[3]{d}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}(bc-ad)}$$

```
[In] Int[x/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -((b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1
/3)*(b*c - a*d))) + (d^(1/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3
))])/(Sqrt[3]*c^(1/3)*(b*c - a*d)) - (b^(1/3)*Log[a^(1/3) + b^(1/3)*x])/(3*
a^(1/3)*(b*c - a*d)) + (d^(1/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(1/3)*(b*c -
 a*d)) + (b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3
)*(b*c - a*d)) - (d^(1/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(
6*c^(1/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\int \frac{x}{a+bx^3}\,dx}{bc-ad} - \frac{d\int \frac{x}{c+dx^3}\,dx}{bc-ad} \\
&= -\frac{b^{2/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}(bc-ad)} + \frac{b^{2/3}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3\sqrt[3]{a}(bc-ad)} \\
&\quad + \frac{d^{2/3}\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3\sqrt[3]{c}(bc-ad)} - \frac{d^{2/3}\int \frac{\sqrt[3]{c}+\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{3\sqrt[3]{c}(bc-ad)} \\
&= -\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}(bc-ad)} + \frac{\sqrt[3]{d}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}(bc-ad)} \\
&\quad + \frac{\sqrt[3]{b}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6\sqrt[3]{a}(bc-ad)} + \frac{b^{2/3}\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2(bc-ad)} \\
&\quad - \frac{\sqrt[3]{d}\int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{6\sqrt[3]{c}(bc-ad)} - \frac{d^{2/3}\int \frac{1}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{2(bc-ad)}
\end{aligned}$$

$$= -\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}(bc-ad)} + \frac{\sqrt[3]{d}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}(bc-ad)} + \frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}(bc-ad)} - \frac{\sqrt[3]{d}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6\sqrt[3]{c}(bc-ad)} + \frac{\sqrt[3]{b}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}(bc-ad)} - \frac{\sqrt[3]{d}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{\sqrt[3]{c}(bc-ad)}$$

$$= -\frac{\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}(bc-ad)} + \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}(bc-ad)} + \frac{\sqrt[3]{d}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}(bc-ad)} + \frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}(bc-ad)} - \frac{\sqrt[3]{d}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6\sqrt[3]{c}(bc-ad)}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.78

$$\int \frac{x}{(a+bx^3)(c+dx^3)}\,dx = \frac{\frac{2\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} - \frac{2\sqrt{3}\sqrt[3]{d}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{c}} + \frac{2\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}} - \frac{2\sqrt[3]{d}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt[3]{c}} - \frac{\sqrt[3]{b}\log\left(a^{2/3}-\right.}{}}{-6bc+6ad}$$

```
[In] Integrate[x/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((2*Sqrt[3]*b^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) -
(2*Sqrt[3]*d^(1/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(1/3) + (
2*b^(1/3)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) - (2*d^(1/3)*Log[c^(1/3) + d^(1
/3)*x])/c^(1/3) - (b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
a^(1/3) + (d^(1/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(1/3))
/(-6*b*c + 6*a*d)
```

Maple [A] (verified)

Time = 4.17 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.72

method	result
default	$\frac{\left(-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)d}{ad-bc}-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\ldots\right.}{ad-bc}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^{3}c\,d^{3}-3a^{2}b\,c^{2}d^{2}+3a\,b^{2}c^{3}d-c^{4}b^{3}\right)\_Z^{3}+d\right)}\_R\ln\left(\left(\left(-a^{4}d^{4}+2a^{3}bc\,d^{3}-2a^{2}b^{2}c^{2}d^{2}+2a\,b^{3}c^{3}d-b^{4}c^{4}\right)\_R^{3}+bd\right)x+(-\ldots\right.}{3}$

```
[In] int(x/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/3/d/(c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/6/d/(c/d)^(1/3)*ln(x^2-(c/d)^(1/3)*
x+(c/d)^(2/3))+1/3*3^(1/2)/d/(c/d)^(1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*
x-1)))*d/(a*d-b*c)-(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*
ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1
/2)*(2/(a/b)^(1/3)*x-1)))*b/(a*d-b*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.70

$$\int \frac{x}{(a+bx^3)(c+dx^3)}\,dx$$
$$=\frac{2\sqrt{3}\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-2\sqrt{3}\left(-\frac{d}{c}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{d}{c}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^{2}-a\ldots\right.}{}$$

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3))
- 2*sqrt(3)*(-d/c)^(1/3)*arctan(2/3*sqrt(3)*x*(-d/c)^(1/3) + 1/3*sqrt(3)) +
 (b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) + (-d/c)^(1/3)*lo
g(d*x^2 - c*x*(-d/c)^(2/3) - c*(-d/c)^(1/3)) - 2*(b/a)^(1/3)*log(b*x + a*(b
/a)^(2/3)) - 2*(-d/c)^(1/3)*log(d*x + c*(-d/c)^(2/3)))/(b*c - a*d)
```

Sympy [A] (verification not implemented)

Time = 69.18 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.79

$$\int \frac{x}{(a+bx^3)(c+dx^3)}\,dx$$
$$= \text{RootSum}\left(t^3\cdot\left(27a^4d^3-81a^3bcd^2+81a^2b^2c^2d-27ab^3c^3\right)-b,\left(t\mapsto t\log\left(x+\frac{243t^5a^7cd^6-1458t^5a^6bc^2\right.\right.\right.$$
$$+\text{RootSum}\left(t^3\cdot\left(27a^3cd^3-81a^2bc^2d^2+81ab^2c^3d-27b^3c^4\right)+d,\left(t\mapsto t\log\left(x+\frac{243t^5a^7cd^6-1458t^5a^6b\right.\right.\right.$$

```
[In] integrate(x/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] RootSum(_t**3*(27*a**4*d**3 - 81*a**3*b*c*d**2 + 81*a**2*b**2*c**2*d - 27*a
*b**3*c**3) - b, Lambda(_t, _t*log(x + (243*_t**5*a**7*c*d**6 - 1458*_t**5*
a**6*b*c**2*d**5 + 3645*_t**5*a**5*b**2*c**3*d**4 - 4860*_t**5*a**4*b**3*c*
*4*d**3 + 3645*_t**5*a**3*b**4*c**5*d**2 - 1458*_t**5*a**2*b**5*c**6*d + 24
3*_t**5*a*b**6*c**7 + 9*_t**2*a**4*d**4 - 18*_t**2*a**3*b*c*d**3 + 18*_t**2
*a**2*b**2*c**2*d**2 - 18*_t**2*a*b**3*c**3*d + 9*_t**2*b**4*c**4)/(a*b*d**
2 + b**2*c*d))) + RootSum(_t**3*(27*a**3*c*d**3 - 81*a**2*b*c**2*d**2 + 81
*a*b**2*c**3*d - 27*b**3*c**4) + d, Lambda(_t, _t*log(x + (243*_t**5*a**7*c
*d**6 - 1458*_t**5*a**6*b*c**2*d**5 + 3645*_t**5*a**5*b**2*c**3*d**4 - 4860
*_t**5*a**4*b**3*c**4*d**3 + 3645*_t**5*a**3*b**4*c**5*d**2 - 1458*_t**5*a*
*2*b**5*c**6*d + 243*_t**5*a*b**6*c**7 + 9*_t**2*a**4*d**4 - 18*_t**2*a**3*
b*c*d**3 + 18*_t**2*a**2*b**2*c**2*d**2 - 18*_t**2*a*b**3*c**3*d + 9*_t**2*
b**4*c**4)/(a*b*d**2 + b**2*c*d))))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.92

$$\int \frac{x}{(a+bx^3)(c+dx^3)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3(bc-ad)\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3(bc-ad)\left(\frac{c}{d}\right)^{\frac{1}{3}}} + \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} - \frac{\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} - \frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} + \frac{\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}$$

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b*c - a*d
)*(a/b)^(1/3)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(
1/3))/((b*c - a*d)*(c/d)^(1/3)) + 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)
)/(b*c*(a/b)^(1/3) - a*d*(a/b)^(1/3)) - 1/6*log(x^2 - x*(c/d)^(1/3) + (c/d)
^(2/3))/(b*c*(c/d)^(1/3) - a*d*(c/d)^(1/3)) - 1/3*log(x + (a/b)^(1/3))/(b*c
*(a/b)^(1/3) - a*d*(a/b)^(1/3)) + 1/3*log(x + (c/d)^(1/3))/(b*c*(c/d)^(1/3)
 - a*d*(c/d)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.01

$$\int \frac{x}{(a+bx^3)(c+dx^3)}\,dx = -\frac{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(abc-a^2d)} + \frac{d\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2-acd)} - \frac{(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}ab^2c-\sqrt{3}a^2bd} + \frac{(-cd^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^2d-\sqrt{3}acd^2} + \frac{(-ab^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(ab^2c-a^2bd)} - \frac{(-cd^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(bc^2d-acd^2)}$$

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) + 1/3*d*(-c/
d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) - (-a*b^2)^(2/3)*arctan
(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a*b^2*c - sqrt(3)*
a^2*b*d) + (-c*d^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1
/3))/(sqrt(3)*b*c^2*d - sqrt(3)*a*c*d^2) + 1/6*(-a*b^2)^(2/3)*log(x^2 + x*(
-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2*c - a^2*b*d) - 1/6*(-c*d^2)^(2/3)*log(x^
2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^2*d - a*c*d^2)
```

Mupad [B] (verification not implemented)

Time = 10.07 (sec) , antiderivative size = 982, normalized size of antiderivative = 3.41

$$\int \frac{x}{(a+bx^3)(c+dx^3)}\,dx = \ln\left(b\,x + a^3\,d^2\left(\frac{b}{a\,(a\,d-b\,c)^3}\right)^{2/3} + a\,b^2\,c^2\left(\frac{b}{a\,(a\,d-b\,c)^3}\right)^{2/3}\right.$$

$$\left.-2\,a^2\,b\,c\,d\left(\frac{b}{a\,(a\,d-b\,c)^3}\right)^{2/3}\right)\left(\frac{b}{27\,a^4\,d^3-81\,a^3\,b\,c\,d^2+81\,a^2\,b^2\,c^2\,d-27\,a\,b^3\,c^3}\right)^{1/3}+\ln\left(d\,x+b^2\,c^3\left(-\right.\right.$$

```
[In] int(x/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(b*x + a^3*d^2*(b/(a*(a*d - b*c)^3))^(2/3) + a*b^2*c^2*(b/(a*(a*d - b*c)
^3))^(2/3) - 2*a^2*b*c*d*(b/(a*(a*d - b*c)^3))^(2/3))*(b/(27*a^4*d^3 - 27*a
*b^3*c^3 + 81*a^2*b^2*c^2*d - 81*a^3*b*c*d^2))^(1/3) + log(d*x + b^2*c^3*(-
d/(c*(a*d - b*c)^3))^(2/3) + a^2*c*d^2*(-d/(c*(a*d - b*c)^3))^(2/3) - 2*a*b
*c^2*d*(-d/(c*(a*d - b*c)^3))^(2/3))*(d/(27*b^3*c^4 - 27*a^3*c*d^3 + 81*a^2
*b*c^2*d^2 - 81*a*b^2*c^3*d))^(1/3) + (log(b^4*d^4*x - (b*(3^(1/2)*1i - 1)^
3*(27*b^3*d^3*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2
)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(b/(a*(a*d - b*c)^3))^(2/3))/4))/(216
*a*(a*d - b*c)^3))*(3^(1/2)*1i - 1)*(b/(27*a^4*d^3 - 27*a*b^3*c^3 + 81*a^2*
b^2*c^2*d - 81*a^3*b*c*d^2))^(1/3))/2 - (log(b^4*d^4*x + (b*(3^(1/2)*1i + 1
)^3*(27*b^3*d^3*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1
/2)*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^4*(b/(a*(a*d - b*c)^3))^(2/3))/4))/(2
16*a*(a*d - b*c)^3))*(3^(1/2)*1i + 1)*(b/(27*a^4*d^3 - 27*a*b^3*c^3 + 81*a^
2*b^2*c^2*d - 81*a^3*b*c*d^2))^(1/3))/2 + (log(b^4*d^4*x + (d*(3^(1/2)*1i -
 1)^3*(27*b^3*d^3*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^
(1/2)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-d/(c*(a*d - b*c)^3))^(2/3))/4))
/(216*c*(a*d - b*c)^3))*(3^(1/2)*1i - 1)*(d/(27*b^3*c^4 - 27*a^3*c*d^3 + 81
*a^2*b*c^2*d^2 - 81*a*b^2*c^3*d))^(1/3))/2 - (log(b^4*d^4*x - (d*(3^(1/2)*1
i + 1)^3*(27*b^3*d^3*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*
(3^(1/2)*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-d/(c*(a*d - b*c)^3))^(2/3))/
4))/(216*c*(a*d - b*c)^3))*(3^(1/2)*1i + 1)*(d/(27*b^3*c^4 - 27*a^3*c*d^3 +
 81*a^2*b*c^2*d^2 - 81*a*b^2*c^3*d))^(1/3))/2
```

3.115 $\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 288

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = -\frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)} + \frac{d^{2/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)} + \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)} - \frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)} - \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)} + \frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)}$$

```
[Out] 1/3*b^(2/3)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/(-a*d+b*c)-1/3*d^(2/3)*ln(c^(1/3)
+d^(1/3)*x)/c^(2/3)/(-a*d+b*c)-1/6*b^(2/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/a^(2/3)/(-a*d+b*c)+1/6*d^(2/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/
3)*x^2)/c^(2/3)/(-a*d+b*c)-1/3*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(
1/3)*3^(1/2))/a^(2/3)/(-a*d+b*c)*3^(1/2)+1/3*d^(2/3)*arctan(1/3*(c^(1/3)-2*
d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(2/3)/(-a*d+b*c)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {400, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = -\frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)} - \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)} + \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)} + \frac{d^{2/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)} + \frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)} - \frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)}$$

```
[In] Int[1/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -((b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2
/3)*(b*c - a*d))) + (d^(2/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3
))])/(Sqrt[3]*c^(2/3)*(b*c - a*d)) + (b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/(3*
a^(2/3)*(b*c - a*d)) - (d^(2/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*(b*c -
 a*d)) - (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3
)*(b*c - a*d)) + (d^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(
6*c^(2/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b\int \frac{1}{a+bx^3}\,dx}{bc-ad}-\frac{d\int \frac{1}{c+dx^3}\,dx}{bc-ad}\\
&= \frac{b\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}(bc-ad)}+\frac{b\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}(bc-ad)}-\frac{d\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3c^{2/3}(bc-ad)}-\frac{d\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{3c^{2/3}(bc-ad)}\\
&= \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)}-\frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)}-\frac{b^{2/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}(bc-ad)}\\
&\quad+\frac{b\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{a}(bc-ad)}+\frac{d^{2/3}\int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{6c^{2/3}(bc-ad)}-\frac{d\int \frac{1}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{2\sqrt[3]{c}(bc-ad)}
\end{aligned}
$$

$$= \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)} - \frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)} - \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)} + \frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)} + \frac{b^{2/3}\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{2/3}(bc-ad)} - \frac{d^{2/3}\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{c^{2/3}(bc-ad)}$$

$$= -\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)} + \frac{d^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)} + \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)} - \frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)} - \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)} + \frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.78

$$\int\frac{1}{(a+bx^3)(c+dx^3)}\,dx$$

$$= \frac{\frac{2\sqrt{3}b^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} - \frac{2\sqrt{3}d^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{2/3}} - \frac{2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}} + \frac{2d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{2/3}} + \frac{b^{2/3}\log\left(a^{2/3}-\right.}{}}{-6bc+6ad}$$

```
[In] Integrate[1/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((2*Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) -
(2*Sqrt[3]*d^(2/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(2/3) - (
2*b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (2*d^(2/3)*Log[c^(1/3) + d^(1
/3)*x])/c^(2/3) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
a^(2/3) - (d^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(2/3))
/(-6*b*c + 6*a*d)
```

Maple [A] (verified)

Time = 4.24 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.72

method	result
default	$\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)d}{ad-bc}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\text{arcta}}{}\right.}{ad-bc}$
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(a^{5}d^{3}-3a^{4}bc\,d^{2}+3a^{3}b^{2}c^{2}d-a^{2}b^{3}c^{3}\right)\_Z^{3}+b^{2}\right)}\_R\ln\left(\left(\left(-a^{5}d^{5}+3a^{4}bc\,d^{4}-2a^{3}b^{2}c^{2}d^{3}-2a^{2}b^{3}c^{3}d^{2}+3a\,b^{4}c^{4}d-b^{5}c^{5}\right)\_R\right.\right.\right.}{3}$

[In] int(1/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] (1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x
+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x
-1)))*d/(a*d-b*c)-(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln
(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2
)*(2/(a/b)^(1/3)*x-1)))*b/(a*d-b*c)

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx =$$
$$-\frac{2\sqrt{3}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)+2\sqrt{3}\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}cx\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}-\sqrt{3}d}{3d}\right)-\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2\right.}{}$$

[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")

[Out] -1/6*(2*sqrt(3)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3)
 - sqrt(3)*b)/b) + 2*sqrt(3)*(d^2/c^2)^(1/3)*arctan(1/3*(2*sqrt(3)*c*x*(d^2
/c^2)^(2/3) - sqrt(3)*d)/d) - (-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^
2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - (d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*(d^2
/c^2)^(1/3) + c^2*(d^2/c^2)^(2/3)) + 2*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a
^2)^(1/3)) + 2*(d^2/c^2)^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)))/(b*c - a*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx &= \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} \\ &\quad - \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} + \frac{\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} \\ &\quad + \frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} - \frac{\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}\end{aligned}$$

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b*c*(a/b)
^(1/3) - a*d*(a/b)^(1/3))*(a/b)^(1/3)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*
x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c*(c/d)^(1/3) - a*d*(c/d)^(1/3))*(c/d)^(1
/3)) - 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*c*(a/b)^(2/3) - a*d*(a
/b)^(2/3)) + 1/6*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c*(c/d)^(2/3) -
a*d*(c/d)^(2/3)) + 1/3*log(x + (a/b)^(1/3))/(b*c*(a/b)^(2/3) - a*d*(a/b)^(2
/3)) - 1/3*log(x + (c/d)^(1/3))/(b*c*(c/d)^(2/3) - a*d*(c/d)^(2/3))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = -\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(abc-a^2d)} + \frac{d\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2-acd)} + \frac{(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}abc-\sqrt{3}a^2d} - \frac{(-cd^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^2-\sqrt{3}acd} + \frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(abc-a^2d)} - \frac{(-cd^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(bc^2-acd)}$$

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) + 1/3*d*(-c/
d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) + (-a*b^2)^(1/3)*arctan
(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a*b*c - sqrt(3)*a^
2*d) - (-c*d^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))
/(sqrt(3)*b*c^2 - sqrt(3)*a*c*d) + 1/6*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1
/3) + (-a/b)^(2/3))/(a*b*c - a^2*d) - 1/6*(-c*d^2)^(1/3)*log(x^2 + x*(-c/d)
^(1/3) + (-c/d)^(2/3))/(b*c^2 - a*c*d)
```

Mupad [B] (verification not implemented)

Time = 13.94 (sec) , antiderivative size = 1364, normalized size of antiderivative = 4.74

$$\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx = \text{Too large to display}$$

```
[In] int(1/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(((-b^2/(a^2*(a*d - b*c)^3))^(1/3)*(9*a^2*b^4*d^6 + 9*b^6*c^2*d^4 - 18*a
*b^5*c*d^5 - 9*b^3*d^3*(x + a*c*(-b^2/(a^2*(a*d - b*c)^3))^(1/3))*(a*d + b*
c)*(a*d - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^(2/3)))/3 - 6*b^5*d^5*x)*(-b^2/
(27*a^5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81*a^4*b*c*d^2))^(1/3) +
```

```
log(((d^2/(c^2*(a*d - b*c)^3))^(1/3)*(9*a^2*b^4*d^6 + 9*b^6*c^2*d^4 - 18*a*
b^5*c*d^5 - 9*b^3*d^3*(x + a*c*(d^2/(c^2*(a*d - b*c)^3))^(1/3))*(a*d + b*c)
*(a*d - b*c)^4*(d^2/(c^2*(a*d - b*c)^3))^(2/3)))/3 - 6*b^5*d^5*x)*(-d^2/(27
*b^3*c^5 - 27*a^3*c^2*d^3 + 81*a^2*b*c^3*d^2 - 81*a*b^2*c^4*d))^(1/3) + (lo
g(6*b^5*d^5*x + ((3^(1/2)*1i - 1)*(-b^2/(a^2*(a*d - b*c)^3))^(1/3)*(((3^(1/
2)*1i - 1)^2*(81*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 + (81*a*b^3*c*d^3*(3^(
1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^(1/3))/2)
*(-b^2/(a^2*(a*d - b*c)^3))^(2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6*c^2*d^4 + 18*
a*b^5*c*d^5))/6)*(-b^2/(27*a^5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81
*a^4*b*c*d^2))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(6*b^5*d^5*x - ((3^(1/2)*1i
+ 1)*(-b^2/(a^2*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81*b^3*d^3*x*(a
*d + b*c)*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)*(a*d
 - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^(1/3))/2)*(-b^2/(a^2*(a*d - b*c)^3))^(
2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6*c^2*d^4 + 18*a*b^5*c*d^5))/6)*(-b^2/(27*a^
5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81*a^4*b*c*d^2))^(1/3)*(3^(1/2)
*1i + 1))/2 + (log(6*b^5*d^5*x + ((3^(1/2)*1i - 1)*(d^2/(c^2*(a*d - b*c)^3)
)^(1/3)*(((3^(1/2)*1i - 1)^2*(81*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 + (81*
a*b^3*c*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(d^2/(c^2*(a*d - b*c
)^3))^(1/3))/2)*(d^2/(c^2*(a*d - b*c)^3))^(2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6
*c^2*d^4 + 18*a*b^5*c*d^5))/6)*(-d^2/(27*b^3*c^5 - 27*a^3*c^2*d^3 + 81*a^2*
b*c^3*d^2 - 81*a*b^2*c^4*d))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(6*b^5*d^5*x -
 ((3^(1/2)*1i + 1)*(d^2/(c^2*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81
*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*
d + b*c)*(a*d - b*c)^4*(d^2/(c^2*(a*d - b*c)^3))^(1/3))/2)*(d^2/(c^2*(a*d -
 b*c)^3))^(2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6*c^2*d^4 + 18*a*b^5*c*d^5))/6)*(
-d^2/(27*b^3*c^5 - 27*a^3*c^2*d^3 + 81*a^2*b*c^3*d^2 - 81*a*b^2*c^4*d))^(1/
3)*(3^(1/2)*1i + 1))/2
```

3.116 $\int \frac{1}{x(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 62

$$\int \frac{1}{x(a+bx^3)(c+dx^3)}\,dx = \frac{\log(x)}{ac} - \frac{b\log(a+bx^3)}{3a(bc-ad)} + \frac{d\log(c+dx^3)}{3c(bc-ad)}$$

```
[Out] ln(x)/a/c-1/3*b*ln(b*x^3+a)/a/(-a*d+b*c)+1/3*d*ln(d*x^3+c)/c/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 84}

$$\int \frac{1}{x(a+bx^3)(c+dx^3)}\,dx = -\frac{b\log(a+bx^3)}{3a(bc-ad)} + \frac{d\log(c+dx^3)}{3c(bc-ad)} + \frac{\log(x)}{ac}$$

```
[In] Int[1/(x*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] Log[x]/(a*c) - (b*Log[a + b*x^3])/(3*a*(b*c - a*d)) + (d*Log[c + d*x^3])/(3
*c*(b*c - a*d))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)}\,dx, x, x^3\right) \\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{acx} + \frac{b^2}{a(-bc+ad)(a+bx)} + \frac{d^2}{c(bc-ad)(c+dx)}\right)\,dx, x, x^3\right) \\
&= \frac{\log(x)}{ac} - \frac{b\log\left(a+bx^3\right)}{3a(bc-ad)} + \frac{d\log\left(c+dx^3\right)}{3c(bc-ad)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = \frac{3bc\log(x) - 3ad\log(x) - bc\log\left(a+bx^3\right) + ad\log\left(c+dx^3\right)}{3abc^2 - 3a^2cd}$$

```
[In] Integrate[1/(x*(a + b*x^3)*(c + d*x^3)),x]
[Out] (3*b*c*Log[x] - 3*a*d*Log[x] - b*c*Log[a + b*x^3] + a*d*Log[c + d*x^3])/(3*
a*b*c^2 - 3*a^2*c*d)
```

Maple [A] (verified)

Time = 4.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

method	result	size
parallelrisch	$\frac{3\ln(x)ad-3\ln(x)bc+b\ln(bx^3+a)c-d\ln(dx^3+c)a}{3ac(ad-bc)}$	55
default	$\frac{\ln(x)}{ac}-\frac{d\ln(dx^3+c)}{3(ad-bc)c}+\frac{b\ln(bx^3+a)}{3(ad-bc)a}$	59
norman	$\frac{\ln(x)}{ac}-\frac{d\ln(dx^3+c)}{3(ad-bc)c}+\frac{b\ln(bx^3+a)}{3(ad-bc)a}$	59
risch	$\frac{\ln(x)}{ac}-\frac{d\ln(-dx^3-c)}{3c(ad-bc)}+\frac{b\ln(-bx^3-a)}{3(ad-bc)a}$	65

```
[In] int(1/x/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
[Out] 1/3*(3*ln(x)*a*d-3*ln(x)*b*c+b*ln(b*x^3+a)*c-d*ln(d*x^3+c)*a)/a/c/(a*d-b*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{1}{x(a+bx^3)(c+dx^3)}\,dx = -\frac{bc\log(bx^3+a) - ad\log(dx^3+c) - 3(bc-ad)\log(x)}{3(abc^2-a^2cd)}$$

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
[Out] -1/3*(b*c*log(b*x^3 + a) - a*d*log(d*x^3 + c) - 3*(b*c - a*d)*log(x))/(a*b*
c^2 - a^2*c*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x(a+bx^3)(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/(b*x**3+a)/(d*x**3+c),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{1}{x(a+bx^3)(c+dx^3)}\,dx = -\frac{b\log(bx^3+a)}{3(abc-a^2d)} + \frac{d\log(dx^3+c)}{3(bc^2-acd)} + \frac{\log(x^3)}{3\,ac}$$

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
[Out] -1/3*b*log(b*x^3 + a)/(a*b*c - a^2*d) + 1/3*d*log(d*x^3 + c)/(b*c^2 - a*c*d
) + 1/3*log(x^3)/(a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.15

$$\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = -\frac{b^2\log\left(\left|bx^3+a\right|\right)}{3\left(ab^2c-a^2bd\right)}+\frac{d^2\log\left(\left|dx^3+c\right|\right)}{3\left(bc^2d-acd^2\right)}+\frac{\log\left(\left|x\right|\right)}{ac}$$

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b^2*log(abs(b*x^3 + a))/(a*b^2*c - a^2*b*d) + 1/3*d^2*log(abs(d*x^3 +
c))/(b*c^2*d - a*c*d^2) + log(abs(x))/(a*c)
```

Mupad [B] (verification not implemented)

Time = 7.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = \frac{b\,\ln\left(b\,x^3+a\right)}{3\,a^2\,d-3\,a\,b\,c}+\frac{d\,\ln\left(d\,x^3+c\right)}{3\,b\,c^2-3\,a\,c\,d}+\frac{\ln\left(x\right)}{a\,c}$$

```
[In] int(1/(x*(a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] (b*log(a + b*x^3))/(3*a^2*d - 3*a*b*c) + (d*log(c + d*x^3))/(3*b*c^2 - 3*a*
c*d) + log(x)/(a*c)
```

3.117 $\int \frac{1}{x^2(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 299

$$\int \frac{1}{x^2(a+bx^3)(c+dx^3)}\,dx = -\frac{1}{acx}+\frac{b^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}(bc-ad)}-\frac{d^{4/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{4/3}(bc-ad)}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}(bc-ad)}-\frac{d^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{4/3}(bc-ad)}-\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}(bc-ad)}+\frac{d^{4/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{4/3}(bc-ad)}$$

```
[Out] -1/a/c/x+1/3*b^(4/3)*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)/(-a*d+b*c)-1/3*d^(4/3)*l
n(c^(1/3)+d^(1/3)*x)/c^(4/3)/(-a*d+b*c)-1/6*b^(4/3)*ln(a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/a^(4/3)/(-a*d+b*c)+1/6*d^(4/3)*ln(c^(2/3)-c^(1/3)*d^(1/3
)*x+d^(2/3)*x^2)/c^(4/3)/(-a*d+b*c)+1/3*b^(4/3)*arctan(1/3*(a^(1/3)-2*b^(1/
3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/(-a*d+b*c)*3^(1/2)-1/3*d^(4/3)*arctan(1/3*(c
^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(4/3)/(-a*d+b*c)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {491, 598, 298, 31, 648, 631, 210, 642}

$$\int \frac{1}{x^2(a+bx^3)(c+dx^3)}\,dx = \frac{b^{4/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}(bc-ad)} - \frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}(bc-ad)} + \frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}(bc-ad)} - \frac{d^{4/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{4/3}(bc-ad)} + \frac{d^{4/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{4/3}(bc-ad)} - \frac{d^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{4/3}(bc-ad)} - \frac{1}{acx}$$

```
[In] Int[1/(x^2*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -(1/(a*c*x)) + (b^(4/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
(Sqrt[3]*a^(4/3)*(b*c - a*d)) - (d^(4/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sq
rt[3]*c^(1/3))])/(Sqrt[3]*c^(4/3)*(b*c - a*d)) + (b^(4/3)*Log[a^(1/3) + b^(
1/3)*x])/(3*a^(4/3)*(b*c - a*d)) - (d^(4/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^
(4/3)*(b*c - a*d)) - (b^(4/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(6*a^(4/3)*(b*c - a*d)) + (d^(4/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(
2/3)*x^2])/(6*c^(4/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{acx}+\frac{\int \frac{x(-bc-ad-bdx^3)}{(a+bx^3)(c+dx^3)}\,dx}{ac}\\
&= -\frac{1}{acx}+\frac{\int \left(-\frac{b^2cx}{(bc-ad)(a+bx^3)}-\frac{ad^2x}{(-bc+ad)(c+dx^3)}\right)dx}{ac}\\
&= -\frac{1}{acx}-\frac{b^2\int \frac{x}{a+bx^3}\,dx}{a(bc-ad)}+\frac{d^2\int \frac{x}{c+dx^3}\,dx}{c(bc-ad)}
\end{aligned}
$$

$$= -\frac{1}{acx} + \frac{b^{5/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{4/3}(bc-ad)} - \frac{b^{5/3}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{4/3}(bc-ad)} - \frac{d^{5/3}\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3c^{4/3}(bc-ad)} + \frac{d^{5/3}\int \frac{\sqrt[3]{c}+\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{3c^{4/3}(bc-ad)}$$

$$= -\frac{1}{acx} + \frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}(bc-ad)} - \frac{d^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{4/3}(bc-ad)} - \frac{b^{4/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{4/3}(bc-ad)} - \frac{b^{5/3}\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a(bc-ad)} + \frac{d^{4/3}\int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{6c^{4/3}(bc-ad)} + \frac{d^{5/3}\int \frac{1}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{2c(bc-ad)}$$

$$= -\frac{1}{acx} + \frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}(bc-ad)} - \frac{d^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{4/3}(bc-ad)} - \frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}(bc-ad)} + \frac{d^{4/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{4/3}(bc-ad)} - \frac{b^{4/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{4/3}(bc-ad)} + \frac{d^{4/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{c^{4/3}(bc-ad)}$$

$$= -\frac{1}{acx} + \frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}(bc-ad)} - \frac{d^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{4/3}(bc-ad)} + \frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}(bc-ad)} - \frac{d^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{4/3}(bc-ad)} - \frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}(bc-ad)} + \frac{d^{4/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{4/3}(bc-ad)}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2(a+bx^3)(c+dx^3)}\,dx = \frac{\frac{6b}{a}-\frac{6d}{c}-\frac{2\sqrt{3}b^{4/3}x\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}}+\frac{2\sqrt{3}d^{4/3}x\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{4/3}}-\frac{2b^{4/3}x\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{4/3}}+\frac{2d^{4/3}x\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{4/3}}}{-6bcx+6adx}$$

```
[In] Integrate[1/(x^2*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((6*b)/a - (6*d)/c - (2*Sqrt[3]*b^(4/3)*x*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3)
)/Sqrt[3]])/a^(4/3) + (2*Sqrt[3]*d^(4/3)*x*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3
))/Sqrt[3]])/c^(4/3) - (2*b^(4/3)*x*Log[a^(1/3) + b^(1/3)*x])/a^(4/3) + (2*
d^(4/3)*x*Log[c^(1/3) + d^(1/3)*x])/c^(4/3) + (b^(4/3)*x*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3) - (d^(4/3)*x*Log[c^(2/3) - c^(1/3)*d^
(1/3)*x + d^(2/3)*x^2])/c^(4/3))/(-6*b*c*x + 6*a*d*x)
```

Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.76

method	result
default	$-\frac{\left(-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)d^2}{(ad-bc)c}+\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\right.}{(ad-bc)a}$
risch	$-\frac{1}{acx}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(d^3a^7-3a^6bcd^2+3a^5b^2c^2d-a^4b^3c^3\right)\_Z^3+b^4\right)}\_R\ln\left(\left(\left(-4a^{10}c^4d^6+22a^9bc^5d^5-52a^8b^2c^6d^4+68a^7b^3c^7d^3-\right.\right.\right.\right.}{}$

```
[In] int(1/x^2/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -(-1/3/d/(c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/6/d/(c/d)^(1/3)*ln(x^2-(c/d)^(1/3)
*x+(c/d)^(2/3))+1/3*3^(1/2)/d/(c/d)^(1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)
*x-1)))*d^2/(a*d-b*c)/c+(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(
```

```
1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*b^2/(a*d-b*c)/a-1/a/c/x
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^2\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = -\frac{2\sqrt{3}bcx\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-2\sqrt{3}adx\left(\frac{d}{c}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{d}{c}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-bcx\left(-\frac{b}{a}\right)^{\frac{1}{3}}}{}$$

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*b*c*x*(-b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*
sqrt(3)) - 2*sqrt(3)*a*d*x*(d/c)^(1/3)*arctan(2/3*sqrt(3)*x*(d/c)^(1/3) - 1
/3*sqrt(3)) - b*c*x*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a*(-b/a)^(1
/3)) - a*d*x*(d/c)^(1/3)*log(d*x^2 - c*x*(d/c)^(2/3) + c*(d/c)^(1/3)) + 2*b
*c*x*(-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)) + 2*a*d*x*(d/c)^(1/3)*log(d*x +
 c*(d/c)^(2/3)) + 6*b*c - 6*a*d)/((a*b*c^2 - a^2*c*d)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x**2/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+bx^3)(c+dx^3)}\,dx = -\frac{\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3(abc-a^2d)\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\sqrt{3}d\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3(bc^2-acd)\left(\frac{c}{d}\right)^{\frac{1}{3}}} - \frac{b\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(abc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} + \frac{d\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^2\left(\frac{c}{d}\right)^{\frac{1}{3}}-acd\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} + \frac{b\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(abc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)} - \frac{d\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc^2\left(\frac{c}{d}\right)^{\frac{1}{3}}-acd\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} - \frac{1}{acx}$$

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((a*b*c
- a^2*d)*(a/b)^(1/3)) + 1/3*sqrt(3)*d*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3)
)/(c/d)^(1/3))/((b*c^2 - a*c*d)*(c/d)^(1/3)) - 1/6*b*log(x^2 - x*(a/b)^(1/3
) + (a/b)^(2/3))/(a*b*c*(a/b)^(1/3) - a^2*d*(a/b)^(1/3)) + 1/6*d*log(x^2 -
x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c^2*(c/d)^(1/3) - a*c*d*(c/d)^(1/3)) + 1/3*
b*log(x + (a/b)^(1/3))/(a*b*c*(a/b)^(1/3) - a^2*d*(a/b)^(1/3)) - 1/3*d*log(
x + (c/d)^(1/3))/(b*c^2*(c/d)^(1/3) - a*c*d*(c/d)^(1/3)) - 1/(a*c*x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^2(a+bx^3)(c+dx^3)}\,dx = \frac{b^2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3(a^2bc-a^3d)} - \frac{d^2\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3(bc^3-ac^2d)} + \frac{(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}a^2bc-\sqrt{3}a^3d} - \frac{(-cd^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^3-\sqrt{3}ac^2d} - \frac{(-ab^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(a^2bc-a^3d)} + \frac{(-cd^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6(bc^3-ac^2d)} - \frac{1}{acx}$$

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b*c - a^3*d) - 1/3*d^2
*(-c/d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^3 - a*c^2*d) + (-a*b^2)^(2/3)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^2*b*c - s
qrt(3)*a^3*d) - (-c*d^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/
d)^(1/3))/(sqrt(3)*b*c^3 - sqrt(3)*a*c^2*d) - 1/6*(-a*b^2)^(2/3)*log(x^2 +
x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b*c - a^3*d) + 1/6*(-c*d^2)^(2/3)*log(x
^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^3 - a*c^2*d) - 1/(a*c*x)
```

Mupad [B] (verification not implemented)

Time = 8.43 (sec) , antiderivative size = 716, normalized size of antiderivative = 2.39

$$\int \frac{1}{x^2\,(a+bx^3)\,(c+dx^3)}\,dx = \ln\left(b-a^2\,d\,x\,\left(-\frac{b^4}{a^4\,(a\,d-b\,c)^3}\right)^{1/3} + a\,b\,c\,x\,\left(-\frac{b^4}{a^4\,(a\,d-b\,c)^3}\right)^{1/3}\right)\,\left(-\frac{b^4}{27\,a^7\,d^3-81\,a^6\,b\,c\,d^2+81\,a^5\,b^2\,c^2\,d-27\,a^4\,b^3\,c^3}\right)^{1/3} + \ln\left(d-b\,c^2\,x\,\left(\frac{d^4}{c^4\,(a\,d-b\,c)^3}\right)^{1/3}+a\,c\,d\,x\,\left(\frac{d^4}{c^4\,(a\,d-b\,c)^3}\right)^{1/3}\right)\,\left(-\frac{d^4}{-27\,a^3\,c^4\,d^3+81\,a^2\,b\,c^5\,d^2-81\,a\,b^2\,c^6\ldots}\right.$$

```
[In] int(1/(x^2*(a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(b - a^2*d*x*(-b^4/(a^4*(a*d - b*c)^3))^(1/3) + a*b*c*x*(-b^4/(a^4*(a*d
- b*c)^3))^(1/3))*(-b^4/(27*a^7*d^3 - 27*a^4*b^3*c^3 + 81*a^5*b^2*c^2*d - 8
1*a^6*b*c*d^2))^(1/3) + log(d - b*c^2*x*(d^4/(c^4*(a*d - b*c)^3))^(1/3) + a
*c*d*x*(d^4/(c^4*(a*d - b*c)^3))^(1/3))*(-d^4/(27*b^3*c^7 - 27*a^3*c^4*d^3
+ 81*a^2*b*c^5*d^2 - 81*a*b^2*c^6*d))^(1/3) - 1/(a*c*x) - (log(b - 3^(1/2)*
b*1i + 2*a^2*d*x*(-b^4/(a^4*(a*d - b*c)^3))^(1/3) - 2*a*b*c*x*(-b^4/(a^4*(a
*d - b*c)^3))^(1/3))*(-b^4/(27*a^7*d^3 - 27*a^4*b^3*c^3 + 81*a^5*b^2*c^2*d
- 81*a^6*b*c*d^2))^(1/3)*(3^(1/2)*1i + 1))/2 + (log(b + 3^(1/2)*b*1i + 2*a^
2*d*x*(-b^4/(a^4*(a*d - b*c)^3))^(1/3) - 2*a*b*c*x*(-b^4/(a^4*(a*d - b*c)^3
))^(1/3))*(-b^4/(27*a^7*d^3 - 27*a^4*b^3*c^3 + 81*a^5*b^2*c^2*d - 81*a^6*b*
c*d^2))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(d - 3^(1/2)*d*1i + 2*b*c^2*x*(d^4/
(c^4*(a*d - b*c)^3))^(1/3) - 2*a*c*d*x*(d^4/(c^4*(a*d - b*c)^3))^(1/3))*(-d
^4/(27*b^3*c^7 - 27*a^3*c^4*d^3 + 81*a^2*b*c^5*d^2 - 81*a*b^2*c^6*d))^(1/3)
*(3^(1/2)*1i + 1))/2 + (log(d + 3^(1/2)*d*1i + 2*b*c^2*x*(d^4/(c^4*(a*d - b
*c)^3))^(1/3) - 2*a*c*d*x*(d^4/(c^4*(a*d - b*c)^3))^(1/3))*(-d^4/(27*b^3*c^
7 - 27*a^3*c^4*d^3 + 81*a^2*b*c^5*d^2 - 81*a*b^2*c^6*d))^(1/3)*(3^(1/2)*1i
- 1))/2
```

3.118 $\int \frac{1}{x^3(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 301

$$\int \frac{1}{x^3(a+bx^3)(c+dx^3)}\,dx = -\frac{1}{2acx^2}+\frac{b^{5/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}(bc-ad)} - \frac{d^{5/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{5/3}(bc-ad)} - \frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}(bc-ad)} + \frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{5/3}(bc-ad)} + \frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}(bc-ad)} - \frac{d^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{5/3}(bc-ad)}$$

```
[Out] -1/2/a/c/x^2-1/3*b^(5/3)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/(-a*d+b*c)+1/3*d^(5/
3)*ln(c^(1/3)+d^(1/3)*x)/c^(5/3)/(-a*d+b*c)+1/6*b^(5/3)*ln(a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/(-a*d+b*c)-1/6*d^(5/3)*ln(c^(2/3)-c^(1/3)*d^
(1/3)*x+d^(2/3)*x^2)/c^(5/3)/(-a*d+b*c)+1/3*b^(5/3)*arctan(1/3*(a^(1/3)-2*b
^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/(-a*d+b*c)*3^(1/2)-1/3*d^(5/3)*arctan(1/
3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(5/3)/(-a*d+b*c)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {491, 536, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{x^3(a+bx^3)(c+dx^3)}\,dx = \frac{b^{5/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}(bc-ad)}+\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}(bc-ad)} - \frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}(bc-ad)} - \frac{d^{5/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{5/3}(bc-ad)} - \frac{d^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{5/3}(bc-ad)} + \frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{5/3}(bc-ad)} - \frac{1}{2acx^2}$$

```
[In] Int[1/(x^3*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/2*1/(a*c*x^2) + (b^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)])/(Sqrt[3]*a^(5/3)*(b*c - a*d)) - (d^(5/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)
/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(5/3)*(b*c - a*d)) - (b^(5/3)*Log[a^(1/3) +
 b^(1/3)*x])/(3*a^(5/3)*(b*c - a*d)) + (d^(5/3)*Log[c^(1/3) + d^(1/3)*x])/(
3*c^(5/3)*(b*c - a*d)) + (b^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2])/(6*a^(5/3)*(b*c - a*d)) - (d^(5/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x +
 d^(2/3)*x^2])/(6*c^(5/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{2acx^2}+\frac{\int \frac{-2(bc+ad)-2bdx^3}{(a+bx^3)(c+dx^3)}\,dx}{2ac}\\ &= -\frac{1}{2acx^2}-\frac{b^2\int \frac{1}{a+bx^3}\,dx}{a(bc-ad)}+\frac{d^2\int \frac{1}{c+dx^3}\,dx}{c(bc-ad)}\end{aligned}$$

$$= -\frac{1}{2acx^2} - \frac{b^2 \int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{5/3}(bc-ad)} - \frac{b^2 \int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{5/3}(bc-ad)} + \frac{d^2 \int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3c^{5/3}(bc-ad)} + \frac{d^2 \int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{3c^{5/3}(bc-ad)}$$

$$= -\frac{1}{2acx^2} - \frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}(bc-ad)} + \frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{5/3}(bc-ad)} + \frac{b^{5/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{5/3}(bc-ad)} - \frac{b^2 \int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a^{4/3}(bc-ad)} - \frac{d^{5/3}\int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{6c^{5/3}(bc-ad)} + \frac{d^2 \int \frac{1}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{2c^{4/3}(bc-ad)}$$

$$= -\frac{1}{2acx^2} - \frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}(bc-ad)} + \frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{5/3}(bc-ad)} + \frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}(bc-ad)} - \frac{d^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{5/3}(bc-ad)} - \frac{b^{5/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{5/3}(bc-ad)} + \frac{d^{5/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{c^{5/3}(bc-ad)}$$

$$= -\frac{1}{2acx^2} + \frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}(bc-ad)} - \frac{d^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{5/3}(bc-ad)} - \frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}(bc-ad)} + \frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{5/3}(bc-ad)} + \frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}(bc-ad)} - \frac{d^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{5/3}(bc-ad)}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^3(a+bx^3)(c+dx^3)}\,dx$$

$$= \frac{\frac{3b}{a} - \frac{3d}{c} - \frac{2\sqrt{3}b^{5/3}x^2\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}} + \frac{2\sqrt{3}d^{5/3}x^2\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{5/3}} + \frac{2b^{5/3}x^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}} - \frac{2d^{5/3}x^2\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{5/3}}}{6(-bc+ad)x^2}$$

```
[In] Integrate[1/(x^3*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((3*b)/a - (3*d)/c - (2*Sqrt[3]*b^(5/3)*x^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]])/a^(5/3) + (2*Sqrt[3]*d^(5/3)*x^2*ArcTan[(1 - (2*d^(1/3)*x)/c^
(1/3))/Sqrt[3]])/c^(5/3) + (2*b^(5/3)*x^2*Log[a^(1/3) + b^(1/3)*x])/a^(5/3)
 - (2*d^(5/3)*x^2*Log[c^(1/3) + d^(1/3)*x])/c^(5/3) - (b^(5/3)*x^2*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3) + (d^(5/3)*x^2*Log[c^(2/3)
- c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(5/3))/(6*(-(b*c) + a*d)*x^2)
```

Maple [A] (verified)

Time = 4.45 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.76

method	result
default	$-\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)d^{2}}{c(ad-bc)}+\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\ldots}{\ldots}\right.}{a(ad-bc)}$
risch	$-\frac{1}{2acx^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(d^{3}a^{8}-3c\,d^{2}a^{7}b+3c^{2}d\,a^{6}b^{2}-a^{5}b^{3}c^{3}\right)\_Z^{3}-b^{5}\right)}\_R\ln\left(\left(\left(-4a^{11}c^{5}d^{6}+22a^{10}b\,c^{6}d^{5}-52a^{9}b^{2}c^{7}d^{4}+68a^{8}b^{3}c^{8}\ldots\right.\right.\right.\right.}{\ldots}$

```
[In] int(1/x^3/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*
x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*
x-1)))/c*d^2/(a*d-b*c)+(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/
3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2/(a/b)^(1/3)*x-1)))/a*b^2/(a*d-b*c)-1/2/a/c/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.78 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00

$$\int\frac{1}{x^{3}\left(a+bx^{3}\right)\left(c+dx^{3}\right)}\,dx=$$

$$-\frac{2\sqrt{3}bcx^{2}\left(\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)+2\sqrt{3}adx^{2}\left(-\frac{d^{2}}{c^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}cx\left(-\frac{d^{2}}{c^{2}}\right)^{\frac{2}{3}}-\sqrt{3}d}{3d}\right)-bcx^{2}\left(\frac{b^{2}}{a^{2}}\right.\ldots}{\ldots}$$

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*b*c*x^2*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)
^(2/3) - sqrt(3)*b)/b) + 2*sqrt(3)*a*d*x^2*(-d^2/c^2)^(1/3)*arctan(1/3*(2*s
qrt(3)*c*x*(-d^2/c^2)^(2/3) - sqrt(3)*d)/d) - b*c*x^2*(b^2/a^2)^(1/3)*log(b
^2*x^2 - a*b*x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) - a*d*x^2*(-d^2/c^2)^
(1/3)*log(d^2*x^2 + c*d*x*(-d^2/c^2)^(1/3) + c^2*(-d^2/c^2)^(2/3)) + 2*b*c*
x^2*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)) + 2*a*d*x^2*(-d^2/c^2)^(1/
3)*log(d*x - c*(-d^2/c^2)^(1/3)) + 3*b*c - 3*a*d)/((a*b*c^2 - a^2*c*d)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(a+bx^3\right)\left(c+dx^3\right)}\,dx=\text{Timed out}$$

```
[In] integrate(1/x**3/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.09

$$\begin{aligned}\int \frac{1}{x^3\left(a+bx^3\right)\left(c+dx^3\right)}\,dx= & -\frac{\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(abc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} \\ & +\frac{\sqrt{3}d\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc^2\left(\frac{c}{d}\right)^{\frac{1}{3}}-acd\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} \\ & +\frac{b\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(abc\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} \\ & -\frac{d\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-acd\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}-\frac{b\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(abc\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} \\ & +\frac{d\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-acd\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}-\frac{1}{2\,acx^2}\end{aligned}$$

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((a*b*c*
(a/b)^(1/3) - a^2*d*(a/b)^(1/3))*(a/b)^(1/3)) + 1/3*sqrt(3)*d*arctan(1/3*sq
rt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c^2*(c/d)^(1/3) - a*c*d*(c/d)^(1
/3))*(c/d)^(1/3)) + 1/6*b*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b*c*(a/
b)^(2/3) - a^2*d*(a/b)^(2/3)) - 1/6*d*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3)
)/(b*c^2*(c/d)^(2/3) - a*c*d*(c/d)^(2/3)) - 1/3*b*log(x + (a/b)^(1/3))/(a*b
*c*(a/b)^(2/3) - a^2*d*(a/b)^(2/3)) + 1/3*d*log(x + (c/d)^(1/3))/(b*c^2*(c/
d)^(2/3) - a*c*d*(c/d)^(2/3)) - 1/2/(a*c*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^3\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = \frac{b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(a^2bc-a^3d\right)} - \frac{d^2\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^3-ac^2d\right)} - \frac{\left(-ab^2\right)^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}a^2bc-\sqrt{3}a^3d} + \frac{\left(-cd^2\right)^{\frac{1}{3}}d\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^3-\sqrt{3}ac^2d} - \frac{\left(-ab^2\right)^{\frac{1}{3}}b\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a^2bc-a^3d\right)} + \frac{\left(-cd^2\right)^{\frac{1}{3}}d\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^3-ac^2d\right)} - \frac{1}{2\,acx^2}$$

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b*c - a^3*d) - 1/3*d^2
*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^3 - a*c^2*d) - (-a*b^2)^(1/3)
*b*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^2*b*c -
 sqrt(3)*a^3*d) + (-c*d^2)^(1/3)*d*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/
(-c/d)^(1/3))/(sqrt(3)*b*c^3 - sqrt(3)*a*c^2*d) - 1/6*(-a*b^2)^(1/3)*b*log(
x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b*c - a^3*d) + 1/6*(-c*d^2)^(1/3)
*d*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^3 - a*c^2*d) - 1/2/(a*c*x^
2)
```

Mupad [B] (verification not implemented)

Time = 16.87 (sec) , antiderivative size = 1829, normalized size of antiderivative = 6.08

$$\int \frac{1}{x^3 (a + bx^3) (c + dx^3)} dx = \text{Too large to display}$$

```
[In] int(1/(x^3*(a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(((b^5/(a^5*(a*d - b*c)^3))^(1/3)*(((81*a^10*b^3*c^10*d^3*(a*d + b*c)*(a
*d - b*c)^4*(b^5/(a^5*(a*d - b*c)^3))^(1/3) - 81*a^8*b^3*c^8*d^3*x*(a*d - b
*c)^4*(a^2*d^2 + b^2*c^2 + a*b*c*d))*(b^5/(a^5*(a*d - b*c)^3))^(2/3))/9 + 9
*a^6*b^9*c^11*d^4 - 9*a^7*b^8*c^10*d^5 - 9*a^10*b^5*c^7*d^8 + 9*a^11*b^4*c^
6*d^9))/3 + 3*a^6*b^6*c^6*d^6*x*(a^2*d^2 + b^2*c^2))*(b^5/(27*a^8*d^3 - 27*
a^5*b^3*c^3 + 81*a^6*b^2*c^2*d - 81*a^7*b*c*d^2))^(1/3) + log(((-d^5/(c^5*(
a*d - b*c)^3))^(1/3)*(((81*a^10*b^3*c^10*d^3*(a*d + b*c)*(a*d - b*c)^4*(-d^
5/(c^5*(a*d - b*c)^3))^(1/3) - 81*a^8*b^3*c^8*d^3*x*(a*d - b*c)^4*(a^2*d^2
+ b^2*c^2 + a*b*c*d))*(-d^5/(c^5*(a*d - b*c)^3))^(2/3))/9 + 9*a^6*b^9*c^11*
d^4 - 9*a^7*b^8*c^10*d^5 - 9*a^10*b^5*c^7*d^8 + 9*a^11*b^4*c^6*d^9))/3 + 3*
a^6*b^6*c^6*d^6*x*(a^2*d^2 + b^2*c^2))*(d^5/(27*b^3*c^8 - 27*a^3*c^5*d^3 +
81*a^2*b*c^6*d^2 - 81*a*b^2*c^7*d))^(1/3) + (log(((3^(1/2)*1i - 1)*(b^5/(a^
5*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i - 1)^2*(81*a^8*b^3*c^8*d^3*x*(a*d - b
*c)^4*(a^2*d^2 + b^2*c^2 + a*b*c*d) - (81*a^10*b^3*c^10*d^3*(3^(1/2)*1i - 1
)*(a*d + b*c)*(a*d - b*c)^4*(b^5/(a^5*(a*d - b*c)^3))^(1/3))/2)*(b^5/(a^5*(
a*d - b*c)^3))^(2/3))/36 - 9*a^6*b^9*c^11*d^4 + 9*a^7*b^8*c^10*d^5 + 9*a^10
*b^5*c^7*d^8 - 9*a^11*b^4*c^6*d^9))/6 - 3*a^6*b^6*c^6*d^6*x*(a^2*d^2 + b^2*
c^2))*(b^5/(27*a^8*d^3 - 27*a^5*b^3*c^3 + 81*a^6*b^2*c^2*d - 81*a^7*b*c*d^2
))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(((3^(1/2)*1i + 1)*(b^5/(a^5*(a*d - b*c)
^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81*a^8*b^3*c^8*d^3*x*(a*d - b*c)^4*(a^2*d^
2 + b^2*c^2 + a*b*c*d) + (81*a^10*b^3*c^10*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)
*(a*d - b*c)^4*(b^5/(a^5*(a*d - b*c)^3))^(1/3))/2)*(b^5/(a^5*(a*d - b*c)^3)
)^(2/3))/36 - 9*a^6*b^9*c^11*d^4 + 9*a^7*b^8*c^10*d^5 + 9*a^10*b^5*c^7*d^8
- 9*a^11*b^4*c^6*d^9))/6 + 3*a^6*b^6*c^6*d^6*x*(a^2*d^2 + b^2*c^2))*(b^5/(2
7*a^8*d^3 - 27*a^5*b^3*c^3 + 81*a^6*b^2*c^2*d - 81*a^7*b*c*d^2))^(1/3)*(3^(
1/2)*1i + 1))/2 + (log(((3^(1/2)*1i - 1)*(-d^5/(c^5*(a*d - b*c)^3))^(1/3)*(
((3^(1/2)*1i - 1)^2*(81*a^8*b^3*c^8*d^3*x*(a*d - b*c)^4*(a^2*d^2 + b^2*c^2
+ a*b*c*d) - (81*a^10*b^3*c^10*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)
^4*(-d^5/(c^5*(a*d - b*c)^3))^(1/3))/2)*(-d^5/(c^5*(a*d - b*c)^3))^(2/3))/3
6 - 9*a^6*b^9*c^11*d^4 + 9*a^7*b^8*c^10*d^5 + 9*a^10*b^5*c^7*d^8 - 9*a^11*b
^4*c^6*d^9))/6 - 3*a^6*b^6*c^6*d^6*x*(a^2*d^2 + b^2*c^2))*(d^5/(27*b^3*c^8
- 27*a^3*c^5*d^3 + 81*a^2*b*c^6*d^2 - 81*a*b^2*c^7*d))^(1/3)*(3^(1/2)*1i -
1))/2 - (log(((3^(1/2)*1i + 1)*(-d^5/(c^5*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*
1i + 1)^2*(81*a^8*b^3*c^8*d^3*x*(a*d - b*c)^4*(a^2*d^2 + b^2*c^2 + a*b*c*d)
 + (81*a^10*b^3*c^10*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)*(a*d - b*c)^4*(-d^5/(
c^5*(a*d - b*c)^3))^(1/3))/2)*(-d^5/(c^5*(a*d - b*c)^3))^(2/3))/36 - 9*a^6*
```

```
b^9*c^11*d^4 + 9*a^7*b^8*c^10*d^5 + 9*a^10*b^5*c^7*d^8 - 9*a^11*b^4*c^6*d^9
))/6 + 3*a^6*b^6*c^6*d^6*x*(a^2*d^2 + b^2*c^2))*(d^5/(27*b^3*c^8 - 27*a^3*c
^5*d^3 + 81*a^2*b*c^6*d^2 - 81*a*b^2*c^7*d))^(1/3)*(3^(1/2)*1i + 1))/2 - 1/
(2*a*c*x^2)
```

3.119 $\int \frac{1}{x^4(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 87

$$\int \frac{1}{x^4(a+bx^3)(c+dx^3)}\,dx=-\frac{1}{3acx^3}-\frac{(bc+ad)\log(x)}{a^2c^2}+\frac{b^2\log(a+bx^3)}{3a^2(bc-ad)}-\frac{d^2\log(c+dx^3)}{3c^2(bc-ad)}$$

```
[Out] -1/3/a/c/x^3-(a*d+b*c)*ln(x)/a^2/c^2+1/3*b^2*ln(b*x^3+a)/a^2/(-a*d+b*c)-1/3
*d^2*ln(d*x^3+c)/c^2/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 84}

$$\int \frac{1}{x^4(a+bx^3)(c+dx^3)}\,dx=\frac{b^2\log(a+bx^3)}{3a^2(bc-ad)}-\frac{\log(x)(ad+bc)}{a^2c^2}-\frac{d^2\log(c+dx^3)}{3c^2(bc-ad)}-\frac{1}{3acx^3}$$

```
[In] Int[1/(x^4*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/3*1/(a*c*x^3) - ((b*c + a*d)*Log[x])/(a^2*c^2) + (b^2*Log[a + b*x^3])/(3
*a^2*(b*c - a*d)) - (d^2*Log[c + d*x^3])/(3*c^2*(b*c - a*d))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2(a+bx)(c+dx)}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{acx^2}+\frac{-bc-ad}{a^2c^2x}-\frac{b^3}{a^2(-bc+ad)(a+bx)}-\frac{d^3}{c^2(bc-ad)(c+dx)}\right)\,dx, x, x^3\right)\\ &= -\frac{1}{3acx^3}-\frac{(bc+ad)\log(x)}{a^2c^2}+\frac{b^2\log\left(a+bx^3\right)}{3a^2(bc-ad)}-\frac{d^2\log\left(c+dx^3\right)}{3c^2(bc-ad)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^4\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = -\frac{1}{3acx^3}+\frac{(-bc-ad)\log(x)}{a^2c^2}-\frac{b^2\log\left(a+bx^3\right)}{3a^2(-bc+ad)}-\frac{d^2\log\left(c+dx^3\right)}{3c^2(bc-ad)}$$

```
[In] Integrate[1/(x^4*(a + b*x^3)*(c + d*x^3)),x]
[Out] -1/3*1/(a*c*x^3) + ((-(b*c) - a*d)*Log[x])/(a^2*c^2) - (b^2*Log[a + b*x^3])
/(3*a^2*(-(b*c) + a*d)) - (d^2*Log[c + d*x^3])/(3*c^2*(b*c - a*d))
```

Maple [A] (verified)

Time = 4.31 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

method	result	size
norman	$-\frac{1}{3ac\,x^3}-\frac{b^2\ln\left(b\,x^3+a\right)}{3a^2(ad-bc)}+\frac{d^2\ln\left(d\,x^3+c\right)}{3c^2(ad-bc)}-\frac{(ad+bc)\ln(x)}{a^2c^2}$	82
default	$-\frac{1}{3ac\,x^3}+\frac{(-ad-bc)\ln(x)}{a^2c^2}+\frac{d^2\ln\left(d\,x^3+c\right)}{3c^2(ad-bc)}-\frac{b^2\ln\left(b\,x^3+a\right)}{3a^2(ad-bc)}$	83
risch	$-\frac{1}{3ac\,x^3}-\frac{\ln(x)d}{a\,c^2}-\frac{\ln(x)b}{a^2c}-\frac{b^2\ln\left(-b\,x^3-a\right)}{3(ad-bc)a^2}+\frac{d^2\ln\left(d\,x^3+c\right)}{3c^2(ad-bc)}$	90
parallelrisch	$-\frac{3\ln(x)x^3a^2d^2-3\ln(x)x^3b^2c^2+b^2\ln\left(b\,x^3+a\right)c^2x^3-d^2\ln\left(d\,x^3+c\right)a^2x^3+a^2cd-b\,c^2a}{3a^2c^2x^3(ad-bc)}$	99

```
[In] int(1/x^4/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a/c/x^3-1/3*b^2/a^2/(a*d-b*c)*ln(b*x^3+a)+1/3*d^2/c^2/(a*d-b*c)*ln(d*x
^3+c)-(a*d+b*c)*ln(x)/a^2/c^2
```

Fricas [A] (verification not implemented)

none

Time = 1.34 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^4 (a+bx^3)(c+dx^3)}\,dx = \frac{b^2c^2x^3\log(bx^3+a) - a^2d^2x^3\log(dx^3+c) - 3(b^2c^2-a^2d^2)x^3\log(x) - abc^2 + a^2cd}{3(a^2bc^3-a^3c^2d)x^3}$$

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*c^2*x^3*log(b*x^3 + a) - a^2*d^2*x^3*log(d*x^3 + c) - 3*(b^2*c^2 -
 a^2*d^2)*x^3*log(x) - a*b*c^2 + a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d)*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^3)(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x**4/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (a+bx^3)(c+dx^3)}\,dx = \frac{b^2\log(bx^3+a)}{3(a^2bc-a^3d)} - \frac{d^2\log(dx^3+c)}{3(bc^3-ac^2d)} - \frac{(bc+ad)\log(x^3)}{3a^2c^2} - \frac{1}{3acx^3}$$

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*b^2*log(b*x^3 + a)/(a^2*b*c - a^3*d) - 1/3*d^2*log(d*x^3 + c)/(b*c^3 -
a*c^2*d) - 1/3*(b*c + a*d)*log(x^3)/(a^2*c^2) - 1/3/(a*c*x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^4(a+bx^3)(c+dx^3)}\,dx = \frac{b^3\log(|bx^3+a|)}{3(a^2b^2c-a^3bd)} - \frac{d^3\log(|dx^3+c|)}{3(bc^3d-ac^2d^2)} - \frac{(bc+ad)\log(|x|)}{a^2c^2} + \frac{bcx^3+adx^3-ac}{3a^2c^2x^3}$$

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^3*log(abs(b*x^3 + a))/(a^2*b^2*c - a^3*b*d) - 1/3*d^3*log(abs(d*x^3 +
 c))/(b*c^3*d - a*c^2*d^2) - (b*c + a*d)*log(abs(x))/(a^2*c^2) + 1/3*(b*c*x
^3 + a*d*x^3 - a*c)/(a^2*c^2*x^3)
```

Mupad [B] (verification not implemented)

Time = 7.68 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4(a+bx^3)(c+dx^3)}\,dx = -\frac{b^2\ln(b\,x^3+a)}{3\,(a^3\,d-a^2\,b\,c)} - \frac{d^2\ln(d\,x^3+c)}{3\,(b\,c^3-a\,c^2\,d)} - \frac{1}{3\,a\,c\,x^3} - \frac{\ln(x)\,(a\,d+b\,c)}{a^2\,c^2}$$

```
[In] int(1/(x^4*(a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] - (b^2*log(a + b*x^3))/(3*(a^3*d - a^2*b*c)) - (d^2*log(c + d*x^3))/(3*(b*c
^3 - a*c^2*d)) - 1/(3*a*c*x^3) - (log(x)*(a*d + b*c))/(a^2*c^2)
```

3.120 $\int \frac{1}{x^5(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 318

$$\int \frac{1}{x^5\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = -\frac{1}{4acx^4}+\frac{bc+ad}{a^2c^2x}-\frac{b^{7/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}(bc-ad)}+\frac{d^{7/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{7/3}(bc-ad)}-\frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}(bc-ad)}+\frac{d^{7/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{7/3}(bc-ad)}+\frac{b^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{7/3}(bc-ad)}-\frac{d^{7/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{7/3}(bc-ad)}$$

```
[Out] -1/4/a/c/x^4+(a*d+b*c)/a^2/c^2/x-1/3*b^(7/3)*ln(a^(1/3)+b^(1/3)*x)/a^(7/3)/
(-a*d+b*c)+1/3*d^(7/3)*ln(c^(1/3)+d^(1/3)*x)/c^(7/3)/(-a*d+b*c)+1/6*b^(7/3)
*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(7/3)/(-a*d+b*c)-1/6*d^(7/3)*l
n(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(7/3)/(-a*d+b*c)-1/3*b^(7/3)*arc
tan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(7/3)/(-a*d+b*c)*3^(1/2)+1
/3*d^(7/3)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(7/3)/(-a*d+
b*c)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {491, 597, 598, 298, 31, 648, 631, 210, 642}

$$\int \frac{1}{x^5(a+bx^3)(c+dx^3)}\,dx = -\frac{b^{7/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}(bc-ad)} + \frac{b^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{7/3}(bc-ad)} - \frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}(bc-ad)} + \frac{ad+bc}{a^2c^2x} + \frac{d^{7/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{7/3}(bc-ad)} - \frac{d^{7/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{7/3}(bc-ad)} + \frac{d^{7/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{7/3}(bc-ad)} - \frac{1}{4acx^4}$$

```
[In] Int[1/(x^5*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/4*1/(a*c*x^4) + (b*c + a*d)/(a^2*c^2*x) - (b^(7/3)*ArcTan[(a^(1/3) - 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)*(b*c - a*d)) + (d^(7/3)*ArcT
an[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(7/3)*(b*c - a*d)
) - (b^(7/3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(7/3)*(b*c - a*d)) + (d^(7/3)*L
og[c^(1/3) + d^(1/3)*x])/(3*c^(7/3)*(b*c - a*d)) + (b^(7/3)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(7/3)*(b*c - a*d)) - (d^(7/3)*Log[c^(
2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(7/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
```

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = -\frac{1}{4acx^4} + \frac{\int \frac{-4(bc+ad)-4bdx^3}{x^2(a+bx^3)(c+dx^3)}\,dx}{4ac}$$

$$= -\frac{1}{4acx^4} + \frac{bc+ad}{a^2c^2x} - \frac{\int \frac{x(-4(b^2c^2+abcd+a^2d^2)-4bd(bc+ad)x^3)}{(a+bx^3)(c+dx^3)}\,dx}{4a^2c^2}$$

$$= -\frac{1}{4acx^4} + \frac{bc+ad}{a^2c^2x} - \frac{\int \left(-\frac{4b^3c^2x}{(bc-ad)(a+bx^3)} - \frac{4a^2d^3x}{(-bc+ad)(c+dx^3)}\right)dx}{4a^2c^2}$$

$$= -\frac{1}{4acx^4} + \frac{bc+ad}{a^2c^2x} + \frac{b^3\int \frac{x}{a+bx^3}\,dx}{a^2(bc-ad)} - \frac{d^3\int \frac{x}{c+dx^3}\,dx}{c^2(bc-ad)}$$

$$= -\frac{1}{4acx^4} + \frac{bc+ad}{a^2c^2x} - \frac{b^{8/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{7/3}(bc-ad)} + \frac{b^{8/3}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{7/3}(bc-ad)} + \frac{d^{8/3}\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3c^{7/3}(bc-ad)} - \frac{d^{8/3}\int \frac{\sqrt[3]{c}+\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{3c^{7/3}(bc-ad)}$$

$$= -\frac{1}{4acx^4} + \frac{bc+ad}{a^2c^2x} - \frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}(bc-ad)} + \frac{d^{7/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{7/3}(bc-ad)} + \frac{b^{7/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{7/3}(bc-ad)} + \frac{b^{8/3}\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a^2(bc-ad)} - \frac{d^{7/3}\int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{6c^{7/3}(bc-ad)} - \frac{d^{8/3}\int \frac{1}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{2c^2(bc-ad)}$$

$$= -\frac{1}{4acx^4} + \frac{bc+ad}{a^2c^2x} - \frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}(bc-ad)} + \frac{d^{7/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{7/3}(bc-ad)} + \frac{b^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{7/3}(bc-ad)} - \frac{d^{7/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{7/3}(bc-ad)} + \frac{b^{7/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{7/3}(bc-ad)} - \frac{d^{7/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{c^{7/3}(bc-ad)}$$

$$= -\frac{1}{4acx^4} + \frac{bc+ad}{a^2c^2x} - \frac{b^{7/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}(bc-ad)} + \frac{d^{7/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{7/3}(bc-ad)} - \frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}(bc-ad)} + \frac{d^{7/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{7/3}(bc-ad)} + \frac{b^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{7/3}(bc-ad)} - \frac{d^{7/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{7/3}(bc-ad)}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^5(a+bx^3)(c+dx^3)}\,dx$$

$$= \frac{\frac{3b}{a} - \frac{3d}{c} - \frac{12b^2x^3}{a^2} + \frac{12d^2x^3}{c^2} + \frac{4\sqrt{3}b^{7/3}x^4\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{7/3}} - \frac{4\sqrt{3}d^{7/3}x^4\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{7/3}} + \frac{4b^{7/3}x^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{7/3}} -}{12(-bc+ad)x^4}$$

```
[In] Integrate[1/(x^5*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((3*b)/a - (3*d)/c - (12*b^2*x^3)/a^2 + (12*d^2*x^3)/c^2 + (4*Sqrt[3]*b^(7/
3)*x^4*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(7/3) - (4*Sqrt[3]*d^
(7/3)*x^4*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(7/3) + (4*b^(7/3)
*x^4*Log[a^(1/3) + b^(1/3)*x])/a^(7/3) - (4*d^(7/3)*x^4*Log[c^(1/3) + d^(1/
3)*x])/c^(7/3) - (2*b^(7/3)*x^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/a^(7/3) + (2*d^(7/3)*x^4*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2
])/c^(7/3))/(12*(-(b*c) + a*d)*x^4)
```

Maple [A] (verified)

Time = 4.38 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.78

method	result
default	$-\frac{1}{4acx^4}-\frac{-ad-bc}{a^2c^2x}+\frac{\left(-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)d^3}{c^2(ad-bc)}-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln}{}\right.}{}$
risch	$\frac{\frac{(ad+bc)x^3}{a^2c^2}-\frac{1}{4ac}}{x^4}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(d^3c^7a^3-3d^2c^8a^2b+3dc^9ab^2-b^3c^{10}\right)\_Z^3+d^7\right)}\_R\ln\left(\left(\left(-4a^{13}c^7d^6+22a^{12}bc^8d^5-52a^{11}b^2c^9d^4+\right.\right.\right.\right.}{}$

[In] int(1/x^5/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] -1/4/a/c/x^4-1/a^2/c^2*(-a*d-b*c)/x+(-1/3/d/(c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/6/d/(c/d)^(1/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3*3^(1/2)/d/(c/d)^(1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1)))*d^3/c^2/(a*d-b*c)-(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*b^3/a^2/(a*d-b*c)

Fricas [A] (verification not implemented)

none

Time = 0.79 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.96

$$\int\frac{1}{x^5\left(a+bx^3\right)\left(c+dx^3\right)}\,dx$$
$$=\frac{4\sqrt{3}b^2c^2x^4\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-4\sqrt{3}a^2d^2x^4\left(-\frac{d}{c}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{d}{c}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+2b^2c}{}$$

[In] integrate(1/x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")

[Out] 1/12*(4*sqrt(3)*b^2*c^2*x^4*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3)) - 4*sqrt(3)*a^2*d^2*x^4*(-d/c)^(1/3)*arctan(2/3*sqrt(3)*x*(-d/c)^(1/3) + 1/3*sqrt(3)) + 2*b^2*c^2*x^4*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) + 2*a^2*d^2*x^4*(-d/c)^(1/3)*log(d*x^2 - c*x*(-d/c)^(2/3) - c*(-d/c)^(1/3)) - 4*b^2*c^2*x^4*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3)) - 4*a^2*d^2*x^4*(-d/c)^(1/3)*log(d*x + c*(-d/c)^(2/3)) - 3*a*b*c^2 + 3*a^2*c*d + 12*(b^2*c^2 - a^2*d^2)*x^3)/((a^2*b*c^3 - a^3*c^2*d)*x^4)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^5\left(a+bx^3\right)\left(c+dx^3\right)}\,dx=\text{Timed out}$$

```
[In] integrate(1/x**5/(b*x**3+a)/(d*x**3+c),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^5\left(a+bx^3\right)\left(c+dx^3\right)}\,dx=\frac{\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(a^2bc-a^3d\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}d^2\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc^3-ac^2d\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{b^2\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a^2bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^3d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}-\frac{d^2\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^3\left(\frac{c}{d}\right)^{\frac{1}{3}}-ac^2d\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}-\frac{b^2\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(a^2bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^3d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}+\frac{d^2\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc^3\left(\frac{c}{d}\right)^{\frac{1}{3}}-ac^2d\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}+\frac{4\,(bc+ad)x^3-ac}{4\,a^2c^2x^4}$$

```
[In] integrate(1/x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((a^2*b
*c - a^3*d)*(a/b)^(1/3)) - 1/3*sqrt(3)*d^2*arctan(1/3*sqrt(3)*(2*x - (c/d)^
(1/3))/(c/d)^(1/3))/((b*c^3 - a*c^2*d)*(c/d)^(1/3)) + 1/6*b^2*log(x^2 - x*(
a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*c*(a/b)^(1/3) - a^3*d*(a/b)^(1/3)) - 1/6*d
^2*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c^3*(c/d)^(1/3) - a*c^2*d*(c/d
)^(1/3)) - 1/3*b^2*log(x + (a/b)^(1/3))/(a^2*b*c*(a/b)^(1/3) - a^3*d*(a/b)^
(1/3)) + 1/3*d^2*log(x + (c/d)^(1/3))/(b*c^3*(c/d)^(1/3) - a*c^2*d*(c/d)^(1
/3)) + 1/4*(4*(b*c + a*d)*x^3 - a*c)/(a^2*c^2*x^4)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^5 (a+bx^3)(c+dx^3)}\,dx = -\frac{b^3\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3(a^3bc-a^4d)} + \frac{d^3\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3(bc^4-ac^3d)} - \frac{(-ab^2)^{\frac{2}{3}}\,b\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}a^3bc-\sqrt{3}a^4d} + \frac{(-cd^2)^{\frac{2}{3}}\,d\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^4-\sqrt{3}ac^3d} + \frac{(-ab^2)^{\frac{2}{3}}\,b\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(a^3bc-a^4d)} - \frac{(-cd^2)^{\frac{2}{3}}\,d\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6(bc^4-ac^3d)} + \frac{4\,bcx^3+4\,adx^3-ac}{4\,a^2c^2x^4}$$

```
[In] integrate(1/x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
[Out] -1/3*b^3*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b*c - a^4*d) + 1/3*d^
3*(-c/d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^4 - a*c^3*d) - (-a*b^2)^(2/3
)*b*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^3*b*c
- sqrt(3)*a^4*d) + (-c*d^2)^(2/3)*d*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))
/(-c/d)^(1/3))/(sqrt(3)*b*c^4 - sqrt(3)*a*c^3*d) + 1/6*(-a*b^2)^(2/3)*b*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b*c - a^4*d) - 1/6*(-c*d^2)^(2/3
)*d*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^4 - a*c^3*d) + 1/4*(4*b*c
*x^3 + 4*a*d*x^3 - a*c)/(a^2*c^2*x^4)
```

Mupad [B] (verification not implemented)

Time = 16.47 (sec) , antiderivative size = 1734, normalized size of antiderivative = 5.45

$$\int \frac{1}{x^5 (a+bx^3)(c+dx^3)}\,dx = \text{Too large to display}$$

```
[In] int(1/(x^5*(a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(((b^7/(a^7*(a*d - b*c)^3))^(2/3)*(((27*a^14*b^3*c^14*d^3*x*(a^6*d^6 + b
^6*c^6)*(a*d - b*c)^2 + 27*a^19*b^3*c^19*d^3*(a*d + b*c)*(a*d - b*c)^4*(b^7
/(a^7*(a*d - b*c)^3))^(2/3))*(b^7/(a^7*(a*d - b*c)^3))^(1/3))/3 + 9*a^13*b^
11*c^20*d^4 - 9*a^14*b^10*c^19*d^5 - 9*a^19*b^5*c^14*d^10 + 9*a^20*b^4*c^13
*d^11))/9 + a^13*b^9*c^13*d^9*x)*(b^7/(27*a^10*d^3 - 27*a^7*b^3*c^3 + 81*a^
8*b^2*c^2*d - 81*a^9*b*c*d^2))^(1/3) + log(((-d^7/(c^7*(a*d - b*c)^3))^(2/3
)*(((27*a^14*b^3*c^14*d^3*x*(a^6*d^6 + b^6*c^6)*(a*d - b*c)^2 + 27*a^19*b^3
*c^19*d^3*(a*d + b*c)*(a*d - b*c)^4*(-d^7/(c^7*(a*d - b*c)^3))^(2/3))*(-d^7
/(c^7*(a*d - b*c)^3))^(1/3))/3 + 9*a^13*b^11*c^20*d^4 - 9*a^14*b^10*c^19*d^
5 - 9*a^19*b^5*c^14*d^10 + 9*a^20*b^4*c^13*d^11))/9 + a^13*b^9*c^13*d^9*x)*
(d^7/(27*b^3*c^10 - 27*a^3*c^7*d^3 + 81*a^2*b*c^8*d^2 - 81*a*b^2*c^9*d))^(1
/3) - (1/(4*a*c) - (x^3*(a*d + b*c))/(a^2*c^2))/x^4 + (log(((3^(1/2)*1i - 1
)^2*(b^7/(a^7*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i - 1)*(27*a^14*b^3*c^14*d^
3*x*(a^6*d^6 + b^6*c^6)*(a*d - b*c)^2 + (27*a^19*b^3*c^19*d^3*(3^(1/2)*1i -
 1)^2*(a*d + b*c)*(a*d - b*c)^4*(b^7/(a^7*(a*d - b*c)^3))^(2/3))/4)*(b^7/(a
^7*(a*d - b*c)^3))^(1/3))/6 + 9*a^13*b^11*c^20*d^4 - 9*a^14*b^10*c^19*d^5 -
 9*a^19*b^5*c^14*d^10 + 9*a^20*b^4*c^13*d^11))/36 + a^13*b^9*c^13*d^9*x)*(b
^7/(27*a^10*d^3 - 27*a^7*b^3*c^3 + 81*a^8*b^2*c^2*d - 81*a^9*b*c*d^2))^(1/3
)*(3^(1/2)*1i - 1))/2 - (log(((3^(1/2)*1i + 1)^2*(b^7/(a^7*(a*d - b*c)^3))^
(2/3)*(((3^(1/2)*1i + 1)*(27*a^14*b^3*c^14*d^3*x*(a^6*d^6 + b^6*c^6)*(a*d -
 b*c)^2 + (27*a^19*b^3*c^19*d^3*(3^(1/2)*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^
4*(b^7/(a^7*(a*d - b*c)^3))^(2/3))/4)*(b^7/(a^7*(a*d - b*c)^3))^(1/3))/6 -
9*a^13*b^11*c^20*d^4 + 9*a^14*b^10*c^19*d^5 + 9*a^19*b^5*c^14*d^10 - 9*a^20
*b^4*c^13*d^11))/36 - a^13*b^9*c^13*d^9*x)*(b^7/(27*a^10*d^3 - 27*a^7*b^3*c
^3 + 81*a^8*b^2*c^2*d - 81*a^9*b*c*d^2))^(1/3)*(3^(1/2)*1i + 1))/2 + (log((
(3^(1/2)*1i - 1)^2*(-d^7/(c^7*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i - 1)*(27*
a^14*b^3*c^14*d^3*x*(a^6*d^6 + b^6*c^6)*(a*d - b*c)^2 + (27*a^19*b^3*c^19*d
^3*(3^(1/2)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-d^7/(c^7*(a*d - b*c)^3))^
(2/3))/4)*(-d^7/(c^7*(a*d - b*c)^3))^(1/3))/6 + 9*a^13*b^11*c^20*d^4 - 9*a^
14*b^10*c^19*d^5 - 9*a^19*b^5*c^14*d^10 + 9*a^20*b^4*c^13*d^11))/36 + a^13*
b^9*c^13*d^9*x)*(d^7/(27*b^3*c^10 - 27*a^3*c^7*d^3 + 81*a^2*b*c^8*d^2 - 81*
a*b^2*c^9*d))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(((3^(1/2)*1i + 1)^2*(-d^7/(c
^7*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i + 1)*(27*a^14*b^3*c^14*d^3*x*(a^6*d^
6 + b^6*c^6)*(a*d - b*c)^2 + (27*a^19*b^3*c^19*d^3*(3^(1/2)*1i + 1)^2*(a*d
+ b*c)*(a*d - b*c)^4*(-d^7/(c^7*(a*d - b*c)^3))^(2/3))/4)*(-d^7/(c^7*(a*d -
 b*c)^3))^(1/3))/6 - 9*a^13*b^11*c^20*d^4 + 9*a^14*b^10*c^19*d^5 + 9*a^19*b
^5*c^14*d^10 - 9*a^20*b^4*c^13*d^11))/36 - a^13*b^9*c^13*d^9*x)*(d^7/(27*b^
3*c^10 - 27*a^3*c^7*d^3 + 81*a^2*b*c^8*d^2 - 81*a*b^2*c^9*d))^(1/3)*(3^(1/2
)*1i + 1))/2
```

3.121 $\int \frac{1}{x^6(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 321

$$\int \frac{1}{x^6(a+bx^3)(c+dx^3)}\,dx = -\frac{1}{5acx^5}+\frac{bc+ad}{2a^2c^2x^2}-\frac{b^{8/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}(bc-ad)}+\frac{d^{8/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{8/3}(bc-ad)}+\frac{b^{8/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}(bc-ad)}-\frac{d^{8/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{8/3}(bc-ad)}-\frac{b^{8/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{8/3}(bc-ad)}+\frac{d^{8/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{8/3}(bc-ad)}$$

```
[Out] -1/5/a/c/x^5+1/2*(a*d+b*c)/a^2/c^2/x^2+1/3*b^(8/3)*ln(a^(1/3)+b^(1/3)*x)/a^
(8/3)/(-a*d+b*c)-1/3*d^(8/3)*ln(c^(1/3)+d^(1/3)*x)/c^(8/3)/(-a*d+b*c)-1/6*b
^(8/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/(-a*d+b*c)+1/6*d^(
8/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(8/3)/(-a*d+b*c)-1/3*b^(8/
3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/(-a*d+b*c)*3^(
1/2)+1/3*d^(8/3)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(8/3)/
(-a*d+b*c)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {491, 597, 536, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{x^6 (a+bx^3)(c+dx^3)}\,dx = -\frac{b^{8/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}(bc-ad)} - \frac{b^{8/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{8/3}(bc-ad)} + \frac{b^{8/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}(bc-ad)} + \frac{ad+bc}{2a^2c^2x^2} + \frac{d^{8/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{8/3}(bc-ad)} + \frac{d^{8/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{8/3}(bc-ad)} - \frac{d^{8/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{8/3}(bc-ad)} - \frac{1}{5acx^5}$$

```
[In] Int[1/(x^6*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/5*1/(a*c*x^5) + (b*c + a*d)/(2*a^2*c^2*x^2) - (b^(8/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(8/3)*(b*c - a*d)) + (d^(8/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(8/3)*(b*c - a*d)) + (b^(8/3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(8/3)*(b*c - a*d)) - (d^(8/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(8/3)*(b*c - a*d)) - (b^(8/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(8/3)*(b*c - a*d)) + (d^(8/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(8/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{5acx^5}+\frac{\int \frac{-5(bc+ad)-5bdx^3}{x^3(a+bx^3)(c+dx^3)}\,dx}{5ac}\\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{2a^2c^2x^2}-\frac{\int \frac{-10(b^2c^2+abcd+a^2d^2)-10bd(bc+ad)x^3}{(a+bx^3)(c+dx^3)}\,dx}{10a^2c^2}\\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{2a^2c^2x^2}+\frac{b^3\int \frac{1}{a+bx^3}\,dx}{a^2(bc-ad)}-\frac{d^3\int \frac{1}{c+dx^3}\,dx}{c^2(bc-ad)}\\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{2a^2c^2x^2}+\frac{b^3\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{8/3}(bc-ad)}+\frac{b^3\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{8/3}(bc-ad)}\\
&\quad -\frac{d^3\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3c^{8/3}(bc-ad)}-\frac{d^3\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{3c^{8/3}(bc-ad)}\\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{2a^2c^2x^2}+\frac{b^{8/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}(bc-ad)}-\frac{d^{8/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{8/3}(bc-ad)}\\
&\quad -\frac{b^{8/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{8/3}(bc-ad)}+\frac{b^3\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a^{7/3}(bc-ad)}\\
&\quad +\frac{d^{8/3}\int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{6c^{8/3}(bc-ad)}-\frac{d^3\int \frac{1}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{2c^{7/3}(bc-ad)}\\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{2a^2c^2x^2}+\frac{b^{8/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}(bc-ad)}-\frac{d^{8/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{8/3}(bc-ad)}\\
&\quad -\frac{b^{8/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{8/3}(bc-ad)}+\frac{d^{8/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{8/3}(bc-ad)}\\
&\quad +\frac{b^{8/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{8/3}(bc-ad)}-\frac{d^{8/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{c^{8/3}(bc-ad)}
\end{aligned}
$$

$$= -\frac{1}{5acx^5} + \frac{bc+ad}{2a^2c^2x^2} - \frac{b^{8/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}(bc-ad)} + \frac{d^{8/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{8/3}(bc-ad)} + \frac{b^{8/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}(bc-ad)} - \frac{d^{8/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{8/3}(bc-ad)} - \frac{b^{8/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{8/3}(bc-ad)} + \frac{d^{8/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{8/3}(bc-ad)}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^6(a+bx^3)(c+dx^3)}\,dx$$

$$= \frac{\frac{6b}{a} - \frac{6d}{c} - \frac{15b^2x^3}{a^2} + \frac{15d^2x^3}{c^2} + \frac{10\sqrt{3}b^{8/3}x^5\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{8/3}} - \frac{10\sqrt{3}d^{8/3}x^5\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{8/3}} - \frac{10b^{8/3}x^5\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{8/3}}}{30(-bc+ad)x^5}$$

```
[In] Integrate[1/(x^6*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((6*b)/a - (6*d)/c - (15*b^2*x^3)/a^2 + (15*d^2*x^3)/c^2 + (10*Sqrt[3]*b^(8
/3)*x^5*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(8/3) - (10*Sqrt[3]*
d^(8/3)*x^5*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(8/3) - (10*b^(8
/3)*x^5*Log[a^(1/3) + b^(1/3)*x])/a^(8/3) + (10*d^(8/3)*x^5*Log[c^(1/3) + d
^(1/3)*x])/c^(8/3) + (5*b^(8/3)*x^5*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/a^(8/3) - (5*d^(8/3)*x^5*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)
*x^2])/c^(8/3))/(30*(-(b*c) + a*d)*x^5)
```

Maple [A] (verified)

Time = 4.38 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.77

method	result
default	$-\frac{1}{5ac\,x^{5}}-\frac{-ad-bc}{2x^{2}a^{2}c^{2}}+\frac{\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)d^{3}}{c^{2}(ad-bc)}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\right.}{}\right.}{}$
risch	$\frac{\frac{(ad+bc)x^{3}}{2a^{2}c^{2}}-\frac{1}{5ac}}{x^{5}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(d^{3}c^{8}a^{3}-3a^{2}b\,c^{9}d^{2}+3a\,b^{2}c^{10}d-b^{3}c^{11}\right)\_Z^{3}-d^{8}\right)}\_R\ln\left(\left(\left(-4a^{14}c^{8}d^{6}+22a^{13}b\,c^{9}d^{5}-52a^{12}b^{2}c^{10}d^{4}+\right.\right.\right.\right.}{}$

```
[In] int(1/x^6/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/a/c/x^5-1/2*(-a*d-b*c)/x^2/a^2/c^2+(1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3)
)-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3/d/(c/d)^(2/3)*3^(
1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1)))/c^2*d^3/(a*d-b*c)-(1/3/b/(a/b
)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3
))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))/a^2*b
^3/(a*d-b*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^{6}\left(a+bx^{3}\right)\left(c+dx^{3}\right)}\,dx = -\frac{10\sqrt{3}b^{2}c^{2}x^{5}\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)+10\sqrt{3}a^{2}d^{2}x^{5}\left(\frac{d^{2}}{c^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}cx\left(\frac{d^{2}}{c^{2}}\right)^{\frac{2}{3}}-\sqrt{3}d}{3d}\right)-5\,b^{2}c}{}$$

```
[In] integrate(1/x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/30*(10*sqrt(3)*b^2*c^2*x^5*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-
b^2/a^2)^(2/3) - sqrt(3)*b)/b) + 10*sqrt(3)*a^2*d^2*x^5*(d^2/c^2)^(1/3)*arc
tan(1/3*(2*sqrt(3)*c*x*(d^2/c^2)^(2/3) - sqrt(3)*d)/d) - 5*b^2*c^2*x^5*(-b^
2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) -
```

```
5*a^2*d^2*x^5*(d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*(d^2/c^2)^(1/3) + c^2*(d
^2/c^2)^(2/3)) + 10*b^2*c^2*x^5*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/
3)) + 10*a^2*d^2*x^5*(d^2/c^2)^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)) + 6*a*b*c
^2 - 6*a^2*c*d - 15*(b^2*c^2 - a^2*d^2)*x^3)/((a^2*b*c^3 - a^3*c^2*d)*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^6\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x**6/(b*x**3+a)/(d*x**3+c),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.15

$$\begin{aligned}\int \frac{1}{x^6\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = &\frac{\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(a^2bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^3d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} \\ &- \frac{\sqrt{3}d^2\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc^3\left(\frac{c}{d}\right)^{\frac{1}{3}}-ac^2d\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} \\ &- \frac{b^2\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a^2bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^3d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} \\ &+ \frac{d^2\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^3\left(\frac{c}{d}\right)^{\frac{2}{3}}-ac^2d\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} + \frac{b^2\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(a^2bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^3d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} \\ &- \frac{d^2\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc^3\left(\frac{c}{d}\right)^{\frac{2}{3}}-ac^2d\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)} + \frac{5\,(bc+ad)x^3-2\,ac}{10\,a^2c^2x^5}\end{aligned}$$

```
[In] integrate(1/x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((a^2*b
*c*(a/b)^(1/3) - a^3*d*(a/b)^(1/3))*(a/b)^(1/3)) - 1/3*sqrt(3)*d^2*arctan(1
```

```
/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c^3*(c/d)^(1/3) - a*c^2*d*(
c/d)^(1/3))*(c/d)^(1/3)) - 1/6*b^2*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(
a^2*b*c*(a/b)^(2/3) - a^3*d*(a/b)^(2/3)) + 1/6*d^2*log(x^2 - x*(c/d)^(1/3)
+ (c/d)^(2/3))/(b*c^3*(c/d)^(2/3) - a*c^2*d*(c/d)^(2/3)) + 1/3*b^2*log(x +
(a/b)^(1/3))/(a^2*b*c*(a/b)^(2/3) - a^3*d*(a/b)^(2/3)) - 1/3*d^2*log(x + (c
/d)^(1/3))/(b*c^3*(c/d)^(2/3) - a*c^2*d*(c/d)^(2/3)) + 1/10*(5*(b*c + a*d)*
x^3 - 2*a*c)/(a^2*c^2*x^5)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^6 (a+bx^3)(c+dx^3)}\,dx = -\frac{b^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,(a^3bc-a^4d)} + \frac{d^3\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^4-ac^3d)} + \frac{(-ab^2)^{\frac{1}{3}}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}a^3bc-\sqrt{3}a^4d} - \frac{(-cd^2)^{\frac{1}{3}}\,d^2\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^4-\sqrt{3}ac^3d} + \frac{(-ab^2)^{\frac{1}{3}}\,b^2\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(a^3bc-a^4d)} - \frac{(-cd^2)^{\frac{1}{3}}\,d^2\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,(bc^4-ac^3d)} + \frac{5\,bcx^3+5\,adx^3-2\,ac}{10\,a^2c^2x^5}$$

```
[In] integrate(1/x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b^3*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b*c - a^4*d) + 1/3*d^
3*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^4 - a*c^3*d) + (-a*b^2)^(1/3
)*b^2*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^3*b*
c - sqrt(3)*a^4*d) - (-c*d^2)^(1/3)*d^2*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1
/3))/(-c/d)^(1/3))/(sqrt(3)*b*c^4 - sqrt(3)*a*c^3*d) + 1/6*(-a*b^2)^(1/3)*b
^2*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b*c - a^4*d) - 1/6*(-c*d^2
)^(1/3)*d^2*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^4 - a*c^3*d) + 1/
10*(5*b*c*x^3 + 5*a*d*x^3 - 2*a*c)/(a^2*c^2*x^5)
```

Mupad [B] (verification not implemented)

Time = 16.54 (sec) , antiderivative size = 1860, normalized size of antiderivative = 5.79

$$\int \frac{1}{x^6 (a + bx^3) (c + dx^3)} dx = \text{Too large to display}$$

```
[In] int(1/(x^6*(a + b*x^3)*(c + d*x^3)),x)
[Out] log(((-b^8/(a^8*(a*d - b*c)^3))^(1/3)*(9*a^13*b^11*c^19*d^5 - 9*a^12*b^12*c
^20*d^4 + 9*a^19*b^5*c^13*d^11 - 9*a^20*b^4*c^12*d^12 + 9*a^16*b^3*c^16*d^3
*(a*d + b*c)*(a*d - b*c)^4*(a^3*c^3*(-b^8/(a^8*(a*d - b*c)^3))^(1/3) + a^2*
d^2*x + b^2*c^2*x)*(-b^8/(a^8*(a*d - b*c)^3))^(2/3)))/3 + 3*a^12*b^7*c^12*d
^7*x*(a^4*d^4 + b^4*c^4))*(-b^8/(27*a^11*d^3 - 27*a^8*b^3*c^3 + 81*a^9*b^2*
c^2*d - 81*a^10*b*c*d^2))^(1/3) - (1/(5*a*c) - (x^3*(a*d + b*c))/(2*a^2*c^2
))/x^5 + log(((d^8/(c^8*(a*d - b*c)^3))^(1/3)*(9*a^13*b^11*c^19*d^5 - 9*a^1
2*b^12*c^20*d^4 + 9*a^19*b^5*c^13*d^11 - 9*a^20*b^4*c^12*d^12 + 9*a^16*b^3*
c^16*d^3*(a*d + b*c)*(a*d - b*c)^4*(a^3*c^3*(d^8/(c^8*(a*d - b*c)^3))^(1/3)
 + a^2*d^2*x + b^2*c^2*x)*(d^8/(c^8*(a*d - b*c)^3))^(2/3)))/3 + 3*a^12*b^7*
c^12*d^7*x*(a^4*d^4 + b^4*c^4))*(-d^8/(27*b^3*c^11 - 27*a^3*c^8*d^3 + 81*a^
2*b*c^9*d^2 - 81*a*b^2*c^10*d))^(1/3) + (log(((3^(1/2)*1i - 1)*(-b^8/(a^8*(
a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i - 1)^2*(81*a^16*b^3*c^16*d^3*x*(a*d - b*
c)^4*(a^3*d^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2) + (81*a^19*b^3*c^19*d^
3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(-b^8/(a^8*(a*d - b*c)^3))^(1/
3))/2)*(-b^8/(a^8*(a*d - b*c)^3))^(2/3))/36 - 9*a^12*b^12*c^20*d^4 + 9*a^13
*b^11*c^19*d^5 + 9*a^19*b^5*c^13*d^11 - 9*a^20*b^4*c^12*d^12))/6 + 3*a^12*b
^7*c^12*d^7*x*(a^4*d^4 + b^4*c^4))*(-b^8/(27*a^11*d^3 - 27*a^8*b^3*c^3 + 81
*a^9*b^2*c^2*d - 81*a^10*b*c*d^2))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(((3^(1/
2)*1i + 1)*(-b^8/(a^8*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81*a^16*b
^3*c^16*d^3*x*(a*d - b*c)^4*(a^3*d^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2)
 - (81*a^19*b^3*c^19*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)*(a*d - b*c)^4*(-b^8/(
a^8*(a*d - b*c)^3))^(1/3))/2)*(-b^8/(a^8*(a*d - b*c)^3))^(2/3))/36 - 9*a^12
*b^12*c^20*d^4 + 9*a^13*b^11*c^19*d^5 + 9*a^19*b^5*c^13*d^11 - 9*a^20*b^4*c
^12*d^12))/6 - 3*a^12*b^7*c^12*d^7*x*(a^4*d^4 + b^4*c^4))*(-b^8/(27*a^11*d^
3 - 27*a^8*b^3*c^3 + 81*a^9*b^2*c^2*d - 81*a^10*b*c*d^2))^(1/3)*(3^(1/2)*1i
 + 1))/2 + (log(((3^(1/2)*1i - 1)*(d^8/(c^8*(a*d - b*c)^3))^(1/3)*(((3^(1/2
)*1i - 1)^2*(81*a^16*b^3*c^16*d^3*x*(a*d - b*c)^4*(a^3*d^3 + b^3*c^3 + a*b^
2*c^2*d + a^2*b*c*d^2) + (81*a^19*b^3*c^19*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)
*(a*d - b*c)^4*(d^8/(c^8*(a*d - b*c)^3))^(1/3))/2)*(d^8/(c^8*(a*d - b*c)^3)
)^(2/3))/36 - 9*a^12*b^12*c^20*d^4 + 9*a^13*b^11*c^19*d^5 + 9*a^19*b^5*c^13
*d^11 - 9*a^20*b^4*c^12*d^12))/6 + 3*a^12*b^7*c^12*d^7*x*(a^4*d^4 + b^4*c^4
))*(-d^8/(27*b^3*c^11 - 27*a^3*c^8*d^3 + 81*a^2*b*c^9*d^2 - 81*a*b^2*c^10*d
))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(((3^(1/2)*1i + 1)*(d^8/(c^8*(a*d - b*c)
^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81*a^16*b^3*c^16*d^3*x*(a*d - b*c)^4*(a^3*
d^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2) - (81*a^19*b^3*c^19*d^3*(3^(1/2)
```

```
*1i + 1)*(a*d + b*c)*(a*d - b*c)^4*(d^8/(c^8*(a*d - b*c)^3))^(1/3))/2)*(d^8
/(c^8*(a*d - b*c)^3))^(2/3))/36 - 9*a^12*b^12*c^20*d^4 + 9*a^13*b^11*c^19*d
^5 + 9*a^19*b^5*c^13*d^11 - 9*a^20*b^4*c^12*d^12))/6 - 3*a^12*b^7*c^12*d^7*
x*(a^4*d^4 + b^4*c^4))*(-d^8/(27*b^3*c^11 - 27*a^3*c^8*d^3 + 81*a^2*b*c^9*d
^2 - 81*a*b^2*c^10*d))^(1/3)*(3^(1/2)*1i + 1))/2
```

3.122 $\int \frac{1}{x^7(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 119

$$\int \frac{1}{x^7 (a+bx^3) (c+dx^3)}\,dx = -\frac{1}{6acx^6}+\frac{bc+ad}{3a^2c^2x^3}+\frac{(b^2c^2+abcd+a^2d^2)\log(x)}{a^3c^3} - \frac{b^3\log(a+bx^3)}{3a^3(bc-ad)}+\frac{d^3\log(c+dx^3)}{3c^3(bc-ad)}$$

```
[Out] -1/6/a/c/x^6+1/3*(a*d+b*c)/a^2/c^2/x^3+(a^2*d^2+a*b*c*d+b^2*c^2)*ln(x)/a^3/
c^3-1/3*b^3*ln(b*x^3+a)/a^3/(-a*d+b*c)+1/3*d^3*ln(d*x^3+c)/c^3/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 84}

$$\int \frac{1}{x^7 (a+bx^3) (c+dx^3)}\,dx = -\frac{b^3\log(a+bx^3)}{3a^3(bc-ad)}+\frac{ad+bc}{3a^2c^2x^3} + \frac{\log(x)(a^2d^2+abcd+b^2c^2)}{a^3c^3}+\frac{d^3\log(c+dx^3)}{3c^3(bc-ad)}-\frac{1}{6acx^6}$$

```
[In] Int[1/(x^7*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/6*1/(a*c*x^6) + (b*c + a*d)/(3*a^2*c^2*x^3) + ((b^2*c^2 + a*b*c*d + a^2*
d^2)*Log[x])/(a^3*c^3) - (b^3*Log[a + b*x^3])/(3*a^3*(b*c - a*d)) + (d^3*Lo
g[c + d*x^3])/(3*c^3*(b*c - a*d))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
```

```
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^3(a+bx)(c+dx)}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{acx^3}+\frac{-bc-ad}{a^2c^2x^2}+\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{b^4}{a^3(-bc+ad)(a+bx)}\right.\right.\\ &\qquad\left.\left.+\frac{d^4}{c^3(bc-ad)(c+dx)}\right)dx, x, x^3\right)\\ &= -\frac{1}{6acx^6}+\frac{bc+ad}{3a^2c^2x^3}+\frac{(b^2c^2+abcd+a^2d^2)\log(x)}{a^3c^3}-\frac{b^3\log(a+bx^3)}{3a^3(bc-ad)}+\frac{d^3\log(c+dx^3)}{3c^3(bc-ad)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^7(a+bx^3)(c+dx^3)}\,dx = \frac{-ac(-bc+ad)(-2bcx^3+a(c-2dx^3))+6(-b^3c^3+a^3d^3)x^6\log(x)+2b^3c^3x^6\log(a+bx^3)-2a^3d^3x^6\log(}{6a^3c^3(-bc+ad)x^6}$$

```
[In] Integrate[1/(x^7*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] (-(a*c*(-(b*c) + a*d)*(-2*b*c*x^3 + a*(c - 2*d*x^3))) + 6*(-(b^3*c^3) + a^3
*d^3)*x^6*Log[x] + 2*b^3*c^3*x^6*Log[a + b*x^3] - 2*a^3*d^3*x^6*Log[c + d*x
^3])/(6*a^3*c^3*(-(b*c) + a*d)*x^6)
```

Maple [A] (verified)

Time = 4.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{1}{6acx^{6}}-\frac{-ad-bc}{3x^{3}a^{2}c^{2}}+\frac{\left(a^{2}d^{2}+abcd+b^{2}c^{2}\right)\ln \left(x\right)}{a^{3}c^{3}}-\frac{d^{3}\ln \left(d\,x^{3}+c\right)}{3c^{3}\left(ad-bc\right)}+\frac{b^{3}\ln \left(b\,x^{3}+a\right)}{3a^{3}\left(ad-bc\right)}$	114
norman	$\frac{-\frac{1}{6ac}+\frac{\left(ad+bc\right)x^{3}}{3a^{2}c^{2}}}{x^{6}}+\frac{\left(a^{2}d^{2}+abcd+b^{2}c^{2}\right)\ln \left(x\right)}{a^{3}c^{3}}+\frac{b^{3}\ln \left(b\,x^{3}+a\right)}{3a^{3}\left(ad-bc\right)}-\frac{d^{3}\ln \left(d\,x^{3}+c\right)}{3c^{3}\left(ad-bc\right)}$	114
risch	$\frac{-\frac{1}{6ac}+\frac{\left(ad+bc\right)x^{3}}{3a^{2}c^{2}}}{x^{6}}+\frac{\ln \left(x\right)d^{2}}{a\,c^{3}}+\frac{\ln \left(x\right)bd}{a^{2}c^{2}}+\frac{\ln \left(x\right)b^{2}}{a^{3}c}-\frac{d^{3}\ln \left(d\,x^{3}+c\right)}{3c^{3}\left(ad-bc\right)}+\frac{b^{3}\ln \left(-b\,x^{3}-a\right)}{3\left(ad-bc\right)a^{3}}$	123
parallelrisch	$\frac{6\ln \left(x\right)x^{6}a^{3}d^{3}-6\ln \left(x\right)x^{6}b^{3}c^{3}+2b^{3}\ln \left(b\,x^{3}+a\right)c^{3}x^{6}-2d^{3}\ln \left(d\,x^{3}+c\right)a^{3}x^{6}+2d^{2}a^{3}c\,x^{3}-2a\,b^{2}c^{3}x^{3}-a^{3}c^{2}d+a^{2}b\,c^{3}}{6a^{3}c^{3}x^{6}\left(ad-bc\right)}$	128

```
[In] int(1/x^7/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/a/c/x^6-1/3*(-a*d-b*c)/x^3/a^2/c^2+(a^2*d^2+a*b*c*d+b^2*c^2)*ln(x)/a^3
/c^3-1/3*d^3/c^3/(a*d-b*c)*ln(d*x^3+c)+1/3*b^3/a^3/(a*d-b*c)*ln(b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 3.73 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^{7}\left(a+bx^{3}\right)\left(c+dx^{3}\right)}\,dx = -\frac{2\,b^{3}c^{3}x^{6}\log \left(bx^{3}+a\right)-2\,a^{3}d^{3}x^{6}\log \left(dx^{3}+c\right)-6\,(b^{3}c^{3}-a^{3}d^{3})x^{6}\log \left(x\right)+a^{2}bc^{3}-a^{3}c^{2}d-2\,(ab^{2}c^{3}-a}{6\,(a^{3}bc^{4}-a^{4}c^{3}d)x^{6}}$$

```
[In] integrate(1/x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*b^3*c^3*x^6*log(b*x^3 + a) - 2*a^3*d^3*x^6*log(d*x^3 + c) - 6*(b^3*
c^3 - a^3*d^3)*x^6*log(x) + a^2*b*c^3 - a^3*c^2*d - 2*(a*b^2*c^3 - a^3*c*d^
2)*x^3)/((a^3*b*c^4 - a^4*c^3*d)*x^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{7}\left(a+bx^{3}\right)\left(c+dx^{3}\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x**7/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^7 (a+bx^3) (c+dx^3)} dx = -\frac{b^3 \log (bx^3+a)}{3 (a^3bc-a^4d)} + \frac{d^3 \log (dx^3+c)}{3 (bc^4-ac^3d)} + \frac{(b^2c^2+abcd+a^2d^2) \log (x^3)}{3 a^3c^3} + \frac{2 (bc+ad)x^3-ac}{6 a^2c^2x^6}$$

```
[In] integrate(1/x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*b^3*log(b*x^3 + a)/(a^3*b*c - a^4*d) + 1/3*d^3*log(d*x^3 + c)/(b*c^4 -
 a*c^3*d) + 1/3*(b^2*c^2 + a*b*c*d + a^2*d^2)*log(x^3)/(a^3*c^3) + 1/6*(2*(
b*c + a*d)*x^3 - a*c)/(a^2*c^2*x^6)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^7 (a+bx^3) (c+dx^3)} dx = -\frac{b^4 \log (|bx^3+a|)}{3 (a^3b^2c-a^4bd)} + \frac{d^4 \log (|dx^3+c|)}{3 (bc^4d-ac^3d^2)} + \frac{(b^2c^2+abcd+a^2d^2) \log (|x|)}{a^3c^3} - \frac{3 b^2c^2x^6+3 abcdx^6+3 a^2d^2x^6-2 abc^2x^3-2 a^2cdx^3+a^2c^2}{6 a^3c^3x^6}$$

```
[In] integrate(1/x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b^4*log(abs(b*x^3 + a))/(a^3*b^2*c - a^4*b*d) + 1/3*d^4*log(abs(d*x^3
+ c))/(b*c^4*d - a*c^3*d^2) + (b^2*c^2 + a*b*c*d + a^2*d^2)*log(abs(x))/(a^
3*c^3) - 1/6*(3*b^2*c^2*x^6 + 3*a*b*c*d*x^6 + 3*a^2*d^2*x^6 - 2*a*b*c^2*x^3
 - 2*a^2*c*d*x^3 + a^2*c^2)/(a^3*c^3*x^6)
```

Mupad [B] (verification not implemented)

Time = 7.64 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^7 (a+bx^3) (c+dx^3)} dx = \frac{b^3 \ln (b\,x^3+a)}{3\,a^4\,d-3\,a^3\,b\,c} - \frac{\frac{1}{6\,a\,c}-\frac{x^3\,(a\,d+b\,c)}{3\,a^2\,c^2}}{x^6} + \frac{d^3 \ln (d\,x^3+c)}{3\,b\,c^4-3\,a\,c^3\,d} + \frac{\ln (x)\,\left(a^2\,d^2+a\,b\,c\,d+b^2\,c^2\right)}{a^3\,c^3}$$

```
[In] int(1/(x^7*(a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] (b^3*log(a + b*x^3))/(3*a^4*d - 3*a^3*b*c) - (1/(6*a*c) - (x^3*(a*d + b*c))
/(3*a^2*c^2))/x^6 + (d^3*log(c + d*x^3))/(3*b*c^4 - 3*a*c^3*d) + (log(x)*(a
^2*d^2 + b^2*c^2 + a*b*c*d))/(a^3*c^3)
```

3.123 $\int \frac{1}{x^8(a+bx^3)(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 352

$$\begin{aligned}\int \frac{1}{x^8(a+bx^3)(c+dx^3)}\,dx = &-\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}\\ &+\frac{b^{10/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{10/3}(bc-ad)}-\frac{d^{10/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{10/3}(bc-ad)}\\ &+\frac{b^{10/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}(bc-ad)}-\frac{d^{10/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{10/3}(bc-ad)}\\ &-\frac{b^{10/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{10/3}(bc-ad)}\\ &+\frac{d^{10/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{10/3}(bc-ad)}\end{aligned}$$

```
[Out] -1/7/a/c/x^7+1/4*(a*d+b*c)/a^2/c^2/x^4+(-a^2*d^2-a*b*c*d-b^2*c^2)/a^3/c^3/x
+1/3*b^(10/3)*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)/(-a*d+b*c)-1/3*d^(10/3)*ln(c^(
1/3)+d^(1/3)*x)/c^(10/3)/(-a*d+b*c)-1/6*b^(10/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)
*x+b^(2/3)*x^2)/a^(10/3)/(-a*d+b*c)+1/6*d^(10/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)
*x+d^(2/3)*x^2)/c^(10/3)/(-a*d+b*c)+1/3*b^(10/3)*arctan(1/3*(a^(1/3)-2*b^(1
/3)*x)/a^(1/3)*3^(1/2))/a^(10/3)/(-a*d+b*c)*3^(1/2)-1/3*d^(10/3)*arctan(1/3
*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(10/3)/(-a*d+b*c)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {491, 597, 598, 298, 31, 648, 631, 210, 642}

$$\int \frac{1}{x^8 (a+bx^3)(c+dx^3)}\,dx = \frac{b^{10/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{10/3}(bc-ad)} - \frac{b^{10/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{10/3}(bc-ad)} + \frac{b^{10/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}(bc-ad)} + \frac{ad+bc}{4a^2c^2x^4} - \frac{a^2d^2+abcd+b^2c^2}{a^3c^3x} - \frac{d^{10/3}\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{10/3}(bc-ad)} + \frac{d^{10/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{10/3}(bc-ad)} - \frac{d^{10/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{10/3}(bc-ad)} - \frac{1}{7acx^7}$$

```
[In] Int[1/(x^8*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/7*1/(a*c*x^7) + (b*c + a*d)/(4*a^2*c^2*x^4) - (b^2*c^2 + a*b*c*d + a^2*d
^2)/(a^3*c^3*x) + (b^(10/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)])/(Sqrt[3]*a^(10/3)*(b*c - a*d)) - (d^(10/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*
x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(10/3)*(b*c - a*d)) + (b^(10/3)*Log[a^(1/
3) + b^(1/3)*x])/(3*a^(10/3)*(b*c - a*d)) - (d^(10/3)*Log[c^(1/3) + d^(1/3)
*x])/(3*c^(10/3)*(b*c - a*d)) - (b^(10/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/(6*a^(10/3)*(b*c - a*d)) + (d^(10/3)*Log[c^(2/3) - c^(1/3)*d
^(1/3)*x + d^(2/3)*x^2])/(6*c^(10/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
```

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = -\frac{1}{7acx^7} + \frac{\int \frac{-7(bc+ad)-7bdx^3}{x^5(a+bx^3)(c+dx^3)}\,dx}{7ac}$$

$$= -\frac{1}{7acx^7} + \frac{bc+ad}{4a^2c^2x^4} - \frac{\int \frac{-28(b^2c^2+abcd+a^2d^2)-28bd(bc+ad)x^3}{x^2(a+bx^3)(c+dx^3)}\,dx}{28a^2c^2}$$

$$= -\frac{1}{7acx^7} + \frac{bc+ad}{4a^2c^2x^4} - \frac{b^2c^2+abcd+a^2d^2}{a^3c^3x} + \frac{\int \frac{x\left(-28(bc+ad)(b^2c^2+a^2d^2)-28bd(b^2c^2+abcd+a^2d^2)x^3\right)}{(a+bx^3)(c+dx^3)}\,dx}{28a^3c^3}$$

$$= -\frac{1}{7acx^7} + \frac{bc+ad}{4a^2c^2x^4} - \frac{b^2c^2+abcd+a^2d^2}{a^3c^3x} + \frac{\int \left(-\frac{28b^4c^3x}{(bc-ad)(a+bx^3)} - \frac{28a^3d^4x}{(-bc+ad)(c+dx^3)}\right)dx}{28a^3c^3}$$

$$= -\frac{1}{7acx^7} + \frac{bc+ad}{4a^2c^2x^4} - \frac{b^2c^2+abcd+a^2d^2}{a^3c^3x} - \frac{b^4\int \frac{x}{a+bx^3}\,dx}{a^3(bc-ad)} + \frac{d^4\int \frac{x}{c+dx^3}\,dx}{c^3(bc-ad)}$$

$$= -\frac{1}{7acx^7} + \frac{bc+ad}{4a^2c^2x^4} - \frac{b^2c^2+abcd+a^2d^2}{a^3c^3x} + \frac{b^{11/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{10/3}(bc-ad)} - \frac{b^{11/3}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{10/3}(bc-ad)} - \frac{d^{11/3}\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3c^{10/3}(bc-ad)} + \frac{d^{11/3}\int \frac{\sqrt[3]{c}+\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{3c^{10/3}(bc-ad)}$$

$$= -\frac{1}{7acx^7} + \frac{bc+ad}{4a^2c^2x^4} - \frac{b^2c^2+abcd+a^2d^2}{a^3c^3x} + \frac{b^{10/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}(bc-ad)} - \frac{d^{10/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{10/3}(bc-ad)} - \frac{b^{10/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{10/3}(bc-ad)} - \frac{b^{11/3}\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a^3(bc-ad)} + \frac{d^{10/3}\int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{6c^{10/3}(bc-ad)} + \frac{d^{11/3}\int \frac{1}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{2c^3(bc-ad)}$$

$$= -\frac{1}{7acx^7} + \frac{bc+ad}{4a^2c^2x^4} - \frac{b^2c^2+abcd+a^2d^2}{a^3c^3x} + \frac{b^{10/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}(bc-ad)} - \frac{d^{10/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{10/3}(bc-ad)} - \frac{b^{10/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{10/3}(bc-ad)} + \frac{d^{10/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{10/3}(bc-ad)} - \frac{b^{10/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{10/3}(bc-ad)} + \frac{d^{10/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{c^{10/3}(bc-ad)}$$

$$= -\frac{1}{7acx^7} + \frac{bc+ad}{4a^2c^2x^4} - \frac{b^2c^2+abcd+a^2d^2}{a^3c^3x} + \frac{b^{10/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{10/3}(bc-ad)} - \frac{d^{10/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{10/3}(bc-ad)} + \frac{b^{10/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}(bc-ad)} - \frac{d^{10/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{10/3}(bc-ad)} - \frac{b^{10/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{10/3}(bc-ad)} + \frac{d^{10/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{10/3}(bc-ad)}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^8(a+bx^3)(c+dx^3)}\,dx$$

$$= \frac{\frac{12b}{a} - \frac{12d}{c} - \frac{21b^2x^3}{a^2} + \frac{21d^2x^3}{c^2} + \frac{84b^3x^6}{a^3} - \frac{84d^3x^6}{c^3} - \frac{28\sqrt{3}b^{10/3}x^7\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{10/3}} + \frac{28\sqrt{3}d^{10/3}x^7\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{10/3}} - \frac{28b}{}}{84(-}$$

```
[In] Integrate[1/(x^8*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((12*b)/a - (12*d)/c - (21*b^2*x^3)/a^2 + (21*d^2*x^3)/c^2 + (84*b^3*x^6)/a
^3 - (84*d^3*x^6)/c^3 - (28*Sqrt[3]*b^(10/3)*x^7*ArcTan[(1 - (2*b^(1/3)*x)/
a^(1/3))/Sqrt[3]])/a^(10/3) + (28*Sqrt[3]*d^(10/3)*x^7*ArcTan[(1 - (2*d^(1/
3)*x)/c^(1/3))/Sqrt[3]])/c^(10/3) - (28*b^(10/3)*x^7*Log[a^(1/3) + b^(1/3)*
x]])/a^(10/3) + (28*d^(10/3)*x^7*Log[c^(1/3) + d^(1/3)*x]])/c^(10/3) + (14*b^
(10/3)*x^7*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]])/a^(10/3) - (14*d
^(10/3)*x^7*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]])/c^(10/3))/(84*(
-(b*c) + a*d)*x^7)
```

Maple [A] (verified)

Time = 4.36 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.79

method	result
default	$-\frac{1}{7acx^7}-\frac{-ad-bc}{4x^4a^2c^2}-\frac{a^2d^2+abcd+b^2c^2}{c^3a^3x}-\frac{\left(-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)d^4}{c^3(ad-bc)}+$
risch	$\frac{-\frac{\left(a^2d^2+abcd+b^2c^2\right)x^6}{c^3a^3}+\frac{(ad+bc)x^3}{4a^2c^2}-\frac{1}{7ac}}{x^7}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(d^3c^{10}a^3-3a^2b\,c^{11}d^2+3a\,b^2c^{12}d-b^3c^{13}\right)\_Z^3-d^{10}\right)}\_R\ln\left(\left(\left(-4a^{16}c^{10}d^6\right.\right.\right.\right.}{}$

[In] int(1/x^8/(b*x^3+a)/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] -1/7/a/c/x^7-1/4*(-a*d-b*c)/x^4/a^2/c^2-(a^2*d^2+a*b*c*d+b^2*c^2)/c^3/a^3/x -(-1/3/d/(c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/6/d/(c/d)^(1/3)*ln(x^2-(c/d)^(1/3) *x+(c/d)^(2/3))+1/3*3^(1/2)/d/(c/d)^(1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3) *x-1)))*d^4/c^3/(a*d-b*c)+(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b) ^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1 /3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*b^4/a^3/(a*d-b*c)

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.94

$$\int\frac{1}{x^8\left(a+bx^3\right)\left(c+dx^3\right)}\,dx=$$
$$-\frac{28\sqrt{3}b^3c^3x^7\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-28\sqrt{3}a^3d^3x^7\left(\frac{d}{c}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{d}{c}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-}{}$$

[In] integrate(1/x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")

[Out] -1/84*(28*sqrt(3)*b^3*c^3*x^7*(-b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqrt(3)) - 28*sqrt(3)*a^3*d^3*x^7*(d/c)^(1/3)*arctan(2/3*sqrt(3)*x* (d/c)^(1/3) - 1/3*sqrt(3)) - 14*b^3*c^3*x^7*(-b/a)^(1/3)*log(b*x^2 - a*x*(- b/a)^(2/3) - a*(-b/a)^(1/3)) - 14*a^3*d^3*x^7*(d/c)^(1/3)*log(d*x^2 - c*x*(d/c)^(2/3) + c*(d/c)^(1/3)) + 28*b^3*c^3*x^7*(-b/a)^(1/3)*log(b*x + a*(-b/a

```
)^(2/3)) + 28*a^3*d^3*x^7*(d/c)^(1/3)*log(d*x + c*(d/c)^(2/3)) + 84*(b^3*c^
3 - a^3*d^3)*x^6 + 12*a^2*b*c^3 - 12*a^3*c^2*d - 21*(a*b^2*c^3 - a^3*c*d^2)
*x^3)/((a^3*b*c^4 - a^4*c^3*d)*x^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^8\,(a+bx^3)\,(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x**8/(b*x**3+a)/(d*x**3+c),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.07

$$\begin{aligned}\int \frac{1}{x^8\,(a+bx^3)\,(c+dx^3)}\,dx = &-\frac{\sqrt{3}b^3\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,(a^3bc-a^4d)\left(\frac{a}{b}\right)^{\frac{1}{3}}} \\ &+\frac{\sqrt{3}d^3\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,(bc^4-ac^3d)\left(\frac{c}{d}\right)^{\frac{1}{3}}} \\ &-\frac{b^3\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a^3bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^4d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}+\frac{d^3\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc^4\left(\frac{c}{d}\right)^{\frac{1}{3}}-ac^3d\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} \\ &+\frac{b^3\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(a^3bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^4d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}-\frac{d^3\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(bc^4\left(\frac{c}{d}\right)^{\frac{1}{3}}-ac^3d\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)} \\ &-\frac{28\,(b^2c^2+abcd+a^2d^2)x^6+4\,a^2c^2-7\,(abc^2+a^2cd)x^3}{28\,a^3c^3x^7}\end{aligned}$$

```
[In] integrate(1/x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((a^3*
b*c - a^4*d)*(a/b)^(1/3)) + 1/3*sqrt(3)*d^3*arctan(1/3*sqrt(3)*(2*x - (c/d)
^(1/3))/(c/d)^(1/3))/((b*c^4 - a*c^3*d)*(c/d)^(1/3)) - 1/6*b^3*log(x^2 - x*
(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*c*(a/b)^(1/3) - a^4*d*(a/b)^(1/3)) + 1/6*
d^3*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c^4*(c/d)^(1/3) - a*c^3*d*(c/
```

```
d)^(1/3)) + 1/3*b^3*log(x + (a/b)^(1/3))/(a^3*b*c*(a/b)^(1/3) - a^4*d*(a/b)
^(1/3)) - 1/3*d^3*log(x + (c/d)^(1/3))/(b*c^4*(c/d)^(1/3) - a*c^3*d*(c/d)^(
1/3)) - 1/28*(28*(b^2*c^2 + a*b*c*d + a^2*d^2)*x^6 + 4*a^2*c^2 - 7*(a*b*c^2
 + a^2*c*d)*x^3)/(a^3*c^3*x^7)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^8 (a+bx^3) (c+dx^3)} dx = \frac{b^4 \left(-\frac{a}{b}\right)^{\frac{2}{3}} \log \left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3 (a^4bc-a^5d)} - \frac{d^4 \left(-\frac{c}{d}\right)^{\frac{2}{3}} \log \left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3 (bc^5-ac^4d)} + \frac{(-ab^2)^{\frac{2}{3}} b^2 \arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}a^4bc-\sqrt{3}a^5d} - \frac{(-cd^2)^{\frac{2}{3}} d^2 \arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^5-\sqrt{3}ac^4d} - \frac{(-ab^2)^{\frac{2}{3}} b^2 \log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 (a^4bc-a^5d)} + \frac{(-cd^2)^{\frac{2}{3}} d^2 \log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6 (bc^5-ac^4d)} - \frac{28 b^2c^2x^6+28 abcdx^6+28 a^2d^2x^6-7 abc^2x^3-7 a^2cdx^3+4 a^2c^2}{28 a^3c^3x^7}$$

```
[In] integrate(1/x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^4*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^4*b*c - a^5*d) - 1/3*d^4
*(-c/d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^5 - a*c^4*d) + (-a*b^2)^(2/3)
*b^2*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^4*b*c
 - sqrt(3)*a^5*d) - (-c*d^2)^(2/3)*d^2*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/
3))/(-c/d)^(1/3))/(sqrt(3)*b*c^5 - sqrt(3)*a*c^4*d) - 1/6*(-a*b^2)^(2/3)*b^
2*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b*c - a^5*d) + 1/6*(-c*d^2)
^(2/3)*d^2*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^5 - a*c^4*d) - 1/2
8*(28*b^2*c^2*x^6 + 28*a*b*c*d*x^6 + 28*a^2*d^2*x^6 - 7*a*b*c^2*x^3 - 7*a^2
*c*d*x^3 + 4*a^2*c^2)/(a^3*c^3*x^7)
```

Mupad [B] (verification not implemented)

Time = 16.93 (sec) , antiderivative size = 1814, normalized size of antiderivative = 5.15

$$\int \frac{1}{x^8 (a+bx^3)(c+dx^3)} \, dx = \text{Too large to display}$$

```
[In] int(1/(x^8*(a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(((-b^10/(a^10*(a*d - b*c)^3))^(2/3)*(((27*a^21*b^3*c^21*d^3*x*(a^8*d^8
+ b^8*c^8)*(a*d - b*c)^2 + 27*a^28*b^3*c^28*d^3*(a*d + b*c)*(a*d - b*c)^4*(
-b^10/(a^10*(a*d - b*c)^3))^(2/3))*(-b^10/(a^10*(a*d - b*c)^3))^(1/3))/3 -
9*a^19*b^14*c^29*d^4 + 9*a^20*b^13*c^28*d^5 + 9*a^28*b^5*c^20*d^13 - 9*a^29
*b^4*c^19*d^14))/9 - a^19*b^11*c^19*d^11*x*(a*d + b*c))*(-b^10/(27*a^13*d^3
 - 27*a^10*b^3*c^3 + 81*a^11*b^2*c^2*d - 81*a^12*b*c*d^2))^(1/3) + log(((d^
10/(c^10*(a*d - b*c)^3))^(2/3)*(((27*a^21*b^3*c^21*d^3*x*(a^8*d^8 + b^8*c^8
)*(a*d - b*c)^2 + 27*a^28*b^3*c^28*d^3*(a*d + b*c)*(a*d - b*c)^4*(d^10/(c^1
0*(a*d - b*c)^3))^(2/3))*(d^10/(c^10*(a*d - b*c)^3))^(1/3))/3 - 9*a^19*b^14
*c^29*d^4 + 9*a^20*b^13*c^28*d^5 + 9*a^28*b^5*c^20*d^13 - 9*a^29*b^4*c^19*d
^14))/9 - a^19*b^11*c^19*d^11*x*(a*d + b*c))*(-d^10/(27*b^3*c^13 - 27*a^3*c
^10*d^3 + 81*a^2*b*c^11*d^2 - 81*a*b^2*c^12*d))^(1/3) - (1/(7*a*c) - (x^3*(
a*d + b*c))/(4*a^2*c^2) + (x^6*(a^2*d^2 + b^2*c^2 + a*b*c*d))/(a^3*c^3))/x^
7 - (log(((3^(1/2)*1i + 1)^2*(-b^10/(a^10*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*
1i + 1)*(27*a^21*b^3*c^21*d^3*x*(a^8*d^8 + b^8*c^8)*(a*d - b*c)^2 + (27*a^2
8*b^3*c^28*d^3*(3^(1/2)*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-b^10/(a^10*(a
*d - b*c)^3))^(2/3))/4)*(-b^10/(a^10*(a*d - b*c)^3))^(1/3))/6 + 9*a^19*b^14
*c^29*d^4 - 9*a^20*b^13*c^28*d^5 - 9*a^28*b^5*c^20*d^13 + 9*a^29*b^4*c^19*d
^14))/36 + a^19*b^11*c^19*d^11*x*(a*d + b*c))*(-b^10/(27*a^13*d^3 - 27*a^10
*b^3*c^3 + 81*a^11*b^2*c^2*d - 81*a^12*b*c*d^2))^(1/3)*(3^(1/2)*1i + 1))/2
+ (log(((3^(1/2)*1i - 1)^2*(-b^10/(a^10*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i
 - 1)*(27*a^21*b^3*c^21*d^3*x*(a^8*d^8 + b^8*c^8)*(a*d - b*c)^2 + (27*a^28*
b^3*c^28*d^3*(3^(1/2)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-b^10/(a^10*(a*d
 - b*c)^3))^(2/3))/4)*(-b^10/(a^10*(a*d - b*c)^3))^(1/3))/6 - 9*a^19*b^14*c
^29*d^4 + 9*a^20*b^13*c^28*d^5 + 9*a^28*b^5*c^20*d^13 - 9*a^29*b^4*c^19*d^1
4))/36 - a^19*b^11*c^19*d^11*x*(a*d + b*c))*(-b^10/(27*a^13*d^3 - 27*a^10*b
^3*c^3 + 81*a^11*b^2*c^2*d - 81*a^12*b*c*d^2))^(1/3)*(3^(1/2)*1i - 1))/2 -
(log(((3^(1/2)*1i + 1)^2*(d^10/(c^10*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i +
1)*(27*a^21*b^3*c^21*d^3*x*(a^8*d^8 + b^8*c^8)*(a*d - b*c)^2 + (27*a^28*b^3
*c^28*d^3*(3^(1/2)*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^4*(d^10/(c^10*(a*d - b
*c)^3))^(2/3))/4)*(d^10/(c^10*(a*d - b*c)^3))^(1/3))/6 + 9*a^19*b^14*c^29*d
^4 - 9*a^20*b^13*c^28*d^5 - 9*a^28*b^5*c^20*d^13 + 9*a^29*b^4*c^19*d^14))/3
6 + a^19*b^11*c^19*d^11*x*(a*d + b*c))*(-d^10/(27*b^3*c^13 - 27*a^3*c^10*d^
3 + 81*a^2*b*c^11*d^2 - 81*a*b^2*c^12*d))^(1/3)*(3^(1/2)*1i + 1))/2 + (log(
((3^(1/2)*1i - 1)^2*(d^10/(c^10*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i - 1)*(2
7*a^21*b^3*c^21*d^3*x*(a^8*d^8 + b^8*c^8)*(a*d - b*c)^2 + (27*a^28*b^3*c^28
```

```
*d^3*(3^(1/2)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(d^10/(c^10*(a*d - b*c)^3
))^(2/3))/4)*(d^10/(c^10*(a*d - b*c)^3))^(1/3))/6 - 9*a^19*b^14*c^29*d^4 +
9*a^20*b^13*c^28*d^5 + 9*a^28*b^5*c^20*d^13 - 9*a^29*b^4*c^19*d^14))/36 - a
^19*b^11*c^19*d^11*x*(a*d + b*c))*(-d^10/(27*b^3*c^13 - 27*a^3*c^10*d^3 + 8
1*a^2*b*c^11*d^2 - 81*a*b^2*c^12*d))^(1/3)*(3^(1/2)*1i - 1))/2
```

3.124 $\int x^m(a+bx^3)^5\left(A+Bx^3\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 148

$$\int x^m\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=\frac{a^5Ax^{1+m}}{1+m}+\frac{a^4(5Ab+aB)x^{4+m}}{4+m}+\frac{5a^3b(2Ab+aB)x^{7+m}}{7+m}+\frac{10a^2b^2(Ab+aB)x^{10+m}}{10+m}+\frac{5ab^3(Ab+2aB)x^{13+m}}{13+m}+\frac{b^4(Ab+5aB)x^{16+m}}{16+m}+\frac{b^5Bx^{19+m}}{19+m}$$

```
[Out] a^5*A*x^(1+m)/(1+m)+a^4*(5*A*b+B*a)*x^(4+m)/(4+m)+5*a^3*b*(2*A*b+B*a)*x^(7+
m)/(7+m)+10*a^2*b^2*(A*b+B*a)*x^(10+m)/(10+m)+5*a*b^3*(A*b+2*B*a)*x^(13+m)/
(13+m)+b^4*(A*b+5*B*a)*x^(16+m)/(16+m)+b^5*B*x^(19+m)/(19+m)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int x^m\left(a+bx^3\right)^5\left(A+Bx^3\right)\,dx=\frac{a^5Ax^{m+1}}{m+1}+\frac{a^4x^{m+4}(aB+5Ab)}{m+4}+\frac{5a^3bx^{m+7}(aB+2Ab)}{m+7}+\frac{10a^2b^2x^{m+10}(aB+Ab)}{m+10}+\frac{b^4x^{m+16}(5aB+Ab)}{m+16}+\frac{5ab^3x^{m+13}(2aB+Ab)}{m+13}+\frac{b^5Bx^{m+19}}{m+19}$$

```
[In] Int[x^m*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^(1 + m))/(1 + m) + (a^4*(5*A*b + a*B)*x^(4 + m))/(4 + m) + (5*a^3*
b*(2*A*b + a*B)*x^(7 + m))/(7 + m) + (10*a^2*b^2*(A*b + a*B)*x^(10 + m))/(1
```

```
0 + m) + (5*a*b^3*(A*b + 2*a*B)*x^(13 + m))/(13 + m) + (b^4*(A*b + 5*a*B)*x
^(16 + m))/(16 + m) + (b^5*B*x^(19 + m))/(19 + m)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^5 A x^m + a^4(5Ab + aB)x^{3+m} + 5a^3b(2Ab + aB)x^{6+m} + 10a^2b^2(Ab + aB)x^{9+m} \right. \\
&\quad \left. + 5ab^3(Ab + 2aB)x^{12+m} + b^4(Ab + 5aB)x^{15+m} + b^5Bx^{18+m}\right) dx \\
&= \frac{a^5 A x^{1+m}}{1+m} + \frac{a^4(5Ab + aB)x^{4+m}}{4+m} + \frac{5a^3b(2Ab + aB)x^{7+m}}{7+m} + \frac{10a^2b^2(Ab + aB)x^{10+m}}{10+m} \\
&\quad + \frac{5ab^3(Ab + 2aB)x^{13+m}}{13+m} + \frac{b^4(Ab + 5aB)x^{16+m}}{16+m} + \frac{b^5Bx^{19+m}}{19+m}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.93

$$\int x^m \left(a + bx^3\right)^5 \left(A + Bx^3\right) dx = x^{1+m} \left(\frac{a^5 A}{1+m} + \frac{a^4(5Ab + aB)x^3}{4+m} + \frac{5a^3b(2Ab + aB)x^6}{7+m} + \frac{10a^2b^2(Ab + aB)x^9}{10+m} + \frac{5ab^3(Ab + 2aB)x^{12}}{13+m} + \frac{b^4(Ab + 5aB)x^{15}}{16+m} + \frac{b^5Bx^{18}}{19+m}\right)$$

```
[In] Integrate[x^m*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] x^(1 + m)*((a^5*A)/(1 + m) + (a^4*(5*A*b + a*B)*x^3)/(4 + m) + (5*a^3*b*(2*
A*b + a*B)*x^6)/(7 + m) + (10*a^2*b^2*(A*b + a*B)*x^9)/(10 + m) + (5*a*b^3*
(A*b + 2*a*B)*x^12)/(13 + m) + (b^4*(A*b + 5*a*B)*x^15)/(16 + m) + (b^5*B*x
^18)/(19 + m))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1076 vs. $2(148) = 296$.

Time = 4.74 (sec) , antiderivative size = 1077, normalized size of antiderivative = 7.28

method	result	size
risch	Expression too large to display	1077
gosper	Expression too large to display	1078
parallelrisch	Expression too large to display	1332

```
[In] int(x^m*(b*x^3+a)^5*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] x*(B*b^5*m^6*x^18+51*B*b^5*m^5*x^18+1005*B*b^5*m^4*x^18+A*b^5*m^6*x^15+5*B*
a*b^4*m^6*x^15+9605*B*b^5*m^3*x^18+54*A*b^5*m^5*x^15+270*B*a*b^4*m^5*x^15+4
5474*B*b^5*m^2*x^18+1110*A*b^5*m^4*x^15+5550*B*a*b^4*m^4*x^15+95064*B*b^5*m
*x^18+5*A*a*b^4*m^6*x^12+10940*A*b^5*m^3*x^15+10*B*a^2*b^3*m^6*x^12+54700*B
*a*b^4*m^3*x^15+58240*B*b^5*x^18+285*A*a*b^4*m^5*x^12+52929*A*b^5*m^2*x^15+
570*B*a^2*b^3*m^5*x^12+264645*B*a*b^4*m^2*x^15+6165*A*a*b^4*m^4*x^12+112206
*A*b^5*m*x^15+12330*B*a^2*b^3*m^4*x^12+561030*B*a*b^4*m*x^15+10*A*a^2*b^3*m
^6*x^9+63355*A*a*b^4*m^3*x^12+69160*A*b^5*x^15+10*B*a^3*b^2*m^6*x^9+126710*
B*a^2*b^3*m^3*x^12+345800*B*a*b^4*x^15+600*A*a^2*b^3*m^5*x^9+316230*A*a*b^4
*m^2*x^12+600*B*a^3*b^2*m^5*x^9+632460*B*a^2*b^3*m^2*x^12+13740*A*a^2*b^3*m
^4*x^9+684360*A*a*b^4*m*x^12+13740*B*a^3*b^2*m^4*x^9+1368720*B*a^2*b^3*m*x^
12+10*A*a^3*b^2*m^6*x^6+149600*A*a^2*b^3*m^3*x^9+425600*A*a*b^4*x^12+5*B*a^
4*b*m^6*x^6+149600*B*a^3*b^2*m^3*x^9+851200*B*a^2*b^3*x^12+630*A*a^3*b^2*m^
5*x^6+783690*A*a^2*b^3*m^2*x^9+315*B*a^4*b*m^5*x^6+783690*B*a^3*b^2*m^2*x^9
+15330*A*a^3*b^2*m^4*x^6+1753800*A*a^2*b^3*m*x^9+7665*B*a^4*b*m^4*x^6+17538
00*B*a^3*b^2*m*x^9+5*A*a^4*b*m^6*x^3+179690*A*a^3*b^2*m^3*x^6+1106560*A*a^2
*b^3*x^9+B*a^5*m^6*x^3+89845*B*a^4*b*m^3*x^6+1106560*B*a^3*b^2*x^9+330*A*a^
4*b*m^5*x^3+1021860*A*a^3*b^2*m^2*x^6+66*B*a^5*m^5*x^3+510930*B*a^4*b*m^2*x
^6+8550*A*a^4*b*m^4*x^3+2437680*A*a^3*b^2*m*x^6+1710*B*a^5*m^4*x^3+1218840*
B*a^4*b*m*x^6+A*a^5*m^6+109300*A*a^4*b*m^3*x^3+1580800*A*a^3*b^2*x^6+21860*
B*a^5*m^3*x^3+790400*B*a^4*b*x^6+69*A*a^5*m^5+702645*A*a^4*b*m^2*x^3+140529
*B*a^5*m^2*x^3+1905*A*a^5*m^4+1984770*A*a^4*b*m*x^3+396954*B*a^5*m*x^3+2679
5*A*a^5*m^3+1383200*A*a^4*b*x^3+276640*B*a^5*x^3+201174*A*a^5*m^2+757896*A*
a^5*m+1106560*A*a^5)*x^m/(1+m)/(4+m)/(7+m)/(10+m)/(13+m)/(16+m)/(19+m)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 851 vs. $2(148) = 296$.

Time = 0.27 (sec) , antiderivative size = 851, normalized size of antiderivative = 5.75

$$\int x^m \left(a+bx^3\right)^5 \left(A+Bx^3\right)\,dx = \frac{\left(\left(Bb^5m^6+51\,Bb^5m^5+1005\,Bb^5m^4+9605\,Bb^5m^3+45474\,Bb^5m^2+95064\,Bb^5m+58240\,Bb^5\right)x^{19}+((5\right.}{}$$

```
[In] integrate(x^m*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] ((B*b^5*m^6 + 51*B*b^5*m^5 + 1005*B*b^5*m^4 + 9605*B*b^5*m^3 + 45474*B*b^5*
m^2 + 95064*B*b^5*m + 58240*B*b^5)*x^19 + ((5*B*a*b^4 + A*b^5)*m^6 + 345800
*B*a*b^4 + 69160*A*b^5 + 54*(5*B*a*b^4 + A*b^5)*m^5 + 1110*(5*B*a*b^4 + A*b
^5)*m^4 + 10940*(5*B*a*b^4 + A*b^5)*m^3 + 52929*(5*B*a*b^4 + A*b^5)*m^2 + 1
12206*(5*B*a*b^4 + A*b^5)*m)*x^16 + 5*((2*B*a^2*b^3 + A*a*b^4)*m^6 + 170240
*B*a^2*b^3 + 85120*A*a*b^4 + 57*(2*B*a^2*b^3 + A*a*b^4)*m^5 + 1233*(2*B*a^2
*b^3 + A*a*b^4)*m^4 + 12671*(2*B*a^2*b^3 + A*a*b^4)*m^3 + 63246*(2*B*a^2*b^
3 + A*a*b^4)*m^2 + 136872*(2*B*a^2*b^3 + A*a*b^4)*m)*x^13 + 10*((B*a^3*b^2
+ A*a^2*b^3)*m^6 + 110656*B*a^3*b^2 + 110656*A*a^2*b^3 + 60*(B*a^3*b^2 + A*
a^2*b^3)*m^5 + 1374*(B*a^3*b^2 + A*a^2*b^3)*m^4 + 14960*(B*a^3*b^2 + A*a^2*
b^3)*m^3 + 78369*(B*a^3*b^2 + A*a^2*b^3)*m^2 + 175380*(B*a^3*b^2 + A*a^2*b^
3)*m)*x^10 + 5*((B*a^4*b + 2*A*a^3*b^2)*m^6 + 158080*B*a^4*b + 316160*A*a^3
*b^2 + 63*(B*a^4*b + 2*A*a^3*b^2)*m^5 + 1533*(B*a^4*b + 2*A*a^3*b^2)*m^4 +
17969*(B*a^4*b + 2*A*a^3*b^2)*m^3 + 102186*(B*a^4*b + 2*A*a^3*b^2)*m^2 + 24
3768*(B*a^4*b + 2*A*a^3*b^2)*m)*x^7 + ((B*a^5 + 5*A*a^4*b)*m^6 + 276640*B*a
^5 + 1383200*A*a^4*b + 66*(B*a^5 + 5*A*a^4*b)*m^5 + 1710*(B*a^5 + 5*A*a^4*b
)*m^4 + 21860*(B*a^5 + 5*A*a^4*b)*m^3 + 140529*(B*a^5 + 5*A*a^4*b)*m^2 + 39
6954*(B*a^5 + 5*A*a^4*b)*m)*x^4 + (A*a^5*m^6 + 69*A*a^5*m^5 + 1905*A*a^5*m^
4 + 26795*A*a^5*m^3 + 201174*A*a^5*m^2 + 757896*A*a^5*m + 1106560*A*a^5)*x)
*x^m/(m^7 + 70*m^6 + 1974*m^5 + 28700*m^4 + 227969*m^3 + 959070*m^2 + 18644
56*m + 1106560)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5418 vs. $2(138) = 276$.

Time = 1.65 (sec) , antiderivative size = 5418, normalized size of antiderivative = 36.61

$$\int x^m \left(a+bx^3\right)^5 \left(A+Bx^3\right)\,dx = \text{Too large to display}$$

```
[In] integrate(x**m*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] Piecewise((-A*a**5/(18*x**18) - A*a**4*b/(3*x**15) - 5*A*a**3*b**2/(6*x**12
) - 10*A*a**2*b**3/(9*x**9) - 5*A*a*b**4/(6*x**6) - A*b**5/(3*x**3) - B*a**
5/(15*x**15) - 5*B*a**4*b/(12*x**12) - 10*B*a**3*b**2/(9*x**9) - 5*B*a**2*b
**3/(3*x**6) - 5*B*a*b**4/(3*x**3) + B*b**5*log(x), Eq(m, -19)), (-A*a**5/(
15*x**15) - 5*A*a**4*b/(12*x**12) - 10*A*a**3*b**2/(9*x**9) - 5*A*a**2*b**3
/(3*x**6) - 5*A*a*b**4/(3*x**3) + A*b**5*log(x) - B*a**5/(12*x**12) - 5*B*a
**4*b/(9*x**9) - 5*B*a**3*b**2/(3*x**6) - 10*B*a**2*b**3/(3*x**3) + 5*B*a*b
**4*log(x) + B*b**5*x**3/3, Eq(m, -16)), (-A*a**5/(12*x**12) - 5*A*a**4*b/(
9*x**9) - 5*A*a**3*b**2/(3*x**6) - 10*A*a**2*b**3/(3*x**3) + 5*A*a*b**4*log
(x) + A*b**5*x**3/3 - B*a**5/(9*x**9) - 5*B*a**4*b/(6*x**6) - 10*B*a**3*b**
2/(3*x**3) + 10*B*a**2*b**3*log(x) + 5*B*a*b**4*x**3/3 + B*b**5*x**6/6, Eq(
m, -13)), (-A*a**5/(9*x**9) - 5*A*a**4*b/(6*x**6) - 10*A*a**3*b**2/(3*x**3)
 + 10*A*a**2*b**3*log(x) + 5*A*a*b**4*x**3/3 + A*b**5*x**6/6 - B*a**5/(6*x*
*6) - 5*B*a**4*b/(3*x**3) + 10*B*a**3*b**2*log(x) + 10*B*a**2*b**3*x**3/3 +
 5*B*a*b**4*x**6/6 + B*b**5*x**9/9, Eq(m, -10)), (-A*a**5/(6*x**6) - 5*A*a*
*4*b/(3*x**3) + 10*A*a**3*b**2*log(x) + 10*A*a**2*b**3*x**3/3 + 5*A*a*b**4*
x**6/6 + A*b**5*x**9/9 - B*a**5/(3*x**3) + 5*B*a**4*b*log(x) + 10*B*a**3*b*
*2*x**3/3 + 5*B*a**2*b**3*x**6/3 + 5*B*a*b**4*x**9/9 + B*b**5*x**12/12, Eq(
m, -7)), (-A*a**5/(3*x**3) + 5*A*a**4*b*log(x) + 10*A*a**3*b**2*x**3/3 + 5*
A*a**2*b**3*x**6/3 + 5*A*a*b**4*x**9/9 + A*b**5*x**12/12 + B*a**5*log(x) +
5*B*a**4*b*x**3/3 + 5*B*a**3*b**2*x**6/3 + 10*B*a**2*b**3*x**9/9 + 5*B*a*b*
*4*x**12/12 + B*b**5*x**15/15, Eq(m, -4)), (A*a**5*log(x) + 5*A*a**4*b*x**3
/3 + 5*A*a**3*b**2*x**6/3 + 10*A*a**2*b**3*x**9/9 + 5*A*a*b**4*x**12/12 + A
*b**5*x**15/15 + B*a**5*x**3/3 + 5*B*a**4*b*x**6/6 + 10*B*a**3*b**2*x**9/9
+ 5*B*a**2*b**3*x**12/6 + B*a*b**4*x**15/3 + B*b**5*x**18/18, Eq(m, -1)), (
A*a**5*m**6*x*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 +
 959070*m**2 + 1864456*m + 1106560) + 69*A*a**5*m**5*x*x**m/(m**7 + 70*m**6
 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560
) + 1905*A*a**5*m**4*x*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 2279
69*m**3 + 959070*m**2 + 1864456*m + 1106560) + 26795*A*a**5*m**3*x*x**m/(m*
*7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456
*m + 1106560) + 201174*A*a**5*m**2*x*x**m/(m**7 + 70*m**6 + 1974*m**5 + 287
00*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 757896*A*a**5*
m*x*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m*
*2 + 1864456*m + 1106560) + 1106560*A*a**5*x*x**m/(m**7 + 70*m**6 + 1974*m*
*5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 5*A*a*
*4*b*m**6*x**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3
+ 959070*m**2 + 1864456*m + 1106560) + 330*A*a**4*b*m**5*x**4*x**m/(m**7 +
70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m +
1106560) + 8550*A*a**4*b*m**4*x**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700
*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 109300*A*a**4*b*
m**3*x**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959
070*m**2 + 1864456*m + 1106560) + 702645*A*a**4*b*m**2*x**4*x**m/(m**7 + 70
*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 11
06560) + 1984770*A*a**4*b*m*x**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m
```

```
**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1383200*A*a**4*b*x
**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m*
*2 + 1864456*m + 1106560) + 10*A*a**3*b**2*m**6*x**7*x**m/(m**7 + 70*m**6 +
 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560)
+ 630*A*a**3*b**2*m**5*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 +
 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 15330*A*a**3*b**2*m**4*
x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m
**2 + 1864456*m + 1106560) + 179690*A*a**3*b**2*m**3*x**7*x**m/(m**7 + 70*m
**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106
560) + 1021860*A*a**3*b**2*m**2*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 287
00*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 2437680*A*a**3
*b**2*m*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 +
959070*m**2 + 1864456*m + 1106560) + 1580800*A*a**3*b**2*x**7*x**m/(m**7 +
70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m +
1106560) + 10*A*a**2*b**3*m**6*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 287
00*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 600*A*a**2*b**
3*m**5*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 +
959070*m**2 + 1864456*m + 1106560) + 13740*A*a**2*b**3*m**4*x**10*x**m/(m**
7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*
m + 1106560) + 149600*A*a**2*b**3*m**3*x**10*x**m/(m**7 + 70*m**6 + 1974*m*
*5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 783690
*A*a**2*b**3*m**2*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227
969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1753800*A*a**2*b**3*m*x**10
*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2
+ 1864456*m + 1106560) + 1106560*A*a**2*b**3*x**10*x**m/(m**7 + 70*m**6 + 1
974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) +
5*A*a*b**4*m**6*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 22796
9*m**3 + 959070*m**2 + 1864456*m + 1106560) + 285*A*a*b**4*m**5*x**13*x**m/
(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864
456*m + 1106560) + 6165*A*a*b**4*m**4*x**13*x**m/(m**7 + 70*m**6 + 1974*m**
5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 63355*A
*a*b**4*m**3*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m
**3 + 959070*m**2 + 1864456*m + 1106560) + 316230*A*a*b**4*m**2*x**13*x**m/
(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864
456*m + 1106560) + 684360*A*a*b**4*m*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5
 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 425600*A
*a*b**4*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 +
 959070*m**2 + 1864456*m + 1106560) + A*b**5*m**6*x**16*x**m/(m**7 + 70*m**
6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 110656
0) + 54*A*b**5*m**5*x**16*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 2
27969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1110*A*b**5*m**4*x**16*x*
*m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1
864456*m + 1106560) + 10940*A*b**5*m**3*x**16*x**m/(m**7 + 70*m**6 + 1974*m
**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 52929
*A*b**5*m**2*x**16*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m
```

```
**3 + 959070*m**2 + 1864456*m + 1106560) + 112206*A*b**5*m*x**16*x**m/(m**7
 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m
 + 1106560) + 69160*A*b**5*x**16*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m
**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + B*a**5*m**6*x**4*x
**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 +
1864456*m + 1106560) + 66*B*a**5*m**5*x**4*x**m/(m**7 + 70*m**6 + 1974*m**5
 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1710*B*a
**5*m**4*x**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 +
 959070*m**2 + 1864456*m + 1106560) + 21860*B*a**5*m**3*x**4*x**m/(m**7 + 7
0*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1
106560) + 140529*B*a**5*m**2*x**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*
m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 396954*B*a**5*m*x
**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m*
*2 + 1864456*m + 1106560) + 276640*B*a**5*x**4*x**m/(m**7 + 70*m**6 + 1974*
m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 5*B*
a**4*b*m**6*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**
3 + 959070*m**2 + 1864456*m + 1106560) + 315*B*a**4*b*m**5*x**7*x**m/(m**7
+ 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m
+ 1106560) + 7665*B*a**4*b*m**4*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 287
00*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 89845*B*a**4*b
*m**3*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 95
9070*m**2 + 1864456*m + 1106560) + 510930*B*a**4*b*m**2*x**7*x**m/(m**7 + 7
0*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1
106560) + 1218840*B*a**4*b*m*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*
m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 790400*B*a**4*b*x
**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m*
*2 + 1864456*m + 1106560) + 10*B*a**3*b**2*m**6*x**10*x**m/(m**7 + 70*m**6
+ 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560)
 + 600*B*a**3*b**2*m**5*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4
 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 13740*B*a**3*b**2*m**
4*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 95907
0*m**2 + 1864456*m + 1106560) + 149600*B*a**3*b**2*m**3*x**10*x**m/(m**7 +
70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m +
1106560) + 783690*B*a**3*b**2*m**2*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 +
 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1753800*B*
a**3*b**2*m*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m*
*3 + 959070*m**2 + 1864456*m + 1106560) + 1106560*B*a**3*b**2*x**10*x**m/(m
**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 186445
6*m + 1106560) + 10*B*a**2*b**3*m**6*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5
 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 570*B*a*
*2*b**3*m**5*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m
**3 + 959070*m**2 + 1864456*m + 1106560) + 12330*B*a**2*b**3*m**4*x**13*x**
m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 18
64456*m + 1106560) + 126710*B*a**2*b**3*m**3*x**13*x**m/(m**7 + 70*m**6 + 1
974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) +
```

```
632460*B*a**2*b**3*m**2*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4
 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1368720*B*a**2*b**3*m
*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070
*m**2 + 1864456*m + 1106560) + 851200*B*a**2*b**3*x**13*x**m/(m**7 + 70*m**
6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 110656
0) + 5*B*a*b**4*m**6*x**16*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 +
227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 270*B*a*b**4*m**5*x**16*
x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 +
 1864456*m + 1106560) + 5550*B*a*b**4*m**4*x**16*x**m/(m**7 + 70*m**6 + 197
4*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 54
700*B*a*b**4*m**3*x**16*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227
969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 264645*B*a*b**4*m**2*x**16*
x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 +
 1864456*m + 1106560) + 561030*B*a*b**4*m*x**16*x**m/(m**7 + 70*m**6 + 1974
*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 345
800*B*a*b**4*x**16*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m
**3 + 959070*m**2 + 1864456*m + 1106560) + B*b**5*m**6*x**19*x**m/(m**7 + 7
0*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1
106560) + 51*B*b**5*m**5*x**19*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**
4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1005*B*b**5*m**4*x**
19*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**
2 + 1864456*m + 1106560) + 9605*B*b**5*m**3*x**19*x**m/(m**7 + 70*m**6 + 19
74*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 4
5474*B*b**5*m**2*x**19*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 2279
69*m**3 + 959070*m**2 + 1864456*m + 1106560) + 95064*B*b**5*m*x**19*x**m/(m
**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 186445
6*m + 1106560) + 58240*B*b**5*x**19*x**m/(m**7 + 70*m**6 + 1974*m**5 + 2870
0*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.39

$$\int x^m \left(a + bx^3\right)^5 \left(A + Bx^3\right) \, dx = \frac{Bb^5x^{m+19}}{m+19} + \frac{5\,Bab^4x^{m+16}}{m+16} + \frac{Ab^5x^{m+16}}{m+16} + \frac{10\,Ba^2b^3x^{m+13}}{m+13} + \frac{5\,Aab^4x^{m+13}}{m+13} + \frac{10\,Ba^3b^2x^{m+10}}{m+10} + \frac{10\,Aa^2b^3x^{m+10}}{m+10} + \frac{5\,Ba^4bx^{m+7}}{m+7} + \frac{10\,Aa^3b^2x^{m+7}}{m+7} + \frac{Ba^5x^{m+4}}{m+4} + \frac{5\,Aa^4bx^{m+4}}{m+4} + \frac{Aa^5x^{m+1}}{m+1}$$

```
[In] integrate(x^m*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

[Out] B*b^5*x^(m + 19)/(m + 19) + 5*B*a*b^4*x^(m + 16)/(m + 16) + A*b^5*x^(m + 16
)/(m + 16) + 10*B*a^2*b^3*x^(m + 13)/(m + 13) + 5*A*a*b^4*x^(m + 13)/(m + 1
3) + 10*B*a^3*b^2*x^(m + 10)/(m + 10) + 10*A*a^2*b^3*x^(m + 10)/(m + 10) +
5*B*a^4*b*x^(m + 7)/(m + 7) + 10*A*a^3*b^2*x^(m + 7)/(m + 7) + B*a^5*x^(m +
 4)/(m + 4) + 5*A*a^4*b*x^(m + 4)/(m + 4) + A*a^5*x^(m + 1)/(m + 1)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1331 vs. $2(148) = 296$.

Time = 0.33 (sec) , antiderivative size = 1331, normalized size of antiderivative = 8.99

$$\int x^m (a+bx^3)^5 (A+Bx^3)\,dx = \text{Too large to display}$$

[In] integrate(x^m*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")

[Out] (B*b^5*m^6*x^19*x^m + 51*B*b^5*m^5*x^19*x^m + 1005*B*b^5*m^4*x^19*x^m + 5*B
*a*b^4*m^6*x^16*x^m + A*b^5*m^6*x^16*x^m + 9605*B*b^5*m^3*x^19*x^m + 270*B*
a*b^4*m^5*x^16*x^m + 54*A*b^5*m^5*x^16*x^m + 45474*B*b^5*m^2*x^19*x^m + 555
0*B*a*b^4*m^4*x^16*x^m + 1110*A*b^5*m^4*x^16*x^m + 95064*B*b^5*m*x^19*x^m +
 10*B*a^2*b^3*m^6*x^13*x^m + 5*A*a*b^4*m^6*x^13*x^m + 54700*B*a*b^4*m^3*x^1
6*x^m + 10940*A*b^5*m^3*x^16*x^m + 58240*B*b^5*x^19*x^m + 570*B*a^2*b^3*m^5
*x^13*x^m + 285*A*a*b^4*m^5*x^13*x^m + 264645*B*a*b^4*m^2*x^16*x^m + 52929*
A*b^5*m^2*x^16*x^m + 12330*B*a^2*b^3*m^4*x^13*x^m + 6165*A*a*b^4*m^4*x^13*x
^m + 561030*B*a*b^4*m*x^16*x^m + 112206*A*b^5*m*x^16*x^m + 10*B*a^3*b^2*m^6
*x^10*x^m + 10*A*a^2*b^3*m^6*x^10*x^m + 126710*B*a^2*b^3*m^3*x^13*x^m + 633
55*A*a*b^4*m^3*x^13*x^m + 345800*B*a*b^4*x^16*x^m + 69160*A*b^5*x^16*x^m +
600*B*a^3*b^2*m^5*x^10*x^m + 600*A*a^2*b^3*m^5*x^10*x^m + 632460*B*a^2*b^3*
m^2*x^13*x^m + 316230*A*a*b^4*m^2*x^13*x^m + 13740*B*a^3*b^2*m^4*x^10*x^m +
 13740*A*a^2*b^3*m^4*x^10*x^m + 1368720*B*a^2*b^3*m*x^13*x^m + 684360*A*a*b
^4*m*x^13*x^m + 5*B*a^4*b*m^6*x^7*x^m + 10*A*a^3*b^2*m^6*x^7*x^m + 149600*B
*a^3*b^2*m^3*x^10*x^m + 149600*A*a^2*b^3*m^3*x^10*x^m + 851200*B*a^2*b^3*x^
13*x^m + 425600*A*a*b^4*x^13*x^m + 315*B*a^4*b*m^5*x^7*x^m + 630*A*a^3*b^2*
m^5*x^7*x^m + 783690*B*a^3*b^2*m^2*x^10*x^m + 783690*A*a^2*b^3*m^2*x^10*x^m
 + 7665*B*a^4*b*m^4*x^7*x^m + 15330*A*a^3*b^2*m^4*x^7*x^m + 1753800*B*a^3*b
^2*m*x^10*x^m + 1753800*A*a^2*b^3*m*x^10*x^m + B*a^5*m^6*x^4*x^m + 5*A*a^4*
b*m^6*x^4*x^m + 89845*B*a^4*b*m^3*x^7*x^m + 179690*A*a^3*b^2*m^3*x^7*x^m +
1106560*B*a^3*b^2*x^10*x^m + 1106560*A*a^2*b^3*x^10*x^m + 66*B*a^5*m^5*x^4*
x^m + 330*A*a^4*b*m^5*x^4*x^m + 510930*B*a^4*b*m^2*x^7*x^m + 1021860*A*a^3*
b^2*m^2*x^7*x^m + 1710*B*a^5*m^4*x^4*x^m + 8550*A*a^4*b*m^4*x^4*x^m + 12188
40*B*a^4*b*m*x^7*x^m + 2437680*A*a^3*b^2*m*x^7*x^m + A*a^5*m^6*x*x^m + 2186
0*B*a^5*m^3*x^4*x^m + 109300*A*a^4*b*m^3*x^4*x^m + 790400*B*a^4*b*x^7*x^m +
 1580800*A*a^3*b^2*x^7*x^m + 69*A*a^5*m^5*x*x^m + 140529*B*a^5*m^2*x^4*x^m
+ 702645*A*a^4*b*m^2*x^4*x^m + 1905*A*a^5*m^4*x*x^m + 396954*B*a^5*m*x^4*x^
m + 1984770*A*a^4*b*m*x^4*x^m + 26795*A*a^5*m^3*x*x^m + 276640*B*a^5*x^4*x^

```
m + 1383200*A*a^4*b*x^4*x^m + 201174*A*a^5*m^2*x*x^m + 757896*A*a^5*m*x*x^m
 + 1106560*A*a^5*x*x^m)/(m^7 + 70*m^6 + 1974*m^5 + 28700*m^4 + 227969*m^3 +
 959070*m^2 + 1864456*m + 1106560)
```

Mupad [B] (verification not implemented)

Time = 7.63 (sec) , antiderivative size = 559, normalized size of antiderivative = 3.78

$$\int x^m(a+bx^3)^5(A+Bx^3)\,dx$$
$$= \frac{B\,b^5\,x^m\,x^{19}\,(m^6+51\,m^5+1005\,m^4+9605\,m^3+45474\,m^2+95064\,m+58240)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560}$$
$$+\frac{a^4\,x^m\,x^4\,(5\,A\,b+B\,a)\,(m^6+66\,m^5+1710\,m^4+21860\,m^3+140529\,m^2+396954\,m+276640)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560}$$
$$+\frac{b^4\,x^m\,x^{16}\,(A\,b+5\,B\,a)\,(m^6+54\,m^5+1110\,m^4+10940\,m^3+52929\,m^2+112206\,m+69160)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560}$$
$$+\frac{A\,a^5\,x\,x^m\,(m^6+69\,m^5+1905\,m^4+26795\,m^3+201174\,m^2+757896\,m+1106560)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560}$$
$$+\frac{10\,a^2\,b^2\,x^m\,x^{10}\,(A\,b+B\,a)\,(m^6+60\,m^5+1374\,m^4+14960\,m^3+78369\,m^2+175380\,m+110656)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560}$$
$$+\frac{5\,a\,b^3\,x^m\,x^{13}\,(A\,b+2\,B\,a)\,(m^6+57\,m^5+1233\,m^4+12671\,m^3+63246\,m^2+136872\,m+85120)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560}$$
$$+\frac{5\,a^3\,b\,x^m\,x^7\,(2\,A\,b+B\,a)\,(m^6+63\,m^5+1533\,m^4+17969\,m^3+102186\,m^2+243768\,m+158080)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560}$$

```
[In] int(x^m*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] (B*b^5*x^m*x^19*(95064*m + 45474*m^2 + 9605*m^3 + 1005*m^4 + 51*m^5 + m^6 +
 58240))/(1864456*m + 959070*m^2 + 227969*m^3 + 28700*m^4 + 1974*m^5 + 70*m
^6 + m^7 + 1106560) + (a^4*x^m*x^4*(5*A*b + B*a)*(396954*m + 140529*m^2 + 2
1860*m^3 + 1710*m^4 + 66*m^5 + m^6 + 276640))/(1864456*m + 959070*m^2 + 227
969*m^3 + 28700*m^4 + 1974*m^5 + 70*m^6 + m^7 + 1106560) + (b^4*x^m*x^16*(A
*b + 5*B*a)*(112206*m + 52929*m^2 + 10940*m^3 + 1110*m^4 + 54*m^5 + m^6 + 6
9160))/(1864456*m + 959070*m^2 + 227969*m^3 + 28700*m^4 + 1974*m^5 + 70*m^6
 + m^7 + 1106560) + (A*a^5*x*x^m*(757896*m + 201174*m^2 + 26795*m^3 + 1905*
m^4 + 69*m^5 + m^6 + 1106560))/(1864456*m + 959070*m^2 + 227969*m^3 + 28700
*m^4 + 1974*m^5 + 70*m^6 + m^7 + 1106560) + (10*a^2*b^2*x^m*x^10*(A*b + B*a
)*(175380*m + 78369*m^2 + 14960*m^3 + 1374*m^4 + 60*m^5 + m^6 + 110656))/(1
864456*m + 959070*m^2 + 227969*m^3 + 28700*m^4 + 1974*m^5 + 70*m^6 + m^7 +
1106560) + (5*a*b^3*x^m*x^13*(A*b + 2*B*a)*(136872*m + 63246*m^2 + 12671*m^
3 + 1233*m^4 + 57*m^5 + m^6 + 85120))/(1864456*m + 959070*m^2 + 227969*m^3
+ 28700*m^4 + 1974*m^5 + 70*m^6 + m^7 + 1106560) + (5*a^3*b*x^m*x^7*(2*A*b
+ B*a)*(243768*m + 102186*m^2 + 17969*m^3 + 1533*m^4 + 63*m^5 + m^6 + 15808
0))/(1864456*m + 959070*m^2 + 227969*m^3 + 28700*m^4 + 1974*m^5 + 70*m^6 +
m^7 + 1106560)
```

3.125 $\int x^m(a+bx^3)^2 (A+Bx^3) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 71

$$\int x^m (a+bx^3)^2 (A+Bx^3) \, dx = \frac{a^2Ax^{1+m}}{1+m} + \frac{a(2Ab+aB)x^{4+m}}{4+m} + \frac{b(Ab+2aB)x^{7+m}}{7+m} + \frac{b^2Bx^{10+m}}{10+m}$$

```
[Out] a^2*A*x^(1+m)/(1+m)+a*(2*A*b+B*a)*x^(4+m)/(4+m)+b*(A*b+2*B*a)*x^(7+m)/(7+m)
+b^2*B*x^(10+m)/(10+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int x^m (a+bx^3)^2 (A+Bx^3) \, dx = \frac{a^2Ax^{m+1}}{m+1} + \frac{ax^{m+4}(aB+2Ab)}{m+4} + \frac{bx^{m+7}(2aB+Ab)}{m+7} + \frac{b^2Bx^{m+10}}{m+10}$$

```
[In] Int[x^m*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (a^2*A*x^(1 + m))/(1 + m) + (a*(2*A*b + a*B)*x^(4 + m))/(4 + m) + (b*(A*b +
 2*a*B)*x^(7 + m))/(7 + m) + (b^2*B*x^(10 + m))/(10 + m)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(a^2 A x^m + a(2Ab + aB)x^{3+m} + b(Ab + 2aB)x^{6+m} + b^2 B x^{9+m}\right) dx \\ &= \frac{a^2 A x^{1+m}}{1+m} + \frac{a(2Ab + aB)x^{4+m}}{4+m} + \frac{b(Ab + 2aB)x^{7+m}}{7+m} + \frac{b^2 B x^{10+m}}{10+m} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int x^m \left(a+bx^3\right)^2 \left(A+Bx^3\right) dx = x^{1+m}\left(\frac{a^2 A}{1+m} + \frac{a(2Ab+aB)x^3}{4+m} + \frac{b(Ab+2aB)x^6}{7+m} + \frac{b^2 B x^9}{10+m}\right)$$

```
[In] Integrate[x^m*(a + b*x^3)^2*(A + B*x^3),x]
[Out] x^(1 + m)*((a^2*A)/(1 + m) + (a*(2*A*b + a*B)*x^3)/(4 + m) + (b*(A*b + 2*a*
B)*x^6)/(7 + m) + (b^2*B*x^9)/(10 + m))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(71) = 142.

Time = 4.07 (sec) , antiderivative size = 261, normalized size of antiderivative = 3.68

method	result
risch	$x\left(B\,b^2 m^3 x^9+12B\,b^2 m^2 x^9+39m\,x^9 B\,b^2+A\,b^2 m^3 x^6+2Bab\,m^3 x^6+28b^2 B\,x^9+15A\,b^2 m^2 x^6+30Bab\,m^2 x^6+54A\,b^2 x^6 m+108B\right.$
gosper	$x^{1+m}\left(B\,b^2 m^3 x^9+12B\,b^2 m^2 x^9+39m\,x^9 B\,b^2+A\,b^2 m^3 x^6+2Bab\,m^3 x^6+28b^2 B\,x^9+15A\,b^2 m^2 x^6+30Bab\,m^2 x^6+54A\,b^2 x^6 m+1\right.$
parallelrisch	$174A\,x^4 x^m abm+B\,x^{10} x^m b^2 m^3+12B\,x^{10} x^m b^2 m^2+2B\,x^7 x^m ab\,m^3+30B\,x^7 x^m ab\,m^2+108B\,x^7 x^m abm+2A\,x^4 x^m ab\,m^3+36A$

```
[In] int(x^m*(b*x^3+a)^2*(B*x^3+A),x,method=_RETURNVERBOSE)
[Out] x*(B*b^2*m^3*x^9+12*B*b^2*m^2*x^9+39*B*b^2*m*x^9+A*b^2*m^3*x^6+2*B*a*b*m^3*
x^6+28*B*b^2*x^9+15*A*b^2*m^2*x^6+30*B*a*b*m^2*x^6+54*A*b^2*m*x^6+108*B*a*b
*m*x^6+2*A*a*b*m^3*x^3+40*A*b^2*x^6+B*a^2*m^3*x^3+80*B*a*b*x^6+36*A*a*b*m^2
*x^3+18*B*a^2*m^2*x^3+174*A*a*b*m*x^3+87*B*a^2*m*x^3+A*a^2*m^3+140*A*a*b*x^
3+70*B*a^2*x^3+21*A*a^2*m^2+138*A*a^2*m+280*A*a^2)*x^m/(10+m)/(7+m)/(4+m)/(
1+m)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(71) = 142$.

Time = 0.26 (sec) , antiderivative size = 215, normalized size of antiderivative = 3.03

$$\int x^m (a+bx^3)^2 (A+Bx^3)\, dx = \frac{((Bb^2m^3 + 12\,Bb^2m^2 + 39\,Bb^2m + 28\,Bb^2)x^{10} + ((2\,Bab + Ab^2)m^3 + 80\,Bab + 40\,Ab^2 + 15\,(2\,Bab + Ab^2)}{}$$

```
[In] integrate(x^m*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] ((B*b^2*m^3 + 12*B*b^2*m^2 + 39*B*b^2*m + 28*B*b^2)*x^10 + ((2*B*a*b + A*b^
2)*m^3 + 80*B*a*b + 40*A*b^2 + 15*(2*B*a*b + A*b^2)*m^2 + 54*(2*B*a*b + A*b
^2)*m)*x^7 + ((B*a^2 + 2*A*a*b)*m^3 + 70*B*a^2 + 140*A*a*b + 18*(B*a^2 + 2*
A*a*b)*m^2 + 87*(B*a^2 + 2*A*a*b)*m)*x^4 + (A*a^2*m^3 + 21*A*a^2*m^2 + 138*
A*a^2*m + 280*A*a^2)*x)*x^m/(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1057 vs. $2(63) = 126$.

Time = 0.60 (sec) , antiderivative size = 1057, normalized size of antiderivative = 14.89

$$\int x^m (a+bx^3)^2 (A+Bx^3)\, dx = \begin{cases} -\frac{Aa^2}{9x^9} - \frac{Aab}{3x^6} - \frac{Ab^2}{3x^3} - \frac{Ba^2}{6x^6} - \frac{2Bab}{3x^3} + Bb^2\log{(x)} \\ -\frac{Aa^2}{6x^6} - \frac{2Aab}{3x^3} + Ab^2\log{(x)} - \frac{Ba^2}{3x^3} + 2Bab\log{(x)} + \frac{Bb^2x^3}{3} \\ -\frac{Aa^2}{3x^3} + 2Aab\log{(x)} + \frac{Ab^2x^3}{3} + Ba^2\log{(x)} + \frac{2Babx^3}{3} + \frac{Bb^2x^6}{6} \\ Aa^2\log{(x)} + \frac{2Aabx^3}{3} + \frac{Ab^2x^6}{6} + \frac{Ba^2x^3}{3} + \frac{Babx^6}{3} + \frac{Bb^2x^9}{9} \\ \frac{Aa^2m^3xx^m}{m^4+22m^3+159m^2+418m+280} + \frac{21Aa^2m^2xx^m}{m^4+22m^3+159m^2+418m+280} + \frac{138Aa^2mxx^m}{m^4+22m^3+159m^2+418m+280} + \frac{280Aa^2xx^m}{m^4+22m^3+159m^2+418m+280} \end{cases}$$

```
[In] integrate(x**m*(b*x**3+a)**2*(B*x**3+A),x)
```

```
[Out] Piecewise((-A*a**2/(9*x**9) - A*a*b/(3*x**6) - A*b**2/(3*x**3) - B*a**2/(6*
x**6) - 2*B*a*b/(3*x**3) + B*b**2*log(x), Eq(m, -10)), (-A*a**2/(6*x**6) -
2*A*a*b/(3*x**3) + A*b**2*log(x) - B*a**2/(3*x**3) + 2*B*a*b*log(x) + B*b**
2*x**3/3, Eq(m, -7)), (-A*a**2/(3*x**3) + 2*A*a*b*log(x) + A*b**2*x**3/3 +
B*a**2*log(x) + 2*B*a*b*x**3/3 + B*b**2*x**6/6, Eq(m, -4)), (A*a**2*log(x)
+ 2*A*a*b*x**3/3 + A*b**2*x**6/6 + B*a**2*x**3/3 + B*a*b*x**6/3 + B*b**2*x*
*9/9, Eq(m, -1)), (A*a**2*m**3*x*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m +
280) + 21*A*a**2*m**2*x*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 13
8*A*a**2*m*x*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 280*A*a**2*x*
```

```
x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 2*A*a*b*m**3*x**4*x**m/(m*
*4 + 22*m**3 + 159*m**2 + 418*m + 280) + 36*A*a*b*m**2*x**4*x**m/(m**4 + 22
*m**3 + 159*m**2 + 418*m + 280) + 174*A*a*b*m*x**4*x**m/(m**4 + 22*m**3 + 1
59*m**2 + 418*m + 280) + 140*A*a*b*x**4*x**m/(m**4 + 22*m**3 + 159*m**2 + 4
18*m + 280) + A*b**2*m**3*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 28
0) + 15*A*b**2*m**2*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 5
4*A*b**2*m*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 40*A*b**2*
x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + B*a**2*m**3*x**4*x**m
/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 18*B*a**2*m**2*x**4*x**m/(m**4
 + 22*m**3 + 159*m**2 + 418*m + 280) + 87*B*a**2*m*x**4*x**m/(m**4 + 22*m**
3 + 159*m**2 + 418*m + 280) + 70*B*a**2*x**4*x**m/(m**4 + 22*m**3 + 159*m**
2 + 418*m + 280) + 2*B*a*b*m**3*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*
m + 280) + 30*B*a*b*m**2*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280
) + 108*B*a*b*m*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 80*B*
a*b*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + B*b**2*m**3*x**10
*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 12*B*b**2*m**2*x**10*x**m
/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 39*B*b**2*m*x**10*x**m/(m**4 +
 22*m**3 + 159*m**2 + 418*m + 280) + 28*B*b**2*x**10*x**m/(m**4 + 22*m**3 +
 159*m**2 + 418*m + 280), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.28

$$\int x^m(a+bx^3)^2(A+Bx^3)\,dx = \frac{Bb^2x^{m+10}}{m+10}+\frac{2\,Babx^{m+7}}{m+7}+\frac{Ab^2x^{m+7}}{m+7}+\frac{Ba^2x^{m+4}}{m+4}+\frac{2\,Aabx^{m+4}}{m+4}+\frac{Aa^2x^{m+1}}{m+1}$$

```
[In] integrate(x^m*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] B*b^2*x^(m + 10)/(m + 10) + 2*B*a*b*x^(m + 7)/(m + 7) + A*b^2*x^(m + 7)/(m
+ 7) + B*a^2*x^(m + 4)/(m + 4) + 2*A*a*b*x^(m + 4)/(m + 4) + A*a^2*x^(m + 1
)/(m + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. 2(71) = 142.

Time = 0.28 (sec) , antiderivative size = 332, normalized size of antiderivative = 4.68

$$\int x^m \left(a+bx^3\right)^2 \left(A+Bx^3\right)\,dx$$
$$=\frac{Bb^2m^3x^{10}x^m+12\,Bb^2m^2x^{10}x^m+39\,Bb^2mx^{10}x^m+2\,Babm^3x^7x^m+Ab^2m^3x^7x^m+28\,Bb^2x^{10}x^m+30\,Bab\cdots}{\cdots}$$

```
[In] integrate(x^m*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] (B*b^2*m^3*x^10*x^m + 12*B*b^2*m^2*x^10*x^m + 39*B*b^2*m*x^10*x^m + 2*B*a*b
*m^3*x^7*x^m + A*b^2*m^3*x^7*x^m + 28*B*b^2*x^10*x^m + 30*B*a*b*m^2*x^7*x^m
 + 15*A*b^2*m^2*x^7*x^m + 108*B*a*b*m*x^7*x^m + 54*A*b^2*m*x^7*x^m + B*a^2*
m^3*x^4*x^m + 2*A*a*b*m^3*x^4*x^m + 80*B*a*b*x^7*x^m + 40*A*b^2*x^7*x^m + 1
8*B*a^2*m^2*x^4*x^m + 36*A*a*b*m^2*x^4*x^m + 87*B*a^2*m*x^4*x^m + 174*A*a*b
*m*x^4*x^m + A*a^2*m^3*x*x^m + 70*B*a^2*x^4*x^m + 140*A*a*b*x^4*x^m + 21*A*
a^2*m^2*x*x^m + 138*A*a^2*m*x*x^m + 280*A*a^2*x*x^m)/(m^4 + 22*m^3 + 159*m^
2 + 418*m + 280)
```

Mupad [B] (verification not implemented)

Time = 6.91 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.49

$$\int x^m \left(a+bx^3\right)^2 \left(A+Bx^3\right)\,dx = x^m \left(\frac{B\,b^2\,x^{10}\,(m^3+12\,m^2+39\,m+28)}{m^4+22\,m^3+159\,m^2+418\,m+280} + \frac{A\,a^2\,x\,(m^3+21\,m^2+138\,m+280)}{m^4+22\,m^3+159\,m^2+418\,m+280} + \frac{a\,x^4\,(2\,A\,b+B\,a)\,(m^3+18\,m^2+87\,m+70)}{m^4+22\,m^3+159\,m^2+418\,m+280} + \frac{b\,x^7\,(A\,b+2\,B\,a)\,(m^3+15\,m^2+54\,m+40)}{m^4+22\,m^3+159\,m^2+418\,m+280}\right)$$

```
[In] int(x^m*(A + B*x^3)*(a + b*x^3)^2,x)
```

```
[Out] x^m*((B*b^2*x^10*(39*m + 12*m^2 + m^3 + 28))/(418*m + 159*m^2 + 22*m^3 + m^
4 + 280) + (A*a^2*x*(138*m + 21*m^2 + m^3 + 280))/(418*m + 159*m^2 + 22*m^3
 + m^4 + 280) + (a*x^4*(2*A*b + B*a)*(87*m + 18*m^2 + m^3 + 70))/(418*m + 1
59*m^2 + 22*m^3 + m^4 + 280) + (b*x^7*(A*b + 2*B*a)*(54*m + 15*m^2 + m^3 +
40))/(418*m + 159*m^2 + 22*m^3 + m^4 + 280))
```

3.126 $\int x^m(a+bx^3)(A+Bx^3)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 45

$$\int x^m(a+bx^3)(A+Bx^3)\,dx=\frac{aAx^{1+m}}{1+m}+\frac{(Ab+aB)x^{4+m}}{4+m}+\frac{bBx^{7+m}}{7+m}$$

```
[Out] a*A*x^(1+m)/(1+m)+(A*b+B*a)*x^(4+m)/(4+m)+b*B*x^(7+m)/(7+m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {459}

$$\int x^m(a+bx^3)(A+Bx^3)\,dx=\frac{x^{m+4}(aB+Ab)}{m+4}+\frac{aAx^{m+1}}{m+1}+\frac{bBx^{m+7}}{m+7}$$

```
[In] Int[x^m*(a + b*x^3)*(A + B*x^3),x]
[Out] (a*A*x^(1 + m))/(1 + m) + ((A*b + a*B)*x^(4 + m))/(4 + m) + (b*B*x^(7 + m))
/(7 + m)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int\left(aAx^m+(Ab+aB)x^{3+m}+bBx^{6+m}\right)dx\\&=\frac{aAx^{1+m}}{1+m}+\frac{(Ab+aB)x^{4+m}}{4+m}+\frac{bBx^{7+m}}{7+m}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int x^m\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=x^{1+m}\left(\frac{aA}{1+m}+\frac{(Ab+aB)x^3}{4+m}+\frac{bBx^6}{7+m}\right)$$

```
[In] Integrate[x^m*(a + b*x^3)*(A + B*x^3),x]
[Out] x^(1 + m)*((a*A)/(1 + m) + ((A*b + a*B)*x^3)/(4 + m) + (b*B*x^6)/(7 + m))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

method	result
norman	$\frac{(Ab+Ba)x^4\mathrm{e}^{m\ln(x)}}{4+m}+\frac{Aax\,\mathrm{e}^{m\ln(x)}}{1+m}+\frac{Bb\,x^7\mathrm{e}^{m\ln(x)}}{7+m}$
risch	$\frac{x\left(Bb\,m^2x^6+5Bbm\,x^6+4bB\,x^6+Ab\,m^2x^3+Ba\,m^2x^3+8Abm\,x^3+8Bam\,x^3+7Ab\,x^3+7Ba\,x^3+Aa\,m^2+11Aam+28Aa\right)x^m}{(7+m)(4+m)(1+m)}$
gosper	$\frac{x^{1+m}\left(Bb\,m^2x^6+5Bbm\,x^6+4bB\,x^6+Ab\,m^2x^3+Ba\,m^2x^3+8Abm\,x^3+8Bam\,x^3+7Ab\,x^3+7Ba\,x^3+Aa\,m^2+11Aam+28Aa\right)}{(1+m)(4+m)(7+m)}$
parallelrisch	$\frac{B\,x^7x^mb\,m^2+5B\,x^7x^mbm+4B\,x^7x^mb+A\,x^4x^mb\,m^2+B\,x^4x^ma\,m^2+8A\,x^4x^mbm+8B\,x^4x^mam+7A\,x^4x^mb+7B\,x^4x^ma+Ax\,x}{(7+m)(4+m)(1+m)}$

```
[In] int(x^m*(b*x^3+a)*(B*x^3+A),x,method=_RETURNVERBOSE)
[Out] (A*b+B*a)/(4+m)*x^4*exp(m*ln(x))+A*a/(1+m)*x*exp(m*ln(x))+B*b/(7+m)*x^7*exp
(m*ln(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(45) = 90$.

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.04

$$\int x^m\left(a+bx^3\right)\left(A+Bx^3\right)\,dx$$
$$=\frac{((Bbm^2+5\,Bbm+4\,Bb)x^7+((Ba+Ab)m^2+7\,Ba+7\,Ab+8\,(Ba+Ab)m)x^4+(Aam^2+11\,Aam+28}{m^3+12\,m^2+39\,m+28}$$

```
[In] integrate(x^m*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
[Out] ((B*b*m^2 + 5*B*b*m + 4*B*b)*x^7 + ((B*a + A*b)*m^2 + 7*B*a + 7*A*b + 8*(B*
a + A*b)*m)*x^4 + (A*a*m^2 + 11*A*a*m + 28*A*a)*x)*x^m/(m^3 + 12*m^2 + 39*m
 + 28)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(37) = 74$.

Time = 0.40 (sec) , antiderivative size = 410, normalized size of antiderivative = 9.11

$$\int x^m \left(a+bx^3\right)\left(A+Bx^3\right)\,dx = \begin{cases} -\frac{Aa}{6x^6} - \frac{Ab}{3x^3} - \frac{Ba}{3x^3} + Bb\log(x) \\ -\frac{Aa}{3x^3} + Ab\log(x) + Ba\log(x) + \frac{Bbx^3}{3} \\ Aa\log(x) + \frac{Abx^3}{3} + \frac{Bax^3}{3} + \frac{Bbx^6}{6} \\ \frac{Aam^2xx^m}{m^3+12m^2+39m+28} + \frac{11Aamxx^m}{m^3+12m^2+39m+28} + \frac{28Aaxx^m}{m^3+12m^2+39m+28} + \frac{Abm^2x^4x^m}{m^3+12m^2+39m+28} + \frac{8Abmx^4x^m}{m^3+12m^2+39m+28} + \frac{7Abx^4x}{m^3+12m^2+3} \end{cases}$$

```
[In] integrate(x**m*(b*x**3+a)*(B*x**3+A),x)
```

```
[Out] Piecewise((-A*a/(6*x**6) - A*b/(3*x**3) - B*a/(3*x**3) + B*b*log(x), Eq(m,
-7)), (-A*a/(3*x**3) + A*b*log(x) + B*a*log(x) + B*b*x**3/3, Eq(m, -4)), (A
*a*log(x) + A*b*x**3/3 + B*a*x**3/3 + B*b*x**6/6, Eq(m, -1)), (A*a*m**2*x*x
**m/(m**3 + 12*m**2 + 39*m + 28) + 11*A*a*m*x*x**m/(m**3 + 12*m**2 + 39*m +
 28) + 28*A*a*x*x**m/(m**3 + 12*m**2 + 39*m + 28) + A*b*m**2*x**4*x**m/(m**
3 + 12*m**2 + 39*m + 28) + 8*A*b*m*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) +
 7*A*b*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + B*a*m**2*x**4*x**m/(m**3 +
12*m**2 + 39*m + 28) + 8*B*a*m*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + 7*B
*a*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + B*b*m**2*x**7*x**m/(m**3 + 12*m
**2 + 39*m + 28) + 5*B*b*m*x**7*x**m/(m**3 + 12*m**2 + 39*m + 28) + 4*B*b*x
**7*x**m/(m**3 + 12*m**2 + 39*m + 28), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int x^m \left(a+bx^3\right)\left(A+Bx^3\right)\,dx = \frac{Bbx^{m+7}}{m+7} + \frac{Bax^{m+4}}{m+4} + \frac{Abx^{m+4}}{m+4} + \frac{Aax^{m+1}}{m+1}$$

```
[In] integrate(x^m*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] B*b*x^(m + 7)/(m + 7) + B*a*x^(m + 4)/(m + 4) + A*b*x^(m + 4)/(m + 4) + A*a
*x^(m + 1)/(m + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(45) = 90.

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.18

$$\int x^m(a+bx^3)(A+Bx^3)\,dx = \frac{Bbm^2x^7x^m + 5\,Bbmx^7x^m + 4\,Bbx^7x^m + Bam^2x^4x^m + Abm^2x^4x^m + 8\,Bamx^4x^m + 8\,Abmx^4x^m + 7\,Bax^4}{m^3+12\,m^2+39\,m+28}$$

```
[In] integrate(x^m*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] (B*b*m^2*x^7*x^m + 5*B*b*m*x^7*x^m + 4*B*b*x^7*x^m + B*a*m^2*x^4*x^m + A*b*
m^2*x^4*x^m + 8*B*a*m*x^4*x^m + 8*A*b*m*x^4*x^m + 7*B*a*x^4*x^m + 7*A*b*x^4
*x^m + A*a*m^2*x*x^m + 11*A*a*m*x*x^m + 28*A*a*x*x^m)/(m^3 + 12*m^2 + 39*m
+ 28)
```

Mupad [B] (verification not implemented)

Time = 6.84 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.11

$$\int x^m(a+bx^3)(A+Bx^3)\,dx = x^m\left(\frac{x^4\,(A\,b+B\,a)\,(m^2+8\,m+7)}{m^3+12\,m^2+39\,m+28} + \frac{B\,b\,x^7\,(m^2+5\,m+4)}{m^3+12\,m^2+39\,m+28} + \frac{A\,a\,x\,(m^2+11\,m+28)}{m^3+12\,m^2+39\,m+28}\right)$$

```
[In] int(x^m*(A + B*x^3)*(a + b*x^3),x)
```

```
[Out] x^m*((x^4*(A*b + B*a)*(8*m + m^2 + 7))/(39*m + 12*m^2 + m^3 + 28) + (B*b*x^
7*(5*m + m^2 + 4))/(39*m + 12*m^2 + m^3 + 28) + (A*a*x*(11*m + m^2 + 28))/(
39*m + 12*m^2 + m^3 + 28))
```

3.127 $\int \frac{x^m(A+Bx^3)}{a+bx^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 66

$$\int \frac{x^m(A+Bx^3)}{a+bx^3}\,dx = \frac{Bx^{1+m}}{b(1+m)} + \frac{(Ab-aB)x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{ab(1+m)}$$

```
[Out] B*x^(1+m)/b/(1+m)+(A*b-B*a)*x^(1+m)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-b
*x^3/a)/a/b/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {470, 371}

$$\int \frac{x^m(A+Bx^3)}{a+bx^3}\,dx = \frac{x^{m+1}(Ab-aB)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{ab(m+1)} + \frac{Bx^{m+1}}{b(m+1)}$$

```
[In] Int[(x^m*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (B*x^(1 + m))/(b*(1 + m)) + ((A*b - a*B)*x^(1 + m)*Hypergeometric2F1[1, (1
+ m)/3, (4 + m)/3, -((b*x^3)/a)])/(a*b*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{Bx^{1+m}}{b(1+m)} - \frac{(-Ab(1+m)+aB(1+m))\int \frac{x^m}{a+bx^3}\,dx}{b(1+m)}\\ &= \frac{Bx^{1+m}}{b(1+m)} + \frac{(Ab-aB)x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{ab(1+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

$$\int \frac{x^m(A+Bx^3)}{a+bx^3}\,dx = \frac{x^{1+m}\left(aB+(Ab-aB)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)\right)}{ab(1+m)}$$

```
[In] Integrate[(x^m*(A + B*x^3))/(a + b*x^3),x]
[Out] (x^(1 + m)*(a*B + (A*b - a*B)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -(
(b*x^3)/a)]))/(a*b*(1 + m))
```

Maple [F]

$$\int \frac{x^m(x^3B+A)}{b\,x^3+a}dx$$

```
[In] int(x^m*(B*x^3+A)/(b*x^3+a),x)
[Out] int(x^m*(B*x^3+A)/(b*x^3+a),x)
```

Fricas [F]

$$\int \frac{x^m(A+Bx^3)}{a+bx^3}\,dx=\int \frac{(Bx^3+A)x^m}{bx^3+a}\,dx$$

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
[Out] integral((B*x^3 + A)*x^m/(b*x^3 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.59 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.83

$$\begin{aligned}\int \frac{x^m(A+Bx^3)}{a+bx^3}\,dx=&\frac{Amx^{m+1}\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{1}{3}\right)\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{9a\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}\\&+\frac{Ax^{m+1}\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{1}{3}\right)\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{9a\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}\\&+\frac{Bmx^{m+4}\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{4}{3}\right)\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}{9a\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}\\&+\frac{4Bx^{m+4}\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{4}{3}\right)\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}{9a\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}\end{aligned}$$

```
[In] integrate(x**m*(B*x**3+A)/(b*x**3+a),x)
[Out] A*m*x**(m + 1)*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/3 +
 1/3)/(9*a*gamma(m/3 + 4/3)) + A*x**(m + 1)*lerchphi(b*x**3*exp_polar(I*pi)
/a, 1, m/3 + 1/3)*gamma(m/3 + 1/3)/(9*a*gamma(m/3 + 4/3)) + B*m*x**(m + 4)*
lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 4/3)*gamma(m/3 + 4/3)/(9*a*gamm
a(m/3 + 7/3)) + 4*B*x**(m + 4)*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 +
4/3)*gamma(m/3 + 4/3)/(9*a*gamma(m/3 + 7/3))
```

Maxima [F]

$$\int \frac{x^m(A+Bx^3)}{a+bx^3}\,dx = \int \frac{(Bx^3+A)x^m}{bx^3+a}\,dx$$

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*x^m/(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^m(A+Bx^3)}{a+bx^3}\,dx = \int \frac{(Bx^3+A)x^m}{bx^3+a}\,dx$$

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*x^m/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(A+Bx^3)}{a+bx^3}\,dx = \int \frac{x^m\,(B\,x^3+A)}{b\,x^3+a}\,dx$$

```
[In] int((x^m*(A + B*x^3))/(a + b*x^3),x)
[Out] int((x^m*(A + B*x^3))/(a + b*x^3), x)
```

3.128 $\int \frac{x^m(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 93

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(Ab-aB)x^{1+m}}{3ab\,(a+bx^3)} + \frac{(Ab(2-m)+aB(1+m))x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{3a^2b(1+m)}$$

```
[Out] 1/3*(A*b-B*a)*x^(1+m)/a/b/(b*x^3+a)+1/3*(A*b*(-m+2)+a*B*(1+m))*x^(1+m)*hype
rgeom([1, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a^2/b/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {468, 371}

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{x^{m+1}(aB(m+1)+Ab(2-m))\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{3a^2b(m+1)} + \frac{x^{m+1}(Ab-aB)}{3ab\,(a+bx^3)}$$

```
[In] Int[(x^m*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((A*b - a*B)*x^(1 + m))/(3*a*b*(a + b*x^3)) + ((A*b*(2 - m) + a*B*(1 + m))*
x^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(3*a^2*
b*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\text{integral} = \frac{(Ab-aB)x^{1+m}}{3ab\,(a+bx^3)} + \frac{(-Ab(-2+m)+aB(1+m))\int \frac{x^m}{a+bx^3}\,dx}{3ab}$$
$$= \frac{(Ab-aB)x^{1+m}}{3ab\,(a+bx^3)} + \frac{(Ab(2-m)+aB(1+m))x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{3a^2b(1+m)}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{x^{1+m}\left(aB\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)+(Ab-aB)\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)\right)}{a^2b(1+m)}$$

```
[In] Integrate[(x^m*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (x^(1 + m)*(a*B*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)] +
(A*b - a*B)*Hypergeometric2F1[2, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)]))/(a^2
*b*(1 + m))
```

Maple [F]

$$\int \frac{x^m (x^3 B + A)}{(b\,x^3 + a)^2} dx$$

```
[In] int(x^m*(B*x^3+A)/(b*x^3+a)^2,x)
[Out] int(x^m*(B*x^3+A)/(b*x^3+a)^2,x)
```

Fricas [F]

$$\int \frac{x^m (A + Bx^3)}{(a + bx^3)^2}\, dx = \int \frac{(Bx^3 + A)x^m}{(bx^3 + a)^2}\, dx$$

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
[Out] integral((B*x^3 + A)*x^m/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 140.65 (sec) , antiderivative size = 1049, normalized size of antiderivative = 11.28

$$\int \frac{x^m (A + Bx^3)}{(a + bx^3)^2}\, dx = \text{Too large to display}$$

```
[In] integrate(x**m*(B*x**3+A)/(b*x**3+a)**2,x)
[Out] A*(-a*m**2*x**(m + 1)*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamm
a(m/3 + 1/3)/(27*a**3*gamma(m/3 + 4/3) + 27*a**2*b*x**3*gamma(m/3 + 4/3)) +
 a*m*x**(m + 1)*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/3
+ 1/3)/(27*a**3*gamma(m/3 + 4/3) + 27*a**2*b*x**3*gamma(m/3 + 4/3)) + 3*a*m
*x**(m + 1)*gamma(m/3 + 1/3)/(27*a**3*gamma(m/3 + 4/3) + 27*a**2*b*x**3*gam
ma(m/3 + 4/3)) + 2*a*x**(m + 1)*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 +
 1/3)*gamma(m/3 + 1/3)/(27*a**3*gamma(m/3 + 4/3) + 27*a**2*b*x**3*gamma(m/3
 + 4/3)) + 3*a*x**(m + 1)*gamma(m/3 + 1/3)/(27*a**3*gamma(m/3 + 4/3) + 27*a
**2*b*x**3*gamma(m/3 + 4/3)) - b*m**2*x**3*x**(m + 1)*lerchphi(b*x**3*exp_p
olar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/3 + 1/3)/(27*a**3*gamma(m/3 + 4/3) + 27
*a**2*b*x**3*gamma(m/3 + 4/3)) + b*m*x**3*x**(m + 1)*lerchphi(b*x**3*exp_po
lar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/3 + 1/3)/(27*a**3*gamma(m/3 + 4/3) + 27*
a**2*b*x**3*gamma(m/3 + 4/3)) + 2*b*x**3*x**(m + 1)*lerchphi(b*x**3*exp_pol
ar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/3 + 1/3)/(27*a**3*gamma(m/3 + 4/3) + 27*a
**2*b*x**3*gamma(m/3 + 4/3))) + B*(-a*m**2*x**(m + 4)*lerchphi(b*x**3*exp_p
olar(I*pi)/a, 1, m/3 + 4/3)*gamma(m/3 + 4/3)/(27*a**3*gamma(m/3 + 7/3) + 27
```

```
*a**2*b*x**3*gamma(m/3 + 7/3)) - 5*a*m*x**(m + 4)*lerchphi(b*x**3*exp_polar
(I*pi)/a, 1, m/3 + 4/3)*gamma(m/3 + 4/3)/(27*a**3*gamma(m/3 + 7/3) + 27*a**
2*b*x**3*gamma(m/3 + 7/3)) + 3*a*m*x**(m + 4)*gamma(m/3 + 4/3)/(27*a**3*gam
ma(m/3 + 7/3) + 27*a**2*b*x**3*gamma(m/3 + 7/3)) - 4*a*x**(m + 4)*lerchphi(
b*x**3*exp_polar(I*pi)/a, 1, m/3 + 4/3)*gamma(m/3 + 4/3)/(27*a**3*gamma(m/3
 + 7/3) + 27*a**2*b*x**3*gamma(m/3 + 7/3)) + 12*a*x**(m + 4)*gamma(m/3 + 4/
3)/(27*a**3*gamma(m/3 + 7/3) + 27*a**2*b*x**3*gamma(m/3 + 7/3)) - b*m**2*x*
*3*x**(m + 4)*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 4/3)*gamma(m/3 +
4/3)/(27*a**3*gamma(m/3 + 7/3) + 27*a**2*b*x**3*gamma(m/3 + 7/3)) - 5*b*m*x
**3*x**(m + 4)*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 4/3)*gamma(m/3 +
 4/3)/(27*a**3*gamma(m/3 + 7/3) + 27*a**2*b*x**3*gamma(m/3 + 7/3)) - 4*b*x*
*3*x**(m + 4)*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 4/3)*gamma(m/3 +
4/3)/(27*a**3*gamma(m/3 + 7/3) + 27*a**2*b*x**3*gamma(m/3 + 7/3)))
```

Maxima [F]

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^2}\,dx = \int \frac{(Bx^3+A)x^m}{(bx^3+a)^2}\,dx$$

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*x^m/(b*x^3 + a)^2, x)
```

Giac [F]

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^2}\,dx = \int \frac{(Bx^3+A)x^m}{(bx^3+a)^2}\,dx$$

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
[Out] integrate((B*x^3 + A)*x^m/(b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^2}\,dx = \int \frac{x^m\,(B\,x^3+A)}{(b\,x^3+a)^2}\,dx$$

```
[In] int((x^m*(A + B*x^3))/(a + b*x^3)^2,x)
[Out] int((x^m*(A + B*x^3))/(a + b*x^3)^2, x)
```

3.129 $\int \frac{x^m(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 93

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(Ab-aB)x^{1+m}}{6ab\,(a+bx^3)^2} + \frac{(Ab(5-m)+aB(1+m))x^{1+m}\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{6a^3b(1+m)}$$

```
[Out] 1/6*(A*b-B*a)*x^(1+m)/a/b/(b*x^3+a)^2+1/6*(A*b*(5-m)+a*B*(1+m))*x^(1+m)*hyp
ergeom([2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a^3/b/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {468, 371}

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{x^{m+1}(aB(m+1)+Ab(5-m))\,\text{Hypergeometric2F1}\left(2,\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{6a^3b(m+1)} + \frac{x^{m+1}(Ab-aB)}{6ab\,(a+bx^3)^2}$$

```
[In] Int[(x^m*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^(1 + m))/(6*a*b*(a + b*x^3)^2) + ((A*b*(5 - m) + a*B*(1 + m)
)*x^(1 + m)*Hypergeometric2F1[2, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(6*a^
3*b*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(Ab-aB)x^{1+m}}{6ab\left(a+bx^3\right)^2}+\frac{\left(-Ab(-5+m)+aB(1+m)\right)\int\frac{x^m}{\left(a+bx^3\right)^2}\,dx}{6ab}\\ &= \frac{(Ab-aB)x^{1+m}}{6ab\left(a+bx^3\right)^2}+\frac{\left(Ab(5-m)+aB(1+m)\right)x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{6a^3b(1+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86

$$\int\frac{x^m(A+Bx^3)}{\left(a+bx^3\right)^3}\,dx = \frac{x^{1+m}\left(aB\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)+(Ab-aB)\,\text{Hypergeometric2F1}\left(3,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)\right)}{a^3b(1+m)}$$

```
[In] Integrate[(x^m*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (x^(1 + m)*(a*B*Hypergeometric2F1[2, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)] +
(A*b - a*B)*Hypergeometric2F1[3, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)]))/(a^3
*b*(1 + m))
```

Maple [F]

$$\int \frac{x^m(x^3B+A)}{(b\,x^3+a)^3}dx$$

[In] int(x^m*(B*x^3+A)/(b*x^3+a)^3,x)

[Out] int(x^m*(B*x^3+A)/(b*x^3+a)^3,x)

Fricas [F]

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^3}\,dx = \int \frac{(Bx^3+A)x^m}{(bx^3+a)^3}\,dx$$

[In] integrate(x^m*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")

[Out] integral((B*x^3 + A)*x^m/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2*b*x^3 + a^3), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

[In] integrate(x**m*(B*x**3+A)/(b*x**3+a)**3,x)

[Out] Timed out

Maxima [F]

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^3}\,dx = \int \frac{(Bx^3+A)x^m}{(bx^3+a)^3}\,dx$$

[In] integrate(x^m*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")

[Out] integrate((B*x^3 + A)*x^m/(b*x^3 + a)^3, x)

Giac [F]

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^3}\,dx = \int \frac{(Bx^3+A)x^m}{(bx^3+a)^3}\,dx$$

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
[Out] integrate((B*x^3 + A)*x^m/(b*x^3 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(A+Bx^3)}{(a+bx^3)^3}\,dx = \int \frac{x^m\,(B\,x^3+A)}{(b\,x^3+a)^3}\,dx$$

```
[In] int((x^m*(A + B*x^3))/(a + b*x^3)^3,x)
[Out] int((x^m*(A + B*x^3))/(a + b*x^3)^3, x)
```

3.130 $\int \frac{(ex)^m}{(a+bx^3)(c+dx^3)} dx$

Optimal result

Integrand size = 24, antiderivative size = 112

$$\int \frac{(ex)^m}{(a+bx^3)(c+dx^3)} dx = \frac{b(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{a(bc-ad)e(1+m)} - \frac{d(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{dx^3}{c}\right)}{c(bc-ad)e(1+m)}$$

```
[Out] b*(e*x)^(1+m)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a/(-a*d+b*c)/e
/(1+m)-d*(e*x)^(1+m)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-d*x^3/c)/c/(-a*d
+b*c)/e/(1+m)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {493, 371}

$$\int \frac{(ex)^m}{(a+bx^3)(c+dx^3)} dx = \frac{b(ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{ae(m+1)(bc-ad)} - \frac{d(ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{3},\frac{m+4}{3},-\frac{dx^3}{c}\right)}{ce(m+1)(bc-ad)}$$

```
[In] Int[(e*x)^m/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] (b*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/
(a*(b*c - a*d)*e*(1 + m)) - (d*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3
, (4 + m)/3, -((d*x^3)/c)])/(c*(b*c - a*d)*e*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\int\frac{(ex)^m}{a+bx^3}\,dx}{bc-ad}-\frac{d\int\frac{(ex)^m}{c+dx^3}\,dx}{bc-ad}\\ &= \frac{b(ex)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{a(bc-ad)e(1+m)}-\frac{d(ex)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{dx^3}{c}\right)}{c(bc-ad)e(1+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

$$\int\frac{(ex)^m}{(a+bx^3)(c+dx^3)}\,dx = \frac{x(ex)^m\left(-bc\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)+ad\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{3},\frac{4+m}{3},-\frac{dx^3}{c}\right)\right)}{ac(-bc+ad)(1+m)}$$

```
[In] Integrate[(e*x)^m/((a + b*x^3)*(c + d*x^3)),x]
[Out] (x*(e*x)^m*(-(b*c*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])
 + a*d*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((d*x^3)/c)]))/(a*c*(-(b
*c) + a*d)*(1 + m))
```

Maple [F]

$$\int \frac{(ex)^m}{(b\,x^3+a)\,(d\,x^3+c)}dx$$

```
[In] int((e*x)^m/(b*x^3+a)/(d*x^3+c),x)
[Out] int((e*x)^m/(b*x^3+a)/(d*x^3+c),x)
```

Fricas [F]

$$\int \frac{(ex)^m}{(a+bx^3)\,(c+dx^3)}\,dx = \int \frac{(ex)^m}{(bx^3+a)(dx^3+c)}\,dx$$

```
[In] integrate((e*x)^m/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
[Out] integral((e*x)^m/(b*d*x^6 + (b*c + a*d)*x^3 + a*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^3)\,(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate((e*x)**m/(b*x**3+a)/(d*x**3+c),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ex)^m}{(a+bx^3)\,(c+dx^3)}\,dx = \int \frac{(ex)^m}{(bx^3+a)(dx^3+c)}\,dx$$

```
[In] integrate((e*x)^m/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((e*x)^m/((b*x^3 + a)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{(ex)^m}{(a+bx^3)(c+dx^3)}\,dx = \int \frac{(ex)^m}{(bx^3+a)(dx^3+c)}\,dx$$

```
[In] integrate((e*x)^m/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((e*x)^m/((b*x^3 + a)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^3)(c+dx^3)}\,dx = \int \frac{(e\,x)^m}{(b\,x^3+a)\,(d\,x^3+c)}\,dx$$

```
[In] int((e*x)^m/((a + b*x^3)*(c + d*x^3)),x)
[Out] int((e*x)^m/((a + b*x^3)*(c + d*x^3)), x)
```

3.131 $\int x^{7/2}(a+bx^3)\left(A+Bx^3\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 39

$$\int x^{7/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2}{9}aAx^{9/2}+\frac{2}{15}(Ab+aB)x^{15/2}+\frac{2}{21}bBx^{21/2}$$

```
[Out] 2/9*a*A*x^(9/2)+2/15*(A*b+B*a)*x^(15/2)+2/21*b*B*x^(21/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int x^{7/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2}{15}x^{15/2}(aB+Ab)+\frac{2}{9}aAx^{9/2}+\frac{2}{21}bBx^{21/2}$$

```
[In] Int[x^(7/2)*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (2*a*A*x^(9/2))/9 + (2*(A*b + a*B)*x^(15/2))/15 + (2*b*B*x^(21/2))/21
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int\left(aAx^{7/2}+(Ab+aB)x^{13/2}+bBx^{19/2}\right)\,dx\\&=\frac{2}{9}aAx^{9/2}+\frac{2}{15}(Ab+aB)x^{15/2}+\frac{2}{21}bBx^{21/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int x^{7/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2}{315}x^{9/2}\left(35aA+21Abx^3+21aBx^3+15bBx^6\right)$$

```
[In] Integrate[x^(7/2)*(a + b*x^3)*(A + B*x^3),x]
[Out] (2*x^(9/2)*(35*a*A + 21*A*b*x^3 + 21*a*B*x^3 + 15*b*B*x^6))/315
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2aA\,x^{\frac{9}{2}}}{9}+\frac{2(Ab+Ba)x^{\frac{15}{2}}}{15}+\frac{2bB\,x^{\frac{21}{2}}}{21}$	28
default	$\frac{2aA\,x^{\frac{9}{2}}}{9}+\frac{2(Ab+Ba)x^{\frac{15}{2}}}{15}+\frac{2bB\,x^{\frac{21}{2}}}{21}$	28
gosper	$\frac{2x^{\frac{9}{2}}\left(15bB\,x^{6}+21Ab\,x^{3}+21Ba\,x^{3}+35Aa\right)}{315}$	32
trager	$\frac{2x^{\frac{9}{2}}\left(15bB\,x^{6}+21Ab\,x^{3}+21Ba\,x^{3}+35Aa\right)}{315}$	32
risch	$\frac{2x^{\frac{9}{2}}\left(15bB\,x^{6}+21Ab\,x^{3}+21Ba\,x^{3}+35Aa\right)}{315}$	32

```
[In] int(x^(7/2)*(b*x^3+a)*(B*x^3+A),x,method=_RETURNVERBOSE)
[Out] 2/9*a*A*x^(9/2)+2/15*(A*b+B*a)*x^(15/2)+2/21*b*B*x^(21/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int x^{7/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2}{315}\left(15\,Bbx^{10}+21\,(Ba+Ab)x^7+35\,Aax^4\right)\sqrt{x}$$

```
[In] integrate(x^(7/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
[Out] 2/315*(15*B*b*x^10 + 21*(B*a + A*b)*x^7 + 35*A*a*x^4)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int x^{7/2}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2Aax^{\frac{9}{2}}}{9}+\frac{2Abx^{\frac{15}{2}}}{15}+\frac{2Bax^{\frac{15}{2}}}{15}+\frac{2Bbx^{\frac{21}{2}}}{21}$$

```
[In] integrate(x**(7/2)*(b*x**3+a)*(B*x**3+A),x)
[Out] 2*A*a*x**(9/2)/9 + 2*A*b*x**(15/2)/15 + 2*B*a*x**(15/2)/15 + 2*B*b*x**(21/2
)/21
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int x^{7/2}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2}{21}\,Bbx^{\frac{21}{2}}+\frac{2}{15}\,(Ba+Ab)x^{\frac{15}{2}}+\frac{2}{9}\,Aax^{\frac{9}{2}}$$

```
[In] integrate(x^(7/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
[Out] 2/21*B*b*x^(21/2) + 2/15*(B*a + A*b)*x^(15/2) + 2/9*A*a*x^(9/2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int x^{7/2}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2}{21}\,Bbx^{\frac{21}{2}}+\frac{2}{15}\,Bax^{\frac{15}{2}}+\frac{2}{15}\,Abx^{\frac{15}{2}}+\frac{2}{9}\,Aax^{\frac{9}{2}}$$

```
[In] integrate(x^(7/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
[Out] 2/21*B*b*x^(21/2) + 2/15*B*a*x^(15/2) + 2/15*A*b*x^(15/2) + 2/9*A*a*x^(9/2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int x^{7/2}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2\,x^{9/2}\,\left(35\,A\,a+21\,A\,b\,x^3+21\,B\,a\,x^3+15\,B\,b\,x^6\right)}{315}$$

```
[In] int(x^(7/2)*(A + B*x^3)*(a + b*x^3),x)
[Out] (2*x^(9/2)*(35*A*a + 21*A*b*x^3 + 21*B*a*x^3 + 15*B*b*x^6))/315
```

3.132 $\int x^{5/2}(a+bx^3)\left(A+Bx^3\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 39

$$\int x^{5/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2}{7}aAx^{7/2}+\frac{2}{13}(Ab+aB)x^{13/2}+\frac{2}{19}bBx^{19/2}$$

```
[Out] 2/7*a*A*x^(7/2)+2/13*(A*b+B*a)*x^(13/2)+2/19*b*B*x^(19/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int x^{5/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2}{13}x^{13/2}(aB+Ab)+\frac{2}{7}aAx^{7/2}+\frac{2}{19}bBx^{19/2}$$

```
[In] Int[x^(5/2)*(a + b*x^3)*(A + B*x^3),x]
[Out] (2*a*A*x^(7/2))/7 + (2*(A*b + a*B)*x^(13/2))/13 + (2*b*B*x^(19/2))/19
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int\left(aAx^{5/2}+(Ab+aB)x^{11/2}+bBx^{17/2}\right)\,dx\\&=\frac{2}{7}aAx^{7/2}+\frac{2}{13}(Ab+aB)x^{13/2}+\frac{2}{19}bBx^{19/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int x^{5/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2x^{7/2}(247aA+133Abx^3+133aBx^3+91bBx^6)}{1729}$$

```
[In] Integrate[x^(5/2)*(a + b*x^3)*(A + B*x^3),x]
[Out] (2*x^(7/2)*(247*a*A + 133*A*b*x^3 + 133*a*B*x^3 + 91*b*B*x^6))/1729
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2aA\,x^{\frac{7}{2}}}{7}+\frac{2(Ab+Ba)x^{\frac{13}{2}}}{13}+\frac{2bB\,x^{\frac{19}{2}}}{19}$	28
default	$\frac{2aA\,x^{\frac{7}{2}}}{7}+\frac{2(Ab+Ba)x^{\frac{13}{2}}}{13}+\frac{2bB\,x^{\frac{19}{2}}}{19}$	28
gosper	$\frac{2x^{\frac{7}{2}}\left(91bB\,x^{6}+133Ab\,x^{3}+133Ba\,x^{3}+247Aa\right)}{1729}$	32
trager	$\frac{2x^{\frac{7}{2}}\left(91bB\,x^{6}+133Ab\,x^{3}+133Ba\,x^{3}+247Aa\right)}{1729}$	32
risch	$\frac{2x^{\frac{7}{2}}\left(91bB\,x^{6}+133Ab\,x^{3}+133Ba\,x^{3}+247Aa\right)}{1729}$	32

```
[In] int(x^(5/2)*(b*x^3+a)*(B*x^3+A),x,method=_RETURNVERBOSE)
[Out] 2/7*a*A*x^(7/2)+2/13*(A*b+B*a)*x^(13/2)+2/19*b*B*x^(19/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int x^{5/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2}{1729}\left(91\,Bbx^9+133\,(Ba+Ab)x^6+247\,Aax^3\right)\sqrt{x}$$

```
[In] integrate(x^(5/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
[Out] 2/1729*(91*B*b*x^9 + 133*(B*a + A*b)*x^6 + 247*A*a*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int x^{5/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2Aax^{\frac{7}{2}}}{7}+\frac{2Abx^{\frac{13}{2}}}{13}+\frac{2Bax^{\frac{13}{2}}}{13}+\frac{2Bbx^{\frac{19}{2}}}{19}$$

```
[In] integrate(x**(5/2)*(b*x**3+a)*(B*x**3+A),x)
[Out] 2*A*a*x**(7/2)/7 + 2*A*b*x**(13/2)/13 + 2*B*a*x**(13/2)/13 + 2*B*b*x**(19/2
)/19
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int x^{5/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2}{19}\,Bbx^{\frac{19}{2}}+\frac{2}{13}\,(Ba+Ab)x^{\frac{13}{2}}+\frac{2}{7}\,Aax^{\frac{7}{2}}$$

```
[In] integrate(x^(5/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
[Out] 2/19*B*b*x^(19/2) + 2/13*(B*a + A*b)*x^(13/2) + 2/7*A*a*x^(7/2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int x^{5/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2}{19}\,Bbx^{\frac{19}{2}}+\frac{2}{13}\,Bax^{\frac{13}{2}}+\frac{2}{13}\,Abx^{\frac{13}{2}}+\frac{2}{7}\,Aax^{\frac{7}{2}}$$

```
[In] integrate(x^(5/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
[Out] 2/19*B*b*x^(19/2) + 2/13*B*a*x^(13/2) + 2/13*A*b*x^(13/2) + 2/7*A*a*x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 7.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int x^{5/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2\,x^{7/2}\,\left(247\,A\,a+133\,A\,b\,x^3+133\,B\,a\,x^3+91\,B\,b\,x^6\right)}{1729}$$

```
[In] int(x^(5/2)*(A + B*x^3)*(a + b*x^3),x)
[Out] (2*x^(7/2)*(247*A*a + 133*A*b*x^3 + 133*B*a*x^3 + 91*B*b*x^6))/1729
```

3.133 $\int x^{3/2}(a+bx^3)\left(A+Bx^3\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 39

$$\int x^{3/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2}{5}aAx^{5/2}+\frac{2}{11}(Ab+aB)x^{11/2}+\frac{2}{17}bBx^{17/2}$$

```
[Out] 2/5*a*A*x^(5/2)+2/11*(A*b+B*a)*x^(11/2)+2/17*b*B*x^(17/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int x^{3/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2}{11}x^{11/2}(aB+Ab)+\frac{2}{5}aAx^{5/2}+\frac{2}{17}bBx^{17/2}$$

```
[In] Int[x^(3/2)*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (2*a*A*x^(5/2))/5 + (2*(A*b + a*B)*x^(11/2))/11 + (2*b*B*x^(17/2))/17
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int\left(aAx^{3/2}+(Ab+aB)x^{9/2}+bBx^{15/2}\right)\,dx\\&=\frac{2}{5}aAx^{5/2}+\frac{2}{11}(Ab+aB)x^{11/2}+\frac{2}{17}bBx^{17/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int x^{3/2}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2}{935}x^{5/2}\left(187aA+85Abx^3+85aBx^3+55bBx^6\right)$$

```
[In] Integrate[x^(3/2)*(a + b*x^3)*(A + B*x^3),x]
[Out] (2*x^(5/2)*(187*a*A + 85*A*b*x^3 + 85*a*B*x^3 + 55*b*B*x^6))/935
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2aA\,x^{\frac{5}{2}}}{5}+\frac{2(Ab+Ba)x^{\frac{11}{2}}}{11}+\frac{2bB\,x^{\frac{17}{2}}}{17}$	28
default	$\frac{2aA\,x^{\frac{5}{2}}}{5}+\frac{2(Ab+Ba)x^{\frac{11}{2}}}{11}+\frac{2bB\,x^{\frac{17}{2}}}{17}$	28
gosper	$\frac{2x^{\frac{5}{2}}\left(55bB\,x^6+85Ab\,x^3+85Ba\,x^3+187Aa\right)}{935}$	32
trager	$\frac{2x^{\frac{5}{2}}\left(55bB\,x^6+85Ab\,x^3+85Ba\,x^3+187Aa\right)}{935}$	32
risch	$\frac{2x^{\frac{5}{2}}\left(55bB\,x^6+85Ab\,x^3+85Ba\,x^3+187Aa\right)}{935}$	32

```
[In] int(x^(3/2)*(b*x^3+a)*(B*x^3+A),x,method=_RETURNVERBOSE)
[Out] 2/5*a*A*x^(5/2)+2/11*(A*b+B*a)*x^(11/2)+2/17*b*B*x^(17/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int x^{3/2}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2}{935}\left(55\,Bbx^8+85\,(Ba+Ab)x^5+187\,Aax^2\right)\sqrt{x}$$

```
[In] integrate(x^(3/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
[Out] 2/935*(55*B*b*x^8 + 85*(B*a + A*b)*x^5 + 187*A*a*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int x^{3/2}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2Aax^{\frac{5}{2}}}{5}+\frac{2Abx^{\frac{11}{2}}}{11}+\frac{2Bax^{\frac{11}{2}}}{11}+\frac{2Bbx^{\frac{17}{2}}}{17}$$

```
[In] integrate(x**(3/2)*(b*x**3+a)*(B*x**3+A),x)
[Out] 2*A*a*x**(5/2)/5 + 2*A*b*x**(11/2)/11 + 2*B*a*x**(11/2)/11 + 2*B*b*x**(17/2
)/17
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int x^{3/2}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2}{17}\,Bbx^{\frac{17}{2}}+\frac{2}{11}\,(Ba+Ab)x^{\frac{11}{2}}+\frac{2}{5}\,Aax^{\frac{5}{2}}$$

```
[In] integrate(x^(3/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
[Out] 2/17*B*b*x^(17/2) + 2/11*(B*a + A*b)*x^(11/2) + 2/5*A*a*x^(5/2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int x^{3/2}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2}{17}\,Bbx^{\frac{17}{2}}+\frac{2}{11}\,Bax^{\frac{11}{2}}+\frac{2}{11}\,Abx^{\frac{11}{2}}+\frac{2}{5}\,Aax^{\frac{5}{2}}$$

```
[In] integrate(x^(3/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
[Out] 2/17*B*b*x^(17/2) + 2/11*B*a*x^(11/2) + 2/11*A*b*x^(11/2) + 2/5*A*a*x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int x^{3/2}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2\,x^{5/2}\,\left(187\,A\,a+85\,A\,b\,x^3+85\,B\,a\,x^3+55\,B\,b\,x^6\right)}{935}$$

```
[In] int(x^(3/2)*(A + B*x^3)*(a + b*x^3),x)
[Out] (2*x^(5/2)*(187*A*a + 85*A*b*x^3 + 85*B*a*x^3 + 55*B*b*x^6))/935
```

3.134 $\int \sqrt{x}(a+bx^3)\left(A+Bx^3\right) dx$

Optimal result

Integrand size = 20, antiderivative size = 39

$$\int \sqrt{x}(a+bx^3)\left(A+Bx^3\right) dx = \frac{2}{3}aAx^{3/2}+\frac{2}{9}(Ab+aB)x^{9/2}+\frac{2}{15}bBx^{15/2}$$

```
[Out] 2/3*a*A*x^(3/2)+2/9*(A*b+B*a)*x^(9/2)+2/15*b*B*x^(15/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {459}

$$\int \sqrt{x}(a+bx^3)\left(A+Bx^3\right) dx = \frac{2}{9}x^{9/2}(aB+Ab)+\frac{2}{3}aAx^{3/2}+\frac{2}{15}bBx^{15/2}$$

```
[In] Int[Sqrt[x]*(a + b*x^3)*(A + B*x^3),x]
[Out] (2*a*A*x^(3/2))/3 + (2*(A*b + a*B)*x^(9/2))/9 + (2*b*B*x^(15/2))/15
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(aA\sqrt{x}+(Ab+aB)x^{7/2}+bBx^{13/2}\right) dx \\ &= \frac{2}{3}aAx^{3/2}+\frac{2}{9}(Ab+aB)x^{9/2}+\frac{2}{15}bBx^{15/2} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \sqrt{x}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2}{45}x^{3/2}\left(15aA+5Abx^3+5aBx^3+3bBx^6\right)$$

```
[In] Integrate[Sqrt[x]*(a + b*x^3)*(A + B*x^3),x]
[Out] (2*x^(3/2)*(15*a*A + 5*A*b*x^3 + 5*a*B*x^3 + 3*b*B*x^6))/45
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2aA\,x^{\frac{3}{2}}}{3}+\frac{2(Ab+Ba)x^{\frac{9}{2}}}{9}+\frac{2bB\,x^{\frac{15}{2}}}{15}$	28
default	$\frac{2aA\,x^{\frac{3}{2}}}{3}+\frac{2(Ab+Ba)x^{\frac{9}{2}}}{9}+\frac{2bB\,x^{\frac{15}{2}}}{15}$	28
gosper	$\frac{2x^{\frac{3}{2}}\left(3bB\,x^{6}+5Ab\,x^{3}+5Ba\,x^{3}+15Aa\right)}{45}$	32
trager	$\frac{2x^{\frac{3}{2}}\left(3bB\,x^{6}+5Ab\,x^{3}+5Ba\,x^{3}+15Aa\right)}{45}$	32
risch	$\frac{2x^{\frac{3}{2}}\left(3bB\,x^{6}+5Ab\,x^{3}+5Ba\,x^{3}+15Aa\right)}{45}$	32

```
[In] int((b*x^3+a)*(B*x^3+A)*x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*a*A*x^(3/2)+2/9*(A*b+B*a)*x^(9/2)+2/15*b*B*x^(15/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \sqrt{x}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx=\frac{2}{45}\left(3\,Bbx^7+5\,(Ba+Ab)x^4+15\,Aax\right)\sqrt{x}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)*x^(1/2),x, algorithm="fricas")
[Out] 2/45*(3*B*b*x^7 + 5*(B*a + A*b)*x^4 + 15*A*a*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int \sqrt{x}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2Aax^{\frac{3}{2}}}{3}+\frac{2Abx^{\frac{9}{2}}}{9}+\frac{2Bax^{\frac{9}{2}}}{9}+\frac{2Bbx^{\frac{15}{2}}}{15}$$

```
[In] integrate((b*x**3+a)*(B*x**3+A)*x**(1/2),x)
[Out] 2*A*a*x**(3/2)/3 + 2*A*b*x**(9/2)/9 + 2*B*a*x**(9/2)/9 + 2*B*b*x**(15/2)/15
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \sqrt{x}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2}{15}\,Bbx^{\frac{15}{2}}+\frac{2}{9}\,(Ba+Ab)x^{\frac{9}{2}}+\frac{2}{3}\,Aax^{\frac{3}{2}}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)*x^(1/2),x, algorithm="maxima")
[Out] 2/15*B*b*x^(15/2) + 2/9*(B*a + A*b)*x^(9/2) + 2/3*A*a*x^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \sqrt{x}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2}{15}\,Bbx^{\frac{15}{2}}+\frac{2}{9}\,Bax^{\frac{9}{2}}+\frac{2}{9}\,Abx^{\frac{9}{2}}+\frac{2}{3}\,Aax^{\frac{3}{2}}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)*x^(1/2),x, algorithm="giac")
[Out] 2/15*B*b*x^(15/2) + 2/9*B*a*x^(9/2) + 2/9*A*b*x^(9/2) + 2/3*A*a*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \sqrt{x}(a+bx^3)\left(A+Bx^3\right)\,dx=\frac{2\,x^{3/2}\,\left(15\,A\,a+5\,A\,b\,x^3+5\,B\,a\,x^3+3\,B\,b\,x^6\right)}{45}$$

```
[In] int(x^(1/2)*(A + B*x^3)*(a + b*x^3),x)
[Out] (2*x^(3/2)*(15*A*a + 5*A*b*x^3 + 5*B*a*x^3 + 3*B*b*x^6))/45
```

3.135 $\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 37

$$\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}}\,dx = 2aA\sqrt{x} + \frac{2}{7}(Ab+aB)x^{7/2} + \frac{2}{13}bBx^{13/2}$$

```
[Out] 2/7*(A*b+B*a)*x^(7/2)+2/13*b*B*x^(13/2)+2*a*A*x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}}\,dx = \frac{2}{7}x^{7/2}(aB+Ab) + 2aA\sqrt{x} + \frac{2}{13}bBx^{13/2}$$

```
[In] Int[((a + b*x^3)*(A + B*x^3))/Sqrt[x],x]
```

```
[Out] 2*a*A*Sqrt[x] + (2*(A*b + a*B)*x^(7/2))/7 + (2*b*B*x^(13/2))/13
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{aA}{\sqrt{x}} + (Ab+aB)x^{5/2} + bBx^{11/2}\right)dx \\ &= 2aA\sqrt{x} + \frac{2}{7}(Ab+aB)x^{7/2} + \frac{2}{13}bBx^{13/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}}\,dx=\frac{2}{91}\sqrt{x}\left(91aA+13Abx^3+13aBx^3+7bBx^6\right)$$

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/Sqrt[x],x]
[Out] (2*Sqrt[x]*(91*a*A + 13*A*b*x^3 + 13*a*B*x^3 + 7*b*B*x^6))/91
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{2(Ab+Ba)x^{\frac{7}{2}}}{7}+\frac{2bB\,x^{\frac{13}{2}}}{13}+2aA\sqrt{x}$	28
default	$\frac{2(Ab+Ba)x^{\frac{7}{2}}}{7}+\frac{2bB\,x^{\frac{13}{2}}}{13}+2aA\sqrt{x}$	28
trager	$\left(\frac{2}{13}bB\,x^6+\frac{2}{7}Ab\,x^3+\frac{2}{7}Ba\,x^3+2Aa\right)\sqrt{x}$	31
gosper	$\frac{2\sqrt{x}\,(7bB\,x^6+13Ab\,x^3+13Ba\,x^3+91Aa)}{91}$	32
risch	$\frac{2\sqrt{x}\,(7bB\,x^6+13Ab\,x^3+13Ba\,x^3+91Aa)}{91}$	32

```
[In] int((b*x^3+a)*(B*x^3+A)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/7*(A*b+B*a)*x^(7/2)+2/13*b*B*x^(13/2)+2*a*A*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}}\,dx=\frac{2}{91}\left(7\,Bbx^6+13\,(Ba+Ab)x^3+91\,Aa\right)\sqrt{x}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(1/2),x, algorithm="fricas")
[Out] 2/91*(7*B*b*x^6 + 13*(B*a + A*b)*x^3 + 91*A*a)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}}\,dx = 2Aa\sqrt{x} + \frac{2Abx^{\frac{7}{2}}}{7} + \frac{2Bax^{\frac{7}{2}}}{7} + \frac{2Bbx^{\frac{13}{2}}}{13}$$

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**(1/2),x)
[Out] 2*A*a*sqrt(x) + 2*A*b*x**(7/2)/7 + 2*B*a*x**(7/2)/7 + 2*B*b*x**(13/2)/13
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}}\,dx = \frac{2}{13}\,Bbx^{\frac{13}{2}} + \frac{2}{7}\,(Ba+Ab)x^{\frac{7}{2}} + 2\,Aa\sqrt{x}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(1/2),x, algorithm="maxima")
[Out] 2/13*B*b*x^(13/2) + 2/7*(B*a + A*b)*x^(7/2) + 2*A*a*sqrt(x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}}\,dx = \frac{2}{13}\,Bbx^{\frac{13}{2}} + \frac{2}{7}\,Bax^{\frac{7}{2}} + \frac{2}{7}\,Abx^{\frac{7}{2}} + 2\,Aa\sqrt{x}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(1/2),x, algorithm="giac")
[Out] 2/13*B*b*x^(13/2) + 2/7*B*a*x^(7/2) + 2/7*A*b*x^(7/2) + 2*A*a*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 6.98 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}}\,dx = \frac{2\,\sqrt{x}\,(91\,A\,a + 13\,A\,b\,x^3 + 13\,B\,a\,x^3 + 7\,B\,b\,x^6)}{91}$$

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^(1/2),x)
[Out] (2*x^(1/2)*(91*A*a + 13*A*b*x^3 + 13*B*a*x^3 + 7*B*b*x^6))/91
```

3.136 $\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 37

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}}\,dx=-\frac{2aA}{\sqrt{x}}+\frac{2}{5}(Ab+aB)x^{5/2}+\frac{2}{11}bBx^{11/2}$$

```
[Out] 2/5*(A*b+B*a)*x^(5/2)+2/11*b*B*x^(11/2)-2*a*A/x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}}\,dx=\frac{2}{5}x^{5/2}(aB+Ab)-\frac{2aA}{\sqrt{x}}+\frac{2}{11}bBx^{11/2}$$

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^(3/2),x]
[Out] (-2*a*A)/Sqrt[x] + (2*(A*b + a*B)*x^(5/2))/5 + (2*b*B*x^(11/2))/11
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(\frac{aA}{x^{3/2}}+(Ab+aB)x^{3/2}+bBx^{9/2}\right)dx\\&=-\frac{2aA}{\sqrt{x}}+\frac{2}{5}(Ab+aB)x^{5/2}+\frac{2}{11}bBx^{11/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}}\,dx = -\frac{2(55aA-11Abx^3-11aBx^3-5bBx^6)}{55\sqrt{x}}$$

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^(3/2),x]
[Out] (-2*(55*a*A - 11*A*b*x^3 - 11*a*B*x^3 - 5*b*B*x^6))/(55*Sqrt[x])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{2bB\,x^{\frac{11}{2}}}{11}+\frac{2Ab\,x^{\frac{5}{2}}}{5}+\frac{2Ba\,x^{\frac{5}{2}}}{5}-\frac{2aA}{\sqrt{x}}$	30
default	$\frac{2bB\,x^{\frac{11}{2}}}{11}+\frac{2Ab\,x^{\frac{5}{2}}}{5}+\frac{2Ba\,x^{\frac{5}{2}}}{5}-\frac{2aA}{\sqrt{x}}$	30
gosper	$-\frac{2(-5bB\,x^6-11Ab\,x^3-11Ba\,x^3+55Aa)}{55\sqrt{x}}$	32
trager	$-\frac{2(-5bB\,x^6-11Ab\,x^3-11Ba\,x^3+55Aa)}{55\sqrt{x}}$	32
risch	$-\frac{2(-5bB\,x^6-11Ab\,x^3-11Ba\,x^3+55Aa)}{55\sqrt{x}}$	32

```
[In] int((b*x^3+a)*(B*x^3+A)/x^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/11*b*B*x^(11/2)+2/5*A*b*x^(5/2)+2/5*B*a*x^(5/2)-2*a*A/x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}}\,dx = \frac{2\,(5\,Bbx^6+11\,(Ba+Ab)x^3-55\,Aa)}{55\,\sqrt{x}}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(3/2),x, algorithm="fricas")
[Out] 2/55*(5*B*b*x^6 + 11*(B*a + A*b)*x^3 - 55*A*a)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}}\,dx = -\frac{2Aa}{\sqrt{x}} + \frac{2Abx^{\frac{5}{2}}}{5} + \frac{2Bax^{\frac{5}{2}}}{5} + \frac{2Bbx^{\frac{11}{2}}}{11}$$

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**(3/2),x)
[Out] -2*A*a/sqrt(x) + 2*A*b*x**(5/2)/5 + 2*B*a*x**(5/2)/5 + 2*B*b*x**(11/2)/11
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}}\,dx = \frac{2}{11}\,Bbx^{\frac{11}{2}} + \frac{2}{5}\,(Ba+Ab)x^{\frac{5}{2}} - \frac{2\,Aa}{\sqrt{x}}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(3/2),x, algorithm="maxima")
[Out] 2/11*B*b*x^(11/2) + 2/5*(B*a + A*b)*x^(5/2) - 2*A*a/sqrt(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}}\,dx = \frac{2}{11}\,Bbx^{\frac{11}{2}} + \frac{2}{5}\,Bax^{\frac{5}{2}} + \frac{2}{5}\,Abx^{\frac{5}{2}} - \frac{2\,Aa}{\sqrt{x}}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(3/2),x, algorithm="giac")
[Out] 2/11*B*b*x^(11/2) + 2/5*B*a*x^(5/2) + 2/5*A*b*x^(5/2) - 2*A*a/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 7.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}}\,dx = \frac{22\,A\,b\,x^3 - 110\,A\,a + 22\,B\,a\,x^3 + 10\,B\,b\,x^6}{55\,\sqrt{x}}$$

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^(3/2),x)
[Out] (22*A*b*x^3 - 110*A*a + 22*B*a*x^3 + 10*B*b*x^6)/(55*x^(1/2))
```

3.137 $\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 39

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}} \, dx = -\frac{2aA}{3x^{3/2}} + \frac{2}{3}(Ab+aB)x^{3/2} + \frac{2}{9}bBx^{9/2}$$

```
[Out] -2/3*a*A/x^(3/2)+2/3*(A*b+B*a)*x^(3/2)+2/9*b*B*x^(9/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}} \, dx = \frac{2}{3}x^{3/2}(aB+Ab) - \frac{2aA}{3x^{3/2}} + \frac{2}{9}bBx^{9/2}$$

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^(5/2),x]
```

```
[Out] (-2*a*A)/(3*x^(3/2)) + (2*(A*b + a*B)*x^(3/2))/3 + (2*b*B*x^(9/2))/9
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{aA}{x^{5/2}} + (Ab+aB)\sqrt{x} + bBx^{7/2}\right) \, dx \\ &= -\frac{2aA}{3x^{3/2}} + \frac{2}{3}(Ab+aB)x^{3/2} + \frac{2}{9}bBx^{9/2} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}}\,dx = \frac{2(-3aA+3Abx^3+3aBx^3+bBx^6)}{9x^{3/2}}$$

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^(5/2),x]
[Out] (2*(-3*a*A + 3*A*b*x^3 + 3*a*B*x^3 + b*B*x^6))/(9*x^(3/2))
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{2bB\,x^{\frac{9}{2}}}{9}+\frac{2Ab\,x^{\frac{3}{2}}}{3}+\frac{2Ba\,x^{\frac{3}{2}}}{3}-\frac{2aA}{3x^{\frac{3}{2}}}$	30
default	$\frac{2bB\,x^{\frac{9}{2}}}{9}+\frac{2Ab\,x^{\frac{3}{2}}}{3}+\frac{2Ba\,x^{\frac{3}{2}}}{3}-\frac{2aA}{3x^{\frac{3}{2}}}$	30
gosper	$-\frac{2(-bB\,x^6-3Ab\,x^3-3Ba\,x^3+3Aa)}{9x^{\frac{3}{2}}}$	32
trager	$-\frac{2(-bB\,x^6-3Ab\,x^3-3Ba\,x^3+3Aa)}{9x^{\frac{3}{2}}}$	32
risch	$-\frac{2(-bB\,x^6-3Ab\,x^3-3Ba\,x^3+3Aa)}{9x^{\frac{3}{2}}}$	32

```
[In] int((b*x^3+a)*(B*x^3+A)/x^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/9*b*B*x^(9/2)+2/3*A*b*x^(3/2)+2/3*B*a*x^(3/2)-2/3*a*A/x^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}}\,dx = \frac{2\left(Bbx^6+3\,(Ba+Ab)x^3-3\,Aa\right)}{9\,x^{\frac{3}{2}}}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(5/2),x, algorithm="fricas")
[Out] 2/9*(B*b*x^6 + 3*(B*a + A*b)*x^3 - 3*A*a)/x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}}\,dx = -\frac{2Aa}{3x^{\frac{3}{2}}} + \frac{2Abx^{\frac{3}{2}}}{3} + \frac{2Bax^{\frac{3}{2}}}{3} + \frac{2Bbx^{\frac{9}{2}}}{9}$$

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**(5/2),x)
[Out] -2*A*a/(3*x**(3/2)) + 2*A*b*x**(3/2)/3 + 2*B*a*x**(3/2)/3 + 2*B*b*x**(9/2)/
9
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}}\,dx = \frac{2}{9}\,Bbx^{\frac{9}{2}} + \frac{2}{3}\,(Ba+Ab)x^{\frac{3}{2}} - \frac{2\,Aa}{3\,x^{\frac{3}{2}}}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(5/2),x, algorithm="maxima")
[Out] 2/9*B*b*x^(9/2) + 2/3*(B*a + A*b)*x^(3/2) - 2/3*A*a/x^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}}\,dx = \frac{2}{9}\,Bbx^{\frac{9}{2}} + \frac{2}{3}\,Bax^{\frac{3}{2}} + \frac{2}{3}\,Abx^{\frac{3}{2}} - \frac{2\,Aa}{3\,x^{\frac{3}{2}}}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(5/2),x, algorithm="giac")
[Out] 2/9*B*b*x^(9/2) + 2/3*B*a*x^(3/2) + 2/3*A*b*x^(3/2) - 2/3*A*a/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}}\,dx = \frac{6\,A\,b\,x^3 - 6\,A\,a + 6\,B\,a\,x^3 + 2\,B\,b\,x^6}{9\,x^{3/2}}$$

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^(5/2),x)
[Out] (6*A*b*x^3 - 6*A*a + 6*B*a*x^3 + 2*B*b*x^6)/(9*x^(3/2))
```

3.138 $\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}} dx$

Optimal result

Integrand size = 20, antiderivative size = 37

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}} dx = -\frac{2aA}{5x^{5/2}} + 2(Ab+aB)\sqrt{x} + \frac{2}{7}bBx^{7/2}$$

```
[Out] -2/5*a*A/x^(5/2)+2/7*b*B*x^(7/2)+2*(A*b+B*a)*x^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {459}

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}} dx = 2\sqrt{x}(aB+Ab) - \frac{2aA}{5x^{5/2}} + \frac{2}{7}bBx^{7/2}$$

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^(7/2),x]
[Out] (-2*a*A)/(5*x^(5/2)) + 2*(A*b + a*B)*Sqrt[x] + (2*b*B*x^(7/2))/7
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{aA}{x^{7/2}} + \frac{Ab+aB}{\sqrt{x}} + bBx^{5/2}\right) dx \\ &= -\frac{2aA}{5x^{5/2}} + 2(Ab+aB)\sqrt{x} + \frac{2}{7}bBx^{7/2} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}}\,dx = -\frac{2(7aA-35Abx^3-35aBx^3-5bBx^6)}{35x^{5/2}}$$

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^(7/2),x]
[Out] (-2*(7*a*A - 35*A*b*x^3 - 35*a*B*x^3 - 5*b*B*x^6))/(35*x^(5/2))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{2bB\,x^{\frac{7}{2}}}{7}+2Ab\sqrt{x}+2Ba\sqrt{x}-\frac{2aA}{5x^{\frac{5}{2}}}$	30
default	$\frac{2bB\,x^{\frac{7}{2}}}{7}+2Ab\sqrt{x}+2Ba\sqrt{x}-\frac{2aA}{5x^{\frac{5}{2}}}$	30
gosper	$-\frac{2(-5bB\,x^6-35Ab\,x^3-35Ba\,x^3+7Aa)}{35x^{\frac{5}{2}}}$	32
trager	$-\frac{2(-5bB\,x^6-35Ab\,x^3-35Ba\,x^3+7Aa)}{35x^{\frac{5}{2}}}$	32
risch	$-\frac{2(-5bB\,x^6-35Ab\,x^3-35Ba\,x^3+7Aa)}{35x^{\frac{5}{2}}}$	32

```
[In] int((b*x^3+a)*(B*x^3+A)/x^(7/2),x,method=_RETURNVERBOSE)
[Out] 2/7*b*B*x^(7/2)+2*A*b*x^(1/2)+2*B*a*x^(1/2)-2/5*a*A/x^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}}\,dx = \frac{2\left(5\,Bbx^6+35\,(Ba+Ab)x^3-7\,Aa\right)}{35\,x^{\frac{5}{2}}}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(7/2),x, algorithm="fricas")
[Out] 2/35*(5*B*b*x^6 + 35*(B*a + A*b)*x^3 - 7*A*a)/x^(5/2)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}}\,dx = -\frac{2Aa}{5x^{\frac{5}{2}}} + 2Ab\sqrt{x} + 2Ba\sqrt{x} + \frac{2Bbx^{\frac{7}{2}}}{7}$$

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**(7/2),x)
[Out] -2*A*a/(5*x**(5/2)) + 2*A*b*sqrt(x) + 2*B*a*sqrt(x) + 2*B*b*x**(7/2)/7
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}}\,dx = \frac{2}{7}\,Bbx^{\frac{7}{2}} + 2\,(Ba+Ab)\sqrt{x} - \frac{2\,Aa}{5\,x^{\frac{5}{2}}}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(7/2),x, algorithm="maxima")
[Out] 2/7*B*b*x^(7/2) + 2*(B*a + A*b)*sqrt(x) - 2/5*A*a/x^(5/2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}}\,dx = \frac{2}{7}\,Bbx^{\frac{7}{2}} + 2\,Ba\sqrt{x} + 2\,Ab\sqrt{x} - \frac{2\,Aa}{5\,x^{\frac{5}{2}}}$$

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(7/2),x, algorithm="giac")
[Out] 2/7*B*b*x^(7/2) + 2*B*a*sqrt(x) + 2*A*b*sqrt(x) - 2/5*A*a/x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}}\,dx = \frac{2\,A\,b\,x^3 - \frac{2\,A\,a}{5} + 2\,B\,a\,x^3 + \frac{2\,B\,b\,x^6}{7}}{x^{5/2}}$$

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^(7/2),x)
[Out] (2*A*b*x^3 - (2*A*a)/5 + 2*B*a*x^3 + (2*B*b*x^6)/7)/x^(5/2)
```

3.139 $\int x^{7/2}(a+bx^3)^2(A+Bx^3)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 63

$$\int x^{7/2}(a+bx^3)^2(A+Bx^3)\,dx=\frac{2}{9}a^2Ax^{9/2}+\frac{2}{15}a(2Ab+aB)x^{15/2}+\frac{2}{21}b(Ab+2aB)x^{21/2}+\frac{2}{27}b^2Bx^{27/2}$$

```
[Out] 2/9*a^2*A*x^(9/2)+2/15*a*(2*A*b+B*a)*x^(15/2)+2/21*b*(A*b+2*B*a)*x^(21/2)+2
/27*b^2*B*x^(27/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {459}

$$\int x^{7/2}(a+bx^3)^2(A+Bx^3)\,dx=\frac{2}{9}a^2Ax^{9/2}+\frac{2}{21}bx^{21/2}(2aB+Ab)+\frac{2}{15}ax^{15/2}(aB+2Ab)+\frac{2}{27}b^2Bx^{27/2}$$

```
[In] Int[x^(7/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*a^2*A*x^(9/2))/9 + (2*a*(2*A*b + a*B)*x^(15/2))/15 + (2*b*(A*b + 2*a*B)*
x^(21/2))/21 + (2*b^2*B*x^(27/2))/27
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(a^2 A x^{7/2} + a(2Ab + aB)x^{13/2} + b(Ab + 2aB)x^{19/2} + b^2 B x^{25/2}\right) dx$$
$$= \frac{2}{9}a^2 A x^{9/2} + \frac{2}{15}a(2Ab + aB)x^{15/2} + \frac{2}{21}b(Ab + 2aB)x^{21/2} + \frac{2}{27}b^2 B x^{27/2}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int x^{7/2}\left(a + bx^3\right)^2 \left(A + Bx^3\right) dx = \frac{2}{945}x^{9/2}\left(105a^2 A + 126aAbx^3 + 63a^2 Bx^3 + 45Ab^2 x^6 + 90abBx^6 + 35b^2 Bx^9\right)$$

```
[In] Integrate[x^(7/2)*(a + b*x^3)^2*(A + B*x^3),x]
[Out] (2*x^(9/2)*(105*a^2*A + 126*a*A*b*x^3 + 63*a^2*B*x^3 + 45*A*b^2*x^6 + 90*a*
b*B*x^6 + 35*b^2*B*x^9))/945
```

Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2b^2 B x^{\frac{27}{2}}}{27} + \frac{2(b^2 A + 2abB)x^{\frac{21}{2}}}{21} + \frac{2(2abA + a^2 B)x^{\frac{15}{2}}}{15} + \frac{2a^2 A x^{\frac{9}{2}}}{9}$	52
default	$\frac{2b^2 B x^{\frac{27}{2}}}{27} + \frac{2(b^2 A + 2abB)x^{\frac{21}{2}}}{21} + \frac{2(2abA + a^2 B)x^{\frac{15}{2}}}{15} + \frac{2a^2 A x^{\frac{9}{2}}}{9}$	52
gosper	$\frac{2x^{\frac{9}{2}}(35b^2 B x^9 + 45A b^2 x^6 + 90B x^6 ab + 126aAb x^3 + 63a^2 B x^3 + 105a^2 A)}{945}$	56
trager	$\frac{2x^{\frac{9}{2}}(35b^2 B x^9 + 45A b^2 x^6 + 90B x^6 ab + 126aAb x^3 + 63a^2 B x^3 + 105a^2 A)}{945}$	56
risch	$\frac{2x^{\frac{9}{2}}(35b^2 B x^9 + 45A b^2 x^6 + 90B x^6 ab + 126aAb x^3 + 63a^2 B x^3 + 105a^2 A)}{945}$	56

```
[In] int(x^(7/2)*(b*x^3+a)^2*(B*x^3+A),x,method=_RETURNVERBOSE)
[Out] 2/27*b^2*B*x^(27/2)+2/21*(A*b^2+2*B*a*b)*x^(21/2)+2/15*(2*A*a*b+B*a^2)*x^(1
5/2)+2/9*a^2*A*x^(9/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int x^{7/2}(a+bx^3)^2(A+Bx^3)\,dx = \frac{2}{945}\left(35\,Bb^2x^{13}+45\left(2\,Bab+Ab^2\right)x^{10}+63\left(Ba^2+2\,Aab\right)x^7+105\,Aa^2x^4\right)\sqrt{x}$$

```
[In] integrate(x^(7/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
[Out] 2/945*(35*B*b^2*x^13 + 45*(2*B*a*b + A*b^2)*x^10 + 63*(B*a^2 + 2*A*a*b)*x^7
 + 105*A*a^2*x^4)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int x^{7/2}(a+bx^3)^2(A+Bx^3)\,dx = \frac{2Aa^2x^{\frac{9}{2}}}{9}+\frac{4Aabx^{\frac{15}{2}}}{15}+\frac{2Ab^2x^{\frac{21}{2}}}{21}+\frac{2Ba^2x^{\frac{15}{2}}}{15}+\frac{4Babx^{\frac{21}{2}}}{21}+\frac{2Bb^2x^{\frac{27}{2}}}{27}$$

```
[In] integrate(x**(7/2)*(b*x**3+a)**2*(B*x**3+A),x)
[Out] 2*A*a**2*x**(9/2)/9 + 4*A*a*b*x**(15/2)/15 + 2*A*b**2*x**(21/2)/21 + 2*B*a*
*2*x**(15/2)/15 + 4*B*a*b*x**(21/2)/21 + 2*B*b**2*x**(27/2)/27
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{7/2}(a+bx^3)^2(A+Bx^3)\,dx = \frac{2}{27}\,Bb^2x^{\frac{27}{2}}+\frac{2}{21}\left(2\,Bab+Ab^2\right)x^{\frac{21}{2}}+\frac{2}{15}\left(Ba^2+2\,Aab\right)x^{\frac{15}{2}}+\frac{2}{9}\,Aa^2x^{\frac{9}{2}}$$

```
[In] integrate(x^(7/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
[Out] 2/27*B*b^2*x^(27/2) + 2/21*(2*B*a*b + A*b^2)*x^(21/2) + 2/15*(B*a^2 + 2*A*a
*b)*x^(15/2) + 2/9*A*a^2*x^(9/2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int x^{7/2}(a+bx^3)^2\left(A+Bx^3\right)\,dx=\frac{2}{27}\,Bb^2x^{\frac{27}{2}}+\frac{4}{21}\,Babx^{\frac{21}{2}}+\frac{2}{21}\,Ab^2x^{\frac{21}{2}}+\frac{2}{15}\,Ba^2x^{\frac{15}{2}}+\frac{4}{15}\,Aabx^{\frac{15}{2}}+\frac{2}{9}\,Aa^2x^{\frac{9}{2}}$$

```
[In] integrate(x^(7/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/27*B*b^2*x^(27/2) + 4/21*B*a*b*x^(21/2) + 2/21*A*b^2*x^(21/2) + 2/15*B*a^
2*x^(15/2) + 4/15*A*a*b*x^(15/2) + 2/9*A*a^2*x^(9/2)
```

Mupad [B] (verification not implemented)

Time = 6.98 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{7/2}(a+bx^3)^2\left(A+Bx^3\right)\,dx=x^{15/2}\left(\frac{2\,B\,a^2}{15}+\frac{4\,A\,b\,a}{15}\right)+x^{21/2}\left(\frac{2\,A\,b^2}{21}+\frac{4\,B\,a\,b}{21}\right)+\frac{2\,A\,a^2\,x^{9/2}}{9}+\frac{2\,B\,b^2\,x^{27/2}}{27}$$

```
[In] int(x^(7/2)*(A + B*x^3)*(a + b*x^3)^2,x)
```

```
[Out] x^(15/2)*((2*B*a^2)/15 + (4*A*a*b)/15) + x^(21/2)*((2*A*b^2)/21 + (4*B*a*b)
/21) + (2*A*a^2*x^(9/2))/9 + (2*B*b^2*x^(27/2))/27
```

3.140 $\int x^{5/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 63

$$\int x^{5/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2}{7}a^2Ax^{7/2}+\frac{2}{13}a(2Ab+aB)x^{13/2}+\frac{2}{19}b(Ab+2aB)x^{19/2}+\frac{2}{25}b^2Bx^{25/2}$$

```
[Out] 2/7*a^2*A*x^(7/2)+2/13*a*(2*A*b+B*a)*x^(13/2)+2/19*b*(A*b+2*B*a)*x^(19/2)+2
/25*b^2*B*x^(25/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {459}

$$\int x^{5/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2}{7}a^2Ax^{7/2}+\frac{2}{19}bx^{19/2}(2aB+Ab)+\frac{2}{13}ax^{13/2}(aB+2Ab)+\frac{2}{25}b^2Bx^{25/2}$$

```
[In] Int[x^(5/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*a^2*A*x^(7/2))/7 + (2*a*(2*A*b + a*B)*x^(13/2))/13 + (2*b*(A*b + 2*a*B)*
x^(19/2))/19 + (2*b^2*B*x^(25/2))/25
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^2Ax^{5/2} + a(2Ab + aB)x^{11/2} + b(Ab + 2aB)x^{17/2} + b^2Bx^{23/2}\right)\,dx \\
&= \frac{2}{7}a^2Ax^{7/2} + \frac{2}{13}a(2Ab + aB)x^{13/2} + \frac{2}{19}b(Ab + 2aB)x^{19/2} + \frac{2}{25}b^2Bx^{25/2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$
\int x^{5/2}\left(a+bx^3\right)^2\left(A + Bx^3\right)\,dx = \frac{2x^{7/2}(475a^2(13A+7Bx^3)+350abx^3(19A+13Bx^3)+91b^2x^6(25A+19Bx^3))}{43225}
$$

```
[In] Integrate[x^(5/2)*(a + b*x^3)^2*(A + B*x^3),x]
[Out] (2*x^(7/2)*(475*a^2*(13*A + 7*B*x^3) + 350*a*b*x^3*(19*A + 13*B*x^3) + 91*b
^2*x^6*(25*A + 19*B*x^3)))/43225
```

Maple [A] (verified)

Time = 4.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2b^2Bx^{\frac{25}{2}}}{25}+\frac{2(b^2A+2abB)x^{\frac{19}{2}}}{19}+\frac{2(2abA+a^2B)x^{\frac{13}{2}}}{13}+\frac{2a^2Ax^{\frac{7}{2}}}{7}$	52
default	$\frac{2b^2Bx^{\frac{25}{2}}}{25}+\frac{2(b^2A+2abB)x^{\frac{19}{2}}}{19}+\frac{2(2abA+a^2B)x^{\frac{13}{2}}}{13}+\frac{2a^2Ax^{\frac{7}{2}}}{7}$	52
gosper	$\frac{2x^{\frac{7}{2}}(1729b^2Bx^9+2275Ab^2x^6+4550Bx^6ab+6650aAbx^3+3325a^2Bx^3+6175a^2A)}{43225}$	56
trager	$\frac{2x^{\frac{7}{2}}(1729b^2Bx^9+2275Ab^2x^6+4550Bx^6ab+6650aAbx^3+3325a^2Bx^3+6175a^2A)}{43225}$	56
risch	$\frac{2x^{\frac{7}{2}}(1729b^2Bx^9+2275Ab^2x^6+4550Bx^6ab+6650aAbx^3+3325a^2Bx^3+6175a^2A)}{43225}$	56

```
[In] int(x^(5/2)*(b*x^3+a)^2*(B*x^3+A),x,method=_RETURNVERBOSE)
[Out] 2/25*b^2*B*x^(25/2)+2/19*(A*b^2+2*B*a*b)*x^(19/2)+2/13*(2*A*a*b+B*a^2)*x^(1
3/2)+2/7*a^2*A*x^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int x^{5/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2}{43225}\left(1729\,Bb^2x^{12}+2275\left(2\,Bab+Ab^2\right)x^9+3325\left(Ba^2+2\,Aab\right)x^6+6175\,Aa^2x^3\right)\sqrt{x}$$

```
[In] integrate(x^(5/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/43225*(1729*B*b^2*x^12 + 2275*(2*B*a*b + A*b^2)*x^9 + 3325*(B*a^2 + 2*A*a
*b)*x^6 + 6175*A*a^2*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int x^{5/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2Aa^2x^{\frac{7}{2}}}{7}+\frac{4Aabx^{\frac{13}{2}}}{13}+\frac{2Ab^2x^{\frac{19}{2}}}{19}+\frac{2Ba^2x^{\frac{13}{2}}}{13}+\frac{4Babx^{\frac{19}{2}}}{19}+\frac{2Bb^2x^{\frac{25}{2}}}{25}$$

```
[In] integrate(x**(5/2)*(b*x**3+a)**2*(B*x**3+A),x)
```

```
[Out] 2*A*a**2*x**(7/2)/7 + 4*A*a*b*x**(13/2)/13 + 2*A*b**2*x**(19/2)/19 + 2*B*a*
*2*x**(13/2)/13 + 4*B*a*b*x**(19/2)/19 + 2*B*b**2*x**(25/2)/25
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{5/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2}{25}\,Bb^2x^{\frac{25}{2}}+\frac{2}{19}\left(2\,Bab+Ab^2\right)x^{\frac{19}{2}}+\frac{2}{13}\left(Ba^2+2\,Aab\right)x^{\frac{13}{2}}+\frac{2}{7}\,Aa^2x^{\frac{7}{2}}$$

```
[In] integrate(x^(5/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/25*B*b^2*x^(25/2) + 2/19*(2*B*a*b + A*b^2)*x^(19/2) + 2/13*(B*a^2 + 2*A*a
*b)*x^(13/2) + 2/7*A*a^2*x^(7/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int x^{5/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2}{25}\,Bb^2x^{\frac{25}{2}}+\frac{4}{19}\,Babx^{\frac{19}{2}}+\frac{2}{19}\,Ab^2x^{\frac{19}{2}}+\frac{2}{13}\,Ba^2x^{\frac{13}{2}}+\frac{4}{13}\,Aabx^{\frac{13}{2}}+\frac{2}{7}\,Aa^2x^{\frac{7}{2}}$$

```
[In] integrate(x^(5/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/25*B*b^2*x^(25/2) + 4/19*B*a*b*x^(19/2) + 2/19*A*b^2*x^(19/2) + 2/13*B*a^
2*x^(13/2) + 4/13*A*a*b*x^(13/2) + 2/7*A*a^2*x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{5/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=x^{13/2}\left(\frac{2\,B\,a^2}{13}+\frac{4\,A\,b\,a}{13}\right)+x^{19/2}\left(\frac{2\,A\,b^2}{19}+\frac{4\,B\,a\,b}{19}\right)+\frac{2\,A\,a^2\,x^{7/2}}{7}+\frac{2\,B\,b^2\,x^{25/2}}{25}$$

```
[In] int(x^(5/2)*(A + B*x^3)*(a + b*x^3)^2,x)
```

```
[Out] x^(13/2)*((2*B*a^2)/13 + (4*A*a*b)/13) + x^(19/2)*((2*A*b^2)/19 + (4*B*a*b)
/19) + (2*A*a^2*x^(7/2))/7 + (2*B*b^2*x^(25/2))/25
```

3.141 $\int x^{3/2}(a+bx^3)^2\left(A+Bx^3\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 63

$$\int x^{3/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2}{5}a^2Ax^{5/2}+\frac{2}{11}a(2Ab+aB)x^{11/2}+\frac{2}{17}b(Ab+2aB)x^{17/2}+\frac{2}{23}b^2Bx^{23/2}$$

```
[Out] 2/5*a^2*A*x^(5/2)+2/11*a*(2*A*b+B*a)*x^(11/2)+2/17*b*(A*b+2*B*a)*x^(17/2)+2
/23*b^2*B*x^(23/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {459}

$$\int x^{3/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2}{5}a^2Ax^{5/2}+\frac{2}{17}bx^{17/2}(2aB+Ab)+\frac{2}{11}ax^{11/2}(aB+2Ab)+\frac{2}{23}b^2Bx^{23/2}$$

```
[In] Int[x^(3/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*a^2*A*x^(5/2))/5 + (2*a*(2*A*b + a*B)*x^(11/2))/11 + (2*b*(A*b + 2*a*B)*
x^(17/2))/17 + (2*b^2*B*x^(23/2))/23
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(a^2Ax^{3/2} + a(2Ab + aB)x^{9/2} + b(Ab + 2aB)x^{15/2} + b^2Bx^{21/2}\right)\,dx$$
$$= \frac{2}{5}a^2Ax^{5/2} + \frac{2}{11}a(2Ab + aB)x^{11/2} + \frac{2}{17}b(Ab + 2aB)x^{17/2} + \frac{2}{23}b^2Bx^{23/2}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int x^{3/2}\left(a+bx^3\right)^2\left(A + Bx^3\right)\,dx = \frac{2x^{5/2}(391a^2(11A+5Bx^3)+230abx^3(17A+11Bx^3)+55b^2x^6(23A+17Bx^3))}{21505}$$

```
[In] Integrate[x^(3/2)*(a + b*x^3)^2*(A + B*x^3),x]
[Out] (2*x^(5/2)*(391*a^2*(11*A + 5*B*x^3) + 230*a*b*x^3*(17*A + 11*B*x^3) + 55*b
^2*x^6*(23*A + 17*B*x^3)))/21505
```

Maple [A] (verified)

Time = 4.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2b^2Bx^{\frac{23}{2}}}{23}+\frac{2(b^2A+2abB)x^{\frac{17}{2}}}{17}+\frac{2(2abA+a^2B)x^{\frac{11}{2}}}{11}+\frac{2a^2Ax^{\frac{5}{2}}}{5}$	52
default	$\frac{2b^2Bx^{\frac{23}{2}}}{23}+\frac{2(b^2A+2abB)x^{\frac{17}{2}}}{17}+\frac{2(2abA+a^2B)x^{\frac{11}{2}}}{11}+\frac{2a^2Ax^{\frac{5}{2}}}{5}$	52
gosper	$\frac{2x^{\frac{5}{2}}(935b^2Bx^9+1265Ab^2x^6+2530Bx^6ab+3910aAbx^3+1955a^2Bx^3+4301a^2A)}{21505}$	56
trager	$\frac{2x^{\frac{5}{2}}(935b^2Bx^9+1265Ab^2x^6+2530Bx^6ab+3910aAbx^3+1955a^2Bx^3+4301a^2A)}{21505}$	56
risch	$\frac{2x^{\frac{5}{2}}(935b^2Bx^9+1265Ab^2x^6+2530Bx^6ab+3910aAbx^3+1955a^2Bx^3+4301a^2A)}{21505}$	56

```
[In] int(x^(3/2)*(b*x^3+a)^2*(B*x^3+A),x,method=_RETURNVERBOSE)
[Out] 2/23*b^2*B*x^(23/2)+2/17*(A*b^2+2*B*a*b)*x^(17/2)+2/11*(2*A*a*b+B*a^2)*x^(1
1/2)+2/5*a^2*A*x^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int x^{3/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2}{21505}\left(935\,Bb^2x^{11}+1265\left(2\,Bab+Ab^2\right)x^8+1955\left(Ba^2+2\,Aab\right)x^5+4301\,Aa^2x^2\right)\sqrt{x}$$

```
[In] integrate(x^(3/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/21505*(935*B*b^2*x^11 + 1265*(2*B*a*b + A*b^2)*x^8 + 1955*(B*a^2 + 2*A*a*
b)*x^5 + 4301*A*a^2*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int x^{3/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2Aa^2x^{\frac{5}{2}}}{5}+\frac{4Aabx^{\frac{11}{2}}}{11}+\frac{2Ab^2x^{\frac{17}{2}}}{17}+\frac{2Ba^2x^{\frac{11}{2}}}{11}+\frac{4Babx^{\frac{17}{2}}}{17}+\frac{2Bb^2x^{\frac{23}{2}}}{23}$$

```
[In] integrate(x**(3/2)*(b*x**3+a)**2*(B*x**3+A),x)
```

```
[Out] 2*A*a**2*x**(5/2)/5 + 4*A*a*b*x**(11/2)/11 + 2*A*b**2*x**(17/2)/17 + 2*B*a*
*2*x**(11/2)/11 + 4*B*a*b*x**(17/2)/17 + 2*B*b**2*x**(23/2)/23
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{3/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2}{23}\,Bb^2x^{\frac{23}{2}}+\frac{2}{17}\left(2\,Bab+Ab^2\right)x^{\frac{17}{2}}+\frac{2}{11}\left(Ba^2+2\,Aab\right)x^{\frac{11}{2}}+\frac{2}{5}\,Aa^2x^{\frac{5}{2}}$$

```
[In] integrate(x^(3/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/23*B*b^2*x^(23/2) + 2/17*(2*B*a*b + A*b^2)*x^(17/2) + 2/11*(B*a^2 + 2*A*a
*b)*x^(11/2) + 2/5*A*a^2*x^(5/2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int x^{3/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=\frac{2}{23}\,Bb^2x^{\frac{23}{2}}+\frac{4}{17}\,Babx^{\frac{17}{2}}+\frac{2}{17}\,Ab^2x^{\frac{17}{2}}+\frac{2}{11}\,Ba^2x^{\frac{11}{2}}+\frac{4}{11}\,Aabx^{\frac{11}{2}}+\frac{2}{5}\,Aa^2x^{\frac{5}{2}}$$

```
[In] integrate(x^(3/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/23*B*b^2*x^(23/2) + 4/17*B*a*b*x^(17/2) + 2/17*A*b^2*x^(17/2) + 2/11*B*a^
2*x^(11/2) + 4/11*A*a*b*x^(11/2) + 2/5*A*a^2*x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{3/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx=x^{11/2}\left(\frac{2\,B\,a^2}{11}+\frac{4\,A\,b\,a}{11}\right)+x^{17/2}\left(\frac{2\,A\,b^2}{17}+\frac{4\,B\,a\,b}{17}\right)+\frac{2\,A\,a^2\,x^{5/2}}{5}+\frac{2\,B\,b^2\,x^{23/2}}{23}$$

```
[In] int(x^(3/2)*(A + B*x^3)*(a + b*x^3)^2,x)
```

```
[Out] x^(11/2)*((2*B*a^2)/11 + (4*A*a*b)/11) + x^(17/2)*((2*A*b^2)/17 + (4*B*a*b)
/17) + (2*A*a^2*x^(5/2))/5 + (2*B*b^2*x^(23/2))/23
```

3.142 $\int \sqrt{x}(a+bx^3)^2 (A+Bx^3) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 63

$$\int \sqrt{x}(a+bx^3)^2 (A+Bx^3) \, dx = \frac{2}{3}a^2Ax^{3/2} + \frac{2}{9}a(2Ab+aB)x^{9/2} + \frac{2}{15}b(Ab+2aB)x^{15/2} + \frac{2}{21}b^2Bx^{21/2}$$

```
[Out] 2/3*a^2*A*x^(3/2)+2/9*a*(2*A*b+B*a)*x^(9/2)+2/15*b*(A*b+2*B*a)*x^(15/2)+2/2
1*b^2*B*x^(21/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {459}

$$\int \sqrt{x}(a+bx^3)^2 (A+Bx^3) \, dx = \frac{2}{3}a^2Ax^{3/2} + \frac{2}{15}bx^{15/2}(2aB+Ab) + \frac{2}{9}ax^{9/2}(aB+2Ab) + \frac{2}{21}b^2Bx^{21/2}$$

```
[In] Int[Sqrt[x]*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*a^2*A*x^(3/2))/3 + (2*a*(2*A*b + a*B)*x^(9/2))/9 + (2*b*(A*b + 2*a*B)*x^
(15/2))/15 + (2*b^2*B*x^(21/2))/21
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(a^2A\sqrt{x} + a(2Ab+aB)x^{7/2} + b(Ab+2aB)x^{13/2} + b^2Bx^{19/2}\right) dx$$
$$= \frac{2}{3}a^2Ax^{3/2} + \frac{2}{9}a(2Ab+aB)x^{9/2} + \frac{2}{15}b(Ab+2aB)x^{15/2} + \frac{2}{21}b^2Bx^{21/2}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \sqrt{x}\left(a+bx^3\right)^2\left(A+Bx^3\right) dx = \frac{2}{315}x^{3/2}\left(35a^2\left(3A+Bx^3\right)+14abx^3\left(5A+3Bx^3\right) + 3b^2x^6\left(7A+5Bx^3\right)\right)$$

```
[In] Integrate[Sqrt[x]*(a + b*x^3)^2*(A + B*x^3),x]
[Out] (2*x^(3/2)*(35*a^2*(3*A + B*x^3) + 14*a*b*x^3*(5*A + 3*B*x^3) + 3*b^2*x^6*(
7*A + 5*B*x^3)))/315
```

Maple [A] (verified)

Time = 4.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2b^2Bx^{\frac{21}{2}}}{21}+\frac{2(b^2A+2abB)x^{\frac{15}{2}}}{15}+\frac{2(2abA+a^2B)x^{\frac{9}{2}}}{9}+\frac{2a^2Ax^{\frac{3}{2}}}{3}$	52
default	$\frac{2b^2Bx^{\frac{21}{2}}}{21}+\frac{2(b^2A+2abB)x^{\frac{15}{2}}}{15}+\frac{2(2abA+a^2B)x^{\frac{9}{2}}}{9}+\frac{2a^2Ax^{\frac{3}{2}}}{3}$	52
gosper	$\frac{2x^{\frac{3}{2}}(15b^2Bx^9+21Ab^2x^6+42Bx^6ab+70aAbx^3+35a^2Bx^3+105a^2A)}{315}$	56
trager	$\frac{2x^{\frac{3}{2}}(15b^2Bx^9+21Ab^2x^6+42Bx^6ab+70aAbx^3+35a^2Bx^3+105a^2A)}{315}$	56
risch	$\frac{2x^{\frac{3}{2}}(15b^2Bx^9+21Ab^2x^6+42Bx^6ab+70aAbx^3+35a^2Bx^3+105a^2A)}{315}$	56

```
[In] int((b*x^3+a)^2*(B*x^3+A)*x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/21*b^2*B*x^(21/2)+2/15*(A*b^2+2*B*a*b)*x^(15/2)+2/9*(2*A*a*b+B*a^2)*x^(9/
2)+2/3*a^2*A*x^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int \sqrt{x}(a+bx^3)^2(A+Bx^3)\,dx = \frac{2}{315}\left(15\,Bb^2x^{10}+21\left(2\,Bab+Ab^2\right)x^7+35\left(Ba^2+2\,Aab\right)x^4+105\,Aa^2x\right)\sqrt{x}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(15*B*b^2*x^10 + 21*(2*B*a*b + A*b^2)*x^7 + 35*(B*a^2 + 2*A*a*b)*x^4
+ 105*A*a^2*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int \sqrt{x}(a+bx^3)^2(A+Bx^3)\,dx = \frac{2Aa^2x^{\frac{3}{2}}}{3}+\frac{4Aabx^{\frac{9}{2}}}{9}+\frac{2Ab^2x^{\frac{15}{2}}}{15}+\frac{2Ba^2x^{\frac{9}{2}}}{9}+\frac{4Babx^{\frac{15}{2}}}{15}+\frac{2Bb^2x^{\frac{21}{2}}}{21}$$

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)*x**(1/2),x)
```

```
[Out] 2*A*a**2*x**(3/2)/3 + 4*A*a*b*x**(9/2)/9 + 2*A*b**2*x**(15/2)/15 + 2*B*a**2
*x**(9/2)/9 + 4*B*a*b*x**(15/2)/15 + 2*B*b**2*x**(21/2)/21
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \sqrt{x}(a+bx^3)^2(A+Bx^3)\,dx = \frac{2}{21}\,Bb^2x^{\frac{21}{2}}+\frac{2}{15}\left(2\,Bab+Ab^2\right)x^{\frac{15}{2}}+\frac{2}{9}\left(Ba^2+2\,Aab\right)x^{\frac{9}{2}}+\frac{2}{3}\,Aa^2x^{\frac{3}{2}}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/21*B*b^2*x^(21/2) + 2/15*(2*B*a*b + A*b^2)*x^(15/2) + 2/9*(B*a^2 + 2*A*a*
b)*x^(9/2) + 2/3*A*a^2*x^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \sqrt{x}(a+bx^3)^2 (A+Bx^3)\, dx = \frac{2}{21} Bb^2x^{\frac{21}{2}} + \frac{4}{15} Babx^{\frac{15}{2}} + \frac{2}{15} Ab^2x^{\frac{15}{2}} + \frac{2}{9} Ba^2x^{\frac{9}{2}} + \frac{4}{9} Aabx^{\frac{9}{2}} + \frac{2}{3} Aa^2x^{\frac{3}{2}}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/21*B*b^2*x^(21/2) + 4/15*B*a*b*x^(15/2) + 2/15*A*b^2*x^(15/2) + 2/9*B*a^2
*x^(9/2) + 4/9*A*a*b*x^(9/2) + 2/3*A*a^2*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \sqrt{x}(a+bx^3)^2 (A+Bx^3)\, dx = x^{9/2}\left(\frac{2\,B\,a^2}{9}+\frac{4\,A\,b\,a}{9}\right) + x^{15/2}\left(\frac{2\,A\,b^2}{15}+\frac{4\,B\,a\,b}{15}\right)+\frac{2\,A\,a^2\,x^{3/2}}{3}+\frac{2\,B\,b^2\,x^{21/2}}{21}$$

```
[In] int(x^(1/2)*(A + B*x^3)*(a + b*x^3)^2,x)
```

```
[Out] x^(9/2)*((2*B*a^2)/9 + (4*A*a*b)/9) + x^(15/2)*((2*A*b^2)/15 + (4*B*a*b)/15
) + (2*A*a^2*x^(3/2))/3 + (2*B*b^2*x^(21/2))/21
```

3.143 $\int \frac{(a+bx^3)^2(A+Bx^3)}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 61

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{\sqrt{x}}\,dx = 2a^2A\sqrt{x}+\frac{2}{7}a(2Ab+aB)x^{7/2} + \frac{2}{13}b(Ab+2aB)x^{13/2}+\frac{2}{19}b^2Bx^{19/2}$$

```
[Out] 2/7*a*(2*A*b+B*a)*x^(7/2)+2/13*b*(A*b+2*B*a)*x^(13/2)+2/19*b^2*B*x^(19/2)+2
*a^2*A*x^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {459}

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{\sqrt{x}}\,dx = 2a^2A\sqrt{x} + \frac{2}{13}bx^{13/2}(2aB+Ab)+\frac{2}{7}ax^{7/2}(aB+2Ab)+\frac{2}{19}b^2Bx^{19/2}$$

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/Sqrt[x],x]
```

```
[Out] 2*a^2*A*Sqrt[x] + (2*a*(2*A*b + a*B)*x^(7/2))/7 + (2*b*(A*b + 2*a*B)*x^(13/
2))/13 + (2*b^2*B*x^(19/2))/19
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a^2A}{\sqrt{x}} + a(2Ab+aB)x^{5/2} + b(Ab+2aB)x^{11/2} + b^2Bx^{17/2}\right) dx$$
$$= 2a^2A\sqrt{x} + \frac{2}{7}a(2Ab+aB)x^{7/2} + \frac{2}{13}b(Ab+2aB)x^{13/2} + \frac{2}{19}b^2Bx^{19/2}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{\sqrt{x}} dx = \frac{2\sqrt{x}(247a^2(7A+Bx^3)+38abx^3(13A+7Bx^3)+7b^2x^6(19A+13Bx^3))}{1729}$$

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/Sqrt[x],x]
[Out] (2*Sqrt[x]*(247*a^2*(7*A + B*x^3) + 38*a*b*x^3*(13*A + 7*B*x^3) + 7*b^2*x^6
*(19*A + 13*B*x^3)))/1729
```

Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{2b^2Bx^{\frac{19}{2}}}{19}+\frac{2(b^2A+2abB)x^{\frac{13}{2}}}{13}+\frac{2(2abA+a^2B)x^{\frac{7}{2}}}{7}+2a^2A\sqrt{x}$	52
default	$\frac{2b^2Bx^{\frac{19}{2}}}{19}+\frac{2(b^2A+2abB)x^{\frac{13}{2}}}{13}+\frac{2(2abA+a^2B)x^{\frac{7}{2}}}{7}+2a^2A\sqrt{x}$	52
trager	$\left(\frac{2}{19}b^2Bx^9+\frac{2}{13}Ab^2x^6+\frac{4}{13}Bx^6ab+\frac{4}{7}aAbx^3+\frac{2}{7}a^2Bx^3+2a^2A\right)\sqrt{x}$	55
gosper	$\frac{2\sqrt{x}\,(91b^2Bx^9+133Ab^2x^6+266Bx^6ab+494aAbx^3+247a^2Bx^3+1729a^2A)}{1729}$	56
risch	$\frac{2\sqrt{x}\,(91b^2Bx^9+133Ab^2x^6+266Bx^6ab+494aAbx^3+247a^2Bx^3+1729a^2A)}{1729}$	56

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/19*b^2*B*x^(19/2)+2/13*(A*b^2+2*B*a*b)*x^(13/2)+2/7*(2*A*a*b+B*a^2)*x^(7/
2)+2*a^2*A*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{\sqrt{x}}\,dx = \frac{2}{1729}\left(91\,Bb^2x^9+133\left(2\,Bab+Ab^2\right)x^6+247\left(Ba^2+2\,Aab\right)x^3+1729\,Aa^2\right)\sqrt{x}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(1/2),x, algorithm="fricas")
[Out] 2/1729*(91*B*b^2*x^9 + 133*(2*B*a*b + A*b^2)*x^6 + 247*(B*a^2 + 2*A*a*b)*x^
3 + 1729*A*a^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{\sqrt{x}}\,dx = 2Aa^2\sqrt{x}+\frac{4Aabx^{\frac{7}{2}}}{7}+\frac{2Ab^2x^{\frac{13}{2}}}{13}+\frac{2Ba^2x^{\frac{7}{2}}}{7}+\frac{4Babx^{\frac{13}{2}}}{13}+\frac{2Bb^2x^{\frac{19}{2}}}{19}$$

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**(1/2),x)
[Out] 2*A*a**2*sqrt(x) + 4*A*a*b*x**(7/2)/7 + 2*A*b**2*x**(13/2)/13 + 2*B*a**2*x*
*(7/2)/7 + 4*B*a*b*x**(13/2)/13 + 2*B*b**2*x**(19/2)/19
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{\sqrt{x}}\,dx = \frac{2}{19}\,Bb^2x^{\frac{19}{2}}+\frac{2}{13}\left(2\,Bab+Ab^2\right)x^{\frac{13}{2}}+\frac{2}{7}\left(Ba^2+2\,Aab\right)x^{\frac{7}{2}}+2\,Aa^2\sqrt{x}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(1/2),x, algorithm="maxima")
[Out] 2/19*B*b^2*x^(19/2) + 2/13*(2*B*a*b + A*b^2)*x^(13/2) + 2/7*(B*a^2 + 2*A*a*
b)*x^(7/2) + 2*A*a^2*sqrt(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{\sqrt{x}}\,dx = \frac{2}{19}\,Bb^2x^{\frac{19}{2}} + \frac{4}{13}\,Babx^{\frac{13}{2}} + \frac{2}{13}\,Ab^2x^{\frac{13}{2}} + \frac{2}{7}\,Ba^2x^{\frac{7}{2}} + \frac{4}{7}\,Aabx^{\frac{7}{2}} + 2\,Aa^2\sqrt{x}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/19*B*b^2*x^(19/2) + 4/13*B*a*b*x^(13/2) + 2/13*A*b^2*x^(13/2) + 2/7*B*a^2
*x^(7/2) + 4/7*A*a*b*x^(7/2) + 2*A*a^2*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{\sqrt{x}}\,dx = x^{7/2}\left(\frac{2\,B\,a^2}{7}+\frac{4\,A\,b\,a}{7}\right) + x^{13/2}\left(\frac{2\,A\,b^2}{13}+\frac{4\,B\,a\,b}{13}\right) + 2\,A\,a^2\,\sqrt{x} + \frac{2\,B\,b^2\,x^{19/2}}{19}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^(1/2),x)
```

```
[Out] x^(7/2)*((2*B*a^2)/7 + (4*A*a*b)/7) + x^(13/2)*((2*A*b^2)/13 + (4*B*a*b)/13
) + 2*A*a^2*x^(1/2) + (2*B*b^2*x^(19/2))/19
```

3.144 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{3/2}} dx$

Optimal result

Integrand size = 22, antiderivative size = 61

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{3/2}} dx = -\frac{2a^2A}{\sqrt{x}} + \frac{2}{5}a(2Ab+aB)x^{5/2} + \frac{2}{11}b(Ab+2aB)x^{11/2} + \frac{2}{17}b^2Bx^{17/2}$$

```
[Out] 2/5*a*(2*A*b+B*a)*x^(5/2)+2/11*b*(A*b+2*B*a)*x^(11/2)+2/17*b^2*B*x^(17/2)-2
*a^2*A/x^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {459}

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{3/2}} dx = -\frac{2a^2A}{\sqrt{x}} + \frac{2}{11}bx^{11/2}(2aB+Ab) + \frac{2}{5}ax^{5/2}(aB+2Ab) + \frac{2}{17}b^2Bx^{17/2}$$

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^(3/2),x]
```

```
[Out] (-2*a^2*A)/Sqrt[x] + (2*a*(2*A*b + a*B)*x^(5/2))/5 + (2*b*(A*b + 2*a*B)*x^(
11/2))/11 + (2*b^2*B*x^(17/2))/17
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a^2A}{x^{3/2}} + a(2Ab+aB)x^{3/2} + b(Ab+2aB)x^{9/2} + b^2Bx^{15/2}\right)\,dx$$
$$= -\frac{2a^2A}{\sqrt{x}} + \frac{2}{5}a(2Ab+aB)x^{5/2} + \frac{2}{11}b(Ab+2aB)x^{11/2} + \frac{2}{17}b^2Bx^{17/2}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{3/2}}\,dx = -\frac{2(935a^2A - 374aAbx^3 - 187a^2Bx^3 - 85Ab^2x^6 - 170abBx^6 - 55b^2Bx^9)}{935\sqrt{x}}$$

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^(3/2),x]
[Out] (-2*(935*a^2*A - 374*a*A*b*x^3 - 187*a^2*B*x^3 - 85*A*b^2*x^6 - 170*a*b*B*x
^6 - 55*b^2*B*x^9))/(935*Sqrt[x])
```

Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{2b^2Bx^{\frac{17}{2}}}{17} + \frac{2Ab^2x^{\frac{11}{2}}}{11} + \frac{4Babx^{\frac{11}{2}}}{11} + \frac{4Aabx^{\frac{5}{2}}}{5} + \frac{2Ba^2x^{\frac{5}{2}}}{5} - \frac{2a^2A}{\sqrt{x}}$	54
default	$\frac{2b^2Bx^{\frac{17}{2}}}{17} + \frac{2Ab^2x^{\frac{11}{2}}}{11} + \frac{4Babx^{\frac{11}{2}}}{11} + \frac{4Aabx^{\frac{5}{2}}}{5} + \frac{2Ba^2x^{\frac{5}{2}}}{5} - \frac{2a^2A}{\sqrt{x}}$	54
gosper	$-\frac{2(-55b^2Bx^9-85Ab^2x^6-170Bx^6ab-374aAbx^3-187a^2Bx^3+935a^2A)}{935\sqrt{x}}$	56
trager	$-\frac{2(-55b^2Bx^9-85Ab^2x^6-170Bx^6ab-374aAbx^3-187a^2Bx^3+935a^2A)}{935\sqrt{x}}$	56
risch	$-\frac{2(-55b^2Bx^9-85Ab^2x^6-170Bx^6ab-374aAbx^3-187a^2Bx^3+935a^2A)}{935\sqrt{x}}$	56

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/17*b^2*B*x^(17/2)+2/11*A*b^2*x^(11/2)+4/11*B*a*b*x^(11/2)+4/5*A*a*b*x^(5/
2)+2/5*B*a^2*x^(5/2)-2*a^2*A/x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{3/2}}\,dx = \frac{2\,(55\,Bb^2x^9 + 85\,(2\,Bab + Ab^2)x^6 + 187\,(Ba^2 + 2\,Aab)x^3 - 935\,Aa^2)}{935\,\sqrt{x}}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(3/2),x, algorithm="fricas")
[Out] 2/935*(55*B*b^2*x^9 + 85*(2*B*a*b + A*b^2)*x^6 + 187*(B*a^2 + 2*A*a*b)*x^3
- 935*A*a^2)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{3/2}}\,dx = -\frac{2Aa^2}{\sqrt{x}} + \frac{4Aabx^{\frac{5}{2}}}{5} + \frac{2Ab^2x^{\frac{11}{2}}}{11} + \frac{2Ba^2x^{\frac{5}{2}}}{5} + \frac{4Babx^{\frac{11}{2}}}{11} + \frac{2Bb^2x^{\frac{17}{2}}}{17}$$

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**(3/2),x)
[Out] -2*A*a**2/sqrt(x) + 4*A*a*b*x**(5/2)/5 + 2*A*b**2*x**(11/2)/11 + 2*B*a**2*x
**(5/2)/5 + 4*B*a*b*x**(11/2)/11 + 2*B*b**2*x**(17/2)/17
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{3/2}}\,dx = \frac{2}{17}\,Bb^2x^{\frac{17}{2}} + \frac{2}{11}\,\left(2\,Bab + Ab^2\right)x^{\frac{11}{2}} + \frac{2}{5}\,\left(Ba^2 + 2\,Aab\right)x^{\frac{5}{2}} - \frac{2\,Aa^2}{\sqrt{x}}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(3/2),x, algorithm="maxima")
[Out] 2/17*B*b^2*x^(17/2) + 2/11*(2*B*a*b + A*b^2)*x^(11/2) + 2/5*(B*a^2 + 2*A*a*
b)*x^(5/2) - 2*A*a^2/sqrt(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{3/2}}\,dx = \frac{2}{17}\,Bb^2x^{\frac{17}{2}} + \frac{4}{11}\,Babx^{\frac{11}{2}} + \frac{2}{11}\,Ab^2x^{\frac{11}{2}} + \frac{2}{5}\,Ba^2x^{\frac{5}{2}} + \frac{4}{5}\,Aabx^{\frac{5}{2}} - \frac{2\,Aa^2}{\sqrt{x}}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/17*B*b^2*x^(17/2) + 4/11*B*a*b*x^(11/2) + 2/11*A*b^2*x^(11/2) + 2/5*B*a^2
*x^(5/2) + 4/5*A*a*b*x^(5/2) - 2*A*a^2/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{3/2}}\,dx = x^{5/2}\left(\frac{2\,B\,a^2}{5} + \frac{4\,A\,b\,a}{5}\right) + x^{11/2}\left(\frac{2\,A\,b^2}{11} + \frac{4\,B\,a\,b}{11}\right) - \frac{2\,A\,a^2}{\sqrt{x}} + \frac{2\,B\,b^2\,x^{17/2}}{17}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^(3/2),x)
```

```
[Out] x^(5/2)*((2*B*a^2)/5 + (4*A*a*b)/5) + x^(11/2)*((2*A*b^2)/11 + (4*B*a*b)/11
) - (2*A*a^2)/x^(1/2) + (2*B*b^2*x^(17/2))/17
```

3.145 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 63

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{5/2}}\,dx = -\frac{2a^2A}{3x^{3/2}}+\frac{2}{3}a(2Ab+aB)x^{3/2}+\frac{2}{9}b(Ab+2aB)x^{9/2}+\frac{2}{15}b^2Bx^{15/2}$$

```
[Out] -2/3*a^2*A/x^(3/2)+2/3*a*(2*A*b+B*a)*x^(3/2)+2/9*b*(A*b+2*B*a)*x^(9/2)+2/15
*b^2*B*x^(15/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {459}

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{5/2}}\,dx = -\frac{2a^2A}{3x^{3/2}}+\frac{2}{9}bx^{9/2}(2aB+Ab)+\frac{2}{3}ax^{3/2}(aB+2Ab)+\frac{2}{15}b^2Bx^{15/2}$$

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^(5/2),x]
```

```
[Out] (-2*a^2*A)/(3*x^(3/2)) + (2*a*(2*A*b + a*B)*x^(3/2))/3 + (2*b*(A*b + 2*a*B)
*x^(9/2))/9 + (2*b^2*B*x^(15/2))/15
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a^2A}{x^{5/2}} + a(2Ab+aB)\sqrt{x} + b(Ab+2aB)x^{7/2} + b^2Bx^{13/2}\right)\,dx$$
$$= -\frac{2a^2A}{3x^{3/2}} + \frac{2}{3}a(2Ab+aB)x^{3/2} + \frac{2}{9}b(Ab+2aB)x^{9/2} + \frac{2}{15}b^2Bx^{15/2}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{5/2}}\,dx = \frac{2(-15a^2A+30aAbx^3+15a^2Bx^3+5Ab^2x^6+10abBx^6+3b^2Bx^9)}{45x^{3/2}}$$

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^(5/2),x]
```

```
[Out] (2*(-15*a^2*A + 30*a*A*b*x^3 + 15*a^2*B*x^3 + 5*A*b^2*x^6 + 10*a*b*B*x^6 +
3*b^2*B*x^9))/(45*x^(3/2))
```

Maple [A] (verified)

Time = 4.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{2b^2Bx^{\frac{15}{2}}}{15}+\frac{2Ab^2x^{\frac{9}{2}}}{9}+\frac{4Babx^{\frac{9}{2}}}{9}+\frac{4Aabx^{\frac{3}{2}}}{3}+\frac{2Ba^2x^{\frac{3}{2}}}{3}-\frac{2a^2A}{3x^{\frac{3}{2}}}$	54
default	$\frac{2b^2Bx^{\frac{15}{2}}}{15}+\frac{2Ab^2x^{\frac{9}{2}}}{9}+\frac{4Babx^{\frac{9}{2}}}{9}+\frac{4Aabx^{\frac{3}{2}}}{3}+\frac{2Ba^2x^{\frac{3}{2}}}{3}-\frac{2a^2A}{3x^{\frac{3}{2}}}$	54
gosper	$-\frac{2(-3b^2Bx^9-5Ab^2x^6-10Bx^6ab-30aAbx^3-15a^2Bx^3+15a^2A)}{45x^{\frac{3}{2}}}$	56
trager	$-\frac{2(-3b^2Bx^9-5Ab^2x^6-10Bx^6ab-30aAbx^3-15a^2Bx^3+15a^2A)}{45x^{\frac{3}{2}}}$	56
risch	$-\frac{2(-3b^2Bx^9-5Ab^2x^6-10Bx^6ab-30aAbx^3-15a^2Bx^3+15a^2A)}{45x^{\frac{3}{2}}}$	56

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*b^2*B*x^(15/2)+2/9*A*b^2*x^(9/2)+4/9*B*a*b*x^(9/2)+4/3*A*a*b*x^(3/2)+2
/3*B*a^2*x^(3/2)-2/3*a^2*A/x^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{5/2}}\,dx = \frac{2\,(3\,Bb^2x^9 + 5\,(2\,Bab + Ab^2)x^6 + 15\,(Ba^2 + 2\,Aab)x^3 - 15\,Aa^2)}{45\,x^{\frac{3}{2}}}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45*(3*B*b^2*x^9 + 5*(2*B*a*b + A*b^2)*x^6 + 15*(B*a^2 + 2*A*a*b)*x^3 - 15
*A*a^2)/x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{5/2}}\,dx = -\frac{2Aa^2}{3x^{\frac{3}{2}}} + \frac{4Aabx^{\frac{3}{2}}}{3} + \frac{2Ab^2x^{\frac{9}{2}}}{9} + \frac{2Ba^2x^{\frac{3}{2}}}{3} + \frac{4Babx^{\frac{9}{2}}}{9} + \frac{2Bb^2x^{\frac{15}{2}}}{15}$$

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**(5/2),x)
```

```
[Out] -2*A*a**2/(3*x**(3/2)) + 4*A*a*b*x**(3/2)/3 + 2*A*b**2*x**(9/2)/9 + 2*B*a**
2*x**(3/2)/3 + 4*B*a*b*x**(9/2)/9 + 2*B*b**2*x**(15/2)/15
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{5/2}}\,dx = \frac{2}{15}\,Bb^2x^{\frac{15}{2}} + \frac{2}{9}\left(2\,Bab + Ab^2\right)x^{\frac{9}{2}} + \frac{2}{3}\left(Ba^2 + 2\,Aab\right)x^{\frac{3}{2}} - \frac{2\,Aa^2}{3\,x^{\frac{3}{2}}}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*B*b^2*x^(15/2) + 2/9*(2*B*a*b + A*b^2)*x^(9/2) + 2/3*(B*a^2 + 2*A*a*b)
*x^(3/2) - 2/3*A*a^2/x^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{5/2}}\,dx = \frac{2}{15}\,Bb^2x^{\frac{15}{2}} + \frac{4}{9}\,Babx^{\frac{9}{2}} + \frac{2}{9}\,Ab^2x^{\frac{9}{2}} + \frac{2}{3}\,Ba^2x^{\frac{3}{2}} + \frac{4}{3}\,Aabx^{\frac{3}{2}} - \frac{2\,Aa^2}{3\,x^{\frac{3}{2}}}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/15*B*b^2*x^(15/2) + 4/9*B*a*b*x^(9/2) + 2/9*A*b^2*x^(9/2) + 2/3*B*a^2*x^(
3/2) + 4/3*A*a*b*x^(3/2) - 2/3*A*a^2/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{5/2}}\,dx = x^{3/2}\left(\frac{2\,B\,a^2}{3} + \frac{4\,A\,b\,a}{3}\right) + x^{9/2}\left(\frac{2\,A\,b^2}{9} + \frac{4\,B\,a\,b}{9}\right) - \frac{2\,A\,a^2}{3\,x^{3/2}} + \frac{2\,B\,b^2\,x^{15/2}}{15}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^(5/2),x)
```

```
[Out] x^(3/2)*((2*B*a^2)/3 + (4*A*a*b)/3) + x^(9/2)*((2*A*b^2)/9 + (4*B*a*b)/9) -
 (2*A*a^2)/(3*x^(3/2)) + (2*B*b^2*x^(15/2))/15
```

3.146 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{7/2}} dx$

Optimal result

Integrand size = 22, antiderivative size = 61

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{7/2}} dx = -\frac{2a^2A}{5x^{5/2}}+2a(2Ab+aB)\sqrt{x}+\frac{2}{7}b(Ab+2aB)x^{7/2}+\frac{2}{13}b^2Bx^{13/2}$$

```
[Out] -2/5*a^2*A/x^(5/2)+2/7*b*(A*b+2*B*a)*x^(7/2)+2/13*b^2*B*x^(13/2)+2*a*(2*A*b
+B*a)*x^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {459}

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{7/2}} dx = -\frac{2a^2A}{5x^{5/2}}+\frac{2}{7}bx^{7/2}(2aB+Ab)+2a\sqrt{x}(aB+2Ab)+\frac{2}{13}b^2Bx^{13/2}$$

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^(7/2),x]
```

```
[Out] (-2*a^2*A)/(5*x^(5/2)) + 2*a*(2*A*b + a*B)*Sqrt[x] + (2*b*(A*b + 2*a*B)*x^(
7/2))/7 + (2*b^2*B*x^(13/2))/13
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a^2A}{x^{7/2}} + \frac{a(2Ab+aB)}{\sqrt{x}} + b(Ab+2aB)x^{5/2} + b^2Bx^{11/2}\right) dx$$
$$= -\frac{2a^2A}{5x^{5/2}} + 2a(2Ab+aB)\sqrt{x} + \frac{2}{7}b(Ab+2aB)x^{7/2} + \frac{2}{13}b^2Bx^{13/2}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{7/2}} dx = -\frac{2(91a^2A - 910aAbx^3 - 455a^2Bx^3 - 65Ab^2x^6 - 130abBx^6 - 35b^2Bx^9)}{455x^{5/2}}$$

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^(7/2),x]
[Out] (-2*(91*a^2*A - 910*a*A*b*x^3 - 455*a^2*B*x^3 - 65*A*b^2*x^6 - 130*a*b*B*x^
6 - 35*b^2*B*x^9))/(455*x^(5/2))
```

Maple [A] (verified)

Time = 4.34 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{2b^2Bx^{\frac{13}{2}}}{13} + \frac{2Ab^2x^{\frac{7}{2}}}{7} + \frac{4Babx^{\frac{7}{2}}}{7} + 4Aab\sqrt{x} + 2Ba^2\sqrt{x} - \frac{2a^2A}{5x^{\frac{5}{2}}}$	54
default	$\frac{2b^2Bx^{\frac{13}{2}}}{13} + \frac{2Ab^2x^{\frac{7}{2}}}{7} + \frac{4Babx^{\frac{7}{2}}}{7} + 4Aab\sqrt{x} + 2Ba^2\sqrt{x} - \frac{2a^2A}{5x^{\frac{5}{2}}}$	54
gosper	$-\frac{2(-35b^2Bx^9-65Ab^2x^6-130Bx^6ab-910aAbx^3-455a^2Bx^3+91a^2A)}{455x^{\frac{5}{2}}}$	56
trager	$-\frac{2(-35b^2Bx^9-65Ab^2x^6-130Bx^6ab-910aAbx^3-455a^2Bx^3+91a^2A)}{455x^{\frac{5}{2}}}$	56
risch	$-\frac{2(-35b^2Bx^9-65Ab^2x^6-130Bx^6ab-910aAbx^3-455a^2Bx^3+91a^2A)}{455x^{\frac{5}{2}}}$	56

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^(7/2),x,method=_RETURNVERBOSE)
[Out] 2/13*b^2*B*x^(13/2)+2/7*A*b^2*x^(7/2)+4/7*B*a*b*x^(7/2)+4*A*a*b*x^(1/2)+2*B
*a^2*x^(1/2)-2/5*a^2*A/x^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{7/2}} \, dx = \frac{2 \left(35 \, B b^2 x^9 + 65 \, (2 \, B a b + A b^2) x^6 + 455 \, (B a^2 + 2 \, A a b) x^3 - 91 \, A a^2\right)}{455 \, x^{\frac{5}{2}}}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/455*(35*B*b^2*x^9 + 65*(2*B*a*b + A*b^2)*x^6 + 455*(B*a^2 + 2*A*a*b)*x^3
- 91*A*a^2)/x^(5/2)
```

Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{7/2}} \, dx = -\frac{2Aa^2}{5x^{\frac{5}{2}}} + 4Aab\sqrt{x} + \frac{2Ab^2x^{\frac{7}{2}}}{7} + 2Ba^2\sqrt{x} + \frac{4Babx^{\frac{7}{2}}}{7} + \frac{2Bb^2x^{\frac{13}{2}}}{13}$$

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**(7/2),x)
```

```
[Out] -2*A*a**2/(5*x**(5/2)) + 4*A*a*b*sqrt(x) + 2*A*b**2*x**(7/2)/7 + 2*B*a**2*s
qrt(x) + 4*B*a*b*x**(7/2)/7 + 2*B*b**2*x**(13/2)/13
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{7/2}} \, dx = \frac{2}{13} \, B b^2 x^{\frac{13}{2}} + \frac{2}{7} \, \left(2 \, B a b + A b^2\right) x^{\frac{7}{2}} + 2 \, \left(B a^2 + 2 \, A a b\right) \sqrt{x} - \frac{2 \, A a^2}{5 \, x^{\frac{5}{2}}}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/13*B*b^2*x^(13/2) + 2/7*(2*B*a*b + A*b^2)*x^(7/2) + 2*(B*a^2 + 2*A*a*b)*s
qrt(x) - 2/5*A*a^2/x^(5/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{7/2}}\,dx = \frac{2}{13}\,Bb^2x^{\frac{13}{2}} + \frac{4}{7}\,Babx^{\frac{7}{2}} + \frac{2}{7}\,Ab^2x^{\frac{7}{2}} + 2\,Ba^2\sqrt{x} + 4\,Aab\sqrt{x} - \frac{2\,Aa^2}{5\,x^{\frac{5}{2}}}$$

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/13*B*b^2*x^(13/2) + 4/7*B*a*b*x^(7/2) + 2/7*A*b^2*x^(7/2) + 2*B*a^2*sqrt(
x) + 4*A*a*b*sqrt(x) - 2/5*A*a^2/x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{7/2}}\,dx = \sqrt{x}\,\left(2\,B\,a^2 + 4\,A\,b\,a\right) + x^{7/2}\left(\frac{2\,A\,b^2}{7} + \frac{4\,B\,a\,b}{7}\right) - \frac{2\,A\,a^2}{5\,x^{5/2}} + \frac{2\,B\,b^2\,x^{13/2}}{13}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^(7/2),x)
```

```
[Out] x^(1/2)*(2*B*a^2 + 4*A*a*b) + x^(7/2)*((2*A*b^2)/7 + (4*B*a*b)/7) - (2*A*a^
2)/(5*x^(5/2)) + (2*B*b^2*x^(13/2))/13
```

3.147 $\int x^{7/2}(a+bx^3)^3\left(A+Bx^3\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int x^{7/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx=\frac{2}{9}a^3Ax^{9/2}+\frac{2}{15}a^2(3Ab+aB)x^{15/2}+\frac{2}{7}ab(Ab+aB)x^{21/2}+\frac{2}{27}b^2(Ab+3aB)x^{27/2}+\frac{2}{33}b^3Bx^{33/2}$$

```
[Out] 2/9*a^3*A*x^(9/2)+2/15*a^2*(3*A*b+B*a)*x^(15/2)+2/7*a*b*(A*b+B*a)*x^(21/2)+
2/27*b^2*(A*b+3*B*a)*x^(27/2)+2/33*b^3*B*x^(33/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {459}

$$\int x^{7/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx=\frac{2}{9}a^3Ax^{9/2}+\frac{2}{15}a^2x^{15/2}(aB+3Ab)+\frac{2}{27}b^2x^{27/2}(3aB+Ab)+\frac{2}{7}abx^{21/2}(aB+Ab)+\frac{2}{33}b^3Bx^{33/2}$$

```
[In] Int[x^(7/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*a^3*A*x^(9/2))/9 + (2*a^2*(3*A*b + a*B)*x^(15/2))/15 + (2*a*b*(A*b + a*B
)*x^(21/2))/7 + (2*b^2*(A*b + 3*a*B)*x^(27/2))/27 + (2*b^3*B*x^(33/2))/33
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(a^3 A x^{7/2} + a^2(3Ab+aB)x^{13/2} + 3ab(Ab+aB)x^{19/2} + b^2(Ab+3aB)x^{25/2} + b^3 B x^{31/2}\right) dx$$
$$= \frac{2}{9}a^3 A x^{9/2} + \frac{2}{15}a^2(3Ab+aB)x^{15/2} + \frac{2}{7}ab(Ab+aB)x^{21/2} + \frac{2}{27}b^2(Ab+3aB)x^{27/2} + \frac{2}{33}b^3 B x^{33/2}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int x^{7/2}\left(a+bx^3\right)^3 \left(A+Bx^3\right) dx = \frac{2x^{9/2}(231a^3(5A+3Bx^3)+297a^2bx^3(7A+5Bx^3)+165ab^2x^6(9A+7Bx^3)+35b^3x^9(11A+9B$$

```
[In] Integrate[x^(7/2)*(a + b*x^3)^3*(A + B*x^3),x]
[Out] (2*x^(9/2)*(231*a^3*(5*A + 3*B*x^3) + 297*a^2*b*x^3*(7*A + 5*B*x^3) + 165*a
*b^2*x^6*(9*A + 7*B*x^3) + 35*b^3*x^9*(11*A + 9*B*x^3)))/10395
```

Maple [A] (verified)

Time = 4.53 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{2b^3 B x^{\frac{33}{2}}}{33}+\frac{2(b^3A+3a b^2 B)x^{\frac{27}{2}}}{27}+\frac{2(3a b^2 A+3a^2 bB)x^{\frac{21}{2}}}{21}+\frac{2(3a^2bA+a^3B)x^{\frac{15}{2}}}{15}+\frac{2a^3 A x^{\frac{9}{2}}}{9}$	76
default	$\frac{2b^3 B x^{\frac{33}{2}}}{33}+\frac{2(b^3A+3a b^2 B)x^{\frac{27}{2}}}{27}+\frac{2(3a b^2 A+3a^2 bB)x^{\frac{21}{2}}}{21}+\frac{2(3a^2bA+a^3B)x^{\frac{15}{2}}}{15}+\frac{2a^3 A x^{\frac{9}{2}}}{9}$	76
gosper	$\frac{2x^{\frac{9}{2}}(315B b^3x^{12}+385A x^9b^3+1155B x^9a b^2+1485A x^6a b^2+1485B x^6a^2b+2079A x^3a^2b+693a^3B x^3+1155a^3A)}{10395}$	80
trager	$\frac{2x^{\frac{9}{2}}(315B b^3x^{12}+385A x^9b^3+1155B x^9a b^2+1485A x^6a b^2+1485B x^6a^2b+2079A x^3a^2b+693a^3B x^3+1155a^3A)}{10395}$	80
risch	$\frac{2x^{\frac{9}{2}}(315B b^3x^{12}+385A x^9b^3+1155B x^9a b^2+1485A x^6a b^2+1485B x^6a^2b+2079A x^3a^2b+693a^3B x^3+1155a^3A)}{10395}$	80

```
[In] int(x^(7/2)*(b*x^3+a)^3*(B*x^3+A),x,method=_RETURNVERBOSE)
[Out] 2/33*b^3*B*x^(33/2)+2/27*(A*b^3+3*B*a*b^2)*x^(27/2)+2/21*(3*A*a*b^2+3*B*a^2
*b)*x^(21/2)+2/15*(3*A*a^2*b+B*a^3)*x^(15/2)+2/9*a^3*A*x^(9/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int x^{7/2}(a+bx^3)^3\,(A+Bx^3)\,dx=\frac{2}{10395}\left(315\,Bb^3x^{16}+385\left(3\,Bab^2+Ab^3\right)x^{13}+1485\left(Ba^2b+Aab^2\right)x^{10}+1155\,Aa^3x^4+693\,(B$$

```
[In] integrate(x^(7/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/10395*(315*B*b^3*x^16 + 385*(3*B*a*b^2 + A*b^3)*x^13 + 1485*(B*a^2*b + A*
a*b^2)*x^10 + 1155*A*a^3*x^4 + 693*(B*a^3 + 3*A*a^2*b)*x^7)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 2.50 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int x^{7/2}(a+bx^3)^3\,(A+Bx^3)\,dx=\frac{2Aa^3x^{\frac{9}{2}}}{9}+\frac{2Aa^2bx^{\frac{15}{2}}}{5}+\frac{2Aab^2x^{\frac{21}{2}}}{7}+\frac{2Ab^3x^{\frac{27}{2}}}{27}+\frac{2Ba^3x^{\frac{15}{2}}}{15}+\frac{2Ba^2bx^{\frac{21}{2}}}{7}+\frac{2Bab^2x^{\frac{27}{2}}}{9}+\frac{2Bb^3x^{\frac{33}{2}}}{33}$$

```
[In] integrate(x**(7/2)*(b*x**3+a)**3*(B*x**3+A),x)
```

```
[Out] 2*A*a**3*x**(9/2)/9 + 2*A*a**2*b*x**(15/2)/5 + 2*A*a*b**2*x**(21/2)/7 + 2*A
*b**3*x**(27/2)/27 + 2*B*a**3*x**(15/2)/15 + 2*B*a**2*b*x**(21/2)/7 + 2*B*a
*b**2*x**(27/2)/9 + 2*B*b**3*x**(33/2)/33
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int x^{7/2}(a+bx^3)^3\,(A+Bx^3)\,dx=\frac{2}{33}\,Bb^3x^{\frac{33}{2}}+\frac{2}{27}\left(3\,Bab^2+Ab^3\right)x^{\frac{27}{2}}+\frac{2}{7}\left(Ba^2b+Aab^2\right)x^{\frac{21}{2}}+\frac{2}{9}\,Aa^3x^{\frac{9}{2}}+\frac{2}{15}\left(Ba^3+3\,Aa^2b\right)x^{\frac{15}{2}}$$

```
[In] integrate(x^(7/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/33*B*b^3*x^(33/2) + 2/27*(3*B*a*b^2 + A*b^3)*x^(27/2) + 2/7*(B*a^2*b + A*
a*b^2)*x^(21/2) + 2/9*A*a^3*x^(9/2) + 2/15*(B*a^3 + 3*A*a^2*b)*x^(15/2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int x^{7/2}(a+bx^3)^3\left(A+Bx^3\right)\,dx=\frac{2}{33}\,Bb^3x^{\frac{33}{2}}+\frac{2}{9}\,Bab^2x^{\frac{27}{2}}+\frac{2}{27}\,Ab^3x^{\frac{27}{2}}+\frac{2}{7}\,Ba^2bx^{\frac{21}{2}}+\frac{2}{7}\,Aab^2x^{\frac{21}{2}}+\frac{2}{15}\,Ba^3x^{\frac{15}{2}}+\frac{2}{5}\,Aa^2bx^{\frac{15}{2}}+\frac{2}{9}\,Aa^3x^{\frac{9}{2}}$$

```
[In] integrate(x^(7/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/33*B*b^3*x^(33/2) + 2/9*B*a*b^2*x^(27/2) + 2/27*A*b^3*x^(27/2) + 2/7*B*a^
2*b*x^(21/2) + 2/7*A*a*b^2*x^(21/2) + 2/15*B*a^3*x^(15/2) + 2/5*A*a^2*b*x^(
15/2) + 2/9*A*a^3*x^(9/2)
```

Mupad [B] (verification not implemented)

Time = 6.94 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int x^{7/2}(a+bx^3)^3\left(A+Bx^3\right)\,dx=x^{15/2}\left(\frac{2\,B\,a^3}{15}+\frac{2\,A\,b\,a^2}{5}\right)+x^{27/2}\left(\frac{2\,A\,b^3}{27}+\frac{2\,B\,a\,b^2}{9}\right)+\frac{2\,A\,a^3\,x^{9/2}}{9}+\frac{2\,B\,b^3\,x^{33/2}}{33}+\frac{2\,a\,b\,x^{21/2}\,(A\,b+B\,a)}{7}$$

```
[In] int(x^(7/2)*(A + B*x^3)*(a + b*x^3)^3,x)
```

```
[Out] x^(15/2)*((2*B*a^3)/15 + (2*A*a^2*b)/5) + x^(27/2)*((2*A*b^3)/27 + (2*B*a*b
^2)/9) + (2*A*a^3*x^(9/2))/9 + (2*B*b^3*x^(33/2))/33 + (2*a*b*x^(21/2)*(A*b
 + B*a))/7
```

3.148 $\int x^{5/2}(a+bx^3)^3(A+Bx^3)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int x^{5/2}(a+bx^3)^3(A+Bx^3)\,dx=\frac{2}{7}a^3Ax^{7/2}+\frac{2}{13}a^2(3Ab+aB)x^{13/2}+\frac{6}{19}ab(Ab+aB)x^{19/2}+\frac{2}{25}b^2(Ab+3aB)x^{25/2}+\frac{2}{31}b^3Bx^{31/2}$$

```
[Out] 2/7*a^3*A*x^(7/2)+2/13*a^2*(3*A*b+B*a)*x^(13/2)+6/19*a*b*(A*b+B*a)*x^(19/2)
+2/25*b^2*(A*b+3*B*a)*x^(25/2)+2/31*b^3*B*x^(31/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}=0.045$, Rules used = {459}

$$\int x^{5/2}(a+bx^3)^3(A+Bx^3)\,dx=\frac{2}{7}a^3Ax^{7/2}+\frac{2}{13}a^2x^{13/2}(aB+3Ab)+\frac{2}{25}b^2x^{25/2}(3aB+Ab)+\frac{6}{19}abx^{19/2}(aB+Ab)+\frac{2}{31}b^3Bx^{31/2}$$

```
[In] Int[x^(5/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*a^3*A*x^(7/2))/7 + (2*a^2*(3*A*b + a*B)*x^(13/2))/13 + (6*a*b*(A*b + a*B
)*x^(19/2))/19 + (2*b^2*(A*b + 3*a*B)*x^(25/2))/25 + (2*b^3*B*x^(31/2))/31
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^3 A x^{5/2} + a^2(3Ab+aB)x^{11/2} + 3ab(Ab+aB)x^{17/2} + b^2(Ab+3aB)x^{23/2} + b^3 B x^{29/2}\right)\,dx\\ &= \frac{2}{7}a^3 A x^{7/2} + \frac{2}{13}a^2(3Ab+aB)x^{13/2} + \frac{6}{19}ab(Ab+aB)x^{19/2} + \frac{2}{25}b^2(Ab+3aB)x^{25/2} + \frac{2}{31}b^3 B x^{31/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.07

$$\int x^{5/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx = \frac{2}{91}a^3x^{7/2}\left(13A+7Bx^3\right) + \frac{6}{247}a^2bx^{13/2}\left(19A+13Bx^3\right) + \frac{6}{475}ab^2x^{19/2}\left(25A+19Bx^3\right) + \frac{2}{775}b^3x^{25/2}\left(31A+25Bx^3\right)$$

```
[In] Integrate[x^(5/2)*(a + b*x^3)^3*(A + B*x^3),x]
[Out] (2*a^3*x^(7/2)*(13*A + 7*B*x^3))/91 + (6*a^2*b*x^(13/2)*(19*A + 13*B*x^3))/
247 + (6*a*b^2*x^(19/2)*(25*A + 19*B*x^3))/475 + (2*b^3*x^(25/2)*(31*A + 25
*B*x^3))/775
```

Maple [A] (verified)

Time = 4.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2b^3 B x^{\frac{31}{2}}}{31}+\frac{2\left(b^3 A+3a b^2 B\right)x^{\frac{25}{2}}}{25}+\frac{2\left(3a b^2 A+3a^2 bB\right)x^{\frac{19}{2}}}{19}+\frac{2\left(3a^2 bA+a^3 B\right)x^{\frac{13}{2}}}{13}+\frac{2a^3 A x^{\frac{7}{2}}}{7}$
default	$\frac{2b^3 B x^{\frac{31}{2}}}{31}+\frac{2\left(b^3 A+3a b^2 B\right)x^{\frac{25}{2}}}{25}+\frac{2\left(3a b^2 A+3a^2 bB\right)x^{\frac{19}{2}}}{19}+\frac{2\left(3a^2 bA+a^3 B\right)x^{\frac{13}{2}}}{13}+\frac{2a^3 A x^{\frac{7}{2}}}{7}$
gosper	$\frac{2x^{\frac{7}{2}}\left(43225B b^3x^{12}+53599A x^9b^3+160797B x^9a b^2+211575A x^6a b^2+211575B x^6a^2b+309225A x^3a^2b+103075a^3B x^3+\right.}{1339975}$
trager	$\frac{2x^{\frac{7}{2}}\left(43225B b^3x^{12}+53599A x^9b^3+160797B x^9a b^2+211575A x^6a b^2+211575B x^6a^2b+309225A x^3a^2b+103075a^3B x^3+\right.}{1339975}$
risch	$\frac{2x^{\frac{7}{2}}\left(43225B b^3x^{12}+53599A x^9b^3+160797B x^9a b^2+211575A x^6a b^2+211575B x^6a^2b+309225A x^3a^2b+103075a^3B x^3+\right.}{1339975}$

```
[In] int(x^(5/2)*(b*x^3+a)^3*(B*x^3+A),x,method=_RETURNVERBOSE)
[Out] 2/31*b^3*B*x^(31/2)+2/25*(A*b^3+3*B*a*b^2)*x^(25/2)+2/19*(3*A*a*b^2+3*B*a^2
*b)*x^(19/2)+2/13*(3*A*a^2*b+B*a^3)*x^(13/2)+2/7*a^3*A*x^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int x^{5/2}(a+bx^3)^3(A+Bx^3)\,dx=\frac{2}{1339975}\left(43225\,Bb^3x^{15}+53599\left(3\,Bab^2+Ab^3\right)x^{12}+211575\left(Ba^2b+Aab^2\right)x^9+191425\,Aa^3x$$

[In] integrate(x^(5/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="fricas")

[Out] 2/1339975*(43225*B*b^3*x^15 + 53599*(3*B*a*b^2 + A*b^3)*x^12 + 211575*(B*a^2*b + A*a*b^2)*x^9 + 191425*A*a^3*x^3 + 103075*(B*a^3 + 3*A*a^2*b)*x^6)*sqrt(x)

Sympy [A] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int x^{5/2}(a+bx^3)^3(A+Bx^3)\,dx=\frac{2Aa^3x^{\frac{7}{2}}}{7}+\frac{6Aa^2bx^{\frac{13}{2}}}{13}+\frac{6Aab^2x^{\frac{19}{2}}}{19}+\frac{2Ab^3x^{\frac{25}{2}}}{25}+\frac{2Ba^3x^{\frac{13}{2}}}{13}+\frac{6Ba^2bx^{\frac{19}{2}}}{19}+\frac{6Bab^2x^{\frac{25}{2}}}{25}+\frac{2Bb^3x^{\frac{31}{2}}}{31}$$

[In] integrate(x**(5/2)*(b*x**3+a)**3*(B*x**3+A),x)

[Out] 2*A*a**3*x**(7/2)/7 + 6*A*a**2*b*x**(13/2)/13 + 6*A*a*b**2*x**(19/2)/19 + 2*A*b**3*x**(25/2)/25 + 2*B*a**3*x**(13/2)/13 + 6*B*a**2*b*x**(19/2)/19 + 6*B*a*b**2*x**(25/2)/25 + 2*B*b**3*x**(31/2)/31

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int x^{5/2}(a+bx^3)^3(A+Bx^3)\,dx=\frac{2}{31}\,Bb^3x^{\frac{31}{2}}+\frac{2}{25}\left(3\,Bab^2+Ab^3\right)x^{\frac{25}{2}}+\frac{6}{19}\left(Ba^2b+Aab^2\right)x^{\frac{19}{2}}+\frac{2}{7}\,Aa^3x^{\frac{7}{2}}+\frac{2}{13}\left(Ba^3+3\,Aa^2b\right)x^{\frac{13}{2}}$$

[In] integrate(x^(5/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="maxima")

[Out] 2/31*B*b^3*x^(31/2) + 2/25*(3*B*a*b^2 + A*b^3)*x^(25/2) + 6/19*(B*a^2*b + A*a*b^2)*x^(19/2) + 2/7*A*a^3*x^(7/2) + 2/13*(B*a^3 + 3*A*a^2*b)*x^(13/2)

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int x^{5/2}(a+bx^3)^3\left(A+Bx^3\right)\,dx=\frac{2}{31}\,Bb^3x^{\frac{31}{2}}+\frac{6}{25}\,Bab^2x^{\frac{25}{2}}+\frac{2}{25}\,Ab^3x^{\frac{25}{2}}+\frac{6}{19}\,Ba^2bx^{\frac{19}{2}}+\frac{6}{19}\,Aab^2x^{\frac{19}{2}}+\frac{2}{13}\,Ba^3x^{\frac{13}{2}}+\frac{6}{13}\,Aa^2bx^{\frac{13}{2}}+\frac{2}{7}\,Aa^3x^{\frac{7}{2}}$$

```
[In] integrate(x^(5/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/31*B*b^3*x^(31/2) + 6/25*B*a*b^2*x^(25/2) + 2/25*A*b^3*x^(25/2) + 6/19*B*
a^2*b*x^(19/2) + 6/19*A*a*b^2*x^(19/2) + 2/13*B*a^3*x^(13/2) + 6/13*A*a^2*b
*x^(13/2) + 2/7*A*a^3*x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int x^{5/2}(a+bx^3)^3\left(A+Bx^3\right)\,dx=x^{13/2}\left(\frac{2\,B\,a^3}{13}+\frac{6\,A\,b\,a^2}{13}\right)+x^{25/2}\left(\frac{2\,A\,b^3}{25}+\frac{6\,B\,a\,b^2}{25}\right)+\frac{2\,A\,a^3\,x^{7/2}}{7}+\frac{2\,B\,b^3\,x^{31/2}}{31}+\frac{6\,a\,b\,x^{19/2}\,(A\,b+B\,a)}{19}$$

```
[In] int(x^(5/2)*(A + B*x^3)*(a + b*x^3)^3,x)
```

```
[Out] x^(13/2)*((2*B*a^3)/13 + (6*A*a^2*b)/13) + x^(25/2)*((2*A*b^3)/25 + (6*B*a*
b^2)/25) + (2*A*a^3*x^(7/2))/7 + (2*B*b^3*x^(31/2))/31 + (6*a*b*x^(19/2)*(A
*b + B*a))/19
```

3.149 $\int x^{3/2}(a+bx^3)^3 (A+Bx^3)\, dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int x^{3/2}(a+bx^3)^3 (A+Bx^3)\, dx = \frac{2}{5}a^3Ax^{5/2} + \frac{2}{11}a^2(3Ab+aB)x^{11/2} + \frac{6}{17}ab(Ab+aB)x^{17/2} + \frac{2}{23}b^2(Ab+3aB)x^{23/2} + \frac{2}{29}b^3Bx^{29/2}$$

```
[Out] 2/5*a^3*A*x^(5/2)+2/11*a^2*(3*A*b+B*a)*x^(11/2)+6/17*a*b*(A*b+B*a)*x^(17/2)
+2/23*b^2*(A*b+3*B*a)*x^(23/2)+2/29*b^3*B*x^(29/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {459}

$$\int x^{3/2}(a+bx^3)^3 (A+Bx^3)\, dx = \frac{2}{5}a^3Ax^{5/2} + \frac{2}{11}a^2x^{11/2}(aB+3Ab) + \frac{2}{23}b^2x^{23/2}(3aB+Ab) + \frac{6}{17}abx^{17/2}(aB+Ab) + \frac{2}{29}b^3Bx^{29/2}$$

```
[In] Int[x^(3/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*a^3*A*x^(5/2))/5 + (2*a^2*(3*A*b + a*B)*x^(11/2))/11 + (6*a*b*(A*b + a*B
)*x^(17/2))/17 + (2*b^2*(A*b + 3*a*B)*x^(23/2))/23 + (2*b^3*B*x^(29/2))/29
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^3 A x^{3/2} + a^2(3Ab+aB)x^{9/2} + 3ab(Ab+aB)x^{15/2} + b^2(Ab+3aB)x^{21/2} + b^3 B x^{27/2}\right)\,dx \\ &= \frac{2}{5}a^3Ax^{5/2} + \frac{2}{11}a^2(3Ab+aB)x^{11/2} + \frac{6}{17}ab(Ab+aB)x^{17/2} + \frac{2}{23}b^2(Ab+3aB)x^{23/2} + \frac{2}{29}b^3Bx^{29/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int x^{3/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx = \frac{2x^{5/2}(11339a^3(11A+5Bx^3)+10005a^2bx^3(17A+11Bx^3)+4785ab^2x^6(23A+17Bx^3)+935b^3x}{623645}$$

```
[In] Integrate[x^(3/2)*(a + b*x^3)^3*(A + B*x^3),x]
[Out] (2*x^(5/2)*(11339*a^3*(11*A + 5*B*x^3) + 10005*a^2*b*x^3*(17*A + 11*B*x^3)
+ 4785*a*b^2*x^6*(23*A + 17*B*x^3) + 935*b^3*x^9*(29*A + 23*B*x^3)))/623645
```

Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2b^3Bx^{\frac{29}{2}}}{29}+\frac{2(b^3A+3ab^2B)x^{\frac{23}{2}}}{23}+\frac{2(3ab^2A+3a^2bB)x^{\frac{17}{2}}}{17}+\frac{2(3a^2bA+a^3B)x^{\frac{11}{2}}}{11}+\frac{2a^3Ax^{\frac{5}{2}}}{5}$
default	$\frac{2b^3Bx^{\frac{29}{2}}}{29}+\frac{2(b^3A+3ab^2B)x^{\frac{23}{2}}}{23}+\frac{2(3ab^2A+3a^2bB)x^{\frac{17}{2}}}{17}+\frac{2(3a^2bA+a^3B)x^{\frac{11}{2}}}{11}+\frac{2a^3Ax^{\frac{5}{2}}}{5}$
gosper	$\frac{2x^{\frac{5}{2}}(21505Bb^3x^{12}+27115Ax^9b^3+81345Bx^9ab^2+110055Ax^6ab^2+110055Bx^6a^2b+170085Ax^3a^2b+56695a^3Bx^3+124}{623645}$
trager	$\frac{2x^{\frac{5}{2}}(21505Bb^3x^{12}+27115Ax^9b^3+81345Bx^9ab^2+110055Ax^6ab^2+110055Bx^6a^2b+170085Ax^3a^2b+56695a^3Bx^3+124}{623645}$
risch	$\frac{2x^{\frac{5}{2}}(21505Bb^3x^{12}+27115Ax^9b^3+81345Bx^9ab^2+110055Ax^6ab^2+110055Bx^6a^2b+170085Ax^3a^2b+56695a^3Bx^3+124}{623645}$

```
[In] int(x^(3/2)*(b*x^3+a)^3*(B*x^3+A),x,method=_RETURNVERBOSE)
[Out] 2/29*b^3*B*x^(29/2)+2/23*(A*b^3+3*B*a*b^2)*x^(23/2)+2/17*(3*A*a*b^2+3*B*a^2
*b)*x^(17/2)+2/11*(3*A*a^2*b+B*a^3)*x^(11/2)+2/5*a^3*A*x^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int x^{3/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx=\frac{2}{623645}\left(21505\,Bb^3x^{14}+27115\left(3\,Bab^2+Ab^3\right)x^{11}+110055\left(Ba^2b+Aab^2\right)x^8+124729\,Aa^3x^2$$

```
[In] integrate(x^(3/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/623645*(21505*B*b^3*x^14 + 27115*(3*B*a*b^2 + A*b^3)*x^11 + 110055*(B*a^2
*b + A*a*b^2)*x^8 + 124729*A*a^3*x^2 + 56695*(B*a^3 + 3*A*a^2*b)*x^5)*sqrt(
x)
```

Sympy [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int x^{3/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx=\frac{2Aa^3x^{\frac{5}{2}}}{5}+\frac{6Aa^2bx^{\frac{11}{2}}}{11}+\frac{6Aab^2x^{\frac{17}{2}}}{17}+\frac{2Ab^3x^{\frac{23}{2}}}{23}+\frac{2Ba^3x^{\frac{11}{2}}}{11}+\frac{6Ba^2bx^{\frac{17}{2}}}{17}+\frac{6Bab^2x^{\frac{23}{2}}}{23}+\frac{2Bb^3x^{\frac{29}{2}}}{29}$$

```
[In] integrate(x**(3/2)*(b*x**3+a)**3*(B*x**3+A),x)
```

```
[Out] 2*A*a**3*x**(5/2)/5 + 6*A*a**2*b*x**(11/2)/11 + 6*A*a*b**2*x**(17/2)/17 + 2
*A*b**3*x**(23/2)/23 + 2*B*a**3*x**(11/2)/11 + 6*B*a**2*b*x**(17/2)/17 + 6*
B*a*b**2*x**(23/2)/23 + 2*B*b**3*x**(29/2)/29
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int x^{3/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)\,dx=\frac{2}{29}\,Bb^3x^{\frac{29}{2}}+\frac{2}{23}\left(3\,Bab^2+Ab^3\right)x^{\frac{23}{2}}+\frac{6}{17}\left(Ba^2b+Aab^2\right)x^{\frac{17}{2}}+\frac{2}{5}\,Aa^3x^{\frac{5}{2}}+\frac{2}{11}\left(Ba^3+3\,Aa^2b\right)x^{\frac{11}{2}}$$

```
[In] integrate(x^(3/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/29*B*b^3*x^(29/2) + 2/23*(3*B*a*b^2 + A*b^3)*x^(23/2) + 6/17*(B*a^2*b + A
*a*b^2)*x^(17/2) + 2/5*A*a^3*x^(5/2) + 2/11*(B*a^3 + 3*A*a^2*b)*x^(11/2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int x^{3/2}(a+bx^3)^3\left(A+Bx^3\right)\,dx = \frac{2}{29}Bb^3x^{\frac{29}{2}} + \frac{6}{23}Bab^2x^{\frac{23}{2}} + \frac{2}{23}Ab^3x^{\frac{23}{2}} + \frac{6}{17}Ba^2bx^{\frac{17}{2}} + \frac{6}{17}Aab^2x^{\frac{17}{2}} + \frac{2}{11}Ba^3x^{\frac{11}{2}} + \frac{6}{11}Aa^2bx^{\frac{11}{2}} + \frac{2}{5}Aa^3x^{\frac{5}{2}}$$

```
[In] integrate(x^(3/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/29*B*b^3*x^(29/2) + 6/23*B*a*b^2*x^(23/2) + 2/23*A*b^3*x^(23/2) + 6/17*B*
a^2*b*x^(17/2) + 6/17*A*a*b^2*x^(17/2) + 2/11*B*a^3*x^(11/2) + 6/11*A*a^2*b
*x^(11/2) + 2/5*A*a^3*x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int x^{3/2}(a+bx^3)^3\left(A+Bx^3\right)\,dx = x^{11/2}\left(\frac{2\,B\,a^3}{11}+\frac{6\,A\,b\,a^2}{11}\right) + x^{23/2}\left(\frac{2\,A\,b^3}{23}+\frac{6\,B\,a\,b^2}{23}\right) + \frac{2\,A\,a^3\,x^{5/2}}{5} + \frac{2\,B\,b^3\,x^{29/2}}{29} + \frac{6\,a\,b\,x^{17/2}\,(A\,b+B\,a)}{17}$$

```
[In] int(x^(3/2)*(A + B*x^3)*(a + b*x^3)^3,x)
```

```
[Out] x^(11/2)*((2*B*a^3)/11 + (6*A*a^2*b)/11) + x^(23/2)*((2*A*b^3)/23 + (6*B*a*
b^2)/23) + (2*A*a^3*x^(5/2))/5 + (2*B*b^3*x^(29/2))/29 + (6*a*b*x^(17/2)*(A
*b + B*a))/17
```

3.150 $\int \sqrt{x}(a+bx^3)^3 \left(A+Bx^3\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \sqrt{x}\left(a+bx^3\right)^3 \left(A+Bx^3\right) \, dx = \frac{2}{3}a^3Ax^{3/2}+\frac{2}{9}a^2(3Ab+aB)x^{9/2} +\frac{2}{5}ab(Ab+aB)x^{15/2}+\frac{2}{21}b^2(Ab+3aB)x^{21/2}+\frac{2}{27}b^3Bx^{27/2}$$

```
[Out] 2/3*a^3*A*x^(3/2)+2/9*a^2*(3*A*b+B*a)*x^(9/2)+2/5*a*b*(A*b+B*a)*x^(15/2)+2/
21*b^2*(A*b+3*B*a)*x^(21/2)+2/27*b^3*B*x^(27/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {459}

$$\int \sqrt{x}\left(a+bx^3\right)^3 \left(A+Bx^3\right) \, dx = \frac{2}{3}a^3Ax^{3/2}+\frac{2}{9}a^2x^{9/2}(aB+3Ab) +\frac{2}{21}b^2x^{21/2}(3aB+Ab)+\frac{2}{5}abx^{15/2}(aB+Ab)+\frac{2}{27}b^3Bx^{27/2}$$

```
[In] Int[Sqrt[x]*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*a^3*A*x^(3/2))/3 + (2*a^2*(3*A*b + a*B)*x^(9/2))/9 + (2*a*b*(A*b + a*B)*
x^(15/2))/5 + (2*b^2*(A*b + 3*a*B)*x^(21/2))/21 + (2*b^3*B*x^(27/2))/27
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^3A\sqrt{x} + a^2(3Ab+aB)x^{7/2} + 3ab(Ab+aB)x^{13/2} + b^2(Ab+3aB)x^{19/2} + b^3Bx^{25/2}\right) dx \\ &= \frac{2}{3}a^3Ax^{3/2} + \frac{2}{9}a^2(3Ab+aB)x^{9/2} + \frac{2}{5}ab(Ab+aB)x^{15/2} + \frac{2}{21}b^2(Ab+3aB)x^{21/2} + \frac{2}{27}b^3Bx^{27/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int \sqrt{x}\left(a+bx^3\right)^3\left(A+Bx^3\right) dx = \frac{2}{945}x^{3/2}\left(105a^3\left(3A+Bx^3\right) + 63a^2bx^3\left(5A+3Bx^3\right) + 27ab^2x^6\left(7A+5Bx^3\right) + 5b^3x^9\left(9A+7Bx^3\right)\right)$$

```
[In] Integrate[Sqrt[x]*(a + b*x^3)^3*(A + B*x^3),x]
[Out] (2*x^(3/2)*(105*a^3*(3*A + B*x^3) + 63*a^2*b*x^3*(5*A + 3*B*x^3) + 27*a*b^2
*x^6*(7*A + 5*B*x^3) + 5*b^3*x^9*(9*A + 7*B*x^3)))/945
```

Maple [A] (verified)

Time = 4.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{2b^3Bx^{\frac{27}{2}}}{27}+\frac{2(b^3A+3ab^2B)x^{\frac{21}{2}}}{21}+\frac{2(3ab^2A+3a^2bB)x^{\frac{15}{2}}}{15}+\frac{2(3a^2bA+a^3B)x^{\frac{9}{2}}}{9}+\frac{2a^3Ax^{\frac{3}{2}}}{3}$	76
default	$\frac{2b^3Bx^{\frac{27}{2}}}{27}+\frac{2(b^3A+3ab^2B)x^{\frac{21}{2}}}{21}+\frac{2(3ab^2A+3a^2bB)x^{\frac{15}{2}}}{15}+\frac{2(3a^2bA+a^3B)x^{\frac{9}{2}}}{9}+\frac{2a^3Ax^{\frac{3}{2}}}{3}$	76
gosper	$\frac{2x^{\frac{3}{2}}(35Bb^3x^{12}+45Ax^9b^3+135Bx^9ab^2+189Ax^6ab^2+189Bx^6a^2b+315Ax^3a^2b+105a^3Bx^3+315a^3A)}{945}$	80
trager	$\frac{2x^{\frac{3}{2}}(35Bb^3x^{12}+45Ax^9b^3+135Bx^9ab^2+189Ax^6ab^2+189Bx^6a^2b+315Ax^3a^2b+105a^3Bx^3+315a^3A)}{945}$	80
risch	$\frac{2x^{\frac{3}{2}}(35Bb^3x^{12}+45Ax^9b^3+135Bx^9ab^2+189Ax^6ab^2+189Bx^6a^2b+315Ax^3a^2b+105a^3Bx^3+315a^3A)}{945}$	80

```
[In] int((b*x^3+a)^3*(B*x^3+A)*x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/27*b^3*B*x^(27/2)+2/21*(A*b^3+3*B*a*b^2)*x^(21/2)+2/15*(3*A*a*b^2+3*B*a^2
*b)*x^(15/2)+2/9*(3*A*a^2*b+B*a^3)*x^(9/2)+2/3*a^3*A*x^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

$$\int \sqrt{x}(a+bx^3)^3(A+Bx^3)\,dx = \frac{2}{945}\left(35\,Bb^3x^{13}+45\left(3\,Bab^2+Ab^3\right)x^{10}+189\left(Ba^2b+Aab^2\right)x^7+315\,Aa^3x+105\left(Ba^3+3\,Aa^2b\right)x^4\right)\sqrt{x}$$

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/945*(35*B*b^3*x^13 + 45*(3*B*a*b^2 + A*b^3)*x^10 + 189*(B*a^2*b + A*a*b^2
)*x^7 + 315*A*a^3*x + 105*(B*a^3 + 3*A*a^2*b)*x^4)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \sqrt{x}(a+bx^3)^3(A+Bx^3)\,dx = \frac{2Aa^3x^{\frac{3}{2}}}{3}+\frac{2Aa^2bx^{\frac{9}{2}}}{3}+\frac{2Aab^2x^{\frac{15}{2}}}{5}+\frac{2Ab^3x^{\frac{21}{2}}}{21}+\frac{2Ba^3x^{\frac{9}{2}}}{9}+\frac{2Ba^2bx^{\frac{15}{2}}}{5}+\frac{2Bab^2x^{\frac{21}{2}}}{7}+\frac{2Bb^3x^{\frac{27}{2}}}{27}$$

```
[In] integrate((b*x**3+a)**3*(B*x**3+A)*x**(1/2),x)
```

```
[Out] 2*A*a**3*x**(3/2)/3 + 2*A*a**2*b*x**(9/2)/3 + 2*A*a*b**2*x**(15/2)/5 + 2*A*
b**3*x**(21/2)/21 + 2*B*a**3*x**(9/2)/9 + 2*B*a**2*b*x**(15/2)/5 + 2*B*a*b*
*2*x**(21/2)/7 + 2*B*b**3*x**(27/2)/27
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \sqrt{x}(a+bx^3)^3(A+Bx^3)\,dx = \frac{2}{27}\,Bb^3x^{\frac{27}{2}}+\frac{2}{21}\left(3\,Bab^2+Ab^3\right)x^{\frac{21}{2}}+\frac{2}{5}\left(Ba^2b+Aab^2\right)x^{\frac{15}{2}}+\frac{2}{3}\,Aa^3x^{\frac{3}{2}}+\frac{2}{9}\left(Ba^3+3\,Aa^2b\right)x^{\frac{9}{2}}$$

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/27*B*b^3*x^(27/2) + 2/21*(3*B*a*b^2 + A*b^3)*x^(21/2) + 2/5*(B*a^2*b + A*
a*b^2)*x^(15/2) + 2/3*A*a^3*x^(3/2) + 2/9*(B*a^3 + 3*A*a^2*b)*x^(9/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \sqrt{x}(a+bx^3)^3(A+Bx^3)\,dx = \frac{2}{27}Bb^3x^{\frac{27}{2}} + \frac{2}{7}Bab^2x^{\frac{21}{2}} + \frac{2}{21}Ab^3x^{\frac{21}{2}} + \frac{2}{5}Ba^2bx^{\frac{15}{2}} + \frac{2}{5}Aab^2x^{\frac{15}{2}} + \frac{2}{9}Ba^3x^{\frac{9}{2}} + \frac{2}{3}Aa^2bx^{\frac{9}{2}} + \frac{2}{3}Aa^3x^{\frac{3}{2}}$$

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/27*B*b^3*x^(27/2) + 2/7*B*a*b^2*x^(21/2) + 2/21*A*b^3*x^(21/2) + 2/5*B*a^
2*b*x^(15/2) + 2/5*A*a*b^2*x^(15/2) + 2/9*B*a^3*x^(9/2) + 2/3*A*a^2*b*x^(9/
2) + 2/3*A*a^3*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \sqrt{x}(a+bx^3)^3(A+Bx^3)\,dx = x^{9/2}\left(\frac{2\,B\,a^3}{9}+\frac{2\,A\,b\,a^2}{3}\right)+x^{21/2}\left(\frac{2\,A\,b^3}{21}+\frac{2\,B\,a\,b^2}{7}\right)+\frac{2\,A\,a^3\,x^{3/2}}{3}+\frac{2\,B\,b^3\,x^{27/2}}{27}+\frac{2\,a\,b\,x^{15/2}\,(A\,b+B\,a)}{5}$$

```
[In] int(x^(1/2)*(A + B*x^3)*(a + b*x^3)^3,x)
```

```
[Out] x^(9/2)*((2*B*a^3)/9 + (2*A*a^2*b)/3) + x^(21/2)*((2*A*b^3)/21 + (2*B*a*b^2
)/7) + (2*A*a^3*x^(3/2))/3 + (2*B*b^3*x^(27/2))/27 + (2*a*b*x^(15/2)*(A*b +
 B*a))/5
```

3.151 $\int \frac{(a+bx^3)^3(A+Bx^3)}{\sqrt{x}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{\sqrt{x}} \, dx = 2a^3A\sqrt{x} + \frac{2}{7}a^2(3Ab+aB)x^{7/2} + \frac{6}{13}ab(Ab+aB)x^{13/2} + \frac{2}{19}b^2(Ab+3aB)x^{19/2} + \frac{2}{25}b^3Bx^{25/2}$$

```
[Out] 2/7*a^2*(3*A*b+B*a)*x^(7/2)+6/13*a*b*(A*b+B*a)*x^(13/2)+2/19*b^2*(A*b+3*B*a
)*x^(19/2)+2/25*b^3*B*x^(25/2)+2*a^3*A*x^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {459}

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{\sqrt{x}} \, dx = 2a^3A\sqrt{x} + \frac{2}{7}a^2x^{7/2}(aB+3Ab) + \frac{2}{19}b^2x^{19/2}(3aB+Ab) + \frac{6}{13}abx^{13/2}(aB+Ab) + \frac{2}{25}b^3Bx^{25/2}$$

```
[In] Int[((a + b*x^3)^3*(A + B*x^3))/Sqrt[x],x]
```

```
[Out] 2*a^3*A*Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(7/2))/7 + (6*a*b*(A*b + a*B)*x^(1
3/2))/13 + (2*b^2*(A*b + 3*a*B)*x^(19/2))/19 + (2*b^3*B*x^(25/2))/25
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a^3 A}{\sqrt{x}} + a^2(3Ab + aB)x^{5/2} + 3ab(Ab + aB)x^{11/2} + b^2(Ab + 3aB)x^{17/2} + b^3 Bx^{23/2}\right) dx$$
$$= 2a^3 A\sqrt{x} + \frac{2}{7}a^2(3Ab+aB)x^{7/2} + \frac{6}{13}ab(Ab+aB)x^{13/2} + \frac{2}{19}b^2(Ab+3aB)x^{19/2} + \frac{2}{25}b^3 Bx^{25/2}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

$$\int \frac{(a + bx^3)^3 (A + Bx^3)}{\sqrt{x}} dx$$
$$= \frac{2\sqrt{x}(6175a^3(7A + Bx^3) + 1425a^2bx^3(13A + 7Bx^3) + 525ab^2x^6(19A + 13Bx^3) + 91b^3x^9(25A + 19Bx^3))}{43225}$$

```
[In] Integrate[((a + b*x^3)^3*(A + B*x^3))/Sqrt[x],x]
[Out] (2*Sqrt[x]*(6175*a^3*(7*A + B*x^3) + 1425*a^2*b*x^3*(13*A + 7*B*x^3) + 525*
a*b^2*x^6*(19*A + 13*B*x^3) + 91*b^3*x^9*(25*A + 19*B*x^3)))/43225
```

Maple [A] (verified)

Time = 4.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{2b^3 B\,x^{\frac{25}{2}}}{25}+\frac{2\left(b^3 A+3a\,b^2 B\right)x^{\frac{19}{2}}}{19}+\frac{2\left(3a\,b^2 A+3a^2 bB\right)x^{\frac{13}{2}}}{13}+\frac{2\left(3a^2 bA+a^3 B\right)x^{\frac{7}{2}}}{7}+2a^3 A\sqrt{x}$
default	$\frac{2b^3 B\,x^{\frac{25}{2}}}{25}+\frac{2\left(b^3 A+3a\,b^2 B\right)x^{\frac{19}{2}}}{19}+\frac{2\left(3a\,b^2 A+3a^2 bB\right)x^{\frac{13}{2}}}{13}+\frac{2\left(3a^2 bA+a^3 B\right)x^{\frac{7}{2}}}{7}+2a^3 A\sqrt{x}$
trager	$\left(\frac{2}{25}B\,b^3 x^{12}+\frac{2}{19}A\,x^9 b^3+\frac{6}{19}B\,x^9 a\,b^2+\frac{6}{13}A\,x^6 a\,b^2+\frac{6}{13}B\,x^6 a^2 b+\frac{6}{7}A\,x^3 a^2 b+\frac{2}{7}a^3 B\,x^3+2\right.$
gosper	$\frac{2\sqrt{x}\left(1729B\,b^3 x^{12}+2275A\,x^9 b^3+6825B\,x^9 a\,b^2+9975A\,x^6 a\,b^2+9975B\,x^6 a^2 b+18525A\,x^3 a^2 b+6175a^3 B\,x^3+43225a^3 A\right)}{43225}$
risch	$\frac{2\sqrt{x}\left(1729B\,b^3 x^{12}+2275A\,x^9 b^3+6825B\,x^9 a\,b^2+9975A\,x^6 a\,b^2+9975B\,x^6 a^2 b+18525A\,x^3 a^2 b+6175a^3 B\,x^3+43225a^3 A\right)}{43225}$

```
[In] int((b*x^3+a)^3*(B*x^3+A)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/25*b^3*B*x^(25/2)+2/19*(A*b^3+3*B*a*b^2)*x^(19/2)+2/13*(3*A*a*b^2+3*B*a^2
*b)*x^(13/2)+2/7*(3*A*a^2*b+B*a^3)*x^(7/2)+2*a^3*A*x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{\sqrt{x}}\,dx$$
$$= \frac{2}{43225}\left(1729\,Bb^3x^{12} + 2275\left(3\,Bab^2 + Ab^3\right)x^9 + 9975\left(Ba^2b + Aab^2\right)x^6 + 43225\,Aa^3 + 6175\left(Ba^3 + 3\,Aa\right.\right.$$

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/43225*(1729*B*b^3*x^12 + 2275*(3*B*a*b^2 + A*b^3)*x^9 + 9975*(B*a^2*b + A
*a*b^2)*x^6 + 43225*A*a^3 + 6175*(B*a^3 + 3*A*a^2*b)*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{\sqrt{x}}\,dx = 2Aa^3\sqrt{x} + \frac{6Aa^2bx^{\frac{7}{2}}}{7} + \frac{6Aab^2x^{\frac{13}{2}}}{13} + \frac{2Ab^3x^{\frac{19}{2}}}{19} + \frac{2Ba^3x^{\frac{7}{2}}}{7} + \frac{6Ba^2bx^{\frac{13}{2}}}{13} + \frac{6Bab^2x^{\frac{19}{2}}}{19} + \frac{2Bb^3x^{\frac{25}{2}}}{25}$$

```
[In] integrate((b*x**3+a)**3*(B*x**3+A)/x**(1/2),x)
```

```
[Out] 2*A*a**3*sqrt(x) + 6*A*a**2*b*x**(7/2)/7 + 6*A*a*b**2*x**(13/2)/13 + 2*A*b*
*3*x**(19/2)/19 + 2*B*a**3*x**(7/2)/7 + 6*B*a**2*b*x**(13/2)/13 + 6*B*a*b**
2*x**(19/2)/19 + 2*B*b**3*x**(25/2)/25
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{\sqrt{x}}\,dx = \frac{2}{25}\,Bb^3x^{\frac{25}{2}} + \frac{2}{19}\left(3\,Bab^2 + Ab^3\right)x^{\frac{19}{2}} + \frac{6}{13}\left(Ba^2b + Aab^2\right)x^{\frac{13}{2}} + 2\,Aa^3\sqrt{x} + \frac{2}{7}\left(Ba^3 + 3\,Aa^2b\right)x^{\frac{7}{2}}$$

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/25*B*b^3*x^(25/2) + 2/19*(3*B*a*b^2 + A*b^3)*x^(19/2) + 6/13*(B*a^2*b + A
*a*b^2)*x^(13/2) + 2*A*a^3*sqrt(x) + 2/7*(B*a^3 + 3*A*a^2*b)*x^(7/2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{\sqrt{x}}\,dx = \frac{2}{25}\,Bb^3x^{\frac{25}{2}} + \frac{6}{19}\,Bab^2x^{\frac{19}{2}} + \frac{2}{19}\,Ab^3x^{\frac{19}{2}} + \frac{6}{13}\,Ba^2bx^{\frac{13}{2}} + \frac{6}{13}\,Aab^2x^{\frac{13}{2}} + \frac{2}{7}\,Ba^3x^{\frac{7}{2}} + \frac{6}{7}\,Aa^2bx^{\frac{7}{2}} + 2\,Aa^3\sqrt{x}$$

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/25*B*b^3*x^(25/2) + 6/19*B*a*b^2*x^(19/2) + 2/19*A*b^3*x^(19/2) + 6/13*B*
a^2*b*x^(13/2) + 6/13*A*a*b^2*x^(13/2) + 2/7*B*a^3*x^(7/2) + 6/7*A*a^2*b*x^
(7/2) + 2*A*a^3*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{\sqrt{x}}\,dx = x^{7/2}\left(\frac{2\,B\,a^3}{7} + \frac{6\,A\,b\,a^2}{7}\right) + x^{19/2}\left(\frac{2\,A\,b^3}{19} + \frac{6\,B\,a\,b^2}{19}\right) + 2\,A\,a^3\,\sqrt{x} + \frac{2\,B\,b^3\,x^{25/2}}{25} + \frac{6\,a\,b\,x^{13/2}\,(A\,b + B\,a)}{13}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^3)/x^(1/2),x)
```

```
[Out] x^(7/2)*((2*B*a^3)/7 + (6*A*a^2*b)/7) + x^(19/2)*((2*A*b^3)/19 + (6*B*a*b^2
)/19) + 2*A*a^3*x^(1/2) + (2*B*b^3*x^(25/2))/25 + (6*a*b*x^(13/2)*(A*b + B*
a))/13
```

3.152 $\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{3/2}}\,dx = -\frac{2a^3A}{\sqrt{x}} + \frac{2}{5}a^2(3Ab+aB)x^{5/2} + \frac{6}{11}ab(Ab+aB)x^{11/2} + \frac{2}{17}b^2(Ab+3aB)x^{17/2} + \frac{2}{23}b^3Bx^{23/2}$$

```
[Out] 2/5*a^2*(3*A*b+B*a)*x^(5/2)+6/11*a*b*(A*b+B*a)*x^(11/2)+2/17*b^2*(A*b+3*B*a
)*x^(17/2)+2/23*b^3*B*x^(23/2)-2*a^3*A/x^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {459}

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{3/2}}\,dx = -\frac{2a^3A}{\sqrt{x}} + \frac{2}{5}a^2x^{5/2}(aB+3Ab) + \frac{2}{17}b^2x^{17/2}(3aB+Ab) + \frac{6}{11}abx^{11/2}(aB+Ab) + \frac{2}{23}b^3Bx^{23/2}$$

```
[In] Int[((a + b*x^3)^3*(A + B*x^3))/x^(3/2),x]
```

```
[Out] (-2*a^3*A)/Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(5/2))/5 + (6*a*b*(A*b + a*B)*x
^(11/2))/11 + (2*b^2*(A*b + 3*a*B)*x^(17/2))/17 + (2*b^3*B*x^(23/2))/23
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

`Q[p, 0] && IGtQ[q, 0]`

Rubi steps

$$\text{integral} = \int \left(\frac{a^3 A}{x^{3/2}} + a^2(3Ab + aB)x^{3/2} + 3ab(Ab + aB)x^{9/2} + b^2(Ab + 3aB)x^{15/2} + b^3 B x^{21/2} \right) dx$$

$$= -\frac{2a^3 A}{\sqrt{x}} + \frac{2}{5}a^2(3Ab+aB)x^{5/2} + \frac{6}{11}ab(Ab+aB)x^{11/2} + \frac{2}{17}b^2(Ab+3aB)x^{17/2} + \frac{2}{23}b^3 B x^{23/2}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{(a + bx^3)^3 (A + Bx^3)}{x^{3/2}} \, dx = -\frac{2(21505a^3 A - 12903a^2 Abx^3 - 4301a^3 Bx^3 - 5865aAb^2x^6 - 5865a^2bBx^6 - 1265Ab^3x^9 - 3795ab^2Bx^9 - 935}{21505\sqrt{x}}$$

```
[In] Integrate[((a + b*x^3)^3*(A + B*x^3))/x^(3/2),x]
[Out] (-2*(21505*a^3*A - 12903*a^2*A*b*x^3 - 4301*a^3*B*x^3 - 5865*a*A*b^2*x^6 -
5865*a^2*b*B*x^6 - 1265*A*b^3*x^9 - 3795*a*b^2*B*x^9 - 935*b^3*B*x^12))/(21
505*Sqrt[x])
```

Maple [A] (verified)

Time = 4.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{2b^3 B x^{\frac{23}{2}}}{23} + \frac{2A b^3 x^{\frac{17}{2}}}{17} + \frac{6Ba b^2 x^{\frac{17}{2}}}{17} + \frac{6Aa b^2 x^{\frac{11}{2}}}{11} + \frac{6B a^2 b x^{\frac{11}{2}}}{11} + \frac{6A a^2 b x^{\frac{5}{2}}}{5} + \frac{2B a^3 x^{\frac{5}{2}}}{5} - \frac{2a^3 A}{\sqrt{x}}$
default	$\frac{2b^3 B x^{\frac{23}{2}}}{23} + \frac{2A b^3 x^{\frac{17}{2}}}{17} + \frac{6Ba b^2 x^{\frac{17}{2}}}{17} + \frac{6Aa b^2 x^{\frac{11}{2}}}{11} + \frac{6B a^2 b x^{\frac{11}{2}}}{11} + \frac{6A a^2 b x^{\frac{5}{2}}}{5} + \frac{2B a^3 x^{\frac{5}{2}}}{5} - \frac{2a^3 A}{\sqrt{x}}$
gosper	$-\frac{2(-935B b^3 x^{12} - 1265A x^9 b^3 - 3795B x^9 a b^2 - 5865A x^6 a b^2 - 5865B x^6 a^2 b - 12903A x^3 a^2 b - 4301a^3 B x^3 + 21505a^3 A)}{21505\sqrt{x}}$
trager	$-\frac{2(-935B b^3 x^{12} - 1265A x^9 b^3 - 3795B x^9 a b^2 - 5865A x^6 a b^2 - 5865B x^6 a^2 b - 12903A x^3 a^2 b - 4301a^3 B x^3 + 21505a^3 A)}{21505\sqrt{x}}$
risch	$-\frac{2(-935B b^3 x^{12} - 1265A x^9 b^3 - 3795B x^9 a b^2 - 5865A x^6 a b^2 - 5865B x^6 a^2 b - 12903A x^3 a^2 b - 4301a^3 B x^3 + 21505a^3 A)}{21505\sqrt{x}}$

```
[In] int((b*x^3+a)^3*(B*x^3+A)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/23*b^3*B*x^(23/2)+2/17*A*b^3*x^(17/2)+6/17*B*a*b^2*x^(17/2)+6/11*A*a*b^2*
x^(11/2)+6/11*B*a^2*b*x^(11/2)+6/5*A*a^2*b*x^(5/2)+2/5*B*a^3*x^(5/2)-2*a^3*
A/x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{3/2}}\,dx = \frac{2\,(935\,Bb^3x^{12}+1265\,(3\,Bab^2+Ab^3)x^9+5865\,(Ba^2b+Aab^2)x^6-21505\,Aa^3}{21505\,\sqrt{x}}$$

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21505*(935*B*b^3*x^12 + 1265*(3*B*a*b^2 + A*b^3)*x^9 + 5865*(B*a^2*b + A*
a*b^2)*x^6 - 21505*A*a^3 + 4301*(B*a^3 + 3*A*a^2*b)*x^3)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{3/2}}\,dx = -\frac{2Aa^3}{\sqrt{x}}+\frac{6Aa^2bx^{\frac{5}{2}}}{5}+\frac{6Aab^2x^{\frac{11}{2}}}{11} + \frac{2Ab^3x^{\frac{17}{2}}}{17}+\frac{2Ba^3x^{\frac{5}{2}}}{5}+\frac{6Ba^2bx^{\frac{11}{2}}}{11}+\frac{6Bab^2x^{\frac{17}{2}}}{17}+\frac{2Bb^3x^{\frac{23}{2}}}{23}$$

```
[In] integrate((b*x**3+a)**3*(B*x**3+A)/x**(3/2),x)
```

```
[Out] -2*A*a**3/sqrt(x) + 6*A*a**2*b*x**(5/2)/5 + 6*A*a*b**2*x**(11/2)/11 + 2*A*b
**3*x**(17/2)/17 + 2*B*a**3*x**(5/2)/5 + 6*B*a**2*b*x**(11/2)/11 + 6*B*a*b*
*2*x**(17/2)/17 + 2*B*b**3*x**(23/2)/23
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{3/2}}\,dx = \frac{2}{23}\,Bb^3x^{\frac{23}{2}}+\frac{2}{17}\,(3\,Bab^2+Ab^3)x^{\frac{17}{2}} + \frac{6}{11}\,(Ba^2b+Aab^2)x^{\frac{11}{2}}-\frac{2\,Aa^3}{\sqrt{x}}+\frac{2}{5}\,(Ba^3+3\,Aa^2b)x^{\frac{5}{2}}$$

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/23*B*b^3*x^(23/2) + 2/17*(3*B*a*b^2 + A*b^3)*x^(17/2) + 6/11*(B*a^2*b + A
*a*b^2)*x^(11/2) - 2*A*a^3/sqrt(x) + 2/5*(B*a^3 + 3*A*a^2*b)*x^(5/2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{3/2}} dx = \frac{2}{23} Bb^3 x^{\frac{23}{2}} + \frac{6}{17} Bab^2 x^{\frac{17}{2}} + \frac{2}{17} Ab^3 x^{\frac{17}{2}} + \frac{6}{11} Ba^2 bx^{\frac{11}{2}} + \frac{6}{11} Aab^2 x^{\frac{11}{2}} + \frac{2}{5} Ba^3 x^{\frac{5}{2}} + \frac{6}{5} Aa^2 bx^{\frac{5}{2}} - \frac{2 Aa^3}{\sqrt{x}}$$

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/23*B*b^3*x^(23/2) + 6/17*B*a*b^2*x^(17/2) + 2/17*A*b^3*x^(17/2) + 6/11*B*
a^2*b*x^(11/2) + 6/11*A*a*b^2*x^(11/2) + 2/5*B*a^3*x^(5/2) + 6/5*A*a^2*b*x^
(5/2) - 2*A*a^3/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{3/2}} dx = x^{5/2} \left(\frac{2 B a^3}{5} + \frac{6 A b a^2}{5}\right) + x^{17/2} \left(\frac{2 A b^3}{17} + \frac{6 B a b^2}{17}\right) - \frac{2 A a^3}{\sqrt{x}} + \frac{2 B b^3 x^{23/2}}{23} + \frac{6 a b x^{11/2} (A b + B a)}{11}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^3)/x^(3/2),x)
```

```
[Out] x^(5/2)*((2*B*a^3)/5 + (6*A*a^2*b)/5) + x^(17/2)*((2*A*b^3)/17 + (6*B*a*b^2
)/17) - (2*A*a^3)/x^(1/2) + (2*B*b^3*x^(23/2))/23 + (6*a*b*x^(11/2)*(A*b +
B*a))/11
```

3.153 $\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{5/2}}\,dx = -\frac{2a^3A}{3x^{3/2}} + \frac{2}{3}a^2(3Ab+aB)x^{3/2} + \frac{2}{3}ab(Ab+aB)x^{9/2} + \frac{2}{15}b^2(Ab+3aB)x^{15/2} + \frac{2}{21}b^3Bx^{21/2}$$

```
[Out] -2/3*a^3*A/x^(3/2)+2/3*a^2*(3*A*b+B*a)*x^(3/2)+2/3*a*b*(A*b+B*a)*x^(9/2)+2/
15*b^2*(A*b+3*B*a)*x^(15/2)+2/21*b^3*B*x^(21/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {459}

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{5/2}}\,dx = -\frac{2a^3A}{3x^{3/2}} + \frac{2}{3}a^2x^{3/2}(aB+3Ab) + \frac{2}{15}b^2x^{15/2}(3aB+Ab) + \frac{2}{3}abx^{9/2}(aB+Ab) + \frac{2}{21}b^3Bx^{21/2}$$

```
[In] Int[((a + b*x^3)^3*(A + B*x^3))/x^(5/2),x]
```

```
[Out] (-2*a^3*A)/(3*x^(3/2)) + (2*a^2*(3*A*b + a*B)*x^(3/2))/3 + (2*a*b*(A*b + a*
B)*x^(9/2))/3 + (2*b^2*(A*b + 3*a*B)*x^(15/2))/15 + (2*b^3*B*x^(21/2))/21
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a^3A}{x^{5/2}}+a^2(3Ab+aB)\sqrt{x}+3ab(Ab+aB)x^{7/2}+b^2(Ab+3aB)x^{13/2}+b^3Bx^{19/2}\right)\,dx$$

$$= -\frac{2a^3A}{3x^{3/2}}+\frac{2}{3}a^2(3Ab+aB)x^{3/2}+\frac{2}{3}ab(Ab+aB)x^{9/2}+\frac{2}{15}b^2(Ab+3aB)x^{15/2}+\frac{2}{21}b^3Bx^{21/2}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3)^3\,(A+Bx^3)}{x^{5/2}}\,dx = \frac{2(-35a^3(A-Bx^3)+35a^2bx^3(3A+Bx^3)+7ab^2x^6(5A+3Bx^3)+b^3x^9(7A+$$

```
[In] Integrate[((a + b*x^3)^3*(A + B*x^3))/x^(5/2),x]
[Out] (2*(-35*a^3*(A - B*x^3) + 35*a^2*b*x^3*(3*A + B*x^3) + 7*a*b^2*x^6*(5*A + 3
*B*x^3) + b^3*x^9*(7*A + 5*B*x^3)))/(105*x^(3/2))
```

Maple [A] (verified)

Time = 4.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{2b^3Bx^{\frac{21}{2}}}{21}+\frac{2Ab^3x^{\frac{15}{2}}}{15}+\frac{2Bab^2x^{\frac{15}{2}}}{5}+\frac{2Aab^2x^{\frac{9}{2}}}{3}+\frac{2Ba^2bx^{\frac{9}{2}}}{3}+2Aa^2bx^{\frac{3}{2}}+\frac{2Ba^3x^{\frac{3}{2}}}{3}-\frac{2a^3A}{3x^{\frac{3}{2}}}$	78
default	$\frac{2b^3Bx^{\frac{21}{2}}}{21}+\frac{2Ab^3x^{\frac{15}{2}}}{15}+\frac{2Bab^2x^{\frac{15}{2}}}{5}+\frac{2Aab^2x^{\frac{9}{2}}}{3}+\frac{2Ba^2bx^{\frac{9}{2}}}{3}+2Aa^2bx^{\frac{3}{2}}+\frac{2Ba^3x^{\frac{3}{2}}}{3}-\frac{2a^3A}{3x^{\frac{3}{2}}}$	78
gosper	$-\frac{2(-5Bb^3x^{12}-7Ax^9b^3-21Bx^9ab^2-35Ax^6ab^2-35Bx^6a^2b-105Ax^3a^2b-35a^3Bx^3+35a^3A)}{105x^{\frac{3}{2}}}$	80
trager	$-\frac{2(-5Bb^3x^{12}-7Ax^9b^3-21Bx^9ab^2-35Ax^6ab^2-35Bx^6a^2b-105Ax^3a^2b-35a^3Bx^3+35a^3A)}{105x^{\frac{3}{2}}}$	80
risch	$-\frac{2(-5Bb^3x^{12}-7Ax^9b^3-21Bx^9ab^2-35Ax^6ab^2-35Bx^6a^2b-105Ax^3a^2b-35a^3Bx^3+35a^3A)}{105x^{\frac{3}{2}}}$	80

```
[In] int((b*x^3+a)^3*(B*x^3+A)/x^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/21*b^3*B*x^(21/2)+2/15*A*b^3*x^(15/2)+2/5*B*a*b^2*x^(15/2)+2/3*A*a*b^2*x^
(9/2)+2/3*B*a^2*b*x^(9/2)+2*A*a^2*b*x^(3/2)+2/3*B*a^3*x^(3/2)-2/3*a^3*A/x^(
3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^3\,(A+Bx^3)}{x^{5/2}}\,dx = \frac{2\,(5\,Bb^3x^{12}+7\,(3\,Bab^2+Ab^3)x^9+35\,(Ba^2b+Aab^2)x^6-35\,Aa^3+35\,(Ba^3}{105\,x^{\frac{3}{2}}}$$

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/105*(5*B*b^3*x^12 + 7*(3*B*a*b^2 + A*b^3)*x^9 + 35*(B*a^2*b + A*a*b^2)*x^
6 - 35*A*a^3 + 35*(B*a^3 + 3*A*a^2*b)*x^3)/x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^3)^3\,(A+Bx^3)}{x^{5/2}}\,dx = -\frac{2Aa^3}{3x^{\frac{3}{2}}} + 2Aa^2bx^{\frac{3}{2}} + \frac{2Aab^2x^{\frac{9}{2}}}{3} + \frac{2Ab^3x^{\frac{15}{2}}}{15} + \frac{2Ba^3x^{\frac{3}{2}}}{3} + \frac{2Ba^2bx^{\frac{9}{2}}}{3} + \frac{2Bab^2x^{\frac{15}{2}}}{5} + \frac{2Bb^3x^{\frac{21}{2}}}{21}$$

```
[In] integrate((b*x**3+a)**3*(B*x**3+A)/x**(5/2),x)
```

```
[Out] -2*A*a**3/(3*x**(3/2)) + 2*A*a**2*b*x**(3/2) + 2*A*a*b**2*x**(9/2)/3 + 2*A*
b**3*x**(15/2)/15 + 2*B*a**3*x**(3/2)/3 + 2*B*a**2*b*x**(9/2)/3 + 2*B*a*b**
2*x**(15/2)/5 + 2*B*b**3*x**(21/2)/21
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)^3\,(A+Bx^3)}{x^{5/2}}\,dx = \frac{2}{21}\,Bb^3x^{\frac{21}{2}} + \frac{2}{15}\,\left(3\,Bab^2+Ab^3\right)x^{\frac{15}{2}} + \frac{2}{3}\,\left(Ba^2b+Aab^2\right)x^{\frac{9}{2}} - \frac{2\,Aa^3}{3\,x^{\frac{3}{2}}} + \frac{2}{3}\,\left(Ba^3+3\,Aa^2b\right)x^{\frac{3}{2}}$$

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/21*B*b^3*x^(21/2) + 2/15*(3*B*a*b^2 + A*b^3)*x^(15/2) + 2/3*(B*a^2*b + A*
a*b^2)*x^(9/2) - 2/3*A*a^3/x^(3/2) + 2/3*(B*a^3 + 3*A*a^2*b)*x^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{5/2}} \, dx = \frac{2}{21} Bb^3 x^{\frac{21}{2}} + \frac{2}{5} Bab^2 x^{\frac{15}{2}} + \frac{2}{15} Ab^3 x^{\frac{15}{2}} + \frac{2}{3} Ba^2 b x^{\frac{9}{2}} + \frac{2}{3} Aab^2 x^{\frac{9}{2}} + \frac{2}{3} Ba^3 x^{\frac{3}{2}} + 2 Aa^2 b x^{\frac{3}{2}} - \frac{2 Aa^3}{3 x^{\frac{3}{2}}}$$

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/21*B*b^3*x^(21/2) + 2/5*B*a*b^2*x^(15/2) + 2/15*A*b^3*x^(15/2) + 2/3*B*a^
2*b*x^(9/2) + 2/3*A*a*b^2*x^(9/2) + 2/3*B*a^3*x^(3/2) + 2*A*a^2*b*x^(3/2) -
 2/3*A*a^3/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{5/2}} \, dx = x^{3/2} \left(\frac{2\,B\,a^3}{3} + 2\,A\,b\,a^2\right) + x^{15/2} \left(\frac{2\,A\,b^3}{15} + \frac{2\,B\,a\,b^2}{5}\right) - \frac{2\,A\,a^3}{3\,x^{3/2}} + \frac{2\,B\,b^3\,x^{21/2}}{21} + \frac{2\,a\,b\,x^{9/2}\,(A\,b + B\,a)}{3}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^3)/x^(5/2),x)
```

```
[Out] x^(3/2)*((2*B*a^3)/3 + 2*A*a^2*b) + x^(15/2)*((2*A*b^3)/15 + (2*B*a*b^2)/5)
 - (2*A*a^3)/(3*x^(3/2)) + (2*B*b^3*x^(21/2))/21 + (2*a*b*x^(9/2)*(A*b + B*
a))/3
```

3.154 $\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{7/2}} dx$

Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{7/2}} dx = -\frac{2a^3A}{5x^{5/2}} + 2a^2(3Ab+aB)\sqrt{x} + \frac{6}{7}ab(Ab+aB)x^{7/2} + \frac{2}{13}b^2(Ab+3aB)x^{13/2} + \frac{2}{19}b^3Bx^{19/2}$$

```
[Out] -2/5*a^3*A/x^(5/2)+6/7*a*b*(A*b+B*a)*x^(7/2)+2/13*b^2*(A*b+3*B*a)*x^(13/2)+
2/19*b^3*B*x^(19/2)+2*a^2*(3*A*b+B*a)*x^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {459}

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{7/2}} dx = -\frac{2a^3A}{5x^{5/2}} + 2a^2\sqrt{x}(aB+3Ab) + \frac{2}{13}b^2x^{13/2}(3aB+Ab) + \frac{6}{7}abx^{7/2}(aB+Ab) + \frac{2}{19}b^3Bx^{19/2}$$

```
[In] Int[((a + b*x^3)^3*(A + B*x^3))/x^(7/2),x]
```

```
[Out] (-2*a^3*A)/(5*x^(5/2)) + 2*a^2*(3*A*b + a*B)*Sqrt[x] + (6*a*b*(A*b + a*B)*x
^(7/2))/7 + (2*b^2*(A*b + 3*a*B)*x^(13/2))/13 + (2*b^3*B*x^(19/2))/19
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
```

```
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a^3A}{x^{7/2}} + \frac{a^2(3Ab+aB)}{\sqrt{x}} + 3ab(Ab+aB)x^{5/2} + b^2(Ab+3aB)x^{11/2} + b^3Bx^{17/2}\right) dx$$
$$= -\frac{2a^3A}{5x^{5/2}} + 2a^2(3Ab+aB)\sqrt{x} + \frac{6}{7}ab(Ab+aB)x^{7/2} + \frac{2}{13}b^2(Ab+3aB)x^{13/2} + \frac{2}{19}b^3Bx^{19/2}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{7/2}}\,dx = \frac{-3458a^3(A-5Bx^3)+7410a^2bx^3(7A+Bx^3)+570ab^2x^6(13A+7Bx^3)+70b}{8645x^{5/2}}$$

```
[In] Integrate[((a + b*x^3)^3*(A + B*x^3))/x^(7/2),x]
[Out] (-3458*a^3*(A - 5*B*x^3) + 7410*a^2*b*x^3*(7*A + B*x^3) + 570*a*b^2*x^6*(13
*A + 7*B*x^3) + 70*b^3*x^9*(19*A + 13*B*x^3))/(8645*x^(5/2))
```

Maple [A] (verified)

Time = 4.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

method	result	s
derivativedivides	$\frac{2b^3Bx^{\frac{19}{2}}}{19}+\frac{2Ab^3x^{\frac{13}{2}}}{13}+\frac{6Bab^2x^{\frac{13}{2}}}{13}+\frac{6Aab^2x^{\frac{7}{2}}}{7}+\frac{6Ba^2bx^{\frac{7}{2}}}{7}+6Aa^2b\sqrt{x}+2Ba^3\sqrt{x}-\frac{2a^3A}{5x^{\frac{5}{2}}}$	7
default	$\frac{2b^3Bx^{\frac{19}{2}}}{19}+\frac{2Ab^3x^{\frac{13}{2}}}{13}+\frac{6Bab^2x^{\frac{13}{2}}}{13}+\frac{6Aab^2x^{\frac{7}{2}}}{7}+\frac{6Ba^2bx^{\frac{7}{2}}}{7}+6Aa^2b\sqrt{x}+2Ba^3\sqrt{x}-\frac{2a^3A}{5x^{\frac{5}{2}}}$	7
gosper	$-\frac{2(-455Bb^3x^{12}-665Ax^9b^3-1995Bx^9ab^2-3705Ax^6ab^2-3705Bx^6a^2b-25935Ax^3a^2b-8645a^3Bx^3+1729a^3A)}{8645x^{\frac{5}{2}}}$	8
trager	$-\frac{2(-455Bb^3x^{12}-665Ax^9b^3-1995Bx^9ab^2-3705Ax^6ab^2-3705Bx^6a^2b-25935Ax^3a^2b-8645a^3Bx^3+1729a^3A)}{8645x^{\frac{5}{2}}}$	8
risch	$-\frac{2(-455Bb^3x^{12}-665Ax^9b^3-1995Bx^9ab^2-3705Ax^6ab^2-3705Bx^6a^2b-25935Ax^3a^2b-8645a^3Bx^3+1729a^3A)}{8645x^{\frac{5}{2}}}$	8

```
[In] int((b*x^3+a)^3*(B*x^3+A)/x^(7/2),x,method=_RETURNVERBOSE)
[Out] 2/19*b^3*B*x^(19/2)+2/13*A*b^3*x^(13/2)+6/13*B*a*b^2*x^(13/2)+6/7*A*a*b^2*x
^(7/2)+6/7*B*a^2*b*x^(7/2)+6*A*a^2*b*x^(1/2)+2*B*a^3*x^(1/2)-2/5*a^3*A/x^(5
/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{7/2}}\,dx = \frac{2\,(455\,Bb^3x^{12}+665\,(3\,Bab^2+Ab^3)x^9+3705\,(Ba^2b+Aab^2)x^6-1729\,Aa^3+}{8645\,x^{\frac{5}{2}}}$$

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/8645*(455*B*b^3*x^12 + 665*(3*B*a*b^2 + A*b^3)*x^9 + 3705*(B*a^2*b + A*a*
b^2)*x^6 - 1729*A*a^3 + 8645*(B*a^3 + 3*A*a^2*b)*x^3)/x^(5/2)
```

Sympy [A] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{7/2}}\,dx = -\frac{2Aa^3}{5x^{\frac{5}{2}}}+6Aa^2b\sqrt{x}+\frac{6Aab^2x^{\frac{7}{2}}}{7} + \frac{2Ab^3x^{\frac{13}{2}}}{13}+2Ba^3\sqrt{x}+\frac{6Ba^2bx^{\frac{7}{2}}}{7}+\frac{6Bab^2x^{\frac{13}{2}}}{13}+\frac{2Bb^3x^{\frac{19}{2}}}{19}$$

```
[In] integrate((b*x**3+a)**3*(B*x**3+A)/x**(7/2),x)
```

```
[Out] -2*A*a**3/(5*x**(5/2)) + 6*A*a**2*b*sqrt(x) + 6*A*a*b**2*x**(7/2)/7 + 2*A*b
**3*x**(13/2)/13 + 2*B*a**3*sqrt(x) + 6*B*a**2*b*x**(7/2)/7 + 6*B*a*b**2*x*
*(13/2)/13 + 2*B*b**3*x**(19/2)/19
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{7/2}}\,dx = \frac{2}{19}\,Bb^3x^{\frac{19}{2}}+\frac{2}{13}\,(3\,Bab^2+Ab^3)x^{\frac{13}{2}} + \frac{6}{7}\,(Ba^2b+Aab^2)x^{\frac{7}{2}}-\frac{2\,Aa^3}{5\,x^{\frac{5}{2}}}+2\,(Ba^3+3\,Aa^2b)\sqrt{x}$$

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/19*B*b^3*x^(19/2) + 2/13*(3*B*a*b^2 + A*b^3)*x^(13/2) + 6/7*(B*a^2*b + A*
a*b^2)*x^(7/2) - 2/5*A*a^3/x^(5/2) + 2*(B*a^3 + 3*A*a^2*b)*sqrt(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{7/2}}\,dx = \frac{2}{19}Bb^3x^{\frac{19}{2}} + \frac{6}{13}Bab^2x^{\frac{13}{2}} + \frac{2}{13}Ab^3x^{\frac{13}{2}} + \frac{6}{7}Ba^2bx^{\frac{7}{2}} + \frac{6}{7}Aab^2x^{\frac{7}{2}} + 2\,Ba^3\sqrt{x} + 6\,Aa^2b\sqrt{x} - \frac{2\,Aa^3}{5\,x^{\frac{5}{2}}}$$

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/19*B*b^3*x^(19/2) + 6/13*B*a*b^2*x^(13/2) + 2/13*A*b^3*x^(13/2) + 6/7*B*a
^2*b*x^(7/2) + 6/7*A*a*b^2*x^(7/2) + 2*B*a^3*sqrt(x) + 6*A*a^2*b*sqrt(x) -
2/5*A*a^3/x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{7/2}}\,dx = \sqrt{x}\,\left(2\,B\,a^3+6\,A\,b\,a^2\right) + x^{13/2}\left(\frac{2\,A\,b^3}{13}+\frac{6\,B\,a\,b^2}{13}\right) - \frac{2\,A\,a^3}{5\,x^{5/2}} + \frac{2\,B\,b^3\,x^{19/2}}{19} + \frac{6\,a\,b\,x^{7/2}\,(A\,b+B\,a)}{7}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^3)/x^(7/2),x)
```

```
[Out] x^(1/2)*(2*B*a^3 + 6*A*a^2*b) + x^(13/2)*((2*A*b^3)/13 + (6*B*a*b^2)/13) -
(2*A*a^3)/(5*x^(5/2)) + (2*B*b^3*x^(19/2))/19 + (6*a*b*x^(7/2)*(A*b + B*a))
/7
```

3.155 $\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3}\,dx = \frac{2(Ab-aB)x^{3/2}}{3b^2}+\frac{2Bx^{9/2}}{9b}-\frac{2\sqrt{a}(Ab-aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3b^{5/2}}$$

```
[Out] 2/3*(A*b-B*a)*x^(3/2)/b^2+2/9*B*x^(9/2)/b-2/3*(A*b-B*a)*arctan(x^(3/2)*b^(1
/2)/a^(1/2))*a^(1/2)/b^(5/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {470, 327, 335, 281, 211}

$$\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3}\,dx = -\frac{2\sqrt{a}(Ab-aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3b^{5/2}}+\frac{2x^{3/2}(Ab-aB)}{3b^2}+\frac{2Bx^{9/2}}{9b}$$

```
[In] Int[(x^(7/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*(A*b - a*B)*x^(3/2))/(3*b^2) + (2*B*x^(9/2))/(9*b) - (2*Sqrt[a]*(A*b - a
*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*b^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2Bx^{9/2}}{9b} - \frac{\left(2\left(-\frac{9Ab}{2}+\frac{9aB}{2}\right)\right)\int \frac{x^{7/2}}{a+bx^3}\,dx}{9b}\\
&= \frac{2(Ab-aB)x^{3/2}}{3b^2} + \frac{2Bx^{9/2}}{9b} - \frac{(a(Ab-aB))\int \frac{\sqrt{x}}{a+bx^3}\,dx}{b^2}\\
&= \frac{2(Ab-aB)x^{3/2}}{3b^2} + \frac{2Bx^{9/2}}{9b} - \frac{(2a(Ab-aB))\text{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx, x, \sqrt{x}\right)}{b^2}\\
&= \frac{2(Ab-aB)x^{3/2}}{3b^2} + \frac{2Bx^{9/2}}{9b} - \frac{(2a(Ab-aB))\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^{3/2}\right)}{3b^2}\\
&= \frac{2(Ab-aB)x^{3/2}}{3b^2} + \frac{2Bx^{9/2}}{9b} - \frac{2\sqrt{a}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3b^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

$$\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3}\,dx=\frac{2x^{3/2}(3Ab-3aB+bBx^3)}{9b^2}+\frac{2\sqrt{a}(-Ab+aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3b^{5/2}}$$

```
[In] Integrate[(x^(7/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*x^(3/2)*(3*A*b - 3*a*B + b*B*x^3))/(9*b^2) + (2*Sqrt[a]*(-(A*b) + a*B)*A
rcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*b^(5/2))
```

Maple [A] (verified)

Time = 4.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

method	result	size
risch	$\frac{2x^{\frac{3}{2}}\left(bB\,x^{3}+3Ab-3Ba\right)}{9b^{2}}-\frac{2a(Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3b^{2}\sqrt{ab}}$	55
derivativedivides	$\frac{\frac{2bB\,x^{\frac{9}{2}}}{9}+\frac{2Ab\,x^{\frac{3}{2}}}{3}-\frac{2Ba\,x^{\frac{3}{2}}}{3}}{b^{2}}-\frac{2a(Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3b^{2}\sqrt{ab}}$	58
default	$\frac{\frac{2bB\,x^{\frac{9}{2}}}{9}+\frac{2Ab\,x^{\frac{3}{2}}}{3}-\frac{2Ba\,x^{\frac{3}{2}}}{3}}{b^{2}}-\frac{2a(Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3b^{2}\sqrt{ab}}$	58

```
[In] int(x^(7/2)*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*x^(3/2)*(B*b*x^3+3*A*b-3*B*a)/b^2-2/3*a*(A*b-B*a)/b^2/(a*b)^(1/2)*arcta
n(b*x^(3/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.96

$$\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3}\,dx=\left[-\frac{3\,(Ba-Ab)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^3-2\,bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}-a}{bx^3+a}\right)-2\,(Bbx^4-3\,(Ba-Ab)x)\sqrt{x}}{9\,b^2},\ \frac{2\left(3\,(\right.}{}\right.$$

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/9*(3*(B*a - A*b)*sqrt(-a/b)*log((b*x^3 - 2*b*x^(3/2)*sqrt(-a/b) - a)/(b
*x^3 + a)) - 2*(B*b*x^4 - 3*(B*a - A*b)*x)*sqrt(x))/b^2, 2/9*(3*(B*a - A*b)
*sqrt(a/b)*arctan(b*x^(3/2)*sqrt(a/b)/a) + (B*b*x^4 - 3*(B*a - A*b)*x)*sqrt
(x))/b^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(73) = 146$.

Time = 62.16 (sec) , antiderivative size = 428, normalized size of antiderivative = 5.86

$$\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3}\,dx = \begin{cases} \tilde{\infty}\left(\frac{2Ax^{\frac{3}{2}}}{3}+\frac{2Bx^{\frac{9}{2}}}{9}\right) \\ \frac{\frac{2Ax^{\frac{9}{2}}}{9}+\frac{2Bx^{\frac{15}{2}}}{15}}{a} \\ \frac{\frac{2Ax^{\frac{3}{2}}}{3}+\frac{2Bx^{\frac{9}{2}}}{9}}{b} \\ -\frac{Aa\log\left(\sqrt{x}-\sqrt[6]{-\frac{a}{b}}\right)}{3b^2\sqrt{-\frac{a}{b}}}+\frac{Aa\log\left(\sqrt{x}+\sqrt[6]{-\frac{a}{b}}\right)}{3b^2\sqrt{-\frac{a}{b}}}+\frac{Aa\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{3b^2\sqrt{-\frac{a}{b}}}-\frac{Aa\log\left(4\right.}{} \end{cases}$$

```
[In] integrate(x**(7/2)*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] Piecewise((zoo*(2*A*x**(3/2)/3 + 2*B*x**(9/2)/9), Eq(a, 0) & Eq(b, 0)), ((2
*A*x**(9/2)/9 + 2*B*x**(15/2)/15)/a, Eq(b, 0)), ((2*A*x**(3/2)/3 + 2*B*x**(
9/2)/9)/b, Eq(a, 0)), (-A*a*log(sqrt(x) - (-a/b)**(1/6))/(3*b**2*sqrt(-a/b)
) + A*a*log(sqrt(x) + (-a/b)**(1/6))/(3*b**2*sqrt(-a/b)) + A*a*log(-4*sqrt(
x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*b**2*sqrt(-a/b)) - A*a*log(4*s
qrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*b**2*sqrt(-a/b)) + 2*A*x**
(3/2)/(3*b) + B*a**2*log(sqrt(x) - (-a/b)**(1/6))/(3*b**3*sqrt(-a/b)) - B*a
**2*log(sqrt(x) + (-a/b)**(1/6))/(3*b**3*sqrt(-a/b)) - B*a**2*log(-4*sqrt(x
)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*b**3*sqrt(-a/b)) + B*a**2*log(4
*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*b**3*sqrt(-a/b)) - 2*B*a
*x**(3/2)/(3*b**2) + 2*B*x**(9/2)/(9*b), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

$$\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3}\,dx = \frac{2\,(Ba^2-Aab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}b^2}+\frac{2\left(Bbx^{\frac{9}{2}}-3\,(Ba-Ab)x^{\frac{3}{2}}\right)}{9\,b^2}$$

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 2/3*(B*a^2 - A*a*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/9*(B*b*
x^(9/2) - 3*(B*a - A*b)*x^(3/2))/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3}\,dx = \frac{2\,(Ba^2-Aab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}b^2}+\frac{2\left(Bb^2x^{\frac{9}{2}}-3\,Babx^{\frac{3}{2}}+3\,Ab^2x^{\frac{3}{2}}\right)}{9\,b^3}$$

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(B*a^2 - A*a*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/9*(B*b^
2*x^(9/2) - 3*B*a*b*x^(3/2) + 3*A*b^2*x^(3/2))/b^3
```

Mupad [B] (verification not implemented)

Time = 7.02 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.52

$$\int \frac{x^{7/2}(A+Bx^3)}{a+bx^3}\,dx = x^{3/2}\left(\frac{2\,A}{3\,b}-\frac{2\,B\,a}{3\,b^2}\right)+\frac{2\,B\,x^{9/2}}{9\,b} - \frac{2\,\sqrt{a}\,\mathrm{atan}\left(\frac{72\,b^{3/2}\,x^{3/2}\,\left(A^2\,a^2\,b^2-2\,A\,B\,a^3\,b+B^2\,a^4\right)}{\sqrt{a}\,\left(72\,A\,a^2\,b^2-72\,B\,a^3\,b\right)\,\left(A\,b-B\,a\right)}\right)\,\left(A\,b-B\,a\right)}{3\,b^{5/2}}$$

```
[In] int((x^(7/2)*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] x^(3/2)*((2*A)/(3*b) - (2*B*a)/(3*b^2)) + (2*B*x^(9/2))/(9*b) - (2*a^(1/2)*
atan((72*b^(3/2)*x^(3/2)*(B^2*a^4 + A^2*a^2*b^2 - 2*A*B*a^3*b))/(a^(1/2)*(7
2*A*a^2*b^2 - 72*B*a^3*b)*(A*b - B*a)))*(A*b - B*a))/(3*b^(5/2))
```

3.156 $\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 288

$$\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3} dx = \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} + \frac{\sqrt[6]{a}(Ab-aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}} - \frac{\sqrt[6]{a}(Ab-aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}} - \frac{2\sqrt[6]{a}(Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}} + \frac{\sqrt[6]{a}(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}b^{13/6}} - \frac{\sqrt[6]{a}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}b^{13/6}}$$

```
[Out] 2/7*B*x^(7/2)/b-2/3*a^(1/6)*(A*b-B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/b^(13
/6)-1/3*a^(1/6)*(A*b-B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/b^(13/
6)-1/3*a^(1/6)*(A*b-B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/b^(13/6)
+1/6*a^(1/6)*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2)
)/b^(13/6)*3^(1/2)-1/6*a^(1/6)*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/
6)*3^(1/2)*x^(1/2))/b^(13/6)*3^(1/2)+2*(A*b-B*a)*x^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {470, 327, 335, 215, 648, 632, 210, 642, 211}

$$\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3}\,dx = \frac{\sqrt[6]{a}(Ab-aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}} - \frac{\sqrt[6]{a}(Ab-aB)\arctan\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{3b^{13/6}} - \frac{2\sqrt[6]{a}(Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}} + \frac{\sqrt[6]{a}(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}b^{13/6}} - \frac{\sqrt[6]{a}(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}b^{13/6}} + \frac{2\sqrt{x}(Ab-aB)}{b^2} + \frac{2Bx^{7/2}}{7b}$$

```
[In] Int[(x^(5/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[x])/b^2 + (2*B*x^(7/2))/(7*b) + (a^(1/6)*(A*b - a*B)*Ar
cTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(3*b^(13/6)) - (a^(1/6)*(A*b -
 a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(3*b^(13/6)) - (2*a^(1
/6)*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(3*b^(13/6)) + (a^(1/6)*
(A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*
Sqrt[3]*b^(13/6)) - (a^(1/6)*(A*b - a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1
/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*b^(13/6))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
```

```
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{2Bx^{7/2}}{7b} - \frac{\left(2\left(-\frac{7Ab}{2}+\frac{7aB}{2}\right)\right)\int \frac{x^{5/2}}{a+bx^3}\,dx}{7b}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} - \frac{(a(Ab-aB))\int \frac{1}{\sqrt{x}(a+bx^3)}\,dx}{b^2}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} - \frac{(2a(Ab-aB))\text{Subst}\left(\int \frac{1}{a+bx^6}\,dx, x, \sqrt{x}\right)}{b^2}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} - \frac{\left(2\sqrt[6]{a}(Ab-aB)\right)\text{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{3b^2} - \frac{\left(2\sqrt[6]{a}(Ab-aB)\right)\text{Subst}\left(\int \frac{\sqrt[6]{a}+\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{3b^2} - \frac{\left(2\sqrt[3]{a}(Ab-aB)\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{3b^2}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} - \frac{2\sqrt[6]{a}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}} + \frac{\left(\sqrt[6]{a}(Ab-aB)\right)\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{3}b^{13/6}} - \frac{\left(\sqrt[6]{a}(Ab-aB)\right)\text{Subst}\left(\int \frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{3}b^{13/6}} - \frac{\left(\sqrt[3]{a}(Ab-aB)\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{6b^2} - \frac{\left(\sqrt[3]{a}(Ab-aB)\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{6b^2}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} - \frac{2\sqrt[6]{a}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}} + \frac{\sqrt[6]{a}(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}b^{13/6}} - \frac{\sqrt[6]{a}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}b^{13/6}} - \frac{\left(\sqrt[6]{a}(Ab-aB)\right)\text{Subst}\left(\int\frac{1}{-\frac{1}{3}-x^2}\,dx,x,1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{3\sqrt{3}b^{13/6}} + \frac{\left(\sqrt[6]{a}(Ab-aB)\right)\text{Subst}\left(\int\frac{1}{-\frac{1}{3}-x^2}\,dx,x,1+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{3\sqrt{3}b^{13/6}}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} + \frac{\sqrt[6]{a}(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}} - \frac{\sqrt[6]{a}(Ab-aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}} - \frac{2\sqrt[6]{a}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}} + \frac{\sqrt[6]{a}(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}b^{13/6}} - \frac{\sqrt[6]{a}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}b^{13/6}}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.60

$$\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3}\,dx = \frac{6\sqrt[6]{b}\sqrt{x}(7Ab-7aB+bBx^3)+14\sqrt[6]{a}(-Ab+aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)-7\sqrt[6]{a}(-Ab+aB)}{21b^{13/6}}$$

```
[In] Integrate[(x^(5/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (6*b^(1/6)*Sqrt[x]*(7*A*b - 7*a*B + b*B*x^3) + 14*a^(1/6)*(-(A*b) + a*B)*Ar
cTan[(b^(1/6)*Sqrt[x])/a^(1/6)] - 7*a^(1/6)*(-(A*b) + a*B)*ArcTan[(a^(1/3)
- b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])] + 7*Sqrt[3]*a^(1/6)*(-(A*b) + a*B)*
ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/(21*b^(13
/6))
```

Maple [A] (verified)

Time = 4.36 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.72

method	result
risch	$\frac{2(bB\,x^3+7Ab-7Ba)\sqrt{x}}{7b^2}-\frac{a(Ab-Ba)\left(\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{3a}+\frac{2\left(\frac{a}{b}\right)^{\frac{1}{6}}\text{arc}}{}\right.}{b^2}$
derivativedivides	$\frac{\frac{2bB\,x^{\frac{7}{2}}}{7}+2Ab\sqrt{x}-2Ba\sqrt{x}}{b^2}-\frac{2\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(-\sqrt{3}+\frac{2}{\left(\right.}\right.}{6a}\right.}{b^2}$
default	$\frac{\frac{2bB\,x^{\frac{7}{2}}}{7}+2Ab\sqrt{x}-2Ba\sqrt{x}}{b^2}-\frac{2\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(-\sqrt{3}+\frac{2}{\left(\right.}\right.}{6a}\right.}{b^2}$

```
[In] int(x^(5/2)*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*(B*b*x^3+7*A*b-7*B*a)*x^(1/2)/b^2-a*(A*b-B*a)/b^2*(1/6/a*3^(1/2)*(a/b)^
(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/3/a*(a/b)^(1/6)*arcta
n(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))+2/3/a*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/
6))-1/6/a*3^(1/2)*(a/b)^(1/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))
+1/3/a*(a/b)^(1/6)*arctan(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1289 vs. 2(204) = 408.

Time = 0.30 (sec) , antiderivative size = 1289, normalized size of antiderivative = 4.48

$$\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/42*(14*b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*
a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6)*log
(b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 +
 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6) - (B*a - A*b
)*sqrt(x)) - 14*b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^
3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/
```

```
6)*log(-b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^
4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6) - (B*
a - A*b)*sqrt(x)) + 7*(sqrt(-3)*b^2 + b^2)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*
A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5
 + A^6*a*b^6)/b^13)^(1/6)*log(-2*(B*a - A*b)*sqrt(x) + (sqrt(-3)*b^2 + b^2)
*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*
A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6)) - 7*(sqrt(-3)*b
^2 + b^2)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*
b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6)*log(-2*
(B*a - A*b)*sqrt(x) - (sqrt(-3)*b^2 + b^2)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*
A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5
 + A^6*a*b^6)/b^13)^(1/6)) + 7*(sqrt(-3)*b^2 - b^2)*(-(B^6*a^7 - 6*A*B^5*a^
6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*
B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6)*log(-2*(B*a - A*b)*sqrt(x) + (sqrt(-3)*b
^2 - b^2)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*
b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6)) - 7*(s
qrt(-3)*b^2 - b^2)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3
*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6
)*log(-2*(B*a - A*b)*sqrt(x) - (sqrt(-3)*b^2 - b^2)*(-(B^6*a^7 - 6*A*B^5*a^
6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*
B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6)) - 12*(B*b*x^3 - 7*B*a + 7*A*b)*sqrt(x))
/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 605 vs. $2(267) = 534$.

Time = 29.28 (sec) , antiderivative size = 605, normalized size of antiderivative = 2.10

$$\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3}\,dx = \begin{cases} \tilde{\infty}\left(2A\sqrt{x}+\frac{2Bx^{\frac{7}{2}}}{7}\right) \\ \frac{\frac{2Ax^{\frac{7}{2}}}{7}+\frac{2Bx^{\frac{13}{2}}}{13}}{a} \\ \frac{2A\sqrt{x}+\frac{2Bx^{\frac{7}{2}}}{7}}{b} \\ \frac{2A\sqrt{x}}{b}+\frac{A\sqrt[6]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[6]{-\frac{a}{b}}\right)}{3b}-\frac{A\sqrt[6]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[6]{-\frac{a}{b}}\right)}{3b}+\frac{A\sqrt[6]{-\frac{a}{b}}\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4\right.}{6b} \end{cases}$$

```
[In] integrate(x**(5/2)*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] Piecewise((zoo*(2*A*sqrt(x) + 2*B*x**(7/2)/7), Eq(a, 0) & Eq(b, 0)), ((2*A*
x**(7/2)/7 + 2*B*x**(13/2)/13)/a, Eq(b, 0)), ((2*A*sqrt(x) + 2*B*x**(7/2)/7
)/b, Eq(a, 0)), (2*A*sqrt(x)/b + A*(-a/b)**(1/6)*log(sqrt(x) - (-a/b)**(1/6
))/(3*b) - A*(-a/b)**(1/6)*log(sqrt(x) + (-a/b)**(1/6))/(3*b) + A*(-a/b)**(
```

```
1/6)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b) - A*(-a/b)
**(1/6)*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b) - sqrt(3
)*A*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*
b) - sqrt(3)*A*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqr
t(3)/3)/(3*b) - 2*B*a*sqrt(x)/b**2 - B*a*(-a/b)**(1/6)*log(sqrt(x) - (-a/b)
**(1/6))/(3*b**2) + B*a*(-a/b)**(1/6)*log(sqrt(x) + (-a/b)**(1/6))/(3*b**2)
 - B*a*(-a/b)**(1/6)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/
(6*b**2) + B*a*(-a/b)**(1/6)*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**
(1/3))/(6*b**2) + sqrt(3)*B*a*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b
)**(1/6)) - sqrt(3)/3)/(3*b**2) + sqrt(3)*B*a*(-a/b)**(1/6)*atan(2*sqrt(3)*
sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(3*b**2) + 2*B*x**(7/2)/(7*b), True)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.02

$$\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3}\,dx = \frac{\left(\frac{\sqrt{3}(Ba-Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} - \frac{\sqrt{3}(Ba-Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} + \frac{4\left(Bab^{\frac{1}{3}}-Ab^{\frac{4}{3}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}}\right.}{} + \frac{2\left(Bbx^{\frac{7}{2}}-7\,(Ba-Ab)\sqrt{x}\right)}{7\,b^2}$$

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/6*(sqrt(3)*(B*a - A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x +
a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(B*a - A*b)*log(-sqrt(3)*a^(1/6)*b^(1/
6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(B*a*b^(1/3) - A*b^
(4/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(
a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*arctan((sqrt(
3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*s
qrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*arctan(-(
sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1
/3)*sqrt(a^(1/3)*b^(1/3))))*a/b^2 + 2/7*(B*b*x^(7/2) - 7*(B*a - A*b)*sqrt(x
))/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00

$$\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3}\,dx = \frac{\sqrt{3}\left((ab^5)^{\frac{1}{6}}\,Ba-(ab^5)^{\frac{1}{6}}\,Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,b^3} - \frac{\sqrt{3}\left((ab^5)^{\frac{1}{6}}\,Ba-(ab^5)^{\frac{1}{6}}\,Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,b^3} + \frac{\left((ab^5)^{\frac{1}{6}}\,Ba-(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,b^3} + \frac{\left((ab^5)^{\frac{1}{6}}\,Ba-(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,b^3} + \frac{2\left((ab^5)^{\frac{1}{6}}\,Ba-(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,b^3} + \frac{2\left(Bb^6x^{\frac{7}{2}}-7\,Bab^5\sqrt{x}+7\,Ab^6\sqrt{x}\right)}{7\,b^7}$$

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/
b)^(1/6) + x + (a/b)^(1/3))/b^3 - 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5)^
(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/b^3 + 1/3*((
a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(
x))/(a/b)^(1/6))/b^3 + 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(-
(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/b^3 + 2/3*((a*b^5)^(1/6)*B*a
 - (a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/b^3 + 2/7*(B*b^6*x^(7/2)
- 7*B*a*b^5*sqrt(x) + 7*A*b^6*sqrt(x))/b^7
```

Mupad [B] (verification not implemented)

Time = 7.33 (sec) , antiderivative size = 1933, normalized size of antiderivative = 6.71

$$\int \frac{x^{5/2}(A+Bx^3)}{a+bx^3}\,dx = \text{Too large to display}$$

```
[In] int((x^(5/2)*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] x^(1/2)*((2*A)/b - (2*B*a)/b^2) + (2*B*x^(7/2))/(7*b) + ((-a)^(1/6)*atan(((
(-a)^(1/6)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*
b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 - (96*(-a)^(1/6)*(A*b - B*a)*(B
^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6))*1i)/(3*b
```

^(13/6)) + ((-a)^(1/6)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 + (96*(-a)^(1/6)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6))*1i)/(3*b^(13/6)))/(((-a)^(1/6)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 - (96*(-a)^(1/6)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6)))/(3*b^(13/6)) - ((-a)^(1/6)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 + (96*(-a)^(1/6)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6)))/(3*b^(13/6))))*(A*b - B*a)*2i)/(3*b^(13/6)) + ((-a)^(1/6)*atan((((-a)^(1/6)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 - (96*(-a)^(1/6)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6))*1i)/(3*b^(13/6)) + ((-a)^(1/6)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 + (96*(-a)^(1/6)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6))*1i)/(3*b^(13/6)))/(((-a)^(1/6)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 - (96*(-a)^(1/6)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6)))/(3*b^(13/6)) - ((-a)^(1/6)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 + (96*(-a)^(1/6)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6)))/(3*b^(13/6))))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*2i)/(3*b^(13/6)) + ((-a)^(1/6)*atan((((-a)^(1/6)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 - (96*(-a)^(1/6)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6))*1i)/(3*b^(13/6)) + ((-a)^(1/6)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 + (96*(-a)^(1/6)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6))*1i)/(3*b^(13/6)))/(((-a)^(1/6)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 - (96*(-a)^(1/6)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6)))/(3*b^(13/6)) - ((-a)^(1/6)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 + (96*(-a)^(1/6)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6)))/(3*b^(13/6))))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*2i)/(3*b^(13/6))

3.157 $\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 270

$$\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}\,dx = \frac{2Bx^{5/2}}{5b} - \frac{(Ab-aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{11/6}} + \frac{(Ab-aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{11/6}} + \frac{2(Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{11/6}} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}\sqrt[6]{a}b^{11/6}} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}\sqrt[6]{a}b^{11/6}}$$

```
[Out] 2/5*B*x^(5/2)/b+2/3*(A*b-B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(1/6)/b^(11
/6)+1/3*(A*b-B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(1/6)/b^(11/
6)+1/3*(A*b-B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(1/6)/b^(11/6)
+1/6*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(1/6
)/b^(11/6)*3^(1/2)-1/6*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/
2)*x^(1/2))/a^(1/6)/b^(11/6)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {470, 335, 301, 648, 632, 210, 642, 211}

$$\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}\,dx = -\frac{(Ab-aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{11/6}} + \frac{(Ab-aB)\arctan\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{3\sqrt[6]{a}b^{11/6}} + \frac{2(Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{11/6}} + \frac{(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}\sqrt[6]{a}b^{11/6}} - \frac{(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}\sqrt[6]{a}b^{11/6}} + \frac{2Bx^{5/2}}{5b}$$

```
[In] Int[(x^(3/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*B*x^(5/2))/(5*b) - ((A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(
1/6)])/(3*a^(1/6)*b^(11/6)) + ((A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt
[x])/a^(1/6)])/(3*a^(1/6)*b^(11/6)) + (2*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x
])/a^(1/6)])/(3*a^(1/6)*b^(11/6)) + ((A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1
/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(1/6)*b^(11/6)) - ((A*b - a*
B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a
^(1/6)*b^(11/6))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
```

```
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{2Bx^{5/2}}{5b} - \frac{\left(2\left(-\frac{5Ab}{2}+\frac{5aB}{2}\right)\right)\int \frac{x^{3/2}}{a+bx^3}\,dx}{5b}$$

$$= \frac{2Bx^{5/2}}{5b} - \frac{\left(4\left(-\frac{5Ab}{2}+\frac{5aB}{2}\right)\right)\text{Subst}\left(\int \frac{x^4}{a+bx^6}\,dx, x, \sqrt{x}\right)}{5b}$$

$$= \frac{2Bx^{5/2}}{5b} + \frac{(2(Ab-aB))\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{3b^{5/3}} + \frac{(2(Ab-aB))\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2}+\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{3\sqrt[6]{a}b^{5/3}} + \frac{(2(Ab-aB))\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{3\sqrt[6]{a}b^{5/3}}$$

$$= \frac{2Bx^{5/2}}{5b} + \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{11/6}} + \frac{(Ab-aB)\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{3}\sqrt[6]{a}b^{11/6}} - \frac{(Ab-aB)\text{Subst}\left(\int \frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{3}\sqrt[6]{a}b^{11/6}} + \frac{(Ab-aB)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{6b^{5/3}} + \frac{(Ab-aB)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{6b^{5/3}}$$

$$= \frac{2Bx^{5/2}}{5b} + \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{11/6}} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}\sqrt[6]{a}b^{11/6}} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}\sqrt[6]{a}b^{11/6}} + \frac{(Ab-aB)\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{11/6}} - \frac{(Ab-aB)\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{11/6}}$$

$$= \frac{2Bx^{5/2}}{5b} - \frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{11/6}} + \frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{11/6}}$$
$$+ \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{11/6}} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}\sqrt[6]{a}b^{11/6}}$$
$$- \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}\sqrt[6]{a}b^{11/6}}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.57

$$\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}\,dx = \frac{6\sqrt[6]{a}b^{5/6}Bx^{5/2} + 10(Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right) - 5(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right) -}{15\sqrt[6]{a}b^{11/6}}$$

```
[In] Integrate[(x^(3/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (6*a^(1/6)*b^(5/6)*B*x^(5/2) + 10*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1
/6)] - 5*(A*b - a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])
] - 5*Sqrt[3]*(A*b - a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3
) + b^(1/3)*x)])/(15*a^(1/6)*b^(11/6))
```

Maple [A] (verified)

Time = 4.32 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.70

method	result
risch	$\frac{2Bx^{\frac{5}{2}}}{5b}+\frac{(Ab-Ba)\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a}+\frac{\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\frac{2\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{\cdots}\right.}{\cdots}\cdots\right.}{b}$
derivativedivides	$\frac{2Bx^{\frac{5}{2}}}{5b}+\frac{2\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\mathrm{arct}\cdots}{\cdots}\right.}{b}$
default	$\frac{2Bx^{\frac{5}{2}}}{5b}+\frac{2\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\mathrm{arct}\cdots}{\cdots}\right.}{b}$

```
[In] int(x^(3/2)*(B*x^3+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*B*x^(5/2)/b+(A*b-B*a)/b*(-1/6/a*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^
(1/6)*x^(1/2)+(a/b)^(1/3))+1/3/b/(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3
^(1/2))+2/3/b/(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))+1/6/a*3^(1/2)*(a/b)^(
5/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+1/3/b/(a/b)^(1/6)*arctan
(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1603 vs. $2(188) = 376$.

Time = 0.31 (sec) , antiderivative size = 1603, normalized size of antiderivative = 5.94

$$\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/30*(12*B*x^(5/2) + 10*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 -
 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11
))^(1/6)*log(a*b^9*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3
*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(5/6
) - (B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - 10*A^3*B^2*a^2*b^3 + 5*
A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) - 10*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*
B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6
*b^6)/(a*b^11))^(1/6)*log(-a*b^9*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^
4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/
```

```
(a*b^11))^(5/6) - (B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - 10*A^3*B^
2*a^2*b^3 + 5*A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) - 5*(sqrt(-3)*b - b)*(-(B^6*a
^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a
^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(1/6)*log((sqrt(-3)*a*b^9 + a*b
^9)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 +
15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(5/6) - 2*(B^5*a^5
- 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - 10*A^3*B^2*a^2*b^3 + 5*A^4*B*a*b^4 -
 A^5*b^5)*sqrt(x)) + 5*(sqrt(-3)*b - b)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2
*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^
6*b^6)/(a*b^11))^(1/6)*log(-(sqrt(-3)*a*b^9 + a*b^9)*(-(B^6*a^6 - 6*A*B^5*a
^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5
*B*a*b^5 + A^6*b^6)/(a*b^11))^(5/6) - 2*(B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B
^3*a^3*b^2 - 10*A^3*B^2*a^2*b^3 + 5*A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) - 5*(sq
rt(-3)*b + b)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*
a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(1/6)*log
((sqrt(-3)*a*b^9 - a*b^9)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 -
 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11
))^(5/6) - 2*(B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - 10*A^3*B^2*a^2
*b^3 + 5*A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) + 5*(sqrt(-3)*b + b)*(-(B^6*a^6 -
6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^
4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(1/6)*log(-(sqrt(-3)*a*b^9 - a*b^9)*
(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A
^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(5/6) - 2*(B^5*a^5 - 5*
A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - 10*A^3*B^2*a^2*b^3 + 5*A^4*B*a*b^4 - A^5
*b^5)*sqrt(x)))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 581 vs. 2(258) = 516.

Time = 11.67 (sec) , antiderivative size = 581, normalized size of antiderivative = 2.15

$$\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{\sqrt{x}}+\frac{2Bx^{\frac{5}{2}}}{5}\right) \\ \frac{\frac{2Ax^{\frac{5}{2}}}{5}+\frac{2Bx^{\frac{11}{2}}}{11}}{a} \\ \frac{-\frac{2A}{\sqrt{x}}+\frac{2Bx^{\frac{5}{2}}}{5}}{b} \\ \frac{A\log\left(\sqrt{x}-\sqrt[6]{-\frac{a}{b}}\right)}{3b\sqrt[6]{-\frac{a}{b}}} - \frac{A\log\left(\sqrt{x}+\sqrt[6]{-\frac{a}{b}}\right)}{3b\sqrt[6]{-\frac{a}{b}}} + \frac{A\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{6b\sqrt[6]{-\frac{a}{b}}} - \frac{A\log\left(4\sqrt{x}\sqrt[6]{-\ldots\right.}{6b\ldots} \end{cases}$$

```
[In] integrate(x**(3/2)*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] Piecewise((zoo*(-2*A/sqrt(x) + 2*B*x**(5/2)/5), Eq(a, 0) & Eq(b, 0)), ((2*A
*x**(5/2)/5 + 2*B*x**(11/2)/11)/a, Eq(b, 0)), ((-2*A/sqrt(x) + 2*B*x**(5/2)
/5)/b, Eq(a, 0)), (A*log(sqrt(x) - (-a/b)**(1/6))/(3*b*(-a/b)**(1/6)) - A*l
og(sqrt(x) + (-a/b)**(1/6))/(3*b*(-a/b)**(1/6)) + A*log(-4*sqrt(x)*(-a/b)**
(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b*(-a/b)**(1/6)) - A*log(4*sqrt(x)*(-a/b)
**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b*(-a/b)**(1/6)) + sqrt(3)*A*atan(2*sqr
t(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*b*(-a/b)**(1/6)) + sqrt(3)*A
*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(3*b*(-a/b)**(1/6))
- B*a*log(sqrt(x) - (-a/b)**(1/6))/(3*b**2*(-a/b)**(1/6)) + B*a*log(sqrt(x)
 + (-a/b)**(1/6))/(3*b**2*(-a/b)**(1/6)) - B*a*log(-4*sqrt(x)*(-a/b)**(1/6)
 + 4*x + 4*(-a/b)**(1/3))/(6*b**2*(-a/b)**(1/6)) + B*a*log(4*sqrt(x)*(-a/b)
**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b**2*(-a/b)**(1/6)) - sqrt(3)*B*a*atan(
2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*b**2*(-a/b)**(1/6)) - s
qrt(3)*B*a*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(3*b**2*(-
a/b)**(1/6)) + 2*B*x**(5/2)/(5*b), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.79

$$\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}\,dx = \frac{2\,Bx^{\frac{5}{2}}}{5\,b} + \frac{(Ba-Ab)\left(\frac{\sqrt{3}\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}} - \frac{\sqrt{3}\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}} - \frac{2\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} - \frac{2\arctan\left(-\frac{\sqrt{3}a}{}\right.}{b^{\frac{2}{3}}\sqrt{a}}\right.}{6\,b}$$

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 2/5*B*x^(5/2)/b + 1/6*(B*a - A*b)*(sqrt(3)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt
(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6)) - sqrt(3)*log(-sqrt(3)*a^(1/6)
*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6)) - 2*arctan((sqrt(
3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqr
t(a^(1/3)*b^(1/3))) - 2*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x
))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 4*arctan(b^(1/3
)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))))/b
```

Giac [A] (verification not implemented)

none

Time = 0.63 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.96

$$\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}\,dx = \frac{2\,Bx^{\frac{5}{2}}}{5\,b} - \frac{(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,(ab^5)^{\frac{1}{6}}\,b} - \frac{(Ba-Ab)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,(ab^5)^{\frac{1}{6}}\,b} - \frac{2\left(Ba\left(\frac{a}{b}\right)^{\frac{5}{6}}-Ab\left(\frac{a}{b}\right)^{\frac{5}{6}}\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,ab} + \frac{\sqrt{3}\left((ab^5)^{\frac{5}{6}}\,Ba-(ab^5)^{\frac{5}{6}}\,Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,ab^6} - \frac{\sqrt{3}\left((ab^5)^{\frac{5}{6}}\,Ba-(ab^5)^{\frac{5}{6}}\,Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,ab^6}$$

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/5*B*x^(5/2)/b - 1/3*(B*a - A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/
(a/b)^(1/6))/((a*b^5)^(1/6)*b) - 1/3*(B*a - A*b)*arctan(-(sqrt(3)*(a/b)^(1/
6) - 2*sqrt(x))/(a/b)^(1/6))/((a*b^5)^(1/6)*b) - 2/3*(B*a*(a/b)^(5/6) - A*b
*(a/b)^(5/6))*arctan(sqrt(x)/(a/b)^(1/6))/(a*b) + 1/6*sqrt(3)*((a*b^5)^(5/6
)*B*a - (a*b^5)^(5/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3
))/(a*b^6) - 1/6*sqrt(3)*((a*b^5)^(5/6)*B*a - (a*b^5)^(5/6)*A*b)*log(-sqrt(
3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a*b^6)
```

Mupad [B] (verification not implemented)

Time = 7.26 (sec) , antiderivative size = 1640, normalized size of antiderivative = 6.07

$$\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}\,dx = \text{Too large to display}$$

```
[In] int((x^(3/2)*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] (2*B*x^(5/2))/(5*b) + (atan((((A*b - B*a)^2*(32*A^3*a^3*b^3 - 32*B^3*a^6 +
96*A*B^2*a^5*b - 96*A^2*B*a^4*b^2 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 +
 864*B^2*a^5*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6)))*1i)/((-a)^(
1/3)*b^(11/3)) + ((A*b - B*a)^2*(32*B^3*a^6 - 32*A^3*a^3*b^3 - 96*A*B^2*a^5
*b + 96*A^2*B*a^4*b^2 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5
*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6)))*1i)/((-a)^(1/3)*b^(11/3
)))/(((A*b - B*a)^2*(32*A^3*a^3*b^3 - 32*B^3*a^6 + 96*A*B^2*a^5*b - 96*A^2*
B*a^4*b^2 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2 - 1728*
```

```
A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6))))/((-a)^(1/3)*b^(11/3)) - ((A*b - B*
a)^2*(32*B^3*a^6 - 32*A^3*a^3*b^3 - 96*A*B^2*a^5*b + 96*A^2*B*a^4*b^2 + (x^
(1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2 - 1728*A*B*a^4*b^3))/(
27*(-a)^(1/6)*b^(11/6))))/((-a)^(1/3)*b^(11/3))))*(A*b - B*a)*2i)/(3*(-a)^(
1/6)*b^(11/6)) + (atan(((((3^(1/2)*1i)/2 - 1/2)^2*(A*b - B*a)^2*(32*A^3*a^3
*b^3 - 32*B^3*a^6 + 96*A*B^2*a^5*b - 96*A^2*B*a^4*b^2 + (x^(1/2)*((3^(1/2)*
1i)/2 - 1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2 - 1728*A*B*a^4*
b^3))/(27*(-a)^(1/6)*b^(11/6)))*1i)/((-a)^(1/3)*b^(11/3)) + (((3^(1/2)*1i)/
2 - 1/2)^2*(A*b - B*a)^2*(32*B^3*a^6 - 32*A^3*a^3*b^3 - 96*A*B^2*a^5*b + 96
*A^2*B*a^4*b^2 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(864*A^2*a^3*b
^4 + 864*B^2*a^5*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6)))*1i)/((-
a)^(1/3)*b^(11/3)))/((((3^(1/2)*1i)/2 - 1/2)^2*(A*b - B*a)^2*(32*A^3*a^3*b^
3 - 32*B^3*a^6 + 96*A*B^2*a^5*b - 96*A^2*B*a^4*b^2 + (x^(1/2)*((3^(1/2)*1i)
/2 - 1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2 - 1728*A*B*a^4*b^3
))/(27*(-a)^(1/6)*b^(11/6))))/((-a)^(1/3)*b^(11/3)) - (((3^(1/2)*1i)/2 - 1/
2)^2*(A*b - B*a)^2*(32*B^3*a^6 - 32*A^3*a^3*b^3 - 96*A*B^2*a^5*b + 96*A^2*B
*a^4*b^2 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 8
64*B^2*a^5*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6))))/((-a)^(1/3)*
b^(11/3))))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*2i)/(3*(-a)^(1/6)*b^(11/6))
+ (atan(((((3^(1/2)*1i)/2 + 1/2)^2*(A*b - B*a)^2*(32*A^3*a^3*b^3 - 32*B^3*a
^6 + 96*A*B^2*a^5*b - 96*A^2*B*a^4*b^2 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A
*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^
(1/6)*b^(11/6)))*1i)/((-a)^(1/3)*b^(11/3)) + (((3^(1/2)*1i)/2 + 1/2)^2*(A*b
 - B*a)^2*(32*B^3*a^6 - 32*A^3*a^3*b^3 - 96*A*B^2*a^5*b + 96*A^2*B*a^4*b^2
+ (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^
5*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6)))*1i)/((-a)^(1/3)*b^(11/
3)))/((((3^(1/2)*1i)/2 + 1/2)^2*(A*b - B*a)^2*(32*A^3*a^3*b^3 - 32*B^3*a^6
+ 96*A*B^2*a^5*b - 96*A^2*B*a^4*b^2 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b
- B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^(1/
6)*b^(11/6))))/((-a)^(1/3)*b^(11/3)) - (((3^(1/2)*1i)/2 + 1/2)^2*(A*b - B*a
)^2*(32*B^3*a^6 - 32*A^3*a^3*b^3 - 96*A*B^2*a^5*b + 96*A^2*B*a^4*b^2 + (x^(
1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2
- 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6))))/((-a)^(1/3)*b^(11/3))))*((3
^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*2i)/(3*(-a)^(1/6)*b^(11/6))
```

3.158 $\int \frac{\sqrt{x}(A+Bx^3)}{a+bx^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{\sqrt{x}(A+Bx^3)}{a+bx^3} dx = \frac{2Bx^{3/2}}{3b} + \frac{2(Ab-aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{3/2}}$$

```
[Out] 2/3*B*x^(3/2)/b+2/3*(A*b-B*a)*arctan(x^(3/2)*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/
2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {470, 335, 281, 211}

$$\int \frac{\sqrt{x}(A+Bx^3)}{a+bx^3} dx = \frac{2(Ab-aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{3/2}} + \frac{2Bx^{3/2}}{3b}$$

```
[In] Int[(Sqrt[x]*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*B*x^(3/2))/(3*b) + (2*(A*b - a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*
Sqrt[a]*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2Bx^{3/2}}{3b} - \frac{\left(2\left(-\frac{3Ab}{2}+\frac{3aB}{2}\right)\right)\int \frac{\sqrt{x}}{a+bx^3}\,dx}{3b} \\
&= \frac{2Bx^{3/2}}{3b} - \frac{\left(4\left(-\frac{3Ab}{2}+\frac{3aB}{2}\right)\right)\text{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx, x, \sqrt{x}\right)}{3b} \\
&= \frac{2Bx^{3/2}}{3b} + \frac{(2(Ab-aB))\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^{3/2}\right)}{3b} \\
&= \frac{2Bx^{3/2}}{3b} + \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}(A+Bx^3)}{a+bx^3}\,dx = \frac{2Bx^{3/2}}{3b} - \frac{2(-Ab+aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{3/2}}$$

```
[In] Integrate[(Sqrt[x]*(A + B*x^3))/(a + b*x^3),x]
[Out] (2*B*x^(3/2))/(3*b) - (2*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/
(3*Sqrt[a]*b^(3/2))
```

Maple [A] (verified)

Time = 4.45 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{2B\,x^{\frac{3}{2}}}{3b}+\frac{2(Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3b\sqrt{ab}}$	40
default	$\frac{2B\,x^{\frac{3}{2}}}{3b}+\frac{2(Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3b\sqrt{ab}}$	40
risch	$\frac{2B\,x^{\frac{3}{2}}}{3b}+\frac{2(Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3b\sqrt{ab}}$	40

```
[In] int((B*x^3+A)*x^(1/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
[Out] 2/3*B*x^(3/2)/b+2/3*(A*b-B*a)/b/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.04

$$\int\frac{\sqrt{x}(A+Bx^3)}{a+bx^3}\,dx=\left[\frac{2\,Babx^{\frac{3}{2}}+(Ba-Ab)\sqrt{-ab}\log\left(\frac{bx^3-2\,\sqrt{-ab}x^{\frac{3}{2}}-a}{bx^3+a}\right)}{3\,ab^2},\frac{2\left(Babx^{\frac{3}{2}}-(Ba-Ab)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x^{\frac{3}{2}}}{a}\right)\right)}{3\,ab^2}\right]$$

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a),x, algorithm="fricas")
[Out] [1/3*(2*B*a*b*x^(3/2) + (B*a - A*b)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x^
(3/2) - a)/(b*x^3 + a)))/(a*b^2), 2/3*(B*a*b*x^(3/2) - (B*a - A*b)*sqrt(a*b
)*arctan(sqrt(a*b)*x^(3/2)/a))/(a*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(51) = 102$.

Time = 4.68 (sec) , antiderivative size = 381, normalized size of antiderivative = 7.19

$$\int \frac{\sqrt{x}(A+Bx^3)}{a+bx^3}\,dx$$
$$= \begin{cases} \tilde{\infty}\left(-\frac{2A}{3x^{\frac{3}{2}}}+\frac{2Bx^{\frac{3}{2}}}{3}\right) \\ \frac{\frac{2Ax^{\frac{3}{2}}}{3}+\frac{2Bx^{\frac{9}{2}}}{9}}{a} \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}+\frac{2Bx^{\frac{3}{2}}}{3}}{b} \\ \frac{A\log\left(\sqrt{x}-\sqrt[6]{-\frac{a}{b}}\right)}{3b\sqrt{-\frac{a}{b}}}-\frac{A\log\left(\sqrt{x}+\sqrt[6]{-\frac{a}{b}}\right)}{3b\sqrt{-\frac{a}{b}}}-\frac{A\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{3b\sqrt{-\frac{a}{b}}}+\frac{A\log\left(4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{3b\sqrt{-\frac{a}{b}}}-\frac{B}{} \end{cases}$$

```
[In] integrate((B*x**3+A)*x**(1/2)/(b*x**3+a),x)
```

```
[Out] Piecewise((zoo*(-2*A/(3*x**(3/2)) + 2*B*x**(3/2)/3), Eq(a, 0) & Eq(b, 0)),
((2*A*x**(3/2)/3 + 2*B*x**(9/2)/9)/a, Eq(b, 0)), ((-2*A/(3*x**(3/2)) + 2*B*
x**(3/2)/3)/b, Eq(a, 0)), (A*log(sqrt(x) - (-a/b)**(1/6))/(3*b*sqrt(-a/b))
- A*log(sqrt(x) + (-a/b)**(1/6))/(3*b*sqrt(-a/b)) - A*log(-4*sqrt(x)*(-a/b)
**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*b*sqrt(-a/b)) + A*log(4*sqrt(x)*(-a/b)*
*(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*b*sqrt(-a/b)) - B*a*log(sqrt(x) - (-a/b)
**(1/6))/(3*b**2*sqrt(-a/b)) + B*a*log(sqrt(x) + (-a/b)**(1/6))/(3*b**2*sqr
t(-a/b)) + B*a*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*b**
2*sqrt(-a/b)) - B*a*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3
*b**2*sqrt(-a/b)) + 2*B*x**(3/2)/(3*b), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{x}(A+Bx^3)}{a+bx^3}\,dx = \frac{2\,Bx^{\frac{3}{2}}}{3\,b} - \frac{2\,(Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}b}$$

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 2/3*B*x^(3/2)/b - 2/3*(B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{x}(A+Bx^3)}{a+bx^3}\,dx = \frac{2\,Bx^{\frac{3}{2}}}{3\,b} - \frac{2\,(Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}b}$$

[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a),x, algorithm="giac")

[Out] 2/3*B*x^(3/2)/b - 2/3*(B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b)

Mupad [B] (verification not implemented)

Time = 7.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt{x}(A+Bx^3)}{a+bx^3}\,dx = \frac{2\,B\,x^{3/2}}{3\,b} - \frac{2\,\mathrm{atan}\left(\frac{3\,\sqrt{a}\,b^{3/2}\,x^{3/2}\,\left(24\,A^2\,b^3-48\,A\,B\,a\,b^2+24\,B^2\,a^2\,b\right)}{\left(72\,B\,a^2\,b^2-72\,A\,a\,b^3\right)\,\left(A\,b-B\,a\right)}\right)\,\left(A\,b-B\,a\right)}{3\,\sqrt{a}\,b^{3/2}}$$

[In] int((x^(1/2)*(A + B*x^3))/(a + b*x^3),x)

[Out] (2*B*x^(3/2))/(3*b) - (2*atan((3*a^(1/2)*b^(3/2)*x^(3/2)*(24*A^2*b^3 + 24*B^2*a^2*b - 48*A*B*a*b^2))/((72*B*a^2*b^2 - 72*A*a*b^3)*(A*b - B*a)))*(A*b - B*a))/(3*a^(1/2)*b^(3/2))

3.159 $\int \frac{A+Bx^3}{\sqrt{x}(a+bx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 268

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)}\,dx = \frac{2B\sqrt{x}}{b}-\frac{(Ab-aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}}+\frac{(Ab-aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}}+\frac{2(Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}}-\frac{(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{5/6}b^{7/6}}+\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{5/6}b^{7/6}}$$

```
[Out] 2/3*(A*b-B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(5/6)/b^(7/6)+1/3*(A*b-B*a)
*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(5/6)/b^(7/6)+1/3*(A*b-B*a)*a
rctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(5/6)/b^(7/6)-1/6*(A*b-B*a)*ln(a
^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(5/6)/b^(7/6)*3^(1/2)+1
/6*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(5/6)/
b^(7/6)*3^(1/2)+2*B*x^(1/2)/b
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {470, 335, 215, 648, 632, 210, 642, 211}

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)}\,dx = -\frac{(Ab-aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}} + \frac{(Ab-aB)\arctan\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{3a^{5/6}b^{7/6}} + \frac{2(Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}} - \frac{(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{5/6}b^{7/6}} + \frac{(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{5/6}b^{7/6}} + \frac{2B\sqrt{x}}{b}$$

```
[In] Int[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)),x]
```

```
[Out] (2*B*Sqrt[x])/b - ((A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)
])/(3*a^(5/6)*b^(7/6)) + ((A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/
a^(1/6)])/(3*a^(5/6)*b^(7/6)) + (2*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(
1/6)])/(3*a^(5/6)*b^(7/6)) - ((A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(
1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(5/6)*b^(7/6)) + ((A*b - a*B)*Log[a
^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(5/6)*b
^(7/6))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
```

```
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2B\sqrt{x}}{b} - \frac{\left(2\left(-\frac{Ab}{2}+\frac{aB}{2}\right)\right)\int \frac{1}{\sqrt{x}(a+bx^3)}\,dx}{b}\\ &= \frac{2B\sqrt{x}}{b} - \frac{\left(4\left(-\frac{Ab}{2}+\frac{aB}{2}\right)\right)\operatorname{Subst}\left(\int \frac{1}{a+bx^6}\,dx, x, \sqrt{x}\right)}{b}\end{aligned}$$

$$= \frac{2B\sqrt{x}}{b} + \frac{(2(Ab-aB))\text{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{3a^{5/6}b} + \frac{(2(Ab-aB))\text{Subst}\left(\int \frac{\sqrt[6]{a}+\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{3a^{5/6}b} + \frac{(2(Ab-aB))\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{3a^{2/3}b}$$

$$= \frac{2B\sqrt{x}}{b} + \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}} - \frac{(Ab-aB)\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{3}a^{5/6}b^{7/6}} + \frac{(Ab-aB)\text{Subst}\left(\int \frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{3}a^{5/6}b^{7/6}} + \frac{(Ab-aB)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{6a^{2/3}b} + \frac{(Ab-aB)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{6a^{2/3}b}$$

$$= \frac{2B\sqrt{x}}{b} + \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{5/6}b^{7/6}} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{5/6}b^{7/6}} + \frac{(Ab-aB)\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{3\sqrt{3}a^{5/6}b^{7/6}} - \frac{(Ab-aB)\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{3\sqrt{3}a^{5/6}b^{7/6}}$$

$$=\frac{2B\sqrt{x}}{b}-\frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}}+\frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}}+\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}}-\frac{(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{5/6}b^{7/6}}+\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{5/6}b^{7/6}}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.57

$$\int\frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)}\,dx$$
$$=\frac{6a^{5/6}\sqrt[6]{b}B\sqrt{x}+2(Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)-(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)+\sqrt{3}(Ab-aB)\text{arctanh}\left(\right.}{3a^{5/6}b^{7/6}}$$

```
[In] Integrate[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)),x]
```

```
[Out] (6*a^(5/6)*b^(1/6)*B*Sqrt[x] + 2*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/
6)] - (A*b - a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])] +
 Sqrt[3]*(A*b - a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b
^(1/3)*x)])/(3*a^(5/6)*b^(7/6))
```

Maple [A] (verified)

Time = 4.17 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.71

method	result
risch	$\frac{2B\sqrt{x}}{b}+\frac{(Ab-Ba)\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{3a}+\frac{2\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right.}{}\right.}{b}$
derivativedivides	$\frac{2B\sqrt{x}}{b}+\frac{2\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6a}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x\right.}{}\right.}{b}$
default	$\frac{2B\sqrt{x}}{b}+\frac{2\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6a}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x\right.}{}\right.}{b}$

```
[In] int((B*x^3+A)/(b*x^3+a)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*B*x^(1/2)/b+(A*b-B*a)/b*(1/6/a*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/
6)*x^(1/2)+(a/b)^(1/3))+1/3/a*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1
/2))+2/3/a*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))-1/6/a*3^(1/2)*(a/b)^(1/6
)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+1/3/a*(a/b)^(1/6)*arctan(-3
^(1/2)+2*x^(1/2)/(a/b)^(1/6)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1245 vs. 2(188) = 376.

Time = 0.30 (sec) , antiderivative size = 1245, normalized size of antiderivative = 4.65

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x^3+A)/(b*x^3+a)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b
^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)*log(a*b
*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*
A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6) - (B*a - A*b)*s
qrt(x)) - 2*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*
a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)*lo
```

```
g(-a*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3
 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6) - (B*a -
A*b)*sqrt(x)) + (sqrt(-3)*b + b)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^
4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/
(a^5*b^7))^(1/6)*log(-2*(B*a - A*b)*sqrt(x) + (sqrt(-3)*a*b + a*b)*(-(B^6*a
^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a
^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)) - (sqrt(-3)*b + b)*(-(B
^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B
^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)*log(-2*(B*a - A*b)*s
qrt(x) - (sqrt(-3)*a*b + a*b)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b
^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^
5*b^7))^(1/6)) + (sqrt(-3)*b - b)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a
^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)
/(a^5*b^7))^(1/6)*log(-2*(B*a - A*b)*sqrt(x) + (sqrt(-3)*a*b - a*b)*(-(B^6*
a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*
a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)) - (sqrt(-3)*b - b)*(-(
B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*
B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)*log(-2*(B*a - A*b)*
sqrt(x) - (sqrt(-3)*a*b - a*b)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*
b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a
^5*b^7))^(1/6)) + 12*B*sqrt(x))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(257) = 514$.

Time = 5.31 (sec) , antiderivative size = 558, normalized size of antiderivative = 2.08

$$\int \frac{A + Bx^3}{\sqrt{x}\,(a + bx^3)}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{5x^{\frac{5}{2}}} + 2B\sqrt{x}\right) \\ \frac{-\frac{2A}{5x^{\frac{5}{2}}} + 2B\sqrt{x}}{b} \\ \frac{2A\sqrt{x} + \frac{2Bx^{\frac{7}{2}}}{7}}{a} \\ -\frac{A\sqrt[6]{-\frac{a}{b}}\log\left(\sqrt{x} - \sqrt[6]{-\frac{a}{b}}\right)}{3a} + \frac{A\sqrt[6]{-\frac{a}{b}}\log\left(\sqrt{x} + \sqrt[6]{-\frac{a}{b}}\right)}{3a} - \frac{A\sqrt[6]{-\frac{a}{b}}\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}} + 4x + 4\sqrt[3]{-\frac{a}{b}}\right)}{6a} + \frac{A\sqrt[6]{-\frac{a}{b}}\log\left(4\right.}{} \end{cases}$$

```
[In] integrate((B*x**3+A)/(b*x**3+a)/x**(1/2),x)
```

```
[Out] Piecewise((zoo*(-2*A/(5*x**(5/2)) + 2*B*sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((-
2*A/(5*x**(5/2)) + 2*B*sqrt(x))/b, Eq(a, 0)), ((2*A*sqrt(x) + 2*B*x**(7/2)/
7)/a, Eq(b, 0)), (-A*(-a/b)**(1/6)*log(sqrt(x) - (-a/b)**(1/6))/(3*a) + A*(
```

```
-a/b)**(1/6)*log(sqrt(x) + (-a/b)**(1/6))/(3*a) - A*(-a/b)**(1/6)*log(-4*sq
rt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a) + A*(-a/b)**(1/6)*log(4*
sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a) + sqrt(3)*A*(-a/b)**(1
/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*a) + sqrt(3)*A
*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(3*a)
+ 2*B*sqrt(x)/b + B*(-a/b)**(1/6)*log(sqrt(x) - (-a/b)**(1/6))/(3*b) - B*(-
a/b)**(1/6)*log(sqrt(x) + (-a/b)**(1/6))/(3*b) + B*(-a/b)**(1/6)*log(-4*sqr
t(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b) - B*(-a/b)**(1/6)*log(4*s
qrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b) - sqrt(3)*B*(-a/b)**(1/
6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*b) - sqrt(3)*B*
(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(3*b),
True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)}\,dx = \frac{2\,B\sqrt{x}}{b} - \frac{\frac{\sqrt{3}(Ba-Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} - \frac{\sqrt{3}(Ba-Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} + \frac{4\left(Bab^{\frac{1}{3}}-Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} + \frac{2\left(Ba^{\frac{4}{3}}\right.}{}}{6\,b}$$

```
[In] integrate((B*x^3+A)/(b*x^3+a)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*B*sqrt(x)/b - 1/6*(sqrt(3)*(B*a - A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x
) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(B*a - A*b)*log(-sqrt(
3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(B*
a*b^(1/3) - A*b^(4/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/
3)*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3
))*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3
)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^
(4/3))*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b
^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))))/b
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)}\,dx = \frac{2\,B\sqrt{x}}{b} - \frac{\sqrt{3}\Big((ab^5)^{\frac{1}{6}}\,Ba-(ab^5)^{\frac{1}{6}}\,Ab\Big)\log\Big(\sqrt{3}\sqrt{x}\big(\frac{a}{b}\big)^{\frac{1}{6}}+x+\big(\frac{a}{b}\big)^{\frac{1}{3}}\Big)}{6\,ab^2} + \frac{\sqrt{3}\Big((ab^5)^{\frac{1}{6}}\,Ba-(ab^5)^{\frac{1}{6}}\,Ab\Big)\log\Big(-\sqrt{3}\sqrt{x}\big(\frac{a}{b}\big)^{\frac{1}{6}}+x+\big(\frac{a}{b}\big)^{\frac{1}{3}}\Big)}{6\,ab^2} - \frac{\Big((ab^5)^{\frac{1}{6}}\,Ba-(ab^5)^{\frac{1}{6}}\,Ab\Big)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,ab^2} - \frac{\Big((ab^5)^{\frac{1}{6}}\,Ba-(ab^5)^{\frac{1}{6}}\,Ab\Big)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,ab^2} - \frac{2\Big((ab^5)^{\frac{1}{6}}\,Ba-(ab^5)^{\frac{1}{6}}\,Ab\Big)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,ab^2}$$

```
[In] integrate((B*x^3+A)/(b*x^3+a)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/b - 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(sqr
t(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a*b^2) + 1/6*sqrt(3)*((a*b^5)^
(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)
^(1/3))/(a*b^2) - 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan((sqrt(
3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a*b^2) - 1/3*((a*b^5)^(1/6)*B*a -
 (a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/
(a*b^2) - 2/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^
(1/6))/(a*b^2)
```

Mupad [B] (verification not implemented)

Time = 7.34 (sec) , antiderivative size = 1915, normalized size of antiderivative = 7.15

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x^3)/(x^(1/2)*(a + b*x^3)),x)
```

```
[Out] (2*B*x^(1/2))/b + (atan((((x^(1/2)*(96*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2
*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) - ((A*b - B*a)*(288*A^3*a*b
^5 - 288*B^3*a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6
)*b^(7/6)))*(A*b - B*a)*1i)/(3*(-a)^(5/6)*b^(7/6)) + ((x^(1/2)*(96*A^4*b^5
```

```
+ 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2)
 + ((A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*
A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6)))*(A*b - B*a)*1i)/(3*(-a)^(5/6)*b^(7/
6)))/(((x^(1/2)*(96*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*
B*a*b^4 - 384*A*B^3*a^3*b^2) - ((A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^
2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6)))*(A*b -
B*a))/(3*(-a)^(5/6)*b^(7/6)) - ((x^(1/2)*(96*A^4*b^5 + 96*B^4*a^4*b + 576*A
^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) + ((A*b - B*a)*(288*A
^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a
)^(5/6)*b^(7/6)))*(A*b - B*a))/(3*(-a)^(5/6)*b^(7/6))))*(A*b - B*a)*2i)/(3*
(-a)^(5/6)*b^(7/6)) + (atan(((((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(x^(1/2)*(
96*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B
^3*a^3*b^2) - (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*
a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6)))*1
i)/(3*(-a)^(5/6)*b^(7/6)) + (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(x^(1/2)*(9
6*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^
3*a^3*b^2) + (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a
^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6)))*1i
)/(3*(-a)^(5/6)*b^(7/6)))/((((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(x^(1/2)*(96
*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3
*a^3*b^2) - (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^
4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6))))/(3
*(-a)^(5/6)*b^(7/6)) - (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(x^(1/2)*(96*A^4
*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3
*b^2) + (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^
2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6))))/(3*(-a
)^(5/6)*b^(7/6))))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*2i)/(3*(-a)^(5/6)*b^(
7/6)) + (atan(((((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(x^(1/2)*(96*A^4*b^5 + 9
6*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) -
(((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*
A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6)))*1i)/(3*(-a)^(5/
6)*b^(7/6)) + (((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(x^(1/2)*(96*A^4*b^5 + 96
*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) + (
((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A
*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6)))*1i)/(3*(-a)^(5/6
)*b^(7/6)))/((((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(x^(1/2)*(96*A^4*b^5 + 96*
B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) - ((
(3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A*
B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6))))/(3*(-a)^(5/6)*b^
(7/6)) - (((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(x^(1/2)*(96*A^4*b^5 + 96*B^4*
a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) + (((3^(
1/2)*1i)/2 + 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A*B^2*
a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6))))/(3*(-a)^(5/6)*b^(7/6
))))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*2i)/(3*(-a)^(5/6)*b^(7/6))
```

3.160 $\int \frac{A+Bx^3}{x^{3/2}(a+bx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 268

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)}\,dx=-\frac{2A}{a\sqrt{x}}+\frac{(Ab-aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}}-\frac{(Ab-aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}}-\frac{2(Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}}-\frac{(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{7/6}b^{5/6}}+\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{7/6}b^{5/6}}$$

```
[Out] -2/3*(A*b-B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(7/6)/b^(5/6)-1/3*(A*b-B*a
)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(7/6)/b^(5/6)-1/3*(A*b-B*a)*
arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(7/6)/b^(5/6)-1/6*(A*b-B*a)*ln(
a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(7/6)/b^(5/6)*3^(1/2)+
1/6*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(7/6)
/b^(5/6)*3^(1/2)-2*A/a/x^(1/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {464, 335, 301, 648, 632, 210, 642, 211}

$$\int \frac{A+Bx^3}{x^{3/2}(a+bx^3)}\,dx = \frac{(Ab-aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}} - \frac{(Ab-aB)\arctan\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{3a^{7/6}b^{5/6}} - \frac{2(Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}} - \frac{(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{7/6}b^{5/6}} + \frac{(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{7/6}b^{5/6}} - \frac{2A}{a\sqrt{x}}$$

```
[In] Int[(A + B*x^3)/(x^(3/2)*(a + b*x^3)),x]
```

```
[Out] (-2*A)/(a*Sqrt[x]) + ((A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1
/6)])/(3*a^(7/6)*b^(5/6)) - ((A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x
])/a^(1/6)])/(3*a^(7/6)*b^(5/6)) - (2*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/
a^(1/6)])/(3*a^(7/6)*b^(5/6)) - ((A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*
b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(7/6)*b^(5/6)) + ((A*b - a*B)*Lo
g[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(7/6
)*b^(5/6))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
```

```
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = -\frac{2A}{a\sqrt{x}} - \frac{\left(2\left(\frac{Ab}{2} - \frac{aB}{2}\right)\right)\int \frac{x^{3/2}}{a+bx^3}\,dx}{a}$$

$$= -\frac{2A}{a\sqrt{x}} - \frac{\left(4\left(\frac{Ab}{2} - \frac{aB}{2}\right)\right)\text{Subst}\left(\int \frac{x^4}{a+bx^6}\,dx, x, \sqrt{x}\right)}{a}$$

$$= -\frac{2A}{a\sqrt{x}} - \frac{(2(Ab-aB))\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2}+\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{3a^{7/6}b^{2/3}}$$

$$-\frac{(2(Ab-aB))\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{3a^{7/6}b^{2/3}}$$

$$-\frac{(2(Ab-aB))\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{3ab^{2/3}}$$

$$= -\frac{2A}{a\sqrt{x}} - \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}}$$

$$-\frac{(Ab-aB)\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{3}a^{7/6}b^{5/6}}$$

$$+\frac{(Ab-aB)\text{Subst}\left(\int \frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{3}a^{7/6}b^{5/6}}$$

$$-\frac{(Ab-aB)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{6ab^{2/3}}$$

$$-\frac{(Ab-aB)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{6ab^{2/3}}$$

$$= -\frac{2A}{a\sqrt{x}} - \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}}$$

$$-\frac{(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{7/6}b^{5/6}}$$

$$+\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{7/6}b^{5/6}}$$

$$-\frac{(Ab-aB)\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{3\sqrt{3}a^{7/6}b^{5/6}}$$

$$+\frac{(Ab-aB)\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{3\sqrt{3}a^{7/6}b^{5/6}}$$

$$= -\frac{2A}{a\sqrt{x}} + \frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}} - \frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}} - \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{7/6}b^{5/6}} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{7/6}b^{5/6}}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.58

$$\int \frac{A+Bx^3}{x^{3/2}(a+bx^3)}\,dx = \frac{-\frac{6\sqrt[6]{a}A}{\sqrt{x}} + \frac{2(-Ab+aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{b^{5/6}} + \frac{(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{b^{5/6}} + \frac{\sqrt{3}(Ab-aB)\text{arctanh}\left(\cdots\right.}{b^{5/6}}}{3a^{7/6}}$$

```
[In] Integrate[(A + B*x^3)/(x^(3/2)*(a + b*x^3)),x]
```

```
[Out] ((-6*a^(1/6)*A)/Sqrt[x] + (2*(-(A*b) + a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6
)])/b^(5/6) + ((A*b - a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sq
rt[x])])/b^(5/6) + (Sqrt[3]*(A*b - a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sq
rt[x])/(a^(1/3) + b^(1/3)*x)])/b^(5/6))/(3*a^(7/6))
```

Maple [A] (verified)

Time = 4.17 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.71

method	result
derivativedivides	$-\frac{2\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right.}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right.}{a}$
default	$-\frac{2\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right.}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right.}{a}$
risch	$-\frac{2A}{a\sqrt{x}}-\frac{(Ab-Ba)\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a}+\frac{\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\frac{2\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\right.}{}\right.}{a}$

```
[In] int((B*x^3+A)/x^(3/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(1/12/a*3^(1/2)*(a/b)^(5/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3)
)+1/6/b/(a/b)^(1/6)*arctan(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6))-1/12/a*3^(1/2)*(
a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/b/(a/b)^(1/6)*
arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))+1/3/b/(a/b)^(1/6)*arctan(x^(1/2)/(a/b
)^(1/6)))*(A*b-B*a)/a-2*A/a/x^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1636 vs. $2(188) = 376$.

Time = 0.28 (sec) , antiderivative size = 1636, normalized size of antiderivative = 6.10

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/6*(2*a*x*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^
3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/6)*log(
a^6*b^4*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^
3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(5/6) - (B^5*a
^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - 10*A^3*B^2*a^2*b^3 + 5*A^4*B*a*b^
4 - A^5*b^5)*sqrt(x)) - 2*a*x*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b
^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^
7*b^5))^(1/6)*log(-a^6*b^4*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2
- 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b
```

```
^5))^(5/6) - (B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - 10*A^3*B^2*a^2
*b^3 + 5*A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) - (sqrt(-3)*a*x - a*x)*(-(B^6*a^6
- 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*
b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/6)*log((sqrt(-3)*a^6*b^4 + a^6
*b^4)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3
+ 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(5/6) - 2*(B^5*a
^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - 10*A^3*B^2*a^2*b^3 + 5*A^4*B*a*b^
4 - A^5*b^5)*sqrt(x)) + (sqrt(-3)*a*x - a*x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 1
5*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5
 + A^6*b^6)/(a^7*b^5))^(1/6)*log(-(sqrt(-3)*a^6*b^4 + a^6*b^4)*(-(B^6*a^6 -
 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b
^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(5/6) - 2*(B^5*a^5 - 5*A*B^4*a^4*b
 + 10*A^2*B^3*a^3*b^2 - 10*A^3*B^2*a^2*b^3 + 5*A^4*B*a*b^4 - A^5*b^5)*sqrt(
x)) - (sqrt(-3)*a*x + a*x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2
- 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b
^5))^(1/6)*log((sqrt(-3)*a^6*b^4 - a^6*b^4)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15
*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5
+ A^6*b^6)/(a^7*b^5))^(5/6) - 2*(B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b
^2 - 10*A^3*B^2*a^2*b^3 + 5*A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) + (sqrt(-3)*a*x
 + a*x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^
3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/6)*log(-(sq
rt(-3)*a^6*b^4 - a^6*b^4)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 -
 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^
5))^(5/6) - 2*(B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - 10*A^3*B^2*a^
2*b^3 + 5*A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) + 12*A*sqrt(x))/(a*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(257) = 514$.

Time = 9.21 (sec) , antiderivative size = 561, normalized size of antiderivative = 2.09

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{7x^{\frac{7}{2}}}-\frac{2B}{\sqrt{x}}\right) \\ \frac{-\frac{2A}{7x^{\frac{7}{2}}}-\frac{2B}{\sqrt{x}}}{b} \\ \frac{-\frac{2A}{\sqrt{x}}+\frac{2Bx^{\frac{5}{2}}}{5}}{a} \\ -\frac{A\log\left(\sqrt{x}-\sqrt[6]{-\frac{a}{b}}\right)}{3a\sqrt[6]{-\frac{a}{b}}}+\frac{A\log\left(\sqrt{x}+\sqrt[6]{-\frac{a}{b}}\right)}{3a\sqrt[6]{-\frac{a}{b}}}-\frac{A\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{6a\sqrt[6]{-\frac{a}{b}}}+\frac{A\log\left(4\sqrt{x}\right.}{} \end{cases}$$

```
[In] integrate((B*x**3+A)/x**(3/2)/(b*x**3+a),x)
```

```
[Out] Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((-
2*A/(7*x**(7/2)) - 2*B/sqrt(x))/b, Eq(a, 0)), ((-2*A/sqrt(x) + 2*B*x**(5/2)
```

```
/5)/a, Eq(b, 0)), (-A*log(sqrt(x) - (-a/b)**(1/6))/(3*a*(-a/b)**(1/6)) + A*
log(sqrt(x) + (-a/b)**(1/6))/(3*a*(-a/b)**(1/6)) - A*log(-4*sqrt(x)*(-a/b)*
*(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a*(-a/b)**(1/6)) + A*log(4*sqrt(x)*(-a/b
)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a*(-a/b)**(1/6)) - sqrt(3)*A*atan(2*sq
rt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*a*(-a/b)**(1/6)) - sqrt(3)*
A*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(3*a*(-a/b)**(1/6))
 - 2*A/(a*sqrt(x)) + B*log(sqrt(x) - (-a/b)**(1/6))/(3*b*(-a/b)**(1/6)) - B
*log(sqrt(x) + (-a/b)**(1/6))/(3*b*(-a/b)**(1/6)) + B*log(-4*sqrt(x)*(-a/b)
**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b*(-a/b)**(1/6)) - B*log(4*sqrt(x)*(-a/
b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*b*(-a/b)**(1/6)) + sqrt(3)*B*atan(2*s
qrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*b*(-a/b)**(1/6)) + sqrt(3)
*B*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(3*b*(-a/b)**(1/6)
), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)}\,dx =$$

$$-\frac{(Ba-Ab)\left(\frac{\sqrt{3}\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{2\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}+2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{2\arctan\left(-\frac{\sqrt{3}a^{\frac{1}{6}}\cdots}{\sqrt{\cdots}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}\cdots}}\cdots\right)}{6a}$$

$$-\frac{2A}{a\sqrt{x}}$$

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/6*(B*a - A*b)*(sqrt(3)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x +
 a^(1/3))/(a^(1/6)*b^(5/6)) - sqrt(3)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x)
+ b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6)) - 2*arctan((sqrt(3)*a^(1/6)*b^(1/6
) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3)
)) - 2*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b
^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 4*arctan(b^(1/3)*sqrt(x)/sqrt(a^
(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))))/a - 2*A/(a*sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.69 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^3}{x^{3/2}(a+bx^3)}\,dx = \frac{(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\left(ab^5\right)^{\frac{1}{6}}a} + \frac{(Ba-Ab)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\left(ab^5\right)^{\frac{1}{6}}a} + \frac{2\left(Ba\left(\frac{a}{b}\right)^{\frac{5}{6}}-Ab\left(\frac{a}{b}\right)^{\frac{5}{6}}\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a^2} - \frac{2A}{a\sqrt{x}} - \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{5}{6}}Ba-\left(ab^5\right)^{\frac{5}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a^2b^5} + \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{5}{6}}Ba-\left(ab^5\right)^{\frac{5}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a^2b^5}$$

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*(B*a - A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/((a*b
^5)^(1/6)*a) + 1/3*(B*a - A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a
/b)^(1/6))/((a*b^5)^(1/6)*a) + 2/3*(B*a*(a/b)^(5/6) - A*b*(a/b)^(5/6))*arct
an(sqrt(x)/(a/b)^(1/6))/a^2 - 2*A/(a*sqrt(x)) - 1/6*sqrt(3)*((a*b^5)^(5/6)*
B*a - (a*b^5)^(5/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))
/(a^2*b^5) + 1/6*sqrt(3)*((a*b^5)^(5/6)*B*a - (a*b^5)^(5/6)*A*b)*log(-sqrt(
3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^5)
```

Mupad [B] (verification not implemented)

Time = 7.29 (sec) , antiderivative size = 1700, normalized size of antiderivative = 6.34

$$\int \frac{A+Bx^3}{x^{3/2}(a+bx^3)}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x^3)/(x^(3/2)*(a + b*x^3)),x)
```

```
[Out] (atan((((A*b - B*a)^2*(32*B^3*a^12*b^3 - 32*A^3*a^9*b^6 - 96*A*B^2*a^11*b^4
 + 96*A^2*B*a^10*b^5 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^1
2*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6)))*1i)/((-a)^(7/3)*b^(5/3
)) + ((A*b - B*a)^2*(32*A^3*a^9*b^6 - 32*B^3*a^12*b^3 + 96*A*B^2*a^11*b^4 -
 96*A^2*B*a^10*b^5 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*
b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6)))*1i)/((-a)^(7/3)*b^(5/3))
)/(((A*b - B*a)^2*(32*B^3*a^12*b^3 - 32*A^3*a^9*b^6 - 96*A*B^2*a^11*b^4 + 9
```

```
6*A^2*B*a^10*b^5 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^
4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6))))/((-a)^(7/3)*b^(5/3)) - ((
A*b - B*a)^2*(32*A^3*a^9*b^6 - 32*B^3*a^12*b^3 + 96*A*B^2*a^11*b^4 - 96*A^2
*B*a^10*b^5 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^4 - 1
728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6))))/((-a)^(7/3)*b^(5/3))))*(A*b -
B*a)*2i)/(3*(-a)^(7/6)*b^(5/6)) - (2*A)/(a*x^(1/2)) + (atan(((((3^(1/2)*1i)
/2 - 1/2)^2*(A*b - B*a)^2*(32*B^3*a^12*b^3 - 32*A^3*a^9*b^6 - 96*A*B^2*a^11
*b^4 + 96*A^2*B*a^10*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(864
*A^2*a^10*b^6 + 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/
6)))*1i)/((-a)^(7/3)*b^(5/3)) + (((3^(1/2)*1i)/2 - 1/2)^2*(A*b - B*a)^2*(32
*A^3*a^9*b^6 - 32*B^3*a^12*b^3 + 96*A*B^2*a^11*b^4 - 96*A^2*B*a^10*b^5 + (x
^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*
b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6)))*1i)/((-a)^(7/3)*b^(5/3))
)/((((3^(1/2)*1i)/2 - 1/2)^2*(A*b - B*a)^2*(32*B^3*a^12*b^3 - 32*A^3*a^9*b^
6 - 96*A*B^2*a^11*b^4 + 96*A^2*B*a^10*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)
*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^5))/(27
*(-a)^(7/6)*b^(5/6))))/((-a)^(7/3)*b^(5/3)) - (((3^(1/2)*1i)/2 - 1/2)^2*(A*
b - B*a)^2*(32*A^3*a^9*b^6 - 32*B^3*a^12*b^3 + 96*A*B^2*a^11*b^4 - 96*A^2*B
*a^10*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 +
 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6))))/((-a)^(7/
3)*b^(5/3))))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*2i)/(3*(-a)^(7/6)*b^(5/6))
 + (atan(((((3^(1/2)*1i)/2 + 1/2)^2*(A*b - B*a)^2*(32*B^3*a^12*b^3 - 32*A^3
*a^9*b^6 - 96*A*B^2*a^11*b^4 + 96*A^2*B*a^10*b^5 + (x^(1/2)*((3^(1/2)*1i)/2
 + 1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^
5))/(27*(-a)^(7/6)*b^(5/6)))*1i)/((-a)^(7/3)*b^(5/3)) + (((3^(1/2)*1i)/2 +
1/2)^2*(A*b - B*a)^2*(32*A^3*a^9*b^6 - 32*B^3*a^12*b^3 + 96*A*B^2*a^11*b^4
- 96*A^2*B*a^10*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(864*A^2*
a^10*b^6 + 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6)))*
1i)/((-a)^(7/3)*b^(5/3)))/((((3^(1/2)*1i)/2 + 1/2)^2*(A*b - B*a)^2*(32*B^3*
a^12*b^3 - 32*A^3*a^9*b^6 - 96*A*B^2*a^11*b^4 + 96*A^2*B*a^10*b^5 + (x^(1/2
)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^4 -
 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6))))/((-a)^(7/3)*b^(5/3)) - (((3^
(1/2)*1i)/2 + 1/2)^2*(A*b - B*a)^2*(32*A^3*a^9*b^6 - 32*B^3*a^12*b^3 + 96*A
*B^2*a^11*b^4 - 96*A^2*B*a^10*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b -
B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7
/6)*b^(5/6))))/((-a)^(7/3)*b^(5/3))))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*2i
)/(3*(-a)^(7/6)*b^(5/6))
```

3.161 $\int \frac{A+Bx^3}{x^{5/2}(a+bx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{A+Bx^3}{x^{5/2}(a+bx^3)}\,dx = -\frac{2A}{3ax^{3/2}} - \frac{2(Ab-aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}}$$

```
[Out] -2/3*A/a/x^(3/2)-2/3*(A*b-B*a)*arctan(x^(3/2)*b^(1/2)/a^(1/2))/a^(3/2)/b^(1
/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {464, 335, 281, 211}

$$\int \frac{A+Bx^3}{x^{5/2}(a+bx^3)}\,dx = -\frac{2(Ab-aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}} - \frac{2A}{3ax^{3/2}}$$

```
[In] Int[(A + B*x^3)/(x^(5/2)*(a + b*x^3)),x]
```

```
[Out] (-2*A)/(3*a*x^(3/2)) - (2*(A*b - a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3
*a^(3/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2A}{3ax^{3/2}} - \frac{\left(2\left(\frac{3Ab}{2} - \frac{3aB}{2}\right)\right)\int \frac{\sqrt{x}}{a+bx^3}\,dx}{3a}\\
&= -\frac{2A}{3ax^{3/2}} - \frac{\left(4\left(\frac{3Ab}{2} - \frac{3aB}{2}\right)\right)\text{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx, x, \sqrt{x}\right)}{3a}\\
&= -\frac{2A}{3ax^{3/2}} - \frac{(2(Ab-aB))\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^{3/2}\right)}{3a}\\
&= -\frac{2A}{3ax^{3/2}} - \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)}\,dx = -\frac{2A}{3ax^{3/2}} + \frac{2(-Ab+aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}}$$

```
[In] Integrate[(A + B*x^3)/(x^(5/2)*(a + b*x^3)),x]
[Out] (-2*A)/(3*a*x^(3/2)) + (2*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])
/(3*a^(3/2)*Sqrt[b])
```

Maple [A] (verified)

Time = 4.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$-\frac{2A}{3a\,x^{\frac{3}{2}}}-\frac{2(Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3a\sqrt{ab}}$	40
default	$-\frac{2A}{3a\,x^{\frac{3}{2}}}-\frac{2(Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3a\sqrt{ab}}$	40
risch	$-\frac{2A}{3a\,x^{\frac{3}{2}}}-\frac{2(Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3a\sqrt{ab}}$	40

```
[In] int((B*x^3+A)/x^(5/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
[Out] -2/3*A/a/x^(3/2)-2/3*(A*b-B*a)/a/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.26

$$\int \frac{A+Bx^3}{x^{5/2}\,(a+bx^3)}\,dx = \left[\frac{(Ba-Ab)\sqrt{-ab}x^2\log\left(\frac{bx^3+2\sqrt{-ab}x^{\frac{3}{2}}-a}{bx^3+a}\right)-2\,Aab\sqrt{x}}{3\,a^2bx^2}, \frac{2\left((Ba-Ab)\sqrt{ab}x^2\arctan\left(\right.\right.}{3\,a^2bx^2}\right.$$

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a),x, algorithm="fricas")
[Out] [1/3*((B*a - A*b)*sqrt(-a*b)*x^2*log((b*x^3 + 2*sqrt(-a*b)*x^(3/2) - a)/(b*
x^3 + a)) - 2*A*a*b*sqrt(x))/(a^2*b*x^2), 2/3*((B*a - A*b)*sqrt(a*b)*x^2*ar
ctan(sqrt(a*b)*x^(3/2)/a) - A*a*b*sqrt(x))/(a^2*b*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(53) = 106$.

Time = 19.50 (sec) , antiderivative size = 371, normalized size of antiderivative = 7.00

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{9x^{\frac{9}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}\right) \\ \frac{-\frac{2A}{9x^{\frac{9}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}}{b} \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}+\frac{2Bx^{\frac{3}{2}}}{3}}{a} \\ -\frac{A\log\left(\sqrt{x}-\sqrt[6]{-\frac{a}{b}}\right)}{3a\sqrt{-\frac{a}{b}}}+\frac{A\log\left(\sqrt{x}+\sqrt[6]{-\frac{a}{b}}\right)}{3a\sqrt{-\frac{a}{b}}}+\frac{A\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{3a\sqrt{-\frac{a}{b}}}-\frac{A\log\left(4\sqrt{x}\sqrt[6]{\right.}}{3} \end{cases}$$

```
[In] integrate((B*x**3+A)/x**(5/2)/(b*x**3+a),x)
```

```
[Out] Piecewise((zoo*(-2*A/(9*x**(9/2)) - 2*B/(3*x**(3/2))), Eq(a, 0) & Eq(b, 0))
, ((-2*A/(9*x**(9/2)) - 2*B/(3*x**(3/2)))/b, Eq(a, 0)), ((-2*A/(3*x**(3/2))
 + 2*B*x**(3/2)/3)/a, Eq(b, 0)), (-A*log(sqrt(x) - (-a/b)**(1/6))/(3*a*sqrt
(-a/b)) + A*log(sqrt(x) + (-a/b)**(1/6))/(3*a*sqrt(-a/b)) + A*log(-4*sqrt(x
)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*a*sqrt(-a/b)) - A*log(4*sqrt(x)
*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(3*a*sqrt(-a/b)) - 2*A/(3*a*x**(3/2
)) + B*log(sqrt(x) - (-a/b)**(1/6))/(3*b*sqrt(-a/b)) - B*log(sqrt(x) + (-a/
b)**(1/6))/(3*b*sqrt(-a/b)) - B*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/
b)**(1/3))/(3*b*sqrt(-a/b)) + B*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b
)**(1/3))/(3*b*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)}\,dx = \frac{2\,(Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}a} - \frac{2\,A}{3\,ax^{\frac{3}{2}}}$$

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 2/3*(B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a) - 2/3*A/(a*x^(3/2
))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx^3}{x^{5/2}\,(a+bx^3)}\,dx = \frac{2\,(Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}a} - \frac{2\,A}{3\,ax^{\frac{3}{2}}}$$

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a) - 2/3*A/(a*x^(3/2
))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.92

$$\int \frac{A+Bx^3}{x^{5/2}\,(a+bx^3)}\,dx = -\frac{2\,A}{3\,a\,x^{3/2}} - \frac{2\,\mathrm{atan}\left(\frac{3\,a^{3/2}\,\sqrt{b}\,x^{3/2}\,\left(24\,A^2\,a^3\,b^5-48\,A\,B\,a^4\,b^4+24\,B^2\,a^5\,b^3\right)}{(A\,b-B\,a)\,\left(72\,A\,a^5\,b^4-72\,B\,a^6\,b^3\right)}\right)\,(A\,b-B\,a)}{3\,a^{3/2}\,\sqrt{b}}$$

```
[In] int((A + B*x^3)/(x^(5/2)*(a + b*x^3)),x)
```

```
[Out] - (2*A)/(3*a*x^(3/2)) - (2*atan((3*a^(3/2)*b^(1/2)*x^(3/2)*(24*A^2*a^3*b^5
+ 24*B^2*a^5*b^3 - 48*A*B*a^4*b^4))/((A*b - B*a)*(72*A*a^5*b^4 - 72*B*a^6*b
^3)))*(A*b - B*a))/(3*a^(3/2)*b^(1/2))
```

3.162 $\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 270

$$\begin{aligned}\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)}\,dx = & -\frac{2A}{5ax^{5/2}}+\frac{(Ab-aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}} \\ & -\frac{(Ab-aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}}-\frac{2(Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}} \\ & +\frac{(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{11/6}\sqrt[6]{b}} \\ & -\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{11/6}\sqrt[6]{b}}\end{aligned}$$

```
[Out] -2/5*A/a/x^(5/2)-2/3*(A*b-B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(
1/6)-1/3*(A*b-B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(1
/6)-1/3*(A*b-B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(1/6
)+1/6*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(11
/6)/b^(1/6)*3^(1/2)-1/6*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1
/2)*x^(1/2))/a^(11/6)/b^(1/6)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {464, 335, 215, 648, 632, 210, 642, 211}

$$\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)}\,dx = \frac{(Ab-aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}} - \frac{(Ab-aB)\arctan\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{3a^{11/6}\sqrt[6]{b}} - \frac{2(Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}} + \frac{(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{11/6}\sqrt[6]{b}} - \frac{(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{11/6}\sqrt[6]{b}} - \frac{2A}{5ax^{5/2}}$$

```
[In] Int[(A + B*x^3)/(x^(7/2)*(a + b*x^3)),x]
```

```
[Out] (-2*A)/(5*a*x^(5/2)) + ((A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^
(1/6)])/(3*a^(11/6)*b^(1/6)) - ((A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqr
t[x])/a^(1/6)])/(3*a^(11/6)*b^(1/6)) - (2*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[
x])/a^(1/6)])/(3*a^(11/6)*b^(1/6)) + ((A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(
1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(11/6)*b^(1/6)) - ((A*b - a
*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*
a^(11/6)*b^(1/6))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
```

```
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2A}{5ax^{5/2}}-\frac{\left(2\left(\frac{5Ab}{2}-\frac{5aB}{2}\right)\right)\int\frac{1}{\sqrt{x}(a+bx^3)}\,dx}{5a}\\ &= -\frac{2A}{5ax^{5/2}}-\frac{\left(4\left(\frac{5Ab}{2}-\frac{5aB}{2}\right)\right)\text{Subst}\left(\int\frac{1}{a+bx^6}\,dx,x,\sqrt{x}\right)}{5a}\end{aligned}$$

$$= -\frac{2A}{5ax^{5/2}} - \frac{(2(Ab-aB))\text{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{3a^{11/6}}$$
$$- \frac{(2(Ab-aB))\text{Subst}\left(\int \frac{\sqrt[6]{a}+\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{3a^{11/6}}$$
$$- \frac{(2(Ab-aB))\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{3a^{5/3}}$$

$$= -\frac{2A}{5ax^{5/2}} - \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}}$$
$$- \frac{(Ab-aB)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{6a^{5/3}}$$
$$- \frac{(Ab-aB)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{6a^{5/3}}$$
$$+ \frac{(Ab-aB)\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{3}a^{11/6}\sqrt[6]{b}}$$
$$- \frac{(Ab-aB)\text{Subst}\left(\int \frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{3}a^{11/6}\sqrt[6]{b}}$$

$$= -\frac{2A}{5ax^{5/2}} - \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}}$$
$$+ \frac{(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{11/6}\sqrt[6]{b}}$$
$$- \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{11/6}\sqrt[6]{b}}$$
$$- \frac{(Ab-aB)\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{3\sqrt{3}a^{11/6}\sqrt[6]{b}}$$
$$+ \frac{(Ab-aB)\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{3\sqrt{3}a^{11/6}\sqrt[6]{b}}$$

$$= -\frac{2A}{5ax^{5/2}} + \frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}} - \frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}} - \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{11/6}\sqrt[6]{b}} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{11/6}\sqrt[6]{b}}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.59

$$\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)}\,dx = \frac{-\frac{6a^{5/6}A}{x^{5/2}} + \frac{10(-Ab+aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{\sqrt[6]{b}} + \frac{5(Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{\sqrt[6]{b}} + \frac{5\sqrt{3}(-Ab+aB)\text{arctanh}}{\sqrt[6]{b}}}{15a^{11/6}}$$

```
[In] Integrate[(A + B*x^3)/(x^(7/2)*(a + b*x^3)),x]
```

```
[Out] ((-6*a^(5/6)*A)/x^(5/2) + (10*(-(A*b) + a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/
6)])/b^(1/6) + (5*(A*b - a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)
*Sqrt[x])])/b^(1/6) + (5*Sqrt[3]*(-(A*b) + a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^
(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/b^(1/6))/(15*a^(11/6))
```

Maple [A] (verified)

Time = 4.33 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.71

method	result
derivativedivides	$-\frac{2A}{5a\,x^{\frac{5}{2}}}+\frac{2\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6a}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\cdots}{}\right.}{a}$
default	$-\frac{2A}{5a\,x^{\frac{5}{2}}}+\frac{2\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6a}+\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\cdots}{}\right.}{a}$
risch	$-\frac{2A}{5a\,x^{\frac{5}{2}}}-\frac{(Ab-Ba)\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{3a}+\frac{2\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\cdots\right.}{a}$

```
[In] int((B*x^3+A)/x^(7/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*A/a/x^(5/2)+2*(1/3/a*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))-1/12/a*3^
(1/2)*(a/b)^(1/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+1/6/a*(a/b)
^(1/6)*arctan(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6))+1/12/a*3^(1/2)*(a/b)^(1/6)*ln
(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arctan(2*x^(1
/2)/(a/b)^(1/6)+3^(1/2)))*(-A*b+B*a)/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1290 vs. $2(188) = 376$.

Time = 0.34 (sec) , antiderivative size = 1290, normalized size of antiderivative = 4.78

$$\int \frac{A+Bx^3}{x^{7/2}\,(a+bx^3)}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/30*(10*a*x^3*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^
3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6)*l
og(a^2*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3
 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6) - (B*a - A
*b)*sqrt(x)) - 10*a*x^3*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 2
0*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))
^(1/6)*log(-a^2*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^
3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6) -
 (B*a - A*b)*sqrt(x)) + 5*(sqrt(-3)*a*x^3 + a*x^3)*(-(B^6*a^6 - 6*A*B^5*a^5
```

```
*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B
*a*b^5 + A^6*b^6)/(a^11*b))^(1/6)*log(-2*(B*a - A*b)*sqrt(x) + (sqrt(-3)*a^
2 + a^2)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b
^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6)) - 5*(sq
rt(-3)*a*x^3 + a*x^3)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*
A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(
1/6)*log(-2*(B*a - A*b)*sqrt(x) - (sqrt(-3)*a^2 + a^2)*(-(B^6*a^6 - 6*A*B^5
*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A
^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6)) + 5*(sqrt(-3)*a*x^3 - a*x^3)*(-(B^6*
a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*
a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6)*log(-2*(B*a - A*b)*sqrt(
x) + (sqrt(-3)*a^2 - a^2)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 -
 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b
))^(1/6)) - 5*(sqrt(-3)*a*x^3 - a*x^3)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*
B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6
*b^6)/(a^11*b))^(1/6)*log(-2*(B*a - A*b)*sqrt(x) - (sqrt(-3)*a^2 - a^2)*(-(
B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*
B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6)) + 12*A*sqrt(x))/(a*
x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(258) = 516$.

Time = 47.88 (sec) , antiderivative size = 586, normalized size of antiderivative = 2.17

$$\int \frac{A + Bx^3}{x^{7/2}\left(a + bx^3\right)}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{11x^{\frac{11}{2}}} - \frac{2B}{5x^{\frac{5}{2}}}\right) \\ \frac{-\frac{2A}{11x^{\frac{11}{2}}} - \frac{2B}{5x^{\frac{5}{2}}}}{b} \\ \frac{-\frac{2A}{5x^{\frac{5}{2}}} + 2B\sqrt{x}}{a} \\ -\frac{2A}{5ax^{\frac{5}{2}}} + \frac{Ab\sqrt[6]{-\frac{a}{b}}\log\left(\sqrt{x} - \sqrt[6]{-\frac{a}{b}}\right)}{3a^2} - \frac{Ab\sqrt[6]{-\frac{a}{b}}\log\left(\sqrt{x} + \sqrt[6]{-\frac{a}{b}}\right)}{3a^2} + \frac{Ab\sqrt[6]{-\frac{a}{b}}\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}} \cdots\right)}{6a^2} \cdots \end{cases}$$

```
[In] integrate((B*x**3+A)/x**(7/2)/(b*x**3+a),x)
```

```
[Out] Piecewise((zoo*(-2*A/(11*x**(11/2)) - 2*B/(5*x**(5/2))), Eq(a, 0) & Eq(b, 0
)), ((-2*A/(11*x**(11/2)) - 2*B/(5*x**(5/2)))/b, Eq(a, 0)), ((-2*A/(5*x**(5
/2)) + 2*B*sqrt(x))/a, Eq(b, 0)), (-2*A/(5*a*x**(5/2)) + A*b*(-a/b)**(1/6)*
log(sqrt(x) - (-a/b)**(1/6))/(3*a**2) - A*b*(-a/b)**(1/6)*log(sqrt(x) + (-a
/b)**(1/6))/(3*a**2) + A*b*(-a/b)**(1/6)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x
 + 4*(-a/b)**(1/3))/(6*a**2) - A*b*(-a/b)**(1/6)*log(4*sqrt(x)*(-a/b)**(1/6
) + 4*x + 4*(-a/b)**(1/3))/(6*a**2) - sqrt(3)*A*b*(-a/b)**(1/6)*atan(2*sqrt
```

```
(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*a**2) - sqrt(3)*A*b*(-a/b)**(
1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(3*a**2) - B*(-a
/b)**(1/6)*log(sqrt(x) - (-a/b)**(1/6))/(3*a) + B*(-a/b)**(1/6)*log(sqrt(x)
 + (-a/b)**(1/6))/(3*a) - B*(-a/b)**(1/6)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*
x + 4*(-a/b)**(1/3))/(6*a) + B*(-a/b)**(1/6)*log(4*sqrt(x)*(-a/b)**(1/6) +
4*x + 4*(-a/b)**(1/3))/(6*a) + sqrt(3)*B*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(
x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(3*a) + sqrt(3)*B*(-a/b)**(1/6)*atan(2*sq
rt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(3*a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)}\,dx = \frac{\frac{\sqrt{3}(Ba-Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} - \frac{\sqrt{3}(Ba-Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} + \frac{4\left(Bab^{\frac{1}{3}}-Ab^{\frac{4}{3}}\right)\operatorname{arct}}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a}}}{} - \frac{2\,A}{5\,ax^{\frac{5}{2}}}$$

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/6*(sqrt(3)*(B*a - A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x +
a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(B*a - A*b)*log(-sqrt(3)*a^(1/6)*b^(1/
6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(B*a*b^(1/3) - A*b^
(4/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(
a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*arctan((sqrt(
3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*s
qrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*arctan(-(
sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1
/3)*sqrt(a^(1/3)*b^(1/3))))/a - 2/5*A/(a*x^(5/2))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^3}{x^{7/2}\,(a+bx^3)}\,dx = \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba-\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,a^2b}$$
$$-\frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba-\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,a^2b}$$
$$+\frac{\left(\left(ab^5\right)^{\frac{1}{6}}Ba-\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,a^2b}$$
$$+\frac{\left(\left(ab^5\right)^{\frac{1}{6}}Ba-\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,a^2b}$$
$$+\frac{2\left(\left(ab^5\right)^{\frac{1}{6}}Ba-\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,a^2b}-\frac{2\,A}{5\,ax^{\frac{5}{2}}}$$

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/
b)^(1/6) + x + (a/b)^(1/3))/(a^2*b) - 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b
^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b)
+ 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) +
 2*sqrt(x))/(a/b)^(1/6))/(a^2*b) + 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A
*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^2*b) + 2/3*((
a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^2*b) -
 2/5*A/(a*x^(5/2))
```

Mupad [B] (verification not implemented)

Time = 7.38 (sec) , antiderivative size = 2023, normalized size of antiderivative = 7.49

$$\int \frac{A+Bx^3}{x^{7/2}\,(a+bx^3)}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x^3)/(x^(7/2)*(a + b*x^3)),x)
```

```
[Out] - (2*A)/(5*a*x^(5/2)) - (atan((((x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 +
 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) - ((A*b - B*a
)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b
^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b - B*a)*1i)/(3*(-a)^(11/6)*b^(1/6)) + ((x
```

```
^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a
^8*b^6 - 384*A^3*B*a^6*b^8) + ((A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*
b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b
 - B*a)*1i)/(3*(-a)^(11/6)*b^(1/6)))/(((x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^
9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) - ((A*
b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*
B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b - B*a))/(3*(-a)^(11/6)*b^(1/6)) -
 ((x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B
^3*a^8*b^6 - 384*A^3*B*a^6*b^8) + ((A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a
^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*
(A*b - B*a))/(3*(-a)^(11/6)*b^(1/6))))*(A*b - B*a)*2i)/(3*(-a)^(11/6)*b^(1/
6)) - (atan(((((3^(1/2)*1i)/2 - 1/2)*(x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*
b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) - (((3^(
1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B
^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b - B*a)*1i)/(
3*(-a)^(11/6)*b^(1/6)) + (((3^(1/2)*1i)/2 - 1/2)*(x^(1/2)*(96*A^4*a^5*b^9 +
 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b
^8) + (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b
^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b
- B*a)*1i)/(3*(-a)^(11/6)*b^(1/6)))/((((3^(1/2)*1i)/2 - 1/2)*(x^(1/2)*(96*A
^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384
*A^3*B*a^6*b^8) - (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a^7*b^8 - 28
8*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(
1/6)))*(A*b - B*a))/(3*(-a)^(11/6)*b^(1/6)) - (((3^(1/2)*1i)/2 - 1/2)*(x^(1
/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*
b^6 - 384*A^3*B*a^6*b^8) + (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a^7
*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(
11/6)*b^(1/6)))*(A*b - B*a))/(3*(-a)^(11/6)*b^(1/6))))*((3^(1/2)*1i)/2 - 1/
2)*(A*b - B*a)*2i)/(3*(-a)^(11/6)*b^(1/6)) - (atan(((((3^(1/2)*1i)/2 + 1/2)
*(x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^
3*a^8*b^6 - 384*A^3*B*a^6*b^8) - (((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(288*A
^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*
(-a)^(11/6)*b^(1/6)))*(A*b - B*a)*1i)/(3*(-a)^(11/6)*b^(1/6)) + (((3^(1/2)*
1i)/2 + 1/2)*(x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^
7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) + (((3^(1/2)*1i)/2 + 1/2)*(A*b -
 B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a
^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b - B*a)*1i)/(3*(-a)^(11/6)*b^(1/6)))/
((((3^(1/2)*1i)/2 + 1/2)*(x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^
2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) - (((3^(1/2)*1i)/2 +
 1/2)*(A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 -
 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b - B*a))/(3*(-a)^(11/6)*b
^(1/6)) - (((3^(1/2)*1i)/2 + 1/2)*(x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5
 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) + (((3^(1/2
)*1i)/2 + 1/2)*(A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*
a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b - B*a))/(3*(-a)
```

```
^(11/6)*b^(1/6))))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*2i)/(3*(-a)^(11/6)*b^
(1/6))
```

3.163 $\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 95

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = -\frac{(Ab-3aB)x^{3/2}}{3ab^2} + \frac{(Ab-aB)x^{9/2}}{3ab\,(a+bx^3)} + \frac{(Ab-3aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{5/2}}$$

```
[Out] -1/3*(A*b-3*B*a)*x^(3/2)/a/b^2+1/3*(A*b-B*a)*x^(9/2)/a/b/(b*x^3+a)+1/3*(A*b
-3*B*a)*arctan(x^(3/2)*b^(1/2)/a^(1/2))/b^(5/2)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {468, 327, 335, 281, 211}

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(Ab-3aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{5/2}} - \frac{x^{3/2}(Ab-3aB)}{3ab^2} + \frac{x^{9/2}(Ab-aB)}{3ab\,(a+bx^3)}$$

```
[In] Int[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -1/3*((A*b - 3*a*B)*x^(3/2))/(a*b^2) + ((A*b - a*B)*x^(9/2))/(3*a*b*(a + b*
x^3)) + ((A*b - 3*a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*Sqrt[a]*b^(5/2
))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(Ab-aB)x^{9/2}}{3ab\,(a+bx^3)}+\frac{\left(-\frac{3Ab}{2}+\frac{9aB}{2}\right)\int\frac{x^{7/2}}{a+bx^3}\,dx}{3ab}\\
&= -\frac{(Ab-3aB)x^{3/2}}{3ab^2}+\frac{(Ab-aB)x^{9/2}}{3ab\,(a+bx^3)}+\frac{(Ab-3aB)\int\frac{\sqrt{x}}{a+bx^3}\,dx}{2b^2}\\
&= -\frac{(Ab-3aB)x^{3/2}}{3ab^2}+\frac{(Ab-aB)x^{9/2}}{3ab\,(a+bx^3)}+\frac{(Ab-3aB)\text{Subst}\left(\int\frac{x^2}{a+bx^6}\,dx,x,\sqrt{x}\right)}{b^2}\\
&= -\frac{(Ab-3aB)x^{3/2}}{3ab^2}+\frac{(Ab-aB)x^{9/2}}{3ab\,(a+bx^3)}+\frac{(Ab-3aB)\text{Subst}\left(\int\frac{1}{a+bx^2}\,dx,x,x^{3/2}\right)}{3b^2}\\
&= -\frac{(Ab-3aB)x^{3/2}}{3ab^2}+\frac{(Ab-aB)x^{9/2}}{3ab\,(a+bx^3)}+\frac{(Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.81

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2}\,dx=\frac{x^{3/2}(-Ab+3aB+2bBx^3)}{3b^2\,(a+bx^3)}+\frac{(Ab-3aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}b^{5/2}}$$

```
[In] Integrate[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^2,x]
[Out] (x^(3/2)*(-(A*b) + 3*a*B + 2*b*B*x^3))/(3*b^2*(a + b*x^3)) + ((A*b - 3*a*B)
*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*Sqrt[a]*b^(5/2))
```

Maple [A] (verified)

Time = 4.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.68

method	result	size
derivativedivides	$\frac{2B\,x^{\frac{3}{2}}}{3b^{2}}+\frac{\frac{2\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)x^{\frac{3}{2}}}{3\left(b\,x^{3}+a\right)}+\frac{\left(Ab-3Ba\right)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}}}{b^{2}}$	65
default	$\frac{2B\,x^{\frac{3}{2}}}{3b^{2}}+\frac{\frac{2\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)x^{\frac{3}{2}}}{3\left(b\,x^{3}+a\right)}+\frac{\left(Ab-3Ba\right)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}}}{b^{2}}$	65
risch	$\frac{2B\,x^{\frac{3}{2}}}{3b^{2}}+\frac{\frac{2\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)x^{\frac{3}{2}}}{3\left(b\,x^{3}+a\right)}+\frac{\left(Ab-3Ba\right)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}}}{b^{2}}$	65

```
[In] int(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
[Out] 2/3/b^2*B*x^(3/2)+2/3/b^2*((-1/2*A*b+1/2*B*a)*x^(3/2)/(b*x^3+a)+1/2*(A*b-3*
B*a)/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.34

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2}\,dx=\left[\frac{((3\,Bab-Ab^2)x^3+3\,Ba^2-Aab)\sqrt{-ab}\log\left(\frac{bx^3-2\,\sqrt{-ab}x^{\frac{3}{2}}-a}{bx^3+a}\right)+2\,(2\,Bab^2x^4+(3\,B}{6\,(ab^4x^3+a^2b^3)}\right.$$

$$\left.-\frac{((3\,Bab-Ab^2)x^3+3\,Ba^2-Aab)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x^{\frac{3}{2}}}{a}\right)-(2\,Bab^2x^4+(3\,Ba^2b-Aab^2)x)\sqrt{x}}{3\,(ab^4x^3+a^2b^3)}\right]$$

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(((3*B*a*b - A*b^2)*x^3 + 3*B*a^2 - A*a*b)*sqrt(-a*b)*log((b*x^3 - 2*s
qrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)) + 2*(2*B*a*b^2*x^4 + (3*B*a^2*b - A*a*b
^2)*x)*sqrt(x))/(a*b^4*x^3 + a^2*b^3), -1/3*(((3*B*a*b - A*b^2)*x^3 + 3*B*a
^2 - A*a*b)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a) - (2*B*a*b^2*x^4 + (3*B*a
^2*b - A*a*b^2)*x)*sqrt(x))/(a*b^4*x^3 + a^2*b^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**(7/2)*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.72

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(Ba-Ab)x^{\frac{3}{2}}}{3\,(b^3x^3+ab^2)} + \frac{2\,Bx^{\frac{3}{2}}}{3\,b^2} - \frac{(3\,Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}b^2}$$

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(B*a - A*b)*x^(3/2)/(b^3*x^3 + a*b^2) + 2/3*B*x^(3/2)/b^2 - 1/3*(3*B*a
- A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.72

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{2\,Bx^{\frac{3}{2}}}{3\,b^2} - \frac{(3\,Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}b^2} + \frac{Bax^{\frac{3}{2}}-Abx^{\frac{3}{2}}}{3\,(bx^3+a)b^2}$$

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 2/3*B*x^(3/2)/b^2 - 1/3*(3*B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b
)*b^2) + 1/3*(B*a*x^(3/2) - A*b*x^(3/2))/((b*x^3 + a)*b^2)
```

Mupad [B] (verification not implemented)

Time = 7.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.22

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{2\,B\,x^{3/2}}{3\,b^2} - \frac{x^{3/2}\,\left(\frac{A\,b}{3}-\frac{B\,a}{3}\right)}{b^3\,x^3+a\,b^2} + \frac{\mathrm{atan}\left(\frac{36\,\sqrt{a}\,b^{3/2}\,x^{3/2}\,\left(A^2\,b^2-6\,A\,B\,a\,b+9\,B^2\,a^2\right)}{(A\,b-3\,B\,a)\,\left(36\,A\,a\,b^2-108\,B\,a^2\,b\right)}\right)\,(A\,b-3\,B\,a)}{3\,\sqrt{a}\,b^{5/2}}$$

```
[In] int((x^(7/2)*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (2*B*x^(3/2))/(3*b^2) - (x^(3/2)*((A*b)/3 - (B*a)/3))/(a*b^2 + b^3*x^3) + (
atan((36*a^(1/2)*b^(3/2)*x^(3/2)*(A^2*b^2 + 9*B^2*a^2 - 6*A*B*a*b))/((A*b -
 3*B*a)*(36*A*a*b^2 - 108*B*a^2*b)))*(A*b - 3*B*a))/(3*a^(1/2)*b^(5/2))
```

3.164 $\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 312

$$\begin{aligned}\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^2}\,dx &= -\frac{(Ab-7aB)\sqrt{x}}{3ab^2}+\frac{(Ab-aB)x^{7/2}}{3ab\,(a+bx^3)}\\
&-\frac{(Ab-7aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{5/6}b^{13/6}}+\frac{(Ab-7aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{5/6}b^{13/6}}\\
&+\frac{(Ab-7aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{5/6}b^{13/6}}-\frac{(Ab-7aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{5/6}b^{13/6}}\\
&+\frac{(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{5/6}b^{13/6}}\end{aligned}$$

```
[Out] 1/3*(A*b-B*a)*x^(7/2)/a/b/(b*x^3+a)+1/9*(A*b-7*B*a)*arctan(b^(1/6)*x^(1/2)/
a^(1/6))/a^(5/6)/b^(13/6)+1/18*(A*b-7*B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2
)/a^(1/6))/a^(5/6)/b^(13/6)+1/18*(A*b-7*B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/
2)/a^(1/6))/a^(5/6)/b^(13/6)-1/36*(A*b-7*B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*
b^(1/6)*3^(1/2)*x^(1/2))/a^(5/6)/b^(13/6)*3^(1/2)+1/36*(A*b-7*B*a)*ln(a^(1/
3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(5/6)/b^(13/6)*3^(1/2)-1/3*
(A*b-7*B*a)*x^(1/2)/a/b^2
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {468, 327, 335, 215, 648, 632, 210, 642, 211}

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = -\frac{(Ab-7aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{5/6}b^{13/6}} + \frac{(Ab-7aB)\arctan\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{18a^{5/6}b^{13/6}} + \frac{(Ab-7aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{5/6}b^{13/6}} - \frac{(Ab-7aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{5/6}b^{13/6}} + \frac{(Ab-7aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{5/6}b^{13/6}} - \frac{\sqrt{x}(Ab-7aB)}{3ab^2} + \frac{x^{7/2}(Ab-aB)}{3ab\,(a+bx^3)}$$

```
[In] Int[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -1/3*((A*b - 7*a*B)*Sqrt[x])/(a*b^2) + ((A*b - a*B)*x^(7/2))/(3*a*b*(a + b*
x^3)) - ((A*b - 7*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a
^(5/6)*b^(13/6)) + ((A*b - 7*a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1
/6)])/(18*a^(5/6)*b^(13/6)) + ((A*b - 7*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/
6)])/(9*a^(5/6)*b^(13/6)) - ((A*b - 7*a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^
(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(5/6)*b^(13/6)) + ((A*b - 7*a*B)*
Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(
5/6)*b^(13/6))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
```

```
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{(Ab-aB)x^{7/2}}{3ab\,(a+bx^3)} + \frac{\left(-\frac{Ab}{2}+\frac{7aB}{2}\right)\int\frac{x^{5/2}}{a+bx^3}\,dx}{3ab}$$

$$= -\frac{(Ab-7aB)\sqrt{x}}{3ab^2} + \frac{(Ab-aB)x^{7/2}}{3ab\,(a+bx^3)} + \frac{(Ab-7aB)\int\frac{1}{\sqrt{x}(a+bx^3)}\,dx}{6b^2}$$

$$= -\frac{(Ab-7aB)\sqrt{x}}{3ab^2} + \frac{(Ab-aB)x^{7/2}}{3ab\,(a+bx^3)} + \frac{(Ab-7aB)\text{Subst}\left(\int\frac{1}{a+bx^6}\,dx, x, \sqrt{x}\right)}{3b^2}$$

$$= -\frac{(Ab-7aB)\sqrt{x}}{3ab^2} + \frac{(Ab-aB)x^{7/2}}{3ab\,(a+bx^3)} + \frac{(Ab-7aB)\text{Subst}\left(\int\frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{9a^{5/6}b^2} + \frac{(Ab-7aB)\text{Subst}\left(\int\frac{\sqrt[6]{a}+\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{9a^{5/6}b^2} + \frac{(Ab-7aB)\text{Subst}\left(\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{9a^{2/3}b^2}$$

$$= -\frac{(Ab-7aB)\sqrt{x}}{3ab^2} + \frac{(Ab-aB)x^{7/2}}{3ab\,(a+bx^3)} + \frac{(Ab-7aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{5/6}b^{13/6}} - \frac{(Ab-7aB)\text{Subst}\left(\int\frac{-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{12\sqrt{3}a^{5/6}b^{13/6}} + \frac{(Ab-7aB)\text{Subst}\left(\int\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{12\sqrt{3}a^{5/6}b^{13/6}} + \frac{(Ab-7aB)\text{Subst}\left(\int\frac{1}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{36a^{2/3}b^2} + \frac{(Ab-7aB)\text{Subst}\left(\int\frac{1}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{36a^{2/3}b^2}$$

$$= -\frac{(Ab-7aB)\sqrt{x}}{3ab^2} + \frac{(Ab-aB)x^{7/2}}{3ab\,(a+bx^3)} + \frac{(Ab-7aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{5/6}b^{13/6}}$$
$$- \frac{(Ab-7aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{5/6}b^{13/6}}$$
$$+ \frac{(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{5/6}b^{13/6}}$$
$$+ \frac{(Ab-7aB)\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{18\sqrt{3}a^{5/6}b^{13/6}}$$
$$- \frac{(Ab-7aB)\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{18\sqrt{3}a^{5/6}b^{13/6}}$$

$$= -\frac{(Ab-7aB)\sqrt{x}}{3ab^2} + \frac{(Ab-aB)x^{7/2}}{3ab\,(a+bx^3)} - \frac{(Ab-7aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{5/6}b^{13/6}}$$
$$+ \frac{(Ab-7aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{5/6}b^{13/6}} + \frac{(Ab-7aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{5/6}b^{13/6}}$$
$$- \frac{(Ab-7aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{5/6}b^{13/6}}$$
$$+ \frac{(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{5/6}b^{13/6}}$$

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.58

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{\frac{6\sqrt[6]{b}\sqrt{x}(-Ab+7aB+6bBx^3)}{a+bx^3} + \frac{2(Ab-7aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{a^{5/6}} + \frac{(-Ab+7aB)\arctan\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{a^{5/6}} + \sqrt{3}(A}{18b^{13/6}}$$

```
[In] Integrate[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((6*b^(1/6)*Sqrt[x]*(-(A*b) + 7*a*B + 6*b*B*x^3))/(a + b*x^3) + (2*(A*b - 7
*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/a^(5/6) + ((-(A*b) + 7*a*B)*ArcTan
[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])])/a^(5/6) + (Sqrt[3]*(A*b
- 7*a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/
a^(5/6))/(18*b^(13/6))
```

Maple [A] (verified)

Time = 4.22 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.69

method	result
derivativedivides	$\frac{2B\sqrt{x}}{b^2}+\frac{\frac{2\left(-\frac{Ab}{6}+\frac{Ba}{6}\right)\sqrt{x}}{b\,x^3+a}+\frac{(Ab-7Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(-\cdots\right)}{6a}\cdots\right)}{3}\cdots}{b^2}$
default	$\frac{2B\sqrt{x}}{b^2}+\frac{\frac{2\left(-\frac{Ab}{6}+\frac{Ba}{6}\right)\sqrt{x}}{b\,x^3+a}+\frac{(Ab-7Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(-\cdots\right)}{6a}\cdots\right)}{3}\cdots}{b^2}$
risch	$\frac{2B\sqrt{x}}{b^2}+\frac{\frac{2\left(-\frac{Ab}{6}+\frac{Ba}{6}\right)\sqrt{x}}{b\,x^3+a}+\frac{(Ab-7Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(-\cdots\right)}{6a}\cdots\right)}{3}\cdots}{b^2}$

```
[In] int(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*B/b^2*x^(1/2)+2/b^2*((-1/6*A*b+1/6*B*a)*x^(1/2)/(b*x^3+a)+1/6*(A*b-7*B*a)
*(1/3/a*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))-1/12/a*3^(1/2)*(a/b)^(1/6)*
ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arctan(-3^(
1/2)+2*x^(1/2)/(a/b)^(1/6))+1/12/a*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(
1/6)*x^(1/2)+(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^
(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1426 vs. 2(232) = 464.

Time = 0.28 (sec) , antiderivative size = 1426, normalized size of antiderivative = 4.57

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(2*(b^3*x^3 + a*b^2)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^
2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5
 + A^6*b^6)/(a^5*b^13))^(1/6)*log(a*b^2*(-(117649*B^6*a^6 - 100842*A*B^5*a^
5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 -
```

```
42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6) - (7*B*a - A*b)*sqrt(x)) - 2*(b
^3*x^3 + a*b^2)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*
b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6
)/(a^5*b^13))^(1/6)*log(-a*b^2*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 360
15*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*
a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6) - (7*B*a - A*b)*sqrt(x)) + (b^3*x^3 + a*
b^2 + sqrt(-3)*(b^3*x^3 + a*b^2))*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b +
36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5
*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)*log(-(7*B*a - A*b)*sqrt(x) + 1/2*(sqr
t(-3)*a*b^2 + a*b^2)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4
*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^
6*b^6)/(a^5*b^13))^(1/6)) - (b^3*x^3 + a*b^2 + sqrt(-3)*(b^3*x^3 + a*b^2))*
(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B
^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1
/6)*log(-(7*B*a - A*b)*sqrt(x) - 1/2*(sqrt(-3)*a*b^2 + a*b^2)*(-(117649*B^6
*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 +
735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)) - (b^3*x
^3 + a*b^2 - sqrt(-3)*(b^3*x^3 + a*b^2))*(-(117649*B^6*a^6 - 100842*A*B^5*a
^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 -
 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)*log(-(7*B*a - A*b)*sqrt(x) + 1
/2*(sqrt(-3)*a*b^2 - a*b^2)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*
A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b
^5 + A^6*b^6)/(a^5*b^13))^(1/6)) + (b^3*x^3 + a*b^2 - sqrt(-3)*(b^3*x^3 + a
*b^2))*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 686
0*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^
13))^(1/6)*log(-(7*B*a - A*b)*sqrt(x) - 1/2*(sqrt(-3)*a*b^2 - a*b^2)*(-(117
649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3
*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)) +
 12*(6*B*b*x^3 + 7*B*a - A*b)*sqrt(x))/(b^3*x^3 + a*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1658 vs. $2(299) = 598$.

Time = 167.74 (sec) , antiderivative size = 1658, normalized size of antiderivative = 5.31

$$\int \frac{x^{5/2}(A + Bx^3)}{(a + bx^3)^2}\, dx = \text{Too large to display}$$

```
[In] integrate(x**(5/2)*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] Piecewise((zoo*(-2*A/(5*x**(5/2)) + 2*B*sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((2
*A*x**(7/2)/7 + 2*B*x**(13/2)/13)/a**2, Eq(b, 0)), ((-2*A/(5*x**(5/2)) + 2*
B*sqrt(x))/b**2, Eq(a, 0)), (-12*A*a*b*sqrt(x)/(36*a**2*b**2 + 36*a*b**3*x*
*3) - 2*A*a*b*(-a/b)**(1/6)*log(sqrt(x) - (-a/b)**(1/6))/(36*a**2*b**2 + 36
```

```
*a*b**3*x**3) + 2*A*a*b*(-a/b)**(1/6)*log(sqrt(x) + (-a/b)**(1/6))/(36*a**2
*b**2 + 36*a*b**3*x**3) - A*a*b*(-a/b)**(1/6)*log(-4*sqrt(x)*(-a/b)**(1/6)
+ 4*x + 4*(-a/b)**(1/3))/(36*a**2*b**2 + 36*a*b**3*x**3) + A*a*b*(-a/b)**(1
/6)*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**2*b**2 + 36
*a*b**3*x**3) + 2*sqrt(3)*A*a*b*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a
/b)**(1/6)) - sqrt(3)/3)/(36*a**2*b**2 + 36*a*b**3*x**3) + 2*sqrt(3)*A*a*b*
(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(36*a**
2*b**2 + 36*a*b**3*x**3) - 2*A*b**2*x**3*(-a/b)**(1/6)*log(sqrt(x) - (-a/b)
**(1/6))/(36*a**2*b**2 + 36*a*b**3*x**3) + 2*A*b**2*x**3*(-a/b)**(1/6)*log(
sqrt(x) + (-a/b)**(1/6))/(36*a**2*b**2 + 36*a*b**3*x**3) - A*b**2*x**3*(-a/
b)**(1/6)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**2*b*
*2 + 36*a*b**3*x**3) + A*b**2*x**3*(-a/b)**(1/6)*log(4*sqrt(x)*(-a/b)**(1/6
) + 4*x + 4*(-a/b)**(1/3))/(36*a**2*b**2 + 36*a*b**3*x**3) + 2*sqrt(3)*A*b*
*2*x**3*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)
/(36*a**2*b**2 + 36*a*b**3*x**3) + 2*sqrt(3)*A*b**2*x**3*(-a/b)**(1/6)*atan
(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(36*a**2*b**2 + 36*a*b**3
*x**3) + 84*B*a**2*sqrt(x)/(36*a**2*b**2 + 36*a*b**3*x**3) + 14*B*a**2*(-a/
b)**(1/6)*log(sqrt(x) - (-a/b)**(1/6))/(36*a**2*b**2 + 36*a*b**3*x**3) - 14
*B*a**2*(-a/b)**(1/6)*log(sqrt(x) + (-a/b)**(1/6))/(36*a**2*b**2 + 36*a*b**
3*x**3) + 7*B*a**2*(-a/b)**(1/6)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a
/b)**(1/3))/(36*a**2*b**2 + 36*a*b**3*x**3) - 7*B*a**2*(-a/b)**(1/6)*log(4*
sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**2*b**2 + 36*a*b**3*x*
*3) - 14*sqrt(3)*B*a**2*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/
6)) - sqrt(3)/3)/(36*a**2*b**2 + 36*a*b**3*x**3) - 14*sqrt(3)*B*a**2*(-a/b)
**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(36*a**2*b**2
 + 36*a*b**3*x**3) + 72*B*a*b*x**(7/2)/(36*a**2*b**2 + 36*a*b**3*x**3) + 14
*B*a*b*x**3*(-a/b)**(1/6)*log(sqrt(x) - (-a/b)**(1/6))/(36*a**2*b**2 + 36*a
*b**3*x**3) - 14*B*a*b*x**3*(-a/b)**(1/6)*log(sqrt(x) + (-a/b)**(1/6))/(36*
a**2*b**2 + 36*a*b**3*x**3) + 7*B*a*b*x**3*(-a/b)**(1/6)*log(-4*sqrt(x)*(-a
/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**2*b**2 + 36*a*b**3*x**3) - 7*B*a
*b*x**3*(-a/b)**(1/6)*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/
(36*a**2*b**2 + 36*a*b**3*x**3) - 14*sqrt(3)*B*a*b*x**3*(-a/b)**(1/6)*atan(
2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(36*a**2*b**2 + 36*a*b**3*
x**3) - 14*sqrt(3)*B*a*b*x**3*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b
)**(1/6)) + sqrt(3)/3)/(36*a**2*b**2 + 36*a*b**3*x**3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(Ba-Ab)\sqrt{x}}{3\,(b^3x^3+ab^2)} + \frac{2\,B\sqrt{x}}{b^2}$$

$$-\frac{\frac{\sqrt{3}(7\,Ba-Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} - \frac{\sqrt{3}(7\,Ba-Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} + \frac{4\left(7\,Bab^{\frac{1}{3}}-Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} + \frac{2\left(7\,B\right.}{}}{36\,b^2}$$

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(B*a - A*b)*sqrt(x)/(b^3*x^3 + a*b^2) + 2*B*sqrt(x)/b^2 - 1/36*(sqrt(3)
*(7*B*a - A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(
a^(5/6)*b^(1/6)) - sqrt(3)*(7*B*a - A*b)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(
x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(7*B*a*b^(1/3) - A*b^(4/3))
*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(a^(1/3
)*b^(1/3))) + 2*(7*B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*arctan((sqrt(3)*a
^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(
a^(1/3)*b^(1/3))) + 2*(7*B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*arctan(-(sq
rt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3
)*sqrt(a^(1/3)*b^(1/3))))/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{2\,B\sqrt{x}}{b^2} - \frac{\sqrt{3}\left(7\,(ab^5)^{\frac{1}{6}}\,Ba-(ab^5)^{\frac{1}{6}}\,Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,ab^3} + \frac{\sqrt{3}\left(7\,(ab^5)^{\frac{1}{6}}\,Ba-(ab^5)^{\frac{1}{6}}\,Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,ab^3} + \frac{Ba\sqrt{x}-Ab\sqrt{x}}{3\,(bx^3+a)b^2} - \frac{\left(7\,(ab^5)^{\frac{1}{6}}\,Ba-(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,ab^3} - \frac{\left(7\,(ab^5)^{\frac{1}{6}}\,Ba-(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,ab^3} - \frac{\left(7\,(ab^5)^{\frac{1}{6}}\,Ba-(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9\,ab^3}$$

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/b^2 - 1/36*sqrt(3)*(7*(a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*lo
g(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a*b^3) + 1/36*sqrt(3)*(7*
(a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x
 + (a/b)^(1/3))/(a*b^3) + 1/3*(B*a*sqrt(x) - A*b*sqrt(x))/((b*x^3 + a)*b^2)
 - 1/18*(7*(a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/
6) + 2*sqrt(x))/(a/b)^(1/6))/(a*b^3) - 1/18*(7*(a*b^5)^(1/6)*B*a - (a*b^5)^
(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a*b^3) -
 1/9*(7*(a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/
(a*b^3)
```

Mupad [B] (verification not implemented)

Time = 7.35 (sec) , antiderivative size = 1884, normalized size of antiderivative = 6.04

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

```
[In] int((x^(5/2)*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (2*B*x^(1/2))/b^2 - (x^(1/2)*((A*b)/3 - (B*a)/3))/(a*b^2 + b^3*x^3) - (atan
(((((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a
^3*b - 28*A^3*B*a*b^3))/(27*b^3) - (2*(A*b - 7*B*a)*(343*B^3*a^4 - A^3*a*b^
3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^(5/6)*b^(19/6)))*(A*b - 7
```

```
*B*a)*1i)/(18*(-a)^(5/6)*b^(13/6)) + (((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 +
 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a^3*b - 28*A^3*B*a*b^3))/(27*b^3) + (2*(A
*b - 7*B*a)*(343*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))
/(27*(-a)^(5/6)*b^(19/6)))*(A*b - 7*B*a)*1i)/(18*(-a)^(5/6)*b^(13/6)))/((((
2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a^3*b
- 28*A^3*B*a*b^3))/(27*b^3) - (2*(A*b - 7*B*a)*(343*B^3*a^4 - A^3*a*b^3 - 1
47*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^(5/6)*b^(19/6)))*(A*b - 7*B*a)
)/(18*(-a)^(5/6)*b^(13/6)) - (((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2
*B^2*a^2*b^2 - 1372*A*B^3*a^3*b - 28*A^3*B*a*b^3))/(27*b^3) + (2*(A*b - 7*B
*a)*(343*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a
)^(5/6)*b^(19/6)))*(A*b - 7*B*a))/(18*(-a)^(5/6)*b^(13/6))))*(A*b - 7*B*a)*
1i)/(9*(-a)^(5/6)*b^(13/6)) - (atan(((((3^(1/2)*1i)/2 - 1/2)*((2*x^(1/2)*(A
^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a^3*b - 28*A^3*B*a
*b^3))/(27*b^3) - (2*((3^(1/2)*1i)/2 - 1/2)*(A*b - 7*B*a)*(343*B^3*a^4 - A^
3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^(5/6)*b^(19/6)))*(A
*b - 7*B*a)*1i)/(18*(-a)^(5/6)*b^(13/6)) + (((3^(1/2)*1i)/2 - 1/2)*((2*x^(1
/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a^3*b - 28*A
^3*B*a*b^3))/(27*b^3) + (2*((3^(1/2)*1i)/2 - 1/2)*(A*b - 7*B*a)*(343*B^3*a^
4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^(5/6)*b^(19/6
)))*(A*b - 7*B*a)*1i)/(18*(-a)^(5/6)*b^(13/6)))/((((3^(1/2)*1i)/2 - 1/2)*((
2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a^3*b
- 28*A^3*B*a*b^3))/(27*b^3) - (2*((3^(1/2)*1i)/2 - 1/2)*(A*b - 7*B*a)*(343*
B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^(5/6)*b
^(19/6)))*(A*b - 7*B*a))/(18*(-a)^(5/6)*b^(13/6)) - (((3^(1/2)*1i)/2 - 1/2)
*((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a^3
*b - 28*A^3*B*a*b^3))/(27*b^3) + (2*((3^(1/2)*1i)/2 - 1/2)*(A*b - 7*B*a)*(3
43*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^(5/6
)*b^(19/6)))*(A*b - 7*B*a))/(18*(-a)^(5/6)*b^(13/6))))*((3^(1/2)*1i)/2 - 1/
2)*(A*b - 7*B*a)*1i)/(9*(-a)^(5/6)*b^(13/6)) - (atan(((((3^(1/2)*1i)/2 + 1/
2)*((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a
^3*b - 28*A^3*B*a*b^3))/(27*b^3) - (2*((3^(1/2)*1i)/2 + 1/2)*(A*b - 7*B*a)*
(343*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^(5
/6)*b^(19/6)))*(A*b - 7*B*a)*1i)/(18*(-a)^(5/6)*b^(13/6)) + (((3^(1/2)*1i)/
2 + 1/2)*((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A
*B^3*a^3*b - 28*A^3*B*a*b^3))/(27*b^3) + (2*((3^(1/2)*1i)/2 + 1/2)*(A*b - 7
*B*a)*(343*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(
-a)^(5/6)*b^(19/6)))*(A*b - 7*B*a)*1i)/(18*(-a)^(5/6)*b^(13/6)))/((((3^(1/2
)*1i)/2 + 1/2)*((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 -
1372*A*B^3*a^3*b - 28*A^3*B*a*b^3))/(27*b^3) - (2*((3^(1/2)*1i)/2 + 1/2)*(A
*b - 7*B*a)*(343*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))
/(27*(-a)^(5/6)*b^(19/6)))*(A*b - 7*B*a))/(18*(-a)^(5/6)*b^(13/6)) - (((3^(
1/2)*1i)/2 + 1/2)*((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2
 - 1372*A*B^3*a^3*b - 28*A^3*B*a*b^3))/(27*b^3) + (2*((3^(1/2)*1i)/2 + 1/2)
*(A*b - 7*B*a)*(343*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^
2))/(27*(-a)^(5/6)*b^(19/6)))*(A*b - 7*B*a))/(18*(-a)^(5/6)*b^(13/6))))*((3
```

```
^(1/2)*1i)/2 + 1/2)*(A*b - 7*B*a)*1i)/(9*(-a)^(5/6)*b^(13/6))
```

3.165 $\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 289

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(Ab-aB)x^{5/2}}{3ab\,(a+bx^3)} - \frac{(Ab+5aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{7/6}b^{11/6}} + \frac{(Ab+5aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{7/6}b^{11/6}} + \frac{(Ab+5aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{7/6}b^{11/6}} + \frac{(Ab+5aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{7/6}b^{11/6}} - \frac{(Ab+5aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{7/6}b^{11/6}}$$

```
[Out] 1/3*(A*b-B*a)*x^(5/2)/a/b/(b*x^3+a)+1/9*(A*b+5*B*a)*arctan(b^(1/6)*x^(1/2)/
a^(1/6))/a^(7/6)/b^(11/6)+1/18*(A*b+5*B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2
)/a^(1/6))/a^(7/6)/b^(11/6)+1/18*(A*b+5*B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/
2)/a^(1/6))/a^(7/6)/b^(11/6)+1/36*(A*b+5*B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*
b^(1/6)*3^(1/2)*x^(1/2))/a^(7/6)/b^(11/6)*3^(1/2)-1/36*(A*b+5*B*a)*ln(a^(1/
3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(7/6)/b^(11/6)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {468, 335, 301, 648, 632, 210, 642, 211}

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = -\frac{(5aB+Ab)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{7/6}b^{11/6}} + \frac{(5aB+Ab)\arctan\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{18a^{7/6}b^{11/6}} + \frac{(5aB+Ab)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{7/6}b^{11/6}} + \frac{(5aB+Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{7/6}b^{11/6}} - \frac{(5aB+Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{7/6}b^{11/6}} + \frac{x^{5/2}(Ab-aB)}{3ab\,(a+bx^3)}$$

```
[In] Int[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((A*b - a*B)*x^(5/2))/(3*a*b*(a + b*x^3)) - ((A*b + 5*a*B)*ArcTan[Sqrt[3] -
 (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(7/6)*b^(11/6)) + ((A*b + 5*a*B)*ArcTa
n[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(7/6)*b^(11/6)) + ((A*b + 5
*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(9*a^(7/6)*b^(11/6)) + ((A*b + 5*a
*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]
*a^(7/6)*b^(11/6)) - ((A*b + 5*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*S
qrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(7/6)*b^(11/6))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
```

```
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{(Ab-aB)x^{5/2}}{3ab\,(a+bx^3)} + \frac{\left(\frac{Ab}{2}+\frac{5aB}{2}\right)\int \frac{x^{3/2}}{a+bx^3}\,dx}{3ab}$$

$$= \frac{(Ab-aB)x^{5/2}}{3ab\left(a+bx^3\right)} + \frac{\left(2\left(\frac{Ab}{2}+\frac{5aB}{2}\right)\right)\text{Subst}\left(\int \frac{x^4}{a+bx^6}\,dx, x, \sqrt{x}\right)}{3ab}$$

$$= \frac{(Ab-aB)x^{5/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab+5aB)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2}+\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{9a^{7/6}b^{5/3}}$$
$$+ \frac{(Ab+5aB)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{9a^{7/6}b^{5/3}}$$
$$+ \frac{(Ab+5aB)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{9ab^{5/3}}$$

$$= \frac{(Ab-aB)x^{5/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab+5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{7/6}b^{11/6}}$$
$$+ \frac{(Ab+5aB)\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{12\sqrt{3}a^{7/6}b^{11/6}}$$
$$- \frac{(Ab+5aB)\text{Subst}\left(\int \frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{12\sqrt{3}a^{7/6}b^{11/6}}$$
$$+ \frac{(Ab+5aB)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{36ab^{5/3}}$$
$$+ \frac{(Ab+5aB)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{36ab^{5/3}}$$

$$= \frac{(Ab-aB)x^{5/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab+5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{7/6}b^{11/6}} + \frac{(Ab+5aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{7/6}b^{11/6}} - \frac{(Ab+5aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{7/6}b^{11/6}} + \frac{(Ab+5aB)\text{Subst}\left(\int\frac{1}{-\frac{1}{3}-x^2}\,dx,x,1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{18\sqrt{3}a^{7/6}b^{11/6}} - \frac{(Ab+5aB)\text{Subst}\left(\int\frac{1}{-\frac{1}{3}-x^2}\,dx,x,1+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{18\sqrt{3}a^{7/6}b^{11/6}}$$

$$= \frac{(Ab-aB)x^{5/2}}{3ab\left(a+bx^3\right)} - \frac{(Ab+5aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{7/6}b^{11/6}} + \frac{(Ab+5aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{7/6}b^{11/6}} + \frac{(Ab+5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{7/6}b^{11/6}} + \frac{(Ab+5aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{7/6}b^{11/6}} - \frac{(Ab+5aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{7/6}b^{11/6}}$$

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.58

$$\int \frac{x^{3/2}(A+Bx^3)}{\left(a+bx^3\right)^2}\,dx = \frac{-\frac{6\sqrt[6]{a}b^{5/6}(-Ab+aB)x^{5/2}}{a+bx^3} + 2(Ab+5aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right) - (Ab+5aB)\arctan\left(\frac{\sqrt[3]{a}-\sqrt[3]{\ldots}}{\sqrt[6]{a}\sqrt[6]{\ldots}}\right. \ldots}{18a^{7/6}b^{11/6}}$$

```
[In] Integrate[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((-6*a^(1/6)*b^(5/6)*(-(A*b) + a*B)*x^(5/2))/(a + b*x^3) + 2*(A*b + 5*a*B)*
ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)] - (A*b + 5*a*B)*ArcTan[(a^(1/3) - b^(1/3)
*x)/(a^(1/6)*b^(1/6)*Sqrt[x])] - Sqrt[3]*(A*b + 5*a*B)*ArcTanh[(Sqrt[3]*a^(
1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/(18*a^(7/6)*b^(11/6))
```

Maple [A] (verified)

Time = 4.00 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{(Ab-Ba)x^{\frac{5}{2}}}{3ab(bx^3+a)}+\frac{(Ab+5Ba)\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{\right.}}{12a}}{3ab}$
default	$\frac{(Ab-Ba)x^{\frac{5}{2}}}{3ab(bx^3+a)}+\frac{(Ab+5Ba)\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{\right.}}{12a}}{3ab}$

```
[In] int(x^(3/2)*(B*x^3+A)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(A*b-B*a)*x^(5/2)/a/b/(b*x^3+a)+1/3*(A*b+5*B*a)/a/b*(1/12/a*3^(1/2)*(a/
b)^(5/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+1/6/b/(a/b)^(1/6)*ar
ctan(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6))-1/12/a*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2
)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/b/(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)
^(1/6)+3^(1/2))+1/3/b/(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1773 vs. $2(207) = 414$.

Time = 0.29 (sec) , antiderivative size = 1773, normalized size of antiderivative = 6.13

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(12*(B*a - A*b)*x^(5/2) - 2*(a*b^2*x^3 + a^2*b)*(-(15625*B^6*a^6 + 18
750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2
*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6)*log(a^6*b^9*(-(15625
*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3
+ 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(5/6) + (3125
*B^5*a^5 + 3125*A*B^4*a^4*b + 1250*A^2*B^3*a^3*b^2 + 250*A^3*B^2*a^2*b^3 +
25*A^4*B*a*b^4 + A^5*b^5)*sqrt(x)) + 2*(a*b^2*x^3 + a^2*b)*(-(15625*B^6*a^6
 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^
4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6)*log(-a^6*b^9*(-
(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^
3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(5/6) +
```

```
(3125*B^5*a^5 + 3125*A*B^4*a^4*b + 1250*A^2*B^3*a^3*b^2 + 250*A^3*B^2*a^2*
b^3 + 25*A^4*B*a*b^4 + A^5*b^5)*sqrt(x)) - (a*b^2*x^3 + a^2*b - sqrt(-3)*(a
*b^2*x^3 + a^2*b))*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*
b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6
)/(a^7*b^11))^(1/6)*log(1/2*(sqrt(-3)*a^6*b^9 + a^6*b^9)*(-(15625*B^6*a^6 +
 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*
B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(5/6) + (3125*B^5*a^5 +
 3125*A*B^4*a^4*b + 1250*A^2*B^3*a^3*b^2 + 250*A^3*B^2*a^2*b^3 + 25*A^4*B*a
*b^4 + A^5*b^5)*sqrt(x)) + (a*b^2*x^3 + a^2*b - sqrt(-3)*(a*b^2*x^3 + a^2*b
))*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B
^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1
/6)*log(-1/2*(sqrt(-3)*a^6*b^9 + a^6*b^9)*(-(15625*B^6*a^6 + 18750*A*B^5*a^
5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 3
0*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(5/6) + (3125*B^5*a^5 + 3125*A*B^4*a^4
*b + 1250*A^2*B^3*a^3*b^2 + 250*A^3*B^2*a^2*b^3 + 25*A^4*B*a*b^4 + A^5*b^5)
*sqrt(x)) + (a*b^2*x^3 + a^2*b + sqrt(-3)*(a*b^2*x^3 + a^2*b))*(-(15625*B^6
*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 37
5*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6)*log(1/2*(sq
rt(-3)*a^6*b^9 - a^6*b^9)*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B
^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 +
A^6*b^6)/(a^7*b^11))^(5/6) + (3125*B^5*a^5 + 3125*A*B^4*a^4*b + 1250*A^2*B^
3*a^3*b^2 + 250*A^3*B^2*a^2*b^3 + 25*A^4*B*a*b^4 + A^5*b^5)*sqrt(x)) - (a*b
^2*x^3 + a^2*b + sqrt(-3)*(a*b^2*x^3 + a^2*b))*(-(15625*B^6*a^6 + 18750*A*B
^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^
4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6)*log(-1/2*(sqrt(-3)*a^6*b^9
- a^6*b^9)*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 25
00*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b
^11))^(5/6) + (3125*B^5*a^5 + 3125*A*B^4*a^4*b + 1250*A^2*B^3*a^3*b^2 + 250
*A^3*B^2*a^2*b^3 + 25*A^4*B*a*b^4 + A^5*b^5)*sqrt(x)))/(a*b^2*x^3 + a^2*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1885 vs. $2(277) = 554$.

Time = 112.41 (sec) , antiderivative size = 1885, normalized size of antiderivative = 6.52

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x**(3/2)*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((2
*A*x**(5/2)/5 + 2*B*x**(11/2)/11)/a**2, Eq(b, 0)), ((-2*A/(7*x**(7/2)) - 2*
B/sqrt(x))/b**2, Eq(a, 0)), (2*A*a*b*log(sqrt(x) - (-a/b)**(1/6))/(36*a**2*
b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) - 2*A*a*b*log(sqrt(x) +
```

```
(-a/b)**(1/6))/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6))
+ A*a*b*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**2*b**2
*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) - A*a*b*log(4*sqrt(x)*(-a/b)
**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x*
*3*(-a/b)**(1/6)) + 2*sqrt(3)*A*a*b*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)
) - sqrt(3)/3)/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6))
+ 2*sqrt(3)*A*a*b*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(36
*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) + 12*A*b**2*x**(5/
2)*(-a/b)**(1/6)/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)
) + 2*A*b**2*x**3*log(sqrt(x) - (-a/b)**(1/6))/(36*a**2*b**2*(-a/b)**(1/6)
+ 36*a*b**3*x**3*(-a/b)**(1/6)) - 2*A*b**2*x**3*log(sqrt(x) + (-a/b)**(1/6)
)/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) + A*b**2*x**3
*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**2*b**2*(-a/b)
**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) - A*b**2*x**3*log(4*sqrt(x)*(-a/b)*
*(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**
3*(-a/b)**(1/6)) + 2*sqrt(3)*A*b**2*x**3*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**
(1/6)) - sqrt(3)/3)/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1
/6)) + 2*sqrt(3)*A*b**2*x**3*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqr
t(3)/3)/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) + 10*B*
a**2*log(sqrt(x) - (-a/b)**(1/6))/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x
**3*(-a/b)**(1/6)) - 10*B*a**2*log(sqrt(x) + (-a/b)**(1/6))/(36*a**2*b**2*(
-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) + 5*B*a**2*log(-4*sqrt(x)*(-a/
b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*
x**3*(-a/b)**(1/6)) - 5*B*a**2*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)
**(1/3))/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) + 10*s
qrt(3)*B*a**2*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(36*a**
2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) + 10*sqrt(3)*B*a**2*at
an(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(36*a**2*b**2*(-a/b)**(
1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) - 12*B*a*b*x**(5/2)*(-a/b)**(1/6)/(36*
a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) + 10*B*a*b*x**3*log
(sqrt(x) - (-a/b)**(1/6))/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/
b)**(1/6)) - 10*B*a*b*x**3*log(sqrt(x) + (-a/b)**(1/6))/(36*a**2*b**2*(-a/b
)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) + 5*B*a*b*x**3*log(-4*sqrt(x)*(-a/
b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*
x**3*(-a/b)**(1/6)) - 5*B*a*b*x**3*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-
a/b)**(1/3))/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) +
10*sqrt(3)*B*a*b*x**3*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)
/(36*a**2*b**2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)) + 10*sqrt(3)*B
*a*b*x**3*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(36*a**2*b*
*2*(-a/b)**(1/6) + 36*a*b**3*x**3*(-a/b)**(1/6)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.81

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^2}\,dx = -\frac{(Ba-Ab)x^{\frac{5}{2}}}{3\,(ab^2x^3+a^2b)} - \frac{(5\,Ba+Ab)\left(\frac{\sqrt{3}\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}} - \frac{\sqrt{3}\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}} - \frac{2\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} - \frac{2\arctan\left(-\sqrt{3}a\right.}{b^{\frac{2}{3}}\sqrt{a}}\right.}{36\,ab}$$

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(B*a - A*b)*x^(5/2)/(a*b^2*x^3 + a^2*b) - 1/36*(5*B*a + A*b)*(sqrt(3)*
log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6)
) - sqrt(3)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^
(1/6)*b^(5/6)) - 2*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqr
t(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 2*arctan(-(sqrt(3)*a^
(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(
1/3)*b^(1/3))) - 4*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*s
qrt(a^(1/3)*b^(1/3))))/(a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.65 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^2}\,dx &= \frac{(5\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,(ab^5)^{\frac{1}{6}}\,ab} \\ &+ \frac{(5\,Ba+Ab)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,(ab^5)^{\frac{1}{6}}\,ab} + \frac{\left(5\,Ba\left(\frac{a}{b}\right)^{\frac{5}{6}}+Ab\left(\frac{a}{b}\right)^{\frac{5}{6}}\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9\,a^2b} \\ &- \frac{Bax^{\frac{5}{2}}-Abx^{\frac{5}{2}}}{3\,(bx^3+a)ab} - \frac{\sqrt{3}\left(5\,(ab^5)^{\frac{5}{6}}\,Ba+(ab^5)^{\frac{5}{6}}\,Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^2b^6} \\ &+ \frac{\sqrt{3}\left(5\,(ab^5)^{\frac{5}{6}}\,Ba+(ab^5)^{\frac{5}{6}}\,Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^2b^6}\end{aligned}$$

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/18*(5*B*a + A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/((
a*b^5)^(1/6)*a*b) + 1/18*(5*B*a + A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqr
t(x))/(a/b)^(1/6))/((a*b^5)^(1/6)*a*b) + 1/9*(5*B*a*(a/b)^(5/6) + A*b*(a/b)
^(5/6))*arctan(sqrt(x)/(a/b)^(1/6))/(a^2*b) - 1/3*(B*a*x^(5/2) - A*b*x^(5/2
))/((b*x^3 + a)*a*b) - 1/36*sqrt(3)*(5*(a*b^5)^(5/6)*B*a + (a*b^5)^(5/6)*A*
b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^6) + 1/36*sqrt
(3)*(5*(a*b^5)^(5/6)*B*a + (a*b^5)^(5/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1
/6) + x + (a/b)^(1/3))/(a^2*b^6)
```

Mupad [B] (verification not implemented)

Time = 7.29 (sec) , antiderivative size = 1578, normalized size of antiderivative = 5.46

$$\int \frac{x^{3/2}(A+Bx^3)}{\left(a+bx^3\right)^2}\,dx = \text{Too large to display}$$

```
[In] int((x^(3/2)*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (x^(5/2)*(A*b - B*a))/(3*a*b*(a + b*x^3)) - (atan(((((3^(1/2)*1i)/2 - 1/2)^
2*(A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A
^2*B*a*b^2 - (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 +
600*B^2*a^3*b^2 + 240*A*B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6)))*1i)/(324*(-a)
^(7/3)*b^(11/3)) - (((3^(1/2)*1i)/2 - 1/2)^2*(A*b + 5*B*a)^2*((4*A^3*b^3)/3
 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2 + (x^(1/2)*((3^(1/2)*
1i)/2 - 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^
3))/(18*(-a)^(7/6)*b^(11/6)))*1i)/(324*(-a)^(7/3)*b^(11/3)))/((((3^(1/2)*1i
)/2 - 1/2)^2*(A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a
^2*b + 20*A^2*B*a*b^2 - (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b + 5*B*a)*(24*A
^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6))))/(
324*(-a)^(7/3)*b^(11/3)) + (((3^(1/2)*1i)/2 - 1/2)^2*(A*b + 5*B*a)^2*((4*A^
3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2 + (x^(1/2)*((
3^(1/2)*1i)/2 - 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*
B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6))))/(324*(-a)^(7/3)*b^(11/3))))*((3^(1/2
)*1i)/2 - 1/2)*(A*b + 5*B*a)*1i)/(9*(-a)^(7/6)*b^(11/6)) - (atan(((((3^(1/2
)*1i)/2 + 1/2)^2*(A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B
^2*a^2*b + 20*A^2*B*a*b^2 - (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b + 5*B*a)*(
24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6))
)*1i)/(324*(-a)^(7/3)*b^(11/3)) - (((3^(1/2)*1i)/2 + 1/2)^2*(A*b + 5*B*a)^2
*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2 + (x^(
1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 +
 240*A*B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6)))*1i)/(324*(-a)^(7/3)*b^(11/3)))
/((((3^(1/2)*1i)/2 + 1/2)^2*(A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/
3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2 - (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b
 + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^3))/(18*(-a)^(7/6
)*b^(11/6))))/(324*(-a)^(7/3)*b^(11/3)) + (((3^(1/2)*1i)/2 + 1/2)^2*(A*b +
```

```
5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^
2 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a
^3*b^2 + 240*A*B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6))))/(324*(-a)^(7/3)*b^(11
/3))))*((3^(1/2)*1i)/2 + 1/2)*(A*b + 5*B*a)*1i)/(9*(-a)^(7/6)*b^(11/6)) - (
atan((((A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b +
 20*A^2*B*a*b^2 - (x^(1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 +
240*A*B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6)))*1i)/(324*(-a)^(7/3)*b^(11/3)) -
 ((A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A
^2*B*a*b^2 + (x^(1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A
*B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6)))*1i)/(324*(-a)^(7/3)*b^(11/3)))/(((A*
b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*
a*b^2 - (x^(1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^
2*b^3))/(18*(-a)^(7/6)*b^(11/6))))/(324*(-a)^(7/3)*b^(11/3)) + ((A*b + 5*B*
a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2 +
(x^(1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^3))/
(18*(-a)^(7/6)*b^(11/6))))/(324*(-a)^(7/3)*b^(11/3))))*(A*b + 5*B*a)*1i)/(9
*(-a)^(7/6)*b^(11/6))
```

3.166 $\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^2} dx = \frac{(Ab-aB)x^{3/2}}{3ab(a+bx^3)} + \frac{(Ab+aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}b^{3/2}}$$

```
[Out] 1/3*(A*b-B*a)*x^(3/2)/a/b/(b*x^3+a)+1/3*(A*b+B*a)*arctan(x^(3/2)*b^(1/2)/a^
(1/2))/a^(3/2)/b^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {468, 335, 281, 211}

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^2} dx = \frac{(aB+Ab)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}b^{3/2}} + \frac{x^{3/2}(Ab-aB)}{3ab(a+bx^3)}$$

```
[In] Int[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((A*b - a*B)*x^(3/2))/(3*a*b*(a + b*x^3)) + ((A*b + a*B)*ArcTan[(Sqrt[b]*x^
(3/2))/Sqrt[a]])/(3*a^(3/2)*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(Ab-aB)x^{3/2}}{3ab\left(a+bx^3\right)}+\frac{(Ab+aB)\int\frac{\sqrt{x}}{a+bx^3}\,dx}{2ab}\\
&= \frac{(Ab-aB)x^{3/2}}{3ab\left(a+bx^3\right)}+\frac{(Ab+aB)\text{Subst}\left(\int\frac{x^2}{a+bx^6}\,dx,x,\sqrt{x}\right)}{ab}\\
&= \frac{(Ab-aB)x^{3/2}}{3ab\left(a+bx^3\right)}+\frac{(Ab+aB)\text{Subst}\left(\int\frac{1}{a+bx^2}\,dx,x,x^{3/2}\right)}{3ab}\\
&= \frac{(Ab-aB)x^{3/2}}{3ab\left(a+bx^3\right)}+\frac{(Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}b^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$
\int\frac{\sqrt{x}\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx=-\frac{(-Ab+aB)x^{3/2}}{3ab\left(a+bx^3\right)}+\frac{(Ab+aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}b^{3/2}}
$$

```
[In] Integrate[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^2,x]
[Out] -1/3*((-(A*b) + a*B)*x^(3/2))/(a*b*(a + b*x^3)) + ((A*b + a*B)*ArcTan[(Sqrt
[b]*x^(3/2))/Sqrt[a]])/(3*a^(3/2)*b^(3/2))
```

Maple [A] (verified)

Time = 4.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{(Ab-Ba)x^{\frac{3}{2}}}{3ab(b\,x^{3}+a)}+\frac{(Ab+Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3ab\sqrt{ab}}$	61
default	$\frac{(Ab-Ba)x^{\frac{3}{2}}}{3ab(b\,x^{3}+a)}+\frac{(Ab+Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3ab\sqrt{ab}}$	61

```
[In] int((B*x^3+A)*x^(1/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/3*(A*b-B*a)*x^(3/2)/a/b/(b*x^3+a)+1/3*(A*b+B*a)/a/b/(a*b)^(1/2)*arctan(b*
x^(3/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.68

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^2}\,dx = \left[-\frac{2\,(Ba^2b-Aab^2)x^{\frac{3}{2}}+((Bab+Ab^2)x^3+Ba^2+Aab)\sqrt{-ab}\log\left(\frac{bx^3-2\,\sqrt{-ab}x^{\frac{3}{2}}-a}{bx^3+a}\right)}{6\,(a^2b^3x^3+a^3b^2)},\; -\frac{(Ba^2b-Aab^2)x^{\frac{3}{2}}-((Bab+Ab^2)x^3+Ba^2+Aab)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x^{\frac{3}{2}}}{a}\right)}{3\,(a^2b^3x^3+a^3b^2)}\right]$$

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
[Out] [-1/6*(2*(B*a^2*b - A*a*b^2)*x^(3/2) + ((B*a*b + A*b^2)*x^3 + B*a^2 + A*a*b
)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)))/(a^2*b^3*
x^3 + a^3*b^2), -1/3*((B*a^2*b - A*a*b^2)*x^(3/2) - ((B*a*b + A*b^2)*x^3 +
B*a^2 + A*a*b)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a))/(a^2*b^3*x^3 + a^3*b^
2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1042 vs. $2(61) = 122$.

Time = 67.82 (sec) , antiderivative size = 1042, normalized size of antiderivative = 14.68

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^2}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{9x^{\frac{9}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}\right) \\ \frac{\frac{2Ax^{\frac{3}{2}}}{3}+\frac{2Bx^{\frac{9}{2}}}{9}}{a^2} \\ \frac{-\frac{2A}{9x^{\frac{9}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}}{b^2} \\ \frac{2Aabx^{\frac{3}{2}}}{6a^3b+6a^2b^2x^3}-\frac{Aab\sqrt{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[6]{-\frac{a}{b}}\right)}{6a^3b+6a^2b^2x^3}+\frac{Aab\sqrt{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[6]{-\frac{a}{b}}\right)}{6a^3b+6a^2b^2x^3}+\frac{Aab\sqrt{-\frac{a}{b}}\log\left(-4\sqrt{x}\sqrt[6]{-\frac{a}{b}}+4x+4\sqrt[3]{-\frac{a}{b}}\right)}{6a^3b+6a^2b^2x^3}-\ldots \end{cases}$$

```
[In] integrate((B*x**3+A)*x**(1/2)/(b*x**3+a)**2,x)
```

```
[Out] Piecewise((zoo*(-2*A/(9*x**(9/2)) - 2*B/(3*x**(3/2))), Eq(a, 0) & Eq(b, 0))
, ((2*A*x**(3/2)/3 + 2*B*x**(9/2)/9)/a**2, Eq(b, 0)), ((-2*A/(9*x**(9/2)) -
 2*B/(3*x**(3/2)))/b**2, Eq(a, 0)), (2*A*a*b*x**(3/2)/(6*a**3*b + 6*a**2*b*
*2*x**3) - A*a*b*sqrt(-a/b)*log(sqrt(x) - (-a/b)**(1/6))/(6*a**3*b + 6*a**2
*b**2*x**3) + A*a*b*sqrt(-a/b)*log(sqrt(x) + (-a/b)**(1/6))/(6*a**3*b + 6*a
**2*b**2*x**3) + A*a*b*sqrt(-a/b)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-
a/b)**(1/3))/(6*a**3*b + 6*a**2*b**2*x**3) - A*a*b*sqrt(-a/b)*log(4*sqrt(x)
*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a**3*b + 6*a**2*b**2*x**3) - A*b
**2*x**3*sqrt(-a/b)*log(sqrt(x) - (-a/b)**(1/6))/(6*a**3*b + 6*a**2*b**2*x*
*3) + A*b**2*x**3*sqrt(-a/b)*log(sqrt(x) + (-a/b)**(1/6))/(6*a**3*b + 6*a**
2*b**2*x**3) + A*b**2*x**3*sqrt(-a/b)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x +
4*(-a/b)**(1/3))/(6*a**3*b + 6*a**2*b**2*x**3) - A*b**2*x**3*sqrt(-a/b)*log
(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a**3*b + 6*a**2*b**2*x
**3) - 2*B*a**2*x**(3/2)/(6*a**3*b + 6*a**2*b**2*x**3) - B*a**2*sqrt(-a/b)*
log(sqrt(x) - (-a/b)**(1/6))/(6*a**3*b + 6*a**2*b**2*x**3) + B*a**2*sqrt(-a
/b)*log(sqrt(x) + (-a/b)**(1/6))/(6*a**3*b + 6*a**2*b**2*x**3) + B*a**2*sqr
t(-a/b)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a**3*b + 6
*a**2*b**2*x**3) - B*a**2*sqrt(-a/b)*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*
(-a/b)**(1/3))/(6*a**3*b + 6*a**2*b**2*x**3) - B*a*b*x**3*sqrt(-a/b)*log(sq
rt(x) - (-a/b)**(1/6))/(6*a**3*b + 6*a**2*b**2*x**3) + B*a*b*x**3*sqrt(-a/b
)*log(sqrt(x) + (-a/b)**(1/6))/(6*a**3*b + 6*a**2*b**2*x**3) + B*a*b*x**3*s
qrt(-a/b)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(6*a**3*b +
 6*a**2*b**2*x**3) - B*a*b*x**3*sqrt(-a/b)*log(4*sqrt(x)*(-a/b)**(1/6) + 4*
x + 4*(-a/b)**(1/3))/(6*a**3*b + 6*a**2*b**2*x**3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^2}\,dx = -\frac{(Ba-Ab)x^{\frac{3}{2}}}{3\,(ab^2x^3+a^2b)} + \frac{(Ba+Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}ab}$$

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(B*a - A*b)*x^(3/2)/(a*b^2*x^3 + a^2*b) + 1/3*(B*a + A*b)*arctan(b*x^(
3/2)/sqrt(a*b))/(sqrt(a*b)*a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{(Ba+Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}ab} - \frac{Bax^{\frac{3}{2}}-Abx^{\frac{3}{2}}}{3\,(bx^3+a)ab}$$

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(B*a + A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a*b) - 1/3*(B*a*x^(3
/2) - A*b*x^(3/2))/((b*x^3 + a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 7.02 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^2}\,dx = \frac{B\,a^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)+A\,b^2\,x^3\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)+A\,a\,b\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)+A\,\sqrt{a}\,b^{3/2}\,x^{3/2}-B\,a^{3/2}\,\sqrt{b}\,x^{3/2}+B\,a}{3\,a^{5/2}\,b^{3/2}+3\,a^{3/2}\,b^{5/2}\,x^3}$$

```
[In] int((x^(1/2)*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (B*a^2*atan((b^(1/2)*x^(3/2))/a^(1/2)) + A*b^2*x^3*atan((b^(1/2)*x^(3/2))/a
^(1/2)) + A*a*b*atan((b^(1/2)*x^(3/2))/a^(1/2)) + A*a^(1/2)*b^(3/2)*x^(3/2)
 - B*a^(3/2)*b^(1/2)*x^(3/2) + B*a*b*x^3*atan((b^(1/2)*x^(3/2))/a^(1/2)))/(
3*a^(5/2)*b^(3/2) + 3*a^(3/2)*b^(5/2)*x^3)
```

3.167 $\int \frac{A+Bx^3}{\sqrt{x}(a+bx^3)^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 289

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^2}\,dx = \frac{(Ab-aB)\sqrt{x}}{3ab\,(a+bx^3)} - \frac{(5Ab+aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{11/6}b^{7/6}} + \frac{(5Ab+aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{11/6}b^{7/6}} + \frac{(5Ab+aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{11/6}b^{7/6}} - \frac{(5Ab+aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{11/6}b^{7/6}} + \frac{(5Ab+aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{11/6}b^{7/6}}$$

```
[Out] 1/9*(5*A*b+B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(7/6)+1/18*(5*A*
b+B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(7/6)+1/18*(5*
A*b+B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(7/6)-1/36*(5
*A*b+B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(11/6)/b^
(7/6)*3^(1/2)+1/36*(5*A*b+B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/2)
*x^(1/2))/a^(11/6)/b^(7/6)*3^(1/2)+1/3*(A*b-B*a)*x^(1/2)/a/b/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {468, 335, 215, 648, 632, 210, 642, 211}

$$\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^2}\,dx = -\frac{(aB+5Ab)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{11/6}b^{7/6}} + \frac{(aB+5Ab)\arctan\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{18a^{11/6}b^{7/6}} + \frac{(aB+5Ab)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{11/6}b^{7/6}} - \frac{(aB+5Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{11/6}b^{7/6}} + \frac{(aB+5Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{11/6}b^{7/6}} + \frac{\sqrt{x}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

```
[In] Int[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^2),x]
```

```
[Out] ((A*b - a*B)*Sqrt[x])/(3*a*b*(a + b*x^3)) - ((5*A*b + a*B)*ArcTan[Sqrt[3] -
 (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(11/6)*b^(7/6)) + ((5*A*b + a*B)*ArcTa
n[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(11/6)*b^(7/6)) + ((5*A*b +
 a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(9*a^(11/6)*b^(7/6)) - ((5*A*b + a
*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]
*a^(11/6)*b^(7/6)) + ((5*A*b + a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*S
qrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(11/6)*b^(7/6))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
```

```
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{(Ab-aB)\sqrt{x}}{3ab\,(a+bx^3)} + \frac{\left(\frac{5Ab}{2}+\frac{aB}{2}\right)\int \frac{1}{\sqrt{x}(a+bx^3)}\,dx}{3ab}$$

$$= \frac{(Ab-aB)\sqrt{x}}{3ab(a+bx^3)} + \frac{\left(2\left(\frac{5Ab}{2}+\frac{aB}{2}\right)\right)\text{Subst}\left(\int \frac{1}{a+bx^6}\,dx, x, \sqrt{x}\right)}{3ab}$$

$$= \frac{(Ab-aB)\sqrt{x}}{3ab(a+bx^3)} + \frac{(5Ab+aB)\text{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{9a^{11/6}b} + \frac{(5Ab+aB)\text{Subst}\left(\int \frac{\sqrt[6]{a}+\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{9a^{11/6}b} + \frac{(5Ab+aB)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{9a^{5/3}b}$$

$$= \frac{(Ab-aB)\sqrt{x}}{3ab(a+bx^3)} + \frac{(5Ab+aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{11/6}b^{7/6}} - \frac{(5Ab+aB)\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{12\sqrt{3}a^{11/6}b^{7/6}} + \frac{(5Ab+aB)\text{Subst}\left(\int \frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{12\sqrt{3}a^{11/6}b^{7/6}} + \frac{(5Ab+aB)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{36a^{5/3}b} + \frac{(5Ab+aB)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{36a^{5/3}b}$$

$$= \frac{(Ab-aB)\sqrt{x}}{3ab(a+bx^3)} + \frac{(5Ab+aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{11/6}b^{7/6}} - \frac{(5Ab+aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{11/6}b^{7/6}} + \frac{(5Ab+aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{11/6}b^{7/6}} + \frac{(5Ab+aB)\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{18\sqrt{3}a^{11/6}b^{7/6}} - \frac{(5Ab+aB)\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{18\sqrt{3}a^{11/6}b^{7/6}}$$

$$= \frac{(Ab-aB)\sqrt{x}}{3ab(a+bx^3)} - \frac{(5Ab+aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{11/6}b^{7/6}} + \frac{(5Ab+aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{11/6}b^{7/6}} + \frac{(5Ab+aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{11/6}b^{7/6}} - \frac{(5Ab+aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{11/6}b^{7/6}} + \frac{(5Ab+aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{11/6}b^{7/6}}$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.58

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^2}\,dx = \frac{-\frac{6a^{5/6}\sqrt[6]{b}(-Ab+aB)\sqrt{x}}{a+bx^3} + 2(5Ab+aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right) - (5Ab+aB)\arctan\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right) + \sqrt{3}(5Ab+aB)}{18a^{11/6}b^{7/6}}$$

```
[In] Integrate[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^2),x]
```

```
[Out] ((-6*a^(5/6)*b^(1/6)*(-(A*b) + a*B)*Sqrt[x])/(a + b*x^3) + 2*(5*A*b + a*B)*
ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)] - (5*A*b + a*B)*ArcTan[(a^(1/3) - b^(1/3)
*x)/(a^(1/6)*b^(1/6)*Sqrt[x])] + Sqrt[3]*(5*A*b + a*B)*ArcTanh[(Sqrt[3]*a^(
1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/(18*a^(11/6)*b^(7/6))
```

Maple [A] (verified)

Time = 4.27 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{(Ab-Ba)\sqrt{x}}{3ab(b\,x^3+a)}+\frac{(5Ab+Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6a}\right)}{3ab}$
default	$\frac{(Ab-Ba)\sqrt{x}}{3ab(b\,x^3+a)}+\frac{(5Ab+Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6a}\right)}{3ab}$

```
[In] int((B*x^3+A)/(b*x^3+a)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(A*b-B*a)*x^(1/2)/a/b/(b*x^3+a)+1/3*(5*A*b+B*a)/a/b*(1/3/a*(a/b)^(1/6)*
arctan(x^(1/2)/(a/b)^(1/6))-1/12/a*3^(1/2)*(a/b)^(1/6)*ln(3^(1/2)*(a/b)^(1/
6)*x^(1/2)-x-(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arctan(-3^(1/2)+2*x^(1/2)/(a/b)
^(1/6))+1/12/a*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(
1/3))+1/6/a*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1417 vs. 2(207) = 414.

Time = 0.32 (sec) , antiderivative size = 1417, normalized size of antiderivative = 4.90

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x^3+A)/(b*x^3+a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/36*(2*(a*b^2*x^3 + a^2*b)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b
^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 1562
5*A^6*b^6)/(a^11*b^7))^(1/6)*log(a^2*b*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^
2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a
*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6) + (B*a + 5*A*b)*sqrt(x)) - 2*(a*b^2
*x^3 + a^2*b)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*
B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^
11*b^7))^(1/6)*log(-a^2*b*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2
 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*
A^6*b^6)/(a^11*b^7))^(1/6) + (B*a + 5*A*b)*sqrt(x)) + (a*b^2*x^3 + a^2*b +
sqrt(-3)*(a*b^2*x^3 + a^2*b))*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4
```

```
*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15
625*A^6*b^6)/(a^11*b^7))^(1/6)*log((B*a + 5*A*b)*sqrt(x) + 1/2*(sqrt(-3)*a^
2*b + a^2*b)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B
^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^1
1*b^7))^(1/6)) - (a*b^2*x^3 + a^2*b + sqrt(-3)*(a*b^2*x^3 + a^2*b))*(-(B^6*
a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^
4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6)*log((B
*a + 5*A*b)*sqrt(x) - 1/2*(sqrt(-3)*a^2*b + a^2*b)*(-(B^6*a^6 + 30*A*B^5*a^
5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 1
8750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6)) - (a*b^2*x^3 + a^2*b -
 sqrt(-3)*(a*b^2*x^3 + a^2*b))*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^
4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 1
5625*A^6*b^6)/(a^11*b^7))^(1/6)*log((B*a + 5*A*b)*sqrt(x) + 1/2*(sqrt(-3)*a
^2*b - a^2*b)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*
B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^
11*b^7))^(1/6)) + (a*b^2*x^3 + a^2*b - sqrt(-3)*(a*b^2*x^3 + a^2*b))*(-(B^6
*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A
^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6)*log((
B*a + 5*A*b)*sqrt(x) - 1/2*(sqrt(-3)*a^2*b - a^2*b)*(-(B^6*a^6 + 30*A*B^5*a
^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 +
18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6)) - 12*(B*a - A*b)*sqrt
(x))/(a*b^2*x^3 + a^2*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1632 vs. $2(277) = 554$.

Time = 90.34 (sec) , antiderivative size = 1632, normalized size of antiderivative = 5.65

$$\int \frac{A + Bx^3}{\sqrt{x}\,(a + bx^3)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x**3+A)/(b*x**3+a)**2/x**(1/2),x)
```

```
[Out] Piecewise((zoo*(-2*A/(11*x**(11/2)) - 2*B/(5*x**(5/2))), Eq(a, 0) & Eq(b, 0
)), ((2*A*sqrt(x) + 2*B*x**(7/2)/7)/a**2, Eq(b, 0)), ((-2*A/(11*x**(11/2))
- 2*B/(5*x**(5/2)))/b**2, Eq(a, 0)), (12*A*a*b*sqrt(x)/(36*a**3*b + 36*a**2
*b**2*x**3) - 10*A*a*b*(-a/b)**(1/6)*log(sqrt(x) - (-a/b)**(1/6))/(36*a**3*
b + 36*a**2*b**2*x**3) + 10*A*a*b*(-a/b)**(1/6)*log(sqrt(x) + (-a/b)**(1/6)
)/(36*a**3*b + 36*a**2*b**2*x**3) - 5*A*a*b*(-a/b)**(1/6)*log(-4*sqrt(x)*(-
a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**3*b + 36*a**2*b**2*x**3) + 5*A*
a*b*(-a/b)**(1/6)*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*
a**3*b + 36*a**2*b**2*x**3) + 10*sqrt(3)*A*a*b*(-a/b)**(1/6)*atan(2*sqrt(3)
*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(36*a**3*b + 36*a**2*b**2*x**3) + 1
0*sqrt(3)*A*a*b*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sq
```

```
rt(3)/3)/(36*a**3*b + 36*a**2*b**2*x**3) - 10*A*b**2*x**3*(-a/b)**(1/6)*log
(sqrt(x) - (-a/b)**(1/6))/(36*a**3*b + 36*a**2*b**2*x**3) + 10*A*b**2*x**3*
(-a/b)**(1/6)*log(sqrt(x) + (-a/b)**(1/6))/(36*a**3*b + 36*a**2*b**2*x**3)
- 5*A*b**2*x**3*(-a/b)**(1/6)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)
**(1/3))/(36*a**3*b + 36*a**2*b**2*x**3) + 5*A*b**2*x**3*(-a/b)**(1/6)*log(
4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**3*b + 36*a**2*b**2*
x**3) + 10*sqrt(3)*A*b**2*x**3*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/
b)**(1/6)) - sqrt(3)/3)/(36*a**3*b + 36*a**2*b**2*x**3) + 10*sqrt(3)*A*b**2
*x**3*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(
36*a**3*b + 36*a**2*b**2*x**3) - 12*B*a**2*sqrt(x)/(36*a**3*b + 36*a**2*b**
2*x**3) - 2*B*a**2*(-a/b)**(1/6)*log(sqrt(x) - (-a/b)**(1/6))/(36*a**3*b +
36*a**2*b**2*x**3) + 2*B*a**2*(-a/b)**(1/6)*log(sqrt(x) + (-a/b)**(1/6))/(3
6*a**3*b + 36*a**2*b**2*x**3) - B*a**2*(-a/b)**(1/6)*log(-4*sqrt(x)*(-a/b)*
*(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**3*b + 36*a**2*b**2*x**3) + B*a**2*(-
a/b)**(1/6)*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**3*b
 + 36*a**2*b**2*x**3) + 2*sqrt(3)*B*a**2*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(
x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(36*a**3*b + 36*a**2*b**2*x**3) + 2*sqrt(
3)*B*a**2*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/
3)/(36*a**3*b + 36*a**2*b**2*x**3) - 2*B*a*b*x**3*(-a/b)**(1/6)*log(sqrt(x)
 - (-a/b)**(1/6))/(36*a**3*b + 36*a**2*b**2*x**3) + 2*B*a*b*x**3*(-a/b)**(1
/6)*log(sqrt(x) + (-a/b)**(1/6))/(36*a**3*b + 36*a**2*b**2*x**3) - B*a*b*x*
*3*(-a/b)**(1/6)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*
a**3*b + 36*a**2*b**2*x**3) + B*a*b*x**3*(-a/b)**(1/6)*log(4*sqrt(x)*(-a/b)
**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**3*b + 36*a**2*b**2*x**3) + 2*sqrt(3
)*B*a*b*x**3*(-a/b)**(1/6)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(
3)/3)/(36*a**3*b + 36*a**2*b**2*x**3) + 2*sqrt(3)*B*a*b*x**3*(-a/b)**(1/6)*
atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(36*a**3*b + 36*a**2*
b**2*x**3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^2}\,dx = -\frac{(Ba-Ab)\sqrt{x}}{3\left(ab^2x^3+a^2b\right)} + \frac{\frac{\sqrt{3}(Ba+5\,Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} - \frac{\sqrt{3}(Ba+5\,Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} + \frac{4\left(Bab^{\frac{1}{3}}+5\,Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} + \cdots}{36\,ab}$$

```
[In] integrate((B*x^3+A)/(b*x^3+a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(B*a - A*b)*sqrt(x)/(a*b^2*x^3 + a^2*b) + 1/36*(sqrt(3)*(B*a + 5*A*b)*
log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)
```

```
) - sqrt(3)*(B*a + 5*A*b)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x
+ a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(B*a*b^(1/3) + 5*A*b^(4/3))*arctan(b^(1/3)
*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2
*(B*a^(4/3)*b^(1/3) + 5*A*a^(1/3)*b^(4/3))*arctan((sqrt(3)*a^(1/6)*b^(1/6)
+ 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3)
)) + 2*(B*a^(4/3)*b^(1/3) + 5*A*a^(1/3)*b^(4/3))*arctan(-(sqrt(3)*a^(1/6)*b
^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*
b^(1/3))))/(a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^2}\,dx = \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba+5\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^2b^2} - \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba+5\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^2b^2} - \frac{Ba\sqrt{x}-Ab\sqrt{x}}{3\,(bx^3+a)ab} + \frac{\left(\left(ab^5\right)^{\frac{1}{6}}Ba+5\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,a^2b^2} + \frac{\left(\left(ab^5\right)^{\frac{1}{6}}Ba+5\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,a^2b^2} + \frac{\left(\left(ab^5\right)^{\frac{1}{6}}Ba+5\left(ab^5\right)^{\frac{1}{6}}Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9\,a^2b^2}$$

```
[In] integrate((B*x^3+A)/(b*x^3+a)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/36*sqrt(3)*((a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x)*
(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^2) - 1/36*sqrt(3)*((a*b^5)^(1/6)*B*a
+ 5*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/
(a^2*b^2) - 1/3*(B*a*sqrt(x) - A*b*sqrt(x))/((b*x^3 + a)*a*b) + 1/18*((a*b^
5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x)
)/(a/b)^(1/6))/(a^2*b^2) + 1/18*((a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*a
rctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^2*b^2) + 1/9*((a*b
^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 7.32 (sec) , antiderivative size = 1922, normalized size of antiderivative = 6.65

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^2}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x^3)/(x^(1/2)*(a + b*x^3)^2),x)
```

```
[Out] (atan(((((2*x^(1/2)*(625*A^4*b^5 + B^4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A^
3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4) - (2*(5*A*b + B*a)*(125*A^3*b^5 + B
^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))*
(5*A*b + B*a)*1i)/(18*(-a)^(11/6)*b^(7/6)) + (((2*x^(1/2)*(625*A^4*b^5 + B^
4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^
4) + (2*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^
2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))*(5*A*b + B*a)*1i)/(18*(-a)^(11/6)*b^(
7/6)))/((((2*x^(1/2)*(625*A^4*b^5 + B^4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A
^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4) - (2*(5*A*b + B*a)*(125*A^3*b^5 +
B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))
*(5*A*b + B*a))/(18*(-a)^(11/6)*b^(7/6)) - (((2*x^(1/2)*(625*A^4*b^5 + B^4*
a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4)
 + (2*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^2*
a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))*(5*A*b + B*a))/(18*(-a)^(11/6)*b^(7/6))
))*(5*A*b + B*a)*1i)/(9*(-a)^(11/6)*b^(7/6)) + (atan(((((3^(1/2)*1i)/2 - 1/
2)*(5*A*b + B*a)*((2*x^(1/2)*(625*A^4*b^5 + B^4*a^4*b + 150*A^2*B^2*a^2*b^3
 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4) - (2*((3^(1/2)*1i)/2 - 1/2
)*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^2*a^2*
b^3))/(27*(-a)^(23/6)*b^(7/6)))*1i)/(18*(-a)^(11/6)*b^(7/6)) + (((3^(1/2)*1
i)/2 - 1/2)*(5*A*b + B*a)*((2*x^(1/2)*(625*A^4*b^5 + B^4*a^4*b + 150*A^2*B^
2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4) + (2*((3^(1/2)*1i
)/2 - 1/2)*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A
*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))*1i)/(18*(-a)^(11/6)*b^(7/6)))/((((
3^(1/2)*1i)/2 - 1/2)*(5*A*b + B*a)*((2*x^(1/2)*(625*A^4*b^5 + B^4*a^4*b + 1
50*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4) - (2*((3
^(1/2)*1i)/2 - 1/2)*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 75*A^2*B*a*b
^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6))))/(18*(-a)^(11/6)*b^(7/6))
 - (((3^(1/2)*1i)/2 - 1/2)*(5*A*b + B*a)*((2*x^(1/2)*(625*A^4*b^5 + B^4*a^4
*b + 150*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4) +
(2*((3^(1/2)*1i)/2 - 1/2)*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 75*A^2
*B*a*b^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6))))/(18*(-a)^(11/6)*b^
(7/6))))*((3^(1/2)*1i)/2 - 1/2)*(5*A*b + B*a)*1i)/(9*(-a)^(11/6)*b^(7/6)) +
 (atan(((((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*((2*x^(1/2)*(625*A^4*b^5 + B^
4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^
4) - (2*((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 7
5*A^2*B*a*b^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))*1i)/(18*(-a)^(
```

```
11/6)*b^(7/6)) + (((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*((2*x^(1/2)*(625*A^4
*b^5 + B^4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2
))/(27*a^4) + (2*((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^
3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))*1i)/(
18*(-a)^(11/6)*b^(7/6)))/((((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*((2*x^(1/2)
*(625*A^4*b^5 + B^4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^
3*a^3*b^2))/(27*a^4) - (2*((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*(125*A^3*b^5
 + B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6
))))/(18*(-a)^(11/6)*b^(7/6)) - (((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*((2*x
^(1/2)*(625*A^4*b^5 + B^4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 2
0*A*B^3*a^3*b^2))/(27*a^4) + (2*((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*(125*A
^3*b^5 + B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*
b^(7/6))))/(18*(-a)^(11/6)*b^(7/6))))*((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*
1i)/(9*(-a)^(11/6)*b^(7/6)) + (x^(1/2)*(A*b - B*a))/(3*a*b*(a + b*x^3))
```

3.168 $\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 318

$$\begin{aligned}\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}\,dx &= -\frac{7Ab-aB}{3a^2b\sqrt{x}}+\frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)} \\ &+\frac{(7Ab-aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{13/6}b^{5/6}}-\frac{(7Ab-aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{13/6}b^{5/6}} \\ &-\frac{(7Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{13/6}b^{5/6}}-\frac{(7Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{13/6}b^{5/6}} \\ &+\frac{(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{13/6}b^{5/6}}\end{aligned}$$

```
[Out] -1/9*(7*A*b-B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(13/6)/b^(5/6)-1/18*(7*A
*b-B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(13/6)/b^(5/6)-1/18*(7
*A*b-B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(13/6)/b^(5/6)-1/36*(
7*A*b-B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(13/6)/b
^(5/6)*3^(1/2)+1/36*(7*A*b-B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/2
)*x^(1/2))/a^(13/6)/b^(5/6)*3^(1/2)+1/3*(-7*A*b+B*a)/a^2/b/x^(1/2)+1/3*(A*b
-B*a)/a/b/(b*x^3+a)/x^(1/2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {468, 331, 335, 301, 648, 632, 210, 642, 211}

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}\,dx=\frac{(7Ab-aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{13/6}b^{5/6}}-\frac{(7Ab-aB)\arctan\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{18a^{13/6}b^{5/6}}-\frac{(7Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{13/6}b^{5/6}}-\frac{(7Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{13/6}b^{5/6}}+\frac{(7Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{13/6}b^{5/6}}-\frac{7Ab-aB}{3a^2b\sqrt{x}}+\frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)}$$

```
[In] Int[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^2),x]
```

```
[Out] -1/3*(7*A*b - a*B)/(a^2*b*Sqrt[x]) + (A*b - a*B)/(3*a*b*Sqrt[x]*(a + b*x^3)
) + ((7*A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(13
/6)*b^(5/6)) - ((7*A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)]
)/(18*a^(13/6)*b^(5/6)) - ((7*A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])
/(9*a^(13/6)*b^(5/6)) - ((7*A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6
)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(13/6)*b^(5/6)) + ((7*A*b - a*B)*Log[
a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(13/6
)*b^(5/6))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
```

```
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{Ab-aB}{3ab\sqrt{x}\,(a+bx^3)} + \frac{\left(\frac{7Ab}{2}-\frac{aB}{2}\right)\int \frac{1}{x^{3/2}(a+bx^3)}\,dx}{3ab}$$

$$= -\frac{7Ab-aB}{3a^2b\sqrt{x}} + \frac{Ab-aB}{3ab\sqrt{x}\,(a+bx^3)} - \frac{(7Ab-aB)\int \frac{x^{3/2}}{a+bx^3}\,dx}{6a^2}$$

$$= -\frac{7Ab-aB}{3a^2b\sqrt{x}} + \frac{Ab-aB}{3ab\sqrt{x}\,(a+bx^3)} - \frac{(7Ab-aB)\text{Subst}\left(\int \frac{x^4}{a+bx^6}\,dx, x, \sqrt{x}\right)}{3a^2}$$

$$= -\frac{7Ab-aB}{3a^2b\sqrt{x}} + \frac{Ab-aB}{3ab\sqrt{x}\,(a+bx^3)} - \frac{(7Ab-aB)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2}+\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{9a^{13/6}b^{2/3}} - \frac{(7Ab-aB)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{9a^{13/6}b^{2/3}} - \frac{(7Ab-aB)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{9a^2b^{2/3}}$$

$$= -\frac{7Ab-aB}{3a^2b\sqrt{x}} + \frac{Ab-aB}{3ab\sqrt{x}\,(a+bx^3)} - \frac{(7Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{13/6}b^{5/6}} - \frac{(7Ab-aB)\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{12\sqrt{3}a^{13/6}b^{5/6}} + \frac{(7Ab-aB)\text{Subst}\left(\int \frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{12\sqrt{3}a^{13/6}b^{5/6}} - \frac{(7Ab-aB)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{36a^2b^{2/3}} - \frac{(7Ab-aB)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{36a^2b^{2/3}}$$

$$= -\frac{7Ab-aB}{3a^2b\sqrt{x}} + \frac{Ab-aB}{3ab\sqrt{x}\,(a+bx^3)} - \frac{(7Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{13/6}b^{5/6}} - \frac{(7Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{13/6}b^{5/6}} + \frac{(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{13/6}b^{5/6}} - \frac{(7Ab-aB)\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{18\sqrt{3}a^{13/6}b^{5/6}} + \frac{(7Ab-aB)\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{18\sqrt{3}a^{13/6}b^{5/6}}$$

$$= -\frac{7Ab-aB}{3a^2b\sqrt{x}} + \frac{Ab-aB}{3ab\sqrt{x}\,(a+bx^3)} + \frac{(7Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{13/6}b^{5/6}} - \frac{(7Ab-aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{13/6}b^{5/6}} - \frac{(7Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{13/6}b^{5/6}} - \frac{(7Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{13/6}b^{5/6}} + \frac{(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{13/6}b^{5/6}}$$

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.58

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}\,dx = \frac{\frac{6\sqrt[6]{a}(-6aA-7Abx^3+aBx^3)}{\sqrt{x}(a+bx^3)} + \frac{2(-7Ab+aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{b^{5/6}} + \frac{(7Ab-aB)\arctan\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{b^{5/6}} + \frac{\sqrt{3}\ldots}{\ldots}}{18a^{13/6}}$$

```
[In] Integrate[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^2),x]
```

```
[Out] ((6*a^(1/6)*(-6*a*A - 7*A*b*x^3 + a*B*x^3))/(Sqrt[x]*(a + b*x^3)) + (2*(-7*
A*b + a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/b^(5/6) + ((7*A*b - a*B)*ArcT
an[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])])/b^(5/6) + (Sqrt[3]*(7*
A*b - a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)]
)/b^(5/6))/(18*a^(13/6))
```

Maple [A] (verified)

Time = 4.50 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.68

method	result
derivativedivides	$-\frac{2\left(\frac{\left(\frac{Ab}{6}-\frac{Ba}{6}\right)x^{\frac{5}{2}}}{b\,x^{3}+a}+\left(\frac{7Ab}{6}-\frac{Ba}{6}\right)\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)\cdots\right.}{12a}\cdots\right.\right.}{a^{2}}$
default	$-\frac{2\left(\frac{\left(\frac{Ab}{6}-\frac{Ba}{6}\right)x^{\frac{5}{2}}}{b\,x^{3}+a}+\left(\frac{7Ab}{6}-\frac{Ba}{6}\right)\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right)\cdots\right.}{12a}\cdots\right.\right.}{a^{2}}$
risch	$-\frac{2A}{a^{2}\sqrt{x}}-\frac{\frac{2\left(\frac{Ab}{6}-\frac{Ba}{6}\right)x^{\frac{5}{2}}}{b\,x^{3}+a}+2\left(\frac{7Ab}{6}-\frac{Ba}{6}\right)\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\cdots\right.}{\cdots}\cdots\right.}{a^{2}}$

```
[In] int((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/a^2*((1/6*A*b-1/6*B*a)*x^(5/2)/(b*x^3+a)+(7/6*A*b-1/6*B*a)*(1/12/a*3^(1/
2)*(a/b)^(5/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+1/6/b/(a/b)^(1
/6)*arctan(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6))-1/12/a*3^(1/2)*(a/b)^(5/6)*ln(x+
3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/b/(a/b)^(1/6)*arctan(2*x^(1/2)
/(a/b)^(1/6)+3^(1/2))+1/3/b/(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))))-2*A/a
^2/x^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1788 vs. $2(226) = 452$.

Time = 0.37 (sec) , antiderivative size = 1788, normalized size of antiderivative = 5.62

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(2*(a^2*b*x^4 + a^3*x)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*
b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 1
17649*A^6*b^6)/(a^13*b^5))^(1/6)*log(a^11*b^4*(-(B^6*a^6 - 42*A*B^5*a^5*b +
 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 10084
2*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6) - (B^5*a^5 - 35*A*B^4*a^4
```

*b + 490*A^2*B^3*a^3*b^2 - 3430*A^3*B^2*a^2*b^3 + 12005*A^4*B*a*b^4 - 16807
*A^5*b^5)*sqrt(x)) - 2*(a^2*b*x^4 + a^3*x)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 73
5*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A
^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(1/6)*log(-a^11*b^4*(-(B^6*a^6 - 4
2*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*
a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6) - (B^5*a^5
 - 35*A*B^4*a^4*b + 490*A^2*B^3*a^3*b^2 - 3430*A^3*B^2*a^2*b^3 + 12005*A^4*
B*a*b^4 - 16807*A^5*b^5)*sqrt(x)) + (a^2*b*x^4 + a^3*x - sqrt(-3)*(a^2*b*x^
4 + a^3*x))*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^
3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a
^13*b^5))^(1/6)*log(1/2*(sqrt(-3)*a^11*b^4 + a^11*b^4)*(-(B^6*a^6 - 42*A*B^
5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^
4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6) - (B^5*a^5 - 35*
A*B^4*a^4*b + 490*A^2*B^3*a^3*b^2 - 3430*A^3*B^2*a^2*b^3 + 12005*A^4*B*a*b^
4 - 16807*A^5*b^5)*sqrt(x)) - (a^2*b*x^4 + a^3*x - sqrt(-3)*(a^2*b*x^4 + a^
3*x))*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*
b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^
5))^(1/6)*log(-1/2*(sqrt(-3)*a^11*b^4 + a^11*b^4)*(-(B^6*a^6 - 42*A*B^5*a^5
*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 1
00842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6) - (B^5*a^5 - 35*A*B^4
*a^4*b + 490*A^2*B^3*a^3*b^2 - 3430*A^3*B^2*a^2*b^3 + 12005*A^4*B*a*b^4 - 1
6807*A^5*b^5)*sqrt(x)) - (a^2*b*x^4 + a^3*x + sqrt(-3)*(a^2*b*x^4 + a^3*x))
*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 +
 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(
1/6)*log(1/2*(sqrt(-3)*a^11*b^4 - a^11*b^4)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 7
35*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*
A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6) - (B^5*a^5 - 35*A*B^4*a^4*b
 + 490*A^2*B^3*a^3*b^2 - 3430*A^3*B^2*a^2*b^3 + 12005*A^4*B*a*b^4 - 16807*A
^5*b^5)*sqrt(x)) + (a^2*b*x^4 + a^3*x + sqrt(-3)*(a^2*b*x^4 + a^3*x))*(-(B^
6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015
*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(1/6)*l
og(-1/2*(sqrt(-3)*a^11*b^4 - a^11*b^4)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^
2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B
*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6) - (B^5*a^5 - 35*A*B^4*a^4*b + 49
0*A^2*B^3*a^3*b^2 - 3430*A^3*B^2*a^2*b^3 + 12005*A^4*B*a*b^4 - 16807*A^5*b^
5)*sqrt(x)) - 12*((B*a - 7*A*b)*x^3 - 6*A*a)*sqrt(x))/(a^2*b*x^4 + a^3*x)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2200 vs. 2(299) = 598.

Time = 151.56 (sec) , antiderivative size = 2200, normalized size of antiderivative = 6.92

$$\int \frac{A+Bx^3}{x^{3/2}\,(a+bx^3)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x**3+A)/x**(3/2)/(b*x**3+a)**2,x)
```

```
[Out] Piecewise((zoo*(-2*A/(13*x**(13/2)) - 2*B/(7*x**(7/2))), Eq(a, 0) & Eq(b, 0
)), ((-2*A/sqrt(x) + 2*B*x**(5/2)/5)/a**2, Eq(b, 0)), ((-2*A/(13*x**(13/2))
 - 2*B/(7*x**(7/2)))/b**2, Eq(a, 0)), (-14*A*a*b*sqrt(x)*log(sqrt(x) - (-a/
b)**(1/6))/(36*a**3*b*sqrt(x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)*
*(1/6)) + 14*A*a*b*sqrt(x)*log(sqrt(x) + (-a/b)**(1/6))/(36*a**3*b*sqrt(x)*
(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) - 7*A*a*b*sqrt(x)*log(
-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**3*b*sqrt(x)*(-a/b)
**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) + 7*A*a*b*sqrt(x)*log(4*sqrt
(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**3*b*sqrt(x)*(-a/b)**(1/6)
 + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) - 14*sqrt(3)*A*a*b*sqrt(x)*atan(2*s
qrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(36*a**3*b*sqrt(x)*(-a/b)**(1
/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) - 14*sqrt(3)*A*a*b*sqrt(x)*atan(
2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(36*a**3*b*sqrt(x)*(-a/b)*
*(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) - 72*A*a*b*(-a/b)**(1/6)/(36*
a**3*b*sqrt(x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) - 14*A*
b**2*x**(7/2)*log(sqrt(x) - (-a/b)**(1/6))/(36*a**3*b*sqrt(x)*(-a/b)**(1/6)
 + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) + 14*A*b**2*x**(7/2)*log(sqrt(x) +
(-a/b)**(1/6))/(36*a**3*b*sqrt(x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a
/b)**(1/6)) - 7*A*b**2*x**(7/2)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/
b)**(1/3))/(36*a**3*b*sqrt(x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)*
*(1/6)) + 7*A*b**2*x**(7/2)*log(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(
1/3))/(36*a**3*b*sqrt(x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6
)) - 14*sqrt(3)*A*b**2*x**(7/2)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) -
sqrt(3)/3)/(36*a**3*b*sqrt(x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)*
*(1/6)) - 14*sqrt(3)*A*b**2*x**(7/2)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6
)) + sqrt(3)/3)/(36*a**3*b*sqrt(x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-
a/b)**(1/6)) - 84*A*b**2*x**3*(-a/b)**(1/6)/(36*a**3*b*sqrt(x)*(-a/b)**(1/6
) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) + 2*B*a**2*sqrt(x)*log(sqrt(x) - (
-a/b)**(1/6))/(36*a**3*b*sqrt(x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/
b)**(1/6)) - 2*B*a**2*sqrt(x)*log(sqrt(x) + (-a/b)**(1/6))/(36*a**3*b*sqrt(
x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) + B*a**2*sqrt(x)*lo
g(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**3*b*sqrt(x)*(-a/
b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) - B*a**2*sqrt(x)*log(4*sqr
t(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**3*b*sqrt(x)*(-a/b)**(1/6
) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) + 2*sqrt(3)*B*a**2*sqrt(x)*atan(2*
```

```
sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(36*a**3*b*sqrt(x)*(-a/b)**(
1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) + 2*sqrt(3)*B*a**2*sqrt(x)*atan
(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(36*a**3*b*sqrt(x)*(-a/b)
**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) + 2*B*a*b*x**(7/2)*log(sqrt(
x) - (-a/b)**(1/6))/(36*a**3*b*sqrt(x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2
)*(-a/b)**(1/6)) - 2*B*a*b*x**(7/2)*log(sqrt(x) + (-a/b)**(1/6))/(36*a**3*b
*sqrt(x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) + B*a*b*x**(7
/2)*log(-4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**3*b*sqrt(x
)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) - B*a*b*x**(7/2)*log
(4*sqrt(x)*(-a/b)**(1/6) + 4*x + 4*(-a/b)**(1/3))/(36*a**3*b*sqrt(x)*(-a/b)
**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) + 2*sqrt(3)*B*a*b*x**(7/2)*a
tan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) - sqrt(3)/3)/(36*a**3*b*sqrt(x)*(-a
/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) + 2*sqrt(3)*B*a*b*x**(7/2
)*atan(2*sqrt(3)*sqrt(x)/(3*(-a/b)**(1/6)) + sqrt(3)/3)/(36*a**3*b*sqrt(x)*
(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1/6)) + 12*B*a*b*x**3*(-a/b)
**(1/6)/(36*a**3*b*sqrt(x)*(-a/b)**(1/6) + 36*a**2*b**2*x**(7/2)*(-a/b)**(1
/6)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.75

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}\,dx = \frac{(Ba-7\,Ab)x^3-6\,Aa}{3\left(a^2bx^{\frac{7}{2}}+a^3\sqrt{x}\right)} - \frac{(Ba-7\,Ab)\left(\frac{\sqrt{3}\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}} - \frac{\sqrt{3}\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}} - \frac{2\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} - \frac{2\arctan\left(-\cdots\right.}{b^{\frac{2}{3}}\cdots}\right.}{36\,a^2}$$

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((B*a - 7*A*b)*x^3 - 6*A*a)/(a^2*b*x^(7/2) + a^3*sqrt(x)) - 1/36*(B*a -
 7*A*b)*(sqrt(3)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))
/(a^(1/6)*b^(5/6)) - sqrt(3)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)
*x + a^(1/3))/(a^(1/6)*b^(5/6)) - 2*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(
1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 2*ar
ctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/
(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 4*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(
1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))))/a^2
```

Giac [A] (verification not implemented)

none

Time = 0.68 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}\,dx = \frac{(Ba-7\,Ab)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,\left(ab^5\right)^{\frac{1}{6}}\,a^2} + \frac{(Ba-7\,Ab)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,\left(ab^5\right)^{\frac{1}{6}}\,a^2} + \frac{\left(Ba\left(\frac{a}{b}\right)^{\frac{5}{6}}-7\,Ab\left(\frac{a}{b}\right)^{\frac{5}{6}}\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9\,a^3} + \frac{Bax^3-7\,Abx^3-6\,Aa}{3\,\left(bx^{\frac{7}{2}}+a\sqrt{x}\right)a^2} - \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{5}{6}}Ba-7\left(ab^5\right)^{\frac{5}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^3b^5} + \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{5}{6}}Ba-7\left(ab^5\right)^{\frac{5}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^3b^5}$$

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/18*(B*a - 7*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/((
a*b^5)^(1/6)*a^2) + 1/18*(B*a - 7*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqr
t(x))/(a/b)^(1/6))/((a*b^5)^(1/6)*a^2) + 1/9*(B*a*(a/b)^(5/6) - 7*A*b*(a/b)
^(5/6))*arctan(sqrt(x)/(a/b)^(1/6))/a^3 + 1/3*(B*a*x^3 - 7*A*b*x^3 - 6*A*a)
/((b*x^(7/2) + a*sqrt(x))*a^2) - 1/36*sqrt(3)*((a*b^5)^(5/6)*B*a - 7*(a*b^5
)^(5/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b^5) +
 1/36*sqrt(3)*((a*b^5)^(5/6)*B*a - 7*(a*b^5)^(5/6)*A*b)*log(-sqrt(3)*sqrt(x
)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b^5)
```

Mupad [B] (verification not implemented)

Time = 7.17 (sec) , antiderivative size = 1757, normalized size of antiderivative = 5.53

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x^3)/(x^(3/2)*(a + b*x^3)^2),x)
```

```
[Out] (atan((((7*A*b - B*a)^2*(81*B^3*a^18*b^3 - 27783*A^3*a^15*b^6 - 1701*A*B^2*
a^17*b^4 + 11907*A^2*B*a^16*b^5 + (x^(1/2)*(7*A*b - B*a)*(23147208*A^2*a^17
*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/
```

6)))*1i)/((-a)^(13/3)*b^(5/3)) + ((7*A*b - B*a)^2*(27783*A^3*a^15*b^6 - 81*B^3*a^18*b^3 + 1701*A*B^2*a^17*b^4 - 11907*A^2*B*a^16*b^5 + (x^(1/2)*(7*A*b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/6)))*1i)/((-a)^(13/3)*b^(5/3)))/(((7*A*b - B*a)^2*(81*B^3*a^18*b^3 - 27783*A^3*a^15*b^6 - 1701*A*B^2*a^17*b^4 + 11907*A^2*B*a^16*b^5 + (x^(1/2)*(7*A*b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/6))))/((-a)^(13/3)*b^(5/3)) - ((7*A*b - B*a)^2*(27783*A^3*a^15*b^6 - 81*B^3*a^18*b^3 + 1701*A*B^2*a^17*b^4 - 11907*A^2*B*a^16*b^5 + (x^(1/2)*(7*A*b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/6))))/((-a)^(13/3)*b^(5/3))))*(7*A*b - B*a)*1i)/(9*(-a)^(13/6)*b^(5/6)) - ((2*A)/a + (x^3*(7*A*b - B*a))/(3*a^2))/(a*x^(1/2) + b*x^(7/2)) + (atan(((((3^(1/2)*1i)/2 - 1/2)^2*(7*A*b - B*a)^2*(81*B^3*a^18*b^3 - 27783*A^3*a^15*b^6 - 1701*A*B^2*a^17*b^4 + 11907*A^2*B*a^16*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(7*A*b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/6)))*1i)/((-a)^(13/3)*b^(5/3)) + (((3^(1/2)*1i)/2 - 1/2)^2*(7*A*b - B*a)^2*(27783*A^3*a^15*b^6 - 81*B^3*a^18*b^3 + 1701*A*B^2*a^17*b^4 - 11907*A^2*B*a^16*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(7*A*b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/6)))*1i)/((-a)^(13/3)*b^(5/3)))/((((3^(1/2)*1i)/2 - 1/2)^2*(7*A*b - B*a)^2*(81*B^3*a^18*b^3 - 27783*A^3*a^15*b^6 - 1701*A*B^2*a^17*b^4 + 11907*A^2*B*a^16*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(7*A*b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/6))))/((-a)^(13/3)*b^(5/3)) - (((3^(1/2)*1i)/2 - 1/2)^2*(7*A*b - B*a)^2*(27783*A^3*a^15*b^6 - 81*B^3*a^18*b^3 + 1701*A*B^2*a^17*b^4 - 11907*A^2*B*a^16*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(7*A*b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/6))))/((-a)^(13/3)*b^(5/3))))*((3^(1/2)*1i)/2 - 1/2)*(7*A*b - B*a)*1i)/(9*(-a)^(13/6)*b^(5/6)) + (atan(((((3^(1/2)*1i)/2 + 1/2)^2*(7*A*b - B*a)^2*(81*B^3*a^18*b^3 - 27783*A^3*a^15*b^6 - 1701*A*B^2*a^17*b^4 + 11907*A^2*B*a^16*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(7*A*b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/6)))*1i)/((-a)^(13/3)*b^(5/3)) + (((3^(1/2)*1i)/2 + 1/2)^2*(7*A*b - B*a)^2*(27783*A^3*a^15*b^6 - 81*B^3*a^18*b^3 + 1701*A*B^2*a^17*b^4 - 11907*A^2*B*a^16*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(7*A*b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/6)))*1i)/((-a)^(13/3)*b^(5/3)))/((((3^(1/2)*1i)/2 + 1/2)^2*(7*A*b - B*a)^2*(81*B^3*a^18*b^3 - 27783*A^3*a^15*b^6 - 1701*A*B^2*a^17*b^4 + 11907*A^2*B*a^16*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(7*A*b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/6))))/((-a)^(13/3)*b^(5/3)) - (((3^(1/2)*1i)/2 + 1/2)^2*(7*A*b - B*a)^2*(27783*A^3*a^15*b^6 - 81*B^3*a^18*b^3 + 1701*A*B^2*a^17*b^4 - 11907*A^2*B*a^16*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(7*A*b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/6))))/((-a)^(13/3)*b^(5/3))))*((3^(1/2)*1i)/2 + 1/2)*(

```
7*A*b - B*a)*1i)/(9*(-a)^(13/6)*b^(5/6))
```

3.169 $\int \frac{A+Bx^3}{x^{5/2}(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 96

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^2}\,dx = \frac{-3Ab+aB}{3a^2bx^{3/2}} + \frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)} - \frac{(3Ab-aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{5/2}\sqrt{b}}$$

```
[Out] 1/3*(-3*A*b+B*a)/a^2/b/x^(3/2)+1/3*(A*b-B*a)/a/b/x^(3/2)/(b*x^3+a)-1/3*(3*A
*b-B*a)*arctan(x^(3/2)*b^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {468, 331, 335, 281, 211}

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^2}\,dx = -\frac{(3Ab-aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{5/2}\sqrt{b}} - \frac{3Ab-aB}{3a^2bx^{3/2}} + \frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}$$

```
[In] Int[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^2),x]
```

```
[Out] -1/3*(3*A*b - a*B)/(a^2*b*x^(3/2)) + (A*b - a*B)/(3*a*b*x^(3/2)*(a + b*x^3)
) - ((3*A*b - a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*a^(5/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}+\frac{\left(\frac{9Ab}{2}-\frac{3aB}{2}\right)\int \frac{1}{x^{5/2}\left(a+bx^3\right)}\,dx}{3ab}\\
&= -\frac{3Ab-aB}{3a^2bx^{3/2}}+\frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}-\frac{(3Ab-aB)\int \frac{\sqrt{x}}{a+bx^3}\,dx}{2a^2}\\
&= -\frac{3Ab-aB}{3a^2bx^{3/2}}+\frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}-\frac{(3Ab-aB)\text{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx,x,\sqrt{x}\right)}{a^2}\\
&= -\frac{3Ab-aB}{3a^2bx^{3/2}}+\frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}-\frac{(3Ab-aB)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^{3/2}\right)}{3a^2}\\
&= -\frac{3Ab-aB}{3a^2bx^{3/2}}+\frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}-\frac{(3Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{5/2}\sqrt{b}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^2}\,dx=\frac{-2aA-3Abx^3+aBx^3}{3a^2x^{3/2}\left(a+bx^3\right)}+\frac{(-3Ab+aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{3a^{5/2}\sqrt{b}}$$

```
[In] Integrate[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^2),x]
```

```
[Out] (-2*a*A - 3*A*b*x^3 + a*B*x^3)/(3*a^2*x^(3/2)*(a + b*x^3)) + ((-3*A*b + a*B
)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*a^(5/2)*Sqrt[b])
```

Maple [A] (verified)

Time = 4.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$-\frac{2A}{3a^2x^{\frac{3}{2}}}-\frac{2\left(\frac{\left(\frac{Ab}{2}-\frac{Ba}{2}\right)x^{\frac{3}{2}}}{b\,x^3+a}+\frac{(3Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{3a^2}$	66
default	$-\frac{2A}{3a^2x^{\frac{3}{2}}}-\frac{2\left(\frac{\left(\frac{Ab}{2}-\frac{Ba}{2}\right)x^{\frac{3}{2}}}{b\,x^3+a}+\frac{(3Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{3a^2}$	66
risch	$-\frac{2A}{3a^2x^{\frac{3}{2}}}-\frac{\frac{2\left(\frac{Ab}{2}-\frac{Ba}{2}\right)x^{\frac{3}{2}}}{3\left(b\,x^3+a\right)}+\frac{(3Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}}}{a^2}$	67

```
[In] int((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*A/a^2/x^(3/2)-2/3/a^2*((1/2*A*b-1/2*B*a)*x^(3/2)/(b*x^3+a)+1/2*(3*A*b-
B*a)/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.42

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^2}\,dx = \left[\frac{((Bab-3\,Ab^2)x^5+(Ba^2-3\,Aab)x^2)\sqrt{-ab}\log\left(\frac{bx^3+2\sqrt{-ab}x^{\frac{3}{2}}-a}{bx^3+a}\right)-2\,(2\,Aa^2b-(Ba}{6\,(a^3b^2x^5+a^4bx^2)}\right.$$

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(((B*a*b - 3*A*b^2)*x^5 + (B*a^2 - 3*A*a*b)*x^2)*sqrt(-a*b)*log((b*x^3
 + 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)) - 2*(2*A*a^2*b - (B*a^2*b - 3*A*a
*b^2)*x^3)*sqrt(x))/(a^3*b^2*x^5 + a^4*b*x^2), 1/3*(((B*a*b - 3*A*b^2)*x^5
+ (B*a^2 - 3*A*a*b)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a) - (2*A*a^2*b
 - (B*a^2*b - 3*A*a*b^2)*x^3)*sqrt(x))/(a^3*b^2*x^5 + a^4*b*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((B*x**3+A)/x**(5/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^2}\,dx = \frac{(Ba-3\,Ab)x^3-2\,Aa}{3\left(a^2bx^{\frac{9}{2}}+a^3x^{\frac{3}{2}}\right)}+\frac{(Ba-3\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}a^2}$$

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((B*a - 3*A*b)*x^3 - 2*A*a)/(a^2*b*x^(9/2) + a^3*x^(3/2)) + 1/3*(B*a -
3*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^2}\,dx = \frac{(Ba-3\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}a^2} + \frac{Bax^3-3\,Abx^3-2\,Aa}{3\left(bx^{\frac{9}{2}}+ax^{\frac{3}{2}}\right)a^2}$$

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(B*a - 3*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(B*a*x^
3 - 3*A*b*x^3 - 2*A*a)/((b*x^(9/2) + a*x^(3/2))*a^2)
```

Mupad [B] (verification not implemented)

Time = 6.80 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.45

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^2}\,dx =$$

$$-\frac{2\,A\,a^{3/2}\,\sqrt{b}-B\,a^2\,x^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)+3\,A\,b^2\,x^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)+3\,A\,\sqrt{a}\,b^{3/2}\,x^3-B\,a^{3/2}\,\sqrt{b}\,x^3+3\,A\,a\,b}{3\,a^{7/2}\,\sqrt{b}\,x^{3/2}+3\,a^{5/2}\,b^{3/2}\,x^{9/2}}$$

```
[In] int((A + B*x^3)/(x^(5/2)*(a + b*x^3)^2),x)
```

```
[Out] -(2*A*a^(3/2)*b^(1/2) - B*a^2*x^(3/2)*atan((b^(1/2)*x^(3/2))/a^(1/2)) + 3*A
*b^2*x^(9/2)*atan((b^(1/2)*x^(3/2))/a^(1/2)) + 3*A*a^(1/2)*b^(3/2)*x^3 - B*
a^(3/2)*b^(1/2)*x^3 + 3*A*a*b*x^(3/2)*atan((b^(1/2)*x^(3/2))/a^(1/2)) - B*a
*b*x^(9/2)*atan((b^(1/2)*x^(3/2))/a^(1/2)))/(3*a^(7/2)*b^(1/2)*x^(3/2) + 3*
a^(5/2)*b^(3/2)*x^(9/2))
```

3.170 $\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 318

$$\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)^2}\,dx = -\frac{11Ab-5aB}{15a^2bx^{5/2}}+\frac{Ab-aB}{3abx^{5/2}(a+bx^3)} + \frac{(11Ab-5aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{17/6}\sqrt[6]{b}} - \frac{(11Ab-5aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{17/6}\sqrt[6]{b}} - \frac{(11Ab-5aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{17/6}\sqrt[6]{b}} + \frac{(11Ab-5aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{17/6}\sqrt[6]{b}} - \frac{(11Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{17/6}\sqrt[6]{b}}$$

```
[Out] 1/15*(-11*A*b+5*B*a)/a^2/b/x^(5/2)+1/3*(A*b-B*a)/a/b/x^(5/2)/(b*x^3+a)-1/9*
(11*A*b-5*B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(17/6)/b^(1/6)-1/18*(11*A*
b-5*B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(17/6)/b^(1/6)-1/18*(
11*A*b-5*B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(17/6)/b^(1/6)+1/
36*(11*A*b-5*B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(
17/6)/b^(1/6)*3^(1/2)-1/36*(11*A*b-5*B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1
/6)*3^(1/2)*x^(1/2))/a^(17/6)/b^(1/6)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {468, 331, 335, 215, 648, 632, 210, 642, 211}

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^2}\,dx = \frac{(11Ab-5aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{17/6}\sqrt[6]{b}} - \frac{(11Ab-5aB)\arctan\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{18a^{17/6}\sqrt[6]{b}} - \frac{(11Ab-5aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{17/6}\sqrt[6]{b}} + \frac{(11Ab-5aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{17/6}\sqrt[6]{b}} - \frac{(11Ab-5aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{17/6}\sqrt[6]{b}} - \frac{11Ab-5aB}{15a^2bx^{5/2}} + \frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)}$$

```
[In] Int[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^2),x]
```

```
[Out] -1/15*(11*A*b - 5*a*B)/(a^2*b*x^(5/2)) + (A*b - a*B)/(3*a*b*x^(5/2)*(a + b*
x^3)) + ((11*A*b - 5*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(1
8*a^(17/6)*b^(1/6)) - ((11*A*b - 5*a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x]
)/a^(1/6)])/(18*a^(17/6)*b^(1/6)) - ((11*A*b - 5*a*B)*ArcTan[(b^(1/6)*Sqrt[
x])/a^(1/6)])/(9*a^(17/6)*b^(1/6)) + ((11*A*b - 5*a*B)*Log[a^(1/3) - Sqrt[3
]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(17/6)*b^(1/6)) - ((1
1*A*b - 5*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/
(12*Sqrt[3]*a^(17/6)*b^(1/6))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
```

```
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{Ab-aB}{3abx^{5/2}(a+bx^3)} + \frac{\left(\frac{11Ab}{2}-\frac{5aB}{2}\right)\int \frac{1}{x^{7/2}(a+bx^3)}\,dx}{3ab}$$

$$= -\frac{11Ab-5aB}{15a^2bx^{5/2}} + \frac{Ab-aB}{3abx^{5/2}(a+bx^3)} - \frac{(11Ab-5aB)\int \frac{1}{\sqrt{x}(a+bx^3)}\,dx}{6a^2}$$

$$= -\frac{11Ab-5aB}{15a^2bx^{5/2}} + \frac{Ab-aB}{3abx^{5/2}(a+bx^3)} - \frac{(11Ab-5aB)\text{Subst}\left(\int \frac{1}{a+bx^6}\,dx, x, \sqrt{x}\right)}{3a^2}$$

$$= -\frac{11Ab-5aB}{15a^2bx^{5/2}} + \frac{Ab-aB}{3abx^{5/2}(a+bx^3)}$$
$$- \frac{(11Ab-5aB)\text{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{9a^{17/6}}$$
$$- \frac{(11Ab-5aB)\text{Subst}\left(\int \frac{\sqrt[6]{a}+\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{9a^{17/6}}$$
$$- \frac{(11Ab-5aB)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{9a^{8/3}}$$

$$= -\frac{11Ab-5aB}{15a^2bx^{5/2}} + \frac{Ab-aB}{3abx^{5/2}(a+bx^3)} - \frac{(11Ab-5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{17/6}\sqrt[6]{b}}$$
$$- \frac{(11Ab-5aB)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{36a^{8/3}}$$
$$- \frac{(11Ab-5aB)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{36a^{8/3}}$$
$$+ \frac{(11Ab-5aB)\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{12\sqrt{3}a^{17/6}\sqrt[6]{b}}$$
$$- \frac{(11Ab-5aB)\text{Subst}\left(\int \frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{12\sqrt{3}a^{17/6}\sqrt[6]{b}}$$

$$= -\frac{11Ab-5aB}{15a^2bx^{5/2}} + \frac{Ab-aB}{3abx^{5/2}(a+bx^3)} - \frac{(11Ab-5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{17/6}\sqrt[6]{b}} + \frac{(11Ab-5aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{17/6}\sqrt[6]{b}} - \frac{(11Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{17/6}\sqrt[6]{b}} - \frac{(11Ab-5aB)\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{18\sqrt{3}a^{17/6}\sqrt[6]{b}} + \frac{(11Ab-5aB)\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{18\sqrt{3}a^{17/6}\sqrt[6]{b}}$$

$$= -\frac{11Ab-5aB}{15a^2bx^{5/2}} + \frac{Ab-aB}{3abx^{5/2}(a+bx^3)} + \frac{(11Ab-5aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{17/6}\sqrt[6]{b}} - \frac{(11Ab-5aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{17/6}\sqrt[6]{b}} - \frac{(11Ab-5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{17/6}\sqrt[6]{b}} + \frac{(11Ab-5aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{17/6}\sqrt[6]{b}} - \frac{(11Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{17/6}\sqrt[6]{b}}$$

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.59

$$\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)^2}\,dx = \frac{\frac{6a^{5/6}(-6aA-11Abx^3+5aBx^3)}{x^{5/2}(a+bx^3)} + \frac{10(-11Ab+5aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{\sqrt[6]{b}} + \frac{5(11Ab-5aB)\arctan\left(\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}}\right)}{\sqrt[6]{b}}}{90a^{17/6}}$$

```
[In] Integrate[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^2),x]
```

```
[Out] ((6*a^(5/6)*(-6*a*A - 11*A*b*x^3 + 5*a*B*x^3))/(x^(5/2)*(a + b*x^3)) + (10*
(-11*A*b + 5*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/b^(1/6) + (5*(11*A*b -
 5*a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])])/b^(1/6) +
(5*Sqrt[3]*(-11*A*b + 5*a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(
1/3) + b^(1/3)*x)])/b^(1/6))/(90*a^(17/6))
```

Maple [A] (verified)

Time = 4.28 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.68

method	result
derivativedivides	$-\frac{2A}{5a^{2}x^{\frac{5}{2}}}-\frac{2\left(\frac{\left(\frac{Ab}{6}-\frac{Ba}{6}\right)\sqrt{x}}{b\,x^{3}+a}+\frac{(11Ab-5Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\text{arct}}{}\right.}{}\right.}{a^{2}}$
default	$-\frac{2A}{5a^{2}x^{\frac{5}{2}}}-\frac{2\left(\frac{\left(\frac{Ab}{6}-\frac{Ba}{6}\right)\sqrt{x}}{b\,x^{3}+a}+\frac{(11Ab-5Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\text{arct}}{}\right.}{}\right.}{a^{2}}$
risch	$-\frac{2A}{5a^{2}x^{\frac{5}{2}}}-\frac{\frac{2\left(\frac{Ab}{6}-\frac{Ba}{6}\right)\sqrt{x}}{b\,x^{3}+a}+\frac{(11Ab-5Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan}{}\right.}{3}}{a^{2}}$

```
[In] int((B*x^3+A)/x^(7/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*A/a^2/x^(5/2)-2/a^2*((1/6*A*b-1/6*B*a)*x^(1/2)/(b*x^3+a)+1/6*(11*A*b-5
*B*a)*(1/3/a*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))-1/12/a*3^(1/2)*(a/b)^(
1/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arctan
(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6))+1/12/a*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a
/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/
6)+3^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1463 vs. 2(232) = 464.

Time = 0.43 (sec) , antiderivative size = 1463, normalized size of antiderivative = 4.60

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/180*(10*(a^2*b*x^6 + a^3*x^3)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 11
34375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 -
 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6)*log(a^3*(-(15625*B^
6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*
b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^1
7*b))^(1/6) - (5*B*a - 11*A*b)*sqrt(x)) - 10*(a^2*b*x^6 + a^3*x^3)*(-(15625
*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a
^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(
a^17*b))^(1/6)*log(-a^3*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2
*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*
A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6) - (5*B*a - 11*A*b)*sqrt(x))
+ 5*(a^2*b*x^6 + a^3*x^3 + sqrt(-3)*(a^2*b*x^6 + a^3*x^3))*(-(15625*B^6*a^6
 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 +
 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))
^(1/6)*log(-(5*B*a - 11*A*b)*sqrt(x) + 1/2*(sqrt(-3)*a^3 + a^3)*(-(15625*B^
6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*
b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^1
7*b))^(1/6)) - 5*(a^2*b*x^6 + a^3*x^3 + sqrt(-3)*(a^2*b*x^6 + a^3*x^3))*(-(
15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*
B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b
^6)/(a^17*b))^(1/6)*log(-(5*B*a - 11*A*b)*sqrt(x) - 1/2*(sqrt(-3)*a^3 + a^3
)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500
*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*
A^6*b^6)/(a^17*b))^(1/6)) - 5*(a^2*b*x^6 + a^3*x^3 - sqrt(-3)*(a^2*b*x^6 +
a^3*x^3))*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 -
 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 +
1771561*A^6*b^6)/(a^17*b))^(1/6)*log(-(5*B*a - 11*A*b)*sqrt(x) + 1/2*(sqrt(
-3)*a^3 - a^3)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*
b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b
^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6)) + 5*(a^2*b*x^6 + a^3*x^3 - sqrt(-3)*
(a^2*b*x^6 + a^3*x^3))*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*
B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A
^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6)*log(-(5*B*a - 11*A*b)*sqrt(x)
 - 1/2*(sqrt(-3)*a^3 - a^3)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375
*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831
```

```
530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6)) - 12*((5*B*a - 11*A*b)*
x^3 - 6*A*a)*sqrt(x))/(a^2*b*x^6 + a^3*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx^3}{x^{7/2}\left(a + bx^3\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((B*x**3+A)/x**(7/2)/(b*x**3+a)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.98

$$\int \frac{A + Bx^3}{x^{7/2}\left(a + bx^3\right)^2}\,dx = \frac{(5\,Ba - 11\,Ab)x^3 - 6\,Aa}{15\left(a^2bx^{\frac{11}{2}} + a^3x^{\frac{5}{2}}\right)} + \frac{\frac{\sqrt{3}(5\,Ba-11\,Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} - \frac{\sqrt{3}(5\,Ba-11\,Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} + \frac{4\left(5\,Bab^{\frac{1}{3}}-11\,Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}}{36\,a^2}$$

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^2,x, algorithm="maxima")
[Out] 1/15*((5*B*a - 11*A*b)*x^3 - 6*A*a)/(a^2*b*x^(11/2) + a^3*x^(5/2)) + 1/36*(
sqrt(3)*(5*B*a - 11*A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x +
a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(5*B*a - 11*A*b)*log(-sqrt(3)*a^(1/6)*
b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(5*B*a*b^(1/3)
 - 11*A*b^(4/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(
1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(5*B*a^(4/3)*b^(1/3) - 11*A*a^(1/3)*b^(4/3)
)*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)
))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(5*B*a^(4/3)*b^(1/3) - 11*A*a^(1/3
)*b^(4/3))*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/
3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))))/a^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)^2}\,dx = \frac{\sqrt{3}\left(5\,(ab^5)^{\frac{1}{6}}\,Ba-11\,(ab^5)^{\frac{1}{6}}\,Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^3b} - \frac{\sqrt{3}\left(5\,(ab^5)^{\frac{1}{6}}\,Ba-11\,(ab^5)^{\frac{1}{6}}\,Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^3b} + \frac{Ba\sqrt{x}-Ab\sqrt{x}}{3\,(bx^3+a)a^2} + \frac{\left(5\,(ab^5)^{\frac{1}{6}}\,Ba-11\,(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,a^3b} + \frac{\left(5\,(ab^5)^{\frac{1}{6}}\,Ba-11\,(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{18\,a^3b} + \frac{\left(5\,(ab^5)^{\frac{1}{6}}\,Ba-11\,(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9\,a^3b} - \frac{2\,A}{5\,a^2x^{\frac{5}{2}}}$$

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/36*sqrt(3)*(5*(a*b^5)^(1/6)*B*a - 11*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(
x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b) - 1/36*sqrt(3)*(5*(a*b^5)^(1/6)*B
*a - 11*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/
3))/(a^3*b) + 1/3*(B*a*sqrt(x) - A*b*sqrt(x))/((b*x^3 + a)*a^2) + 1/18*(5*(
a*b^5)^(1/6)*B*a - 11*(a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sq
rt(x))/(a/b)^(1/6))/(a^3*b) + 1/18*(5*(a*b^5)^(1/6)*B*a - 11*(a*b^5)^(1/6)*
A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^3*b) + 1/9*(
5*(a*b^5)^(1/6)*B*a - 11*(a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^
3*b) - 2/5*A/(a^2*x^(5/2))
```

Mupad [B] (verification not implemented)

Time = 7.18 (sec) , antiderivative size = 2080, normalized size of antiderivative = 6.54

$$\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)^2}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x^3)/(x^(7/2)*(a + b*x^3)^2),x)
```

```
[Out] - ((2*A)/(5*a) + (x^3*(11*A*b - 5*B*a))/(15*a^2))/(a*x^(5/2) + b*x^(11/2))
- (atan((((x^(1/2)*(21346578*A^4*a^10*b^9 + 911250*B^4*a^14*b^5 + 26462700*
A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6 - 38811960*A^3*B*a^11*b^8) - ((11
```

*A*b - 5*B*a)*(34930764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 47632860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a)*1i)/(18*(-a)^(17/6)*b^(1/6)) + ((x^(1/2)*(21346578*A^4*a^10*b^9 + 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6 - 38811960*A^3*B*a^11*b^8) + ((11*A*b - 5*B*a)*(34930764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 47632860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a)*1i)/(18*(-a)^(17/6)*b^(1/6)))/(((x^(1/2)*(21346578*A^4*a^10*b^9 + 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6 - 38811960*A^3*B*a^11*b^8) - ((11*A*b - 5*B*a)*(34930764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 47632860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a))/(18*(-a)^(17/6)*b^(1/6)) - ((x^(1/2)*(21346578*A^4*a^10*b^9 + 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6 - 38811960*A^3*B*a^11*b^8) + ((11*A*b - 5*B*a)*(34930764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 47632860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a))/(18*(-a)^(17/6)*b^(1/6))))*(11*A*b - 5*B*a)*1i)/(9*(-a)^(17/6)*b^(1/6)) - (atan(((((3^(1/2)*1i)/2 - 1/2)*(x^(1/2)*(21346578*A^4*a^10*b^9 + 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6 - 38811960*A^3*B*a^11*b^8) - (((3^(1/2)*1i)/2 - 1/2)*(11*A*b - 5*B*a)*(34930764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 47632860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a)*1i)/(18*(-a)^(17/6)*b^(1/6)) + (((3^(1/2)*1i)/2 - 1/2)*(x^(1/2)*(21346578*A^4*a^10*b^9 + 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6 - 38811960*A^3*B*a^11*b^8) + (((3^(1/2)*1i)/2 - 1/2)*(11*A*b - 5*B*a)*(34930764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 47632860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a)*1i)/(18*(-a)^(17/6)*b^(1/6)))/((((3^(1/2)*1i)/2 - 1/2)*(x^(1/2)*(21346578*A^4*a^10*b^9 + 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6 - 38811960*A^3*B*a^11*b^8) - (((3^(1/2)*1i)/2 - 1/2)*(11*A*b - 5*B*a)*(34930764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 47632860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a))/(18*(-a)^(17/6)*b^(1/6)) - (((3^(1/2)*1i)/2 - 1/2)*(x^(1/2)*(21346578*A^4*a^10*b^9 + 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6 - 38811960*A^3*B*a^11*b^8) + (((3^(1/2)*1i)/2 - 1/2)*(11*A*b - 5*B*a)*(34930764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 47632860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a))/(18*(-a)^(17/6)*b^(1/6))))*((3^(1/2)*1i)/2 - 1/2)*(11*A*b - 5*B*a)*1i)/(9*(-a)^(17/6)*b^(1/6)) - (atan(((((3^(1/2)*1i)/2 + 1/2)*(x^(1/2)*(21346578*A^4*a^10*b^9 + 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6 - 38811960*A^3*B*a^11*b^8) - (((3^(1/2)*1i)/2 + 1/2)*(11*A*b - 5*B*a)*(34930764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 47632860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a)*1i)/(18*(-a)^(17/6)*b^(1/6)) + (((3^(1/2)*1i)/2 + 1/2)*(x^(1/2)*(21346578*A^4*a^10*b^9 + 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6 - 38811960*A^3*B*a^11*b^8) + (((3^(1/2)*1i)/2 + 1/2)*(11*A*b - 5*B*a)*(3493

0764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 476328
60*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a)*1i)/(18*(-a)
^(17/6)*b^(1/6)))/((((3^(1/2)*1i)/2 + 1/2)*(x^(1/2)*(21346578*A^4*a^10*b^9
+ 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6
- 38811960*A^3*B*a^11*b^8) - (((3^(1/2)*1i)/2 + 1/2)*(11*A*b - 5*B*a)*(3493
0764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 476328
60*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a))/(18*(-a)^(1
7/6)*b^(1/6)) - (((3^(1/2)*1i)/2 + 1/2)*(x^(1/2)*(21346578*A^4*a^10*b^9 + 9
11250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6 - 3
8811960*A^3*B*a^11*b^8) + (((3^(1/2)*1i)/2 + 1/2)*(11*A*b - 5*B*a)*(3493076
4*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 47632860*
A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a))/(18*(-a)^(17/6
)*b^(1/6))))*((3^(1/2)*1i)/2 + 1/2)*(11*A*b - 5*B*a)*1i)/(9*(-a)^(17/6)*b^(
1/6))

3.171 $\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3} \, dx = \frac{(Ab-aB)x^{9/2}}{6ab(a+bx^3)^2} - \frac{(Ab+3aB)x^{3/2}}{12ab^2(a+bx^3)} + \frac{(Ab+3aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{3/2}b^{5/2}}$$

```
[Out] 1/6*(A*b-B*a)*x^(9/2)/a/b/(b*x^3+a)^2-1/12*(A*b+3*B*a)*x^(3/2)/a/b^2/(b*x^3
+a)+1/12*(A*b+3*B*a)*arctan(x^(3/2)*b^(1/2)/a^(1/2))/a^(3/2)/b^(5/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {468, 294, 335, 281, 211}

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3} \, dx = \frac{(3aB+Ab)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{3/2}b^{5/2}} - \frac{x^{3/2}(3aB+Ab)}{12ab^2(a+bx^3)} + \frac{x^{9/2}(Ab-aB)}{6ab(a+bx^3)^2}$$

```
[In] Int[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^(9/2))/(6*a*b*(a + b*x^3)^2) - ((A*b + 3*a*B)*x^(3/2))/(12*a
*b^2*(a + b*x^3)) + ((A*b + 3*a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a
^(3/2)*b^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(Ab-aB)x^{9/2}}{6ab\left(a+bx^3\right)^2}+\frac{\left(\frac{3Ab}{2}+\frac{9aB}{2}\right)\int\frac{x^{7/2}}{\left(a+bx^3\right)^2}\,dx}{6ab}\\
&= \frac{(Ab-aB)x^{9/2}}{6ab\left(a+bx^3\right)^2}-\frac{(Ab+3aB)x^{3/2}}{12ab^2\left(a+bx^3\right)}+\frac{(Ab+3aB)\int\frac{\sqrt{x}}{a+bx^3}\,dx}{8ab^2}\\
&= \frac{(Ab-aB)x^{9/2}}{6ab\left(a+bx^3\right)^2}-\frac{(Ab+3aB)x^{3/2}}{12ab^2\left(a+bx^3\right)}+\frac{(Ab+3aB)\text{Subst}\left(\int\frac{x^2}{a+bx^6}\,dx,x,\sqrt{x}\right)}{4ab^2}\\
&= \frac{(Ab-aB)x^{9/2}}{6ab\left(a+bx^3\right)^2}-\frac{(Ab+3aB)x^{3/2}}{12ab^2\left(a+bx^3\right)}+\frac{(Ab+3aB)\text{Subst}\left(\int\frac{1}{a+bx^2}\,dx,x,x^{3/2}\right)}{12ab^2}
\end{aligned}$$

$$= \frac{(Ab-aB)x^{9/2}}{6ab\,(a+bx^3)^2} - \frac{(Ab+3aB)x^{3/2}}{12ab^2\,(a+bx^3)} + \frac{(Ab+3aB)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{3/2}b^{5/2}}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{x^{3/2}(aAb+3a^2B-Ab^2x^3+5abBx^3)}{12ab^2\,(a+bx^3)^2} + \frac{(Ab+3aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{3/2}b^{5/2}}$$

```
[In] Integrate[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] -1/12*(x^(3/2)*(a*A*b + 3*a^2*B - A*b^2*x^3 + 5*a*b*B*x^3))/(a*b^2*(a + b*x
^3)^2) + ((A*b + 3*a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a^(3/2)*b^(5
/2))
```

Maple [A] (verified)

Time = 4.38 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\frac{(Ab-5Ba)x^{\frac{9}{2}}}{12ab}-\frac{(Ab+3Ba)x^{\frac{3}{2}}}{12b^2}}{(b\,x^3+a)^2}+\frac{(Ab+3Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12b^2a\sqrt{ab}}$	81
default	$\frac{\frac{(Ab-5Ba)x^{\frac{9}{2}}}{12ab}-\frac{(Ab+3Ba)x^{\frac{3}{2}}}{12b^2}}{(b\,x^3+a)^2}+\frac{(Ab+3Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12b^2a\sqrt{ab}}$	81

```
[In] int(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(1/8*(A*b-5*B*a)/a/b*x^(9/2)-1/8*(A*b+3*B*a)/b^2*x^(3/2))/(b*x^3+a)^2+1
/12*(A*b+3*B*a)/b^2/a/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 314, normalized size of antiderivative = 3.02

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \left[-\frac{((3\,Bab^2+Ab^3)x^6+3\,Ba^3+Aa^2b+2\,(3\,Ba^2b+Aab^2)x^3)\sqrt{-ab}\log\left(\frac{bx^3-2\,\sqrt{-ab}}{bx^3+a}\right.}{24\,(a^2b^5x^6+2\,a^3b^4x^3+a^4}\right.$$

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/24*(((3*B*a*b^2 + A*b^3)*x^6 + 3*B*a^3 + A*a^2*b + 2*(3*B*a^2*b + A*a*b
^2)*x^3)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)) + 2
*((5*B*a^2*b^2 - A*a*b^3)*x^4 + (3*B*a^3*b + A*a^2*b^2)*x)*sqrt(x))/(a^2*b^
5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3), 1/12*(((3*B*a*b^2 + A*b^3)*x^6 + 3*B*a^3
+ A*a^2*b + 2*(3*B*a^2*b + A*a*b^2)*x^3)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)
/a) - ((5*B*a^2*b^2 - A*a*b^3)*x^4 + (3*B*a^3*b + A*a^2*b^2)*x)*sqrt(x))/(a
^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**(7/2)*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.92

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{(5\,Bab-Ab^2)x^{\frac{9}{2}}+(3\,Ba^2+Aab)x^{\frac{3}{2}}}{12\,(ab^4x^6+2\,a^2b^3x^3+a^3b^2)}+\frac{(3\,Ba+Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\,\sqrt{ab}ab^2}$$

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/12*((5*B*a*b - A*b^2)*x^(9/2) + (3*B*a^2 + A*a*b)*x^(3/2))/(a*b^4*x^6 +
2*a^2*b^3*x^3 + a^3*b^2) + 1/12*(3*B*a + A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(
sqrt(a*b)*a*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.81

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(3\,Ba+Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\,\sqrt{ab}ab^2}-\frac{5\,Babx^{\frac{9}{2}}-Ab^2x^{\frac{9}{2}}+3\,Ba^2x^{\frac{3}{2}}+Aabx^{\frac{3}{2}}}{12\,(bx^3+a)^2ab^2}$$

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/12*(3*B*a + A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a*b^2) - 1/12*(5*
B*a*b*x^(9/2) - A*b^2*x^(9/2) + 3*B*a^2*x^(3/2) + A*a*b*x^(3/2))/((b*x^3 +
a)^2*a*b^2)
```

Mupad [B] (verification not implemented)

Time = 7.02 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.28

$$\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{\mathrm{atan}\left(\frac{9\,b^{3/2}\,x^{3/2}\,\left(A^2\,b^2+6\,A\,B\,a\,b+9\,B^2\,a^2\right)}{\sqrt{a}\,\left(9\,A\,b^2+27\,B\,a\,b\right)\,\left(A\,b+3\,B\,a\right)}\right)\,\left(A\,b+3\,B\,a\right)}{12\,a^{3/2}\,b^{5/2}} - \frac{\frac{x^{3/2}\,\left(A\,b+3\,B\,a\right)}{12\,b^2}-\frac{x^{9/2}\,\left(A\,b-5\,B\,a\right)}{12\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}$$

```
[In] int((x^(7/2)*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (atan((9*b^(3/2)*x^(3/2)*(A^2*b^2 + 9*B^2*a^2 + 6*A*B*a*b))/(a^(1/2)*(9*A*b
^2 + 27*B*a*b)*(A*b + 3*B*a)))*(A*b + 3*B*a))/(12*a^(3/2)*b^(5/2)) - ((x^(3
/2)*(A*b + 3*B*a))/(12*b^2) - (x^(9/2)*(A*b - 5*B*a))/(12*a*b))/(a^2 + b^2*
x^6 + 2*a*b*x^3)
```

3.172 $\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^3} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 327

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^3} \, dx = \frac{(Ab-aB)x^{7/2}}{6ab\,(a+bx^3)^2} - \frac{(5Ab+7aB)\sqrt{x}}{36ab^2\,(a+bx^3)} - \frac{(5Ab+7aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{11/6}b^{13/6}} + \frac{(5Ab+7aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{11/6}b^{13/6}} + \frac{(5Ab+7aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{11/6}b^{13/6}} - \frac{(5Ab+7aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{11/6}b^{13/6}} + \frac{(5Ab+7aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{11/6}b^{13/6}}$$

```
[Out] 1/6*(A*b-B*a)*x^(7/2)/a/b/(b*x^3+a)^2+1/108*(5*A*b+7*B*a)*arctan(b^(1/6)*x^
(1/2)/a^(1/6))/a^(11/6)/b^(13/6)+1/216*(5*A*b+7*B*a)*arctan(-3^(1/2)+2*b^(1
/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(13/6)+1/216*(5*A*b+7*B*a)*arctan(3^(1/2)+2
*b^(1/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(13/6)-1/432*(5*A*b+7*B*a)*ln(a^(1/3)+
b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(11/6)/b^(13/6)*3^(1/2)+1/432*
(5*A*b+7*B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(11/6
)/b^(13/6)*3^(1/2)-1/36*(5*A*b+7*B*a)*x^(1/2)/a/b^2/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {468, 294, 335, 215, 648, 632, 210, 642, 211}

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^3}\,dx=-\frac{(7aB+5Ab)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{11/6}b^{13/6}}+\frac{(7aB+5Ab)\arctan\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{216a^{11/6}b^{13/6}}+\frac{(7aB+5Ab)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{11/6}b^{13/6}}-\frac{(7aB+5Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{11/6}b^{13/6}}+\frac{(7aB+5Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{11/6}b^{13/6}}-\frac{\sqrt{x}(7aB+5Ab)}{36ab^2(a+bx^3)}+\frac{x^{7/2}(Ab-aB)}{6ab(a+bx^3)^2}$$

```
[In] Int[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^(7/2))/(6*a*b*(a + b*x^3)^2) - ((5*A*b + 7*a*B)*Sqrt[x])/(36
*a*b^2*(a + b*x^3)) - ((5*A*b + 7*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])
/a^(1/6)])/(216*a^(11/6)*b^(13/6)) + ((5*A*b + 7*a*B)*ArcTan[Sqrt[3] + (2*b
^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(11/6)*b^(13/6)) + ((5*A*b + 7*a*B)*ArcTan
[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(11/6)*b^(13/6)) - ((5*A*b + 7*a*B)*Log
[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(11
/6)*b^(13/6)) + ((5*A*b + 7*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt
[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(11/6)*b^(13/6))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
```

```
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{(Ab-aB)x^{7/2}}{6ab\,(a+bx^3)^2} + \frac{\left(\frac{5Ab}{2}+\frac{7aB}{2}\right)\int\frac{x^{5/2}}{(a+bx^3)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^{7/2}}{6ab\,(a+bx^3)^2} - \frac{(5Ab+7aB)\sqrt{x}}{36ab^2\,(a+bx^3)} + \frac{(5Ab+7aB)\int\frac{1}{\sqrt{x}(a+bx^3)}\,dx}{72ab^2}$$

$$= \frac{(Ab-aB)x^{7/2}}{6ab\,(a+bx^3)^2} - \frac{(5Ab+7aB)\sqrt{x}}{36ab^2\,(a+bx^3)} + \frac{(5Ab+7aB)\text{Subst}\left(\int\frac{1}{a+bx^6}\,dx,x,\sqrt{x}\right)}{36ab^2}$$

$$= \frac{(Ab-aB)x^{7/2}}{6ab\,(a+bx^3)^2} - \frac{(5Ab+7aB)\sqrt{x}}{36ab^2\,(a+bx^3)} + \frac{(5Ab+7aB)\text{Subst}\left(\int\frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{108a^{11/6}b^2} + \frac{(5Ab+7aB)\text{Subst}\left(\int\frac{\sqrt[6]{a}+\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{108a^{11/6}b^2} + \frac{(5Ab+7aB)\text{Subst}\left(\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{108a^{5/3}b^2}$$

$$= \frac{(Ab-aB)x^{7/2}}{6ab\,(a+bx^3)^2} - \frac{(5Ab+7aB)\sqrt{x}}{36ab^2\,(a+bx^3)} + \frac{(5Ab+7aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{11/6}b^{13/6}} - \frac{(5Ab+7aB)\text{Subst}\left(\int\frac{-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{144\sqrt{3}a^{11/6}b^{13/6}} + \frac{(5Ab+7aB)\text{Subst}\left(\int\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{144\sqrt{3}a^{11/6}b^{13/6}} + \frac{(5Ab+7aB)\text{Subst}\left(\int\frac{1}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{432a^{5/3}b^2} + \frac{(5Ab+7aB)\text{Subst}\left(\int\frac{1}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{432a^{5/3}b^2}$$

$$= \frac{(Ab-aB)x^{7/2}}{6ab\,(a+bx^3)^2} - \frac{(5Ab+7aB)\sqrt{x}}{36ab^2\,(a+bx^3)} + \frac{(5Ab+7aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{11/6}b^{13/6}} - \frac{(5Ab+7aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{11/6}b^{13/6}} + \frac{(5Ab+7aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{11/6}b^{13/6}} + \frac{(5Ab+7aB)\text{Subst}\left(\int\frac{1}{-\frac{1}{3}-x^2}\,dx,x,1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{216\sqrt{3}a^{11/6}b^{13/6}} - \frac{(5Ab+7aB)\text{Subst}\left(\int\frac{1}{-\frac{1}{3}-x^2}\,dx,x,1+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{216\sqrt{3}a^{11/6}b^{13/6}}$$

$$= \frac{(Ab-aB)x^{7/2}}{6ab\,(a+bx^3)^2} - \frac{(5Ab+7aB)\sqrt{x}}{36ab^2\,(a+bx^3)} - \frac{(5Ab+7aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{11/6}b^{13/6}} + \frac{(5Ab+7aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{11/6}b^{13/6}} + \frac{(5Ab+7aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{11/6}b^{13/6}} - \frac{(5Ab+7aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{11/6}b^{13/6}} + \frac{(5Ab+7aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{11/6}b^{13/6}}$$

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.59

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{-\frac{6a^{5/6}\sqrt[6]{b}\sqrt{x}(7a^2B-Ab^2x^3+ab(5A+13Bx^3))}{(a+bx^3)^2} + 2(5Ab+7aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right) - (5Ab+7aB}{216a^{11/6}b^{13/6}}$$

```
[In] Integrate[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((-6*a^(5/6)*b^(1/6)*Sqrt[x]*(7*a^2*B - A*b^2*x^3 + a*b*(5*A + 13*B*x^3)))/
(a + b*x^3)^2 + 2*(5*A*b + 7*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)] - (5*A*
b + 7*a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])] + Sqrt[3
]*(5*A*b + 7*a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1
/3)*x)])/(216*a^(11/6)*b^(13/6))
```

Maple [A] (verified)

Time = 4.36 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{\frac{(Ab-13Ba)x^{\frac{7}{2}}}{36ab}-\frac{(5Ab+7Ba)\sqrt{x}}{36b^2}}{(bx^3+a)^2}+\frac{(5Ab+7Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\right.}{}$
default	$\frac{\frac{(Ab-13Ba)x^{\frac{7}{2}}}{36ab}-\frac{(5Ab+7Ba)\sqrt{x}}{36b^2}}{(bx^3+a)^2}+\frac{(5Ab+7Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\right.}{}$

```
[In] int(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/72*(A*b-13*B*a)/a/b*x^(7/2)-1/72*(5*A*b+7*B*a)/b^2*x^(1/2))/(b*x^3+a)^
2+1/36*(5*A*b+7*B*a)/b^2/a*(1/3/a*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))-1
/12/a*3^(1/2)*(a/b)^(1/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+1/6
/a*(a/b)^(1/6)*arctan(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6))+1/12/a*3^(1/2)*(a/b)^
(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arcta
n(2*x^(1/2)/(a/b)^(1/6)+3^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1618 vs. 2(241) = 482.

Time = 0.40 (sec) , antiderivative size = 1618, normalized size of antiderivative = 4.95

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/432*(2*(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)*(-(117649*B^6*a^6 + 504210*A
*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B
^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)*log(a^2
*b^2*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 8575
00*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^
6*b^6)/(a^11*b^13))^(1/6) + (7*B*a + 5*A*b)*sqrt(x)) - 2*(a*b^4*x^6 + 2*a^2
*b^3*x^3 + a^3*b^2)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4
*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a
*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)*log(-a^2*b^2*(-(117649*B^6*a^6 + 5
04210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 45937
```

```
5*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)
+ (7*B*a + 5*A*b)*sqrt(x)) + (a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2 + sqrt(-3
)*(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2))*(-(117649*B^6*a^6 + 504210*A*B^5*a
^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2
*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)*log((7*B*a +
5*A*b)*sqrt(x) + 1/2*(sqrt(-3)*a^2*b^2 + a^2*b^2)*(-(117649*B^6*a^6 + 50421
0*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^
4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)) - (
a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2 + sqrt(-3)*(a*b^4*x^6 + 2*a^2*b^3*x^3 +
 a^3*b^2))*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2
+ 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15
625*A^6*b^6)/(a^11*b^13))^(1/6)*log((7*B*a + 5*A*b)*sqrt(x) - 1/2*(sqrt(-3)
*a^2*b^2 + a^2*b^2)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4
*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a
*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)) - (a*b^4*x^6 + 2*a^2*b^3*x^3 + a^
3*b^2 - sqrt(-3)*(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2))*(-(117649*B^6*a^6 +
 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459
375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6
)*log((7*B*a + 5*A*b)*sqrt(x) + 1/2*(sqrt(-3)*a^2*b^2 - a^2*b^2)*(-(117649*
B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*
b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^
13))^(1/6)) + (a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2 - sqrt(-3)*(a*b^4*x^6 +
2*a^2*b^3*x^3 + a^3*b^2))*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A
^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A
^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)*log((7*B*a + 5*A*b)*sqrt(x)
- 1/2*(sqrt(-3)*a^2*b^2 - a^2*b^2)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b +
 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 +
 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)) - 12*((13*B*a*b -
A*b^2)*x^3 + 7*B*a^2 + 5*A*a*b)*sqrt(x))/(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b
^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**(5/2)*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.04

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{(13\,Bab - Ab^2)x^{\frac{7}{2}} + (7\,Ba^2 + 5\,Aab)\sqrt{x}}{36\,(ab^4x^6 + 2\,a^2b^3x^3 + a^3b^2)} + \frac{\frac{\sqrt{3}(7\,Ba+5\,Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} - \frac{\sqrt{3}(7\,Ba+5\,Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} + \frac{4\left(7\,Bab^{\frac{1}{3}}+5\,Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} + \cdots}{432\,ab^2}$$

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/36*((13*B*a*b - A*b^2)*x^(7/2) + (7*B*a^2 + 5*A*a*b)*sqrt(x))/(a*b^4*x^6
 + 2*a^2*b^3*x^3 + a^3*b^2) + 1/432*(sqrt(3)*(7*B*a + 5*A*b)*log(sqrt(3)*a^
(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(7
*B*a + 5*A*b)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(
a^(5/6)*b^(1/6)) + 4*(7*B*a*b^(1/3) + 5*A*b^(4/3))*arctan(b^(1/3)*sqrt(x)/s
qrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(7*B*a^(4
/3)*b^(1/3) + 5*A*a^(1/3)*b^(4/3))*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1
/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(
7*B*a^(4/3)*b^(1/3) + 5*A*a^(1/3)*b^(4/3))*arctan(-(sqrt(3)*a^(1/6)*b^(1/6)
 - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3
))))/(a*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.00

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{\sqrt{3}\left(7\,(ab^5)^{\frac{1}{6}}\,Ba + 5\,(ab^5)^{\frac{1}{6}}\,Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^2b^3} - \frac{\sqrt{3}\left(7\,(ab^5)^{\frac{1}{6}}\,Ba + 5\,(ab^5)^{\frac{1}{6}}\,Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^2b^3} + \frac{\left(7\,(ab^5)^{\frac{1}{6}}\,Ba + 5\,(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^2b^3} + \frac{\left(7\,(ab^5)^{\frac{1}{6}}\,Ba + 5\,(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^2b^3} + \frac{\left(7\,(ab^5)^{\frac{1}{6}}\,Ba + 5\,(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108\,a^2b^3} - \frac{13\,Babx^{\frac{7}{2}} - Ab^2x^{\frac{7}{2}} + 7\,Ba^2\sqrt{x} + 5\,Aab\sqrt{x}}{36\,(bx^3+a)^2ab^2}$$

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/432*sqrt(3)*(7*(a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(
x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^3) - 1/432*sqrt(3)*(7*(a*b^5)^(1/6
)*B*a + 5*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(
1/3))/(a^2*b^3) + 1/216*(7*(a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arctan(
(sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a^2*b^3) + 1/216*(7*(a*b^5)
^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))
/(a/b)^(1/6))/(a^2*b^3) + 1/108*(7*(a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)
*arctan(sqrt(x)/(a/b)^(1/6))/(a^2*b^3) - 1/36*(13*B*a*b*x^(7/2) - A*b^2*x^(
7/2) + 7*B*a^2*sqrt(x) + 5*A*a*b*sqrt(x))/((b*x^3 + a)^2*a*b^2)
```

Mupad [B] (verification not implemented)

Time = 7.39 (sec) , antiderivative size = 1944, normalized size of antiderivative = 5.94

$$\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

```
[In] int((x^(5/2)*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (atan((((((5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 52
5*A^2*B*a*b^2))/(279936*(-a)^(23/6)*b^(19/6)) - (x^(1/2)*(625*A^4*b^4 + 240
1*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(2
79936*a^4*b^3))*(5*A*b + 7*B*a)*1i)/(216*(-a)^(11/6)*b^(13/6)) - ((((5*A*b
+ 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(
279936*(-a)^(23/6)*b^(19/6)) + (x^(1/2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*
A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*a^4*b^3))*(
5*A*b + 7*B*a)*1i)/(216*(-a)^(11/6)*b^(13/6)))/(((((5*A*b + 7*B*a)*(125*A^3
*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(279936*(-a)^(23/6
)*b^(19/6)) - (x^(1/2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 +
 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*a^4*b^3))*(5*A*b + 7*B*a))/(
216*(-a)^(11/6)*b^(13/6)) + ((((5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 +
 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(279936*(-a)^(23/6)*b^(19/6)) + (x^(1/
2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b +
3500*A^3*B*a*b^3))/(279936*a^4*b^3))*(5*A*b + 7*B*a))/(216*(-a)^(11/6)*b^(1
3/6))))*(5*A*b + 7*B*a)*1i)/(108*(-a)^(11/6)*b^(13/6)) - ((x^(1/2)*(5*A*b +
 7*B*a))/(36*b^2) - (x^(7/2)*(A*b - 13*B*a))/(36*a*b))/(a^2 + b^2*x^6 + 2*a
*b*x^3) + (atan(((((3^(1/2)*1i)/2 - 1/2)*((x^(1/2)*(625*A^4*b^4 + 2401*B^4*
a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*
a^4*b^3) - (((3^(1/2)*1i)/2 - 1/2)*(5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a
^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(279936*(-a)^(23/6)*b^(19/6)))*(5*
A*b + 7*B*a)*1i)/(216*(-a)^(11/6)*b^(13/6)) + (((3^(1/2)*1i)/2 - 1/2)*((x^(
1/2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b
+ 3500*A^3*B*a*b^3))/(279936*a^4*b^3) + (((3^(1/2)*1i)/2 - 1/2)*(5*A*b + 7*
```

```
B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(2799
36*(-a)^(23/6)*b^(19/6)))*(5*A*b + 7*B*a)*1i)/(216*(-a)^(11/6)*b^(13/6)))/(
(((3^(1/2)*1i)/2 - 1/2)*((x^(1/2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^
2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*a^4*b^3) - (((3^(
1/2)*1i)/2 - 1/2)*(5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^
2*b + 525*A^2*B*a*b^2))/(279936*(-a)^(23/6)*b^(19/6)))*(5*A*b + 7*B*a))/(21
6*(-a)^(11/6)*b^(13/6)) - (((3^(1/2)*1i)/2 - 1/2)*((x^(1/2)*(625*A^4*b^4 +
2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))
/(279936*a^4*b^3) + (((3^(1/2)*1i)/2 - 1/2)*(5*A*b + 7*B*a)*(125*A^3*b^3 +
343*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(279936*(-a)^(23/6)*b^(19
/6)))*(5*A*b + 7*B*a))/(216*(-a)^(11/6)*b^(13/6))))*((3^(1/2)*1i)/2 - 1/2)*
(5*A*b + 7*B*a)*1i)/(108*(-a)^(11/6)*b^(13/6)) + (atan(((((3^(1/2)*1i)/2 +
1/2)*((x^(1/2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*
B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*a^4*b^3) - (((3^(1/2)*1i)/2 + 1/2)*(
5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b
^2))/(279936*(-a)^(23/6)*b^(19/6)))*(5*A*b + 7*B*a)*1i)/(216*(-a)^(11/6)*b^
(13/6)) + (((3^(1/2)*1i)/2 + 1/2)*((x^(1/2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7
350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*a^4*b^3
) + (((3^(1/2)*1i)/2 + 1/2)*(5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 73
5*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(279936*(-a)^(23/6)*b^(19/6)))*(5*A*b + 7
*B*a)*1i)/(216*(-a)^(11/6)*b^(13/6)))/((((3^(1/2)*1i)/2 + 1/2)*((x^(1/2)*(6
25*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*
A^3*B*a*b^3))/(279936*a^4*b^3) - (((3^(1/2)*1i)/2 + 1/2)*(5*A*b + 7*B*a)*(1
25*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(279936*(-a)
^(23/6)*b^(19/6)))*(5*A*b + 7*B*a))/(216*(-a)^(11/6)*b^(13/6)) - (((3^(1/2)
*1i)/2 + 1/2)*((x^(1/2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2
+ 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*a^4*b^3) + (((3^(1/2)*1i)/2
 + 1/2)*(5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 525*
A^2*B*a*b^2))/(279936*(-a)^(23/6)*b^(19/6)))*(5*A*b + 7*B*a))/(216*(-a)^(11
/6)*b^(13/6))))*((3^(1/2)*1i)/2 + 1/2)*(5*A*b + 7*B*a)*1i)/(108*(-a)^(11/6)
*b^(13/6))
```

3.173 $\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 327

$$\begin{aligned}\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3}\,dx &= \frac{(Ab-aB)x^{5/2}}{6ab\,(a+bx^3)^2}+\frac{(7Ab+5aB)x^{5/2}}{36a^2b\,(a+bx^3)}\\ &\quad-\frac{(7Ab+5aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{13/6}b^{11/6}}+\frac{(7Ab+5aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{13/6}b^{11/6}}\\ &\quad+\frac{(7Ab+5aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{13/6}b^{11/6}}+\frac{(7Ab+5aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{13/6}b^{11/6}}\\ &\quad-\frac{(7Ab+5aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{13/6}b^{11/6}}\end{aligned}$$

```
[Out] 1/6*(A*b-B*a)*x^(5/2)/a/b/(b*x^3+a)^2+1/36*(7*A*b+5*B*a)*x^(5/2)/a^2/b/(b*x
^3+a)+1/108*(7*A*b+5*B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(13/6)/b^(11/6)
+1/216*(7*A*b+5*B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(13/6)/b^
(11/6)+1/216*(7*A*b+5*B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(13/
6)/b^(11/6)+1/432*(7*A*b+5*B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2
)*x^(1/2))/a^(13/6)/b^(11/6)*3^(1/2)-1/432*(7*A*b+5*B*a)*ln(a^(1/3)+b^(1/3)
*x+a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(13/6)/b^(11/6)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {468, 296, 335, 301, 648, 632, 210, 642, 211}

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = -\frac{(5aB+7Ab)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{13/6}b^{11/6}} + \frac{(5aB+7Ab)\arctan\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{216a^{13/6}b^{11/6}} + \frac{(5aB+7Ab)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{13/6}b^{11/6}} + \frac{(5aB+7Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{13/6}b^{11/6}} - \frac{(5aB+7Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{13/6}b^{11/6}} + \frac{x^{5/2}(5aB+7Ab)}{36a^2b\,(a+bx^3)} + \frac{x^{5/2}(Ab-aB)}{6ab\,(a+bx^3)^2}$$

```
[In] Int[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^(5/2))/(6*a*b*(a + b*x^3)^2) + ((7*A*b + 5*a*B)*x^(5/2))/(36
*a^2*b*(a + b*x^3)) - ((7*A*b + 5*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])
/a^(1/6)])/(216*a^(13/6)*b^(11/6)) + ((7*A*b + 5*a*B)*ArcTan[Sqrt[3] + (2*b
^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(13/6)*b^(11/6)) + ((7*A*b + 5*a*B)*ArcTan
[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(13/6)*b^(11/6)) + ((7*A*b + 5*a*B)*Log
[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(13
/6)*b^(11/6)) - ((7*A*b + 5*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt
[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(13/6)*b^(11/6))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
```

```
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{(Ab-aB)x^{5/2}}{6ab\left(a+bx^3\right)^2}+\frac{\left(\frac{7Ab}{2}+\frac{5aB}{2}\right)\int \frac{x^{3/2}}{\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^{5/2}}{6ab\left(a+bx^3\right)^2}+\frac{(7Ab+5aB)x^{5/2}}{36a^2b\left(a+bx^3\right)}+\frac{(7Ab+5aB)\int \frac{x^{3/2}}{a+bx^3}\,dx}{72a^2b}$$

$$= \frac{(Ab-aB)x^{5/2}}{6ab\left(a+bx^3\right)^2}+\frac{(7Ab+5aB)x^{5/2}}{36a^2b\left(a+bx^3\right)}+\frac{(7Ab+5aB)\text{Subst}\left(\int \frac{x^4}{a+bx^6}\,dx,x,\sqrt{x}\right)}{36a^2b}$$

$$= \frac{(Ab-aB)x^{5/2}}{6ab\left(a+bx^3\right)^2}+\frac{(7Ab+5aB)x^{5/2}}{36a^2b\left(a+bx^3\right)}$$
$$+\frac{(7Ab+5aB)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2}+\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{108a^{13/6}b^{5/3}}$$
$$+\frac{(7Ab+5aB)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{108a^{13/6}b^{5/3}}$$
$$+\frac{(7Ab+5aB)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{108a^2b^{5/3}}$$

$$= \frac{(Ab-aB)x^{5/2}}{6ab\left(a+bx^3\right)^2}+\frac{(7Ab+5aB)x^{5/2}}{36a^2b\left(a+bx^3\right)}+\frac{(7Ab+5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{13/6}b^{11/6}}$$
$$+\frac{(7Ab+5aB)\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{144\sqrt{3}a^{13/6}b^{11/6}}$$
$$-\frac{(7Ab+5aB)\text{Subst}\left(\int \frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{144\sqrt{3}a^{13/6}b^{11/6}}$$
$$+\frac{(7Ab+5aB)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{432a^2b^{5/3}}$$
$$+\frac{(7Ab+5aB)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{432a^2b^{5/3}}$$

$$= \frac{(Ab-aB)x^{5/2}}{6ab(a+bx^3)^2} + \frac{(7Ab+5aB)x^{5/2}}{36a^2b(a+bx^3)} + \frac{(7Ab+5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{13/6}b^{11/6}} + \frac{(7Ab+5aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{13/6}b^{11/6}} - \frac{(7Ab+5aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{13/6}b^{11/6}} + \frac{(7Ab+5aB)\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{216\sqrt{3}a^{13/6}b^{11/6}} - \frac{(7Ab+5aB)\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{216\sqrt{3}a^{13/6}b^{11/6}}$$

$$= \frac{(Ab-aB)x^{5/2}}{6ab(a+bx^3)^2} + \frac{(7Ab+5aB)x^{5/2}}{36a^2b(a+bx^3)} - \frac{(7Ab+5aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{13/6}b^{11/6}} + \frac{(7Ab+5aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{13/6}b^{11/6}} + \frac{(7Ab+5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{13/6}b^{11/6}} + \frac{(7Ab+5aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{13/6}b^{11/6}} - \frac{(7Ab+5aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{13/6}b^{11/6}}$$

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.59

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{\frac{6\sqrt[6]{a}b^{5/6}x^{5/2}(-a^2B+7Ab^2x^3+ab(13A+5Bx^3))}{(a+bx^3)^2} + 2(7Ab+5aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right) - (7Ab+5aI}{216a^{13/6}b^{11/6}}$$

```
[In] Integrate[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((6*a^(1/6)*b^(5/6)*x^(5/2)*(-(a^2*B) + 7*A*b^2*x^3 + a*b*(13*A + 5*B*x^3))
)/(a + b*x^3)^2 + 2*(7*A*b + 5*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)] - (7*
A*b + 5*a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])] - Sqrt
[3]*(7*A*b + 5*a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^
(1/3)*x)])/(216*a^(13/6)*b^(11/6))
```

Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{\frac{(7Ab+5Ba)x^{\frac{11}{2}}}{36a^2}+\frac{(13Ab-Ba)x^{\frac{5}{2}}}{36ab}}{(b\,x^3+a)^2}+\frac{(7Ab+5Ba)\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{a}{b}\right.}{}\right.}{36a^2b}$
default	$\frac{\frac{(7Ab+5Ba)x^{\frac{11}{2}}}{36a^2}+\frac{(13Ab-Ba)x^{\frac{5}{2}}}{36ab}}{(b\,x^3+a)^2}+\frac{(7Ab+5Ba)\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(-\sqrt{3}+\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\frac{\sqrt{3}\left(\frac{a}{b}\right.}{}\right.}{36a^2b}$

```
[In] int(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/72*(7*A*b+5*B*a)/a^2*x^(11/2)+1/72*(13*A*b-B*a)/a/b*x^(5/2))/(b*x^3+a)
^2+1/36*(7*A*b+5*B*a)/a^2/b*(1/12/a*3^(1/2)*(a/b)^(5/6)*ln(3^(1/2)*(a/b)^(1
/6)*x^(1/2)-x-(a/b)^(1/3))+1/6/b/(a/b)^(1/6)*arctan(-3^(1/2)+2*x^(1/2)/(a/b
)^(1/6))-1/12/a*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^
(1/3))+1/6/b/(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))+1/3/b/(a/b)^
(1/6)*arctan(x^(1/2)/(a/b)^(1/6)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1959 vs. 2(241) = 482.

Time = 0.43 (sec) , antiderivative size = 1959, normalized size of antiderivative = 5.99

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/432*(2*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(15625*B^6*a^6 + 131250*A*
B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^
2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6)*log(a^1
1*b^9*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 8575
00*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A
^6*b^6)/(a^13*b^11))^(5/6) + (3125*B^5*a^5 + 21875*A*B^4*a^4*b + 61250*A^2*
B^3*a^3*b^2 + 85750*A^3*B^2*a^2*b^3 + 60025*A^4*B*a*b^4 + 16807*A^5*b^5)*sq
rt(x)) - 2*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(15625*B^6*a^6 + 131250*
A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*
B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6)*log(-
```

```
a^11*b^9*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 8
57500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 11764
9*A^6*b^6)/(a^13*b^11))^(5/6) + (3125*B^5*a^5 + 21875*A*B^4*a^4*b + 61250*A
^2*B^3*a^3*b^2 + 85750*A^3*B^2*a^2*b^3 + 60025*A^4*B*a*b^4 + 16807*A^5*b^5)
*sqrt(x)) + (a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b - sqrt(-3)*(a^2*b^3*x^6 +
2*a^3*b^2*x^3 + a^4*b))*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*
B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*
B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6)*log(1/2*(sqrt(-3)*a^11*b^9 + a
^11*b^9)*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 8
57500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 11764
9*A^6*b^6)/(a^13*b^11))^(5/6) + (3125*B^5*a^5 + 21875*A*B^4*a^4*b + 61250*A
^2*B^3*a^3*b^2 + 85750*A^3*B^2*a^2*b^3 + 60025*A^4*B*a*b^4 + 16807*A^5*b^5)
*sqrt(x)) - (a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b - sqrt(-3)*(a^2*b^3*x^6 +
2*a^3*b^2*x^3 + a^4*b))*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*
B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*
B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6)*log(-1/2*(sqrt(-3)*a^11*b^9 +
a^11*b^9)*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 +
857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 1176
49*A^6*b^6)/(a^13*b^11))^(5/6) + (3125*B^5*a^5 + 21875*A*B^4*a^4*b + 61250*
A^2*B^3*a^3*b^2 + 85750*A^3*B^2*a^2*b^3 + 60025*A^4*B*a*b^4 + 16807*A^5*b^5
)*sqrt(x)) - (a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b + sqrt(-3)*(a^2*b^3*x^6 +
 2*a^3*b^2*x^3 + a^4*b))*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2
*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5
*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6)*log(1/2*(sqrt(-3)*a^11*b^9 -
a^11*b^9)*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 +
857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 1176
49*A^6*b^6)/(a^13*b^11))^(5/6) + (3125*B^5*a^5 + 21875*A*B^4*a^4*b + 61250*
A^2*B^3*a^3*b^2 + 85750*A^3*B^2*a^2*b^3 + 60025*A^4*B*a*b^4 + 16807*A^5*b^5
)*sqrt(x)) + (a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b + sqrt(-3)*(a^2*b^3*x^6 +
 2*a^3*b^2*x^3 + a^4*b))*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2
*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5
*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6)*log(-1/2*(sqrt(-3)*a^11*b^9 -
 a^11*b^9)*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 +
 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117
649*A^6*b^6)/(a^13*b^11))^(5/6) + (3125*B^5*a^5 + 21875*A*B^4*a^4*b + 61250
*A^2*B^3*a^3*b^2 + 85750*A^3*B^2*a^2*b^3 + 60025*A^4*B*a*b^4 + 16807*A^5*b^
5)*sqrt(x)) + 12*((5*B*a*b + 7*A*b^2)*x^5 - (B*a^2 - 13*A*a*b)*x^2)*sqrt(x)
)/(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate(x**(3/2)*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.83

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(5\,Bab+7\,Ab^2)x^{\frac{11}{2}}-(Ba^2-13\,Aab)x^{\frac{5}{2}}}{36\,(a^2b^3x^6+2\,a^3b^2x^3+a^4b)}$$
$$-\frac{(5\,Ba+7\,Ab)\left(\frac{\sqrt{3}\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{2\,\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{2\,\arctan\left(-\right.}{b^{\frac{2}{3}}}\right.}{432\,a^2b}$$

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/36*((5*B*a*b + 7*A*b^2)*x^(11/2) - (B*a^2 - 13*A*a*b)*x^(5/2))/(a^2*b^3*x
^6 + 2*a^3*b^2*x^3 + a^4*b) - 1/432*(5*B*a + 7*A*b)*(sqrt(3)*log(sqrt(3)*a^
(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6)) - sqrt(3)*lo
g(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6))
 - 2*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1
/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 2*arctan(-(sqrt(3)*a^(1/6)*b^(1/6)
- 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3)))
 - 4*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^
(1/3))))/(a^2*b)
```

Giac [A] (verification not implemented)

none

Time = 0.65 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.96

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(5\,Ba+7\,Ab)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,(ab^5)^{\frac{1}{6}}\,a^2b} + \frac{(5\,Ba+7\,Ab)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,(ab^5)^{\frac{1}{6}}\,a^2b} + \frac{\left(5\,Ba\left(\frac{a}{b}\right)^{\frac{5}{6}}+7\,Ab\left(\frac{a}{b}\right)^{\frac{5}{6}}\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108\,a^3b} + \frac{5\,Babx^{\frac{11}{2}}+7\,Ab^2x^{\frac{11}{2}}-Ba^2x^{\frac{5}{2}}+13\,Aabx^{\frac{5}{2}}}{36\,(bx^3+a)^2a^2b} - \frac{\sqrt{3}\left(5\,(ab^5)^{\frac{5}{6}}\,Ba+7\,(ab^5)^{\frac{5}{6}}\,Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^3b^6} + \frac{\sqrt{3}\left(5\,(ab^5)^{\frac{5}{6}}\,Ba+7\,(ab^5)^{\frac{5}{6}}\,Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^3b^6}$$

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/216*(5*B*a + 7*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))
/((a*b^5)^(1/6)*a^2*b) + 1/216*(5*B*a + 7*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6)
 - 2*sqrt(x))/(a/b)^(1/6))/((a*b^5)^(1/6)*a^2*b) + 1/108*(5*B*a*(a/b)^(5/6)
 + 7*A*b*(a/b)^(5/6))*arctan(sqrt(x)/(a/b)^(1/6))/(a^3*b) + 1/36*(5*B*a*b*x
^(11/2) + 7*A*b^2*x^(11/2) - B*a^2*x^(5/2) + 13*A*a*b*x^(5/2))/((b*x^3 + a)
^2*a^2*b) - 1/432*sqrt(3)*(5*(a*b^5)^(5/6)*B*a + 7*(a*b^5)^(5/6)*A*b)*log(s
qrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b^6) + 1/432*sqrt(3)*(5*
(a*b^5)^(5/6)*B*a + 7*(a*b^5)^(5/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) +
 x + (a/b)^(1/3))/(a^3*b^6)
```

Mupad [B] (verification not implemented)

Time = 7.35 (sec) , antiderivative size = 1672, normalized size of antiderivative = 5.11

$$\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

```
[In] int((x^(3/2)*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] ((x^(11/2)*(7*A*b + 5*B*a))/(36*a^2) + (x^(5/2)*(13*A*b - B*a))/(36*a*b))/(
a^2 + b^2*x^6 + 2*a*b*x^3) + (atan(((((343*A^3*b^3 + 125*B^3*a^3 + 525*A*B^
2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) - (x^(1/2)*(7*A*b + 5*B*a)*(49*A^2*b^
4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6)))*(7*A*b + 5
*B*a)^2*1i)/(46656*(-a)^(13/3)*b^(11/3)) - (((343*A^3*b^3 + 125*B^3*a^3 + 5
25*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) + (x^(1/2)*(7*A*b + 5*B*a)*(49
```

```
*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6)))*(7*
A*b + 5*B*a)^2*1i)/(46656*(-a)^(13/3)*b^(11/3)))/((((343*A^3*b^3 + 125*B^3*
a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) - (x^(1/2)*(7*A*b + 5*B
*a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6
)))*(7*A*b + 5*B*a)^2)/(46656*(-a)^(13/3)*b^(11/3)) + (((343*A^3*b^3 + 125*
B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) + (x^(1/2)*(7*A*b +
 5*B*a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(
11/6)))*(7*A*b + 5*B*a)^2)/(46656*(-a)^(13/3)*b^(11/3))))*(7*A*b + 5*B*a)*1
i)/(108*(-a)^(13/6)*b^(11/6)) + (atan(((((3^(1/2)*1i)/2 - 1/2)^2*(7*A*b + 5
*B*a)^2*((343*A^3*b^3 + 125*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1
296*a^3) - (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(7*A*b + 5*B*a)*(49*A^2*b^4 + 25
*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6)))*1i)/(46656*(-a)^
(13/3)*b^(11/3)) - (((3^(1/2)*1i)/2 - 1/2)^2*(7*A*b + 5*B*a)^2*((343*A^3*b^
3 + 125*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) + (x^(1/2)*
((3^(1/2)*1i)/2 - 1/2)*(7*A*b + 5*B*a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*
B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6)))*1i)/(46656*(-a)^(13/3)*b^(11/3)))/((
((3^(1/2)*1i)/2 - 1/2)^2*(7*A*b + 5*B*a)^2*((343*A^3*b^3 + 125*B^3*a^3 + 52
5*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) - (x^(1/2)*((3^(1/2)*1i)/2 - 1/
2)*(7*A*b + 5*B*a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)
^(19/6)*b^(11/6))))/(46656*(-a)^(13/3)*b^(11/3)) + (((3^(1/2)*1i)/2 - 1/2)^
2*(7*A*b + 5*B*a)^2*((343*A^3*b^3 + 125*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2
*B*a*b^2)/(1296*a^3) + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(7*A*b + 5*B*a)*(49*
A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6))))/(46
656*(-a)^(13/3)*b^(11/3))))*((3^(1/2)*1i)/2 - 1/2)*(7*A*b + 5*B*a)*1i)/(108
*(-a)^(13/6)*b^(11/6)) + (atan(((((3^(1/2)*1i)/2 + 1/2)^2*(7*A*b + 5*B*a)^2
*((343*A^3*b^3 + 125*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3
) - (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(7*A*b + 5*B*a)*(49*A^2*b^4 + 25*B^2*a^
2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6)))*1i)/(46656*(-a)^(13/3)*
b^(11/3)) - (((3^(1/2)*1i)/2 + 1/2)^2*(7*A*b + 5*B*a)^2*((343*A^3*b^3 + 125
*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) + (x^(1/2)*((3^(1/
2)*1i)/2 + 1/2)*(7*A*b + 5*B*a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3
))/(1296*(-a)^(19/6)*b^(11/6)))*1i)/(46656*(-a)^(13/3)*b^(11/3)))/((((3^(1/
2)*1i)/2 + 1/2)^2*(7*A*b + 5*B*a)^2*((343*A^3*b^3 + 125*B^3*a^3 + 525*A*B^2
*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) - (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(7*A
*b + 5*B*a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)
*b^(11/6))))/(46656*(-a)^(13/3)*b^(11/3)) + (((3^(1/2)*1i)/2 + 1/2)^2*(7*A*
b + 5*B*a)^2*((343*A^3*b^3 + 125*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^
2)/(1296*a^3) + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(7*A*b + 5*B*a)*(49*A^2*b^4
 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6))))/(46656*(-a
)^(13/3)*b^(11/3))))*((3^(1/2)*1i)/2 + 1/2)*(7*A*b + 5*B*a)*1i)/(108*(-a)^(
13/6)*b^(11/6))
```

3.174 $\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^3} dx = \frac{(Ab-aB)x^{3/2}}{6ab(a+bx^3)^2} + \frac{(3Ab+aB)x^{3/2}}{12a^2b(a+bx^3)} + \frac{(3Ab+aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{5/2}b^{3/2}}$$

```
[Out] 1/6*(A*b-B*a)*x^(3/2)/a/b/(b*x^3+a)^2+1/12*(3*A*b+B*a)*x^(3/2)/a^2/b/(b*x^3
+a)+1/12*(3*A*b+B*a)*arctan(x^(3/2)*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {468, 296, 335, 281, 211}

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^3} dx = \frac{(aB+3Ab)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{5/2}b^{3/2}} + \frac{x^{3/2}(aB+3Ab)}{12a^2b(a+bx^3)} + \frac{x^{3/2}(Ab-aB)}{6ab(a+bx^3)^2}$$

```
[In] Int[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^(3/2))/(6*a*b*(a + b*x^3)^2) + ((3*A*b + a*B)*x^(3/2))/(12*a
^2*b*(a + b*x^3)) + ((3*A*b + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a
^(5/2)*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(Ab-aB)x^{3/2}}{6ab\left(a+bx^3\right)^2}+\frac{\left(\frac{9Ab}{2}+\frac{3aB}{2}\right)\int \frac{\sqrt{x}}{\left(a+bx^3\right)^2}\,dx}{6ab}\\
&= \frac{(Ab-aB)x^{3/2}}{6ab\left(a+bx^3\right)^2}+\frac{(3Ab+aB)x^{3/2}}{12a^2b\left(a+bx^3\right)}+\frac{(3Ab+aB)\int \frac{\sqrt{x}}{a+bx^3}\,dx}{8a^2b}\\
&= \frac{(Ab-aB)x^{3/2}}{6ab\left(a+bx^3\right)^2}+\frac{(3Ab+aB)x^{3/2}}{12a^2b\left(a+bx^3\right)}+\frac{(3Ab+aB)\text{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx,x,\sqrt{x}\right)}{4a^2b}\\
&= \frac{(Ab-aB)x^{3/2}}{6ab\left(a+bx^3\right)^2}+\frac{(3Ab+aB)x^{3/2}}{12a^2b\left(a+bx^3\right)}+\frac{(3Ab+aB)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^{3/2}\right)}{12a^2b}
\end{aligned}$$

$$= \frac{(Ab-aB)x^{3/2}}{6ab\left(a+bx^3\right)^2}+\frac{(3Ab+aB)x^{3/2}}{12a^2b\left(a+bx^3\right)}+\frac{(3Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{5/2}b^{3/2}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{x}(A+Bx^3)}{\left(a+bx^3\right)^3}\,dx$$
$$= -\frac{x^{3/2}(-5aAb+a^2B-3Ab^2x^3-abBx^3)}{12a^2b\left(a+bx^3\right)^2}+\frac{(3Ab+aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{5/2}b^{3/2}}$$

```
[In] Integrate[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] -1/12*(x^(3/2)*(-5*a*A*b + a^2*B - 3*A*b^2*x^3 - a*b*B*x^3))/(a^2*b*(a + b*
x^3)^2) + ((3*A*b + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a^(5/2)*b^(
3/2))
```

Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\frac{(3Ab+Ba)x^{\frac{9}{2}}}{12a^2}+\frac{(5Ab-Ba)x^{\frac{3}{2}}}{12ab}}{(b\,x^3+a)^2}+\frac{(3Ab+Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12a^2b\sqrt{ab}}$	82
default	$\frac{\frac{(3Ab+Ba)x^{\frac{9}{2}}}{12a^2}+\frac{(5Ab-Ba)x^{\frac{3}{2}}}{12ab}}{(b\,x^3+a)^2}+\frac{(3Ab+Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12a^2b\sqrt{ab}}$	82

```
[In] int((B*x^3+A)*x^(1/2)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(1/8*(3*A*b+B*a)/a^2*x^(9/2)+1/8*(5*A*b-B*a)/a/b*x^(3/2))/(b*x^3+a)^2+1
/12*(3*A*b+B*a)/a^2/b/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 313, normalized size of antiderivative = 3.01

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^3}\,dx = \left[-\frac{((Bab^2+3\,Ab^3)x^6+Ba^3+3\,Aa^2b+2\,(Ba^2b+3\,Aab^2)x^3)\sqrt{-ab}\log\left(\frac{bx^3-2\,\sqrt{-ab}x^{\frac{3}{2}}-a}{bx^3+a}\right)-2\,((Ba^2b^2+\right.}{24\,(a^3b^4x^6+2\,a^4b^3x^3+a^5b^2)}$$

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/24*(((B*a*b^2 + 3*A*b^3)*x^6 + B*a^3 + 3*A*a^2*b + 2*(B*a^2*b + 3*A*a*b
^2)*x^3)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)) - 2
*((B*a^2*b^2 + 3*A*a*b^3)*x^4 - (B*a^3*b - 5*A*a^2*b^2)*x)*sqrt(x))/(a^3*b^
4*x^6 + 2*a^4*b^3*x^3 + a^5*b^2), 1/12*(((B*a*b^2 + 3*A*b^3)*x^6 + B*a^3 +
3*A*a^2*b + 2*(B*a^2*b + 3*A*a*b^2)*x^3)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)
/a) + ((B*a^2*b^2 + 3*A*a*b^3)*x^4 - (B*a^3*b - 5*A*a^2*b^2)*x)*sqrt(x))/(a
^3*b^4*x^6 + 2*a^4*b^3*x^3 + a^5*b^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate((B*x**3+A)*x**(1/2)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(Bab+3\,Ab^2)x^{\frac{9}{2}}-(Ba^2-5\,Aab)x^{\frac{3}{2}}}{12\,(a^2b^3x^6+2\,a^3b^2x^3+a^4b)}+\frac{(Ba+3\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\,\sqrt{ab}a^2b}$$

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/12*((B*a*b + 3*A*b^2)*x^(9/2) - (B*a^2 - 5*A*a*b)*x^(3/2))/(a^2*b^3*x^6 +
 2*a^3*b^2*x^3 + a^4*b) + 1/12*(B*a + 3*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(s
qrt(a*b)*a^2*b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(Ba+3\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\,\sqrt{ab}a^2b} + \frac{Babx^{\frac{9}{2}}+3\,Ab^2x^{\frac{9}{2}}-Ba^2x^{\frac{3}{2}}+5\,Aabx^{\frac{3}{2}}}{12\,(bx^3+a)^2a^2b}$$

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/12*(B*a + 3*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/12*(B*
a*b*x^(9/2) + 3*A*b^2*x^(9/2) - B*a^2*x^(3/2) + 5*A*a*b*x^(3/2))/((b*x^3 +
a)^2*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 7.16 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{x}(A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{\frac{x^{9/2}\,(3\,A\,b+B\,a)}{12\,a^2}+\frac{x^{3/2}\,(5\,A\,b-B\,a)}{12\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6} + \frac{\mathrm{atan}\left(\frac{b^{3/2}\,x^{3/2}\,(9\,A^2\,b^3+6\,A\,B\,a\,b^2+B^2\,a^2\,b)}{\sqrt{a}\,(3\,A\,b+B\,a)\,(3\,A\,b^3+B\,a\,b^2)}\right)\,(3\,A\,b+B\,a)}{12\,a^{5/2}\,b^{3/2}}$$

```
[In] int((x^(1/2)*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] ((x^(9/2)*(3*A*b + B*a))/(12*a^2) + (x^(3/2)*(5*A*b - B*a))/(12*a*b))/(a^2
+ b^2*x^6 + 2*a*b*x^3) + (atan((b^(3/2)*x^(3/2)*(9*A^2*b^3 + B^2*a^2*b + 6*
A*B*a*b^2))/(a^(1/2)*(3*A*b + B*a)*(3*A*b^3 + B*a*b^2)))*(3*A*b + B*a))/(12
*a^(5/2)*b^(3/2))
```

3.175 $\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 321

$$\begin{aligned}\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^3}\,dx &= \frac{(Ab-aB)\sqrt{x}}{6ab\,(a+bx^3)^2}+\frac{(11Ab+aB)\sqrt{x}}{36a^2b\,(a+bx^3)} \\ &- \frac{5(11Ab+aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{17/6}b^{7/6}} \\ &+ \frac{5(11Ab+aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{17/6}b^{7/6}} \\ &+ \frac{5(11Ab+aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{17/6}b^{7/6}} \\ &- \frac{5(11Ab+aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{17/6}b^{7/6}} \\ &+ \frac{5(11Ab+aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{17/6}b^{7/6}}\end{aligned}$$

```
[Out] 5/108*(11*A*b+B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(17/6)/b^(7/6)+5/216*(
11*A*b+B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(17/6)/b^(7/6)+5/2
16*(11*A*b+B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(17/6)/b^(7/6)-
5/432*(11*A*b+B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^
(17/6)/b^(7/6)*3^(1/2)+5/432*(11*A*b+B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1
/6)*3^(1/2)*x^(1/2))/a^(17/6)/b^(7/6)*3^(1/2)+1/6*(A*b-B*a)*x^(1/2)/a/b/(b*
x^3+a)^2+1/36*(11*A*b+B*a)*x^(1/2)/a^2/b/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {468, 296, 335, 215, 648, 632, 210, 642, 211}

$$\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^3}\,dx = -\frac{5(aB+11Ab)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{17/6}b^{7/6}} + \frac{5(aB+11Ab)\arctan\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{216a^{17/6}b^{7/6}} + \frac{5(aB+11Ab)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{17/6}b^{7/6}} - \frac{5(aB+11Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{17/6}b^{7/6}} + \frac{5(aB+11Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{17/6}b^{7/6}} + \frac{\sqrt{x}(aB+11Ab)}{36a^2b\left(a+bx^3\right)} + \frac{\sqrt{x}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

```
[In] Int[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^3),x]
```

```
[Out] ((A*b - a*B)*Sqrt[x])/(6*a*b*(a + b*x^3)^2) + ((11*A*b + a*B)*Sqrt[x])/(36*
a^2*b*(a + b*x^3)) - (5*(11*A*b + a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])
/a^(1/6)])/(216*a^(17/6)*b^(7/6)) + (5*(11*A*b + a*B)*ArcTan[Sqrt[3] + (2*b
^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(17/6)*b^(7/6)) + (5*(11*A*b + a*B)*ArcTan
[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(17/6)*b^(7/6)) - (5*(11*A*b + a*B)*Log
[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(17
/6)*b^(7/6)) + (5*(11*A*b + a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt
[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(17/6)*b^(7/6))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(Ab-aB)\sqrt{x}}{6ab\left(a+bx^3\right)^2}+\frac{(11Ab+aB)\int \frac{1}{\sqrt{x}\left(a+bx^3\right)^2}\,dx}{12ab}\\
&= \frac{(Ab-aB)\sqrt{x}}{6ab\left(a+bx^3\right)^2}+\frac{(11Ab+aB)\sqrt{x}}{36a^2b\left(a+bx^3\right)}+\frac{(5(11Ab+aB))\int \frac{1}{\sqrt{x}\left(a+bx^3\right)}\,dx}{72a^2b}\\
&= \frac{(Ab-aB)\sqrt{x}}{6ab\left(a+bx^3\right)^2}+\frac{(11Ab+aB)\sqrt{x}}{36a^2b\left(a+bx^3\right)}+\frac{(5(11Ab+aB))\text{Subst}\left(\int \frac{1}{a+bx^6}\,dx,x,\sqrt{x}\right)}{36a^2b}\\
&= \frac{(Ab-aB)\sqrt{x}}{6ab\left(a+bx^3\right)^2}+\frac{(11Ab+aB)\sqrt{x}}{36a^2b\left(a+bx^3\right)}\\
&\quad+\frac{(5(11Ab+aB))\text{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{108a^{17/6}b}\\
&\quad+\frac{(5(11Ab+aB))\text{Subst}\left(\int \frac{\sqrt[6]{a}+\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{108a^{17/6}b}\\
&\quad+\frac{(5(11Ab+aB))\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{108a^{8/3}b}\\
&= \frac{(Ab-aB)\sqrt{x}}{6ab\left(a+bx^3\right)^2}+\frac{(11Ab+aB)\sqrt{x}}{36a^2b\left(a+bx^3\right)}+\frac{5(11Ab+aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{17/6}b^{7/6}}\\
&\quad-\frac{(5(11Ab+aB))\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{144\sqrt{3}a^{17/6}b^{7/6}}\\
&\quad+\frac{(5(11Ab+aB))\text{Subst}\left(\int \frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{144\sqrt{3}a^{17/6}b^{7/6}}\\
&\quad+\frac{(5(11Ab+aB))\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{432a^{8/3}b}\\
&\quad+\frac{(5(11Ab+aB))\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{432a^{8/3}b}
\end{aligned}$$

$$= \frac{(Ab-aB)\sqrt{x}}{6ab\left(a+bx^3\right)^2}+\frac{(11Ab+aB)\sqrt{x}}{36a^2b\left(a+bx^3\right)}+\frac{5(11Ab+aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{17/6}b^{7/6}} - \frac{5(11Ab+aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{17/6}b^{7/6}} + \frac{5(11Ab+aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{17/6}b^{7/6}} + \frac{(5(11Ab+aB))\text{Subst}\left(\int\frac{1}{-\frac{1}{3}-x^2}\,dx,x,1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{216\sqrt{3}a^{17/6}b^{7/6}} - \frac{(5(11Ab+aB))\text{Subst}\left(\int\frac{1}{-\frac{1}{3}-x^2}\,dx,x,1+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{216\sqrt{3}a^{17/6}b^{7/6}}$$

$$= \frac{(Ab-aB)\sqrt{x}}{6ab\left(a+bx^3\right)^2}+\frac{(11Ab+aB)\sqrt{x}}{36a^2b\left(a+bx^3\right)}-\frac{5(11Ab+aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{17/6}b^{7/6}} + \frac{5(11Ab+aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{17/6}b^{7/6}}+\frac{5(11Ab+aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{17/6}b^{7/6}} - \frac{5(11Ab+aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{17/6}b^{7/6}} + \frac{5(11Ab+aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{17/6}b^{7/6}}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.59

$$\int\frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^3}\,dx = \frac{\frac{6a^{5/6}\sqrt[6]{b}\sqrt{x}\left(-5a^2B+11Ab^2x^3+ab\left(17A+Bx^3\right)\right)}{\left(a+bx^3\right)^2}+10(11Ab+aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)-5(11Ab+aB)\arctan\left(\frac{\sqrt[3]{a}-\sqrt[3]{}}{\sqrt[6]{a}\sqrt[6]{}}\right.}{216a^{17/6}b^{7/6}}$$

```
[In] Integrate[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^3),x]
```

```
[Out] ((6*a^(5/6)*b^(1/6)*Sqrt[x]*(-5*a^2*B + 11*A*b^2*x^3 + a*b*(17*A + B*x^3)))
/(a + b*x^3)^2 + 10*(11*A*b + a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)] - 5*(1
1*A*b + a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/6)*b^(1/6)*Sqrt[x])] + 5*Sq
rt[3]*(11*A*b + a*B)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b
^(1/3)*x)])/(216*a^(17/6)*b^(7/6))
```

Maple [A] (verified)

Time = 4.18 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{\frac{(11Ab+Ba)x^{\frac{7}{2}}}{36a^2}+\frac{(17Ab-5Ba)\sqrt{x}}{36ab}}{(b\,x^3+a)^2}+\frac{5(11Ab+Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}a}{}\right.}{3}$
default	$\frac{\frac{(11Ab+Ba)x^{\frac{7}{2}}}{36a^2}+\frac{(17Ab-5Ba)\sqrt{x}}{36ab}}{(b\,x^3+a)^2}+\frac{5(11Ab+Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}a}{}\right.}{3}$

```
[In] int((B*x^3+A)/(b*x^3+a)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/72*(11*A*b+B*a)/a^2*x^(7/2)+1/72*(17*A*b-5*B*a)/a/b*x^(1/2))/(b*x^3+a)
^2+5/36*(11*A*b+B*a)/a^2/b*(1/3/a*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))-1
/12/a*3^(1/2)*(a/b)^(1/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+1/6
/a*(a/b)^(1/6)*arctan(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6))+1/12/a*3^(1/2)*(a/b)^
(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arcta
n(2*x^(1/2)/(a/b)^(1/6)+3^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1588 vs. 2(235) = 470.

Time = 0.42 (sec) , antiderivative size = 1588, normalized size of antiderivative = 4.95

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^3}\,dx=\text{Too large to display}$$

```
[In] integrate((B*x^3+A)/(b*x^3+a)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/432*(10*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(B^6*a^6 + 66*A*B^5*a^5*b
 + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 +
966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6)*log(5*a^3*b*(-(B^6*
a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 21961
5*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6)
 + 5*(B*a + 11*A*b)*sqrt(x)) - 10*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(
B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 2
19615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(
1/6)*log(-5*a^3*b*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 2662
0*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A
```

```
^6*b^6)/(a^17*b^7))^(1/6) + 5*(B*a + 11*A*b)*sqrt(x)) + 5*(a^2*b^3*x^6 + 2*
a^3*b^2*x^3 + a^4*b + sqrt(-3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b))*(-(B^
6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219
615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/
6)*log(5*(B*a + 11*A*b)*sqrt(x) + 5/2*(sqrt(-3)*a^3*b + a^3*b)*(-(B^6*a^6 +
 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4
*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6)) - 5
*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b + sqrt(-3)*(a^2*b^3*x^6 + 2*a^3*b^2*x
^3 + a^4*b))*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3
*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^
6)/(a^17*b^7))^(1/6)*log(5*(B*a + 11*A*b)*sqrt(x) - 5/2*(sqrt(-3)*a^3*b + a
^3*b)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^
3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^1
7*b^7))^(1/6)) - 5*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b - sqrt(-3)*(a^2*b^3
*x^6 + 2*a^3*b^2*x^3 + a^4*b))*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a
^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^
5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6)*log(5*(B*a + 11*A*b)*sqrt(x) + 5/2*(
sqrt(-3)*a^3*b - a^3*b)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2
+ 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 177
1561*A^6*b^6)/(a^17*b^7))^(1/6)) + 5*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b -
 sqrt(-3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b))*(-(B^6*a^6 + 66*A*B^5*a^5*
b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 +
 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6)*log(5*(B*a + 11*A*
b)*sqrt(x) - 5/2*(sqrt(-3)*a^3*b - a^3*b)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 181
5*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306
*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6)) + 12*((B*a*b + 11*A*b^2)
*x^3 - 5*B*a^2 + 17*A*a*b)*sqrt(x))/(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^3}\,dx = \text{Timed out}$$

```
[In] integrate((B*x**3+A)/(b*x**3+a)**3/x**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^3}\,dx = \frac{(Bab+11\,Ab^2)x^{\frac{7}{2}}-(5\,Ba^2-17\,Aab)\sqrt{x}}{36\left(a^2b^3x^6+2\,a^3b^2x^3+a^4b\right)} + \frac{5\left(\frac{\sqrt{3}(Ba+11\,Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}-\frac{\sqrt{3}(Ba+11\,Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}+\frac{4\left(Bab^{\frac{1}{3}}+11\,Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right.}{432\,a^2b}$$

```
[In] integrate((B*x^3+A)/(b*x^3+a)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/36*((B*a*b + 11*A*b^2)*x^(7/2) - (5*B*a^2 - 17*A*a*b)*sqrt(x))/(a^2*b^3*x
^6 + 2*a^3*b^2*x^3 + a^4*b) + 5/432*(sqrt(3)*(B*a + 11*A*b)*log(sqrt(3)*a^(
1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(B*
a + 11*A*b)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^
(5/6)*b^(1/6)) + 4*(B*a*b^(1/3) + 11*A*b^(4/3))*arctan(b^(1/3)*sqrt(x)/sqrt
(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b
^(1/3) + 11*A*a^(1/3)*b^(4/3))*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*
sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(B*a^
(4/3)*b^(1/3) + 11*A*a^(1/3)*b^(4/3))*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*
b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))))/
(a^2*b)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^3}\,dx = \frac{5\sqrt{3}\left((ab^5)^{\frac{1}{6}}\,Ba+11\,(ab^5)^{\frac{1}{6}}\,Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^3b^2} - \frac{5\sqrt{3}\left((ab^5)^{\frac{1}{6}}\,Ba+11\,(ab^5)^{\frac{1}{6}}\,Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^3b^2} + \frac{5\left((ab^5)^{\frac{1}{6}}\,Ba+11\,(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^3b^2} + \frac{5\left((ab^5)^{\frac{1}{6}}\,Ba+11\,(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^3b^2} + \frac{5\left((ab^5)^{\frac{1}{6}}\,Ba+11\,(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108\,a^3b^2} + \frac{Babx^{\frac{7}{2}}+11\,Ab^2x^{\frac{7}{2}}-5\,Ba^2\sqrt{x}+17\,Aab\sqrt{x}}{36\,(bx^3+a)^2a^2b}$$

```
[In] integrate((B*x^3+A)/(b*x^3+a)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 5/432*sqrt(3)*((a*b^5)^(1/6)*B*a + 11*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x
)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b^2) - 5/432*sqrt(3)*((a*b^5)^(1/6)*B
*a + 11*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/
3))/(a^3*b^2) + 5/216*((a*b^5)^(1/6)*B*a + 11*(a*b^5)^(1/6)*A*b)*arctan((sq
rt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a^3*b^2) + 5/216*((a*b^5)^(1/6
)*B*a + 11*(a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/
b)^(1/6))/(a^3*b^2) + 5/108*((a*b^5)^(1/6)*B*a + 11*(a*b^5)^(1/6)*A*b)*arct
an(sqrt(x)/(a/b)^(1/6))/(a^3*b^2) + 1/36*(B*a*b*x^(7/2) + 11*A*b^2*x^(7/2)
- 5*B*a^2*sqrt(x) + 17*A*a*b*sqrt(x))/((b*x^3 + a)^2*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 1952, normalized size of antiderivative = 6.08

$$\int \frac{A+Bx^3}{\sqrt{x}\,(a+bx^3)^3}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x^3)/(x^(1/2)*(a + b*x^3)^3),x)
```

```
[Out] ((x^(7/2)*(11*A*b + B*a))/(36*a^2) + (x^(1/2)*(17*A*b - 5*B*a))/(36*a*b))/(
a^2 + b^2*x^6 + 2*a*b*x^3) - (atan(((((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4
*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^
8) - (625*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33
```

```
*A*B^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6)))*(11*A*b + B*a)*5i)/(216*(-a)
^(17/6)*b^(7/6)) + (((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*
a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) + (625*(11*A*b
 + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/
(279936*(-a)^(47/6)*b^(7/6)))*(11*A*b + B*a)*5i)/(216*(-a)^(17/6)*b^(7/6)))
/((5*((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*
A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) - (625*(11*A*b + B*a)*(1331*A
^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(4
7/6)*b^(7/6)))*(11*A*b + B*a))/(216*(-a)^(17/6)*b^(7/6)) - (5*((625*x^(1/2)
*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A
*B^3*a^3*b^2))/(279936*a^8) + (625*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b
^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6)))*(11
*A*b + B*a))/(216*(-a)^(17/6)*b^(7/6))))*(11*A*b + B*a)*5i)/(108*(-a)^(17/6
)*b^(7/6)) - (atan(((((3^(1/2)*1i)/2 - 1/2)*(11*A*b + B*a)*((625*x^(1/2)*(1
4641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^
3*a^3*b^2))/(279936*a^8) - (625*((3^(1/2)*1i)/2 - 1/2)*(11*A*b + B*a)*(1331
*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^
(47/6)*b^(7/6)))*5i)/(216*(-a)^(17/6)*b^(7/6)) + (((3^(1/2)*1i)/2 - 1/2)*(1
1*A*b + B*a)*((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3
 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) + (625*((3^(1/2)*1i)/
2 - 1/2)*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*
A*B^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6)))*5i)/(216*(-a)^(17/6)*b^(7/6))
)/((5*((3^(1/2)*1i)/2 - 1/2)*(11*A*b + B*a)*((625*x^(1/2)*(14641*A^4*b^5 +
B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(27
9936*a^8) - (625*((3^(1/2)*1i)/2 - 1/2)*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*
a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6))
))/(216*(-a)^(17/6)*b^(7/6)) - (5*((3^(1/2)*1i)/2 - 1/2)*(11*A*b + B*a)*((6
25*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*
b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) + (625*((3^(1/2)*1i)/2 - 1/2)*(11*A*b
 + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/
(279936*(-a)^(47/6)*b^(7/6))))/(216*(-a)^(17/6)*b^(7/6))))*((3^(1/2)*1i)/2
- 1/2)*(11*A*b + B*a)*5i)/(108*(-a)^(17/6)*b^(7/6)) - (atan(((((3^(1/2)*1i)
/2 + 1/2)*(11*A*b + B*a)*((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2
*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) - (625*((
3^(1/2)*1i)/2 + 1/2)*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B
*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6)))*5i)/(216*(-a)^(17
/6)*b^(7/6)) + (((3^(1/2)*1i)/2 + 1/2)*(11*A*b + B*a)*((625*x^(1/2)*(14641*
A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3
*b^2))/(279936*a^8) + (625*((3^(1/2)*1i)/2 + 1/2)*(11*A*b + B*a)*(1331*A^3*
b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(47/6
)*b^(7/6)))*5i)/(216*(-a)^(17/6)*b^(7/6)))/((5*((3^(1/2)*1i)/2 + 1/2)*(11*A
*b + B*a)*((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 +
5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) - (625*((3^(1/2)*1i)/2 +
 1/2)*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B
^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6))))/(216*(-a)^(17/6)*b^(7/6)) - (5*
```

```
((3^(1/2)*1i)/2 + 1/2)*(11*A*b + B*a)*((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^
4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a
^8) + (625*((3^(1/2)*1i)/2 + 1/2)*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b^
2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6))))/(21
6*(-a)^(17/6)*b^(7/6))))*((3^(1/2)*1i)/2 + 1/2)*(11*A*b + B*a)*5i)/(108*(-a
)^(17/6)*b^(7/6))
```

3.176 $\int \frac{A+Bx^3}{x^{3/2}(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 351

$$\int \frac{A+Bx^3}{x^{3/2}(a+bx^3)^3}\,dx = -\frac{7(13Ab-aB)}{36a^3b\sqrt{x}}+\frac{Ab-aB}{6ab\sqrt{x}(a+bx^3)^2}+\frac{13Ab-aB}{36a^2b\sqrt{x}(a+bx^3)} + \frac{7(13Ab-aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{19/6}b^{5/6}}-\frac{7(13Ab-aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{19/6}b^{5/6}} - \frac{7(13Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{19/6}b^{5/6}}-\frac{7(13Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{19/6}b^{5/6}} + \frac{7(13Ab-aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{19/6}b^{5/6}}$$

```
[Out] -7/108*(13*A*b-B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(19/6)/b^(5/6)-7/216*
(13*A*b-B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(19/6)/b^(5/6)-7/
216*(13*A*b-B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(19/6)/b^(5/6)
-7/432*(13*A*b-B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a
^(19/6)/b^(5/6)*3^(1/2)+7/432*(13*A*b-B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(
1/6)*3^(1/2)*x^(1/2))/a^(19/6)/b^(5/6)*3^(1/2)-7/36*(13*A*b-B*a)/a^3/b/x^(1
/2)+1/6*(A*b-B*a)/a/b/(b*x^3+a)^2/x^(1/2)+1/36*(13*A*b-B*a)/a^2/b/(b*x^3+a)
/x^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {468, 296, 331, 335, 301, 648, 632, 210, 642, 211}

$$\int \frac{A+Bx^3}{x^{3/2}(a+bx^3)^3}\,dx = \frac{7(13Ab-aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{19/6}b^{5/6}} - \frac{7(13Ab-aB)\arctan\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{216a^{19/6}b^{5/6}} - \frac{7(13Ab-aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{19/6}b^{5/6}} - \frac{7(13Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{19/6}b^{5/6}} + \frac{7(13Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{19/6}b^{5/6}} - \frac{7(13Ab-aB)}{36a^3b\sqrt{x}} + \frac{13Ab-aB}{36a^2b\sqrt{x}\,(a+bx^3)} + \frac{Ab-aB}{6ab\sqrt{x}\,(a+bx^3)^2}$$

```
[In] Int[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^3),x]
```

```
[Out] (-7*(13*A*b - a*B))/(36*a^3*b*Sqrt[x]) + (A*b - a*B)/(6*a*b*Sqrt[x]*(a + b*
x^3)^2) + (13*A*b - a*B)/(36*a^2*b*Sqrt[x]*(a + b*x^3)) + (7*(13*A*b - a*B)
*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(19/6)*b^(5/6)) - (7
*(13*A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(19/6
)*b^(5/6)) - (7*(13*A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(1
9/6)*b^(5/6)) - (7*(13*A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqr
t[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(19/6)*b^(5/6)) + (7*(13*A*b - a*B)*Log[a
^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(19/6
)*b^(5/6))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{Ab-aB}{6ab\sqrt{x}\,(a+bx^3)^2} + \frac{\left(\frac{13Ab}{2}-\frac{aB}{2}\right)\int \frac{1}{x^{3/2}(a+bx^3)^2}\,dx}{6ab}$$

$$= \frac{Ab-aB}{6ab\sqrt{x}\,(a+bx^3)^2} + \frac{13Ab-aB}{36a^2b\sqrt{x}\,(a+bx^3)} + \frac{(7(13Ab-aB))\int \frac{1}{x^{3/2}(a+bx^3)}\,dx}{72a^2b}$$

$$= -\frac{7(13Ab-aB)}{36a^3b\sqrt{x}} + \frac{Ab-aB}{6ab\sqrt{x}\,(a+bx^3)^2} + \frac{13Ab-aB}{36a^2b\sqrt{x}\,(a+bx^3)} - \frac{(7(13Ab-aB))\int \frac{x^{3/2}}{a+bx^3}\,dx}{72a^3}$$

$$= -\frac{7(13Ab-aB)}{36a^3b\sqrt{x}} + \frac{Ab-aB}{6ab\sqrt{x}\,(a+bx^3)^2} + \frac{13Ab-aB}{36a^2b\sqrt{x}\,(a+bx^3)} - \frac{(7(13Ab-aB))\text{Subst}\left(\int \frac{x^4}{a+bx^6}\,dx, x, \sqrt{x}\right)}{36a^3}$$

$$= -\frac{7(13Ab-aB)}{36a^3b\sqrt{x}} + \frac{Ab-aB}{6ab\sqrt{x}\,(a+bx^3)^2} + \frac{13Ab-aB}{36a^2b\sqrt{x}\,(a+bx^3)} - \frac{(7(13Ab-aB))\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2}+\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{108a^{19/6}b^{2/3}} - \frac{(7(13Ab-aB))\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{108a^{19/6}b^{2/3}} - \frac{(7(13Ab-aB))\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{108a^3b^{2/3}}$$

$$= -\frac{7(13Ab-aB)}{36a^3b\sqrt{x}} + \frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2} + \frac{13Ab-aB}{36a^2b\sqrt{x}\left(a+bx^3\right)} - \frac{7(13Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{19/6}b^{5/6}} - \frac{(7(13Ab-aB))\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{144\sqrt{3}a^{19/6}b^{5/6}} + \frac{(7(13Ab-aB))\text{Subst}\left(\int \frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{144\sqrt{3}a^{19/6}b^{5/6}} - \frac{(7(13Ab-aB))\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{432a^3b^{2/3}} - \frac{(7(13Ab-aB))\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{432a^3b^{2/3}}$$

$$= -\frac{7(13Ab-aB)}{36a^3b\sqrt{x}} + \frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2} + \frac{13Ab-aB}{36a^2b\sqrt{x}\left(a+bx^3\right)} - \frac{7(13Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{19/6}b^{5/6}} - \frac{7(13Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{19/6}b^{5/6}} + \frac{7(13Ab-aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{19/6}b^{5/6}} - \frac{(7(13Ab-aB))\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx,x,1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{216\sqrt{3}a^{19/6}b^{5/6}} + \frac{(7(13Ab-aB))\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx,x,1+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{216\sqrt{3}a^{19/6}b^{5/6}}$$

$$= -\frac{7(13Ab-aB)}{36a^3b\sqrt{x}} + \frac{Ab-aB}{6ab\sqrt{x}\,(a+bx^3)^2} + \frac{13Ab-aB}{36a^2b\sqrt{x}\,(a+bx^3)} + \frac{7(13Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{19/6}b^{5/6}} - \frac{7(13Ab-aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{19/6}b^{5/6}} - \frac{7(13Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{19/6}b^{5/6}} - \frac{7(13Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{19/6}b^{5/6}} + \frac{7(13Ab-aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{19/6}b^{5/6}}$$

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.59

$$\int \frac{A+Bx^3}{x^{3/2}\,(a+bx^3)^3}\,dx = \frac{-\frac{6\sqrt[6]{a}\left(91Ab^2x^6+a^2(72A-13Bx^3)+abx^3(169A-7Bx^3)\right)}{\sqrt{x}(a+bx^3)^2} + \frac{14(-13Ab+aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{b^{5/6}} + \frac{7(13Ab-a}{}}{216a^{19/6}}$$

```
[In] Integrate[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^3),x]
```

```
[Out] ((-6*a^(1/6)*(91*A*b^2*x^6 + a^2*(72*A - 13*B*x^3) + a*b*x^3*(169*A - 7*B*x
^3)))/(Sqrt[x]*(a + b*x^3)^2) + (14*(-13*A*b + a*B)*ArcTan[(b^(1/6)*Sqrt[x]
)/a^(1/6)])/b^(5/6) + (7*(13*A*b - a*B)*ArcTan[(a^(1/3) - b^(1/3)*x)/(a^(1/
6)*b^(1/6)*Sqrt[x])])/b^(5/6) + (7*Sqrt[3]*(13*A*b - a*B)*ArcTanh[(Sqrt[3]*
a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/b^(5/6))/(216*a^(19/6))
```

Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.67

method	result
derivativedivides	$-\frac{2A}{a^3\sqrt{x}}-\frac{2\left(\frac{\left(\frac{19}{72}b^2A-\frac{7}{72}abB\right)x^{\frac{11}{2}}+\frac{a(25Ab-13Ba)x^{\frac{5}{2}}}{72}}{\left(b\,x^3+a\right)^2}+\left(\frac{91Ab}{72}-\frac{7Ba}{72}\right)\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(\cdots\right.}{6\cdots}\cdots\right.\right.}{a^3}$
default	$-\frac{2A}{a^3\sqrt{x}}-\frac{2\left(\frac{\left(\frac{19}{72}b^2A-\frac{7}{72}abB\right)x^{\frac{11}{2}}+\frac{a(25Ab-13Ba)x^{\frac{5}{2}}}{72}}{\left(b\,x^3+a\right)^2}+\left(\frac{91Ab}{72}-\frac{7Ba}{72}\right)\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(\cdots\right.}{6\cdots}\cdots\right.\right.}{a^3}$
risch	$-\frac{2A}{a^3\sqrt{x}}-\frac{\frac{2\left(\frac{19}{72}b^2A-\frac{7}{72}abB\right)x^{\frac{11}{2}}+\frac{a(25Ab-13Ba)x^{\frac{5}{2}}}{36}}{\left(b\,x^3+a\right)^2}+2\left(\frac{91Ab}{72}-\frac{7Ba}{72}\right)\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\arctan\left(-\cdots\right.}{6b\cdots}\cdots\right.}{a^3}$

```
[In] int((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*A/a^3/x^(1/2)-2/a^3*(((19/72*b^2*A-7/72*a*b*B)*x^(11/2)+1/72*a*(25*A*b-1
3*B*a)*x^(5/2))/(b*x^3+a)^2+(91/72*A*b-7/72*B*a)*(1/12/a*3^(1/2)*(a/b)^(5/6
)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))+1/6/b/(a/b)^(1/6)*arctan(-3
^(1/2)+2*x^(1/2)/(a/b)^(1/6))-1/12/a*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)
^(1/6)*x^(1/2)+(a/b)^(1/3))+1/6/b/(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+
3^(1/2))+1/3/b/(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1934 vs. $2(254) = 508$.

Time = 0.37 (sec) , antiderivative size = 1934, normalized size of antiderivative = 5.51

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/432*(14*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*(-(B^6*a^6 - 78*A*B^5*a^5*b
+ 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2
227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(1/6)*log(16807*a^16*b^4*
(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3
+ 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5
```

))^(5/6) - 16807*(B^5*a^5 - 65*A*B^4*a^4*b + 1690*A^2*B^3*a^3*b^2 - 21970*A^3*B^2*a^2*b^3 + 142805*A^4*B*a*b^4 - 371293*A^5*b^5)*sqrt(x)) - 14*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(1/6)*log(-16807*a^16*b^4*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(5/6) - 16807*(B^5*a^5 - 65*A*B^4*a^4*b + 1690*A^2*B^3*a^3*b^2 - 21970*A^3*B^2*a^2*b^3 + 142805*A^4*B*a*b^4 - 371293*A^5*b^5)*sqrt(x)) + 7*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x - sqrt(-3)*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x))*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(1/6)*log(16807/2*(sqrt(-3)*a^16*b^4 + a^16*b^4)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(5/6) - 16807*(B^5*a^5 - 65*A*B^4*a^4*b + 1690*A^2*B^3*a^3*b^2 - 21970*A^3*B^2*a^2*b^3 + 142805*A^4*B*a*b^4 - 371293*A^5*b^5)*sqrt(x)) - 7*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x - sqrt(-3)*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x))*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(1/6)*log(-16807/2*(sqrt(-3)*a^16*b^4 + a^16*b^4)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(5/6) - 16807*(B^5*a^5 - 65*A*B^4*a^4*b + 1690*A^2*B^3*a^3*b^2 - 21970*A^3*B^2*a^2*b^3 + 142805*A^4*B*a*b^4 - 371293*A^5*b^5)*sqrt(x)) - 7*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x + sqrt(-3)*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x))*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(1/6)*log(16807/2*(sqrt(-3)*a^16*b^4 - a^16*b^4)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(5/6) - 16807*(B^5*a^5 - 65*A*B^4*a^4*b + 1690*A^2*B^3*a^3*b^2 - 21970*A^3*B^2*a^2*b^3 + 142805*A^4*B*a*b^4 - 371293*A^5*b^5)*sqrt(x)) + 7*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x + sqrt(-3)*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x))*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(1/6)*log(-16807/2*(sqrt(-3)*a^16*b^4 - a^16*b^4)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(5/6) - 16807*(B^5*a^5 - 65*A*B^4*a^4*b + 1690*A^2*B^3*a^3*b^2 - 21970*A^3*B^2*a^2*b^3 + 142805*A^4*B*a*b^4 - 371293*A^5*b^5)*sqrt(x)) - 12*(7*(B*a*b - 13*A*b^2)*x^6 + 13*(B*a^2 - 13*A*a*b)*x^3 - 72*A*a^2)*sqrt(x))/(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx^3}{x^{3/2} (a + bx^3)^3} \, dx = \text{Timed out}$$

```
[In] integrate((B*x**3+A)/x**(3/2)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.78

$$\int \frac{A + Bx^3}{x^{3/2} (a + bx^3)^3} \, dx = \frac{7 \, (Bab - 13 \, Ab^2) x^6 + 13 \, (Ba^2 - 13 \, Aab) x^3 - 72 \, Aa^2}{36 \, \left(a^3 b^2 x^{\frac{13}{2}} + 2 \, a^4 b x^{\frac{7}{2}} + a^5 \sqrt{x}\right)} - \frac{7 \, (Ba - 13 \, Ab) \left(\frac{\sqrt{3} \log\left(\sqrt{3} a^{\frac{1}{6}} b^{\frac{1}{6}} \sqrt{x} + b^{\frac{1}{3}} x + a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}} b^{\frac{5}{6}}} - \frac{\sqrt{3} \log\left(-\sqrt{3} a^{\frac{1}{6}} b^{\frac{1}{6}} \sqrt{x} + b^{\frac{1}{3}} x + a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}} b^{\frac{5}{6}}} - \frac{2 \arctan\left(\frac{\sqrt{3} a^{\frac{1}{6}} b^{\frac{1}{6}} + 2 \, b^{\frac{1}{3}} \sqrt{x}}{\sqrt{a^{\frac{1}{3}} b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}} \sqrt{a^{\frac{1}{3}} b^{\frac{1}{3}}}} - \frac{2 \arctan\left(-\cdots\right.}{b^{\frac{2}{3}} \cdots}\right.}{432 \, a^3}$$

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/36*(7*(B*a*b - 13*A*b^2)*x^6 + 13*(B*a^2 - 13*A*a*b)*x^3 - 72*A*a^2)/(a^3
*b^2*x^(13/2) + 2*a^4*b*x^(7/2) + a^5*sqrt(x)) - 7/432*(B*a - 13*A*b)*(sqrt
(3)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(
5/6)) - sqrt(3)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))
/(a^(1/6)*b^(5/6)) - 2*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))
/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 2*arctan(-(sqrt(3
)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt
(a^(1/3)*b^(1/3))) - 4*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(b^(2/
3)*sqrt(a^(1/3)*b^(1/3))))/a^3
```

Giac [A] (verification not implemented)

none

Time = 0.69 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^3}\,dx = \frac{7\,(Ba-13\,Ab)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,\left(ab^5\right)^{\frac{1}{6}}a^3} + \frac{7\,(Ba-13\,Ab)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,\left(ab^5\right)^{\frac{1}{6}}a^3} + \frac{7\left(Ba\left(\frac{a}{b}\right)^{\frac{5}{6}}-13\,Ab\left(\frac{a}{b}\right)^{\frac{5}{6}}\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108\,a^4} - \frac{2\,A}{a^3\sqrt{x}} + \frac{7\,Babx^{\frac{11}{2}}-19\,Ab^2x^{\frac{11}{2}}+13\,Ba^2x^{\frac{5}{2}}-25\,Aabx^{\frac{5}{2}}}{36\,\left(bx^3+a\right)^2a^3} - \frac{7\,\sqrt{3}\left(\left(ab^5\right)^{\frac{5}{6}}Ba-13\,\left(ab^5\right)^{\frac{5}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^4b^5} + \frac{7\,\sqrt{3}\left(\left(ab^5\right)^{\frac{5}{6}}Ba-13\,\left(ab^5\right)^{\frac{5}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^4b^5}$$

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 7/216*(B*a - 13*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/
((a*b^5)^(1/6)*a^3) + 7/216*(B*a - 13*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2
*sqrt(x))/(a/b)^(1/6))/((a*b^5)^(1/6)*a^3) + 7/108*(B*a*(a/b)^(5/6) - 13*A*
b*(a/b)^(5/6))*arctan(sqrt(x)/(a/b)^(1/6))/a^4 - 2*A/(a^3*sqrt(x)) + 1/36*(
7*B*a*b*x^(11/2) - 19*A*b^2*x^(11/2) + 13*B*a^2*x^(5/2) - 25*A*a*b*x^(5/2))
/((b*x^3 + a)^2*a^3) - 7/432*sqrt(3)*((a*b^5)^(5/6)*B*a - 13*(a*b^5)^(5/6)*
A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^4*b^5) + 7/432*s
qrt(3)*((a*b^5)^(5/6)*B*a - 13*(a*b^5)^(5/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b
)^(1/6) + x + (a/b)^(1/3))/(a^4*b^5)
```

Mupad [B] (verification not implemented)

Time = 7.38 (sec) , antiderivative size = 1786, normalized size of antiderivative = 5.09

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^3}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x^3)/(x^(3/2)*(a + b*x^3)^3),x)
```

```
[Out] (atan((((13*A*b - B*a)^2*(28229306112*B^3*a^24*b^3 - 62019785528064*A^3*a^2
1*b^6 - 1100942938368*A*B^2*a^23*b^4 + 14312258198784*A^2*B*a^22*b^5 + (343
*x^(1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*
B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/
6))*1i)/((-a)^(19/3)*b^(5/3)) + ((13*A*b - B*a)^2*(62019785528064*A^3*a^21
*b^6 - 28229306112*B^3*a^24*b^3 + 1100942938368*A*B^2*a^23*b^4 - 1431225819
```

8784*A^2*B*a^22*b^5 + (343*x^(1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 2156456413259366 4*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/6)))*1i)/((-a)^(19/3)*b^(5/3)))/(((13*A*b - B*a)^2*(28229306112*B^3*a^24*b^3 - 62019785528064*A^3*a^21*b^6 - 1100942938368*A*B^2*a^23*b^4 + 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/6))))/((-a)^(19/3)*b^(5/3)) - ((13*A*b - B*a)^2*(62019785528064*A^3*a^21*b^6 - 28229306112*B^3*a^24*b^3 + 1100942938368*A*B^2*a^23*b^4 - 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/6))))/((-a)^(19/3)*b^(5/3))))*(13*A*b - B*a)*7i)/(108*(-a)^(19/6)*b^(5/6)) - ((2*A)/a + (13*x^3*(13*A*b - B*a))/(36*a^2) + (7*b*x^6*(13*A*b - B*a))/(36*a^3))/(a^2*x^(1/2) + b^2*x^(13/2) + 2*a*b*x^(7/2)) + (atan(((((3^(1/2)*1i)/2 - 1/2)^2*(13*A*b - B*a)^2*(28229306112*B^3*a^24*b^3 - 62019785528064*A^3*a^21*b^6 - 1100942938368*A*B^2*a^23*b^4 + 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/6)))*1i)/((-a)^(19/3)*b^(5/3)) + (((3^(1/2)*1i)/2 - 1/2)^2*(13*A*b - B*a)^2*(62019785528064*A^3*a^21*b^6 - 28229306112*B^3*a^24*b^3 + 1100942938368*A*B^2*a^23*b^4 - 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/6)))*1i)/((-a)^(19/3)*b^(5/3)))/((((3^(1/2)*1i)/2 - 1/2)^2*(13*A*b - B*a)^2*(28229306112*B^3*a^24*b^3 - 62019785528064*A^3*a^21*b^6 - 1100942938368*A*B^2*a^23*b^4 + 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/6))))/((-a)^(19/3)*b^(5/3)) - (((3^(1/2)*1i)/2 - 1/2)^2*(13*A*b - B*a)^2*(62019785528064*A^3*a^21*b^6 - 28229306112*B^3*a^24*b^3 + 1100942938368*A*B^2*a^23*b^4 - 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/6))))/((-a)^(19/3)*b^(5/3))))*((3^(1/2)*1i)/2 - 1/2)*(13*A*b - B*a)*7i)/(108*(-a)^(19/6)*b^(5/6)) + (atan(((((3^(1/2)*1i)/2 + 1/2)^2*(13*A*b - B*a)^2*(28229306112*B^3*a^24*b^3 - 62019785528064*A^3*a^21*b^6 - 1100942938368*A*B^2*a^23*b^4 + 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/6)))*1i)/((-a)^(19/3)*b^(5/3)) + (((3^(1/2)*1i)/2 + 1/2)^2*(13*A*b - B*a)^2*(62019785528064*A^3*a^21*b^6 - 28229306112*B^3*a^24*b^3 + 1100942938368*A*B^2*a^23*b^4 - 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/6)))*1i)/((-a)^(19/3)*b^(5/3)))/((((3^(1/2)*1i)/2 + 1/2)

```
^2*(13*A*b - B*a)^2*(28229306112*B^3*a^24*b^3 - 62019785528064*A^3*a^21*b^6
 - 1100942938368*A*B^2*a^23*b^4 + 14312258198784*A^2*B*a^22*b^5 + (343*x^(1
/2)*((3^(1/2)*1i)/2 + 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6
+ 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696
*(-a)^(19/6)*b^(5/6))))/((-a)^(19/3)*b^(5/3)) - ((((3^(1/2)*1i)/2 + 1/2)^2*(
13*A*b - B*a)^2*(62019785528064*A^3*a^21*b^6 - 28229306112*B^3*a^24*b^3 + 1
100942938368*A*B^2*a^23*b^4 - 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*
((3^(1/2)*1i)/2 + 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 82
9406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a
)^(19/6)*b^(5/6))))/((-a)^(19/3)*b^(5/3))))*((3^(1/2)*1i)/2 + 1/2)*(13*A*b
- B*a)*7i)/(108*(-a)^(19/6)*b^(5/6))
```

3.177 $\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 129

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3} dx = \frac{-5Ab+aB}{4a^3bx^{3/2}} + \frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2} + \frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)} - \frac{(5Ab-aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{4a^{7/2}\sqrt{b}}$$

```
[Out] 1/4*(-5*A*b+B*a)/a^3/b/x^(3/2)+1/6*(A*b-B*a)/a/b/x^(3/2)/(b*x^3+a)^2+1/12*(
5*A*b-B*a)/a^2/b/x^(3/2)/(b*x^3+a)-1/4*(5*A*b-B*a)*arctan(x^(3/2)*b^(1/2)/a
^(1/2))/a^(7/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {468, 296, 331, 335, 281, 211}

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3} dx = -\frac{(5Ab-aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{4a^{7/2}\sqrt{b}} - \frac{5Ab-aB}{4a^3bx^{3/2}} + \frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)} + \frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}$$

```
[In] Int[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^3),x]
```

```
[Out] -1/4*(5*A*b - a*B)/(a^3*b*x^(3/2)) + (A*b - a*B)/(6*a*b*x^(3/2)*(a + b*x^3)
^2) + (5*A*b - a*B)/(12*a^2*b*x^(3/2)*(a + b*x^3)) - ((5*A*b - a*B)*ArcTan[
(Sqrt[b]*x^(3/2))/Sqrt[a]])/(4*a^(7/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\text{integral} = \frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}+\frac{\left(\frac{15Ab}{2}-\frac{3aB}{2}\right)\int\frac{1}{x^{5/2}(a+bx^3)^2}\,dx}{6ab}$$

$$=\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}+\frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)}+\frac{(3(5Ab-aB))\int\frac{1}{x^{5/2}(a+bx^3)}\,dx}{8a^2b}$$

$$=-\frac{5Ab-aB}{4a^3bx^{3/2}}+\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}+\frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)}-\frac{(3(5Ab-aB))\int\frac{\sqrt{x}}{a+bx^3}\,dx}{8a^3}$$

$$=-\frac{5Ab-aB}{4a^3bx^{3/2}}+\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}+\frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)}-\frac{(3(5Ab-aB))\text{Subst}\left(\int\frac{x^2}{a+bx^6}\,dx,x,\sqrt{x}\right)}{4a^3}$$

$$=-\frac{5Ab-aB}{4a^3bx^{3/2}}+\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}+\frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)}-\frac{(5Ab-aB)\text{Subst}\left(\int\frac{1}{a+bx^2}\,dx,x,x^{3/2}\right)}{4a^3}$$

$$=-\frac{5Ab-aB}{4a^3bx^{3/2}}+\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}+\frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)}-\frac{(5Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{4a^{7/2}\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.79

$$\int\frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3}\,dx=\frac{-8a^2A-25aAbx^3+5a^2Bx^3-15Ab^2x^6+3abBx^6}{12a^3x^{3/2}\left(a+bx^3\right)^2}+\frac{(-5Ab+aB)\arctan\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{4a^{7/2}\sqrt{b}}$$

```
[In] Integrate[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^3),x]
```

```
[Out] (-8*a^2*A - 25*a*A*b*x^3 + 5*a^2*B*x^3 - 15*A*b^2*x^6 + 3*a*b*B*x^6)/(12*a^
3*x^(3/2)*(a + b*x^3)^2) + ((-5*A*b + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]
])/(4*a^(7/2)*Sqrt[b])
```

Maple [A] (verified)

Time = 4.22 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$-\frac{2\left(\frac{\left(\frac{7}{8}b^2A-\frac{3}{8}abB\right)x^{\frac{9}{2}}+\frac{a(9Ab-5Ba)x^{\frac{3}{2}}}{8}}{\left(b\,x^3+a\right)^2}+\frac{3(5Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{3a^3}-\frac{2A}{3a^3x^{\frac{3}{2}}}$	86
default	$-\frac{2\left(\frac{\left(\frac{7}{8}b^2A-\frac{3}{8}abB\right)x^{\frac{9}{2}}+\frac{a(9Ab-5Ba)x^{\frac{3}{2}}}{8}}{\left(b\,x^3+a\right)^2}+\frac{3(5Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{3a^3}-\frac{2A}{3a^3x^{\frac{3}{2}}}$	86
risch	$-\frac{2A}{3a^3x^{\frac{3}{2}}}-\frac{\frac{\frac{2\left(\frac{7}{8}b^2A-\frac{3}{8}abB\right)x^{\frac{9}{2}}}{3}+\frac{a(9Ab-5Ba)x^{\frac{3}{2}}}{12}}{\left(b\,x^3+a\right)^2}+\frac{(5Ab-Ba)\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{4\sqrt{ab}}}{a^3}$	87

```
[In] int((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/a^3*(((7/8*b^2*A-3/8*a*b*B)*x^(9/2)+1/8*a*(9*A*b-5*B*a)*x^(3/2))/(b*x^
3+a)^2+3/8*(5*A*b-B*a)/(a*b)^(1/2)*arctan(b*x^(3/2)/(a*b)^(1/2)))-2/3*A/a^3
/x^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.69

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3}\,dx = \left[\frac{3\left((Bab^2-5\,Ab^3)x^8+2\,(Ba^2b-5\,Aab^2)x^5+(Ba^3-5\,Aa^2b)x^2\right)\sqrt{-ab}\log\left(\frac{bx^3+2\sqrt{}}{b}\right.}{24\left(a^4b^3x^8+2\,a^5b^2\right.}\right.$$

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(3*((B*a*b^2 - 5*A*b^3)*x^8 + 2*(B*a^2*b - 5*A*a*b^2)*x^5 + (B*a^3 -
5*A*a^2*b)*x^2)*sqrt(-a*b)*log((b*x^3 + 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 +
a)) + 2*(3*(B*a^2*b^2 - 5*A*a*b^3)*x^6 - 8*A*a^3*b + 5*(B*a^3*b - 5*A*a^2*b
^2)*x^3)*sqrt(x))/(a^4*b^3*x^8 + 2*a^5*b^2*x^5 + a^6*b*x^2), 1/12*(3*((B*a*
b^2 - 5*A*b^3)*x^8 + 2*(B*a^2*b - 5*A*a*b^2)*x^5 + (B*a^3 - 5*A*a^2*b)*x^2)
*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a) + (3*(B*a^2*b^2 - 5*A*a*b^3)*x^6 - 8
*A*a^3*b + 5*(B*a^3*b - 5*A*a^2*b^2)*x^3)*sqrt(x))/(a^4*b^3*x^8 + 2*a^5*b^2
*x^5 + a^6*b*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate((B*x**3+A)/x**(5/2)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3}\,dx = \frac{3\,(Bab-5\,Ab^2)x^6+5\,(Ba^2-5\,Aab)x^3-8\,Aa^2}{12\,\left(a^3b^2x^{\frac{15}{2}}+2\,a^4bx^{\frac{9}{2}}+a^5x^{\frac{3}{2}}\right)} + \frac{(Ba-5\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^3}$$

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(3*(B*a*b - 5*A*b^2)*x^6 + 5*(B*a^2 - 5*A*a*b)*x^3 - 8*A*a^2)/(a^3*b^2
*x^(15/2) + 2*a^4*b*x^(9/2) + a^5*x^(3/2)) + 1/4*(B*a - 5*A*b)*arctan(b*x^(
3/2)/sqrt(a*b))/(sqrt(a*b)*a^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.68

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3}\,dx = \frac{(Ba-5\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}a^3} - \frac{2\,A}{3\,a^3x^{\frac{3}{2}}} + \frac{3\,Babx^{\frac{9}{2}}-7\,Ab^2x^{\frac{9}{2}}+5\,Ba^2x^{\frac{3}{2}}-9\,Aabx^{\frac{3}{2}}}{12\,\left(bx^3+a\right)^2a^3}$$

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(B*a - 5*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/3*A/(a^3*
x^(3/2)) + 1/12*(3*B*a*b*x^(9/2) - 7*A*b^2*x^(9/2) + 5*B*a^2*x^(3/2) - 9*A*
a*b*x^(3/2))/((b*x^3 + a)^2*a^3)
```

Mupad [B] (verification not implemented)

Time = 7.18 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.26

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3}\,dx = -\frac{\frac{2\,A}{3\,a}+\frac{5\,x^3\,(5\,A\,b-B\,a)}{12\,a^2}+\frac{b\,x^6\,(5\,A\,b-B\,a)}{4\,a^3}}{a^2\,x^{3/2}+b^2\,x^{15/2}+2\,a\,b\,x^{9/2}} - \frac{\mathrm{atan}\left(\frac{8\,a^{7/2}\,\sqrt{b}\,x^{3/2}\,\left(86400\,A^2\,a^9\,b^5-34560\,A\,B\,a^{10}\,b^4+3456\,B^2\,a^{11}\,b^3\right)}{(5\,A\,b-B\,a)\,\left(138240\,A\,a^{13}\,b^4-27648\,B\,a^{14}\,b^3\right)}\right)\,(5\,A\,b-B\,a)}{4\,a^{7/2}\,\sqrt{b}}$$

```
[In] int((A + B*x^3)/(x^(5/2)*(a + b*x^3)^3),x)
```

```
[Out] - ((2*A)/(3*a) + (5*x^3*(5*A*b - B*a))/(12*a^2) + (b*x^6*(5*A*b - B*a))/(4*
a^3))/(a^2*x^(3/2) + b^2*x^(15/2) + 2*a*b*x^(9/2)) - (atan((8*a^(7/2)*b^(1/
2)*x^(3/2)*(86400*A^2*a^9*b^5 + 3456*B^2*a^11*b^3 - 34560*A*B*a^10*b^4))/((
5*A*b - B*a)*(138240*A*a^13*b^4 - 27648*B*a^14*b^3)))*(5*A*b - B*a))/(4*a^(
7/2)*b^(1/2))
```

3.178 $\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 351

$$\begin{aligned}\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^3}\,dx &= -\frac{11(17Ab-5aB)}{180a^3bx^{5/2}}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}\\&\quad+\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}+\frac{11(17Ab-5aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{23/6}\sqrt[6]{b}}\\&\quad-\frac{11(17Ab-5aB)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{23/6}\sqrt[6]{b}}-\frac{11(17Ab-5aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{23/6}\sqrt[6]{b}}\\&\quad+\frac{11(17Ab-5aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{23/6}\sqrt[6]{b}}\\&\quad-\frac{11(17Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{23/6}\sqrt[6]{b}}\end{aligned}$$

```
[Out] -11/180*(17*A*b-5*B*a)/a^3/b/x^(5/2)+1/6*(A*b-B*a)/a/b/x^(5/2)/(b*x^3+a)^2+
1/36*(17*A*b-5*B*a)/a^2/b/x^(5/2)/(b*x^3+a)-11/108*(17*A*b-5*B*a)*arctan(b^
(1/6)*x^(1/2)/a^(1/6))/a^(23/6)/b^(1/6)-11/216*(17*A*b-5*B*a)*arctan(-3^(1/
2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(23/6)/b^(1/6)-11/216*(17*A*b-5*B*a)*arctan
(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(23/6)/b^(1/6)+11/432*(17*A*b-5*B*a)*
ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(23/6)/b^(1/6)*3^(1
/2)-11/432*(17*A*b-5*B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/2)*x^(1
/2))/a^(23/6)/b^(1/6)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {468, 296, 331, 335, 215, 648, 632, 210, 642, 211}

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^3}\,dx = \frac{11(17Ab-5aB)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{23/6}\sqrt[6]{b}} - \frac{11(17Ab-5aB)\arctan\left(\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}+\sqrt{3}\right)}{216a^{23/6}\sqrt[6]{b}} - \frac{11(17Ab-5aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{23/6}\sqrt[6]{b}} + \frac{11(17Ab-5aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{23/6}\sqrt[6]{b}} - \frac{11(17Ab-5aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{23/6}\sqrt[6]{b}} - \frac{11(17Ab-5aB)}{180a^3bx^{5/2}} + \frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)} + \frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}$$

```
[In] Int[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^3),x]
```

```
[Out] (-11*(17*A*b - 5*a*B))/(180*a^3*b*x^(5/2)) + (A*b - a*B)/(6*a*b*x^(5/2)*(a
+ b*x^3)^2) + (17*A*b - 5*a*B)/(36*a^2*b*x^(5/2)*(a + b*x^3)) + (11*(17*A*b
 - 5*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(23/6)*b^(1
/6)) - (11*(17*A*b - 5*a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/
(216*a^(23/6)*b^(1/6)) - (11*(17*A*b - 5*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1
/6)])/(108*a^(23/6)*b^(1/6)) + (11*(17*A*b - 5*a*B)*Log[a^(1/3) - Sqrt[3]*a
^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(23/6)*b^(1/6)) - (11*(
17*A*b - 5*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])
/(144*Sqrt[3]*a^(23/6)*b^(1/6))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}+\frac{\left(\frac{17Ab}{2}-\frac{5aB}{2}\right)\int\frac{1}{x^{7/2}\left(a+bx^3\right)^2}\,dx}{6ab}\\
&= \frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}+\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}+\frac{(11(17Ab-5aB))\int\frac{1}{x^{7/2}\left(a+bx^3\right)}\,dx}{72a^2b}\\
&= -\frac{11(17Ab-5aB)}{180a^3bx^{5/2}}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}\\
&\quad+\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}-\frac{(11(17Ab-5aB))\int\frac{1}{\sqrt{x}\left(a+bx^3\right)}\,dx}{72a^3}\\
&= -\frac{11(17Ab-5aB)}{180a^3bx^{5/2}}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}+\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}\\
&\quad-\frac{(11(17Ab-5aB))\text{Subst}\left(\int\frac{1}{a+bx^6}\,dx,x,\sqrt{x}\right)}{36a^3}\\
&= -\frac{11(17Ab-5aB)}{180a^3bx^{5/2}}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}+\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}\\
&\quad-\frac{(11(17Ab-5aB))\text{Subst}\left(\int\frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{108a^{23/6}}\\
&\quad-\frac{(11(17Ab-5aB))\text{Subst}\left(\int\frac{\sqrt[6]{a}+\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{108a^{23/6}}\\
&\quad-\frac{(11(17Ab-5aB))\text{Subst}\left(\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{108a^{11/3}}
\end{aligned}$$

$$=-\frac{11(17Ab-5aB)}{180a^3bx^{5/2}}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}+\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}-\frac{11(17Ab-5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{23/6}\sqrt[6]{b}}-\frac{(11(17Ab-5aB))\text{Subst}\left(\int\frac{1}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{432a^{11/3}}-\frac{(11(17Ab-5aB))\text{Subst}\left(\int\frac{1}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{432a^{11/3}}+\frac{(11(17Ab-5aB))\text{Subst}\left(\int\frac{-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{144\sqrt{3}a^{23/6}\sqrt[6]{b}}-\frac{(11(17Ab-5aB))\text{Subst}\left(\int\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{144\sqrt{3}a^{23/6}\sqrt[6]{b}}$$

$$=-\frac{11(17Ab-5aB)}{180a^3bx^{5/2}}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}+\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}-\frac{11(17Ab-5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{23/6}\sqrt[6]{b}}+\frac{11(17Ab-5aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{23/6}\sqrt[6]{b}}-\frac{11(17Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{23/6}\sqrt[6]{b}}-\frac{(11(17Ab-5aB))\text{Subst}\left(\int\frac{1}{-\frac{1}{3}-x^2}\,dx,x,1-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{216\sqrt{3}a^{23/6}\sqrt[6]{b}}+\frac{(11(17Ab-5aB))\text{Subst}\left(\int\frac{1}{-\frac{1}{3}-x^2}\,dx,x,1+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt{3}\sqrt[6]{a}}\right)}{216\sqrt{3}a^{23/6}\sqrt[6]{b}}$$

$$= -\frac{11(17Ab-5aB)}{180a^3bx^{5/2}} + \frac{Ab-aB}{6abx^{5/2}(a+bx^3)^2} + \frac{17Ab-5aB}{36a^2bx^{5/2}(a+bx^3)} + \frac{11(17Ab-5aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{23/6}\sqrt[6]{b}} - \frac{11(17Ab-5aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{23/6}\sqrt[6]{b}} - \frac{11(17Ab-5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{23/6}\sqrt[6]{b}} + \frac{11(17Ab-5aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{23/6}\sqrt[6]{b}} - \frac{11(17Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{23/6}\sqrt[6]{b}}$$

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.60

$$\int \frac{A+Bx^3}{x^{7/2}(a+bx^3)^3}\,dx = \frac{-\frac{6a^{5/6}(187Ab^2x^6+a^2(72A-85Bx^3)+abx^3(289A-55Bx^3))}{x^{5/2}(a+bx^3)^2} + \frac{110(-17Ab+5aB)\arctan\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{\sqrt[6]{b}} + \frac{55(17A\cdots}{\cdots}}{1080a^{23/6}}$$

```
[In] Integrate[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^3),x]
```

```
[Out] ((-6*a^(5/6)*(187*A*b^2*x^6 + a^2*(72*A - 85*B*x^3) + a*b*x^3*(289*A - 55*B
*x^3)))/(x^(5/2)*(a + b*x^3)^2) + (110*(-17*A*b + 5*a*B)*ArcTan[(b^(1/6)*Sq
rt[x])/a^(1/6)])/b^(1/6) + (55*(17*A*b - 5*a*B)*ArcTan[(a^(1/3) - b^(1/3)*x
)/(a^(1/6)*b^(1/6)*Sqrt[x])])/b^(1/6) + (55*Sqrt[3]*(-17*A*b + 5*a*B)*ArcTa
nh[(Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x])/(a^(1/3) + b^(1/3)*x)])/b^(1/6))/(1080
*a^(23/6))
```

Maple [A] (verified)

Time = 4.39 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.68

method	result
derivativedivides	$-\frac{2A}{5a^3x^{\frac{5}{2}}}-\frac{2\left(\frac{\left(\frac{23}{72}b^2A-\frac{11}{72}abB\right)x^{\frac{7}{2}}+\frac{a(29Ab-17Ba)\sqrt{x}}{72}}{\left(bx^3+a\right)^2}+\frac{11(17Ab-5Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{\cdots}\right)}{12a}\cdots\right)}{\cdots}\right)}{\cdots}$
default	$-\frac{2A}{5a^3x^{\frac{5}{2}}}-\frac{2\left(\frac{\left(\frac{23}{72}b^2A-\frac{11}{72}abB\right)x^{\frac{7}{2}}+\frac{a(29Ab-17Ba)\sqrt{x}}{72}}{\left(bx^3+a\right)^2}+\frac{11(17Ab-5Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{\cdots}\right)}{12a}\cdots\right)}{\cdots}\right)}{\cdots}$
risch	$-\frac{2A}{5a^3x^{\frac{5}{2}}}-\frac{\frac{2\left(\frac{23}{72}b^2A-\frac{11}{72}abB\right)x^{\frac{7}{2}}+\frac{a(29Ab-17Ba)\sqrt{x}}{36}}{\left(bx^3+a\right)^2}+\frac{11(17Ab-5Ba)\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-\cdots\right)}{12a}\cdots\right)}{\cdots}}{\cdots}$

```
[In] int((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*A/a^3/x^(5/2)-2/a^3*((((23/72*b^2*A-11/72*a*b*B)*x^(7/2)+1/72*a*(29*A*b
-17*B*a)*x^(1/2))/(b*x^3+a)^2+11/72*(17*A*b-5*B*a)*(1/3/a*(a/b)^(1/6)*arcta
n(x^(1/2)/(a/b)^(1/6))-1/12/a*3^(1/2)*(a/b)^(1/6)*ln(3^(1/2)*(a/b)^(1/6)*x^
(1/2)-x-(a/b)^(1/3))+1/6/a*(a/b)^(1/6)*arctan(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6
))+1/12/a*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))
+1/6/a*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b)^(1/6)+3^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1608 vs. $2(261) = 522$.

Time = 0.28 (sec) , antiderivative size = 1608, normalized size of antiderivative = 4.58

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2160*(110*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)*(-(15625*B^6*a^6 - 31875
0*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 313203
75*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/
6)*log(11*a^4*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b
^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a
*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/6) - 11*(5*B*a - 17*A*b)*sqrt(x)) - 1
10*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)*(-(15625*B^6*a^6 - 318750*A*B^5*a^
5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2
*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/6)*log(-11
*a^4*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 1228
2500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24
137569*A^6*b^6)/(a^23*b))^(1/6) - 11*(5*B*a - 17*A*b)*sqrt(x)) + 55*(a^3*b^
2*x^9 + 2*a^4*b*x^6 + a^5*x^3 + sqrt(-3)*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x
^3))*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 1228
2500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24
137569*A^6*b^6)/(a^23*b))^(1/6)*log(-11*(5*B*a - 17*A*b)*sqrt(x) + 11/2*(sq
rt(-3)*a^4 + a^4)*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a
^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5
*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/6)) - 55*(a^3*b^2*x^9 + 2*a^4*b*x
^6 + a^5*x^3 + sqrt(-3)*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3))*(-(15625*B^6
*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*
b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(
a^23*b))^(1/6)*log(-11*(5*B*a - 17*A*b)*sqrt(x) - 11/2*(sqrt(-3)*a^4 + a^4)
*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500
*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 241375
69*A^6*b^6)/(a^23*b))^(1/6)) - 55*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3 - sq
rt(-3)*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3))*(-(15625*B^6*a^6 - 318750*A*B
^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^
4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/6)*lo
g(-11*(5*B*a - 17*A*b)*sqrt(x) + 11/2*(sqrt(-3)*a^4 - a^4)*(-(15625*B^6*a^6
 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3
+ 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23
*b))^(1/6)) + 55*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3 - sqrt(-3)*(a^3*b^2*x
^9 + 2*a^4*b*x^6 + a^5*x^3))*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 270937
5*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 4
2595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/6)*log(-11*(5*B*a - 17
*A*b)*sqrt(x) - 11/2*(sqrt(-3)*a^4 - a^4)*(-(15625*B^6*a^6 - 318750*A*B^5*a
^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^
2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/6)) - 12*
(11*(5*B*a*b - 17*A*b^2)*x^6 + 17*(5*B*a^2 - 17*A*a*b)*x^3 - 72*A*a^2)*sqrt
(x))/(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^3}\,dx=\text{Timed out}$$

```
[In] integrate((B*x**3+A)/x**(7/2)/(b*x**3+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^3}\,dx=\frac{11\,(5\,Bab-17\,Ab^2)x^6+17\,(5\,Ba^2-17\,Aab)x^3-72\,Aa^2}{180\left(a^3b^2x^{\frac{17}{2}}+2\,a^4bx^{\frac{11}{2}}+a^5x^{\frac{5}{2}}\right)}$$

$$+\frac{11\left(\frac{\sqrt{3}(5\,Ba-17\,Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}-\frac{\sqrt{3}(5\,Ba-17\,Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}+\frac{4\left(5\,Bab^{\frac{1}{3}}-17\,Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\cdots\right)}{432\,a^3}$$

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x, algorithm="maxima")
[Out] 1/180*(11*(5*B*a*b - 17*A*b^2)*x^6 + 17*(5*B*a^2 - 17*A*a*b)*x^3 - 72*A*a^2
)/(a^3*b^2*x^(17/2) + 2*a^4*b*x^(11/2) + a^5*x^(5/2)) + 11/432*(sqrt(3)*(5*
B*a - 17*A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a
^(5/6)*b^(1/6)) - sqrt(3)*(5*B*a - 17*A*b)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqr
t(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(5*B*a*b^(1/3) - 17*A*b^(
4/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(a
^(1/3)*b^(1/3))) + 2*(5*B*a^(4/3)*b^(1/3) - 17*A*a^(1/3)*b^(4/3))*arctan((s
qrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/
3)*sqrt(a^(1/3)*b^(1/3))) + 2*(5*B*a^(4/3)*b^(1/3) - 17*A*a^(1/3)*b^(4/3))*
arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3))
)/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))))/a^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^3}\,dx = \frac{11\,\sqrt{3}\left(5\,(ab^5)^{\frac{1}{6}}\,Ba-17\,(ab^5)^{\frac{1}{6}}\,Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^4b}$$
$$-\frac{11\,\sqrt{3}\left(5\,(ab^5)^{\frac{1}{6}}\,Ba-17\,(ab^5)^{\frac{1}{6}}\,Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^4b}$$
$$+\frac{11\left(5\,(ab^5)^{\frac{1}{6}}\,Ba-17\,(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}+2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^4b}$$
$$+\frac{11\left(5\,(ab^5)^{\frac{1}{6}}\,Ba-17\,(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}-2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{216\,a^4b}$$
$$+\frac{11\left(5\,(ab^5)^{\frac{1}{6}}\,Ba-17\,(ab^5)^{\frac{1}{6}}\,Ab\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108\,a^4b}$$
$$+\frac{11\,Babx^{\frac{7}{2}}-23\,Ab^2x^{\frac{7}{2}}+17\,Ba^2\sqrt{x}-29\,Aab\sqrt{x}}{36\,(bx^3+a)^2a^3}-\frac{2\,A}{5\,a^3x^{\frac{5}{2}}}$$

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 11/432*sqrt(3)*(5*(a*b^5)^(1/6)*B*a - 17*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqr
t(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^4*b) - 11/432*sqrt(3)*(5*(a*b^5)^(1/
6)*B*a - 17*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)
^(1/3))/(a^4*b) + 11/216*(5*(a*b^5)^(1/6)*B*a - 17*(a*b^5)^(1/6)*A*b)*arcta
n((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a^4*b) + 11/216*(5*(a*b^5
)^(1/6)*B*a - 17*(a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x
))/(a/b)^(1/6))/(a^4*b) + 11/108*(5*(a*b^5)^(1/6)*B*a - 17*(a*b^5)^(1/6)*A*
b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^4*b) + 1/36*(11*B*a*b*x^(7/2) - 23*A*b^2*
x^(7/2) + 17*B*a^2*sqrt(x) - 29*A*a*b*sqrt(x))/((b*x^3 + a)^2*a^3) - 2/5*A/
(a^3*x^(5/2))
```

Mupad [B] (verification not implemented)

Time = 7.47 (sec) , antiderivative size = 2109, normalized size of antiderivative = 6.01

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^3}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x^3)/(x^(7/2)*(a + b*x^3)^3),x)
```

```
[Out] - ((2*A)/(5*a) + (17*x^3*(17*A*b - 5*B*a))/(180*a^2) + (11*b*x^6*(17*A*b -
5*B*a))/(180*a^3))/(a^2*x^(5/2) + b^2*x^(17/2) + 2*a*b*x^(11/2)) - (atan(((
(x^(1/2)*(443639472636450816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^5 +
 230262702060441600*A^2*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6 - 5
21928791337000960*A^3*B*a^16*b^8) - (11*(17*A*b - 5*B*a)*(51243917694905548
8*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b^5 + 132985945575014400*A*B^2*
a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(1/6)))*(
17*A*b - 5*B*a)*11i)/(216*(-a)^(23/6)*b^(1/6)) + ((x^(1/2)*(443639472636450
816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^5 + 230262702060441600*A^2*B
^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6 - 521928791337000960*A^3*B*a
^16*b^8) + (11*(17*A*b - 5*B*a)*(512439176949055488*A^3*a^19*b^8 - 13037837
801472000*B^3*a^22*b^5 + 132985945575014400*A*B^2*a^21*b^6 - 4521522149550 4
8960*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(1/6)))*(17*A*b - 5*B*a)*11i)/(216
*(-a)^(23/6)*b^(1/6)))/((11*(x^(1/2)*(443639472636450816*A^4*a^15*b^9 + 331
9819810560000*B^4*a^19*b^5 + 230262702060441600*A^2*B^2*a^17*b^7 - 45149549
423616000*A*B^3*a^18*b^6 - 521928791337000960*A^3*B*a^16*b^8) - (11*(17*A*b
 - 5*B*a)*(512439176949055488*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b^5
 + 132985945575014400*A*B^2*a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7))/
(216*(-a)^(23/6)*b^(1/6)))*(17*A*b - 5*B*a))/(216*(-a)^(23/6)*b^(1/6)) - (1
1*(x^(1/2)*(443639472636450816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^5
 + 230262702060441600*A^2*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6 -
 521928791337000960*A^3*B*a^16*b^8) + (11*(17*A*b - 5*B*a)*(512439176949055
488*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b^5 + 132985945575014400*A*B^
2*a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(1/6)))
*(17*A*b - 5*B*a))/(216*(-a)^(23/6)*b^(1/6))))*(17*A*b - 5*B*a)*11i)/(108*(
-a)^(23/6)*b^(1/6)) - (atan(((((3^(1/2)*1i)/2 - 1/2)*(17*A*b - 5*B*a)*(x^(1
/2)*(443639472636450816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^5 + 2302
62702060441600*A^2*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6 - 521928
791337000960*A^3*B*a^16*b^8) - (11*((3^(1/2)*1i)/2 - 1/2)*(17*A*b - 5*B*a)*
(512439176949055488*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b^5 + 1329859
45575014400*A*B^2*a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7))/(216*(-a)^
(23/6)*b^(1/6)))*11i)/(216*(-a)^(23/6)*b^(1/6)) + (((3^(1/2)*1i)/2 - 1/2)*(
17*A*b - 5*B*a)*(x^(1/2)*(443639472636450816*A^4*a^15*b^9 + 331981981056000
0*B^4*a^19*b^5 + 230262702060441600*A^2*B^2*a^17*b^7 - 45149549423616000*A*
B^3*a^18*b^6 - 521928791337000960*A^3*B*a^16*b^8) + (11*((3^(1/2)*1i)/2 - 1
/2)*(17*A*b - 5*B*a)*(512439176949055488*A^3*a^19*b^8 - 13037837801472000*B
```

```
^3*a^22*b^5 + 132985945575014400*A*B^2*a^21*b^6 - 452152214955048960*A^2*B*
a^20*b^7))/(216*(-a)^(23/6)*b^(1/6)))*11i)/(216*(-a)^(23/6)*b^(1/6)))/((11*
((3^(1/2)*1i)/2 - 1/2)*(17*A*b - 5*B*a)*(x^(1/2)*(443639472636450816*A^4*a^
15*b^9 + 3319819810560000*B^4*a^19*b^5 + 230262702060441600*A^2*B^2*a^17*b^
7 - 45149549423616000*A*B^3*a^18*b^6 - 521928791337000960*A^3*B*a^16*b^8) -
 (11*((3^(1/2)*1i)/2 - 1/2)*(17*A*b - 5*B*a)*(512439176949055488*A^3*a^19*b
^8 - 13037837801472000*B^3*a^22*b^5 + 132985945575014400*A*B^2*a^21*b^6 - 4
52152214955048960*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(1/6))))/(216*(-a)^(2
3/6)*b^(1/6)) - (11*((3^(1/2)*1i)/2 - 1/2)*(17*A*b - 5*B*a)*(x^(1/2)*(44363
9472636450816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^5 + 23026270206044
1600*A^2*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6 - 5219287913370009
60*A^3*B*a^16*b^8) + (11*((3^(1/2)*1i)/2 - 1/2)*(17*A*b - 5*B*a)*(512439176
949055488*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b^5 + 13298594557501440
0*A*B^2*a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(
1/6))))/(216*(-a)^(23/6)*b^(1/6))))*((3^(1/2)*1i)/2 - 1/2)*(17*A*b - 5*B*a)
*11i)/(108*(-a)^(23/6)*b^(1/6)) - (atan(((((3^(1/2)*1i)/2 + 1/2)*(17*A*b -
5*B*a)*(x^(1/2)*(443639472636450816*A^4*a^15*b^9 + 3319819810560000*B^4*a^1
9*b^5 + 230262702060441600*A^2*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*
b^6 - 521928791337000960*A^3*B*a^16*b^8) - (11*((3^(1/2)*1i)/2 + 1/2)*(17*A
*b - 5*B*a)*(512439176949055488*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b
^5 + 132985945575014400*A*B^2*a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7)
)/(216*(-a)^(23/6)*b^(1/6)))*11i)/(216*(-a)^(23/6)*b^(1/6)) + (((3^(1/2)*1i
)/2 + 1/2)*(17*A*b - 5*B*a)*(x^(1/2)*(443639472636450816*A^4*a^15*b^9 + 331
9819810560000*B^4*a^19*b^5 + 230262702060441600*A^2*B^2*a^17*b^7 - 45149549
423616000*A*B^3*a^18*b^6 - 521928791337000960*A^3*B*a^16*b^8) + (11*((3^(1/
2)*1i)/2 + 1/2)*(17*A*b - 5*B*a)*(512439176949055488*A^3*a^19*b^8 - 1303783
7801472000*B^3*a^22*b^5 + 132985945575014400*A*B^2*a^21*b^6 - 4521522149550
48960*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(1/6)))*11i)/(216*(-a)^(23/6)*b^(
1/6)))/((11*((3^(1/2)*1i)/2 + 1/2)*(17*A*b - 5*B*a)*(x^(1/2)*(4436394726364
50816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^5 + 230262702060441600*A^2
*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6 - 521928791337000960*A^3*B
*a^16*b^8) - (11*((3^(1/2)*1i)/2 + 1/2)*(17*A*b - 5*B*a)*(51243917694905548
8*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b^5 + 132985945575014400*A*B^2*
a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(1/6))))/
(216*(-a)^(23/6)*b^(1/6)) - (11*((3^(1/2)*1i)/2 + 1/2)*(17*A*b - 5*B*a)*(x^
(1/2)*(443639472636450816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^5 + 23
0262702060441600*A^2*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6 - 5219
28791337000960*A^3*B*a^16*b^8) + (11*((3^(1/2)*1i)/2 + 1/2)*(17*A*b - 5*B*a
)*(512439176949055488*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b^5 + 13298
5945575014400*A*B^2*a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7))/(216*(-a
)^(23/6)*b^(1/6))))/(216*(-a)^(23/6)*b^(1/6))))*((3^(1/2)*1i)/2 + 1/2)*(17*
A*b - 5*B*a)*11i)/(108*(-a)^(23/6)*b^(1/6))
```

3.179 $\int x^8\sqrt{a+bx^3}(A+Bx^3)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int x^8\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2a^2(Ab-aB)\left(a+bx^3\right)^{3/2}}{9b^4}-\frac{2a(2Ab-3aB)\left(a+bx^3\right)^{5/2}}{15b^4}+\frac{2(Ab-3aB)\left(a+bx^3\right)^{7/2}}{21b^4}+\frac{2B\left(a+bx^3\right)^{9/2}}{27b^4}$$

```
[Out] 2/9*a^2*(A*b-B*a)*(b*x^3+a)^(3/2)/b^4-2/15*a*(2*A*b-3*B*a)*(b*x^3+a)^(5/2)/
b^4+2/21*(A*b-3*B*a)*(b*x^3+a)^(7/2)/b^4+2/27*B*(b*x^3+a)^(9/2)/b^4
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 78}

$$\int x^8\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2a^2\left(a+bx^3\right)^{3/2}(Ab-aB)}{9b^4}+\frac{2\left(a+bx^3\right)^{7/2}(Ab-3aB)}{21b^4}-\frac{2a\left(a+bx^3\right)^{5/2}(2Ab-3aB)}{15b^4}+\frac{2B\left(a+bx^3\right)^{9/2}}{27b^4}$$

```
[In] Int[x^8*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*a^2*(A*b - a*B)*(a + b*x^3)^(3/2))/(9*b^4) - (2*a*(2*A*b - 3*a*B)*(a + b
*x^3)^(5/2))/(15*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(7/2))/(21*b^4) + (2*B
*(a + b*x^3)^(9/2))/(27*b^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
```

```
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int x^2\sqrt{a+bx}(A+Bx)\,dx, x, x^3\right) \\
&= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{a^2(-Ab+aB)\sqrt{a+bx}}{b^3}+\frac{a(-2Ab+3aB)(a+bx)^{3/2}}{b^3}\right.\right. \\
&\qquad \left.\left. +\frac{(Ab-3aB)(a+bx)^{5/2}}{b^3}+\frac{B(a+bx)^{7/2}}{b^3}\right)\,dx, x, x^3\right) \\
&= \frac{2a^2(Ab-aB)\left(a+bx^3\right)^{3/2}}{9b^4}-\frac{2a(2Ab-3aB)\left(a+bx^3\right)^{5/2}}{15b^4} \\
&\quad +\frac{2(Ab-3aB)\left(a+bx^3\right)^{7/2}}{21b^4}+\frac{2B\left(a+bx^3\right)^{9/2}}{27b^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.73

$$\begin{aligned}
&\int x^8\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx \\
&= \frac{2\left(a+bx^3\right)^{3/2}\left(-16a^3B+24a^2b(A+Bx^3)-6ab^2x^3(6A+5Bx^3)+5b^3x^6(9A+7Bx^3)\right)}{945b^4}
\end{aligned}$$

```
[In] Integrate[x^8*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*(a + b*x^3)^(3/2)*(-16*a^3*B + 24*a^2*b*(A + B*x^3) - 6*a*b^2*x^3*(6*A +
 5*B*x^3) + 5*b^3*x^6*(9*A + 7*B*x^3)))/(945*b^4)
```

Maple [A] (verified)

Time = 4.48 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$\frac{16\left(b\,x^{3}+a\right)^{\frac{3}{2}}\left(\frac{15x^{6}\left(\frac{7x^{3}B}{9}+A\right)b^{3}}{8}-\frac{3x^{3}\left(\frac{5x^{3}B}{6}+A\right)a\,b^{2}}{2}+a^{2}\left(x^{3}B+A\right)b-\frac{2a^{3}B}{3}\right)}{315b^{4}}$
gosper	$\frac{2\left(b\,x^{3}+a\right)^{\frac{3}{2}}\left(35b^{3}B\,x^{9}+45x^{6}b^{3}A-30B\,x^{6}a\,b^{2}-36aA\,b^{2}x^{3}+24B\,a^{2}b\,x^{3}+24a^{2}bA-16a^{3}B\right)}{945b^{4}}$
trager	$\frac{2\left(35B\,b^{4}x^{12}+45A\,b^{4}x^{9}+5Ba\,b^{3}x^{9}+9Aa\,b^{3}x^{6}-6B\,a^{2}b^{2}x^{6}-12A\,a^{2}b^{2}x^{3}+8B\,a^{3}b\,x^{3}+24A\,a^{3}b-16B\,a^{4}\right)\sqrt{b\,x^{3}+a}}{945b^{4}}$
risch	$\frac{2\left(35B\,b^{4}x^{12}+45A\,b^{4}x^{9}+5Ba\,b^{3}x^{9}+9Aa\,b^{3}x^{6}-6B\,a^{2}b^{2}x^{6}-12A\,a^{2}b^{2}x^{3}+8B\,a^{3}b\,x^{3}+24A\,a^{3}b-16B\,a^{4}\right)\sqrt{b\,x^{3}+a}}{945b^{4}}$
elliptic	$\frac{2B\,x^{12}\sqrt{b\,x^{3}+a}}{27}+\frac{2\left(Ab+\frac{Ba}{9}\right)x^{9}\sqrt{b\,x^{3}+a}}{21b}+\frac{2\left(Aa-\frac{6a\left(Ab+\frac{Ba}{9}\right)}{7b}\right)x^{6}\sqrt{b\,x^{3}+a}}{15b}-\frac{8a\left(Aa-\frac{6a\left(Ab+\frac{Ba}{9}\right)}{7b}\right)x^{3}\sqrt{b\,x^{3}+a}}{45b^{2}}+$
default	$A\left(\frac{2x^{9}\sqrt{b\,x^{3}+a}}{21}+\frac{2a\,x^{6}\sqrt{b\,x^{3}+a}}{105b}-\frac{8a^{2}x^{3}\sqrt{b\,x^{3}+a}}{315b^{2}}+\frac{16a^{3}\sqrt{b\,x^{3}+a}}{315b^{3}}\right)+B\left(\frac{2x^{12}\sqrt{b\,x^{3}+a}}{27}+\frac{2a\,x^{9}\sqrt{b\,x^{3}+a}}{189b}-\frac{4a}{}\right.$

```
[In] int(x^8*(B*x^3+A)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 16/315*(b*x^3+a)^(3/2)*(15/8*x^6*(7/9*x^3*B+A)*b^3-3/2*x^3*(5/6*x^3*B+A)*a*
b^2+a^2*(B*x^3+A)*b-2/3*a^3*B)/b^4
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.96

$$\int x^8\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{2\,(35\,Bb^4x^{12}+5\,(Bab^3+9\,Ab^4)x^9-3\,(2\,Ba^2b^2-3\,Aab^3)x^6-16\,Ba^4+24\,Aa^3b+4\,(2\,Ba^3b-3\,Aa^2b^2)x^3}{945\,b^4}$$

```
[In] integrate(x^8*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/945*(35*B*b^4*x^12 + 5*(B*a*b^3 + 9*A*b^4)*x^9 - 3*(2*B*a^2*b^2 - 3*A*a*b
^3)*x^6 - 16*B*a^4 + 24*A*a^3*b + 4*(2*B*a^3*b - 3*A*a^2*b^2)*x^3)*sqrt(b*x
^3 + a)/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(100) = 200$.

Time = 0.40 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.13

$$\int x^8\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\begin{cases}\frac{16Aa^3\sqrt{a+bx^3}}{315b^3}-\frac{8Aa^2x^3\sqrt{a+bx^3}}{315b^2}+\frac{2Aax^6\sqrt{a+bx^3}}{105b}+\frac{2Ax^9\sqrt{a+bx^3}}{21}-\frac{32Ba^4\sqrt{a+bx^3}}{945b^4}+\frac{16Ba^3x^3\sqrt{a+bx^3}}{945b^3}-\frac{4Ba^2x^6\sqrt{a+bx^3}}{315b^2}+ \\ \sqrt{a}\left(\frac{Ax^9}{9}+\frac{Bx^{12}}{12}\right)\end{cases}$$

```
[In] integrate(x**8*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((16*A*a**3*sqrt(a + b*x**3)/(315*b**3) - 8*A*a**2*x**3*sqrt(a + b
*x**3)/(315*b**2) + 2*A*a*x**6*sqrt(a + b*x**3)/(105*b) + 2*A*x**9*sqrt(a +
 b*x**3)/21 - 32*B*a**4*sqrt(a + b*x**3)/(945*b**4) + 16*B*a**3*x**3*sqrt(a
 + b*x**3)/(945*b**3) - 4*B*a**2*x**6*sqrt(a + b*x**3)/(315*b**2) + 2*B*a*x
**9*sqrt(a + b*x**3)/(189*b) + 2*B*x**12*sqrt(a + b*x**3)/27, Ne(b, 0)), (s
qrt(a)*(A*x**9/9 + B*x**12/12), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.15

$$\int x^8\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\frac{2}{945}B\left(\frac{35\left(bx^3+a\right)^{\frac{9}{2}}}{b^4}-\frac{135\left(bx^3+a\right)^{\frac{7}{2}}a}{b^4}+\frac{189\left(bx^3+a\right)^{\frac{5}{2}}a^2}{b^4}-\frac{105\left(bx^3+a\right)^{\frac{3}{2}}a^3}{b^4}\right)$$
$$+\frac{2}{315}A\left(\frac{15\left(bx^3+a\right)^{\frac{7}{2}}}{b^3}-\frac{42\left(bx^3+a\right)^{\frac{5}{2}}a}{b^3}+\frac{35\left(bx^3+a\right)^{\frac{3}{2}}a^2}{b^3}\right)$$

```
[In] integrate(x^8*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/945*B*(35*(b*x^3 + a)^(9/2)/b^4 - 135*(b*x^3 + a)^(7/2)*a/b^4 + 189*(b*x^
3 + a)^(5/2)*a^2/b^4 - 105*(b*x^3 + a)^(3/2)*a^3/b^4) + 2/315*A*(15*(b*x^3
+ a)^(7/2)/b^3 - 42*(b*x^3 + a)^(5/2)*a/b^3 + 35*(b*x^3 + a)^(3/2)*a^2/b^3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int x^8\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{2\left(35\,(bx^3+a)^{\frac{9}{2}}B-135\,(bx^3+a)^{\frac{7}{2}}Ba+189\,(bx^3+a)^{\frac{5}{2}}Ba^2-105\,(bx^3+a)^{\frac{3}{2}}Ba^3+45\,(bx^3+a)^{\frac{7}{2}}Ab-12\right.}{945\,b^4}$$

```
[In] integrate(x^8*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/945*(35*(b*x^3 + a)^(9/2)*B - 135*(b*x^3 + a)^(7/2)*B*a + 189*(b*x^3 + a)
^(5/2)*B*a^2 - 105*(b*x^3 + a)^(3/2)*B*a^3 + 45*(b*x^3 + a)^(7/2)*A*b - 126
*(b*x^3 + a)^(5/2)*A*a*b + 105*(b*x^3 + a)^(3/2)*A*a^2*b)/b^4
```

Mupad [B] (verification not implemented)

Time = 7.06 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.50

$$\int x^8\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{2\,B\,x^{12}\,\sqrt{b\,x^3+a}}{27}+\frac{x^9\,\sqrt{b\,x^3+a}\,\left(2\,A\,b+\frac{2\,B\,a}{9}\right)}{21\,b} + \frac{8\,a^2\,\left(2\,A\,a-\frac{6\,a\,\left(2\,A\,b+\frac{2\,B\,a}{9}\right)}{7\,b}\right)\,\sqrt{b\,x^3+a}}{45\,b^3} + \frac{x^6\,\left(2\,A\,a-\frac{6\,a\,\left(2\,A\,b+\frac{2\,B\,a}{9}\right)}{7\,b}\right)\,\sqrt{b\,x^3+a}}{15\,b} - \frac{4\,a\,x^3\,\left(2\,A\,a-\frac{6\,a\,\left(2\,A\,b+\frac{2\,B\,a}{9}\right)}{7\,b}\right)\,\sqrt{b\,x^3+a}}{45\,b^2}$$

```
[In] int(x^8*(A + B*x^3)*(a + b*x^3)^(1/2),x)
```

```
[Out] (2*B*x^12*(a + b*x^3)^(1/2))/27 + (x^9*(a + b*x^3)^(1/2)*(2*A*b + (2*B*a)/9
))/(21*b) + (8*a^2*(2*A*a - (6*a*(2*A*b + (2*B*a)/9))/(7*b))*(a + b*x^3)^(1
/2))/(45*b^3) + (x^6*(2*A*a - (6*a*(2*A*b + (2*B*a)/9))/(7*b))*(a + b*x^3)^
(1/2))/(15*b) - (4*a*x^3*(2*A*a - (6*a*(2*A*b + (2*B*a)/9))/(7*b))*(a + b*x
^3)^(1/2))/(45*b^2)
```

3.180 $\int x^5\sqrt{a+bx^3}(A+Bx^3)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int x^5\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=-\frac{2a(Ab-aB)\left(a+bx^3\right)^{3/2}}{9b^3}+\frac{2(Ab-2aB)\left(a+bx^3\right)^{5/2}}{15b^3}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^3}$$

```
[Out] -2/9*a*(A*b-B*a)*(b*x^3+a)^(3/2)/b^3+2/15*(A*b-2*B*a)*(b*x^3+a)^(5/2)/b^3+2
/21*B*(b*x^3+a)^(7/2)/b^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 78}

$$\int x^5\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2\left(a+bx^3\right)^{5/2}(Ab-2aB)}{15b^3}-\frac{2a\left(a+bx^3\right)^{3/2}(Ab-aB)}{9b^3}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^3}$$

```
[In] Int[x^5*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (-2*a*(A*b - a*B)*(a + b*x^3)^(3/2))/(9*b^3) + (2*(A*b - 2*a*B)*(a + b*x^3)
^(5/2))/(15*b^3) + (2*B*(a + b*x^3)^(7/2))/(21*b^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int x\sqrt{a+bx}(A+Bx)\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int\left(\frac{a(-Ab+aB)\sqrt{a+bx}}{b^2}+\frac{(Ab-2aB)(a+bx)^{3/2}}{b^2}+\frac{B(a+bx)^{5/2}}{b^2}\right)\,dx,x,x^3\right)\\ &= -\frac{2a(Ab-aB)\left(a+bx^3\right)^{3/2}}{9b^3}+\frac{2(Ab-2aB)\left(a+bx^3\right)^{5/2}}{15b^3}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int x^5\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{2\left(a+bx^3\right)^{3/2}\left(-14aAb+8a^2B+21Ab^2x^3-12abBx^3+15b^2Bx^6\right)}{315b^3}$$

```
[In] Integrate[x^5*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*(a + b*x^3)^(3/2)*(-14*a*A*b + 8*a^2*B + 21*A*b^2*x^3 - 12*a*b*B*x^3 + 1
5*b^2*B*x^6))/(315*b^3)
```

Maple [A] (verified)

Time = 4.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$-\frac{4(b\,x^{3}+a)^{\frac{3}{2}}\left(-\frac{3x^{3}\left(\frac{5x^{3}B}{7}+A\right)b^{2}}{2}+a\left(\frac{6x^{3}B}{7}+A\right)b-\frac{4a^{2}B}{7}\right)}{45b^{3}}$
gosper	$-\frac{2(b\,x^{3}+a)^{\frac{3}{2}}\left(-15b^{2}B\,x^{6}-21A\,b^{2}x^{3}+12Bab\,x^{3}+14abA-8a^{2}B\right)}{315b^{3}}$
trager	$-\frac{2\left(-15b^{3}B\,x^{9}-21x^{6}b^{3}A-3B\,x^{6}a\,b^{2}-7aA\,b^{2}x^{3}+4B\,a^{2}b\,x^{3}+14a^{2}bA-8a^{3}B\right)\sqrt{b\,x^{3}+a}}{315b^{3}}$
risch	$-\frac{2\left(-15b^{3}B\,x^{9}-21x^{6}b^{3}A-3B\,x^{6}a\,b^{2}-7aA\,b^{2}x^{3}+4B\,a^{2}b\,x^{3}+14a^{2}bA-8a^{3}B\right)\sqrt{b\,x^{3}+a}}{315b^{3}}$
elliptic	$\frac{2B\,x^{9}\sqrt{b\,x^{3}+a}}{21}+\frac{2\left(Ab+\frac{Ba}{7}\right)x^{6}\sqrt{b\,x^{3}+a}}{15b}+\frac{2\left(Aa-\frac{4a\left(Ab+\frac{Ba}{7}\right)}{5b}\right)x^{3}\sqrt{b\,x^{3}+a}}{9b}-\frac{4a\left(Aa-\frac{4a\left(Ab+\frac{Ba}{7}\right)}{5b}\right)\sqrt{b\,x^{3}+a}}{9b^{2}}$
default	$B\left(\frac{2x^{9}\sqrt{b\,x^{3}+a}}{21}+\frac{2a\,x^{6}\sqrt{b\,x^{3}+a}}{105b}-\frac{8a^{2}x^{3}\sqrt{b\,x^{3}+a}}{315b^{2}}+\frac{16a^{3}\sqrt{b\,x^{3}+a}}{315b^{3}}\right)+A\left(\frac{2x^{6}\sqrt{b\,x^{3}+a}}{15}+\frac{2a\,x^{3}\sqrt{b\,x^{3}+a}}{45b}-\right.$ 4a

```
[In] int(x^5*(B*x^3+A)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/45*(b*x^3+a)^(3/2)*(-3/2*x^3*(5/7*x^3*B+A)*b^2+a*(6/7*x^3*B+A)*b-4/7*a^2
*B)/b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int x^{5}\sqrt{a+bx^{3}}\left(A+Bx^{3}\right)\,dx$$
$$=\frac{2\left(15\,Bb^{3}x^{9}+3\left(Bab^{2}+7\,Ab^{3}\right)x^{6}+8\,Ba^{3}-14\,Aa^{2}b-\left(4\,Ba^{2}b-7\,Aab^{2}\right)x^{3}\right)\sqrt{bx^{3}+a}}{315\,b^{3}}$$

```
[In] integrate(x^5*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(15*B*b^3*x^9 + 3*(B*a*b^2 + 7*A*b^3)*x^6 + 8*B*a^3 - 14*A*a^2*b - (4
*B*a^2*b - 7*A*a*b^2)*x^3)*sqrt(b*x^3 + a)/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(70) = 140$.

Time = 0.30 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.30

$$\int x^5\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \begin{cases} -\frac{4Aa^2\sqrt{a+bx^3}}{45b^2}+\frac{2Aax^3\sqrt{a+bx^3}}{45b}+\frac{2Ax^6\sqrt{a+bx^3}}{15}+\frac{16Ba^3\sqrt{a+bx^3}}{315b^3}-\frac{8Ba^2x^3\sqrt{a+bx^3}}{315b^2}+\frac{2Bax^6\sqrt{a+bx^3}}{105b}+\frac{2Bx^9\sqrt{a+bx^3}}{21} & \text{for } b \\ \sqrt{a}\left(\frac{Ax^6}{6}+\frac{Bx^9}{9}\right) & \text{othe} \end{cases}$$

```
[In] integrate(x**5*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((-4*A*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*A*a*x**3*sqrt(a + b*x**
3)/(45*b) + 2*A*x**6*sqrt(a + b*x**3)/15 + 16*B*a**3*sqrt(a + b*x**3)/(315*
b**3) - 8*B*a**2*x**3*sqrt(a + b*x**3)/(315*b**2) + 2*B*a*x**6*sqrt(a + b*x
**3)/(105*b) + 2*B*x**9*sqrt(a + b*x**3)/21, Ne(b, 0)), (sqrt(a)*(A*x**6/6
+ B*x**9/9), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15

$$\int x^5\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{2}{315}\,B\left(\frac{15\,(bx^3+a)^{\frac{7}{2}}}{b^3}-\frac{42\,(bx^3+a)^{\frac{5}{2}}a}{b^3}+\frac{35\,(bx^3+a)^{\frac{3}{2}}a^2}{b^3}\right) + \frac{2}{45}\,A\left(\frac{3\,(bx^3+a)^{\frac{5}{2}}}{b^2}-\frac{5\,(bx^3+a)^{\frac{3}{2}}a}{b^2}\right)$$

```
[In] integrate(x^5*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*B*(15*(b*x^3 + a)^(7/2)/b^3 - 42*(b*x^3 + a)^(5/2)*a/b^3 + 35*(b*x^3
+ a)^(3/2)*a^2/b^3) + 2/45*A*(3*(b*x^3 + a)^(5/2)/b^2 - 5*(b*x^3 + a)^(3/2)
*a/b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int x^5\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{2\left(15\left(bx^3+a\right)^{\frac{7}{2}}B-42\left(bx^3+a\right)^{\frac{5}{2}}Ba+35\left(bx^3+a\right)^{\frac{3}{2}}Ba^2+21\left(bx^3+a\right)^{\frac{5}{2}}Ab-35\left(bx^3+a\right)^{\frac{3}{2}}Aab\right)}{315\,b^3}$$

```
[In] integrate(x^5*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(15*(b*x^3 + a)^(7/2)*B - 42*(b*x^3 + a)^(5/2)*B*a + 35*(b*x^3 + a)^(
3/2)*B*a^2 + 21*(b*x^3 + a)^(5/2)*A*b - 35*(b*x^3 + a)^(3/2)*A*a*b)/b^3
```

Mupad [B] (verification not implemented)

Time = 7.06 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.56

$$\int x^5\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{2\,B\,x^9\,\sqrt{b\,x^3+a}}{21}+\frac{x^6\,\sqrt{b\,x^3+a}\,\left(2\,A\,b+\frac{2\,B\,a}{7}\right)}{15\,b}-\frac{2\,a\,\left(2\,A\,a-\frac{4\,a\,\left(2\,A\,b+\frac{2\,B\,a}{7}\right)}{5\,b}\right)\,\sqrt{b\,x^3+a}}{9\,b^2}+\frac{x^3\,\left(2\,A\,a-\frac{4\,a\,\left(2\,A\,b+\frac{2\,B\,a}{7}\right)}{5\,b}\right)\,\sqrt{b\,x^3+a}}{9\,b}$$

```
[In] int(x^5*(A + B*x^3)*(a + b*x^3)^(1/2),x)
```

```
[Out] (2*B*x^9*(a + b*x^3)^(1/2))/21 + (x^6*(a + b*x^3)^(1/2)*(2*A*b + (2*B*a)/7)
)/(15*b) - (2*a*(2*A*a - (4*a*(2*A*b + (2*B*a)/7))/(5*b))*(a + b*x^3)^(1/2)
)/(9*b^2) + (x^3*(2*A*a - (4*a*(2*A*b + (2*B*a)/7))/(5*b))*(a + b*x^3)^(1/2
))/(9*b)
```

3.181 $\int x^2\sqrt{a+bx^3}(A+Bx^3)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int x^2\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2(Ab-aB)\left(a+bx^3\right)^{3/2}}{9b^2}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^2}$$

```
[Out] 2/9*(A*b-B*a)*(b*x^3+a)^(3/2)/b^2+2/15*B*(b*x^3+a)^(5/2)/b^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {455, 45}

$$\int x^2\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2\left(a+bx^3\right)^{3/2}(Ab-aB)}{9b^2}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^2}$$

```
[In] Int[x^2*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*(A*b - a*B)*(a + b*x^3)^(3/2))/(9*b^2) + (2*B*(a + b*x^3)^(5/2))/(15*b^2
)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \sqrt{a+bx}(A+Bx)\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{(Ab-aB)\sqrt{a+bx}}{b}+\frac{B(a+bx)^{3/2}}{b}\right)\,dx,x,x^3\right)\\
&= \frac{2(Ab-aB)\left(a+bx^3\right)^{3/2}}{9b^2}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int x^2\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2\left(a+bx^3\right)^{3/2}\left(5Ab-2aB+3bBx^3\right)}{45b^2}$$

```
[In] Integrate[x^2*Sqrt[a + b*x^3]*(A + B*x^3),x]
[Out] (2*(a + b*x^3)^(3/2)*(5*A*b - 2*a*B + 3*b*B*x^3))/(45*b^2)
```

Maple [A] (verified)

Time = 4.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{2\left(b\,x^{3}+a\right)^{\frac{3}{2}}\left(3bB\,x^{3}+5Ab-2Ba\right)}{45b^{2}}$	31
pseudoelliptic	$\frac{2\left(\left(3x^{3}B+5A\right)b-2Ba\right)\left(b\,x^{3}+a\right)^{\frac{3}{2}}}{45b^{2}}$	32
trager	$\frac{2\left(3b^{2}B\,x^{6}+5A\,b^{2}x^{3}+Bab\,x^{3}+5abA-2a^{2}B\right)\sqrt{b\,x^{3}+a}}{45b^{2}}$	52
risch	$\frac{2\left(3b^{2}B\,x^{6}+5A\,b^{2}x^{3}+Bab\,x^{3}+5abA-2a^{2}B\right)\sqrt{b\,x^{3}+a}}{45b^{2}}$	52
default	$B\left(\frac{2x^{6}\sqrt{b\,x^{3}+a}}{15}+\frac{2a\,x^{3}\sqrt{b\,x^{3}+a}}{45b}-\frac{4a^{2}\sqrt{b\,x^{3}+a}}{45b^{2}}\right)+\frac{2A\left(b\,x^{3}+a\right)^{\frac{3}{2}}}{9b}$	69
elliptic	$\frac{2B\,x^{6}\sqrt{b\,x^{3}+a}}{15}+\frac{2\left(Ab+\frac{Ba}{5}\right)x^{3}\sqrt{b\,x^{3}+a}}{9b}+\frac{2\left(Aa-\frac{2a\left(Ab+\frac{Ba}{5}\right)}{3b}\right)\sqrt{b\,x^{3}+a}}{3b}$	74

```
[In] int(x^2*(B*x^3+A)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45*(b*x^3+a)^(3/2)*(3*B*b*x^3+5*A*b-2*B*a)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int x^2\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2\left(3\,Bb^2x^6+(Bab+5\,Ab^2)x^3-2\,Ba^2+5\,Aab\right)\sqrt{bx^3+a}}{45\,b^2}$$

```
[In] integrate(x^2*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] 2/45*(3*B*b^2*x^6 + (B*a*b + 5*A*b^2)*x^3 - 2*B*a^2 + 5*A*a*b)*sqrt(b*x^3 +
 a)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(44) = 88.

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.54

$$\int x^2\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\begin{cases}\frac{2Aa\sqrt{a+bx^3}}{9b}+\frac{2Ax^3\sqrt{a+bx^3}}{9}-\frac{4Ba^2\sqrt{a+bx^3}}{45b^2}+\frac{2Bax^3\sqrt{a+bx^3}}{45b}+\frac{2Bx^6\sqrt{a+bx^3}}{15} & \text{for } b\neq 0\\ \sqrt{a}\left(\frac{Ax^3}{3}+\frac{Bx^6}{6}\right) & \text{otherwise}\end{cases}$$

```
[In] integrate(x**2*(B*x**3+A)*(b*x**3+a)**(1/2),x)
[Out] Piecewise((2*A*a*sqrt(a + b*x**3)/(9*b) + 2*A*x**3*sqrt(a + b*x**3)/9 - 4*B
*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*B*a*x**3*sqrt(a + b*x**3)/(45*b) + 2*B
*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*(A*x**3/3 + B*x**6/6), True)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int x^2\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2}{45}\,B\left(\frac{3\,(bx^3+a)^{\frac{5}{2}}}{b^2}-\frac{5\,(bx^3+a)^{\frac{3}{2}}a}{b^2}\right)+\frac{2\,(bx^3+a)^{\frac{3}{2}}A}{9\,b}$$

```
[In] integrate(x^2*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] 2/45*B*(3*(b*x^3 + a)^(5/2)/b^2 - 5*(b*x^3 + a)^(3/2)*a/b^2) + 2/9*(b*x^3 +
 a)^(3/2)*A/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int x^2\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2\left(3\left(bx^3+a\right)^{\frac{5}{2}}B-5\left(bx^3+a\right)^{\frac{3}{2}}Ba+5\left(bx^3+a\right)^{\frac{3}{2}}Ab\right)}{45\,b^2}$$

```
[In] integrate(x^2*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/45*(3*(b*x^3 + a)^(5/2)*B - 5*(b*x^3 + a)^(3/2)*B*a + 5*(b*x^3 + a)^(3/2)
*A*b)/b^2
```

Mupad [B] (verification not implemented)

Time = 6.98 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int x^2\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{6\,B\,\left(b\,x^3+a\right)^{5/2}+10\,A\,b\,\left(b\,x^3+a\right)^{3/2}-10\,B\,a\,\left(b\,x^3+a\right)^{3/2}}{45\,b^2}$$

```
[In] int(x^2*(A + B*x^3)*(a + b*x^3)^(1/2),x)
```

```
[Out] (6*B*(a + b*x^3)^(5/2) + 10*A*b*(a + b*x^3)^(3/2) - 10*B*a*(a + b*x^3)^(3/2
))/(45*b^2)
```

3.182 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x} \, dx = \frac{2}{3}A\sqrt{a+bx^3} + \frac{2B(a+bx^3)^{3/2}}{9b} - \frac{2}{3}\sqrt{a}A\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

```
[Out] 2/9*B*(b*x^3+a)^(3/2)/b-2/3*A*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+2/3*
A*(b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 81, 52, 65, 214}

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x} \, dx = -\frac{2}{3}\sqrt{a}A\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right) + \frac{2}{3}A\sqrt{a+bx^3} + \frac{2B(a+bx^3)^{3/2}}{9b}$$

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x,x]
```

```
[Out] (2*A*Sqrt[a + b*x^3])/3 + (2*B*(a + b*x^3)^(3/2))/(9*b) - (2*Sqrt[a]*A*ArcT
anh[Sqrt[a + b*x^3]/Sqrt[a]])/3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx}(A+Bx)}{x}\,dx, x, x^3\right)\\
&= \frac{2B(a+bx^3)^{3/2}}{9b} + \frac{1}{3}A\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^3\right)\\
&= \frac{2}{3}A\sqrt{a+bx^3} + \frac{2B(a+bx^3)^{3/2}}{9b} + \frac{1}{3}(aA)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)\\
&= \frac{2}{3}A\sqrt{a+bx^3} + \frac{2B(a+bx^3)^{3/2}}{9b} + \frac{(2aA)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{3b}\\
&= \frac{2}{3}A\sqrt{a+bx^3} + \frac{2B(a+bx^3)^{3/2}}{9b} - \frac{2}{3}\sqrt{a}A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x}\,dx = \frac{2\sqrt{a+bx^3}(3Ab+aB+bBx^3)}{9b} - \frac{2}{3}\sqrt{a}A\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x,x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(3*A*b + a*B + b*B*x^3))/(9*b) - (2*Sqrt[a]*A*ArcTanh[Sq
rt[a + b*x^3]/Sqrt[a]])/3
```

Maple [A] (verified)

Time = 4.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{2B(bx^3+a)^{\frac{3}{2}}}{9b}+A\left(-\frac{2\,\text{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)\sqrt{a}}{3}+\frac{2\sqrt{bx^3+a}}{3}\right)$	50
elliptic	$\frac{2Bx^3\sqrt{bx^3+a}}{9}+\frac{2\left(Ab+\frac{Ba}{3}\right)\sqrt{bx^3+a}}{3b}-\frac{2A\,\text{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)\sqrt{a}}{3}$	59
pseudoelliptic	$\frac{2Bbx^3\sqrt{bx^3+a}-6\sqrt{a}\,bA\,\text{arctanh}\left(\frac{\sqrt{bx^3+a}}{\sqrt{a}}\right)+6Ab\sqrt{bx^3+a}+2Ba\sqrt{bx^3+a}}{9b}$	70

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*B*(b*x^3+a)^(3/2)/b+A*(-2/3*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+2/
3*(b*x^3+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.95

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x}\,dx$$
$$= \left[\frac{3\,A\sqrt{a}b\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\,(Bbx^3+Ba+3\,Ab)\sqrt{bx^3+a}}{9\,b}, \frac{2\left(3\,A\sqrt{-a}b\arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right)+(\right.}{9\,b}\right.$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/9*(3*A*sqrt(a)*b*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*
(B*b*x^3 + B*a + 3*A*b)*sqrt(b*x^3 + a))/b, 2/9*(3*A*sqrt(-a)*b*arctan(sqrt
(b*x^3 + a)*sqrt(-a)/a) + (B*b*x^3 + B*a + 3*A*b)*sqrt(b*x^3 + a))/b]
```

Sympy [A] (verification not implemented)

Time = 4.65 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x}\,dx = \frac{A\left(\begin{cases} \frac{2a\operatorname{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2\sqrt{a+bx^3} & \text{for}\ b \neq 0 \\ -\sqrt{a}\log\left(\frac{1}{x^3}\right) & \text{otherwise} \end{cases}\right)}{3} - \frac{B\left(\begin{cases} -\sqrt{a}x^3 & \text{for}\ b = 0 \\ -\frac{2(a+bx^3)^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases}\right)}{3}$$

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x,x)
```

```
[Out] A*Piecewise((2*a*atan(sqrt(a + b*x**3)/sqrt(-a))/sqrt(-a) + 2*sqrt(a + b*x*
*3), Ne(b, 0)), (-sqrt(a)*log(x**(-3)), True))/3 - B*Piecewise((-sqrt(a)*x*
*3, Eq(b, 0)), (-2*(a + b*x**3)**(3/2)/(3*b), True))/3
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x}\,dx = \frac{1}{3}\left(\sqrt{a}\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right) + 2\sqrt{bx^3+a}\right)A + \frac{2\,(bx^3+a)^{\frac{3}{2}}B}{9\,b}$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(a)*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a))) +
 2*sqrt(b*x^3 + a))*A + 2/9*(b*x^3 + a)^(3/2)*B/b
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x}\,dx = \frac{2\,Aa\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}} + \frac{2\left((bx^3+a)^{\frac{3}{2}}Bb^2 + 3\,\sqrt{bx^3+a}Ab^3\right)}{9\,b^3}$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2/3*A*a*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) + 2/9*((b*x^3 + a)^(3/2)*
B*b^2 + 3*sqrt(b*x^3 + a)*A*b^3)/b^3
```

Mupad [B] (verification not implemented)

Time = 7.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x}\,dx = \frac{2\,B\,x^3\,\sqrt{b\,x^3+a}}{9} + \frac{\sqrt{b\,x^3+a}\,\left(2\,A\,b+\frac{2\,B\,a}{3}\right)}{3\,b} + \frac{A\,\sqrt{a}\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x,x)
```

```
[Out] (2*B*x^3*(a + b*x^3)^(1/2))/9 + ((a + b*x^3)^(1/2)*(2*A*b + (2*B*a)/3))/(3*
b) + (A*a^(1/2)*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a
^(1/2)))/x^6))/3
```

3.183 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^4} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^4} \, dx = \frac{(Ab+2aB)\sqrt{a+bx^3}}{3a} - \frac{A(a+bx^3)^{3/2}}{3ax^3} - \frac{(Ab+2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

```
[Out] -1/3*A*(b*x^3+a)^(3/2)/a/x^3-1/3*(A*b+2*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2
))/a^(1/2)+1/3*(A*b+2*B*a)*(b*x^3+a)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {457, 79, 52, 65, 214}

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^4} \, dx = -\frac{(2aB+Ab)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}} + \frac{\sqrt{a+bx^3}(2aB+Ab)}{3a} - \frac{A(a+bx^3)^{3/2}}{3ax^3}$$

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^4,x]
```

```
[Out] ((A*b + 2*a*B)*Sqrt[a + b*x^3])/(3*a) - (A*(a + b*x^3)^(3/2))/(3*a*x^3) - (
(A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx}(A+Bx)}{x^2}\,dx, x, x^3\right)$$

$$= -\frac{A(a+bx^3)^{3/2}}{3ax^3} + \frac{(Ab+2aB)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^3\right)}{6a}$$

$$= \frac{(Ab+2aB)\sqrt{a+bx^3}}{3a} - \frac{A(a+bx^3)^{3/2}}{3ax^3} + \frac{1}{6}(Ab+2aB)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)$$

$$= \frac{(Ab+2aB)\sqrt{a+bx^3}}{3a} - \frac{A(a+bx^3)^{3/2}}{3ax^3} + \frac{(Ab+2aB)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{3b}$$

$$= \frac{(Ab+2aB)\sqrt{a+bx^3}}{3a} - \frac{A(a+bx^3)^{3/2}}{3ax^3} - \frac{(Ab+2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^4}\,dx = \frac{\sqrt{a+bx^3}(-A+2Bx^3)}{3x^3} + \frac{(-Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^4,x]
[Out] (Sqrt[a + b*x^3]*(-A + 2*B*x^3))/(3*x^3) + ((-(A*b) - 2*a*B)*ArcTanh[Sqrt[a
 + b*x^3]/Sqrt[a]])/(3*Sqrt[a])
```

Maple [A] (verified)

Time = 4.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.67

method	result	size
risch	$-\frac{A\sqrt{b\,x^3+a}}{3x^3}+\frac{2B\sqrt{b\,x^3+a}}{3}-\frac{(Ab+2Ba)\,\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}}$	56
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{3x^3}+\frac{2B\sqrt{b\,x^3+a}}{3}-\frac{2\left(\frac{Ab}{2}+Ba\right)\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}}$	56
pseudoelliptic	$-\frac{(Ab+2Ba)\,\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)x^3+\sqrt{b\,x^3+a}\,(-2x^3B+A)\sqrt{a}}{3\sqrt{a}\,x^3}$	57
default	$B\left(-\frac{2\,\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)\sqrt{a}}{3}+\frac{2\sqrt{b\,x^3+a}}{3}\right)+A\left(-\frac{b\,\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}}-\frac{\sqrt{b\,x^3+a}}{3x^3}\right)$	72

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^4,x,method=_RETURNVERBOSE)
[Out] -1/3*A*(b*x^3+a)^(1/2)/x^3+2/3*B*(b*x^3+a)^(1/2)-1/3*(A*b+2*B*a)*arctanh((b
*x^3+a)^(1/2)/a^(1/2))/a^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.70

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^4}\,dx = \left[\frac{(2\,Ba+Ab)\sqrt{a}x^3\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\,(2\,Bax^3-Aa)\sqrt{bx^3+a}}{6\,ax^3}, \frac{(2\,Ba+Ab)\sqrt{-a}x^3\arctan\left(\frac{\sqrt{bx^3}}{}\right.}{3}\right.$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*((2*B*a + A*b)*sqrt(a)*x^3*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*
a)/x^3) + 2*(2*B*a*x^3 - A*a)*sqrt(b*x^3 + a))/(a*x^3), 1/3*((2*B*a + A*b)*
sqrt(-a)*x^3*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (2*B*a*x^3 - A*a)*sqrt(b*
x^3 + a))/(a*x^3)]
```

Sympy [A] (verification not implemented)

Time = 11.83 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.60

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^4}\,dx = -\frac{A\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} - \frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}} - \frac{2B\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{2Ba}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2B\sqrt{b}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}$$

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**4,x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - A*b*asinh(sqrt(a)/(sqrt(b)*x
**(3/2)))/(3*sqrt(a)) - 2*B*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + 2
*B*a/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*B*sqrt(b)*x**(3/2)/(3*sq
rt(a/(b*x**3) + 1))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^4}\,dx = \frac{1}{6}\left(\frac{b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{\sqrt{a}} - \frac{2\sqrt{bx^3+a}}{x^3}\right)A + \frac{1}{3}\left(\sqrt{a}\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)+2\sqrt{bx^3+a}\right)B$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sqrt(a)
 - 2*sqrt(b*x^3 + a)/x^3)*A + 1/3*(sqrt(a)*log((sqrt(b*x^3 + a) - sqrt(a))/
(sqrt(b*x^3 + a) + sqrt(a))) + 2*sqrt(b*x^3 + a))*B
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^4}\,dx = \frac{2\sqrt{bx^3+a}Bb + \frac{(2\,Bab+Ab^2)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\sqrt{bx^3+a}Ab}{x^3}}{3\,b}$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*(2*sqrt(b*x^3 + a)*B*b + (2*B*a*b + A*b^2)*arctan(sqrt(b*x^3 + a)/sqrt(
-a))/sqrt(-a) - sqrt(b*x^3 + a)*A*b/x^3)/b
```

Mupad [B] (verification not implemented)

Time = 7.36 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^4}\,dx = \frac{2\,B\,\sqrt{b\,x^3+a}}{3} - \frac{A\,\sqrt{b\,x^3+a}}{3\,x^3} + \frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)\left(\frac{A\,b}{2}+B\,a\right)}{3\,\sqrt{a}}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^4,x)
```

```
[Out] (2*B*(a + b*x^3)^(1/2))/3 - (A*(a + b*x^3)^(1/2))/(3*x^3) + (log((((a + b*x
^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)*((A*b)/2 + B*a))
/(3*a^(1/2))
```

3.184 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^7} dx$

Optimal result

Integrand size = 22, antiderivative size = 88

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^7} dx = \frac{(Ab-4aB)\sqrt{a+bx^3}}{12ax^3} - \frac{A(a+bx^3)^{3/2}}{6ax^6} + \frac{b(Ab-4aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}$$

```
[Out] -1/6*A*(b*x^3+a)^(3/2)/a/x^6+1/12*b*(A*b-4*B*a)*arctanh((b*x^3+a)^(1/2)/a^(
1/2))/a^(3/2)+1/12*(A*b-4*B*a)*(b*x^3+a)^(1/2)/a/x^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 79, 43, 65, 214}

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^7} dx = \frac{b(Ab-4aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}} + \frac{\sqrt{a+bx^3}(Ab-4aB)}{12ax^3} - \frac{A(a+bx^3)^{3/2}}{6ax^6}$$

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^7,x]
```

```
[Out] ((A*b - 4*a*B)*Sqrt[a + b*x^3])/(12*a*x^3) - (A*(a + b*x^3)^(3/2))/(6*a*x^6
) + (b*(A*b - 4*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx}(A+Bx)}{x^3}\,dx, x, x^3\right)\\ &= -\frac{A(a+bx^3)^{3/2}}{6ax^6} + \frac{\left(-\frac{Ab}{2}+2aB\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, x^3\right)}{6a}\\ &= \frac{(Ab-4aB)\sqrt{a+bx^3}}{12ax^3} - \frac{A(a+bx^3)^{3/2}}{6ax^6} - \frac{(b(Ab-4aB))\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)}{24a}\end{aligned}$$

$$= \frac{(Ab-4aB)\sqrt{a+bx^3}}{12ax^3} - \frac{A(a+bx^3)^{3/2}}{6ax^6} - \frac{(Ab-4aB)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{12a}$$

$$= \frac{(Ab-4aB)\sqrt{a+bx^3}}{12ax^3} - \frac{A(a+bx^3)^{3/2}}{6ax^6} + \frac{b(Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^7}\,dx = \frac{\sqrt{a+bx^3}(-2aA-Abx^3-4aBx^3)}{12ax^6} - \frac{b(-Ab+4aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}$$

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^7,x]
[Out] (Sqrt[a + b*x^3]*(-2*a*A - A*b*x^3 - 4*a*B*x^3))/(12*a*x^6) - (b*(-(A*b) + 4*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a^(3/2))
```

Maple [A] (verified)

Time = 4.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.74

method	result	size
risch	$-\frac{\sqrt{b\,x^3+a}\,(Ab\,x^3+4Ba\,x^3+2Aa)}{12x^6a}+\frac{b(Ab-4Ba)\,\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{12a^{\frac{3}{2}}}$	65
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{6x^6}-\frac{(Ab+4Ba)\sqrt{b\,x^3+a}}{12a\,x^3}+\frac{b(Ab-4Ba)\,\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{12a^{\frac{3}{2}}}$	70
pseudoelliptic	$-\frac{-b\,x^6(Ab-4Ba)\,\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)+\left((4x^3B+2A)a^{\frac{3}{2}}+A\sqrt{a}\,b\,x^3\right)\sqrt{b\,x^3+a}}{12a^{\frac{3}{2}}x^6}$	72
default	$A\left(\frac{b^2\,\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{12a^{\frac{3}{2}}}-\frac{\sqrt{b\,x^3+a}}{6x^6}-\frac{b\sqrt{b\,x^3+a}}{12a\,x^3}\right)+B\left(-\frac{b\,\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}}-\frac{\sqrt{b\,x^3+a}}{3x^3}\right)$	96

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^7,x,method=_RETURNVERBOSE)
[Out] -1/12*(b*x^3+a)^(1/2)*(A*b*x^3+4*B*a*x^3+2*A*a)/x^6/a+1/12*b*(A*b-4*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.95

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^7}\,dx = \left[-\frac{(4\,Bab-Ab^2)\sqrt{a}x^6\log\left(\frac{bx^3+2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\left((4\,Ba^2+Aab)x^3+2\,Aa^2\right)\sqrt{bx^3+a}}{24\,a^2x^6}, \frac{(4\,Bab-Ab^2)\sqrt{}}{}\right.$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/24*((4*B*a*b - A*b^2)*sqrt(a)*x^6*log((b*x^3 + 2*sqrt(b*x^3 + a)*sqrt(a
) + 2*a)/x^3) + 2*((4*B*a^2 + A*a*b)*x^3 + 2*A*a^2)*sqrt(b*x^3 + a))/(a^2*x
^6), 1/12*((4*B*a*b - A*b^2)*sqrt(-a)*x^6*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a
) - ((4*B*a^2 + A*a*b)*x^3 + 2*A*a^2)*sqrt(b*x^3 + a))/(a^2*x^6)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(76) = 152.

Time = 33.57 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.82

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^7}\,dx = -\frac{Aa}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{A\sqrt{b}}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{Ab^{\frac{3}{2}}}{12ax^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{Ab^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{12a^{\frac{3}{2}}} - \frac{B\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} - \frac{Bb\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**7,x)
```

```
[Out] -A*a/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - A*sqrt(b)/(4*x**(9/2)*sqr
t(a/(b*x**3) + 1)) - A*b**(3/2)/(12*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) + A*b*
*2*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(12*a**(3/2)) - B*sqrt(b)*sqrt(a/(b*x*
*3) + 1)/(3*x**(3/2)) - B*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(74) = 148.

Time = 0.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.80

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^7}\,dx = -\frac{1}{24}\left(\frac{b^2\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2\left((bx^3+a)^{\frac{3}{2}}b^2+\sqrt{bx^3+a}ab^2\right)}{(bx^3+a)^2a-2(bx^3+a)a^2+a^3}\right)A + \frac{1}{6}\left(\frac{b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\sqrt{bx^3+a}}{x^3}\right)B$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/24*(b^2*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(
3/2) + 2*((b*x^3 + a)^(3/2)*b^2 + sqrt(b*x^3 + a)*a*b^2)/((b*x^3 + a)^2*a -
 2*(b*x^3 + a)*a^2 + a^3))*A + 1/6*(b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt
(b*x^3 + a) + sqrt(a)))/sqrt(a) - 2*sqrt(b*x^3 + a)/x^3)*B
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^7}\,dx = \frac{\frac{(4Bab^2-Ab^3)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}-\frac{4(bx^3+a)^{\frac{3}{2}}Bab^2-4\sqrt{bx^3+a}Ba^2b^2+(bx^3+a)^{\frac{3}{2}}Ab^3+\sqrt{bx^3+a}Aab^3}{ab^2x^6}}{12b}$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/12*((4*B*a*b^2 - A*b^3)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a) - (
4*(b*x^3 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^3 + a)*B*a^2*b^2 + (b*x^3 + a)^(3/
2)*A*b^3 + sqrt(b*x^3 + a)*A*a*b^3)/(a*b^2*x^6))/b
```

Mupad [B] (verification not implemented)

Time = 7.60 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^7}\,dx = \frac{b\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)^3}{x^6}\right)(A\,b-4\,B\,a)}{24\,a^{3/2}} - \frac{(4\,B\,a^2+A\,b\,a)\,\sqrt{b\,x^3+a}}{12\,a^2\,x^3} - \frac{A\,\sqrt{b\,x^3+a}}{6\,x^6}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^7,x)
```

```
[Out] (b*log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/2))^3)/x^6)
*(A*b - 4*B*a))/(24*a^(3/2)) - ((4*B*a^2 + A*a*b)*(a + b*x^3)^(1/2))/(12*a^
2*x^3) - (A*(a + b*x^3)^(1/2))/(6*x^6)
```

3.185 $\int x^3\sqrt{a+bx^3}(A+Bx^3)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 303

$$\int x^3\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$

$$=\frac{6a(17Ab-8aB)x\sqrt{a+bx^3}}{935b^2}+\frac{2(17Ab-8aB)x^4\sqrt{a+bx^3}}{187b}+\frac{2Bx^4\left(a+bx^3\right)^{3/2}}{17b}$$

$$-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}a^2(17Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{935b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/17*B*x^4*(b*x^3+a)^(3/2)/b+6/935*a*(17*A*b-8*B*a)*x*(b*x^3+a)^(1/2)/b^2+2
/187*(17*A*b-8*B*a)*x^4*(b*x^3+a)^(1/2)/b-4/935*3^(3/4)*a^2*(17*A*b-8*B*a)*
(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(7/3)/
(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {470, 285, 327, 224}

$$\int x^3\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx =$$

$$-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(17Ab-8aB)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{935b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{6ax\sqrt{a+bx^3}(17Ab-8aB)}{935b^2}+\frac{2x^4\sqrt{a+bx^3}(17Ab-8aB)}{187b}+\frac{2Bx^4\left(a+bx^3\right)^{3/2}}{17b}$$

```
[In] Int[x^3*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (6*a*(17*A*b - 8*a*B)*x*Sqrt[a + b*x^3])/(935*b^2) + (2*(17*A*b - 8*a*B)*x^
4*Sqrt[a + b*x^3])/(187*b) + (2*B*x^4*(a + b*x^3)^(3/2))/(17*b) - (4*3^(3/4
)*Sqrt[2 + Sqrt[3]]*a^2*(17*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(935*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{2Bx^4(a+bx^3)^{3/2}}{17b} - \frac{\left(2\left(-\frac{17Ab}{2}+4aB\right)\right)\int x^3\sqrt{a+bx^3}\,dx}{17b}$$

$$= \frac{2(17Ab-8aB)x^4\sqrt{a+bx^3}}{187b} + \frac{2Bx^4(a+bx^3)^{3/2}}{17b} + \frac{(3a(17Ab-8aB))\int \frac{x^3}{\sqrt{a+bx^3}}\,dx}{187b}$$

$$= \frac{6a(17Ab-8aB)x\sqrt{a+bx^3}}{935b^2} + \frac{2(17Ab-8aB)x^4\sqrt{a+bx^3}}{187b} + \frac{2Bx^4(a+bx^3)^{3/2}}{17b} - \frac{(6a^2(17Ab-8aB))\int \frac{1}{\sqrt{a+bx^3}}\,dx}{935b^2}$$

$$= \frac{6a(17Ab-8aB)x\sqrt{a+bx^3}}{935b^2} + \frac{2(17Ab-8aB)x^4\sqrt{a+bx^3}}{187b} + \frac{2Bx^4(a+bx^3)^{3/2}}{17b} - \frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^2(17Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.\Bigg)}{935b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.49 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.29

$$\int x^3\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$

$$= \frac{2x\sqrt{a+bx^3}\left(-\left(\left(a+bx^3\right)\left(-17Ab+8aB-11bBx^3\right)\right)+\frac{a(-17Ab+8aB)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{187b^2}$$

```
[In] Integrate[x^3*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*(-((a + b*x^3)*(-17*A*b + 8*a*B - 11*b*B*x^3)) + (a*(-
17*A*b + 8*a*B)*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x^3)/a)])/Sqrt[1 + (
b*x^3)/a]))/(187*b^2)
```

Maple [A] (verified)

Time = 4.42 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.15

method	result
risch	$\frac{2x(55b^2Bx^6+85Ab^2x^3+15Babx^3+51abA-24a^2B)\sqrt{bx^3+a}}{935b^2}+\frac{4ia^2(17Ab-8Ba)\sqrt{3}\,(-ab^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\cdots}{(-ab^2)^{\frac{1}{3}}}}\cdots}{\cdots}$
elliptic	$\frac{2Bx^7\sqrt{bx^3+a}}{17}+\frac{2\left(Ab+\frac{3Ba}{17}\right)x^4\sqrt{bx^3+a}}{11b}+\frac{2\left(Aa-\frac{8a\left(Ab+\frac{3Ba}{17}\right)}{11b}\right)x\sqrt{bx^3+a}}{5b}+\frac{4ia\left(Aa-\frac{8a\left(Ab+\frac{3Ba}{17}\right)}{11b}\right)\sqrt{3}\,(-ab^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x\cdots\right)}{\cdots}}\cdots}{\cdots}$
default	$B\left(\frac{2x^7\sqrt{bx^3+a}}{17}+\frac{6ax^4\sqrt{bx^3+a}}{187b}-\frac{48a^2x\sqrt{bx^3+a}}{935b^2}-\frac{32ia^3\sqrt{3}\,(-ab^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-ab^2)^{\frac{1}{3}}}}\sqrt{\frac{x\cdots}{-\frac{3(-ab^2)^{\frac{1}{3}}}{2b}\cdots}}\cdots}{\cdots}\right.$

```
[In] int(x^3*(B*x^3+A)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/935*x*(55*B*b^2*x^6+85*A*b^2*x^3+15*B*a*b*x^3+51*A*a*b-24*B*a^2)*(b*x^3+a
)^(1/2)/b^2+4/935*I*a^2*(17*A*b-8*B*a)/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
```

```
^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.30

$$\int x^3\sqrt{a+bx^3}(A+Bx^3)\,dx = \frac{2\left(6\,(8\,Ba^3-17\,Aa^2b)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+(55\,Bb^3x^7+5\,(3\,Bab^2+17\,Ab^3)x^4-3\,(8\,Ba^2b\right.}{935\,b^3}$$

```
[In] integrate(x^3*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/935*(6*(8*B*a^3 - 17*A*a^2*b)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) +
 (55*B*b^3*x^7 + 5*(3*B*a*b^2 + 17*A*b^3)*x^4 - 3*(8*B*a^2*b - 17*A*a*b^2)*
x)*sqrt(b*x^3 + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.27

$$\int x^3\sqrt{a+bx^3}(A+Bx^3)\,dx = \frac{A\sqrt{a}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{B\sqrt{a}x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

```
[In] integrate(x**3*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(7/3)) + B*sqrt(a)*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

Maxima [F]

$$\int x^3\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\sqrt{bx^3+a}\,x^3\,dx$$

```
[In] integrate(x^3*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x^3, x)
```

Giac [F]

$$\int x^3\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\sqrt{bx^3+a}\,x^3\,dx$$

```
[In] integrate(x^3*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int x^3\,\left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}\,dx$$

```
[In] int(x^3*(A + B*x^3)*(a + b*x^3)^(1/2),x)
[Out] int(x^3*(A + B*x^3)*(a + b*x^3)^(1/2), x)
```

3.186 $\int \sqrt{a+bx^3}(A+Bx^3)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 268

$$\int \sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{2(11Ab-2aB)x\sqrt{a+bx^3}}{55b}+\frac{2Bx\left(a+bx^3\right)^{3/2}}{11b}$$
$$+\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a(11Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right)}{55b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/11*B*x*(b*x^3+a)^(3/2)/b+2/55*(11*A*b-2*B*a)*x*(b*x^3+a)^(1/2)/b+2/55*3^(
3/4)*a*(11*A*b-2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3
^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^
(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)/b^(4/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used

$= \{396, 201, 224\}$

$$\int \sqrt{a+bx^3}\left(A+Bx^3\right) dx$$

$$= \frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-2aB)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{55b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2x\sqrt{a+bx^3}(11Ab-2aB)}{55b}+\frac{2Bx\left(a+bx^3\right)^{3/2}}{11b}$$

```
[In] Int[Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*(11*A*b - 2*a*B)*x*Sqrt[a + b*x^3])/(55*b) + (2*B*x*(a + b*x^3)^(3/2))/(
11*b) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(11*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(55*b^(4/3)*Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^
3])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{2Bx(a+bx^3)^{3/2}}{11b} - \frac{\left(2\left(-\frac{11Ab}{2}+aB\right)\right)\int\sqrt{a+bx^3}\,dx}{11b}$$

$$= \frac{2(11Ab-2aB)x\sqrt{a+bx^3}}{55b} + \frac{2Bx(a+bx^3)^{3/2}}{11b} + \frac{(3a(11Ab-2aB))\int\frac{1}{\sqrt{a+bx^3}}\,dx}{55b}$$

$$= \frac{2(11Ab-2aB)x\sqrt{a+bx^3}}{55b} + \frac{2Bx(a+bx^3)^{3/2}}{11b} + \frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a(11Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{55b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.57 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.28

$$\int\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{2x\sqrt{a+bx^3}\left(B(a+bx^3)+\frac{(11Ab-2aB)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}\right)}{11b}$$

```
[In] Integrate[Sqrt[a + b*x^3]*(A + B*x^3),x]
[Out] (2*x*Sqrt[a + b*x^3]*(B*(a + b*x^3) + ((11*A*b - 2*a*B)*Hypergeometric2F1[-
1/2, 1/3, 4/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(11*b)
```

Maple [A] (verified)

Time = 4.39 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.21

method	result
risch	$\frac{2x\left(5bB\,x^{3}+11Ab+3Ba\right)\sqrt{b\,x^{3}+a}}{55b}-\frac{2ia\left(11Ab-2Ba\right)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\right.}{2b}}}}{}$
elliptic	$\frac{2B\,x^{4}\sqrt{b\,x^{3}+a}}{11}+\frac{2\left(Ab+\frac{3Ba}{11}\right)x\sqrt{b\,x^{3}+a}}{5b}-\frac{2i\left(Aa-\frac{2a\left(Ab+\frac{3Ba}{11}\right)}{5b}\right)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{}{-\frac{3\left(-\right.}{}}}}{}$
default	$A\left(\frac{2x\sqrt{b\,x^{3}+a}}{5}-\frac{2ia\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{}}{}\right.}{\left(-a\,b\right.}}}{5b\sqrt{b\,x^{3}+}}\right.$

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/55*x*(5*B*b*x^3+11*A*b+3*B*a)/b*(b*x^3+a)^(1/2)-2/55*I*a*(11*A*b-2*B*a)/b
^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^
3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.25

$$\int \sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = -\frac{2\left(3\left(2\,Ba^2-11\,Aab\right)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-\left(5\,Bb^2x^4+\left(3\,Bab+11\,Ab^2\right)x\right)\sqrt{bx^3+a}\right)}{55\,b^2}$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/55*(3*(2*B*a^2 - 11*A*a*b)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) - (
5*B*b^2*x^4 + (3*B*a*b + 11*A*b^2)*x)*sqrt(b*x^3 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.31

$$\int \sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{A\sqrt{a}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{B\sqrt{a}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(4/3)) + B*sqrt(a)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

Maxima [F]

$$\int \sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \int \left(Bx^3+A\right)\sqrt{bx^3+a}\,dx$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int (Bx^3+A)\sqrt{bx^3+a}\,dx$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}\,dx$$

```
[In] int((A + B*x^3)*(a + b*x^3)^(1/2),x)
[Out] int((A + B*x^3)*(a + b*x^3)^(1/2), x)
```

3.187 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 269

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^3} dx = \frac{(5Ab+4aB)x\sqrt{a+bx^3}}{10a} - \frac{A(a+bx^3)^{3/2}}{2ax^2} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}(5Ab+4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-\right.}{10\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/2*A*(b*x^3+a)^(3/2)/a/x^2+1/10*(5*A*b+4*B*a)*x*(b*x^3+a)^(1/2)/a+1/10*3^
(3/4)*(5*A*b+4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(
1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1
/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
))^2)^(1/2)/b^(1/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used

$= \{464, 201, 224\}$

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^3}\,dx$$

$$= \frac{3^{3/4}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(4aB+5Ab)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7\right.}{10\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{x\sqrt{a+bx^3}(4aB+5Ab)}{10a}-\frac{A(a+bx^3)^{3/2}}{2ax^2}$$

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^3,x]
```

```
[Out] ((5*A*b + 4*a*B)*x*Sqrt[a + b*x^3])/(10*a) - (A*(a + b*x^3)^(3/2))/(2*a*x^2
) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*(5*A*b + 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(10*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3)
+ b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{A(a+bx^3)^{3/2}}{2ax^2} - \frac{\left(-\frac{5Ab}{2}-2aB\right)\int\sqrt{a+bx^3}\,dx}{2a}$$

$$= \frac{(5Ab+4aB)x\sqrt{a+bx^3}}{10a} - \frac{A(a+bx^3)^{3/2}}{2ax^2} + \frac{1}{20}(3(5Ab+4aB))\int\frac{1}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{(5Ab+4aB)x\sqrt{a+bx^3}}{10a} - \frac{A(a+bx^3)^{3/2}}{2ax^2} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}(5Ab+4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7}{10\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.30

$$\int\frac{\sqrt{a+bx^3}(A+Bx^3)}{x^3}\,dx = \frac{\sqrt{a+bx^3}\left(-A(a+bx^3)+\frac{(5Ab+4aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}\right)}{2ax^2}$$

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^3,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-(A*(a + b*x^3)) + ((5*A*b + 4*a*B)*x^3*Hypergeometric2F1
[-1/2, 1/3, 4/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(2*a*x^2)
```

Maple [A] (verified)

Time = 4.41 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.18

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-4x^{3}B+5A\right)}{10x^{2}}-\frac{2i\left(\frac{3Ab}{4}+\frac{3Ba}{5}\right)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\,\sqrt{}$
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{2x^{2}}+\frac{2Bx\sqrt{b\,x^{3}+a}}{5}-\frac{2i\left(\frac{3Ab}{4}+\frac{3Ba}{5}\right)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)}{2b}}}}{}$
default	$B\left(\frac{2x\sqrt{b\,x^{3}+a}}{5}-\frac{2ia\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\,\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\right)}{\left(-a\,b\right)}}}{5b\sqrt{b\,x^{3}+}}\right.$

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*(b*x^3+a)^(1/2)*(-4*B*x^3+5*A)/x^2-2/3*I*(3/4*A*b+3/5*B*a)*3^(1/2)/b*
(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^3}\,dx = \frac{3\,(4\,Ba+5\,Ab)\sqrt{b}x^2\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+(4\,Bbx^3-5\,Ab)\sqrt{bx^3+a}}{10\,bx^2}$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/10*(3*(4*B*a + 5*A*b)*sqrt(b)*x^2*weierstrassPInverse(0, -4*a/b, x) + (4*
B*b*x^3 - 5*A*b)*sqrt(b*x^3 + a))/(b*x^2)
```

Sympy [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.32

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^3}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{B\sqrt{a}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**3,x)
```

```
[Out] A*sqrt(a)*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)
/(3*x**2*gamma(1/3)) + B*sqrt(a)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^3}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^3}\,dx$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^3}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^3}\,dx$$

[In] `integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^3,x, algorithm="giac")`

[Out] `integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^3, x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^3}\,dx = \int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{x^3}\,dx$$

[In] `int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^3,x)`

[Out] `int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^3, x)`

3.188 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^6}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 272

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^6}\,dx = \frac{(Ab-10aB)\sqrt{a+bx^3}}{20ax^2} - \frac{A(a+bx^3)^{3/2}}{5ax^5} - \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}(Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),\ldots\right)}{20a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/5*A*(b*x^3+a)^(3/2)/a/x^5+1/20*(A*b-10*B*a)*(b*x^3+a)^(1/2)/a/x^2-1/20*3
^(3/4)*b^(2/3)*(A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3
)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+
1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2)))^2)^(1/2)/a/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used

= {464, 283, 224}

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^6}\,dx =$$

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-10aB)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right.\right.}{20a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{\sqrt{a+bx^3}(Ab-10aB)}{20ax^2}-\frac{A(a+bx^3)^{3/2}}{5ax^5}$$

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^6,x]
```

```
[Out] ((A*b - 10*a*B)*Sqrt[a + b*x^3])/(20*a*x^2) - (A*(a + b*x^3)^(3/2))/(5*a*x^
5) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(2/3)*(A*b - 10*a*B)*(a^(1/3) + b^(1/3)*x
)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(20*a*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{A(a+bx^3)^{3/2}}{5ax^5} - \frac{\left(\frac{Ab}{2}-5aB\right)\int\frac{\sqrt{a+bx^3}}{x^3}\,dx}{5a}$$
$$= \frac{(Ab-10aB)\sqrt{a+bx^3}}{20ax^2} - \frac{A(a+bx^3)^{3/2}}{5ax^5} - \frac{(3b(Ab-10aB))\int\frac{1}{\sqrt{a+bx^3}}\,dx}{40a}$$
$$= \frac{(Ab-10aB)\sqrt{a+bx^3}}{20ax^2} - \frac{A(a+bx^3)^{3/2}}{5ax^5} - \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}(Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right)}{20a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.29

$$\int\frac{\sqrt{a+bx^3}(A+Bx^3)}{x^6}\,dx = \frac{\sqrt{a+bx^3}\left(-2A(a+bx^3)+\frac{\left(\frac{Ab}{2}-5aB\right)x^3\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{10ax^5}$$

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^6,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-2*A*(a + b*x^3) + (((A*b)/2 - 5*a*B)*x^3*Hypergeometric2
F1[-2/3, -1/2, 1/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(10*a*x^5)
```

Maple [A] (verified)

Time = 4.47 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.21

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,(3Ab\,x^{3}+10Ba\,x^{3}+4Aa)}{20x^{5}a}+\frac{i(Ab-10Ba)\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^{2})^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a}{2b}}}}{}$
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{5x^{5}}-\frac{(3Ab+10Ba)\sqrt{b\,x^{3}+a}}{20a\,x^{2}}-\frac{2i\left(Bb-\frac{b(3Ab+10Ba)}{40a}\right)\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\sqrt{\frac{x}{-\frac{3(-a\,b^{2}}{2b}}}}{}$
default	$B\left(-\frac{\sqrt{b\,x^{3}+a}}{2x^{2}}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^{2})^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}}{}\right)}{(-a\,b^{2})}}}{2\sqrt{b\,x^{3}+a}}\right.$

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*(b*x^3+a)^(1/2)*(3*A*b*x^3+10*B*a*x^3+4*A*a)/x^5/a+1/20*I*(A*b-10*B*a
)/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x
^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/
3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.24

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^6}\,dx$$
$$= \frac{3\,(10\,Ba-Ab)\sqrt{b}x^5\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-((10\,Ba+3\,Ab)x^3+4\,Aa)\sqrt{bx^3+a}}{20\,ax^5}$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/20*(3*(10*B*a - A*b)*sqrt(b)*x^5*weierstrassPInverse(0, -4*a/b, x) - ((10
*B*a + 3*A*b)*x^3 + 4*A*a)*sqrt(b*x^3 + a))/(a*x^5)
```

Sympy [A] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.35

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^6}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{B\sqrt{a}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**6,x)
```

```
[Out] A*sqrt(a)*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**5*gamma(-2/3)) + B*sqrt(a)*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,),
b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^6}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^6}\,dx$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^6}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^6}\,dx$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^6,x, algorithm="giac")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^6}\,dx = \int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{x^6}\,dx$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^6,x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^6, x)
```

3.189 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^9}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 305

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^9}\,dx = \frac{(7Ab-16aB)\sqrt{a+bx^3}}{80ax^5}+\frac{3b(7Ab-16aB)\sqrt{a+bx^3}}{320a^2x^2}-\frac{A(a+bx^3)^{3/2}}{8ax^8} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{5/3}(7Ab-16aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{320a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/8*A*(b*x^3+a)^(3/2)/a/x^8+1/80*(7*A*b-16*B*a)*(b*x^3+a)^(1/2)/a/x^5+3/32
0*b*(7*A*b-16*B*a)*(b*x^3+a)^(1/2)/a^2/x^2+1/320*3^(3/4)*b^(5/3)*(7*A*b-16*
B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^2
/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {464, 283, 331, 224}

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^9}\,dx = \frac{3b\sqrt{a+bx^3}(7Ab-16aB)}{320a^2x^2} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(7Ab-16aB)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{320a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{\sqrt{a+bx^3}(7Ab-16aB)}{80ax^5} - \frac{A(a+bx^3)^{3/2}}{8ax^8}$$

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^9,x]
```

```
[Out] ((7*A*b - 16*a*B)*Sqrt[a + b*x^3])/(80*a*x^5) + (3*b*(7*A*b - 16*a*B)*Sqrt[
a + b*x^3])/(320*a^2*x^2) - (A*(a + b*x^3)^(3/2))/(8*a*x^8) + (3^(3/4)*Sqrt
[2 + Sqrt[3]]*b^(5/3)*(7*A*b - 16*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(320*a^2*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x
))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{A(a+bx^3)^{3/2}}{8ax^8} - \frac{\left(\frac{7Ab}{2} - 8aB\right)\int \frac{\sqrt{a+bx^3}}{x^6}\,dx}{8a}$$

$$= \frac{(7Ab-16aB)\sqrt{a+bx^3}}{80ax^5} - \frac{A(a+bx^3)^{3/2}}{8ax^8} - \frac{(3b(7Ab-16aB))\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx}{160a}$$

$$= \frac{(7Ab-16aB)\sqrt{a+bx^3}}{80ax^5} + \frac{3b(7Ab-16aB)\sqrt{a+bx^3}}{320a^2x^2} - \frac{A(a+bx^3)^{3/2}}{8ax^8} + \frac{(3b^2(7Ab-16aB))\int \frac{1}{\sqrt{a+bx^3}}\,dx}{640a^2}$$

$$= \frac{(7Ab-16aB)\sqrt{a+bx^3}}{80ax^5} + \frac{3b(7Ab-16aB)\sqrt{a+bx^3}}{320a^2x^2} - \frac{A(a+bx^3)^{3/2}}{8ax^8} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{5/3}(7Ab-16aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{320a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.26

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^9}\,dx = \frac{\sqrt{a+bx^3}\left(-5A(a+bx^3)+\frac{\left(\frac{7Ab}{2}-8aB\right)x^3\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{1}{2},-\frac{2}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{40ax^8}$$

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^9,x]
[Out] (Sqrt[a + b*x^3]*(-5*A*(a + b*x^3) + (((7*A*b)/2 - 8*a*B)*x^3*Hypergeometri
c2F1[-5/3, -1/2, -2/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(40*a*x^8)
```

Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.15

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-21 A\,b^{2} x^{6}+48 B\,x^{6} a b+12 a A b\,x^{3}+64 a^{2} B\,x^{3}+40 a^{2} A\right)}{320 x^{8} a^{2}}-\frac{i b (7 A b-16 B a) \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{}$
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{8 x^{8}}-\frac{(3 A b+16 B a)\sqrt{b\,x^{3}+a}}{80 a\,x^{5}}+\frac{3 b (7 A b-16 B a)\sqrt{b\,x^{3}+a}}{320 a^{2} x^{2}}-\frac{i b (7 A b-16 B a) \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{}$
default	$A\left(-\frac{\sqrt{b\,x^{3}+a}}{8 x^{8}}-\frac{3 b\sqrt{b\,x^{3}+a}}{80 a\,x^{5}}+\frac{21 b^{2}\sqrt{b\,x^{3}+a}}{320 a^{2} x^{2}}-\frac{7 i b^{2} \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right)\sqrt{3}\, b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i\sqrt{3}\,(}{}}}}{}\right.$

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/320*(b*x^3+a)^(1/2)*(-21*A*b^2*x^6+48*B*a*b*x^6+12*A*a*b*x^3+64*B*a^2*x^
3+40*A*a^2)/x^8/a^2-1/320*I*b*(7*A*b-16*B*a)/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1
/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-
a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.29

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^9}\,dx = -\frac{3\,(16\,Bab-7\,Ab^2)\sqrt{b}x^8\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+\left(3\,(16\,Bab-7\,Ab^2)x^6+4\,(16\,Ba^2+3\,Aab)x\right.}{320\,a^2x^8}$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/320*(3*(16*B*a*b - 7*A*b^2)*sqrt(b)*x^8*weierstrassPInverse(0, -4*a/b, x
) + (3*(16*B*a*b - 7*A*b^2)*x^6 + 4*(16*B*a^2 + 3*A*a*b)*x^3 + 40*A*a^2)*sq
rt(b*x^3 + a))/(a^2*x^8)
```

Sympy [A] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.32

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^9}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{8}{3},-\frac{1}{2}\\-\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^8\Gamma\left(-\frac{5}{3}\right)} + \frac{B\sqrt{a}\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}$$

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**9,x)
```

```
[Out] A*sqrt(a)*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**8*gamma(-5/3)) + B*sqrt(a)*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^9}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^9}\,dx$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^9, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^9}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^9}\,dx$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^9,x, algorithm="giac")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^9, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^9}\,dx = \int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{x^9}\,dx$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^9,x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^9, x)
```

3.190 $\int x^4\sqrt{a+bx^3}(A+Bx^3)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 581

$$\int x^4\sqrt{a+bx^3}(A+Bx^3)\,dx = \frac{6a(19Ab-10aB)x^2\sqrt{a+bx^3}}{1729b^2} + \frac{2(19Ab-10aB)x^5\sqrt{a+bx^3}}{247b} - \frac{24a^2(19Ab-10aB)\sqrt{a+bx^3}}{1729b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{2Bx^5(a+bx^3)^{3/2}}{19b} + \frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}(19Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|\right.}{1729b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{8\sqrt{2}3^{3/4}a^{7/3}(19Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{1729b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/19*B*x^5*(b*x^3+a)^(3/2)/b+6/1729*a*(19*A*b-10*B*a)*x^2*(b*x^3+a)^(1/2)/b
^2+2/247*(19*A*b-10*B*a)*x^5*(b*x^3+a)^(1/2)/b-24/1729*a^2*(19*A*b-10*B*a)*
(b*x^3+a)^(1/2)/b^(8/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-8/1729*3^(3/4)*a^(7
/3)*(19*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(
1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(8/3
)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))^2)^(1/2)+12/1729*3^(1/4)*a^(7/3)*(19*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*El
```

```
lipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3
^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/
3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {470, 285, 327, 309, 224, 1891}

$$\int x^4\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx =$$

$$-\frac{8\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(19Ab-10aB)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-\right.}{1729b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(19Ab-10aB)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7\right.}{1729b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{24a^2\sqrt{a+bx^3}(19Ab-10aB)}{1729b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{6ax^2\sqrt{a+bx^3}(19Ab-10aB)}{1729b^2}$$

$$+\frac{2x^5\sqrt{a+bx^3}(19Ab-10aB)}{247b}+\frac{2Bx^5\left(a+bx^3\right)^{3/2}}{19b}$$

```
[In] Int[x^4*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (6*a*(19*A*b - 10*a*B)*x^2*Sqrt[a + b*x^3])/(1729*b^2) + (2*(19*A*b - 10*a*
B)*x^5*Sqrt[a + b*x^3])/(247*b) - (24*a^2*(19*A*b - 10*a*B)*Sqrt[a + b*x^3]
)/(1729*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*B*x^5*(a + b*x^3)
^(3/2))/(19*b) + (12*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*(19*A*b - 10*a*B)*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(
8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3]) - (8*Sqrt[2]*3^(3/4)*a^(7/3)*(19*A*b - 10*a*B)*(a^(1
/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(
```

```
1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(8/3
)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{2Bx^5(a+bx^3)^{3/2}}{19b} - \frac{\left(2\left(-\frac{19Ab}{2}+5aB\right)\right)\int x^4\sqrt{a+bx^3}\,dx}{19b}$$

$$= \frac{2(19Ab-10aB)x^5\sqrt{a+bx^3}}{247b} + \frac{2Bx^5(a+bx^3)^{3/2}}{19b} + \frac{(3a(19Ab-10aB))\int \frac{x^4}{\sqrt{a+bx^3}}\,dx}{247b}$$

$$= \frac{6a(19Ab-10aB)x^2\sqrt{a+bx^3}}{1729b^2} + \frac{2(19Ab-10aB)x^5\sqrt{a+bx^3}}{247b} + \frac{2Bx^5(a+bx^3)^{3/2}}{19b} - \frac{(12a^2(19Ab-10aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{1729b^2}$$

$$= \frac{6a(19Ab-10aB)x^2\sqrt{a+bx^3}}{1729b^2} + \frac{2(19Ab-10aB)x^5\sqrt{a+bx^3}}{247b} + \frac{2Bx^5(a+bx^3)^{3/2}}{19b} - \frac{(12a^2(19Ab-10aB))\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{1729b^{7/3}} + \frac{\left(12\left(1-\sqrt{3}\right)a^{7/3}(19Ab-10aB)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{1729b^{7/3}}$$

$$= \frac{6a(19Ab-10aB)x^2\sqrt{a+bx^3}}{1729b^2} + \frac{2(19Ab-10aB)x^5\sqrt{a+bx^3}}{247b} - \frac{24a^2(19Ab-10aB)\sqrt{a+bx^3}}{1729b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{2Bx^5(a+bx^3)^{3/2}}{19b} + \frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}(19Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{\cdot}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{\cdot}}\right.\right.}{1729b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{8\sqrt{2}3^{3/4}a^{7/3}(19Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-\right.}{1729b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.82 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.16

$$\int x^4\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{2x^2\sqrt{a+bx^3}\left(-\left(\left(a+bx^3\right)\left(-19Ab+10aB-13bBx^3\right)\right)+\frac{a(-19Ab+10aB)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{247b^2}$$

```
[In] Integrate[x^4*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*x^2*Sqrt[a + b*x^3]*(-((a + b*x^3)*(-19*A*b + 10*a*B - 13*b*B*x^3)) + (a
*(-19*A*b + 10*a*B)*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*x^3)/a)])/Sqrt[1
 + (b*x^3)/a]))/(247*b^2)
```

Maple [A] (verified)

Time = 4.64 (sec) , antiderivative size = 503, normalized size of antiderivative = 0.87

method	result
risch	$\frac{2x^{2}\left(91b^{2}B\,x^{6}+133A\,b^{2}x^{3}+21Bab\,x^{3}+57abA-30a^{2}B\right)\sqrt{b\,x^{3}+a}}{1729b^{2}}+\frac{8ia^{2}(19Ab-10Ba)\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right.}{(-a\,b^{2})^{\frac{1}{3}}}}}{}$
elliptic	$\frac{2B\,x^{8}\sqrt{b\,x^{3}+a}}{19}+\frac{2\left(Ab+\frac{3Ba}{19}\right)x^{5}\sqrt{b\,x^{3}+a}}{13b}+\frac{2\left(Aa-\frac{10a\left(Ab+\frac{3Ba}{19}\right)}{13b}\right)x^{2}\sqrt{b\,x^{3}+a}}{7b}+\frac{8ia\left(Aa-\frac{10a\left(Ab+\frac{3Ba}{19}\right)}{13b}\right)\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(\right.}{}}}{}$
default	Expression too large to display

```
[In] int(x^4*(B*x^3+A)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/1729*x^2*(91*B*b^2*x^6+133*A*b^2*x^3+21*B*a*b*x^3+57*A*a*b-30*B*a^2)/b^2*
(b*x^3+a)^(1/2)+8/1729*I*a^2*(19*A*b-10*B*a)/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(
```

```
I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.18

$$\int x^4\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = -\frac{2\left(12\left(10\,Ba^3-19\,Aa^2b\right)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-\left(91\,Bb^3x^8+7\right.\right.}{1729\,b^3}$$

```
[In] integrate(x^4*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/1729*(12*(10*B*a^3 - 19*A*a^2*b)*sqrt(b)*weierstrassZeta(0, -4*a/b, weie
rstrassPInverse(0, -4*a/b, x)) - (91*B*b^3*x^8 + 7*(3*B*a*b^2 + 19*A*b^3)*x
^5 - 3*(10*B*a^2*b - 19*A*a*b^2)*x^2)*sqrt(b*x^3 + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.14

$$\int x^4\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{A\sqrt{a}x^5\Gamma\left(\frac{5}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{B\sqrt{a}x^8\Gamma\left(\frac{8}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

```
[In] integrate(x**4*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(8/3)) + B*sqrt(a)*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3))
```

Maxima [F]

$$\int x^4\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int\left(Bx^3+A\right)\sqrt{bx^3+a}x^4\,dx$$

[In] integrate(x^4*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x^4, x)

Giac [F]

$$\int x^4\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int\left(Bx^3+A\right)\sqrt{bx^3+a}x^4\,dx$$

[In] integrate(x^4*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")

[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x^4, x)

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int x^4\,\left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}\,dx$$

[In] int(x^4*(A + B*x^3)*(a + b*x^3)^(1/2),x)

[Out] int(x^4*(A + B*x^3)*(a + b*x^3)^(1/2), x)

3.191 $\int x\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 548

$$\begin{aligned}\int x\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx &= \frac{2(13Ab-4aB)x^2\sqrt{a+bx^3}}{91b}+\frac{6a(13Ab-4aB)\sqrt{a+bx^3}}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2Bx^2\left(a+bx^3\right)^{3/2}}{13b} \\ &-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}(13Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} \\ &+\frac{2\sqrt{2}3^{3/4}a^{4/3}(13Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\end{aligned}$$

```
[Out] 2/13*B*x^2*(b*x^3+a)^(3/2)/b+2/91*(13*A*b-4*B*a)*x^2*(b*x^3+a)^(1/2)/b+6/91
*a*(13*A*b-4*B*a)*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+2
/91*3^(3/4)*a^(4/3)*(13*A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x
+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2
)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^
2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))^2)^(1/2)-3/91*3^(1/4)*a^(4/3)*(13*A*b-4*B*a)*(a^(1/3)+b^
(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*
```

```
x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(
1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {470, 285, 309, 224, 1891}

$$\int x\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$= \frac{2\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(13Ab-4aB)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-\right.}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(13Ab-4aB)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-\right.}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{6a\sqrt{a+bx^3}(13Ab-4aB)}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2x^2\sqrt{a+bx^3}(13Ab-4aB)}{91b}+\frac{2Bx^2\left(a+bx^3\right)^{3/2}}{13b}$$

```
[In] Int[x*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*(13*A*b - 4*a*B)*x^2*Sqrt[a + b*x^3])/(91*b) + (6*a*(13*A*b - 4*a*B)*Sqr
t[a + b*x^3])/(91*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*B*x^2*(
a + b*x^3)^(3/2))/(13*b) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13*A*b - 4
*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^
(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
91*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2]*3^(3/4)*a^(4/3)*(13*A*b - 4*a*B)*
(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3)
+ b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(
5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{2Bx^2(a+bx^3)^{3/2}}{13b} - \frac{\left(2\left(-\frac{13Ab}{2}+2aB\right)\right)\int x\sqrt{a+bx^3}\,dx}{13b}$$

$$= \frac{2(13Ab-4aB)x^2\sqrt{a+bx^3}}{91b} + \frac{2Bx^2(a+bx^3)^{3/2}}{13b} + \frac{(3a(13Ab-4aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{91b}$$

$$= \frac{2(13Ab-4aB)x^2\sqrt{a+bx^3}}{91b} + \frac{2Bx^2(a+bx^3)^{3/2}}{13b} + \frac{(3a(13Ab-4aB))\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{91b^{4/3}} - \frac{\left(3\left(1-\sqrt{3}\right)a^{4/3}(13Ab-4aB)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{91b^{4/3}}$$

$$= \frac{2(13Ab-4aB)x^2\sqrt{a+bx^3}}{91b} + \frac{6a(13Ab-4aB)\sqrt{a+bx^3}}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{2Bx^2(a+bx^3)^{3/2}}{13b} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}(13Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{2\sqrt{2}3^{3/4}a^{4/3}(13Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7\right.}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.56 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.14

$$\int x\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{x^2\sqrt{a+bx^3}\left(4B(a+bx^3)+\frac{(13Ab-4aB)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{26b}$$

```
[In] Integrate[x*Sqrt[a + b*x^3]*(A + B*x^3),x]
[Out] (x^2*Sqrt[a + b*x^3]*(4*B*(a + b*x^3) + ((13*A*b - 4*a*B)*Hypergeometric2F1
[-1/2, 2/3, 5/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(26*b)
```

Maple [A] (verified)

Time = 4.61 (sec) , antiderivative size = 479, normalized size of antiderivative = 0.87

method	result
risch	$\frac{2x^2(7bBx^3+13Ab+3Ba)\sqrt{bx^3+a}}{91b} - \frac{2ia(13Ab-4Ba)\sqrt{3}\,(-ab^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-ab^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-ab^2)^{\frac{1}{3}}}{b}}{-\frac{3(-ab^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-}{2}}}}{}$
elliptic	$\frac{2Bx^5\sqrt{bx^3+a}}{13}+\frac{2\left(Ab+\frac{3Ba}{13}\right)x^2\sqrt{bx^3+a}}{7b} - \frac{2i\left(Aa-\frac{4a\left(Ab+\frac{3Ba}{13}\right)}{7b}\right)\sqrt{3}\,(-ab^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-ab^2)^{\frac{1}{3}}}}\sqrt{-\frac{3(}{}}}{}$
default	Expression too large to display

```
[In] int(x*(B*x^3+A)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/91*x^2*(7*B*b*x^3+13*A*b+3*B*a)/b*(b*x^3+a)^(1/2)-2/91*I*a*(13*A*b-4*B*a)
/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*
x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ellipt
icE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2
```

```
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.14

$$\int x\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{2\left(3\left(4\,Ba^2-13\,Aab\right)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+\left(7\,Bb^2x^5+\left(3\,Bab+1\right.\right.\right.}{91\,b^2}$$

```
[In] integrate(x*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/91*(3*(4*B*a^2 - 13*A*a*b)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrass
PInverse(0, -4*a/b, x)) + (7*B*b^2*x^5 + (3*B*a*b + 13*A*b^2)*x^2)*sqrt(b*x
^3 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.15

$$\int x\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{A\sqrt{a}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{B\sqrt{a}x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\\frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

```
[In] integrate(x*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(5/3)) + B*sqrt(a)*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b
*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3))
```

Maxima [F]

$$\int x\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\sqrt{bx^3+a}x\,dx$$

```
[In] integrate(x*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x, x)
```

Giac [F]

$$\int x\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\sqrt{bx^3+a}x\,dx$$

```
[In] integrate(x*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int x\,\left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}\,dx$$

```
[In] int(x*(A + B*x^3)*(a + b*x^3)^(1/2),x)
```

```
[Out] int(x*(A + B*x^3)*(a + b*x^3)^(1/2), x)
```

3.192 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 545

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^2}\,dx = \frac{(7Ab+2aB)x^2\sqrt{a+bx^3}}{7a} + \frac{3(7Ab+2aB)\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{A(a+bx^3)^{3/2}}{ax} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}(7Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\dots\right)}{14b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{\sqrt{2}3^{3/4}\sqrt[3]{a}(7Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-\dots\right)}{7b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -A*(b*x^3+a)^(3/2)/a/x+1/7*(7*A*b+2*B*a)*x^2*(b*x^3+a)^(1/2)/a+3/7*(7*A*b+2
*B*a)*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+1/7*3^(3/4)*a
^(1/3)*(7*A*b+2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^
(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/
3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)-3/14*3^(1/4)*a^(1/3)*(7*A*b+2*B*a)*(a^(1/3)+b^(1/3)*x)*Ellipt
icE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/
```

```
2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/
(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(
a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {464, 285, 309, 224, 1891}

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^2}\,dx$$

$$= \frac{\sqrt{2}3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2aB+7Ab)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-\right.}{7b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2aB+7Ab)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-\right.}{14b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{3\sqrt{a+bx^3}(2aB+7Ab)}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{x^2\sqrt{a+bx^3}(2aB+7Ab)}{7a}-\frac{A(a+bx^3)^{3/2}}{ax}$$

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^2,x]
```

```
[Out] ((7*A*b + 2*a*B)*x^2*Sqrt[a + b*x^3])/(7*a) + (3*(7*A*b + 2*a*B)*Sqrt[a + b
*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (A*(a + b*x^3)^(3/
2))/(a*x) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(7*A*b + 2*a*B)*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*
x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(14*b^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3]) + (Sqrt[2]*3^(3/4)*a^(1/3)*(7*A*b + 2*a*B)*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*b^(2/3)*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3
])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{A(a+bx^3)^{3/2}}{ax} - \frac{\left(-\frac{7Ab}{2} - aB\right)\int x\sqrt{a+bx^3}\,dx}{a}$$

$$= \frac{(7Ab+2aB)x^2\sqrt{a+bx^3}}{7a} - \frac{A(a+bx^3)^{3/2}}{ax} + \frac{1}{14}(3(7Ab+2aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{(7Ab+2aB)x^2\sqrt{a+bx^3}}{7a} - \frac{A(a+bx^3)^{3/2}}{ax} + \frac{(3(7Ab+2aB))\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{14\sqrt[3]{b}} - \frac{\left(3\left(1-\sqrt{3}\right)\sqrt[3]{a}(7Ab+2aB)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{14\sqrt[3]{b}}$$

$$= \frac{(7Ab+2aB)x^2\sqrt{a+bx^3}}{7a} + \frac{3(7Ab+2aB)\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{A(a+bx^3)^{3/2}}{ax} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}(7Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{14b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{\sqrt{2}3^{3/4}\sqrt[3]{a}(7Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\middle|-7-\right.}{7b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.15

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^2}\,dx = \frac{\sqrt{a+bx^3}\left(-2A(a+bx^3) + \frac{(7Ab+2aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}\right)}{2ax}$$

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^2,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-2*A*(a + b*x^3) + ((7*A*b + 2*a*B)*x^3*Hypergeometric2F1
[-1/2, 2/3, 5/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(2*a*x)
```

Maple [A] (verified)

Time = 4.35 (sec) , antiderivative size = 469, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-2x^{3}B+7A\right)}{7x}-\frac{2i\left(\frac{3Ab}{2}+\frac{3Ba}{7}\right)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-}}{}$
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{x}+\frac{2B\sqrt{b\,x^{3}+a}\,x^{2}}{7}-$
default	Expression too large to display

[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)

[Out] -1/7*(b*x^3+a)^(1/2)*(-2*B*x^3+7*A)/x-2/3*I*(3/2*A*b+3/7*B*a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^

```
(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.12

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^2}\,dx = -\frac{3\,(2\,Ba+7\,Ab)\sqrt{b}x\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-(2\,Bbx^3-7\,Ab)\sqrt{bx^3+}}{7\,bx}$$

[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^2,x, algorithm="fricas")

[Out] -1/7*(3*(2*B*a + 7*A*b)*sqrt(b)*x*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - (2*B*b*x^3 - 7*A*b)*sqrt(b*x^3 + a))/(b*x)

Sympy [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.16

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^2}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} + \frac{B\sqrt{a}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**2,x)

[Out] A*sqrt(a)*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) + B*sqrt(a)*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3))

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^2}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^2}\,dx$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^2}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^2}\,dx$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^2,x, algorithm="giac")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^2}\,dx = \int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{x^2}\,dx$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^2,x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^2, x)
```

3.193 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^5}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 546

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^5}\,dx$$

$$= -\frac{(Ab+8aB)\sqrt{a+bx^3}}{8ax} + \frac{3\sqrt[3]{b}(Ab+8aB)\sqrt{a+bx^3}}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{A(a+bx^3)^{3/2}}{4ax^4}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}(Ab+8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\right.}{16a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{3^{3/4}\sqrt[3]{b}(Ab+8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{\right.}}{4\sqrt{2}a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/4*A*(b*x^3+a)^(3/2)/a/x^4-1/8*(A*b+8*B*a)*(b*x^3+a)^(1/2)/a/x+3/8*b^(1/3
)*(A*b+8*B*a)*(b*x^3+a)^(1/2)/a/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+1/8*3^(3/4)
*b^(1/3)*(A*b+8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^
(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^
(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)*2^(1/
2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)-3/16*3^(1/4)*b^(1/3)*(A*b+8*B*a)*(a^(1/3)+b^(1/3)*x)*Elliptic
E((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)
```

```
+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b
^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^
(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {464, 283, 309, 224, 1891}

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^5}\,dx$$

$$= \frac{3^{3/4}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(8aB+Ab)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{4\sqrt{2}a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(8aB+Ab)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{16a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{\sqrt{a+bx^3}(8aB+Ab)}{8ax}+\frac{3\sqrt[3]{b}\sqrt{a+bx^3}(8aB+Ab)}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{A\left(a+bx^3\right)^{3/2}}{4ax^4}$$

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^5,x]
```

```
[Out] -1/8*((A*b + 8*a*B)*Sqrt[a + b*x^3])/(a*x) + (3*b^(1/3)*(A*b + 8*a*B)*Sqrt[
a + b*x^3])/(8*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (A*(a + b*x^3)^(3/2
))/(4*a*x^4) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(A*b + 8*a*B)*(a^(1/3)
+ b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*a^(2/3)*Sqrt
[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqr
t[a + b*x^3]) + (3^(3/4)*b^(1/3)*(A*b + 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*a^(2/3)*Sqrt[(a^(1/3)*(a
^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]
)
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{A(a+bx^3)^{3/2}}{4ax^4} - \frac{\left(-\frac{Ab}{2} - 4aB\right)\int \frac{\sqrt{a+bx^3}}{x^2}\,dx}{4a}$$

$$= -\frac{(Ab+8aB)\sqrt{a+bx^3}}{8ax} - \frac{A(a+bx^3)^{3/2}}{4ax^4} + \frac{(3b(Ab+8aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{16a}$$

$$= -\frac{(Ab+8aB)\sqrt{a+bx^3}}{8ax} - \frac{A(a+bx^3)^{3/2}}{4ax^4} + \frac{\left(3b^{2/3}(Ab+8aB)\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{16a} - \frac{\left(3\left(1-\sqrt{3}\right)b^{2/3}(Ab+8aB)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{16a^{2/3}}$$

$$= -\frac{(Ab+8aB)\sqrt{a+bx^3}}{8ax} + \frac{3\sqrt[3]{b}(Ab+8aB)\sqrt{a+bx^3}}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{A(a+bx^3)^{3/2}}{4ax^4} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}(Ab+8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|\right.}{16a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{3^{3/4}\sqrt[3]{b}(Ab+8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.15

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^5}\,dx = \frac{\sqrt{a+bx^3}\left(-A(a+bx^3) - \frac{(Ab+8aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{3},\frac{2}{3},-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}\right)}{4ax^4}$$

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^5,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-(A*(a + b*x^3)) - ((A*b + 8*a*B)*x^3*Hypergeometric2F1[-
1/2, -1/3, 2/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(4*a*x^4)
```

Maple [A] (verified)

Time = 4.25 (sec) , antiderivative size = 480, normalized size of antiderivative = 0.88

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(3 A b\,x^{3}+8 B a\,x^{3}+2 A a\right)}{8 x^{4} a}-\frac{i(Ab+8Ba)\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i\sqrt{3}\left(-a\,b^{2\cdots}\right.}{2 b}}}\cdots}{\cdots}$
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{4 x^{4}}-\frac{\left(3 A b+8 B a\right)\sqrt{b\,x^{3}+a}}{8 a x}-\frac{2i\left(Bb+\frac{b(3Ab+8Ba)}{16a}\right)\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\cdots}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\cdots}}\cdots}{\cdots}$
default	Expression too large to display

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(b*x^3+a)^(1/2)*(3*A*b*x^3+8*B*a*x^3+2*A*a)/x^4/a-1/8*I*(A*b+8*B*a)/a*
3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a
)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1
/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/
```

```
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3
*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.13

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^5}\,dx = -\frac{3\,(8\,Ba+Ab)\sqrt{b}x^4\text{weierstrassZeta}\big(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\big(0,-\frac{4\,a}{b},x\big)\big)+((8\,Ba+3\,Ab)x^3+2\,Aa)}{8\,ax^4}$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/8*(3*(8*B*a + A*b)*sqrt(b)*x^4*weierstrassZeta(0, -4*a/b, weierstrassPIn
verse(0, -4*a/b, x)) + ((8*B*a + 3*A*b)*x^3 + 2*A*a)*sqrt(b*x^3 + a))/(a*x^
4)
```

Sympy [A] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^5}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{B\sqrt{a}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**5,x)
```

```
[Out] A*sqrt(a)*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**4*gamma(-1/3)) + B*sqrt(a)*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,),
b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^5}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^5}\,dx$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^5,x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^5, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^5}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^5}\,dx$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^5,x, algorithm="giac")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^5}\,dx = \int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{x^5}\,dx$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^5,x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^5, x)
```

3.194 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^8}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 581

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^8}\,dx = \frac{(5Ab-14aB)\sqrt{a+bx^3}}{56ax^4} + \frac{3b(5Ab-14aB)\sqrt{a+bx^3}}{112a^2x} - \frac{3b^{4/3}(5Ab-14aB)\sqrt{a+bx^3}}{112a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{A(a+bx^3)^{3/2}}{7ax^7}$$

$$+ \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{4/3}(5Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\right.}{224a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$- \frac{3^{3/4}b^{4/3}(5Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\right.}{56\sqrt{2}a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/7*A*(b*x^3+a)^(3/2)/a/x^7+1/56*(5*A*b-14*B*a)*(b*x^3+a)^(1/2)/a/x^4+3/11
2*b*(5*A*b-14*B*a)*(b*x^3+a)^(1/2)/a^2/x-3/112*b^(4/3)*(5*A*b-14*B*a)*(b*x^
3+a)^(1/2)/a^2/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-1/112*3^(3/4)*b^(4/3)*(5*A*b
-14*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2
/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(5/3)*2^(1/2)/(b*x^3+a)
^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2
)+3/224*3^(1/4)*b^(4/3)*(5*A*b-14*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/
```

```
3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1
/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^
(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {464, 283, 331, 309, 224, 1891}

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^8}\,dx =$$

$$-\frac{3^{3/4}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-14aB)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-\right.}{56\sqrt{2}a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-14aB)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7\right.}{224a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3b^{4/3}\sqrt{a+bx^3}(5Ab-14aB)}{112a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{3b\sqrt{a+bx^3}(5Ab-14aB)}{112a^2x}$$

$$+\frac{\sqrt{a+bx^3}(5Ab-14aB)}{56ax^4}-\frac{A(a+bx^3)^{3/2}}{7ax^7}$$

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^8,x]
```

```
[Out] ((5*A*b - 14*a*B)*Sqrt[a + b*x^3])/(56*a*x^4) + (3*b*(5*A*b - 14*a*B)*Sqrt[
a + b*x^3])/(112*a^2*x) - (3*b^(4/3)*(5*A*b - 14*a*B)*Sqrt[a + b*x^3])/(112
*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (A*(a + b*x^3)^(3/2))/(7*a*x^7)
 + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*(5*A*b - 14*a*B)*(a^(1/3) + b^(1/3)
*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(224*a^(5/3)*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*
x^3]) - (3^(3/4)*b^(4/3)*(5*A*b - 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
```

```
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(56*Sqrt[2]*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{A(a+bx^3)^{3/2}}{7ax^7} - \frac{\left(\frac{5Ab}{2}-7aB\right)\int \frac{\sqrt{a+bx^3}}{x^5}\,dx}{7a}$$

$$= \frac{(5Ab-14aB)\sqrt{a+bx^3}}{56ax^4} - \frac{A(a+bx^3)^{3/2}}{7ax^7} - \frac{(3b(5Ab-14aB))\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx}{112a}$$

$$= \frac{(5Ab-14aB)\sqrt{a+bx^3}}{56ax^4} + \frac{3b(5Ab-14aB)\sqrt{a+bx^3}}{112a^2x} - \frac{A(a+bx^3)^{3/2}}{7ax^7} - \frac{(3b^2(5Ab-14aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{224a^2}$$

$$= \frac{(5Ab-14aB)\sqrt{a+bx^3}}{56ax^4} + \frac{3b(5Ab-14aB)\sqrt{a+bx^3}}{112a^2x} - \frac{A(a+bx^3)^{3/2}}{7ax^7} - \frac{\left(3b^{5/3}(5Ab-14aB)\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{224a^2} + \frac{\left(3\left(1-\sqrt{3}\right)b^{5/3}(5Ab-14aB)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{224a^{5/3}}$$

$$= \frac{(5Ab-14aB)\sqrt{a+bx^3}}{56ax^4} + \frac{3b(5Ab-14aB)\sqrt{a+bx^3}}{112a^2x} - \frac{3b^{4/3}(5Ab-14aB)\sqrt{a+bx^3}}{112a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{A(a+bx^3)^{3/2}}{7ax^7} + \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}(5Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{224a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{3^{3/4}b^{4/3}(5Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\right.}{56\sqrt{2}a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.14

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^8}\,dx = \frac{\sqrt{a+bx^3}\left(-4A(a+bx^3)+\frac{\left(\frac{5Ab}{2}-7aB\right)x^3\,\text{Hypergeometric2F1}\left(-\frac{4}{3},-\frac{1}{2},-\frac{1}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{28ax^7}$$

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^8,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-4*A*(a + b*x^3) + (((5*A*b)/2 - 7*a*B)*x^3*Hypergeometri
c2F1[-4/3, -1/2, -1/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(28*a*x^7)
```

Maple [A] (verified)

Time = 4.33 (sec) , antiderivative size = 504, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-15 A\,b^{2} x^{6}+42 B\,x^{6} a b+6 a A b\,x^{3}+28 a^{2} B\,x^{3}+16 a^{2} A\right)}{112 x^{7} a^{2}}+\frac{i b(5Ab-14Ba)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right.}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{}$
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{7x^{7}}-\frac{(3Ab+14Ba)\sqrt{b\,x^{3}+a}}{56a\,x^{4}}+\frac{3b(5Ab-14Ba)\sqrt{b\,x^{3}+a}}{112a^{2}x}+\frac{i b(5Ab-14Ba)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right.}{2b}\right.}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{}$
default	Expression too large to display

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/112*(b*x^3+a)^(1/2)*(-15*A*b^2*x^6+42*B*a*b*x^6+6*A*a*b*x^3+28*B*a^2*x^3
+16*A*a^2)/x^7/a^2+1/112*I*b*(5*A*b-14*B*a)/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(
x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),
(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I
```

```
*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^8}\,dx = -\frac{3\,(14\,Bab-5\,Ab^2)\sqrt{b}x^7\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+(3\,(14\,Bab-5\,Ab^2)x}{112\,a^2x^7}$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/112*(3*(14*B*a*b - 5*A*b^2)*sqrt(b)*x^7*weierstrassZeta(0, -4*a/b, weier
strassPInverse(0, -4*a/b, x)) + (3*(14*B*a*b - 5*A*b^2)*x^6 + 2*(14*B*a^2 +
 3*A*a*b)*x^3 + 16*A*a^2)*sqrt(b*x^3 + a))/(a^2*x^7)
```

Sympy [A] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^8}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)} + \frac{B\sqrt{a}\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}$$

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**8,x)
```

```
[Out] A*sqrt(a)*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**7*gamma(-4/3)) + B*sqrt(a)*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^8}\,dx=\int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^8}\,dx$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^8,x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^8, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^8}\,dx=\int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^8}\,dx$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^8,x, algorithm="giac")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^8}\,dx=\int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{x^8}\,dx$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^8,x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^8, x)
```

3.195 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 614

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11}}\,dx = \frac{(11Ab-20aB)\sqrt{a+bx^3}}{140ax^7} + \frac{3b(11Ab-20aB)\sqrt{a+bx^3}}{1120a^2x^4} - \frac{3b^2(11Ab-20aB)\sqrt{a+bx^3}}{448a^3x} + \frac{3b^{7/3}(11Ab-20aB)\sqrt{a+bx^3}}{448a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{A(a+bx^3)^{3/2}}{10ax^{10}}$$

$$- \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{7/3}(11Ab-20aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7}{896a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{3^{3/4}b^{7/3}(11Ab-20aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-}{224\sqrt{2}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/10*A*(b*x^3+a)^(3/2)/a/x^10+1/140*(11*A*b-20*B*a)*(b*x^3+a)^(1/2)/a/x^7+
3/1120*b*(11*A*b-20*B*a)*(b*x^3+a)^(1/2)/a^2/x^4-3/448*b^2*(11*A*b-20*B*a)*
(b*x^3+a)^(1/2)/a^3/x+3/448*b^(7/3)*(11*A*b-20*B*a)*(b*x^3+a)^(1/2)/a^3/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))+1/448*3^(3/4)*b^(7/3)*(11*A*b-20*B*a)*(a^(1/3)
+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(8/3)*2^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(
a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-3/896*3^(1/4)*b
```

```
^(7/3)*(11*A*b-20*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-
3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2
^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))^2)^(1/2)/a^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 614, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {464, 283, 331, 309, 224, 1891}

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11}}\,dx$$

$$= \frac{3^{3/4}b^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-20aB)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\right.}{224\sqrt{2}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-20aB)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7\right.}{896a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{3b^{7/3}\sqrt{a+bx^3}(11Ab-20aB)}{448a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{3b^2\sqrt{a+bx^3}(11Ab-20aB)}{448a^3x}$$

$$+\frac{3b\sqrt{a+bx^3}(11Ab-20aB)}{1120a^2x^4}+\frac{\sqrt{a+bx^3}(11Ab-20aB)}{140ax^7}-\frac{A(a+bx^3)^{3/2}}{10ax^{10}}$$

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^11,x]
```

```
[Out] ((11*A*b - 20*a*B)*Sqrt[a + b*x^3])/(140*a*x^7) + (3*b*(11*A*b - 20*a*B)*Sq
rt[a + b*x^3])/(1120*a^2*x^4) - (3*b^2*(11*A*b - 20*a*B)*Sqrt[a + b*x^3])/(
448*a^3*x) + (3*b^(7/3)*(11*A*b - 20*a*B)*Sqrt[a + b*x^3])/(448*a^3*((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)) - (A*(a + b*x^3)^(3/2))/(10*a*x^10) - (3*3^(1
/4)*Sqrt[2 - Sqrt[3]]*b^(7/3)*(11*A*b - 20*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(896*a^(8/3)*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (
3^(3/4)*b^(7/3)*(11*A*b - 20*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
```

```
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(224*Sqrt[2]*a^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{A(a+bx^3)^{3/2}}{10ax^{10}} - \frac{\left(\frac{11Ab}{2}-10aB\right)\int \frac{\sqrt{a+bx^3}}{x^8}\,dx}{10a} \\
&= \frac{(11Ab-20aB)\sqrt{a+bx^3}}{140ax^7} - \frac{A(a+bx^3)^{3/2}}{10ax^{10}} - \frac{(3b(11Ab-20aB))\int \frac{1}{x^5\sqrt{a+bx^3}}\,dx}{280a} \\
&= \frac{(11Ab-20aB)\sqrt{a+bx^3}}{140ax^7} + \frac{3b(11Ab-20aB)\sqrt{a+bx^3}}{1120a^2x^4} \\
&\quad - \frac{A(a+bx^3)^{3/2}}{10ax^{10}} + \frac{(3b^2(11Ab-20aB))\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx}{448a^2} \\
&= \frac{(11Ab-20aB)\sqrt{a+bx^3}}{140ax^7} + \frac{3b(11Ab-20aB)\sqrt{a+bx^3}}{1120a^2x^4} \\
&\quad - \frac{3b^2(11Ab-20aB)\sqrt{a+bx^3}}{448a^3x} - \frac{A(a+bx^3)^{3/2}}{10ax^{10}} + \frac{(3b^3(11Ab-20aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{896a^3} \\
&= \frac{(11Ab-20aB)\sqrt{a+bx^3}}{140ax^7} + \frac{3b(11Ab-20aB)\sqrt{a+bx^3}}{1120a^2x^4} \\
&\quad - \frac{3b^2(11Ab-20aB)\sqrt{a+bx^3}}{448a^3x} - \frac{A(a+bx^3)^{3/2}}{10ax^{10}} \\
&\quad + \frac{\left(3b^{8/3}(11Ab-20aB)\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{896a^3} \\
&\quad - \frac{\left(3\left(1-\sqrt{3}\right)b^{8/3}(11Ab-20aB)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{896a^{8/3}}
\end{aligned}$$

$$= \frac{(11Ab-20aB)\sqrt{a+bx^3}}{140ax^7} + \frac{3b(11Ab-20aB)\sqrt{a+bx^3}}{1120a^2x^4} - \frac{3b^2(11Ab-20aB)\sqrt{a+bx^3}}{448a^3x} + \frac{3b^{7/3}(11Ab-20aB)\sqrt{a+bx^3}}{448a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{A(a+bx^3)^{3/2}}{10ax^{10}} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{7/3}(11Ab-20aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{896a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{3^{3/4}b^{7/3}(11Ab-20aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\right.}{224\sqrt{2}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.13

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11}}\,dx = \frac{\sqrt{a+bx^3}\left(-7A(a+bx^3)+\frac{\left(\frac{11Ab}{2}-10aB\right)x^3\,\text{Hypergeometric2F1}\left(-\frac{7}{3},-\frac{1}{2},-\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{70ax^{10}}$$

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^11,x]
[Out] (Sqrt[a + b*x^3]*(-7*A*(a + b*x^3) + (((11*A*b)/2 - 10*a*B)*x^3*Hypergeomet
ric2F1[-7/3, -1/2, -4/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(70*a*x^10)
```

Maple [A] (verified)

Time = 4.49 (sec) , antiderivative size = 530, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,(165A\,x^9b^3-300B\,x^9a\,b^2-66A\,x^6a\,b^2+120B\,x^6a^2b+48A\,x^3a^2b+320a^3B\,x^3+224a^3A)}{2240x^{10}a^3}-\frac{ib^2(11Ab-20Ba)\sqrt{3}\,(-a\,b^2)}{}$ $ib^2(11Ab-20Ba)\sqrt{3}\,(-a\,b$
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{10x^{10}}-\frac{(3Ab+20Ba)\sqrt{b\,x^3+a}}{140a\,x^7}+\frac{3b(11Ab-20Ba)\sqrt{b\,x^3+a}}{1120a^2x^4}-\frac{3b^2(11Ab-20Ba)\sqrt{b\,x^3+a}}{448a^3x}-$
default	Expression too large to display

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2240*(b*x^3+a)^(1/2)*(165*A*b^3*x^9-300*B*a*b^2*x^9-66*A*a*b^2*x^6+120*B
*a^2*b*x^6+48*A*a^2*b*x^3+320*B*a^3*x^3+224*A*a^3)/x^10/a^3-1/448*I*b^2*(11
*A*b-20*B*a)/a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1
/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1
/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
```

```
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11}}\,dx$$
$$= \frac{15\,(20\,Bab^2-11\,Ab^3)\sqrt{b}x^{10}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+(15\,(20\,Bab^2-11\,A}{2240\,a^3x^{10}}$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^11,x, algorithm="fricas")
```

```
[Out] 1/2240*(15*(20*B*a*b^2 - 11*A*b^3)*sqrt(b)*x^10*weierstrassZeta(0, -4*a/b,
weierstrassPInverse(0, -4*a/b, x)) + (15*(20*B*a*b^2 - 11*A*b^3)*x^9 - 6*(2
0*B*a^2*b - 11*A*a*b^2)*x^6 - 224*A*a^3 - 16*(20*B*a^3 + 3*A*a^2*b)*x^3)*sq
rt(b*x^3 + a))/(a^3*x^10)
```

Sympy [A] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.16

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11}}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{10}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{10}{3},-\frac{1}{2}\\-\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{10}\Gamma\left(-\frac{7}{3}\right)} + \frac{B\sqrt{a}\Gamma\left(-\frac{7}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)}$$

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**11,x)
```

```
[Out] A*sqrt(a)*gamma(-10/3)*hyper((-10/3, -1/2), (-7/3,), b*x**3*exp_polar(I*pi)
/a)/(3*x**10*gamma(-7/3)) + B*sqrt(a)*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3
,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^{11}}\,dx$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^11,x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^11, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^{11}}\,dx$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^11,x, algorithm="giac")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^11, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11}}\,dx = \int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{x^{11}}\,dx$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^11,x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^11, x)
```

3.196 $\int x^8(a+bx^3)^{3/2}(A+Bx^3)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int x^8(a+bx^3)^{3/2}(A+Bx^3)\,dx=\frac{2a^2(Ab-aB)(a+bx^3)^{5/2}}{15b^4}-\frac{2a(2Ab-3aB)(a+bx^3)^{7/2}}{21b^4}+\frac{2(Ab-3aB)(a+bx^3)^{9/2}}{27b^4}+\frac{2B(a+bx^3)^{11/2}}{33b^4}$$

```
[Out] 2/15*a^2*(A*b-B*a)*(b*x^3+a)^(5/2)/b^4-2/21*a*(2*A*b-3*B*a)*(b*x^3+a)^(7/2)
/b^4+2/27*(A*b-3*B*a)*(b*x^3+a)^(9/2)/b^4+2/33*B*(b*x^3+a)^(11/2)/b^4
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 78}

$$\int x^8(a+bx^3)^{3/2}(A+Bx^3)\,dx=\frac{2a^2(a+bx^3)^{5/2}(Ab-aB)}{15b^4}+\frac{2(a+bx^3)^{9/2}(Ab-3aB)}{27b^4}-\frac{2a(a+bx^3)^{7/2}(2Ab-3aB)}{21b^4}+\frac{2B(a+bx^3)^{11/2}}{33b^4}$$

```
[In] Int[x^8*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*a^2*(A*b - a*B)*(a + b*x^3)^(5/2))/(15*b^4) - (2*a*(2*A*b - 3*a*B)*(a +
b*x^3)^(7/2))/(21*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(9/2))/(27*b^4) + (2*
B*(a + b*x^3)^(11/2))/(33*b^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int x^2(a+bx)^{3/2}(A+Bx)\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int\left(-\frac{a^2(-Ab+aB)(a+bx)^{3/2}}{b^3}+\frac{a(-2Ab+3aB)(a+bx)^{5/2}}{b^3}\right.\right.\\ &\qquad\left.\left.+\frac{(Ab-3aB)(a+bx)^{7/2}}{b^3}+\frac{B(a+bx)^{9/2}}{b^3}\right)\,dx,x,x^3\right)\\ &= \frac{2a^2(Ab-aB)\left(a+bx^3\right)^{5/2}}{15b^4}-\frac{2a(2Ab-3aB)\left(a+bx^3\right)^{7/2}}{21b^4}\\ &\quad+\frac{2(Ab-3aB)\left(a+bx^3\right)^{9/2}}{27b^4}+\frac{2B\left(a+bx^3\right)^{11/2}}{33b^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.78

$$\int x^8\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{2\left(a+bx^3\right)^{5/2}\left(88a^2Ab-48a^3B-220aAb^2x^3+120a^2bBx^3+385Ab^3x^6-210ab^2Bx^6+315b^3B\right.}{10395b^4}$$

```
[In] Integrate[x^8*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*(a + b*x^3)^(5/2)*(88*a^2*A*b - 48*a^3*B - 220*a*A*b^2*x^3 + 120*a^2*b*B
*x^3 + 385*A*b^3*x^6 - 210*a*b^2*B*x^6 + 315*b^3*B*x^9))/(10395*b^4)
```

Maple [A] (verified)

Time = 4.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{16(b\,x^3+a)^{\frac{5}{2}}\left(\frac{35x^6\left(\frac{9x^3B}{11}+A\right)b^3}{8}-\frac{5x^3\left(\frac{21x^3B}{22}+A\right)a\,b^2}{2}+a^2\left(\frac{15x^3B}{11}+A\right)b-\frac{6a^3B}{11}\right)}{945b^4}$
gosper	$\frac{2(b\,x^3+a)^{\frac{5}{2}}(315b^3B\,x^9+385x^6b^3A-210B\,x^6a\,b^2-220aA\,b^2x^3+120B\,a^2b\,x^3+88a^2bA-48a^3B)}{10395b^4}$
trager	$\frac{2(315b^5B\,x^{15}+385b^5A\,x^{12}+420a\,b^4B\,x^{12}+550a\,b^4A\,x^9+15a^2b^3B\,x^9+33a^2A\,b^3x^6-18a^3b^2B\,x^6-44a^3A\,b^2x^3+24B\,a^4b\,x^3+8}{10395b^4}$
risch	$\frac{2(315b^5B\,x^{15}+385b^5A\,x^{12}+420a\,b^4B\,x^{12}+550a\,b^4A\,x^9+15a^2b^3B\,x^9+33a^2A\,b^3x^6-18a^3b^2B\,x^6-44a^3A\,b^2x^3+24B\,a^4b\,x^3+8}{10395b^4}$
default	$A\left(\frac{2b\,x^{12}\sqrt{b\,x^3+a}}{27}+\frac{20a\,x^9\sqrt{b\,x^3+a}}{189}+\frac{2a^2x^6\sqrt{b\,x^3+a}}{315b}-\frac{8a^3x^3\sqrt{b\,x^3+a}}{945b^2}+\frac{16a^4\sqrt{b\,x^3+a}}{945b^3}\right)+B\left(\frac{2b\,x^{15}\sqrt{b\,x^3+a}}{33}\right.$
elliptic	$\frac{2Bb\,x^{15}\sqrt{b\,x^3+a}}{33}+\frac{2(b^2A+\frac{12}{11}abB)x^{12}\sqrt{b\,x^3+a}}{27b}+\frac{2\left(2abA+a^2B-\frac{8a\left(b^2A+\frac{12}{11}abB\right)}{9b}\right)x^9\sqrt{b\,x^3+a}}{21b}+\frac{2\left(a^2A-\frac{6a\left(2abA+\right.}{}\right.}{}$

[In] int(x^8*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)

[Out] 16/945*(b*x^3+a)^(5/2)*(35/8*x^6*(9/11*x^3*B+A)*b^3-5/2*x^3*(21/22*x^3*B+A)
*a*b^2+a^2*(15/11*x^3*B+A)*b-6/11*a^3*B)/b^4

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.20

$$\int x^8(a+bx^3)^{3/2}(A+Bx^3)\,dx = \frac{2\,(315\,Bb^5x^{15}+35\,(12\,Bab^4+11\,Ab^5)x^{12}+5\,(3\,Ba^2b^3+110\,Aab^4)x^9-3\,(6\,Ba^3b^2-11\,Aa^2b^3)x}{10395\,b^4}$$

[In] integrate(x^8*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")

[Out] 2/10395*(315*B*b^5*x^15 + 35*(12*B*a*b^4 + 11*A*b^5)*x^12 + 5*(3*B*a^2*b^3
+ 110*A*a*b^4)*x^9 - 3*(6*B*a^3*b^2 - 11*A*a^2*b^3)*x^6 - 48*B*a^5 + 88*A*a
^4*b + 4*(6*B*a^4*b - 11*A*a^3*b^2)*x^3)*sqrt(b*x^3 + a)/b^4

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(100) = 200$.

Time = 0.58 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.59

$$\int x^8 (a+bx^3)^{3/2} (A+Bx^3)\, dx = \begin{cases} \frac{16Aa^4\sqrt{a+bx^3}}{945b^3} - \frac{8Aa^3x^3\sqrt{a+bx^3}}{945b^2} + \frac{2Aa^2x^6\sqrt{a+bx^3}}{315b} + \frac{20Aax^9\sqrt{a+bx^3}}{189} + \frac{2Abx^{12}\sqrt{a+bx^3}}{27} - \frac{32Ba^5\sqrt{a+bx^3}}{3465b^4} + \frac{16B}{} \\ a^{\frac{3}{2}}\left(\frac{Ax^9}{9} + \frac{Bx^{12}}{12}\right) \end{cases}$$

```
[In] integrate(x**8*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] Piecewise((16*A*a**4*sqrt(a + b*x**3)/(945*b**3) - 8*A*a**3*x**3*sqrt(a + b
*x**3)/(945*b**2) + 2*A*a**2*x**6*sqrt(a + b*x**3)/(315*b) + 20*A*a*x**9*sq
rt(a + b*x**3)/189 + 2*A*b*x**12*sqrt(a + b*x**3)/27 - 32*B*a**5*sqrt(a + b
*x**3)/(3465*b**4) + 16*B*a**4*x**3*sqrt(a + b*x**3)/(3465*b**3) - 4*B*a**3
*x**6*sqrt(a + b*x**3)/(1155*b**2) + 2*B*a**2*x**9*sqrt(a + b*x**3)/(693*b)
 + 8*B*a*x**12*sqrt(a + b*x**3)/99 + 2*B*b*x**15*sqrt(a + b*x**3)/33, Ne(b,
 0)), (a**(3/2)*(A*x**9/9 + B*x**12/12), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.15

$$\int x^8 (a+bx^3)^{3/2} (A+Bx^3)\, dx = \frac{2}{945}\left(\frac{35\,(bx^3+a)^{\frac{9}{2}}}{b^3} - \frac{90\,(bx^3+a)^{\frac{7}{2}}a}{b^3} + \frac{63\,(bx^3+a)^{\frac{5}{2}}a^2}{b^3}\right)A + \frac{2}{3465}\left(\frac{105\,(bx^3+a)^{\frac{11}{2}}}{b^4} - \frac{385\,(bx^3+a)^{\frac{9}{2}}a}{b^4} + \frac{495\,(bx^3+a)^{\frac{7}{2}}a^2}{b^4} - \frac{231\,(bx^3+a)^{\frac{5}{2}}a^3}{b^4}\right)B$$

```
[In] integrate(x^8*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/945*(35*(b*x^3 + a)^(9/2)/b^3 - 90*(b*x^3 + a)^(7/2)*a/b^3 + 63*(b*x^3 +
a)^(5/2)*a^2/b^3)*A + 2/3465*(105*(b*x^3 + a)^(11/2)/b^4 - 385*(b*x^3 + a)^
(9/2)*a/b^4 + 495*(b*x^3 + a)^(7/2)*a^2/b^4 - 231*(b*x^3 + a)^(5/2)*a^3/b^4
)*B
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int x^8 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right)\, dx = \frac{2\left(315\left(bx^3+a\right)^{\frac{11}{2}}B-1155\left(bx^3+a\right)^{\frac{9}{2}}Ba+1485\left(bx^3+a\right)^{\frac{7}{2}}Ba^2-693\left(bx^3+a\right)^{\frac{5}{2}}Ba^3+385\left(bx\right.\right.}{10395\,b^4}$$

```
[In] integrate(x^8*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/10395*(315*(b*x^3 + a)^(11/2)*B - 1155*(b*x^3 + a)^(9/2)*B*a + 1485*(b*x^
3 + a)^(7/2)*B*a^2 - 693*(b*x^3 + a)^(5/2)*B*a^3 + 385*(b*x^3 + a)^(9/2)*A*
b - 990*(b*x^3 + a)^(7/2)*A*a*b + 693*(b*x^3 + a)^(5/2)*A*a^2*b)/b^4
```

Mupad [B] (verification not implemented)

Time = 6.99 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.00

$$\int x^8 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right)\, dx = \frac{20\,A\,a\,x^9\,\sqrt{b\,x^3+a}}{189}+\frac{2\,A\,b\,x^{12}\,\sqrt{b\,x^3+a}}{27}+\frac{8\,B\,a\,x^{12}\,\sqrt{b\,x^3+a}}{99}+\frac{2\,B\,b\,x^{15}\,\sqrt{b\,x^3+a}}{33}+\frac{16\,A\,a^4\,\sqrt{b\,x^3+a}}{945\,b^3}-\frac{32\,B\,a^5\,\sqrt{b\,x^3+a}}{3465\,b^4}-\frac{8\,A\,a^3\,x^3\,\sqrt{b\,x^3+a}}{945\,b^2}+\frac{2\,A\,a^2\,x^6\,\sqrt{b\,x^3+a}}{315\,b}+\frac{16\,B\,a^4\,x^3\,\sqrt{b\,x^3+a}}{3465\,b^3}-\frac{4\,B\,a^3\,x^6\,\sqrt{b\,x^3+a}}{1155\,b^2}+\frac{2\,B\,a^2\,x^9\,\sqrt{b\,x^3+a}}{693\,b}$$

```
[In] int(x^8*(A + B*x^3)*(a + b*x^3)^(3/2),x)
```

```
[Out] (20*A*a*x^9*(a + b*x^3)^(1/2))/189 + (2*A*b*x^12*(a + b*x^3)^(1/2))/27 + (8
*B*a*x^12*(a + b*x^3)^(1/2))/99 + (2*B*b*x^15*(a + b*x^3)^(1/2))/33 + (16*A
*a^4*(a + b*x^3)^(1/2))/(945*b^3) - (32*B*a^5*(a + b*x^3)^(1/2))/(3465*b^4)
 - (8*A*a^3*x^3*(a + b*x^3)^(1/2))/(945*b^2) + (2*A*a^2*x^6*(a + b*x^3)^(1/
2))/(315*b) + (16*B*a^4*x^3*(a + b*x^3)^(1/2))/(3465*b^3) - (4*B*a^3*x^6*(a
 + b*x^3)^(1/2))/(1155*b^2) + (2*B*a^2*x^9*(a + b*x^3)^(1/2))/(693*b)
```

3.197 $\int x^5(a+bx^3)^{3/2}(A+Bx^3)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int x^5(a+bx^3)^{3/2}(A+Bx^3)\,dx = -\frac{2a(Ab-aB)(a+bx^3)^{5/2}}{15b^3} + \frac{2(Ab-2aB)(a+bx^3)^{7/2}}{21b^3} + \frac{2B(a+bx^3)^{9/2}}{27b^3}$$

```
[Out] -2/15*a*(A*b-B*a)*(b*x^3+a)^(5/2)/b^3+2/21*(A*b-2*B*a)*(b*x^3+a)^(7/2)/b^3+
2/27*B*(b*x^3+a)^(9/2)/b^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 78}

$$\int x^5(a+bx^3)^{3/2}(A+Bx^3)\,dx = \frac{2(a+bx^3)^{7/2}(Ab-2aB)}{21b^3} - \frac{2a(a+bx^3)^{5/2}(Ab-aB)}{15b^3} + \frac{2B(a+bx^3)^{9/2}}{27b^3}$$

```
[In] Int[x^5*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (-2*a*(A*b - a*B)*(a + b*x^3)^(5/2))/(15*b^3) + (2*(A*b - 2*a*B)*(a + b*x^3
)^(7/2))/(21*b^3) + (2*B*(a + b*x^3)^(9/2))/(27*b^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int x(a+bx)^{3/2}(A+Bx)\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)(a+bx)^{3/2}}{b^2}+\frac{(Ab-2aB)(a+bx)^{5/2}}{b^2}+\frac{B(a+bx)^{7/2}}{b^2}\right)\,dx, x, x^3\right)\\
&= -\frac{2a(Ab-aB)\left(a+bx^3\right)^{5/2}}{15b^3}+\frac{2(Ab-2aB)\left(a+bx^3\right)^{7/2}}{21b^3}+\frac{2B\left(a+bx^3\right)^{9/2}}{27b^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int x^5\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{2\left(a+bx^3\right)^{5/2}\left(-18aAb+8a^2B+45Ab^2x^3-20abBx^3+35b^2Bx^6\right)}{945b^3}$$

```
[In] Integrate[x^5*(a + b*x^3)^(3/2)*(A + B*x^3),x]
[Out] (2*(a + b*x^3)^(5/2)*(-18*a*A*b + 8*a^2*B + 45*A*b^2*x^3 - 20*a*b*B*x^3 + 3
5*b^2*B*x^6))/(945*b^3)
```

Maple [A] (verified)

Time = 4.34 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$-\frac{4(b\,x^3+a)^{\frac{5}{2}}\left(-\frac{5x^3\left(\frac{7x^3B}{9}+A\right)b^2}{2}+a\left(\frac{10x^3B}{9}+A\right)b-\frac{4a^2B}{9}\right)}{105b^3}$
gosper	$-\frac{2(b\,x^3+a)^{\frac{5}{2}}\left(-35b^2B\,x^6-45A\,b^2x^3+20Bab\,x^3+18abA-8a^2B\right)}{945b^3}$
trager	$-\frac{2\left(-35B\,b^4x^{12}-45A\,b^4x^9-50Ba\,b^3x^9-72Aa\,b^3x^6-3B\,a^2b^2x^6-9A\,a^2b^2x^3+4B\,a^3b\,x^3+18A\,a^3b-8B\,a^4\right)\sqrt{b\,x^3+a}}{945b^3}$
risch	$-\frac{2\left(-35B\,b^4x^{12}-45A\,b^4x^9-50Ba\,b^3x^9-72Aa\,b^3x^6-3B\,a^2b^2x^6-9A\,a^2b^2x^3+4B\,a^3b\,x^3+18A\,a^3b-8B\,a^4\right)\sqrt{b\,x^3+a}}{945b^3}$
default	$B\left(\frac{2b\,x^{12}\sqrt{b\,x^3+a}}{27}+\frac{20a\,x^9\sqrt{b\,x^3+a}}{189}+\frac{2a^2x^6\sqrt{b\,x^3+a}}{315b}-\frac{8a^3x^3\sqrt{b\,x^3+a}}{945b^2}+\frac{16a^4\sqrt{b\,x^3+a}}{945b^3}\right)+A\left(\frac{2b\,x^9\sqrt{b\,x^3+a}}{21}\right.$
elliptic	$\frac{2Bb\,x^{12}\sqrt{b\,x^3+a}}{27}+\frac{2(b^2A+\frac{10}{9}abB)x^9\sqrt{b\,x^3+a}}{21b}+\frac{2\left(2abA+a^2B-\frac{6a\left(b^2A+\frac{10}{9}abB\right)}{7b}\right)x^6\sqrt{b\,x^3+a}}{15b}+\frac{2\left(a^2A-\frac{4a\left(2abA+\right.}{}\right.}{}$

```
[In] int(x^5*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] -4/105*(b*x^3+a)^(5/2)*(-5/2*x^3*(7/9*x^3*B+A)*b^2+a*(10/9*x^3*B+A)*b-4/9*a
^2*B)/b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.36

$$\int x^5\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{2\left(35\,Bb^4x^{12}+5\,(10\,Bab^3+9\,Ab^4)x^9+3\,(Ba^2b^2+24\,Aab^3)x^6+8\,Ba^4-18\,Aa^3b-(4\,Ba^3b-\right.}{945\,b^3}$$

```
[In] integrate(x^5*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/945*(35*B*b^4*x^12 + 5*(10*B*a*b^3 + 9*A*b^4)*x^9 + 3*(B*a^2*b^2 + 24*A*a
*b^3)*x^6 + 8*B*a^4 - 18*A*a^3*b - (4*B*a^3*b - 9*A*a^2*b^2)*x^3)*sqrt(b*x^
3 + a)/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(70) = 140.

Time = 0.42 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.96

$$\int x^5\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\begin{cases}-\frac{4Aa^3\sqrt{a+bx^3}}{105b^2}+\frac{2Aa^2x^3\sqrt{a+bx^3}}{105b}+\frac{16Aax^6\sqrt{a+bx^3}}{105}+\frac{2Abx^9\sqrt{a+bx^3}}{21}+\frac{16Ba^4\sqrt{a+bx^3}}{945b^3}-\frac{8Ba^3x^3\sqrt{a+bx^3}}{945b^2}+\frac{2Ba^2 \ldots \\ a^{\frac{3}{2}}\left(\frac{Ax^6}{6}+\frac{Bx^9}{9}\right)\end{cases}$$

```
[In] integrate(x**5*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] Piecewise((-4*A*a**3*sqrt(a + b*x**3)/(105*b**2) + 2*A*a**2*x**3*sqrt(a + b
*x**3)/(105*b) + 16*A*a*x**6*sqrt(a + b*x**3)/105 + 2*A*b*x**9*sqrt(a + b*x
**3)/21 + 16*B*a**4*sqrt(a + b*x**3)/(945*b**3) - 8*B*a**3*x**3*sqrt(a + b*
x**3)/(945*b**2) + 2*B*a**2*x**6*sqrt(a + b*x**3)/(315*b) + 20*B*a*x**9*sqr
t(a + b*x**3)/189 + 2*B*b*x**12*sqrt(a + b*x**3)/27, Ne(b, 0)), (a**(3/2)*(
A*x**6/6 + B*x**9/9), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15

$$\int x^5\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{2}{105}\left(\frac{5\left(bx^3+a\right)^{\frac{7}{2}}}{b^2}-\frac{7\left(bx^3+a\right)^{\frac{5}{2}}a}{b^2}\right)A+\frac{2}{945}\left(\frac{35\left(bx^3+a\right)^{\frac{9}{2}}}{b^3}-\frac{90\left(bx^3+a\right)^{\frac{7}{2}}a}{b^3}+\frac{63\left(bx^3+a\right)^{\frac{5}{2}}a^2}{b^3}\right)B$$

```
[In] integrate(x^5*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/105*(5*(b*x^3 + a)^(7/2)/b^2 - 7*(b*x^3 + a)^(5/2)*a/b^2)*A + 2/945*(35*(
b*x^3 + a)^(9/2)/b^3 - 90*(b*x^3 + a)^(7/2)*a/b^3 + 63*(b*x^3 + a)^(5/2)*a^
2/b^3)*B
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int x^5 (a+bx^3)^{3/2} (A+Bx^3)\,dx = \frac{2\left(35\,(bx^3+a)^{\frac{9}{2}}B - 90\,(bx^3+a)^{\frac{7}{2}}Ba + 63\,(bx^3+a)^{\frac{5}{2}}Ba^2 + 45\,(bx^3+a)^{\frac{7}{2}}Ab - 63\,(bx^3+a)^{\frac{5}{2}}A\right.}{945\,b^3}$$

```
[In] integrate(x^5*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/945*(35*(b*x^3 + a)^(9/2)*B - 90*(b*x^3 + a)^(7/2)*B*a + 63*(b*x^3 + a)^(
5/2)*B*a^2 + 45*(b*x^3 + a)^(7/2)*A*b - 63*(b*x^3 + a)^(5/2)*A*a*b)/b^3
```

Mupad [B] (verification not implemented)

Time = 6.83 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.89

$$\int x^5 (a+bx^3)^{3/2} (A+Bx^3)\,dx = \frac{x^6\sqrt{b\,x^3+a}\left(2\,B\,a^2+4\,A\,a\,b-\frac{6\,a\left(2\,A\,b^2+\frac{20\,B\,a\,b}{9}\right)}{7\,b}\right)}{15\,b} - \frac{2\,a\left(2\,A\,a^2-\frac{4\,a\left(2\,B\,a^2+4\,A\,a\,b-\frac{6\,a\left(2\,A\,b^2+\frac{20\,B\,a\,b}{9}\right)}{7\,b}\right)}{5\,b}\right)\sqrt{b\,x^3+a}}{9\,b^2} + \frac{2\,B\,b\,x^{12}\sqrt{b\,x^3+a}}{27} + \frac{x^3\left(2\,A\,a^2-\frac{4\,a\left(2\,B\,a^2+4\,A\,a\,b-\frac{6\,a\left(2\,A\,b^2+\frac{20\,B\,a\,b}{9}\right)}{7\,b}\right)}{5\,b}\right)\sqrt{b\,x^3+a}}{9\,b} + \frac{x^9\left(2\,A\,b^2+\frac{20\,B\,a\,b}{9}\right)\sqrt{b\,x^3+a}}{21\,b}$$

```
[In] int(x^5*(A + B*x^3)*(a + b*x^3)^(3/2),x)
```

```
[Out] (x^6*(a + b*x^3)^(1/2)*(2*B*a^2 + 4*A*a*b - (6*a*(2*A*b^2 + (20*B*a*b)/9))/
(7*b)))/(15*b) - (2*a*(2*A*a^2 - (4*a*(2*B*a^2 + 4*A*a*b - (6*a*(2*A*b^2 +
(20*B*a*b)/9))/(7*b)))/(5*b))*(a + b*x^3)^(1/2))/(9*b^2) + (2*B*b*x^12*(a +
 b*x^3)^(1/2))/27 + (x^3*(2*A*a^2 - (4*a*(2*B*a^2 + 4*A*a*b - (6*a*(2*A*b^2
 + (20*B*a*b)/9))/(7*b)))/(5*b))*(a + b*x^3)^(1/2))/(9*b) + (x^9*(2*A*b^2 +
 (20*B*a*b)/9)*(a + b*x^3)^(1/2))/(21*b)
```

3.198 $\int x^2(a+bx^3)^{3/2}(A+Bx^3)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int x^2(a+bx^3)^{3/2}\left(A+Bx^3\right)\,dx=\frac{2(Ab-aB)\left(a+bx^3\right)^{5/2}}{15b^2}+\frac{2B(a+bx^3)^{7/2}}{21b^2}$$

```
[Out] 2/15*(A*b-B*a)*(b*x^3+a)^(5/2)/b^2+2/21*B*(b*x^3+a)^(7/2)/b^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {455, 45}

$$\int x^2(a+bx^3)^{3/2}\left(A+Bx^3\right)\,dx=\frac{2(a+bx^3)^{5/2}(Ab-aB)}{15b^2}+\frac{2B(a+bx^3)^{7/2}}{21b^2}$$

```
[In] Int[x^2*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*(A*b - a*B)*(a + b*x^3)^(5/2))/(15*b^2) + (2*B*(a + b*x^3)^(7/2))/(21*b^
2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int (a+bx)^{3/2}(A+Bx)\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{(Ab-aB)(a+bx)^{3/2}}{b}+\frac{B(a+bx)^{5/2}}{b}\right)\,dx, x, x^3\right)\\ &= \frac{2(Ab-aB)\left(a+bx^3\right)^{5/2}}{15b^2}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int x^2\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{2\left(a+bx^3\right)^{5/2}\left(7Ab-2aB+5bBx^3\right)}{105b^2}$$

```
[In] Integrate[x^2*(a + b*x^3)^(3/2)*(A + B*x^3),x]
[Out] (2*(a + b*x^3)^(5/2)*(7*A*b - 2*a*B + 5*b*B*x^3))/(105*b^2)
```

Maple [A] (verified)

Time = 4.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

method	result
gosper	$\frac{2\left(b\,x^{3}+a\right)^{\frac{5}{2}}\left(5bB\,x^{3}+7Ab-2Ba\right)}{105b^{2}}$
pseudoelliptic	$\frac{2\left(\left(5x^{3}B+7A\right)b-2Ba\right)\left(b\,x^{3}+a\right)^{\frac{5}{2}}}{105b^{2}}$
trager	$\frac{2\left(5b^{3}B\,x^{9}+7x^{6}b^{3}A+8B\,x^{6}a\,b^{2}+14aA\,b^{2}x^{3}+B\,a^{2}b\,x^{3}+7a^{2}bA-2a^{3}B\right)\sqrt{b\,x^{3}+a}}{105b^{2}}$
risch	$\frac{2\left(5b^{3}B\,x^{9}+7x^{6}b^{3}A+8B\,x^{6}a\,b^{2}+14aA\,b^{2}x^{3}+B\,a^{2}b\,x^{3}+7a^{2}bA-2a^{3}B\right)\sqrt{b\,x^{3}+a}}{105b^{2}}$
default	$B\left(\frac{2b\,x^{9}\sqrt{b\,x^{3}+a}}{21}+\frac{16a\,x^{6}\sqrt{b\,x^{3}+a}}{105}+\frac{2a^{2}x^{3}\sqrt{b\,x^{3}+a}}{105b}-\frac{4a^{3}\sqrt{b\,x^{3}+a}}{105b^{2}}\right)+\frac{2A\left(b\,x^{3}+a\right)^{\frac{5}{2}}}{15b}$
elliptic	$\frac{2Bb\,x^{9}\sqrt{b\,x^{3}+a}}{21}+\frac{2\left(b^{2}A+\frac{8}{7}abB\right)x^{6}\sqrt{b\,x^{3}+a}}{15b}+\frac{2\left(2abA+a^{2}B-\frac{4a\left(b^{2}A+\frac{8}{7}abB\right)}{5b}\right)x^{3}\sqrt{b\,x^{3}+a}}{9b}+\frac{2\left(a^{2}A-\frac{2a\left(2abA+a^{2}B-\right.}{}\right.}{}$

[In] int(x^2*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)

[Out] 2/105*(b*x^3+a)^(5/2)*(5*B*b*x^3+7*A*b-2*B*a)/b^2

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.59

$$\int x^2\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{2\left(5\,Bb^3x^9+(8\,Bab^2+7\,Ab^3)x^6-2\,Ba^3+7\,Aa^2b+(Ba^2b+14\,Aab^2)x^3\right)\sqrt{bx^3+a}}{105\,b^2}$$

[In] integrate(x^2*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")

[Out] 2/105*(5*B*b^3*x^9 + (8*B*a*b^2 + 7*A*b^3)*x^6 - 2*B*a^3 + 7*A*a^2*b + (B*a^2*b + 14*A*a*b^2)*x^3)*sqrt(b*x^3 + a)/b^2

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(44) = 88$.

Time = 0.32 (sec) , antiderivative size = 165, normalized size of antiderivative = 3.59

$$\int x^2 (a + bx^3)^{3/2} (A + Bx^3) \, dx = \begin{cases} \frac{2Aa^2\sqrt{a+bx^3}}{15b} + \frac{4Aax^3\sqrt{a+bx^3}}{15} + \frac{2Abx^6\sqrt{a+bx^3}}{15} - \frac{4Ba^3\sqrt{a+bx^3}}{105b^2} + \frac{2Ba^2x^3\sqrt{a+bx^3}}{105b} + \frac{16Bax^6\sqrt{a+bx^3}}{105} + \frac{2Bbx^9\sqrt{}}{21} \\ a^{\frac{3}{2}}\left(\frac{Ax^3}{3} + \frac{Bx^6}{6}\right) \end{cases}$$

```
[In] integrate(x**2*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] Piecewise((2*A*a**2*sqrt(a + b*x**3)/(15*b) + 4*A*a*x**3*sqrt(a + b*x**3)/1
5 + 2*A*b*x**6*sqrt(a + b*x**3)/15 - 4*B*a**3*sqrt(a + b*x**3)/(105*b**2) +
 2*B*a**2*x**3*sqrt(a + b*x**3)/(105*b) + 16*B*a*x**6*sqrt(a + b*x**3)/105
+ 2*B*b*x**9*sqrt(a + b*x**3)/21, Ne(b, 0)), (a**(3/2)*(A*x**3/3 + B*x**6/6
), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int x^2 (a + bx^3)^{3/2} (A + Bx^3) \, dx = \frac{2 (bx^3 + a)^{\frac{5}{2}} A}{15 \, b} + \frac{2}{105} \left(\frac{5 (bx^3 + a)^{\frac{7}{2}}}{b^2} - \frac{7 (bx^3 + a)^{\frac{5}{2}} a}{b^2} \right) B$$

```
[In] integrate(x^2*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/15*(b*x^3 + a)^(5/2)*A/b + 2/105*(5*(b*x^3 + a)^(7/2)/b^2 - 7*(b*x^3 + a)
^(5/2)*a/b^2)*B
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int x^2 (a + bx^3)^{3/2} (A + Bx^3) \, dx = \frac{2 \left(5 (bx^3 + a)^{\frac{7}{2}} B - 7 (bx^3 + a)^{\frac{5}{2}} Ba + 7 (bx^3 + a)^{\frac{5}{2}} Ab\right)}{105 \, b^2}$$

```
[In] integrate(x^2*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/105*(5*(b*x^3 + a)^(7/2)*B - 7*(b*x^3 + a)^(5/2)*B*a + 7*(b*x^3 + a)^(5/2
)*A*b)/b^2
```

Mupad [B] (verification not implemented)

Time = 6.91 (sec) , antiderivative size = 150, normalized size of antiderivative = 3.26

$$\int x^2 (a + bx^3)^{3/2} (A + Bx^3) \, dx = \frac{\left(2 A a^2 - \frac{2 a \left(2 B a^2 + 4 A a b - \frac{4 a \left(2 A b^2 + \frac{16 B a b}{7}\right)}{5 b}\right)}{3 b}\right) \sqrt{b x^3 + a}}{3 b} + \frac{x^3 \sqrt{b x^3 + a} \left(2 B a^2 + 4 A a b - \frac{4 a \left(2 A b^2 + \frac{16 B a b}{7}\right)}{5 b}\right)}{9 b} + \frac{2 B b x^9 \sqrt{b x^3 + a}}{21} + \frac{x^6 \left(2 A b^2 + \frac{16 B a b}{7}\right) \sqrt{b x^3 + a}}{15 b}$$

```
[In] int(x^2*(A + B*x^3)*(a + b*x^3)^(3/2),x)
```

```
[Out] ((2*A*a^2 - (2*a*(2*B*a^2 + 4*A*a*b - (4*a*(2*A*b^2 + (16*B*a*b)/7))/(5*b))
)/(3*b))*(a + b*x^3)^(1/2))/(3*b) + (x^3*(a + b*x^3)^(1/2)*(2*B*a^2 + 4*A*a
*b - (4*a*(2*A*b^2 + (16*B*a*b)/7))/(5*b)))/(9*b) + (2*B*b*x^9*(a + b*x^3)^
(1/2))/21 + (x^6*(2*A*b^2 + (16*B*a*b)/7)*(a + b*x^3)^(1/2))/(15*b)
```

3.199 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x} dx$

Optimal result

Integrand size = 22, antiderivative size = 81

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x} dx = \frac{2}{3}aA\sqrt{a+bx^3} + \frac{2}{9}A(a+bx^3)^{3/2} + \frac{2B(a+bx^3)^{5/2}}{15b} - \frac{2}{3}a^{3/2}A\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

```
[Out] 2/9*A*(b*x^3+a)^(3/2)+2/15*B*(b*x^3+a)^(5/2)/b-2/3*a^(3/2)*A*arctanh((b*x^3
+a)^(1/2)/a^(1/2))+2/3*a*A*(b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 81, 52, 65, 214}

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x} dx = -\frac{2}{3}a^{3/2}A\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right) + \frac{2}{9}A(a+bx^3)^{3/2} + \frac{2}{3}aA\sqrt{a+bx^3} + \frac{2B(a+bx^3)^{5/2}}{15b}$$

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x,x]
```

```
[Out] (2*a*A*Sqrt[a + b*x^3])/3 + (2*A*(a + b*x^3)^(3/2))/9 + (2*B*(a + b*x^3)^(5
/2))/(15*b) - (2*a^(3/2)*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x}\,dx, x, x^3\right)\\
&= \frac{2B(a+bx^3)^{5/2}}{15b} + \frac{1}{3}A\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, x^3\right)\\
&= \frac{2}{9}A(a+bx^3)^{3/2} + \frac{2B(a+bx^3)^{5/2}}{15b} + \frac{1}{3}(aA)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^3\right)
\end{aligned}$$

$$= \frac{2}{3} a A \sqrt{a + bx^3} + \frac{2}{9} A \left(a + bx^3\right)^{3/2} + \frac{2B(a + bx^3)^{5/2}}{15b} + \frac{1}{3} \left(a^2 A\right) \text{Subst}\left(\int \frac{1}{x\sqrt{a + bx}} \, dx, x, x^3\right)$$

$$= \frac{2}{3} a A \sqrt{a + bx^3} + \frac{2}{9} A \left(a + bx^3\right)^{3/2} + \frac{2B(a + bx^3)^{5/2}}{15b} + \frac{(2a^2 A) \text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} \, dx, x, \sqrt{a + bx^3}\right)}{3b}$$

$$= \frac{2}{3} a A \sqrt{a + bx^3} + \frac{2}{9} A \left(a + bx^3\right)^{3/2} + \frac{2B(a + bx^3)^{5/2}}{15b} - \frac{2}{3} a^{3/2} A \tanh^{-1}\left(\frac{\sqrt{a + bx^3}}{\sqrt{a}}\right)$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int \frac{(a + bx^3)^{3/2} (A + Bx^3)}{x} \, dx = \frac{2\sqrt{a + bx^3}(20aAb + 3a^2 B + 5Ab^2 x^3 + 6abBx^3 + 3b^2 Bx^6)}{45b} - \frac{2}{3} a^{3/2} A \text{arctanh}\left(\frac{\sqrt{a + bx^3}}{\sqrt{a}}\right)$$

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x,x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(20*a*A*b + 3*a^2*B + 5*A*b^2*x^3 + 6*a*b*B*x^3 + 3*b^2*
B*x^6))/(45*b) - (2*a^(3/2)*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3
```

Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.81

method	result
default	$\frac{2B\left(b\,x^3+a\right)^{\frac{5}{2}}}{15b} + A\left(\frac{2b\,x^3\sqrt{b\,x^3+a}}{9} + \frac{8a\sqrt{b\,x^3+a}}{9} - \frac{2a^{\frac{3}{2}}\,\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3}\right)$
pseudoelliptic	$\frac{-\frac{2a^{\frac{3}{2}}\,bA\,\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3} + \frac{8\left(\frac{x^3\left(\frac{3x^3B}{5}+A\right)b^2}{4}+a\left(\frac{3x^3B}{10}+A\right)b+\frac{3a^2B}{20}\right)\sqrt{b\,x^3+a}}{9}}{b}$
elliptic	$\frac{2Bb\,x^6\sqrt{b\,x^3+a}}{15} + \frac{2\left(b^2A+\frac{6}{5}abB\right)x^3\sqrt{b\,x^3+a}}{9b} + \frac{2\left(2abA+a^2B-\frac{2\left(b^2A+\frac{6}{5}abB\right)a}{3b}\right)\sqrt{b\,x^3+a}}{3b} - \frac{2a^{\frac{3}{2}}\,A\,\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3}$

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*B*(b*x^3+a)^(5/2)/b+A*(2/9*b*x^3*(b*x^3+a)^(1/2)+8/9*a*(b*x^3+a)^(1/2)
-2/3*a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.12

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x}\,dx = \left[\frac{15\,Aa^{\frac{3}{2}}b\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\,(3\,Bb^2x^6+(6\,Bab+5\,Ab^2)x^3+3\,Ba^2}{45\,b}\right.$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x,x, algorithm="fricas")
```

```
[Out] [1/45*(15*A*a^(3/2)*b*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) +
2*(3*B*b^2*x^6 + (6*B*a*b + 5*A*b^2)*x^3 + 3*B*a^2 + 20*A*a*b)*sqrt(b*x^3 +
 a))/b, 2/45*(15*A*sqrt(-a)*a*b*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (3*B*b
^2*x^6 + (6*B*a*b + 5*A*b^2)*x^3 + 3*B*a^2 + 20*A*a*b)*sqrt(b*x^3 + a))/b]
```

Sympy [A] (verification not implemented)

Time = 10.22 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x}\,dx = \frac{\begin{cases} \frac{2Aa^2\operatorname{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2Aa\sqrt{a+bx^3}+\frac{2A(a+bx^3)^{\frac{3}{2}}}{3}+\frac{2B(a+bx^3)^{\frac{5}{2}}}{5b} & \text{for } b\neq 0\\ Aa^{\frac{3}{2}}\log\left(Ba^{\frac{3}{2}}x^3\right)+Ba^{\frac{3}{2}}x^3 & \text{otherwise}\end{cases}}{3}$$

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x,x)
```

```
[Out] Piecewise((2*A*a**2*atan(sqrt(a + b*x**3)/sqrt(-a))/sqrt(-a) + 2*A*a*sqrt(a
 + b*x**3) + 2*A*(a + b*x**3)**(3/2)/3 + 2*B*(a + b*x**3)**(5/2)/(5*b), Ne(
b, 0)), (A*a**(3/2)*log(B*a**(3/2)*x**3) + B*a**(3/2)*x**3, True))/3
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x}\,dx = \frac{2\,(bx^3+a)^{\frac{5}{2}}B}{15\,b} + \frac{1}{9}\left(3\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)+2\,(bx^3+a)^{\frac{3}{2}}+6\,\sqrt{bx^3+a}a\right)A$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x,x, algorithm="maxima")
```

```
[Out] 2/15*(b*x^3 + a)^(5/2)*B/b + 1/9*(3*a^(3/2)*log((sqrt(b*x^3 + a) - sqrt(a))
/(sqrt(b*x^3 + a) + sqrt(a))) + 2*(b*x^3 + a)^(3/2) + 6*sqrt(b*x^3 + a)*a)*
A
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x}\,dx = \frac{2\,Aa^2\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}} + \frac{2\left(3\,(bx^3+a)^{\frac{5}{2}}Bb^4+5\,(bx^3+a)^{\frac{3}{2}}Ab^5+15\,\sqrt{bx^3+a}Aab^5\right)}{45\,b^5}$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x,x, algorithm="giac")
```

```
[Out] 2/3*A*a^2*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) + 2/45*(3*(b*x^3 + a)^(
5/2)*B*b^4 + 5*(b*x^3 + a)^(3/2)*A*b^5 + 15*sqrt(b*x^3 + a)*A*a*b^5)/b^5
```

Mupad [B] (verification not implemented)

Time = 6.91 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.62

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x}\,dx = \frac{A\,a^{3/2}\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3} + \frac{\sqrt{b\,x^3+a}\left(2\,B\,a^2+4\,A\,a\,b-\frac{2\,a\left(2\,A\,b^2+\frac{12\,B\,a\,b}{5}\right)}{3\,b}\right)}{3\,b} + \frac{2\,B\,b\,x^6\,\sqrt{b\,x^3+a}}{15} + \frac{x^3\left(2\,A\,b^2+\frac{12\,B\,a\,b}{5}\right)\sqrt{b\,x^3+a}}{9\,b}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x,x)
```

```
[Out] (A*a^(3/2)*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2
)))/x^6))/3 + ((a + b*x^3)^(1/2)*(2*B*a^2 + 4*A*a*b - (2*a*(2*A*b^2 + (12*B
*a*b)/5))/(3*b)))/(3*b) + (2*B*b*x^6*(a + b*x^3)^(1/2))/15 + (x^3*(2*A*b^2
+ (12*B*a*b)/5)*(a + b*x^3)^(1/2))/(9*b)
```

3.200 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 110

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^4}\,dx = \frac{1}{3}(3Ab+2aB)\sqrt{a+bx^3}+\frac{(3Ab+2aB)(a+bx^3)^{3/2}}{9a} - \frac{A(a+bx^3)^{5/2}}{3ax^3}-\frac{1}{3}\sqrt{a}(3Ab+2aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

```
[Out] 1/9*(3*A*b+2*B*a)*(b*x^3+a)^(3/2)/a-1/3*A*(b*x^3+a)^(5/2)/a/x^3-1/3*(3*A*b+
2*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+1/3*(3*A*b+2*B*a)*(b*x^3+a)
^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 79, 52, 65, 214}

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^4}\,dx = -\frac{1}{3}\sqrt{a}(2aB+3Ab)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right) + \frac{(a+bx^3)^{3/2}(2aB+3Ab)}{9a}+\frac{1}{3}\sqrt{a+bx^3}(2aB+3Ab)-\frac{A(a+bx^3)^{5/2}}{3ax^3}$$

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^4,x]
```

```
[Out] ((3*A*b + 2*a*B)*Sqrt[a + b*x^3])/3 + ((3*A*b + 2*a*B)*(a + b*x^3)^(3/2))/(
9*a) - (A*(a + b*x^3)^(5/2))/(3*a*x^3) - (Sqrt[a]*(3*A*b + 2*a*B)*ArcTanh[S
qrt[a + b*x^3]/Sqrt[a]])/3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x^2}\,dx, x, x^3\right)$$

$$= -\frac{A(a+bx^3)^{5/2}}{3ax^3} + \frac{\left(\frac{3Ab}{2}+aB\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, x^3\right)}{3a}$$

$$=\frac{(3Ab+2aB)\left(a+bx^3\right)^{3/2}}{9a}-\frac{A\left(a+bx^3\right)^{5/2}}{3ax^3}+\frac{1}{6}(3Ab+2aB)\text{Subst}\left(\int\frac{\sqrt{a+bx}}{x}\,dx,x,x^3\right)$$

$$=\frac{1}{3}(3Ab+2aB)\sqrt{a+bx^3}+\frac{(3Ab+2aB)\left(a+bx^3\right)^{3/2}}{9a}-\frac{A\left(a+bx^3\right)^{5/2}}{3ax^3}+\frac{1}{6}(a(3Ab+2aB))\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)$$

$$=\frac{1}{3}(3Ab+2aB)\sqrt{a+bx^3}+\frac{(3Ab+2aB)\left(a+bx^3\right)^{3/2}}{9a}-\frac{A\left(a+bx^3\right)^{5/2}}{3ax^3}+\frac{(a(3Ab+2aB))\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{3b}$$

$$=\frac{1}{3}(3Ab+2aB)\sqrt{a+bx^3}+\frac{(3Ab+2aB)\left(a+bx^3\right)^{3/2}}{9a}-\frac{A\left(a+bx^3\right)^{5/2}}{3ax^3}-\frac{1}{3}\sqrt{a}(3Ab+2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.74

$$\int\frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^4}\,dx=\frac{\sqrt{a+bx^3}\left(-3aA+6Abx^3+8aBx^3+2bBx^6\right)}{9x^3}-\frac{1}{3}\sqrt{a}(3Ab+2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^4,x]
[Out] (Sqrt[a + b*x^3]*(-3*a*A + 6*A*b*x^3 + 8*a*B*x^3 + 2*b*B*x^6))/(9*x^3) - (S
qrt[a]*(3*A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3
```

Maple [A] (verified)

Time = 4.40 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$-\frac{a\,x^{3} \left(A b+\frac{2 B a}{3}\right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)-\frac{2 \sqrt{b\,x^{3}+a}\,\left(\left(\frac{4 x^{3} B}{3}-\frac{A}{2}\right) a^{\frac{3}{2}}+b\,x^{3} \sqrt{a}\,\left(\frac{x^{3} B}{3}+A\right)\right)}{3}}{\sqrt{a}\,x^{3}}$
elliptic	$-\frac{a A \sqrt{b\,x^{3}+a}}{3 x^{3}}+\frac{2 B b\,x^{3} \sqrt{b\,x^{3}+a}}{9}+\frac{2 \left(b^{2} A+\frac{4}{3} a b B\right) \sqrt{b\,x^{3}+a}}{3 b}-\frac{2 \left(\frac{3}{2} a b A+a^{2} B\right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3 \sqrt{a}}$
default	$B \left(\frac{2 b\,x^{3} \sqrt{b\,x^{3}+a}}{9}+\frac{8 a \sqrt{b\,x^{3}+a}}{9}-\frac{2 a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3}\right)+A \left(-\frac{a \sqrt{b\,x^{3}+a}}{3 x^{3}}+\frac{2 b \sqrt{b\,x^{3}+a}}{3}-b \operatorname{arctanh}\right.$
risch	$-\frac{a A \sqrt{b\,x^{3}+a}}{3 x^{3}}+B\,b^{2} \left(\frac{2 x^{3} \sqrt{b\,x^{3}+a}}{9 b}-\frac{4 a \sqrt{b\,x^{3}+a}}{9 b^{2}}\right)+\frac{2 A b \sqrt{b\,x^{3}+a}}{3}+\frac{4 B a \sqrt{b\,x^{3}+a}}{3}-\frac{\left(3 A b+2 B a\right) \operatorname{arctanh}\left(\frac{\sqrt{b}}{}\right.}{3}$

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^(1/2)*(a*x^3*(A*b+2/3*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))-2/3*(b*x^3
+a)^(1/2)*((4/3*x^3*B-1/2*A)*a^(3/2)+b*x^3*a^(1/2)*(1/3*x^3*B+A)))/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.54

$$\int \frac{(a+bx^3)^{3/2}\,(A+Bx^3)}{x^4}\,dx = \left[\frac{3\,(2\,Ba+3\,Ab)\sqrt{a}x^3\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\,(2\,Bbx^6+2\,(4\,Ba+3\,A}{18\,x^3}\right.$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x, algorithm="fricas")
```

```
[Out] [1/18*(3*(2*B*a + 3*A*b)*sqrt(a)*x^3*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a)
 + 2*a)/x^3) + 2*(2*B*b*x^6 + 2*(4*B*a + 3*A*b)*x^3 - 3*A*a)*sqrt(b*x^3 + a
))/x^3, 1/9*(3*(2*B*a + 3*A*b)*sqrt(-a)*x^3*arctan(sqrt(b*x^3 + a)*sqrt(-a)
/a) + (2*B*b*x^6 + 2*(4*B*a + 3*A*b)*x^3 - 3*A*a)*sqrt(b*x^3 + a))/x^3]
```

Sympy [A] (verification not implemented)

Time = 14.42 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.03

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^4}\,dx = -A\sqrt{a}b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right) - \frac{Aa\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} + \frac{2Aa\sqrt{b}}{3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2Ab^{\frac{3}{2}}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} - \frac{2Ba^{\frac{3}{2}}\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3} + \frac{2Ba^2}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2Ba\sqrt{b}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}} + Bb\left(\begin{cases}\frac{\sqrt{a}x^3}{3} & \text{for}\ b=0\\ \frac{2(a+bx^3)^{\frac{3}{2}}}{9b} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**4,x)
```

```
[Out] -A*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) - A*a*sqrt(b)*sqrt(a/(b*x**3
) + 1)/(3*x**(3/2)) + 2*A*a*sqrt(b)/(3*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*A
*b**(3/2)*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) - 2*B*a**(3/2)*asinh(sqrt(a)/(s
qrt(b)*x**(3/2)))/3 + 2*B*a**2/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) +
2*B*a*sqrt(b)*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + B*b*Piecewise((sqrt(a)*x*
*3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^4}\,dx = \frac{1}{6}\left(3\sqrt{a}b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right) + 4\sqrt{bx^3+a}b - \frac{2\sqrt{bx^3+a}a}{x^3}\right)A + \frac{1}{9}\left(3a^{\frac{3}{2}}\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right) + 2(bx^3+a)^{\frac{3}{2}} + 6\sqrt{bx^3+a}a\right)B$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(3*sqrt(a)*b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)
)) + 4*sqrt(b*x^3 + a)*b - 2*sqrt(b*x^3 + a)*a/x^3)*A + 1/9*(3*a^(3/2)*log(
(sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a))) + 2*(b*x^3 + a)^(3
/2) + 6*sqrt(b*x^3 + a)*a)*B
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^4}\,dx = \frac{2\,(bx^3+a)^{\frac{3}{2}}Bb + 6\,\sqrt{bx^3+a}Bab + 6\,\sqrt{bx^3+a}Ab^2 + \frac{3\,(2\,Ba^2b+3\,Aab^2)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \dots}{9\,b}$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/9*(2*(b*x^3 + a)^(3/2)*B*b + 6*sqrt(b*x^3 + a)*B*a*b + 6*sqrt(b*x^3 + a)*
A*b^2 + 3*(2*B*a^2*b + 3*A*a*b^2)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a)
 - 3*sqrt(b*x^3 + a)*A*a*b/x^3)/b
```

Mupad [B] (verification not implemented)

Time = 7.37 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^4}\,dx = \frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)\,(3\,A\,b+2\,B\,a)\,\sqrt{\frac{a}{4}}}{3} + \frac{\left(2\,A\,b^2+\frac{8\,B\,a\,b}{3}\right)\,\sqrt{b\,x^3+a}}{3\,b} - \frac{A\,a\,\sqrt{b\,x^3+a}}{3\,x^3} + \frac{2\,B\,b\,x^3\,\sqrt{b\,x^3+a}}{9}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^4,x)
```

```
[Out] (log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)*(
3*A*b + 2*B*a)*(a/4)^(1/2))/3 + ((2*A*b^2 + (8*B*a*b)/3)*(a + b*x^3)^(1/2))
/(3*b) - (A*a*(a + b*x^3)^(1/2))/(3*x^3) + (2*B*b*x^3*(a + b*x^3)^(1/2))/9
```

3.201 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^7} dx$

Optimal result

Integrand size = 22, antiderivative size = 115

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^7} dx = \frac{b(Ab+4aB)\sqrt{a+bx^3}}{4a} - \frac{(Ab+4aB)(a+bx^3)^{3/2}}{12ax^3} - \frac{A(a+bx^3)^{5/2}}{6ax^6} - \frac{b(Ab+4aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

```
[Out] -1/12*(A*b+4*B*a)*(b*x^3+a)^(3/2)/a/x^3-1/6*A*(b*x^3+a)^(5/2)/a/x^6-1/4*b*(
A*b+4*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+1/4*b*(A*b+4*B*a)*(b*x^
3+a)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {457, 79, 43, 52, 65, 214}

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^7} dx = -\frac{b(4aB+Ab)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}} - \frac{(a+bx^3)^{3/2}(4aB+Ab)}{12ax^3} + \frac{b\sqrt{a+bx^3}(4aB+Ab)}{4a} - \frac{A(a+bx^3)^{5/2}}{6ax^6}$$

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^7,x]
```

```
[Out] (b*(A*b + 4*a*B)*Sqrt[a + b*x^3])/(4*a) - ((A*b + 4*a*B)*(a + b*x^3)^(3/2))
/(12*a*x^3) - (A*(a + b*x^3)^(5/2))/(6*a*x^6) - (b*(A*b + 4*a*B)*ArcTanh[Sq
rt[a + b*x^3]/Sqrt[a]])/(4*Sqrt[a])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x^3}\,dx, x, x^3\right)$$
$$= -\frac{A(a+bx^3)^{5/2}}{6ax^6} + \frac{(Ab+4aB)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\,dx, x, x^3\right)}{12a}$$
$$= -\frac{(Ab+4aB)\,(a+bx^3)^{3/2}}{12ax^3} - \frac{A(a+bx^3)^{5/2}}{6ax^6} + \frac{(b(Ab+4aB))\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^3\right)}{8a}$$
$$= \frac{b(Ab+4aB)\sqrt{a+bx^3}}{4a} - \frac{(Ab+4aB)\,(a+bx^3)^{3/2}}{12ax^3} - \frac{A(a+bx^3)^{5/2}}{6ax^6} + \frac{1}{8}(b(Ab+4aB))\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)$$
$$= \frac{b(Ab+4aB)\sqrt{a+bx^3}}{4a} - \frac{(Ab+4aB)\,(a+bx^3)^{3/2}}{12ax^3} - \frac{A(a+bx^3)^{5/2}}{6ax^6} + \frac{1}{4}(Ab+4aB)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)$$
$$= \frac{b(Ab+4aB)\sqrt{a+bx^3}}{4a} - \frac{(Ab+4aB)\,(a+bx^3)^{3/2}}{12ax^3} - \frac{A(a+bx^3)^{5/2}}{6ax^6} - \frac{b(Ab+4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^3)^{3/2}\,(A+Bx^3)}{x^7}\,dx = \frac{\sqrt{a+bx^3}(-2aA-5Abx^3-4aBx^3+8bBx^6)}{12x^6} - \frac{b(Ab+4aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^7,x]
[Out] (Sqrt[a + b*x^3]*(-2*a*A - 5*A*b*x^3 - 4*a*B*x^3 + 8*b*B*x^6))/(12*x^6) - (
b*(A*b + 4*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(4*Sqrt[a])
```

Maple [A] (verified)

Time = 4.40 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$-\frac{b\,x^{6}(Ab+4Ba)\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)+\frac{5\sqrt{b\,x^{3}+a}\left(\frac{2\left(2x^{3}B+A\right)a^{\frac{3}{2}}}{5}+b\,x^{3}\sqrt{a}\left(-\frac{8x^{3}B}{5}+A\right)\right)}{3}}{4\sqrt{a}\,x^{6}}$
risch	$-\frac{\sqrt{b\,x^{3}+a}\,(5Ab\,x^{3}+4Ba\,x^{3}+2Aa)}{12x^{6}}+\frac{b\left(\frac{16B\sqrt{b\,x^{3}+a}}{3}-\frac{2(3Ab+12Ba)\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{a}}\right)}{8}$
elliptic	$-\frac{Aa\sqrt{b\,x^{3}+a}}{6x^{6}}-\frac{\left(\frac{5Ab}{4}+Ba\right)\sqrt{b\,x^{3}+a}}{3x^{3}}+\frac{2Bb\sqrt{b\,x^{3}+a}}{3}-\frac{2\left(\frac{3}{8}b^{2}A+\frac{3}{2}abB\right)\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{a}}$
default	$A\left(-\frac{a\sqrt{b\,x^{3}+a}}{6x^{6}}-\frac{5b\sqrt{b\,x^{3}+a}}{12x^{3}}-\frac{b^{2}\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{4\sqrt{a}}\right)+B\left(-\frac{a\sqrt{b\,x^{3}+a}}{3x^{3}}+\frac{2b\sqrt{b\,x^{3}+a}}{3}-b\,\mathrm{arctanh}\left(\right.\right.$

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a^(1/2)*(b*x^6*(A*b+4*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))+5/3*(b*x^3
+a)^(1/2)*(2/5*(2*B*x^3+A)*a^(3/2)+b*x^3*a^(1/2)*(-8/5*x^3*B+A)))/x^6
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.66

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^7}\,dx = \left[\frac{3\left(4\,Bab+Ab^2\right)\sqrt{a}x^6\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\left(8\,Babx^6-\left(4\,Ba^2+5\,A\right.\right.}{24\,ax^6}\right.$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x, algorithm="fricas")
```

```
[Out] [1/24*(3*(4*B*a*b + A*b^2)*sqrt(a)*x^6*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(
a) + 2*a)/x^3) + 2*(8*B*a*b*x^6 - (4*B*a^2 + 5*A*a*b)*x^3 - 2*A*a^2)*sqrt(b
*x^3 + a))/(a*x^6), 1/12*(3*(4*B*a*b + A*b^2)*sqrt(-a)*x^6*arctan(sqrt(b*x^
3 + a)*sqrt(-a)/a) + (8*B*a*b*x^6 - (4*B*a^2 + 5*A*a*b)*x^3 - 2*A*a^2)*sqrt
(b*x^3 + a))/(a*x^6)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(102) = 204.

Time = 38.80 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.11

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^7}\,dx = -\frac{Aa^2}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{Aa\sqrt{b}}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} - \frac{Ab^{\frac{3}{2}}}{12x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{Ab^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{4\sqrt{a}} - B\sqrt{a}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right) - \frac{Ba\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} + \frac{2Ba\sqrt{b}}{3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2Bb^{\frac{3}{2}}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}$$

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**7,x)
```

```
[Out] -A*a**2/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - A*a*sqrt(b)/(4*x**(9/2
)*sqrt(a/(b*x**3) + 1)) - A*b**(3/2)*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - A*
b**(3/2)/(12*x**(3/2)*sqrt(a/(b*x**3) + 1)) - A*b**2*asinh(sqrt(a)/(sqrt(b)
*x**(3/2)))/(4*sqrt(a)) - B*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) - B
*a*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*B*a*sqrt(b)/(3*x**(3/2)*sq
rt(a/(b*x**3) + 1)) + 2*B*b**(3/2)*x**(3/2)/(3*sqrt(a/(b*x**3) + 1))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.49

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^7}\,dx = \frac{1}{24}\left(\frac{3\,b^2\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{\sqrt{a}} - \frac{2\left(5\,(bx^3+a)^{\frac{3}{2}}b^2 - 3\,\sqrt{bx^3+a}ab^2\right)}{(bx^3+a)^2 - 2\,(bx^3+a)a + a^2}\right)A + \frac{1}{6}\left(3\,\sqrt{a}b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right) + 4\,\sqrt{bx^3+a}b - \frac{2\,\sqrt{bx^3+a}a}{x^3}\right)B$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x, algorithm="maxima")
```

```
[Out] 1/24*(3*b^2*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sq
rt(a) - 2*(5*(b*x^3 + a)^(3/2)*b^2 - 3*sqrt(b*x^3 + a)*a*b^2)/((b*x^3 + a)^
2 - 2*(b*x^3 + a)*a + a^2))*A + 1/6*(3*sqrt(a)*b*log((sqrt(b*x^3 + a) - sqr
t(a))/(sqrt(b*x^3 + a) + sqrt(a))) + 4*sqrt(b*x^3 + a)*b - 2*sqrt(b*x^3 + a
)*a/x^3)*B
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^7}\,dx = \frac{8\sqrt{bx^3+a}Bb^2 + \frac{3\left(4\,Bab^2+Ab^3\right)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{4\left(bx^3+a\right)^{\frac{3}{2}}Bab^2-4\sqrt{bx^3+a}Ba^2b^2+5}{b^2x^6}}{12\,b}$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x, algorithm="giac")
```

```
[Out] 1/12*(8*sqrt(b*x^3 + a)*B*b^2 + 3*(4*B*a*b^2 + A*b^3)*arctan(sqrt(b*x^3 + a
)/sqrt(-a))/sqrt(-a) - (4*(b*x^3 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^3 + a)*B*a
^2*b^2 + 5*(b*x^3 + a)^(3/2)*A*b^3 - 3*sqrt(b*x^3 + a)*A*a*b^3)/(b^2*x^6))/
b
```

Mupad [B] (verification not implemented)

Time = 7.56 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^7}\,dx = \frac{2\,B\,b\,\sqrt{b\,x^3+a}}{3} - \frac{\sqrt{b\,x^3+a}\,\left(4\,B\,a^3+5\,A\,b\,a^2\right)}{12\,a^2\,x^3} - \frac{A\,a\,\sqrt{b\,x^3+a}}{6\,x^6} + \frac{b\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)\left(A\,b+4\,B\,a\right)}{8\,\sqrt{a}}$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^7,x)
```

```
[Out] (2*B*b*(a + b*x^3)^(1/2))/3 - ((a + b*x^3)^(1/2)*(4*B*a^3 + 5*A*a^2*b))/(12
*a^2*x^3) - (A*a*(a + b*x^3)^(1/2))/(6*x^6) + (b*log((((a + b*x^3)^(1/2) -
a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)*(A*b + 4*B*a))/(8*a^(1/2))
```

3.202 $\int x^3(a+bx^3)^{3/2}(A+Bx^3)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 336

$$\int x^3(a+bx^3)^{3/2}(A+Bx^3)\,dx = \frac{54a^2(23Ab-8aB)x\sqrt{a+bx^3}}{21505b^2} + \frac{18a(23Ab-8aB)x^4\sqrt{a+bx^3}}{4301b} + \frac{2(23Ab-8aB)x^4(a+bx^3)^{3/2}}{391b} + \frac{2Bx^4(a+bx^3)^{5/2}}{23b} - \frac{36\,3^{3/4}\sqrt{2+\sqrt{3}}a^3(23Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{21505b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/391*(23*A*b-8*B*a)*x^4*(b*x^3+a)^(3/2)/b+2/23*B*x^4*(b*x^3+a)^(5/2)/b+54/
21505*a^2*(23*A*b-8*B*a)*x*(b*x^3+a)^(1/2)/b^2+18/4301*a*(23*A*b-8*B*a)*x^4
*(b*x^3+a)^(1/2)/b-36/21505*3^(3/4)*a^3*(23*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*
EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I
*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)
*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(7/3)/(b*x^3+a)^(1/2)/(a^(
1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {470, 285, 327, 224}

$$\int x^3\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{54a^2x\sqrt{a+bx^3}(23Ab-8aB)}{21505b^2}$$

$$-\frac{36\ 3^{3/4}\sqrt{2+\sqrt{3}}a^3\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(23Ab-8aB)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right.\right.}{21505b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2x^4\left(a+bx^3\right)^{3/2}(23Ab-8aB)}{391b}+\frac{18ax^4\sqrt{a+bx^3}(23Ab-8aB)}{4301b}+\frac{2Bx^4\left(a+bx^3\right)^{5/2}}{23b}$$

```
[In] Int[x^3*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (54*a^2*(23*A*b - 8*a*B)*x*Sqrt[a + b*x^3])/(21505*b^2) + (18*a*(23*A*b - 8
*a*B)*x^4*Sqrt[a + b*x^3])/(4301*b) + (2*(23*A*b - 8*a*B)*x^4*(a + b*x^3)^(
3/2))/(391*b) + (2*B*x^4*(a + b*x^3)^(5/2))/(23*b) - (36*3^(3/4)*Sqrt[2 + S
qrt[3]]*a^3*(23*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[A
rcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)], -7 - 4*Sqrt[3]])/(21505*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{2Bx^4(a+bx^3)^{5/2}}{23b} - \frac{\left(2\left(-\frac{23Ab}{2}+4aB\right)\right)\int x^3(a+bx^3)^{3/2}\,dx}{23b}$$

$$= \frac{2(23Ab-8aB)x^4(a+bx^3)^{3/2}}{391b} + \frac{2Bx^4(a+bx^3)^{5/2}}{23b} + \frac{(9a(23Ab-8aB))\int x^3\sqrt{a+bx^3}\,dx}{391b}$$

$$= \frac{18a(23Ab-8aB)x^4\sqrt{a+bx^3}}{4301b} + \frac{2(23Ab-8aB)x^4(a+bx^3)^{3/2}}{391b} + \frac{2Bx^4(a+bx^3)^{5/2}}{23b} + \frac{(27a^2(23Ab-8aB))\int\frac{x^3}{\sqrt{a+bx^3}}\,dx}{4301b}$$

$$= \frac{54a^2(23Ab-8aB)x\sqrt{a+bx^3}}{21505b^2} + \frac{18a(23Ab-8aB)x^4\sqrt{a+bx^3}}{4301b} + \frac{2(23Ab-8aB)x^4(a+bx^3)^{3/2}}{391b} + \frac{2Bx^4(a+bx^3)^{5/2}}{23b} - \frac{(54a^3(23Ab-8aB))\int\frac{1}{\sqrt{a+bx^3}}\,dx}{21505b^2}$$

$$= \frac{54a^2(23Ab-8aB)x\sqrt{a+bx^3}}{21505b^2} + \frac{18a(23Ab-8aB)x^4\sqrt{a+bx^3}}{4301b} + \frac{2(23Ab-8aB)x^4(a+bx^3)^{3/2}}{391b} + \frac{2Bx^4(a+bx^3)^{5/2}}{23b} - \frac{36\ 3^{3/4}\sqrt{2+\sqrt{3}}a^3(23Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{21505b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.54 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.28

$$\int x^3(a+bx^3)^{3/2}(A+Bx^3)\,dx = \frac{2x\sqrt{a+bx^3}\left(-(a+bx^3)^2(-23Ab+8aB-17bBx^3)+\frac{a^2(-23Ab+8aB)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{391b^2}$$

```
[In] Integrate[x^3*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*(-((a + b*x^3)^2*(-23*A*b + 8*a*B - 17*b*B*x^3)) + (a^
2*(-23*A*b + 8*a*B)*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)])/Sqrt[1
 + (b*x^3)/a]))/(391*b^2)
```

Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.11

method	result
risch	$\frac{2x(935b^3Bx^9+1265x^6b^3A+1430Bx^6ab^2+2300aAb^2x^3+135Ba^2bx^3+621a^2bA-216a^3B)\sqrt{bx^3+a}}{21505b^2}+\frac{36ia^3(23Ab-8Ba)\sqrt{3}\,(-a\,b^2 \cdots}{\cdots}$
elliptic	$\frac{2Bbx^{10}\sqrt{bx^3+a}}{23}+\frac{2\left(b^2A+\frac{26}{23}abB\right)x^7\sqrt{bx^3+a}}{17b}+\frac{2\left(2abA+a^2B-\frac{14a\left(b^2A+\frac{26}{23}abB\right)}{17b}\right)x^4\sqrt{bx^3+a}}{11b}+\frac{2\left(a^2A-\frac{8a\left(2abA+a^2B-\frac{14a \cdots}{\cdots}\right)}{11b}\cdots\right.}{5\cdots}$
default	$B\left(\frac{2bx^{10}\sqrt{bx^3+a}}{23}+\frac{52ax^7\sqrt{bx^3+a}}{391}+\frac{54a^2x^4\sqrt{bx^3+a}}{4301b}-\frac{432a^3x\sqrt{bx^3+a}}{21505b^2}-\frac{288ia^4\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(- \cdots}{2b}\cdots\right.}{(-a\,b^2)^{\frac{1}{3}}}}\cdots}{\cdots}\right.$

```
[In] int(x^3*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21505/b^2*x*(935*B*b^3*x^9+1265*A*b^3*x^6+1430*B*a*b^2*x^6+2300*A*a*b^2*x
^3+135*B*a^2*b*x^3+621*A*a^2*b-216*B*a^3)*(b*x^3+a)^(1/2)+36/21505*I*a^3*(2
3*A*b-8*B*a)/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1
/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.34

$$\int x^3(a+bx^3)^{3/2}(A+Bx^3)\,dx = \frac{2\left(54\,(8\,Ba^4-23\,Aa^3b)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+(935\,Bb^4x^{10}+55\,(26\,Bab^3+23\,A\right.}{21505\,b^3}$$

```
[In] integrate(x^3*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/21505*(54*(8*B*a^4 - 23*A*a^3*b)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x
) + (935*B*b^4*x^10 + 55*(26*B*a*b^3 + 23*A*b^4)*x^7 + 5*(27*B*a^2*b^2 + 46
0*A*a*b^3)*x^4 - 27*(8*B*a^3*b - 23*A*a^2*b^2)*x)*sqrt(b*x^3 + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.51

$$\int x^3(a+bx^3)^{3/2}(A+Bx^3)\,dx = \frac{Aa^{\frac{3}{2}}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{A\sqrt{a}bx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{Ba^{\frac{3}{2}}x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{B\sqrt{a}bx^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{3}\right)}$$

```
[In] integrate(x**3*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] A*a**(3/2)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(7/3)) + A*sqrt(a)*b*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,
), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + B*a**(3/2)*x**7*gamma(7/3)*h
yper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + B*sq
rt(a)*b*x**10*gamma(10/3)*hyper((-1/2, 10/3), (13/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(13/3))
```

Maxima [F]

$$\int x^3(a+bx^3)^{3/2}(A+Bx^3)\,dx=\int(Bx^3+A)(bx^3+a)^{\frac{3}{2}}x^3\,dx$$

```
[In] integrate(x^3*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x^3, x)
```

Giac [F]

$$\int x^3(a+bx^3)^{3/2}(A+Bx^3)\,dx=\int(Bx^3+A)(bx^3+a)^{\frac{3}{2}}x^3\,dx$$

```
[In] integrate(x^3*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(a+bx^3)^{3/2}(A+Bx^3)\,dx=\int x^3\,(B\,x^3+A)\,(b\,x^3+a)^{3/2}\,dx$$

```
[In] int(x^3*(A + B*x^3)*(a + b*x^3)^(3/2),x)
[Out] int(x^3*(A + B*x^3)*(a + b*x^3)^(3/2), x)
```

3.203 $\int \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 299

$$\int \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \frac{18a(17Ab-2aB)x\sqrt{a+bx^3}}{935b} + \frac{2(17Ab-2aB)x\left(a+bx^3\right)^{3/2}}{187b} + \frac{2Bx\left(a+bx^3\right)^{5/2}}{17b} + \frac{18\, 3^{3/4}\sqrt{2+\sqrt{3}}a^2(17Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{935b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/187*(17*A*b-2*B*a)*x*(b*x^3+a)^(3/2)/b+2/17*B*x*(b*x^3+a)^(5/2)/b+18/935*
a*(17*A*b-2*B*a)*x*(b*x^3+a)^(1/2)/b+18/935*3^(3/4)*a^2*(17*A*b-2*B*a)*(a^(
1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3
)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(4/3)/(b*x
^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)
^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {396, 201, 224}

$$\int \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx = \frac{18\, 3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(17Ab-2aB)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+(\ldots}{\sqrt[3]{b}x+(\ldots}\right.\right.}{935b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{2x(a+bx^3)^{3/2}(17Ab-2aB)}{187b} + \frac{18ax\sqrt{a+bx^3}(17Ab-2aB)}{935b} + \frac{2Bx(a+bx^3)^{5/2}}{17b}$$

```
[In] Int[(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (18*a*(17*A*b - 2*a*B)*x*Sqrt[a + b*x^3])/(935*b) + (2*(17*A*b - 2*a*B)*x*(
a + b*x^3)^(3/2))/(187*b) + (2*B*x*(a + b*x^3)^(5/2))/(17*b) + (18*3^(3/4)*
Sqrt[2 + Sqrt[3]]*a^2*(17*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(935*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/
3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{2Bx(a+bx^3)^{5/2}}{17b} - \frac{\left(2\left(-\frac{17Ab}{2}+aB\right)\right)\int (a+bx^3)^{3/2}\,dx}{17b}$$

$$= \frac{2(17Ab-2aB)x(a+bx^3)^{3/2}}{187b} + \frac{2Bx(a+bx^3)^{5/2}}{17b} + \frac{(9a(17Ab-2aB))\int \sqrt{a+bx^3}\,dx}{187b}$$

$$= \frac{18a(17Ab-2aB)x\sqrt{a+bx^3}}{935b} + \frac{2(17Ab-2aB)x(a+bx^3)^{3/2}}{187b} + \frac{2Bx(a+bx^3)^{5/2}}{17b} + \frac{(27a^2(17Ab-2aB))\int \frac{1}{\sqrt{a+bx^3}}\,dx}{935b}$$

$$= \frac{18a(17Ab-2aB)x\sqrt{a+bx^3}}{935b} + \frac{2(17Ab-2aB)x(a+bx^3)^{3/2}}{187b} + \frac{2Bx(a+bx^3)^{5/2}}{17b} + \frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a^2(17Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\ldots}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\ldots}\right.\right.}{935b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.45 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.26

$$\int (a+bx^3)^{3/2}(A+Bx^3)\,dx = \frac{2x\sqrt{a+bx^3}\left(B(a+bx^3)^2 - \frac{a\left(-\frac{17Ab}{2}+aB\right)\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{17b}$$

```
[In] Integrate[(a + b*x^3)^(3/2)*(A + B*x^3),x]
[Out] (2*x*Sqrt[a + b*x^3]*(B*(a + b*x^3)^2 - (a*((-17*A*b)/2 + a*B)*Hypergeometr
ic2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(17*b)
```

Maple [A] (verified)

Time = 4.60 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.17

method	result
risch	$\frac{2x\left(55b^{2}B\,x^{6}+85A\,b^{2}x^{3}+100Bab\,x^{3}+238abA+27a^{2}B\right)\sqrt{b\,x^{3}+a}}{935b}-\frac{18ia^{2}(17Ab-2Ba)\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right.}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{}$
elliptic	$\frac{2Bb\,x^{7}\sqrt{b\,x^{3}+a}}{17}+\frac{2\left(b^{2}A+\frac{20}{17}abB\right)x^{4}\sqrt{b\,x^{3}+a}}{11b}+\frac{2\left(2abA+a^{2}B-\frac{8a\left(b^{2}A+\frac{20}{17}abB\right)}{11b}\right)x\sqrt{b\,x^{3}+a}}{5b}-\frac{2i\left(a^{2}A-\frac{2a\left(2abA+a^{2}B-\frac{8a\left(b^{2}\right.}{}\right)}{5b}\right.}{}$
default	$A\left(\frac{2b\,x^{4}\sqrt{b\,x^{3}+a}}{11}+\frac{28ax\sqrt{b\,x^{3}+a}}{55}-\frac{18ia^{2}\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}}{}\right.$

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 2/935/b*x*(55*B*b^2*x^6+85*A*b^2*x^3+100*B*a*b*x^3+238*A*a*b+27*B*a^2)*(b*x
^3+a)^(1/2)-18/935*I*a^2*(17*A*b-2*B*a)/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.30

$$\int \left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx = -\frac{2\left(27\left(2\,Ba^3-17\,Aa^2b\right)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-\left(55\,Bb^3x^7+5\left(20\,Bab^2+17\,Ab^3\right)x^4+\left(27\,B\right.\right.\right.}{935\,b^2}$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] -2/935*(27*(2*B*a^3 - 17*A*a^2*b)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x)
 - (55*B*b^3*x^7 + 5*(20*B*a*b^2 + 17*A*b^3)*x^4 + (27*B*a^2*b + 238*A*a*b^
2)*x)*sqrt(b*x^3 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.57

$$\int \left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx = \frac{Aa^{\frac{3}{2}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{3}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{A\sqrt{a}bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{Ba^{\frac{3}{2}}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{B\sqrt{a}bx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{3}\\\frac{10}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] A*a**(3/2)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + A*sqrt(a)*b*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b
*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + B*a**(3/2)*x**4*gamma(4/3)*hyper(
(-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + B*sqrt(a)*b
*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*g
amma(10/3))
```

Maxima [F]

$$\int \left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int \left(B\,x^3+A\right)\,\left(b\,x^3+a\right)^{3/2}\,dx$$

```
[In] int((A + B*x^3)*(a + b*x^3)^(3/2),x)
[Out] int((A + B*x^3)*(a + b*x^3)^(3/2), x)
```

3.204 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 295

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^3}\,dx = \frac{9}{110}(11Ab+4aB)x\sqrt{a+bx^3} + \frac{(11Ab+4aB)x(a+bx^3)^{3/2}}{22a} - \frac{A(a+bx^3)^{5/2}}{2ax^2} + \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,a(11Ab+4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{110\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 1/22*(11*A*b+4*B*a)*x*(b*x^3+a)^(3/2)/a-1/2*A*(b*x^3+a)^(5/2)/a/x^2+9/110*(
11*A*b+4*B*a)*x*(b*x^3+a)^(1/2)+9/110*3^(3/4)*a*(11*A*b+4*B*a)*(a^(1/3)+b^(
1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(
1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x
+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(1/3)/(b*x^3+a)^(1
/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {464, 201, 224}

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^3}\,dx = \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(4aB+11Ab)\,\text{EllipticF}}{110\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx}}$$

$$+\frac{x(a+bx^3)^{3/2}(4aB+11Ab)}{22a}+\frac{9}{110}x\sqrt{a+bx^3}(4aB+11Ab)-\frac{A(a+bx^3)^{5/2}}{2ax^2}$$

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^3,x]
```

```
[Out] (9*(11*A*b + 4*a*B)*x*Sqrt[a + b*x^3])/110 + ((11*A*b + 4*a*B)*x*(a + b*x^3
)^(3/2))/(22*a) - (A*(a + b*x^3)^(5/2))/(2*a*x^2) + (9*3^(3/4)*Sqrt[2 + Sqr
t[3]]*a*(11*A*b + 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(110*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
```

```
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{A(a+bx^3)^{5/2}}{2ax^2} - \frac{\left(-\frac{11Ab}{2}-2aB\right)\int (a+bx^3)^{3/2}\,dx}{2a}$$

$$= \frac{(11Ab+4aB)x(a+bx^3)^{3/2}}{22a} - \frac{A(a+bx^3)^{5/2}}{2ax^2} + \frac{1}{44}(9(11Ab+4aB))\int \sqrt{a+bx^3}\,dx$$

$$= \frac{9}{110}(11Ab+4aB)x\sqrt{a+bx^3} + \frac{(11Ab+4aB)x(a+bx^3)^{3/2}}{22a} - \frac{A(a+bx^3)^{5/2}}{2ax^2} + \frac{1}{220}(27a(11Ab+4aB))\int \frac{1}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{9}{110}(11Ab+4aB)x\sqrt{a+bx^3} + \frac{(11Ab+4aB)x(a+bx^3)^{3/2}}{22a} - \frac{A(a+bx^3)^{5/2}}{2ax^2} + \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}a(11Ab+4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{110\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.58 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.28

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^3}\,dx = -\frac{A(a+bx^3)^{5/2}}{2ax^2} - \frac{\left(-\frac{11Ab}{2}-2aB\right)x\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}$$

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^3,x]
[Out] -1/2*(A*(a + b*x^3)^(5/2))/(a*x^2) - (((-11*A*b)/2 - 2*a*B)*x*Sqrt[a + b*x^
3]*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])
```

Maple [A] (verified)

Time = 4.25 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-20 b B\,x^{6}-44 A b\,x^{3}-56 B a\,x^{3}+55 A a\right)}{110 x^{2}}-\frac{9 i a (11 A b+4 B a)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{\cdots}{-\frac{3\left(-\cdots\right.}{}}}}{}$
elliptic	$-\frac{A a\sqrt{b\,x^{3}+a}}{2 x^{2}}+\frac{2 B b\,x^{4}\sqrt{b\,x^{3}+a}}{11}+\frac{2\left(b^{2} A+\frac{14}{11} a b B\right) x\sqrt{b\,x^{3}+a}}{5 b}-\frac{2 i\left(\frac{7 a b A}{4}+a^{2} B-\frac{2 a\left(b^{2} A+\frac{14}{11} a b B\right)}{5 b}\right)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\cdots\right.}{2 b}\cdots\right.}{}}}{}$
default	$B\left(\frac{2 b\,x^{4}\sqrt{b\,x^{3}+a}}{11}+\frac{28 a x\sqrt{b\,x^{3}+a}}{55}-\frac{18 i a^{2}\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}}}\cdots}{}\right.$

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/110*(b*x^3+a)^(1/2)*(-20*B*b*x^6-44*A*b*x^3-56*B*a*x^3+55*A*a)/x^2-9/110
*I*a*(11*A*b+4*B*a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^
2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I
*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1
/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
)^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.27

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^3}\,dx = \frac{27\,(4\,Ba^2+11\,Aab)\sqrt{b}x^2\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+(20\,Bb^2x^6+4\,(1\cdots}{110\,bx^2}$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/110*(27*(4*B*a^2 + 11*A*a*b)*sqrt(b)*x^2*weierstrassPInverse(0, -4*a/b, x
) + (20*B*b^2*x^6 + 4*(14*B*a*b + 11*A*b^2)*x^3 - 55*A*a*b)*sqrt(b*x^3 + a)
)/(b*x^2)
```

Sympy [A] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^3}\,dx = \frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{A\sqrt{a}bx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{Ba^{\frac{3}{2}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{B\sqrt{a}bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**3,x)
```

```
[Out] A*a**(3/2)*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**2*gamma(1/3)) + A*sqrt(a)*b*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + B*a**(3/2)*x*gamma(1/3)*hyper((
-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + B*sqrt(a)*b*
x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gam
ma(7/3))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^3}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^3,x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^3, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^3}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^3,x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^3}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{3/2}}{x^3}\,dx$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^3,x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^3, x)
```

3.205 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^6} dx$

Optimal result

Integrand size = 22, antiderivative size = 297

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^6} dx = \frac{9b(Ab+2aB)x\sqrt{a+bx^3}}{20a} - \frac{(Ab+2aB)(a+bx^3)^{3/2}}{4ax^2} - \frac{A(a+bx^3)^{5/2}}{5ax^5} + \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}(Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right), \ldots\right)}{20\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/4*(A*b+2*B*a)*(b*x^3+a)^(3/2)/a/x^2-1/5*A*(b*x^3+a)^(5/2)/a/x^5+9/20*b*(
A*b+2*B*a)*x*(b*x^3+a)^(1/2)/a+9/20*3^(3/4)*b^(2/3)*(A*b+2*B*a)*(a^(1/3)+b^
(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/(b*x^3+a)^(1/2)/(a^
(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {464, 283, 201, 224}

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^6}\,dx = \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2aB+Ab)\,\text{EllipticF}}{20\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{9bx\sqrt{a+bx^3}(2aB+Ab)}{20a}-\frac{(a+bx^3)^{3/2}(2aB+Ab)}{4ax^2}-\frac{A(a+bx^3)^{5/2}}{5ax^5}$$

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^6,x]
```

```
[Out] (9*b*(A*b + 2*a*B)*x*Sqrt[a + b*x^3])/(20*a) - ((A*b + 2*a*B)*(a + b*x^3)^(
3/2))/(4*a*x^2) - (A*(a + b*x^3)^(5/2))/(5*a*x^5) + (9*3^(3/4)*Sqrt[2 + Sqr
t[3]]*b^(2/3)*(A*b + 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[Ar
cSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(20*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{A(a+bx^3)^{5/2}}{5ax^5}+-\frac{\left(-\frac{5Ab}{2}-5aB\right)\int\frac{(a+bx^3)^{3/2}}{x^3}\,dx}{5a}\\
&= -\frac{(Ab+2aB)\,(a+bx^3)^{3/2}}{4ax^2}-\frac{A(a+bx^3)^{5/2}}{5ax^5}+\frac{(9b(Ab+2aB))\int\sqrt{a+bx^3}\,dx}{8a}\\
&= \frac{9b(Ab+2aB)x\sqrt{a+bx^3}}{20a}-\frac{(Ab+2aB)\,(a+bx^3)^{3/2}}{4ax^2}\\
&\quad -\frac{A(a+bx^3)^{5/2}}{5ax^5}+\frac{1}{40}(27b(Ab+2aB))\int\frac{1}{\sqrt{a+bx^3}}\,dx\\
&= \frac{9b(Ab+2aB)x\sqrt{a+bx^3}}{20a}-\frac{(Ab+2aB)\,(a+bx^3)^{3/2}}{4ax^2}-\frac{A(a+bx^3)^{5/2}}{5ax^5}\\
&\quad +\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}(Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{20\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.28

$$
\int\frac{(a+bx^3)^{3/2}\,(A+Bx^3)}{x^6}\,dx=\frac{\sqrt{a+bx^3}\left(-\frac{2A(a+bx^3)^2}{a}-\frac{5(Ab+2aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{2}{3},\frac{1}{3},-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}\right)}{10x^5}
$$

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^6,x]
```

```
[Out] (Sqrt[a + b*x^3]*((-2*A*(a + b*x^3)^2)/a - (5*(A*b + 2*a*B)*x^3*Hypergeomet
ric2F1[-3/2, -2/3, 1/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(10*x^5)
```

Maple [A] (verified)

Time = 4.49 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.11

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,\left(-8bB\,x^6+13Ab\,x^3+10Ba\,x^3+4Aa\right)}{20x^5}-\frac{9i(Ab+2Ba)\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\left(\cdots\right.}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\cdots}}\cdots}{\cdots}$
elliptic	$-\frac{Aa\sqrt{b\,x^3+a}}{5x^5}-\frac{\left(\frac{13Ab}{10}+Ba\right)\sqrt{b\,x^3+a}}{2x^2}+\frac{2Bbx\sqrt{b\,x^3+a}}{5}-\frac{2i\left(b^2A+\frac{8abB}{5}-\frac{b(13Ab+10Ba)}{40}\right)\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(\cdots\right.}{\cdots}\right)\cdots}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\cdots}{\cdots}$
default	$B\left(-\frac{a\sqrt{b\,x^3+a}}{2x^2}+\frac{2bx\sqrt{b\,x^3+a}}{5}-\frac{9ia\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(\cdots\right.}{\cdots}}\cdots}{\cdots}\right.$

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*(b*x^3+a)^(1/2)*(-8*B*b*x^6+13*A*b*x^3+10*B*a*x^3+4*A*a)/x^5-9/20*I*(
A*b+2*B*a)*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
```

```
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b
^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.23

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^6}\,dx = \frac{27\,(2\,Ba+Ab)\sqrt{b}x^5\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+(8\,Bbx^6-(10\,Ba+\cdots}{20\,x^5}$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^6,x, algorithm="fricas")
```

```
[Out] 1/20*(27*(2*B*a + A*b)*sqrt(b)*x^5*weierstrassPInverse(0, -4*a/b, x) + (8*B
*b*x^6 - (10*B*a + 13*A*b)*x^3 - 4*A*a)*sqrt(b*x^3 + a))/x^5
```

Sympy [A] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^6}\,dx = \frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{A\sqrt{a}b\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{Ba^{\frac{3}{2}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)} + \frac{B\sqrt{a}bx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{3}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**6,x)
```

```
[Out] A*a**(3/2)*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**5*gamma(-2/3)) + A*sqrt(a)*b*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + B*a**(3/2)*gamma(-2/3)*h
yper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) +
B*sqrt(a)*b*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{(a + bx^3)^{3/2} (A + Bx^3)}{x^6} \, dx = \int \frac{(Bx^3 + A)(bx^3 + a)^{\frac{3}{2}}}{x^6} \, dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^6,x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^6, x)
```

Giac [F]

$$\int \frac{(a + bx^3)^{3/2} (A + Bx^3)}{x^6} \, dx = \int \frac{(Bx^3 + A)(bx^3 + a)^{\frac{3}{2}}}{x^6} \, dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^6,x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + bx^3)^{3/2} (A + Bx^3)}{x^6} \, dx = \int \frac{(B\,x^3 + A)\,(b\,x^3 + a)^{3/2}}{x^6} \, dx$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^6,x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^6, x)
```

3.206 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^9}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 302

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^9}\,dx = \frac{9b(Ab-16aB)\sqrt{a+bx^3}}{320ax^2} + \frac{(Ab-16aB)(a+bx^3)^{3/2}}{80ax^5} - \frac{A(a+bx^3)^{5/2}}{8ax^8}$$
$$- \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{5/3}(Ab-16aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right)}{320a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 1/80*(A*b-16*B*a)*(b*x^3+a)^(3/2)/a/x^5-1/8*A*(b*x^3+a)^(5/2)/a/x^8+9/320*b
*(A*b-16*B*a)*(b*x^3+a)^(1/2)/a/x^2-9/320*3^(3/4)*b^(5/3)*(A*b-16*B*a)*(a^(
1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3
)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a/(b*x^3+a)^
(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {464, 283, 224}

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^9}\,dx =$$

$$-\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-16aB)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{320a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{(a+bx^3)^{3/2}(Ab-16aB)}{80ax^5}+\frac{9b\sqrt{a+bx^3}(Ab-16aB)}{320ax^2}-\frac{A(a+bx^3)^{5/2}}{8ax^8}$$

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^9,x]
```

```
[Out] (9*b*(A*b - 16*a*B)*Sqrt[a + b*x^3])/(320*a*x^2) + ((A*b - 16*a*B)*(a + b*x
^3)^(3/2))/(80*a*x^5) - (A*(a + b*x^3)^(5/2))/(8*a*x^8) - (9*3^(3/4)*Sqrt[2
 + Sqrt[3]]*b^(5/3)*(A*b - 16*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellip
ticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(320*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{A(a+bx^3)^{5/2}}{8ax^8} - \frac{\left(\frac{Ab}{2}-8aB\right)\int \frac{(a+bx^3)^{3/2}}{x^6}\,dx}{8a}$$

$$= \frac{(Ab-16aB)\,(a+bx^3)^{3/2}}{80ax^5} - \frac{A(a+bx^3)^{5/2}}{8ax^8} - \frac{(9b(Ab-16aB))\int \frac{\sqrt{a+bx^3}}{x^3}\,dx}{160a}$$

$$= \frac{9b(Ab-16aB)\sqrt{a+bx^3}}{320ax^2} + \frac{(Ab-16aB)\,(a+bx^3)^{3/2}}{80ax^5} - \frac{A(a+bx^3)^{5/2}}{8ax^8} - \frac{(27b^2(Ab-16aB))\int \frac{1}{\sqrt{a+bx^3}}\,dx}{640a}$$

$$= \frac{9b(Ab-16aB)\sqrt{a+bx^3}}{320ax^2} + \frac{(Ab-16aB)\,(a+bx^3)^{3/2}}{80ax^5} - \frac{A(a+bx^3)^{5/2}}{8ax^8} - \frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{5/3}(Ab-16aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\ldots}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\ldots}\right.\right.}{320a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.27

$$\int \frac{(a+bx^3)^{3/2}\,(A+Bx^3)}{x^9}\,dx = \frac{\sqrt{a+bx^3}\left(-\frac{5A(a+bx^3)^2}{a} + \frac{\left(\frac{Ab}{2}-8aB\right)x^3\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{3}{2},-\frac{2}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{40x^8}$$

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^9,x]
```

```
[Out] (Sqrt[a + b*x^3]*((-5*A*(a + b*x^3)^2)/a + (((A*b)/2 - 8*a*B)*x^3*Hypergeom
etric2F1[-5/3, -3/2, -2/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(40*x^8)
```

Maple [A] (verified)

Time = 4.52 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.16

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(27A\,b^{2}x^{6}+208B\,x^{6}ab+76aAb\,x^{3}+64a^{2}B\,x^{3}+40a^{2}A\right)}{320x^{8}a}+\frac{9ib(Ab-16Ba)\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{}$
elliptic	$-\frac{Aa\sqrt{b\,x^{3}+a}}{8x^{8}}-\frac{\left(\frac{19Ab}{16}+Ba\right)\sqrt{b\,x^{3}+a}}{5x^{5}}-\frac{b(27Ab+208Ba)\sqrt{b\,x^{3}+a}}{320a\,x^{2}}-\frac{2i\left(B\,b^{2}-\frac{b^{2}(27Ab+208Ba)}{640a}\right)\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)}{}}}{}$
default	$A\left(-\frac{a\sqrt{b\,x^{3}+a}}{8x^{8}}-\frac{19b\sqrt{b\,x^{3}+a}}{80x^{5}}-\frac{27b^{2}\sqrt{b\,x^{3}+a}}{320a\,x^{2}}+\frac{9ib^{2}\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{}}{}}}}{}\right.$

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/320*(b*x^3+a)^(1/2)*(27*A*b^2*x^6+208*B*a*b*x^6+76*A*a*b*x^3+64*B*a^2*x^
3+40*A*a^2)/x^8/a+9/320*I*b*(A*b-16*B*a)/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.29

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^9}\,dx = \frac{27\,(16\,Bab-Ab^2)\sqrt{b}x^8\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-((208\,Bab+27\,A\dots}{320\,ax^8}$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^9,x, algorithm="fricas")
```

```
[Out] 1/320*(27*(16*B*a*b - A*b^2)*sqrt(b)*x^8*weierstrassPInverse(0, -4*a/b, x)
- ((208*B*a*b + 27*A*b^2)*x^6 + 4*(16*B*a^2 + 19*A*a*b)*x^3 + 40*A*a^2)*sqr
t(b*x^3 + a))/(a*x^8)
```

Sympy [A] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.65

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^9}\,dx = \frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{8}{3},-\frac{1}{2}\\-\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^8\Gamma\left(-\frac{5}{3}\right)} + \frac{A\sqrt{a}b\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{Ba^{\frac{3}{2}}\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{B\sqrt{a}b\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**9,x)
```

```
[Out] A*a**(3/2)*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**8*gamma(-5/3)) + A*sqrt(a)*b*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3
,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + B*a**(3/2)*gamma(-5/3)
*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)
) + B*sqrt(a)*b*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*
pi)/a)/(3*x**2*gamma(1/3))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}\,(A+Bx^3)}{x^9}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^9}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^9,x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^9, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}\,(A+Bx^3)}{x^9}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^9}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^9,x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^9, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}\,(A+Bx^3)}{x^9}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{3/2}}{x^9}\,dx$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^9,x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^9, x)
```

3.207 $\int x^4(a+bx^3)^{3/2}\left(A+Bx^3\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 614

$$\int x^4(a+bx^3)^{3/2}\left(A+Bx^3\right)\,dx=\frac{54a^2(5Ab-2aB)x^2\sqrt{a+bx^3}}{8645b^2}+\frac{18a(5Ab-2aB)x^5\sqrt{a+bx^3}}{1235b}-\frac{216a^3(5Ab-2aB)\sqrt{a+bx^3}}{8645b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2(5Ab-2aB)x^5(a+bx^3)^{3/2}}{95b}+\frac{2Bx^5(a+bx^3)^{5/2}}{25b}+\frac{108\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{10/3}(5Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7\right.}{8645b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{72\sqrt{2}3^{3/4}a^{10/3}(5Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-\right.}{8645b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/95*(5*A*b-2*B*a)*x^5*(b*x^3+a)^(3/2)/b+2/25*B*x^5*(b*x^3+a)^(5/2)/b+54/86
45*a^2*(5*A*b-2*B*a)*x^2*(b*x^3+a)^(1/2)/b^2+18/1235*a*(5*A*b-2*B*a)*x^5*(b
*x^3+a)^(1/2)/b-216/8645*a^3*(5*A*b-2*B*a)*(b*x^3+a)^(1/2)/b^(8/3)/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))-72/8645*3^(3/4)*a^(10/3)*(5*A*b-2*B*a)*(a^(1/3)+b^(
1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(
1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(
```

```
1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1
/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+108/8645*3^(1/4)*a^
(10/3)*(5*A*b-2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^
(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(
1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))^2)^(1/2)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*
x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 614, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {470, 285, 327, 309, 224, 1891}

$$\int x^4\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx =$$

$$-\frac{72\sqrt{2}3^{3/4}a^{10/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-2aB)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-\right.}{8645b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{108\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{10/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-2aB)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-\right.}{8645b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{216a^3\sqrt{a+bx^3}(5Ab-2aB)}{8645b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{54a^2x^2\sqrt{a+bx^3}(5Ab-2aB)}{8645b^2}$$

$$+\frac{2x^5\left(a+bx^3\right)^{3/2}(5Ab-2aB)}{95b}+\frac{18ax^5\sqrt{a+bx^3}(5Ab-2aB)}{1235b}+\frac{2Bx^5\left(a+bx^3\right)^{5/2}}{25b}$$

```
[In] Int[x^4*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (54*a^2*(5*A*b - 2*a*B)*x^2*Sqrt[a + b*x^3])/(8645*b^2) + (18*a*(5*A*b - 2*
a*B)*x^5*Sqrt[a + b*x^3])/(1235*b) - (216*a^3*(5*A*b - 2*a*B)*Sqrt[a + b*x^
3])/(8645*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*(5*A*b - 2*a*B)
*x^5*(a + b*x^3)^(3/2))/(95*b) + (2*B*x^5*(a + b*x^3)^(5/2))/(25*b) + (108*
3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(10/3)*(5*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(8645*b^(8/3)*Sqrt[(a^(1/3)*(a^
```

```
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
 - (72*Sqrt[2]*3^(3/4)*a^(10/3)*(5*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(8645*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2Bx^5(a+bx^3)^{5/2}}{25b} - \frac{\left(2\left(-\frac{25Ab}{2}+5aB\right)\right)\int x^4(a+bx^3)^{3/2}\,dx}{25b} \\
&= \frac{2(5Ab-2aB)x^5(a+bx^3)^{3/2}}{95b} + \frac{2Bx^5(a+bx^3)^{5/2}}{25b} + \frac{(9a(5Ab-2aB))\int x^4\sqrt{a+bx^3}\,dx}{95b} \\
&= \frac{18a(5Ab-2aB)x^5\sqrt{a+bx^3}}{1235b} + \frac{2(5Ab-2aB)x^5(a+bx^3)^{3/2}}{95b} \\
&\quad + \frac{2Bx^5(a+bx^3)^{5/2}}{25b} + \frac{(27a^2(5Ab-2aB))\int \frac{x^4}{\sqrt{a+bx^3}}\,dx}{1235b} \\
&= \frac{54a^2(5Ab-2aB)x^2\sqrt{a+bx^3}}{8645b^2} + \frac{18a(5Ab-2aB)x^5\sqrt{a+bx^3}}{1235b} \\
&\quad + \frac{2(5Ab-2aB)x^5(a+bx^3)^{3/2}}{95b} + \frac{2Bx^5(a+bx^3)^{5/2}}{25b} \\
&\quad - \frac{(108a^3(5Ab-2aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{8645b^2} \\
&= \frac{54a^2(5Ab-2aB)x^2\sqrt{a+bx^3}}{8645b^2} + \frac{18a(5Ab-2aB)x^5\sqrt{a+bx^3}}{1235b} \\
&\quad + \frac{2(5Ab-2aB)x^5(a+bx^3)^{3/2}}{95b} + \frac{2Bx^5(a+bx^3)^{5/2}}{25b} \\
&\quad - \frac{(108a^3(5Ab-2aB))\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{8645b^{7/3}} \\
&\quad + \frac{\left(108\left(1-\sqrt{3}\right)a^{10/3}(5Ab-2aB)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{8645b^{7/3}}
\end{aligned}$$

$$= \frac{54a^2(5Ab-2aB)x^2\sqrt{a+bx^3}}{8645b^2} + \frac{18a(5Ab-2aB)x^5\sqrt{a+bx^3}}{1235b} - \frac{216a^3(5Ab-2aB)\sqrt{a+bx^3}}{8645b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{2(5Ab-2aB)x^5(a+bx^3)^{3/2}}{95b} + \frac{2Bx^5(a+bx^3)^{5/2}}{25b} + \frac{108\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{10/3}(5Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{8645b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{72\sqrt{2}3^{3/4}a^{10/3}(5Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-\right.}{8645b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.81 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.16

$$\int x^4\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx = \frac{2x^2\sqrt{a+bx^3}\left(-\left(a+bx^3\right)^2\left(-25Ab+10aB-19bBx^3\right)+\frac{5a^2(-5Ab+2aB)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2}{3},\frac{5}{3},-\right.}{\sqrt{1+\frac{bx^3}{a}}}\right.}{475b^2}$$

```
[In] Integrate[x^4*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*x^2*Sqrt[a + b*x^3]*(-((a + b*x^3)^2*(-25*A*b + 10*a*B - 19*b*B*x^3)) +
(5*a^2*(-5*A*b + 2*a*B)*Hypergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)])/Sq
rt[1 + (b*x^3)/a]))/(475*b^2)
```

Maple [A] (verified)

Time = 4.51 (sec) , antiderivative size = 527, normalized size of antiderivative = 0.86

method	result
risch	$\frac{2x^{2}\left(1729b^{3}B\,x^{9}+2275x^{6}b^{3}A+2548B\,x^{6}a\,b^{2}+3850aA\,b^{2}x^{3}+189B\,a^{2}b\,x^{3}+675a^{2}bA-270a^{3}B\right)\sqrt{b\,x^{3}+a}}{43225b^{2}}+\frac{72ia^{3}(5Ab-2Ba)\sqrt{3}\,(-a\,b \ldots}{\ldots}$
elliptic	$\frac{2Bb\,x^{11}\sqrt{b\,x^{3}+a}}{25}+\frac{2\left(b^{2}A+\frac{28}{25}abB\right)x^{8}\sqrt{b\,x^{3}+a}}{19b}+\frac{2\left(2abA+a^{2}B-\frac{16a\left(b^{2}A+\frac{28}{25}abB\right)}{19b}\right)x^{5}\sqrt{b\,x^{3}+a}}{13b}+\frac{2\left(a^{2}A-\frac{10a\left(2abA+a^{2}B-\ldots\right)}{13b}\ldots\right.}{\ldots}$
default	Expression too large to display

```
[In] int(x^4*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 2/43225/b^2*x^2*(1729*B*b^3*x^9+2275*A*b^3*x^6+2548*B*a*b^2*x^6+3850*A*a*b^
2*x^3+189*B*a^2*b*x^3+675*A*a^2*b-270*B*a^3)*(b*x^3+a)^(1/2)+72/8645*I*a^3*
(5*A*b-2*B*a)/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(
1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
```

```
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(
1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.21

$$\int x^4 (a+bx^3)^{3/2} (A+Bx^3)\, dx = -\frac{2\left(540\,(2\,Ba^4-5\,Aa^3b)\sqrt{b}\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-(1729\,Bb^4x^{11}+91}{43225\,b^3}$$

```
[In] integrate(x^4*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] -2/43225*(540*(2*B*a^4 - 5*A*a^3*b)*sqrt(b)*weierstrassZeta(0, -4*a/b, weie
rstrassPInverse(0, -4*a/b, x)) - (1729*B*b^4*x^11 + 91*(28*B*a*b^3 + 25*A*b
^4)*x^8 + 7*(27*B*a^2*b^2 + 550*A*a*b^3)*x^5 - 135*(2*B*a^3*b - 5*A*a^2*b^2
)*x^2)*sqrt(b*x^3 + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.28

$$\int x^4 (a+bx^3)^{3/2} (A+Bx^3)\, dx = \frac{Aa^{\frac{3}{2}}x^5\Gamma\left(\frac{5}{3}\right)\, {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{3} \\ \frac{8}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{A\sqrt{a}bx^8\Gamma\left(\frac{8}{3}\right)\, {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{8}{3} \\ \frac{11}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)} + \frac{Ba^{\frac{3}{2}}x^8\Gamma\left(\frac{8}{3}\right)\, {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{8}{3} \\ \frac{11}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)} + \frac{B\sqrt{a}bx^{11}\Gamma\left(\frac{11}{3}\right)\, {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{11}{3} \\ \frac{14}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{14}{3}\right)}$$

```
[In] integrate(x**4*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] A*a**(3/2)*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(8/3)) + A*sqrt(a)*b*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,
```

```
), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) + B*a**(3/2)*x**8*gamma(8/3)*h
yper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) + B*sq
rt(a)*b*x**11*gamma(11/3)*hyper((-1/2, 11/3), (14/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(14/3))
```

Maxima [F]

$$\int x^4 \left(a + bx^3\right)^{3/2} \left(A + Bx^3\right) \, dx = \int \left(Bx^3 + A\right) \left(bx^3 + a\right)^{\frac{3}{2}} x^4 \, dx$$

```
[In] integrate(x^4*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x^4, x)
```

Giac [F]

$$\int x^4 \left(a + bx^3\right)^{3/2} \left(A + Bx^3\right) \, dx = \int \left(Bx^3 + A\right) \left(bx^3 + a\right)^{\frac{3}{2}} x^4 \, dx$$

```
[In] integrate(x^4*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \left(a + bx^3\right)^{3/2} \left(A + Bx^3\right) \, dx = \int x^4 \left(B\,x^3 + A\right) \, \left(b\,x^3 + a\right)^{3/2} \, dx$$

```
[In] int(x^4*(A + B*x^3)*(a + b*x^3)^(3/2),x)
[Out] int(x^4*(A + B*x^3)*(a + b*x^3)^(3/2), x)
```

3.208 $\int x(a+bx^3)^{3/2}(A+Bx^3)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 581

$$\int x(a+bx^3)^{3/2}(A+Bx^3)\,dx = \frac{18a(19Ab-4aB)x^2\sqrt{a+bx^3}}{1729b} + \frac{54a^2(19Ab-4aB)\sqrt{a+bx^3}}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{2(19Ab-4aB)x^2(a+bx^3)^{3/2}}{247b} + \frac{2Bx^2(a+bx^3)^{5/2}}{19b}$$
$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}(19Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7\right.}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{18\sqrt{2}3^{3/4}a^{7/3}(19Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-\right.}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/247*(19*A*b-4*B*a)*x^2*(b*x^3+a)^(3/2)/b+2/19*B*x^2*(b*x^3+a)^(5/2)/b+18/
1729*a*(19*A*b-4*B*a)*x^2*(b*x^3+a)^(1/2)/b+54/1729*a^2*(19*A*b-4*B*a)*(b*x
^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+18/1729*3^(3/4)*a^(7/3)
*(19*A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)
))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)
*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b
*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^
2)^(1/2)-27/1729*3^(1/4)*a^(7/3)*(19*A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*Ellipti
```

```
cE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2
)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(
b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a
^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {470, 285, 309, 224, 1891}

$$\int x\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{18\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(19Ab-4aB)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(19Ab-4aB)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-\right.}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{54a^2\sqrt{a+bx^3}(19Ab-4aB)}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2x^2\left(a+bx^3\right)^{3/2}(19Ab-4aB)}{247b}$$
$$+\frac{18ax^2\sqrt{a+bx^3}(19Ab-4aB)}{1729b}+\frac{2Bx^2\left(a+bx^3\right)^{5/2}}{19b}$$

```
[In] Int[x*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (18*a*(19*A*b - 4*a*B)*x^2*Sqrt[a + b*x^3])/(1729*b) + (54*a^2*(19*A*b - 4*
a*B)*Sqrt[a + b*x^3])/(1729*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) +
(2*(19*A*b - 4*a*B)*x^2*(a + b*x^3)^(3/2))/(247*b) + (2*B*x^2*(a + b*x^3)^(
5/2))/(19*b) - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*(19*A*b - 4*a*B)*(a^(1
/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(5/3
)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*Sqrt[a + b*x^3]) + (18*Sqrt[2]*3^(3/4)*a^(7/3)*(19*A*b - 4*a*B)*(a^(1/3)
 + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3
```

```
)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(5/3)*S
qrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{2Bx^2(a+bx^3)^{5/2}}{19b} - \frac{\left(2\left(-\frac{19Ab}{2}+2aB\right)\right)\int x(a+bx^3)^{3/2}\,dx}{19b}$$

$$= \frac{2(19Ab-4aB)x^2(a+bx^3)^{3/2}}{247b} + \frac{2Bx^2(a+bx^3)^{5/2}}{19b} + \frac{(9a(19Ab-4aB))\int x\sqrt{a+bx^3}\,dx}{247b}$$

$$= \frac{18a(19Ab-4aB)x^2\sqrt{a+bx^3}}{1729b} + \frac{2(19Ab-4aB)x^2(a+bx^3)^{3/2}}{247b} + \frac{2Bx^2(a+bx^3)^{5/2}}{19b} + \frac{(27a^2(19Ab-4aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{1729b}$$

$$= \frac{18a(19Ab-4aB)x^2\sqrt{a+bx^3}}{1729b} + \frac{2(19Ab-4aB)x^2(a+bx^3)^{3/2}}{247b} + \frac{2Bx^2(a+bx^3)^{5/2}}{19b} + \frac{(27a^2(19Ab-4aB))\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{1729b^{4/3}} - \frac{\left(27(1-\sqrt{3})\,a^{7/3}(19Ab-4aB)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{1729b^{4/3}}$$

$$= \frac{18a(19Ab-4aB)x^2\sqrt{a+bx^3}}{1729b} + \frac{54a^2(19Ab-4aB)\sqrt{a+bx^3}}{1729b^{5/3}\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{2(19Ab-4aB)x^2(a+bx^3)^{3/2}}{247b} + \frac{2Bx^2(a+bx^3)^{5/2}}{19b} - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}(19Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{18\sqrt{2}3^{3/4}a^{7/3}(19Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-\right.}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.47 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.13

$$\int x\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{x^2\sqrt{a+bx^3}\left(4B\left(a+bx^3\right)^2+\frac{a(19Ab-4aB)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{38b}$$

```
[In] Integrate[x*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (x^2*Sqrt[a + b*x^3]*(4*B*(a + b*x^3)^2 + (a*(19*A*b - 4*a*B)*Hypergeometri
c2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(38*b)
```

Maple [A] (verified)

Time = 4.29 (sec) , antiderivative size = 503, normalized size of antiderivative = 0.87

method	result
risch	$\frac{2x^2(91b^2Bx^6+133Ab^2x^3+154Babx^3+304abA+27a^2B)\sqrt{bx^3+a}}{1729b} - \frac{18ia^2(19Ab-4Ba)\sqrt{3}\,(-ab^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-ab^2)}{2b}\right.}{(-ab^2)^{\frac{1}{3}}}}}{}$
elliptic	$\frac{2Bbx^8\sqrt{bx^3+a}}{19} + \frac{2(b^2A+\frac{22}{19}abB)x^5\sqrt{bx^3+a}}{13b} + \frac{2\left(2abA+a^2B-\frac{10a(b^2A+\frac{22}{19}abB)}{13b}\right)x^2\sqrt{bx^3+a}}{7b} - \frac{2i\left(a^2A-\frac{4a\left(2abA+a^2B-\frac{10a}{}\right.}{7b}\right.}{}$
default	Expression too large to display

```
[In] int(x*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 2/1729/b*x^2*(91*B*b^2*x^6+133*A*b^2*x^3+154*B*a*b*x^3+304*A*a*b+27*B*a^2)*
(b*x^3+a)^(1/2)-18/1729*I*a^2*(19*A*b-4*B*a)/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(
```

```
I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.17

$$\int x(a+bx^3)^{3/2}(A+Bx^3)\,dx = \frac{2\left(27\,(4\,Ba^3-19\,Aa^2b)\sqrt{b}\,\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+(91\,Bb^3\right.}{1729\,b^2}$$

```
[In] integrate(x*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/1729*(27*(4*B*a^3 - 19*A*a^2*b)*sqrt(b)*weierstrassZeta(0, -4*a/b, weiers
trassPInverse(0, -4*a/b, x)) + (91*B*b^3*x^8 + 7*(22*B*a*b^2 + 19*A*b^3)*x^
5 + (27*B*a^2*b + 304*A*a*b^2)*x^2)*sqrt(b*x^3 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.30

$$\int x(a+bx^3)^{3/2}(A+Bx^3)\,dx = \frac{Aa^{\frac{3}{2}}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{A\sqrt{a}bx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\\frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{Ba^{\frac{3}{2}}x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\\frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{B\sqrt{a}bx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\\frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

```
[In] integrate(x*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] A*a**(3/2)*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(5/3)) + A*sqrt(a)*b*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + B*a**(3/2)*x**5*gamma(5/3)*hyp
er((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + B*sqrt(a
)*b*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(11/3))
```

Maxima [F]

$$\int x(a+bx^3)^{3/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}x\,dx$$

```
[In] integrate(x*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x, x)
```

Giac [F]

$$\int x(a+bx^3)^{3/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}x\,dx$$

```
[In] integrate(x*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+bx^3)^{3/2}\left(A+Bx^3\right)\,dx=\int x\left(B\,x^3+A\right)\,\left(b\,x^3+a\right)^{3/2}\,dx$$

```
[In] int(x*(A + B*x^3)*(a + b*x^3)^(3/2),x)
[Out] int(x*(A + B*x^3)*(a + b*x^3)^(3/2), x)
```

3.209 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 573

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^2}\,dx = \frac{9}{91}(13Ab+2aB)x^2\sqrt{a+bx^3} + \frac{27a(13Ab+2aB)\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{(13Ab+2aB)x^2(a+bx^3)^{3/2}}{13a} - \frac{A(a+bx^3)^{5/2}}{ax} - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}(13Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7 \right.}{182b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{9\sqrt{2}3^{3/4}a^{4/3}(13Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7 \right.}{91b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 1/13*(13*A*b+2*B*a)*x^2*(b*x^3+a)^(3/2)/a-A*(b*x^3+a)^(5/2)/a/x+9/91*(13*A*
b+2*B*a)*x^2*(b*x^3+a)^(1/2)+27/91*a*(13*A*b+2*B*a)*(b*x^3+a)^(1/2)/b^(2/3)
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+9/91*3^(3/4)*a^(4/3)*(13*A*b+2*B*a)*(a^(1/
3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2
)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)
*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-27/182*3^(1/4
)*a^(4/3)*(13*A*b+2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(
```

```
1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2
*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 573, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {464, 285, 309, 224, 1891}

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^2}\,dx = \frac{9\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2aB+13Ab)\,\text{EllipticF}\left(\text{arc}\right.}{91b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2aB+13Ab)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-\right.}{182b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{27a\sqrt{a+bx^3}(2aB+13Ab)}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{x^2(a+bx^3)^{3/2}(2aB+13Ab)}{13a}$$
$$+\frac{9}{91}x^2\sqrt{a+bx^3}(2aB+13Ab)-\frac{A(a+bx^3)^{5/2}}{ax}$$

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^2,x]
```

```
[Out] (9*(13*A*b + 2*a*B)*x^2*Sqrt[a + b*x^3])/91 + (27*a*(13*A*b + 2*a*B)*Sqrt[a
 + b*x^3])/(91*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + ((13*A*b + 2*
a*B)*x^2*(a + b*x^3)^(3/2))/(13*a) - (A*(a + b*x^3)^(5/2))/(a*x) - (27*3^(1
/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13*A*b + 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(182*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3)
 + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9
*Sqrt[2]*3^(3/4)*a^(4/3)*(13*A*b + 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{A(a+bx^3)^{5/2}}{ax} - \frac{\left(-\frac{13Ab}{2}-aB\right)\int x(a+bx^3)^{3/2}\,dx}{a}$$

$$= \frac{(13Ab+2aB)x^2(a+bx^3)^{3/2}}{13a} - \frac{A(a+bx^3)^{5/2}}{ax} + \frac{1}{26}(9(13Ab+2aB))\int x\sqrt{a+bx^3}\,dx$$

$$= \frac{9}{91}(13Ab+2aB)x^2\sqrt{a+bx^3} + \frac{(13Ab+2aB)x^2(a+bx^3)^{3/2}}{13a} - \frac{A(a+bx^3)^{5/2}}{ax} + \frac{1}{182}(27a(13Ab+2aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{9}{91}(13Ab+2aB)x^2\sqrt{a+bx^3} + \frac{(13Ab+2aB)x^2(a+bx^3)^{3/2}}{13a} - \frac{A(a+bx^3)^{5/2}}{ax} + \frac{(27a(13Ab+2aB))\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{182\sqrt[3]{b}} - \frac{\left(27\left(1-\sqrt{3}\right)a^{4/3}(13Ab+2aB)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{182\sqrt[3]{b}}$$

$$= \frac{9}{91}(13Ab+2aB)x^2\sqrt{a+bx^3} + \frac{27a(13Ab+2aB)\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{(13Ab+2aB)x^2(a+bx^3)^{3/2}}{13a} - \frac{A(a+bx^3)^{5/2}}{ax} - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}(13Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{182b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{9\sqrt{2}3^{3/4}a^{4/3}(13Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7\right.}{91b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.63 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.14

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^2}\,dx = -\frac{A(a+bx^3)^{5/2}}{ax} - \frac{\left(-\frac{13Ab}{2}-aB\right)x^2\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}$$

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^2,x]
```

```
[Out] -((A*(a + b*x^3)^(5/2))/(a*x)) - (((-13*A*b)/2 - a*B)*x^2*Sqrt[a + b*x^3]*H
ypergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])
```

Maple [A] (verified)

Time = 4.44 (sec) , antiderivative size = 486, normalized size of antiderivative = 0.85

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,(-14bB\,x^6-26Ab\,x^3-32Ba\,x^3+91Aa)}{91x}-\frac{9ia(13Ab+2Ba)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\ldots}}{\ldots}$
elliptic	$-\frac{Aa\sqrt{b\,x^3+a}}{x}+\frac{2Bb\,x^5\sqrt{b\,x^3+a}}{13}+\frac{2(b^2A+\frac{16}{13}abB)x^2\sqrt{b\,x^3+a}}{7b}-\frac{2i\left(\frac{5abA}{2}+a^2B-\frac{4a\left(b^2A+\frac{16}{13}abB\right)}{7b}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\ldots}}{\ldots}$
default	Expression too large to display

[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^2,x,method=_RETURNVERBOSE)

[Out] -1/91*(b*x^3+a)^(1/2)*(-14*B*b*x^6-26*A*b*x^3-32*B*a*x^3+91*A*a)/x-9/91*I*a*(13*A*b+2*B*a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/

```
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.15

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^2}\,dx = -\frac{27\,(2\,Ba^2+13\,Aab)\sqrt{b}x\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-(14\,Bb^2x^6+2\,(16\,Ba}{91\,bx}$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^2,x, algorithm="fricas")
```

```
[Out] -1/91*(27*(2*B*a^2 + 13*A*a*b)*sqrt(b)*x*weierstrassZeta(0, -4*a/b, weierst
rassPInverse(0, -4*a/b, x)) - (14*B*b^2*x^6 + 2*(16*B*a*b + 13*A*b^2)*x^3 -
 91*A*a*b)*sqrt(b*x^3 + a))/(b*x)
```

Sympy [A] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.30

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^2}\,dx = \frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} + \frac{A\sqrt{a}bx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{Ba^{\frac{3}{2}}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{B\sqrt{a}bx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**2,x)
```

```
[Out] A*a**(3/2)*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x*gamma(2/3)) + A*sqrt(a)*b*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + B*a**(3/2)*x**2*gamma(2/3)*hype
r((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + B*sqrt(a)
*b*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*
gamma(8/3))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^2}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^2,x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^2}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^2,x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^2}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{3/2}}{x^2}\,dx$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^2,x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^2, x)
```

3.210 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^5}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 578

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^5}\,dx = \frac{9b(7Ab+8aB)x^2\sqrt{a+bx^3}}{56a}$$

$$+\frac{27\sqrt[3]{b}(7Ab+8aB)\sqrt{a+bx^3}}{56\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{(7Ab+8aB)\left(a+bx^3\right)^{3/2}}{8ax}-\frac{A(a+bx^3)^{5/2}}{4ax^4}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}(7Ab+8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7}{112\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{9\,3^{3/4}\sqrt[3]{a}\sqrt[3]{b}(7Ab+8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-}{28\sqrt{2}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/8*(7*A*b+8*B*a)*(b*x^3+a)^(3/2)/a/x-1/4*A*(b*x^3+a)^(5/2)/a/x^4+9/56*b*(
7*A*b+8*B*a)*x^2*(b*x^3+a)^(1/2)/a+27/56*b^(1/3)*(7*A*b+8*B*a)*(b*x^3+a)^(1
/2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+9/56*3^(3/4)*a^(1/3)*b^(1/3)*(7*A*b+8*B
*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*
x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*2^(1/2)/(b*x^3+a)^(1/2)/(a^(1/
3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-27/112*3^(1
/4)*a^(1/3)*b^(1/3)*(7*A*b+8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+
```

```
a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^
(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2)))^2)^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 578, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {464, 283, 285, 309, 224, 1891}

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^5}\,dx = \frac{9\,3^{3/4}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(8aB+7Ab)\,\text{EllipticF}\left(\text{arcs}\right.}{28\sqrt{2}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(8aB+7Ab)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-\right.}{112\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{(a+bx^3)^{3/2}(8aB+7Ab)}{8ax}+\frac{27\sqrt[3]{b}\sqrt{a+bx^3}(8aB+7Ab)}{56\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{9bx^2\sqrt{a+bx^3}(8aB+7Ab)}{56a}-\frac{A(a+bx^3)^{5/2}}{4ax^4}$$

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^5,x]
```

```
[Out] (9*b*(7*A*b + 8*a*B)*x^2*Sqrt[a + b*x^3])/(56*a) + (27*b^(1/3)*(7*A*b + 8*a
*B)*Sqrt[a + b*x^3])/(56*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - ((7*A*b + 8
*a*B)*(a + b*x^3)^(3/2))/(8*a*x) - (A*(a + b*x^3)^(5/2))/(4*a*x^4) - (27*3^
(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*b^(1/3)*(7*A*b + 8*a*B)*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(112*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (
9*3^(3/4)*a^(1/3)*b^(1/3)*(7*A*b + 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(28*Sqrt[2]*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
```

```
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{A(a+bx^3)^{5/2}}{4ax^4} - \frac{\left(-\frac{7Ab}{2}-4aB\right)\int \frac{(a+bx^3)^{3/2}}{x^2}\,dx}{4a}$$

$$= -\frac{(7Ab+8aB)\,(a+bx^3)^{3/2}}{8ax} - \frac{A(a+bx^3)^{5/2}}{4ax^4} + \frac{(9b(7Ab+8aB))\int x\sqrt{a+bx^3}\,dx}{16a}$$

$$= \frac{9b(7Ab+8aB)x^2\sqrt{a+bx^3}}{56a} - \frac{(7Ab+8aB)\,(a+bx^3)^{3/2}}{8ax} - \frac{A(a+bx^3)^{5/2}}{4ax^4} + \frac{1}{112}(27b(7Ab+8aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{9b(7Ab+8aB)x^2\sqrt{a+bx^3}}{56a} - \frac{(7Ab+8aB)\,(a+bx^3)^{3/2}}{8ax} - \frac{A(a+bx^3)^{5/2}}{4ax^4} + \frac{1}{112}(27b^{2/3}(7Ab+8aB))\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx - \frac{1}{112}\left(27\left(1-\sqrt{3}\right)\sqrt[3]{a}b^{2/3}(7Ab+8aB)\right)\int$$

$$= \frac{9b(7Ab+8aB)x^2\sqrt{a+bx^3}}{56a} + \frac{27\sqrt[3]{b}(7Ab+8aB)\sqrt{a+bx^3}}{56\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(7Ab+8aB)\,(a+bx^3)^{3/2}}{8ax} - \frac{A(a+bx^3)^{5/2}}{4ax^4}$$

$$- \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}(7Ab+8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}}\right.\right.}{112\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{9\;3^{3/4}\sqrt[3]{a}\sqrt[3]{b}(7Ab+8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\middle|-7-\right.}{28\sqrt{2}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.15

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^5}\,dx = -\frac{A(a+bx^3)^{5/2}}{4ax^4} + \frac{\left(-\frac{7Ab}{2}-4aB\right)\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{3},\frac{2}{3},-\frac{bx^3}{a}\right)}{4x\sqrt{1+\frac{bx^3}{a}}}$$

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^5,x]
```

```
[Out] -1/4*(A*(a + b*x^3)^(5/2))/(a*x^4) + (((-7*A*b)/2 - 4*a*B)*Sqrt[a + b*x^3]*
Hypergeometric2F1[-3/2, -1/3, 2/3, -((b*x^3)/a)])/(4*x*Sqrt[1 + (b*x^3)/a])
```

Maple [A] (verified)

Time = 4.49 (sec) , antiderivative size = 482, normalized size of antiderivative = 0.83

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,(-16bB\,x^6+77Ab\,x^3+56Ba\,x^3+14Aa)}{56x^4}-\frac{9i(7Ab+8Ba)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x\ldots}{-\frac{3(-a\,b^2\ldots}{2b}}}}{\ldots}$
elliptic	$-\frac{Aa\sqrt{b\,x^3+a}}{4x^4}-\frac{\left(\frac{11Ab}{8}+Ba\right)\sqrt{b\,x^3+a}}{x}+\frac{2Bb\,x^2\sqrt{b\,x^3+a}}{7}-\frac{2i\left(\frac{27}{16}b^2A+\frac{27}{14}abB\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\ldots}{(-a\,b^2)^{\frac{1}{3}}}}}{\ldots}$
default	Expression too large to display

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/56*(b*x^3+a)^(1/2)*(-16*B*b*x^6+77*A*b*x^3+56*B*a*x^3+14*A*a)/x^4-9/56*I
*(7*A*b+8*B*a)*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3
))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b
```

```
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.13

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^5}\,dx = -\frac{27\,(8\,Ba+7\,Ab)\sqrt{b}x^4\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-(16\,Bbx^6-7\,(8\,Ba+11}{56\,x^4}$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^5,x, algorithm="fricas")
```

```
[Out] -1/56*(27*(8*B*a + 7*A*b)*sqrt(b)*x^4*weierstrassZeta(0, -4*a/b, weierstras
sPInverse(0, -4*a/b, x)) - (16*B*b*x^6 - 7*(8*B*a + 11*A*b)*x^3 - 14*A*a)*s
qrt(b*x^3 + a))/x^4
```

Sympy [A] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.31

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^5}\,dx = \frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{A\sqrt{a}b\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} + \frac{Ba^{\frac{3}{2}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} + \frac{B\sqrt{a}bx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**5,x)
```

```
[Out] A*a**(3/2)*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**4*gamma(-1/3)) + A*sqrt(a)*b*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) + B*a**(3/2)*gamma(-1/3)*hype
r((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) + B*sqrt
(a)*b*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(5/3))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^5}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^5}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^5,x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^5, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^5}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^5}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^5,x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^5}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{3/2}}{x^5}\,dx$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^5,x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^5, x)
```

3.211 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^8}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 576

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^8}\,dx = -\frac{9b(Ab+14aB)\sqrt{a+bx^3}}{112ax} + \frac{27b^{4/3}(Ab+14aB)\sqrt{a+bx^3}}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(Ab+14aB)\left(a+bx^3\right)^{3/2}}{56ax^4} - \frac{A(a+bx^3)^{5/2}}{7ax^7}$$
$$- \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}(Ab+14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\right.}{224a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+ \frac{9\ 3^{3/4}b^{4/3}(Ab+14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\right.}{56\sqrt{2}a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/56*(A*b+14*B*a)*(b*x^3+a)^(3/2)/a/x^4-1/7*A*(b*x^3+a)^(5/2)/a/x^7-9/112*
b*(A*b+14*B*a)*(b*x^3+a)^(1/2)/a/x+27/112*b^(4/3)*(A*b+14*B*a)*(b*x^3+a)^(1
/2)/a/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+9/112*3^(3/4)*b^(4/3)*(A*b+14*B*a)*(a
^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b
^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)*2^(1/2)/(b*x^3+a)^(1/2)/(a^(
1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-27/224*3^
(1/4)*b^(4/3)*(A*b+14*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)
```

```
*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1
/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {464, 283, 309, 224, 1891}

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^8}\,dx = \frac{9\,3^{3/4}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(14aB+Ab)\,\text{EllipticF}\left(\arcsin\right.}{56\sqrt{2}a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(14aB+Ab)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\right.}{224a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{27b^{4/3}\sqrt{a+bx^3}(14aB+Ab)}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{9b\sqrt{a+bx^3}(14aB+Ab)}{112ax}$$

$$-\frac{(a+bx^3)^{3/2}(14aB+Ab)}{56ax^4}-\frac{A(a+bx^3)^{5/2}}{7ax^7}$$

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^8,x]
```

```
[Out] (-9*b*(A*b + 14*a*B)*Sqrt[a + b*x^3])/(112*a*x) + (27*b^(4/3)*(A*b + 14*a*B
)*Sqrt[a + b*x^3])/(112*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - ((A*b + 14
*a*B)*(a + b*x^3)^(3/2))/(56*a*x^4) - (A*(a + b*x^3)^(5/2))/(7*a*x^7) - (27
*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*(A*b + 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(224*a^(2/3)*Sqrt[(a^(1/3)*(a^(1
/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) +
 (9*3^(3/4)*b^(4/3)*(A*b + 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellip
ticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(56*Sqrt[2]*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{A(a+bx^3)^{5/2}}{7ax^7} - \frac{\left(-\frac{Ab}{2}-7aB\right)\int \frac{(a+bx^3)^{3/2}}{x^5}\,dx}{7a}$$

$$= -\frac{(Ab+14aB)(a+bx^3)^{3/2}}{56ax^4} - \frac{A(a+bx^3)^{5/2}}{7ax^7} + \frac{(9b(Ab+14aB))\int \frac{\sqrt{a+bx^3}}{x^2}\,dx}{112a}$$

$$= -\frac{9b(Ab+14aB)\sqrt{a+bx^3}}{112ax} - \frac{(Ab+14aB)(a+bx^3)^{3/2}}{56ax^4} - \frac{A(a+bx^3)^{5/2}}{7ax^7} + \frac{(27b^2(Ab+14aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{224a}$$

$$= -\frac{9b(Ab+14aB)\sqrt{a+bx^3}}{112ax} - \frac{(Ab+14aB)(a+bx^3)^{3/2}}{56ax^4} - \frac{A(a+bx^3)^{5/2}}{7ax^7} + \frac{\left(27b^{5/3}(Ab+14aB)\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{224a} - \frac{\left(27\left(1-\sqrt{3}\right)b^{5/3}(Ab+14aB)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{224a^{2/3}}$$

$$= -\frac{9b(Ab+14aB)\sqrt{a+bx^3}}{112ax} + \frac{27b^{4/3}(Ab+14aB)\sqrt{a+bx^3}}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(Ab+14aB)(a+bx^3)^{3/2}}{56ax^4} - \frac{A(a+bx^3)^{5/2}}{7ax^7} - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}(Ab+14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{224a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{9\ 3^{3/4}b^{4/3}(Ab+14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\right.}{56\sqrt{2}a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.14

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^8}\,dx = \frac{\sqrt{a+bx^3}\left(-\frac{4A(a+bx^3)^2}{a}-\frac{(Ab+14aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{4}{3},-\frac{1}{3},-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}\right)}{28x^7}$$

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^8,x]
```

```
[Out] (Sqrt[a + b*x^3]*((-4*A*(a + b*x^3)^2)/a - ((A*b + 14*a*B)*x^3*Hypergeometr
ic2F1[-3/2, -4/3, -1/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(28*x^7)
```

Maple [A] (verified)

Time = 4.50 (sec) , antiderivative size = 503, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(27A\,b^{2}x^{6}+154B\,x^{6}ab+34aAb\,x^{3}+28a^{2}B\,x^{3}+16a^{2}A\right)}{112x^{7}a}-\frac{9ib(Ab+14Ba)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{}$ $2i\left(B\,b^{2}+\frac{b^{2}(27Ab+154Ba)}{224a}\right)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right.}{(}}$
elliptic	$-\frac{Aa\sqrt{b\,x^{3}+a}}{7x^{7}}-\frac{\left(\frac{17Ab}{14}+Ba\right)\sqrt{b\,x^{3}+a}}{4x^{4}}-\frac{b(27Ab+154Ba)\sqrt{b\,x^{3}+a}}{112ax}-$
default	Expression too large to display

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/112*(b*x^3+a)^(1/2)*(27*A*b^2*x^6+154*B*a*b*x^6+34*A*a*b*x^3+28*B*a^2*x^
3+16*A*a^2)/x^7/a-9/112*I*b*(A*b+14*B*a)/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1
```

```
/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
)^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.16

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^8}\,dx = -\frac{27\,(14\,Bab+Ab^2)\sqrt{b}x^7\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+((154\,Bab+27\,Ab^2)x^6}{112\,ax^7}$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^8,x, algorithm="fricas")
```

```
[Out] -1/112*(27*(14*B*a*b + A*b^2)*sqrt(b)*x^7*weierstrassZeta(0, -4*a/b, weiers
trassPInverse(0, -4*a/b, x)) + ((154*B*a*b + 27*A*b^2)*x^6 + 2*(14*B*a^2 +
17*A*a*b)*x^3 + 16*A*a^2)*sqrt(b*x^3 + a))/(a*x^7)
```

Sympy [A] (verification not implemented)

Time = 2.44 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.34

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^8}\,dx = \frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)} + \frac{A\sqrt{a}b\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{Ba^{\frac{3}{2}}\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{B\sqrt{a}b\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**8,x)
```

```
[Out] A*a**(3/2)*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**7*gamma(-4/3)) + A*sqrt(a)*b*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3
,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + B*a**(3/2)*gamma(-4/3)
*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)
) + B*sqrt(a)*b*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*
pi)/a)/(3*x*gamma(2/3))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^8}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^8}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^8,x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^8, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^8}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^8}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^8,x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^8}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{3/2}}{x^8}\,dx$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^8,x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^8, x)
```

3.212 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^{11}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 608

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^{11}} \, dx = \frac{9b(Ab-4aB)\sqrt{a+bx^3}}{224ax^4} + \frac{27b^2(Ab-4aB)\sqrt{a+bx^3}}{448a^2x}$$
$$- \frac{27b^{7/3}(Ab-4aB)\sqrt{a+bx^3}}{448a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{(Ab-4aB)(a+bx^3)^{3/2}}{28ax^7} - \frac{A(a+bx^3)^{5/2}}{10ax^{10}}$$
$$+ \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{7/3}(Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4}{896a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$- \frac{9\ 3^{3/4}b^{7/3}(Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4}{224\sqrt{2}a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 1/28*(A*b-4*B*a)*(b*x^3+a)^(3/2)/a/x^7-1/10*A*(b*x^3+a)^(5/2)/a/x^10+9/224*
b*(A*b-4*B*a)*(b*x^3+a)^(1/2)/a/x^4+27/448*b^2*(A*b-4*B*a)*(b*x^3+a)^(1/2)/
a^2/x-27/448*b^(7/3)*(A*b-4*B*a)*(b*x^3+a)^(1/2)/a^2/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))-9/448*3^(3/4)*b^(7/3)*(A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((
b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*
I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
^2)^(1/2)/a^(5/3)*2^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+27/896*3^(1/4)*b^(7/3)*(A*b-4*B*a)*(a^
```

```
(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(5/3)/(b*
x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2
)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 608, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {464, 283, 331, 309, 224, 1891}

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^{11}}\,dx =$$

$$-\frac{9\,3^{3/4}b^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-4aB)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right.}{224\sqrt{2}a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-4aB)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{\right.}{896a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{27b^{7/3}\sqrt{a+bx^3}(Ab-4aB)}{448a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{27b^2\sqrt{a+bx^3}(Ab-4aB)}{448a^2x}$$

$$+\frac{(a+bx^3)^{3/2}(Ab-4aB)}{28ax^7}+\frac{9b\sqrt{a+bx^3}(Ab-4aB)}{224ax^4}-\frac{A(a+bx^3)^{5/2}}{10ax^{10}}$$

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^11,x]
```

```
[Out] (9*b*(A*b - 4*a*B)*Sqrt[a + b*x^3])/(224*a*x^4) + (27*b^2*(A*b - 4*a*B)*Sqr
t[a + b*x^3])/(448*a^2*x) - (27*b^(7/3)*(A*b - 4*a*B)*Sqrt[a + b*x^3])/(448
*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + ((A*b - 4*a*B)*(a + b*x^3)^(3/2
))/(28*a*x^7) - (A*(a + b*x^3)^(5/2))/(10*a*x^10) + (27*3^(1/4)*Sqrt[2 - Sq
rt[3]]*b^(7/3)*(A*b - 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[A
rcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)], -7 - 4*Sqrt[3]])/(896*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (9*3^(3/4)*b^(7/3)*(
A*b - 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
```

```
/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt
[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt
[3]])/(224*Sqrt[2]*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{A(a+bx^3)^{5/2}}{10ax^{10}}-\frac{\left(\frac{5Ab}{2}-10aB\right)\int\frac{(a+bx^3)^{3/2}}{x^8}\,dx}{10a}\\
&= \frac{(Ab-4aB)\,(a+bx^3)^{3/2}}{28ax^7}-\frac{A(a+bx^3)^{5/2}}{10ax^{10}}-\frac{(9b(Ab-4aB))\int\frac{\sqrt{a+bx^3}}{x^5}\,dx}{56a}\\
&= \frac{9b(Ab-4aB)\sqrt{a+bx^3}}{224ax^4}+\frac{(Ab-4aB)\,(a+bx^3)^{3/2}}{28ax^7}\\
&\quad-\frac{A(a+bx^3)^{5/2}}{10ax^{10}}-\frac{(27b^2(Ab-4aB))\int\frac{1}{x^2\sqrt{a+bx^3}}\,dx}{448a}\\
&= \frac{9b(Ab-4aB)\sqrt{a+bx^3}}{224ax^4}+\frac{27b^2(Ab-4aB)\sqrt{a+bx^3}}{448a^2x}\\
&\quad+\frac{(Ab-4aB)\,(a+bx^3)^{3/2}}{28ax^7}-\frac{A(a+bx^3)^{5/2}}{10ax^{10}}-\frac{(27b^3(Ab-4aB))\int\frac{x}{\sqrt{a+bx^3}}\,dx}{896a^2}\\
&= \frac{9b(Ab-4aB)\sqrt{a+bx^3}}{224ax^4}+\frac{27b^2(Ab-4aB)\sqrt{a+bx^3}}{448a^2x}+\frac{(Ab-4aB)\,(a+bx^3)^{3/2}}{28ax^7}\\
&\quad-\frac{A(a+bx^3)^{5/2}}{10ax^{10}}-\frac{\left(27b^{8/3}(Ab-4aB)\right)\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{896a^2}\\
&\quad+\frac{\left(27\left(1-\sqrt{3}\right)b^{8/3}(Ab-4aB)\right)\int\frac{1}{\sqrt{a+bx^3}}\,dx}{896a^{5/3}}
\end{aligned}$$

$$= \frac{9b(Ab-4aB)\sqrt{a+bx^3}}{224ax^4} + \frac{27b^2(Ab-4aB)\sqrt{a+bx^3}}{448a^2x} - \frac{27b^{7/3}(Ab-4aB)\sqrt{a+bx^3}}{448a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{(Ab-4aB)\left(a+bx^3\right)^{3/2}}{28ax^7} - \frac{A\left(a+bx^3\right)^{5/2}}{10ax^{10}}$$

$$+ \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{7/3}(Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{896a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$- \frac{9\ 3^{3/4}b^{7/3}(Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\right.}{224\sqrt{2}a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.13

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^{11}}\,dx = \frac{\sqrt{a+bx^3}\left(-\frac{7A\left(a+bx^3\right)^2}{a} + \frac{5(Ab-4aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{7}{3},-\frac{3}{2},-\frac{4}{3},-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}\right)}{70x^{10}}$$

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^11,x]
[Out] (Sqrt[a + b*x^3]*((-7*A*(a + b*x^3)^2)/a + (5*(A*b - 4*a*B)*x^3*Hypergeomet
ric2F1[-7/3, -3/2, -4/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(70*x^10)
```

Maple [A] (verified)

Time = 4.55 (sec) , antiderivative size = 529, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-135A\,x^{9}b^{3}+540B\,x^{9}a\,b^{2}+54A\,x^{6}a\,b^{2}+680B\,x^{6}a^{2}b+368A\,x^{3}a^{2}b+320a^{3}B\,x^{3}+224a^{3}A\right)}{2240x^{10}a^{2}}+\frac{9ib^{2}(Ab-4Ba)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\cdots}{\cdots}$
elliptic	$-\frac{Aa\sqrt{b\,x^{3}+a}}{10x^{10}}-\frac{\left(\frac{23Ab}{20}+Ba\right)\sqrt{b\,x^{3}+a}}{7x^{7}}-\frac{b(27Ab+340Ba)\sqrt{b\,x^{3}+a}}{1120a\,x^{4}}+\frac{27b^{2}(Ab-4Ba)\sqrt{b\,x^{3}+a}}{448a^{2}x}+\frac{9ib^{2}(Ab-4Ba)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\cdots}{\cdots}$
default	Expression too large to display

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2240*(b*x^3+a)^(1/2)*(-135*A*b^3*x^9+540*B*a*b^2*x^9+54*A*a*b^2*x^6+680*
B*a^2*b*x^6+368*A*a^2*b*x^3+320*B*a^3*x^3+224*A*a^3)/x^10/a^2+9/448*I*b^2*(
A*b-4*B*a)/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3
))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
```

```
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.20

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^{11}}\,dx =$$
$$-\frac{135\,(4\,Bab^2-Ab^3)\sqrt{b}x^{10}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+(135\,(4\,Bab^2-Ab^3)}{2240\,a^2x^{10}}$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^11,x, algorithm="fricas")
```

```
[Out] -1/2240*(135*(4*B*a*b^2 - A*b^3)*sqrt(b)*x^10*weierstrassZeta(0, -4*a/b, we
ierstrassPInverse(0, -4*a/b, x)) + (135*(4*B*a*b^2 - A*b^3)*x^9 + 2*(340*B*
a^2*b + 27*A*a*b^2)*x^6 + 224*A*a^3 + 16*(20*B*a^3 + 23*A*a^2*b)*x^3)*sqrt(
b*x^3 + a))/(a^2*x^10)
```

Sympy [A] (verification not implemented)

Time = 2.74 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.33

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^{11}}\,dx = \frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{10}{3},-\frac{1}{2}\\-\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{10}\Gamma\left(-\frac{7}{3}\right)}$$
$$+\frac{A\sqrt{a}b\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)}+\frac{Ba^{\frac{3}{2}}\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)}$$
$$+\frac{B\sqrt{a}b\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}$$

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**11,x)
```

```
[Out] A*a**(3/2)*gamma(-10/3)*hyper((-10/3, -1/2), (-7/3,), b*x**3*exp_polar(I*pi
)/a)/(3*x**10*gamma(-7/3)) + A*sqrt(a)*b*gamma(-7/3)*hyper((-7/3, -1/2), (-
4/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3)) + B*a**(3/2)*gamma(-7
/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4
/3)) + B*sqrt(a)*b*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_pola
r(I*pi)/a)/(3*x**4*gamma(-1/3))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^{11}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^{11}}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^11,x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^11, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^{11}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^{11}}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^11,x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^11, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^{11}}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{3/2}}{x^{11}}\,dx$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^11,x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^11, x)
```

3.213 $\int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}} dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}} dx = \frac{2a^2(Ab-aB)\sqrt{a+bx^3}}{3b^4} - \frac{2a(2Ab-3aB)\left(a+bx^3\right)^{3/2}}{9b^4} + \frac{2(Ab-3aB)\left(a+bx^3\right)^{5/2}}{15b^4} + \frac{2B\left(a+bx^3\right)^{7/2}}{21b^4}$$

```
[Out] -2/9*a*(2*A*b-3*B*a)*(b*x^3+a)^(3/2)/b^4+2/15*(A*b-3*B*a)*(b*x^3+a)^(5/2)/b
^4+2/21*B*(b*x^3+a)^(7/2)/b^4+2/3*a^2*(A*b-B*a)*(b*x^3+a)^(1/2)/b^4
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 78}

$$\int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}} dx = \frac{2a^2\sqrt{a+bx^3}(Ab-aB)}{3b^4} + \frac{2\left(a+bx^3\right)^{5/2}(Ab-3aB)}{15b^4} - \frac{2a\left(a+bx^3\right)^{3/2}(2Ab-3aB)}{9b^4} + \frac{2B\left(a+bx^3\right)^{7/2}}{21b^4}$$

```
[In] Int[(x^8*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*a^2*(A*b - a*B)*Sqrt[a + b*x^3])/(3*b^4) - (2*a*(2*A*b - 3*a*B)*(a + b*x
^3)^(3/2))/(9*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(5/2))/(15*b^4) + (2*B*(a
 + b*x^3)^(7/2))/(21*b^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2(A+Bx)}{\sqrt{a+bx}}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{a^2(-Ab+aB)}{b^3\sqrt{a+bx}}+\frac{a(-2Ab+3aB)\sqrt{a+bx}}{b^3}\right.\right.\\
&\qquad\left.\left.+\frac{(Ab-3aB)(a+bx)^{3/2}}{b^3}+\frac{B(a+bx)^{5/2}}{b^3}\right)\,dx, x, x^3\right)\\
&= \frac{2a^2(Ab-aB)\sqrt{a+bx^3}}{3b^4}-\frac{2a(2Ab-3aB)\left(a+bx^3\right)^{3/2}}{9b^4}\\
&\quad+\frac{2(Ab-3aB)\left(a+bx^3\right)^{5/2}}{15b^4}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.78

$$
\int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{a+bx^3}(56a^2Ab-48a^3B-28aAb^2x^3+24a^2bBx^3+21Ab^3x^6-18ab^2Bx^6+15b^3Bx^9)}{315b^4}
$$

```
[In] Integrate[(x^8*(A + B*x^3))/Sqrt[a + b*x^3],x]
[Out] (2*Sqrt[a + b*x^3]*(56*a^2*A*b - 48*a^3*B - 28*a*A*b^2*x^3 + 24*a^2*b*B*x^3
 + 21*A*b^3*x^6 - 18*a*b^2*B*x^6 + 15*b^3*B*x^9))/(315*b^4)
```

Maple [A] (verified)

Time = 4.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{16\sqrt{b\,x^{3}+a}\,\left(\frac{3x^{6}\left(\frac{5x^{3}B}{7}+A\right)b^{3}}{8}-\frac{x^{3}\left(\frac{9x^{3}B}{14}+A\right)a\,b^{2}}{2}+a^{2}\left(\frac{3x^{3}B}{7}+A\right)b-\frac{6a^{3}B}{7}\right)}{45b^{4}}$
gosper	$\frac{2\sqrt{b\,x^{3}+a}\,\left(15b^{3}B\,x^{9}+21x^{6}b^{3}A-18B\,x^{6}a\,b^{2}-28aA\,b^{2}x^{3}+24B\,a^{2}b\,x^{3}+56a^{2}bA-48a^{3}B\right)}{315b^{4}}$
trager	$\frac{2\sqrt{b\,x^{3}+a}\,\left(15b^{3}B\,x^{9}+21x^{6}b^{3}A-18B\,x^{6}a\,b^{2}-28aA\,b^{2}x^{3}+24B\,a^{2}b\,x^{3}+56a^{2}bA-48a^{3}B\right)}{315b^{4}}$
risch	$\frac{2\sqrt{b\,x^{3}+a}\,\left(15b^{3}B\,x^{9}+21x^{6}b^{3}A-18B\,x^{6}a\,b^{2}-28aA\,b^{2}x^{3}+24B\,a^{2}b\,x^{3}+56a^{2}bA-48a^{3}B\right)}{315b^{4}}$
elliptic	$\frac{2B\,x^{9}\sqrt{b\,x^{3}+a}}{21b}+\frac{2\left(A-\frac{6aB}{7b}\right)x^{6}\sqrt{b\,x^{3}+a}}{15b}-\frac{8a\left(A-\frac{6aB}{7b}\right)x^{3}\sqrt{b\,x^{3}+a}}{45b^{2}}+\frac{16a^{2}\left(A-\frac{6aB}{7b}\right)\sqrt{b\,x^{3}+a}}{45b^{3}}$
default	$A\left(\frac{2x^{6}\sqrt{b\,x^{3}+a}}{15b}-\frac{8a\,x^{3}\sqrt{b\,x^{3}+a}}{45b^{2}}+\frac{16a^{2}\sqrt{b\,x^{3}+a}}{45b^{3}}\right)+B\left(\frac{2x^{9}\sqrt{b\,x^{3}+a}}{21b}-\frac{4a\,x^{6}\sqrt{b\,x^{3}+a}}{35b^{2}}+\frac{16a^{2}x^{3}\sqrt{b\,x^{3}+a}}{105b^{3}}-\right.$

```
[In] int(x^8*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 16/45*(b*x^3+a)^(1/2)*(3/8*x^6*(5/7*x^3*B+A)*b^3-1/2*x^3*(9/14*x^3*B+A)*a*b
^2+a^2*(3/7*x^3*B+A)*b-6/7*a^3*B)/b^4
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.74

$$\int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\,(15\,Bb^3x^9-3\,(6\,Bab^2-7\,Ab^3)x^6-48\,Ba^3+56\,Aa^2b+4\,(6\,Ba^2b-7\,Aab^2)x^3)\sqrt{bx^3+a}}{315\,b^4}$$

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(15*B*b^3*x^9 - 3*(6*B*a*b^2 - 7*A*b^3)*x^6 - 48*B*a^3 + 56*A*a^2*b +
 4*(6*B*a^2*b - 7*A*a*b^2)*x^3)*sqrt(b*x^3 + a)/b^4
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.70

$$\int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \begin{cases} \frac{16Aa^2\sqrt{a+bx^3}}{45b^3} - \frac{8Aax^3\sqrt{a+bx^3}}{45b^2} + \frac{2Ax^6\sqrt{a+bx^3}}{15b} - \frac{32Ba^3\sqrt{a+bx^3}}{105b^4} + \frac{16Ba^2x^3\sqrt{a+bx^3}}{105b^3} - \frac{4Bax^6\sqrt{a+bx^3}}{35b^2} + \frac{2Bx^9\sqrt{a+bx^3}}{21b} & \text{for } b \neq 0 \\ \frac{\frac{Ax^9}{9}+\frac{Bx^{12}}{12}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**8*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((16*A*a**2*sqrt(a + b*x**3)/(45*b**3) - 8*A*a*x**3*sqrt(a + b*x**
3)/(45*b**2) + 2*A*x**6*sqrt(a + b*x**3)/(15*b) - 32*B*a**3*sqrt(a + b*x**3
)/(105*b**4) + 16*B*a**2*x**3*sqrt(a + b*x**3)/(105*b**3) - 4*B*a*x**6*sqrt
(a + b*x**3)/(35*b**2) + 2*B*x**9*sqrt(a + b*x**3)/(21*b), Ne(b, 0)), ((A*x
**9/9 + B*x**12/12)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.15

$$\int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2}{105}\,B\left(\frac{5\,(bx^3+a)^{\frac{7}{2}}}{b^4} - \frac{21\,(bx^3+a)^{\frac{5}{2}}a}{b^4} + \frac{35\,(bx^3+a)^{\frac{3}{2}}a^2}{b^4} - \frac{35\,\sqrt{bx^3+a}a^3}{b^4}\right) + \frac{2}{45}\,A\left(\frac{3\,(bx^3+a)^{\frac{5}{2}}}{b^3} - \frac{10\,(bx^3+a)^{\frac{3}{2}}a}{b^3} + \frac{15\,\sqrt{bx^3+a}a^2}{b^3}\right)$$

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*B*(5*(b*x^3 + a)^(7/2)/b^4 - 21*(b*x^3 + a)^(5/2)*a/b^4 + 35*(b*x^3 +
 a)^(3/2)*a^2/b^4 - 35*sqrt(b*x^3 + a)*a^3/b^4) + 2/45*A*(3*(b*x^3 + a)^(5/
2)/b^3 - 10*(b*x^3 + a)^(3/2)*a/b^3 + 15*sqrt(b*x^3 + a)*a^2/b^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98

$$\int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = -\frac{2\,(Ba^3-Aa^2b)\sqrt{bx^3+a}}{3\,b^4} + \frac{2\left(15\,(bx^3+a)^{\frac{7}{2}}B-63\,(bx^3+a)^{\frac{5}{2}}Ba+105\,(bx^3+a)^{\frac{3}{2}}Ba^2+21\,(bx^3+a)^{\frac{5}{2}}Ab-70\,(bx^3+a)^{\frac{3}{2}}Aab\right)}{315\,b^4}$$

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(B*a^3 - A*a^2*b)*sqrt(b*x^3 + a)/b^4 + 2/315*(15*(b*x^3 + a)^(7/2)*B
- 63*(b*x^3 + a)^(5/2)*B*a + 105*(b*x^3 + a)^(3/2)*B*a^2 + 21*(b*x^3 + a)^(
5/2)*A*b - 70*(b*x^3 + a)^(3/2)*A*a*b)/b^4
```

Mupad [B] (verification not implemented)

Time = 7.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{8\,a^2\,\sqrt{b\,x^3+a}\,\left(2\,A-\frac{12\,B\,a}{7\,b}\right)}{45\,b^3} + \frac{x^6\,\sqrt{b\,x^3+a}\,\left(2\,A-\frac{12\,B\,a}{7\,b}\right)}{15\,b} + \frac{2\,B\,x^9\,\sqrt{b\,x^3+a}}{21\,b} - \frac{4\,a\,x^3\,\sqrt{b\,x^3+a}\,\left(2\,A-\frac{12\,B\,a}{7\,b}\right)}{45\,b^2}$$

```
[In] int((x^8*(A + B*x^3))/(a + b*x^3)^(1/2),x)
```

```
[Out] (8*a^2*(a + b*x^3)^(1/2)*(2*A - (12*B*a)/(7*b)))/(45*b^3) + (x^6*(a + b*x^3
)^(1/2)*(2*A - (12*B*a)/(7*b)))/(15*b) + (2*B*x^9*(a + b*x^3)^(1/2))/(21*b)
 - (4*a*x^3*(a + b*x^3)^(1/2)*(2*A - (12*B*a)/(7*b)))/(45*b^2)
```

3.214 $\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = -\frac{2a(Ab-aB)\sqrt{a+bx^3}}{3b^3}+\frac{2(Ab-2aB)\left(a+bx^3\right)^{3/2}}{9b^3}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^3}$$

```
[Out] 2/9*(A*b-2*B*a)*(b*x^3+a)^(3/2)/b^3+2/15*B*(b*x^3+a)^(5/2)/b^3-2/3*a*(A*b-B
*a)*(b*x^3+a)^(1/2)/b^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 78}

$$\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\left(a+bx^3\right)^{3/2}(Ab-2aB)}{9b^3}-\frac{2a\sqrt{a+bx^3}(Ab-aB)}{3b^3}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^3}$$

```
[In] Int[(x^5*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (-2*a*(A*b - a*B)*Sqrt[a + b*x^3])/(3*b^3) + (2*(A*b - 2*a*B)*(a + b*x^3)^(
3/2))/(9*b^3) + (2*B*(a + b*x^3)^(5/2))/(15*b^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x(A+Bx)}{\sqrt{a+bx}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^2\sqrt{a+bx}}+\frac{(Ab-2aB)\sqrt{a+bx}}{b^2}+\frac{B(a+bx)^{3/2}}{b^2}\right)dx, x, x^3\right)\\ &= -\frac{2a(Ab-aB)\sqrt{a+bx^3}}{3b^3}+\frac{2(Ab-2aB)\left(a+bx^3\right)^{3/2}}{9b^3}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{a+bx^3}(-10aAb+8a^2B+5Ab^2x^3-4abBx^3+3b^2Bx^6)}{45b^3}$$

```
[In] Integrate[(x^5*(A + B*x^3))/Sqrt[a + b*x^3],x]
[Out] (2*Sqrt[a + b*x^3]*(-10*a*A*b + 8*a^2*B + 5*A*b^2*x^3 - 4*a*b*B*x^3 + 3*b^2
*B*x^6))/(45*b^3)
```

Maple [A] (verified)

Time = 4.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$-\frac{4\left(-\frac{x^{3}\left(\frac{3x^{3}B}{5}+A\right)b^{2}}{2}+a\left(\frac{2x^{3}B}{5}+A\right)b-\frac{4a^{2}B}{5}\right)\sqrt{b\,x^{3}+a}}{9b^{3}}$	49
gosper	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(-3b^{2}B\,x^{6}-5A\,b^{2}x^{3}+4Bab\,x^{3}+10abA-8a^{2}B\right)}{45b^{3}}$	53
trager	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(-3b^{2}B\,x^{6}-5A\,b^{2}x^{3}+4Bab\,x^{3}+10abA-8a^{2}B\right)}{45b^{3}}$	53
risch	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(-3b^{2}B\,x^{6}-5A\,b^{2}x^{3}+4Bab\,x^{3}+10abA-8a^{2}B\right)}{45b^{3}}$	53
elliptic	$\frac{2B\,x^{6}\sqrt{b\,x^{3}+a}}{15b}+\frac{2\left(A-\frac{4aB}{5b}\right)x^{3}\sqrt{b\,x^{3}+a}}{9b}-\frac{4a\left(A-\frac{4aB}{5b}\right)\sqrt{b\,x^{3}+a}}{9b^{2}}$	70
default	$B\left(\frac{2x^{6}\sqrt{b\,x^{3}+a}}{15b}-\frac{8a\,x^{3}\sqrt{b\,x^{3}+a}}{45b^{2}}+\frac{16a^{2}\sqrt{b\,x^{3}+a}}{45b^{3}}\right)+A\left(\frac{2x^{3}\sqrt{b\,x^{3}+a}}{9b}-\frac{4a\sqrt{b\,x^{3}+a}}{9b^{2}}\right)$	92

```
[In] int(x^5*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/9*(-1/2*x^3*(3/5*x^3*B+A)*b^2+a*(2/5*x^3*B+A)*b-4/5*a^2*B)*(b*x^3+a)^(1/
2)/b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\,(3\,Bb^2x^6-(4\,Bab-5\,Ab^2)x^3+8\,Ba^2-10\,Aab)\sqrt{bx^3+a}}{45\,b^3}$$

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(3*B*b^2*x^6 - (4*B*a*b - 5*A*b^2)*x^3 + 8*B*a^2 - 10*A*a*b)*sqrt(b*x^
3 + a)/b^3
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.70

$$\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \begin{cases} -\frac{4Aa\sqrt{a+bx^3}}{9b^2}+\frac{2Ax^3\sqrt{a+bx^3}}{9b}+\frac{16Ba^2\sqrt{a+bx^3}}{45b^3}-\frac{8Bax^3\sqrt{a+bx^3}}{45b^2}+\frac{2Bx^6\sqrt{a+bx^3}}{15b} & \text{for } b\neq 0 \\ \frac{\frac{Ax^6}{6}+\frac{Bx^9}{9}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**5*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((-4*A*a*sqrt(a + b*x**3)/(9*b**2) + 2*A*x**3*sqrt(a + b*x**3)/(9*
b) + 16*B*a**2*sqrt(a + b*x**3)/(45*b**3) - 8*B*a*x**3*sqrt(a + b*x**3)/(45
*b**2) + 2*B*x**6*sqrt(a + b*x**3)/(15*b), Ne(b, 0)), ((A*x**6/6 + B*x**9/9
)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.14

$$\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2}{45}\,B\left(\frac{3\,(bx^3+a)^{\frac{5}{2}}}{b^3} - \frac{10\,(bx^3+a)^{\frac{3}{2}}a}{b^3} + \frac{15\,\sqrt{bx^3+a}a^2}{b^3}\right) + \frac{2}{9}\,A\left(\frac{(bx^3+a)^{\frac{3}{2}}}{b^2} - \frac{3\,\sqrt{bx^3+a}a}{b^2}\right)$$

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/45*B*(3*(b*x^3 + a)^(5/2)/b^3 - 10*(b*x^3 + a)^(3/2)*a/b^3 + 15*sqrt(b*x^
3 + a)*a^2/b^3) + 2/9*A*((b*x^3 + a)^(3/2)/b^2 - 3*sqrt(b*x^3 + a)*a/b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\,\sqrt{bx^3+a}(Ba^2-Aab)}{3\,b^3} + \frac{2\left(3\,(bx^3+a)^{\frac{5}{2}}B - 10\,(bx^3+a)^{\frac{3}{2}}Ba + 5\,(bx^3+a)^{\frac{3}{2}}Ab\right)}{45\,b^3}$$

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(b*x^3 + a)*(B*a^2 - A*a*b)/b^3 + 2/45*(3*(b*x^3 + a)^(5/2)*B - 10*
(b*x^3 + a)^(3/2)*B*a + 5*(b*x^3 + a)^(3/2)*A*b)/b^3
```

Mupad [B] (verification not implemented)

Time = 7.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\,\sqrt{b\,x^3+a}\,(8\,B\,a^2-4\,B\,a\,b\,x^3-10\,A\,a\,b+3\,B\,b^2\,x^6+5\,A\,b^2\,x^3)}{45\,b^3}$$

```
[In] int((x^5*(A + B*x^3))/(a + b*x^3)^(1/2),x)
```

```
[Out] (2*(a + b*x^3)^(1/2)*(8*B*a^2 + 5*A*b^2*x^3 + 3*B*b^2*x^6 - 10*A*a*b - 4*B*
a*b*x^3))/(45*b^3)
```

3.215 $\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2(Ab-aB)\sqrt{a+bx^3}}{3b^2} + \frac{2B(a+bx^3)^{3/2}}{9b^2}$$

```
[Out] 2/9*B*(b*x^3+a)^(3/2)/b^2+2/3*(A*b-B*a)*(b*x^3+a)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {455, 45}

$$\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{a+bx^3}(Ab-aB)}{3b^2} + \frac{2B(a+bx^3)^{3/2}}{9b^2}$$

```
[In] Int[(x^2*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[a + b*x^3])/(3*b^2) + (2*B*(a + b*x^3)^(3/2))/(9*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{\sqrt{a+bx}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{Ab-aB}{b\sqrt{a+bx}}+\frac{B\sqrt{a+bx}}{b}\right)\,dx, x, x^3\right)\\ &= \frac{2(Ab-aB)\sqrt{a+bx^3}}{3b^2}+\frac{2B(a+bx^3)^{3/2}}{9b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{a+bx^3}(3Ab-2aB+bBx^3)}{9b^2}$$

```
[In] Integrate[(x^2*(A + B*x^3))/Sqrt[a + b*x^3],x]
[Out] (2*Sqrt[a + b*x^3]*(3*A*b - 2*a*B + b*B*x^3))/(9*b^2)
```

Maple [A] (verified)

Time = 4.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{2\sqrt{b\,x^3+a}\,(bB\,x^3+3Ab-2Ba)}{9b^2}$	30
trager	$\frac{2\sqrt{b\,x^3+a}\,(bB\,x^3+3Ab-2Ba)}{9b^2}$	30
risch	$\frac{2\sqrt{b\,x^3+a}\,(bB\,x^3+3Ab-2Ba)}{9b^2}$	30
pseudoelliptic	$\frac{2\sqrt{b\,x^3+a}\,\left(\left(\frac{x^3B}{3}+A\right)b-\frac{2Ba}{3}\right)}{3b^2}$	30
elliptic	$\frac{2B\,x^3\sqrt{b\,x^3+a}}{9b}+\frac{2\left(A-\frac{2aB}{3b}\right)\sqrt{b\,x^3+a}}{3b}$	43
default	$B\left(\frac{2x^3\sqrt{b\,x^3+a}}{9b}-\frac{4a\sqrt{b\,x^3+a}}{9b^2}\right)+\frac{2A\sqrt{b\,x^3+a}}{3b}$	52

```
[In] int(x^2*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/9*(b*x^3+a)^(1/2)*(B*b*x^3+3*A*b-2*B*a)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.63

$$\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\,(Bbx^3-2\,Ba+3\,Ab)\sqrt{bx^3+a}}{9\,b^2}$$

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*(B*b*x^3 - 2*B*a + 3*A*b)*sqrt(b*x^3 + a)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.63

$$\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \begin{cases} \frac{2A\sqrt{a+bx^3}}{3b} - \frac{4Ba\sqrt{a+bx^3}}{9b^2} + \frac{2Bx^3\sqrt{a+bx^3}}{9b} & \text{for } b\neq 0 \\ \frac{\frac{Ax^3}{3}+\frac{Bx^6}{6}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((2*A*sqrt(a + b*x**3)/(3*b) - 4*B*a*sqrt(a + b*x**3)/(9*b**2) + 2
*B*x**3*sqrt(a + b*x**3)/(9*b), Ne(b, 0)), ((A*x**3/3 + B*x**6/6)/sqrt(a),
True))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2}{9}\,B\left(\frac{(bx^3+a)^{\frac{3}{2}}}{b^2} - \frac{3\,\sqrt{bx^3+a}a}{b^2}\right) + \frac{2\,\sqrt{bx^3+a}A}{3\,b}$$

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*B*((b*x^3 + a)^(3/2)/b^2 - 3*sqrt(b*x^3 + a)*a/b^2) + 2/3*sqrt(b*x^3 +
a)*A/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\,(bx^3+a)^{\frac{3}{2}}B}{9\,b^2} - \frac{2\,\sqrt{bx^3+a}(Ba-Ab)}{3\,b^2}$$

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] 2/9*(b*x^3 + a)^(3/2)*B/b^2 - 2/3*sqrt(b*x^3 + a)*(B*a - A*b)/b^2
```

Mupad [B] (verification not implemented)

Time = 7.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.63

$$\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\,\sqrt{b\,x^3+a}\,(B\,b\,x^3+3\,A\,b-2\,B\,a)}{9\,b^2}$$

```
[In] int((x^2*(A + B*x^3))/(a + b*x^3)^(1/2),x)
[Out] (2*(a + b*x^3)^(1/2)*(3*A*b - 2*B*a + B*b*x^3))/(9*b^2)
```

3.216 $\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 48

$$\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx=\frac{2B\sqrt{a+bx^3}}{3b}-\frac{2A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

```
[Out] -2/3*A*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+2/3*B*(b*x^3+a)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {457, 81, 65, 214}

$$\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx=\frac{2B\sqrt{a+bx^3}}{3b}-\frac{2A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

```
[In] Int[(A + B*x^3)/(x*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*B*Sqrt[a + b*x^3])/(3*b) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqr
t[a])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x\sqrt{a+bx}}\,dx, x, x^3\right)\\
&= \frac{2B\sqrt{a+bx^3}}{3b}+\frac{1}{3}A\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)\\
&= \frac{2B\sqrt{a+bx^3}}{3b}+\frac{(2A)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{3b}\\
&= \frac{2B\sqrt{a+bx^3}}{3b}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx=\frac{2B\sqrt{a+bx^3}}{3b}-\frac{2A\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

```
[In] Integrate[(A + B*x^3)/(x*Sqrt[a + b*x^3]),x]
[Out] (2*B*Sqrt[a + b*x^3])/(3*b) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqr
t[a])
```

Maple [A] (verified)

Time = 4.48 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{2A\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{a}}+\frac{2B\sqrt{b\,x^{3}+a}}{3b}$	37
elliptic	$-\frac{2A\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{a}}+\frac{2B\sqrt{b\,x^{3}+a}}{3b}$	37
pseudoelliptic	$-\frac{2\left(Ab\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)-B\sqrt{b\,x^{3}+a}\,\sqrt{a}\right)}{3\sqrt{a}\,b}$	42

```
[In] int((B*x^3+A)/x/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*A*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+2/3*B*(b*x^3+a)^(1/2)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.19

$$\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx = \left[\frac{A\sqrt{a}b\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2a}{x^3}\right)+2\sqrt{bx^3+a}Ba}{3ab}, \frac{2\left(A\sqrt{-a}b\arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right)+\sqrt{bx^3+a}Ba\right)}{3ab}\right]$$

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] [1/3*(A*sqrt(a)*b*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*sq
rt(b*x^3 + a)*B*a)/(a*b), 2/3*(A*sqrt(-a)*b*arctan(sqrt(b*x^3 + a)*sqrt(-a)
/a) + sqrt(b*x^3 + a)*B*a)/(a*b)]
```

Sympy [A] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.48

$$\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx = \frac{A\left(\begin{cases}\frac{2\operatorname{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{\sqrt{-a}} & \text{for}\ b\neq 0\\ -\frac{\log\left(\frac{1}{x^3}\right)}{\sqrt{a}} & \text{otherwise}\end{cases}\right)}{3} - \frac{B\left(\begin{cases}-\frac{x^3}{\sqrt{a}} & \text{for}\ b=0\\ -\frac{2\sqrt{a+bx^3}}{b} & \text{otherwise}\end{cases}\right)}{3}$$

```
[In] integrate((B*x**3+A)/x/(b*x**3+a)**(1/2),x)
```

```
[Out] A*Piecewise((2*atan(sqrt(a + b*x**3)/sqrt(-a))/sqrt(-a), Ne(b, 0)), (-log(x
**(-3))/sqrt(a), True))/3 - B*Piecewise((-x**3/sqrt(a), Eq(b, 0)), (-2*sqrt
(a + b*x**3)/b, True))/3
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx = \frac{A\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{3\sqrt{a}} + \frac{2\sqrt{bx^3+a}B}{3b}$$

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*A*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sqrt(a)
+ 2/3*sqrt(b*x^3 + a)*B/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx = \frac{2A\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}} + \frac{2\sqrt{bx^3+a}B}{3b}$$

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*A*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) + 2/3*sqrt(b*x^3 + a)*B/b
```

Mupad [B] (verification not implemented)

Time = 7.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx = \frac{2B\sqrt{b\,x^3+a}}{3b} + \frac{A\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3\sqrt{a}}$$

```
[In] int((A + B*x^3)/(x*(a + b*x^3)^(1/2)),x)
```

```
[Out] (2*B*(a + b*x^3)^(1/2))/(3*b) + (A*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a
 + b*x^3)^(1/2) + a^(1/2)))/x^6))/(3*a^(1/2))
```

3.217 $\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 58

$$\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}\,dx=-\frac{A\sqrt{a+bx^3}}{3ax^3}+\frac{(Ab-2aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

```
[Out] 1/3*(A*b-2*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/3*A*(b*x^3+a)^(1
/2)/a/x^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {457, 79, 65, 214}

$$\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}\,dx=\frac{(Ab-2aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{A\sqrt{a+bx^3}}{3ax^3}$$

```
[In] Int[(A + B*x^3)/(x^4*Sqrt[a + b*x^3]),x]
```

```
[Out] -1/3*(A*Sqrt[a + b*x^3])/(a*x^3) + ((A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/S
qrt[a]])/(3*a^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^2\sqrt{a+bx}}\,dx,x,x^3\right)\\
&= -\frac{A\sqrt{a+bx^3}}{3ax^3}+\frac{\left(-\frac{Ab}{2}+aB\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)}{3a}\\
&= -\frac{A\sqrt{a+bx^3}}{3ax^3}+\frac{\left(2\left(-\frac{Ab}{2}+aB\right)\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{3ab}\\
&= -\frac{A\sqrt{a+bx^3}}{3ax^3}+\frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}\,dx = -\frac{A\sqrt{a+bx^3}}{3ax^3} + \frac{(Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

```
[In] Integrate[(A + B*x^3)/(x^4*Sqrt[a + b*x^3]),x]
[Out] -1/3*(A*Sqrt[a + b*x^3])/(a*x^3) + ((A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/S
qrt[a]])/(3*a^(3/2))
```

Maple [A] (verified)

Time = 4.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{(Ab-2Ba)\,\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}-\frac{A\sqrt{b\,x^3+a}}{3a\,x^3}$	47
elliptic	$\frac{(Ab-2Ba)\,\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}-\frac{A\sqrt{b\,x^3+a}}{3a\,x^3}$	47
pseudoelliptic	$\frac{(Ab-2Ba)\,\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}-\frac{A\sqrt{b\,x^3+a}}{3a\,x^3}$	47
default	$-\frac{2B\ \text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}}+A\left(\frac{b\ \text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}-\frac{\sqrt{b\,x^3+a}}{3a\,x^3}\right)$	62

```
[In] int((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(A*b-2*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/3*A*(b*x^3+a)^(1
/2)/a/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.17

$$\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}\,dx$$
$$= \left[-\frac{(2\,Ba-Ab)\sqrt{a}x^3\log\left(\frac{bx^3+2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\,\sqrt{bx^3+a}Aa}{6\,a^2x^3}, \frac{(2\,Ba-Ab)\sqrt{-a}x^3\arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right)}{3\,a^2x^3}\right.$$

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/6*((2*B*a - A*b)*sqrt(a)*x^3*log((b*x^3 + 2*sqrt(b*x^3 + a)*sqrt(a) + 2
*a)/x^3) + 2*sqrt(b*x^3 + a)*A*a)/(a^2*x^3), 1/3*((2*B*a - A*b)*sqrt(-a)*x^
3*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) - sqrt(b*x^3 + a)*A*a)/(a^2*x^3)]
```

Sympy [A] (verification not implemented)

Time = 5.76 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.38

$$\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}\,dx = -\frac{A\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3ax^{\frac{3}{2}}} + \frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3a^{\frac{3}{2}}} - \frac{2B\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

```
[In] integrate((B*x**3+A)/x**4/(b*x**3+a)**(1/2),x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*a*x**(3/2)) + A*b*asinh(sqrt(a)/(sqrt(b)
*x**(3/2)))/(3*a**(3/2)) - 2*B*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a)
)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(47) = 94.

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.88

$$\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}\,dx = -\frac{1}{6}A\left(\frac{2\sqrt{bx^3+a}b}{(bx^3+a)a-a^2} + \frac{b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right) + \frac{B\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{3\sqrt{a}}$$

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*A*(2*sqrt(b*x^3 + a)*b/((b*x^3 + a)*a - a^2) + b*log((sqrt(b*x^3 + a)
- sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(3/2)) + 1/3*B*log((sqrt(b*x^3 +
a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sqrt(a)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}\,dx = \frac{\frac{(2Bab-Ab^2)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} - \frac{\sqrt{bx^3+a}Ab}{ax^3}}{3b}$$

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*((2*B*a*b - A*b^2)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a) - sqrt
(b*x^3 + a)*A*b/(a*x^3))/b
```

Mupad [B] (verification not implemented)

Time = 7.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16

$$\int \frac{A + Bx^3}{x^4\sqrt{a + bx^3}}\, dx = \frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\,{\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}^3}{x^6}\right)\,\left(A\,b-2\,B\,a\right)}{6\,a^{3/2}} - \frac{A\,\sqrt{b\,x^3+a}}{3\,a\,x^3}$$

```
[In] int((A + B*x^3)/(x^4*(a + b*x^3)^(1/2)),x)
```

```
[Out] (log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/2))^3)/x^6)*(
A*b - 2*B*a))/(6*a^(3/2)) - (A*(a + b*x^3)^(1/2))/(3*a*x^3)
```

3.218 $\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx=-\frac{A\sqrt{a+bx^3}}{6ax^6}+\frac{(3Ab-4aB)\sqrt{a+bx^3}}{12a^2x^3}-\frac{b(3Ab-4aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{5/2}}$$

```
[Out] -1/12*b*(3*A*b-4*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(5/2)-1/6*A*(b*x^3
+a)^(1/2)/a/x^6+1/12*(3*A*b-4*B*a)*(b*x^3+a)^(1/2)/a^2/x^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 79, 44, 65, 214}

$$\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx=-\frac{b(3Ab-4aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{5/2}}+\frac{\sqrt{a+bx^3}(3Ab-4aB)}{12a^2x^3}-\frac{A\sqrt{a+bx^3}}{6ax^6}$$

```
[In] Int[(A + B*x^3)/(x^7*Sqrt[a + b*x^3]),x]
```

```
[Out] -1/6*(A*Sqrt[a + b*x^3])/(a*x^6) + ((3*A*b - 4*a*B)*Sqrt[a + b*x^3])/(12*a^
2*x^3) - (b*(3*A*b - 4*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a^(5/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
```

```
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^3\sqrt{a+bx}}\,dx,x,x^3\right)\\
&= -\frac{A\sqrt{a+bx^3}}{6ax^6}+\frac{\left(-\frac{3Ab}{2}+2aB\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,x,x^3\right)}{6a}\\
&= -\frac{A\sqrt{a+bx^3}}{6ax^6}+\frac{(3Ab-4aB)\sqrt{a+bx^3}}{12a^2x^3}+\frac{(b(3Ab-4aB))\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)}{24a^2}\\
&= -\frac{A\sqrt{a+bx^3}}{6ax^6}+\frac{(3Ab-4aB)\sqrt{a+bx^3}}{12a^2x^3}+\frac{(3Ab-4aB)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{12a^2}
\end{aligned}$$

$$= -\frac{A\sqrt{a+bx^3}}{6ax^6} + \frac{(3Ab-4aB)\sqrt{a+bx^3}}{12a^2x^3} - \frac{b(3Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{5/2}}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx = \frac{\sqrt{a+bx^3}(-2aA+3Abx^3-4aBx^3)}{12a^2x^6} + \frac{b(-3Ab+4aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{5/2}}$$

```
[In] Integrate[(A + B*x^3)/(x^7*Sqrt[a + b*x^3]),x]
[Out] (Sqrt[a + b*x^3]*(-2*a*A + 3*A*b*x^3 - 4*a*B*x^3))/(12*a^2*x^6) + (b*(-3*A*
b + 4*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a^(5/2))
```

Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

method	result	size
risch	$-\frac{\sqrt{b\,x^3+a}\,(-3Ab\,x^3+4Ba\,x^3+2Aa)}{12a^2x^6}-\frac{b(3Ab-4Ba)\,\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{12a^{\frac{5}{2}}}$	67
pseudoelliptic	$\frac{-x^6\left(Ab-\frac{4Ba}{3}\right)b\,\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)+\left(\frac{2\left(-2x^3B-A\right)a^{\frac{3}{2}}}{3}+A\sqrt{a}\,b\,x^3\right)\sqrt{b\,x^3+a}}{4a^{\frac{5}{2}}\,x^6}$	73
elliptic	$-\frac{b(3Ab-4Ba)\,\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{12a^{\frac{5}{2}}}-\frac{A\sqrt{b\,x^3+a}}{6a\,x^6}+\frac{(3Ab-4Ba)\sqrt{b\,x^3+a}}{12a^2x^3}$	75
default	$A\left(-\frac{b^2\,\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{4a^{\frac{5}{2}}}-\frac{\sqrt{b\,x^3+a}}{6a\,x^6}+\frac{b\sqrt{b\,x^3+a}}{4a^2x^3}\right)+B\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}-\frac{\sqrt{b\,x^3+a}}{3a\,x^3}\right)$	102

```
[In] int((B*x^3+A)/x^7/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/12*(b*x^3+a)^(1/2)*(-3*A*b*x^3+4*B*a*x^3+2*A*a)/a^2/x^6-1/12*b*(3*A*b-4*
B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.92

$$\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx = \left[-\frac{(4\,Bab-3\,Ab^2)\sqrt{a}x^6\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\,((4\,Ba^2-3\,Aab)x^3+2\,Aa^2)\sqrt{bx^3+a}}{24\,a^3x^6}, -\frac{(4\,Bab-3\,Ab^2)\sqrt{-a}x^6\arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right)+((4\,Ba^2-3\,Aab)x^3+2\,Aa^2)\sqrt{bx^3+a}}{12\,a^3x^6}\right]$$

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/24*((4*B*a*b - 3*A*b^2)*sqrt(a)*x^6*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt
(a) + 2*a)/x^3) + 2*((4*B*a^2 - 3*A*a*b)*x^3 + 2*A*a^2)*sqrt(b*x^3 + a))/(a
^3*x^6), -1/12*((4*B*a*b - 3*A*b^2)*sqrt(-a)*x^6*arctan(sqrt(b*x^3 + a)*sqr
t(-a)/a) + ((4*B*a^2 - 3*A*a*b)*x^3 + 2*A*a^2)*sqrt(b*x^3 + a))/(a^3*x^6)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(80) = 160$.

Time = 14.46 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.81

$$\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx = -\frac{A}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{A\sqrt{b}}{12ax^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{Ab^{\frac{3}{2}}}{4a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{Ab^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{4a^{\frac{5}{2}}}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3ax^{\frac{3}{2}}}+\frac{Bb\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3a^{\frac{3}{2}}}$$

```
[In] integrate((B*x**3+A)/x**7/(b*x**3+a)**(1/2),x)
```

```
[Out] -A/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) + A*sqrt(b)/(12*a*x**(9/2)*sq
rt(a/(b*x**3) + 1)) + A*b**(3/2)/(4*a**2*x**(3/2)*sqrt(a/(b*x**3) + 1)) - A
*b**2*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*a**(5/2)) - B*sqrt(b)*sqrt(a/(b*
x**3) + 1)/(3*a*x**(3/2)) + B*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*a**(3/
2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(74) = 148$.

Time = 0.26 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.98

$$\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx = -\frac{1}{6}\,B\left(\frac{2\,\sqrt{bx^3+a}b}{(bx^3+a)a-a^2}+\frac{b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right) + \frac{1}{24}\,A\left(\frac{3\,b^2\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(3\,(bx^3+a)^{\frac{3}{2}}b^2-5\,\sqrt{bx^3+a}ab^2\right)}{(bx^3+a)^2a^2-2\,(bx^3+a)a^3+a^4}\right)$$

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*B*(2*sqrt(b*x^3 + a)*b/((b*x^3 + a)*a - a^2) + b*log((sqrt(b*x^3 + a)
- sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(3/2)) + 1/24*A*(3*b^2*log((sqrt(
b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(5/2) + 2*(3*(b*x^3 +
a)^(3/2)*b^2 - 5*sqrt(b*x^3 + a)*a*b^2)/((b*x^3 + a)^2*a^2 - 2*(b*x^3 + a)*
a^3 + a^4))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.34

$$\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx = -\frac{\frac{(4\,Bab^2-3\,Ab^3)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{4\,(bx^3+a)^{\frac{3}{2}}Bab^2-4\,\sqrt{bx^3+a}Ba^2b^2-3\,(bx^3+a)^{\frac{3}{2}}Ab^3+5\,\sqrt{bx^3+a}Aab^3}{a^2b^2x^6}}{12\,b}$$

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/12*((4*B*a*b^2 - 3*A*b^3)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^2
) + (4*(b*x^3 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^3 + a)*B*a^2*b^2 - 3*(b*x^3 +
 a)^(3/2)*A*b^3 + 5*sqrt(b*x^3 + a)*A*a*b^3)/(a^2*b^2*x^6))/b
```

Mupad [B] (verification not implemented)

Time = 7.39 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx=\frac{\sqrt{b\,x^3+a}\,\left(3\,A\,b-4\,B\,a\right)}{12\,a^2\,x^3}-\frac{A\,\sqrt{b\,x^3+a}}{6\,a\,x^6}+\frac{b\,\ln\left(\frac{{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)\,\left(3\,A\,b-4\,B\,a\right)}{24\,a^{5/2}}$$

```
[In] int((A + B*x^3)/(x^7*(a + b*x^3)^(1/2)),x)
```

```
[Out] ((a + b*x^3)^(1/2)*(3*A*b - 4*B*a))/(12*a^2*x^3) - (A*(a + b*x^3)^(1/2))/(6
*a*x^6) + (b*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1
/2)))/x^6)*(3*A*b - 4*B*a))/(24*a^(5/2))
```

3.219 $\int \frac{x^3(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 270

$$\int \frac{x^3(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2(11Ab-8aB)x\sqrt{a+bx^3}}{55b^2}+\frac{2Bx^4\sqrt{a+bx^3}}{11b} - \frac{4\sqrt{2+\sqrt{3}}a(11Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-}{55\sqrt[4]{3}b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/55*(11*A*b-8*B*a)*x*(b*x^3+a)^(1/2)/b^2+2/11*B*x^4*(b*x^3+a)^(1/2)/b-4/16
5*a*(11*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1
/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/
2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)^2)^(1/2)*3^(3/4)/b^(7/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used

$= \{470, 327, 224\}$

$$\int \frac{x^3(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = -\frac{4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-8aB)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right), -\ldots}{55\sqrt[4]{3}b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{2x\sqrt{a+bx^3}(11Ab-8aB)}{55b^2} + \frac{2Bx^4\sqrt{a+bx^3}}{11b}$$

```
[In] Int[(x^3*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*(11*A*b - 8*a*B)*x*Sqrt[a + b*x^3])/(55*b^2) + (2*B*x^4*Sqrt[a + b*x^3])
/(11*b) - (4*Sqrt[2 + Sqrt[3]]*a*(11*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(55*3^(1/4)*b^(7/3)*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*
x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
```

```
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{2Bx^4\sqrt{a+bx^3}}{11b} - \frac{\left(2\left(-\frac{11Ab}{2}+4aB\right)\right)\int \frac{x^3}{\sqrt{a+bx^3}}\,dx}{11b}$$
$$= \frac{2(11Ab-8aB)x\sqrt{a+bx^3}}{55b^2} + \frac{2Bx^4\sqrt{a+bx^3}}{11b} - \frac{(2a(11Ab-8aB))\int \frac{1}{\sqrt{a+bx^3}}\,dx}{55b^2}$$
$$= \frac{2(11Ab-8aB)x\sqrt{a+bx^3}}{55b^2} + \frac{2Bx^4\sqrt{a+bx^3}}{11b} - \frac{4\sqrt{2+\sqrt{3}}a(11Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7}{55\sqrt[4]{3}b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.33

$$\int \frac{x^3(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$$
$$= \frac{2x\left(-\left((a+bx^3)\left(-11Ab+8aB-5bBx^3\right)\right)+a(-11Ab+8aB)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx}{a}\right.}{55b^2\sqrt{a+bx^3}}$$

```
[In] Integrate[(x^3*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*x*(-((a + b*x^3)*(-11*A*b + 8*a*B - 5*b*B*x^3)) + a*(-11*A*b + 8*a*B)*Sq
rt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]))/(55*b^2*
Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.20

method	result
risch	$\frac{2x(5bB\,x^3+11Ab-8Ba)\sqrt{b\,x^3+a}}{55b^2}+\frac{4i(11Ab-8Ba)a\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\cdots}{2b}}}\cdots}{\cdots}$
elliptic	$\frac{2B\,x^4\sqrt{b\,x^3+a}}{11b}+\frac{2\left(A-\frac{8aB}{11b}\right)x\sqrt{b\,x^3+a}}{5b}+\frac{4ia\left(A-\frac{8aB}{11b}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b\cdots}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\cdots}}\cdots}{\cdots}$
default	$B\left(\frac{2x^4\sqrt{b\,x^3+a}}{11b}-\frac{16ax\sqrt{b\,x^3+a}}{55b^2}-\frac{32ia^2\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\cdots}{\cdots}\cdots\right.$

```
[In] int(x^3*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/55*x*(5*B*b*x^3+11*A*b-8*B*a)*(b*x^3+a)^(1/2)/b^2+4/165*I*(11*A*b-8*B*a)*
a/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b
*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(
1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.25

$$\int \frac{x^3(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\left(2\,(8\,Ba^2-11\,Aab)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+(5\,Bb^2x^4-(8\,Bab-11\,Ab^2)x)\sqrt{bx^3+a}\right)}{55\,b^3}$$

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/55*(2*(8*B*a^2 - 11*A*a*b)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + (5
*B*b^2*x^4 - (8*B*a*b - 11*A*b^2)*x)*sqrt(b*x^3 + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.30

$$\int \frac{x^3(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{Ax^4\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{7}{3}\right)} + \frac{Bx^7\Gamma\left(\frac{7}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{10}{3}\right)}$$

```
[In] integrate(x**3*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*sq
rt(a)*gamma(7/3)) + B*x**7*gamma(7/3)*hyper((1/2, 7/3), (10/3,), b*x**3*exp
_polar(I*pi)/a)/(3*sqrt(a)*gamma(10/3))
```

Maxima [F]

$$\int \frac{x^3(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \int \frac{(Bx^3+A)x^3}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^3/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^3(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \int \frac{(Bx^3+A)x^3}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*x^3/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \int \frac{x^3\,(B\,x^3+A)}{\sqrt{b\,x^3+a}}\,dx$$

```
[In] int((x^3*(A + B*x^3))/(a + b*x^3)^(1/2),x)
[Out] int((x^3*(A + B*x^3))/(a + b*x^3)^(1/2), x)
```

3.220 $\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 239

$$\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx = \frac{2Bx\sqrt{a+bx^3}}{5b} + \frac{2\sqrt{2+\sqrt{3}}(5Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7 -}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/5*B*x*(b*x^3+a)^(1/2)/b+2/15*(5*A*b-2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF(
(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2
*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(4/3)/(b*x^3+a)^(1/2)/(a^(1/
3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used

$= \{396,\ 224\}$

$$\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-2aB)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-\right.}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2Bx\sqrt{a+bx^3}}{5b}$$

```
[In] Int[(A + B*x^3)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*B*x*Sqrt[a + b*x^3])/(5*b) + (2*Sqrt[2 + Sqrt[3]]*(5*A*b - 2*a*B)*(a^(1/
3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*b^
(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{2Bx\sqrt{a+bx^3}}{5b} - \frac{\left(2\left(-\frac{5Ab}{2}+aB\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{5b}$$

$$= \frac{2Bx\sqrt{a+bx^3}}{5b} + \frac{2\sqrt{2+\sqrt{3}}(5Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.31

$$\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx = \frac{2Bx(a+bx^3)+(5Ab-2aB)x\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)}{5b\sqrt{a+bx^3}}$$

```
[In] Integrate[(A + B*x^3)/Sqrt[a + b*x^3],x]
[Out] (2*B*x*(a + b*x^3) + (5*A*b - 2*a*B)*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F
1[1/3, 1/2, 4/3, -((b*x^3)/a)])/(5*b*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.29

method	result
risch	$\frac{2Bx\sqrt{b\,x^{3}+a}}{5b}-\frac{2i(5Ab-2Ba)\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^{2})^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}\right.}{}}}{15b^{2}\sqrt{b\,x}}$
elliptic	$\frac{2Bx\sqrt{b\,x^{3}+a}}{5b}-\frac{2i\left(A-\frac{2aB}{5b}\right)\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^{2})^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a\,b^{2})}{2b}\right.}{}}}{3b\sqrt{b\,x^{3}}}$
default	$-\frac{2iA\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^{2})^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}}{3b\sqrt{b\,x^{3}+a}}$

```
[In] int((B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*B*x*(b*x^3+a)^(1/2)/b-2/15*I*(5*A*b-2*B*a)/b^2*3^(1/2)*(-a*b^2)^(1/3)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.18

$$\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx = \frac{2\left(\sqrt{bx^3+a}Bbx-(2\,Ba-5\,Ab)\sqrt{b}\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)}{5\,b^2}$$

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(sqrt(b*x^3 + a)*B*b*x - (2*B*a - 5*A*b)*sqrt(b)*weierstrassPInverse(0,
 -4*a/b, x))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.33

$$\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx = \frac{Ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{Bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{7}{3}\right)}$$

```
[In] integrate((B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*gamma(4/3)) + B*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*sqrt(a)*gamma(7/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx = \int \frac{Bx^3+A}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx = \int \frac{Bx^3+A}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx = \int \frac{B\,x^3+A}{\sqrt{b\,x^3+a}}\,dx$$

```
[In] int((A + B*x^3)/(a + b*x^3)^(1/2),x)
[Out] int((A + B*x^3)/(a + b*x^3)^(1/2), x)
```

3.221 $\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 243

$$\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx=-\frac{A\sqrt{a+bx^3}}{2ax^2}-\frac{\sqrt{2+\sqrt{3}}(Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\ldots\right)}{2\sqrt[4]{3}a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/2*A*(b*x^3+a)^(1/2)/a/x^2-1/6*(A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF(
(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2
*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a/b^(1/3)/(b*x^3+a)^(1/2)/(a^(
1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used

= {464, 224}

$$\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx = -\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-4aB)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-\right.}{2\sqrt[4]{3}a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{A\sqrt{a+bx^3}}{2ax^2}$$

```
[In] Int[(A + B*x^3)/(x^3*Sqrt[a + b*x^3]),x]
```

```
[Out] -1/2*(A*Sqrt[a + b*x^3])/(a*x^2) - (Sqrt[2 + Sqrt[3]]*(A*b - 4*a*B)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*a*b
^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{A\sqrt{a+bx^3}}{2ax^2} - \frac{\left(\frac{Ab}{2}-2aB\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{2a}$$

$$= -\frac{A\sqrt{a+bx^3}}{2ax^2} - \frac{\sqrt{2+\sqrt{3}}(Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.32

$$\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx = \frac{-2A(a+bx^3)+(-Ab+4aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)}{4ax^2\sqrt{a+bx^3}}$$

```
[In] Integrate[(A + B*x^3)/(x^3*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*A*(a + b*x^3) + (-(A*b) + 4*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric
2F1[1/3, 1/2, 4/3, -((b*x^3)/a)])/(4*a*x^2*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 4.32 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.28

method	result
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{2a\,x^{2}}-\frac{2i\left(B-\frac{Ab}{4a}\right)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\cdots\right.}{\cdots}}\cdots}{3b\sqrt{b\,x^{3}+\cdots}}$
risch	$-\frac{A\sqrt{b\,x^{3}+a}}{2a\,x^{2}}+\frac{i(Ab-4Ba)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\cdots\right.}{\cdots}}\cdots}{6ab\sqrt{b\,x^{3}\cdots}}$
default	$-\frac{2iB\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\cdots}{3b\sqrt{b\,x^{3}+a}}$

```
[In] int((B*x^3+A)/x^3/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*A*(b*x^3+a)^(1/2)/a/x^2-2/3*I*(B-1/4*A/a*b)*3^(1/2)/b*(-a*b^2)^(1/3)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.21

$$\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx = \frac{(4\,Ba-Ab)\sqrt{b}x^2\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-\sqrt{bx^3+a}Ab}{2\,abx^2}$$

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((4*B*a - A*b)*sqrt(b)*x^2*weierstrassPInverse(0, -4*a/b, x) - sqrt(b*x
^3 + a)*A*b)/(a*b*x^2)
```

Sympy [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.34

$$\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx = \frac{A\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{1}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x^2\Gamma\left(\frac{1}{3}\right)}+\frac{Bx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((B*x**3+A)/x**3/(b*x**3+a)**(1/2),x)
```

```
[Out] A*gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*x**2*gamma(1/3)) + B*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_p
olar(I*pi)/a)/(3*sqrt(a)*gamma(4/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx = \int \frac{Bx^3+A}{\sqrt{bx^3+a}x^3}\,dx$$

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^3), x)
```

Giac [F]

$$\int \frac{A + Bx^3}{x^3\sqrt{a + bx^3}}\,dx = \int \frac{Bx^3 + A}{\sqrt{bx^3 + a}x^3}\,dx$$

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^3}{x^3\sqrt{a + bx^3}}\,dx = \int \frac{B\,x^3 + A}{x^3\,\sqrt{b\,x^3 + a}}\,dx$$

```
[In] int((A + B*x^3)/(x^3*(a + b*x^3)^(1/2)),x)
[Out] int((A + B*x^3)/(x^3*(a + b*x^3)^(1/2)), x)
```

3.222 $\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 274

$$\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx = -\frac{A\sqrt{a+bx^3}}{5ax^5}+\frac{(7Ab-10aB)\sqrt{a+bx^3}}{20a^2x^2}$$
$$+\frac{\sqrt{2+\sqrt{3}}\,b^{2/3}(7Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-\right.}{20\sqrt[4]{3}a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/5*A*(b*x^3+a)^(1/2)/a/x^5+1/20*(7*A*b-10*B*a)*(b*x^3+a)^(1/2)/a^2/x^2+1/
60*b^(2/3)*(7*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*
(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/
2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3
^(1/2)))^2)^(1/2)*3^(3/4)/a^2/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/
(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used

$= \{464, 331, 224\}$

$$\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx = \frac{\sqrt{a+bx^3}(7Ab-10aB)}{20a^2x^2} + \frac{\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(7Ab-10aB)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),\right.}{20\sqrt[4]{3}a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{A\sqrt{a+bx^3}}{5ax^5}$$

```
[In] Int[(A + B*x^3)/(x^6*Sqrt[a + b*x^3]),x]
```

```
[Out] -1/5*(A*Sqrt[a + b*x^3])/(a*x^5) + ((7*A*b - 10*a*B)*Sqrt[a + b*x^3])/(20*a
^2*x^2) + (Sqrt[2 + Sqrt[3]]*b^(2/3)*(7*A*b - 10*a*B)*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(20*3^(1/4)*a^2*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*
x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
```

```
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{A\sqrt{a+bx^3}}{5ax^5} - \frac{\left(\frac{7Ab}{2}-5aB\right)\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx}{5a}$$

$$= -\frac{A\sqrt{a+bx^3}}{5ax^5} + \frac{(7Ab-10aB)\sqrt{a+bx^3}}{20a^2x^2} + \frac{(b(7Ab-10aB))\int \frac{1}{\sqrt{a+bx^3}}\,dx}{40a^2}$$

$$= -\frac{A\sqrt{a+bx^3}}{5ax^5} + \frac{(7Ab-10aB)\sqrt{a+bx^3}}{20a^2x^2} + \frac{\sqrt{2+\sqrt{3}}\,b^{2/3}(7Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-}{20\sqrt[4]{3}a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.28

$$\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx = \frac{-4A(a+bx^3)+(7Ab-10aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a}\right)}{20ax^5\sqrt{a+bx^3}}$$

```
[In] Integrate[(A + B*x^3)/(x^6*Sqrt[a + b*x^3]),x]
```

```
[Out] (-4*A*(a + b*x^3) + (7*A*b - 10*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric
2F1[-2/3, 1/2, 1/3, -((b*x^3)/a)])/(20*a*x^5*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 4.39 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.20

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-7A b\,x^{3}+10B a\,x^{3}+4A a\right)}{20a^{2}x^{5}}-\frac{i(7Ab-10Ba)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(\ldots}{\ldots}}}\ldots}{\ldots}$
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{5a\,x^{5}}+\frac{(7Ab-10Ba)\sqrt{b\,x^{3}+a}}{20a^{2}x^{2}}-\frac{i(7Ab-10Ba)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)\ldots}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\ldots}{\ldots}}}\ldots}{\ldots}$
default	$B\left(-\frac{\sqrt{b\,x^{3}+a}}{2a\,x^{2}}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\ldots}{\ldots}\right)\ldots}{\left(-a\,b^{2}\right)\ldots}}\ldots}{6a\sqrt{b\,x^{3}+a}\ldots}\right.$

```
[In] int((B*x^3+A)/x^6/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*(b*x^3+a)^(1/2)*(-7*A*b*x^3+10*B*a*x^3+4*A*a)/a^2/x^5-1/60*I*(7*A*b-1
0*B*a)/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b
^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.23

$$\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx = -\frac{(10\,Ba-7\,Ab)\sqrt{b}x^5\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+((10\,Ba-7\,Ab)x^3+4\,Aa)\sqrt{bx^3+a}}{20\,a^2x^5}$$

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/20*((10*B*a - 7*A*b)*sqrt(b)*x^5*weierstrassPInverse(0, -4*a/b, x) + ((1
0*B*a - 7*A*b)*x^3 + 4*A*a)*sqrt(b*x^3 + a))/(a^2*x^5)
```

Sympy [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.33

$$\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx = \frac{A\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x^5\Gamma\left(-\frac{2}{3}\right)} + \frac{B\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x^2\Gamma\left(\frac{1}{3}\right)}$$

```
[In] integrate((B*x**3+A)/x**6/(b*x**3+a)**(1/2),x)
```

```
[Out] A*gamma(-5/3)*hyper((-5/3, 1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*x**5*gamma(-2/3)) + B*gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), b*x**3*exp
_polar(I*pi)/a)/(3*sqrt(a)*x**2*gamma(1/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx = \int \frac{Bx^3+A}{\sqrt{bx^3+a}x^6}\,dx$$

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^6), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx = \int \frac{Bx^3+A}{\sqrt{bx^3+ax^6}}\,dx$$

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx = \int \frac{B\,x^3+A}{x^6\,\sqrt{b\,x^3+a}}\,dx$$

```
[In] int((A + B*x^3)/(x^6*(a + b*x^3)^(1/2)),x)
[Out] int((A + B*x^3)/(x^6*(a + b*x^3)^(1/2)), x)
```

3.223 $\int \frac{x^4(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 548

$$\int \frac{x^4(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2(13Ab-10aB)x^2\sqrt{a+bx^3}}{91b^2}+\frac{2Bx^5\sqrt{a+bx^3}}{13b}-\frac{8a(13Ab-10aB)\sqrt{a+bx^3}}{91b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}(13Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7}{91b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{8\sqrt{2}a^{4/3}(13Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-}{91\sqrt[4]{3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/91*(13*A*b-10*B*a)*x^2*(b*x^3+a)^(1/2)/b^2+2/13*B*x^5*(b*x^3+a)^(1/2)/b-8
/91*a*(13*A*b-10*B*a)*(b*x^3+a)^(1/2)/b^(8/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
))-8/273*a^(4/3)*(13*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a
^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*
((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)
^(1/2)*3^(3/4)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+4/91*3^(1/4)*a^(4/3)*(13*A*b-10*B*a)*(a^(
1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3
```

```
)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(8/3)/(b*x
^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)
^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {470, 327, 309, 224, 1891}

$$\int \frac{x^4(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = -\frac{8\sqrt{2}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(13Ab-10aB)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7\right.}{91\sqrt[4]{3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(13Ab-10aB)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right|-}{91b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{8a\sqrt{a+bx^3}(13Ab-10aB)}{91b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{2x^2\sqrt{a+bx^3}(13Ab-10aB)}{91b^2} + \frac{2Bx^5\sqrt{a+bx^3}}{13b}$$

```
[In] Int[(x^4*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*(13*A*b - 10*a*B)*x^2*Sqrt[a + b*x^3])/(91*b^2) + (2*B*x^5*Sqrt[a + b*x^
3])/(13*b) - (8*a*(13*A*b - 10*a*B)*Sqrt[a + b*x^3])/(91*b^(8/3)*((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13*A*b -
 10*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])
*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]
)/(91*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (8*Sqrt[2]*a^(4/3)*(13*A*b - 10*a*B)*(a^(
1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*3^(1/4)
*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{2Bx^5\sqrt{a+bx^3}}{13b} - \frac{\left(2\left(-\frac{13Ab}{2}+5aB\right)\right)\int\frac{x^4}{\sqrt{a+bx^3}}\,dx}{13b}$$

$$=\frac{2(13Ab-10aB)x^2\sqrt{a+bx^3}}{91b^2}+\frac{2Bx^5\sqrt{a+bx^3}}{13b}-\frac{(4a(13Ab-10aB))\int\frac{x}{\sqrt{a+bx^3}}dx}{91b^2}$$

$$=\frac{2(13Ab-10aB)x^2\sqrt{a+bx^3}}{91b^2}+\frac{2Bx^5\sqrt{a+bx^3}}{13b}-\frac{(4a(13Ab-10aB))\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}dx}{91b^{7/3}}+\frac{\left(4\left(1-\sqrt{3}\right)a^{4/3}(13Ab-10aB)\right)\int\frac{1}{\sqrt{a+bx^3}}dx}{91b^{7/3}}$$

$$=\frac{2(13Ab-10aB)x^2\sqrt{a+bx^3}}{91b^2}+\frac{2Bx^5\sqrt{a+bx^3}}{13b}-\frac{8a(13Ab-10aB)\sqrt{a+bx^3}}{91b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}(13Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}}\right.\right.}{91b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{8\sqrt{2}a^{4/3}(13Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\right.}{91\sqrt[4]{3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.17

$$\int\frac{x^4(A+Bx^3)}{\sqrt{a+bx^3}}dx=\frac{2x^2\left(-\left((a+bx^3)\left(-13Ab+10aB-7bBx^3\right)\right)+a(-13Ab+10aB)\sqrt{1+\frac{bx^3}{a}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3}\right.\right.}{91b^2\sqrt{a+bx^3}}$$

```
[In] Integrate[(x^4*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*x^2*(-((a + b*x^3)*(-13*A*b + 10*a*B - 7*b*B*x^3)) + a*(-13*A*b + 10*a*B
)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)]))/(91*
b^2*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 4.26 (sec) , antiderivative size = 479, normalized size of antiderivative = 0.87

method	result
risch	$\frac{2x^{2}\left(7bB\,x^{3}+13Ab-10Ba\right)\sqrt{b\,x^{3}+a}}{91b^{2}}+\frac{8i(13Ab-10Ba)a\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-\right.}{}}}}{}$
elliptic	$\frac{2B\,x^{5}\sqrt{b\,x^{3}+a}}{13b}+\frac{2\left(A-\frac{10aB}{13b}\right)x^{2}\sqrt{b\,x^{3}+a}}{7b}+\frac{8ia\left(A-\frac{10aB}{13b}\right)\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\right.}{}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+}}}{}$
default	Expression too large to display

```
[In] int(x^4*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/91*x^2*(7*B*b*x^3+13*A*b-10*B*a)/b^2*(b*x^3+a)^(1/2)+8/273*I*(13*A*b-10*B
*a)*a/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*E
llipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-
```

```
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.14

$$\int \frac{x^4(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = -\frac{2\left(4\,(10\,Ba^2-13\,Aab)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-(7\,Bb^2x^5-(10\,B$$
$$91\,b^3$$

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/91*(4*(10*B*a^2 - 13*A*a*b)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstra
ssPInverse(0, -4*a/b, x)) - (7*B*b^2*x^5 - (10*B*a*b - 13*A*b^2)*x^2)*sqrt(
b*x^3 + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.15

$$\int \frac{x^4(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{Ax^5\Gamma\left(\frac{5}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{3}\\\frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{8}{3}\right)}+\frac{Bx^8\Gamma\left(\frac{8}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{8}{3}\\\frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{11}{3}\right)}$$

```
[In] integrate(x**4*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*x**5*gamma(5/3)*hyper((1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*sq
rt(a)*gamma(8/3)) + B*x**8*gamma(8/3)*hyper((1/2, 8/3), (11/3,), b*x**3*exp
_polar(I*pi)/a)/(3*sqrt(a)*gamma(11/3))
```

Maxima [F]

$$\int \frac{x^4(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\int \frac{(Bx^3+A)x^4}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*x^4/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^4(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\int \frac{(Bx^3+A)x^4}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*x^4/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\int \frac{x^4\,(B\,x^3+A)}{\sqrt{b\,x^3+a}}\,dx$$

```
[In] int((x^4*(A + B*x^3))/(a + b*x^3)^(1/2),x)
[Out] int((x^4*(A + B*x^3))/(a + b*x^3)^(1/2), x)
```

3.224 $\int \frac{x(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 517

$$\int \frac{x(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2Bx^2\sqrt{a+bx^3}}{7b} + \frac{2(7Ab-4aB)\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}(7Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\ldots}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2\sqrt{2}\sqrt[3]{a}(7Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-\ldots}{7\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/7*B*x^2*(b*x^3+a)^(1/2)/b+2/7*(7*A*b-4*B*a)*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))+2/21*a^(1/3)*(7*A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*E
llipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*
3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/
3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/7*3^(1/4)*a^(1/3)*
(7*A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*(
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {470, 309, 224, 1891}

$$\int \frac{x(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(7Ab-4aB)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right.}{7\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(7Ab-4aB)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\right.}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2\sqrt{a+bx^3}(7Ab-4aB)}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2Bx^2\sqrt{a+bx^3}}{7b}$$

```
[In] Int[(x*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*B*x^2*Sqrt[a + b*x^3])/(7*b) + (2*(7*A*b - 4*a*B)*Sqrt[a + b*x^3])/(7*b^
(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(
1/3)*(7*A*b - 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((
1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(7*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2]*a^(1/3)*(7*A*b - 4*
a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(
1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7
*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2Bx^2\sqrt{a+bx^3}}{7b} - \frac{\left(2\left(-\frac{7Ab}{2}+2aB\right)\right)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{7b} \\
&= \frac{2Bx^2\sqrt{a+bx^3}}{7b} + \frac{(7Ab-4aB)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{7b^{4/3}} \\
&\quad - \frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}(7Ab-4aB)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{7b^{4/3}}
\end{aligned}$$

$$= \frac{2Bx^2\sqrt{a+bx^3}}{7b} + \frac{2(7Ab-4aB)\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}(7Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|\right.}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{2\sqrt{2}\sqrt[3]{a}(7Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{\right.}{7\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.15

$$\int \frac{x(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$$
$$= \frac{x^2\left(4B(a+bx^3)+(7Ab-4aB)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{14b\sqrt{a+bx^3}}$$

```
[In] Integrate[(x*(A + B*x^3))/Sqrt[a + b*x^3],x]
[Out] (x^2*(4*B*(a + b*x^3) + (7*A*b - 4*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2
F1[1/2, 2/3, 5/3, -((b*x^3)/a)]))/(14*b*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 4.35 (sec) , antiderivative size = 463, normalized size of antiderivative = 0.90

method	result
risch	$\frac{2B\,x^{2}\sqrt{b\,x^{3}+a}}{7b}-\frac{2i(7Ab-4Ba)\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^{2})^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a}{2}\right.}{}}}{}$
elliptic	$\frac{2B\,x^{2}\sqrt{b\,x^{3}+a}}{7b}-\frac{2i\left(A-\frac{4aB}{7b}\right)\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^{2})^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a\,b^{2}}{2b}\right.}{}}}{}$
default	Expression too large to display

```
[In] int(x*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*B*x^2*(b*x^3+a)^(1/2)/b-2/21*I*(7*A*b-4*B*a)/b^2*3^(1/2)*(-a*b^2)^(1/3)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
```

```
^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.10

$$\int \frac{x(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2\left(\sqrt{bx^3+a}Bbx^2+(4\,Ba-7\,Ab)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)\right)}{7\,b^2}$$

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] 2/7*(sqrt(b*x^3 + a)*B*b*x^2 + (4*B*a - 7*A*b)*sqrt(b)*weierstrassZeta(0, -
4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b^2
```

Sympy [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.15

$$\int \frac{x(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{Ax^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}+\frac{Bx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{8}{3}\right)}$$

```
[In] integrate(x*(B*x**3+A)/(b*x**3+a)**(1/2),x)
[Out] A*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*sq
rt(a)*gamma(5/3)) + B*x**5*gamma(5/3)*hyper((1/2, 5/3), (8/3,), b*x**3*exp_
polar(I*pi)/a)/(3*sqrt(a)*gamma(8/3))
```

Maxima [F]

$$\int \frac{x(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \int \frac{(Bx^3+A)x}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*x/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \int \frac{(Bx^3+A)x}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*x/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \int \frac{x\,(B\,x^3+A)}{\sqrt{b\,x^3+a}}\,dx$$

```
[In] int((x*(A + B*x^3))/(a + b*x^3)^(1/2),x)
[Out] int((x*(A + B*x^3))/(a + b*x^3)^(1/2), x)
```

3.225 $\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 509

$$\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx=-\frac{A\sqrt{a+bx^3}}{ax}+\frac{(Ab+2aB)\sqrt{a+bx^3}}{ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{2a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{\sqrt{2}(Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -A*(b*x^3+a)^(1/2)/a/x+(A*b+2*B*a)*(b*x^3+a)^(1/2)/a/b^(2/3)/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2)))+1/3*(A*b+2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+
a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)
*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2
)^(1/2)*3^(3/4)/a^(2/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x
)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/2*3^(1/4)*(A*b+2*B*a)*(a^(1/3)
+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/
3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/b^(2/3)/
(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {464, 309, 224, 1891}

$$\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx$$
$$= \frac{\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2aB+Ab)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2aB+Ab)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{\sqrt{a+bx^3}(2aB+Ab)}{ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{A\sqrt{a+bx^3}}{ax}$$

```
[In] Int[(A + B*x^3)/(x^2*Sqrt[a + b*x^3]),x]
```

```
[Out] -((A*Sqrt[a + b*x^3])/(a*x)) + ((A*b + 2*a*B)*Sqrt[a + b*x^3])/(a*b^(2/3)*(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(A*b + 2*a
*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*
a^(2/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (Sqrt[2]*(A*b + 2*a*B)*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*a^(2/3)*b^(2/3
)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{A\sqrt{a+bx^3}}{ax} - \frac{\left(-\frac{Ab}{2}-aB\right)\int\frac{x}{\sqrt{a+bx^3}}\,dx}{a}$$

$$= -\frac{A\sqrt{a+bx^3}}{ax} + \frac{(Ab+2aB)\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{2a\sqrt[3]{b}} - \frac{\left(\left(1-\sqrt{3}\right)(Ab+2aB)\right)\int\frac{1}{\sqrt{a+bx^3}}\,dx}{2a^{2/3}\sqrt[3]{b}}$$

$$= -\frac{A\sqrt{a+bx^3}}{ax} + \frac{(Ab+2aB)\sqrt{a+bx^3}}{ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7}{2a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{\sqrt{2}(Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.15

$$\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx$$
$$= \frac{-4A(a+bx^3)+(Ab+2aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{4ax\sqrt{a+bx^3}}$$

```
[In] Integrate[(A + B*x^3)/(x^2*Sqrt[a + b*x^3]),x]
```

```
[Out] (-4*A*(a + b*x^3) + (A*b + 2*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1
[1/2, 2/3, 5/3, -((b*x^3)/a)])/(4*a*x*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 463, normalized size of antiderivative = 0.91

method	result
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{ax}-\frac{2i\left(B+\frac{Ab}{2a}\right)\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right.}{\left(-\right.}}}{}$
risch	$-\frac{A\sqrt{b\,x^3+a}}{ax}-\frac{i(Ab+2Ba)\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right.}{\left(-\right.}}}{}$
default	Expression too large to display

```
[In] int((B*x^3+A)/x^2/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -A*(b*x^3+a)^(1/2)/a/x-2/3*I*(B+1/2*A/a*b)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*
3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
```

```
)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.11

$$\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx = -\frac{(2\,Ba+Ab)\sqrt{b}x\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+\sqrt{bx^3+a}Ab}{abx}$$

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] -((2*B*a + A*b)*sqrt(b)*x*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0,
 -4*a/b, x)) + sqrt(b*x^3 + a)*A*b)/(a*b*x)
```

Sympy [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.16

$$\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx = \frac{A\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{2}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x\Gamma\left(\frac{2}{3}\right)}+\frac{Bx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

```
[In] integrate((B*x**3+A)/x**2/(b*x**3+a)**(1/2),x)
[Out] A*gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*x*gamma(2/3)) + B*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_p
olar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx = \int \frac{Bx^3+A}{\sqrt{bx^3+a}x^2}\,dx$$

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx = \int \frac{Bx^3+A}{\sqrt{bx^3+a}x^2}\,dx$$

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx = \int \frac{B\,x^3+A}{x^2\,\sqrt{b\,x^3+a}}\,dx$$

```
[In] int((A + B*x^3)/(x^2*(a + b*x^3)^(1/2)),x)
[Out] int((A + B*x^3)/(x^2*(a + b*x^3)^(1/2)), x)
```

3.226 $\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 550

$$\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx = -\frac{A\sqrt{a+bx^3}}{4ax^4}+\frac{(5Ab-8aB)\sqrt{a+bx^3}}{8a^2x}-\frac{\sqrt[3]{b}(5Ab-8aB)\sqrt{a+bx^3}}{8a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}(5Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\cdots\right.}{16a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{\sqrt[3]{b}(5Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\cdots\right.}{4\sqrt{2}\sqrt[4]{3}a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/4*A*(b*x^3+a)^(1/2)/a/x^4+1/8*(5*A*b-8*B*a)*(b*x^3+a)^(1/2)/a^2/x-1/8*b^
(1/3)*(5*A*b-8*B*a)*(b*x^3+a)^(1/2)/a^2/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-1/2
4*b^(1/3)*(5*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1
-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)
*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^
(5/3)*2^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)+1/16*3^(1/4)*b^(1/3)*(5*A*b-8*B*a)*(a^(1/3)+b^(1/3
)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^
(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(5/3)/(b*x^3+a)^(1/2)
/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {464, 331, 309, 224, 1891}

$$\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx = -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-8aB)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-8aB)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{16a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{\sqrt{a+bx^3}(5Ab-8aB)}{8a^2x} - \frac{\sqrt[3]{b}\sqrt{a+bx^3}(5Ab-8aB)}{8a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{A\sqrt{a+bx^3}}{4ax^4}$$

```
[In] Int[(A + B*x^3)/(x^5*Sqrt[a + b*x^3]),x]
```

```
[Out] -1/4*(A*Sqrt[a + b*x^3])/(a*x^4) + ((5*A*b - 8*a*B)*Sqrt[a + b*x^3])/(8*a^2
*x) - (b^(1/3)*(5*A*b - 8*a*B)*Sqrt[a + b*x^3])/(8*a^2*((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(5*A*b - 8*a*B)*(a^(1
/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*a^(5/3)*
Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*Sqrt[a + b*x^3]) - (b^(1/3)*(5*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*3^(1/4)*a^(5/3)*Sqrt[(a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
```

```
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{A\sqrt{a+bx^3}}{4ax^4} - \frac{\left(\frac{5Ab}{2}-4aB\right)\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx}{4a}\\ &= -\frac{A\sqrt{a+bx^3}}{4ax^4} + \frac{(5Ab-8aB)\sqrt{a+bx^3}}{8a^2x} - \frac{(b(5Ab-8aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{16a^2}\end{aligned}$$

$$= -\frac{A\sqrt{a+bx^3}}{4ax^4} + \frac{(5Ab-8aB)\sqrt{a+bx^3}}{8a^2x} - \frac{\left(b^{2/3}(5Ab-8aB)\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{16a^2} + \frac{\left(\left(1-\sqrt{3}\right)b^{2/3}(5Ab-8aB)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{16a^{5/3}}$$

$$= -\frac{A\sqrt{a+bx^3}}{4ax^4} + \frac{(5Ab-8aB)\sqrt{a+bx^3}}{8a^2x} - \frac{\sqrt[3]{b}(5Ab-8aB)\sqrt{a+bx^3}}{8a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}(5Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-\right.}{16a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{\sqrt[3]{b}(5Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.14

$$\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx = \frac{-2A(a+bx^3)+(5Ab-8aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a}\right)}{8ax^4\sqrt{a+bx^3}}$$

```
[In] Integrate[(A + B*x^3)/(x^5*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*A*(a + b*x^3) + (5*A*b - 8*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2
F1[-1/3, 1/2, 2/3, -((b*x^3)/a)])/(8*a*x^4*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 4.39 (sec) , antiderivative size = 481, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,(-5Ab\,x^3+8Ba\,x^3+2Aa)}{8a^2x^4}+\frac{i(5Ab-8Ba)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-}{2}}}}{\;}$
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{4a\,x^4}+\frac{(5Ab-8Ba)\sqrt{b\,x^3+a}}{8a^2x}+$
default	Expression too large to display

```
[In] int((B*x^3+A)/x^5/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(b*x^3+a)^(1/2)*(-5*A*b*x^3+8*B*a*x^3+2*A*a)/a^2/x^4+1/24*I*(5*A*b-8*B
*a)/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/
(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ell
ipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*
```

```
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.13

$$\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx = -\frac{(8\,Ba-5\,Ab)\sqrt{b}x^4\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+((8\,Ba-5\,Ab)x^3+2\,Aa)}{8\,a^2x^4}$$

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*((8*B*a - 5*A*b)*sqrt(b)*x^4*weierstrassZeta(0, -4*a/b, weierstrassPIn
verse(0, -4*a/b, x)) + ((8*B*a - 5*A*b)*x^3 + 2*A*a)*sqrt(b*x^3 + a))/(a^2*
x^4)
```

Sympy [A] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.16

$$\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx = \frac{A\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x^4\Gamma\left(-\frac{1}{3}\right)}+\frac{B\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{2}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x\Gamma\left(\frac{2}{3}\right)}$$

```
[In] integrate((B*x**3+A)/x**5/(b*x**3+a)**(1/2),x)
```

```
[Out] A*gamma(-4/3)*hyper((-4/3, 1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*x**4*gamma(-1/3)) + B*gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), b*x**3*exp
_polar(I*pi)/a)/(3*sqrt(a)*x*gamma(2/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx = \int \frac{Bx^3+A}{\sqrt{bx^3+ax^5}}\,dx$$

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^5), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx = \int \frac{Bx^3+A}{\sqrt{bx^3+ax^5}}\,dx$$

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx = \int \frac{B\,x^3+A}{x^5\,\sqrt{b\,x^3+a}}\,dx$$

```
[In] int((A + B*x^3)/(x^5*(a + b*x^3)^(1/2)),x)
[Out] int((A + B*x^3)/(x^5*(a + b*x^3)^(1/2)), x)
```

3.227 $\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 581

$$\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx = -\frac{A\sqrt{a+bx^3}}{7ax^7}+\frac{(11Ab-14aB)\sqrt{a+bx^3}}{56a^2x^4} - \frac{5b(11Ab-14aB)\sqrt{a+bx^3}}{112a^3x}+\frac{5b^{4/3}(11Ab-14aB)\sqrt{a+bx^3}}{112a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{5\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}(11Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7\right.}{224a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{5b^{4/3}(11Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\right.}{56\sqrt{2}\sqrt[4]{3}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/7*A*(b*x^3+a)^(1/2)/a/x^7+1/56*(11*A*b-14*B*a)*(b*x^3+a)^(1/2)/a^2/x^4-5
/112*b*(11*A*b-14*B*a)*(b*x^3+a)^(1/2)/a^3/x+5/112*b^(4/3)*(11*A*b-14*B*a)*
(b*x^3+a)^(1/2)/a^3/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+5/336*b^(4/3)*(11*A*b-1
4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/
3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3
)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(8/3)*2^(1/2)/(b*
x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2
)^(1/2)-5/224*3^(1/4)*b^(4/3)*(11*A*b-14*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE
```

```
((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+
2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(
1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {464, 331, 309, 224, 1891}

$$\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx = \frac{5b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-14aB)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{56\sqrt{2}\sqrt[4]{3}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{5\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-14aB)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{224a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{5b^{4/3}\sqrt{a+bx^3}(11Ab-14aB)}{112a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{5b\sqrt{a+bx^3}(11Ab-14aB)}{112a^3x} + \frac{\sqrt{a+bx^3}(11Ab-14aB)}{56a^2x^4} - \frac{A\sqrt{a+bx^3}}{7ax^7}$$

```
[In] Int[(A + B*x^3)/(x^8*Sqrt[a + b*x^3]),x]
```

```
[Out] -1/7*(A*Sqrt[a + b*x^3])/(a*x^7) + ((11*A*b - 14*a*B)*Sqrt[a + b*x^3])/(56*
a^2*x^4) - (5*b*(11*A*b - 14*a*B)*Sqrt[a + b*x^3])/(112*a^3*x) + (5*b^(4/3)
*(11*A*b - 14*a*B)*Sqrt[a + b*x^3])/(112*a^3*((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)) - (5*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*(11*A*b - 14*a*B)*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(224*a^(8/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3]) + (5*b^(4/3)*(11*A*b - 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(
```

```
1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(56*Sqrt[2]*3^(1/4)*a^(8/3)*Sqrt[(a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
```

```
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{A\sqrt{a+bx^3}}{7ax^7} - \frac{\left(\frac{11Ab}{2}-7aB\right)\int \frac{1}{x^5\sqrt{a+bx^3}}\,dx}{7a}$$

$$= -\frac{A\sqrt{a+bx^3}}{7ax^7} + \frac{(11Ab-14aB)\sqrt{a+bx^3}}{56a^2x^4} + \frac{(5b(11Ab-14aB))\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx}{112a^2}$$

$$= -\frac{A\sqrt{a+bx^3}}{7ax^7} + \frac{(11Ab-14aB)\sqrt{a+bx^3}}{56a^2x^4} - \frac{5b(11Ab-14aB)\sqrt{a+bx^3}}{112a^3x} + \frac{(5b^2(11Ab-14aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{224a^3}$$

$$= -\frac{A\sqrt{a+bx^3}}{7ax^7} + \frac{(11Ab-14aB)\sqrt{a+bx^3}}{56a^2x^4} - \frac{5b(11Ab-14aB)\sqrt{a+bx^3}}{112a^3x} + \frac{\left(5b^{5/3}(11Ab-14aB)\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{224a^3} - \frac{\left(5\left(1-\sqrt{3}\right)b^{5/3}(11Ab-14aB)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{224a^{8/3}}$$

$$= -\frac{A\sqrt{a+bx^3}}{7ax^7} + \frac{(11Ab-14aB)\sqrt{a+bx^3}}{56a^2x^4} - \frac{5b(11Ab-14aB)\sqrt{a+bx^3}}{112a^3x} + \frac{5b^{4/3}(11Ab-14aB)\sqrt{a+bx^3}}{112a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$- \frac{5\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{4/3}(11Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{\cdots}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{\cdots}}\right.\right.}{224a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{5b^{4/3}(11Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{\cdots}\right.}{56\sqrt{2}\sqrt[4]{3}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.13

$$\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx = \frac{-8A(a+bx^3)+(11Ab-14aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{2},-\frac{1}{3},-\frac{bx^3}{a}\right)}{56ax^7\sqrt{a+bx^3}}$$

```
[In] Integrate[(A + B*x^3)/(x^8*Sqrt[a + b*x^3]),x]
```

```
[Out] (-8*A*(a + b*x^3) + (11*A*b - 14*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometri
c2F1[-4/3, 1/2, -1/3, -((b*x^3)/a)])/(56*a*x^7*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 4.45 (sec) , antiderivative size = 504, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,(55A\,b^{2}x^{6}-70B\,x^{6}ab-22aAb\,x^{3}+28a^{2}B\,x^{3}+16a^{2}A)}{112a^{3}x^{7}}-\frac{5ib(11Ab-14Ba)\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\cdots\right.}{(-a\,b^{2})^{\frac{1}{3}}}}\cdots}{\cdots}$ … $5ib(11Ab-14Ba)\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(\cdots}{\cdots}\right.}{(-a\,b^{2})^{\frac{1}{3}}}}\cdots$
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{7a\,x^{7}}+\frac{(11Ab-14Ba)\sqrt{b\,x^{3}+a}}{56a^{2}x^{4}}-\frac{5b(11Ab-14Ba)\sqrt{b\,x^{3}+a}}{112a^{3}x}-\cdots$
default	Expression too large to display

```
[In] int((B*x^3+A)/x^8/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/112*(b*x^3+a)^(1/2)*(55*A*b^2*x^6-70*B*a*b*x^6-22*A*a*b*x^3+28*B*a^2*x^3
+16*A*a^2)/a^3/x^7-5/336*I*b*(11*A*b-14*B*a)/a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(
```

```
I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.17

$$\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx$$
$$=\frac{5\,(14\,Bab-11\,Ab^2)\sqrt{b}x^7\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+(5\,(14\,Bab-11\,Ab^2)x}{112\,a^3x^7}$$

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/112*(5*(14*B*a*b - 11*A*b^2)*sqrt(b)*x^7*weierstrassZeta(0, -4*a/b, weier
strassPInverse(0, -4*a/b, x)) + (5*(14*B*a*b - 11*A*b^2)*x^6 - 2*(14*B*a^2
- 11*A*a*b)*x^3 - 16*A*a^2)*sqrt(b*x^3 + a))/(a^3*x^7)
```

Sympy [A] (verification not implemented)

Time = 1.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.16

$$\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx=\frac{A\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x^7\Gamma\left(-\frac{4}{3}\right)}+\frac{B\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x^4\Gamma\left(-\frac{1}{3}\right)}$$

```
[In] integrate((B*x**3+A)/x**8/(b*x**3+a)**(1/2),x)
```

```
[Out] A*gamma(-7/3)*hyper((-7/3, 1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*x**7*gamma(-4/3)) + B*gamma(-4/3)*hyper((-4/3, 1/2), (-1/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*sqrt(a)*x**4*gamma(-1/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx=\int \frac{Bx^3+A}{\sqrt{bx^3+a}x^8}\,dx$$

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^8), x)
```

Giac [F]

$$\int \frac{A + Bx^3}{x^8\sqrt{a + bx^3}}\,dx = \int \frac{Bx^3 + A}{\sqrt{bx^3 + a}x^8}\,dx$$

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^3}{x^8\sqrt{a + bx^3}}\,dx = \int \frac{B\,x^3 + A}{x^8\,\sqrt{b\,x^3 + a}}\,dx$$

```
[In] int((A + B*x^3)/(x^8*(a + b*x^3)^(1/2)),x)
[Out] int((A + B*x^3)/(x^8*(a + b*x^3)^(1/2)), x)
```

3.228 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = -\frac{2a^2(Ab-aB)}{3b^4\sqrt{a+bx^3}} - \frac{2a(2Ab-3aB)\sqrt{a+bx^3}}{3b^4} + \frac{2(Ab-3aB)\left(a+bx^3\right)^{3/2}}{9b^4} + \frac{2B(a+bx^3)^{5/2}}{15b^4}$$

```
[Out] 2/9*(A*b-3*B*a)*(b*x^3+a)^(3/2)/b^4+2/15*B*(b*x^3+a)^(5/2)/b^4-2/3*a^2*(A*b
-B*a)/b^4/(b*x^3+a)^(1/2)-2/3*a*(2*A*b-3*B*a)*(b*x^3+a)^(1/2)/b^4
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 78}

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = -\frac{2a^2(Ab-aB)}{3b^4\sqrt{a+bx^3}} + \frac{2(a+bx^3)^{3/2}(Ab-3aB)}{9b^4} - \frac{2a\sqrt{a+bx^3}(2Ab-3aB)}{3b^4} + \frac{2B(a+bx^3)^{5/2}}{15b^4}$$

```
[In] Int[(x^8*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*a^2*(A*b - a*B))/(3*b^4*Sqrt[a + b*x^3]) - (2*a*(2*A*b - 3*a*B)*Sqrt[a
+ b*x^3])/(3*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(3/2))/(9*b^4) + (2*B*(a +
 b*x^3)^(5/2))/(15*b^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx)^{3/2}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{a^2(-Ab+aB)}{b^3(a+bx)^{3/2}}+\frac{a(-2Ab+3aB)}{b^3\sqrt{a+bx}}+\frac{(Ab-3aB)\sqrt{a+bx}}{b^3}+\frac{B(a+bx)^{3/2}}{b^3}\right)dx, x, x^3\right)\\ &= -\frac{2a^2(Ab-aB)}{3b^4\sqrt{a+bx^3}}-\frac{2a(2Ab-3aB)\sqrt{a+bx^3}}{3b^4}+\frac{2(Ab-3aB)\left(a+bx^3\right)^{3/2}}{9b^4}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.75

$$\int \frac{x^8(A+Bx^3)}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{2(48a^3B-8a^2b(5A-3Bx^3)+b^3x^6(5A+3Bx^3)-2ab^2x^3(10A+3Bx^3))}{45b^4\sqrt{a+bx^3}}$$

```
[In] Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^(3/2),x]
[Out] (2*(48*a^3*B - 8*a^2*b*(5*A - 3*B*x^3) + b^3*x^6*(5*A + 3*B*x^3) - 2*a*b^2*
x^3*(10*A + 3*B*x^3)))/(45*b^4*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 4.40 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$-\frac{16\left(-\frac{x^{6}\left(\frac{3x^{3}B}{5}+A\right)b^{3}}{8}+\frac{x^{3}\left(\frac{3x^{3}B}{10}+A\right)a\,b^{2}}{2}+a^{2}\left(-\frac{3x^{3}B}{5}+A\right)b-\frac{6a^{3}B}{5}\right)}{9\sqrt{b\,x^{3}+a}\,b^{4}}$
gosper	$-\frac{2\left(-3b^{3}B\,x^{9}-5x^{6}b^{3}A+6B\,x^{6}a\,b^{2}+20aA\,b^{2}x^{3}-24B\,a^{2}b\,x^{3}+40a^{2}bA-48a^{3}B\right)}{45\sqrt{b\,x^{3}+a}\,b^{4}}$
trager	$-\frac{2\left(-3b^{3}B\,x^{9}-5x^{6}b^{3}A+6B\,x^{6}a\,b^{2}+20aA\,b^{2}x^{3}-24B\,a^{2}b\,x^{3}+40a^{2}bA-48a^{3}B\right)}{45\sqrt{b\,x^{3}+a}\,b^{4}}$
risch	$-\frac{2\left(-3b^{2}B\,x^{6}-5A\,b^{2}x^{3}+9Bab\,x^{3}+25abA-33a^{2}B\right)\sqrt{b\,x^{3}+a}}{45b^{4}}-\frac{2a^{2}(Ab-Ba)}{3b^{4}\sqrt{b\,x^{3}+a}}$
default	$A\left(-\frac{2a^{2}}{3b^{3}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2x^{3}\sqrt{b\,x^{3}+a}}{9b^{2}}-\frac{10a\sqrt{b\,x^{3}+a}}{9b^{3}}\right)+B\left(\frac{2a^{3}}{3b^{4}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2x^{6}\sqrt{b\,x^{3}+a}}{15b^{2}}-\frac{2a\,x^{3}\sqrt{b\,x^{3}+a}}{5b^{3}}+\right.$
elliptic	$-\frac{2a^{2}(Ab-Ba)}{3b^{4}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2B\,x^{6}\sqrt{b\,x^{3}+a}}{15b^{2}}+\frac{2\left(\frac{Ab-Ba}{b^{2}}-\frac{4Ba}{5b^{2}}\right)x^{3}\sqrt{b\,x^{3}+a}}{9b}+\frac{2\left(-\frac{a(Ab-Ba)}{b^{3}}-\frac{2\left(\frac{Ab-Ba}{b^{2}}-\frac{4Ba}{5b^{2}}\right)a}{3b}\right)\sqrt{b\,x^{3}+a}}{3b}$

[In] int(x^8*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)

[Out] -16/9*(-1/8*x^6*(3/5*x^3*B+A)*b^3+1/2*x^3*(3/10*x^3*B+A)*a*b^2+a^2*(-3/5*x^3*B+A)*b-6/5*a^3*B)/(b*x^3+a)^(1/2)/b^4

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int \frac{x^{8}(A+Bx^{3})}{(a+bx^{3})^{3/2}}\,dx=\frac{2\left(3\,Bb^{3}x^{9}-(6\,Bab^{2}-5\,Ab^{3})x^{6}+48\,Ba^{3}-40\,Aa^{2}b+4\,(6\,Ba^{2}b-5\,Aab^{2})x^{3}\right)\sqrt{bx^{3}+}}{45\,(b^{5}x^{3}+ab^{4})}$$

[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] 2/45*(3*B*b^3*x^9 - (6*B*a*b^2 - 5*A*b^3)*x^6 + 48*B*a^3 - 40*A*a^2*b + 4*(6*B*a^2*b - 5*A*a*b^2)*x^3)*sqrt(b*x^3 + a)/(b^5*x^3 + a*b^4)

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.70

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \begin{cases} -\frac{16Aa^2}{9b^3\sqrt{a+bx^3}} - \frac{8Aax^3}{9b^2\sqrt{a+bx^3}} + \frac{2Ax^6}{9b\sqrt{a+bx^3}} + \frac{32Ba^3}{15b^4\sqrt{a+bx^3}} + \frac{16Ba^2x^3}{15b^3\sqrt{a+bx^3}} - \frac{4Bax^6}{15b^2\sqrt{a+bx^3}} + \frac{2Bx^9}{15b\sqrt{a+bx^3}} \\ \frac{\frac{Ax^9}{9}+\frac{Bx^{12}}{12}}{a^{\frac{3}{2}}} \end{cases}$$

```
[In] integrate(x**8*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] Piecewise((-16*A*a**2/(9*b**3*sqrt(a + b*x**3)) - 8*A*a*x**3/(9*b**2*sqrt(a
 + b*x**3)) + 2*A*x**6/(9*b*sqrt(a + b*x**3)) + 32*B*a**3/(15*b**4*sqrt(a +
 b*x**3)) + 16*B*a**2*x**3/(15*b**3*sqrt(a + b*x**3)) - 4*B*a*x**6/(15*b**2
*sqrt(a + b*x**3)) + 2*B*x**9/(15*b*sqrt(a + b*x**3)), Ne(b, 0)), ((A*x**9/
9 + B*x**12/12)/a**(3/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.13

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2}{15}\,B\left(\frac{(bx^3+a)^{\frac{5}{2}}}{b^4} - \frac{5\,(bx^3+a)^{\frac{3}{2}}a}{b^4} + \frac{15\,\sqrt{bx^3+a}a^2}{b^4} + \frac{5\,a^3}{\sqrt{bx^3+a}b^4}\right) + \frac{2}{9}\,A\left(\frac{(bx^3+a)^{\frac{3}{2}}}{b^3} - \frac{6\,\sqrt{bx^3+a}a}{b^3} - \frac{3\,a^2}{\sqrt{bx^3+a}b^3}\right)$$

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*B*((b*x^3 + a)^(5/2)/b^4 - 5*(b*x^3 + a)^(3/2)*a/b^4 + 15*sqrt(b*x^3 +
 a)*a^2/b^4 + 5*a^3/(sqrt(b*x^3 + a)*b^4)) + 2/9*A*((b*x^3 + a)^(3/2)/b^3 -
 6*sqrt(b*x^3 + a)*a/b^3 - 3*a^2/(sqrt(b*x^3 + a)*b^3))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.11

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2\,(Ba^3-Aa^2b)}{3\,\sqrt{bx^3+a}b^4} + \frac{2\left(3\,(bx^3+a)^{\frac{5}{2}}Bb^{16} - 15\,(bx^3+a)^{\frac{3}{2}}Bab^{16} + 45\,\sqrt{bx^3+a}Ba^2b^{16} + 5\,(bx^3+a)^{\frac{3}{2}}Ab^{17} - 30\,\sqrt{bx^3+a}Aab^{17}\right)}{45\,b^{20}}$$

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*(B*a^3 - A*a^2*b)/(sqrt(b*x^3 + a)*b^4) + 2/45*(3*(b*x^3 + a)^(5/2)*B*b
^16 - 15*(b*x^3 + a)^(3/2)*B*a*b^16 + 45*sqrt(b*x^3 + a)*B*a^2*b^16 + 5*(b*
x^3 + a)^(3/2)*A*b^17 - 30*sqrt(b*x^3 + a)*A*a*b^17)/b^20
```

Mupad [B] (verification not implemented)

Time = 7.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.48

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{\sqrt{b\,x^3+a}\,\left(\frac{2\,\left(B\,a^2-A\,a\,b\right)}{b^3}-\frac{2\,a\,\left(\frac{2\,\left(A\,b^2-B\,a\,b\right)}{b^3}-\frac{8\,B\,a}{5\,b^2}\right)}{3\,b}\right)}{3\,b} + \frac{x^3\,\sqrt{b\,x^3+a}\,\left(\frac{2\,\left(A\,b^2-B\,a\,b\right)}{b^3}-\frac{8\,B\,a}{5\,b^2}\right)}{9\,b} - \frac{a^2\,\left(\frac{2\,A}{3\,b}-\frac{2\,B\,a}{3\,b^2}\right)}{b^2\,\sqrt{b\,x^3+a}} + \frac{2\,B\,x^6\,\sqrt{b\,x^3+a}}{15\,b^2}$$

```
[In] int((x^8*(A + B*x^3))/(a + b*x^3)^(3/2),x)
```

```
[Out] ((a + b*x^3)^(1/2)*((2*(B*a^2 - A*a*b))/b^3 - (2*a*((2*(A*b^2 - B*a*b))/b^3
 - (8*B*a)/(5*b^2)))/(3*b)))/(3*b) + (x^3*(a + b*x^3)^(1/2)*((2*(A*b^2 - B*
a*b))/b^3 - (8*B*a)/(5*b^2)))/(9*b) - (a^2*((2*A)/(3*b) - (2*B*a)/(3*b^2)))
/(b^2*(a + b*x^3)^(1/2)) + (2*B*x^6*(a + b*x^3)^(1/2))/(15*b^2)
```

3.229 $\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2a(Ab-aB)}{3b^3\sqrt{a+bx^3}}+\frac{2(Ab-2aB)\sqrt{a+bx^3}}{3b^3}+\frac{2B(a+bx^3)^{3/2}}{9b^3}$$

```
[Out] 2/9*B*(b*x^3+a)^(3/2)/b^3+2/3*a*(A*b-B*a)/b^3/(b*x^3+a)^(1/2)+2/3*(A*b-2*B*
a)*(b*x^3+a)^(1/2)/b^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 78}

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2\sqrt{a+bx^3}(Ab-2aB)}{3b^3}+\frac{2a(Ab-aB)}{3b^3\sqrt{a+bx^3}}+\frac{2B(a+bx^3)^{3/2}}{9b^3}$$

```
[In] Int[(x^5*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*a*(A*b - a*B))/(3*b^3*Sqrt[a + b*x^3]) + (2*(A*b - 2*a*B)*Sqrt[a + b*x^3
])/(3*b^3) + (2*B*(a + b*x^3)^(3/2))/(9*b^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x(A+Bx)}{(a+bx)^{3/2}}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^2(a+bx)^{3/2}} + \frac{Ab-2aB}{b^2\sqrt{a+bx}} + \frac{B\sqrt{a+bx}}{b^2}\right)\,dx, x, x^3\right)\\
&= \frac{2a(Ab-aB)}{3b^3\sqrt{a+bx^3}} + \frac{2(Ab-2aB)\sqrt{a+bx^3}}{3b^3} + \frac{2B(a+bx^3)^{3/2}}{9b^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

$$
\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2(6aAb-8a^2B+3Ab^2x^3-4abBx^3+b^2Bx^6)}{9b^3\sqrt{a+bx^3}}
$$

```
[In] Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^(3/2),x]
[Out] (2*(6*a*A*b - 8*a^2*B + 3*A*b^2*x^3 - 4*a*b*B*x^3 + b^2*B*x^6))/(9*b^3*Sqrt
[a + b*x^3])
```

Maple [A] (verified)

Time = 4.34 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$\frac{\frac{2x^3\left(\frac{x^3B}{3}+A\right)b^2}{3}+\frac{4a\left(-\frac{2x^3B}{3}+A\right)b}{3}-\frac{16a^2B}{9}}{\sqrt{b\,x^3+a}\,b^3}$	49
gosper	$\frac{\frac{2}{9}b^2B\,x^6+\frac{2}{3}A\,b^2x^3-\frac{8}{9}Bab\,x^3+\frac{4}{3}abA-\frac{16}{9}a^2B}{\sqrt{b\,x^3+a}\,b^3}$	52
trager	$\frac{\frac{2}{9}b^2B\,x^6+\frac{2}{3}A\,b^2x^3-\frac{8}{9}Bab\,x^3+\frac{4}{3}abA-\frac{16}{9}a^2B}{\sqrt{b\,x^3+a}\,b^3}$	52
risch	$\frac{2(bB\,x^3+3Ab-5Ba)\sqrt{b\,x^3+a}}{9b^3}+\frac{2a(Ab-Ba)}{3b^3\sqrt{b\,x^3+a}}$	54
elliptic	$\frac{2a(Ab-Ba)}{3b^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2B\,x^3\sqrt{b\,x^3+a}}{9b^2}+\frac{2\left(\frac{Ab-Ba}{b^2}-\frac{2Ba}{3b^2}\right)\sqrt{b\,x^3+a}}{3b}$	81
default	$B\left(-\frac{2a^2}{3b^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2x^3\sqrt{b\,x^3+a}}{9b^2}-\frac{10a\sqrt{b\,x^3+a}}{9b^3}\right)+A\left(\frac{2a}{3b^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2\sqrt{b\,x^3+a}}{3b^2}\right)$	94

```
[In] int(x^5*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] 4/3*(1/2*x^3*(1/3*x^3*B+A)*b^2+a*(-2/3*x^3*B+A)*b-4/3*a^2*B)/(b*x^3+a)^(1/2
)/b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx=\frac{2\,(Bb^2x^6-(4\,Bab-3\,Ab^2)x^3-8\,Ba^2+6\,Aab)\sqrt{bx^3+a}}{9\,(b^4x^3+ab^3)}$$

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
[Out] 2/9*(B*b^2*x^6 - (4*B*a*b - 3*A*b^2)*x^3 - 8*B*a^2 + 6*A*a*b)*sqrt(b*x^3 +
a)/(b^4*x^3 + a*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.70

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx=\begin{cases}\frac{4Aa}{3b^2\sqrt{a+bx^3}}+\frac{2Ax^3}{3b\sqrt{a+bx^3}}-\frac{16Ba^2}{9b^3\sqrt{a+bx^3}}-\frac{8Bax^3}{9b^2\sqrt{a+bx^3}}+\frac{2Bx^6}{9b\sqrt{a+bx^3}} & \text{for } b\neq 0\\ \frac{\frac{Ax^6}{6}+\frac{Bx^9}{9}}{a^{\frac{3}{2}}} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**5*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] Piecewise((4*A*a/(3*b**2*sqrt(a + b*x**3)) + 2*A*x**3/(3*b*sqrt(a + b*x**3)
) - 16*B*a**2/(9*b**3*sqrt(a + b*x**3)) - 8*B*a*x**3/(9*b**2*sqrt(a + b*x**
3)) + 2*B*x**6/(9*b*sqrt(a + b*x**3)), Ne(b, 0)), ((A*x**6/6 + B*x**9/9)/a*
*(3/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.11

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2}{9}\,B\left(\frac{(bx^3+a)^{\frac{3}{2}}}{b^3} - \frac{6\,\sqrt{bx^3+a}a}{b^3} - \frac{3\,a^2}{\sqrt{bx^3+a}b^3}\right) + \frac{2}{3}\,A\left(\frac{\sqrt{bx^3+a}}{b^2} + \frac{a}{\sqrt{bx^3+a}b^2}\right)$$

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/9*B*((b*x^3 + a)^(3/2)/b^3 - 6*sqrt(b*x^3 + a)*a/b^3 - 3*a^2/(sqrt(b*x^3
+ a)*b^3)) + 2/3*A*(sqrt(b*x^3 + a)/b^2 + a/(sqrt(b*x^3 + a)*b^2))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = -\frac{2\,(Ba^2-Aab)}{3\,\sqrt{bx^3+a}b^3} + \frac{2\left((bx^3+a)^{\frac{3}{2}}Bb^6 - 6\,\sqrt{bx^3+a}Bab^6 + 3\,\sqrt{bx^3+a}Ab^7\right)}{9\,b^9}$$

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*(B*a^2 - A*a*b)/(sqrt(b*x^3 + a)*b^3) + 2/9*((b*x^3 + a)^(3/2)*B*b^6 -
 6*sqrt(b*x^3 + a)*B*a*b^6 + 3*sqrt(b*x^3 + a)*A*b^7)/b^9
```

Mupad [B] (verification not implemented)

Time = 7.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2\,B\,(b\,x^3+a)^2-6\,B\,a^2+6\,A\,b\,(b\,x^3+a)-12\,B\,a\,(b\,x^3+a)+6\,A\,a\,b}{9\,b^3\,\sqrt{b\,x^3+a}}$$

```
[In] int((x^5*(A + B*x^3))/(a + b*x^3)^(3/2),x)
[Out] (2*B*(a + b*x^3)^2 - 6*B*a^2 + 6*A*b*(a + b*x^3) - 12*B*a*(a + b*x^3) + 6*A
*a*b)/(9*b^3*(a + b*x^3)^(1/2))
```

3.230 $\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = -\frac{2(Ab-aB)}{3b^2\sqrt{a+bx^3}}+\frac{2B\sqrt{a+bx^3}}{3b^2}$$

```
[Out] -2/3*(A*b-B*a)/b^2/(b*x^3+a)^(1/2)+2/3*B*(b*x^3+a)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {455, 45}

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2B\sqrt{a+bx^3}}{3b^2}-\frac{2(Ab-aB)}{3b^2\sqrt{a+bx^3}}$$

```
[In] Int[(x^2*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*(A*b - a*B))/(3*b^2*Sqrt[a + b*x^3]) + (2*B*Sqrt[a + b*x^3])/(3*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{(a+bx)^{3/2}}\,dx, x, x^3\right) \\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{Ab-aB}{b(a+bx)^{3/2}}+\frac{B}{b\sqrt{a+bx}}\right)\,dx, x, x^3\right) \\
&= -\frac{2(Ab-aB)}{3b^2\sqrt{a+bx^3}}+\frac{2B\sqrt{a+bx^3}}{3b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2(-Ab+2aB+bBx^3)}{3b^2\sqrt{a+bx^3}}$$

```
[In] Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^(3/2),x]
[Out] (2*(-(A*b) + 2*a*B + b*B*x^3))/(3*b^2*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 4.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.65

method	result	size
gosper	$-\frac{2(-bB\,x^3+Ab-2Ba)}{3\sqrt{b\,x^3+a}\,b^2}$	30
trager	$-\frac{2(-bB\,x^3+Ab-2Ba)}{3\sqrt{b\,x^3+a}\,b^2}$	30
pseudoelliptic	$-\frac{2((-x^3B+A)b-2Ba)}{3\sqrt{b\,x^3+a}\,b^2}$	30
risch	$-\frac{2(Ab-Ba)}{3b^2\sqrt{b\,x^3+a}}+\frac{2B\sqrt{b\,x^3+a}}{3b^2}$	39
elliptic	$-\frac{2(Ab-Ba)}{3b^2\sqrt{(x^3+\frac{a}{b})b}}+\frac{2B\sqrt{b\,x^3+a}}{3b^2}$	43
default	$B\left(\frac{2a}{3b^2\sqrt{(x^3+\frac{a}{b})b}}+\frac{2\sqrt{b\,x^3+a}}{3b^2}\right)-\frac{2A}{3b\sqrt{b\,x^3+a}}$	53

```
[In] int(x^2*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(b*x^3+a)^(1/2)*(-B*b*x^3+A*b-2*B*a)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2\,(Bbx^3+2\,Ba-Ab)\sqrt{bx^3+a}}{3\,(b^3x^3+ab^2)}$$

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(B*b*x^3 + 2*B*a - A*b)*sqrt(b*x^3 + a)/(b^3*x^3 + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.63

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \begin{cases} -\frac{2A}{3b\sqrt{a+bx^3}} + \frac{4Ba}{3b^2\sqrt{a+bx^3}} + \frac{2Bx^3}{3b\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{\frac{Ax^3}{3}+\frac{Bx^6}{6}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] Piecewise((-2*A/(3*b*sqrt(a + b*x**3)) + 4*B*a/(3*b**2*sqrt(a + b*x**3)) +
2*B*x**3/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), ((A*x**3/3 + B*x**6/6)/a**(3/2)
, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2}{3}\,B\left(\frac{\sqrt{bx^3+a}}{b^2}+\frac{a}{\sqrt{bx^3+a}b^2}\right)-\frac{2\,A}{3\,\sqrt{bx^3+a}b}$$

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*B*(sqrt(b*x^3 + a)/b^2 + a/(sqrt(b*x^3 + a)*b^2)) - 2/3*A/(sqrt(b*x^3 +
 a)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2\sqrt{bx^3+a}B}{3\,b^2} + \frac{2\,(Ba-Ab)}{3\,\sqrt{bx^3+a}b^2}$$

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
[Out] 2/3*sqrt(b*x^3 + a)*B/b^2 + 2/3*(B*a - A*b)/(sqrt(b*x^3 + a)*b^2)
```

Mupad [B] (verification not implemented)

Time = 7.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2\,B\,a-2\,A\,b+2\,B\,(b\,x^3+a)}{3\,b^2\,\sqrt{b\,x^3+a}}$$

```
[In] int((x^2*(A + B*x^3))/(a + b*x^3)^(3/2),x)
[Out] (2*B*a - 2*A*b + 2*B*(a + b*x^3))/(3*b^2*(a + b*x^3)^(1/2))
```

3.231 $\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 58

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{3/2}}\,dx=\frac{2(Ab-aB)}{3ab\sqrt{a+bx^3}}-\frac{2A\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

```
[Out] -2/3*A*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)+2/3*(A*b-B*a)/a/b/(b*x^3+a)
^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {457, 79, 65, 214}

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{3/2}}\,dx=\frac{2(Ab-aB)}{3ab\sqrt{a+bx^3}}-\frac{2A\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

```
[In] Int[(A + B*x^3)/(x*(a + b*x^3)^(3/2)),x]
```

```
[Out] (2*(A*b - a*B))/(3*a*b*Sqrt[a + b*x^3]) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt
[a]])/(3*a^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x(a+bx)^{3/2}}\,dx, x, x^3\right) \\
&= \frac{2(Ab-aB)}{3ab\sqrt{a+bx^3}} + \frac{A\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)}{3a} \\
&= \frac{2(Ab-aB)}{3ab\sqrt{a+bx^3}} + \frac{(2A)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{3ab} \\
&= \frac{2(Ab-aB)}{3ab\sqrt{a+bx^3}} - \frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{3/2}}\,dx = \frac{2(Ab-aB)}{3ab\sqrt{a+bx^3}} - \frac{2A\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

```
[In] Integrate[(A + B*x^3)/(x*(a + b*x^3)^(3/2)),x]
```

```
[Out] (2*(A*b - a*B))/(3*a*b*Sqrt[a + b*x^3]) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt
[a]])/(3*a^(3/2))
```

Maple [A] (verified)

Time = 4.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

method	result	size
elliptic	$\frac{\frac{2Ab}{3}-\frac{2Ba}{3}}{ba\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2A\,\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}$	51
default	$-\frac{2B}{3b\sqrt{b\,x^3+a}}+A\left(\frac{2}{3a\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2\,\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}\right)$	57
pseudoelliptic	$-\frac{2\left(A\,\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)b\sqrt{b\,x^3+a}-Ab\sqrt{a}+B\,a^{\frac{3}{2}}\right)}{3a^{\frac{3}{2}}\sqrt{b\,x^3+a}\,b}$	57

```
[In] int((B*x^3+A)/x/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/b*(A*b-B*a)/a/((x^3+a/b)*b)^(1/2)-2/3*A*arctanh((b*x^3+a)^(1/2)/a^(1/2)
)/a^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.93

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{3/2}}\,dx = \left[\frac{(Ab^2x^3+Aab)\sqrt{a}\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2a}{x^3}\right)-2\sqrt{bx^3+a}(Ba^2-Aab)}{3\left(a^2b^2x^3+a^3b\right)}, \frac{2\left(\left(Ab^2x^3+A\right.\right.}{}\right.$$

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/3*((A*b^2*x^3 + A*a*b)*sqrt(a)*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) +
2*a)/x^3) - 2*sqrt(b*x^3 + a)*(B*a^2 - A*a*b))/(a^2*b^2*x^3 + a^3*b), 2/3*(
```

```
(A*b^2*x^3 + A*a*b)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) - sqrt(b*x^
3 + a)*(B*a^2 - A*a*b))/(a^2*b^2*x^3 + a^3*b)]
```

Sympy [A] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.34

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{Ab\operatorname{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{3a\sqrt{-a}} - \frac{-Ab+Ba}{3a\sqrt{a+bx^3}}\right)}{b} & \text{for } b \neq 0 \\ \frac{A\log\left(Bx^3\right)+Bx^3}{3a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate((B*x**3+A)/x/(b*x**3+a)**(3/2),x)
```

```
[Out] Piecewise((2*(A*b*atan(sqrt(a + b*x**3)/sqrt(-a))/(3*a*sqrt(-a)) - (-A*b +
B*a)/(3*a*sqrt(a + b*x**3)))/b, Ne(b, 0)), ((A*log(B*x**3) + B*x**3)/(3*a**
(3/2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{3/2}}\,dx = \frac{1}{3}A\left(\frac{\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{2}{\sqrt{bx^3+a}a}\right) - \frac{2B}{3\sqrt{bx^3+a}b}$$

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*A*(log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(3/2)
 + 2/(sqrt(b*x^3 + a)*a)) - 2/3*B/(sqrt(b*x^3 + a)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{3/2}}\,dx = \frac{2A\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}a} - \frac{2(Ba-Ab)}{3\sqrt{bx^3+a}ab}$$

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*A*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a) - 2/3*(B*a - A*b)/(sqrt
(b*x^3 + a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 7.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{3/2}}\,dx=\frac{\frac{2\,A}{3\,a}-\frac{2\,B}{3\,b}}{\sqrt{b\,x^3+a}}+\frac{A\,\ln\left(\frac{{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3\,a^{3/2}}$$

```
[In] int((A + B*x^3)/(x*(a + b*x^3)^(3/2)),x)
[Out] ((2*A)/(3*a) - (2*B)/(3*b))/(a + b*x^3)^(1/2) + (A*log((((a + b*x^3)^(1/2)
- a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6))/(3*a^(3/2))
```

3.232 $\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{3/2}}\,dx=\frac{-3Ab+2aB}{3a^2\sqrt{a+bx^3}}-\frac{A}{3ax^3\sqrt{a+bx^3}}+\frac{(3Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}$$

```
[Out] 1/3*(3*A*b-2*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(5/2)+1/3*(-3*A*b+2*B*
a)/a^2/(b*x^3+a)^(1/2)-1/3*A/a/x^3/(b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 79, 53, 65, 214}

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{3/2}}\,dx=\frac{(3Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}-\frac{3Ab-2aB}{3a^2\sqrt{a+bx^3}}-\frac{A}{3ax^3\sqrt{a+bx^3}}$$

```
[In] Int[(A + B*x^3)/(x^4*(a + b*x^3)^(3/2)),x]
```

```
[Out] -1/3*(3*A*b - 2*a*B)/(a^2*Sqrt[a + b*x^3]) - A/(3*a*x^3*Sqrt[a + b*x^3]) +
((3*A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(5/2))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
```

```
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)^{3/2}}\,dx, x, x^3\right)\\
&= -\frac{A}{3ax^3\sqrt{a+bx^3}} + \frac{\left(-\frac{3Ab}{2}+aB\right)\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, x^3\right)}{3a}\\
&= -\frac{3Ab-2aB}{3a^2\sqrt{a+bx^3}} - \frac{A}{3ax^3\sqrt{a+bx^3}} - \frac{(3Ab-2aB)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)}{6a^2}
\end{aligned}$$

$$= -\frac{3Ab-2aB}{3a^2\sqrt{a+bx^3}} - \frac{A}{3ax^3\sqrt{a+bx^3}} - \frac{(3Ab-2aB)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{3a^2b}$$

$$= -\frac{3Ab-2aB}{3a^2\sqrt{a+bx^3}} - \frac{A}{3ax^3\sqrt{a+bx^3}} + \frac{(3Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^3}{x^4(a+bx^3)^{3/2}}\,dx = \frac{-aA-3Abx^3+2aBx^3}{3a^2x^3\sqrt{a+bx^3}} + \frac{(3Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}$$

```
[In] Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^(3/2)),x]
[Out] (-(a*A) - 3*A*b*x^3 + 2*a*B*x^3)/(3*a^2*x^3*Sqrt[a + b*x^3]) + ((3*A*b - 2*
a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(5/2))
```

Maple [A] (verified)

Time = 4.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

method	result
elliptic	$-\frac{A\sqrt{b\,x^3+a}}{3a^2x^3}-\frac{2(Ab-Ba)}{3a^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{(3Ab-2Ba)\,\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{5}{2}}}$
pseudoelliptic	$\frac{\sqrt{b\,x^3+a}\,x^3\left(Ab-\frac{2Ba}{3}\right)\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)+\frac{(2x^3B-A)a^{\frac{3}{2}}}{3}-A\sqrt{a}\,b\,x^3}{\sqrt{b\,x^3+a}\,a^{\frac{5}{2}}x^3}$
risch	$-\frac{A\sqrt{b\,x^3+a}}{3a^2x^3}-\frac{-\frac{2bA}{3\sqrt{b\,x^3+a}}+a(3Ab-2Ba)\left(\frac{2}{3a\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2\,\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}\right)}{2a^2}$
default	$B\left(\frac{2}{3a\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2\,\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}\right)+A\left(-\frac{\sqrt{b\,x^3+a}}{3a^2x^3}-\frac{2b}{3a^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{b\,\text{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right)$

```
[In] int((B*x^3+A)/x^4/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/3/a^2*A*(b*x^3+a)^(1/2)/x^3-2/3*(A*b-B*a)/a^2/((x^3+a/b)*b)^(1/2)+1/3*(3
*A*b-2*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.71

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{3/2}}\,dx = \left[-\frac{((2\,Bab-3\,Ab^2)x^6+(2\,Ba^2-3\,Aab)x^3)\sqrt{a}\log\left(\frac{bx^3+2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)-2\,((2\,Ba^2-}{6\,(a^3bx^6+a^4x^3)}\right.$$

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(((2*B*a*b - 3*A*b^2)*x^6 + (2*B*a^2 - 3*A*a*b)*x^3)*sqrt(a)*log((b*x
^3 + 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) - 2*((2*B*a^2 - 3*A*a*b)*x^3 - A
*a^2)*sqrt(b*x^3 + a))/(a^3*b*x^6 + a^4*x^3), 1/3*(((2*B*a*b - 3*A*b^2)*x^6
 + (2*B*a^2 - 3*A*a*b)*x^3)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (
(2*B*a^2 - 3*A*a*b)*x^3 - A*a^2)*sqrt(b*x^3 + a))/(a^3*b*x^6 + a^4*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(73) = 146.

Time = 23.53 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.07

$$\begin{aligned}\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{3/2}}\,dx &= A\left(-\frac{1}{3a\sqrt{b}x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{\sqrt{b}}{a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{a^{\frac{5}{2}}}\right)\\ &\quad+B\left(\frac{2a^3\sqrt{1+\frac{bx^3}{a}}}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}+\frac{a^3\log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^3}{a}}+1\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}\right.\\ &\quad\left.+\frac{a^2bx^3\log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}-\frac{2a^2bx^3\log\left(\sqrt{1+\frac{bx^3}{a}}+1\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}\right)\end{aligned}$$

```
[In] integrate((B*x**3+A)/x**4/(b*x**3+a)**(3/2),x)
```

```
[Out] A*(-1/(3*a*sqrt(b)*x**(9/2)*sqrt(a/(b*x**3) + 1)) - sqrt(b)/(a**2*x**(3/2)*
sqrt(a/(b*x**3) + 1)) + b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/a**(5/2)) + B*(
2*a**3*sqrt(1 + b*x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**3*log(b*x**
3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) - 2*a**3*log(sqrt(1 + b*x**3/a) + 1)/
(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**2*b*x**3*log(b*x**3/a)/(3*a**(9/2) +
3*a**(7/2)*b*x**3) - 2*a**2*b*x**3*log(sqrt(1 + b*x**3/a) + 1)/(3*a**(9/2)
+ 3*a**(7/2)*b*x**3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(70) = 140$.

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.67

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{3/2}}\,dx = -\frac{1}{6}\,A\left(\frac{2\,(3\,(bx^3+a)b-2\,ab)}{(bx^3+a)^{\frac{3}{2}}a^2-\sqrt{bx^3+a}a^3}+\frac{3\,b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right) + \frac{1}{3}\,B\left(\frac{\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{bx^3+a}a}\right)$$

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/6*A*(2*(3*(b*x^3 + a)*b - 2*a*b)/((b*x^3 + a)^(3/2)*a^2 - sqrt(b*x^3 + a
)*a^3) + 3*b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a
^(5/2)) + 1/3*B*(log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)
))/a^(3/2) + 2/(sqrt(b*x^3 + a)*a))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.15

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{3/2}}\,dx = \frac{(2\,Ba-3\,Ab)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}a^2} + \frac{2\,(bx^3+a)Ba-2\,Ba^2-3\,(bx^3+a)Ab+2\,Aab}{3\left((bx^3+a)^{\frac{3}{2}}-\sqrt{bx^3+a}a\right)a^2}$$

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*B*a - 3*A*b)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^2) + 1/3*(
2*(b*x^3 + a)*B*a - 2*B*a^2 - 3*(b*x^3 + a)*A*b + 2*A*a*b)/(((b*x^3 + a)^(3
/2) - sqrt(b*x^3 + a)*a)*a^2)
```

Mupad [B] (verification not implemented)

Time = 7.47 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.52

$$\int \frac{A+Bx^3}{x^4\,(a+bx^3)^{3/2}}\,dx=\frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)^3}{x^6}\right)\,\left(3\,A\,b-2\,B\,a\right)}{6\,a^{5/2}}-\frac{\frac{2\,B\,a^2-3\,A\,a\,b}{2\,a^3}-\frac{a\,\left(\frac{A\,b^2}{3\,a^3}+\frac{5\,b\,\left(2\,B\,a^2-3\,A\,a\,b\right)}{6\,a^4}\right)}{b}}{\sqrt{b\,x^3+a}}-\frac{A\,\sqrt{b\,x^3+a}}{3\,a^2\,x^3}$$

```
[In] int((A + B*x^3)/(x^4*(a + b*x^3)^(3/2)),x)
```

```
[Out] (log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/2))^3)/x^6)*(
3*A*b - 2*B*a))/(6*a^(5/2)) - ((2*B*a^2 - 3*A*a*b)/(2*a^3) - (a*((A*b^2)/(3
*a^3) + (5*b*(2*B*a^2 - 3*A*a*b))/(6*a^4)))/b)/(a + b*x^3)^(1/2) - (A*(a +
b*x^3)^(1/2))/(3*a^2*x^3)
```

3.233 $\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 118

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{3/2}}\,dx = \frac{b(5Ab-4aB)}{4a^3\sqrt{a+bx^3}}-\frac{A}{6ax^6\sqrt{a+bx^3}}+\frac{5Ab-4aB}{12a^2x^3\sqrt{a+bx^3}}-\frac{b(5Ab-4aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4a^{7/2}}$$

```
[Out] -1/4*b*(5*A*b-4*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(7/2)+1/4*b*(5*A*b-
4*B*a)/a^3/(b*x^3+a)^(1/2)-1/6*A/a/x^6/(b*x^3+a)^(1/2)+1/12*(5*A*b-4*B*a)/a
^2/x^3/(b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {457, 79, 44, 53, 65, 214}

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{3/2}}\,dx = -\frac{b(5Ab-4aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4a^{7/2}}+\frac{b(5Ab-4aB)}{4a^3\sqrt{a+bx^3}}+\frac{5Ab-4aB}{12a^2x^3\sqrt{a+bx^3}}-\frac{A}{6ax^6\sqrt{a+bx^3}}$$

```
[In] Int[(A + B*x^3)/(x^7*(a + b*x^3)^(3/2)),x]
```

```
[Out] (b*(5*A*b - 4*a*B))/(4*a^3*Sqrt[a + b*x^3]) - A/(6*a*x^6*Sqrt[a + b*x^3]) +
 (5*A*b - 4*a*B)/(12*a^2*x^3*Sqrt[a + b*x^3]) - (b*(5*A*b - 4*a*B)*ArcTanh[
Sqrt[a + b*x^3]/Sqrt[a]])/(4*a^(7/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)^{3/2}}\,dx, x, x^3\right)\\
&= -\frac{A}{6ax^6\sqrt{a+bx^3}} + \frac{\left(-\frac{5Ab}{2}+2aB\right)\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, x^3\right)}{6a}\\
&= -\frac{A}{6ax^6\sqrt{a+bx^3}} + \frac{5Ab-4aB}{12a^2x^3\sqrt{a+bx^3}} + \frac{(b(5Ab-4aB))\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, x^3\right)}{8a^2}\\
&= \frac{b(5Ab-4aB)}{4a^3\sqrt{a+bx^3}} - \frac{A}{6ax^6\sqrt{a+bx^3}} + \frac{5Ab-4aB}{12a^2x^3\sqrt{a+bx^3}}\\
&\quad + \frac{(b(5Ab-4aB))\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)}{8a^3}\\
&= \frac{b(5Ab-4aB)}{4a^3\sqrt{a+bx^3}} - \frac{A}{6ax^6\sqrt{a+bx^3}} + \frac{5Ab-4aB}{12a^2x^3\sqrt{a+bx^3}}\\
&\quad + \frac{(5Ab-4aB)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{4a^3}\\
&= \frac{b(5Ab-4aB)}{4a^3\sqrt{a+bx^3}} - \frac{A}{6ax^6\sqrt{a+bx^3}} + \frac{5Ab-4aB}{12a^2x^3\sqrt{a+bx^3}} - \frac{b(5Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4a^{7/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.85

$$
\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{3/2}}\,dx = \frac{-2a^2A+5aAbx^3-4a^2Bx^3+15Ab^2x^6-12abBx^6}{12a^3x^6\sqrt{a+bx^3}} + \frac{b(-5Ab+4aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4a^{7/2}}
$$

```
[In] Integrate[(A + B*x^3)/(x^7*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-2*a^2*A + 5*a*A*b*x^3 - 4*a^2*B*x^3 + 15*A*b^2*x^6 - 12*a*b*B*x^6)/(12*a^
3*x^6*Sqrt[a + b*x^3]) + (b*(-5*A*b + 4*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a
]])/(4*a^(7/2))
```

Maple [A] (verified)

Time = 4.35 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$-\frac{5\left(-\frac{x^{3} b\left(-\frac{12 x^{3} B}{5}+A\right) a^{\frac{3}{2}}}{3}+\frac{2\left(2 x^{3} B+A\right) a^{\frac{5}{2}}}{15}+x^{6}\left(-A b \sqrt{a}+\left(A b-\frac{4 B a}{5}\right) \sqrt{b\,x^{3}+a}\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)\right) b\right)}{4 \sqrt{b\,x^{3}+a}\, a^{\frac{7}{2}} x^{6}}$
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{6a^{2}x^{6}}+\frac{(7Ab-4Ba)\sqrt{b\,x^{3}+a}}{12a^{3}x^{3}}+\frac{2b(Ab-Ba)}{3a^{3}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{b(5Ab-4Ba)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{4a^{\frac{7}{2}}}$
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-7Ab\,x^{3}+4Ba\,x^{3}+2Aa\right)}{12a^{3}x^{6}}+\frac{b\left(-\frac{2(7Ab-4Ba)}{3\sqrt{b\,x^{3}+a}}+3a(5Ab-4Ba)\left(\frac{2}{3a\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}\right)\right)}{8a^{3}}$
default	$A\left(-\frac{\sqrt{b\,x^{3}+a}}{6a^{2}x^{6}}+\frac{7b\sqrt{b\,x^{3}+a}}{12a^{3}x^{3}}+\frac{2b^{2}}{3a^{3}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{5b^{2}\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{4a^{\frac{7}{2}}}\right)+B\left(-\frac{\sqrt{b\,x^{3}+a}}{3a^{2}x^{3}}-\frac{2b}{3a^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}\right.$

```
[In] int((B*x^3+A)/x^7/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -5/4*(-1/3*x^3*b*(-12/5*x^3*B+A)*a^(3/2)+2/15*(2*B*x^3+A)*a^(5/2)+x^6*(-A*b
*a^(1/2)+(A*b-4/5*B*a)*(b*x^3+a)^(1/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))*b)
/(b*x^3+a)^(1/2)/a^(7/2)/x^6
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.45

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{3/2}}\,dx = \left[-\frac{3\left((4\,Bab^2-5\,Ab^3)x^9+(4\,Ba^2b-5\,Aab^2)x^6\right)\sqrt{a}\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\,(3\,(4}{24\left(a^4bx^9+a^5x^6\right)}\right.$$

$$-\frac{3\left((4\,Bab^2-5\,Ab^3)x^9+(4\,Ba^2b-5\,Aab^2)x^6\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+a}\sqrt{-a}}{a}\right)+\left(3\,(4\,Ba^2b-5\,Aab^2)x^6+2\,Aa^3-\right.}{12\left(a^4bx^9+a^5x^6\right)}$$

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((4*B*a*b^2 - 5*A*b^3)*x^9 + (4*B*a^2*b - 5*A*a*b^2)*x^6)*sqrt(a)
*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*(3*(4*B*a^2*b - 5*A
*a*b^2)*x^6 + 2*A*a^3 + (4*B*a^3 - 5*A*a^2*b)*x^3)*sqrt(b*x^3 + a))/(a^4*b*
x^9 + a^5*x^6), -1/12*(3*((4*B*a*b^2 - 5*A*b^3)*x^9 + (4*B*a^2*b - 5*A*a*b^
2)*x^6)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (3*(4*B*a^2*b - 5*A*a
*b^2)*x^6 + 2*A*a^3 + (4*B*a^3 - 5*A*a^2*b)*x^3)*sqrt(b*x^3 + a))/(a^4*b*x^
9 + a^5*x^6)]
```

Sympy [A] (verification not implemented)

Time = 59.40 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.63

$$\int \frac{A+Bx^3}{x^7 (a+bx^3)^{3/2}}\,dx = A\left(-\frac{1}{6a\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{5\sqrt{b}}{12a^2x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{5b^{\frac{3}{2}}}{4a^3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{5b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{4a^{\frac{7}{2}}}\right)+B\left(-\frac{1}{3a\sqrt{b}x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{\sqrt{b}}{a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{a^{\frac{5}{2}}}\right)$$

```
[In] integrate((B*x**3+A)/x**7/(b*x**3+a)**(3/2),x)
```

```
[Out] A*(-1/(6*a*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) + 5*sqrt(b)/(12*a**2*x**
(9/2)*sqrt(a/(b*x**3) + 1)) + 5*b**(3/2)/(4*a**3*x**(3/2)*sqrt(a/(b*x**3) +
 1)) - 5*b**2*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*a**(7/2))) + B*(-1/(3*a*
sqrt(b)*x**(9/2)*sqrt(a/(b*x**3) + 1)) - sqrt(b)/(a**2*x**(3/2)*sqrt(a/(b*x
**3) + 1)) + b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/a**(5/2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(98) = 196.

Time = 0.29 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.82

$$\int \frac{A+Bx^3}{x^7 (a+bx^3)^{3/2}}\,dx = \frac{1}{24}A\left(\frac{2\left(15(bx^3+a)^2b^2-25(bx^3+a)ab^2+8a^2b^2\right)}{(bx^3+a)^{\frac{5}{2}}a^3-2(bx^3+a)^{\frac{3}{2}}a^4+\sqrt{bx^3+a}a^5}+\frac{15b^2\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right)-\frac{1}{6}B\left(\frac{2(3(bx^3+a)b-2ab)}{(bx^3+a)^{\frac{3}{2}}a^2-\sqrt{bx^3+a}a^3}+\frac{3b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right)$$

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/24*A*(2*(15*(b*x^3 + a)^2*b^2 - 25*(b*x^3 + a)*a*b^2 + 8*a^2*b^2)/((b*x^3
 + a)^(5/2)*a^3 - 2*(b*x^3 + a)^(3/2)*a^4 + sqrt(b*x^3 + a)*a^5) + 15*b^2*l
og((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(7/2)) - 1/6*
B*(2*(3*(b*x^3 + a)*b - 2*a*b)/((b*x^3 + a)^(3/2)*a^2 - sqrt(b*x^3 + a)*a^3
) + 3*b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(5/2
))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{3/2}}\,dx = -\frac{\left(4\,Bab-5\,Ab^2\right)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a^3}-\frac{2\,\left(Bab-Ab^2\right)}{3\,\sqrt{bx^3+a}a^3}-\frac{4\,\left(bx^3+a\right)^{\frac{3}{2}}Bab-4\,\sqrt{bx^3+a}Ba^2b-7\,\left(bx^3+a\right)^{\frac{3}{2}}Ab^2+9\,\sqrt{bx^3+a}Aab^2}{12\,a^3b^2x^6}$$

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*(4*B*a*b - 5*A*b^2)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^3) -
2/3*(B*a*b - A*b^2)/(sqrt(b*x^3 + a)*a^3) - 1/12*(4*(b*x^3 + a)^(3/2)*B*a*b
 - 4*sqrt(b*x^3 + a)*B*a^2*b - 7*(b*x^3 + a)^(3/2)*A*b^2 + 9*sqrt(b*x^3 + a
)*A*a*b^2)/(a^3*b^2*x^6)
```

Mupad [B] (verification not implemented)

Time = 7.69 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.42

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{3/2}}\,dx = \frac{b\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)\,\left(5\,A\,b-4\,B\,a\right)}{8\,a^{7/2}}-\frac{\left(4\,B\,a^2-7\,A\,a\,b\right)\,\sqrt{b\,x^3+a}}{12\,a^4\,x^3}-\frac{A\,\sqrt{b\,x^3+a}}{6\,a^2\,x^6}-\frac{\frac{a\,\left(\frac{7\,A\,b^3-4\,B\,a\,b^2}{12\,a^4}-\frac{5\,b^2\,\left(5\,A\,b-4\,B\,a\right)}{8\,a^4}\right)}{b}+\frac{3\,b\,\left(5\,A\,b-4\,B\,a\right)}{8\,a^3}}{\sqrt{b\,x^3+a}}$$

```
[In] int((A + B*x^3)/(x^7*(a + b*x^3)^(3/2)),x)
```

```
[Out] (b*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)
*(5*A*b - 4*B*a))/(8*a^(7/2)) - ((4*B*a^2 - 7*A*a*b)*(a + b*x^3)^(1/2))/(12
*a^4*x^3) - (A*(a + b*x^3)^(1/2))/(6*a^2*x^6) - ((a*((7*A*b^3 - 4*B*a*b^2)/
(12*a^4) - (5*b^2*(5*A*b - 4*B*a))/(8*a^4)))/b + (3*b*(5*A*b - 4*B*a))/(8*a
^3))/(a + b*x^3)^(1/2)
```

3.234 $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 300

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = -\frac{2(11Ab-14aB)x^4}{33b^2\sqrt{a+bx^3}}+\frac{2Bx^7}{11b\sqrt{a+bx^3}}+\frac{16(11Ab-14aB)x\sqrt{a+bx^3}}{165b^3}$$

$$-\frac{32\sqrt{2+\sqrt{3}}a(11Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),\cdots\right.}{165\sqrt[4]{3}b^{10/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -2/33*(11*A*b-14*B*a)*x^4/b^2/(b*x^3+a)^(1/2)+2/11*B*x^7/b/(b*x^3+a)^(1/2)+
16/165*(11*A*b-14*B*a)*x*(b*x^3+a)^(1/2)/b^3-32/495*a*(11*A*b-14*B*a)*(a^(1
/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)
*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^
(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(10
/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {470, 294, 327, 224}

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx =$$

$$-\frac{32\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-14aB)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7}{165\sqrt[4]{3}b^{10/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{16x\sqrt{a+bx^3}(11Ab-14aB)}{165b^3}-\frac{2x^4(11Ab-14aB)}{33b^2\sqrt{a+bx^3}}+\frac{2Bx^7}{11b\sqrt{a+bx^3}}$$

```
[In] Int[(x^6*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*(11*A*b - 14*a*B)*x^4)/(33*b^2*Sqrt[a + b*x^3]) + (2*B*x^7)/(11*b*Sqrt[
a + b*x^3]) + (16*(11*A*b - 14*a*B)*x*Sqrt[a + b*x^3])/(165*b^3) - (32*Sqrt
[2 + Sqrt[3]]*a*(11*A*b - 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(165*3^(1/4)*b^(10/3)*Sqrt[(a^(1/3)*(a^(1/3) + b
^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2Bx^7}{11b\sqrt{a+bx^3}} - \frac{\left(2\left(-\frac{11Ab}{2}+7aB\right)\right)\int \frac{x^6}{(a+bx^3)^{3/2}}\,dx}{11b} \\
&= -\frac{2(11Ab-14aB)x^4}{33b^2\sqrt{a+bx^3}} + \frac{2Bx^7}{11b\sqrt{a+bx^3}} + \frac{(8(11Ab-14aB))\int \frac{x^3}{\sqrt{a+bx^3}}\,dx}{33b^2} \\
&= -\frac{2(11Ab-14aB)x^4}{33b^2\sqrt{a+bx^3}} + \frac{2Bx^7}{11b\sqrt{a+bx^3}} \\
&\quad + \frac{16(11Ab-14aB)x\sqrt{a+bx^3}}{165b^3} - \frac{(16a(11Ab-14aB))\int \frac{1}{\sqrt{a+bx^3}}\,dx}{165b^3} \\
&= -\frac{2(11Ab-14aB)x^4}{33b^2\sqrt{a+bx^3}} + \frac{2Bx^7}{11b\sqrt{a+bx^3}} + \frac{16(11Ab-14aB)x\sqrt{a+bx^3}}{165b^3} \\
&\quad - \frac{32\sqrt{2+\sqrt{3}}a(11Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{165\sqrt[4]{3}b^{10/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.34

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2x\left(-112a^2B+3b^2x^3(11A+5Bx^3)+a(88Ab-42bBx^3)+8a(-11Ab+14aB)\sqrt{1+\cdots}\right.}{165b^3\sqrt{a+bx^3}}$$

[In] Integrate[(x^6*(A + B*x^3))/(a + b*x^3)^(3/2),x]

[Out] (2*x*(-112*a^2*B + 3*b^2*x^3*(11*A + 5*B*x^3) + a*(88*A*b - 42*b*B*x^3) + 8
a(-11*A*b + 14*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3,
-((b*x^3)/a)]))/(165*b^3*Sqrt[a + b*x^3])

Maple [A] (verified)

Time = 4.81 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.35

method	result
elliptic	$\frac{2xa(Ab-Ba)}{3b^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2Bx^4\sqrt{bx^3+a}}{11b^2}+\frac{2\left(\frac{Ab-Ba}{b^2}-\frac{8Ba}{11b^2}\right)x\sqrt{bx^3+a}}{5b}-\frac{2i\left(-\frac{2a(Ab-Ba)}{3b^3}-\frac{2\left(\frac{Ab-Ba}{b^2}-\frac{8Ba}{11b^2}\right)a}{5b}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\cdots\right.}{\cdots}}\cdots}{\cdots}$
default	$B\left(-\frac{2a^2x}{3b^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2x^4\sqrt{bx^3+a}}{11b^2}-\frac{38ax\sqrt{bx^3+a}}{55b^3}-\frac{448ia^2\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{\cdots}{-\frac{3(-a\,b\cdots}{2b}}}\cdots}{\cdots}\right.$
risch	Expression too large to display

[In] int(x^6*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)

```
[Out] 2/3/b^3*x*a*(A*b-B*a)/((x^3+a/b)*b)^(1/2)+2/11*B/b^2*x^4*(b*x^3+a)^(1/2)+2/
5*((A*b-B*a)/b^2-8/11*B/b^2*a)/b*x*(b*x^3+a)^(1/2)-2/3*I*(-2/3*a*(A*b-B*a)/
b^3-2/5*((A*b-B*a)/b^2-8/11*B/b^2*a)/b*a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.41

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2\left(16\left(14\,Ba^3-11\,Aa^2b+(14\,Ba^2b-11\,Aab^2)x^3\right)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right.}{165\,(b^5x^3+a}$$

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/165*(16*(14*B*a^3 - 11*A*a^2*b + (14*B*a^2*b - 11*A*a*b^2)*x^3)*sqrt(b)*w
eierstrassPInverse(0, -4*a/b, x) + (15*B*b^3*x^7 - 3*(14*B*a*b^2 - 11*A*b^3
)*x^4 - 8*(14*B*a^2*b - 11*A*a*b^2)*x)*sqrt(b*x^3 + a))/(b^5*x^3 + a*b^4)
```

Sympy [A] (verification not implemented)

Time = 8.75 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.27

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{Ax^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{10}{3}\right)}+\frac{Bx^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{13}{3}\right)}$$

```
[In] integrate(x**6*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] A*x**7*gamma(7/3)*hyper((3/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(3/2)*gamma(10/3)) + B*x**10*gamma(10/3)*hyper((3/2, 10/3), (13/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(13/3))
```

Maxima [F]

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)x^6}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*x^6/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)x^6}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*x^6/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{x^6\,(B\,x^3+A)}{(b\,x^3+a)^{3/2}}\,dx$$

```
[In] int((x^6*(A + B*x^3))/(a + b*x^3)^(3/2),x)
[Out] int((x^6*(A + B*x^3))/(a + b*x^3)^(3/2), x)
```

3.235 $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 269

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = -\frac{2(5Ab-8aB)x}{15b^2\sqrt{a+bx^3}} + \frac{2Bx^4}{5b\sqrt{a+bx^3}} + \frac{4\sqrt{2+\sqrt{3}}(5Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-\right.}{15\sqrt[4]{3}b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -2/15*(5*A*b-8*B*a)*x/b^2/(b*x^3+a)^(1/2)+2/5*B*x^4/b/(b*x^3+a)^(1/2)+4/45*
(5*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*(
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)*3^(3/4)/b^(7/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used

$= \{470, 294, 224\}$

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-8aB)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+(}{\sqrt[3]{b}x+(}\right.\right.}{15\sqrt[4]{3}b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2x(5Ab-8aB)}{15b^2\sqrt{a+bx^3}}+\frac{2Bx^4}{5b\sqrt{a+bx^3}}$$

```
[In] Int[(x^3*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*(5*A*b - 8*a*B)*x)/(15*b^2*Sqrt[a + b*x^3]) + (2*B*x^4)/(5*b*Sqrt[a + b
*x^3]) + (4*Sqrt[2 + Sqrt[3]]*(5*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(15*3^(1/4)*b^(7/3)*Sqrt[(a^(1/3)*(a
^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]
)
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{2Bx^4}{5b\sqrt{a+bx^3}} - \frac{\left(2\left(-\frac{5Ab}{2}+4aB\right)\right)\int \frac{x^3}{(a+bx^3)^{3/2}}\,dx}{5b}$$

$$= -\frac{2(5Ab-8aB)x}{15b^2\sqrt{a+bx^3}} + \frac{2Bx^4}{5b\sqrt{a+bx^3}} + \frac{(2(5Ab-8aB))\int \frac{1}{\sqrt{a+bx^3}}\,dx}{15b^2}$$

$$= -\frac{2(5Ab-8aB)x}{15b^2\sqrt{a+bx^3}} + \frac{2Bx^4}{5b\sqrt{a+bx^3}} + \frac{4\sqrt{2+\sqrt{3}}(5Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-}{15\sqrt[4]{3}b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.29

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2x\left(-5Ab+8aB+3bBx^3+(5Ab-8aB)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx}{a}\right.\right.}{15b^2\sqrt{a+bx^3}}$$

```
[In] Integrate[(x^3*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(-5*A*b + 8*a*B + 3*b*B*x^3 + (5*A*b - 8*a*B)*Sqrt[1 + (b*x^3)/a]*Hype
rgeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]))/(15*b^2*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 4.68 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.29

method	result
elliptic	$-\frac{2x(Ab-Ba)}{3b^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2Bx\sqrt{b\,x^3+a}}{5b^2}-\frac{2i\left(\frac{\frac{2Ab}{3}-\frac{2Ba}{3}}{b^2}-\frac{2Ba}{5b^2}\right)\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b\right.}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i}{}}}}{}$
default	$B\left(\frac{2ax}{3b^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2x\sqrt{b\,x^3+a}}{5b^2}+\frac{32ia\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-}}{}\right.$
risch	$\frac{2Bx\sqrt{b\,x^3+a}}{5b^2}+\frac{-2a^2B\left(\frac{2x}{3a\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x\right.}{}}}{}\right.}{}$

```
[In] int(x^3*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/b^2*x*(A*b-B*a)/((x^3+a/b)*b)^(1/2)+2/5*B*x/b^2*(b*x^3+a)^(1/2)-2/3*I*
(2/3*(A*b-B*a)/b^2-2/5*B/b^2*a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((
x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
```

)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.35

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx =$$
$$-\frac{2\left(2\left((8\,Bab-5\,Ab^2)x^3+8\,Ba^2-5\,Aab\right)\sqrt{b}\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-\left(3\,Bb^2x^4+(8\,Bab-5\,Ab^2\right.\right.}{15\,(b^4x^3+ab^3)}$$

[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] -2/15*(2*((8*B*a*b - 5*A*b^2)*x^3 + 8*B*a^2 - 5*A*a*b)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) - (3*B*b^2*x^4 + (8*B*a*b - 5*A*b^2)*x)*sqrt(b*x^3 + a))/(b^4*x^3 + a*b^3)

Sympy [A] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.30

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{Ax^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)}+\frac{Bx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{10}{3}\right)}$$

[In] integrate(x**3*(B*x**3+A)/(b*x**3+a)**(3/2),x)

[Out] A*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(7/3)) + B*x**7*gamma(7/3)*hyper((3/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(10/3))

Maxima [F]

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)x^3}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*x^3/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)x^3}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*x^3/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{x^3\,(B\,x^3+A)}{(b\,x^3+a)^{3/2}}\,dx$$

```
[In] int((x^3*(A + B*x^3))/(a + b*x^3)^(3/2),x)
[Out] int((x^3*(A + B*x^3))/(a + b*x^3)^(3/2), x)
```

3.236 $\int \frac{A+Bx^3}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 251

$$\int \frac{A+Bx^3}{(a+bx^3)^{3/2}}\,dx = \frac{2(Ab-aB)x}{3ab\sqrt{a+bx^3}} + \frac{2\sqrt{2+\sqrt{3}}(Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4}{3\sqrt[4]{3}ab^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/3*(A*b-B*a)*x/a/b/(b*x^3+a)^(1/2)+2/9*(A*b+2*B*a)*(a^(1/3)+b^(1/3)*x)*Ell
ipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^
(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a/b^(4/3)/(b*x^3+a)^(1/
2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used

$= \{393, 224\}$

$$\int \frac{A+Bx^3}{(a+bx^3)^{3/2}}\,dx = \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2aB+Ab)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\right.}{\sqrt[3]{b}x+\left(1+\right.}\right.\right.}{3\sqrt[4]{3}ab^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2x(Ab-aB)}{3ab\sqrt{a+bx^3}}$$

```
[In] Int[(A + B*x^3)/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(A*b - a*B)*x)/(3*a*b*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(A*b + 2*a
*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*
3^(1/4)*a*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\text{integral} = \frac{2(Ab-aB)x}{3ab\sqrt{a+bx^3}} + \frac{\left(2\left(\frac{Ab}{2}+aB\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{3ab}$$

$$= \frac{2(Ab-aB)x}{3ab\sqrt{a+bx^3}} + \frac{2\sqrt{2+\sqrt{3}}(Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\right.}{3\sqrt[4]{3}ab^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.29

$$\int \frac{A+Bx^3}{(a+bx^3)^{3/2}}\,dx = \frac{x\left(2Ab-2aB+(Ab+2aB)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)\right)}{3ab\sqrt{a+bx^3}}$$

```
[In] Integrate[(A + B*x^3)/(a + b*x^3)^(3/2),x]
[Out] (x*(2*A*b - 2*a*B + (A*b + 2*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3
, 1/2, 4/3, -((b*x^3)/a)]))/(3*a*b*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 4.30 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.34

method	result
elliptic	$\frac{2x(Ab-Ba)}{3ba\sqrt{(x^3+\frac{a}{b})b}}-\frac{2i\left(\frac{B}{b}+\frac{Ab-Ba}{3ab}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a}{}\right.}{}}}{3b\sqrt{b\,x}}$
default	$A\left(\frac{2x}{3a\sqrt{(x^3+\frac{a}{b})b}}-\frac{2i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{}}{}\right.}{(-a\,b^2}}}{9ab\sqrt{b\,x^3+}}\right.$

```
[In] int((B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/b*x/a*(A*b-B*a)/((x^3+a/b)*b)^(1/2)-2/3*I*(B/b+1/3*(A*b-B*a)/a/b)*3^(1/
2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(
1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.33

$$\int \frac{A+Bx^3}{(a+bx^3)^{3/2}}\,dx =$$
$$-\frac{2\left(\sqrt{bx^3+a}(Bab-Ab^2)x-((2\,Bab+Ab^2)x^3+2\,Ba^2+Aab)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)}{3\,(ab^3x^3+a^2b^2)}$$

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(b*x^3 + a)*(B*a*b - A*b^2)*x - ((2*B*a*b + A*b^2)*x^3 + 2*B*a^2
+ A*a*b)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x))/(a*b^3*x^3 + a^2*b^2)
```

Sympy [A] (verification not implemented)

Time = 2.79 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.31

$$\int \frac{A+Bx^3}{(a+bx^3)^{3/2}}\,dx = \frac{Ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{Bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)}$$

```
[In] integrate((B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] A*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3
/2)*gamma(4/3)) + B*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(3/2)*gamma(7/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{(a+bx^3)^{3/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{(a+bx^3)^{3/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{B\,x^3+A}{\left(b\,x^3+a\right)^{3/2}}\,dx$$

```
[In] int((A + B*x^3)/(a + b*x^3)^(3/2),x)
[Out] int((A + B*x^3)/(a + b*x^3)^(3/2), x)
```

3.237 $\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 272

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{3/2}}\,dx=-\frac{A}{2ax^2\sqrt{a+bx^3}}-\frac{(7Ab-4aB)x}{6a^2\sqrt{a+bx^3}}$$
$$-\frac{\sqrt{2+\sqrt{3}}(7Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4}{6\sqrt[4]{3}a^2\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/2*A/a/x^2/(b*x^3+a)^(1/2)-1/6*(7*A*b-4*B*a)*x/a^2/(b*x^3+a)^(1/2)-1/18*(
7*A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/
(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(
1/2)*3^(3/4)/a^2/b^(1/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used

$= \{464, 205, 224\}$

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{3/2}}\,dx = -\frac{x(7Ab-4aB)}{6a^2\sqrt{a+bx^3}} - \frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(7Ab-4aB)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{6\sqrt[4]{3}a^2\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{A}{2ax^2\sqrt{a+bx^3}}$$

```
[In] Int[(A + B*x^3)/(x^3*(a + b*x^3)^(3/2)),x]
```

```
[Out] -1/2*A/(a*x^2*Sqrt[a + b*x^3]) - ((7*A*b - 4*a*B)*x)/(6*a^2*Sqrt[a + b*x^3]
) - (Sqrt[2 + Sqrt[3]]*(7*A*b - 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(6*3^(1/4)*a^2*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{A}{2ax^2\sqrt{a+bx^3}} - \frac{\left(\frac{7Ab}{2}-2aB\right)\int \frac{1}{(a+bx^3)^{3/2}}\,dx}{2a}$$

$$= -\frac{A}{2ax^2\sqrt{a+bx^3}} - \frac{(7Ab-4aB)x}{6a^2\sqrt{a+bx^3}} - \frac{(7Ab-4aB)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{12a^2}$$

$$= -\frac{A}{2ax^2\sqrt{a+bx^3}} - \frac{(7Ab-4aB)x}{6a^2\sqrt{a+bx^3}} - \frac{\sqrt{2+\sqrt{3}}(7Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\right.}{6\sqrt[4]{3}a^2\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.32

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{3/2}}\,dx = \frac{-6aA-14Abx^3+8aBx^3+(-7Ab+4aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3}\right.}{12a^2x^2\sqrt{a+bx^3}}$$

```
[In] Integrate[(A + B*x^3)/(x^3*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-6*a*A - 14*A*b*x^3 + 8*a*B*x^3 + (-7*A*b + 4*a*B)*x^3*Sqrt[1 + (b*x^3)/a]
*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)])/(12*a^2*x^2*Sqrt[a + b*x^3
])
```

Maple [A] (verified)

Time = 4.67 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.29

method	result
elliptic	$-\frac{2x(Ab-Ba)}{3a^2\sqrt{(x^3+\frac{a}{b})b}}-\frac{A\sqrt{bx^3+a}}{2a^2x^2}-\frac{2i\left(-\frac{Ab-Ba}{3a^2}-\frac{Ab}{4a^2}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-}{2}}}}{}$
default	$B\left(\frac{2x}{3a\sqrt{(x^3+\frac{a}{b})b}}-\frac{2i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{}}{}\right.}{(-a\,b^2}}}{9ab\sqrt{bx^3-}}\right.$
risch	Expression too large to display

```
[In] int((B*x^3+A)/x^3/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*x/a^2*(A*b-B*a)/((x^3+a/b)*b)^(1/2)-1/2/a^2*A*(b*x^3+a)^(1/2)/x^2-2/3*
I*(-1/3*(A*b-B*a)/a^2-1/4/a^2*A*b)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.38

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{3/2}}\,dx = \frac{((4\,Bab-7\,Ab^2)x^5+(4\,Ba^2-7\,Aab)x^2)\sqrt{b}\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+((4\,B}{6\,(a^2b^2x^5+a^3bx^2)}$$

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(((4*B*a*b - 7*A*b^2)*x^5 + (4*B*a^2 - 7*A*a*b)*x^2)*sqrt(b)*weierstras
sPInverse(0, -4*a/b, x) + ((4*B*a*b - 7*A*b^2)*x^3 - 3*A*a*b)*sqrt(b*x^3 +
a))/(a^2*b^2*x^5 + a^3*b*x^2)
```

Sympy [A] (verification not implemented)

Time = 8.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.30

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{3/2}}\,dx = \frac{A\Gamma\left(-\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{3}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^2\Gamma\left(\frac{1}{3}\right)}+\frac{Bx\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((B*x**3+A)/x**3/(b*x**3+a)**(3/2),x)
```

```
[Out] A*gamma(-2/3)*hyper((-2/3, 3/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3
/2)*x**2*gamma(1/3)) + B*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(3/2)*gamma(4/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{3/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{3}{2}}x^3}\,dx$$

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{3/2}}\,dx = \int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}x^3}\,dx$$

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{3/2}}\,dx = \int \frac{B\,x^3+A}{x^3\,\left(b\,x^3+a\right)^{3/2}}\,dx$$

```
[In] int((A + B*x^3)/(x^3*(a + b*x^3)^(3/2)),x)
[Out] int((A + B*x^3)/(x^3*(a + b*x^3)^(3/2)), x)
```

3.238 $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 304

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{3/2}}\,dx=-\frac{A}{5ax^5\sqrt{a+bx^3}}-\frac{13Ab-10aB}{15a^2x^2\sqrt{a+bx^3}}+\frac{7(13Ab-10aB)\sqrt{a+bx^3}}{60a^3x^2}+\frac{7\sqrt{2+\sqrt{3}}b^{2/3}(13Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{60\sqrt[4]{3}a^3\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/5*A/a/x^5/(b*x^3+a)^(1/2)+1/15*(-13*A*b+10*B*a)/a^2/x^2/(b*x^3+a)^(1/2)+
7/60*(13*A*b-10*B*a)*(b*x^3+a)^(1/2)/a^3/x^2+7/180*b^(2/3)*(13*A*b-10*B*a)*
(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/
a^3/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {464, 296, 331, 224}

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{3/2}}\,dx=\frac{7\sqrt{a+bx^3}(13Ab-10aB)}{60a^3x^2}-\frac{13Ab-10aB}{15a^2x^2\sqrt{a+bx^3}}+\frac{7\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(13Ab-10aB)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-\right.}{60\sqrt[4]{3}a^3\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{A}{5ax^5\sqrt{a+bx^3}}$$

```
[In] Int[(A + B*x^3)/(x^6*(a + b*x^3)^(3/2)),x]
```

```
[Out] -1/5*A/(a*x^5*Sqrt[a + b*x^3]) - (13*A*b - 10*a*B)/(15*a^2*x^2*Sqrt[a + b*x
^3]) + (7*(13*A*b - 10*a*B)*Sqrt[a + b*x^3])/(60*a^3*x^2) + (7*Sqrt[2 + Sqr
t[3]]*b^(2/3)*(13*A*b - 10*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elliptic
F[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)], -7 - 4*Sqrt[3]])/(60*3^(1/4)*a^3*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x
))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{A}{5ax^5\sqrt{a+bx^3}} - \frac{\left(\frac{13Ab}{2}-5aB\right)\int \frac{1}{x^3(a+bx^3)^{3/2}}\,dx}{5a} \\
&= -\frac{A}{5ax^5\sqrt{a+bx^3}} - \frac{13Ab-10aB}{15a^2x^2\sqrt{a+bx^3}} - \frac{(7(13Ab-10aB))\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx}{30a^2} \\
&= -\frac{A}{5ax^5\sqrt{a+bx^3}} - \frac{13Ab-10aB}{15a^2x^2\sqrt{a+bx^3}} \\
&\quad + \frac{7(13Ab-10aB)\sqrt{a+bx^3}}{60a^3x^2} + \frac{(7b(13Ab-10aB))\int \frac{1}{\sqrt{a+bx^3}}\,dx}{120a^3} \\
&= -\frac{A}{5ax^5\sqrt{a+bx^3}} - \frac{13Ab-10aB}{15a^2x^2\sqrt{a+bx^3}} + \frac{7(13Ab-10aB)\sqrt{a+bx^3}}{60a^3x^2} \\
&\quad + \frac{7\sqrt{2+\sqrt{3}}b^{2/3}(13Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{60\sqrt[4]{3}a^3\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.24

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{3/2}}\,dx = \frac{-4aA+(13Ab-10aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{3}{2},\frac{1}{3},-\frac{bx^3}{a}\right)}{20a^2x^5\sqrt{a+bx^3}}$$

```
[In] Integrate[(A + B*x^3)/(x^6*(a + b*x^3)^(3/2)),x]
[Out] (-4*a*A + (13*A*b - 10*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-2/3,
 3/2, 1/3, -((b*x^3)/a)])/(20*a^2*x^5*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 4.84 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.27

method	result
elliptic	$\frac{2bx(Ab-Ba)}{3a^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{A\sqrt{b\,x^3+a}}{5a^2x^5}+\frac{(17Ab-10Ba)\sqrt{b\,x^3+a}}{20a^3x^2}-\frac{2i\left(\frac{(Ab-Ba)b}{3a^3}+\frac{b(17Ab-10Ba)}{40a^3}\right)\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-\cdots\right)}{2b}\cdots\right)}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\cdots}{\cdots}$
default	$B\left(-\frac{2bx}{3a^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{\sqrt{b\,x^3+a}}{2a^2x^2}+\frac{7i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{\cdots}}{\cdots}\cdots\right.$
risch	Expression too large to display

```
[In] int((B*x^3+A)/x^6/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*b*x/a^3*(A*b-B*a)/((x^3+a/b)*b)^(1/2)-1/5*A/a^2*(b*x^3+a)^(1/2)/x^5+1/2
0/a^3*(17*A*b-10*B*a)*(b*x^3+a)^(1/2)/x^2-2/3*I*(1/3*(A*b-B*a)*b/a^3+1/40*b
*(17*A*b-10*B*a)/a^3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*
b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(
-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.39

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{3/2}}\,dx = -\frac{7\left((10\,Bab-13\,Ab^2)x^8+(10\,Ba^2-13\,Aab)x^5\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+\left(7\left(10\,Bab-13\,Ab^2\right)\right.}{60\left(a^3bx^8+a^4x^5\right)}$$

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/60*(7*((10*B*a*b - 13*A*b^2)*x^8 + (10*B*a^2 - 13*A*a*b)*x^5)*sqrt(b)*we
ierstrassPInverse(0, -4*a/b, x) + (7*(10*B*a*b - 13*A*b^2)*x^6 + 3*(10*B*a^
2 - 13*A*a*b)*x^3 + 12*A*a^2)*sqrt(b*x^3 + a))/(a^3*b*x^8 + a^4*x^5)
```

Sympy [A] (verification not implemented)

Time = 24.21 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.30

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{3/2}}\,dx = \frac{A\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},\frac{3}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^5\Gamma\left(-\frac{2}{3}\right)}+\frac{B\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{3}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^2\Gamma\left(\frac{1}{3}\right)}$$

```
[In] integrate((B*x**3+A)/x**6/(b*x**3+a)**(3/2),x)
```

```
[Out] A*gamma(-5/3)*hyper((-5/3, 3/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
3/2)*x**5*gamma(-2/3)) + B*gamma(-2/3)*hyper((-2/3, 3/2), (1/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(3/2)*x**2*gamma(1/3))
```

Maxima [F]

$$\int \frac{A + Bx^3}{x^6 (a + bx^3)^{3/2}}\,dx = \int \frac{Bx^3 + A}{(bx^3 + a)^{\frac{3}{2}} x^6}\,dx$$

[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(3/2),x, algorithm="maxima")

[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^6), x)

Giac [F]

$$\int \frac{A + Bx^3}{x^6 (a + bx^3)^{3/2}}\,dx = \int \frac{Bx^3 + A}{(bx^3 + a)^{\frac{3}{2}} x^6}\,dx$$

[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(3/2),x, algorithm="giac")

[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^6), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^3}{x^6 (a + bx^3)^{3/2}}\,dx = \int \frac{B\,x^3 + A}{x^6\,(b\,x^3 + a)^{3/2}}\,dx$$

[In] int((A + B*x^3)/(x^6*(a + b*x^3)^(3/2)),x)

[Out] int((A + B*x^3)/(x^6*(a + b*x^3)^(3/2)), x)

3.239 $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 547

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = -\frac{2(7Ab-10aB)x^2}{21b^2\sqrt{a+bx^3}}+\frac{2Bx^5}{7b\sqrt{a+bx^3}}+\frac{8(7Ab-10aB)\sqrt{a+bx^3}}{21b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{4\sqrt{2-\sqrt{3}}\sqrt[3]{a}(7Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{\ldots}\right.}{7\,3^{3/4}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{8\sqrt{2}\sqrt[3]{a}(7Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{\ldots}\right.}{21\sqrt[4]{3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -2/21*(7*A*b-10*B*a)*x^2/b^2/(b*x^3+a)^(1/2)+2/7*B*x^5/b/(b*x^3+a)^(1/2)+8/
21*(7*A*b-10*B*a)*(b*x^3+a)^(1/2)/b^(8/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+8
/63*a^(1/3)*(7*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)
*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2
/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
*3^(3/4)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))^2)^(1/2)-4/21*a^(1/3)*(7*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)
*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),
I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3
```

)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/b^(8/3)/(b*x^3+a)^(
1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {470, 294, 309, 224, 1891}

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{8\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(7Ab-10aB)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{21\sqrt[4]{3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{4\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(7Ab-10aB)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7\ 3^{3/4}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{8\sqrt{a+bx^3}(7Ab-10aB)}{21b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{2x^2(7Ab-10aB)}{21b^2\sqrt{a+bx^3}} + \frac{2Bx^5}{7b\sqrt{a+bx^3}}$$

```
[In] Int[(x^4*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*(7*A*b - 10*a*B)*x^2)/(21*b^2*Sqrt[a + b*x^3]) + (2*B*x^5)/(7*b*Sqrt[a
+ b*x^3]) + (8*(7*A*b - 10*a*B)*Sqrt[a + b*x^3])/(21*b^(8/3)*((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)) - (4*Sqrt[2 - Sqrt[3]]*a^(1/3)*(7*A*b - 10*a*B)*(a^(
1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(3/4)*
b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*Sqrt[a + b*x^3]) + (8*Sqrt[2]*a^(1/3)*(7*A*b - 10*a*B)*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(21*3^(1/4)*b^(8/3
)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2Bx^5}{7b\sqrt{a+bx^3}} - \frac{\left(2\left(-\frac{7Ab}{2}+5aB\right)\right)\int \frac{x^4}{(a+bx^3)^{3/2}}\,dx}{7b} \\ &= -\frac{2(7Ab-10aB)x^2}{21b^2\sqrt{a+bx^3}} + \frac{2Bx^5}{7b\sqrt{a+bx^3}} + \frac{(4(7Ab-10aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{21b^2}\end{aligned}$$

$$= -\frac{2(7Ab-10aB)x^2}{21b^2\sqrt{a+bx^3}} + \frac{2Bx^5}{7b\sqrt{a+bx^3}} + \frac{(4(7Ab-10aB))\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{21b^{7/3}} - \frac{\left(4\left(1-\sqrt{3}\right)\sqrt[3]{a}(7Ab-10aB)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{21b^{7/3}}$$

$$= -\frac{2(7Ab-10aB)x^2}{21b^2\sqrt{a+bx^3}} + \frac{2Bx^5}{7b\sqrt{a+bx^3}} + \frac{8(7Ab-10aB)\sqrt{a+bx^3}}{21b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{4\sqrt{2-\sqrt{3}}\sqrt[3]{a}(7Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-}{7\ 3^{3/4}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{8\sqrt{2}\sqrt[3]{a}(7Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4}{21\sqrt[4]{3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.14

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2x^2\left(7Ab-10aB+bBx^3+(-7Ab+10aB)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{3}{2},\frac{5}{3},-\frac{b}{}\right.\right.}{7b^2\sqrt{a+bx^3}}$$

```
[In] Integrate[(x^4*(A + B*x^3))/(a + b*x^3)^(3/2),x]
[Out] (2*x^2*(7*A*b - 10*a*B + b*B*x^3 + (-7*A*b + 10*a*B)*Sqrt[1 + (b*x^3)/a]*Hy
pergeometric2F1[2/3, 3/2, 5/3, -((b*x^3)/a)]))/(7*b^2*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 5.06 (sec) , antiderivative size = 504, normalized size of antiderivative = 0.92

method	result
	$2i\left(\frac{\frac{4Ab}{3}-\frac{4Ba}{3}}{b^{2}}-\frac{4Ba}{7b^{2}}\right)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\left(\right.}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}$
elliptic	$-\frac{2x^{2}(Ab-Ba)}{3b^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2B\,x^{2}\sqrt{b\,x^{3}+a}}{7b^{2}}-$
default	Expression too large to display
risch	Expression too large to display

```
[In] int(x^4*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/b^2*x^2*(A*b-B*a)/((x^3+a/b)*b)^(1/2)+2/7*B*x^2/b^2*(b*x^3+a)^(1/2)-2/
3*I*(4/3*(A*b-B*a)/b^2-4/7*B/b^2*a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)
/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.19

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2\left(4\left((10\,Bab-7\,Ab^2)x^3+10\,Ba^2-7\,Aab\right)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInv}\right.}{21\,(b^4x^3+ab^3)}$$

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*(4*((10*B*a*b - 7*A*b^2)*x^3 + 10*B*a^2 - 7*A*a*b)*sqrt(b)*weierstrass
Zeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (3*B*b^2*x^5 + (10*B*a
*b - 7*A*b^2)*x^2)*sqrt(b*x^3 + a))/(b^4*x^3 + a*b^3)
```

Sympy [A] (verification not implemented)

Time = 4.84 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.15

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{Ax^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)}+\frac{Bx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{11}{3}\right)}$$

```
[In] integrate(x**4*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] A*x**5*gamma(5/3)*hyper((3/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(3/2)*gamma(8/3)) + B*x**8*gamma(8/3)*hyper((3/2, 8/3), (11/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(3/2)*gamma(11/3))
```

Maxima [F]

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)x^4}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^4/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)x^4}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*x^4/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{x^4\,(B\,x^3+A)}{(b\,x^3+a)^{3/2}}\,dx$$

```
[In] int((x^4*(A + B*x^3))/(a + b*x^3)^(3/2),x)
[Out] int((x^4*(A + B*x^3))/(a + b*x^3)^(3/2), x)
```

3.240 $\int \frac{x(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 524

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2(Ab-aB)x^2}{3ab\sqrt{a+bx^3}} - \frac{2(Ab-4aB)\sqrt{a+bx^3}}{3ab^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{\sqrt{2-\sqrt{3}}(Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{2\sqrt{2}(Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/3*(A*b-B*a)*x^2/a/b/(b*x^3+a)^(1/2)-2/3*(A*b-4*B*a)*(b*x^3+a)^(1/2)/a/b^(
5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-2/9*(A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*El
lipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3
^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(2/3)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)
*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/3*(A*b-4*B*
a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/
```

```
4)/a^(2/3)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {468, 309, 224, 1891}

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx =$$

$$-\frac{2\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-4aB)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-4aB)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2\sqrt{a+bx^3}(Ab-4aB)}{3ab^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2x^2(Ab-aB)}{3ab\sqrt{a+bx^3}}$$

```
[In] Int[(x*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(A*b - a*B)*x^2)/(3*a*b*Sqrt[a + b*x^3]) - (2*(A*b - 4*a*B)*Sqrt[a + b*x
^3])/(3*a*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (Sqrt[2 - Sqrt[3]]
*(A*b - 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sq
rt[3]])/(3^(3/4)*a^(2/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (2*Sqrt[2]*(A*b - 4*a*
B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/
3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3
^(1/4)*a^(2/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{2(Ab-aB)x^2}{3ab\sqrt{a+bx^3}} + \frac{\left(2\left(-\frac{Ab}{2}+2aB\right)\right)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{3ab}$$

$$= \frac{2(Ab-aB)x^2}{3ab\sqrt{a+bx^3}} - \frac{(Ab-4aB)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{3ab^{4/3}} + \frac{\left(\left(1-\sqrt{3}\right)(Ab-4aB)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{3a^{2/3}b^{4/3}}$$

$$= \frac{2(Ab-aB)x^2}{3ab\sqrt{a+bx^3}} - \frac{2(Ab-4aB)\sqrt{a+bx^3}}{3ab^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$+ \frac{\sqrt{2-\sqrt{3}}(Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4}{3^{3/4}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$- \frac{2\sqrt{2}(Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.14

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{x^2\left(4aB+(Ab-4aB)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{2ab\sqrt{a+bx^3}}$$

```
[In] Integrate[(x*(A + B*x^3))/(a + b*x^3)^(3/2),x]
[Out] (x^2*(4*a*B + (A*b - 4*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2,
 5/3, -((b*x^3)/a)]))/(2*a*b*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 4.18 (sec) , antiderivative size = 490, normalized size of antiderivative = 0.94

method	result
	$2i\left(\frac{B}{b}-\frac{Ab-Ba}{3ab}\right)\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\,\sqrt{-\frac{i\left(x+\frac{\left(-a\right.}{}\right.}{}}$
elliptic	$\frac{2x^{2}(Ab-Ba)}{3ba\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-$
default	Expression too large to display

```
[In] int(x*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/3/b*x^2/a*(A*b-B*a)/((x^3+a/b)*b)^(1/2)-2/3*I*(B/b-1/3*(A*b-B*a)/a/b)*3^(
1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)
^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*
3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.18

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx =$$

$$-\frac{2\left(\sqrt{bx^3+a}(Bab-Ab^2)x^2+((4\,Bab-Ab^2)x^3+4\,Ba^2-Aab)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPIn}\right.\right.}{3\left(ab^3x^3+a^2b^2\right)}$$

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(b*x^3 + a)*(B*a*b - A*b^2)*x^2 + ((4*B*a*b - A*b^2)*x^3 + 4*B*a^
2 - A*a*b)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b
, x)))/(a*b^3*x^3 + a^2*b^2)
```

Sympy [A] (verification not implemented)

Time = 2.80 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.15

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{Ax^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\left.\begin{matrix}\frac{2}{3}, \frac{3}{2}\\ \frac{5}{3}\end{matrix}\right| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)} + \frac{Bx^5\Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\left.\begin{matrix}\frac{3}{2}, \frac{5}{3}\\ \frac{8}{3}\end{matrix}\right| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)}$$

```
[In] integrate(x*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] A*x**2*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(3/2)*gamma(5/3)) + B*x**5*gamma(5/3)*hyper((3/2, 5/3), (8/3,), b*x**3*exp
_polar(I*pi)/a)/(3*a**(3/2)*gamma(8/3))
```

Maxima [F]

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)x}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)x}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx=\int \frac{x\,(B\,x^3+A)}{(b\,x^3+a)^{3/2}}\,dx$$

```
[In] int((x*(A + B*x^3))/(a + b*x^3)^(3/2),x)
[Out] int((x*(A + B*x^3))/(a + b*x^3)^(3/2), x)
```

3.241 $\int \frac{A+Bx^3}{x^2(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 548

$$\int \frac{A+Bx^3}{x^2(a+bx^3)^{3/2}}\,dx = -\frac{A}{ax\sqrt{a+bx^3}} - \frac{(5Ab-2aB)x^2}{3a^2\sqrt{a+bx^3}} + \frac{(5Ab-2aB)\sqrt{a+bx^3}}{3a^2b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt{2-\sqrt{3}}(5Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{2\ 3^{3/4}a^{5/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{\sqrt{2}(5Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a^{5/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -A/a/x/(b*x^3+a)^(1/2)-1/3*(5*A*b-2*B*a)*x^2/a^2/(b*x^3+a)^(1/2)+1/3*(5*A*b
-2*B*a)*(b*x^3+a)^(1/2)/a^2/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+1/9*(5*
A*b-2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b
^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(5/3)
/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2)))^2)^(1/2)-1/6*(5*A*b-2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3
)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/
2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(5/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1
/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {464, 296, 309, 224, 1891}

$$\int \frac{A+Bx^3}{x^2(a+bx^3)^{3/2}}\,dx = \frac{\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-2aB)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{3\sqrt[4]{3}a^{5/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-2aB)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2\ 3^{3/4}a^{5/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{\sqrt{a+bx^3}(5Ab-2aB)}{3a^2b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{x^2(5Ab-2aB)}{3a^2\sqrt{a+bx^3}}-\frac{A}{ax\sqrt{a+bx^3}}$$

```
[In] Int[(A + B*x^3)/(x^2*(a + b*x^3)^(3/2)),x]
```

```
[Out] -(A/(a*x*Sqrt[a + b*x^3])) - ((5*A*b - 2*a*B)*x^2)/(3*a^2*Sqrt[a + b*x^3])
+ ((5*A*b - 2*a*B)*Sqrt[a + b*x^3])/(3*a^2*b^(2/3)*((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)) - (Sqrt[2 - Sqrt[3]]*(5*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(3/4)*a^(5/3)*b^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3]) + (Sqrt[2]*(5*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(5/3)*b^(2/3)*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3
])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{A}{ax\sqrt{a+bx^3}} - \frac{\left(\frac{5Ab}{2}-aB\right)\int \frac{x}{(a+bx^3)^{3/2}}\,dx}{a} \\
&= -\frac{A}{ax\sqrt{a+bx^3}} - \frac{(5Ab-2aB)x^2}{3a^2\sqrt{a+bx^3}} + \frac{(5Ab-2aB)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{6a^2}
\end{aligned}$$

$$= -\frac{A}{ax\sqrt{a+bx^3}} - \frac{(5Ab-2aB)x^2}{3a^2\sqrt{a+bx^3}} + \frac{(5Ab-2aB)\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{6a^2\sqrt[3]{b}} - \frac{\left((1-\sqrt{3})\,(5Ab-2aB)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{6a^{5/3}\sqrt[3]{b}}$$

$$= -\frac{A}{ax\sqrt{a+bx^3}} - \frac{(5Ab-2aB)x^2}{3a^2\sqrt{a+bx^3}} + \frac{(5Ab-2aB)\sqrt{a+bx^3}}{3a^2b^{2/3}\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt{2-\sqrt{3}}(5Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{2\,3^{3/4}a^{5/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{\sqrt{2}(5Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a^{5/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.13

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{3/2}}\,dx = \frac{-4aA+(-5Ab+2aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a}\right)}{4a^2x\sqrt{a+bx^3}}$$

```
[In] Integrate[(A + B*x^3)/(x^2*(a + b*x^3)^(3/2)),x]
[Out] (-4*a*A + (-5*A*b + 2*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3
/2, 5/3, -((b*x^3)/a)])/(4*a^2*x*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 4.95 (sec) , antiderivative size = 504, normalized size of antiderivative = 0.92

method	result
	$2i\left(\frac{Ab-Ba}{3a^2}+\frac{Ab}{2a^2}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-}{2}}}$
elliptic	$-\frac{2x^2(Ab-Ba)}{3a^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{A\sqrt{b\,x^3+a}}{a^2x}-$
default	Expression too large to display
risch	Expression too large to display

```
[In] int((B*x^3+A)/x^2/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*x^2/a^2*(A*b-B*a)/((x^3+a/b)*b)^(1/2)-1/a^2*A*(b*x^3+a)^(1/2)/x-2/3*I*
(1/3*(A*b-B*a)/a^2+1/2/a^2*A*b)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((
x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(
-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
)+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a
*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.19

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{3/2}}\,dx = \frac{((2\,Bab-5\,Ab^2)x^4+(2\,Ba^2-5\,Aab)x)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInve}\right.}{3\,(a^2b^2x^4+a^3bx)}$$

[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] 1/3*(((2*B*a*b - 5*A*b^2)*x^4 + (2*B*a^2 - 5*A*a*b)*x)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + ((2*B*a*b - 5*A*b^2)*x^3 - 3*A*a*b)*sqrt(b*x^3 + a))/(a^2*b^2*x^4 + a^3*b*x)

Sympy [A] (verification not implemented)

Time = 6.36 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.15

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{3/2}}\,dx = \frac{A\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{3}{2}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x\Gamma\left(\frac{2}{3}\right)}+\frac{Bx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{3}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)}$$

[In] integrate((B*x**3+A)/x**2/(b*x**3+a)**(3/2),x)

[Out] A*gamma(-1/3)*hyper((-1/3, 3/2), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*x*gamma(2/3)) + B*x**2*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(5/3))

Maxima [F]

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{3/2}}\,dx = \int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}x^2}\,dx$$

[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(3/2),x, algorithm="maxima")

[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^2), x)

Giac [F]

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{3/2}}\,dx = \int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{3/2}}\,dx = \int \frac{B\,x^3+A}{x^2\,\left(b\,x^3+a\right)^{3/2}}\,dx$$

```
[In] int((A + B*x^3)/(x^2*(a + b*x^3)^(3/2)),x)
[Out] int((A + B*x^3)/(x^2*(a + b*x^3)^(3/2)), x)
```

3.242 $\int \frac{A+Bx^3}{x^5(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 580

$$\int \frac{A+Bx^3}{x^5(a+bx^3)^{3/2}}\,dx = -\frac{A}{4ax^4\sqrt{a+bx^3}}-\frac{11Ab-8aB}{12a^2x\sqrt{a+bx^3}}+\frac{5(11Ab-8aB)\sqrt{a+bx^3}}{24a^3x}-\frac{5\sqrt[3]{b}(11Ab-8aB)\sqrt{a+bx^3}}{24a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{5\sqrt{2-\sqrt{3}}\sqrt[3]{b}(11Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{16\,3^{3/4}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{5\sqrt[3]{b}(11Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{12\sqrt{2}\sqrt[4]{3}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/4*A/a/x^4/(b*x^3+a)^(1/2)+1/12*(-11*A*b+8*B*a)/a^2/x/(b*x^3+a)^(1/2)+5/2
4*(11*A*b-8*B*a)*(b*x^3+a)^(1/2)/a^3/x-5/24*b^(1/3)*(11*A*b-8*B*a)*(b*x^3+a
)^(1/2)/a^3/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-5/72*b^(1/3)*(11*A*b-8*B*a)*(a^
(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(8/3)*2^(1/2)/(b*x^3+a)^(1/
2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+5/
```

```
48*b^(1/3)*(11*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*
(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/
2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3
^(1/2)))^2)^(1/2)*3^(1/4)/a^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)
*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {464, 296, 331, 309, 224, 1891}

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{3/2}}\,dx = -\frac{5\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-8aB)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{12\sqrt{2}\sqrt[4]{3}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{5\sqrt{2-\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-8aB)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{16\,3^{3/4}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{5\sqrt{a+bx^3}(11Ab-8aB)}{24a^3x}-\frac{5\sqrt[3]{b}\sqrt{a+bx^3}(11Ab-8aB)}{24a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{11Ab-8aB}{12a^2x\sqrt{a+bx^3}}-\frac{A}{4ax^4\sqrt{a+bx^3}}$$

```
[In] Int[(A + B*x^3)/(x^5*(a + b*x^3)^(3/2)),x]
```

```
[Out] -1/4*A/(a*x^4*Sqrt[a + b*x^3]) - (11*A*b - 8*a*B)/(12*a^2*x*Sqrt[a + b*x^3]
) + (5*(11*A*b - 8*a*B)*Sqrt[a + b*x^3])/(24*a^3*x) - (5*b^(1/3)*(11*A*b -
8*a*B)*Sqrt[a + b*x^3])/(24*a^3*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (5*S
qrt[2 - Sqrt[3]]*b^(1/3)*(11*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*3^(3/4)*a^(8/3)*Sqrt[(a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) -
(5*b^(1/3)*(11*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[Ar
cSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
```

```
)], -7 - 4*Sqrt[3]])/(12*Sqrt[2]*3^(1/4)*a^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b
^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{A}{4ax^4\sqrt{a+bx^3}} - \frac{\left(\frac{11Ab}{2}-4aB\right)\int \frac{1}{x^2(a+bx^3)^{3/2}}\,dx}{4a}$$

$$= -\frac{A}{4ax^4\sqrt{a+bx^3}} - \frac{11Ab-8aB}{12a^2x\sqrt{a+bx^3}} - \frac{(5(11Ab-8aB))\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx}{24a^2}$$

$$= -\frac{A}{4ax^4\sqrt{a+bx^3}} - \frac{11Ab-8aB}{12a^2x\sqrt{a+bx^3}} + \frac{5(11Ab-8aB)\sqrt{a+bx^3}}{24a^3x} - \frac{(5b(11Ab-8aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{48a^3}$$

$$= -\frac{A}{4ax^4\sqrt{a+bx^3}} - \frac{11Ab-8aB}{12a^2x\sqrt{a+bx^3}} + \frac{5(11Ab-8aB)\sqrt{a+bx^3}}{24a^3x} - \frac{\left(5b^{2/3}(11Ab-8aB)\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{48a^3} + \frac{\left(5\left(1-\sqrt{3}\right)b^{2/3}(11Ab-8aB)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{48a^{8/3}}$$

$$= -\frac{A}{4ax^4\sqrt{a+bx^3}} - \frac{11Ab-8aB}{12a^2x\sqrt{a+bx^3}} + \frac{5(11Ab-8aB)\sqrt{a+bx^3}}{24a^3x} - \frac{5\sqrt[3]{b}(11Ab-8aB)\sqrt{a+bx^3}}{24a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+ \frac{5\sqrt{2-\sqrt{3}}\sqrt[3]{b}(11Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|\right.}{16\ 3^{3/4}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$- \frac{5\sqrt[3]{b}(11Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right.}{12\sqrt{2}\sqrt[4]{3}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.12

$$\int \frac{A+Bx^3}{x^5 (a+bx^3)^{3/2}}\,dx = \frac{-2aA+(11Ab-8aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{3}{2},\frac{2}{3},-\frac{bx^3}{a}\right)}{8a^2x^4\sqrt{a+bx^3}}$$

```
[In] Integrate[(A + B*x^3)/(x^5*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-2*a*A + (11*A*b - 8*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/3,
3/2, 2/3, -((b*x^3)/a)])/(8*a^2*x^4*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 5.36 (sec) , antiderivative size = 540, normalized size of antiderivative = 0.93

method	result
elliptic	$\frac{2b x^2 (Ab-Ba)}{3a^3\sqrt{(x^3+\frac{a}{b})b}} - \frac{A\sqrt{b x^3+a}}{4a^2x^4} + \frac{(13Ab-8Ba)\sqrt{b x^3+a}}{8a^3 x} - \frac{2i\left(-\frac{(Ab-Ba)b}{3a^3}-\frac{b(13Ab-8Ba)}{16a^3}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a}{2b}\right.}{(-a\,b^2)^{\frac{1}{3}}}}}{}$
default	Expression too large to display
risch	Expression too large to display

```
[In] int((B*x^3+A)/x^5/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*b*x^2/a^3*(A*b-B*a)/((x^3+a/b)*b)^(1/2)-1/4/a^2*A*(b*x^3+a)^(1/2)/x^4+1
/8/a^3*(13*A*b-8*B*a)*(b*x^3+a)^(1/2)/x-2/3*I*(-1/3*(A*b-B*a)*b/a^3-1/16*b*
(13*A*b-8*B*a)/a^3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^
2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I
*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
```

```
^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^
2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.22

$$\int \frac{A+Bx^3}{x^5\,(a+bx^3)^{3/2}}\,dx = -\frac{5\,((8\,Bab-11\,Ab^2)x^7+(8\,Ba^2-11\,Aab)x^4)\sqrt{b}\,\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)}{24\,(a^3bx^7+a^4x^4)}$$

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/24*(5*((8*B*a*b - 11*A*b^2)*x^7 + (8*B*a^2 - 11*A*a*b)*x^4)*sqrt(b)*weie
rstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (5*(8*B*a*b - 1
1*A*b^2)*x^6 + 3*(8*B*a^2 - 11*A*a*b)*x^3 + 6*A*a^2)*sqrt(b*x^3 + a))/(a^3*
b*x^7 + a^4*x^4)
```

Sympy [A] (verification not implemented)

Time = 16.46 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.15

$$\int \frac{A+Bx^3}{x^5\,(a+bx^3)^{3/2}}\,dx = \frac{A\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{4}{3},\frac{3}{2}\\-\frac{1}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^4\Gamma\left(-\frac{1}{3}\right)}+\frac{B\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{3}{2}\\\frac{2}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x\Gamma\left(\frac{2}{3}\right)}$$

```
[In] integrate((B*x**3+A)/x**5/(b*x**3+a)**(3/2),x)
```

```
[Out] A*gamma(-4/3)*hyper((-4/3, 3/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
3/2)*x**4*gamma(-1/3)) + B*gamma(-1/3)*hyper((-1/3, 3/2), (2/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(3/2)*x*gamma(2/3))
```

Maxima [F]

$$\int \frac{A + Bx^3}{x^5 (a + bx^3)^{3/2}} \, dx = \int \frac{Bx^3 + A}{(bx^3 + a)^{\frac{3}{2}} x^5} \, dx$$

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^5), x)
```

Giac [F]

$$\int \frac{A + Bx^3}{x^5 (a + bx^3)^{3/2}} \, dx = \int \frac{Bx^3 + A}{(bx^3 + a)^{\frac{3}{2}} x^5} \, dx$$

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^3}{x^5 (a + bx^3)^{3/2}} \, dx = \int \frac{B\,x^3 + A}{x^5\,(b\,x^3 + a)^{3/2}} \, dx$$

```
[In] int((A + B*x^3)/(x^5*(a + b*x^3)^(3/2)),x)
[Out] int((A + B*x^3)/(x^5*(a + b*x^3)^(3/2)), x)
```

3.243 $\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 611

$$\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)^{3/2}}\,dx=-\frac{A}{7ax^7\sqrt{a+bx^3}}-\frac{17Ab-14aB}{21a^2x^4\sqrt{a+bx^3}}+\frac{11(17Ab-14aB)\sqrt{a+bx^3}}{168a^3x^4}$$
$$-\frac{55b(17Ab-14aB)\sqrt{a+bx^3}}{336a^4x}+\frac{55b^{4/3}(17Ab-14aB)\sqrt{a+bx^3}}{336a^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$-\frac{55\sqrt{2-\sqrt{3}}b^{4/3}(17Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\right.}{224\ 3^{3/4}a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{55b^{4/3}(17Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{}\right.}{168\sqrt{2}\sqrt[4]{3}a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/7*A/a/x^7/(b*x^3+a)^(1/2)+1/21*(-17*A*b+14*B*a)/a^2/x^4/(b*x^3+a)^(1/2)+
11/168*(17*A*b-14*B*a)*(b*x^3+a)^(1/2)/a^3/x^4-55/336*b*(17*A*b-14*B*a)*(b*
x^3+a)^(1/2)/a^4/x+55/336*b^(4/3)*(17*A*b-14*B*a)*(b*x^3+a)^(1/2)/a^4/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))+55/1008*b^(4/3)*(17*A*b-14*B*a)*(a^(1/3)+b^(1/3)
*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
)),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(11/3)*2^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*
```

```
(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-55/672*b^(4/3)
*(17*A*b-14*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2
)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2)
)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^
2)^(1/2)*3^(1/4)/a^(11/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 611, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {464, 296, 331, 309, 224, 1891}

$$\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)^{3/2}}\,dx = \frac{55b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(17Ab-14aB)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1\right.}{\sqrt[3]{b}x+\left(1\right.}\right.\right.}{168\sqrt{2}\sqrt[4]{3}a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{55\sqrt{2-\sqrt{3}}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(17Ab-14aB)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{\right.}{224\ 3^{3/4}a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{55b^{4/3}\sqrt{a+bx^3}(17Ab-14aB)}{336a^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{55b\sqrt{a+bx^3}(17Ab-14aB)}{336a^4x}$$

$$+\frac{11\sqrt{a+bx^3}(17Ab-14aB)}{168a^3x^4}-\frac{17Ab-14aB}{21a^2x^4\sqrt{a+bx^3}}-\frac{A}{7ax^7\sqrt{a+bx^3}}$$

```
[In] Int[(A + B*x^3)/(x^8*(a + b*x^3)^(3/2)),x]
```

```
[Out] -1/7*A/(a*x^7*Sqrt[a + b*x^3]) - (17*A*b - 14*a*B)/(21*a^2*x^4*Sqrt[a + b*x
^3]) + (11*(17*A*b - 14*a*B)*Sqrt[a + b*x^3])/(168*a^3*x^4) - (55*b*(17*A*b
 - 14*a*B)*Sqrt[a + b*x^3])/(336*a^4*x) + (55*b^(4/3)*(17*A*b - 14*a*B)*Sqr
t[a + b*x^3])/(336*a^4*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (55*Sqrt[2 -
Sqrt[3]]*b^(4/3)*(17*A*b - 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellip
ticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(224*3^(3/4)*a^(11/3)*Sqrt[(a^(1/3)*(a^(1/3) +
b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (55*b
^(4/3)*(17*A*b - 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSi
```

```
n[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(168*Sqrt[2]*3^(1/4)*a^(11/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{A}{7ax^7\sqrt{a+bx^3}} - \frac{\left(\frac{17Ab}{2}-7aB\right)\int \frac{1}{x^5(a+bx^3)^{3/2}}\,dx}{7a} \\
&= -\frac{A}{7ax^7\sqrt{a+bx^3}} - \frac{17Ab-14aB}{21a^2x^4\sqrt{a+bx^3}} - \frac{(11(17Ab-14aB))\int \frac{1}{x^5\sqrt{a+bx^3}}\,dx}{42a^2} \\
&= -\frac{A}{7ax^7\sqrt{a+bx^3}} - \frac{17Ab-14aB}{21a^2x^4\sqrt{a+bx^3}} + \frac{11(17Ab-14aB)\sqrt{a+bx^3}}{168a^3x^4} \\
&\quad + \frac{(55b(17Ab-14aB))\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx}{336a^3} \\
&= -\frac{A}{7ax^7\sqrt{a+bx^3}} - \frac{17Ab-14aB}{21a^2x^4\sqrt{a+bx^3}} + \frac{11(17Ab-14aB)\sqrt{a+bx^3}}{168a^3x^4} \\
&\quad - \frac{55b(17Ab-14aB)\sqrt{a+bx^3}}{336a^4x} + \frac{(55b^2(17Ab-14aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{672a^4} \\
&= -\frac{A}{7ax^7\sqrt{a+bx^3}} - \frac{17Ab-14aB}{21a^2x^4\sqrt{a+bx^3}} + \frac{11(17Ab-14aB)\sqrt{a+bx^3}}{168a^3x^4} \\
&\quad - \frac{55b(17Ab-14aB)\sqrt{a+bx^3}}{336a^4x} + \frac{\left(55b^{5/3}(17Ab-14aB)\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{672a^4} \\
&\quad - \frac{\left(55\left(1-\sqrt{3}\right)b^{5/3}(17Ab-14aB)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{672a^{11/3}}
\end{aligned}$$

$$= -\frac{A}{7ax^7\sqrt{a+bx^3}} - \frac{17Ab-14aB}{21a^2x^4\sqrt{a+bx^3}} + \frac{11(17Ab-14aB)\sqrt{a+bx^3}}{168a^3x^4} - \frac{55b(17Ab-14aB)\sqrt{a+bx^3}}{336a^4x} + \frac{55b^{4/3}(17Ab-14aB)\sqrt{a+bx^3}}{336a^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$- \frac{55\sqrt{2-\sqrt{3}}b^{4/3}(17Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{224\ 3^{3/4}a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{55b^{4/3}(17Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\middle|-7-4\right.}{168\sqrt{2}\sqrt[4]{3}a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.12

$$\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)^{3/2}}\,dx = \frac{-8aA+(17Ab-14aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{3}{2},-\frac{1}{3},-\frac{bx^3}{a}\right)}{56a^2x^7\sqrt{a+bx^3}}$$

```
[In] Integrate[(A + B*x^3)/(x^8*(a + b*x^3)^(3/2)),x]
[Out] (-8*a*A + (17*A*b - 14*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-4/3,
 3/2, -1/3, -((b*x^3)/a)])/(56*a^2*x^7*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 5.22 (sec) , antiderivative size = 573, normalized size of antiderivative = 0.94

method	result
elliptic	$-\frac{2b^2x^2(Ab-Ba)}{3a^4\sqrt{(x^3+\frac{a}{b})b}}-\frac{A\sqrt{b\,x^3+a}}{7a^2x^7}+\frac{(25Ab-14Ba)\sqrt{b\,x^3+a}}{56a^3x^4}-\frac{(237Ab-182Ba)b\sqrt{b\,x^3+a}}{112a^4x}-\frac{2i\left(\frac{b^2(Ab-Ba)}{3a^4}+\frac{b^2(237Ab-182Ba)}{224a^4}\right)\sqrt{}}{}$
risch	Expression too large to display
default	Expression too large to display

```
[In] int((B*x^3+A)/x^8/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*b^2*x^2/a^4*(A*b-B*a)/((x^3+a/b)*b)^(1/2)-1/7*A/a^2*(b*x^3+a)^(1/2)/x^
7+1/56/a^3*(25*A*b-14*B*a)*(b*x^3+a)^(1/2)/x^4-1/112/a^4*(237*A*b-182*B*a)*
b*(b*x^3+a)^(1/2)/x-2/3*I*(1/3*b^2*(A*b-B*a)/a^4+1/224*b^2*(237*A*b-182*B*a
)/a^4)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*El
lipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.26

$$\int \frac{A+Bx^3}{x^8\,(a+bx^3)^{3/2}}\,dx = \frac{55\,((14\,Bab^2-17\,Ab^3)x^{10}+(14\,Ba^2b-17\,Aab^2)x^7)\sqrt{b}\text{weierstrassZeta}\big(0,-\frac{4\,a}{b},\text{weier}}{}$$

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/336*(55*((14*B*a*b^2 - 17*A*b^3)*x^10 + (14*B*a^2*b - 17*A*a*b^2)*x^7)*sq
rt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (55*(
14*B*a*b^2 - 17*A*b^3)*x^9 + 33*(14*B*a^2*b - 17*A*a*b^2)*x^6 - 48*A*a^3 -
6*(14*B*a^3 - 17*A*a^2*b)*x^3)*sqrt(b*x^3 + a))/(a^4*b*x^10 + a^5*x^7)
```

Sympy [A] (verification not implemented)

Time = 43.68 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.15

$$\int \frac{A+Bx^3}{x^8(a+bx^3)^{3/2}}\,dx = \frac{A\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},\frac{3}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^7\Gamma\left(-\frac{4}{3}\right)}+\frac{B\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},\frac{3}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^4\Gamma\left(-\frac{1}{3}\right)}$$

```
[In] integrate((B*x**3+A)/x**8/(b*x**3+a)**(3/2),x)
```

```
[Out] A*gamma(-7/3)*hyper((-7/3, 3/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
3/2)*x**7*gamma(-4/3)) + B*gamma(-4/3)*hyper((-4/3, 3/2), (-1/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*a**(3/2)*x**4*gamma(-1/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{x^8(a+bx^3)^{3/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{3}{2}}x^8}\,dx$$

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^8), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^8(a+bx^3)^{3/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{3}{2}}x^8}\,dx$$

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{B\,x^3+A}{x^8\,{\left(b\,x^3+a\right)}^{3/2}}\,dx$$

```
[In] int((A + B*x^3)/(x^8*(a + b*x^3)^(3/2)),x)
[Out] int((A + B*x^3)/(x^8*(a + b*x^3)^(3/2)), x)
```

3.244 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2a^2(Ab-aB)}{9b^4(a+bx^3)^{3/2}}+\frac{2a(2Ab-3aB)}{3b^4\sqrt{a+bx^3}}+\frac{2(Ab-3aB)\sqrt{a+bx^3}}{3b^4}+\frac{2B(a+bx^3)^{3/2}}{9b^4}$$

```
[Out] -2/9*a^2*(A*b-B*a)/b^4/(b*x^3+a)^(3/2)+2/9*B*(b*x^3+a)^(3/2)/b^4+2/3*a*(2*A
*b-3*B*a)/b^4/(b*x^3+a)^(1/2)+2/3*(A*b-3*B*a)*(b*x^3+a)^(1/2)/b^4
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 78}

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2a^2(Ab-aB)}{9b^4(a+bx^3)^{3/2}}+\frac{2a(2Ab-3aB)}{3b^4\sqrt{a+bx^3}}+\frac{2\sqrt{a+bx^3}(Ab-3aB)}{3b^4}+\frac{2B(a+bx^3)^{3/2}}{9b^4}$$

```
[In] Int[(x^8*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (-2*a^2*(A*b - a*B))/(9*b^4*(a + b*x^3)^(3/2)) + (2*a*(2*A*b - 3*a*B))/(3*b
^4*Sqrt[a + b*x^3]) + (2*(A*b - 3*a*B)*Sqrt[a + b*x^3])/(3*b^4) + (2*B*(a +
 b*x^3)^(3/2))/(9*b^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx)^{5/2}}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{a^2(-Ab+aB)}{b^3(a+bx)^{5/2}}+\frac{a(-2Ab+3aB)}{b^3(a+bx)^{3/2}}+\frac{Ab-3aB}{b^3\sqrt{a+bx}}+\frac{B\sqrt{a+bx}}{b^3}\right)\,dx, x, x^3\right)\\
&= -\frac{2a^2(Ab-aB)}{9b^4\left(a+bx^3\right)^{3/2}}+\frac{2a(2Ab-3aB)}{3b^4\sqrt{a+bx^3}}+\frac{2(Ab-3aB)\sqrt{a+bx^3}}{3b^4}+\frac{2B\left(a+bx^3\right)^{3/2}}{9b^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.71

$$\int \frac{x^8(A+Bx^3)}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{2(-16a^3B+8a^2b(A-3Bx^3)-6ab^2x^3(-2A+Bx^3)+b^3x^6(3A+Bx^3))}{9b^4\left(a+bx^3\right)^{3/2}}$$

```
[In] Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^(5/2),x]
[Out] (2*(-16*a^3*B + 8*a^2*b*(A - 3*B*x^3) - 6*a*b^2*x^3*(-2*A + B*x^3) + b^3*x^
6*(3*A + B*x^3)))/(9*b^4*(a + b*x^3)^(3/2))
```

Maple [A] (verified)

Time = 4.39 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$\frac{\left(2B\,x^{9}+6A\,x^{6}\right)b^{3}+24x^{3}\left(-\frac{x^{3}B}{2}+A\right)a\,b^{2}+16a^{2}\left(-3x^{3}B+A\right)b-32a^{3}B}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}}b^{4}}$
risch	$\frac{2\left(bB\,x^{3}+3Ab-8Ba\right)\sqrt{b\,x^{3}+a}}{9b^{4}}+\frac{2a\left(6A\,b^{2}x^{3}-9Bab\,x^{3}+5abA-8a^{2}B\right)}{9b^{4}\left(b\,x^{3}+a\right)^{\frac{3}{2}}}$
gosper	$\frac{\frac{2}{9}b^{3}B\,x^{9}+\frac{2}{3}x^{6}b^{3}A-\frac{4}{3}B\,x^{6}a\,b^{2}+\frac{8}{3}aA\,b^{2}x^{3}-\frac{16}{3}B\,a^{2}b\,x^{3}+\frac{16}{9}a^{2}bA-\frac{32}{9}a^{3}B}{\left(b\,x^{3}+a\right)^{\frac{3}{2}}b^{4}}$
trager	$\frac{\frac{2}{9}b^{3}B\,x^{9}+\frac{2}{3}x^{6}b^{3}A-\frac{4}{3}B\,x^{6}a\,b^{2}+\frac{8}{3}aA\,b^{2}x^{3}-\frac{16}{3}B\,a^{2}b\,x^{3}+\frac{16}{9}a^{2}bA-\frac{32}{9}a^{3}B}{\left(b\,x^{3}+a\right)^{\frac{3}{2}}b^{4}}$
elliptic	$-\frac{2a^{2}\left(Ab-Ba\right)\sqrt{b\,x^{3}+a}}{9b^{6}\left(x^{3}+\frac{a}{b}\right)^{2}}+\frac{2\left(2Ab-3Ba\right)a}{3b^{4}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2B\,x^{3}\sqrt{b\,x^{3}+a}}{9b^{3}}+\frac{2\left(\frac{Ab-2Ba}{b^{3}}-\frac{2Ba}{3b^{3}}\right)\sqrt{b\,x^{3}+a}}{3b}$
default	$A\left(-\frac{2a^{2}\sqrt{b\,x^{3}+a}}{9b^{5}\left(x^{3}+\frac{a}{b}\right)^{2}}+\frac{4a}{3b^{3}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2\sqrt{b\,x^{3}+a}}{3b^{3}}\right)+B\left(\frac{2a^{3}\sqrt{b\,x^{3}+a}}{9b^{6}\left(x^{3}+\frac{a}{b}\right)^{2}}-\frac{2a^{2}}{b^{4}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{2x^{3}\sqrt{b\,x^{3}+a}}{9b^{3}}-16a\right.$

```
[In] int(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*((2*B*x^9+6*A*x^6)*b^3+24*x^3*(-1/2*x^3*B+A)*a*b^2+16*a^2*(-3*B*x^3+A)*
b-32*a^3*B)/(b*x^3+a)^(3/2)/b^4
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.95

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx=\frac{2\left(Bb^3x^9-3\left(2\,Bab^2-Ab^3\right)x^6-16\,Ba^3+8\,Aa^2b-12\left(2\,Ba^2b-Aab^2\right)x^3\right)\sqrt{bx^3+a}}{9\left(b^6x^6+2\,ab^5x^3+a^2b^4\right)}$$

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9*(B*b^3*x^9 - 3*(2*B*a*b^2 - A*b^3)*x^6 - 16*B*a^3 + 8*A*a^2*b - 12*(2*B
*a^2*b - A*a*b^2)*x^3)*sqrt(b*x^3 + a)/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(99) = 198.

Time = 0.59 (sec) , antiderivative size = 338, normalized size of antiderivative = 3.28

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \begin{cases} \frac{16Aa^2b}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}} + \frac{24Aab^2x^3}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}} + \frac{6Ab^3x^6}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}} - \frac{32B}{9ab^4\sqrt{a+bx^3}+} \\ \frac{\frac{Ax^9}{9}+\frac{Bx^{12}}{12}}{a^{\frac{5}{2}}} \end{cases}$$

```
[In] integrate(x**8*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Piecewise((16*A*a**2*b/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a + b*
x**3)) + 24*A*a*b**2*x**3/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a +
 b*x**3)) + 6*A*b**3*x**6/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a +
 b*x**3)) - 32*B*a**3/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a + b*x
**3)) - 48*B*a**2*b*x**3/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a +
b*x**3)) - 12*B*a*b**2*x**6/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a
 + b*x**3)) + 2*B*b**3*x**9/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a
 + b*x**3)), Ne(b, 0)), ((A*x**9/9 + B*x**12/12)/a**(5/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.13

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2}{9}\,B\left(\frac{(bx^3+a)^{\frac{3}{2}}}{b^4} - \frac{9\sqrt{bx^3+a}a}{b^4} - \frac{9\,a^2}{\sqrt{bx^3+a}b^4} + \frac{a^3}{(bx^3+a)^{\frac{3}{2}}b^4}\right) + \frac{2}{9}\,A\left(\frac{3\sqrt{bx^3+a}}{b^3} + \frac{6\,a}{\sqrt{bx^3+a}b^3} - \frac{a^2}{(bx^3+a)^{\frac{3}{2}}b^3}\right)$$

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9*B*((b*x^3 + a)^(3/2)/b^4 - 9*sqrt(b*x^3 + a)*a/b^4 - 9*a^2/(sqrt(b*x^3
+ a)*b^4) + a^3/((b*x^3 + a)^(3/2)*b^4)) + 2/9*A*(3*sqrt(b*x^3 + a)/b^3 + 6
*a/(sqrt(b*x^3 + a)*b^3) - a^2/((b*x^3 + a)^(3/2)*b^3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2\left(9\,(bx^3+a)Ba^2 - Ba^3 - 6\,(bx^3+a)Aab + Aa^2b\right)}{9\,(bx^3+a)^{\frac{3}{2}}b^4} + \frac{2\left((bx^3+a)^{\frac{3}{2}}Bb^8 - 9\,\sqrt{bx^3+a}Bab^8 + 3\,\sqrt{bx^3+a}Ab^9\right)}{9\,b^{12}}$$

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2/9*(9*(b*x^3 + a)*B*a^2 - B*a^3 - 6*(b*x^3 + a)*A*a*b + A*a^2*b)/((b*x^3
+ a)^(3/2)*b^4) + 2/9*((b*x^3 + a)^(3/2)*B*b^8 - 9*sqrt(b*x^3 + a)*B*a*b^8
+ 3*sqrt(b*x^3 + a)*A*b^9)/b^12
```

Mupad [B] (verification not implemented)

Time = 7.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.41

$$\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{\sqrt{b\,x^3+a}\,\left(\frac{2\,(A\,b-2\,B\,a)}{b^3}-\frac{4\,B\,a}{3\,b^3}\right)}{3\,b} - \frac{\frac{2\,B\,a^2-2\,A\,a\,b}{3\,b^4}-\frac{a\,\left(\frac{2\,A\,b^2-2\,B\,a\,b}{3\,b^4}-\frac{2\,B\,a}{3\,b^3}\right)}{b}}{\sqrt{b\,x^3+a}} - \frac{a^2\,\left(\frac{2\,A}{9\,b}-\frac{2\,B\,a}{9\,b^2}\right)}{b^2\,{\left(b\,x^3+a\right)}^{3/2}} + \frac{2\,B\,x^3\,\sqrt{b\,x^3+a}}{9\,b^3}$$

```
[In] int((x^8*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

```
[Out] ((a + b*x^3)^(1/2)*((2*(A*b - 2*B*a))/b^3 - (4*B*a)/(3*b^3)))/(3*b) - ((2*B
*a^2 - 2*A*a*b)/(3*b^4) - (a*((2*A*b^2 - 2*B*a*b)/(3*b^4) - (2*B*a)/(3*b^3)
))/b)/(a + b*x^3)^(1/2) - (a^2*((2*A)/(9*b) - (2*B*a)/(9*b^2)))/(b^2*(a + b
*x^3)^(3/2)) + (2*B*x^3*(a + b*x^3)^(1/2))/(9*b^3)
```

3.245 $\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx=\frac{2a(Ab-aB)}{9b^3(a+bx^3)^{3/2}}-\frac{2(Ab-2aB)}{3b^3\sqrt{a+bx^3}}+\frac{2B\sqrt{a+bx^3}}{3b^3}$$

```
[Out] 2/9*a*(A*b-B*a)/b^3/(b*x^3+a)^(3/2)-2/3*(A*b-2*B*a)/b^3/(b*x^3+a)^(1/2)+2/3
*B*(b*x^3+a)^(1/2)/b^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 78}

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx=-\frac{2(Ab-2aB)}{3b^3\sqrt{a+bx^3}}+\frac{2a(Ab-aB)}{9b^3(a+bx^3)^{3/2}}+\frac{2B\sqrt{a+bx^3}}{3b^3}$$

```
[In] Int[(x^5*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*a*(A*b - a*B))/(9*b^3*(a + b*x^3)^(3/2)) - (2*(A*b - 2*a*B))/(3*b^3*Sqrt
[a + b*x^3]) + (2*B*Sqrt[a + b*x^3])/(3*b^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x(A+Bx)}{(a+bx)^{5/2}}\,dx, x, x^3\right) \\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^2(a+bx)^{5/2}}+\frac{Ab-2aB}{b^2(a+bx)^{3/2}}+\frac{B}{b^2\sqrt{a+bx}}\right)\,dx, x, x^3\right) \\
&= \frac{2a(Ab-aB)}{9b^3\left(a+bx^3\right)^{3/2}}-\frac{2(Ab-2aB)}{3b^3\sqrt{a+bx^3}}+\frac{2B\sqrt{a+bx^3}}{3b^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int \frac{x^5(A+Bx^3)}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{2(-2aAb+8a^2B-3Ab^2x^3+12abBx^3+3b^2Bx^6)}{9b^3\left(a+bx^3\right)^{3/2}}$$

```
[In] Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^(5/2),x]
[Out] (2*(-2*a*A*b + 8*a^2*B - 3*A*b^2*x^3 + 12*a*b*B*x^3 + 3*b^2*B*x^6))/(9*b^3*
(a + b*x^3)^(3/2))
```

Maple [A] (verified)

Time = 4.37 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$-\frac{4\left(\frac{3x^3\left(-x^3B+A\right)b^2}{2}+a\left(-6x^3B+A\right)b-4a^2B\right)}{9\left(b\,x^3+a\right)^{\frac{3}{2}}b^3}$	49
gosper	$-\frac{2\left(-3b^2B\,x^6+3A\,b^2x^3-12Bab\,x^3+2abA-8a^2B\right)}{9\left(b\,x^3+a\right)^{\frac{3}{2}}b^3}$	53
trager	$-\frac{2\left(-3b^2B\,x^6+3A\,b^2x^3-12Bab\,x^3+2abA-8a^2B\right)}{9\left(b\,x^3+a\right)^{\frac{3}{2}}b^3}$	53
risch	$\frac{2B\sqrt{b\,x^3+a}}{3b^3}-\frac{2\left(3A\,b^2x^3-6Bab\,x^3+2abA-5a^2B\right)}{9b^3\left(b\,x^3+a\right)^{\frac{3}{2}}}$	60
elliptic	$\frac{2(Ab-Ba)a\sqrt{b\,x^3+a}}{9b^5\left(x^3+\frac{a}{b}\right)^2}-\frac{2(Ab-2Ba)}{3b^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2B\sqrt{b\,x^3+a}}{3b^3}$	77
default	$B\left(-\frac{2a^2\sqrt{b\,x^3+a}}{9b^5\left(x^3+\frac{a}{b}\right)^2}+\frac{4a}{3b^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2\sqrt{b\,x^3+a}}{3b^3}\right)+A\left(\frac{2a\sqrt{b\,x^3+a}}{9b^4\left(x^3+\frac{a}{b}\right)^2}-\frac{2}{3b^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}\right)$	113

```
[In] int(x^5*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] -4/9/(b*x^3+a)^(3/2)*(3/2*x^3*(-B*x^3+A)*b^2+a*(-6*B*x^3+A)*b-4*a^2*B)/b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2\,(3\,Bb^2x^6+3\,(4\,Bab-Ab^2)x^3+8\,Ba^2-2\,Aab)\sqrt{bx^3+a}}{9\,(b^5x^6+2\,ab^4x^3+a^2b^3)}$$

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
[Out] 2/9*(3*B*b^2*x^6 + 3*(4*B*a*b - A*b^2)*x^3 + 8*B*a^2 - 2*A*a*b)*sqrt(b*x^3
+ a)/(b^5*x^6 + 2*a*b^4*x^3 + a^2*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(70) = 140.

Time = 0.44 (sec) , antiderivative size = 240, normalized size of antiderivative = 3.29

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \begin{cases} -\frac{4Aab}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+bx^3}}-\frac{6Ab^2x^3}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+bx^3}}+\frac{16Ba^2}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+bx^3}}+\frac{24}{9ab^3\sqrt{a+bx}} \\ \frac{\frac{Ax^6}{6}+\frac{Bx^9}{9}}{a^{\frac{5}{2}}} \end{cases}$$

```
[In] integrate(x**5*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Piecewise((-4*A*a*b/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**
3)) - 6*A*b**2*x**3/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**
3)) + 16*B*a**2/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**3))
+ 24*B*a*b*x**3/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**3))
+ 6*B*b**2*x**6/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**3)),
 Ne(b, 0)), ((A*x**6/6 + B*x**9/9)/a**(5/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2}{9}\,B\left(\frac{3\,\sqrt{bx^3+a}}{b^3} + \frac{6\,a}{\sqrt{bx^3+a}b^3} - \frac{a^2}{(bx^3+a)^{\frac{3}{2}}b^3}\right) - \frac{2}{9}\,A\left(\frac{3}{\sqrt{bx^3+a}b^2} - \frac{a}{(bx^3+a)^{\frac{3}{2}}b^2}\right)$$

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9*B*(3*sqrt(b*x^3 + a)/b^3 + 6*a/(sqrt(b*x^3 + a)*b^3) - a^2/((b*x^3 + a)
^(3/2)*b^3)) - 2/9*A*(3/(sqrt(b*x^3 + a)*b^2) - a/((b*x^3 + a)^(3/2)*b^2))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2\,\sqrt{bx^3+a}B}{3\,b^3} + \frac{2\,(6\,(bx^3+a)Ba - Ba^2 - 3\,(bx^3+a)Ab + Aab)}{9\,(bx^3+a)^{\frac{3}{2}}b^3}$$

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(b*x^3 + a)*B/b^3 + 2/9*(6*(b*x^3 + a)*B*a - B*a^2 - 3*(b*x^3 + a)*
A*b + A*a*b)/((b*x^3 + a)^(3/2)*b^3)
```

Mupad [B] (verification not implemented)

Time = 7.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{6\,B\,(b\,x^3+a)^2-2\,B\,a^2-6\,A\,b\,(b\,x^3+a)+12\,B\,a\,(b\,x^3+a)+2\,A\,a\,b}{9\,b^3\,(b\,x^3+a)^{3/2}}$$

```
[In] int((x^5*(A + B*x^3))/(a + b*x^3)^(5/2),x)
[Out] (6*B*(a + b*x^3)^2 - 2*B*a^2 - 6*A*b*(a + b*x^3) + 12*B*a*(a + b*x^3) + 2*A
*a*b)/(9*b^3*(a + b*x^3)^(3/2))
```

3.246 $\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2(Ab-aB)}{9b^2(a+bx^3)^{3/2}} - \frac{2B}{3b^2\sqrt{a+bx^3}}$$

```
[Out] -2/9*(A*b-B*a)/b^2/(b*x^3+a)^(3/2)-2/3*B/b^2/(b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {455, 45}

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2(Ab-aB)}{9b^2(a+bx^3)^{3/2}} - \frac{2B}{3b^2\sqrt{a+bx^3}}$$

```
[In] Int[(x^2*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (-2*(A*b - a*B))/(9*b^2*(a + b*x^3)^(3/2)) - (2*B)/(3*b^2*Sqrt[a + b*x^3])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{(a+bx)^{5/2}}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{Ab-aB}{b(a+bx)^{5/2}}+\frac{B}{b(a+bx)^{3/2}}\right)\,dx, x, x^3\right)\\
&= -\frac{2(Ab-aB)}{9b^2\left(a+bx^3\right)^{3/2}}-\frac{2B}{3b^2\sqrt{a+bx^3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \frac{x^2(A+Bx^3)}{\left(a+bx^3\right)^{5/2}}\,dx = -\frac{2(Ab+2aB+3bBx^3)}{9b^2\left(a+bx^3\right)^{3/2}}$$

```
[In] Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^(5/2),x]
[Out] (-2*(A*b + 2*a*B + 3*b*B*x^3))/(9*b^2*(a + b*x^3)^(3/2))
```

Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.65

method	result	size
gosper	$-\frac{2\left(3bB\,x^{3}+Ab+2Ba\right)}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}} b^{2}}$	30
trager	$-\frac{2\left(3bB\,x^{3}+Ab+2Ba\right)}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}} b^{2}}$	30
pseudoelliptic	$-\frac{2\left(\left(3x^{3}B+A\right)b+2Ba\right)}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}} b^{2}}$	30
elliptic	$-\frac{2(Ab-Ba)\sqrt{b\,x^{3}+a}}{9b^{4}\left(x^{3}+\frac{a}{b}\right)^{2}}-\frac{2B}{3b^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}$	54
default	$B\left(\frac{2a\sqrt{b\,x^{3}+a}}{9b^{4}\left(x^{3}+\frac{a}{b}\right)^{2}}-\frac{2}{3b^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}\right)-\frac{2A}{9b\left(b\,x^{3}+a\right)^{\frac{3}{2}}}$	64

```
[In] int(x^2*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/9/(b*x^3+a)^(3/2)*(3*B*b*x^3+A*b+2*B*a)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2\,(3\,Bbx^3+2\,Ba+Ab)\sqrt{bx^3+a}}{9\,(b^4x^6+2\,ab^3x^3+a^2b^2)}$$

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/9*(3*B*b*x^3 + 2*B*a + A*b)*sqrt(b*x^3 + a)/(b^4*x^6 + 2*a*b^3*x^3 + a^2
*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(44) = 88$.

Time = 0.34 (sec) , antiderivative size = 144, normalized size of antiderivative = 3.13

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \begin{cases} -\frac{2Ab}{9ab^2\sqrt{a+bx^3}+9b^3x^3\sqrt{a+bx^3}} - \frac{4Ba}{9ab^2\sqrt{a+bx^3}+9b^3x^3\sqrt{a+bx^3}} - \frac{6Bbx^3}{9ab^2\sqrt{a+bx^3}+9b^3x^3\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{\frac{Ax^3}{3}+\frac{Bx^6}{6}}{a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Piecewise((-2*A*b/(9*a*b**2*sqrt(a + b*x**3) + 9*b**3*x**3*sqrt(a + b*x**3)
) - 4*B*a/(9*a*b**2*sqrt(a + b*x**3) + 9*b**3*x**3*sqrt(a + b*x**3)) - 6*B*
b*x**3/(9*a*b**2*sqrt(a + b*x**3) + 9*b**3*x**3*sqrt(a + b*x**3)), Ne(b, 0)
), ((A*x**3/3 + B*x**6/6)/a**(5/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2}{9}\,B\left(\frac{3}{\sqrt{bx^3+a}b^2} - \frac{a}{(bx^3+a)^{\frac{3}{2}}b^2}\right) - \frac{2\,A}{9\,(bx^3+a)^{\frac{3}{2}}b}$$

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/9*B*(3/(sqrt(b*x^3 + a)*b^2) - a/((b*x^3 + a)^(3/2)*b^2)) - 2/9*A/((b*x^
3 + a)^(3/2)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \frac{x^2(A + Bx^3)}{(a + bx^3)^{5/2}}\,dx = -\frac{2\,(3\,(bx^3 + a)B - Ba + Ab)}{9\,(bx^3 + a)^{\frac{3}{2}}b^2}$$

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
[Out] -2/9*(3*(b*x^3 + a)*B - B*a + A*b)/((b*x^3 + a)^(3/2)*b^2)
```

Mupad [B] (verification not implemented)

Time = 6.96 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \frac{x^2(A + Bx^3)}{(a + bx^3)^{5/2}}\,dx = -\frac{2\,A\,b - 2\,B\,a + 6\,B\,(b\,x^3 + a)}{9\,b^2\,(b\,x^3 + a)^{3/2}}$$

```
[In] int((x^2*(A + B*x^3))/(a + b*x^3)^(5/2),x)
[Out] -(2*A*b - 2*B*a + 6*B*(a + b*x^3))/(9*b^2*(a + b*x^3)^(3/2))
```

3.247 $\int \frac{A+Bx^3}{x(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 77

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{5/2}}\,dx=\frac{2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}+\frac{2A}{3a^2\sqrt{a+bx^3}}-\frac{2A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}$$

```
[Out] 2/9*(A*b-B*a)/a/b/(b*x^3+a)^(3/2)-2/3*A*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^
(5/2)+2/3*A/a^2/(b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 79, 53, 65, 214}

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{5/2}}\,dx=-\frac{2A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}+\frac{2A}{3a^2\sqrt{a+bx^3}}+\frac{2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}$$

```
[In] Int[(A + B*x^3)/(x*(a + b*x^3)^(5/2)),x]
```

```
[Out] (2*(A*b - a*B))/(9*a*b*(a + b*x^3)^(3/2)) + (2*A)/(3*a^2*Sqrt[a + b*x^3]) -
 (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(5/2))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
```

```
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n])) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x(a+bx)^{5/2}}\,dx, x, x^3\right)$$

$$= \frac{2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}} + \frac{A\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, x^3\right)}{3a}$$

$$= \frac{2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}} + \frac{2A}{3a^2\sqrt{a+bx^3}} + \frac{A\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)}{3a^2}$$

$$= \frac{2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}} + \frac{2A}{3a^2\sqrt{a+bx^3}} + \frac{(2A)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{3a^2b}$$

$$= \frac{2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}} + \frac{2A}{3a^2\sqrt{a+bx^3}} - \frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{5/2}}\,dx = -\frac{2(-4aAb+a^2B-3Ab^2x^3)}{9a^2b\left(a+bx^3\right)^{3/2}} - \frac{2A\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}$$

```
[In] Integrate[(A + B*x^3)/(x*(a + b*x^3)^(5/2)),x]
[Out] (-2*(-4*a*A*b + a^2*B - 3*A*b^2*x^3))/(9*a^2*b*(a + b*x^3)^(3/2)) - (2*A*Ar
cTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(5/2))
```

Maple [A] (verified)

Time = 4.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

method	result	size
pseudoelliptic	$-\frac{2\left(-3A\sqrt{a}\,b^2x^3+3A\left(b\,x^3+a\right)^{\frac{3}{2}}b\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)-4A\,a^{\frac{3}{2}}b+B\,a^{\frac{5}{2}}\right)}{9\left(b\,x^3+a\right)^{\frac{3}{2}}a^{\frac{5}{2}}b}$	70
elliptic	$\frac{2(Ab-Ba)\sqrt{b\,x^3+a}}{9b^3a\left(x^3+\frac{a}{b}\right)^2}+\frac{2A}{3a^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2A\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{5}{2}}}$	77
default	$-\frac{2B}{9b\left(b\,x^3+a\right)^{\frac{3}{2}}}+A\left(\frac{2\sqrt{b\,x^3+a}}{9a\,b^2\left(x^3+\frac{a}{b}\right)^2}+\frac{2}{3a^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{5}{2}}}\right)$	85

```
[In] int((B*x^3+A)/x/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/9*(-3*A*a^(1/2)*b^2*x^3+3*A*(b*x^3+a)^(3/2)*b*arctanh((b*x^3+a)^(1/2)/a^
(1/2))-4*A*a^(3/2)*b+B*a^(5/2))/(b*x^3+a)^(3/2)/a^(5/2)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.16

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{5/2}}\,dx = \left[\frac{3\left(Ab^3x^6+2\,Aab^2x^3+Aa^2b\right)\sqrt{a}\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\left(3\,Aab^2x^3-Ba^3+4\,Aa\right.}{9\left(a^3b^3x^6+2\,a^4b^2x^3+a^5b\right)}\right.$$

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/9*(3*(A*b^3*x^6 + 2*A*a*b^2*x^3 + A*a^2*b)*sqrt(a)*log((b*x^3 - 2*sqrt(b
*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*(3*A*a*b^2*x^3 - B*a^3 + 4*A*a^2*b)*sqrt(
b*x^3 + a))/(a^3*b^3*x^6 + 2*a^4*b^2*x^3 + a^5*b), 2/9*(3*(A*b^3*x^6 + 2*A*
a*b^2*x^3 + A*a^2*b)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (3*A*a*b
^2*x^3 - B*a^3 + 4*A*a^2*b)*sqrt(b*x^3 + a))/(a^3*b^3*x^6 + 2*a^4*b^2*x^3 +
 a^5*b)]
```

Sympy [A] (verification not implemented)

Time = 6.74 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.30

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{5/2}}\,dx = \begin{cases} \frac{2\left(\frac{Ab}{3a^2\sqrt{a+bx^3}}+\frac{Ab\,\mathrm{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{3a^2\sqrt{-a}}-\frac{-Ab+Ba}{9a\left(a+bx^3\right)^{\frac{3}{2}}}\right)}{b} & \text{for } b\neq 0 \\ \frac{A\log\left(Bx^3\right)+Bx^3}{3a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate((B*x**3+A)/x/(b*x**3+a)**(5/2),x)
```

```
[Out] Piecewise((2*(A*b/(3*a**2*sqrt(a + b*x**3)) + A*b*atan(sqrt(a + b*x**3)/sqr
t(-a))/(3*a**2*sqrt(-a)) - (-A*b + B*a)/(9*a*(a + b*x**3)**(3/2)))/b, Ne(b,
 0)), ((A*log(B*x**3) + B*x**3)/(3*a**(5/2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{5/2}}\,dx = \frac{1}{9}\,A\left(\frac{3\,\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(3\,bx^3+4\,a\right)}{\left(bx^3+a\right)^{\frac{3}{2}}a^2}\right)-\frac{2\,B}{9\left(bx^3+a\right)^{\frac{3}{2}}b}$$

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/9*A*(3*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(5/
2) + 2*(3*b*x^3 + 4*a)/((b*x^3 + a)^(3/2)*a^2)) - 2/9*B/((b*x^3 + a)^(3/2)*
b)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{5/2}}\,dx=\frac{2\,A\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}a^2}-\frac{2\,(Ba^2-3\,(bx^3+a)Ab-Aab)}{9\,(bx^3+a)^{\frac{3}{2}}a^2b}$$

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*A*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^2) - 2/9*(B*a^2 - 3*(b*x
^3 + a)*A*b - A*a*b)/((b*x^3 + a)^(3/2)*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 7.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{5/2}}\,dx=\frac{\frac{2\,A}{9\,a}-\frac{2\,B}{9\,b}}{\left(b\,x^3+a\right)^{3/2}}+\frac{2\,A}{3\,a^2\,\sqrt{b\,x^3+a}}+\frac{A\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3\,a^{5/2}}$$

```
[In] int((A + B*x^3)/(x*(a + b*x^3)^(5/2)),x)
```

```
[Out] ((2*A)/(9*a) - (2*B)/(9*b))/(a + b*x^3)^(3/2) + (2*A)/(3*a^2*(a + b*x^3)^(1
/2)) + (A*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)
))/x^6))/(3*a^(5/2))
```

3.248 $\int \frac{A+Bx^3}{x^4(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 113

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{5/2}}\,dx = \frac{-5Ab+2aB}{9a^2\left(a+bx^3\right)^{3/2}} - \frac{A}{3ax^3\left(a+bx^3\right)^{3/2}} - \frac{5Ab-2aB}{3a^3\sqrt{a+bx^3}} + \frac{(5Ab-2aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{7/2}}$$

```
[Out] 1/9*(-5*A*b+2*B*a)/a^2/(b*x^3+a)^(3/2)-1/3*A/a/x^3/(b*x^3+a)^(3/2)+1/3*(5*A
*b-2*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(7/2)+1/3*(-5*A*b+2*B*a)/a^3/(
b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 79, 53, 65, 214}

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{5/2}}\,dx = \frac{(5Ab-2aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{7/2}} - \frac{5Ab-2aB}{3a^3\sqrt{a+bx^3}} - \frac{5Ab-2aB}{9a^2\left(a+bx^3\right)^{3/2}} - \frac{A}{3ax^3\left(a+bx^3\right)^{3/2}}$$

```
[In] Int[(A + B*x^3)/(x^4*(a + b*x^3)^(5/2)),x]
```

```
[Out] -1/9*(5*A*b - 2*a*B)/(a^2*(a + b*x^3)^(3/2)) - A/(3*a*x^3*(a + b*x^3)^(3/2)
) - (5*A*b - 2*a*B)/(3*a^3*Sqrt[a + b*x^3]) + ((5*A*b - 2*a*B)*ArcTanh[Sqrt
[a + b*x^3]/Sqrt[a]])/(3*a^(7/2))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)^{5/2}}\,dx, x, x^3\right)$$

$$= -\frac{A}{3ax^3\left(a+bx^3\right)^{3/2}} + \frac{\left(-\frac{5Ab}{2}+aB\right)\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, x^3\right)}{3a}$$

$$=-\frac{5Ab-2aB}{9a^2\left(a+bx^3\right)^{3/2}}-\frac{A}{3ax^3\left(a+bx^3\right)^{3/2}}-\frac{(5Ab-2aB)\text{Subst}\left(\int\frac{1}{x(a+bx)^{3/2}}\,dx,x,x^3\right)}{6a^2}$$

$$=-\frac{5Ab-2aB}{9a^2\left(a+bx^3\right)^{3/2}}-\frac{A}{3ax^3\left(a+bx^3\right)^{3/2}}-\frac{5Ab-2aB}{3a^3\sqrt{a+bx^3}}-\frac{(5Ab-2aB)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)}{6a^3}$$

$$=-\frac{5Ab-2aB}{9a^2\left(a+bx^3\right)^{3/2}}-\frac{A}{3ax^3\left(a+bx^3\right)^{3/2}}-\frac{5Ab-2aB}{3a^3\sqrt{a+bx^3}}-\frac{(5Ab-2aB)\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{3a^3b}$$

$$=-\frac{5Ab-2aB}{9a^2\left(a+bx^3\right)^{3/2}}-\frac{A}{3ax^3\left(a+bx^3\right)^{3/2}}-\frac{5Ab-2aB}{3a^3\sqrt{a+bx^3}}+\frac{(5Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{7/2}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int\frac{A+Bx^3}{x^4\left(a+bx^3\right)^{5/2}}\,dx=\frac{-3a^2A-20aAbx^3+8a^2Bx^3-15Ab^2x^6+6abBx^6}{9a^3x^3\left(a+bx^3\right)^{3/2}}+\frac{(5Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{7/2}}$$

```
[In] Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^(5/2)),x]
[Out] (-3*a^2*A - 20*a*A*b*x^3 + 8*a^2*B*x^3 - 15*A*b^2*x^6 + 6*a*b*B*x^6)/(9*a^3
*x^3*(a + b*x^3)^(3/2)) + ((5*A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]
)/(3*a^(7/2))
```

Maple [A] (verified)

Time = 4.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.83

method	result
risch	$-\frac{A\sqrt{b\,x^{3}+a}}{3a^{3}x^{3}}-\frac{-\frac{2(5Ab-2Ba)\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{a}}+\frac{\frac{8Ab}{3}-\frac{4Ba}{3}}{\sqrt{b\,x^{3}+a}}+\frac{4a(Ab-Ba)}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}}}}{2a^{3}}$
pseudoelliptic	$-\frac{-5\left(b\,x^{3}+a\right)^{\frac{3}{2}}x^{3}\left(Ab-\frac{2Ba}{5}\right)\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)+\frac{20x^{3}b\left(-\frac{3x^{3}B}{10}+A\right)a^{\frac{3}{2}}}{3}+\left(-\frac{8x^{3}B}{3}+A\right)a^{\frac{5}{2}}+5A\sqrt{a}\,b^{2}x^{6}}{3\left(b\,x^{3}+a\right)^{\frac{3}{2}}a^{\frac{7}{2}}x^{3}}$
elliptic	$-\frac{A\sqrt{b\,x^{3}+a}}{3a^{3}x^{3}}-\frac{2(Ab-Ba)\sqrt{b\,x^{3}+a}}{9a^{2}b^{2}\left(x^{3}+\frac{a}{b}\right)^{2}}-\frac{2(2Ab-Ba)}{3a^{3}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{(5Ab-2Ba)\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{7}{2}}}$
default	$B\left(\frac{2\sqrt{b\,x^{3}+a}}{9a\,b^{2}\left(x^{3}+\frac{a}{b}\right)^{2}}+\frac{2}{3a^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{5}{2}}}\right)+A\left(-\frac{\sqrt{b\,x^{3}+a}}{3a^{3}x^{3}}-\frac{2\sqrt{b\,x^{3}+a}}{9a^{2}b\left(x^{3}+\frac{a}{b}\right)^{2}}-\frac{4b}{3a^{3}\sqrt{(x^{3}}}\right.$

```
[In] int((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a^3*A*(b*x^3+a)^(1/2)/x^3-1/2/a^3*(-2/3*(5*A*b-2*B*a)*arctanh((b*x^3+a
)^(1/2)/a^(1/2))/a^(1/2)+4/3*(2*A*b-B*a)/(b*x^3+a)^(1/2)+4/9*a*(A*b-B*a)/(b
*x^3+a)^(3/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 351, normalized size of antiderivative = 3.11

$$\int \frac{A+Bx^3}{x^4\,(a+bx^3)^{5/2}}\,dx=\left[-\frac{3\,((2\,Bab^2-5\,Ab^3)x^9+2\,(2\,Ba^2b-5\,Aab^2)x^6+(2\,Ba^3-5\,Aa^2b)x^3)\sqrt{a}\log\left(\frac{bx^3}{}\right.}{18\,(a^4b^2x^9}\right.$$

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/18*(3*((2*B*a*b^2 - 5*A*b^3)*x^9 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^6 + (2*B
*a^3 - 5*A*a^2*b)*x^3)*sqrt(a)*log((b*x^3 + 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a
)/x^3) - 2*(3*(2*B*a^2*b - 5*A*a*b^2)*x^6 - 3*A*a^3 + 4*(2*B*a^3 - 5*A*a^2*
b)*x^3)*sqrt(b*x^3 + a))/(a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^3), 1/9*(3*((2*
B*a*b^2 - 5*A*b^3)*x^9 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^6 + (2*B*a^3 - 5*A*a^2
*b)*x^3)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (3*(2*B*a^2*b - 5*A*
a*b^2)*x^6 - 3*A*a^3 + 4*(2*B*a^3 - 5*A*a^2*b)*x^3)*sqrt(b*x^3 + a))/(a^4*b
^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1608 vs. 2(107) = 214.

Time = 126.77 (sec) , antiderivative size = 1608, normalized size of antiderivative = 14.23

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x**3+A)/x**4/(b*x**3+a)**(5/2),x)
```

```
[Out] A*(-6*a**17*sqrt(1 + b*x**3/a)/(18*a**(39/2)*x**3 + 54*a**(37/2)*b*x**6 + 5
4*a**(35/2)*b**2*x**9 + 18*a**(33/2)*b**3*x**12) - 46*a**16*b*x**3*sqrt(1 +
 b*x**3/a)/(18*a**(39/2)*x**3 + 54*a**(37/2)*b*x**6 + 54*a**(35/2)*b**2*x**
9 + 18*a**(33/2)*b**3*x**12) - 15*a**16*b*x**3*log(b*x**3/a)/(18*a**(39/2)*
x**3 + 54*a**(37/2)*b*x**6 + 54*a**(35/2)*b**2*x**9 + 18*a**(33/2)*b**3*x**
12) + 30*a**16*b*x**3*log(sqrt(1 + b*x**3/a) + 1)/(18*a**(39/2)*x**3 + 54*a
**(37/2)*b*x**6 + 54*a**(35/2)*b**2*x**9 + 18*a**(33/2)*b**3*x**12) - 70*a*
*15*b**2*x**6*sqrt(1 + b*x**3/a)/(18*a**(39/2)*x**3 + 54*a**(37/2)*b*x**6 +
 54*a**(35/2)*b**2*x**9 + 18*a**(33/2)*b**3*x**12) - 45*a**15*b**2*x**6*log
(b*x**3/a)/(18*a**(39/2)*x**3 + 54*a**(37/2)*b*x**6 + 54*a**(35/2)*b**2*x**
9 + 18*a**(33/2)*b**3*x**12) + 90*a**15*b**2*x**6*log(sqrt(1 + b*x**3/a) +
1)/(18*a**(39/2)*x**3 + 54*a**(37/2)*b*x**6 + 54*a**(35/2)*b**2*x**9 + 18*a
**(33/2)*b**3*x**12) - 30*a**14*b**3*x**9*sqrt(1 + b*x**3/a)/(18*a**(39/2)*
x**3 + 54*a**(37/2)*b*x**6 + 54*a**(35/2)*b**2*x**9 + 18*a**(33/2)*b**3*x**
12) - 45*a**14*b**3*x**9*log(b*x**3/a)/(18*a**(39/2)*x**3 + 54*a**(37/2)*b*
x**6 + 54*a**(35/2)*b**2*x**9 + 18*a**(33/2)*b**3*x**12) + 90*a**14*b**3*x*
*9*log(sqrt(1 + b*x**3/a) + 1)/(18*a**(39/2)*x**3 + 54*a**(37/2)*b*x**6 + 5
4*a**(35/2)*b**2*x**9 + 18*a**(33/2)*b**3*x**12) - 15*a**13*b**4*x**12*log(
b*x**3/a)/(18*a**(39/2)*x**3 + 54*a**(37/2)*b*x**6 + 54*a**(35/2)*b**2*x**9
 + 18*a**(33/2)*b**3*x**12) + 30*a**13*b**4*x**12*log(sqrt(1 + b*x**3/a) +
1)/(18*a**(39/2)*x**3 + 54*a**(37/2)*b*x**6 + 54*a**(35/2)*b**2*x**9 + 18*a
**(33/2)*b**3*x**12)) + B*(8*a**7*sqrt(1 + b*x**3/a)/(9*a**(19/2) + 27*a**(
17/2)*b*x**3 + 27*a**(15/2)*b**2*x**6 + 9*a**(13/2)*b**3*x**9) + 3*a**7*log
(b*x**3/a)/(9*a**(19/2) + 27*a**(17/2)*b*x**3 + 27*a**(15/2)*b**2*x**6 + 9*
a**(13/2)*b**3*x**9) - 6*a**7*log(sqrt(1 + b*x**3/a) + 1)/(9*a**(19/2) + 27
*a**(17/2)*b*x**3 + 27*a**(15/2)*b**2*x**6 + 9*a**(13/2)*b**3*x**9) + 14*a*
*6*b*x**3*sqrt(1 + b*x**3/a)/(9*a**(19/2) + 27*a**(17/2)*b*x**3 + 27*a**(15
/2)*b**2*x**6 + 9*a**(13/2)*b**3*x**9) + 9*a**6*b*x**3*log(b*x**3/a)/(9*a**
(19/2) + 27*a**(17/2)*b*x**3 + 27*a**(15/2)*b**2*x**6 + 9*a**(13/2)*b**3*x*
*9) - 18*a**6*b*x**3*log(sqrt(1 + b*x**3/a) + 1)/(9*a**(19/2) + 27*a**(17/2
)*b*x**3 + 27*a**(15/2)*b**2*x**6 + 9*a**(13/2)*b**3*x**9) + 6*a**5*b**2*x*
*6*sqrt(1 + b*x**3/a)/(9*a**(19/2) + 27*a**(17/2)*b*x**3 + 27*a**(15/2)*b**
2*x**6 + 9*a**(13/2)*b**3*x**9) + 9*a**5*b**2*x**6*log(b*x**3/a)/(9*a**(19/
2) + 27*a**(17/2)*b*x**3 + 27*a**(15/2)*b**2*x**6 + 9*a**(13/2)*b**3*x**9)
- 18*a**5*b**2*x**6*log(sqrt(1 + b*x**3/a) + 1)/(9*a**(19/2) + 27*a**(17/2)
```

```
*b*x**3 + 27*a**(15/2)*b**2*x**6 + 9*a**(13/2)*b**3*x**9) + 3*a**4*b**3*x**
9*log(b*x**3/a)/(9*a**(19/2) + 27*a**(17/2)*b*x**3 + 27*a**(15/2)*b**2*x**6
 + 9*a**(13/2)*b**3*x**9) - 6*a**4*b**3*x**9*log(sqrt(1 + b*x**3/a) + 1)/(9
*a**(19/2) + 27*a**(17/2)*b*x**3 + 27*a**(15/2)*b**2*x**6 + 9*a**(13/2)*b**
3*x**9))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.50

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{5/2}}\,dx = -\frac{1}{18}\,A\left(\frac{2\left(15\,(bx^3+a)^2b-10\,(bx^3+a)ab-2\,a^2b\right)}{(bx^3+a)^{\frac{5}{2}}a^3-(bx^3+a)^{\frac{3}{2}}a^4}+\frac{15\,b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right) + \frac{1}{9}\,B\left(\frac{3\,\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\,(3\,bx^3+4\,a)}{(bx^3+a)^{\frac{3}{2}}a^2}\right)$$

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/18*A*(2*(15*(b*x^3 + a)^2*b - 10*(b*x^3 + a)*a*b - 2*a^2*b)/((b*x^3 + a)
^(5/2)*a^3 - (b*x^3 + a)^(3/2)*a^4) + 15*b*log((sqrt(b*x^3 + a) - sqrt(a))/
(sqrt(b*x^3 + a) + sqrt(a)))/a^(7/2)) + 1/9*B*(3*log((sqrt(b*x^3 + a) - sqr
t(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(5/2) + 2*(3*b*x^3 + 4*a)/((b*x^3 + a)
^(3/2)*a^2))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{5/2}}\,dx = \frac{(2\,Ba-5\,Ab)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}a^3} + \frac{2\,(3\,(bx^3+a)Ba+Ba^2-6\,(bx^3+a)Ab-Aab)}{9\,(bx^3+a)^{\frac{3}{2}}a^3} - \frac{\sqrt{bx^3+a}A}{3\,a^3x^3}$$

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*B*a - 5*A*b)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^3) + 2/9*(
3*(b*x^3 + a)*B*a + B*a^2 - 6*(b*x^3 + a)*A*b - A*a*b)/((b*x^3 + a)^(3/2)*a
^3) - 1/3*sqrt(b*x^3 + a)*A/(a^3*x^3)
```

Mupad [B] (verification not implemented)

Time = 7.23 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.75

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{5/2}}\,dx=\frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)^3}{x^6}\right)\,(5\,A\,b-2\,B\,a)}{6\,a^{7/2}}-\frac{\frac{2\,B\,a^2-5\,A\,a\,b}{2\,a^4}-\frac{a\left(\frac{A\,b^2}{3\,a^4}+\frac{5\,b\left(2\,B\,a^2-5\,A\,a\,b\right)}{6\,a^5}\right)}{b}}{\sqrt{b\,x^3+a}}-\frac{\frac{2\,B\,a^3-5\,A\,a^2\,b}{4\,a^4}-\frac{a\left(\frac{13\,b\left(2\,B\,a^3-5\,A\,a^2\,b\right)}{36\,a^5}+\frac{A\,b^2}{3\,a^3}\right)}{b}}{\left(b\,x^3+a\right)^{3/2}}-\frac{A\,\sqrt{b\,x^3+a}}{3\,a^3\,x^3}$$

```
[In] int((A + B*x^3)/(x^4*(a + b*x^3)^(5/2)),x)
[Out] (log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/2))^3)/x^6)*(
5*A*b - 2*B*a))/(6*a^(7/2)) - ((2*B*a^2 - 5*A*a*b)/(2*a^4) - (a*((A*b^2)/(3
*a^4) + (5*b*(2*B*a^2 - 5*A*a*b))/(6*a^5)))/b)/(a + b*x^3)^(1/2) - ((2*B*a^
3 - 5*A*a^2*b)/(4*a^4) - (a*((13*b*(2*B*a^3 - 5*A*a^2*b))/(36*a^5) + (A*b^2
)/(3*a^3)))/b)/(a + b*x^3)^(3/2) - (A*(a + b*x^3)^(1/2))/(3*a^3*x^3)
```

3.249 $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 299

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2(5Ab-14aB)x^4}{45b^2(a+bx^3)^{3/2}}+\frac{2Bx^7}{5b(a+bx^3)^{3/2}}-\frac{16(5Ab-14aB)x}{135b^3\sqrt{a+bx^3}} + \frac{32\sqrt{2+\sqrt{3}}(5Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7}{135\sqrt[4]{3}b^{10/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -2/45*(5*A*b-14*B*a)*x^4/b^2/(b*x^3+a)^(3/2)+2/5*B*x^7/b/(b*x^3+a)^(3/2)-16
/135*(5*A*b-14*B*a)*x/b^3/(b*x^3+a)^(1/2)+32/405*(5*A*b-14*B*a)*(a^(1/3)+b^
(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(10/3)/(b
*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^
2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {470, 294, 224}

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{32\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-14aB)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{b}x}\right.\right.}{135\sqrt[4]{3}b^{10/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{16x(5Ab-14aB)}{135b^3\sqrt{a+bx^3}}-\frac{2x^4(5Ab-14aB)}{45b^2\left(a+bx^3\right)^{3/2}}+\frac{2Bx^7}{5b\left(a+bx^3\right)^{3/2}}$$

```
[In] Int[(x^6*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (-2*(5*A*b - 14*a*B)*x^4)/(45*b^2*(a + b*x^3)^(3/2)) + (2*B*x^7)/(5*b*(a +
b*x^3)^(3/2)) - (16*(5*A*b - 14*a*B)*x)/(135*b^3*Sqrt[a + b*x^3]) + (32*Sqr
t[2 + Sqrt[3]]*(5*A*b - 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elliptic
F[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)], -7 - 4*Sqrt[3]])/(135*3^(1/4)*b^(10/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
```

```
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{2Bx^7}{5b(a+bx^3)^{3/2}} - \frac{\left(2\left(-\frac{5Ab}{2}+7aB\right)\right)\int \frac{x^6}{(a+bx^3)^{5/2}}\,dx}{5b}$$

$$= -\frac{2(5Ab-14aB)x^4}{45b^2(a+bx^3)^{3/2}} + \frac{2Bx^7}{5b(a+bx^3)^{3/2}} + \frac{(8(5Ab-14aB))\int \frac{x^3}{(a+bx^3)^{3/2}}\,dx}{45b^2}$$

$$= -\frac{2(5Ab-14aB)x^4}{45b^2(a+bx^3)^{3/2}} + \frac{2Bx^7}{5b(a+bx^3)^{3/2}} - \frac{16(5Ab-14aB)x}{135b^3\sqrt{a+bx^3}} + \frac{(16(5Ab-14aB))\int \frac{1}{\sqrt{a+bx^3}}\,dx}{135b^3}$$

$$= -\frac{2(5Ab-14aB)x^4}{45b^2(a+bx^3)^{3/2}} + \frac{2Bx^7}{5b(a+bx^3)^{3/2}} - \frac{16(5Ab-14aB)x}{135b^3\sqrt{a+bx^3}} + \frac{32\sqrt{2+\sqrt{3}}(5Ab-14aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-\right.}{135\sqrt[4]{3}b^{10/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.36

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2x\left(112a^2B+b^2x^3(-55A+27Bx^3)+a(-40Ab+154bBx^3)+8(5Ab-14aB)(a+b\right.}{135b^3(a+bx^3)^{3/2}}$$

```
[In] Integrate[(x^6*(A + B*x^3))/(a + b*x^3)^(5/2),x]
[Out] (2*x*(112*a^2*B + b^2*x^3*(-55*A + 27*B*x^3) + a*(-40*A*b + 154*b*B*x^3) +
8*(5*A*b - 14*a*B)*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1
/2, 4/3, -((b*x^3)/a)]))/(135*b^3*(a + b*x^3)^(3/2))
```

Maple [A] (verified)

Time = 5.36 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.33

method	result
elliptic	$\frac{2xa(Ab-Ba)\sqrt{b\,x^3+a}}{9b^5\left(x^3+\frac{a}{b}\right)^2}-\frac{2x(11Ab-20Ba)}{27b^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2Bx\sqrt{b\,x^3+a}}{5b^3}-\frac{2i\left(\frac{Ab-2Ba}{b^3}-\frac{11Ab-20Ba}{27b^3}-\frac{2Ba}{5b^3}\right)\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{}}{}\right.}{\left(-a\,b\right.}}}{}$
default	$B\left(-\frac{2x\,a^2\sqrt{b\,x^3+a}}{9b^5\left(x^3+\frac{a}{b}\right)^2}+\frac{40ax}{27b^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2x\sqrt{b\,x^3+a}}{5b^3}+\frac{448ia\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x}{-\frac{3\left(-a\,b^2\right.}{2b}}}}{}\right.$
risch	Expression too large to display

[In] int(x^6*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)

[Out] 2/9*x*a/b^5*(A*b-B*a)*(b*x^3+a)^(1/2)/(x^3+a/b)^2-2/27/b^3*x*(11*A*b-20*B*a)/((x^3+a/b)*b)^(1/2)+2/5*B/b^3*x*(b*x^3+a)^(1/2)-2/3*I*((A*b-2*B*a)/b^3-1/27/b^3*(11*A*b-20*B*a)-2/5*B/b^3*a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.51

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2\left(16\left(\left(14\,Bab^2-5\,Ab^3\right)x^6+14\,Ba^3-5\,Aa^2b+2\left(14\,Ba^2b-5\,Aab^2\right)x^3\right)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4a}{b},\right.\right.}{135\left(b^6x^6+2\,ab^5x^3+a^2b^4\right)}$$

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/135*(16*((14*B*a*b^2 - 5*A*b^3)*x^6 + 14*B*a^3 - 5*A*a^2*b + 2*(14*B*a^2
*b - 5*A*a*b^2)*x^3)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) - (27*B*b^3*
x^7 + 11*(14*B*a*b^2 - 5*A*b^3)*x^4 + 8*(14*B*a^2*b - 5*A*a*b^2)*x)*sqrt(b*
x^3 + a))/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)
```

Sympy [A] (verification not implemented)

Time = 61.00 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.27

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{Ax^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{7}{3},\frac{5}{2}\\\frac{10}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{10}{3}\right)}+\frac{Bx^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{2},\frac{10}{3}\\\frac{13}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{13}{3}\right)}$$

```
[In] integrate(x**6*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] A*x**7*gamma(7/3)*hyper((7/3, 5/2), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(5/2)*gamma(10/3)) + B*x**10*gamma(10/3)*hyper((5/2, 10/3), (13/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(13/3))
```

Maxima [F]

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)x^6}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^6/(b*x^3 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)x^6}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*x^6/(b*x^3 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{x^6\,(B\,x^3+A)}{(b\,x^3+a)^{5/2}}\,dx$$

```
[In] int((x^6*(A + B*x^3))/(a + b*x^3)^(5/2),x)
[Out] int((x^6*(A + B*x^3))/(a + b*x^3)^(5/2), x)
```

3.250 $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 283

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2(Ab-aB)x^4}{9ab\,(a+bx^3)^{3/2}} - \frac{2(Ab+8aB)x}{27ab^2\sqrt{a+bx^3}} + \frac{4\sqrt{2+\sqrt{3}}(Ab+8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right), -7-4$$
$$27\sqrt[4]{3}ab^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}$$

```
[Out] 2/9*(A*b-B*a)*x^4/a/b/(b*x^3+a)^(3/2)-2/27*(A*b+8*B*a)*x/a/b^2/(b*x^3+a)^(1
/2)+4/81*(A*b+8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^
(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(
1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))^2)^(1/2)*3^(3/4)/a/b^(7/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used

$= \{468, 294, 224\}$

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(8aB+Ab)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+(1\cdots}{\sqrt[3]{b}x+(1\cdots}\right.\right.}{27\sqrt[4]{3}ab^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2x(8aB+Ab)}{27ab^2\sqrt{a+bx^3}}+\frac{2x^4(Ab-aB)}{9ab\,(a+bx^3)^{3/2}}$$

```
[In] Int[(x^3*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*x^4)/(9*a*b*(a + b*x^3)^(3/2)) - (2*(A*b + 8*a*B)*x)/(27*a*b
^2*Sqrt[a + b*x^3]) + (4*Sqrt[2 + Sqrt[3]]*(A*b + 8*a*B)*(a^(1/3) + b^(1/3)
*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a*b^(7/3)*Sqr
t[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sq
rt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
```

```
m, (-n)*(p + 1)]))
```

Rubi steps

$$\text{integral} = \frac{2(Ab-aB)x^4}{9ab\left(a+bx^3\right)^{3/2}} + \frac{\left(2\left(\frac{Ab}{2}+4aB\right)\right)\int \frac{x^3}{(a+bx^3)^{3/2}}\,dx}{9ab}$$

$$= \frac{2(Ab-aB)x^4}{9ab\left(a+bx^3\right)^{3/2}} - \frac{2(Ab+8aB)x}{27ab^2\sqrt{a+bx^3}} + \frac{(2(Ab+8aB))\int \frac{1}{\sqrt{a+bx^3}}\,dx}{27ab^2}$$

$$= \frac{2(Ab-aB)x^4}{9ab\left(a+bx^3\right)^{3/2}} - \frac{2(Ab+8aB)x}{27ab^2\sqrt{a+bx^3}} + \frac{4\sqrt{2+\sqrt{3}}(Ab+8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-}{27\sqrt[4]{3}ab^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.35

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2x\left(-8a^2B+2Ab^2x^3-ab(A+11Bx^3)+(Ab+8aB)\left(a+bx^3\right)\sqrt{1+\frac{bx^3}{a}}\text{Hypergeon}\right.}{27ab^2\left(a+bx^3\right)^{3/2}}$$

```
[In] Integrate[(x^3*(A + B*x^3))/(a + b*x^3)^(5/2),x]
[Out] (2*x*(-8*a^2*B + 2*A*b^2*x^3 - a*b*(A + 11*B*x^3) + (A*b + 8*a*B)*(a + b*x^
3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]))/(27
*a*b^2*(a + b*x^3)^(3/2))
```

Maple [A] (verified)

Time = 4.43 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.31

method	result
elliptic	$-\frac{2x(Ab-Ba)\sqrt{b\,x^3+a}}{9b^4\left(x^3+\frac{a}{b}\right)^2}+\frac{2x(2Ab-11Ba)}{27b^2a\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2i\left(\frac{B}{b^2}+\frac{2Ab-11Ba}{27b^2a}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x \cdots}{-\frac{3(-a\,b \cdots}{2b}}}\cdots}{\cdots}$
default	$B\left(\frac{2ax\sqrt{b\,x^3+a}}{9b^4\left(x^3+\frac{a}{b}\right)^2}-\frac{22x}{27b^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{32i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{\cdots}}{\cdots}\right.$

```
[In] int(x^3*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/9*x/b^4*(A*b-B*a)*(b*x^3+a)^(1/2)/(x^3+a/b)^2+2/27/b^2*x/a*(2*A*b-11*B*a
)/((x^3+a/b)*b)^(1/2)-2/3*I*(B/b^2+1/27/b^2/a*(2*A*b-11*B*a))*3^(1/2)/b*(-a
*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.50

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx=\frac{2\left(2\left((8\,Bab^2+Ab^3)x^6+8\,Ba^3+Aa^2b+2\,(8\,Ba^2b+Aab^2)x^3\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\cdots\right.}{27\,(ab^5x^6+2\,a^2b^4x^3+\cdots}$$

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/27*(2*((8*B*a*b^2 + A*b^3)*x^6 + 8*B*a^3 + A*a^2*b + 2*(8*B*a^2*b + A*a*b
^2)*x^3)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) - ((11*B*a*b^2 - 2*A*b^3
)*x^4 + (8*B*a^2*b + A*a*b^2)*x)*sqrt(b*x^3 + a))/(a*b^5*x^6 + 2*a^2*b^4*x^
3 + a^3*b^3)
```

Sympy [A] (verification not implemented)

Time = 38.55 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.28

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{Ax^4\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{5}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{Bx^7\Gamma\left(\frac{7}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{7}{3},\frac{5}{2}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{10}{3}\right)}$$

```
[In] integrate(x**3*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] A*x**4*gamma(4/3)*hyper((4/3, 5/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(5/2)*gamma(7/3)) + B*x**7*gamma(7/3)*hyper((7/3, 5/2), (10/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(5/2)*gamma(10/3))
```

Maxima [F]

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)x^3}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^3/(b*x^3 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)x^3}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^3/(b*x^3 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{x^3\,(B\,x^3+A)}{(b\,x^3+a)^{5/2}}\,dx$$

```
[In] int((x^3*(A + B*x^3))/(a + b*x^3)^(5/2),x)
[Out] int((x^3*(A + B*x^3))/(a + b*x^3)^(5/2), x)
```

3.251 $\int \frac{A+Bx^3}{(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 283

$$\int \frac{A+Bx^3}{(a+bx^3)^{5/2}}\,dx=\frac{2(Ab-aB)x}{9ab\,(a+bx^3)^{3/2}}+\frac{2(7Ab+2aB)x}{27a^2b\sqrt{a+bx^3}}$$
$$+\frac{2\sqrt{2+\sqrt{3}}(7Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-\right.}{27\sqrt[4]{3}a^2b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/9*(A*b-B*a)*x/a/b/(b*x^3+a)^(3/2)+2/27*(7*A*b+2*B*a)*x/a^2/b/(b*x^3+a)^(1
/2)+2/81*(7*A*b+2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-
3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2
^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))^2)^(1/2)*3^(3/4)/a^2/b^(4/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3
)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used

= {393, 205, 224}

$$\int \frac{A+Bx^3}{(a+bx^3)^{5/2}}\,dx = \frac{2x(2aB+7Ab)}{27a^2b\sqrt{a+bx^3}} + \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2aB+7Ab)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}a^2b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{2x(Ab-aB)}{9ab\,(a+bx^3)^{3/2}}$$

```
[In] Int[(A + B*x^3)/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*x)/(9*a*b*(a + b*x^3)^(3/2)) + (2*(7*A*b + 2*a*B)*x)/(27*a^2
*b*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(7*A*b + 2*a*B)*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a^2*b^(4/3)
*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*Sqrt[a + b*x^3])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
```

```
 + p, 0])
```

Rubi steps

$$\text{integral} = \frac{2(Ab-aB)x}{9ab\,(a+bx^3)^{3/2}} + \frac{\left(2\left(\frac{7Ab}{2}+aB\right)\right)\int\frac{1}{(a+bx^3)^{3/2}}\,dx}{9ab}$$

$$= \frac{2(Ab-aB)x}{9ab\,(a+bx^3)^{3/2}} + \frac{2(7Ab+2aB)x}{27a^2b\sqrt{a+bx^3}} + \frac{(7Ab+2aB)\int\frac{1}{\sqrt{a+bx^3}}\,dx}{27a^2b}$$

$$= \frac{2(Ab-aB)x}{9ab\,(a+bx^3)^{3/2}} + \frac{2(7Ab+2aB)x}{27a^2b\sqrt{a+bx^3}} + \frac{2\sqrt{2+\sqrt{3}}(7Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\right.}{27\sqrt[4]{3}a^2b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.36

$$\int\frac{A+Bx^3}{(a+bx^3)^{5/2}}\,dx = \frac{-2a^2Bx+14Ab^2x^4+4abx(5A+Bx^3)+(7Ab+2aB)x(a+bx^3)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeo}}{27a^2b\,(a+bx^3)^{3/2}}$$

```
[In] Integrate[(A + B*x^3)/(a + b*x^3)^(5/2),x]
[Out] (-2*a^2*B*x + 14*A*b^2*x^4 + 4*a*b*x*(5*A + B*x^3) + (7*A*b + 2*a*B)*x*(a +
 b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)])
/(27*a^2*b*(a + b*x^3)^(3/2))
```

Maple [A] (verified)

Time = 4.38 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.29

method	result
elliptic	$\frac{2x(Ab-Ba)\sqrt{b\,x^3+a}}{9a\,b^3\left(x^3+\frac{a}{b}\right)^2}+\frac{2x(7Ab+2Ba)}{27b\,a^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2i(7Ab+2Ba)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{}}{}}}}{}$
default	$A\left(\frac{2x\sqrt{b\,x^3+a}}{9a\,b^2\left(x^3+\frac{a}{b}\right)^2}+\frac{14x}{27a^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{14i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}}{}\right.$

```
[In] int((B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*x/a/b^3*(A*b-B*a)*(b*x^3+a)^(1/2)/(x^3+a/b)^2+2/27/b*x/a^2*(7*A*b+2*B*a
)/((x^3+a/b)*b)^(1/2)-2/81*I*(7*A*b+2*B*a)/a^2/b^2*3^(1/2)*(-a*b^2)^(1/3)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.51

$$\int \frac{A+Bx^3}{(a+bx^3)^{5/2}}\,dx=\frac{2\left(((2\,Bab^2+7\,Ab^3)x^6+2\,Ba^3+7\,Aa^2b+2\,(2\,Ba^2b+7\,Aab^2)x^3)\sqrt{b}\text{weierstrassPInvers}\right.}{27\,(a^2b^4x^6+2\,a^3b^3x^3+}$$

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/27*(((2*B*a*b^2 + 7*A*b^3)*x^6 + 2*B*a^3 + 7*A*a^2*b + 2*(2*B*a^2*b + 7*A
*a*b^2)*x^3)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + ((2*B*a*b^2 + 7*A*
b^3)*x^4 - (B*a^2*b - 10*A*a*b^2)*x)*sqrt(b*x^3 + a))/(a^2*b^4*x^6 + 2*a^3*
b^3*x^3 + a^4*b^2)
```

Sympy [A] (verification not implemented)

Time = 24.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.28

$$\int \frac{A+Bx^3}{(a+bx^3)^{5/2}}\,dx = \frac{Ax\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{5}{2} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{Bx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} \frac{4}{3}, \frac{5}{2} \\ \frac{7}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{7}{3}\right)}$$

```
[In] integrate((B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] A*x*gamma(1/3)*hyper((1/3, 5/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5
/2)*gamma(4/3)) + B*x**4*gamma(4/3)*hyper((4/3, 5/2), (7/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(5/2)*gamma(7/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{(a+bx^3)^{5/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/(b*x^3 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{(a+bx^3)^{5/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/(b*x^3 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{B\,x^3+A}{\left(b\,x^3+a\right)^{5/2}}\,dx$$

```
[In] int((A + B*x^3)/(a + b*x^3)^(5/2),x)
[Out] int((A + B*x^3)/(a + b*x^3)^(5/2), x)
```

3.252 $\int \frac{A+Bx^3}{x^3(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 300

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{5/2}}\,dx = -\frac{A}{2ax^2\left(a+bx^3\right)^{3/2}}-\frac{(13Ab-4aB)x}{18a^2\left(a+bx^3\right)^{3/2}}-\frac{7(13Ab-4aB)x}{54a^3\sqrt{a+bx^3}}$$
$$-\frac{7\sqrt{2+\sqrt{3}}(13Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-}{54\sqrt[4]{3}a^3\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/2*A/a/x^2/(b*x^3+a)^(3/2)-1/18*(13*A*b-4*B*a)*x/a^2/(b*x^3+a)^(3/2)-7/54
*(13*A*b-4*B*a)*x/a^3/(b*x^3+a)^(1/2)-7/162*(13*A*b-4*B*a)*(a^(1/3)+b^(1/3)
*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^3/b^(1/3)/(b*x
^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)
^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {464, 205, 224}

$$\int \frac{A+Bx^3}{x^3(a+bx^3)^{5/2}}\,dx = -\frac{7x(13Ab-4aB)}{54a^3\sqrt{a+bx^3}} - \frac{x(13Ab-4aB)}{18a^2(a+bx^3)^{3/2}}$$
$$-\frac{7\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(13Ab-4aB)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-\right.}{54\sqrt[4]{3}a^3\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{A}{2ax^2(a+bx^3)^{3/2}}$$

```
[In] Int[(A + B*x^3)/(x^3*(a + b*x^3)^(5/2)),x]
```

```
[Out] -1/2*A/(a*x^2*(a + b*x^3)^(3/2)) - ((13*A*b - 4*a*B)*x)/(18*a^2*(a + b*x^3)
^(3/2)) - (7*(13*A*b - 4*a*B)*x)/(54*a^3*Sqrt[a + b*x^3]) - (7*Sqrt[2 + Sqr
t[3]]*(13*A*b - 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -
7 - 4*Sqrt[3]])/(54*3^(1/4)*a^3*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x)
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{A}{2ax^2\left(a+bx^3\right)^{3/2}} - \frac{\left(\frac{13Ab}{2}-2aB\right)\int \frac{1}{(a+bx^3)^{5/2}}\,dx}{2a}$$

$$= -\frac{A}{2ax^2\left(a+bx^3\right)^{3/2}} - \frac{(13Ab-4aB)x}{18a^2\left(a+bx^3\right)^{3/2}} - \frac{(7(13Ab-4aB))\int \frac{1}{(a+bx^3)^{3/2}}\,dx}{36a^2}$$

$$= -\frac{A}{2ax^2\left(a+bx^3\right)^{3/2}} - \frac{(13Ab-4aB)x}{18a^2\left(a+bx^3\right)^{3/2}} - \frac{7(13Ab-4aB)x}{54a^3\sqrt{a+bx^3}} - \frac{(7(13Ab-4aB))\int \frac{1}{\sqrt{a+bx^3}}\,dx}{108a^3}$$

$$= -\frac{A}{2ax^2\left(a+bx^3\right)^{3/2}} - \frac{(13Ab-4aB)x}{18a^2\left(a+bx^3\right)^{3/2}} - \frac{7(13Ab-4aB)x}{54a^3\sqrt{a+bx^3}} - \frac{7\sqrt{2+\sqrt{3}}(13Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7\right.}{54\sqrt[4]{3}a^3\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.39

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{5/2}}\,dx = \frac{-182Ab^2x^6+a^2(-54A+80Bx^3)+a(-260Abx^3+56bBx^6)+7(-13Ab+4aB)x^3(}{108a^3x^2\left(a+bx^3\right)^{3/2}}$$

```
[In] Integrate[(A + B*x^3)/(x^3*(a + b*x^3)^(5/2)),x]
```

```
[Out] (-182*A*b^2*x^6 + a^2*(-54*A + 80*B*x^3) + a*(-260*A*b*x^3 + 56*b*B*x^6) +
7*(-13*A*b + 4*a*B)*x^3*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1
/3, 1/2, 4/3, -((b*x^3)/a)])/(108*a^3*x^2*(a + b*x^3)^(3/2))
```

Maple [A] (verified)

Time = 5.54 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.30

method	result
elliptic	$-\frac{2x(Ab-Ba)\sqrt{b\,x^3+a}}{9a^2b^2\left(x^3+\frac{a}{b}\right)^2}-\frac{2x(16Ab-7Ba)}{27a^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{A\sqrt{b\,x^3+a}}{2a^3x^2}-\frac{2i\left(-\frac{16Ab-7Ba}{27a^3}-\frac{Ab}{4a^3}\right)\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right.}{\left(-a\,b^2\right)^{\frac{1}{3}}}}}{}$
default	$B\left(\frac{2x\sqrt{b\,x^3+a}}{9a\,b^2\left(x^3+\frac{a}{b}\right)^2}+\frac{14x}{27a^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{14i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}}{}\right.$
risch	Expression too large to display

[In] int((B*x^3+A)/x^3/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)

[Out] -2/9*x/a^2/b^2*(A*b-B*a)*(b*x^3+a)^(1/2)/(x^3+a/b)^2-2/27*x/a^3*(16*A*b-7*B*a)/((x^3+a/b)*b)^(1/2)-1/2/a^3*A*(b*x^3+a)^(1/2)/x^2-2/3*I*(-1/27/a^3*(16*A*b-7*B*a)-1/4/a^3*A*b)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.54

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{5/2}}\,dx = \frac{7\left((4\,Bab^2-13\,Ab^3)x^8+2\,(4\,Ba^2b-13\,Aab^2)x^5+(4\,Ba^3-13\,Aa^2b)x^2\right)\sqrt{b}\text{weierstr}}{54\,(a^3b^3}$$

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/54*(7*((4*B*a*b^2 - 13*A*b^3)*x^8 + 2*(4*B*a^2*b - 13*A*a*b^2)*x^5 + (4*B
*a^3 - 13*A*a^2*b)*x^2)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + (7*(4*B
*a*b^2 - 13*A*b^3)*x^6 - 27*A*a^2*b + 10*(4*B*a^2*b - 13*A*a*b^2)*x^3)*sqrt
(b*x^3 + a))/(a^3*b^3*x^8 + 2*a^4*b^2*x^5 + a^5*b*x^2)
```

Sympy [A] (verification not implemented)

Time = 81.00 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.27

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{5/2}}\,dx = \frac{A\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{5}{2}\\\frac{1}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}x^2\Gamma\left(\frac{1}{3}\right)} + \frac{Bx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{5}{2}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{4}{3}\right)}$$

```
[In] integrate((B*x**3+A)/x**3/(b*x**3+a)**(5/2),x)
```

```
[Out] A*gamma(-2/3)*hyper((-2/3, 5/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5
/2)*x**2*gamma(1/3)) + B*x*gamma(1/3)*hyper((1/3, 5/2), (4/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(5/2)*gamma(4/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{5/2}}\,dx = \int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}x^3}\,dx$$

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^3), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^3(a+bx^3)^{5/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{5}{2}}x^3}\,dx$$

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(5/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^3(a+bx^3)^{5/2}}\,dx = \int \frac{B\,x^3+A}{x^3\,(b\,x^3+a)^{5/2}}\,dx$$

```
[In] int((A + B*x^3)/(x^3*(a + b*x^3)^(5/2)),x)
[Out] int((A + B*x^3)/(x^3*(a + b*x^3)^(5/2)), x)
```

3.253 $\int \frac{A+Bx^3}{x^6(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 334

$$\int \frac{A+Bx^3}{x^6(a+bx^3)^{5/2}}\,dx = -\frac{A}{5ax^5(a+bx^3)^{3/2}} - \frac{19Ab-10aB}{45a^2x^2(a+bx^3)^{3/2}} - \frac{13(19Ab-10aB)}{135a^3x^2\sqrt{a+bx^3}} + \frac{91(19Ab-10aB)\sqrt{a+bx^3}}{540a^4x^2} + \frac{91\sqrt{2+\sqrt{3}}\,b^{2/3}(19Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{540\sqrt[4]{3}a^4\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/5*A/a/x^5/(b*x^3+a)^(3/2)+1/45*(-19*A*b+10*B*a)/a^2/x^2/(b*x^3+a)^(3/2)-
13/135*(19*A*b-10*B*a)/a^3/x^2/(b*x^3+a)^(1/2)+91/540*(19*A*b-10*B*a)*(b*x^
3+a)^(1/2)/a^4/x^2+91/1620*b^(2/3)*(19*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*Elli
pticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(
1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2
)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^4/(b*x^3+a)^(1/2)/(a^(
1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {464, 296, 331, 224}

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{5/2}}\,dx=\frac{91\sqrt{a+bx^3}(19Ab-10aB)}{540a^4x^2}-\frac{13(19Ab-10aB)}{135a^3x^2\sqrt{a+bx^3}}-\frac{19Ab-10aB}{45a^2x^2\left(a+bx^3\right)^{3/2}}+\frac{91\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(19Ab-10aB)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-\right.}{540\sqrt[4]{3}a^4\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{A}{5ax^5\left(a+bx^3\right)^{3/2}}$$

```
[In] Int[(A + B*x^3)/(x^6*(a + b*x^3)^(5/2)),x]
```

```
[Out] -1/5*A/(a*x^5*(a + b*x^3)^(3/2)) - (19*A*b - 10*a*B)/(45*a^2*x^2*(a + b*x^3
)^(3/2)) - (13*(19*A*b - 10*a*B))/(135*a^3*x^2*Sqrt[a + b*x^3]) + (91*(19*A
*b - 10*a*B)*Sqrt[a + b*x^3])/(540*a^4*x^2) + (91*Sqrt[2 + Sqrt[3]]*b^(2/3)
*(19*A*b - 10*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(540*3^(1/4)*a^4*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{A}{5ax^5\left(a+bx^3\right)^{3/2}} - \frac{\left(\frac{19Ab}{2}-5aB\right)\int\frac{1}{x^3\left(a+bx^3\right)^{5/2}}\,dx}{5a}$$

$$= -\frac{A}{5ax^5\left(a+bx^3\right)^{3/2}} - \frac{19Ab-10aB}{45a^2x^2\left(a+bx^3\right)^{3/2}} - \frac{\left(13(19Ab-10aB)\right)\int\frac{1}{x^3\left(a+bx^3\right)^{3/2}}\,dx}{90a^2}$$

$$= -\frac{A}{5ax^5\left(a+bx^3\right)^{3/2}} - \frac{19Ab-10aB}{45a^2x^2\left(a+bx^3\right)^{3/2}} - \frac{13(19Ab-10aB)}{135a^3x^2\sqrt{a+bx^3}} - \frac{\left(91(19Ab-10aB)\right)\int\frac{1}{x^3\sqrt{a+bx^3}}\,dx}{270a^3}$$

$$= -\frac{A}{5ax^5\left(a+bx^3\right)^{3/2}} - \frac{19Ab-10aB}{45a^2x^2\left(a+bx^3\right)^{3/2}} - \frac{13(19Ab-10aB)}{135a^3x^2\sqrt{a+bx^3}} + \frac{91(19Ab-10aB)\sqrt{a+bx^3}}{540a^4x^2} + \frac{\left(91b(19Ab-10aB)\right)\int\frac{1}{\sqrt{a+bx^3}}\,dx}{1080a^4}$$

$$= -\frac{A}{5ax^5\left(a+bx^3\right)^{3/2}} - \frac{19Ab-10aB}{45a^2x^2\left(a+bx^3\right)^{3/2}} - \frac{13(19Ab-10aB)}{135a^3x^2\sqrt{a+bx^3}} + \frac{91(19Ab-10aB)\sqrt{a+bx^3}}{540a^4x^2} + \frac{91\sqrt{2+\sqrt{3}}\,b^{2/3}(19Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{540\sqrt[4]{3}a^4\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.25

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{5/2}}\,dx=\frac{-2a^2A+\left(\frac{19Ab}{2}-5aB\right)x^3\left(a+bx^3\right)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{5}{2},\frac{1}{3},-\frac{bx^3}{a}\right)}{10a^3x^5\left(a+bx^3\right)^{3/2}}$$

```
[In] Integrate[(A + B*x^3)/(x^6*(a + b*x^3)^(5/2)),x]
```

```
[Out] (-2*a^2*A + ((19*A*b)/2 - 5*a*B)*x^3*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hyperg
eometric2F1[-2/3, 5/2, 1/3, -((b*x^3)/a)])/(10*a^3*x^5*(a + b*x^3)^(3/2))
```

Maple [A] (verified)

Time = 5.58 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.27

method	result
elliptic	$\frac{2x(Ab-Ba)\sqrt{b\,x^3+a}}{9a^3b\left(x^3+\frac{a}{b}\right)^2}+\frac{2bx(25Ab-16Ba)}{27a^4\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{A\sqrt{b\,x^3+a}}{5a^3x^5}+\frac{(27Ab-10Ba)\sqrt{b\,x^3+a}}{20a^4x^2}-\frac{2i\left(\frac{b(25Ab-16Ba)}{27a^4}+\frac{b(27Ab-10Ba)}{40a^4}\right)\sqrt{3}\,(-a\ldots}{\ldots}$
default	$B\left(-\frac{2x\sqrt{b\,x^3+a}}{9a^2b\left(x^3+\frac{a}{b}\right)^2}-\frac{32bx}{27a^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{\sqrt{b\,x^3+a}}{2a^3x^2}+\frac{91i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-\ldots}{\ldots}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\ldots}}\ldots}{\ldots}\right.$
risch	Expression too large to display

```
[In] int((B*x^3+A)/x^6/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*x/a^3/b*(A*b-B*a)*(b*x^3+a)^(1/2)/(x^3+a/b)^2+2/27*b*x/a^4*(25*A*b-16*B
*a)/((x^3+a/b)*b)^(1/2)-1/5*A/a^3*(b*x^3+a)^(1/2)/x^5+1/20/a^4*(27*A*b-10*B
*a)*(b*x^3+a)^(1/2)/x^2-2/3*I*(1/27*b/a^4*(25*A*b-16*B*a)+1/40*b*(27*A*b-10
*B*a)/a^4)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a
*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.53

$$\int \frac{A+Bx^3}{x^6 (a+bx^3)^{5/2}}\,dx = -\frac{91\left((10\,Bab^2-19\,Ab^3)x^{11}+2\,(10\,Ba^2b-19\,Aab^2)x^8+(10\,Ba^3-19\,Aa^2b)x^5\right)\sqrt{b}\,\mathrm{weierstrassPInverse}(0,}{540\,(a^4b^2x^1}$$

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/540*(91*((10*B*a*b^2 - 19*A*b^3)*x^11 + 2*(10*B*a^2*b - 19*A*a*b^2)*x^8
+ (10*B*a^3 - 19*A*a^2*b)*x^5)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) +
(91*(10*B*a*b^2 - 19*A*b^3)*x^9 + 130*(10*B*a^2*b - 19*A*a*b^2)*x^6 + 108*A
*a^3 + 27*(10*B*a^3 - 19*A*a^2*b)*x^3)*sqrt(b*x^3 + a))/(a^4*b^2*x^11 + 2*a
^5*b*x^8 + a^6*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^6 (a+bx^3)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((B*x**3+A)/x**6/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}x^6}\,dx$$

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(5/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^6), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}x^6}\,dx$$

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(5/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{B\,x^3+A}{x^6\,\left(b\,x^3+a\right)^{5/2}}\,dx$$

```
[In] int((A + B*x^3)/(x^6*(a + b*x^3)^(5/2)),x)
[Out] int((A + B*x^3)/(x^6*(a + b*x^3)^(5/2)), x)
```

3.254 $\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 577

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2(7Ab-16aB)x^5}{63b^2(a+bx^3)^{3/2}} + \frac{2Bx^8}{7b(a+bx^3)^{3/2}} - \frac{20(7Ab-16aB)x^2}{189b^3\sqrt{a+bx^3}} + \frac{80(7Ab-16aB)\sqrt{a+bx^3}}{189b^{11/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{40\sqrt{2-\sqrt{3}}\sqrt[3]{a}(7Ab-16aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{63\ 3^{3/4}b^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{80\sqrt{2}\sqrt[3]{a}(7Ab-16aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{189\sqrt[4]{3}b^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -2/63*(7*A*b-16*B*a)*x^5/b^2/(b*x^3+a)^(3/2)+2/7*B*x^8/b/(b*x^3+a)^(3/2)-20
/189*(7*A*b-16*B*a)*x^2/b^3/(b*x^3+a)^(1/2)+80/189*(7*A*b-16*B*a)*(b*x^3+a)
^(1/2)/b^(11/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+80/567*a^(1/3)*(7*A*b-16*B*
a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^
(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(11/3)/(b*x^3
+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(
```

```
1/2)-40/189*a^(1/3)*(7*A*b-16*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x
+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6
^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/b^(11/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/
3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 577, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {470, 294, 309, 224, 1891}

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{80\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(7Ab-16aB)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+(1\cdots}{\sqrt[3]{b}x+(1\cdots}\right.\right.}{189\sqrt[4]{3}b^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{40\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(7Ab-16aB)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\cdots\right.}{63\ 3^{3/4}b^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{80\sqrt{a+bx^3}(7Ab-16aB)}{189b^{11/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{20x^2(7Ab-16aB)}{189b^3\sqrt{a+bx^3}}-\frac{2x^5(7Ab-16aB)}{63b^2\left(a+bx^3\right)^{3/2}}+\frac{2Bx^8}{7b\left(a+bx^3\right)^{3/2}}$$

```
[In] Int[(x^7*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (-2*(7*A*b - 16*a*B)*x^5)/(63*b^2*(a + b*x^3)^(3/2)) + (2*B*x^8)/(7*b*(a +
b*x^3)^(3/2)) - (20*(7*A*b - 16*a*B)*x^2)/(189*b^3*Sqrt[a + b*x^3]) + (80*(
7*A*b - 16*a*B)*Sqrt[a + b*x^3])/(189*b^(11/3)*((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)) - (40*Sqrt[2 - Sqrt[3]]*a^(1/3)*(7*A*b - 16*a*B)*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(63*3^(3/4)*b^(11/3)*Sqr
t[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sq
rt[a + b*x^3]) + (80*Sqrt[2]*a^(1/3)*(7*A*b - 16*a*B)*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(189*3^(1/4)*b^(11/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{2Bx^8}{7b\left(a+bx^3\right)^{3/2}} - \frac{\left(2\left(-\frac{7Ab}{2}+8aB\right)\right)\int \frac{x^7}{\left(a+bx^3\right)^{5/2}}\,dx}{7b}$$

$$= -\frac{2(7Ab-16aB)x^5}{63b^2\left(a+bx^3\right)^{3/2}} + \frac{2Bx^8}{7b\left(a+bx^3\right)^{3/2}} + \frac{(10(7Ab-16aB))\int \frac{x^4}{\left(a+bx^3\right)^{3/2}}\,dx}{63b^2}$$

$$= -\frac{2(7Ab-16aB)x^5}{63b^2\left(a+bx^3\right)^{3/2}} + \frac{2Bx^8}{7b\left(a+bx^3\right)^{3/2}} - \frac{20(7Ab-16aB)x^2}{189b^3\sqrt{a+bx^3}} + \frac{(40(7Ab-16aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{189b^3}$$

$$= -\frac{2(7Ab-16aB)x^5}{63b^2\left(a+bx^3\right)^{3/2}} + \frac{2Bx^8}{7b\left(a+bx^3\right)^{3/2}} - \frac{20(7Ab-16aB)x^2}{189b^3\sqrt{a+bx^3}} + \frac{(40(7Ab-16aB))\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{189b^{10/3}} - \frac{\left(40\left(1-\sqrt{3}\right)\sqrt[3]{a}(7Ab-16aB)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{189b^{10/3}}$$

$$= -\frac{2(7Ab-16aB)x^5}{63b^2\left(a+bx^3\right)^{3/2}} + \frac{2Bx^8}{7b\left(a+bx^3\right)^{3/2}} - \frac{20(7Ab-16aB)x^2}{189b^3\sqrt{a+bx^3}} + \frac{80(7Ab-16aB)\sqrt{a+bx^3}}{189b^{11/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{40\sqrt{2-\sqrt{3}}\sqrt[3]{a}(7Ab-16aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|\right.}{63\ 3^{3/4}b^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{80\sqrt{2}\sqrt[3]{a}(7Ab-16aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\right.}{189\sqrt[4]{3}b^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.19

$$\int \frac{x^7(A+Bx^3)}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{2x^2\left(-32a^2B+2ab(7A-8Bx^3)+b^2x^3(7A+Bx^3)+2(-7Ab+16aB)\left(a+bx^3\right)\sqrt{1+\right.}{7b^3\left(a+bx^3\right)^{3/2}}$$

```
[In] Integrate[(x^7*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

[Out] (2*x^2*(-32*a^2*B + 2*a*b*(7*A - 8*B*x^3) + b^2*x^3*(7*A + B*x^3) + 2*(-7*A
*b + 16*a*B)*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 5/2, 5/
3, -((b*x^3)/a)]))/(7*b^3*(a + b*x^3)^(3/2))

Maple [A] (verified)

Time = 5.56 (sec) , antiderivative size = 555, normalized size of antiderivative = 0.96

method	result
elliptic	$\frac{2x^2a(Ab-Ba)\sqrt{b\,x^3+a}}{9b^5\left(x^3+\frac{a}{b}\right)^2}-\frac{2x^2(13Ab-22Ba)}{27b^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2B\,x^2\sqrt{b\,x^3+a}}{7b^3}-\frac{2i\left(\frac{Ab-2Ba}{b^3}+\frac{13Ab-22Ba}{27b^3}-\frac{4Ba}{7b^3}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)}{2b}\right.}{}}}{}$
default	Expression too large to display
risch	Expression too large to display

[In] int(x^7*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)

[Out] 2/9*x^2*a/b^5*(A*b-B*a)*(b*x^3+a)^(1/2)/(x^3+a/b)^2-2/27/b^3*x^2*(13*A*b-22
*B*a)/((x^3+a/b)*b)^(1/2)+2/7*B/b^3*x^2*(b*x^3+a)^(1/2)-2/3*I*((A*b-2*B*a)/
b^3+1/27/b^3*(13*A*b-22*B*a)-4/7*B/b^3*a)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3
^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(
1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2)))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.28

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2\left(40\left((16\,Bab^2-7\,Ab^3)x^6+16\,Ba^3-7\,Aa^2b+2\,(16\,Ba^2b-7\,Aab^2)x^3\right)\sqrt{b}\text{weierstrass}\right.}{}$$

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/189*(40*((16*B*a*b^2 - 7*A*b^3)*x^6 + 16*B*a^3 - 7*A*a^2*b + 2*(16*B*a^2*
b - 7*A*a*b^2)*x^3)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(
0, -4*a/b, x)) + (27*B*b^3*x^8 + 13*(16*B*a*b^2 - 7*A*b^3)*x^5 + 10*(16*B*a
^2*b - 7*A*a*b^2)*x^2)*sqrt(b*x^3 + a))/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)
```

Sympy [A] (verification not implemented)

Time = 79.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.14

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{Ax^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{11}{3}\right)} + \frac{Bx^{11}\Gamma\left(\frac{11}{3}\right){}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{11}{3}\\ \frac{14}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{14}{3}\right)}$$

```
[In] integrate(x**7*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] A*x**8*gamma(8/3)*hyper((5/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(5/2)*gamma(11/3)) + B*x**11*gamma(11/3)*hyper((5/2, 11/3), (14/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(14/3))
```

Maxima [F]

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)x^7}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^7/(b*x^3 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)x^7}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*x^7/(b*x^3 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{x^7\,(B\,x^3+A)}{(b\,x^3+a)^{5/2}}\,dx$$

```
[In] int((x^7*(A + B*x^3))/(a + b*x^3)^(5/2),x)
[Out] int((x^7*(A + B*x^3))/(a + b*x^3)^(5/2), x)
```

3.255 $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 559

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2(Ab-aB)x^5}{9ab\,(a+bx^3)^{3/2}}+\frac{2(Ab-10aB)x^2}{27ab^2\sqrt{a+bx^3}}-\frac{8(Ab-10aB)\sqrt{a+bx^3}}{27ab^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{4\sqrt{2-\sqrt{3}}(Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{9\,3^{3/4}a^{2/3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{8\sqrt{2}(Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}a^{2/3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/9*(A*b-B*a)*x^5/a/b/(b*x^3+a)^(3/2)+2/27*(A*b-10*B*a)*x^2/a/b^2/(b*x^3+a)
^(1/2)-8/27*(A*b-10*B*a)*(b*x^3+a)^(1/2)/a/b^(8/3)/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2)))-8/81*(A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*
(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/
3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*
3^(3/4)/a^(2/3)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/
3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+4/27*(A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*El
lipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3
^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(2/3)/b^(8/3)/(b*x^3
```

```
+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(
1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {468, 294, 309, 224, 1891}

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx =$$

$$-\frac{8\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-10aB)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}a^{2/3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{4\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-10aB)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}a^{2/3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{8\sqrt{a+bx^3}(Ab-10aB)}{27ab^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2x^2(Ab-10aB)}{27ab^2\sqrt{a+bx^3}}+\frac{2x^5(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}$$

```
[In] Int[(x^4*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*x^5)/(9*a*b*(a + b*x^3)^(3/2)) + (2*(A*b - 10*a*B)*x^2)/(27*
a*b^2*Sqrt[a + b*x^3]) - (8*(A*b - 10*a*B)*Sqrt[a + b*x^3])/(27*a*b^(8/3)*(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (4*Sqrt[2 - Sqrt[3]]*(A*b - 10*a*B)*(
a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(9*3^(3/
4)*a^(2/3)*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (8*Sqrt[2]*(A*b - 10*a*B)*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*
x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a^(2/
3)*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{2(Ab-aB)x^5}{9ab\left(a+bx^3\right)^{3/2}} + \frac{\left(2\left(-\frac{Ab}{2}+5aB\right)\right)\int \frac{x^4}{\left(a+bx^3\right)^{3/2}}\,dx}{9ab}$$

$$= \frac{2(Ab-aB)x^5}{9ab\left(a+bx^3\right)^{3/2}} + \frac{2(Ab-10aB)x^2}{27ab^2\sqrt{a+bx^3}} - \frac{(4(Ab-10aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{27ab^2}$$

$$= \frac{2(Ab-aB)x^5}{9ab\left(a+bx^3\right)^{3/2}} + \frac{2(Ab-10aB)x^2}{27ab^2\sqrt{a+bx^3}} - \frac{(4(Ab-10aB))\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{27ab^{7/3}} + \frac{\left(4\left(1-\sqrt{3}\right)(Ab-10aB)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{27a^{2/3}b^{7/3}}$$

$$= \frac{2(Ab-aB)x^5}{9ab\left(a+bx^3\right)^{3/2}} + \frac{2(Ab-10aB)x^2}{27ab^2\sqrt{a+bx^3}} - \frac{8(Ab-10aB)\sqrt{a+bx^3}}{27ab^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{4\sqrt{2-\sqrt{3}}(Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\cdots\right.}{9\,3^{3/4}a^{2/3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{8\sqrt{2}(Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\cdots\right.}{27\sqrt[4]{3}a^{2/3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.16

$$\int \frac{x^4\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{2x^2\left(-aAb+5aB\left(2a+bx^3\right)+(Ab-10aB)\left(a+bx^3\right)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\cdots\right.}{5ab^2\left(a+bx^3\right)^{3/2}}$$

```
[In] Integrate[(x^4*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*x^2*(-(a*A*b) + 5*a*B*(2*a + b*x^3) + (A*b - 10*a*B)*(a + b*x^3)*Sqrt[1
+ (b*x^3)/a]*Hypergeometric2F1[2/3, 5/2, 5/3, -((b*x^3)/a)]))/(5*a*b^2*(a +
 b*x^3)^(3/2))
```

Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 528, normalized size of antiderivative = 0.94

method	result
	$2i\left(\frac{B}{b^2}-\frac{4Ab-13Ba}{27a\,b^2}\right)\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{-\frac{3(-a}{2}}$
elliptic	$-\frac{2x^2(Ab-Ba)\sqrt{b\,x^3+a}}{9b^4\left(x^3+\frac{a}{b}\right)^2}+\frac{2x^2(4Ab-13Ba)}{27b^2a\sqrt{\left(x^3+\frac{a}{b}\right)b}}-$
default	Expression too large to display

[In] int(x^4*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)

[Out] -2/9*x^2/b^4*(A*b-B*a)*(b*x^3+a)^(1/2)/(x^3+a/b)^2+2/27/b^2*x^2/a*(4*A*b-13*B*a)/((x^3+a/b)*b)^(1/2)-2/3*I*(B/b^2-1/27*(4*A*b-13*B*a)/a/b^2)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.28

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2\left(4\left((10\,Bab^2-Ab^3)x^6+10\,Ba^3-Aa^2b+2\,(10\,Ba^2b-Aab^2)x^3\right)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrass}\right.\right.}{27\,(ab^5x^6+2\,a^2b^4x^3+a^3b}$$

[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")

[Out] -2/27*(4*((10*B*a*b^2 - A*b^3)*x^6 + 10*B*a^3 - A*a^2*b + 2*(10*B*a^2*b - A*a*b^2)*x^3)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + ((13*B*a*b^2 - 4*A*b^3)*x^5 + (10*B*a^2*b - A*a*b^2)*x^2)*sqrt(b*x^3 + a))/(a*b^5*x^6 + 2*a^2*b^4*x^3 + a^3*b^3)

Sympy [A] (verification not implemented)

Time = 39.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.14

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{Ax^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{3},\frac{5}{2}\\\frac{8}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{8}{3}\right)} + \frac{Bx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{2},\frac{8}{3}\\\frac{11}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{11}{3}\right)}$$

[In] integrate(x**4*(B*x**3+A)/(b*x**3+a)**(5/2),x)

[Out] A*x**5*gamma(5/3)*hyper((5/3, 5/2), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(8/3)) + B*x**8*gamma(8/3)*hyper((5/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(11/3))

Maxima [F]

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)x^4}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")

[Out] integrate((B*x^3 + A)*x^4/(b*x^3 + a)^(5/2), x)

Giac [F]

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)x^4}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*x^4/(b*x^3 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{x^4\,(B\,x^3+A)}{(b\,x^3+a)^{5/2}}\,dx$$

```
[In] int((x^4*(A + B*x^3))/(a + b*x^3)^(5/2),x)
[Out] int((x^4*(A + B*x^3))/(a + b*x^3)^(5/2), x)
```

3.256 $\int \frac{x(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 563

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2(Ab-aB)x^2}{9ab\,(a+bx^3)^{3/2}} + \frac{2(5Ab+4aB)x^2}{27a^2b\sqrt{a+bx^3}} - \frac{2(5Ab+4aB)\sqrt{a+bx^3}}{27a^2b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{\sqrt{2-\sqrt{3}}(5Ab+4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{9\,3^{3/4}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{2\sqrt{2}(5Ab+4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/9*(A*b-B*a)*x^2/a/b/(b*x^3+a)^(3/2)+2/27*(5*A*b+4*B*a)*x^2/a^2/b/(b*x^3+a
)^(1/2)-2/27*(5*A*b+4*B*a)*(b*x^3+a)^(1/2)/a^2/b^(5/3)/(b^(1/3)*x+a^(1/3)*(
1+3^(1/2)))-2/81*(5*A*b+4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(
1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(
1/2)*3^(3/4)/a^(5/3)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(
b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/27*(5*A*b+4*B*a)*(a^(1/3)+b^(1/3)
*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(5/3)/b^(5/3)/
```

```
(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {468, 296, 309, 224, 1891}

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(4aB+5Ab)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(4aB+5Ab)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{2\sqrt{a+bx^3}(4aB+5Ab)}{27a^2b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{2x^2(4aB+5Ab)}{27a^2b\sqrt{a+bx^3}} + \frac{2x^2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}$$

```
[In] Int[(x*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*x^2)/(9*a*b*(a + b*x^3)^(3/2)) + (2*(5*A*b + 4*a*B)*x^2)/(27
*a^2*b*Sqrt[a + b*x^3]) - (2*(5*A*b + 4*a*B)*Sqrt[a + b*x^3])/(27*a^2*b^(5/
3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (Sqrt[2 - Sqrt[3]]*(5*A*b + 4*a*B
)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3
) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(9*3^
(3/4)*a^(5/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (2*Sqrt[2]*(5*A*b + 4*a*B)*(a^(1/
3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a
^(5/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{2(Ab-aB)x^2}{9ab\left(a+bx^3\right)^{3/2}} + \frac{\left(2\left(\frac{5Ab}{2}+2aB\right)\right)\int \frac{x}{\left(a+bx^3\right)^{3/2}}\,dx}{9ab}$$

$$= \frac{2(Ab-aB)x^2}{9ab\left(a+bx^3\right)^{3/2}} + \frac{2(5Ab+4aB)x^2}{27a^2b\sqrt{a+bx^3}} - \frac{(5Ab+4aB)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{27a^2b}$$

$$= \frac{2(Ab-aB)x^2}{9ab\left(a+bx^3\right)^{3/2}} + \frac{2(5Ab+4aB)x^2}{27a^2b\sqrt{a+bx^3}} - \frac{(5Ab+4aB)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{27a^2b^{4/3}} + \frac{\left(\left(1-\sqrt{3}\right)(5Ab+4aB)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{27a^{5/3}b^{4/3}}$$

$$= \frac{2(Ab-aB)x^2}{9ab\left(a+bx^3\right)^{3/2}} + \frac{2(5Ab+4aB)x^2}{27a^2b\sqrt{a+bx^3}} - \frac{2(5Ab+4aB)\sqrt{a+bx^3}}{27a^2b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sqrt{2-\sqrt{3}}(5Ab+4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4}{9\ 3^{3/4}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{2\sqrt{2}(5Ab+4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.14

$$\int \frac{x(A+Bx^3)}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{x^2\left(-4a^2B+(5Ab+4aB)\left(a+bx^3\right)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{2},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{10a^2b\left(a+bx^3\right)^{3/2}}$$

```
[In] Integrate[(x*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (x^2*(-4*a^2*B + (5*A*b + 4*a*B)*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeome
tric2F1[2/3, 5/2, 5/3, -((b*x^3)/a)]))/(10*a^2*b*(a + b*x^3)^(3/2))
```

Maple [A] (verified)

Time = 4.28 (sec) , antiderivative size = 520, normalized size of antiderivative = 0.92

method	result
	$2i(5Ab+4Ba)\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-}{}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-}}$
elliptic	$\frac{2x^2(Ab-Ba)\sqrt{b\,x^3+a}}{9a\,b^3\left(x^3+\frac{a}{b}\right)^2}+\frac{2x^2(5Ab+4Ba)}{27b\,a^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}+$
default	Expression too large to display

```
[In] int(x*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*x^2/a/b^3*(A*b-B*a)*(b*x^3+a)^(1/2)/(x^3+a/b)^2+2/27/b*x^2/a^2*(5*A*b+4
*B*a)/((x^3+a/b)*b)^(1/2)+2/81*I/b^2/a^2*(5*A*b+4*B*a)*3^(1/2)*(-a*b^2)^(1/
3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.27

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2\left(((4\,Bab^2+5\,Ab^3)x^6+4\,Ba^3+5\,Aa^2b+2\,(4\,Ba^2b+5\,Aab^2)x^3)\sqrt{b}\text{weierstrassZeta}(0\right.}{27\,(a^2b^4x^6}$$

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/27*(((4*B*a*b^2 + 5*A*b^3)*x^6 + 4*B*a^3 + 5*A*a^2*b + 2*(4*B*a^2*b + 5*A
*a*b^2)*x^3)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a
/b, x)) + ((4*B*a*b^2 + 5*A*b^3)*x^5 + (B*a^2*b + 8*A*a*b^2)*x^2)*sqrt(b*x^
3 + a))/(a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2)
```

Sympy [A] (verification not implemented)

Time = 24.60 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.14

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{Ax^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{5}{2}\\\frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{5}{3}\right)} + \frac{Bx^5\Gamma\left(\frac{5}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{5}{3},\frac{5}{2}\\\frac{8}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{8}{3}\right)}$$

```
[In] integrate(x*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] A*x**2*gamma(2/3)*hyper((2/3, 5/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(5/2)*gamma(5/3)) + B*x**5*gamma(5/3)*hyper((5/3, 5/2), (8/3,), b*x**3*exp
_polar(I*pi)/a)/(3*a**(5/2)*gamma(8/3))
```

Maxima [F]

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)x}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x/(b*x^3 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)x}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*x/(b*x^3 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{x\,(B\,x^3+A)}{(b\,x^3+a)^{5/2}}\,dx$$

```
[In] int((x*(A + B*x^3))/(a + b*x^3)^(5/2),x)
[Out] int((x*(A + B*x^3))/(a + b*x^3)^(5/2), x)
```

3.257 $\int \frac{A+Bx^3}{x^2(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 578

$$\int \frac{A+Bx^3}{x^2(a+bx^3)^{5/2}}\,dx = -\frac{A}{ax(a+bx^3)^{3/2}} - \frac{(11Ab-2aB)x^2}{9a^2(a+bx^3)^{3/2}} - \frac{5(11Ab-2aB)x^2}{27a^3\sqrt{a+bx^3}} + \frac{5(11Ab-2aB)\sqrt{a+bx^3}}{27a^3b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{5\sqrt{2-\sqrt{3}}(11Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{18\,3^{3/4}a^{8/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{5\sqrt{2}(11Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}a^{8/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -A/a/x/(b*x^3+a)^(3/2)-1/9*(11*A*b-2*B*a)*x^2/a^2/(b*x^3+a)^(3/2)-5/27*(11*
A*b-2*B*a)*x^2/a^3/(b*x^3+a)^(1/2)+5/27*(11*A*b-2*B*a)*(b*x^3+a)^(1/2)/a^3/
b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+5/81*(11*A*b-2*B*a)*(a^(1/3)+b^(1/3
)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(8/3)/b^(2/3)/(b*x^3+a)^(1/2)/(
a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-5/54*(
```

```
11*A*b-2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*(
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)*3^(1/4)/a^(8/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/
(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 578, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {464, 296, 309, 224, 1891}

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{5/2}}\,dx = \frac{5\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-2aB)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}a^{8/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{5\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-2aB)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{18\ 3^{3/4}a^{8/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{5\sqrt{a+bx^3}(11Ab-2aB)}{27a^3b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{5x^2(11Ab-2aB)}{27a^3\sqrt{a+bx^3}}-\frac{x^2(11Ab-2aB)}{9a^2\left(a+bx^3\right)^{3/2}}-\frac{A}{ax\left(a+bx^3\right)^{3/2}}$$

```
[In] Int[(A + B*x^3)/(x^2*(a + b*x^3)^(5/2)),x]
```

```
[Out] -(A/(a*x*(a + b*x^3)^(3/2))) - ((11*A*b - 2*a*B)*x^2)/(9*a^2*(a + b*x^3)^(3
/2)) - (5*(11*A*b - 2*a*B)*x^2)/(27*a^3*Sqrt[a + b*x^3]) + (5*(11*A*b - 2*a
*B)*Sqrt[a + b*x^3])/(27*a^3*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) -
 (5*Sqrt[2 - Sqrt[3]]*(11*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(18*3^(3/4)*a^(8/3)*b^(2/3)*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3
]) + (5*Sqrt[2]*(11*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a^(8/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{A}{ax\,(a+bx^3)^{3/2}} - \frac{\left(\frac{11Ab}{2}-aB\right)\int \frac{x}{(a+bx^3)^{5/2}}\,dx}{a}$$

$$= -\frac{A}{ax(a+bx^3)^{3/2}} - \frac{(11Ab-2aB)x^2}{9a^2(a+bx^3)^{3/2}} - \frac{(5(11Ab-2aB))\int \frac{x}{(a+bx^3)^{3/2}}\,dx}{18a^2}$$

$$= -\frac{A}{ax(a+bx^3)^{3/2}} - \frac{(11Ab-2aB)x^2}{9a^2(a+bx^3)^{3/2}} - \frac{5(11Ab-2aB)x^2}{27a^3\sqrt{a+bx^3}} + \frac{(5(11Ab-2aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{54a^3}$$

$$= -\frac{A}{ax(a+bx^3)^{3/2}} - \frac{(11Ab-2aB)x^2}{9a^2(a+bx^3)^{3/2}} - \frac{5(11Ab-2aB)x^2}{27a^3\sqrt{a+bx^3}} + \frac{(5(11Ab-2aB))\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{54a^3\sqrt[3]{b}} - \frac{\left(5\left(1-\sqrt{3}\right)(11Ab-2aB)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{54a^{8/3}\sqrt[3]{b}}$$

$$= -\frac{A}{ax(a+bx^3)^{3/2}} - \frac{(11Ab-2aB)x^2}{9a^2(a+bx^3)^{3/2}} - \frac{5(11Ab-2aB)x^2}{27a^3\sqrt{a+bx^3}} + \frac{5(11Ab-2aB)\sqrt{a+bx^3}}{27a^3b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$- \frac{5\sqrt{2-\sqrt{3}}(11Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7}{18\ 3^{3/4}a^{8/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{5\sqrt{2}(11Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}}{27\sqrt[4]{3}a^{8/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.15

$$\int \frac{A+Bx^3}{x^2(a+bx^3)^{5/2}}\,dx = -\frac{A}{ax(a+bx^3)^{3/2}} - \frac{\left(\frac{11Ab}{2}-aB\right)x^2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{2},\frac{5}{3},-\frac{bx^3}{a}\right)}{2a^3\sqrt{a+bx^3}}$$

```
[In] Integrate[(A + B*x^3)/(x^2*(a + b*x^3)^(5/2)),x]
[Out] -(A/(a*x*(a + b*x^3)^(3/2))) - (((11*A*b)/2 - a*B)*x^2*Sqrt[1 + (b*x^3)/a]*
Hypergeometric2F1[2/3, 5/2, 5/3, -((b*x^3)/a)])/(2*a^3*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 5.22 (sec) , antiderivative size = 545, normalized size of antiderivative = 0.94

method	result
	$2i\left(\frac{14Ab-5Ba}{27a^3}+\frac{Ab}{2a^3}\right)\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right.}{\left(-a\,b^2\right)^{\frac{1}{3}}}}$
elliptic	$-\frac{2x^2(Ab-Ba)\sqrt{b\,x^3+a}}{9a^2b^2\left(x^3+\frac{a}{b}\right)^2}-\frac{2x^2(14Ab-5Ba)}{27a^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{A\sqrt{b\,x^3+a}}{a^3x}-$
default	Expression too large to display
risch	Expression too large to display

```
[In] int((B*x^3+A)/x^2/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/9*x^2/a^2/b^2*(A*b-B*a)*(b*x^3+a)^(1/2)/(x^3+a/b)^2-2/27*x^2/a^3*(14*A*b
-5*B*a)/((x^3+a/b)*b)^(1/2)-1/a^3*A*(b*x^3+a)^(1/2)/x-2/3*I*(1/27/a^3*(14*A
*b-5*B*a)+1/2/a^3*A*b)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a
*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*
(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(
1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a
*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/
3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.29

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{5/2}}\,dx = \frac{5\left((2\,Bab^2-11\,Ab^3)x^7+2\,(2\,Ba^2b-11\,Aab^2)x^4+(2\,Ba^3-11\,Aa^2b)x\right)\sqrt{b}\text{weierstra}}{}$$

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/27*(5*((2*B*a*b^2 - 11*A*b^3)*x^7 + 2*(2*B*a^2*b - 11*A*a*b^2)*x^4 + (2*B
*a^3 - 11*A*a^2*b)*x)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInvers
e(0, -4*a/b, x)) + (5*(2*B*a*b^2 - 11*A*b^3)*x^6 - 27*A*a^2*b + 8*(2*B*a^2*
b - 11*A*a*b^2)*x^3)*sqrt(b*x^3 + a))/(a^3*b^3*x^7 + 2*a^4*b^2*x^4 + a^5*b*
x)
```

Sympy [A] (verification not implemented)

Time = 52.47 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.14

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{5/2}}\,dx = \frac{A\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{5}{2}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}x\Gamma\left(\frac{2}{3}\right)}+\frac{Bx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{5}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{5}{3}\right)}$$

```
[In] integrate((B*x**3+A)/x**2/(b*x**3+a)**(5/2),x)
```

```
[Out] A*gamma(-1/3)*hyper((-1/3, 5/2), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5
/2)*x*gamma(2/3)) + B*x**2*gamma(2/3)*hyper((2/3, 5/2), (5/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(5/2)*gamma(5/3))
```

Maxima [F]

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{5/2}}\,dx = \int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}x^2}\,dx$$

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}x^2}\,dx$$

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(5/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{B\,x^3+A}{x^2\,\left(b\,x^3+a\right)^{5/2}}\,dx$$

```
[In] int((A + B*x^3)/(x^2*(a + b*x^3)^(5/2)),x)
[Out] int((A + B*x^3)/(x^2*(a + b*x^3)^(5/2)), x)
```

3.258 $\int \frac{A+Bx^3}{x^5(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 610

$$\int \frac{A+Bx^3}{x^5(a+bx^3)^{5/2}}\,dx = -\frac{A}{4ax^4(a+bx^3)^{3/2}} - \frac{17Ab-8aB}{36a^2x(a+bx^3)^{3/2}} - \frac{11(17Ab-8aB)}{108a^3x\sqrt{a+bx^3}} + \frac{55(17Ab-8aB)\sqrt{a+bx^3}}{216a^4x} - \frac{55\sqrt[3]{b}(17Ab-8aB)\sqrt{a+bx^3}}{216a^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{55\sqrt{2-\sqrt{3}}\sqrt[3]{b}(17Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{144\ 3^{3/4}a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{55\sqrt[3]{b}(17Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{108\sqrt{2}\sqrt[4]{3}a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/4*A/a/x^4/(b*x^3+a)^(3/2)+1/36*(-17*A*b+8*B*a)/a^2/x/(b*x^3+a)^(3/2)-11/
108*(17*A*b-8*B*a)/a^3/x/(b*x^3+a)^(1/2)+55/216*(17*A*b-8*B*a)*(b*x^3+a)^(1
/2)/a^4/x-55/216*b^(1/3)*(17*A*b-8*B*a)*(b*x^3+a)^(1/2)/a^4/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2)))-55/648*b^(1/3)*(17*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*Elliptic
F((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)
+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))^2)^(1/2)*3^(3/4)/a^(11/3)*2^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(
```

```
1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+55/432*b^(1/3)*(17*A*b-8*B
*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*
x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a
^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1
/4)/a^(11/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3
)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {464, 296, 331, 309, 224, 1891}

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{5/2}}\,dx = -\frac{55\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(17Ab-8aB)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{108\sqrt{2}\sqrt[4]{3}a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{55\sqrt{2-\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(17Ab-8aB)E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{144\ 3^{3/4}a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{55\sqrt{a+bx^3}(17Ab-8aB)}{216a^4x} - \frac{55\sqrt[3]{b}\sqrt{a+bx^3}(17Ab-8aB)}{216a^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{11(17Ab-8aB)}{108a^3x\sqrt{a+bx^3}} - \frac{17Ab-8aB}{36a^2x\left(a+bx^3\right)^{3/2}} - \frac{A}{4ax^4\left(a+bx^3\right)^{3/2}}$$

```
[In] Int[(A + B*x^3)/(x^5*(a + b*x^3)^(5/2)),x]
```

```
[Out] -1/4*A/(a*x^4*(a + b*x^3)^(3/2)) - (17*A*b - 8*a*B)/(36*a^2*x*(a + b*x^3)^(
3/2)) - (11*(17*A*b - 8*a*B))/(108*a^3*x*Sqrt[a + b*x^3]) + (55*(17*A*b - 8
*a*B)*Sqrt[a + b*x^3])/(216*a^4*x) - (55*b^(1/3)*(17*A*b - 8*a*B)*Sqrt[a +
b*x^3])/(216*a^4*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (55*Sqrt[2 - Sqrt[3
]]*b^(1/3)*(17*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[Ar
cSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(144*3^(3/4)*a^(11/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)
*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (55*b^(1/3)*
```

```
(17*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*
Sqrt[3]])/(108*Sqrt[2]*3^(1/4)*a^(11/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x)
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}}-\frac{\left(\frac{17Ab}{2}-4aB\right)\int\frac{1}{x^2\left(a+bx^3\right)^{5/2}}\,dx}{4a}\\
&= -\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}}-\frac{17Ab-8aB}{36a^2x\left(a+bx^3\right)^{3/2}}-\frac{(11(17Ab-8aB))\int\frac{1}{x^2\left(a+bx^3\right)^{3/2}}\,dx}{72a^2}\\
&= -\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}}-\frac{17Ab-8aB}{36a^2x\left(a+bx^3\right)^{3/2}}\\
&\quad-\frac{11(17Ab-8aB)}{108a^3x\sqrt{a+bx^3}}-\frac{(55(17Ab-8aB))\int\frac{1}{x^2\sqrt{a+bx^3}}\,dx}{216a^3}\\
&= -\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}}-\frac{17Ab-8aB}{36a^2x\left(a+bx^3\right)^{3/2}}-\frac{11(17Ab-8aB)}{108a^3x\sqrt{a+bx^3}}\\
&\quad+\frac{55(17Ab-8aB)\sqrt{a+bx^3}}{216a^4x}-\frac{(55b(17Ab-8aB))\int\frac{x}{\sqrt{a+bx^3}}\,dx}{432a^4}\\
&= -\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}}-\frac{17Ab-8aB}{36a^2x\left(a+bx^3\right)^{3/2}}-\frac{11(17Ab-8aB)}{108a^3x\sqrt{a+bx^3}}\\
&\quad+\frac{55(17Ab-8aB)\sqrt{a+bx^3}}{216a^4x}-\frac{\left(55b^{2/3}(17Ab-8aB)\right)\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{432a^4}\\
&\quad+\frac{\left(55\left(1-\sqrt{3}\right)b^{2/3}(17Ab-8aB)\right)\int\frac{1}{\sqrt{a+bx^3}}\,dx}{432a^{11/3}}
\end{aligned}
$$

$$= -\frac{A}{4ax^4(a+bx^3)^{3/2}} - \frac{17Ab-8aB}{36a^2x(a+bx^3)^{3/2}} - \frac{11(17Ab-8aB)}{108a^3x\sqrt{a+bx^3}} + \frac{55(17Ab-8aB)\sqrt{a+bx^3}}{216a^4x} - \frac{55\sqrt[3]{b}(17Ab-8aB)\sqrt{a+bx^3}}{216a^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{55\sqrt{2-\sqrt{3}}\sqrt[3]{b}(17Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\right.}{144\,3^{3/4}a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{55\sqrt[3]{b}(17Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{\right.}{108\sqrt{2}\sqrt[4]{3}a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.14

$$\int \frac{A+Bx^3}{x^5(a+bx^3)^{5/2}}\,dx = \frac{-a^2A+\left(\frac{17Ab}{2}-4aB\right)x^3(a+bx^3)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{5}{2},\frac{2}{3},-\frac{bx^3}{a}\right)}{4a^3x^4(a+bx^3)^{3/2}}$$

```
[In] Integrate[(A + B*x^3)/(x^5*(a + b*x^3)^(5/2)),x]
[Out] (-(a^2*A) + ((17*A*b)/2 - 4*a*B)*x^3*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hyperg
eometric2F1[-1/3, 5/2, 2/3, -((b*x^3)/a)])/(4*a^3*x^4*(a + b*x^3)^(3/2))
```

Maple [A] (verified)

Time = 5.88 (sec) , antiderivative size = 581, normalized size of antiderivative = 0.95

method	result
	$2i\left(-\frac{b(23Ab-14Ba)}{27a^4}-\frac{b(21Ab-8Ba)}{16a^4}\right)\sqrt{3}\,($
elliptic	$\frac{2x^2(Ab-Ba)\sqrt{b\,x^3+a}}{9a^3b\left(x^3+\frac{a}{b}\right)^2}+\frac{2b\,x^2(23Ab-14Ba)}{27a^4\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{A\sqrt{b\,x^3+a}}{4a^3x^4}+\frac{(21Ab-8Ba)\sqrt{b\,x^3+a}}{8a^4x}-$
default	Expression too large to display
risch	Expression too large to display

```
[In] int((B*x^3+A)/x^5/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*x^2/a^3/b*(A*b-B*a)*(b*x^3+a)^(1/2)/(x^3+a/b)^2+2/27*b*x^2/a^4*(23*A*b-
14*B*a)/((x^3+a/b)*b)^(1/2)-1/4/a^3*A*(b*x^3+a)^(1/2)/x^4+1/8/a^4*(21*A*b-8
*B*a)*(b*x^3+a)^(1/2)/x-2/3*I*(-1/27*b/a^4*(23*A*b-14*B*a)-1/16*b*(21*A*b-8
*B*a)/a^4)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.30

$$\int \frac{A+Bx^3}{x^5\,(a+bx^3)^{5/2}}\,dx =$$

$$-\frac{55\left((8\,Bab^2-17\,Ab^3)x^{10}+2\,(8\,Ba^2b-17\,Aab^2)x^7+(8\,Ba^3-17\,Aa^2b)x^4\right)\sqrt{b}\,\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weie}\right.}{216\,(}$$

[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(5/2),x, algorithm="fricas")

[Out] -1/216*(55*((8*B*a*b^2 - 17*A*b^3)*x^10 + 2*(8*B*a^2*b - 17*A*a*b^2)*x^7 + (8*B*a^3 - 17*A*a^2*b)*x^4)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (55*(8*B*a*b^2 - 17*A*b^3)*x^9 + 88*(8*B*a^2*b - 17*A*a*b^2)*x^6 + 54*A*a^3 + 27*(8*B*a^3 - 17*A*a^2*b)*x^3)*sqrt(b*x^3 + a))/(a^4*b^2*x^10 + 2*a^5*b*x^7 + a^6*x^4)

Sympy [A] (verification not implemented)

Time = 156.89 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.14

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{5/2}}\,dx = \frac{A\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},\frac{5}{2}\\ -\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{B\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{5}{2}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}x\Gamma\left(\frac{2}{3}\right)}$$

[In] integrate((B*x**3+A)/x**5/(b*x**3+a)**(5/2),x)

[Out] A*gamma(-4/3)*hyper((-4/3, 5/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5/2)*x**4*gamma(-1/3)) + B*gamma(-1/3)*hyper((-1/3, 5/2), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5/2)*x*gamma(2/3))

Maxima [F]

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{5/2}}\,dx = \int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}x^5}\,dx$$

[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(5/2),x, algorithm="maxima")

[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^5), x)

Giac [F]

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{5/2}}\,dx = \int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}x^5}\,dx$$

[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(5/2),x, algorithm="giac")

[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^5), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{B\,x^3+A}{x^5\,\left(b\,x^3+a\right)^{5/2}}\,dx$$

```
[In] int((A + B*x^3)/(x^5*(a + b*x^3)^(5/2)),x)
[Out] int((A + B*x^3)/(x^5*(a + b*x^3)^(5/2)), x)
```

3.259 $\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 97

$$\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{32c^2\sqrt{c+dx^3}}{3d^3} - \frac{10c(c+dx^3)^{3/2}}{9d^3} + \frac{2(c+dx^3)^{5/2}}{15d^3} - \frac{32c^{5/2}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^3}$$

```
[Out] -10/9*c*(d*x^3+c)^(3/2)/d^3+2/15*(d*x^3+c)^(5/2)/d^3-32/3*c^(5/2)*arctan(1/
3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))/d^3*3^(1/2)+32/3*c^2*(d*x^3+c)^(1/2)/d^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {457, 90, 52, 65, 209}

$$\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx = -\frac{32c^{5/2}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^3} + \frac{32c^2\sqrt{c+dx^3}}{3d^3} - \frac{10c(c+dx^3)^{3/2}}{9d^3} + \frac{2(c+dx^3)^{5/2}}{15d^3}$$

```
[In] Int[(x^8*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (32*c^2*Sqrt[c + d*x^3])/(3*d^3) - (10*c*(c + d*x^3)^(3/2))/(9*d^3) + (2*(c
 + d*x^3)^(5/2))/(15*d^3) - (32*c^(5/2)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqr
t[c]])])/(Sqrt[3]*d^3)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2\sqrt{c+dx}}{4c+dx}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{5c\sqrt{c+dx}}{d^2}+\frac{(c+dx)^{3/2}}{d^2}+\frac{16c^2\sqrt{c+dx}}{d^2(4c+dx)}\right)dx, x, x^3\right)\\
&= -\frac{10c(c+dx^3)^{3/2}}{9d^3}+\frac{2(c+dx^3)^{5/2}}{15d^3}+\frac{(16c^2)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{4c+dx}\,dx, x, x^3\right)}{3d^2}
\end{aligned}$$

$$= \frac{32c^2\sqrt{c+dx^3}}{3d^3} - \frac{10c(c+dx^3)^{3/2}}{9d^3} + \frac{2(c+dx^3)^{5/2}}{15d^3} - \frac{(16c^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}(4c+dx)}\,dx, x, x^3\right)}{d^2}$$

$$= \frac{32c^2\sqrt{c+dx^3}}{3d^3} - \frac{10c(c+dx^3)^{3/2}}{9d^3} + \frac{2(c+dx^3)^{5/2}}{15d^3} - \frac{(32c^3)\,\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d^3}$$

$$= \frac{32c^2\sqrt{c+dx^3}}{3d^3} - \frac{10c(c+dx^3)^{3/2}}{9d^3} + \frac{2(c+dx^3)^{5/2}}{15d^3} - \frac{32c^{5/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^3}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

$$\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{2\sqrt{c+dx^3}(218c^2-19cdx^3+3d^2x^6)}{45d^3} - \frac{32c^{5/2}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^3}$$

```
[In] Integrate[(x^8*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
[Out] (2*Sqrt[c + d*x^3]*(218*c^2 - 19*c*d*x^3 + 3*d^2*x^6))/(45*d^3) - (32*c^(5/
2)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(Sqrt[3]*d^3)
```

Maple [A] (verified)

Time = 8.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{-480c^{\frac{5}{2}}\sqrt{3}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{3}}{3\sqrt{c}}\right)+(6d^2x^6-38cdx^3+436c^2)\sqrt{dx^3+c}}{45d^3}$
risch	$\frac{2(3d^2x^6-19cdx^3+218c^2)\sqrt{dx^3+c}}{45d^3}-\frac{32c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{3d^3}$
default	$\frac{\frac{2x^6\sqrt{dx^3+c}}{15}+\frac{2cx^3\sqrt{dx^3+c}}{45d}-\frac{4c^2\sqrt{dx^3+c}}{45d^2}}{d}-\frac{8c(dx^3+c)^{\frac{3}{2}}}{9d^3}+\frac{16c^2\left(2\sqrt{dx^3+c}-2\sqrt{c}\sqrt{3}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{3}}{3\sqrt{c}}\right)\right)}{3d^3}$
elliptic	$\frac{2x^6\sqrt{dx^3+c}}{15d}-\frac{38cx^3\sqrt{dx^3+c}}{45d^2}+\frac{436c^2\sqrt{dx^3+c}}{45d^3}+\frac{16ic^2\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3+4c\right)}\frac{\left(-cd^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right.}{}\right.}{\left(-cd^2\right)}}}{}\right.}{}$

[In] int(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)

[Out] 1/45*(-480*c^(5/2)*3^(1/2)*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))+(6*d
^2*x^6-38*c*d*x^3+436*c^2)*(d*x^3+c)^(1/2))/d^3

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.61

$$\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx = \left[\frac{2\left(120\sqrt{3}\sqrt{-c}c^2\log\left(\frac{dx^3-2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)+(3d^2x^6-19cdx^3+218c^2)\sqrt{dx^3+c}\right)}{45d^3}, -\frac{2\left(240\sqrt{3}c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)-(3d^2x^6-19cdx^3+218c^2)\sqrt{dx^3+c}\right)}{45d^3}\right]$$

[In] integrate(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")

```
[Out] [2/45*(120*sqrt(3)*sqrt(-c)*c^2*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt
(-c) - 2*c)/(d*x^3 + 4*c)) + (3*d^2*x^6 - 19*c*d*x^3 + 218*c^2)*sqrt(d*x^3
+ c))/d^3, -2/45*(240*sqrt(3)*c^(5/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sq
rt(c)) - (3*d^2*x^6 - 19*c*d*x^3 + 218*c^2)*sqrt(d*x^3 + c))/d^3]
```

Sympy [A] (verification not implemented)

Time = 8.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98

$$\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx = \begin{cases} \frac{2\left(-\frac{16\sqrt{3}c^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3}+\frac{16c^2\sqrt{c+dx^3}}{3}-\frac{5c\left(c+dx^3\right)^{\frac{3}{2}}}{9}+\frac{\left(c+dx^3\right)^{\frac{5}{2}}}{15}\right)}{d^3} & \text{for } d\neq 0 \\ \frac{x^9}{36\sqrt{c}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**8*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

```
[Out] Piecewise((2*(-16*sqrt(3)*c**(5/2)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)
))/3 + 16*c**2*sqrt(c + d*x**3)/3 - 5*c*(c + d*x**3)**(3/2)/9 + (c + d*x**3
)**(5/2)/15)/d**3, Ne(d, 0)), (x**9/(36*sqrt(c)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.71

$$\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx = -\frac{2\left(240\sqrt{3}c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)-3\left(dx^3+c\right)^{\frac{5}{2}}+25\left(dx^3+c\right)^{\frac{3}{2}}c-240\sqrt{dx^3+c}c^2\right)}{45\,d^3}$$

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] -2/45*(240*sqrt(3)*c^(5/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - 3*
(d*x^3 + c)^(5/2) + 25*(d*x^3 + c)^(3/2)*c - 240*sqrt(d*x^3 + c)*c^2)/d^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx = -\frac{32\sqrt{3}c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{3\,d^3} + \frac{2\left(3\left(dx^3+c\right)^{\frac{5}{2}}d^{12}-25\left(dx^3+c\right)^{\frac{3}{2}}cd^{12}+240\sqrt{dx^3+c}c^2d^{12}\right)}{45\,d^{15}}$$

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] -32/3*sqrt(3)*c^(5/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d^3 + 2/4
5*(3*(d*x^3 + c)^(5/2)*d^12 - 25*(d*x^3 + c)^(3/2)*c*d^12 + 240*sqrt(d*x^3
+ c)*c^2*d^12)/d^15
```

Mupad [B] (verification not implemented)

Time = 8.91 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.12

$$\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{436\,c^2\,\sqrt{d\,x^3+c}}{45\,d^3} + \frac{2\,x^6\,\sqrt{d\,x^3+c}}{15\,d} - \frac{38\,c\,x^3\,\sqrt{d\,x^3+c}}{45\,d^2} + \frac{\sqrt{3}\,c^{5/2}\,\ln\left(\frac{2\,\sqrt{3}\,c-\sqrt{3}\,d\,x^3+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)\,16\mathrm{i}}{3\,d^3}$$

```
[In] int((x^8*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)
```

```
[Out] (436*c^2*(c + d*x^3)^(1/2))/(45*d^3) + (2*x^6*(c + d*x^3)^(1/2))/(15*d) - (
38*c*x^3*(c + d*x^3)^(1/2))/(45*d^2) + (3^(1/2)*c^(5/2)*log((2*3^(1/2)*c +
c^(1/2)*(c + d*x^3)^(1/2)*6i - 3^(1/2)*d*x^3)/(4*c + d*x^3))*16i)/(3*d^3)
```

3.260 $\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 76

$$\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx=-\frac{8c\sqrt{c+dx^3}}{3d^2}+\frac{2(c+dx^3)^{3/2}}{9d^2}+\frac{8c^{3/2}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^2}$$

```
[Out] 2/9*(d*x^3+c)^(3/2)/d^2+8/3*c^(3/2)*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1
/2))/d^2*3^(1/2)-8/3*c*(d*x^3+c)^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {457, 81, 52, 65, 209}

$$\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx=\frac{8c^{3/2}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^2}-\frac{8c\sqrt{c+dx^3}}{3d^2}+\frac{2(c+dx^3)^{3/2}}{9d^2}$$

```
[In] Int[(x^5*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (-8*c*Sqrt[c + d*x^3])/(3*d^2) + (2*(c + d*x^3)^(3/2))/(9*d^2) + (8*c^(3/2)
*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(Sqrt[3]*d^2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x\sqrt{c+dx}}{4c+dx}\,dx, x, x^3\right)\\
&= \frac{2(c+dx^3)^{3/2}}{9d^2} - \frac{(4c)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{4c+dx}\,dx, x, x^3\right)}{3d}\\
&= -\frac{8c\sqrt{c+dx^3}}{3d^2} + \frac{2(c+dx^3)^{3/2}}{9d^2} + \frac{(4c^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}(4c+dx)}\,dx, x, x^3\right)}{d}\\
&= -\frac{8c\sqrt{c+dx^3}}{3d^2} + \frac{2(c+dx^3)^{3/2}}{9d^2} + \frac{(8c^2)\,\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d^2}\\
&= -\frac{8c\sqrt{c+dx^3}}{3d^2} + \frac{2(c+dx^3)^{3/2}}{9d^2} + \frac{8c^{3/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx=\frac{2(-11c+dx^3)\sqrt{c+dx^3}}{9d^2}+\frac{8c^{3/2}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^2}$$

```
[In] Integrate[(x^5*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (2*(-11*c + d*x^3)*Sqrt[c + d*x^3])/(9*d^2) + (8*c^(3/2)*ArcTan[Sqrt[c + d*
x^3]/(Sqrt[3]*Sqrt[c])])/(Sqrt[3]*d^2)
```

Maple [A] (verified)

Time = 4.54 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$\frac{24c^{\frac{3}{2}}\sqrt{3}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{3}}{3\sqrt{c}}\right)-2\sqrt{dx^3+c}\,(-dx^3+11c)}{9d^2}$
risch	$-\frac{2(-dx^3+11c)\sqrt{dx^3+c}}{9d^2}+\frac{8c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{3d^2}$
default	$\frac{2(dx^3+c)^{\frac{3}{2}}}{9d^2}-\frac{4c\left(2\sqrt{dx^3+c}-2\sqrt{c}\sqrt{3}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{3}}{3\sqrt{c}}\right)\right)}{3d^2}$
elliptic	$\frac{2x^3\sqrt{dx^3+c}}{9d}-\frac{22c\sqrt{dx^3+c}}{9d^2}-\frac{4ic\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}(d\_Z^3+4c)}\frac{(-cd^2)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}(-cd^2)^{\frac{1}{3}}+(-cd^2)^{\frac{1}{3}}}{d}\right)}{(-cd^2)^{\frac{1}{3}}}}\sqrt{\frac{\cdots}{-3(\cdots}}\cdots}{\cdots}\right.}{\cdots}$

```
[In] int(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*(24*c^(3/2)*3^(1/2)*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))-2*(d*x^
3+c)^(1/2)*(-d*x^3+11*c))/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.70

$$\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx = \left[\frac{2\left(6\sqrt{3}\sqrt{-c}c\log\left(\frac{dx^3+2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)+\sqrt{dx^3+c}(dx^3-11c)\right)}{9d^2}, \frac{2\left(12\sqrt{3}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)+\sqrt{d}\right.}{9d^2}\right.$$

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] [2/9*(6*sqrt(3)*sqrt(-c)*c*log((d*x^3 + 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c)
- 2*c)/(d*x^3 + 4*c)) + sqrt(d*x^3 + c)*(d*x^3 - 11*c))/d^2, 2/9*(12*sqrt(3
)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) + sqrt(d*x^3 + c)*(d*
x^3 - 11*c))/d^2]
```

Sympy [A] (verification not implemented)

Time = 3.41 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx = \begin{cases} \frac{2\cdot\left(\frac{4\sqrt{3}c^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3}-\frac{4c\sqrt{c+dx^3}}{3}+\frac{(c+dx^3)^{\frac{3}{2}}}{9}\right)}{d^2} & \text{for } d\neq 0 \\ \frac{x^6}{24\sqrt{c}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**5*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

```
[Out] Piecewise((2*(4*sqrt(3)*c**(3/2)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))
/3 - 4*c*sqrt(c + d*x**3)/3 + (c + d*x**3)**(3/2)/9)/d**2, Ne(d, 0)), (x**6
/(24*sqrt(c)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

$$\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{2\left(12\sqrt{3}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)+(dx^3+c)^{\frac{3}{2}}-12\sqrt{dx^3+c}c\right)}{9d^2}$$

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] 2/9*(12*sqrt(3)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) + (d*x^
3 + c)^(3/2) - 12*sqrt(d*x^3 + c)*c)/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{8\sqrt{3}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{3\,d^2} + \frac{2\left((dx^3+c)^{\frac{3}{2}}d^4 - 12\sqrt{dx^3+c}cd^4\right)}{9\,d^6}$$

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] 8/3*sqrt(3)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d^2 + 2/9*(
(d*x^3 + c)^(3/2)*d^4 - 12*sqrt(d*x^3 + c)*c*d^4)/d^6
```

Mupad [B] (verification not implemented)

Time = 8.53 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16

$$\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d} - \frac{22\,c\,\sqrt{d\,x^3+c}}{9\,d^2} + \frac{\sqrt{3}\,c^{3/2}\,\ln\left(\frac{\sqrt{3}\,d\,x^3-2\,\sqrt{3}\,c+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)\,4\mathrm{i}}{3\,d^2}$$

```
[In] int((x^5*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)
```

```
[Out] (2*x^3*(c + d*x^3)^(1/2))/(9*d) - (22*c*(c + d*x^3)^(1/2))/(9*d^2) + (3^(1/
2)*c^(3/2)*log((c^(1/2)*(c + d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3^(1/2)*d*x^3)
/(4*c + d*x^3))*4i)/(3*d^2)
```

3.261 $\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 57

$$\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx=\frac{2\sqrt{c+dx^3}}{3d}-\frac{2\sqrt{c}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d}$$

```
[Out] -2/3*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))*c^(1/2)/d*3^(1/2)+2/3*(d*x
^3+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {455, 52, 65, 209}

$$\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx=\frac{2\sqrt{c+dx^3}}{3d}-\frac{2\sqrt{c}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d}$$

```
[In] Int[(x^2*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*d) - (2*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt
[c])])/(Sqrt[3]*d)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{4c+dx}\,dx, x, x^3\right)\\
&= \frac{2\sqrt{c+dx^3}}{3d} - c\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}(4c+dx)}\,dx, x, x^3\right)\\
&= \frac{2\sqrt{c+dx^3}}{3d} - \frac{(2c)\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d}\\
&= \frac{2\sqrt{c+dx^3}}{3d} - \frac{2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{2\left(\sqrt{c+dx^3}-\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)\right)}{3d}$$

[In] Integrate[(x^2*Sqrt[c + d*x^3])/(4*c + d*x^3),x]

[Out] (2*(Sqrt[c + d*x^3] - Sqrt[3]*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])]))/(3*d)

Maple [A] (verified)

Time = 4.34 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

method	result
default	$\frac{2\sqrt{d\,x^3+c}-2\sqrt{c}\,\sqrt{3}\,\arctan\left(\frac{\sqrt{d\,x^3+c}\,\sqrt{3}}{3\sqrt{c}}\right)}{3d}$
pseudoelliptic	$\frac{2\sqrt{d\,x^3+c}-2\sqrt{c}\,\sqrt{3}\,\arctan\left(\frac{\sqrt{d\,x^3+c}\,\sqrt{3}}{3\sqrt{c}}\right)}{3d}$
risch	$-\frac{2\arctan\left(\frac{\sqrt{d\,x^3+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{c}\,\sqrt{3}}{3d}+\frac{2\sqrt{d\,x^3+c}}{3d}$
elliptic	$\frac{2\sqrt{d\,x^3+c}}{3d}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3+4c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{i d\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\cdots}{\cdots}\right)}{\cdots}$

[In] int(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)

[Out] 1/3/d*(2*(d*x^3+c)^(1/2)-2*c^(1/2)*3^(1/2)*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.93

$$\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx = \left[\frac{\sqrt{3}\sqrt{-c}\log\left(\frac{dx^3-2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)+2\sqrt{dx^3+c}}{3d}, -\frac{2\left(\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)-\sqrt{dx^3+c}\right)}{3d}\right]$$

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] [1/3*(sqrt(3)*sqrt(-c)*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*
c)/(d*x^3 + 4*c)) + 2*sqrt(d*x^3 + c))/d, -2/3*(sqrt(3)*sqrt(c)*arctan(1/3*
sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - sqrt(d*x^3 + c))/d]
```

Sympy [A] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx = \begin{cases} \frac{2\left(-\frac{\sqrt{3}\sqrt{c}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3}+\frac{\sqrt{c+dx^3}}{3}\right)}{d} & \text{for}\ d\neq 0 \\ \frac{x^3}{12\sqrt{c}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

```
[Out] Piecewise((2*(-sqrt(3)*sqrt(c)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))/3
 + sqrt(c + d*x**3)/3)/d, Ne(d, 0)), (x**3/(12*sqrt(c)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx = -\frac{2\left(\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)-\sqrt{dx^3+c}\right)}{3d}$$

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] -2/3*(sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - sqrt(d*
x^3 + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx = -\frac{2\,\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)}{3\,d} + \frac{2\,\sqrt{dx^3+c}}{3\,d}$$

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d + 2/3*sq
rt(d*x^3 + c)/d
```

Mupad [B] (verification not implemented)

Time = 8.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.25

$$\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{2\,\sqrt{d\,x^3+c}}{3\,d} + \frac{\sqrt{3}\,\sqrt{c}\,\ln\left(\frac{2\,\sqrt{3}\,c-\sqrt{3}\,d\,x^3+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)\,1\mathrm{i}}{3\,d}$$

```
[In] int((x^2*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*d) + (3^(1/2)*c^(1/2)*log((2*3^(1/2)*c + c^(1/2)*(
c + d*x^3)^(1/2)*6i - 3^(1/2)*d*x^3)/(4*c + d*x^3))*1i)/(3*d)
```

3.262 $\int \frac{\sqrt{c+dx^3}}{x(4c+dx^3)} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 65

$$\int \frac{\sqrt{c+dx^3}}{x(4c+dx^3)} \, dx = \frac{\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2\sqrt{3}\sqrt{c}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6\sqrt{c}}$$

```
[Out] -1/6*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)+1/6*arctan(1/3*(d*x^3+c)^(1/2
)*3^(1/2)/c^(1/2))*3^(1/2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {457, 85, 65, 214, 209}

$$\int \frac{\sqrt{c+dx^3}}{x(4c+dx^3)} \, dx = \frac{\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2\sqrt{3}\sqrt{c}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6\sqrt{c}}$$

```
[In] Int[Sqrt[c + d*x^3]/(x*(4*c + d*x^3)),x]
```

```
[Out] ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])]/(2*Sqrt[3]*Sqrt[c]) - ArcTanh[Sqr
t[c + d*x^3]/Sqrt[c]]/(6*Sqrt[c])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x(4c+dx)}\,dx, x, x^3\right)\\
&= \frac{1}{12}\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right) + \frac{1}{4}d\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}(4c+dx)}\,dx, x, x^3\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right) + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{6d}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2\sqrt{3}\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6\sqrt{c}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{c+dx^3}}{x\,(4c+dx^3)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)-\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6\sqrt{c}}$$

```
[In] Integrate[Sqrt[c + d*x^3]/(x*(4*c + d*x^3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])] - ArcTanh[Sqrt[c + d*x^3
]/Sqrt[c]])/(6*Sqrt[c])
```

Maple [A] (verified)

Time = 4.70 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

method	result	size
pseudoelliptic	$\frac{\arctan\left(\frac{\sqrt{d\,x^3+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}-\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{6\sqrt{c}}$	45
default	$\frac{\frac{2\sqrt{d\,x^3+c}}{3}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)\sqrt{c}}{3}}{4c}-\frac{2\sqrt{d\,x^3+c}-2\sqrt{c}\,\sqrt{3}\,\arctan\left(\frac{\sqrt{d\,x^3+c}\,\sqrt{3}}{3\sqrt{c}}\right)}{12c}$	81
elliptic	Expression too large to display	1502

```
[In] int((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))*3^(1/2)-arctanh((d*x^3+c)^
(1/2)/c^(1/2)))/c^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.26

$$\int \frac{\sqrt{c+dx^3}}{x\,(4c+dx^3)}\,dx = \left[\frac{2\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)+\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)}{12\,c}, -\frac{\sqrt{3}\sqrt{-c}\log\left(\frac{dx^3-2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2\,c}{dx^3+4\,c}\right)-2\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)}{12\,c}\right]$$

```
[In] integrate((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] [1/12*(2*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) + sqrt
(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/c, -1/12*(sqrt(3)*s
```

```
qrt(-c)*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3 + 4*c
)) - 2*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c))/c]
```

Sympy [A] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{c+dx^3}}{x(4c+dx^3)}\,dx = \begin{cases} \frac{2\left(\frac{d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{12\sqrt{-c}}+\frac{\sqrt{3}d\,\mathrm{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{12\sqrt{c}}\right)}{d} & \text{for } d\neq 0 \\ \frac{\log(x^3)}{12\sqrt{c}} & \text{otherwise} \end{cases}$$

```
[In] integrate((d*x**3+c)**(1/2)/x/(d*x**3+4*c),x)
```

```
[Out] Piecewise((2*(d*atan(sqrt(c + d*x**3)/sqrt(-c))/(12*sqrt(-c)) + sqrt(3)*d*a
tan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))/(12*sqrt(c)))/d, Ne(d, 0)), (log(
x**3)/(12*sqrt(c)), True))
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x(4c+dx^3)}\,dx = \int \frac{\sqrt{dx^3+c}}{(dx^3+4\,c)x}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{c+dx^3}}{x(4c+dx^3)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{6\sqrt{c}} + \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{6\sqrt{-c}}$$

```
[In] integrate((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/sqrt(c) + 1/6*arcta
n(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c)
```

Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{c+dx^3}}{x\,(4c+dx^3)}\,dx = \frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{12\,\sqrt{c}} + \frac{\sqrt{3}\,\ln\left(\frac{\sqrt{3}\,d\,x^3-2\,\sqrt{3}\,c+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)\,1\mathrm{i}}{12\,\sqrt{c}}$$

```
[In] int((c + d*x^3)^(1/2)/(x*(4*c + d*x^3)),x)
```

```
[Out] log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(1
2*c^(1/2)) + (3^(1/2)*log((c^(1/2)*(c + d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3^(
1/2)*d*x^3)/(4*c + d*x^3))*1i)/(12*c^(1/2))
```

3.263 $\int \frac{\sqrt{c+dx^3}}{x^4(4c+dx^3)} dx$

Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \frac{\sqrt{c+dx^3}}{x^4(4c+dx^3)} dx = -\frac{\sqrt{c+dx^3}}{12cx^3} - \frac{d\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{8\sqrt{3}c^{3/2}} - \frac{d\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{24c^{3/2}}$$

```
[Out] -1/24*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-1/24*d*arctan(1/3*(d*x^3+c
)^(1/2)*3^(1/2)/c^(1/2))/c^(3/2)*3^(1/2)-1/12*(d*x^3+c)^(1/2)/c/x^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {457, 101, 162, 65, 214, 209}

$$\int \frac{\sqrt{c+dx^3}}{x^4(4c+dx^3)} dx = -\frac{d\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{8\sqrt{3}c^{3/2}} - \frac{d\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{24c^{3/2}} - \frac{\sqrt{c+dx^3}}{12cx^3}$$

```
[In] Int[Sqrt[c + d*x^3]/(x^4*(4*c + d*x^3)),x]
```

```
[Out] -1/12*Sqrt[c + d*x^3]/(c*x^3) - (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])
])/(8*Sqrt[3]*c^(3/2)) - (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(24*c^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(4c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{12cx^3} + \frac{\text{Subst}\left(\int \frac{cd-\frac{d^2x}{2}}{x\sqrt{c+dx}(4c+dx)}\,dx, x, x^3\right)}{12c}$$

$$= -\frac{\sqrt{c+dx^3}}{12cx^3} + \frac{d\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{48c} - \frac{d^2\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}(4c+dx)}\,dx, x, x^3\right)}{16c}$$

$$= -\frac{\sqrt{c+dx^3}}{12cx^3} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{24c} - \frac{d\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right)}{8c}$$

$$= -\frac{\sqrt{c+dx^3}}{12cx^3} - \frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{8\sqrt{3}c^{3/2}} - \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{24c^{3/2}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+dx^3}}{x^4(4c+dx^3)}\,dx = -\frac{\sqrt{c+dx^3}}{12cx^3} - \frac{d\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{8\sqrt{3}c^{3/2}} - \frac{d\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{24c^{3/2}}$$

```
[In] Integrate[Sqrt[c + d*x^3]/(x^4*(4*c + d*x^3)),x]
[Out] -1/12*Sqrt[c + d*x^3]/(c*x^3) - (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])
])/(8*Sqrt[3]*c^(3/2)) - (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(24*c^(3/2))
```

Maple [A] (verified)

Time = 4.47 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{d\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{24c^{\frac{3}{2}}} - \frac{d\arctan\left(\frac{\sqrt{d\,x^3+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{24c^{\frac{3}{2}}} - \frac{\sqrt{d\,x^3+c}}{12c\,x^3}$
pseudoelliptic	$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{d\,x^3+c}\,\sqrt{3}}{3\sqrt{c}}\right)d\,x^3+\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)d\,x^3+2\sqrt{d\,x^3+c}\,\sqrt{c}}{24c^{\frac{3}{2}}\,x^3}$
default	$\frac{-\frac{\sqrt{d\,x^3+c}}{3x^3}-\frac{d\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3\sqrt{c}}}{4c} - \frac{d\left(\frac{2\sqrt{d\,x^3+c}}{3}-\frac{2\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)\sqrt{c}}{3}\right)}{16c^2} + \frac{d\left(2\sqrt{d\,x^3+c}-2\sqrt{c}\,\sqrt{3}\,\arctan\left(\frac{\sqrt{d\,x^3+c}\,\sqrt{3}}{3\sqrt{c}}\right.\right.}{48c^2}$
elliptic	Expression too large to display

```
[In] int((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x,method=_RETURNVERBOSE)
[Out] -1/24*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-1/24*d*arctan(1/3*(d*x^3+c
)^(1/2)*3^(1/2)/c^(1/2))/c^(3/2)*3^(1/2)-1/12*(d*x^3+c)^(1/2)/c/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.20

$$\int \frac{\sqrt{c+dx^3}}{x^4\,(4c+dx^3)}\,dx = \left[-\frac{2\,\sqrt{3}\sqrt{c}dx^3\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)-\sqrt{c}dx^3\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)+4\,\sqrt{dx^3+c}c}{48\,c^2x^3},\ -\frac{\sqrt{3}\sqrt{-c}dx^3\log\left(\frac{dx^3+2\,\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2\,c}{dx^3+4\,c}\right)-2\,\sqrt{-c}dx^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+4\,\sqrt{dx^3+c}c}{48\,c^2x^3}\right]$$

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] [-1/48*(2*sqrt(3)*sqrt(c)*d*x^3*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))
 - sqrt(c)*d*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 4*sqr
t(d*x^3 + c)*c)/(c^2*x^3), -1/48*(sqrt(3)*sqrt(-c)*d*x^3*log((d*x^3 + 2*sqr
t(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3 + 4*c)) - 2*sqrt(-c)*d*x^3*arct
an(sqrt(d*x^3 + c)*sqrt(-c)/c) + 4*sqrt(d*x^3 + c)*c)/(c^2*x^3)]
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4\,(4c+dx^3)}\,dx = \int \frac{\sqrt{c+dx^3}}{x^4\cdot(4c+dx^3)}\,dx$$

```
[In] integrate((d*x**3+c)**(1/2)/x**4/(d*x**3+4*c),x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x**4*(4*c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4\,(4c+dx^3)}\,dx = \int \frac{\sqrt{dx^3+c}}{(dx^3+4\,c)x^4}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{c+dx^3}}{x^4\,(4c+dx^3)}\,dx = -\frac{\sqrt{3}d\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{24\,c^{\frac{3}{2}}} + \frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{24\sqrt{-c}c} - \frac{\sqrt{dx^3+c}}{12\,cx^3}$$

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(3)*d*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/c^(3/2) + 1/24*
d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/12*sqrt(d*x^3 + c)/(c*x
^3)
```

Mupad [B] (verification not implemented)

Time = 9.33 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{c+dx^3}}{x^4\,(4c+dx^3)}\,dx = \frac{d\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{48\,c^{3/2}} - \frac{\sqrt{d\,x^3+c}}{12\,c\,x^3} + \frac{\sqrt{3}\,d\,\ln\left(\frac{2\sqrt{3}\,c-\sqrt{3}\,d\,x^3+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)\,1\mathrm{i}}{48\,c^{3/2}}$$

```
[In] int((c + d*x^3)^(1/2)/(x^4*(4*c + d*x^3)),x)
```

```
[Out] (d*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)
)/(48*c^(3/2)) - (c + d*x^3)^(1/2)/(12*c*x^3) + (3^(1/2)*d*log((2*3^(1/2)*c
 + c^(1/2)*(c + d*x^3)^(1/2)*6i - 3^(1/2)*d*x^3)/(4*c + d*x^3))*1i)/(48*c^(
3/2))
```

3.264 $\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 689

$$\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{50c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{2\sqrt[3]{2}c^{7/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{\sqrt{3}d^{5/3}} + \frac{2\sqrt[3]{2}c^{7/6}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^{5/3}} - \frac{2\sqrt[3]{2}c^{7/6}\text{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}} + \frac{2\sqrt[3]{2}c^{7/6}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3d^{5/3}} + \frac{25\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{50\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{7\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] -2*2^(1/3)*c^(7/6)*arctanh(c^(1/6)*(c^(1/3)-2^(1/3)*d^(1/3)*x)/(d*x^3+c)^(1
/2))/d^(5/3)+2/3*2^(1/3)*c^(7/6)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/d^(5/3)-2
```

```
/3*2^(1/3)*c^(7/6)*arctan(c^(1/6)*(c^(1/3)+2^(1/3)*d^(1/3)*x)*3^(1/2)/(d*x^
3+c)^(1/2))/d^(5/3)*3^(1/2)+2/3*2^(1/3)*c^(7/6)*arctan(1/3*(d*x^3+c)^(1/2)*
3^(1/2)/c^(1/2))/d^(5/3)*3^(1/2)+2/7*x^2*(d*x^3+c)^(1/2)/d-50/7*c*(d*x^3+c)
^(1/2)/d^(5/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-50/21*c^(4/3)*(c^(1/3)+d^(1/
3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/
2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/
3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)
*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+25/7*3^(1/4)*
c^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1
/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/
3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/
d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+
3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 689, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used

= {489, 598, 309, 224, 1891, 497}

$$\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx =$$

$$-\frac{50\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{7\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{25\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{7d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{2\sqrt[3]{2}c^{7/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{\sqrt{3}d^{5/3}}+\frac{2\sqrt[3]{2}c^{7/6}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^{5/3}}$$

$$-\frac{2\sqrt[3]{2}c^{7/6}\text{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}}+\frac{2\sqrt[3]{2}c^{7/6}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3d^{5/3}}$$

$$-\frac{50c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{2x^2\sqrt{c+dx^3}}{7d}$$

```
[In] Int[(x^4*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (2*x^2*Sqrt[c + d*x^3])/(7*d) - (50*c*Sqrt[c + d*x^3])/(7*d^(5/3)*((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)) - (2*2^(1/3)*c^(7/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c
^(1/3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(Sqrt[3]*d^(5/3)) + (2*2^(1/
3)*c^(7/6)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(Sqrt[3]*d^(5/3)) - (
2*2^(1/3)*c^(7/6)*ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3)*d^(1/3)*x))/Sqrt[c +
d*x^3]])/d^(5/3) + (2*2^(1/3)*c^(7/6)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*
d^(5/3)) + (25*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt
[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3)
 + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (5
0*Sqrt[2]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x +
 d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
 Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4
```

```
*Sqrt[3]])/(7*3^(1/4)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 497

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[q*(ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]]/(9*2^(2/3)*b
*Rt[c, 2])), x] + (-Simp[q*(ArcTanh[Rt[c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c + d*
x^3])]/(3*2^(2/3)*b*Rt[c, 2])), x] + Simp[q*(ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])), x] - Simp[q*(ArcTan[Sqrt[3]*R
t[c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{2\int \frac{x\left(8c^2+\frac{25}{2}cdx^3\right)}{\sqrt{c+dx^3}(4c+dx^3)}\,dx}{7d} \\
&= \frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{2\int \left(\frac{25cx}{2\sqrt{c+dx^3}} - \frac{42c^2x}{\sqrt{c+dx^3}(4c+dx^3)}\right)dx}{7d} \\
&= \frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{(25c)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{7d} + \frac{(12c^2)\int \frac{x}{\sqrt{c+dx^3}(4c+dx^3)}\,dx}{d} \\
&= \frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{2\sqrt[3]{2}c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{\sqrt{3}d^{5/3}} + \frac{2\sqrt[3]{2}c^{7/6}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^{5/3}} \\
&\quad - \frac{2\sqrt[3]{2}c^{7/6}\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}} + \frac{2\sqrt[3]{2}c^{7/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3d^{5/3}} \\
&\quad - \frac{(25c)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{7d^{4/3}} + \frac{\left(25\left(1-\sqrt{3}\right)c^{4/3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{7d^{4/3}}
\end{aligned}
$$

$$= \frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{50c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{2\sqrt[3]{2}c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{\sqrt{3}d^{5/3}} + \frac{2\sqrt[3]{2}c^{7/6}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}d^{5/3}} - \frac{2\sqrt[3]{2}c^{7/6}\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}} + \frac{2\sqrt[3]{2}c^{7/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3d^{5/3}} + \frac{25\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{50\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.02 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.19

$$\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{8x^2(c+dx^3) - 8cx^2\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right) - 5dx^5\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c}\right.}{28d\sqrt{c+dx^3}}$$

```
[In] Integrate[(x^4*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (8*x^2*(c + d*x^3) - 8*c*x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3,
 -((d*x^3)/c), -1/4*(d*x^3)/c] - 5*d*x^5*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3,
1/2, 1, 8/3, -((d*x^3)/c), -1/4*(d*x^3)/c])/(28*d*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.85 (sec) , antiderivative size = 867, normalized size of antiderivative = 1.26

method	result	size
elliptic	Expression too large to display	867
risch	Expression too large to display	872
default	Expression too large to display	1309

```
[In] int(x^4*(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*x^2*(d*x^3+c)^(1/2)/d+50/21*I*c/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*
(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/
3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/
2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))
^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)
-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)
/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(
1/2)))-4/3*I*c/d^4*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(
-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*
d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3
)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2
)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^
(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.12 (sec) , antiderivative size = 2442, normalized size of antiderivative = 3.54

$$\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] 1/42*(12*sqrt(d*x^3 + c)*d*x^2 - 14*(4/27)^(1/6)*d^2*(-c^7/d^10)^(1/6)*log(
32*(9*(4/27)^(5/6)*(d^11*x^9 - 66*c*d^10*x^6 - 72*c^2*d^9*x^3 - 32*c^3*d^8)
*(-c^7/d^10)^(5/6) - 96*sqrt(1/3)*(c^3*d^7*x^7 - c^4*d^6*x^4 - 2*c^5*d^5*x)
*sqrt(-c^7/d^10) + 4*(9*4^(2/3)*c^2*d^8*x^5*(-c^7/d^10)^(2/3) + 2*c^6*d^2*x
^7 - 32*c^7*d*x^4 - 16*c^8*x + 4^(1/3)*(5*c^4*d^5*x^6 - 20*c^5*d^4*x^3 - 16
*c^6*d^3)*(-c^7/d^10)^(1/3))*sqrt(d*x^3 + c) - 24*(4/27)^(1/6)*(c^5*d^4*x^8
 - 7*c^6*d^3*x^5 - 8*c^7*d^2*x^2)*(-c^7/d^10)^(1/6))/(d^3*x^9 + 12*c*d^2*x^
6 + 48*c^2*d*x^3 + 64*c^3)) + 14*(4/27)^(1/6)*d^2*(-c^7/d^10)^(1/6)*log(-32
*(9*(4/27)^(5/6)*(d^11*x^9 - 66*c*d^10*x^6 - 72*c^2*d^9*x^3 - 32*c^3*d^8)*(
-c^7/d^10)^(5/6) - 96*sqrt(1/3)*(c^3*d^7*x^7 - c^4*d^6*x^4 - 2*c^5*d^5*x)*s
qrt(-c^7/d^10) - 4*(9*4^(2/3)*c^2*d^8*x^5*(-c^7/d^10)^(2/3) + 2*c^6*d^2*x^7
 - 32*c^7*d*x^4 - 16*c^8*x + 4^(1/3)*(5*c^4*d^5*x^6 - 20*c^5*d^4*x^3 - 16*c
^6*d^3)*(-c^7/d^10)^(1/3))*sqrt(d*x^3 + c) - 24*(4/27)^(1/6)*(c^5*d^4*x^8 -
 7*c^6*d^3*x^5 - 8*c^7*d^2*x^2)*(-c^7/d^10)^(1/6))/(d^3*x^9 + 12*c*d^2*x^6
+ 48*c^2*d*x^3 + 64*c^3)) + 300*c*sqrt(d)*weierstrassZeta(0, -4*c/d, weiers
trassPInverse(0, -4*c/d, x)) + 7*(4/27)^(1/6)*(sqrt(-3)*d^2 - d^2)*(-c^7/d^
10)^(1/6)*log(32*(9*(4/27)^(5/6)*(d^11*x^9 - 66*c*d^10*x^6 - 72*c^2*d^9*x^3
 - 32*c^3*d^8 + sqrt(-3)*(d^11*x^9 - 66*c*d^10*x^6 - 72*c^2*d^9*x^3 - 32*c^
3*d^8))*(-c^7/d^10)^(5/6) + 192*sqrt(1/3)*(c^3*d^7*x^7 - c^4*d^6*x^4 - 2*c^
5*d^5*x)*sqrt(-c^7/d^10) + 4*(4*c^6*d^2*x^7 - 64*c^7*d*x^4 - 32*c^8*x + 9*4
^(2/3)*(sqrt(-3)*c^2*d^8*x^5 - c^2*d^8*x^5)*(-c^7/d^10)^(2/3) - 4^(1/3)*(5*
c^4*d^5*x^6 - 20*c^5*d^4*x^3 - 16*c^6*d^3 + sqrt(-3)*(5*c^4*d^5*x^6 - 20*c^
5*d^4*x^3 - 16*c^6*d^3))*(-c^7/d^10)^(1/3))*sqrt(d*x^3 + c) - 24*(4/27)^(1/
6)*(c^5*d^4*x^8 - 7*c^6*d^3*x^5 - 8*c^7*d^2*x^2 - sqrt(-3)*(c^5*d^4*x^8 - 7
*c^6*d^3*x^5 - 8*c^7*d^2*x^2))*(-c^7/d^10)^(1/6))/(d^3*x^9 + 12*c*d^2*x^6 +
 48*c^2*d*x^3 + 64*c^3)) - 7*(4/27)^(1/6)*(sqrt(-3)*d^2 - d^2)*(-c^7/d^10)^
(1/6)*log(-32*(9*(4/27)^(5/6)*(d^11*x^9 - 66*c*d^10*x^6 - 72*c^2*d^9*x^3 -
32*c^3*d^8 + sqrt(-3)*(d^11*x^9 - 66*c*d^10*x^6 - 72*c^2*d^9*x^3 - 32*c^3*d
^8))*(-c^7/d^10)^(5/6) + 192*sqrt(1/3)*(c^3*d^7*x^7 - c^4*d^6*x^4 - 2*c^5*d
^5*x)*sqrt(-c^7/d^10) - 4*(4*c^6*d^2*x^7 - 64*c^7*d*x^4 - 32*c^8*x + 9*4^(2
/3)*(sqrt(-3)*c^2*d^8*x^5 - c^2*d^8*x^5)*(-c^7/d^10)^(2/3) - 4^(1/3)*(5*c^4
*d^5*x^6 - 20*c^5*d^4*x^3 - 16*c^6*d^3 + sqrt(-3)*(5*c^4*d^5*x^6 - 20*c^5*d
^4*x^3 - 16*c^6*d^3))*(-c^7/d^10)^(1/3))*sqrt(d*x^3 + c) - 24*(4/27)^(1/6)*
(c^5*d^4*x^8 - 7*c^6*d^3*x^5 - 8*c^7*d^2*x^2 - sqrt(-3)*(c^5*d^4*x^8 - 7*c^
6*d^3*x^5 - 8*c^7*d^2*x^2))*(-c^7/d^10)^(1/6))/(d^3*x^9 + 12*c*d^2*x^6 + 48
*c^2*d*x^3 + 64*c^3)) - 7*(4/27)^(1/6)*(sqrt(-3)*d^2 + d^2)*(-c^7/d^10)^(1/
6)*log(32*(9*(4/27)^(5/6)*(d^11*x^9 - 66*c*d^10*x^6 - 72*c^2*d^9*x^3 - 32*c
^3*d^8 - sqrt(-3)*(d^11*x^9 - 66*c*d^10*x^6 - 72*c^2*d^9*x^3 - 32*c^3*d^8))
*(-c^7/d^10)^(5/6) + 192*sqrt(1/3)*(c^3*d^7*x^7 - c^4*d^6*x^4 - 2*c^5*d^5*x
)*sqrt(-c^7/d^10) + 4*(4*c^6*d^2*x^7 - 64*c^7*d*x^4 - 32*c^8*x - 9*4^(2/3)*
(sqrt(-3)*c^2*d^8*x^5 + c^2*d^8*x^5)*(-c^7/d^10)^(2/3) - 4^(1/3)*(5*c^4*d^5
*x^6 - 20*c^5*d^4*x^3 - 16*c^6*d^3 - sqrt(-3)*(5*c^4*d^5*x^6 - 20*c^5*d^4*x
^3 - 16*c^6*d^3))*(-c^7/d^10)^(1/3))*sqrt(d*x^3 + c) - 24*(4/27)^(1/6)*(c^5
*d^4*x^8 - 7*c^6*d^3*x^5 - 8*c^7*d^2*x^2 + sqrt(-3)*(c^5*d^4*x^8 - 7*c^6*d^
3*x^5 - 8*c^7*d^2*x^2))*(-c^7/d^10)^(1/6))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2
```

```
*d*x^3 + 64*c^3)) + 7*(4/27)^(1/6)*(sqrt(-3)*d^2 + d^2)*(-c^7/d^10)^(1/6)*l
og(-32*(9*(4/27)^(5/6)*(d^11*x^9 - 66*c*d^10*x^6 - 72*c^2*d^9*x^3 - 32*c^3*
d^8 - sqrt(-3)*(d^11*x^9 - 66*c*d^10*x^6 - 72*c^2*d^9*x^3 - 32*c^3*d^8))*(-
c^7/d^10)^(5/6) + 192*sqrt(1/3)*(c^3*d^7*x^7 - c^4*d^6*x^4 - 2*c^5*d^5*x)*s
qrt(-c^7/d^10) - 4*(4*c^6*d^2*x^7 - 64*c^7*d*x^4 - 32*c^8*x - 9*4^(2/3)*(sq
rt(-3)*c^2*d^8*x^5 + c^2*d^8*x^5)*(-c^7/d^10)^(2/3) - 4^(1/3)*(5*c^4*d^5*x^
6 - 20*c^5*d^4*x^3 - 16*c^6*d^3 - sqrt(-3)*(5*c^4*d^5*x^6 - 20*c^5*d^4*x^3
- 16*c^6*d^3))*(-c^7/d^10)^(1/3))*sqrt(d*x^3 + c) - 24*(4/27)^(1/6)*(c^5*d^
4*x^8 - 7*c^6*d^3*x^5 - 8*c^7*d^2*x^2 + sqrt(-3)*(c^5*d^4*x^8 - 7*c^6*d^3*x
^5 - 8*c^7*d^2*x^2))*(-c^7/d^10)^(1/6))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*
x^3 + 64*c^3)))/d^2
```

Sympy [F]

$$\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx = \int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx$$

```
[In] integrate(x**4*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
[Out] Integral(x**4*sqrt(c + d*x**3)/(4*c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx = \int \frac{\sqrt{dx^3+c}x^4}{dx^3+4\,c}\,dx$$

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)*x^4/(d*x^3 + 4*c), x)
```

Giac [F]

$$\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx = \int \frac{\sqrt{dx^3+c}x^4}{dx^3+4\,c}\,dx$$

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)*x^4/(d*x^3 + 4*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx = \int \frac{x^4\,\sqrt{d\,x^3+c}}{d\,x^3+4\,c}\,dx$$

```
[In] int((x^4*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)
[Out] int((x^4*(c + d*x^3)^(1/2))/(4*c + d*x^3), x)
```

3.265 $\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 659

$$\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{2\sqrt{c+dx^3}}{d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt[6]{c}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{2^{2/3}\sqrt{3}d^{2/3}}$$
$$- \frac{\sqrt[6]{c}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2^{2/3}\sqrt{3}d^{2/3}} + \frac{\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{2^{2/3}d^{2/3}} - \frac{\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3\ 2^{2/3}d^{2/3}}$$
$$- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+ \frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 1/2*c^(1/6)*arctanh(c^(1/6)*(c^(1/3)-2^(1/3)*d^(1/3)*x)/(d*x^3+c)^(1/2))*2^
(1/3)/d^(2/3)-1/6*c^(1/6)*arctanh((d*x^3+c)^(1/2)/c^(1/2))*2^(1/3)/d^(2/3)+
1/6*c^(1/6)*arctan(c^(1/6)*(c^(1/3)+2^(1/3)*d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1
/2))*2^(1/3)/d^(2/3)*3^(1/2)-1/6*c^(1/6)*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)
```

```
/c^(1/2))*2^(1/3)/d^(2/3)*3^(1/2)+2*(d*x^3+c)^(1/2)/d^(2/3)/(d^(1/3)*x+c^(1
/3)*(1+3^(1/2)))+2/3*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/
3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^
(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/
2)*3^(3/4)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+
c^(1/3)*(1+3^(1/2)))^2)^(1/2)-3^(1/4)*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE
((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+
2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^
(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(
1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 659, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {495, 309, 224, 1891, 497}

$$\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx$$

$$= \frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+\frac{\sqrt[6]{c}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{2^{2/3}\sqrt{3}d^{2/3}}-\frac{\sqrt[6]{c}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2^{2/3}\sqrt{3}d^{2/3}}$$
$$+\frac{\sqrt[6]{c}\,\text{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{2^{2/3}d^{2/3}}-\frac{\sqrt[6]{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3\ 2^{2/3}d^{2/3}}+\frac{2\sqrt{c+dx^3}}{d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

```
[In] Int[(x*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (c^(1/6
)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/
(2^(2/3)*Sqrt[3]*d^(2/3)) - (c^(1/6)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c
])])/(2^(2/3)*Sqrt[3]*d^(2/3)) + (c^(1/6)*ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/
3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(2^(2/3)*d^(2/3)) - (c^(1/6)*ArcTanh[Sqrt[
c + d*x^3]/Sqrt[c]])/(3*2^(2/3)*d^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/
3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/(
(1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/
3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(
2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(
2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)
^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(
1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3
) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 497

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[q*(ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]]/(9*2^(2/3)*b
*Rt[c, 2])), x] + (-Simp[q*(ArcTanh[Rt[c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c + d*
x^3])]/(3*2^(2/3)*b*Rt[c, 2])), x] + Simp[q*(ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])), x] - Simp[q*(ArcTan[Sqrt[3]*R
```

```
t[c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left((3c)\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx\right)+\int \frac{x}{\sqrt{c+dx^3}}\,dx\\
&= \frac{\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{2^{2/3}\sqrt{3}d^{2/3}}-\frac{\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2^{2/3}\sqrt{3}d^{2/3}}\\
&\quad+\frac{\sqrt[6]{c}\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{2^{2/3}d^{2/3}}-\frac{\sqrt[6]{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3\ 2^{2/3}d^{2/3}}\\
&\quad+\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{\sqrt[3]{d}}-\frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{c}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{\sqrt[3]{d}}
\end{aligned}$$

$$= \frac{2\sqrt{c+dx^3}}{d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{2^{2/3}\sqrt{3}d^{2/3}}$$

$$- \frac{\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2^{2/3}\sqrt{3}d^{2/3}} + \frac{\sqrt[6]{c}\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{2^{2/3}d^{2/3}} - \frac{\sqrt[6]{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3\ 2^{2/3}d^{2/3}}$$

$$- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 8.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.10

$$\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{x^2\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{8\sqrt{c+dx^3}}$$

```
[In] Integrate[(x*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
[Out] (x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, -1/2, 1, 5/3, -((d*x^3)/c), -1/4*(d*
x^3)/c])/(8*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.21 (sec) , antiderivative size = 848, normalized size of antiderivative = 1.29

method	result	size
default	Expression too large to display	848
elliptic	Expression too large to display	848

```
[In] int(x*(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*El
lipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d
^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I/d^3*2^(1/2)*sum(1/
_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)
+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1
/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3
)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_
alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(
-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*
c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4
*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.84 (sec) , antiderivative size = 2202, normalized size of antiderivative = 3.34

$$\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

[Out] -1/12*((1/432)^(1/6)*(sqrt(-3)*d - d)*(-c/d^4)^(1/6)*log(1/2*(36*(1/432)^(5/6)*(d^6*x^9 - 66*c*d^5*x^6 - 72*c^2*d^4*x^3 - 32*c^3*d^3 + sqrt(-3)*(d^6*x^9 - 66*c*d^5*x^6 - 72*c^2*d^4*x^3 - 32*c^3*d^3))*(-c/d^4)^(5/6) + 24*sqrt(1/3)*(c*d^4*x^7 - c^2*d^3*x^4 - 2*c^3*d^2*x)*sqrt(-c/d^4) + (2*c*d^2*x^7 - 32*c^2*d*x^4 - 16*c^3*x + 18*(1/2)^(2/3)*(sqrt(-3)*c*d^4*x^5 - c*d^4*x^5)*(-c/d^4)^(2/3) - (1/2)^(1/3)*(5*c*d^3*x^6 - 20*c^2*d^2*x^3 - 16*c^3*d + sqrt(-3)*(5*c*d^3*x^6 - 20*c^2*d^2*x^3 - 16*c^3*d))*(-c/d^4)^(1/3))*sqrt(d*x^3 + c) - 6*(1/432)^(1/6)*(c*d^3*x^8 - 7*c^2*d^2*x^5 - 8*c^3*d*x^2 - sqrt(-3)*(c*d^3*x^8 - 7*c^2*d^2*x^5 - 8*c^3*d*x^2))*(-c/d^4)^(1/6))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - (1/432)^(1/6)*(sqrt(-3)*d - d)*(-c/d^4)^(1/6)*log(-1/2*(36*(1/432)^(5/6)*(d^6*x^9 - 66*c*d^5*x^6 - 72*c^2*d^4*x^3 - 32*c^3*d^3 + sqrt(-3)*(d^6*x^9 - 66*c*d^5*x^6 - 72*c^2*d^4*x^3 - 32*c^3*d^3))*(-c/d^4)^(5/6) + 24*sqrt(1/3)*(c*d^4*x^7 - c^2*d^3*x^4 - 2*c^3*d^2*x)*sqrt(-c/d^4) - (2*c*d^2*x^7 - 32*c^2*d*x^4 - 16*c^3*x + 18*(1/2)^(2/3)*(sqrt(-3)*c*d^4*x^5 - c*d^4*x^5)*(-c/d^4)^(2/3) - (1/2)^(1/3)*(5*c*d^3*x^6 - 20*c^2*d^2*x^3 - 16*c^3*d + sqrt(-3)*(5*c*d^3*x^6 - 20*c^2*d^2*x^3 - 16*c^3*d))*(-c/d^4)^(1/3))*sqrt(d*x^3 + c) - 6*(1/432)^(1/6)*(c*d^3*x^8 - 7*c^2*d^2*x^5 - 8*c^3*d*x^2 - sqrt(-3)*(c*d^3*x^8 - 7*c^2*d^2*x^5 - 8*c^3*d*x^2))*(-c/d^4)^(1/6))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - (1/432)^(1/6)*(sqrt(-3)*d + d)*(-c/d^4)^(1/6)*log(1/2*(36*(1/432)^(5/6)*(d^6*x^9 - 66*c*d^5*x^6 - 72*c^2*d^4*x^3 - 32*c^3*d^3 - sqrt(-3)*(d^6*x^9 - 66*c*d^5*x^6 - 72*c^2*d^4*x^3 - 32*c^3*d^3))*(-c/d^4)^(5/6) + 24*sqrt(1/3)*(c*d^4*x^7 - c^2*d^3*x^4 - 2*c^3*d^2*x)*sqrt(-c/d^4) + (2*c*d^2*x^7 - 32*c^2*d*x^4 - 16*c^3*x - 18*(1/2)^(2/3)*(sqrt(-3)*c*d^4*x^5 + c*d^4*x^5)*(-c/d^4)^(2/3) - (1/2)^(1/3)*(5*c*d^3*x^6 - 20*c^2*d^2*x^3 - 16*c^3*d - sqrt(-3)*(5*c*d^3*x^6 - 20*c^2*d^2*x^3 - 16*c^3*d))*(-c/d^4)^(1/3))*sqrt(d*x^3 + c) - 6*(1/432)^(1/6)*(c*d^3*x^8 - 7*c^2*d^2*x^5 - 8*c^3*d*x^2 + sqrt(-3)*(c*d^3*x^8 - 7*c^2*d^2*x^5 - 8*c^3*d*x^2))*(-c/d^4)^(1/6))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + (1/432)^(1/6)*(sqrt(-3)*d + d)*(-c/d^4)^(1/6)*log(-1/2*(36*(1/432)^(5/6)*(d^6*x^9 - 66*c*d^5*x^6 - 72*c^2*d^4*x^3 - 32*c^3*d^3 - sqrt(-3)*(d^6*x^9 - 66*c*d^5*x^6 - 72*c^2*d^4*x^3 - 32*c^3*d^3))*(-c/d^4)^(5/6) + 24*sqrt(1/3)*(c*d^4*x^7 - c^2*d^3*x^4 - 2*c^3*d^2*x)*sqrt(-c/d^4) - (2*c*d^2*x^7 - 32*c^2*d*x^4 - 16*c^3*x - 18*(1/2)^(2/3)*(sqrt(-3)*c*d^4*x^5 + c*d^4*x^5)*(-c/d^4)^(2/3) - (1/2)^(1/3)*(5*c*d^3*x^6 - 20*c^2*d^2*x^3 - 16*c^3*d - sqrt(-3)*(5*c*d^3*x^6 - 20*c^2*d^2*x^3 - 16*c^3*d))*(-c/d^4)^(1/3))*sqrt(d*x^3 + c) - 6*(1/432)^(1/6)*(c*d^3*x^8 - 7*c^2*d^2*x^5 - 8*c^3*d*x^2 + sqrt(-3)*(c*d^3*x^8 - 7*c^2*d^2*x^5 - 8*c^3*d*x^2))*(-c/d^4)^(1/6))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 2*(1/432)^(1/6)*d*(-c/d^4)^(1/6)*log((36*(1/432)^(5/6)*(d^6*x^9 - 66*c*d^5*x^6 - 72*c^2*d^4*x^3 - 32*c^3*d^3)*(-c/d^4)^(5/6) - 12*sqrt(1/3)*(c*d^4*x^7 - c^2*d^3*x^4 - 2*c^3*d^2*x)*sqrt(-c/d^4) + (18*(1/2)^(2/3)*c*d^4*x^5*(-c/d^4)^(2/3) + c*d^2*x^7 - 16*c^2*d*x^4 - 8*c^3*x + (1/2)^(1/3)*(5*c*d^3*x^6 - 20*c^2*d^2*x^3 - 16*c^3*d)*(-c/d^4)^(1/3))*sqrt(d*x^3 + c) - 6*(1/432)^(1/6)*(c*d^3*x^8 - 7*c^2*d^2*x^5 - 8*c^3*d*x^2)*(-c/d^4)^(1/6))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + 2*(1/432)^(1/6)*d*(-c/d^4)^(1/6)*log(-(36*(1/432)^(5/6)*(d^6*

```
x^9 - 66*c*d^5*x^6 - 72*c^2*d^4*x^3 - 32*c^3*d^3)*(-c/d^4)^(5/6) - 12*sqrt(
1/3)*(c*d^4*x^7 - c^2*d^3*x^4 - 2*c^3*d^2*x)*sqrt(-c/d^4) - (18*(1/2)^(2/3)
*c*d^4*x^5*(-c/d^4)^(2/3) + c*d^2*x^7 - 16*c^2*d*x^4 - 8*c^3*x + (1/2)^(1/3
)*(5*c*d^3*x^6 - 20*c^2*d^2*x^3 - 16*c^3*d)*(-c/d^4)^(1/3))*sqrt(d*x^3 + c)
 - 6*(1/432)^(1/6)*(c*d^3*x^8 - 7*c^2*d^2*x^5 - 8*c^3*d*x^2)*(-c/d^4)^(1/6)
)/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + 24*sqrt(d)*weierstras
sZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)))/d
```

Sympy [F]

$$\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx = \int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx$$

```
[In] integrate(x*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
[Out] Integral(x*sqrt(c + d*x**3)/(4*c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx = \int \frac{\sqrt{dx^3+c}x}{dx^3+4\,c}\,dx$$

```
[In] integrate(x*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)*x/(d*x^3 + 4*c), x)
```

Giac [F]

$$\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx = \int \frac{\sqrt{dx^3+c}x}{dx^3+4\,c}\,dx$$

```
[In] integrate(x*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)*x/(d*x^3 + 4*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx=\int \frac{x\,\sqrt{d\,x^3+c}}{d\,x^3+4\,c}\,dx$$

```
[In] int((x*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)
[Out] int((x*(c + d*x^3)^(1/2))/(4*c + d*x^3), x)
```

3.266 $\int \frac{\sqrt{c+dx^3}}{x^2(4c+dx^3)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 697

$$\int \frac{\sqrt{c+dx^3}}{x^2(4c+dx^3)}\,dx$$

$$= -\frac{\sqrt{c+dx^3}}{4cx} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{4c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{4\ 2^{2/3}\sqrt{3}c^{5/6}}$$

$$+ \frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{4\ 2^{2/3}\sqrt{3}c^{5/6}} - \frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{4\ 2^{2/3}c^{5/6}} + \frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\ 2^{2/3}c^{5/6}}$$

$$- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{8c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{2\sqrt{2}\sqrt[4]{3}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] -1/8*d^(1/3)*arctanh(c^(1/6)*(c^(1/3)-2^(1/3)*d^(1/3)*x)/(d*x^3+c)^(1/2))*2
^(1/3)/c^(5/6)+1/24*d^(1/3)*arctanh((d*x^3+c)^(1/2)/c^(1/2))*2^(1/3)/c^(5/6
```

```
)-1/24*d^(1/3)*arctan(c^(1/6)*(c^(1/3)+2^(1/3)*d^(1/3)*x)*3^(1/2)/(d*x^3+c)
^(1/2))*2^(1/3)/c^(5/6)*3^(1/2)+1/24*d^(1/3)*arctan(1/3*(d*x^3+c)^(1/2)*3^(
1/2)/c^(1/2))*2^(1/3)/c^(5/6)*3^(1/2)-1/4*(d*x^3+c)^(1/2)/c/x+1/4*d^(1/3)*(
d*x^3+c)^(1/2)/c/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/12*d^(1/3)*(c^(1/3)+d^(1
/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1
/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^
(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(2/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3
)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/8*3^(1/4)*
d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1
/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/
3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/
c^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+
3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 697, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used

$= \{486, 598, 309, 224, 1891, 497\}$

$$\int \frac{\sqrt{c+dx^3}}{x^2(4c+dx^3)}\,dx$$

$$= \frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{2\sqrt{2}\sqrt[4]{3}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{8c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{4\ 2^{2/3}\sqrt{3}c^{5/6}}+\frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{4\ 2^{2/3}\sqrt{3}c^{5/6}}$$
$$-\frac{\sqrt[3]{d}\text{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{4\ 2^{2/3}c^{5/6}}+\frac{\sqrt[3]{d}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\ 2^{2/3}c^{5/6}}$$
$$-\frac{\sqrt{c+dx^3}}{4cx}+\frac{\sqrt[3]{d}\sqrt{c+dx^3}}{4c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

```
[In] Int[Sqrt[c + d*x^3]/(x^2*(4*c + d*x^3)),x]
```

```
[Out] -1/4*Sqrt[c + d*x^3]/(c*x) + (d^(1/3)*Sqrt[c + d*x^3])/(4*c*((1 + Sqrt[3])*
c^(1/3) + d^(1/3)*x)) - (d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(1/3)
*d^(1/3)*x))/Sqrt[c + d*x^3]])/(4*2^(2/3)*Sqrt[3]*c^(5/6)) + (d^(1/3)*ArcTa
n[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(4*2^(2/3)*Sqrt[3]*c^(5/6)) - (d^(1/3
)*ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(4*2^(2
/3)*c^(5/6)) + (d^(1/3)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(12*2^(2/3)*c^(5/
6)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3
) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]
*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(8*c^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/
3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^(1/3)*(
c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) +
 d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*Sqrt[
```

```
2]*3^(1/4)*c^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1
/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 497

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[q*(ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]]/(9*2^(2/3)*b
*Rt[c, 2])), x] + (-Simp[q*(ArcTanh[Rt[c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c + d*
x^3])]/(3*2^(2/3)*b*Rt[c, 2])), x] + Simp[q*(ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])), x] - Simp[q*(ArcTan[Sqrt[3]*R
t[c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{c+dx^3}}{4cx}+\frac{\int \frac{x\left(5cd+\frac{d^2x^3}{2}\right)}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx}{4c}\\
&= -\frac{\sqrt{c+dx^3}}{4cx}+\frac{\int \left(\frac{dx}{2\sqrt{c+dx^3}}+\frac{3cdx}{\sqrt{c+dx^3}\,(4c+dx^3)}\right)dx}{4c}\\
&= -\frac{\sqrt{c+dx^3}}{4cx}+\frac{1}{4}(3d)\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx+\frac{d\int \frac{x}{\sqrt{c+dx^3}}\,dx}{8c}\\
&= -\frac{\sqrt{c+dx^3}}{4cx}-\frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{4\ 2^{2/3}\sqrt{3}c^{5/6}}+\frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{4\ 2^{2/3}\sqrt{3}c^{5/6}}\\
&\quad -\frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{4\ 2^{2/3}c^{5/6}}+\frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\ 2^{2/3}c^{5/6}}\\
&\quad +\frac{d^{2/3}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{8c}-\frac{\left(\left(1-\sqrt{3}\right)d^{2/3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{8c^{2/3}}
\end{aligned}$$

$$= -\frac{\sqrt{c+dx^3}}{4cx} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{4c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{4\ 2^{2/3}\sqrt{3}c^{5/6}}$$
$$+ \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{4\ 2^{2/3}\sqrt{3}c^{5/6}} - \frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{4\ 2^{2/3}c^{5/6}} + \frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\ 2^{2/3}c^{5/6}}$$
$$- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{8c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+ \frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{2\sqrt{2}\sqrt[4]{3}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(4c+dx^3\right)}\,dx$$
$$= \frac{-40c(c+dx^3)+25cdx^3\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+d^2x^6\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3}\right.}{160c^2x\sqrt{c+dx^3}}$$

```
[In] Integrate[Sqrt[c + d*x^3]/(x^2*(4*c + d*x^3)),x]
```

```
[Out] (-40*c*(c + d*x^3) + 25*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5
/3, -((d*x^3)/c), -1/4*(d*x^3)/c] + d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[5/
3, 1/2, 1, 8/3, -((d*x^3)/c), -1/4*(d*x^3)/c])/(160*c^2*x*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.01 (sec) , antiderivative size = 868, normalized size of antiderivative = 1.25

method	result	size
elliptic	Expression too large to display	868
risch	Expression too large to display	872
default	Expression too large to display	1306

```
[In] int((d*x^3+c)^(1/2)/x^2/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(d*x^3+c)^(1/2)/c/x-1/12*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^
2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((
x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(
-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)
)+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c
*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))
-1/12*I/d^2/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^
2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*
x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x
^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_
alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/
3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)
/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(
1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.53 (sec) , antiderivative size = 2253, normalized size of antiderivative = 3.23

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(4c+dx^3\right)}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] 1/48*(2*(1/432)^(1/6)*c*x*(-d^2/c^5)^(1/6)*log((d^4*x^9 - 66*c*d^3*x^6 - 72
*c^2*d^2*x^3 - 32*c^3*d + 48*(1/2)^(2/3)*(c^4*d^2*x^7 - c^5*d*x^4 - 2*c^6*x
)*(-d^2/c^5)^(2/3) + 12*(1/2)^(1/3)*(c^2*d^3*x^8 - 7*c^3*d^2*x^5 - 8*c^4*d*
x^2)*(-d^2/c^5)^(1/3) + 6*(1296*(1/432)^(5/6)*c^5*d*x^5*(-d^2/c^5)^(5/6) +
sqrt(1/3)*(5*c^3*d^2*x^6 - 20*c^4*d*x^3 - 16*c^5)*sqrt(-d^2/c^5) + 2*(1/432
)^(1/6)*(c*d^3*x^7 - 16*c^2*d^2*x^4 - 8*c^3*d*x)*(-d^2/c^5)^(1/6))*sqrt(d*x
^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 2*(1/432)^(1/6
)*c*x*(-d^2/c^5)^(1/6)*log((d^4*x^9 - 66*c*d^3*x^6 - 72*c^2*d^2*x^3 - 32*c^
3*d + 48*(1/2)^(2/3)*(c^4*d^2*x^7 - c^5*d*x^4 - 2*c^6*x)*(-d^2/c^5)^(2/3) +
 12*(1/2)^(1/3)*(c^2*d^3*x^8 - 7*c^3*d^2*x^5 - 8*c^4*d*x^2)*(-d^2/c^5)^(1/3
) - 6*(1296*(1/432)^(5/6)*c^5*d*x^5*(-d^2/c^5)^(5/6) + sqrt(1/3)*(5*c^3*d^2
*x^6 - 20*c^4*d*x^3 - 16*c^5)*sqrt(-d^2/c^5) + 2*(1/432)^(1/6)*(c*d^3*x^7 -
 16*c^2*d^2*x^4 - 8*c^3*d*x)*(-d^2/c^5)^(1/6))*sqrt(d*x^3 + c))/(d^3*x^9 +
12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 12*sqrt(d)*x*weierstrassZeta(0, -4
*c/d, weierstrassPInverse(0, -4*c/d, x)) - (1/432)^(1/6)*(sqrt(-3)*c*x + c*
x)*(-d^2/c^5)^(1/6)*log((d^4*x^9 - 66*c*d^3*x^6 - 72*c^2*d^2*x^3 - 32*c^3*d
 - 24*(1/2)^(2/3)*(c^4*d^2*x^7 - c^5*d*x^4 - 2*c^6*x + sqrt(-3)*(c^4*d^2*x^
7 - c^5*d*x^4 - 2*c^6*x))*(-d^2/c^5)^(2/3) - 6*(1/2)^(1/3)*(c^2*d^3*x^8 - 7
*c^3*d^2*x^5 - 8*c^4*d*x^2 - sqrt(-3)*(c^2*d^3*x^8 - 7*c^3*d^2*x^5 - 8*c^4*
d*x^2))*(-d^2/c^5)^(1/3) + 6*sqrt(d*x^3 + c)*(648*(1/432)^(5/6)*(sqrt(-3)*c
^5*d*x^5 - c^5*d*x^5)*(-d^2/c^5)^(5/6) + sqrt(1/3)*(5*c^3*d^2*x^6 - 20*c^4*
d*x^3 - 16*c^5)*sqrt(-d^2/c^5) - (1/432)^(1/6)*(c*d^3*x^7 - 16*c^2*d^2*x^4
- 8*c^3*d*x + sqrt(-3)*(c*d^3*x^7 - 16*c^2*d^2*x^4 - 8*c^3*d*x))*(-d^2/c^5)
^(1/6)))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + (1/432)^(1/6)*
(sqrt(-3)*c*x + c*x)*(-d^2/c^5)^(1/6)*log((d^4*x^9 - 66*c*d^3*x^6 - 72*c^2*
d^2*x^3 - 32*c^3*d - 24*(1/2)^(2/3)*(c^4*d^2*x^7 - c^5*d*x^4 - 2*c^6*x + sq
rt(-3)*(c^4*d^2*x^7 - c^5*d*x^4 - 2*c^6*x))*(-d^2/c^5)^(2/3) - 6*(1/2)^(1/3
)*(c^2*d^3*x^8 - 7*c^3*d^2*x^5 - 8*c^4*d*x^2 - sqrt(-3)*(c^2*d^3*x^8 - 7*c^
3*d^2*x^5 - 8*c^4*d*x^2))*(-d^2/c^5)^(1/3) - 6*sqrt(d*x^3 + c)*(648*(1/432)
^(5/6)*(sqrt(-3)*c^5*d*x^5 - c^5*d*x^5)*(-d^2/c^5)^(5/6) + sqrt(1/3)*(5*c^3
*d^2*x^6 - 20*c^4*d*x^3 - 16*c^5)*sqrt(-d^2/c^5) - (1/432)^(1/6)*(c*d^3*x^7
 - 16*c^2*d^2*x^4 - 8*c^3*d*x + sqrt(-3)*(c*d^3*x^7 - 16*c^2*d^2*x^4 - 8*c^
3*d*x))*(-d^2/c^5)^(1/6)))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)
) - (1/432)^(1/6)*(sqrt(-3)*c*x - c*x)*(-d^2/c^5)^(1/6)*log((d^4*x^9 - 66*c
*d^3*x^6 - 72*c^2*d^2*x^3 - 32*c^3*d - 24*(1/2)^(2/3)*(c^4*d^2*x^7 - c^5*d*
x^4 - 2*c^6*x - sqrt(-3)*(c^4*d^2*x^7 - c^5*d*x^4 - 2*c^6*x))*(-d^2/c^5)^(2
/3) - 6*(1/2)^(1/3)*(c^2*d^3*x^8 - 7*c^3*d^2*x^5 - 8*c^4*d*x^2 + sqrt(-3)*(
c^2*d^3*x^8 - 7*c^3*d^2*x^5 - 8*c^4*d*x^2))*(-d^2/c^5)^(1/3) + 6*sqrt(d*x^3
 + c)*(648*(1/432)^(5/6)*(sqrt(-3)*c^5*d*x^5 + c^5*d*x^5)*(-d^2/c^5)^(5/6)
- sqrt(1/3)*(5*c^3*d^2*x^6 - 20*c^4*d*x^3 - 16*c^5)*sqrt(-d^2/c^5) + (1/432
)^(1/6)*(c*d^3*x^7 - 16*c^2*d^2*x^4 - 8*c^3*d*x - sqrt(-3)*(c*d^3*x^7 - 16*
c^2*d^2*x^4 - 8*c^3*d*x))*(-d^2/c^5)^(1/6)))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c
^2*d*x^3 + 64*c^3)) + (1/432)^(1/6)*(sqrt(-3)*c*x - c*x)*(-d^2/c^5)^(1/6)*l
og((d^4*x^9 - 66*c*d^3*x^6 - 72*c^2*d^2*x^3 - 32*c^3*d - 24*(1/2)^(2/3)*(c^
4*d^2*x^7 - c^5*d*x^4 - 2*c^6*x - sqrt(-3)*(c^4*d^2*x^7 - c^5*d*x^4 - 2*c^6
```

```
*x))*(-d^2/c^5)^(2/3) - 6*(1/2)^(1/3)*(c^2*d^3*x^8 - 7*c^3*d^2*x^5 - 8*c^4*
d*x^2 + sqrt(-3)*(c^2*d^3*x^8 - 7*c^3*d^2*x^5 - 8*c^4*d*x^2))*(-d^2/c^5)^(1
/3) - 6*sqrt(d*x^3 + c)*(648*(1/432)^(5/6)*(sqrt(-3)*c^5*d*x^5 + c^5*d*x^5)
*(-d^2/c^5)^(5/6) - sqrt(1/3)*(5*c^3*d^2*x^6 - 20*c^4*d*x^3 - 16*c^5)*sqrt(
-d^2/c^5) + (1/432)^(1/6)*(c*d^3*x^7 - 16*c^2*d^2*x^4 - 8*c^3*d*x - sqrt(-3
)*(c*d^3*x^7 - 16*c^2*d^2*x^4 - 8*c^3*d*x))*(-d^2/c^5)^(1/6)))/(d^3*x^9 + 1
2*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 12*sqrt(d*x^3 + c))/(c*x)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(4c+dx^3\right)}\,dx=\int \frac{\sqrt{c+dx^3}}{x^2\cdot\left(4c+dx^3\right)}\,dx$$

```
[In] integrate((d*x**3+c)**(1/2)/x**2/(d*x**3+4*c),x)
[Out] Integral(sqrt(c + d*x**3)/(x**2*(4*c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(4c+dx^3\right)}\,dx=\int \frac{\sqrt{dx^3+c}}{(dx^3+4\,c)x^2}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(d*x^3+4*c),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(4c+dx^3\right)}\,dx=\int \frac{\sqrt{dx^3+c}}{(dx^3+4\,c)x^2}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(d*x^3+4*c),x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^2\,(4c+dx^3)}\,dx=\int \frac{\sqrt{d\,x^3+c}}{x^2\,\left(d\,x^3+4\,c\right)}\,dx$$

```
[In] int((c + d*x^3)^(1/2)/(x^2*(4*c + d*x^3)),x)
[Out] int((c + d*x^3)^(1/2)/(x^2*(4*c + d*x^3)), x)
```

3.267 $\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 66

$$\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx=\frac{x^4\sqrt{c+dx^3}\operatorname{AppellF1}\left(\frac{4}{3},1,-\frac{1}{2},\frac{7}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{16c\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] 1/16*x^4*AppellF1(4/3,-1/2,1,7/3,-d*x^3/c,-1/4*d*x^3/c)*(d*x^3+c)^(1/2)/c/(
1+d*x^3/c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {525, 524}

$$\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx=\frac{x^4\sqrt{c+dx^3}\operatorname{AppellF1}\left(\frac{4}{3},1,-\frac{1}{2},\frac{7}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{16c\sqrt{\frac{dx^3}{c}+1}}$$

```
[In] Int[(x^3*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 1, -1/2, 7/3, -1/4*(d*x^3)/c, -((d*x^3)/
c)])/(16*c*Sqrt[1 + (d*x^3)/c])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c+dx^3}\int \frac{x^3\sqrt{1+\frac{dx^3}{c}}}{4c+dx^3}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$

$$= \frac{x^4\sqrt{c+dx^3}F_1\left(\frac{4}{3};1,-\frac{1}{2};\frac{7}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{16c\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 236 vs. 2(66) = 132.

Time = 6.34 (sec) , antiderivative size = 236, normalized size of antiderivative = 3.58

$$\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx$$
$$= \frac{x\left(-17x^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+32\left(\frac{c}{d}+x^3+\frac{6\cdots}{d(4c+dx^3)\left(-16c\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{dx\cdots}{4c\cdots}\right.\right.}\right.\right.}{80\sqrt{c+dx^3}}$$

```
[In] Integrate[(x^3*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (x*(-17*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -1
/4*(d*x^3)/c] + 32*(c/d + x^3 + (64*c^3*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3
)/c), -1/4*(d*x^3)/c])/(d*(4*c + d*x^3)*(-16*c*AppellF1[1/3, 1/2, 1, 4/3, -
((d*x^3)/c), -1/4*(d*x^3)/c] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3
)/c), -1/4*(d*x^3)/c] + 2*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -1/4*(d*
x^3)/c]))))))/(80*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.74 (sec) , antiderivative size = 713, normalized size of antiderivative = 10.80

method	result
elliptic	$\frac{2x\sqrt{d\,x^3+c}}{5d}+\frac{34ic\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}}}\sqrt{-\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\right.}{2d}\right.}{\left(-c\,d^2\right)^{\frac{1}{3}}}}}{15d^2\sqrt{d\,x^3+c}}$
risch	Expression too large to display
default	Expression too large to display

```
[In] int(x^3*(d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*x*(d*x^3+c)^(1/2)/d+34/15*I*c/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-
c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2))-4/3*I*c/d^4*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*
(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3
))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1
/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(
-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3
^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3)
)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*
_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_
alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2387 vs. $2(52) = 104$.

Time = 1.87 (sec) , antiderivative size = 2387, normalized size of antiderivative = 36.17

$$\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] -1/60*(10*(16/27)^(1/6)*d^2*(-c^5/d^8)^(1/6)*log((27*(16/27)^(5/6)*(d^9*x^8
 - 7*c*d^8*x^5 - 8*c^2*d^7*x^2)*(-c^5/d^8)^(5/6) + 96*sqrt(1/3)*(c^2*d^6*x^
7 - c^3*d^5*x^4 - 2*c^4*d^4*x)*sqrt(-c^5/d^8) + 4*(2*c^4*d^2*x^7 - 18*2^(1/
3)*c^3*d^4*x^5*(-c^5/d^8)^(1/3) - 32*c^5*d*x^4 - 16*c^6*x - 2^(2/3)*(5*c*d^
7*x^6 - 20*c^2*d^6*x^3 - 16*c^3*d^5)*(-c^5/d^8)^(2/3))*sqrt(d*x^3 + c) - 2*
(16/27)^(1/6)*(c^3*d^4*x^9 - 66*c^4*d^3*x^6 - 72*c^5*d^2*x^3 - 32*c^6*d)*(-
c^5/d^8)^(1/6))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 10*(16/
27)^(1/6)*d^2*(-c^5/d^8)^(1/6)*log(-(27*(16/27)^(5/6)*(d^9*x^8 - 7*c*d^8*x^
5 - 8*c^2*d^7*x^2)*(-c^5/d^8)^(5/6) + 96*sqrt(1/3)*(c^2*d^6*x^7 - c^3*d^5*x
^4 - 2*c^4*d^4*x)*sqrt(-c^5/d^8) - 4*(2*c^4*d^2*x^7 - 18*2^(1/3)*c^3*d^4*x^
5*(-c^5/d^8)^(1/3) - 32*c^5*d*x^4 - 16*c^6*x - 2^(2/3)*(5*c*d^7*x^6 - 20*c^
2*d^6*x^3 - 16*c^3*d^5)*(-c^5/d^8)^(2/3))*sqrt(d*x^3 + c) - 2*(16/27)^(1/6)
*(c^3*d^4*x^9 - 66*c^4*d^3*x^6 - 72*c^5*d^2*x^3 - 32*c^6*d)*(-c^5/d^8)^(1/6
))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 24*sqrt(d*x^3 + c)*d
*x + 168*c*sqrt(d)*weierstrassPInverse(0, -4*c/d, x) - 5*(16/27)^(1/6)*(sqr
t(-3)*d^2 - d^2)*(-c^5/d^8)^(1/6)*log((27*(16/27)^(5/6)*(d^9*x^8 - 7*c*d^8*
x^5 - 8*c^2*d^7*x^2 + sqrt(-3)*(d^9*x^8 - 7*c*d^8*x^5 - 8*c^2*d^7*x^2))*(-c
^5/d^8)^(5/6) - 192*sqrt(1/3)*(c^2*d^6*x^7 - c^3*d^5*x^4 - 2*c^4*d^4*x)*sqr
t(-c^5/d^8) + 4*(4*c^4*d^2*x^7 - 64*c^5*d*x^4 - 32*c^6*x + 2^(2/3)*(5*c*d^7
*x^6 - 20*c^2*d^6*x^3 - 16*c^3*d^5 - sqrt(-3)*(5*c*d^7*x^6 - 20*c^2*d^6*x^3
 - 16*c^3*d^5))*(-c^5/d^8)^(2/3) + 18*2^(1/3)*(sqrt(-3)*c^3*d^4*x^5 + c^3*d
^4*x^5)*(-c^5/d^8)^(1/3))*sqrt(d*x^3 + c) - 2*(16/27)^(1/6)*(c^3*d^4*x^9 -
66*c^4*d^3*x^6 - 72*c^5*d^2*x^3 - 32*c^6*d - sqrt(-3)*(c^3*d^4*x^9 - 66*c^4
*d^3*x^6 - 72*c^5*d^2*x^3 - 32*c^6*d))*(-c^5/d^8)^(1/6))/(d^3*x^9 + 12*c*d^
2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + 5*(16/27)^(1/6)*(sqrt(-3)*d^2 - d^2)*(-c^
5/d^8)^(1/6)*log(-(27*(16/27)^(5/6)*(d^9*x^8 - 7*c*d^8*x^5 - 8*c^2*d^7*x^2
+ sqrt(-3)*(d^9*x^8 - 7*c*d^8*x^5 - 8*c^2*d^7*x^2))*(-c^5/d^8)^(5/6) - 192*
sqrt(1/3)*(c^2*d^6*x^7 - c^3*d^5*x^4 - 2*c^4*d^4*x)*sqrt(-c^5/d^8) - 4*(4*c
^4*d^2*x^7 - 64*c^5*d*x^4 - 32*c^6*x + 2^(2/3)*(5*c*d^7*x^6 - 20*c^2*d^6*x^
3 - 16*c^3*d^5 - sqrt(-3)*(5*c*d^7*x^6 - 20*c^2*d^6*x^3 - 16*c^3*d^5))*(-c^
5/d^8)^(2/3) + 18*2^(1/3)*(sqrt(-3)*c^3*d^4*x^5 + c^3*d^4*x^5)*(-c^5/d^8)^(
1/3))*sqrt(d*x^3 + c) - 2*(16/27)^(1/6)*(c^3*d^4*x^9 - 66*c^4*d^3*x^6 - 72*
c^5*d^2*x^3 - 32*c^6*d - sqrt(-3)*(c^3*d^4*x^9 - 66*c^4*d^3*x^6 - 72*c^5*d^
2*x^3 - 32*c^6*d))*(-c^5/d^8)^(1/6))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3
 + 64*c^3)) + 5*(16/27)^(1/6)*(sqrt(-3)*d^2 + d^2)*(-c^5/d^8)^(1/6)*log((27
```

```
*(16/27)^(5/6)*(d^9*x^8 - 7*c*d^8*x^5 - 8*c^2*d^7*x^2 - sqrt(-3)*(d^9*x^8 -
 7*c*d^8*x^5 - 8*c^2*d^7*x^2))*(-c^5/d^8)^(5/6) - 192*sqrt(1/3)*(c^2*d^6*x^
7 - c^3*d^5*x^4 - 2*c^4*d^4*x)*sqrt(-c^5/d^8) + 4*(4*c^4*d^2*x^7 - 64*c^5*d
*x^4 - 32*c^6*x + 2^(2/3)*(5*c*d^7*x^6 - 20*c^2*d^6*x^3 - 16*c^3*d^5 + sqrt
(-3)*(5*c*d^7*x^6 - 20*c^2*d^6*x^3 - 16*c^3*d^5))*(-c^5/d^8)^(2/3) - 18*2^(
1/3)*(sqrt(-3)*c^3*d^4*x^5 - c^3*d^4*x^5)*(-c^5/d^8)^(1/3))*sqrt(d*x^3 + c)
 - 2*(16/27)^(1/6)*(c^3*d^4*x^9 - 66*c^4*d^3*x^6 - 72*c^5*d^2*x^3 - 32*c^6*
d + sqrt(-3)*(c^3*d^4*x^9 - 66*c^4*d^3*x^6 - 72*c^5*d^2*x^3 - 32*c^6*d))*(-
c^5/d^8)^(1/6))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 5*(16/2
7)^(1/6)*(sqrt(-3)*d^2 + d^2)*(-c^5/d^8)^(1/6)*log(-(27*(16/27)^(5/6)*(d^9*
x^8 - 7*c*d^8*x^5 - 8*c^2*d^7*x^2 - sqrt(-3)*(d^9*x^8 - 7*c*d^8*x^5 - 8*c^2
*d^7*x^2))*(-c^5/d^8)^(5/6) - 192*sqrt(1/3)*(c^2*d^6*x^7 - c^3*d^5*x^4 - 2*
c^4*d^4*x)*sqrt(-c^5/d^8) - 4*(4*c^4*d^2*x^7 - 64*c^5*d*x^4 - 32*c^6*x + 2^
(2/3)*(5*c*d^7*x^6 - 20*c^2*d^6*x^3 - 16*c^3*d^5 + sqrt(-3)*(5*c*d^7*x^6 -
20*c^2*d^6*x^3 - 16*c^3*d^5))*(-c^5/d^8)^(2/3) - 18*2^(1/3)*(sqrt(-3)*c^3*d
^4*x^5 - c^3*d^4*x^5)*(-c^5/d^8)^(1/3))*sqrt(d*x^3 + c) - 2*(16/27)^(1/6)*(
c^3*d^4*x^9 - 66*c^4*d^3*x^6 - 72*c^5*d^2*x^3 - 32*c^6*d + sqrt(-3)*(c^3*d^
4*x^9 - 66*c^4*d^3*x^6 - 72*c^5*d^2*x^3 - 32*c^6*d))*(-c^5/d^8)^(1/6))/(d^3
*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)))/d^2
```

Sympy [F]

$$\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx = \int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx$$

```
[In] integrate(x**3*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
[Out] Integral(x**3*sqrt(c + d*x**3)/(4*c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx = \int \frac{\sqrt{dx^3+c}x^3}{dx^3+4\,c}\,dx$$

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)*x^3/(d*x^3 + 4*c), x)
```

Giac [F]

$$\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx = \int \frac{\sqrt{dx^3+c}x^3}{dx^3+4\,c}\,dx$$

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)*x^3/(d*x^3 + 4*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx = \int \frac{x^3\,\sqrt{d\,x^3+c}}{d\,x^3+4\,c}\,dx$$

```
[In] int((x^3*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)
[Out] int((x^3*(c + d*x^3)^(1/2))/(4*c + d*x^3), x)
```

3.268 $\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 64

$$\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{x\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},1,-\frac{1}{2},\frac{4}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{4c\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] 1/4*x*AppellF1(1/3,-1/2,1,4/3,-d*x^3/c,-1/4*d*x^3/c)*(d*x^3+c)^(1/2)/c/(1+d
*x^3/c)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {441, 440}

$$\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{x\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},1,-\frac{1}{2},\frac{4}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{4c\sqrt{\frac{dx^3}{c}+1}}$$

```
[In] Int[Sqrt[c + d*x^3]/(4*c + d*x^3),x]
```

```
[Out] (x*Sqrt[c + d*x^3]*AppellF1[1/3, 1, -1/2, 4/3, -1/4*(d*x^3)/c, -((d*x^3)/c)
])/(4*c*Sqrt[1 + (d*x^3)/c])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c+dx^3}\int \frac{\sqrt{1+\frac{dx^3}{c}}}{4c+dx^3}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$

$$= \frac{x\sqrt{c+dx^3}F_1\left(\frac{1}{3};1,-\frac{1}{2};\frac{4}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{4c\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 165 vs. 2(64) = 128.

Time = 10.16 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.58

$$\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx$$

$$= \frac{16cx\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{(4c+dx^3)\left(16c\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)-3dx^3\left(\text{AppellF1}\left(\frac{4}{3},-\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)-2\,\text{Appell}\right.\right.}$$

```
[In] Integrate[Sqrt[c + d*x^3]/(4*c + d*x^3),x]
```

```
[Out] (16*c*x*Sqrt[c + d*x^3]*AppellF1[1/3, -1/2, 1, 4/3, -((d*x^3)/c), -1/4*(d*x
^3)/c])/((4*c + d*x^3)*(16*c*AppellF1[1/3, -1/2, 1, 4/3, -((d*x^3)/c), -1/4
*(d*x^3)/c] - 3*d*x^3*(AppellF1[4/3, -1/2, 2, 7/3, -((d*x^3)/c), -1/4*(d*x^
3)/c] - 2*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -1/4*(d*x^3)/c])))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.65 (sec) , antiderivative size = 696, normalized size of antiderivative = 10.88

method	result
default	$-\frac{2 i \sqrt{3}\, \left(-c\,d^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i \left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2 d}-\frac{i \sqrt{3}\, \left(-c\,d^{2}\right)^{\frac{1}{3}}}{2 d}\right) \sqrt{3}\, d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\, \sqrt{\frac{x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}}{-\frac{3 \left(-c\,d^{2}\right)^{\frac{1}{3}}}{2 d}+\frac{i \sqrt{3}\, \left(-c\,d^{2}\right)^{\frac{1}{3}}}{2 d}}}\, \sqrt{-\frac{i \left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2 d}+\frac{i \sqrt{3}\, \left(-c\,d^{2}\right)^{\frac{1}{3}}}{2 d}\right) \sqrt{3}\, d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\, F\left(\right.}{3 d \sqrt{d\,x^{3}+c}}$
elliptic	$-\frac{2 i \sqrt{3}\, \left(-c\,d^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i \left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2 d}-\frac{i \sqrt{3}\, \left(-c\,d^{2}\right)^{\frac{1}{3}}}{2 d}\right) \sqrt{3}\, d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\, \sqrt{\frac{x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}}{-\frac{3 \left(-c\,d^{2}\right)^{\frac{1}{3}}}{2 d}+\frac{i \sqrt{3}\, \left(-c\,d^{2}\right)^{\frac{1}{3}}}{2 d}}}\, \sqrt{-\frac{i \left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2 d}+\frac{i \sqrt{3}\, \left(-c\,d^{2}\right)^{\frac{1}{3}}}{2 d}\right) \sqrt{3}\, d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\, F\left(\right.}{3 d \sqrt{d\,x^{3}+c}}$

```
[In] int((d*x^3+c)^(1/2)/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2
)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/3*
I/d^3*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/
3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*
(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^
(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^
2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+
```

```
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(
1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c
*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),
_alpha=RootOf(_Z^3*d+4*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2240 vs. $2(50) = 100$.

Time = 0.51 (sec) , antiderivative size = 2240, normalized size of antiderivative = 35.00

$$\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] 1/24*((1/108)^(1/6)*(sqrt(-3)*d + d)*(-1/(c*d^2))^(1/6)*log((d^3*x^9 - 66*c
*d^2*x^6 - 72*c^2*d*x^3 - 32*c^3 + 12*(1/4)^(2/3)*(c*d^4*x^8 - 7*c^2*d^3*x^
5 - 8*c^3*d^2*x^2 + sqrt(-3)*(c*d^4*x^8 - 7*c^2*d^3*x^5 - 8*c^3*d^2*x^2))*(
-1/(c*d^2))^(2/3) + 24*(1/4)^(1/3)*(c*d^3*x^7 - c^2*d^2*x^4 - 2*c^3*d*x - s
qrt(-3)*(c*d^3*x^7 - c^2*d^2*x^4 - 2*c^3*d*x))*(-1/(c*d^2))^(1/3) + 6*sqrt(
d*x^3 + c)*(18*(1/108)^(5/6)*(c*d^4*x^7 - 16*c^2*d^3*x^4 - 8*c^3*d^2*x - sq
rt(-3)*(c*d^4*x^7 - 16*c^2*d^3*x^4 - 8*c^3*d^2*x))*(-1/(c*d^2))^(5/6) - sqr
t(1/3)*(5*c*d^3*x^6 - 20*c^2*d^2*x^3 - 16*c^3*d)*sqrt(-1/(c*d^2)) + 9*(1/10
8)^(1/6)*(sqrt(-3)*c*d^2*x^5 + c*d^2*x^5)*(-1/(c*d^2))^(1/6)))/(d^3*x^9 + 1
2*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - (1/108)^(1/6)*(sqrt(-3)*d + d)*(-1/
(c*d^2))^(1/6)*log((d^3*x^9 - 66*c*d^2*x^6 - 72*c^2*d*x^3 - 32*c^3 + 12*(1/
4)^(2/3)*(c*d^4*x^8 - 7*c^2*d^3*x^5 - 8*c^3*d^2*x^2 + sqrt(-3)*(c*d^4*x^8 -
 7*c^2*d^3*x^5 - 8*c^3*d^2*x^2))*(-1/(c*d^2))^(2/3) + 24*(1/4)^(1/3)*(c*d^3
*x^7 - c^2*d^2*x^4 - 2*c^3*d*x - sqrt(-3)*(c*d^3*x^7 - c^2*d^2*x^4 - 2*c^3*
d*x))*(-1/(c*d^2))^(1/3) - 6*sqrt(d*x^3 + c)*(18*(1/108)^(5/6)*(c*d^4*x^7 -
 16*c^2*d^3*x^4 - 8*c^3*d^2*x - sqrt(-3)*(c*d^4*x^7 - 16*c^2*d^3*x^4 - 8*c^
3*d^2*x))*(-1/(c*d^2))^(5/6) - sqrt(1/3)*(5*c*d^3*x^6 - 20*c^2*d^2*x^3 - 16
*c^3*d)*sqrt(-1/(c*d^2)) + 9*(1/108)^(1/6)*(sqrt(-3)*c*d^2*x^5 + c*d^2*x^5)
*(-1/(c*d^2))^(1/6)))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - (
1/108)^(1/6)*(sqrt(-3)*d - d)*(-1/(c*d^2))^(1/6)*log((d^3*x^9 - 66*c*d^2*x^
6 - 72*c^2*d*x^3 - 32*c^3 + 12*(1/4)^(2/3)*(c*d^4*x^8 - 7*c^2*d^3*x^5 - 8*c
^3*d^2*x^2 - sqrt(-3)*(c*d^4*x^8 - 7*c^2*d^3*x^5 - 8*c^3*d^2*x^2))*(-1/(c*d
^2))^(2/3) + 24*(1/4)^(1/3)*(c*d^3*x^7 - c^2*d^2*x^4 - 2*c^3*d*x + sqrt(-3)
*(c*d^3*x^7 - c^2*d^2*x^4 - 2*c^3*d*x))*(-1/(c*d^2))^(1/3) + 6*sqrt(d*x^3 +
 c)*(18*(1/108)^(5/6)*(c*d^4*x^7 - 16*c^2*d^3*x^4 - 8*c^3*d^2*x + sqrt(-3)*
(c*d^4*x^7 - 16*c^2*d^3*x^4 - 8*c^3*d^2*x))*(-1/(c*d^2))^(5/6) - sqrt(1/3)*
(5*c*d^3*x^6 - 20*c^2*d^2*x^3 - 16*c^3*d)*sqrt(-1/(c*d^2)) - 9*(1/108)^(1/6
)*(sqrt(-3)*c*d^2*x^5 - c*d^2*x^5)*(-1/(c*d^2))^(1/6)))/(d^3*x^9 + 12*c*d^2
```

```
*x^6 + 48*c^2*d*x^3 + 64*c^3)) + (1/108)^(1/6)*(sqrt(-3)*d - d)*(-1/(c*d^2)
)^(1/6)*log((d^3*x^9 - 66*c*d^2*x^6 - 72*c^2*d*x^3 - 32*c^3 + 12*(1/4)^(2/3
)*(c*d^4*x^8 - 7*c^2*d^3*x^5 - 8*c^3*d^2*x^2 - sqrt(-3)*(c*d^4*x^8 - 7*c^2*
d^3*x^5 - 8*c^3*d^2*x^2))*(-1/(c*d^2))^(2/3) + 24*(1/4)^(1/3)*(c*d^3*x^7 -
c^2*d^2*x^4 - 2*c^3*d*x + sqrt(-3)*(c*d^3*x^7 - c^2*d^2*x^4 - 2*c^3*d*x))*(
-1/(c*d^2))^(1/3) - 6*sqrt(d*x^3 + c)*(18*(1/108)^(5/6)*(c*d^4*x^7 - 16*c^2
*d^3*x^4 - 8*c^3*d^2*x + sqrt(-3)*(c*d^4*x^7 - 16*c^2*d^3*x^4 - 8*c^3*d^2*x
))*(-1/(c*d^2))^(5/6) - sqrt(1/3)*(5*c*d^3*x^6 - 20*c^2*d^2*x^3 - 16*c^3*d)
*sqrt(-1/(c*d^2)) - 9*(1/108)^(1/6)*(sqrt(-3)*c*d^2*x^5 - c*d^2*x^5)*(-1/(c
*d^2))^(1/6)))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + 2*(1/108
)^(1/6)*d*(-1/(c*d^2))^(1/6)*log((d^3*x^9 - 66*c*d^2*x^6 - 72*c^2*d*x^3 - 3
2*c^3 - 24*(1/4)^(2/3)*(c*d^4*x^8 - 7*c^2*d^3*x^5 - 8*c^3*d^2*x^2)*(-1/(c*d
^2))^(2/3) - 48*(1/4)^(1/3)*(c*d^3*x^7 - c^2*d^2*x^4 - 2*c^3*d*x)*(-1/(c*d^
2))^(1/3) + 6*(18*(1/108)^(1/6)*c*d^2*x^5*(-1/(c*d^2))^(1/6) + 36*(1/108)^(
5/6)*(c*d^4*x^7 - 16*c^2*d^3*x^4 - 8*c^3*d^2*x)*(-1/(c*d^2))^(5/6) + sqrt(1
/3)*(5*c*d^3*x^6 - 20*c^2*d^2*x^3 - 16*c^3*d)*sqrt(-1/(c*d^2)))*sqrt(d*x^3
+ c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 2*(1/108)^(1/6)*d
*(-1/(c*d^2))^(1/6)*log((d^3*x^9 - 66*c*d^2*x^6 - 72*c^2*d*x^3 - 32*c^3 - 2
4*(1/4)^(2/3)*(c*d^4*x^8 - 7*c^2*d^3*x^5 - 8*c^3*d^2*x^2)*(-1/(c*d^2))^(2/3
) - 48*(1/4)^(1/3)*(c*d^3*x^7 - c^2*d^2*x^4 - 2*c^3*d*x)*(-1/(c*d^2))^(1/3)
 - 6*(18*(1/108)^(1/6)*c*d^2*x^5*(-1/(c*d^2))^(1/6) + 36*(1/108)^(5/6)*(c*d
^4*x^7 - 16*c^2*d^3*x^4 - 8*c^3*d^2*x)*(-1/(c*d^2))^(5/6) + sqrt(1/3)*(5*c*
d^3*x^6 - 20*c^2*d^2*x^3 - 16*c^3*d)*sqrt(-1/(c*d^2)))*sqrt(d*x^3 + c))/(d^
3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + 24*sqrt(d)*weierstrassPInv
erse(0, -4*c/d, x))/d
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx = \int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx$$

```
[In] integrate((d*x**3+c)**(1/2)/(d*x**3+4*c),x)
[Out] Integral(sqrt(c + d*x**3)/(4*c + d*x**3), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx = \int \frac{\sqrt{dx^3+c}}{dx^3+4\,c}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)/(d*x^3 + 4*c), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx = \int \frac{\sqrt{dx^3+c}}{dx^3+4\,c}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)/(d*x^3 + 4*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx = \int \frac{\sqrt{d\,x^3+c}}{d\,x^3+4\,c}\,dx$$

```
[In] int((c + d*x^3)^(1/2)/(4*c + d*x^3),x)
[Out] int((c + d*x^3)^(1/2)/(4*c + d*x^3), x)
```

3.269 $\int \frac{\sqrt{c+dx^3}}{x^3(4c+dx^3)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 66

$$\int \frac{\sqrt{c+dx^3}}{x^3(4c+dx^3)}\,dx=-\frac{\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{2}{3},1,-\frac{1}{2},\frac{1}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{8cx^2\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] -1/8*AppellF1(-2/3,-1/2,1,1/3,-d*x^3/c,-1/4*d*x^3/c)*(d*x^3+c)^(1/2)/c/x^2/
(1+d*x^3/c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {525, 524}

$$\int \frac{\sqrt{c+dx^3}}{x^3(4c+dx^3)}\,dx=-\frac{\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{2}{3},1,-\frac{1}{2},\frac{1}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{8cx^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[In] Int[Sqrt[c + d*x^3]/(x^3*(4*c + d*x^3)),x]
```

```
[Out] -1/8*(Sqrt[c + d*x^3]*AppellF1[-2/3, 1, -1/2, 1/3, -1/4*(d*x^3)/c, -((d*x^3
)/c)])/(c*x^2*Sqrt[1 + (d*x^3)/c])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c+dx^3}\int \frac{\sqrt{1+\frac{dx^3}{c}}}{x^3(4c+dx^3)}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$

$$= -\frac{\sqrt{c+dx^3}F_1\left(-\frac{2}{3};1,-\frac{1}{2};\frac{1}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{8cx^2\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 244 vs. $2(66) = 132$.

Time = 11.15 (sec) , antiderivative size = 244, normalized size of antiderivative = 3.70

$$\int \frac{\sqrt{c+dx^3}}{x^3(4c+dx^3)}\,dx$$

$$= \frac{-32c(c+dx^3) - d^2x^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right) + \frac{2048c^3 d\cdots}{(4c+dx^3)\left(16c\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)-\cdots\right.}}{256c^2x^2\sqrt{c+dx^3}}$$

```
[In] Integrate[Sqrt[c + d*x^3]/(x^3*(4*c + d*x^3)),x]
```

```
[Out] (-32*c*(c + d*x^3) - d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3,
 -((d*x^3)/c), -1/4*(d*x^3)/c] + (2048*c^3*d*x^3*AppellF1[1/3, 1/2, 1, 4/3,
 -((d*x^3)/c), -1/4*(d*x^3)/c])/((4*c + d*x^3)*(16*c*AppellF1[1/3, 1/2, 1,
4/3, -((d*x^3)/c), -1/4*(d*x^3)/c] - 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -(
(d*x^3)/c), -1/4*(d*x^3)/c] + 2*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -1
/4*(d*x^3)/c]))))/(256*c^2*x^2*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.64 (sec) , antiderivative size = 716, normalized size of antiderivative = 10.85

method	result
elliptic	$-\frac{\sqrt{d\,x^{3}+c}}{8c\,x^{2}}+\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}}}\sqrt{-\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)}{2d}\right.}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}}{24c\sqrt{d\,x^{3}+c}}$
risch	Expression too large to display
default	Expression too large to display

```
[In] int((d*x^3+c)^(1/2)/x^3/(d*x^3+4*c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(d*x^3+c)^(1/2)/c/x^2+1/24*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*
d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*
((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2))-1/12*I/d^2/c*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(
1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3)
)^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/
3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-
c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^
(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))
*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_
alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_a
lpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2361 vs. 2(52) = 104.

Time = 0.99 (sec) , antiderivative size = 2361, normalized size of antiderivative = 35.77

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(4c+dx^3\right)}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(d*x^3+4*c),x, algorithm="fricas")
[Out] -1/96*(2*(1/108)^(1/6)*c*x^2*(-d^4/c^7)^(1/6)*log((d^6*x^9 - 66*c*d^5*x^6 -
 72*c^2*d^4*x^3 - 32*c^3*d^3 - 24*(1/4)^(2/3)*(c^5*d^3*x^8 - 7*c^6*d^2*x^5
- 8*c^7*d*x^2)*(-d^4/c^7)^(2/3) - 48*(1/4)^(1/3)*(c^3*d^4*x^7 - c^4*d^3*x^4
 - 2*c^5*d^2*x)*(-d^4/c^7)^(1/3) + 6*(18*(1/108)^(1/6)*c^2*d^4*x^5*(-d^4/c^
7)^(1/6) + 36*(1/108)^(5/6)*(c^6*d^2*x^7 - 16*c^7*d*x^4 - 8*c^8*x)*(-d^4/c^
7)^(5/6) + sqrt(1/3)*(5*c^4*d^3*x^6 - 20*c^5*d^2*x^3 - 16*c^6*d)*sqrt(-d^4/
c^7))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) -
2*(1/108)^(1/6)*c*x^2*(-d^4/c^7)^(1/6)*log((d^6*x^9 - 66*c*d^5*x^6 - 72*c^2
*d^4*x^3 - 32*c^3*d^3 - 24*(1/4)^(2/3)*(c^5*d^3*x^8 - 7*c^6*d^2*x^5 - 8*c^7
*d*x^2)*(-d^4/c^7)^(2/3) - 48*(1/4)^(1/3)*(c^3*d^4*x^7 - c^4*d^3*x^4 - 2*c^
5*d^2*x)*(-d^4/c^7)^(1/3) - 6*(18*(1/108)^(1/6)*c^2*d^4*x^5*(-d^4/c^7)^(1/6
) + 36*(1/108)^(5/6)*(c^6*d^2*x^7 - 16*c^7*d*x^4 - 8*c^8*x)*(-d^4/c^7)^(5/6
) + sqrt(1/3)*(5*c^4*d^3*x^6 - 20*c^5*d^2*x^3 - 16*c^6*d)*sqrt(-d^4/c^7))*s
qrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 12*sqrt
(d)*x^2*weierstrassPInverse(0, -4*c/d, x) + (1/108)^(1/6)*(sqrt(-3)*c*x^2 +
 c*x^2)*(-d^4/c^7)^(1/6)*log((d^6*x^9 - 66*c*d^5*x^6 - 72*c^2*d^4*x^3 - 32*
c^3*d^3 + 12*(1/4)^(2/3)*(c^5*d^3*x^8 - 7*c^6*d^2*x^5 - 8*c^7*d*x^2 + sqrt(
-3)*(c^5*d^3*x^8 - 7*c^6*d^2*x^5 - 8*c^7*d*x^2))*(-d^4/c^7)^(2/3) + 24*(1/4
)^(1/3)*(c^3*d^4*x^7 - c^4*d^3*x^4 - 2*c^5*d^2*x - sqrt(-3)*(c^3*d^4*x^7 -
c^4*d^3*x^4 - 2*c^5*d^2*x))*(-d^4/c^7)^(1/3) + 6*sqrt(d*x^3 + c)*(18*(1/108
)^(5/6)*(c^6*d^2*x^7 - 16*c^7*d*x^4 - 8*c^8*x - sqrt(-3)*(c^6*d^2*x^7 - 16*
c^7*d*x^4 - 8*c^8*x))*(-d^4/c^7)^(5/6) - sqrt(1/3)*(5*c^4*d^3*x^6 - 20*c^5*
d^2*x^3 - 16*c^6*d)*sqrt(-d^4/c^7) + 9*(1/108)^(1/6)*(sqrt(-3)*c^2*d^4*x^5
+ c^2*d^4*x^5)*(-d^4/c^7)^(1/6)))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 +
64*c^3)) - (1/108)^(1/6)*(sqrt(-3)*c*x^2 + c*x^2)*(-d^4/c^7)^(1/6)*log((d^6
*x^9 - 66*c*d^5*x^6 - 72*c^2*d^4*x^3 - 32*c^3*d^3 + 12*(1/4)^(2/3)*(c^5*d^3
*x^8 - 7*c^6*d^2*x^5 - 8*c^7*d*x^2 + sqrt(-3)*(c^5*d^3*x^8 - 7*c^6*d^2*x^5
- 8*c^7*d*x^2))*(-d^4/c^7)^(2/3) + 24*(1/4)^(1/3)*(c^3*d^4*x^7 - c^4*d^3*x^
4 - 2*c^5*d^2*x - sqrt(-3)*(c^3*d^4*x^7 - c^4*d^3*x^4 - 2*c^5*d^2*x))*(-d^4
/c^7)^(1/3) - 6*sqrt(d*x^3 + c)*(18*(1/108)^(5/6)*(c^6*d^2*x^7 - 16*c^7*d*x
^4 - 8*c^8*x - sqrt(-3)*(c^6*d^2*x^7 - 16*c^7*d*x^4 - 8*c^8*x))*(-d^4/c^7)^
(5/6) - sqrt(1/3)*(5*c^4*d^3*x^6 - 20*c^5*d^2*x^3 - 16*c^6*d)*sqrt(-d^4/c^7
) + 9*(1/108)^(1/6)*(sqrt(-3)*c^2*d^4*x^5 + c^2*d^4*x^5)*(-d^4/c^7)^(1/6)))
/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - (1/108)^(1/6)*(sqrt(-3
)*c*x^2 - c*x^2)*(-d^4/c^7)^(1/6)*log((d^6*x^9 - 66*c*d^5*x^6 - 72*c^2*d^4*
```

```
x^3 - 32*c^3*d^3 + 12*(1/4)^(2/3)*(c^5*d^3*x^8 - 7*c^6*d^2*x^5 - 8*c^7*d*x^
2 - sqrt(-3)*(c^5*d^3*x^8 - 7*c^6*d^2*x^5 - 8*c^7*d*x^2))*(-d^4/c^7)^(2/3)
+ 24*(1/4)^(1/3)*(c^3*d^4*x^7 - c^4*d^3*x^4 - 2*c^5*d^2*x + sqrt(-3)*(c^3*d
^4*x^7 - c^4*d^3*x^4 - 2*c^5*d^2*x))*(-d^4/c^7)^(1/3) + 6*sqrt(d*x^3 + c)*(
18*(1/108)^(5/6)*(c^6*d^2*x^7 - 16*c^7*d*x^4 - 8*c^8*x + sqrt(-3)*(c^6*d^2*
x^7 - 16*c^7*d*x^4 - 8*c^8*x))*(-d^4/c^7)^(5/6) - sqrt(1/3)*(5*c^4*d^3*x^6
- 20*c^5*d^2*x^3 - 16*c^6*d)*sqrt(-d^4/c^7) - 9*(1/108)^(1/6)*(sqrt(-3)*c^2
*d^4*x^5 - c^2*d^4*x^5)*(-d^4/c^7)^(1/6)))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2
*d*x^3 + 64*c^3)) + (1/108)^(1/6)*(sqrt(-3)*c*x^2 - c*x^2)*(-d^4/c^7)^(1/6)
*log((d^6*x^9 - 66*c*d^5*x^6 - 72*c^2*d^4*x^3 - 32*c^3*d^3 + 12*(1/4)^(2/3)
*(c^5*d^3*x^8 - 7*c^6*d^2*x^5 - 8*c^7*d*x^2 - sqrt(-3)*(c^5*d^3*x^8 - 7*c^6
*d^2*x^5 - 8*c^7*d*x^2))*(-d^4/c^7)^(2/3) + 24*(1/4)^(1/3)*(c^3*d^4*x^7 - c
^4*d^3*x^4 - 2*c^5*d^2*x + sqrt(-3)*(c^3*d^4*x^7 - c^4*d^3*x^4 - 2*c^5*d^2*
x))*(-d^4/c^7)^(1/3) - 6*sqrt(d*x^3 + c)*(18*(1/108)^(5/6)*(c^6*d^2*x^7 - 1
6*c^7*d*x^4 - 8*c^8*x + sqrt(-3)*(c^6*d^2*x^7 - 16*c^7*d*x^4 - 8*c^8*x))*(-
d^4/c^7)^(5/6) - sqrt(1/3)*(5*c^4*d^3*x^6 - 20*c^5*d^2*x^3 - 16*c^6*d)*sqrt
(-d^4/c^7) - 9*(1/108)^(1/6)*(sqrt(-3)*c^2*d^4*x^5 - c^2*d^4*x^5)*(-d^4/c^7
)^(1/6)))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + 12*sqrt(d*x^3
 + c))/(c*x^2)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(4c+dx^3\right)}\,dx=\int \frac{\sqrt{c+dx^3}}{x^3\cdot\left(4c+dx^3\right)}\,dx$$

```
[In] integrate((d*x**3+c)**(1/2)/x**3/(d*x**3+4*c),x)
[Out] Integral(sqrt(c + d*x**3)/(x**3*(4*c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(4c+dx^3\right)}\,dx=\int \frac{\sqrt{dx^3+c}}{(dx^3+4\,c)x^3}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(d*x^3+4*c),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x^3), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^3\,(4c+dx^3)}\,dx = \int \frac{\sqrt{dx^3+c}}{(dx^3+4\,c)x^3}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(d*x^3+4*c),x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^3\,(4c+dx^3)}\,dx = \int \frac{\sqrt{d\,x^3+c}}{x^3\,\left(d\,x^3+4\,c\right)}\,dx$$

```
[In] int((c + d*x^3)^(1/2)/(x^3*(4*c + d*x^3)),x)
[Out] int((c + d*x^3)^(1/2)/(x^3*(4*c + d*x^3)), x)
```

3.270 $\int \frac{x^8}{\sqrt{c+dx^3}(4c+dx^3)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 78

$$\int \frac{x^8}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=-\frac{10c\sqrt{c+dx^3}}{3d^3}+\frac{2(c+dx^3)^{3/2}}{9d^3}+\frac{32c^{3/2}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}d^3}$$

```
[Out] 2/9*(d*x^3+c)^(3/2)/d^3+32/9*c^(3/2)*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(
1/2))/d^3*3^(1/2)-10/3*c*(d*x^3+c)^(1/2)/d^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {457, 90, 65, 209}

$$\int \frac{x^8}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\frac{32c^{3/2}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}d^3}-\frac{10c\sqrt{c+dx^3}}{3d^3}+\frac{2(c+dx^3)^{3/2}}{9d^3}$$

```
[In] Int[x^8/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (-10*c*Sqrt[c + d*x^3])/(3*d^3) + (2*(c + d*x^3)^(3/2))/(9*d^3) + (32*c^(3/
2)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*d^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{\sqrt{c+dx}(4c+dx)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{5c}{d^2\sqrt{c+dx}}+\frac{\sqrt{c+dx}}{d^2}+\frac{16c^2}{d^2\sqrt{c+dx}(4c+dx)}\right)dx, x, x^3\right)\\
&= -\frac{10c\sqrt{c+dx^3}}{3d^3}+\frac{2(c+dx^3)^{3/2}}{9d^3}+\frac{(16c^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}(4c+dx)}\,dx, x, x^3\right)}{3d^2}\\
&= -\frac{10c\sqrt{c+dx^3}}{3d^3}+\frac{2(c+dx^3)^{3/2}}{9d^3}+\frac{(32c^2)\,\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d^3}\\
&= -\frac{10c\sqrt{c+dx^3}}{3d^3}+\frac{2(c+dx^3)^{3/2}}{9d^3}+\frac{32c^{3/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}d^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.83

$$\int \frac{x^8}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{2(-14c+dx^3)\sqrt{c+dx^3}+32\sqrt{3}c^{3/2}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{9d^3}$$

[In] Integrate[x^8/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]

[Out] (2*(-14*c + d*x^3)*Sqrt[c + d*x^3] + 32*Sqrt[3]*c^(3/2)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(9*d^3)

Maple [A] (verified)

Time = 4.39 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$\frac{32c^{\frac{3}{2}}\sqrt{3}\,\arctan\left(\frac{\sqrt{d\,x^3+c}\,\sqrt{3}}{3\sqrt{c}}\right)-2\sqrt{d\,x^3+c}\,(-d\,x^3+14c)}{9d^3}$
risch	$-\frac{2(-d\,x^3+14c)\sqrt{d\,x^3+c}}{9d^3}+\frac{32c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{d\,x^3+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{9d^3}$
default	$\frac{\frac{2x^3\sqrt{d\,x^3+c}}{9d}-\frac{4c\sqrt{d\,x^3+c}}{9d^2}}{d}-\frac{8c\sqrt{d\,x^3+c}}{3d^3}+\frac{32c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{d\,x^3+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{9d^3}$
elliptic	$\frac{2x^3\sqrt{d\,x^3+c}}{9d^2}-\frac{28c\sqrt{d\,x^3+c}}{9d^3}-\frac{16ic\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3+4c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\cdots}{-3(-\cdots}}\cdots}{\cdots}\right)}{\cdots}$

[In] int(x^8/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/9*(32*c^(3/2)*3^(1/2)*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))-2*(d*x^3+c)^(1/2)*(-d*x^3+14*c))/d^3

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.65

$$\int \frac{x^8}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \left[\frac{2\left(8\sqrt{3}\sqrt{-c}c\log\left(\frac{dx^3+2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)+\sqrt{dx^3+c}(dx^3-14c)\right)}{9d^3}, \frac{2\left(16\sqrt{3}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)+\sqrt{}\right.}{9d^3}\right.$$

```
[In] integrate(x^8/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/9*(8*sqrt(3)*sqrt(-c)*c*log((d*x^3 + 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c)
- 2*c)/(d*x^3 + 4*c)) + sqrt(d*x^3 + c)*(d*x^3 - 14*c))/d^3, 2/9*(16*sqrt(3
)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) + sqrt(d*x^3 + c)*(d*
x^3 - 14*c))/d^3]
```

Sympy [A] (verification not implemented)

Time = 8.73 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int \frac{x^8}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \begin{cases} \frac{2\cdot\left(\frac{16\sqrt{3}c^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9}-\frac{5c\sqrt{c+dx^3}}{3}+\frac{(c+dx^3)^{\frac{3}{2}}}{9}\right)}{d^3} & \text{for } d\neq 0\\ \frac{x^9}{36c^{\frac{3}{2}}} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**8/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Piecewise((2*(16*sqrt(3)*c**(3/2)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c))
)/9 - 5*c*sqrt(c + d*x**3)/3 + (c + d*x**3)**(3/2)/9)/d**3, Ne(d, 0)), (x**
9/(36*c**(3/2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.68

$$\int \frac{x^8}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{2\left(16\sqrt{3}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)+(dx^3+c)^{\frac{3}{2}}-15\sqrt{dx^3+c}c\right)}{9d^3}$$

```
[In] integrate(x^8/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*(16*sqrt(3)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) + (d*x^
3 + c)^(3/2) - 15*sqrt(d*x^3 + c)*c)/d^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int \frac{x^8}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{32\,\sqrt{3}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)}{9\,d^3} + \frac{2\left((dx^3+c)^{\frac{3}{2}}d^6 - 15\,\sqrt{dx^3+c}cd^6\right)}{9\,d^9}$$

```
[In] integrate(x^8/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 32/9*sqrt(3)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d^3 + 2/9*
((d*x^3 + c)^(3/2)*d^6 - 15*sqrt(d*x^3 + c)*c*d^6)/d^9
```

Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.13

$$\int \frac{x^8}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d^2} - \frac{28\,c\,\sqrt{d\,x^3+c}}{9\,d^3} + \frac{\sqrt{3}\,c^{3/2}\,\ln\left(\frac{\sqrt{3}\,d\,x^3-2\,\sqrt{3}\,c+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)\,16\mathrm{i}}{9\,d^3}$$

```
[In] int(x^8/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

```
[Out] (2*x^3*(c + d*x^3)^(1/2))/(9*d^2) - (28*c*(c + d*x^3)^(1/2))/(9*d^3) + (3^(
1/2)*c^(3/2)*log((c^(1/2)*(c + d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3^(1/2)*d*x^
3)/(4*c + d*x^3))*16i)/(9*d^3)
```

3.271 $\int \frac{x^5}{\sqrt{c+dx^3}(4c+dx^3)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 59

$$\int \frac{x^5}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\frac{2\sqrt{c+dx^3}}{3d^2}-\frac{8\sqrt{c}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}d^2}$$

```
[Out] -8/9*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))*c^(1/2)/d^2*3^(1/2)+2/3*(d
*x^3+c)^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {457, 81, 65, 209}

$$\int \frac{x^5}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\frac{2\sqrt{c+dx^3}}{3d^2}-\frac{8\sqrt{c}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}d^2}$$

```
[In] Int[x^5/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*d^2) - (8*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sq
rt[c])])/(3*Sqrt[3]*d^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{\sqrt{c+dx}(4c+dx)}\,dx, x, x^3\right)\\
&= \frac{2\sqrt{c+dx^3}}{3d^2} - \frac{(4c)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}(4c+dx)}\,dx, x, x^3\right)}{3d}\\
&= \frac{2\sqrt{c+dx^3}}{3d^2} - \frac{(8c)\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d^2}\\
&= \frac{2\sqrt{c+dx^3}}{3d^2} - \frac{8\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}d^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{x^5}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\frac{6\sqrt{c+dx^3}-8\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{9d^2}$$

```
[In] Integrate[x^5/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
[Out] (6*Sqrt[c + d*x^3] - 8*Sqrt[3]*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt
[c])])/(9*d^2)
```

Maple [A] (verified)

Time = 4.43 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$\frac{-8\sqrt{c}\,\sqrt{3}\,\arctan\left(\frac{\sqrt{d\,x^3+c}\,\sqrt{3}}{3\sqrt{c}}\right)+6\sqrt{d\,x^3+c}}{9d^2}$
default	$-\frac{8\arctan\left(\frac{\sqrt{d\,x^3+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{c}\,\sqrt{3}}{9d^2}+\frac{2\sqrt{d\,x^3+c}}{3d^2}$
risch	$-\frac{8\arctan\left(\frac{\sqrt{d\,x^3+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{c}\,\sqrt{3}}{9d^2}+\frac{2\sqrt{d\,x^3+c}}{3d^2}$
elliptic	$\frac{2\sqrt{d\,x^3+c}}{3d^2}+\frac{4i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3+4c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)}}}{}\right)}{}$

```
[In] int(x^5/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/9*(-8*c^(1/2)*3^(1/2)*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))+6*(d*x^
3+c)^(1/2))/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.90

$$\int \frac{x^5}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \left[\frac{2\left(2\,\sqrt{3}\sqrt{-c}\log\left(\frac{dx^3-2\,\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2\,c}{dx^3+4\,c}\right)+3\,\sqrt{dx^3+c}\right)}{9\,d^2},\ -\frac{2\left(4\,\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)-3\,\sqrt{dx^3+c}\right)}{9\,d^2}\right]$$

```
[In] integrate(x^5/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/9*(2*sqrt(3)*sqrt(-c)*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) -
2*c)/(d*x^3 + 4*c)) + 3*sqrt(d*x^3 + c))/d^2, -2/9*(4*sqrt(3)*sqrt(c)*arcta
n(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - 3*sqrt(d*x^3 + c))/d^2]
```

Sympy [A] (verification not implemented)

Time = 4.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07

$$\int \frac{x^5}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \begin{cases} \frac{2\left(-\frac{4\sqrt{3}\sqrt{c}\,\text{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9}+\frac{\sqrt{c+dx^3}}{3}\right)}{d^2} & \text{for}\ d\neq 0 \\ \frac{x^6}{24c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**5/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Piecewise((2*(-4*sqrt(3)*sqrt(c)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))
/9 + sqrt(c + d*x**3)/3)/d**2, Ne(d, 0)), (x**6/(24*c**(3/2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int \frac{x^5}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = -\frac{2\left(4\,\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)-3\,\sqrt{dx^3+c}\right)}{9\,d^2}$$

```
[In] integrate(x^5/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/9*(4*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - 3*sqr
t(d*x^3 + c))/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \frac{x^5}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = -\frac{2\left(\frac{4\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{d} - \frac{3\sqrt{dx^3+c}}{d}\right)}{9\,d}$$

[In] integrate(x^5/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] -2/9*(4*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d - 3*sqrt(d*x^3 + c)/d)/d

Mupad [B] (verification not implemented)

Time = 9.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.20

$$\int \frac{x^5}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{2\sqrt{d\,x^3+c}}{3\,d^2} + \frac{\sqrt{3}\,\sqrt{c}\,\ln\left(\frac{2\sqrt{3}\,c-\sqrt{3}\,d\,x^3+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)\,4\mathrm{i}}{9\,d^2}$$

[In] int(x^5/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)

[Out] (2*(c + d*x^3)^(1/2))/(3*d^2) + (3^(1/2)*c^(1/2)*log((2*3^(1/2)*c + c^(1/2)*(c + d*x^3)^(1/2)*6i - 3^(1/2)*d*x^3)/(4*c + d*x^3))*4i)/(9*d^2)

3.272 $\int \frac{x^2}{\sqrt{c+dx^3}(4c+dx^3)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 40

$$\int \frac{x^2}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\frac{2\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\sqrt{c}d}$$

```
[Out] 2/9*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))/d*3^(1/2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {455, 65, 209}

$$\int \frac{x^2}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\frac{2\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\sqrt{c}d}$$

```
[In] Int[x^2/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (2*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*Sqrt[c]*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}(4c+dx)}\,dx, x, x^3\right)\\ &= \frac{2\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d}\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\sqrt{c}d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{2\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\sqrt{c}d}$$

```
[In] Integrate[x^2/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
[Out] (2*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*Sqrt[c]*d)
```

Maple [A] (verified)

Time = 4.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

method	result
default	$\frac{2\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{9d\sqrt{c}}$
pseudoelliptic	$\frac{2\arctan\left(\frac{\sqrt{d\,x^{3}+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{9d\sqrt{c}}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}+4c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\cdots\right)}{\cdots}}\cdots}{\cdots}\right)}{\cdots}$

[In] int(x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/9*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))/d*3^(1/2)/c^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.18

$$\int \frac{x^2}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \left[-\frac{\sqrt{3}\sqrt{-c}\log\left(\frac{dx^3-2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2\,c}{dx^3+4\,c}\right)}{9\,cd}, \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{9\sqrt{c}d}\right]$$

[In] integrate(x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] [-1/9*sqrt(3)*sqrt(-c)*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3 + 4*c))/(c*d), 2/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/(sqrt(c)*d)]

Sympy [A] (verification not implemented)

Time = 2.83 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \frac{x^2}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \begin{cases} \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}d} & \text{for } d \neq 0 \\ \frac{x^3}{12c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
[Out] Piecewise((2*sqrt(3)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))/(9*sqrt(c)*
d), Ne(d, 0)), (x**3/(12*c**(3/2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int \frac{x^2}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{9\sqrt{c}d}$$

```
[In] integrate(x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/(sqrt(c)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int \frac{x^2}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{9\sqrt{c}d}$$

```
[In] integrate(x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/(sqrt(c)*d)
```

Mupad [B] (verification not implemented)

Time = 8.42 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.40

$$\int \frac{x^2}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\frac{\sqrt{3}\,\ln\left(\frac{\sqrt{3}\,d\,x^3-2\,\sqrt{3}\,c+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{2\,d\,x^3+8\,c}\right)\,1\mathrm{i}}{9\,\sqrt{c}\,d}$$

```
[In] int(x^2/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

```
[Out] (3^(1/2)*log((c^(1/2)*(c + d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3^(1/2)*d*x^3)/(
8*c + 2*d*x^3))*1i)/(9*c^(1/2)*d)
```

3.273 $\int \frac{1}{x\sqrt{c+dx^3}(4c+dx^3)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 65

$$\int \frac{1}{x\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=-\frac{\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{6\sqrt{3}c^{3/2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6c^{3/2}}$$

```
[Out] -1/6*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-1/18*arctan(1/3*(d*x^3+c)^(1/
2)*3^(1/2)/c^(1/2))/c^(3/2)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {457, 88, 65, 214, 209}

$$\int \frac{1}{x\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=-\frac{\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{6\sqrt{3}c^{3/2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6c^{3/2}}$$

```
[In] Int[1/(x*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] -1/6*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])]/(Sqrt[3]*c^(3/2)) - ArcTanh[
Sqrt[c + d*x^3]/Sqrt[c]]/(6*c^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}(4c+dx)}\,dx,x,x^3\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{12c}-\frac{d\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}(4c+dx)}\,dx,x,x^3\right)}{12c}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx,x,\sqrt{c+dx^3}\right)}{6c}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{6cd}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{6\sqrt{3}c^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6c^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)+3\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{18c^{3/2}}$$

```
[In] Integrate[1/(x*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
[Out] -1/18*(Sqrt[3]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])] + 3*ArcTanh[Sqrt[c
 + d*x^3]/Sqrt[c]])/c^(3/2)
```

Maple [A] (verified)

Time = 4.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

method	result	size
pseudoelliptic	$-\frac{\arctan\left(\frac{\sqrt{d\,x^3+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}+3\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{18c^{\frac{3}{2}}}$	45
default	$-\frac{\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{6c^{\frac{3}{2}}}-\frac{\arctan\left(\frac{\sqrt{d\,x^3+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{18c^{\frac{3}{2}}}$	47
elliptic	Expression too large to display	1508

```
[In] int(1/x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/18*(arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))*3^(1/2)+3*arctanh((d*x^3
+c)^(1/2)/c^(1/2)))/c^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.28

$$\int \frac{1}{x\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \left[-\frac{2\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)-3\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)}{36c^2}, -\frac{\sqrt{3}\sqrt{-c}\log\left(\frac{dx^3+2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)-6\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)}{36c^2}\right]$$

```
[In] integrate(1/x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/36*(2*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - 3*s
qrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/c^2, -1/36*(sqrt
(3)*sqrt(-c)*log((d*x^3 + 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3
+ 4*c)) - 6*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c))/c^2]
```

Sympy [A] (verification not implemented)

Time = 3.45 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.23

$$\int \frac{1}{x\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \begin{cases} \frac{2\left(\frac{d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{12c\sqrt{-c}} - \frac{\sqrt{3}d\,\mathrm{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{36c^{\frac{3}{2}}}\right)}{d} & \text{for } d \neq 0 \\ \frac{\log(x^3)}{12c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/x/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Piecewise((2*(d*atan(sqrt(c + d*x**3)/sqrt(-c))/(12*c*sqrt(-c)) - sqrt(3)*d
*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))/(36*c**(3/2)))/d, Ne(d, 0)), (l
og(x**3)/(12*c**(3/2)), True))
```

Maxima [F]

$$\int \frac{1}{x\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{(dx^3+4\,c)\sqrt{dx^3+c}x}\,dx$$

```
[In] integrate(1/x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{18\,c^{\frac{3}{2}}} + \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{6\sqrt{-c}c}$$

```
[In] integrate(1/x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/18*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/c^(3/2) + 1/6*arc
tan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c)
```

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.45

$$\int \frac{1}{x\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{12\,c^{3/2}} + \frac{\sqrt{3}\,\ln\left(\frac{2\,\sqrt{3}\,c-\sqrt{3}\,d\,x^3+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)\,1\mathrm{i}}{36\,c^{3/2}}$$

```
[In] int(1/(x*(c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

```
[Out] log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(1
2*c^(3/2)) + (3^(1/2)*log((2*3^(1/2)*c + c^(1/2)*(c + d*x^3)^(1/2)*6i - 3^(
1/2)*d*x^3)/(4*c + d*x^3))*1i)/(36*c^(3/2))
```

3.274 $\int \frac{1}{x^4\sqrt{c+dx^3}(4c+dx^3)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \frac{1}{x^4\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = -\frac{\sqrt{c+dx^3}}{12c^2x^3}+\frac{d\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{24\sqrt{3}c^{5/2}}+\frac{d\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{8c^{5/2}}$$

```
[Out] 1/8*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)+1/72*d*arctan(1/3*(d*x^3+c)^
(1/2)*3^(1/2)/c^(1/2))/c^(5/2)*3^(1/2)-1/12*(d*x^3+c)^(1/2)/c^2/x^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {457, 105, 162, 65, 214, 209}

$$\int \frac{1}{x^4\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{d\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{24\sqrt{3}c^{5/2}}+\frac{d\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{8c^{5/2}}-\frac{\sqrt{c+dx^3}}{12c^2x^3}$$

```
[In] Int[1/(x^4*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] -1/12*Sqrt[c + d*x^3]/(c^2*x^3) + (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c
])])/(24*Sqrt[3]*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(8*c^(5/2)
)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2\sqrt{c+dx}(4c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{12c^2x^3} - \frac{\text{Subst}\left(\int \frac{3cd+\frac{d^2x}{2}}{x\sqrt{c+dx}(4c+dx)}\,dx, x, x^3\right)}{12c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{12c^2x^3} - \frac{d\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{16c^2} + \frac{d^2\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}(4c+dx)}\,dx, x, x^3\right)}{48c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{12c^2x^3} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{8c^2} + \frac{d\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right)}{24c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{12c^2x^3} + \frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{24\sqrt{3}c^{5/2}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{8c^{5/2}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = -\frac{\sqrt{c+dx^3}}{12c^2x^3} + \frac{d\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{24\sqrt{3}c^{5/2}} + \frac{d\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{8c^{5/2}}$$

```
[In] Integrate[1/(x^4*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] -1/12*Sqrt[c + d*x^3]/(c^2*x^3) + (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c
])])/(24*Sqrt[3]*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(8*c^(5/2)
)
```

Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

method	result	size
risch	$\frac{d\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{8c^{\frac{5}{2}}}+\frac{d\arctan\left(\frac{\sqrt{d\,x^3+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{72c^{\frac{5}{2}}}-\frac{\sqrt{d\,x^3+c}}{12c^2x^3}$	66
pseudoelliptic	$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{d\,x^3+c}\,\sqrt{3}}{3\sqrt{c}}\right)d\,x^3+9\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)d\,x^3-6\sqrt{d\,x^3+c}\,\sqrt{c}}{72c^{\frac{5}{2}}\,x^3}$	70
default	$\frac{-\frac{\sqrt{d\,x^3+c}}{3c\,x^3}+\frac{d\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}}{4c}+\frac{d\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{24c^{\frac{5}{2}}}+\frac{d\arctan\left(\frac{\sqrt{d\,x^3+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{3}}{72c^{\frac{5}{2}}}$	92
elliptic	Expression too large to display	1523

```
[In] int(1/x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)+1/72*d*arctan(1/3*(d*x^3+c)^
(1/2)*3^(1/2)/c^(1/2))/c^(5/2)*3^(1/2)-1/12*(d*x^3+c)^(1/2)/c^2/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.20

$$\int \frac{1}{x^4\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \left[\frac{2\sqrt{3}\sqrt{c}dx^3\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)+9\sqrt{c}dx^3\log\left(\frac{dx^3+2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)-12\sqrt{dx^3+c}c}{144c^3x^3}, -\frac{\sqrt{3}\sqrt{-c}dx^3\log\left(\frac{dx^3-2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)+18\sqrt{-c}dx^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+12\sqrt{dx^3+c}c}{144c^3x^3}\right]$$

```
[In] integrate(1/x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
[Out] [1/144*(2*sqrt(3)*sqrt(c)*d*x^3*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))
 + 9*sqrt(c)*d*x^3*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 12*
sqrt(d*x^3 + c)*c)/(c^3*x^3), -1/144*(sqrt(3)*sqrt(-c)*d*x^3*log((d*x^3 - 2
*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3 + 4*c)) + 18*sqrt(-c)*d*x^3
*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*sqrt(d*x^3 + c)*c)/(c^3*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{x^4\sqrt{c+dx^3}\cdot(4c+dx^3)}\,dx$$

```
[In] integrate(1/x**4/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(x**4*sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{(dx^3+4c)\sqrt{dx^3+c}x^4}\,dx$$

```
[In] integrate(1/x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^4\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{\sqrt{3}d\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{72\,c^{\frac{5}{2}}} - \frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{8\sqrt{-c}c^2} - \frac{\sqrt{dx^3+c}}{12\,c^2x^3}$$

```
[In] integrate(1/x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/72*sqrt(3)*d*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/c^(5/2) - 1/8*d*
arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/12*sqrt(d*x^3 + c)/(c^2
*x^3)
```

Mupad [B] (verification not implemented)

Time = 9.50 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.27

$$\int \frac{1}{x^4\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{d\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)^3}{x^6}\right)}{16\,c^{5/2}} - \frac{\sqrt{d\,x^3+c}}{12\,c^2\,x^3} + \frac{\sqrt{3}\,d\,\ln\left(\frac{\sqrt{3}\,d\,x^3-2\,\sqrt{3}\,c+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)\,1\mathrm{i}}{144\,c^{5/2}}$$

```
[In] int(1/(x^4*(c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

```
[Out] (d*log((((c + d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2) + c^(1/2))^3)/x^6)
)/(16*c^(5/2)) - (c + d*x^3)^(1/2)/(12*c^2*x^3) + (3^(1/2)*d*log((c^(1/2)*(
c + d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3^(1/2)*d*x^3)/(4*c + d*x^3))*1i)/(144*
c^(5/2))
```

3.275 $\int \frac{x^4}{\sqrt{c+dx^3}(4c+dx^3)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 667

$$
\begin{aligned}
\int \frac{x^4}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx &= \frac{2\sqrt{c+dx^3}}{d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} \\
&+ \frac{2\sqrt[3]{2}\sqrt[6]{c}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{5/3}} - \frac{2\sqrt[3]{2}\sqrt[6]{c}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}d^{5/3}} \\
&+ \frac{2\sqrt[3]{2}\sqrt[6]{c}\,\text{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3d^{5/3}} - \frac{2\sqrt[3]{2}\sqrt[6]{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{9d^{5/3}} \\
&- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+ \frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}
$$

```
[Out] 2/3*2^(1/3)*c^(1/6)*arctanh(c^(1/6)*(c^(1/3)-2^(1/3)*d^(1/3)*x)/(d*x^3+c)^(
1/2))/d^(5/3)-2/9*2^(1/3)*c^(1/6)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/d^(5/3)+
```

```
2/9*2^(1/3)*c^(1/6)*arctan(c^(1/6)*(c^(1/3)+2^(1/3)*d^(1/3)*x)*3^(1/2)/(d*x
^3+c)^(1/2))/d^(5/3)*3^(1/2)-2/9*2^(1/3)*c^(1/6)*arctan(1/3*(d*x^3+c)^(1/2)
*3^(1/2)/c^(1/2))/d^(5/3)*3^(1/2)+2*(d*x^3+c)^(1/2)/d^(5/3)/(d^(1/3)*x+c^(1
/3)*(1+3^(1/2)))+2/3*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/
3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^
(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/
2)*3^(3/4)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+
c^(1/3)*(1+3^(1/2)))^2)^(1/2)-3^(1/4)*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE
((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+
2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^
(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(
1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 667, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used

$= \{494, 309, 224, 1891, 497\}$

$$\int \frac{x^4}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$$

$$= \frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{2\sqrt[3]{2}\sqrt[6]{c}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{5/3}}-\frac{2\sqrt[3]{2}\sqrt[6]{c}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}d^{5/3}}$$

$$+\frac{2\sqrt[3]{2}\sqrt[6]{c}\,\text{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3d^{5/3}}$$

$$-\frac{2\sqrt[3]{2}\sqrt[6]{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{9d^{5/3}}+\frac{2\sqrt{c+dx^3}}{d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

```
[In] Int[x^4/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(d^(5/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (2*2^(1
/3)*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c +
 d*x^3]])/(3*Sqrt[3]*d^(5/3)) - (2*2^(1/3)*c^(1/6)*ArcTan[Sqrt[c + d*x^3]/(
Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*d^(5/3)) + (2*2^(1/3)*c^(1/6)*ArcTanh[(c^(1/6
)*(c^(1/3) - 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(3*d^(5/3)) - (2*2^(1/3)
*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(9*d^(5/3)) - (3^(1/4)*Sqrt[2 -
Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x +
d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 -
Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*
Sqrt[3]])/(d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1
/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3
)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1
```

```
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(5/3)*Sqrt[(c
^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c
 + d*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 497

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[q*(ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]]/(9*2^(2/3)*b
*Rt[c, 2])), x] + (-Simp[q*(ArcTanh[Rt[c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c + d*
x^3])]/(3*2^(2/3)*b*Rt[c, 2])), x] + Simp[q*(ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])), x] - Simp[q*(ArcTan[Sqrt[3]*R
t[c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
```

```
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{x}{\sqrt{c+dx^3}}\,dx}{d} - \frac{(4c)\int \frac{x}{\sqrt{c+dx^3}(4c+dx^3)}\,dx}{d}$$

$$= \frac{2\sqrt[3]{2}\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{5/3}} - \frac{2\sqrt[3]{2}\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}d^{5/3}} + \frac{2\sqrt[3]{2}\sqrt[6]{c}\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3d^{5/3}} - \frac{2\sqrt[3]{2}\sqrt[6]{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{9d^{5/3}} + \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{d^{4/3}} - \frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{c}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{d^{4/3}}$$

$$= \frac{2\sqrt{c+dx^3}}{d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{2\sqrt[3]{2}\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{5/3}} - \frac{2\sqrt[3]{2}\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}d^{5/3}} + \frac{2\sqrt[3]{2}\sqrt[6]{c}\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3d^{5/3}} - \frac{2\sqrt[3]{2}\sqrt[6]{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{9d^{5/3}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.10

$$\int \frac{x^4}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{x^5\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{20c\sqrt{c+dx^3}}$$

```
[In] Integrate[x^4/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (x^5*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -1/4*(d*x
^3)/c])/(20*c*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.38 (sec) , antiderivative size = 848, normalized size of antiderivative = 1.27

method	result	size
default	Expression too large to display	848
elliptic	Expression too large to display	848

```
[In] int(x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+4/9*I/d^4*2^(1/2)*sum(
1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/
3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1
/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)
*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)
-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I
*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2
```

```
)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d
+4*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.64 (sec) , antiderivative size = 2268, normalized size of antiderivative = 3.40

$$\int \frac{x^4}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/18*(2*(4/27)^(1/6)*d^2*(-c/d^10)^(1/6)*log(32*(9*(4/27)^(5/6)*(d^11*x^9 -
 66*c*d^10*x^6 - 72*c^2*d^9*x^3 - 32*c^3*d^8)*(-c/d^10)^(5/6) - 96*sqrt(1/3
)*(c*d^7*x^7 - c^2*d^6*x^4 - 2*c^3*d^5*x)*sqrt(-c/d^10) + 4*(9*4^(2/3)*c*d^
8*x^5*(-c/d^10)^(2/3) + 2*c*d^2*x^7 - 32*c^2*d*x^4 - 16*c^3*x + 4^(1/3)*(5*
c*d^5*x^6 - 20*c^2*d^4*x^3 - 16*c^3*d^3)*(-c/d^10)^(1/3))*sqrt(d*x^3 + c) -
 24*(4/27)^(1/6)*(c*d^4*x^8 - 7*c^2*d^3*x^5 - 8*c^3*d^2*x^2)*(-c/d^10)^(1/6
))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 2*(4/27)^(1/6)*d^2*(
-c/d^10)^(1/6)*log(-32*(9*(4/27)^(5/6)*(d^11*x^9 - 66*c*d^10*x^6 - 72*c^2*d
^9*x^3 - 32*c^3*d^8)*(-c/d^10)^(5/6) - 96*sqrt(1/3)*(c*d^7*x^7 - c^2*d^6*x^
4 - 2*c^3*d^5*x)*sqrt(-c/d^10) - 4*(9*4^(2/3)*c*d^8*x^5*(-c/d^10)^(2/3) + 2
*c*d^2*x^7 - 32*c^2*d*x^4 - 16*c^3*x + 4^(1/3)*(5*c*d^5*x^6 - 20*c^2*d^4*x^
3 - 16*c^3*d^3)*(-c/d^10)^(1/3))*sqrt(d*x^3 + c) - 24*(4/27)^(1/6)*(c*d^4*x
^8 - 7*c^2*d^3*x^5 - 8*c^3*d^2*x^2)*(-c/d^10)^(1/6))/(d^3*x^9 + 12*c*d^2*x^
6 + 48*c^2*d*x^3 + 64*c^3)) - (4/27)^(1/6)*(sqrt(-3)*d^2 - d^2)*(-c/d^10)^(
1/6)*log(32*(9*(4/27)^(5/6)*(d^11*x^9 - 66*c*d^10*x^6 - 72*c^2*d^9*x^3 - 32
*c^3*d^8 + sqrt(-3)*(d^11*x^9 - 66*c*d^10*x^6 - 72*c^2*d^9*x^3 - 32*c^3*d^8
))*(-c/d^10)^(5/6) + 192*sqrt(1/3)*(c*d^7*x^7 - c^2*d^6*x^4 - 2*c^3*d^5*x)*
sqrt(-c/d^10) + 4*(4*c*d^2*x^7 - 64*c^2*d*x^4 - 32*c^3*x + 9*4^(2/3)*(sqrt(
-3)*c*d^8*x^5 - c*d^8*x^5)*(-c/d^10)^(2/3) - 4^(1/3)*(5*c*d^5*x^6 - 20*c^2*
d^4*x^3 - 16*c^3*d^3 + sqrt(-3)*(5*c*d^5*x^6 - 20*c^2*d^4*x^3 - 16*c^3*d^3)
)*(-c/d^10)^(1/3))*sqrt(d*x^3 + c) - 24*(4/27)^(1/6)*(c*d^4*x^8 - 7*c^2*d^3
*x^5 - 8*c^3*d^2*x^2 - sqrt(-3)*(c*d^4*x^8 - 7*c^2*d^3*x^5 - 8*c^3*d^2*x^2)
)*(-c/d^10)^(1/6))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + (4/2
7)^(1/6)*(sqrt(-3)*d^2 - d^2)*(-c/d^10)^(1/6)*log(-32*(9*(4/27)^(5/6)*(d^11
*x^9 - 66*c*d^10*x^6 - 72*c^2*d^9*x^3 - 32*c^3*d^8 + sqrt(-3)*(d^11*x^9 - 6
6*c*d^10*x^6 - 72*c^2*d^9*x^3 - 32*c^3*d^8))*(-c/d^10)^(5/6) + 192*sqrt(1/3
)*(c*d^7*x^7 - c^2*d^6*x^4 - 2*c^3*d^5*x)*sqrt(-c/d^10) - 4*(4*c*d^2*x^7 -
64*c^2*d*x^4 - 32*c^3*x + 9*4^(2/3)*(sqrt(-3)*c*d^8*x^5 - c*d^8*x^5)*(-c/d^
10)^(2/3) - 4^(1/3)*(5*c*d^5*x^6 - 20*c^2*d^4*x^3 - 16*c^3*d^3 + sqrt(-3)*(
5*c*d^5*x^6 - 20*c^2*d^4*x^3 - 16*c^3*d^3))*(-c/d^10)^(1/3))*sqrt(d*x^3 + c
```

```
) - 24*(4/27)^(1/6)*(c*d^4*x^8 - 7*c^2*d^3*x^5 - 8*c^3*d^2*x^2 - sqrt(-3)*(
c*d^4*x^8 - 7*c^2*d^3*x^5 - 8*c^3*d^2*x^2))*(-c/d^10)^(1/6))/(d^3*x^9 + 12*
c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + (4/27)^(1/6)*(sqrt(-3)*d^2 + d^2)*(-c
/d^10)^(1/6)*log(32*(9*(4/27)^(5/6)*(d^11*x^9 - 66*c*d^10*x^6 - 72*c^2*d^9*
x^3 - 32*c^3*d^8 - sqrt(-3)*(d^11*x^9 - 66*c*d^10*x^6 - 72*c^2*d^9*x^3 - 32
*c^3*d^8))*(-c/d^10)^(5/6) + 192*sqrt(1/3)*(c*d^7*x^7 - c^2*d^6*x^4 - 2*c^3
*d^5*x)*sqrt(-c/d^10) + 4*(4*c*d^2*x^7 - 64*c^2*d*x^4 - 32*c^3*x - 9*4^(2/3
)*(sqrt(-3)*c*d^8*x^5 + c*d^8*x^5)*(-c/d^10)^(2/3) - 4^(1/3)*(5*c*d^5*x^6 -
 20*c^2*d^4*x^3 - 16*c^3*d^3 - sqrt(-3)*(5*c*d^5*x^6 - 20*c^2*d^4*x^3 - 16*
c^3*d^3))*(-c/d^10)^(1/3))*sqrt(d*x^3 + c) - 24*(4/27)^(1/6)*(c*d^4*x^8 - 7
*c^2*d^3*x^5 - 8*c^3*d^2*x^2 + sqrt(-3)*(c*d^4*x^8 - 7*c^2*d^3*x^5 - 8*c^3*
d^2*x^2))*(-c/d^10)^(1/6))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)
) - (4/27)^(1/6)*(sqrt(-3)*d^2 + d^2)*(-c/d^10)^(1/6)*log(-32*(9*(4/27)^(5/
6)*(d^11*x^9 - 66*c*d^10*x^6 - 72*c^2*d^9*x^3 - 32*c^3*d^8 - sqrt(-3)*(d^11
*x^9 - 66*c*d^10*x^6 - 72*c^2*d^9*x^3 - 32*c^3*d^8))*(-c/d^10)^(5/6) + 192*
sqrt(1/3)*(c*d^7*x^7 - c^2*d^6*x^4 - 2*c^3*d^5*x)*sqrt(-c/d^10) - 4*(4*c*d^
2*x^7 - 64*c^2*d*x^4 - 32*c^3*x - 9*4^(2/3)*(sqrt(-3)*c*d^8*x^5 + c*d^8*x^5
)*(-c/d^10)^(2/3) - 4^(1/3)*(5*c*d^5*x^6 - 20*c^2*d^4*x^3 - 16*c^3*d^3 - sq
rt(-3)*(5*c*d^5*x^6 - 20*c^2*d^4*x^3 - 16*c^3*d^3))*(-c/d^10)^(1/3))*sqrt(d
*x^3 + c) - 24*(4/27)^(1/6)*(c*d^4*x^8 - 7*c^2*d^3*x^5 - 8*c^3*d^2*x^2 + sq
rt(-3)*(c*d^4*x^8 - 7*c^2*d^3*x^5 - 8*c^3*d^2*x^2))*(-c/d^10)^(1/6))/(d^3*x
^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 36*sqrt(d)*weierstrassZeta(0,
 -4*c/d, weierstrassPInverse(0, -4*c/d, x)))/d^2
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{x^4}{\sqrt{c+dx^3}\cdot(4c+dx^3)}\,dx$$

```
[In] integrate(x**4/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
[Out] Integral(x**4/(sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{x^4}{(dx^3+4\,c)\sqrt{dx^3+c}}\,dx$$

```
[In] integrate(x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^4/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{x^4}{(dx^3+4\,c)\sqrt{dx^3+c}}\,dx$$

```
[In] integrate(x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{x^4}{\sqrt{d\,x^3+c}\,(d\,x^3+4\,c)}\,dx$$

```
[In] int(x^4/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
[Out] int(x^4/((c + d*x^3)^(1/2)*(4*c + d*x^3)), x)
```

3.276 $\int \frac{x}{\sqrt{c+dx^3}(4c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 206

$$\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = -\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}+\frac{\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}c^{5/6}d^{2/3}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{9\ 2^{2/3}c^{5/6}d^{2/3}}$$

```
[Out] -1/6*arctanh(c^(1/6)*(c^(1/3)-2^(1/3)*d^(1/3)*x)/(d*x^3+c)^(1/2))*2^(1/3)/c
^(5/6)/d^(2/3)+1/18*arctanh((d*x^3+c)^(1/2)/c^(1/2))*2^(1/3)/c^(5/6)/d^(2/3
)-1/18*arctan(c^(1/6)*(c^(1/3)+2^(1/3)*d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*
2^(1/3)/c^(5/6)/d^(2/3)*3^(1/2)+1/18*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(
1/2))*2^(1/3)/c^(5/6)/d^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used

$= \{497\}$

$$\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = -\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}} + \frac{\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}c^{5/6}d^{2/3}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{9\ 2^{2/3}c^{5/6}d^{2/3}}$$

```
[In] Int[x/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] -1/3*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]
]/(2^(2/3)*Sqrt[3]*c^(5/6)*d^(2/3)) + ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[
c])]/(3*2^(2/3)*Sqrt[3]*c^(5/6)*d^(2/3)) - ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1
/3)*d^(1/3)*x))/Sqrt[c + d*x^3]]/(3*2^(2/3)*c^(5/6)*d^(2/3)) + ArcTanh[Sqrt
[c + d*x^3]/Sqrt[c]]/(9*2^(2/3)*c^(5/6)*d^(2/3))
```

Rule 497

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[q*(ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]]/(9*2^(2/3)*b
*Rt[c, 2])), x] + (-Simp[q*(ArcTanh[Rt[c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c + d*
x^3])]/(3*2^(2/3)*b*Rt[c, 2])), x] + Simp[q*(ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])), x] - Simp[q*(ArcTan[Sqrt[3]*R
t[c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rubi steps

$$\text{integral} = -\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}} + \frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}c^{5/6}d^{2/3}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{9\ 2^{2/3}c^{5/6}d^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.33

$$\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\frac{x^2\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{8c\sqrt{c+dx^3}}$$

```
[In] Integrate[x/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -1/4*(d*x
^3)/c])/(8*c*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.35 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.02

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3+4c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-c\cdots\right.}{}\right.}{2\left(-c\cdots\right.}}\cdots}{\cdots}\cdots\right)}{\cdots}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3+4c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-c\cdots\right.}{}\right.}{2\left(-c\cdots\right.}}\cdots}{\cdots}\cdots\right)}{\cdots}$

```
[In] int(x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*I/d^3/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1
/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2
)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x
+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^
3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_a
lpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(
I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3
)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/
d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1
/2)),_alpha=RootOf(_Z^3*d+4*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2289 vs. $2(141) = 282$.

Time = 0.67 (sec) , antiderivative size = 2289, normalized size of antiderivative = 11.11

$$\int \frac{x}{\sqrt{c + dx^3}\,(4c + dx^3)}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/36*(1/432)^(1/6)*(sqrt(-3) + 1)*(-1/(c^5*d^4))^(1/6)*log((d^3*x^9 - 66*c
*d^2*x^6 - 72*c^2*d*x^3 - 32*c^3 - 24*(1/2)^(2/3)*(c^4*d^5*x^7 - c^5*d^4*x^
4 - 2*c^6*d^3*x + sqrt(-3)*(c^4*d^5*x^7 - c^5*d^4*x^4 - 2*c^6*d^3*x))*(-1/(
c^5*d^4))^(2/3) - 6*(1/2)^(1/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^
2 - sqrt(-3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2))*(-1/(c^5*d^4))^
(1/3) + 6*sqrt(d*x^3 + c)*(648*(1/432)^(5/6)*(sqrt(-3)*c^5*d^5*x^5 - c^5*d^
5*x^5)*(-1/(c^5*d^4))^(5/6) + sqrt(1/3)*(5*c^3*d^4*x^6 - 20*c^4*d^3*x^3 - 1
6*c^5*d^2)*sqrt(-1/(c^5*d^4)) - (1/432)^(1/6)*(c*d^3*x^7 - 16*c^2*d^2*x^4 -
 8*c^3*d*x + sqrt(-3)*(c*d^3*x^7 - 16*c^2*d^2*x^4 - 8*c^3*d*x))*(-1/(c^5*d^
4))^(1/6)))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + 1/36*(1/432
)^(1/6)*(sqrt(-3) + 1)*(-1/(c^5*d^4))^(1/6)*log((d^3*x^9 - 66*c*d^2*x^6 - 7
2*c^2*d*x^3 - 32*c^3 - 24*(1/2)^(2/3)*(c^4*d^5*x^7 - c^5*d^4*x^4 - 2*c^6*d^
3*x + sqrt(-3)*(c^4*d^5*x^7 - c^5*d^4*x^4 - 2*c^6*d^3*x))*(-1/(c^5*d^4))^(2
/3) - 6*(1/2)^(1/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2 - sqrt(-3)
*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2))*(-1/(c^5*d^4))^(1/3) - 6*sq
rt(d*x^3 + c)*(648*(1/432)^(5/6)*(sqrt(-3)*c^5*d^5*x^5 - c^5*d^5*x^5)*(-1/(
c^5*d^4))^(5/6) + sqrt(1/3)*(5*c^3*d^4*x^6 - 20*c^4*d^3*x^3 - 16*c^5*d^2)*s
qrt(-1/(c^5*d^4)) - (1/432)^(1/6)*(c*d^3*x^7 - 16*c^2*d^2*x^4 - 8*c^3*d*x +
 sqrt(-3)*(c*d^3*x^7 - 16*c^2*d^2*x^4 - 8*c^3*d*x))*(-1/(c^5*d^4))^(1/6)))/
(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 1/36*(1/432)^(1/6)*(sqr
t(-3) - 1)*(-1/(c^5*d^4))^(1/6)*log((d^3*x^9 - 66*c*d^2*x^6 - 72*c^2*d*x^3
- 32*c^3 - 24*(1/2)^(2/3)*(c^4*d^5*x^7 - c^5*d^4*x^4 - 2*c^6*d^3*x - sqrt(-
3)*(c^4*d^5*x^7 - c^5*d^4*x^4 - 2*c^6*d^3*x))*(-1/(c^5*d^4))^(2/3) - 6*(1/2
)^(1/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2 + sqrt(-3)*(c^2*d^4*x^
8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2))*(-1/(c^5*d^4))^(1/3) + 6*sqrt(d*x^3 + c
)*(648*(1/432)^(5/6)*(sqrt(-3)*c^5*d^5*x^5 + c^5*d^5*x^5)*(-1/(c^5*d^4))^(5
/6) - sqrt(1/3)*(5*c^3*d^4*x^6 - 20*c^4*d^3*x^3 - 16*c^5*d^2)*sqrt(-1/(c^5*
d^4)) + (1/432)^(1/6)*(c*d^3*x^7 - 16*c^2*d^2*x^4 - 8*c^3*d*x - sqrt(-3)*(c
*d^3*x^7 - 16*c^2*d^2*x^4 - 8*c^3*d*x))*(-1/(c^5*d^4))^(1/6)))/(d^3*x^9 + 1
2*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + 1/36*(1/432)^(1/6)*(sqrt(-3) - 1)*(
-1/(c^5*d^4))^(1/6)*log((d^3*x^9 - 66*c*d^2*x^6 - 72*c^2*d*x^3 - 32*c^3 - 2
4*(1/2)^(2/3)*(c^4*d^5*x^7 - c^5*d^4*x^4 - 2*c^6*d^3*x - sqrt(-3)*(c^4*d^5*
x^7 - c^5*d^4*x^4 - 2*c^6*d^3*x))*(-1/(c^5*d^4))^(2/3) - 6*(1/2)^(1/3)*(c^2
*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2 + sqrt(-3)*(c^2*d^4*x^8 - 7*c^3*d^
3*x^5 - 8*c^4*d^2*x^2))*(-1/(c^5*d^4))^(1/3) - 6*sqrt(d*x^3 + c)*(648*(1/43
```

```
2)^(5/6)*(sqrt(-3)*c^5*d^5*x^5 + c^5*d^5*x^5)*(-1/(c^5*d^4))^(5/6) - sqrt(1
/3)*(5*c^3*d^4*x^6 - 20*c^4*d^3*x^3 - 16*c^5*d^2)*sqrt(-1/(c^5*d^4)) + (1/4
32)^(1/6)*(c*d^3*x^7 - 16*c^2*d^2*x^4 - 8*c^3*d*x - sqrt(-3)*(c*d^3*x^7 - 1
6*c^2*d^2*x^4 - 8*c^3*d*x))*(-1/(c^5*d^4))^(1/6)))/(d^3*x^9 + 12*c*d^2*x^6
+ 48*c^2*d*x^3 + 64*c^3)) + 1/18*(1/432)^(1/6)*(-1/(c^5*d^4))^(1/6)*log((d^
3*x^9 - 66*c*d^2*x^6 - 72*c^2*d*x^3 - 32*c^3 + 48*(1/2)^(2/3)*(c^4*d^5*x^7
- c^5*d^4*x^4 - 2*c^6*d^3*x)*(-1/(c^5*d^4))^(2/3) + 12*(1/2)^(1/3)*(c^2*d^4
*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(-1/(c^5*d^4))^(1/3) + 6*(1296*(1/432
)^(5/6)*c^5*d^5*x^5*(-1/(c^5*d^4))^(5/6) + sqrt(1/3)*(5*c^3*d^4*x^6 - 20*c^
4*d^3*x^3 - 16*c^5*d^2)*sqrt(-1/(c^5*d^4)) + 2*(1/432)^(1/6)*(c*d^3*x^7 - 1
6*c^2*d^2*x^4 - 8*c^3*d*x)*(-1/(c^5*d^4))^(1/6))*sqrt(d*x^3 + c))/(d^3*x^9
+ 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 1/18*(1/432)^(1/6)*(-1/(c^5*d^4)
)^(1/6)*log((d^3*x^9 - 66*c*d^2*x^6 - 72*c^2*d*x^3 - 32*c^3 + 48*(1/2)^(2/3
)*(c^4*d^5*x^7 - c^5*d^4*x^4 - 2*c^6*d^3*x)*(-1/(c^5*d^4))^(2/3) + 12*(1/2)
^(1/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(-1/(c^5*d^4))^(1/3) -
 6*(1296*(1/432)^(5/6)*c^5*d^5*x^5*(-1/(c^5*d^4))^(5/6) + sqrt(1/3)*(5*c^3*
d^4*x^6 - 20*c^4*d^3*x^3 - 16*c^5*d^2)*sqrt(-1/(c^5*d^4)) + 2*(1/432)^(1/6)
*(c*d^3*x^7 - 16*c^2*d^2*x^4 - 8*c^3*d*x)*(-1/(c^5*d^4))^(1/6))*sqrt(d*x^3
+ c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3))
```

Sympy [F]

$$\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\int \frac{x}{\sqrt{c+dx^3}\cdot(4c+dx^3)}\,dx$$

```
[In] integrate(x/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
[Out] Integral(x/(sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\int \frac{x}{(dx^3+4\,c)\sqrt{dx^3+c}}\,dx$$

```
[In] integrate(x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\int \frac{x}{(dx^3+4\,c)\sqrt{dx^3+c}}\,dx$$

```
[In] integrate(x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

Mupad [B] (verification not implemented)

Time = 29.70 (sec) , antiderivative size = 453, normalized size of antiderivative = 2.20

$$\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{\sqrt{3}\,314928^{1/3}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}+\sqrt{3}\,\sqrt{-c}-2^{1/3}\,\sqrt{3}\,(-c)^{1/6}\,d^{1/3}\,x\right)^3\left(54\,\sqrt{d\,x^3+c}-54\,\sqrt{3}\,\sqrt{-c}+54\,2^{1/3}\,\sqrt{3}\,(-c)^{1/6}\,d^{1/3}\,x\right)}{\left(d^{1/3}\,x-2^{2/3}\,(-c)^{1/3}\right)^6}\right)}{2916\,(-c)^{5/6}\,d^{2/3}} + \frac{\sqrt{3}\,314928^{1/3}\,\ln\left(\frac{\left(2\,\sqrt{3}\,\sqrt{-c}-2\,\sqrt{d\,x^3+c}+2^{1/3}\,\sqrt{3}\,(-c)^{1/6}\,d^{1/3}\,x+2^{1/3}\,(-c)^{1/6}\,d^{1/3}\,x\,3\mathrm{i}\right)^3\left(108\,\sqrt{d\,x^3+c}+108\,\sqrt{3}\,\sqrt{-c}+54\,2^{1/3}\,\sqrt{3}\,(-\cdots\right.}{\left(2\,d^{1/3}\,x+2^{2/3}\,(-c)^{1/3}-2^{2/3}\,\sqrt{3}\,(-c)^{1/3}\,1\mathrm{i}\right)^6}\cdots}{2916\,(-c)^{5/6}\,d^{2/3}} + \frac{\sqrt{3}\,314928^{1/3}\,\ln\left(\frac{\left(2\,\sqrt{d\,x^3+c}+2\,\sqrt{3}\,\sqrt{-c}+2^{1/3}\,\sqrt{3}\,(-c)^{1/6}\,d^{1/3}\,x-2^{1/3}\,(-c)^{1/6}\,d^{1/3}\,x\,3\mathrm{i}\right)^3\left(108\,\sqrt{d\,x^3+c}-108\,\sqrt{3}\,\sqrt{-c}-54\,2^{1/3}\,\sqrt{3}\,(-\cdots\right.}{\left(2\,d^{1/3}\,x+2^{2/3}\,(-c)^{1/3}+2^{2/3}\,\sqrt{3}\,(-c)^{1/3}\,1\mathrm{i}\right)^6}\cdots}{2916\,(-c)^{5/6}\,d^{2/3}}$$

```
[In] int(x/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

```
[Out] (3^(1/2)*314928^(1/3)*log((((c + d*x^3)^(1/2) + 3^(1/2)*(-c)^(1/2) - 2^(1/3
)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x)^3*(54*(c + d*x^3)^(1/2) - 54*3^(1/2)*(-c)^(
1/2) + 54*2^(1/3)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x))/(d^(1/3)*x - 2^(2/3)*(-c)^
(1/3))^6))/(2916*(-c)^(5/6)*d^(2/3)) + (3^(1/2)*314928^(1/3)*log(((2*3^(1/2
)*(-c)^(1/2) - 2*(c + d*x^3)^(1/2) + 2^(1/3)*(-c)^(1/6)*d^(1/3)*x*3i + 2^(1
/3)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x)^3*(108*(c + d*x^3)^(1/2) + 108*3^(1/2)*(-
c)^(1/2) + 2^(1/3)*(-c)^(1/6)*d^(1/3)*x*162i + 54*2^(1/3)*3^(1/2)*(-c)^(1/6
)*d^(1/3)*x))/(2*d^(1/3)*x + 2^(2/3)*(-c)^(1/3) - 2^(2/3)*3^(1/2)*(-c)^(1/3
)*1i)^6)*((3^(1/2)*1i)/2 - 1/2)^(1/2))/(2916*(-c)^(5/6)*d^(2/3)) + (3^(1/2)
*314928^(1/3)*log(((2*(c + d*x^3)^(1/2) + 2*3^(1/2)*(-c)^(1/2) - 2^(1/3)*(-
c)^(1/6)*d^(1/3)*x*3i + 2^(1/3)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x)^3*(108*(c + d
*x^3)^(1/2) - 108*3^(1/2)*(-c)^(1/2) + 2^(1/3)*(-c)^(1/6)*d^(1/3)*x*162i -
54*2^(1/3)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x))/(2*d^(1/3)*x + 2^(2/3)*(-c)^(1/3)
 + 2^(2/3)*3^(1/2)*(-c)^(1/3)*1i)^6)*((3^(1/2)*1i)/2 + 1/2)^(1/2)*1i)/(2916
*(-c)^(5/6)*d^(2/3))
```

3.277 $\int \frac{1}{x^2\sqrt{c+dx^3}(4c+dx^3)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 697

$$\int \frac{1}{x^2\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = -\frac{\sqrt{c+dx^3}}{4c^2x}+\frac{\sqrt[3]{d}\sqrt{c+dx^3}}{4c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{12\ 2^{2/3}\sqrt{3}c^{11/6}}$$
$$-\frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{12\ 2^{2/3}\sqrt{3}c^{11/6}}+\frac{\sqrt[3]{d}\text{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{12\ 2^{2/3}c^{11/6}}-\frac{\sqrt[3]{d}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{36\ 2^{2/3}c^{11/6}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{8c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{2\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 1/24*d^(1/3)*arctanh(c^(1/6)*(c^(1/3)-2^(1/3)*d^(1/3)*x)/(d*x^3+c)^(1/2))*2
^(1/3)/c^(11/6)-1/72*d^(1/3)*arctanh((d*x^3+c)^(1/2)/c^(1/2))*2^(1/3)/c^(11
```

```
/6)+1/72*d^(1/3)*arctan(c^(1/6)*(c^(1/3)+2^(1/3)*d^(1/3)*x)*3^(1/2)/(d*x^3+
c)^(1/2))*2^(1/3)/c^(11/6)*3^(1/2)-1/72*d^(1/3)*arctan(1/3*(d*x^3+c)^(1/2)*
3^(1/2)/c^(1/2))*2^(1/3)/c^(11/6)*3^(1/2)-1/4*(d*x^3+c)^(1/2)/c^2/x+1/4*d^(
1/3)*(d*x^3+c)^(1/2)/c^2/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/12*d^(1/3)*(c^(1
/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)
*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1
/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(5/3)*2^(1/2)/(d*x^3+c)^(1/2)
/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/8*
3^(1/4)*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)
))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))
*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2
)^(1/2)/c^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(
1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 697, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used

$= \{491, 598, 309, 224, 1891, 497\}$

$$\int \frac{1}{x^2\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$$

$$= \frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{2\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{8c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+\frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{12\ 2^{2/3}\sqrt{3}c^{11/6}}-\frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{12\ 2^{2/3}\sqrt{3}c^{11/6}}$$
$$+\frac{\sqrt[3]{d}\text{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{12\ 2^{2/3}c^{11/6}}-\frac{\sqrt[3]{d}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{36\ 2^{2/3}c^{11/6}}$$
$$-\frac{\sqrt{c+dx^3}}{4c^2x}+\frac{\sqrt[3]{d}\sqrt{c+dx^3}}{4c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

```
[In] Int[1/(x^2*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] -1/4*Sqrt[c + d*x^3]/(c^2*x) + (d^(1/3)*Sqrt[c + d*x^3])/(4*c^2*((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)) + (d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(
1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(12*2^(2/3)*Sqrt[3]*c^(11/6)) - (d^(1/3)
*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(12*2^(2/3)*Sqrt[3]*c^(11/6)) +
 (d^(1/3)*ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])
/(12*2^(2/3)*c^(11/6)) - (d^(1/3)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(36*2^(
2/3)*c^(11/6)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*S
qrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(8*c^(5/3)*Sqrt[(c^(1/3)*(c^(1
/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) +
 (d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)
*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3]
)*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]
```

```
])/(2*Sqrt[2]*3^(1/4)*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 497

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[q*(ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]]/(9*2^(2/3)*b
*Rt[c, 2])), x] + (-Simp[q*(ArcTanh[Rt[c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c + d*
x^3])]/(3*2^(2/3)*b*Rt[c, 2])), x] + Simp[q*(ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])), x] - Simp[q*(ArcTan[Sqrt[3]*R
t[c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{c+dx^3}}{4c^2x}+\frac{\int \frac{x\left(cd+\frac{d^2x^3}{2}\right)}{\sqrt{c+dx^3}(4c+dx^3)}\,dx}{4c^2}\\
&= -\frac{\sqrt{c+dx^3}}{4c^2x}+\frac{\int \left(\frac{dx}{2\sqrt{c+dx^3}}-\frac{cdx}{\sqrt{c+dx^3}(4c+dx^3)}\right)dx}{4c^2}\\
&= -\frac{\sqrt{c+dx^3}}{4c^2x}+\frac{d\int \frac{x}{\sqrt{c+dx^3}}\,dx}{8c^2}-\frac{d\int \frac{x}{\sqrt{c+dx^3}(4c+dx^3)}\,dx}{4c}\\
&= -\frac{\sqrt{c+dx^3}}{4c^2x}+\frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{12\ 2^{2/3}\sqrt{3}c^{11/6}}-\frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{12\ 2^{2/3}\sqrt{3}c^{11/6}}\\
&\quad+\frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{12\ 2^{2/3}c^{11/6}}-\frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{36\ 2^{2/3}c^{11/6}}\\
&\quad+\frac{d^{2/3}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{8c^2}-\frac{\left(\left(1-\sqrt{3}\right)d^{2/3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{8c^{5/3}}
\end{aligned}$$

$$= -\frac{\sqrt{c+dx^3}}{4c^2x} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{4c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{12\ 2^{2/3}\sqrt{3}c^{11/6}} - \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{12\ 2^{2/3}\sqrt{3}c^{11/6}} + \frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{12\ 2^{2/3}c^{11/6}} - \frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{36\ 2^{2/3}c^{11/6}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{8c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{2\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.20

$$\int \frac{1}{x^2\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{-40c(c+dx^3)+5cdx^3\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+d^2x^6\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\right.}{160c^3x\sqrt{c+dx^3}}$$

```
[In] Integrate[1/(x^2*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (-40*c*(c + d*x^3) + 5*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/
3, -((d*x^3)/c), -1/4*(d*x^3)/c] + d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3
, 1/2, 1, 8/3, -((d*x^3)/c), -1/4*(d*x^3)/c])/(160*c^3*x*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.94 (sec) , antiderivative size = 868, normalized size of antiderivative = 1.25

method	result	size
elliptic	Expression too large to display	868
risch	Expression too large to display	872
default	Expression too large to display	874

```
[In] int(1/x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(d*x^3+c)^(1/2)/c^2/x-1/12*I/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-
c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)
-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)
/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(
1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d
*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/
2)))+1/36*I/d^2/c^2*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(
-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*
(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I
*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2
)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/
3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^
(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^
2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3
^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.70 (sec) , antiderivative size = 2293, normalized size of antiderivative = 3.29

$$\int \frac{1}{x^2\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/144*(2*(1/432)^(1/6)*c^2*x*(-d^2/c^11)^(1/6)*log((d^4*x^9 - 66*c*d^3*x^6
 - 72*c^2*d^2*x^3 - 32*c^3*d + 48*(1/2)^(2/3)*(c^8*d^2*x^7 - c^9*d*x^4 - 2*
c^10*x)*(-d^2/c^11)^(2/3) + 12*(1/2)^(1/3)*(c^4*d^3*x^8 - 7*c^5*d^2*x^5 - 8
*c^6*d*x^2)*(-d^2/c^11)^(1/3) + 6*(1296*(1/432)^(5/6)*c^10*d*x^5*(-d^2/c^11
)^(5/6) + sqrt(1/3)*(5*c^6*d^2*x^6 - 20*c^7*d*x^3 - 16*c^8)*sqrt(-d^2/c^11)
 + 2*(1/432)^(1/6)*(c^2*d^3*x^7 - 16*c^3*d^2*x^4 - 8*c^4*d*x)*(-d^2/c^11)^(
1/6))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) -
2*(1/432)^(1/6)*c^2*x*(-d^2/c^11)^(1/6)*log((d^4*x^9 - 66*c*d^3*x^6 - 72*c^
2*d^2*x^3 - 32*c^3*d + 48*(1/2)^(2/3)*(c^8*d^2*x^7 - c^9*d*x^4 - 2*c^10*x)*
(-d^2/c^11)^(2/3) + 12*(1/2)^(1/3)*(c^4*d^3*x^8 - 7*c^5*d^2*x^5 - 8*c^6*d*x
^2)*(-d^2/c^11)^(1/3) - 6*(1296*(1/432)^(5/6)*c^10*d*x^5*(-d^2/c^11)^(5/6)
+ sqrt(1/3)*(5*c^6*d^2*x^6 - 20*c^7*d*x^3 - 16*c^8)*sqrt(-d^2/c^11) + 2*(1/
432)^(1/6)*(c^2*d^3*x^7 - 16*c^3*d^2*x^4 - 8*c^4*d*x)*(-d^2/c^11)^(1/6))*sq
rt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + 36*sqrt(
d)*x*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) - (1/432
)^(1/6)*(sqrt(-3)*c^2*x + c^2*x)*(-d^2/c^11)^(1/6)*log((d^4*x^9 - 66*c*d^3*
x^6 - 72*c^2*d^2*x^3 - 32*c^3*d - 24*(1/2)^(2/3)*(c^8*d^2*x^7 - c^9*d*x^4 -
 2*c^10*x + sqrt(-3)*(c^8*d^2*x^7 - c^9*d*x^4 - 2*c^10*x))*(-d^2/c^11)^(2/3
) - 6*(1/2)^(1/3)*(c^4*d^3*x^8 - 7*c^5*d^2*x^5 - 8*c^6*d*x^2 - sqrt(-3)*(c^
4*d^3*x^8 - 7*c^5*d^2*x^5 - 8*c^6*d*x^2))*(-d^2/c^11)^(1/3) + 6*sqrt(d*x^3
+ c)*(648*(1/432)^(5/6)*(sqrt(-3)*c^10*d*x^5 - c^10*d*x^5)*(-d^2/c^11)^(5/6
) + sqrt(1/3)*(5*c^6*d^2*x^6 - 20*c^7*d*x^3 - 16*c^8)*sqrt(-d^2/c^11) - (1/
432)^(1/6)*(c^2*d^3*x^7 - 16*c^3*d^2*x^4 - 8*c^4*d*x + sqrt(-3)*(c^2*d^3*x^
7 - 16*c^3*d^2*x^4 - 8*c^4*d*x))*(-d^2/c^11)^(1/6)))/(d^3*x^9 + 12*c*d^2*x^
6 + 48*c^2*d*x^3 + 64*c^3)) + (1/432)^(1/6)*(sqrt(-3)*c^2*x + c^2*x)*(-d^2/
c^11)^(1/6)*log((d^4*x^9 - 66*c*d^3*x^6 - 72*c^2*d^2*x^3 - 32*c^3*d - 24*(1
/2)^(2/3)*(c^8*d^2*x^7 - c^9*d*x^4 - 2*c^10*x + sqrt(-3)*(c^8*d^2*x^7 - c^9
*d*x^4 - 2*c^10*x))*(-d^2/c^11)^(2/3) - 6*(1/2)^(1/3)*(c^4*d^3*x^8 - 7*c^5*
d^2*x^5 - 8*c^6*d*x^2 - sqrt(-3)*(c^4*d^3*x^8 - 7*c^5*d^2*x^5 - 8*c^6*d*x^2
))*(-d^2/c^11)^(1/3) - 6*sqrt(d*x^3 + c)*(648*(1/432)^(5/6)*(sqrt(-3)*c^10*
d*x^5 - c^10*d*x^5)*(-d^2/c^11)^(5/6) + sqrt(1/3)*(5*c^6*d^2*x^6 - 20*c^7*d
*x^3 - 16*c^8)*sqrt(-d^2/c^11) - (1/432)^(1/6)*(c^2*d^3*x^7 - 16*c^3*d^2*x^
4 - 8*c^4*d*x + sqrt(-3)*(c^2*d^3*x^7 - 16*c^3*d^2*x^4 - 8*c^4*d*x))*(-d^2/
c^11)^(1/6)))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - (1/432)^(
1/6)*(sqrt(-3)*c^2*x - c^2*x)*(-d^2/c^11)^(1/6)*log((d^4*x^9 - 66*c*d^3*x^6
 - 72*c^2*d^2*x^3 - 32*c^3*d - 24*(1/2)^(2/3)*(c^8*d^2*x^7 - c^9*d*x^4 - 2*
c^10*x - sqrt(-3)*(c^8*d^2*x^7 - c^9*d*x^4 - 2*c^10*x))*(-d^2/c^11)^(2/3) -
 6*(1/2)^(1/3)*(c^4*d^3*x^8 - 7*c^5*d^2*x^5 - 8*c^6*d*x^2 + sqrt(-3)*(c^4*d
^3*x^8 - 7*c^5*d^2*x^5 - 8*c^6*d*x^2))*(-d^2/c^11)^(1/3) + 6*sqrt(d*x^3 + c
)*(648*(1/432)^(5/6)*(sqrt(-3)*c^10*d*x^5 + c^10*d*x^5)*(-d^2/c^11)^(5/6) -
 sqrt(1/3)*(5*c^6*d^2*x^6 - 20*c^7*d*x^3 - 16*c^8)*sqrt(-d^2/c^11) + (1/432
)^(1/6)*(c^2*d^3*x^7 - 16*c^3*d^2*x^4 - 8*c^4*d*x - sqrt(-3)*(c^2*d^3*x^7 -
 16*c^3*d^2*x^4 - 8*c^4*d*x))*(-d^2/c^11)^(1/6)))/(d^3*x^9 + 12*c*d^2*x^6 +
 48*c^2*d*x^3 + 64*c^3)) + (1/432)^(1/6)*(sqrt(-3)*c^2*x - c^2*x)*(-d^2/c^1
1)^(1/6)*log((d^4*x^9 - 66*c*d^3*x^6 - 72*c^2*d^2*x^3 - 32*c^3*d - 24*(1/2)
```

```
^(2/3)*(c^8*d^2*x^7 - c^9*d*x^4 - 2*c^10*x - sqrt(-3)*(c^8*d^2*x^7 - c^9*d*
x^4 - 2*c^10*x))*(-d^2/c^11)^(2/3) - 6*(1/2)^(1/3)*(c^4*d^3*x^8 - 7*c^5*d^2
*x^5 - 8*c^6*d*x^2 + sqrt(-3)*(c^4*d^3*x^8 - 7*c^5*d^2*x^5 - 8*c^6*d*x^2))*
(-d^2/c^11)^(1/3) - 6*sqrt(d*x^3 + c)*(648*(1/432)^(5/6)*(sqrt(-3)*c^10*d*x
^5 + c^10*d*x^5)*(-d^2/c^11)^(5/6) - sqrt(1/3)*(5*c^6*d^2*x^6 - 20*c^7*d*x^
3 - 16*c^8)*sqrt(-d^2/c^11) + (1/432)^(1/6)*(c^2*d^3*x^7 - 16*c^3*d^2*x^4 -
 8*c^4*d*x - sqrt(-3)*(c^2*d^3*x^7 - 16*c^3*d^2*x^4 - 8*c^4*d*x))*(-d^2/c^1
1)^(1/6)))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + 36*sqrt(d*x^
3 + c))/(c^2*x)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{x^2\sqrt{c+dx^3}\cdot(4c+dx^3)}\,dx$$

```
[In] integrate(1/x**2/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(x**2*sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{(dx^3+4\,c)\sqrt{dx^3+c}x^2}\,dx$$

```
[In] integrate(1/x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{(dx^3+4\,c)\sqrt{dx^3+c}x^2}\,dx$$

```
[In] integrate(1/x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{x^2\,\sqrt{d\,x^3+c}\,\left(d\,x^3+4\,c\right)}\,dx$$

```
[In] int(1/(x^2*(c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
[Out] int(1/(x^2*(c + d*x^3)^(1/2)*(4*c + d*x^3)), x)
```

3.278 $\int \frac{x^3}{\sqrt{c+dx^3}(4c+dx^3)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 66

$$\int \frac{x^3}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\frac{x^4\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},1,\frac{1}{2},\frac{7}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{16c\sqrt{c+dx^3}}$$

```
[Out] 1/16*x^4*AppellF1(4/3,1/2,1,7/3,-d*x^3/c,-1/4*d*x^3/c)*(1+d*x^3/c)^(1/2)/c/
(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {525, 524}

$$\int \frac{x^3}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{4}{3},1,\frac{1}{2},\frac{7}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{16c\sqrt{c+dx^3}}$$

```
[In] Int[x^3/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 1/2, 7/3, -1/4*(d*x^3)/c, -((d*x^
3)/c)])/(16*c*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{(4c+dx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$

$$= \frac{x^4\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{16c\sqrt{c+dx^3}}$$

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{x^4\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{16c\sqrt{c+dx^3}}$$

```
[In] Integrate[x^3/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
[Out] (x^4*Sqrt[(c + d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -1/4*(d*x
^3)/c])/(16*c*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.36 (sec) , antiderivative size = 696, normalized size of antiderivative = 10.55

method	result
default	$-\frac{2i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}}}\sqrt{-\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,F}{3d^{2}\sqrt{d\,x^{3}+c}}$
elliptic	$-\frac{2i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}}}\sqrt{-\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,F}{3d^{2}\sqrt{d\,x^{3}+c}}$

```
[In] int(x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d
^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+4/
9*I/d^4*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(
1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/
d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c
)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alph
a^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
```

```
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3
^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(
-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)
),_alpha=RootOf(_Z^3*d+4*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2300 vs. $2(52) = 104$.

Time = 0.71 (sec) , antiderivative size = 2300, normalized size of antiderivative = 34.85

$$\int \frac{x^3}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/36*(2*(16/27)^(1/6)*d^2*(-1/(c*d^8))^(1/6)*log((4*d^3*x^9 - 264*c*d^2*x^6
 - 288*c^2*d*x^3 - 128*c^3 - 24*2^(2/3)*(c*d^8*x^8 - 7*c^2*d^7*x^5 - 8*c^3*
d^6*x^2)*(-1/(c*d^8))^(2/3) - 96*2^(1/3)*(c*d^5*x^7 - c^2*d^4*x^4 - 2*c^3*d
^3*x)*(-1/(c*d^8))^(1/3) + 3*(72*(16/27)^(1/6)*c*d^3*x^5*(-1/(c*d^8))^(1/6)
 + 9*(16/27)^(5/6)*(c*d^9*x^7 - 16*c^2*d^8*x^4 - 8*c^3*d^7*x)*(-1/(c*d^8))^
(5/6) + 8*sqrt(1/3)*(5*c*d^6*x^6 - 20*c^2*d^5*x^3 - 16*c^3*d^4)*sqrt(-1/(c*
d^8)))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) -
 2*(16/27)^(1/6)*d^2*(-1/(c*d^8))^(1/6)*log((4*d^3*x^9 - 264*c*d^2*x^6 - 28
8*c^2*d*x^3 - 128*c^3 - 24*2^(2/3)*(c*d^8*x^8 - 7*c^2*d^7*x^5 - 8*c^3*d^6*x
^2)*(-1/(c*d^8))^(2/3) - 96*2^(1/3)*(c*d^5*x^7 - c^2*d^4*x^4 - 2*c^3*d^3*x)
*(-1/(c*d^8))^(1/3) - 3*(72*(16/27)^(1/6)*c*d^3*x^5*(-1/(c*d^8))^(1/6) + 9*
(16/27)^(5/6)*(c*d^9*x^7 - 16*c^2*d^8*x^4 - 8*c^3*d^7*x)*(-1/(c*d^8))^(5/6)
 + 8*sqrt(1/3)*(5*c*d^6*x^6 - 20*c^2*d^5*x^3 - 16*c^3*d^4)*sqrt(-1/(c*d^8))
)*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + (16/
27)^(1/6)*(sqrt(-3)*d^2 + d^2)*(-1/(c*d^8))^(1/6)*log((8*d^3*x^9 - 528*c*d^
2*x^6 - 576*c^2*d*x^3 - 256*c^3 + 24*2^(2/3)*(c*d^8*x^8 - 7*c^2*d^7*x^5 - 8
*c^3*d^6*x^2 + sqrt(-3)*(c*d^8*x^8 - 7*c^2*d^7*x^5 - 8*c^3*d^6*x^2))*(-1/(c
*d^8))^(2/3) + 96*2^(1/3)*(c*d^5*x^7 - c^2*d^4*x^4 - 2*c^3*d^3*x - sqrt(-3)
*(c*d^5*x^7 - c^2*d^4*x^4 - 2*c^3*d^3*x))*(-1/(c*d^8))^(1/3) + 3*sqrt(d*x^3
 + c)*(9*(16/27)^(5/6)*(c*d^9*x^7 - 16*c^2*d^8*x^4 - 8*c^3*d^7*x - sqrt(-3)
*(c*d^9*x^7 - 16*c^2*d^8*x^4 - 8*c^3*d^7*x))*(-1/(c*d^8))^(5/6) - 16*sqrt(1
/3)*(5*c*d^6*x^6 - 20*c^2*d^5*x^3 - 16*c^3*d^4)*sqrt(-1/(c*d^8)) + 72*(16/2
7)^(1/6)*(sqrt(-3)*c*d^3*x^5 + c*d^3*x^5)*(-1/(c*d^8))^(1/6)))/(d^3*x^9 + 1
2*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - (16/27)^(1/6)*(sqrt(-3)*d^2 + d^2)*
(-1/(c*d^8))^(1/6)*log((8*d^3*x^9 - 528*c*d^2*x^6 - 576*c^2*d*x^3 - 256*c^3
 + 24*2^(2/3)*(c*d^8*x^8 - 7*c^2*d^7*x^5 - 8*c^3*d^6*x^2 + sqrt(-3)*(c*d^8*
x^8 - 7*c^2*d^7*x^5 - 8*c^3*d^6*x^2))*(-1/(c*d^8))^(2/3) + 96*2^(1/3)*(c*d^
5*x^7 - c^2*d^4*x^4 - 2*c^3*d^3*x - sqrt(-3)*(c*d^5*x^7 - c^2*d^4*x^4 - 2*c
```

```
^3*d^3*x))*(-1/(c*d^8))^(1/3) - 3*sqrt(d*x^3 + c)*(9*(16/27)^(5/6)*(c*d^9*x
^7 - 16*c^2*d^8*x^4 - 8*c^3*d^7*x - sqrt(-3)*(c*d^9*x^7 - 16*c^2*d^8*x^4 -
8*c^3*d^7*x))*(-1/(c*d^8))^(5/6) - 16*sqrt(1/3)*(5*c*d^6*x^6 - 20*c^2*d^5*x
^3 - 16*c^3*d^4)*sqrt(-1/(c*d^8)) + 72*(16/27)^(1/6)*(sqrt(-3)*c*d^3*x^5 +
c*d^3*x^5)*(-1/(c*d^8))^(1/6)))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64
*c^3)) - (16/27)^(1/6)*(sqrt(-3)*d^2 - d^2)*(-1/(c*d^8))^(1/6)*log((8*d^3*x
^9 - 528*c*d^2*x^6 - 576*c^2*d*x^3 - 256*c^3 + 24*2^(2/3)*(c*d^8*x^8 - 7*c^
2*d^7*x^5 - 8*c^3*d^6*x^2 - sqrt(-3)*(c*d^8*x^8 - 7*c^2*d^7*x^5 - 8*c^3*d^6
*x^2))*(-1/(c*d^8))^(2/3) + 96*2^(1/3)*(c*d^5*x^7 - c^2*d^4*x^4 - 2*c^3*d^3
*x + sqrt(-3)*(c*d^5*x^7 - c^2*d^4*x^4 - 2*c^3*d^3*x))*(-1/(c*d^8))^(1/3) +
 3*sqrt(d*x^3 + c)*(9*(16/27)^(5/6)*(c*d^9*x^7 - 16*c^2*d^8*x^4 - 8*c^3*d^7
*x + sqrt(-3)*(c*d^9*x^7 - 16*c^2*d^8*x^4 - 8*c^3*d^7*x))*(-1/(c*d^8))^(5/6
) - 16*sqrt(1/3)*(5*c*d^6*x^6 - 20*c^2*d^5*x^3 - 16*c^3*d^4)*sqrt(-1/(c*d^8
)) - 72*(16/27)^(1/6)*(sqrt(-3)*c*d^3*x^5 - c*d^3*x^5)*(-1/(c*d^8))^(1/6)))
/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + (16/27)^(1/6)*(sqrt(-3
)*d^2 - d^2)*(-1/(c*d^8))^(1/6)*log((8*d^3*x^9 - 528*c*d^2*x^6 - 576*c^2*d*
x^3 - 256*c^3 + 24*2^(2/3)*(c*d^8*x^8 - 7*c^2*d^7*x^5 - 8*c^3*d^6*x^2 - sqr
t(-3)*(c*d^8*x^8 - 7*c^2*d^7*x^5 - 8*c^3*d^6*x^2))*(-1/(c*d^8))^(2/3) + 96*
2^(1/3)*(c*d^5*x^7 - c^2*d^4*x^4 - 2*c^3*d^3*x + sqrt(-3)*(c*d^5*x^7 - c^2*
d^4*x^4 - 2*c^3*d^3*x))*(-1/(c*d^8))^(1/3) - 3*sqrt(d*x^3 + c)*(9*(16/27)^(
5/6)*(c*d^9*x^7 - 16*c^2*d^8*x^4 - 8*c^3*d^7*x + sqrt(-3)*(c*d^9*x^7 - 16*c
^2*d^8*x^4 - 8*c^3*d^7*x))*(-1/(c*d^8))^(5/6) - 16*sqrt(1/3)*(5*c*d^6*x^6 -
 20*c^2*d^5*x^3 - 16*c^3*d^4)*sqrt(-1/(c*d^8)) - 72*(16/27)^(1/6)*(sqrt(-3)
*c*d^3*x^5 - c*d^3*x^5)*(-1/(c*d^8))^(1/6)))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c
^2*d*x^3 + 64*c^3)) + 24*sqrt(d)*weierstrassPInverse(0, -4*c/d, x))/d^2
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\int \frac{x^3}{\sqrt{c+dx^3}\cdot(4c+dx^3)}\,dx$$

```
[In] integrate(x**3/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
[Out] Integral(x**3/(sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{x^3}{(dx^3+4\,c)\sqrt{dx^3+c}}\,dx$$

```
[In] integrate(x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{x^3}{(dx^3+4\,c)\sqrt{dx^3+c}}\,dx$$

```
[In] integrate(x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^3/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{x^3}{\sqrt{d\,x^3+c}\,(d\,x^3+4\,c)}\,dx$$

```
[In] int(x^3/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
[Out] int(x^3/((c + d*x^3)^(1/2)*(4*c + d*x^3)), x)
```

3.279 $\int \frac{1}{\sqrt{c+dx^3}(4c+dx^3)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 64

$$\int \frac{1}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\frac{x\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{1}{3},1,\frac{1}{2},\frac{4}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{4c\sqrt{c+dx^3}}$$

```
[Out] 1/4*x*AppellF1(1/3,1/2,1,4/3,-d*x^3/c,-1/4*d*x^3/c)*(1+d*x^3/c)^(1/2)/c/(d*
x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {441, 440}

$$\int \frac{1}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\frac{x\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},1,\frac{1}{2},\frac{4}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{4c\sqrt{c+dx^3}}$$

```
[In] Int[1/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 1/2, 4/3, -1/4*(d*x^3)/c, -((d*x^3)
/c)])/(4*c*Sqrt[c + d*x^3])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{(4c+dx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$

$$= \frac{x\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{4c\sqrt{c+dx^3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 165 vs. 2(64) = 128.

Time = 10.05 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.58

$$\int \frac{1}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$$

$$= \frac{16cx\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{\sqrt{c+dx^3}\,(4c+dx^3)\left(16c\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)-3dx^3\left(\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+2\,\text{A}\right.\right.}$$

```
[In] Integrate[1/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (16*c*x*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -1/4*(d*x^3)/c])/(Sqrt[c +
 d*x^3]*(4*c + d*x^3)*(16*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -1/4*(
d*x^3)/c] - 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -1/4*(d*x^3)/
c] + 2*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -1/4*(d*x^3)/c])))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.33 (sec) , antiderivative size = 416, normalized size of antiderivative = 6.50

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}+4c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{i d\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{i d\left(2x+\frac{i\sqrt{3}\left(-c\right.}{}\right)}{2\left(-c\right.}}}{}\right.}{}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}+4c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{i d\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{i d\left(2x+\frac{i\sqrt{3}\left(-c\right.}{}\right)}{2\left(-c\right.}}}{}\right.}{}$

```
[In] int(1/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/9*I/d^3/c*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d
^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2
*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*
x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*
_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)
*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2
/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2
)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^
(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2347 vs. $2(50) = 100$.

Time = 0.72 (sec) , antiderivative size = 2347, normalized size of antiderivative = 36.67

$$\int \frac{1}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/72*(2*(1/108)^(1/6)*c*d*(-1/(c^7*d^2))^(1/6)*log((d^3*x^9 - 66*c*d^2*x^6
 - 72*c^2*d*x^3 - 32*c^3 - 24*(1/4)^(2/3)*(c^5*d^4*x^8 - 7*c^6*d^3*x^5 - 8*
c^7*d^2*x^2)*(-1/(c^7*d^2))^(2/3) - 48*(1/4)^(1/3)*(c^3*d^3*x^7 - c^4*d^2*x
^4 - 2*c^5*d*x)*(-1/(c^7*d^2))^(1/3) + 6*(18*(1/108)^(1/6)*c^2*d^2*x^5*(-1/
(c^7*d^2))^(1/6) + 36*(1/108)^(5/6)*(c^6*d^4*x^7 - 16*c^7*d^3*x^4 - 8*c^8*d
^2*x)*(-1/(c^7*d^2))^(5/6) + sqrt(1/3)*(5*c^4*d^3*x^6 - 20*c^5*d^2*x^3 - 16
*c^6*d)*sqrt(-1/(c^7*d^2)))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c
^2*d*x^3 + 64*c^3)) - 2*(1/108)^(1/6)*c*d*(-1/(c^7*d^2))^(1/6)*log((d^3*x^9
 - 66*c*d^2*x^6 - 72*c^2*d*x^3 - 32*c^3 - 24*(1/4)^(2/3)*(c^5*d^4*x^8 - 7*c
^6*d^3*x^5 - 8*c^7*d^2*x^2)*(-1/(c^7*d^2))^(2/3) - 48*(1/4)^(1/3)*(c^3*d^3*
x^7 - c^4*d^2*x^4 - 2*c^5*d*x)*(-1/(c^7*d^2))^(1/3) - 6*(18*(1/108)^(1/6)*c
^2*d^2*x^5*(-1/(c^7*d^2))^(1/6) + 36*(1/108)^(5/6)*(c^6*d^4*x^7 - 16*c^7*d^
3*x^4 - 8*c^8*d^2*x)*(-1/(c^7*d^2))^(5/6) + sqrt(1/3)*(5*c^4*d^3*x^6 - 20*c
^5*d^2*x^3 - 16*c^6*d)*sqrt(-1/(c^7*d^2)))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c
*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + (1/108)^(1/6)*(sqrt(-3)*c*d + c*d)*(-1
/(c^7*d^2))^(1/6)*log((d^3*x^9 - 66*c*d^2*x^6 - 72*c^2*d*x^3 - 32*c^3 + 12*
(1/4)^(2/3)*(c^5*d^4*x^8 - 7*c^6*d^3*x^5 - 8*c^7*d^2*x^2 + sqrt(-3)*(c^5*d^
4*x^8 - 7*c^6*d^3*x^5 - 8*c^7*d^2*x^2))*(-1/(c^7*d^2))^(2/3) + 24*(1/4)^(1/
3)*(c^3*d^3*x^7 - c^4*d^2*x^4 - 2*c^5*d*x - sqrt(-3)*(c^3*d^3*x^7 - c^4*d^2
*x^4 - 2*c^5*d*x))*(-1/(c^7*d^2))^(1/3) + 6*sqrt(d*x^3 + c)*(18*(1/108)^(5/
6)*(c^6*d^4*x^7 - 16*c^7*d^3*x^4 - 8*c^8*d^2*x - sqrt(-3)*(c^6*d^4*x^7 - 16
*c^7*d^3*x^4 - 8*c^8*d^2*x))*(-1/(c^7*d^2))^(5/6) - sqrt(1/3)*(5*c^4*d^3*x^
6 - 20*c^5*d^2*x^3 - 16*c^6*d)*sqrt(-1/(c^7*d^2)) + 9*(1/108)^(1/6)*(sqrt(-
3)*c^2*d^2*x^5 + c^2*d^2*x^5)*(-1/(c^7*d^2))^(1/6)))/(d^3*x^9 + 12*c*d^2*x^
6 + 48*c^2*d*x^3 + 64*c^3)) - (1/108)^(1/6)*(sqrt(-3)*c*d + c*d)*(-1/(c^7*d
^2))^(1/6)*log((d^3*x^9 - 66*c*d^2*x^6 - 72*c^2*d*x^3 - 32*c^3 + 12*(1/4)^(
2/3)*(c^5*d^4*x^8 - 7*c^6*d^3*x^5 - 8*c^7*d^2*x^2 + sqrt(-3)*(c^5*d^4*x^8 -
 7*c^6*d^3*x^5 - 8*c^7*d^2*x^2))*(-1/(c^7*d^2))^(2/3) + 24*(1/4)^(1/3)*(c^3
*d^3*x^7 - c^4*d^2*x^4 - 2*c^5*d*x - sqrt(-3)*(c^3*d^3*x^7 - c^4*d^2*x^4 -
2*c^5*d*x))*(-1/(c^7*d^2))^(1/3) - 6*sqrt(d*x^3 + c)*(18*(1/108)^(5/6)*(c^6
*d^4*x^7 - 16*c^7*d^3*x^4 - 8*c^8*d^2*x - sqrt(-3)*(c^6*d^4*x^7 - 16*c^7*d^
3*x^4 - 8*c^8*d^2*x))*(-1/(c^7*d^2))^(5/6) - sqrt(1/3)*(5*c^4*d^3*x^6 - 20*
c^5*d^2*x^3 - 16*c^6*d)*sqrt(-1/(c^7*d^2)) + 9*(1/108)^(1/6)*(sqrt(-3)*c^2*
d^2*x^5 + c^2*d^2*x^5)*(-1/(c^7*d^2))^(1/6)))/(d^3*x^9 + 12*c*d^2*x^6 + 48*
c^2*d*x^3 + 64*c^3)) - (1/108)^(1/6)*(sqrt(-3)*c*d - c*d)*(-1/(c^7*d^2))^(1
```

```
/6)*log((d^3*x^9 - 66*c*d^2*x^6 - 72*c^2*d*x^3 - 32*c^3 + 12*(1/4)^(2/3)*(c
^5*d^4*x^8 - 7*c^6*d^3*x^5 - 8*c^7*d^2*x^2 - sqrt(-3)*(c^5*d^4*x^8 - 7*c^6*
d^3*x^5 - 8*c^7*d^2*x^2))*(-1/(c^7*d^2))^(2/3) + 24*(1/4)^(1/3)*(c^3*d^3*x^
7 - c^4*d^2*x^4 - 2*c^5*d*x + sqrt(-3)*(c^3*d^3*x^7 - c^4*d^2*x^4 - 2*c^5*d
*x))*(-1/(c^7*d^2))^(1/3) + 6*sqrt(d*x^3 + c)*(18*(1/108)^(5/6)*(c^6*d^4*x^
7 - 16*c^7*d^3*x^4 - 8*c^8*d^2*x + sqrt(-3)*(c^6*d^4*x^7 - 16*c^7*d^3*x^4 -
 8*c^8*d^2*x))*(-1/(c^7*d^2))^(5/6) - sqrt(1/3)*(5*c^4*d^3*x^6 - 20*c^5*d^2
*x^3 - 16*c^6*d)*sqrt(-1/(c^7*d^2)) - 9*(1/108)^(1/6)*(sqrt(-3)*c^2*d^2*x^5
 - c^2*d^2*x^5)*(-1/(c^7*d^2))^(1/6)))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x
^3 + 64*c^3)) + (1/108)^(1/6)*(sqrt(-3)*c*d - c*d)*(-1/(c^7*d^2))^(1/6)*log
((d^3*x^9 - 66*c*d^2*x^6 - 72*c^2*d*x^3 - 32*c^3 + 12*(1/4)^(2/3)*(c^5*d^4*
x^8 - 7*c^6*d^3*x^5 - 8*c^7*d^2*x^2 - sqrt(-3)*(c^5*d^4*x^8 - 7*c^6*d^3*x^5
 - 8*c^7*d^2*x^2))*(-1/(c^7*d^2))^(2/3) + 24*(1/4)^(1/3)*(c^3*d^3*x^7 - c^4
*d^2*x^4 - 2*c^5*d*x + sqrt(-3)*(c^3*d^3*x^7 - c^4*d^2*x^4 - 2*c^5*d*x))*(-
1/(c^7*d^2))^(1/3) - 6*sqrt(d*x^3 + c)*(18*(1/108)^(5/6)*(c^6*d^4*x^7 - 16*
c^7*d^3*x^4 - 8*c^8*d^2*x + sqrt(-3)*(c^6*d^4*x^7 - 16*c^7*d^3*x^4 - 8*c^8*
d^2*x))*(-1/(c^7*d^2))^(5/6) - sqrt(1/3)*(5*c^4*d^3*x^6 - 20*c^5*d^2*x^3 -
16*c^6*d)*sqrt(-1/(c^7*d^2)) - 9*(1/108)^(1/6)*(sqrt(-3)*c^2*d^2*x^5 - c^2*
d^2*x^5)*(-1/(c^7*d^2))^(1/6)))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64
*c^3)) - 24*sqrt(d)*weierstrassPInverse(0, -4*c/d, x))/(c*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\int \frac{1}{\sqrt{c+dx^3}\cdot(4c+dx^3)}\,dx$$

```
[In] integrate(1/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=\int \frac{1}{(dx^3+4\,c)\sqrt{dx^3+c}}\,dx$$

```
[In] integrate(1/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{(dx^3+4\,c)\sqrt{dx^3+c}}\,dx$$

```
[In] integrate(1/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{\sqrt{d\,x^3+c}\,(d\,x^3+4\,c)}\,dx$$

```
[In] int(1/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
[Out] int(1/((c + d*x^3)^(1/2)*(4*c + d*x^3)), x)
```

3.280 $\int \frac{1}{x^3\sqrt{c+dx^3}(4c+dx^3)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 66

$$\int \frac{1}{x^3\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=-\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{8cx^2\sqrt{c+dx^3}}$$

```
[Out] -1/8*AppellF1(-2/3,1/2,1,1/3,-d*x^3/c,-1/4*d*x^3/c)*(1+d*x^3/c)^(1/2)/c/x^2
/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {525, 524}

$$\int \frac{1}{x^3\sqrt{c+dx^3}\,(4c+dx^3)}\,dx=-\frac{\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{8cx^2\sqrt{c+dx^3}}$$

```
[In] Int[1/(x^3*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] -1/8*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 1/2, 1/3, -1/4*(d*x^3)/c, -((d*
x^3)/c)])/(c*x^2*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^3(4c+dx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$

$$= -\frac{\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{8cx^2\sqrt{c+dx^3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 243 vs. $2(66) = 132$.

Time = 11.18 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.68

$$\int \frac{1}{x^3\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$$

$$= \frac{-\frac{32(c+dx^3)}{c^2} - \frac{d^2x^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{c^3} + \frac{2048dx^3\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\right.}{(4c+dx^3)\left(-16c\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+3dx^3\left(\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},2\right.\right.\right.}}{256x^2\sqrt{c+dx^3}}$$

```
[In] Integrate[1/(x^3*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] ((-32*(c + d*x^3))/c^2 - (d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1,
 7/3, -((d*x^3)/c), -1/4*(d*x^3)/c])/c^3 + (2048*d*x^3*AppellF1[1/3, 1/2, 1
, 4/3, -((d*x^3)/c), -1/4*(d*x^3)/c])/((4*c + d*x^3)*(-16*c*AppellF1[1/3, 1
/2, 1, 4/3, -((d*x^3)/c), -1/4*(d*x^3)/c] + 3*d*x^3*(AppellF1[4/3, 1/2, 2,
7/3, -((d*x^3)/c), -1/4*(d*x^3)/c] + 2*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)
/c), -1/4*(d*x^3)/c]))))/(256*x^2*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 5.04 (sec) , antiderivative size = 716, normalized size of antiderivative = 10.85

method	result
elliptic	$-\frac{\sqrt{d\,x^3+c}}{8c^2x^2}+\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}}}\sqrt{-\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^{2}\right.}{2d}\right.}{\left(-c\,d^2\right)^{\frac{1}{3}}}}}{24c^2\sqrt{d\,x^3+c}}$
risch	Expression too large to display
default	$\frac{-\frac{\sqrt{d\,x^3+c}}{2c\,x^2}+\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}}}\sqrt{-\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}}{6c\sqrt{d\,x^3+c}}}{4c}$

```
[In] int(1/x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(d*x^3+c)^(1/2)/c^2/x^2+1/24*I/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*
(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
```

```
/d*(-c*d^2)^(1/3)))^(1/2))+1/36*I/d^2/c^2*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(
1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)
^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^
2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3
)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*
d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^
(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(
1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2
/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2381 vs. $2(52) = 104$.

Time = 1.42 (sec) , antiderivative size = 2381, normalized size of antiderivative = 36.08

$$\int \frac{1}{x^3\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/288*(2*(1/108)^(1/6)*c^2*x^2*(-d^4/c^13)^(1/6)*log((d^6*x^9 - 66*c*d^5*x^
6 - 72*c^2*d^4*x^3 - 32*c^3*d^3 - 24*(1/4)^(2/3)*(c^9*d^3*x^8 - 7*c^10*d^2*
x^5 - 8*c^11*d*x^2)*(-d^4/c^13)^(2/3) - 48*(1/4)^(1/3)*(c^5*d^4*x^7 - c^6*d
^3*x^4 - 2*c^7*d^2*x)*(-d^4/c^13)^(1/3) + 6*(18*(1/108)^(1/6)*c^3*d^4*x^5*(
-d^4/c^13)^(1/6) + 36*(1/108)^(5/6)*(c^11*d^2*x^7 - 16*c^12*d*x^4 - 8*c^13*
x)*(-d^4/c^13)^(5/6) + sqrt(1/3)*(5*c^7*d^3*x^6 - 20*c^8*d^2*x^3 - 16*c^9*d
)*sqrt(-d^4/c^13))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3
+ 64*c^3)) - 2*(1/108)^(1/6)*c^2*x^2*(-d^4/c^13)^(1/6)*log((d^6*x^9 - 66*c*
d^5*x^6 - 72*c^2*d^4*x^3 - 32*c^3*d^3 - 24*(1/4)^(2/3)*(c^9*d^3*x^8 - 7*c^1
0*d^2*x^5 - 8*c^11*d*x^2)*(-d^4/c^13)^(2/3) - 48*(1/4)^(1/3)*(c^5*d^4*x^7 -
 c^6*d^3*x^4 - 2*c^7*d^2*x)*(-d^4/c^13)^(1/3) - 6*(18*(1/108)^(1/6)*c^3*d^4
*x^5*(-d^4/c^13)^(1/6) + 36*(1/108)^(5/6)*(c^11*d^2*x^7 - 16*c^12*d*x^4 - 8
*c^13*x)*(-d^4/c^13)^(5/6) + sqrt(1/3)*(5*c^7*d^3*x^6 - 20*c^8*d^2*x^3 - 16
*c^9*d)*sqrt(-d^4/c^13))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*
d*x^3 + 64*c^3)) - 60*sqrt(d)*x^2*weierstrassPInverse(0, -4*c/d, x) + (1/10
8)^(1/6)*(sqrt(-3)*c^2*x^2 + c^2*x^2)*(-d^4/c^13)^(1/6)*log((d^6*x^9 - 66*c
*d^5*x^6 - 72*c^2*d^4*x^3 - 32*c^3*d^3 + 12*(1/4)^(2/3)*(c^9*d^3*x^8 - 7*c^
10*d^2*x^5 - 8*c^11*d*x^2 + sqrt(-3)*(c^9*d^3*x^8 - 7*c^10*d^2*x^5 - 8*c^11
*d*x^2))*(-d^4/c^13)^(2/3) + 24*(1/4)^(1/3)*(c^5*d^4*x^7 - c^6*d^3*x^4 - 2*
c^7*d^2*x - sqrt(-3)*(c^5*d^4*x^7 - c^6*d^3*x^4 - 2*c^7*d^2*x))*(-d^4/c^13)
^(1/3) + 6*sqrt(d*x^3 + c)*(18*(1/108)^(5/6)*(c^11*d^2*x^7 - 16*c^12*d*x^4
- 8*c^13*x - sqrt(-3)*(c^11*d^2*x^7 - 16*c^12*d*x^4 - 8*c^13*x))*(-d^4/c^13
```

```
)^(5/6) - sqrt(1/3)*(5*c^7*d^3*x^6 - 20*c^8*d^2*x^3 - 16*c^9*d)*sqrt(-d^4/c
^13) + 9*(1/108)^(1/6)*(sqrt(-3)*c^3*d^4*x^5 + c^3*d^4*x^5)*(-d^4/c^13)^(1/
6)))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - (1/108)^(1/6)*(sqr
t(-3)*c^2*x^2 + c^2*x^2)*(-d^4/c^13)^(1/6)*log((d^6*x^9 - 66*c*d^5*x^6 - 72
*c^2*d^4*x^3 - 32*c^3*d^3 + 12*(1/4)^(2/3)*(c^9*d^3*x^8 - 7*c^10*d^2*x^5 -
8*c^11*d*x^2 + sqrt(-3)*(c^9*d^3*x^8 - 7*c^10*d^2*x^5 - 8*c^11*d*x^2))*(-d^
4/c^13)^(2/3) + 24*(1/4)^(1/3)*(c^5*d^4*x^7 - c^6*d^3*x^4 - 2*c^7*d^2*x - s
qrt(-3)*(c^5*d^4*x^7 - c^6*d^3*x^4 - 2*c^7*d^2*x))*(-d^4/c^13)^(1/3) - 6*sq
rt(d*x^3 + c)*(18*(1/108)^(5/6)*(c^11*d^2*x^7 - 16*c^12*d*x^4 - 8*c^13*x -
sqrt(-3)*(c^11*d^2*x^7 - 16*c^12*d*x^4 - 8*c^13*x))*(-d^4/c^13)^(5/6) - sqr
t(1/3)*(5*c^7*d^3*x^6 - 20*c^8*d^2*x^3 - 16*c^9*d)*sqrt(-d^4/c^13) + 9*(1/1
08)^(1/6)*(sqrt(-3)*c^3*d^4*x^5 + c^3*d^4*x^5)*(-d^4/c^13)^(1/6)))/(d^3*x^9
 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - (1/108)^(1/6)*(sqrt(-3)*c^2*x^2
 - c^2*x^2)*(-d^4/c^13)^(1/6)*log((d^6*x^9 - 66*c*d^5*x^6 - 72*c^2*d^4*x^3
- 32*c^3*d^3 + 12*(1/4)^(2/3)*(c^9*d^3*x^8 - 7*c^10*d^2*x^5 - 8*c^11*d*x^2
- sqrt(-3)*(c^9*d^3*x^8 - 7*c^10*d^2*x^5 - 8*c^11*d*x^2))*(-d^4/c^13)^(2/3)
 + 24*(1/4)^(1/3)*(c^5*d^4*x^7 - c^6*d^3*x^4 - 2*c^7*d^2*x + sqrt(-3)*(c^5*
d^4*x^7 - c^6*d^3*x^4 - 2*c^7*d^2*x))*(-d^4/c^13)^(1/3) + 6*sqrt(d*x^3 + c)
*(18*(1/108)^(5/6)*(c^11*d^2*x^7 - 16*c^12*d*x^4 - 8*c^13*x + sqrt(-3)*(c^1
1*d^2*x^7 - 16*c^12*d*x^4 - 8*c^13*x))*(-d^4/c^13)^(5/6) - sqrt(1/3)*(5*c^7
*d^3*x^6 - 20*c^8*d^2*x^3 - 16*c^9*d)*sqrt(-d^4/c^13) - 9*(1/108)^(1/6)*(sq
rt(-3)*c^3*d^4*x^5 - c^3*d^4*x^5)*(-d^4/c^13)^(1/6)))/(d^3*x^9 + 12*c*d^2*x
^6 + 48*c^2*d*x^3 + 64*c^3)) + (1/108)^(1/6)*(sqrt(-3)*c^2*x^2 - c^2*x^2)*(
-d^4/c^13)^(1/6)*log((d^6*x^9 - 66*c*d^5*x^6 - 72*c^2*d^4*x^3 - 32*c^3*d^3
+ 12*(1/4)^(2/3)*(c^9*d^3*x^8 - 7*c^10*d^2*x^5 - 8*c^11*d*x^2 - sqrt(-3)*(c
^9*d^3*x^8 - 7*c^10*d^2*x^5 - 8*c^11*d*x^2))*(-d^4/c^13)^(2/3) + 24*(1/4)^(
1/3)*(c^5*d^4*x^7 - c^6*d^3*x^4 - 2*c^7*d^2*x + sqrt(-3)*(c^5*d^4*x^7 - c^6
*d^3*x^4 - 2*c^7*d^2*x))*(-d^4/c^13)^(1/3) - 6*sqrt(d*x^3 + c)*(18*(1/108)^
(5/6)*(c^11*d^2*x^7 - 16*c^12*d*x^4 - 8*c^13*x + sqrt(-3)*(c^11*d^2*x^7 - 1
6*c^12*d*x^4 - 8*c^13*x))*(-d^4/c^13)^(5/6) - sqrt(1/3)*(5*c^7*d^3*x^6 - 20
*c^8*d^2*x^3 - 16*c^9*d)*sqrt(-d^4/c^13) - 9*(1/108)^(1/6)*(sqrt(-3)*c^3*d^
4*x^5 - c^3*d^4*x^5)*(-d^4/c^13)^(1/6)))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d
*x^3 + 64*c^3)) - 36*sqrt(d*x^3 + c))/(c^2*x^2)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{x^3\sqrt{c+dx^3}\cdot(4c+dx^3)}\,dx$$

```
[In] integrate(1/x**3/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{(dx^3+4\,c)\sqrt{dx^3+c}x^3}\,dx$$

```
[In] integrate(1/x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{(dx^3+4\,c)\sqrt{dx^3+c}x^3}\,dx$$

```
[In] integrate(1/x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \int \frac{1}{x^3\,\sqrt{d\,x^3+c}\,\left(d\,x^3+4\,c\right)}\,dx$$

```
[In] int(1/(x^3*(c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
[Out] int(1/(x^3*(c + d*x^3)^(1/2)*(4*c + d*x^3)), x)
```

3.281 $\int \frac{x}{\sqrt{1-x^3}(4-x^3)} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)} \, dx = -\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\operatorname{arctanh}\left(\frac{1+\sqrt[3]{2}x}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\operatorname{arctanh}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

```
[Out] -1/6*arctanh((1+2^(1/3)*x)/(-x^3+1)^(1/2))*2^(1/3)+1/18*arctanh((-x^3+1)^(1
/2))*2^(1/3)-1/18*arctan((1-2^(1/3)*x)*3^(1/2)/(-x^3+1)^(1/2))*2^(1/3)*3^(1
/2)+1/18*arctan(1/3*(-x^3+1)^(1/2)*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {497}

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)} \, dx = -\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{2}x+1}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\operatorname{arctanh}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

```
[In] Int[x/(Sqrt[1 - x^3]*(4 - x^3)),x]
```

```
[Out] -1/3*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]]/(2^(2/3)*Sqrt[3]) + Ar
cTan[Sqrt[1 - x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3]) - ArcTanh[(1 + 2^(1/3)*x)/S
qrt[1 - x^3]]/(3*2^(2/3)) + ArcTanh[Sqrt[1 - x^3]]/(9*2^(2/3))
```

Rule 497

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[q*(ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]]/(9*2^(2/3)*b
*Rt[c, 2])), x] + (-Simp[q*(ArcTanh[Rt[c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c + d*
x^3])]/(3*2^(2/3)*b*Rt[c, 2])), x] + Simp[q*(ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])), x] - Simp[q*(ArcTan[Sqrt[3]*R
t[c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rubi steps

$$\text{integral} = -\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{1+\sqrt[3]{2}x}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.22

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = \frac{1}{8}x^2\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},x^3,\frac{x^3}{4}\right)$$

```
[In] Integrate[x/(Sqrt[1 - x^3]*(4 - x^3)),x]
[Out] (x^2*AppellF1[2/3, 1/2, 1, 5/3, x^3, x^3/4])/8
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 13.97 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.29

method	result
default	$\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^{3}-4\right)}\frac{\_\alpha^{2}\sqrt{2}\,\sqrt{i\left(-i\sqrt{3}+2x+1\right)}\,\sqrt{\frac{-1+x}{i\sqrt{3}-3}}\,\sqrt{-\frac{i\left(i\sqrt{3}+2x+1\right)}{2}}\,\left(-2\_\alpha^{2}+\_\alpha+1+i\sqrt{3}\,(1-\_\alpha)\right)\Pi\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}\right.}{2\sqrt{-x^{3}+1}}\right.}{36}$
elliptic	$\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^{3}-4\right)}\frac{\_\alpha^{2}\sqrt{2}\,\sqrt{i\left(-i\sqrt{3}+2x+1\right)}\,\sqrt{\frac{-1+x}{i\sqrt{3}-3}}\,\sqrt{-\frac{i\left(i\sqrt{3}+2x+1\right)}{2}}\,\left(-2\_\alpha^{2}+\_\alpha+1+i\sqrt{3}\,(1-\_\alpha)\right)\Pi\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}\right.}{2\sqrt{-x^{3}+1}}\right.}{36}$
trager	Expression too large to display

```
[In] int(x/(-x^3+4)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/36*I*2^(1/2)*sum(_alpha^2*(1/2*I*(-I*3^(1/2)+2*x+1))^(1/2)*((-1+x)/(I*3^(
1/2)-3))^(1/2)*(-1/2*I*(I*3^(1/2)+2*x+1))^(1/2)/(-x^3+1)^(1/2)*(-2*_alpha^2
+_alpha+1+I*3^(1/2)*(1-_alpha))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1
/2))*3^(1/2))^(1/2),1/2*_alpha-1/3*I*_alpha^2*3^(1/2)-1/2+1/6*I*_alpha*3^(1
/2)+1/6*I*3^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^
3-4))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 1116, normalized size of antiderivative = 8.79

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15552*432^(5/6)*(-1)^(1/6)*(sqrt(-3) - 1)*log(-(72*x^9 + 4752*x^6 - 5184
*x^3 + 216*2^(2/3)*(-1)^(1/3)*(x^8 + 7*x^5 - 8*x^2 + sqrt(-3)*(x^8 + 7*x^5
- 8*x^2)) - 864*2^(1/3)*(-1)^(2/3)*(x^7 + x^4 - sqrt(-3)*(x^7 + x^4 - 2*x)
- 2*x) + sqrt(-x^3 + 1)*(432^(5/6)*(-1)^(1/6)*(x^7 + 16*x^4 - sqrt(-3)*(x^7
 + 16*x^4 - 8*x) - 8*x) + 648*432^(1/6)*(-1)^(5/6)*(sqrt(-3)*x^5 + x^5) + 1
44*sqrt(3)*(5*I*x^6 + 20*I*x^3 - 16*I)) + 2304)/(x^9 - 12*x^6 + 48*x^3 - 64
)) + 1/15552*432^(5/6)*(-1)^(1/6)*(sqrt(-3) - 1)*log(-(72*x^9 + 4752*x^6 -
5184*x^3 + 216*2^(2/3)*(-1)^(1/3)*(x^8 + 7*x^5 - 8*x^2 + sqrt(-3)*(x^8 + 7*
x^5 - 8*x^2)) - 864*2^(1/3)*(-1)^(2/3)*(x^7 + x^4 - sqrt(-3)*(x^7 + x^4 - 2
*x) - 2*x) - sqrt(-x^3 + 1)*(432^(5/6)*(-1)^(1/6)*(x^7 + 16*x^4 - sqrt(-3)*
```

```
(x^7 + 16*x^4 - 8*x) - 8*x) + 648*432^(1/6)*(-1)^(5/6)*(sqrt(-3)*x^5 + x^5)
 - 144*sqrt(3)*(-5*I*x^6 - 20*I*x^3 + 16*I)) + 2304)/(x^9 - 12*x^6 + 48*x^3
 - 64)) + 1/15552*432^(5/6)*(-1)^(1/6)*(sqrt(-3) + 1)*log(-(72*x^9 + 4752*x
^6 - 5184*x^3 + 216*2^(2/3)*(-1)^(1/3)*(x^8 + 7*x^5 - 8*x^2 - sqrt(-3)*(x^8
 + 7*x^5 - 8*x^2)) - 864*2^(1/3)*(-1)^(2/3)*(x^7 + x^4 + sqrt(-3)*(x^7 + x^
4 - 2*x) - 2*x) + sqrt(-x^3 + 1)*(432^(5/6)*(-1)^(1/6)*(x^7 + 16*x^4 + sqrt
(-3)*(x^7 + 16*x^4 - 8*x) - 8*x) - 648*432^(1/6)*(-1)^(5/6)*(sqrt(-3)*x^5 -
 x^5) + 144*sqrt(3)*(5*I*x^6 + 20*I*x^3 - 16*I)) + 2304)/(x^9 - 12*x^6 + 48
*x^3 - 64)) - 1/15552*432^(5/6)*(-1)^(1/6)*(sqrt(-3) + 1)*log(-(72*x^9 + 47
52*x^6 - 5184*x^3 + 216*2^(2/3)*(-1)^(1/3)*(x^8 + 7*x^5 - 8*x^2 - sqrt(-3)*
(x^8 + 7*x^5 - 8*x^2)) - 864*2^(1/3)*(-1)^(2/3)*(x^7 + x^4 + sqrt(-3)*(x^7
+ x^4 - 2*x) - 2*x) - sqrt(-x^3 + 1)*(432^(5/6)*(-1)^(1/6)*(x^7 + 16*x^4 +
sqrt(-3)*(x^7 + 16*x^4 - 8*x) - 8*x) - 648*432^(1/6)*(-1)^(5/6)*(sqrt(-3)*x
^5 - x^5) - 144*sqrt(3)*(-5*I*x^6 - 20*I*x^3 + 16*I)) + 2304)/(x^9 - 12*x^6
 + 48*x^3 - 64)) - 1/7776*432^(5/6)*(-1)^(1/6)*log(-(36*x^9 + 2376*x^6 - 25
92*x^3 - 216*2^(2/3)*(-1)^(1/3)*(x^8 + 7*x^5 - 8*x^2) + 864*2^(1/3)*(-1)^(2
/3)*(x^7 + x^4 - 2*x) - (648*432^(1/6)*(-1)^(5/6)*x^5 + 432^(5/6)*(-1)^(1/6
)*(x^7 + 16*x^4 - 8*x) - 72*sqrt(3)*(5*I*x^6 + 20*I*x^3 - 16*I))*sqrt(-x^3
+ 1) + 1152)/(x^9 - 12*x^6 + 48*x^3 - 64)) + 1/7776*432^(5/6)*(-1)^(1/6)*lo
g(-(36*x^9 + 2376*x^6 - 2592*x^3 - 216*2^(2/3)*(-1)^(1/3)*(x^8 + 7*x^5 - 8*
x^2) + 864*2^(1/3)*(-1)^(2/3)*(x^7 + x^4 - 2*x) + (648*432^(1/6)*(-1)^(5/6)
*x^5 + 432^(5/6)*(-1)^(1/6)*(x^7 + 16*x^4 - 8*x) + 72*sqrt(3)*(-5*I*x^6 - 2
0*I*x^3 + 16*I))*sqrt(-x^3 + 1) + 1152)/(x^9 - 12*x^6 + 48*x^3 - 64))
```

Sympy [F]

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = -\int \frac{x}{x^3\sqrt{1-x^3}-4\sqrt{1-x^3}}\,dx$$

```
[In] integrate(x/(-x**3+4)/(-x**3+1)**(1/2),x)
[Out] -Integral(x/(x**3*sqrt(1 - x**3) - 4*sqrt(1 - x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = \int -\frac{x}{(x^3-4)\sqrt{-x^3+1}}\,dx$$

```
[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="maxima")
[Out] -integrate(x/((x^3 - 4)*sqrt(-x^3 + 1)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = \int -\frac{x}{(x^3-4)\sqrt{-x^3+1}}\,dx$$

[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="giac")

[Out] integrate(-x/((x^3 - 4)*sqrt(-x^3 + 1)), x)

Mupad [B] (verification not implemented)

Time = 7.16 (sec) , antiderivative size = 653, normalized size of antiderivative = 5.14

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx = \text{Too large to display}$$

[In] int(-x/((1 - x^3)^(1/2)*(x^3 - 4)),x)

```
[Out] - (2^(1/3)*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1
/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-((3^(1
/2)*1i)/2 + 3/2)/(2^(2/3) - 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(1 - x^3)^(1/2)*(2^
(2/3) - 1)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i
)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (2^(1/3)*((3^(1/2)*1
i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 -
3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x
- 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(2^(2/
3)*((3^(1/2)*1i)/2 + 1/2) + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*((3^(1/2)*1i)/2 + 1
/2)*(1 - x^3)^(1/2)*(2^(2/3)*((3^(1/2)*1i)/2 + 1/2) + 1)*(((3^(1/2)*1i)/2 -
 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
1/2) + 1) + x^3)^(1/2)) - (2^(1/3)*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(
-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i
)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)*ellipticPi(-((3^(1/2)*1i)/2 + 3/2)/(2^(2/3)*((3^(1/2)*1i)/2 - 1/2) -
1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/
((3^(1/2)*1i)/2 - 3/2)))/(3*((3^(1/2)*1i)/2 - 1/2)*(1 - x^3)^(1/2)*(2^(2/3)
*((3^(1/2)*1i)/2 - 1/2) - 1)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.282 $\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 111

$$\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{1024c^3\sqrt{c+dx^3}}{3d^4}-\frac{38c^2(c+dx^3)^{3/2}}{3d^4}-\frac{4c(c+dx^3)^{5/2}}{5d^4}-\frac{2(c+dx^3)^{7/2}}{21d^4}+\frac{1024c^{7/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}$$

```
[Out] -38/3*c^2*(d*x^3+c)^(3/2)/d^4-4/5*c*(d*x^3+c)^(5/2)/d^4-2/21*(d*x^3+c)^(7/2
)/d^4+1024*c^(7/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^4-1024/3*c^3*(d*x
^3+c)^(1/2)/d^4
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {457, 90, 52, 65, 212}

$$\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx = \frac{1024c^{7/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}-\frac{1024c^3\sqrt{c+dx^3}}{3d^4}-\frac{38c^2(c+dx^3)^{3/2}}{3d^4}-\frac{4c(c+dx^3)^{5/2}}{5d^4}-\frac{2(c+dx^3)^{7/2}}{21d^4}$$

```
[In] Int[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-1024*c^3*Sqrt[c + d*x^3])/(3*d^4) - (38*c^2*(c + d*x^3)^(3/2))/(3*d^4) -
(4*c*(c + d*x^3)^(5/2))/(5*d^4) - (2*(c + d*x^3)^(7/2))/(21*d^4) + (1024*c^
(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^4
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^3\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{57c^2\sqrt{c+dx}}{d^3}+\frac{512c^3\sqrt{c+dx}}{d^3(8c-dx)}-\frac{6c(c+dx)^{3/2}}{d^3}-\frac{(c+dx)^{5/2}}{d^3}\right)\,dx, x, x^3\right)\\
&= -\frac{38c^2(c+dx^3)^{3/2}}{3d^4}-\frac{4c(c+dx^3)^{5/2}}{5d^4}-\frac{2(c+dx^3)^{7/2}}{21d^4}+\frac{(512c^3)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)}{3d^3}
\end{aligned}$$

$$= -\frac{1024c^3\sqrt{c+dx^3}}{3d^4} - \frac{38c^2(c+dx^3)^{3/2}}{3d^4} - \frac{4c(c+dx^3)^{5/2}}{5d^4} - \frac{2(c+dx^3)^{7/2}}{21d^4} + \frac{(1536c^4)\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{d^3}$$

$$= -\frac{1024c^3\sqrt{c+dx^3}}{3d^4} - \frac{38c^2(c+dx^3)^{3/2}}{3d^4} - \frac{4c(c+dx^3)^{5/2}}{5d^4} - \frac{2(c+dx^3)^{7/2}}{21d^4} + \frac{(3072c^4)\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d^4}$$

$$= -\frac{1024c^3\sqrt{c+dx^3}}{3d^4} - \frac{38c^2(c+dx^3)^{3/2}}{3d^4} - \frac{4c(c+dx^3)^{5/2}}{5d^4} - \frac{2(c+dx^3)^{7/2}}{21d^4} + \frac{1024c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.74

$$\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2\sqrt{c+dx^3}(18632c^3+764c^2dx^3+57cd^2x^6+5d^3x^9)}{105d^4} + \frac{1024c^{7/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}$$

```
[In] Integrate[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
[Out] (-2*Sqrt[c + d*x^3]*(18632*c^3 + 764*c^2*d*x^3 + 57*c*d^2*x^6 + 5*d^3*x^9))
/(105*d^4) + (1024*c^(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^4
```

Maple [A] (verified)

Time = 3.65 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.63

method	result
pseudoelliptic	$\frac{1024c^{\frac{7}{2}} \operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)-\frac{2\left(5d^3x^9+57c\,d^2x^6+764c^2d\,x^3+18632c^3\right)\sqrt{d\,x^3+c}}{105}}{d^4}$
risch	$-\frac{2\left(5d^3x^9+57c\,d^2x^6+764c^2d\,x^3+18632c^3\right)\sqrt{d\,x^3+c}}{105d^4}+\frac{1024c^{\frac{7}{2}} \operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{d^4}$
default	$-\frac{\frac{2x^9\sqrt{d\,x^3+c}}{21}+\frac{2c\,x^6\sqrt{d\,x^3+c}}{105d}-\frac{8c^2x^3\sqrt{d\,x^3+c}}{315d^2}+\frac{16c^3\sqrt{d\,x^3+c}}{315d^3}}{d}-\frac{8c\left(\frac{2x^6\sqrt{d\,x^3+c}}{15}+\frac{2c\,x^3\sqrt{d\,x^3+c}}{45d}-\frac{4c^2\sqrt{d\,x^3+c}}{45d^2}\right)}{d^2}-\frac{128c^2(}{}$
	$512ic^3\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^3-8c\right)} (- \right.$
elliptic	$-\frac{2x^9\sqrt{d\,x^3+c}}{21d}-\frac{38c\,x^6\sqrt{d\,x^3+c}}{35d^2}-\frac{1528c^2x^3\sqrt{d\,x^3+c}}{105d^3}-\frac{37264c^3\sqrt{d\,x^3+c}}{105d^4}-$

[In] int(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)

[Out] 2/105*(53760*c^(7/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))-(5*d^3*x^9+57*c*d^2*x^6+764*c^2*d*x^3+18632*c^3)*(d*x^3+c)^(1/2))/d^4

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.52

$$\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx$$
$$=\left[\frac{2\left(26880\,c^{\frac{7}{2}}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-(5\,d^3x^9+57\,cd^2x^6+764\,c^2dx^3+18632\,c^3)\sqrt{dx^3+c}\right)}{105\,d^4},\right.$$
$$\left.-\frac{2\left(53760\sqrt{-c}c^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+(5\,d^3x^9+57\,cd^2x^6+764\,c^2dx^3+18632\,c^3)\sqrt{dx^3+c}\right)}{105\,d^4}\right]$$

[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")

```
[Out] [2/105*(26880*c^(7/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3
 - 8*c)) - (5*d^3*x^9 + 57*c*d^2*x^6 + 764*c^2*d*x^3 + 18632*c^3)*sqrt(d*x^
3 + c))/d^4, -2/105*(53760*sqrt(-c)*c^3*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)
/c) + (5*d^3*x^9 + 57*c*d^2*x^6 + 764*c^2*d*x^3 + 18632*c^3)*sqrt(d*x^3 + c
))/d^4]
```

Sympy [A] (verification not implemented)

Time = 18.48 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.98

$$\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx = \begin{cases} \frac{2\left(-\frac{512c^4\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{512c^3\sqrt{c+dx^3}}{3}-\frac{19c^2\left(c+dx^3\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+dx^3\right)^{\frac{5}{2}}}{5}-\frac{\left(c+dx^3\right)^{\frac{7}{2}}}{21}\right)}{d^4} & \text{for } d\neq 0 \\ \frac{x^{12}}{96\sqrt{c}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**11*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

```
[Out] Piecewise((2*(-512*c**4*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/sqrt(-c) - 512*
c**3*sqrt(c + d*x**3)/3 - 19*c**2*(c + d*x**3)**(3/2)/3 - 2*c*(c + d*x**3)*
*(5/2)/5 - (c + d*x**3)**(7/2)/21)/d**4, Ne(d, 0)), (x**12/(96*sqrt(c)), Tr
ue))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.86

$$\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2\left(26880\,c^{\frac{7}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+5\left(dx^3+c\right)^{\frac{7}{2}}+42\left(dx^3+c\right)^{\frac{5}{2}}c+665\left(dx^3+c\right)^{\frac{3}{2}}c^2+17920\sqrt{dx^3+c}c^3\right)}{105\,d^4}$$

```
[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -2/105*(26880*c^(7/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) +
3*sqrt(c))) + 5*(d*x^3 + c)^(7/2) + 42*(d*x^3 + c)^(5/2)*c + 665*(d*x^3 + c
)^(3/2)*c^2 + 17920*sqrt(d*x^3 + c)*c^3)/d^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.90

$$\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx$$
$$= -\frac{1024\,c^4\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d^4}$$
$$- \frac{2\left(5\,(dx^3+c)^{\frac{7}{2}}d^{24}+42\,(dx^3+c)^{\frac{5}{2}}cd^{24}+665\,(dx^3+c)^{\frac{3}{2}}c^2d^{24}+17920\,\sqrt{dx^3+c}c^3d^{24}\right)}{105\,d^{28}}$$

```
[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -1024*c^4*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 2/105*(5*(d
*x^3 + c)^(7/2)*d^24 + 42*(d*x^3 + c)^(5/2)*c*d^24 + 665*(d*x^3 + c)^(3/2)*
c^2*d^24 + 17920*sqrt(d*x^3 + c)*c^3*d^24)/d^28
```

Mupad [B] (verification not implemented)

Time = 7.23 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.06

$$\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx = \frac{512\,c^{7/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^4} - \frac{37264\,c^3\,\sqrt{d\,x^3+c}}{105\,d^4} - \frac{2\,x^9\,\sqrt{d\,x^3+c}}{21\,d} - \frac{38\,c\,x^6\,\sqrt{d\,x^3+c}}{35\,d^2} - \frac{1528\,c^2\,x^3\,\sqrt{d\,x^3+c}}{105\,d^3}$$

```
[In] int((x^11*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
```

```
[Out] (512*c^(7/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)
))/d^4 - (37264*c^3*(c + d*x^3)^(1/2))/(105*d^4) - (2*x^9*(c + d*x^3)^(1/2)
)/(21*d) - (38*c*x^6*(c + d*x^3)^(1/2))/(35*d^2) - (1528*c^2*x^3*(c + d*x^3
)^(1/2))/(105*d^3)
```

3.283 $\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 90

$$\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx=-\frac{128c^2\sqrt{c+dx^3}}{3d^3}-\frac{14c(c+dx^3)^{3/2}}{9d^3}-\frac{2(c+dx^3)^{5/2}}{15d^3}+\frac{128c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}$$

```
[Out] -14/9*c*(d*x^3+c)^(3/2)/d^3-2/15*(d*x^3+c)^(5/2)/d^3+128*c^(5/2)*arctanh(1/
3*(d*x^3+c)^(1/2)/c^(1/2))/d^3-128/3*c^2*(d*x^3+c)^(1/2)/d^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {457, 90, 52, 65, 212}

$$\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx=\frac{128c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}-\frac{128c^2\sqrt{c+dx^3}}{3d^3}-\frac{14c(c+dx^3)^{3/2}}{9d^3}-\frac{2(c+dx^3)^{5/2}}{15d^3}$$

```
[In] Int[(x^8*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-128*c^2*Sqrt[c + d*x^3])/(3*d^3) - (14*c*(c + d*x^3)^(3/2))/(9*d^3) - (2*
(c + d*x^3)^(5/2))/(15*d^3) + (128*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[
c])])/d^3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{7c\sqrt{c+dx}}{d^2}+\frac{64c^2\sqrt{c+dx}}{d^2(8c-dx)}-\frac{(c+dx)^{3/2}}{d^2}\right)\,dx, x, x^3\right)\\
&= -\frac{14c(c+dx^3)^{3/2}}{9d^3}-\frac{2(c+dx^3)^{5/2}}{15d^3}+\frac{(64c^2)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)}{3d^2}
\end{aligned}
$$

$$= -\frac{128c^2\sqrt{c+dx^3}}{3d^3} - \frac{14c(c+dx^3)^{3/2}}{9d^3} - \frac{2(c+dx^3)^{5/2}}{15d^3} + \frac{(192c^3)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{d^2}$$

$$= -\frac{128c^2\sqrt{c+dx^3}}{3d^3} - \frac{14c(c+dx^3)^{3/2}}{9d^3} - \frac{2(c+dx^3)^{5/2}}{15d^3} + \frac{(384c^3)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d^3}$$

$$= -\frac{128c^2\sqrt{c+dx^3}}{3d^3} - \frac{14c(c+dx^3)^{3/2}}{9d^3} - \frac{2(c+dx^3)^{5/2}}{15d^3} + \frac{128c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.79

$$\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2\sqrt{c+dx^3}(998c^2+41cdx^3+3d^2x^6)}{45d^3} + \frac{128c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}$$

```
[In] Integrate[(x^8*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
[Out] (-2*Sqrt[c + d*x^3]*(998*c^2 + 41*c*d*x^3 + 3*d^2*x^6))/(45*d^3) + (128*c^(
5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^3
```

Maple [A] (verified)

Time = 4.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$\frac{\frac{2\left(-3d^2x^6-41cd\,x^3-998c^2\right)\sqrt{d\,x^3+c}}{45}+128c^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{d^3}$
risch	$-\frac{2\left(3d^2x^6+41cd\,x^3+998c^2\right)\sqrt{d\,x^3+c}}{45d^3}+\frac{128c^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{d^3}$
default	$-\frac{\frac{2x^6\sqrt{d\,x^3+c}}{15}+\frac{2c\,x^3\sqrt{d\,x^3+c}}{45d}-\frac{4c^2\sqrt{d\,x^3+c}}{45d^2}}{d}-\frac{16c\left(d\,x^3+c\right)^{\frac{3}{2}}}{9d^3}+\frac{64c^2\left(-2\sqrt{d\,x^3+c}+6\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)\sqrt{c}\right)}{3d^3}$
elliptic	$-\frac{2x^6\sqrt{d\,x^3+c}}{15d}-\frac{82c\,x^3\sqrt{d\,x^3+c}}{45d^2}-\frac{1996c^2\sqrt{d\,x^3+c}}{45d^3}-\frac{64ic^2\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(\cdots\right.}{\cdots}\right)}{\left(\cdots\right.}}\cdots}{\cdots}\right.}{\cdots}$

[In] int(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)

[Out] 2/45*((-3*d^2*x^6-41*c*d*x^3-998*c^2)*(d*x^3+c)^(1/2)+2880*c^(5/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2)))/d^3

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.63

$$\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx = \left[\frac{2\left(1440\,c^{\frac{5}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-\left(3\,d^2x^6+41\,cdx^3+998\,c^2\right)\sqrt{dx^3+c}\right)}{45\,d^3},\ -\frac{2\left(2880\,\sqrt{-c}c^2\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+\left(3\,d^2x^6+41\,cdx^3+998\,c^2\right)\sqrt{dx^3+c}\right)}{45\,d^3}\right]$$

[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")

```
[Out] [2/45*(1440*c^(5/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 -
 8*c)) - (3*d^2*x^6 + 41*c*d*x^3 + 998*c^2)*sqrt(d*x^3 + c))/d^3, -2/45*(28
80*sqrt(-c)*c^2*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (3*d^2*x^6 + 41*c*
d*x^3 + 998*c^2)*sqrt(d*x^3 + c))/d^3]
```

Sympy [A] (verification not implemented)

Time = 8.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx = \begin{cases} \frac{2\left(-\frac{64c^3\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{64c^2\sqrt{c+dx^3}}{3}-\frac{7c\left(c+dx^3\right)^{\frac{3}{2}}}{9}-\frac{\left(c+dx^3\right)^{\frac{5}{2}}}{15}\right)}{d^3} & \text{for } d\neq 0 \\ \frac{x^9}{72\sqrt{c}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**8*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

```
[Out] Piecewise((2*(-64*c**3*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/sqrt(-c) - 64*c*
*2*sqrt(c + d*x**3)/3 - 7*c*(c + d*x**3)**(3/2)/9 - (c + d*x**3)**(5/2)/15)
/d**3, Ne(d, 0)), (x**9/(72*sqrt(c)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2\left(1440\,c^{\frac{5}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+3\left(dx^3+c\right)^{\frac{5}{2}}+35\left(dx^3+c\right)^{\frac{3}{2}}c+960\sqrt{dx^3+c}c^2\right)}{45\,d^3}$$

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -2/45*(1440*c^(5/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*
sqrt(c))) + 3*(d*x^3 + c)^(5/2) + 35*(d*x^3 + c)^(3/2)*c + 960*sqrt(d*x^3 +
 c)*c^2)/d^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92

$$\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{128\,c^3\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d^3} - \frac{2\left(3\,(dx^3+c)^{\frac{5}{2}}d^{12}+35\,(dx^3+c)^{\frac{3}{2}}cd^{12}+960\,\sqrt{dx^3+c}c^2d^{12}\right)}{45\,d^{15}}$$

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -128*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/45*(3*(d*x
^3 + c)^(5/2)*d^12 + 35*(d*x^3 + c)^(3/2)*c*d^12 + 960*sqrt(d*x^3 + c)*c^2*
d^12)/d^15
```

Mupad [B] (verification not implemented)

Time = 7.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.09

$$\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx = \frac{64\,c^{5/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^3} - \frac{1996\,c^2\,\sqrt{d\,x^3+c}}{45\,d^3} - \frac{2\,x^6\,\sqrt{d\,x^3+c}}{15\,d} - \frac{82\,c\,x^3\,\sqrt{d\,x^3+c}}{45\,d^2}$$

```
[In] int((x^8*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
```

```
[Out] (64*c^(5/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))
)/d^3 - (1996*c^2*(c + d*x^3)^(1/2))/(45*d^3) - (2*x^6*(c + d*x^3)^(1/2))/(
15*d) - (82*c*x^3*(c + d*x^3)^(1/2))/(45*d^2)
```

3.284 $\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 69

$$\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx=-\frac{16c\sqrt{c+dx^3}}{3d^2}-\frac{2(c+dx^3)^{3/2}}{9d^2}+\frac{16c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}$$

```
[Out] -2/9*(d*x^3+c)^(3/2)/d^2+16*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^
2-16/3*c*(d*x^3+c)^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {457, 81, 52, 65, 212}

$$\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx=\frac{16c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}-\frac{16c\sqrt{c+dx^3}}{3d^2}-\frac{2(c+dx^3)^{3/2}}{9d^2}$$

```
[In] Int[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-16*c*Sqrt[c + d*x^3])/(3*d^2) - (2*(c + d*x^3)^(3/2))/(9*d^2) + (16*c^(3/
2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^2
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)\\
&= -\frac{2(c+dx^3)^{3/2}}{9d^2} + \frac{(8c)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)}{3d}\\
&= -\frac{16c\sqrt{c+dx^3}}{3d^2} - \frac{2(c+dx^3)^{3/2}}{9d^2} + \frac{(24c^2)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{d}\\
&= -\frac{16c\sqrt{c+dx^3}}{3d^2} - \frac{2(c+dx^3)^{3/2}}{9d^2} + \frac{(48c^2)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d^2}\\
&= -\frac{16c\sqrt{c+dx^3}}{3d^2} - \frac{2(c+dx^3)^{3/2}}{9d^2} + \frac{16c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2\sqrt{c+dx^3}(25c+dx^3)}{9d^2}+\frac{16c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}$$

```
[In] Integrate[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(25*c + d*x^3))/(9*d^2) + (16*c^(3/2)*ArcTanh[Sqrt[c +
d*x^3]/(3*Sqrt[c])])/d^2
```

Maple [A] (verified)

Time = 4.43 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$\frac{16c^{\frac{3}{2}}\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)-\frac{2\left(d\,x^3+25c\right)\sqrt{d\,x^3+c}}{9}}{d^2}$
risch	$-\frac{2\left(d\,x^3+25c\right)\sqrt{d\,x^3+c}}{9d^2}+\frac{16c^{\frac{3}{2}}\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{d^2}$
default	$-\frac{2\left(d\,x^3+c\right)^{\frac{3}{2}}}{9d^2}+\frac{8c\left(-2\sqrt{d\,x^3+c}+6\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)\sqrt{c}\right)}{3d^2}$
elliptic	$-\frac{2x^3\sqrt{d\,x^3+c}}{9d}-\frac{50c\sqrt{d\,x^3+c}}{9d^2}-\frac{8ic\sqrt{2}\left(\sum_{\_\alpha=\text{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\cdots}{-3(\cdots}}\cdots}{\cdots}\right.}{\cdots}$

```
[In] int(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*(72*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))-(d*x^3+25*c)*(d*x^3+c)
^(1/2))/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.75

$$\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx = \left[\frac{2\left(36\,c^{\frac{3}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-(dx^3+25\,c)\sqrt{dx^3+c}\right)}{9\,d^2},\ -\frac{2\left(72\,\sqrt{-c}c\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+(dx^3+25\,c)\sqrt{dx^3+c}\right)}{9\,d^2}\right]$$

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [2/9*(36*c^(3/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*
c)) - (d*x^3 + 25*c)*sqrt(d*x^3 + c))/d^2, -2/9*(72*sqrt(-c)*c*arctan(1/3*s
qrt(d*x^3 + c)*sqrt(-c)/c) + (d*x^3 + 25*c)*sqrt(d*x^3 + c))/d^2]
```

Sympy [A] (verification not implemented)

Time = 3.74 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09

$$\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx = \begin{cases} \frac{2\left(-\frac{8c^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{8c\sqrt{c+dx^3}}{3}-\frac{\left(c+dx^3\right)^{\frac{3}{2}}}{9}\right)}{d^2} & \text{for } d\neq 0 \\ \frac{x^6}{48\sqrt{c}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**5*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

```
[Out] Piecewise((2*(-8*c**2*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/sqrt(-c) - 8*c*sq
rt(c + d*x**3)/3 - (c + d*x**3)**(3/2)/9)/d**2, Ne(d, 0)), (x**6/(48*sqrt(c
)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2\left(36\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+(dx^3+c)^{\frac{3}{2}}+24\,\sqrt{dx^3+c}c\right)}{9\,d^2}$$

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -2/9*(36*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqr
t(c))) + (d*x^3 + c)^(3/2) + 24*sqrt(d*x^3 + c)*c)/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{16\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d^2} - \frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}d^4 + 24\sqrt{dx^3+c}cd^4\right)}{9\,d^6}$$

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -16*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^2) - 2/9*((d*x^3 +
 c)^(3/2)*d^4 + 24*sqrt(d*x^3 + c)*c*d^4)/d^6
```

Mupad [B] (verification not implemented)

Time = 7.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx = \frac{8\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^2} - \frac{50\,c\,\sqrt{d\,x^3+c}}{9\,d^2} - \frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d}$$

```
[In] int((x^5*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
```

```
[Out] (8*c^(3/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))
/d^2 - (50*c*(c + d*x^3)^(1/2))/(9*d^2) - (2*x^3*(c + d*x^3)^(1/2))/(9*d)
```

3.285 $\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 50

$$\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2\sqrt{c+dx^3}}{3d} + \frac{2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

```
[Out] 2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/d-2/3*(d*x^3+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {455, 52, 65, 212}

$$\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx = \frac{2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d} - \frac{2\sqrt{c+dx^3}}{3d}$$

```
[In] Int[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3])/(3*d) + (2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])
])/d
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)\\
&= -\frac{2\sqrt{c+dx^3}}{3d} + (3c)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= -\frac{2\sqrt{c+dx^3}}{3d} + \frac{(6c)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d}\\
&= -\frac{2\sqrt{c+dx^3}}{3d} + \frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2\left(\sqrt{c+dx^3} - 3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{3d}$$

```
[In] Integrate[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
[Out] (-2*(Sqrt[c + d*x^3] - 3*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(3*
d)
```

Maple [A] (verified)

Time = 4.34 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

method	result
default	$\frac{-2\sqrt{d\,x^{3}+c}+6\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\sqrt{c}}{3d}$
pseudoelliptic	$\frac{-2\sqrt{d\,x^{3}+c}+6\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\sqrt{c}}{3d}$
risch	$\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\sqrt{c}}{d}-\frac{2\sqrt{d\,x^{3}+c}}{3d}$
elliptic	$-\frac{2\sqrt{d\,x^{3}+c}}{3d}-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right.}}}{}\right)}{}$

[In] int(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)

[Out] 1/3/d*(-2*(d*x^3+c)^(1/2)+6*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.02

$$\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx=\left[\frac{3\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-2\sqrt{dx^3+c}}{3\,d},\ -\frac{2\left(3\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+\sqrt{dx^3+c}\right)}{3\,d}\right]$$

[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")

```
[Out] [1/3*(3*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c
)) - 2*sqrt(d*x^3 + c))/d, -2/3*(3*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt
(-c)/c) + sqrt(d*x^3 + c))/d]
```

Sympy [A] (verification not implemented)

Time = 1.90 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12

$$\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx = \begin{cases} \frac{2\left(-\frac{c\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{\sqrt{c+dx^3}}{3}\right)}{d} & \text{for } d\neq 0 \\ \frac{x^3}{24\sqrt{c}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

```
[Out] Piecewise((2*(-c*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/sqrt(-c) - sqrt(c + d*
x**3)/3)/d, Ne(d, 0)), (x**3/(24*sqrt(c)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12

$$\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{3\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+2\sqrt{dx^3+c}}{3\,d}$$

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt
(c))) + 2*sqrt(d*x^3 + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d}-\frac{2\sqrt{dx^3+c}}{3\,d}$$

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -2*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 2/3*sqrt(d*x^3 + c
)/d
```

Mupad [B] (verification not implemented)

Time = 7.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.18

$$\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx = \frac{\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d} - \frac{2\,\sqrt{d\,x^3+c}}{3\,d}$$

```
[In] int((x^2*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
[Out] (c^(1/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/d
 - (2*(c + d*x^3)^(1/2))/(3*d)
```

3.286 $\int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 58

$$\int \frac{\sqrt{c+dx^3}}{x\,(8c-dx^3)}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\sqrt{c}}$$

```
[Out] 1/4*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)-1/12*arctanh((d*x^3+c)^(1/
2)/c^(1/2))/c^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {457, 85, 65, 214, 212}

$$\int \frac{\sqrt{c+dx^3}}{x\,(8c-dx^3)}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\sqrt{c}}$$

```
[In] Int[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)),x]
```

```
[Out] ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(4*Sqrt[c]) - ArcTanh[Sqrt[c + d*x^3]/
Sqrt[c]]/(12*Sqrt[c])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x(8c-dx)}\,dx, x, x^3\right)\\
&= \frac{1}{24}\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right) + \frac{1}{8}(3d)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= \frac{3}{4}\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right) + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{12d}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\sqrt{c}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{c+dx^3}}{x\,(8c-dx^3)}\,dx=\frac{3\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\sqrt{c}}$$

```
[In] Integrate[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)),x]
[Out] (3*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])
/(12*Sqrt[c])
```

Maple [A] (verified)

Time = 4.55 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.66

method	result	size
pseudoelliptic	$-\frac{-3\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{12\sqrt{c}}$	38
default	$\frac{\frac{2\sqrt{d\,x^3+c}}{3}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)\sqrt{c}}{3}}{8c}+\frac{-2\sqrt{d\,x^3+c}+6\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)\sqrt{c}}{24c}$	75
elliptic	Expression too large to display	1502

```
[In] int((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
[Out] -1/12*(-3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))+arctanh((d*x^3+c)^(1/2)/c^(1
/2)))/c^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.38

$$\int \frac{\sqrt{c+dx^3}}{x\,(8c-dx^3)}\,dx = \left[\frac{3\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)}{24\,c}, \frac{\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-3\sqrt{-c}\arctan\left(\right.}{12\,c}\right.$$

```
[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x, algorithm="fricas")
[Out] [1/24*(3*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*
c)) + sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/c, 1/12*(
sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 3*sqrt(-c)*arctan(1/3*sqrt(d*
x^3 + c)*sqrt(-c)/c))/c]
```

Sympy [A] (verification not implemented)

Time = 2.77 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{c+dx^3}}{x\,(8c-dx^3)}\,dx = \begin{cases} \frac{2\left(-\frac{d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{8\sqrt{-c}}+\frac{d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{24\sqrt{-c}}\right)}{d} & \text{for } d\neq 0 \\ \frac{\log\left(x^3\right)}{24\sqrt{c}} & \text{otherwise} \end{cases}$$

```
[In] integrate((d*x**3+c)**(1/2)/x/(-d*x**3+8*c),x)
[Out] Piecewise((2*(-d*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(8*sqrt(-c)) + d*atan(
sqrt(c + d*x**3)/sqrt(-c))/(24*sqrt(-c)))/d, Ne(d, 0)), (log(x**3)/(24*sqrt
(c)), True))
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x\,(8c-dx^3)}\,dx = \int -\frac{\sqrt{dx^3+c}}{(dx^3-8\,c)x}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x, algorithm="maxima")
[Out] -integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{c+dx^3}}{x\,(8c-dx^3)}\,dx = \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{12\,\sqrt{-c}} - \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{4\,\sqrt{-c}}$$

```
[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x, algorithm="giac")
[Out] 1/12*arctan(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) - 1/4*arctan(1/3*sqrt(d*x^3
+ c)/sqrt(-c))/sqrt(-c)
```

Mupad [B] (verification not implemented)

Time = 8.42 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.16

$$\int \frac{\sqrt{c+dx^3}}{x\,(8c-dx^3)}\,dx = \frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)\,\left(6\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}\right)^3\,\left(24\,c^2-24\,c^{3/2}\,\sqrt{d\,x^3+c}+d^2\,x^6-20\,c\,d\,x^3\right)^3}{x^{15}\,\left(8\,c-d\,x^3\right)^3\,\left(24\,c-d\,x^3\right)^3}\right)}{24\,\sqrt{c}}$$

```
[In] int((c + d*x^3)^(1/2)/(x*(8*c - d*x^3)),x)
```

```
[Out] log(((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))*(6*c + d
*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))^3*(24*c^2 - 24*c^(3/2)*(c + d*x^3)^(1/2
) + d^2*x^6 - 20*c*d*x^3)^3)/(x^15*(8*c - d*x^3)^3*(24*c - d*x^3)^3))/(24*c
^(1/2))
```

3.287 $\int \frac{\sqrt{c+dx^3}}{x^4(8c-dx^3)} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 81

$$\int \frac{\sqrt{c+dx^3}}{x^4(8c-dx^3)} \, dx = -\frac{\sqrt{c+dx^3}}{24cx^3} + \frac{d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32c^{3/2}} - \frac{5d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}$$

```
[Out] 1/32*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-5/96*d*arctanh((d*x^3+c
)^(1/2)/c^(1/2))/c^(3/2)-1/24*(d*x^3+c)^(1/2)/c/x^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {457, 101, 162, 65, 214, 212}

$$\int \frac{\sqrt{c+dx^3}}{x^4(8c-dx^3)} \, dx = \frac{d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32c^{3/2}} - \frac{5d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}} - \frac{\sqrt{c+dx^3}}{24cx^3}$$

```
[In] Int[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)),x]
```

```
[Out] -1/24*Sqrt[c + d*x^3]/(c*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3
2*c^(3/2)) - (5*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*c^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(8c-dx)}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{24cx^3} + \frac{\text{Subst}\left(\int \frac{5cd+\frac{d^2x}{2}}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{24c}$$

$$= -\frac{\sqrt{c+dx^3}}{24cx^3} + \frac{(5d)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{192c} + \frac{(3d^2)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{64c}$$

$$= -\frac{\sqrt{c+dx^3}}{24cx^3} + \frac{5\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{96c} + \frac{(3d)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{32c}$$

$$= -\frac{\sqrt{c+dx^3}}{24cx^3} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32c^{3/2}} - \frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+dx^3}}{x^4\,(8c-dx^3)}\,dx = -\frac{\sqrt{c+dx^3}}{24cx^3} + \frac{d\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32c^{3/2}} - \frac{5d\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}$$

```
[In] Integrate[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)),x]
```

```
[Out] -1/24*Sqrt[c + d*x^3]/(c*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3
2*c^(3/2)) - (5*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*c^(3/2))
```

Maple [A] (verified)

Time = 4.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{\sqrt{d\,x^3+c}}{24c\,x^3}+\frac{d\left(-\frac{5\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{6\sqrt{c}}+\frac{\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{2\sqrt{c}}\right)}{16c}$
pseudoelliptic	$\frac{-5\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)d\,x^3+3\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)d\,x^3-4\sqrt{d\,x^3+c}\,\sqrt{c}}{96c^{\frac{3}{2}}\,x^3}$
default	$\frac{-\frac{\sqrt{d\,x^3+c}}{3x^3}-\frac{d\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3\sqrt{c}}}{8c}+\frac{d\left(\frac{2\sqrt{d\,x^3+c}}{3}-\frac{2\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)\sqrt{c}}{3}\right)}{64c^2}+\frac{d\left(-2\sqrt{d\,x^3+c}+6\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)\sqrt{}\right.}{192c^2}$
elliptic	Expression too large to display

```
[In] int((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(d*x^3+c)^(1/2)/c/x^3+1/16*d/c*(-5/6*arctanh((d*x^3+c)^(1/2)/c^(1/2))
/c^(1/2)+1/2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.30

$$\int \frac{\sqrt{c+dx^3}}{x^4(8c-dx^3)}\,dx = \left[\frac{3\sqrt{c}dx^3\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+5\sqrt{c}dx^3\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)-8\sqrt{dx^3+c}c}{192c^2x^3}, \frac{5\sqrt{-c}dx^3\arctan\left(\sqrt{\ldots}\right.}{}\right.$$

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [1/192*(3*sqrt(c)*d*x^3*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x
^3 - 8*c)) + 5*sqrt(c)*d*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/
x^3) - 8*sqrt(d*x^3 + c)*c)/(c^2*x^3), 1/96*(5*sqrt(-c)*d*x^3*arctan(sqrt(d
*x^3 + c)*sqrt(-c)/c) - 3*sqrt(-c)*d*x^3*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c
)/c) - 4*sqrt(d*x^3 + c)*c)/(c^2*x^3)]
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4(8c-dx^3)}\,dx = -\int \frac{\sqrt{c+dx^3}}{-8cx^4+dx^7}\,dx$$

```
[In] integrate((d*x**3+c)**(1/2)/x**4/(-d*x**3+8*c),x)
```

```
[Out] -Integral(sqrt(c + d*x**3)/(-8*c*x**4 + d*x**7), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4(8c-dx^3)}\,dx = \int -\frac{\sqrt{dx^3+c}}{(dx^3-8\,c)x^4}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{c+dx^3}}{x^4\,(8c-dx^3)}\,dx = \frac{5\,d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\,\sqrt{-c}c} - \frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{32\,\sqrt{-c}c} - \frac{\sqrt{dx^3+c}}{24\,cx^3}$$

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] 5/96*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/32*d*arctan(1/3*sq
rt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/24*sqrt(d*x^3 + c)/(c*x^3)
```

Mupad [B] (verification not implemented)

Time = 7.41 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{c+dx^3}}{x^4\,(8c-dx^3)}\,dx = \frac{d\,\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^3}}\right)}{32\,\sqrt{c^3}} - \frac{5\,d\,\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{\sqrt{c^3}}\right)}{96\,\sqrt{c^3}} - \frac{\sqrt{d\,x^3+c}}{24\,c\,x^3}$$

```
[In] int((c + d*x^3)^(1/2)/(x^4*(8*c - d*x^3)),x)
```

```
[Out] (d*atanh((c*(c + d*x^3)^(1/2))/(3*(c^3)^(1/2))))/(32*(c^3)^(1/2)) - (5*d*at
anh((c*(c + d*x^3)^(1/2))/(c^3)^(1/2)))/(96*(c^3)^(1/2)) - (c + d*x^3)^(1/2
)/(24*c*x^3)
```

3.288 $\int \frac{\sqrt{c+dx^3}}{x^7(8c-dx^3)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 107

$$\int \frac{\sqrt{c+dx^3}}{x^7(8c-dx^3)}\,dx=-\frac{\sqrt{c+dx^3}}{48cx^6}-\frac{d\sqrt{c+dx^3}}{64c^2x^3}+\frac{d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{5/2}}+\frac{d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{5/2}}$$

```
[Out] 1/256*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)+1/256*d^2*arctanh((d
*x^3+c)^(1/2)/c^(1/2))/c^(5/2)-1/48*(d*x^3+c)^(1/2)/c/x^6-1/64*d*(d*x^3+c)^
(1/2)/c^2/x^3
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {457, 101, 156, 162, 65, 214, 212}

$$\int \frac{\sqrt{c+dx^3}}{x^7(8c-dx^3)}\,dx=\frac{d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{5/2}}+\frac{d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{5/2}}-\frac{d\sqrt{c+dx^3}}{64c^2x^3}-\frac{\sqrt{c+dx^3}}{48cx^6}$$

```
[In] Int[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)),x]
```

```
[Out] -1/48*Sqrt[c + d*x^3]/(c*x^6) - (d*Sqrt[c + d*x^3])/(64*c^2*x^3) + (d^2*Arc
Tanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(256*c^(5/2)) + (d^2*ArcTanh[Sqrt[c + d*
x^3]/Sqrt[c]])/(256*c^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^3(8c-dx)}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{48cx^6} + \frac{\text{Subst}\left(\int \frac{6cd+\frac{3d^2x}{2}}{x^2(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{48c}$$

$$= -\frac{\sqrt{c+dx^3}}{48cx^6} - \frac{d\sqrt{c+dx^3}}{64c^2x^3} - \frac{\text{Subst}\left(\int \frac{6c^2d^2-3cd^3x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{384c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{48cx^6} - \frac{d\sqrt{c+dx^3}}{64c^2x^3} - \frac{d^2\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{512c^2} + \frac{(3d^3)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{512c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{48cx^6} - \frac{d\sqrt{c+dx^3}}{64c^2x^3} - \frac{d\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{256c^2} + \frac{(3d^2)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{256c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{48cx^6} - \frac{d\sqrt{c+dx^3}}{64c^2x^3} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{5/2}} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{5/2}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{c+dx^3}}{x^7(8c-dx^3)}\,dx = \frac{(-4c-3dx^3)\sqrt{c+dx^3}}{192c^2x^6} + \frac{d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{5/2}} + \frac{d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{5/2}}$$

```
[In] Integrate[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)),x]
[Out] ((-4*c - 3*d*x^3)*Sqrt[c + d*x^3])/(192*c^2*x^6) + (d^2*ArcTanh[Sqrt[c + d*
x^3]/(3*Sqrt[c])])/(256*c^(5/2)) + (d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(
256*c^(5/2))
```

Maple [A] (verified)

Time = 4.45 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{\sqrt{d\,x^{3}+c}\,\left(3d\,x^{3}+4c\right)}{192c^{2}x^{6}}-\frac{3d^{2}\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{6\sqrt{c}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{6\sqrt{c}}\right)}{128c^{2}}$
pseudoelliptic	$\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)d^{2}x^{6}+3\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)d^{2}x^{6}-12d\,x^{3}\sqrt{d\,x^{3}+c}\,\sqrt{c}-16\sqrt{d\,x^{3}+c}\,c^{\frac{3}{2}}}{768c^{\frac{5}{2}}x^{6}}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{6x^{6}}-\frac{d\sqrt{d\,x^{3}+c}}{12c\,x^{3}}+\frac{d^{2}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{12c^{\frac{3}{2}}}}{8c}+\frac{d\left(-\frac{\sqrt{d\,x^{3}+c}}{3x^{3}}-\frac{d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3\sqrt{c}}\right)}{64c^{2}}+\frac{d^{2}\left(\frac{2\sqrt{d\,x^{3}+c}}{3}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d}}{}\right.}{3}\right.}{512c^{3}}$
elliptic	Expression too large to display

```
[In] int((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/192*(d*x^3+c)^(1/2)*(3*d*x^3+4*c)/c^2/x^6-3/128*d^2/c^2*(-1/6*arctanh((d
*x^3+c)^(1/2)/c^(1/2))/c^(1/2)-1/6*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(
1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.76

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)}\,dx = \left[\frac{3\sqrt{c}d^2x^6\log\left(\frac{d^2x^6+24\,cdx^3+8\,(dx^3+4\,c)\sqrt{dx^3+c}\sqrt{c}+32\,c^2}{dx^6-8\,cx^3}\right)-8\,(3\,cdx^3+4\,c^2)\sqrt{dx^3+c}}{1536\,c^3x^6},\ -\frac{3\sqrt{-c}d^2x^6\arctan\left(\frac{(dx^3+4\,c)\sqrt{dx^3+c}\sqrt{-c}}{4\,(cdx^3+c^2)}\right)+4\,(3\,cdx^3+4\,c^2)\sqrt{dx^3+c}}{768\,c^3x^6}\right]$$

```
[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [1/1536*(3*sqrt(c)*d^2*x^6*log((d^2*x^6 + 24*c*d*x^3 + 8*(d*x^3 + 4*c)*sqrt
(d*x^3 + c)*sqrt(c) + 32*c^2)/(d*x^6 - 8*c*x^3)) - 8*(3*c*d*x^3 + 4*c^2)*sq
rt(d*x^3 + c))/(c^3*x^6), -1/768*(3*sqrt(-c)*d^2*x^6*arctan(1/4*(d*x^3 + 4*
c)*sqrt(d*x^3 + c)*sqrt(-c)/(c*d*x^3 + c^2)) + 4*(3*c*d*x^3 + 4*c^2)*sqrt(d
*x^3 + c))/(c^3*x^6)]
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^7(8c-dx^3)}\,dx = -\int \frac{\sqrt{c+dx^3}}{-8cx^7+dx^{10}}\,dx$$

```
[In] integrate((d*x**3+c)**(1/2)/x**7/(-d*x**3+8*c),x)
[Out] -Integral(sqrt(c + d*x**3)/(-8*c*x**7 + d*x**10), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^7(8c-dx^3)}\,dx = \int -\frac{\sqrt{dx^3+c}}{(dx^3-8\,c)x^7}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x, algorithm="maxima")
[Out] -integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^7), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c+dx^3}}{x^7(8c-dx^3)}\,dx = -\frac{d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{256\sqrt{-c}c^2} - \frac{d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{256\sqrt{-c}c^2} - \frac{3\,(dx^3+c)^{\frac{3}{2}}d^2+\sqrt{dx^3+c}cd^2}{192\,c^2d^2x^6}$$

```
[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x, algorithm="giac")
[Out] -1/256*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/256*d^2*arct
an(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/192*(3*(d*x^3 + c)^(3/2
)*d^2 + sqrt(d*x^3 + c)*c*d^2)/(c^2*d^2*x^6)
```

Mupad [B] (verification not implemented)

Time = 7.57 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)}\,dx=\frac{d^2\,\mathrm{atanh}\left(\frac{d^4\,\sqrt{d\,x^3+c}}{2048\,c^{7/2}\,\left(\frac{d^4}{2048\,c^3}+\frac{d^5\,x^3}{8192\,c^4}\right)}\right)}{256\,c^{5/2}}-\frac{\sqrt{d\,x^3+c}}{192\,c\,x^6}-\frac{{\left(d\,x^3+c\right)}^{3/2}}{64\,c^2\,x^6}$$

```
[In] int((c + d*x^3)^(1/2)/(x^7*(8*c - d*x^3)),x)
```

```
[Out] (d^2*atanh((d^4*(c + d*x^3)^(1/2))/(2048*c^(7/2)*(d^4/(2048*c^3) + (d^5*x^3
)/(8192*c^4)))))/(256*c^(5/2)) - (c + d*x^3)^(1/2)/(192*c*x^6) - (c + d*x^3
)^(3/2)/(64*c^2*x^6)
```

3.289 $\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 648

$$\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{214cx^2\sqrt{c+dx^3}}{91d^2}-\frac{2x^5\sqrt{c+dx^3}}{13d} - \frac{12248c^2\sqrt{c+dx^3}}{91d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{32\sqrt{3}c^{13/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{8/3}} + \frac{32c^{13/6}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{8/3}} - \frac{32c^{13/6}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{8/3}} + \frac{6124\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{91d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{12248\sqrt{2}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{91\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 32*c^(13/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(8
/3)-32*c^(13/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(8/3)-32*c^(13/6)*ar
```

```
ctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/d^(8/3)-2
14/91*c*x^2*(d*x^3+c)^(1/2)/d^2-2/13*x^5*(d*x^3+c)^(1/2)/d-12248/91*c^2*(d*
x^3+c)^(1/2)/d^(8/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-12248/273*c^(7/3)*(c^(
1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3
)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x
^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^(8/3)/(d*x^3+c)^(1/2
)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+612
4/91*3^(1/4)*c^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^
(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(
1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2
)))^2)^(1/2)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*
x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 648, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {489, 596, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{12248\sqrt{2}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{91\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{6124\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{91d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{32\sqrt{3}c^{13/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{8/3}}+\frac{32c^{13/6}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{8/3}}-\frac{32c^{13/6}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{8/3}}-\frac{12248c^2\sqrt{c+dx^3}}{91d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{214cx^2\sqrt{c+dx^3}}{91d^2}-\frac{2x^5\sqrt{c+dx^3}}{13d}$$

```
[In] Int[(x^7*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-214*c*x^2*Sqrt[c + d*x^3])/(91*d^2) - (2*x^5*Sqrt[c + d*x^3])/(13*d) - (1
2248*c^2*Sqrt[c + d*x^3])/(91*d^(8/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x))
- (32*Sqrt[3]*c^(13/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[
c + d*x^3]])/d^(8/3) + (32*c^(13/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1
/6)*Sqrt[c + d*x^3])])/d^(8/3) - (32*c^(13/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sq
rt[c])])/d^(8/3) + (6124*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(7/3)*(c^(1/3) + d^(1/
3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*d^(8/3)*Sqrt[(c^(1/
3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d
*x^3]) - (12248*Sqrt[2]*c^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/
3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Elliptic
F[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)], -7 - 4*Sqrt[3]])/(91*3^(1/4)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/
3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2x^5\sqrt{c+dx^3}}{13d}+\frac{2\int \frac{x^4\left(40c^2+\frac{107}{2}cdx^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{13d}\\
&= -\frac{214cx^2\sqrt{c+dx^3}}{91d^2}-\frac{2x^5\sqrt{c+dx^3}}{13d}+\frac{4\int \frac{x(856c^3d+1531c^2d^2x^3)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{91d^3}\\
&= -\frac{214cx^2\sqrt{c+dx^3}}{91d^2}-\frac{2x^5\sqrt{c+dx^3}}{13d}+\frac{4\int \left(-\frac{1531c^2dx}{\sqrt{c+dx^3}}+\frac{13104c^3dx}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{91d^3}\\
&= -\frac{214cx^2\sqrt{c+dx^3}}{91d^2}-\frac{2x^5\sqrt{c+dx^3}}{13d}-\frac{(6124c^2)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{91d^2}+\frac{(576c^3)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{d^2}
\end{aligned}$$

$$= -\frac{214cx^2\sqrt{c+dx^3}}{91d^2} - \frac{2x^5\sqrt{c+dx^3}}{13d} - \frac{(48c^2)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{d^3}$$

$$-\frac{(6124c^2)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{91d^{7/3}} + \frac{\left(48c^{7/3}\right)\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{d^{7/3}}$$

$$+\frac{\left(6124\left(1-\sqrt{3}\right)c^{7/3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{91d^{7/3}} - \frac{\left(144c^{8/3}\right)\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{d^{5/3}}$$

$$= -\frac{214cx^2\sqrt{c+dx^3}}{91d^2} - \frac{2x^5\sqrt{c+dx^3}}{13d} - \frac{12248c^2\sqrt{c+dx^3}}{91d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$+\frac{6124\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4}{91d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{12248\sqrt{2}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{91\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{\left(96c^{8/3}\right)\text{Subst}\left(\int \frac{1}{9-cx^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{d^{8/3}}$$

$$-\frac{\left(48c^{8/3}\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{d^{5/3}}$$

$$+\frac{\left(192c^{5/3}\right)\text{Subst}\left(\int \frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{d^{2/3}}$$

$$= -\frac{214cx^2\sqrt{c+dx^3}}{91d^2} - \frac{2x^5\sqrt{c+dx^3}}{13d} - \frac{12248c^2\sqrt{c+dx^3}}{91d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$-\frac{32\sqrt{3}c^{13/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{8/3}} + \frac{32c^{13/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{8/3}}$$

$$+\frac{6124\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{91d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{12248\sqrt{2}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{91\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{\left(96c^{8/3}\right)\mathrm{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{d^{8/3}}$$

$$= -\frac{214cx^2\sqrt{c+dx^3}}{91d^2} - \frac{2x^5\sqrt{c+dx^3}}{13d} - \frac{12248c^2\sqrt{c+dx^3}}{91d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$- \frac{32\sqrt{3}c^{13/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{8/3}}$$

$$+ \frac{32c^{13/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{8/3}} - \frac{32c^{13/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{8/3}}$$

$$+ \frac{6124\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4}{91d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$- \frac{12248\sqrt{2}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{91\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.11 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.23

$$\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx$$

$$= \frac{-20(107c^2x^2+114cdx^5+7d^2x^8)+2140c^2x^2\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+1531cdx^5\sqrt{1+}}{910d^2\sqrt{c+dx^3}}$$

```
[In] Integrate[(x^7*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-20*(107*c^2*x^2 + 114*c*d*x^5 + 7*d^2*x^8) + 2140*c^2*x^2*Sqrt[1 + (d*x^3
)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 1531*c*d*x^5
*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)
])/(910*d^2*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.91 (sec) , antiderivative size = 884, normalized size of antiderivative = 1.36

method	result	size
risch	Expression too large to display	884
elliptic	Expression too large to display	889
default	Expression too large to display	1788

```
[In] int(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] -2/91*x^2*(7*d*x^3+107*c)*(d*x^3+c)^(1/2)/d^2-4/91/d^2*c^2*(-3062/3*I*3^(1/
2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(
1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*
3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1456/3*I/d^3*2^(1/2)*sum(1/_alpha*(-
c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/
(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2
)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d
^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*
3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(
-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)
^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(
-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 13.58 (sec) , antiderivative size = 2442, normalized size of antiderivative = 3.77

$$\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] 2/273*(728*d^3*(c^13/d^16)^(1/6)*log(33554432*((d^16*x^9 + 318*c*d^15*x^6 +
 1200*c^2*d^14*x^3 + 640*c^3*d^13)*(c^13/d^16)^(5/6) + 6*(c^11*d^2*x^7 + 80
*c^12*d*x^4 + 160*c^13*x + 6*(5*c^3*d^12*x^5 + 32*c^4*d^11*x^2)*(c^13/d^16)
^(2/3) + (7*c^7*d^7*x^6 + 152*c^8*d^6*x^3 + 64*c^9*d^5)*(c^13/d^16)^(1/3))*
sqrt(d*x^3 + c) + 18*(5*c^5*d^10*x^7 + 64*c^6*d^9*x^4 + 32*c^7*d^8*x)*sqrt(
c^13/d^16) + 18*(c^9*d^5*x^8 + 38*c^10*d^4*x^5 + 64*c^11*d^3*x^2)*(c^13/d^1
6)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 728*d^3*(c^
13/d^16)^(1/6)*log(-33554432*((d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^
3 + 640*c^3*d^13)*(c^13/d^16)^(5/6) - 6*(c^11*d^2*x^7 + 80*c^12*d*x^4 + 160
*c^13*x + 6*(5*c^3*d^12*x^5 + 32*c^4*d^11*x^2)*(c^13/d^16)^(2/3) + (7*c^7*d
^7*x^6 + 152*c^8*d^6*x^3 + 64*c^9*d^5)*(c^13/d^16)^(1/3))*sqrt(d*x^3 + c) +
 18*(5*c^5*d^10*x^7 + 64*c^6*d^9*x^4 + 32*c^7*d^8*x)*sqrt(c^13/d^16) + 18*(
c^9*d^5*x^8 + 38*c^10*d^4*x^5 + 64*c^11*d^3*x^2)*(c^13/d^16)^(1/6))/(d^3*x^
9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 18372*c^2*sqrt(d)*weierstras
sZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) - 364*(sqrt(-3)*d^3 - d
^3)*(c^13/d^16)^(1/6)*log(33554432*((d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d
^14*x^3 + 640*c^3*d^13 + sqrt(-3)*(d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^1
4*x^3 + 640*c^3*d^13))*(c^13/d^16)^(5/6) + 6*(2*c^11*d^2*x^7 + 160*c^12*d*x
^4 + 320*c^13*x - 6*(5*c^3*d^12*x^5 + 32*c^4*d^11*x^2 - sqrt(-3)*(5*c^3*d^1
2*x^5 + 32*c^4*d^11*x^2))*(c^13/d^16)^(2/3) - (7*c^7*d^7*x^6 + 152*c^8*d^6*
x^3 + 64*c^9*d^5 + sqrt(-3)*(7*c^7*d^7*x^6 + 152*c^8*d^6*x^3 + 64*c^9*d^5))
*(c^13/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^5*d^10*x^7 + 64*c^6*d^9*x^4 +
 32*c^7*d^8*x)*sqrt(c^13/d^16) + 18*(c^9*d^5*x^8 + 38*c^10*d^4*x^5 + 64*c^1
1*d^3*x^2 - sqrt(-3)*(c^9*d^5*x^8 + 38*c^10*d^4*x^5 + 64*c^11*d^3*x^2))*(c^
13/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 364*(
sqrt(-3)*d^3 - d^3)*(c^13/d^16)^(1/6)*log(-33554432*((d^16*x^9 + 318*c*d^15
*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13 + sqrt(-3)*(d^16*x^9 + 318*c*d^15*x
^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c^13/d^16)^(5/6) - 6*(2*c^11*d^2*x
^7 + 160*c^12*d*x^4 + 320*c^13*x - 6*(5*c^3*d^12*x^5 + 32*c^4*d^11*x^2 - sq
rt(-3)*(5*c^3*d^12*x^5 + 32*c^4*d^11*x^2))*(c^13/d^16)^(2/3) - (7*c^7*d^7*x
^6 + 152*c^8*d^6*x^3 + 64*c^9*d^5 + sqrt(-3)*(7*c^7*d^7*x^6 + 152*c^8*d^6*x
^3 + 64*c^9*d^5))*(c^13/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^5*d^10*x^7 +
 64*c^6*d^9*x^4 + 32*c^7*d^8*x)*sqrt(c^13/d^16) + 18*(c^9*d^5*x^8 + 38*c^10
*d^4*x^5 + 64*c^11*d^3*x^2 - sqrt(-3)*(c^9*d^5*x^8 + 38*c^10*d^4*x^5 + 64*c
^11*d^3*x^2))*(c^13/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 -
512*c^3)) + 364*(sqrt(-3)*d^3 + d^3)*(c^13/d^16)^(1/6)*log(33554432*((d^16*
x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13 - sqrt(-3)*(d^16*x^
9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c^13/d^16)^(5/6) +
 6*(2*c^11*d^2*x^7 + 160*c^12*d*x^4 + 320*c^13*x - 6*(5*c^3*d^12*x^5 + 32*c
^4*d^11*x^2 + sqrt(-3)*(5*c^3*d^12*x^5 + 32*c^4*d^11*x^2))*(c^13/d^16)^(2/3
) - (7*c^7*d^7*x^6 + 152*c^8*d^6*x^3 + 64*c^9*d^5 - sqrt(-3)*(7*c^7*d^7*x^6
 + 152*c^8*d^6*x^3 + 64*c^9*d^5))*(c^13/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(
5*c^5*d^10*x^7 + 64*c^6*d^9*x^4 + 32*c^7*d^8*x)*sqrt(c^13/d^16) + 18*(c^9*d
^5*x^8 + 38*c^10*d^4*x^5 + 64*c^11*d^3*x^2 + sqrt(-3)*(c^9*d^5*x^8 + 38*c^1
0*d^4*x^5 + 64*c^11*d^3*x^2))*(c^13/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 +
```

```
192*c^2*d*x^3 - 512*c^3)) - 364*(sqrt(-3)*d^3 + d^3)*(c^13/d^16)^(1/6)*log(
-33554432*((d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13 -
sqrt(-3)*(d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c
^13/d^16)^(5/6) - 6*(2*c^11*d^2*x^7 + 160*c^12*d*x^4 + 320*c^13*x - 6*(5*c^
3*d^12*x^5 + 32*c^4*d^11*x^2 + sqrt(-3)*(5*c^3*d^12*x^5 + 32*c^4*d^11*x^2))
*(c^13/d^16)^(2/3) - (7*c^7*d^7*x^6 + 152*c^8*d^6*x^3 + 64*c^9*d^5 - sqrt(-
3)*(7*c^7*d^7*x^6 + 152*c^8*d^6*x^3 + 64*c^9*d^5))*(c^13/d^16)^(1/3))*sqrt(
d*x^3 + c) - 36*(5*c^5*d^10*x^7 + 64*c^6*d^9*x^4 + 32*c^7*d^8*x)*sqrt(c^13/
d^16) + 18*(c^9*d^5*x^8 + 38*c^10*d^4*x^5 + 64*c^11*d^3*x^2 + sqrt(-3)*(c^9
*d^5*x^8 + 38*c^10*d^4*x^5 + 64*c^11*d^3*x^2))*(c^13/d^16)^(1/6))/(d^3*x^9
- 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 3*(7*d^2*x^5 + 107*c*d*x^2)*sq
rt(d*x^3 + c))/d^3
```

Sympy [F]

$$\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\int \frac{x^7\sqrt{c+dx^3}}{-8c+dx^3}\,dx$$

```
[In] integrate(x**7*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
[Out] -Integral(x**7*sqrt(c + d*x**3)/(-8*c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx = \int -\frac{\sqrt{dx^3+c}x^7}{dx^3-8\,c}\,dx$$

```
[In] integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
[Out] -integrate(sqrt(d*x^3 + c)*x^7/(d*x^3 - 8*c), x)
```

Giac [F]

$$\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx = \int -\frac{\sqrt{dx^3+c}x^7}{dx^3-8\,c}\,dx$$

```
[In] integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
[Out] integrate(-sqrt(d*x^3 + c)*x^7/(d*x^3 - 8*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx = \int \frac{x^7\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\,dx$$

```
[In] int((x^7*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
[Out] int((x^7*(c + d*x^3)^(1/2))/(8*c - d*x^3), x)
```

3.290 $\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 624

$$\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2x^2\sqrt{c+dx^3}}{7d}-\frac{118c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{4\sqrt{3}c^{7/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}}+\frac{4c^{7/6}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{5/3}}-\frac{4c^{7/6}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{5/3}}+\frac{59\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{118\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{7\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 4*c^(7/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(5/3
)-4*c^(7/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(5/3)-4*c^(7/6)*arctan(c
```

```
^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/d^(5/3)-2/7*x^2
*(d*x^3+c)^(1/2)/d-118/7*c*(d*x^3+c)^(1/2)/d^(5/3)/(d^(1/3)*x+c^(1/3)*(1+3^
(1/2)))-118/21*c^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-
3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-
c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(
3/4)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3
)*(1+3^(1/2)))^2)^(1/2)+59/7*3^(1/4)*c^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE(
(d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2
*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(
1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1
/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 624, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {489, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{118\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{7\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{59\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{7d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{4\sqrt{3}c^{7/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}}+\frac{4c^{7/6}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{5/3}}-\frac{4c^{7/6}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{5/3}}-\frac{118c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{2x^2\sqrt{c+dx^3}}{7d}$$

```
[In] Int[(x^4*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*x^2*Sqrt[c + d*x^3])/(7*d) - (118*c*Sqrt[c + d*x^3])/(7*d^(5/3)*((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)) - (4*Sqrt[3]*c^(7/6)*ArcTan[(Sqrt[3]*c^(1/6)*
```

```
(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/d^(5/3) + (4*c^(7/6)*ArcTanh[(c^(1
/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]])/d^(5/3) - (4*c^(7/6)*ArcTa
nh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^(5/3) + (59*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^
(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2
)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^
(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(
7*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)^2]*Sqrt[c + d*x^3]) - (118*Sqrt[2]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sq
rt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(1/4)*d^(5/3)*Sqrt[(c^(1/3
)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*
x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
```

```
/; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2x^2\sqrt{c+dx^3}}{7d} + \frac{2\int \frac{x\left(16c^2+\frac{59}{2}cdx^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{7d} \\
&= -\frac{2x^2\sqrt{c+dx^3}}{7d} + \frac{2\int \left(-\frac{59cx}{2\sqrt{c+dx^3}} + \frac{252c^2x}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{7d} \\
&= -\frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{(59c)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{7d} + \frac{(72c^2)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{d} \\
&= -\frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{(6c)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{d^2} \\
&\quad - \frac{(59c)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{7d^{4/3}} + \frac{\left(6c^{4/3}\right)\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{d^{4/3}} \\
&\quad + \frac{\left(59\left(1-\sqrt{3}\right)c^{4/3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{7d^{4/3}} - \frac{\left(18c^{5/3}\right)\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{d^{2/3}}
\end{aligned}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{118c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$+ \frac{59\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$- \frac{118\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{\left(12c^{5/3}\right)\text{Subst}\left(\int \frac{1}{9-cx^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{d^{5/3}}$$

$$- \frac{\left(6c^{5/3}\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{d^{2/3}}$$

$$+ \left(24c^{2/3}\sqrt[3]{d}\right)\text{Subst}\left(\int \frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)$$

$$
\begin{aligned}
&= -\frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{118c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} \\
&- \frac{4\sqrt{3}c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}} + \frac{4c^{7/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{5/3}} \\
&+ \frac{59\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&- \frac{118\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&- \frac{\left(12c^{5/3}\right)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d^{5/3}}
\end{aligned}
$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{118c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{4\sqrt{3}c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}} + \frac{4c^{7/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{5/3}} - \frac{4c^{7/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{5/3}} + \frac{59\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{118\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.21 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.21

$$\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx = \frac{x^2\left(-80(c+dx^3)+80c\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+59dx^3\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},\right)\right)}{280d\sqrt{c+dx^3}}$$

```
[In] Integrate[(x^4*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (x^2*(-80*(c + d*x^3) + 80*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3,
 -((d*x^3)/c), (d*x^3)/(8*c)] + 59*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3,
1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(280*d*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.84 (sec) , antiderivative size = 867, normalized size of antiderivative = 1.39

method	result	size
elliptic	Expression too large to display	867
risch	Expression too large to display	872
default	Expression too large to display	1310

```
[In] int(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] -2/7*x^2*(d*x^3+c)^(1/2)/d+118/21*I*c/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/
d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(
1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(
1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/
3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/
2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))
^(1/2)))-8/3*I*c/d^4*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*
(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d
*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*
I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2
/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3
^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c
*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(
I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.95 (sec) , antiderivative size = 2428, normalized size of antiderivative = 3.89

$$\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] -1/21*(6*sqrt(d*x^3 + c)*d*x^2 - 14*d^2*(c^7/d^10)^(1/6)*log(1024*((d^11*x^
9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8)*(c^7/d^10)^(5/6) + 6*(
c^6*d^2*x^7 + 80*c^7*d*x^4 + 160*c^8*x + 6*(5*c^2*d^8*x^5 + 32*c^3*d^7*x^2)
*(c^7/d^10)^(2/3) + (7*c^4*d^5*x^6 + 152*c^5*d^4*x^3 + 64*c^6*d^3)*(c^7/d^1
0)^(1/3))*sqrt(d*x^3 + c) + 18*(5*c^3*d^7*x^7 + 64*c^4*d^6*x^4 + 32*c^5*d^5
*x)*sqrt(c^7/d^10) + 18*(c^5*d^4*x^8 + 38*c^6*d^3*x^5 + 64*c^7*d^2*x^2)*(c^
7/d^10)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 14*d^2
*(c^7/d^10)^(1/6)*log(-1024*((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3
+ 640*c^3*d^8)*(c^7/d^10)^(5/6) - 6*(c^6*d^2*x^7 + 80*c^7*d*x^4 + 160*c^8*x
 + 6*(5*c^2*d^8*x^5 + 32*c^3*d^7*x^2)*(c^7/d^10)^(2/3) + (7*c^4*d^5*x^6 + 1
52*c^5*d^4*x^3 + 64*c^6*d^3)*(c^7/d^10)^(1/3))*sqrt(d*x^3 + c) + 18*(5*c^3*
d^7*x^7 + 64*c^4*d^6*x^4 + 32*c^5*d^5*x)*sqrt(c^7/d^10) + 18*(c^5*d^4*x^8 +
 38*c^6*d^3*x^5 + 64*c^7*d^2*x^2)*(c^7/d^10)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6
 + 192*c^2*d*x^3 - 512*c^3)) - 354*c*sqrt(d)*weierstrassZeta(0, -4*c/d, wei
erstrassPInverse(0, -4*c/d, x)) + 7*(sqrt(-3)*d^2 - d^2)*(c^7/d^10)^(1/6)*l
og(1024*((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 + sqrt
(-3)*(d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8))*(c^7/d^1
0)^(5/6) + 6*(2*c^6*d^2*x^7 + 160*c^7*d*x^4 + 320*c^8*x - 6*(5*c^2*d^8*x^5
+ 32*c^3*d^7*x^2 - sqrt(-3)*(5*c^2*d^8*x^5 + 32*c^3*d^7*x^2))*(c^7/d^10)^(2
/3) - (7*c^4*d^5*x^6 + 152*c^5*d^4*x^3 + 64*c^6*d^3 + sqrt(-3)*(7*c^4*d^5*x
^6 + 152*c^5*d^4*x^3 + 64*c^6*d^3))*(c^7/d^10)^(1/3))*sqrt(d*x^3 + c) - 36*
(5*c^3*d^7*x^7 + 64*c^4*d^6*x^4 + 32*c^5*d^5*x)*sqrt(c^7/d^10) + 18*(c^5*d^
4*x^8 + 38*c^6*d^3*x^5 + 64*c^7*d^2*x^2 - sqrt(-3)*(c^5*d^4*x^8 + 38*c^6*d^
3*x^5 + 64*c^7*d^2*x^2))*(c^7/d^10)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^
2*d*x^3 - 512*c^3)) - 7*(sqrt(-3)*d^2 - d^2)*(c^7/d^10)^(1/6)*log(-1024*((d
^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 + sqrt(-3)*(d^11*
x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8))*(c^7/d^10)^(5/6) -
6*(2*c^6*d^2*x^7 + 160*c^7*d*x^4 + 320*c^8*x - 6*(5*c^2*d^8*x^5 + 32*c^3*d^
7*x^2 - sqrt(-3)*(5*c^2*d^8*x^5 + 32*c^3*d^7*x^2))*(c^7/d^10)^(2/3) - (7*c^
4*d^5*x^6 + 152*c^5*d^4*x^3 + 64*c^6*d^3 + sqrt(-3)*(7*c^4*d^5*x^6 + 152*c^
5*d^4*x^3 + 64*c^6*d^3))*(c^7/d^10)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^3*d^7*
x^7 + 64*c^4*d^6*x^4 + 32*c^5*d^5*x)*sqrt(c^7/d^10) + 18*(c^5*d^4*x^8 + 38*
c^6*d^3*x^5 + 64*c^7*d^2*x^2 - sqrt(-3)*(c^5*d^4*x^8 + 38*c^6*d^3*x^5 + 64*
c^7*d^2*x^2))*(c^7/d^10)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 5
12*c^3)) - 7*(sqrt(-3)*d^2 + d^2)*(c^7/d^10)^(1/6)*log(1024*((d^11*x^9 + 31
8*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 - sqrt(-3)*(d^11*x^9 + 318*c*
d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8))*(c^7/d^10)^(5/6) + 6*(2*c^6*d^2
*x^7 + 160*c^7*d*x^4 + 320*c^8*x - 6*(5*c^2*d^8*x^5 + 32*c^3*d^7*x^2 + sqrt
(-3)*(5*c^2*d^8*x^5 + 32*c^3*d^7*x^2))*(c^7/d^10)^(2/3) - (7*c^4*d^5*x^6 +
152*c^5*d^4*x^3 + 64*c^6*d^3 - sqrt(-3)*(7*c^4*d^5*x^6 + 152*c^5*d^4*x^3 +
64*c^6*d^3))*(c^7/d^10)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^3*d^7*x^7 + 64*c^4
*d^6*x^4 + 32*c^5*d^5*x)*sqrt(c^7/d^10) + 18*(c^5*d^4*x^8 + 38*c^6*d^3*x^5
+ 64*c^7*d^2*x^2 + sqrt(-3)*(c^5*d^4*x^8 + 38*c^6*d^3*x^5 + 64*c^7*d^2*x^2)
)*(c^7/d^10)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 7
*(sqrt(-3)*d^2 + d^2)*(c^7/d^10)^(1/6)*log(-1024*((d^11*x^9 + 318*c*d^10*x^
```

```
6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 - sqrt(-3)*(d^11*x^9 + 318*c*d^10*x^6 +
1200*c^2*d^9*x^3 + 640*c^3*d^8))*(c^7/d^10)^(5/6) - 6*(2*c^6*d^2*x^7 + 160*
c^7*d*x^4 + 320*c^8*x - 6*(5*c^2*d^8*x^5 + 32*c^3*d^7*x^2 + sqrt(-3)*(5*c^2
*d^8*x^5 + 32*c^3*d^7*x^2))*(c^7/d^10)^(2/3) - (7*c^4*d^5*x^6 + 152*c^5*d^4
*x^3 + 64*c^6*d^3 - sqrt(-3)*(7*c^4*d^5*x^6 + 152*c^5*d^4*x^3 + 64*c^6*d^3)
)*(c^7/d^10)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^3*d^7*x^7 + 64*c^4*d^6*x^4 +
32*c^5*d^5*x)*sqrt(c^7/d^10) + 18*(c^5*d^4*x^8 + 38*c^6*d^3*x^5 + 64*c^7*d^
2*x^2 + sqrt(-3)*(c^5*d^4*x^8 + 38*c^6*d^3*x^5 + 64*c^7*d^2*x^2))*(c^7/d^10
)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/d^2
```

Sympy [F]

$$\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\int \frac{x^4\sqrt{c+dx^3}}{-8c+dx^3}\,dx$$

```
[In] integrate(x**4*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
[Out] -Integral(x**4*sqrt(c + d*x**3)/(-8*c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx = \int -\frac{\sqrt{dx^3+c}x^4}{dx^3-8\,c}\,dx$$

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
[Out] -integrate(sqrt(d*x^3 + c)*x^4/(d*x^3 - 8*c), x)
```

Giac [F]

$$\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx = \int -\frac{\sqrt{dx^3+c}x^4}{dx^3-8\,c}\,dx$$

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
[Out] integrate(-sqrt(d*x^3 + c)*x^4/(d*x^3 - 8*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx = \int \frac{x^4\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\,dx$$

```
[In] int((x^4*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
[Out] int((x^4*(c + d*x^3)^(1/2))/(8*c - d*x^3), x)
```

3.291 $\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 601

$$\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2\sqrt{c+dx^3}}{d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt{3}\sqrt[6]{c}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{2d^{2/3}}+\frac{\sqrt[6]{c}\,\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{2d^{2/3}}-\frac{\sqrt[6]{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2d^{2/3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 1/2*c^(1/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(2
/3)-1/2*c^(1/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(2/3)-1/2*c^(1/6)*ar
ctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/d^(2/3)-2
*(d*x^3+c)^(1/2)/d^(2/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-2/3*c^(1/3)*(c^(1/
```

```
3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*
(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2
)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^(2/3)/(d*x^3+c)^(1/2)/
(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+3^(1/
4)*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d
^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^
(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/
2)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*
(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 601, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {495, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{\sqrt{3}\sqrt[6]{c}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{2d^{2/3}} + \frac{\sqrt[6]{c}\,\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{2d^{2/3}} - \frac{\sqrt[6]{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2d^{2/3}} - \frac{2\sqrt{c+dx^3}}{d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

```
[In] Int[(x*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (Sqrt[
3]*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])
/(2*d^(2/3)) + (c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c +
```

```
 d*x^3])])/(2*d^(2/3)) - (c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2*
d^(2/3)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c
^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c
^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^
(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (2*Sqrt
[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/
3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[
3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[
3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c
^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
```

```
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\text{integral} = (9c)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx - \int \frac{x}{\sqrt{c+dx^3}}\,dx$$

$$= -\frac{3\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{4d} - \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{\sqrt[3]{d}}$$

$$+ \frac{\left(3\sqrt[3]{c}\right)\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{4\sqrt[3]{d}} + \frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{c}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{\sqrt[3]{d}}$$

$$- \frac{1}{4}\left(9c^{2/3}\sqrt[3]{d}\right)\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$$

$$= -\frac{2\sqrt{c+dx^3}}{d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{\left(3c^{2/3}\right)\text{Subst}\left(\int\frac{1}{9-cx^2}\,dx,x,\frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{2d^{2/3}}$$

$$-\frac{1}{4}\left(3c^{2/3}\sqrt[3]{d}\right)\text{Subst}\left(\int\frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)+\frac{\left(3d^{4/3}\right)\text{Subst}\left(\int\frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{d}}{\sqrt[3]{c}}}{\sqrt{c+dx}}\right.}{\sqrt[3]{c}}$$

$$= -\frac{2\sqrt{c+dx^3}}{d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt{3}\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{2d^{2/3}} + \frac{\sqrt[6]{c}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{2d^{2/3}} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{\left(3c^{2/3}\right)\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{2d^{2/3}}$$

$$= -\frac{2\sqrt{c+dx^3}}{d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt{3}\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{2d^{2/3}} + \frac{\sqrt[6]{c}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{2d^{2/3}} - \frac{\sqrt[6]{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2d^{2/3}} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 8.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.10

$$\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx=\frac{x^2\sqrt{1+\frac{dx^3}{c}}\ \text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{16\sqrt{c+dx^3}}$$

```
[In] Integrate[(x*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, -1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/
(8*c)])/(16*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.20 (sec) , antiderivative size = 848, normalized size of antiderivative = 1.41

method	result	size
default	Expression too large to display	848
elliptic	Expression too large to display	848

```
[In] int(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/
(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*Ell
ipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-1/3*I/d^3*2^(1/2)*sum(1/_
alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^
(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+
I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)
*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_a
lpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*
(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)
```

```
*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-
8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.76 (sec) , antiderivative size = 2194, normalized size of antiderivative = 3.65

$$\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] -1/24*((sqrt(-3)*d - d)*(c/d^4)^(1/6)*log(1/4*((d^6*x^9 + 318*c*d^5*x^6 + 1
200*c^2*d^4*x^3 + 640*c^3*d^3 + sqrt(-3)*(d^6*x^9 + 318*c*d^5*x^6 + 1200*c^
2*d^4*x^3 + 640*c^3*d^3))*(c/d^4)^(5/6) + 6*(2*c*d^2*x^7 + 160*c^2*d*x^4 +
320*c^3*x - 6*(5*c*d^4*x^5 + 32*c^2*d^3*x^2 - sqrt(-3)*(5*c*d^4*x^5 + 32*c^
2*d^3*x^2))*(c/d^4)^(2/3) - (7*c*d^3*x^6 + 152*c^2*d^2*x^3 + 64*c^3*d + sqr
t(-3)*(7*c*d^3*x^6 + 152*c^2*d^2*x^3 + 64*c^3*d))*(c/d^4)^(1/3))*sqrt(d*x^3
 + c) - 36*(5*c*d^4*x^7 + 64*c^2*d^3*x^4 + 32*c^3*d^2*x)*sqrt(c/d^4) + 18*(
c*d^3*x^8 + 38*c^2*d^2*x^5 + 64*c^3*d*x^2 - sqrt(-3)*(c*d^3*x^8 + 38*c^2*d^
2*x^5 + 64*c^3*d*x^2))*(c/d^4)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x
^3 - 512*c^3)) - (sqrt(-3)*d - d)*(c/d^4)^(1/6)*log(-1/4*((d^6*x^9 + 318*c*
d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 + sqrt(-3)*(d^6*x^9 + 318*c*d^5*x^
6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3))*(c/d^4)^(5/6) - 6*(2*c*d^2*x^7 + 160*c
^2*d*x^4 + 320*c^3*x - 6*(5*c*d^4*x^5 + 32*c^2*d^3*x^2 - sqrt(-3)*(5*c*d^4*
x^5 + 32*c^2*d^3*x^2))*(c/d^4)^(2/3) - (7*c*d^3*x^6 + 152*c^2*d^2*x^3 + 64*
c^3*d + sqrt(-3)*(7*c*d^3*x^6 + 152*c^2*d^2*x^3 + 64*c^3*d))*(c/d^4)^(1/3))
*sqrt(d*x^3 + c) - 36*(5*c*d^4*x^7 + 64*c^2*d^3*x^4 + 32*c^3*d^2*x)*sqrt(c/
d^4) + 18*(c*d^3*x^8 + 38*c^2*d^2*x^5 + 64*c^3*d*x^2 - sqrt(-3)*(c*d^3*x^8
+ 38*c^2*d^2*x^5 + 64*c^3*d*x^2))*(c/d^4)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 +
192*c^2*d*x^3 - 512*c^3)) - (sqrt(-3)*d + d)*(c/d^4)^(1/6)*log(1/4*((d^6*x^
9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 - sqrt(-3)*(d^6*x^9 + 31
8*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3))*(c/d^4)^(5/6) + 6*(2*c*d^2*x
^7 + 160*c^2*d*x^4 + 320*c^3*x - 6*(5*c*d^4*x^5 + 32*c^2*d^3*x^2 + sqrt(-3)
*(5*c*d^4*x^5 + 32*c^2*d^3*x^2))*(c/d^4)^(2/3) - (7*c*d^3*x^6 + 152*c^2*d^2
*x^3 + 64*c^3*d - sqrt(-3)*(7*c*d^3*x^6 + 152*c^2*d^2*x^3 + 64*c^3*d))*(c/d
^4)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c*d^4*x^7 + 64*c^2*d^3*x^4 + 32*c^3*d^2*
x)*sqrt(c/d^4) + 18*(c*d^3*x^8 + 38*c^2*d^2*x^5 + 64*c^3*d*x^2 + sqrt(-3)*(
c*d^3*x^8 + 38*c^2*d^2*x^5 + 64*c^3*d*x^2))*(c/d^4)^(1/6))/(d^3*x^9 - 24*c*
d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (sqrt(-3)*d + d)*(c/d^4)^(1/6)*log(-1
/4*((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 - sqrt(-3)*(d
^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3))*(c/d^4)^(5/6) - 6
```

```
*(2*c*d^2*x^7 + 160*c^2*d*x^4 + 320*c^3*x - 6*(5*c*d^4*x^5 + 32*c^2*d^3*x^2
 + sqrt(-3)*(5*c*d^4*x^5 + 32*c^2*d^3*x^2))*(c/d^4)^(2/3) - (7*c*d^3*x^6 +
152*c^2*d^2*x^3 + 64*c^3*d - sqrt(-3)*(7*c*d^3*x^6 + 152*c^2*d^2*x^3 + 64*c
^3*d))*(c/d^4)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c*d^4*x^7 + 64*c^2*d^3*x^4 +
32*c^3*d^2*x)*sqrt(c/d^4) + 18*(c*d^3*x^8 + 38*c^2*d^2*x^5 + 64*c^3*d*x^2 +
 sqrt(-3)*(c*d^3*x^8 + 38*c^2*d^2*x^5 + 64*c^3*d*x^2))*(c/d^4)^(1/6))/(d^3*
x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*d*(c/d^4)^(1/6)*log(1/2*
((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3)*(c/d^4)^(5/6) +
 6*(c*d^2*x^7 + 80*c^2*d*x^4 + 160*c^3*x + 6*(5*c*d^4*x^5 + 32*c^2*d^3*x^2)
*(c/d^4)^(2/3) + (7*c*d^3*x^6 + 152*c^2*d^2*x^3 + 64*c^3*d)*(c/d^4)^(1/3))*
sqrt(d*x^3 + c) + 18*(5*c*d^4*x^7 + 64*c^2*d^3*x^4 + 32*c^3*d^2*x)*sqrt(c/d
^4) + 18*(c*d^3*x^8 + 38*c^2*d^2*x^5 + 64*c^3*d*x^2)*(c/d^4)^(1/6))/(d^3*x^
9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 2*d*(c/d^4)^(1/6)*log(-1/2*(
(d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3)*(c/d^4)^(5/6) -
6*(c*d^2*x^7 + 80*c^2*d*x^4 + 160*c^3*x + 6*(5*c*d^4*x^5 + 32*c^2*d^3*x^2)*
(c/d^4)^(2/3) + (7*c*d^3*x^6 + 152*c^2*d^2*x^3 + 64*c^3*d)*(c/d^4)^(1/3))*s
qrt(d*x^3 + c) + 18*(5*c*d^4*x^7 + 64*c^2*d^3*x^4 + 32*c^3*d^2*x)*sqrt(c/d^
4) + 18*(c*d^3*x^8 + 38*c^2*d^2*x^5 + 64*c^3*d*x^2)*(c/d^4)^(1/6))/(d^3*x^9
 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 48*sqrt(d)*weierstrassZeta(0,
 -4*c/d, weierstrassPInverse(0, -4*c/d, x)))/d
```

Sympy [F]

$$\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\int \frac{x\sqrt{c+dx^3}}{-8c+dx^3}\,dx$$

```
[In] integrate(x*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
[Out] -Integral(x*sqrt(c + d*x**3)/(-8*c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx = \int -\frac{\sqrt{dx^3+c}x}{dx^3-8\,c}\,dx$$

```
[In] integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
[Out] -integrate(sqrt(d*x^3 + c)*x/(d*x^3 - 8*c), x)
```

Giac [F]

$$\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx = \int -\frac{\sqrt{dx^3+c}x}{dx^3-8\,c}\,dx$$

```
[In] integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
[Out] integrate(-sqrt(d*x^3 + c)*x/(d*x^3 - 8*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx = \int \frac{x\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\,dx$$

```
[In] int((x*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
[Out] int((x*(c + d*x^3)^(1/2))/(8*c - d*x^3), x)
```

3.292 $\int \frac{\sqrt{c+dx^3}}{x^2(8c-dx^3)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 632

$$\int \frac{\sqrt{c+dx^3}}{x^2(8c-dx^3)}\,dx = -\frac{\sqrt{c+dx^3}}{8cx}+\frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt{3}\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{16c^{5/6}}+\frac{\sqrt[3]{d}\,\mathrm{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{16c^{5/6}}-\frac{\sqrt[3]{d}\,\mathrm{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{16c^{5/6}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{16c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 1/16*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(
5/6)-1/16*d^(1/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/6)-1/16*d^(1/3)
```

```
*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/c^(5/6
)-1/8*(d*x^3+c)^(1/2)/c/x+1/8*d^(1/3)*(d*x^3+c)^(1/2)/c/(d^(1/3)*x+c^(1/3)*
(1+3^(1/2)))+1/24*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*
(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/
3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/
c^(2/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(
1/3)*(1+3^(1/2)))^2)^(1/2)-1/16*3^(1/4)*d^(1/3)*(c^(1/3)+d^(1/3)*x)*Ellipti
cE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2
)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(
d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c
^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 632, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {486, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{\sqrt{c+dx^3}}{x^2(8c-dx^3)}\,dx$$
$$= \frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{16c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{\sqrt{3}\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{16c^{5/6}}+\frac{\sqrt[3]{d}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{16c^{5/6}}$$
$$-\frac{\sqrt[3]{d}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{16c^{5/6}}-\frac{\sqrt{c+dx^3}}{8cx}+\frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

```
[In] Int[Sqrt[c + d*x^3]/(x^2*(8*c - d*x^3)),x]
```

```
[Out] -1/8*Sqrt[c + d*x^3]/(c*x) + (d^(1/3)*Sqrt[c + d*x^3])/(8*c*((1 + Sqrt[3])*
c^(1/3) + d^(1/3)*x)) - (Sqrt[3]*d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) +
```

```
 d^(1/3)*x))/Sqrt[c + d*x^3]])/(16*c^(5/6)) + (d^(1/3)*ArcTanh[(c^(1/3) + d
^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(16*c^(5/6)) - (d^(1/3)*ArcTanh[S
qrt[c + d*x^3]/(3*Sqrt[c])])/(16*c^(5/6)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(1
/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1
/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(16
*c^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)^2]*Sqrt[c + d*x^3]) + (d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) -
 c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) +
 d^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*3^(1/4)*c^(2/3)*Sqrt[(c^(1/3)*(c^
(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
```

```
/; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{c+dx^3}}{8cx} + \frac{\int \frac{x\left(13cd-\frac{d^2x^3}{2}\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{8c}$$

$$= -\frac{\sqrt{c+dx^3}}{8cx} + \frac{\int \left(\frac{dx}{2\sqrt{c+dx^3}} + \frac{9cdx}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{8c}$$

$$= -\frac{\sqrt{c+dx^3}}{8cx} + \frac{1}{8}(9d)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx + \frac{d\int \frac{x}{\sqrt{c+dx^3}}\,dx}{16c}$$

$$= -\frac{\sqrt{c+dx^3}}{8cx} - \frac{3\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{32c}$$

$$+ \frac{d^{2/3}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{16c} + \frac{\left(3d^{2/3}\right)\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{32c^{2/3}}$$

$$- \frac{\left(\left(1-\sqrt{3}\right)d^{2/3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{16c^{2/3}} - \frac{\left(9d^{4/3}\right)\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{32\sqrt[3]{c}}$$

$$= -\frac{\sqrt{c+dx^3}}{8cx} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{16c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{\left(3\sqrt[3]{d}\right)\text{Subst}\left(\int\frac{1}{9-cx^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{16\sqrt[3]{c}}$$

$$- \frac{\left(3d^{4/3}\right)\text{Subst}\left(\int\frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{32\sqrt[3]{c}}$$

$$+ \frac{\left(3d^{7/3}\right)\text{Subst}\left(\int\frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{8c^{4/3}}$$

$$= -\frac{\sqrt{c+dx^3}}{8cx} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt{3}\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{16c^{5/6}} + \frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{16c^{5/6}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{16c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{\left(3\sqrt[3]{d}\right)\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{16\sqrt[3]{c}}$$

$$= -\frac{\sqrt{c+dx^3}}{8cx} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt{3}\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{16c^{5/6}} + \frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{16c^{5/6}} - \frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{16c^{5/6}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{16c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.22

$$\int \frac{\sqrt{c+dx^3}}{x^2(8c-dx^3)}\,dx$$
$$= \frac{-80c(c+dx^3)+65cdx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-d^2x^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\right.}{640c^2x\sqrt{c+dx^3}}$$

```
[In] Integrate[Sqrt[c + d*x^3]/(x^2*(8*c - d*x^3)),x]
```

```
[Out] (-80*c*(c + d*x^3) + 65*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5
/3, -((d*x^3)/c), (d*x^3)/(8*c)] - d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3
, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(640*c^2*x*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.11 (sec) , antiderivative size = 868, normalized size of antiderivative = 1.37

method	result	size
elliptic	Expression too large to display	868
risch	Expression too large to display	872
default	Expression too large to display	1306

```
[In] int((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(d*x^3+c)^(1/2)/c/x-1/24*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^
2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((
x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(
-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)
)+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c
*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))
-1/24*I/d^2/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^
2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*
x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x
```

```
^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_
alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(
2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/
2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))
^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.36 (sec) , antiderivative size = 2219, normalized size of antiderivative = 3.51

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(8c-dx^3\right)}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] 1/192*(2*c*x*(d^2/c^5)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^
3 + 640*c^3*d + 18*(5*c^4*d^2*x^7 + 64*c^5*d*x^4 + 32*c^6*x)*(d^2/c^5)^(2/3
) + 6*sqrt(d*x^3 + c)*(6*(5*c^5*d*x^5 + 32*c^6*x^2)*(d^2/c^5)^(5/6) + (7*c^
3*d^2*x^6 + 152*c^4*d*x^3 + 64*c^5)*sqrt(d^2/c^5) + (c*d^3*x^7 + 80*c^2*d^2
*x^4 + 160*c^3*d*x)*(d^2/c^5)^(1/6)) + 18*(c^2*d^3*x^8 + 38*c^3*d^2*x^5 + 6
4*c^4*d*x^2)*(d^2/c^5)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512
*c^3)) - 2*c*x*(d^2/c^5)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*
x^3 + 640*c^3*d + 18*(5*c^4*d^2*x^7 + 64*c^5*d*x^4 + 32*c^6*x)*(d^2/c^5)^(2
/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^5*d*x^5 + 32*c^6*x^2)*(d^2/c^5)^(5/6) + (7*
c^3*d^2*x^6 + 152*c^4*d*x^3 + 64*c^5)*sqrt(d^2/c^5) + (c*d^3*x^7 + 80*c^2*d
^2*x^4 + 160*c^3*d*x)*(d^2/c^5)^(1/6)) + 18*(c^2*d^3*x^8 + 38*c^3*d^2*x^5 +
 64*c^4*d*x^2)*(d^2/c^5)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 5
12*c^3)) - 24*sqrt(d)*x*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -
4*c/d, x)) + (sqrt(-3)*c*x + c*x)*(d^2/c^5)^(1/6)*log((d^4*x^9 + 318*c*d^3*
x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d - 9*(5*c^4*d^2*x^7 + 64*c^5*d*x^4 + 32*c
^6*x + sqrt(-3)*(5*c^4*d^2*x^7 + 64*c^5*d*x^4 + 32*c^6*x))*(d^2/c^5)^(2/3)
+ 3*sqrt(d*x^3 + c)*(6*(5*c^5*d*x^5 + 32*c^6*x^2 - sqrt(-3)*(5*c^5*d*x^5 +
32*c^6*x^2))*(d^2/c^5)^(5/6) - 2*(7*c^3*d^2*x^6 + 152*c^4*d*x^3 + 64*c^5)*s
qrt(d^2/c^5) + (c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x + sqrt(-3)*(c*d^3*
x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x))*(d^2/c^5)^(1/6)) - 9*(c^2*d^3*x^8 + 38
*c^3*d^2*x^5 + 64*c^4*d*x^2 - sqrt(-3)*(c^2*d^3*x^8 + 38*c^3*d^2*x^5 + 64*c
^4*d*x^2))*(d^2/c^5)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c
^3)) - (sqrt(-3)*c*x + c*x)*(d^2/c^5)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 +
1200*c^2*d^2*x^3 + 640*c^3*d - 9*(5*c^4*d^2*x^7 + 64*c^5*d*x^4 + 32*c^6*x +
 sqrt(-3)*(5*c^4*d^2*x^7 + 64*c^5*d*x^4 + 32*c^6*x))*(d^2/c^5)^(2/3) - 3*sq
rt(d*x^3 + c)*(6*(5*c^5*d*x^5 + 32*c^6*x^2 - sqrt(-3)*(5*c^5*d*x^5 + 32*c^6
```

```
*x^2))*(d^2/c^5)^(5/6) - 2*(7*c^3*d^2*x^6 + 152*c^4*d*x^3 + 64*c^5)*sqrt(d^
2/c^5) + (c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x + sqrt(-3)*(c*d^3*x^7 +
80*c^2*d^2*x^4 + 160*c^3*d*x))*(d^2/c^5)^(1/6)) - 9*(c^2*d^3*x^8 + 38*c^3*d
^2*x^5 + 64*c^4*d*x^2 - sqrt(-3)*(c^2*d^3*x^8 + 38*c^3*d^2*x^5 + 64*c^4*d*x
^2))*(d^2/c^5)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) -
 (sqrt(-3)*c*x - c*x)*(d^2/c^5)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c
^2*d^2*x^3 + 640*c^3*d - 9*(5*c^4*d^2*x^7 + 64*c^5*d*x^4 + 32*c^6*x - sqrt(
-3)*(5*c^4*d^2*x^7 + 64*c^5*d*x^4 + 32*c^6*x))*(d^2/c^5)^(2/3) + 3*sqrt(d*x
^3 + c)*(6*(5*c^5*d*x^5 + 32*c^6*x^2 + sqrt(-3)*(5*c^5*d*x^5 + 32*c^6*x^2))
*(d^2/c^5)^(5/6) - 2*(7*c^3*d^2*x^6 + 152*c^4*d*x^3 + 64*c^5)*sqrt(d^2/c^5)
 + (c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x - sqrt(-3)*(c*d^3*x^7 + 80*c^2
*d^2*x^4 + 160*c^3*d*x))*(d^2/c^5)^(1/6)) - 9*(c^2*d^3*x^8 + 38*c^3*d^2*x^5
 + 64*c^4*d*x^2 + sqrt(-3)*(c^2*d^3*x^8 + 38*c^3*d^2*x^5 + 64*c^4*d*x^2))*(
d^2/c^5)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (sqrt
(-3)*c*x - c*x)*(d^2/c^5)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2
*x^3 + 640*c^3*d - 9*(5*c^4*d^2*x^7 + 64*c^5*d*x^4 + 32*c^6*x - sqrt(-3)*(5
*c^4*d^2*x^7 + 64*c^5*d*x^4 + 32*c^6*x))*(d^2/c^5)^(2/3) - 3*sqrt(d*x^3 + c
)*(6*(5*c^5*d*x^5 + 32*c^6*x^2 + sqrt(-3)*(5*c^5*d*x^5 + 32*c^6*x^2))*(d^2/
c^5)^(5/6) - 2*(7*c^3*d^2*x^6 + 152*c^4*d*x^3 + 64*c^5)*sqrt(d^2/c^5) + (c*
d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x - sqrt(-3)*(c*d^3*x^7 + 80*c^2*d^2*x
^4 + 160*c^3*d*x))*(d^2/c^5)^(1/6)) - 9*(c^2*d^3*x^8 + 38*c^3*d^2*x^5 + 64*
c^4*d*x^2 + sqrt(-3)*(c^2*d^3*x^8 + 38*c^3*d^2*x^5 + 64*c^4*d*x^2))*(d^2/c^
5)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 24*sqrt(d*x
^3 + c))/(c*x)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(8c-dx^3\right)}\,dx=-\int \frac{\sqrt{c+dx^3}}{-8cx^2+dx^5}\,dx$$

```
[In] integrate((d*x**3+c)**(1/2)/x**2/(-d*x**3+8*c),x)
[Out] -Integral(sqrt(c + d*x**3)/(-8*c*x**2 + d*x**5), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(8c-dx^3\right)}\,dx=\int -\frac{\sqrt{dx^3+c}}{(dx^3-8\,c)x^2}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c),x, algorithm="maxima")
[Out] -integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(8c-dx^3\right)}\,dx = \int -\frac{\sqrt{dx^3+c}}{(dx^3-8\,c)x^2}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c),x, algorithm="giac")
[Out] integrate(-sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(8c-dx^3\right)}\,dx = \int \frac{\sqrt{d\,x^3+c}}{x^2\,\left(8\,c-d\,x^3\right)}\,dx$$

```
[In] int((c + d*x^3)^(1/2)/(x^2*(8*c - d*x^3)),x)
[Out] int((c + d*x^3)^(1/2)/(x^2*(8*c - d*x^3)), x)
```

3.293 $\int \frac{\sqrt{c+dx^3}}{x^5(8c-dx^3)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 654

$$\begin{aligned}
\int \frac{\sqrt{c+dx^3}}{x^5(8c-dx^3)}\,dx &= -\frac{\sqrt{c+dx^3}}{32cx^4}-\frac{d\sqrt{c+dx^3}}{16c^2x} \\
&+\frac{d^{4/3}\sqrt{c+dx^3}}{16c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt{3}d^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{128c^{11/6}} \\
&+\frac{d^{4/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{128c^{11/6}}-\frac{d^{4/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{11/6}} \\
&-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+\frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{8\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}$$

```
[Out] 1/128*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^
(11/6)-1/128*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)-1/128*d^
```

```
(4/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/c
^(11/6)-1/32*(d*x^3+c)^(1/2)/c/x^4-1/16*d*(d*x^3+c)^(1/2)/c^2/x+1/16*d^(4/3
)*(d*x^3+c)^(1/2)/c^2/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/48*d^(4/3)*(c^(1/3)
+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1
+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)
*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(5/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c
^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/32*3^
(1/4)*d^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))
/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*(
(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^
(1/2)/c^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/
3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 654, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {486, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{\sqrt{c+dx^3}}{x^5(8c-dx^3)}\,dx$$
$$= \frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{8\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{32c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{\sqrt{3}d^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{128c^{11/6}}+\frac{d^{4/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{128c^{11/6}}$$
$$-\frac{d^{4/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{11/6}}+\frac{d^{4/3}\sqrt{c+dx^3}}{16c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{d\sqrt{c+dx^3}}{16c^2x}-\frac{\sqrt{c+dx^3}}{32cx^4}$$

```
[In] Int[Sqrt[c + d*x^3]/(x^5*(8*c - d*x^3)),x]
```

```
[Out] -1/32*Sqrt[c + d*x^3]/(c*x^4) - (d*Sqrt[c + d*x^3])/(16*c^2*x) + (d^(4/3)*S
qrt[c + d*x^3])/(16*c^2*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (Sqrt[3]*d^(
4/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(128*
c^(11/6)) + (d^(4/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*
x^3])])/(128*c^(11/6)) - (d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(12
8*c^(11/6)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt
[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(32*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3
) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (
d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x
^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])
/(8*Sqrt[2]*3^(1/4)*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
```

```
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{c+dx^3}}{32cx^4} + \frac{\int \frac{16cd+\frac{5d^2x^3}{2}}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{32c} \\
&= -\frac{\sqrt{c+dx^3}}{32cx^4} - \frac{d\sqrt{c+dx^3}}{16c^2x} - \frac{\int \frac{x(-100c^2d^2+8cd^3x^3)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{256c^3} \\
&= -\frac{\sqrt{c+dx^3}}{32cx^4} - \frac{d\sqrt{c+dx^3}}{16c^2x} - \frac{\int \left(-\frac{8cd^2x}{\sqrt{c+dx^3}} - \frac{36c^2d^2x}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{256c^3} \\
&= -\frac{\sqrt{c+dx^3}}{32cx^4} - \frac{d\sqrt{c+dx^3}}{16c^2x} + \frac{d^2\int \frac{x}{\sqrt{c+dx^3}}\,dx}{32c^2} + \frac{(9d^2)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{64c}
\end{aligned}$$

$$= -\frac{\sqrt{c+dx^3}}{32cx^4} - \frac{d\sqrt{c+dx^3}}{16c^2x} - \frac{(3d)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{256c^2}$$

$$+ \frac{d^{5/3}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{32c^2} + \frac{\left(3d^{5/3}\right)\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{256c^{5/3}}$$

$$- \frac{\left(\left(1-\sqrt{3}\right)d^{5/3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{32c^{5/3}} - \frac{\left(9d^{7/3}\right)\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{256c^{4/3}}$$

$$= -\frac{\sqrt{c+dx^3}}{32cx^4} - \frac{d\sqrt{c+dx^3}}{16c^2x} + \frac{d^{4/3}\sqrt{c+dx^3}}{16c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{8\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{\left(3d^{4/3}\right)\text{Subst}\left(\int \frac{1}{9-cx^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{128c^{4/3}}$$

$$- \frac{\left(3d^{7/3}\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{256c^{4/3}}$$

$$+ \frac{\left(3d^{10/3}\right)\text{Subst}\left(\int \frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{64c^{7/3}}$$

$$= -\frac{\sqrt{c+dx^3}}{32cx^4} - \frac{d\sqrt{c+dx^3}}{16c^2x} + \frac{d^{4/3}\sqrt{c+dx^3}}{16c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt{3}d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{128c^{11/6}} + \frac{d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{128c^{11/6}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{8\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{\left(3d^{4/3}\right)\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{128c^{4/3}}$$

$$= -\frac{\sqrt{c+dx^3}}{32cx^4} - \frac{d\sqrt{c+dx^3}}{16c^2x} + \frac{d^{4/3}\sqrt{c+dx^3}}{16c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt{3}d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{128c^{11/6}} + \frac{d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{128c^{11/6}} - \frac{d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{11/6}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{8\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.10 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.23

$$\int \frac{\sqrt{c+dx^3}}{x^5(8c-dx^3)}\,dx = \frac{125cd^2x^6\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right) - 4\left(40c(c^2+3cdx^3+2d^2x^6)+d^3x^9\sqrt{1+\frac{dx^3}{c}}\operatorname{Appell}\right.}{5120c^3x^4\sqrt{c+dx^3}}$$

```
[In] Integrate[Sqrt[c + d*x^3]/(x^5*(8*c - d*x^3)),x]
```

```
[Out] (125*c*d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c),
 (d*x^3)/(8*c)] - 4*(40*c*(c^2 + 3*c*d*x^3 + 2*d^2*x^6) + d^3*x^9*Sqrt[1 +
(d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(5120*
c^3*x^4*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.79 (sec) , antiderivative size = 882, normalized size of antiderivative = 1.35

method	result	size
risch	Expression too large to display	882
elliptic	Expression too large to display	887
default	Expression too large to display	1782

```
[In] int((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*(d*x^3+c)^(1/2)*(2*d*x^3+c)/c^2/x^4+1/64*d^2/c^2*(-4/3*I*3^(1/2)/d*(-
c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(
1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)
*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(
I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-1/3*I/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/
3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(
1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)
^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))
)/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-
I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2
/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(
1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2
/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.74 (sec) , antiderivative size = 2401, normalized size of antiderivative = 3.67

$$\int \frac{\sqrt{c+dx^3}}{x^5\,(8c-dx^3)}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] 1/1536*(2*c^2*x^4*(d^8/c^11)^(1/6)*log((d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*
d^7*x^3 + 640*c^3*d^6 + 18*(5*c^8*d^3*x^7 + 64*c^9*d^2*x^4 + 32*c^10*d*x)*(
d^8/c^11)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2)*(d^8/c^
11)^(5/6) + (7*c^6*d^4*x^6 + 152*c^7*d^3*x^3 + 64*c^8*d^2)*sqrt(d^8/c^11) +
 (c^2*d^7*x^7 + 80*c^3*d^6*x^4 + 160*c^4*d^5*x)*(d^8/c^11)^(1/6)) + 18*(c^4
*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2)*(d^8/c^11)^(1/3))/(d^3*x^9 - 24
*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*c^2*x^4*(d^8/c^11)^(1/6)*log((d^
9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 + 18*(5*c^8*d^3*x^7
+ 64*c^9*d^2*x^4 + 32*c^10*d*x)*(d^8/c^11)^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*
c^10*d*x^5 + 32*c^11*x^2)*(d^8/c^11)^(5/6) + (7*c^6*d^4*x^6 + 152*c^7*d^3*x
^3 + 64*c^8*d^2)*sqrt(d^8/c^11) + (c^2*d^7*x^7 + 80*c^3*d^6*x^4 + 160*c^4*d
^5*x)*(d^8/c^11)^(1/6)) + 18*(c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2
)*(d^8/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 9
6*d^(3/2)*x^4*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x))
 + (sqrt(-3)*c^2*x^4 + c^2*x^4)*(d^8/c^11)^(1/6)*log((d^9*x^9 + 318*c*d^8*x
^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^8*d^3*x^7 + 64*c^9*d^2*x^4 + 3
2*c^10*d*x + sqrt(-3)*(5*c^8*d^3*x^7 + 64*c^9*d^2*x^4 + 32*c^10*d*x))*(d^8/
c^11)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2 - sqrt(-3)*(
5*c^10*d*x^5 + 32*c^11*x^2))*(d^8/c^11)^(5/6) - 2*(7*c^6*d^4*x^6 + 152*c^7*
d^3*x^3 + 64*c^8*d^2)*sqrt(d^8/c^11) + (c^2*d^7*x^7 + 80*c^3*d^6*x^4 + 160*
c^4*d^5*x + sqrt(-3)*(c^2*d^7*x^7 + 80*c^3*d^6*x^4 + 160*c^4*d^5*x))*(d^8/c
^11)^(1/6)) - 9*(c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2 - sqrt(-3)*(
c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2))*(d^8/c^11)^(1/3))/(d^3*x^9
- 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (sqrt(-3)*c^2*x^4 + c^2*x^4)*(
d^8/c^11)^(1/6)*log((d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d
^6 - 9*(5*c^8*d^3*x^7 + 64*c^9*d^2*x^4 + 32*c^10*d*x + sqrt(-3)*(5*c^8*d^3*
x^7 + 64*c^9*d^2*x^4 + 32*c^10*d*x))*(d^8/c^11)^(2/3) - 3*sqrt(d*x^3 + c)*(
6*(5*c^10*d*x^5 + 32*c^11*x^2 - sqrt(-3)*(5*c^10*d*x^5 + 32*c^11*x^2))*(d^8
/c^11)^(5/6) - 2*(7*c^6*d^4*x^6 + 152*c^7*d^3*x^3 + 64*c^8*d^2)*sqrt(d^8/c^
11) + (c^2*d^7*x^7 + 80*c^3*d^6*x^4 + 160*c^4*d^5*x + sqrt(-3)*(c^2*d^7*x^7
 + 80*c^3*d^6*x^4 + 160*c^4*d^5*x))*(d^8/c^11)^(1/6)) - 9*(c^4*d^6*x^8 + 38
*c^5*d^5*x^5 + 64*c^6*d^4*x^2 - sqrt(-3)*(c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64
*c^6*d^4*x^2))*(d^8/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 -
512*c^3)) - (sqrt(-3)*c^2*x^4 - c^2*x^4)*(d^8/c^11)^(1/6)*log((d^9*x^9 + 31
8*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^8*d^3*x^7 + 64*c^9*d^
2*x^4 + 32*c^10*d*x - sqrt(-3)*(5*c^8*d^3*x^7 + 64*c^9*d^2*x^4 + 32*c^10*d*
x))*(d^8/c^11)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2 + s
qrt(-3)*(5*c^10*d*x^5 + 32*c^11*x^2))*(d^8/c^11)^(5/6) - 2*(7*c^6*d^4*x^6 +
 152*c^7*d^3*x^3 + 64*c^8*d^2)*sqrt(d^8/c^11) + (c^2*d^7*x^7 + 80*c^3*d^6*x
^4 + 160*c^4*d^5*x - sqrt(-3)*(c^2*d^7*x^7 + 80*c^3*d^6*x^4 + 160*c^4*d^5*x
))*(d^8/c^11)^(1/6)) - 9*(c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2 + s
qrt(-3)*(c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2))*(d^8/c^11)^(1/3))/
(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (sqrt(-3)*c^2*x^4 - c
^2*x^4)*(d^8/c^11)^(1/6)*log((d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 +
640*c^3*d^6 - 9*(5*c^8*d^3*x^7 + 64*c^9*d^2*x^4 + 32*c^10*d*x - sqrt(-3)*(5
```

```
*c^8*d^3*x^7 + 64*c^9*d^2*x^4 + 32*c^10*d*x))*(d^8/c^11)^(2/3) - 3*sqrt(d*x
^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2 + sqrt(-3)*(5*c^10*d*x^5 + 32*c^11*x
^2))*(d^8/c^11)^(5/6) - 2*(7*c^6*d^4*x^6 + 152*c^7*d^3*x^3 + 64*c^8*d^2)*sq
rt(d^8/c^11) + (c^2*d^7*x^7 + 80*c^3*d^6*x^4 + 160*c^4*d^5*x - sqrt(-3)*(c^
2*d^7*x^7 + 80*c^3*d^6*x^4 + 160*c^4*d^5*x))*(d^8/c^11)^(1/6)) - 9*(c^4*d^6
*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2 + sqrt(-3)*(c^4*d^6*x^8 + 38*c^5*d^5
*x^5 + 64*c^6*d^4*x^2))*(d^8/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2
*d*x^3 - 512*c^3)) - 48*(2*d*x^3 + c)*sqrt(d*x^3 + c))/(c^2*x^4)
```

Sympy [F]

$$\int \frac{\sqrt{c + dx^3}}{x^5 (8c - dx^3)}\, dx = -\int \frac{\sqrt{c + dx^3}}{-8cx^5 + dx^8}\, dx$$

```
[In] integrate((d*x**3+c)**(1/2)/x**5/(-d*x**3+8*c),x)
[Out] -Integral(sqrt(c + d*x**3)/(-8*c*x**5 + d*x**8), x)
```

Maxima [F]

$$\int \frac{\sqrt{c + dx^3}}{x^5 (8c - dx^3)}\, dx = \int -\frac{\sqrt{dx^3 + c}}{(dx^3 - 8\,c)x^5}\, dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c),x, algorithm="maxima")
[Out] -integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^5), x)
```

Giac [F]

$$\int \frac{\sqrt{c + dx^3}}{x^5 (8c - dx^3)}\, dx = \int -\frac{\sqrt{dx^3 + c}}{(dx^3 - 8\,c)x^5}\, dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c),x, algorithm="giac")
[Out] integrate(-sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^5\,(8c-dx^3)}\,dx = \int \frac{\sqrt{d\,x^3+c}}{x^5\,\left(8\,c-d\,x^3\right)}\,dx$$

```
[In] int((c + d*x^3)^(1/2)/(x^5*(8*c - d*x^3)),x)
[Out] int((c + d*x^3)^(1/2)/(x^5*(8*c - d*x^3)), x)
```

3.294 $\int \frac{\sqrt{c+dx^3}}{x^8(8c-dx^3)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 678

$$\begin{aligned}\int \frac{\sqrt{c+dx^3}}{x^8(8c-dx^3)}\,dx &= -\frac{\sqrt{c+dx^3}}{56cx^7}-\frac{19d\sqrt{c+dx^3}}{1792c^2x^4}+\frac{d^2\sqrt{c+dx^3}}{112c^3x}\\
&-\frac{d^{7/3}\sqrt{c+dx^3}}{112c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt{3}d^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1024c^{17/6}}\\
&+\frac{d^{7/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1024c^{17/6}}-\frac{d^{7/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1024c^{17/6}}\\
&+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{224c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}\\
&-\frac{d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{56\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}\end{aligned}$$

```
[Out] 1/1024*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c
^(17/6)-1/1024*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)-1/1024
```

```
*d^(7/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2
)/c^(17/6)-1/56*(d*x^3+c)^(1/2)/c/x^7-19/1792*d*(d*x^3+c)^(1/2)/c^2/x^4+1/1
12*d^2*(d*x^3+c)^(1/2)/c^3/x-1/112*d^(7/3)*(d*x^3+c)^(1/2)/c^3/(d^(1/3)*x+c
^(1/3)*(1+3^(1/2)))-1/336*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+
c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/
3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*
3^(3/4)/c^(8/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/
3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/224*3^(1/4)*d^(7/3)*(c^(1/3)+d^(1/3)*x
)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))
,I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/
3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(8/3)/(d*x^3+c)^(1/2)/(c
^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 678, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {486, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{\sqrt{c+dx^3}}{x^8(8c-dx^3)}\,dx = -\frac{d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{56\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{224c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{\sqrt{3}d^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1024c^{17/6}} + \frac{d^{7/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1024c^{17/6}} - \frac{d^{7/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1024c^{17/6}} - \frac{d^{7/3}\sqrt{c+dx^3}}{112c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{d^2\sqrt{c+dx^3}}{112c^3x} - \frac{19d\sqrt{c+dx^3}}{1792c^2x^4} - \frac{\sqrt{c+dx^3}}{56cx^7}$$

```
[In] Int[Sqrt[c + d*x^3]/(x^8*(8*c - d*x^3)),x]
```

```
[Out] -1/56*Sqrt[c + d*x^3]/(c*x^7) - (19*d*Sqrt[c + d*x^3])/(1792*c^2*x^4) + (d^
2*Sqrt[c + d*x^3])/(112*c^3*x) - (d^(7/3)*Sqrt[c + d*x^3])/(112*c^3*((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)) - (Sqrt[3]*d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c
^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(1024*c^(17/6)) + (d^(7/3)*ArcTanh[(
c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(1024*c^(17/6)) - (d^(
7/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1024*c^(17/6)) + (3^(1/4)*Sqrt[
2 - Sqrt[3]]*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*
x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((
1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7
- 4*Sqrt[3]])/(224*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (d^(7/3)*(c^(1/3) + d^(1/3)*
x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3)
+ d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(56*Sqrt[2]*3^(1/4)*c^(8/3
)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^
2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
```

```
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{c+dx^3}}{56cx^7}+\frac{\int \frac{19cd+\frac{11d^2x^3}{2}}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx}{56c}\\
&= -\frac{\sqrt{c+dx^3}}{56cx^7}-\frac{19d\sqrt{c+dx^3}}{1792c^2x^4}-\frac{\int \frac{128c^2d^2-\frac{95}{2}cd^3x^3}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{1792c^3}\\
&= -\frac{\sqrt{c+dx^3}}{56cx^7}-\frac{19d\sqrt{c+dx^3}}{1792c^2x^4}+\frac{d^2\sqrt{c+dx^3}}{112c^3x}+\frac{\int \frac{x(-260c^3d^3+64c^2d^4x^3)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{14336c^5}\\
&= -\frac{\sqrt{c+dx^3}}{56cx^7}-\frac{19d\sqrt{c+dx^3}}{1792c^2x^4}+\frac{d^2\sqrt{c+dx^3}}{112c^3x}+\frac{\int \left(-\frac{64c^2d^3x}{\sqrt{c+dx^3}}+\frac{252c^3d^3x}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{14336c^5}\\
&= -\frac{\sqrt{c+dx^3}}{56cx^7}-\frac{19d\sqrt{c+dx^3}}{1792c^2x^4}+\frac{d^2\sqrt{c+dx^3}}{112c^3x}-\frac{d^3\int \frac{x}{\sqrt{c+dx^3}}\,dx}{224c^3}+\frac{(9d^3)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{512c^2}
\end{aligned}
$$

$$= -\frac{\sqrt{c+dx^3}}{56cx^7} - \frac{19d\sqrt{c+dx^3}}{1792c^2x^4} + \frac{d^2\sqrt{c+dx^3}}{112c^3x} - \frac{(3d^2)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{2048c^3}$$

$$- \frac{d^{8/3}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{224c^3} + \frac{(3d^{8/3})\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{2048c^{8/3}}$$

$$+ \frac{\left(\left(1-\sqrt{3}\right)d^{8/3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{224c^{8/3}} - \frac{\left(9d^{10/3}\right)\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{2048c^{7/3}}$$

$$= -\frac{\sqrt{c+dx^3}}{56cx^7} - \frac{19d\sqrt{c+dx^3}}{1792c^2x^4} + \frac{d^2\sqrt{c+dx^3}}{112c^3x} - \frac{d^{7/3}\sqrt{c+dx^3}}{112c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$+ \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{224c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$- \frac{d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{56\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{\left(3d^{7/3}\right)\text{Subst}\left(\int \frac{1}{9-cx^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{1024c^{7/3}}$$

$$- \frac{\left(3d^{10/3}\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{2048c^{7/3}}$$

$$+ \frac{\left(3d^{13/3}\right)\text{Subst}\left(\int \frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{512c^{10/3}}$$

$$= -\frac{\sqrt{c+dx^3}}{56cx^7} - \frac{19d\sqrt{c+dx^3}}{1792c^2x^4} + \frac{d^2\sqrt{c+dx^3}}{112c^3x} - \frac{d^{7/3}\sqrt{c+dx^3}}{112c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$- \frac{\sqrt{3}d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1024c^{17/6}} + \frac{d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1024c^{17/6}}$$

$$+ \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{224c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$- \frac{d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{56\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$- \frac{\left(3d^{7/3}\right)\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{1024c^{7/3}}$$

$$= -\frac{\sqrt{c+dx^3}}{56cx^7} - \frac{19d\sqrt{c+dx^3}}{1792c^2x^4} + \frac{d^2\sqrt{c+dx^3}}{112c^3x}$$

$$- \frac{d^{7/3}\sqrt{c+dx^3}}{112c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt{3}d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1024c^{17/6}}$$

$$+ \frac{d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1024c^{17/6}} - \frac{d^{7/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1024c^{17/6}}$$

$$+ \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{224c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$- \frac{d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{56\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.24

$$\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)}\,dx$$

$$= \frac{-160c(32c^3+51c^2dx^3+3cd^2x^6-16d^3x^9)-325cd^3x^9\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+32d^4x}{286720c^4x^7\sqrt{c+dx^3}}$$

```
[In] Integrate[Sqrt[c + d*x^3]/(x^8*(8*c - d*x^3)),x]
```

```
[Out] (-160*c*(32*c^3 + 51*c^2*d*x^3 + 3*c*d^2*x^6 - 16*d^3*x^9) - 325*c*d^3*x^9*
Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)]
 + 32*d^4*x^12*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c),
 (d*x^3)/(8*c)])/(286720*c^4*x^7*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.03 (sec) , antiderivative size = 895, normalized size of antiderivative = 1.32

method	result	size
risch	Expression too large to display	895
elliptic	Expression too large to display	906
default	Expression too large to display	2280

```
[In] int((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/1792*(d*x^3+c)^(1/2)*(-16*d^2*x^6+19*c*d*x^3+32*c^2)/c^3/x^7-1/3584*d^3/
c^3*(-32/3*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3
))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+7/3*I/d^3*2^(1/2)*
sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)
^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2
)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(
1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(
1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/
d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*
3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(
_Z^3*d-8*c)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.92 (sec) , antiderivative size = 2436, normalized size of antiderivative = 3.59

$$\int \frac{\sqrt{c+dx^3}}{x^8 (8c-dx^3)} dx = \text{Too large to display}$$

```
[In] integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] 1/86016*(14*c^3*x^7*(d^14/c^17)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6 + 1200
*c^2*d^12*x^3 + 640*c^3*d^11 + 18*(5*c^12*d^4*x^7 + 64*c^13*d^3*x^4 + 32*c^
14*d^2*x)*(d^14/c^17)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*
x^2)*(d^14/c^17)^(5/6) + (7*c^9*d^6*x^6 + 152*c^10*d^5*x^3 + 64*c^11*d^4)*s
qrt(d^14/c^17) + (c^3*d^11*x^7 + 80*c^4*d^10*x^4 + 160*c^5*d^9*x)*(d^14/c^1
7)^(1/6)) + 18*(c^6*d^9*x^8 + 38*c^7*d^8*x^5 + 64*c^8*d^7*x^2)*(d^14/c^17)^
(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 14*c^3*x^7*(d^
14/c^17)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3
*d^11 + 18*(5*c^12*d^4*x^7 + 64*c^13*d^3*x^4 + 32*c^14*d^2*x)*(d^14/c^17)^(
2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2)*(d^14/c^17)^(5/6)
+ (7*c^9*d^6*x^6 + 152*c^10*d^5*x^3 + 64*c^11*d^4)*sqrt(d^14/c^17) + (c^3*d
^11*x^7 + 80*c^4*d^10*x^4 + 160*c^5*d^9*x)*(d^14/c^17)^(1/6)) + 18*(c^6*d^9
*x^8 + 38*c^7*d^8*x^5 + 64*c^8*d^7*x^2)*(d^14/c^17)^(1/3))/(d^3*x^9 - 24*c*
d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 768*d^(5/2)*x^7*weierstrassZeta(0, -4
*c/d, weierstrassPInverse(0, -4*c/d, x)) + 7*(sqrt(-3)*c^3*x^7 + c^3*x^7)*(
d^14/c^17)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c
^3*d^11 - 9*(5*c^12*d^4*x^7 + 64*c^13*d^3*x^4 + 32*c^14*d^2*x + sqrt(-3)*(5
*c^12*d^4*x^7 + 64*c^13*d^3*x^4 + 32*c^14*d^2*x))*(d^14/c^17)^(2/3) + 3*sqr
t(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2 - sqrt(-3)*(5*c^15*d*x^5 + 32*c
^16*x^2))*(d^14/c^17)^(5/6) - 2*(7*c^9*d^6*x^6 + 152*c^10*d^5*x^3 + 64*c^11
*d^4)*sqrt(d^14/c^17) + (c^3*d^11*x^7 + 80*c^4*d^10*x^4 + 160*c^5*d^9*x + s
qrt(-3)*(c^3*d^11*x^7 + 80*c^4*d^10*x^4 + 160*c^5*d^9*x))*(d^14/c^17)^(1/6)
) - 9*(c^6*d^9*x^8 + 38*c^7*d^8*x^5 + 64*c^8*d^7*x^2 - sqrt(-3)*(c^6*d^9*x^
8 + 38*c^7*d^8*x^5 + 64*c^8*d^7*x^2))*(d^14/c^17)^(1/3))/(d^3*x^9 - 24*c*d^
2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 7*(sqrt(-3)*c^3*x^7 + c^3*x^7)*(d^14/c^
17)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11
 - 9*(5*c^12*d^4*x^7 + 64*c^13*d^3*x^4 + 32*c^14*d^2*x + sqrt(-3)*(5*c^12*d
^4*x^7 + 64*c^13*d^3*x^4 + 32*c^14*d^2*x))*(d^14/c^17)^(2/3) - 3*sqrt(d*x^3
 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2 - sqrt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2
))*(d^14/c^17)^(5/6) - 2*(7*c^9*d^6*x^6 + 152*c^10*d^5*x^3 + 64*c^11*d^4)*s
qrt(d^14/c^17) + (c^3*d^11*x^7 + 80*c^4*d^10*x^4 + 160*c^5*d^9*x + sqrt(-3)
*(c^3*d^11*x^7 + 80*c^4*d^10*x^4 + 160*c^5*d^9*x))*(d^14/c^17)^(1/6)) - 9*(
c^6*d^9*x^8 + 38*c^7*d^8*x^5 + 64*c^8*d^7*x^2 - sqrt(-3)*(c^6*d^9*x^8 + 38*
c^7*d^8*x^5 + 64*c^8*d^7*x^2))*(d^14/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 +
 192*c^2*d*x^3 - 512*c^3)) - 7*(sqrt(-3)*c^3*x^7 - c^3*x^7)*(d^14/c^17)^(1/
```

```
6)*log((d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 - 9*(5
*c^12*d^4*x^7 + 64*c^13*d^3*x^4 + 32*c^14*d^2*x - sqrt(-3)*(5*c^12*d^4*x^7
+ 64*c^13*d^3*x^4 + 32*c^14*d^2*x))*(d^14/c^17)^(2/3) + 3*sqrt(d*x^3 + c)*(
6*(5*c^15*d*x^5 + 32*c^16*x^2 + sqrt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2))*(d^1
4/c^17)^(5/6) - 2*(7*c^9*d^6*x^6 + 152*c^10*d^5*x^3 + 64*c^11*d^4)*sqrt(d^1
4/c^17) + (c^3*d^11*x^7 + 80*c^4*d^10*x^4 + 160*c^5*d^9*x - sqrt(-3)*(c^3*d
^11*x^7 + 80*c^4*d^10*x^4 + 160*c^5*d^9*x))*(d^14/c^17)^(1/6)) - 9*(c^6*d^9
*x^8 + 38*c^7*d^8*x^5 + 64*c^8*d^7*x^2 + sqrt(-3)*(c^6*d^9*x^8 + 38*c^7*d^8
*x^5 + 64*c^8*d^7*x^2))*(d^14/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^
2*d*x^3 - 512*c^3)) + 7*(sqrt(-3)*c^3*x^7 - c^3*x^7)*(d^14/c^17)^(1/6)*log(
(d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 - 9*(5*c^12*d
^4*x^7 + 64*c^13*d^3*x^4 + 32*c^14*d^2*x - sqrt(-3)*(5*c^12*d^4*x^7 + 64*c^
13*d^3*x^4 + 32*c^14*d^2*x))*(d^14/c^17)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^
15*d*x^5 + 32*c^16*x^2 + sqrt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2))*(d^14/c^17)
^(5/6) - 2*(7*c^9*d^6*x^6 + 152*c^10*d^5*x^3 + 64*c^11*d^4)*sqrt(d^14/c^17)
 + (c^3*d^11*x^7 + 80*c^4*d^10*x^4 + 160*c^5*d^9*x - sqrt(-3)*(c^3*d^11*x^7
 + 80*c^4*d^10*x^4 + 160*c^5*d^9*x))*(d^14/c^17)^(1/6)) - 9*(c^6*d^9*x^8 +
38*c^7*d^8*x^5 + 64*c^8*d^7*x^2 + sqrt(-3)*(c^6*d^9*x^8 + 38*c^7*d^8*x^5 +
64*c^8*d^7*x^2))*(d^14/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3
 - 512*c^3)) + 48*(16*d^2*x^6 - 19*c*d*x^3 - 32*c^2)*sqrt(d*x^3 + c))/(c^3*
x^7)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^8\,(8c-dx^3)}\,dx = -\int \frac{\sqrt{c+dx^3}}{-8cx^8+dx^{11}}\,dx$$

```
[In] integrate((d*x**3+c)**(1/2)/x**8/(-d*x**3+8*c),x)
[Out] -Integral(sqrt(c + d*x**3)/(-8*c*x**8 + d*x**11), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^8\,(8c-dx^3)}\,dx = \int -\frac{\sqrt{dx^3+c}}{(dx^3-8\,c)x^8}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c),x, algorithm="maxima")
[Out] -integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^8), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)}\,dx=\int -\frac{\sqrt{dx^3+c}}{(dx^3-8\,c)x^8}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c),x, algorithm="giac")
[Out] integrate(-sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)}\,dx=\int \frac{\sqrt{d\,x^3+c}}{x^8\,\left(8\,c-d\,x^3\right)}\,dx$$

```
[In] int((c + d*x^3)^(1/2)/(x^8*(8*c - d*x^3)),x)
[Out] int((c + d*x^3)^(1/2)/(x^8*(8*c - d*x^3)), x)
```

3.295 $\int \frac{x^{11}(c+dx^3)^{3/2}}{8c-dx^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 130

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{8c-dx^3}\,dx=-\frac{3072c^4\sqrt{c+dx^3}}{d^4}-\frac{1024c^3(c+dx^3)^{3/2}}{9d^4}-\frac{38c^2(c+dx^3)^{5/2}}{5d^4}-\frac{4c(c+dx^3)^{7/2}}{7d^4}-\frac{2(c+dx^3)^{9/2}}{27d^4}+\frac{9216c^{9/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}$$

```
[Out] -1024/9*c^3*(d*x^3+c)^(3/2)/d^4-38/5*c^2*(d*x^3+c)^(5/2)/d^4-4/7*c*(d*x^3+c
)^(7/2)/d^4-2/27*(d*x^3+c)^(9/2)/d^4+9216*c^(9/2)*arctanh(1/3*(d*x^3+c)^(1/
2)/c^(1/2))/d^4-3072*c^4*(d*x^3+c)^(1/2)/d^4
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {457, 90, 52, 65, 212}

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{8c-dx^3}\,dx=\frac{9216c^{9/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}-\frac{3072c^4\sqrt{c+dx^3}}{d^4}-\frac{1024c^3(c+dx^3)^{3/2}}{9d^4}-\frac{38c^2(c+dx^3)^{5/2}}{5d^4}-\frac{4c(c+dx^3)^{7/2}}{7d^4}-\frac{2(c+dx^3)^{9/2}}{27d^4}$$

```
[In] Int[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-3072*c^4*Sqrt[c + d*x^3])/d^4 - (1024*c^3*(c + d*x^3)^(3/2))/(9*d^4) - (3
8*c^2*(c + d*x^3)^(5/2))/(5*d^4) - (4*c*(c + d*x^3)^(7/2))/(7*d^4) - (2*(c
+ d*x^3)^(9/2))/(27*d^4) + (9216*c^(9/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c]
)])/d^4
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^3(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{57c^2(c+dx)^{3/2}}{d^3}+\frac{512c^3(c+dx)^{3/2}}{d^3(8c-dx)}-\frac{6c(c+dx)^{5/2}}{d^3}-\frac{(c+dx)^{7/2}}{d^3}\right)\,dx, x, x^3\right)\\
&= -\frac{38c^2(c+dx^3)^{5/2}}{5d^4}-\frac{4c(c+dx^3)^{7/2}}{7d^4}-\frac{2(c+dx^3)^{9/2}}{27d^4}+\frac{(512c^3)\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)}{3d^3}
\end{aligned}$$

$$= -\frac{1024c^3(c+dx^3)^{3/2}}{9d^4} - \frac{38c^2(c+dx^3)^{5/2}}{5d^4} - \frac{4c(c+dx^3)^{7/2}}{7d^4} - \frac{2(c+dx^3)^{9/2}}{27d^4} + \frac{(1536c^4)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)}{d^3}$$

$$= -\frac{3072c^4\sqrt{c+dx^3}}{d^4} - \frac{1024c^3(c+dx^3)^{3/2}}{9d^4} - \frac{38c^2(c+dx^3)^{5/2}}{5d^4} - \frac{4c(c+dx^3)^{7/2}}{7d^4} - \frac{2(c+dx^3)^{9/2}}{27d^4} + \frac{(13824c^5)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{d^3}$$

$$= -\frac{3072c^4\sqrt{c+dx^3}}{d^4} - \frac{1024c^3(c+dx^3)^{3/2}}{9d^4} - \frac{38c^2(c+dx^3)^{5/2}}{5d^4} - \frac{4c(c+dx^3)^{7/2}}{7d^4} - \frac{2(c+dx^3)^{9/2}}{27d^4} + \frac{(27648c^5)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d^4}$$

$$= -\frac{3072c^4\sqrt{c+dx^3}}{d^4} - \frac{1024c^3(c+dx^3)^{3/2}}{9d^4} - \frac{38c^2(c+dx^3)^{5/2}}{5d^4} - \frac{4c(c+dx^3)^{7/2}}{7d^4} - \frac{2(c+dx^3)^{9/2}}{27d^4} + \frac{9216c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.72

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{2\sqrt{c+dx^3}(1509176c^4+61892c^3dx^3+4611c^2d^2x^6+410cd^3x^9+35d^4x^{12})}{945d^4} + \frac{9216c^{9/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}$$

```
[In] Integrate[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(1509176*c^4 + 61892*c^3*d*x^3 + 4611*c^2*d^2*x^6 + 410
*c*d^3*x^9 + 35*d^4*x^12))/(945*d^4) + (9216*c^(9/2)*ArcTanh[Sqrt[c + d*x^3
]/(3*Sqrt[c])])/d^4
```

Maple [A] (verified)

Time = 4.40 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.62

method	result
pseudoelliptic	$\frac{\frac{2\left(-35 d^{4} x^{12}-410 d^{3} x^{9} c-4611 c^{2} x^{6} d^{2}-61892 c^{3} d \,x^{3}-1509176 c^{4}\right) \sqrt{d \,x^{3}+c}}{945}+9216 c^{\frac{9}{2}} \operatorname{arctanh}\left(\frac{\sqrt{d \,x^{3}+c}}{3 \sqrt{c}}\right)}{d^{4}}$
risch	$-\frac{2\left(35 d^{4} x^{12}+410 d^{3} x^{9} c+4611 c^{2} x^{6} d^{2}+61892 c^{3} d \,x^{3}+1509176 c^{4}\right) \sqrt{d \,x^{3}+c}}{945 d^{4}}+\frac{9216 c^{\frac{9}{2}} \operatorname{arctanh}\left(\frac{\sqrt{d \,x^{3}+c}}{3 \sqrt{c}}\right)}{d^{4}}$
default	$-\frac{\frac{2 d \,x^{12} \sqrt{d \,x^{3}+c}}{27}+\frac{20 c \,x^{9} \sqrt{d \,x^{3}+c}}{189}+\frac{2 c^{2} x^{6} \sqrt{d \,x^{3}+c}}{315 d}-\frac{8 c^{3} x^{3} \sqrt{d \,x^{3}+c}}{945 d^{2}}+\frac{16 c^{4} \sqrt{d \,x^{3}+c}}{945 d^{3}}}{d}-\frac{8 c\left(\frac{2 d \,x^{9} \sqrt{d \,x^{3}+c}}{21}+\frac{16 c \,x^{6} \sqrt{d \,x^{3}+c}}{105}+\frac{2 c^{2}}{}\right.}{d^{2}}$ … $1536 i c^{4} \sqrt{}$
elliptic	$-\frac{2 x^{12} \sqrt{d \,x^{3}+c}}{27}-\frac{164 c \,x^{9} \sqrt{d \,x^{3}+c}}{189 d}-\frac{3074 c^{2} x^{6} \sqrt{d \,x^{3}+c}}{315 d^{2}}-\frac{123784 c^{3} x^{3} \sqrt{d \,x^{3}+c}}{945 d^{3}}-\frac{3018352 c^{4} \sqrt{d \,x^{3}+c}}{945 d^{4}}-$

```
[In] int(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] 2/945*((-35*d^4*x^12-410*c*d^3*x^9-4611*c^2*d^2*x^6-61892*c^3*d*x^3-1509176
*c^4)*(d*x^3+c)^(1/2)+4354560*c^(9/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2)))
/d^4
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.47

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \left[\frac{2\left(2177280\,c^{\frac{9}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-(35\,d^4x^{12}+410\,cd^3x^9+4611\,c^2d^2x^6+6\right.}{945\,d^4}\right.$$

$$-\frac{2\left(4354560\,\sqrt{-c}c^4\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+(35\,d^4x^{12}+410\,cd^3x^9+4611\,c^2d^2x^6+61892\,c^3dx^3+1509176\,c^4\right.}{945\,d^4}$$

```
[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [2/945*(2177280*c^(9/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x
^3 - 8*c)) - (35*d^4*x^12 + 410*c*d^3*x^9 + 4611*c^2*d^2*x^6 + 61892*c^3*d*
x^3 + 1509176*c^4)*sqrt(d*x^3 + c))/d^4, -2/945*(4354560*sqrt(-c)*c^4*arcta
n(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (35*d^4*x^12 + 410*c*d^3*x^9 + 4611*c^2
*d^2*x^6 + 61892*c^3*d*x^3 + 1509176*c^4)*sqrt(d*x^3 + c))/d^4]
```

Sympy [A] (verification not implemented)

Time = 49.71 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.10

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \begin{cases} \frac{2\left(-\frac{4608c^5\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{d^3\sqrt{-c}}-\frac{1536c^4\sqrt{c+dx^3}}{d^3}-\frac{512c^3\left(c+dx^3\right)^{\frac{3}{2}}}{9d^3}-\frac{19c^2\left(c+dx^3\right)^{\frac{5}{2}}}{5d^3}-\frac{2c\left(c+dx^3\right)^{\frac{7}{2}}}{7d^3}-\frac{\left(c+dx^3\right)^{\frac{9}{2}}}{27d^3}\right)}{d} & \text{fo} \\ \frac{\sqrt{c}x^{12}}{96} & \text{ot} \end{cases}$$

```
[In] integrate(x**11*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

```
[Out] Piecewise((2*(-4608*c**5*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(d**3*sqrt(-c)
) - 1536*c**4*sqrt(c + d*x**3)/d**3 - 512*c**3*(c + d*x**3)**(3/2)/(9*d**3)
 - 19*c**2*(c + d*x**3)**(5/2)/(5*d**3) - 2*c*(c + d*x**3)**(7/2)/(7*d**3)
- (c + d*x**3)**(9/2)/(27*d**3))/d, Ne(d, 0)), (sqrt(c)*x**12/96, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.85

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{2\left(2177280\,c^{\frac{9}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+35\left(dx^3+c\right)^{\frac{9}{2}}+270\left(dx^3+c\right)^{\frac{7}{2}}c+3591\left(dx^3+c\right)^{\frac{5}{2}}c^2+53760\left(dx^3+c\right)^{\frac{3}{2}}\right.}{945\,d^4}$$

```
[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -2/945*(2177280*c^(9/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c)
+ 3*sqrt(c))) + 35*(d*x^3 + c)^(9/2) + 270*(d*x^3 + c)^(7/2)*c + 3591*(d*x^
3 + c)^(5/2)*c^2 + 53760*(d*x^3 + c)^(3/2)*c^3 + 1451520*sqrt(d*x^3 + c)*c^
4)/d^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.90

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{9216\,c^5 \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d^4} - \frac{2\left(35\,(dx^3+c)^{\frac{9}{2}}d^{32}+270\,(dx^3+c)^{\frac{7}{2}}cd^{32}+3591\,(dx^3+c)^{\frac{5}{2}}c^2d^{32}+53760\,(dx^3+c)^{\frac{3}{2}}c^3d^{32}+1451520\,\sqrt{dx}\right.}{945\,d^{36}}$$

```
[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -9216*c^5*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 2/945*(35*(
d*x^3 + c)^(9/2)*d^32 + 270*(d*x^3 + c)^(7/2)*c*d^32 + 3591*(d*x^3 + c)^(5/
2)*c^2*d^32 + 53760*(d*x^3 + c)^(3/2)*c^3*d^32 + 1451520*sqrt(d*x^3 + c)*c^
4*d^32)/d^36
```

Mupad [B] (verification not implemented)

Time = 7.46 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.04

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \frac{4608\,c^{9/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^4} - \frac{2\,x^{12}\,\sqrt{d\,x^3+c}}{27} - \frac{3018352\,c^4\,\sqrt{d\,x^3+c}}{945\,d^4} - \frac{164\,c\,x^9\,\sqrt{d\,x^3+c}}{189\,d} - \frac{123784\,c^3\,x^3\,\sqrt{d\,x^3+c}}{945\,d^3} - \frac{3074\,c^2\,x^6\,\sqrt{d\,x^3+c}}{315\,d^2}$$

```
[In] int((x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

```
[Out] (4608*c^(9/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3
)))/d^4 - (2*x^12*(c + d*x^3)^(1/2))/27 - (3018352*c^4*(c + d*x^3)^(1/2))/(
945*d^4) - (164*c*x^9*(c + d*x^3)^(1/2))/(189*d) - (123784*c^3*x^3*(c + d*x
^3)^(1/2))/(945*d^3) - (3074*c^2*x^6*(c + d*x^3)^(1/2))/(315*d^2)
```

3.296 $\int \frac{x^8(c+dx^3)^{3/2}}{8c-dx^3} dx$

Optimal result

Integrand size = 27, antiderivative size = 109

$$\int \frac{x^8(c+dx^3)^{3/2}}{8c-dx^3} dx = -\frac{384c^3\sqrt{c+dx^3}}{d^3} - \frac{128c^2(c+dx^3)^{3/2}}{9d^3} - \frac{14c(c+dx^3)^{5/2}}{15d^3} - \frac{2(c+dx^3)^{7/2}}{21d^3} + \frac{1152c^{7/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}$$

```
[Out] -128/9*c^2*(d*x^3+c)^(3/2)/d^3-14/15*c*(d*x^3+c)^(5/2)/d^3-2/21*(d*x^3+c)^(
7/2)/d^3+1152*c^(7/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^3-384*c^3*(d*x
^3+c)^(1/2)/d^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {457, 90, 52, 65, 212}

$$\int \frac{x^8(c+dx^3)^{3/2}}{8c-dx^3} dx = \frac{1152c^{7/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3} - \frac{384c^3\sqrt{c+dx^3}}{d^3} - \frac{128c^2(c+dx^3)^{3/2}}{9d^3} - \frac{14c(c+dx^3)^{5/2}}{15d^3} - \frac{2(c+dx^3)^{7/2}}{21d^3}$$

```
[In] Int[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-384*c^3*Sqrt[c + d*x^3])/d^3 - (128*c^2*(c + d*x^3)^(3/2))/(9*d^3) - (14*
c*(c + d*x^3)^(5/2))/(15*d^3) - (2*(c + d*x^3)^(7/2))/(21*d^3) + (1152*c^(7
/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{7c(c+dx)^{3/2}}{d^2}+\frac{64c^2(c+dx)^{3/2}}{d^2(8c-dx)}-\frac{(c+dx)^{5/2}}{d^2}\right)\,dx, x, x^3\right)\\
&= -\frac{14c(c+dx^3)^{5/2}}{15d^3}-\frac{2(c+dx^3)^{7/2}}{21d^3}+\frac{(64c^2)\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)}{3d^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{128c^2(c+dx^3)^{3/2}}{9d^3} - \frac{14c(c+dx^3)^{5/2}}{15d^3} - \frac{2(c+dx^3)^{7/2}}{21d^3} + \frac{(192c^3)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)}{d^2} \\
&= -\frac{384c^3\sqrt{c+dx^3}}{d^3} - \frac{128c^2(c+dx^3)^{3/2}}{9d^3} - \frac{14c(c+dx^3)^{5/2}}{15d^3} \\
&\quad - \frac{2(c+dx^3)^{7/2}}{21d^3} + \frac{(1728c^4)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{d^2} \\
&= -\frac{384c^3\sqrt{c+dx^3}}{d^3} - \frac{128c^2(c+dx^3)^{3/2}}{9d^3} - \frac{14c(c+dx^3)^{5/2}}{15d^3} \\
&\quad - \frac{2(c+dx^3)^{7/2}}{21d^3} + \frac{(3456c^4)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d^3} \\
&= -\frac{384c^3\sqrt{c+dx^3}}{d^3} - \frac{128c^2(c+dx^3)^{3/2}}{9d^3} - \frac{14c(c+dx^3)^{5/2}}{15d^3} \\
&\quad - \frac{2(c+dx^3)^{7/2}}{21d^3} + \frac{1152c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.75

$$
\begin{aligned}
\int \frac{x^8(c+dx^3)^{3/2}}{8c-dx^3}\,dx &= -\frac{2\sqrt{c+dx^3}(62882c^3+2579c^2dx^3+192cd^2x^6+15d^3x^9)}{315d^3} \\
&\quad + \frac{1152c^{7/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}
\end{aligned}
$$

```
[In] Integrate[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
[Out] (-2*Sqrt[c + d*x^3]*(62882*c^3 + 2579*c^2*d*x^3 + 192*c*d^2*x^6 + 15*d^3*x^
9))/(315*d^3) + (1152*c^(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^3
```

Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.63

method	result
pseudoelliptic	$\frac{\frac{2\left(-15d^3x^9-192c\,d^2x^6-2579c^2d\,x^3-62882c^3\right)\sqrt{d\,x^3+c}}{315}+1152c^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{d^3}$
risch	$-\frac{2\left(15d^3x^9+192c\,d^2x^6+2579c^2d\,x^3+62882c^3\right)\sqrt{d\,x^3+c}}{315d^3}+\frac{1152c^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{d^3}$
default	$-\frac{\frac{2d\,x^9\sqrt{d\,x^3+c}}{21}+\frac{16c\,x^6\sqrt{d\,x^3+c}}{105}+\frac{2c^2x^3\sqrt{d\,x^3+c}}{105d}-\frac{4c^3\sqrt{d\,x^3+c}}{105d^2}}{d}-\frac{16c\left(d\,x^3+c\right)^{\frac{5}{2}}}{15d^3}+\frac{128c^2\left(81c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)-(d\right.}{9d^3}$
elliptic	$-\frac{2x^9\sqrt{d\,x^3+c}}{21}-\frac{128c\,x^6\sqrt{d\,x^3+c}}{105d}-\frac{5158c^2x^3\sqrt{d\,x^3+c}}{315d^2}-\frac{125764c^3\sqrt{d\,x^3+c}}{315d^3}-\frac{192ic^3\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^3-8c\right)}\right.}{}$

```
[In] int(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] 2/315*((-15*d^3*x^9-192*c*d^2*x^6-2579*c^2*d*x^3-62882*c^3)*(d*x^3+c)^(1/2)
+181440*c^(7/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2)))/d^3
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.55

$$\int \frac{x^8(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \left[\frac{2\left(90720\,c^{\frac{7}{2}}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-(15\,d^3x^9+192\,cd^2x^6+2579\,c^2dx^3+62882\right.}{315\,d^3}\right.$$

$$\left.-\frac{2\left(181440\sqrt{-c}c^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+(15\,d^3x^9+192\,cd^2x^6+2579\,c^2dx^3+62882\,c^3)\sqrt{dx^3+c}\right)}{315\,d^3}\right]$$

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [2/315*(90720*c^(7/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3
 - 8*c)) - (15*d^3*x^9 + 192*c*d^2*x^6 + 2579*c^2*d*x^3 + 62882*c^3)*sqrt(d
```

```
*x^3 + c))/d^3, -2/315*(181440*sqrt(-c)*c^3*arctan(1/3*sqrt(d*x^3 + c)*sqrt
(-c)/c) + (15*d^3*x^9 + 192*c*d^2*x^6 + 2579*c^2*d*x^3 + 62882*c^3)*sqrt(d*
x^3 + c))/d^3]
```

Sympy [A] (verification not implemented)

Time = 27.49 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.12

$$\int \frac{x^8(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \begin{cases} \frac{2\left(-\frac{576c^4\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{d^2\sqrt{-c}}-\frac{192c^3\sqrt{c+dx^3}}{d^2}-\frac{64c^2\left(c+dx^3\right)^{\frac{3}{2}}}{9d^2}-\frac{7c\left(c+dx^3\right)^{\frac{5}{2}}}{15d^2}-\frac{\left(c+dx^3\right)^{\frac{7}{2}}}{21d^2}\right)}{d} & \text{for } d\neq 0 \\ \frac{\sqrt{c}x^9}{72} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**8*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

```
[Out] Piecewise((2*(-576*c**4*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(d**2*sqrt(-c))
 - 192*c**3*sqrt(c + d*x**3)/d**2 - 64*c**2*(c + d*x**3)**(3/2)/(9*d**2) -
7*c*(c + d*x**3)**(5/2)/(15*d**2) - (c + d*x**3)**(7/2)/(21*d**2))/d, Ne(d,
 0)), (sqrt(c)*x**9/72, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.88

$$\int \frac{x^8(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{2\left(90720\,c^{\frac{7}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+15\,(dx^3+c)^{\frac{7}{2}}+147\,(dx^3+c)^{\frac{5}{2}}c+2240\,(dx^3+c)^{\frac{3}{2}}c^2+60480\,\sqrt{dx^3+c}c^3\right)}{315\,d^3}$$

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -2/315*(90720*c^(7/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) +
3*sqrt(c))) + 15*(d*x^3 + c)^(7/2) + 147*(d*x^3 + c)^(5/2)*c + 2240*(d*x^3
+ c)^(3/2)*c^2 + 60480*sqrt(d*x^3 + c)*c^3)/d^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.92

$$\int \frac{x^8(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{1152\,c^4\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d^3} - \frac{2\left(15\,(dx^3+c)^{\frac{7}{2}}d^{18}+147\,(dx^3+c)^{\frac{5}{2}}cd^{18}+2240\,(dx^3+c)^{\frac{3}{2}}c^2d^{18}+60480\,\sqrt{dx^3+c}c^3d^{18}\right)}{315\,d^{21}}$$

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -1152*c^4*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/315*(15*(
d*x^3 + c)^(7/2)*d^18 + 147*(d*x^3 + c)^(5/2)*c*d^18 + 2240*(d*x^3 + c)^(3/
2)*c^2*d^18 + 60480*sqrt(d*x^3 + c)*c^3*d^18)/d^21
```

Mupad [B] (verification not implemented)

Time = 7.42 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06

$$\int \frac{x^8(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \frac{576\,c^{7/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^3} - \frac{2\,x^9\,\sqrt{d\,x^3+c}}{21} - \frac{125764\,c^3\,\sqrt{d\,x^3+c}}{315\,d^3} - \frac{128\,c\,x^6\,\sqrt{d\,x^3+c}}{105\,d} - \frac{5158\,c^2\,x^3\,\sqrt{d\,x^3+c}}{315\,d^2}$$

```
[In] int((x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

```
[Out] (576*c^(7/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)
))/d^3 - (2*x^9*(c + d*x^3)^(1/2))/21 - (125764*c^3*(c + d*x^3)^(1/2))/(315
*d^3) - (128*c*x^6*(c + d*x^3)^(1/2))/(105*d) - (5158*c^2*x^3*(c + d*x^3)^(
1/2))/(315*d^2)
```

3.297 $\int \frac{x^5(c+dx^3)^{3/2}}{8c-dx^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 88

$$\int \frac{x^5(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{48c^2\sqrt{c+dx^3}}{d^2}-\frac{16c(c+dx^3)^{3/2}}{9d^2}-\frac{2(c+dx^3)^{5/2}}{15d^2}+\frac{144c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}$$

```
[Out] -16/9*c*(d*x^3+c)^(3/2)/d^2-2/15*(d*x^3+c)^(5/2)/d^2+144*c^(5/2)*arctanh(1/
3*(d*x^3+c)^(1/2)/c^(1/2))/d^2-48*c^2*(d*x^3+c)^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {457, 81, 52, 65, 212}

$$\int \frac{x^5(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \frac{144c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}-\frac{48c^2\sqrt{c+dx^3}}{d^2}-\frac{16c(c+dx^3)^{3/2}}{9d^2}-\frac{2(c+dx^3)^{5/2}}{15d^2}$$

```
[In] Int[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-48*c^2*Sqrt[c + d*x^3])/d^2 - (16*c*(c + d*x^3)^(3/2))/(9*d^2) - (2*(c +
d*x^3)^(5/2))/(15*d^2) + (144*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])
/d^2
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x(c+dx)^{3/2}}{8c-dx}\,dx,x,x^3\right)\\
&= -\frac{2(c+dx^3)^{5/2}}{15d^2}+\frac{(8c)\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{8c-dx}\,dx,x,x^3\right)}{3d}\\
&= -\frac{16c(c+dx^3)^{3/2}}{9d^2}-\frac{2(c+dx^3)^{5/2}}{15d^2}+\frac{(24c^2)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx,x,x^3\right)}{d}
\end{aligned}
$$

$$= -\frac{48c^2\sqrt{c+dx^3}}{d^2} - \frac{16c(c+dx^3)^{3/2}}{9d^2} - \frac{2(c+dx^3)^{5/2}}{15d^2} + \frac{(216c^3)\,\mathrm{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{d}$$

$$= -\frac{48c^2\sqrt{c+dx^3}}{d^2} - \frac{16c(c+dx^3)^{3/2}}{9d^2} - \frac{2(c+dx^3)^{5/2}}{15d^2} + \frac{(432c^3)\,\mathrm{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d^2}$$

$$= -\frac{48c^2\sqrt{c+dx^3}}{d^2} - \frac{16c(c+dx^3)^{3/2}}{9d^2} - \frac{2(c+dx^3)^{5/2}}{15d^2} + \frac{144c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.81

$$\int \frac{x^5(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{2\sqrt{c+dx^3}(1123c^2+46cdx^3+3d^2x^6)}{45d^2} + \frac{144c^{5/2}\mathrm{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}$$

```
[In] Integrate[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
[Out] (-2*Sqrt[c + d*x^3]*(1123*c^2 + 46*c*d*x^3 + 3*d^2*x^6))/(45*d^2) + (144*c^
(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^2
```

Maple [A] (verified)

Time = 4.36 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{6480 c^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{d x^{3}+c}}{3 \sqrt{c}}\right)+\left(-6 d^{2} x^{6}-92 c d x^{3}-2246 c^{2}\right) \sqrt{d x^{3}+c}}{45 d^{2}}$
risch	$-\frac{2\left(3 d^{2} x^{6}+46 c d x^{3}+1123 c^{2}\right) \sqrt{d x^{3}+c}}{45 d^{2}}+\frac{144 c^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{d x^{3}+c}}{3 \sqrt{c}}\right)}{d^{2}}$
default	$-\frac{2\left(d x^{3}+c\right)^{\frac{5}{2}}}{15 d^{2}}+\frac{16 c\left(81 c^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{d x^{3}+c}}{3 \sqrt{c}}\right)-\left(d x^{3}+28 c\right) \sqrt{d x^{3}+c}\right)}{9 d^{2}}$
elliptic	$-\frac{2 x^{6} \sqrt{d x^{3}+c}}{15}-\frac{92 c x^{3} \sqrt{d x^{3}+c}}{45 d}-\frac{2246 c^{2} \sqrt{d x^{3}+c}}{45 d^{2}}-\frac{24 i c^{2} \sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d \_Z^{3}-8 c\right)} \frac{\left(-c d^{2}\right)^{\frac{1}{3}} \sqrt{2} \sqrt{\frac{i d\left(2 x+\frac{-i \sqrt{3}\left(\cdots\right.}{\cdots}\right)}{\left(\cdots\right.}}\cdots}{\cdots}\right.}{\cdots}$

[In] int(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)

[Out] 1/45*(6480*c^(5/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))+(-6*d^2*x^6-92*c*d*x^3-2246*c^2)*(d*x^3+c)^(1/2))/d^2

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.67

$$\int \frac{x^5 (c+dx^3)^{3/2}}{8c-dx^3}\,dx = \left[\frac{2\left(1620\,c^{\frac{5}{2}}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-(3\,d^2x^6+46\,cdx^3+1123\,c^2)\sqrt{dx^3+c}\right)}{45\,d^2},\right.$$
$$\left.-\frac{2\left(3240\,\sqrt{-c}c^2\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+(3\,d^2x^6+46\,cdx^3+1123\,c^2)\sqrt{dx^3+c}\right)}{45\,d^2}\right]$$

[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")

[Out] [2/45*(1620*c^(5/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - (3*d^2*x^6 + 46*c*d*x^3 + 1123*c^2)*sqrt(d*x^3 + c))/d^2, -2/45*(3240*sqrt(-c)*c^2*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (3*d^2*x^6 + 46*c*d*x^3 + 1123*c^2)*sqrt(d*x^3 + c))/d^2]

Sympy [A] (verification not implemented)

Time = 13.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08

$$\int \frac{x^5(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \begin{cases} \frac{2\left(-\frac{72c^3\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{d\sqrt{-c}}-\frac{24c^2\sqrt{c+dx^3}}{d}-\frac{8c\left(c+dx^3\right)^{\frac{3}{2}}}{9d}-\frac{\left(c+dx^3\right)^{\frac{5}{2}}}{15d}\right)}{d} & \text{for } d\neq 0\\ \frac{\sqrt{c}x^6}{48} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**5*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
[Out] Piecewise((2*(-72*c**3*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(d*sqrt(-c)) - 2
4*c**2*sqrt(c + d*x**3)/d - 8*c*(c + d*x**3)**(3/2)/(9*d) - (c + d*x**3)**(
5/2)/(15*d))/d, Ne(d, 0)), (sqrt(c)*x**6/48, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.93

$$\int \frac{x^5(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{2\left(1620\,c^{\frac{5}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+3\left(dx^3+c\right)^{\frac{5}{2}}+40\left(dx^3+c\right)^{\frac{3}{2}}c+1080\sqrt{dx^3+c}c^2\right)}{45\,d^2}$$

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
[Out] -2/45*(1620*c^(5/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*
sqrt(c))) + 3*(d*x^3 + c)^(5/2) + 40*(d*x^3 + c)^(3/2)*c + 1080*sqrt(d*x^3
+ c)*c^2)/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.94

$$\int \frac{x^5(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{144\,c^3\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d^2} - \frac{2\left(3\left(dx^3+c\right)^{\frac{5}{2}}d^8+40\left(dx^3+c\right)^{\frac{3}{2}}cd^8+1080\sqrt{dx^3+c}c^2d^8\right)}{45\,d^{10}}$$

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
[Out] -144*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^2) - 2/45*(3*(d*x
^3 + c)^(5/2)*d^8 + 40*(d*x^3 + c)^(3/2)*c*d^8 + 1080*sqrt(d*x^3 + c)*c^2*d
^8)/d^10
```

Mupad [B] (verification not implemented)

Time = 7.44 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08

$$\int \frac{x^5(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \frac{72\,c^{5/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^2} - \frac{2\,x^6\,\sqrt{d\,x^3+c}}{15} - \frac{2246\,c^2\,\sqrt{d\,x^3+c}}{45\,d^2} - \frac{92\,c\,x^3\,\sqrt{d\,x^3+c}}{45\,d}$$

```
[In] int((x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

```
[Out] (72*c^(5/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))
)/d^2 - (2*x^6*(c + d*x^3)^(1/2))/15 - (2246*c^2*(c + d*x^3)^(1/2))/(45*d^2
) - (92*c*x^3*(c + d*x^3)^(1/2))/(45*d)
```

3.298 $\int \frac{x^2(c+dx^3)^{3/2}}{8c-dx^3} dx$

Optimal result

Integrand size = 27, antiderivative size = 67

$$\int \frac{x^2(c+dx^3)^{3/2}}{8c-dx^3} dx = -\frac{6c\sqrt{c+dx^3}}{d} - \frac{2(c+dx^3)^{3/2}}{9d} + \frac{18c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

```
[Out] -2/9*(d*x^3+c)^(3/2)/d+18*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d-6*
c*(d*x^3+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {455, 52, 65, 212}

$$\int \frac{x^2(c+dx^3)^{3/2}}{8c-dx^3} dx = \frac{18c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d} - \frac{6c\sqrt{c+dx^3}}{d} - \frac{2(c+dx^3)^{3/2}}{9d}$$

```
[In] Int[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-6*c*Sqrt[c + d*x^3])/d - (2*(c + d*x^3)^(3/2))/(9*d) + (18*c^(3/2)*ArcTan
h[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)$$

$$= -\frac{2(c+dx^3)^{3/2}}{9d} + (3c)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)$$

$$= -\frac{6c\sqrt{c+dx^3}}{d} - \frac{2(c+dx^3)^{3/2}}{9d} + \left(27c^2\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= -\frac{6c\sqrt{c+dx^3}}{d} - \frac{2(c+dx^3)^{3/2}}{9d} + \frac{\left(54c^2\right)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d}$$

$$= -\frac{6c\sqrt{c+dx^3}}{d} - \frac{2(c+dx^3)^{3/2}}{9d} + \frac{18c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{x^2(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{2\sqrt{c+dx^3}(28c+dx^3)}{9d} + \frac{18c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

```
[In] Integrate[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(28*c + d*x^3))/(9*d) + (18*c^(3/2)*ArcTanh[Sqrt[c + d*
x^3]/(3*Sqrt[c])])/d
```

Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

method	result
default	$\frac{18c^{\frac{3}{2}}\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)-\frac{2\left(d\,x^3+28c\right)\sqrt{d\,x^3+c}}{9}}{d}$
pseudoelliptic	$\frac{18c^{\frac{3}{2}}\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)-\frac{2\left(d\,x^3+28c\right)\sqrt{d\,x^3+c}}{9}}{d}$
risch	$-\frac{2(d\,x^3+28c)\sqrt{d\,x^3+c}}{9d}+\frac{18c^{\frac{3}{2}}\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{d}$
elliptic	$-\frac{2x^3\sqrt{d\,x^3+c}}{9}-\frac{56c\sqrt{d\,x^3+c}}{9d}-\frac{3ic\sqrt{2}\left(\sum_{\_\alpha=\text{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\cdots}{-3(\cdots}}}{\cdots}\right.}{\cdots}$

```
[In] int(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*(81*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))-(d*x^3+28*c)*(d*x^3+c)
^(1/2))/d
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.81

$$\int \frac{x^2(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \left[\frac{81\,c^{\frac{3}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-2\,(dx^3+28\,c)\sqrt{dx^3+c}}{9\,d},\right.$$
$$\left.-\frac{2\left(81\,\sqrt{-c}c\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+(dx^3+28\,c)\sqrt{dx^3+c}\right)}{9\,d}\right]$$

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [1/9*(81*c^(3/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*
c)) - 2*(d*x^3 + 28*c)*sqrt(d*x^3 + c))/d, -2/9*(81*sqrt(-c)*c*arctan(1/3*s
qrt(d*x^3 + c)*sqrt(-c)/c) + (d*x^3 + 28*c)*sqrt(d*x^3 + c))/d]
```

Sympy [A] (verification not implemented)

Time = 6.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06

$$\int \frac{x^2(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \begin{cases} \frac{2\left(-\frac{9c^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}}-3c\sqrt{c+dx^3}-\frac{(c+dx^3)^{\frac{3}{2}}}{9}\right)}{d} & \text{for } d\neq 0 \\ \frac{\sqrt{c}x^3}{24} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

```
[Out] Piecewise((2*(-9*c**2*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/sqrt(-c) - 3*c*sq
rt(c + d*x**3) - (c + d*x**3)**(3/2)/9)/d, Ne(d, 0)), (sqrt(c)*x**3/24, Tru
e))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int \frac{x^2(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{81\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+2\,(dx^3+c)^{\frac{3}{2}}+54\,\sqrt{dx^3+c}c}{9\,d}$$

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -1/9*(81*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqr
t(c))) + 2*(d*x^3 + c)^(3/2) + 54*sqrt(d*x^3 + c)*c)/d
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

$$\int \frac{x^2(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{18\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d}-\frac{2\left((dx^3+c)^{\frac{3}{2}}d^2+27\sqrt{dx^3+c}cd^2\right)}{9\,d^3}$$

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -18*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 2/9*((d*x^3 + c
)^(3/2)*d^2 + 27*sqrt(d*x^3 + c)*c*d^2)/d^3
```

Mupad [B] (verification not implemented)

Time = 7.43 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12

$$\int \frac{x^2(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \frac{9\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d}-\frac{56\,c\,\sqrt{d\,x^3+c}}{9\,d}-\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9}$$

```
[In] int((x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

```
[Out] (9*c^(3/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))
/d - (56*c*(c + d*x^3)^(1/2))/(9*d) - (2*x^3*(c + d*x^3)^(1/2))/9
```

3.299 $\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 73

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)}\,dx=-\frac{2}{3}\sqrt{c+dx^3}+\frac{9}{4}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-\frac{1}{12}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)$$

```
[Out] 9/4*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)-1/12*arctanh((d*x^3+c)^(1/
2)/c^(1/2))*c^(1/2)-2/3*(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {457, 86, 162, 65, 214, 212}

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)}\,dx=\frac{9}{4}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-\frac{1}{12}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)-\frac{2}{3}\sqrt{c+dx^3}$$

```
[In] Int[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)),x]
```

```
[Out] (-2*Sqrt[c + d*x^3])/3 + (9*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/4
 - (Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/12
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Dist[1/(b*d), I
nt[(b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x(8c-dx)}\,dx, x, x^3\right) \\
&= -\frac{2}{3}\sqrt{c+dx^3} - \frac{\text{Subst}\left(\int \frac{-c^2d-10cd^2x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3d} \\
&= -\frac{2}{3}\sqrt{c+dx^3} + \frac{1}{24}c\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right) \\
&\quad + \frac{1}{8}(27cd)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)
\end{aligned}$$

$$= -\frac{2}{3}\sqrt{c+dx^3} + \frac{1}{4}(27c)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right) + \frac{c\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{12d}$$

$$= -\frac{2}{3}\sqrt{c+dx^3} + \frac{9}{4}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - \frac{1}{12}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)}\,dx = -\frac{2}{3}\sqrt{c+dx^3} + \frac{9}{4}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - \frac{1}{12}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)$$

```
[In] Integrate[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)),x]
```

```
[Out] (-2*Sqrt[c + d*x^3])/3 + (9*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/4
 - (Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/12
```

Maple [A] (verified)

Time = 4.41 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

method	result	size
pseudoelliptic	$\frac{9\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)\sqrt{c}}{4}-\frac{\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)\sqrt{c}}{12}-\frac{2\sqrt{d\,x^3+c}}{3}$	52
default	$\frac{\frac{2d\,x^3\sqrt{d\,x^3+c}}{9}+\frac{8c\sqrt{d\,x^3+c}}{9}-\frac{2c^{\frac{3}{2}}\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3}}{8c}+\frac{81c^{\frac{3}{2}}\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)-(d\,x^3+28c)\sqrt{d\,x^3+c}}{36c}$	100
elliptic	Expression too large to display	1506

```
[In] int((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] 9/4*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)-1/12*arctanh((d*x^3+c)^(1/
2)/c^(1/2))*c^(1/2)-2/3*(d*x^3+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.08

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)}\,dx = \left[\frac{9}{8}\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right) + \frac{1}{24}\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right) - \frac{2}{3}\sqrt{dx^3+c}, \frac{1}{12}\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) - \frac{9}{4}\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right) - \frac{2}{3}\sqrt{dx^3+c}\right]$$

```
[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [9/8*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c))
+ 1/24*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2/3*sqr
t(d*x^3 + c), 1/12*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 9/4*sqrt(-
c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 2/3*sqrt(d*x^3 + c)]
```

Sympy [A] (verification not implemented)

Time = 4.39 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.26

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)}\,dx = \begin{cases} \frac{2\left(-\frac{9cd\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{8\sqrt{-c}} + \frac{cd\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{24\sqrt{-c}} - \frac{d\sqrt{c+dx^3}}{3}\right)}{d} & \text{for } d \neq 0 \\ \frac{\sqrt{c}\log\left(x^3\right)}{24} & \text{otherwise} \end{cases}$$

```
[In] integrate((d*x**3+c)**(3/2)/x/(-d*x**3+8*c),x)
```

```
[Out] Piecewise((2*(-9*c*d*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(8*sqrt(-c)) + c*d
*atan(sqrt(c + d*x**3)/sqrt(-c))/(24*sqrt(-c)) - d*sqrt(c + d*x**3)/3)/d, N
e(d, 0)), (sqrt(c)*log(x**3)/24, True))
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)x}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x, algorithm="maxima")
[Out] -integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)}\,dx = \frac{c\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{12\sqrt{-c}} - \frac{9\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{4\sqrt{-c}} - \frac{2}{3}\sqrt{dx^3+c}$$

```
[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x, algorithm="giac")
[Out] 1/12*c*arctan(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) - 9/4*c*arctan(1/3*sqrt(d*
x^3 + c)/sqrt(-c))/sqrt(-c) - 2/3*sqrt(d*x^3 + c)
```

Mupad [B] (verification not implemented)

Time = 9.97 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.22

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)}\,dx = \frac{\sqrt{c}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)\left(10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}\right)^{27}}{x^6\,(8\,c-d\,x^3)^{27}}\right)}{24} - \frac{2\,\sqrt{d\,x^3+c}}{3}$$

```
[In] int((c + d*x^3)^(3/2)/(x*(8*c - d*x^3)),x)
[Out] (c^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))
*(10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))^27)/(x^6*(8*c - d*x^3)^27)))/
24 - (2*(c + d*x^3)^(1/2))/3
```

3.300 $\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)} dx$

Optimal result

Integrand size = 27, antiderivative size = 78

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)} dx = -\frac{\sqrt{c+dx^3}}{24x^3} + \frac{9d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}} - \frac{13d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

```
[Out] 9/32*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)-13/96*d*arctanh((d*x^3+
c)^(1/2)/c^(1/2))/c^(1/2)-1/24*(d*x^3+c)^(1/2)/x^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {457, 100, 162, 65, 214, 212}

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)} dx = \frac{9d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}} - \frac{13d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}} - \frac{\sqrt{c+dx^3}}{24x^3}$$

```
[In] Int[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)),x]
```

```
[Out] -1/24*Sqrt[c + d*x^3]/x^3 + (9*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(32*
Sqrt[c]) - (13*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*Sqrt[c])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(8c-dx)}\,dx, x, x^3\right)\\ &= -\frac{\sqrt{c+dx^3}}{24x^3} - \frac{\text{Subst}\left(\int \frac{-13c^2d-\frac{17}{2}cd^2x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{24c}\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\sqrt{c+dx^3}}{24x^3}+\frac{1}{192}(13d)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right) \\
&\quad+\frac{1}{64}\left(27d^2\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right) \\
&= -\frac{\sqrt{c+dx^3}}{24x^3}+\frac{13}{96}\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right) \\
&\quad+\frac{1}{32}(27d)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right) \\
&= -\frac{\sqrt{c+dx^3}}{24x^3}+\frac{9d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}}-\frac{13d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$
\int \frac{\left(c+dx^3\right)^{3/2}}{x^4\left(8c-dx^3\right)}\,dx=-\frac{\sqrt{c+dx^3}}{24x^3}+\frac{9d\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}}-\frac{13d\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}
$$

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)),x]
[Out] -1/24*Sqrt[c + d*x^3]/x^3 + (9*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(32*
Sqrt[c]) - (13*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*Sqrt[c])
```

Maple [A] (verified)

Time = 4.57 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{\sqrt{d\,x^{3}+c}}{24x^{3}}+\frac{d\left(-\frac{13\,\text{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{6\sqrt{c}}+\frac{9\,\text{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{2\sqrt{c}}\right)}{16}$
pseudoelliptic	$\frac{-13\,\text{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)d\,x^{3}+27\,\text{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)d\,x^{3}-4\sqrt{d\,x^{3}+c}\,\sqrt{c}}{96x^{3}\sqrt{c}}$
default	$\frac{-\frac{c\sqrt{d\,x^{3}+c}}{3x^{3}}+\frac{2d\sqrt{d\,x^{3}+c}}{3}-\sqrt{c}\,d\,\text{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{8c}+\frac{d\left(\frac{2d\,x^{3}\sqrt{d\,x^{3}+c}}{9}+\frac{8c\sqrt{d\,x^{3}+c}}{9}-\frac{2c^{\frac{3}{2}}\,\text{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3}\right)}{64c^{2}}+\frac{d\left(81c^{\frac{3}{2}}\right.}{}$
elliptic	Expression too large to display

```
[In] int((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

[Out] -1/24*(d*x^3+c)^(1/2)/x^3+1/16*d*(-13/6*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)+9/2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.38

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)}\,dx = \left[\frac{27\sqrt{c}dx^3\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+13\sqrt{c}dx^3\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-8\sqrt{dx^3+c}}{192\,cx^3}\right.$$

[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c),x, algorithm="fricas")

[Out] [1/192*(27*sqrt(c)*d*x^3*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 13*sqrt(c)*d*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 8*sqrt(d*x^3 + c)*c)/(c*x^3), 1/96*(13*sqrt(-c)*d*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 27*sqrt(-c)*d*x^3*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 4*sqrt(d*x^3 + c)*c)/(c*x^3)]

Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)}\,dx = -\int \frac{c\sqrt{c+dx^3}}{-8cx^4+dx^7}\,dx - \int \frac{dx^3\sqrt{c+dx^3}}{-8cx^4+dx^7}\,dx$$

[In] integrate((d*x**3+c)**(3/2)/x**4/(-d*x**3+8*c),x)

[Out] -Integral(c*sqrt(c + d*x**3)/(-8*c*x**4 + d*x**7), x) - Integral(d*x**3*sqrt(c + d*x**3)/(-8*c*x**4 + d*x**7), x)

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)x^4}\,dx$$

[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c),x, algorithm="maxima")

[Out] -integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^4), x)

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int \frac{(c+dx^3)^{3/2}}{x^4\,(8c-dx^3)}\,dx = \frac{13\,d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\,\sqrt{-c}} - \frac{9\,d\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{32\,\sqrt{-c}} - \frac{\sqrt{dx^3+c}}{24\,x^3}$$

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] 13/96*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) - 9/32*d*arctan(1/3*sqrt(
d*x^3 + c)/sqrt(-c))/sqrt(-c) - 1/24*sqrt(d*x^3 + c)/x^3
```

Mupad [B] (verification not implemented)

Time = 7.73 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.72

$$\int \frac{(c+dx^3)^{3/2}}{x^4\,(8c-dx^3)}\,dx = \frac{9\,d\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^3+c}}{3\,\sqrt{c}}\right)}{32\,\sqrt{c}} - \frac{13\,d\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{96\,\sqrt{c}} - \frac{\sqrt{d\,x^3+c}}{24\,x^3}$$

```
[In] int((c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)),x)
```

```
[Out] (9*d*atanh((c + d*x^3)^(1/2)/(3*c^(1/2))))/(32*c^(1/2)) - (13*d*atanh((c +
d*x^3)^(1/2)/c^(1/2)))/(96*c^(1/2)) - (c + d*x^3)^(1/2)/(24*x^3)
```

3.301 $\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 104

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)}\,dx=-\frac{\sqrt{c+dx^3}}{48x^6}-\frac{11d\sqrt{c+dx^3}}{192cx^3}+\frac{9d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{3/2}}-\frac{37d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{3/2}}$$

```
[Out] 9/256*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-37/768*d^2*arctanh((
d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-1/48*(d*x^3+c)^(1/2)/x^6-11/192*d*(d*x^3+c)
^(1/2)/c/x^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {457, 100, 156, 162, 65, 214, 212}

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)}\,dx=\frac{9d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{3/2}}-\frac{37d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{3/2}}-\frac{11d\sqrt{c+dx^3}}{192cx^3}-\frac{\sqrt{c+dx^3}}{48x^6}$$

```
[In] Int[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)),x]
```

```
[Out] -1/48*Sqrt[c + d*x^3]/x^6 - (11*d*Sqrt[c + d*x^3])/(192*c*x^3) + (9*d^2*Arc
Tanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(256*c^(3/2)) - (37*d^2*ArcTanh[Sqrt[c +
 d*x^3]/Sqrt[c]])/(768*c^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^3(8c-dx)}\,dx, x, x^3\right) \\
&= -\frac{\sqrt{c+dx^3}}{48x^6} - \frac{\text{Subst}\left(\int \frac{-22c^2d-\frac{35}{2}cd^2x}{x^2(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{48c} \\
&= -\frac{\sqrt{c+dx^3}}{48x^6} - \frac{11d\sqrt{c+dx^3}}{192cx^3} + \frac{\text{Subst}\left(\int \frac{74c^3d^2+11c^2d^3x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{384c^3} \\
&= -\frac{\sqrt{c+dx^3}}{48x^6} - \frac{11d\sqrt{c+dx^3}}{192cx^3} + \frac{(37d^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{1536c} \\
&\quad + \frac{(27d^3)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{512c} \\
&= -\frac{\sqrt{c+dx^3}}{48x^6} - \frac{11d\sqrt{c+dx^3}}{192cx^3} + \frac{(37d)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{768c} \\
&\quad + \frac{(27d^2)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{256c} \\
&= -\frac{\sqrt{c+dx^3}}{48x^6} - \frac{11d\sqrt{c+dx^3}}{192cx^3} + \frac{9d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{3/2}} - \frac{37d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)}\,dx = \frac{(-4c-11dx^3)\sqrt{c+dx^3}}{192cx^6} + \frac{9d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{3/2}} - \frac{37d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{3/2}}$$

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)),x]
```

```
[Out] ((-4*c - 11*d*x^3)*Sqrt[c + d*x^3])/(192*c*x^6) + (9*d^2*ArcTanh[Sqrt[c + d
*x^3]/(3*Sqrt[c])])/(256*c^(3/2)) - (37*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]
])/(768*c^(3/2))
```

Maple [A] (verified)

Time = 4.74 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{\sqrt{d\,x^{3}+c}\,(11d\,x^{3}+4c)}{192x^{6}c}+\frac{d^{2}\left(-\frac{37\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{6\sqrt{c}}+\frac{9\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{2\sqrt{c}}\right)}{128c}$
pseudoelliptic	$\frac{-37\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)d^{2}x^{6}+27\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)d^{2}x^{6}-44d\,x^{3}\sqrt{d\,x^{3}+c}\,\sqrt{c}-16\sqrt{d\,x^{3}+c}\,c^{\frac{3}{2}}}{768c^{\frac{3}{2}}x^{6}}$
default	$\frac{-\frac{c\sqrt{d\,x^{3}+c}}{6x^{6}}-\frac{5d\sqrt{d\,x^{3}+c}}{12x^{3}}-\frac{d^{2}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{4\sqrt{c}}}{8c}+\frac{d\left(-\frac{c\sqrt{d\,x^{3}+c}}{3x^{3}}+\frac{2d\sqrt{d\,x^{3}+c}}{3}-\sqrt{c}\,d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)\right)}{64c^{2}}+\frac{d^{2}\left(\frac{2d\,x^{3}\sqrt{c}}{9}\right.}{}$
elliptic	Expression too large to display

```
[In] int((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/192*(d*x^3+c)^(1/2)*(11*d*x^3+4*c)/x^6/c+1/128*d^2/c*(-37/6*arctanh((d*x
^3+c)^(1/2)/c^(1/2))/c^(1/2)+9/2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/
2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.10

$$\int \frac{(c+dx^3)^{3/2}}{x^7\,(8c-dx^3)}\,dx = \left[\frac{27\,\sqrt{c}d^2x^6\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+37\,\sqrt{c}d^2x^6\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-8\,(11\,cdx^3}{1536\,c^2x^6}\right.$$

```
[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [1/1536*(27*sqrt(c)*d^2*x^6*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/
(d*x^3 - 8*c)) + 37*sqrt(c)*d^2*x^6*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c)
+ 2*c)/x^3) - 8*(11*c*d*x^3 + 4*c^2)*sqrt(d*x^3 + c))/(c^2*x^6), 1/768*(37*
sqrt(-c)*d^2*x^6*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 27*sqrt(-c)*d^2*x^6*a
rctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 4*(11*c*d*x^3 + 4*c^2)*sqrt(d*x^3 +
 c))/(c^2*x^6)]
```

Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)}\,dx = -\int \frac{c\sqrt{c+dx^3}}{-8cx^7+dx^{10}}\,dx - \int \frac{dx^3\sqrt{c+dx^3}}{-8cx^7+dx^{10}}\,dx$$

```
[In] integrate((d*x**3+c)**(3/2)/x**7/(-d*x**3+8*c),x)
[Out] -Integral(c*sqrt(c + d*x**3)/(-8*c*x**7 + d*x**10), x) - Integral(d*x**3*sq
rt(c + d*x**3)/(-8*c*x**7 + d*x**10), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)x^7}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x, algorithm="maxima")
[Out] -integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^7), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)}\,dx = \frac{37\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{768\,\sqrt{-c}c} - \frac{9\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{256\,\sqrt{-c}c} - \frac{11\,(dx^3+c)^{\frac{3}{2}}d^2 - 7\,\sqrt{dx^3+c}cd^2}{192\,cd^2x^6}$$

```
[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x, algorithm="giac")
[Out] 37/768*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 9/256*d^2*arctan
(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/192*(11*(d*x^3 + c)^(3/2)*d
^2 - 7*sqrt(d*x^3 + c)*c*d^2)/(c*d^2*x^6)
```

Mupad [B] (verification not implemented)

Time = 7.91 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \frac{(c+dx^3)^{3/2}}{x^7\,(8c-dx^3)}\,dx = \frac{7\,\sqrt{d\,x^3+c}}{192\,x^6} - \frac{37\,d^2\,\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{\sqrt{c^3}}\right)}{768\,\sqrt{c^3}} + \frac{9\,d^2\,\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^3}}\right)}{256\,\sqrt{c^3}} - \frac{11\,(d\,x^3+c)^{3/2}}{192\,c\,x^6}$$

```
[In] int((c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)),x)
```

```
[Out] (7*(c + d*x^3)^(1/2))/(192*x^6) - (37*d^2*atanh((c*(c + d*x^3)^(1/2))/(c^3)
^(1/2)))/(768*(c^3)^(1/2)) + (9*d^2*atanh((c*(c + d*x^3)^(1/2))/(3*(c^3)^(1
/2))))/(256*(c^3)^(1/2)) - (11*(c + d*x^3)^(3/2))/(192*c*x^6)
```

3.302 $\int \frac{x^7(c+dx^3)^{3/2}}{8c-dx^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 669

$$\begin{aligned}
\int \frac{x^7(c+dx^3)^{3/2}}{8c-dx^3}\,dx &= -\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2}-\frac{348cx^5\sqrt{c+dx^3}}{247d}-\frac{2}{19}x^8\sqrt{c+dx^3} \\
&-\frac{2094648c^3\sqrt{c+dx^3}}{1729d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{288\sqrt{3}c^{19/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{8/3}} \\
&+\frac{288c^{19/6}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{8/3}}-\frac{288c^{19/6}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{8/3}} \\
&+\frac{1047324\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{10/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{1729d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&-\frac{698216\sqrt{2}3^{3/4}c^{10/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{1729d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}$$

```
[Out] 288*c^(19/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(
8/3)-288*c^(19/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(8/3)-288*c^(19/6)
```

```
*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/d^(8/3
)-36534/1729*c^2*x^2*(d*x^3+c)^(1/2)/d^2-348/247*c*x^5*(d*x^3+c)^(1/2)/d-2/
19*x^8*(d*x^3+c)^(1/2)-2094648/1729*c^3*(d*x^3+c)^(1/2)/d^(8/3)/(d^(1/3)*x+
c^(1/3)*(1+3^(1/2)))-698216/1729*3^(3/4)*c^(10/3)*(c^(1/3)+d^(1/3)*x)*Ellip
ticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1
/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3
)*(1+3^(1/2)))^2)^(1/2)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x
)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+1047324/1729*3^(1/4)*c^(10/3)*(c
^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1
/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)
*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(8/3)/(d
*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^
2)^(1/2)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 669, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {488, 596, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{x^7 (c+dx^3)^{3/2}}{8c-dx^3}\,dx =$$

$$-\frac{698216\sqrt{2}3^{3/4}c^{10/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{1729d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{1047324\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{10/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{1729d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{288\sqrt{3}c^{19/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{8/3}}+\frac{288c^{19/6}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{8/3}}$$

$$-\frac{288c^{19/6}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{8/3}}-\frac{2094648c^3\sqrt{c+dx^3}}{1729d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$-\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2}-\frac{2}{19}x^8\sqrt{c+dx^3}-\frac{348cx^5\sqrt{c+dx^3}}{247d}$$

```
[In] Int[(x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-36534*c^2*x^2*Sqrt[c + d*x^3])/(1729*d^2) - (348*c*x^5*Sqrt[c + d*x^3])/(
247*d) - (2*x^8*Sqrt[c + d*x^3])/19 - (2094648*c^3*Sqrt[c + d*x^3])/(1729*d
^(8/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (288*Sqrt[3]*c^(19/6)*ArcTan[
(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/d^(8/3) + (288*c^
(19/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]])/d^(8/3
) - (288*c^(19/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]])/d^(8/3) + (1047324*
3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(10/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^
(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Ellip
ticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*
x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (698216*Sqrt[
2]*3^(3/4)*c^(10/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x
 + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1
 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 -
 4*Sqrt[3]])/(1729*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2}{19}x^8\sqrt{c+dx^3} - \frac{2\int \frac{x^7\left(-\frac{147c^2d}{2}-87cd^2x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{19d} \\
&= -\frac{348cx^5\sqrt{c+dx^3}}{247d} - \frac{2}{19}x^8\sqrt{c+dx^3} - \frac{4\int \frac{x^4\left(-3480c^3d^2-\frac{18267}{4}c^2d^3x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{247d^3} \\
&= -\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2} - \frac{348cx^5\sqrt{c+dx^3}}{247d} - \frac{2}{19}x^8\sqrt{c+dx^3} - \frac{8\int \frac{x\left(-73068c^4d^3-\frac{261831}{2}c^3d^4x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{1729d^5} \\
&= -\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2} - \frac{348cx^5\sqrt{c+dx^3}}{247d} \\
&\quad - \frac{2}{19}x^8\sqrt{c+dx^3} - \frac{8\int \left(\frac{261831c^3d^3x}{2\sqrt{c+dx^3}} - \frac{1120392c^4d^3x}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{1729d^5}
\end{aligned}$$

$$= -\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2} - \frac{348cx^5\sqrt{c+dx^3}}{247d} - \frac{2}{19}x^8\sqrt{c+dx^3} - \frac{(1047324c^3)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{1729d^2} + \frac{(5184c^4)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{d^2}$$

$$= -\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2} - \frac{348cx^5\sqrt{c+dx^3}}{247d} - \frac{2}{19}x^8\sqrt{c+dx^3} - \frac{(432c^3)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{d^3} - \frac{(1047324c^3)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{1729d^{7/3}} + \frac{\left(432c^{10/3}\right)\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{d^{7/3}} + \frac{\left(1047324\left(1-\sqrt{3}\right)c^{10/3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{1729d^{7/3}} - \frac{\left(1296c^{11/3}\right)\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{d^{5/3}}$$

$$= -\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2} - \frac{348cx^5\sqrt{c+dx^3}}{247d} - \frac{2}{19}x^8\sqrt{c+dx^3} - \frac{2094648c^3\sqrt{c+dx^3}}{1729d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$+ \frac{1047324\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{10/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7\right.}{1729d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$- \frac{698216\sqrt{2}3^{3/4}c^{10/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right.}{1729d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{\left(864c^{11/3}\right)\text{Subst}\left(\int\frac{1}{9-cx^2}\,dx,x,\frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{d^{8/3}} - \frac{\left(432c^{11/3}\right)\text{Subst}\left(\int\frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{d^{5/3}}$$

$$+ \frac{\left(1728c^{8/3}\right)\text{Subst}\left(\int\frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{d^{2/3}}$$

$$= -\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2} - \frac{348cx^5\sqrt{c+dx^3}}{247d} - \frac{2}{19}x^8\sqrt{c+dx^3} - \frac{2094648c^3\sqrt{c+dx^3}}{1729d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{288\sqrt{3}c^{19/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{8/3}} + \frac{288c^{19/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{8/3}} + \frac{1047324\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{10/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-\right.}{1729d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{698216\sqrt{2}3^{3/4}c^{10/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{1729d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{\left(864c^{11/3}\right)\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{d^{8/3}}$$

$$= -\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2} - \frac{348cx^5\sqrt{c+dx^3}}{247d} - \frac{2}{19}x^8\sqrt{c+dx^3}$$
$$- \frac{2094648c^3\sqrt{c+dx^3}}{1729d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{288\sqrt{3}c^{19/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{8/3}}$$
$$+ \frac{288c^{19/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{8/3}} - \frac{288c^{19/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{8/3}}$$
$$+ \frac{1047324\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{10/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7\right.}{1729d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$- \frac{698216\sqrt{2}3^{3/4}c^{10/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right.}{1729d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 7.83 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.24

$$\int \frac{x^7(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \frac{-20x^2(18267c^3+19485c^2dx^3+1309cd^2x^6+91d^3x^9)+365340c^3x^2\sqrt{1+\frac{dx^3}{c}}\,\text{Appel}}{17290d^2\sqrt{}}$$

```
[In] Integrate[(x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-20*x^2*(18267*c^3 + 19485*c^2*d*x^3 + 1309*c*d^2*x^6 + 91*d^3*x^9) + 3653
40*c^3*x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*
x^3)/(8*c)] + 261831*c^2*d*x^5*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/
3, -((d*x^3)/c), (d*x^3)/(8*c)])/(17290*d^2*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.75 (sec) , antiderivative size = 895, normalized size of antiderivative = 1.34

method	result	size
risch	Expression too large to display	895
elliptic	Expression too large to display	906
default	Expression too large to display	1840

```
[In] int(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] -2/1729*x^2*(91*d^2*x^6+1218*c*d*x^3+18267*c^2)*(d*x^3+c)^(1/2)/d^2-12/1729
*c^3/d^2*(-174554/3*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*
d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(
-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1
/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*
d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3
)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+27664*I/d
^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^
2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(
-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)
*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2
-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d
*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2
)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^
2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_al
pha=RootOf(_Z^3*d-8*c)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 24.90 (sec) , antiderivative size = 2453, normalized size of antiderivative = 3.67

$$\int \frac{x^7(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] 2/1729*(1047324*c^3*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrassPInverse(
0, -4*c/d, x)) + 41496*(c^19/d^16)^(1/6)*d^3*log(1981355655168*((d^16*x^9 +
 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13)*(c^19/d^16)^(5/6) + 6*(
c^16*d^2*x^7 + 80*c^17*d*x^4 + 160*c^18*x + 6*(5*c^4*d^12*x^5 + 32*c^5*d^11
*x^2)*(c^19/d^16)^(2/3) + (7*c^10*d^7*x^6 + 152*c^11*d^6*x^3 + 64*c^12*d^5)
*(c^19/d^16)^(1/3))*sqrt(d*x^3 + c) + 18*(5*c^7*d^10*x^7 + 64*c^8*d^9*x^4 +
 32*c^9*d^8*x)*sqrt(c^19/d^16) + 18*(c^13*d^5*x^8 + 38*c^14*d^4*x^5 + 64*c^
15*d^3*x^2)*(c^19/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 51
2*c^3)) - 41496*(c^19/d^16)^(1/6)*d^3*log(-1981355655168*((d^16*x^9 + 318*c
*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13)*(c^19/d^16)^(5/6) - 6*(c^16*d
^2*x^7 + 80*c^17*d*x^4 + 160*c^18*x + 6*(5*c^4*d^12*x^5 + 32*c^5*d^11*x^2)*
(c^19/d^16)^(2/3) + (7*c^10*d^7*x^6 + 152*c^11*d^6*x^3 + 64*c^12*d^5)*(c^19
/d^16)^(1/3))*sqrt(d*x^3 + c) + 18*(5*c^7*d^10*x^7 + 64*c^8*d^9*x^4 + 32*c^
9*d^8*x)*sqrt(c^19/d^16) + 18*(c^13*d^5*x^8 + 38*c^14*d^4*x^5 + 64*c^15*d^3
*x^2)*(c^19/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)
) - 20748*(sqrt(-3)*d^3 - d^3)*(c^19/d^16)^(1/6)*log(1981355655168*((d^16*x
^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13 + sqrt(-3)*(d^16*x^9
 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c^19/d^16)^(5/6) +
6*(2*c^16*d^2*x^7 + 160*c^17*d*x^4 + 320*c^18*x - 6*(5*c^4*d^12*x^5 + 32*c^
5*d^11*x^2 - sqrt(-3)*(5*c^4*d^12*x^5 + 32*c^5*d^11*x^2))*(c^19/d^16)^(2/3)
 - (7*c^10*d^7*x^6 + 152*c^11*d^6*x^3 + 64*c^12*d^5 + sqrt(-3)*(7*c^10*d^7*
x^6 + 152*c^11*d^6*x^3 + 64*c^12*d^5))*(c^19/d^16)^(1/3))*sqrt(d*x^3 + c) -
 36*(5*c^7*d^10*x^7 + 64*c^8*d^9*x^4 + 32*c^9*d^8*x)*sqrt(c^19/d^16) + 18*(
c^13*d^5*x^8 + 38*c^14*d^4*x^5 + 64*c^15*d^3*x^2 - sqrt(-3)*(c^13*d^5*x^8 +
 38*c^14*d^4*x^5 + 64*c^15*d^3*x^2))*(c^19/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2
*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 20748*(sqrt(-3)*d^3 - d^3)*(c^19/d^16)^(
1/6)*log(-1981355655168*((d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 6
40*c^3*d^13 + sqrt(-3)*(d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640
*c^3*d^13))*(c^19/d^16)^(5/6) - 6*(2*c^16*d^2*x^7 + 160*c^17*d*x^4 + 320*c^
18*x - 6*(5*c^4*d^12*x^5 + 32*c^5*d^11*x^2 - sqrt(-3)*(5*c^4*d^12*x^5 + 32*
c^5*d^11*x^2))*(c^19/d^16)^(2/3) - (7*c^10*d^7*x^6 + 152*c^11*d^6*x^3 + 64*
c^12*d^5 + sqrt(-3)*(7*c^10*d^7*x^6 + 152*c^11*d^6*x^3 + 64*c^12*d^5))*(c^1
9/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^7*d^10*x^7 + 64*c^8*d^9*x^4 + 32*c
^9*d^8*x)*sqrt(c^19/d^16) + 18*(c^13*d^5*x^8 + 38*c^14*d^4*x^5 + 64*c^15*d^
3*x^2 - sqrt(-3)*(c^13*d^5*x^8 + 38*c^14*d^4*x^5 + 64*c^15*d^3*x^2))*(c^19/
```

```
d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 20748*(s
qrt(-3)*d^3 + d^3)*(c^19/d^16)^(1/6)*log(1981355655168*((d^16*x^9 + 318*c*d
^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13 - sqrt(-3)*(d^16*x^9 + 318*c*d^1
5*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c^19/d^16)^(5/6) + 6*(2*c^16*d^
2*x^7 + 160*c^17*d*x^4 + 320*c^18*x - 6*(5*c^4*d^12*x^5 + 32*c^5*d^11*x^2 +
 sqrt(-3)*(5*c^4*d^12*x^5 + 32*c^5*d^11*x^2))*(c^19/d^16)^(2/3) - (7*c^10*d
^7*x^6 + 152*c^11*d^6*x^3 + 64*c^12*d^5 - sqrt(-3)*(7*c^10*d^7*x^6 + 152*c^
11*d^6*x^3 + 64*c^12*d^5))*(c^19/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^7*d
^10*x^7 + 64*c^8*d^9*x^4 + 32*c^9*d^8*x)*sqrt(c^19/d^16) + 18*(c^13*d^5*x^8
 + 38*c^14*d^4*x^5 + 64*c^15*d^3*x^2 + sqrt(-3)*(c^13*d^5*x^8 + 38*c^14*d^4
*x^5 + 64*c^15*d^3*x^2))*(c^19/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c
^2*d*x^3 - 512*c^3)) - 20748*(sqrt(-3)*d^3 + d^3)*(c^19/d^16)^(1/6)*log(-19
81355655168*((d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13
- sqrt(-3)*(d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13))*
(c^19/d^16)^(5/6) - 6*(2*c^16*d^2*x^7 + 160*c^17*d*x^4 + 320*c^18*x - 6*(5*
c^4*d^12*x^5 + 32*c^5*d^11*x^2 + sqrt(-3)*(5*c^4*d^12*x^5 + 32*c^5*d^11*x^2
))*(c^19/d^16)^(2/3) - (7*c^10*d^7*x^6 + 152*c^11*d^6*x^3 + 64*c^12*d^5 - s
qrt(-3)*(7*c^10*d^7*x^6 + 152*c^11*d^6*x^3 + 64*c^12*d^5))*(c^19/d^16)^(1/3
))*sqrt(d*x^3 + c) - 36*(5*c^7*d^10*x^7 + 64*c^8*d^9*x^4 + 32*c^9*d^8*x)*sq
rt(c^19/d^16) + 18*(c^13*d^5*x^8 + 38*c^14*d^4*x^5 + 64*c^15*d^3*x^2 + sqrt
(-3)*(c^13*d^5*x^8 + 38*c^14*d^4*x^5 + 64*c^15*d^3*x^2))*(c^19/d^16)^(1/6))
/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (91*d^3*x^8 + 1218*c
*d^2*x^5 + 18267*c^2*d*x^2)*sqrt(d*x^3 + c))/d^3
```

Sympy [F]

$$\int \frac{x^7(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\int \frac{cx^7\sqrt{c+dx^3}}{-8c+dx^3}\,dx - \int \frac{dx^{10}\sqrt{c+dx^3}}{-8c+dx^3}\,dx$$

```
[In] integrate(x**7*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
[Out] -Integral(c*x**7*sqrt(c + d*x**3)/(-8*c + d*x**3), x) - Integral(d*x**10*sq
rt(c + d*x**3)/(-8*c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^7(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}x^7}{dx^3-8\,c}\,dx$$

```
[In] integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
[Out] -integrate((d*x^3 + c)^(3/2)*x^7/(d*x^3 - 8*c), x)
```

Giac [F]

$$\int \frac{x^7(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}x^7}{dx^3-8\,c}\,dx$$

```
[In] integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
[Out] integrate(-(d*x^3 + c)^(3/2)*x^7/(d*x^3 - 8*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \int \frac{x^7\,(d\,x^3+c)^{3/2}}{8\,c-d\,x^3}\,dx$$

```
[In] int((x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
[Out] int((x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3), x)
```

3.303 $\int \frac{x^4(c+dx^3)^{3/2}}{8c-dx^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 645

$$\int \frac{x^4(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{240cx^2\sqrt{c+dx^3}}{91d} - \frac{2}{13}x^5\sqrt{c+dx^3} - \frac{13782c^2\sqrt{c+dx^3}}{91d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{36\sqrt{3}c^{13/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}} + \frac{36c^{13/6}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{5/3}} - \frac{36c^{13/6}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{5/3}} + \frac{6891\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{91d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{4594\sqrt{2}3^{3/4}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{91d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 36*c^(13/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(5
/3)-36*c^(13/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(5/3)-36*c^(13/6)*ar
```

```
ctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/d^(5/3)-2
40/91*c*x^2*(d*x^3+c)^(1/2)/d-2/13*x^5*(d*x^3+c)^(1/2)-13782/91*c^2*(d*x^3+
c)^(1/2)/d^(5/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-4594/91*3^(3/4)*c^(7/3)*(c
^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1
/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)
*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(
1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+6891/91*3
^(1/4)*c^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2))
)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*
((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)
^(1/2)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1
/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 645, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.481, Rules used = {488, 596, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{x^4(c+dx^3)^{3/2}}{8c-dx^3}\,dx =$$

$$-\frac{4594\sqrt{2}3^{3/4}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{91d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{6891\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{91d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{36\sqrt{3}c^{13/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}}+\frac{36c^{13/6}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{5/3}}$$

$$-\frac{36c^{13/6}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{5/3}}-\frac{13782c^2\sqrt{c+dx^3}}{91d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$-\frac{2}{13}x^5\sqrt{c+dx^3}-\frac{240cx^2\sqrt{c+dx^3}}{91d}$$

```
[In] Int[(x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-240*c*x^2*Sqrt[c + d*x^3])/(91*d) - (2*x^5*Sqrt[c + d*x^3])/13 - (13782*c
^2*Sqrt[c + d*x^3])/(91*d^(5/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (36*
Sqrt[3]*c^(13/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*
x^3]])/d^(5/3) + (36*c^(13/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sq
rt[c + d*x^3])])/d^(5/3) - (36*c^(13/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])
])/d^(5/3) + (6891*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(7/3)*(c^(1/3) + d^(1/3)*x)*
Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d
^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*d^(5/3)*Sqrt[(c^(1/3)*(c^
(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
 - (4594*Sqrt[2]*3^(3/4)*c^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1
/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)], -7 - 4*Sqrt[3]])/(91*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
```

```
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2}{13}x^5\sqrt{c+dx^3} - \frac{2\int \frac{x^4\left(-\frac{93c^2d}{2}-60cd^2x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{13d} \\
&= -\frac{240cx^2\sqrt{c+dx^3}}{91d} - \frac{2}{13}x^5\sqrt{c+dx^3} - \frac{4\int \frac{x\left(-960c^3d^2-\frac{6891}{4}c^2d^3x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{91d^3} \\
&= -\frac{240cx^2\sqrt{c+dx^3}}{91d} - \frac{2}{13}x^5\sqrt{c+dx^3} - \frac{4\int \left(\frac{6891c^2d^2x}{4\sqrt{c+dx^3}} - \frac{14742c^3d^2x}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{91d^3} \\
&= -\frac{240cx^2\sqrt{c+dx^3}}{91d} - \frac{2}{13}x^5\sqrt{c+dx^3} - \frac{(6891c^2)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{91d} + \frac{(648c^3)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{d}
\end{aligned}$$

$$= -\frac{240cx^2\sqrt{c+dx^3}}{91d} - \frac{2}{13}x^5\sqrt{c+dx^3} - \frac{(54c^2)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{d^2}$$

$$-\frac{(6891c^2)\int \frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{91d^{4/3}} + \frac{\left(54c^{7/3}\right)\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{d^{4/3}}$$

$$+\frac{\left(6891\left(1-\sqrt{3}\right)c^{7/3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{91d^{4/3}} - \frac{\left(162c^{8/3}\right)\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{d^{2/3}}$$

$$= -\frac{240cx^2\sqrt{c+dx^3}}{91d} - \frac{2}{13}x^5\sqrt{c+dx^3} - \frac{13782c^2\sqrt{c+dx^3}}{91d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$+\frac{6891\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4}{91d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{4594\sqrt{2}3^{3/4}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{91d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{\left(108c^{8/3}\right)\text{Subst}\left(\int \frac{1}{9-cx^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{d^{5/3}}$$

$$-\frac{\left(54c^{8/3}\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{d^{2/3}}$$

$$+\left(216c^{5/3}\sqrt[3]{d}\right)\text{Subst}\left(\int \frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)$$

$$= -\frac{240cx^2\sqrt{c+dx^3}}{91d} - \frac{2}{13}x^5\sqrt{c+dx^3} - \frac{13782c^2\sqrt{c+dx^3}}{91d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$- \frac{36\sqrt{3}c^{13/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}} + \frac{36c^{13/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{5/3}}$$

$$+ \frac{6891\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{}\right.}{91d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$- \frac{4594\sqrt{2}3^{3/4}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{91d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$- \frac{\left(108c^{8/3}\right)\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d^{5/3}}$$

$$= -\frac{240cx^2\sqrt{c+dx^3}}{91d} - \frac{2}{13}x^5\sqrt{c+dx^3} - \frac{13782c^2\sqrt{c+dx^3}}{91d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$- \frac{36\sqrt{3}c^{13/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}}$$

$$+ \frac{36c^{13/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{5/3}} - \frac{36c^{13/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{5/3}}$$

$$+ \frac{6891\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4}{91d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$- \frac{4594\sqrt{2}3^{3/4}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{91d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 7.59 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.23

$$\int \frac{x^4(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \frac{-80(120c^2x^2+127cdx^5+7d^2x^8)+9600c^2x^2\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx}{8c}\right.}{3640d\sqrt{c+dx^3}}$$

```
[In] Integrate[(x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-80*(120*c^2*x^2 + 127*c*d*x^5 + 7*d^2*x^8) + 9600*c^2*x^2*Sqrt[1 + (d*x^3
)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 6891*c*d*x^5
*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)
])/(3640*d*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.89 (sec) , antiderivative size = 884, normalized size of antiderivative = 1.37

method	result	size
risch	Expression too large to display	884
elliptic	Expression too large to display	886
default	Expression too large to display	1344

```
[In] int(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] -2/91*x^2*(7*d*x^3+120*c)/d*(d*x^3+c)^(1/2)-3/91*c^2/d*(-4594/3*I*3^(1/2)/d
*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)
*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1
/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-
c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+728*I/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^
(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2
)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d
^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/
3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)
*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)
^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*
3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)
^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 8.32 (sec) , antiderivative size = 2442, normalized size of antiderivative = 3.79

$$\int \frac{x^4(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

[Out] 1/91*(13782*c^2*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -
4*c/d, x)) + 546*(c^13/d^10)^(1/6)*d^2*log(60466176*((d^11*x^9 + 318*c*d^10
*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8)*(c^13/d^10)^(5/6) + 6*(c^11*d^2*x^7
+ 80*c^12*d*x^4 + 160*c^13*x + 6*(5*c^3*d^8*x^5 + 32*c^4*d^7*x^2)*(c^13/d^1
0)^(2/3) + (7*c^7*d^5*x^6 + 152*c^8*d^4*x^3 + 64*c^9*d^3)*(c^13/d^10)^(1/3)
)*sqrt(d*x^3 + c) + 18*(5*c^5*d^7*x^7 + 64*c^6*d^6*x^4 + 32*c^7*d^5*x)*sqrt
(c^13/d^10) + 18*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2)*(c^13/d^
10)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 546*(c^13/
d^10)^(1/6)*d^2*log(-60466176*((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^
3 + 640*c^3*d^8)*(c^13/d^10)^(5/6) - 6*(c^11*d^2*x^7 + 80*c^12*d*x^4 + 160*
c^13*x + 6*(5*c^3*d^8*x^5 + 32*c^4*d^7*x^2)*(c^13/d^10)^(2/3) + (7*c^7*d^5*
x^6 + 152*c^8*d^4*x^3 + 64*c^9*d^3)*(c^13/d^10)^(1/3))*sqrt(d*x^3 + c) + 18
*(5*c^5*d^7*x^7 + 64*c^6*d^6*x^4 + 32*c^7*d^5*x)*sqrt(c^13/d^10) + 18*(c^9*
d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2)*(c^13/d^10)^(1/6))/(d^3*x^9 -
24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 273*(c^13/d^10)^(1/6)*(sqrt(-3)*
d^2 - d^2)*log(60466176*((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 64
0*c^3*d^8 + sqrt(-3)*(d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^
3*d^8))*(c^13/d^10)^(5/6) + 6*(2*c^11*d^2*x^7 + 160*c^12*d*x^4 + 320*c^13*x
 - 6*(5*c^3*d^8*x^5 + 32*c^4*d^7*x^2 - sqrt(-3)*(5*c^3*d^8*x^5 + 32*c^4*d^7
x^2))(c^13/d^10)^(2/3) - (7*c^7*d^5*x^6 + 152*c^8*d^4*x^3 + 64*c^9*d^3 +
sqrt(-3)*(7*c^7*d^5*x^6 + 152*c^8*d^4*x^3 + 64*c^9*d^3))*(c^13/d^10)^(1/3))
*sqrt(d*x^3 + c) - 36*(5*c^5*d^7*x^7 + 64*c^6*d^6*x^4 + 32*c^7*d^5*x)*sqrt(
c^13/d^10) + 18*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2 - sqrt(-3)
*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2))*(c^13/d^10)^(1/6))/(d^3
*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 273*(c^13/d^10)^(1/6)*(sq
rt(-3)*d^2 - d^2)*log(-60466176*((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*
x^3 + 640*c^3*d^8 + sqrt(-3)*(d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3
+ 640*c^3*d^8))*(c^13/d^10)^(5/6) - 6*(2*c^11*d^2*x^7 + 160*c^12*d*x^4 + 32
0*c^13*x - 6*(5*c^3*d^8*x^5 + 32*c^4*d^7*x^2 - sqrt(-3)*(5*c^3*d^8*x^5 + 32
*c^4*d^7*x^2))*(c^13/d^10)^(2/3) - (7*c^7*d^5*x^6 + 152*c^8*d^4*x^3 + 64*c^
9*d^3 + sqrt(-3)*(7*c^7*d^5*x^6 + 152*c^8*d^4*x^3 + 64*c^9*d^3))*(c^13/d^10
)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^5*d^7*x^7 + 64*c^6*d^6*x^4 + 32*c^7*d^5*
x)*sqrt(c^13/d^10) + 18*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2 -
sqrt(-3)*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2))*(c^13/d^10)^(1/
6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 273*(c^13/d^10)^(
1/6)*(sqrt(-3)*d^2 + d^2)*log(60466176*((d^11*x^9 + 318*c*d^10*x^6 + 1200*c
^2*d^9*x^3 + 640*c^3*d^8 - sqrt(-3)*(d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d
^9*x^3 + 640*c^3*d^8))*(c^13/d^10)^(5/6) + 6*(2*c^11*d^2*x^7 + 160*c^12*d*x
^4 + 320*c^13*x - 6*(5*c^3*d^8*x^5 + 32*c^4*d^7*x^2 + sqrt(-3)*(5*c^3*d^8*x
^5 + 32*c^4*d^7*x^2))*(c^13/d^10)^(2/3) - (7*c^7*d^5*x^6 + 152*c^8*d^4*x^3
+ 64*c^9*d^3 - sqrt(-3)*(7*c^7*d^5*x^6 + 152*c^8*d^4*x^3 + 64*c^9*d^3))*(c^
13/d^10)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^5*d^7*x^7 + 64*c^6*d^6*x^4 + 32*c
^7*d^5*x)*sqrt(c^13/d^10) + 18*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2
x^2 + sqrt(-3)(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2))*(c^13/d^
10)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 273*(c^13/

```
d^10)^(1/6)*(sqrt(-3)*d^2 + d^2)*log(-60466176*((d^11*x^9 + 318*c*d^10*x^6
+ 1200*c^2*d^9*x^3 + 640*c^3*d^8 - sqrt(-3)*(d^11*x^9 + 318*c*d^10*x^6 + 12
00*c^2*d^9*x^3 + 640*c^3*d^8))*(c^13/d^10)^(5/6) - 6*(2*c^11*d^2*x^7 + 160*
c^12*d*x^4 + 320*c^13*x - 6*(5*c^3*d^8*x^5 + 32*c^4*d^7*x^2 + sqrt(-3)*(5*c
^3*d^8*x^5 + 32*c^4*d^7*x^2))*(c^13/d^10)^(2/3) - (7*c^7*d^5*x^6 + 152*c^8*
d^4*x^3 + 64*c^9*d^3 - sqrt(-3)*(7*c^7*d^5*x^6 + 152*c^8*d^4*x^3 + 64*c^9*d
^3))*(c^13/d^10)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^5*d^7*x^7 + 64*c^6*d^6*x^
4 + 32*c^7*d^5*x)*sqrt(c^13/d^10) + 18*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*
c^11*d^2*x^2 + sqrt(-3)*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2))*
(c^13/d^10)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*
(7*d^2*x^5 + 120*c*d*x^2)*sqrt(d*x^3 + c))/d^2
```

Sympy [F]

$$\int \frac{x^4(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\int \frac{cx^4\sqrt{c+dx^3}}{-8c+dx^3}\,dx - \int \frac{dx^7\sqrt{c+dx^3}}{-8c+dx^3}\,dx$$

```
[In] integrate(x**4*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
[Out] -Integral(c*x**4*sqrt(c + d*x**3)/(-8*c + d*x**3), x) - Integral(d*x**7*sqr
t(c + d*x**3)/(-8*c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^4(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}x^4}{dx^3-8\,c}\,dx$$

```
[In] integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
[Out] -integrate((d*x^3 + c)^(3/2)*x^4/(d*x^3 - 8*c), x)
```

Giac [F]

$$\int \frac{x^4(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}x^4}{dx^3-8\,c}\,dx$$

```
[In] integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
[Out] integrate(-(d*x^3 + c)^(3/2)*x^4/(d*x^3 - 8*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \int \frac{x^4\,(d\,x^3+c)^{3/2}}{8\,c-d\,x^3}\,dx$$

```
[In] int((x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
[Out] int((x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3), x)
```

3.304 $\int \frac{x(c+dx^3)^{3/2}}{8c-dx^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 627

$$\int \frac{x(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\frac{2}{7}x^2\sqrt{c+dx^3} - \frac{132c\sqrt{c+dx^3}}{7d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{9\sqrt{3}c^{7/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{2d^{2/3}} + \frac{9c^{7/6}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{2d^{2/3}} - \frac{9c^{7/6}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2d^{2/3}} + \frac{66\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{44\sqrt{2}3^{3/4}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{7d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 9/2*c^(7/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(2
/3)-9/2*c^(7/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(2/3)-9/2*c^(7/6)*ar
```

```
ctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/d^(2/3)-2
/7*x^2*(d*x^3+c)^(1/2)-132/7*c*(d*x^3+c)^(1/2)/d^(2/3)/(d^(1/3)*x+c^(1/3)*(
1+3^(1/2)))-44/7*3^(3/4)*c^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c
^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*
((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)
^(1/2)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1
/3)*(1+3^(1/2)))^2)^(1/2)+66/7*3^(1/4)*c^(4/3)*(c^(1/3)+d^(1/3)*x)*Elliptic
E((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)
+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d
^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^
(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 627, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {488, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{x(c+dx^3)^{3/2}}{8c-dx^3}\,dx =$$

$$-\frac{44\sqrt{2}3^{3/4}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{7d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{66\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{7d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{9\sqrt{3}c^{7/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{2d^{2/3}}+\frac{9c^{7/6}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{2d^{2/3}}$$

$$-\frac{9c^{7/6}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2d^{2/3}}-\frac{132c\sqrt{c+dx^3}}{7d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{2}{7}x^2\sqrt{c+dx^3}$$

```
[In] Int[(x*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-2*x^2*Sqrt[c + d*x^3])/7 - (132*c*Sqrt[c + d*x^3])/(7*d^(2/3)*((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)) - (9*Sqrt[3]*c^(7/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(
```

```
1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(2*d^(2/3)) + (9*c^(7/6)*ArcTanh[(c^(1
/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(2*d^(2/3)) - (9*c^(7/6)*A
rcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2*d^(2/3)) + (66*3^(1/4)*Sqrt[2 - Sqr
t[3]]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(
2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqr
t[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqr
t[3]])/(7*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (44*Sqrt[2]*3^(3/4)*c^(4/3)*(c^(1/3)
+ d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)
*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*d^(2/3)*Sqrt[
(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt
[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
```

```
/; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
```

```
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\text{integral} = -\frac{2}{7}x^2\sqrt{c+dx^3} - \frac{2\int \frac{x\left(-\frac{39c^2d}{2}-33cd^2x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{7d}$$

$$= -\frac{2}{7}x^2\sqrt{c+dx^3} - \frac{2\int \left(\frac{33cdx}{\sqrt{c+dx^3}} - \frac{567c^2dx}{2(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{7d}$$

$$= -\frac{2}{7}x^2\sqrt{c+dx^3} - \frac{1}{7}(66c)\int \frac{x}{\sqrt{c+dx^3}}\,dx + \left(81c^2\right)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$$

$$= -\frac{2}{7}x^2\sqrt{c+dx^3} - \frac{(27c)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{4d}$$

$$- \frac{(66c)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{7\sqrt[3]{d}} + \frac{\left(27c^{4/3}\right)\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{4\sqrt[3]{d}}$$

$$+ \frac{\left(66\left(1-\sqrt{3}\right)c^{4/3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{7\sqrt[3]{d}} - \frac{1}{4}\left(81c^{5/3}\sqrt[3]{d}\right)\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$$

$$= -\frac{2}{7}x^2\sqrt{c+dx^3} - \frac{132c\sqrt{c+dx^3}}{7d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$+\frac{66\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right.}{7d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{44\sqrt{2}3^{3/4}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{\left(27c^{5/3}\right)\text{Subst}\left(\int\frac{1}{9-cx^2}\,dx,x,\frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{2d^{2/3}}$$

$$-\frac{1}{4}\left(27c^{5/3}\sqrt[3]{d}\right)\text{Subst}\left(\int\frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)+\left(27c^{2/3}d^{4/3}\right)\text{Subst}\left(\int\frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx\right.$$

$$= -\frac{2}{7}x^2\sqrt{c+dx^3} - \frac{132c\sqrt{c+dx^3}}{7d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$-\frac{9\sqrt{3}c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{2d^{2/3}} + \frac{9c^{7/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{2d^{2/3}}$$

$$+\frac{66\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{44\sqrt{2}3^{3/4}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{\left(27c^{5/3}\right)\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{2d^{2/3}}$$

$$= -\frac{2}{7}x^2\sqrt{c+dx^3} - \frac{132c\sqrt{c+dx^3}}{7d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{9\sqrt{3}c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{2d^{2/3}} + \frac{9c^{7/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{2d^{2/3}} - \frac{9c^{7/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2d^{2/3}} + \frac{66\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{44\sqrt{2}3^{3/4}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 9.98 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.20

$$\int \frac{x(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \frac{x^2\left(-160(c+dx^3)+195c\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+132dx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}{560\sqrt{c+dx^3}}$$

```
[In] Integrate[(x*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (x^2*(-160*(c + d*x^3) + 195*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/
3, -((d*x^3)/c), (d*x^3)/(8*c)] + 132*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/
3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(560*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.85 (sec) , antiderivative size = 864, normalized size of antiderivative = 1.38

method	result	size
default	Expression too large to display	864
elliptic	Expression too large to display	864
risch	Expression too large to display	866

```
[In] int(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] -2/7*x^2*(d*x^3+c)^(1/2)+44/7*I*c*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*
d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*
((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d
*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/
2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(
-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)
))-3*I*c/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1
/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2
)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x
+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^
3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_a
lpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(
I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2
/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2
)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^
(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.81 (sec) , antiderivative size = 2368, normalized size of antiderivative = 3.78

$$\int \frac{x(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] -1/56*(16*sqrt(d*x^3 + c)*d*x^2 - 1056*c*sqrt(d)*weierstrassZeta(0, -4*c/d,
 weierstrassPInverse(0, -4*c/d, x)) + 21*(c^7/d^4)^(1/6)*(sqrt(-3)*d - d)*l
og(59049/4*((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 + sqr
t(-3)*(d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3))*(c^7/d^4)
^(5/6) + 6*(2*c^6*d^2*x^7 + 160*c^7*d*x^4 + 320*c^8*x - 6*(5*c^2*d^4*x^5 +
32*c^3*d^3*x^2 - sqrt(-3)*(5*c^2*d^4*x^5 + 32*c^3*d^3*x^2))*(c^7/d^4)^(2/3)
 - (7*c^4*d^3*x^6 + 152*c^5*d^2*x^3 + 64*c^6*d + sqrt(-3)*(7*c^4*d^3*x^6 +
152*c^5*d^2*x^3 + 64*c^6*d))*(c^7/d^4)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^3*d
^4*x^7 + 64*c^4*d^3*x^4 + 32*c^5*d^2*x)*sqrt(c^7/d^4) + 18*(c^5*d^3*x^8 + 3
8*c^6*d^2*x^5 + 64*c^7*d*x^2 - sqrt(-3)*(c^5*d^3*x^8 + 38*c^6*d^2*x^5 + 64*
c^7*d*x^2))*(c^7/d^4)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*
c^3)) - 21*(c^7/d^4)^(1/6)*(sqrt(-3)*d - d)*log(-59049/4*((d^6*x^9 + 318*c*
d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 + sqrt(-3)*(d^6*x^9 + 318*c*d^5*x^
6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3))*(c^7/d^4)^(5/6) - 6*(2*c^6*d^2*x^7 + 1
60*c^7*d*x^4 + 320*c^8*x - 6*(5*c^2*d^4*x^5 + 32*c^3*d^3*x^2 - sqrt(-3)*(5*
c^2*d^4*x^5 + 32*c^3*d^3*x^2))*(c^7/d^4)^(2/3) - (7*c^4*d^3*x^6 + 152*c^5*d
^2*x^3 + 64*c^6*d + sqrt(-3)*(7*c^4*d^3*x^6 + 152*c^5*d^2*x^3 + 64*c^6*d))*
(c^7/d^4)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^3*d^4*x^7 + 64*c^4*d^3*x^4 + 32*
c^5*d^2*x)*sqrt(c^7/d^4) + 18*(c^5*d^3*x^8 + 38*c^6*d^2*x^5 + 64*c^7*d*x^2
 - sqrt(-3)*(c^5*d^3*x^8 + 38*c^6*d^2*x^5 + 64*c^7*d*x^2))*(c^7/d^4)^(1/6))/
(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 21*(c^7/d^4)^(1/6)*(s
qrt(-3)*d + d)*log(59049/4*((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 6
40*c^3*d^3 - sqrt(-3)*(d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3
*d^3))*(c^7/d^4)^(5/6) + 6*(2*c^6*d^2*x^7 + 160*c^7*d*x^4 + 320*c^8*x - 6*(
5*c^2*d^4*x^5 + 32*c^3*d^3*x^2 + sqrt(-3)*(5*c^2*d^4*x^5 + 32*c^3*d^3*x^2))
*(c^7/d^4)^(2/3) - (7*c^4*d^3*x^6 + 152*c^5*d^2*x^3 + 64*c^6*d - sqrt(-3)*(
7*c^4*d^3*x^6 + 152*c^5*d^2*x^3 + 64*c^6*d))*(c^7/d^4)^(1/3))*sqrt(d*x^3 +
c) - 36*(5*c^3*d^4*x^7 + 64*c^4*d^3*x^4 + 32*c^5*d^2*x)*sqrt(c^7/d^4) + 18*
(c^5*d^3*x^8 + 38*c^6*d^2*x^5 + 64*c^7*d*x^2 + sqrt(-3)*(c^5*d^3*x^8 + 38*c
^6*d^2*x^5 + 64*c^7*d*x^2))*(c^7/d^4)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*
c^2*d*x^3 - 512*c^3)) + 21*(c^7/d^4)^(1/6)*(sqrt(-3)*d + d)*log(-59049/4*((
d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 - sqrt(-3)*(d^6*x^
9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3))*(c^7/d^4)^(5/6) - 6*(2
*c^6*d^2*x^7 + 160*c^7*d*x^4 + 320*c^8*x - 6*(5*c^2*d^4*x^5 + 32*c^3*d^3*x^
2 + sqrt(-3)*(5*c^2*d^4*x^5 + 32*c^3*d^3*x^2))*(c^7/d^4)^(2/3) - (7*c^4*d^3
*x^6 + 152*c^5*d^2*x^3 + 64*c^6*d - sqrt(-3)*(7*c^4*d^3*x^6 + 152*c^5*d^2*x
^3 + 64*c^6*d))*(c^7/d^4)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^3*d^4*x^7 + 64*c
^4*d^3*x^4 + 32*c^5*d^2*x)*sqrt(c^7/d^4) + 18*(c^5*d^3*x^8 + 38*c^6*d^2*x^5
 + 64*c^7*d*x^2 + sqrt(-3)*(c^5*d^3*x^8 + 38*c^6*d^2*x^5 + 64*c^7*d*x^2))*(
c^7/d^4)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 42*(c
^7/d^4)^(1/6)*d*log(59049/2*((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 +
640*c^3*d^3)*(c^7/d^4)^(5/6) + 6*(c^6*d^2*x^7 + 80*c^7*d*x^4 + 160*c^8*x +
6*(5*c^2*d^4*x^5 + 32*c^3*d^3*x^2)*(c^7/d^4)^(2/3) + (7*c^4*d^3*x^6 + 152*c
^5*d^2*x^3 + 64*c^6*d)*(c^7/d^4)^(1/3))*sqrt(d*x^3 + c) + 18*(5*c^3*d^4*x^7
 + 64*c^4*d^3*x^4 + 32*c^5*d^2*x)*sqrt(c^7/d^4) + 18*(c^5*d^3*x^8 + 38*c^6*
```

```
d^2*x^5 + 64*c^7*d*x^2)*(c^7/d^4)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*
d*x^3 - 512*c^3)) + 42*(c^7/d^4)^(1/6)*d*log(-59049/2*((d^6*x^9 + 318*c*d^5
*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3)*(c^7/d^4)^(5/6) - 6*(c^6*d^2*x^7 + 8
0*c^7*d*x^4 + 160*c^8*x + 6*(5*c^2*d^4*x^5 + 32*c^3*d^3*x^2)*(c^7/d^4)^(2/3
) + (7*c^4*d^3*x^6 + 152*c^5*d^2*x^3 + 64*c^6*d)*(c^7/d^4)^(1/3))*sqrt(d*x^
3 + c) + 18*(5*c^3*d^4*x^7 + 64*c^4*d^3*x^4 + 32*c^5*d^2*x)*sqrt(c^7/d^4) +
 18*(c^5*d^3*x^8 + 38*c^6*d^2*x^5 + 64*c^7*d*x^2)*(c^7/d^4)^(1/6))/(d^3*x^9
 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/d
```

Sympy [F]

$$\int \frac{x(c+dx^3)^{3/2}}{8c-dx^3}\,dx = -\int \frac{cx\sqrt{c+dx^3}}{-8c+dx^3}\,dx - \int \frac{dx^4\sqrt{c+dx^3}}{-8c+dx^3}\,dx$$

```
[In] integrate(x*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
[Out] -Integral(c*x*sqrt(c + d*x**3)/(-8*c + d*x**3), x) - Integral(d*x**4*sqrt(c
 + d*x**3)/(-8*c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}x}{dx^3-8\,c}\,dx$$

```
[In] integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
[Out] -integrate((d*x^3 + c)^(3/2)*x/(d*x^3 - 8*c), x)
```

Giac [F]

$$\int \frac{x(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}x}{dx^3-8\,c}\,dx$$

```
[In] integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
[Out] integrate(-(d*x^3 + c)^(3/2)*x/(d*x^3 - 8*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx^3)^{3/2}}{8c-dx^3}\,dx = \int \frac{x\,(d\,x^3+c)^{3/2}}{8\,c-d\,x^3}\,dx$$

```
[In] int((x*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
[Out] int((x*(c + d*x^3)^(3/2))/(8*c - d*x^3), x)
```

3.305 $\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 626

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)}\,dx = -\frac{\sqrt{c+dx^3}}{8x} - \frac{15\sqrt[3]{d}\sqrt{c+dx^3}}{8\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{9}{16}\sqrt{3}\sqrt[6]{c}\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right) + \frac{9}{16}\sqrt[6]{c}\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right) - \frac{9}{16}\sqrt[6]{c}\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) + \frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{16\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{5\,3^{3/4}\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 9/16*c^(1/6)*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1
/2))-9/16*c^(1/6)*d^(1/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))-9/16*c^(1/6)
*d^(1/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2
```

```
)-1/8*(d*x^3+c)^(1/2)/x-15/8*d^(1/3)*(d*x^3+c)^(1/2)/(d^(1/3)*x+c^(1/3)*(1+
3^(1/2)))-5/8*3^(3/4)*c^(1/3)*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3
)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c
^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1
/2)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)
*(1+3^(1/2)))^2)^(1/2)+15/16*3^(1/4)*c^(1/3)*d^(1/3)*(c^(1/3)+d^(1/3)*x)*El
lipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3
^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x
^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1
/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 626, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {485, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)}\,dx =$$

$$-\frac{5\,3^{3/4}\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{16\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{9}{16}\sqrt{3}\sqrt[6]{c}\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)+\frac{9}{16}\sqrt[6]{c}\sqrt[3]{d}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)-\frac{9}{16}\sqrt[6]{c}\sqrt[3]{d}\text{arctanh}\left(\right.$$

```
[In] Int[(c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)),x]
```

```
[Out] -1/8*Sqrt[c + d*x^3]/x - (15*d^(1/3)*Sqrt[c + d*x^3])/(8*((1 + Sqrt[3])*c^(
1/3) + d^(1/3)*x)) - (9*Sqrt[3]*c^(1/6)*d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^
(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/16 + (9*c^(1/6)*d^(1/3)*ArcTanh[(c^(1
/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/16 - (9*c^(1/6)*d^(1/3)*Ar
cTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/16 + (15*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1
/3)*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/
```

```
3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[
3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[
3]])/(16*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)^2]*Sqrt[c + d*x^3]) - (5*3^(3/4)*c^(1/3)*d^(1/3)*(c^(1/3) + d^(1/3)*
x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3)
+ d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*Sqrt[(c^(1/3)*(
c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3
])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
```

```
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{c+dx^3}}{8x}+\frac{\int \frac{x\left(21c^2d+\frac{15}{2}cd^2x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{8c}$$

$$= -\frac{\sqrt{c+dx^3}}{8x}+\frac{\int \left(-\frac{15cdx}{2\sqrt{c+dx^3}}+\frac{81c^2dx}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{8c}$$

$$= -\frac{\sqrt{c+dx^3}}{8x}-\frac{1}{16}(15d)\int \frac{x}{\sqrt{c+dx^3}}\,dx+\frac{1}{8}(81cd)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$$

$$= -\frac{\sqrt{c+dx^3}}{8x}-\frac{27}{32}\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx$$

$$-\frac{1}{16}\left(15d^{2/3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx+\frac{1}{32}\left(27\sqrt[3]{c}d^{2/3}\right)\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx+\frac{1}{16}\Big(15\Big(1-$$

$$= -\frac{\sqrt{c+dx^3}}{8x} - \frac{15\sqrt[3]{d}\sqrt{c+dx^3}}{8\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$+\frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4}{16\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{5\ 3^{3/4}\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{1}{16}\left(27c^{2/3}\sqrt[3]{d}\right)\text{Subst}\left(\int\frac{1}{9-cx^2}\,dx,x,\frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)-\frac{1}{32}\left(27c^{2/3}d^{4/3}\right)\text{Subst}\left(\int\frac{1}{(8c-dx)\sqrt{}}\right.$$

$$= -\frac{\sqrt{c+dx^3}}{8x} - \frac{15\sqrt[3]{d}\sqrt{c+dx^3}}{8\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$- \frac{9}{16}\sqrt{3}\sqrt[6]{c}\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)$$

$$+ \frac{9}{16}\sqrt[6]{c}\sqrt[3]{d}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)$$

$$+ \frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{}\right.}{16\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$- \frac{5\ 3^{3/4}\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$- \frac{1}{16}\left(27c^{2/3}\sqrt[3]{d}\right)\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)$$

$$= -\frac{\sqrt{c+dx^3}}{8x} - \frac{15\sqrt[3]{d}\sqrt{c+dx^3}}{8\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$
$$-\frac{9}{16}\sqrt{3}\sqrt[6]{c}\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)$$
$$+\frac{9}{16}\sqrt[6]{c}\sqrt[3]{d}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right) - \frac{9}{16}\sqrt[6]{c}\sqrt[3]{d}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)$$
$$+\frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4}{16\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{5\,3^{3/4}\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.22

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)}\,dx = \frac{-16c(c+dx^3)+21cdx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3d^2x^6\sqrt{1+\frac{dx^3}{c}}\,A}{128cx\sqrt{c+dx^3}}$$

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)),x]
```

```
[Out] (-16*c*(c + d*x^3) + 21*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5
/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[5
/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(128*c*x*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.89 (sec) , antiderivative size = 859, normalized size of antiderivative = 1.37

method	result	size
elliptic	Expression too large to display	859
risch	Expression too large to display	866
default	Expression too large to display	1339

```
[In] int((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(d*x^3+c)^(1/2)/x+5/8*I*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1
/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d
*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1
/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^
2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d
*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)
^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-3/8*
I/d^2*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c
*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3)
)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1
/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*
d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/
2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(
1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c
*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),
_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)}\,dx = -\int \frac{c\sqrt{c+dx^3}}{-8cx^2+dx^5}\,dx - \int \frac{dx^3\sqrt{c+dx^3}}{-8cx^2+dx^5}\,dx$$

```
[In] integrate((d*x**3+c)**(3/2)/x**2/(-d*x**3+8*c),x)
[Out] -Integral(c*sqrt(c + d*x**3)/(-8*c*x**2 + d*x**5), x) - Integral(d*x**3*sqr
t(c + d*x**3)/(-8*c*x**2 + d*x**5), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)x^2}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c),x, algorithm="maxima")
[Out] -integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^2), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)x^2}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c),x, algorithm="giac")
[Out] integrate(-(d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)}\,dx = \int \frac{(d\,x^3+c)^{3/2}}{x^2\,(8\,c-d\,x^3)}\,dx$$

```
[In] int((c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)),x)
[Out] int((c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)), x)
```

3.306 $\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 651

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)}\,dx = -\frac{\sqrt{c+dx^3}}{32x^4}-\frac{3d\sqrt{c+dx^3}}{16cx} + \frac{3d^{4/3}\sqrt{c+dx^3}}{16c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{9\sqrt{3}d^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{128c^{5/6}} + \frac{9d^{4/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{128c^{5/6}} - \frac{9d^{4/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{5/6}} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{3^{3/4}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{8\sqrt{2}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 9/128*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^
(5/6)-9/128*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/6)-9/128*d^(4
```

```
/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/c^(
5/6)-1/32*(d*x^3+c)^(1/2)/x^4-3/16*d*(d*x^3+c)^(1/2)/c/x+3/16*d^(4/3)*(d*x^
3+c)^(1/2)/c/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/16*3^(3/4)*d^(4/3)*(c^(1/3)+
d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+
3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*
x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(2/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c
^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-3/32*3^(1/4)*d^(
4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)
*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-
c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(
2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(
1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 651, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {485, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^5\left(8c-dx^3\right)}dx = \frac{3^{3/4}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-\right.}{8\sqrt{2}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{32c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{9\sqrt{3}d^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{128c^{5/6}}+\frac{9d^{4/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{128c^{5/6}}$$

$$-\frac{9d^{4/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{5/6}}+\frac{3d^{4/3}\sqrt{c+dx^3}}{16c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{3d\sqrt{c+dx^3}}{16cx}-\frac{\sqrt{c+dx^3}}{32x^4}$$

```
[In] Int[(c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)),x]

[Out] -1/32*Sqrt[c + d*x^3]/x^4 - (3*d*Sqrt[c + d*x^3])/(16*c*x) + (3*d^(4/3)*Sqr
t[c + d*x^3])/(16*c*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (9*Sqrt[3]*d^(4/
3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(128*c^
```

```
(5/6)) + (9*d^(4/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x
^3])])/(128*c^(5/6)) - (9*d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(12
8*c^(5/6)) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqr
t[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(32*c^(2/3)*Sqrt[(c^(1/3)*(c^(1/
3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) +
(3^(3/4)*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x +
d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*
Sqrt[3]])/(8*Sqrt[2]*c^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
```

```
/; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{c+dx^3}}{32x^4}+\frac{\int \frac{48c^2d+\frac{69}{2}cd^2x^3}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{32c} \\
&= -\frac{\sqrt{c+dx^3}}{32x^4}-\frac{3d\sqrt{c+dx^3}}{16cx}-\frac{\int \frac{x(-516c^3d^2+24c^2d^3x^3)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{256c^3} \\
&= -\frac{\sqrt{c+dx^3}}{32x^4}-\frac{3d\sqrt{c+dx^3}}{16cx}-\frac{\int \left(-\frac{24c^2d^2x}{\sqrt{c+dx^3}}-\frac{324c^3d^2x}{(8c-dx^3)\sqrt{c+dx^3}}\right)\,dx}{256c^3} \\
&= -\frac{\sqrt{c+dx^3}}{32x^4}-\frac{3d\sqrt{c+dx^3}}{16cx}+\frac{1}{64}\left(81d^2\right)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx+\frac{(3d^2)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{32c}
\end{aligned}$$

$$= -\frac{\sqrt{c+dx^3}}{32x^4} - \frac{3d\sqrt{c+dx^3}}{16cx} - \frac{(27d)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{256c}$$

$$+ \frac{\left(3d^{5/3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{32c} + \frac{\left(27d^{5/3}\right)\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{256c^{2/3}}$$

$$- \frac{\left(3\left(1-\sqrt{3}\right)d^{5/3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{32c^{2/3}} - \frac{\left(81d^{7/3}\right)\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{256\sqrt[3]{c}}$$

$$= -\frac{\sqrt{c+dx^3}}{32x^4} - \frac{3d\sqrt{c+dx^3}}{16cx} + \frac{3d^{4/3}\sqrt{c+dx^3}}{16c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$- \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{3^{3/4}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{8\sqrt{2}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{\left(27d^{4/3}\right)\text{Subst}\left(\int \frac{1}{9-cx^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{128\sqrt[3]{c}}$$

$$- \frac{\left(27d^{7/3}\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{256\sqrt[3]{c}}$$

$$+ \frac{\left(27d^{10/3}\right)\text{Subst}\left(\int \frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{64c^{4/3}}$$

$$= -\frac{\sqrt{c+dx^3}}{32x^4} - \frac{3d\sqrt{c+dx^3}}{16cx} + \frac{3d^{4/3}\sqrt{c+dx^3}}{16c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$- \frac{9\sqrt{3}d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{128c^{5/6}} + \frac{9d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{128c^{5/6}}$$

$$- \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{3^{3/4}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{8\sqrt{2}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$- \frac{\left(27d^{4/3}\right)\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{128\sqrt[3]{c}}$$

$$= -\frac{\sqrt{c+dx^3}}{32x^4} - \frac{3d\sqrt{c+dx^3}}{16cx} + \frac{3d^{4/3}\sqrt{c+dx^3}}{16c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$-\frac{9\sqrt{3}d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{128c^{5/6}}$$

$$+\frac{9d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{128c^{5/6}} - \frac{9d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{5/6}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{3^{3/4}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{8\sqrt{2}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.24

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^5\left(8c-dx^3\right)}\,dx = \frac{645cd^2x^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right) - 4\left(40c\left(c^2+7cdx^3+6d^2x^6\right)+3\right.}{5120c^2x^4\sqrt{c+dx^3}}$$

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)),x]
```

```
[Out] (645*c*d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c),
 (d*x^3)/(8*c)] - 4*(40*c*(c^2 + 7*c*d*x^3 + 6*d^2*x^6) + 3*d^3*x^9*Sqrt[1
+ (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(512
0*c^2*x^4*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.96 (sec) , antiderivative size = 882, normalized size of antiderivative = 1.35

method	result	size
risch	Expression too large to display	882
elliptic	Expression too large to display	884
default	Expression too large to display	1810

```
[In] int((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*(d*x^3+c)^(1/2)*(6*d*x^3+c)/x^4/c+3/64*d^2/c*(-4/3*I*3^(1/2)/d*(-c*d^
2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*
(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3)))^(1/2)))-I/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*
I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1
/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))
^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^
2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2
)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ell
ipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_al
pha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alp
ha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.56 (sec) , antiderivative size = 2369, normalized size of antiderivative = 3.64

$$\int \frac{(c+dx^3)^{3/2}}{x^5\,(8c-dx^3)}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c),x, algorithm="fricas")
```

[Out] -1/512*(96*d^(3/2)*x^4*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) - 6*(d^8/c^5)^(1/6)*c*x^4*log(6561*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 + 18*(5*c^4*d^3*x^7 + 64*c^5*d^2*x^4 + 32*c^6*d*x)*(d^8/c^5)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^5*d*x^5 + 32*c^6*x^2)*(d^8/c^5)^(5/6) + (7*c^3*d^4*x^6 + 152*c^4*d^3*x^3 + 64*c^5*d^2)*sqrt(d^8/c^5) + (c*d^7*x^7 + 80*c^2*d^6*x^4 + 160*c^3*d^5*x)*(d^8/c^5)^(1/6)) + 18*(c^2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4*x^2)*(d^8/c^5)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 6*(d^8/c^5)^(1/6)*c*x^4*log(6561*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 + 18*(5*c^4*d^3*x^7 + 64*c^5*d^2*x^4 + 32*c^6*d*x)*(d^8/c^5)^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^5*d*x^5 + 32*c^6*x^2)*(d^8/c^5)^(5/6) + (7*c^3*d^4*x^6 + 152*c^4*d^3*x^3 + 64*c^5*d^2)*sqrt(d^8/c^5) + (c*d^7*x^7 + 80*c^2*d^6*x^4 + 160*c^3*d^5*x)*(d^8/c^5)^(1/6)) + 18*(c^2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4*x^2)*(d^8/c^5)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 3*(sqrt(-3)*c*x^4 + c*x^4)*(d^8/c^5)^(1/6)*log(6561*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^4*d^3*x^7 + 64*c^5*d^2*x^4 + 32*c^6*d*x + sqrt(-3)*(5*c^4*d^3*x^7 + 64*c^5*d^2*x^4 + 32*c^6*d*x))*(d^8/c^5)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^5*d*x^5 + 32*c^6*x^2 - sqrt(-3)*(5*c^5*d*x^5 + 32*c^6*x^2))*(d^8/c^5)^(5/6) - 2*(7*c^3*d^4*x^6 + 152*c^4*d^3*x^3 + 64*c^5*d^2)*sqrt(d^8/c^5) + (c*d^7*x^7 + 80*c^2*d^6*x^4 + 160*c^3*d^5*x + sqrt(-3)*(c*d^7*x^7 + 80*c^2*d^6*x^4 + 160*c^3*d^5*x))*(d^8/c^5)^(1/6)) - 9*(c^2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4*x^2 - sqrt(-3)*(c^2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4*x^2))*(d^8/c^5)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 3*(sqrt(-3)*c*x^4 + c*x^4)*(d^8/c^5)^(1/6)*log(6561*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^4*d^3*x^7 + 64*c^5*d^2*x^4 + 32*c^6*d*x + sqrt(-3)*(5*c^4*d^3*x^7 + 64*c^5*d^2*x^4 + 32*c^6*d*x))*(d^8/c^5)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^5*d*x^5 + 32*c^6*x^2 - sqrt(-3)*(5*c^5*d*x^5 + 32*c^6*x^2))*(d^8/c^5)^(5/6) - 2*(7*c^3*d^4*x^6 + 152*c^4*d^3*x^3 + 64*c^5*d^2)*sqrt(d^8/c^5) + (c*d^7*x^7 + 80*c^2*d^6*x^4 + 160*c^3*d^5*x + sqrt(-3)*(c*d^7*x^7 + 80*c^2*d^6*x^4 + 160*c^3*d^5*x))*(d^8/c^5)^(1/6)) - 9*(c^2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4*x^2 - sqrt(-3)*(c^2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4*x^2))*(d^8/c^5)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 3*(sqrt(-3)*c*x^4 - c*x^4)*(d^8/c^5)^(1/6)*log(6561*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^4*d^3*x^7 + 64*c^5*d^2*x^4 + 32*c^6*d*x - sqrt(-3)*(5*c^4*d^3*x^7 + 64*c^5*d^2*x^4 + 32*c^6*d*x))*(d^8/c^5)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^5*d*x^5 + 32*c^6*x^2 + sqrt(-3)*(5*c^5*d*x^5 + 32*c^6*x^2))*(d^8/c^5)^(5/6) - 2*(7*c^3*d^4*x^6 + 152*c^4*d^3*x^3 + 64*c^5*d^2)*sqrt(d^8/c^5) + (c*d^7*x^7 + 80*c^2*d^6*x^4 + 160*c^3*d^5*x - sqrt(-3)*(c*d^7*x^7 + 80*c^2*d^6*x^4 + 160*c^3*d^5*x))*(d^8/c^5)^(1/6)) - 9*(c^2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4*x^2 + sqrt(-3)*(c^2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4*x^2))*(d^8/c^5)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 3*(sqrt(-3)*c*x^4 - c*x^4)*(d^8/c^5)^(1/6)*log(6561*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^4*d^3*x^7 + 64*c^5*d^2*x^4 + 32*c^6*d*x - sqrt(-3)*(5*c^4*d^3*x^7 + 64*c^5*d^2*x^4 + 32*c^6*d*x))*(d^8/c^5)

```
^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^5*d*x^5 + 32*c^6*x^2 + sqrt(-3)*(5*c^5*d
*x^5 + 32*c^6*x^2))*(d^8/c^5)^(5/6) - 2*(7*c^3*d^4*x^6 + 152*c^4*d^3*x^3 +
64*c^5*d^2)*sqrt(d^8/c^5) + (c*d^7*x^7 + 80*c^2*d^6*x^4 + 160*c^3*d^5*x - s
qrt(-3)*(c*d^7*x^7 + 80*c^2*d^6*x^4 + 160*c^3*d^5*x))*(d^8/c^5)^(1/6)) - 9*
(c^2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4*x^2 + sqrt(-3)*(c^2*d^6*x^8 + 38
*c^3*d^5*x^5 + 64*c^4*d^4*x^2))*(d^8/c^5)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 +
192*c^2*d*x^3 - 512*c^3)) + 16*(6*d*x^3 + c)*sqrt(d*x^3 + c))/(c*x^4)
```

Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)}\,dx = -\int \frac{c\sqrt{c+dx^3}}{-8cx^5+dx^8}\,dx - \int \frac{dx^3\sqrt{c+dx^3}}{-8cx^5+dx^8}\,dx$$

```
[In] integrate((d*x**3+c)**(3/2)/x**5/(-d*x**3+8*c),x)
[Out] -Integral(c*sqrt(c + d*x**3)/(-8*c*x**5 + d*x**8), x) - Integral(d*x**3*sqr
t(c + d*x**3)/(-8*c*x**5 + d*x**8), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)x^5}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c),x, algorithm="maxima")
[Out] -integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^5), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)x^5}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c),x, algorithm="giac")
[Out] integrate(-(d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c + dx^3)^{3/2}}{x^5 (8c - dx^3)} \, dx = \int \frac{(d\,x^3 + c)^{3/2}}{x^5 \, (8\,c - d\,x^3)} \, dx$$

```
[In] int((c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)),x)
[Out] int((c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)), x)
```

3.307 $\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 675

$$\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)}\,dx = -\frac{\sqrt{c+dx^3}}{56x^7}-\frac{75d\sqrt{c+dx^3}}{1792cx^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^2x} + \frac{3d^{7/3}\sqrt{c+dx^3}}{56c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{9\sqrt{3}d^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1024c^{11/6}} + \frac{9d^{7/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1024c^{11/6}} - \frac{9d^{7/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1024c^{11/6}} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{112c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{3^{3/4}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{28\sqrt{2}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 9/1024*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c
^(11/6)-9/1024*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)-9/1024
```

```
*d^(7/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2
)/c^(11/6)-1/56*(d*x^3+c)^(1/2)/x^7-75/1792*d*(d*x^3+c)^(1/2)/c/x^4-3/56*d^
2*(d*x^3+c)^(1/2)/c^2/x+3/56*d^(7/3)*(d*x^3+c)^(1/2)/c^2/(d^(1/3)*x+c^(1/3)
*(1+3^(1/2)))+1/56*3^(3/4)*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x
+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2
/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
/c^(5/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^
(1/3)*(1+3^(1/2)))^2)^(1/2)-3/112*3^(1/4)*d^(7/3)*(c^(1/3)+d^(1/3)*x)*Ellip
ticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1
/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)
/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*
(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 675, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {485, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)}dx = \frac{3^{3/4}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-\right.}{28\sqrt{2}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{112c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{9\sqrt{3}d^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1024c^{11/6}}+\frac{9d^{7/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1024c^{11/6}}$$

$$-\frac{9d^{7/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1024c^{11/6}}+\frac{3d^{7/3}\sqrt{c+dx^3}}{56c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$-\frac{3d^2\sqrt{c+dx^3}}{56c^2x}-\frac{\sqrt{c+dx^3}}{56x^7}-\frac{75d\sqrt{c+dx^3}}{1792cx^4}$$

```
[In] Int[(c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)),x]
```

```
[Out] -1/56*Sqrt[c + d*x^3]/x^7 - (75*d*Sqrt[c + d*x^3])/(1792*c*x^4) - (3*d^2*Sq
rt[c + d*x^3])/(56*c^2*x) + (3*d^(7/3)*Sqrt[c + d*x^3])/(56*c^2*((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)) - (9*Sqrt[3]*d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(
1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(1024*c^(11/6)) + (9*d^(7/3)*ArcTanh[(
c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(1024*c^(11/6)) - (9*d
^(7/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1024*c^(11/6)) - (3*3^(1/4)*S
qrt[2 - Sqrt[3]]*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1
/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSi
n[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(112*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (3^(3/4)*d^(7/3)*(c^(1/3
) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/
3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(28*Sqrt[2]*c^
(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{c+dx^3}}{56x^7} + \frac{\int \frac{75c^2d+\frac{123}{2}cd^2x^3}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx}{56c}$$

$$= -\frac{\sqrt{c+dx^3}}{56x^7} - \frac{75d\sqrt{c+dx^3}}{1792cx^4} - \frac{\int \frac{-768c^3d^2-\frac{375}{2}c^2d^3x^3}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{1792c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{56x^7} - \frac{75d\sqrt{c+dx^3}}{1792cx^4} - \frac{3d^2\sqrt{c+dx^3}}{56c^2x} + \frac{\int \frac{x(5340c^4d^3-384c^3d^4x^3)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{14336c^5}$$

$$= -\frac{\sqrt{c+dx^3}}{56x^7} - \frac{75d\sqrt{c+dx^3}}{1792cx^4} - \frac{3d^2\sqrt{c+dx^3}}{56c^2x} + \frac{\int \left(\frac{384c^3d^3x}{\sqrt{c+dx^3}} + \frac{2268c^4d^3x}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{14336c^5}$$

$$= -\frac{\sqrt{c+dx^3}}{56x^7} - \frac{75d\sqrt{c+dx^3}}{1792cx^4} - \frac{3d^2\sqrt{c+dx^3}}{56c^2x} + \frac{(3d^3)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{112c^2} + \frac{(81d^3)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{512c}$$

$$= -\frac{\sqrt{c+dx^3}}{56x^7} - \frac{75d\sqrt{c+dx^3}}{1792cx^4} - \frac{3d^2\sqrt{c+dx^3}}{56c^2x} - \frac{(27d^2)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{2048c^2}$$
$$+ \frac{(3d^{8/3})\int \frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{112c^2} + \frac{(27d^{8/3})\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{2048c^{5/3}}$$
$$- \frac{\left(3(1-\sqrt{3})\,d^{8/3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{112c^{5/3}} - \frac{(81d^{10/3})\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{2048c^{4/3}}$$
$$= -\frac{\sqrt{c+dx^3}}{56x^7} - \frac{75d\sqrt{c+dx^3}}{1792cx^4} - \frac{3d^2\sqrt{c+dx^3}}{56c^2x} + \frac{3d^{7/3}\sqrt{c+dx^3}}{56c^2\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$
$$- \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{(1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{112c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+ \frac{3^{3/4}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{(1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{28\sqrt{2}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+ \frac{(27d^{7/3})\,\text{Subst}\left(\int \frac{1}{9-cx^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{1024c^{4/3}}$$
$$- \frac{(27d^{10/3})\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{2048c^{4/3}}$$
$$+ \frac{(27d^{13/3})\,\text{Subst}\left(\int \frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{512c^{7/3}}$$

$$= -\frac{\sqrt{c+dx^3}}{56x^7} - \frac{75d\sqrt{c+dx^3}}{1792cx^4} - \frac{3d^2\sqrt{c+dx^3}}{56c^2x} + \frac{3d^{7/3}\sqrt{c+dx^3}}{56c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$- \frac{9\sqrt{3}d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1024c^{11/6}} + \frac{9d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1024c^{11/6}}$$

$$- \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{112c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{3^{3/4}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{28\sqrt{2}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$- \frac{\left(27d^{7/3}\right)\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{1024c^{4/3}}$$

$$= -\frac{\sqrt{c+dx^3}}{56x^7} - \frac{75d\sqrt{c+dx^3}}{1792cx^4} - \frac{3d^2\sqrt{c+dx^3}}{56c^2x} + \frac{3d^{7/3}\sqrt{c+dx^3}}{56c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{9\sqrt{3}d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1024c^{11/6}} + \frac{9d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1024c^{11/6}} - \frac{9d^{7/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1024c^{11/6}} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right.}{112c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{3^{3/4}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{28\sqrt{2}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.25

$$\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)}\,dx = \frac{6675cd^3x^9\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right) - 32\left(5c(32c^3+107c^2dx^3+171c\right.}{286720c^3x^7\sqrt{c+d}}$$

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)),x]
```

```
[Out] (6675*c*d^3*x^9*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c)
, (d*x^3)/(8*c)] - 32*(5*c*(32*c^3 + 107*c^2*d*x^3 + 171*c*d^2*x^6 + 96*d^3
*x^9) + 6*d^4*x^12*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)
/c), (d*x^3)/(8*c)]))/(286720*c^3*x^7*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.93 (sec) , antiderivative size = 895, normalized size of antiderivative = 1.33

method	result	size
risch	Expression too large to display	895
elliptic	Expression too large to display	903
default	Expression too large to display	2306

```
[In] int((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/1792*(d*x^3+c)^(1/2)*(96*d^2*x^6+75*c*d*x^3+32*c^2)/x^7/c^2+3/3584*d^3/c
^2*(-64/3*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3)
)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-7*I/d^3*2^(1/2)*sum
(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d
^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1
/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)
^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(
1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3
)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3
)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(
2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(
1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^
3*d-8*c)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.51 (sec) , antiderivative size = 2442, normalized size of antiderivative = 3.62

$$\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)}\,dx = \text{Too large to display}$$

[In] integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c),x, algorithm="fricas")

```
[Out] -1/28672*(1536*d^(5/2)*x^7*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0
, -4*c/d, x)) - 42*(d^14/c^11)^(1/6)*c^2*x^7*log(6561*(d^14*x^9 + 318*c*d^1
3*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 + 18*(5*c^8*d^4*x^7 + 64*c^9*d^3*x
^4 + 32*c^10*d^2*x)*(d^14/c^11)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5
+ 32*c^11*x^2)*(d^14/c^11)^(5/6) + (7*c^6*d^6*x^6 + 152*c^7*d^5*x^3 + 64*c^
8*d^4)*sqrt(d^14/c^11) + (c^2*d^11*x^7 + 80*c^3*d^10*x^4 + 160*c^4*d^9*x)*(
d^14/c^11)^(1/6)) + 18*(c^4*d^9*x^8 + 38*c^5*d^8*x^5 + 64*c^6*d^7*x^2)*(d^1
4/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 42*(d^
14/c^11)^(1/6)*c^2*x^7*log(6561*(d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*
x^3 + 640*c^3*d^11 + 18*(5*c^8*d^4*x^7 + 64*c^9*d^3*x^4 + 32*c^10*d^2*x)*(d
^14/c^11)^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2)*(d^14/c
^11)^(5/6) + (7*c^6*d^6*x^6 + 152*c^7*d^5*x^3 + 64*c^8*d^4)*sqrt(d^14/c^11)
 + (c^2*d^11*x^7 + 80*c^3*d^10*x^4 + 160*c^4*d^9*x)*(d^14/c^11)^(1/6)) + 18
*(c^4*d^9*x^8 + 38*c^5*d^8*x^5 + 64*c^6*d^7*x^2)*(d^14/c^11)^(1/3))/(d^3*x^
9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 21*(sqrt(-3)*c^2*x^7 + c^2*x
^7)*(d^14/c^11)^(1/6)*log(6561*(d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x
^3 + 640*c^3*d^11 - 9*(5*c^8*d^4*x^7 + 64*c^9*d^3*x^4 + 32*c^10*d^2*x + sqr
t(-3)*(5*c^8*d^4*x^7 + 64*c^9*d^3*x^4 + 32*c^10*d^2*x))*(d^14/c^11)^(2/3) +
 3*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2 - sqrt(-3)*(5*c^10*d*x^5
+ 32*c^11*x^2))*(d^14/c^11)^(5/6) - 2*(7*c^6*d^6*x^6 + 152*c^7*d^5*x^3 + 64
*c^8*d^4)*sqrt(d^14/c^11) + (c^2*d^11*x^7 + 80*c^3*d^10*x^4 + 160*c^4*d^9*x
 + sqrt(-3)*(c^2*d^11*x^7 + 80*c^3*d^10*x^4 + 160*c^4*d^9*x))*(d^14/c^11)^(
1/6)) - 9*(c^4*d^9*x^8 + 38*c^5*d^8*x^5 + 64*c^6*d^7*x^2 - sqrt(-3)*(c^4*d^
9*x^8 + 38*c^5*d^8*x^5 + 64*c^6*d^7*x^2))*(d^14/c^11)^(1/3))/(d^3*x^9 - 24*
c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 21*(sqrt(-3)*c^2*x^7 + c^2*x^7)*(d^
14/c^11)^(1/6)*log(6561*(d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 64
0*c^3*d^11 - 9*(5*c^8*d^4*x^7 + 64*c^9*d^3*x^4 + 32*c^10*d^2*x + sqrt(-3)*(
5*c^8*d^4*x^7 + 64*c^9*d^3*x^4 + 32*c^10*d^2*x))*(d^14/c^11)^(2/3) - 3*sqrt
(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2 - sqrt(-3)*(5*c^10*d*x^5 + 32*c^
11*x^2))*(d^14/c^11)^(5/6) - 2*(7*c^6*d^6*x^6 + 152*c^7*d^5*x^3 + 64*c^8*d^
4)*sqrt(d^14/c^11) + (c^2*d^11*x^7 + 80*c^3*d^10*x^4 + 160*c^4*d^9*x + sqrt
(-3)*(c^2*d^11*x^7 + 80*c^3*d^10*x^4 + 160*c^4*d^9*x))*(d^14/c^11)^(1/6)) -
 9*(c^4*d^9*x^8 + 38*c^5*d^8*x^5 + 64*c^6*d^7*x^2 - sqrt(-3)*(c^4*d^9*x^8 +
 38*c^5*d^8*x^5 + 64*c^6*d^7*x^2))*(d^14/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x
^6 + 192*c^2*d*x^3 - 512*c^3)) + 21*(sqrt(-3)*c^2*x^7 - c^2*x^7)*(d^14/c^11
```

```
)^(1/6)*log(6561*(d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d
^11 - 9*(5*c^8*d^4*x^7 + 64*c^9*d^3*x^4 + 32*c^10*d^2*x - sqrt(-3)*(5*c^8*d
^4*x^7 + 64*c^9*d^3*x^4 + 32*c^10*d^2*x))*(d^14/c^11)^(2/3) + 3*sqrt(d*x^3
+ c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2 + sqrt(-3)*(5*c^10*d*x^5 + 32*c^11*x^2)
)*(d^14/c^11)^(5/6) - 2*(7*c^6*d^6*x^6 + 152*c^7*d^5*x^3 + 64*c^8*d^4)*sqrt
(d^14/c^11) + (c^2*d^11*x^7 + 80*c^3*d^10*x^4 + 160*c^4*d^9*x - sqrt(-3)*(c
^2*d^11*x^7 + 80*c^3*d^10*x^4 + 160*c^4*d^9*x))*(d^14/c^11)^(1/6)) - 9*(c^4
*d^9*x^8 + 38*c^5*d^8*x^5 + 64*c^6*d^7*x^2 + sqrt(-3)*(c^4*d^9*x^8 + 38*c^5
*d^8*x^5 + 64*c^6*d^7*x^2))*(d^14/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 19
2*c^2*d*x^3 - 512*c^3)) - 21*(sqrt(-3)*c^2*x^7 - c^2*x^7)*(d^14/c^11)^(1/6)
*log(6561*(d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 - 9
*(5*c^8*d^4*x^7 + 64*c^9*d^3*x^4 + 32*c^10*d^2*x - sqrt(-3)*(5*c^8*d^4*x^7
+ 64*c^9*d^3*x^4 + 32*c^10*d^2*x))*(d^14/c^11)^(2/3) - 3*sqrt(d*x^3 + c)*(6
*(5*c^10*d*x^5 + 32*c^11*x^2 + sqrt(-3)*(5*c^10*d*x^5 + 32*c^11*x^2))*(d^14
/c^11)^(5/6) - 2*(7*c^6*d^6*x^6 + 152*c^7*d^5*x^3 + 64*c^8*d^4)*sqrt(d^14/c
^11) + (c^2*d^11*x^7 + 80*c^3*d^10*x^4 + 160*c^4*d^9*x - sqrt(-3)*(c^2*d^11
*x^7 + 80*c^3*d^10*x^4 + 160*c^4*d^9*x))*(d^14/c^11)^(1/6)) - 9*(c^4*d^9*x^
8 + 38*c^5*d^8*x^5 + 64*c^6*d^7*x^2 + sqrt(-3)*(c^4*d^9*x^8 + 38*c^5*d^8*x^
5 + 64*c^6*d^7*x^2))*(d^14/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d
*x^3 - 512*c^3)) + 16*(96*d^2*x^6 + 75*c*d*x^3 + 32*c^2)*sqrt(d*x^3 + c))/(
c^2*x^7)
```

Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^8\,(8c-dx^3)}\,dx = -\int \frac{c\sqrt{c+dx^3}}{-8cx^8+dx^{11}}\,dx - \int \frac{dx^3\sqrt{c+dx^3}}{-8cx^8+dx^{11}}\,dx$$

```
[In] integrate((d*x**3+c)**(3/2)/x**8/(-d*x**3+8*c),x)
[Out] -Integral(c*sqrt(c + d*x**3)/(-8*c*x**8 + d*x**11), x) - Integral(d*x**3*sq
rt(c + d*x**3)/(-8*c*x**8 + d*x**11), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^8\,(8c-dx^3)}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)x^8}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c),x, algorithm="maxima")
[Out] -integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^8), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)}\,dx = \int -\frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)x^8}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c),x, algorithm="giac")
[Out] integrate(-(d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)}\,dx = \int \frac{(d\,x^3+c)^{3/2}}{x^8\,(8\,c-d\,x^3)}\,dx$$

```
[In] int((c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)),x)
[Out] int((c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)), x)
```

3.308 $\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 90

$$\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{38c^2\sqrt{c+dx^3}}{d^4} - \frac{4c(c+dx^3)^{3/2}}{3d^4} - \frac{2(c+dx^3)^{5/2}}{15d^4} + \frac{1024c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4}$$

```
[Out] -4/3*c*(d*x^3+c)^(3/2)/d^4-2/15*(d*x^3+c)^(5/2)/d^4+1024/9*c^(5/2)*arctanh(
1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^4-38*c^2*(d*x^3+c)^(1/2)/d^4
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {457, 90, 65, 212}

$$\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{1024c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4} - \frac{38c^2\sqrt{c+dx^3}}{d^4} - \frac{4c(c+dx^3)^{3/2}}{3d^4} - \frac{2(c+dx^3)^{5/2}}{15d^4}$$

```
[In] Int[x^11/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-38*c^2*Sqrt[c + d*x^3])/d^4 - (4*c*(c + d*x^3)^(3/2))/(3*d^4) - (2*(c + d
*x^3)^(5/2))/(15*d^4) + (1024*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])
/(9*d^4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{57c^2}{d^3\sqrt{c+dx}}+\frac{512c^3}{d^3(8c-dx)\sqrt{c+dx}}-\frac{6c\sqrt{c+dx}}{d^3}-\frac{(c+dx)^{3/2}}{d^3}\right)\,dx, x, x^3\right)\\
&= -\frac{38c^2\sqrt{c+dx^3}}{d^4}-\frac{4c(c+dx^3)^{3/2}}{3d^4}-\frac{2(c+dx^3)^{5/2}}{15d^4}+\frac{(512c^3)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3d^3}\\
&= -\frac{38c^2\sqrt{c+dx^3}}{d^4}-\frac{4c(c+dx^3)^{3/2}}{3d^4}-\frac{2(c+dx^3)^{5/2}}{15d^4}+\frac{(1024c^3)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d^4}\\
&= -\frac{38c^2\sqrt{c+dx^3}}{d^4}-\frac{4c(c+dx^3)^{3/2}}{3d^4}-\frac{2(c+dx^3)^{5/2}}{15d^4}+\frac{1024c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.77

$$\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{-6\sqrt{c+dx^3}(296c^2+12cdx^3+d^2x^6)+5120c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{45d^4}$$

[In] Integrate[x^11/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]

[Out] (-6*Sqrt[c + d*x^3]*(296*c^2 + 12*c*d*x^3 + d^2*x^6) + 5120*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(45*d^4)

Maple [A] (verified)

Time = 4.50 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$\frac{5120c^{\frac{5}{2}}\,\text{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)-6\sqrt{d\,x^{3}+c}\,(d^{2}x^{6}+12cd\,x^{3}+296c^{2})}{45d^{4}}$
risch	$-\frac{2(d^{2}x^{6}+12cd\,x^{3}+296c^{2})\sqrt{d\,x^{3}+c}}{15d^{4}}+\frac{1024c^{\frac{5}{2}}\,\text{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{9d^{4}}$
default	$-\frac{\frac{2x^{6}\sqrt{d\,x^{3}+c}}{15d}-\frac{8c\,x^{3}\sqrt{d\,x^{3}+c}}{45d^{2}}+\frac{16c^{2}\sqrt{d\,x^{3}+c}}{45d^{3}}}{d}-\frac{8c\left(\frac{2x^{3}\sqrt{d\,x^{3}+c}}{9d}-\frac{4c\sqrt{d\,x^{3}+c}}{9d^{2}}\right)}{d^{2}}-\frac{128c^{2}\sqrt{d\,x^{3}+c}}{3d^{4}}+\frac{1024c^{\frac{5}{2}}\,\text{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{9d^{4}}$
elliptic	$-\frac{2x^{6}\sqrt{d\,x^{3}+c}}{15d^{2}}-\frac{8c\,x^{3}\sqrt{d\,x^{3}+c}}{5d^{3}}-\frac{592c^{2}\sqrt{d\,x^{3}+c}}{15d^{4}}-\frac{512ic^{2}\sqrt{2}\left(\sum_{\_\alpha=\text{RootOf}(d\_Z^{3}-8c)}\frac{(-c\,d^{2})^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}(-c\ldots}{\ldots}\right)}{(-c\ldots}}\ldots}{\ldots}\right)}{\ldots}$

[In] int(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/45*(5120*c^(5/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))-6*(d*x^3+c)^(1/2)*(d^2*x^6+12*c*d*x^3+296*c^2))/d^4

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.62

$$\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \left[\frac{2\left(1280\,c^{\frac{5}{2}}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-3\,(d^2x^6+12\,cdx^3+296\,c^2)\sqrt{dx^3+c}\right)}{45\,d^4}, -\frac{2\left(2560\sqrt{-c}c^2\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+3\,(d^2x^6+12\,cdx^3+296\,c^2)\sqrt{dx^3+c}\right)}{45\,d^4}\right]$$

```
[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/45*(1280*c^(5/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 -
 8*c)) - 3*(d^2*x^6 + 12*c*d*x^3 + 296*c^2)*sqrt(d*x^3 + c))/d^4, -2/45*(25
60*sqrt(-c)*c^2*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(d^2*x^6 + 12*c*
d*x^3 + 296*c^2)*sqrt(d*x^3 + c))/d^4]
```

Sympy [A] (verification not implemented)

Time = 16.60 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \begin{cases} \frac{2\left(-\frac{512c^3\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{9\sqrt{-c}}-19c^2\sqrt{c+dx^3}-\frac{2c\left(c+dx^3\right)^{\frac{3}{2}}}{3}-\frac{\left(c+dx^3\right)^{\frac{5}{2}}}{15}\right)}{d^4} & \text{for } d\neq 0 \\ \frac{x^{12}}{96c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**11/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Piecewise((2*(-512*c**3*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(9*sqrt(-c)) -
19*c**2*sqrt(c + d*x**3) - 2*c*(c + d*x**3)**(3/2)/3 - (c + d*x**3)**(5/2)/
15)/d**4, Ne(d, 0)), (x**12/(96*c**(3/2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{2\left(1280\,c^{\frac{5}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+3\,(dx^3+c)^{\frac{5}{2}}+30\,(dx^3+c)^{\frac{3}{2}}c+855\sqrt{dx^3+c}c^2\right)}{45\,d^4}$$

```
[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/45*(1280*c^(5/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*
sqrt(c))) + 3*(d*x^3 + c)^(5/2) + 30*(d*x^3 + c)^(3/2)*c + 855*sqrt(d*x^3 +
 c)*c^2)/d^4
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{1024\,c^3\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}d^4} - \frac{2\left((dx^3+c)^{\frac{5}{2}}d^{16}+10\,(dx^3+c)^{\frac{3}{2}}cd^{16}+285\sqrt{dx^3+c}c^2d^{16}\right)}{15\,d^{20}}$$

```
[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1024/9*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 2/15*((d*
x^3 + c)^(5/2)*d^16 + 10*(d*x^3 + c)^(3/2)*c*d^16 + 285*sqrt(d*x^3 + c)*c^2
*d^16)/d^20
```

Mupad [B] (verification not implemented)

Time = 7.47 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.09

$$\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{512\,c^{5/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^4} - \frac{592\,c^2\,\sqrt{d\,x^3+c}}{15\,d^4} - \frac{2\,x^6\,\sqrt{d\,x^3+c}}{15\,d^2} - \frac{8\,c\,x^3\,\sqrt{d\,x^3+c}}{5\,d^3}$$

```
[In] int(x^11/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] (512*c^(5/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)
))/(9*d^4) - (592*c^2*(c + d*x^3)^(1/2))/(15*d^4) - (2*x^6*(c + d*x^3)^(1/2
))/(15*d^2) - (8*c*x^3*(c + d*x^3)^(1/2))/(5*d^3)
```

3.309 $\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 71

$$\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}\,dx=-\frac{14c\sqrt{c+dx^3}}{3d^3}-\frac{2(c+dx^3)^{3/2}}{9d^3}+\frac{128c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}$$

```
[Out] -2/9*(d*x^3+c)^(3/2)/d^3+128/9*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))
/d^3-14/3*c*(d*x^3+c)^(1/2)/d^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {457, 90, 65, 212}

$$\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}\,dx=\frac{128c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}-\frac{14c\sqrt{c+dx^3}}{3d^3}-\frac{2(c+dx^3)^{3/2}}{9d^3}$$

```
[In] Int[x^8/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-14*c*Sqrt[c + d*x^3])/(3*d^3) - (2*(c + d*x^3)^(3/2))/(9*d^3) + (128*c^(3
/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{7c}{d^2\sqrt{c+dx}}+\frac{64c^2}{d^2(8c-dx)\sqrt{c+dx}}-\frac{\sqrt{c+dx}}{d^2}\right)dx, x, x^3\right)\\
&= -\frac{14c\sqrt{c+dx^3}}{3d^3}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^3}+\frac{\left(64c^2\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3d^2}\\
&= -\frac{14c\sqrt{c+dx^3}}{3d^3}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^3}+\frac{\left(128c^2\right)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d^3}\\
&= -\frac{14c\sqrt{c+dx^3}}{3d^3}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^3}+\frac{128c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.82

$$\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{-2\sqrt{c+dx^3}(22c+dx^3)+128c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}$$

```
[In] Integrate[x^8/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(22*c + d*x^3) + 128*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3
*Sqrt[c])])/(9*d^3)
```

Maple [A] (verified)

Time = 4.45 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{\frac{128c^{\frac{3}{2}}\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{9}-\frac{2\left(d\,x^3+22c\right)\sqrt{d\,x^3+c}}{9}}{d^3}$
risch	$-\frac{2(d\,x^3+22c)\sqrt{d\,x^3+c}}{9d^3}+\frac{128c^{\frac{3}{2}}\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{9d^3}$
default	$-\frac{\frac{2x^3\sqrt{d\,x^3+c}}{9d}-\frac{4c\sqrt{d\,x^3+c}}{9d^2}}{d}-\frac{16c\sqrt{d\,x^3+c}}{3d^3}+\frac{128c^{\frac{3}{2}}\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{9d^3}$
elliptic	$-\frac{2x^3\sqrt{d\,x^3+c}}{9d^2}-\frac{44c\sqrt{d\,x^3+c}}{9d^3}-\frac{64ic\sqrt{2}\left(\sum_{\_\alpha=\text{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\cdots}{-3\cdots}}\cdots}{\cdots}\right)}{\cdots}$

```
[In] int(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*(64*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))-(d*x^3+22*c)*(d*x^3+c)
^(1/2))/d^3
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.70

$$\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \left[\frac{2\left(32\,c^{\frac{3}{2}}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-(dx^3+22\,c)\sqrt{dx^3+c}\right)}{9\,d^3}, -\frac{2\left(64\sqrt{-c}c\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+(dx^3+22\,c)\sqrt{dx^3+c}\right)}{9\,d^3}\right]$$

```
[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
[Out] [2/9*(32*c^(3/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*
c)) - (d*x^3 + 22*c)*sqrt(d*x^3 + c))/d^3, -2/9*(64*sqrt(-c)*c*arctan(1/3*s
qrt(d*x^3 + c)*sqrt(-c)/c) + (d*x^3 + 22*c)*sqrt(d*x^3 + c))/d^3]
```

Sympy [A] (verification not implemented)

Time = 8.64 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

$$\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \begin{cases} \frac{2\left(-\frac{64c^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{9\sqrt{-c}}-\frac{7c\sqrt{c+dx^3}}{3}-\frac{\left(c+dx^3\right)^{\frac{3}{2}}}{9}\right)}{d^3} & \text{for } d\neq 0 \\ \frac{x^9}{72c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**8/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
[Out] Piecewise((2*(-64*c**2*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(9*sqrt(-c)) - 7
*c*sqrt(c + d*x**3)/3 - (c + d*x**3)**(3/2)/9)/d**3, Ne(d, 0)), (x**9/(72*c
**(3/2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{2\left(32\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+(dx^3+c)^{\frac{3}{2}}+21\sqrt{dx^3+c}c\right)}{9\,d^3}$$

```
[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] -2/9*(32*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqr
t(c))) + (d*x^3 + c)^(3/2) + 21*sqrt(d*x^3 + c)*c)/d^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{128\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}d^3} - \frac{2\left((dx^3+c)^{\frac{3}{2}}d^6+21\sqrt{dx^3+c}cd^6\right)}{9\,d^9}$$

```
[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -128/9*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/9*((d*x^
3 + c)^(3/2)*d^6 + 21*sqrt(d*x^3 + c)*c*d^6)/d^9
```

Mupad [B] (verification not implemented)

Time = 7.44 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.10

$$\int \frac{x^8}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{64\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^3} - \frac{44\,c\,\sqrt{d\,x^3+c}}{9\,d^3} - \frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d^2}$$

```
[In] int(x^8/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] (64*c^(3/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))
)/(9*d^3) - (44*c*(c + d*x^3)^(1/2))/(9*d^3) - (2*x^3*(c + d*x^3)^(1/2))/(9
*d^2)
```

3.310 $\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 52

$$\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{2\sqrt{c+dx^3}}{3d^2}+\frac{16\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}$$

```
[Out] 16/9*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/d^2-2/3*(d*x^3+c)^(1/2)/d
^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {457, 81, 65, 212}

$$\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{16\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}-\frac{2\sqrt{c+dx^3}}{3d^2}$$

```
[In] Int[x^5/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*Sqrt[c + d*x^3])/(3*d^2) + (16*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[
c])])/(9*d^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right) \\
&= -\frac{2\sqrt{c+dx^3}}{3d^2} + \frac{(8c)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3d} \\
&= -\frac{2\sqrt{c+dx^3}}{3d^2} + \frac{(16c)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d^2} \\
&= -\frac{2\sqrt{c+dx^3}}{3d^2} + \frac{16\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{2\left(3\sqrt{c+dx^3}-8\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{9d^2}$$

```
[In] Integrate[x^5/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
[Out] (-2*(3*Sqrt[c + d*x^3] - 8*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(
9*d^2)
```

Maple [A] (verified)

Time = 4.54 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$\frac{-\frac{2\sqrt{d\,x^{3}+c}}{3}+\frac{16\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\sqrt{c}}{9}}{d^{2}}$
default	$\frac{16\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\sqrt{c}}{9d^{2}}-\frac{2\sqrt{d\,x^{3}+c}}{3d^{2}}$
risch	$\frac{16\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\sqrt{c}}{9d^{2}}-\frac{2\sqrt{d\,x^{3}+c}}{3d^{2}}$
elliptic	$-\frac{2\sqrt{d\,x^{3}+c}}{3d^{2}}-\frac{8i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\,\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d\cdots\right.}}}{\cdots}\right.}{\cdots}$

```
[In] int(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/9*(8*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)-3*(d*x^3+c)^(1/2))/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.98

$$\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \left[\frac{2\left(4\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-3\sqrt{dx^3+c}\right)}{9\,d^2}, -\frac{2\left(8\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+3\sqrt{dx^3+c}\right)}{9\,d^2}\right]$$

```
[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/9*(4*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c
)) - 3*sqrt(d*x^3 + c))/d^2, -2/9*(8*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sq
rt(-c)/c) + 3*sqrt(d*x^3 + c))/d^2]
```

Sympy [A] (verification not implemented)

Time = 4.87 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15

$$\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \begin{cases} \frac{2\left(-\frac{8c\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{9\sqrt{-c}}-\frac{\sqrt{c+dx^3}}{3}\right)}{d^2} & \text{for } d \neq 0 \\ \frac{x^6}{48c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**5/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Piecewise((2*(-8*c*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(9*sqrt(-c)) - sqrt(
c + d*x**3)/3)/d**2, Ne(d, 0)), (x**6/(48*c**(3/2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{2\left(4\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+3\sqrt{dx^3+c}\right)}{9\,d^2}$$

```
[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/9*(4*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt
(c))) + 3*sqrt(d*x^3 + c))/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{2\left(\frac{8\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d}+\frac{3\sqrt{dx^3+c}}{d}\right)}{9\,d}$$

```
[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] -2/9*(8*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) + 3*sqrt(d*x^3
+ c)/d)/d
```

Mupad [B] (verification not implemented)

Time = 7.49 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15

$$\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{8\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3+6\sqrt{c}\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^2}-\frac{2\sqrt{d\,x^3+c}}{3\,d^2}$$

```
[In] int(x^5/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
[Out] (8*c^(1/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))
/(9*d^2) - (2*(c + d*x^3)^(1/2))/(3*d^2)
```

3.311 $\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 33

$$\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}d}$$

```
[Out] 2/9*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d/c^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {455, 65, 212}

$$\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}d}$$

```
[In] Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*Sqrt[c]*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)\\ &= \frac{2\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}d}$$

```
[In] Integrate[x^2/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
[Out] (2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*Sqrt[c]*d)
```

Maple [A] (verified)

Time = 4.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

method	result
default	$\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{9d\sqrt{c}}$
pseudoelliptic	$\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{9d\sqrt{c}}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\frac{i\sqrt{}}{}\right.}{}}}{}\right)}{}$

[In] int(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/9*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d/c^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.36

$$\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \left[\frac{\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)}{9\sqrt{c}d}, -\frac{2\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)}{9\,cd}\right]$$

[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] [1/9*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c))/(sqrt(c)*d), -2/9*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c)/(c*d)]

Sympy [A] (verification not implemented)

Time = 3.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \begin{cases} -\frac{2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{9d\sqrt{-c}} & \text{for } d\neq 0 \\ \frac{x^3}{24c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
[Out] Piecewise((-2*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(9*d*sqrt(-c)), Ne(d, 0))
, (x**3/(24*c**(3/2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{9\sqrt{c}d}$$

```
[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] -1/9*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/(sqrt
(c)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{2\,\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}d}$$

```
[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] -2/9*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d)
```

Mupad [B] (verification not implemented)

Time = 7.50 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.36

$$\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx=\frac{\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,\sqrt{c}\,d}$$

```
[In] int(x^2/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(9*c^(1/2)*
d)
```

3.312 $\int \frac{1}{x(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 58

$$\int \frac{1}{x(8c-dx^3)\sqrt{c+dx^3}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{36c^{3/2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{3/2}}$$

```
[Out] 1/36*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-1/12*arctanh((d*x^3+c)^(1
/2)/c^(1/2))/c^(3/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {457, 88, 65, 214, 212}

$$\int \frac{1}{x(8c-dx^3)\sqrt{c+dx^3}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{36c^{3/2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{3/2}}$$

```
[In] Int[1/(x*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(36*c^(3/2)) - ArcTanh[Sqrt[c + d*x^3]
/Sqrt[c]]/(12*c^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{24c} + \frac{d\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{24c}\\
&= \frac{\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{12c} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{12cd}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{36c^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-3\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{36c^{3/2}}$$

```
[In] Integrate[1/(x*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
[Out] (ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 3*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])
/(36*c^(3/2))
```

Maple [A] (verified)

Time = 4.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.66

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)-3\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{36c^{\frac{3}{2}}}$	38
default	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{36c^{\frac{3}{2}}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{12c^{\frac{3}{2}}}$	41
elliptic	Expression too large to display	1508

```
[In] int(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/36*(arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))-3*arctanh((d*x^3+c)^(1/2)/c^(1/2
)))/c^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.40

$$\int \frac{1}{x\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx$$
$$=\left[\frac{\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+3\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)}{72\,c^2},\frac{3\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-\sqrt{-c}\arctan\left(\right.}{36\,c^2}\right.$$

```
[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
[Out] [1/72*(sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)
) + 3*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/c^2, 1/36
```

```
*(3*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - sqrt(-c)*arctan(1/3*sqrt(
d*x^3 + c)*sqrt(-c)/c))/c^2]
```

Sympy [A] (verification not implemented)

Time = 3.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.29

$$\int \frac{1}{x\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = \begin{cases} \frac{2\left(-\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{72c\sqrt{-c}}+\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{24c\sqrt{-c}}\right)}{d} & \text{for } d\neq 0 \\ \frac{\log\left(x^3\right)}{24c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/x/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
[Out] Piecewise((2*(-d*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(72*c*sqrt(-c)) + d*at
an(sqrt(c + d*x**3)/sqrt(-c))/(24*c*sqrt(-c)))/d, Ne(d, 0)), (log(x**3)/(24
*c**(3/2)), True))
```

Maxima [F]

$$\int \frac{1}{x\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = \int -\frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)x}\,dx$$

```
[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{12\sqrt{-c}c} - \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{36\sqrt{-c}c}$$

```
[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] 1/12*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/36*arctan(1/3*sqrt(d
*x^3 + c)/sqrt(-c))/(sqrt(-c)*c)
```

Mupad [B] (verification not implemented)

Time = 7.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int \frac{1}{x\,(8c-dx^3)\,\sqrt{c+dx^3}}\,dx=-\frac{3\,\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{\sqrt{c^3}}\right)-\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^3}}\right)}{36\,\sqrt{c^3}}$$

```
[In] int(1/(x*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
[Out] -(3*atanh((c*(c + d*x^3)^(1/2))/(c^3)^(1/2)) - atanh((c*(c + d*x^3)^(1/2))/
(3*(c^3)^(1/2))))/(36*(c^3)^(1/2))
```

3.313 $\int \frac{1}{x^4(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 81

$$\int \frac{1}{x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=-\frac{\sqrt{c+dx^3}}{24c^2x^3}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{5/2}}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{32c^{5/2}}$$

```
[Out] 1/288*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)+1/32*d*arctanh((d*x^3+
c)^(1/2)/c^(1/2))/c^(5/2)-1/24*(d*x^3+c)^(1/2)/c^2/x^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {457, 105, 162, 65, 214, 212}

$$\int \frac{1}{x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=\frac{d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{5/2}}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{32c^{5/2}}-\frac{\sqrt{c+dx^3}}{24c^2x^3}$$

```
[In] Int[1/(x^4*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/24*Sqrt[c + d*x^3]/(c^2*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/
(288*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(32*c^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{24c^2x^3} - \frac{\text{Subst}\left(\int \frac{3cd-\frac{d^2x}{2}}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{24c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{24c^2x^3} - \frac{d\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{64c^2} + \frac{d^2\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{192c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{24c^2x^3} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{32c^2} + \frac{d\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{96c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{24c^2x^3} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{5/2}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{32c^{5/2}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{c+dx^3}}{24c^2x^3} + \frac{d\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{5/2}} + \frac{d\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{32c^{5/2}}$$

```
[In] Integrate[1/(x^4*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/24*Sqrt[c + d*x^3]/(c^2*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/
(288*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(32*c^(5/2))
```

Maple [A] (verified)

Time = 4.48 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.79

method	result	size
pseudoelliptic	$\frac{9\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)d\,x^3+\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)d\,x^3-12\sqrt{d\,x^3+c}\,\sqrt{c}}{288c^{\frac{5}{2}}\,x^3}$	64
risch	$-\frac{\sqrt{d\,x^3+c}}{24c^2x^3}-\frac{d\left(-\frac{\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{2\sqrt{c}}-\frac{\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{18\sqrt{c}}\right)}{16c^2}$	65
default	$\frac{-\frac{\sqrt{d\,x^3+c}}{3c\,x^3}+\frac{d\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}}{8c}-\frac{d\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{96c^{\frac{5}{2}}}+\frac{d\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{288c^{\frac{5}{2}}}$	86
elliptic	Expression too large to display	1523

```
[In] int(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/288/c^(5/2)*(9*arctanh((d*x^3+c)^(1/2)/c^(1/2))*d*x^3+arctanh(1/3*(d*x^3+
c)^(1/2)/c^(1/2))*d*x^3-12*(d*x^3+c)^(1/2)*c^(1/2))/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.27

$$\int \frac{1}{x^4 (8c - dx^3) \sqrt{c + dx^3}} dx = \left[\frac{\sqrt{c}dx^3 \log \left(\frac{dx^3 + 6\sqrt{dx^3 + c}\sqrt{c} + 10c}{dx^3 - 8c} \right) + 9\sqrt{c}dx^3 \log \left(\frac{dx^3 + 2\sqrt{dx^3 + c}\sqrt{c} + 2c}{x^3} \right) - 24\sqrt{dx^3 + c}c}{576c^3x^3}, -\frac{9\sqrt{-c}dx^3 \arctan \left(\frac{\sqrt{dx^3 + c}\sqrt{-c}}{c} \right) + \sqrt{-c}dx^3 \arctan \left(\frac{\sqrt{dx^3 + c}\sqrt{-c}}{3c} \right) + 12\sqrt{dx^3 + c}c}{288c^3x^3} \right]$$

```
[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/576*(sqrt(c)*d*x^3*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3
 - 8*c)) + 9*sqrt(c)*d*x^3*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^
3) - 24*sqrt(d*x^3 + c)*c)/(c^3*x^3), -1/288*(9*sqrt(-c)*d*x^3*arctan(sqrt(
d*x^3 + c)*sqrt(-c)/c) + sqrt(-c)*d*x^3*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)
/c) + 12*sqrt(d*x^3 + c)*c)/(c^3*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4 (8c - dx^3) \sqrt{c + dx^3}} dx = -\int \frac{1}{-8cx^4\sqrt{c + dx^3} + dx^7\sqrt{c + dx^3}} dx$$

```
[In] integrate(1/x**4/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(1/(-8*c*x**4*sqrt(c + d*x**3) + d*x**7*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (8c - dx^3) \sqrt{c + dx^3}} dx = \int -\frac{1}{\sqrt{dx^3 + c}(dx^3 - 8c)x^4} dx$$

```
[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^4(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{32\sqrt{-c}c^2} - \frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{288\sqrt{-c}c^2} - \frac{\sqrt{dx^3+c}}{24\,c^2x^3}$$

```
[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/32*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/288*d*arctan(1/
3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/24*sqrt(d*x^3 + c)/(c^2*x^3)
```

Mupad [B] (verification not implemented)

Time = 7.59 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^4(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{d\,\mathrm{atanh}\left(\frac{c^2\sqrt{d\,x^3+c}}{\sqrt{c^5}}\right)}{32\sqrt{c^5}} + \frac{d\,\mathrm{atanh}\left(\frac{c^2\sqrt{d\,x^3+c}}{3\sqrt{c^5}}\right)}{288\sqrt{c^5}} - \frac{\sqrt{d\,x^3+c}}{24\,c^2\,x^3}$$

```
[In] int(1/(x^4*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] (d*atanh((c^2*(c + d*x^3)^(1/2))/(c^5)^(1/2)))/(32*(c^5)^(1/2)) + (d*atanh(
(c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2))))/(288*(c^5)^(1/2)) - (c + d*x^3)^(
1/2)/(24*c^2*x^3)
```

3.314 $\int \frac{1}{x^7(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 107

$$\int \frac{1}{x^7(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{c+dx^3}}{48c^2x^6}+\frac{5d\sqrt{c+dx^3}}{192c^3x^3} + \frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2304c^{7/2}}-\frac{7d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{7/2}}$$

```
[Out] 1/2304*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)-7/256*d^2*arctanh((
d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)-1/48*(d*x^3+c)^(1/2)/c^2/x^6+5/192*d*(d*x^3
+c)^(1/2)/c^3/x^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {457, 105, 156, 162, 65, 214, 212}

$$\int \frac{1}{x^7(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2304c^{7/2}}-\frac{7d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{7/2}} + \frac{5d\sqrt{c+dx^3}}{192c^3x^3}-\frac{\sqrt{c+dx^3}}{48c^2x^6}$$

```
[In] Int[1/(x^7*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/48*Sqrt[c + d*x^3]/(c^2*x^6) + (5*d*Sqrt[c + d*x^3])/(192*c^3*x^3) + (d^
2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2304*c^(7/2)) - (7*d^2*ArcTanh[Sqr
t[c + d*x^3]/Sqrt[c]])/(256*c^(7/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^3(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x^6} - \frac{\text{Subst}\left(\int \frac{10cd-\frac{3d^2x}{2}}{x^2(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{48c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x^6} + \frac{5d\sqrt{c+dx^3}}{192c^3x^3} + \frac{\text{Subst}\left(\int \frac{42c^2d^2-5cd^3x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{384c^4}$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x^6} + \frac{5d\sqrt{c+dx^3}}{192c^3x^3} + \frac{(7d^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{512c^3} + \frac{d^3\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{1536c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x^6} + \frac{5d\sqrt{c+dx^3}}{192c^3x^3} + \frac{(7d)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{256c^3} + \frac{d^2\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{768c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x^6} + \frac{5d\sqrt{c+dx^3}}{192c^3x^3} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2304c^{7/2}} - \frac{7d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{7/2}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^7(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{\sqrt{c+dx^3}(-4c+5dx^3)}{192c^3x^6} + \frac{d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2304c^{7/2}} - \frac{7d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{7/2}}$$

```
[In] Integrate[1/(x^7*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
[Out] (Sqrt[c + d*x^3]*(-4*c + 5*d*x^3))/(192*c^3*x^6) + (d^2*ArcTanh[Sqrt[c + d*
x^3]/(3*Sqrt[c])])/(2304*c^(7/2)) - (7*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]
)/(256*c^(7/2))
```

Maple [A] (verified)

Time = 4.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{\sqrt{d\,x^{3}+c}\,\left(-5d\,x^{3}+4c\right)}{192c^{3}x^{6}}+\frac{d^{2}\left(-\frac{7\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{2\sqrt{c}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{18\sqrt{c}}\right)}{128c^{3}}$
pseudoelliptic	$\frac{-63\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)d^{2}x^{6}+\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)d^{2}x^{6}+60d\,x^{3}\sqrt{d\,x^{3}+c}\,\sqrt{c}-48\sqrt{d\,x^{3}+c}\,c^{\frac{3}{2}}}{2304c^{\frac{7}{2}}x^{6}}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{6c\,x^{6}}+\frac{d\sqrt{d\,x^{3}+c}}{4c^{2}x^{3}}-\frac{d^{2}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{4c^{\frac{5}{2}}}}{8c}+\frac{d\left(-\frac{\sqrt{d\,x^{3}+c}}{3c\,x^{3}}+\frac{d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{64c^{2}}-\frac{d^{2}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{768c^{\frac{7}{2}}}+\frac{d^{2}\,\mathrm{a}}{}$
elliptic	Expression too large to display

```
[In] int(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/192*(d*x^3+c)^(1/2)*(-5*d*x^3+4*c)/c^3/x^6+1/128*d^2/c^3*(-7/2*arctanh((
d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)+1/18*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c
^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.03

$$\int \frac{1}{x^7\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = \left[\frac{\sqrt{c}d^2x^6\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+63\sqrt{c}d^2x^6\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)+24\left(5\,cdx^3-4\,c^2\right)\sqrt{dx^3+c}}{4608\,c^4x^6}, \frac{63\,\sqrt{}}{}\right.$$

```
[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4608*(sqrt(c)*d^2*x^6*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*
x^3 - 8*c)) + 63*sqrt(c)*d^2*x^6*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2
*c)/x^3) + 24*(5*c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c))/(c^4*x^6), 1/2304*(63*sq
rt(-c)*d^2*x^6*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - sqrt(-c)*d^2*x^6*arctan
(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*(5*c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c))/
(c^4*x^6)]
```

Sympy [F]

$$\int \frac{1}{x^7 (8c-dx^3)\sqrt{c+dx^3}}\,dx = -\int \frac{1}{-8cx^7\sqrt{c+dx^3}+dx^{10}\sqrt{c+dx^3}}\,dx$$

[In] integrate(1/x**7/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)

[Out] -Integral(1/(-8*c*x**7*sqrt(c + d*x**3) + d*x**10*sqrt(c + d*x**3)), x)

Maxima [F]

$$\int \frac{1}{x^7 (8c-dx^3)\sqrt{c+dx^3}}\,dx = \int -\frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)x^7}\,dx$$

[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^7), x)

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^7 (8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{7\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{256\,\sqrt{-c}c^3} - \frac{d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{2304\,\sqrt{-c}c^3} + \frac{5\,(dx^3+c)^{\frac{3}{2}}d^2 - 9\,\sqrt{dx^3+c}cd^2}{192\,c^3d^2x^6}$$

[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] 7/256*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/2304*d^2*arct
an(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) + 1/192*(5*(d*x^3 + c)^(3/2
)*d^2 - 9*sqrt(d*x^3 + c)*c*d^2)/(c^3*d^2*x^6)

Mupad [B] (verification not implemented)

Time = 7.67 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^7 (8c - dx^3) \sqrt{c + dx^3}} \, dx = \frac{d^2 \operatorname{atanh}\left(\frac{c^3 \sqrt{d x^3 + c}}{3 \sqrt{c^7}}\right)}{2304 \sqrt{c^7}} - \frac{7 d^2 \operatorname{atanh}\left(\frac{c^3 \sqrt{d x^3 + c}}{\sqrt{c^7}}\right)}{256 \sqrt{c^7}} - \frac{3 \sqrt{d x^3 + c}}{64 c^2 x^6} + \frac{5 (d x^3 + c)^{3/2}}{192 c^3 x^6}$$

```
[In] int(1/(x^7*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] (d^2*atanh((c^3*(c + d*x^3)^(1/2))/(3*(c^7)^(1/2))))/(2304*(c^7)^(1/2)) - (
7*d^2*atanh((c^3*(c + d*x^3)^(1/2))/(c^7)^(1/2)))/(256*(c^7)^(1/2)) - (3*(c
 + d*x^3)^(1/2))/(64*c^2*x^6) + (5*(c + d*x^3)^(3/2))/(192*c^3*x^6)
```

3.315 $\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 630

$$\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{2x^2\sqrt{c+dx^3}}{7d^2} - \frac{104c\sqrt{c+dx^3}}{7d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{32c^{7/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{8/3}}$$
$$+ \frac{32c^{7/6}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{8/3}} - \frac{32c^{7/6}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^{8/3}}$$
$$+ \frac{52\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$- \frac{104\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{7\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 32/9*c^(7/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(
8/3)-32/9*c^(7/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(8/3)-32/9*c^(7/6)
```

```
*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/d^(8/3)*3^(1/2
)-2/7*x^2*(d*x^3+c)^(1/2)/d^2-104/7*c*(d*x^3+c)^(1/2)/d^(8/3)/(d^(1/3)*x+c^
(1/3)*(1+3^(1/2)))-104/21*c^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+
c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)
*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2
)^(1/2)*3^(3/4)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/
3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+52/7*3^(1/4)*c^(4/3)*(c^(1/3)+d^(1/3)*x)
*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),
I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3
)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(8/3)/(d*x^3+c)^(1/2)/(c^
(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 630, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {490, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{104\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{7\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{52\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{7d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{32c^{7/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{8/3}}+\frac{32c^{7/6}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{8/3}}-\frac{32c^{7/6}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^{8/3}}-\frac{104c\sqrt{c+dx^3}}{7d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{2x^2\sqrt{c+dx^3}}{7d^2}$$

```
[In] Int[x^7/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*x^2*Sqrt[c + d*x^3])/(7*d^2) - (104*c*Sqrt[c + d*x^3])/(7*d^(8/3)*((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (32*c^(7/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1
```

/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(3*Sqrt[3]*d^(8/3)) + (32*c^(7/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]])/(9*d^(8/3)) - (32*c^(7/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]])/(9*d^(8/3)) + (52*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (104*Sqrt[2]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(1/4)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
```

```
/; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2x^2\sqrt{c+dx^3}}{7d^2}+\frac{2\int \frac{x(16c^2+26cdx^3)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{7d^2}\\
&= -\frac{2x^2\sqrt{c+dx^3}}{7d^2}+\frac{2\int \left(-\frac{26cx}{\sqrt{c+dx^3}}+\frac{224c^2x}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{7d^2}\\
&= -\frac{2x^2\sqrt{c+dx^3}}{7d^2}-\frac{(52c)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{7d^2}+\frac{(64c^2)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{d^2}\\
&= -\frac{2x^2\sqrt{c+dx^3}}{7d^2}-\frac{(16c)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{3d^3}\\
&\quad -\frac{(52c)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{7d^{7/3}}+\frac{\left(16c^{4/3}\right)\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{3d^{7/3}}\\
&\quad +\frac{\left(52\left(1-\sqrt{3}\right)c^{4/3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{7d^{7/3}}-\frac{\left(16c^{5/3}\right)\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{d^{5/3}}
\end{aligned}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d^2} - \frac{104c\sqrt{c+dx^3}}{7d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$+ \frac{52\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$- \frac{104\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{\left(32c^{5/3}\right)\text{Subst}\left(\int\frac{1}{9-cx^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{3d^{8/3}}$$

$$- \frac{\left(16c^{5/3}\right)\text{Subst}\left(\int\frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3d^{5/3}}$$

$$+ \frac{\left(64c^{2/3}\right)\text{Subst}\left(\int\frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{3d^{2/3}}$$

$$=-\frac{2x^2\sqrt{c+dx^3}}{7d^2}-\frac{104c\sqrt{c+dx^3}}{7d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{32c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{8/3}}+\frac{32c^{7/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{8/3}}+\frac{52\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{104\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\left(32c^{5/3}\right)\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{3d^{8/3}}$$

$$=-\frac{2x^2\sqrt{c+dx^3}}{7d^2}-\frac{104c\sqrt{c+dx^3}}{7d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{32c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{8/3}}+\frac{32c^{7/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{8/3}}-\frac{32c^{7/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^{8/3}}+\frac{52\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{104\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.21

$$\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{x^2\left(-20(c+dx^3)+20c\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+13dx^3\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\right.\right.}{70d^2\sqrt{c+dx^3}}$$

```
[In] Integrate[x^7/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^2*(-20*(c + d*x^3) + 20*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3,
 -((d*x^3)/c), (d*x^3)/(8*c)] + 13*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3,
1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(70*d^2*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.87 (sec) , antiderivative size = 867, normalized size of antiderivative = 1.38

method	result	size
elliptic	Expression too large to display	867
risch	Expression too large to display	872
default	Expression too large to display	1308

```
[In] int(x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/7*x^2*(d*x^3+c)^(1/2)/d^2+104/21*I/d^3*c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/
2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3
^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(
1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(
1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
))^(1/2)))-64/27*I*c/d^5*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+
1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x
-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-
1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))
```

```
^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2
)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1
/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I
*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)
/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 8.43 (sec) , antiderivative size = 2428, normalized size of antiderivative = 3.85

$$\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/189*(56*d^3*(c^7/d^16)^(1/6)*log(33554432/3*((d^16*x^9 + 318*c*d^15*x^6 +
 1200*c^2*d^14*x^3 + 640*c^3*d^13)*(c^7/d^16)^(5/6) + 6*(c^6*d^2*x^7 + 80*c
^7*d*x^4 + 160*c^8*x + 6*(5*c^2*d^12*x^5 + 32*c^3*d^11*x^2)*(c^7/d^16)^(2/3
) + (7*c^4*d^7*x^6 + 152*c^5*d^6*x^3 + 64*c^6*d^5)*(c^7/d^16)^(1/3))*sqrt(d
*x^3 + c) + 18*(5*c^3*d^10*x^7 + 64*c^4*d^9*x^4 + 32*c^5*d^8*x)*sqrt(c^7/d^
16) + 18*(c^5*d^5*x^8 + 38*c^6*d^4*x^5 + 64*c^7*d^3*x^2)*(c^7/d^16)^(1/6))/
(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 56*d^3*(c^7/d^16)^(1/
6)*log(-33554432/3*((d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^
3*d^13)*(c^7/d^16)^(5/6) - 6*(c^6*d^2*x^7 + 80*c^7*d*x^4 + 160*c^8*x + 6*(5
*c^2*d^12*x^5 + 32*c^3*d^11*x^2)*(c^7/d^16)^(2/3) + (7*c^4*d^7*x^6 + 152*c^
5*d^6*x^3 + 64*c^6*d^5)*(c^7/d^16)^(1/3))*sqrt(d*x^3 + c) + 18*(5*c^3*d^10*
x^7 + 64*c^4*d^9*x^4 + 32*c^5*d^8*x)*sqrt(c^7/d^16) + 18*(c^5*d^5*x^8 + 38*
c^6*d^4*x^5 + 64*c^7*d^3*x^2)*(c^7/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 1
92*c^2*d*x^3 - 512*c^3)) - 27*sqrt(d*x^3 + c)*d*x^2 + 1404*c*sqrt(d)*weiers
trassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) - 28*(sqrt(-3)*d^3
- d^3)*(c^7/d^16)^(1/6)*log(33554432/3*((d^16*x^9 + 318*c*d^15*x^6 + 1200*c
^2*d^14*x^3 + 640*c^3*d^13 + sqrt(-3)*(d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2
*d^14*x^3 + 640*c^3*d^13))*(c^7/d^16)^(5/6) + 6*(2*c^6*d^2*x^7 + 160*c^7*d*
x^4 + 320*c^8*x - 6*(5*c^2*d^12*x^5 + 32*c^3*d^11*x^2 - sqrt(-3)*(5*c^2*d^1
2*x^5 + 32*c^3*d^11*x^2))*(c^7/d^16)^(2/3) - (7*c^4*d^7*x^6 + 152*c^5*d^6*x
^3 + 64*c^6*d^5 + sqrt(-3)*(7*c^4*d^7*x^6 + 152*c^5*d^6*x^3 + 64*c^6*d^5))*
(c^7/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^3*d^10*x^7 + 64*c^4*d^9*x^4 + 3
2*c^5*d^8*x)*sqrt(c^7/d^16) + 18*(c^5*d^5*x^8 + 38*c^6*d^4*x^5 + 64*c^7*d^3
*x^2 - sqrt(-3)*(c^5*d^5*x^8 + 38*c^6*d^4*x^5 + 64*c^7*d^3*x^2))*(c^7/d^16)
^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 28*(sqrt(-3)*
d^3 - d^3)*(c^7/d^16)^(1/6)*log(-33554432/3*((d^16*x^9 + 318*c*d^15*x^6 + 1
```

```
200*c^2*d^14*x^3 + 640*c^3*d^13 + sqrt(-3)*(d^16*x^9 + 318*c*d^15*x^6 + 120
0*c^2*d^14*x^3 + 640*c^3*d^13))*(c^7/d^16)^(5/6) - 6*(2*c^6*d^2*x^7 + 160*c
^7*d*x^4 + 320*c^8*x - 6*(5*c^2*d^12*x^5 + 32*c^3*d^11*x^2 - sqrt(-3)*(5*c^
2*d^12*x^5 + 32*c^3*d^11*x^2))*(c^7/d^16)^(2/3) - (7*c^4*d^7*x^6 + 152*c^5*
d^6*x^3 + 64*c^6*d^5 + sqrt(-3)*(7*c^4*d^7*x^6 + 152*c^5*d^6*x^3 + 64*c^6*d
^5))*(c^7/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^3*d^10*x^7 + 64*c^4*d^9*x^
4 + 32*c^5*d^8*x)*sqrt(c^7/d^16) + 18*(c^5*d^5*x^8 + 38*c^6*d^4*x^5 + 64*c^
7*d^3*x^2 - sqrt(-3)*(c^5*d^5*x^8 + 38*c^6*d^4*x^5 + 64*c^7*d^3*x^2))*(c^7/
d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 28*(sqrt
(-3)*d^3 + d^3)*(c^7/d^16)^(1/6)*log(33554432/3*((d^16*x^9 + 318*c*d^15*x^6
 + 1200*c^2*d^14*x^3 + 640*c^3*d^13 - sqrt(-3)*(d^16*x^9 + 318*c*d^15*x^6 +
 1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c^7/d^16)^(5/6) + 6*(2*c^6*d^2*x^7 + 1
60*c^7*d*x^4 + 320*c^8*x - 6*(5*c^2*d^12*x^5 + 32*c^3*d^11*x^2 + sqrt(-3)*(
5*c^2*d^12*x^5 + 32*c^3*d^11*x^2))*(c^7/d^16)^(2/3) - (7*c^4*d^7*x^6 + 152*
c^5*d^6*x^3 + 64*c^6*d^5 - sqrt(-3)*(7*c^4*d^7*x^6 + 152*c^5*d^6*x^3 + 64*c
^6*d^5))*(c^7/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^3*d^10*x^7 + 64*c^4*d^
9*x^4 + 32*c^5*d^8*x)*sqrt(c^7/d^16) + 18*(c^5*d^5*x^8 + 38*c^6*d^4*x^5 + 6
4*c^7*d^3*x^2 + sqrt(-3)*(c^5*d^5*x^8 + 38*c^6*d^4*x^5 + 64*c^7*d^3*x^2))*(
c^7/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 28*(
sqrt(-3)*d^3 + d^3)*(c^7/d^16)^(1/6)*log(-33554432/3*((d^16*x^9 + 318*c*d^1
5*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13 - sqrt(-3)*(d^16*x^9 + 318*c*d^15*
x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c^7/d^16)^(5/6) - 6*(2*c^6*d^2*x^
7 + 160*c^7*d*x^4 + 320*c^8*x - 6*(5*c^2*d^12*x^5 + 32*c^3*d^11*x^2 + sqrt(
-3)*(5*c^2*d^12*x^5 + 32*c^3*d^11*x^2))*(c^7/d^16)^(2/3) - (7*c^4*d^7*x^6 +
 152*c^5*d^6*x^3 + 64*c^6*d^5 - sqrt(-3)*(7*c^4*d^7*x^6 + 152*c^5*d^6*x^3 +
 64*c^6*d^5))*(c^7/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^3*d^10*x^7 + 64*c
^4*d^9*x^4 + 32*c^5*d^8*x)*sqrt(c^7/d^16) + 18*(c^5*d^5*x^8 + 38*c^6*d^4*x^
5 + 64*c^7*d^3*x^2 + sqrt(-3)*(c^5*d^5*x^8 + 38*c^6*d^4*x^5 + 64*c^7*d^3*x^
2))*(c^7/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/
d^3
```

Sympy [F]

$$\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\int \frac{x^7}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(x**7/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(x**7/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int -\frac{x^7}{\sqrt{dx^3+c}(dx^3-8\,c)}\,dx$$

```
[In] integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] -integrate(x^7/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Giac [F]

$$\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int -\frac{x^7}{\sqrt{dx^3+c}(dx^3-8\,c)}\,dx$$

```
[In] integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(-x^7/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x^7}{\sqrt{d\,x^3+c}\,(8\,c-d\,x^3)}\,dx$$

```
[In] int(x^7/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
[Out] int(x^7/((c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

3.316 $\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 601

$$\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{2\sqrt{c+dx^3}}{d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{4\sqrt[6]{c}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{5/3}} + \frac{4\sqrt[6]{c}\,\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{5/3}} - \frac{4\sqrt[6]{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^{5/3}} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 4/9*c^(1/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(5
/3)-4/9*c^(1/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(5/3)-4/9*c^(1/6)*ar
```

```
ctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/d^(5/3)*3^(1/2)-2
*(d*x^3+c)^(1/2)/d^(5/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-2/3*c^(1/3)*(c^(1/
3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*
(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2
)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^(5/3)/(d*x^3+c)^(1/2)/
(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+3^(1/
4)*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d
^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^
(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/
2)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*
(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 601, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {494, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{4\sqrt[6]{c}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{5/3}}+\frac{4\sqrt[6]{c}\,\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{5/3}}-\frac{4\sqrt[6]{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^{5/3}}-\frac{2\sqrt{c+dx^3}}{d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

```
[In] Int[x^4/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]

[Out] (-2*Sqrt[c + d*x^3])/(d^(5/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (4*c^(
1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(3*Sq
```

```
rt[3]*d^(5/3)) + (4*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt
[c + d*x^3])])/(9*d^(5/3)) - (4*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])
])/(9*d^(5/3)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*S
qrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3
) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (
2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x +
 d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
 Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4
*Sqrt[3]])/(3^(1/4)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
```

```
/; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\int \frac{x}{\sqrt{c+dx^3}}\,dx}{d}+\frac{(8c)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{d}\\
&= -\frac{2\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{3d^2}\\
&\quad -\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{d^{4/3}}+\frac{\left(2\sqrt[3]{c}\right)\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{3d^{4/3}}\\
&\quad +\frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{c}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{d^{4/3}}-\frac{\left(2c^{2/3}\right)\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{d^{2/3}}
\end{aligned}$$

$$= -\frac{2\sqrt{c+dx^3}}{d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{\left(4c^{2/3}\right)\text{Subst}\left(\int\frac{1}{9-cx^2}\,dx,x,\frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{3d^{5/3}}$$

$$-\frac{\left(2c^{2/3}\right)\text{Subst}\left(\int\frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3d^{2/3}}$$

$$+\frac{\left(8\sqrt[3]{d}\right)\text{Subst}\left(\int\frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{3\sqrt[3]{c}}$$

$$= -\frac{2\sqrt{c+dx^3}}{d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{4\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{5/3}} + \frac{4\sqrt[6]{c}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{5/3}} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{\left(4c^{2/3}\right)\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{3d^{5/3}}$$

$$= -\frac{2\sqrt{c+dx^3}}{d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{4\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{5/3}} + \frac{4\sqrt[6]{c}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{5/3}} - \frac{4\sqrt[6]{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^{5/3}} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.11

$$\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{x^5\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{40c\sqrt{c+dx^3}}$$

```
[In] Integrate[x^4/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^5*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(
8*c)])/(40*c*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.45 (sec) , antiderivative size = 848, normalized size of antiderivative = 1.41

method	result	size
default	Expression too large to display	848
elliptic	Expression too large to display	848

```
[In] int(x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*E
llipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-8/27*I/d^4*2^(1/2)*sum(
1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/
3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1
/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)
*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)
-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2
*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1
```

```
/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3
*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.63 (sec) , antiderivative size = 2254, normalized size of antiderivative = 3.75

$$\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/27*(2*d^2*(c/d^10)^(1/6)*log(1024/3*((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^
2*d^9*x^3 + 640*c^3*d^8)*(c/d^10)^(5/6) + 6*(c*d^2*x^7 + 80*c^2*d*x^4 + 160
*c^3*x + 6*(5*c*d^8*x^5 + 32*c^2*d^7*x^2)*(c/d^10)^(2/3) + (7*c*d^5*x^6 + 1
52*c^2*d^4*x^3 + 64*c^3*d^3)*(c/d^10)^(1/3))*sqrt(d*x^3 + c) + 18*(5*c*d^7*
x^7 + 64*c^2*d^6*x^4 + 32*c^3*d^5*x)*sqrt(c/d^10) + 18*(c*d^4*x^8 + 38*c^2*
d^3*x^5 + 64*c^3*d^2*x^2)*(c/d^10)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2
*d*x^3 - 512*c^3)) - 2*d^2*(c/d^10)^(1/6)*log(-1024/3*((d^11*x^9 + 318*c*d^
10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8)*(c/d^10)^(5/6) - 6*(c*d^2*x^7 + 80
*c^2*d*x^4 + 160*c^3*x + 6*(5*c*d^8*x^5 + 32*c^2*d^7*x^2)*(c/d^10)^(2/3) +
(7*c*d^5*x^6 + 152*c^2*d^4*x^3 + 64*c^3*d^3)*(c/d^10)^(1/3))*sqrt(d*x^3 + c
) + 18*(5*c*d^7*x^7 + 64*c^2*d^6*x^4 + 32*c^3*d^5*x)*sqrt(c/d^10) + 18*(c*d
^4*x^8 + 38*c^2*d^3*x^5 + 64*c^3*d^2*x^2)*(c/d^10)^(1/6))/(d^3*x^9 - 24*c*d
^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (sqrt(-3)*d^2 - d^2)*(c/d^10)^(1/6)*lo
g(1024/3*((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 + sqr
t(-3)*(d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8))*(c/d^10
)^(5/6) + 6*(2*c*d^2*x^7 + 160*c^2*d*x^4 + 320*c^3*x - 6*(5*c*d^8*x^5 + 32*
c^2*d^7*x^2 - sqrt(-3)*(5*c*d^8*x^5 + 32*c^2*d^7*x^2))*(c/d^10)^(2/3) - (7*
c*d^5*x^6 + 152*c^2*d^4*x^3 + 64*c^3*d^3 + sqrt(-3)*(7*c*d^5*x^6 + 152*c^2*
d^4*x^3 + 64*c^3*d^3))*(c/d^10)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c*d^7*x^7 +
64*c^2*d^6*x^4 + 32*c^3*d^5*x)*sqrt(c/d^10) + 18*(c*d^4*x^8 + 38*c^2*d^3*x^
5 + 64*c^3*d^2*x^2 - sqrt(-3)*(c*d^4*x^8 + 38*c^2*d^3*x^5 + 64*c^3*d^2*x^2)
)*(c/d^10)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (sq
rt(-3)*d^2 - d^2)*(c/d^10)^(1/6)*log(-1024/3*((d^11*x^9 + 318*c*d^10*x^6 +
1200*c^2*d^9*x^3 + 640*c^3*d^8 + sqrt(-3)*(d^11*x^9 + 318*c*d^10*x^6 + 1200
*c^2*d^9*x^3 + 640*c^3*d^8))*(c/d^10)^(5/6) - 6*(2*c*d^2*x^7 + 160*c^2*d*x^
4 + 320*c^3*x - 6*(5*c*d^8*x^5 + 32*c^2*d^7*x^2 - sqrt(-3)*(5*c*d^8*x^5 + 3
2*c^2*d^7*x^2))*(c/d^10)^(2/3) - (7*c*d^5*x^6 + 152*c^2*d^4*x^3 + 64*c^3*d^
3 + sqrt(-3)*(7*c*d^5*x^6 + 152*c^2*d^4*x^3 + 64*c^3*d^3))*(c/d^10)^(1/3))*
sqrt(d*x^3 + c) - 36*(5*c*d^7*x^7 + 64*c^2*d^6*x^4 + 32*c^3*d^5*x)*sqrt(c/d
^10) + 18*(c*d^4*x^8 + 38*c^2*d^3*x^5 + 64*c^3*d^2*x^2 - sqrt(-3)*(c*d^4*x^
```

```
8 + 38*c^2*d^3*x^5 + 64*c^3*d^2*x^2))*(c/d^10)^(1/6))/(d^3*x^9 - 24*c*d^2*x
^6 + 192*c^2*d*x^3 - 512*c^3)) + (sqrt(-3)*d^2 + d^2)*(c/d^10)^(1/6)*log(10
24/3*((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 - sqrt(-3
)*(d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8))*(c/d^10)^(5
/6) + 6*(2*c*d^2*x^7 + 160*c^2*d*x^4 + 320*c^3*x - 6*(5*c*d^8*x^5 + 32*c^2*
d^7*x^2 + sqrt(-3)*(5*c*d^8*x^5 + 32*c^2*d^7*x^2))*(c/d^10)^(2/3) - (7*c*d^
5*x^6 + 152*c^2*d^4*x^3 + 64*c^3*d^3 - sqrt(-3)*(7*c*d^5*x^6 + 152*c^2*d^4*
x^3 + 64*c^3*d^3))*(c/d^10)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c*d^7*x^7 + 64*c
^2*d^6*x^4 + 32*c^3*d^5*x)*sqrt(c/d^10) + 18*(c*d^4*x^8 + 38*c^2*d^3*x^5 +
64*c^3*d^2*x^2 + sqrt(-3)*(c*d^4*x^8 + 38*c^2*d^3*x^5 + 64*c^3*d^2*x^2))*(c
/d^10)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (sqrt(-
3)*d^2 + d^2)*(c/d^10)^(1/6)*log(-1024/3*((d^11*x^9 + 318*c*d^10*x^6 + 1200
*c^2*d^9*x^3 + 640*c^3*d^8 - sqrt(-3)*(d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2
*d^9*x^3 + 640*c^3*d^8))*(c/d^10)^(5/6) - 6*(2*c*d^2*x^7 + 160*c^2*d*x^4 +
320*c^3*x - 6*(5*c*d^8*x^5 + 32*c^2*d^7*x^2 + sqrt(-3)*(5*c*d^8*x^5 + 32*c^
2*d^7*x^2))*(c/d^10)^(2/3) - (7*c*d^5*x^6 + 152*c^2*d^4*x^3 + 64*c^3*d^3 -
sqrt(-3)*(7*c*d^5*x^6 + 152*c^2*d^4*x^3 + 64*c^3*d^3))*(c/d^10)^(1/3))*sqrt
(d*x^3 + c) - 36*(5*c*d^7*x^7 + 64*c^2*d^6*x^4 + 32*c^3*d^5*x)*sqrt(c/d^10)
 + 18*(c*d^4*x^8 + 38*c^2*d^3*x^5 + 64*c^3*d^2*x^2 + sqrt(-3)*(c*d^4*x^8 +
38*c^2*d^3*x^5 + 64*c^3*d^2*x^2))*(c/d^10)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 +
 192*c^2*d*x^3 - 512*c^3)) + 54*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstr
assPInverse(0, -4*c/d, x)))/d^2
```

Sympy [F]

$$\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\int \frac{x^4}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(x**4/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
[Out] -Integral(x**4/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int -\frac{x^4}{\sqrt{dx^3+c}(dx^3-8\,c)}\,dx$$

```
[In] integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] -integrate(x^4/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Giac [F]

$$\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int -\frac{x^4}{\sqrt{dx^3+c}(dx^3-8\,c)}\,dx$$

```
[In] integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(-x^4/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x^4}{\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)}\,dx$$

```
[In] int(x^4/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
[Out] int(x^4/((c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

3.317 $\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 141

$$\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{6\sqrt{3}c^{5/6}d^{2/3}} + \frac{\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}$$

```
[Out] 1/18*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(5/6)/d^(
2/3)-1/18*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/6)/d^(2/3)-1/18*arctan(
c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(5/6)/d^(2/3)*3^(1/2
)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{6\sqrt{3}c^{5/6}d^{2/3}} + \frac{\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}$$

```
[In] Int[x/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/6*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]]/(Sqrt[
3]*c^(5/6)*d^(2/3)) + ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d
*x^3])]/(18*c^(5/6)*d^(2/3)) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(18*c^(
5/6)*d^(2/3))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
1, 0]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{12cd}+\frac{\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{12c^{2/3}\sqrt[3]{d}}-\frac{\sqrt[3]{d}\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{4\sqrt[3]{c}}$$

$$=\frac{\text{Subst}\left(\int \frac{1}{9-cx^2}\,dx,x,\frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{6\sqrt[3]{c}d^{2/3}}-\frac{\sqrt[3]{d}\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{12\sqrt[3]{c}}$$

$$+\frac{d^{4/3}\text{Subst}\left(\int \frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{3c^{4/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{6\sqrt{3}c^{5/6}d^{2/3}} + \frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{6\sqrt[3]{c}d^{2/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{6\sqrt{3}c^{5/6}d^{2/3}} + \frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.48

$$\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{x^2\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{16c\sqrt{c+dx^3}}$$

```
[In] Integrate[x/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
[Out] (x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(
8*c)])/(16*c*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.23 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.95

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-c\right.}{}\right)}{2\left(-c\right.}}}{}\right)}{}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-c\right.}{}\right)}{2\left(-c\right.}}}{}\right)}{}$

```
[In] int(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/27*I/d^3/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^
2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*
x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x
^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_
alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(
2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/
2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))
^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2285 vs. 2(95) = 190.

Time = 0.54 (sec) , antiderivative size = 2285, normalized size of antiderivative = 16.21

$$\int \frac{x}{(8c - dx^3)\sqrt{c + dx^3}}\, dx = \text{Too large to display}$$

```
[In] integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/216*(sqrt(-3) + 1)*(1/(c^5*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 120
0*c^2*d*x^3 + 640*c^3 - 9*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 32*c^6*d^3*x +
sqrt(-3)*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 32*c^6*d^3*x))*(1/(c^5*d^4))^(2/
3) + 3*sqrt(d*x^3 + c)*(6*(5*c^5*d^5*x^5 + 32*c^6*d^4*x^2 - sqrt(-3)*(5*c^5
*d^5*x^5 + 32*c^6*d^4*x^2))*(1/(c^5*d^4))^(5/6) - 2*(7*c^3*d^4*x^6 + 152*c^
4*d^3*x^3 + 64*c^5*d^2)*sqrt(1/(c^5*d^4)) + (c*d^3*x^7 + 80*c^2*d^2*x^4 + 1
60*c^3*d*x + sqrt(-3)*(c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x))*(1/(c^5*d
^4))^(1/6)) - 9*(c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*x^2 - sqrt(-3)*(
c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*x^2))*(1/(c^5*d^4))^(1/3))/(d^3*x
^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 1/216*(sqrt(-3) + 1)*(1/(c^
5*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(
5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 32*c^6*d^3*x + sqrt(-3)*(5*c^4*d^5*x^7 + 6
4*c^5*d^4*x^4 + 32*c^6*d^3*x))*(1/(c^5*d^4))^(2/3) - 3*sqrt(d*x^3 + c)*(6*(
5*c^5*d^5*x^5 + 32*c^6*d^4*x^2 - sqrt(-3)*(5*c^5*d^5*x^5 + 32*c^6*d^4*x^2))
*(1/(c^5*d^4))^(5/6) - 2*(7*c^3*d^4*x^6 + 152*c^4*d^3*x^3 + 64*c^5*d^2)*sqr
t(1/(c^5*d^4)) + (c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x + sqrt(-3)*(c*d^
3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x))*(1/(c^5*d^4))^(1/6)) - 9*(c^2*d^4*x^
8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*x^2 - sqrt(-3)*(c^2*d^4*x^8 + 38*c^3*d^3*x^
5 + 64*c^4*d^2*x^2))*(1/(c^5*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2
*d*x^3 - 512*c^3)) - 1/216*(sqrt(-3) - 1)*(1/(c^5*d^4))^(1/6)*log((d^3*x^9
+ 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^4*d^5*x^7 + 64*c^5*d^4*
x^4 + 32*c^6*d^3*x - sqrt(-3)*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 32*c^6*d^3*
x))*(1/(c^5*d^4))^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^5*d^5*x^5 + 32*c^6*d^4*
x^2 + sqrt(-3)*(5*c^5*d^5*x^5 + 32*c^6*d^4*x^2))*(1/(c^5*d^4))^(5/6) - 2*(7
*c^3*d^4*x^6 + 152*c^4*d^3*x^3 + 64*c^5*d^2)*sqrt(1/(c^5*d^4)) + (c*d^3*x^7
 + 80*c^2*d^2*x^4 + 160*c^3*d*x - sqrt(-3)*(c*d^3*x^7 + 80*c^2*d^2*x^4 + 16
0*c^3*d*x))*(1/(c^5*d^4))^(1/6)) - 9*(c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4
*d^2*x^2 + sqrt(-3)*(c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*x^2))*(1/(c^
5*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 1/216*
(sqrt(-3) - 1)*(1/(c^5*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*
d*x^3 + 640*c^3 - 9*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 32*c^6*d^3*x - sqrt(-
3)*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 32*c^6*d^3*x))*(1/(c^5*d^4))^(2/3) - 3
*sqrt(d*x^3 + c)*(6*(5*c^5*d^5*x^5 + 32*c^6*d^4*x^2 + sqrt(-3)*(5*c^5*d^5*x
^5 + 32*c^6*d^4*x^2))*(1/(c^5*d^4))^(5/6) - 2*(7*c^3*d^4*x^6 + 152*c^4*d^3*
x^3 + 64*c^5*d^2)*sqrt(1/(c^5*d^4)) + (c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3
```

```
*d*x - sqrt(-3)*(c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x))*(1/(c^5*d^4))^(
1/6)) - 9*(c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*x^2 + sqrt(-3)*(c^2*d^
4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*x^2))*(1/(c^5*d^4))^(1/3))/(d^3*x^9 - 2
4*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 1/108*(1/(c^5*d^4))^(1/6)*log((d^
3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(5*c^4*d^5*x^7 + 64*c
^5*d^4*x^4 + 32*c^6*d^3*x)*(1/(c^5*d^4))^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^
5*d^5*x^5 + 32*c^6*d^4*x^2)*(1/(c^5*d^4))^(5/6) + (7*c^3*d^4*x^6 + 152*c^4*
d^3*x^3 + 64*c^5*d^2)*sqrt(1/(c^5*d^4)) + (c*d^3*x^7 + 80*c^2*d^2*x^4 + 160
*c^3*d*x)*(1/(c^5*d^4))^(1/6)) + 18*(c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*
d^2*x^2)*(1/(c^5*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512
*c^3)) - 1/108*(1/(c^5*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*
d*x^3 + 640*c^3 + 18*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 32*c^6*d^3*x)*(1/(c^
5*d^4))^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^5*d^5*x^5 + 32*c^6*d^4*x^2)*(1/(c
^5*d^4))^(5/6) + (7*c^3*d^4*x^6 + 152*c^4*d^3*x^3 + 64*c^5*d^2)*sqrt(1/(c^5
*d^4)) + (c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x)*(1/(c^5*d^4))^(1/6)) +
18*(c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*x^2)*(1/(c^5*d^4))^(1/3))/(d^
3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3))
```

Sympy [F]

$$\int \frac{x}{(8c - dx^3)\sqrt{c + dx^3}}\,dx = -\int \frac{x}{-8c\sqrt{c + dx^3} + dx^3\sqrt{c + dx^3}}\,dx$$

```
[In] integrate(x/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
[Out] -Integral(x/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{(8c - dx^3)\sqrt{c + dx^3}}\,dx = \int -\frac{x}{\sqrt{dx^3 + c}(dx^3 - 8\,c)}\,dx$$

```
[In] integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] -integrate(x/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Giac [F]

$$\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int -\frac{x}{\sqrt{dx^3+c}(dx^3-8\,c)}\,dx$$

```
[In] integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Mupad [B] (verification not implemented)

Time = 45.12 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.93

$$\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)\left(\sqrt{d\,x^3+c}-\sqrt{c}+2\,c^{1/6}\,d^{1/3}\,x\right)^3}{x^3\,\left(d^{1/3}\,x-2\,c^{1/3}\right)^3}\right)}{54\,c^{5/6}\,d^{2/3}} + \frac{\sqrt{2}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)\left(-\sqrt{3}\,c^{1/6}\,d^{1/3}\,x+\sqrt{d\,x^3+c}\,1\mathrm{i}+\sqrt{c}\,1\mathrm{i}+c^{1/6}\,d^{1/3}\,x\,1\mathrm{i}\right)^3}{x^3\,\left(d^{1/3}\,x+c^{1/3}-\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)^3}\right)\sqrt{-1+\sqrt{3}\,1\mathrm{i}}}{108\,c^{5/6}\,d^{2/3}} + \frac{\sqrt{2}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)\left(\sqrt{3}\,c^{1/6}\,d^{1/3}\,x-\sqrt{d\,x^3+c}\,1\mathrm{i}+\sqrt{c}\,1\mathrm{i}+c^{1/6}\,d^{1/3}\,x\,1\mathrm{i}\right)^3}{x^3\,\left(d^{1/3}\,x+c^{1/3}+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)^3}\right)\sqrt{1+\sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{108\,c^{5/6}\,d^{2/3}}$$

```
[In] int(x/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] log((((c + d*x^3)^(1/2) + c^(1/2))*((c + d*x^3)^(1/2) - c^(1/2) + 2*c^(1/6)
*d^(1/3)*x)^3)/(x^3*(d^(1/3)*x - 2*c^(1/3))^3))/(54*c^(5/6)*d^(2/3)) + (2^(
1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2)*1i + c^(1/2)*1i
+ c^(1/6)*d^(1/3)*x*1i - 3^(1/2)*c^(1/6)*d^(1/3)*x)^3)/(x^3*(d^(1/3)*x - 3^
(1/2)*c^(1/3)*1i + c^(1/3))^3))*(3^(1/2)*1i - 1)^(1/2))/(108*c^(5/6)*d^(2/3
)) + (2^(1/2)*log((((c + d*x^3)^(1/2) + c^(1/2))*(c^(1/2)*1i - (c + d*x^3)^
(1/2)*1i + c^(1/6)*d^(1/3)*x*1i + 3^(1/2)*c^(1/6)*d^(1/3)*x)^3)/(x^3*(3^(1/
2)*c^(1/3)*1i + d^(1/3)*x + c^(1/3))^3))*(3^(1/2)*1i + 1)^(1/2)*1i)/(108*c^
(5/6)*d^(2/3))
```

3.318 $\int \frac{1}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 632

$$\int \frac{1}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{c+dx^3}}{8c^2x} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}c^{11/6}} + \frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{144c^{11/6}} - \frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{144c^{11/6}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{16c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 1/144*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^
(11/6)-1/144*d^(1/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)-1/144*d^
(1/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(11/6)*
3^(1/2)-1/8*(d*x^3+c)^(1/2)/c^2/x+1/8*d^(1/3)*(d*x^3+c)^(1/2)/c^2/(d^(1/3)*
x+c^(1/3)*(1+3^(1/2)))+1/24*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*
x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(
```

```
2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2
)*3^(3/4)/c^(5/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(
1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/16*3^(1/4)*d^(1/3)*(c^(1/3)+d^(1/3)*
x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))
),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2
/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(5/3)/(d*x^3+c)^(1/2)/(
c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 632, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {491, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{1}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx$$

$$= \frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{16c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}c^{11/6}}+\frac{\sqrt[3]{d}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{144c^{11/6}}$$
$$-\frac{\sqrt[3]{d}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{144c^{11/6}}-\frac{\sqrt{c+dx^3}}{8c^2x}+\frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

```
[In] Int[1/(x^2*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/8*Sqrt[c + d*x^3]/(c^2*x) + (d^(1/3)*Sqrt[c + d*x^3])/(8*c^2*((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)) - (d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(
1/3)*x))/Sqrt[c + d*x^3]])/(48*Sqrt[3]*c^(11/6)) + (d^(1/3)*ArcTanh[(c^(1/3
) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(144*c^(11/6)) - (d^(1/3)*Ar
cTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(144*c^(11/6)) - (3^(1/4)*Sqrt[2 - Sqrt
```

```
[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2
/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt
[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt
[3]])/(16*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(
c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*
c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*3^(1/4)*c^(5/3)*Sqrt[(c^
(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c
+ d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
```

```
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{c+dx^3}}{8c^2x} + \frac{\int \frac{x\left(5cd-\frac{d^2x^3}{2}\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{8c^2} \\
&= -\frac{\sqrt{c+dx^3}}{8c^2x} + \frac{\int \left(\frac{dx}{2\sqrt{c+dx^3}} + \frac{cdx}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{8c^2} \\
&= -\frac{\sqrt{c+dx^3}}{8c^2x} + \frac{d\int \frac{x}{\sqrt{c+dx^3}}\,dx}{16c^2} + \frac{d\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{8c} \\
&= -\frac{\sqrt{c+dx^3}}{8c^2x} - \frac{\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{96c^2} \\
&\quad + \frac{d^{2/3}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{16c^2} + \frac{d^{2/3}\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{96c^{5/3}} \\
&\quad - \frac{\left(\left(1-\sqrt{3}\right)d^{2/3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{16c^{5/3}} - \frac{d^{4/3}\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{32c^{4/3}}
\end{aligned}$$

$$= -\frac{\sqrt{c+dx^3}}{8c^2x} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{16c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{\sqrt[3]{d}\text{Subst}\left(\int \frac{1}{9-cx^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{48c^{4/3}} - \frac{d^{4/3}\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{96c^{4/3}} + \frac{d^{7/3}\text{Subst}\left(\int \frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{24c^{7/3}}$$

$$= -\frac{\sqrt{c+dx^3}}{8c^2x} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}c^{11/6}} + \frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{144c^{11/6}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{16c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{\sqrt[3]{d}\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{48c^{4/3}}$$

$$= -\frac{\sqrt{c+dx^3}}{8c^2x} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}c^{11/6}} + \frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{144c^{11/6}} - \frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{144c^{11/6}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{16c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.22

$$\int \frac{1}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx$$
$$= \frac{-80c(c+dx^3)+25cdx^3\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-d^2x^6\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\right.}{640c^3x\sqrt{c+dx^3}}$$

```
[In] Integrate[1/(x^2*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-80*c*(c + d*x^3) + 25*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5
/3, -((d*x^3)/c), (d*x^3)/(8*c)] - d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3
, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(640*c^3*x*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.89 (sec) , antiderivative size = 868, normalized size of antiderivative = 1.37

method	result	size
elliptic	Expression too large to display	868
risch	Expression too large to display	872
default	Expression too large to display	874

```
[In] int(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(d*x^3+c)^(1/2)/c^2/x-1/24*I/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-
c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)
-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)
/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(
1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d
*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/
2)))-1/216*I/d^2/c^2*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*
(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d
*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*
I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
```

```
2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2
/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3
^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c
*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(
I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.46 (sec) , antiderivative size = 2259, normalized size of antiderivative = 3.57

$$\int \frac{1}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/1728*(2*c^2*x*(d^2/c^11)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^
2*x^3 + 640*c^3*d + 18*(5*c^8*d^2*x^7 + 64*c^9*d*x^4 + 32*c^10*x)*(d^2/c^11
)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2)*(d^2/c^11)^(5/6
) + (7*c^6*d^2*x^6 + 152*c^7*d*x^3 + 64*c^8)*sqrt(d^2/c^11) + (c^2*d^3*x^7
+ 80*c^3*d^2*x^4 + 160*c^4*d*x)*(d^2/c^11)^(1/6)) + 18*(c^4*d^3*x^8 + 38*c^
5*d^2*x^5 + 64*c^6*d*x^2)*(d^2/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c
^2*d*x^3 - 512*c^3)) - 2*c^2*x*(d^2/c^11)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^
6 + 1200*c^2*d^2*x^3 + 640*c^3*d + 18*(5*c^8*d^2*x^7 + 64*c^9*d*x^4 + 32*c^
10*x)*(d^2/c^11)^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2)*
(d^2/c^11)^(5/6) + (7*c^6*d^2*x^6 + 152*c^7*d*x^3 + 64*c^8)*sqrt(d^2/c^11)
+ (c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x)*(d^2/c^11)^(1/6)) + 18*(c^4*
d^3*x^8 + 38*c^5*d^2*x^5 + 64*c^6*d*x^2)*(d^2/c^11)^(1/3))/(d^3*x^9 - 24*c*
d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 216*sqrt(d)*x*weierstrassZeta(0, -4*c
/d, weierstrassPInverse(0, -4*c/d, x)) + (sqrt(-3)*c^2*x + c^2*x)*(d^2/c^11
)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d - 9*(5*
c^8*d^2*x^7 + 64*c^9*d*x^4 + 32*c^10*x + sqrt(-3)*(5*c^8*d^2*x^7 + 64*c^9*d
*x^4 + 32*c^10*x))*(d^2/c^11)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 +
32*c^11*x^2 - sqrt(-3)*(5*c^10*d*x^5 + 32*c^11*x^2))*(d^2/c^11)^(5/6) - 2*(
7*c^6*d^2*x^6 + 152*c^7*d*x^3 + 64*c^8)*sqrt(d^2/c^11) + (c^2*d^3*x^7 + 80*
c^3*d^2*x^4 + 160*c^4*d*x + sqrt(-3)*(c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^
4*d*x))*(d^2/c^11)^(1/6)) - 9*(c^4*d^3*x^8 + 38*c^5*d^2*x^5 + 64*c^6*d*x^2
- sqrt(-3)*(c^4*d^3*x^8 + 38*c^5*d^2*x^5 + 64*c^6*d*x^2))*(d^2/c^11)^(1/3))
/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (sqrt(-3)*c^2*x + c^
2*x)*(d^2/c^11)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640
*c^3*d - 9*(5*c^8*d^2*x^7 + 64*c^9*d*x^4 + 32*c^10*x + sqrt(-3)*(5*c^8*d^2*
x^7 + 64*c^9*d*x^4 + 32*c^10*x))*(d^2/c^11)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5
```

```
*c^10*d*x^5 + 32*c^11*x^2 - sqrt(-3)*(5*c^10*d*x^5 + 32*c^11*x^2))*(d^2/c^1
1)^(5/6) - 2*(7*c^6*d^2*x^6 + 152*c^7*d*x^3 + 64*c^8)*sqrt(d^2/c^11) + (c^2
*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x + sqrt(-3)*(c^2*d^3*x^7 + 80*c^3*d^
2*x^4 + 160*c^4*d*x))*(d^2/c^11)^(1/6)) - 9*(c^4*d^3*x^8 + 38*c^5*d^2*x^5 +
 64*c^6*d*x^2 - sqrt(-3)*(c^4*d^3*x^8 + 38*c^5*d^2*x^5 + 64*c^6*d*x^2))*(d^
2/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (sqrt(
-3)*c^2*x - c^2*x)*(d^2/c^11)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2
*d^2*x^3 + 640*c^3*d - 9*(5*c^8*d^2*x^7 + 64*c^9*d*x^4 + 32*c^10*x - sqrt(-
3)*(5*c^8*d^2*x^7 + 64*c^9*d*x^4 + 32*c^10*x))*(d^2/c^11)^(2/3) + 3*sqrt(d*
x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2 + sqrt(-3)*(5*c^10*d*x^5 + 32*c^11*
x^2))*(d^2/c^11)^(5/6) - 2*(7*c^6*d^2*x^6 + 152*c^7*d*x^3 + 64*c^8)*sqrt(d^
2/c^11) + (c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x - sqrt(-3)*(c^2*d^3*x
^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x))*(d^2/c^11)^(1/6)) - 9*(c^4*d^3*x^8 + 38
*c^5*d^2*x^5 + 64*c^6*d*x^2 + sqrt(-3)*(c^4*d^3*x^8 + 38*c^5*d^2*x^5 + 64*c
^6*d*x^2))*(d^2/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*
c^3)) + (sqrt(-3)*c^2*x - c^2*x)*(d^2/c^11)^(1/6)*log((d^4*x^9 + 318*c*d^3*
x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d - 9*(5*c^8*d^2*x^7 + 64*c^9*d*x^4 + 32*c
^10*x - sqrt(-3)*(5*c^8*d^2*x^7 + 64*c^9*d*x^4 + 32*c^10*x))*(d^2/c^11)^(2/
3) - 3*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2 + sqrt(-3)*(5*c^10*d*
x^5 + 32*c^11*x^2))*(d^2/c^11)^(5/6) - 2*(7*c^6*d^2*x^6 + 152*c^7*d*x^3 + 6
4*c^8)*sqrt(d^2/c^11) + (c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x - sqrt(
-3)*(c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x))*(d^2/c^11)^(1/6)) - 9*(c^
4*d^3*x^8 + 38*c^5*d^2*x^5 + 64*c^6*d*x^2 + sqrt(-3)*(c^4*d^3*x^8 + 38*c^5*
d^2*x^5 + 64*c^6*d*x^2))*(d^2/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^
2*d*x^3 - 512*c^3)) - 216*sqrt(d*x^3 + c))/(c^2*x)
```

Sympy [F]

$$\int \frac{1}{x^2 (8c - dx^3) \sqrt{c + dx^3}} \, dx = -\int \frac{1}{-8cx^2\sqrt{c + dx^3} + dx^5\sqrt{c + dx^3}} \, dx$$

```
[In] integrate(1/x**2/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
[Out] -Integral(1/(-8*c*x**2*sqrt(c + d*x**3) + d*x**5*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (8c - dx^3) \sqrt{c + dx^3}} \, dx = \int -\frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)x^2} \, dx$$

```
[In] integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (8c - dx^3) \sqrt{c + dx^3}} \, dx = \int -\frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)x^2} \, dx$$

```
[In] integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (8c - dx^3) \sqrt{c + dx^3}} \, dx = \int \frac{1}{x^2 \sqrt{d\,x^3 + c}\,(8\,c - d\,x^3)} \, dx$$

```
[In] int(1/(x^2*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
[Out] int(1/(x^2*(c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

3.319 $\int \frac{1}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 654

$$\begin{aligned}
&\int \frac{1}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{c+dx^3}}{32c^2x^4}+\frac{d\sqrt{c+dx^3}}{16c^3x}-\frac{d^{4/3}\sqrt{c+dx^3}}{16c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} \\
&-\frac{d^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{384\sqrt{3}c^{17/6}}+\frac{d^{4/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1152c^{17/6}}-\frac{d^{4/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1152c^{17/6}} \\
&+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&-\frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{8\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}$$

```
[Out] 1/1152*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c
^(17/6)-1/1152*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)-1/1152
*d^(4/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(17/
6)*3^(1/2)-1/32*(d*x^3+c)^(1/2)/c^2/x^4+1/16*d*(d*x^3+c)^(1/2)/c^3/x-1/16*d
^(4/3)*(d*x^3+c)^(1/2)/c^3/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-1/48*d^(4/3)*(c^
(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/
```

```
3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^
(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(8/3)*2^(1/2)/(d*x^3+c)^(1/
2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/
32*3^(1/4)*d^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1
/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/
2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))
)^2)^(1/2)/c^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+
c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 654, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {491, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{1}{x^5 (8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{8\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{32c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{d^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{384\sqrt{3}c^{17/6}} + \frac{d^{4/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1152c^{17/6}} - \frac{d^{4/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1152c^{17/6}} - \frac{d^{4/3}\sqrt{c+dx^3}}{16c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{d\sqrt{c+dx^3}}{16c^3x} - \frac{\sqrt{c+dx^3}}{32c^2x^4}$$

```
[In] Int[1/(x^5*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/32*Sqrt[c + d*x^3]/(c^2*x^4) + (d*Sqrt[c + d*x^3])/(16*c^3*x) - (d^(4/3)
*Sqrt[c + d*x^3])/(16*c^3*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (d^(4/3)*A
rcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(384*Sqrt[3
]*c^(17/6)) + (d^(4/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c +
```

```
d*x^3])])/(1152*c^(17/6)) - (d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/
(1152*c^(17/6)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*
Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d
^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(32*c^(8/3)*Sqrt[(c^(1/3)*(c^
(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
 - (d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/
3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[
3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[
3]])/(8*Sqrt[2]*3^(1/4)*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{c+dx^3}}{32c^2x^4}+\frac{\int \frac{-16cd+\frac{5d^2x^3}{2}}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{32c^2}\\
&= -\frac{\sqrt{c+dx^3}}{32c^2x^4}+\frac{d\sqrt{c+dx^3}}{16c^3x}-\frac{\int \frac{x(60c^2d^2-8cd^3x^3)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{256c^4}\\
&= -\frac{\sqrt{c+dx^3}}{32c^2x^4}+\frac{d\sqrt{c+dx^3}}{16c^3x}-\frac{\int \left(\frac{8cd^2x}{\sqrt{c+dx^3}}-\frac{4c^2d^2x}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{256c^4}\\
&= -\frac{\sqrt{c+dx^3}}{32c^2x^4}+\frac{d\sqrt{c+dx^3}}{16c^3x}-\frac{d^2\int \frac{x}{\sqrt{c+dx^3}}\,dx}{32c^3}+\frac{d^2\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{64c^2}\\
&= -\frac{\sqrt{c+dx^3}}{32c^2x^4}+\frac{d\sqrt{c+dx^3}}{16c^3x}-\frac{d\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{768c^3}-\frac{d^{5/3}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{32c^3}\\
&\quad+\frac{d^{5/3}\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{768c^{8/3}}+\frac{\left(\left(1-\sqrt{3}\right)d^{5/3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{32c^{8/3}}-\frac{d^{7/3}\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{256c^{7/3}}
\end{aligned}$$

$$= -\frac{\sqrt{c+dx^3}}{32c^2x^4} + \frac{d\sqrt{c+dx^3}}{16c^3x} - \frac{d^{4/3}\sqrt{c+dx^3}}{16c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$+ \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$- \frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{8\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{d^{4/3}\text{Subst}\left(\int \frac{1}{9-cx^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{384c^{7/3}} - \frac{d^{7/3}\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{768c^{7/3}}$$

$$+ \frac{d^{10/3}\text{Subst}\left(\int \frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{192c^{10/3}}$$

$$= -\frac{\sqrt{c+dx^3}}{32c^2x^4} + \frac{d\sqrt{c+dx^3}}{16c^3x} - \frac{d^{4/3}\sqrt{c+dx^3}}{16c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$-\frac{d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{384\sqrt{3}c^{17/6}} + \frac{d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1152c^{17/6}}$$

$$+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{8\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{d^{4/3}\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{384c^{7/3}}$$

$$= -\frac{\sqrt{c+dx^3}}{32c^2x^4} + \frac{d\sqrt{c+dx^3}}{16c^3x} - \frac{d^{4/3}\sqrt{c+dx^3}}{16c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{384\sqrt{3}c^{17/6}} + \frac{d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1152c^{17/6}} - \frac{d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1152c^{17/6}} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{8\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.23

$$\int \frac{1}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = \frac{160c(-c^2+cdx^3+2d^2x^6) - 75cd^2x^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right) + 4d^3x^9\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF}}{5120c^4x^4\sqrt{c+dx^3}}$$

```
[In] Integrate[1/(x^5*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (160*c*(-c^2 + c*d*x^3 + 2*d^2*x^6) - 75*c*d^2*x^6*Sqrt[1 + (d*x^3)/c]*Appe
llF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 4*d^3*x^9*Sqrt[1 + (d
*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(5120*c^4
*x^4*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.93 (sec) , antiderivative size = 882, normalized size of antiderivative = 1.35

method	result	size
risch	Expression too large to display	882
elliptic	Expression too large to display	887
default	Expression too large to display	1351

```
[In] int(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*(d*x^3+c)^(1/2)*(-2*d*x^3+c)/c^3/x^4-1/64/c^3*d^2*(-4/3*I*3^(1/2)/d*(
-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3
)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(
(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2
)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/27*I/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(
1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)
^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^
2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3
)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*
d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^
(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3
^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^
(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.11 (sec) , antiderivative size = 2403, normalized size of antiderivative = 3.67

$$\int \frac{1}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/13824*(2*c^3*x^4*(d^8/c^17)^(1/6)*log((d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2
*d^7*x^3 + 640*c^3*d^6 + 18*(5*c^12*d^3*x^7 + 64*c^13*d^2*x^4 + 32*c^14*d*x
)*(d^8/c^17)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2)*(d^8
/c^17)^(5/6) + (7*c^9*d^4*x^6 + 152*c^10*d^3*x^3 + 64*c^11*d^2)*sqrt(d^8/c^
17) + (c^3*d^7*x^7 + 80*c^4*d^6*x^4 + 160*c^5*d^5*x)*(d^8/c^17)^(1/6)) + 18
*(c^6*d^6*x^8 + 38*c^7*d^5*x^5 + 64*c^8*d^4*x^2)*(d^8/c^17)^(1/3))/(d^3*x^9
 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*c^3*x^4*(d^8/c^17)^(1/6)*lo
g((d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 + 18*(5*c^12*d^
3*x^7 + 64*c^13*d^2*x^4 + 32*c^14*d*x)*(d^8/c^17)^(2/3) - 6*sqrt(d*x^3 + c)
*(6*(5*c^15*d*x^5 + 32*c^16*x^2)*(d^8/c^17)^(5/6) + (7*c^9*d^4*x^6 + 152*c^
10*d^3*x^3 + 64*c^11*d^2)*sqrt(d^8/c^17) + (c^3*d^7*x^7 + 80*c^4*d^6*x^4 +
160*c^5*d^5*x)*(d^8/c^17)^(1/6)) + 18*(c^6*d^6*x^8 + 38*c^7*d^5*x^5 + 64*c^
8*d^4*x^2)*(d^8/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*
c^3)) + 864*d^(3/2)*x^4*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -
4*c/d, x)) + (sqrt(-3)*c^3*x^4 + c^3*x^4)*(d^8/c^17)^(1/6)*log((d^9*x^9 + 3
18*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^12*d^3*x^7 + 64*c^13
*d^2*x^4 + 32*c^14*d*x + sqrt(-3)*(5*c^12*d^3*x^7 + 64*c^13*d^2*x^4 + 32*c^
14*d*x))*(d^8/c^17)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^
2 - sqrt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2))*(d^8/c^17)^(5/6) - 2*(7*c^9*d^4*
x^6 + 152*c^10*d^3*x^3 + 64*c^11*d^2)*sqrt(d^8/c^17) + (c^3*d^7*x^7 + 80*c^
4*d^6*x^4 + 160*c^5*d^5*x + sqrt(-3)*(c^3*d^7*x^7 + 80*c^4*d^6*x^4 + 160*c^
5*d^5*x))*(d^8/c^17)^(1/6)) - 9*(c^6*d^6*x^8 + 38*c^7*d^5*x^5 + 64*c^8*d^4*
x^2 - sqrt(-3)*(c^6*d^6*x^8 + 38*c^7*d^5*x^5 + 64*c^8*d^4*x^2))*(d^8/c^17)^
(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (sqrt(-3)*c^3*
x^4 + c^3*x^4)*(d^8/c^17)^(1/6)*log((d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7
*x^3 + 640*c^3*d^6 - 9*(5*c^12*d^3*x^7 + 64*c^13*d^2*x^4 + 32*c^14*d*x + sq
rt(-3)*(5*c^12*d^3*x^7 + 64*c^13*d^2*x^4 + 32*c^14*d*x))*(d^8/c^17)^(2/3) -
 3*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2 - sqrt(-3)*(5*c^15*d*x^5
+ 32*c^16*x^2))*(d^8/c^17)^(5/6) - 2*(7*c^9*d^4*x^6 + 152*c^10*d^3*x^3 + 64
*c^11*d^2)*sqrt(d^8/c^17) + (c^3*d^7*x^7 + 80*c^4*d^6*x^4 + 160*c^5*d^5*x +
 sqrt(-3)*(c^3*d^7*x^7 + 80*c^4*d^6*x^4 + 160*c^5*d^5*x))*(d^8/c^17)^(1/6))
 - 9*(c^6*d^6*x^8 + 38*c^7*d^5*x^5 + 64*c^8*d^4*x^2 - sqrt(-3)*(c^6*d^6*x^8
 + 38*c^7*d^5*x^5 + 64*c^8*d^4*x^2))*(d^8/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*
x^6 + 192*c^2*d*x^3 - 512*c^3)) - (sqrt(-3)*c^3*x^4 - c^3*x^4)*(d^8/c^17)^(
1/6)*log((d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c
^12*d^3*x^7 + 64*c^13*d^2*x^4 + 32*c^14*d*x - sqrt(-3)*(5*c^12*d^3*x^7 + 64
*c^13*d^2*x^4 + 32*c^14*d*x))*(d^8/c^17)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^
15*d*x^5 + 32*c^16*x^2 + sqrt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2))*(d^8/c^17)^
(5/6) - 2*(7*c^9*d^4*x^6 + 152*c^10*d^3*x^3 + 64*c^11*d^2)*sqrt(d^8/c^17) +
 (c^3*d^7*x^7 + 80*c^4*d^6*x^4 + 160*c^5*d^5*x - sqrt(-3)*(c^3*d^7*x^7 + 80
*c^4*d^6*x^4 + 160*c^5*d^5*x))*(d^8/c^17)^(1/6)) - 9*(c^6*d^6*x^8 + 38*c^7*
d^5*x^5 + 64*c^8*d^4*x^2 + sqrt(-3)*(c^6*d^6*x^8 + 38*c^7*d^5*x^5 + 64*c^8*
d^4*x^2))*(d^8/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c
^3)) + (sqrt(-3)*c^3*x^4 - c^3*x^4)*(d^8/c^17)^(1/6)*log((d^9*x^9 + 318*c*d
^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^12*d^3*x^7 + 64*c^13*d^2*x
```

```
^4 + 32*c^14*d*x - sqrt(-3)*(5*c^12*d^3*x^7 + 64*c^13*d^2*x^4 + 32*c^14*d*x
))*(d^8/c^17)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2 + sq
rt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2))*(d^8/c^17)^(5/6) - 2*(7*c^9*d^4*x^6 +
152*c^10*d^3*x^3 + 64*c^11*d^2)*sqrt(d^8/c^17) + (c^3*d^7*x^7 + 80*c^4*d^6*
x^4 + 160*c^5*d^5*x - sqrt(-3)*(c^3*d^7*x^7 + 80*c^4*d^6*x^4 + 160*c^5*d^5*
x))*(d^8/c^17)^(1/6)) - 9*(c^6*d^6*x^8 + 38*c^7*d^5*x^5 + 64*c^8*d^4*x^2 +
sqrt(-3)*(c^6*d^6*x^8 + 38*c^7*d^5*x^5 + 64*c^8*d^4*x^2))*(d^8/c^17)^(1/3))
/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 432*(2*d*x^3 - c)*sq
rt(d*x^3 + c))/(c^3*x^4)
```

Sympy [F]

$$\int \frac{1}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=-\int \frac{1}{-8cx^5\sqrt{c+dx^3}+dx^8\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(1/x**5/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
[Out] -Integral(1/(-8*c*x**5*sqrt(c + d*x**3) + d*x**8*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=\int -\frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)x^5}\,dx$$

```
[In] integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=\int -\frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)x^5}\,dx$$

```
[In] integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (8c-dx^3)\sqrt{c+dx^3}}\,dx = \int \frac{1}{x^5\,\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)} \,dx$$

```
[In] int(1/(x^5*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
[Out] int(1/(x^5*(c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

3.320 $\int \frac{1}{x^8(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 678

$$
\begin{aligned}
\int \frac{1}{x^8\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx &= -\frac{\sqrt{c+dx^3}}{56c^2x^7}+\frac{37d\sqrt{c+dx^3}}{1792c^3x^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^4x} \\
&+\frac{3d^{7/3}\sqrt{c+dx^3}}{56c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{d^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3072\sqrt{3}c^{23/6}} \\
&+\frac{d^{7/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9216c^{23/6}}-\frac{d^{7/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9216c^{23/6}} \\
&-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{112c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+\frac{3^{3/4}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{28\sqrt{2}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}
$$

```
[Out] 1/9216*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c
^(23/6)-1/9216*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(23/6)-1/9216
```

```
*d^(7/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(23/
6)*3^(1/2)-1/56*(d*x^3+c)^(1/2)/c^2/x^7+37/1792*d*(d*x^3+c)^(1/2)/c^3/x^4-3
/56*d^2*(d*x^3+c)^(1/2)/c^4/x+3/56*d^(7/3)*(d*x^3+c)^(1/2)/c^4/(d^(1/3)*x+c
^(1/3)*(1+3^(1/2)))+1/56*3^(3/4)*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(
1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*
((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)
^(1/2)/c^(11/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/
3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-3/112*3^(1/4)*d^(7/3)*(c^(1/3)+d^(1/3)*x
)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))
,I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/
3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(11/3)/(d*x^3+c)^(1/2)/(
c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 678, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {491, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{1}{x^8 (8c - dx^3) \sqrt{c + dx^3}} \, dx$$

$$= \frac{3^{3/4} d^{7/3} \left(\sqrt[3]{c} + \sqrt[3]{d}x\right) \sqrt{\frac{c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c} + \sqrt[3]{d}x\right)^2}} \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x + \left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x + \left(1+\sqrt{3}\right)\sqrt[3]{c}}\right), -7 - 4\sqrt{3}\right)}{28\sqrt{2} c^{11/3} \sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c} + \sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c} + \sqrt[3]{d}x\right)^2}} \sqrt{c + dx^3}}$$

$$- \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}} d^{7/3} \left(\sqrt[3]{c} + \sqrt[3]{d}x\right) \sqrt{\frac{c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c} + \sqrt[3]{d}x\right)^2}} E\left(\arcsin\left(\frac{\sqrt[3]{d}x + \left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x + \left(1+\sqrt{3}\right)\sqrt[3]{c}}\right) | -7 - 4\sqrt{3}\right)}{112 c^{11/3} \sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c} + \sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c} + \sqrt[3]{d}x\right)^2}} \sqrt{c + dx^3}}$$

$$- \frac{d^{7/3} \arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c} + \sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3072\sqrt{3} c^{23/6}} + \frac{d^{7/3} \operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c} + \sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9216 c^{23/6}}$$

$$- \frac{d^{7/3} \operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9216 c^{23/6}} + \frac{3 d^{7/3} \sqrt{c + dx^3}}{56 c^4 \left(\left(1+\sqrt{3}\right)\sqrt[3]{c} + \sqrt[3]{d}x\right)}$$

$$- \frac{3 d^2 \sqrt{c + dx^3}}{56 c^4 x} + \frac{37 d \sqrt{c + dx^3}}{1792 c^3 x^4} - \frac{\sqrt{c + dx^3}}{56 c^2 x^7}$$

```
[In] Int[1/(x^8*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/56*Sqrt[c + d*x^3]/(c^2*x^7) + (37*d*Sqrt[c + d*x^3])/(1792*c^3*x^4) - (
3*d^2*Sqrt[c + d*x^3])/(56*c^4*x) + (3*d^(7/3)*Sqrt[c + d*x^3])/(56*c^4*((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/
3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(3072*Sqrt[3]*c^(23/6)) + (d^(7/3)*ArcTa
nh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(9216*c^(23/6)) -
(d^(7/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9216*c^(23/6)) - (3*3^(1/4)
*Sqrt[2 - Sqrt[3]]*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^
(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[Arc
Sin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)
], -7 - 4*Sqrt[3]])/(112*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (3^(3/4)*d^(7/3)*(c^(
1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^
(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(28*Sqrt[2]
*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
```

```
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{c+dx^3}}{56c^2x^7}+\frac{\int \frac{-37cd+\frac{11d^2x^3}{2}}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx}{56c^2}\\
&= -\frac{\sqrt{c+dx^3}}{56c^2x^7}+\frac{37d\sqrt{c+dx^3}}{1792c^3x^4}-\frac{\int \frac{-768c^2d^2+\frac{185}{2}cd^3x^3}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{1792c^4}\\
&= -\frac{\sqrt{c+dx^3}}{56c^2x^7}+\frac{37d\sqrt{c+dx^3}}{1792c^3x^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^4x}+\frac{\int \frac{x(3100c^3d^3-384c^2d^4x^3)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{14336c^6}\\
&= -\frac{\sqrt{c+dx^3}}{56c^2x^7}+\frac{37d\sqrt{c+dx^3}}{1792c^3x^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^4x}+\frac{\int \left(\frac{384c^2d^3x}{\sqrt{c+dx^3}}+\frac{28c^3d^3x}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{14336c^6}\\
&= -\frac{\sqrt{c+dx^3}}{56c^2x^7}+\frac{37d\sqrt{c+dx^3}}{1792c^3x^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^4x}+\frac{(3d^3)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{112c^4}+\frac{d^3\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{512c^3}
\end{aligned}
$$

$$=-\frac{\sqrt{c+dx^3}}{56c^2x^7}+\frac{37d\sqrt{c+dx^3}}{1792c^3x^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^4x}-\frac{d^2\int\frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}dx}{6144c^4}$$

$$+\frac{\left(3d^{8/3}\right)\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}dx}{112c^4}+\frac{d^{8/3}\int\frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}dx}{6144c^{11/3}}$$

$$-\frac{\left(3\left(1-\sqrt{3}\right)d^{8/3}\right)\int\frac{1}{\sqrt{c+dx^3}}dx}{112c^{11/3}}-\frac{d^{10/3}\int\frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}dx}{2048c^{10/3}}$$

$$=-\frac{\sqrt{c+dx^3}}{56c^2x^7}+\frac{37d\sqrt{c+dx^3}}{1792c^3x^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^4x}+\frac{3d^{7/3}\sqrt{c+dx^3}}{56c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{112c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{3^{3/4}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{28\sqrt{2}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{d^{7/3}\text{Subst}\left(\int\frac{1}{9-cx^2}dx,x,\frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{3072c^{10/3}}-\frac{d^{10/3}\text{Subst}\left(\int\frac{1}{(8c-dx)\sqrt{c+dx}}dx,x,x^3\right)}{6144c^{10/3}}$$

$$+\frac{d^{13/3}\text{Subst}\left(\int\frac{1}{-\frac{2d^2}{c}-6d^2x^2}dx,x,\frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{1536c^{13/3}}$$

$$= -\frac{\sqrt{c+dx^3}}{56c^2x^7} + \frac{37d\sqrt{c+dx^3}}{1792c^3x^4} - \frac{3d^2\sqrt{c+dx^3}}{56c^4x} + \frac{3d^{7/3}\sqrt{c+dx^3}}{56c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$-\frac{d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3072\sqrt{3}c^{23/6}} + \frac{d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9216c^{23/6}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{112c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{3^{3/4}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{28\sqrt{2}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{d^{7/3}\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{3072c^{10/3}}$$

$$= -\frac{\sqrt{c+dx^3}}{56c^2x^7} + \frac{37d\sqrt{c+dx^3}}{1792c^3x^4} - \frac{3d^2\sqrt{c+dx^3}}{56c^4x} + \frac{3d^{7/3}\sqrt{c+dx^3}}{56c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$-\frac{d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3072\sqrt{3}c^{23/6}} + \frac{d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9216c^{23/6}}$$

$$-\frac{d^{7/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9216c^{23/6}} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)}\right.\right.}{112c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{3^{3/4}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{28\sqrt{2}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.25

$$\int \frac{1}{x^8\,(8c-dx^3)\,\sqrt{c+dx^3}}\,dx$$
$$= \frac{3875cd^3x^9\sqrt{1+\frac{dx^3}{c}}\,\mathrm{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-32\Bigl(5c(32c^3-5c^2dx^3+59cd^2x^6+96d^3x^9)+6d^4x}{286720c^5x^7\sqrt{c+dx^3}}$$

```
[In] Integrate[1/(x^8*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (3875*c*d^3*x^9*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c)
, (d*x^3)/(8*c)] - 32*(5*c*(32*c^3 - 5*c^2*d*x^3 + 59*c*d^2*x^6 + 96*d^3*x^
9) + 6*d^4*x^12*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c)
, (d*x^3)/(8*c)]))/(286720*c^5*x^7*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.02 (sec) , antiderivative size = 895, normalized size of antiderivative = 1.32

method	result	size
risch	Expression too large to display	895
elliptic	Expression too large to display	906
default	Expression too large to display	1849

```
[In] int(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/1792*(d*x^3+c)^(1/2)*(96*d^2*x^6-37*c*d*x^3+32*c^2)/c^4/x^7+1/3584*d^3/c
^4*(-64*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/
(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-7/27*I/d^3*2^(1/2)*su
m(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*
d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(
```

```
1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^
(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/
3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/
3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*
(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^
(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z
^3*d-8*c)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.62 (sec) , antiderivative size = 2436, normalized size of antiderivative = 3.59

$$\int \frac{1}{x^8\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/774144*(14*c^4*x^7*(d^14/c^23)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6 + 120
0*c^2*d^12*x^3 + 640*c^3*d^11 + 18*(5*c^16*d^4*x^7 + 64*c^17*d^3*x^4 + 32*c
^18*d^2*x)*(d^14/c^23)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21
*x^2)*(d^14/c^23)^(5/6) + (7*c^12*d^6*x^6 + 152*c^13*d^5*x^3 + 64*c^14*d^4)
*sqrt(d^14/c^23) + (c^4*d^11*x^7 + 80*c^5*d^10*x^4 + 160*c^6*d^9*x)*(d^14/c
^23)^(1/6)) + 18*(c^8*d^9*x^8 + 38*c^9*d^8*x^5 + 64*c^10*d^7*x^2)*(d^14/c^2
3)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 14*c^4*x^7*
(d^14/c^23)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*
c^3*d^11 + 18*(5*c^16*d^4*x^7 + 64*c^17*d^3*x^4 + 32*c^18*d^2*x)*(d^14/c^23
)^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2)*(d^14/c^23)^(5/
6) + (7*c^12*d^6*x^6 + 152*c^13*d^5*x^3 + 64*c^14*d^4)*sqrt(d^14/c^23) + (c
^4*d^11*x^7 + 80*c^5*d^10*x^4 + 160*c^6*d^9*x)*(d^14/c^23)^(1/6)) + 18*(c^8
*d^9*x^8 + 38*c^9*d^8*x^5 + 64*c^10*d^7*x^2)*(d^14/c^23)^(1/3))/(d^3*x^9 -
24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 41472*d^(5/2)*x^7*weierstrassZet
a(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 7*(sqrt(-3)*c^4*x^7 + c^4
*x^7)*(d^14/c^23)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3
+ 640*c^3*d^11 - 9*(5*c^16*d^4*x^7 + 64*c^17*d^3*x^4 + 32*c^18*d^2*x + sqrt
(-3)*(5*c^16*d^4*x^7 + 64*c^17*d^3*x^4 + 32*c^18*d^2*x))*(d^14/c^23)^(2/3)
+ 3*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2 - sqrt(-3)*(5*c^20*d*x^5
 + 32*c^21*x^2))*(d^14/c^23)^(5/6) - 2*(7*c^12*d^6*x^6 + 152*c^13*d^5*x^3 +
 64*c^14*d^4)*sqrt(d^14/c^23) + (c^4*d^11*x^7 + 80*c^5*d^10*x^4 + 160*c^6*d
^9*x + sqrt(-3)*(c^4*d^11*x^7 + 80*c^5*d^10*x^4 + 160*c^6*d^9*x))*(d^14/c^2
3)^(1/6)) - 9*(c^8*d^9*x^8 + 38*c^9*d^8*x^5 + 64*c^10*d^7*x^2 - sqrt(-3)*(c
^8*d^9*x^8 + 38*c^9*d^8*x^5 + 64*c^10*d^7*x^2))*(d^14/c^23)^(1/3))/(d^3*x^9
```

```
 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 7*(sqrt(-3)*c^4*x^7 + c^4*x^7
)*(d^14/c^23)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 64
0*c^3*d^11 - 9*(5*c^16*d^4*x^7 + 64*c^17*d^3*x^4 + 32*c^18*d^2*x + sqrt(-3)
*(5*c^16*d^4*x^7 + 64*c^17*d^3*x^4 + 32*c^18*d^2*x))*(d^14/c^23)^(2/3) - 3*
sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2 - sqrt(-3)*(5*c^20*d*x^5 + 3
2*c^21*x^2))*(d^14/c^23)^(5/6) - 2*(7*c^12*d^6*x^6 + 152*c^13*d^5*x^3 + 64*
c^14*d^4)*sqrt(d^14/c^23) + (c^4*d^11*x^7 + 80*c^5*d^10*x^4 + 160*c^6*d^9*x
 + sqrt(-3)*(c^4*d^11*x^7 + 80*c^5*d^10*x^4 + 160*c^6*d^9*x))*(d^14/c^23)^(
1/6)) - 9*(c^8*d^9*x^8 + 38*c^9*d^8*x^5 + 64*c^10*d^7*x^2 - sqrt(-3)*(c^8*d
^9*x^8 + 38*c^9*d^8*x^5 + 64*c^10*d^7*x^2))*(d^14/c^23)^(1/3))/(d^3*x^9 - 2
4*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 7*(sqrt(-3)*c^4*x^7 - c^4*x^7)*(d
^14/c^23)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^
3*d^11 - 9*(5*c^16*d^4*x^7 + 64*c^17*d^3*x^4 + 32*c^18*d^2*x - sqrt(-3)*(5*
c^16*d^4*x^7 + 64*c^17*d^3*x^4 + 32*c^18*d^2*x))*(d^14/c^23)^(2/3) + 3*sqrt
(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2 + sqrt(-3)*(5*c^20*d*x^5 + 32*c^
21*x^2))*(d^14/c^23)^(5/6) - 2*(7*c^12*d^6*x^6 + 152*c^13*d^5*x^3 + 64*c^14
*d^4)*sqrt(d^14/c^23) + (c^4*d^11*x^7 + 80*c^5*d^10*x^4 + 160*c^6*d^9*x - s
qrt(-3)*(c^4*d^11*x^7 + 80*c^5*d^10*x^4 + 160*c^6*d^9*x))*(d^14/c^23)^(1/6)
) - 9*(c^8*d^9*x^8 + 38*c^9*d^8*x^5 + 64*c^10*d^7*x^2 + sqrt(-3)*(c^8*d^9*x
^8 + 38*c^9*d^8*x^5 + 64*c^10*d^7*x^2))*(d^14/c^23)^(1/3))/(d^3*x^9 - 24*c*
d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 7*(sqrt(-3)*c^4*x^7 - c^4*x^7)*(d^14/
c^23)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^
11 - 9*(5*c^16*d^4*x^7 + 64*c^17*d^3*x^4 + 32*c^18*d^2*x - sqrt(-3)*(5*c^16
*d^4*x^7 + 64*c^17*d^3*x^4 + 32*c^18*d^2*x))*(d^14/c^23)^(2/3) - 3*sqrt(d*x
^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2 + sqrt(-3)*(5*c^20*d*x^5 + 32*c^21*x
^2))*(d^14/c^23)^(5/6) - 2*(7*c^12*d^6*x^6 + 152*c^13*d^5*x^3 + 64*c^14*d^4
)*sqrt(d^14/c^23) + (c^4*d^11*x^7 + 80*c^5*d^10*x^4 + 160*c^6*d^9*x - sqrt(
-3)*(c^4*d^11*x^7 + 80*c^5*d^10*x^4 + 160*c^6*d^9*x))*(d^14/c^23)^(1/6)) -
9*(c^8*d^9*x^8 + 38*c^9*d^8*x^5 + 64*c^10*d^7*x^2 + sqrt(-3)*(c^8*d^9*x^8 +
 38*c^9*d^8*x^5 + 64*c^10*d^7*x^2))*(d^14/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*
x^6 + 192*c^2*d*x^3 - 512*c^3)) - 432*(96*d^2*x^6 - 37*c*d*x^3 + 32*c^2)*sq
rt(d*x^3 + c))/(c^4*x^7)
```

Sympy [F]

$$\int \frac{1}{x^8 (8c - dx^3) \sqrt{c + dx^3}} \, dx = - \int \frac{1}{-8cx^8\sqrt{c + dx^3} + dx^{11}\sqrt{c + dx^3}} \, dx$$

```
[In] integrate(1/x**8/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(1/(-8*c*x**8*sqrt(c + d*x**3) + d*x**11*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^8\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=\int -\frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)x^8}\,dx$$

[In] integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^8), x)

Giac [F]

$$\int \frac{1}{x^8\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=\int -\frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)x^8}\,dx$$

[In] integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^8), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^8\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=\int \frac{1}{x^8\,\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)}\,dx$$

[In] int(1/(x^8*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)

[Out] int(1/(x^8*(c + d*x^3)^(1/2)*(8*c - d*x^3)), x)

3.321 $\int \frac{x^3}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{x^3}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{x^4\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},1,\frac{1}{2},\frac{7}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{32c\sqrt{c+dx^3}}$$

```
[Out] 1/32*x^4*AppellF1(4/3,1/2,1,7/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c/(
d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {525, 524}

$$\int \frac{x^3}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{x^4\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{4}{3},1,\frac{1}{2},\frac{7}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{32c\sqrt{c+dx^3}}$$

```
[In] Int[x^3/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 1/2, 7/3, (d*x^3)/(8*c), -((d*x^3
)/c)])/(32*c*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}\\ &= \frac{x^4\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{32c\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{x^4\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{32c\sqrt{c+dx^3}}$$

```
[In] Integrate[x^3/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^4*Sqrt[(c + d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(
8*c)])/(32*c*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.36 (sec) , antiderivative size = 696, normalized size of antiderivative = 10.55

method	result
default	$\frac{2i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}}}\sqrt{-\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,F\Bigg(}{3d^{2}\sqrt{d\,x^{3}+c}}$
elliptic	$\frac{2i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}}}\sqrt{-\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,F\Bigg(}{3d^{2}\sqrt{d\,x^{3}+c}}$

```
[In] int(x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^
2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-8/2
7*I/d^4*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(
1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/
d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c
)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alph
a^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
```

```
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)
*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d
*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/
2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2284 vs. $2(52) = 104$.

Time = 0.56 (sec) , antiderivative size = 2284, normalized size of antiderivative = 34.61

$$\int \frac{x^3}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/54*(2*d^2*(1/(c*d^8))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3
 + 640*c^3 + 18*(c*d^8*x^8 + 38*c^2*d^7*x^5 + 64*c^3*d^6*x^2)*(1/(c*d^8))^(
2/3) + 6*sqrt(d*x^3 + c)*((c*d^9*x^7 + 80*c^2*d^8*x^4 + 160*c^3*d^7*x)*(1/(
c*d^8))^(5/6) + (7*c*d^6*x^6 + 152*c^2*d^5*x^3 + 64*c^3*d^4)*sqrt(1/(c*d^8)
) + 6*(5*c*d^3*x^5 + 32*c^2*d^2*x^2)*(1/(c*d^8))^(1/6)) + 18*(5*c*d^5*x^7 +
 64*c^2*d^4*x^4 + 32*c^3*d^3*x)*(1/(c*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6
+ 192*c^2*d*x^3 - 512*c^3)) - 2*d^2*(1/(c*d^8))^(1/6)*log((d^3*x^9 + 318*c*
d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(c*d^8*x^8 + 38*c^2*d^7*x^5 + 64*c^
3*d^6*x^2)*(1/(c*d^8))^(2/3) - 6*sqrt(d*x^3 + c)*((c*d^9*x^7 + 80*c^2*d^8*x
^4 + 160*c^3*d^7*x)*(1/(c*d^8))^(5/6) + (7*c*d^6*x^6 + 152*c^2*d^5*x^3 + 64
*c^3*d^4)*sqrt(1/(c*d^8)) + 6*(5*c*d^3*x^5 + 32*c^2*d^2*x^2)*(1/(c*d^8))^(1
/6)) + 18*(5*c*d^5*x^7 + 64*c^2*d^4*x^4 + 32*c^3*d^3*x)*(1/(c*d^8))^(1/3))/
(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (sqrt(-3)*d^2 + d^2)*
(1/(c*d^8))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 -
 9*(c*d^8*x^8 + 38*c^2*d^7*x^5 + 64*c^3*d^6*x^2 + sqrt(-3)*(c*d^8*x^8 + 38*
c^2*d^7*x^5 + 64*c^3*d^6*x^2))*(1/(c*d^8))^(2/3) + 3*sqrt(d*x^3 + c)*((c*d^
9*x^7 + 80*c^2*d^8*x^4 + 160*c^3*d^7*x - sqrt(-3)*(c*d^9*x^7 + 80*c^2*d^8*x
^4 + 160*c^3*d^7*x))*(1/(c*d^8))^(5/6) - 2*(7*c*d^6*x^6 + 152*c^2*d^5*x^3 +
 64*c^3*d^4)*sqrt(1/(c*d^8)) + 6*(5*c*d^3*x^5 + 32*c^2*d^2*x^2 + sqrt(-3)*(
5*c*d^3*x^5 + 32*c^2*d^2*x^2))*(1/(c*d^8))^(1/6)) - 9*(5*c*d^5*x^7 + 64*c^2
*d^4*x^4 + 32*c^3*d^3*x - sqrt(-3)*(5*c*d^5*x^7 + 64*c^2*d^4*x^4 + 32*c^3*d
^3*x))*(1/(c*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3
)) - (sqrt(-3)*d^2 + d^2)*(1/(c*d^8))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 +
1200*c^2*d*x^3 + 640*c^3 - 9*(c*d^8*x^8 + 38*c^2*d^7*x^5 + 64*c^3*d^6*x^2 +
 sqrt(-3)*(c*d^8*x^8 + 38*c^2*d^7*x^5 + 64*c^3*d^6*x^2))*(1/(c*d^8))^(2/3)
- 3*sqrt(d*x^3 + c)*((c*d^9*x^7 + 80*c^2*d^8*x^4 + 160*c^3*d^7*x - sqrt(-3)
*(c*d^9*x^7 + 80*c^2*d^8*x^4 + 160*c^3*d^7*x))*(1/(c*d^8))^(5/6) - 2*(7*c*d
^6*x^6 + 152*c^2*d^5*x^3 + 64*c^3*d^4)*sqrt(1/(c*d^8)) + 6*(5*c*d^3*x^5 + 3
```

```
2*c^2*d^2*x^2 + sqrt(-3)*(5*c*d^3*x^5 + 32*c^2*d^2*x^2))*(1/(c*d^8))^(1/6))
 - 9*(5*c*d^5*x^7 + 64*c^2*d^4*x^4 + 32*c^3*d^3*x - sqrt(-3)*(5*c*d^5*x^7 +
 64*c^2*d^4*x^4 + 32*c^3*d^3*x))*(1/(c*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6
 + 192*c^2*d*x^3 - 512*c^3)) - (sqrt(-3)*d^2 - d^2)*(1/(c*d^8))^(1/6)*log((
d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c*d^8*x^8 + 38*c^2*
d^7*x^5 + 64*c^3*d^6*x^2 - sqrt(-3)*(c*d^8*x^8 + 38*c^2*d^7*x^5 + 64*c^3*d^
6*x^2))*(1/(c*d^8))^(2/3) + 3*sqrt(d*x^3 + c)*((c*d^9*x^7 + 80*c^2*d^8*x^4
+ 160*c^3*d^7*x + sqrt(-3)*(c*d^9*x^7 + 80*c^2*d^8*x^4 + 160*c^3*d^7*x))*(1
/(c*d^8))^(5/6) - 2*(7*c*d^6*x^6 + 152*c^2*d^5*x^3 + 64*c^3*d^4)*sqrt(1/(c*
d^8)) + 6*(5*c*d^3*x^5 + 32*c^2*d^2*x^2 - sqrt(-3)*(5*c*d^3*x^5 + 32*c^2*d^
2*x^2))*(1/(c*d^8))^(1/6)) - 9*(5*c*d^5*x^7 + 64*c^2*d^4*x^4 + 32*c^3*d^3*x
 + sqrt(-3)*(5*c*d^5*x^7 + 64*c^2*d^4*x^4 + 32*c^3*d^3*x))*(1/(c*d^8))^(1/3
))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (sqrt(-3)*d^2 - d^
2)*(1/(c*d^8))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^
3 - 9*(c*d^8*x^8 + 38*c^2*d^7*x^5 + 64*c^3*d^6*x^2 - sqrt(-3)*(c*d^8*x^8 +
38*c^2*d^7*x^5 + 64*c^3*d^6*x^2))*(1/(c*d^8))^(2/3) - 3*sqrt(d*x^3 + c)*((c
*d^9*x^7 + 80*c^2*d^8*x^4 + 160*c^3*d^7*x + sqrt(-3)*(c*d^9*x^7 + 80*c^2*d^
8*x^4 + 160*c^3*d^7*x))*(1/(c*d^8))^(5/6) - 2*(7*c*d^6*x^6 + 152*c^2*d^5*x^
3 + 64*c^3*d^4)*sqrt(1/(c*d^8)) + 6*(5*c*d^3*x^5 + 32*c^2*d^2*x^2 - sqrt(-3
)*(5*c*d^3*x^5 + 32*c^2*d^2*x^2))*(1/(c*d^8))^(1/6)) - 9*(5*c*d^5*x^7 + 64*
c^2*d^4*x^4 + 32*c^3*d^3*x + sqrt(-3)*(5*c*d^5*x^7 + 64*c^2*d^4*x^4 + 32*c^
3*d^3*x))*(1/(c*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*
c^3)) - 36*sqrt(d)*weierstrassPInverse(0, -4*c/d, x))/d^2
```

Sympy [F]

$$\int \frac{x^3}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\int \frac{x^3}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(x**3/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
[Out] -Integral(x**3/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^3}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int -\frac{x^3}{\sqrt{dx^3+c}(dx^3-8\,c)}\,dx$$

```
[In] integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] -integrate(x^3/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Giac [F]

$$\int \frac{x^3}{(8c - dx^3)\sqrt{c + dx^3}}\,dx = \int -\frac{x^3}{\sqrt{dx^3 + c}(dx^3 - 8\,c)}\,dx$$

```
[In] integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(-x^3/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(8c - dx^3)\sqrt{c + dx^3}}\,dx = \int \frac{x^3}{\sqrt{d\,x^3 + c}\,(8\,c - d\,x^3)}\,dx$$

```
[In] int(x^3/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
[Out] int(x^3/((c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

3.322 $\int \frac{1}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{(8c-dx^3)\sqrt{c+dx^3}}\,dx=\frac{x\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{1}{3},1,\frac{1}{2},\frac{4}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{8c\sqrt{c+dx^3}}$$

```
[Out] 1/8*x*AppellF1(1/3,1/2,1,4/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c/(d*x
^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {441, 440}

$$\int \frac{1}{(8c-dx^3)\sqrt{c+dx^3}}\,dx=\frac{x\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},1,\frac{1}{2},\frac{4}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{8c\sqrt{c+dx^3}}$$

```
[In] Int[1/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 1/2, 4/3, (d*x^3)/(8*c), -((d*x^3)/
c)])/(8*c*Sqrt[c + d*x^3])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$

$$= \frac{x\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{8c\sqrt{c+dx^3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 166 vs. 2(64) = 128.

Time = 10.16 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.59

$$\int \frac{1}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$$

$$= \frac{32cx\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{(8c-dx^3)\sqrt{c+dx^3}\left(32c\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\left(\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4\,\text{Appe}\right.\right.}$$

```
[In] Integrate[1/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (32*c*x*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((8*c - d*
x^3)*Sqrt[c + d*x^3]*(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)
/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]
- 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.22 (sec) , antiderivative size = 416, normalized size of antiderivative = 6.50

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-c\right.}{}\right.}{2\left(-c\right.}}}{}\right)}{}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-c\right.}{}\right.}{2\left(-c\right.}}}{}\right)}{}$

```
[In] int(1/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/27*I/d^3/c*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*
d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(
2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d
*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2
*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2
)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)
^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(
1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2319 vs. $2(50) = 100$.

Time = 0.59 (sec) , antiderivative size = 2319, normalized size of antiderivative = 36.23

$$\int \frac{1}{(8c - dx^3)\sqrt{c + dx^3}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/432*(2*c*d*(1/(c^7*d^2))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*
x^3 + 640*c^3 + 18*(c^5*d^4*x^8 + 38*c^6*d^3*x^5 + 64*c^7*d^2*x^2)*(1/(c^7*
d^2))^(2/3) + 6*sqrt(d*x^3 + c)*((c^6*d^4*x^7 + 80*c^7*d^3*x^4 + 160*c^8*d^
2*x)*(1/(c^7*d^2))^(5/6) + (7*c^4*d^3*x^6 + 152*c^5*d^2*x^3 + 64*c^6*d)*sqr
t(1/(c^7*d^2)) + 6*(5*c^2*d^2*x^5 + 32*c^3*d*x^2)*(1/(c^7*d^2))^(1/6)) + 18
*(5*c^3*d^3*x^7 + 64*c^4*d^2*x^4 + 32*c^5*d*x)*(1/(c^7*d^2))^(1/3))/(d^3*x^
9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*c*d*(1/(c^7*d^2))^(1/6)*lo
g((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(c^5*d^4*x^8 + 3
8*c^6*d^3*x^5 + 64*c^7*d^2*x^2)*(1/(c^7*d^2))^(2/3) - 6*sqrt(d*x^3 + c)*((c
^6*d^4*x^7 + 80*c^7*d^3*x^4 + 160*c^8*d^2*x)*(1/(c^7*d^2))^(5/6) + (7*c^4*d
^3*x^6 + 152*c^5*d^2*x^3 + 64*c^6*d)*sqrt(1/(c^7*d^2)) + 6*(5*c^2*d^2*x^5 +
 32*c^3*d*x^2)*(1/(c^7*d^2))^(1/6)) + 18*(5*c^3*d^3*x^7 + 64*c^4*d^2*x^4 +
32*c^5*d*x)*(1/(c^7*d^2))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 -
512*c^3)) + (sqrt(-3)*c*d + c*d)*(1/(c^7*d^2))^(1/6)*log((d^3*x^9 + 318*c*d
^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c^5*d^4*x^8 + 38*c^6*d^3*x^5 + 64*c^
7*d^2*x^2 + sqrt(-3)*(c^5*d^4*x^8 + 38*c^6*d^3*x^5 + 64*c^7*d^2*x^2))*(1/(c
^7*d^2))^(2/3) + 3*sqrt(d*x^3 + c)*((c^6*d^4*x^7 + 80*c^7*d^3*x^4 + 160*c^8
*d^2*x - sqrt(-3)*(c^6*d^4*x^7 + 80*c^7*d^3*x^4 + 160*c^8*d^2*x))*(1/(c^7*d
^2))^(5/6) - 2*(7*c^4*d^3*x^6 + 152*c^5*d^2*x^3 + 64*c^6*d)*sqrt(1/(c^7*d^2
)) + 6*(5*c^2*d^2*x^5 + 32*c^3*d*x^2 + sqrt(-3)*(5*c^2*d^2*x^5 + 32*c^3*d*x
^2))*(1/(c^7*d^2))^(1/6)) - 9*(5*c^3*d^3*x^7 + 64*c^4*d^2*x^4 + 32*c^5*d*x
- sqrt(-3)*(5*c^3*d^3*x^7 + 64*c^4*d^2*x^4 + 32*c^5*d*x))*(1/(c^7*d^2))^(1/
3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (sqrt(-3)*c*d + c
*d)*(1/(c^7*d^2))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640
*c^3 - 9*(c^5*d^4*x^8 + 38*c^6*d^3*x^5 + 64*c^7*d^2*x^2 + sqrt(-3)*(c^5*d^4
*x^8 + 38*c^6*d^3*x^5 + 64*c^7*d^2*x^2))*(1/(c^7*d^2))^(2/3) - 3*sqrt(d*x^3
 + c)*((c^6*d^4*x^7 + 80*c^7*d^3*x^4 + 160*c^8*d^2*x - sqrt(-3)*(c^6*d^4*x^
7 + 80*c^7*d^3*x^4 + 160*c^8*d^2*x))*(1/(c^7*d^2))^(5/6) - 2*(7*c^4*d^3*x^6
 + 152*c^5*d^2*x^3 + 64*c^6*d)*sqrt(1/(c^7*d^2)) + 6*(5*c^2*d^2*x^5 + 32*c^
3*d*x^2 + sqrt(-3)*(5*c^2*d^2*x^5 + 32*c^3*d*x^2))*(1/(c^7*d^2))^(1/6)) - 9
*(5*c^3*d^3*x^7 + 64*c^4*d^2*x^4 + 32*c^5*d*x - sqrt(-3)*(5*c^3*d^3*x^7 + 6
4*c^4*d^2*x^4 + 32*c^5*d*x))*(1/(c^7*d^2))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 +
 192*c^2*d*x^3 - 512*c^3)) - (sqrt(-3)*c*d - c*d)*(1/(c^7*d^2))^(1/6)*log((
d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c^5*d^4*x^8 + 38*c^
6*d^3*x^5 + 64*c^7*d^2*x^2 - sqrt(-3)*(c^5*d^4*x^8 + 38*c^6*d^3*x^5 + 64*c^
```

```
7*d^2*x^2))*(1/(c^7*d^2))^(2/3) + 3*sqrt(d*x^3 + c)*((c^6*d^4*x^7 + 80*c^7*
d^3*x^4 + 160*c^8*d^2*x + sqrt(-3)*(c^6*d^4*x^7 + 80*c^7*d^3*x^4 + 160*c^8*
d^2*x))*(1/(c^7*d^2))^(5/6) - 2*(7*c^4*d^3*x^6 + 152*c^5*d^2*x^3 + 64*c^6*d
)*sqrt(1/(c^7*d^2)) + 6*(5*c^2*d^2*x^5 + 32*c^3*d*x^2 - sqrt(-3)*(5*c^2*d^2
*x^5 + 32*c^3*d*x^2))*(1/(c^7*d^2))^(1/6)) - 9*(5*c^3*d^3*x^7 + 64*c^4*d^2*
x^4 + 32*c^5*d*x + sqrt(-3)*(5*c^3*d^3*x^7 + 64*c^4*d^2*x^4 + 32*c^5*d*x))*
(1/(c^7*d^2))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) +
(sqrt(-3)*c*d - c*d)*(1/(c^7*d^2))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 120
0*c^2*d*x^3 + 640*c^3 - 9*(c^5*d^4*x^8 + 38*c^6*d^3*x^5 + 64*c^7*d^2*x^2 -
sqrt(-3)*(c^5*d^4*x^8 + 38*c^6*d^3*x^5 + 64*c^7*d^2*x^2))*(1/(c^7*d^2))^(2/
3) - 3*sqrt(d*x^3 + c)*((c^6*d^4*x^7 + 80*c^7*d^3*x^4 + 160*c^8*d^2*x + sqr
t(-3)*(c^6*d^4*x^7 + 80*c^7*d^3*x^4 + 160*c^8*d^2*x))*(1/(c^7*d^2))^(5/6) -
 2*(7*c^4*d^3*x^6 + 152*c^5*d^2*x^3 + 64*c^6*d)*sqrt(1/(c^7*d^2)) + 6*(5*c^
2*d^2*x^5 + 32*c^3*d*x^2 - sqrt(-3)*(5*c^2*d^2*x^5 + 32*c^3*d*x^2))*(1/(c^7
*d^2))^(1/6)) - 9*(5*c^3*d^3*x^7 + 64*c^4*d^2*x^4 + 32*c^5*d*x + sqrt(-3)*(
5*c^3*d^3*x^7 + 64*c^4*d^2*x^4 + 32*c^5*d*x))*(1/(c^7*d^2))^(1/3))/(d^3*x^9
 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 72*sqrt(d)*weierstrassPInvers
e(0, -4*c/d, x))/(c*d)
```

Sympy [F]

$$\int \frac{1}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\int \frac{1}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(1/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
[Out] -Integral(1/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int -\frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)}\,dx$$

```
[In] integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Giac [F]

$$\int \frac{1}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int -\frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)}\,dx$$

```
[In] integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{d\,x^3+c}\,(8\,c-d\,x^3)}\,dx$$

```
[In] int(1/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
[Out] int(1/((c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

3.323 $\int \frac{1}{x^3(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{1}{x^3(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{16cx^2\sqrt{c+dx^3}}$$

```
[Out] -1/16*AppellF1(-2/3,1/2,1,1/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c/x^2
/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {525, 524}

$$\int \frac{1}{x^3(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{16cx^2\sqrt{c+dx^3}}$$

```
[In] Int[1/(x^3*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/16*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 1/2, 1/3, (d*x^3)/(8*c), -((d*
x^3)/c)])/(c*x^2*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^3(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$

$$= -\frac{\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{16cx^2\sqrt{c+dx^3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 242 vs. $2(66) = 132$.

Time = 11.24 (sec) , antiderivative size = 242, normalized size of antiderivative = 3.67

$$\int \frac{1}{x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx$$
$$= \frac{-\frac{64(c+dx^3)}{c^2}+\frac{d^2x^6\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^3}+\frac{4096dx^3\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c}\right.}{(-8c+dx^3)\left(32c\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\left(\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\right.\right.\right.}}{1024x^2\sqrt{c+dx^3}}$$

```
[In] Integrate[1/(x^3*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] ((-64*(c + d*x^3))/c^2 + (d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1,
 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])/c^3 + (4096*d*x^3*AppellF1[1/3, 1/2, 1,
 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((-8*c + d*x^3)*(32*c*AppellF1[1/3, 1/2
, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3
, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c),
 (d*x^3)/(8*c)]))))/(1024*x^2*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.90 (sec) , antiderivative size = 716, normalized size of antiderivative = 10.85

method	result
elliptic	$-\frac{\sqrt{d\,x^{3}+c}}{16c^{2}x^{2}}+\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}}}\sqrt{-\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right.}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}}{48c^{2}\sqrt{d\,x^{3}+c}}$
risch	Expression too large to display
default	Expression too large to display

```
[In] int(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(d*x^3+c)^(1/2)/c^2/x^2+1/48*I/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d
*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3)))^(1/2))-1/216*I/d^2/c^2*2^(1/2)*sum(1/_alpha^2*(-c*d^2)
^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^
2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*
d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1
/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2
)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2
)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)
*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2
)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2373 vs. 2(52) = 104.

Time = 0.99 (sec) , antiderivative size = 2373, normalized size of antiderivative = 35.95

$$\int \frac{1}{x^3 (8c - dx^3) \sqrt{c + dx^3}} dx = \text{Too large to display}$$

```
[In] integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
[Out] 1/3456*(2*c^2*x^2*(d^4/c^13)^(1/6)*log((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*
d^4*x^3 + 640*c^3*d^3 + 18*(c^9*d^3*x^8 + 38*c^10*d^2*x^5 + 64*c^11*d*x^2)*
(d^4/c^13)^(2/3) + 6*sqrt(d*x^3 + c)*((c^11*d^2*x^7 + 80*c^12*d*x^4 + 160*c
^13*x)*(d^4/c^13)^(5/6) + (7*c^7*d^3*x^6 + 152*c^8*d^2*x^3 + 64*c^9*d)*sqrt
(d^4/c^13) + 6*(5*c^3*d^4*x^5 + 32*c^4*d^3*x^2)*(d^4/c^13)^(1/6)) + 18*(5*c
^5*d^4*x^7 + 64*c^6*d^3*x^4 + 32*c^7*d^2*x)*(d^4/c^13)^(1/3))/(d^3*x^9 - 24
*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*c^2*x^2*(d^4/c^13)^(1/6)*log((d^
6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 + 18*(c^9*d^3*x^8 +
38*c^10*d^2*x^5 + 64*c^11*d*x^2)*(d^4/c^13)^(2/3) - 6*sqrt(d*x^3 + c)*((c^1
1*d^2*x^7 + 80*c^12*d*x^4 + 160*c^13*x)*(d^4/c^13)^(5/6) + (7*c^7*d^3*x^6 +
 152*c^8*d^2*x^3 + 64*c^9*d)*sqrt(d^4/c^13) + 6*(5*c^3*d^4*x^5 + 32*c^4*d^3
*x^2)*(d^4/c^13)^(1/6)) + 18*(5*c^5*d^4*x^7 + 64*c^6*d^3*x^4 + 32*c^7*d^2*x
)*(d^4/c^13)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 1
44*sqrt(d)*x^2*weierstrassPInverse(0, -4*c/d, x) + (sqrt(-3)*c^2*x^2 + c^2*
x^2)*(d^4/c^13)^(1/6)*log((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640
*c^3*d^3 - 9*(c^9*d^3*x^8 + 38*c^10*d^2*x^5 + 64*c^11*d*x^2 + sqrt(-3)*(c^9
*d^3*x^8 + 38*c^10*d^2*x^5 + 64*c^11*d*x^2))*(d^4/c^13)^(2/3) + 3*sqrt(d*x^
3 + c)*((c^11*d^2*x^7 + 80*c^12*d*x^4 + 160*c^13*x - sqrt(-3)*(c^11*d^2*x^7
 + 80*c^12*d*x^4 + 160*c^13*x))*(d^4/c^13)^(5/6) - 2*(7*c^7*d^3*x^6 + 152*c
^8*d^2*x^3 + 64*c^9*d)*sqrt(d^4/c^13) + 6*(5*c^3*d^4*x^5 + 32*c^4*d^3*x^2 +
 sqrt(-3)*(5*c^3*d^4*x^5 + 32*c^4*d^3*x^2))*(d^4/c^13)^(1/6)) - 9*(5*c^5*d^
4*x^7 + 64*c^6*d^3*x^4 + 32*c^7*d^2*x - sqrt(-3)*(5*c^5*d^4*x^7 + 64*c^6*d^
3*x^4 + 32*c^7*d^2*x))*(d^4/c^13)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*
d*x^3 - 512*c^3)) - (sqrt(-3)*c^2*x^2 + c^2*x^2)*(d^4/c^13)^(1/6)*log((d^6*
x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 - 9*(c^9*d^3*x^8 + 38*
c^10*d^2*x^5 + 64*c^11*d*x^2 + sqrt(-3)*(c^9*d^3*x^8 + 38*c^10*d^2*x^5 + 64
*c^11*d*x^2))*(d^4/c^13)^(2/3) - 3*sqrt(d*x^3 + c)*((c^11*d^2*x^7 + 80*c^12
*d*x^4 + 160*c^13*x - sqrt(-3)*(c^11*d^2*x^7 + 80*c^12*d*x^4 + 160*c^13*x))
*(d^4/c^13)^(5/6) - 2*(7*c^7*d^3*x^6 + 152*c^8*d^2*x^3 + 64*c^9*d)*sqrt(d^4
/c^13) + 6*(5*c^3*d^4*x^5 + 32*c^4*d^3*x^2 + sqrt(-3)*(5*c^3*d^4*x^5 + 32*c
^4*d^3*x^2))*(d^4/c^13)^(1/6)) - 9*(5*c^5*d^4*x^7 + 64*c^6*d^3*x^4 + 32*c^7
*d^2*x - sqrt(-3)*(5*c^5*d^4*x^7 + 64*c^6*d^3*x^4 + 32*c^7*d^2*x))*(d^4/c^1
3)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (sqrt(-3)*c
^2*x^2 - c^2*x^2)*(d^4/c^13)^(1/6)*log((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*
d^4*x^3 + 640*c^3*d^3 - 9*(c^9*d^3*x^8 + 38*c^10*d^2*x^5 + 64*c^11*d*x^2 -
```

```
sqrt(-3)*(c^9*d^3*x^8 + 38*c^10*d^2*x^5 + 64*c^11*d*x^2))*(d^4/c^13)^(2/3)
+ 3*sqrt(d*x^3 + c)*((c^11*d^2*x^7 + 80*c^12*d*x^4 + 160*c^13*x + sqrt(-3)*
(c^11*d^2*x^7 + 80*c^12*d*x^4 + 160*c^13*x))*(d^4/c^13)^(5/6) - 2*(7*c^7*d^
3*x^6 + 152*c^8*d^2*x^3 + 64*c^9*d)*sqrt(d^4/c^13) + 6*(5*c^3*d^4*x^5 + 32*
c^4*d^3*x^2 - sqrt(-3)*(5*c^3*d^4*x^5 + 32*c^4*d^3*x^2))*(d^4/c^13)^(1/6))
- 9*(5*c^5*d^4*x^7 + 64*c^6*d^3*x^4 + 32*c^7*d^2*x + sqrt(-3)*(5*c^5*d^4*x^
7 + 64*c^6*d^3*x^4 + 32*c^7*d^2*x))*(d^4/c^13)^(1/3))/(d^3*x^9 - 24*c*d^2*x
^6 + 192*c^2*d*x^3 - 512*c^3)) + (sqrt(-3)*c^2*x^2 - c^2*x^2)*(d^4/c^13)^(1
/6)*log((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 - 9*(c^9*
d^3*x^8 + 38*c^10*d^2*x^5 + 64*c^11*d*x^2 - sqrt(-3)*(c^9*d^3*x^8 + 38*c^10
*d^2*x^5 + 64*c^11*d*x^2))*(d^4/c^13)^(2/3) - 3*sqrt(d*x^3 + c)*((c^11*d^2*
x^7 + 80*c^12*d*x^4 + 160*c^13*x + sqrt(-3)*(c^11*d^2*x^7 + 80*c^12*d*x^4 +
 160*c^13*x))*(d^4/c^13)^(5/6) - 2*(7*c^7*d^3*x^6 + 152*c^8*d^2*x^3 + 64*c^
9*d)*sqrt(d^4/c^13) + 6*(5*c^3*d^4*x^5 + 32*c^4*d^3*x^2 - sqrt(-3)*(5*c^3*d
^4*x^5 + 32*c^4*d^3*x^2))*(d^4/c^13)^(1/6)) - 9*(5*c^5*d^4*x^7 + 64*c^6*d^3
*x^4 + 32*c^7*d^2*x + sqrt(-3)*(5*c^5*d^4*x^7 + 64*c^6*d^3*x^4 + 32*c^7*d^2
*x))*(d^4/c^13)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3))
- 216*sqrt(d*x^3 + c))/(c^2*x^2)
```

Sympy [F]

$$\int \frac{1}{x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=-\int \frac{1}{-8cx^3\sqrt{c+dx^3}+dx^6\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(1/x**3/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
[Out] -Integral(1/(-8*c*x**3*sqrt(c + d*x**3) + d*x**6*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=\int -\frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)x^3}\,dx$$

```
[In] integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=\int -\frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)x^3}\,dx$$

```
[In] integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx=\int \frac{1}{x^3\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)}\,dx$$

```
[In] int(1/(x^3*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
[Out] int(1/(x^3*(c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

3.324 $\int \frac{1}{x^6(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{1}{x^6 (8c-dx^3) \sqrt{c+dx^3}}\,dx = -\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{5}{3},1,\frac{1}{2},-\frac{2}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{40cx^5\sqrt{c+dx^3}}$$

```
[Out] -1/40*AppellF1(-5/3,1/2,1,-2/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c/x^
5/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {525, 524}

$$\int \frac{1}{x^6 (8c-dx^3) \sqrt{c+dx^3}}\,dx = -\frac{\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{5}{3},1,\frac{1}{2},-\frac{2}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{40cx^5\sqrt{c+dx^3}}$$

```
[In] Int[1/(x^6*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/40*(Sqrt[1 + (d*x^3)/c]*AppellF1[-5/3, 1, 1/2, -2/3, (d*x^3)/(8*c), -((d
*x^3)/c)])/(c*x^5*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^6(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$= -\frac{\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{5}{3};1,\frac{1}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{40cx^5\sqrt{c+dx^3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 261 vs. $2(66)=132$.

Time = 11.22 (sec) , antiderivative size = 261, normalized size of antiderivative = 3.95

$$\int \frac{1}{x^6\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx$$
$$= \frac{-23d^3x^9\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+64c\left(-16c^2+7cdx^3+23d^2x^6+\frac{\cdots}{(8c-dx^3)\left(32c\,\text{AppellF1}\left(\frac{1}{3},\cdots\right.\right.}\right.}{40960c^4x^5\sqrt{c+dx^3}}$$

```
[In] Integrate[1/(x^6*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-23*d^3*x^9*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (
d*x^3)/(8*c)] + 64*c*(-16*c^2 + 7*c*d*x^3 + 23*d^2*x^6 + (3264*c^2*d^2*x^6*
AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((8*c - d*x^3)*(32
*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(Appel
lF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1
, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])))))/(40960*c^4*x^5*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.92 (sec) , antiderivative size = 732, normalized size of antiderivative = 11.09

method	result	size
risch	Expression too large to display	732
elliptic	Expression too large to display	735
default	Expression too large to display	1047

```
[In] int(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/640*(d*x^3+c)^(1/2)*(-23*d*x^3+16*c)/c^3/x^5+1/1280/c^3*d^2*(-46/3*I*3^(
1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d
^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3
)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)
^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-20/27*I/d^3*2^
(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*
(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/
2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I
*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-
c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-
c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_
alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^
(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha
=RootOf(_Z^3*d-8*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2417 vs. 2(52) = 104.

Time = 2.51 (sec) , antiderivative size = 2417, normalized size of antiderivative = 36.62

$$\int \frac{1}{x^6 (8c-dx^3) \sqrt{c+dx^3}} \, dx = \text{Too large to display}$$

```
[In] integrate(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/138240*(10*c^3*x^5*(d^10/c^19)^(1/6)*log((d^11*x^9 + 318*c*d^10*x^6 + 120
0*c^2*d^9*x^3 + 640*c^3*d^8 + 18*(c^13*d^4*x^8 + 38*c^14*d^3*x^5 + 64*c^15*
```

d^2*x^2)*(d^10/c^19)^(2/3) + 6*sqrt(d*x^3 + c)*((c^16*d^2*x^7 + 80*c^17*d*x^4 + 160*c^18*x)*(d^10/c^19)^(5/6) + (7*c^10*d^5*x^6 + 152*c^11*d^4*x^3 + 64*c^12*d^3)*sqrt(d^10/c^19) + 6*(5*c^4*d^8*x^5 + 32*c^5*d^7*x^2)*(d^10/c^19)^(1/6)) + 18*(5*c^7*d^7*x^7 + 64*c^8*d^6*x^4 + 32*c^9*d^5*x)*(d^10/c^19)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 10*c^3*x^5*(d^10/c^19)^(1/6)*log((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 + 18*(c^13*d^4*x^8 + 38*c^14*d^3*x^5 + 64*c^15*d^2*x^2)*(d^10/c^19)^(2/3) - 6*sqrt(d*x^3 + c)*((c^16*d^2*x^7 + 80*c^17*d*x^4 + 160*c^18*x)*(d^10/c^19)^(5/6) + (7*c^10*d^5*x^6 + 152*c^11*d^4*x^3 + 64*c^12*d^3)*sqrt(d^10/c^19) + 6*(5*c^4*d^8*x^5 + 32*c^5*d^7*x^2)*(d^10/c^19)^(1/6)) + 18*(5*c^7*d^7*x^7 + 64*c^8*d^6*x^4 + 32*c^9*d^5*x)*(d^10/c^19)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 5328*d^(3/2)*x^5*weierstrassPInverse(0, -4*c/d, x) + 5*(sqrt(-3)*c^3*x^5 + c^3*x^5)*(d^10/c^19)^(1/6)*log((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 - 9*(c^13*d^4*x^8 + 38*c^14*d^3*x^5 + 64*c^15*d^2*x^2 + sqrt(-3)*(c^13*d^4*x^8 + 38*c^14*d^3*x^5 + 64*c^15*d^2*x^2))*(d^10/c^19)^(2/3) + 3*sqrt(d*x^3 + c)*((c^16*d^2*x^7 + 80*c^17*d*x^4 + 160*c^18*x - sqrt(-3)*(c^16*d^2*x^7 + 80*c^17*d*x^4 + 160*c^18*x))*(d^10/c^19)^(5/6) - 2*(7*c^10*d^5*x^6 + 152*c^11*d^4*x^3 + 64*c^12*d^3)*sqrt(d^10/c^19) + 6*(5*c^4*d^8*x^5 + 32*c^5*d^7*x^2 + sqrt(-3)*(5*c^4*d^8*x^5 + 32*c^5*d^7*x^2))*(d^10/c^19)^(1/6)) - 9*(5*c^7*d^7*x^7 + 64*c^8*d^6*x^4 + 32*c^9*d^5*x - sqrt(-3)*(5*c^7*d^7*x^7 + 64*c^8*d^6*x^4 + 32*c^9*d^5*x))*(d^10/c^19)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 5*(sqrt(-3)*c^3*x^5 + c^3*x^5)*(d^10/c^19)^(1/6)*log((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 - 9*(c^13*d^4*x^8 + 38*c^14*d^3*x^5 + 64*c^15*d^2*x^2 + sqrt(-3)*(c^13*d^4*x^8 + 38*c^14*d^3*x^5 + 64*c^15*d^2*x^2))*(d^10/c^19)^(2/3) - 3*sqrt(d*x^3 + c)*((c^16*d^2*x^7 + 80*c^17*d*x^4 + 160*c^18*x - sqrt(-3)*(c^16*d^2*x^7 + 80*c^17*d*x^4 + 160*c^18*x))*(d^10/c^19)^(5/6) - 2*(7*c^10*d^5*x^6 + 152*c^11*d^4*x^3 + 64*c^12*d^3)*sqrt(d^10/c^19) + 6*(5*c^4*d^8*x^5 + 32*c^5*d^7*x^2 + sqrt(-3)*(5*c^4*d^8*x^5 + 32*c^5*d^7*x^2))*(d^10/c^19)^(1/6)) - 9*(5*c^7*d^7*x^7 + 64*c^8*d^6*x^4 + 32*c^9*d^5*x - sqrt(-3)*(5*c^7*d^7*x^7 + 64*c^8*d^6*x^4 + 32*c^9*d^5*x))*(d^10/c^19)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 5*(sqrt(-3)*c^3*x^5 - c^3*x^5)*(d^10/c^19)^(1/6)*log((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 - 9*(c^13*d^4*x^8 + 38*c^14*d^3*x^5 + 64*c^15*d^2*x^2 - sqrt(-3)*(c^13*d^4*x^8 + 38*c^14*d^3*x^5 + 64*c^15*d^2*x^2))*(d^10/c^19)^(2/3) + 3*sqrt(d*x^3 + c)*((c^16*d^2*x^7 + 80*c^17*d*x^4 + 160*c^18*x + sqrt(-3)*(c^16*d^2*x^7 + 80*c^17*d*x^4 + 160*c^18*x))*(d^10/c^19)^(5/6) - 2*(7*c^10*d^5*x^6 + 152*c^11*d^4*x^3 + 64*c^12*d^3)*sqrt(d^10/c^19) + 6*(5*c^4*d^8*x^5 + 32*c^5*d^7*x^2 - sqrt(-3)*(5*c^4*d^8*x^5 + 32*c^5*d^7*x^2))*(d^10/c^19)^(1/6)) - 9*(5*c^7*d^7*x^7 + 64*c^8*d^6*x^4 + 32*c^9*d^5*x + sqrt(-3)*(5*c^7*d^7*x^7 + 64*c^8*d^6*x^4 + 32*c^9*d^5*x))*(d^10/c^19)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 5*(sqrt(-3)*c^3*x^5 - c^3*x^5)*(d^10/c^19)^(1/6)*log((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 - 9*(c^13*d^4*x^8 + 38*c^14*d^3*x^5 + 64*c^15*d^2*x^2 - sqrt(-3)*(c^13*d^4*x^8 + 38*c^14*d^3*x^5 + 64*c^15*d^2*x^2))*(d^10/c^19)^

```
(2/3) - 3*sqrt(d*x^3 + c)*((c^16*d^2*x^7 + 80*c^17*d*x^4 + 160*c^18*x + sqr
t(-3)*(c^16*d^2*x^7 + 80*c^17*d*x^4 + 160*c^18*x))*(d^10/c^19)^(5/6) - 2*(7
*c^10*d^5*x^6 + 152*c^11*d^4*x^3 + 64*c^12*d^3)*sqrt(d^10/c^19) + 6*(5*c^4*
d^8*x^5 + 32*c^5*d^7*x^2 - sqrt(-3)*(5*c^4*d^8*x^5 + 32*c^5*d^7*x^2))*(d^10
/c^19)^(1/6)) - 9*(5*c^7*d^7*x^7 + 64*c^8*d^6*x^4 + 32*c^9*d^5*x + sqrt(-3)
*(5*c^7*d^7*x^7 + 64*c^8*d^6*x^4 + 32*c^9*d^5*x))*(d^10/c^19)^(1/3))/(d^3*x
^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 216*(23*d*x^3 - 16*c)*sqrt(
d*x^3 + c))/(c^3*x^5)
```

Sympy [F]

$$\int \frac{1}{x^6\,(8c-dx^3)\,\sqrt{c+dx^3}}\,dx = -\int \frac{1}{-8cx^6\sqrt{c+dx^3}+dx^9\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(1/x**6/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
[Out] -Integral(1/(-8*c*x**6*sqrt(c + d*x**3) + d*x**9*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^6\,(8c-dx^3)\,\sqrt{c+dx^3}}\,dx = \int -\frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)x^6}\,dx$$

```
[In] integrate(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6\,(8c-dx^3)\,\sqrt{c+dx^3}}\,dx = \int -\frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)x^6}\,dx$$

```
[In] integrate(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (8c - dx^3) \sqrt{c + dx^3}} \, dx = \int \frac{1}{x^6 \sqrt{d x^3 + c} \, (8 c - d x^3)} \, dx$$

```
[In] int(1/(x^6*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
[Out] int(1/(x^6*(c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

3.325 $\int \frac{x^{11}}{(8c-dx^3)(c+dx^3)^{3/2}} dx$

Optimal result

Integrand size = 27, antiderivative size = 90

$$\int \frac{x^{11}}{(8c-dx^3)(c+dx^3)^{3/2}} dx = \frac{2c^2}{27d^4\sqrt{c+dx^3}} - \frac{4c\sqrt{c+dx^3}}{d^4} - \frac{2(c+dx^3)^{3/2}}{9d^4} + \frac{1024c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4}$$

```
[Out] -2/9*(d*x^3+c)^(3/2)/d^4+1024/81*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2
))/d^4+2/27*c^2/d^4/(d*x^3+c)^(1/2)-4*c*(d*x^3+c)^(1/2)/d^4
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {457, 89, 45, 65, 212}

$$\int \frac{x^{11}}{(8c-dx^3)(c+dx^3)^{3/2}} dx = \frac{1024c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4} + \frac{2c^2}{27d^4\sqrt{c+dx^3}} - \frac{4c\sqrt{c+dx^3}}{d^4} - \frac{2(c+dx^3)^{3/2}}{9d^4}$$

```
[In] Int[x^11/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*c^2)/(27*d^4*Sqrt[c + d*x^3]) - (4*c*Sqrt[c + d*x^3])/d^4 - (2*(c + d*x^
3)^(3/2))/(9*d^4) + (1024*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81
*d^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{c^2}{9d^3(c+dx)^{3/2}} - \frac{7c}{d^3\sqrt{c+dx}} - \frac{x}{d^2\sqrt{c+dx}} + \frac{512c^2}{9d^3(8c-dx)\sqrt{c+dx}}\right)\,dx, x, x^3\right)\end{aligned}$$

$$= \frac{2c^2}{27d^4\sqrt{c+dx^3}} - \frac{14c\sqrt{c+dx^3}}{3d^4} + \frac{(512c^2)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27d^3} - \frac{\text{Subst}\left(\int \frac{x}{\sqrt{c+dx}}\,dx, x, x^3\right)}{3d^2}$$

$$= \frac{2c^2}{27d^4\sqrt{c+dx^3}} - \frac{14c\sqrt{c+dx^3}}{3d^4} + \frac{(1024c^2)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{27d^4} - \frac{\text{Subst}\left(\int \left(-\frac{c}{d\sqrt{c+dx}} + \frac{\sqrt{c+dx}}{d}\right)\,dx, x, x^3\right)}{3d^2}$$

$$= \frac{2c^2}{27d^4\sqrt{c+dx^3}} - \frac{4c\sqrt{c+dx^3}}{d^4} - \frac{2(c+dx^3)^{3/2}}{9d^4} + \frac{1024c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.78

$$\int \frac{x^{11}}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \frac{2\left(-\frac{3(56c^2+60cdx^3+3d^2x^6)}{\sqrt{c+dx^3}} + 512c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{81d^4}$$

```
[In] Integrate[x^11/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
[Out] (2*((-3*(56*c^2 + 60*c*d*x^3 + 3*d^2*x^6))/Sqrt[c + d*x^3] + 512*c^(3/2)*Ar
cTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(81*d^4)
```

Maple [A] (verified)

Time = 4.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$\frac{-\frac{2d^2x^6}{9}+\frac{1024c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)\sqrt{dx^3+c}}{81}-\frac{40cdx^3}{9}-\frac{112c^2}{27}}{\sqrt{dx^3+c}\,d^4}$
risch	$-\frac{2(dx^3+19c)\sqrt{dx^3+c}}{9d^4}-\frac{c^2\left(-\frac{2}{27d\sqrt{dx^3+c}}-\frac{1024\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{81d\sqrt{c}}\right)}{d^3}$
default	$-\frac{-\frac{2c^2}{3d^3\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{2x^3\sqrt{dx^3+c}}{9d^2}-\frac{10c\sqrt{dx^3+c}}{9d^3}}{d}-\frac{8c\left(\frac{2c}{3d^2\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{2\sqrt{dx^3+c}}{3d^2}\right)}{d^2}+\frac{128c^2}{3d^4\sqrt{dx^3+c}}-\frac{1024c^{\frac{3}{2}}\left(-\frac{\operatorname{arctanh}(\cdots}{\cdots}\right.}{27d\cdots}$
elliptic	$\frac{2c^2}{27d^4\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{2x^3\sqrt{dx^3+c}}{9d^3}-\frac{38c\sqrt{dx^3+c}}{9d^4}-\frac{512ic\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-cd^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right)\cdots}{d}\right)}{\left(-cd^2\right)\cdots}}\cdots}{\cdots}\right.}{\cdots}$

[In] int(x^11/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/81*(-9*d^2*x^6+512*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*(d*x^3+c)^(1/2)-180*c*d*x^3-168*c^2)/(d*x^3+c)^(1/2)/d^4

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.10

$$\int \frac{x^{11}}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \left[\frac{2\left(256\,(cdx^3+c^2)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-3\,(3\,d^2x^6+60\,cdx^3+56\,c^2\cdots\right.}{81\,(d^5x^3+cd^4)}\right.$$

$$\left.-\frac{2\left(512\,(cdx^3+c^2)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+3\,(3\,d^2x^6+60\,cdx^3+56\,c^2)\sqrt{dx^3+c}\right)}{81\,(d^5x^3+cd^4)}\right]$$

```
[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [2/81*(256*(c*d*x^3 + c^2)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) +
 10*c)/(d*x^3 - 8*c)) - 3*(3*d^2*x^6 + 60*c*d*x^3 + 56*c^2)*sqrt(d*x^3 + c)
)/(d^5*x^3 + c*d^4), -2/81*(512*(c*d*x^3 + c^2)*sqrt(-c)*arctan(1/3*sqrt(d*
x^3 + c)*sqrt(-c)/c) + 3*(3*d^2*x^6 + 60*c*d*x^3 + 56*c^2)*sqrt(d*x^3 + c))
/(d^5*x^3 + c*d^4)]
```

Sympy [A] (verification not implemented)

Time = 21.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{c^2}{27\sqrt{c+dx^3}} - \frac{512c^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{81\sqrt{-c}} - 2c\sqrt{c+dx^3} - \frac{\left(c+dx^3\right)^{\frac{3}{2}}}{9}\right)}{d^4} & \text{for } d \neq 0 \\ \frac{x^{12}}{96c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**11/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Piecewise((2*(c**2/(27*sqrt(c + d*x**3)) - 512*c**2*atan(sqrt(c + d*x**3)/(
3*sqrt(-c)))/(81*sqrt(-c)) - 2*c*sqrt(c + d*x**3) - (c + d*x**3)**(3/2)/9)/
d**4, Ne(d, 0)), (x**12/(96*c**(5/2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int \frac{x^{11}}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = -\frac{2\left(256\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right) + 9\left(dx^3+c\right)^{\frac{3}{2}} + 162\sqrt{dx^3+c}c - \frac{3c^2}{\sqrt{dx^3+c}}\right)}{81\,d^4}$$

```
[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/81*(256*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*s
qrt(c))) + 9*(d*x^3 + c)^(3/2) + 162*sqrt(d*x^3 + c)*c - 3*c^2/sqrt(d*x^3 +
 c))/d^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int \frac{x^{11}}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = -\frac{1024\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}d^4} + \frac{2\,c^2}{27\sqrt{dx^3+c}d^4} - \frac{2\left((dx^3+c)^{\frac{3}{2}}d^8+18\sqrt{dx^3+c}cd^8\right)}{9\,d^{12}}$$

```
[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1024/81*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) + 2/27*c^2
/(sqrt(d*x^3 + c)*d^4) - 2/9*((d*x^3 + c)^(3/2)*d^8 + 18*sqrt(d*x^3 + c)*c*
d^8)/d^12
```

Mupad [B] (verification not implemented)

Time = 8.73 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.06

$$\int \frac{x^{11}}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{512\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,d^4} - \frac{38\,c\,\sqrt{d\,x^3+c}}{9\,d^4} + \frac{2\,c^2}{27\,d^4\,\sqrt{d\,x^3+c}} - \frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d^3}$$

```
[In] int(x^11/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] (512*c^(3/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)
))/(81*d^4) - (38*c*(c + d*x^3)^(1/2))/(9*d^4) + (2*c^2)/(27*d^4*(c + d*x^3
)^(1/2)) - (2*x^3*(c + d*x^3)^(1/2))/(9*d^3)
```

3.326 $\int \frac{x^8}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 71

$$\int \frac{x^8}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = -\frac{2c}{27d^3\sqrt{c+dx^3}}-\frac{2\sqrt{c+dx^3}}{3d^3}+\frac{128\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

```
[Out] 128/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/d^3-2/27*c/d^3/(d*x^3+c
)^(1/2)-2/3*(d*x^3+c)^(1/2)/d^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {457, 89, 65, 212}

$$\int \frac{x^8}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{128\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}-\frac{2c}{27d^3\sqrt{c+dx^3}}-\frac{2\sqrt{c+dx^3}}{3d^3}$$

```
[In] Int[x^8/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*c)/(27*d^3*Sqrt[c + d*x^3]) - (2*Sqrt[c + d*x^3])/(3*d^3) + (128*Sqrt[c
]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c}{9d^2(c+dx)^{3/2}} - \frac{1}{d^2\sqrt{c+dx}} + \frac{64c}{9d^2(8c-dx)\sqrt{c+dx}}\right)dx, x, x^3\right)\\
&= -\frac{2c}{27d^3\sqrt{c+dx^3}} - \frac{2\sqrt{c+dx^3}}{3d^3} + \frac{(64c)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27d^2}\\
&= -\frac{2c}{27d^3\sqrt{c+dx^3}} - \frac{2\sqrt{c+dx^3}}{3d^3} + \frac{(128c)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{27d^3}\\
&= -\frac{2c}{27d^3\sqrt{c+dx^3}} - \frac{2\sqrt{c+dx^3}}{3d^3} + \frac{128\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

$$\int \frac{x^8}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{2\left(-\frac{3(10c+9dx^3)}{\sqrt{c+dx^3}}+64\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{81d^3}$$

```
[In] Integrate[x^8/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
[Out] (2*((-3*(10*c + 9*d*x^3))/Sqrt[c + d*x^3] + 64*Sqrt[c]*ArcTanh[Sqrt[c + d*x
^3]/(3*Sqrt[c])]))/(81*d^3)
```

Maple [A] (verified)

Time = 4.47 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$-\frac{2\left(-64\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)\sqrt{c}\,\sqrt{d\,x^3+c}+27d\,x^3+30c\right)}{81\sqrt{d\,x^3+c}\,d^3}$
risch	$-\frac{2\sqrt{d\,x^3+c}}{3d^3}-\frac{c\left(\frac{2}{27d\sqrt{d\,x^3+c}}-\frac{128\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{81d\sqrt{c}}\right)}{d^2}$
default	$-\frac{\frac{2c}{3d^2\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{2\sqrt{d\,x^3+c}}{3d^2}}{d}+\frac{16c}{3d^3\sqrt{d\,x^3+c}}-\frac{128\sqrt{c}\left(-\frac{\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)\sqrt{d\,x^3+c}}{3}+\sqrt{c}\right)}{27d^3\sqrt{d\,x^3+c}}$
elliptic	$-\frac{2c}{27d^3\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{2\sqrt{d\,x^3+c}}{3d^3}-\frac{64i\sqrt{2}\left(\sum_{\_\alpha=\text{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\cdots}}{\cdots}\right.}{\cdots}$

```
[In] int(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/81*(-64*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)*(d*x^3+c)^(1/2)+27*
d*x^3+30*c)/(d*x^3+c)^(1/2)/d^3
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.27

$$\int \frac{x^8}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \left[\frac{2\left(32\,(dx^3+c)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-3\,(9\,dx^3+10\,c)\sqrt{dx^3+c}\right)}{81\,(d^4x^3+cd^3)},\right.$$
$$\left.-\frac{2\left(64\,(dx^3+c)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+3\,(9\,dx^3+10\,c)\sqrt{dx^3+c}\right)}{81\,(d^4x^3+cd^3)}\right]$$

```
[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [2/81*(32*(d*x^3 + c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c
)/(d*x^3 - 8*c)) - 3*(9*d*x^3 + 10*c)*sqrt(d*x^3 + c))/(d^4*x^3 + c*d^3), -
2/81*(64*(d*x^3 + c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(9
*d*x^3 + 10*c)*sqrt(d*x^3 + c))/(d^4*x^3 + c*d^3)]
```

Sympy [A] (verification not implemented)

Time = 11.70 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06

$$\int \frac{x^8}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \begin{cases} \frac{2\left(-\frac{c}{27\sqrt{c+dx^3}}-\frac{64c\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{81\sqrt{-c}}-\frac{\sqrt{c+dx^3}}{3}\right)}{d^3} & \text{for } d\neq 0 \\ \frac{x^9}{72c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**8/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Piecewise((2*(-c/(27*sqrt(c + d*x**3)) - 64*c*atan(sqrt(c + d*x**3)/(3*sqrt
(-c)))/(81*sqrt(-c)) - sqrt(c + d*x**3)/3)/d**3, Ne(d, 0)), (x**9/(72*c**(5
/2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

$$\int \frac{x^8}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = -\frac{2\left(32\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+27\sqrt{dx^3+c}+\frac{3\,c}{\sqrt{dx^3+c}}\right)}{81\,d^3}$$

```
[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/81*(32*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sq
rt(c))) + 27*sqrt(d*x^3 + c) + 3*c/sqrt(d*x^3 + c))/d^3
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.82

$$\int \frac{x^8}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = -\frac{128\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\,\sqrt{-c}d^3} - \frac{2\,\sqrt{dx^3+c}}{3\,d^3} - \frac{2\,c}{27\,\sqrt{dx^3+c}d^3}$$

```
[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -128/81*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/3*sqrt(d*
x^3 + c)/d^3 - 2/27*c/(sqrt(d*x^3 + c)*d^3)
```

Mupad [B] (verification not implemented)

Time = 7.92 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06

$$\int \frac{x^8}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \frac{64\,\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,d^3} - \frac{2\,c}{27\,d^3\,\sqrt{d\,x^3+c}} - \frac{2\,\sqrt{d\,x^3+c}}{3\,d^3}$$

```
[In] int(x^8/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] (64*c^(1/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))
)/(81*d^3) - (2*c)/(27*d^3*(c + d*x^3)^(1/2)) - (2*(c + d*x^3)^(1/2))/(3*d^
3)
```

3.327 $\int \frac{x^5}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 52

$$\int \frac{x^5}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx=\frac{2}{27d^2\sqrt{c+dx^3}}+\frac{16\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}d^2}$$

```
[Out] 16/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^2/c^(1/2)+2/27/d^2/(d*x^3+c)^(
1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {457, 79, 65, 212}

$$\int \frac{x^5}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx=\frac{16\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}d^2}+\frac{2}{27d^2\sqrt{c+dx^3}}$$

```
[In] Int[x^5/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] 2/(27*d^2*Sqrt[c + d*x^3]) + (16*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*
Sqrt[c]*d^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)\\
&= \frac{2}{27d^2\sqrt{c+dx^3}} + \frac{8\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27d}\\
&= \frac{2}{27d^2\sqrt{c+dx^3}} + \frac{16\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{27d^2}\\
&= \frac{2}{27d^2\sqrt{c+dx^3}} + \frac{16\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}d^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{2\left(\frac{3}{\sqrt{c+dx^3}}+\frac{8\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{\sqrt{c}}\right)}{81d^2}$$

```
[In] Integrate[x^5/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
[Out] (2*(3/Sqrt[c + d*x^3] + (8*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/Sqrt[c]))/
(81*d^2)
```

Maple [A] (verified)

Time = 4.70 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

method	result
pseudoelliptic	$\frac{\frac{16\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)\sqrt{d\,x^3+c}}{81}+\frac{2\sqrt{c}}{27}}{\sqrt{d\,x^3+c}\,\sqrt{c}\,d^2}$
default	$\frac{2}{3d^2\sqrt{d\,x^3+c}}-\frac{16\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)\sqrt{d\,x^3+c}}{3}+\sqrt{c}\right)}{27\sqrt{c}\,d^2\sqrt{d\,x^3+c}}$
elliptic	$\frac{2}{27d^2\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{8i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\cdots\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-\cdots\right)}}\cdots}{\cdots}\right)}{\cdots}$

```
[In] int(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/27*(8/3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*(d*x^3+c)^(1/2)+c^(1/2))/(d*
x^3+c)^(1/2)/c^(1/2)/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.87

$$\int \frac{x^5}{(8c-dx^3)\left(c+dx^3\right)^{3/2}}\,dx = \left[\frac{2\left(4\left(dx^3+c\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+3\sqrt{dx^3+c}c\right)}{81\left(cd^3x^3+c^2d^2\right)}, -\frac{2\left(8\left(dx^3+c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)-3\sqrt{dx^3+c}c\right)}{81\left(cd^3x^3+c^2d^2\right)}\right]$$

```
[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
[Out] [2/81*(4*(d*x^3 + c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)
/(d*x^3 - 8*c)) + 3*sqrt(d*x^3 + c)*c)/(c*d^3*x^3 + c^2*d^2), -2/81*(8*(d*x
^3 + c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 3*sqrt(d*x^3 + c)
*c)/(c*d^3*x^3 + c^2*d^2)]
```

Sympy [A] (verification not implemented)

Time = 7.90 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{(8c-dx^3)\left(c+dx^3\right)^{3/2}}\,dx = \begin{cases} \frac{2\cdot\left(\frac{1}{27\sqrt{c+dx^3}}-\frac{8\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{81\sqrt{-c}}\right)}{d^2} & \text{for}\ d\neq 0 \\ \frac{x^6}{48c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**5/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
[Out] Piecewise((2*(1/(27*sqrt(c + d*x**3)) - 8*atan(sqrt(c + d*x**3)/(3*sqrt(-c)
))/(81*sqrt(-c)))/d**2, Ne(d, 0)), (x**6/(48*c**(5/2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{(8c-dx^3)\left(c+dx^3\right)^{3/2}}\,dx = -\frac{2\left(\frac{4\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{\sqrt{c}}-\frac{3}{\sqrt{dx^3+c}}\right)}{81\,d^2}$$

```
[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] -2/81*(4*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/s
qrt(c) - 3/sqrt(d*x^3 + c))/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.90

$$\int \frac{x^5}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = -\frac{2\left(\frac{8\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d} - \frac{3}{\sqrt{dx^3+c}d}\right)}{81\,d}$$

```
[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/81*(8*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 3/(sqrt(d*x^3
+ c)*d))/d
```

Mupad [B] (verification not implemented)

Time = 7.89 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15

$$\int \frac{x^5}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{2}{27\,d^2\,\sqrt{d\,x^3+c}} + \frac{8\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,\sqrt{c}\,d^2}$$

```
[In] int(x^5/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] 2/(27*d^2*(c + d*x^3)^(1/2)) + (8*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)
^(1/2))/(8*c - d*x^3)))/(81*c^(1/2)*d^2)
```

3.328 $\int \frac{x^2}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 55

$$\int \frac{x^2}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = -\frac{2}{27cd\sqrt{c+dx^3}}+\frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}$$

```
[Out] 2/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)/d-2/27/c/d/(d*x^3+c)^(1/2
)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {455, 53, 65, 212}

$$\int \frac{x^2}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}-\frac{2}{27cd\sqrt{c+dx^3}}$$

```
[In] Int[x^2/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] -2/(27*c*d*Sqrt[c + d*x^3]) + (2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*
c^(3/2)*d)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
```

```
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{(8c-dx)(c+dx)^{3/2}}\,dx,x,x^3\right)\\
&= -\frac{2}{27cd\sqrt{c+dx^3}}+\frac{\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{27c}\\
&= -\frac{2}{27cd\sqrt{c+dx^3}}+\frac{2\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{27cd}\\
&= -\frac{2}{27cd\sqrt{c+dx^3}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx=\frac{2\left(-\frac{3\sqrt{c}}{\sqrt{c+dx^3}}+\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{81c^{3/2}d}$$

[In] Integrate[x^2/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]

[Out] (2*((-3*Sqrt[c])/Sqrt[c + d*x^3] + ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(
81*c^(3/2)*d)

Maple [A] (verified)

Time = 4.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

method	result
default	$-\frac{2\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)\sqrt{d\,x^3+c}}{3}+\sqrt{c}\right)}{27\sqrt{d\,x^3+c}\,c^{\frac{3}{2}}\,d}$
pseudoelliptic	$-\frac{2\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)\sqrt{d\,x^3+c}}{3}+\sqrt{c}\right)}{27\sqrt{d\,x^3+c}\,c^{\frac{3}{2}}\,d}$
elliptic	$-\frac{2}{27dc\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right.}{d}\right.}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\ldots}}\ldots}{\ldots}\right)}{\ldots}$

[In] int(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] -2/27/(d*x^3+c)^(1/2)/c^(3/2)*(-1/3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*(d
*x^3+c)^(1/2)+c^(1/2))/d

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.67

$$\int \frac{x^2}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \left[\frac{(dx^3+c)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-6\sqrt{dx^3+c}c}{81\,(c^2d^2x^3+c^3d)}, -\frac{2\left((dx^3+c)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+3\sqrt{dx^3+c}c\right)}{81\,(c^2d^2x^3+c^3d)}\right]$$

```
[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/81*((d*x^3 + c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(
d*x^3 - 8*c)) - 6*sqrt(d*x^3 + c)*c)/(c^2*d^2*x^3 + c^3*d), -2/81*((d*x^3 +
 c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*sqrt(d*x^3 + c)*c)/
(c^2*d^2*x^3 + c^3*d)]
```

Sympy [A] (verification not implemented)

Time = 6.51 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \begin{cases} \frac{2\left(-\frac{1}{27c\sqrt{c+dx^3}}-\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{81c\sqrt{-c}}\right)}{d} & \text{for } d\neq 0 \\ \frac{x^3}{24c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Piecewise((2*(-1/(27*c*sqrt(c + d*x**3)) - atan(sqrt(c + d*x**3)/(3*sqrt(-c
)))/(81*c*sqrt(-c)))/d, Ne(d, 0)), (x**3/(24*c**(5/2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = -\frac{\frac{\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{6}{\sqrt{dx^3+c}c}}{81\,d}$$

```
[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/81*(log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/c^(
3/2) + 6/(sqrt(d*x^3 + c)*c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}cd} - \frac{2}{27\sqrt{dx^3+c}cd}$$

```
[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/81*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c*d) - 2/27/(sqrt(d*x^
3 + c)*c*d)
```

Mupad [B] (verification not implemented)

Time = 7.89 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int \frac{x^2}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,c^{3/2}\,d} - \frac{2}{27\,c\,d\,\sqrt{d\,x^3+c}}$$

```
[In] int(x^2/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(81*c^(3/2)
*d) - 2/(27*c*d*(c + d*x^3)^(1/2))
```

3.329 $\int \frac{1}{x(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 76

$$\int \frac{1}{x(8c-dx^3)(c+dx^3)^{3/2}}\,dx=\frac{2}{27c^2\sqrt{c+dx^3}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{324c^{5/2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{5/2}}$$

```
[Out] 1/324*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)-1/12*arctanh((d*x^3+c)^(
1/2)/c^(1/2))/c^(5/2)+2/27/c^2/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {457, 87, 162, 65, 214, 212}

$$\int \frac{1}{x(8c-dx^3)(c+dx^3)^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{324c^{5/2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{5/2}}+\frac{2}{27c^2\sqrt{c+dx^3}}$$

```
[In] Int[1/(x*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] 2/(27*c^2*Sqrt[c + d*x^3]) + ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(324*c^(5
/2)) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(12*c^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x
)*((e + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right) \\
&= \frac{2}{27c^2\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{-9cd+d^2x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27c^2d} \\
&= \frac{2}{27c^2\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{24c^2} + \frac{d\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{216c^2}
\end{aligned}$$

$$= \frac{2}{27c^2\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{108c^2} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{12c^2d}$$

$$= \frac{2}{27c^2\sqrt{c+dx^3}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{324c^{5/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{5/2}}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.91

$$\int \frac{1}{x(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{\frac{24\sqrt{c}}{\sqrt{c+dx^3}} + \text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 27\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{324c^{5/2}}$$

```
[In] Integrate[1/(x*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] ((24*Sqrt[c])/Sqrt[c + d*x^3] + ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 27*A
rcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(324*c^(5/2))
```

Maple [A] (verified)

Time = 4.43 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.93

method	result	size
pseudoelliptic	$\frac{\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)\sqrt{d\,x^3+c}-27\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)\sqrt{d\,x^3+c}+24\sqrt{c}}{324c^{\frac{5}{2}}\sqrt{d\,x^3+c}}$	71
default	$\frac{\frac{2}{3c\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{2\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}}{8c}-\frac{-\frac{\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)\sqrt{d\,x^3+c}}{3}+\sqrt{c}}{108c^{\frac{5}{2}}\sqrt{d\,x^3+c}}$	89
elliptic	Expression too large to display	1526

```
[In] int(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/324/c^(5/2)*(arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*(d*x^3+c)^(1/2)-27*arct
anh((d*x^3+c)^(1/2)/c^(1/2))*(d*x^3+c)^(1/2)+24*c^(1/2))/(d*x^3+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.80

$$\int \frac{1}{x\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \left[\frac{(dx^3+c)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+27\,(dx^3+c)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}}{x^3}\right.}{648\,(c^3dx^3+c^4)}\right.$$

```
[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/648*((d*x^3 + c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/
(d*x^3 - 8*c)) + 27*(d*x^3 + c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt
(c) + 2*c)/x^3) + 48*sqrt(d*x^3 + c)*c)/(c^3*d*x^3 + c^4), 1/324*(27*(d*x^3
 + c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - (d*x^3 + c)*sqrt(-c)*ar
ctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 24*sqrt(d*x^3 + c)*c)/(c^3*d*x^3 + c
^4)]
```

Sympy [A] (verification not implemented)

Time = 4.40 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.28

$$\int \frac{1}{x\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{d}{27c^2\sqrt{c+dx^3}}-\frac{d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{648c^2\sqrt{-c}}+\frac{d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{24c^2\sqrt{-c}}\right)}{d} & \text{for}\ d\neq 0 \\ \frac{\log\left(x^3\right)}{24c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/x/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Piecewise((2*(d/(27*c**2*sqrt(c + d*x**3)) - d*atan(sqrt(c + d*x**3)/(3*sqr
t(-c)))/(648*c**2*sqrt(-c)) + d*atan(sqrt(c + d*x**3)/sqrt(-c))/(24*c**2*sq
rt(-c)))/d, Ne(d, 0)), (log(x**3)/(24*c**(5/2)), True))
```

Maxima [F]

$$\int \frac{1}{x\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \int -\frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)x}\,dx$$

```
[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{12\sqrt{-c}c^2}-\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{324\sqrt{-c}c^2}+\frac{2}{27\sqrt{dx^3+c}c^2}$$

```
[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/12*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/324*arctan(1/3*sqr
t(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) + 2/27/(sqrt(d*x^3 + c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 7.94 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\frac{2}{27\,c^2\,\sqrt{d\,x^3+c}}-\frac{\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{\sqrt{c^5}}\right)}{12\,\sqrt{c^5}}+\frac{\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^5}}\right)}{324\,\sqrt{c^5}}$$

```
[In] int(1/(x*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] 2/(27*c^2*(c + d*x^3)^(1/2)) - atanh((c^2*(c + d*x^3)^(1/2))/(c^5)^(1/2))/(
12*(c^5)^(1/2)) + atanh((c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2)))/(324*(c^5)
^(1/2))
```

3.330 $\int \frac{1}{x^4(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 100

$$\int \frac{1}{x^4(8c-dx^3)(c+dx^3)^{3/2}}\,dx = -\frac{25d}{216c^3\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\sqrt{c+dx^3}} + \frac{d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{7/2}} + \frac{11d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}}$$

```
[Out] 1/2592*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)+11/96*d*arctanh((d*x^
3+c)^(1/2)/c^(1/2))/c^(7/2)-25/216*d/c^3/(d*x^3+c)^(1/2)-1/24/c^2/x^3/(d*x^
3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {457, 105, 157, 162, 65, 214, 212}

$$\int \frac{1}{x^4(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{7/2}} + \frac{11d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}} - \frac{25d}{216c^3\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\sqrt{c+dx^3}}$$

```
[In] Int[1/(x^4*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-25*d)/(216*c^3*Sqrt[c + d*x^3]) - 1/(24*c^2*x^3*Sqrt[c + d*x^3]) + (d*Arc
Tanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2592*c^(7/2)) + (11*d*ArcTanh[Sqrt[c +
d*x^3]/Sqrt[c]])/(96*c^(7/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)\\
&= -\frac{1}{24c^2x^3\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{11cd-\frac{3d^2x}{2}}{x(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{24c^2}\\
&= -\frac{25d}{216c^3\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{\frac{99c^2d^2}{2}-\frac{25}{4}cd^3x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{108c^4d}\\
&= -\frac{25d}{216c^3\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\sqrt{c+dx^3}} - \frac{(11d)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{192c^3}\\
&\quad + \frac{d^2\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{1728c^3}\\
&= -\frac{25d}{216c^3\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\sqrt{c+dx^3}}\\
&\quad - \frac{11\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{96c^3} + \frac{d\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{864c^3}\\
&= -\frac{25d}{216c^3\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\sqrt{c+dx^3}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{7/2}} + \frac{11d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^4(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{-\frac{12\sqrt{c}(9c+25dx^3)}{x^3\sqrt{c+dx^3}} + d\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) + 297d\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2592c^{7/2}}$$

```
[In] Integrate[1/(x^4*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] ((-12*Sqrt[c]*(9*c + 25*d*x^3))/(x^3*Sqrt[c + d*x^3]) + d*ArcTanh[Sqrt[c +
d*x^3]/(3*Sqrt[c])] + 297*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2592*c^(7/2)
)
```

Maple [A] (verified)

Time = 4.51 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{\sqrt{d\,x^{3}+c}}{24c^{3}x^{3}}-\frac{d\left(-\frac{11\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{6\sqrt{c}}+\frac{32}{27\sqrt{d\,x^{3}+c}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{162\sqrt{c}}\right)}{16c^{3}}$
pseudoelliptic	$\frac{297\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)d\,x^{3}\sqrt{d\,x^{3}+c}+\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)d\,x^{3}\sqrt{d\,x^{3}+c}-300d\,x^{3}\sqrt{c}-108c^{\frac{3}{2}}}{2592c^{\frac{7}{2}}x^{3}\sqrt{d\,x^{3}+c}}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{3c^{2}x^{3}}-\frac{2d}{3c^{2}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}+\frac{d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}}{8c}+\frac{d\left(\frac{2}{3c\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{64c^{2}}-\frac{d\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\sqrt{\ }}{3}\right.}{864c^{\frac{7}{2}}\sqrt{d\,x^{3}}}$
elliptic	Expression too large to display

```
[In] int(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24/c^3*(d*x^3+c)^(1/2)/x^3-1/16/c^3*d*(-11/6*arctanh((d*x^3+c)^(1/2)/c^(
1/2))/c^(1/2)+32/27/(d*x^3+c)^(1/2)-1/162*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/
2))/c^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.72

$$\int \frac{1}{x^{4}\left(8c-dx^{3}\right)\left(c+dx^{3}\right)^{3/2}}\,dx=\left[\frac{(d^{2}x^{6}+cdx^{3})\sqrt{c}\log\left(\frac{dx^{3}+6\sqrt{dx^{3}+c}\sqrt{c}+10c}{dx^{3}-8c}\right)+297\left(d^{2}x^{6}+cdx^{3}\right)\sqrt{c}\log\left(\frac{dx}{\ }\right.}{5184\left(c^{4}dx^{6}+c^{5}x^{3}\right)}\right.$$
$$-\frac{297\left(d^{2}x^{6}+cdx^{3}\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^{3}+c}\sqrt{-c}}{c}\right)+\left(d^{2}x^{6}+cdx^{3}\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^{3}+c}\sqrt{-c}}{3c}\right)+12\left(25cdx^{3}+9c^{2}\right)}{2592\left(c^{4}dx^{6}+c^{5}x^{3}\right)}$$

```
[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/5184*((d^2*x^6 + c*d*x^3)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c)
 + 10*c)/(d*x^3 - 8*c)) + 297*(d^2*x^6 + c*d*x^3)*sqrt(c)*log((d*x^3 + 2*sq
rt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*(25*c*d*x^3 + 9*c^2)*sqrt(d*x^3 + c)
)/(c^4*d*x^6 + c^5*x^3), -1/2592*(297*(d^2*x^6 + c*d*x^3)*sqrt(-c)*arctan(s
qrt(d*x^3 + c)*sqrt(-c)/c) + (d^2*x^6 + c*d*x^3)*sqrt(-c)*arctan(1/3*sqrt(d
*x^3 + c)*sqrt(-c)/c) + 12*(25*c*d*x^3 + 9*c^2)*sqrt(d*x^3 + c))/(c^4*d*x^6
 + c^5*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4 (8c - dx^3) (c + dx^3)^{3/2}} dx = -\int \frac{1}{-8c^2x^4\sqrt{c + dx^3} - 7cdx^7\sqrt{c + dx^3} + d^2x^{10}\sqrt{c + dx^3}} dx$$

[In] integrate(1/x**4/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)

[Out] -Integral(1/(-8*c**2*x**4*sqrt(c + d*x**3) - 7*c*d*x**7*sqrt(c + d*x**3) + d**2*x**10*sqrt(c + d*x**3)), x)

Maxima [F]

$$\int \frac{1}{x^4 (8c - dx^3) (c + dx^3)^{3/2}} dx = \int -\frac{1}{(dx^3 + c)^{\frac{3}{2}} (dx^3 - 8c)x^4} dx$$

[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")

[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^4), x)

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (8c - dx^3) (c + dx^3)^{3/2}} dx = -\frac{11 d \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96 \sqrt{-c}c^3} - \frac{d \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{2592 \sqrt{-c}c^3} - \frac{25 (dx^3 + c)d - 16 cd}{216 \left((dx^3 + c)^{\frac{3}{2}} - \sqrt{dx^3 + c}c\right)c^3}$$

[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")

[Out] -11/96*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/2592*d*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/216*(25*(d*x^3 + c)*d - 16*c*d)/(((d*x^3 + c)^(3/2) - sqrt(d*x^3 + c)*c)*c^3)

Mupad [B] (verification not implemented)

Time = 8.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^4 (8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{11\,d\,\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{\sqrt{c^7}}\right)}{96\,\sqrt{c^7}} - \frac{25\,d}{216\,c^3\,\sqrt{d\,x^3+c}} + \frac{d\,\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^7}}\right)}{2592\,\sqrt{c^7}} - \frac{1}{24\,c^2\,x^3\,\sqrt{d\,x^3+c}}$$

```
[In] int(1/(x^4*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] (11*d*atanh((c^3*(c + d*x^3)^(1/2))/(c^7)^(1/2)))/(96*(c^7)^(1/2)) - (25*d)
/(216*c^3*(c + d*x^3)^(1/2)) + (d*atanh((c^3*(c + d*x^3)^(1/2))/(3*(c^7)^(1
/2))))/(2592*(c^7)^(1/2)) - 1/(24*c^2*x^3*(c + d*x^3)^(1/2))
```

3.331 $\int \frac{1}{x^7(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 128

$$\int \frac{1}{x^7(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{245d^2}{1728c^4\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\sqrt{c+dx^3}} + \frac{3d}{64c^3x^3\sqrt{c+dx^3}} + \frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{20736c^{9/2}} - \frac{109d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{9/2}}$$

```
[Out] 1/20736*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(9/2)-109/768*d^2*arctan
h((d*x^3+c)^(1/2)/c^(1/2))/c^(9/2)+245/1728*d^2/c^4/(d*x^3+c)^(1/2)-1/48/c^
2/x^6/(d*x^3+c)^(1/2)+3/64*d/c^3/x^3/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {457, 105, 156, 157, 162, 65, 214, 212}

$$\int \frac{1}{x^7(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{20736c^{9/2}} - \frac{109d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{9/2}} + \frac{245d^2}{1728c^4\sqrt{c+dx^3}} + \frac{3d}{64c^3x^3\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\sqrt{c+dx^3}}$$

```
[In] Int[1/(x^7*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (245*d^2)/(1728*c^4*Sqrt[c + d*x^3]) - 1/(48*c^2*x^6*Sqrt[c + d*x^3]) + (3*
d)/(64*c^3*x^3*Sqrt[c + d*x^3]) + (d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]
```

```
)/(20736*c^(9/2)) - (109*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(768*c^(9/2)
)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^3(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)\\
&= -\frac{1}{48c^2x^6\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{18cd-\frac{5d^2x}{2}}{x^2(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{48c^2}\\
&= -\frac{1}{48c^2x^6\sqrt{c+dx^3}} + \frac{3d}{64c^3x^3\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{218c^2d^2-27cd^3x}{x(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{384c^4}\\
&= \frac{245d^2}{1728c^4\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\sqrt{c+dx^3}} + \frac{3d}{64c^3x^3\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{981c^3d^3-\frac{245}{2}c^2d^4x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{1728c^6d}\\
&= \frac{245d^2}{1728c^4\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\sqrt{c+dx^3}} + \frac{3d}{64c^3x^3\sqrt{c+dx^3}}\\
&\quad + \frac{(109d^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{1536c^4} + \frac{d^3\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{13824c^4}\\
&= \frac{245d^2}{1728c^4\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\sqrt{c+dx^3}} + \frac{3d}{64c^3x^3\sqrt{c+dx^3}}\\
&\quad + \frac{(109d)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{768c^4} + \frac{d^2\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{6912c^4}
\end{aligned}$$

$$= \frac{245d^2}{1728c^4\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\sqrt{c+dx^3}} + \frac{3d}{64c^3x^3\sqrt{c+dx^3}} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{20736c^{9/2}} - \frac{109d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{9/2}}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^7(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{\frac{12\sqrt{c}(-36c^2+81cdx^3+245d^2x^6)}{x^6\sqrt{c+dx^3}} + d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 2943d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{20736c^{9/2}}$$

```
[In] Integrate[1/(x^7*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] ((12*Sqrt[c]*(-36*c^2 + 81*c*d*x^3 + 245*d^2*x^6))/(x^6*Sqrt[c + d*x^3]) +
d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 2943*d^2*ArcTanh[Sqrt[c + d*x^3]
/Sqrt[c]])/(20736*c^(9/2))
```

Maple [A] (verified)

Time = 4.52 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{\sqrt{d\,x^3+c}\,(-13d\,x^3+4c)}{192c^4x^6}+\frac{d^2\left(-\frac{109\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{6\sqrt{c}}+\frac{\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{162\sqrt{c}}+\frac{256}{27\sqrt{d\,x^3+c}}\right)}{128c^4}$
pseudoelliptic	$\frac{-2943\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)d^2x^6\sqrt{d\,x^3+c}+\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)d^2x^6\sqrt{d\,x^3+c}+2940d^2x^6\sqrt{c}+972c^{\frac{3}{2}}d\,x^3-432c^{\frac{5}{2}}}{20736c^{\frac{9}{2}}x^6\sqrt{d\,x^3+c}}$
default	$\frac{-\frac{\sqrt{d\,x^3+c}}{6c^2x^6}+\frac{7d\sqrt{d\,x^3+c}}{12c^3x^3}+\frac{2d^2}{3c^3\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{5d^2\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{4c^{\frac{7}{2}}}}{8c}+\frac{d\left(-\frac{\sqrt{d\,x^3+c}}{3c^2x^3}-\frac{2d}{3c^2\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{d\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right)}{64c^2}$
elliptic	Expression too large to display

```
[In] int(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/192*(d*x^3+c)^(1/2)*(-13*d*x^3+4*c)/c^4/x^6+1/128*d^2/c^4*(-109/6*arctan
h((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)+1/162*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2
))/c^(1/2)+256/27/(d*x^3+c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.37

$$\int \frac{1}{x^7 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \left[\frac{(d^3x^9 + cd^2x^6)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right) + 2943\,(d^3x^9 + cd^2x^6)\sqrt{c}\log}{41472\,(c^5dx^9}\right.$$

```
[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/41472*((d^3*x^9 + c*d^2*x^6)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt
(c) + 10*c)/(d*x^3 - 8*c)) + 2943*(d^3*x^9 + c*d^2*x^6)*sqrt(c)*log((d*x^3
- 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 24*(245*c*d^2*x^6 + 81*c^2*d*x^3
- 36*c^3)*sqrt(d*x^3 + c))/(c^5*d*x^9 + c^6*x^6), 1/20736*(2943*(d^3*x^9 +
c*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - (d^3*x^9 + c*d^2*x
^6)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*(245*c*d^2*x^6 + 8
1*c^2*d*x^3 - 36*c^3)*sqrt(d*x^3 + c))/(c^5*d*x^9 + c^6*x^6)]
```

Sympy [F]

$$\int \frac{1}{x^7 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = -\int \frac{1}{-8c^2x^7\sqrt{c + dx^3} - 7cdx^{10}\sqrt{c + dx^3} + d^2x^{13}\sqrt{c + dx^3}} \, dx$$

```
[In] integrate(1/x**7/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] -Integral(1/(-8*c**2*x**7*sqrt(c + d*x**3) - 7*c*d*x**10*sqrt(c + d*x**3) +
 d**2*x**13*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^7 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \int -\frac{1}{(dx^3 + c)^{\frac{3}{2}}(dx^3 - 8\,c)x^7} \, dx$$

```
[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^7), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^7 (8c-dx^3) (c+dx^3)^{3/2}}\, dx = \frac{109\, d^2 \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{768\, \sqrt{-c}c^4} - \frac{d^2 \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{20736\, \sqrt{-c}c^4} + \frac{2\, d^2}{27\, \sqrt{dx^3+c}c^4} + \frac{13\, (dx^3+c)^{\frac{3}{2}} d^2 - 17\, \sqrt{dx^3+c}cd^2}{192\, c^4 d^2 x^6}$$

```
[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 109/768*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 1/20736*d^2*a
rctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) + 2/27*d^2/(sqrt(d*x^3 +
 c)*c^4) + 1/192*(13*(d*x^3 + c)^(3/2)*d^2 - 17*sqrt(d*x^3 + c)*c*d^2)/(c^4
*d^2*x^6)
```

Mupad [B] (verification not implemented)

Time = 8.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^7 (8c-dx^3) (c+dx^3)^{3/2}}\, dx = \frac{245\, d^2}{1728\, c^4\, \sqrt{d\, x^3+c}} - \frac{109\, d^2\, \mathrm{atanh}\left(\frac{c^4\, \sqrt{d\, x^3+c}}{\sqrt{c^9}}\right)}{768\, \sqrt{c^9}} + \frac{d^2\, \mathrm{atanh}\left(\frac{c^4\, \sqrt{d\, x^3+c}}{3\, \sqrt{c^9}}\right)}{20736\, \sqrt{c^9}} - \frac{1}{48\, c^2\, x^6\, \sqrt{d\, x^3+c}} + \frac{3\, d}{64\, c^3\, x^3\, \sqrt{d\, x^3+c}}$$

```
[In] int(1/(x^7*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] (245*d^2)/(1728*c^4*(c + d*x^3)^(1/2)) - (109*d^2*atanh((c^4*(c + d*x^3)^(1
/2))/(c^9)^(1/2)))/(768*(c^9)^(1/2)) + (d^2*atanh((c^4*(c + d*x^3)^(1/2))/(
3*(c^9)^(1/2))))/(20736*(c^9)^(1/2)) - 1/(48*c^2*x^6*(c + d*x^3)^(1/2)) + (
3*d)/(64*c^3*x^3*(c + d*x^3)^(1/2))
```

3.332 $\int \frac{x^7}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 629

$$\int \frac{x^7}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{2x^2}{27d^2\sqrt{c+dx^3}} - \frac{56\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{32\sqrt[6]{c}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}d^{8/3}} + \frac{32\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{81d^{8/3}} - \frac{32\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^{8/3}} + \frac{28\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{56\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 32/81*c^(1/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^
(8/3)-32/81*c^(1/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(8/3)-32/81*c^(1
```

```
/6)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/d^(8/3)*3^(
1/2)+2/27*x^2/d^2/(d*x^3+c)^(1/2)-56/27*(d*x^3+c)^(1/2)/d^(8/3)/(d^(1/3)*x+
c^(1/3)*(1+3^(1/2)))-56/81*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x
+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2
)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^
2)^(1/2)*3^(3/4)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1
/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+28/27*c^(1/3)*(c^(1/3)+d^(1/3)*x)*Ellip
ticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1
/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)
/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/d^(8/3)/(d*x^3+c)^(1/2)/(
c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {481, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{x^7}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = -\frac{56\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{28\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{32\sqrt[6]{c}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}d^{8/3}} + \frac{32\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{81d^{8/3}} - \frac{32\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^{8/3}} - \frac{56\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{2x^2}{27d^2\sqrt{c+dx^3}}$$

```
[In] Int[x^7/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*x^2)/(27*d^2*Sqrt[c + d*x^3]) - (56*Sqrt[c + d*x^3])/(27*d^(8/3)*((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)) - (32*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3
```

```
) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(27*Sqrt[3]*d^(8/3)) + (32*c^(1/6)*ArcTan
h[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(81*d^(8/3)) - (32*
c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^(8/3)) + (28*Sqrt[2 - S
qrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d
^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - S
qrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*S
qrt[3]])/(9*3^(3/4)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (56*Sqrt[2]*c^(1/3)*(c^(1/3
) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/
3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*d^
(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
```

```
/; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
```

```
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x^2}{27d^2\sqrt{c+dx^3}} - \frac{2\int \frac{x(16c^2-14cdx^3)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{27cd^2} \\
&= \frac{2x^2}{27d^2\sqrt{c+dx^3}} - \frac{2\int \left(\frac{14cx}{\sqrt{c+dx^3}} - \frac{96c^2x}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{27cd^2} \\
&= \frac{2x^2}{27d^2\sqrt{c+dx^3}} - \frac{28\int \frac{x}{\sqrt{c+dx^3}}\,dx}{27d^2} + \frac{(64c)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{9d^2} \\
&= \frac{2x^2}{27d^2\sqrt{c+dx^3}} - \frac{16\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{27d^3} \\
&\quad - \frac{28\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{27d^{7/3}} + \frac{\left(16\sqrt[3]{c}\right)\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{27d^{7/3}} \\
&\quad + \frac{\left(28\left(1-\sqrt{3}\right)\sqrt[3]{c}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{27d^{7/3}} - \frac{\left(16c^{2/3}\right)\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{9d^{5/3}}
\end{aligned}$$

$$= \frac{2x^2}{27d^2\sqrt{c+dx^3}} - \frac{56\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$+ \frac{28\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$- \frac{56\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{\left(32c^{2/3}\right)\text{Subst}\left(\int\frac{1}{9-cx^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{27d^{8/3}}$$

$$- \frac{\left(16c^{2/3}\right)\text{Subst}\left(\int\frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27d^{5/3}}$$

$$+ \frac{64\text{Subst}\left(\int\frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{27\sqrt[3]{c}d^{2/3}}$$

$$= \frac{2x^2}{27d^2\sqrt{c+dx^3}} - \frac{56\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{32\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}d^{8/3}} + \frac{32\sqrt[6]{c}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{81d^{8/3}} + \frac{28\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{56\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{\left(32c^{2/3}\right)\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{27d^{8/3}}$$

$$= \frac{2x^2}{27d^2\sqrt{c+dx^3}} - \frac{56\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{32\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}d^{8/3}} + \frac{32\sqrt[6]{c}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{81d^{8/3}} - \frac{32\sqrt[6]{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^{8/3}} + \frac{28\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{56\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 8.54 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.20

$$\int \frac{x^7}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{x^2\left(20c-20c\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+7dx^3\sqrt{1+\frac{dx^3}{c}}\operatorname{App}\right.}{270cd^2\sqrt{c+dx^3}}$$

```
[In] Integrate[x^7/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^2*(20*c - 20*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/
c), (d*x^3)/(8*c)] + 7*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3,
 -((d*x^3)/c), (d*x^3)/(8*c)]))/(270*c*d^2*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.32 (sec) , antiderivative size = 869, normalized size of antiderivative = 1.38

method	result	size
elliptic	Expression too large to display	869
default	Expression too large to display	1810

```
[In] int(x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/27/d^2*x^2/((x^3+c/d)*d)^(1/2)+56/81*I/d^3*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3
)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2
)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*
3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^
(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^
(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
)))^(1/2)))-64/243*I/d^5*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+
1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x
-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-
1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))
^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2
)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1
/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
```

```
2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I
*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)
/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.57 (sec) , antiderivative size = 2384, normalized size of antiderivative = 3.79

$$\int \frac{x^7}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/243*(9*sqrt(d*x^3 + c)*d*x^2 + 252*(d*x^3 + c)*sqrt(d)*weierstrassZeta(0,
 -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 4*(d^4*x^3 + c*d^3 - sqrt(-3)
*(d^4*x^3 + c*d^3))*(c/d^16)^(1/6)*log(33554432/3*((d^16*x^9 + 318*c*d^15*x
^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13 + sqrt(-3)*(d^16*x^9 + 318*c*d^15*x^6
 + 1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c/d^16)^(5/6) + 6*(2*c*d^2*x^7 + 160
*c^2*d*x^4 + 320*c^3*x - 6*(5*c*d^12*x^5 + 32*c^2*d^11*x^2 - sqrt(-3)*(5*c*
d^12*x^5 + 32*c^2*d^11*x^2))*(c/d^16)^(2/3) - (7*c*d^7*x^6 + 152*c^2*d^6*x^
3 + 64*c^3*d^5 + sqrt(-3)*(7*c*d^7*x^6 + 152*c^2*d^6*x^3 + 64*c^3*d^5))*(c/
d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c*d^10*x^7 + 64*c^2*d^9*x^4 + 32*c^3*d
^8*x)*sqrt(c/d^16) + 18*(c*d^5*x^8 + 38*c^2*d^4*x^5 + 64*c^3*d^3*x^2 - sqrt
(-3)*(c*d^5*x^8 + 38*c^2*d^4*x^5 + 64*c^3*d^3*x^2))*(c/d^16)^(1/6))/(d^3*x^
9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 4*(d^4*x^3 + c*d^3 - sqrt(-3
)*(d^4*x^3 + c*d^3))*(c/d^16)^(1/6)*log(-33554432/3*((d^16*x^9 + 318*c*d^15
*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13 + sqrt(-3)*(d^16*x^9 + 318*c*d^15*x
^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c/d^16)^(5/6) - 6*(2*c*d^2*x^7 + 1
60*c^2*d*x^4 + 320*c^3*x - 6*(5*c*d^12*x^5 + 32*c^2*d^11*x^2 - sqrt(-3)*(5*
c*d^12*x^5 + 32*c^2*d^11*x^2))*(c/d^16)^(2/3) - (7*c*d^7*x^6 + 152*c^2*d^6*
x^3 + 64*c^3*d^5 + sqrt(-3)*(7*c*d^7*x^6 + 152*c^2*d^6*x^3 + 64*c^3*d^5))*(
c/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c*d^10*x^7 + 64*c^2*d^9*x^4 + 32*c^3
*d^8*x)*sqrt(c/d^16) + 18*(c*d^5*x^8 + 38*c^2*d^4*x^5 + 64*c^3*d^3*x^2 - sq
rt(-3)*(c*d^5*x^8 + 38*c^2*d^4*x^5 + 64*c^3*d^3*x^2))*(c/d^16)^(1/6))/(d^3*
x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 4*(d^4*x^3 + c*d^3 + sqrt(
-3)*(d^4*x^3 + c*d^3))*(c/d^16)^(1/6)*log(33554432/3*((d^16*x^9 + 318*c*d^1
5*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13 - sqrt(-3)*(d^16*x^9 + 318*c*d^15*
x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c/d^16)^(5/6) + 6*(2*c*d^2*x^7 +
160*c^2*d*x^4 + 320*c^3*x - 6*(5*c*d^12*x^5 + 32*c^2*d^11*x^2 + sqrt(-3)*(5
*c*d^12*x^5 + 32*c^2*d^11*x^2))*(c/d^16)^(2/3) - (7*c*d^7*x^6 + 152*c^2*d^6
*x^3 + 64*c^3*d^5 - sqrt(-3)*(7*c*d^7*x^6 + 152*c^2*d^6*x^3 + 64*c^3*d^5))*
(c/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c*d^10*x^7 + 64*c^2*d^9*x^4 + 32*c^
```

```
3*d^8*x)*sqrt(c/d^16) + 18*(c*d^5*x^8 + 38*c^2*d^4*x^5 + 64*c^3*d^3*x^2 + s
qrt(-3)*(c*d^5*x^8 + 38*c^2*d^4*x^5 + 64*c^3*d^3*x^2))*(c/d^16)^(1/6))/(d^3
*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 4*(d^4*x^3 + c*d^3 + sqrt
(-3)*(d^4*x^3 + c*d^3))*(c/d^16)^(1/6)*log(-33554432/3*((d^16*x^9 + 318*c*d
^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13 - sqrt(-3)*(d^16*x^9 + 318*c*d^1
5*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c/d^16)^(5/6) - 6*(2*c*d^2*x^7
+ 160*c^2*d*x^4 + 320*c^3*x - 6*(5*c*d^12*x^5 + 32*c^2*d^11*x^2 + sqrt(-3)*
(5*c*d^12*x^5 + 32*c^2*d^11*x^2))*(c/d^16)^(2/3) - (7*c*d^7*x^6 + 152*c^2*d
^6*x^3 + 64*c^3*d^5 - sqrt(-3)*(7*c*d^7*x^6 + 152*c^2*d^6*x^3 + 64*c^3*d^5)
)*(c/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c*d^10*x^7 + 64*c^2*d^9*x^4 + 32*
c^3*d^8*x)*sqrt(c/d^16) + 18*(c*d^5*x^8 + 38*c^2*d^4*x^5 + 64*c^3*d^3*x^2 +
 sqrt(-3)*(c*d^5*x^8 + 38*c^2*d^4*x^5 + 64*c^3*d^3*x^2))*(c/d^16)^(1/6))/(d
^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 8*(d^4*x^3 + c*d^3)*(c/
d^16)^(1/6)*log(33554432/3*((d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3
+ 640*c^3*d^13)*(c/d^16)^(5/6) + 6*(c*d^2*x^7 + 80*c^2*d*x^4 + 160*c^3*x +
6*(5*c*d^12*x^5 + 32*c^2*d^11*x^2)*(c/d^16)^(2/3) + (7*c*d^7*x^6 + 152*c^2*
d^6*x^3 + 64*c^3*d^5)*(c/d^16)^(1/3))*sqrt(d*x^3 + c) + 18*(5*c*d^10*x^7 +
64*c^2*d^9*x^4 + 32*c^3*d^8*x)*sqrt(c/d^16) + 18*(c*d^5*x^8 + 38*c^2*d^4*x^
5 + 64*c^3*d^3*x^2)*(c/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3
 - 512*c^3)) - 8*(d^4*x^3 + c*d^3)*(c/d^16)^(1/6)*log(-33554432/3*((d^16*x^
9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13)*(c/d^16)^(5/6) - 6*(
c*d^2*x^7 + 80*c^2*d*x^4 + 160*c^3*x + 6*(5*c*d^12*x^5 + 32*c^2*d^11*x^2)*(
c/d^16)^(2/3) + (7*c*d^7*x^6 + 152*c^2*d^6*x^3 + 64*c^3*d^5)*(c/d^16)^(1/3)
)*sqrt(d*x^3 + c) + 18*(5*c*d^10*x^7 + 64*c^2*d^9*x^4 + 32*c^3*d^8*x)*sqrt(
c/d^16) + 18*(c*d^5*x^8 + 38*c^2*d^4*x^5 + 64*c^3*d^3*x^2)*(c/d^16)^(1/6))/
(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^4*x^3 + c*d^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**7/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^7}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \int -\frac{x^7}{(dx^3+c)^{\frac{3}{2}}(dx^3-8c)}\,dx$$

```
[In] integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] -integrate(x^7/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Giac [F]

$$\int \frac{x^7}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \int -\frac{x^7}{(dx^3+c)^{\frac{3}{2}}(dx^3-8c)}\,dx$$

```
[In] integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(-x^7/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \int \frac{x^7}{(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)}\,dx$$

```
[In] int(x^7/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
[Out] int(x^7/((c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

3.333 $\int \frac{x^4}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 635

$$\int \frac{x^4}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = -\frac{2x^2}{27cd\sqrt{c+dx^3}}+\frac{2\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{4\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}c^{5/6}d^{5/3}} + \frac{4\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{81c^{5/6}d^{5/3}} - \frac{4\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{5/6}d^{5/3}} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 4/81*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(5/6)/d^(
5/3)-4/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/6)/d^(5/3)-4/81*arctan(
c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(5/6)/d^(5/3)*3^(1/2
)-2/27*x^2/c/d/(d*x^3+c)^(1/2)+2/27*(d*x^3+c)^(1/2)/c/d^(5/3)/(d^(1/3)*x+c^
(1/3)*(1+3^(1/2)))+2/81*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1
-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)
```

```
-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^
(3/4)/c^(2/3)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)
*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/27*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3
)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/
2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+
c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(2/3)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1
/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {482, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{x^4}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right.\right.}{27\sqrt[4]{3}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{4\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}c^{5/6}d^{5/3}}+\frac{4\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{81c^{5/6}d^{5/3}}$$

$$-\frac{4\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{5/6}d^{5/3}}+\frac{2\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{2x^2}{27cd\sqrt{c+dx^3}}$$

```
[In] Int[x^4/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*x^2)/(27*c*d*Sqrt[c + d*x^3]) + (2*Sqrt[c + d*x^3])/(27*c*d^(5/3)*((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (4*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(
1/3)*x))/Sqrt[c + d*x^3]])/(27*Sqrt[3]*c^(5/6)*d^(5/3)) + (4*ArcTanh[(c^(1/
3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]])/(81*c^(5/6)*d^(5/3)) - (4*A
rcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]])/(81*c^(5/6)*d^(5/3)) - (Sqrt[2 - Sqrt[
3]]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1
```

```
/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(9*
3^(3/4)*c^(2/3)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2]*(c^(1/3) + d^(1/3)*x
)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) +
 d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*c^(2/3)*d^(5/3)
*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2
]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
```

```
EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/((((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\text{integral} = -\frac{2x^2}{27cd\sqrt{c+dx^3}} + \frac{2\int \frac{x\left(16c-\frac{dx^3}{2}\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{27cd}$$

$$= -\frac{2x^2}{27cd\sqrt{c+dx^3}} + \frac{2\int \left(\frac{x}{2\sqrt{c+dx^3}} + \frac{12cx}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{27cd}$$

$$= -\frac{2x^2}{27cd\sqrt{c+dx^3}} + \frac{8\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{9d} + \frac{\int \frac{x}{\sqrt{c+dx^3}}\,dx}{27cd}$$

$$= -\frac{2x^2}{27cd\sqrt{c+dx^3}} - \frac{2\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{27cd^2} + \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{27cd^{4/3}}$$

$$+ \frac{2\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{27c^{2/3}d^{4/3}} - \frac{\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{27c^{2/3}d^{4/3}} - \frac{2\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{9\sqrt[3]{c}d^{2/3}}$$

$$= -\frac{2x^2}{27cd\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$- \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{4\text{Subst}\left(\int \frac{1}{9-cx^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{27\sqrt[3]{c}d^{5/3}} - \frac{2\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27\sqrt[3]{c}d^{2/3}}$$

$$+ \frac{\left(8\sqrt[3]{d}\right)\text{Subst}\left(\int \frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{27c^{4/3}}$$

$$= -\frac{2x^2}{27cd\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{4\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}c^{5/6}d^{5/3}} + \frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{81c^{5/6}d^{5/3}} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{4\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{27\sqrt[3]{c}d^{5/3}}$$

$$= -\frac{2x^2}{27cd\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{4\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}c^{5/6}d^{5/3}} + \frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{81c^{5/6}d^{5/3}} - \frac{4\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{5/6}d^{5/3}} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 8.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.20

$$\int \frac{x^4}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = -\frac{x^2\left(80c-80c\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+dx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}{1080c^2d\sqrt{c+dx^3}}$$

```
[In] Integrate[x^4/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/1080*(x^2*(80*c - 80*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -(
(d*x^3)/c), (d*x^3)/(8*c)] + d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1
, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(c^2*d*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.36 (sec) , antiderivative size = 878, normalized size of antiderivative = 1.38

method	result	size
elliptic	Expression too large to display	878
default	Expression too large to display	1346

```
[In] int(x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/27/d*x^2/c/((x^3+c/d)*d)^(1/2)-2/81*I/d^2/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x
+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d
^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(
I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2
)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*
3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)))^(1/2)))-8/243*I/d^4/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2
*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d
*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)
*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/
3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*
```

```
d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticP
i(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*
d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c
*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.45 (sec) , antiderivative size = 2525, normalized size of antiderivative = 3.98

$$\int \frac{x^4}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/243*(18*sqrt(d*x^3 + c)*d*x^2 + 18*(d*x^3 + c)*sqrt(d)*weierstrassZeta(0
, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) - (c*d^3*x^3 + c^2*d^2 + sqrt(
-3)*(c*d^3*x^3 + c^2*d^2))*(1/(c^5*d^10))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^
6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^4*d^9*x^7 + 64*c^5*d^8*x^4 + 32*c^6*d
^7*x + sqrt(-3)*(5*c^4*d^9*x^7 + 64*c^5*d^8*x^4 + 32*c^6*d^7*x))*(1/(c^5*d^
10))^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^5*d^10*x^5 + 32*c^6*d^9*x^2 - sqrt(-
3)*(5*c^5*d^10*x^5 + 32*c^6*d^9*x^2))*(1/(c^5*d^10))^(5/6) - 2*(7*c^3*d^7*x
^6 + 152*c^4*d^6*x^3 + 64*c^5*d^5)*sqrt(1/(c^5*d^10)) + (c*d^4*x^7 + 80*c^2
*d^3*x^4 + 160*c^3*d^2*x + sqrt(-3)*(c*d^4*x^7 + 80*c^2*d^3*x^4 + 160*c^3*d
^2*x))*(1/(c^5*d^10))^(1/6)) - 9*(c^2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4
*x^2 - sqrt(-3)*(c^2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4*x^2))*(1/(c^5*d^
10))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (c*d^3*x^
3 + c^2*d^2 + sqrt(-3)*(c*d^3*x^3 + c^2*d^2))*(1/(c^5*d^10))^(1/6)*log((d^3
*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^4*d^9*x^7 + 64*c^5
*d^8*x^4 + 32*c^6*d^7*x + sqrt(-3)*(5*c^4*d^9*x^7 + 64*c^5*d^8*x^4 + 32*c^6
*d^7*x))*(1/(c^5*d^10))^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^5*d^10*x^5 + 32*c
^6*d^9*x^2 - sqrt(-3)*(5*c^5*d^10*x^5 + 32*c^6*d^9*x^2))*(1/(c^5*d^10))^(5/
6) - 2*(7*c^3*d^7*x^6 + 152*c^4*d^6*x^3 + 64*c^5*d^5)*sqrt(1/(c^5*d^10)) +
(c*d^4*x^7 + 80*c^2*d^3*x^4 + 160*c^3*d^2*x + sqrt(-3)*(c*d^4*x^7 + 80*c^2*
d^3*x^4 + 160*c^3*d^2*x))*(1/(c^5*d^10))^(1/6)) - 9*(c^2*d^6*x^8 + 38*c^3*d
^5*x^5 + 64*c^4*d^4*x^2 - sqrt(-3)*(c^2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d
^4*x^2))*(1/(c^5*d^10))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 51
2*c^3)) - (c*d^3*x^3 + c^2*d^2 - sqrt(-3)*(c*d^3*x^3 + c^2*d^2))*(1/(c^5*d^
10))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c
^4*d^9*x^7 + 64*c^5*d^8*x^4 + 32*c^6*d^7*x - sqrt(-3)*(5*c^4*d^9*x^7 + 64*c
^5*d^8*x^4 + 32*c^6*d^7*x))*(1/(c^5*d^10))^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*
c^5*d^10*x^5 + 32*c^6*d^9*x^2 + sqrt(-3)*(5*c^5*d^10*x^5 + 32*c^6*d^9*x^2))
```

```
*(1/(c^5*d^10))^(5/6) - 2*(7*c^3*d^7*x^6 + 152*c^4*d^6*x^3 + 64*c^5*d^5)*sq
rt(1/(c^5*d^10)) + (c*d^4*x^7 + 80*c^2*d^3*x^4 + 160*c^3*d^2*x - sqrt(-3)*(
c*d^4*x^7 + 80*c^2*d^3*x^4 + 160*c^3*d^2*x))*(1/(c^5*d^10))^(1/6)) - 9*(c^2
*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4*x^2 + sqrt(-3)*(c^2*d^6*x^8 + 38*c^3
*d^5*x^5 + 64*c^4*d^4*x^2))*(1/(c^5*d^10))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 +
 192*c^2*d*x^3 - 512*c^3)) + (c*d^3*x^3 + c^2*d^2 - sqrt(-3)*(c*d^3*x^3 + c
^2*d^2))*(1/(c^5*d^10))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3
 + 640*c^3 - 9*(5*c^4*d^9*x^7 + 64*c^5*d^8*x^4 + 32*c^6*d^7*x - sqrt(-3)*(5
*c^4*d^9*x^7 + 64*c^5*d^8*x^4 + 32*c^6*d^7*x))*(1/(c^5*d^10))^(2/3) - 3*sqr
t(d*x^3 + c)*(6*(5*c^5*d^10*x^5 + 32*c^6*d^9*x^2 + sqrt(-3)*(5*c^5*d^10*x^5
 + 32*c^6*d^9*x^2))*(1/(c^5*d^10))^(5/6) - 2*(7*c^3*d^7*x^6 + 152*c^4*d^6*x
^3 + 64*c^5*d^5)*sqrt(1/(c^5*d^10)) + (c*d^4*x^7 + 80*c^2*d^3*x^4 + 160*c^3
*d^2*x - sqrt(-3)*(c*d^4*x^7 + 80*c^2*d^3*x^4 + 160*c^3*d^2*x))*(1/(c^5*d^1
0))^(1/6)) - 9*(c^2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4*x^2 + sqrt(-3)*(c
^2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4*x^2))*(1/(c^5*d^10))^(1/3))/(d^3*x
^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*(c*d^3*x^3 + c^2*d^2)*(1/
(c^5*d^10))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 +
 18*(5*c^4*d^9*x^7 + 64*c^5*d^8*x^4 + 32*c^6*d^7*x)*(1/(c^5*d^10))^(2/3) +
6*sqrt(d*x^3 + c)*(6*(5*c^5*d^10*x^5 + 32*c^6*d^9*x^2)*(1/(c^5*d^10))^(5/6)
 + (7*c^3*d^7*x^6 + 152*c^4*d^6*x^3 + 64*c^5*d^5)*sqrt(1/(c^5*d^10)) + (c*d
^4*x^7 + 80*c^2*d^3*x^4 + 160*c^3*d^2*x)*(1/(c^5*d^10))^(1/6)) + 18*(c^2*d^
6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4*x^2)*(1/(c^5*d^10))^(1/3))/(d^3*x^9 - 2
4*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 2*(c*d^3*x^3 + c^2*d^2)*(1/(c^5*d
^10))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(5
*c^4*d^9*x^7 + 64*c^5*d^8*x^4 + 32*c^6*d^7*x)*(1/(c^5*d^10))^(2/3) - 6*sqrt
(d*x^3 + c)*(6*(5*c^5*d^10*x^5 + 32*c^6*d^9*x^2)*(1/(c^5*d^10))^(5/6) + (7*
c^3*d^7*x^6 + 152*c^4*d^6*x^3 + 64*c^5*d^5)*sqrt(1/(c^5*d^10)) + (c*d^4*x^7
 + 80*c^2*d^3*x^4 + 160*c^3*d^2*x)*(1/(c^5*d^10))^(1/6)) + 18*(c^2*d^6*x^8
+ 38*c^3*d^5*x^5 + 64*c^4*d^4*x^2)*(1/(c^5*d^10))^(1/3))/(d^3*x^9 - 24*c*d^
2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(c*d^3*x^3 + c^2*d^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**4/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^4}{(8c - dx^3)(c + dx^3)^{3/2}} dx = \int -\frac{x^4}{(dx^3 + c)^{\frac{3}{2}}(dx^3 - 8c)} dx$$

```
[In] integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] -integrate(x^4/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Giac [F]

$$\int \frac{x^4}{(8c - dx^3)(c + dx^3)^{3/2}} dx = \int -\frac{x^4}{(dx^3 + c)^{\frac{3}{2}}(dx^3 - 8c)} dx$$

```
[In] integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(-x^4/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(8c - dx^3)(c + dx^3)^{3/2}} dx = \int \frac{x^4}{(d x^3 + c)^{3/2} (8 c - d x^3)} dx$$

```
[In] int(x^4/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
[Out] int(x^4/((c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

3.334 $\int \frac{x}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 632

$$\int \frac{x}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{2x^2}{27c^2\sqrt{c+dx^3}} - \frac{2\sqrt{c+dx^3}}{27c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{54\sqrt{3}c^{11/6}d^{2/3}} + \frac{\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{162c^{11/6}d^{2/3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{162c^{11/6}d^{2/3}} + \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{9\,3^{3/4}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 1/162*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(11/6)/d
^(2/3)-1/162*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)/d^(2/3)-1/162*ar
ctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(11/6)/d^(2/3)*
3^(1/2)+2/27*x^2/c^2/(d*x^3+c)^(1/2)-2/27*(d*x^3+c)^(1/2)/c^2/d^(2/3)/(d^(1
/3)*x+c^(1/3)*(1+3^(1/2)))-2/81*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^
(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*(
```

```
(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^
(1/2)*3^(3/4)/c^(5/3)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/
(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/27*(c^(1/3)+d^(1/3)*x)*EllipticE
((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+
2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^
(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(5/3)/d^(2/3)/(d*x^3+c)^(1/
2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 632, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {483, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{x}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = -\frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{54\sqrt{3}c^{11/6}d^{2/3}} + \frac{\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{162c^{11/6}d^{2/3}} - \frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{162c^{11/6}d^{2/3}} - \frac{2\sqrt{c+dx^3}}{27c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{2x^2}{27c^2\sqrt{c+dx^3}}$$

```
[In] Int[x/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*x^2)/(27*c^2*Sqrt[c + d*x^3]) - (2*Sqrt[c + d*x^3])/(27*c^2*d^(2/3)*((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)) - ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/
3)*x))/Sqrt[c + d*x^3]]/(54*Sqrt[3]*c^(11/6)*d^(2/3)) + ArcTanh[(c^(1/3) +
d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]/(162*c^(11/6)*d^(2/3)) - ArcTanh[
Sqrt[c + d*x^3]/(3*Sqrt[c])]/(162*c^(11/6)*d^(2/3)) + (Sqrt[2 - Sqrt[3]]*(c
```

```
^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) +
d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(9*3^(3/4
)*c^(5/3)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (2*Sqrt[2]*(c^(1/3) + d^(1/3)*x)*Sqrt
[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*c^(5/3)*d^(2/3)*Sqrt[
(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt
[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
```

```
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x^2}{27c^2\sqrt{c+dx^3}} - \frac{2\int \frac{x\left(\frac{5cd}{2}-\frac{d^2x^3}{2}\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{27c^2d} \\
&= \frac{2x^2}{27c^2\sqrt{c+dx^3}} - \frac{2\int \left(\frac{dx}{2\sqrt{c+dx^3}}-\frac{3cdx}{2(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{27c^2d} \\
&= \frac{2x^2}{27c^2\sqrt{c+dx^3}} - \frac{\int \frac{x}{\sqrt{c+dx^3}}\,dx}{27c^2} + \frac{\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{9c} \\
&= \frac{2x^2}{27c^2\sqrt{c+dx^3}} - \frac{\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{108c^2d} - \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{27c^2\sqrt[3]{d}} \\
&\quad + \frac{\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{108c^{5/3}\sqrt[3]{d}} + \frac{\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{27c^{5/3}\sqrt[3]{d}} - \frac{\sqrt[3]{d}\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{36c^{4/3}}
\end{aligned}$$

$$= \frac{2x^2}{27c^2\sqrt{c+dx^3}} - \frac{2\sqrt{c+dx^3}}{27c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$+ \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$- \frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{\text{Subst}\left(\int \frac{1}{9-cx^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{54c^{4/3}d^{2/3}} - \frac{\sqrt[3]{d}\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{108c^{4/3}}$$

$$+ \frac{d^{4/3}\,\text{Subst}\left(\int \frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{27c^{7/3}}$$

$$= \frac{2x^2}{27c^2\sqrt{c+dx^3}} - \frac{2\sqrt{c+dx^3}}{27c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{54\sqrt{3}c^{11/6}d^{2/3}} + \frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{162c^{11/6}d^{2/3}} + \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{54c^{4/3}d^{2/3}}$$

$$= \frac{2x^2}{27c^2\sqrt{c+dx^3}} - \frac{2\sqrt{c+dx^3}}{27c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{54\sqrt{3}c^{11/6}d^{2/3}} + \frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{162c^{11/6}d^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{162c^{11/6}d^{2/3}} + \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.20

$$\int \frac{x}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{x^2\left(160c-25c\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+2dx^3\sqrt{1+\frac{dx^3}{c}}\,\text{Ap}\right.}{2160c^3\sqrt{c+dx^3}}$$

```
[In] Integrate[x/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^2*(160*c - 25*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)
/c), (d*x^3)/(8*c)] + 2*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3
, -((d*x^3)/c), (d*x^3)/(8*c)]))/(2160*c^3*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.55 (sec) , antiderivative size = 875, normalized size of antiderivative = 1.38

method	result	size
default	Expression too large to display	875
elliptic	Expression too large to display	875

```
[In] int(x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/27*x^2/c^2/((x^3+c/d)*d)^(1/2)+2/81*I/c^2*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^
2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I
*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3
^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2)))-1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(
2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(
d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2
)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1
/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c
*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Elliptic
Pi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3
```

```
^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2
*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*
c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.43 (sec) , antiderivative size = 2525, normalized size of antiderivative = 4.00

$$\int \frac{x}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1944*(144*sqrt(d*x^3 + c)*d*x^2 + 144*(d*x^3 + c)*sqrt(d)*weierstrassZeta
(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + (c^2*d^2*x^3 + c^3*d + sqr
t(-3)*(c^2*d^2*x^3 + c^3*d))*(1/(c^11*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*
x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^8*d^5*x^7 + 64*c^9*d^4*x^4 + 32*c^1
0*d^3*x + sqrt(-3)*(5*c^8*d^5*x^7 + 64*c^9*d^4*x^4 + 32*c^10*d^3*x))*(1/(c^
11*d^4))^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^10*d^5*x^5 + 32*c^11*d^4*x^2 - s
qrt(-3)*(5*c^10*d^5*x^5 + 32*c^11*d^4*x^2))*(1/(c^11*d^4))^(5/6) - 2*(7*c^6
*d^4*x^6 + 152*c^7*d^3*x^3 + 64*c^8*d^2)*sqrt(1/(c^11*d^4)) + (c^2*d^3*x^7
+ 80*c^3*d^2*x^4 + 160*c^4*d*x + sqrt(-3)*(c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 1
60*c^4*d*x))*(1/(c^11*d^4))^(1/6)) - 9*(c^4*d^4*x^8 + 38*c^5*d^3*x^5 + 64*c
^6*d^2*x^2 - sqrt(-3)*(c^4*d^4*x^8 + 38*c^5*d^3*x^5 + 64*c^6*d^2*x^2))*(1/(
c^11*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c^
2*d^2*x^3 + c^3*d + sqrt(-3)*(c^2*d^2*x^3 + c^3*d))*(1/(c^11*d^4))^(1/6)*lo
g((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^8*d^5*x^7 +
64*c^9*d^4*x^4 + 32*c^10*d^3*x + sqrt(-3)*(5*c^8*d^5*x^7 + 64*c^9*d^4*x^4 +
 32*c^10*d^3*x))*(1/(c^11*d^4))^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^10*d^5*x^
5 + 32*c^11*d^4*x^2 - sqrt(-3)*(5*c^10*d^5*x^5 + 32*c^11*d^4*x^2))*(1/(c^11
*d^4))^(5/6) - 2*(7*c^6*d^4*x^6 + 152*c^7*d^3*x^3 + 64*c^8*d^2)*sqrt(1/(c^1
1*d^4)) + (c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x + sqrt(-3)*(c^2*d^3*x
^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x))*(1/(c^11*d^4))^(1/6)) - 9*(c^4*d^4*x^8
+ 38*c^5*d^3*x^5 + 64*c^6*d^2*x^2 - sqrt(-3)*(c^4*d^4*x^8 + 38*c^5*d^3*x^5
+ 64*c^6*d^2*x^2))*(1/(c^11*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*
d*x^3 - 512*c^3)) + (c^2*d^2*x^3 + c^3*d - sqrt(-3)*(c^2*d^2*x^3 + c^3*d))*
(1/(c^11*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^
3 - 9*(5*c^8*d^5*x^7 + 64*c^9*d^4*x^4 + 32*c^10*d^3*x - sqrt(-3)*(5*c^8*d^5
*x^7 + 64*c^9*d^4*x^4 + 32*c^10*d^3*x))*(1/(c^11*d^4))^(2/3) + 3*sqrt(d*x^3
 + c)*(6*(5*c^10*d^5*x^5 + 32*c^11*d^4*x^2 + sqrt(-3)*(5*c^10*d^5*x^5 + 32*
c^11*d^4*x^2))*(1/(c^11*d^4))^(5/6) - 2*(7*c^6*d^4*x^6 + 152*c^7*d^3*x^3 +
64*c^8*d^2)*sqrt(1/(c^11*d^4)) + (c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*
```

```
x - sqrt(-3)*(c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x))*(1/(c^11*d^4))^(
1/6)) - 9*(c^4*d^4*x^8 + 38*c^5*d^3*x^5 + 64*c^6*d^2*x^2 + sqrt(-3)*(c^4*d^
4*x^8 + 38*c^5*d^3*x^5 + 64*c^6*d^2*x^2))*(1/(c^11*d^4))^(1/3))/(d^3*x^9 -
24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c^2*d^2*x^3 + c^3*d - sqrt(-3)*
(c^2*d^2*x^3 + c^3*d))*(1/(c^11*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 +
1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^8*d^5*x^7 + 64*c^9*d^4*x^4 + 32*c^10*d^3*
x - sqrt(-3)*(5*c^8*d^5*x^7 + 64*c^9*d^4*x^4 + 32*c^10*d^3*x))*(1/(c^11*d^4
))^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^10*d^5*x^5 + 32*c^11*d^4*x^2 + sqrt(-3
)*(5*c^10*d^5*x^5 + 32*c^11*d^4*x^2))*(1/(c^11*d^4))^(5/6) - 2*(7*c^6*d^4*x
^6 + 152*c^7*d^3*x^3 + 64*c^8*d^2)*sqrt(1/(c^11*d^4)) + (c^2*d^3*x^7 + 80*c
^3*d^2*x^4 + 160*c^4*d*x - sqrt(-3)*(c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4
*d*x))*(1/(c^11*d^4))^(1/6)) - 9*(c^4*d^4*x^8 + 38*c^5*d^3*x^5 + 64*c^6*d^2
*x^2 + sqrt(-3)*(c^4*d^4*x^8 + 38*c^5*d^3*x^5 + 64*c^6*d^2*x^2))*(1/(c^11*d
^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 2*(c^2*d^
2*x^3 + c^3*d)*(1/(c^11*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2
*d*x^3 + 640*c^3 + 18*(5*c^8*d^5*x^7 + 64*c^9*d^4*x^4 + 32*c^10*d^3*x)*(1/(
c^11*d^4))^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^10*d^5*x^5 + 32*c^11*d^4*x^2)*
(1/(c^11*d^4))^(5/6) + (7*c^6*d^4*x^6 + 152*c^7*d^3*x^3 + 64*c^8*d^2)*sqrt(
1/(c^11*d^4)) + (c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x)*(1/(c^11*d^4))
^(1/6)) + 18*(c^4*d^4*x^8 + 38*c^5*d^3*x^5 + 64*c^6*d^2*x^2)*(1/(c^11*d^4))
^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*(c^2*d^2*x^
3 + c^3*d)*(1/(c^11*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x
^3 + 640*c^3 + 18*(5*c^8*d^5*x^7 + 64*c^9*d^4*x^4 + 32*c^10*d^3*x)*(1/(c^11
*d^4))^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^10*d^5*x^5 + 32*c^11*d^4*x^2)*(1/(
c^11*d^4))^(5/6) + (7*c^6*d^4*x^6 + 152*c^7*d^3*x^3 + 64*c^8*d^2)*sqrt(1/(c
^11*d^4)) + (c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x)*(1/(c^11*d^4))^(1/
6)) + 18*(c^4*d^4*x^8 + 38*c^5*d^3*x^5 + 64*c^6*d^2*x^2)*(1/(c^11*d^4))^(1/
3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(c^2*d^2*x^3 + c^3
*d)
```

Sympy [F]

$$\int \frac{x}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = -\int \frac{x}{-8c^2\sqrt{c+dx^3}-7cdx^3\sqrt{c+dx^3}+d^2x^6\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(x/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] -Integral(x/(-8*c**2*sqrt(c + d*x**3) - 7*c*d*x**3*sqrt(c + d*x**3) + d**2*
x**6*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \int -\frac{x}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)}\,dx$$

```
[In] integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] -integrate(x/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Giac [F]

$$\int \frac{x}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \int -\frac{x}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)}\,dx$$

```
[In] integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(-x/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \int \frac{x}{(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)}\,dx$$

```
[In] int(x/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
[Out] int(x/((c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

3.335 $\int \frac{1}{x^2(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 653

$$\int \frac{1}{x^2(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{43\sqrt{c+dx^3}}{216c^3x} + \frac{43\sqrt[3]{d}\sqrt{c+dx^3}}{216c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{432\sqrt{3}c^{17/6}} + \frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1296c^{17/6}} - \frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1296c^{17/6}} - \frac{43\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{144\ 3^{3/4}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{43\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{108\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 1/1296*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c
^(17/6)-1/1296*d^(1/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)-1/1296
```

```
*d^(1/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(17/
6)*3^(1/2)+2/27/c^2/x/(d*x^3+c)^(1/2)-43/216*(d*x^3+c)^(1/2)/c^3/x+43/216*d
^(1/3)*(d*x^3+c)^(1/2)/c^3/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+43/648*d^(1/3)*(
c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(
1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(
d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(8/3)*2^(1/2)/(d*x^3+c)^(
1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-
43/432*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2))
)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*
((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)
^(1/2)*3^(1/4)/c^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3
)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 653, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {483, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{1}{x^2\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{43\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right.}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right.}\right.\right.}{108\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{43\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{144\ 3^{3/4}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{432\sqrt{3}c^{17/6}}+\frac{\sqrt[3]{d}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1296c^{17/6}}-\frac{\sqrt[3]{d}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1296c^{17/6}}$$

$$-\frac{43\sqrt{c+dx^3}}{216c^3x}+\frac{43\sqrt[3]{d}\sqrt{c+dx^3}}{216c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{2}{27c^2x\sqrt{c+dx^3}}$$

```
[In] Int[1/(x^2*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] 2/(27*c^2*x*Sqrt[c + d*x^3]) - (43*Sqrt[c + d*x^3])/(216*c^3*x) + (43*d^(1/
3)*Sqrt[c + d*x^3])/(216*c^3*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (d^(1/3
)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(432*Sqr
```

```
t[3]*c^(17/6)) + (d^(1/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c
 + d*x^3])])/(1296*c^(17/6)) - (d^(1/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])
])/(1296*c^(17/6)) - (43*Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sq
rt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(144*3^(3/4)*c^(8/3)*Sqrt[(c^(1
/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c +
d*x^3]) + (43*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)
*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[(
(1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7
 - 4*Sqrt[3]])/(108*Sqrt[2]*3^(1/4)*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3
)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
```

```
/; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{2\int \frac{-\frac{43cd}{2}+\frac{5d^2x^3}{2}}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{27c^2d} \\
&= \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{43\sqrt{c+dx^3}}{216c^3x} + \frac{\int \frac{x\left(\frac{175c^2d^2}{2}-\frac{43}{4}cd^3x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{108c^4d} \\
&= \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{43\sqrt{c+dx^3}}{216c^3x} + \frac{\int \left(\frac{43cd^2x}{4\sqrt{c+dx^3}} + \frac{3c^2d^2x}{2(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{108c^4d} \\
&= \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{43\sqrt{c+dx^3}}{216c^3x} + \frac{(43d)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{432c^3} + \frac{d\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{72c^2}
\end{aligned}$$

$$= \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{43\sqrt{c+dx^3}}{216c^3x} - \frac{\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{864c^3} + \frac{\left(43d^{2/3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{432c^3} + \frac{d^{2/3}\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{864c^{8/3}} - \frac{\left(43\left(1-\sqrt{3}\right)d^{2/3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{432c^{8/3}} - \frac{d^{4/3}\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{288c^{7/3}}$$

$$= \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{43\sqrt{c+dx^3}}{216c^3x} + \frac{43\sqrt[3]{d}\sqrt{c+dx^3}}{216c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{43\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{144\ 3^{3/4}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{43\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{108\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{\sqrt[3]{d}\text{Subst}\left(\int \frac{1}{9-cx^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{432c^{7/3}} - \frac{d^{4/3}\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{864c^{7/3}} + \frac{d^{7/3}\text{Subst}\left(\int \frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{216c^{10/3}}$$

$$= \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{43\sqrt{c+dx^3}}{216c^3x} + \frac{43\sqrt[3]{d}\sqrt{c+dx^3}}{216c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{432\sqrt{3}c^{17/6}} + \frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1296c^{17/6}} - \frac{43\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{144\ 3^{3/4}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{43\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{108\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{\sqrt[3]{d}\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{432c^{7/3}}$$

$$= \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{43\sqrt{c+dx^3}}{216c^3x} + \frac{43\sqrt[3]{d}\sqrt{c+dx^3}}{216c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{432\sqrt{3}c^{17/6}} + \frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1296c^{17/6}} - \frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1296c^{17/6}} - \frac{43\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{144\ 3^{3/4}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{43\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{108\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.21

$$\int \frac{1}{x^2\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{-80c(27c+43dx^3)+875cdx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{17280c^4x\sqrt{c+dx^3}}$$

```
[In] Integrate[1/(x^2*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-80*c*(27*c + 43*d*x^3) + 875*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/
2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 43*d^2*x^6*Sqrt[1 + (d*x^3)/c]*Ap
pellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(17280*c^4*x*Sqrt[c
+ d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.42 (sec) , antiderivative size = 890, normalized size of antiderivative = 1.36

method	result	size
elliptic	Expression too large to display	890
risch	Expression too large to display	1334
default	Expression too large to display	1361

```
[In] int(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/27*d/c^3*x^2/((x^3+c/d)*d)^(1/2)-1/8*(d*x^3+c)^(1/2)/c^3/x-43/648*I/c^3*
3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c
)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1
/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-1/1944*I/c^3/d^2*2^(1/2)*sum(1/_a
lpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(
1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I
*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3
)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*
_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_al
pha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(
-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*
c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8
*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.64 (sec) , antiderivative size = 2379, normalized size of antiderivative = 3.64

$$\int \frac{1}{x^2 (8c - dx^3) (c + dx^3)^{3/2}} dx = \text{Too large to display}$$

```
[In] integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15552*(3096*(d*x^4 + c*x)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrass
PInverse(0, -4*c/d, x)) - (c^3*d*x^4 + c^4*x + sqrt(-3)*(c^3*d*x^4 + c^4*x)
)*(d^2/c^17)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^
3*d - 9*(5*c^12*d^2*x^7 + 64*c^13*d*x^4 + 32*c^14*x + sqrt(-3)*(5*c^12*d^2*
x^7 + 64*c^13*d*x^4 + 32*c^14*x))*(d^2/c^17)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(
5*c^15*d*x^5 + 32*c^16*x^2 - sqrt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2))*(d^2/c^
17)^(5/6) - 2*(7*c^9*d^2*x^6 + 152*c^10*d*x^3 + 64*c^11)*sqrt(d^2/c^17) + (
c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x + sqrt(-3)*(c^3*d^3*x^7 + 80*c^4
*d^2*x^4 + 160*c^5*d*x))*(d^2/c^17)^(1/6)) - 9*(c^6*d^3*x^8 + 38*c^7*d^2*x^
5 + 64*c^8*d*x^2 - sqrt(-3)*(c^6*d^3*x^8 + 38*c^7*d^2*x^5 + 64*c^8*d*x^2))*
(d^2/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (c^
3*d*x^4 + c^4*x + sqrt(-3)*(c^3*d*x^4 + c^4*x))*(d^2/c^17)^(1/6)*log((d^4*x
^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d - 9*(5*c^12*d^2*x^7 + 64*
c^13*d*x^4 + 32*c^14*x + sqrt(-3)*(5*c^12*d^2*x^7 + 64*c^13*d*x^4 + 32*c^14
*x))*(d^2/c^17)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2 -
sqrt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2))*(d^2/c^17)^(5/6) - 2*(7*c^9*d^2*x^6
+ 152*c^10*d*x^3 + 64*c^11)*sqrt(d^2/c^17) + (c^3*d^3*x^7 + 80*c^4*d^2*x^4
+ 160*c^5*d*x + sqrt(-3)*(c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x))*(d^2
/c^17)^(1/6)) - 9*(c^6*d^3*x^8 + 38*c^7*d^2*x^5 + 64*c^8*d*x^2 - sqrt(-3)*(
c^6*d^3*x^8 + 38*c^7*d^2*x^5 + 64*c^8*d*x^2))*(d^2/c^17)^(1/3))/(d^3*x^9 -
24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c^3*d*x^4 + c^4*x - sqrt(-3)*(c
^3*d*x^4 + c^4*x))*(d^2/c^17)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2
*d^2*x^3 + 640*c^3*d - 9*(5*c^12*d^2*x^7 + 64*c^13*d*x^4 + 32*c^14*x - sqrt
(-3)*(5*c^12*d^2*x^7 + 64*c^13*d*x^4 + 32*c^14*x))*(d^2/c^17)^(2/3) + 3*sqr
t(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2 + sqrt(-3)*(5*c^15*d*x^5 + 32*c
^16*x^2))*(d^2/c^17)^(5/6) - 2*(7*c^9*d^2*x^6 + 152*c^10*d*x^3 + 64*c^11)*s
qrt(d^2/c^17) + (c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x - sqrt(-3)*(c^3
*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x))*(d^2/c^17)^(1/6)) - 9*(c^6*d^3*x^
8 + 38*c^7*d^2*x^5 + 64*c^8*d*x^2 + sqrt(-3)*(c^6*d^3*x^8 + 38*c^7*d^2*x^5
+ 64*c^8*d*x^2))*(d^2/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3
- 512*c^3)) + (c^3*d*x^4 + c^4*x - sqrt(-3)*(c^3*d*x^4 + c^4*x))*(d^2/c^17)
^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d - 9*(5*c
^12*d^2*x^7 + 64*c^13*d*x^4 + 32*c^14*x - sqrt(-3)*(5*c^12*d^2*x^7 + 64*c^1
3*d*x^4 + 32*c^14*x))*(d^2/c^17)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5
 + 32*c^16*x^2 + sqrt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2))*(d^2/c^17)^(5/6) -
```

```
2*(7*c^9*d^2*x^6 + 152*c^10*d*x^3 + 64*c^11)*sqrt(d^2/c^17) + (c^3*d^3*x^7
+ 80*c^4*d^2*x^4 + 160*c^5*d*x - sqrt(-3)*(c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 1
60*c^5*d*x))*(d^2/c^17)^(1/6)) - 9*(c^6*d^3*x^8 + 38*c^7*d^2*x^5 + 64*c^8*d
*x^2 + sqrt(-3)*(c^6*d^3*x^8 + 38*c^7*d^2*x^5 + 64*c^8*d*x^2))*(d^2/c^17)^(
1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*(c^3*d*x^4 +
c^4*x)*(d^2/c^17)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 6
40*c^3*d + 18*(5*c^12*d^2*x^7 + 64*c^13*d*x^4 + 32*c^14*x)*(d^2/c^17)^(2/3)
 + 6*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2)*(d^2/c^17)^(5/6) + (7*
c^9*d^2*x^6 + 152*c^10*d*x^3 + 64*c^11)*sqrt(d^2/c^17) + (c^3*d^3*x^7 + 80*
c^4*d^2*x^4 + 160*c^5*d*x)*(d^2/c^17)^(1/6)) + 18*(c^6*d^3*x^8 + 38*c^7*d^2
*x^5 + 64*c^8*d*x^2)*(d^2/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*
x^3 - 512*c^3)) + 2*(c^3*d*x^4 + c^4*x)*(d^2/c^17)^(1/6)*log((d^4*x^9 + 318
*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d + 18*(5*c^12*d^2*x^7 + 64*c^13*d*
x^4 + 32*c^14*x)*(d^2/c^17)^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32
*c^16*x^2)*(d^2/c^17)^(5/6) + (7*c^9*d^2*x^6 + 152*c^10*d*x^3 + 64*c^11)*sq
rt(d^2/c^17) + (c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x)*(d^2/c^17)^(1/6
)) + 18*(c^6*d^3*x^8 + 38*c^7*d^2*x^5 + 64*c^8*d*x^2)*(d^2/c^17)^(1/3))/(d^
3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 72*(43*d*x^3 + 27*c)*sqr
t(d*x^3 + c))/(c^3*d*x^4 + c^4*x)
```

Sympy [F]

$$\int \frac{1}{x^2(8c-dx^3)(c+dx^3)^{3/2}}\,dx = -\int \frac{1}{-8c^2x^2\sqrt{c+dx^3}-7cdx^5\sqrt{c+dx^3}+d^2x^8\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(1/x**2/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
[Out] -Integral(1/(-8*c**2*x**2*sqrt(c + d*x**3) - 7*c*d*x**5*sqrt(c + d*x**3) +
d**2*x**8*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^2(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \int -\frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)x^2}\,dx$$

```
[In] integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \int -\frac{1}{(dx^3 + c)^{\frac{3}{2}} (dx^3 - 8c)x^2} \, dx$$

```
[In] integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \int \frac{1}{x^2 (d x^3 + c)^{3/2} (8 c - d x^3)} \, dx$$

```
[In] int(1/(x^2*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
[Out] int(1/(x^2*(c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

3.336 $\int \frac{1}{x^5(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 675

$$\int \frac{1}{x^5(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{113d\sqrt{c+dx^3}}{432c^4x} - \frac{113d^{4/3}\sqrt{c+dx^3}}{432c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{d^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3456\sqrt{3}c^{23/6}} + \frac{d^{4/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{10368c^{23/6}} - \frac{d^{4/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{10368c^{23/6}} + \frac{113\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{288\ 3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{113d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{216\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 1/10368*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/
c^(23/6)-1/10368*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(23/6)-1/10
```

```
368*d^(4/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(
23/6)*3^(1/2)+2/27/c^2/x^4/(d*x^3+c)^(1/2)-91/864*(d*x^3+c)^(1/2)/c^3/x^4+1
13/432*d*(d*x^3+c)^(1/2)/c^4/x-113/432*d^(4/3)*(d*x^3+c)^(1/2)/c^4/(d^(1/3)
*x+c^(1/3)*(1+3^(1/2)))-113/1296*d^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(
1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*
((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)
^(1/2)*3^(3/4)/c^(11/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x
)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+113/864*d^(4/3)*(c^(1/3)+d^(1/3)
*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)
)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(
2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(11/3)/(d*x^3+
c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 675, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {483, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{1}{x^5 (8c-dx^3) (c+dx^3)^{3/2}} \, dx =$$

$$-\frac{113 d^{4/3} \left(\sqrt[3]{c}+\sqrt[3]{d}x\right) \sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{216\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+\frac{113\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{288\ 3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{d^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3456\sqrt{3}c^{23/6}}+\frac{d^{4/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{10368c^{23/6}}-\frac{d^{4/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{10368c^{23/6}}$$
$$-\frac{113d^{4/3}\sqrt{c+dx^3}}{432c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{113d\sqrt{c+dx^3}}{432c^4x}-\frac{91\sqrt{c+dx^3}}{864c^3x^4}+\frac{2}{27c^2x^4\sqrt{c+dx^3}}$$

```
[In] Int[1/(x^5*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] 2/(27*c^2*x^4*Sqrt[c + d*x^3]) - (91*Sqrt[c + d*x^3])/(864*c^3*x^4) + (113*
d*Sqrt[c + d*x^3])/(432*c^4*x) - (113*d^(4/3)*Sqrt[c + d*x^3])/(432*c^4*((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (d^(4/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/
3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(3456*Sqrt[3]*c^(23/6)) + (d^(4/3)*ArcTa
nh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]])/(10368*c^(23/6)) -
 (d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]])/(10368*c^(23/6)) + (113*Sqr
t[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3
)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[
((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -
7 - 4*Sqrt[3]])/(288*3^(3/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (113*d^(4/3)*(c^(
1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^
(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(216*Sqrt[2
]*3^(1/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1
/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{2\int \frac{-\frac{91cd}{2}+\frac{11d^2x^3}{2}}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx}{27c^2d} \\
&= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{\int \frac{-904c^2d^2+\frac{455}{4}cd^3x^3}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{432c^4d} \\
&= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{113d\sqrt{c+dx^3}}{432c^4x} - \frac{\int \frac{x(3610c^3d^3-452c^2d^4x^3)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{3456c^6d} \\
&= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{113d\sqrt{c+dx^3}}{432c^4x} - \frac{\int \left(\frac{452c^2d^3x}{\sqrt{c+dx^3}} - \frac{6c^3d^3x}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{3456c^6d}
\end{aligned}$$

$$= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{113d\sqrt{c+dx^3}}{432c^4x} - \frac{(113d^2)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{864c^4} + \frac{d^2\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{576c^3}$$

$$= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{113d\sqrt{c+dx^3}}{432c^4x} - \frac{d\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{6912c^4} - \frac{(113d^{5/3})\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{864c^4} + \frac{d^{5/3}\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{6912c^{11/3}} + \frac{\left(113\left(1-\sqrt{3}\right)d^{5/3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{864c^{11/3}} - \frac{d^{7/3}\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{2304c^{10/3}}$$

$$= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{113d\sqrt{c+dx^3}}{432c^4x} - \frac{113d^{4/3}\sqrt{c+dx^3}}{432c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{113\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{288\ 3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{113d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{216\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{d^{4/3}\text{Subst}\left(\int \frac{1}{9-cx^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{3456c^{10/3}} - \frac{d^{7/3}\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{6912c^{10/3}} + \frac{d^{10/3}\text{Subst}\left(\int \frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{1728c^{13/3}}$$

$$
= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{113d\sqrt{c+dx^3}}{432c^4x} - \frac{113d^{4/3}\sqrt{c+dx^3}}{432c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3456\sqrt{3}c^{23/6}} + \frac{d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{10368c^{23/6}} + \frac{113\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{288\ 3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{113d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{216\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{d^{4/3}\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{3456c^{10/3}}
$$

$$
= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{113d\sqrt{c+dx^3}}{432c^4x} - \frac{113d^{4/3}\sqrt{c+dx^3}}{432c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3456\sqrt{3}c^{23/6}} + \frac{d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{10368c^{23/6}} - \frac{d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{10368c^{23/6}} + \frac{113\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{288\ 3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{113d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{216\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.10 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.23

$$\int \frac{1}{x^5 (8c - dx^3) (c + dx^3)^{3/2}} dx = \frac{160c(-27c^2 + 135cdx^3 + 226d^2x^6) - 9025cd^2x^6\sqrt{1 + \frac{dx^3}{c}} \text{AppellF1}\left(\frac{2}{3}, \frac{1}{2}\right.}{138240c^5x^4\sqrt{}}$$

```
[In] Integrate[1/(x^5*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (160*c*(-27*c^2 + 135*c*d*x^3 + 226*d^2*x^6) - 9025*c*d^2*x^6*Sqrt[1 + (d*x
^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 452*d^3*x^
9*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c
)])/(138240*c^5*x^4*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.48 (sec) , antiderivative size = 911, normalized size of antiderivative = 1.35

method	result	size
elliptic	Expression too large to display	911
risch	Expression too large to display	1344
default	Expression too large to display	1864

```
[In] int(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*(d*x^3+c)^(1/2)/c^3/x^4+3/16*d*(d*x^3+c)^(1/2)/c^4/x+2/27*d^2/c^4*x^2
/((x^3+c/d)*d)^(1/2)+113/1296*I*d/c^4*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-
c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)
-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)
/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(
1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d
*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/
2)))-1/15552*I/d/c^4*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*
(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d
*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*
I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
```

```
2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2
/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3
^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c
*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(
I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.74 (sec) , antiderivative size = 2529, normalized size of antiderivative = 3.75

$$\int \frac{1}{x^5 (8c-dx^3) (c+dx^3)^{3/2}} dx = \text{Too large to display}$$

```
[In] integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/124416*(32544*(d^2*x^7 + c*d*x^4)*sqrt(d)*weierstrassZeta(0, -4*c/d, weie
rstrassPInverse(0, -4*c/d, x)) + (c^4*d*x^7 + c^5*x^4 + sqrt(-3)*(c^4*d*x^7
 + c^5*x^4))*(d^8/c^23)^(1/6)*log((d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x
^3 + 640*c^3*d^6 - 9*(5*c^16*d^3*x^7 + 64*c^17*d^2*x^4 + 32*c^18*d*x + sqrt
(-3)*(5*c^16*d^3*x^7 + 64*c^17*d^2*x^4 + 32*c^18*d*x))*(d^8/c^23)^(2/3) + 3
*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2 - sqrt(-3)*(5*c^20*d*x^5 +
32*c^21*x^2))*(d^8/c^23)^(5/6) - 2*(7*c^12*d^4*x^6 + 152*c^13*d^3*x^3 + 64*
c^14*d^2)*sqrt(d^8/c^23) + (c^4*d^7*x^7 + 80*c^5*d^6*x^4 + 160*c^6*d^5*x +
sqrt(-3)*(c^4*d^7*x^7 + 80*c^5*d^6*x^4 + 160*c^6*d^5*x))*(d^8/c^23)^(1/6))
- 9*(c^8*d^6*x^8 + 38*c^9*d^5*x^5 + 64*c^10*d^4*x^2 - sqrt(-3)*(c^8*d^6*x^8
 + 38*c^9*d^5*x^5 + 64*c^10*d^4*x^2))*(d^8/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2
*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c^4*d*x^7 + c^5*x^4 + sqrt(-3)*(c^4*d*x
^7 + c^5*x^4))*(d^8/c^23)^(1/6)*log((d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7
*x^3 + 640*c^3*d^6 - 9*(5*c^16*d^3*x^7 + 64*c^17*d^2*x^4 + 32*c^18*d*x + sq
rt(-3)*(5*c^16*d^3*x^7 + 64*c^17*d^2*x^4 + 32*c^18*d*x))*(d^8/c^23)^(2/3) -
 3*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2 - sqrt(-3)*(5*c^20*d*x^5
+ 32*c^21*x^2))*(d^8/c^23)^(5/6) - 2*(7*c^12*d^4*x^6 + 152*c^13*d^3*x^3 + 6
4*c^14*d^2)*sqrt(d^8/c^23) + (c^4*d^7*x^7 + 80*c^5*d^6*x^4 + 160*c^6*d^5*x
+ sqrt(-3)*(c^4*d^7*x^7 + 80*c^5*d^6*x^4 + 160*c^6*d^5*x))*(d^8/c^23)^(1/6)
) - 9*(c^8*d^6*x^8 + 38*c^9*d^5*x^5 + 64*c^10*d^4*x^2 - sqrt(-3)*(c^8*d^6*x
^8 + 38*c^9*d^5*x^5 + 64*c^10*d^4*x^2))*(d^8/c^23)^(1/3))/(d^3*x^9 - 24*c*d
^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (c^4*d*x^7 + c^5*x^4 - sqrt(-3)*(c^4*d
*x^7 + c^5*x^4))*(d^8/c^23)^(1/6)*log((d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d
^7*x^3 + 640*c^3*d^6 - 9*(5*c^16*d^3*x^7 + 64*c^17*d^2*x^4 + 32*c^18*d*x -
sqrt(-3)*(5*c^16*d^3*x^7 + 64*c^17*d^2*x^4 + 32*c^18*d*x))*(d^8/c^23)^(2/3)
 + 3*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2 + sqrt(-3)*(5*c^20*d*x^
```

```
5 + 32*c^21*x^2))*(d^8/c^23)^(5/6) - 2*(7*c^12*d^4*x^6 + 152*c^13*d^3*x^3 +
 64*c^14*d^2)*sqrt(d^8/c^23) + (c^4*d^7*x^7 + 80*c^5*d^6*x^4 + 160*c^6*d^5*
x - sqrt(-3)*(c^4*d^7*x^7 + 80*c^5*d^6*x^4 + 160*c^6*d^5*x))*(d^8/c^23)^(1/
6)) - 9*(c^8*d^6*x^8 + 38*c^9*d^5*x^5 + 64*c^10*d^4*x^2 + sqrt(-3)*(c^8*d^6
*x^8 + 38*c^9*d^5*x^5 + 64*c^10*d^4*x^2))*(d^8/c^23)^(1/3))/(d^3*x^9 - 24*c
*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c^4*d*x^7 + c^5*x^4 - sqrt(-3)*(c^4
*d*x^7 + c^5*x^4))*(d^8/c^23)^(1/6)*log((d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2
*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^16*d^3*x^7 + 64*c^17*d^2*x^4 + 32*c^18*d*x
- sqrt(-3)*(5*c^16*d^3*x^7 + 64*c^17*d^2*x^4 + 32*c^18*d*x))*(d^8/c^23)^(2/
3) - 3*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2 + sqrt(-3)*(5*c^20*d*
x^5 + 32*c^21*x^2))*(d^8/c^23)^(5/6) - 2*(7*c^12*d^4*x^6 + 152*c^13*d^3*x^3
 + 64*c^14*d^2)*sqrt(d^8/c^23) + (c^4*d^7*x^7 + 80*c^5*d^6*x^4 + 160*c^6*d^
5*x - sqrt(-3)*(c^4*d^7*x^7 + 80*c^5*d^6*x^4 + 160*c^6*d^5*x))*(d^8/c^23)^(
1/6)) - 9*(c^8*d^6*x^8 + 38*c^9*d^5*x^5 + 64*c^10*d^4*x^2 + sqrt(-3)*(c^8*d
^6*x^8 + 38*c^9*d^5*x^5 + 64*c^10*d^4*x^2))*(d^8/c^23)^(1/3))/(d^3*x^9 - 24
*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 2*(c^4*d*x^7 + c^5*x^4)*(d^8/c^23)
^(1/6)*log((d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 + 18*(
5*c^16*d^3*x^7 + 64*c^17*d^2*x^4 + 32*c^18*d*x)*(d^8/c^23)^(2/3) + 6*sqrt(d
*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2)*(d^8/c^23)^(5/6) + (7*c^12*d^4*x^
6 + 152*c^13*d^3*x^3 + 64*c^14*d^2)*sqrt(d^8/c^23) + (c^4*d^7*x^7 + 80*c^5*
d^6*x^4 + 160*c^6*d^5*x)*(d^8/c^23)^(1/6)) + 18*(c^8*d^6*x^8 + 38*c^9*d^5*x
^5 + 64*c^10*d^4*x^2)*(d^8/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d
*x^3 - 512*c^3)) - 2*(c^4*d*x^7 + c^5*x^4)*(d^8/c^23)^(1/6)*log((d^9*x^9 +
318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 + 18*(5*c^16*d^3*x^7 + 64*c^
17*d^2*x^4 + 32*c^18*d*x)*(d^8/c^23)^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^20*d
*x^5 + 32*c^21*x^2)*(d^8/c^23)^(5/6) + (7*c^12*d^4*x^6 + 152*c^13*d^3*x^3 +
 64*c^14*d^2)*sqrt(d^8/c^23) + (c^4*d^7*x^7 + 80*c^5*d^6*x^4 + 160*c^6*d^5*
x)*(d^8/c^23)^(1/6)) + 18*(c^8*d^6*x^8 + 38*c^9*d^5*x^5 + 64*c^10*d^4*x^2)*
(d^8/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 144
*(226*d^2*x^6 + 135*c*d*x^3 - 27*c^2)*sqrt(d*x^3 + c))/(c^4*d*x^7 + c^5*x^4
)
```

Sympy [F]

$$\int \frac{1}{x^5\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = -\int \frac{1}{-8c^2x^5\sqrt{c+dx^3}-7cdx^8\sqrt{c+dx^3}+d^2x^{11}\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(1/x**5/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] -Integral(1/(-8*c**2*x**5*sqrt(c + d*x**3) - 7*c*d*x**8*sqrt(c + d*x**3) +
d**2*x**11*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^5 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \int -\frac{1}{(dx^3 + c)^{\frac{3}{2}} (dx^3 - 8\,c)x^5} \, dx$$

```
[In] integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \int -\frac{1}{(dx^3 + c)^{\frac{3}{2}} (dx^3 - 8\,c)x^5} \, dx$$

```
[In] integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \int \frac{1}{x^5 \, (d\,x^3 + c)^{3/2} \, (8\,c - d\,x^3)} \, dx$$

```
[In] int(1/(x^5*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
[Out] int(1/(x^5*(c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

3.337 $\int \frac{1}{x^8(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 699

$$\int \frac{1}{x^8(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} - \frac{953d^2\sqrt{c+dx^3}}{3024c^5x} + \frac{953d^{7/3}\sqrt{c+dx^3}}{3024c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{d^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27648\sqrt{3}c^{29/6}} + \frac{d^{7/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{82944c^{29/6}} - \frac{d^{7/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{82944c^{29/6}} - \frac{953\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{2016\ 3^{3/4}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{953d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{1512\sqrt{2}\sqrt[4]{3}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 1/82944*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/
c^(29/6)-1/82944*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(29/6)-1/82
944*d^(7/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(
```

```
29/6)*3^(1/2)+2/27/c^2/x^7/(d*x^3+c)^(1/2)-139/1512*(d*x^3+c)^(1/2)/c^3/x^7
+6095/48384*d*(d*x^3+c)^(1/2)/c^4/x^4-953/3024*d^2*(d*x^3+c)^(1/2)/c^5/x+95
3/3024*d^(7/3)*(d*x^3+c)^(1/2)/c^5/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+953/9072
*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(
1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2
/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(14/3)*2^(1/2)/
(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))
)^2)^(1/2)-953/6048*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3
)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-
1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1
+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(14/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1
/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 699, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {483, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{1}{x^8\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{953d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)\ldots\right)}{1512\sqrt{2}\sqrt[4]{3}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{953\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{2016\ 3^{3/4}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{d^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27648\sqrt{3}c^{29/6}}+\frac{d^{7/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{82944c^{29/6}}$$

$$-\frac{d^{7/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{82944c^{29/6}}+\frac{953d^{7/3}\sqrt{c+dx^3}}{3024c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$-\frac{953d^2\sqrt{c+dx^3}}{3024c^5x}+\frac{6095d\sqrt{c+dx^3}}{48384c^4x^4}-\frac{139\sqrt{c+dx^3}}{1512c^3x^7}+\frac{2}{27c^2x^7\sqrt{c+dx^3}}$$

```
[In] Int[1/(x^8*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] 2/(27*c^2*x^7*Sqrt[c + d*x^3]) - (139*Sqrt[c + d*x^3])/(1512*c^3*x^7) + (60
95*d*Sqrt[c + d*x^3])/(48384*c^4*x^4) - (953*d^2*Sqrt[c + d*x^3])/(3024*c^5
*x) + (953*d^(7/3)*Sqrt[c + d*x^3])/(3024*c^5*((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)) - (d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c +
d*x^3]])/(27648*Sqrt[3]*c^(29/6)) + (d^(7/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^
2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(82944*c^(29/6)) - (d^(7/3)*ArcTanh[Sqrt[c
+ d*x^3]/(3*Sqrt[c])])/(82944*c^(29/6)) - (953*Sqrt[2 - Sqrt[3]]*d^(7/3)*(c
^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) +
d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(2016*3^(
3/4)*c^(14/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) +
 d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (953*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(
c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*
c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(1512*Sqrt[2]*3^(1/4)*c^(14/3)*Sqrt
[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqr
t[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{2\int \frac{-\frac{139cd}{2}+\frac{17d^2x^3}{2}}{x^8(8c-dx^3)\sqrt{c+dx^3}}\,dx}{27c^2d} \\
&= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{\int \frac{-\frac{6095}{2}c^2d^2+\frac{1529}{4}cd^3x^3}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx}{756c^4d} \\
&= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} - \frac{\int \frac{-60992c^3d^3+\frac{30475}{4}c^2d^4x^3}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{24192c^6d} \\
&= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} \\
&\quad - \frac{953d^2\sqrt{c+dx^3}}{3024c^5x} + \frac{\int \frac{x(244010c^4d^4-30496c^3d^5x^3)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{193536c^8d}
\end{aligned}$$

$$= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} - \frac{953d^2\sqrt{c+dx^3}}{3024c^5x} + \frac{\int \left(\frac{30496c^3d^4x}{\sqrt{c+dx^3}} + \frac{42c^4d^4x}{(8c-dx^3)\sqrt{c+dx^3}}\right) dx}{193536c^8d}$$

$$= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} - \frac{953d^2\sqrt{c+dx^3}}{3024c^5x} + \frac{(953d^3)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{6048c^5} + \frac{d^3\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{4608c^4}$$

$$= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} - \frac{953d^2\sqrt{c+dx^3}}{3024c^5x} - \frac{d^2\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{55296c^5} + \frac{(953d^{8/3})\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{6048c^5} + \frac{d^{8/3}\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{55296c^{14/3}} - \frac{\left(953\left(1-\sqrt{3}\right)d^{8/3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{6048c^{14/3}} - \frac{d^{10/3}\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{18432c^{13/3}}$$

$$= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} - \frac{953d^2\sqrt{c+dx^3}}{3024c^5x} + \frac{953d^{7/3}\sqrt{c+dx^3}}{3024c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$- \frac{953\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{2016\ 3^{3/4}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{953d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{1512\sqrt{2}\sqrt[4]{3}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{d^{7/3}\text{Subst}\left(\int \frac{1}{9-cx^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{27648c^{13/3}} - \frac{d^{10/3}\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{55296c^{13/3}}$$

$$+ \frac{d^{13/3}\text{Subst}\left(\int \frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{13824c^{16/3}}$$

$$
\begin{aligned}
&= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} \\
&- \frac{953d^2\sqrt{c+dx^3}}{3024c^5x} + \frac{953d^{7/3}\sqrt{c+dx^3}}{3024c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} \\
&- \frac{d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27648\sqrt{3}c^{29/6}} + \frac{d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{82944c^{29/6}} \\
&- \frac{953\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{2016\ 3^{3/4}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+ \frac{953d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{1512\sqrt{2}\sqrt[4]{3}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&- \frac{d^{7/3}\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{27648c^{13/3}}
\end{aligned}
$$

$$= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} - \frac{953d^2\sqrt{c+dx^3}}{3024c^5x} + \frac{953d^{7/3}\sqrt{c+dx^3}}{3024c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27648\sqrt{3}c^{29/6}} + \frac{d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{82944c^{29/6}} - \frac{d^{7/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{82944c^{29/6}} - \frac{953\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{2016\ 3^{3/4}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{953d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{1512\sqrt{2}\sqrt[4]{3}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.24

$$\int \frac{1}{x^8\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{610025cd^3x^9\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right) - 32\left(5c(864c^3 - \right.}{7}$$

```
[In] Integrate[1/(x^8*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (610025*c*d^3*x^9*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/
c), (d*x^3)/(8*c)] - 32*(5*c*(864*c^3 - 1647*c^2*d*x^3 + 9153*c*d^2*x^6 + 1
5248*d^3*x^9) + 953*d^4*x^12*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3,
 -((d*x^3)/c), (d*x^3)/(8*c)]))/(7741440*c^6*x^7*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.48 (sec) , antiderivative size = 930, normalized size of antiderivative = 1.33

method	result	size
elliptic	Expression too large to display	930
risch	Expression too large to display	1357
default	Expression too large to display	2389

```
[In] int(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/27*d^3*x^2/c^5/((x^3+c/d)*d)^(1/2)-1/56*(d*x^3+c)^(1/2)/c^3/x^7+93/1792*
d*(d*x^3+c)^(1/2)/c^4/x^4-27/112*d^2*(d*x^3+c)^(1/2)/c^5/x-953/9072*I*d^2/c
^5*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^
3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*Elliptic
E(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(
1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-1/124416*I/c^5*2^(1/2)*sum(1/_
alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^
(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+
I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)
*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_a
lpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*
(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)
*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-
8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.26 (sec) , antiderivative size = 2564, normalized size of antiderivative = 3.67

$$\int \frac{1}{x^8 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \text{Too large to display}$$

```
[In] integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6967296*(2195712*(d^3*x^10 + c*d^2*x^7)*sqrt(d)*weierstrassZeta(0, -4*c/
d, weierstrassPInverse(0, -4*c/d, x)) - 7*(c^5*d*x^10 + c^6*x^7 + sqrt(-3)*
(c^5*d*x^10 + c^6*x^7))*(d^14/c^29)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6 +
1200*c^2*d^12*x^3 + 640*c^3*d^11 - 9*(5*c^20*d^4*x^7 + 64*c^21*d^3*x^4 + 32
*c^22*d^2*x + sqrt(-3)*(5*c^20*d^4*x^7 + 64*c^21*d^3*x^4 + 32*c^22*d^2*x))*
(d^14/c^29)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^25*d*x^5 + 32*c^26*x^2 - sqrt
(-3)*(5*c^25*d*x^5 + 32*c^26*x^2))*(d^14/c^29)^(5/6) - 2*(7*c^15*d^6*x^6 +
152*c^16*d^5*x^3 + 64*c^17*d^4)*sqrt(d^14/c^29) + (c^5*d^11*x^7 + 80*c^6*d^
10*x^4 + 160*c^7*d^9*x + sqrt(-3)*(c^5*d^11*x^7 + 80*c^6*d^10*x^4 + 160*c^7
*d^9*x))*(d^14/c^29)^(1/6)) - 9*(c^10*d^9*x^8 + 38*c^11*d^8*x^5 + 64*c^12*d
^7*x^2 - sqrt(-3)*(c^10*d^9*x^8 + 38*c^11*d^8*x^5 + 64*c^12*d^7*x^2))*(d^14
/c^29)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 7*(c^5*
d*x^10 + c^6*x^7 + sqrt(-3)*(c^5*d*x^10 + c^6*x^7))*(d^14/c^29)^(1/6)*log((
d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 - 9*(5*c^20*d^
4*x^7 + 64*c^21*d^3*x^4 + 32*c^22*d^2*x + sqrt(-3)*(5*c^20*d^4*x^7 + 64*c^2
1*d^3*x^4 + 32*c^22*d^2*x))*(d^14/c^29)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^2
5*d*x^5 + 32*c^26*x^2 - sqrt(-3)*(5*c^25*d*x^5 + 32*c^26*x^2))*(d^14/c^29)^
(5/6) - 2*(7*c^15*d^6*x^6 + 152*c^16*d^5*x^3 + 64*c^17*d^4)*sqrt(d^14/c^29)
 + (c^5*d^11*x^7 + 80*c^6*d^10*x^4 + 160*c^7*d^9*x + sqrt(-3)*(c^5*d^11*x^7
 + 80*c^6*d^10*x^4 + 160*c^7*d^9*x))*(d^14/c^29)^(1/6)) - 9*(c^10*d^9*x^8 +
 38*c^11*d^8*x^5 + 64*c^12*d^7*x^2 - sqrt(-3)*(c^10*d^9*x^8 + 38*c^11*d^8*x
^5 + 64*c^12*d^7*x^2))*(d^14/c^29)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2
*d*x^3 - 512*c^3)) - 7*(c^5*d*x^10 + c^6*x^7 - sqrt(-3)*(c^5*d*x^10 + c^6*x
^7))*(d^14/c^29)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 +
 640*c^3*d^11 - 9*(5*c^20*d^4*x^7 + 64*c^21*d^3*x^4 + 32*c^22*d^2*x - sqrt(
-3)*(5*c^20*d^4*x^7 + 64*c^21*d^3*x^4 + 32*c^22*d^2*x))*(d^14/c^29)^(2/3) +
 3*sqrt(d*x^3 + c)*(6*(5*c^25*d*x^5 + 32*c^26*x^2 + sqrt(-3)*(5*c^25*d*x^5
+ 32*c^26*x^2))*(d^14/c^29)^(5/6) - 2*(7*c^15*d^6*x^6 + 152*c^16*d^5*x^3 +
64*c^17*d^4)*sqrt(d^14/c^29) + (c^5*d^11*x^7 + 80*c^6*d^10*x^4 + 160*c^7*d^
9*x - sqrt(-3)*(c^5*d^11*x^7 + 80*c^6*d^10*x^4 + 160*c^7*d^9*x))*(d^14/c^29
)^(1/6)) - 9*(c^10*d^9*x^8 + 38*c^11*d^8*x^5 + 64*c^12*d^7*x^2 + sqrt(-3)*(
c^10*d^9*x^8 + 38*c^11*d^8*x^5 + 64*c^12*d^7*x^2))*(d^14/c^29)^(1/3))/(d^3*
x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 7*(c^5*d*x^10 + c^6*x^7 -
sqrt(-3)*(c^5*d*x^10 + c^6*x^7))*(d^14/c^29)^(1/6)*log((d^14*x^9 + 318*c*d^
13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 - 9*(5*c^20*d^4*x^7 + 64*c^21*d^3
```

```
*x^4 + 32*c^22*d^2*x - sqrt(-3)*(5*c^20*d^4*x^7 + 64*c^21*d^3*x^4 + 32*c^22
*d^2*x))*(d^14/c^29)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^25*d*x^5 + 32*c^26*x
^2 + sqrt(-3)*(5*c^25*d*x^5 + 32*c^26*x^2))*(d^14/c^29)^(5/6) - 2*(7*c^15*d
^6*x^6 + 152*c^16*d^5*x^3 + 64*c^17*d^4)*sqrt(d^14/c^29) + (c^5*d^11*x^7 +
80*c^6*d^10*x^4 + 160*c^7*d^9*x - sqrt(-3)*(c^5*d^11*x^7 + 80*c^6*d^10*x^4
+ 160*c^7*d^9*x))*(d^14/c^29)^(1/6)) - 9*(c^10*d^9*x^8 + 38*c^11*d^8*x^5 +
64*c^12*d^7*x^2 + sqrt(-3)*(c^10*d^9*x^8 + 38*c^11*d^8*x^5 + 64*c^12*d^7*x^
2))*(d^14/c^29)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3))
- 14*(c^5*d*x^10 + c^6*x^7)*(d^14/c^29)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^
6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 + 18*(5*c^20*d^4*x^7 + 64*c^21*d^3*x^4
 + 32*c^22*d^2*x)*(d^14/c^29)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^25*d*x^5 +
32*c^26*x^2)*(d^14/c^29)^(5/6) + (7*c^15*d^6*x^6 + 152*c^16*d^5*x^3 + 64*c^
17*d^4)*sqrt(d^14/c^29) + (c^5*d^11*x^7 + 80*c^6*d^10*x^4 + 160*c^7*d^9*x)*
(d^14/c^29)^(1/6)) + 18*(c^10*d^9*x^8 + 38*c^11*d^8*x^5 + 64*c^12*d^7*x^2)*
(d^14/c^29)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 14
*(c^5*d*x^10 + c^6*x^7)*(d^14/c^29)^(1/6)*log((d^14*x^9 + 318*c*d^13*x^6 +
1200*c^2*d^12*x^3 + 640*c^3*d^11 + 18*(5*c^20*d^4*x^7 + 64*c^21*d^3*x^4 + 3
2*c^22*d^2*x)*(d^14/c^29)^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^25*d*x^5 + 32*c
^26*x^2)*(d^14/c^29)^(5/6) + (7*c^15*d^6*x^6 + 152*c^16*d^5*x^3 + 64*c^17*d
^4)*sqrt(d^14/c^29) + (c^5*d^11*x^7 + 80*c^6*d^10*x^4 + 160*c^7*d^9*x)*(d^1
4/c^29)^(1/6)) + 18*(c^10*d^9*x^8 + 38*c^11*d^8*x^5 + 64*c^12*d^7*x^2)*(d^1
4/c^29)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 144*(1
5248*d^3*x^9 + 9153*c*d^2*x^6 - 1647*c^2*d*x^3 + 864*c^3)*sqrt(d*x^3 + c))/
(c^5*d*x^10 + c^6*x^7)
```

Sympy [F]

$$\int \frac{1}{x^8\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=-\int \frac{1}{-8c^2x^8\sqrt{c+dx^3}-7cdx^{11}\sqrt{c+dx^3}+d^2x^{14}\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(1/x**8/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
[Out] -Integral(1/(-8*c**2*x**8*sqrt(c + d*x**3) - 7*c*d*x**11*sqrt(c + d*x**3) +
 d**2*x**14*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^8 (8c - dx^3) (c + dx^3)^{3/2}} dx = \int -\frac{1}{(dx^3 + c)^{\frac{3}{2}} (dx^3 - 8c)x^8} dx$$

```
[In] integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^8), x)
```

Giac [F]

$$\int \frac{1}{x^8 (8c - dx^3) (c + dx^3)^{3/2}} dx = \int -\frac{1}{(dx^3 + c)^{\frac{3}{2}} (dx^3 - 8c)x^8} dx$$

```
[In] integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^8 (8c - dx^3) (c + dx^3)^{3/2}} dx = \int \frac{1}{x^8 (d x^3 + c)^{3/2} (8c - d x^3)} dx$$

```
[In] int(1/(x^8*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
[Out] int(1/(x^8*(c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

3.338 $\int \frac{x^3}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{x^3}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx=\frac{x^4\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},1,\frac{3}{2},\frac{7}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{32c^2\sqrt{c+dx^3}}$$

```
[Out] 1/32*x^4*AppellF1(4/3,3/2,1,7/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^2
/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {525, 524}

$$\int \frac{x^3}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx=\frac{x^4\sqrt{\frac{dx^3}{c}+1}\operatorname{AppellF1}\left(\frac{4}{3},1,\frac{3}{2},\frac{7}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{32c^2\sqrt{c+dx^3}}$$

```
[In] Int[x^3/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 3/2, 7/3, (d*x^3)/(8*c), -((d*x^3
)/c)])/(32*c^2*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{(8c-dx^3)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$

$$= \frac{x^4\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{4}{3};1,\frac{3}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{32c^2\sqrt{c+dx^3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 233 vs. $2(66) = 132$.

Time = 8.11 (sec) , antiderivative size = 233, normalized size of antiderivative = 3.53

$$\int \frac{x^3}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \frac{x\left(x^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+\frac{64c\left(-1+\frac{256c^2\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{(8c-dx^3)\left(32c\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\left(\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4\,\text{AppellF1}\left(\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)\right)}\right)}{d}\right)}{864c^2\sqrt{c+dx^3}}$$

```
[In] Integrate[x^3/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*(x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d*x^3
)/(8*c)] + (64*c*(-1 + (256*c^2*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d
*x^3)/(8*c)])/((8*c - d*x^3)*(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c),
 (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)
/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]))))))/d)
)/(864*c^2*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.24 (sec) , antiderivative size = 724, normalized size of antiderivative = 10.97

method	result	size
elliptic	Expression too large to display	724
default	Expression too large to display	1038

```
[In] int(x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/27/d*x/c/((x^3+c/d)*d)^(1/2)+2/81*I/d^2/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-8/243*I/d^4/c*2^(1/2)*sum(1/_alpha^2*(-c*d^
2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*
d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-
c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^
(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1
/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d
^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/
3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d
^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3
)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2535 vs. 2(52) = 104.

Time = 0.78 (sec) , antiderivative size = 2535, normalized size of antiderivative = 38.41

$$\int \frac{x^3}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/486*(36*sqrt(d*x^3 + c)*d*x - 36*(d*x^3 + c)*sqrt(d)*weierstrassPInverse
(0, -4*c/d, x) - (c*d^3*x^3 + c^2*d^2 + sqrt(-3)*(c*d^3*x^3 + c^2*d^2))*(1/
(c^7*d^8))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 -
9*(c^5*d^8*x^8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x^2 + sqrt(-3)*(c^5*d^8*x^8 +
```

38*c^6*d^7*x^5 + 64*c^7*d^6*x^2))*(1/(c^7*d^8))^(2/3) + 3*sqrt(d*x^3 + c)*((c^6*d^9*x^7 + 80*c^7*d^8*x^4 + 160*c^8*d^7*x - sqrt(-3)*(c^6*d^9*x^7 + 80*c^7*d^8*x^4 + 160*c^8*d^7*x))*(1/(c^7*d^8))^(5/6) - 2*(7*c^4*d^6*x^6 + 152*c^5*d^5*x^3 + 64*c^6*d^4)*sqrt(1/(c^7*d^8)) + 6*(5*c^2*d^3*x^5 + 32*c^3*d^2*x^2 + sqrt(-3)*(5*c^2*d^3*x^5 + 32*c^3*d^2*x^2))*(1/(c^7*d^8))^(1/6)) - 9*(5*c^3*d^5*x^7 + 64*c^4*d^4*x^4 + 32*c^5*d^3*x - sqrt(-3)*(5*c^3*d^5*x^7 + 64*c^4*d^4*x^4 + 32*c^5*d^3*x))*(1/(c^7*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (c*d^3*x^3 + c^2*d^2 + sqrt(-3)*(c*d^3*x^3 + c^2*d^2))*(1/(c^7*d^8))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c^5*d^8*x^8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x^2 + sqrt(-3)*(c^5*d^8*x^8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x^2))*(1/(c^7*d^8))^(2/3) - 3*sqrt(d*x^3 + c)*((c^6*d^9*x^7 + 80*c^7*d^8*x^4 + 160*c^8*d^7*x - sqrt(-3)*(c^6*d^9*x^7 + 80*c^7*d^8*x^4 + 160*c^8*d^7*x))*(1/(c^7*d^8))^(5/6) - 2*(7*c^4*d^6*x^6 + 152*c^5*d^5*x^3 + 64*c^6*d^4)*sqrt(1/(c^7*d^8)) + 6*(5*c^2*d^3*x^5 + 32*c^3*d^2*x^2 + sqrt(-3)*(5*c^2*d^3*x^5 + 32*c^3*d^2*x^2))*(1/(c^7*d^8))^(1/6)) - 9*(5*c^3*d^5*x^7 + 64*c^4*d^4*x^4 + 32*c^5*d^3*x - sqrt(-3)*(5*c^3*d^5*x^7 + 64*c^4*d^4*x^4 + 32*c^5*d^3*x))*(1/(c^7*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c*d^3*x^3 + c^2*d^2 - sqrt(-3)*(c*d^3*x^3 + c^2*d^2))*(1/(c^7*d^8))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c^5*d^8*x^8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x^2 - sqrt(-3)*(c^5*d^8*x^8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x^2))*(1/(c^7*d^8))^(2/3) + 3*sqrt(d*x^3 + c)*((c^6*d^9*x^7 + 80*c^7*d^8*x^4 + 160*c^8*d^7*x + sqrt(-3)*(c^6*d^9*x^7 + 80*c^7*d^8*x^4 + 160*c^8*d^7*x))*(1/(c^7*d^8))^(5/6) - 2*(7*c^4*d^6*x^6 + 152*c^5*d^5*x^3 + 64*c^6*d^4)*sqrt(1/(c^7*d^8)) + 6*(5*c^2*d^3*x^5 + 32*c^3*d^2*x^2 - sqrt(-3)*(5*c^2*d^3*x^5 + 32*c^3*d^2*x^2))*(1/(c^7*d^8))^(1/6)) - 9*(5*c^3*d^5*x^7 + 64*c^4*d^4*x^4 + 32*c^5*d^3*x + sqrt(-3)*(5*c^3*d^5*x^7 + 64*c^4*d^4*x^4 + 32*c^5*d^3*x))*(1/(c^7*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (c*d^3*x^3 + c^2*d^2 - sqrt(-3)*(c*d^3*x^3 + c^2*d^2))*(1/(c^7*d^8))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c^5*d^8*x^8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x^2 - sqrt(-3)*(c^5*d^8*x^8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x^2))*(1/(c^7*d^8))^(2/3) - 3*sqrt(d*x^3 + c)*((c^6*d^9*x^7 + 80*c^7*d^8*x^4 + 160*c^8*d^7*x + sqrt(-3)*(c^6*d^9*x^7 + 80*c^7*d^8*x^4 + 160*c^8*d^7*x))*(1/(c^7*d^8))^(5/6) - 2*(7*c^4*d^6*x^6 + 152*c^5*d^5*x^3 + 64*c^6*d^4)*sqrt(1/(c^7*d^8)) + 6*(5*c^2*d^3*x^5 + 32*c^3*d^2*x^2 - sqrt(-3)*(5*c^2*d^3*x^5 + 32*c^3*d^2*x^2))*(1/(c^7*d^8))^(1/6)) - 9*(5*c^3*d^5*x^7 + 64*c^4*d^4*x^4 + 32*c^5*d^3*x + sqrt(-3)*(5*c^3*d^5*x^7 + 64*c^4*d^4*x^4 + 32*c^5*d^3*x))*(1/(c^7*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*(c*d^3*x^3 + c^2*d^2)*(1/(c^7*d^8))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(c^5*d^8*x^8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x^2)*(1/(c^7*d^8))^(2/3) + 6*sqrt(d*x^3 + c)*((c^6*d^9*x^7 + 80*c^7*d^8*x^4 + 160*c^8*d^7*x)*(1/(c^7*d^8))^(5/6) + (7*c^4*d^6*x^6 + 152*c^5*d^5*x^3 + 64*c^6*d^4)*sqrt(1/(c^7*d^8)) + 6*(5*c^2*d^3*x^5 + 32*c^3*d^2*x^2)*(1/(c^7*d^8))^(1/6)) + 18*(5*c^3*d^5*x^7 + 64*c^4*d^4*x^4 + 32*c^5*d^3*x)*(1/(c^7*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 2*(c*d^3*

```
x^3 + c^2*d^2)*(1/(c^7*d^8))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*
d*x^3 + 640*c^3 + 18*(c^5*d^8*x^8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x^2)*(1/(c^
7*d^8))^(2/3) - 6*sqrt(d*x^3 + c)*((c^6*d^9*x^7 + 80*c^7*d^8*x^4 + 160*c^8*
d^7*x)*(1/(c^7*d^8))^(5/6) + (7*c^4*d^6*x^6 + 152*c^5*d^5*x^3 + 64*c^6*d^4)
*sqrt(1/(c^7*d^8)) + 6*(5*c^2*d^3*x^5 + 32*c^3*d^2*x^2)*(1/(c^7*d^8))^(1/6)
) + 18*(5*c^3*d^5*x^7 + 64*c^4*d^4*x^4 + 32*c^5*d^3*x)*(1/(c^7*d^8))^(1/3))
/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(c*d^3*x^3 + c^2*d^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**3/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \int -\frac{x^3}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)}\,dx$$

```
[In] integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] -integrate(x^3/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Giac [F]

$$\int \frac{x^3}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \int -\frac{x^3}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)}\,dx$$

```
[In] integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(-x^3/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \int \frac{x^3}{(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)}\,dx$$

```
[In] int(x^3/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
[Out] int(x^3/((c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

3.339 $\int \frac{1}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{x\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{1}{3},1,\frac{3}{2},\frac{4}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{8c^2\sqrt{c+dx^3}}$$

```
[Out] 1/8*x*AppellF1(1/3,3/2,1,4/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^2/(d
*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {441, 440}

$$\int \frac{1}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{x\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},1,\frac{3}{2},\frac{4}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{8c^2\sqrt{c+dx^3}}$$

```
[In] Int[1/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 3/2, 4/3, (d*x^3)/(8*c), -((d*x^3)/
c)])/(8*c^2*Sqrt[c + d*x^3])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{(8c-dx^3)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$

$$= \frac{x\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{1}{3};1,\frac{3}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{8c^2\sqrt{c+dx^3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 230 vs. 2(64) = 128.

Time = 10.16 (sec) , antiderivative size = 230, normalized size of antiderivative = 3.59

$$\int \frac{1}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \frac{x\left(-\frac{dx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^3}+64\left(\frac{1}{c^2}+\frac{}{(8c-dx^3)\left(32c\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},\right.\right.}\right.\right.}{864\sqrt{c+}}$$

```
[In] Integrate[1/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*(-((d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (
d*x^3)/(8*c)])/c^3) + 64*(c^(-2) + (176*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3
)/c), (d*x^3)/(8*c)])/((8*c - d*x^3)*(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*
x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c),
 (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]
))))))/(864*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.41 (sec) , antiderivative size = 721, normalized size of antiderivative = 11.27

method	result	size
default	Expression too large to display	721
elliptic	Expression too large to display	721

```
[In] int(1/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/27*x/c^2/((x^3+c/d)*d)^(1/2)-2/81*I/c^2*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/
2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(
I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha^2*(-c*d
^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c
*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(
-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(
1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*
d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1
/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*
d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2483 vs. $2(50) = 100$.

Time = 0.92 (sec) , antiderivative size = 2483, normalized size of antiderivative = 38.80

$$\int \frac{1}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3888*(288*sqrt(d*x^3 + c)*d*x + 360*(d*x^3 + c)*sqrt(d)*weierstrassPInver
se(0, -4*c/d, x) + (c^2*d^2*x^3 + c^3*d + sqrt(-3)*(c^2*d^2*x^3 + c^3*d))*(
1/(c^13*d^2))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3
 - 9*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2 + sqrt(-3)*(c^9*d^4*x
^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2))*(1/(c^13*d^2))^(2/3) + 3*sqrt(d*x^
3 + c)*((c^11*d^4*x^7 + 80*c^12*d^3*x^4 + 160*c^13*d^2*x - sqrt(-3)*(c^11*d
^4*x^7 + 80*c^12*d^3*x^4 + 160*c^13*d^2*x))*(1/(c^13*d^2))^(5/6) - 2*(7*c^7
*d^3*x^6 + 152*c^8*d^2*x^3 + 64*c^9*d)*sqrt(1/(c^13*d^2)) + 6*(5*c^3*d^2*x^
5 + 32*c^4*d*x^2 + sqrt(-3)*(5*c^3*d^2*x^5 + 32*c^4*d*x^2))*(1/(c^13*d^2))^
(1/6)) - 9*(5*c^5*d^3*x^7 + 64*c^6*d^2*x^4 + 32*c^7*d*x - sqrt(-3)*(5*c^5*d
^3*x^7 + 64*c^6*d^2*x^4 + 32*c^7*d*x))*(1/(c^13*d^2))^(1/3))/(d^3*x^9 - 24*
c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c^2*d^2*x^3 + c^3*d + sqrt(-3)*(c^
2*d^2*x^3 + c^3*d))*(1/(c^13*d^2))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 120
0*c^2*d*x^3 + 640*c^3 - 9*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2
```

+ sqrt(-3)*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2))*(1/(c^13*d^2)
)^(2/3) - 3*sqrt(d*x^3 + c)*((c^11*d^4*x^7 + 80*c^12*d^3*x^4 + 160*c^13*d^2
x - sqrt(-3)(c^11*d^4*x^7 + 80*c^12*d^3*x^4 + 160*c^13*d^2*x))*(1/(c^13*d
^2))^(5/6) - 2*(7*c^7*d^3*x^6 + 152*c^8*d^2*x^3 + 64*c^9*d)*sqrt(1/(c^13*d^
2)) + 6*(5*c^3*d^2*x^5 + 32*c^4*d*x^2 + sqrt(-3)*(5*c^3*d^2*x^5 + 32*c^4*d*
x^2))*(1/(c^13*d^2))^(1/6)) - 9*(5*c^5*d^3*x^7 + 64*c^6*d^2*x^4 + 32*c^7*d*
x - sqrt(-3)*(5*c^5*d^3*x^7 + 64*c^6*d^2*x^4 + 32*c^7*d*x))*(1/(c^13*d^2))^
(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (c^2*d^2*x^3 +
 c^3*d - sqrt(-3)*(c^2*d^2*x^3 + c^3*d))*(1/(c^13*d^2))^(1/6)*log((d^3*x^9
+ 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c^9*d^4*x^8 + 38*c^10*d^3*x
^5 + 64*c^11*d^2*x^2 - sqrt(-3)*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^
2*x^2))*(1/(c^13*d^2))^(2/3) + 3*sqrt(d*x^3 + c)*((c^11*d^4*x^7 + 80*c^12*d
^3*x^4 + 160*c^13*d^2*x + sqrt(-3)*(c^11*d^4*x^7 + 80*c^12*d^3*x^4 + 160*c^
13*d^2*x))*(1/(c^13*d^2))^(5/6) - 2*(7*c^7*d^3*x^6 + 152*c^8*d^2*x^3 + 64*c
^9*d)*sqrt(1/(c^13*d^2)) + 6*(5*c^3*d^2*x^5 + 32*c^4*d*x^2 - sqrt(-3)*(5*c^
3*d^2*x^5 + 32*c^4*d*x^2))*(1/(c^13*d^2))^(1/6)) - 9*(5*c^5*d^3*x^7 + 64*c^
6*d^2*x^4 + 32*c^7*d*x + sqrt(-3)*(5*c^5*d^3*x^7 + 64*c^6*d^2*x^4 + 32*c^7*
d*x))*(1/(c^13*d^2))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c
^3)) - (c^2*d^2*x^3 + c^3*d - sqrt(-3)*(c^2*d^2*x^3 + c^3*d))*(1/(c^13*d^2)
)^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c^9*d^
4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2 - sqrt(-3)*(c^9*d^4*x^8 + 38*c^10
*d^3*x^5 + 64*c^11*d^2*x^2))*(1/(c^13*d^2))^(2/3) - 3*sqrt(d*x^3 + c)*((c^1
1*d^4*x^7 + 80*c^12*d^3*x^4 + 160*c^13*d^2*x + sqrt(-3)*(c^11*d^4*x^7 + 80*
c^12*d^3*x^4 + 160*c^13*d^2*x))*(1/(c^13*d^2))^(5/6) - 2*(7*c^7*d^3*x^6 + 1
52*c^8*d^2*x^3 + 64*c^9*d)*sqrt(1/(c^13*d^2)) + 6*(5*c^3*d^2*x^5 + 32*c^4*d
x^2 - sqrt(-3)(5*c^3*d^2*x^5 + 32*c^4*d*x^2))*(1/(c^13*d^2))^(1/6)) - 9*(
5*c^5*d^3*x^7 + 64*c^6*d^2*x^4 + 32*c^7*d*x + sqrt(-3)*(5*c^5*d^3*x^7 + 64*
c^6*d^2*x^4 + 32*c^7*d*x))*(1/(c^13*d^2))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 +
192*c^2*d*x^3 - 512*c^3)) + 2*(c^2*d^2*x^3 + c^3*d)*(1/(c^13*d^2))^(1/6)*lo
g((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(c^9*d^4*x^8 + 3
8*c^10*d^3*x^5 + 64*c^11*d^2*x^2)*(1/(c^13*d^2))^(2/3) + 6*sqrt(d*x^3 + c)*
((c^11*d^4*x^7 + 80*c^12*d^3*x^4 + 160*c^13*d^2*x)*(1/(c^13*d^2))^(5/6) + (
7*c^7*d^3*x^6 + 152*c^8*d^2*x^3 + 64*c^9*d)*sqrt(1/(c^13*d^2)) + 6*(5*c^3*d
^2*x^5 + 32*c^4*d*x^2)*(1/(c^13*d^2))^(1/6)) + 18*(5*c^5*d^3*x^7 + 64*c^6*d
^2*x^4 + 32*c^7*d*x)*(1/(c^13*d^2))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^
2*d*x^3 - 512*c^3)) - 2*(c^2*d^2*x^3 + c^3*d)*(1/(c^13*d^2))^(1/6)*log((d^3
*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(c^9*d^4*x^8 + 38*c^10
*d^3*x^5 + 64*c^11*d^2*x^2)*(1/(c^13*d^2))^(2/3) - 6*sqrt(d*x^3 + c)*((c^11
*d^4*x^7 + 80*c^12*d^3*x^4 + 160*c^13*d^2*x)*(1/(c^13*d^2))^(5/6) + (7*c^7*
d^3*x^6 + 152*c^8*d^2*x^3 + 64*c^9*d)*sqrt(1/(c^13*d^2)) + 6*(5*c^3*d^2*x^5
 + 32*c^4*d*x^2)*(1/(c^13*d^2))^(1/6)) + 18*(5*c^5*d^3*x^7 + 64*c^6*d^2*x^4
 + 32*c^7*d*x)*(1/(c^13*d^2))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^
3 - 512*c^3)))/(c^2*d^2*x^3 + c^3*d)

Sympy [F]

$$\int \frac{1}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = -\int \frac{1}{-8c^2\sqrt{c+dx^3}-7cdx^3\sqrt{c+dx^3}+d^2x^6\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(1/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
[Out] -Integral(1/(-8*c**2*sqrt(c + d*x**3) - 7*c*d*x**3*sqrt(c + d*x**3) + d**2*
x**6*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \int -\frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)}\,dx$$

```
[In] integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Giac [F]

$$\int \frac{1}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \int -\frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)}\,dx$$

```
[In] integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \int \frac{1}{(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)}\,dx$$

```
[In] int(1/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
[Out] int(1/((c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

3.340 $\int \frac{1}{x^3(8c-dx^3)(c+dx^3)^{3/2}} dx$

Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{1}{x^3 (8c-dx^3) (c+dx^3)^{3/2}} dx = -\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{2}{3},1,\frac{3}{2},\frac{1}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{16c^2x^2\sqrt{c+dx^3}}$$

```
[Out] -1/16*AppellF1(-2/3,3/2,1,1/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^2/x
^2/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {525, 524}

$$\int \frac{1}{x^3 (8c-dx^3) (c+dx^3)^{3/2}} dx = -\frac{\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{2}{3},1,\frac{3}{2},\frac{1}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{16c^2x^2\sqrt{c+dx^3}}$$

```
[In] Int[1/(x^3*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/16*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 3/2, 1/3, (d*x^3)/(8*c), -((d*
x^3)/c)])/(c^2*x^2*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^3(8c-dx^3)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$

$$= -\frac{\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{2}{3};1,\frac{3}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{16c^2x^2\sqrt{c+dx^3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 248 vs. $2(66) = 132$.

Time = 11.19 (sec) , antiderivative size = 248, normalized size of antiderivative = 3.76

$$\int \frac{1}{x^3(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{59d^2x^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+64c\left(-27c-59dx^3-\cdots\right.}{2\cdots}$$

```
[In] Integrate[1/(x^3*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (59*d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d
*x^3)/(8*c)] + 64*c*(-27*c - 59*d*x^3 - (7360*c^2*d*x^3*AppellF1[1/3, 1/2,
1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((8*c - d*x^3)*(32*c*AppellF1[1/3, 1/
2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/
3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c)
, (d*x^3)/(8*c)])))))/(27648*c^4*x^2*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 5.20 (sec) , antiderivative size = 736, normalized size of antiderivative = 11.15

method	result	size
elliptic	Expression too large to display	736
risch	Expression too large to display	1028
default	Expression too large to display	1053

```
[In] int(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/27*d/c^3*x/((x^3+c/d)*d)^(1/2)-1/16*(d*x^3+c)^(1/2)/c^3/x^2+59/1296*I/c^
3*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(
1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3
+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)
/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/1944*I/c^
3/d^2*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/
3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*
(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^
(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^
2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3
^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(
-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)
),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2495 vs. 2(52) = 104.

Time = 1.59 (sec) , antiderivative size = 2495, normalized size of antiderivative = 37.80

$$\int \frac{1}{x^3 (8c-dx^3) (c+dx^3)^{3/2}} \, dx = \text{Too large to display}$$

```
[In] integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/31104*(4176*(d*x^5 + c*x^2)*sqrt(d)*weierstrassPInverse(0, -4*c/d, x) -
(c^3*d*x^5 + c^4*x^2 + sqrt(-3)*(c^3*d*x^5 + c^4*x^2))*(d^4/c^19)^(1/6)*log
```

((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 - 9*(c^13*d^3*x^
8 + 38*c^14*d^2*x^5 + 64*c^15*d*x^2 + sqrt(-3)*(c^13*d^3*x^8 + 38*c^14*d^2*
x^5 + 64*c^15*d*x^2))*(d^4/c^19)^(2/3) + 3*sqrt(d*x^3 + c)*((c^16*d^2*x^7 +
 80*c^17*d*x^4 + 160*c^18*x - sqrt(-3)*(c^16*d^2*x^7 + 80*c^17*d*x^4 + 160*
c^18*x))*(d^4/c^19)^(5/6) - 2*(7*c^10*d^3*x^6 + 152*c^11*d^2*x^3 + 64*c^12*
d)*sqrt(d^4/c^19) + 6*(5*c^4*d^4*x^5 + 32*c^5*d^3*x^2 + sqrt(-3)*(5*c^4*d^4
*x^5 + 32*c^5*d^3*x^2))*(d^4/c^19)^(1/6)) - 9*(5*c^7*d^4*x^7 + 64*c^8*d^3*x
^4 + 32*c^9*d^2*x - sqrt(-3)*(5*c^7*d^4*x^7 + 64*c^8*d^3*x^4 + 32*c^9*d^2*x
))*(d^4/c^19)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) +
(c^3*d*x^5 + c^4*x^2 + sqrt(-3)*(c^3*d*x^5 + c^4*x^2))*(d^4/c^19)^(1/6)*log
((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 - 9*(c^13*d^3*x^
8 + 38*c^14*d^2*x^5 + 64*c^15*d*x^2 + sqrt(-3)*(c^13*d^3*x^8 + 38*c^14*d^2*
x^5 + 64*c^15*d*x^2))*(d^4/c^19)^(2/3) - 3*sqrt(d*x^3 + c)*((c^16*d^2*x^7 +
 80*c^17*d*x^4 + 160*c^18*x - sqrt(-3)*(c^16*d^2*x^7 + 80*c^17*d*x^4 + 160*
c^18*x))*(d^4/c^19)^(5/6) - 2*(7*c^10*d^3*x^6 + 152*c^11*d^2*x^3 + 64*c^12*
d)*sqrt(d^4/c^19) + 6*(5*c^4*d^4*x^5 + 32*c^5*d^3*x^2 + sqrt(-3)*(5*c^4*d^4
*x^5 + 32*c^5*d^3*x^2))*(d^4/c^19)^(1/6)) - 9*(5*c^7*d^4*x^7 + 64*c^8*d^3*x
^4 + 32*c^9*d^2*x - sqrt(-3)*(5*c^7*d^4*x^7 + 64*c^8*d^3*x^4 + 32*c^9*d^2*x
))*(d^4/c^19)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) -
(c^3*d*x^5 + c^4*x^2 - sqrt(-3)*(c^3*d*x^5 + c^4*x^2))*(d^4/c^19)^(1/6)*log
((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 - 9*(c^13*d^3*x^
8 + 38*c^14*d^2*x^5 + 64*c^15*d*x^2 - sqrt(-3)*(c^13*d^3*x^8 + 38*c^14*d^2*
x^5 + 64*c^15*d*x^2))*(d^4/c^19)^(2/3) + 3*sqrt(d*x^3 + c)*((c^16*d^2*x^7 +
 80*c^17*d*x^4 + 160*c^18*x + sqrt(-3)*(c^16*d^2*x^7 + 80*c^17*d*x^4 + 160*
c^18*x))*(d^4/c^19)^(5/6) - 2*(7*c^10*d^3*x^6 + 152*c^11*d^2*x^3 + 64*c^12*
d)*sqrt(d^4/c^19) + 6*(5*c^4*d^4*x^5 + 32*c^5*d^3*x^2 - sqrt(-3)*(5*c^4*d^4
*x^5 + 32*c^5*d^3*x^2))*(d^4/c^19)^(1/6)) - 9*(5*c^7*d^4*x^7 + 64*c^8*d^3*x
^4 + 32*c^9*d^2*x + sqrt(-3)*(5*c^7*d^4*x^7 + 64*c^8*d^3*x^4 + 32*c^9*d^2*x
))*(d^4/c^19)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) +
(c^3*d*x^5 + c^4*x^2 - sqrt(-3)*(c^3*d*x^5 + c^4*x^2))*(d^4/c^19)^(1/6)*log
((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 - 9*(c^13*d^3*x^
8 + 38*c^14*d^2*x^5 + 64*c^15*d*x^2 - sqrt(-3)*(c^13*d^3*x^8 + 38*c^14*d^2*
x^5 + 64*c^15*d*x^2))*(d^4/c^19)^(2/3) - 3*sqrt(d*x^3 + c)*((c^16*d^2*x^7 +
 80*c^17*d*x^4 + 160*c^18*x + sqrt(-3)*(c^16*d^2*x^7 + 80*c^17*d*x^4 + 160*
c^18*x))*(d^4/c^19)^(5/6) - 2*(7*c^10*d^3*x^6 + 152*c^11*d^2*x^3 + 64*c^12*
d)*sqrt(d^4/c^19) + 6*(5*c^4*d^4*x^5 + 32*c^5*d^3*x^2 - sqrt(-3)*(5*c^4*d^4
*x^5 + 32*c^5*d^3*x^2))*(d^4/c^19)^(1/6)) - 9*(5*c^7*d^4*x^7 + 64*c^8*d^3*x
^4 + 32*c^9*d^2*x + sqrt(-3)*(5*c^7*d^4*x^7 + 64*c^8*d^3*x^4 + 32*c^9*d^2*x
))*(d^4/c^19)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) -
2*(c^3*d*x^5 + c^4*x^2)*(d^4/c^19)^(1/6)*log((d^6*x^9 + 318*c*d^5*x^6 + 120
0*c^2*d^4*x^3 + 640*c^3*d^3 + 18*(c^13*d^3*x^8 + 38*c^14*d^2*x^5 + 64*c^15*
d*x^2)*(d^4/c^19)^(2/3) + 6*sqrt(d*x^3 + c)*((c^16*d^2*x^7 + 80*c^17*d*x^4
+ 160*c^18*x)*(d^4/c^19)^(5/6) + (7*c^10*d^3*x^6 + 152*c^11*d^2*x^3 + 64*c^
12*d)*sqrt(d^4/c^19) + 6*(5*c^4*d^4*x^5 + 32*c^5*d^3*x^2)*(d^4/c^19)^(1/6))
 + 18*(5*c^7*d^4*x^7 + 64*c^8*d^3*x^4 + 32*c^9*d^2*x)*(d^4/c^19)^(1/3))/(d^

```
3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 2*(c^3*d*x^5 + c^4*x^2)*
(d^4/c^19)^(1/6)*log((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*
d^3 + 18*(c^13*d^3*x^8 + 38*c^14*d^2*x^5 + 64*c^15*d*x^2)*(d^4/c^19)^(2/3)
- 6*sqrt(d*x^3 + c)*((c^16*d^2*x^7 + 80*c^17*d*x^4 + 160*c^18*x)*(d^4/c^19)
^(5/6) + (7*c^10*d^3*x^6 + 152*c^11*d^2*x^3 + 64*c^12*d)*sqrt(d^4/c^19) + 6
*(5*c^4*d^4*x^5 + 32*c^5*d^3*x^2)*(d^4/c^19)^(1/6)) + 18*(5*c^7*d^4*x^7 + 6
4*c^8*d^3*x^4 + 32*c^9*d^2*x)*(d^4/c^19)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 1
92*c^2*d*x^3 - 512*c^3)) + 72*(59*d*x^3 + 27*c)*sqrt(d*x^3 + c))/(c^3*d*x^5
 + c^4*x^2)
```

Sympy [F]

$$\int \frac{1}{x^3\,(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = -\int \frac{1}{-8c^2x^3\sqrt{c+dx^3}-7cdx^6\sqrt{c+dx^3}+d^2x^9\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(1/x**3/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
[Out] -Integral(1/(-8*c**2*x**3*sqrt(c + d*x**3) - 7*c*d*x**6*sqrt(c + d*x**3) +
d**2*x**9*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^3\,(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \int -\frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)x^3}\,dx$$

```
[In] integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\,(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx = \int -\frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)x^3}\,dx$$

```
[In] integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (8c - dx^3) (c + dx^3)^{3/2}} \, dx = \int \frac{1}{x^3 \left(d x^3 + c\right)^{3/2} \left(8 c - d x^3\right)} \, dx$$

```
[In] int(1/(x^3*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
[Out] int(1/(x^3*(c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

3.341 $\int \frac{1}{x^6(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{1}{x^6(8c-dx^3)(c+dx^3)^{3/2}}\,dx = -\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{5}{3},1,\frac{3}{2},-\frac{2}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{40c^2x^5\sqrt{c+dx^3}}$$

```
[Out] -1/40*AppellF1(-5/3,3/2,1,-2/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^2/
x^5/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {525, 524}

$$\int \frac{1}{x^6(8c-dx^3)(c+dx^3)^{3/2}}\,dx = -\frac{\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{5}{3},1,\frac{3}{2},-\frac{2}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{40c^2x^5\sqrt{c+dx^3}}$$

```
[In] Int[1/(x^6*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/40*(Sqrt[1 + (d*x^3)/c]*AppellF1[-5/3, 1, 3/2, -2/3, (d*x^3)/(8*c), -((d
*x^3)/c)])/(c^2*x^5*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^6(8c-dx^3)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$
$$= -\frac{\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{5}{3};1,\frac{3}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{40c^2x^5\sqrt{c+dx^3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 261 vs. $2(66) = 132$.

Time = 11.26 (sec) , antiderivative size = 261, normalized size of antiderivative = 3.95

$$\int \frac{1}{x^6(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{-2981d^3x^9\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+64c\Big(-432c^2+126}{}$$

```
[In] Integrate[1/(x^6*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2981*d^3*x^9*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c),
 (d*x^3)/(8*c)] + 64*c*(-432*c^2 + 1269*c*d*x^3 + 2981*d^2*x^6 + (382528*c^
2*d^2*x^6*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((8*c -
d*x^3)*(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*
x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4
/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])))))/(1105920*c^5*x^5*Sqrt[c
+ d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 5.29 (sec) , antiderivative size = 757, normalized size of antiderivative = 11.47

method	result	size
elliptic	Expression too large to display	757
risch	Expression too large to display	1040
default	Expression too large to display	1402

```
[In] int(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/27*d^2/c^4*x/((x^3+c/d)*d)^(1/2)-1/40*(d*x^3+c)^(1/2)/c^3/x^5+63/640*d*(d
*x^3+c)^(1/2)/c^4/x^2-2981/51840*I*d/c^4*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d
*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3)))^(1/2))-1/15552*I/d/c^4*2^(1/2)*sum(1/_alpha^2*(-c*d^2)
^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^
2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*
d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1
/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2
)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2
)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)
*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2
)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2543 vs. 2(52) = 104.

Time = 4.22 (sec) , antiderivative size = 2543, normalized size of antiderivative = 38.53

$$\int \frac{1}{x^6 (8c - dx^3) (c + dx^3)^{3/2}} dx = \text{Too large to display}$$

```
[In] integrate(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1244160*(214992*(d^2*x^8 + c*d*x^5)*sqrt(d)*weierstrassPInverse(0, -4*c/d
, x) + 5*(c^4*d*x^8 + c^5*x^5 + sqrt(-3)*(c^4*d*x^8 + c^5*x^5))*(d^10/c^25)
```

^(1/6)*log((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 - 9*
(c^17*d^4*x^8 + 38*c^18*d^3*x^5 + 64*c^19*d^2*x^2 + sqrt(-3)*(c^17*d^4*x^8
+ 38*c^18*d^3*x^5 + 64*c^19*d^2*x^2))*(d^10/c^25)^(2/3) + 3*sqrt(d*x^3 + c)
*((c^21*d^2*x^7 + 80*c^22*d*x^4 + 160*c^23*x - sqrt(-3)*(c^21*d^2*x^7 + 80*
c^22*d*x^4 + 160*c^23*x))*(d^10/c^25)^(5/6) - 2*(7*c^13*d^5*x^6 + 152*c^14*
d^4*x^3 + 64*c^15*d^3)*sqrt(d^10/c^25) + 6*(5*c^5*d^8*x^5 + 32*c^6*d^7*x^2
+ sqrt(-3)*(5*c^5*d^8*x^5 + 32*c^6*d^7*x^2))*(d^10/c^25)^(1/6)) - 9*(5*c^9*
d^7*x^7 + 64*c^10*d^6*x^4 + 32*c^11*d^5*x - sqrt(-3)*(5*c^9*d^7*x^7 + 64*c^
10*d^6*x^4 + 32*c^11*d^5*x))*(d^10/c^25)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 1
92*c^2*d*x^3 - 512*c^3)) - 5*(c^4*d*x^8 + c^5*x^5 + sqrt(-3)*(c^4*d*x^8 + c
^5*x^5))*(d^10/c^25)^(1/6)*log((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^
3 + 640*c^3*d^8 - 9*(c^17*d^4*x^8 + 38*c^18*d^3*x^5 + 64*c^19*d^2*x^2 + sqr
t(-3)*(c^17*d^4*x^8 + 38*c^18*d^3*x^5 + 64*c^19*d^2*x^2))*(d^10/c^25)^(2/3)
 - 3*sqrt(d*x^3 + c)*((c^21*d^2*x^7 + 80*c^22*d*x^4 + 160*c^23*x - sqrt(-3)
*(c^21*d^2*x^7 + 80*c^22*d*x^4 + 160*c^23*x))*(d^10/c^25)^(5/6) - 2*(7*c^13
*d^5*x^6 + 152*c^14*d^4*x^3 + 64*c^15*d^3)*sqrt(d^10/c^25) + 6*(5*c^5*d^8*x
^5 + 32*c^6*d^7*x^2 + sqrt(-3)*(5*c^5*d^8*x^5 + 32*c^6*d^7*x^2))*(d^10/c^25
)^(1/6)) - 9*(5*c^9*d^7*x^7 + 64*c^10*d^6*x^4 + 32*c^11*d^5*x - sqrt(-3)*(5
*c^9*d^7*x^7 + 64*c^10*d^6*x^4 + 32*c^11*d^5*x))*(d^10/c^25)^(1/3))/(d^3*x^
9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 5*(c^4*d*x^8 + c^5*x^5 - sqr
t(-3)*(c^4*d*x^8 + c^5*x^5))*(d^10/c^25)^(1/6)*log((d^11*x^9 + 318*c*d^10*x
^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 - 9*(c^17*d^4*x^8 + 38*c^18*d^3*x^5 + 6
4*c^19*d^2*x^2 - sqrt(-3)*(c^17*d^4*x^8 + 38*c^18*d^3*x^5 + 64*c^19*d^2*x^2
))*(d^10/c^25)^(2/3) + 3*sqrt(d*x^3 + c)*((c^21*d^2*x^7 + 80*c^22*d*x^4 + 1
60*c^23*x + sqrt(-3)*(c^21*d^2*x^7 + 80*c^22*d*x^4 + 160*c^23*x))*(d^10/c^2
5)^(5/6) - 2*(7*c^13*d^5*x^6 + 152*c^14*d^4*x^3 + 64*c^15*d^3)*sqrt(d^10/c^
25) + 6*(5*c^5*d^8*x^5 + 32*c^6*d^7*x^2 - sqrt(-3)*(5*c^5*d^8*x^5 + 32*c^6*
d^7*x^2))*(d^10/c^25)^(1/6)) - 9*(5*c^9*d^7*x^7 + 64*c^10*d^6*x^4 + 32*c^11
*d^5*x + sqrt(-3)*(5*c^9*d^7*x^7 + 64*c^10*d^6*x^4 + 32*c^11*d^5*x))*(d^10/
c^25)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 5*(c^4*d
*x^8 + c^5*x^5 - sqrt(-3)*(c^4*d*x^8 + c^5*x^5))*(d^10/c^25)^(1/6)*log((d^1
1*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 - 9*(c^17*d^4*x^8 +
 38*c^18*d^3*x^5 + 64*c^19*d^2*x^2 - sqrt(-3)*(c^17*d^4*x^8 + 38*c^18*d^3*x
^5 + 64*c^19*d^2*x^2))*(d^10/c^25)^(2/3) - 3*sqrt(d*x^3 + c)*((c^21*d^2*x^7
 + 80*c^22*d*x^4 + 160*c^23*x + sqrt(-3)*(c^21*d^2*x^7 + 80*c^22*d*x^4 + 16
0*c^23*x))*(d^10/c^25)^(5/6) - 2*(7*c^13*d^5*x^6 + 152*c^14*d^4*x^3 + 64*c^
15*d^3)*sqrt(d^10/c^25) + 6*(5*c^5*d^8*x^5 + 32*c^6*d^7*x^2 - sqrt(-3)*(5*c
^5*d^8*x^5 + 32*c^6*d^7*x^2))*(d^10/c^25)^(1/6)) - 9*(5*c^9*d^7*x^7 + 64*c^
10*d^6*x^4 + 32*c^11*d^5*x + sqrt(-3)*(5*c^9*d^7*x^7 + 64*c^10*d^6*x^4 + 32
*c^11*d^5*x))*(d^10/c^25)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 -
512*c^3)) + 10*(c^4*d*x^8 + c^5*x^5)*(d^10/c^25)^(1/6)*log((d^11*x^9 + 318*
c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 + 18*(c^17*d^4*x^8 + 38*c^18*d^
3*x^5 + 64*c^19*d^2*x^2)*(d^10/c^25)^(2/3) + 6*sqrt(d*x^3 + c)*((c^21*d^2*x
^7 + 80*c^22*d*x^4 + 160*c^23*x)*(d^10/c^25)^(5/6) + (7*c^13*d^5*x^6 + 152*
c^14*d^4*x^3 + 64*c^15*d^3)*sqrt(d^10/c^25) + 6*(5*c^5*d^8*x^5 + 32*c^6*d^7

```
*x^2)*(d^10/c^25)^(1/6)) + 18*(5*c^9*d^7*x^7 + 64*c^10*d^6*x^4 + 32*c^11*d^
5*x)*(d^10/c^25)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3))
 - 10*(c^4*d*x^8 + c^5*x^5)*(d^10/c^25)^(1/6)*log((d^11*x^9 + 318*c*d^10*x^
6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 + 18*(c^17*d^4*x^8 + 38*c^18*d^3*x^5 + 6
4*c^19*d^2*x^2)*(d^10/c^25)^(2/3) - 6*sqrt(d*x^3 + c)*((c^21*d^2*x^7 + 80*c
^22*d*x^4 + 160*c^23*x)*(d^10/c^25)^(5/6) + (7*c^13*d^5*x^6 + 152*c^14*d^4*
x^3 + 64*c^15*d^3)*sqrt(d^10/c^25) + 6*(5*c^5*d^8*x^5 + 32*c^6*d^7*x^2)*(d^
10/c^25)^(1/6)) + 18*(5*c^9*d^7*x^7 + 64*c^10*d^6*x^4 + 32*c^11*d^5*x)*(d^1
0/c^25)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 72*(29
81*d^2*x^6 + 1269*c*d*x^3 - 432*c^2)*sqrt(d*x^3 + c))/(c^4*d*x^8 + c^5*x^5)
```

Sympy [F]

$$\int \frac{1}{x^6\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=-\int \frac{1}{-8c^2x^6\sqrt{c+dx^3}-7cdx^9\sqrt{c+dx^3}+d^2x^{12}\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(1/x**6/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
[Out] -Integral(1/(-8*c**2*x**6*sqrt(c + d*x**3) - 7*c*d*x**9*sqrt(c + d*x**3) +
d**2*x**12*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^6\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\int -\frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)x^6}\,dx$$

```
[In] integrate(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\int -\frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)x^6}\,dx$$

```
[In] integrate(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6\,(8c-dx^3)\,(c+dx^3)^{3/2}}\,dx=\int \frac{1}{x^6\,{\left(d\,x^3+c\right)}^{3/2}\,\left(8\,c-d\,x^3\right)} \,d x$$

```
[In] int(1/(x^6*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
[Out] int(1/(x^6*(c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

3.342 $\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 737

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx$$

$$= \frac{2\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{3^{3/4}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$+\frac{\sqrt[6]{a}\arctan\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}+\frac{\sqrt[4]{3}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{\sqrt{2}b^{2/3}}$$
$$+\frac{\sqrt[4]{3}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 1/4*3^(3/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1+3^(1/
2))*2^(1/2)/(b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)+1/6*a^(1/6)*arctan(1/6*(1-3^(1
/2))*(b*x^3+a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*3^(3/4)/b^(2/3)*2^(1/2)+1/4*3
^(1/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1-3^(1/2))*
2^(1/2)/(b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)+1/2*3^(1/4)*a^(1/6)*arctanh(1/2*3^
(1/4)*a^(1/6)*(-2*b^(1/3)*x+a^(1/3)*(1+3^(1/2)))*2^(1/2)/(b*x^3+a)^(1/2))/b
^(2/3)*2^(1/2)+2*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+2/
3*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/
3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/
(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))
```

```
)^2)^(1/2)-3^(1/4)*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)
*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1
/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 737, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {495, 309, 224, 1891, 500}

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx$$

$$= \frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{3^{3/4}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}+\frac{\sqrt[6]{a}\arctan\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}$$
$$+\frac{\sqrt[4]{3}\sqrt[6]{a}\text{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{\sqrt{2}b^{2/3}}$$
$$+\frac{\sqrt[4]{3}\sqrt[6]{a}\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}+\frac{2\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

```
[In] Int[(x*Sqrt[a + b*x^3])/(2*(5 + 3*Sqrt[3])*a + b*x^3),x]
```

```
[Out] (2*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (3^(3/4
)*a^(1/6)*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqr
```

```
t[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*b^(2/3)) + (a^(1/6)*ArcTan[((1 - Sqrt[3]
)*Sqrt[a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(Sqrt[2]*3^(1/4)*b^(2/3)) +
(3^(1/4)*a^(1/6)*ArcTanh[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) - 2*b^(1/3
)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(Sqrt[2]*b^(2/3)) + (3^(1/4)*a^(1/6)*ArcT
anh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a +
 b*x^3])])/(2*Sqrt[2]*b^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3]) + (2*Sqrt[2]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3
)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 500

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
```

```
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\left(\left(3\left(3+2\sqrt{3}\right)a\right)\int\frac{x}{\sqrt{a+bx^3}\,\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx\right)+\int\frac{x}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{3^{3/4}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}+\frac{\sqrt[6]{a}\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}$$
$$+\frac{\sqrt[4]{3}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{\sqrt{2}b^{2/3}}$$
$$+\frac{\sqrt[4]{3}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$+\frac{\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{\sqrt[3]{b}}-\frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)\int\frac{1}{\sqrt{a+bx^3}}\,dx}{\sqrt[3]{b}}$$

$$= \frac{2\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{3^{3/4}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$+\frac{\sqrt[6]{a}\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}+\frac{\sqrt[4]{3}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{\sqrt{2}b^{2/3}}$$
$$+\frac{\sqrt[4]{3}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.11

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx=\frac{x^2\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bx^3}{10a+6\sqrt{3}a}\right)}{\left(20+12\sqrt{3}\right)\sqrt{a+bx^3}}$$

```
[In] Integrate[(x*Sqrt[a + b*x^3])/(2*(5 + 3*Sqrt[3])*a + b*x^3),x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, -1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3
)/(10*a + 6*Sqrt[3]*a))])/((20 + 12*Sqrt[3])*Sqrt[a + b*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.02 (sec) , antiderivative size = 977, normalized size of antiderivative = 1.33

method	result	size
elliptic	Expression too large to display	977
default	Expression too large to display	995

```
[In] int(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5+3*3^(1/2))),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*El
lipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+1/9*I/b^3*2^(1/2)*sum(1/
_alpha*(3+2*3^(1/2))*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(
1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*(
-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)
^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(-3
*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)+3*I*(-a*b^2)^(2/3
)*3^(1/2)-2*3^(1/2)*(-a*b^2)^(1/3)*_alpha*b+6*I*(-a*b^2)^(1/3)*_alpha*b-6*b
^2*_alpha^2-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(-a*b^2)^(2/3)+3*(-a*b^2)^(1/3)*_a
lpha*b+3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/6/b*(2*I
*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b-I*3^(1/2)*(-a*b^2)^(2/3)*_alpha-4*I*(-a*
b^2)^(1/3)*_alpha^2*b+2*3^(1/2)*(-a*b^2)^(2/3)*_alpha+I*3^(1/2)*a*b+2*I*(-a
*b^2)^(2/3)*_alpha+2*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-2*I*a*b-3*a*b)/a,(
I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3+6*3^(1/2)*a+10*a))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.43 (sec) , antiderivative size = 4847, normalized size of antiderivative = 6.58

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5+3*3^(1/2))),x, algorithm="fricas"
)
```

```
[Out] -1/8*((1/72)^(1/6)*(sqrt(-3)*b - b)*(-sqrt(3)*a/b^4)^(1/6)*log((72*(1/72)^(
5/6)*(7*b^6*x^10 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x + sqrt(-3
)*(7*b^6*x^10 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x) + 3*sqrt(3)
*(b^6*x^10 - 12*a*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x + sqrt(-3)*(b^6*x
^10 - 12*a*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x)))*(-sqrt(3)*a/b^4)^(5/6
) - 4*sqrt(1/2)*(3*b^5*x^11 - 18*a*b^4*x^8 + 360*a^2*b^3*x^5 + 624*a^3*b^2*
x^2 + sqrt(3)*(b^5*x^11 - 42*a*b^4*x^8 - 168*a^2*b^3*x^5 - 368*a^3*b^2*x^2)
)*sqrt(-sqrt(3)*a/b^4) + 6*(12*a*b^2*x^8 - 48*a^2*b*x^5 - 384*a^3*x^2 + 2*(
1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 + 48*a^3*b^2 + sqrt(3)*(b^5*x^9 - 3
0*a*b^4*x^6 - 144*a^2*b^3*x^3 - 32*a^3*b^2 - sqrt(-3)*(b^5*x^9 - 30*a*b^4*x
^6 - 144*a^2*b^3*x^3 - 32*a^3*b^2)) - 3*sqrt(-3)*(b^5*x^9 + 96*a^2*b^3*x^3
+ 16*a^3*b^2))*(-sqrt(3)*a/b^4)^(2/3) + (1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2
*x^4 + 160*a^3*b*x + sqrt(-3)*(b^4*x^10 + 240*a^2*b^2*x^4 + 160*a^3*b*x) -
24*sqrt(3)*(a*b^3*x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x + sqrt(-3)*(a*b^3*x^7 + 5
*a^2*b^2*x^4 + 4*a^3*b*x)))*(-sqrt(3)*a/b^4)^(1/3) + 8*sqrt(3)*(a*b^2*x^8 +
 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(b*x^3 + a) + (1/72)^(1/6)*(3*b^4*x^12 - 12
*a*b^3*x^9 + 1080*a^2*b^2*x^6 + 2208*a^3*b*x^3 + 384*a^4 - sqrt(3)*(b^4*x^1
2 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*b*x^3 + 256*a^4 - sqrt(-3)*(
b^4*x^12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*b*x^3 + 256*a^4)) - 3
*sqrt(-3)*(b^4*x^12 - 4*a*b^3*x^9 + 360*a^2*b^2*x^6 + 736*a^3*b*x^3 + 128*a
^4))*(-sqrt(3)*a/b^4)^(1/6))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 3
20*a^3*b*x^3 + 64*a^4)) - (1/72)^(1/6)*(sqrt(-3)*b - b)*(-sqrt(3)*a/b^4)^(1
/6)*log(-(72*(1/72)^(5/6)*(7*b^6*x^10 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 16
0*a^3*b^3*x + sqrt(-3)*(7*b^6*x^10 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a
^3*b^3*x) + 3*sqrt(3)*(b^6*x^10 - 12*a*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^
3*x + sqrt(-3)*(b^6*x^10 - 12*a*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x)))*
(-sqrt(3)*a/b^4)^(5/6) - 4*sqrt(1/2)*(3*b^5*x^11 - 18*a*b^4*x^8 + 360*a^2*b
^3*x^5 + 624*a^3*b^2*x^2 + sqrt(3)*(b^5*x^11 - 42*a*b^4*x^8 - 168*a^2*b^3*x
^5 - 368*a^3*b^2*x^2))*sqrt(-sqrt(3)*a/b^4) - 6*(12*a*b^2*x^8 - 48*a^2*b*x^
5 - 384*a^3*x^2 + 2*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 + 48*a^3*b^2 +
 sqrt(3)*(b^5*x^9 - 30*a*b^4*x^6 - 144*a^2*b^3*x^3 - 32*a^3*b^2 - sqrt(-3)*
(b^5*x^9 - 30*a*b^4*x^6 - 144*a^2*b^3*x^3 - 32*a^3*b^2)) - 3*sqrt(-3)*(b^5*
x^9 + 96*a^2*b^3*x^3 + 16*a^3*b^2))*(-sqrt(3)*a/b^4)^(2/3) + (1/9)^(1/3)*(b
^4*x^10 + 240*a^2*b^2*x^4 + 160*a^3*b*x + sqrt(-3)*(b^4*x^10 + 240*a^2*b^2*
```

```
x^4 + 160*a^3*b*x) - 24*sqrt(3)*(a*b^3*x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x + sq
rt(-3)*(a*b^3*x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x)))*(-sqrt(3)*a/b^4)^(1/3) + 8
*sqrt(3)*(a*b^2*x^8 + 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(b*x^3 + a) + (1/72)^(
1/6)*(3*b^4*x^12 - 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 + 2208*a^3*b*x^3 + 384*a
^4 - sqrt(3)*(b^4*x^12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*b*x^3 +
 256*a^4 - sqrt(-3)*(b^4*x^12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*
b*x^3 + 256*a^4)) - 3*sqrt(-3)*(b^4*x^12 - 4*a*b^3*x^9 + 360*a^2*b^2*x^6 +
736*a^3*b*x^3 + 128*a^4))*(-sqrt(3)*a/b^4)^(1/6))/(b^4*x^12 + 40*a*b^3*x^9
+ 384*a^2*b^2*x^6 - 320*a^3*b*x^3 + 64*a^4)) - (1/72)^(1/6)*(sqrt(-3)*b + b
)*(-sqrt(3)*a/b^4)^(1/6)*log((72*(1/72)^(5/6)*(7*b^6*x^10 + 12*a*b^5*x^7 +
408*a^2*b^4*x^4 + 160*a^3*b^3*x - sqrt(-3)*(7*b^6*x^10 + 12*a*b^5*x^7 + 408
*a^2*b^4*x^4 + 160*a^3*b^3*x) + 3*sqrt(3)*(b^6*x^10 - 12*a*b^5*x^7 - 72*a^2
*b^4*x^4 - 32*a^3*b^3*x - sqrt(-3)*(b^6*x^10 - 12*a*b^5*x^7 - 72*a^2*b^4*x^
4 - 32*a^3*b^3*x)))*(-sqrt(3)*a/b^4)^(5/6) - 4*sqrt(1/2)*(3*b^5*x^11 - 18*a
*b^4*x^8 + 360*a^2*b^3*x^5 + 624*a^3*b^2*x^2 + sqrt(3)*(b^5*x^11 - 42*a*b^4
*x^8 - 168*a^2*b^3*x^5 - 368*a^3*b^2*x^2))*sqrt(-sqrt(3)*a/b^4) + 6*(12*a*b
^2*x^8 - 48*a^2*b*x^5 - 384*a^3*x^2 + 2*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^
3*x^3 + 48*a^3*b^2 + sqrt(3)*(b^5*x^9 - 30*a*b^4*x^6 - 144*a^2*b^3*x^3 - 32
*a^3*b^2 + sqrt(-3)*(b^5*x^9 - 30*a*b^4*x^6 - 144*a^2*b^3*x^3 - 32*a^3*b^2)
) + 3*sqrt(-3)*(b^5*x^9 + 96*a^2*b^3*x^3 + 16*a^3*b^2))*(-sqrt(3)*a/b^4)^(2
/3) + (1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 + 160*a^3*b*x - sqrt(-3)*(b^4
*x^10 + 240*a^2*b^2*x^4 + 160*a^3*b*x) - 24*sqrt(3)*(a*b^3*x^7 + 5*a^2*b^2*
x^4 + 4*a^3*b*x - sqrt(-3)*(a*b^3*x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x)))*(-sqrt
(3)*a/b^4)^(1/3) + 8*sqrt(3)*(a*b^2*x^8 + 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(b
*x^3 + a) + (1/72)^(1/6)*(3*b^4*x^12 - 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 + 22
08*a^3*b*x^3 + 384*a^4 - sqrt(3)*(b^4*x^12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^
6 + 1120*a^3*b*x^3 + 256*a^4 + sqrt(-3)*(b^4*x^12 + 124*a*b^3*x^9 + 744*a^2
*b^2*x^6 + 1120*a^3*b*x^3 + 256*a^4)) + 3*sqrt(-3)*(b^4*x^12 - 4*a*b^3*x^9
+ 360*a^2*b^2*x^6 + 736*a^3*b*x^3 + 128*a^4))*(-sqrt(3)*a/b^4)^(1/6))/(b^4*
x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*x^3 + 64*a^4)) + (1/72)^(
1/6)*(sqrt(-3)*b + b)*(-sqrt(3)*a/b^4)^(1/6)*log(-(72*(1/72)^(5/6)*(7*b^6*x
^10 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x - sqrt(-3)*(7*b^6*x^10
 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x) + 3*sqrt(3)*(b^6*x^10 -
12*a*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x - sqrt(-3)*(b^6*x^10 - 12*a*b^
5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x)))*(-sqrt(3)*a/b^4)^(5/6) - 4*sqrt(1/
2)*(3*b^5*x^11 - 18*a*b^4*x^8 + 360*a^2*b^3*x^5 + 624*a^3*b^2*x^2 + sqrt(3)
*(b^5*x^11 - 42*a*b^4*x^8 - 168*a^2*b^3*x^5 - 368*a^3*b^2*x^2))*sqrt(-sqrt(
3)*a/b^4) - 6*(12*a*b^2*x^8 - 48*a^2*b*x^5 - 384*a^3*x^2 + 2*(1/9)^(2/3)*(3
*b^5*x^9 + 288*a^2*b^3*x^3 + 48*a^3*b^2 + sqrt(3)*(b^5*x^9 - 30*a*b^4*x^6 -
 144*a^2*b^3*x^3 - 32*a^3*b^2 + sqrt(-3)*(b^5*x^9 - 30*a*b^4*x^6 - 144*a^2*
b^3*x^3 - 32*a^3*b^2)) + 3*sqrt(-3)*(b^5*x^9 + 96*a^2*b^3*x^3 + 16*a^3*b^2)
)*(-sqrt(3)*a/b^4)^(2/3) + (1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 + 160*a^
3*b*x - sqrt(-3)*(b^4*x^10 + 240*a^2*b^2*x^4 + 160*a^3*b*x) - 24*sqrt(3)*(a
*b^3*x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x - sqrt(-3)*(a*b^3*x^7 + 5*a^2*b^2*x^4
+ 4*a^3*b*x)))*(-sqrt(3)*a/b^4)^(1/3) + 8*sqrt(3)*(a*b^2*x^8 + 2*a^2*b*x^5
```

```
+ 28*a^3*x^2))*sqrt(b*x^3 + a) + (1/72)^(1/6)*(3*b^4*x^12 - 12*a*b^3*x^9 +
1080*a^2*b^2*x^6 + 2208*a^3*b*x^3 + 384*a^4 - sqrt(3)*(b^4*x^12 + 124*a*b^3
*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*b*x^3 + 256*a^4 + sqrt(-3)*(b^4*x^12 + 12
4*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*b*x^3 + 256*a^4)) + 3*sqrt(-3)*(b^
4*x^12 - 4*a*b^3*x^9 + 360*a^2*b^2*x^6 + 736*a^3*b*x^3 + 128*a^4))*(-sqrt(3
)*a/b^4)^(1/6))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*x^3
+ 64*a^4)) - 2*(1/72)^(1/6)*b*(-sqrt(3)*a/b^4)^(1/6)*log((72*(1/72)^(5/6)*(
7*b^6*x^10 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x + 3*sqrt(3)*(b^
6*x^10 - 12*a*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x))*(-sqrt(3)*a/b^4)^(5
/6) + 2*sqrt(1/2)*(3*b^5*x^11 - 18*a*b^4*x^8 + 360*a^2*b^3*x^5 + 624*a^3*b^
2*x^2 + sqrt(3)*(b^5*x^11 - 42*a*b^4*x^8 - 168*a^2*b^3*x^5 - 368*a^3*b^2*x^
2))*sqrt(-sqrt(3)*a/b^4) + 6*(6*a*b^2*x^8 - 24*a^2*b*x^5 - 192*a^3*x^2 - 2*
(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 + 48*a^3*b^2 + sqrt(3)*(b^5*x^9 -
30*a*b^4*x^6 - 144*a^2*b^3*x^3 - 32*a^3*b^2))*(-sqrt(3)*a/b^4)^(2/3) - (1/9
)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 + 160*a^3*b*x - 24*sqrt(3)*(a*b^3*x^7 +
 5*a^2*b^2*x^4 + 4*a^3*b*x))*(-sqrt(3)*a/b^4)^(1/3) + 4*sqrt(3)*(a*b^2*x^8
+ 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(b*x^3 + a) + (1/72)^(1/6)*(3*b^4*x^12 - 1
2*a*b^3*x^9 + 1080*a^2*b^2*x^6 + 2208*a^3*b*x^3 + 384*a^4 - sqrt(3)*(b^4*x^
12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*b*x^3 + 256*a^4))*(-sqrt(3)
*a/b^4)^(1/6))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*x^3 +
 64*a^4)) + 2*(1/72)^(1/6)*b*(-sqrt(3)*a/b^4)^(1/6)*log(-(72*(1/72)^(5/6)*(
7*b^6*x^10 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x + 3*sqrt(3)*(b^
6*x^10 - 12*a*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x))*(-sqrt(3)*a/b^4)^(5
/6) + 2*sqrt(1/2)*(3*b^5*x^11 - 18*a*b^4*x^8 + 360*a^2*b^3*x^5 + 624*a^3*b^
2*x^2 + sqrt(3)*(b^5*x^11 - 42*a*b^4*x^8 - 168*a^2*b^3*x^5 - 368*a^3*b^2*x^
2))*sqrt(-sqrt(3)*a/b^4) - 6*(6*a*b^2*x^8 - 24*a^2*b*x^5 - 192*a^3*x^2 - 2*
(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 + 48*a^3*b^2 + sqrt(3)*(b^5*x^9 -
30*a*b^4*x^6 - 144*a^2*b^3*x^3 - 32*a^3*b^2))*(-sqrt(3)*a/b^4)^(2/3) - (1/9
)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 + 160*a^3*b*x - 24*sqrt(3)*(a*b^3*x^7 +
 5*a^2*b^2*x^4 + 4*a^3*b*x))*(-sqrt(3)*a/b^4)^(1/3) + 4*sqrt(3)*(a*b^2*x^8
+ 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(b*x^3 + a) + (1/72)^(1/6)*(3*b^4*x^12 - 1
2*a*b^3*x^9 + 1080*a^2*b^2*x^6 + 2208*a^3*b*x^3 + 384*a^4 - sqrt(3)*(b^4*x^
12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*b*x^3 + 256*a^4))*(-sqrt(3)
*a/b^4)^(1/6))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*x^3 +
 64*a^4)) + 16*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4
*a/b, x)))/b
```

Sympy [F]

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx = \int \frac{x\sqrt{a+bx^3}}{10a+6\sqrt{3}a+bx^3}\,dx$$

```
[In] integrate(x*(b*x**3+a)**(1/2)/(b*x**3+2*a*(5+3*3**(1/2))),x)
[Out] Integral(x*sqrt(a + b*x**3)/(10*a + 6*sqrt(3)*a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx = \int \frac{\sqrt{bx^3+a}x}{bx^3+2a\left(3\sqrt{3}+5\right)}\,dx$$

```
[In] integrate(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5+3*3^(1/2))),x, algorithm="maxima"
)
[Out] integrate(sqrt(b*x^3 + a)*x/(b*x^3 + 2*a*(3*sqrt(3) + 5)), x)
```

Giac [F]

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx = \int \frac{\sqrt{bx^3+a}x}{bx^3+2a\left(3\sqrt{3}+5\right)}\,dx$$

```
[In] integrate(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5+3*3^(1/2))),x, algorithm="giac")
[Out] integrate(sqrt(b*x^3 + a)*x/(b*x^3 + 2*a*(3*sqrt(3) + 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx = \int \frac{x\sqrt{b\,x^3+a}}{b\,x^3+2\,a\,\left(3\sqrt{3}+5\right)}\,dx$$

```
[In] int((x*(a + b*x^3)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) + 5)),x)
[Out] int((x*(a + b*x^3)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) + 5)), x)
```

3.343 $\int \frac{x\sqrt{a-bx^3}}{2(5+3\sqrt{3})a-bx^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 757

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx$$

$$= \frac{2\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}+\frac{3^{3/4}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$+\frac{\sqrt[6]{a}\arctan\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}+\frac{\sqrt[4]{3}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$+\frac{\sqrt[4]{3}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{\sqrt{2}b^{2/3}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$
$$+\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

```
[Out] 1/4*3^(3/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1+3^(1/
2))*2^(1/2)/(-b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)+1/6*a^(1/6)*arctan(1/6*(1-3^(
1/2))*(-b*x^3+a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*3^(3/4)/b^(2/3)*2^(1/2)+1/4
*3^(1/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1-3^(1/2)
)*2^(1/2)/(-b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)+1/2*3^(1/4)*a^(1/6)*arctanh(1/2
*3^(1/4)*a^(1/6)*(2*b^(1/3)*x+a^(1/3)*(1+3^(1/2)))*2^(1/2)/(-b*x^3+a)^(1/2)
)/b^(2/3)*2^(1/2)+2*(-b*x^3+a)^(1/2)/b^(2/3)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)
))+2/3*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)
))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)+a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/
b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(
```

```
1+3^(1/2)))^2)^(1/2)-3^(1/4)*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/3
)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1
/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*
x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-
b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 757, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {495, 309, 224, 1891, 500}

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx$$
$$= \frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$
$$+\frac{3^{3/4}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}+\frac{\sqrt[6]{a}\arctan\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}$$
$$+\frac{\sqrt[4]{3}\sqrt[6]{a}\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$+\frac{\sqrt[4]{3}\sqrt[6]{a}\text{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{\sqrt{2}b^{2/3}}+\frac{2\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

```
[In] Int[(x*Sqrt[a - b*x^3])/(2*(5 + 3*Sqrt[3])*a - b*x^3),x]
```

```
[Out] (2*Sqrt[a - b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) + (3^(3/4
)*a^(1/6)*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqr
```

```
t[2]*Sqrt[a - b*x^3])])/(2*Sqrt[2]*b^(2/3)) + (a^(1/6)*ArcTan[((1 - Sqrt[3]
)*Sqrt[a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(Sqrt[2]*3^(1/4)*b^(2/3)) +
(3^(1/4)*a^(1/6)*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*
x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(2*Sqrt[2]*b^(2/3)) + (3^(1/4)*a^(1/6)*ArcT
anh[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a
 - b*x^3])])/(Sqrt[2]*b^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3
) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt
[a - b*x^3]) + (2*Sqrt[2]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3
)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 500

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
```

```
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\left(\left(3\left(3+2\sqrt{3}\right)a\right)\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx\right)+\int \frac{x}{\sqrt{a-bx^3}}\,dx$$

$$= \frac{3^{3/4}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}+\frac{\sqrt[6]{a}\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}$$
$$+\frac{\sqrt[4]{3}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$+\frac{\sqrt[4]{3}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{\sqrt{2}b^{2/3}}$$
$$-\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx}{\sqrt[3]{b}}+\frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)\int \frac{1}{\sqrt{a-bx^3}}\,dx}{\sqrt[3]{b}}$$

$$
\begin{aligned}
&= \frac{2\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} + \frac{3^{3/4}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}b^{2/3}} \\
&+ \frac{\sqrt[6]{a}\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}} + \frac{\sqrt[4]{3}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}b^{2/3}} \\
&+ \frac{\sqrt[4]{3}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{\sqrt{2}b^{2/3}} \\
&- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}} \\
&+ \frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.11

$$
\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = \frac{x^2\sqrt{1-\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right)}{\left(20+12\sqrt{3}\right)\sqrt{a-bx^3}}
$$

```
[In] Integrate[(x*Sqrt[a - b*x^3])/(2*(5 + 3*Sqrt[3])*a - b*x^3),x]
[Out] (x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, -1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10
*a + 6*Sqrt[3]*a)])/((20 + 12*Sqrt[3])*Sqrt[a - b*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.28 (sec) , antiderivative size = 924, normalized size of antiderivative = 1.22

method	result	size
elliptic	Expression too large to display	924
default	Expression too large to display	942

```
[In] int(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5+3*3^(1/2))),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a
*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*
(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)
^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*
3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b
/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)
*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2
)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(a*b^2)^(1/3)))^(1/2)))-1/9*I/b^3*2^(1/2)*sum(1/_alpha*(3+2*3^(1/2))*
(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(
a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1/3))/(-3*(a*b^2)^(1/3)-I*3^(1/2)*(a
*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/
3)))/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*(3*I*(a*b^2)^(1/3)*_alpha*3^(1/2
)*b+4*b^2*_alpha^2*3^(1/2)-3*I*(a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(a*b^2)^(1/3
)*_alpha*b-6*I*(a*b^2)^(1/3)*_alpha*b-6*b^2*_alpha^2-2*3^(1/2)*(a*b^2)^(2/3
)+6*I*(a*b^2)^(2/3)+3*(a*b^2)^(1/3)*_alpha*b+3*(a*b^2)^(2/3))*EllipticPi(1/
3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)
*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*3^(1/2)*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1
/2)*(a*b^2)^(2/3)*_alpha+4*I*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1/2)*a*b+2*3^(1/
2)*(a*b^2)^(2/3)*_alpha-2*I*(a*b^2)^(2/3)*_alpha-2*3^(1/2)*a*b-2*I*a*b-3*(a
*b^2)^(2/3)*_alpha+3*a*b)/a,(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3-6*3^(1/2)*a-
10*a))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.37 (sec) , antiderivative size = 4855, normalized size of antiderivative = 6.41

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5+3*3^(1/2))),x, algorithm="frica
s")
```

```
[Out] -1/8*((1/72)^(1/6)*(sqrt(-3)*b - b)*(-sqrt(3)*a/b^4)^(1/6)*log((72*(1/72)^(
5/6)*(7*b^6*x^10 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x + sqrt(-3
)*(7*b^6*x^10 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x) + 3*sqrt(3)
*(b^6*x^10 + 12*a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x + sqrt(-3)*(b^6*x
^10 + 12*a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x)))*(-sqrt(3)*a/b^4)^(5/6
) + 4*sqrt(1/2)*(3*b^5*x^11 + 18*a*b^4*x^8 + 360*a^2*b^3*x^5 - 624*a^3*b^2*
x^2 + sqrt(3)*(b^5*x^11 + 42*a*b^4*x^8 - 168*a^2*b^3*x^5 + 368*a^3*b^2*x^2)
)*sqrt(-sqrt(3)*a/b^4) + 6*(12*a*b^2*x^8 + 48*a^2*b*x^5 - 384*a^3*x^2 - 2*(
1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 - 48*a^3*b^2 + sqrt(3)*(b^5*x^9 + 3
0*a*b^4*x^6 - 144*a^2*b^3*x^3 + 32*a^3*b^2 - sqrt(-3)*(b^5*x^9 + 30*a*b^4*x
^6 - 144*a^2*b^3*x^3 + 32*a^3*b^2)) - 3*sqrt(-3)*(b^5*x^9 + 96*a^2*b^3*x^3
 - 16*a^3*b^2))*(-sqrt(3)*a/b^4)^(2/3) + (1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2
*x^4 - 160*a^3*b*x + sqrt(-3)*(b^4*x^10 + 240*a^2*b^2*x^4 - 160*a^3*b*x) +
24*sqrt(3)*(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x + sqrt(-3)*(a*b^3*x^7 - 5
*a^2*b^2*x^4 + 4*a^3*b*x)))*(-sqrt(3)*a/b^4)^(1/3) + 8*sqrt(3)*(a*b^2*x^8 -
 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(-b*x^3 + a) + (1/72)^(1/6)*(3*b^4*x^12 + 1
2*a*b^3*x^9 + 1080*a^2*b^2*x^6 - 2208*a^3*b*x^3 + 384*a^4 - 3*sqrt(-3)*(b^4
*x^12 + 4*a*b^3*x^9 + 360*a^2*b^2*x^6 - 736*a^3*b*x^3 + 128*a^4) - sqrt(3)*
(b^4*x^12 - 124*a*b^3*x^9 + 744*a^2*b^2*x^6 - 1120*a^3*b*x^3 + 256*a^4 - sq
rt(-3)*(b^4*x^12 - 124*a*b^3*x^9 + 744*a^2*b^2*x^6 - 1120*a^3*b*x^3 + 256*a
^4)))*(-sqrt(3)*a/b^4)^(1/6))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2*x^6 +
320*a^3*b*x^3 + 64*a^4)) - (1/72)^(1/6)*(sqrt(-3)*b - b)*(-sqrt(3)*a/b^4)^(
1/6)*log(-(72*(1/72)^(5/6)*(7*b^6*x^10 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 1
60*a^3*b^3*x + sqrt(-3)*(7*b^6*x^10 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*
a^3*b^3*x) + 3*sqrt(3)*(b^6*x^10 + 12*a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b
^3*x + sqrt(-3)*(b^6*x^10 + 12*a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x)))
*(-sqrt(3)*a/b^4)^(5/6) + 4*sqrt(1/2)*(3*b^5*x^11 + 18*a*b^4*x^8 + 360*a^2*
b^3*x^5 - 624*a^3*b^2*x^2 + sqrt(3)*(b^5*x^11 + 42*a*b^4*x^8 - 168*a^2*b^3*
x^5 + 368*a^3*b^2*x^2))*sqrt(-sqrt(3)*a/b^4) - 6*(12*a*b^2*x^8 + 48*a^2*b*x
^5 - 384*a^3*x^2 - 2*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 - 48*a^3*b^2
+ sqrt(3)*(b^5*x^9 + 30*a*b^4*x^6 - 144*a^2*b^3*x^3 + 32*a^3*b^2 - sqrt(-3)
*(b^5*x^9 + 30*a*b^4*x^6 - 144*a^2*b^3*x^3 + 32*a^3*b^2)) - 3*sqrt(-3)*(b^5
*x^9 + 96*a^2*b^3*x^3 - 16*a^3*b^2))*(-sqrt(3)*a/b^4)^(2/3) + (1/9)^(1/3)*(
b^4*x^10 + 240*a^2*b^2*x^4 - 160*a^3*b*x + sqrt(-3)*(b^4*x^10 + 240*a^2*b^2
```

```
*x^4 - 160*a^3*b*x) + 24*sqrt(3)*(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x + s
qrt(-3)*(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x)))*(-sqrt(3)*a/b^4)^(1/3) +
8*sqrt(3)*(a*b^2*x^8 - 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(-b*x^3 + a) + (1/72)
^(1/6)*(3*b^4*x^12 + 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 - 2208*a^3*b*x^3 + 384
*a^4 - 3*sqrt(-3)*(b^4*x^12 + 4*a*b^3*x^9 + 360*a^2*b^2*x^6 - 736*a^3*b*x^3
 + 128*a^4) - sqrt(3)*(b^4*x^12 - 124*a*b^3*x^9 + 744*a^2*b^2*x^6 - 1120*a^
3*b*x^3 + 256*a^4 - sqrt(-3)*(b^4*x^12 - 124*a*b^3*x^9 + 744*a^2*b^2*x^6 -
1120*a^3*b*x^3 + 256*a^4)))*(-sqrt(3)*a/b^4)^(1/6))/(b^4*x^12 - 40*a*b^3*x^
9 + 384*a^2*b^2*x^6 + 320*a^3*b*x^3 + 64*a^4)) - (1/72)^(1/6)*(sqrt(-3)*b +
 b)*(-sqrt(3)*a/b^4)^(1/6)*log((72*(1/72)^(5/6)*(7*b^6*x^10 - 12*a*b^5*x^7
+ 408*a^2*b^4*x^4 - 160*a^3*b^3*x - sqrt(-3)*(7*b^6*x^10 - 12*a*b^5*x^7 + 4
08*a^2*b^4*x^4 - 160*a^3*b^3*x) + 3*sqrt(3)*(b^6*x^10 + 12*a*b^5*x^7 - 72*a
^2*b^4*x^4 + 32*a^3*b^3*x - sqrt(-3)*(b^6*x^10 + 12*a*b^5*x^7 - 72*a^2*b^4*
x^4 + 32*a^3*b^3*x)))*(-sqrt(3)*a/b^4)^(5/6) + 4*sqrt(1/2)*(3*b^5*x^11 + 18
*a*b^4*x^8 + 360*a^2*b^3*x^5 - 624*a^3*b^2*x^2 + sqrt(3)*(b^5*x^11 + 42*a*b
^4*x^8 - 168*a^2*b^3*x^5 + 368*a^3*b^2*x^2))*sqrt(-sqrt(3)*a/b^4) + 6*(12*a
*b^2*x^8 + 48*a^2*b*x^5 - 384*a^3*x^2 - 2*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*
b^3*x^3 - 48*a^3*b^2 + sqrt(3)*(b^5*x^9 + 30*a*b^4*x^6 - 144*a^2*b^3*x^3 +
32*a^3*b^2 + sqrt(-3)*(b^5*x^9 + 30*a*b^4*x^6 - 144*a^2*b^3*x^3 + 32*a^3*b^
2)) + 3*sqrt(-3)*(b^5*x^9 + 96*a^2*b^3*x^3 - 16*a^3*b^2))*(-sqrt(3)*a/b^4)^
(2/3) + (1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 - 160*a^3*b*x - sqrt(-3)*(b
^4*x^10 + 240*a^2*b^2*x^4 - 160*a^3*b*x) + 24*sqrt(3)*(a*b^3*x^7 - 5*a^2*b^
2*x^4 + 4*a^3*b*x - sqrt(-3)*(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x)))*(-sq
rt(3)*a/b^4)^(1/3) + 8*sqrt(3)*(a*b^2*x^8 - 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt
(-b*x^3 + a) + (1/72)^(1/6)*(3*b^4*x^12 + 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 -
 2208*a^3*b*x^3 + 384*a^4 + 3*sqrt(-3)*(b^4*x^12 + 4*a*b^3*x^9 + 360*a^2*b^
2*x^6 - 736*a^3*b*x^3 + 128*a^4) - sqrt(3)*(b^4*x^12 - 124*a*b^3*x^9 + 744*
a^2*b^2*x^6 - 1120*a^3*b*x^3 + 256*a^4 + sqrt(-3)*(b^4*x^12 - 124*a*b^3*x^9
 + 744*a^2*b^2*x^6 - 1120*a^3*b*x^3 + 256*a^4)))*(-sqrt(3)*a/b^4)^(1/6))/(b
^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2*x^6 + 320*a^3*b*x^3 + 64*a^4)) + (1/72
)^(1/6)*(sqrt(-3)*b + b)*(-sqrt(3)*a/b^4)^(1/6)*log(-(72*(1/72)^(5/6)*(7*b^
6*x^10 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x - sqrt(-3)*(7*b^6*x
^10 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x) + 3*sqrt(3)*(b^6*x^10
 + 12*a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x - sqrt(-3)*(b^6*x^10 + 12*a
*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x)))*(-sqrt(3)*a/b^4)^(5/6) + 4*sqrt
(1/2)*(3*b^5*x^11 + 18*a*b^4*x^8 + 360*a^2*b^3*x^5 - 624*a^3*b^2*x^2 + sqrt
(3)*(b^5*x^11 + 42*a*b^4*x^8 - 168*a^2*b^3*x^5 + 368*a^3*b^2*x^2))*sqrt(-sq
rt(3)*a/b^4) - 6*(12*a*b^2*x^8 + 48*a^2*b*x^5 - 384*a^3*x^2 - 2*(1/9)^(2/3)
*(3*b^5*x^9 + 288*a^2*b^3*x^3 - 48*a^3*b^2 + sqrt(3)*(b^5*x^9 + 30*a*b^4*x^
6 - 144*a^2*b^3*x^3 + 32*a^3*b^2 + sqrt(-3)*(b^5*x^9 + 30*a*b^4*x^6 - 144*a
^2*b^3*x^3 + 32*a^3*b^2)) + 3*sqrt(-3)*(b^5*x^9 + 96*a^2*b^3*x^3 - 16*a^3*b
^2))*(-sqrt(3)*a/b^4)^(2/3) + (1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 - 160
*a^3*b*x - sqrt(-3)*(b^4*x^10 + 240*a^2*b^2*x^4 - 160*a^3*b*x) + 24*sqrt(3)
*(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x - sqrt(-3)*(a*b^3*x^7 - 5*a^2*b^2*x
^4 + 4*a^3*b*x)))*(-sqrt(3)*a/b^4)^(1/3) + 8*sqrt(3)*(a*b^2*x^8 - 2*a^2*b*x
```

```
^5 + 28*a^3*x^2))*sqrt(-b*x^3 + a) + (1/72)^(1/6)*(3*b^4*x^12 + 12*a*b^3*x^
9 + 1080*a^2*b^2*x^6 - 2208*a^3*b*x^3 + 384*a^4 + 3*sqrt(-3)*(b^4*x^12 + 4*
a*b^3*x^9 + 360*a^2*b^2*x^6 - 736*a^3*b*x^3 + 128*a^4) - sqrt(3)*(b^4*x^12
- 124*a*b^3*x^9 + 744*a^2*b^2*x^6 - 1120*a^3*b*x^3 + 256*a^4 + sqrt(-3)*(b^
4*x^12 - 124*a*b^3*x^9 + 744*a^2*b^2*x^6 - 1120*a^3*b*x^3 + 256*a^4)))*(-sq
rt(3)*a/b^4)^(1/6))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2*x^6 + 320*a^3*b*
x^3 + 64*a^4)) - 2*(1/72)^(1/6)*b*(-sqrt(3)*a/b^4)^(1/6)*log((72*(1/72)^(5/
6)*(7*b^6*x^10 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x + 3*sqrt(3)
*(b^6*x^10 + 12*a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x))*(-sqrt(3)*a/b^4
)^(5/6) - 2*sqrt(1/2)*(3*b^5*x^11 + 18*a*b^4*x^8 + 360*a^2*b^3*x^5 - 624*a^
3*b^2*x^2 + sqrt(3)*(b^5*x^11 + 42*a*b^4*x^8 - 168*a^2*b^3*x^5 + 368*a^3*b^
2*x^2))*sqrt(-sqrt(3)*a/b^4) + 6*(6*a*b^2*x^8 + 24*a^2*b*x^5 - 192*a^3*x^2
+ 2*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 - 48*a^3*b^2 + sqrt(3)*(b^5*x^
9 + 30*a*b^4*x^6 - 144*a^2*b^3*x^3 + 32*a^3*b^2))*(-sqrt(3)*a/b^4)^(2/3) -
(1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 - 160*a^3*b*x + 24*sqrt(3)*(a*b^3*x
^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x))*(-sqrt(3)*a/b^4)^(1/3) + 4*sqrt(3)*(a*b^2*
x^8 - 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(-b*x^3 + a) + (1/72)^(1/6)*(3*b^4*x^1
2 + 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 - 2208*a^3*b*x^3 + 384*a^4 - sqrt(3)*(b
^4*x^12 - 124*a*b^3*x^9 + 744*a^2*b^2*x^6 - 1120*a^3*b*x^3 + 256*a^4))*(-sq
rt(3)*a/b^4)^(1/6))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2*x^6 + 320*a^3*b*
x^3 + 64*a^4)) + 2*(1/72)^(1/6)*b*(-sqrt(3)*a/b^4)^(1/6)*log(-(72*(1/72)^(5
/6)*(7*b^6*x^10 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x + 3*sqrt(3
)*(b^6*x^10 + 12*a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x))*(-sqrt(3)*a/b^
4)^(5/6) - 2*sqrt(1/2)*(3*b^5*x^11 + 18*a*b^4*x^8 + 360*a^2*b^3*x^5 - 624*a
^3*b^2*x^2 + sqrt(3)*(b^5*x^11 + 42*a*b^4*x^8 - 168*a^2*b^3*x^5 + 368*a^3*b
^2*x^2))*sqrt(-sqrt(3)*a/b^4) - 6*(6*a*b^2*x^8 + 24*a^2*b*x^5 - 192*a^3*x^2
 + 2*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 - 48*a^3*b^2 + sqrt(3)*(b^5*x
^9 + 30*a*b^4*x^6 - 144*a^2*b^3*x^3 + 32*a^3*b^2))*(-sqrt(3)*a/b^4)^(2/3) -
 (1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 - 160*a^3*b*x + 24*sqrt(3)*(a*b^3*
x^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x))*(-sqrt(3)*a/b^4)^(1/3) + 4*sqrt(3)*(a*b^2
*x^8 - 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(-b*x^3 + a) + (1/72)^(1/6)*(3*b^4*x^
12 + 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 - 2208*a^3*b*x^3 + 384*a^4 - sqrt(3)*(
b^4*x^12 - 124*a*b^3*x^9 + 744*a^2*b^2*x^6 - 1120*a^3*b*x^3 + 256*a^4))*(-s
qrt(3)*a/b^4)^(1/6))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2*x^6 + 320*a^3*b
*x^3 + 64*a^4)) - 16*sqrt(-b)*weierstrassZeta(0, 4*a/b, weierstrassPInverse
(0, 4*a/b, x)))/b
```

Sympy [F]

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = -\int \frac{x\sqrt{a-bx^3}}{-6\sqrt{3}a-10a+bx^3}\,dx$$

```
[In] integrate(x*(-b*x**3+a)**(1/2)/(-b*x**3+2*a*(5+3*3**(1/2))),x)
[Out] -Integral(x*sqrt(a - b*x**3)/(-6*sqrt(3)*a - 10*a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = \int -\frac{\sqrt{-bx^3+ax}}{bx^3-2\,a\left(3\,\sqrt{3}+5\right)}\,dx$$

```
[In] integrate(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5+3*3^(1/2))),x, algorithm="maxim
a")
[Out] -integrate(sqrt(-b*x^3 + a)*x/(b*x^3 - 2*a*(3*sqrt(3) + 5)), x)
```

Giac [F]

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = \int -\frac{\sqrt{-bx^3+ax}}{bx^3-2\,a\left(3\,\sqrt{3}+5\right)}\,dx$$

```
[In] integrate(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5+3*3^(1/2))),x, algorithm="giac"
)
[Out] integrate(-sqrt(-b*x^3 + a)*x/(b*x^3 - 2*a*(3*sqrt(3) + 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = -\int \frac{x\,\sqrt{a-b\,x^3}}{b\,x^3-2\,a\,\left(3\,\sqrt{3}+5\right)}\,dx$$

```
[In] int(-(x*(a - b*x^3)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) + 5)),x)
[Out] -int((x*(a - b*x^3)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) + 5)), x)
```

3.344 $\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 774

$$\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx$$
$$= -\frac{2\sqrt{-a+bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}+\frac{\sqrt[4]{3}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$+\frac{\sqrt[4]{3}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{\sqrt{2}b^{2/3}}$$
$$+\frac{3^{3/4}\sqrt[6]{a}\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}-\frac{\sqrt[6]{a}\text{arctanh}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{-a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}$$
$$+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$
$$-\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

```
[Out] 1/4*3^(1/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1-3^(1/
2))*2^(1/2)/(b*x^3-a)^(1/2))/b^(2/3)*2^(1/2)+1/2*3^(1/4)*a^(1/6)*arctan(1/2
*3^(1/4)*a^(1/6)*(2*b^(1/3)*x+a^(1/3)*(1+3^(1/2)))*2^(1/2)/(b*x^3-a)^(1/2))
/b^(2/3)*2^(1/2)+1/4*3^(3/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b
^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(b*x^3-a)^(1/2))/b^(2/3)*2^(1/2)-1/6*a^(1/6)*
arctanh(1/6*(1-3^(1/2))*(b*x^3-a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*3^(3/4)/b^
(2/3)*2^(1/2)-2*(b*x^3-a)^(1/2)/b^(2/3)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))-2/
3*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-
b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/
3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(
```

```
1/2)))^2)^(1/2)+3^(1/4)*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/3)*x+a
^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/
3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
*(1/2*6^(1/2)+1/2*2^(1/2))/b^(2/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/
3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 774, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {495, 310, 225, 1893, 501}

$$\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx = -\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}} + \frac{\sqrt[4]{3}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{2\sqrt{2}b^{2/3}} + \frac{\sqrt[4]{3}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{\sqrt{2}b^{2/3}} + \frac{3^{3/4}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{2\sqrt{2}b^{2/3}} - \frac{\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{bx^3-a}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}} - \frac{2\sqrt{bx^3-a}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

```
[In] Int[(x*Sqrt[-a + b*x^3])/(-2*(5 + 3*Sqrt[3])*a + b*x^3),x]
```

```
[Out] (-2*Sqrt[-a + b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) + (3^(1
/4)*a^(1/6)*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(S
qrt[2]*Sqrt[-a + b*x^3])])/(2*Sqrt[2]*b^(2/3)) + (3^(1/4)*a^(1/6)*ArcTan[(3
^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*
x^3])])/(Sqrt[2]*b^(2/3)) + (3^(3/4)*a^(1/6)*ArcTanh[(3^(1/4)*(1 + Sqrt[3])
*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])])/(2*Sqrt[2]*b^(
2/3)) - (a^(1/6)*ArcTanh[((1 - Sqrt[3])*Sqrt[-a + b*x^3])/(Sqrt[2]*3^(3/4)*
Sqrt[a])])/(Sqrt[2]*3^(1/4)*b^(2/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(
a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 -
 Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) -
 b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)
*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)
^2)]*Sqrt[-a + b*x^3]) - (2*Sqrt[2]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(
2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)
^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(
1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1
/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 501

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
```

```
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \left(3\left(3+2\sqrt{3}\right)a\right)\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx+\int \frac{x}{\sqrt{-a+bx^3}}\,dx$$

$$= \frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}b^{2/3}} + \frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{\sqrt{2}b^{2/3}} + \frac{3^{3/4}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}b^{2/3}} - \frac{\sqrt[6]{a}\tanh^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{-a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}} - \frac{\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx}{\sqrt[3]{b}} + \frac{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)\int \frac{1}{\sqrt{-a+bx^3}}\,dx}{\sqrt[3]{b}}$$

$$= -\frac{2\sqrt{-a+bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}+\frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$+\frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{\sqrt{2}b^{2/3}}$$
$$+\frac{3^{3/4}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}-\frac{\sqrt[6]{a}\tanh^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{-a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}$$
$$+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$
$$-\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.11

$$\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx = -\frac{x^2\sqrt{-a+bx^3}\,\mathrm{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right)}{4\left(5+3\sqrt{3}\right)a\sqrt{1-\frac{bx^3}{a}}}$$

```
[In] Integrate[(x*Sqrt[-a + b*x^3])/(-2*(5 + 3*Sqrt[3])*a + b*x^3),x]
```

```
[Out] -1/4*(x^2*Sqrt[-a + b*x^3]*AppellF1[2/3, -1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(
10*a + 6*Sqrt[3]*a)])/((5 + 3*Sqrt[3])*a*Sqrt[1 - (b*x^3)/a])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.97 (sec) , antiderivative size = 926, normalized size of antiderivative = 1.20

method	result	size
elliptic	Expression too large to display	926
default	Expression too large to display	944

```
[In] int(x*(b*x^3-a)^(1/2)/(b*x^3-2*a*(5+3*3^(1/2))),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a
*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*
(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^
(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3
^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/
(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*
(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)
^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(a*b^2)^(1/3)))^(1/2)))-1/9*I/b^3*2^(1/2)*sum(1/_alpha*(3+2*3^(1/2))*(
a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a
*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1/3))/(-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*
b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3
)))/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*(3*I*(a*b^2)^(1/3)*_alpha*3^(1/2)*
b+4*b^2*_alpha^2*3^(1/2)-3*I*(a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(a*b^2)^(1/3)*
_alpha*b-6*I*(a*b^2)^(1/3)*_alpha*b-6*b^2*_alpha^2-2*3^(1/2)*(a*b^2)^(2/3)+
6*I*(a*b^2)^(2/3)+3*(a*b^2)^(1/3)*_alpha*b+3*(a*b^2)^(2/3))*EllipticPi(1/3*
3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b
/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*3^(1/2)*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1/2
)*(a*b^2)^(2/3)*_alpha+4*I*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1/2)*a*b+2*3^(1/2)
*(a*b^2)^(2/3)*_alpha-2*I*(a*b^2)^(2/3)*_alpha-2*3^(1/2)*a*b-2*I*a*b-3*(a*b
^2)^(2/3)*_alpha+3*a*b)/a,(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3-6*3^(1/2)*a-10
*a))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.74 (sec) , antiderivative size = 4963, normalized size of antiderivative = 6.41

$$\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(b*x^3-a)^(1/2)/(b*x^3-2*a*(5+3*3^(1/2))),x, algorithm="fricas"
)
```

```
[Out] 1/8*((1/72)^(1/6)*(sqrt(-3)*b + b)*(sqrt(3)*a/b^4)^(1/6)*log(-(12*(1/9)^(2/
3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 - 48*a^3*b^2 + sqrt(3)*(b^5*x^9 + 30*a*b^4*
x^6 - 144*a^2*b^3*x^3 + 32*a^3*b^2 + sqrt(-3)*(b^5*x^9 + 30*a*b^4*x^6 - 144
*a^2*b^3*x^3 + 32*a^3*b^2)) + 3*sqrt(-3)*(b^5*x^9 + 96*a^2*b^3*x^3 - 16*a^3
*b^2))*sqrt(b*x^3 - a)*(sqrt(3)*a/b^4)^(2/3) + 72*(1/72)^(5/6)*(7*b^6*x^10
- 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x - sqrt(-3)*(7*b^6*x^10 - 1
2*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x) + 3*sqrt(3)*(b^6*x^10 + 12*a
*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x - sqrt(-3)*(b^6*x^10 + 12*a*b^5*x^
7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x)))*(sqrt(3)*a/b^4)^(5/6) + 6*(1/9)^(1/3)*
(b^4*x^10 + 240*a^2*b^2*x^4 - 160*a^3*b*x - sqrt(-3)*(b^4*x^10 + 240*a^2*b^
2*x^4 - 160*a^3*b*x) + 24*sqrt(3)*(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x -
sqrt(-3)*(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x)))*sqrt(b*x^3 - a)*(sqrt(3)
*a/b^4)^(1/3) - 4*sqrt(1/2)*(3*b^5*x^11 + 18*a*b^4*x^8 + 360*a^2*b^3*x^5 -
624*a^3*b^2*x^2 + sqrt(3)*(b^5*x^11 + 42*a*b^4*x^8 - 168*a^2*b^3*x^5 + 368*
a^3*b^2*x^2))*sqrt(sqrt(3)*a/b^4) - 24*(3*a*b^2*x^8 + 12*a^2*b*x^5 - 96*a^3
*x^2 + 2*sqrt(3)*(a*b^2*x^8 - 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(b*x^3 - a) +
(1/72)^(1/6)*(3*b^4*x^12 + 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 - 2208*a^3*b*x^3
 + 384*a^4 + 3*sqrt(-3)*(b^4*x^12 + 4*a*b^3*x^9 + 360*a^2*b^2*x^6 - 736*a^3
*b*x^3 + 128*a^4) - sqrt(3)*(b^4*x^12 - 124*a*b^3*x^9 + 744*a^2*b^2*x^6 - 1
120*a^3*b*x^3 + 256*a^4 + sqrt(-3)*(b^4*x^12 - 124*a*b^3*x^9 + 744*a^2*b^2*
x^6 - 1120*a^3*b*x^3 + 256*a^4)))*(sqrt(3)*a/b^4)^(1/6))/(b^4*x^12 - 40*a*b
^3*x^9 + 384*a^2*b^2*x^6 + 320*a^3*b*x^3 + 64*a^4)) - (1/72)^(1/6)*(sqrt(-3
)*b + b)*(sqrt(3)*a/b^4)^(1/6)*log(-(12*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^
3*x^3 - 48*a^3*b^2 + sqrt(3)*(b^5*x^9 + 30*a*b^4*x^6 - 144*a^2*b^3*x^3 + 32
*a^3*b^2 + sqrt(-3)*(b^5*x^9 + 30*a*b^4*x^6 - 144*a^2*b^3*x^3 + 32*a^3*b^2)
) + 3*sqrt(-3)*(b^5*x^9 + 96*a^2*b^3*x^3 - 16*a^3*b^2))*sqrt(b*x^3 - a)*(sq
rt(3)*a/b^4)^(2/3) - 72*(1/72)^(5/6)*(7*b^6*x^10 - 12*a*b^5*x^7 + 408*a^2*b
^4*x^4 - 160*a^3*b^3*x - sqrt(-3)*(7*b^6*x^10 - 12*a*b^5*x^7 + 408*a^2*b^4*
x^4 - 160*a^3*b^3*x) + 3*sqrt(3)*(b^6*x^10 + 12*a*b^5*x^7 - 72*a^2*b^4*x^4
+ 32*a^3*b^3*x - sqrt(-3)*(b^6*x^10 + 12*a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^
3*b^3*x)))*(sqrt(3)*a/b^4)^(5/6) + 6*(1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^
4 - 160*a^3*b*x - sqrt(-3)*(b^4*x^10 + 240*a^2*b^2*x^4 - 160*a^3*b*x) + 24*
sqrt(3)*(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x - sqrt(-3)*(a*b^3*x^7 - 5*a^
2*b^2*x^4 + 4*a^3*b*x)))*sqrt(b*x^3 - a)*(sqrt(3)*a/b^4)^(1/3) + 4*sqrt(1/2
```

)*(3*b^5*x^11 + 18*a*b^4*x^8 + 360*a^2*b^3*x^5 - 624*a^3*b^2*x^2 + sqrt(3)*
(b^5*x^11 + 42*a*b^4*x^8 - 168*a^2*b^3*x^5 + 368*a^3*b^2*x^2))*sqrt(sqrt(3)
a/b^4) - 24(3*a*b^2*x^8 + 12*a^2*b*x^5 - 96*a^3*x^2 + 2*sqrt(3)*(a*b^2*x^
8 - 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(b*x^3 - a) - (1/72)^(1/6)*(3*b^4*x^12 +
 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 - 2208*a^3*b*x^3 + 384*a^4 + 3*sqrt(-3)*(b
^4*x^12 + 4*a*b^3*x^9 + 360*a^2*b^2*x^6 - 736*a^3*b*x^3 + 128*a^4) - sqrt(3
)*(b^4*x^12 - 124*a*b^3*x^9 + 744*a^2*b^2*x^6 - 1120*a^3*b*x^3 + 256*a^4 +
sqrt(-3)*(b^4*x^12 - 124*a*b^3*x^9 + 744*a^2*b^2*x^6 - 1120*a^3*b*x^3 + 256
a^4)))(sqrt(3)*a/b^4)^(1/6))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2*x^6 +
 320*a^3*b*x^3 + 64*a^4)) - (1/72)^(1/6)*(sqrt(-3)*b - b)*(sqrt(3)*a/b^4)^(
1/6)*log(-(12*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 - 48*a^3*b^2 + sqrt(
3)*(b^5*x^9 + 30*a*b^4*x^6 - 144*a^2*b^3*x^3 + 32*a^3*b^2 - sqrt(-3)*(b^5*x
^9 + 30*a*b^4*x^6 - 144*a^2*b^3*x^3 + 32*a^3*b^2)) - 3*sqrt(-3)*(b^5*x^9 +
96*a^2*b^3*x^3 - 16*a^3*b^2))*sqrt(b*x^3 - a)*(sqrt(3)*a/b^4)^(2/3) + 72*(1
/72)^(5/6)*(7*b^6*x^10 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x + s
qrt(-3)*(7*b^6*x^10 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x) + 3*s
qrt(3)*(b^6*x^10 + 12*a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x + sqrt(-3)*
(b^6*x^10 + 12*a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x)))*(sqrt(3)*a/b^4)
^(5/6) + 6*(1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 - 160*a^3*b*x + sqrt(-3)
*(b^4*x^10 + 240*a^2*b^2*x^4 - 160*a^3*b*x) + 24*sqrt(3)*(a*b^3*x^7 - 5*a^2
*b^2*x^4 + 4*a^3*b*x + sqrt(-3)*(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x)))*s
qrt(b*x^3 - a)*(sqrt(3)*a/b^4)^(1/3) - 4*sqrt(1/2)*(3*b^5*x^11 + 18*a*b^4*x
^8 + 360*a^2*b^3*x^5 - 624*a^3*b^2*x^2 + sqrt(3)*(b^5*x^11 + 42*a*b^4*x^8 -
 168*a^2*b^3*x^5 + 368*a^3*b^2*x^2))*sqrt(sqrt(3)*a/b^4) - 24*(3*a*b^2*x^8
+ 12*a^2*b*x^5 - 96*a^3*x^2 + 2*sqrt(3)*(a*b^2*x^8 - 2*a^2*b*x^5 + 28*a^3*x
^2))*sqrt(b*x^3 - a) + (1/72)^(1/6)*(3*b^4*x^12 + 12*a*b^3*x^9 + 1080*a^2*b
^2*x^6 - 2208*a^3*b*x^3 + 384*a^4 - 3*sqrt(-3)*(b^4*x^12 + 4*a*b^3*x^9 + 36
0*a^2*b^2*x^6 - 736*a^3*b*x^3 + 128*a^4) - sqrt(3)*(b^4*x^12 - 124*a*b^3*x^
9 + 744*a^2*b^2*x^6 - 1120*a^3*b*x^3 + 256*a^4 - sqrt(-3)*(b^4*x^12 - 124*a
*b^3*x^9 + 744*a^2*b^2*x^6 - 1120*a^3*b*x^3 + 256*a^4)))*(sqrt(3)*a/b^4)^(1
/6))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2*x^6 + 320*a^3*b*x^3 + 64*a^4))
+ (1/72)^(1/6)*(sqrt(-3)*b - b)*(sqrt(3)*a/b^4)^(1/6)*log(-(12*(1/9)^(2/3)*
(3*b^5*x^9 + 288*a^2*b^3*x^3 - 48*a^3*b^2 + sqrt(3)*(b^5*x^9 + 30*a*b^4*x^6
 - 144*a^2*b^3*x^3 + 32*a^3*b^2 - sqrt(-3)*(b^5*x^9 + 30*a*b^4*x^6 - 144*a^
2*b^3*x^3 + 32*a^3*b^2)) - 3*sqrt(-3)*(b^5*x^9 + 96*a^2*b^3*x^3 - 16*a^3*b^
2))*sqrt(b*x^3 - a)*(sqrt(3)*a/b^4)^(2/3) - 72*(1/72)^(5/6)*(7*b^6*x^10 - 1
2*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x + sqrt(-3)*(7*b^6*x^10 - 12*a
*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x) + 3*sqrt(3)*(b^6*x^10 + 12*a*b^
5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x + sqrt(-3)*(b^6*x^10 + 12*a*b^5*x^7 -
 72*a^2*b^4*x^4 + 32*a^3*b^3*x)))*(sqrt(3)*a/b^4)^(5/6) + 6*(1/9)^(1/3)*(b^
4*x^10 + 240*a^2*b^2*x^4 - 160*a^3*b*x + sqrt(-3)*(b^4*x^10 + 240*a^2*b^2*x
^4 - 160*a^3*b*x) + 24*sqrt(3)*(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x + sqr
t(-3)*(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x)))*sqrt(b*x^3 - a)*(sqrt(3)*a/
b^4)^(1/3) + 4*sqrt(1/2)*(3*b^5*x^11 + 18*a*b^4*x^8 + 360*a^2*b^3*x^5 - 624
*a^3*b^2*x^2 + sqrt(3)*(b^5*x^11 + 42*a*b^4*x^8 - 168*a^2*b^3*x^5 + 368*a^3

*b^2*x^2))*sqrt(sqrt(3)*a/b^4) - 24*(3*a*b^2*x^8 + 12*a^2*b*x^5 - 96*a^3*x^2 + 2*sqrt(3)*(a*b^2*x^8 - 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(b*x^3 - a) - (1/72)^(1/6)*(3*b^4*x^12 + 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 - 2208*a^3*b*x^3 + 384*a^4 - 3*sqrt(-3)*(b^4*x^12 + 4*a*b^3*x^9 + 360*a^2*b^2*x^6 - 736*a^3*b*x^3 + 128*a^4) - sqrt(3)*(b^4*x^12 - 124*a*b^3*x^9 + 744*a^2*b^2*x^6 - 1120*a^3*b*x^3 + 256*a^4 - sqrt(-3)*(b^4*x^12 - 124*a*b^3*x^9 + 744*a^2*b^2*x^6 - 1120*a^3*b*x^3 + 256*a^4)))*(sqrt(3)*a/b^4)^(1/6))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2*x^6 + 320*a^3*b*x^3 + 64*a^4)) - 2*(1/72)^(1/6)*b*(sqrt(3)*a/b^4)^(1/6)*log((12*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 - 48*a^3*b^2 + sqrt(3)*(b^5*x^9 + 30*a*b^4*x^6 - 144*a^2*b^3*x^3 + 32*a^3*b^2))*sqrt(b*x^3 - a)*(sqrt(3)*a/b^4)^(2/3) + 72*(1/72)^(5/6)*(7*b^6*x^10 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x + 3*sqrt(3)*(b^6*x^10 + 12*a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x))*(sqrt(3)*a/b^4)^(5/6) + 6*(1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 - 160*a^3*b*x + 24*sqrt(3)*(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x))*sqrt(b*x^3 - a)*(sqrt(3)*a/b^4)^(1/3) + 2*sqrt(1/2)*(3*b^5*x^11 + 18*a*b^4*x^8 + 360*a^2*b^3*x^5 - 624*a^3*b^2*x^2 + sqrt(3)*(b^5*x^11 + 42*a*b^4*x^8 - 168*a^2*b^3*x^5 + 368*a^3*b^2*x^2))*sqrt(sqrt(3)*a/b^4) + 12*(3*a*b^2*x^8 + 12*a^2*b*x^5 - 96*a^3*x^2 + 2*sqrt(3)*(a*b^2*x^8 - 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(b*x^3 - a) + (1/72)^(1/6)*(3*b^4*x^12 + 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 - 2208*a^3*b*x^3 + 384*a^4 - sqrt(3)*(b^4*x^12 - 124*a*b^3*x^9 + 744*a^2*b^2*x^6 - 1120*a^3*b*x^3 + 256*a^4))*(sqrt(3)*a/b^4)^(1/6))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2*x^6 + 320*a^3*b*x^3 + 64*a^4)) + 2*(1/72)^(1/6)*b*(sqrt(3)*a/b^4)^(1/6)*log((12*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 - 48*a^3*b^2 + sqrt(3)*(b^5*x^9 + 30*a*b^4*x^6 - 144*a^2*b^3*x^3 + 32*a^3*b^2))*sqrt(b*x^3 - a)*(sqrt(3)*a/b^4)^(2/3) - 72*(1/72)^(5/6)*(7*b^6*x^10 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x + 3*sqrt(3)*(b^6*x^10 + 12*a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x))*(sqrt(3)*a/b^4)^(5/6) + 6*(1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 - 160*a^3*b*x + 24*sqrt(3)*(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x))*sqrt(b*x^3 - a)*(sqrt(3)*a/b^4)^(1/3) - 2*sqrt(1/2)*(3*b^5*x^11 + 18*a*b^4*x^8 + 360*a^2*b^3*x^5 - 624*a^3*b^2*x^2 + sqrt(3)*(b^5*x^11 + 42*a*b^4*x^8 - 168*a^2*b^3*x^5 + 368*a^3*b^2*x^2))*sqrt(sqrt(3)*a/b^4) + 12*(3*a*b^2*x^8 + 12*a^2*b*x^5 - 96*a^3*x^2 + 2*sqrt(3)*(a*b^2*x^8 - 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(b*x^3 - a) - (1/72)^(1/6)*(3*b^4*x^12 + 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 - 2208*a^3*b*x^3 + 384*a^4 - sqrt(3)*(b^4*x^12 - 124*a*b^3*x^9 + 744*a^2*b^2*x^6 - 1120*a^3*b*x^3 + 256*a^4))*(sqrt(3)*a/b^4)^(1/6))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2*x^6 + 320*a^3*b*x^3 + 64*a^4)) - 16*sqrt(b)*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b

Sympy [F]

$$\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{x\sqrt{-a+bx^3}}{-6\sqrt{3}a-10a+bx^3}\,dx$$

```
[In] integrate(x*(b*x**3-a)**(1/2)/(b*x**3-2*a*(5+3*3**(1/2))),x)
[Out] Integral(x*sqrt(-a + b*x**3)/(-6*sqrt(3)*a - 10*a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{\sqrt{bx^3-a}x}{bx^3-2a\left(3\sqrt{3}+5\right)}\,dx$$

```
[In] integrate(x*(b*x^3-a)^(1/2)/(b*x^3-2*a*(5+3*3^(1/2))),x, algorithm="maxima"
)
[Out] integrate(sqrt(b*x^3 - a)*x/(b*x^3 - 2*a*(3*sqrt(3) + 5)), x)
```

Giac [F]

$$\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{\sqrt{bx^3-a}x}{bx^3-2a\left(3\sqrt{3}+5\right)}\,dx$$

```
[In] integrate(x*(b*x^3-a)^(1/2)/(b*x^3-2*a*(5+3*3^(1/2))),x, algorithm="giac")
[Out] integrate(sqrt(b*x^3 - a)*x/(b*x^3 - 2*a*(3*sqrt(3) + 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{x\,\sqrt{b\,x^3-a}}{b\,x^3-2\,a\,\left(3\,\sqrt{3}+5\right)}\,dx$$

```
[In] int((x*(b*x^3 - a)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) + 5)),x)
[Out] int((x*(b*x^3 - a)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) + 5)), x)
```

3.345 $\int \frac{x\sqrt{-a-bx^3}}{-2(5+3\sqrt{3})a-bx^3}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 768

$$\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx$$
$$= -\frac{2\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\sqrt[4]{3}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{\sqrt{2}b^{2/3}}$$
$$+\frac{\sqrt[4]{3}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$+\frac{3^{3/4}\sqrt[6]{a}\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}-\frac{\sqrt[6]{a}\text{arctanh}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}$$
$$+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$
$$-\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

```
[Out] 1/4*3^(1/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1-3^(1/
2))*2^(1/2)/(-b*x^3-a)^(1/2))/b^(2/3)*2^(1/2)+1/2*3^(1/4)*a^(1/6)*arctan(1/
2*3^(1/4)*a^(1/6)*(-2*b^(1/3)*x+a^(1/3)*(1+3^(1/2)))*2^(1/2)/(-b*x^3-a)^(1/
2))/b^(2/3)*2^(1/2)+1/4*3^(3/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3
)+b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(-b*x^3-a)^(1/2))/b^(2/3)*2^(1/2)-1/6*a^(1
/6)*arctanh(1/6*(1-3^(1/2))*(-b*x^3-a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*3^(3/
4)/b^(2/3)*2^(1/2)-2*(-b*x^3-a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)
))-2/3*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/b^(
2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3
```

```
^(1/2)))^2)^(1/2)+3^(1/4)*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/
3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*
(1/2*6^(1/2)+1/2*2^(1/2))/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/
3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 768, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {495, 310, 225, 1893, 501}

$$\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = -\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{\sqrt[4]{3}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{\sqrt{2}b^{2/3}}+\frac{\sqrt[4]{3}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}+\frac{3^{3/4}\sqrt[6]{a}\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}-\frac{\sqrt[6]{a}\text{arctanh}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}-\frac{2\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

```
[In] Int[(x*Sqrt[-a - b*x^3])/(-2*(5 + 3*Sqrt[3])*a - b*x^3),x]
```

```
[Out] (-2*Sqrt[-a - b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (3^(1
/4)*a^(1/6)*ArcTan[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/
(Sqrt[2]*Sqrt[-a - b*x^3])])/(Sqrt[2]*b^(2/3)) + (3^(1/4)*a^(1/6)*ArcTan[(3
^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^
3])])/(2*Sqrt[2]*b^(2/3)) + (3^(3/4)*a^(1/6)*ArcTanh[(3^(1/4)*(1 + Sqrt[3])
*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(2*Sqrt[2]*b^(
2/3)) - (a^(1/6)*ArcTanh[((1 - Sqrt[3])*Sqrt[-a - b*x^3])/(Sqrt[2]*3^(3/4)*
Sqrt[a])])/(Sqrt[2]*3^(1/4)*b^(2/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(
a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 -
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)
*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2)]*Sqrt[-a - b*x^3]) - (2*Sqrt[2]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(
1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1
/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 501

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
```

```
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \left(3\left(3+2\sqrt{3}\right)a\right)\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx+\int \frac{x}{\sqrt{-a-bx^3}}\,dx$$

$$= \frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{\sqrt{2}b^{2/3}} + \frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}b^{2/3}} + \frac{3^{3/4}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}b^{2/3}} - \frac{\sqrt[6]{a}\tanh^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}} + \frac{\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx}{\sqrt[3]{b}} - \frac{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)\int \frac{1}{\sqrt{-a-bx^3}}\,dx}{\sqrt[3]{b}}$$

$$= -\frac{2\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{\sqrt{2}b^{2/3}} + \frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}b^{2/3}} + \frac{3^{3/4}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}b^{2/3}} - \frac{\sqrt[6]{a}\tanh^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}} - \frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.12

$$\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = -\frac{x^2\sqrt{-a-bx^3}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bx^3}{10a+6\sqrt{3}a}\right)}{4\left(5+3\sqrt{3}\right)a\sqrt{\frac{a+bx^3}{a}}}$$

```
[In] Integrate[(x*Sqrt[-a - b*x^3])/(-2*(5 + 3*Sqrt[3])*a - b*x^3),x]
```

```
[Out] -1/4*(x^2*Sqrt[-a - b*x^3]*AppellF1[2/3, -1/2, 1, 5/3, -((b*x^3)/a), -((b*x
^3)/(10*a + 6*Sqrt[3]*a))])/((5 + 3*Sqrt[3])*a*Sqrt[(a + b*x^3)/a])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.09 (sec) , antiderivative size = 983, normalized size of antiderivative = 1.28

method	result	size
elliptic	Expression too large to display	983
default	Expression too large to display	1001

```
[In] int(x*(-b*x^3-a)^(1/2)/(-b*x^3-2*a*(5+3*3^(1/2))),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
/(-b*x^3-a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*E
llipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+1/9*I/b^3*2^(1/2)*sum(1
/_alpha*(3+2*3^(1/2))*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^
(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*
(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2
)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*(
-3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)+3*I*(-a*b^2)^(2
/3)*3^(1/2)-2*3^(1/2)*(-a*b^2)^(1/3)*_alpha*b+6*I*(-a*b^2)^(1/3)*_alpha*b-6
*b^2*_alpha^2-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(-a*b^2)^(2/3)+3*(-a*b^2)^(1/3)*
_alpha*b+3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/6/b*(2
*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b-I*3^(1/2)*(-a*b^2)^(2/3)*_alpha-4*I*(-
a*b^2)^(1/3)*_alpha^2*b+2*3^(1/2)*(-a*b^2)^(2/3)*_alpha+I*3^(1/2)*a*b+2*I*(
-a*b^2)^(2/3)*_alpha+2*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-2*I*a*b-3*a*b)/a
,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3+6*3^(1/2)*a+10*a))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.28 (sec) , antiderivative size = 4981, normalized size of antiderivative = 6.49

$$\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(-b*x^3-a)^(1/2)/(-b*x^3-2*a*(5+3*3^(1/2))),x, algorithm="frica
s")
```

```
[Out] -1/8*((1/72)^(1/6)*(sqrt(-3)*b + b)*(sqrt(3)*a/b^4)^(1/6)*log((12*(1/9)^(2/
3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 + 48*a^3*b^2 + sqrt(3)*(b^5*x^9 - 30*a*b^4*
x^6 - 144*a^2*b^3*x^3 - 32*a^3*b^2 + sqrt(-3)*(b^5*x^9 - 30*a*b^4*x^6 - 144
*a^2*b^3*x^3 - 32*a^3*b^2)) + 3*sqrt(-3)*(b^5*x^9 + 96*a^2*b^3*x^3 + 16*a^3
*b^2))*sqrt(-b*x^3 - a)*(sqrt(3)*a/b^4)^(2/3) + 72*(1/72)^(5/6)*(7*b^6*x^10
 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x - sqrt(-3)*(7*b^6*x^10 +
12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x) + 3*sqrt(3)*(b^6*x^10 - 12*
a*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x - sqrt(-3)*(b^6*x^10 - 12*a*b^5*x
^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x)))*(sqrt(3)*a/b^4)^(5/6) - 6*(1/9)^(1/3)
*(b^4*x^10 + 240*a^2*b^2*x^4 + 160*a^3*b*x - sqrt(-3)*(b^4*x^10 + 240*a^2*b
^2*x^4 + 160*a^3*b*x) - 24*sqrt(3)*(a*b^3*x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x -
 sqrt(-3)*(a*b^3*x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x)))*sqrt(-b*x^3 - a)*(sqrt(
3)*a/b^4)^(1/3) + 4*sqrt(1/2)*(3*b^5*x^11 - 18*a*b^4*x^8 + 360*a^2*b^3*x^5
+ 624*a^3*b^2*x^2 + sqrt(3)*(b^5*x^11 - 42*a*b^4*x^8 - 168*a^2*b^3*x^5 - 36
8*a^3*b^2*x^2))*sqrt(sqrt(3)*a/b^4) + 24*(3*a*b^2*x^8 - 12*a^2*b*x^5 - 96*a
^3*x^2 + 2*sqrt(3)*(a*b^2*x^8 + 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(-b*x^3 - a)
 + (1/72)^(1/6)*(3*b^4*x^12 - 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 + 2208*a^3*b*
x^3 + 384*a^4 - sqrt(3)*(b^4*x^12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*
a^3*b*x^3 + 256*a^4 + sqrt(-3)*(b^4*x^12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6
+ 1120*a^3*b*x^3 + 256*a^4)) + 3*sqrt(-3)*(b^4*x^12 - 4*a*b^3*x^9 + 360*a^2
*b^2*x^6 + 736*a^3*b*x^3 + 128*a^4))*(sqrt(3)*a/b^4)^(1/6))/(b^4*x^12 + 40*
a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*x^3 + 64*a^4)) - (1/72)^(1/6)*(sqrt
(-3)*b + b)*(sqrt(3)*a/b^4)^(1/6)*log((12*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*
b^3*x^3 + 48*a^3*b^2 + sqrt(3)*(b^5*x^9 - 30*a*b^4*x^6 - 144*a^2*b^3*x^3 -
32*a^3*b^2 + sqrt(-3)*(b^5*x^9 - 30*a*b^4*x^6 - 144*a^2*b^3*x^3 - 32*a^3*b^
2)) + 3*sqrt(-3)*(b^5*x^9 + 96*a^2*b^3*x^3 + 16*a^3*b^2))*sqrt(-b*x^3 - a)*
(sqrt(3)*a/b^4)^(2/3) - 72*(1/72)^(5/6)*(7*b^6*x^10 + 12*a*b^5*x^7 + 408*a^
2*b^4*x^4 + 160*a^3*b^3*x - sqrt(-3)*(7*b^6*x^10 + 12*a*b^5*x^7 + 408*a^2*b
^4*x^4 + 160*a^3*b^3*x) + 3*sqrt(3)*(b^6*x^10 - 12*a*b^5*x^7 - 72*a^2*b^4*x
^4 - 32*a^3*b^3*x - sqrt(-3)*(b^6*x^10 - 12*a*b^5*x^7 - 72*a^2*b^4*x^4 - 32
*a^3*b^3*x)))*(sqrt(3)*a/b^4)^(5/6) - 6*(1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2
*x^4 + 160*a^3*b*x - sqrt(-3)*(b^4*x^10 + 240*a^2*b^2*x^4 + 160*a^3*b*x) -
24*sqrt(3)*(a*b^3*x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x - sqrt(-3)*(a*b^3*x^7 + 5
*a^2*b^2*x^4 + 4*a^3*b*x)))*sqrt(-b*x^3 - a)*(sqrt(3)*a/b^4)^(1/3) - 4*sqrt
```

(1/2)*(3*b^5*x^11 - 18*a*b^4*x^8 + 360*a^2*b^3*x^5 + 624*a^3*b^2*x^2 + sqrt(3)*(b^5*x^11 - 42*a*b^4*x^8 - 168*a^2*b^3*x^5 - 368*a^3*b^2*x^2))*sqrt(sqrt(3)*a/b^4) + 24*(3*a*b^2*x^8 - 12*a^2*b*x^5 - 96*a^3*x^2 + 2*sqrt(3)*(a*b^2*x^8 + 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(-b*x^3 - a) - (1/72)^(1/6)*(3*b^4*x^12 - 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 + 2208*a^3*b*x^3 + 384*a^4 - sqrt(3)*(b^4*x^12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*b*x^3 + 256*a^4 + sqrt(-3)*(b^4*x^12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*b*x^3 + 256*a^4)) + 3*sqrt(-3)*(b^4*x^12 - 4*a*b^3*x^9 + 360*a^2*b^2*x^6 + 736*a^3*b*x^3 + 128*a^4))*(sqrt(3)*a/b^4)^(1/6))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*x^3 + 64*a^4)) - (1/72)^(1/6)*(sqrt(-3)*b - b)*(sqrt(3)*a/b^4)^(1/6)*log((12*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 + 48*a^3*b^2 + sqrt(3)*(b^5*x^9 - 30*a*b^4*x^6 - 144*a^2*b^3*x^3 - 32*a^3*b^2 - sqrt(-3)*(b^5*x^9 - 30*a*b^4*x^6 - 144*a^2*b^3*x^3 - 32*a^3*b^2)) - 3*sqrt(-3)*(b^5*x^9 + 96*a^2*b^3*x^3 + 16*a^3*b^2))*sqrt(-b*x^3 - a)*(sqrt(3)*a/b^4)^(2/3) + 72*(1/72)^(5/6)*(7*b^6*x^10 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x + sqrt(-3)*(7*b^6*x^10 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x) + 3*sqrt(3)*(b^6*x^10 - 12*a*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x + sqrt(-3)*(b^6*x^10 - 12*a*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x)))*(sqrt(3)*a/b^4)^(5/6) - 6*(1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 + 160*a^3*b*x + sqrt(-3)*(b^4*x^10 + 240*a^2*b^2*x^4 + 160*a^3*b*x) - 24*sqrt(3)*(a*b^3*x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x + sqrt(-3)*(a*b^3*x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x)))*sqrt(-b*x^3 - a)*(sqrt(3)*a/b^4)^(1/3) + 4*sqrt(1/2)*(3*b^5*x^11 - 18*a*b^4*x^8 + 360*a^2*b^3*x^5 + 624*a^3*b^2*x^2 + sqrt(3)*(b^5*x^11 - 42*a*b^4*x^8 - 168*a^2*b^3*x^5 - 368*a^3*b^2*x^2))*sqrt(sqrt(3)*a/b^4) + 24*(3*a*b^2*x^8 - 12*a^2*b*x^5 - 96*a^3*x^2 + 2*sqrt(3)*(a*b^2*x^8 + 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(-b*x^3 - a) + (1/72)^(1/6)*(3*b^4*x^12 - 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 + 2208*a^3*b*x^3 + 384*a^4 - sqrt(3)*(b^4*x^12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*b*x^3 + 256*a^4 - sqrt(-3)*(b^4*x^12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*b*x^3 + 256*a^4)) - 3*sqrt(-3)*(b^4*x^12 - 4*a*b^3*x^9 + 360*a^2*b^2*x^6 + 736*a^3*b*x^3 + 128*a^4))*(sqrt(3)*a/b^4)^(1/6))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*x^3 + 64*a^4)) + (1/72)^(1/6)*(sqrt(-3)*b - b)*(sqrt(3)*a/b^4)^(1/6)*log((12*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 + 48*a^3*b^2 + sqrt(3)*(b^5*x^9 - 30*a*b^4*x^6 - 144*a^2*b^3*x^3 - 32*a^3*b^2 - sqrt(-3)*(b^5*x^9 - 30*a*b^4*x^6 - 144*a^2*b^3*x^3 - 32*a^3*b^2)) - 3*sqrt(-3)*(b^5*x^9 + 96*a^2*b^3*x^3 + 16*a^3*b^2))*sqrt(-b*x^3 - a)*(sqrt(3)*a/b^4)^(2/3) - 72*(1/72)^(5/6)*(7*b^6*x^10 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x + sqrt(-3)*(7*b^6*x^10 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x) + 3*sqrt(3)*(b^6*x^10 - 12*a*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x + sqrt(-3)*(b^6*x^10 - 12*a*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x)))*(sqrt(3)*a/b^4)^(5/6) - 6*(1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 + 160*a^3*b*x + sqrt(-3)*(b^4*x^10 + 240*a^2*b^2*x^4 + 160*a^3*b*x) - 24*sqrt(3)*(a*b^3*x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x + sqrt(-3)*(a*b^3*x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x)))*sqrt(-b*x^3 - a)*(sqrt(3)*a/b^4)^(1/3) - 4*sqrt(1/2)*(3*b^5*x^11 - 18*a*b^4*x^8 + 360*a^2*b^3*x^5 + 624*a^3*b^2*x^2 + sqrt(3)*(b^5*x^11 - 42*a*b^4*x^8 - 168*a^2*b^3*x^5 -

368*a^3*b^2*x^2))*sqrt(sqrt(3)*a/b^4) + 24*(3*a*b^2*x^8 - 12*a^2*b*x^5 - 96*a^3*x^2 + 2*sqrt(3)*(a*b^2*x^8 + 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(-b*x^3 - a) - (1/72)^(1/6)*(3*b^4*x^12 - 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 + 2208*a^3*b*x^3 + 384*a^4 - sqrt(3)*(b^4*x^12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*b*x^3 + 256*a^4 - sqrt(-3)*(b^4*x^12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*b*x^3 + 256*a^4)) - 3*sqrt(-3)*(b^4*x^12 - 4*a*b^3*x^9 + 360*a^2*b^2*x^6 + 736*a^3*b*x^3 + 128*a^4))*(sqrt(3)*a/b^4)^(1/6))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*x^3 + 64*a^4)) - 2*(1/72)^(1/6)*b*(sqrt(3)*a/b^4)^(1/6)*log(-(12*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 + 48*a^3*b^2 + sqrt(3)*(b^5*x^9 - 30*a*b^4*x^6 - 144*a^2*b^3*x^3 - 32*a^3*b^2))*sqrt(-b*x^3 - a)*(sqrt(3)*a/b^4)^(2/3) + 72*(1/72)^(5/6)*(7*b^6*x^10 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x + 3*sqrt(3)*(b^6*x^10 - 12*a*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x))*(sqrt(3)*a/b^4)^(5/6) - 6*(1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 + 160*a^3*b*x - 24*sqrt(3)*(a*b^3*x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x))*sqrt(-b*x^3 - a)*(sqrt(3)*a/b^4)^(1/3) - 2*sqrt(1/2)*(3*b^5*x^11 - 18*a*b^4*x^8 + 360*a^2*b^3*x^5 + 624*a^3*b^2*x^2 + sqrt(3)*(b^5*x^11 - 42*a*b^4*x^8 - 168*a^2*b^3*x^5 - 368*a^3*b^2*x^2))*sqrt(sqrt(3)*a/b^4) - 12*(3*a*b^2*x^8 - 12*a^2*b*x^5 - 96*a^3*x^2 + 2*sqrt(3)*(a*b^2*x^8 + 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(-b*x^3 - a) + (1/72)^(1/6)*(3*b^4*x^12 - 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 + 2208*a^3*b*x^3 + 384*a^4 - sqrt(3)*(b^4*x^12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*b*x^3 + 256*a^4))*(sqrt(3)*a/b^4)^(1/6))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*x^3 + 64*a^4)) + 2*(1/72)^(1/6)*b*(sqrt(3)*a/b^4)^(1/6)*log(-(12*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 + 48*a^3*b^2 + sqrt(3)*(b^5*x^9 - 30*a*b^4*x^6 - 144*a^2*b^3*x^3 - 32*a^3*b^2))*sqrt(-b*x^3 - a)*(sqrt(3)*a/b^4)^(2/3) - 72*(1/72)^(5/6)*(7*b^6*x^10 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x + 3*sqrt(3)*(b^6*x^10 - 12*a*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x))*(sqrt(3)*a/b^4)^(5/6) - 6*(1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 + 160*a^3*b*x - 24*sqrt(3)*(a*b^3*x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x))*sqrt(-b*x^3 - a)*(sqrt(3)*a/b^4)^(1/3) + 2*sqrt(1/2)*(3*b^5*x^11 - 18*a*b^4*x^8 + 360*a^2*b^3*x^5 + 624*a^3*b^2*x^2 + sqrt(3)*(b^5*x^11 - 42*a*b^4*x^8 - 168*a^2*b^3*x^5 - 368*a^3*b^2*x^2))*sqrt(sqrt(3)*a/b^4) - 12*(3*a*b^2*x^8 - 12*a^2*b*x^5 - 96*a^3*x^2 + 2*sqrt(3)*(a*b^2*x^8 + 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(-b*x^3 - a) - (1/72)^(1/6)*(3*b^4*x^12 - 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 + 2208*a^3*b*x^3 + 384*a^4 - sqrt(3)*(b^4*x^12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*b*x^3 + 256*a^4))*(sqrt(3)*a/b^4)^(1/6))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*x^3 + 64*a^4)) - 16*sqrt(-b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b

Sympy [F]

$$\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = -\int \frac{x\sqrt{-a-bx^3}}{10a+6\sqrt{3}a+bx^3}\,dx$$

```
[In] integrate(x*(-b*x**3-a)**(1/2)/(-b*x**3-2*a*(5+3*3**(1/2))),x)
[Out] -Integral(x*sqrt(-a - b*x**3)/(10*a + 6*sqrt(3)*a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = \int -\frac{\sqrt{-bx^3-a}\,x}{bx^3+2\,a\left(3\,\sqrt{3}+5\right)}\,dx$$

```
[In] integrate(x*(-b*x^3-a)^(1/2)/(-b*x^3-2*a*(5+3*3^(1/2))),x, algorithm="maxim
a")
[Out] -integrate(sqrt(-b*x^3 - a)*x/(b*x^3 + 2*a*(3*sqrt(3) + 5)), x)
```

Giac [F]

$$\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = \int -\frac{\sqrt{-bx^3-a}\,x}{bx^3+2\,a\left(3\,\sqrt{3}+5\right)}\,dx$$

```
[In] integrate(x*(-b*x^3-a)^(1/2)/(-b*x^3-2*a*(5+3*3^(1/2))),x, algorithm="giac"
)
[Out] integrate(-sqrt(-b*x^3 - a)*x/(b*x^3 + 2*a*(3*sqrt(3) + 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = \int -\frac{x\,\sqrt{-b\,x^3-a}}{b\,x^3+2\,a\,\left(3\,\sqrt{3}+5\right)}\,dx$$

```
[In] int(-(x*(- a - b*x^3)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) + 5)),x)
[Out] int(-(x*(- a - b*x^3)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) + 5)), x)
```

3.346 $\int \frac{x\sqrt{a+bx^3}}{2(5-3\sqrt{3})a+bx^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 738

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx$$

$$= \frac{2\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{\sqrt{2}b^{2/3}}$$
$$- \frac{\sqrt[4]{3}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$+ \frac{3^{3/4}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}b^{2/3}} + \frac{\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}$$
$$- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+ \frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/2*3^(1/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(-2*b^(1/3)*x+a^(1/3)*(1-3^
(1/2)))*2^(1/2)/(b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)-1/4*3^(1/4)*a^(1/6)*arctan
(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(b*x^3+a)^(1/2
))/b^(2/3)*2^(1/2)+1/4*3^(3/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3)
+b^(1/3)*x)*(1-3^(1/2))*2^(1/2)/(b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)+1/6*a^(1/6
)*arctanh(1/6*(1+3^(1/2))*(b*x^3+a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*3^(3/4)/
b^(2/3)*2^(1/2)+2*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+2
/3*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b
^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)
/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
```

```
))^2)^(1/2)-3^(1/4)*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3
)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-
1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b
^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 738, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {495, 309, 224, 1891, 500}

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx$$

$$= \frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{\sqrt[4]{3}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{\sqrt{2}b^{2/3}}$$
$$-\frac{\sqrt[4]{3}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$+\frac{3^{3/4}\sqrt[6]{a}\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$+\frac{\sqrt[6]{a}\text{arctanh}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}+\frac{2\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

```
[In] Int[(x*Sqrt[a + b*x^3])/(2*(5 - 3*Sqrt[3])*a + b*x^3),x]
```

```
[Out] (2*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4
)*a^(1/6)*ArcTan[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/(S
qrt[2]*Sqrt[a + b*x^3])])/(Sqrt[2]*b^(2/3)) - (3^(1/4)*a^(1/6)*ArcTan[(3^(1
/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])]
)/(2*Sqrt[2]*b^(2/3)) + (3^(3/4)*a^(1/6)*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(
1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*b^(2/3))
 + (a^(1/6)*ArcTanh[((1 + Sqrt[3])*Sqrt[a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a
])])/(Sqrt[2]*3^(1/4)*b^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3]) + (2*Sqrt[2]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3
)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 500

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
```

```
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\left(\left(3\left(3-2\sqrt{3}\right)a\right)\int\frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx\right)+\int\frac{x}{\sqrt{a+bx^3}}\,dx$$

$$= -\frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{\sqrt{2}b^{2/3}} - \frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}b^{2/3}} + \frac{3^{3/4}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}b^{2/3}} + \frac{\sqrt[6]{a}\tanh^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}} + \frac{\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{\sqrt[3]{b}} - \frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)\int\frac{1}{\sqrt{a+bx^3}}\,dx}{\sqrt[3]{b}}$$

$$= \frac{2\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{\sqrt{2}b^{2/3}}$$
$$- \frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$+ \frac{3^{3/4}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}b^{2/3}} + \frac{\sqrt[6]{a}\tanh^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}$$
$$- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+ \frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.11

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx = \frac{x^2\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20-12\sqrt{3}\right)\sqrt{a+bx^3}}$$

```
[In] Integrate[(x*Sqrt[a + b*x^3])/(2*(5 - 3*Sqrt[3])*a + b*x^3),x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, -1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3
)/(10*a - 6*Sqrt[3]*a))])/((20 - 12*Sqrt[3])*Sqrt[a + b*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.29 (sec) , antiderivative size = 977, normalized size of antiderivative = 1.32

method	result	size
elliptic	Expression too large to display	977
default	Expression too large to display	995

```
[In] int(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*El
lipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+1/9*I/b^3*2^(1/2)*sum(1/
_alpha*(2*3^(1/2)-3)*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(
1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*(
-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)
^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(3*
I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)-3*I*(-a*b^2)^(2/3)
*3^(1/2)+6*I*(-a*b^2)^(1/3)*_alpha*b-2*3^(1/2)*(-a*b^2)^(1/3)*_alpha*b+6*b^
2*_alpha^2-6*I*(-a*b^2)^(2/3)-2*3^(1/2)*(-a*b^2)^(2/3)-3*(-a*b^2)^(1/3)*_al
pha*b-3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/6/b*(2*I*
3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b-I*3^(1/2)*(-a*b^2)^(2/3)*_alpha+4*I*(-a*b
^2)^(1/3)*_alpha^2*b-2*I*(-a*b^2)^(2/3)*_alpha-2*3^(1/2)*(-a*b^2)^(2/3)*_al
pha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha+2*I*a*b-2*3^(1/2)*a*b-3*a*b)/a,(I
*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)),_alpha=RootOf(b*_Z^3-6*3^(1/2)*a+10*a))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.39 (sec) , antiderivative size = 4931, normalized size of antiderivative = 6.68

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="fricas"
)
```

```
[Out] 1/8*((1/72)^(1/6)*(sqrt(-3)*b + b)*(sqrt(3)*a/b^4)^(1/6)*log((12*(1/9)^(2/3
)*(3*b^5*x^9 + 288*a^2*b^3*x^3 + 48*a^3*b^2 - sqrt(3)*(b^5*x^9 - 30*a*b^4*x
^6 - 144*a^2*b^3*x^3 - 32*a^3*b^2 + sqrt(-3)*(b^5*x^9 - 30*a*b^4*x^6 - 144*
a^2*b^3*x^3 - 32*a^3*b^2)) + 3*sqrt(-3)*(b^5*x^9 + 96*a^2*b^3*x^3 + 16*a^3*
b^2))*sqrt(b*x^3 + a)*(sqrt(3)*a/b^4)^(2/3) + 72*(1/72)^(5/6)*(7*b^6*x^10 +
 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x - sqrt(-3)*(7*b^6*x^10 + 12
*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x) - 3*sqrt(3)*(b^6*x^10 - 12*a*
b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x - sqrt(-3)*(b^6*x^10 - 12*a*b^5*x^7
 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x)))*(sqrt(3)*a/b^4)^(5/6) + 6*(1/9)^(1/3)*(
b^4*x^10 + 240*a^2*b^2*x^4 + 160*a^3*b*x - sqrt(-3)*(b^4*x^10 + 240*a^2*b^2
*x^4 + 160*a^3*b*x) + 24*sqrt(3)*(a*b^3*x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x - s
qrt(-3)*(a*b^3*x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x)))*sqrt(b*x^3 + a)*(sqrt(3)*
a/b^4)^(1/3) - 4*sqrt(1/2)*(3*b^5*x^11 - 18*a*b^4*x^8 + 360*a^2*b^3*x^5 + 6
24*a^3*b^2*x^2 - sqrt(3)*(b^5*x^11 - 42*a*b^4*x^8 - 168*a^2*b^3*x^5 - 368*a
^3*b^2*x^2))*sqrt(sqrt(3)*a/b^4) + 24*(3*a*b^2*x^8 - 12*a^2*b*x^5 - 96*a^3*
x^2 - 2*sqrt(3)*(a*b^2*x^8 + 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(b*x^3 + a) + (
1/72)^(1/6)*(3*b^4*x^12 - 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 + 2208*a^3*b*x^3
+ 384*a^4 + sqrt(3)*(b^4*x^12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*
b*x^3 + 256*a^4 + sqrt(-3)*(b^4*x^12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 11
20*a^3*b*x^3 + 256*a^4)) + 3*sqrt(-3)*(b^4*x^12 - 4*a*b^3*x^9 + 360*a^2*b^2
*x^6 + 736*a^3*b*x^3 + 128*a^4))*(sqrt(3)*a/b^4)^(1/6))/(b^4*x^12 + 40*a*b^
3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*x^3 + 64*a^4)) - (1/72)^(1/6)*(sqrt(-3)
*b + b)*(sqrt(3)*a/b^4)^(1/6)*log((12*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*
x^3 + 48*a^3*b^2 - sqrt(3)*(b^5*x^9 - 30*a*b^4*x^6 - 144*a^2*b^3*x^3 - 32*a
^3*b^2 + sqrt(-3)*(b^5*x^9 - 30*a*b^4*x^6 - 144*a^2*b^3*x^3 - 32*a^3*b^2))
+ 3*sqrt(-3)*(b^5*x^9 + 96*a^2*b^3*x^3 + 16*a^3*b^2))*sqrt(b*x^3 + a)*(sqrt
(3)*a/b^4)^(2/3) - 72*(1/72)^(5/6)*(7*b^6*x^10 + 12*a*b^5*x^7 + 408*a^2*b^4
*x^4 + 160*a^3*b^3*x - sqrt(-3)*(7*b^6*x^10 + 12*a*b^5*x^7 + 408*a^2*b^4*x^
4 + 160*a^3*b^3*x) - 3*sqrt(3)*(b^6*x^10 - 12*a*b^5*x^7 - 72*a^2*b^4*x^4 -
32*a^3*b^3*x - sqrt(-3)*(b^6*x^10 - 12*a*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*
b^3*x)))*(sqrt(3)*a/b^4)^(5/6) + 6*(1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4
+ 160*a^3*b*x - sqrt(-3)*(b^4*x^10 + 240*a^2*b^2*x^4 + 160*a^3*b*x) + 24*sq
rt(3)*(a*b^3*x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x - sqrt(-3)*(a*b^3*x^7 + 5*a^2*
b^2*x^4 + 4*a^3*b*x)))*sqrt(b*x^3 + a)*(sqrt(3)*a/b^4)^(1/3) + 4*sqrt(1/2)*
```

(3*b^5*x^11 - 18*a*b^4*x^8 + 360*a^2*b^3*x^5 + 624*a^3*b^2*x^2 - sqrt(3)*(b
^5*x^11 - 42*a*b^4*x^8 - 168*a^2*b^3*x^5 - 368*a^3*b^2*x^2))*sqrt(sqrt(3)*a
/b^4) + 24*(3*a*b^2*x^8 - 12*a^2*b*x^5 - 96*a^3*x^2 - 2*sqrt(3)*(a*b^2*x^8
+ 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(b*x^3 + a) - (1/72)^(1/6)*(3*b^4*x^12 - 1
2*a*b^3*x^9 + 1080*a^2*b^2*x^6 + 2208*a^3*b*x^3 + 384*a^4 + sqrt(3)*(b^4*x^
12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*b*x^3 + 256*a^4 + sqrt(-3)*
(b^4*x^12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*b*x^3 + 256*a^4)) +
3*sqrt(-3)*(b^4*x^12 - 4*a*b^3*x^9 + 360*a^2*b^2*x^6 + 736*a^3*b*x^3 + 128*
a^4))*(sqrt(3)*a/b^4)^(1/6))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 3
20*a^3*b*x^3 + 64*a^4)) - (1/72)^(1/6)*(sqrt(-3)*b - b)*(sqrt(3)*a/b^4)^(1/
6)*log((12*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 + 48*a^3*b^2 - sqrt(3)*
(b^5*x^9 - 30*a*b^4*x^6 - 144*a^2*b^3*x^3 - 32*a^3*b^2 - sqrt(-3)*(b^5*x^9
- 30*a*b^4*x^6 - 144*a^2*b^3*x^3 - 32*a^3*b^2)) - 3*sqrt(-3)*(b^5*x^9 + 96*
a^2*b^3*x^3 + 16*a^3*b^2))*sqrt(b*x^3 + a)*(sqrt(3)*a/b^4)^(2/3) + 72*(1/72
)^(5/6)*(7*b^6*x^10 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x + sqrt
(-3)*(7*b^6*x^10 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x) - 3*sqrt
(3)*(b^6*x^10 - 12*a*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x + sqrt(-3)*(b^
6*x^10 - 12*a*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x)))*(sqrt(3)*a/b^4)^(5
/6) + 6*(1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 + 160*a^3*b*x + sqrt(-3)*(b
^4*x^10 + 240*a^2*b^2*x^4 + 160*a^3*b*x) + 24*sqrt(3)*(a*b^3*x^7 + 5*a^2*b^
2*x^4 + 4*a^3*b*x + sqrt(-3)*(a*b^3*x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x)))*sqrt
(b*x^3 + a)*(sqrt(3)*a/b^4)^(1/3) - 4*sqrt(1/2)*(3*b^5*x^11 - 18*a*b^4*x^8
+ 360*a^2*b^3*x^5 + 624*a^3*b^2*x^2 - sqrt(3)*(b^5*x^11 - 42*a*b^4*x^8 - 16
8*a^2*b^3*x^5 - 368*a^3*b^2*x^2))*sqrt(sqrt(3)*a/b^4) + 24*(3*a*b^2*x^8 - 1
2*a^2*b*x^5 - 96*a^3*x^2 - 2*sqrt(3)*(a*b^2*x^8 + 2*a^2*b*x^5 + 28*a^3*x^2)
)*sqrt(b*x^3 + a) + (1/72)^(1/6)*(3*b^4*x^12 - 12*a*b^3*x^9 + 1080*a^2*b^2*
x^6 + 2208*a^3*b*x^3 + 384*a^4 + sqrt(3)*(b^4*x^12 + 124*a*b^3*x^9 + 744*a^
2*b^2*x^6 + 1120*a^3*b*x^3 + 256*a^4 - sqrt(-3)*(b^4*x^12 + 124*a*b^3*x^9 +
744*a^2*b^2*x^6 + 1120*a^3*b*x^3 + 256*a^4)) - 3*sqrt(-3)*(b^4*x^12 - 4*a*
b^3*x^9 + 360*a^2*b^2*x^6 + 736*a^3*b*x^3 + 128*a^4))*(sqrt(3)*a/b^4)^(1/6)
)/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*x^3 + 64*a^4)) + (
1/72)^(1/6)*(sqrt(-3)*b - b)*(sqrt(3)*a/b^4)^(1/6)*log((12*(1/9)^(2/3)*(3*b
^5*x^9 + 288*a^2*b^3*x^3 + 48*a^3*b^2 - sqrt(3)*(b^5*x^9 - 30*a*b^4*x^6 - 1
44*a^2*b^3*x^3 - 32*a^3*b^2 - sqrt(-3)*(b^5*x^9 - 30*a*b^4*x^6 - 144*a^2*b^
3*x^3 - 32*a^3*b^2)) - 3*sqrt(-3)*(b^5*x^9 + 96*a^2*b^3*x^3 + 16*a^3*b^2))*
sqrt(b*x^3 + a)*(sqrt(3)*a/b^4)^(2/3) - 72*(1/72)^(5/6)*(7*b^6*x^10 + 12*a*
b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x + sqrt(-3)*(7*b^6*x^10 + 12*a*b^5
*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x) - 3*sqrt(3)*(b^6*x^10 - 12*a*b^5*x^
7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x + sqrt(-3)*(b^6*x^10 - 12*a*b^5*x^7 - 72*
a^2*b^4*x^4 - 32*a^3*b^3*x)))*(sqrt(3)*a/b^4)^(5/6) + 6*(1/9)^(1/3)*(b^4*x^
10 + 240*a^2*b^2*x^4 + 160*a^3*b*x + sqrt(-3)*(b^4*x^10 + 240*a^2*b^2*x^4 +
160*a^3*b*x) + 24*sqrt(3)*(a*b^3*x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x + sqrt(-3
)*(a*b^3*x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x)))*sqrt(b*x^3 + a)*(sqrt(3)*a/b^4)
^(1/3) + 4*sqrt(1/2)*(3*b^5*x^11 - 18*a*b^4*x^8 + 360*a^2*b^3*x^5 + 624*a^3
*b^2*x^2 - sqrt(3)*(b^5*x^11 - 42*a*b^4*x^8 - 168*a^2*b^3*x^5 - 368*a^3*b^2

```
*x^2))*sqrt(sqrt(3)*a/b^4) + 24*(3*a*b^2*x^8 - 12*a^2*b*x^5 - 96*a^3*x^2 -
2*sqrt(3)*(a*b^2*x^8 + 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(b*x^3 + a) - (1/72)^
(1/6)*(3*b^4*x^12 - 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 + 2208*a^3*b*x^3 + 384*
a^4 + sqrt(3)*(b^4*x^12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*b*x^3
+ 256*a^4 - sqrt(-3)*(b^4*x^12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3
*b*x^3 + 256*a^4)) - 3*sqrt(-3)*(b^4*x^12 - 4*a*b^3*x^9 + 360*a^2*b^2*x^6 +
 736*a^3*b*x^3 + 128*a^4))*(sqrt(3)*a/b^4)^(1/6))/(b^4*x^12 + 40*a*b^3*x^9
+ 384*a^2*b^2*x^6 - 320*a^3*b*x^3 + 64*a^4)) - 2*(1/72)^(1/6)*b*(sqrt(3)*a/
b^4)^(1/6)*log(-(12*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 + 48*a^3*b^2 -
 sqrt(3)*(b^5*x^9 - 30*a*b^4*x^6 - 144*a^2*b^3*x^3 - 32*a^3*b^2))*sqrt(b*x^
3 + a)*(sqrt(3)*a/b^4)^(2/3) + 72*(1/72)^(5/6)*(7*b^6*x^10 + 12*a*b^5*x^7 +
 408*a^2*b^4*x^4 + 160*a^3*b^3*x - 3*sqrt(3)*(b^6*x^10 - 12*a*b^5*x^7 - 72*
a^2*b^4*x^4 - 32*a^3*b^3*x))*(sqrt(3)*a/b^4)^(5/6) + 6*(1/9)^(1/3)*(b^4*x^1
0 + 240*a^2*b^2*x^4 + 160*a^3*b*x + 24*sqrt(3)*(a*b^3*x^7 + 5*a^2*b^2*x^4 +
 4*a^3*b*x))*sqrt(b*x^3 + a)*(sqrt(3)*a/b^4)^(1/3) + 2*sqrt(1/2)*(3*b^5*x^1
1 - 18*a*b^4*x^8 + 360*a^2*b^3*x^5 + 624*a^3*b^2*x^2 - sqrt(3)*(b^5*x^11 -
42*a*b^4*x^8 - 168*a^2*b^3*x^5 - 368*a^3*b^2*x^2))*sqrt(sqrt(3)*a/b^4) - 12
*(3*a*b^2*x^8 - 12*a^2*b*x^5 - 96*a^3*x^2 - 2*sqrt(3)*(a*b^2*x^8 + 2*a^2*b*
x^5 + 28*a^3*x^2))*sqrt(b*x^3 + a) + (1/72)^(1/6)*(3*b^4*x^12 - 12*a*b^3*x^
9 + 1080*a^2*b^2*x^6 + 2208*a^3*b*x^3 + 384*a^4 + sqrt(3)*(b^4*x^12 + 124*a
*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*b*x^3 + 256*a^4))*(sqrt(3)*a/b^4)^(1/
6))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*x^3 + 64*a^4)) +
 2*(1/72)^(1/6)*b*(sqrt(3)*a/b^4)^(1/6)*log(-(12*(1/9)^(2/3)*(3*b^5*x^9 + 2
88*a^2*b^3*x^3 + 48*a^3*b^2 - sqrt(3)*(b^5*x^9 - 30*a*b^4*x^6 - 144*a^2*b^3
*x^3 - 32*a^3*b^2))*sqrt(b*x^3 + a)*(sqrt(3)*a/b^4)^(2/3) - 72*(1/72)^(5/6)
*(7*b^6*x^10 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x - 3*sqrt(3)*(
b^6*x^10 - 12*a*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x))*(sqrt(3)*a/b^4)^(
5/6) + 6*(1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 + 160*a^3*b*x + 24*sqrt(3)
*(a*b^3*x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x))*sqrt(b*x^3 + a)*(sqrt(3)*a/b^4)^(
1/3) - 2*sqrt(1/2)*(3*b^5*x^11 - 18*a*b^4*x^8 + 360*a^2*b^3*x^5 + 624*a^3*b
^2*x^2 - sqrt(3)*(b^5*x^11 - 42*a*b^4*x^8 - 168*a^2*b^3*x^5 - 368*a^3*b^2*x
^2))*sqrt(sqrt(3)*a/b^4) - 12*(3*a*b^2*x^8 - 12*a^2*b*x^5 - 96*a^3*x^2 - 2*
sqrt(3)*(a*b^2*x^8 + 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(b*x^3 + a) - (1/72)^(1
/6)*(3*b^4*x^12 - 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 + 2208*a^3*b*x^3 + 384*a^
4 + sqrt(3)*(b^4*x^12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*b*x^3 +
256*a^4))*(sqrt(3)*a/b^4)^(1/6))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6
 - 320*a^3*b*x^3 + 64*a^4)) - 16*sqrt(b)*weierstrassZeta(0, -4*a/b, weierst
rassPInverse(0, -4*a/b, x)))/b
```

Sympy [F]

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{x\sqrt{a+bx^3}}{-6\sqrt{3}a+10a+bx^3}\,dx$$

```
[In] integrate(x*(b*x**3+a)**(1/2)/(b*x**3+2*a*(5-3*3**(1/2))),x)
[Out] Integral(x*sqrt(a + b*x**3)/(-6*sqrt(3)*a + 10*a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{\sqrt{bx^3+a}x}{bx^3-2\,a\left(3\sqrt{3}-5\right)}\,dx$$

```
[In] integrate(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="maxima"
)
[Out] integrate(sqrt(b*x^3 + a)*x/(b*x^3 - 2*a*(3*sqrt(3) - 5)), x)
```

Giac [F]

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{\sqrt{bx^3+a}x}{bx^3-2\,a\left(3\sqrt{3}-5\right)}\,dx$$

```
[In] integrate(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="giac")
[Out] integrate(sqrt(b*x^3 + a)*x/(b*x^3 - 2*a*(3*sqrt(3) - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{x\,\sqrt{b\,x^3+a}}{b\,x^3-2\,a\,\left(3\,\sqrt{3}-5\right)}\,dx$$

```
[In] int((x*(a + b*x^3)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) - 5)),x)
[Out] int((x*(a + b*x^3)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) - 5)), x)
```

3.347 $\int \frac{x\sqrt{a-bx^3}}{2(5-3\sqrt{3})a-bx^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 758

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx$$

$$= \frac{2\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$- \frac{\sqrt[4]{3}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{\sqrt{2}b^{2/3}}$$
$$+ \frac{3^{3/4}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}b^{2/3}} + \frac{\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}$$
$$- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$
$$+ \frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

```
[Out] -1/2*3^(1/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(2*b^(1/3)*x+a^(1/3)*(1-3^(
1/2)))*2^(1/2)/(-b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)-1/4*3^(1/4)*a^(1/6)*arctan
(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(-b*x^3+a)^(1/
2))/b^(2/3)*2^(1/2)+1/4*3^(3/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3
)-b^(1/3)*x)*(1-3^(1/2))*2^(1/2)/(-b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)+1/6*a^(1
/6)*arctanh(1/6*(1+3^(1/2))*(-b*x^3+a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*3^(3/
4)/b^(2/3)*2^(1/2)+2*(-b*x^3+a)^(1/2)/b^(2/3)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))+2/3*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1-3^(1/2
)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)+a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)
/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*
```

```
(1+3^(1/2)))^2)^(1/2)-3^(1/4)*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/
3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(
1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)
-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 758, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {495, 309, 224, 1891, 500}

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx$$

$$= \frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$
$$-\frac{\sqrt[4]{3}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$-\frac{\sqrt[4]{3}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{\sqrt{2}b^{2/3}}$$
$$+\frac{3^{3/4}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$+\frac{\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}+\frac{2\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

```
[In] Int[(x*Sqrt[a - b*x^3])/(2*(5 - 3*Sqrt[3])*a - b*x^3),x]
```

```
[Out] (2*Sqrt[a - b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1/4
)*a^(1/6)*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqr
t[2]*Sqrt[a - b*x^3])])/(2*Sqrt[2]*b^(2/3)) - (3^(1/4)*a^(1/6)*ArcTan[(3^(1
/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3]
)])/(Sqrt[2]*b^(2/3)) + (3^(3/4)*a^(1/6)*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(
1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(2*Sqrt[2]*b^(2/3))
 + (a^(1/6)*ArcTanh[((1 + Sqrt[3])*Sqrt[a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a
])])/(Sqrt[2]*3^(1/4)*b^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3
) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt
[a - b*x^3]) + (2*Sqrt[2]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3
)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 500

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
```

```
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\left(\left(3\left(3-2\sqrt{3}\right)a\right)\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx\right)+\int \frac{x}{\sqrt{a-bx^3}}\,dx$$

$$= -\frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}b^{2/3}} - \frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{\sqrt{2}b^{2/3}} + \frac{3^{3/4}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}b^{2/3}} + \frac{\sqrt[6]{a}\tanh^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}} - \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx}{\sqrt[3]{b}} + \frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)\int \frac{1}{\sqrt{a-bx^3}}\,dx}{\sqrt[3]{b}}$$

$$= \frac{2\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$

$$- \frac{\sqrt[4]{3}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{\sqrt{2}b^{2/3}}$$

$$+ \frac{3^{3/4}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}b^{2/3}} + \frac{\sqrt[6]{a}\tanh^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}$$

$$- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$+ \frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.11

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx = \frac{x^2\sqrt{1-\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20-12\sqrt{3}\right)\sqrt{a-bx^3}}$$

```
[In] Integrate[(x*Sqrt[a - b*x^3])/(2*(5 - 3*Sqrt[3])*a - b*x^3),x]
[Out] (x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, -1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10
*a - 6*Sqrt[3]*a)])/((20 - 12*Sqrt[3])*Sqrt[a - b*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.00 (sec) , antiderivative size = 924, normalized size of antiderivative = 1.22

method	result	size
elliptic	Expression too large to display	924
default	Expression too large to display	942

```
[In] int(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a
*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*
(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)
^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*
3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b
/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)
*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2
)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(a*b^2)^(1/3)))^(1/2)))-1/9*I/b^3*2^(1/2)*sum(1/_alpha*(2*3^(1/2)-3)*
(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(
a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1/3))/(-3*(a*b^2)^(1/3)-I*3^(1/2)*(a
*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/
3)))/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*(-3*I*(a*b^2)^(1/3)*_alpha*3^(1/
2)*b+4*b^2*_alpha^2*3^(1/2)+3*I*(a*b^2)^(2/3)*3^(1/2)-6*I*(a*b^2)^(1/3)*_al
pha*b-2*3^(1/2)*(a*b^2)^(1/3)*_alpha*b+6*b^2*_alpha^2+6*I*(a*b^2)^(2/3)-2*3
^(1/2)*(a*b^2)^(2/3)-3*(a*b^2)^(1/3)*_alpha*b-3*(a*b^2)^(2/3))*EllipticPi(1
/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2
)*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*3^(1/2)*(a*b^2)^(1/3)*_alpha^2*b+I*3^(
1/2)*(a*b^2)^(2/3)*_alpha-4*I*(a*b^2)^(1/3)*_alpha^2*b+2*I*(a*b^2)^(2/3)*_a
lpha-2*3^(1/2)*(a*b^2)^(2/3)*_alpha+I*3^(1/2)*a*b-3*(a*b^2)^(2/3)*_alpha+2*
I*a*b+2*3^(1/2)*a*b+3*a*b)/a,(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3+6*3^(1/2)*a
-10*a))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.38 (sec) , antiderivative size = 4953, normalized size of antiderivative = 6.53

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="frica
s")
```

```
[Out] -1/8*((1/72)^(1/6)*(sqrt(-3)*b + b)*(sqrt(3)*a/b^4)^(1/6)*log(-(12*(1/9)^(2
/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 - 48*a^3*b^2 - sqrt(3)*(b^5*x^9 + 30*a*b^4
*x^6 - 144*a^2*b^3*x^3 + 32*a^3*b^2 + sqrt(-3)*(b^5*x^9 + 30*a*b^4*x^6 - 14
4*a^2*b^3*x^3 + 32*a^3*b^2)) + 3*sqrt(-3)*(b^5*x^9 + 96*a^2*b^3*x^3 - 16*a^
3*b^2))*sqrt(-b*x^3 + a)*(sqrt(3)*a/b^4)^(2/3) + 72*(1/72)^(5/6)*(7*b^6*x^1
0 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x - sqrt(-3)*(7*b^6*x^10 -
 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x) - 3*sqrt(3)*(b^6*x^10 + 12
*a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x - sqrt(-3)*(b^6*x^10 + 12*a*b^5*
x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x)))*(sqrt(3)*a/b^4)^(5/6) - 6*(1/9)^(1/3
)*(b^4*x^10 + 240*a^2*b^2*x^4 - 160*a^3*b*x - sqrt(-3)*(b^4*x^10 + 240*a^2*
b^2*x^4 - 160*a^3*b*x) - 24*sqrt(3)*(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x
- sqrt(-3)*(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x)))*sqrt(-b*x^3 + a)*(sqrt
(3)*a/b^4)^(1/3) + 4*sqrt(1/2)*(3*b^5*x^11 + 18*a*b^4*x^8 + 360*a^2*b^3*x^5
 - 624*a^3*b^2*x^2 - sqrt(3)*(b^5*x^11 + 42*a*b^4*x^8 - 168*a^2*b^3*x^5 + 3
68*a^3*b^2*x^2))*sqrt(sqrt(3)*a/b^4) - 24*(3*a*b^2*x^8 + 12*a^2*b*x^5 - 96*
a^3*x^2 - 2*sqrt(3)*(a*b^2*x^8 - 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(-b*x^3 + a
) + (1/72)^(1/6)*(3*b^4*x^12 + 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 - 2208*a^3*b
*x^3 + 384*a^4 + 3*sqrt(-3)*(b^4*x^12 + 4*a*b^3*x^9 + 360*a^2*b^2*x^6 - 736
*a^3*b*x^3 + 128*a^4) + sqrt(3)*(b^4*x^12 - 124*a*b^3*x^9 + 744*a^2*b^2*x^6
 - 1120*a^3*b*x^3 + 256*a^4 + sqrt(-3)*(b^4*x^12 - 124*a*b^3*x^9 + 744*a^2*
b^2*x^6 - 1120*a^3*b*x^3 + 256*a^4)))*(sqrt(3)*a/b^4)^(1/6))/(b^4*x^12 - 40
*a*b^3*x^9 + 384*a^2*b^2*x^6 + 320*a^3*b*x^3 + 64*a^4)) - (1/72)^(1/6)*(sqr
t(-3)*b + b)*(sqrt(3)*a/b^4)^(1/6)*log(-(12*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^
2*b^3*x^3 - 48*a^3*b^2 - sqrt(3)*(b^5*x^9 + 30*a*b^4*x^6 - 144*a^2*b^3*x^3
+ 32*a^3*b^2 + sqrt(-3)*(b^5*x^9 + 30*a*b^4*x^6 - 144*a^2*b^3*x^3 + 32*a^3*
b^2)) + 3*sqrt(-3)*(b^5*x^9 + 96*a^2*b^3*x^3 - 16*a^3*b^2))*sqrt(-b*x^3 + a
)*(sqrt(3)*a/b^4)^(2/3) - 72*(1/72)^(5/6)*(7*b^6*x^10 - 12*a*b^5*x^7 + 408*
a^2*b^4*x^4 - 160*a^3*b^3*x - sqrt(-3)*(7*b^6*x^10 - 12*a*b^5*x^7 + 408*a^2
*b^4*x^4 - 160*a^3*b^3*x) - 3*sqrt(3)*(b^6*x^10 + 12*a*b^5*x^7 - 72*a^2*b^4
*x^4 + 32*a^3*b^3*x - sqrt(-3)*(b^6*x^10 + 12*a*b^5*x^7 - 72*a^2*b^4*x^4 +
32*a^3*b^3*x)))*(sqrt(3)*a/b^4)^(5/6) - 6*(1/9)^(1/3)*(b^4*x^10 + 240*a^2*b
^2*x^4 - 160*a^3*b*x - sqrt(-3)*(b^4*x^10 + 240*a^2*b^2*x^4 - 160*a^3*b*x)
- 24*sqrt(3)*(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x - sqrt(-3)*(a*b^3*x^7 -
 5*a^2*b^2*x^4 + 4*a^3*b*x)))*sqrt(-b*x^3 + a)*(sqrt(3)*a/b^4)^(1/3) - 4*sq
```

rt(1/2)*(3*b^5*x^11 + 18*a*b^4*x^8 + 360*a^2*b^3*x^5 - 624*a^3*b^2*x^2 - sq
rt(3)*(b^5*x^11 + 42*a*b^4*x^8 - 168*a^2*b^3*x^5 + 368*a^3*b^2*x^2))*sqrt(s
qrt(3)*a/b^4) - 24*(3*a*b^2*x^8 + 12*a^2*b*x^5 - 96*a^3*x^2 - 2*sqrt(3)*(a*
b^2*x^8 - 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(-b*x^3 + a) - (1/72)^(1/6)*(3*b^4
*x^12 + 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 - 2208*a^3*b*x^3 + 384*a^4 + 3*sqrt
(-3)*(b^4*x^12 + 4*a*b^3*x^9 + 360*a^2*b^2*x^6 - 736*a^3*b*x^3 + 128*a^4) +
 sqrt(3)*(b^4*x^12 - 124*a*b^3*x^9 + 744*a^2*b^2*x^6 - 1120*a^3*b*x^3 + 256
a^4 + sqrt(-3)(b^4*x^12 - 124*a*b^3*x^9 + 744*a^2*b^2*x^6 - 1120*a^3*b*x^
3 + 256*a^4)))*(sqrt(3)*a/b^4)^(1/6))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^
2*x^6 + 320*a^3*b*x^3 + 64*a^4)) - (1/72)^(1/6)*(sqrt(-3)*b - b)*(sqrt(3)*a
/b^4)^(1/6)*log(-(12*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 - 48*a^3*b^2
- sqrt(3)*(b^5*x^9 + 30*a*b^4*x^6 - 144*a^2*b^3*x^3 + 32*a^3*b^2 - sqrt(-3)
*(b^5*x^9 + 30*a*b^4*x^6 - 144*a^2*b^3*x^3 + 32*a^3*b^2)) - 3*sqrt(-3)*(b^5
*x^9 + 96*a^2*b^3*x^3 - 16*a^3*b^2))*sqrt(-b*x^3 + a)*(sqrt(3)*a/b^4)^(2/3)
 + 72*(1/72)^(5/6)*(7*b^6*x^10 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b
^3*x + sqrt(-3)*(7*b^6*x^10 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*
x) - 3*sqrt(3)*(b^6*x^10 + 12*a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x + s
qrt(-3)*(b^6*x^10 + 12*a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x)))*(sqrt(3
)*a/b^4)^(5/6) - 6*(1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 - 160*a^3*b*x +
sqrt(-3)*(b^4*x^10 + 240*a^2*b^2*x^4 - 160*a^3*b*x) - 24*sqrt(3)*(a*b^3*x^7
 - 5*a^2*b^2*x^4 + 4*a^3*b*x + sqrt(-3)*(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^3*
b*x)))*sqrt(-b*x^3 + a)*(sqrt(3)*a/b^4)^(1/3) + 4*sqrt(1/2)*(3*b^5*x^11 + 1
8*a*b^4*x^8 + 360*a^2*b^3*x^5 - 624*a^3*b^2*x^2 - sqrt(3)*(b^5*x^11 + 42*a*
b^4*x^8 - 168*a^2*b^3*x^5 + 368*a^3*b^2*x^2))*sqrt(sqrt(3)*a/b^4) - 24*(3*a
*b^2*x^8 + 12*a^2*b*x^5 - 96*a^3*x^2 - 2*sqrt(3)*(a*b^2*x^8 - 2*a^2*b*x^5 +
 28*a^3*x^2))*sqrt(-b*x^3 + a) + (1/72)^(1/6)*(3*b^4*x^12 + 12*a*b^3*x^9 +
1080*a^2*b^2*x^6 - 2208*a^3*b*x^3 + 384*a^4 - 3*sqrt(-3)*(b^4*x^12 + 4*a*b^
3*x^9 + 360*a^2*b^2*x^6 - 736*a^3*b*x^3 + 128*a^4) + sqrt(3)*(b^4*x^12 - 12
4*a*b^3*x^9 + 744*a^2*b^2*x^6 - 1120*a^3*b*x^3 + 256*a^4 - sqrt(-3)*(b^4*x^
12 - 124*a*b^3*x^9 + 744*a^2*b^2*x^6 - 1120*a^3*b*x^3 + 256*a^4)))*(sqrt(3)
*a/b^4)^(1/6))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2*x^6 + 320*a^3*b*x^3 +
 64*a^4)) + (1/72)^(1/6)*(sqrt(-3)*b - b)*(sqrt(3)*a/b^4)^(1/6)*log(-(12*(1
/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 - 48*a^3*b^2 - sqrt(3)*(b^5*x^9 + 30
*a*b^4*x^6 - 144*a^2*b^3*x^3 + 32*a^3*b^2 - sqrt(-3)*(b^5*x^9 + 30*a*b^4*x^
6 - 144*a^2*b^3*x^3 + 32*a^3*b^2)) - 3*sqrt(-3)*(b^5*x^9 + 96*a^2*b^3*x^3 -
 16*a^3*b^2))*sqrt(-b*x^3 + a)*(sqrt(3)*a/b^4)^(2/3) - 72*(1/72)^(5/6)*(7*b
^6*x^10 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x + sqrt(-3)*(7*b^6*
x^10 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x) - 3*sqrt(3)*(b^6*x^1
0 + 12*a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x + sqrt(-3)*(b^6*x^10 + 12*
a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x)))*(sqrt(3)*a/b^4)^(5/6) - 6*(1/9
)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 - 160*a^3*b*x + sqrt(-3)*(b^4*x^10 + 24
0*a^2*b^2*x^4 - 160*a^3*b*x) - 24*sqrt(3)*(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^
3*b*x + sqrt(-3)*(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x)))*sqrt(-b*x^3 + a)
*(sqrt(3)*a/b^4)^(1/3) - 4*sqrt(1/2)*(3*b^5*x^11 + 18*a*b^4*x^8 + 360*a^2*b
^3*x^5 - 624*a^3*b^2*x^2 - sqrt(3)*(b^5*x^11 + 42*a*b^4*x^8 - 168*a^2*b^3*x

^5 + 368*a^3*b^2*x^2))*sqrt(sqrt(3)*a/b^4) - 24*(3*a*b^2*x^8 + 12*a^2*b*x^5
 - 96*a^3*x^2 - 2*sqrt(3)*(a*b^2*x^8 - 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(-b*x
^3 + a) - (1/72)^(1/6)*(3*b^4*x^12 + 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 - 2208
*a^3*b*x^3 + 384*a^4 - 3*sqrt(-3)*(b^4*x^12 + 4*a*b^3*x^9 + 360*a^2*b^2*x^6
 - 736*a^3*b*x^3 + 128*a^4) + sqrt(3)*(b^4*x^12 - 124*a*b^3*x^9 + 744*a^2*b
^2*x^6 - 1120*a^3*b*x^3 + 256*a^4 - sqrt(-3)*(b^4*x^12 - 124*a*b^3*x^9 + 74
4*a^2*b^2*x^6 - 1120*a^3*b*x^3 + 256*a^4)))*(sqrt(3)*a/b^4)^(1/6))/(b^4*x^1
2 - 40*a*b^3*x^9 + 384*a^2*b^2*x^6 + 320*a^3*b*x^3 + 64*a^4)) - 2*(1/72)^(1
/6)*b*(sqrt(3)*a/b^4)^(1/6)*log((12*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^
3 - 48*a^3*b^2 - sqrt(3)*(b^5*x^9 + 30*a*b^4*x^6 - 144*a^2*b^3*x^3 + 32*a^3
*b^2))*sqrt(-b*x^3 + a)*(sqrt(3)*a/b^4)^(2/3) + 72*(1/72)^(5/6)*(7*b^6*x^10
 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x - 3*sqrt(3)*(b^6*x^10 + 1
2*a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x))*(sqrt(3)*a/b^4)^(5/6) - 6*(1/
9)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 - 160*a^3*b*x - 24*sqrt(3)*(a*b^3*x^7
 - 5*a^2*b^2*x^4 + 4*a^3*b*x))*sqrt(-b*x^3 + a)*(sqrt(3)*a/b^4)^(1/3) - 2*sq
rt(1/2)*(3*b^5*x^11 + 18*a*b^4*x^8 + 360*a^2*b^3*x^5 - 624*a^3*b^2*x^2 - sq
rt(3)*(b^5*x^11 + 42*a*b^4*x^8 - 168*a^2*b^3*x^5 + 368*a^3*b^2*x^2))*sqrt(s
qrt(3)*a/b^4) + 12*(3*a*b^2*x^8 + 12*a^2*b*x^5 - 96*a^3*x^2 - 2*sqrt(3)*(a*
b^2*x^8 - 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(-b*x^3 + a) + (1/72)^(1/6)*(3*b^4
*x^12 + 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 - 2208*a^3*b*x^3 + 384*a^4 + sqrt(3
)*(b^4*x^12 - 124*a*b^3*x^9 + 744*a^2*b^2*x^6 - 1120*a^3*b*x^3 + 256*a^4))*
(sqrt(3)*a/b^4)^(1/6))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2*x^6 + 320*a^3
*b*x^3 + 64*a^4)) + 2*(1/72)^(1/6)*b*(sqrt(3)*a/b^4)^(1/6)*log((12*(1/9)^(2
/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 - 48*a^3*b^2 - sqrt(3)*(b^5*x^9 + 30*a*b^4
*x^6 - 144*a^2*b^3*x^3 + 32*a^3*b^2))*sqrt(-b*x^3 + a)*(sqrt(3)*a/b^4)^(2/3
) - 72*(1/72)^(5/6)*(7*b^6*x^10 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*
b^3*x - 3*sqrt(3)*(b^6*x^10 + 12*a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x)
)*(sqrt(3)*a/b^4)^(5/6) - 6*(1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 - 160*a
^3*b*x - 24*sqrt(3)*(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x))*sqrt(-b*x^3 +
a)*(sqrt(3)*a/b^4)^(1/3) + 2*sqrt(1/2)*(3*b^5*x^11 + 18*a*b^4*x^8 + 360*a^2
*b^3*x^5 - 624*a^3*b^2*x^2 - sqrt(3)*(b^5*x^11 + 42*a*b^4*x^8 - 168*a^2*b^3
*x^5 + 368*a^3*b^2*x^2))*sqrt(sqrt(3)*a/b^4) + 12*(3*a*b^2*x^8 + 12*a^2*b*x
^5 - 96*a^3*x^2 - 2*sqrt(3)*(a*b^2*x^8 - 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(-b
x^3 + a) - (1/72)^(1/6)(3*b^4*x^12 + 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 - 22
08*a^3*b*x^3 + 384*a^4 + sqrt(3)*(b^4*x^12 - 124*a*b^3*x^9 + 744*a^2*b^2*x^
6 - 1120*a^3*b*x^3 + 256*a^4))*(sqrt(3)*a/b^4)^(1/6))/(b^4*x^12 - 40*a*b^3*
x^9 + 384*a^2*b^2*x^6 + 320*a^3*b*x^3 + 64*a^4)) - 16*sqrt(-b)*weierstrassZ
eta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b

Sympy [F]

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx = -\int \frac{x\sqrt{a-bx^3}}{-10a+6\sqrt{3}a+bx^3}\,dx$$

```
[In] integrate(x*(-b*x**3+a)**(1/2)/(-b*x**3+2*a*(5-3*3**(1/2))),x)
[Out] -Integral(x*sqrt(a - b*x**3)/(-10*a + 6*sqrt(3)*a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx = \int -\frac{\sqrt{-bx^3+a}x}{bx^3+2\,a\left(3\,\sqrt{3}-5\right)}\,dx$$

```
[In] integrate(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="maxim
a")
[Out] -integrate(sqrt(-b*x^3 + a)*x/(b*x^3 + 2*a*(3*sqrt(3) - 5)), x)
```

Giac [F]

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx = \int -\frac{\sqrt{-bx^3+a}x}{bx^3+2\,a\left(3\,\sqrt{3}-5\right)}\,dx$$

```
[In] integrate(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="giac"
)
[Out] integrate(-sqrt(-b*x^3 + a)*x/(b*x^3 + 2*a*(3*sqrt(3) - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx = \int -\frac{x\,\sqrt{a-b\,x^3}}{b\,x^3+2\,a\,\left(3\,\sqrt{3}-5\right)}\,dx$$

```
[In] int(-(x*(a - b*x^3)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) - 5)),x)
[Out] int(-(x*(a - b*x^3)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) - 5)), x)
```

3.348 $\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 774

$$\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx$$

$$= \frac{2\sqrt{-a+bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{3^{3/4}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$+ \frac{\sqrt[6]{a}\arctan\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}} + \frac{\sqrt[4]{3}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$+ \frac{\sqrt[4]{3}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{\sqrt{2}b^{2/3}}$$
$$- \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$
$$+ \frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

```
[Out] -1/4*3^(3/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1-3^(1
/2))*2^(1/2)/(b*x^3-a)^(1/2))/b^(2/3)*2^(1/2)+1/6*a^(1/6)*arctan(1/6*(1+3^(
1/2))*(b*x^3-a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*3^(3/4)/b^(2/3)*2^(1/2)+1/2*
3^(1/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*(2*b^(1/3)*x+a^(1/3)*(1-3^(1/2)
))*2^(1/2)/(b*x^3-a)^(1/2))/b^(2/3)*2^(1/2)+1/4*3^(1/4)*a^(1/6)*arctanh(1/2
*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(b*x^3-a)^(1/2))/b
^(2/3)*2^(1/2)+2*(b*x^3-a)^(1/2)/b^(2/3)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))+2
/3*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*b^
(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2
/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^
```

```
(1/2)))^2)^(1/2)-3^(1/4)*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2
/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2
)*(1/2*6^(1/2)+1/2*2^(1/2))/b^(2/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1
/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 774, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {495, 310, 225, 1893, 501}

$$\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx$$
$$= \frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$
$$-\frac{3^{3/4}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{2\sqrt{2}b^{2/3}}+\frac{\sqrt[6]{a}\arctan\left(\frac{\left(1+\sqrt{3}\right)\sqrt{bx^3-a}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}$$
$$+\frac{\sqrt[4]{3}\sqrt[6]{a}\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{2\sqrt{2}b^{2/3}}$$
$$+\frac{\sqrt[4]{3}\sqrt[6]{a}\text{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{\sqrt{2}b^{2/3}}+\frac{2\sqrt{bx^3-a}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

```
[In] Int[(x*Sqrt[-a + b*x^3])/(2*(5 - 3*Sqrt[3])*a - b*x^3),x]
```

```
[Out] (2*Sqrt[-a + b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(3/
4)*a^(1/6)*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sq
```

```
rt[2]*Sqrt[-a + b*x^3])])/(2*Sqrt[2]*b^(2/3)) + (a^(1/6)*ArcTan[((1 + Sqrt[
3])*Sqrt[-a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(Sqrt[2]*3^(1/4)*b^(2/3))
 + (3^(1/4)*a^(1/6)*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/
3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])])/(2*Sqrt[2]*b^(2/3)) + (3^(1/4)*a^(1/6)*
ArcTanh[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sq
rt[-a + b*x^3])])/(Sqrt[2]*b^(2/3)) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a
^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 -
Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) -
b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)*
Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^
2)]*Sqrt[-a + b*x^3]) + (2*Sqrt[2]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2
/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^
2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1
/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/
3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 501

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
```

```
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \left(3\left(3-2\sqrt{3}\right)a\right)\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx-\int \frac{x}{\sqrt{-a+bx^3}}\,dx$$

$$= -\frac{3^{3/4}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}+\frac{\sqrt[6]{a}\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}$$
$$+\frac{\sqrt[4]{3}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$+\frac{\sqrt[4]{3}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{\sqrt{2}b^{2/3}}$$
$$+\frac{\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx}{\sqrt[3]{b}}-\frac{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)\int \frac{1}{\sqrt{-a+bx^3}}\,dx}{\sqrt[3]{b}}$$

$$= \frac{2\sqrt{-a+bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{3^{3/4}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$

$$+ \frac{\sqrt[6]{a}\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}} + \frac{\sqrt[4]{3}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$

$$+ \frac{\sqrt[4]{3}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{\sqrt{2}b^{2/3}}$$

$$- \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

$$+ \frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.11

$$\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx = -\frac{x^2\sqrt{-a+bx^3}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},-\frac{bx^3}{-10a+6\sqrt{3}a}\right)}{4\left(-5+3\sqrt{3}\right)a\sqrt{\frac{a-bx^3}{a}}}$$

```
[In] Integrate[(x*Sqrt[-a + b*x^3])/(2*(5 - 3*Sqrt[3])*a - b*x^3),x]
```

```
[Out] -1/4*(x^2*Sqrt[-a + b*x^3]*AppellF1[2/3, -1/2, 1, 5/3, (b*x^3)/a, -((b*x^3)
/(-10*a + 6*Sqrt[3]*a))])/((-5 + 3*Sqrt[3])*a*Sqrt[(a - b*x^3)/a])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.74 (sec) , antiderivative size = 926, normalized size of antiderivative = 1.20

method	result	size
elliptic	Expression too large to display	926
default	Expression too large to display	944

```
[In] int(x*(b*x^3-a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b
*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)
^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*
3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b
/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)
*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2
)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(a*b^2)^(1/3)))^(1/2)))+1/9*I/b^3*2^(1/2)*sum(1/_alpha*(2*3^(1/2)-3)*
(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(
a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1/3))/(-3*(a*b^2)^(1/3)-I*3^(1/2)*(a
*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/
3)))/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*(-3*I*(a*b^2)^(1/3)*_alpha*3^(1/2
)*b+4*b^2*_alpha^2*3^(1/2)+3*I*(a*b^2)^(2/3)*3^(1/2)-6*I*(a*b^2)^(1/3)*_alp
ha*b-2*3^(1/2)*(a*b^2)^(1/3)*_alpha*b+6*b^2*_alpha^2+6*I*(a*b^2)^(2/3)-2*3^
(1/2)*(a*b^2)^(2/3)-3*(a*b^2)^(1/3)*_alpha*b-3*(a*b^2)^(2/3))*EllipticPi(1/
3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)
*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*3^(1/2)*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1
/2)*(a*b^2)^(2/3)*_alpha-4*I*(a*b^2)^(1/3)*_alpha^2*b+2*I*(a*b^2)^(2/3)*_al
pha-2*3^(1/2)*(a*b^2)^(2/3)*_alpha+I*3^(1/2)*a*b-3*(a*b^2)^(2/3)*_alpha+2*I
*a*b+2*3^(1/2)*a*b+3*a*b)/a,(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3+6*3^(1/2)*a-
10*a))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.39 (sec) , antiderivative size = 4867, normalized size of antiderivative = 6.29

$$\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(b*x^3-a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="fricas
")
```

```
[Out] -1/8*((1/72)^(1/6)*(sqrt(-3)*b - b)*(-sqrt(3)*a/b^4)^(1/6)*log((72*(1/72)^(
5/6)*(7*b^6*x^10 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x + sqrt(-3
)*(7*b^6*x^10 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x) - 3*sqrt(3)
*(b^6*x^10 + 12*a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x + sqrt(-3)*(b^6*x
^10 + 12*a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x)))*(-sqrt(3)*a/b^4)^(5/6
) - 4*sqrt(1/2)*(3*b^5*x^11 + 18*a*b^4*x^8 + 360*a^2*b^3*x^5 - 624*a^3*b^2*
x^2 - sqrt(3)*(b^5*x^11 + 42*a*b^4*x^8 - 168*a^2*b^3*x^5 + 368*a^3*b^2*x^2)
)*sqrt(-sqrt(3)*a/b^4) + 6*(12*a*b^2*x^8 + 48*a^2*b*x^5 - 384*a^3*x^2 - 2*(
1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 - 48*a^3*b^2 - sqrt(3)*(b^5*x^9 + 3
0*a*b^4*x^6 - 144*a^2*b^3*x^3 + 32*a^3*b^2 - sqrt(-3)*(b^5*x^9 + 30*a*b^4*x
^6 - 144*a^2*b^3*x^3 + 32*a^3*b^2)) - 3*sqrt(-3)*(b^5*x^9 + 96*a^2*b^3*x^3
- 16*a^3*b^2))*(-sqrt(3)*a/b^4)^(2/3) - (1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2
*x^4 - 160*a^3*b*x + sqrt(-3)*(b^4*x^10 + 240*a^2*b^2*x^4 - 160*a^3*b*x) -
24*sqrt(3)*(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x + sqrt(-3)*(a*b^3*x^7 - 5
*a^2*b^2*x^4 + 4*a^3*b*x)))*(-sqrt(3)*a/b^4)^(1/3) - 8*sqrt(3)*(a*b^2*x^8 -
 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(b*x^3 - a) + (1/72)^(1/6)*(3*b^4*x^12 + 12
*a*b^3*x^9 + 1080*a^2*b^2*x^6 - 2208*a^3*b*x^3 + 384*a^4 - 3*sqrt(-3)*(b^4*
x^12 + 4*a*b^3*x^9 + 360*a^2*b^2*x^6 - 736*a^3*b*x^3 + 128*a^4) + sqrt(3)*(
b^4*x^12 - 124*a*b^3*x^9 + 744*a^2*b^2*x^6 - 1120*a^3*b*x^3 + 256*a^4 - sqr
t(-3)*(b^4*x^12 - 124*a*b^3*x^9 + 744*a^2*b^2*x^6 - 1120*a^3*b*x^3 + 256*a^
4)))*(-sqrt(3)*a/b^4)^(1/6))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2*x^6 + 3
20*a^3*b*x^3 + 64*a^4)) - (1/72)^(1/6)*(sqrt(-3)*b - b)*(-sqrt(3)*a/b^4)^(1
/6)*log(-(72*(1/72)^(5/6)*(7*b^6*x^10 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 16
0*a^3*b^3*x + sqrt(-3)*(7*b^6*x^10 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a
^3*b^3*x) - 3*sqrt(3)*(b^6*x^10 + 12*a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^
3*x + sqrt(-3)*(b^6*x^10 + 12*a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x)))*
(-sqrt(3)*a/b^4)^(5/6) - 4*sqrt(1/2)*(3*b^5*x^11 + 18*a*b^4*x^8 + 360*a^2*b
^3*x^5 - 624*a^3*b^2*x^2 - sqrt(3)*(b^5*x^11 + 42*a*b^4*x^8 - 168*a^2*b^3*x
^5 + 368*a^3*b^2*x^2))*sqrt(-sqrt(3)*a/b^4) - 6*(12*a*b^2*x^8 + 48*a^2*b*x^
5 - 384*a^3*x^2 - 2*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 - 48*a^3*b^2 -
 sqrt(3)*(b^5*x^9 + 30*a*b^4*x^6 - 144*a^2*b^3*x^3 + 32*a^3*b^2 - sqrt(-3)*
(b^5*x^9 + 30*a*b^4*x^6 - 144*a^2*b^3*x^3 + 32*a^3*b^2)) - 3*sqrt(-3)*(b^5*
x^9 + 96*a^2*b^3*x^3 - 16*a^3*b^2))*(-sqrt(3)*a/b^4)^(2/3) - (1/9)^(1/3)*(b
^4*x^10 + 240*a^2*b^2*x^4 - 160*a^3*b*x + sqrt(-3)*(b^4*x^10 + 240*a^2*b^2*
```

```
x^4 - 160*a^3*b*x) - 24*sqrt(3)*(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x + sq
rt(-3)*(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x)))*(-sqrt(3)*a/b^4)^(1/3) - 8
*sqrt(3)*(a*b^2*x^8 - 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(b*x^3 - a) + (1/72)^(
1/6)*(3*b^4*x^12 + 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 - 2208*a^3*b*x^3 + 384*a
^4 - 3*sqrt(-3)*(b^4*x^12 + 4*a*b^3*x^9 + 360*a^2*b^2*x^6 - 736*a^3*b*x^3 +
 128*a^4) + sqrt(3)*(b^4*x^12 - 124*a*b^3*x^9 + 744*a^2*b^2*x^6 - 1120*a^3*
b*x^3 + 256*a^4 - sqrt(-3)*(b^4*x^12 - 124*a*b^3*x^9 + 744*a^2*b^2*x^6 - 11
20*a^3*b*x^3 + 256*a^4)))*(-sqrt(3)*a/b^4)^(1/6))/(b^4*x^12 - 40*a*b^3*x^9
+ 384*a^2*b^2*x^6 + 320*a^3*b*x^3 + 64*a^4)) - (1/72)^(1/6)*(sqrt(-3)*b + b
)*(-sqrt(3)*a/b^4)^(1/6)*log((72*(1/72)^(5/6)*(7*b^6*x^10 - 12*a*b^5*x^7 +
408*a^2*b^4*x^4 - 160*a^3*b^3*x - sqrt(-3)*(7*b^6*x^10 - 12*a*b^5*x^7 + 408
*a^2*b^4*x^4 - 160*a^3*b^3*x) - 3*sqrt(3)*(b^6*x^10 + 12*a*b^5*x^7 - 72*a^2
*b^4*x^4 + 32*a^3*b^3*x - sqrt(-3)*(b^6*x^10 + 12*a*b^5*x^7 - 72*a^2*b^4*x^
4 + 32*a^3*b^3*x)))*(-sqrt(3)*a/b^4)^(5/6) - 4*sqrt(1/2)*(3*b^5*x^11 + 18*a
*b^4*x^8 + 360*a^2*b^3*x^5 - 624*a^3*b^2*x^2 - sqrt(3)*(b^5*x^11 + 42*a*b^4
*x^8 - 168*a^2*b^3*x^5 + 368*a^3*b^2*x^2))*sqrt(-sqrt(3)*a/b^4) + 6*(12*a*b
^2*x^8 + 48*a^2*b*x^5 - 384*a^3*x^2 - 2*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^
3*x^3 - 48*a^3*b^2 - sqrt(3)*(b^5*x^9 + 30*a*b^4*x^6 - 144*a^2*b^3*x^3 + 32
*a^3*b^2 + sqrt(-3)*(b^5*x^9 + 30*a*b^4*x^6 - 144*a^2*b^3*x^3 + 32*a^3*b^2)
) + 3*sqrt(-3)*(b^5*x^9 + 96*a^2*b^3*x^3 - 16*a^3*b^2))*(-sqrt(3)*a/b^4)^(2
/3) - (1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 - 160*a^3*b*x - sqrt(-3)*(b^4
*x^10 + 240*a^2*b^2*x^4 - 160*a^3*b*x) - 24*sqrt(3)*(a*b^3*x^7 - 5*a^2*b^2*
x^4 + 4*a^3*b*x - sqrt(-3)*(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x)))*(-sqrt
(3)*a/b^4)^(1/3) - 8*sqrt(3)*(a*b^2*x^8 - 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(b
*x^3 - a) + (1/72)^(1/6)*(3*b^4*x^12 + 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 - 22
08*a^3*b*x^3 + 384*a^4 + 3*sqrt(-3)*(b^4*x^12 + 4*a*b^3*x^9 + 360*a^2*b^2*x
^6 - 736*a^3*b*x^3 + 128*a^4) + sqrt(3)*(b^4*x^12 - 124*a*b^3*x^9 + 744*a^2
*b^2*x^6 - 1120*a^3*b*x^3 + 256*a^4 + sqrt(-3)*(b^4*x^12 - 124*a*b^3*x^9 +
744*a^2*b^2*x^6 - 1120*a^3*b*x^3 + 256*a^4)))*(-sqrt(3)*a/b^4)^(1/6))/(b^4*
x^12 - 40*a*b^3*x^9 + 384*a^2*b^2*x^6 + 320*a^3*b*x^3 + 64*a^4)) + (1/72)^(
1/6)*(sqrt(-3)*b + b)*(-sqrt(3)*a/b^4)^(1/6)*log(-(72*(1/72)^(5/6)*(7*b^6*x
^10 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x - sqrt(-3)*(7*b^6*x^10
 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x) - 3*sqrt(3)*(b^6*x^10 +
12*a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x - sqrt(-3)*(b^6*x^10 + 12*a*b^
5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x)))*(-sqrt(3)*a/b^4)^(5/6) - 4*sqrt(1/
2)*(3*b^5*x^11 + 18*a*b^4*x^8 + 360*a^2*b^3*x^5 - 624*a^3*b^2*x^2 - sqrt(3)
*(b^5*x^11 + 42*a*b^4*x^8 - 168*a^2*b^3*x^5 + 368*a^3*b^2*x^2))*sqrt(-sqrt(
3)*a/b^4) - 6*(12*a*b^2*x^8 + 48*a^2*b*x^5 - 384*a^3*x^2 - 2*(1/9)^(2/3)*(3
*b^5*x^9 + 288*a^2*b^3*x^3 - 48*a^3*b^2 - sqrt(3)*(b^5*x^9 + 30*a*b^4*x^6 -
 144*a^2*b^3*x^3 + 32*a^3*b^2 + sqrt(-3)*(b^5*x^9 + 30*a*b^4*x^6 - 144*a^2*
b^3*x^3 + 32*a^3*b^2)) + 3*sqrt(-3)*(b^5*x^9 + 96*a^2*b^3*x^3 - 16*a^3*b^2)
)*(-sqrt(3)*a/b^4)^(2/3) - (1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 - 160*a^
3*b*x - sqrt(-3)*(b^4*x^10 + 240*a^2*b^2*x^4 - 160*a^3*b*x) - 24*sqrt(3)*(a
*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x - sqrt(-3)*(a*b^3*x^7 - 5*a^2*b^2*x^4
+ 4*a^3*b*x)))*(-sqrt(3)*a/b^4)^(1/3) - 8*sqrt(3)*(a*b^2*x^8 - 2*a^2*b*x^5
```

+ 28*a^3*x^2))*sqrt(b*x^3 - a) + (1/72)^(1/6)*(3*b^4*x^12 + 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 - 2208*a^3*b*x^3 + 384*a^4 + 3*sqrt(-3)*(b^4*x^12 + 4*a*b^3*x^9 + 360*a^2*b^2*x^6 - 736*a^3*b*x^3 + 128*a^4) + sqrt(3)*(b^4*x^12 - 124*a*b^3*x^9 + 744*a^2*b^2*x^6 - 1120*a^3*b*x^3 + 256*a^4 + sqrt(-3)*(b^4*x^12 - 124*a*b^3*x^9 + 744*a^2*b^2*x^6 - 1120*a^3*b*x^3 + 256*a^4)))*(-sqrt(3)*a/b^4)^(1/6))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2*x^6 + 320*a^3*b*x^3 + 64*a^4)) - 2*(1/72)^(1/6)*b*(-sqrt(3)*a/b^4)^(1/6)*log((72*(1/72)^(5/6)*(7*b^6*x^10 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x - 3*sqrt(3)*(b^6*x^10 + 12*a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x))*(-sqrt(3)*a/b^4)^(5/6) + 2*sqrt(1/2)*(3*b^5*x^11 + 18*a*b^4*x^8 + 360*a^2*b^3*x^5 - 624*a^3*b^2*x^2 - sqrt(3)*(b^5*x^11 + 42*a*b^4*x^8 - 168*a^2*b^3*x^5 + 368*a^3*b^2*x^2))*sqrt(-sqrt(3)*a/b^4) + 6*(6*a*b^2*x^8 + 24*a^2*b*x^5 - 192*a^3*x^2 + 2*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 - 48*a^3*b^2 - sqrt(3)*(b^5*x^9 + 30*a*b^4*x^6 - 144*a^2*b^3*x^3 + 32*a^3*b^2))*(-sqrt(3)*a/b^4)^(2/3) + (1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 - 160*a^3*b*x - 24*sqrt(3)*(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x))*(-sqrt(3)*a/b^4)^(1/3) - 4*sqrt(3)*(a*b^2*x^8 - 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(b*x^3 - a) + (1/72)^(1/6)*(3*b^4*x^12 + 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 - 2208*a^3*b*x^3 + 384*a^4 + sqrt(3)*(b^4*x^12 - 124*a*b^3*x^9 + 744*a^2*b^2*x^6 - 1120*a^3*b*x^3 + 256*a^4))*(-sqrt(3)*a/b^4)^(1/6))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2*x^6 + 320*a^3*b*x^3 + 64*a^4)) + 2*(1/72)^(1/6)*b*(-sqrt(3)*a/b^4)^(1/6)*log(-(72*(1/72)^(5/6)*(7*b^6*x^10 - 12*a*b^5*x^7 + 408*a^2*b^4*x^4 - 160*a^3*b^3*x - 3*sqrt(3)*(b^6*x^10 + 12*a*b^5*x^7 - 72*a^2*b^4*x^4 + 32*a^3*b^3*x))*(-sqrt(3)*a/b^4)^(5/6) + 2*sqrt(1/2)*(3*b^5*x^11 + 18*a*b^4*x^8 + 360*a^2*b^3*x^5 - 624*a^3*b^2*x^2 - sqrt(3)*(b^5*x^11 + 42*a*b^4*x^8 - 168*a^2*b^3*x^5 + 368*a^3*b^2*x^2))*sqrt(-sqrt(3)*a/b^4) - 6*(6*a*b^2*x^8 + 24*a^2*b*x^5 - 192*a^3*x^2 + 2*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 - 48*a^3*b^2 - sqrt(3)*(b^5*x^9 + 30*a*b^4*x^6 - 144*a^2*b^3*x^3 + 32*a^3*b^2))*(-sqrt(3)*a/b^4)^(2/3) + (1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 - 160*a^3*b*x - 24*sqrt(3)*(a*b^3*x^7 - 5*a^2*b^2*x^4 + 4*a^3*b*x))*(-sqrt(3)*a/b^4)^(1/3) - 4*sqrt(3)*(a*b^2*x^8 - 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(b*x^3 - a) + (1/72)^(1/6)*(3*b^4*x^12 + 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 - 2208*a^3*b*x^3 + 384*a^4 + sqrt(3)*(b^4*x^12 - 124*a*b^3*x^9 + 744*a^2*b^2*x^6 - 1120*a^3*b*x^3 + 256*a^4))*(-sqrt(3)*a/b^4)^(1/6))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2*x^6 + 320*a^3*b*x^3 + 64*a^4)) - 16*sqrt(b)*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b

Sympy [F]

$$\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx=-\int \frac{x\sqrt{-a+bx^3}}{-10a+6\sqrt{3}a+bx^3}\,dx$$

```
[In] integrate(x*(b*x**3-a)**(1/2)/(-b*x**3+2*a*(5-3*3**(1/2))),x)
[Out] -Integral(x*sqrt(-a + b*x**3)/(-10*a + 6*sqrt(3)*a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx=\int -\frac{\sqrt{bx^3-a}x}{bx^3+2\,a\left(3\,\sqrt{3}-5\right)}\,dx$$

```
[In] integrate(x*(b*x^3-a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="maxima
")
[Out] -integrate(sqrt(b*x^3 - a)*x/(b*x^3 + 2*a*(3*sqrt(3) - 5)), x)
```

Giac [F]

$$\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx=\int -\frac{\sqrt{bx^3-a}x}{bx^3+2\,a\left(3\,\sqrt{3}-5\right)}\,dx$$

```
[In] integrate(x*(b*x^3-a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="giac")
[Out] integrate(-sqrt(b*x^3 - a)*x/(b*x^3 + 2*a*(3*sqrt(3) - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx=\int -\frac{x\,\sqrt{b\,x^3-a}}{b\,x^3+2\,a\,\left(3\,\sqrt{3}-5\right)}\,dx$$

```
[In] int(-(x*(b*x^3 - a)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) - 5)),x)
[Out] int(-(x*(b*x^3 - a)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) - 5)), x)
```

3.349 $\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 768

$$\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx$$

$$= \frac{2\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{3^{3/4}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$+ \frac{\sqrt[6]{a}\arctan\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}} + \frac{\sqrt[4]{3}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{\sqrt{2}b^{2/3}}$$
$$+ \frac{\sqrt[4]{3}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}$$
$$- \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$
$$+ \frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

```
[Out] -1/4*3^(3/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1-3^(1
/2))*2^(1/2)/(-b*x^3-a)^(1/2))/b^(2/3)*2^(1/2)+1/6*a^(1/6)*arctan(1/6*(1+3^
(1/2))*(-b*x^3-a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*3^(3/4)/b^(2/3)*2^(1/2)+1/
2*3^(1/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*(-2*b^(1/3)*x+a^(1/3)*(1-3^(1
/2)))*2^(1/2)/(-b*x^3-a)^(1/2))/b^(2/3)*2^(1/2)+1/4*3^(1/4)*a^(1/6)*arctanh
(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(-b*x^3-a)^(1/
2))/b^(2/3)*2^(1/2)+2*(-b*x^3-a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2
)))+2/3*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1+3^(1/2)
))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)-a^(1/3)
*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/b^
(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-
```

```
3^(1/2)))^2)^(1/2)-3^(1/4)*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2
/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
*(1/2*6^(1/2)+1/2*2^(1/2))/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1
/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 768, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {495, 310, 225, 1893, 501}

$$\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx$$
$$= \frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$
$$-\frac{3^{3/4}\sqrt[6]{a}\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}+\frac{\sqrt[6]{a}\arctan\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}}$$
$$+\frac{\sqrt[4]{3}\sqrt[6]{a}\text{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{\sqrt{2}b^{2/3}}$$
$$+\frac{\sqrt[4]{3}\sqrt[6]{a}\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}+\frac{2\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

```
[In] Int[(x*Sqrt[-a - b*x^3])/(2*(5 - 3*Sqrt[3])*a + b*x^3),x]
```

```
[Out] (2*Sqrt[-a - b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(3/
4)*a^(1/6)*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sq
```

```
rt[2]*Sqrt[-a - b*x^3])])/(2*Sqrt[2]*b^(2/3)) + (a^(1/6)*ArcTan[((1 + Sqrt[
3])*Sqrt[-a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(Sqrt[2]*3^(1/4)*b^(2/3))
 + (3^(1/4)*a^(1/6)*ArcTanh[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) - 2*b^(
1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(Sqrt[2]*b^(2/3)) + (3^(1/4)*a^(1/6)*
ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt
[-a - b*x^3])])/(2*Sqrt[2]*b^(2/3)) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)*
Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2)]*Sqrt[-a - b*x^3]) + (2*Sqrt[2]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 501

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
```

```
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(3\left(3-2\sqrt{3}\right)a\right)\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx-\int \frac{x}{\sqrt{-a-bx^3}}\,dx \\
&= -\frac{3^{3/4}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}b^{2/3}}+\frac{\sqrt[6]{a}\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}} \\
&\quad+\frac{\sqrt[4]{3}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{\sqrt{2}b^{2/3}} \\
&\quad+\frac{\sqrt[4]{3}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}b^{2/3}} \\
&\quad-\frac{\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx}{\sqrt[3]{b}}+\frac{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)\int \frac{1}{\sqrt{-a-bx^3}}\,dx}{\sqrt[3]{b}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{3^{3/4}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}b^{2/3}} \\
&+ \frac{\sqrt[6]{a}\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}} + \frac{\sqrt[4]{3}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{\sqrt{2}b^{2/3}} \\
&+ \frac{\sqrt[4]{3}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}b^{2/3}} \\
&- \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}} \\
&+ \frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.12

$$
\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx = -\frac{x^2\sqrt{-a-bx^3}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)}{4\left(-5+3\sqrt{3}\right)a\sqrt{1+\frac{bx^3}{a}}}
$$

```
[In] Integrate[(x*Sqrt[-a - b*x^3])/(2*(5 - 3*Sqrt[3])*a + b*x^3),x]
```

```
[Out] -1/4*(x^2*Sqrt[-a - b*x^3]*AppellF1[2/3, -1/2, 1, 5/3, -((b*x^3)/a), -((b*x
^3)/(10*a - 6*Sqrt[3]*a))])/((-5 + 3*Sqrt[3])*a*Sqrt[1 + (b*x^3)/a])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.77 (sec) , antiderivative size = 983, normalized size of antiderivative = 1.28

method	result	size
elliptic	Expression too large to display	983
default	Expression too large to display	1001

```
[In] int(x*(-b*x^3-a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/
(-b*x^3-a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*El
lipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-1/9*I/b^3*2^(1/2)*sum(1/
_alpha*(2*3^(1/2)-3)*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(
1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*(
-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)
^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*(3
*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)-3*I*(-a*b^2)^(2/3
)*3^(1/2)+6*I*(-a*b^2)^(1/3)*_alpha*b-2*3^(1/2)*(-a*b^2)^(1/3)*_alpha*b+6*b
^2*_alpha^2-6*I*(-a*b^2)^(2/3)-2*3^(1/2)*(-a*b^2)^(2/3)-3*(-a*b^2)^(1/3)*_a
lpha*b-3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/6/b*(2*I
*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b-I*3^(1/2)*(-a*b^2)^(2/3)*_alpha+4*I*(-a*
b^2)^(1/3)*_alpha^2*b-2*I*(-a*b^2)^(2/3)*_alpha-2*3^(1/2)*(-a*b^2)^(2/3)*_a
lpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha+2*I*a*b-2*3^(1/2)*a*b-3*a*b)/a,(
I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3-6*3^(1/2)*a+10*a))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.41 (sec) , antiderivative size = 4875, normalized size of antiderivative = 6.35

$$\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(-b*x^3-a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="fricas
")
```

```
[Out] -1/8*((1/72)^(1/6)*(sqrt(-3)*b - b)*(-sqrt(3)*a/b^4)^(1/6)*log((72*(1/72)^(
5/6)*(7*b^6*x^10 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x + sqrt(-3
)*(7*b^6*x^10 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x) - 3*sqrt(3)
*(b^6*x^10 - 12*a*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x + sqrt(-3)*(b^6*x
^10 - 12*a*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x)))*(-sqrt(3)*a/b^4)^(5/6
) + 4*sqrt(1/2)*(3*b^5*x^11 - 18*a*b^4*x^8 + 360*a^2*b^3*x^5 + 624*a^3*b^2*
x^2 - sqrt(3)*(b^5*x^11 - 42*a*b^4*x^8 - 168*a^2*b^3*x^5 - 368*a^3*b^2*x^2)
)*sqrt(-sqrt(3)*a/b^4) + 6*(12*a*b^2*x^8 - 48*a^2*b*x^5 - 384*a^3*x^2 + 2*(
1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 + 48*a^3*b^2 - sqrt(3)*(b^5*x^9 - 3
0*a*b^4*x^6 - 144*a^2*b^3*x^3 - 32*a^3*b^2 - sqrt(-3)*(b^5*x^9 - 30*a*b^4*x
^6 - 144*a^2*b^3*x^3 - 32*a^3*b^2)) - 3*sqrt(-3)*(b^5*x^9 + 96*a^2*b^3*x^3
+ 16*a^3*b^2))*(-sqrt(3)*a/b^4)^(2/3) - (1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2
*x^4 + 160*a^3*b*x + sqrt(-3)*(b^4*x^10 + 240*a^2*b^2*x^4 + 160*a^3*b*x) +
24*sqrt(3)*(a*b^3*x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x + sqrt(-3)*(a*b^3*x^7 + 5
*a^2*b^2*x^4 + 4*a^3*b*x)))*(-sqrt(3)*a/b^4)^(1/3) - 8*sqrt(3)*(a*b^2*x^8 +
 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(-b*x^3 - a) + (1/72)^(1/6)*(3*b^4*x^12 - 1
2*a*b^3*x^9 + 1080*a^2*b^2*x^6 + 2208*a^3*b*x^3 + 384*a^4 + sqrt(3)*(b^4*x^
12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*b*x^3 + 256*a^4 - sqrt(-3)*
(b^4*x^12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*b*x^3 + 256*a^4)) -
3*sqrt(-3)*(b^4*x^12 - 4*a*b^3*x^9 + 360*a^2*b^2*x^6 + 736*a^3*b*x^3 + 128*
a^4))*(-sqrt(3)*a/b^4)^(1/6))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 -
320*a^3*b*x^3 + 64*a^4)) - (1/72)^(1/6)*(sqrt(-3)*b - b)*(-sqrt(3)*a/b^4)^(
1/6)*log(-(72*(1/72)^(5/6)*(7*b^6*x^10 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 1
60*a^3*b^3*x + sqrt(-3)*(7*b^6*x^10 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*
a^3*b^3*x) - 3*sqrt(3)*(b^6*x^10 - 12*a*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b
^3*x + sqrt(-3)*(b^6*x^10 - 12*a*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x)))
*(-sqrt(3)*a/b^4)^(5/6) + 4*sqrt(1/2)*(3*b^5*x^11 - 18*a*b^4*x^8 + 360*a^2*
b^3*x^5 + 624*a^3*b^2*x^2 - sqrt(3)*(b^5*x^11 - 42*a*b^4*x^8 - 168*a^2*b^3*
x^5 - 368*a^3*b^2*x^2))*sqrt(-sqrt(3)*a/b^4) - 6*(12*a*b^2*x^8 - 48*a^2*b*x
^5 - 384*a^3*x^2 + 2*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 + 48*a^3*b^2
- sqrt(3)*(b^5*x^9 - 30*a*b^4*x^6 - 144*a^2*b^3*x^3 - 32*a^3*b^2 - sqrt(-3)
*(b^5*x^9 - 30*a*b^4*x^6 - 144*a^2*b^3*x^3 - 32*a^3*b^2)) - 3*sqrt(-3)*(b^5
*x^9 + 96*a^2*b^3*x^3 + 16*a^3*b^2))*(-sqrt(3)*a/b^4)^(2/3) - (1/9)^(1/3)*(
b^4*x^10 + 240*a^2*b^2*x^4 + 160*a^3*b*x + sqrt(-3)*(b^4*x^10 + 240*a^2*b^2
```

```
*x^4 + 160*a^3*b*x) + 24*sqrt(3)*(a*b^3*x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x + s
qrt(-3)*(a*b^3*x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x)))*(-sqrt(3)*a/b^4)^(1/3) -
8*sqrt(3)*(a*b^2*x^8 + 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(-b*x^3 - a) + (1/72)
^(1/6)*(3*b^4*x^12 - 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 + 2208*a^3*b*x^3 + 384
*a^4 + sqrt(3)*(b^4*x^12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*b*x^3
 + 256*a^4 - sqrt(-3)*(b^4*x^12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^
3*b*x^3 + 256*a^4)) - 3*sqrt(-3)*(b^4*x^12 - 4*a*b^3*x^9 + 360*a^2*b^2*x^6
+ 736*a^3*b*x^3 + 128*a^4))*(-sqrt(3)*a/b^4)^(1/6))/(b^4*x^12 + 40*a*b^3*x^
9 + 384*a^2*b^2*x^6 - 320*a^3*b*x^3 + 64*a^4)) - (1/72)^(1/6)*(sqrt(-3)*b +
 b)*(-sqrt(3)*a/b^4)^(1/6)*log((72*(1/72)^(5/6)*(7*b^6*x^10 + 12*a*b^5*x^7
+ 408*a^2*b^4*x^4 + 160*a^3*b^3*x - sqrt(-3)*(7*b^6*x^10 + 12*a*b^5*x^7 + 4
08*a^2*b^4*x^4 + 160*a^3*b^3*x) - 3*sqrt(3)*(b^6*x^10 - 12*a*b^5*x^7 - 72*a
^2*b^4*x^4 - 32*a^3*b^3*x - sqrt(-3)*(b^6*x^10 - 12*a*b^5*x^7 - 72*a^2*b^4*
x^4 - 32*a^3*b^3*x)))*(-sqrt(3)*a/b^4)^(5/6) + 4*sqrt(1/2)*(3*b^5*x^11 - 18
*a*b^4*x^8 + 360*a^2*b^3*x^5 + 624*a^3*b^2*x^2 - sqrt(3)*(b^5*x^11 - 42*a*b
^4*x^8 - 168*a^2*b^3*x^5 - 368*a^3*b^2*x^2))*sqrt(-sqrt(3)*a/b^4) + 6*(12*a
*b^2*x^8 - 48*a^2*b*x^5 - 384*a^3*x^2 + 2*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*
b^3*x^3 + 48*a^3*b^2 - sqrt(3)*(b^5*x^9 - 30*a*b^4*x^6 - 144*a^2*b^3*x^3 -
32*a^3*b^2 + sqrt(-3)*(b^5*x^9 - 30*a*b^4*x^6 - 144*a^2*b^3*x^3 - 32*a^3*b^
2)) + 3*sqrt(-3)*(b^5*x^9 + 96*a^2*b^3*x^3 + 16*a^3*b^2))*(-sqrt(3)*a/b^4)^
(2/3) - (1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 + 160*a^3*b*x - sqrt(-3)*(b
^4*x^10 + 240*a^2*b^2*x^4 + 160*a^3*b*x) + 24*sqrt(3)*(a*b^3*x^7 + 5*a^2*b^
2*x^4 + 4*a^3*b*x - sqrt(-3)*(a*b^3*x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x)))*(-sq
rt(3)*a/b^4)^(1/3) - 8*sqrt(3)*(a*b^2*x^8 + 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt
(-b*x^3 - a) + (1/72)^(1/6)*(3*b^4*x^12 - 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 +
 2208*a^3*b*x^3 + 384*a^4 + sqrt(3)*(b^4*x^12 + 124*a*b^3*x^9 + 744*a^2*b^2
*x^6 + 1120*a^3*b*x^3 + 256*a^4 + sqrt(-3)*(b^4*x^12 + 124*a*b^3*x^9 + 744*
a^2*b^2*x^6 + 1120*a^3*b*x^3 + 256*a^4)) + 3*sqrt(-3)*(b^4*x^12 - 4*a*b^3*x
^9 + 360*a^2*b^2*x^6 + 736*a^3*b*x^3 + 128*a^4))*(-sqrt(3)*a/b^4)^(1/6))/(b
^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*x^3 + 64*a^4)) + (1/72
)^(1/6)*(sqrt(-3)*b + b)*(-sqrt(3)*a/b^4)^(1/6)*log(-(72*(1/72)^(5/6)*(7*b^
6*x^10 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x - sqrt(-3)*(7*b^6*x
^10 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x) - 3*sqrt(3)*(b^6*x^10
 - 12*a*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x - sqrt(-3)*(b^6*x^10 - 12*a
*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x)))*(-sqrt(3)*a/b^4)^(5/6) + 4*sqrt
(1/2)*(3*b^5*x^11 - 18*a*b^4*x^8 + 360*a^2*b^3*x^5 + 624*a^3*b^2*x^2 - sqrt
(3)*(b^5*x^11 - 42*a*b^4*x^8 - 168*a^2*b^3*x^5 - 368*a^3*b^2*x^2))*sqrt(-sq
rt(3)*a/b^4) - 6*(12*a*b^2*x^8 - 48*a^2*b*x^5 - 384*a^3*x^2 + 2*(1/9)^(2/3)
*(3*b^5*x^9 + 288*a^2*b^3*x^3 + 48*a^3*b^2 - sqrt(3)*(b^5*x^9 - 30*a*b^4*x^
6 - 144*a^2*b^3*x^3 - 32*a^3*b^2 + sqrt(-3)*(b^5*x^9 - 30*a*b^4*x^6 - 144*a
^2*b^3*x^3 - 32*a^3*b^2)) + 3*sqrt(-3)*(b^5*x^9 + 96*a^2*b^3*x^3 + 16*a^3*b
^2))*(-sqrt(3)*a/b^4)^(2/3) - (1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 + 160
*a^3*b*x - sqrt(-3)*(b^4*x^10 + 240*a^2*b^2*x^4 + 160*a^3*b*x) + 24*sqrt(3)
*(a*b^3*x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x - sqrt(-3)*(a*b^3*x^7 + 5*a^2*b^2*x
^4 + 4*a^3*b*x)))*(-sqrt(3)*a/b^4)^(1/3) - 8*sqrt(3)*(a*b^2*x^8 + 2*a^2*b*x
```

```
^5 + 28*a^3*x^2))*sqrt(-b*x^3 - a) + (1/72)^(1/6)*(3*b^4*x^12 - 12*a*b^3*x^
9 + 1080*a^2*b^2*x^6 + 2208*a^3*b*x^3 + 384*a^4 + sqrt(3)*(b^4*x^12 + 124*a
*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*b*x^3 + 256*a^4 + sqrt(-3)*(b^4*x^12
+ 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*b*x^3 + 256*a^4)) + 3*sqrt(-3)
*(b^4*x^12 - 4*a*b^3*x^9 + 360*a^2*b^2*x^6 + 736*a^3*b*x^3 + 128*a^4))*(-sq
rt(3)*a/b^4)^(1/6))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*
x^3 + 64*a^4)) - 2*(1/72)^(1/6)*b*(-sqrt(3)*a/b^4)^(1/6)*log((72*(1/72)^(5/
6)*(7*b^6*x^10 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x - 3*sqrt(3)
*(b^6*x^10 - 12*a*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x))*(-sqrt(3)*a/b^4
)^(5/6) - 2*sqrt(1/2)*(3*b^5*x^11 - 18*a*b^4*x^8 + 360*a^2*b^3*x^5 + 624*a^
3*b^2*x^2 - sqrt(3)*(b^5*x^11 - 42*a*b^4*x^8 - 168*a^2*b^3*x^5 - 368*a^3*b^
2*x^2))*sqrt(-sqrt(3)*a/b^4) + 6*(6*a*b^2*x^8 - 24*a^2*b*x^5 - 192*a^3*x^2
- 2*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 + 48*a^3*b^2 - sqrt(3)*(b^5*x^
9 - 30*a*b^4*x^6 - 144*a^2*b^3*x^3 - 32*a^3*b^2))*(-sqrt(3)*a/b^4)^(2/3) +
(1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 + 160*a^3*b*x + 24*sqrt(3)*(a*b^3*x
^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x))*(-sqrt(3)*a/b^4)^(1/3) - 4*sqrt(3)*(a*b^2*
x^8 + 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(-b*x^3 - a) + (1/72)^(1/6)*(3*b^4*x^1
2 - 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 + 2208*a^3*b*x^3 + 384*a^4 + sqrt(3)*(b
^4*x^12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*b*x^3 + 256*a^4))*(-sq
rt(3)*a/b^4)^(1/6))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*
x^3 + 64*a^4)) + 2*(1/72)^(1/6)*b*(-sqrt(3)*a/b^4)^(1/6)*log(-(72*(1/72)^(5
/6)*(7*b^6*x^10 + 12*a*b^5*x^7 + 408*a^2*b^4*x^4 + 160*a^3*b^3*x - 3*sqrt(3
)*(b^6*x^10 - 12*a*b^5*x^7 - 72*a^2*b^4*x^4 - 32*a^3*b^3*x))*(-sqrt(3)*a/b^
4)^(5/6) - 2*sqrt(1/2)*(3*b^5*x^11 - 18*a*b^4*x^8 + 360*a^2*b^3*x^5 + 624*a
^3*b^2*x^2 - sqrt(3)*(b^5*x^11 - 42*a*b^4*x^8 - 168*a^2*b^3*x^5 - 368*a^3*b
^2*x^2))*sqrt(-sqrt(3)*a/b^4) - 6*(6*a*b^2*x^8 - 24*a^2*b*x^5 - 192*a^3*x^2
 - 2*(1/9)^(2/3)*(3*b^5*x^9 + 288*a^2*b^3*x^3 + 48*a^3*b^2 - sqrt(3)*(b^5*x
^9 - 30*a*b^4*x^6 - 144*a^2*b^3*x^3 - 32*a^3*b^2))*(-sqrt(3)*a/b^4)^(2/3) +
 (1/9)^(1/3)*(b^4*x^10 + 240*a^2*b^2*x^4 + 160*a^3*b*x + 24*sqrt(3)*(a*b^3*
x^7 + 5*a^2*b^2*x^4 + 4*a^3*b*x))*(-sqrt(3)*a/b^4)^(1/3) - 4*sqrt(3)*(a*b^2
*x^8 + 2*a^2*b*x^5 + 28*a^3*x^2))*sqrt(-b*x^3 - a) + (1/72)^(1/6)*(3*b^4*x^
12 - 12*a*b^3*x^9 + 1080*a^2*b^2*x^6 + 2208*a^3*b*x^3 + 384*a^4 + sqrt(3)*(
b^4*x^12 + 124*a*b^3*x^9 + 744*a^2*b^2*x^6 + 1120*a^3*b*x^3 + 256*a^4))*(-s
qrt(3)*a/b^4)^(1/6))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b
*x^3 + 64*a^4)) + 16*sqrt(-b)*weierstrassZeta(0, -4*a/b, weierstrassPInvers
e(0, -4*a/b, x)))/b
```

Sympy [F]

$$\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{x\sqrt{-a-bx^3}}{-6\sqrt{3}a+10a+bx^3}\,dx$$

```
[In] integrate(x*(-b*x**3-a)**(1/2)/(b*x**3+2*a*(5-3*3**(1/2))),x)
[Out] Integral(x*sqrt(-a - b*x**3)/(-6*sqrt(3)*a + 10*a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{\sqrt{-bx^3-a}\,x}{bx^3-2\,a\left(3\sqrt{3}-5\right)}\,dx$$

```
[In] integrate(x*(-b*x^3-a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="maxima
")
[Out] integrate(sqrt(-b*x^3 - a)*x/(b*x^3 - 2*a*(3*sqrt(3) - 5)), x)
```

Giac [F]

$$\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{\sqrt{-bx^3-a}\,x}{bx^3-2\,a\left(3\sqrt{3}-5\right)}\,dx$$

```
[In] integrate(x*(-b*x^3-a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="giac")
[Out] integrate(sqrt(-b*x^3 - a)*x/(b*x^3 - 2*a*(3*sqrt(3) - 5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx=\int \frac{x\,\sqrt{-b\,x^3-a}}{b\,x^3-2\,a\,\left(3\,\sqrt{3}-5\right)}\,dx$$

```
[In] int((x*(- a - b*x^3)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) - 5)),x)
[Out] int((x*(- a - b*x^3)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) - 5)), x)
```

3.350 $\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 318

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx = -\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

```
[Out] -1/12*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(b
*x^3+a)^(1/2))*(2-3^(1/2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)-1/18*arctan(1/6*
(1-3^(1/2))*(b*x^3+a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*(2-3^(1/2))*3^(1/4)/a^
(5/6)/b^(2/3)*2^(1/2)-1/36*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*
(1-3^(1/2))*2^(1/2)/(b*x^3+a)^(1/2))*(2-3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^
(1/2)-1/18*arctanh(1/2*3^(1/4)*a^(1/6)*(-2*b^(1/3)*x+a^(1/3)*(1+3^(1/2)))*2
^(1/2)/(b*x^3+a)^(1/2))*(2-3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {500}

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx = -\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

```
[In] Int[x/(Sqrt[a + b*x^3]*(2*(5 + 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] -1/2*((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3
)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 -
 Sqrt[3])*ArcTan[((1 - Sqrt[3])*Sqrt[a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])]
)/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*a^(
1/6)*((1 + Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(3*
Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqr
t[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]
*3^(1/4)*a^(5/6)*b^(2/3))
```

Rule 500

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\text{integral} = -\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.26

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx = \frac{x^2\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bx^3}{10a+6\sqrt{3}a}\right)}{\left(20a+12\sqrt{3}a\right)\sqrt{a+bx^3}}$$

```
[In] Integrate[x/(Sqrt[a + b*x^3]*(2*(5 + 3*Sqrt[3])*a + b*x^3)),x]
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3)
/(10*a + 6*Sqrt[3]*a))])/((20*a + 12*Sqrt[3]*a)*Sqrt[a + b*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.83 (sec) , antiderivative size = 538, normalized size of antiderivative = 1.69

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+6\sqrt{3}\,a+10a\right)}\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{ib\left(2x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{b\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(-a\,b^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{ib\left(2x+\frac{(-a}{}\right.}{}}}{}\right.}{}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+6\sqrt{3}\,a+10a\right)}\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{ib\left(2x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{b\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(-a\,b^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{ib\left(2x+\frac{(-a}{}\right.}{}}}{}\right.}{}$

```
[In] int(x/(b*x^3+2*a*(5+3*3^(1/2)))/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/27*I/a/b^3*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^
2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2
)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x
+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^
3+a)^(1/2)*(-3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)+3*I
*(-a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(-a*b^2)^(1/3)*_alpha*b+6*I*(-a*b^2)^(1/3
)*_alpha*b-6*b^2*_alpha^2-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(-a*b^2)^(2/3)+3*(-a
*b^2)^(1/3)*_alpha*b+3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2),-1/6/b*(2*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b-I*3^(1/2)*(-a*b^2)^(2/3)*_
alpha-4*I*(-a*b^2)^(1/3)*_alpha^2*b+2*3^(1/2)*(-a*b^2)^(2/3)*_alpha+I*3^(1/
2)*a*b+2*I*(-a*b^2)^(2/3)*_alpha+2*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-2*I*
a*b-3*a*b)/a,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
```

```
2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3+6*3^(1/2)*a+10*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5563 vs. $2(211) = 422$.

Time = 3.56 (sec) , antiderivative size = 5563, normalized size of antiderivative = 17.49

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(b*x^3+2*a*(5+3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="fricas"
)
[Out] Too large to include
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx = \int \frac{x}{\sqrt{a+bx^3}\cdot\left(10a+6\sqrt{3}a+bx^3\right)}\,dx$$

```
[In] integrate(x/(b*x**3+2*a*(5+3*3**(1/2)))/(b*x**3+a)**(1/2),x)
[Out] Integral(x/(sqrt(a + b*x**3)*(10*a + 6*sqrt(3)*a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx = \int \frac{x}{\left(bx^3+2\,a\left(3\,\sqrt{3}+5\right)\right)\sqrt{bx^3+a}}\,dx$$

```
[In] integrate(x/(b*x^3+2*a*(5+3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="maxima"
)
[Out] integrate(x/((b*x^3 + 2*a*(3*sqrt(3) + 5))*sqrt(b*x^3 + a)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x/(b*x^3+2*a*(5+3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx = \int \frac{x}{\sqrt{b\,x^3+a}\,\left(b\,x^3+2\,a\,\left(3\,\sqrt{3}+5\right)\right)}\,dx$$

```
[In] int(x/((a + b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) + 5))),x)
```

```
[Out] int(x/((a + b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) + 5))), x)
```

3.351 $\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 324

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = -\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

```
[Out] -1/12*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(-
b*x^3+a)^(1/2))*(2-3^(1/2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)-1/18*arctan(1/6
*(1-3^(1/2))*(-b*x^3+a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*(2-3^(1/2))*3^(1/4)/
a^(5/6)/b^(2/3)*2^(1/2)-1/36*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x
)*(1-3^(1/2))*2^(1/2)/(-b*x^3+a)^(1/2))*(2-3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)
*2^(1/2)-1/18*arctanh(1/2*3^(1/4)*a^(1/6)*(2*b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
*2^(1/2)/(-b*x^3+a)^(1/2))*(2-3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {500}

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = -\frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

```
[In] Int[x/(Sqrt[a - b*x^3]*(2*(5 + 3*Sqrt[3])*a - b*x^3)),x]
```

```
[Out] -1/2*((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3
)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 -
 Sqrt[3])*ArcTan[((1 - Sqrt[3])*Sqrt[a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])]
)/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1
- Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(6*Sq
rt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*a^(1/6)*((
1 + Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(3*Sqrt[2]
*3^(1/4)*a^(5/6)*b^(2/3))
```

Rule 500

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\text{integral} = -\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.26

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = \frac{x^2\sqrt{1-\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right)}{\left(20a+12\sqrt{3}a\right)\sqrt{a-bx^3}}$$

```
[In] Integrate[x/(Sqrt[a - b*x^3]*(2*(5 + 3*Sqrt[3])*a - b*x^3)),x]
[Out] (x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*
a + 6*Sqrt[3]*a)])/((20*a + 12*Sqrt[3]*a)*Sqrt[a - b*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.69 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.57

method	result
default	$\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}-6\sqrt{3}\,a-10a\right)}\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{ib\left(2x+\frac{i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}+\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{2\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{-i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}+}{b}\right.}{\left(a\,b^{2}\right)^{\frac{1}{3}}}}}{}\right.}{}$
elliptic	$\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}-6\sqrt{3}\,a-10a\right)}\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{ib\left(2x+\frac{i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}+\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{2\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{-i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}+}{b}\right.}{\left(a\,b^{2}\right)^{\frac{1}{3}}}}}{}\right.}{}$

```
[In] int(x/(-b*x^3+2*a*(5+3*3^(1/2)))/(-b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/27*I/a/b^3*2^(1/2)*sum(1/_alpha*(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1/
2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1/
3))/(-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-I
*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2
)*(3*I*(a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)-3*I*(a*b^2)^(2
/3)*3^(1/2)-2*3^(1/2)*(a*b^2)^(1/3)*_alpha*b-6*I*(a*b^2)^(1/3)*_alpha*b-6*b
^2*_alpha^2-2*3^(1/2)*(a*b^2)^(2/3)+6*I*(a*b^2)^(2/3)+3*(a*b^2)^(1/3)*_alph
a*b+3*(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*3^(1/
2)*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1/2)*(a*b^2)^(2/3)*_alpha+4*I*(a*b^2)^(1/3
)*_alpha^2*b+I*3^(1/2)*a*b+2*3^(1/2)*(a*b^2)^(2/3)*_alpha-2*I*(a*b^2)^(2/3)
*_alpha-2*3^(1/2)*a*b-2*I*a*b-3*(a*b^2)^(2/3)*_alpha+3*a*b)/a,(-I*3^(1/2)/b
*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))
```

```
,_alpha=RootOf(b*_Z^3-6*3^(1/2)*a-10*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5587 vs. $2(218) = 436$.

Time = 3.50 (sec) , antiderivative size = 5587, normalized size of antiderivative = 17.24

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(-b*x^3+2*a*(5+3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="frica
s")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = -\int \frac{x}{-6\sqrt{3}a\sqrt{a-bx^3}-10a\sqrt{a-bx^3}+bx^3\sqrt{a-bx^3}}\,dx$$

```
[In] integrate(x/(-b*x**3+2*a*(5+3*3**(1/2)))/(-b*x**3+a)**(1/2),x)
[Out] -Integral(x/(-6*sqrt(3)*a*sqrt(a - b*x**3) - 10*a*sqrt(a - b*x**3) + b*x**3
*sqrt(a - b*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = \int -\frac{x}{\left(bx^3-2\,a\left(3\sqrt{3}+5\right)\right)\sqrt{-bx^3+a}}\,dx$$

```
[In] integrate(x/(-b*x^3+2*a*(5+3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="maxim
a")
[Out] -integrate(x/((b*x^3 - 2*a*(3*sqrt(3) + 5))*sqrt(-b*x^3 + a)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a-bx^3}\,\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x/(-b*x^3+2*a*(5+3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a-bx^3}\,\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = -\int \frac{x}{\sqrt{a-b\,x^3}\,\left(b\,x^3-2\,a\,\left(3\,\sqrt{3}+5\right)\right)}\,dx$$

```
[In] int(-x/((a - b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) + 5))),x)
```

```
[Out] -int(x/((a - b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) + 5))), x)
```

3.352 $\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 328

$$\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx = \frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{\left(2-\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\text{arctanh}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{-a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

```
[Out] 1/36*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1-3^(1/2))*2^(1/2)/(b*
x^3-a)^(1/2))*(2-3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)+1/18*arctan(1/2*3
^(1/4)*a^(1/6)*(2*b^(1/3)*x+a^(1/3)*(1+3^(1/2)))*2^(1/2)/(b*x^3-a)^(1/2))*(
2-3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)+1/12*arctanh(1/2*3^(1/4)*a^(1/6)
*(a^(1/3)-b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(b*x^3-a)^(1/2))*(2-3^(1/2))*3^(1/
4)/a^(5/6)/b^(2/3)*2^(1/2)-1/18*arctanh(1/6*(1-3^(1/2))*(b*x^3-a)^(1/2)*3^(
1/4)*2^(1/2)/a^(1/2))*(2-3^(1/2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {501}

$$\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx = \frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{bx^3-a}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

```
[In] Int[x/(Sqrt[-a + b*x^3]*(-2*(5 + 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] ((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))
/(Sqrt[2]*Sqrt[-a + b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 - S
qrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqr
t[2]*Sqrt[-a + b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 - Sqrt[3
])*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*S
qrt[-a + b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*Arc
Tanh[((1 - Sqrt[3])*Sqrt[-a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2
]*3^(3/4)*a^(5/6)*b^(2/3))
```

Rule 501

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\text{integral} = \frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{-a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.26

$$\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx = -\frac{x^2\sqrt{1-\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right)}{\left(20a+12\sqrt{3}a\right)\sqrt{-a+bx^3}}$$

```
[In] Integrate[x/(Sqrt[-a + b*x^3]*(-2*(5 + 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] -((x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(1
0*a + 6*Sqrt[3]*a)])/((20*a + 12*Sqrt[3]*a)*Sqrt[-a + b*x^3]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.86 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.55

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}-6\sqrt{3}\,a-10a\right)}\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{ib\left(2x+\frac{i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}+\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{2\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{-i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right.}{\left(a\,b^{2}\right)^{\frac{1}{3}}}}}{}\right.}{}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}-6\sqrt{3}\,a-10a\right)}\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{ib\left(2x+\frac{i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}+\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{2\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{-i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right.}{\left(a\,b^{2}\right)^{\frac{1}{3}}}}}{}\right.}{}$

```
[In] int(x/(b*x^3-2*a*(5+3*3^(1/2)))/(b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/27*I/a/b^3*2^(1/2)*sum(1/_alpha*(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1
/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1
/3))/(-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-
I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2
)*(3*I*(a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)-3*I*(a*b^2)^(2
/3)*3^(1/2)-2*3^(1/2)*(a*b^2)^(1/3)*_alpha*b-6*I*(a*b^2)^(1/3)*_alpha*b-6*b
^2*_alpha^2-2*3^(1/2)*(a*b^2)^(2/3)+6*I*(a*b^2)^(2/3)+3*(a*b^2)^(1/3)*_alph
a*b+3*(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*3^(1/
2)*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1/2)*(a*b^2)^(2/3)*_alpha+4*I*(a*b^2)^(1/3
)*_alpha^2*b+I*3^(1/2)*a*b+2*3^(1/2)*(a*b^2)^(2/3)*_alpha-2*I*(a*b^2)^(2/3)
*_alpha-2*3^(1/2)*a*b-2*I*a*b-3*(a*b^2)^(2/3)*_alpha+3*a*b)/a,(-I*3^(1/2)/b
*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))
```

```
,_alpha=RootOf(b*_Z^3-6*3^(1/2)*a-10*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5667 vs. 2(222) = 444.

Time = 3.69 (sec) , antiderivative size = 5667, normalized size of antiderivative = 17.28

$$\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(b*x^3-2*a*(5+3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="fricas"
)
[Out] Too large to include
```

Sympy [F]

$$\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx = \int \frac{x}{\sqrt{-a+bx^3}\left(-6\sqrt{3}a-10a+bx^3\right)}\,dx$$

```
[In] integrate(x/(b*x**3-2*a*(5+3*3**(1/2)))/(b*x**3-a)**(1/2),x)
[Out] Integral(x/(sqrt(-a + b*x**3)*(-6*sqrt(3)*a - 10*a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx = \int \frac{x}{\left(bx^3-2\,a\left(3\,\sqrt{3}+5\right)\right)\sqrt{bx^3-a}}\,dx$$

```
[In] integrate(x/(b*x^3-2*a*(5+3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="maxima"
)
[Out] integrate(x/((b*x^3 - 2*a*(3*sqrt(3) + 5))*sqrt(b*x^3 - a)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x/(b*x^3-2*a*(5+3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx = \int \frac{x}{\sqrt{b\,x^3-a}\,\left(b\,x^3-2\,a\,\left(3\,\sqrt{3}+5\right)\right)}\,dx$$

```
[In] int(x/((b*x^3 - a)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) + 5))),x)
```

```
[Out] int(x/((b*x^3 - a)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) + 5))), x)
```

3.353 $\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 330

$$\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = \frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

```
[Out] 1/36*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1-3^(1/2))*2^(1/2)/(-b
*x^3-a)^(1/2))*(2-3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)+1/18*arctan(1/2*
3^(1/4)*a^(1/6)*(-2*b^(1/3)*x+a^(1/3)*(1+3^(1/2)))*2^(1/2)/(-b*x^3-a)^(1/2)
)*(2-3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)+1/12*arctanh(1/2*3^(1/4)*a^(1
/6)*(a^(1/3)+b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(-b*x^3-a)^(1/2))*(2-3^(1/2))*3
^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)-1/18*arctanh(1/6*(1-3^(1/2))*(-b*x^3-a)^(1/2
)*3^(1/4)*2^(1/2)/a^(1/2))*(2-3^(1/2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {501}

$$\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = \frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{\left(2-\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{\left(2-\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\text{arctanh}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

```
[In] Int[x/(Sqrt[-a - b*x^3]*(-2*(5 + 3*Sqrt[3])*a - b*x^3)),x]
```

```
[Out] ((2 - Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) - 2*b^(1/3)*x
))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 -
 Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqr
t[2]*Sqrt[-a - b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 - Sqrt[3
])*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*S
qrt[-a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*Arc
Tanh[((1 - Sqrt[3])*Sqrt[-a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2
]*3^(3/4)*a^(5/6)*b^(2/3))
```

Rule 501

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\text{integral} = \frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.26

$$\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = -\frac{x^2\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bx^3}{10a+6\sqrt{3}a}\right)}{\left(20a+12\sqrt{3}a\right)\sqrt{-a-bx^3}}$$

```
[In] Integrate[x/(Sqrt[-a - b*x^3]*(-2*(5 + 3*Sqrt[3])*a - b*x^3)),x]
[Out] -((x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^
3)/(10*a + 6*Sqrt[3]*a))])/((20*a + 12*Sqrt[3]*a)*Sqrt[-a - b*x^3]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.75 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.64

method	result
default	$\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+6\sqrt{3}\,a+10a\right)}\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{ib\left(2x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{b\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(-a\,b^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{ib\left(2x+\frac{\left(-a\,b^{2}\right.}{}\right.}{2(}}}{}\right.}{}$
elliptic	$\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+6\sqrt{3}\,a+10a\right)}\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{ib\left(2x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{b\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(-a\,b^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{ib\left(2x+\frac{\left(-a\,b^{2}\right.}{}\right.}{2(}}}{}\right.}{}$

```
[In] int(x/(-b*x^3-2*a*(5+3*3^(1/2)))/(-b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/27*I/a/b^3*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2
)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)
^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+
1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(-b*x^
3-a)^(1/2)*(-3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)+3*I
*(-a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(-a*b^2)^(1/3)*_alpha*b+6*I*(-a*b^2)^(1/3
)*_alpha*b-6*b^2*_alpha^2-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(-a*b^2)^(2/3)+3*(-a
*b^2)^(1/3)*_alpha*b+3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2),-1/6/b*(2*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b-I*3^(1/2)*(-a*b^2)^(2/3)*_
alpha-4*I*(-a*b^2)^(1/3)*_alpha^2*b+2*3^(1/2)*(-a*b^2)^(2/3)*_alpha+I*3^(1/
2)*a*b+2*I*(-a*b^2)^(2/3)*_alpha+2*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-2*I*
a*b-3*a*b)/a,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
```

```
2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3+6*3^(1/2)*a+10*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5679 vs. 2(223) = 446.

Time = 3.55 (sec) , antiderivative size = 5679, normalized size of antiderivative = 17.21

$$\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(-b*x^3-2*a*(5+3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="frica
s")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx$$
$$= -\int \frac{x}{10a\sqrt{-a-bx^3}+6\sqrt{3}a\sqrt{-a-bx^3}+bx^3\sqrt{-a-bx^3}}\,dx$$

```
[In] integrate(x/(-b*x**3-2*a*(5+3*3**(1/2)))/(-b*x**3-a)**(1/2),x)
[Out] -Integral(x/(10*a*sqrt(-a - b*x**3) + 6*sqrt(3)*a*sqrt(-a - b*x**3) + b*x**
3*sqrt(-a - b*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = \int -\frac{x}{\left(bx^3+2\,a\left(3\sqrt{3}+5\right)\right)\sqrt{-bx^3-a}}\,dx$$

```
[In] integrate(x/(-b*x^3-2*a*(5+3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="maxim
a")
[Out] -integrate(x/((b*x^3 + 2*a*(3*sqrt(3) + 5))*sqrt(-b*x^3 - a)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x/(-b*x^3-2*a*(5+3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx = \int -\frac{x}{\sqrt{-b\,x^3-a}\,\left(b\,x^3+2\,a\,\left(3\,\sqrt{3}+5\right)\right)}\,dx$$

```
[In] int(-x/((- a - b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) + 5))),x)
```

```
[Out] int(-x/((- a - b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) + 5))), x)
```

3.354 $\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 310

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = -\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{\left(2+\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} + \frac{\left(2+\sqrt{3}\right)\text{arctanh}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

```
[Out] -1/18*arctan(1/2*3^(1/4)*a^(1/6)*(-2*b^(1/3)*x+a^(1/3)*(1-3^(1/2)))*2^(1/2)
/(b*x^3+a)^(1/2))*(2+3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)-1/36*arctan(1
/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(b*x^3+a)^(1/2))
*(2+3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)+1/12*arctanh(1/2*3^(1/4)*a^(1/
6)*(a^(1/3)+b^(1/3)*x)*(1-3^(1/2))*2^(1/2)/(b*x^3+a)^(1/2))*(2+3^(1/2))*3^(
1/4)/a^(5/6)/b^(2/3)*2^(1/2)+1/18*arctanh(1/6*(1+3^(1/2))*(b*x^3+a)^(1/2)*3
^(1/4)*2^(1/2)/a^(1/2))*(2+3^(1/2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {500}

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = -\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{\left(2+\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} + \frac{\left(2+\sqrt{3}\right)\text{arctanh}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

```
[In] Int[x/(Sqrt[a + b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] -1/3*((2 + Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) - 2*b^(1
/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])]])/(Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2
 + Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(S
qrt[2]*Sqrt[a + b*x^3])]])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 + Sqrt[
3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*
Sqrt[a + b*x^3])]])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) + ((2 + Sqrt[3])*Arc
Tanh[((1 + Sqrt[3])*Sqrt[a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])]])/(3*Sqrt[2]
*3^(3/4)*a^(5/6)*b^(2/3))
```

Rule 500

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
[a + b*x^3]))]]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\text{integral} = -\frac{(2+\sqrt{3})\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{(2+\sqrt{3})\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{(2+\sqrt{3})\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} + \frac{(2+\sqrt{3})\tanh^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.27

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = \frac{x^2\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20a-12\sqrt{3}a\right)\sqrt{a+bx^3}}$$

```
[In] Integrate[x/(Sqrt[a + b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3)
/(10*a - 6*Sqrt[3]*a))])/((20*a - 12*Sqrt[3]*a)*Sqrt[a + b*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.72 (sec) , antiderivative size = 538, normalized size of antiderivative = 1.74

method	result
default	$\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}-6\sqrt{3}\,a+10a\right)}\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(-a\,b^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{ib\left(2x+\frac{\left(-a\,b^{2}\right.}{}\right.}{2\left(\cdot\right.}}}{}\right.}{}$
elliptic	$\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}-6\sqrt{3}\,a+10a\right)}\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(-a\,b^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{ib\left(2x+\frac{\left(-a\,b^{2}\right.}{}\right.}{2\left(\cdot\right.}}}{}\right.}{}$

```
[In] int(x/(b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/27*I/a/b^3*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2
)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)
^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+
1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3
+a)^(1/2)*(3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)-3*I*(
-a*b^2)^(2/3)*3^(1/2)+6*I*(-a*b^2)^(1/3)*_alpha*b-2*3^(1/2)*(-a*b^2)^(1/3)*
_alpha*b+6*b^2*_alpha^2-6*I*(-a*b^2)^(2/3)-2*3^(1/2)*(-a*b^2)^(2/3)-3*(-a*b
^2)^(1/3)*_alpha*b-3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
,-1/6/b*(2*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b-I*3^(1/2)*(-a*b^2)^(2/3)*_al
pha+4*I*(-a*b^2)^(1/3)*_alpha^2*b-2*I*(-a*b^2)^(2/3)*_alpha-2*3^(1/2)*(-a*b
^2)^(2/3)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha+2*I*a*b-2*3^(1/2)*a*
b-3*a*b)/a,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
```

```
/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3-6*3^(1/2)*a+10*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5631 vs. $2(210) = 420$.

Time = 3.43 (sec) , antiderivative size = 5631, normalized size of antiderivative = 18.16

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="fricas"
)
[Out] Too large to include
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = \int \frac{x}{\sqrt{a+bx^3}\left(-6\sqrt{3}a+10a+bx^3\right)}\,dx$$

```
[In] integrate(x/(b*x**3+2*a*(5-3*3**(1/2)))/(b*x**3+a)**(1/2),x)
[Out] Integral(x/(sqrt(a + b*x**3)*(-6*sqrt(3)*a + 10*a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = \int \frac{x}{\left(bx^3-2\,a\left(3\,\sqrt{3}-5\right)\right)\sqrt{bx^3+a}}\,dx$$

```
[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="maxima"
)
[Out] integrate(x/((b*x^3 - 2*a*(3*sqrt(3) - 5))*sqrt(b*x^3 + a)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = \int \frac{x}{\sqrt{b\,x^3+a}\,\left(b\,x^3-2\,a\,\left(3\,\sqrt{3}-5\right)\right)}\,dx$$

```
[In] int(x/((a + b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) - 5))),x)
```

```
[Out] int(x/((a + b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) - 5))), x)
```

3.355 $\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 316

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx = -\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{\left(2+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} + \frac{\left(2+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

```
[Out] -1/18*arctan(1/2*3^(1/4)*a^(1/6)*(2*b^(1/3)*x+a^(1/3)*(1-3^(1/2)))*2^(1/2)/
(-b*x^3+a)^(1/2))*(2+3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)-1/36*arctan(1
/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(-b*x^3+a)^(1/2)
)*(2+3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)+1/12*arctanh(1/2*3^(1/4)*a^(1
/6)*(a^(1/3)-b^(1/3)*x)*(1-3^(1/2))*2^(1/2)/(-b*x^3+a)^(1/2))*(2+3^(1/2))*3
^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)+1/18*arctanh(1/6*(1+3^(1/2))*(-b*x^3+a)^(1/2
)*3^(1/4)*2^(1/2)/a^(1/2))*(2+3^(1/2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {500}

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx = -\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{\left(2+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} + \frac{\left(2+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

```
[In] Int[x/(Sqrt[a - b*x^3]*(2*(5 - 3*Sqrt[3])*a - b*x^3)),x]
```

```
[Out] -1/6*((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3
)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 +
 Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(S
qrt[2]*Sqrt[a - b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 + Sqrt[
3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*
Sqrt[a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) + ((2 + Sqrt[3])*Arc
Tanh[((1 + Sqrt[3])*Sqrt[a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2]
*3^(3/4)*a^(5/6)*b^(2/3))
```

Rule 500

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\text{integral} = -\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} + \frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} + \frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.26

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx = \frac{x^2\sqrt{1-\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20a-12\sqrt{3}a\right)\sqrt{a-bx^3}}$$

```
[In] Integrate[x/(Sqrt[a - b*x^3]*(2*(5 - 3*Sqrt[3])*a - b*x^3)),x]
[Out] (x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*
a - 6*Sqrt[3]*a)])/((20*a - 12*Sqrt[3]*a)*Sqrt[a - b*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.99 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.61

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+6\sqrt{3}\,a-10a\right)}\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{ib\left(2x+\frac{i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}+\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{2\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{-i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}\cdots}{b}\cdots\right)}{\left(a\,b^{2}\right)^{\frac{1}{3}}\cdots}}\cdots}{\cdots}\cdots\right)}{\cdots}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+6\sqrt{3}\,a-10a\right)}\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{ib\left(2x+\frac{i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}+\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{2\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{-i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}\cdots}{b}\cdots\right)}{\left(a\,b^{2}\right)^{\frac{1}{3}}\cdots}}\cdots}{\cdots}\cdots\right)}{\cdots}$

```
[In] int(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/27*I/a/b^3*2^(1/2)*sum(1/_alpha*(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1
/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1
/3))/(-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-
I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/
2)*(-3*I*(a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)+3*I*(a*b^2)^
(2/3)*3^(1/2)-6*I*(a*b^2)^(1/3)*_alpha*b-2*3^(1/2)*(a*b^2)^(1/3)*_alpha*b+6
*b^2*_alpha^2+6*I*(a*b^2)^(2/3)-2*3^(1/2)*(a*b^2)^(2/3)-3*(a*b^2)^(1/3)*_al
pha*b-3*(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*3^(
1/2)*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1/2)*(a*b^2)^(2/3)*_alpha-4*I*(a*b^2)^(1
/3)*_alpha^2*b+2*I*(a*b^2)^(2/3)*_alpha-2*3^(1/2)*(a*b^2)^(2/3)*_alpha+I*3^
(1/2)*a*b-3*(a*b^2)^(2/3)*_alpha+2*I*a*b+2*3^(1/2)*a*b+3*a*b)/a,(-I*3^(1/2)
/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2
```

```
)),_alpha=RootOf(b*_Z^3+6*3^(1/2)*a-10*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5667 vs. $2(219) = 438$.

Time = 3.44 (sec) , antiderivative size = 5667, normalized size of antiderivative = 17.93

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="frica
s")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx$$
$$= -\int \frac{x}{-10a\sqrt{a-bx^3}+6\sqrt{3}a\sqrt{a-bx^3}+bx^3\sqrt{a-bx^3}}\,dx$$

```
[In] integrate(x/(-b*x**3+2*a*(5-3*3**(1/2)))/(-b*x**3+a)**(1/2),x)
[Out] -Integral(x/(-10*a*sqrt(a - b*x**3) + 6*sqrt(3)*a*sqrt(a - b*x**3) + b*x**3
*sqrt(a - b*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx = \int -\frac{x}{\left(bx^3+2\,a\left(3\sqrt{3}-5\right)\right)\sqrt{-bx^3+a}}\,dx$$

```
[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="maxim
a")
[Out] -integrate(x/((b*x^3 + 2*a*(3*sqrt(3) - 5))*sqrt(-b*x^3 + a)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx = \int -\frac{x}{\sqrt{a-b\,x^3}\,\left(b\,x^3+2\,a\,\left(3\,\sqrt{3}-5\right)\right)}\,dx$$

```
[In] int(-x/((a - b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) - 5))),x)
```

```
[Out] int(-x/((a - b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) - 5))), x)
```

3.356 $\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 320

$$\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx = \frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

```
[Out] 1/12*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1-3^(1/2))*2^(1/2)/(b*
x^3-a)^(1/2))*(2+3^(1/2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)-1/18*arctan(1/6*(
1+3^(1/2))*(b*x^3-a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*(2+3^(1/2))*3^(1/4)/a^(
5/6)/b^(2/3)*2^(1/2)-1/18*arctanh(1/2*3^(1/4)*a^(1/6)*(2*b^(1/3)*x+a^(1/3)*
(1-3^(1/2)))*2^(1/2)/(b*x^3-a)^(1/2))*(2+3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2
^(1/2)-1/36*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1+3^(1/2))*2^(
1/2)/(b*x^3-a)^(1/2))*(2+3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {501}

$$\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx$$
$$=\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\left(1+\sqrt{3}\right)\sqrt{bx^3-a}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}}$$
$$-\frac{\left(2+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$
$$-\frac{\left(2+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

```
[In] Int[x/((2*(5 - 3*Sqrt[3])*a - b*x^3)*Sqrt[-a + b*x^3]),x]
```

```
[Out] ((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))
/(Sqrt[2]*Sqrt[-a + b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + S
qrt[3])*ArcTan[((1 + Sqrt[3])*Sqrt[-a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])
/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 +
 Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])])/(6*Sq
rt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*a^(1/6)*((
1 - Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])])/(3*Sqrt[2
]*3^(1/4)*a^(5/6)*b^(2/3))
```

Rule 501

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\text{integral} = \frac{(2+\sqrt{3})\tan^{-1}\left(\frac{\sqrt[4]{3}(1-\sqrt{3})\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{(2+\sqrt{3})\tan^{-1}\left(\frac{(1+\sqrt{3})\sqrt{-a+bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{(2+\sqrt{3})\tanh^{-1}\left(\frac{\sqrt[4]{3}(1+\sqrt{3})\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{(2+\sqrt{3})\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left((1-\sqrt{3})\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.26

$$\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx = \frac{x^2\sqrt{1-\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20a-12\sqrt{3}a\right)\sqrt{-a+bx^3}}$$

```
[In] Integrate[x/((2*(5 - 3*Sqrt[3])*a - b*x^3)*Sqrt[-a + b*x^3]),x]
[Out] (x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*
a - 6*Sqrt[3]*a)])/((20*a - 12*Sqrt[3]*a)*Sqrt[-a + b*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.85 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.59

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+6\sqrt{3}\,a-10a\right)}\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{ib\left(2x+\frac{i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}+\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{2\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{-i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{\left(a\,b^{2}\right)^{\frac{1}{3}}}}}{}\right)}{}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+6\sqrt{3}\,a-10a\right)}\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{ib\left(2x+\frac{i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}+\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{2\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(x-\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{-i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{\left(a\,b^{2}\right)^{\frac{1}{3}}}}}{}\right)}{}$

```
[In] int(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/27*I/a/b^3*2^(1/2)*sum(1/_alpha*(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1
/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1
/3))/(-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-
I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2
)*(-3*I*(a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)+3*I*(a*b^2)^(
2/3)*3^(1/2)-6*I*(a*b^2)^(1/3)*_alpha*b-2*3^(1/2)*(a*b^2)^(1/3)*_alpha*b+6*
b^2*_alpha^2+6*I*(a*b^2)^(2/3)-2*3^(1/2)*(a*b^2)^(2/3)-3*(a*b^2)^(1/3)*_alp
ha*b-3*(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*3^(1
/2)*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1/2)*(a*b^2)^(2/3)*_alpha-4*I*(a*b^2)^(1/
3)*_alpha^2*b+2*I*(a*b^2)^(2/3)*_alpha-2*3^(1/2)*(a*b^2)^(2/3)*_alpha+I*3^(
1/2)*a*b-3*(a*b^2)^(2/3)*_alpha+2*I*a*b+2*3^(1/2)*a*b+3*a*b)/a,(-I*3^(1/2)/
b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)
```

```
),_alpha=RootOf(b*_Z^3+6*3^(1/2)*a-10*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5599 vs. 2(223) = 446.

Time = 3.57 (sec) , antiderivative size = 5599, normalized size of antiderivative = 17.50

$$\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="fricas
")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx$$
$$= -\int \frac{x}{-10a\sqrt{-a+bx^3}+6\sqrt{3}a\sqrt{-a+bx^3}+bx^3\sqrt{-a+bx^3}}\,dx$$

```
[In] integrate(x/(-b*x**3+2*a*(5-3*3**(1/2)))/(b*x**3-a)**(1/2),x)
[Out] -Integral(x/(-10*a*sqrt(-a + b*x**3) + 6*sqrt(3)*a*sqrt(-a + b*x**3) + b*x*
*3*sqrt(-a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx = \int -\frac{x}{\left(bx^3+2\,a\left(3\,\sqrt{3}-5\right)\right)\sqrt{bx^3-a}}\,dx$$

```
[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="maxima
")
[Out] -integrate(x/((b*x^3 + 2*a*(3*sqrt(3) - 5))*sqrt(b*x^3 - a)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx = \int -\frac{x}{\sqrt{b\,x^3-a}\,\left(b\,x^3+2\,a\,\left(3\,\sqrt{3}-5\right)\right)}\,dx$$

```
[In] int(-x/((b*x^3 - a)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) - 5))),x)
```

```
[Out] int(-x/((b*x^3 - a)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) - 5))), x)
```

3.357 $\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 322

$$\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = \frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

```
[Out] 1/12*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1-3^(1/2))*2^(1/2)/(-b
*x^3-a)^(1/2))*(2+3^(1/2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)-1/18*arctan(1/6*
(1+3^(1/2))*(-b*x^3-a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*(2+3^(1/2))*3^(1/4)/a
^(5/6)/b^(2/3)*2^(1/2)-1/18*arctanh(1/2*3^(1/4)*a^(1/6)*(-2*b^(1/3)*x+a^(1/
3)*(1-3^(1/2)))*2^(1/2)/(-b*x^3-a)^(1/2))*(2+3^(1/2))*3^(3/4)/a^(5/6)/b^(2/
3)*2^(1/2)-1/36*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1+3^(1/2))
*2^(1/2)/(-b*x^3-a)^(1/2))*(2+3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {501}

$$\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = \frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\arctan\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

```
[In] Int[x/(Sqrt[-a - b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] ((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))
/(Sqrt[2]*Sqrt[-a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + S
qrt[3])*ArcTan[((1 + Sqrt[3])*Sqrt[-a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])
/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*a^(1
/6)*((1 - Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(3*
Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 + Sqr
t[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(6*Sqrt[2
]*3^(1/4)*a^(5/6)*b^(2/3))
```

Rule 501

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\text{integral} = \frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}3^{3/4}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}} - \frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.27

$$\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = \frac{x^2\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20a-12\sqrt{3}a\right)\sqrt{-a-bx^3}}$$

```
[In] Integrate[x/(Sqrt[-a - b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3)
/(10*a - 6*Sqrt[3]*a))])/((20*a - 12*Sqrt[3]*a)*Sqrt[-a - b*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.97 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.68

method	result
default	$\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}-6\sqrt{3}\,a+10a\right)}\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{ib\left(2x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{b\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(-a\,b^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{ib\left(2x+\frac{\left(-a\,b^{2}\right)\cdots}{\cdots}\right)}{2\left(\cdots\right)}}\cdots}{\cdots}\right)}{\cdots}$
elliptic	$\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}-6\sqrt{3}\,a+10a\right)}\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{ib\left(2x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{b\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{-3\left(-a\,b^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{ib\left(2x+\frac{\left(-a\,b^{2}\right)\cdots}{\cdots}\right)}{2\left(\cdots\right)}}\cdots}{\cdots}\right)}{\cdots}$

```
[In] int(x/(b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/27*I/a/b^3*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2
)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)
^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+
1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(-b*x^
3-a)^(1/2)*(3*I*(-a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)-3*I*
(-a*b^2)^(2/3)*3^(1/2)+6*I*(-a*b^2)^(1/3)*_alpha*b-2*3^(1/2)*(-a*b^2)^(1/3)
*_alpha*b+6*b^2*_alpha^2-6*I*(-a*b^2)^(2/3)-2*3^(1/2)*(-a*b^2)^(2/3)-3*(-a*
b^2)^(1/3)*_alpha*b-3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
),-1/6/b*(2*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b-I*3^(1/2)*(-a*b^2)^(2/3)*_a
lpha+4*I*(-a*b^2)^(1/3)*_alpha^2*b-2*I*(-a*b^2)^(2/3)*_alpha-2*3^(1/2)*(-a*
b^2)^(2/3)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha+2*I*a*b-2*3^(1/2)*a
*b-3*a*b)/a,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
```

```
)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(b*_Z^3-6*3^(1/2)*a+10*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5599 vs. $2(222) = 444$.

Time = 3.60 (sec) , antiderivative size = 5599, normalized size of antiderivative = 17.39

$$\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="fricas
")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = \int \frac{x}{\sqrt{-a-bx^3}\left(-6\sqrt{3}a+10a+bx^3\right)}\,dx$$

```
[In] integrate(x/(b*x**3+2*a*(5-3*3**(1/2)))/(-b*x**3-a)**(1/2),x)
[Out] Integral(x/(sqrt(-a - b*x**3)*(-6*sqrt(3)*a + 10*a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx = \int \frac{x}{\left(bx^3-2\,a\left(3\,\sqrt{3}-5\right)\right)\sqrt{-bx^3-a}}\,dx$$

```
[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="maxima
")
[Out] integrate(x/((b*x^3 - 2*a*(3*sqrt(3) - 5))*sqrt(-b*x^3 - a)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{-a-bx^3}\,(2\,(5-3\sqrt{3})\,a+bx^3)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{-a-bx^3}\,(2\,(5-3\sqrt{3})\,a+bx^3)}\,dx = \int \frac{x}{\sqrt{-b\,x^3-a}\,\left(b\,x^3-2\,a\,\left(3\,\sqrt{3}-5\right)\right)}\,dx$$

```
[In] int(x/((- a - b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) - 5))),x)
```

```
[Out] int(x/((- a - b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) - 5))), x)
```

3.358 $\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 125

$$\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{2a^2\sqrt{c+dx^3}}{3b^3} - \frac{2(bc+ad)\left(c+dx^3\right)^{3/2}}{9b^2d^2} + \frac{2(c+dx^3)^{5/2}}{15bd^2} - \frac{2a^2\sqrt{bc-ad}\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}$$

```
[Out] -2/9*(a*d+b*c)*(d*x^3+c)^(3/2)/b^2/d^2+2/15*(d*x^3+c)^(5/2)/b/d^2-2/3*a^2*a
rctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(7/2)+2
/3*a^2*(d*x^3+c)^(1/2)/b^3
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 90, 52, 65, 214}

$$\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx = -\frac{2a^2\sqrt{bc-ad}\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}} + \frac{2a^2\sqrt{c+dx^3}}{3b^3} - \frac{2(c+dx^3)^{3/2}(ad+bc)}{9b^2d^2} + \frac{2(c+dx^3)^{5/2}}{15bd^2}$$

```
[In] Int[(x^8*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (2*a^2*Sqrt[c + d*x^3])/(3*b^3) - (2*(b*c + a*d)*(c + d*x^3)^(3/2))/(9*b^2*
d^2) + (2*(c + d*x^3)^(5/2))/(15*b*d^2) - (2*a^2*Sqrt[b*c - a*d]*ArcTanh[(S
qrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{(-bc-ad)\sqrt{c+dx}}{b^2d} + \frac{a^2\sqrt{c+dx}}{b^2(a+bx)} + \frac{(c+dx)^{3/2}}{bd}\right)dx, x, x^3\right)\\
&= -\frac{2(bc+ad)\left(c+dx^3\right)^{3/2}}{9b^2d^2} + \frac{2\left(c+dx^3\right)^{5/2}}{15bd^2} + \frac{a^2\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)}{3b^2}
\end{aligned}$$

$$= \frac{2a^2\sqrt{c+dx^3}}{3b^3} - \frac{2(bc+ad)\left(c+dx^3\right)^{3/2}}{9b^2d^2} + \frac{2\left(c+dx^3\right)^{5/2}}{15bd^2} + \frac{\left(a^2(bc-ad)\right)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3b^3}$$

$$= \frac{2a^2\sqrt{c+dx^3}}{3b^3} - \frac{2(bc+ad)\left(c+dx^3\right)^{3/2}}{9b^2d^2} + \frac{2\left(c+dx^3\right)^{5/2}}{15bd^2} + \frac{\left(2a^2(bc-ad)\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3b^3d}$$

$$= \frac{2a^2\sqrt{c+dx^3}}{3b^3} - \frac{2(bc+ad)\left(c+dx^3\right)^{3/2}}{9b^2d^2} + \frac{2\left(c+dx^3\right)^{5/2}}{15bd^2} - \frac{2a^2\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.97

$$\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{2\sqrt{c+dx^3}\left(15a^2d^2-5abd(c+dx^3)+b^2(-2c^2+cdx^3+3d^2x^6)\right)}{45b^3d^2} - \frac{2a^2\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{7/2}}$$

```
[In] Integrate[(x^8*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(15*a^2*d^2 - 5*a*b*d*(c + d*x^3) + b^2*(-2*c^2 + c*d*x^
3 + 3*d^2*x^6)))/(45*b^3*d^2) - (2*a^2*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*S
qrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(3*b^(7/2))
```

Maple [A] (verified)

Time = 5.61 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.99

method	result
risch	$\frac{2\left(3 b^{2} d^{2} x^{6}-5 x^{3} a b \,d^{2}+x^{3} b^{2} c d +15 a^{2} d^{2}-5 a b c d -2 b^{2} c^{2}\right) \sqrt{d \,x^{3}+c}}{45 d^{2} b^{3}}-\frac{2 a^{2} \left(a d -b c \right) \arctan \left(\frac{b \sqrt{d \,x^{3}+c}}{\sqrt{\left(a d -b c \right) b}}\right)}{3 b^{3} \sqrt{\left(a d -b c \right) b}}$
pseudoelliptic	$-\frac{2 \left(-\sqrt{\left(a d -b c \right) b}\, \left(-\frac{2 \left(d \,x^{3}+c \right) \left(-\frac{3 d \,x^{3}}{2}+c \right) b^{2}}{15}-\frac{\left(d \,x^{3}+c \right) a b d}{3}+a^{2} d^{2}\right) \sqrt{d \,x^{3}+c}+a^{2} d^{2} \left(a d -b c \right) \arctan \left(\frac{b \sqrt{d \,x^{3}+c}}{\sqrt{\left(a d -b c \right) b}}\right)\right)}{3 \sqrt{\left(a d -b c \right) b}\, d^{2} b^{3}}$
default	$\frac{\frac{2 x^{6} \sqrt{d \,x^{3}+c}}{15}+\frac{2 c \,x^{3} \sqrt{d \,x^{3}+c}}{45 d}-\frac{4 c^{2} \sqrt{d \,x^{3}+c}}{45 d^{2}}}{b}-\frac{2 a \left(d \,x^{3}+c \right)^{\frac{3}{2}}}{9 b^{2} d}+\frac{2 a^{2} \left(\sqrt{d \,x^{3}+c}-\frac{\left(a d -b c \right) \arctan \left(\frac{b \sqrt{d \,x^{3}+c}}{\sqrt{\left(a d -b c \right) b}}\right)}{\sqrt{\left(a d -b c \right) b}}\right)}{3 b^{3}}$
elliptic	$\frac{2 x^{6} \sqrt{d \,x^{3}+c}}{15 b}+\frac{2 \left(-\frac{a d -b c}{b^{2}}-\frac{4 c}{5 b}\right) x^{3} \sqrt{d \,x^{3}+c}}{9 d}+\frac{2 \left(\frac{\left(a d -b c \right) a}{b^{3}}-\frac{2 \left(-\frac{a d -b c}{b^{2}}-\frac{4 c}{5 b}\right) c}{3 d}\right) \sqrt{d \,x^{3}+c}}{3 d}+\frac{i a^{2} \sqrt{2}\, \left(\sum_{\_\alpha =\mathrm{RootOf}\left(b \_Z^{3}+\right.} \right.}{}$

[In] int(x^8*(d*x^3+c)^(1/2)/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] 2/45*(3*b^2*d^2*x^6-5*a*b*d^2*x^3+b^2*c*d*x^3+15*a^2*d^2-5*a*b*c*d-2*b^2*c^2)*(d*x^3+c)^(1/2)/d^2/b^3-2/3*a^2*(a*d-b*c)/b^3/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.24

$$\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx$$
$$= \left[\frac{15\,a^2d^2\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\,(3\,b^2d^2x^6-2\,b^2c^2-5\,abcd+15\,a^2d^2+(b^2cd-5\,a}{45\,b^3d^2}\right.$$
$$-\frac{2\left(15\,a^2d^2\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-(3\,b^2d^2x^6-2\,b^2c^2-5\,abcd+15\,a^2d^2+(b^2cd-5\,abc\right.}{45\,b^3d^2}$$

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/45*(15*a^2*d^2*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d
*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*(3*b^2*d^2*x^6 - 2*b^2*c^
2 - 5*a*b*c*d + 15*a^2*d^2 + (b^2*c*d - 5*a*b*d^2)*x^3)*sqrt(d*x^3 + c))/(b
^3*d^2), -2/45*(15*a^2*d^2*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*s
qrt(-(b*c - a*d)/b)/(b*c - a*d)) - (3*b^2*d^2*x^6 - 2*b^2*c^2 - 5*a*b*c*d +
 15*a^2*d^2 + (b^2*c*d - 5*a*b*d^2)*x^3)*sqrt(d*x^3 + c))/(b^3*d^2)]
```

Sympy [A] (verification not implemented)

Time = 8.96 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.38

$$\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx$$
$$= \begin{cases} \frac{2\left(\frac{a^2d^3\sqrt{c+dx^3}}{3b^3}-\frac{a^2d^3(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^4\sqrt{\frac{ad-bc}{b}}}+\frac{d\left(c+dx^3\right)^{\frac{5}{2}}}{15b}+\frac{\left(c+dx^3\right)^{\frac{3}{2}}\left(-ad^2-bcd\right)}{9b^2}\right)}{d^3} & \text{for } d\neq 0 \\ \sqrt{c}\left(\frac{a^2\left(\begin{cases}\frac{x^3}{a} & \text{for } b=0 \\ \frac{\log\left(a+bx^3\right)}{b} & \text{otherwise}\end{cases}\right)}{3b^2}-\frac{ax^3}{3b^2}+\frac{x^6}{6b}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(x**8*(d*x**3+c)**(1/2)/(b*x**3+a),x)
```

```
[Out] Piecewise((2*(a**2*d**3*sqrt(c + d*x**3)/(3*b**3) - a**2*d**3*(a*d - b*c)*a
tan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*b**4*sqrt((a*d - b*c)/b)) + d*
(c + d*x**3)**(5/2)/(15*b) + (c + d*x**3)**(3/2)*(-a*d**2 - b*c*d)/(9*b**2)
)/d**3, Ne(d, 0)), (sqrt(c)*(a**2*Piecewise((x**3/a, Eq(b, 0)), (log(a + b*
x**3)/b, True))/(3*b**2) - a*x**3/(3*b**2) + x**6/(6*b)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.11

$$\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{2\left(a^2bc-a^3d\right)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}b^3} + \frac{2\left(3\left(dx^3+c\right)^{\frac{5}{2}}b^4d^8-5\left(dx^3+c\right)^{\frac{3}{2}}b^4cd^8-5\left(dx^3+c\right)^{\frac{3}{2}}ab^3d^9+15\sqrt{dx^3+c}a^2b^2d^{10}\right)}{45\,b^5d^{10}}$$

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(a^2*b*c - a^3*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(
-b^2*c + a*b*d)*b^3) + 2/45*(3*(d*x^3 + c)^(5/2)*b^4*d^8 - 5*(d*x^3 + c)^(3
/2)*b^4*c*d^8 - 5*(d*x^3 + c)^(3/2)*a*b^3*d^9 + 15*sqrt(d*x^3 + c)*a^2*b^2*
d^10)/(b^5*d^10)
```

Mupad [B] (verification not implemented)

Time = 10.53 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.41

$$\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{2\,a^2\,\sqrt{d\,x^3+c}}{3\,b^3}+\frac{2\,{\left(d\,x^3+c\right)}^{5/2}}{15\,b\,d^2}-\frac{2\,a\,{\left(d\,x^3+c\right)}^{3/2}}{9\,b^2\,d}-\frac{2\,c\,{\left(d\,x^3+c\right)}^{3/2}}{9\,b\,d^2}+\frac{a^2\,\ln\left(\frac{a^2\,d^2\,1\mathrm{i}+b^2\,c^2\,2\mathrm{i}-2\,\sqrt{b}\,\sqrt{d\,x^3+c}\,{\left(a\,d-b\,c\right)}^{3/2}-a\,b\,d^2\,x^3\,1\mathrm{i}+b^2\,c\,d\,x^3\,1\mathrm{i}-a\,b\,c\,d\,3\mathrm{i}}{2\,b\,x^3+2\,a}\right)\,\sqrt{a\,d-b\,c}\,1\mathrm{i}}{3\,b^{7/2}}$$

```
[In] int((x^8*(c + d*x^3)^(1/2))/(a + b*x^3),x)
```

```
[Out] (2*a^2*(c + d*x^3)^(1/2))/(3*b^3) + (2*(c + d*x^3)^(5/2))/(15*b*d^2) - (2*a
*(c + d*x^3)^(3/2))/(9*b^2*d) - (2*c*(c + d*x^3)^(3/2))/(9*b*d^2) + (a^2*lo
g((a^2*d^2*1i + b^2*c^2*2i - 2*b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)
- a*b*d^2*x^3*1i + b^2*c*d*x^3*1i - a*b*c*d*3i)/(2*a + 2*b*x^3))*(a*d - b*c
)^(1/2)*1i)/(3*b^(7/2))
```

3.359 $\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx=-\frac{2a\sqrt{c+dx^3}}{3b^2}+\frac{2(c+dx^3)^{3/2}}{9bd}+\frac{2a\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}$$

```
[Out] 2/9*(d*x^3+c)^(3/2)/b/d+2/3*a*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1
/2))*(-a*d+b*c)^(1/2)/b^(5/2)-2/3*a*(d*x^3+c)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 81, 52, 65, 214}

$$\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx=\frac{2a\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}-\frac{2a\sqrt{c+dx^3}}{3b^2}+\frac{2(c+dx^3)^{3/2}}{9bd}$$

```
[In] Int[(x^5*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (-2*a*Sqrt[c + d*x^3])/(3*b^2) + (2*(c + d*x^3)^(3/2))/(9*b*d) + (2*a*Sqrt[
b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)\\
&= \frac{2(c+dx^3)^{3/2}}{9bd} - \frac{a\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)}{3b}\\
&= -\frac{2a\sqrt{c+dx^3}}{3b^2} + \frac{2(c+dx^3)^{3/2}}{9bd} - \frac{(a(bc-ad))\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3b^2}\\
&= -\frac{2a\sqrt{c+dx^3}}{3b^2} + \frac{2(c+dx^3)^{3/2}}{9bd} - \frac{(2a(bc-ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3b^2d}\\
&= -\frac{2a\sqrt{c+dx^3}}{3b^2} + \frac{2(c+dx^3)^{3/2}}{9bd} + \frac{2a\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

$$\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx=\frac{2\sqrt{c+dx^3}(-3ad+b(c+dx^3))}{9b^2d}+\frac{2a\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{5/2}}$$

```
[In] Integrate[(x^5*Sqrt[c + d*x^3])/(a + b*x^3),x]
[Out] (2*Sqrt[c + d*x^3]*(-3*a*d + b*(c + d*x^3)))/(9*b^2*d) + (2*a*Sqrt[-(b*c) +
 a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(3*b^(5/2))
```

Maple [A] (verified)

Time = 4.37 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.87

method	result
default	$\frac{2(dx^3+c)^{\frac{3}{2}}}{9bd}-\frac{2a\left(\sqrt{dx^3+c}-\frac{(ad-bc)\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}\right)}{3b^2}$
pseudoelliptic	$\frac{-\frac{2\sqrt{dx^3+c}\left(-bdx^3+3ad-bc\right)}{9}+\frac{2ad(ad-bc)\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}}{b^2d}$
risch	$-\frac{2(-bdx^3+3ad-bc)\sqrt{dx^3+c}}{9db^2}+\frac{2(ad-bc)a\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)}{3b^2\sqrt{(ad-bc)b}}$
elliptic	$\frac{2x^3\sqrt{dx^3+c}}{9b}+\frac{2\left(-\frac{ad-bc}{b^2}-\frac{2c}{3b}\right)\sqrt{dx^3+c}}{3d}-\frac{ia\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-cd^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right.}{\left(-cd^2\right)^{\frac{1}{3}}}}}{\cdots}\right.}{\cdots}$

```
[In] int(x^5*(d*x^3+c)^(1/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
[Out] 2/9*(d*x^3+c)^(3/2)/b/d-2/3*a/b^2*((d*x^3+c)^(1/2)-(a*d-b*c)*arctan(b*(d*x^
3+c)^(1/2)/((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.10

$$\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx = \left[\frac{3\,ad\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\,(bdx^3+bc-3\,ad)\sqrt{dx^3+c}}{9\,b^2d}, \frac{2\left(3\,ad\sqrt{-\frac{bc-ad}{b}}\arctan\right.}{}\right.$$

[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")

[Out] [1/9*(3*a*d*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*(b*d*x^3 + b*c - 3*a*d)*sqrt(d*x^3 + c))/(b^2*d), 2/9*(3*a*d*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (b*d*x^3 + b*c - 3*a*d)*sqrt(d*x^3 + c))/(b^2*d)]

Sympy [A] (verification not implemented)

Time = 4.21 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.38

$$\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx = \begin{cases} \frac{2\left(-\frac{ad^2\sqrt{c+dx^3}}{3b^2}+\frac{ad^2(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^3\sqrt{\frac{ad-bc}{b}}}+\frac{d\left(c+dx^3\right)^{\frac{3}{2}}}{9b}\right)}{d^2} & \text{for } d\neq 0 \\ \sqrt{c}\left(-\frac{a\left(\begin{cases}\frac{x^3}{a} & \text{for } b=0 \\ \frac{\log\left(a+bx^3\right)}{b} & \text{otherwise}\end{cases}\right)}{3b}+\frac{x^3}{3b}\right) & \text{otherwise} \end{cases}$$

[In] integrate(x**5*(d*x**3+c)**(1/2)/(b*x**3+a),x)

[Out] Piecewise((2*(-a*d**2*sqrt(c + d*x**3)/(3*b**2) + a*d**2*(a*d - b*c)*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*b**3*sqrt((a*d - b*c)/b)) + d*(c + d*x**3)**(3/2)/(9*b))/d**2, Ne(d, 0)), (sqrt(c)*(-a*Piecewise((x**3/a, Eq(b, 0)), (log(a + b*x**3)/b, True))/(3*b) + x**3/(3*b)), True))

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.03

$$\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx = -\frac{2\,(abc-a^2d)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}b^2} + \frac{2\left((dx^3+c)^{\frac{3}{2}}b^2d^2-3\sqrt{dx^3+c}abd^3\right)}{9\,b^3d^3}$$

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -2/3*(a*b*c - a^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-
b^2*c + a*b*d)*b^2) + 2/9*((d*x^3 + c)^(3/2)*b^2*d^2 - 3*sqrt(d*x^3 + c)*a*
b*d^3)/(b^3*d^3)
```

Mupad [B] (verification not implemented)

Time = 10.42 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.46

$$\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{2\,(d\,x^3+c)^{3/2}}{9\,b\,d} - \frac{2\,a\,\sqrt{d\,x^3+c}}{3\,b^2} + \frac{a\,\ln\left(\frac{a^2\,d^2\,1\mathrm{i}+b^2\,c^2\,2\mathrm{i}+2\,\sqrt{b}\,\sqrt{d\,x^3+c}\,(a\,d-b\,c)^{3/2}-a\,b\,d^2\,x^3\,1\mathrm{i}+b^2\,c\,d\,x^3\,1\mathrm{i}-a\,b\,c\,d\,3\mathrm{i}}{2\,b\,x^3+2\,a}\right)\,\sqrt{a\,d-b\,c}\,1\mathrm{i}}{3\,b^{5/2}}$$

```
[In] int((x^5*(c + d*x^3)^(1/2))/(a + b*x^3),x)
```

```
[Out] (2*(c + d*x^3)^(3/2))/(9*b*d) - (2*a*(c + d*x^3)^(1/2))/(3*b^2) + (a*log((a
^2*d^2*1i + b^2*c^2*2i + 2*b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2) - a*
b*d^2*x^3*1i + b^2*c*d*x^3*1i - a*b*c*d*3i)/(2*a + 2*b*x^3))*(a*d - b*c)^(1
/2)*1i)/(3*b^(5/2))
```

3.360 $\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 70

$$\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx=\frac{2\sqrt{c+dx^3}}{3b}-\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}}$$

```
[Out] -2/3*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(
3/2)+2/3*(d*x^3+c)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {455, 52, 65, 214}

$$\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx=\frac{2\sqrt{c+dx^3}}{3b}-\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}}$$

```
[In] Int[(x^2*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*b) - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*
x^3])/Sqrt[b*c - a*d]])/(3*b^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)\\
&= \frac{2\sqrt{c+dx^3}}{3b} + \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3b}\\
&= \frac{2\sqrt{c+dx^3}}{3b} + \frac{(2(bc-ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3bd}\\
&= \frac{2\sqrt{c+dx^3}}{3b} - \frac{2\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx=\frac{1}{3}\left(\frac{2\sqrt{c+dx^3}}{b}-\frac{2\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{b^{3/2}}\right)$$

```
[In] Integrate[(x^2*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] ((2*Sqrt[c + d*x^3])/b - (2*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x
^3])/Sqrt[-(b*c) + a*d]])/b^(3/2))/3
```

Maple [A] (verified)

Time = 4.38 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

method	result
default	$\frac{\frac{2\sqrt{d\,x^3+c}}{3}-\frac{2(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}}{b}$
pseudoelliptic	$\frac{\frac{2\sqrt{d\,x^3+c}}{3}-\frac{2(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}}{b}$
risch	$\frac{2\sqrt{d\,x^3+c}}{3b}-\frac{2(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3b\sqrt{(ad-bc)b}}$
elliptic	$\frac{2\sqrt{d\,x^3+c}}{3b}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}}{}\right.}{}$

```
[In] int(x^2*(d*x^3+c)^(1/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/b*((d*x^3+c)^(1/2)-(a*d-b*c)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/
2))/((a*d-b*c)*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.23

$$\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx = \left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\,\sqrt{dx^3+c}}{3\,b}, -\frac{2\left(\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\sqrt{dx^3+c}\right)}{3\,b}\right]$$

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/3*(sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*b*
sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*sqrt(d*x^3 + c))/b, -2/3*(sqrt(-(b*c
- a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - sqr
t(d*x^3 + c))/b]
```

Sympy [A] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.36

$$\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx = \begin{cases} \frac{2\left(\frac{d\sqrt{c+dx^3}}{3b}-\frac{d(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^2\sqrt{\frac{ad-bc}{b}}}\right)}{d} & \text{for } d\neq 0 \\ \sqrt{c}\left(\begin{cases} \frac{x^3}{3a} & \text{for } b=0 \\ \frac{\log(3a+3bx^3)}{3b} & \text{otherwise} \end{cases}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2*(d*x**3+c)**(1/2)/(b*x**3+a),x)
```

```
[Out] Piecewise((2*(d*sqrt(c + d*x**3)/(3*b) - d*(a*d - b*c)*atan(sqrt(c + d*x**3
)/sqrt((a*d - b*c)/b))/(3*b**2*sqrt((a*d - b*c)/b)))/d, Ne(d, 0)), (sqrt(c)
*Piecewise((x**3/(3*a), Eq(b, 0)), (log(3*a + 3*b*x**3)/(3*b), True)), True
))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{2\,(bc-ad)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}b} + \frac{2\sqrt{dx^3+c}}{3\,b}$$

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(b*c - a*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c
 + a*b*d)*b) + 2/3*sqrt(d*x^3 + c)/b
```

Mupad [B] (verification not implemented)

Time = 10.44 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.17

$$\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{2\sqrt{d\,x^3+c}}{3\,b} + \frac{\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\sqrt{a\,d-b\,c}\,1{}\mathrm{i}}{3\,b^{3/2}}$$

```
[In] int((x^2*(c + d*x^3)^(1/2))/(a + b*x^3),x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*b) + (log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)
*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*1i)/(3*b^(3
/2))
```

3.361 $\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 85

$$\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)}\,dx=-\frac{2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a}+\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{b}}$$

```
[Out] -2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/a+2/3*arctanh(b^(1/2)*(d*x^3+
c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/a/b^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {457, 85, 65, 214}

$$\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)}\,dx=\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{b}}-\frac{2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a}$$

```
[In] Int[Sqrt[c + d*x^3]/(x*(a + b*x^3)),x]
```

```
[Out] (-2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a) + (2*Sqrt[b*c - a*d]*Ar
cTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a*Sqrt[b])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x(a+bx)}\,dx, x, x^3\right) \\
&= \frac{c\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{3a} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a} \\
&= \frac{(2c)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3ad} - \frac{(2(bc-ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3ad} \\
&= -\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a} + \frac{2\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{b}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)}\,dx = \frac{2\left(\frac{\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}} - \sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)\right)}{3a}$$

```
[In] Integrate[Sqrt[c + d*x^3]/(x*(a + b*x^3)),x]
[Out] (2*((Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]
])/Sqrt[b] - Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]))/(3*a)
```

Maple [A] (verified)

Time = 4.50 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{\frac{2\sqrt{d x^3+c}}{3}-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)\sqrt{c}}{3}}{a}-\frac{2\left(\sqrt{d x^3+c}-\frac{(ad-bc)\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}\right)}{3a}$	98
pseudoelliptic	$\frac{2\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)ad-2\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)bc-2\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)\sqrt{(ad-bc)b}}{3a\sqrt{(ad-bc)b}}$	106
elliptic	Expression too large to display	1543

```
[In] int((d*x^3+c)^(1/2)/x/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(2/3*(d*x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))-2/3/
a*((d*x^3+c)^(1/2)-(a*d-b*c)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))/
((a*d-b*c)*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 383, normalized size of antiderivative = 4.51

$$\int \frac{\sqrt{c+dx^3}}{x\,(a+bx^3)}\,dx$$
$$=\left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)}{3\,a},\ \frac{2\,\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{-}}{bc-ad}\right.}{}\right.$$

```
[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/3*(sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*
sqrt((b*c - a*d)/b))/(b*x^3 + a)) + sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*
sqrt(c) + 2*c)/x^3))/a, 1/3*(2*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)
*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 +
c)*sqrt(c) + 2*c)/x^3))/a, 1/3*(2*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/
c) + sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*s
qrt((b*c - a*d)/b))/(b*x^3 + a)))/a, 2/3*(sqrt(-(b*c - a*d)/b)*arctan(-sqrt
(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + sqrt(-c)*arctan(sqrt(d*x^
3 + c)*sqrt(-c)/c))/a]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(73) = 146$.

Time = 4.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.94

$$\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)}\,dx = \begin{cases} \frac{2\left(\frac{cd\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}+\frac{d(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3ab\sqrt{\frac{ad-bc}{b}}}\right)}{d} & \text{for}\ d\neq 0 \\ \sqrt{c}\left(-\frac{2b\left(\begin{cases}\frac{\frac{a}{2b}+x^3}{a} & \text{for}\ b=0\\ -\frac{\log\left(a-2b\left(\frac{a}{2b}+x^3\right)\right)}{2b} & \text{otherwise}\end{cases}\right)}{3a}-\frac{2b\left(\begin{cases}\frac{\frac{a}{2b}+x^3}{a} & \text{for}\ b=0\\ \frac{\log\left(a+2b\left(\frac{a}{2b}+x^3\right)\right)}{2b} & \text{otherwise}\end{cases}\right)}{3a}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((d*x**3+c)**(1/2)/x/(b*x**3+a),x)
```

```
[Out] Piecewise((2*(c*d*atan(sqrt(c + d*x**3)/sqrt(-c))/(3*a*sqrt(-c)) + d*(a*d -
 b*c)*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*a*b*sqrt((a*d - b*c)/b)
))/d, Ne(d, 0)), (sqrt(c)*(-2*b*Piecewise(((a/(2*b) + x**3)/a, Eq(b, 0)), (
-log(a - 2*b*(a/(2*b) + x**3))/(2*b), True))/(3*a) - 2*b*Piecewise(((a/(2*b
) + x**3)/a, Eq(b, 0)), (log(a + 2*b*(a/(2*b) + x**3))/(2*b), True))/(3*a))
, True))
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)}\,dx = \int \frac{\sqrt{dx^3+c}}{(bx^3+a)x}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)}\,dx = -\frac{2(bc-ad)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}a} + \frac{2c\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}$$

```
[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -2/3*(b*c - a*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*
c + a*b*d)*a) + 2/3*c*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a*sqrt(-c))
```

Mupad [B] (verification not implemented)

Time = 12.46 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)}\,dx = \frac{\sqrt{c}\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a} + \frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\sqrt{a\,d-b\,c}\,1\mathrm{i}}{3\,a\,\sqrt{b}}$$

```
[In] int((c + d*x^3)^(1/2)/(x*(a + b*x^3)),x)
```

```
[Out] (c^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))
)/x^6))/(3*a) + (log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(
1/2)*2i + b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*1i)/(3*a*b^(1/2))
```

3.362 $\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 115

$$\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)}\,dx = -\frac{\sqrt{c+dx^3}}{3ax^3} + \frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} - \frac{2\sqrt{b}\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2}$$

```
[Out] 1/3*(-a*d+2*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^2/c^(1/2)-2/3*arctanh(b
^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)*(-a*d+b*c)^(1/2)/a^2-1/3*(
d*x^3+c)^(1/2)/a/x^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 101, 162, 65, 214}

$$\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)}\,dx = \frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} - \frac{2\sqrt{b}\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2} - \frac{\sqrt{c+dx^3}}{3ax^3}$$

```
[In] Int[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)),x]
```

```
[Out] -1/3*Sqrt[c + d*x^3]/(a*x^3) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[
c]])/(3*a^2*Sqrt[c]) - (2*Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^3])/Sqrt[b*c - a*d]])/(3*a^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(a+bx)}\,dx, x, x^3\right)\\ &= -\frac{\sqrt{c+dx^3}}{3ax^3} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-2bc+ad)-\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a}\end{aligned}$$

$$= -\frac{\sqrt{c+dx^3}}{3ax^3} + \frac{(b(bc-ad))\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a^2} - \frac{(2bc-ad)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{6a^2}$$

$$= -\frac{\sqrt{c+dx^3}}{3ax^3} + \frac{(2b(bc-ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^2d} - \frac{(2bc-ad)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^2d}$$

$$= -\frac{\sqrt{c+dx^3}}{3ax^3} + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} - \frac{2\sqrt{b}\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)}\,dx = \frac{-\frac{a\sqrt{c+dx^3}}{x^3} - 2\sqrt{b}\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right) + \frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a^2}$$

```
[In] Integrate[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)),x]
[Out] (-((a*Sqrt[c + d*x^3])/x^3) - 2*Sqrt[b]*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*
Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]] + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^
3]/Sqrt[c]])/Sqrt[c])/(3*a^2)
```

Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$\frac{-\frac{2(ad-bc)b\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}-\frac{\sqrt{d\,x^3+c}\,a}{x^3}-\frac{(ad-2bc)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a^2}$
risch	$-\frac{\sqrt{d\,x^3+c}}{3a\,x^3}-\frac{-\frac{2(-ad+2bc)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3a\sqrt{c}}+\frac{4(ad-bc)b\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3a\sqrt{(ad-bc)b}}}{2a}$
default	$\frac{-\frac{\sqrt{d\,x^3+c}}{3x^3}-\frac{d\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3\sqrt{c}}}{a}-\frac{b\left(\frac{2\sqrt{d\,x^3+c}}{3}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)\sqrt{c}}{3}\right)}{a^2}+\frac{2b\left(\sqrt{d\,x^3+c}-\frac{(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}\right.}{3a^2}$
elliptic	Expression too large to display

```
[In] int((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a^2*(-2*(a*d-b*c)*b/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^3+c)^(1/2)/((a*d-
b*c)*b)^(1/2))-(d*x^3+c)^(1/2)*a/x^3-(a*d-2*b*c)/c^(1/2)*arctanh((d*x^3+c)^
(1/2)/c^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 513, normalized size of antiderivative = 4.46

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)}\,dx$$

$$= \left[\frac{2\sqrt{b^2c-abd}cx^3\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)-(2\,bc-ad)\sqrt{c}x^3\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-2\,\sqrt{dx^3+}}{6\,a^2cx^3}\right.$$

$$-\frac{(2\,bc-ad)\sqrt{-c}x^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)-\sqrt{b^2c-abd}cx^3\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)+\sqrt{dx^3+c}a}{3\,a^2cx^3}$$

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*sqrt(b^2*c - a*b*d)*c*x^3*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3
 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) - (2*b*c - a*d)*sqrt(c)*x^3*log((d*
x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*sqrt(d*x^3 + c)*a*c)/(a^2*c
*x^3), 1/6*(4*sqrt(-b^2*c + a*b*d)*c*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c
 + a*b*d)/(b*d*x^3 + b*c)) - (2*b*c - a*d)*sqrt(c)*x^3*log((d*x^3 - 2*sqrt(
d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*sqrt(d*x^3 + c)*a*c)/(a^2*c*x^3), -1/3*(
(2*b*c - a*d)*sqrt(-c)*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - sqrt(b^2*c
- a*b*d)*c*x^3*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c -
a*b*d))/(b*x^3 + a)) + sqrt(d*x^3 + c)*a*c)/(a^2*c*x^3), 1/3*(2*sqrt(-b^2*c
 + a*b*d)*c*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)
```

```
) - (2*b*c - a*d)*sqrt(-c)*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - sqrt(d*
x^3 + c)*a*c)/(a^2*c*x^3)]
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4\,(a+bx^3)}\,dx=\int \frac{\sqrt{c+dx^3}}{x^4\,(a+bx^3)}\,dx$$

```
[In] integrate((d*x**3+c)**(1/2)/x**4/(b*x**3+a),x)
[Out] Integral(sqrt(c + d*x**3)/(x**4*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4\,(a+bx^3)}\,dx=\int \frac{\sqrt{dx^3+c}}{(bx^3+a)x^4}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c+dx^3}}{x^4\,(a+bx^3)}\,dx=\frac{2\,(b^2c-abd)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}a^2}-\frac{(2\,bc-ad)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}}-\frac{\sqrt{dx^3+c}}{3\,ax^3}$$

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x, algorithm="giac")
[Out] 2/3*(b^2*c - a*b*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b
^2*c + a*b*d)*a^2) - 1/3*(2*b*c - a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^
2*sqrt(-c)) - 1/3*sqrt(d*x^3 + c)/(a*x^3)
```

Mupad [B] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{c+dx^3}}{x^4\,(a+bx^3)}\,dx = \frac{\ln\left(\frac{a\,d-2\,b\,c+2\,\sqrt{d\,x^3+c}\,\sqrt{b^2\,c-a\,b\,d}-b\,d\,x^3}{b\,x^3+a}\right)\,\sqrt{b^2\,c-a\,b\,d}}{3\,a^2} - \frac{\sqrt{d\,x^3+c}}{3\,a\,x^3} + \frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)\,(a\,d-2\,b\,c)}{6\,a^2\,\sqrt{c}}$$

```
[In] int((c + d*x^3)^(1/2)/(x^4*(a + b*x^3)),x)
```

```
[Out] (log((a*d - 2*b*c + 2*(c + d*x^3)^(1/2)*(b^2*c - a*b*d)^(1/2) - b*d*x^3)/(a
 + b*x^3))*(b^2*c - a*b*d)^(1/2))/(3*a^2) - (c + d*x^3)^(1/2)/(3*a*x^3) + (
log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)*(a
*d - 2*b*c))/(6*a^2*c^(1/2))
```

3.363 $\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx=\frac{x^4\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{4}{3},1,-\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] 1/4*x^4*AppellF1(4/3,1,-1/2,7/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/(1+d*x
^3/c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx=\frac{x^4\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{4}{3},1,-\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{\frac{dx^3}{c}+1}}$$

```
[In] Int[(x^3*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 1, -1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)
])/(4*a*Sqrt[1 + (d*x^3)/c])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{c+dx^3}\int \frac{x^3\sqrt{1+\frac{dx^3}{c}}}{a+bx^3}\,dx}{\sqrt{1+\frac{dx^3}{c}}}\\ &= \frac{x^4\sqrt{c+dx^3}F_1\left(\frac{4}{3};1,-\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{1+\frac{dx^3}{c}}}\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 241 vs. 2(64) = 128.

Time = 6.50 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.77

$$\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{x\left(\frac{(3bc-5ad)x^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a}+8\left(c+dx^3+\frac{8a^2c^2\,\text{AppellF1}}{(a+bx^3)\left(-8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3\left(2bc\,\text{App}\right.\right.}\right.\right.}{20b\sqrt{c+dx^3}}$$

```
[In] Integrate[(x^3*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (x*(((3*b*c - 5*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((
d*x^3)/c), -((b*x^3)/a)])/a + 8*(c + d*x^3 + (8*a^2*c^2*AppellF1[1/3, 1/2,
1, 4/3, -((d*x^3)/c), -((b*x^3)/a)])/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2
, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2,
7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)
/c), -((b*x^3)/a)]))))))/(20*b*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 5.09 (sec) , antiderivative size = 741, normalized size of antiderivative = 11.58

method	result	size
elliptic	Expression too large to display	741
risch	Expression too large to display	757
default	Expression too large to display	1012

```
[In] int(x^3*(d*x^3+c)^(1/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*x/b*(d*x^3+c)^(1/2)-2/3*I*(-(a*d-b*c)/b^2-2/5*c/b)*3^(1/2)/d*(-c*d^2)^(
1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(
1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/3*I*a/b^2/d^2*2^(1/2)*sum(1/_
alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2
)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3
)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(
1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/
3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*
_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*
I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/
2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)
/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=Root
Of(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx = \text{Timed out}$$

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx = \int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx$$

```
[In] integrate(x**3*(d*x**3+c)**(1/2)/(b*x**3+a),x)
[Out] Integral(x**3*sqrt(c + d*x**3)/(a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx = \int \frac{\sqrt{dx^3+c}x^3}{bx^3+a}\,dx$$

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)*x^3/(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx = \int \frac{\sqrt{dx^3+c}x^3}{bx^3+a}\,dx$$

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)*x^3/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx = \int \frac{x^3\,\sqrt{d\,x^3+c}}{b\,x^3+a}\,dx$$

```
[In] int((x^3*(c + d*x^3)^(1/2))/(a + b*x^3),x)
[Out] int((x^3*(c + d*x^3)^(1/2))/(a + b*x^3), x)
```

3.364 $\int \frac{x\sqrt{c+dx^3}}{a+bx^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{x\sqrt{c+dx^3}}{a+bx^3} dx = \frac{x^2\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{2}{3},1,-\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] 1/2*x^2*AppellF1(2/3,1,-1/2,5/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/(1+d*x
^3/c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {525, 524}

$$\int \frac{x\sqrt{c+dx^3}}{a+bx^3} dx = \frac{x^2\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{2}{3},1,-\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{\frac{dx^3}{c}+1}}$$

```
[In] Int[(x*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (x^2*Sqrt[c + d*x^3]*AppellF1[2/3, 1, -1/2, 5/3, -((b*x^3)/a), -((d*x^3)/c)
])/(2*a*Sqrt[1 + (d*x^3)/c])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c+dx^3}\int \frac{x\sqrt{1+\frac{dx^3}{c}}}{a+bx^3}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$

$$= \frac{x^2\sqrt{c+dx^3}F_1\left(\frac{2}{3};1,-\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [A] (verified)

Time = 9.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{x^2\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{2}{3},-\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{2a\sqrt{\frac{c+dx^3}{c}}}$$

```
[In] Integrate[(x*Sqrt[c + d*x^3])/(a + b*x^3),x]
[Out] (x^2*Sqrt[c + d*x^3]*AppellF1[2/3, -1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)
])/(2*a*Sqrt[(c + d*x^3)/c])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.44 (sec) , antiderivative size = 857, normalized size of antiderivative = 13.39

method	result	size
default	Expression too large to display	857
elliptic	Expression too large to display	857

```
[In] int(x*(d*x^3+c)^(1/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*El
lipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d
^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I/b/d^2*2^(1/2)*sum(
1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/
3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1
/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)
*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)
-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2
*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1
/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3
)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=Roo
tOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx = \text{Timed out}$$

```
[In] integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx = \int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx$$

```
[In] integrate(x*(d*x**3+c)**(1/2)/(b*x**3+a),x)
```

```
[Out] Integral(x*sqrt(c + d*x**3)/(a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx = \int \frac{\sqrt{dx^3+cx}}{bx^3+a}\,dx$$

```
[In] integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)*x/(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx = \int \frac{\sqrt{dx^3+cx}}{bx^3+a}\,dx$$

```
[In] integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)*x/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx = \int \frac{x\,\sqrt{d\,x^3+c}}{b\,x^3+a}\,dx$$

```
[In] int((x*(c + d*x^3)^(1/2))/(a + b*x^3),x)
[Out] int((x*(c + d*x^3)^(1/2))/(a + b*x^3), x)
```

3.365 $\int \frac{\sqrt{c+dx^3}}{a+bx^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\sqrt{c+dx^3}}{a+bx^3} dx = \frac{x\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},1,-\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] x*AppellF1(1/3,1,-1/2,4/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/(1+d*x^3/c)^
(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {441, 440}

$$\int \frac{\sqrt{c+dx^3}}{a+bx^3} dx = \frac{x\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},1,-\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{\frac{dx^3}{c}+1}}$$

```
[In] Int[Sqrt[c + d*x^3]/(a + b*x^3),x]
```

```
[Out] (x*Sqrt[c + d*x^3]*AppellF1[1/3, 1, -1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)])
/(a*Sqrt[1 + (d*x^3)/c])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c+dx^3}\int \frac{\sqrt{1+\frac{dx^3}{c}}}{a+bx^3}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$

$$= \frac{x\sqrt{c+dx^3}F_1\left(\frac{1}{3};1,-\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 161 vs. 2(59) = 118.

Time = 10.17 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.73

$$\int \frac{\sqrt{c+dx^3}}{a+bx^3}\,dx$$

$$= \frac{8acx\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{(a+bx^3)\left(8ac\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3\left(-2bc\,\text{AppellF1}\left(\frac{4}{3},-\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+ad\,\text{App}\right.\right.}$$

```
[In] Integrate[Sqrt[c + d*x^3]/(a + b*x^3),x]
```

```
[Out] (8*a*c*x*Sqrt[c + d*x^3]*AppellF1[1/3, -1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3
)/a)])/((a + b*x^3)*(8*a*c*AppellF1[1/3, -1/2, 1, 4/3, -((d*x^3)/c), -((b*x
^3)/a)] + 3*x^3*(-2*b*c*AppellF1[4/3, -1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)
/a)] + a*d*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.28 (sec) , antiderivative size = 705, normalized size of antiderivative = 11.95

method	result
default	$-\frac{2i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}}}\,\sqrt{-\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,F}{3b\sqrt{d\,x^{3}+c}}$
elliptic	$-\frac{2i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}}}\,\sqrt{-\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,F}{3b\sqrt{d\,x^{3}+c}}$

```
[In] int((d*x^3+c)^(1/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2
)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/3*
I/b/d^2*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(
1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/
d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c
)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alph
a^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
```

```
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)
*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3
^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{a+bx^3}\,dx = \text{Timed out}$$

```
[In] integrate((d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{a+bx^3}\,dx = \int \frac{\sqrt{c+dx^3}}{a+bx^3}\,dx$$

```
[In] integrate((d*x**3+c)**(1/2)/(b*x**3+a),x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(a + b*x**3), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{a+bx^3}\,dx = \int \frac{\sqrt{dx^3+c}}{bx^3+a}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{a+bx^3}\,dx = \int \frac{\sqrt{dx^3+c}}{bx^3+a}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{a+bx^3}\,dx = \int \frac{\sqrt{d\,x^3+c}}{b\,x^3+a}\,dx$$

```
[In] int((c + d*x^3)^(1/2)/(a + b*x^3),x)
[Out] int((c + d*x^3)^(1/2)/(a + b*x^3), x)
```

3.366 $\int \frac{\sqrt{c+dx^3}}{x^2(a+bx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{\sqrt{c+dx^3}}{x^2(a+bx^3)}\,dx = -\frac{\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{1}{3},1,-\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] -AppellF1(-1/3,1,-1/2,2/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/x/(1+d*x^3/c
)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{\sqrt{c+dx^3}}{x^2(a+bx^3)}\,dx = -\frac{\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{1}{3},1,-\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{\frac{dx^3}{c}+1}}$$

```
[In] Int[Sqrt[c + d*x^3]/(x^2*(a + b*x^3)),x]
```

```
[Out] -((Sqrt[c + d*x^3]*AppellF1[-1/3, 1, -1/2, 2/3, -((b*x^3)/a), -((d*x^3)/c)]
)/(a*x*Sqrt[1 + (d*x^3)/c]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c+dx^3}\int \frac{\sqrt{1+\frac{dx^3}{c}}}{x^2(a+bx^3)}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$

$$= -\frac{\sqrt{c+dx^3}F_1\left(-\frac{1}{3};1,-\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 139 vs. $2(62) = 124$.

Time = 10.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.24

$$\int \frac{\sqrt{c+dx^3}}{x^2(a+bx^3)}\,dx$$
$$= \frac{-20a(c+dx^3)+5(-2bc+3ad)x^3\sqrt{1+\frac{dx^3}{c}}\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+2bdx^6\sqrt{1+\frac{dx^3}{c}}\text{AppellF}}{20a^2x\sqrt{c+dx^3}}$$

```
[In] Integrate[Sqrt[c + d*x^3]/(x^2*(a + b*x^3)),x]
```

```
[Out] (-20*a*(c + d*x^3) + 5*(-2*b*c + 3*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/
3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*b*d*x^6*Sqrt[1 + (d*x^3)/c]
*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(20*a^2*x*Sqrt[c +
 d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 5.24 (sec) , antiderivative size = 892, normalized size of antiderivative = 14.39

method	result	size
elliptic	Expression too large to display	892
risch	Expression too large to display	893
default	Expression too large to display	1314

```
[In] int((d*x^3+c)^(1/2)/x^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a*(d*x^3+c)^(1/2)/x-1/3*I/a*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^
(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1
/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^
(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*
d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1
/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^
2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/
3*I/a/d^2*2^(1/2)*sum((-a*d+b*c)/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(
2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(
d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2
)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1
/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c
*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Elliptic
Pi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3
^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2
*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*
c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   Not i
ntegrable (provided residues have no relations)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2(a+bx^3)}\,dx=\int \frac{\sqrt{c+dx^3}}{x^2(a+bx^3)}\,dx$$

```
[In] integrate((d*x**3+c)**(1/2)/x**2/(b*x**3+a),x)
[Out] Integral(sqrt(c + d*x**3)/(x**2*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2(a+bx^3)}\,dx=\int \frac{\sqrt{dx^3+c}}{(bx^3+a)x^2}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2(a+bx^3)}\,dx=\int \frac{\sqrt{dx^3+c}}{(bx^3+a)x^2}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^2(a+bx^3)}\,dx=\int \frac{\sqrt{d\,x^3+c}}{x^2\,(b\,x^3+a)}\,dx$$

```
[In] int((c + d*x^3)^(1/2)/(x^2*(a + b*x^3)),x)
[Out] int((c + d*x^3)^(1/2)/(x^2*(a + b*x^3)), x)
```

3.367 $\int \frac{\sqrt{c+dx^3}}{x^3(a+bx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{\sqrt{c+dx^3}}{x^3(a+bx^3)}\,dx=-\frac{\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{2}{3},1,-\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] -1/2*AppellF1(-2/3,1,-1/2,1/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/x^2/(1+d
*x^3/c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{\sqrt{c+dx^3}}{x^3(a+bx^3)}\,dx=-\frac{\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{2}{3},1,-\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[In] Int[Sqrt[c + d*x^3]/(x^3*(a + b*x^3)),x]
```

```
[Out] -1/2*(Sqrt[c + d*x^3]*AppellF1[-2/3, 1, -1/2, 1/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a*x^2*Sqrt[1 + (d*x^3)/c])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c+dx^3}\int \frac{\sqrt{1+\frac{dx^3}{c}}}{x^3(a+bx^3)}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$

$$= -\frac{\sqrt{c+dx^3}F_1\left(-\frac{2}{3};1,-\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 335 vs. $2(64) = 128$.

Time = 10.27 (sec) , antiderivative size = 335, normalized size of antiderivative = 5.23

$$\int \frac{\sqrt{c+dx^3}}{x^3\,(a+bx^3)}\,dx$$

$$= \frac{-bdx^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{a\left(32ac\left(2ac+6bcx^3-adx^3+2bdx^6\right)\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-2\cdots}{(a+bx^3)\left(-8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3a\cdots}}{16a^2x^2\sqrt{c+dx^3}}$$

```
[In] Integrate[Sqrt[c + d*x^3]/(x^3*(a + b*x^3)),x]
```

```
[Out] (-(b*d*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((
b*x^3)/a)]) + (a*(32*a*c*(2*a*c + 6*b*c*x^3 - a*d*x^3 + 2*b*d*x^6)*AppellF1
[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 24*x^3*(a + b*x^3)*(c + d*
x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*Ap
pellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3)*(-8*a*
c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b*c*App
ellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2
, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(16*a^2*x^2*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 5.35 (sec) , antiderivative size = 740, normalized size of antiderivative = 11.56

method	result	size
elliptic	Expression too large to display	740
risch	Expression too large to display	741
default	Expression too large to display	1010

```
[In] int((d*x^3+c)^(1/2)/x^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a*(d*x^3+c)^(1/2)/x^2+1/6*I/a*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d
^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*(
(x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/
d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2))+1/3*I/a/d^2*2^(1/2)*sum((-a*d+b*c)/_alpha^2/(a*d-b*c)
*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(
1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alp
ha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*
d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d
^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-
3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3
*b+a))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   Not i
ntegrable (provided residues have no relations)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^3\,(a+bx^3)}\,dx = \int \frac{\sqrt{c+dx^3}}{x^3\,(a+bx^3)}\,dx$$

```
[In] integrate((d*x**3+c)**(1/2)/x**3/(b*x**3+a),x)
[Out] Integral(sqrt(c + d*x**3)/(x**3*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^3\,(a+bx^3)}\,dx = \int \frac{\sqrt{dx^3+c}}{(bx^3+a)x^3}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x^3), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^3\,(a+bx^3)}\,dx = \int \frac{\sqrt{dx^3+c}}{(bx^3+a)x^3}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^3\,(a+bx^3)}\,dx = \int \frac{\sqrt{d\,x^3+c}}{x^3\,\left(b\,x^3+a\right)}\,dx$$

```
[In] int((c + d*x^3)^(1/2)/(x^3*(a + b*x^3)),x)
[Out] int((c + d*x^3)^(1/2)/(x^3*(a + b*x^3)), x)
```

3.368 $\int \frac{x^8(c+dx^3)^{3/2}}{a+bx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 154

$$\int \frac{x^8(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{2a^2(bc-ad)\sqrt{c+dx^3}}{3b^4} + \frac{2a^2(c+dx^3)^{3/2}}{9b^3} - \frac{2(bc+ad)\,(c+dx^3)^{5/2}}{15b^2d^2} + \frac{2(c+dx^3)^{7/2}}{21bd^2} - \frac{2a^2(bc-ad)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{9/2}}$$

```
[Out] 2/9*a^2*(d*x^3+c)^(3/2)/b^3-2/15*(a*d+b*c)*(d*x^3+c)^(5/2)/b^2/d^2+2/21*(d*
x^3+c)^(7/2)/b/d^2-2/3*a^2*(-a*d+b*c)^(3/2)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)
/(-a*d+b*c)^(1/2))/b^(9/2)+2/3*a^2*(-a*d+b*c)*(d*x^3+c)^(1/2)/b^4
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 90, 52, 65, 214}

$$\int \frac{x^8(c+dx^3)^{3/2}}{a+bx^3}\,dx = -\frac{2a^2(bc-ad)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{9/2}} + \frac{2a^2\sqrt{c+dx^3}(bc-ad)}{3b^4} + \frac{2a^2(c+dx^3)^{3/2}}{9b^3} - \frac{2(c+dx^3)^{5/2}\,(ad+bc)}{15b^2d^2} + \frac{2(c+dx^3)^{7/2}}{21bd^2}$$

```
[In] Int[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (2*a^2*(b*c - a*d)*Sqrt[c + d*x^3])/(3*b^4) + (2*a^2*(c + d*x^3)^(3/2))/(9*
b^3) - (2*(b*c + a*d)*(c + d*x^3)^(5/2))/(15*b^2*d^2) + (2*(c + d*x^3)^(7/2
))/(21*b*d^2) - (2*a^2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/
Sqrt[b*c - a*d]])/(3*b^(9/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2(c+dx)^{3/2}}{a+bx}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{(-bc-ad)(c+dx)^{3/2}}{b^2d}+\frac{a^2(c+dx)^{3/2}}{b^2(a+bx)}+\frac{(c+dx)^{5/2}}{bd}\right)\,dx, x, x^3\right)\\
&= -\frac{2(bc+ad)\left(c+dx^3\right)^{5/2}}{15b^2d^2}+\frac{2\left(c+dx^3\right)^{7/2}}{21bd^2}+\frac{a^2\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx, x, x^3\right)}{3b^2}
\end{aligned}$$

$$= \frac{2a^2(c+dx^3)^{3/2}}{9b^3} - \frac{2(bc+ad)\,(c+dx^3)^{5/2}}{15b^2d^2} + \frac{2(c+dx^3)^{7/2}}{21bd^2} + \frac{(a^2(bc-ad))\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)}{3b^3}$$

$$= \frac{2a^2(bc-ad)\sqrt{c+dx^3}}{3b^4} + \frac{2a^2(c+dx^3)^{3/2}}{9b^3} - \frac{2(bc+ad)\,(c+dx^3)^{5/2}}{15b^2d^2} + \frac{2(c+dx^3)^{7/2}}{21bd^2} + \frac{(a^2(bc-ad)^2)\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3b^4}$$

$$= \frac{2a^2(bc-ad)\sqrt{c+dx^3}}{3b^4} + \frac{2a^2(c+dx^3)^{3/2}}{9b^3} - \frac{2(bc+ad)\,(c+dx^3)^{5/2}}{15b^2d^2} + \frac{2(c+dx^3)^{7/2}}{21bd^2} + \frac{(2a^2(bc-ad)^2)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3b^4d}$$

$$= \frac{2a^2(bc-ad)\sqrt{c+dx^3}}{3b^4} + \frac{2a^2(c+dx^3)^{3/2}}{9b^3} - \frac{2(bc+ad)\,(c+dx^3)^{5/2}}{15b^2d^2} + \frac{2(c+dx^3)^{7/2}}{21bd^2} - \frac{2a^2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{9/2}}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.93

$$\int \frac{x^8(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{2\sqrt{c+dx^3}\left(-105a^3d^3 - 21ab^2d(c+dx^3)^2 - 3b^3(2c-5dx^3)\,(c+dx^3)^2 + 35a^2bd^2(4c+\right.}{315b^4d^2} + \frac{2a^2(-bc+ad)^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{9/2}}$$

```
[In] Integrate[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(-105*a^3*d^3 - 21*a*b^2*d*(c + d*x^3)^2 - 3*b^3*(2*c -
5*d*x^3)*(c + d*x^3)^2 + 35*a^2*b*d^2*(4*c + d*x^3)))/(315*b^4*d^2) + (2*a^
2*(-(b*c) + a*d)^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]]
)/(3*b^(9/2))
```

Maple [A] (verified)

Time = 4.40 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$-\frac{2\left(\left(\left(\frac{2\left(-\frac{5d x^3}{2}+c\right)\left(d x^3+c\right)^2 b^3}{35}+\frac{a d \left(d x^3+c\right)^2 b^2}{5}-\frac{4\left(\frac{d x^3}{4}+c\right) d^2 a^2 b}{3}+a^3 d^3\right)\sqrt{(ad-bc)b}\,\sqrt{d x^3+c}-a^2 d^2 (ad-bc)^2 \arctan\left(\cdots\right.\right.\right.}{3\sqrt{(ad-bc)b}\, d^2 b^4}$
default	$\frac{\frac{2d x^9\sqrt{d x^3+c}}{21}+\frac{16 c x^6\sqrt{d x^3+c}}{105}+\frac{2c^2x^3\sqrt{d x^3+c}}{105d}-\frac{4c^3\sqrt{d x^3+c}}{105d^2}}{b}-\frac{2a\left(d x^3+c\right)^{\frac{5}{2}}}{15b^2 d}-\frac{2a^2\left(-(ad-bc)^2\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)+\left(\cdots\right.\right.}{3b^4\sqrt{(\cdots}}$
risch	$-\frac{2\left(-15b^3d^3x^9+21a b^2d^3x^6-24b^3c d^2x^6-35a^2b d^3x^3+42a b^2c d^2x^3-3b^3c^2d x^3+105a^3d^3-140a^2bc d^2+21a b^2c^2d+6b^3c^3\right)\cdots}{315d^2b^4}$
elliptic	$\frac{2d x^9\sqrt{d x^3+c}}{21b}+\frac{2\left(-\frac{d(ad-2bc)}{b^2}-\frac{6cd}{7b}\right)x^6\sqrt{d x^3+c}}{15d}+\frac{2\left(\frac{a^2d^2-2abcd+b^2c^2}{b^3}-\frac{4\left(-\frac{d(ad-2bc)}{b^2}-\frac{6cd}{7b}\right)c}{5d}\right)x^3\sqrt{d x^3+c}}{9d}+\cdots$

```
[In] int(x^8*(d*x^3+c)^(3/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*((2/35*(-5/2*d*x^3+c)*(d*x^3+c)^2*b^3+1/5*a*d*(d*x^3+c)^2*b^2-4/3*(1/4
*d*x^3+c)*d^2*a^2*b+a^3*d^3)*((a*d-b*c)*b)^(1/2)*(d*x^3+c)^(1/2)-a^2*d^2*(a
*d-b*c)^2*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2)))/((a*d-b*c)*b)^(1/2
)/d^2/b^4
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 410, normalized size of antiderivative = 2.66

$$\int \frac{x^8(c+dx^3)^{3/2}}{a+bx^3}\,dx = \left[-\frac{105\,(a^2bcd^2-a^3d^3)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)-2\,(15\,b^3d^3x^9+3\,(8\cdots}{\cdots}\right.$$

$$-\frac{2\left(105\,(a^2bcd^2-a^3d^3)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-(15\,b^3d^3x^9+3\,(8\,b^3cd^2-7\,ab^2d^3)x^6-6\,b^3c^3\cdots\right.}{315\,b^4d^2}$$

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/315*(105*(a^2*b*c*d^2 - a^3*d^3)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b
*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) - 2*(15*b^
3*d^3*x^9 + 3*(8*b^3*c*d^2 - 7*a*b^2*d^3)*x^6 - 6*b^3*c^3 - 21*a*b^2*c^2*d
+ 140*a^2*b*c*d^2 - 105*a^3*d^3 + (3*b^3*c^2*d - 42*a*b^2*c*d^2 + 35*a^2*b*
d^3)*x^3)*sqrt(d*x^3 + c))/(b^4*d^2), -2/315*(105*(a^2*b*c*d^2 - a^3*d^3)*s
qrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a
*d)) - (15*b^3*d^3*x^9 + 3*(8*b^3*c*d^2 - 7*a*b^2*d^3)*x^6 - 6*b^3*c^3 - 21
*a*b^2*c^2*d + 140*a^2*b*c*d^2 - 105*a^3*d^3 + (3*b^3*c^2*d - 42*a*b^2*c*d^
2 + 35*a^2*b*d^3)*x^3)*sqrt(d*x^3 + c))/(b^4*d^2)]
```

Sympy [A] (verification not implemented)

Time = 33.41 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.31

$$\int \frac{x^8(c+dx^3)^{3/2}}{a+bx^3}\,dx = \begin{cases} \frac{2\left(\frac{a^2d\left(c+dx^3\right)^{\frac{3}{2}}}{9b^3}+\frac{a^2d(ad-bc)^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^5\sqrt{\frac{ad-bc}{b}}}+\frac{\left(c+dx^3\right)^{\frac{7}{2}}}{21bd}+\frac{\left(c+dx^3\right)^{\frac{5}{2}}(-ad-bc)}{15b^2d}+\frac{\sqrt{c+dx^3}\left(-a^3d^2+a^2bcd\right)}{3b^4}\right)}{d} & \text{fo}\cdots \\ c^{\frac{3}{2}}\left(\frac{a^2\left(\begin{cases}\frac{x^3}{a} & \text{for } b=0\\ \frac{\log\left(a+bx^3\right)}{b} & \text{otherwise}\end{cases}\right)}{3b^2}-\frac{ax^3}{3b^2}+\frac{x^6}{6b}\right) & \text{o}\cdots \end{cases}$$

```
[In] integrate(x**8*(d*x**3+c)**(3/2)/(b*x**3+a),x)
```

```
[Out] Piecewise((2*(a**2*d*(c + d*x**3)**(3/2)/(9*b**3) + a**2*d*(a*d - b*c)**2*a
tan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*b**5*sqrt((a*d - b*c)/b)) + (c
 + d*x**3)**(7/2)/(21*b*d) + (c + d*x**3)**(5/2)*(-a*d - b*c)/(15*b**2*d) +
 sqrt(c + d*x**3)*(-a**3*d**2 + a**2*b*c*d)/(3*b**4))/d, Ne(d, 0)), (c**(3/
2)*(a**2*Piecewise((x**3/a, Eq(b, 0)), (log(a + b*x**3)/b, True))/(3*b**2)
- a*x**3/(3*b**2) + x**6/(6*b)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8(c+dx^3)^{3/2}}{a+bx^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.25

$$\int \frac{x^8(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{2\,(a^2b^2c^2-2\,a^3bcd+a^4d^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}b^4} + \frac{2\left(15\,(dx^3+c)^{\frac{7}{2}}b^6d^{12}-21\,(dx^3+c)^{\frac{5}{2}}b^6cd^{12}-21\,(dx^3+c)^{\frac{5}{2}}ab^5d^{13}+35\,(dx^3+c)^{\frac{3}{2}}a^2b^4d^{14}+105\,\sqrt{dx^3+c}\right.}{315\,b^7d^{14}}$$

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^
2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^4) + 2/315*(15*(d*x^3 + c)^(7/2)*b^6*
d^12 - 21*(d*x^3 + c)^(5/2)*b^6*c*d^12 - 21*(d*x^3 + c)^(5/2)*a*b^5*d^13 +
35*(d*x^3 + c)^(3/2)*a^2*b^4*d^14 + 105*sqrt(d*x^3 + c)*a^2*b^4*c*d^14 - 10
5*sqrt(d*x^3 + c)*a^3*b^3*d^15)/(b^7*d^14)
```

Mupad [B] (verification not implemented)

Time = 10.39 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.14

$$\int \frac{x^8(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{2\,d\,x^9\,\sqrt{d\,x^3+c}}{21\,b} - \frac{\left(\frac{2\,a\,\left(\frac{c^2}{b}+\frac{a\,\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{b}\right)}{b}+\frac{2\,c\,\left(\frac{2\,c^2}{b}+\frac{2\,a\,\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{b}+\frac{4\,c\,\left(\frac{2\,a\,d^2}{b^2}-\frac{16\,c\,d}{7\,b}\right)}{5\,d}\right)}{3\,d}\right)\sqrt{d\,x^3+c}}{3\,d} + \frac{x^3\,\sqrt{d\,x^3+c}\,\left(\frac{2\,c^2}{b}+\frac{2\,a\,\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{b}+\frac{4\,c\,\left(\frac{2\,a\,d^2}{b^2}-\frac{16\,c\,d}{7\,b}\right)}{5\,d}\right)}{9\,d} - \frac{x^6\,\sqrt{d\,x^3+c}\,\left(\frac{2\,a\,d^2}{b^2}-\frac{16\,c\,d}{7\,b}\right)}{15\,d} + \frac{a^2\,\ln\left(\frac{a^2\,d^2+2\,b^2\,c^2-a\,b\,d^2\,x^3+b^2\,c\,d\,x^3-3\,a\,b\,c\,d-\sqrt{b}\,\sqrt{d\,x^3+c}\,{\left(a\,d-b\,c\right)}^{3/2}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,{\left(a\,d-b\,c\right)}^{3/2}\,1{}\mathrm{i}}{3\,b^{9/2}}$$

```
[In] int((x^8*(c + d*x^3)^(3/2))/(a + b*x^3),x)
```

```
[Out] (2*d*x^9*(c + d*x^3)^(1/2))/(21*b) - (((2*a*(c^2/b + (a*((a*d^2)/b^2 - (2*c
*d)/b))/b))/b + (2*c*((2*c^2)/b + (2*a*((a*d^2)/b^2 - (2*c*d)/b))/b + (4*c*
((2*a*d^2)/b^2 - (16*c*d)/(7*b)))/(5*d)))/(3*d))*(c + d*x^3)^(1/2))/(3*d) +
 (x^3*(c + d*x^3)^(1/2)*((2*c^2)/b + (2*a*((a*d^2)/b^2 - (2*c*d)/b))/b + (4
*c*((2*a*d^2)/b^2 - (16*c*d)/(7*b)))/(5*d)))/(9*d) - (x^6*(c + d*x^3)^(1/2)
*((2*a*d^2)/b^2 - (16*c*d)/(7*b)))/(15*d) + (a^2*log((a^2*d^2 + 2*b^2*c^2 -
 b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)*2i - a*b*d^2*x^3 + b^2*c*d*x^3
 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3/2)*1i)/(3*b^(9/2))
```

3.369 $\int \frac{x^5(c+dx^3)^{3/2}}{a+bx^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 120

$$\int \frac{x^5(c+dx^3)^{3/2}}{a+bx^3} dx = -\frac{2a(bc-ad)\sqrt{c+dx^3}}{3b^3} - \frac{2a(c+dx^3)^{3/2}}{9b^2} + \frac{2(c+dx^3)^{5/2}}{15bd} + \frac{2a(bc-ad)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}$$

```
[Out] -2/9*a*(d*x^3+c)^(3/2)/b^2+2/15*(d*x^3+c)^(5/2)/b/d+2/3*a*(-a*d+b*c)^(3/2)*
arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(7/2)-2/3*a*(-a*d+b*c)*
(d*x^3+c)^(1/2)/b^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 81, 52, 65, 214}

$$\int \frac{x^5(c+dx^3)^{3/2}}{a+bx^3} dx = \frac{2a(bc-ad)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}} - \frac{2a\sqrt{c+dx^3}(bc-ad)}{3b^3} - \frac{2a(c+dx^3)^{3/2}}{9b^2} + \frac{2(c+dx^3)^{5/2}}{15bd}$$

```
[In] Int[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (-2*a*(b*c - a*d)*Sqrt[c + d*x^3])/(3*b^3) - (2*a*(c + d*x^3)^(3/2))/(9*b^2
) + (2*(c + d*x^3)^(5/2))/(15*b*d) + (2*a*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b
]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x(c+dx)^{3/2}}{a+bx}\,dx, x, x^3\right)$$

$$= \frac{2(c+dx^3)^{5/2}}{15bd} - \frac{a\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx, x, x^3\right)}{3b}$$

$$= -\frac{2a(c+dx^3)^{3/2}}{9b^2} + \frac{2(c+dx^3)^{5/2}}{15bd} - \frac{(a(bc-ad))\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)}{3b^2}$$

$$= -\frac{2a(bc-ad)\sqrt{c+dx^3}}{3b^3} - \frac{2a(c+dx^3)^{3/2}}{9b^2} + \frac{2(c+dx^3)^{5/2}}{15bd} - \frac{(a(bc-ad)^2)\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3b^3}$$

$$= -\frac{2a(bc-ad)\sqrt{c+dx^3}}{3b^3} - \frac{2a(c+dx^3)^{3/2}}{9b^2} + \frac{2(c+dx^3)^{5/2}}{15bd} - \frac{(2a(bc-ad)^2)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3b^3d}$$

$$= -\frac{2a(bc-ad)\sqrt{c+dx^3}}{3b^3} - \frac{2a(c+dx^3)^{3/2}}{9b^2} + \frac{2(c+dx^3)^{5/2}}{15bd} + \frac{2a(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.92

$$\int \frac{x^5(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{2\sqrt{c+dx^3}\left(15a^2d^2+3b^2(c+dx^3)^2-5abd(4c+dx^3)\right)}{45b^3d} - \frac{2a(-bc+ad)^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{7/2}}$$

```
[In] Integrate[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(15*a^2*d^2 + 3*b^2*(c + d*x^3)^2 - 5*a*b*d*(4*c + d*x^3
)))/(45*b^3*d) - (2*a*(-(b*c) + a*d)^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])
/Sqrt[-(b*c) + a*d]])/(3*b^(7/2))
```

Maple [A] (verified)

Time = 4.68 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.94

method	result
default	$\frac{2\left(d\,x^{3}+c\right)^{\frac{5}{2}}}{15bd}+\frac{2a\left(-\left(ad-bc\right)^{2}\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(ad-bc\right)b}}\right)+\left(\frac{\left(-d\,x^{3}-4c\right)b}{3}+ad\right)\sqrt{d\,x^{3}+c}\,\sqrt{\left(ad-bc\right)b}\right)}{3b^{3}\sqrt{\left(ad-bc\right)b}}$
pseudoelliptic	$\frac{\frac{2\left(\frac{\left(d\,x^{3}+c\right)^{2}b^{2}}{5}-\frac{4\left(\frac{d\,x^{3}}{4}+c\right)dab}{3}+a^{2}d^{2}\right)\sqrt{\left(ad-bc\right)b}\,\sqrt{d\,x^{3}+c}}{3}-\frac{2ad\left(ad-bc\right)^{2}\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(ad-bc\right)b}}\right)}{3}}{d\,b^{3}\sqrt{\left(ad-bc\right)b}}$
risch	$\frac{2\left(3b^{2}d^{2}x^{6}-5x^{3}ab\,d^{2}+6x^{3}b^{2}cd+15a^{2}d^{2}-20abcd+3b^{2}c^{2}\right)\sqrt{d\,x^{3}+c}}{45d\,b^{3}}-\frac{2a\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(ad-bc\right)b}}\right)}{3b^{3}\sqrt{\left(ad-bc\right)b}}$
elliptic	$\frac{2d\,x^{6}\sqrt{d\,x^{3}+c}}{15b}+\frac{2\left(-\frac{d\left(ad-2bc\right)}{b^{2}}-\frac{4cd}{5b}\right)x^{3}\sqrt{d\,x^{3}+c}}{9d}+\frac{2\left(\frac{a^{2}d^{2}-2abcd+b^{2}c^{2}}{b^{3}}-\frac{2\left(-\frac{d\left(ad-2bc\right)}{b^{2}}-\frac{4cd}{5b}\right)c}{3d}\right)\sqrt{d\,x^{3}+c}}{3d}+\frac{ia\sqrt{2}\left(\cdots\right.}{}$

[In] int(x^5*(d*x^3+c)^(3/2)/(b*x^3+a),x,method=_RETURNVERBOSE)

[Out] 2/15*(d*x^3+c)^(5/2)/b/d+2/3*a/b^3*(-(a*d-b*c)^2*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))+(1/3*(-d*x^3-4*c)*b+a*d)*(d*x^3+c)^(1/2)*((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.48

$$\int \frac{x^5(c+dx^3)^{3/2}}{a+bx^3}\,dx = \left[-\frac{15\left(abcd-a^2d^2\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)-2\left(3\,b^2d^2x^6+3\,b^2c^2-\cdots\right.}{45\,b^3d}\right.$$

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/45*(15*(a*b*c*d - a^2*d^2)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a
*d - 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) - 2*(3*b^2*d^2*x
^6 + 3*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 + (6*b^2*c*d - 5*a*b*d^2)*x^3)*sqr
t(d*x^3 + c))/(b^3*d), 2/45*(15*(a*b*c*d - a^2*d^2)*sqrt(-(b*c - a*d)/b)*ar
ctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (3*b^2*d^2*x^6
+ 3*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 + (6*b^2*c*d - 5*a*b*d^2)*x^3)*sqrt(d
*x^3 + c))/(b^3*d)]
```

Sympy [A] (verification not implemented)

Time = 16.10 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.28

$$\int \frac{x^5(c+dx^3)^{3/2}}{a+bx^3}\,dx = \begin{cases} \frac{2\left(-\frac{ad\left(c+dx^3\right)^{\frac{3}{2}}}{9b^2}-\frac{ad(ad-bc)^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^4\sqrt{\frac{ad-bc}{b}}}+\frac{\left(c+dx^3\right)^{\frac{5}{2}}}{15b}+\frac{\sqrt{c+dx^3}\left(a^2d^2-abcd\right)}{3b^3}\right)}{d} & \text{for } d\neq 0 \\ c^{\frac{3}{2}}\left(-\frac{a\left(\begin{cases}\frac{x^3}{a} & \text{for } b=0\\ \frac{\log\left(a+bx^3\right)}{b} & \text{otherwise}\end{cases}\right)}{3b}+\frac{x^3}{3b}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(x**5*(d*x**3+c)**(3/2)/(b*x**3+a),x)
```

```
[Out] Piecewise((2*(-a*d*(c + d*x**3)**(3/2)/(9*b**2) - a*d*(a*d - b*c)**2*atan(s
qrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*b**4*sqrt((a*d - b*c)/b)) + (c + d*
x**3)**(5/2)/(15*b) + sqrt(c + d*x**3)*(a**2*d**2 - a*b*c*d)/(3*b**3))/d, N
e(d, 0)), (c**(3/2)*(-a*Piecewise((x**3/a, Eq(b, 0)), (log(a + b*x**3)/b, T
rue))/(3*b) + x**3/(3*b)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(c+dx^3)^{3/2}}{a+bx^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.26

$$\int \frac{x^5(c+dx^3)^{3/2}}{a+bx^3}\,dx = -\frac{2\,(ab^2c^2-2\,a^2bcd+a^3d^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}b^3} + \frac{2\left(3\,(dx^3+c)^{\frac{5}{2}}b^4d^4-5\,(dx^3+c)^{\frac{3}{2}}ab^3d^5-15\,\sqrt{dx^3+c}ab^3cd^5+15\,\sqrt{dx^3+c}a^2b^2d^6\right)}{45\,b^5d^5}$$

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -2/3*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2
*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 2/45*(3*(d*x^3 + c)^(5/2)*b^4*d^4
 - 5*(d*x^3 + c)^(3/2)*a*b^3*d^5 - 15*sqrt(d*x^3 + c)*a*b^3*c*d^5 + 15*sqrt
(d*x^3 + c)*a^2*b^2*d^6)/(b^5*d^5)
```

Mupad [B] (verification not implemented)

Time = 10.36 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.79

$$\int \frac{x^5(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{\sqrt{d\,x^3+c}\,\left(\frac{2\,c^2}{b}+\frac{2\,a\,\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{b}+\frac{2\,c\,\left(\frac{2\,a\,d^2}{b^2}-\frac{12\,c\,d}{5\,b}\right)}{3\,d}\right)}{3\,d} + \frac{2\,d\,x^6\,\sqrt{d\,x^3+c}}{15\,b} - \frac{x^3\,\sqrt{d\,x^3+c}\,\left(\frac{2\,a\,d^2}{b^2}-\frac{12\,c\,d}{5\,b}\right)}{9\,d} + \frac{a\,\ln\left(\frac{a^2\,d^2+2\,b^2\,c^2-a\,b\,d^2\,x^3+b^2\,c\,d\,x^3-3\,a\,b\,c\,d+\sqrt{b}\,\sqrt{d\,x^3+c}\,{\left(a\,d-b\,c\right)}^{3/2}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,{\left(a\,d-b\,c\right)}^{3/2}\,1{}\mathrm{i}}{3\,b^{7/2}}$$

```
[In] int((x^5*(c + d*x^3)^(3/2))/(a + b*x^3),x)
```

```
[Out] ((c + d*x^3)^(1/2)*((2*c^2)/b + (2*a*((a*d^2)/b^2 - (2*c*d)/b))/b + (2*c*((
2*a*d^2)/b^2 - (12*c*d)/(5*b)))/(3*d)))/(3*d) + (2*d*x^6*(c + d*x^3)^(1/2))
/(15*b) - (x^3*(c + d*x^3)^(1/2)*((2*a*d^2)/b^2 - (12*c*d)/(5*b)))/(9*d) +
(a*log((a^2*d^2 + 2*b^2*c^2 + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)*2
i - a*b*d^2*x^3 + b^2*c*d*x^3 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3/2)*1
i)/(3*b^(7/2))
```

3.370 $\int \frac{x^2(c+dx^3)^{3/2}}{a+bx^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 96

$$\int \frac{x^2(c+dx^3)^{3/2}}{a+bx^3} dx = \frac{2(bc-ad)\sqrt{c+dx^3}}{3b^2} + \frac{2(c+dx^3)^{3/2}}{9b} - \frac{2(bc-ad)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}$$

```
[Out] 2/9*(d*x^3+c)^(3/2)/b-2/3*(-a*d+b*c)^(3/2)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/
(-a*d+b*c)^(1/2))/b^(5/2)+2/3*(-a*d+b*c)*(d*x^3+c)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {455, 52, 65, 214}

$$\int \frac{x^2(c+dx^3)^{3/2}}{a+bx^3} dx = -\frac{2(bc-ad)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}} + \frac{2\sqrt{c+dx^3}(bc-ad)}{3b^2} + \frac{2(c+dx^3)^{3/2}}{9b}$$

```
[In] Int[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (2*(b*c - a*d)*Sqrt[c + d*x^3])/(3*b^2) + (2*(c + d*x^3)^(3/2))/(9*b) - (2*
(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^
(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx, x, x^3\right)\\
&= \frac{2(c+dx^3)^{3/2}}{9b} + \frac{(bc-ad)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)}{3b}\\
&= \frac{2(bc-ad)\sqrt{c+dx^3}}{3b^2} + \frac{2(c+dx^3)^{3/2}}{9b} + \frac{(bc-ad)^2\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3b^2}\\
&= \frac{2(bc-ad)\sqrt{c+dx^3}}{3b^2} + \frac{2(c+dx^3)^{3/2}}{9b} + \frac{\left(2(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3b^2d}\\
&= \frac{2(bc-ad)\sqrt{c+dx^3}}{3b^2} + \frac{2(c+dx^3)^{3/2}}{9b} - \frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \frac{x^2(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{2\sqrt{c+dx^3}(4bc-3ad+bdx^3)}{9b^2} + \frac{2(-bc+ad)^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{5/2}}$$

```
[In] Integrate[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(4*b*c - 3*a*d + b*d*x^3))/(9*b^2) + (2*(-(b*c) + a*d)^(
3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(3*b^(5/2))
```

Maple [A] (verified)

Time = 4.42 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.98

method	result
default	$-\frac{2\left(-(ad-bc)^2\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)+\left(\frac{\left(-d\,x^3-4c\right)b}{3}+ad\right)\sqrt{d\,x^3+c}\,\sqrt{(ad-bc)b}\right)}{3\sqrt{(ad-bc)b}\,b^2}$
risch	$-\frac{2\sqrt{d\,x^3+c}\,(-bd\,x^3+3ad-4bc)}{9b^2}+\frac{2(a^2d^2-2abcd+b^2c^2)\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3b^2\sqrt{(ad-bc)b}}$
pseudoelliptic	$-\frac{2\left(-(ad-bc)^2\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)+\left(\frac{\left(-d\,x^3-4c\right)b}{3}+ad\right)\sqrt{d\,x^3+c}\,\sqrt{(ad-bc)b}\right)}{3\sqrt{(ad-bc)b}\,b^2}$
elliptic	$\frac{2d\,x^3\sqrt{d\,x^3+c}}{9b}+\frac{2\left(-\frac{d(ad-2bc)}{b^2}-\frac{2cd}{3b}\right)\sqrt{d\,x^3+c}}{3d}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\text{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-a^2d^2+2abcd-b^2c^2\right)\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x\right.}{}}}{}\right.}{}$

```
[In] int(x^2*(d*x^3+c)^(3/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(-(a*d-b*c)^2*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))+(1/3*(-d*x
^3-4*c)*b+a*d)*(d*x^3+c)^(1/2)*((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.12

$$\int \frac{x^2(c+dx^3)^{3/2}}{a+bx^3}\,dx = \left[-\frac{3\,(bc-ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)-2\,(bdx^3+4\,bc-3\,ad)\sqrt{dx^3}}{9\,b^2}\right.$$

$$\left.-\frac{2\left(3\,(bc-ad)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-(bdx^3+4\,bc-3\,ad)\sqrt{dx^3+c}\right)}{9\,b^2}\right]$$

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/9*(3*(b*c - a*d)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqr
t(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) - 2*(b*d*x^3 + 4*b*c - 3*a
*d)*sqrt(d*x^3 + c))/b^2, -2/9*(3*(b*c - a*d)*sqrt(-(b*c - a*d)/b)*arctan(-
sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - (b*d*x^3 + 4*b*c - 3*
a*d)*sqrt(d*x^3 + c))/b^2]
```

Sympy [A] (verification not implemented)

Time = 7.95 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.29

$$\int \frac{x^2(c+dx^3)^{3/2}}{a+bx^3}\,dx = \begin{cases} \frac{2\left(\frac{d(c+dx^3)^{\frac{3}{2}}}{9b}+\frac{\sqrt{c+dx^3}(-ad^2+bcd)}{3b^2}+\frac{d(ad-bc)^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^3\sqrt{\frac{ad-bc}{b}}}\right)}{d} & \text{for } d\neq 0 \\ c^{\frac{3}{2}}\left(\begin{cases}\frac{x^3}{3a} & \text{for } b=0 \\ \frac{\log(3a+3bx^3)}{3b} & \text{otherwise}\end{cases}\right) & \text{otherwise}\end{cases}$$

```
[In] integrate(x**2*(d*x**3+c)**(3/2)/(b*x**3+a),x)
```

```
[Out] Piecewise((2*(d*(c + d*x**3)**(3/2)/(9*b) + sqrt(c + d*x**3)*(-a*d**2 + b*c
*d)/(3*b**2) + d*(a*d - b*c)**2*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/
(3*b**3*sqrt((a*d - b*c)/b)))/d, Ne(d, 0)), (c**(3/2)*Piecewise((x**3/(3*a)
, Eq(b, 0)), (log(3*a + 3*b*x**3)/(3*b), True)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(c+dx^3)^{3/2}}{a+bx^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.18

$$\int \frac{x^2(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{2\,(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}b^2} + \frac{2\left((dx^3+c)^{\frac{3}{2}}b^2+3\,\sqrt{dx^3+c}b^2c-3\,\sqrt{dx^3+c}abd\right)}{9\,b^3}$$

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c +
a*b*d))/(sqrt(-b^2*c + a*b*d)*b^2) + 2/9*((d*x^3 + c)^(3/2)*b^2 + 3*sqrt(d*
x^3 + c)*b^2*c - 3*sqrt(d*x^3 + c)*a*b*d)/b^3
```

Mupad [B] (verification not implemented)

Time = 10.18 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.49

$$\int \frac{x^2(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{2\,d\,x^3\,\sqrt{d\,x^3+c}}{9\,b} - \frac{\sqrt{d\,x^3+c}\,\left(\frac{2\,a\,d^2}{b^2}-\frac{8\,c\,d}{3\,b}\right)}{3\,d} + \frac{\ln\left(\frac{a^2\,d^2+2\,b^2\,c^2-a\,b\,d^2\,x^3+b^2\,c\,d\,x^3-3\,a\,b\,c\,d-\sqrt{b}\,\sqrt{d\,x^3+c}\,{(a\,d-b\,c)}^{3/2}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,{(a\,d-b\,c)}^{3/2}\,1{}\mathrm{i}}{3\,b^{5/2}}$$

```
[In] int((x^2*(c + d*x^3)^(3/2))/(a + b*x^3),x)
```

```
[Out] (log((a^2*d^2 + 2*b^2*c^2 - b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)*2i
- a*b*d^2*x^3 + b^2*c*d*x^3 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3/2)*1i)
/(3*b^(5/2)) - ((c + d*x^3)^(1/2)*((2*a*d^2)/b^2 - (8*c*d)/(3*b)))/(3*d) +
(2*d*x^3*(c + d*x^3)^(1/2))/(9*b)
```

3.371 $\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)}\,dx=\frac{2d\sqrt{c+dx^3}}{3b}-\frac{2c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a}+\frac{2(bc-ad)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3ab^{3/2}}$$

```
[Out] -2/3*c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a+2/3*(-a*d+b*c)^(3/2)*arctan
h(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a/b^(3/2)+2/3*d*(d*x^3+c)^(1/2)
/b
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 86, 162, 65, 214}

$$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)}\,dx=\frac{2(bc-ad)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3ab^{3/2}}-\frac{2c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a}+\frac{2d\sqrt{c+dx^3}}{3b}$$

```
[In] Int[(c + d*x^3)^(3/2)/(x*(a + b*x^3)),x]
```

```
[Out] (2*d*Sqrt[c + d*x^3])/(3*b) - (2*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/
(3*a) + (2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a
*d]])/(3*a*b^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Dist[1/(b*d), I
nt[(b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x(a+bx)}\,dx, x, x^3\right)\\
&= \frac{2d\sqrt{c+dx^3}}{3b} + \frac{\text{Subst}\left(\int \frac{bc^2+d(2bc-ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3b}\\
&= \frac{2d\sqrt{c+dx^3}}{3b} + \frac{c^2\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{3a} - \frac{(bc-ad)^2\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3ab}\\
&= \frac{2d\sqrt{c+dx^3}}{3b} + \frac{(2c^2)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3ad}\\
&\quad - \frac{(2(bc-ad)^2)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3abd}
\end{aligned}
$$

$$= \frac{2d\sqrt{c+dx^3}}{3b} - \frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a} + \frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3ab^{3/2}}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.02

$$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)}\,dx = \frac{2\left(a\sqrt{b}d\sqrt{c+dx^3} - (-bc+ad)^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right) - b^{3/2}c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)\right)}{3ab^{3/2}}$$

```
[In] Integrate[(c + d*x^3)^(3/2)/(x*(a + b*x^3)),x]
```

```
[Out] (2*(a*Sqrt[b]*d*Sqrt[c + d*x^3] - (-(b*c) + a*d)^(3/2)*ArcTan[(Sqrt[b]*Sqrt
[c + d*x^3])/Sqrt[-(b*c) + a*d]] - b^(3/2)*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/
Sqrt[c]]))/(3*a*b^(3/2))
```

Maple [A] (verified)

Time = 4.59 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$\frac{-\frac{2(ad-bc)^{2}\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3}+\frac{2\left(-\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)c^{\frac{3}{2}}b+ad\sqrt{d\,x^{3}+c}\right)\sqrt{(ad-bc)b}}{3}}{ab\sqrt{(ad-bc)b}}$
default	$\frac{\frac{2d\,x^{3}\sqrt{d\,x^{3}+c}}{9}+\frac{8c\sqrt{d\,x^{3}+c}}{9}-\frac{2c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3}}{a}+\frac{-\frac{2(ad-bc)^{2}\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3}+\frac{2\left(\frac{\left(-d\,x^{3}-4c\right)b}{3}+ad\right)\sqrt{d\,x^{3}+c}\,\sqrt{(ad-}{3}}{ab\sqrt{(ad-bc)b}}$
elliptic	Expression too large to display

```
[In] int((d*x^3+c)^(3/2)/x/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/((a*d-b*c)*b)^(1/2)*(-(a*d-b*c)^2*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b
)^(1/2))+(-arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(3/2)*b+a*d*(d*x^3+c)^(1/2))*
((a*d-b*c)*b)^(1/2))/b/a
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 486, normalized size of antiderivative = 4.67

$$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)}\,dx = \left[\frac{bc^{\frac{3}{2}}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)+2\sqrt{dx^3+c}ad-(bc-ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2bc-ad-2\sqrt{dx^3}}{bx^3+a}\right.}{3ab}\right.$$

```
[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/3*(b*c^(3/2)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt
(d*x^3 + c)*a*d - (b*c - a*d)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*
d - 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)))/(a*b), 1/3*(b*c^
(3/2)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt(d*x^3 + c
)*a*d + 2*(b*c - a*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-
(b*c - a*d)/b)/(b*c - a*d)))/(a*b), 1/3*(2*b*sqrt(-c)*c*arctan(sqrt(d*x^3 +
 c)*sqrt(-c)/c) + 2*sqrt(d*x^3 + c)*a*d - (b*c - a*d)*sqrt((b*c - a*d)/b)*l
og((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3
 + a)))/(a*b), 2/3*(b*sqrt(-c)*c*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + sqrt(
d*x^3 + c)*a*d + (b*c - a*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b
*sqrt(-(b*c - a*d)/b)/(b*c - a*d)))/(a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(90) = 180$.

Time = 6.57 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.82

$$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)}\,dx = \begin{cases} \frac{2\left(\frac{d^2\sqrt{c+dx^3}}{3b}+\frac{c^2d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}-\frac{d(ad-bc)^2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3ab^2\sqrt{\frac{ad-bc}{b}}}\right)}{d} & \text{fo} \\ c^{\frac{3}{2}}\left(-\frac{2b\left(\begin{cases}\frac{\frac{a}{2b}+x^3}{a} & \text{for } b=0\\ -\frac{\log\left(a-2b\left(\frac{a}{2b}+x^3\right)\right)}{2b} & \text{otherwise}\end{cases}\right)}{3a}-\frac{2b\left(\begin{cases}\frac{\frac{a}{2b}+x^3}{a} & \text{for } b=0\\ \frac{\log\left(a+2b\left(\frac{a}{2b}+x^3\right)\right)}{2b} & \text{otherwise}\end{cases}\right)}{3a}\right) & \text{o} \end{cases}$$

```
[In] integrate((d*x**3+c)**(3/2)/x/(b*x**3+a),x)
```

```
[Out] Piecewise((2*(d**2*sqrt(c + d*x**3)/(3*b) + c**2*d*atan(sqrt(c + d*x**3)/sq
rt(-c))/(3*a*sqrt(-c)) - d*(a*d - b*c)**2*atan(sqrt(c + d*x**3)/sqrt((a*d -
```

```
b*c)/b))/(3*a*b**2*sqrt((a*d - b*c)/b)))/d, Ne(d, 0)), (c**(3/2)*(-2*b*Pie
cewise(((a/(2*b) + x**3)/a, Eq(b, 0)), (-log(a - 2*b*(a/(2*b) + x**3))/(2*b
), True))/(3*a) - 2*b*Piecewise(((a/(2*b) + x**3)/a, Eq(b, 0)), (log(a + 2*
b*(a/(2*b) + x**3))/(2*b), True))/(3*a)), True))
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)x}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)}\,dx = \frac{2\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a\sqrt{-c}} + \frac{2\,\sqrt{dx^3+c}d}{3\,b} - \frac{2\,(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}ab}$$

```
[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a),x, algorithm="giac")
[Out] 2/3*c^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a*sqrt(-c)) + 2/3*sqrt(d*x^3 + c)
*d/b - 2/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b
^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a*b)
```

Mupad [B] (verification not implemented)

Time = 12.33 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.49

$$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)}\,dx = \frac{c^{3/2}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a} + \frac{2\,d\,\sqrt{d\,x^3+c}}{3\,b} + \frac{\ln\left(\frac{a^2\,d^2+2\,b^2\,c^2-a\,b\,d^2\,x^3+b^2\,c\,d\,x^3-3\,a\,b\,c\,d+\sqrt{b}\,\sqrt{d\,x^3+c}\,(a\,d-b\,c)^{3/2}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,(a\,d-b\,c)^{3/2}\,1{}\mathrm{i}}{3\,a\,b^{3/2}}$$

```
[In] int((c + d*x^3)^(3/2)/(x*(a + b*x^3)),x)
```

```
[Out] (c^(3/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))
)/x^6))/(3*a) + (2*d*(c + d*x^3)^(1/2))/(3*b) + (log((a^2*d^2 + 2*b^2*c^2 +
 b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)*2i - a*b*d^2*x^3 + b^2*c*d*x^3
 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3/2)*1i)/(3*a*b^(3/2))
```

3.372 $\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)} dx$

Optimal result

Integrand size = 24, antiderivative size = 116

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)} dx = -\frac{c\sqrt{c+dx^3}}{3ax^3} + \frac{\sqrt{c}(2bc-3ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2} - \frac{2(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}}$$

```
[Out] -2/3*(-a*d+b*c)^(3/2)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2
/b^(1/2)+1/3*(-3*a*d+2*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/a^2-1/
3*c*(d*x^3+c)^(1/2)/a/x^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 100, 162, 65, 214}

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)} dx = -\frac{2(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}} + \frac{\sqrt{c}(2bc-3ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2} - \frac{c\sqrt{c+dx^3}}{3ax^3}$$

```
[In] Int[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)),x]
```

```
[Out] -1/3*(c*Sqrt[c + d*x^3])/(a*x^3) + (Sqrt[c]*(2*b*c - 3*a*d)*ArcTanh[Sqrt[c
+ d*x^3]/Sqrt[c]])/(3*a^2) - (2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^3])/Sqrt[b*c - a*d]])/(3*a^2*Sqrt[b])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(a+bx)}\,dx, x, x^3\right)$$

$$= -\frac{c\sqrt{c+dx^3}}{3ax^3} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}c(2bc-3ad)+\frac{1}{2}d(bc-2ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a}$$

$$= -\frac{c\sqrt{c+dx^3}}{3ax^3} - \frac{(c(2bc-3ad))\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{6a^2} + \frac{(bc-ad)^2\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a^2}$$

$$= -\frac{c\sqrt{c+dx^3}}{3ax^3} - \frac{(c(2bc-3ad))\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^2d} + \frac{(2(bc-ad)^2)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^2d}$$

$$= -\frac{c\sqrt{c+dx^3}}{3ax^3} + \frac{\sqrt{c}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2} - \frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)}\,dx = \frac{-\frac{ac\sqrt{c+dx^3}}{x^3} + \frac{2(-bc+ad)^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}} + \sqrt{c}(2bc-3ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}$$

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)),x]
```

```
[Out] (-((a*c*Sqrt[c + d*x^3])/x^3) + (2*(-(b*c) + a*d)^(3/2)*ArcTan[(Sqrt[b]*Sqr
t[c + d*x^3])/Sqrt[-(b*c) + a*d]])/Sqrt[b] + Sqrt[c]*(2*b*c - 3*a*d)*ArcTan
h[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2)
```

Maple [A] (verified)

Time = 4.43 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$\frac{\frac{2(ad-bc)^2\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)x^3}{3}+\frac{2\left(x^3\left(c^{\frac{3}{2}}b-\frac{3ad\sqrt{c}}{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)-\frac{\sqrt{d\,x^3+c}\,ca}{2}\right)\sqrt{(ad-bc)b}}{3}}{a^2\sqrt{(ad-bc)b}\,x^3}$
risch	$-\frac{c\sqrt{d\,x^3+c}}{3a\,x^3}+\frac{-\frac{2\sqrt{c}\,(3ad-2bc)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3a}+\frac{4\left(a^2d^2-2abcd+b^2c^2\right)\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3a\sqrt{(ad-bc)b}}}{2a}$
default	$\frac{-\frac{c\sqrt{d\,x^3+c}}{3x^3}+\frac{2d\sqrt{d\,x^3+c}}{3}-\sqrt{c}\,d\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{a}-\frac{b\left(\frac{2d\,x^3\sqrt{d\,x^3+c}}{9}+\frac{8c\sqrt{d\,x^3+c}}{9}-\frac{2c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3}\right)}{a^2}-\frac{2\left(-(\right.}{}$
elliptic	Expression too large to display

```
[In] int((d*x^3+c)^(3/2)/x^4/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/((a*d-b*c)*b)^(1/2)*((a*d-b*c)^2*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)
^(1/2))*x^3+(x^3*(c^(3/2)*b-3/2*a*d*c^(1/2))*arctanh((d*x^3+c)^(1/2)/c^(1/2
))-1/2*(d*x^3+c)^(1/2)*c*a)*((a*d-b*c)*b)^(1/2))/a^2/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 538, normalized size of antiderivative = 4.64

$$\int\frac{(c+dx^3)^{3/2}}{x^4\,(a+bx^3)}\,dx=\left[-\frac{2\,(bc-ad)x^3\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+(2\,bc-3\,ad)\sqrt{c}x^3\log\left(\frac{dx^3}{}\right.}{6\,a^2x^3}\right.$$

$$-\frac{4\,(bc-ad)x^3\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+(2\,bc-3\,ad)\sqrt{c}x^3\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)+2\,\sqrt{dx^3}}{6\,a^2x^3}$$

$$-\frac{(2\,bc-3\,ad)\sqrt{-c}x^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+(bc-ad)x^3\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+\sqrt{dx^3}}{3\,a^2x^3}$$

$$-\frac{2\,(bc-ad)x^3\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+(2\,bc-3\,ad)\sqrt{-c}x^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+\sqrt{dx^3+c}}{3\,a^2x^3}$$

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*(b*c - a*d)*x^3*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2
*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + (2*b*c - 3*a*d)*sqrt
(c)*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt(d*x^3 +
 c)*a*c)/(a^2*x^3), -1/6*(4*(b*c - a*d)*x^3*sqrt(-(b*c - a*d)/b)*arctan(-sq
rt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (2*b*c - 3*a*d)*sqrt(c)
*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt(d*x^3 + c)
*a*c)/(a^2*x^3), -1/3*((2*b*c - 3*a*d)*sqrt(-c)*x^3*arctan(sqrt(d*x^3 + c)*
sqrt(-c)/c) + (b*c - a*d)*x^3*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*
d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + sqrt(d*x^3 + c)
*a*c)/(a^2*x^3), -1/3*(2*(b*c - a*d)*x^3*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(
d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (2*b*c - 3*a*d)*sqrt(-c)*x
^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + sqrt(d*x^3 + c)*a*c)/(a^2*x^3)]
```

Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)}\,dx = \int \frac{(c+dx^3)^{\frac{3}{2}}}{x^4(a+bx^3)}\,dx$$

```
[In] integrate((d*x**3+c)**(3/2)/x**4/(b*x**3+a),x)
[Out] Integral((c + d*x**3)**(3/2)/(x**4*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)x^4}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)}\,dx = \frac{2\,(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}a^2} - \frac{(2\,bc^2-3\,acd)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}} - \frac{\sqrt{dx^3+c}c}{3\,ax^3}$$

[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a),x, algorithm="giac")

[Out] 2/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2) - 1/3*(2*b*c^2 - 3*a*c*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)) - 1/3*sqrt(d*x^3 + c)*c/(a*x^3)

Mupad [B] (verification not implemented)

Time = 13.46 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.44

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)}\,dx = \frac{\sqrt{c}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)(3\,a\,d-2\,b\,c)}{6\,a^2} - \frac{c\,\sqrt{d\,x^3+c}}{3\,a\,x^3} + \frac{\ln\left(\frac{a^2\,d^2+2\,b^2\,c^2-a\,b\,d^2\,x^3+b^2\,c\,d\,x^3-3\,a\,b\,c\,d-\sqrt{b}\,\sqrt{d\,x^3+c}\,(a\,d-b\,c)^{3/2}\,2\mathrm{i}}{b\,x^3+a}\right)(a\,d-b\,c)^{3/2}\,1\mathrm{i}}{3\,a^2\,\sqrt{b}}$$

[In] int((c + d*x^3)^(3/2)/(x^4*(a + b*x^3)),x)

[Out] (c^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)*(3*a*d - 2*b*c))/(6*a^2) - (c*(c + d*x^3)^(1/2))/(3*a*x^3) + (log((a^2*d^2 + 2*b^2*c^2 - b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)*2i - a*b*d^2*x^3 + b^2*c*d*x^3 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3/2)*1i)/(3*a^2*b^(1/2))

3.373 $\int \frac{x^3(c+dx^3)^{3/2}}{a+bx^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{x^3(c+dx^3)^{3/2}}{a+bx^3} dx = \frac{cx^4\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{4}{3},1,-\frac{3}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] 1/4*c*x^4*AppellF1(4/3,1,-3/2,7/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/(1+d
*x^3/c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{x^3(c+dx^3)^{3/2}}{a+bx^3} dx = \frac{cx^4\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{4}{3},1,-\frac{3}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{\frac{dx^3}{c}+1}}$$

```
[In] Int[(x^3*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (c*x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 1, -3/2, 7/3, -((b*x^3)/a), -((d*x^3)/
c)])/(4*a*Sqrt[1 + (d*x^3)/c])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{x^3\left(1+\frac{dx^3}{c}\right)^{3/2}}{a+bx^3}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$

$$= \frac{cx^4\sqrt{c+dx^3}F_1\left(\frac{4}{3};1,-\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 280 vs. 2(65) = 130.

Time = 8.32 (sec) , antiderivative size = 280, normalized size of antiderivative = 4.31

$$\int \frac{x^3(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{x\left(8(c+dx^3)\left(14bc-11ad+5bdx^3\right)+\frac{(27b^2c^2-88abcd+55a^2d^2)x^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\right.}{a}}{}$$

```
[In] Integrate[(x^3*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (x*(8*(c + d*x^3)*(14*b*c - 11*a*d + 5*b*d*x^3) + ((27*b^2*c^2 - 88*a*b*c*d
 + 55*a^2*d^2)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)
/c), -((b*x^3)/a)])/a - (64*a^2*c^2*(-14*b*c + 11*a*d)*AppellF1[1/3, 1/2, 1
, 4/3, -((d*x^3)/c), -((b*x^3)/a)])/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2,
 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7
/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/
c), -((b*x^3)/a)]))))/(220*b^2*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 5.24 (sec) , antiderivative size = 800, normalized size of antiderivative = 12.31

method	result	size
risch	Expression too large to display	800
elliptic	Expression too large to display	846
default	Expression too large to display	1101

```
[In] int(x^3*(d*x^3+c)^(3/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2/55*x*(-5*b*d*x^3+11*a*d-14*b*c)*(d*x^3+c)^(1/2)/b^2+1/55/b^2*(-2/3*I*(55
*a^2*d^2-88*a*b*c*d+27*b^2*c^2)/b*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*
d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*
((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2))+55/3*I*a*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b/d^2*2^(1/2)*s
um(1/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^
2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(
-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)
*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2
-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d
*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2
)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/
d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1
/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx^3)^{3/2}}{a+bx^3}\,dx = \text{Timed out}$$

```
[In] integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^3(c+dx^3)^{3/2}}{a+bx^3}\,dx=\int \frac{x^3(c+dx^3)^{\frac{3}{2}}}{a+bx^3}\,dx$$

```
[In] integrate(x**3*(d*x**3+c)**(3/2)/(b*x**3+a),x)
[Out] Integral(x**3*(c + d*x**3)**(3/2)/(a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x^3(c+dx^3)^{3/2}}{a+bx^3}\,dx=\int \frac{(dx^3+c)^{\frac{3}{2}}x^3}{bx^3+a}\,dx$$

```
[In] integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)*x^3/(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^3(c+dx^3)^{3/2}}{a+bx^3}\,dx=\int \frac{(dx^3+c)^{\frac{3}{2}}x^3}{bx^3+a}\,dx$$

```
[In] integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
[Out] integrate((d*x^3 + c)^(3/2)*x^3/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx^3)^{3/2}}{a+bx^3}\,dx=\int \frac{x^3\,(d\,x^3+c)^{3/2}}{b\,x^3+a}\,dx$$

```
[In] int((x^3*(c + d*x^3)^(3/2))/(a + b*x^3),x)
[Out] int((x^3*(c + d*x^3)^(3/2))/(a + b*x^3), x)
```

3.374 $\int \frac{x\left(c+dx^3\right)^{3/2}}{a+bx^3} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 65

$$\int \frac{x(c+dx^3)^{3/2}}{a+bx^3} \, dx = \frac{cx^2\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{2}{3},1,-\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] 1/2*c*x^2*AppellF1(2/3,1,-3/2,5/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/(1+d
*x^3/c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {525, 524}

$$\int \frac{x(c+dx^3)^{3/2}}{a+bx^3} \, dx = \frac{cx^2\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{2}{3},1,-\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{\frac{dx^3}{c}+1}}$$

```
[In] Int[(x*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (c*x^2*Sqrt[c + d*x^3]*AppellF1[2/3, 1, -3/2, 5/3, -((b*x^3)/a), -((d*x^3)/
c)])/(2*a*Sqrt[1 + (d*x^3)/c])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{x\left(1+\frac{dx^3}{c}\right)^{3/2}}{a+bx^3}\,dx}{\sqrt{1+\frac{dx^3}{c}}} \\ &= \frac{cx^2\sqrt{c+dx^3}F_1\left(\frac{2}{3};1,-\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{1+\frac{dx^3}{c}}} \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 149 vs. 2(65) = 130.

Time = 10.16 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.29

$$\int \frac{x(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{x^2\left(20ad(c+dx^3)+5c(7bc-4ad)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+2d(1\right.}{70ab\sqrt{c+dx^3}}$$

```
[In] Integrate[(x*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (x^2*(20*a*d*(c + d*x^3) + 5*c*(7*b*c - 4*a*d)*Sqrt[1 + (d*x^3)/c]*AppellF1
[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*d*(10*b*c - 7*a*d)*x^3*S
qrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)]))
/(70*a*b*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 5.57 (sec) , antiderivative size = 921, normalized size of antiderivative = 14.17

method	result	size
risch	Expression too large to display	921
default	Expression too large to display	930
elliptic	Expression too large to display	930

```
[In] int(x*(d*x^3+c)^(3/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*d/b*x^2*(d*x^3+c)^(1/2)-1/7/b*(-2/3*I*(7*a*d-10*b*c)/b*3^(1/2)*(-c*d^2)
^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x
+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2)))+7/3*I*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b/d^2*2^(1/2)*s
um(1/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)
^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3
*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1
/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(
I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(
-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*
_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*
(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2
)),_alpha=RootOf(_Z^3*b+a)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x(c+dx^3)^{3/2}}{a+bx^3}\,dx = \text{Timed out}$$

```
[In] integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x(c+dx^3)^{3/2}}{a+bx^3}\,dx = \int \frac{x(c+dx^3)^{\frac{3}{2}}}{a+bx^3}\,dx$$

```
[In] integrate(x*(d*x**3+c)**(3/2)/(b*x**3+a),x)
[Out] Integral(x*(c + d*x**3)**(3/2)/(a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x(c+dx^3)^{3/2}}{a+bx^3}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}x}{bx^3+a}\,dx$$

```
[In] integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)*x/(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x(c+dx^3)^{3/2}}{a+bx^3}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}x}{bx^3+a}\,dx$$

```
[In] integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
[Out] integrate((d*x^3 + c)^(3/2)*x/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx^3)^{3/2}}{a+bx^3}\,dx = \int \frac{x\,(d\,x^3+c)^{3/2}}{b\,x^3+a}\,dx$$

```
[In] int((x*(c + d*x^3)^(3/2))/(a + b*x^3),x)
[Out] int((x*(c + d*x^3)^(3/2))/(a + b*x^3), x)
```

3.375 $\int \frac{(c+dx^3)^{3/2}}{a+bx^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{(c+dx^3)^{3/2}}{a+bx^3} dx = \frac{cx\sqrt{c+dx^3} \operatorname{AppellF1}\left(\frac{1}{3},1,-\frac{3}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] c*x*AppellF1(1/3,1,-3/2,4/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/(1+d*x^3/c
)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {441, 440}

$$\int \frac{(c+dx^3)^{3/2}}{a+bx^3} dx = \frac{cx\sqrt{c+dx^3} \operatorname{AppellF1}\left(\frac{1}{3},1,-\frac{3}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{\frac{dx^3}{c}+1}}$$

```
[In] Int[(c + d*x^3)^(3/2)/(a + b*x^3),x]
```

```
[Out] (c*x*Sqrt[c + d*x^3]*AppellF1[1/3, 1, -3/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(a*Sqrt[1 + (d*x^3)/c])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
```

```
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{\left(1+\frac{dx^3}{c}\right)^{3/2}}{a+bx^3}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$

$$= \frac{cx\sqrt{c+dx^3}F_1\left(\frac{1}{3};1,-\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 351 vs. 2(60) = 120.

Time = 10.33 (sec) , antiderivative size = 351, normalized size of antiderivative = 5.85

$$\int \frac{(c+dx^3)^{3/2}}{a+bx^3}\,dx = \frac{x\left(\frac{d(8bc-5ad)x^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a}+\frac{8\left(-4ac(2ad^2x^3+b(5c^2+2cdx^3+2d^2x^6))\,\text{AppellF1}\left(\frac{1}{3}\right.\right.}{(a+bx^3)\left(-8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1\right.\right.}}{20b\sqrt{}}$$

```
[In] Integrate[(c + d*x^3)^(3/2)/(a + b*x^3),x]
```

```
[Out] (x*((d*(8*b*c - 5*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -
((d*x^3)/c), -((b*x^3)/a)])/a + (8*(-4*a*c*(2*a*d^2*x^3 + b*(5*c^2 + 2*c*d*
x^3 + 2*d^2*x^6))*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] +
3*d*x^3*(a + b*x^3)*(c + d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)
/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)
/a)])))/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*
x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/
a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))))/(20*b
*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 5.17 (sec) , antiderivative size = 769, normalized size of antiderivative = 12.82

method	result	size
risch	Expression too large to display	769
default	Expression too large to display	776
elliptic	Expression too large to display	776

```
[In] int((d*x^3+c)^(3/2)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*d/b*x*(d*x^3+c)^(1/2)-1/5/b*(-2/3*I*(5*a*d-8*b*c)/b*3^(1/2)*(-c*d^2)^(1
/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-
c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1
/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/3*I*(-5*a^2*d^2+10*a*b*c*d-5*b
^2*c^2)/b/d^2*2^(1/2)*sum(1/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x
+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(
x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(
-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3)
)^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^
2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(
1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-
I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d
)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{a+bx^3}\,dx = \text{Timed out}$$

```
[In] integrate((d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{a+bx^3}\,dx = \int \frac{(c+dx^3)^{\frac{3}{2}}}{a+bx^3}\,dx$$

```
[In] integrate((d*x**3+c)**(3/2)/(b*x**3+a),x)
[Out] Integral((c + d*x**3)**(3/2)/(a + b*x**3), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{a+bx^3}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{bx^3+a}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)/(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{a+bx^3}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{bx^3+a}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
[Out] integrate((d*x^3 + c)^(3/2)/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{a+bx^3}\,dx = \int \frac{(d\,x^3+c)^{3/2}}{b\,x^3+a}\,dx$$

```
[In] int((c + d*x^3)^(3/2)/(a + b*x^3),x)
[Out] int((c + d*x^3)^(3/2)/(a + b*x^3), x)
```

3.376 $\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)} dx$

Optimal result

Integrand size = 24, antiderivative size = 63

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)} dx = -\frac{c\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{1}{3},1,-\frac{3}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] -c*AppellF1(-1/3,1,-3/2,2/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/x/(1+d*x^3
/c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)} dx = -\frac{c\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{1}{3},1,-\frac{3}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{\frac{dx^3}{c}+1}}$$

```
[In] Int[(c + d*x^3)^(3/2)/(x^2*(a + b*x^3)),x]
```

```
[Out] -((c*Sqrt[c + d*x^3]*AppellF1[-1/3, 1, -3/2, 2/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a*x*Sqrt[1 + (d*x^3)/c]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{\left(1+\frac{dx^3}{c}\right)^{3/2}}{x^2(a+bx^3)}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$
$$= -\frac{c\sqrt{c+dx^3}F_1\left(-\frac{1}{3};1,-\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 148 vs. 2(63) = 126.

Time = 10.13 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.35

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)}\,dx = \frac{-20ac(c+dx^3)+5c(-2bc+5ad)x^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+2d(bc}{20a^2x\sqrt{c+dx^3}}$$

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^2*(a + b*x^3)),x]
```

```
[Out] (-20*a*c*(c + d*x^3) + 5*c*(-2*b*c + 5*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF
1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*d*(b*c + 2*a*d)*x^6*Sqr
t[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(2
0*a^2*x*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 5.70 (sec) , antiderivative size = 920, normalized size of antiderivative = 14.60

method	result	size
risch	Expression too large to display	920
elliptic	Expression too large to display	924
default	Expression too large to display	1404

```
[In] int((d*x^3+c)^(3/2)/x^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -c/a*(d*x^3+c)^(1/2)/x+1/2/a*(-2/3*I*(2*a*d+b*c)/b*3^(1/2)*(-c*d^2)^(1/3)*(
I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c
*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3)))^(1/2)))+2/3*I*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b/d^2*2^(1/2)*sum(1/_al
pha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(
-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2
)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^
2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^
(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^
(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b
/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I
*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)
^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alph
a=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)}\,dx = \text{Timed out}$$

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)}\,dx = \int \frac{(c+dx^3)^{\frac{3}{2}}}{x^2(a+bx^3)}\,dx$$

```
[In] integrate((d*x**3+c)**(3/2)/x**2/(b*x**3+a),x)
[Out] Integral((c + d*x**3)**(3/2)/(x**2*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)x^2}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x^2), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)x^2}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a),x, algorithm="giac")
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)}\,dx = \int \frac{(d\,x^3+c)^{3/2}}{x^2\,(b\,x^3+a)}\,dx$$

```
[In] int((c + d*x^3)^(3/2)/(x^2*(a + b*x^3)),x)
[Out] int((c + d*x^3)^(3/2)/(x^2*(a + b*x^3)), x)
```

3.377 $\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)} dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)} dx = -\frac{c\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{2}{3},1,-\frac{3}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] -1/2*c*AppellF1(-2/3,1,-3/2,1/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/x^2/(1
+d*x^3/c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)} dx = -\frac{c\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{2}{3},1,-\frac{3}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[In] Int[(c + d*x^3)^(3/2)/(x^3*(a + b*x^3)),x]
```

```
[Out] -1/2*(c*Sqrt[c + d*x^3]*AppellF1[-2/3, 1, -3/2, 1/3, -((b*x^3)/a), -((d*x^3
)/c)])/(a*x^2*Sqrt[1 + (d*x^3)/c])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{\left(1+\frac{dx^3}{c}\right)^{3/2}}{x^3(a+bx^3)}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$

$$= -\frac{c\sqrt{c+dx^3}F_1\left(-\frac{2}{3};1,-\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 343 vs. $2(65) = 130$.

Time = 10.35 (sec) , antiderivative size = 343, normalized size of antiderivative = 5.28

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)}\,dx =$$

$$-\frac{d(bc-4ad)x^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{8ac\left(-4ac(2ac+6bcx^3-5adx^3+2bdx^6)\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c}\right.\right.}{(a+bx^3)\left(-8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\right.\right.}}{16a^2x^2\sqrt{c+dx^3}}$$

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^3*(a + b*x^3)),x]
```

```
[Out] -1/16*(d*(b*c - 4*a*d)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -
((d*x^3)/c), -((b*x^3)/a)] + (8*a*c*(-4*a*c*(2*a*c + 6*b*c*x^3 - 5*a*d*x^3
+ 2*b*d*x^6)*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3
*(a + b*x^3)*(c + d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -(
(b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))
/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)
] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a
*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(a^2*x^2*Sqrt
[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 5.26 (sec) , antiderivative size = 771, normalized size of antiderivative = 11.86

method	result	size
risch	Expression too large to display	771
elliptic	Expression too large to display	772
default	Expression too large to display	1096

```
[In] int((d*x^3+c)^(3/2)/x^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*c/a*(d*x^3+c)^(1/2)/x^2+1/4/a*(-2/3*I*(4*a*d-b*c)/b*3^(1/2)*(-c*d^2)^(
1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(
1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/3*I*(-4*a^2*d^2+8*a*b*c*d-4*b
^2*c^2)/b/d^2*2^(1/2)*sum(1/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x
+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(
x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(
-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3)
)^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^
2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(
1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-
I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d
)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)}\,dx = \text{Timed out}$$

```
[In] integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^3\,(a+bx^3)}\,dx = \int \frac{(c+dx^3)^{\frac{3}{2}}}{x^3\,(a+bx^3)}\,dx$$

```
[In] integrate((d*x**3+c)**(3/2)/x**3/(b*x**3+a),x)
[Out] Integral((c + d*x**3)**(3/2)/(x**3*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^3\,(a+bx^3)}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)x^3}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x^3), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^3\,(a+bx^3)}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)x^3}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a),x, algorithm="giac")
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^3\,(a+bx^3)}\,dx = \int \frac{(d\,x^3+c)^{3/2}}{x^3\,(b\,x^3+a)}\,dx$$

```
[In] int((c + d*x^3)^(3/2)/(x^3*(a + b*x^3)),x)
[Out] int((c + d*x^3)^(3/2)/(x^3*(a + b*x^3)), x)
```

3.378 $\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx=-\frac{2(bc+ad)\sqrt{c+dx^3}}{3b^2d^2}+\frac{2(c+dx^3)^{3/2}}{9bd^2}-\frac{2a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}$$

```
[Out] 2/9*(d*x^3+c)^(3/2)/b/d^2-2/3*a^2*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c
)^(1/2))/b^(5/2)/(-a*d+b*c)^(1/2)-2/3*(a*d+b*c)*(d*x^3+c)^(1/2)/b^2/d^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {457, 90, 65, 214}

$$\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx=-\frac{2a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}-\frac{2\sqrt{c+dx^3}(ad+bc)}{3b^2d^2}+\frac{2(c+dx^3)^{3/2}}{9bd^2}$$

```
[In] Int[x^8/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*(b*c + a*d)*Sqrt[c + d*x^3])/(3*b^2*d^2) + (2*(c + d*x^3)^(3/2))/(9*b*d
^2) - (2*a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2)
*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{-bc-ad}{b^2d\sqrt{c+dx}}+\frac{a^2}{b^2(a+bx)\sqrt{c+dx}}+\frac{\sqrt{c+dx}}{bd}\right)dx, x, x^3\right)\\
&= -\frac{2(bc+ad)\sqrt{c+dx^3}}{3b^2d^2}+\frac{2\left(c+dx^3\right)^{3/2}}{9bd^2}+\frac{a^2\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3b^2}\\
&= -\frac{2(bc+ad)\sqrt{c+dx^3}}{3b^2d^2}+\frac{2\left(c+dx^3\right)^{3/2}}{9bd^2}+\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3b^2d}\\
&= -\frac{2(bc+ad)\sqrt{c+dx^3}}{3b^2d^2}+\frac{2\left(c+dx^3\right)^{3/2}}{9bd^2}-\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{2\sqrt{c+dx^3}(-2bc-3ad+bdx^3)}{9b^2d^2} + \frac{2a^2\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{5/2}\sqrt{-bc+ad}}$$

```
[In] Integrate[x^8/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(-2*b*c - 3*a*d + b*d*x^3))/(9*b^2*d^2) + (2*a^2*ArcTan[
(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(3*b^(5/2)*Sqrt[-(b*c) + a*d
])
```

Maple [A] (verified)

Time = 4.34 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{2(-bd\,x^3+3ad+2bc)\sqrt{d\,x^3+c}}{9d^2b^2}+\frac{2a^2\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3b^2\sqrt{(ad-bc)b}}$
pseudoelliptic	$\frac{\frac{2\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)a^2d^2}{3}-\frac{2\left(\left(-\frac{b\,x^3}{3}+a\right)d+\frac{2bc}{3}\right)\sqrt{d\,x^3+c}\,\sqrt{(ad-bc)b}}{3}}{b^2d^2\sqrt{(ad-bc)b}}$
default	$\frac{\frac{2x^3\sqrt{d\,x^3+c}}{9d}-\frac{4c\sqrt{d\,x^3+c}}{9d^2}}{b}-\frac{2a\sqrt{d\,x^3+c}}{3b^2d}+\frac{2a^2\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3b^2\sqrt{(ad-bc)b}}$
elliptic	$\frac{2x^3\sqrt{d\,x^3+c}}{9bd}+\frac{2\left(-\frac{a}{b^2}-\frac{2c}{3db}\right)\sqrt{d\,x^3+c}}{3d}-\frac{ia^2\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\cdots\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}\cdots}}\cdots}{\cdots}\right.}{\cdots}$

```
[In] int(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/9*(-b*d*x^3+3*a*d+2*b*c)*(d*x^3+c)^(1/2)/d^2/b^2+2/3*a^2/b^2/((a*d-b*c)*
b)^(1/2)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.78

$$\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$
$$= \left[\frac{3\sqrt{b^2c-abd}a^2d^2\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)-2\,(2\,b^3c^2+ab^2cd-3\,a^2bd^2-(b^3cd-ab^2d^2)x^3)\sqrt{dx}}{9\,(b^4cd^2-ab^3d^3)}\right.$$

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/9*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x
^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) - 2*(2*b^3*c^2 + a*b^2*c*d - 3*a^
2*b*d^2 - (b^3*c*d - a*b^2*d^2)*x^3)*sqrt(d*x^3 + c))/(b^4*c*d^2 - a*b^3*d^
3), 2/9*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c
+ a*b*d)/(b*d*x^3 + b*c)) - (2*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2 - (b^3*c*d
 - a*b^2*d^2)*x^3)*sqrt(d*x^3 + c))/(b^4*c*d^2 - a*b^3*d^3)]
```

Sympy [F]

$$\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(x**8/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**8/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.02

$$\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{2\,a^2\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}b^2} + \frac{2\left((dx^3+c)^{\frac{3}{2}}b^2d^4-3\,\sqrt{dx^3+c}b^2cd^4-3\,\sqrt{dx^3+c}abd^5\right)}{9\,b^3d^6}$$

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*a^2*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*b^2) + 2/9*((d*x^3 + c)^(3/2)*b^2*d^4 - 3*sqrt(d*x^3 + c)*b^2*c*d^4 - 3*s
qrt(d*x^3 + c)*a*b*d^5)/(b^3*d^6)
```

Mupad [B] (verification not implemented)

Time = 9.69 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.16

$$\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,b\,d} - \frac{\left(\frac{2\,a}{b^2}+\frac{4\,c}{3\,b\,d}\right)\,\sqrt{d\,x^3+c}}{3\,d} + \frac{a^2\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,b^{5/2}\,\sqrt{a\,d-b\,c}}$$

```
[In] int(x^8/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

```
[Out] (2*x^3*(c + d*x^3)^(1/2))/(9*b*d) - (((2*a)/b^2 + (4*c)/(3*b*d))*(c + d*x^3
)^(1/2))/(3*d) + (a^2*log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b
*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*1i)/(3*b^(5/2)*(a*d - b*c)^(1/2))
```

3.379 $\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx=\frac{2\sqrt{c+dx^3}}{3bd}+\frac{2a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}$$

```
[Out] 2/3*a*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)^
(1/2)+2/3*(d*x^3+c)^(1/2)/b/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {457, 81, 65, 214}

$$\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx=\frac{2a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}+\frac{2\sqrt{c+dx^3}}{3bd}$$

```
[In] Int[x^5/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*b*d) + (2*a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b
*c - a*d]])/(3*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= \frac{2\sqrt{c+dx^3}}{3bd} - \frac{a\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3b}\\
&= \frac{2\sqrt{c+dx^3}}{3bd} - \frac{(2a)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3bd}\\
&= \frac{2\sqrt{c+dx^3}}{3bd} + \frac{2a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

$$\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx=\frac{2\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{d}-\frac{a\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}\right)}{3b^{3/2}}$$

```
[In] Integrate[x^5/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*((Sqrt[b]*Sqrt[c + d*x^3])/d - (a*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[
-(b*c) + a*d]])/Sqrt[-(b*c) + a*d]))/(3*b^(3/2))
```

Maple [A] (verified)

Time = 4.53 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

method	result
default	$\frac{2\sqrt{d\,x^3+c}}{3bd}-\frac{2a\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3b\sqrt{(ad-bc)b}}$
risch	$\frac{2\sqrt{d\,x^3+c}}{3bd}-\frac{2a\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3b\sqrt{(ad-bc)b}}$
pseudoelliptic	$-\frac{2\left(\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)ad-\sqrt{d\,x^3+c}\,\sqrt{(ad-bc)b}\right)}{3bd\sqrt{(ad-bc)b}}$
elliptic	$\frac{2\sqrt{d\,x^3+c}}{3bd}+\frac{ia\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}}{}\right.}{}$

```
[In] int(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(d*x^3+c)^(1/2)/b/d-2/3*a/b/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^3+c)^(1/2
)/((a*d-b*c)*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.77

$$\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$
$$= \left[\frac{\sqrt{b^2c-abd}ad\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)+2\,\sqrt{dx^3+c}(b^2c-abd)}{3\,(b^3cd-ab^2d^2)},\right.$$
$$\left.-\frac{2\left(\sqrt{-b^2c+abd}ad\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-b^2c+abd}}{bdx^3+bc}\right)-\sqrt{dx^3+c}(b^2c-abd)\right)}{3\,(b^3cd-ab^2d^2)}\right]$$

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(sqrt(b^2*c - a*b*d)*a*d*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c
)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*sqrt(d*x^3 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2), -2/3*(sqrt(-b^2*c + a*b*d)*a*d*arctan(sqrt(d*x^3 + c)*
sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) - sqrt(d*x^3 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2)]
```

Sympy [F]

$$\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(x**5/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx = -\frac{2\left(\frac{ad\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b} - \frac{\sqrt{dx^3+c}}{b}\right)}{3\,d}$$

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(a*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b
*d)*b) - sqrt(d*x^3 + c)/b)/d
```

Mupad [B] (verification not implemented)

Time = 9.41 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.16

$$\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{2\sqrt{d\,x^3+c}}{3\,b\,d} + \frac{a\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,b^{3/2}\,\sqrt{a\,d-b\,c}}$$

```
[In] int(x^5/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*b*d) + (a*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(
1/2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*1i)/(3*b^(3/2)*(a*d - b*c
)^(1/2))
```

3.380 $\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

```
[Out] -2/3*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(1/2)/(-a*d+b*c)^(
1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {455, 65, 214}

$$\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

```
[In] Int[x^2/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*Sqrt[b]*Sqrt[b*c
 - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)\\ &= \frac{2\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3d}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{2\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3\sqrt{b}\sqrt{-bc+ad}}$$

```
[In] Integrate[x^2/((a + b*x^3)*Sqrt[c + d*x^3]),x]
[Out] (2*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(3*Sqrt[b]*Sqrt[-(
b*c) + a*d])
```

Maple [A] (verified)

Time = 4.67 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

method	result
default	$\frac{2\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}$
pseudoelliptic	$\frac{2\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\cdots\right.}{\cdots}}}{\cdots}\right.}{\cdots}$

[In] int(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/3/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.55

$$\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \left[\frac{\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)}{3\,\sqrt{b^2c-abd}}, \frac{2\,\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-b^2c+abd}}{bdx^3+bc}\right)}{3\,(b^2c-abd)}\right]$$

[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] [1/3*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a))/sqrt(b^2*c - a*b*d), 2/3*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c))/(b^2*c - a*b*d)]

Sympy [A] (verification not implemented)

Time = 3.93 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.73

$$\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \begin{cases} \frac{2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b\sqrt{\frac{ad-bc}{b}}} & \text{for } d\neq 0 \\ \begin{cases} \frac{x^3}{3a\sqrt{c}} & \text{for } b=0 \\ \tilde{\infty}x^3 & \text{for } \sqrt{c}=0 \\ \frac{\log(3a\sqrt{c}+3b\sqrt{c}x^3)}{3b\sqrt{c}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] Piecewise((2*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*b*sqrt((a*d - b*
c)/b)), Ne(d, 0)), (Piecewise((x**3/(3*a*sqrt(c)), Eq(b, 0)), (zoo*x**3, Eq
(sqrt(c), 0)), (log(3*a*sqrt(c) + 3*b*sqrt(c)*x**3)/(3*b*sqrt(c)), True)),
True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{2\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}}$$

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

Mupad [B] (verification not implemented)

Time = 10.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.37

$$\int \frac{x^2}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{\ln\left(\frac{a\,d\,1\mathrm{i}-b\,c\,2\mathrm{i}+2\,\sqrt{d\,x^3+c}\,\sqrt{a\,b\,d-b^2\,c}-b\,d\,x^3\,1\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,\sqrt{a\,b\,d-b^2\,c}}$$

```
[In] int(x^2/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

```
[Out] (log((a*d*1i - b*c*2i + 2*(c + d*x^3)^(1/2)*(a*b*d - b^2*c)^(1/2) - b*d*x^3
*1i)/(a + b*x^3))*1i)/(3*(a*b*d - b^2*c)^(1/2))
```

3.381 $\int \frac{1}{x(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 85

$$\int \frac{1}{x\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a\sqrt{c}}+\frac{2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}$$

```
[Out] -2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a/c^(1/2)+2/3*arctanh(b^(1/2)*(d*x^3+
c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/a/(-a*d+b*c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {457, 88, 65, 214}

$$\int \frac{1}{x\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx=\frac{2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a\sqrt{c}}$$

```
[In] Int[1/(x*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a*Sqrt[c]) + (2*Sqrt[b]*ArcTanh[(S
qrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{3a} - \frac{b\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a}\\
&= \frac{2\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3ad} - \frac{(2b)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3ad}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a\sqrt{c}} + \frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

$$\int \frac{1}{x(a+bx^3)\sqrt{c+dx^3}}\,dx = -\frac{\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}} + \frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a}$$

```
[In] Integrate[1/(x*(a + b*x^3)*Sqrt[c + d*x^3]),x]
[Out] -1/3*((2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/Sqrt
[-(b*c) + a*d] + (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/Sqrt[c])/a
```

Maple [A] (verified)

Time = 4.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3a\sqrt{c}}-\frac{2b\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3a\sqrt{(ad-bc)b}}$	66
pseudoelliptic	$-\frac{2\left(b\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)\sqrt{c}+\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)\sqrt{(ad-bc)b}\right)}{3a\sqrt{c}\,\sqrt{(ad-bc)b}}$	78
elliptic	Expression too large to display	1598

```
[In] int(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a/c^(1/2)-2/3*b/a/((a*d-b*c)*b)^(1/2)
*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 431, normalized size of antiderivative = 5.07

$$\int \frac{1}{x\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx = \left[\frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)+\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)}{3\,ac}, \frac{2\,c\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{d}}{}\right.}{}\right.$$

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(c*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*
(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + sqrt(c)*log((d*x^3 - 2*sqrt
(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a*c), 1/3*(2*c*sqrt(-b/(b*c - a*d))*arcta
n(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) + sqrt
(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a*c), 1/3*(c*sqrt(
b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*s
qrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-
c)/c))/(a*c), 2/3*(c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*
d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) + sqrt(-c)*arctan(sqrt(d*x^3 + c)*
sqrt(-c)/c))/(a*c)]
```

Sympy [A] (verification not implemented)

Time = 5.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.40

$$\int \frac{1}{x\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx = \begin{cases} \frac{2\left(-\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3a\sqrt{\frac{ad-bc}{b}}}+\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}\right)}{d} & \text{for}\: d\neq 0 \\ \frac{2\operatorname{atan}\left(\frac{2\left(\frac{a}{2b}+x^3\right)}{\sqrt{-\frac{a^2}{b^2}}}\right)}{3b\sqrt{c}\sqrt{-\frac{a^2}{b^2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/x/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] Piecewise((2*(-d*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*a*sqrt((a*d
- b*c)/b)) + d*atan(sqrt(c + d*x**3)/sqrt(-c))/(3*a*sqrt(-c)))/d, Ne(d, 0))
, (2*atan(2*(a/(2*b) + x**3)/sqrt(-a**2/b**2))/(3*b*sqrt(c)*sqrt(-a**2/b**2
)), True))
```

Maxima [F]

$$\int \frac{1}{x\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx = \int \frac{1}{(bx^3+a)\sqrt{dx^3+cx}}\,dx$$

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int \frac{1}{x\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx = -\frac{2\,b\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}a}+\frac{2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a\sqrt{-c}}$$

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*b*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*a) + 2/3*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a*sqrt(-c))
```

Mupad [B] (verification not implemented)

Time = 11.77 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{1}{x\,(a+bx^3)\,\sqrt{c+dx^3}}\,dx=\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a\,\sqrt{c}}+\frac{\sqrt{b}\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,a\,\sqrt{a\,d-b\,c}}$$

```
[In] int(1/(x*(a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

```
[Out] log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(3
*a*c^(1/2)) + (b^(1/2)*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d -
b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*1i)/(3*a*(a*d - b*c)^(1/2))
```

3.382 $\int \frac{1}{x^4(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 117

$$\int \frac{1}{x^4\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{c+dx^3}}{3acx^3}+\frac{(2bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}} - \frac{2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}}$$

```
[Out] 1/3*(a*d+2*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^2/c^(3/2)-2/3*b^(3/2)*ar
ctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(1/2)-1/3*(d
*x^3+c)^(1/2)/a/c/x^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 105, 162, 65, 214}

$$\int \frac{1}{x^4\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx = -\frac{2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}} + \frac{(ad+2bc)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}} - \frac{\sqrt{c+dx^3}}{3acx^3}$$

```
[In] Int[1/(x^4*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/3*Sqrt[c + d*x^3]/(a*c*x^3) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqr
t[c]])/(3*a^2*c^(3/2)) - (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[
b*c - a*d]])/(3*a^2*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{3acx^3} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2bc+ad)+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{\sqrt{c+dx^3}}{3acx^3} + \frac{b^2\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a^2} - \frac{(2bc+ad)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{6a^2c}$$

$$= -\frac{\sqrt{c+dx^3}}{3acx^3} + \frac{(2b^2)\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^2d} - \frac{(2bc+ad)\mathrm{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^2cd}$$

$$= -\frac{\sqrt{c+dx^3}}{3acx^3} + \frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}} - \frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^4(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{-\frac{a\sqrt{c+dx^3}}{cx^3} + \frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}} + \frac{(2bc+ad)\mathrm{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{c^{3/2}}}{3a^2}$$

```
[In] Integrate[1/(x^4*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-((a*Sqrt[c + d*x^3])/(c*x^3)) + (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3
])/Sqrt[-(b*c) + a*d]])/Sqrt[-(b*c) + a*d] + ((2*b*c + a*d)*ArcTanh[Sqrt[c
+ d*x^3]/Sqrt[c]])/c^(3/2))/(3*a^2)
```

Maple [A] (verified)

Time = 4.61 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.79

method	result	size
pseudoelliptic	$\frac{\frac{2b^2\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}} - \frac{a\sqrt{dx^3+c}}{cx^3} + \frac{(ad+2bc)\,\mathrm{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}}{3a^2}$	92
risch	$-\frac{\sqrt{dx^3+c}}{3acx^3} - \frac{-\frac{2(ad+2bc)\,\mathrm{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)}{3a\sqrt{c}} - \frac{4b^2c\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)}{3a\sqrt{(ad-bc)b}}}{2ac}$	106
default	$\frac{-\frac{\sqrt{dx^3+c}}{3cx^3} + \frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}}{a} + \frac{2b\,\mathrm{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} + \frac{2b^2\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)}{3a^2\sqrt{(ad-bc)b}}$	111
elliptic	Expression too large to display	1652

```
[In] int(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a^2*(2*b^2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(
1/2))-a/c*(d*x^3+c)^(1/2)/x^3+(a*d+2*b*c)/c^(3/2)*arctanh((d*x^3+c)^(1/2)/c
^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 565, normalized size of antiderivative = 4.83

$$\int \frac{1}{x^4 (a+bx^3) \sqrt{c+dx^3}} \, dx$$
$$= \left[\frac{2\, bc^2x^3\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)+(2\,bc+ad)\sqrt{c}x^3\log\left(\frac{dx^3+2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-2\,\sqrt{dx}}{6\,a^2c^2x^3} \right.$$
$$-\frac{4\,bc^2x^3\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right)-(2\,bc+ad)\sqrt{c}x^3\log\left(\frac{dx^3+2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)+2\,\sqrt{dx^3+}}{6\,a^2c^2x^3}$$
$$-\frac{2\,bc^2x^3\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right)+(2\,bc+ad)\sqrt{-c}x^3\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+\sqrt{dx^3+ca}}{3\,a^2c^2x^3}$$

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b*c^2*x^3*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d
*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + (2*b*c + a*d)*sqr
t(c)*x^3*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*sqrt(d*x^3
+ c)*a*c)/(a^2*c^2*x^3), -1/6*(4*b*c^2*x^3*sqrt(-b/(b*c - a*d))*arctan(-sqr
t(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) - (2*b*c + a
*d)*sqrt(c)*x^3*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt
(d*x^3 + c)*a*c)/(a^2*c^2*x^3), 1/3*(b*c^2*x^3*sqrt(b/(b*c - a*d))*log((b*d
*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*
x^3 + a)) - (2*b*c + a*d)*sqrt(-c)*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) -
 sqrt(d*x^3 + c)*a*c)/(a^2*c^2*x^3), -1/3*(2*b*c^2*x^3*sqrt(-b/(b*c - a*d))
*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c))
+ (2*b*c + a*d)*sqrt(-c)*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + sqrt(d*x^
3 + c)*a*c)/(a^2*c^2*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4 (a + bx^3) \sqrt{c + dx^3}}\, dx = \int \frac{1}{x^4 (a + bx^3) \sqrt{c + dx^3}}\, dx$$

```
[In] integrate(1/x**4/(b*x**3+a)/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(x**4*(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (a + bx^3) \sqrt{c + dx^3}}\, dx = \int \frac{1}{(bx^3 + a)\sqrt{dx^3 + c}x^4}\, dx$$

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^4 (a + bx^3) \sqrt{c + dx^3}}\, dx = \frac{2\, b^2 \arctan\left(\frac{\sqrt{dx^3 + c}b}{\sqrt{-b^2c + abd}}\right)}{3\, \sqrt{-b^2c + abd}a^2} - \frac{(2\, bc + ad) \arctan\left(\frac{\sqrt{dx^3 + c}}{\sqrt{-c}}\right)}{3\, a^2\sqrt{-c}c} - \frac{\sqrt{dx^3 + c}}{3\, acx^3}$$

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] 2/3*b^2*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*a^2) - 1/3*(2*b*c + a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)*c
) - 1/3*sqrt(d*x^3 + c)/(a*c*x^3)
```

Mupad [B] (verification not implemented)

Time = 12.83 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^4\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx=\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)^3}{x^6}\right)\,(a\,d+2\,b\,c)}{6\,a^2\,c^{3/2}}-\frac{\sqrt{d\,x^3+c}}{3\,a\,c\,x^3}+\frac{b^{3/2}\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,a^2\,\sqrt{a\,d-b\,c}}$$

```
[In] int(1/(x^4*(a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

```
[Out] (log((((c + d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2) + c^(1/2))^3)/x^6)*(
a*d + 2*b*c))/(6*a^2*c^(3/2)) - (c + d*x^3)^(1/2)/(3*a*c*x^3) + (b^(3/2)*lo
g((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/
(a + b*x^3))*1i)/(3*a^2*(a*d - b*c)^(1/2))
```

3.383 $\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{x^4\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},1,\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{c+dx^3}}$$

```
[Out] 1/4*x^4*AppellF1(4/3,1,1/2,7/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/(d*x^
3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{x^4\sqrt{\frac{dx^3}{c}+1}\operatorname{AppellF1}\left(\frac{4}{3},1,\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{c+dx^3}}$$

```
[In] Int[x^3/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)])/(4*a*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{(a+bx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$= \frac{x^4\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{c+dx^3}}$$

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{x^4\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{4a\sqrt{c+dx^3}}$$

```
[In] Integrate[x^3/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^4*Sqrt[(c + d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)
/a)])/(4*a*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.28 (sec) , antiderivative size = 719, normalized size of antiderivative = 11.23

method	result	size
default	Expression too large to display	719
elliptic	Expression too large to display	719

```
[In] int(x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I/b*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(
```

```
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d
^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/
3*I*a/b/d^2*2^(1/2)*sum(1/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1
/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-
1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1
/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)
^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/
3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*
(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/
(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \text{Timed out}$$

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(x**3/(b*x**3+a)/(d*x**3+c)**(1/2),x)
[Out] Integral(x**3/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{(bx^3+a)\sqrt{dx^3+c}}\,dx$$

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{(bx^3+a)\sqrt{dx^3+c}}\,dx$$

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^3/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{(b\,x^3+a)\,\sqrt{d\,x^3+c}}\,dx$$

```
[In] int(x^3/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
[Out] int(x^3/((a + b*x^3)*(c + d*x^3)^(1/2)), x)
```

3.384 $\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx=\frac{x^2\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},1,\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{c+dx^3}}$$

```
[Out] 1/2*x^2*AppellF1(2/3,1,1/2,5/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/(d*x^
3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {525, 524}

$$\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx=\frac{x^2\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{2}{3},1,\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{c+dx^3}}$$

```
[In] Int[x/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1, 1/2, 5/3, -((b*x^3)/a), -((d*x^3)
/c)])/(2*a*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x}{(a+bx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$= \frac{x^2\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{2}{3};1,\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{c+dx^3}}$$

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{x^2\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{2a\sqrt{c+dx^3}}$$

```
[In] Integrate[x/((a + b*x^3)*Sqrt[c + d*x^3]),x]
[Out] (x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)
/a)])/(2*a*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.26 (sec) , antiderivative size = 429, normalized size of antiderivative = 6.70

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-c\,d^2\right.}{}\right.}{2\left(-c\right.}}}{}\right.}{}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{id\left(2x+\frac{i\sqrt{3}\left(-c\,d^2\right.}{}\right.}{2\left(-c\right.}}}{}\right.}{}$

```
[In] int(x/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*I/d^2*2^(1/2)*sum(1/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*
(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d
*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*
I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2
/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3
^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c
*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*
d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \text{Timed out}$$

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(x/(b*x**3+a)/(d*x**3+c)**(1/2),x)
[Out] Integral(x/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x}{(bx^3+a)\sqrt{dx^3+c}}\,dx$$

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x}{(bx^3+a)\sqrt{dx^3+c}}\,dx$$

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(x/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{x}{(b\,x^3+a)\,\sqrt{d\,x^3+c}}\,dx$$

```
[In] int(x/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
[Out] int(x/((a + b*x^3)*(c + d*x^3)^(1/2)), x)
```

3.385 $\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{x\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{1}{3},1,\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{c+dx^3}}$$

```
[Out] x*AppellF1(1/3,1,1/2,4/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/(d*x^3+c)^(
1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {441, 440}

$$\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{x\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},1,\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{c+dx^3}}$$

```
[In] Int[1/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a*Sqrt[c + d*x^3])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{(a+bx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$

$$= \frac{x\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{c+dx^3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 161 vs. 2(59) = 118.

Time = 10.06 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.73

$$\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx = -\frac{8acx\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{(a+bx^3)\sqrt{c+dx^3}\left(-8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3\left(2bc\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right.\right.}$$

```
[In] Integrate[1/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-8*a*c*x*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)])/((a + b*x
^3)*Sqrt[c + d*x^3]*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x
^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a
)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.42 (sec) , antiderivative size = 429, normalized size of antiderivative = 7.27

method	result
default	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{i d\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{i d\left(2x+\frac{i\sqrt{3}\left(-c\,d^{2}\right)\cdots}{}\right)}{2\left(-c\,d^{2}\cdots\right.}}\cdots}{\cdots}\right.}{\cdots}$
elliptic	$-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{i d\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{i d\left(2x+\frac{i\sqrt{3}\left(-c\,d^{2}\right)\cdots}{}\right)}{2\left(-c\,d^{2}\cdots\right.}}\cdots}{\cdots}\right.}{\cdots}$

```
[In] int(1/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*I/d^2*2^(1/2)*sum(1/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/
d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1
/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/
2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(
1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^
(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(
-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(
a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(1/(b*x**3+a)/(d*x**3+c)**(1/2),x)
[Out] Integral(1/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}\,dx$$

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{1}{(bx^3+a)\sqrt{dx^3+c}}\,dx$$

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \int \frac{1}{(b\,x^3+a)\,\sqrt{d\,x^3+c}}\,dx$$

```
[In] int(1/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
[Out] int(1/((a + b*x^3)*(c + d*x^3)^(1/2)), x)
```

3.386 $\int \frac{1}{x^2(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{1}{x^2(a+bx^3)\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{1}{3},1,\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{c+dx^3}}$$

```
[Out] -AppellF1(-1/3,1,1/2,2/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/x/(d*x^3+c)
^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\int \frac{1}{x^2(a+bx^3)\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{1}{3},1,\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{c+dx^3}}$$

```
[In] Int[1/(x^2*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -((Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 1, 1/2, 2/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a*x*Sqrt[c + d*x^3]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^2(a+bx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$= -\frac{\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{1}{3};1,\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{c+dx^3}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(62) = 124.

Time = 10.11 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.27

$$\int \frac{1}{x^2\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx = \frac{-20a(c+dx^3)+5(-2bc+ad)x^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+2bdx^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\right.}{20a^2cx\sqrt{c+dx^3}}$$

```
[In] Integrate[1/(x^2*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-20*a*(c + d*x^3) + 5*(-2*b*c + a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3,
 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*b*d*x^6*Sqrt[1 + (d*x^3)/c]*A
ppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(20*a^2*c*x*Sqrt[c +
 d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.93 (sec) , antiderivative size = 890, normalized size of antiderivative = 14.35

method	result	size
default	Expression too large to display	890
elliptic	Expression too large to display	891
risch	Expression too large to display	892

```
[In] int(1/x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-(d*x^3+c)^(1/2)/c/x-1/3*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^
2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((
x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(
-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)
)+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c
*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))
)+1/3*I*b/a/d^2*2^(1/2)*sum(1/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x
+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(
x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(
-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3)
)^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^
2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(
1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-
I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d
)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^3) \sqrt{c+dx^3}}\, dx = \text{Timed out}$$

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^2 (a + bx^3) \sqrt{c + dx^3}} \, dx = \int \frac{1}{x^2 (a + bx^3) \sqrt{c + dx^3}} \, dx$$

```
[In] integrate(1/x**2/(b*x**3+a)/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(x**2*(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a + bx^3) \sqrt{c + dx^3}} \, dx = \int \frac{1}{(bx^3 + a)\sqrt{dx^3 + c}x^2} \, dx$$

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a + bx^3) \sqrt{c + dx^3}} \, dx = \int \frac{1}{(bx^3 + a)\sqrt{dx^3 + c}x^2} \, dx$$

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + bx^3) \sqrt{c + dx^3}} \, dx = \int \frac{1}{x^2 \left(b\,x^3 + a\right) \sqrt{d\,x^3 + c}} \, dx$$

```
[In] int(1/(x^2*(a + b*x^3)*(c + d*x^3)^(1/2)),x)
[Out] int(1/(x^2*(a + b*x^3)*(c + d*x^3)^(1/2)), x)
```

3.387 $\int \frac{1}{x^3(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{x^3 (a+bx^3) \sqrt{c+dx^3}}\,dx = -\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{c+dx^3}}$$

```
[Out] -1/2*AppellF1(-2/3,1,1/2,1/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/x^2/(d*
x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{1}{x^3 (a+bx^3) \sqrt{c+dx^3}}\,dx = -\frac{\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{c+dx^3}}$$

```
[In] Int[1/(x^3*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/2*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 1/2, 1/3, -((b*x^3)/a), -((d*x^
3)/c)])/(a*x^2*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^3(a+bx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}\\ &= -\frac{\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 339 vs. $2(64) = 128$.

Time = 10.30 (sec) , antiderivative size = 339, normalized size of antiderivative = 5.30

$$\int \frac{1}{x^3(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{-bdx^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{8a\left(-4ac(2ac+6bcx^3+3adx^3+2bdx^6)\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3\cdots\right.}{(a+bx^3)\left(8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-3x^3\cdots\right.}}{16a^2cx^2\sqrt{c+dx^3}}$$

```
[In] Integrate[1/(x^3*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-(b*d*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((
b*x^3)/a)]) + (8*a*(-4*a*c*(2*a*c + 6*b*c*x^3 + 3*a*d*x^3 + 2*b*d*x^6)*Appe
llF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(a + b*x^3)*(c +
 d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d
*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3)*(8*
a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 3*x^3*(2*b*c*A
ppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3
/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(16*a^2*c*x^2*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.88 (sec) , antiderivative size = 738, normalized size of antiderivative = 11.53

method	result	size
default	Expression too large to display	738
elliptic	Expression too large to display	739
risch	Expression too large to display	740

```
[In] int(1/x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/2*(d*x^3+c)^(1/2)/c/x^2+1/6*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*
(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I*b/a/d^2*2^(1/2)*sum(1/_alpha^2/(a*d-b*c)*
(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))
)/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1
/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c
*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alph
a*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d
-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^
2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3
*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*
b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(a+bx^3\right) \sqrt{c+dx^3}}\, dx = \text{Timed out}$$

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^3 (a + bx^3) \sqrt{c + dx^3}} \, dx = \int \frac{1}{x^3 (a + bx^3) \sqrt{c + dx^3}} \, dx$$

```
[In] integrate(1/x**3/(b*x**3+a)/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(x**3*(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (a + bx^3) \sqrt{c + dx^3}} \, dx = \int \frac{1}{(bx^3 + a)\sqrt{dx^3 + c}x^3} \, dx$$

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a + bx^3) \sqrt{c + dx^3}} \, dx = \int \frac{1}{(bx^3 + a)\sqrt{dx^3 + c}x^3} \, dx$$

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a + bx^3) \sqrt{c + dx^3}} \, dx = \int \frac{1}{x^3 \left(b\,x^3 + a\right) \sqrt{d\,x^3 + c}} \, dx$$

```
[In] int(1/(x^3*(a + b*x^3)*(c + d*x^3)^(1/2)),x)
[Out] int(1/(x^3*(a + b*x^3)*(c + d*x^3)^(1/2)), x)
```

3.388 $\int \frac{x^8}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 107

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)^{3/2}}\,dx=\frac{2c^2}{3d^2(bc-ad)\sqrt{c+dx^3}}+\frac{2\sqrt{c+dx^3}}{3bd^2}-\frac{2a^2\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}$$

```
[Out] -2/3*a^2*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*
c)^(3/2)+2/3*c^2/d^2/(-a*d+b*c)/(d*x^3+c)^(1/2)+2/3*(d*x^3+c)^(1/2)/b/d^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {457, 89, 65, 214}

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)^{3/2}}\,dx=-\frac{2a^2\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}+\frac{2c^2}{3d^2\sqrt{c+dx^3}(bc-ad)}+\frac{2\sqrt{c+dx^3}}{3bd^2}$$

```
[In] Int[x^8/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*c^2)/(3*d^2*(b*c - a*d)*Sqrt[c + d*x^3]) + (2*Sqrt[c + d*x^3])/(3*b*d^2)
 - (2*a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(3/2)*(b
*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{c^2}{d(-bc+ad)(c+dx)^{3/2}}+\frac{1}{bd\sqrt{c+dx}}+\frac{a^2}{b(bc-ad)(a+bx)\sqrt{c+dx}}\right)\,dx, x, x^3\right)\\
&= \frac{2c^2}{3d^2(bc-ad)\sqrt{c+dx^3}}+\frac{2\sqrt{c+dx^3}}{3bd^2}+\frac{a^2\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3b(bc-ad)}\\
&= \frac{2c^2}{3d^2(bc-ad)\sqrt{c+dx^3}}+\frac{2\sqrt{c+dx^3}}{3bd^2}+\frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3bd(bc-ad)}\\
&= \frac{2c^2}{3d^2(bc-ad)\sqrt{c+dx^3}}+\frac{2\sqrt{c+dx^3}}{3bd^2}-\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.04

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{2\left(\frac{\sqrt{b}(ad(c+dx^3)-bc(2c+dx^3))}{d^2(-bc+ad)\sqrt{c+dx^3}}-\frac{a^2\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}\right)}{3b^{3/2}}$$

```
[In] Integrate[x^8/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
[Out] (2*((Sqrt[b]*(a*d*(c + d*x^3) - b*c*(2*c + d*x^3)))/(d^2*(-(b*c) + a*d)*Sqr
t[c + d*x^3]) - (a^2*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/
(-(b*c) + a*d)^(3/2)))/(3*b^(3/2))
```

Maple [A] (verified)

Time = 4.54 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

method	result
risch	$\frac{2\sqrt{dx^3+c}}{3bd^2}-\frac{2a^2\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)}{3b(ad-bc)\sqrt{(ad-bc)b}}-\frac{2c^2}{3d^2(ad-bc)\sqrt{dx^3+c}}$
pseudoelliptic	$\frac{2\sqrt{dx^3+c}}{3bd^2}-\frac{2a^2\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)}{3b(ad-bc)\sqrt{(ad-bc)b}}-\frac{2c^2}{3d^2(ad-bc)\sqrt{dx^3+c}}$
default	$\frac{\frac{2c}{3d^2\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{2\sqrt{dx^3+c}}{3d^2}}{b}+\frac{2a}{3b^2d\sqrt{dx^3+c}}-\frac{2a^2\left(b\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)\sqrt{dx^3+c}+\sqrt{(ad-bc)b}\right)}{b^2\sqrt{(ad-bc)b}\sqrt{dx^3+c}\,(3ad-3bc)}$
elliptic	$-\frac{2c^2}{3d^2(ad-bc)\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{2\sqrt{dx^3+c}}{3bd^2}+\frac{ia^2\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-cd^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\cdots\right)}{\left(-cd^2\right)^{\frac{1}{3}}}}\cdots}{\cdots}\right)}{\cdots}$

```
[In] int(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(d*x^3+c)^(1/2)/b/d^2-2/3/b*a^2/(a*d-b*c)/((a*d-b*c)*b)^(1/2)*arctan(b*
(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))-2/3/d^2*c^2/(a*d-b*c)/(d*x^3+c)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(87) = 174$.

Time = 0.39 (sec) , antiderivative size = 440, normalized size of antiderivative = 4.11

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \left[-\frac{(a^2d^3x^3+a^2cd^2)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)-2\,(2\,b^3c^3-}{3\,(b^4c^3d^2-2\,ab^3c^2d^3+a^2b^2cd^4+(b^4c^2d}\right.$$

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/3*((a^2*d^3*x^3 + a^2*c*d^2)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c -
 a*d + 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) - 2*(2*b^3*c^3 -
 3*a*b^2*c^2*d + a^2*b*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^3)
*sqrt(d*x^3 + c))/(b^4*c^3*d^2 - 2*a*b^3*c^2*d^3 + a^2*b^2*c*d^4 + (b^4*c^2
*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^3), 2/3*((a^2*d^3*x^3 + a^2*c*d^2)*sq
rt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b
*c)) + (2*b^3*c^3 - 3*a*b^2*c^2*d + a^2*b*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^
2 + a^2*b*d^3)*x^3)*sqrt(d*x^3 + c))/(b^4*c^3*d^2 - 2*a*b^3*c^2*d^3 + a^2*b
^2*c*d^4 + (b^4*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^3)]
```

Sympy [F]

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \int \frac{x^8}{(a+bx^3)(c+dx^3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**8/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**8/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.96

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{2\,a^2\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,(b^2c-abd)\sqrt{-b^2c+abd}} + \frac{2\,c^2}{3\,(bcd^2-ad^3)\sqrt{dx^3+c}} + \frac{2\,\sqrt{dx^3+c}}{3\,bd^2}$$

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*a^2*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c - a*b*d)*sqr
t(-b^2*c + a*b*d)) + 2/3*c^2/((b*c*d^2 - a*d^3)*sqrt(d*x^3 + c)) + 2/3*sqrt
(d*x^3 + c)/(b*d^2)
```

Mupad [B] (verification not implemented)

Time = 10.81 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.07

$$\int \frac{x^8}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{2\,\sqrt{d\,x^3+c}}{3\,b\,d^2} - \frac{2\,c^2}{3\,d^2\,\sqrt{d\,x^3+c}\,(a\,d-b\,c)} + \frac{a^2\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,b^{3/2}\,(a\,d-b\,c)^{3/2}}$$

```
[In] int(x^8/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*b*d^2) - (2*c^2)/(3*d^2*(c + d*x^3)^(1/2)*(a*d - b
*c)) + (a^2*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*
2i - b*d*x^3)/(a + b*x^3))*1i)/(3*b^(3/2)*(a*d - b*c)^(3/2))
```

3.389 $\int \frac{x^5}{(a+bx^3)(c+dx^3)^{3/2}} dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)^{3/2}} dx = -\frac{2c}{3d(bc-ad)\sqrt{c+dx^3}} + \frac{2a\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{3/2}}$$

```
[Out] 2/3*a*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(3/2)/b^
(1/2)-2/3*c/d/(-a*d+b*c)/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {457, 79, 65, 214}

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)^{3/2}} dx = \frac{2a\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{3/2}} - \frac{2c}{3d\sqrt{c+dx^3}(bc-ad)}$$

```
[In] Int[x^5/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*c)/(3*d*(b*c - a*d)*Sqrt[c + d*x^3]) + (2*a*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^3])/Sqrt[b*c - a*d]])/(3*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)\\
&= -\frac{2c}{3d(bc-ad)\sqrt{c+dx^3}} - \frac{a\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3(bc-ad)}\\
&= -\frac{2c}{3d(bc-ad)\sqrt{c+dx^3}} - \frac{(2a)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3d(bc-ad)}\\
&= -\frac{2c}{3d(bc-ad)\sqrt{c+dx^3}} + \frac{2a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)^{3/2}}\,dx=\frac{2}{3}\left(\frac{c}{d(-bc+ad)\sqrt{c+dx^3}}+\frac{a\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}(-bc+ad)^{3/2}}\right)$$

```
[In] Integrate[x^5/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*(c/(d*(-(b*c) + a*d)*Sqrt[c + d*x^3]) + (a*ArcTan[(Sqrt[b]*Sqrt[c + d*x^
3])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2))))/3
```

Maple [A] (verified)

Time = 4.35 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$\frac{\frac{2a\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)d\sqrt{d\,x^3+c}}{3}+\frac{2c\sqrt{(ad-bc)b}}{3}}{(ad-bc)\sqrt{(ad-bc)b}\,d\sqrt{d\,x^3+c}}$
default	$-\frac{2}{3bd\sqrt{d\,x^3+c}}+\frac{2a\left(b\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)\sqrt{d\,x^3+c}+\sqrt{(ad-bc)b}\right)}{b\sqrt{(ad-bc)b}\,\sqrt{d\,x^3+c}\,(3ad-3bc)}$
elliptic	$\frac{2c}{3d(ad-bc)\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{ia\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{\ldots}}}\cdots}{\cdots}\right.}{\cdots}$

```
[In] int(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(a*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))*d*(d*x^3+c)^(1/2)+c*((
a*d-b*c)*b)^(1/2))/(a*d-b*c)/((a*d-b*c)*b)^(1/2)/d/(d*x^3+c)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(66) = 132.

Time = 0.34 (sec) , antiderivative size = 326, normalized size of antiderivative = 3.98

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \left[-\frac{(ad^2x^3+acd)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)+2\,(b^2c^2-ab}{3\,(b^3c^3d-2\,ab^2c^2d^2+a^2bcd^3+(b^3c^2d^2-2\,ab^2cd^3+a^2bd^4)x^3}\right.$$
$$\left.-\frac{2\left((ad^2x^3+acd)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-b^2c+abd}}{bdx^3+bc}\right)+(b^2c^2-abcd)\sqrt{dx^3+c}\right)}{3\,(b^3c^3d-2\,ab^2c^2d^2+a^2bcd^3+(b^3c^2d^2-2\,ab^2cd^3+a^2bd^4)x^3)}\right]$$

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/3*((a*d^2*x^3 + a*c*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d -
 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*(b^2*c^2 - a*b*c*d
)*sqrt(d*x^3 + c))/(b^3*c^3*d - 2*a*b^2*c^2*d^2 + a^2*b*c*d^3 + (b^3*c^2*d^
2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3), -2/3*((a*d^2*x^3 + a*c*d)*sqrt(-b^2*c
+ a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) + (b^
2*c^2 - a*b*c*d)*sqrt(d*x^3 + c))/(b^3*c^3*d - 2*a*b^2*c^2*d^2 + a^2*b*c*d^
3 + (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3)]
```

Sympy [F]

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \int \frac{x^5}{(a+bx^3)(c+dx^3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**5/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**5/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int \frac{x^5}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = -\frac{2\left(\frac{ad\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}(bc-ad)}+\frac{c}{\sqrt{dx^3+c}(bc-ad)}\right)}{3\,d}$$

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*(a*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b
*d)*(b*c - a*d)) + c/(sqrt(d*x^3 + c)*(b*c - a*d)))/d
```

Mupad [B] (verification not implemented)

Time = 10.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.15

$$\int \frac{x^5}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \frac{2\,c}{3\,d\,\sqrt{d\,x^3+c}\,\left(a\,d-b\,c\right)} + \frac{a\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,1{}\mathrm{i}}{3\,\sqrt{b}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

```
[In] int(x^5/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

```
[Out] (2*c)/(3*d*(c + d*x^3)^(1/2)*(a*d - b*c)) + (a*log((2*b*c - a*d + b^(1/2)*(
c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*1i)/(3*b^(1/2
)*(a*d - b*c)^(3/2))
```

3.390 $\int \frac{x^2}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 77

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)^{3/2}}\,dx=\frac{2}{3(bc-ad)\sqrt{c+dx^3}}-\frac{2\sqrt{b}\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3(bc-ad)^{3/2}}$$

```
[Out] -2/3*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/(-a*d+b*c)^(
3/2)+2/3/(-a*d+b*c)/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {455, 53, 65, 214}

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)^{3/2}}\,dx=\frac{2}{3\sqrt{c+dx^3}(bc-ad)}-\frac{2\sqrt{b}\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3(bc-ad)^{3/2}}$$

```
[In] Int[x^2/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] 2/(3*(b*c - a*d)*Sqrt[c + d*x^3]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*
x^3])/Sqrt[b*c - a*d]])/(3*(b*c - a*d)^(3/2))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
```

```
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)\\
&= \frac{2}{3(bc-ad)\sqrt{c+dx^3}} + \frac{b\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3(bc-ad)}\\
&= \frac{2}{3(bc-ad)\sqrt{c+dx^3}} + \frac{(2b)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3d(bc-ad)}\\
&= \frac{2}{3(bc-ad)\sqrt{c+dx^3}} - \frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{2}{(3bc-3ad)\sqrt{c+dx^3}} - \frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3(-bc+ad)^{3/2}}$$

```
[In] Integrate[x^2/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] 2/((3*b*c - 3*a*d)*Sqrt[c + d*x^3]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[c + d
*x^3])/Sqrt[-(b*c) + a*d]])/(3*(-(b*c) + a*d)^(3/2))
```

Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.08

method	result
default	$-\frac{2\left(b\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)\sqrt{d\,x^3+c}+\sqrt{(ad-bc)b}\right)}{\sqrt{(ad-bc)b}\,\sqrt{d\,x^3+c}\,(3ad-3bc)}$
pseudoelliptic	$-\frac{2\left(b\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)\sqrt{d\,x^3+c}+\sqrt{(ad-bc)b}\right)}{\sqrt{(ad-bc)b}\,\sqrt{d\,x^3+c}\,(3ad-3bc)}$
elliptic	$-\frac{2}{3(ad-bc)\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{ib\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{(\cdots}{}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}\cdots}}\cdots}{\cdots}\right)}{\cdots}$

```
[In] int(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(b*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))*(d*x^3+c)^(1/2)+((a*d-b
*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)/(d*x^3+c)^(1/2)/(3*a*d-3*b*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 236, normalized size of antiderivative = 3.06

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \left[-\frac{(dx^3+c)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)-2\,\sqrt{dx^3+c}}{3\,((bcd-ad^2)x^3+bc^2-acd)},\right.$$
$$\left.-\frac{2\left((dx^3+c)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right)-\sqrt{dx^3+c}\right)}{3\,((bcd-ad^2)x^3+bc^2-acd)}\right]$$

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/3*((d*x^3 + c)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(
d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) - 2*sqrt(d*x^3 + c
))/((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d), -2/3*((d*x^3 + c)*sqrt(-b/(b*c -
a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b
*c)) - sqrt(d*x^3 + c))/((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d)]
```

Sympy [A] (verification not implemented)

Time = 8.69 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.57

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \begin{cases} \frac{2\left(-\frac{d}{3\sqrt{c+dx^3}(ad-bc)}-\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3\sqrt{\frac{ad-bc}{b}}(ad-bc)}\right)}{d} & \text{for}\ d\neq 0\\ \begin{cases} \frac{x^3}{3ac^{\frac{3}{2}}} & \text{for}\ b=0\\ \tilde{\infty}x^3 & \text{for}\ c^{\frac{3}{2}}=0\\ \frac{\log\left(3ac^{\frac{3}{2}}+3bc^{\frac{3}{2}}x^3\right)}{3bc^{\frac{3}{2}}} & \text{otherwise}\end{cases} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**2/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] Piecewise((2*(-d/(3*sqrt(c + d*x**3)*(a*d - b*c)) - d*atan(sqrt(c + d*x**3)
/sqrt((a*d - b*c)/b))/(3*sqrt((a*d - b*c)/b)*(a*d - b*c)))/d, Ne(d, 0)), (P
iecewise((x**3/(3*a*c**(3/2)), Eq(b, 0)), (zoo*x**3, Eq(c**(3/2), 0)), (log
(3*a*c**(3/2) + 3*b*c**(3/2)*x**3)/(3*b*c**(3/2)), True)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{2\,b\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}(bc-ad)} + \frac{2}{3\sqrt{dx^3+c}(bc-ad)}$$

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*b*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*
(b*c - a*d)) + 2/3/(sqrt(d*x^3 + c)*(b*c - a*d))
```

Mupad [B] (verification not implemented)

Time = 10.22 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.16

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = -\frac{2}{3\sqrt{d\,x^3+c}\,(a\,d-b\,c)} + \frac{\sqrt{b}\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,(a\,d-b\,c)^{3/2}}$$

```
[In] int(x^2/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

```
[Out] (b^(1/2)*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i
- b*d*x^3)/(a + b*x^3))*1i)/(3*(a*d - b*c)^(3/2)) - 2/(3*(c + d*x^3)^(1/2)*
(a*d - b*c))
```

3.391 $\int \frac{1}{x(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 114

$$\int \frac{1}{x(a+bx^3)(c+dx^3)^{3/2}}\,dx = -\frac{2d}{3c(bc-ad)\sqrt{c+dx^3}} - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3ac^{3/2}} + \frac{2b^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a(bc-ad)^{3/2}}$$

```
[Out] -2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a/c^(3/2)+2/3*b^(3/2)*arctanh(b^(1/2)
*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a/(-a*d+b*c)^(3/2)-2/3*d/c/(-a*d+b*c)/(d
*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 87, 162, 65, 214}

$$\int \frac{1}{x(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{2b^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a(bc-ad)^{3/2}} - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3ac^{3/2}} - \frac{2d}{3c\sqrt{c+dx^3}(bc-ad)}$$

```
[In] Int[1/(x*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*d)/(3*c*(b*c - a*d)*Sqrt[c + d*x^3]) - (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[
c]])/(3*a*c^(3/2)) + (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c
- a*d]])/(3*a*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x
)*((e + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= -\frac{2d}{3c(bc-ad)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{bc-ad-bdx}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3c(bc-ad)}$$

$$= -\frac{2d}{3c(bc-ad)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{3ac} - \frac{b^2\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a(bc-ad)}$$

$$= -\frac{2d}{3c(bc-ad)\sqrt{c+dx^3}} + \frac{2\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3acd} - \frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3ad(bc-ad)}$$

$$= -\frac{2d}{3c(bc-ad)\sqrt{c+dx^3}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3ac^{3/2}} + \frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a(bc-ad)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\,(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \frac{2}{3}\left(\frac{d}{c(-bc+ad)\sqrt{c+dx^3}} + \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{a(-bc+ad)^{3/2}} - \frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{ac^{3/2}}\right)$$

```
[In] Integrate[1/(x*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
[Out] (2*(d/(c*(-(b*c) + a*d)*Sqrt[c + d*x^3]) + (b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c
+ d*x^3])/Sqrt[-(b*c) + a*d]])/(a*(-(b*c) + a*d)^(3/2)) - ArcTanh[Sqrt[c +
d*x^3]/Sqrt[c]]/(a*c^(3/2))))/3
```

Maple [A] (verified)

Time = 4.91 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

method	result	size
pseudoelliptic	$\frac{2b^2 \arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3(ad-bc)a\sqrt{(ad-bc)b}} + \frac{2d}{3(ad-bc)c\sqrt{d\,x^3+c}} - \frac{2\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3a\,c^{\frac{3}{2}}}$	103
default	$\frac{\frac{2}{3c\sqrt{\left(x^3+\frac{c}{d}\right)d}} - \frac{2\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}}{a} + \frac{2b\left(b\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)\sqrt{d\,x^3+c}+\sqrt{(ad-bc)b}\right)}{a\sqrt{(ad-bc)b}\,\sqrt{d\,x^3+c}\,(3ad-3bc)}$	130
elliptic	Expression too large to display	1637

```
[In] int(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/(a*d-b*c)*b^2/a/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)
*b)^(1/2))+2/3*d/(a*d-b*c)/c/(d*x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^
(1/2))/a/c^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(90) = 180$.

Time = 0.38 (sec) , antiderivative size = 790, normalized size of antiderivative = 6.93

$$\int \frac{1}{x\,(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \left[-\frac{2\sqrt{dx^3+c}acd + (bc^2dx^3+bc^3)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}(bc-ad)\sqrt{\cdots}}{bx^3+a}\right.\cdots}{3\,(abc^4-a^2c^3d+(abc\cdots}\right.$$

$$-\frac{2\sqrt{dx^3+c}acd - 2\,(bc^2dx^3+bc^3)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right) - ((bcd-ad^2)x^3+bc^2-acd\cdots}{3\,(abc^4-a^2c^3d+(abc^3d-a^2c^2d^2)x^3)}$$

$$-\frac{2\sqrt{dx^3+c}acd - 2\,((bcd-ad^2)x^3+bc^2-acd)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) + (bc^2dx^3+bc^3)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx\cdots}{\cdots}\right.}{3\,(abc^4-a^2c^3d+(abc^3d-a^2c^2d^2)x^3)}$$

$$-\frac{2\left(\sqrt{dx^3+c}acd - (bc^2dx^3+bc^3)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right) - ((bcd-ad^2)x^3+bc^2-acd\cdots\right.}{3\,(abc^4-a^2c^3d+(abc^3d-a^2c^2d^2)x^3)}$$

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(2*sqrt(d*x^3 + c)*a*c*d + (b*c^2*d*x^3 + b*c^3)*sqrt(b/(b*c - a*d))*
log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*
d)))/(b*x^3 + a)) - ((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d)*sqrt(c)*log((d*x^
3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a*b*c^4 - a^2*c^3*d + (a*b*c^3*
d - a^2*c^2*d^2)*x^3), -1/3*(2*sqrt(d*x^3 + c)*a*c*d - 2*(b*c^2*d*x^3 + b*c
^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c -
 a*d))/(b*d*x^3 + b*c)) - ((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d)*sqrt(c)*log
((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a*b*c^4 - a^2*c^3*d + (a*
b*c^3*d - a^2*c^2*d^2)*x^3), -1/3*(2*sqrt(d*x^3 + c)*a*c*d - 2*((b*c*d - a*
d^2)*x^3 + b*c^2 - a*c*d)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (b*
c^2*d*x^3 + b*c^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(
d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)))/(a*b*c^4 - a^2*c^
3*d + (a*b*c^3*d - a^2*c^2*d^2)*x^3), -2/3*(sqrt(d*x^3 + c)*a*c*d - (b*c^2*
d*x^3 + b*c^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqr
t(-b/(b*c - a*d))/(b*d*x^3 + b*c)) - ((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d)*
sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c))/(a*b*c^4 - a^2*c^3*d + (a*b*c^
3*d - a^2*c^2*d^2)*x^3)]
```

Sympy [A] (verification not implemented)

Time = 6.87 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.36

$$\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{d^2}{3c\sqrt{c+dx^3}(ad-bc)}+\frac{bd\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3a\sqrt{\frac{ad-bc}{b}}(ad-bc)}+\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{3ac\sqrt{-c}}\right)}{d} & \text{for } d\neq 0 \\ \frac{2\operatorname{atan}\left(\frac{2\left(\frac{a}{2b}+x^3\right)}{\sqrt{-\frac{a^2}{b^2}}}\right)}{3bc^{\frac{3}{2}}\sqrt{-\frac{a^2}{b^2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/x/(b*x**3+a)/(d*x**3+c)**(3/2),x)
[Out] Piecewise((2*(d**2/(3*c*sqrt(c + d*x**3)*(a*d - b*c)) + b*d*atan(sqrt(c + d
*x**3)/sqrt((a*d - b*c)/b))/(3*a*sqrt((a*d - b*c)/b)*(a*d - b*c)) + d*atan(
sqrt(c + d*x**3)/sqrt(-c))/(3*a*c*sqrt(-c)))/d, Ne(d, 0)), (2*atan(2*(a/(2*
b) + x**3)/sqrt(-a**2/b**2))/(3*b*c**(3/2)*sqrt(-a**2/b**2)), True))
```

Maxima [F]

$$\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

$$\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = -\frac{2\,b^2\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,(abc-a^2d)\sqrt{-b^2c+abd}} - \frac{2\,d}{3\,\sqrt{dx^3+c}(bc^2-acd)} + \frac{2\,\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a\sqrt{-c}c}$$

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] -2/3*b^2*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((a*b*c - a^2*d)*sq
rt(-b^2*c + a*b*d)) - 2/3*d/(sqrt(d*x^3 + c)*(b*c^2 - a*c*d)) + 2/3*arctan(
sqrt(d*x^3 + c)/sqrt(-c))/(a*sqrt(-c)*c)
```

Mupad [B] (verification not implemented)

Time = 12.98 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.22

$$\int \frac{1}{x\,(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a\,c^{3/2}} + \frac{2\,d}{3\,c\,\sqrt{d\,x^3+c}\,\left(a\,d-b\,c\right)} + \frac{b^{3/2}\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,a\,\left(a\,d-b\,c\right)^{3/2}}$$

```
[In] int(1/(x*(a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

```
[Out] log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(3
*a*c^(3/2)) + (2*d)/(3*c*(c + d*x^3)^(1/2)*(a*d - b*c)) + (b^(3/2)*log((2*b
*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b
*x^3))*1i)/(3*a*(a*d - b*c)^(3/2))
```

3.392 $\int \frac{1}{x^4(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 158

$$\int \frac{1}{x^4(a+bx^3)(c+dx^3)^{3/2}}\,dx = -\frac{d(bc-3ad)}{3ac^2(bc-ad)\sqrt{c+dx^3}} - \frac{1}{3acx^3\sqrt{c+dx^3}} + \frac{(2bc+3ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{5/2}} - \frac{2b^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}}$$

```
[Out] 1/3*(3*a*d+2*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^2/c^(5/2)-2/3*b^(5/2)*
arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(3/2)-1/3*
d*(-3*a*d+b*c)/a/c^2/(-a*d+b*c)/(d*x^3+c)^(1/2)-1/3/a/c/x^3/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 105, 157, 162, 65, 214}

$$\int \frac{1}{x^4(a+bx^3)(c+dx^3)^{3/2}}\,dx = -\frac{2b^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}} + \frac{(3ad+2bc)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{5/2}} - \frac{d(bc-3ad)}{3ac^2\sqrt{c+dx^3}(bc-ad)} - \frac{1}{3acx^3\sqrt{c+dx^3}}$$

```
[In] Int[1/(x^4*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/3*(d*(b*c - 3*a*d))/(a*c^2*(b*c - a*d)*Sqrt[c + d*x^3]) - 1/(3*a*c*x^3*S
qrt[c + d*x^3]) + ((2*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2
```

```
*c^(5/2)) - (2*b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/
(3*a^2*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= -\frac{1}{3acx^3\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2bc+3ad)+\frac{3bdx}{2}}{x(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{d(bc-3ad)}{3ac^2(bc-ad)\sqrt{c+dx^3}} - \frac{1}{3acx^3\sqrt{c+dx^3}} + \frac{2\text{Subst}\left(\int \frac{-\frac{1}{4}(bc-ad)(2bc+3ad)-\frac{1}{4}bd(bc-3ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3ac^2(bc-ad)}$$

$$= -\frac{d(bc-3ad)}{3ac^2(bc-ad)\sqrt{c+dx^3}} - \frac{1}{3acx^3\sqrt{c+dx^3}} + \frac{b^3\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a^2(bc-ad)} - \frac{(2bc+3ad)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{6a^2c^2}$$

$$= -\frac{d(bc-3ad)}{3ac^2(bc-ad)\sqrt{c+dx^3}} - \frac{1}{3acx^3\sqrt{c+dx^3}} + \frac{(2b^3)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^2d(bc-ad)} - \frac{(2bc+3ad)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^2c^2d}$$

$$= -\frac{d(bc-3ad)}{3ac^2(bc-ad)\sqrt{c+dx^3}} - \frac{1}{3acx^3\sqrt{c+dx^3}} + \frac{(2bc+3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{5/2}} - \frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^4 (a+bx^3) (c+dx^3)^{3/2}} \, dx = \frac{\frac{a(-bc(c+dx^3)+ad(c+3dx^3))}{c^2(bc-ad)x^3\sqrt{c+dx^3}} - \frac{2b^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}} + \frac{(2bc+3ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{c^{5/2}}}{3a^2}$$

```
[In] Integrate[1/(x^4*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] ((a*(-(b*c*(c + d*x^3)) + a*d*(c + 3*d*x^3)))/(c^2*(b*c - a*d)*x^3*Sqrt[c +
 d*x^3]) - (2*b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])
/(-(b*c) + a*d)^(3/2) + ((2*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/
c^(5/2))/(3*a^2)
```

Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{\sqrt{d\,x^3+c}}{3c^2 a\,x^3}-\frac{-\frac{2(3ad+2bc)\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3a\sqrt{c}}+\frac{4a\,d^2}{3(ad-bc)\sqrt{d\,x^3+c}}+\frac{4b^3c^2\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3(ad-bc)a\sqrt{(ad-bc)b}}}{2a\,c^2}$
pseudoelliptic	$\frac{d^2\left(-\frac{2b^3\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{(ad-bc)a^2d^2\sqrt{(ad-bc)b}}-\frac{-3\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)ad\,x^3-2\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)bc\,x^3+\sqrt{d\,x^3+c}\,a\sqrt{c}}{x^3c^{\frac{5}{2}}a^2d^2}-\frac{2}{(ad-bc)c^2\sqrt{d\,x^3+c}}\right)}{3}$
default	$\frac{-\frac{\sqrt{d\,x^3+c}}{3c^2x^3}-\frac{2d}{3c^2\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{d\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}}{a}-\frac{b\left(\frac{2}{3c\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{2\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{a^2}-\frac{2b^2\left(b\arctan\left(\frac{b\sqrt{d\,x^3+}}{\sqrt{(ad-b}}\right.\right.}{a^2\sqrt{(ad-bc)b}}$
elliptic	Expression too large to display

```
[In] int(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/c^2/a*(d*x^3+c)^(1/2)/x^3-1/2/a/c^2*(-2/3*(3*a*d+2*b*c)/a*arctanh((d*x
^3+c)^(1/2)/c^(1/2))/c^(1/2)+4/3*a*d^2/(a*d-b*c)/(d*x^3+c)^(1/2)+4/3*b^3*c^
2/(a*d-b*c)/a/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1
/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(130) = 260$.

Time = 0.45 (sec) , antiderivative size = 1120, normalized size of antiderivative = 7.09

$$\int \frac{1}{x^4 (a+bx^3) (c+dx^3)^{3/2}}\,dx = \left[-\frac{2\,(b^2c^3dx^6+b^2c^4x^3)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)-((2\cdots}{\cdots} \right.$$

$$-\frac{4\,(b^2c^3dx^6+b^2c^4x^3)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right)-((2\,b^2c^2d+abcd^2-3\,a^2d^3)x^6+(2\,b^2c^3+\cdots}{6\,((a^2bc^4d-a^3c^3d^2)x^6+(\cdots}$$

$$-\frac{((2\,b^2c^2d+abcd^2-3\,a^2d^3)x^6+(2\,b^2c^3+abc^2d-3\,a^2cd^2)x^3)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+(b^2c^3dx^6+b^2c^4x^3\cdots}{3\,((a^2bc^4d-a^3c^3d^2)x^6+(\cdots}$$

$$-\frac{2\,(b^2c^3dx^6+b^2c^4x^3)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right)+((2\,b^2c^2d+abcd^2-3\,a^2d^3)x^6+(2\,b^2c^3+\cdots}{3\,((a^2bc^4d-a^3c^3d^2)x^6+(a^2\cdots}$$

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*(b^2*c^3*d*x^6 + b^2*c^4*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2
*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)
) - ((2*b^2*c^2*d + a*b*c*d^2 - 3*a^2*d^3)*x^6 + (2*b^2*c^3 + a*b*c^2*d - 3
*a^2*c*d^2)*x^3)*sqrt(c)*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3)
 + 2*(a*b*c^3 - a^2*c^2*d + (a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(d*x^3 + c))
/((a^2*b*c^4*d - a^3*c^3*d^2)*x^6 + (a^2*b*c^5 - a^3*c^4*d)*x^3), -1/6*(4*(
b^2*c^3*d*x^6 + b^2*c^4*x^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(
b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) - ((2*b^2*c^2*d + a*b*c*d^
2 - 3*a^2*d^3)*x^6 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(c)*log
((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*(a*b*c^3 - a^2*c^2*d +
(a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(d*x^3 + c))/((a^2*b*c^4*d - a^3*c^3*d^2
)*x^6 + (a^2*b*c^5 - a^3*c^4*d)*x^3), -1/3*(((2*b^2*c^2*d + a*b*c*d^2 - 3*a
^2*d^3)*x^6 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(-c)*arctan(sq
rt(d*x^3 + c)*sqrt(-c)/c) + (b^2*c^3*d*x^6 + b^2*c^4*x^3)*sqrt(b/(b*c - a*d
))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c -
 a*d)))/(b*x^3 + a)) + (a*b*c^3 - a^2*c^2*d + (a*b*c^2*d - 3*a^2*c*d^2)*x^3
)*sqrt(d*x^3 + c))/((a^2*b*c^4*d - a^3*c^3*d^2)*x^6 + (a^2*b*c^5 - a^3*c^4*
d)*x^3), -1/3*(2*(b^2*c^3*d*x^6 + b^2*c^4*x^3)*sqrt(-b/(b*c - a*d))*arctan(
-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) + ((2*b^
2*c^2*d + a*b*c*d^2 - 3*a^2*d^3)*x^6 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2
)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (a*b*c^3 - a^2*c^2*d +
```

```
(a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(d*x^3 + c))/((a^2*b*c^4*d - a^3*c^3*d^
2)*x^6 + (a^2*b*c^5 - a^3*c^4*d)*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4 (a+bx^3) (c+dx^3)^{3/2}}\,dx = \int \frac{1}{x^4 (a+bx^3) (c+dx^3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**4/(b*x**3+a)/(d*x**3+c)**(3/2),x)
[Out] Integral(1/(x**4*(a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (a+bx^3) (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}x^4}\,dx$$

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4 (a+bx^3) (c+dx^3)^{3/2}}\,dx = \frac{2\,b^3 \arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,(a^2bc-a^3d)\sqrt{-b^2c+abd}} - \frac{(dx^3+c)bcd-3\,(dx^3+c)ad^2+2\,acd^2}{3\,(abc^3-a^2c^2d)\left((dx^3+c)^{\frac{3}{2}}-\sqrt{dx^3+c}c\right)} - \frac{(2\,bc+3\,ad)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}c^2}$$

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] 2/3*b^3*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b*c - a^3*d)*s
qrt(-b^2*c + a*b*d)) - 1/3*((d*x^3 + c)*b*c*d - 3*(d*x^3 + c)*a*d^2 + 2*a*c
*d^2)/((a*b*c^3 - a^2*c^2*d)*((d*x^3 + c)^(3/2) - sqrt(d*x^3 + c)*c)) - 1/3
*(2*b*c + 3*a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 15.21 (sec) , antiderivative size = 597, normalized size of antiderivative = 3.78

$$\int \frac{1}{x^4 (a+bx^3) (c+dx^3)^{3/2}}\,dx = \frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)^3}{x^6}\right)(3\,a\,d+2\,b\,c)}{6\,a^2\,c^{5/2}} - \frac{\sqrt{d\,x^3+c}}{3\,a\,c^2\,x^3}$$

$$- \frac{c\left(\frac{c\left(\frac{c\left(\frac{3\,a^2\,d^4+24\,a\,b\,c\,d^3+15\,b^2\,c^2\,d^2}{8\,a^3\,c^5}+\frac{c\left(\frac{c\left(\frac{3\,b^2\,d^4}{8\,a^3\,c^5}+\frac{b^2\,d^4\,(5\,a\,d-3\,b\,c)}{8\,a^3\,c^4\,\left(b\,c^2-a\,c\,d\right)}-\frac{b\,d^4\,(a\,d+2\,b\,c)\,(5\,a\,d-3\,b\,c)}{4\,a^3\,c^5\,\left(b\,c^2-a\,c\,d\right)}\right)}{d}-\frac{3\,b\,d^3\,(a\,d+2\,b\,c)}{4\,a^3\,c^5}+\frac{d\,(5\,a\,d-3\,b\,c)\,\left(3\,a^2\,d^4+24\,a\,b\,c \cdots\right.}{24\,a^3\,c^5\,\left(b\,c^2-a \cdots\right.}\cdots}{d}\cdots\right)}{d}\cdots\right)}{d}\cdots\right)}{\cdots}$$

$$+ \frac{b^{5/2}\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,a^2\,(a\,d-b\,c)^{3/2}}$$

```
[In] int(1/(x^4*(a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

```
[Out] (log((((c + d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2) + c^(1/2))^3)/x^6)*(
3*a*d + 2*b*c))/(6*a^2*c^(5/2)) - (c + d*x^3)^(1/2)/(3*a*c^2*x^3) - ((c*((c
*((c*((3*a^2*d^4 + 15*b^2*c^2*d^2 + 24*a*b*c*d^3)/(8*a^3*c^5) + (c*((c*((3*
b^2*d^4)/(8*a^3*c^5) + (b^2*d^4*(5*a*d - 3*b*c))/(8*a^3*c^4*(b*c^2 - a*c*d)
) - (b*d^4*(a*d + 2*b*c)*(5*a*d - 3*b*c))/(4*a^3*c^5*(b*c^2 - a*c*d))))/d -
 (3*b*d^3*(a*d + 2*b*c))/(4*a^3*c^5) + (d*(5*a*d - 3*b*c)*(3*a^2*d^4 + 15*b
```

```
^2*c^2*d^2 + 24*a*b*c*d^3))/(24*a^3*c^5*(b*c^2 - a*c*d))))/d - (d^2*(5*a*d
- 3*b*c)*(6*a^2*d^2 + 3*b^2*c^2 + 14*a*b*c*d))/(12*a^3*c^4*(b*c^2 - a*c*d))
))/d - (d*(6*a^2*d^2 + 3*b^2*c^2 + 14*a*b*c*d))/(4*a^3*c^4) + (d^2*(5*a*d -
 3*b*c)*(13*a*d + 18*b*c))/(24*a^2*c^3*(b*c^2 - a*c*d))))/d + (d*(13*a*d +
18*b*c))/(8*a^2*c^3) - (d*(3*a*d + 2*b*c)*(5*a*d - 3*b*c))/(6*a^2*c^2*(b*c^
2 - a*c*d))))/d - (3*a*d + 2*b*c)/(2*a^2*c^2))/(c + d*x^3)^(1/2) + (b^(5/2)
*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i - b*d*x^
3)/(a + b*x^3))*1i)/(3*a^2*(a*d - b*c)^(3/2))
```

3.393 $\int \frac{x^3}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{x^4\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},1,\frac{3}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4ac\sqrt{c+dx^3}}$$

```
[Out] 1/4*x^4*AppellF1(4/3,1,3/2,7/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/c/(d*
x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{x^4\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{4}{3},1,\frac{3}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4ac\sqrt{c+dx^3}}$$

```
[In] Int[x^3/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 3/2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)])/(4*a*c*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{(a+bx^3)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$

$$= \frac{x^4\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{4}{3};1,\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4ac\sqrt{c+dx^3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 231 vs. 2(67) = 134.

Time = 10.19 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.45

$$\int \frac{x^3}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \frac{x\left(-8-\frac{bx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a}-\frac{\cdots}{(a+bx^3)\left(-8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c}\cdots\right.\right.}\right.}{12(-bc+ad)\sqrt{\cdots}}$$

```
[In] Integrate[x^3/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*(-8 - (b*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c)
, -((b*x^3)/a)])/a - (64*a^2*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((
b*x^3)/a)])/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -
((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x
^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))))/(
12*(-(b*c) + a*d)*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.45 (sec) , antiderivative size = 749, normalized size of antiderivative = 11.18

method	result	size
elliptic	Expression too large to display	749
default	Expression too large to display	1069

```
[In] int(x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*x/(a*d-b*c)/((x^3+c/d)*d)^(1/2)+2/9*I/(a*d-b*c)*3^(1/2)/d*(-c*d^2)^(1/
3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c
*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3
)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/3*I*a/d^2*2^(1/2)*sum(1/(a*d-b*
c)^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(
-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2
)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^
2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^
(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^
(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b
/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I
*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)
^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alph
a=RootOf(_Z^3*b+a))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+bx^3)(c+dx^3)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   Not i
ntegrable (provided residues have no relations)
```

Sympy [F]

$$\int \frac{x^3}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \int \frac{x^3}{(a+bx^3)\,(c+dx^3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**3/(b*x**3+a)/(d*x**3+c)**(3/2),x)
[Out] Integral(x**3/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^3}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \int \frac{x^3}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(x^3/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{x^3}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \int \frac{x^3}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(x^3/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \int \frac{x^3}{(b\,x^3+a)\,\left(d\,x^3+c\right)^{3/2}}\,dx$$

```
[In] int(x^3/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
[Out] int(x^3/((a + b*x^3)*(c + d*x^3)^(3/2)), x)
```

3.394 $\int \frac{x}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 67

$$\int \frac{x}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{x^2\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},1,\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ac\sqrt{c+dx^3}}$$

```
[Out] 1/2*x^2*AppellF1(2/3,1,3/2,5/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/c/(d*
x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {525, 524}

$$\int \frac{x}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{x^2\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{2}{3},1,\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ac\sqrt{c+dx^3}}$$

```
[In] Int[x/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1, 3/2, 5/3, -((b*x^3)/a), -((d*x^3)
/c)])/(2*a*c*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x}{(a+bx^3)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$

$$= \frac{x^2\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{2}{3};1,\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ac\sqrt{c+dx^3}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 142 vs. 2(67) = 134.

Time = 10.13 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.12

$$\int \frac{x}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \frac{x^2\left(-20ad+5(3bc+ad)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+2bdx^3\right.}{30ac(bc-ad)\sqrt{c+dx^3}}$$

```
[In] Integrate[x/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^2*(-20*a*d + 5*(3*b*c + a*d)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5
/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*b*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/
3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)]))/(30*a*c*(b*c - a*d)*Sqrt[c +
d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.43 (sec) , antiderivative size = 907, normalized size of antiderivative = 13.54

method	result	size
default	Expression too large to display	907
elliptic	Expression too large to display	907

[In] int(x/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/3*d*x^2/c/(a*d-b*c)/((x^3+c/d)*d)^(1/2)+2/9*I/c/(a*d-b*c)*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/3*I*b/d^2*2^(1/2)*sum(1/(a*d-b*c)^2/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: Not integrable (provided residues have no relations)

Sympy [F]

$$\int \frac{x}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \int \frac{x}{(a+bx^3)(c+dx^3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(b*x**3+a)/(d*x**3+c)**(3/2),x)
[Out] Integral(x/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \int \frac{x}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(x/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{x}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \int \frac{x}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(x/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \int \frac{x}{(b\,x^3+a)\,(d\,x^3+c)^{3/2}}\,dx$$

```
[In] int(x/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
[Out] int(x/((a + b*x^3)*(c + d*x^3)^(3/2)), x)
```

3.395 $\int \frac{1}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{1}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{x\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{1}{3},1,\frac{3}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac\sqrt{c+dx^3}}$$

```
[Out] x*AppellF1(1/3,1,3/2,4/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/c/(d*x^3+c)
^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {441, 440}

$$\int \frac{1}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{x\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},1,\frac{3}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac\sqrt{c+dx^3}}$$

```
[In] Int[1/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 3/2, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a*c*Sqrt[c + d*x^3])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{(a+bx^3)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}} \\ &= \frac{x\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{1}{3};1,\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 338 vs. 2(62) = 124.

Time = 10.32 (sec) , antiderivative size = 338, normalized size of antiderivative = 5.45

$$\int \frac{1}{(a+bx^3)\left(c+dx^3\right)^{3/2}}\,dx = \frac{x\left(\frac{bdx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a(-bc+ad)} + \frac{32ac(-3bc+3ad+2bdx^3)\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-d\cdots\right.}{(bc-ad)(a+bx^3)\left(-8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3}\cdots\right.\right.}\cdots\right.}{12c\sqrt{\cdots}}$$

```
[In] Integrate[1/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*((b*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -
((b*x^3)/a)])/(a*(-(b*c) + a*d)) + (32*a*c*(-3*b*c + 3*a*d + 2*b*d*x^3)*App
ellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 24*d*x^3*(a + b*x^3)*
(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF
1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))/((b*c - a*d)*(a + b*x^3)*
(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b
*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/
3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))))/(12*c*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.26 (sec) , antiderivative size = 753, normalized size of antiderivative = 12.15

method	result	size
default	Expression too large to display	753
elliptic	Expression too large to display	753

```
[In] int(1/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*d*x/c/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-2/9*I/c/(a*d-b*c)*3^(1/2)*(-c*d^2)^
(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF
(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(
1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(
1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/3*I*b/d^2*2^(1/2)*sum(1/(a*d
-b*c)^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3
)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*
d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c
*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^
2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^
2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/
2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alph
a+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d
^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_a
lpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx=\int \frac{1}{(a+bx^3)\,(c+dx^3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x**3+a)/(d*x**3+c)**(3/2),x)
[Out] Integral(1/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx=\int \frac{1}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx=\int \frac{1}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)\,(c+dx^3)^{3/2}}\,dx=\int \frac{1}{(b\,x^3+a)\,\left(d\,x^3+c\right)^{3/2}}\,dx$$

```
[In] int(1/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
[Out] int(1/((a + b*x^3)*(c + d*x^3)^(3/2)), x)
```

3.396 $\int \frac{1}{x^2(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{1}{x^2\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = -\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{1}{3},1,\frac{3}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{acx\sqrt{c+dx^3}}$$

```
[Out] -AppellF1(-1/3,1,3/2,2/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/c/x/(d*x^3+
c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{1}{x^2\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = -\frac{\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{1}{3},1,\frac{3}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{acx\sqrt{c+dx^3}}$$

```
[In] Int[1/(x^2*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] -((Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 1, 3/2, 2/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a*c*x*Sqrt[c + d*x^3]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^2(a+bx^3)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$

$$= -\frac{\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{1}{3};1,\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{acx\sqrt{c+dx^3}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 193 vs. 2(65) = 130.

Time = 10.21 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.97

$$\int \frac{1}{x^2\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{20a(-3bc(c+dx^3)+ad(3c+5dx^3))-5(6b^2c^2-3abcd+5a^2d^2)\,x^3\sqrt{1+}}{}$$

```
[In] Integrate[1/(x^2*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (20*a*(-3*b*c*(c + d*x^3) + a*d*(3*c + 5*d*x^3)) - 5*(6*b^2*c^2 - 3*a*b*c*d
 + 5*a^2*d^2)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/
c), -((b*x^3)/a)] + 2*b*d*(3*b*c - 5*a*d)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[
5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(60*a^2*c^2*(b*c - a*d)*x*Sq
rt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 5.86 (sec) , antiderivative size = 952, normalized size of antiderivative = 14.65

method	result	size
elliptic	Expression too large to display	952
risch	Expression too large to display	1382
default	Expression too large to display	1392

```
[In] int(1/x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*d^2*x^2/c^2/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-1/c^2/a*(d*x^3+c)^(1/2)/x-2/
3*I*(1/3*d^2/c^2/(a*d-b*c)+1/2/a/c^2*d)*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/
d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(
1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(
1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/
3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/
2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))
^(1/2)))-1/3*I*b^2/a/d^2*2^(1/2)*sum(1/(a*d-b*c)^2/_alpha*(-c*d^2)^(1/3)*(1
/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))
^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3
)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c
*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(
1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*
EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*
_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_
alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + bx^3) (c + dx^3)^{3/2}} dx = \text{Timed out}$$

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^2 (a+bx^3) (c+dx^3)^{3/2}} dx = \int \frac{1}{x^2 (a+bx^3) (c+dx^3)^{\frac{3}{2}}} dx$$

```
[In] integrate(1/x**2/(b*x**3+a)/(d*x**3+c)**(3/2),x)
[Out] Integral(1/(x**2*(a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a+bx^3) (c+dx^3)^{3/2}} dx = \int \frac{1}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}x^2} dx$$

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a+bx^3) (c+dx^3)^{3/2}} dx = \int \frac{1}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}x^2} dx$$

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^3) (c+dx^3)^{3/2}} dx = \int \frac{1}{x^2 (b x^3+a) (d x^3+c)^{3/2}} dx$$

```
[In] int(1/(x^2*(a + b*x^3)*(c + d*x^3)^(3/2)),x)
[Out] int(1/(x^2*(a + b*x^3)*(c + d*x^3)^(3/2)), x)
```

3.397 $\int \frac{1}{x^3(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{1}{x^3\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=-\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{2}{3},1,\frac{3}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2acx^2\sqrt{c+dx^3}}$$

```
[Out] -1/2*AppellF1(-2/3,1,3/2,1/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/c/x^2/(
d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{1}{x^3\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=-\frac{\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{2}{3},1,\frac{3}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2acx^2\sqrt{c+dx^3}}$$

```
[In] Int[1/(x^3*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/2*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 3/2, 1/3, -((b*x^3)/a), -((d*x^
3)/c)])/(a*c*x^2*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^3(a+bx^3)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$

$$= -\frac{\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{2}{3};1,\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2acx^2\sqrt{c+dx^3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 408 vs. $2(67) = 134$.

Time = 10.58 (sec) , antiderivative size = 408, normalized size of antiderivative = 6.09

$$\int \frac{1}{x^3(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{bd(3bc-7ad)x^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{8a\left(-4ac(-6b^2c\right.}{}}{}$$

```
[In] Integrate[1/(x^3*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (b*d*(3*b*c - 7*a*d)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((
d*x^3)/c), -((b*x^3)/a)] + (8*a*(-4*a*c*(-6*b^2*c*x^3*(3*c + d*x^3) + 3*a^2
*d*(2*c + 7*d*x^3) + a*b*(-6*c^2 - 3*c*d*x^3 + 14*d^2*x^6))*AppellF1[1/3, 1
/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(a + b*x^3)*(-3*b*c*(c + d*
x^3) + a*d*(3*c + 7*d*x^3))*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c),
 -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]
)))/((a + b*x^3)*(8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/
a)] - 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] +
 a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(48*a^2*c^2
*(-(b*c) + a*d)*x^2*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 6.05 (sec) , antiderivative size = 798, normalized size of antiderivative = 11.91

method	result	size
elliptic	Expression too large to display	798
risch	Expression too large to display	1076
default	Expression too large to display	1084

```
[In] int(1/x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*d^2*x/c^2/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-1/2/c^2/a*(d*x^3+c)^(1/2)/x^2-
2/3*I*(-1/3*d^2/c^2/(a*d-b*c)-1/4/a/c^2*d)*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/3*I*b^2/a/d^2*2^(1/2)*sum(1/(a*d-b*c)^2/_
alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2
)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3
)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(
1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/
3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*
_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*
I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/
2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)
/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=Root
Of(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3\,(a+bx^3)\,(c+dx^3)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^3 (a+bx^3) (c+dx^3)^{3/2}}\,dx = \int \frac{1}{x^3 (a+bx^3) (c+dx^3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**3/(b*x**3+a)/(d*x**3+c)**(3/2),x)
[Out] Integral(1/(x**3*(a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (a+bx^3) (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}x^3}\,dx$$

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a+bx^3) (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}x^3}\,dx$$

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^3) (c+dx^3)^{3/2}}\,dx = \int \frac{1}{x^3 \left(b\,x^3+a\right) \left(d\,x^3+c\right)^{3/2}}\,dx$$

```
[In] int(1/(x^3*(a + b*x^3)*(c + d*x^3)^(3/2)),x)
[Out] int(1/(x^3*(a + b*x^3)*(c + d*x^3)^(3/2)), x)
```

3.398 $\int \frac{x^{11}\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 117

$$\int \frac{x^{11}\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{7x^6\sqrt{c+dx^3}}{15d^2}+\frac{x^9\sqrt{c+dx^3}}{3d\,(8c-dx^3)}+\frac{2c\sqrt{c+dx^3}(1146c+47dx^3)}{15d^4}-\frac{3968c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4}$$

```
[Out] -3968/9*c^(5/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^4+7/15*x^6*(d*x^3+c)
^(1/2)/d^2+1/3*x^9*(d*x^3+c)^(1/2)/d/(-d*x^3+8*c)+2/15*c*(47*d*x^3+1146*c)*
(d*x^3+c)^(1/2)/d^4
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {457, 99, 158, 152, 65, 212}

$$\int \frac{x^{11}\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = -\frac{3968c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4}+\frac{2c\sqrt{c+dx^3}(1146c+47dx^3)}{15d^4}+\frac{7x^6\sqrt{c+dx^3}}{15d^2}+\frac{x^9\sqrt{c+dx^3}}{3d\,(8c-dx^3)}$$

```
[In] Int[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (7*x^6*Sqrt[c + d*x^3])/(15*d^2) + (x^9*Sqrt[c + d*x^3])/(3*d*(8*c - d*x^3)
) + (2*c*Sqrt[c + d*x^3]*(1146*c + 47*d*x^3))/(15*d^4) - (3968*c^(5/2)*ArcT
anh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x^3\sqrt{c+dx}}{(8c-dx)^2}\,dx, x, x^3\right)$$

$$= \frac{x^9\sqrt{c+dx^3}}{3d\,(8c-dx^3)} - \frac{\text{Subst}\left(\int \frac{x^2\left(3c+\frac{7dx}{2}\right)}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3d}$$

$$= \frac{7x^6\sqrt{c+dx^3}}{15d^2} + \frac{x^9\sqrt{c+dx^3}}{3d\,(8c-dx^3)} + \frac{2\text{Subst}\left(\int \frac{x\left(-56c^2d-\frac{141}{2}cd^2x\right)}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{15d^3}$$

$$= \frac{7x^6\sqrt{c+dx^3}}{15d^2} + \frac{x^9\sqrt{c+dx^3}}{3d\,(8c-dx^3)} + \frac{2c\sqrt{c+dx^3}(1146c+47dx^3)}{15d^4} - \frac{(1984c^3)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3d^3}$$

$$= \frac{7x^6\sqrt{c+dx^3}}{15d^2} + \frac{x^9\sqrt{c+dx^3}}{3d\,(8c-dx^3)} + \frac{2c\sqrt{c+dx^3}(1146c+47dx^3)}{15d^4} - \frac{(3968c^3)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d^4}$$

$$= \frac{7x^6\sqrt{c+dx^3}}{15d^2} + \frac{x^9\sqrt{c+dx^3}}{3d\,(8c-dx^3)} + \frac{2c\sqrt{c+dx^3}(1146c+47dx^3)}{15d^4} - \frac{3968c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.78

$$\int \frac{x^{11}\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{2\left(\frac{3\sqrt{c+dx^3}(-9168c^3+770c^2dx^3+19cd^2x^6+d^3x^9)}{-8c+dx^3} - 9920c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{45d^4}$$

```
[In] Integrate[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (2*((3*Sqrt[c + d*x^3]*(-9168*c^3 + 770*c^2*d*x^3 + 19*c*d^2*x^6 + d^3*x^9)
)/(-8*c + d*x^3) - 9920*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(45*
d^4)
```

Maple [A] (verified)

Time = 4.41 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$-\frac{158720\left(c^{3}\left(c-\frac{d\,x^{3}}{8}\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)+\frac{3\sqrt{d\,x^{3}+c}\left(\sqrt{c}\,d^{3}x^{9}+19c^{\frac{3}{2}}d^{2}x^{6}+770c^{\frac{5}{2}}d\,x^{3}-9168c^{\frac{7}{2}}\right)}{79360}\right)}{\sqrt{c}\left(-45d^{5}x^{3}+360c\,d^{4}\right)}$
risch	$\frac{2\left(d^{2}x^{6}+27cd\,x^{3}+986c^{2}\right)\sqrt{d\,x^{3}+c}}{15d^{4}}+\frac{64c^{3}\left(-\frac{70\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{9d\sqrt{c}}+\frac{8c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{3d}\right)}{d^{3}}$
default	$\frac{d\left(\frac{2x^{6}\sqrt{d\,x^{3}+c}}{15}+\frac{2c\,x^{3}\sqrt{d\,x^{3}+c}}{45d}-\frac{4c^{2}\sqrt{d\,x^{3}+c}}{45d^{2}}\right)+\frac{32c\left(d\,x^{3}+c\right)^{\frac{3}{2}}}{9d}}{d^{3}}-\frac{64c^{2}\left(-2\sqrt{d\,x^{3}+c}+6\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\sqrt{c}\right)}{d^{4}}+\frac{512c^{3}}{}$
elliptic	$\frac{512c^{3}\sqrt{d\,x^{3}+c}}{3d^{4}\left(-d\,x^{3}+8c\right)}+\frac{2x^{6}\sqrt{d\,x^{3}+c}}{15d^{2}}+\frac{18c\,x^{3}\sqrt{d\,x^{3}+c}}{5d^{3}}+\frac{1972c^{2}\sqrt{d\,x^{3}+c}}{15d^{4}}+\frac{1984ic^{2}\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{}\right)}{}$

[In] int(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)

[Out] -158720*(c^3*(c-1/8*d*x^3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))+3/79360*(d*x^3+c)^(1/2)*(c^(1/2)*d^3*x^9+19*c^(3/2)*d^2*x^6+770*c^(5/2)*d*x^3-9168*c^(7/2)))/c^(1/2)/(-45*d^5*x^3+360*c*d^4)

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.87

$$\int \frac{x^{11}\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \left[\frac{2\left(4960\left(c^2dx^3-8c^3\right)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+3\left(d^3x^9+19cd^2x^6+770c^2dx^3-9168c^3\right)\sqrt{dx^3+c}\right)}{45\left(d^5x^3-8cd^4\right)}\right.$$

```
[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [2/45*(4960*(c^2*d*x^3 - 8*c^3)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt
(c) + 10*c)/(d*x^3 - 8*c)) + 3*(d^3*x^9 + 19*c*d^2*x^6 + 770*c^2*d*x^3 - 91
68*c^3)*sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4), 2/45*(9920*(c^2*d*x^3 - 8*c^3
)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(d^3*x^9 + 19*c*d^2*x
^6 + 770*c^2*d*x^3 - 9168*c^3)*sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4)]
```

Sympy [F]

$$\int \frac{x^{11}\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{x^{11}\sqrt{c+dx^3}}{(-8c+dx^3)^2}\,dx$$

```
[In] integrate(x**11*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(x**11*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int \frac{x^{11}\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{2\left(4960c^{\frac{5}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+3\left(dx^3+c\right)^{\frac{5}{2}}+75\left(dx^3+c\right)^{\frac{3}{2}}c+2880\sqrt{dx^3+c}c^2-\frac{3840\sqrt{dx^3+c}c^3}{dx^3-8c}\right)}{45d^4}$$

```
[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] 2/45*(4960*c^(5/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*s
qrt(c))) + 3*(d*x^3 + c)^(5/2) + 75*(d*x^3 + c)^(3/2)*c + 2880*sqrt(d*x^3 +
 c)*c^2 - 3840*sqrt(d*x^3 + c)*c^3/(d*x^3 - 8*c))/d^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.94

$$\int \frac{x^{11}\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{3968\,c^3\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}d^4} - \frac{512\sqrt{dx^3+c}c^3}{3\,(dx^3-8\,c)d^4} + \frac{2\left((dx^3+c)^{\frac{5}{2}}d^{16}+25\,(dx^3+c)^{\frac{3}{2}}cd^{16}+960\sqrt{dx^3+c}c^2d^{16}\right)}{15\,d^{20}}$$

```
[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 3968/9*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 512/3*sqrt
(d*x^3 + c)*c^3/((d*x^3 - 8*c)*d^4) + 2/15*((d*x^3 + c)^(5/2)*d^16 + 25*(d*
x^3 + c)^(3/2)*c*d^16 + 960*sqrt(d*x^3 + c)*c^2*d^16)/d^20
```

Mupad [B] (verification not implemented)

Time = 8.32 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09

$$\int \frac{x^{11}\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{1984\,c^{5/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^4} + \frac{1972\,c^2\,\sqrt{d\,x^3+c}}{15\,d^4} + \frac{2\,x^6\,\sqrt{d\,x^3+c}}{15\,d^2} + \frac{18\,c\,x^3\,\sqrt{d\,x^3+c}}{5\,d^3} + \frac{512\,c^3\,\sqrt{d\,x^3+c}}{3\,d^4\,\left(8\,c-d\,x^3\right)}$$

```
[In] int((x^11*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
```

```
[Out] (1984*c^(5/2)*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3
)))/(9*d^4) + (1972*c^2*(c + d*x^3)^(1/2))/(15*d^4) + (2*x^6*(c + d*x^3)^(1
/2))/(15*d^2) + (18*c*x^3*(c + d*x^3)^(1/2))/(5*d^3) + (512*c^3*(c + d*x^3)
^(1/2))/(3*d^4*(8*c - d*x^3))
```

3.399 $\int \frac{x^8\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 102

$$\int \frac{x^8\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx=\frac{352c\sqrt{c+dx^3}}{27d^3}+\frac{2(c+dx^3)^{3/2}}{9d^3}+\frac{64c(c+dx^3)^{3/2}}{27d^3(8c-dx^3)}-\frac{352c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}$$

```
[Out] 2/9*(d*x^3+c)^(3/2)/d^3+64/27*c*(d*x^3+c)^(3/2)/d^3/(-d*x^3+8*c)-352/9*c^(3
/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^3+352/27*c*(d*x^3+c)^(1/2)/d^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {457, 91, 81, 52, 65, 212}

$$\int \frac{x^8\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx=-\frac{352c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}+\frac{64c(c+dx^3)^{3/2}}{27d^3(8c-dx^3)}+\frac{352c\sqrt{c+dx^3}}{27d^3}+\frac{2(c+dx^3)^{3/2}}{9d^3}$$

```
[In] Int[(x^8*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (352*c*Sqrt[c + d*x^3])/(27*d^3) + (2*(c + d*x^3)^(3/2))/(9*d^3) + (64*c*(c
 + d*x^3)^(3/2))/(27*d^3*(8*c - d*x^3)) - (352*c^(3/2)*ArcTanh[Sqrt[c + d*x
^3]/(3*Sqrt[c])])/(9*d^3)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x^2\sqrt{c+dx}}{(8c-dx)^2}\,dx, x, x^3\right)$$
$$= \frac{64c(c+dx^3)^{3/2}}{27d^3(8c-dx^3)} - \frac{\text{Subst}\left(\int \frac{\sqrt{c+dx}(104c^2d+9cd^2x)}{8c-dx}\,dx, x, x^3\right)}{27cd^3}$$
$$= \frac{2(c+dx^3)^{3/2}}{9d^3} + \frac{64c(c+dx^3)^{3/2}}{27d^3(8c-dx^3)} - \frac{(176c)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)}{27d^2}$$
$$= \frac{352c\sqrt{c+dx^3}}{27d^3} + \frac{2(c+dx^3)^{3/2}}{9d^3} + \frac{64c(c+dx^3)^{3/2}}{27d^3(8c-dx^3)} - \frac{(176c^2)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3d^2}$$
$$= \frac{352c\sqrt{c+dx^3}}{27d^3} + \frac{2(c+dx^3)^{3/2}}{9d^3} + \frac{64c(c+dx^3)^{3/2}}{27d^3(8c-dx^3)} - \frac{(352c^2)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d^3}$$
$$= \frac{352c\sqrt{c+dx^3}}{27d^3} + \frac{2(c+dx^3)^{3/2}}{9d^3} + \frac{64c(c+dx^3)^{3/2}}{27d^3(8c-dx^3)} - \frac{352c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.77

$$\int \frac{x^8\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{2\left(\frac{\sqrt{c+dx^3}(-488c^2+41cdx^3+d^2x^6)}{-8c+dx^3} - 176c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{9d^3}$$

```
[In] Integrate[(x^8*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (2*((Sqrt[c + d*x^3]*(-488*c^2 + 41*c*d*x^3 + d^2*x^6))/(-8*c + d*x^3) - 17
6*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(9*d^3)
```

Maple [A] (verified)

Time = 4.67 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.77

method	result
pseudoelliptic	$\frac{\frac{2\left(d\,x^{3}+c\right)^{\frac{3}{2}}}{9}+\frac{32c\sqrt{d\,x^{3}+c}}{3}+\frac{32c^{2}\left(\frac{2\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}-\frac{11\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3\sqrt{c}}\right)}{3}}{d^{3}}$
default	$\frac{2\left(d\,x^{3}+c\right)^{\frac{3}{2}}}{9d^{3}}-\frac{16c\left(-2\sqrt{d\,x^{3}+c}+6\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\sqrt{c}\right)}{3d^{3}}+\frac{64c^{2}\left(\frac{\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3\sqrt{c}}\right)}{3d^{3}}$
risch	$\frac{2\left(d\,x^{3}+49c\right)\sqrt{d\,x^{3}+c}}{9d^{3}}+\frac{16c^{2}\left(-\frac{26\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{9d\sqrt{c}}+\frac{4c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{3d}\right)}{d^{2}}$
elliptic	$\frac{64c^{2}\sqrt{d\,x^{3}+c}}{3d^{3}\left(-d\,x^{3}+8c\right)}+\frac{2x^{3}\sqrt{d\,x^{3}+c}}{9d^{2}}+\frac{98c\sqrt{d\,x^{3}+c}}{9d^{3}}+\frac{176ic\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)\ldots}{\ldots}\right)}{\left(-c\,d^{2}\right)\ldots}}\ldots}{\ldots}\right)}{\ldots}$

[In] int(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)

[Out] 2/3*(1/3*(d*x^3+c)^(3/2)+16*c*(d*x^3+c)^(1/2)+16*c^2*(2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-11/3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)))/d^3

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.87

$$\int \frac{x^8\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \left[\frac{2\left(88\left(cdx^3-8\,c^2\right)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+\left(d^2x^6+41\,cdx^3-488\,c^2\right)\sqrt{dx^3+c}\right)}{9\left(d^4x^3-8\,cd^3\right)}, \frac{2\left(176\left(cdx^3-8\,c\right.\right.}{}\right.$$

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [2/9*(88*(c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) +
 10*c)/(d*x^3 - 8*c)) + (d^2*x^6 + 41*c*d*x^3 - 488*c^2)*sqrt(d*x^3 + c))/(
d^4*x^3 - 8*c*d^3), 2/9*(176*(c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/3*sqrt(d*x
^3 + c)*sqrt(-c)/c) + (d^2*x^6 + 41*c*d*x^3 - 488*c^2)*sqrt(d*x^3 + c))/(d^
4*x^3 - 8*c*d^3)]
```

Sympy [F]

$$\int \frac{x^8\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{x^8\sqrt{c+dx^3}}{(-8c+dx^3)^2}\,dx$$

```
[In] integrate(x**8*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(x**8*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.89

$$\int \frac{x^8\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{2\left(88\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+\left(dx^3+c\right)^{\frac{3}{2}}+48\,\sqrt{dx^3+c}c-\frac{96\,\sqrt{dx^3+c}c^2}{dx^3-8\,c}\right)}{9\,d^3}$$

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] 2/9*(88*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt
(c))) + (d*x^3 + c)^(3/2) + 48*sqrt(d*x^3 + c)*c - 96*sqrt(d*x^3 + c)*c^2/(
d*x^3 - 8*c))/d^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.91

$$\int \frac{x^8\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{352\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}d^3} - \frac{64\sqrt{dx^3+c}c^2}{3\,(dx^3-8\,c)d^3} + \frac{2\left((dx^3+c)^{\frac{3}{2}}d^6+48\sqrt{dx^3+c}cd^6\right)}{9\,d^9}$$

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 352/9*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 64/3*sqrt(d
*x^3 + c)*c^2/((d*x^3 - 8*c)*d^3) + 2/9*((d*x^3 + c)^(3/2)*d^6 + 48*sqrt(d*
x^3 + c)*c*d^6)/d^9
```

Mupad [B] (verification not implemented)

Time = 8.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.05

$$\int \frac{x^8\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{98\,c\,\sqrt{d\,x^3+c}}{9\,d^3} + \frac{176\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^3} + \frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d^2} + \frac{64\,c^2\,\sqrt{d\,x^3+c}}{3\,d^3\,\left(8\,c-d\,x^3\right)}$$

```
[In] int((x^8*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
```

```
[Out] (98*c*(c + d*x^3)^(1/2))/(9*d^3) + (176*c^(3/2)*log((10*c + d*x^3 - 6*c^(1/
2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(9*d^3) + (2*x^3*(c + d*x^3)^(1/2))/(
9*d^2) + (64*c^2*(c + d*x^3)^(1/2))/(3*d^3*(8*c - d*x^3))
```

3.400 $\int \frac{x^5\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 82

$$\int \frac{x^5\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx=\frac{26\sqrt{c+dx^3}}{27d^2}+\frac{8\left(c+dx^3\right)^{3/2}}{27d^2\left(8c-dx^3\right)}-\frac{26\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}$$

```
[Out] 8/27*(d*x^3+c)^(3/2)/d^2/(-d*x^3+8*c)-26/9*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1
/2))*c^(1/2)/d^2+26/27*(d*x^3+c)^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {457, 79, 52, 65, 212}

$$\int \frac{x^5\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx=-\frac{26\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}+\frac{8\left(c+dx^3\right)^{3/2}}{27d^2\left(8c-dx^3\right)}+\frac{26\sqrt{c+dx^3}}{27d^2}$$

```
[In] Int[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (26*Sqrt[c + d*x^3])/(27*d^2) + (8*(c + d*x^3)^(3/2))/(27*d^2*(8*c - d*x^3)
) - (26*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x\sqrt{c+dx}}{(8c-dx)^2}\,dx, x, x^3\right)\\
&= \frac{8(c+dx^3)^{3/2}}{27d^2(8c-dx^3)} - \frac{13\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)}{27d}\\
&= \frac{26\sqrt{c+dx^3}}{27d^2} + \frac{8(c+dx^3)^{3/2}}{27d^2(8c-dx^3)} - \frac{(13c)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3d}
\end{aligned}$$

$$= \frac{26\sqrt{c+dx^3}}{27d^2} + \frac{8(c+dx^3)^{3/2}}{27d^2\,(8c-dx^3)} - \frac{(26c)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d^2}$$

$$= \frac{26\sqrt{c+dx^3}}{27d^2} + \frac{8(c+dx^3)^{3/2}}{27d^2\,(8c-dx^3)} - \frac{26\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

$$\int \frac{x^5\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{2\left(\frac{3(-12c+dx^3)\sqrt{c+dx^3}}{-8c+dx^3} - 13\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{9d^2}$$

```
[In] Integrate[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
[Out] (2*((3*(-12*c + d*x^3)*Sqrt[c + d*x^3])/(-8*c + d*x^3) - 13*Sqrt[c]*ArcTanh
[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(9*d^2)
```

Maple [A] (verified)

Time = 4.88 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{\frac{2\sqrt{d\,x^{3}+c}}{3}+\frac{2c\left(\frac{4\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}-\frac{13\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3\sqrt{c}}\right)}{3}}{d^{2}}$
default	$-\frac{-2\sqrt{d\,x^{3}+c}+6\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\sqrt{c}}{3d^{2}}+\frac{8c\left(\frac{\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3\sqrt{c}}\right)}{3d^{2}}$
risch	$\frac{2\sqrt{d\,x^{3}+c}}{3d^{2}}+\frac{c\left(-\frac{34\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{9d\sqrt{c}}+\frac{8c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{3d}\right)}{d}$
elliptic	$\frac{8c\sqrt{d\,x^{3}+c}}{3d^{2}(-d\,x^{3}+8c)}+\frac{2\sqrt{d\,x^{3}+c}}{3d^{2}}+\frac{13i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{\cdots}{-3(\cdots}}\cdots}{\cdots}\right.}{\cdots}$

[In] int(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)

[Out] 2/3*((d*x^3+c)^(1/2)+c*(4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-13/3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)))/d^2

Fricas [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.01

$$\int \frac{x^5\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$$
$$=\left[\frac{13\,(dx^3-8\,c)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+6\,\sqrt{dx^3+c}(dx^3-12\,c)}{9\,(d^3x^3-8\,cd^2)},\frac{2\left(13\,(dx^3-8\,c)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3}\cdots}{\cdots}\right.\right.}{9\,(d^3x^3\cdots}\right.$$

[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")

```
[Out] [1/9*(13*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*
c)/(d*x^3 - 8*c)) + 6*sqrt(d*x^3 + c)*(d*x^3 - 12*c))/(d^3*x^3 - 8*c*d^2),
2/9*(13*(d*x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*s
qrt(d*x^3 + c)*(d*x^3 - 12*c))/(d^3*x^3 - 8*c*d^2)]
```

Sympy [F]

$$\int \frac{x^5\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{x^5\sqrt{c+dx^3}}{(-8c+dx^3)^2}\,dx$$

```
[In] integrate(x**5*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
[Out] Integral(x**5*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int \frac{x^5\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{13\,\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+6\,\sqrt{dx^3+c}-\frac{24\,\sqrt{dx^3+c}c}{dx^3-8\,c}}{9\,d^2}$$

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
[Out] 1/9*(13*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt
(c))) + 6*sqrt(d*x^3 + c) - 24*sqrt(d*x^3 + c)*c/(d*x^3 - 8*c))/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

$$\int \frac{x^5\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{26\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{9\,\sqrt{-c}d^2}+\frac{2\,\sqrt{dx^3+c}}{3\,d^2}-\frac{8\,\sqrt{dx^3+c}c}{3\,(dx^3-8\,c)d^2}$$

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
[Out] 26/9*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^2) + 2/3*sqrt(d*x^3
 + c)/d^2 - 8/3*sqrt(d*x^3 + c)*c/((d*x^3 - 8*c)*d^2)
```

Mupad [B] (verification not implemented)

Time = 8.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.06

$$\int \frac{x^5\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{2\sqrt{d\,x^3+c}}{3\,d^2} + \frac{13\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3-6\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^2} + \frac{8\,c\,\sqrt{d\,x^3+c}}{3\,d^2\,\left(8\,c-d\,x^3\right)}$$

```
[In] int((x^5*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*d^2) + (13*c^(1/2)*log((10*c + d*x^3 - 6*c^(1/2)*(
c + d*x^3)^(1/2))/(8*c - d*x^3)))/(9*d^2) + (8*c*(c + d*x^3)^(1/2))/(3*d^2*
(8*c - d*x^3))
```

3.401 $\int \frac{x^2\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 64

$$\int \frac{x^2\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx=\frac{\sqrt{c+dx^3}}{3d\,(8c-dx^3)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}d}$$

```
[Out] -1/9*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d/c^(1/2)+1/3*(d*x^3+c)^(1/2)/d/(
-d*x^3+8*c)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {455, 43, 65, 212}

$$\int \frac{x^2\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx=\frac{\sqrt{c+dx^3}}{3d\,(8c-dx^3)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}d}$$

```
[In] Int[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] Sqrt[c + d*x^3]/(3*d*(8*c - d*x^3)) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/
(9*Sqrt[c]*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{(8c-dx)^2}\,dx, x, x^3\right)\\
&= \frac{\sqrt{c+dx^3}}{3d\,(8c-dx^3)} - \frac{1}{6}\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= \frac{\sqrt{c+dx^3}}{3d\,(8c-dx^3)} - \frac{\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d}\\
&= \frac{\sqrt{c+dx^3}}{3d\,(8c-dx^3)} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{x^2\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{\frac{3\sqrt{c+dx^3}}{8c-dx^3} - \frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{\sqrt{c}}}{9d}$$

```
[In] Integrate[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] ((3*Sqrt[c + d*x^3])/(8*c - d*x^3) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/S
qrt[c])/(9*d)
```

Maple [A] (verified)

Time = 4.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.77

method	result
default	$\frac{\frac{\sqrt{d x^3+c}}{-d x^3+8c}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{3\sqrt{c}}\right)}{3\sqrt{c}}}{3d}$
pseudoelliptic	$\frac{\frac{\sqrt{d x^3+c}}{-d x^3+8c}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{3\sqrt{c}}\right)}{3\sqrt{c}}}{3d}$
elliptic	$\frac{\sqrt{d x^3+c}}{3d(-d x^3+8c)}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c d^2\right. \cdots}}\cdots}{\cdots}\right.}{\cdots}$

[In] int(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)

[Out] 1/3*((d*x^3+c)^(1/2)/(-d*x^3+8*c)-1/3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))/d

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.33

$$\int \frac{x^2\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \left[\frac{(dx^3-8\,c)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-6\,\sqrt{dx^3+c}c}{18\,(cd^2x^3-8\,c^2d)}, \frac{(dx^3-8\,c)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)-3\,\sqrt{dx^3+c}\cdots}{9\,(cd^2x^3-8\,c^2d)}\right.$$

[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")

[Out] [1/18*((d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - 6*sqrt(d*x^3 + c)*c)/(c*d^2*x^3 - 8*c^2*d), 1/9*((d*x^3 -

```
8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 3*sqrt(d*x^3 + c)*c
)/(c*d^2*x^3 - 8*c^2*d)]
```

Sympy [F]

$$\int \frac{x^2\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{x^2\sqrt{c+dx^3}}{(-8c+dx^3)^2}\,dx$$

```
[In] integrate(x**2*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
[Out] Integral(x**2*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03

$$\int \frac{x^2\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{\frac{\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{\sqrt{c}} - \frac{6\sqrt{dx^3+c}}{dx^3-8c}}{18\,d}$$

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
[Out] 1/18*(log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/sqrt
(c) - 6*sqrt(d*x^3 + c)/(d*x^3 - 8*c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int \frac{x^2\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}d} - \frac{\sqrt{dx^3+c}}{3\,(dx^3-8\,c)d}$$

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
[Out] 1/9*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 1/3*sqrt(d*x^3 + c)
/((d*x^3 - 8*c)*d)
```

Mupad [B] (verification not implemented)

Time = 8.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.12

$$\int \frac{x^2\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{18\,\sqrt{c}\,d}+\frac{\sqrt{d\,x^3+c}}{3\,d\,\left(8\,c-d\,x^3\right)}$$

```
[In] int((x^2*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
```

```
[Out] log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(18*c^(1/2)
*d) + (c + d*x^3)^(1/2)/(3*d*(8*c - d*x^3))
```

3.402 $\int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 88

$$\int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)^2}\,dx=\frac{\sqrt{c+dx^3}}{24c(8c-dx^3)}+\frac{5\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{3/2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}$$

```
[Out] 5/288*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-1/96*arctanh((d*x^3+c)^(
1/2)/c^(1/2))/c^(3/2)+1/24*(d*x^3+c)^(1/2)/c/(-d*x^3+8*c)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {457, 101, 162, 65, 214, 212}

$$\int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)^2}\,dx=\frac{5\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{3/2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}+\frac{\sqrt{c+dx^3}}{24c(8c-dx^3)}$$

```
[In] Int[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)^2),x]
```

```
[Out] Sqrt[c + d*x^3]/(24*c*(8*c - d*x^3)) + (5*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c
])])/(288*c^(3/2)) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(96*c^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x(8c-dx)^2}\,dx, x, x^3\right) \\
&= \frac{\sqrt{c+dx^3}}{24c\,(8c-dx^3)} - \frac{\text{Subst}\left(\int \frac{-c-\frac{dx}{2}}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{24c}
\end{aligned}
$$

$$= \frac{\sqrt{c+dx^3}}{24c(8c-dx^3)} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{192c} + \frac{(5d)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{192c}$$

$$= \frac{\sqrt{c+dx^3}}{24c(8c-dx^3)} + \frac{5\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{96c} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{96cd}$$

$$= \frac{\sqrt{c+dx^3}}{24c(8c-dx^3)} + \frac{5\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)^2}\,dx = \frac{\frac{12\sqrt{c}\sqrt{c+dx^3}}{8c-dx^3} + 5\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 3\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{288c^{3/2}}$$

```
[In] Integrate[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)^2),x]
[Out] ((12*Sqrt[c]*Sqrt[c + d*x^3])/(8*c - d*x^3) + 5*ArcTanh[Sqrt[c + d*x^3]/(3*
Sqrt[c])] - 3*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(288*c^(3/2))
```

Maple [A] (verified)

Time = 4.52 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89

method	result	size
pseudoelliptic	$-\frac{\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{96c^{\frac{3}{2}}}+\frac{\frac{5\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)(d\,x^3-8c)}{\sqrt{c}}-12\sqrt{d\,x^3+c}}{288(d\,x^3-8c)c}$	78
default	$\frac{\frac{2\sqrt{d\,x^3+c}}{3}-\frac{2\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)\sqrt{c}}{3}}{64c^2}+\frac{\frac{\sqrt{d\,x^3+c}}{-d\,x^3+8c}-\frac{\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{3\sqrt{c}}}{24c}+\frac{-2\sqrt{d\,x^3+c}+6\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)\sqrt{c}}{192c^2}$	123
elliptic	Expression too large to display	1534

```
[In] int((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
[Out] -1/96*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)+1/288*(5*arctanh(1/3*(d*x^3+
c)^(1/2)/c^(1/2))/c^(1/2)*(d*x^3-8*c)-12*(d*x^3+c)^(1/2))/(d*x^3-8*c)/c
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.57

$$\int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)^2}\,dx = \left[\frac{5\,(dx^3-8\,c)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+3\,(dx^3-8\,c)\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-24\,\sqrt{dx^3+c}c}{576\,(c^2dx^3-8\,c^3)}, \frac{3\,(dx\ldots}{\ldots}\right.$$

```
[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/576*(5*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10
*c)/(d*x^3 - 8*c)) + 3*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)
*sqrt(c) + 2*c)/x^3) - 24*sqrt(d*x^3 + c)*c)/(c^2*d*x^3 - 8*c^3), 1/288*(3*
(d*x^3 - 8*c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 5*(d*x^3 - 8*c)
*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*sqrt(d*x^3 + c)*c)/(c
^2*d*x^3 - 8*c^3)]
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)^2}\,dx = \int \frac{\sqrt{c+dx^3}}{x(-8c+dx^3)^2}\,dx$$

```
[In] integrate((d*x**3+c)**(1/2)/x/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x*(-8*c + d*x**3)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}}{(dx^3-8\,c)^2x}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{c+dx^3}}{x\left(8c-dx^3\right)^2}\,dx = \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}c} - \frac{5\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{288\sqrt{-c}c} - \frac{\sqrt{dx^3+c}}{24\left(dx^3-8c\right)c}$$

```
[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 1/96*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 5/288*arctan(1/3*sqrt(
d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/24*sqrt(d*x^3 + c)/((d*x^3 - 8*c)*c)
```

Mupad [B] (verification not implemented)

Time = 8.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{c+dx^3}}{x\left(8c-dx^3\right)^2}\,dx = \frac{5\,\mathrm{atanh}\left(\frac{c\sqrt{d\,x^3+c}}{3\sqrt{c^3}}\right)}{288\sqrt{c^3}} - \frac{\mathrm{atanh}\left(\frac{c\sqrt{d\,x^3+c}}{\sqrt{c^3}}\right)}{96\sqrt{c^3}} + \frac{\sqrt{d\,x^3+c}}{8\,c\,\left(24\,c-3\,d\,x^3\right)}$$

```
[In] int((c + d*x^3)^(1/2)/(x*(8*c - d*x^3)^2),x)
```

```
[Out] (5*atanh((c*(c + d*x^3)^(1/2))/(3*(c^3)^(1/2))))/(288*(c^3)^(1/2)) - atanh(
(c*(c + d*x^3)^(1/2))/(c^3)^(1/2))/(96*(c^3)^(1/2)) + (c + d*x^3)^(1/2)/(8*
c*(24*c - 3*d*x^3))
```

3.403 $\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 124

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)^2}\,dx=\frac{d\sqrt{c+dx^3}}{96c^2\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)}+\frac{7d\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1152c^{5/2}}-\frac{d\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{128c^{5/2}}$$

```
[Out] 7/1152*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)-1/128*d*arctanh((d*x^
3+c)^(1/2)/c^(1/2))/c^(5/2)+1/96*d*(d*x^3+c)^(1/2)/c^2/(-d*x^3+8*c)-1/24*(d
*x^3+c)^(1/2)/c/x^3/(-d*x^3+8*c)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {457, 101, 156, 162, 65, 214, 212}

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)^2}\,dx=\frac{7d\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1152c^{5/2}}-\frac{d\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{128c^{5/2}}+\frac{d\sqrt{c+dx^3}}{96c^2\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)}$$

```
[In] Int[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)^2),x]
```

```
[Out] (d*Sqrt[c + d*x^3])/(96*c^2*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(24*c*x^3*(8*c
 - d*x^3)) + (7*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1152*c^(5/2)) - (d
*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(128*c^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(8c-dx)^2}\,dx, x, x^3\right) \\
&= -\frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{6cd+\frac{3d^2x}{2}}{x(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{24c} \\
&= \frac{d\sqrt{c+dx^3}}{96c^2\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{-54c^2d^2-9cd^3x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{1728c^3d} \\
&= \frac{d\sqrt{c+dx^3}}{96c^2\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)} + \frac{d\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{256c^2} \\
&\quad + \frac{\left(7d^2\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{768c^2} \\
&= \frac{d\sqrt{c+dx^3}}{96c^2\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{128c^2} \\
&\quad + \frac{(7d)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{384c^2} \\
&= \frac{d\sqrt{c+dx^3}}{96c^2\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)} + \frac{7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1152c^{5/2}} - \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{128c^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.78

$$
\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)^2}\,dx = \frac{\frac{12\sqrt{c}\left(4c-dx^3\right)\sqrt{c+dx^3}}{-8cx^3+dx^6} + 7d\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 9d\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{1152c^{5/2}}
$$

```
[In] Integrate[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)^2),x]
```

```
[Out] ((12*Sqrt[c]*(4*c - d*x^3)*Sqrt[c + d*x^3])/(-8*c*x^3 + d*x^6) + 7*d*ArcTan
h[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 9*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(115
2*c^(5/2))
```

Maple [A] (verified)

Time = 4.63 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{d\left(-\frac{3\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)d\,x^{3}+2\sqrt{d\,x^{3}+c}\,\sqrt{c}}{2d\,x^{3}c^{\frac{5}{2}}}+\frac{\frac{\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}+\frac{7\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{6\sqrt{c}}}{c^{2}}\right)}{192}$
risch	$-\frac{\sqrt{d\,x^{3}+c}}{192c^{2}x^{3}}-\frac{d\left(\frac{\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{9\sqrt{c}}-\frac{2c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{3}\right)}{128c^{2}}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{3x^{3}}-\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3\sqrt{c}}}{64c^{2}}+\frac{d\left(\frac{2\sqrt{d\,x^{3}+c}}{3}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)\sqrt{c}}{3}\right)}{256c^{3}}+\frac{d\left(\frac{\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}-\frac{\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3\sqrt{c}}\right)}{192c^{2}}+$
elliptic	Expression too large to display

```
[In] int((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/192*d*(-1/2*(3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*d*x^3+2*(d*x^3+c)^(1/2)*c
^(1/2))/d/x^3/c^(5/2)+((d*x^3+c)^(1/2)/(-d*x^3+8*c)+7/6*arctanh(1/3*(d*x^3+
c)^(1/2)/c^(1/2))/c^(1/2))/c^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.24

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)^2}\,dx$$
$$=\left[\frac{7\left(d^2x^6-8\,cdx^3\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+9\left(d^2x^6-8\,cdx^3\right)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-24\left(cdx^3\right.}{2304\left(c^3dx^6-8\,c^4x^3\right)}\right.$$

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/2304*(7*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqr
t(c) + 10*c)/(d*x^3 - 8*c)) + 9*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 -
2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*(c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c
))/(c^3*d*x^6 - 8*c^4*x^3), 1/1152*(9*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*arctan
(sqrt(d*x^3 + c)*sqrt(-c)/c) - 7*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*arctan(1/3*
sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*(c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c))/(c^3*d*
x^6 - 8*c^4*x^3)]
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)^2}\,dx=\int \frac{\sqrt{c+dx^3}}{x^4\left(-8c+dx^3\right)^2}\,dx$$

```
[In] integrate((d*x**3+c)**(1/2)/x**4/(-d*x**3+8*c)**2,x)
[Out] Integral(sqrt(c + d*x**3)/(x**4*(-8*c + d*x**3)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)^2}\,dx=\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)^2x^4}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)^2}\,dx=\frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{128\sqrt{-c}c^2}-\frac{7\,d\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{1152\sqrt{-c}c^2}-\frac{(dx^3+c)^{\frac{3}{2}}d-5\sqrt{dx^3+c}cd}{96\left((dx^3+c)^2-10\,(dx^3+c)c+9\,c^2\right)c^2}$$

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="giac")
[Out] 1/128*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 7/1152*d*arctan(1
/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/96*((d*x^3 + c)^(3/2)*d - 5
*sqrt(d*x^3 + c)*c*d)/(((d*x^3 + c)^2 - 10*(d*x^3 + c)*c + 9*c^2)*c^2)
```

Mupad [B] (verification not implemented)

Time = 8.43 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{c+dx^3}}{x^4\,(8c-dx^3)^2}\,dx = \frac{\frac{5\,d\,\sqrt{d\,x^3+c}}{32\,c}-\frac{d\,{\left(d\,x^3+c\right)}^{3/2}}{32\,c^2}}{3\,{\left(d\,x^3+c\right)}^2-30\,c\,\left(d\,x^3+c\right)+27\,c^2}+\frac{d\,\left(\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{\sqrt{c^5}}\right)\,1{}\mathrm{i}-\frac{\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^5}}\right)\,7{}\mathrm{i}}{9}\right)\,1{}\mathrm{i}}{128\,\sqrt{c^5}}$$

```
[In] int((c + d*x^3)^(1/2)/(x^4*(8*c - d*x^3)^2),x)
```

```
[Out] ((5*d*(c + d*x^3)^(1/2))/(32*c) - (d*(c + d*x^3)^(3/2))/(32*c^2))/(3*(c + d
*x^3)^2 - 30*c*(c + d*x^3) + 27*c^2) + (d*(atanh((c^2*(c + d*x^3)^(1/2))/(c
^5)^(1/2))*1i - (atanh((c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2)))*7i)/9)*1i)/
(128*(c^5)^(1/2))
```

3.404 $\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 164

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)^2}\,dx = \frac{5d^2\sqrt{c+dx^3}}{1536c^3\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)} - \frac{7d\sqrt{c+dx^3}}{384c^2x^3\left(8c-dx^3\right)} + \frac{23d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18432c^{7/2}} - \frac{d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{7/2}}$$

```
[Out] 23/18432*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)-1/2048*d^2*arctan
h((d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)+5/1536*d^2*(d*x^3+c)^(1/2)/c^3/(-d*x^3+8
*c)-1/48*(d*x^3+c)^(1/2)/c/x^6/(-d*x^3+8*c)-7/384*d*(d*x^3+c)^(1/2)/c^2/x^3
/(-d*x^3+8*c)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {457, 101, 156, 162, 65, 214, 212}

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)^2}\,dx = \frac{23d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18432c^{7/2}} - \frac{d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{7/2}} + \frac{5d^2\sqrt{c+dx^3}}{1536c^3\left(8c-dx^3\right)} - \frac{7d\sqrt{c+dx^3}}{384c^2x^3\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)}$$

```
[In] Int[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)^2),x]
```

```
[Out] (5*d^2*Sqrt[c + d*x^3])/(1536*c^3*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(48*c*x^
6*(8*c - d*x^3)) - (7*d*Sqrt[c + d*x^3])/(384*c^2*x^3*(8*c - d*x^3)) + (23*
```

```
d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(18432*c^(7/2)) - (d^2*ArcTanh[Sq
rt[c + d*x^3]/Sqrt[c]])/(2048*c^(7/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^3(8c-dx)^2}\,dx, x, x^3\right) \\
&= -\frac{\sqrt{c+dx^3}}{48cx^6\,(8c-dx^3)} + \frac{\text{Subst}\left(\int \frac{7cd+\frac{5d^2x}{2}}{x^2(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{48c} \\
&= -\frac{\sqrt{c+dx^3}}{48cx^6\,(8c-dx^3)} - \frac{7d\sqrt{c+dx^3}}{384c^2x^3\,(8c-dx^3)} - \frac{\text{Subst}\left(\int \frac{-6c^2d^2-\frac{21}{2}cd^3x}{x(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{384c^3} \\
&= \frac{5d^2\sqrt{c+dx^3}}{1536c^3\,(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{48cx^6\,(8c-dx^3)} \\
&\quad - \frac{7d\sqrt{c+dx^3}}{384c^2x^3\,(8c-dx^3)} + \frac{\text{Subst}\left(\int \frac{54c^3d^3+45c^2d^4x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27648c^5d} \\
&= \frac{5d^2\sqrt{c+dx^3}}{1536c^3\,(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{48cx^6\,(8c-dx^3)} - \frac{7d\sqrt{c+dx^3}}{384c^2x^3\,(8c-dx^3)} \\
&\quad + \frac{d^2\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{4096c^3} + \frac{(23d^3)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{12288c^3} \\
&= \frac{5d^2\sqrt{c+dx^3}}{1536c^3\,(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{48cx^6\,(8c-dx^3)} - \frac{7d\sqrt{c+dx^3}}{384c^2x^3\,(8c-dx^3)} \\
&\quad + \frac{d\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{2048c^3} + \frac{(23d^2)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{6144c^3} \\
&= \frac{5d^2\sqrt{c+dx^3}}{1536c^3\,(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{48cx^6\,(8c-dx^3)} - \frac{7d\sqrt{c+dx^3}}{384c^2x^3\,(8c-dx^3)} \\
&\quad + \frac{23d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18432c^{7/2}} - \frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{7/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)^2}\,dx = \frac{\frac{12\sqrt{c}\sqrt{c+dx^3}\left(32c^2+28cdx^3-5d^2x^6\right)}{-8cx^6+dx^9}+23d^2\mathrm{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-9d^2\mathrm{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{18432c^{7/2}}$$

```
[In] Integrate[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)^2),x]
```

```
[Out] ((12*Sqrt[c]*Sqrt[c + d*x^3]*(32*c^2 + 28*c*d*x^3 - 5*d^2*x^6))/(-8*c*x^6 +
 d*x^9) + 23*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 9*d^2*ArcTanh[Sqrt[
c + d*x^3]/Sqrt[c]])/(18432*c^(7/2))
```

Maple [A] (verified)

Time = 4.64 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{\left(d\,x^{3}+c\right)^{\frac{3}{2}}}{384c^{3}x^{6}}-\frac{d^{2}\left(\frac{\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{8\sqrt{c}}-\frac{19\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{72\sqrt{c}}-\frac{c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{6}\right)}{256c^{3}}$
pseudoelliptic	$-\frac{14\sqrt{d\,x^{3}+c}\,c^{\frac{3}{2}}\,d\,x^{3}+16\sqrt{d\,x^{3}+c}\,c^{\frac{5}{2}}+\frac{\left(-5\sqrt{d\,x^{3}+c}\,\sqrt{c}+\frac{\left(-d\,x^{3}+8c\right)\left(9\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)-23\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\right)}{12}\right)d^{2}x^{6}}{2}}{768c^{\frac{7}{2}}\left(-d\,x^{9}+8c\,x^{6}\right)}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{6x^{6}}-\frac{d\sqrt{d\,x^{3}+c}}{12c\,x^{3}}+\frac{d^{2}\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{12c^{\frac{3}{2}}}}{64c^{2}}+\frac{d\left(-\frac{\sqrt{d\,x^{3}+c}}{3x^{3}}-\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3\sqrt{c}}\right)}{256c^{3}}+\frac{3d^{2}\left(\frac{2\sqrt{d\,x^{3}+c}}{3}-\frac{2\,\mathrm{arctanh}\left(\sqrt{\ldots}\right)}{3\ldots}\right.}{4096c^{4}}$
elliptic	Expression too large to display

```
[In] int((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/384*(d*x^3+c)^(3/2)/c^3/x^6-1/256/c^3*d^2*(1/8*arctanh((d*x^3+c)^(1/2)/c
^(1/2))/c^(1/2)-19/72*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)-1/6*c*(-
(d*x^3+c)^(1/2)/c/(d*x^3-8*c)+1/3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3
/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.89

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)^2}\,dx$$
$$= \left[\frac{23\left(d^3x^9-8\,cd^2x^6\right)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+9\left(d^3x^9-8\,cd^2x^6\right)\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-24\left(5\,cd\right.}{36864\left(c^4dx^9-8\,c^5x^6\right)}\right.$$

```
[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/36864*(23*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)
*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 9*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d*
x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*(5*c*d^2*x^6 - 28*c^2*d*x^
3 - 32*c^3)*sqrt(d*x^3 + c))/(c^4*d*x^9 - 8*c^5*x^6), 1/18432*(9*(d^3*x^9 -
 8*c*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 23*(d^3*x^9 - 8
*c*d^2*x^6)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*(5*c*d^2*x
^6 - 28*c^2*d*x^3 - 32*c^3)*sqrt(d*x^3 + c))/(c^4*d*x^9 - 8*c^5*x^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*x**3+c)**(1/2)/x**7/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)^2}\,dx = \int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)^2x^7}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^7), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)^2}\,dx = \frac{d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{2048\sqrt{-c}c^3} - \frac{23\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{18432\sqrt{-c}c^3} - \frac{\sqrt{dx^3+c}d^2}{1536\,(dx^3-8\,c)c^3} - \frac{(dx^3+c)^{\frac{3}{2}}}{384\,c^3x^6}$$

```
[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 1/2048*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 23/18432*d^2*a
rctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/1536*sqrt(d*x^3 + c)
*d^2/((d*x^3 - 8*c)*c^3) - 1/384*(d*x^3 + c)^(3/2)/(c^3*x^6)
```

Mupad [B] (verification not implemented)

Time = 8.70 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)^2}\,dx = \frac{\frac{d^2\sqrt{d\,x^3+c}}{512\,c} - \frac{19\,d^2\,\left(d\,x^3+c\right)^{3/2}}{256\,c^2} + \frac{5\,d^2\,\left(d\,x^3+c\right)^{5/2}}{512\,c^3}}{33\,c\,\left(d\,x^3+c\right)^2 - 57\,c^2\,\left(d\,x^3+c\right) - 3\,\left(d\,x^3+c\right)^3 + 27\,c^3} + \frac{d^2\left(\mathrm{atanh}\left(\frac{c^3\sqrt{d\,x^3+c}}{\sqrt{c^7}}\right)\,1\mathrm{i} - \frac{\mathrm{atanh}\left(\frac{c^3\sqrt{d\,x^3+c}}{3\sqrt{c^7}}\right)\,23\mathrm{i}}{9}\right)\,1\mathrm{i}}{2048\sqrt{c^7}}$$

```
[In] int((c + d*x^3)^(1/2)/(x^7*(8*c - d*x^3)^2),x)
```

```
[Out] ((d^2*(c + d*x^3)^(1/2))/(512*c) - (19*d^2*(c + d*x^3)^(3/2))/(256*c^2) + (
5*d^2*(c + d*x^3)^(5/2))/(512*c^3))/(33*c*(c + d*x^3)^2 - 57*c^2*(c + d*x^3
) - 3*(c + d*x^3)^3 + 27*c^3) + (d^2*(atanh((c^3*(c + d*x^3)^(1/2))/(c^7)^(
1/2))*1i - (atanh((c^3*(c + d*x^3)^(1/2))/(3*(c^7)^(1/2)))*23i)/9)*1i)/(204
8*(c^7)^(1/2))
```

3.405 $\int \frac{x^7\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 663

$$\int \frac{x^7\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{13x^2\sqrt{c+dx^3}}{21d^2}+\frac{746c\sqrt{c+dx^3}}{21d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^5\sqrt{c+dx^3}}{3d\,(8c-dx^3)}+\frac{76c^{7/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{8/3}} - \frac{76c^{7/6}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{8/3}}+\frac{76c^{7/6}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^{8/3}} - \frac{373\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7\ 3^{3/4}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{746\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{21\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] -76/9*c^(7/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^
(8/3)+76/9*c^(7/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(8/3)+76/9*c^(7/6
```

```
)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/d^(8/3)*3^(1/
2)+13/21*x^2*(d*x^3+c)^(1/2)/d^2+1/3*x^5*(d*x^3+c)^(1/2)/d/(-d*x^3+8*c)+746
/21*c*(d*x^3+c)^(1/2)/d^(8/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+746/63*c^(4/3
)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+
c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(
2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^(8/3)/(d*x^3+c
)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/
2)-373/21*3^(1/4)*c^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*
(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/
2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3
^(1/2)))^2)^(1/2)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(
1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 663, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {478, 596, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{x^7\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$$

$$= \frac{746\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{21\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{373\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{7\ 3^{3/4}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+\frac{76c^{7/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{8/3}}-\frac{76c^{7/6}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{8/3}}$$
$$+\frac{76c^{7/6}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^{8/3}}+\frac{746c\sqrt{c+dx^3}}{21d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$
$$+\frac{13x^2\sqrt{c+dx^3}}{21d^2}+\frac{x^5\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)}$$

```
[In] Int[(x^7*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (13*x^2*Sqrt[c + d*x^3])/(21*d^2) + (746*c*Sqrt[c + d*x^3])/(21*d^(8/3)*((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (x^5*Sqrt[c + d*x^3])/(3*d*(8*c - d*x^3
)) + (76*c^(7/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*
x^3]])/(3*Sqrt[3]*d^(8/3)) - (76*c^(7/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3
*c^(1/6)*Sqrt[c + d*x^3])])/(9*d^(8/3)) + (76*c^(7/6)*ArcTanh[Sqrt[c + d*x^
3]/(3*Sqrt[c])])/(9*d^(8/3)) - (373*Sqrt[2 - Sqrt[3]]*c^(4/3)*(c^(1/3) + d^
(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^
(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(3/4)*d^(8/3)*S
qrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
Sqrt[c + d*x^3]) + (746*Sqrt[2]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)
 - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(21*3^(1/4)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3)
 + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^5\sqrt{c+dx^3}}{3d\,(8c-dx^3)} - \frac{\int \frac{x^4\left(5c+\frac{13dx^3}{2}\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{3d} \\
&= \frac{13x^2\sqrt{c+dx^3}}{21d^2} + \frac{x^5\sqrt{c+dx^3}}{3d\,(8c-dx^3)} - \frac{2\int \frac{x\left(104c^2d+\frac{373}{2}cd^2x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{21d^3} \\
&= \frac{13x^2\sqrt{c+dx^3}}{21d^2} + \frac{x^5\sqrt{c+dx^3}}{3d\,(8c-dx^3)} - \frac{2\int \left(-\frac{373cdx}{2\sqrt{c+dx^3}}+\frac{1596c^2dx}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{21d^3} \\
&= \frac{13x^2\sqrt{c+dx^3}}{21d^2} + \frac{x^5\sqrt{c+dx^3}}{3d\,(8c-dx^3)} + \frac{(373c)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{21d^2} - \frac{(152c^2)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{d^2}
\end{aligned}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d^2} + \frac{x^5\sqrt{c+dx^3}}{3d(8c-dx^3)} + \frac{(38c)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{3d^3}$$

$$+ \frac{(373c)\int \frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{21d^{7/3}} - \frac{\left(38c^{4/3}\right)\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{3d^{7/3}}$$

$$- \frac{\left(373(1-\sqrt{3})\,c^{4/3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{21d^{7/3}} + \frac{\left(38c^{5/3}\right)\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{d^{5/3}}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d^2} + \frac{746c\sqrt{c+dx^3}}{21d^{8/3}\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^5\sqrt{c+dx^3}}{3d(8c-dx^3)}$$

$$- \frac{373\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{(1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7\ 3^{3/4}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{746\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{(1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{21\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$- \frac{\left(76c^{5/3}\right)\text{Subst}\left(\int \frac{1}{9-cx^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{3d^{8/3}}$$

$$+ \frac{\left(38c^{5/3}\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3d^{5/3}}$$

$$- \frac{\left(152c^{2/3}\right)\text{Subst}\left(\int \frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{3d^{2/3}}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d^2} + \frac{746c\sqrt{c+dx^3}}{21d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^5\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)}$$

$$+ \frac{76c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{8/3}} - \frac{76c^{7/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{8/3}}$$

$$- \frac{373\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7\ 3^{3/4}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{746\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{21\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{\left(76c^{5/3}\right)\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{3d^{8/3}}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d^2} + \frac{746c\sqrt{c+dx^3}}{21d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^5\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{76c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{8/3}} - \frac{76c^{7/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{8/3}} + \frac{76c^{7/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^{8/3}} - \frac{373\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7\ 3^{3/4}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{746\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{21\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 7.09 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.27

$$\int \frac{x^7\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx = -\frac{80(52c^2x^2+49cdx^5-3d^2x^8)+520cx^2(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+373dx^5}{840d^2\left(-8c+dx^3\right)\sqrt{c+dx^3}}$$

```
[In] Integrate[(x^7*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] -1/840*(80*(52*c^2*x^2 + 49*c*d*x^5 - 3*d^2*x^8) + 520*c*x^2*(-8*c + d*x^3)
*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)
] + 373*d*x^5*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3,
 -((d*x^3)/c), (d*x^3)/(8*c)])/(d^2*(-8*c + d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.54 (sec) , antiderivative size = 897, normalized size of antiderivative = 1.35

method	result	size
elliptic	Expression too large to display	897
risch	Expression too large to display	1758
default	Expression too large to display	2199

```
[In] int(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 8/3*x^2*c/d^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+2/7*x^2*(d*x^3+c)^(1/2)/d^2-746/
63*I/d^3*c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+152/27*I*c/d^5*2^(1/2)
*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(
-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2
)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^
2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^
(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^
(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18
/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I
*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf
(_Z^3*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 8.83 (sec) , antiderivative size = 2568, normalized size of antiderivative = 3.87

$$\int \frac{x^7\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] -1/189*(6714*(c*d*x^3 - 8*c^2)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstra
ssPInverse(0, -4*c/d, x)) + 133*(d^4*x^3 - 8*c*d^3 - sqrt(-3)*(d^4*x^3 - 8*
c*d^3))*(c^7/d^16)^(1/6)*log(2535525376/3*((d^16*x^9 + 318*c*d^15*x^6 + 120
0*c^2*d^14*x^3 + 640*c^3*d^13 + sqrt(-3)*(d^16*x^9 + 318*c*d^15*x^6 + 1200*
c^2*d^14*x^3 + 640*c^3*d^13))*(c^7/d^16)^(5/6) + 6*(2*c^6*d^2*x^7 + 160*c^7
*d*x^4 + 320*c^8*x - 6*(5*c^2*d^12*x^5 + 32*c^3*d^11*x^2 - sqrt(-3)*(5*c^2*
d^12*x^5 + 32*c^3*d^11*x^2))*(c^7/d^16)^(2/3) - (7*c^4*d^7*x^6 + 152*c^5*d^
6*x^3 + 64*c^6*d^5 + sqrt(-3)*(7*c^4*d^7*x^6 + 152*c^5*d^6*x^3 + 64*c^6*d^5
))*(c^7/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^3*d^10*x^7 + 64*c^4*d^9*x^4
+ 32*c^5*d^8*x)*sqrt(c^7/d^16) + 18*(c^5*d^5*x^8 + 38*c^6*d^4*x^5 + 64*c^7*
d^3*x^2 - sqrt(-3)*(c^5*d^5*x^8 + 38*c^6*d^4*x^5 + 64*c^7*d^3*x^2))*(c^7/d^
16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 133*(d^4*x
^3 - 8*c*d^3 - sqrt(-3)*(d^4*x^3 - 8*c*d^3))*(c^7/d^16)^(1/6)*log(-25355253
76/3*((d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13 + sqrt(
-3)*(d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c^7/d^
16)^(5/6) - 6*(2*c^6*d^2*x^7 + 160*c^7*d*x^4 + 320*c^8*x - 6*(5*c^2*d^12*x^
5 + 32*c^3*d^11*x^2 - sqrt(-3)*(5*c^2*d^12*x^5 + 32*c^3*d^11*x^2))*(c^7/d^1
6)^(2/3) - (7*c^4*d^7*x^6 + 152*c^5*d^6*x^3 + 64*c^6*d^5 + sqrt(-3)*(7*c^4*
d^7*x^6 + 152*c^5*d^6*x^3 + 64*c^6*d^5))*(c^7/d^16)^(1/3))*sqrt(d*x^3 + c)
- 36*(5*c^3*d^10*x^7 + 64*c^4*d^9*x^4 + 32*c^5*d^8*x)*sqrt(c^7/d^16) + 18*(
c^5*d^5*x^8 + 38*c^6*d^4*x^5 + 64*c^7*d^3*x^2 - sqrt(-3)*(c^5*d^5*x^8 + 38*
c^6*d^4*x^5 + 64*c^7*d^3*x^2))*(c^7/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 +
192*c^2*d*x^3 - 512*c^3)) + 133*(d^4*x^3 - 8*c*d^3 + sqrt(-3)*(d^4*x^3 - 8*
c*d^3))*(c^7/d^16)^(1/6)*log(2535525376/3*((d^16*x^9 + 318*c*d^15*x^6 + 120
0*c^2*d^14*x^3 + 640*c^3*d^13 - sqrt(-3)*(d^16*x^9 + 318*c*d^15*x^6 + 1200*
c^2*d^14*x^3 + 640*c^3*d^13))*(c^7/d^16)^(5/6) + 6*(2*c^6*d^2*x^7 + 160*c^7
*d*x^4 + 320*c^8*x - 6*(5*c^2*d^12*x^5 + 32*c^3*d^11*x^2 + sqrt(-3)*(5*c^2*
d^12*x^5 + 32*c^3*d^11*x^2))*(c^7/d^16)^(2/3) - (7*c^4*d^7*x^6 + 152*c^5*d^
6*x^3 + 64*c^6*d^5 - sqrt(-3)*(7*c^4*d^7*x^6 + 152*c^5*d^6*x^3 + 64*c^6*d^5
))*(c^7/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^3*d^10*x^7 + 64*c^4*d^9*x^4
+ 32*c^5*d^8*x)*sqrt(c^7/d^16) + 18*(c^5*d^5*x^8 + 38*c^6*d^4*x^5 + 64*c^7*
d^3*x^2 + sqrt(-3)*(c^5*d^5*x^8 + 38*c^6*d^4*x^5 + 64*c^7*d^3*x^2))*(c^7/d^
16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 133*(d^4*x
^3 - 8*c*d^3 + sqrt(-3)*(d^4*x^3 - 8*c*d^3))*(c^7/d^16)^(1/6)*log(-25355253
76/3*((d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13 - sqrt(
```

```
-3)*(d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c^7/d^
16)^(5/6) - 6*(2*c^6*d^2*x^7 + 160*c^7*d*x^4 + 320*c^8*x - 6*(5*c^2*d^12*x^
5 + 32*c^3*d^11*x^2 + sqrt(-3)*(5*c^2*d^12*x^5 + 32*c^3*d^11*x^2))*(c^7/d^1
6)^(2/3) - (7*c^4*d^7*x^6 + 152*c^5*d^6*x^3 + 64*c^6*d^5 - sqrt(-3)*(7*c^4*
d^7*x^6 + 152*c^5*d^6*x^3 + 64*c^6*d^5))*(c^7/d^16)^(1/3))*sqrt(d*x^3 + c)
- 36*(5*c^3*d^10*x^7 + 64*c^4*d^9*x^4 + 32*c^5*d^8*x)*sqrt(c^7/d^16) + 18*(
c^5*d^5*x^8 + 38*c^6*d^4*x^5 + 64*c^7*d^3*x^2 + sqrt(-3)*(c^5*d^5*x^8 + 38*
c^6*d^4*x^5 + 64*c^7*d^3*x^2))*(c^7/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 +
192*c^2*d*x^3 - 512*c^3)) + 266*(d^4*x^3 - 8*c*d^3)*(c^7/d^16)^(1/6)*log(25
35525376/3*((d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13)*
(c^7/d^16)^(5/6) + 6*(c^6*d^2*x^7 + 80*c^7*d*x^4 + 160*c^8*x + 6*(5*c^2*d^1
2*x^5 + 32*c^3*d^11*x^2)*(c^7/d^16)^(2/3) + (7*c^4*d^7*x^6 + 152*c^5*d^6*x^
3 + 64*c^6*d^5)*(c^7/d^16)^(1/3))*sqrt(d*x^3 + c) + 18*(5*c^3*d^10*x^7 + 64
*c^4*d^9*x^4 + 32*c^5*d^8*x)*sqrt(c^7/d^16) + 18*(c^5*d^5*x^8 + 38*c^6*d^4*
x^5 + 64*c^7*d^3*x^2)*(c^7/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d
*x^3 - 512*c^3)) - 266*(d^4*x^3 - 8*c*d^3)*(c^7/d^16)^(1/6)*log(-2535525376
/3*((d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13)*(c^7/d^1
6)^(5/6) - 6*(c^6*d^2*x^7 + 80*c^7*d*x^4 + 160*c^8*x + 6*(5*c^2*d^12*x^5 +
32*c^3*d^11*x^2)*(c^7/d^16)^(2/3) + (7*c^4*d^7*x^6 + 152*c^5*d^6*x^3 + 64*c
^6*d^5)*(c^7/d^16)^(1/3))*sqrt(d*x^3 + c) + 18*(5*c^3*d^10*x^7 + 64*c^4*d^9
*x^4 + 32*c^5*d^8*x)*sqrt(c^7/d^16) + 18*(c^5*d^5*x^8 + 38*c^6*d^4*x^5 + 64
*c^7*d^3*x^2)*(c^7/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 5
12*c^3)) - 18*(3*d^2*x^5 - 52*c*d*x^2)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^3)
```

Sympy [F]

$$\int \frac{x^7\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{x^7\sqrt{c+dx^3}}{(-8c+dx^3)^2}\,dx$$

```
[In] integrate(x**7*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
[Out] Integral(x**7*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

Maxima [F]

$$\int \frac{x^7\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}\,x^7}{(dx^3-8\,c)^2}\,dx$$

```
[In] integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)*x^7/(d*x^3 - 8*c)^2, x)
```

Giac [F]

$$\int \frac{x^7\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}x^7}{(dx^3-8\,c)^2}\,dx$$

```
[In] integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)*x^7/(d*x^3 - 8*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{x^7\,\sqrt{d\,x^3+c}}{(8\,c-d\,x^3)^2}\,dx$$

```
[In] int((x^7*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
[Out] int((x^7*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2, x)
```

3.406 $\int \frac{x^4\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 641

$$\int \frac{x^4\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{7\sqrt{c+dx^3}}{3d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{x^2\sqrt{c+dx^3}}{3d\,(8c-dx^3)}+\frac{5\sqrt[6]{c}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{5/3}} - \frac{5\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{5/3}}+\frac{5\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^{5/3}} - \frac{7\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{2\ 3^{3/4}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{7\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] -5/9*c^(1/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(
5/3)+5/9*c^(1/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(5/3)+5/9*c^(1/6)*a
```

```
rctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/d^(5/3)*3^(1/2)+
1/3*x^2*(d*x^3+c)^(1/2)/d/(-d*x^3+8*c)+7/3*(d*x^3+c)^(1/2)/d^(5/3)/(d^(1/3)
*x+c^(1/3)*(1+3^(1/2)))+7/9*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*
x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/
2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))
^2)^(1/2)*3^(3/4)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(
1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-7/6*3^(1/4)*c^(1/3)*(c^(1/3)+d^(1/3)*x
)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))
,I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/
3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(5/3)/(d*x^3+c)^(1/2)/(c
^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {478, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{x^4\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$$
$$= \frac{7\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{7\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{2\ 3^{3/4}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+\frac{5\sqrt[6]{c}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{5/3}}-\frac{5\sqrt[6]{c}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{5/3}}$$
$$+\frac{5\sqrt[6]{c}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^{5/3}}+\frac{7\sqrt{c+dx^3}}{3d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)}$$

```
[In] Int[(x^4*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (7*Sqrt[c + d*x^3])/(3*d^(5/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (x^2*
Sqrt[c + d*x^3])/(3*d*(8*c - d*x^3)) + (5*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(
```

```
c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(3*Sqrt[3]*d^(5/3)) - (5*c^(1/6)*Ar
cTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(9*d^(5/3)) + (
5*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^(5/3)) - (7*Sqrt[2 - S
qrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d
^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - S
qrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*S
qrt[3]])/(2*3^(3/4)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (7*Sqrt[2]*c^(1/3)*(c^(1/3)
 + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3
)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*d^(5
/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x
)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
```

```
/; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} - \frac{\int \frac{x\left(2c+\frac{7dx^3}{2}\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{3d} \\
&= \frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} - \frac{\int \left(-\frac{7x}{2\sqrt{c+dx^3}} + \frac{30cx}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{3d} \\
&= \frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{7\int \frac{x}{\sqrt{c+dx^3}}\,dx}{6d} - \frac{(10c)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{d} \\
&= \frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{5\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{6d^2} \\
&\quad + \frac{7\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{6d^{4/3}} - \frac{\left(5\sqrt[3]{c}\right)\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{6d^{4/3}} \\
&\quad - \frac{\left(7\left(1-\sqrt{3}\right)\sqrt[3]{c}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{6d^{4/3}} + \frac{\left(5c^{2/3}\right)\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{2d^{2/3}}
\end{aligned}$$

$$= \frac{7\sqrt{c+dx^3}}{3d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)}$$

$$- \frac{7\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{2\ 3^{3/4}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{7\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$- \frac{\left(5c^{2/3}\right)\text{Subst}\left(\int \frac{1}{9-cx^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{3d^{5/3}}$$

$$+ \frac{\left(5c^{2/3}\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{6d^{2/3}}$$

$$- \frac{\left(10\sqrt[3]{d}\right)\text{Subst}\left(\int \frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{3\sqrt[3]{c}}$$

$$=\frac{7\sqrt{c+dx^3}}{3d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)}+\frac{5\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{5/3}}-\frac{5\sqrt[6]{c}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{5/3}}-\frac{7\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{2\,3^{3/4}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{7\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{\left(5c^{2/3}\right)\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{3d^{5/3}}$$

$$=\frac{7\sqrt{c+dx^3}}{3d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)}+\frac{5\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{5/3}}-\frac{5\sqrt[6]{c}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{5/3}}+\frac{5\sqrt[6]{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^{5/3}}-\frac{7\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{2\,3^{3/4}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{7\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.26

$$\int \frac{x^4\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$$
$$= \frac{80cx^2(c+dx^3)+10cx^2(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+7dx^5(-8c+dx^3)\sqrt{1+\cdots}}{240cd\,(8c-dx^3)\sqrt{c+dx^3}}$$

```
[In] Integrate[(x^4*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (80*c*x^2*(c + d*x^3) + 10*c*x^2*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF
1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 7*d*x^5*(-8*c + d*x^3)*S
qrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])
/(240*c*d*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.57 (sec) , antiderivative size = 877, normalized size of antiderivative = 1.37

method	result	size
elliptic	Expression too large to display	877
default	Expression too large to display	1741

```
[In] int(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^2*(d*x^3+c)^(1/2)/d/(-d*x^3+8*c)-7/9*I/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*
(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c
*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d
^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(
I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2)))+10/27*I/d^4*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2
*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d
*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)
*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/
```

```
3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*
d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticP
i(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*
d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c
*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.73 (sec) , antiderivative size = 2397, normalized size of antiderivative = 3.74

$$\int \frac{x^4\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] -1/108*(36*sqrt(d*x^3 + c)*d*x^2 + 252*(d*x^3 - 8*c)*sqrt(d)*weierstrassZet
a(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 5*(d^3*x^3 - 8*c*d^2 - sq
rt(-3)*(d^3*x^3 - 8*c*d^2))*(c/d^10)^(1/6)*log(3125/3*((d^11*x^9 + 318*c*d^
10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 + sqrt(-3)*(d^11*x^9 + 318*c*d^10*x
^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8))*(c/d^10)^(5/6) + 6*(2*c*d^2*x^7 + 160
*c^2*d*x^4 + 320*c^3*x - 6*(5*c*d^8*x^5 + 32*c^2*d^7*x^2 - sqrt(-3)*(5*c*d^
8*x^5 + 32*c^2*d^7*x^2))*(c/d^10)^(2/3) - (7*c*d^5*x^6 + 152*c^2*d^4*x^3 +
64*c^3*d^3 + sqrt(-3)*(7*c*d^5*x^6 + 152*c^2*d^4*x^3 + 64*c^3*d^3))*(c/d^10
)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c*d^7*x^7 + 64*c^2*d^6*x^4 + 32*c^3*d^5*x)
*sqrt(c/d^10) + 18*(c*d^4*x^8 + 38*c^2*d^3*x^5 + 64*c^3*d^2*x^2 - sqrt(-3)*
(c*d^4*x^8 + 38*c^2*d^3*x^5 + 64*c^3*d^2*x^2))*(c/d^10)^(1/6))/(d^3*x^9 - 2
4*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 5*(d^3*x^3 - 8*c*d^2 - sqrt(-3)*(
d^3*x^3 - 8*c*d^2))*(c/d^10)^(1/6)*log(-3125/3*((d^11*x^9 + 318*c*d^10*x^6
+ 1200*c^2*d^9*x^3 + 640*c^3*d^8 + sqrt(-3)*(d^11*x^9 + 318*c*d^10*x^6 + 12
00*c^2*d^9*x^3 + 640*c^3*d^8))*(c/d^10)^(5/6) - 6*(2*c*d^2*x^7 + 160*c^2*d*
x^4 + 320*c^3*x - 6*(5*c*d^8*x^5 + 32*c^2*d^7*x^2 - sqrt(-3)*(5*c*d^8*x^5 +
 32*c^2*d^7*x^2))*(c/d^10)^(2/3) - (7*c*d^5*x^6 + 152*c^2*d^4*x^3 + 64*c^3*
d^3 + sqrt(-3)*(7*c*d^5*x^6 + 152*c^2*d^4*x^3 + 64*c^3*d^3))*(c/d^10)^(1/3)
)*sqrt(d*x^3 + c) - 36*(5*c*d^7*x^7 + 64*c^2*d^6*x^4 + 32*c^3*d^5*x)*sqrt(c
/d^10) + 18*(c*d^4*x^8 + 38*c^2*d^3*x^5 + 64*c^3*d^2*x^2 - sqrt(-3)*(c*d^4*
x^8 + 38*c^2*d^3*x^5 + 64*c^3*d^2*x^2))*(c/d^10)^(1/6))/(d^3*x^9 - 24*c*d^2
*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 5*(d^3*x^3 - 8*c*d^2 + sqrt(-3)*(d^3*x^3
 - 8*c*d^2))*(c/d^10)^(1/6)*log(3125/3*((d^11*x^9 + 318*c*d^10*x^6 + 1200*c
^2*d^9*x^3 + 640*c^3*d^8 - sqrt(-3)*(d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d
^9*x^3 + 640*c^3*d^8))*(c/d^10)^(5/6) + 6*(2*c*d^2*x^7 + 160*c^2*d*x^4 + 32
0*c^3*x - 6*(5*c*d^8*x^5 + 32*c^2*d^7*x^2 + sqrt(-3)*(5*c*d^8*x^5 + 32*c^2*
```

```
d^7*x^2))*(c/d^10)^(2/3) - (7*c*d^5*x^6 + 152*c^2*d^4*x^3 + 64*c^3*d^3 - sq
rt(-3)*(7*c*d^5*x^6 + 152*c^2*d^4*x^3 + 64*c^3*d^3))*(c/d^10)^(1/3))*sqrt(d
*x^3 + c) - 36*(5*c*d^7*x^7 + 64*c^2*d^6*x^4 + 32*c^3*d^5*x)*sqrt(c/d^10) +
 18*(c*d^4*x^8 + 38*c^2*d^3*x^5 + 64*c^3*d^2*x^2 + sqrt(-3)*(c*d^4*x^8 + 38
*c^2*d^3*x^5 + 64*c^3*d^2*x^2))*(c/d^10)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 1
92*c^2*d*x^3 - 512*c^3)) - 5*(d^3*x^3 - 8*c*d^2 + sqrt(-3)*(d^3*x^3 - 8*c*d
^2))*(c/d^10)^(1/6)*log(-3125/3*((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*
x^3 + 640*c^3*d^8 - sqrt(-3)*(d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3
+ 640*c^3*d^8))*(c/d^10)^(5/6) - 6*(2*c*d^2*x^7 + 160*c^2*d*x^4 + 320*c^3*x
 - 6*(5*c*d^8*x^5 + 32*c^2*d^7*x^2 + sqrt(-3)*(5*c*d^8*x^5 + 32*c^2*d^7*x^2
))*(c/d^10)^(2/3) - (7*c*d^5*x^6 + 152*c^2*d^4*x^3 + 64*c^3*d^3 - sqrt(-3)*
(7*c*d^5*x^6 + 152*c^2*d^4*x^3 + 64*c^3*d^3))*(c/d^10)^(1/3))*sqrt(d*x^3 +
c) - 36*(5*c*d^7*x^7 + 64*c^2*d^6*x^4 + 32*c^3*d^5*x)*sqrt(c/d^10) + 18*(c*
d^4*x^8 + 38*c^2*d^3*x^5 + 64*c^3*d^2*x^2 + sqrt(-3)*(c*d^4*x^8 + 38*c^2*d^
3*x^5 + 64*c^3*d^2*x^2))*(c/d^10)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*
d*x^3 - 512*c^3)) + 10*(d^3*x^3 - 8*c*d^2)*(c/d^10)^(1/6)*log(3125/3*((d^11
*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8)*(c/d^10)^(5/6) + 6*
(c*d^2*x^7 + 80*c^2*d*x^4 + 160*c^3*x + 6*(5*c*d^8*x^5 + 32*c^2*d^7*x^2)*(c
/d^10)^(2/3) + (7*c*d^5*x^6 + 152*c^2*d^4*x^3 + 64*c^3*d^3)*(c/d^10)^(1/3))
*sqrt(d*x^3 + c) + 18*(5*c*d^7*x^7 + 64*c^2*d^6*x^4 + 32*c^3*d^5*x)*sqrt(c/
d^10) + 18*(c*d^4*x^8 + 38*c^2*d^3*x^5 + 64*c^3*d^2*x^2)*(c/d^10)^(1/6))/(d
^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 10*(d^3*x^3 - 8*c*d^2)*
(c/d^10)^(1/6)*log(-3125/3*((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 +
 640*c^3*d^8)*(c/d^10)^(5/6) - 6*(c*d^2*x^7 + 80*c^2*d*x^4 + 160*c^3*x + 6*
(5*c*d^8*x^5 + 32*c^2*d^7*x^2)*(c/d^10)^(2/3) + (7*c*d^5*x^6 + 152*c^2*d^4*
x^3 + 64*c^3*d^3)*(c/d^10)^(1/3))*sqrt(d*x^3 + c) + 18*(5*c*d^7*x^7 + 64*c^
2*d^6*x^4 + 32*c^3*d^5*x)*sqrt(c/d^10) + 18*(c*d^4*x^8 + 38*c^2*d^3*x^5 + 6
4*c^3*d^2*x^2)*(c/d^10)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 51
2*c^3)))/(d^3*x^3 - 8*c*d^2)
```

Sympy [F]

$$\int \frac{x^4\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx=\int \frac{x^4\sqrt{c+dx^3}}{\left(-8c+dx^3\right)^2}\,dx$$

```
[In] integrate(x**4*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(x**4*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

Maxima [F]

$$\int \frac{x^4\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}x^4}{(dx^3-8\,c)^2}\,dx$$

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)*x^4/(d*x^3 - 8*c)^2, x)
```

Giac [F]

$$\int \frac{x^4\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}x^4}{(dx^3-8\,c)^2}\,dx$$

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)*x^4/(d*x^3 - 8*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{x^4\,\sqrt{d\,x^3+c}}{(8\,c-d\,x^3)^2}\,dx$$

```
[In] int((x^4*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
[Out] int((x^4*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2, x)
```

3.407 $\int \frac{x\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 644

$$\int \frac{x\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \frac{\sqrt{c+dx^3}}{24cd^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{x^2\sqrt{c+dx^3}}{24c(8c-dx^3)}$$
$$+\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}c^{5/6}d^{2/3}}-\frac{\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{144c^{5/6}d^{2/3}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{144c^{5/6}d^{2/3}}$$
$$-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{16\ 3^{3/4}c^{2/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{12\sqrt{2}\sqrt[4]{3}c^{2/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] -1/144*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(5/6)/d
^(2/3)+1/144*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/6)/d^(2/3)+1/144*arc
tan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(5/6)/d^(2/3)*3^
(1/2)+1/24*x^2*(d*x^3+c)^(1/2)/c/(-d*x^3+8*c)+1/24*(d*x^3+c)^(1/2)/c/d^(2/3
)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/72*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/
3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((
```

```
c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(
1/2)*3^(3/4)/c^(2/3)/d^(2/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1
/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/48*(c^(1/3)+d^(1/3)*x)*El
lipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3
^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x
^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(2/3)/d^(2/3)/(d*x^3
+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(
1/2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 644, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {480, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{x\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$$

$$= \frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{12\sqrt{2}\sqrt[4]{3}c^{2/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{16\ 3^{3/4}c^{2/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}c^{5/6}d^{2/3}}-\frac{\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{144c^{5/6}d^{2/3}}$$
$$+\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{144c^{5/6}d^{2/3}}+\frac{\sqrt{c+dx^3}}{24cd^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{x^2\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)}$$

```
[In] Int[(x*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] Sqrt[c + d*x^3]/(24*c*d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (x^2*S
qrt[c + d*x^3])/(24*c*(8*c - d*x^3)) + ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d
^(1/3)*x))/Sqrt[c + d*x^3]]/(48*Sqrt[3]*c^(5/6)*d^(2/3)) - ArcTanh[(c^(1/3)
 + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]/(144*c^(5/6)*d^(2/3)) + ArcTan
```

```
h[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(144*c^(5/6)*d^(2/3)) - (Sqrt[2 - Sqrt[3]]*(
c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) +
 d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*3^(3
/4)*c^(2/3)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(
1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + ((c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3
) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]
*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(12*Sqrt[2]*3^(1/4)*c^(2/3)*d^(2/3)*Sqrt[
(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt
[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 480

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p
 + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\text{integral} = \frac{x^2\sqrt{c+dx^3}}{24c(8c-dx^3)} - \frac{\int \frac{x\left(-c+\frac{dx^3}{2}\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{24c}$$

$$= \frac{x^2\sqrt{c+dx^3}}{24c(8c-dx^3)} - \frac{\int \left(-\frac{x}{2\sqrt{c+dx^3}} + \frac{3cx}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{24c}$$

$$= \frac{x^2\sqrt{c+dx^3}}{24c(8c-dx^3)} - \frac{1}{8}\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx + \frac{\int \frac{x}{\sqrt{c+dx^3}}\,dx}{48c}$$

$$= \frac{x^2\sqrt{c+dx^3}}{24c(8c-dx^3)} + \frac{\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{96cd} + \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{48c\sqrt[3]{d}}$$

$$- \frac{\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{96c^{2/3}\sqrt[3]{d}} - \frac{\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{48c^{2/3}\sqrt[3]{d}} + \frac{\sqrt[3]{d}\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{32\sqrt[3]{c}}$$

$$= \frac{\sqrt{c+dx^3}}{24cd^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{x^2\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)}$$

$$-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{16\,3^{3/4}c^{2/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{12\sqrt{2}\sqrt[4]{3}c^{2/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{\text{Subst}\left(\int\frac{1}{9-cx^2}\,dx,x,\frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{48\sqrt[3]{c}d^{2/3}}+\frac{\sqrt[3]{d}\text{Subst}\left(\int\frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{96\sqrt[3]{c}}$$

$$-\frac{d^{4/3}\text{Subst}\left(\int\frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{24c^{4/3}}$$

$$= \frac{\sqrt{c+dx^3}}{24cd^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^2\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}c^{5/6}d^{2/3}} - \frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{144c^{5/6}d^{2/3}} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{16\,3^{3/4}c^{2/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{12\sqrt{2}\sqrt[4]{3}c^{2/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{48\sqrt[3]{c}d^{2/3}}$$

$$= \frac{\sqrt{c+dx^3}}{24cd^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^2\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}c^{5/6}d^{2/3}} - \frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{144c^{5/6}d^{2/3}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{144c^{5/6}d^{2/3}} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{16\,3^{3/4}c^{2/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{12\sqrt{2}\sqrt[4]{3}c^{2/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.25

$$\int \frac{x\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx$$
$$= \frac{80cx^2(c+dx^3)+5cx^2(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+dx^5(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{Ap}}{1920c^2(8c-dx^3)\sqrt{c+dx^3}}$$

```
[In] Integrate[(x*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (80*c*x^2*(c + d*x^3) + 5*c*x^2*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[
2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(-8*c + d*x^3)*Sqrt[
1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(19
20*c^2*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.49 (sec) , antiderivative size = 883, normalized size of antiderivative = 1.37

method	result	size
default	Expression too large to display	883
elliptic	Expression too large to display	883

```
[In] int(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*x^2*(d*x^3+c)^(1/2)/c/(-d*x^3+8*c)-1/72*I/c*3^(1/2)/d*(-c*d^2)^(1/3)*(
I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c
*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3)))^(1/2)))+1/216*I/d^3/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*
d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2
)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(
1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)
```

```
^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*
(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellip
ticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alph
a^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha
-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.40 (sec) , antiderivative size = 2496, normalized size of antiderivative = 3.88

$$\int \frac{x\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] -1/1728*(72*sqrt(d*x^3 + c)*d*x^2 + 72*(d*x^3 - 8*c)*sqrt(d)*weierstrassZet
a(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + (c*d^2*x^3 - 8*c^2*d + sq
rt(-3)*(c*d^2*x^3 - 8*c^2*d))*(1/(c^5*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*
x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 32*c^6
*d^3*x + sqrt(-3)*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 32*c^6*d^3*x))*(1/(c^5*
d^4))^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^5*d^5*x^5 + 32*c^6*d^4*x^2 - sqrt(-
3)*(5*c^5*d^5*x^5 + 32*c^6*d^4*x^2))*(1/(c^5*d^4))^(5/6) - 2*(7*c^3*d^4*x^6
 + 152*c^4*d^3*x^3 + 64*c^5*d^2)*sqrt(1/(c^5*d^4)) + (c*d^3*x^7 + 80*c^2*d^
2*x^4 + 160*c^3*d*x + sqrt(-3)*(c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x))*
(1/(c^5*d^4))^(1/6)) - 9*(c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*x^2 - s
qrt(-3)*(c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*x^2))*(1/(c^5*d^4))^(1/3
))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c*d^2*x^3 - 8*c^2
*d + sqrt(-3)*(c*d^2*x^3 - 8*c^2*d))*(1/(c^5*d^4))^(1/6)*log((d^3*x^9 + 318
*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 +
 32*c^6*d^3*x + sqrt(-3)*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 32*c^6*d^3*x))*(
1/(c^5*d^4))^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^5*d^5*x^5 + 32*c^6*d^4*x^2 -
 sqrt(-3)*(5*c^5*d^5*x^5 + 32*c^6*d^4*x^2))*(1/(c^5*d^4))^(5/6) - 2*(7*c^3*
d^4*x^6 + 152*c^4*d^3*x^3 + 64*c^5*d^2)*sqrt(1/(c^5*d^4)) + (c*d^3*x^7 + 80
*c^2*d^2*x^4 + 160*c^3*d*x + sqrt(-3)*(c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3
*d*x))*(1/(c^5*d^4))^(1/6)) - 9*(c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*
x^2 - sqrt(-3)*(c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*x^2))*(1/(c^5*d^4
))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (c*d^2*x^3
- 8*c^2*d - sqrt(-3)*(c*d^2*x^3 - 8*c^2*d))*(1/(c^5*d^4))^(1/6)*log((d^3*x^
9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^4*d^5*x^7 + 64*c^5*d^
4*x^4 + 32*c^6*d^3*x - sqrt(-3)*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 32*c^6*d^
3*x))*(1/(c^5*d^4))^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^5*d^5*x^5 + 32*c^6*d^
```

```
4*x^2 + sqrt(-3)*(5*c^5*d^5*x^5 + 32*c^6*d^4*x^2))*(1/(c^5*d^4))^(5/6) - 2*
(7*c^3*d^4*x^6 + 152*c^4*d^3*x^3 + 64*c^5*d^2)*sqrt(1/(c^5*d^4)) + (c*d^3*x
^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x - sqrt(-3)*(c*d^3*x^7 + 80*c^2*d^2*x^4 +
160*c^3*d*x))*(1/(c^5*d^4))^(1/6)) - 9*(c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c
^4*d^2*x^2 + sqrt(-3)*(c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*x^2))*(1/(
c^5*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c*d
^2*x^3 - 8*c^2*d - sqrt(-3)*(c*d^2*x^3 - 8*c^2*d))*(1/(c^5*d^4))^(1/6)*log(
(d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^4*d^5*x^7 + 64
*c^5*d^4*x^4 + 32*c^6*d^3*x - sqrt(-3)*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 32
*c^6*d^3*x))*(1/(c^5*d^4))^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^5*d^5*x^5 + 32
*c^6*d^4*x^2 + sqrt(-3)*(5*c^5*d^5*x^5 + 32*c^6*d^4*x^2))*(1/(c^5*d^4))^(5/
6) - 2*(7*c^3*d^4*x^6 + 152*c^4*d^3*x^3 + 64*c^5*d^2)*sqrt(1/(c^5*d^4)) + (
c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x - sqrt(-3)*(c*d^3*x^7 + 80*c^2*d^2
*x^4 + 160*c^3*d*x))*(1/(c^5*d^4))^(1/6)) - 9*(c^2*d^4*x^8 + 38*c^3*d^3*x^5
 + 64*c^4*d^2*x^2 + sqrt(-3)*(c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*x^2
))*(1/(c^5*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3))
 + 2*(c*d^2*x^3 - 8*c^2*d)*(1/(c^5*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6
 + 1200*c^2*d*x^3 + 640*c^3 + 18*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 32*c^6*d
^3*x)*(1/(c^5*d^4))^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^5*d^5*x^5 + 32*c^6*d^
4*x^2)*(1/(c^5*d^4))^(5/6) + (7*c^3*d^4*x^6 + 152*c^4*d^3*x^3 + 64*c^5*d^2)
*sqrt(1/(c^5*d^4)) + (c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x)*(1/(c^5*d^4
))^(1/6)) + 18*(c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*x^2)*(1/(c^5*d^4)
)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*(c*d^2*x^3
 - 8*c^2*d)*(1/(c^5*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x
^3 + 640*c^3 + 18*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 32*c^6*d^3*x)*(1/(c^5*d
^4))^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^5*d^5*x^5 + 32*c^6*d^4*x^2)*(1/(c^5*
d^4))^(5/6) + (7*c^3*d^4*x^6 + 152*c^4*d^3*x^3 + 64*c^5*d^2)*sqrt(1/(c^5*d^
4)) + (c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x)*(1/(c^5*d^4))^(1/6)) + 18*
(c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*x^2)*(1/(c^5*d^4))^(1/3))/(d^3*x
^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(c*d^2*x^3 - 8*c^2*d)
```

Sympy [F]

$$\int \frac{x\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{x\sqrt{c+dx^3}}{(-8c+dx^3)^2}\,dx$$

```
[In] integrate(x*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(x*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

Maxima [F]

$$\int \frac{x\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}x}{(dx^3-8\,c)^2}\,dx$$

```
[In] integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)*x/(d*x^3 - 8*c)^2, x)
```

Giac [F]

$$\int \frac{x\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}x}{(dx^3-8\,c)^2}\,dx$$

```
[In] integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)*x/(d*x^3 - 8*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{c+dx^3}}{(8c-dx^3)^2}\,dx = \int \frac{x\,\sqrt{d\,x^3+c}}{(8\,c-d\,x^3)^2}\,dx$$

```
[In] int((x*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
[Out] int((x*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2, x)
```

3.408 $\int \frac{\sqrt{c+dx^3}}{x^2(8c-dx^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 665

$$\int \frac{\sqrt{c+dx^3}}{x^2(8c-dx^3)^2}\,dx = -\frac{\sqrt{c+dx^3}}{48c^2x}+\frac{\sqrt[3]{d}\sqrt{c+dx^3}}{48c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{\sqrt{c+dx^3}}{24cx(8c-dx^3)} - \frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}c^{11/6}}+\frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{144c^{11/6}}-\frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{144c^{11/6}} - \frac{\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32\ 3^{3/4}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{24\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 1/144*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^
(11/6)-1/144*d^(1/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)-1/144*d^
(1/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(11/6)*
3^(1/2)-1/48*(d*x^3+c)^(1/2)/c^2/x+1/24*(d*x^3+c)^(1/2)/c/x/(-d*x^3+8*c)+1/
48*d^(1/3)*(d*x^3+c)^(1/2)/c^2/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/144*d^(1/3
)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+
```

```
c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2
)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(5/3)*2^(1/2)/(d*x^3+c
)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/
2)-1/96*3^(1/4)*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1
-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*
2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(
1/2)))^2)^(1/2)/c^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/
3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 665, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {480, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(8c-dx^3\right)^2}\,dx$$

$$= \frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{24\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{32\ 3^{3/4}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}c^{11/6}}+\frac{\sqrt[3]{d}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{144c^{11/6}}$$
$$-\frac{\sqrt[3]{d}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{144c^{11/6}}-\frac{\sqrt{c+dx^3}}{48c^2x}+\frac{\sqrt[3]{d}\sqrt{c+dx^3}}{48c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{\sqrt{c+dx^3}}{24cx\left(8c-dx^3\right)}$$

```
[In] Int[Sqrt[c + d*x^3]/(x^2*(8*c - d*x^3)^2),x]
```

```
[Out] -1/48*Sqrt[c + d*x^3]/(c^2*x) + (d^(1/3)*Sqrt[c + d*x^3])/(48*c^2*((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)) + Sqrt[c + d*x^3]/(24*c*x*(8*c - d*x^3)) - (d^(
1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(48*S
qrt[3]*c^(11/6)) + (d^(1/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt
```

```
[c + d*x^3])])/(144*c^(11/6)) - (d^(1/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c]
)])/(144*c^(11/6)) - (Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[
(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(32*3^(3/4)*c^(5/3)*Sqrt[(c^(1/3)*
(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^
3]) + (d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^
(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sq
rt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sq
rt[3]])/(24*Sqrt[2]*3^(1/4)*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 480

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p
 + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c+dx^3}}{24cx\,(8c-dx^3)} - \frac{\int \frac{-4c-\frac{5dx^3}{2}}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{24c}\\
&= -\frac{\sqrt{c+dx^3}}{48c^2x} + \frac{\sqrt{c+dx^3}}{24cx\,(8c-dx^3)} + \frac{\int \frac{x(40c^2d-2cd^2x^3)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{192c^3}\\
&= -\frac{\sqrt{c+dx^3}}{48c^2x} + \frac{\sqrt{c+dx^3}}{24cx\,(8c-dx^3)} + \frac{\int \left(\frac{2cdx}{\sqrt{c+dx^3}} + \frac{24c^2dx}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{192c^3}\\
&= -\frac{\sqrt{c+dx^3}}{48c^2x} + \frac{\sqrt{c+dx^3}}{24cx\,(8c-dx^3)} + \frac{d\int \frac{x}{\sqrt{c+dx^3}}\,dx}{96c^2} + \frac{d\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{8c}\\
&= -\frac{\sqrt{c+dx^3}}{48c^2x} + \frac{\sqrt{c+dx^3}}{24cx\,(8c-dx^3)} - \frac{\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{96c^2} + \frac{d^{2/3}\int \frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{96c^2}\\
&\quad + \frac{d^{2/3}\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{96c^{5/3}} - \frac{\left((1-\sqrt{3})\,d^{2/3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{96c^{5/3}} - \frac{d^{4/3}\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{32c^{4/3}}
\end{aligned}$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{48c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt{c+dx^3}}{24cx\left(8c-dx^3\right)}$$

$$- \frac{\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32\ 3^{3/4}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{24\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{\sqrt[3]{d}\text{Subst}\left(\int \frac{1}{9-cx^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{48c^{4/3}} - \frac{d^{4/3}\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{96c^{4/3}}$$

$$+ \frac{d^{7/3}\text{Subst}\left(\int \frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{24c^{7/3}}$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{48c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt{c+dx^3}}{24cx\left(8c-dx^3\right)}$$

$$-\frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}c^{11/6}} + \frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{144c^{11/6}}$$

$$-\frac{\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32\ 3^{3/4}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{24\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{\sqrt[3]{d}\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{48c^{4/3}}$$

$$
\begin{aligned}
&= -\frac{\sqrt{c+dx^3}}{48c^2x} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{48c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} \\
&+ \frac{\sqrt{c+dx^3}}{24cx\,(8c-dx^3)} - \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}c^{11/6}} \\
&+ \frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{144c^{11/6}} - \frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{144c^{11/6}} \\
&- \frac{\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32\;3^{3/4}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+ \frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{24\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.13 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.27

$$
\int \frac{\sqrt{c+dx^3}}{x^2\,(8c-dx^3)^2}\,dx
$$

$$
= \frac{-80c(6c^2+5cdx^3-d^2x^6)+50cdx^3(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+d^2x^6(-8c+d}{3840c^3\sqrt{c+dx^3}\,(8cx-dx^4)}
$$

```
[In] Integrate[Sqrt[c + d*x^3]/(x^2*(8*c - d*x^3)^2),x]
```

```
[Out] (-80*c*(6*c^2 + 5*c*d*x^3 - d^2*x^6) + 50*c*d*x^3*(8*c - d*x^3)*Sqrt[1 + (d
*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d^2*x^6*
(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c),
 (d*x^3)/(8*c)])/(3840*c^3*Sqrt[c + d*x^3]*(8*c*x - d*x^4))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.67 (sec) , antiderivative size = 898, normalized size of antiderivative = 1.35

method	result	size
elliptic	Expression too large to display	898
risch	Expression too large to display	1758
default	Expression too large to display	2194

```
[In] int((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/192*x^2/c^2*d*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-1/64*(d*x^3+c)^(1/2)/c^2/x-1/1
44*I/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*El
lipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d
^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-1/216*I/d^2/c^2*2^(1/2)*
sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)
^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2
)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(
1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(
1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/
d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*
3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(
_Z^3*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.50 (sec) , antiderivative size = 2390, normalized size of antiderivative = 3.59

$$\int \frac{\sqrt{c+dx^3}}{x^2 (8c-dx^3)^2} \, dx = \text{Too large to display}$$

[In] integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="fricas")

[Out] -1/1728*(36*(d*x^4 - 8*c*x)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) - (c^2*d*x^4 - 8*c^3*x + sqrt(-3)*(c^2*d*x^4 - 8*c^3*x))*(d^2/c^11)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d - 9*(5*c^8*d^2*x^7 + 64*c^9*d*x^4 + 32*c^10*x + sqrt(-3)*(5*c^8*d^2*x^7 + 64*c^9*d*x^4 + 32*c^10*x))*(d^2/c^11)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2 - sqrt(-3)*(5*c^10*d*x^5 + 32*c^11*x^2))*(d^2/c^11)^(5/6) - 2*(7*c^6*d^2*x^6 + 152*c^7*d*x^3 + 64*c^8)*sqrt(d^2/c^11) + (c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x + sqrt(-3)*(c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x))*(d^2/c^11)^(1/6)) - 9*(c^4*d^3*x^8 + 38*c^5*d^2*x^5 + 64*c^6*d*x^2 - sqrt(-3)*(c^4*d^3*x^8 + 38*c^5*d^2*x^5 + 64*c^6*d*x^2))*(d^2/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (c^2*d*x^4 - 8*c^3*x + sqrt(-3)*(c^2*d*x^4 - 8*c^3*x))*(d^2/c^11)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d - 9*(5*c^8*d^2*x^7 + 64*c^9*d*x^4 + 32*c^10*x + sqrt(-3)*(5*c^8*d^2*x^7 + 64*c^9*d*x^4 + 32*c^10*x))*(d^2/c^11)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2 - sqrt(-3)*(5*c^10*d*x^5 + 32*c^11*x^2))*(d^2/c^11)^(5/6) - 2*(7*c^6*d^2*x^6 + 152*c^7*d*x^3 + 64*c^8)*sqrt(d^2/c^11) + (c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x + sqrt(-3)*(c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x))*(d^2/c^11)^(1/6)) - 9*(c^4*d^3*x^8 + 38*c^5*d^2*x^5 + 64*c^6*d*x^2 - sqrt(-3)*(c^4*d^3*x^8 + 38*c^5*d^2*x^5 + 64*c^6*d*x^2))*(d^2/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c^2*d*x^4 - 8*c^3*x - sqrt(-3)*(c^2*d*x^4 - 8*c^3*x))*(d^2/c^11)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d - 9*(5*c^8*d^2*x^7 + 64*c^9*d*x^4 + 32*c^10*x - sqrt(-3)*(5*c^8*d^2*x^7 + 64*c^9*d*x^4 + 32*c^10*x))*(d^2/c^11)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2 + sqrt(-3)*(5*c^10*d*x^5 + 32*c^11*x^2))*(d^2/c^11)^(5/6) - 2*(7*c^6*d^2*x^6 + 152*c^7*d*x^3 + 64*c^8)*sqrt(d^2/c^11) + (c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x - sqrt(-3)*(c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x))*(d^2/c^11)^(1/6)) - 9*(c^4*d^3*x^8 + 38*c^5*d^2*x^5 + 64*c^6*d*x^2 + sqrt(-3)*(c^4*d^3*x^8 + 38*c^5*d^2*x^5 + 64*c^6*d*x^2))*(d^2/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (c^2*d*x^4 - 8*c^3*x - sqrt(-3)*(c^2*d*x^4 - 8*c^3*x))*(d^2/c^11)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d - 9*(5*c^8*d^2*x^7 + 64*c^9*d*x^4 + 32*c^10*x - sqrt(-3)*(5*c^8*d^2*x^7 + 64*c^9*d*x^4 + 32*c^10*x))*(d^2/c^11)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2 + sqrt(-3)*(5*c^10*d*x^5 + 32*c^11*x^2))*(d^2/c^11)^(5/6) - 2*(7*c^

```
6*d^2*x^6 + 152*c^7*d*x^3 + 64*c^8)*sqrt(d^2/c^11) + (c^2*d^3*x^7 + 80*c^3*
d^2*x^4 + 160*c^4*d*x - sqrt(-3)*(c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*
x))*(d^2/c^11)^(1/6)) - 9*(c^4*d^3*x^8 + 38*c^5*d^2*x^5 + 64*c^6*d*x^2 + sq
rt(-3)*(c^4*d^3*x^8 + 38*c^5*d^2*x^5 + 64*c^6*d*x^2))*(d^2/c^11)^(1/3))/(d^
3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*(c^2*d*x^4 - 8*c^3*x)*
(d^2/c^11)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*
d + 18*(5*c^8*d^2*x^7 + 64*c^9*d*x^4 + 32*c^10*x)*(d^2/c^11)^(2/3) + 6*sqrt
(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2)*(d^2/c^11)^(5/6) + (7*c^6*d^2*x
^6 + 152*c^7*d*x^3 + 64*c^8)*sqrt(d^2/c^11) + (c^2*d^3*x^7 + 80*c^3*d^2*x^4
 + 160*c^4*d*x)*(d^2/c^11)^(1/6)) + 18*(c^4*d^3*x^8 + 38*c^5*d^2*x^5 + 64*c
^6*d*x^2)*(d^2/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c
^3)) + 2*(c^2*d*x^4 - 8*c^3*x)*(d^2/c^11)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^
6 + 1200*c^2*d^2*x^3 + 640*c^3*d + 18*(5*c^8*d^2*x^7 + 64*c^9*d*x^4 + 32*c^
10*x)*(d^2/c^11)^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2)*
(d^2/c^11)^(5/6) + (7*c^6*d^2*x^6 + 152*c^7*d*x^3 + 64*c^8)*sqrt(d^2/c^11)
+ (c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x)*(d^2/c^11)^(1/6)) + 18*(c^4*
d^3*x^8 + 38*c^5*d^2*x^5 + 64*c^6*d*x^2)*(d^2/c^11)^(1/3))/(d^3*x^9 - 24*c*
d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 36*sqrt(d*x^3 + c)*(d*x^3 - 6*c))/(c^
2*d*x^4 - 8*c^3*x)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(8c-dx^3\right)^2}\,dx=\int \frac{\sqrt{c+dx^3}}{x^2\left(-8c+dx^3\right)^2}\,dx$$

```
[In] integrate((d*x**3+c)**(1/2)/x**2/(-d*x**3+8*c)**2,x)
[Out] Integral(sqrt(c + d*x**3)/(x**2*(-8*c + d*x**3)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(8c-dx^3\right)^2}\,dx=\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)^2x^2}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2(8c-dx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}}{(dx^3-8\,c)^2x^2}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^2(8c-dx^3)^2}\,dx = \int \frac{\sqrt{d\,x^3+c}}{x^2\,(8\,c-d\,x^3)^2}\,dx$$

```
[In] int((c + d*x^3)^(1/2)/(x^2*(8*c - d*x^3)^2),x)
[Out] int((c + d*x^3)^(1/2)/(x^2*(8*c - d*x^3)^2), x)
```

3.409 $\int \frac{\sqrt{c+dx^3}}{x^5(8c-dx^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 687

$$\int \frac{\sqrt{c+dx^3}}{x^5(8c-dx^3)^2}\,dx = -\frac{7\sqrt{c+dx^3}}{768c^2x^4}-\frac{d\sqrt{c+dx^3}}{96c^3x}+\frac{d^{4/3}\sqrt{c+dx^3}}{96c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$
$$+\frac{\sqrt{c+dx^3}}{24cx^4(8c-dx^3)}-\frac{17d^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3072\sqrt{3}c^{17/6}}$$
$$+\frac{17d^{4/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9216c^{17/6}}-\frac{17d^{4/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9216c^{17/6}}$$
$$-\frac{\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{64\ 3^{3/4}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+\frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{48\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 17/9216*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/
c^(17/6)-17/9216*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)-17/9
```

```
216*d^(4/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(
17/6)*3^(1/2)-7/768*(d*x^3+c)^(1/2)/c^2/x^4-1/96*d*(d*x^3+c)^(1/2)/c^3/x+1/
24*(d*x^3+c)^(1/2)/c/x^4/(-d*x^3+8*c)+1/96*d^(4/3)*(d*x^3+c)^(1/2)/c^3/(d^(
1/3)*x+c^(1/3)*(1+3^(1/2)))+1/288*d^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^
(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)
*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2
)^(1/2)*3^(3/4)/c^(8/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x
)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/192*3^(1/4)*d^(4/3)*(c^(1/3)+d
^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3
^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)
*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(8/3)/(d*x^3+c)^
(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 687, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {480, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{\sqrt{c+dx^3}}{x^5 \left(8c-dx^3\right)^2} \, dx$$
$$= \frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{48\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{64\ 3^{3/4}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{17d^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3072\sqrt{3}c^{17/6}}+\frac{17d^{4/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9216c^{17/6}}$$
$$-\frac{17d^{4/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9216c^{17/6}}+\frac{d^{4/3}\sqrt{c+dx^3}}{96c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$
$$-\frac{d\sqrt{c+dx^3}}{96c^3x}-\frac{7\sqrt{c+dx^3}}{768c^2x^4}+\frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)}$$

```
[In] Int[Sqrt[c + d*x^3]/(x^5*(8*c - d*x^3)^2),x]
```

```
[Out] (-7*Sqrt[c + d*x^3])/(768*c^2*x^4) - (d*Sqrt[c + d*x^3])/(96*c^3*x) + (d^(4
/3)*Sqrt[c + d*x^3])/(96*c^3*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + Sqrt[c
+ d*x^3]/(24*c*x^4*(8*c - d*x^3)) - (17*d^(4/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^
(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(3072*Sqrt[3]*c^(17/6)) + (17*d^(4/3)
*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(9216*c^(17/
6)) - (17*d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9216*c^(17/6)) - (
Sqrt[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(
1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcS
in[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(64*3^(3/4)*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))
/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^(4/3)*(c^(1/3
) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/
3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(48*Sqrt[2]*3^
(1/4)*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) +
 d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 480

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p
 + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
```

```
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)}-\frac{\int \frac{-7c-\frac{11dx^3}{2}}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx}{24c}\\
&= -\frac{7\sqrt{c+dx^3}}{768c^2x^4}+\frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)}+\frac{\int \frac{64c^2d+\frac{35}{2}cd^2x^3}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{768c^3}\\
&= -\frac{7\sqrt{c+dx^3}}{768c^2x^4}-\frac{d\sqrt{c+dx^3}}{96c^3x}+\frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)}-\frac{\int \frac{x\left(-460c^3d^2+32c^2d^3x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{6144c^5}\\
&= -\frac{7\sqrt{c+dx^3}}{768c^2x^4}-\frac{d\sqrt{c+dx^3}}{96c^3x}+\frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)}-\frac{\int \left(-\frac{32c^2d^2x}{\sqrt{c+dx^3}}-\frac{204c^3d^2x}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{6144c^5}\\
&= -\frac{7\sqrt{c+dx^3}}{768c^2x^4}-\frac{d\sqrt{c+dx^3}}{96c^3x}+\frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)}+\frac{d^2\int \frac{x}{\sqrt{c+dx^3}}\,dx}{192c^3}+\frac{\left(17d^2\right)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{512c^2}
\end{aligned}$$

$$= -\frac{7\sqrt{c+dx^3}}{768c^2x^4} - \frac{d\sqrt{c+dx^3}}{96c^3x} + \frac{\sqrt{c+dx^3}}{24cx^4(8c-dx^3)} - \frac{(17d)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{6144c^3}$$

$$+ \frac{d^{5/3}\int \frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{192c^3} + \frac{(17d^{5/3})\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{6144c^{8/3}}$$

$$- \frac{\left((1-\sqrt{3})\,d^{5/3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{192c^{8/3}} - \frac{(17d^{7/3})\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{2048c^{7/3}}$$

$$= -\frac{7\sqrt{c+dx^3}}{768c^2x^4} - \frac{d\sqrt{c+dx^3}}{96c^3x} + \frac{d^{4/3}\sqrt{c+dx^3}}{96c^3\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt{c+dx^3}}{24cx^4(8c-dx^3)}$$

$$- \frac{\sqrt{2-\sqrt{3}}\,d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{(1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{64\ 3^{3/4}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{(1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{48\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{(17d^{4/3})\,\text{Subst}\left(\int \frac{1}{9-cx^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{3072c^{7/3}}$$

$$- \frac{(17d^{7/3})\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{6144c^{7/3}}$$

$$+ \frac{(17d^{10/3})\,\text{Subst}\left(\int \frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{1536c^{10/3}}$$

$$= -\frac{7\sqrt{c+dx^3}}{768c^2x^4} - \frac{d\sqrt{c+dx^3}}{96c^3x} + \frac{d^{4/3}\sqrt{c+dx^3}}{96c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)}$$

$$- \frac{17d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3072\sqrt{3}c^{17/6}} + \frac{17d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9216c^{17/6}}$$

$$- \frac{\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{64\ 3^{3/4}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{48\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$- \frac{\left(17d^{4/3}\right)\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{3072c^{7/3}}$$

$$= -\frac{7\sqrt{c+dx^3}}{768c^2x^4} - \frac{d\sqrt{c+dx^3}}{96c^3x} + \frac{d^{4/3}\sqrt{c+dx^3}}{96c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$
$$+ \frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)} - \frac{17d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3072\sqrt{3}c^{17/6}}$$
$$+ \frac{17d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9216c^{17/6}} - \frac{17d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9216c^{17/6}}$$
$$- \frac{\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{64\ 3^{3/4}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+ \frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{48\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.29

$$\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)^2}\,dx = \sqrt{c+dx^3}\left(-\frac{1}{256c^2x^4} - \frac{5d}{512c^3x} - \frac{d^2x^2}{1536c^3\left(-8c+dx^3\right)}\right)$$
$$+ \frac{115d^2x^2\sqrt{\frac{c+dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{24576c^3\sqrt{c+dx^3}}$$
$$- \frac{d^3x^5\sqrt{\frac{c+dx^3}{c}}\operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{7680c^4\sqrt{c+dx^3}}$$

```
[In] Integrate[Sqrt[c + d*x^3]/(x^5*(8*c - d*x^3)^2),x]
```

```
[Out] Sqrt[c + d*x^3]*(-1/256*1/(c^2*x^4) - (5*d)/(512*c^3*x) - (d^2*x^2)/(1536*c
^3*(-8*c + d*x^3))) + (115*d^2*x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1
, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(24576*c^3*Sqrt[c + d*x^3]) - (d^3*x^5
*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)
])/(7680*c^4*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.65 (sec) , antiderivative size = 919, normalized size of antiderivative = 1.34

method	result	size
elliptic	Expression too large to display	919
risch	Expression too large to display	1770
default	Expression too large to display	2672

```
[In] int((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/1536*d^2*x^2/c^3*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-1/256*(d*x^3+c)^(1/2)/c^2/x
^4-5/512*d*(d*x^3+c)^(1/2)/c^3/x-1/288*I*d/c^3*3^(1/2)*(-c*d^2)^(1/3)*(I*(x
+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d
^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(
I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2
)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*
3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)))^(1/2)))-17/13824*I/d/c^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d
*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)
*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1
/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^
(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(
-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellipt
icPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha
^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-
3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.43 (sec) , antiderivative size = 2549, normalized size of antiderivative = 3.71

$$\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] -1/110592*(1152*(d^2*x^7 - 8*c*d*x^4)*sqrt(d)*weierstrassZeta(0, -4*c/d, we
ierstrassPInverse(0, -4*c/d, x)) - 17*(c^3*d*x^7 - 8*c^4*x^4 + sqrt(-3)*(c^
3*d*x^7 - 8*c^4*x^4))*(d^8/c^17)^(1/6)*log(1419857*(d^9*x^9 + 318*c*d^8*x^6
 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^12*d^3*x^7 + 64*c^13*d^2*x^4 + 3
2*c^14*d*x + sqrt(-3)*(5*c^12*d^3*x^7 + 64*c^13*d^2*x^4 + 32*c^14*d*x))*(d^
8/c^17)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2 - sqrt(-3)
*(5*c^15*d*x^5 + 32*c^16*x^2))*(d^8/c^17)^(5/6) - 2*(7*c^9*d^4*x^6 + 152*c^
10*d^3*x^3 + 64*c^11*d^2)*sqrt(d^8/c^17) + (c^3*d^7*x^7 + 80*c^4*d^6*x^4 +
160*c^5*d^5*x + sqrt(-3)*(c^3*d^7*x^7 + 80*c^4*d^6*x^4 + 160*c^5*d^5*x))*(d
^8/c^17)^(1/6)) - 9*(c^6*d^6*x^8 + 38*c^7*d^5*x^5 + 64*c^8*d^4*x^2 - sqrt(-
3)*(c^6*d^6*x^8 + 38*c^7*d^5*x^5 + 64*c^8*d^4*x^2))*(d^8/c^17)^(1/3))/(d^3*
x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 17*(c^3*d*x^7 - 8*c^4*x^4
+ sqrt(-3)*(c^3*d*x^7 - 8*c^4*x^4))*(d^8/c^17)^(1/6)*log(1419857*(d^9*x^9 +
 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^12*d^3*x^7 + 64*c^
13*d^2*x^4 + 32*c^14*d*x + sqrt(-3)*(5*c^12*d^3*x^7 + 64*c^13*d^2*x^4 + 32*
c^14*d*x))*(d^8/c^17)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*
x^2 - sqrt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2))*(d^8/c^17)^(5/6) - 2*(7*c^9*d^
4*x^6 + 152*c^10*d^3*x^3 + 64*c^11*d^2)*sqrt(d^8/c^17) + (c^3*d^7*x^7 + 80*
c^4*d^6*x^4 + 160*c^5*d^5*x + sqrt(-3)*(c^3*d^7*x^7 + 80*c^4*d^6*x^4 + 160*
c^5*d^5*x))*(d^8/c^17)^(1/6)) - 9*(c^6*d^6*x^8 + 38*c^7*d^5*x^5 + 64*c^8*d^
4*x^2 - sqrt(-3)*(c^6*d^6*x^8 + 38*c^7*d^5*x^5 + 64*c^8*d^4*x^2))*(d^8/c^17
)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 17*(c^3*d*x^
7 - 8*c^4*x^4 - sqrt(-3)*(c^3*d*x^7 - 8*c^4*x^4))*(d^8/c^17)^(1/6)*log(1419
857*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^12*d
^3*x^7 + 64*c^13*d^2*x^4 + 32*c^14*d*x - sqrt(-3)*(5*c^12*d^3*x^7 + 64*c^13
*d^2*x^4 + 32*c^14*d*x))*(d^8/c^17)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^15*d*
x^5 + 32*c^16*x^2 + sqrt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2))*(d^8/c^17)^(5/6)
 - 2*(7*c^9*d^4*x^6 + 152*c^10*d^3*x^3 + 64*c^11*d^2)*sqrt(d^8/c^17) + (c^3
*d^7*x^7 + 80*c^4*d^6*x^4 + 160*c^5*d^5*x - sqrt(-3)*(c^3*d^7*x^7 + 80*c^4*
d^6*x^4 + 160*c^5*d^5*x))*(d^8/c^17)^(1/6)) - 9*(c^6*d^6*x^8 + 38*c^7*d^5*x
^5 + 64*c^8*d^4*x^2 + sqrt(-3)*(c^6*d^6*x^8 + 38*c^7*d^5*x^5 + 64*c^8*d^4*x
^2))*(d^8/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3))
+ 17*(c^3*d*x^7 - 8*c^4*x^4 - sqrt(-3)*(c^3*d*x^7 - 8*c^4*x^4))*(d^8/c^17)^
(1/6)*log(1419857*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6
 - 9*(5*c^12*d^3*x^7 + 64*c^13*d^2*x^4 + 32*c^14*d*x - sqrt(-3)*(5*c^12*d^3
```

```
*x^7 + 64*c^13*d^2*x^4 + 32*c^14*d*x))*(d^8/c^17)^(2/3) - 3*sqrt(d*x^3 + c)
*(6*(5*c^15*d*x^5 + 32*c^16*x^2 + sqrt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2))*(d
^8/c^17)^(5/6) - 2*(7*c^9*d^4*x^6 + 152*c^10*d^3*x^3 + 64*c^11*d^2)*sqrt(d^
8/c^17) + (c^3*d^7*x^7 + 80*c^4*d^6*x^4 + 160*c^5*d^5*x - sqrt(-3)*(c^3*d^7
*x^7 + 80*c^4*d^6*x^4 + 160*c^5*d^5*x))*(d^8/c^17)^(1/6)) - 9*(c^6*d^6*x^8
+ 38*c^7*d^5*x^5 + 64*c^8*d^4*x^2 + sqrt(-3)*(c^6*d^6*x^8 + 38*c^7*d^5*x^5
+ 64*c^8*d^4*x^2))*(d^8/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^
3 - 512*c^3)) - 34*(c^3*d*x^7 - 8*c^4*x^4)*(d^8/c^17)^(1/6)*log(1419857*(d^
9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 + 18*(5*c^12*d^3*x^7
 + 64*c^13*d^2*x^4 + 32*c^14*d*x)*(d^8/c^17)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(
5*c^15*d*x^5 + 32*c^16*x^2)*(d^8/c^17)^(5/6) + (7*c^9*d^4*x^6 + 152*c^10*d^
3*x^3 + 64*c^11*d^2)*sqrt(d^8/c^17) + (c^3*d^7*x^7 + 80*c^4*d^6*x^4 + 160*c
^5*d^5*x)*(d^8/c^17)^(1/6)) + 18*(c^6*d^6*x^8 + 38*c^7*d^5*x^5 + 64*c^8*d^4
*x^2)*(d^8/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3))
 + 34*(c^3*d*x^7 - 8*c^4*x^4)*(d^8/c^17)^(1/6)*log(1419857*(d^9*x^9 + 318*c
*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 + 18*(5*c^12*d^3*x^7 + 64*c^13*d^
2*x^4 + 32*c^14*d*x)*(d^8/c^17)^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5
+ 32*c^16*x^2)*(d^8/c^17)^(5/6) + (7*c^9*d^4*x^6 + 152*c^10*d^3*x^3 + 64*c^
11*d^2)*sqrt(d^8/c^17) + (c^3*d^7*x^7 + 80*c^4*d^6*x^4 + 160*c^5*d^5*x)*(d^
8/c^17)^(1/6)) + 18*(c^6*d^6*x^8 + 38*c^7*d^5*x^5 + 64*c^8*d^4*x^2)*(d^8/c^
17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 144*(8*d^2
*x^6 - 57*c*d*x^3 - 24*c^2)*sqrt(d*x^3 + c))/(c^3*d*x^7 - 8*c^4*x^4)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)^2}\,dx=\int \frac{\sqrt{c+dx^3}}{x^5\left(-8c+dx^3\right)^2}\,dx$$

```
[In] integrate((d*x**3+c)**(1/2)/x**5/(-d*x**3+8*c)**2,x)
[Out] Integral(sqrt(c + d*x**3)/(x**5*(-8*c + d*x**3)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)^2}\,dx=\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)^2x^5}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^5), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^5 (8c-dx^3)^2} \, dx = \int \frac{\sqrt{dx^3+c}}{(dx^3-8c)^2 x^5} \, dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^5 (8c-dx^3)^2} \, dx = \int \frac{\sqrt{d\,x^3+c}}{x^5\,(8\,c-d\,x^3)^2} \, dx$$

```
[In] int((c + d*x^3)^(1/2)/(x^5*(8*c - d*x^3)^2),x)
[Out] int((c + d*x^3)^(1/2)/(x^5*(8*c - d*x^3)^2), x)
```

3.410 $\int \frac{\sqrt{c+dx^3}}{x^8(8c-dx^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 711

$$\int \frac{\sqrt{c+dx^3}}{x^8(8c-dx^3)^2}\,dx = -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} - \frac{d^2\sqrt{c+dx^3}}{5376c^4x} + \frac{d^{7/3}\sqrt{c+dx^3}}{5376c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt{c+dx^3}}{24cx^7(8c-dx^3)} - \frac{13d^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{12288\sqrt{3}c^{23/6}} + \frac{13d^{7/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{36864c^{23/6}} - \frac{13d^{7/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{36864c^{23/6}} - \frac{\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{3584\ 3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{2688\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 13/36864*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))
/c^(23/6)-13/36864*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(23/6)-13
```

```
/36864*d^(7/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/
c^(23/6)*3^(1/2)-5/672*(d*x^3+c)^(1/2)/c^2/x^7-53/21504*d*(d*x^3+c)^(1/2)/c
^3/x^4-1/5376*d^2*(d*x^3+c)^(1/2)/c^4/x+1/24*(d*x^3+c)^(1/2)/c/x^7/(-d*x^3+
8*c)+1/5376*d^(7/3)*(d*x^3+c)^(1/2)/c^4/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/1
6128*3^(3/4)*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^
(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^
(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(11/3)*2^(1
/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1
/2)))^2)^(1/2)-1/10752*3^(1/4)*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/
3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1
/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x
+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(11/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d
^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 711, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.481, Rules used = {480, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)^2}\,dx$$

$$=\frac{d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{2688\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{3584\ 3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{13d^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{12288\sqrt{3}c^{23/6}}+\frac{13d^{7/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{36864c^{23/6}}$$
$$-\frac{13d^{7/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{36864c^{23/6}}+\frac{d^{7/3}\sqrt{c+dx^3}}{5376c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$
$$-\frac{d^2\sqrt{c+dx^3}}{5376c^4x}-\frac{53d\sqrt{c+dx^3}}{21504c^3x^4}-\frac{5\sqrt{c+dx^3}}{672c^2x^7}+\frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)}$$

```
[In] Int[Sqrt[c + d*x^3]/(x^8*(8*c - d*x^3)^2),x]
```

```
[Out] (-5*Sqrt[c + d*x^3])/(672*c^2*x^7) - (53*d*Sqrt[c + d*x^3])/(21504*c^3*x^4)
 - (d^2*Sqrt[c + d*x^3])/(5376*c^4*x) + (d^(7/3)*Sqrt[c + d*x^3])/(5376*c^4
*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + Sqrt[c + d*x^3]/(24*c*x^7*(8*c - d*
x^3)) - (13*d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c +
 d*x^3]])/(12288*Sqrt[3]*c^(23/6)) + (13*d^(7/3)*ArcTanh[(c^(1/3) + d^(1/3)
*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(36864*c^(23/6)) - (13*d^(7/3)*ArcTanh[
Sqrt[c + d*x^3]/(3*Sqrt[c])])/(36864*c^(23/6)) - (Sqrt[2 - Sqrt[3]]*d^(7/3)
*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3)
 + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3584*
3^(3/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c
^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c
^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(2688*Sqrt[2]*3^(1/4)*c^(11/3)*Sqrt[
(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt
[c + d*x^3])
```

<u>Rule 65</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

<u>Rule 211</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

<u>Rule 212</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

<u>Rule 224</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 480

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p
 + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
```

```
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)} - \frac{\int \frac{-10c-\frac{17dx^3}{2}}{x^8(8c-dx^3)\sqrt{c+dx^3}}\,dx}{24c}$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} + \frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)} + \frac{\int \frac{106c^2d+55cd^2x^3}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx}{1344c^3}$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} + \frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)} - \frac{\int \frac{-64c^3d^2-265c^2d^3x^3}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{43008c^5}$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} - \frac{d^2\sqrt{c+dx^3}}{5376c^4x} + \frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)} + \frac{\int \frac{x\left(2440c^4d^3-32c^3d^4x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{344064c^7}$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} - \frac{d^2\sqrt{c+dx^3}}{5376c^4x} + \frac{\sqrt{c+dx^3}}{24cx^7(8c-dx^3)} + \frac{\int \left(\frac{32c^3d^3x}{\sqrt{c+dx^3}} + \frac{2184c^4d^3x}{(8c-dx^3)\sqrt{c+dx^3}}\right) dx}{344064c^7}$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} - \frac{d^2\sqrt{c+dx^3}}{5376c^4x} + \frac{\sqrt{c+dx^3}}{24cx^7(8c-dx^3)} + \frac{d^3\int \frac{x}{\sqrt{c+dx^3}}\,dx}{10752c^4} + \frac{(13d^3)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{2048c^3}$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} - \frac{d^2\sqrt{c+dx^3}}{5376c^4x} + \frac{\sqrt{c+dx^3}}{24cx^7(8c-dx^3)} - \frac{(13d^2)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{24576c^4} + \frac{d^{8/3}\int \frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{10752c^4} + \frac{(13d^{8/3})\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{24576c^{11/3}} - \frac{\left((1-\sqrt{3})\,d^{8/3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{10752c^{11/3}} - \frac{(13d^{10/3})\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{8192c^{10/3}}$$

$$
\begin{aligned}
&= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} - \frac{d^2\sqrt{c+dx^3}}{5376c^4x} \\
&+ \frac{d^{7/3}\sqrt{c+dx^3}}{5376c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)} \\
&- \frac{\sqrt{2-\sqrt{3}}\,d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{3584\ 3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+ \frac{d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{2688\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+ \frac{\left(13d^{7/3}\right)\text{Subst}\left(\int\frac{1}{9-cx^2}\,dx,x,\frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{12288c^{10/3}} \\
&- \frac{\left(13d^{10/3}\right)\text{Subst}\left(\int\frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{24576c^{10/3}} \\
&+ \frac{\left(13d^{13/3}\right)\text{Subst}\left(\int\frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{6144c^{13/3}}
\end{aligned}
$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} - \frac{d^2\sqrt{c+dx^3}}{5376c^4x} + \frac{d^{7/3}\sqrt{c+dx^3}}{5376c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)} - \frac{13d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{12288\sqrt{3}c^{23/6}} + \frac{13d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{36864c^{23/6}} - \frac{\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{3584\ 3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{2688\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{\left(13d^{7/3}\right)\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{12288c^{10/3}}$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} - \frac{d^2\sqrt{c+dx^3}}{5376c^4x} + \frac{d^{7/3}\sqrt{c+dx^3}}{5376c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)} - \frac{13d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{12288\sqrt{3}c^{23/6}} + \frac{13d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{36864c^{23/6}} - \frac{13d^{7/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{36864c^{23/6}} - \frac{\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{3584\ 3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{2688\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.29

$$\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)^2}\,dx$$

$$= \frac{1525cd^3x^9(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right) - 8\Big(20c(384c^4+648c^3dx^3+243c^2d^2x^6-}{3440640c^5x^7\left(8c-dx^3\right)\sqrt{c+}}$$

```
[In] Integrate[Sqrt[c + d*x^3]/(x^8*(8*c - d*x^3)^2),x]
```

```
[Out] (1525*c*d^3*x^9*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3
, -((d*x^3)/c), (d*x^3)/(8*c)] - 8*(20*c*(384*c^4 + 648*c^3*d*x^3 + 243*c^2
*d^2*x^6 - 25*c*d^3*x^9 - 4*d^4*x^12) + d^4*x^12*(8*c - d*x^3)*Sqrt[1 + (d*
x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(3440640*
c^5*x^7*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.51 (sec) , antiderivative size = 938, normalized size of antiderivative = 1.32

method	result	size
elliptic	Expression too large to display	938
risch	Expression too large to display	1781
default	Expression too large to display	3170

```
[In] int((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12288*d^3*x^2/c^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-1/448*(d*x^3+c)^(1/2)/c^2/
x^7-13/7168*d*(d*x^3+c)^(1/2)/c^3/x^4-3/28672*d^2*(d*x^3+c)^(1/2)/c^4/x-1/1
6128*I*d^2/c^4*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3
))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-13/55296*I/c^4*2^(
1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c
*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*
(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-
c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d
^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d
^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-
1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alp
ha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/
3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=Ro
otOf(_Z^3*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.94 (sec) , antiderivative size = 2582, normalized size of antiderivative = 3.63

$$\int \frac{\sqrt{c+dx^3}}{x^8 (8c-dx^3)^2} \, dx = \text{Too large to display}$$

```
[In] integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="fricas")
[Out] -1/3096576*(576*(d^3*x^10 - 8*c*d^2*x^7)*sqrt(d)*weierstrassZeta(0, -4*c/d,
 weierstrassPInverse(0, -4*c/d, x)) - 91*(c^4*d*x^10 - 8*c^5*x^7 + sqrt(-3)
*(c^4*d*x^10 - 8*c^5*x^7))*(d^14/c^23)^(1/6)*log(371293*(d^14*x^9 + 318*c*d
^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 - 9*(5*c^16*d^4*x^7 + 64*c^17*d^
3*x^4 + 32*c^18*d^2*x + sqrt(-3)*(5*c^16*d^4*x^7 + 64*c^17*d^3*x^4 + 32*c^1
8*d^2*x))*(d^14/c^23)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*
x^2 - sqrt(-3)*(5*c^20*d*x^5 + 32*c^21*x^2))*(d^14/c^23)^(5/6) - 2*(7*c^12*
d^6*x^6 + 152*c^13*d^5*x^3 + 64*c^14*d^4)*sqrt(d^14/c^23) + (c^4*d^11*x^7 +
 80*c^5*d^10*x^4 + 160*c^6*d^9*x + sqrt(-3)*(c^4*d^11*x^7 + 80*c^5*d^10*x^4
 + 160*c^6*d^9*x))*(d^14/c^23)^(1/6)) - 9*(c^8*d^9*x^8 + 38*c^9*d^8*x^5 + 6
4*c^10*d^7*x^2 - sqrt(-3)*(c^8*d^9*x^8 + 38*c^9*d^8*x^5 + 64*c^10*d^7*x^2))
*(d^14/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 9
1*(c^4*d*x^10 - 8*c^5*x^7 + sqrt(-3)*(c^4*d*x^10 - 8*c^5*x^7))*(d^14/c^23)^
(1/6)*log(371293*(d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d
^11 - 9*(5*c^16*d^4*x^7 + 64*c^17*d^3*x^4 + 32*c^18*d^2*x + sqrt(-3)*(5*c^1
6*d^4*x^7 + 64*c^17*d^3*x^4 + 32*c^18*d^2*x))*(d^14/c^23)^(2/3) - 3*sqrt(d*
x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2 - sqrt(-3)*(5*c^20*d*x^5 + 32*c^21*
x^2))*(d^14/c^23)^(5/6) - 2*(7*c^12*d^6*x^6 + 152*c^13*d^5*x^3 + 64*c^14*d^
4)*sqrt(d^14/c^23) + (c^4*d^11*x^7 + 80*c^5*d^10*x^4 + 160*c^6*d^9*x + sqrt
(-3)*(c^4*d^11*x^7 + 80*c^5*d^10*x^4 + 160*c^6*d^9*x))*(d^14/c^23)^(1/6)) -
 9*(c^8*d^9*x^8 + 38*c^9*d^8*x^5 + 64*c^10*d^7*x^2 - sqrt(-3)*(c^8*d^9*x^8
+ 38*c^9*d^8*x^5 + 64*c^10*d^7*x^2))*(d^14/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2
*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 91*(c^4*d*x^10 - 8*c^5*x^7 - sqrt(-3)*(c
^4*d*x^10 - 8*c^5*x^7))*(d^14/c^23)^(1/6)*log(371293*(d^14*x^9 + 318*c*d^13
*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 - 9*(5*c^16*d^4*x^7 + 64*c^17*d^3*x
^4 + 32*c^18*d^2*x - sqrt(-3)*(5*c^16*d^4*x^7 + 64*c^17*d^3*x^4 + 32*c^18*d
^2*x))*(d^14/c^23)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2
 + sqrt(-3)*(5*c^20*d*x^5 + 32*c^21*x^2))*(d^14/c^23)^(5/6) - 2*(7*c^12*d^6
*x^6 + 152*c^13*d^5*x^3 + 64*c^14*d^4)*sqrt(d^14/c^23) + (c^4*d^11*x^7 + 80
*c^5*d^10*x^4 + 160*c^6*d^9*x - sqrt(-3)*(c^4*d^11*x^7 + 80*c^5*d^10*x^4 +
160*c^6*d^9*x))*(d^14/c^23)^(1/6)) - 9*(c^8*d^9*x^8 + 38*c^9*d^8*x^5 + 64*c
^10*d^7*x^2 + sqrt(-3)*(c^8*d^9*x^8 + 38*c^9*d^8*x^5 + 64*c^10*d^7*x^2))*(d
^14/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 91*(
c^4*d*x^10 - 8*c^5*x^7 - sqrt(-3)*(c^4*d*x^10 - 8*c^5*x^7))*(d^14/c^23)^(1/
6)*log(371293*(d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11
```

```
 - 9*(5*c^16*d^4*x^7 + 64*c^17*d^3*x^4 + 32*c^18*d^2*x - sqrt(-3)*(5*c^16*d
^4*x^7 + 64*c^17*d^3*x^4 + 32*c^18*d^2*x))*(d^14/c^23)^(2/3) - 3*sqrt(d*x^3
 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2 + sqrt(-3)*(5*c^20*d*x^5 + 32*c^21*x^2
))*(d^14/c^23)^(5/6) - 2*(7*c^12*d^6*x^6 + 152*c^13*d^5*x^3 + 64*c^14*d^4)*
sqrt(d^14/c^23) + (c^4*d^11*x^7 + 80*c^5*d^10*x^4 + 160*c^6*d^9*x - sqrt(-3
)*(c^4*d^11*x^7 + 80*c^5*d^10*x^4 + 160*c^6*d^9*x))*(d^14/c^23)^(1/6)) - 9*
(c^8*d^9*x^8 + 38*c^9*d^8*x^5 + 64*c^10*d^7*x^2 + sqrt(-3)*(c^8*d^9*x^8 + 3
8*c^9*d^8*x^5 + 64*c^10*d^7*x^2))*(d^14/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^
6 + 192*c^2*d*x^3 - 512*c^3)) - 182*(c^4*d*x^10 - 8*c^5*x^7)*(d^14/c^23)^(1
/6)*log(371293*(d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^1
1 + 18*(5*c^16*d^4*x^7 + 64*c^17*d^3*x^4 + 32*c^18*d^2*x)*(d^14/c^23)^(2/3)
 + 6*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2)*(d^14/c^23)^(5/6) + (7
*c^12*d^6*x^6 + 152*c^13*d^5*x^3 + 64*c^14*d^4)*sqrt(d^14/c^23) + (c^4*d^11
*x^7 + 80*c^5*d^10*x^4 + 160*c^6*d^9*x)*(d^14/c^23)^(1/6)) + 18*(c^8*d^9*x^
8 + 38*c^9*d^8*x^5 + 64*c^10*d^7*x^2)*(d^14/c^23)^(1/3))/(d^3*x^9 - 24*c*d^
2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 182*(c^4*d*x^10 - 8*c^5*x^7)*(d^14/c^23
)^(1/6)*log(371293*(d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3
*d^11 + 18*(5*c^16*d^4*x^7 + 64*c^17*d^3*x^4 + 32*c^18*d^2*x)*(d^14/c^23)^(
2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2)*(d^14/c^23)^(5/6)
+ (7*c^12*d^6*x^6 + 152*c^13*d^5*x^3 + 64*c^14*d^4)*sqrt(d^14/c^23) + (c^4*
d^11*x^7 + 80*c^5*d^10*x^4 + 160*c^6*d^9*x)*(d^14/c^23)^(1/6)) + 18*(c^8*d^
9*x^8 + 38*c^9*d^8*x^5 + 64*c^10*d^7*x^2)*(d^14/c^23)^(1/3))/(d^3*x^9 - 24*
c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 144*(4*d^3*x^9 + 21*c*d^2*x^6 - 264
*c^2*d*x^3 - 384*c^3)*sqrt(d*x^3 + c))/(c^4*d*x^10 - 8*c^5*x^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^8 (8c-dx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*x**3+c)**(1/2)/x**8/(-d*x**3+8*c)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^8 (8c-dx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}}{(dx^3-8\,c)^2x^8}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^8), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)^2}\,dx=\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)^2x^8}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)^2}\,dx=\int \frac{\sqrt{d\,x^3+c}}{x^8\,\left(8\,c-d\,x^3\right)^2}\,dx$$

```
[In] int((c + d*x^3)^(1/2)/(x^8*(8*c - d*x^3)^2),x)
[Out] int((c + d*x^3)^(1/2)/(x^8*(8*c - d*x^3)^2), x)
```

3.411 $\int \frac{x^{11}(c+dx^3)^{3/2}}{(8c-dx^3)^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 134

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{(8c-dx^3)^2} dx = \frac{1664c^3\sqrt{c+dx^3}}{d^4} + \frac{3x^6(c+dx^3)^{3/2}}{7d^2} + \frac{x^9(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} + \frac{2c(c+dx^3)^{3/2}\,(694c+51dx^3)}{21d^4} - \frac{4992c^{7/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}$$

```
[Out] 3/7*x^6*(d*x^3+c)^(3/2)/d^2+1/3*x^9*(d*x^3+c)^(3/2)/d/(-d*x^3+8*c)+2/21*c*(
d*x^3+c)^(3/2)*(51*d*x^3+694*c)/d^4-4992*c^(7/2)*arctanh(1/3*(d*x^3+c)^(1/2
)/c^(1/2))/d^4+1664*c^3*(d*x^3+c)^(1/2)/d^4
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {457, 99, 158, 152, 52, 65, 212}

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{(8c-dx^3)^2} dx = -\frac{4992c^{7/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4} + \frac{1664c^3\sqrt{c+dx^3}}{d^4} + \frac{2c(c+dx^3)^{3/2}\,(694c+51dx^3)}{21d^4} + \frac{3x^6(c+dx^3)^{3/2}}{7d^2} + \frac{x^9(c+dx^3)^{3/2}}{3d\,(8c-dx^3)}$$

```
[In] Int[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (1664*c^3*Sqrt[c + d*x^3])/d^4 + (3*x^6*(c + d*x^3)^(3/2))/(7*d^2) + (x^9*(
c + d*x^3)^(3/2))/(3*d*(8*c - d*x^3)) + (2*c*(c + d*x^3)^(3/2)*(694*c + 51*
d*x^3))/(21*d^4) - (4992*c^(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^4
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
```

```
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^3(c+dx)^{3/2}}{(8c-dx)^2}\,dx, x, x^3\right)\\
&= \frac{x^9(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} - \frac{\text{Subst}\left(\int \frac{x^2\sqrt{c+dx}\left(3c+\frac{9dx}{2}\right)}{8c-dx}\,dx, x, x^3\right)}{3d}\\
&= \frac{3x^6(c+dx^3)^{3/2}}{7d^2} + \frac{x^9(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} + \frac{2\text{Subst}\left(\int \frac{x\sqrt{c+dx}\left(-72c^2d-\frac{255}{2}cd^2x\right)}{8c-dx}\,dx, x, x^3\right)}{21d^3}\\
&= \frac{3x^6(c+dx^3)^{3/2}}{7d^2} + \frac{x^9(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} + \frac{2c(c+dx^3)^{3/2}\,(694c+51dx^3)}{21d^4}\\
&\quad - \frac{(832c^3)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)}{d^3}\\
&= \frac{1664c^3\sqrt{c+dx^3}}{d^4} + \frac{3x^6(c+dx^3)^{3/2}}{7d^2} + \frac{x^9(c+dx^3)^{3/2}}{3d\,(8c-dx^3)}\\
&\quad + \frac{2c(c+dx^3)^{3/2}\,(694c+51dx^3)}{21d^4} - \frac{(7488c^4)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{d^3}\\
&= \frac{1664c^3\sqrt{c+dx^3}}{d^4} + \frac{3x^6(c+dx^3)^{3/2}}{7d^2} + \frac{x^9(c+dx^3)^{3/2}}{3d\,(8c-dx^3)}\\
&\quad + \frac{2c(c+dx^3)^{3/2}\,(694c+51dx^3)}{21d^4} - \frac{(14976c^4)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d^4}
\end{aligned}$$

$$= \frac{1664c^3\sqrt{c+dx^3}}{d^4} + \frac{3x^6(c+dx^3)^{3/2}}{7d^2} + \frac{x^9(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} + \frac{2c(c+dx^3)^{3/2}\,(694c+51dx^3)}{21d^4} - \frac{4992c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.77

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{2\sqrt{c+dx^3}(-145328c^4+12206c^3dx^3+301c^2d^2x^6+16cd^3x^9+d^4x^{12})}{21d^4\,(-8c+dx^3)} - \frac{4992c^{7/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}$$

```
[In] Integrate[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
[Out] (2*Sqrt[c + d*x^3]*(-145328*c^4 + 12206*c^3*d*x^3 + 301*c^2*d^2*x^6 + 16*c*
d^3*x^9 + d^4*x^12))/(21*d^4*(-8*c + d*x^3)) - (4992*c^(7/2)*ArcTanh[Sqrt[c
 + d*x^3]/(3*Sqrt[c])])/d^4
```

Maple [A] (verified)

Time = 4.78 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.81

method	result
pseudoelliptic	$-\frac{39936\left(c^{4}\left(c-\frac{d\,x^{3}}{8}\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)+\frac{\left(\sqrt{c}\,d^{4}x^{12}+16c^{\frac{3}{2}}d^{3}x^{9}+301c^{\frac{5}{2}}d^{2}x^{6}+12206c^{\frac{7}{2}}d\,x^{3}-145328c^{\frac{9}{2}}\right)\sqrt{d\,x^{3}+c}}{419328}\right)}{\sqrt{c}\,(-d^{5}x^{3}+8c\,d^{4})}$
risch	$\frac{2(d^{3}x^{9}+24c\,d^{2}x^{6}+493c^{2}d\,x^{3}+16150c^{3})\sqrt{d\,x^{3}+c}}{21d^{4}}+\frac{576c^{4}\left(-\frac{86\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{9d\sqrt{c}}+\frac{8c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right.}{3d}\right.}{d^{3}}$
default	$\frac{d\left(\frac{2d\,x^{9}\sqrt{d\,x^{3}+c}}{21}+\frac{16c\,x^{6}\sqrt{d\,x^{3}+c}}{105}+\frac{2c^{2}x^{3}\sqrt{d\,x^{3}+c}}{105d}-\frac{4c^{3}\sqrt{d\,x^{3}+c}}{105d^{2}}\right)+\frac{32c\left(d\,x^{3}+c\right)^{\frac{5}{2}}}{15d}}{d^{3}}-\frac{128c^{2}\left(81c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)-(d\,x\right.}{3d^{4}}$
elliptic	$\frac{1536c^{4}\sqrt{d\,x^{3}+c}}{d^{4}(-d\,x^{3}+8c)}+\frac{2x^{9}\sqrt{d\,x^{3}+c}}{21d}+\frac{16c\,x^{6}\sqrt{d\,x^{3}+c}}{7d^{2}}+\frac{986c^{2}x^{3}\sqrt{d\,x^{3}+c}}{21d^{3}}+\frac{32300c^{3}\sqrt{d\,x^{3}+c}}{21d^{4}}+\frac{832ic^{3}\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootO}}\right.}{}$

```
[In] int(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -39936/c^(1/2)*(c^4*(c-1/8*d*x^3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))+1/41
9328*(c^(1/2)*d^4*x^12+16*c^(3/2)*d^3*x^9+301*c^(5/2)*d^2*x^6+12206*c^(7/2)
*d*x^3-145328*c^(9/2))*(d*x^3+c)^(1/2))/(-d^5*x^3+8*c*d^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.78

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx=\left[\frac{2\left(26208\,(c^3dx^3-8\,c^4)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+(d^4x^{12}+16\,cd^3x^9+301\,c^2d^2\right.}{21\,(d^5x^3-8\,cd^4)}\right.$$

```
[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [2/21*(26208*(c^3*d*x^3 - 8*c^4)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqr
t(c) + 10*c)/(d*x^3 - 8*c)) + (d^4*x^12 + 16*c*d^3*x^9 + 301*c^2*d^2*x^6 +
12206*c^3*d*x^3 - 145328*c^4)*sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4), 2/21*(5
2416*(c^3*d*x^3 - 8*c^4)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) +
(d^4*x^12 + 16*c*d^3*x^9 + 301*c^2*d^2*x^6 + 12206*c^3*d*x^3 - 145328*c^4)*
sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**11*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.89

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{2\left(26208\,c^{\frac{7}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right) + (dx^3+c)^{\frac{7}{2}} + 21\,(dx^3+c)^{\frac{5}{2}}c + 448\,(dx^3+c)^{\frac{3}{2}}c^2 + 15}{21\,d^4}$$

```
[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] 2/21*(26208*c^(7/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*
sqrt(c))) + (d*x^3 + c)^(7/2) + 21*(d*x^3 + c)^(5/2)*c + 448*(d*x^3 + c)^(3
/2)*c^2 + 15680*sqrt(d*x^3 + c)*c^3 - 16128*sqrt(d*x^3 + c)*c^4/(d*x^3 - 8*
c))/d^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.95

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{4992\,c^4\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d^4} - \frac{1536\,\sqrt{dx^3+c}c^4}{(dx^3-8\,c)d^4} + \frac{2\left((dx^3+c)^{\frac{7}{2}}d^{24} + 21\,(dx^3+c)^{\frac{5}{2}}cd^{24} + 448\,(dx^3+c)^{\frac{3}{2}}c^2d^{24} + 15680\,\sqrt{dx^3+c}c^3d^{24}\right)}{21\,d^{28}}$$

```
[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 4992*c^4*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 1536*sqrt(d*
x^3 + c)*c^4/((d*x^3 - 8*c)*d^4) + 2/21*((d*x^3 + c)^(7/2)*d^24 + 21*(d*x^3
 + c)^(5/2)*c*d^24 + 448*(d*x^3 + c)^(3/2)*c^2*d^24 + 15680*sqrt(d*x^3 + c)
*c^3*d^24)/d^28
```

Mupad [B] (verification not implemented)

Time = 8.32 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.10

$$\int \frac{x^{11}(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{2496\,c^{7/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^4}+\frac{32300\,c^3\,\sqrt{d\,x^3+c}}{21\,d^4}+\frac{2\,x^9\,\sqrt{d\,x^3+c}}{21\,d}+\frac{16\,c\,x^6\,\sqrt{d\,x^3+c}}{7\,d^2}+\frac{986\,c^2\,x^3\,\sqrt{d\,x^3+c}}{21\,d^3}+\frac{1536\,c^4\,\sqrt{d\,x^3+c}}{d^4\,\left(8\,c-d\,x^3\right)}$$

```
[In] int((x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
```

```
[Out] (2496*c^(7/2)*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3
)))/d^4 + (32300*c^3*(c + d*x^3)^(1/2))/(21*d^4) + (2*x^9*(c + d*x^3)^(1/2)
)/(21*d) + (16*c*x^6*(c + d*x^3)^(1/2))/(7*d^2) + (986*c^2*x^3*(c + d*x^3)^
(1/2))/(21*d^3) + (1536*c^4*(c + d*x^3)^(1/2))/(d^4*(8*c - d*x^3))
```

3.412 $\int \frac{x^8(c+dx^3)^{3/2}}{(8c-dx^3)^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 119

$$\int \frac{x^8(c+dx^3)^{3/2}}{(8c-dx^3)^2} dx = \frac{160c^2\sqrt{c+dx^3}}{d^3} + \frac{160c(c+dx^3)^{3/2}}{27d^3} + \frac{2(c+dx^3)^{5/2}}{15d^3} + \frac{64c(c+dx^3)^{5/2}}{27d^3(8c-dx^3)} - \frac{480c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}$$

```
[Out] 160/27*c*(d*x^3+c)^(3/2)/d^3+2/15*(d*x^3+c)^(5/2)/d^3+64/27*c*(d*x^3+c)^(5/
2)/d^3/(-d*x^3+8*c)-480*c^(5/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^3+16
0*c^2*(d*x^3+c)^(1/2)/d^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {457, 91, 81, 52, 65, 212}

$$\int \frac{x^8(c+dx^3)^{3/2}}{(8c-dx^3)^2} dx = -\frac{480c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3} + \frac{160c^2\sqrt{c+dx^3}}{d^3} + \frac{64c(c+dx^3)^{5/2}}{27d^3(8c-dx^3)} + \frac{160c(c+dx^3)^{3/2}}{27d^3} + \frac{2(c+dx^3)^{5/2}}{15d^3}$$

```
[In] Int[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (160*c^2*Sqrt[c + d*x^3])/d^3 + (160*c*(c + d*x^3)^(3/2))/(27*d^3) + (2*(c
+ d*x^3)^(5/2))/(15*d^3) + (64*c*(c + d*x^3)^(5/2))/(27*d^3*(8*c - d*x^3))
- (480*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x^2(c+dx)^{3/2}}{(8c-dx)^2}\,dx, x, x^3\right)$$

$$= \frac{64c(c+dx^3)^{5/2}}{27d^3(8c-dx^3)} - \frac{\text{Subst}\left(\int \frac{(c+dx)^{3/2}(168c^2d+9cd^2x)}{8c-dx}\,dx, x, x^3\right)}{27cd^3}$$

$$= \frac{2(c+dx^3)^{5/2}}{15d^3} + \frac{64c(c+dx^3)^{5/2}}{27d^3(8c-dx^3)} - \frac{(80c)\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)}{9d^2}$$

$$= \frac{160c(c+dx^3)^{3/2}}{27d^3} + \frac{2(c+dx^3)^{5/2}}{15d^3} + \frac{64c(c+dx^3)^{5/2}}{27d^3(8c-dx^3)} - \frac{(80c^2)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)}{d^2}$$

$$= \frac{160c^2\sqrt{c+dx^3}}{d^3} + \frac{160c(c+dx^3)^{3/2}}{27d^3} + \frac{2(c+dx^3)^{5/2}}{15d^3} + \frac{64c(c+dx^3)^{5/2}}{27d^3(8c-dx^3)} - \frac{(720c^3)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{d^2}$$

$$= \frac{160c^2\sqrt{c+dx^3}}{d^3} + \frac{160c(c+dx^3)^{3/2}}{27d^3} + \frac{2(c+dx^3)^{5/2}}{15d^3} + \frac{64c(c+dx^3)^{5/2}}{27d^3(8c-dx^3)} - \frac{(1440c^3)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d^3}$$

$$= \frac{160c^2\sqrt{c+dx^3}}{d^3} + \frac{160c(c+dx^3)^{3/2}}{27d^3} + \frac{2(c+dx^3)^{5/2}}{15d^3} + \frac{64c(c+dx^3)^{5/2}}{27d^3(8c-dx^3)} - \frac{480c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.78

$$\int \frac{x^8(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{2\sqrt{c+dx^3}(-29944c^3+2515c^2dx^3+62cd^2x^6+3d^3x^9)}{45d^3(-8c+dx^3)} - \frac{480c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}$$

```
[In] Integrate[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

[Out] (2*Sqrt[c + d*x^3]*(-29944*c^3 + 2515*c^2*d*x^3 + 62*c*d^2*x^6 + 3*d^3*x^9)
)/(45*d^3*(-8*c + d*x^3)) - (480*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c]
)])/d^3

Maple [A] (verified)

Time = 4.64 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$-\frac{3840\left(c^3\left(c-\frac{d x^3}{8}\right)\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{3\sqrt{c}}\right)+\frac{\sqrt{d x^3+c}\left(\sqrt{c}\,d^3x^9+\frac{62c^{\frac{3}{2}}d^2x^6}{3}+\frac{2515c^{\frac{5}{2}}d x^3}{3}-\frac{29944c^{\frac{7}{2}}}{3}\right)}{28800}\right)}{\sqrt{c}\,(-d^4x^3+8d^3c)}$
risch	$\frac{2(3d^2x^6+86cd x^3+3203c^2)\sqrt{d x^3+c}}{45d^3}+\frac{144c^3\left(-\frac{34\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{3\sqrt{c}}\right)}{9d\sqrt{c}}+\frac{4c\left(-\frac{\sqrt{d x^3+c}}{c\left(d x^3-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{3d}\right)}{d^2}$
default	$\frac{2(d x^3+c)^{\frac{5}{2}}}{15d^3}-\frac{32c\left(81c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{3\sqrt{c}}\right)-(d x^3+28c)\sqrt{d x^3+c}\right)}{9d^3}+\frac{64c^2\left(\frac{2\sqrt{d x^3+c}}{3}-3c\left(-\frac{\sqrt{d x^3+c}}{-d x^3+8c}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}}{\ldots}\right)}{\sqrt{c}}\right.\right.}{d^3}$
elliptic	$\frac{192c^3\sqrt{d x^3+c}}{d^3(-d x^3+8c)}+\frac{2x^6\sqrt{d x^3+c}}{15d}+\frac{172c x^3\sqrt{d x^3+c}}{45d^2}+\frac{6406c^2\sqrt{d x^3+c}}{45d^3}+\frac{80ic^2\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c d^2\right)^{\frac{1}{3}}\sqrt{\ldots}}{\ldots}\right)}{\ldots}$

[In] int(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)

[Out] -3840*(c^3*(c-1/8*d*x^3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))+1/28800*(d*x^
3+c)^(1/2)*(c^(1/2)*d^3*x^9+62/3*c^(3/2)*d^2*x^6+2515/3*c^(5/2)*d*x^3-29944
/3*c^(7/2)))/c^(1/2)/(-d^4*x^3+8*c*d^3)

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.84

$$\int \frac{x^8(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \left[\frac{2\left(5400\,(c^2dx^3-8\,c^3)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+(3\,d^3x^9+62\,cd^2x^6+2515\,c^2dx^3\right.}{45\,(d^4x^3-8\,cd^3)}\right.$$

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [2/45*(5400*(c^2*d*x^3 - 8*c^3)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt
(c) + 10*c)/(d*x^3 - 8*c)) + (3*d^3*x^9 + 62*c*d^2*x^6 + 2515*c^2*d*x^3 - 2
9944*c^3)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^3), 2/45*(10800*(c^2*d*x^3 - 8*
c^3)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (3*d^3*x^9 + 62*c*d^
2*x^6 + 2515*c^2*d*x^3 - 29944*c^3)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^8(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**8*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.90

$$\int \frac{x^8(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{2\left(5400\,c^{\frac{5}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+3\,(dx^3+c)^{\frac{5}{2}}+80\,(dx^3+c)^{\frac{3}{2}}c+3120\,\sqrt{dx^3+c}c^2-\frac{432}{}\right.}{45\,d^3}$$

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] 2/45*(5400*c^(5/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*s
qrt(c))) + 3*(d*x^3 + c)^(5/2) + 80*(d*x^3 + c)^(3/2)*c + 3120*sqrt(d*x^3 +
 c)*c^2 - 4320*sqrt(d*x^3 + c)*c^3/(d*x^3 - 8*c))/d^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.93

$$\int \frac{x^8(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{480\,c^3\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d^3} - \frac{192\sqrt{dx^3+c}c^3}{(dx^3-8\,c)d^3} + \frac{2\left(3\,(dx^3+c)^{\frac{5}{2}}d^{12}+80\,(dx^3+c)^{\frac{3}{2}}cd^{12}+3120\sqrt{dx^3+c}c^2d^{12}\right)}{45\,d^{15}}$$

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 480*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 192*sqrt(d*x^
3 + c)*c^3/((d*x^3 - 8*c)*d^3) + 2/45*(3*(d*x^3 + c)^(5/2)*d^12 + 80*(d*x^3
 + c)^(3/2)*c*d^12 + 3120*sqrt(d*x^3 + c)*c^2*d^12)/d^15
```

Mupad [B] (verification not implemented)

Time = 8.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.07

$$\int \frac{x^8(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{240\,c^{5/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^3} + \frac{6406\,c^2\,\sqrt{d\,x^3+c}}{45\,d^3} + \frac{2\,x^6\,\sqrt{d\,x^3+c}}{15\,d} + \frac{172\,c\,x^3\,\sqrt{d\,x^3+c}}{45\,d^2} + \frac{192\,c^3\,\sqrt{d\,x^3+c}}{d^3\,\left(8\,c-d\,x^3\right)}$$

```
[In] int((x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
```

```
[Out] (240*c^(5/2)*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)
))/d^3 + (6406*c^2*(c + d*x^3)^(1/2))/(45*d^3) + (2*x^6*(c + d*x^3)^(1/2))/
(15*d) + (172*c*x^3*(c + d*x^3)^(1/2))/(45*d^2) + (192*c^3*(c + d*x^3)^(1/2
))/(d^3*(8*c - d*x^3))
```

3.413 $\int \frac{x^5(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 97

$$\int \frac{x^5(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{14c\sqrt{c+dx^3}}{d^2} + \frac{14(c+dx^3)^{3/2}}{27d^2} + \frac{8(c+dx^3)^{5/2}}{27d^2(8c-dx^3)} - \frac{42c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}$$

```
[Out] 14/27*(d*x^3+c)^(3/2)/d^2+8/27*(d*x^3+c)^(5/2)/d^2/(-d*x^3+8*c)-42*c^(3/2)*
arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^2+14*c*(d*x^3+c)^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {457, 79, 52, 65, 212}

$$\int \frac{x^5(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = -\frac{42c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2} + \frac{8(c+dx^3)^{5/2}}{27d^2(8c-dx^3)} + \frac{14(c+dx^3)^{3/2}}{27d^2} + \frac{14c\sqrt{c+dx^3}}{d^2}$$

```
[In] Int[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (14*c*Sqrt[c + d*x^3])/d^2 + (14*(c + d*x^3)^(3/2))/(27*d^2) + (8*(c + d*x^
3)^(5/2))/(27*d^2*(8*c - d*x^3)) - (42*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*S
qrt[c])])/d^2
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x(c+dx)^{3/2}}{(8c-dx)^2}\,dx, x, x^3\right)$$

$$= \frac{8(c+dx^3)^{5/2}}{27d^2\,(8c-dx^3)} - \frac{7\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)}{9d}$$

$$=\frac{14(c+dx^3)^{3/2}}{27d^2}+\frac{8(c+dx^3)^{5/2}}{27d^2(8c-dx^3)}-\frac{(7c)\text{Subst}\left(\int\frac{\sqrt{c+dx}}{8c-dx}\,dx,x,x^3\right)}{d}$$

$$=\frac{14c\sqrt{c+dx^3}}{d^2}+\frac{14(c+dx^3)^{3/2}}{27d^2}+\frac{8(c+dx^3)^{5/2}}{27d^2(8c-dx^3)}-\frac{(63c^2)\,\text{Subst}\left(\int\frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{d}$$

$$=\frac{14c\sqrt{c+dx^3}}{d^2}+\frac{14(c+dx^3)^{3/2}}{27d^2}+\frac{8(c+dx^3)^{5/2}}{27d^2(8c-dx^3)}-\frac{(126c^2)\,\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{d^2}$$

$$=\frac{14c\sqrt{c+dx^3}}{d^2}+\frac{14(c+dx^3)^{3/2}}{27d^2}+\frac{8(c+dx^3)^{5/2}}{27d^2(8c-dx^3)}-\frac{42c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.84

$$\int\frac{x^5(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx=\frac{2\sqrt{c+dx^3}(-524c^2+44cdx^3+d^2x^6)}{9d^2(-8c+dx^3)}-\frac{42c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}$$

```
[In] Integrate[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(-524*c^2 + 44*c*d*x^3 + d^2*x^6))/(9*d^2*(-8*c + d*x^3)
) - (42*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^2
```

Maple [A] (verified)

Time = 4.41 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

method	result
pseudoelliptic	$\frac{\frac{2(dx^3+c)^{\frac{3}{2}}}{9}+\frac{34c\sqrt{dx^3+c}}{3}+6c^2\left(\frac{4\sqrt{dx^3+c}}{-dx^3+8c}-\frac{7\,\mathrm{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{\sqrt{c}}\right)}{d^2}$
risch	$\frac{2(dx^3+52c)\sqrt{dx^3+c}}{9d^2}+\frac{9c^2\left(-\frac{50\,\mathrm{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{9d\sqrt{c}}+\frac{8c\left(-\frac{\sqrt{dx^3+c}}{c(dx^3-8c)}+\frac{\mathrm{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{3d}\right)}{d}$
default	$-\frac{2\left(81c^{\frac{3}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)-(dx^3+28c)\sqrt{dx^3+c}\right)}{9d^2}+\frac{8c\left(\frac{2\sqrt{dx^3+c}}{3}-3c\left(-\frac{\sqrt{dx^3+c}}{-dx^3+8c}+\frac{\mathrm{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{\sqrt{c}}\right)\right)}{d^2}$
elliptic	$\frac{24c^2\sqrt{dx^3+c}}{d^2(-dx^3+8c)}+\frac{2x^3\sqrt{dx^3+c}}{9d}+\frac{104c\sqrt{dx^3+c}}{9d^2}+\frac{7ic\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{(-cd^2)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}(-cd^2)^{\frac{1}{3}}}{d}\right.}{(-cd^2)^{\frac{1}{3}}}}}{}\right.}{}$

```
[In] int(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/9*(d*x^3+c)^(3/2)+17/3*c*(d*x^3+c)^(1/2)+3*c^2*(4*(d*x^3+c)^(1/2)/(-d*
x^3+8*c)-7*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)))/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.98

$$\int \frac{x^5(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \left[\frac{189\,(cdx^3-8\,c^2)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+2\,(d^2x^6+44\,cdx^3-524\,c^2)\sqrt{dx^3+}}{9\,(d^3x^3-8\,cd^2)}\right.$$

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/9*(189*(c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c)
+ 10*c)/(d*x^3 - 8*c)) + 2*(d^2*x^6 + 44*c*d*x^3 - 524*c^2)*sqrt(d*x^3 + c)
)/(d^3*x^3 - 8*c*d^2), 2/9*(189*(c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/3*sqrt(
d*x^3 + c)*sqrt(-c)/c) + (d^2*x^6 + 44*c*d*x^3 - 524*c^2)*sqrt(d*x^3 + c))/
(d^3*x^3 - 8*c*d^2)]
```

Sympy [F]

$$\int \frac{x^5(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{x^5(c+dx^3)^{\frac{3}{2}}}{(-8c+dx^3)^2}\,dx$$

```
[In] integrate(x**5*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(x**5*(c + d*x**3)**(3/2)/(-8*c + d*x**3)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.96

$$\int \frac{x^5(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{189\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+2\,(dx^3+c)^{\frac{3}{2}}+102\,\sqrt{dx^3+c}c-\frac{216\,\sqrt{dx^3+c}c^2}{dx^3-8\,c}}{9\,d^2}$$

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] 1/9*(189*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqr
t(c))) + 2*(d*x^3 + c)^(3/2) + 102*sqrt(d*x^3 + c)*c - 216*sqrt(d*x^3 + c)*
c^2/(d*x^3 - 8*c))/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.96

$$\int \frac{x^5(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{42\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}d^2} - \frac{24\,\sqrt{dx^3+c}c^2}{(dx^3-8\,c)d^2} + \frac{2\left((dx^3+c)^{\frac{3}{2}}d^4+51\,\sqrt{dx^3+c}cd^4\right)}{9\,d^6}$$

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 42*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^2) - 24*sqrt(d*x^3
+ c)*c^2/((d*x^3 - 8*c)*d^2) + 2/9*((d*x^3 + c)^(3/2)*d^4 + 51*sqrt(d*x^3 +
 c)*c*d^4)/d^6
```

Mupad [B] (verification not implemented)

Time = 8.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10

$$\int \frac{x^5(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{104\,c\,\sqrt{d\,x^3+c}}{9\,d^2} + \frac{21\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^2} + \frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d} + \frac{24\,c^2\,\sqrt{d\,x^3+c}}{d^2\,\left(8\,c-d\,x^3\right)}$$

```
[In] int((x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
```

```
[Out] (104*c*(c + d*x^3)^(1/2))/(9*d^2) + (21*c^(3/2)*log((10*c + d*x^3 - 6*c^(1/
2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/d^2 + (2*x^3*(c + d*x^3)^(1/2))/(9*d)
 + (24*c^2*(c + d*x^3)^(1/2))/(d^2*(8*c - d*x^3))
```

3.414 $\int \frac{x^2(c+dx^3)^{3/2}}{(8c-dx^3)^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 77

$$\int \frac{x^2(c+dx^3)^{3/2}}{(8c-dx^3)^2} dx = \frac{\sqrt{c+dx^3}}{d} + \frac{(c+dx^3)^{3/2}}{3d(8c-dx^3)} - \frac{3\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

```
[Out] 1/3*(d*x^3+c)^(3/2)/d/(-d*x^3+8*c)-3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c
^(1/2)/d+(d*x^3+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {455, 43, 52, 65, 212}

$$\int \frac{x^2(c+dx^3)^{3/2}}{(8c-dx^3)^2} dx = -\frac{3\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d} + \frac{(c+dx^3)^{3/2}}{3d(8c-dx^3)} + \frac{\sqrt{c+dx^3}}{d}$$

```
[In] Int[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] Sqrt[c + d*x^3]/d + (c + d*x^3)^(3/2)/(3*d*(8*c - d*x^3)) - (3*Sqrt[c]*ArcT
anh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{(8c-dx)^2}\,dx, x, x^3\right) \\
&= \frac{\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} - \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right) \\
&= \frac{\sqrt{c+dx^3}}{d} + \frac{\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} - \frac{1}{2}(9c)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right) \\
&= \frac{\sqrt{c+dx^3}}{d} + \frac{\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} - \frac{(9c)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d}
\end{aligned}$$

$$= \frac{\sqrt{c+dx^3}}{d} + \frac{(c+dx^3)^{3/2}}{3d(8c-dx^3)} - \frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

$$\int \frac{x^2(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{(25c-2dx^3)\sqrt{c+dx^3}}{3d(8c-dx^3)} - \frac{3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

```
[In] Integrate[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
[Out] ((25*c - 2*d*x^3)*Sqrt[c + d*x^3])/(3*d*(8*c - d*x^3)) - (3*Sqrt[c]*ArcTanh
[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d
```

Maple [A] (verified)

Time = 4.49 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.82

method	result
default	$\frac{\frac{2\sqrt{d\,x^{3}+c}}{3}-3c\left(-\frac{\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{\sqrt{c}}\right)}{d}$
pseudoelliptic	$\frac{\frac{2\sqrt{d\,x^{3}+c}}{3}+3c\left(\frac{\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{\sqrt{c}}\right)}{d}$
risch	$\frac{2\sqrt{d\,x^{3}+c}}{3d}+9c\left(-\frac{4\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{9d\sqrt{c}}+\frac{c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{3d}\right)$
elliptic	$\frac{3c\sqrt{d\,x^{3}+c}}{d(-d\,x^{3}+8c)}+\frac{2\sqrt{d\,x^{3}+c}}{3d}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x\right.}{-3\left(-c\,d^{2}\right.}}}{}\right.}{}$

[In] int(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)

[Out] (2/3*(d*x^3+c)^(1/2)-3*c*(-(d*x^3+c)^(1/2)/(-d*x^3+8*c)+arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)))/d

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.10

$$\int \frac{x^2(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \left[\frac{9\,(dx^3-8\,c)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+2\,(2\,dx^3-25\,c)\sqrt{dx^3+c}}{6\,(d^2x^3-8\,cd)},\ \frac{9\,(dx^3-8\,c)\cdot}{}\right.$$

[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")

[Out] [1/6*(9*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 2*(2*d*x^3 - 25*c)*sqrt(d*x^3 + c))/(d^2*x^3 - 8*c*d), 1

```
/3*(9*(d*x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (2*d*
x^3 - 25*c)*sqrt(d*x^3 + c))/(d^2*x^3 - 8*c*d)]
```

Sympy [F]

$$\int \frac{x^2(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{x^2(c+dx^3)^{\frac{3}{2}}}{(-8c+dx^3)^2}\,dx$$

```
[In] integrate(x**2*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
[Out] Integral(x**2*(c + d*x**3)**(3/2)/(-8*c + d*x**3)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03

$$\int \frac{x^2(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{9\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+4\sqrt{dx^3+c}-\frac{18\sqrt{dx^3+c}c}{dx^3-8c}}{6d}$$

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
[Out] 1/6*(9*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(
c))) + 4*sqrt(d*x^3 + c) - 18*sqrt(d*x^3 + c)*c/(d*x^3 - 8*c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

$$\int \frac{x^2(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{3c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d}+\frac{2\sqrt{dx^3+c}}{3d}-\frac{3\sqrt{dx^3+c}c}{(dx^3-8c)d}$$

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
[Out] 3*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) + 2/3*sqrt(d*x^3 + c)
/d - 3*sqrt(d*x^3 + c)*c/((d*x^3 - 8*c)*d)
```

Mupad [B] (verification not implemented)

Time = 8.00 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.13

$$\int \frac{x^2(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{2\sqrt{d\,x^3+c}}{3\,d}+\frac{3\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{2\,d}+\frac{3\,c\,\sqrt{d\,x^3+c}}{d\,\left(8\,c-d\,x^3\right)}$$

```
[In] int((x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
[Out] (2*(c + d*x^3)^(1/2))/(3*d) + (3*c^(1/2)*log((10*c + d*x^3 - 6*c^(1/2)*(c +
 d*x^3)^(1/2))/(8*c - d*x^3)))/(2*d) + (3*c*(c + d*x^3)^(1/2))/(d*(8*c - d*
x^3))
```

3.415 $\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 85

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)^2} dx = \frac{3\sqrt{c+dx^3}}{8(8c-dx^3)} - \frac{3\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}} - \frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

```
[Out] -3/32*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)-1/96*arctanh((d*x^3+c)^(
1/2)/c^(1/2))/c^(1/2)+3/8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {457, 100, 162, 65, 214, 212}

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)^2} dx = -\frac{3\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}} - \frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}} + \frac{3\sqrt{c+dx^3}}{8(8c-dx^3)}$$

```
[In] Int[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)^2),x]
```

```
[Out] (3*Sqrt[c + d*x^3])/(8*(8*c - d*x^3)) - (3*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[
c])])/(32*Sqrt[c]) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(96*Sqrt[c])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x(8c-dx)^2}\,dx, x, x^3\right)$$

$$= \frac{3\sqrt{c+dx^3}}{8\,(8c-dx^3)} - \frac{\text{Subst}\left(\int \frac{-c^2d+\frac{7}{2}cd^2x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{24cd}$$

$$= \frac{3\sqrt{c+dx^3}}{8(8c-dx^3)} + \frac{1}{192}\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right) - \frac{1}{64}(9d)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= \frac{3\sqrt{c+dx^3}}{8(8c-dx^3)} - \frac{9}{32}\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right) + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{96d}$$

$$= \frac{3\sqrt{c+dx^3}}{8(8c-dx^3)} - \frac{3\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)^2}\,dx = \frac{3\sqrt{c+dx^3}}{8(8c-dx^3)} - \frac{3\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}} - \frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

```
[In] Integrate[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)^2),x]
[Out] (3*Sqrt[c + d*x^3])/(8*(8*c - d*x^3)) - (3*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[
c])])/(32*Sqrt[c]) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(96*Sqrt[c])
```

Maple [A] (verified)

Time = 4.53 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$-\frac{3\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{32\sqrt{c}} - \frac{\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{96\sqrt{c}} + \frac{3\sqrt{d\,x^3+c}}{-8d\,x^3+64c}$
default	$\frac{\frac{2d\,x^3\sqrt{d\,x^3+c}}{9}+\frac{8c\sqrt{d\,x^3+c}}{9}-\frac{2c^{\frac{3}{2}}\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3}}{64c^2} + \frac{\frac{2\sqrt{d\,x^3+c}}{3}-3c\left(-\frac{\sqrt{d\,x^3+c}}{-d\,x^3+8c}+\frac{\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{\sqrt{c}}\right)}{8c} + \frac{81c^{\frac{3}{2}}\,\text{arctanh}(}{}$
elliptic	Expression too large to display

```
[In] int((d*x^3+c)^(3/2)/x/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
[Out] -3/32*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)-1/96*arctanh((d*x^3+c)^(
1/2)/c^(1/2))/c^(1/2)+3*(d*x^3+c)^(1/2)/(-8*d*x^3+64*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.59

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)^2}\,dx = \left[\frac{9\,(dx^3-8\,c)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+(dx^3-8\,c)\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-}{192\,(cdx^3-8\,c^2)}\right.$$

```
[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/192*(9*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10
*c)/(d*x^3 - 8*c)) + (d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*s
qrt(c) + 2*c)/x^3) - 72*sqrt(d*x^3 + c)*c)/(c*d*x^3 - 8*c^2), 1/96*((d*x^3
- 8*c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + 9*(d*x^3 - 8*c)*sqrt(-
c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 36*sqrt(d*x^3 + c)*c)/(c*d*x^3
- 8*c^2)]
```

Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)^2}\,dx = \int \frac{(c+dx^3)^{\frac{3}{2}}}{x(-8c+dx^3)^2}\,dx$$

```
[In] integrate((d*x**3+c)**(3/2)/x/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral((c + d*x**3)**(3/2)/(x*(-8*c + d*x**3)**2), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)^2x}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.82

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)^2}\,dx = \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}} + \frac{3\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{32\sqrt{-c}} - \frac{3\sqrt{dx^3+c}}{8(dx^3-8c)}$$

```
[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 1/96*arctan(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) + 3/32*arctan(1/3*sqrt(d*x^3
 + c)/sqrt(-c))/sqrt(-c) - 3/8*sqrt(d*x^3 + c)/(d*x^3 - 8*c)
```

Mupad [B] (verification not implemented)

Time = 8.75 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.19

$$\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)^2}\,dx = \frac{3\sqrt{d\,x^3+c}}{8\,(8\,c-d\,x^3)} + \frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)\,\left(10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}\right)^9}{x^6\,(8\,c-d\,x^3)^9}\right)}{192\,\sqrt{c}}$$

```
[In] int((c + d*x^3)^(3/2)/(x*(8*c - d*x^3)^2),x)
```

```
[Out] (3*(c + d*x^3)^(1/2))/(8*(8*c - d*x^3)) + log((((c + d*x^3)^(1/2) - c^(1/2)
)^3*((c + d*x^3)^(1/2) + c^(1/2))*(10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/
2))^9)/(x^6*(8*c - d*x^3)^9))/(192*c^(1/2))
```

3.416 $\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 121

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)^2}\,dx = \frac{5d\sqrt{c+dx^3}}{96c(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{24x^3(8c-dx^3)} + \frac{3d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{3/2}} - \frac{7d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{3/2}}$$

```
[Out] 3/128*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-7/384*d*arctanh((d*x^3
+c)^(1/2)/c^(1/2))/c^(3/2)+5/96*d*(d*x^3+c)^(1/2)/c/(-d*x^3+8*c)-1/24*(d*x^
3+c)^(1/2)/x^3/(-d*x^3+8*c)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {457, 100, 156, 162, 65, 214, 212}

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)^2}\,dx = \frac{3d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{3/2}} - \frac{7d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{3/2}} + \frac{5d\sqrt{c+dx^3}}{96c(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{24x^3(8c-dx^3)}$$

```
[In] Int[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)^2),x]
```

```
[Out] (5*d*Sqrt[c + d*x^3])/(96*c*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(24*x^3*(8*c -
 d*x^3)) + (3*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(128*c^(3/2)) - (7*d*
ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(384*c^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(8c-dx)^2}\,dx,x,x^3\right)\\
&= -\frac{\sqrt{c+dx^3}}{24x^3\left(8c-dx^3\right)}-\frac{\text{Subst}\left(\int \frac{-14c^2d-\frac{19}{2}cd^2x}{x(8c-dx)^2\sqrt{c+dx}}\,dx,x,x^3\right)}{24c}\\
&= \frac{5d\sqrt{c+dx^3}}{96c\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24x^3\left(8c-dx^3\right)}+\frac{\text{Subst}\left(\int \frac{126c^3d^2+45c^2d^3x}{x(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{1728c^3d}\\
&= \frac{5d\sqrt{c+dx^3}}{96c\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24x^3\left(8c-dx^3\right)}+\frac{(7d)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{768c}\\
&\quad+\frac{\left(9d^2\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{256c}\\
&= \frac{5d\sqrt{c+dx^3}}{96c\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24x^3\left(8c-dx^3\right)}+\frac{7\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{384c}\\
&\quad+\frac{(9d)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{128c}\\
&= \frac{5d\sqrt{c+dx^3}}{96c\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24x^3\left(8c-dx^3\right)}+\frac{3d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{3/2}}-\frac{7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.80

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^4\left(8c-dx^3\right)^2}\,dx=\frac{\frac{4\sqrt{c}\left(4c-5dx^3\right)\sqrt{c+dx^3}}{-8cx^3+dx^6}+9d\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-7d\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{3/2}}$$

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)^2),x]
```

```
[Out] ((4*Sqrt[c]*(4*c - 5*d*x^3)*Sqrt[c + d*x^3])/(-8*c*x^3 + d*x^6) + 9*d*ArcTa
nh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 7*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(38
4*c^(3/2))
```

Maple [A] (verified)

Time = 4.64 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$\frac{d\left(-\frac{7\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)d\,x^3+2\sqrt{d\,x^3+c}\,\sqrt{c}}{2d\,x^3c^{\frac{3}{2}}}+\frac{\frac{9\sqrt{d\,x^3+c}}{-d\,x^3+8c}+\frac{9\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{2\sqrt{c}}}{c}\right)}{192}$
risch	$-\frac{\sqrt{d\,x^3+c}}{192c\,x^3}-\frac{d\left(\frac{7\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3\sqrt{c}}-\frac{\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{\sqrt{c}}-6c\left(-\frac{\sqrt{d\,x^3+c}}{c\left(d\,x^3-8c\right)}+\frac{\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)\right)}{128c}$
default	$\frac{-\frac{c\sqrt{d\,x^3+c}}{3x^3}+\frac{2d\sqrt{d\,x^3+c}}{3}-\sqrt{c}\,d\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{64c^2}+\frac{d\left(\frac{2d\,x^3\sqrt{d\,x^3+c}}{9}+\frac{8c\sqrt{d\,x^3+c}}{9}-\frac{2c^{\frac{3}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3}\right)}{256c^3}+\frac{d\left(2\sqrt{d\right.}{}}{}$
elliptic	Expression too large to display

```
[In] int((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/192*d*(-1/2*(7*arctanh((d*x^3+c)^(1/2)/c^(1/2))*d*x^3+2*(d*x^3+c)^(1/2)*c
^(1/2))/d/x^3/c^(3/2)+9*((d*x^3+c)^(1/2)/(-d*x^3+8*c)+1/2*arctanh(1/3*(d*x^
3+c)^(1/2)/c^(1/2))/c^(1/2))/c)
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.31

$$\int \frac{(c+dx^3)^{3/2}}{x^4\,(8c-dx^3)^2}\,dx = \left[\frac{9\,(d^2x^6-8\,cdx^3)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+7\,(d^2x^6-8\,cdx^3)\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}}{x^3}\right.}{768\,(c^2dx^6-8\,c^3x^3)}\right.$$

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/768*(9*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt
(c) + 10*c)/(d*x^3 - 8*c)) + 7*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 - 2
*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 8*(5*c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c
))/(c^2*d*x^6 - 8*c^3*x^3), 1/384*(7*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*arctan(
sqrt(d*x^3 + c)*sqrt(-c)/c) - 9*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*arctan(1/3*s
qrt(d*x^3 + c)*sqrt(-c)/c) - 4*(5*c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c))/(c^2*d*
x^6 - 8*c^3*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*x**3+c)**(3/2)/x**4/(-d*x**3+8*c)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)^2x^4}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)^2}\,dx = \frac{7\,d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{384\,\sqrt{-c}c} - \frac{3\,d\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{128\,\sqrt{-c}c} - \frac{5\,(dx^3+c)^{\frac{3}{2}}d-9\,\sqrt{dx^3+c}cd}{96\,\left((dx^3+c)^2-10\,(dx^3+c)c+9\,c^2\right)c}$$

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="giac")
[Out] 7/384*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 3/128*d*arctan(1/3*
sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/96*(5*(d*x^3 + c)^(3/2)*d - 9*sq
rt(d*x^3 + c)*c*d)/(((d*x^3 + c)^2 - 10*(d*x^3 + c)*c + 9*c^2)*c)
```

Mupad [B] (verification not implemented)

Time = 8.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx^3)^{3/2}}{x^4 (8c-dx^3)^2}\,dx = \frac{\frac{9\,d\,\sqrt{d\,x^3+c}}{32}-\frac{5\,d\,\left(d\,x^3+c\right)^{3/2}}{32\,c}}{3\,\left(d\,x^3+c\right)^2-30\,c\,\left(d\,x^3+c\right)+27\,c^2} + \frac{d\,\left(\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{\sqrt{c^3}}\right)\,1\mathrm{i}-\frac{\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^3}}\right)\,9\mathrm{i}}{7}\right)\,7\mathrm{i}}{384\,\sqrt{c^3}}$$

```
[In] int((c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)^2),x)
```

```
[Out] ((9*d*(c + d*x^3)^(1/2))/32 - (5*d*(c + d*x^3)^(3/2))/(32*c))/(3*(c + d*x^3
)^2 - 30*c*(c + d*x^3) + 27*c^2) + (d*(atanh((c*(c + d*x^3)^(1/2))/(c^3)^(1
/2))*1i - (atanh((c*(c + d*x^3)^(1/2))/(3*(c^3)^(1/2)))*9i)/7)*7i)/(384*(c^
3)^(1/2))
```

3.417 $\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 161

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)^2}\,dx = \frac{7d^2\sqrt{c+dx^3}}{512c^2(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{48x^6(8c-dx^3)} - \frac{23d\sqrt{c+dx^3}}{384cx^3(8c-dx^3)} + \frac{15d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2048c^{5/2}} - \frac{17d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{5/2}}$$

```
[Out] 15/2048*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)-17/2048*d^2*arctan
h((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)+7/512*d^2*(d*x^3+c)^(1/2)/c^2/(-d*x^3+8*
c)-1/48*(d*x^3+c)^(1/2)/x^6/(-d*x^3+8*c)-23/384*d*(d*x^3+c)^(1/2)/c/x^3/(-d
*x^3+8*c)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {457, 100, 156, 162, 65, 214, 212}

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)^2}\,dx = \frac{15d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2048c^{5/2}} - \frac{17d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{5/2}} + \frac{7d^2\sqrt{c+dx^3}}{512c^2(8c-dx^3)} - \frac{23d\sqrt{c+dx^3}}{384cx^3(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{48x^6(8c-dx^3)}$$

```
[In] Int[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)^2),x]
```

```
[Out] (7*d^2*Sqrt[c + d*x^3])/(512*c^2*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(48*x^6*(
8*c - d*x^3)) - (23*d*Sqrt[c + d*x^3])/(384*c*x^3*(8*c - d*x^3)) + (15*d^2*
```

```
ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2048*c^(5/2)) - (17*d^2*ArcTanh[Sqrt
[c + d*x^3]/Sqrt[c]])/(2048*c^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^3(8c-dx)^2}\,dx,x,x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{48x^6\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{-23c^2d-\frac{37}{2}cd^2x}{x^2(8c-dx)^2\sqrt{c+dx}}\,dx,x,x^3\right)}{48c}$$

$$= -\frac{\sqrt{c+dx^3}}{48x^6\left(8c-dx^3\right)} - \frac{23d\sqrt{c+dx^3}}{384cx^3\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{102c^3d^2+\frac{69}{2}c^2d^3x}{x(8c-dx)^2\sqrt{c+dx}}\,dx,x,x^3\right)}{384c^3}$$

$$= \frac{7d^2\sqrt{c+dx^3}}{512c^2\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48x^6\left(8c-dx^3\right)} - \frac{23d\sqrt{c+dx^3}}{384cx^3\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{-918c^4d^3-189c^3d^4x}{x(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{27648c^5d}$$

$$= \frac{7d^2\sqrt{c+dx^3}}{512c^2\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48x^6\left(8c-dx^3\right)} - \frac{23d\sqrt{c+dx^3}}{384cx^3\left(8c-dx^3\right)} + \frac{\left(17d^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{4096c^2} + \frac{\left(45d^3\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{4096c^2}$$

$$= \frac{7d^2\sqrt{c+dx^3}}{512c^2\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48x^6\left(8c-dx^3\right)} - \frac{23d\sqrt{c+dx^3}}{384cx^3\left(8c-dx^3\right)} + \frac{(17d)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{2048c^2} + \frac{\left(45d^2\right)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{2048c^2}$$

$$= \frac{7d^2\sqrt{c+dx^3}}{512c^2\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48x^6\left(8c-dx^3\right)} - \frac{23d\sqrt{c+dx^3}}{384cx^3\left(8c-dx^3\right)} + \frac{15d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2048c^{5/2}} - \frac{17d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{5/2}}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.70

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^7\left(8c-dx^3\right)^2}\,dx=\frac{\frac{4\sqrt{c}\sqrt{c+dx^3}\left(32c^2+92cdx^3-21d^2x^6\right)}{-8cx^6+dx^9}+45d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-51d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6144c^{5/2}}$$

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)^2),x]
```

```
[Out] ((4*Sqrt[c]*Sqrt[c + d*x^3]*(32*c^2 + 92*c*d*x^3 - 21*d^2*x^6))/(-8*c*x^6 +
 d*x^9) + 45*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 51*d^2*ArcTanh[Sqrt
[c + d*x^3]/Sqrt[c]])/(6144*c^(5/2))
```

Maple [A] (verified)

Time = 4.65 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$-\frac{408\left(-\frac{15\left(c-\frac{d x^3}{8}\right) d^2 x^6 \operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{3\sqrt{c}}\right)}{17}+\left(c-\frac{d x^3}{8}\right) d^2 x^6 \operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)-\frac{7\left(d^2 x^6 \sqrt{c}-\frac{92 d x^3 c^{\frac{3}{2}}}{21}-\frac{32 c^{\frac{5}{2}}}{21}\right)\sqrt{d x^3+c}}{34}\right)}{c^{\frac{5}{2}}\left(-6144 d x^9+49152 c x^6\right)}$
risch	$-\frac{\sqrt{d x^3+c}\,\left(3 d x^3+c\right)}{384 c^2 x^6}-\frac{3 d^2\left(\frac{17 \operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{24\sqrt{c}}-\frac{11 \operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{3\sqrt{c}}\right)}{24\sqrt{c}}-\frac{c\left(-\frac{\sqrt{d x^3+c}}{c\left(d x^3-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{3\sqrt{c}}\right)}{3 c^{\frac{3}{2}}}\right)}{2}\right)}{256 c^2}$
default	$\frac{-\frac{c\sqrt{d x^3+c}}{6x^6}-\frac{5d\sqrt{d x^3+c}}{12x^3}-\frac{d^2 \operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{4\sqrt{c}}}{64c^2}+\frac{d\left(-\frac{c\sqrt{d x^3+c}}{3x^3}+\frac{2d\sqrt{d x^3+c}}{3}-\sqrt{c}\,d \operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)\right)}{256c^3}+$ $3d^2\left(\frac{2 d x^3 \sqrt{\ }}{9}\right.$
elliptic	Expression too large to display

```
[In] int((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -408/c^(5/2)*(-15/17*(c-1/8*d*x^3)*d^2*x^6*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1
/2))+(c-1/8*d*x^3)*d^2*x^6*arctanh((d*x^3+c)^(1/2)/c^(1/2))-7/34*(d^2*x^6*c
^(1/2)-92/21*d*x^3*c^(3/2)-32/21*c^(5/2))*(d*x^3+c)^(1/2))/(-6144*d*x^9+491
52*c*x^6)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.93

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)^2}\,dx = \left[\frac{45\,(d^3x^9-8\,cd^2x^6)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+51\,(d^3x^9-8\,cd^2x^6)\sqrt{c}\log\left(\frac{dx^3-2}{}\right.}{12288\,(c^3dx^9-8\,c^4x^6)}\right.$$

```
[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/12288*(45*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)
*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 51*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d
*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 8*(21*c*d^2*x^6 - 92*c^2*d*x
^3 - 32*c^3)*sqrt(d*x^3 + c))/(c^3*d*x^9 - 8*c^4*x^6), 1/6144*(51*(d^3*x^9
- 8*c*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 45*(d^3*x^9 -
8*c*d^2*x^6)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 4*(21*c*d^2*
x^6 - 92*c^2*d*x^3 - 32*c^3)*sqrt(d*x^3 + c))/(c^3*d*x^9 - 8*c^4*x^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*x**3+c)**(3/2)/x**7/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)^2x^7}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^7), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.80

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)^2}\,dx = \frac{17\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{2048\,\sqrt{-c}c^2} - \frac{15\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{2048\,\sqrt{-c}c^2} - \frac{3\,\sqrt{dx^3+c}d^2}{512\,(dx^3-8\,c)c^2} - \frac{3\,(dx^3+c)^{\frac{3}{2}}d^2-2\,\sqrt{dx^3+c}cd^2}{384\,c^2d^2x^6}$$

```
[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 17/2048*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 15/2048*d^2*a
rctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 3/512*sqrt(d*x^3 + c)*
d^2/((d*x^3 - 8*c)*c^2) - 1/384*(3*(d*x^3 + c)^(3/2)*d^2 - 2*sqrt(d*x^3 + c
)*c*d^2)/(c^2*d^2*x^6)
```

Mupad [B] (verification not implemented)

Time = 8.59 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)^2}\,dx = \frac{\frac{81\,d^2\,\sqrt{d\,x^3+c}}{512} - \frac{67\,d^2\,(d\,x^3+c)^{3/2}}{256\,c} + \frac{21\,d^2\,(d\,x^3+c)^{5/2}}{512\,c^2}}{33\,c\,(d\,x^3+c)^2 - 57\,c^2\,(d\,x^3+c) - 3\,(d\,x^3+c)^3 + 27\,c^3} + \frac{d^2\left(\operatorname{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{\sqrt{c^5}}\right)\,1\mathrm{i} - \frac{\operatorname{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^5}}\right)\,15\mathrm{i}}{17}\right)\,17\mathrm{i}}{2048\,\sqrt{c^5}}$$

```
[In] int((c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)^2),x)
```

```
[Out] ((81*d^2*(c + d*x^3)^(1/2))/512 - (67*d^2*(c + d*x^3)^(3/2))/(256*c) + (21*
d^2*(c + d*x^3)^(5/2))/(512*c^2))/(33*c*(c + d*x^3)^2 - 57*c^2*(c + d*x^3)
- 3*(c + d*x^3)^3 + 27*c^3) + (d^2*(atanh((c^2*(c + d*x^3)^(1/2))/(c^5)^(1/
2))*1i - (atanh((c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2)))*15i)/17)*17i)/(204
8*(c^5)^(1/2))
```

3.418 $\int \frac{x^7(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 681

$$\int \frac{x^7(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{103cx^2\sqrt{c+dx^3}}{13d^2}+\frac{19x^5\sqrt{c+dx^3}}{39d}+\frac{5906c^2\sqrt{c+dx^3}}{13d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{x^5(c+dx^3)^{3/2}}{3d(8c-dx^3)}+\frac{108\sqrt{3}c^{13/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{8/3}}-\frac{108c^{13/6}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{8/3}}+\frac{108c^{13/6}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{8/3}}-\frac{2953\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{13d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{5906\sqrt{2}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{13\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 1/3*x^5*(d*x^3+c)^(3/2)/d/(-d*x^3+8*c)-108*c^(13/6)*arctanh(1/3*(c^(1/3)+d^
(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(8/3)+108*c^(13/6)*arctanh(1/3*(d*x^3
```

```
+c)^(1/2)/c^(1/2))/d^(8/3)+108*c^(13/6)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*
3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/d^(8/3)+103/13*c*x^2*(d*x^3+c)^(1/2)/d^2+1
9/39*x^5*(d*x^3+c)^(1/2)/d+5906/13*c^2*(d*x^3+c)^(1/2)/d^(8/3)/(d^(1/3)*x+c
^(1/3)*(1+3^(1/2)))+5906/39*c^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*
x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/
2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))
^2)^(1/2)*3^(3/4)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(
1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-2953/13*3^(1/4)*c^(7/3)*(c^(1/3)+d^(1/
3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/
2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d
^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(8/3)/(d*x^3+c)^(1/2
)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 681, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.519$, Rules used = {478, 595, 596, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{x^7\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx = \frac{5906\sqrt{2}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)\right.}{13\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{2953\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{13d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{108\sqrt{3}c^{13/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{8/3}}-\frac{108c^{13/6}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{8/3}}$$

$$+\frac{108c^{13/6}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{8/3}}+\frac{5906c^2\sqrt{c+dx^3}}{13d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$+\frac{103cx^2\sqrt{c+dx^3}}{13d^2}+\frac{x^5\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}+\frac{19x^5\sqrt{c+dx^3}}{39d}$$

```
[In] Int[(x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (103*c*x^2*Sqrt[c + d*x^3])/(13*d^2) + (19*x^5*Sqrt[c + d*x^3])/(39*d) + (5
906*c^2*Sqrt[c + d*x^3])/(13*d^(8/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) +
 (x^5*(c + d*x^3)^(3/2))/(3*d*(8*c - d*x^3)) + (108*Sqrt[3]*c^(13/6)*ArcTan
[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/d^(8/3) - (108*c
^(13/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]])/d^(8/
3) + (108*c^(13/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^(8/3) - (2953*3^
(1/4)*Sqrt[2 - Sqrt[3]]*c^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/
3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)], -7 - 4*Sqrt[3]])/(13*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/(
(1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (5906*Sqrt[2]*c^(7
/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1
/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(13
*3^(1/4)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 595

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
```

```
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\text{integral} = \frac{x^5(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} - \frac{\int \frac{x^4\sqrt{c+dx^3}\left(5c+\frac{19dx^3}{2}\right)}{8c-dx^3}\,dx}{3d}$$

$$= \frac{19x^5\sqrt{c+dx^3}}{39d} + \frac{x^5(c+dx^3)^{3/2}}{3d(8c-dx^3)} + \frac{2\int \frac{x^4\left(-\frac{825c^2d}{2}-\frac{2163}{4}cd^2x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{39d^2}$$

$$= \frac{103cx^2\sqrt{c+dx^3}}{13d^2} + \frac{19x^5\sqrt{c+dx^3}}{39d} + \frac{x^5(c+dx^3)^{3/2}}{3d(8c-dx^3)} + \frac{4\int \frac{x\left(-8652c^3d^2-\frac{62013}{4}c^2d^3x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{273d^4}$$

$$= \frac{103cx^2\sqrt{c+dx^3}}{13d^2} + \frac{19x^5\sqrt{c+dx^3}}{39d} + \frac{x^5(c+dx^3)^{3/2}}{3d(8c-dx^3)} + \frac{4\int \left(\frac{62013c^2d^2x}{4\sqrt{c+dx^3}} - \frac{132678c^3d^2x}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{273d^4}$$

$$= \frac{103cx^2\sqrt{c+dx^3}}{13d^2} + \frac{19x^5\sqrt{c+dx^3}}{39d} + \frac{x^5(c+dx^3)^{3/2}}{3d(8c-dx^3)} + \frac{(2953c^2)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{13d^2} - \frac{(1944c^3)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{d^2}$$

$$= \frac{103cx^2\sqrt{c+dx^3}}{13d^2} + \frac{19x^5\sqrt{c+dx^3}}{39d} + \frac{x^5(c+dx^3)^{3/2}}{3d(8c-dx^3)} + \frac{(162c^2)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{d^3}$$

$$+ \frac{(2953c^2)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{13d^{7/3}} - \frac{\left(162c^{7/3}\right)\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{d^{7/3}} - \frac{\left(2953\left(1-\sqrt{3}\right)c^{7/3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{13d^{7/3}} + \frac{\left(486c^{8/3}\right)\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{d^{5/3}}$$

$$= \frac{103cx^2\sqrt{c+dx^3}}{13d^2} + \frac{19x^5\sqrt{c+dx^3}}{39d} + \frac{5906c^2\sqrt{c+dx^3}}{13d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^5\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}$$

$$- \frac{2953\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4}{13d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{5906\sqrt{2}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{13\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$- \frac{\left(324c^{8/3}\right)\text{Subst}\left(\int\frac{1}{9-cx^2}\,dx,x,\frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{d^{8/3}}$$

$$+ \frac{\left(162c^{8/3}\right)\text{Subst}\left(\int\frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{d^{5/3}}$$

$$- \frac{\left(648c^{5/3}\right)\text{Subst}\left(\int\frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{d^{2/3}}$$

$$= \frac{103cx^2\sqrt{c+dx^3}}{13d^2} + \frac{19x^5\sqrt{c+dx^3}}{39d} + \frac{5906c^2\sqrt{c+dx^3}}{13d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^5\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}$$

$$+ \frac{108\sqrt{3}c^{13/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{8/3}} - \frac{108c^{13/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{8/3}}$$

$$- \frac{2953\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{}\right.}{13d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{5906\sqrt{2}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{13\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{\left(324c^{8/3}\right)\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d^{8/3}}$$

$$= \frac{103cx^2\sqrt{c+dx^3}}{13d^2} + \frac{19x^5\sqrt{c+dx^3}}{39d} + \frac{5906c^2\sqrt{c+dx^3}}{13d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^5(c+dx^3)^{3/2}}{3d(8c-dx^3)} + \frac{108\sqrt{3}c^{13/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{8/3}} - \frac{108c^{13/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{8/3}} + \frac{108c^{13/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{8/3}} - \frac{2953\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4}{13d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{5906\sqrt{2}c^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{13\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 8.44 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.28

$$\int \frac{x^7(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{80x^2(-412c^3-388c^2dx^3+25cd^2x^6+d^3x^9)+4120c^2x^2(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellI}}{520d^2(-8c}$$

```
[In] Integrate[(x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (80*x^2*(-412*c^3 - 388*c^2*d*x^3 + 25*c*d^2*x^6 + d^3*x^9) + 4120*c^2*x^2*
(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c),
(d*x^3)/(8*c)] + 2953*c*d*x^5*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/
3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(520*d^2*(-8*c + d*x^3)*Sqrt[
c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.51 (sec) , antiderivative size = 921, normalized size of antiderivative = 1.35

method	result	size
elliptic	Expression too large to display	921
risch	Expression too large to display	1769
default	Expression too large to display	2224

```
[In] int(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 24*c^2/d^2*x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+2/13*x^5*(d*x^3+c)^(1/2)/d+64/1
3*c*x^2*(d*x^3+c)^(1/2)/d^2-5906/39*I*c^2/d^3*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^
2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I
*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3
^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2)))+72*I*c^2/d^5*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x
+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(
x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(
-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3)
)^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^
2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(
1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-
I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d
)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 17.75 (sec) , antiderivative size = 2580, normalized size of antiderivative = 3.79

$$\int \frac{x^7(c+dx^3)^{3/2}}{(8c-dx^3)^2} dx = \text{Too large to display}$$

```
[In] integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
[Out] -1/13*(5906*(c^2*d*x^3 - 8*c^3)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstr
assPInverse(0, -4*c/d, x)) + 117*(d^4*x^3 - 8*c*d^3 - sqrt(-3)*(d^4*x^3 - 8
*c*d^3))*(c^13/d^16)^(1/6)*log(14693280768*((d^16*x^9 + 318*c*d^15*x^6 + 12
00*c^2*d^14*x^3 + 640*c^3*d^13 + sqrt(-3)*(d^16*x^9 + 318*c*d^15*x^6 + 1200
*c^2*d^14*x^3 + 640*c^3*d^13))*(c^13/d^16)^(5/6) + 6*(2*c^11*d^2*x^7 + 160*
c^12*d*x^4 + 320*c^13*x - 6*(5*c^3*d^12*x^5 + 32*c^4*d^11*x^2 - sqrt(-3)*(5
*c^3*d^12*x^5 + 32*c^4*d^11*x^2))*(c^13/d^16)^(2/3) - (7*c^7*d^7*x^6 + 152*
c^8*d^6*x^3 + 64*c^9*d^5 + sqrt(-3)*(7*c^7*d^7*x^6 + 152*c^8*d^6*x^3 + 64*c
^9*d^5))*(c^13/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^5*d^10*x^7 + 64*c^6*d
^9*x^4 + 32*c^7*d^8*x)*sqrt(c^13/d^16) + 18*(c^9*d^5*x^8 + 38*c^10*d^4*x^5
+ 64*c^11*d^3*x^2 - sqrt(-3)*(c^9*d^5*x^8 + 38*c^10*d^4*x^5 + 64*c^11*d^3*x
^2))*(c^13/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3))
 - 117*(d^4*x^3 - 8*c*d^3 - sqrt(-3)*(d^4*x^3 - 8*c*d^3))*(c^13/d^16)^(1/6)
*log(-14693280768*((d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3
*d^13 + sqrt(-3)*(d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d
^13))*(c^13/d^16)^(5/6) - 6*(2*c^11*d^2*x^7 + 160*c^12*d*x^4 + 320*c^13*x -
 6*(5*c^3*d^12*x^5 + 32*c^4*d^11*x^2 - sqrt(-3)*(5*c^3*d^12*x^5 + 32*c^4*d^
11*x^2))*(c^13/d^16)^(2/3) - (7*c^7*d^7*x^6 + 152*c^8*d^6*x^3 + 64*c^9*d^5
+ sqrt(-3)*(7*c^7*d^7*x^6 + 152*c^8*d^6*x^3 + 64*c^9*d^5))*(c^13/d^16)^(1/3
))*sqrt(d*x^3 + c) - 36*(5*c^5*d^10*x^7 + 64*c^6*d^9*x^4 + 32*c^7*d^8*x)*sq
rt(c^13/d^16) + 18*(c^9*d^5*x^8 + 38*c^10*d^4*x^5 + 64*c^11*d^3*x^2 - sqrt(
-3)*(c^9*d^5*x^8 + 38*c^10*d^4*x^5 + 64*c^11*d^3*x^2))*(c^13/d^16)^(1/6))/(
d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 117*(d^4*x^3 - 8*c*d^3
 + sqrt(-3)*(d^4*x^3 - 8*c*d^3))*(c^13/d^16)^(1/6)*log(14693280768*((d^16*x
^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13 - sqrt(-3)*(d^16*x^9
 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c^13/d^16)^(5/6) +
6*(2*c^11*d^2*x^7 + 160*c^12*d*x^4 + 320*c^13*x - 6*(5*c^3*d^12*x^5 + 32*c^
4*d^11*x^2 + sqrt(-3)*(5*c^3*d^12*x^5 + 32*c^4*d^11*x^2))*(c^13/d^16)^(2/3)
 - (7*c^7*d^7*x^6 + 152*c^8*d^6*x^3 + 64*c^9*d^5 - sqrt(-3)*(7*c^7*d^7*x^6
+ 152*c^8*d^6*x^3 + 64*c^9*d^5))*(c^13/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5
*c^5*d^10*x^7 + 64*c^6*d^9*x^4 + 32*c^7*d^8*x)*sqrt(c^13/d^16) + 18*(c^9*d^
5*x^8 + 38*c^10*d^4*x^5 + 64*c^11*d^3*x^2 + sqrt(-3)*(c^9*d^5*x^8 + 38*c^10
*d^4*x^5 + 64*c^11*d^3*x^2))*(c^13/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 1
92*c^2*d*x^3 - 512*c^3)) - 117*(d^4*x^3 - 8*c*d^3 + sqrt(-3)*(d^4*x^3 - 8*c
*d^3))*(c^13/d^16)^(1/6)*log(-14693280768*((d^16*x^9 + 318*c*d^15*x^6 + 120
```

```
0*c^2*d^14*x^3 + 640*c^3*d^13 - sqrt(-3)*(d^16*x^9 + 318*c*d^15*x^6 + 1200*
c^2*d^14*x^3 + 640*c^3*d^13))*(c^13/d^16)^(5/6) - 6*(2*c^11*d^2*x^7 + 160*c
^12*d*x^4 + 320*c^13*x - 6*(5*c^3*d^12*x^5 + 32*c^4*d^11*x^2 + sqrt(-3)*(5*
c^3*d^12*x^5 + 32*c^4*d^11*x^2))*(c^13/d^16)^(2/3) - (7*c^7*d^7*x^6 + 152*c
^8*d^6*x^3 + 64*c^9*d^5 - sqrt(-3)*(7*c^7*d^7*x^6 + 152*c^8*d^6*x^3 + 64*c^
9*d^5))*(c^13/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^5*d^10*x^7 + 64*c^6*d^
9*x^4 + 32*c^7*d^8*x)*sqrt(c^13/d^16) + 18*(c^9*d^5*x^8 + 38*c^10*d^4*x^5 +
 64*c^11*d^3*x^2 + sqrt(-3)*(c^9*d^5*x^8 + 38*c^10*d^4*x^5 + 64*c^11*d^3*x^
2))*(c^13/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3))
+ 234*(d^4*x^3 - 8*c*d^3)*(c^13/d^16)^(1/6)*log(14693280768*((d^16*x^9 + 31
8*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13)*(c^13/d^16)^(5/6) + 6*(c^1
1*d^2*x^7 + 80*c^12*d*x^4 + 160*c^13*x + 6*(5*c^3*d^12*x^5 + 32*c^4*d^11*x^
2)*(c^13/d^16)^(2/3) + (7*c^7*d^7*x^6 + 152*c^8*d^6*x^3 + 64*c^9*d^5)*(c^13
/d^16)^(1/3))*sqrt(d*x^3 + c) + 18*(5*c^5*d^10*x^7 + 64*c^6*d^9*x^4 + 32*c^
7*d^8*x)*sqrt(c^13/d^16) + 18*(c^9*d^5*x^8 + 38*c^10*d^4*x^5 + 64*c^11*d^3*
x^2)*(c^13/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3))
 - 234*(d^4*x^3 - 8*c*d^3)*(c^13/d^16)^(1/6)*log(-14693280768*((d^16*x^9 +
318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13)*(c^13/d^16)^(5/6) - 6*(c
^11*d^2*x^7 + 80*c^12*d*x^4 + 160*c^13*x + 6*(5*c^3*d^12*x^5 + 32*c^4*d^11*
x^2)*(c^13/d^16)^(2/3) + (7*c^7*d^7*x^6 + 152*c^8*d^6*x^3 + 64*c^9*d^5)*(c^
13/d^16)^(1/3))*sqrt(d*x^3 + c) + 18*(5*c^5*d^10*x^7 + 64*c^6*d^9*x^4 + 32*
c^7*d^8*x)*sqrt(c^13/d^16) + 18*(c^9*d^5*x^8 + 38*c^10*d^4*x^5 + 64*c^11*d^
3*x^2)*(c^13/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3
)) - 2*(d^3*x^8 + 24*c*d^2*x^5 - 412*c^2*d*x^2)*sqrt(d*x^3 + c))/(d^4*x^3 -
 8*c*d^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7 (c + dx^3)^{3/2}}{(8c - dx^3)^2} \, dx = \text{Timed out}$$

```
[In] integrate(x**7*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^7(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}x^7}{(dx^3-8\,c)^2}\,dx$$

```
[In] integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)*x^7/(d*x^3 - 8*c)^2, x)
```

Giac [F]

$$\int \frac{x^7(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}x^7}{(dx^3-8\,c)^2}\,dx$$

```
[In] integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
[Out] integrate((d*x^3 + c)^(3/2)*x^7/(d*x^3 - 8*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{x^7\,(d\,x^3+c)^{3/2}}{(8\,c-d\,x^3)^2}\,dx$$

```
[In] int((x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
[Out] int((x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2, x)
```

3.419 $\int \frac{x^4(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 657

$$\int \frac{x^4(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{13x^2\sqrt{c+dx^3}}{21d} + \frac{265c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^2(c+dx^3)^{3/2}}{3d(8c-dx^3)} + \frac{9\sqrt{3}c^{7/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}} - \frac{9c^{7/6}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{5/3}} + \frac{9c^{7/6}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{5/3}} - \frac{265\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{14d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{265\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{7\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 1/3*x^2*(d*x^3+c)^(3/2)/d/(-d*x^3+8*c)-9*c^(7/6)*arctanh(1/3*(c^(1/3)+d^(1/
3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(5/3)+9*c^(7/6)*arctanh(1/3*(d*x^3+c)^(1
```

```
/2)/c^(1/2))/d^(5/3)+9*c^(7/6)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(
d*x^3+c)^(1/2))*3^(1/2)/d^(5/3)+13/21*x^2*(d*x^3+c)^(1/2)/d+265/7*c*(d*x^3+
c)^(1/2)/d^(5/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+265/21*c^(4/3)*(c^(1/3)+d^
(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^
(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^
(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1
/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-265/14*3^(
1/4)*c^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/
(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((
c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(
1/2)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3
)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 657, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {478, 595, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{x^4(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{265\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)\right.}{7\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{265\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{14d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{9\sqrt{3}c^{7/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}}-\frac{9c^{7/6}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{5/3}}$$

$$+\frac{9c^{7/6}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{5/3}}+\frac{265c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{x^2(c+dx^3)^{3/2}}{3d\,(8c-dx^3)}+\frac{13x^2\sqrt{c+dx^3}}{21d}$$

```
[In] Int[(x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (13*x^2*Sqrt[c + d*x^3])/(21*d) + (265*c*Sqrt[c + d*x^3])/(7*d^(5/3)*((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (x^2*(c + d*x^3)^(3/2))/(3*d*(8*c - d*x^3)
) + (9*Sqrt[3]*c^(7/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[
```

```
c + d*x^3]])/d^(5/3) - (9*c^(7/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6
)*Sqrt[c + d*x^3])]])/d^(5/3) + (9*c^(7/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c
])])/d^(5/3) - (265*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(4/3)*(c^(1/3) + d^(1/3)*x)
*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) +
d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(14*d^(5/3)*Sqrt[(c^(1/3)*(c
^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]
) + (265*Sqrt[2]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1
/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(7*3^(1/4)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/(
(1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
```

```
/; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 595

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^2(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} - \frac{\int \frac{x\sqrt{c+dx^3}\left(2c+\frac{13dx^3}{2}\right)}{8c-dx^3}\,dx}{3d} \\
&= \frac{13x^2\sqrt{c+dx^3}}{21d} + \frac{x^2(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} + \frac{2\int \frac{x\left(-111c^2d-\frac{795}{4}cd^2x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{21d^2} \\
&= \frac{13x^2\sqrt{c+dx^3}}{21d} + \frac{x^2(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} + \frac{2\int \left(\frac{795cdx}{4\sqrt{c+dx^3}} - \frac{1701c^2dx}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{21d^2} \\
&= \frac{13x^2\sqrt{c+dx^3}}{21d} + \frac{x^2(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} + \frac{(265c)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{14d} - \frac{(162c^2)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{d}
\end{aligned}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d} + \frac{x^2(c+dx^3)^{3/2}}{3d(8c-dx^3)} + \frac{(27c)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{2d^2}$$

$$+ \frac{(265c)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{14d^{4/3}} - \frac{\left(27c^{4/3}\right)\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{2d^{4/3}}$$

$$- \frac{\left(265\left(1-\sqrt{3}\right)c^{4/3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{14d^{4/3}} + \frac{\left(81c^{5/3}\right)\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{2d^{2/3}}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d} + \frac{265c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^2(c+dx^3)^{3/2}}{3d(8c-dx^3)}$$

$$- \frac{265\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{}\right.}{14d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{265\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$- \frac{\left(27c^{5/3}\right)\text{Subst}\left(\int \frac{1}{9-cx^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{d^{5/3}}$$

$$+ \frac{\left(27c^{5/3}\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{2d^{2/3}}$$

$$- \left(54c^{2/3}\sqrt[3]{d}\right)\text{Subst}\left(\int \frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d} + \frac{265c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^2\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}$$

$$+ \frac{9\sqrt{3}c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}} - \frac{9c^{7/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{5/3}}$$

$$- \frac{265\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right.}{14d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{265\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{\left(27c^{5/3}\right)\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{d^{5/3}}$$

$$
= \frac{13x^2\sqrt{c+dx^3}}{21d} + \frac{265c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^2(c+dx^3)^{3/2}}{3d\,(8c-dx^3)} + \frac{9\sqrt{3}c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}} - \frac{9c^{7/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{5/3}} + \frac{9c^{7/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{5/3}} - \frac{265\sqrt[4]{3}\sqrt{2-\sqrt{3}}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{\vphantom{3}}\right.}{14d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{265\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.27

$$
\int \frac{x^4(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = -\frac{16x^2(37c^2+35cdx^3-2d^2x^6)+74cx^2(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+53dx^5(-8c}{112d\left(-8c+dx^3\right)\sqrt{c+dx^3}}
$$

```
[In] Integrate[(x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] -1/112*(16*x^2*(37*c^2 + 35*c*d*x^3 - 2*d^2*x^6) + 74*c*x^2*(-8*c + d*x^3)*
Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)]
 + 53*d*x^5*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -
((d*x^3)/c), (d*x^3)/(8*c)])/(d*(-8*c + d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.52 (sec) , antiderivative size = 897, normalized size of antiderivative = 1.37

method	result	size
elliptic	Expression too large to display	897
default	Expression too large to display	1748
risch	Expression too large to display	1758

```
[In] int(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3*c/d*x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+2/7*x^2*(d*x^3+c)^(1/2)/d-265/21*I*c
/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*
x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*Ellipt
icE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+6*I*c/d^4*2^(1/2)*sum(1/_alp
ha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/
3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3
^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+
(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_a
lpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alph
a*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c
*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*
d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c
))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.79 (sec) , antiderivative size = 2568, normalized size of antiderivative = 3.91

$$\int \frac{x^4(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] -1/28*(1060*(c*d*x^3 - 8*c^2)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstras
sPInverse(0, -4*c/d, x)) + 21*(d^3*x^3 - 8*c*d^2 - sqrt(-3)*(d^3*x^3 - 8*c*
d^2))*(c^7/d^10)^(1/6)*log(59049*((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9
*x^3 + 640*c^3*d^8 + sqrt(-3)*(d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3
 + 640*c^3*d^8))*(c^7/d^10)^(5/6) + 6*(2*c^6*d^2*x^7 + 160*c^7*d*x^4 + 320*
c^8*x - 6*(5*c^2*d^8*x^5 + 32*c^3*d^7*x^2 - sqrt(-3)*(5*c^2*d^8*x^5 + 32*c^
3*d^7*x^2))*(c^7/d^10)^(2/3) - (7*c^4*d^5*x^6 + 152*c^5*d^4*x^3 + 64*c^6*d^
3 + sqrt(-3)*(7*c^4*d^5*x^6 + 152*c^5*d^4*x^3 + 64*c^6*d^3))*(c^7/d^10)^(1/
3))*sqrt(d*x^3 + c) - 36*(5*c^3*d^7*x^7 + 64*c^4*d^6*x^4 + 32*c^5*d^5*x)*sq
rt(c^7/d^10) + 18*(c^5*d^4*x^8 + 38*c^6*d^3*x^5 + 64*c^7*d^2*x^2 - sqrt(-3)
*(c^5*d^4*x^8 + 38*c^6*d^3*x^5 + 64*c^7*d^2*x^2))*(c^7/d^10)^(1/6))/(d^3*x^
9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 21*(d^3*x^3 - 8*c*d^2 - sqrt
(-3)*(d^3*x^3 - 8*c*d^2))*(c^7/d^10)^(1/6)*log(-59049*((d^11*x^9 + 318*c*d^
10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 + sqrt(-3)*(d^11*x^9 + 318*c*d^10*x
^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8))*(c^7/d^10)^(5/6) - 6*(2*c^6*d^2*x^7 +
 160*c^7*d*x^4 + 320*c^8*x - 6*(5*c^2*d^8*x^5 + 32*c^3*d^7*x^2 - sqrt(-3)*(
5*c^2*d^8*x^5 + 32*c^3*d^7*x^2))*(c^7/d^10)^(2/3) - (7*c^4*d^5*x^6 + 152*c^
5*d^4*x^3 + 64*c^6*d^3 + sqrt(-3)*(7*c^4*d^5*x^6 + 152*c^5*d^4*x^3 + 64*c^6
*d^3))*(c^7/d^10)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^3*d^7*x^7 + 64*c^4*d^6*x
^4 + 32*c^5*d^5*x)*sqrt(c^7/d^10) + 18*(c^5*d^4*x^8 + 38*c^6*d^3*x^5 + 64*c
^7*d^2*x^2 - sqrt(-3)*(c^5*d^4*x^8 + 38*c^6*d^3*x^5 + 64*c^7*d^2*x^2))*(c^7
/d^10)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 21*(d^3
*x^3 - 8*c*d^2 + sqrt(-3)*(d^3*x^3 - 8*c*d^2))*(c^7/d^10)^(1/6)*log(59049*(
(d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 - sqrt(-3)*(d^1
1*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8))*(c^7/d^10)^(5/6)
+ 6*(2*c^6*d^2*x^7 + 160*c^7*d*x^4 + 320*c^8*x - 6*(5*c^2*d^8*x^5 + 32*c^3*
d^7*x^2 + sqrt(-3)*(5*c^2*d^8*x^5 + 32*c^3*d^7*x^2))*(c^7/d^10)^(2/3) - (7*
c^4*d^5*x^6 + 152*c^5*d^4*x^3 + 64*c^6*d^3 - sqrt(-3)*(7*c^4*d^5*x^6 + 152*
c^5*d^4*x^3 + 64*c^6*d^3))*(c^7/d^10)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c^3*d^
7*x^7 + 64*c^4*d^6*x^4 + 32*c^5*d^5*x)*sqrt(c^7/d^10) + 18*(c^5*d^4*x^8 + 3
8*c^6*d^3*x^5 + 64*c^7*d^2*x^2 + sqrt(-3)*(c^5*d^4*x^8 + 38*c^6*d^3*x^5 + 6
4*c^7*d^2*x^2))*(c^7/d^10)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 -
 512*c^3)) - 21*(d^3*x^3 - 8*c*d^2 + sqrt(-3)*(d^3*x^3 - 8*c*d^2))*(c^7/d^1
0)^(1/6)*log(-59049*((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^
3*d^8 - sqrt(-3)*(d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^
```

```
8))*(c^7/d^10)^(5/6) - 6*(2*c^6*d^2*x^7 + 160*c^7*d*x^4 + 320*c^8*x - 6*(5*
c^2*d^8*x^5 + 32*c^3*d^7*x^2 + sqrt(-3)*(5*c^2*d^8*x^5 + 32*c^3*d^7*x^2))*(
c^7/d^10)^(2/3) - (7*c^4*d^5*x^6 + 152*c^5*d^4*x^3 + 64*c^6*d^3 - sqrt(-3)*
(7*c^4*d^5*x^6 + 152*c^5*d^4*x^3 + 64*c^6*d^3))*(c^7/d^10)^(1/3))*sqrt(d*x^
3 + c) - 36*(5*c^3*d^7*x^7 + 64*c^4*d^6*x^4 + 32*c^5*d^5*x)*sqrt(c^7/d^10)
+ 18*(c^5*d^4*x^8 + 38*c^6*d^3*x^5 + 64*c^7*d^2*x^2 + sqrt(-3)*(c^5*d^4*x^8
 + 38*c^6*d^3*x^5 + 64*c^7*d^2*x^2))*(c^7/d^10)^(1/6))/(d^3*x^9 - 24*c*d^2*
x^6 + 192*c^2*d*x^3 - 512*c^3)) + 42*(d^3*x^3 - 8*c*d^2)*(c^7/d^10)^(1/6)*l
og(59049*((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8)*(c^7
/d^10)^(5/6) + 6*(c^6*d^2*x^7 + 80*c^7*d*x^4 + 160*c^8*x + 6*(5*c^2*d^8*x^5
 + 32*c^3*d^7*x^2)*(c^7/d^10)^(2/3) + (7*c^4*d^5*x^6 + 152*c^5*d^4*x^3 + 64
*c^6*d^3)*(c^7/d^10)^(1/3))*sqrt(d*x^3 + c) + 18*(5*c^3*d^7*x^7 + 64*c^4*d^
6*x^4 + 32*c^5*d^5*x)*sqrt(c^7/d^10) + 18*(c^5*d^4*x^8 + 38*c^6*d^3*x^5 + 6
4*c^7*d^2*x^2)*(c^7/d^10)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 -
512*c^3)) - 42*(d^3*x^3 - 8*c*d^2)*(c^7/d^10)^(1/6)*log(-59049*((d^11*x^9 +
 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8)*(c^7/d^10)^(5/6) - 6*(c^6
*d^2*x^7 + 80*c^7*d*x^4 + 160*c^8*x + 6*(5*c^2*d^8*x^5 + 32*c^3*d^7*x^2)*(c
^7/d^10)^(2/3) + (7*c^4*d^5*x^6 + 152*c^5*d^4*x^3 + 64*c^6*d^3)*(c^7/d^10)^
(1/3))*sqrt(d*x^3 + c) + 18*(5*c^3*d^7*x^7 + 64*c^4*d^6*x^4 + 32*c^5*d^5*x)
*sqrt(c^7/d^10) + 18*(c^5*d^4*x^8 + 38*c^6*d^3*x^5 + 64*c^7*d^2*x^2)*(c^7/d
^10)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 4*(2*d^2*
x^5 - 37*c*d*x^2)*sqrt(d*x^3 + c))/(d^3*x^3 - 8*c*d^2)
```

Sympy [F]

$$\int \frac{x^4(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{x^4(c+dx^3)^{\frac{3}{2}}}{(-8c+dx^3)^2}\,dx$$

```
[In] integrate(x**4*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
[Out] Integral(x**4*(c + d*x**3)**(3/2)/(-8*c + d*x**3)**2, x)
```

Maxima [F]

$$\int \frac{x^4(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}x^4}{(dx^3-8\,c)^2}\,dx$$

```
[In] integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)*x^4/(d*x^3 - 8*c)^2, x)
```

Giac [F]

$$\int \frac{x^4(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}x^4}{(dx^3-8\,c)^2}\,dx$$

```
[In] integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
[Out] integrate((d*x^3 + c)^(3/2)*x^4/(d*x^3 - 8*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{x^4\,(d\,x^3+c)^{3/2}}{(8\,c-d\,x^3)^2}\,dx$$

```
[In] int((x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
[Out] int((x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2, x)
```

3.420 $\int \frac{x(c+dx^3)^{3/2}}{(8c-dx^3)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 638

$$\begin{aligned}
&\int \frac{x(c+dx^3)^{3/2}}{(8c-dx^3)^2} dx = \frac{19\sqrt{c+dx^3}}{8d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} \\
&+\frac{3x^2\sqrt{c+dx^3}}{8(8c-dx^3)}+\frac{9\sqrt{3}\sqrt[6]{c}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{16d^{2/3}} \\
&-\frac{9\sqrt[6]{c}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{16d^{2/3}}+\frac{9\sqrt[6]{c}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{16d^{2/3}} \\
&-\frac{19\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{16d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+\frac{19\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}$$

```
[Out] -9/16*c^(1/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^
(2/3)+9/16*c^(1/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(2/3)+9/16*c^(1/6
```

```
)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/d^(2/
3)+3/8*x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+19/8*(d*x^3+c)^(1/2)/d^(2/3)/(d^(1/
3)*x+c^(1/3)*(1+3^(1/2)))+19/24*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1
/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2
^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/
2)))^2)^(1/2)*3^(3/4)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/
(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-19/16*3^(1/4)*c^(1/3)*(c^(1/3)+d^(
1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(
1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x
+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(2/3)/(d*x^3+c)^(1
/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {479, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{x(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{19\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-\right.}{4\sqrt{2}\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{19\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{16d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{9\sqrt{3}\sqrt[6]{c}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{16d^{2/3}}-\frac{9\sqrt[6]{c}\,\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{16d^{2/3}}$$

$$+\frac{9\sqrt[6]{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{16d^{2/3}}+\frac{19\sqrt{c+dx^3}}{8d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{3x^2\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}$$

```
[In] Int[(x*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]

[Out] (19*Sqrt[c + d*x^3])/(8*d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (3*x
^2*Sqrt[c + d*x^3])/(8*(8*c - d*x^3)) + (9*Sqrt[3]*c^(1/6)*ArcTan[(Sqrt[3]*
c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(16*d^(2/3)) - (9*c^(1/6)*
ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(16*d^(2/3))
```

```
+ (9*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(16*d^(2/3)) - (19*3^(1/
4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*
d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[A
rcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)], -7 - 4*Sqrt[3]])/(16*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (19*c^(1/3)*(c^(1/3)
+ d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)
*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*3^(1/
4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3x^2\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}+\frac{\int \frac{x\left(-15c^2d-\frac{57}{2}cd^2x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{24cd}\\
&= \frac{3x^2\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}+\frac{\int \left(\frac{57cdx}{2\sqrt{c+dx^3}}-\frac{243c^2dx}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{24cd}\\
&= \frac{3x^2\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}+\frac{19}{16}\int \frac{x}{\sqrt{c+dx^3}}\,dx-\frac{1}{8}(81c)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx\\
&= \frac{3x^2\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}+\frac{27\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{32d}\\
&\quad+\frac{19\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{16\sqrt[3]{d}}-\frac{\left(27\sqrt[3]{c}\right)\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{32\sqrt[3]{d}}\\
&\quad-\frac{\left(19\left(1-\sqrt{3}\right)\sqrt[3]{c}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{16\sqrt[3]{d}}+\frac{1}{32}\left(81c^{2/3}\sqrt[3]{d}\right)\int \frac{x^2}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx
\end{aligned}$$

$$= \frac{19\sqrt{c+dx^3}}{8d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{3x^2\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}$$

$$- \frac{19\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right.}{16d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{19\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$- \frac{\left(27c^{2/3}\right)\text{Subst}\left(\int \frac{1}{9-cx^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{16d^{2/3}}$$

$$+ \frac{1}{32}\left(27c^{2/3}\sqrt[3]{d}\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right) - \frac{\left(27d^{4/3}\right)\text{Subst}\left(\int \frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx, x, \frac{1+}{\sqrt{c}}\right.}{8\sqrt[3]{c}}$$

$$= \frac{19\sqrt{c+dx^3}}{8d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{3x^2\sqrt{c+dx^3}}{8\left(8c-dx^3\right)} + \frac{9\sqrt{3}\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{16d^{2/3}} - \frac{9\sqrt[6]{c}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{16d^{2/3}} - \frac{19\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{16d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{19\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{\left(27c^{2/3}\right)\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{16d^{2/3}}$$

$$= \frac{19\sqrt{c+dx^3}}{8d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{3x^2\sqrt{c+dx^3}}{8\left(8c-dx^3\right)} + \frac{9\sqrt{3}\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{16d^{2/3}} - \frac{9\sqrt[6]{c}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{16d^{2/3}} + \frac{9\sqrt[6]{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{16d^{2/3}} - \frac{19\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{16d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{19\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.19 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.22

$$\int \frac{x(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \frac{x^2\left(\frac{240(c+dx^3)}{8c-dx^3}-25\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-\frac{19dx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{5}{3},\right.}{c}\right.}{640\sqrt{c+dx^3}}$$

```
[In] Integrate[(x*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (x^2*((240*(c + d*x^3))/(8*c - d*x^3) - 25*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3
, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] - (19*d*x^3*Sqrt[1 + (d*x^3)/c]
*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/c))/(640*Sqrt[c +
 d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.49 (sec) , antiderivative size = 874, normalized size of antiderivative = 1.37

method	result	size
default	Expression too large to display	874
elliptic	Expression too large to display	874

```
[In] int(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-19/24*I*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x
+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d
^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(
I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2
)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*
3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)))^(1/2)))+3/8*I/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1
/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-
1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1
/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)
```

```
^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/
3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*
(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/
c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.99 (sec) , antiderivative size = 2335, normalized size of antiderivative = 3.66

$$\int \frac{x(c+dx^3)^{3/2}}{(8c-dx^3)^2} dx = \text{Too large to display}$$

```
[In] integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] -1/64*(24*sqrt(d*x^3 + c)*d*x^2 + 152*(d*x^3 - 8*c)*sqrt(d)*weierstrassZeta
(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 3*(d^2*x^3 - 8*c*d - sqrt(
-3)*(d^2*x^3 - 8*c*d))*(c/d^4)^(1/6)*log(59049/4*((d^6*x^9 + 318*c*d^5*x^6
+ 1200*c^2*d^4*x^3 + 640*c^3*d^3 + sqrt(-3)*(d^6*x^9 + 318*c*d^5*x^6 + 1200
*c^2*d^4*x^3 + 640*c^3*d^3))*(c/d^4)^(5/6) + 6*(2*c*d^2*x^7 + 160*c^2*d*x^4
 + 320*c^3*x - 6*(5*c*d^4*x^5 + 32*c^2*d^3*x^2 - sqrt(-3)*(5*c*d^4*x^5 + 32
*c^2*d^3*x^2))*(c/d^4)^(2/3) - (7*c*d^3*x^6 + 152*c^2*d^2*x^3 + 64*c^3*d +
sqrt(-3)*(7*c*d^3*x^6 + 152*c^2*d^2*x^3 + 64*c^3*d))*(c/d^4)^(1/3))*sqrt(d*
x^3 + c) - 36*(5*c*d^4*x^7 + 64*c^2*d^3*x^4 + 32*c^3*d^2*x)*sqrt(c/d^4) + 1
8*(c*d^3*x^8 + 38*c^2*d^2*x^5 + 64*c^3*d*x^2 - sqrt(-3)*(c*d^3*x^8 + 38*c^2
*d^2*x^5 + 64*c^3*d*x^2))*(c/d^4)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*
d*x^3 - 512*c^3)) - 3*(d^2*x^3 - 8*c*d - sqrt(-3)*(d^2*x^3 - 8*c*d))*(c/d^4
)^(1/6)*log(-59049/4*((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3
*d^3 + sqrt(-3)*(d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3))
*(c/d^4)^(5/6) - 6*(2*c*d^2*x^7 + 160*c^2*d*x^4 + 320*c^3*x - 6*(5*c*d^4*x^
5 + 32*c^2*d^3*x^2 - sqrt(-3)*(5*c*d^4*x^5 + 32*c^2*d^3*x^2))*(c/d^4)^(2/3)
 - (7*c*d^3*x^6 + 152*c^2*d^2*x^3 + 64*c^3*d + sqrt(-3)*(7*c*d^3*x^6 + 152*
c^2*d^2*x^3 + 64*c^3*d))*(c/d^4)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c*d^4*x^7 +
 64*c^2*d^3*x^4 + 32*c^3*d^2*x)*sqrt(c/d^4) + 18*(c*d^3*x^8 + 38*c^2*d^2*x^
5 + 64*c^3*d*x^2 - sqrt(-3)*(c*d^3*x^8 + 38*c^2*d^2*x^5 + 64*c^3*d*x^2))*(c
/d^4)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 3*(d^2*x
^3 - 8*c*d + sqrt(-3)*(d^2*x^3 - 8*c*d))*(c/d^4)^(1/6)*log(59049/4*((d^6*x^
9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 - sqrt(-3)*(d^6*x^9 + 31
8*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3))*(c/d^4)^(5/6) + 6*(2*c*d^2*x
^7 + 160*c^2*d*x^4 + 320*c^3*x - 6*(5*c*d^4*x^5 + 32*c^2*d^3*x^2 + sqrt(-3)
*(5*c*d^4*x^5 + 32*c^2*d^3*x^2))*(c/d^4)^(2/3) - (7*c*d^3*x^6 + 152*c^2*d^2
*x^3 + 64*c^3*d - sqrt(-3)*(7*c*d^3*x^6 + 152*c^2*d^2*x^3 + 64*c^3*d))*(c/d
```

```
^4)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c*d^4*x^7 + 64*c^2*d^3*x^4 + 32*c^3*d^2*
x)*sqrt(c/d^4) + 18*(c*d^3*x^8 + 38*c^2*d^2*x^5 + 64*c^3*d*x^2 + sqrt(-3)*(
c*d^3*x^8 + 38*c^2*d^2*x^5 + 64*c^3*d*x^2))*(c/d^4)^(1/6))/(d^3*x^9 - 24*c*
d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 3*(d^2*x^3 - 8*c*d + sqrt(-3)*(d^2*x^
3 - 8*c*d))*(c/d^4)^(1/6)*log(-59049/4*((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2
*d^4*x^3 + 640*c^3*d^3 - sqrt(-3)*(d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x
^3 + 640*c^3*d^3))*(c/d^4)^(5/6) - 6*(2*c*d^2*x^7 + 160*c^2*d*x^4 + 320*c^3
*x - 6*(5*c*d^4*x^5 + 32*c^2*d^3*x^2 + sqrt(-3)*(5*c*d^4*x^5 + 32*c^2*d^3*x
^2))*(c/d^4)^(2/3) - (7*c*d^3*x^6 + 152*c^2*d^2*x^3 + 64*c^3*d - sqrt(-3)*(
7*c*d^3*x^6 + 152*c^2*d^2*x^3 + 64*c^3*d))*(c/d^4)^(1/3))*sqrt(d*x^3 + c) -
 36*(5*c*d^4*x^7 + 64*c^2*d^3*x^4 + 32*c^3*d^2*x)*sqrt(c/d^4) + 18*(c*d^3*x
^8 + 38*c^2*d^2*x^5 + 64*c^3*d*x^2 + sqrt(-3)*(c*d^3*x^8 + 38*c^2*d^2*x^5 +
 64*c^3*d*x^2))*(c/d^4)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 51
2*c^3)) + 6*(d^2*x^3 - 8*c*d)*(c/d^4)^(1/6)*log(59049/2*((d^6*x^9 + 318*c*d
^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3)*(c/d^4)^(5/6) + 6*(c*d^2*x^7 + 80*
c^2*d*x^4 + 160*c^3*x + 6*(5*c*d^4*x^5 + 32*c^2*d^3*x^2)*(c/d^4)^(2/3) + (7
*c*d^3*x^6 + 152*c^2*d^2*x^3 + 64*c^3*d)*(c/d^4)^(1/3))*sqrt(d*x^3 + c) + 1
8*(5*c*d^4*x^7 + 64*c^2*d^3*x^4 + 32*c^3*d^2*x)*sqrt(c/d^4) + 18*(c*d^3*x^8
 + 38*c^2*d^2*x^5 + 64*c^3*d*x^2)*(c/d^4)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 +
192*c^2*d*x^3 - 512*c^3)) - 6*(d^2*x^3 - 8*c*d)*(c/d^4)^(1/6)*log(-59049/2*
((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3)*(c/d^4)^(5/6) -
 6*(c*d^2*x^7 + 80*c^2*d*x^4 + 160*c^3*x + 6*(5*c*d^4*x^5 + 32*c^2*d^3*x^2)
*(c/d^4)^(2/3) + (7*c*d^3*x^6 + 152*c^2*d^2*x^3 + 64*c^3*d)*(c/d^4)^(1/3))*
sqrt(d*x^3 + c) + 18*(5*c*d^4*x^7 + 64*c^2*d^3*x^4 + 32*c^3*d^2*x)*sqrt(c/d
^4) + 18*(c*d^3*x^8 + 38*c^2*d^2*x^5 + 64*c^3*d*x^2)*(c/d^4)^(1/6))/(d^3*x^
9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^2*x^3 - 8*c*d)
```

Sympy [F]

$$\int \frac{x(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{x(c+dx^3)^{\frac{3}{2}}}{(-8c+dx^3)^2}\,dx$$

```
[In] integrate(x*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
[Out] Integral(x*(c + d*x**3)**(3/2)/(-8*c + d*x**3)**2, x)
```

Maxima [F]

$$\int \frac{x(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}x}{(dx^3-8\,c)^2}\,dx$$

```
[In] integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)*x/(d*x^3 - 8*c)^2, x)
```

Giac [F]

$$\int \frac{x(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}x}{(dx^3-8\,c)^2}\,dx$$

```
[In] integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
[Out] integrate((d*x^3 + c)^(3/2)*x/(d*x^3 - 8*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx = \int \frac{x\,(d\,x^3+c)^{3/2}}{(8\,c-d\,x^3)^2}\,dx$$

```
[In] int((x*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
[Out] int((x*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2, x)
```

3.421 $\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 522

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)^2}\,dx = -\frac{\sqrt{c+dx^3}}{16cx}+\frac{\sqrt[3]{d}\sqrt{c+dx^3}}{16c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{3\sqrt{c+dx^3}}{8x(8c-dx^3)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{8\sqrt{2}\sqrt[4]{3}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] -1/16*(d*x^3+c)^(1/2)/c/x+3/8*(d*x^3+c)^(1/2)/x/(-d*x^3+8*c)+1/16*d^(1/3)*(
d*x^3+c)^(1/2)/c/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/48*d^(1/3)*(c^(1/3)+d^(1
/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1
/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^
(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(2/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3
)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/32*3^(1/4)
*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(
1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2
/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

```
/c^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1
+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {479, 21, 331, 309, 224, 1891}

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)^2}\,dx = \frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4}{8\sqrt{2}\sqrt[4]{3}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{32c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{\sqrt{c+dx^3}}{16cx}+\frac{\sqrt[3]{d}\sqrt{c+dx^3}}{16c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{3\sqrt{c+dx^3}}{8x(8c-dx^3)}$$

```
[In] Int[(c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)^2),x]
```

```
[Out] -1/16*Sqrt[c + d*x^3]/(c*x) + (d^(1/3)*Sqrt[c + d*x^3])/(16*c*((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)) + (3*Sqrt[c + d*x^3])/(8*x*(8*c - d*x^3)) - (3^(1/4
)*Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d
^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[Ar
cSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x
)], -7 - 4*Sqrt[3]])/(32*c^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^(1/3)*(c^(1/3) + d^
(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^
(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(8*Sqrt[2]*3^(1/4)*c
^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)^2]*Sqrt[c + d*x^3])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
 a + b*x])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
```

```
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{3\sqrt{c+dx^3}}{8x(8c-dx^3)} + \frac{\int \frac{12c^2d-\frac{3}{2}cd^2x^3}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{24cd}$$

$$= \frac{3\sqrt{c+dx^3}}{8x(8c-dx^3)} + \frac{1}{16}\int \frac{1}{x^2\sqrt{c+dx^3}}\,dx$$

$$= -\frac{\sqrt{c+dx^3}}{16cx} + \frac{3\sqrt{c+dx^3}}{8x(8c-dx^3)} + \frac{d\int \frac{x}{\sqrt{c+dx^3}}\,dx}{32c}$$

$$= -\frac{\sqrt{c+dx^3}}{16cx} + \frac{3\sqrt{c+dx^3}}{8x(8c-dx^3)} + \frac{d^{2/3}\int \frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{32c} - \frac{\left((1-\sqrt{3})\,d^{2/3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{32c^{2/3}}$$

$$= -\frac{\sqrt{c+dx^3}}{16cx} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{16c\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{3\sqrt{c+dx^3}}{8x(8c-dx^3)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{(1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{(1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{8\sqrt{2}\sqrt[4]{3}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.74 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.46

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)^2}\,dx = \frac{(2c-dx^3)\sqrt{c+dx^3}}{16cx(-8c+dx^3)} - \frac{\sqrt[6]{-1}\sqrt[3]{-d}\sqrt{(-1)^{5/6}\left(-1+\frac{\sqrt[3]{-d}x}{\sqrt[3]{c}}\right)}\sqrt{1+\frac{\sqrt[3]{-d}x}{\sqrt[3]{c}}+\frac{(-d)^{2/3}x^2}{c^{2/3}}}\left(-i\sqrt{3}E\left(\arcsin\left(\frac{\sqrt{-(-1)^{5/6}-\frac{i\sqrt[3]{-d}x}{\sqrt[3]{c}}}}{\sqrt[4]{3}}\right)|\sqrt[3]{-\cdots}\right.\right.}{16\sqrt[4]{3}\sqrt[3]{c}\sqrt{c+dx^3}}$$

[In] Integrate[(c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)^2),x]

[Out] ((2*c - d*x^3)*Sqrt[c + d*x^3])/(16*c*x*(-8*c + d*x^3)) - ((-1)^(1/6)*(-d)^(1/3)*Sqrt[(-1)^(5/6)*(-1 + ((-d)^(1/3)*x)/c^(1/3))]*Sqrt[1 + ((-d)^(1/3)*x)/c^(1/3) + ((-d)^(2/3)*x^2)/c^(2/3)]*((-I)*Sqrt[3]*EllipticE[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-d)^(1/3)*x)/c^(1/3)]/3^(1/4)], (-1)^(1/3)] + (-1)^(1/3)*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-d)^(1/3)*x)/c^(1/3)]/3^(1/4)], (-1)^(1/3)]))/(16*3^(1/4)*c^(1/3)*Sqrt[c + d*x^3])

Maple [A] (verified)

Time = 5.35 (sec) , antiderivative size = 483, normalized size of antiderivative = 0.93

method	result
elliptic	$\frac{3 d\,x^{2} \sqrt{d\,x^{3}+c}}{64 c \left(-d\,x^{3}+8 c\right)}-\frac{\sqrt{d\,x^{3}+c}}{64 c x}-\frac{i \sqrt{3}\, \left(-c\,d^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i \left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2 d}-\frac{i \sqrt{3}\, \left(-c\,d^{2}\right)^{\frac{1}{3}}}{2 d}\right) \sqrt{3}\, d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\, \sqrt{\frac{x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}}{-\frac{3 \left(-c\,d^{2}\right)^{\frac{1}{3}}}{2 d}+\frac{i \sqrt{3}\, \left(-c\,d^{2}\right)^{\frac{1}{3}}}{2 d}}}\, \sqrt{-\frac{i \left(x+\frac{\left(-c \cdots\right.}{}\right.}{}}\cdots}{\cdots}$
risch	Expression too large to display
default	Expression too large to display

[In] int((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)

[Out] 3/64*d*x^2/c*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-1/64*(d*x^3+c)^(1/2)/c/x-1/48*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.13

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)^2}\,dx = -\frac{(dx^4-8\,cx)\sqrt{d}\text{weierstrassZeta}\left(0,-\frac{4\,c}{d},\text{weierstrassPInverse}\left(0,-\frac{4\,c}{d},x\right)\right)+\sqrt{dx^3+c}(dx^3-2\,c)}{16\,(cdx^4-8\,c^2x)}$$

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] -1/16*((d*x^4 - 8*c*x)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrassPInver
se(0, -4*c/d, x)) + sqrt(d*x^3 + c)*(d*x^3 - 2*c))/(c*d*x^4 - 8*c^2*x)
```

Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)^2}\,dx = \int \frac{(c+dx^3)^{\frac{3}{2}}}{x^2(-8c+dx^3)^2}\,dx$$

```
[In] integrate((d*x**3+c)**(3/2)/x**2/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral((c + d*x**3)**(3/2)/(x**2*(-8*c + d*x**3)**2), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)^2x^2}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^2), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)^2x^2}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="giac")
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)^2}\,dx = \int \frac{(d\,x^3+c)^{3/2}}{x^2\,(8\,c-d\,x^3)^2}\,dx$$

```
[In] int((c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)^2),x)
[Out] int((c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)^2), x)
```

3.422 $\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 684

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)^2}\,dx = -\frac{13\sqrt{c+dx^3}}{256cx^4}-\frac{d\sqrt{c+dx^3}}{32c^2x}+\frac{d^{4/3}\sqrt{c+dx^3}}{32c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$
$$+\frac{3\sqrt{c+dx^3}}{8x^4(8c-dx^3)}-\frac{9\sqrt{3}d^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1024c^{11/6}}$$
$$+\frac{9d^{4/3}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1024c^{11/6}}-\frac{9d^{4/3}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1024c^{11/6}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{64c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+\frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{16\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 9/1024*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c
^(11/6)-9/1024*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)-9/1024
```

```
*d^(4/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2
)/c^(11/6)-13/256*(d*x^3+c)^(1/2)/c/x^4-1/32*d*(d*x^3+c)^(1/2)/c^2/x+3/8*(d
*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)+1/32*d^(4/3)*(d*x^3+c)^(1/2)/c^2/(d^(1/3)*x+
c^(1/3)*(1+3^(1/2)))+1/96*d^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+
c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/
3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*
3^(3/4)/c^(5/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/
3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/64*3^(1/4)*d^(4/3)*(c^(1/3)+d^(1/3)*x)
*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),
I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3
)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(5/3)/(d*x^3+c)^(1/2)/(c^
(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 684, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {479, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)^2}\,dx = \frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-\right.}{16\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{64c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{9\sqrt{3}d^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1024c^{11/6}}+\frac{9d^{4/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1024c^{11/6}}$$

$$-\frac{9d^{4/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1024c^{11/6}}+\frac{d^{4/3}\sqrt{c+dx^3}}{32c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$-\frac{d\sqrt{c+dx^3}}{32c^2x}+\frac{3\sqrt{c+dx^3}}{8x^4(8c-dx^3)}-\frac{13\sqrt{c+dx^3}}{256cx^4}$$

```
[In] Int[(c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)^2),x]
```

```
[Out] (-13*Sqrt[c + d*x^3])/(256*c*x^4) - (d*Sqrt[c + d*x^3])/(32*c^2*x) + (d^(4/
3)*Sqrt[c + d*x^3])/(32*c^2*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (3*Sqrt[
c + d*x^3])/(8*x^4*(8*c - d*x^3)) - (9*Sqrt[3]*d^(4/3)*ArcTan[(Sqrt[3]*c^(1
/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(1024*c^(11/6)) + (9*d^(4/3)*A
rcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(1024*c^(11/6)
) - (9*d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1024*c^(11/6)) - (3^(
1/4)*Sqrt[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3
)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE
[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)], -7 - 4*Sqrt[3]])/(64*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^(4/3)*(c^(1/3) +
 d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*
x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*Sqrt[2]*3^(1/
4)*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
```

```
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\text{integral} = \frac{3\sqrt{c+dx^3}}{8x^4\left(8c-dx^3\right)}+\frac{\int \frac{39c^2d+\frac{51}{2}cd^2x^3}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{24cd}$$

$$=-\frac{13\sqrt{c+dx^3}}{256cx^4}+\frac{3\sqrt{c+dx^3}}{8x^4\left(8c-dx^3\right)}-\frac{\int \frac{-192c^3d^2-\frac{195}{2}c^2d^3x^3}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{768c^3d}$$

$$=-\frac{13\sqrt{c+dx^3}}{256cx^4}-\frac{d\sqrt{c+dx^3}}{32c^2x}+\frac{3\sqrt{c+dx^3}}{8x^4\left(8c-dx^3\right)}+\frac{\int \frac{x\left(1740c^4d^3-96c^3d^4x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{6144c^5d}$$

$$=-\frac{13\sqrt{c+dx^3}}{256cx^4}-\frac{d\sqrt{c+dx^3}}{32c^2x}+\frac{3\sqrt{c+dx^3}}{8x^4\left(8c-dx^3\right)}+\frac{\int \left(\frac{96c^3d^3x}{\sqrt{c+dx^3}}+\frac{972c^4d^3x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{6144c^5d}$$

$$=-\frac{13\sqrt{c+dx^3}}{256cx^4}-\frac{d\sqrt{c+dx^3}}{32c^2x}+\frac{3\sqrt{c+dx^3}}{8x^4\left(8c-dx^3\right)}+\frac{d^2\int \frac{x}{\sqrt{c+dx^3}}\,dx}{64c^2}+\frac{\left(81d^2\right)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{512c}$$

$$= -\frac{13\sqrt{c+dx^3}}{256cx^4} - \frac{d\sqrt{c+dx^3}}{32c^2x} + \frac{3\sqrt{c+dx^3}}{8x^4(8c-dx^3)} - \frac{(27d)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{2048c^2}$$

$$+ \frac{d^{5/3}\int \frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{64c^2} + \frac{(27d^{5/3})\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{2048c^{5/3}}$$

$$- \frac{((1-\sqrt{3})\,d^{5/3})\int \frac{1}{\sqrt{c+dx^3}}\,dx}{64c^{5/3}} - \frac{(81d^{7/3})\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{2048c^{4/3}}$$

$$= -\frac{13\sqrt{c+dx^3}}{256cx^4} - \frac{d\sqrt{c+dx^3}}{32c^2x} + \frac{d^{4/3}\sqrt{c+dx^3}}{32c^2\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{3\sqrt{c+dx^3}}{8x^4(8c-dx^3)}$$

$$- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{(1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{64c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{(1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{16\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{(27d^{4/3})\,\text{Subst}\left(\int \frac{1}{9-cx^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{1024c^{4/3}}$$

$$- \frac{(27d^{7/3})\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{2048c^{4/3}}$$

$$+ \frac{(27d^{10/3})\,\text{Subst}\left(\int \frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{512c^{7/3}}$$

$$=-\frac{13\sqrt{c+dx^3}}{256cx^4}-\frac{d\sqrt{c+dx^3}}{32c^2x}+\frac{d^{4/3}\sqrt{c+dx^3}}{32c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{3\sqrt{c+dx^3}}{8x^4\left(8c-dx^3\right)}$$
$$-\frac{9\sqrt{3}d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1024c^{11/6}}+\frac{9d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1024c^{11/6}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{64c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+\frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{16\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{\left(27d^{4/3}\right)\operatorname{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{1024c^{4/3}}$$

$$= -\frac{13\sqrt{c+dx^3}}{256cx^4} - \frac{d\sqrt{c+dx^3}}{32c^2x} + \frac{d^{4/3}\sqrt{c+dx^3}}{32c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$
$$+ \frac{3\sqrt{c+dx^3}}{8x^4(8c-dx^3)} - \frac{9\sqrt{3}d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1024c^{11/6}}$$
$$+ \frac{9d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1024c^{11/6}} - \frac{9d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1024c^{11/6}}$$
$$- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{64c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+ \frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{16\sqrt{2}\sqrt[4]{3}c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.29

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)^2}\,dx = \sqrt{c+dx^3}\left(-\frac{1}{256cx^4} - \frac{13d}{512c^2x} - \frac{3d^2x^2}{512c^2(-8c+dx^3)}\right)$$
$$+ \frac{145d^2x^2\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{8192c^2\sqrt{c+dx^3}}$$
$$- \frac{d^3x^5\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{2560c^3\sqrt{c+dx^3}}$$

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)^2),x]
```

```
[Out] Sqrt[c + d*x^3]*(-1/256*1/(c*x^4) - (13*d)/(512*c^2*x) - (3*d^2*x^2)/(512*c
^2*(-8*c + d*x^3))) + (145*d^2*x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1
, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(8192*c^2*Sqrt[c + d*x^3]) - (d^3*x^5*
Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]
)/(2560*c^3*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.62 (sec) , antiderivative size = 919, normalized size of antiderivative = 1.34

method	result	size
elliptic	Expression too large to display	919
risch	Expression too large to display	1770
default	Expression too large to display	2691

```
[In] int((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3/512/c^2*x^2*d^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-1/256*(d*x^3+c)^(1/2)/c/x^4-
13/512*d*(d*x^3+c)^(1/2)/c^2/x-1/96*I*d/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/
2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3
^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(
1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(
1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
))^(1/2)))-3/512*I/d/c^2*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+
1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x
-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-
1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))
^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2
)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1
/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I
*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)
/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.90 (sec) , antiderivative size = 2549, normalized size of antiderivative = 3.73

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] -1/4096*(128*(d^2*x^7 - 8*c*d*x^4)*sqrt(d)*weierstrassZeta(0, -4*c/d, weier
strassPInverse(0, -4*c/d, x)) - 3*(c^2*d*x^7 - 8*c^3*x^4 + sqrt(-3)*(c^2*d*
x^7 - 8*c^3*x^4))*(d^8/c^11)^(1/6)*log(6561*(d^9*x^9 + 318*c*d^8*x^6 + 1200
*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^8*d^3*x^7 + 64*c^9*d^2*x^4 + 32*c^10*d*
x + sqrt(-3)*(5*c^8*d^3*x^7 + 64*c^9*d^2*x^4 + 32*c^10*d*x))*(d^8/c^11)^(2/
3) + 3*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2 - sqrt(-3)*(5*c^10*d*
x^5 + 32*c^11*x^2))*(d^8/c^11)^(5/6) - 2*(7*c^6*d^4*x^6 + 152*c^7*d^3*x^3 +
 64*c^8*d^2)*sqrt(d^8/c^11) + (c^2*d^7*x^7 + 80*c^3*d^6*x^4 + 160*c^4*d^5*x
 + sqrt(-3)*(c^2*d^7*x^7 + 80*c^3*d^6*x^4 + 160*c^4*d^5*x))*(d^8/c^11)^(1/6
)) - 9*(c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2 - sqrt(-3)*(c^4*d^6*x
^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2))*(d^8/c^11)^(1/3))/(d^3*x^9 - 24*c*d^
2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 3*(c^2*d*x^7 - 8*c^3*x^4 + sqrt(-3)*(c^
2*d*x^7 - 8*c^3*x^4))*(d^8/c^11)^(1/6)*log(6561*(d^9*x^9 + 318*c*d^8*x^6 +
1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^8*d^3*x^7 + 64*c^9*d^2*x^4 + 32*c^1
0*d*x + sqrt(-3)*(5*c^8*d^3*x^7 + 64*c^9*d^2*x^4 + 32*c^10*d*x))*(d^8/c^11)
^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2 - sqrt(-3)*(5*c^1
0*d*x^5 + 32*c^11*x^2))*(d^8/c^11)^(5/6) - 2*(7*c^6*d^4*x^6 + 152*c^7*d^3*x
^3 + 64*c^8*d^2)*sqrt(d^8/c^11) + (c^2*d^7*x^7 + 80*c^3*d^6*x^4 + 160*c^4*d
^5*x + sqrt(-3)*(c^2*d^7*x^7 + 80*c^3*d^6*x^4 + 160*c^4*d^5*x))*(d^8/c^11)^
(1/6)) - 9*(c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2 - sqrt(-3)*(c^4*d
^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2))*(d^8/c^11)^(1/3))/(d^3*x^9 - 24*
c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 3*(c^2*d*x^7 - 8*c^3*x^4 - sqrt(-3)
*(c^2*d*x^7 - 8*c^3*x^4))*(d^8/c^11)^(1/6)*log(6561*(d^9*x^9 + 318*c*d^8*x^
6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^8*d^3*x^7 + 64*c^9*d^2*x^4 + 32
*c^10*d*x - sqrt(-3)*(5*c^8*d^3*x^7 + 64*c^9*d^2*x^4 + 32*c^10*d*x))*(d^8/c
^11)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2 + sqrt(-3)*(5
*c^10*d*x^5 + 32*c^11*x^2))*(d^8/c^11)^(5/6) - 2*(7*c^6*d^4*x^6 + 152*c^7*d
^3*x^3 + 64*c^8*d^2)*sqrt(d^8/c^11) + (c^2*d^7*x^7 + 80*c^3*d^6*x^4 + 160*c
^4*d^5*x - sqrt(-3)*(c^2*d^7*x^7 + 80*c^3*d^6*x^4 + 160*c^4*d^5*x))*(d^8/c^
11)^(1/6)) - 9*(c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2 + sqrt(-3)*(c
^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2))*(d^8/c^11)^(1/3))/(d^3*x^9 -
 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 3*(c^2*d*x^7 - 8*c^3*x^4 - sqrt
(-3)*(c^2*d*x^7 - 8*c^3*x^4))*(d^8/c^11)^(1/6)*log(6561*(d^9*x^9 + 318*c*d^
8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^8*d^3*x^7 + 64*c^9*d^2*x^4
+ 32*c^10*d*x - sqrt(-3)*(5*c^8*d^3*x^7 + 64*c^9*d^2*x^4 + 32*c^10*d*x))*(d
```

```
^8/c^11)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2 + sqrt(-3
)*(5*c^10*d*x^5 + 32*c^11*x^2))*(d^8/c^11)^(5/6) - 2*(7*c^6*d^4*x^6 + 152*c
^7*d^3*x^3 + 64*c^8*d^2)*sqrt(d^8/c^11) + (c^2*d^7*x^7 + 80*c^3*d^6*x^4 + 1
60*c^4*d^5*x - sqrt(-3)*(c^2*d^7*x^7 + 80*c^3*d^6*x^4 + 160*c^4*d^5*x))*(d^
8/c^11)^(1/6)) - 9*(c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2 + sqrt(-3
)*(c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2))*(d^8/c^11)^(1/3))/(d^3*x
^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 6*(c^2*d*x^7 - 8*c^3*x^4)*(
d^8/c^11)^(1/6)*log(6561*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*
c^3*d^6 + 18*(5*c^8*d^3*x^7 + 64*c^9*d^2*x^4 + 32*c^10*d*x)*(d^8/c^11)^(2/3
) + 6*sqrt(d*x^3 + c)*(6*(5*c^10*d*x^5 + 32*c^11*x^2)*(d^8/c^11)^(5/6) + (7
*c^6*d^4*x^6 + 152*c^7*d^3*x^3 + 64*c^8*d^2)*sqrt(d^8/c^11) + (c^2*d^7*x^7
+ 80*c^3*d^6*x^4 + 160*c^4*d^5*x)*(d^8/c^11)^(1/6)) + 18*(c^4*d^6*x^8 + 38*
c^5*d^5*x^5 + 64*c^6*d^4*x^2)*(d^8/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 1
92*c^2*d*x^3 - 512*c^3)) + 6*(c^2*d*x^7 - 8*c^3*x^4)*(d^8/c^11)^(1/6)*log(6
561*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 + 18*(5*c^8*d
^3*x^7 + 64*c^9*d^2*x^4 + 32*c^10*d*x)*(d^8/c^11)^(2/3) - 6*sqrt(d*x^3 + c)
*(6*(5*c^10*d*x^5 + 32*c^11*x^2)*(d^8/c^11)^(5/6) + (7*c^6*d^4*x^6 + 152*c^
7*d^3*x^3 + 64*c^8*d^2)*sqrt(d^8/c^11) + (c^2*d^7*x^7 + 80*c^3*d^6*x^4 + 16
0*c^4*d^5*x)*(d^8/c^11)^(1/6)) + 18*(c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*
d^4*x^2)*(d^8/c^11)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^
3)) + 16*(8*d^2*x^6 - 51*c*d*x^3 - 8*c^2)*sqrt(d*x^3 + c))/(c^2*d*x^7 - 8*c
^3*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^5\,(8c-dx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*x**3+c)**(3/2)/x**5/(-d*x**3+8*c)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^5\,(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)^2x^5}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^5), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)^2x^5}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="giac")
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)^2}\,dx = \int \frac{(d\,x^3+c)^{3/2}}{x^5\,(8\,c-d\,x^3)^2}\,dx$$

```
[In] int((c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)^2),x)
[Out] int((c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)^2), x)
```

3.423 $\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 708

$$\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)^2}\,dx = -\frac{11\sqrt{c+dx^3}}{224cx^7}-\frac{83d\sqrt{c+dx^3}}{7168c^2x^4}-\frac{19d^2\sqrt{c+dx^3}}{1792c^3x}+\frac{19d^{7/3}\sqrt{c+dx^3}}{1792c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{3\sqrt{c+dx^3}}{8x^7(8c-dx^3)}-\frac{9\sqrt{3}d^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{4096c^{17/6}}+\frac{9d^{7/3}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{4096c^{17/6}}-\frac{9d^{7/3}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4096c^{17/6}}-\frac{19\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{3584c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{19d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{896\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 9/4096*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c
^(17/6)-9/4096*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)-9/4096
```

```
*d^(7/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2
)/c^(17/6)-11/224*(d*x^3+c)^(1/2)/c/x^7-83/7168*d*(d*x^3+c)^(1/2)/c^2/x^4-1
9/1792*d^2*(d*x^3+c)^(1/2)/c^3/x+3/8*(d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)+19/17
92*d^(7/3)*(d*x^3+c)^(1/2)/c^3/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+19/5376*d^(7
/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*
x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x
^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(8/3)*2^(1/2)/(d*x^3
+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(
1/2)-19/3584*3^(1/4)*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/
3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)
-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(
1+3^(1/2)))^2)^(1/2)/c^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(
d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 708, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.481, Rules used = {479, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^8\left(8c-dx^3\right)^2}\,dx=\frac{19d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7\right.}{896\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{19\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{3584c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{9\sqrt{3}d^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{4096c^{17/6}}+\frac{9d^{7/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{4096c^{17/6}}$$

$$-\frac{9d^{7/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4096c^{17/6}}+\frac{19d^{7/3}\sqrt{c+dx^3}}{1792c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$-\frac{19d^2\sqrt{c+dx^3}}{1792c^3x}-\frac{83d\sqrt{c+dx^3}}{7168c^2x^4}+\frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)}-\frac{11\sqrt{c+dx^3}}{224cx^7}$$

```
[In] Int[(c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)^2),x]
```

```
[Out] (-11*Sqrt[c + d*x^3])/(224*c*x^7) - (83*d*Sqrt[c + d*x^3])/(7168*c^2*x^4) -
 (19*d^2*Sqrt[c + d*x^3])/(1792*c^3*x) + (19*d^(7/3)*Sqrt[c + d*x^3])/(1792
*c^3*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (3*Sqrt[c + d*x^3])/(8*x^7*(8*c
 - d*x^3)) - (9*Sqrt[3]*d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*
x))/Sqrt[c + d*x^3]])/(4096*c^(17/6)) + (9*d^(7/3)*ArcTanh[(c^(1/3) + d^(1/
3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(4096*c^(17/6)) - (9*d^(7/3)*ArcTanh[
Sqrt[c + d*x^3]/(3*Sqrt[c])])/(4096*c^(17/6)) - (19*3^(1/4)*Sqrt[2 - Sqrt[3
]]*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3
)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3
])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3
]])/(3584*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (19*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqr
t[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(896*Sqrt[2]*3^(1/4)*c^(8/3)*Sqr
t[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sq
rt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
```

```
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3\sqrt{c+dx^3}}{8x^7\,(8c-dx^3)}+\frac{\int \frac{66c^2d+\frac{105}{2}cd^2x^3}{x^8(8c-dx^3)\sqrt{c+dx^3}}\,dx}{24cd}\\
&= -\frac{11\sqrt{c+dx^3}}{224cx^7}+\frac{3\sqrt{c+dx^3}}{8x^7\,(8c-dx^3)}-\frac{\int \frac{-498c^3d^2-363c^2d^3x^3}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx}{1344c^3d}\\
&= -\frac{11\sqrt{c+dx^3}}{224cx^7}-\frac{83d\sqrt{c+dx^3}}{7168c^2x^4}+\frac{3\sqrt{c+dx^3}}{8x^7\,(8c-dx^3)}+\frac{\int \frac{3648c^4d^3+1245c^3d^4x^3}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{43008c^5d}\\
&= -\frac{11\sqrt{c+dx^3}}{224cx^7}-\frac{83d\sqrt{c+dx^3}}{7168c^2x^4}-\frac{19d^2\sqrt{c+dx^3}}{1792c^3x}\\
&\quad+\frac{3\sqrt{c+dx^3}}{8x^7\,(8c-dx^3)}-\frac{\int \frac{x\left(-28200c^5d^4+1824c^4d^5x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{344064c^7d}
\end{aligned}$$

$$= -\frac{11\sqrt{c+dx^3}}{224cx^7} - \frac{83d\sqrt{c+dx^3}}{7168c^2x^4} - \frac{19d^2\sqrt{c+dx^3}}{1792c^3x} + \frac{3\sqrt{c+dx^3}}{8x^7(8c-dx^3)} - \frac{\int \left(-\frac{1824c^4d^4x}{\sqrt{c+dx^3}} - \frac{13608c^5d^4x}{(8c-dx^3)\sqrt{c+dx^3}}\right) dx}{344064c^7d}$$

$$= -\frac{11\sqrt{c+dx^3}}{224cx^7} - \frac{83d\sqrt{c+dx^3}}{7168c^2x^4} - \frac{19d^2\sqrt{c+dx^3}}{1792c^3x} + \frac{3\sqrt{c+dx^3}}{8x^7(8c-dx^3)} + \frac{(19d^3)\int \frac{x}{\sqrt{c+dx^3}} dx}{3584c^3} + \frac{(81d^3)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}} dx}{2048c^2}$$

$$= -\frac{11\sqrt{c+dx^3}}{224cx^7} - \frac{83d\sqrt{c+dx^3}}{7168c^2x^4} - \frac{19d^2\sqrt{c+dx^3}}{1792c^3x} + \frac{3\sqrt{c+dx^3}}{8x^7(8c-dx^3)} - \frac{(27d^2)\int \frac{2\sqrt[3]{c}d^{2/3} - 2dx - \frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4 + \frac{2\sqrt[3]{d}x}{\sqrt[3]{c}} + \frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}} dx}{8192c^3}$$

$$+ \frac{\left(19d^{8/3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c} + \sqrt[3]{d}x}{\sqrt{c+dx^3}} dx}{3584c^3} + \frac{\left(27d^{8/3}\right)\int \frac{1 + \frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2 - \frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}} dx}{8192c^{8/3}} - \frac{\left(19\left(1-\sqrt{3}\right)d^{8/3}\right)\int \frac{1}{\sqrt{c+dx^3}} dx}{3584c^{8/3}} - \frac{\left(81d^{10/3}\right)\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}} dx}{8192c^{7/3}}$$

$$
\begin{aligned}
&= -\frac{11\sqrt{c+dx^3}}{224cx^7} - \frac{83d\sqrt{c+dx^3}}{7168c^2x^4} - \frac{19d^2\sqrt{c+dx^3}}{1792c^3x} \\
&+ \frac{19d^{7/3}\sqrt{c+dx^3}}{1792c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)} \\
&- \frac{19\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{3584c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+ \frac{19d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{896\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+ \frac{\left(27d^{7/3}\right)\text{Subst}\left(\int\frac{1}{9-cx^2}\,dx,x,\frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{4096c^{7/3}} \\
&- \frac{\left(27d^{10/3}\right)\text{Subst}\left(\int\frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{8192c^{7/3}} \\
&+ \frac{\left(27d^{13/3}\right)\text{Subst}\left(\int\frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{2048c^{10/3}}
\end{aligned}
$$

$$= -\frac{11\sqrt{c+dx^3}}{224cx^7} - \frac{83d\sqrt{c+dx^3}}{7168c^2x^4} - \frac{19d^2\sqrt{c+dx^3}}{1792c^3x} + \frac{19d^{7/3}\sqrt{c+dx^3}}{1792c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)} - \frac{9\sqrt{3}d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{4096c^{17/6}} + \frac{9d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{4096c^{17/6}} - \frac{19\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right.}{3584c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{19d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{896\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{\left(27d^{7/3}\right)\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{4096c^{7/3}}$$

$$= -\frac{11\sqrt{c+dx^3}}{224cx^7} - \frac{83d\sqrt{c+dx^3}}{7168c^2x^4} - \frac{19d^2\sqrt{c+dx^3}}{1792c^3x} + \frac{19d^{7/3}\sqrt{c+dx^3}}{1792c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$
$$+ \frac{3\sqrt{c+dx^3}}{8x^7(8c-dx^3)} - \frac{9\sqrt{3}d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{4096c^{17/6}}$$
$$+ \frac{9d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{4096c^{17/6}} - \frac{9d^{7/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4096c^{17/6}}$$
$$- \frac{19\sqrt[4]{3}\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{3584c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+ \frac{19d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{896\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.30

$$\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)^2}\,dx = \sqrt{c+dx^3}\left(-\frac{1}{448cx^7} - \frac{41d}{7168c^2x^4} - \frac{283d^2}{28672c^3x} - \frac{3d^3x^2}{4096c^3(-8c+dx^3)}\right) + \frac{1175d^3x^2\sqrt{\frac{c+dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{229376c^3\sqrt{c+dx^3}} - \frac{19d^4x^5\sqrt{\frac{c+dx^3}{c}}\operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{143360c^4\sqrt{c+dx^3}}$$

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)^2),x]
```

```
[Out] Sqrt[c + d*x^3]*(-1/448*1/(c*x^7) - (41*d)/(7168*c^2*x^4) - (283*d^2)/(2867
2*c^3*x) - (3*d^3*x^2)/(4096*c^3*(-8*c + d*x^3))) + (1175*d^3*x^2*Sqrt[(c +
 d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(229376
*c^3*Sqrt[c + d*x^3]) - (19*d^4*x^5*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2,
1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(143360*c^4*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.84 (sec) , antiderivative size = 938, normalized size of antiderivative = 1.32

method	result	size
elliptic	Expression too large to display	938
risch	Expression too large to display	1781
default	Expression too large to display	3187

```
[In] int((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3/4096/c^3*x^2*d^3*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-1/448*(d*x^3+c)^(1/2)/c/x^7
-41/7168*d*(d*x^3+c)^(1/2)/c^2/x^4-283/28672*d^2*(d*x^3+c)^(1/2)/c^3/x-19/5
376*I/c^3*d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3)
)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-3/2048*I/c^3*2^(1/2
)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+
(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^
2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c
*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d
^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)
^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/1
8/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+
I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/
(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootO
f(_Z^3*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.95 (sec) , antiderivative size = 2582, normalized size of antiderivative = 3.65

$$\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] -1/114688*(1216*(d^3*x^10 - 8*c*d^2*x^7)*sqrt(d)*weierstrassZeta(0, -4*c/d,
 weierstrassPInverse(0, -4*c/d, x)) - 21*(c^3*d*x^10 - 8*c^4*x^7 + sqrt(-3)
*(c^3*d*x^10 - 8*c^4*x^7))*(d^14/c^17)^(1/6)*log(6561*(d^14*x^9 + 318*c*d^1
3*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 - 9*(5*c^12*d^4*x^7 + 64*c^13*d^3*
x^4 + 32*c^14*d^2*x + sqrt(-3)*(5*c^12*d^4*x^7 + 64*c^13*d^3*x^4 + 32*c^14*
d^2*x))*(d^14/c^17)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^
2 - sqrt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2))*(d^14/c^17)^(5/6) - 2*(7*c^9*d^6
*x^6 + 152*c^10*d^5*x^3 + 64*c^11*d^4)*sqrt(d^14/c^17) + (c^3*d^11*x^7 + 80
*c^4*d^10*x^4 + 160*c^5*d^9*x + sqrt(-3)*(c^3*d^11*x^7 + 80*c^4*d^10*x^4 +
160*c^5*d^9*x))*(d^14/c^17)^(1/6)) - 9*(c^6*d^9*x^8 + 38*c^7*d^8*x^5 + 64*c
^8*d^7*x^2 - sqrt(-3)*(c^6*d^9*x^8 + 38*c^7*d^8*x^5 + 64*c^8*d^7*x^2))*(d^1
4/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 21*(c^
3*d*x^10 - 8*c^4*x^7 + sqrt(-3)*(c^3*d*x^10 - 8*c^4*x^7))*(d^14/c^17)^(1/6)
*log(6561*(d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 - 9
*(5*c^12*d^4*x^7 + 64*c^13*d^3*x^4 + 32*c^14*d^2*x + sqrt(-3)*(5*c^12*d^4*x
^7 + 64*c^13*d^3*x^4 + 32*c^14*d^2*x))*(d^14/c^17)^(2/3) - 3*sqrt(d*x^3 + c
)*(6*(5*c^15*d*x^5 + 32*c^16*x^2 - sqrt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2))*(
d^14/c^17)^(5/6) - 2*(7*c^9*d^6*x^6 + 152*c^10*d^5*x^3 + 64*c^11*d^4)*sqrt(
d^14/c^17) + (c^3*d^11*x^7 + 80*c^4*d^10*x^4 + 160*c^5*d^9*x + sqrt(-3)*(c^
3*d^11*x^7 + 80*c^4*d^10*x^4 + 160*c^5*d^9*x))*(d^14/c^17)^(1/6)) - 9*(c^6*
d^9*x^8 + 38*c^7*d^8*x^5 + 64*c^8*d^7*x^2 - sqrt(-3)*(c^6*d^9*x^8 + 38*c^7*
d^8*x^5 + 64*c^8*d^7*x^2))*(d^14/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192
*c^2*d*x^3 - 512*c^3)) - 21*(c^3*d*x^10 - 8*c^4*x^7 - sqrt(-3)*(c^3*d*x^10
- 8*c^4*x^7))*(d^14/c^17)^(1/6)*log(6561*(d^14*x^9 + 318*c*d^13*x^6 + 1200*
c^2*d^12*x^3 + 640*c^3*d^11 - 9*(5*c^12*d^4*x^7 + 64*c^13*d^3*x^4 + 32*c^14
*d^2*x - sqrt(-3)*(5*c^12*d^4*x^7 + 64*c^13*d^3*x^4 + 32*c^14*d^2*x))*(d^14
/c^17)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2 + sqrt(-3)*
(5*c^15*d*x^5 + 32*c^16*x^2))*(d^14/c^17)^(5/6) - 2*(7*c^9*d^6*x^6 + 152*c^
10*d^5*x^3 + 64*c^11*d^4)*sqrt(d^14/c^17) + (c^3*d^11*x^7 + 80*c^4*d^10*x^4
 + 160*c^5*d^9*x - sqrt(-3)*(c^3*d^11*x^7 + 80*c^4*d^10*x^4 + 160*c^5*d^9*x
))*(d^14/c^17)^(1/6)) - 9*(c^6*d^9*x^8 + 38*c^7*d^8*x^5 + 64*c^8*d^7*x^2 +
sqrt(-3)*(c^6*d^9*x^8 + 38*c^7*d^8*x^5 + 64*c^8*d^7*x^2))*(d^14/c^17)^(1/3)
)/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 21*(c^3*d*x^10 - 8*
c^4*x^7 - sqrt(-3)*(c^3*d*x^10 - 8*c^4*x^7))*(d^14/c^17)^(1/6)*log(6561*(d^
14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 - 9*(5*c^12*d^4*
```

```
x^7 + 64*c^13*d^3*x^4 + 32*c^14*d^2*x - sqrt(-3)*(5*c^12*d^4*x^7 + 64*c^13*
d^3*x^4 + 32*c^14*d^2*x))*(d^14/c^17)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^15*
d*x^5 + 32*c^16*x^2 + sqrt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2))*(d^14/c^17)^(5
/6) - 2*(7*c^9*d^6*x^6 + 152*c^10*d^5*x^3 + 64*c^11*d^4)*sqrt(d^14/c^17) +
(c^3*d^11*x^7 + 80*c^4*d^10*x^4 + 160*c^5*d^9*x - sqrt(-3)*(c^3*d^11*x^7 +
80*c^4*d^10*x^4 + 160*c^5*d^9*x))*(d^14/c^17)^(1/6)) - 9*(c^6*d^9*x^8 + 38*
c^7*d^8*x^5 + 64*c^8*d^7*x^2 + sqrt(-3)*(c^6*d^9*x^8 + 38*c^7*d^8*x^5 + 64*
c^8*d^7*x^2))*(d^14/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 -
512*c^3)) - 42*(c^3*d*x^10 - 8*c^4*x^7)*(d^14/c^17)^(1/6)*log(6561*(d^14*x^
9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 + 18*(5*c^12*d^4*x^7
+ 64*c^13*d^3*x^4 + 32*c^14*d^2*x)*(d^14/c^17)^(2/3) + 6*sqrt(d*x^3 + c)*(6
*(5*c^15*d*x^5 + 32*c^16*x^2)*(d^14/c^17)^(5/6) + (7*c^9*d^6*x^6 + 152*c^10
*d^5*x^3 + 64*c^11*d^4)*sqrt(d^14/c^17) + (c^3*d^11*x^7 + 80*c^4*d^10*x^4 +
 160*c^5*d^9*x)*(d^14/c^17)^(1/6)) + 18*(c^6*d^9*x^8 + 38*c^7*d^8*x^5 + 64*
c^8*d^7*x^2)*(d^14/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 5
12*c^3)) + 42*(c^3*d*x^10 - 8*c^4*x^7)*(d^14/c^17)^(1/6)*log(6561*(d^14*x^9
 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 + 18*(5*c^12*d^4*x^7 +
 64*c^13*d^3*x^4 + 32*c^14*d^2*x)*(d^14/c^17)^(2/3) - 6*sqrt(d*x^3 + c)*(6*
(5*c^15*d*x^5 + 32*c^16*x^2)*(d^14/c^17)^(5/6) + (7*c^9*d^6*x^6 + 152*c^10*
d^5*x^3 + 64*c^11*d^4)*sqrt(d^14/c^17) + (c^3*d^11*x^7 + 80*c^4*d^10*x^4 +
160*c^5*d^9*x)*(d^14/c^17)^(1/6)) + 18*(c^6*d^9*x^8 + 38*c^7*d^8*x^5 + 64*c
^8*d^7*x^2)*(d^14/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 51
2*c^3)) + 16*(76*d^3*x^9 - 525*c*d^2*x^6 - 312*c^2*d*x^3 - 128*c^3)*sqrt(d*
x^3 + c))/(c^3*d*x^10 - 8*c^4*x^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*x**3+c)**(3/2)/x**8/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)^2x^8}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^8), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^8\left(8c-dx^3\right)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(dx^3-8\,c)^2x^8}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="giac")
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^8\left(8c-dx^3\right)^2}\,dx = \int \frac{\left(d\,x^3+c\right)^{3/2}}{x^8\left(8\,c-d\,x^3\right)^2}\,dx$$

```
[In] int((c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)^2),x)
[Out] int((c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)^2), x)
```

3.424 $\int \frac{x^{11}}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 95

$$\int \frac{x^{11}}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{8x^6\sqrt{c+dx^3}}{27d^2(8c-dx^3)} + \frac{2\sqrt{c+dx^3}(170c+7dx^3)}{27d^4} - \frac{2944c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4}$$

```
[Out] -2944/81*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^4+8/27*x^6*(d*x^3+c
)^(1/2)/d^2/(-d*x^3+8*c)+2/27*(7*d*x^3+170*c)*(d*x^3+c)^(1/2)/d^4
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {457, 100, 152, 65, 212}

$$\int \frac{x^{11}}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = -\frac{2944c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4} + \frac{2\sqrt{c+dx^3}(170c+7dx^3)}{27d^4} + \frac{8x^6\sqrt{c+dx^3}}{27d^2(8c-dx^3)}$$

```
[In] Int[x^11/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (8*x^6*Sqrt[c + d*x^3])/(27*d^2*(8*c - d*x^3)) + (2*Sqrt[c + d*x^3]*(170*c
+ 7*d*x^3))/(27*d^4) - (2944*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/
(81*d^4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= \frac{8x^6\sqrt{c+dx^3}}{27d^2(8c-dx^3)} - \frac{\text{Subst}\left(\int \frac{x(16c^2+21cdx)}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27cd^2}$$

$$= \frac{8x^6\sqrt{c+dx^3}}{27d^2(8c-dx^3)} + \frac{2\sqrt{c+dx^3}(170c+7dx^3)}{27d^4} - \frac{(1472c^2)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27d^3}$$

$$= \frac{8x^6\sqrt{c+dx^3}}{27d^2(8c-dx^3)} + \frac{2\sqrt{c+dx^3}(170c+7dx^3)}{27d^4} - \frac{(2944c^2)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{27d^4}$$

$$= \frac{8x^6\sqrt{c+dx^3}}{27d^2(8c-dx^3)} + \frac{2\sqrt{c+dx^3}(170c+7dx^3)}{27d^4} - \frac{2944c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.85

$$\int \frac{x^{11}}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{2\left(\frac{3\sqrt{c+dx^3}(-1360c^2+114cdx^3+3d^2x^6)}{-8c+dx^3} - 1472c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{81d^4}$$

```
[In] Integrate[x^11/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
[Out] (2*((3*Sqrt[c + d*x^3]*(-1360*c^2 + 114*c*d*x^3 + 3*d^2*x^6))/(-8*c + d*x^3
) - 1472*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(81*d^4)
```

Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$\frac{\frac{2\left(d\,x^{3}+c\right)^{\frac{3}{2}}}{9}+10c\sqrt{d\,x^{3}+c}+\frac{128c^{2}\left(\frac{4\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}-\frac{23\ \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3\sqrt{c}}\right)}{27}}{d^{4}}$
risch	$\frac{2\left(d\,x^{3}+46c\right)\sqrt{d\,x^{3}+c}}{9d^{4}}+\frac{64c^{2}\left(-\frac{2\ \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3d\sqrt{c}}+\frac{8c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{27d}\right)}{d^{3}}$
default	$\frac{d\left(\frac{2x^{3}\sqrt{d\,x^{3}+c}}{9d}-\frac{4c\sqrt{d\,x^{3}+c}}{9d^{2}}\right)+\frac{32c\sqrt{d\,x^{3}+c}}{3d}}{d^{3}}-\frac{128c^{\frac{3}{2}}\ \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3d^{4}}+\frac{512c^{3}\left(\frac{\sqrt{d\,x^{3}+c}}{c\left(-d\,x^{3}+8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{27d^{4}}$
elliptic	$\frac{512c^{2}\sqrt{d\,x^{3}+c}}{27d^{4}\left(-d\,x^{3}+8c\right)}+\frac{2x^{3}\sqrt{d\,x^{3}+c}}{9d^{3}}+\frac{92c\sqrt{d\,x^{3}+c}}{9d^{4}}+\frac{1472ic\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\cdots\right.}{\cdots}\right)}{\left(-c\,d^{2}\cdots\right.}}\cdots}{\cdots}\right)}{\cdots}$

[In] int(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2*(1/9*(d*x^3+c)^(3/2)+5*c*(d*x^3+c)^(1/2)+64/27*c^2*(4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-23/3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)))/d^4

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.05

$$\int \frac{x^{11}}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \left[\frac{2\left(736\,(cdx^3-8\,c^2)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+3\,(3\,d^2x^6+114\,cdx^3-1360\,c^2)\sqrt{dx^3+c}\right)}{81\,(d^5x^3-8\,cd^4)}, \frac{2\left(1472\,(c\right.}{}\right.$$

```
[In] integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/81*(736*(c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c)
 + 10*c)/(d*x^3 - 8*c)) + 3*(3*d^2*x^6 + 114*c*d*x^3 - 1360*c^2)*sqrt(d*x^3
 + c))/(d^5*x^3 - 8*c*d^4), 2/81*(1472*(c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/
3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(3*d^2*x^6 + 114*c*d*x^3 - 1360*c^2)*sqrt
(d*x^3 + c))/(d^5*x^3 - 8*c*d^4)]
```

Sympy [F]

$$\int \frac{x^{11}}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^{11}}{(-8c+dx^3)^2\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(x**11/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**11/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.98

$$\int \frac{x^{11}}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{2\left(736\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+9\,(dx^3+c)^{\frac{3}{2}}+405\,\sqrt{dx^3+c}c-\frac{768\,\sqrt{dx^3+c}c^2}{dx^3-8\,c}\right)}{81\,d^4}$$

```
[In] integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/81*(736*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sq
rt(c))) + 9*(d*x^3 + c)^(3/2) + 405*sqrt(d*x^3 + c)*c - 768*sqrt(d*x^3 + c)
*c^2/(d*x^3 - 8*c))/d^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.98

$$\int \frac{x^{11}}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{2944\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}d^4} - \frac{512\sqrt{dx^3+c}c^2}{27\,(dx^3-8\,c)d^4} + \frac{2\left((dx^3+c)^{\frac{3}{2}}d^8+45\sqrt{dx^3+c}cd^8\right)}{9\,d^{12}}$$

```
[In] integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2944/81*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 512/27*sq
rt(d*x^3 + c)*c^2/((d*x^3 - 8*c)*d^4) + 2/9*((d*x^3 + c)^(3/2)*d^8 + 45*sqr
t(d*x^3 + c)*c*d^8)/d^12
```

Mupad [B] (verification not implemented)

Time = 8.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13

$$\int \frac{x^{11}}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{92\,c\sqrt{d\,x^3+c}}{9\,d^4} + \frac{1472\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,d^4} + \frac{2\,x^3\sqrt{d\,x^3+c}}{9\,d^3} + \frac{512\,c^2\sqrt{d\,x^3+c}}{27\,d^4\,\left(8\,c-d\,x^3\right)}$$

```
[In] int(x^11/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] (92*c*(c + d*x^3)^(1/2))/(9*d^4) + (1472*c^(3/2)*log((10*c + d*x^3 - 6*c^(1
/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(81*d^4) + (2*x^3*(c + d*x^3)^(1/2))
/(9*d^3) + (512*c^2*(c + d*x^3)^(1/2))/(27*d^4*(8*c - d*x^3))
```

3.425 $\int \frac{x^8}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 83

$$\int \frac{x^8}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\frac{2\sqrt{c+dx^3}}{3d^3}+\frac{64c\sqrt{c+dx^3}}{27d^3(8c-dx^3)}-\frac{224\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

```
[Out] -224/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/d^3+2/3*(d*x^3+c)^(1/2
)/d^3+64/27*c*(d*x^3+c)^(1/2)/d^3/(-d*x^3+8*c)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {457, 91, 81, 65, 212}

$$\int \frac{x^8}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=-\frac{224\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}+\frac{64c\sqrt{c+dx^3}}{27d^3(8c-dx^3)}+\frac{2\sqrt{c+dx^3}}{3d^3}$$

```
[In] Int[x^8/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*d^3) + (64*c*Sqrt[c + d*x^3])/(27*d^3*(8*c - d*x^3))
 - (224*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right) \\
&= \frac{64c\sqrt{c+dx^3}}{27d^3\,(8c-dx^3)} - \frac{\text{Subst}\left(\int \frac{40c^2d+9cd^2x}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27cd^3} \\
&= \frac{2\sqrt{c+dx^3}}{3d^3} + \frac{64c\sqrt{c+dx^3}}{27d^3\,(8c-dx^3)} - \frac{(112c)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27d^2}
\end{aligned}
$$

$$= \frac{2\sqrt{c+dx^3}}{3d^3} + \frac{64c\sqrt{c+dx^3}}{27d^3(8c-dx^3)} - \frac{(224c)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{27d^3}$$

$$= \frac{2\sqrt{c+dx^3}}{3d^3} + \frac{64c\sqrt{c+dx^3}}{27d^3(8c-dx^3)} - \frac{224\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.84

$$\int \frac{x^8}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{2\left(\frac{3\sqrt{c+dx^3}(-104c+9dx^3)}{-8c+dx^3} - 112\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{81d^3}$$

```
[In] Integrate[x^8/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
[Out] (2*((3*Sqrt[c + d*x^3]*(-104*c + 9*d*x^3))/(-8*c + d*x^3) - 112*Sqrt[c]*Arc
Tanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(81*d^3)
```

Maple [A] (verified)

Time = 4.39 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{\frac{2\sqrt{d\,x^{3}+c}}{3}+\frac{32c\left(\frac{2\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}-\frac{7\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3\sqrt{c}}\right)}{27}}{d^{3}}$
default	$\frac{2\sqrt{d\,x^{3}+c}}{3d^{3}}-\frac{32\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\sqrt{c}}{9d^{3}}+\frac{64c^{2}\left(\frac{\sqrt{d\,x^{3}+c}}{c\left(-d\,x^{3}+8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{27d^{3}}$
risch	$\frac{2\sqrt{d\,x^{3}+c}}{3d^{3}}+\frac{16c\left(-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{9d\sqrt{c}}+\frac{4c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{27d}\right)}{d^{2}}$
elliptic	$\frac{64c\sqrt{d\,x^{3}+c}}{27d^{3}\left(-d\,x^{3}+8c\right)}+\frac{2\sqrt{d\,x^{3}+c}}{3d^{3}}+\frac{112i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{\cdots}{-3(\cdots}}\cdots}{\cdots}\right)}{\cdots}$

[In] int(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/3*((d*x^3+c)^(1/2)+16/9*c*(2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-7/3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)))/d^3

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.01

$$\int \frac{x^{8}}{\left(8c-dx^{3}\right)^{2}\sqrt{c+dx^{3}}}\,dx$$
$$=\left[\frac{2\left(56\left(dx^{3}-8\,c\right)\sqrt{c}\log\left(\frac{dx^{3}-6\,\sqrt{dx^{3}+c}\sqrt{c}+10\,c}{dx^{3}-8\,c}\right)+3\left(9\,dx^{3}-104\,c\right)\sqrt{dx^{3}+c}\right)}{81\left(d^{4}x^{3}-8\,cd^{3}\right)},\frac{2\left(112\left(dx^{3}-8\,c\right)\sqrt{-c}\,\text{arcta}\cdots\right.}{8\cdots}\right.$$

[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")

```
[Out] [2/81*(56*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10
*c)/(d*x^3 - 8*c)) + 3*(9*d*x^3 - 104*c)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^
3), 2/81*(112*(d*x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c)
 + 3*(9*d*x^3 - 104*c)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^3)]
```

Sympy [F]

$$\int \frac{x^8}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^8}{(-8c+dx^3)^2\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(x**8/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**8/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

$$\int \frac{x^8}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{2\left(56\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+27\sqrt{dx^3+c}-\frac{96\sqrt{dx^3+c}c}{dx^3-8c}\right)}{81d^3}$$

```
[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/81*(56*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqr
t(c))) + 27*sqrt(d*x^3 + c) - 96*sqrt(d*x^3 + c)*c/(d*x^3 - 8*c))/d^3
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{x^8}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{224\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}d^3}+\frac{2\sqrt{dx^3+c}}{3d^3}-\frac{64\sqrt{dx^3+c}c}{27(dx^3-8c)d^3}$$

```
[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 224/81*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) + 2/3*sqrt(d*x
^3 + c)/d^3 - 64/27*sqrt(d*x^3 + c)*c/((d*x^3 - 8*c)*d^3)
```

Mupad [B] (verification not implemented)

Time = 8.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \frac{x^8}{(8c-dx^3)^2\,\sqrt{c+dx^3}}\,dx=\frac{2\,\sqrt{d\,x^3+c}}{3\,d^3}+\frac{112\,\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,d^3}+\frac{64\,c\,\sqrt{d\,x^3+c}}{27\,d^3\,\left(8\,c-d\,x^3\right)}$$

```
[In] int(x^8/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*d^3) + (112*c^(1/2)*log((10*c + d*x^3 - 6*c^(1/2)*
(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(81*d^3) + (64*c*(c + d*x^3)^(1/2))/(27*
d^3*(8*c - d*x^3))
```

3.426 $\int \frac{x^5}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 64

$$\int \frac{x^5}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{8\sqrt{c+dx^3}}{27d^2(8c-dx^3)} - \frac{10\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}d^2}$$

```
[Out] -10/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^2/c^(1/2)+8/27*(d*x^3+c)^(1/2
)/d^2/(-d*x^3+8*c)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {457, 79, 65, 212}

$$\int \frac{x^5}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{8\sqrt{c+dx^3}}{27d^2(8c-dx^3)} - \frac{10\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}d^2}$$

```
[In] Int[x^5/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (8*Sqrt[c + d*x^3])/(27*d^2*(8*c - d*x^3)) - (10*ArcTanh[Sqrt[c + d*x^3]/(3
*Sqrt[c])])/(81*Sqrt[c]*d^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{(8c-dx)^2\sqrt{c+dx}}\,dx,x,x^3\right)\\ &= \frac{8\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)}-\frac{5\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{27d}\\ &= \frac{8\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)}-\frac{10\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{27d^2}\\ &= \frac{8\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)}-\frac{10\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}d^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int \frac{x^5}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=-\frac{8\sqrt{c+dx^3}}{27d^2\left(-8c+dx^3\right)}-\frac{10\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}d^2}$$

```
[In] Integrate[x^5/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-8*Sqrt[c + d*x^3])/(27*d^2*(-8*c + d*x^3)) - (10*ArcTanh[Sqrt[c + d*x^3]/
(3*Sqrt[c])])/(81*Sqrt[c]*d^2)
```

Maple [A] (verified)

Time = 4.35 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$\frac{\frac{8\sqrt{d\,x^{3}+c}}{27\left(-d\,x^{3}+8c\right)}-\frac{10\,\text{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{81\sqrt{c}}}{d^{2}}$
default	$-\frac{2\,\text{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{9d^{2}\sqrt{c}}+\frac{8c\left(\frac{\sqrt{d\,x^{3}+c}}{c\left(-d\,x^{3}+8c\right)}+\frac{\text{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{27d^{2}}$
elliptic	$\frac{8\sqrt{d\,x^{3}+c}}{27d^{2}(-d\,x^{3}+8c)}+\frac{5i\sqrt{2}\left(\sum_{\_\alpha=\text{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^{2}\right)}{d}\right.}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(\right.}}}{\ }\right.}{\ }$

```
[In] int(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/27*(4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-5/3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2
))/c^(1/2))/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.42

$$\int \frac{x^5}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \left[\frac{5\,(dx^3-8\,c)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-24\,\sqrt{dx^3+c}c}{81\,(cd^3x^3-8\,c^2d^2)}, \frac{2\left(5\,(dx^3-8\,c)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)-12\right.}{81\,(cd^3x^3-8\,c^2d^2)}\right.$$

```
[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/81*(5*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*
c)/(d*x^3 - 8*c)) - 24*sqrt(d*x^3 + c)*c)/(c*d^3*x^3 - 8*c^2*d^2), 2/81*(5*
(d*x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*sqrt(d*x
^3 + c)*c)/(c*d^3*x^3 - 8*c^2*d^2)]
```

Sympy [F]

$$\int \frac{x^5}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^5}{(-8c+dx^3)^2\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(x**5/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**5/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05

$$\int \frac{x^5}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{\frac{5\,\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)}{\sqrt{c}}-\frac{24\,\sqrt{dx^3+c}}{dx^3-8\,c}}{81\,d^2}$$

```
[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/81*(5*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/sq
rt(c) - 24*sqrt(d*x^3 + c)/(d*x^3 - 8*c))/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.91

$$\int \frac{x^5}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{2\left(\frac{5\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}d} - \frac{12\sqrt{dx^3+c}}{(dx^3-8c)d}\right)}{81\,d}$$

```
[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/81*(5*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 12*sqrt(d*x^3 +
 c)/((d*x^3 - 8*c)*d))/d
```

Mupad [B] (verification not implemented)

Time = 7.96 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.12

$$\int \frac{x^5}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{5\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,\sqrt{c}\,d^2} + \frac{8\,\sqrt{d\,x^3+c}}{27\,d^2\,\left(8\,c-d\,x^3\right)}$$

```
[In] int(x^5/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] (5*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(81*c^(
1/2)*d^2) + (8*(c + d*x^3)^(1/2))/(27*d^2*(8*c - d*x^3))
```

3.427 $\int \frac{x^2}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 67

$$\int \frac{x^2}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\frac{\sqrt{c+dx^3}}{27cd\,(8c-dx^3)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}$$

```
[Out] 1/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)/d+1/27*(d*x^3+c)^(1/2)/c/
d/(-d*x^3+8*c)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {455, 44, 65, 212}

$$\int \frac{x^2}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}+\frac{\sqrt{c+dx^3}}{27cd\,(8c-dx^3)}$$

```
[In] Int[x^2/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] Sqrt[c + d*x^3]/(27*c*d*(8*c - d*x^3)) + ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c]
)]/(81*c^(3/2)*d)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !Int
```

```
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= \frac{\sqrt{c+dx^3}}{27cd\,(8c-dx^3)}+\frac{\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{54c}\\
&= \frac{\sqrt{c+dx^3}}{27cd\,(8c-dx^3)}+\frac{\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{27cd}\\
&= \frac{\sqrt{c+dx^3}}{27cd\,(8c-dx^3)}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\frac{\frac{3\sqrt{c}\sqrt{c+dx^3}}{8c-dx^3}+\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}$$

```
[In] Integrate[x^2/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((3*Sqrt[c]*Sqrt[c + d*x^3])/(8*c - d*x^3) + ArcTanh[Sqrt[c + d*x^3]/(3*Sqr
t[c])])/(81*c^(3/2)*d)
```

Maple [A] (verified)

Time = 4.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

method	result
default	$\frac{\frac{\sqrt{d\,x^3+c}}{c\left(-d\,x^3+8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}}{27d}$
pseudoelliptic	$\frac{\frac{\sqrt{d\,x^3+c}}{c\left(-d\,x^3+8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}}{27d}$
elliptic	$\frac{\sqrt{d\,x^3+c}}{27cd\left(-d\,x^3+8c\right)}-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\right.}}}{}\right.}{}$

```
[In] int(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/27*((d*x^3+c)^(1/2)/c/(-d*x^3+8*c)+1/3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2
))/c^(3/2))/d
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.28

$$\int \frac{x^2}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \left[\frac{(dx^3-8\,c)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-6\,\sqrt{dx^3+c}c}{162\,(c^2d^2x^3-8\,c^3d)}, -\frac{(dx^3-8\,c)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+3\,\sqrt{dx^3+c}c}{81\,(c^2d^2x^3-8\,c^3d)}\right]$$

```
[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/162*((d*x^3 - 8*c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c
)/(d*x^3 - 8*c)) - 6*sqrt(d*x^3 + c)*c)/(c^2*d^2*x^3 - 8*c^3*d), -1/81*((d*
x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*sqrt(d*x^3 +
 c)*c)/(c^2*d^2*x^3 - 8*c^3*d)]
```

Sympy [F]

$$\int \frac{x^2}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^2}{(-8c+dx^3)^2\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(x**2/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**2/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = -\frac{\frac{6\,\sqrt{dx^3+c}}{(dx^3+c)c-9\,c^2}+\frac{\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)}{c^{\frac{3}{2}}}}{162\,d}$$

```
[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/162*(6*sqrt(d*x^3 + c)/((d*x^3 + c)*c - 9*c^2) + log((sqrt(d*x^3 + c) -
3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/c^(3/2))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = -\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}cd} - \frac{\sqrt{dx^3+c}}{27\,(dx^3-8\,c)cd}$$

```
[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/81*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c*d) - 1/27*sqrt(d*x^3
 + c)/((d*x^3 - 8*c)*c*d)
```

Mupad [B] (verification not implemented)

Time = 7.92 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{162\,c^{3/2}\,d} + \frac{\sqrt{d\,x^3+c}}{27\,c\,d\,\left(8\,c-d\,x^3\right)}$$

```
[In] int(x^2/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(162*c^(3/2
)*d) + (c + d*x^3)^(1/2)/(27*c*d*(8*c - d*x^3))
```

3.428 $\int \frac{1}{x(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 88

$$\int \frac{1}{x\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx=\frac{\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)}+\frac{13\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{5/2}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{5/2}}$$

```
[Out] 13/2592*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)-1/96*arctanh((d*x^3+c)
^(1/2)/c^(1/2))/c^(5/2)+1/216*(d*x^3+c)^(1/2)/c^2/(-d*x^3+8*c)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {457, 105, 162, 65, 214, 212}

$$\int \frac{1}{x\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx=\frac{13\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{5/2}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{5/2}}+\frac{\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)}$$

```
[In] Int[1/(x*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] Sqrt[c + d*x^3]/(216*c^2*(8*c - d*x^3)) + (13*ArcTanh[Sqrt[c + d*x^3]/(3*Sq
rt[c])])/(2592*c^(5/2)) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(96*c^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)\\ &= \frac{\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{-9cd-\frac{d^2x}{2}}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{216c^2d}\end{aligned}$$

$$= \frac{\sqrt{c+dx^3}}{216c^2\,(8c-dx^3)} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{192c^2} + \frac{(13d)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{1728c^2}$$

$$= \frac{\sqrt{c+dx^3}}{216c^2\,(8c-dx^3)} + \frac{13\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{864c^2} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{96c^2d}$$

$$= \frac{\sqrt{c+dx^3}}{216c^2\,(8c-dx^3)} + \frac{13\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{5/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{5/2}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\,(8c-dx^3)^2\,\sqrt{c+dx^3}}\,dx = \frac{\frac{12\sqrt{c}\sqrt{c+dx^3}}{8c-dx^3} + 13\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 27\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2592c^{5/2}}$$

```
[In] Integrate[1/(x*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((12*Sqrt[c]*Sqrt[c + d*x^3])/(8*c - d*x^3) + 13*ArcTanh[Sqrt[c + d*x^3]/(3
*Sqrt[c])] - 27*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2592*c^(5/2))
```

Maple [A] (verified)

Time = 4.51 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89

method	result	size
pseudoelliptic	$-\frac{\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{96c^{\frac{5}{2}}} + \frac{\frac{13\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)\left(d\,x^3-8c\right)}{\sqrt{c}} - 12\sqrt{d\,x^3+c}}{2592(d\,x^3-8c)c^2}$	78
default	$-\frac{\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{96c^{\frac{5}{2}}} + \frac{\frac{\sqrt{d\,x^3+c}}{c\left(-d\,x^3+8c\right)} + \frac{\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}}{216c} + \frac{\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{288c^{\frac{5}{2}}}$	92
elliptic	Expression too large to display	1534

```
[In] int(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/96*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)+1/2592*(13*arctanh(1/3*(d*x^
3+c)^(1/2)/c^(1/2))/c^(1/2)*(d*x^3-8*c)-12*(d*x^3+c)^(1/2))/(d*x^3-8*c)/c^2
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.57

$$\int \frac{1}{x\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$$
$$=\left[\frac{13\left(dx^3-8\,c\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+27\left(dx^3-8\,c\right)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-24\sqrt{dx^3+c}c}{5184\left(c^3dx^3-8\,c^4\right)},\,\frac{27}{\ldots}\right.$$

```
[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/5184*(13*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) +
10*c)/(d*x^3 - 8*c)) + 27*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 +
 c)*sqrt(c) + 2*c)/x^3) - 24*sqrt(d*x^3 + c)*c)/(c^3*d*x^3 - 8*c^4), 1/2592
*(27*(d*x^3 - 8*c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 13*(d*x^3
- 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*sqrt(d*x^3 + c)
*c)/(c^3*d*x^3 - 8*c^4)]
```

Sympy [F]

$$\int \frac{1}{x\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx=\int \frac{1}{x\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(1/x/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx=\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2x}\,dx$$

```
[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}c^2} - \frac{13\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{2592\sqrt{-c}c^2} - \frac{\sqrt{dx^3+c}}{216\,(dx^3-8\,c)c^2}$$

```
[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 13/2592*arctan(1/3*s
qrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/216*sqrt(d*x^3 + c)/((d*x^3 - 8
*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 8.00 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{13\,\mathrm{atanh}\left(\frac{c^2\sqrt{d\,x^3+c}}{3\sqrt{c^5}}\right)}{2592\sqrt{c^5}} - \frac{\mathrm{atanh}\left(\frac{c^2\sqrt{d\,x^3+c}}{\sqrt{c^5}}\right)}{96\sqrt{c^5}} + \frac{\sqrt{d\,x^3+c}}{72\,c^2\,\left(24\,c-3\,d\,x^3\right)}$$

```
[In] int(1/(x*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] (13*atanh((c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2))))/(2592*(c^5)^(1/2)) - at
anh((c^2*(c + d*x^3)^(1/2))/(c^5)^(1/2))/(96*(c^5)^(1/2)) + (c + d*x^3)^(1/
2)/(72*c^2*(24*c - 3*d*x^3))
```

3.429 $\int \frac{1}{x^4(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 124

$$\int \frac{1}{x^4 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = \frac{5d\sqrt{c+dx^3}}{864c^3(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{24c^2x^3(8c-dx^3)} + \frac{11d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{10368c^{7/2}} + \frac{d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{7/2}}$$

```
[Out] 11/10368*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)+1/384*d*arctanh((d*
x^3+c)^(1/2)/c^(1/2))/c^(7/2)+5/864*d*(d*x^3+c)^(1/2)/c^3/(-d*x^3+8*c)-1/24
*(d*x^3+c)^(1/2)/c^2/x^3/(-d*x^3+8*c)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {457, 105, 156, 162, 65, 214, 212}

$$\int \frac{1}{x^4 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = \frac{11d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{10368c^{7/2}} + \frac{d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{7/2}} + \frac{5d\sqrt{c+dx^3}}{864c^3(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{24c^2x^3(8c-dx^3)}$$

```
[In] Int[1/(x^4*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (5*d*Sqrt[c + d*x^3])/(864*c^3*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(24*c^2*x^3
*(8*c - d*x^3)) + (11*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(10368*c^(7/2
)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(384*c^(7/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2(8c-dx)^2\sqrt{c+dx}}\,dx,x,x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{24c^2x^3\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{2cd-\frac{3d^2x}{2}}{x(8c-dx)^2\sqrt{c+dx}}\,dx,x,x^3\right)}{24c^2}$$

$$= \frac{5d\sqrt{c+dx^3}}{864c^3\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24c^2x^3\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{-18c^2d^2+5cd^3x}{x(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{1728c^4d}$$

$$= \frac{5d\sqrt{c+dx^3}}{864c^3\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24c^2x^3\left(8c-dx^3\right)} - \frac{d\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{768c^3} + \frac{(11d^2)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{6912c^3}$$

$$= \frac{5d\sqrt{c+dx^3}}{864c^3\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24c^2x^3\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{384c^3} + \frac{(11d)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{3456c^3}$$

$$= \frac{5d\sqrt{c+dx^3}}{864c^3\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24c^2x^3\left(8c-dx^3\right)} + \frac{11d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{10368c^{7/2}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{7/2}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^4\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{\frac{12\sqrt{c}(36c-5dx^3)\sqrt{c+dx^3}}{-8cx^3+dx^6} + 11d\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) + 27d\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{10368c^{7/2}}$$

```
[In] Integrate[1/(x^4*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

[Out] ((12*Sqrt[c]*(36*c - 5*d*x^3)*Sqrt[c + d*x^3])/(-8*c*x^3 + d*x^6) + 11*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] + 27*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(10368*c^(7/2))

Maple [A] (verified)

Time = 4.64 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{d\left(-\frac{-\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)d\,x^{3}+2\sqrt{d\,x^{3}+c}\,\sqrt{c}}{2d\,x^{3}c^{\frac{7}{2}}}+\frac{\frac{\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}+\frac{11\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{6\sqrt{c}}}{9c^{3}}\right)}{192}$
risch	$-\frac{\sqrt{d\,x^{3}+c}}{192c^{3}x^{3}}-\frac{d\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3\sqrt{c}}-\frac{2c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{27}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{9\sqrt{c}}\right)}{128c^{3}}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{3c\,x^{3}}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}}{64c^{2}}-\frac{d\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{384c^{\frac{7}{2}}}+\frac{d\left(\frac{\sqrt{d\,x^{3}+c}}{c\left(-d\,x^{3}+8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{1728c^{2}}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{d}}{3}\right.}{1152c^{\frac{7}{2}}}$
elliptic	Expression too large to display

[In] int(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/192*d*(-1/2*(-arctanh((d*x^3+c)^(1/2)/c^(1/2))*d*x^3+2*(d*x^3+c)^(1/2)*c^(1/2))/d/x^3/c^(7/2)+1/9*((d*x^3+c)^(1/2)/(-d*x^3+8*c)+11/6*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))/c^3)

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.26

$$\int \frac{1}{x^4\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$$
$$=\left[\frac{11\left(d^2x^6-8\,cdx^3\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+27\left(d^2x^6-8\,cdx^3\right)\sqrt{c}\log\left(\frac{dx^3+2\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-24\left(5\,c\right.}{20736\left(c^4dx^6-8\,c^5x^3\right)}\right.$$
$$-\frac{27\left(d^2x^6-8\,cdx^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+11\left(d^2x^6-8\,cdx^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+12\left(5\,cdx^3\right.}{10368\left(c^4dx^6-8\,c^5x^3\right)}$$

[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")

```
[Out] [1/20736*(11*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*s
qrt(c) + 10*c)/(d*x^3 - 8*c)) + 27*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3
 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*(5*c*d*x^3 - 36*c^2)*sqrt(d*x
^3 + c))/(c^4*d*x^6 - 8*c^5*x^3), -1/10368*(27*(d^2*x^6 - 8*c*d*x^3)*sqrt(-
c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + 11*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*a
rctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*(5*c*d*x^3 - 36*c^2)*sqrt(d*x^3
+ c))/(c^4*d*x^6 - 8*c^5*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{x^4 (-8c+dx^3)^2 \sqrt{c+dx^3}}\,dx$$

```
[In] integrate(1/x**4/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(x**4*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)^2x^4}\,dx$$

```
[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = -\frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{384\,\sqrt{-c}c^3} - \frac{11\,d\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{10368\,\sqrt{-c}c^3} - \frac{5\,(dx^3+c)^{\frac{3}{2}}d - 41\,\sqrt{dx^3+c}cd}{864\,\left((dx^3+c)^2 - 10\,(dx^3+c)c + 9\,c^2\right)c^3}$$

```
[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] -1/384*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 11/10368*d*arcta
n(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/864*(5*(d*x^3 + c)^(3/2)
*d - 41*sqrt(d*x^3 + c)*c*d)/(((d*x^3 + c)^2 - 10*(d*x^3 + c)*c + 9*c^2)*c^
3)
```

Mupad [B] (verification not implemented)

Time = 8.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^4 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = \frac{\frac{41\,d\,\sqrt{d\,x^3+c}}{288\,c^2}-\frac{5\,d\,{\left(d\,x^3+c\right)}^{3/2}}{288\,c^3}}{3\,{\left(d\,x^3+c\right)}^2-30\,c\,\left(d\,x^3+c\right)+27\,c^2} - \frac{d\,\left(\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{\sqrt{c^7}}\right)\,1{}\mathrm{i}+\frac{\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^7}}\right)\,11{}\mathrm{i}}{27}\right)\,1{}\mathrm{i}}{384\,\sqrt{c^7}}$$

```
[In] int(1/(x^4*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] ((41*d*(c + d*x^3)^(1/2))/(288*c^2) - (5*d*(c + d*x^3)^(3/2))/(288*c^3))/(3
*(c + d*x^3)^2 - 30*c*(c + d*x^3) + 27*c^2) - (d*(atanh((c^3*(c + d*x^3)^(1
/2))/(c^7)^(1/2))*1i + (atanh((c^3*(c + d*x^3)^(1/2))/(3*(c^7)^(1/2)))*11i)
/27)*1i)/(384*(c^7)^(1/2))
```

3.430 $\int \frac{1}{x^7(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 164

$$\int \frac{1}{x^7(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = -\frac{35d^2\sqrt{c+dx^3}}{13824c^4(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{48c^2x^6(8c-dx^3)} + \frac{3d\sqrt{c+dx^3}}{128c^3x^3(8c-dx^3)} + \frac{31d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{165888c^{9/2}} - \frac{19d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6144c^{9/2}}$$

```
[Out] 31/165888*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(9/2)-19/6144*d^2*arct
anh((d*x^3+c)^(1/2)/c^(1/2))/c^(9/2)-35/13824*d^2*(d*x^3+c)^(1/2)/c^4/(-d*x
^3+8*c)-1/48*(d*x^3+c)^(1/2)/c^2/x^6/(-d*x^3+8*c)+3/128*d*(d*x^3+c)^(1/2)/c
^3/x^3/(-d*x^3+8*c)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {457, 105, 156, 162, 65, 214, 212}

$$\int \frac{1}{x^7(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{31d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{165888c^{9/2}} - \frac{19d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6144c^{9/2}} - \frac{35d^2\sqrt{c+dx^3}}{13824c^4(8c-dx^3)} + \frac{3d\sqrt{c+dx^3}}{128c^3x^3(8c-dx^3)} - \frac{\sqrt{c+dx^3}}{48c^2x^6(8c-dx^3)}$$

```
[In] Int[1/(x^7*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-35*d^2*Sqrt[c + d*x^3])/(13824*c^4*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(48*c
^2*x^6*(8*c - d*x^3)) + (3*d*Sqrt[c + d*x^3])/(128*c^3*x^3*(8*c - d*x^3)) +
 (31*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(165888*c^(9/2)) - (19*d^2*A
rcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(6144*c^(9/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^3(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right) \\
&= -\frac{\sqrt{c+dx^3}}{48c^2x^6\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{9cd-\frac{5d^2x}{2}}{x^2(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{48c^2} \\
&= -\frac{\sqrt{c+dx^3}}{48c^2x^6\left(8c-dx^3\right)} + \frac{3d\sqrt{c+dx^3}}{128c^3x^3\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{38c^2d^2-\frac{27}{2}cd^3x}{x(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{384c^4} \\
&= -\frac{35d^2\sqrt{c+dx^3}}{13824c^4\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48c^2x^6\left(8c-dx^3\right)} \\
&\quad + \frac{3d\sqrt{c+dx^3}}{128c^3x^3\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{-342c^3d^3+35c^2d^4x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27648c^6d} \\
&= -\frac{35d^2\sqrt{c+dx^3}}{13824c^4\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48c^2x^6\left(8c-dx^3\right)} + \frac{3d\sqrt{c+dx^3}}{128c^3x^3\left(8c-dx^3\right)} \\
&\quad + \frac{\left(19d^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{12288c^4} + \frac{\left(31d^3\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{110592c^4} \\
&= -\frac{35d^2\sqrt{c+dx^3}}{13824c^4\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48c^2x^6\left(8c-dx^3\right)} + \frac{3d\sqrt{c+dx^3}}{128c^3x^3\left(8c-dx^3\right)} \\
&\quad + \frac{(19d)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{6144c^4} + \frac{\left(31d^2\right)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{55296c^4} \\
&= -\frac{35d^2\sqrt{c+dx^3}}{13824c^4\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48c^2x^6\left(8c-dx^3\right)} + \frac{3d\sqrt{c+dx^3}}{128c^3x^3\left(8c-dx^3\right)} \\
&\quad + \frac{31d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{165888c^{9/2}} - \frac{19d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6144c^{9/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^7 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = \frac{\frac{12\sqrt{c}\sqrt{c+dx^3}(288c^2-324cdx^3+35d^2x^6)}{-8cx^6+dx^9}+31d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-513d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{165888c^{9/2}}$$

```
[In] Integrate[1/(x^7*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((12*Sqrt[c]*Sqrt[c + d*x^3]*(288*c^2 - 324*c*d*x^3 + 35*d^2*x^6))/(-8*c*x^
6 + d*x^9) + 31*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 513*d^2*ArcTanh[
Sqrt[c + d*x^3]/Sqrt[c]])/(165888*c^(9/2))
```

Maple [A] (verified)

Time = 4.64 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$-\frac{4104\left(-\frac{31\left(c-\frac{d\,x^{3}}{8}\right)d^{2}x^{6}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{513}+\left(c-\frac{d\,x^{3}}{8}\right)d^{2}x^{6}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)+\frac{35\left(d^{2}x^{6}\sqrt{c}-\frac{324d\,x^{3}c^{\frac{3}{2}}}{35}+\frac{288c^{\frac{5}{2}}}{35}\right)\sqrt{d\,x^{3}+c}}{342}\right)}{c^{\frac{9}{2}}\left(-165888d\,x^{9}+1327104c\,x^{6}\right)}$
risch	$-\frac{\sqrt{d\,x^{3}+c}\,\left(-d\,x^{3}+c\right)}{384c^{4}x^{6}}+\frac{d^{2}\left(-\frac{19\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{24\sqrt{c}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{24\sqrt{c}}+\frac{c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{54}\right)}{256c^{4}}$
default	$\frac{-\frac{\sqrt{d\,x^{3}+c}}{6c\,x^{6}}+\frac{d\sqrt{d\,x^{3}+c}}{4c^{2}x^{3}}-\frac{d^{2}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{4c^{\frac{5}{2}}}}{64c^{2}}+\frac{d\left(-\frac{\sqrt{d\,x^{3}+c}}{3c\,x^{3}}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{256c^{3}}-\frac{d^{2}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{2048c^{\frac{9}{2}}}+$
elliptic	Expression too large to display

```
[In] int(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4104*(-31/513*(c-1/8*d*x^3)*d^2*x^6*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))+(
c-1/8*d*x^3)*d^2*x^6*arctanh((d*x^3+c)^(1/2)/c^(1/2))+35/342*(d^2*x^6*c^(1/
2)-324/35*d*x^3*c^(3/2)+288/35*c^(5/2))*(d*x^3+c)^(1/2))/c^(9/2)/(-165888*d
*x^9+1327104*c*x^6)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.89

$$\int \frac{1}{x^7 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx$$
$$= \left[\frac{31 (d^3x^9 - 8cd^2x^6)\sqrt{c} \log \left(\frac{dx^3 + 6\sqrt{dx^3 + c}\sqrt{c} + 10c}{dx^3 - 8c}\right) + 513 (d^3x^9 - 8cd^2x^6)\sqrt{c} \log \left(\frac{dx^3 - 2\sqrt{dx^3 + c}\sqrt{c} + 2c}{x^3}\right) + 24 (3}{331776 (c^5dx^9 - 8c^6x^6)} \right.$$

```
[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/331776*(31*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c
)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 513*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log(
(d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 24*(35*c*d^2*x^6 - 324*c^2
*d*x^3 + 288*c^3)*sqrt(d*x^3 + c))/(c^5*d*x^9 - 8*c^6*x^6), 1/165888*(513*(
d^3*x^9 - 8*c*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 31*(d^
3*x^9 - 8*c*d^2*x^6)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*(
35*c*d^2*x^6 - 324*c^2*d*x^3 + 288*c^3)*sqrt(d*x^3 + c))/(c^5*d*x^9 - 8*c^6
*x^6)]
```

Sympy [F]

$$\int \frac{1}{x^7 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{1}{x^7 (-8c + dx^3)^2 \sqrt{c + dx^3}} \, dx$$

```
[In] integrate(1/x**7/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**7*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^7 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{1}{\sqrt{dx^3 + c}(dx^3 - 8c)^2 x^7} \, dx$$

```
[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^7), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^7 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = \frac{19\,d^2 \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{6144\,\sqrt{-c}c^4} - \frac{31\,d^2 \arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{165888\,\sqrt{-c}c^4} - \frac{\sqrt{dx^3+c}d^2}{13824\,(dx^3-8\,c)c^4} + \frac{(dx^3+c)^{\frac{3}{2}}d^2 - 2\,\sqrt{dx^3+c}cd^2}{384\,c^4d^2x^6}$$

```
[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 19/6144*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 31/165888*d^2
*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 1/13824*sqrt(d*x^3 +
 c)*d^2/((d*x^3 - 8*c)*c^4) + 1/384*((d*x^3 + c)^(3/2)*d^2 - 2*sqrt(d*x^3 +
 c)*c*d^2)/(c^4*d^2*x^6)
```

Mupad [B] (verification not implemented)

Time = 8.43 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^7 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = -\frac{\frac{647\,d^2\,\sqrt{d\,x^3+c}}{4608\,c^2} - \frac{197\,d^2\,{\left(d\,x^3+c\right)}^{3/2}}{2304\,c^3} + \frac{35\,d^2\,{\left(d\,x^3+c\right)}^{5/2}}{4608\,c^4}}{33\,c\,{\left(d\,x^3+c\right)}^2 - 57\,c^2\,\left(d\,x^3+c\right) - 3\,{\left(d\,x^3+c\right)}^3 + 27\,c^3} + \frac{d^2\,\left(\mathrm{atanh}\left(\frac{c^4\,\sqrt{d\,x^3+c}}{\sqrt{c^9}}\right)\,1{}\mathrm{i} - \frac{\mathrm{atanh}\left(\frac{c^4\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^9}}\right)\,31{}\mathrm{i}}{513}\right)\,19{}\mathrm{i}}{6144\,\sqrt{c^9}}$$

```
[In] int(1/(x^7*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] (d^2*(atanh((c^4*(c + d*x^3)^(1/2))/(c^9)^(1/2))*1i - (atanh((c^4*(c + d*x^
3)^(1/2))/(3*(c^9)^(1/2)))*31i)/513)*19i)/(6144*(c^9)^(1/2)) - ((647*d^2*(c
 + d*x^3)^(1/2))/(4608*c^2) - (197*d^2*(c + d*x^3)^(3/2))/(2304*c^3) + (35*
d^2*(c + d*x^3)^(5/2))/(4608*c^4))/(33*c*(c + d*x^3)^2 - 57*c^2*(c + d*x^3)
 - 3*(c + d*x^3)^3 + 27*c^3)
```

3.431 $\int \frac{x^7}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 641

$$\int \frac{x^7}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{62\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{8x^2\sqrt{c+dx^3}}{27d^2(8c-dx^3)}+\frac{44\sqrt[6]{c}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}d^{8/3}}$$
$$-\frac{44\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{81d^{8/3}}+\frac{44\sqrt[6]{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^{8/3}}$$
$$-\frac{31\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)\middle|-7-4\sqrt{3}\right)}{9\ 3^{3/4}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+\frac{62\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] -44/81*c^(1/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d
^(8/3)+44/81*c^(1/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(8/3)+44/81*c^(
```

```
1/6)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/d^(8/3)*3^
(1/2)+8/27*x^2*(d*x^3+c)^(1/2)/d^2/(-d*x^3+8*c)+62/27*(d*x^3+c)^(1/2)/d^(8/
3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+62/81*c^(1/3)*(c^(1/3)+d^(1/3)*x)*Ellipt
icF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/
2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)
*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^
(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-31/27*c^(1/3)*(c^(1/3)+d^
(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^
(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*
x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/d^(8/3)/(d*
x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2
)^(1/2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {481, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{x^7}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$$
$$= \frac{62\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{31\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+\frac{44\sqrt[6]{c}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}d^{8/3}}-\frac{44\sqrt[6]{c}\,\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{81d^{8/3}}$$
$$+\frac{44\sqrt[6]{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^{8/3}}+\frac{62\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{8x^2\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)}$$

```
[In] Int[x^7/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (62*Sqrt[c + d*x^3])/(27*d^(8/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (8*
x^2*Sqrt[c + d*x^3])/(27*d^2*(8*c - d*x^3)) + (44*c^(1/6)*ArcTan[(Sqrt[3]*c
^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(27*Sqrt[3]*d^(8/3)) - (44*
c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(81*d
^(8/3)) + (44*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^(8/3)) -
(31*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)
*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[
ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)], -7 - 4*Sqrt[3]])/(9*3^(3/4)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*
x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (62*Sqrt[2]*c
^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^
2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c
^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/
(27*3^(1/4)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(
1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
```

```
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
```

```
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\text{integral} = \frac{8x^2\sqrt{c+dx^3}}{27d^2(8c-dx^3)} - \frac{\int \frac{x(16c^2+31cdx^3)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{27cd^2}$$

$$= \frac{8x^2\sqrt{c+dx^3}}{27d^2(8c-dx^3)} - \frac{\int \left(-\frac{31cx}{\sqrt{c+dx^3}} + \frac{264c^2x}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{27cd^2}$$

$$= \frac{8x^2\sqrt{c+dx^3}}{27d^2(8c-dx^3)} + \frac{31\int \frac{x}{\sqrt{c+dx^3}}\,dx}{27d^2} - \frac{(88c)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{9d^2}$$

$$= \frac{8x^2\sqrt{c+dx^3}}{27d^2(8c-dx^3)} + \frac{22\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{27d^3}$$

$$+ \frac{31\int \frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{27d^{7/3}} - \frac{(22\sqrt[3]{c})\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{27d^{7/3}}$$

$$- \frac{(31(1-\sqrt{3})\sqrt[3]{c})\int \frac{1}{\sqrt{c+dx^3}}\,dx}{27d^{7/3}} + \frac{(22c^{2/3})\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{9d^{5/3}}$$

$$= \frac{62\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{8x^2\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} - \frac{31\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{62\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{\left(44c^{2/3}\right)\text{Subst}\left(\int \frac{1}{9-cx^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{27d^{8/3}} + \frac{\left(22c^{2/3}\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27d^{5/3}} - \frac{88\text{Subst}\left(\int \frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{27\sqrt[3]{c}d^{2/3}}$$

$$= \frac{62\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{8x^2\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)}$$

$$+\frac{44\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}d^{8/3}}-\frac{44\sqrt[6]{c}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{81d^{8/3}}$$

$$-\frac{31\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{62\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{\left(44c^{2/3}\right)\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{27d^{8/3}}$$

$$
\begin{aligned}
&= \frac{62\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{8x^2\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} \\
&\quad + \frac{44\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}d^{8/3}} \\
&\quad - \frac{44\sqrt[6]{c}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{81d^{8/3}} + \frac{44\sqrt[6]{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^{8/3}} \\
&\quad - \frac{31\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&\quad + \frac{62\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.26

$$
\int \frac{x^7}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx
$$

$$
= \frac{320cx^2(c+dx^3)+40cx^2(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+31dx^5(-8c+dx^3)\sqrt{1+\cdots}}{1080cd^2\left(8c-dx^3\right)\sqrt{c+dx^3}}
$$

```
[In] Integrate[x^7/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] (320*c*x^2*(c + d*x^3) + 40*c*x^2*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*Appell
F1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 31*d*x^5*(-8*c + d*x^3)
*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)
])/(1080*c*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.69 (sec) , antiderivative size = 877, normalized size of antiderivative = 1.37

method	result	size
elliptic	Expression too large to display	877
default	Expression too large to display	1738

```
[In] int(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 8/27*x^2*(d*x^3+c)^(1/2)/d^2/(-d*x^3+8*c)-62/81*I/d^3*3^(1/2)*(-c*d^2)^(1/3
)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/
d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*
(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3)))^(1/2)))+88/243*I/d^5*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*
I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1
/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))
^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^
2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2
)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ell
ipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_al
pha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alp
ha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.67 (sec) , antiderivative size = 2397, normalized size of antiderivative = 3.74

$$\int \frac{x^7}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/243*(72*sqrt(d*x^3 + c)*d*x^2 + 558*(d*x^3 - 8*c)*sqrt(d)*weierstrassZet
a(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 11*(d^4*x^3 - 8*c*d^3 - s
qrt(-3)*(d^4*x^3 - 8*c*d^3))*(c/d^16)^(1/6)*log(164916224/3*((d^16*x^9 + 31
8*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13 + sqrt(-3)*(d^16*x^9 + 318*
c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c/d^16)^(5/6) + 6*(2*c*d^2
*x^7 + 160*c^2*d*x^4 + 320*c^3*x - 6*(5*c*d^12*x^5 + 32*c^2*d^11*x^2 - sqrt
(-3)*(5*c*d^12*x^5 + 32*c^2*d^11*x^2))*(c/d^16)^(2/3) - (7*c*d^7*x^6 + 152*
c^2*d^6*x^3 + 64*c^3*d^5 + sqrt(-3)*(7*c*d^7*x^6 + 152*c^2*d^6*x^3 + 64*c^3
*d^5))*(c/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c*d^10*x^7 + 64*c^2*d^9*x^4
+ 32*c^3*d^8*x)*sqrt(c/d^16) + 18*(c*d^5*x^8 + 38*c^2*d^4*x^5 + 64*c^3*d^3*
x^2 - sqrt(-3)*(c*d^5*x^8 + 38*c^2*d^4*x^5 + 64*c^3*d^3*x^2))*(c/d^16)^(1/6
))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 11*(d^4*x^3 - 8*c*
d^3 - sqrt(-3)*(d^4*x^3 - 8*c*d^3))*(c/d^16)^(1/6)*log(-164916224/3*((d^16*
x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13 + sqrt(-3)*(d^16*x^
9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c/d^16)^(5/6) - 6*
(2*c*d^2*x^7 + 160*c^2*d*x^4 + 320*c^3*x - 6*(5*c*d^12*x^5 + 32*c^2*d^11*x^
2 - sqrt(-3)*(5*c*d^12*x^5 + 32*c^2*d^11*x^2))*(c/d^16)^(2/3) - (7*c*d^7*x^
6 + 152*c^2*d^6*x^3 + 64*c^3*d^5 + sqrt(-3)*(7*c*d^7*x^6 + 152*c^2*d^6*x^3
+ 64*c^3*d^5))*(c/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c*d^10*x^7 + 64*c^2*
d^9*x^4 + 32*c^3*d^8*x)*sqrt(c/d^16) + 18*(c*d^5*x^8 + 38*c^2*d^4*x^5 + 64*
c^3*d^3*x^2 - sqrt(-3)*(c*d^5*x^8 + 38*c^2*d^4*x^5 + 64*c^3*d^3*x^2))*(c/d^
16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 11*(d^4*x^
3 - 8*c*d^3 + sqrt(-3)*(d^4*x^3 - 8*c*d^3))*(c/d^16)^(1/6)*log(164916224/3*
((d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13 - sqrt(-3)*(
d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c/d^16)^(5/
6) + 6*(2*c*d^2*x^7 + 160*c^2*d*x^4 + 320*c^3*x - 6*(5*c*d^12*x^5 + 32*c^2*
d^11*x^2 + sqrt(-3)*(5*c*d^12*x^5 + 32*c^2*d^11*x^2))*(c/d^16)^(2/3) - (7*c
*d^7*x^6 + 152*c^2*d^6*x^3 + 64*c^3*d^5 - sqrt(-3)*(7*c*d^7*x^6 + 152*c^2*d
^6*x^3 + 64*c^3*d^5))*(c/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c*d^10*x^7 +
64*c^2*d^9*x^4 + 32*c^3*d^8*x)*sqrt(c/d^16) + 18*(c*d^5*x^8 + 38*c^2*d^4*x^
5 + 64*c^3*d^3*x^2 + sqrt(-3)*(c*d^5*x^8 + 38*c^2*d^4*x^5 + 64*c^3*d^3*x^2)
)*(c/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 11*
(d^4*x^3 - 8*c*d^3 + sqrt(-3)*(d^4*x^3 - 8*c*d^3))*(c/d^16)^(1/6)*log(-1649
16224/3*((d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13 - sq
rt(-3)*(d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13))*(c/d
^16)^(5/6) - 6*(2*c*d^2*x^7 + 160*c^2*d*x^4 + 320*c^3*x - 6*(5*c*d^12*x^5 +
 32*c^2*d^11*x^2 + sqrt(-3)*(5*c*d^12*x^5 + 32*c^2*d^11*x^2))*(c/d^16)^(2/3
) - (7*c*d^7*x^6 + 152*c^2*d^6*x^3 + 64*c^3*d^5 - sqrt(-3)*(7*c*d^7*x^6 + 1
52*c^2*d^6*x^3 + 64*c^3*d^5))*(c/d^16)^(1/3))*sqrt(d*x^3 + c) - 36*(5*c*d^1
0*x^7 + 64*c^2*d^9*x^4 + 32*c^3*d^8*x)*sqrt(c/d^16) + 18*(c*d^5*x^8 + 38*c^
2*d^4*x^5 + 64*c^3*d^3*x^2 + sqrt(-3)*(c*d^5*x^8 + 38*c^2*d^4*x^5 + 64*c^3*
d^3*x^2))*(c/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3
)) + 22*(d^4*x^3 - 8*c*d^3)*(c/d^16)^(1/6)*log(164916224/3*((d^16*x^9 + 318
*c*d^15*x^6 + 1200*c^2*d^14*x^3 + 640*c^3*d^13)*(c/d^16)^(5/6) + 6*(c*d^2*x
^7 + 80*c^2*d*x^4 + 160*c^3*x + 6*(5*c*d^12*x^5 + 32*c^2*d^11*x^2)*(c/d^16)
```

```
^(2/3) + (7*c*d^7*x^6 + 152*c^2*d^6*x^3 + 64*c^3*d^5)*(c/d^16)^(1/3))*sqrt(
d*x^3 + c) + 18*(5*c*d^10*x^7 + 64*c^2*d^9*x^4 + 32*c^3*d^8*x)*sqrt(c/d^16)
 + 18*(c*d^5*x^8 + 38*c^2*d^4*x^5 + 64*c^3*d^3*x^2)*(c/d^16)^(1/6))/(d^3*x^
9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 22*(d^4*x^3 - 8*c*d^3)*(c/d^
16)^(1/6)*log(-164916224/3*((d^16*x^9 + 318*c*d^15*x^6 + 1200*c^2*d^14*x^3
+ 640*c^3*d^13)*(c/d^16)^(5/6) - 6*(c*d^2*x^7 + 80*c^2*d*x^4 + 160*c^3*x +
6*(5*c*d^12*x^5 + 32*c^2*d^11*x^2)*(c/d^16)^(2/3) + (7*c*d^7*x^6 + 152*c^2*
d^6*x^3 + 64*c^3*d^5)*(c/d^16)^(1/3))*sqrt(d*x^3 + c) + 18*(5*c*d^10*x^7 +
64*c^2*d^9*x^4 + 32*c^3*d^8*x)*sqrt(c/d^16) + 18*(c*d^5*x^8 + 38*c^2*d^4*x^
5 + 64*c^3*d^3*x^2)*(c/d^16)^(1/6))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3
 - 512*c^3)))/(d^4*x^3 - 8*c*d^3)
```

Sympy [F]

$$\int \frac{x^7}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{x^7}{(-8c+dx^3)^2\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(x**7/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(x**7/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^7}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{x^7}{\sqrt{dx^3+c}(dx^3-8\,c)^2}\,dx$$

```
[In] integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^7/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Giac [F]

$$\int \frac{x^7}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{x^7}{\sqrt{dx^3+c}(dx^3-8\,c)^2}\,dx$$

```
[In] integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^7/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^7}{\sqrt{d\,x^3+c}\,(8\,c-d\,x^3)^2}\,dx$$

```
[In] int(x^7/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
[Out] int(x^7/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.432 $\int \frac{x^4}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 647

$$\int \frac{x^4}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{x^2\sqrt{c+dx^3}}{27cd\,(8c-dx^3)}$$
$$+\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}c^{5/6}d^{5/3}}-\frac{\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{81c^{5/6}d^{5/3}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{5/6}d^{5/3}}$$
$$-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{18\;3^{3/4}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+\frac{\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] -1/81*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(5/6)/d^
(5/3)+1/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/6)/d^(5/3)+1/81*arctan
(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(5/6)/d^(5/3)*3^(1/
2)+1/27*x^2*(d*x^3+c)^(1/2)/c/d/(-d*x^3+8*c)+1/27*(d*x^3+c)^(1/2)/c/d^(5/3)
/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/81*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3
)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(
```

```
1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)
))^2)^(1/2)*3^(3/4)/c^(2/3)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/
3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/54*(c^(1/3)+d^(1/3)*x)*Ell
ipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^
(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^
2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(2/3)/d^(5/3)/(d*x^3+
c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 647, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {482, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{x^4}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$$
$$= \frac{\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{18\ 3^{3/4}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}c^{5/6}d^{5/3}}-\frac{\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{81c^{5/6}d^{5/3}}$$
$$+\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{5/6}d^{5/3}}+\frac{\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{x^2\sqrt{c+dx^3}}{27cd\,(8c-dx^3)}$$

```
[In] Int[x^4/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] Sqrt[c + d*x^3]/(27*c*d^(5/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (x^2*S
qrt[c + d*x^3])/(27*c*d*(8*c - d*x^3)) + ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) +
 d^(1/3)*x))/Sqrt[c + d*x^3]]/(27*Sqrt[3]*c^(5/6)*d^(5/3)) - ArcTanh[(c^(1/
3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]/(81*c^(5/6)*d^(5/3)) + ArcTa
```

```
nh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(81*c^(5/6)*d^(5/3)) - (Sqrt[2 - Sqrt[3]]*(
c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) +
 d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(18*3^(3
/4)*c^(2/3)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(
1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (Sqrt[2]*(c^(1/3) + d^(1/3)*x)*Sqrt
[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*c^(2/3)*d^(5/3)*Sqrt[
(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt
[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^2\sqrt{c+dx^3}}{27cd\,(8c-dx^3)} - \frac{\int \frac{x\left(2c+\frac{dx^3}{2}\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{27cd} \\
&= \frac{x^2\sqrt{c+dx^3}}{27cd\,(8c-dx^3)} - \frac{\int \left(-\frac{x}{2\sqrt{c+dx^3}} + \frac{6cx}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{27cd} \\
&= \frac{x^2\sqrt{c+dx^3}}{27cd\,(8c-dx^3)} - \frac{2\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{9d} + \frac{\int \frac{x}{\sqrt{c+dx^3}}\,dx}{54cd} \\
&= \frac{x^2\sqrt{c+dx^3}}{27cd\,(8c-dx^3)} + \frac{\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{54cd^2} + \frac{\int \frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{54cd^{4/3}} \\
&\quad - \frac{\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{54c^{2/3}d^{4/3}} - \frac{(1-\sqrt{3})\int \frac{1}{\sqrt{c+dx^3}}\,dx}{54c^{2/3}d^{4/3}} + \frac{\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{18\sqrt[3]{c}d^{2/3}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{x^2\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)} \\
&- \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{18\ 3^{3/4}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+ \frac{\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&- \frac{\text{Subst}\left(\int\frac{1}{9-cx^2}\,dx,x,\frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{27\sqrt[3]{c}d^{5/3}}+\frac{\text{Subst}\left(\int\frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{54\sqrt[3]{c}d^{2/3}} \\
&- \frac{\left(2\sqrt[3]{d}\right)\text{Subst}\left(\int\frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{27c^{4/3}}
\end{aligned}
$$

$$= \frac{\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^2\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}c^{5/6}d^{5/3}} - \frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{81c^{5/6}d^{5/3}} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{18\ 3^{3/4}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{27\sqrt[3]{c}d^{5/3}}$$

$$= \frac{\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^2\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}c^{5/6}d^{5/3}} - \frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{81c^{5/6}d^{5/3}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{5/6}d^{5/3}} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{18\ 3^{3/4}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.26

$$\int \frac{x^4}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$$
$$= \frac{80cx^2(c+dx^3)+10cx^2(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+dx^5(-8c+dx^3)\sqrt{1+\frac{d}{c}}}{2160c^2d\,(8c-dx^3)\sqrt{c+dx^3}}$$

```
[In] Integrate[x^4/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (80*c*x^2*(c + d*x^3) + 10*c*x^2*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF
1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(-8*c + d*x^3)*Sqr
t[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(
2160*c^2*d*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.89 (sec) , antiderivative size = 886, normalized size of antiderivative = 1.37

method	result	size
elliptic	Expression too large to display	886
default	Expression too large to display	1305

```
[In] int(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/27*x^2*(d*x^3+c)^(1/2)/c/d/(-d*x^3+8*c)-1/81*I/d^2/c*3^(1/2)*(-c*d^2)^(1/
3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c
*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d
*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3)))^(1/2)))+2/243*I/d^4/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/
2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^
(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)
))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*
```

```
d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1
/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*E
llipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_
alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_a
lpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.45 (sec) , antiderivative size = 2538, normalized size of antiderivative = 3.92

$$\int \frac{x^4}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/972*(36*sqrt(d*x^3 + c)*d*x^2 + 36*(d*x^3 - 8*c)*sqrt(d)*weierstrassZeta
(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + (c*d^3*x^3 - 8*c^2*d^2 + s
qrt(-3)*(c*d^3*x^3 - 8*c^2*d^2))*(1/(c^5*d^10))^(1/6)*log((d^3*x^9 + 318*c*
d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^4*d^9*x^7 + 64*c^5*d^8*x^4 + 32
*c^6*d^7*x + sqrt(-3)*(5*c^4*d^9*x^7 + 64*c^5*d^8*x^4 + 32*c^6*d^7*x))*(1/(
c^5*d^10))^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^5*d^10*x^5 + 32*c^6*d^9*x^2 -
sqrt(-3)*(5*c^5*d^10*x^5 + 32*c^6*d^9*x^2))*(1/(c^5*d^10))^(5/6) - 2*(7*c^3
*d^7*x^6 + 152*c^4*d^6*x^3 + 64*c^5*d^5)*sqrt(1/(c^5*d^10)) + (c*d^4*x^7 +
80*c^2*d^3*x^4 + 160*c^3*d^2*x + sqrt(-3)*(c*d^4*x^7 + 80*c^2*d^3*x^4 + 160
*c^3*d^2*x))*(1/(c^5*d^10))^(1/6)) - 9*(c^2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c
^4*d^4*x^2 - sqrt(-3)*(c^2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4*x^2))*(1/(
c^5*d^10))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c*
d^3*x^3 - 8*c^2*d^2 + sqrt(-3)*(c*d^3*x^3 - 8*c^2*d^2))*(1/(c^5*d^10))^(1/6
)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^4*d^9*x^
7 + 64*c^5*d^8*x^4 + 32*c^6*d^7*x + sqrt(-3)*(5*c^4*d^9*x^7 + 64*c^5*d^8*x^
4 + 32*c^6*d^7*x))*(1/(c^5*d^10))^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^5*d^10*
x^5 + 32*c^6*d^9*x^2 - sqrt(-3)*(5*c^5*d^10*x^5 + 32*c^6*d^9*x^2))*(1/(c^5*
d^10))^(5/6) - 2*(7*c^3*d^7*x^6 + 152*c^4*d^6*x^3 + 64*c^5*d^5)*sqrt(1/(c^5
*d^10)) + (c*d^4*x^7 + 80*c^2*d^3*x^4 + 160*c^3*d^2*x + sqrt(-3)*(c*d^4*x^7
 + 80*c^2*d^3*x^4 + 160*c^3*d^2*x))*(1/(c^5*d^10))^(1/6)) - 9*(c^2*d^6*x^8
+ 38*c^3*d^5*x^5 + 64*c^4*d^4*x^2 - sqrt(-3)*(c^2*d^6*x^8 + 38*c^3*d^5*x^5
+ 64*c^4*d^4*x^2))*(1/(c^5*d^10))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*
d*x^3 - 512*c^3)) + (c*d^3*x^3 - 8*c^2*d^2 - sqrt(-3)*(c*d^3*x^3 - 8*c^2*d^
2))*(1/(c^5*d^10))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 64
0*c^3 - 9*(5*c^4*d^9*x^7 + 64*c^5*d^8*x^4 + 32*c^6*d^7*x - sqrt(-3)*(5*c^4*
d^9*x^7 + 64*c^5*d^8*x^4 + 32*c^6*d^7*x))*(1/(c^5*d^10))^(2/3) + 3*sqrt(d*x
```

```
^3 + c)*(6*(5*c^5*d^10*x^5 + 32*c^6*d^9*x^2 + sqrt(-3)*(5*c^5*d^10*x^5 + 32
*c^6*d^9*x^2))*(1/(c^5*d^10))^(5/6) - 2*(7*c^3*d^7*x^6 + 152*c^4*d^6*x^3 +
64*c^5*d^5)*sqrt(1/(c^5*d^10)) + (c*d^4*x^7 + 80*c^2*d^3*x^4 + 160*c^3*d^2*
x - sqrt(-3)*(c*d^4*x^7 + 80*c^2*d^3*x^4 + 160*c^3*d^2*x))*(1/(c^5*d^10))^(
1/6)) - 9*(c^2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4*x^2 + sqrt(-3)*(c^2*d^
6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4*x^2))*(1/(c^5*d^10))^(1/3))/(d^3*x^9 -
24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c*d^3*x^3 - 8*c^2*d^2 - sqrt(-3
)*(c*d^3*x^3 - 8*c^2*d^2))*(1/(c^5*d^10))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^
6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^4*d^9*x^7 + 64*c^5*d^8*x^4 + 32*c^6*d
^7*x - sqrt(-3)*(5*c^4*d^9*x^7 + 64*c^5*d^8*x^4 + 32*c^6*d^7*x))*(1/(c^5*d^
10))^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^5*d^10*x^5 + 32*c^6*d^9*x^2 + sqrt(-
3)*(5*c^5*d^10*x^5 + 32*c^6*d^9*x^2))*(1/(c^5*d^10))^(5/6) - 2*(7*c^3*d^7*x
^6 + 152*c^4*d^6*x^3 + 64*c^5*d^5)*sqrt(1/(c^5*d^10)) + (c*d^4*x^7 + 80*c^2
*d^3*x^4 + 160*c^3*d^2*x - sqrt(-3)*(c*d^4*x^7 + 80*c^2*d^3*x^4 + 160*c^3*d
^2*x))*(1/(c^5*d^10))^(1/6)) - 9*(c^2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4
*x^2 + sqrt(-3)*(c^2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4*x^2))*(1/(c^5*d^
10))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 2*(c*d^3*
x^3 - 8*c^2*d^2)*(1/(c^5*d^10))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c
^2*d*x^3 + 640*c^3 + 18*(5*c^4*d^9*x^7 + 64*c^5*d^8*x^4 + 32*c^6*d^7*x)*(1/
(c^5*d^10))^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^5*d^10*x^5 + 32*c^6*d^9*x^2)*
(1/(c^5*d^10))^(5/6) + (7*c^3*d^7*x^6 + 152*c^4*d^6*x^3 + 64*c^5*d^5)*sqrt(
1/(c^5*d^10)) + (c*d^4*x^7 + 80*c^2*d^3*x^4 + 160*c^3*d^2*x)*(1/(c^5*d^10))
^(1/6)) + 18*(c^2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4*x^2)*(1/(c^5*d^10))
^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*(c*d^3*x^3
- 8*c^2*d^2)*(1/(c^5*d^10))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d
*x^3 + 640*c^3 + 18*(5*c^4*d^9*x^7 + 64*c^5*d^8*x^4 + 32*c^6*d^7*x)*(1/(c^5
*d^10))^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^5*d^10*x^5 + 32*c^6*d^9*x^2)*(1/(
c^5*d^10))^(5/6) + (7*c^3*d^7*x^6 + 152*c^4*d^6*x^3 + 64*c^5*d^5)*sqrt(1/(c
^5*d^10)) + (c*d^4*x^7 + 80*c^2*d^3*x^4 + 160*c^3*d^2*x)*(1/(c^5*d^10))^(1/
6)) + 18*(c^2*d^6*x^8 + 38*c^3*d^5*x^5 + 64*c^4*d^4*x^2)*(1/(c^5*d^10))^(1/
3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(c*d^3*x^3 - 8*c^2
*d^2)
```

Sympy [F]

$$\int \frac{x^4}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{x^4}{(-8c+dx^3)^2\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(x**4/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**4/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^4}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^4}{\sqrt{dx^3+c}(dx^3-8\,c)^2}\,dx$$

```
[In] integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^4/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Giac [F]

$$\int \frac{x^4}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^4}{\sqrt{dx^3+c}(dx^3-8\,c)^2}\,dx$$

```
[In] integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^4}{\sqrt{d\,x^3+c}\,(8\,c-d\,x^3)^2}\,dx$$

```
[In] int(x^4/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
[Out] int(x^4/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.433 $\int \frac{x}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 644

$$\int \frac{x}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{\sqrt{c+dx^3}}{216c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{x^2\sqrt{c+dx^3}}{216c^2(8c-dx^3)}$$
$$-\frac{7\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{432\sqrt{3}c^{11/6}d^{2/3}}+\frac{7\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1296c^{11/6}d^{2/3}}-\frac{7\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1296c^{11/6}d^{2/3}}$$
$$-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{144\,3^{3/4}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{108\sqrt{2}\sqrt[4]{3}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 7/1296*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(11/6)/
d^(2/3)-7/1296*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)/d^(2/3)-7/1296
*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(11/6)/d^(2/
3)*3^(1/2)+1/216*x^2*(d*x^3+c)^(1/2)/c^2/(-d*x^3+8*c)+1/216*(d*x^3+c)^(1/2)
/c^2/d^(2/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/648*(c^(1/3)+d^(1/3)*x)*Elli
pticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(
```

```
1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/
3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(5/3)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(
c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/432*(c^(1/3)+
d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+
3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3
)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(5/3)/d
^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3
^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 644, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {483, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{x}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$$

$$= \frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{108\sqrt{2}\sqrt[4]{3}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{144\ 3^{3/4}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{7\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{432\sqrt{3}c^{11/6}d^{2/3}}+\frac{7\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1296c^{11/6}d^{2/3}}$$
$$-\frac{7\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1296c^{11/6}d^{2/3}}+\frac{\sqrt{c+dx^3}}{216c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{x^2\sqrt{c+dx^3}}{216c^2(8c-dx^3)}$$

```
[In] Int[x/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] Sqrt[c + d*x^3]/(216*c^2*d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (x^
2*Sqrt[c + d*x^3])/(216*c^2*(8*c - d*x^3)) - (7*ArcTan[(Sqrt[3]*c^(1/6)*(c^
(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(432*Sqrt[3]*c^(11/6)*d^(2/3)) + (7*A
rcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(1296*c^(11/6)
```

```
*d^(2/3)) - (7*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1296*c^(11/6)*d^(2/3)
) - (Sqrt[2 - Sqrt[3]]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3
)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[
((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -
7 - 4*Sqrt[3]])/(144*3^(3/4)*c^(5/3)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/
3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + ((c^(1/3)
+ d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)
*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(108*Sqrt[2]*3^(
1/4)*c^(5/3)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^
(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\text{integral} = \frac{x^2\sqrt{c+dx^3}}{216c^2(8c-dx^3)} + \frac{\int \frac{x\left(25cd-\frac{d^2x^3}{2}\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{216c^2d}$$

$$= \frac{x^2\sqrt{c+dx^3}}{216c^2(8c-dx^3)} + \frac{\int \left(\frac{dx}{2\sqrt{c+dx^3}} + \frac{21cdx}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{216c^2d}$$

$$= \frac{x^2\sqrt{c+dx^3}}{216c^2(8c-dx^3)} + \frac{\int \frac{x}{\sqrt{c+dx^3}}\,dx}{432c^2} + \frac{7\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{72c}$$

$$= \frac{x^2\sqrt{c+dx^3}}{216c^2(8c-dx^3)} - \frac{7\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{864c^2d} + \frac{\int \frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{432c^2\sqrt[3]{d}}$$

$$+ \frac{7\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{864c^{5/3}\sqrt[3]{d}} - \frac{(1-\sqrt{3})\int \frac{1}{\sqrt{c+dx^3}}\,dx}{432c^{5/3}\sqrt[3]{d}} - \frac{\left(7\sqrt[3]{d}\right)\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{288c^{4/3}}$$

$$= \frac{\sqrt{c+dx^3}}{216c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{x^2\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)}$$

$$-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{144\ 3^{3/4}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{108\sqrt{2}\sqrt[4]{3}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{7\text{Subst}\left(\int\frac{1}{9-cx^2}\,dx,x,\frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{432c^{4/3}d^{2/3}}-\frac{\left(7\sqrt[3]{d}\right)\text{Subst}\left(\int\frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{864c^{4/3}}$$

$$+\frac{\left(7d^{4/3}\right)\text{Subst}\left(\int\frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{216c^{7/3}}$$

$$= \frac{\sqrt{c+dx^3}}{216c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{x^2\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} - \frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{432\sqrt{3}c^{11/6}d^{2/3}}+\frac{7\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1296c^{11/6}d^{2/3}} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{144\ 3^{3/4}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{108\sqrt{2}\sqrt[4]{3}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{7\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{432c^{4/3}d^{2/3}}$$

$$= \frac{\sqrt{c+dx^3}}{216c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{x^2\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} - \frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{432\sqrt{3}c^{11/6}d^{2/3}}+\frac{7\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1296c^{11/6}d^{2/3}}-\frac{7\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1296c^{11/6}d^{2/3}} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{144\ 3^{3/4}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{108\sqrt{2}\sqrt[4]{3}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.25

$$\int \frac{x}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$$
$$=\frac{80cx^2(c+dx^3)+125cx^2(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+dx^5(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}}{17280c^3\,(8c-dx^3)\sqrt{c+dx^3}}$$

```
[In] Integrate[x/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (80*c*x^2*(c + d*x^3) + 125*c*x^2*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF
1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(-8*c + d*x^3)*Sqr
t[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(
17280*c^3*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.32 (sec) , antiderivative size = 883, normalized size of antiderivative = 1.37

method	result	size
default	Expression too large to display	883
elliptic	Expression too large to display	883

```
[In] int(x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/216*x^2*(d*x^3+c)^(1/2)/c^2/(-d*x^3+8*c)-1/648*I/c^2*3^(1/2)/d*(-c*d^2)^(
1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2)))-7/1944*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3
)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1
/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^
(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))
```

```
/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I
*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/
3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1
/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/
3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.57 (sec) , antiderivative size = 2540, normalized size of antiderivative = 3.94

$$\int \frac{x}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15552*(72*sqrt(d*x^3 + c)*d*x^2 + 72*(d*x^3 - 8*c)*sqrt(d)*weierstrassZe
ta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) - 7*(c^2*d^2*x^3 - 8*c^3*d
 + sqrt(-3)*(c^2*d^2*x^3 - 8*c^3*d))*(1/(c^11*d^4))^(1/6)*log((d^3*x^9 + 31
8*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^8*d^5*x^7 + 64*c^9*d^4*x^4
+ 32*c^10*d^3*x + sqrt(-3)*(5*c^8*d^5*x^7 + 64*c^9*d^4*x^4 + 32*c^10*d^3*x)
)*(1/(c^11*d^4))^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^10*d^5*x^5 + 32*c^11*d^4
*x^2 - sqrt(-3)*(5*c^10*d^5*x^5 + 32*c^11*d^4*x^2))*(1/(c^11*d^4))^(5/6) -
2*(7*c^6*d^4*x^6 + 152*c^7*d^3*x^3 + 64*c^8*d^2)*sqrt(1/(c^11*d^4)) + (c^2*
d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x + sqrt(-3)*(c^2*d^3*x^7 + 80*c^3*d^2
*x^4 + 160*c^4*d*x))*(1/(c^11*d^4))^(1/6)) - 9*(c^4*d^4*x^8 + 38*c^5*d^3*x^
5 + 64*c^6*d^2*x^2 - sqrt(-3)*(c^4*d^4*x^8 + 38*c^5*d^3*x^5 + 64*c^6*d^2*x^
2))*(1/(c^11*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3
)) + 7*(c^2*d^2*x^3 - 8*c^3*d + sqrt(-3)*(c^2*d^2*x^3 - 8*c^3*d))*(1/(c^11*
d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*
c^8*d^5*x^7 + 64*c^9*d^4*x^4 + 32*c^10*d^3*x + sqrt(-3)*(5*c^8*d^5*x^7 + 64
*c^9*d^4*x^4 + 32*c^10*d^3*x))*(1/(c^11*d^4))^(2/3) - 3*sqrt(d*x^3 + c)*(6*
(5*c^10*d^5*x^5 + 32*c^11*d^4*x^2 - sqrt(-3)*(5*c^10*d^5*x^5 + 32*c^11*d^4*
x^2))*(1/(c^11*d^4))^(5/6) - 2*(7*c^6*d^4*x^6 + 152*c^7*d^3*x^3 + 64*c^8*d^
2)*sqrt(1/(c^11*d^4)) + (c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x + sqrt(
-3)*(c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x))*(1/(c^11*d^4))^(1/6)) - 9
*(c^4*d^4*x^8 + 38*c^5*d^3*x^5 + 64*c^6*d^2*x^2 - sqrt(-3)*(c^4*d^4*x^8 + 3
8*c^5*d^3*x^5 + 64*c^6*d^2*x^2))*(1/(c^11*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*
x^6 + 192*c^2*d*x^3 - 512*c^3)) - 7*(c^2*d^2*x^3 - 8*c^3*d - sqrt(-3)*(c^2*
d^2*x^3 - 8*c^3*d))*(1/(c^11*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 120
0*c^2*d*x^3 + 640*c^3 - 9*(5*c^8*d^5*x^7 + 64*c^9*d^4*x^4 + 32*c^10*d^3*x -
 sqrt(-3)*(5*c^8*d^5*x^7 + 64*c^9*d^4*x^4 + 32*c^10*d^3*x))*(1/(c^11*d^4))^
```

```
(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^10*d^5*x^5 + 32*c^11*d^4*x^2 + sqrt(-3)*(
5*c^10*d^5*x^5 + 32*c^11*d^4*x^2))*(1/(c^11*d^4))^(5/6) - 2*(7*c^6*d^4*x^6
+ 152*c^7*d^3*x^3 + 64*c^8*d^2)*sqrt(1/(c^11*d^4)) + (c^2*d^3*x^7 + 80*c^3*
d^2*x^4 + 160*c^4*d*x - sqrt(-3)*(c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*
x))*(1/(c^11*d^4))^(1/6)) - 9*(c^4*d^4*x^8 + 38*c^5*d^3*x^5 + 64*c^6*d^2*x^
2 + sqrt(-3)*(c^4*d^4*x^8 + 38*c^5*d^3*x^5 + 64*c^6*d^2*x^2))*(1/(c^11*d^4)
)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 7*(c^2*d^2*x
^3 - 8*c^3*d - sqrt(-3)*(c^2*d^2*x^3 - 8*c^3*d))*(1/(c^11*d^4))^(1/6)*log((
d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^8*d^5*x^7 + 64*
c^9*d^4*x^4 + 32*c^10*d^3*x - sqrt(-3)*(5*c^8*d^5*x^7 + 64*c^9*d^4*x^4 + 32
*c^10*d^3*x))*(1/(c^11*d^4))^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^10*d^5*x^5 +
 32*c^11*d^4*x^2 + sqrt(-3)*(5*c^10*d^5*x^5 + 32*c^11*d^4*x^2))*(1/(c^11*d^
4))^(5/6) - 2*(7*c^6*d^4*x^6 + 152*c^7*d^3*x^3 + 64*c^8*d^2)*sqrt(1/(c^11*d
^4)) + (c^2*d^3*x^7 + 80*c^3*d^2*x^4 + 160*c^4*d*x - sqrt(-3)*(c^2*d^3*x^7
+ 80*c^3*d^2*x^4 + 160*c^4*d*x))*(1/(c^11*d^4))^(1/6)) - 9*(c^4*d^4*x^8 + 3
8*c^5*d^3*x^5 + 64*c^6*d^2*x^2 + sqrt(-3)*(c^4*d^4*x^8 + 38*c^5*d^3*x^5 + 6
4*c^6*d^2*x^2))*(1/(c^11*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x
^3 - 512*c^3)) - 14*(c^2*d^2*x^3 - 8*c^3*d)*(1/(c^11*d^4))^(1/6)*log((d^3*x
^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(5*c^8*d^5*x^7 + 64*c^9*
d^4*x^4 + 32*c^10*d^3*x)*(1/(c^11*d^4))^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^1
0*d^5*x^5 + 32*c^11*d^4*x^2)*(1/(c^11*d^4))^(5/6) + (7*c^6*d^4*x^6 + 152*c^
7*d^3*x^3 + 64*c^8*d^2)*sqrt(1/(c^11*d^4)) + (c^2*d^3*x^7 + 80*c^3*d^2*x^4
+ 160*c^4*d*x)*(1/(c^11*d^4))^(1/6)) + 18*(c^4*d^4*x^8 + 38*c^5*d^3*x^5 + 6
4*c^6*d^2*x^2)*(1/(c^11*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^
3 - 512*c^3)) + 14*(c^2*d^2*x^3 - 8*c^3*d)*(1/(c^11*d^4))^(1/6)*log((d^3*x^
9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(5*c^8*d^5*x^7 + 64*c^9*d
^4*x^4 + 32*c^10*d^3*x)*(1/(c^11*d^4))^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^10
*d^5*x^5 + 32*c^11*d^4*x^2)*(1/(c^11*d^4))^(5/6) + (7*c^6*d^4*x^6 + 152*c^7
*d^3*x^3 + 64*c^8*d^2)*sqrt(1/(c^11*d^4)) + (c^2*d^3*x^7 + 80*c^3*d^2*x^4 +
 160*c^4*d*x)*(1/(c^11*d^4))^(1/6)) + 18*(c^4*d^4*x^8 + 38*c^5*d^3*x^5 + 64
*c^6*d^2*x^2)*(1/(c^11*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3
 - 512*c^3)))/(c^2*d^2*x^3 - 8*c^3*d)
```

Sympy [F]

$$\int \frac{x}{\left(8c - dx^3\right)^2 \sqrt{c + dx^3}}\, dx = \int \frac{x}{\left(-8c + dx^3\right)^2 \sqrt{c + dx^3}}\, dx$$

```
[In] integrate(x/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x}{\sqrt{dx^3+c}(dx^3-8\,c)^2}\,dx$$

```
[In] integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Giac [F]

$$\int \frac{x}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x}{\sqrt{dx^3+c}(dx^3-8\,c)^2}\,dx$$

```
[In] integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(x/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x}{\sqrt{d\,x^3+c}\,(8\,c-d\,x^3)^2}\,dx$$

```
[In] int(x/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
[Out] int(x/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.434 $\int \frac{1}{x^2(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 665

$$\begin{aligned}
&\int \frac{1}{x^2\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx \\
&= -\frac{7\sqrt{c+dx^3}}{432c^3x}+\frac{7\sqrt[3]{d}\sqrt{c+dx^3}}{432c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)} \\
&\quad-\frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{216\sqrt{3}c^{17/6}}+\frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{648c^{17/6}}-\frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{648c^{17/6}} \\
&\quad-\frac{7\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{288\ 3^{3/4}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&\quad+\frac{7\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{216\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}$$

```
[Out] 1/648*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^
(17/6)-1/648*d^(1/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)-1/648*d^
(1/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(17/6)*
3^(1/2)-7/432*(d*x^3+c)^(1/2)/c^3/x+1/216*(d*x^3+c)^(1/2)/c^2/x/(-d*x^3+8*c
```

```
)+7/432*d^(1/3)*(d*x^3+c)^(1/2)/c^3/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+7/1296*
d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1
/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/
3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(8/3)*2^(1/2)/(d
*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^
2)^(1/2)-7/864*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-
3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2
^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1
/2)))^2)^(1/2)*3^(1/4)/c^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)
/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 665, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {483, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{1}{x^2 \left(8c-dx^3\right)^2 \sqrt{c+dx^3}}\,dx$$
$$= \frac{7\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{216\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{7\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{288\ 3^{3/4}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$-\frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{216\sqrt{3}c^{17/6}}+\frac{\sqrt[3]{d}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{648c^{17/6}}$$
$$-\frac{\sqrt[3]{d}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{648c^{17/6}}-\frac{7\sqrt{c+dx^3}}{432c^3x}+\frac{7\sqrt[3]{d}\sqrt{c+dx^3}}{432c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)}$$

```
[In] Int[1/(x^2*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] (-7*Sqrt[c + d*x^3])/(432*c^3*x) + (7*d^(1/3)*Sqrt[c + d*x^3])/(432*c^3*((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + Sqrt[c + d*x^3]/(216*c^2*x*(8*c - d*x^3
```

```
)) - (d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3
]])/(216*Sqrt[3]*c^(17/6)) + (d^(1/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^
(1/6)*Sqrt[c + d*x^3])]])/(648*c^(17/6)) - (d^(1/3)*ArcTanh[Sqrt[c + d*x^3]/
(3*Sqrt[c])])/(648*c^(17/6)) - (7*Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1
/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1
/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(288*3^(3/4)*c^(8/3)*S
qrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
Sqrt[c + d*x^3]) + (7*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)
*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[
ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)], -7 - 4*Sqrt[3]])/(216*Sqrt[2]*3^(1/4)*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3)
+ d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
```

```
/; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)}+\frac{\int \frac{28cd+\frac{5d^2x^3}{2}}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{216c^2d}\\
&= -\frac{7\sqrt{c+dx^3}}{432c^3x}+\frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)}-\frac{\int \frac{x\left(-160c^2d^2+14cd^3x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{1728c^4d}\\
&= -\frac{7\sqrt{c+dx^3}}{432c^3x}+\frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)}-\frac{\int \left(-\frac{14cd^2x}{\sqrt{c+dx^3}}-\frac{48c^2d^2x}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{1728c^4d}\\
&= -\frac{7\sqrt{c+dx^3}}{432c^3x}+\frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)}+\frac{(7d)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{864c^3}+\frac{d\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{36c^2}
\end{aligned}$$

$$= -\frac{7\sqrt{c+dx^3}}{432c^3x} + \frac{\sqrt{c+dx^3}}{216c^2x(8c-dx^3)} - \frac{\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{432c^3} + \frac{(7d^{2/3})\int \frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{864c^3} + \frac{d^{2/3}\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{432c^{8/3}} - \frac{\left(7(1-\sqrt{3})\,d^{2/3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{864c^{8/3}} - \frac{d^{4/3}\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{144c^{7/3}}$$

$$= -\frac{7\sqrt{c+dx^3}}{432c^3x} + \frac{7\sqrt[3]{d}\sqrt{c+dx^3}}{432c^3\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt{c+dx^3}}{216c^2x(8c-dx^3)} - \frac{7\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{(1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{288\ 3^{3/4}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{7\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{(1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{216\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{\sqrt[3]{d}\,\mathrm{Subst}\left(\int \frac{1}{9-cx^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{216c^{7/3}} - \frac{d^{4/3}\mathrm{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{432c^{7/3}} + \frac{d^{7/3}\mathrm{Subst}\left(\int \frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{108c^{10/3}}$$

$$= -\frac{7\sqrt{c+dx^3}}{432c^3x} + \frac{7\sqrt[3]{d}\sqrt{c+dx^3}}{432c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)}$$

$$- \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{216\sqrt{3}c^{17/6}} + \frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{648c^{17/6}}$$

$$- \frac{7\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{288\ 3^{3/4}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{7\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{216\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$- \frac{\sqrt[3]{d}\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{216c^{7/3}}$$

$$= -\frac{7\sqrt{c+dx^3}}{432c^3x} + \frac{7\sqrt[3]{d}\sqrt{c+dx^3}}{432c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)} - \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{216\sqrt{3}c^{17/6}} + \frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{648c^{17/6}} - \frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{648c^{17/6}} - \frac{7\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{288\ 3^{3/4}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{7\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{216\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.13 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.27

$$\int \frac{1}{x^2\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{-80c(54c^2+47cdx^3-7d^2x^6)+200cdx^3(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+7d^2x^6(-8}{34560c^4\sqrt{c+dx^3}\left(8cx-dx^4\right)}$$

```
[In] Integrate[1/(x^2*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-80*c*(54*c^2 + 47*c*d*x^3 - 7*d^2*x^6) + 200*c*d*x^3*(8*c - d*x^3)*Sqrt[1
 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 7*d
^2*x^6*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x
^3)/c), (d*x^3)/(8*c)])/(34560*c^4*Sqrt[c + d*x^3]*(8*c*x - d*x^4))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.49 (sec) , antiderivative size = 898, normalized size of antiderivative = 1.35

method	result	size
elliptic	Expression too large to display	898
risch	Expression too large to display	1758
default	Expression too large to display	1762

```
[In] int(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1728*x^2/c^3*d*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-1/64*(d*x^3+c)^(1/2)/c^3/x-7/
1296*I/c^3*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-1/972*I/c^3/d^2*2^(1/2
)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+
(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^
2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c
*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d
^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)
^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/1
8/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+
I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/
(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootO
f(_Z^3*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.68 (sec) , antiderivative size = 2391, normalized size of antiderivative = 3.60

$$\int \frac{1}{x^2 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/7776*(126*(d*x^4 - 8*c*x)*sqrt(d)*weierstrassZeta(0, -4*c/d, weierstrass
PInverse(0, -4*c/d, x)) - (c^3*d*x^4 - 8*c^4*x + sqrt(-3)*(c^3*d*x^4 - 8*c^
4*x))*(d^2/c^17)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 64
0*c^3*d - 9*(5*c^12*d^2*x^7 + 64*c^13*d*x^4 + 32*c^14*x + sqrt(-3)*(5*c^12*
d^2*x^7 + 64*c^13*d*x^4 + 32*c^14*x))*(d^2/c^17)^(2/3) + 3*sqrt(d*x^3 + c)*
(6*(5*c^15*d*x^5 + 32*c^16*x^2 - sqrt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2))*(d^
2/c^17)^(5/6) - 2*(7*c^9*d^2*x^6 + 152*c^10*d*x^3 + 64*c^11)*sqrt(d^2/c^17)
 + (c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x + sqrt(-3)*(c^3*d^3*x^7 + 80
*c^4*d^2*x^4 + 160*c^5*d*x))*(d^2/c^17)^(1/6)) - 9*(c^6*d^3*x^8 + 38*c^7*d^
2*x^5 + 64*c^8*d*x^2 - sqrt(-3)*(c^6*d^3*x^8 + 38*c^7*d^2*x^5 + 64*c^8*d*x^
2))*(d^2/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) +
 (c^3*d*x^4 - 8*c^4*x + sqrt(-3)*(c^3*d*x^4 - 8*c^4*x))*(d^2/c^17)^(1/6)*lo
g((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d - 9*(5*c^12*d^2*x
^7 + 64*c^13*d*x^4 + 32*c^14*x + sqrt(-3)*(5*c^12*d^2*x^7 + 64*c^13*d*x^4 +
 32*c^14*x))*(d^2/c^17)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^1
6*x^2 - sqrt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2))*(d^2/c^17)^(5/6) - 2*(7*c^9*
d^2*x^6 + 152*c^10*d*x^3 + 64*c^11)*sqrt(d^2/c^17) + (c^3*d^3*x^7 + 80*c^4*
d^2*x^4 + 160*c^5*d*x + sqrt(-3)*(c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*
x))*(d^2/c^17)^(1/6)) - 9*(c^6*d^3*x^8 + 38*c^7*d^2*x^5 + 64*c^8*d*x^2 - sq
rt(-3)*(c^6*d^3*x^8 + 38*c^7*d^2*x^5 + 64*c^8*d*x^2))*(d^2/c^17)^(1/3))/(d^
3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c^3*d*x^4 - 8*c^4*x - s
qrt(-3)*(c^3*d*x^4 - 8*c^4*x))*(d^2/c^17)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^
6 + 1200*c^2*d^2*x^3 + 640*c^3*d - 9*(5*c^12*d^2*x^7 + 64*c^13*d*x^4 + 32*c
^14*x - sqrt(-3)*(5*c^12*d^2*x^7 + 64*c^13*d*x^4 + 32*c^14*x))*(d^2/c^17)^(
2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2 + sqrt(-3)*(5*c^15*
d*x^5 + 32*c^16*x^2))*(d^2/c^17)^(5/6) - 2*(7*c^9*d^2*x^6 + 152*c^10*d*x^3
+ 64*c^11)*sqrt(d^2/c^17) + (c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x - s
qrt(-3)*(c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x))*(d^2/c^17)^(1/6)) - 9
*(c^6*d^3*x^8 + 38*c^7*d^2*x^5 + 64*c^8*d*x^2 + sqrt(-3)*(c^6*d^3*x^8 + 38*
c^7*d^2*x^5 + 64*c^8*d*x^2))*(d^2/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 19
2*c^2*d*x^3 - 512*c^3)) + (c^3*d*x^4 - 8*c^4*x - sqrt(-3)*(c^3*d*x^4 - 8*c^
4*x))*(d^2/c^17)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 64
0*c^3*d - 9*(5*c^12*d^2*x^7 + 64*c^13*d*x^4 + 32*c^14*x - sqrt(-3)*(5*c^12*
d^2*x^7 + 64*c^13*d*x^4 + 32*c^14*x))*(d^2/c^17)^(2/3) - 3*sqrt(d*x^3 + c)*
(6*(5*c^15*d*x^5 + 32*c^16*x^2 + sqrt(-3)*(5*c^15*d*x^5 + 32*c^16*x^2))*(d^
```

```
2/c^17)^(5/6) - 2*(7*c^9*d^2*x^6 + 152*c^10*d*x^3 + 64*c^11)*sqrt(d^2/c^17)
 + (c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x - sqrt(-3)*(c^3*d^3*x^7 + 80
*c^4*d^2*x^4 + 160*c^5*d*x))*(d^2/c^17)^(1/6)) - 9*(c^6*d^3*x^8 + 38*c^7*d^
2*x^5 + 64*c^8*d*x^2 + sqrt(-3)*(c^6*d^3*x^8 + 38*c^7*d^2*x^5 + 64*c^8*d*x^
2))*(d^2/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) -
 2*(c^3*d*x^4 - 8*c^4*x)*(d^2/c^17)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 12
00*c^2*d^2*x^3 + 640*c^3*d + 18*(5*c^12*d^2*x^7 + 64*c^13*d*x^4 + 32*c^14*x
)*(d^2/c^17)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^15*d*x^5 + 32*c^16*x^2)*(d^2
/c^17)^(5/6) + (7*c^9*d^2*x^6 + 152*c^10*d*x^3 + 64*c^11)*sqrt(d^2/c^17) +
(c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x)*(d^2/c^17)^(1/6)) + 18*(c^6*d^
3*x^8 + 38*c^7*d^2*x^5 + 64*c^8*d*x^2)*(d^2/c^17)^(1/3))/(d^3*x^9 - 24*c*d^
2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 2*(c^3*d*x^4 - 8*c^4*x)*(d^2/c^17)^(1/6
)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d + 18*(5*c^12*
d^2*x^7 + 64*c^13*d*x^4 + 32*c^14*x)*(d^2/c^17)^(2/3) - 6*sqrt(d*x^3 + c)*(
6*(5*c^15*d*x^5 + 32*c^16*x^2)*(d^2/c^17)^(5/6) + (7*c^9*d^2*x^6 + 152*c^10
*d*x^3 + 64*c^11)*sqrt(d^2/c^17) + (c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*
d*x)*(d^2/c^17)^(1/6)) + 18*(c^6*d^3*x^8 + 38*c^7*d^2*x^5 + 64*c^8*d*x^2)*(
d^2/c^17)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 18*(
7*d*x^3 - 54*c)*sqrt(d*x^3 + c))/(c^3*d*x^4 - 8*c^4*x)
```

Sympy [F]

$$\int \frac{1}{x^2\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx=\int \frac{1}{x^2\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

[In] integrate(1/x**2/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)

[Out] Integral(1/(x**2*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)

Maxima [F]

$$\int \frac{1}{x^2\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx=\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2x^2}\,dx$$

[In] integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^2), x)

Giac [F]

$$\int \frac{1}{x^2 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)^2 x^2} \, dx$$

```
[In] integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{1}{x^2 \sqrt{d\,x^3 + c}\,(8\,c - d\,x^3)^2} \, dx$$

```
[In] int(1/(x^2*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
[Out] int(1/(x^2*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.435 $\int \frac{1}{x^5(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 687

$$\int \frac{1}{x^5(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = -\frac{31\sqrt{c+dx^3}}{6912c^3x^4}+\frac{5d\sqrt{c+dx^3}}{864c^4x}-\frac{5d^{4/3}\sqrt{c+dx^3}}{864c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{\sqrt{c+dx^3}}{216c^2x^4(8c-dx^3)}-\frac{25d^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27648\sqrt{3}c^{23/6}}+\frac{25d^{4/3}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{82944c^{23/6}}-\frac{25d^{4/3}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{82944c^{23/6}}+\frac{5\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{576\,3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{5d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{432\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 25/82944*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))
/c^(23/6)-25/82944*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(23/6)-25
/82944*d^(4/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/
c^(23/6)*3^(1/2)-31/6912*(d*x^3+c)^(1/2)/c^3/x^4+5/864*d*(d*x^3+c)^(1/2)/c^
4/x+1/216*(d*x^3+c)^(1/2)/c^2/x^4/(-d*x^3+8*c)-5/864*d^(4/3)*(d*x^3+c)^(1/2
)/c^4/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-5/2592*d^(4/3)*(c^(1/3)+d^(1/3)*x)*El
lipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3
^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+
3^(1/2)))^2)^(1/2)*3^(3/4)/c^(11/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/
3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+5/1728*d^(4/3)*(c^(1
/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)
*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^
(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(11
/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1
/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 687, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules

used = {483, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{1}{x^5 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = -\frac{5d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{432\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{5\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{576\ 3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{25d^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27648\sqrt{3}c^{23/6}} + \frac{25d^{4/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{82944c^{23/6}} - \frac{25d^{4/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{82944c^{23/6}} - \frac{5d^{4/3}\sqrt{c+dx^3}}{864c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{5d\sqrt{c+dx^3}}{864c^4x} - \frac{31\sqrt{c+dx^3}}{6912c^3x^4} + \frac{\sqrt{c+dx^3}}{216c^2x^4(8c-dx^3)}$$

```
[In] Int[1/(x^5*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-31*Sqrt[c + d*x^3])/(6912*c^3*x^4) + (5*d*Sqrt[c + d*x^3])/(864*c^4*x) -
(5*d^(4/3)*Sqrt[c + d*x^3])/(864*c^4*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) +
 Sqrt[c + d*x^3]/(216*c^2*x^4*(8*c - d*x^3)) - (25*d^(4/3)*ArcTan[(Sqrt[3]*
c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(27648*Sqrt[3]*c^(23/6)) +
 (25*d^(4/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/
(82944*c^(23/6)) - (25*d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(82944
*c^(23/6)) + (5*Sqrt[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/
3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2
]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(576*3^(3/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(
1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
- (5*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2
/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt
[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt
[3]])/(432*Sqrt[2]*3^(1/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
```

```
1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
```

```
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c+dx^3}}{216c^2x^4(8c-dx^3)} + \frac{\int \frac{31cd+\frac{11d^2x^3}{2}}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx}{216c^2d} \\
&= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4} + \frac{\sqrt{c+dx^3}}{216c^2x^4(8c-dx^3)} - \frac{\int \frac{320c^2d^2-\frac{155}{2}cd^3x^3}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{6912c^4d} \\
&= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4} + \frac{5d\sqrt{c+dx^3}}{864c^4x} + \frac{\sqrt{c+dx^3}}{216c^2x^4(8c-dx^3)} + \frac{\int \frac{x(-980c^3d^3+160c^2d^4x^3)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{55296c^6d} \\
&= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4} + \frac{5d\sqrt{c+dx^3}}{864c^4x} + \frac{\sqrt{c+dx^3}}{216c^2x^4(8c-dx^3)} + \frac{\int \left(-\frac{160c^2d^3x}{\sqrt{c+dx^3}} + \frac{300c^3d^3x}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{55296c^6d} \\
&= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4} + \frac{5d\sqrt{c+dx^3}}{864c^4x} + \frac{\sqrt{c+dx^3}}{216c^2x^4(8c-dx^3)} \\
&\quad - \frac{(5d^2)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{1728c^4} + \frac{(25d^2)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{4608c^3} \\
&= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4} + \frac{5d\sqrt{c+dx^3}}{864c^4x} + \frac{\sqrt{c+dx^3}}{216c^2x^4(8c-dx^3)} \\
&\quad - \frac{(25d)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{55296c^4} \\
&\quad - \frac{(5d^{5/3})\int \frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{1728c^4} + \frac{(25d^{5/3})\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{55296c^{11/3}} \\
&\quad + \frac{(5(1-\sqrt{3})d^{5/3})\int \frac{1}{\sqrt{c+dx^3}}\,dx}{1728c^{11/3}} - \frac{(25d^{7/3})\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{18432c^{10/3}}
\end{aligned}$$

$$=-\frac{31\sqrt{c+dx^3}}{6912c^3x^4}+\frac{5d\sqrt{c+dx^3}}{864c^4x}-\frac{5d^{4/3}\sqrt{c+dx^3}}{864c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{\sqrt{c+dx^3}}{216c^2x^4\left(8c-dx^3\right)}$$

$$+\frac{5\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{576\ 3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{5d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{432\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+\frac{\left(25d^{4/3}\right)\text{Subst}\left(\int\frac{1}{9-cx^2}\,dx,x,\frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{27648c^{10/3}}$$

$$-\frac{\left(25d^{7/3}\right)\text{Subst}\left(\int\frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{55296c^{10/3}}$$

$$+\frac{\left(25d^{10/3}\right)\text{Subst}\left(\int\frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{13824c^{13/3}}$$

$$= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4} + \frac{5d\sqrt{c+dx^3}}{864c^4x} - \frac{5d^{4/3}\sqrt{c+dx^3}}{864c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt{c+dx^3}}{216c^2x^4\left(8c-dx^3\right)}$$

$$-\frac{25d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27648\sqrt{3}c^{23/6}} + \frac{25d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{82944c^{23/6}}$$

$$+\frac{5\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{576\ 3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{5d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{432\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{\left(25d^{4/3}\right)\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{27648c^{10/3}}$$

$$= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4} + \frac{5d\sqrt{c+dx^3}}{864c^4x} - \frac{5d^{4/3}\sqrt{c+dx^3}}{864c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$
$$+ \frac{\sqrt{c+dx^3}}{216c^2x^4\left(8c-dx^3\right)} - \frac{25d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27648\sqrt{3}c^{23/6}}$$
$$+ \frac{25d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{82944c^{23/6}} - \frac{25d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{82944c^{23/6}}$$
$$+ \frac{5\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{576\ 3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$- \frac{5d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{432\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.29

$$\int \frac{1}{x^5\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$$
$$= \frac{245cd^2x^6(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-16\Big(2c(216c^3-135c^2dx^3-311cd^2x^6+4}{221184c^5x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

```
[In] Integrate[1/(x^5*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (245*c*d^2*x^6*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3
, -((d*x^3)/c), (d*x^3)/(8*c)] - 16*(2*c*(216*c^3 - 135*c^2*d*x^3 - 311*c*d
^2*x^6 + 40*d^3*x^9) + d^3*x^9*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[
5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(221184*c^5*x^4*(8*c - d*x
^3)*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.60 (sec) , antiderivative size = 919, normalized size of antiderivative = 1.34

method	result	size
elliptic	Expression too large to display	919
risch	Expression too large to display	1770
default	Expression too large to display	2241

```
[In] int(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/13824*d^2*x^2/c^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-1/256*(d*x^3+c)^(1/2)/c^3/
x^4+3/512*d*(d*x^3+c)^(1/2)/c^4/x+5/2592*I*d/c^4*3^(1/2)*(-c*d^2)^(1/3)*(I*
(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c
*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d
^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(
I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2)))-25/124416*I/d/c^4*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*
I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1
/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))
^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^
2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2
)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ell
ipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_al
pha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alp
ha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.07 (sec) , antiderivative size = 2549, normalized size of antiderivative = 3.71

$$\int \frac{1}{x^5 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \text{Too large to display}$$

```
[In] integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/995328*(5760*(d^2*x^7 - 8*c*d*x^4)*sqrt(d)*weierstrassZeta(0, -4*c/d, wei
erstrassPInverse(0, -4*c/d, x)) + 25*(c^4*d*x^7 - 8*c^5*x^4 + sqrt(-3)*(c^4
*d*x^7 - 8*c^5*x^4))*(d^8/c^23)^(1/6)*log(9765625*(d^9*x^9 + 318*c*d^8*x^6
+ 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^16*d^3*x^7 + 64*c^17*d^2*x^4 + 32
*c^18*d*x + sqrt(-3)*(5*c^16*d^3*x^7 + 64*c^17*d^2*x^4 + 32*c^18*d*x))*(d^8
/c^23)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2 - sqrt(-3)*
(5*c^20*d*x^5 + 32*c^21*x^2))*(d^8/c^23)^(5/6) - 2*(7*c^12*d^4*x^6 + 152*c^
13*d^3*x^3 + 64*c^14*d^2)*sqrt(d^8/c^23) + (c^4*d^7*x^7 + 80*c^5*d^6*x^4 +
160*c^6*d^5*x + sqrt(-3)*(c^4*d^7*x^7 + 80*c^5*d^6*x^4 + 160*c^6*d^5*x))*(d
^8/c^23)^(1/6)) - 9*(c^8*d^6*x^8 + 38*c^9*d^5*x^5 + 64*c^10*d^4*x^2 - sqrt(
-3)*(c^8*d^6*x^8 + 38*c^9*d^5*x^5 + 64*c^10*d^4*x^2))*(d^8/c^23)^(1/3))/(d^
3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 25*(c^4*d*x^7 - 8*c^5*x^
4 + sqrt(-3)*(c^4*d*x^7 - 8*c^5*x^4))*(d^8/c^23)^(1/6)*log(9765625*(d^9*x^9
 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^16*d^3*x^7 + 64*
c^17*d^2*x^4 + 32*c^18*d*x + sqrt(-3)*(5*c^16*d^3*x^7 + 64*c^17*d^2*x^4 + 3
2*c^18*d*x))*(d^8/c^23)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^2
1*x^2 - sqrt(-3)*(5*c^20*d*x^5 + 32*c^21*x^2))*(d^8/c^23)^(5/6) - 2*(7*c^12
*d^4*x^6 + 152*c^13*d^3*x^3 + 64*c^14*d^2)*sqrt(d^8/c^23) + (c^4*d^7*x^7 +
80*c^5*d^6*x^4 + 160*c^6*d^5*x + sqrt(-3)*(c^4*d^7*x^7 + 80*c^5*d^6*x^4 + 1
60*c^6*d^5*x))*(d^8/c^23)^(1/6)) - 9*(c^8*d^6*x^8 + 38*c^9*d^5*x^5 + 64*c^1
0*d^4*x^2 - sqrt(-3)*(c^8*d^6*x^8 + 38*c^9*d^5*x^5 + 64*c^10*d^4*x^2))*(d^8
/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 25*(c^4
*d*x^7 - 8*c^5*x^4 - sqrt(-3)*(c^4*d*x^7 - 8*c^5*x^4))*(d^8/c^23)^(1/6)*log
(9765625*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c
^16*d^3*x^7 + 64*c^17*d^2*x^4 + 32*c^18*d*x - sqrt(-3)*(5*c^16*d^3*x^7 + 64
*c^17*d^2*x^4 + 32*c^18*d*x))*(d^8/c^23)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^
20*d*x^5 + 32*c^21*x^2 + sqrt(-3)*(5*c^20*d*x^5 + 32*c^21*x^2))*(d^8/c^23)^
(5/6) - 2*(7*c^12*d^4*x^6 + 152*c^13*d^3*x^3 + 64*c^14*d^2)*sqrt(d^8/c^23)
+ (c^4*d^7*x^7 + 80*c^5*d^6*x^4 + 160*c^6*d^5*x - sqrt(-3)*(c^4*d^7*x^7 + 8
0*c^5*d^6*x^4 + 160*c^6*d^5*x))*(d^8/c^23)^(1/6)) - 9*(c^8*d^6*x^8 + 38*c^9
*d^5*x^5 + 64*c^10*d^4*x^2 + sqrt(-3)*(c^8*d^6*x^8 + 38*c^9*d^5*x^5 + 64*c^
10*d^4*x^2))*(d^8/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 51
2*c^3)) - 25*(c^4*d*x^7 - 8*c^5*x^4 - sqrt(-3)*(c^4*d*x^7 - 8*c^5*x^4))*(d^
8/c^23)^(1/6)*log(9765625*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640
*c^3*d^6 - 9*(5*c^16*d^3*x^7 + 64*c^17*d^2*x^4 + 32*c^18*d*x - sqrt(-3)*(5*
```

```
c^16*d^3*x^7 + 64*c^17*d^2*x^4 + 32*c^18*d*x))*(d^8/c^23)^(2/3) - 3*sqrt(d*
x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2 + sqrt(-3)*(5*c^20*d*x^5 + 32*c^21*
x^2))*(d^8/c^23)^(5/6) - 2*(7*c^12*d^4*x^6 + 152*c^13*d^3*x^3 + 64*c^14*d^2
)*sqrt(d^8/c^23) + (c^4*d^7*x^7 + 80*c^5*d^6*x^4 + 160*c^6*d^5*x - sqrt(-3)
*(c^4*d^7*x^7 + 80*c^5*d^6*x^4 + 160*c^6*d^5*x))*(d^8/c^23)^(1/6)) - 9*(c^8
*d^6*x^8 + 38*c^9*d^5*x^5 + 64*c^10*d^4*x^2 + sqrt(-3)*(c^8*d^6*x^8 + 38*c^
9*d^5*x^5 + 64*c^10*d^4*x^2))*(d^8/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 1
92*c^2*d*x^3 - 512*c^3)) + 50*(c^4*d*x^7 - 8*c^5*x^4)*(d^8/c^23)^(1/6)*log(
9765625*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 + 18*(5*c
^16*d^3*x^7 + 64*c^17*d^2*x^4 + 32*c^18*d*x)*(d^8/c^23)^(2/3) + 6*sqrt(d*x^
3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2)*(d^8/c^23)^(5/6) + (7*c^12*d^4*x^6 +
 152*c^13*d^3*x^3 + 64*c^14*d^2)*sqrt(d^8/c^23) + (c^4*d^7*x^7 + 80*c^5*d^6
*x^4 + 160*c^6*d^5*x)*(d^8/c^23)^(1/6)) + 18*(c^8*d^6*x^8 + 38*c^9*d^5*x^5
+ 64*c^10*d^4*x^2)*(d^8/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^
3 - 512*c^3)) - 50*(c^4*d*x^7 - 8*c^5*x^4)*(d^8/c^23)^(1/6)*log(9765625*(d^
9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 + 18*(5*c^16*d^3*x^7
 + 64*c^17*d^2*x^4 + 32*c^18*d*x)*(d^8/c^23)^(2/3) - 6*sqrt(d*x^3 + c)*(6*(
5*c^20*d*x^5 + 32*c^21*x^2)*(d^8/c^23)^(5/6) + (7*c^12*d^4*x^6 + 152*c^13*d
^3*x^3 + 64*c^14*d^2)*sqrt(d^8/c^23) + (c^4*d^7*x^7 + 80*c^5*d^6*x^4 + 160*
c^6*d^5*x)*(d^8/c^23)^(1/6)) + 18*(c^8*d^6*x^8 + 38*c^9*d^5*x^5 + 64*c^10*d
^4*x^2)*(d^8/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3
)) + 144*(40*d^2*x^6 - 351*c*d*x^3 + 216*c^2)*sqrt(d*x^3 + c))/(c^4*d*x^7 -
 8*c^5*x^4)
```

Sympy [F]

$$\int \frac{1}{x^5 (8c - dx^3)^2 \sqrt{c + dx^3}}\, dx = \int \frac{1}{x^5 (-8c + dx^3)^2 \sqrt{c + dx^3}}\, dx$$

```
[In] integrate(1/x**5/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(x**5*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^5 (8c - dx^3)^2 \sqrt{c + dx^3}}\, dx = \int \frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)^2 x^5}\, dx$$

```
[In] integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)^2 x^5} \, dx$$

```
[In] integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{1}{x^5 \sqrt{d\,x^3 + c}\,(8\,c - d\,x^3)^2} \, dx$$

```
[In] int(1/(x^5*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
[Out] int(1/(x^5*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.436 $\int \frac{1}{x^8(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 711

$$\begin{aligned}
&\int \frac{1}{x^8(8c-dx^3)^2\sqrt{c+dx^3}}\,dx \\
&= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7}+\frac{391d\sqrt{c+dx^3}}{193536c^4x^4}-\frac{289d^2\sqrt{c+dx^3}}{48384c^5x}+\frac{289d^{7/3}\sqrt{c+dx^3}}{48384c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} \\
&\quad+\frac{\sqrt{c+dx^3}}{216c^2x^7(8c-dx^3)}-\frac{17d^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{110592\sqrt{3}c^{29/6}} \\
&\quad+\frac{17d^{7/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{331776c^{29/6}}-\frac{17d^{7/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{331776c^{29/6}} \\
&\quad-\frac{289\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32256\ 3^{3/4}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&\quad+\frac{289d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{24192\sqrt{2}\sqrt[4]{3}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}$$

```
[Out] 17/331776*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2)
)/c^(29/6)-17/331776*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(29/6)-
17/331776*d^(7/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2
))/c^(29/6)*3^(1/2)-17/6048*(d*x^3+c)^(1/2)/c^3/x^7+391/193536*d*(d*x^3+c)^
(1/2)/c^4/x^4-289/48384*d^2*(d*x^3+c)^(1/2)/c^5/x+1/216*(d*x^3+c)^(1/2)/c^2
/x^7/(-d*x^3+8*c)+289/48384*d^(7/3)*(d*x^3+c)^(1/2)/c^5/(d^(1/3)*x+c^(1/3)*
(1+3^(1/2)))+289/145152*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^
(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)
-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^
(3/4)/c^(14/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3
)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-289/96768*d^(7/3)*(c^(1/3)+d^(1/3)*x)*Ell
ipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^
(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^
2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(14/3)/(d*x^3+c)^(1/2
)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 711, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules

used = {483, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{1}{x^8 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx$$

$$= \frac{289 d^{7/3} \left(\sqrt[3]{c}+\sqrt[3]{d}x\right) \sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right), -7-4\sqrt{3}\right)}{24192\sqrt{2}\sqrt[4]{3}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$- \frac{289\sqrt{2-\sqrt{3}}d^{7/3} \left(\sqrt[3]{c}+\sqrt[3]{d}x\right) \sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right) |-7-4\sqrt{3}\right)}{32256\ 3^{3/4}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$- \frac{17 d^{7/3} \arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{110592\sqrt{3}c^{29/6}} + \frac{17 d^{7/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{331776 c^{29/6}}$$

$$- \frac{17 d^{7/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{331776 c^{29/6}} + \frac{289 d^{7/3}\sqrt{c+dx^3}}{48384 c^5 \left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$- \frac{289 d^2\sqrt{c+dx^3}}{48384 c^5 x} + \frac{391 d\sqrt{c+dx^3}}{193536 c^4 x^4} - \frac{17\sqrt{c+dx^3}}{6048 c^3 x^7} + \frac{\sqrt{c+dx^3}}{216 c^2 x^7 (8c-dx^3)}$$

```
[In] Int[1/(x^8*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-17*Sqrt[c + d*x^3])/(6048*c^3*x^7) + (391*d*Sqrt[c + d*x^3])/(193536*c^4*
x^4) - (289*d^2*Sqrt[c + d*x^3])/(48384*c^5*x) + (289*d^(7/3)*Sqrt[c + d*x^
3])/(48384*c^5*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + Sqrt[c + d*x^3]/(216*
c^2*x^7*(8*c - d*x^3)) - (17*d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(
1/3)*x))/Sqrt[c + d*x^3]])/(110592*Sqrt[3]*c^(29/6)) + (17*d^(7/3)*ArcTanh[
(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(331776*c^(29/6)) - (
17*d^(7/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(331776*c^(29/6)) - (289*S
qrt[2 - Sqrt[3]]*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1
/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSi
n[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(32256*3^(3/4)*c^(14/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*
x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (289*d^(7/3)*
(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3)
+ d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(24192*
```

```
Sqrt[2]*3^(1/4)*c^(14/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{c+dx^3}}{216c^2x^7(8c-dx^3)} + \frac{\int \frac{34cd+\frac{17d^2x^3}{2}}{x^8(8c-dx^3)\sqrt{c+dx^3}}\,dx}{216c^2d} \\
&= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{\sqrt{c+dx^3}}{216c^2x^7(8c-dx^3)} - \frac{\int \frac{782c^2d^2-187cd^3x^3}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx}{12096c^4d} \\
&= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} + \frac{\sqrt{c+dx^3}}{216c^2x^7(8c-dx^3)} + \frac{\int \frac{18496c^3d^3-1955c^2d^4x^3}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{387072c^6d} \\
&= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} - \frac{289d^2\sqrt{c+dx^3}}{48384c^5x} \\
&\quad + \frac{\sqrt{c+dx^3}}{216c^2x^7(8c-dx^3)} - \frac{\int \frac{x(-76840c^4d^4+9248c^3d^5x^3)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{3096576c^8d} \\
&= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} - \frac{289d^2\sqrt{c+dx^3}}{48384c^5x} \\
&\quad + \frac{\sqrt{c+dx^3}}{216c^2x^7(8c-dx^3)} - \frac{\int \left(-\frac{9248c^3d^4x}{\sqrt{c+dx^3}} - \frac{2856c^4d^4x}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{3096576c^8d} \\
&= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} - \frac{289d^2\sqrt{c+dx^3}}{48384c^5x} + \frac{\sqrt{c+dx^3}}{216c^2x^7(8c-dx^3)} \\
&\quad + \frac{(289d^3)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{96768c^5} + \frac{(17d^3)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{18432c^4}
\end{aligned}$$

$$= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} - \frac{289d^2\sqrt{c+dx^3}}{48384c^5x} + \frac{\sqrt{c+dx^3}}{216c^2x^7(8c-dx^3)} - \frac{(17d^2)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{221184c^5}$$

$$+ \frac{(289d^{8/3})\int \frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{96768c^5} + \frac{(17d^{8/3})\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{221184c^{14/3}} - \frac{\left(289(1-\sqrt{3})d^{8/3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{96768c^{14/3}} - \frac{(17d^{10/3})\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{73728c^{13/3}}$$

$$= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} - \frac{289d^2\sqrt{c+dx^3}}{48384c^5x} + \frac{289d^{7/3}\sqrt{c+dx^3}}{48384c^5\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt{c+dx^3}}{216c^2x^7(8c-dx^3)}$$

$$- \frac{289\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{(1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32256\ 3^{3/4}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{289d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{(1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{24192\sqrt{2}\sqrt[4]{3}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{(17d^{7/3})\,\text{Subst}\left(\int \frac{1}{9-cx^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{110592c^{13/3}} - \frac{(17d^{10/3})\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{221184c^{13/3}} + \frac{(17d^{13/3})\,\text{Subst}\left(\int \frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{55296c^{16/3}}$$

$$= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} - \frac{289d^2\sqrt{c+dx^3}}{48384c^5x} + \frac{289d^{7/3}\sqrt{c+dx^3}}{48384c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt{c+dx^3}}{216c^2x^7\left(8c-dx^3\right)} - \frac{17d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{110592\sqrt{3}c^{29/6}} + \frac{17d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{331776c^{29/6}} - \frac{289\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32256\ 3^{3/4}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{289d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{24192\sqrt{2}\sqrt[4]{3}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{\left(17d^{7/3}\right)\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{110592c^{13/3}}$$

$$= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} - \frac{289d^2\sqrt{c+dx^3}}{48384c^5x} + \frac{289d^{7/3}\sqrt{c+dx^3}}{48384c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$+ \frac{\sqrt{c+dx^3}}{216c^2x^7(8c-dx^3)} - \frac{17d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{110592\sqrt{3}c^{29/6}}$$

$$+ \frac{17d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{331776c^{29/6}} - \frac{17d^{7/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{331776c^{29/6}}$$

$$- \frac{289\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{32256\ 3^{3/4}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{289d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{24192\sqrt{2}\sqrt[4]{3}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.30

$$\int \frac{1}{x^8(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \sqrt{c+dx^3}\left(-\frac{1}{448c^3x^7} + \frac{15d}{7168c^4x^4} - \frac{171d^2}{28672c^5x} - \frac{d^3x^2}{110592c^5(-8c+dx^3)}\right) + \frac{9605d^3x^2\sqrt{\frac{c+dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{6193152c^5\sqrt{c+dx^3}} - \frac{289d^4x^5\sqrt{\frac{c+dx^3}{c}}\operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{2},1,\frac{8}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{3870720c^6\sqrt{c+dx^3}}$$

```
[In] Integrate[1/(x^8*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
[Out] Sqrt[c + d*x^3]*(-1/448*1/(c^3*x^7) + (15*d)/(7168*c^4*x^4) - (171*d^2)/(28
672*c^5*x) - (d^3*x^2)/(110592*c^5*(-8*c + d*x^3))) + (9605*d^3*x^2*Sqrt[(c
```

```
 + d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(6193
152*c^5*Sqrt[c + d*x^3]) - (289*d^4*x^5*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1
/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(3870720*c^6*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.80 (sec) , antiderivative size = 938, normalized size of antiderivative = 1.32

method	result	size
elliptic	Expression too large to display	938
risch	Expression too large to display	1781
default	Expression too large to display	2739

```
[In] int(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/110592*d^3*x^2/c^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-1/448*(d*x^3+c)^(1/2)/c^3
/x^7+15/7168*d*(d*x^3+c)^(1/2)/c^4/x^4-171/28672*d^2*(d*x^3+c)^(1/2)/c^5/x-
289/145152*I*d^2/c^5*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2
)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*
(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)
/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2
)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(
-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-17/497664*I/
c^5*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/
(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2
)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^
2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/
d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/
2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d
^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_a
lpha=RootOf(_Z^3*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.29 (sec) , antiderivative size = 2582, normalized size of antiderivative = 3.63

$$\int \frac{1}{x^8 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/27869184*(166464*(d^3*x^10 - 8*c*d^2*x^7)*sqrt(d)*weierstrassZeta(0, -4*
c/d, weierstrassPInverse(0, -4*c/d, x)) - 119*(c^5*d*x^10 - 8*c^6*x^7 + sqr
t(-3)*(c^5*d*x^10 - 8*c^6*x^7))*(d^14/c^29)^(1/6)*log(1419857*(d^14*x^9 + 3
18*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 - 9*(5*c^20*d^4*x^7 + 64*c
^21*d^3*x^4 + 32*c^22*d^2*x + sqrt(-3)*(5*c^20*d^4*x^7 + 64*c^21*d^3*x^4 +
32*c^22*d^2*x))*(d^14/c^29)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^25*d*x^5 + 32
*c^26*x^2 - sqrt(-3)*(5*c^25*d*x^5 + 32*c^26*x^2))*(d^14/c^29)^(5/6) - 2*(7
*c^15*d^6*x^6 + 152*c^16*d^5*x^3 + 64*c^17*d^4)*sqrt(d^14/c^29) + (c^5*d^11
*x^7 + 80*c^6*d^10*x^4 + 160*c^7*d^9*x + sqrt(-3)*(c^5*d^11*x^7 + 80*c^6*d^
10*x^4 + 160*c^7*d^9*x))*(d^14/c^29)^(1/6)) - 9*(c^10*d^9*x^8 + 38*c^11*d^8
*x^5 + 64*c^12*d^7*x^2 - sqrt(-3)*(c^10*d^9*x^8 + 38*c^11*d^8*x^5 + 64*c^12
*d^7*x^2))*(d^14/c^29)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512
*c^3)) + 119*(c^5*d*x^10 - 8*c^6*x^7 + sqrt(-3)*(c^5*d*x^10 - 8*c^6*x^7))*(
d^14/c^29)^(1/6)*log(1419857*(d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3
 + 640*c^3*d^11 - 9*(5*c^20*d^4*x^7 + 64*c^21*d^3*x^4 + 32*c^22*d^2*x + sqr
t(-3)*(5*c^20*d^4*x^7 + 64*c^21*d^3*x^4 + 32*c^22*d^2*x))*(d^14/c^29)^(2/3)
 - 3*sqrt(d*x^3 + c)*(6*(5*c^25*d*x^5 + 32*c^26*x^2 - sqrt(-3)*(5*c^25*d*x^
5 + 32*c^26*x^2))*(d^14/c^29)^(5/6) - 2*(7*c^15*d^6*x^6 + 152*c^16*d^5*x^3
+ 64*c^17*d^4)*sqrt(d^14/c^29) + (c^5*d^11*x^7 + 80*c^6*d^10*x^4 + 160*c^7*
d^9*x + sqrt(-3)*(c^5*d^11*x^7 + 80*c^6*d^10*x^4 + 160*c^7*d^9*x))*(d^14/c^
29)^(1/6)) - 9*(c^10*d^9*x^8 + 38*c^11*d^8*x^5 + 64*c^12*d^7*x^2 - sqrt(-3)
*(c^10*d^9*x^8 + 38*c^11*d^8*x^5 + 64*c^12*d^7*x^2))*(d^14/c^29)^(1/3))/(d^
3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 119*(c^5*d*x^10 - 8*c^6*
x^7 - sqrt(-3)*(c^5*d*x^10 - 8*c^6*x^7))*(d^14/c^29)^(1/6)*log(1419857*(d^1
4*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 - 9*(5*c^20*d^4*x
^7 + 64*c^21*d^3*x^4 + 32*c^22*d^2*x - sqrt(-3)*(5*c^20*d^4*x^7 + 64*c^21*d
^3*x^4 + 32*c^22*d^2*x))*(d^14/c^29)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^25*d
*x^5 + 32*c^26*x^2 + sqrt(-3)*(5*c^25*d*x^5 + 32*c^26*x^2))*(d^14/c^29)^(5/
6) - 2*(7*c^15*d^6*x^6 + 152*c^16*d^5*x^3 + 64*c^17*d^4)*sqrt(d^14/c^29) +
(c^5*d^11*x^7 + 80*c^6*d^10*x^4 + 160*c^7*d^9*x - sqrt(-3)*(c^5*d^11*x^7 +
80*c^6*d^10*x^4 + 160*c^7*d^9*x))*(d^14/c^29)^(1/6)) - 9*(c^10*d^9*x^8 + 38
*c^11*d^8*x^5 + 64*c^12*d^7*x^2 + sqrt(-3)*(c^10*d^9*x^8 + 38*c^11*d^8*x^5
+ 64*c^12*d^7*x^2))*(d^14/c^29)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*
x^3 - 512*c^3)) + 119*(c^5*d*x^10 - 8*c^6*x^7 - sqrt(-3)*(c^5*d*x^10 - 8*c^
6*x^7))*(d^14/c^29)^(1/6)*log(1419857*(d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2
```

```
*d^12*x^3 + 640*c^3*d^11 - 9*(5*c^20*d^4*x^7 + 64*c^21*d^3*x^4 + 32*c^22*d^
2*x - sqrt(-3)*(5*c^20*d^4*x^7 + 64*c^21*d^3*x^4 + 32*c^22*d^2*x))*(d^14/c^
29)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^25*d*x^5 + 32*c^26*x^2 + sqrt(-3)*(5*
c^25*d*x^5 + 32*c^26*x^2))*(d^14/c^29)^(5/6) - 2*(7*c^15*d^6*x^6 + 152*c^16
*d^5*x^3 + 64*c^17*d^4)*sqrt(d^14/c^29) + (c^5*d^11*x^7 + 80*c^6*d^10*x^4 +
 160*c^7*d^9*x - sqrt(-3)*(c^5*d^11*x^7 + 80*c^6*d^10*x^4 + 160*c^7*d^9*x))
*(d^14/c^29)^(1/6)) - 9*(c^10*d^9*x^8 + 38*c^11*d^8*x^5 + 64*c^12*d^7*x^2 +
 sqrt(-3)*(c^10*d^9*x^8 + 38*c^11*d^8*x^5 + 64*c^12*d^7*x^2))*(d^14/c^29)^(
1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 238*(c^5*d*x^10
 - 8*c^6*x^7)*(d^14/c^29)^(1/6)*log(1419857*(d^14*x^9 + 318*c*d^13*x^6 + 12
00*c^2*d^12*x^3 + 640*c^3*d^11 + 18*(5*c^20*d^4*x^7 + 64*c^21*d^3*x^4 + 32*
c^22*d^2*x)*(d^14/c^29)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^25*d*x^5 + 32*c^2
6*x^2)*(d^14/c^29)^(5/6) + (7*c^15*d^6*x^6 + 152*c^16*d^5*x^3 + 64*c^17*d^4
)*sqrt(d^14/c^29) + (c^5*d^11*x^7 + 80*c^6*d^10*x^4 + 160*c^7*d^9*x)*(d^14/
c^29)^(1/6)) + 18*(c^10*d^9*x^8 + 38*c^11*d^8*x^5 + 64*c^12*d^7*x^2)*(d^14/
c^29)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 238*(c^5
*d*x^10 - 8*c^6*x^7)*(d^14/c^29)^(1/6)*log(1419857*(d^14*x^9 + 318*c*d^13*x
^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 + 18*(5*c^20*d^4*x^7 + 64*c^21*d^3*x^
4 + 32*c^22*d^2*x)*(d^14/c^29)^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^25*d*x^5 +
 32*c^26*x^2)*(d^14/c^29)^(5/6) + (7*c^15*d^6*x^6 + 152*c^16*d^5*x^3 + 64*c
^17*d^4)*sqrt(d^14/c^29) + (c^5*d^11*x^7 + 80*c^6*d^10*x^4 + 160*c^7*d^9*x)
*(d^14/c^29)^(1/6)) + 18*(c^10*d^9*x^8 + 38*c^11*d^8*x^5 + 64*c^12*d^7*x^2)
*(d^14/c^29)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 1
44*(1156*d^3*x^9 - 9639*c*d^2*x^6 + 3672*c^2*d*x^3 - 3456*c^3)*sqrt(d*x^3 +
 c))/(c^5*d*x^10 - 8*c^6*x^7)
```

Sympy [F]

$$\int \frac{1}{x^8 \left(8c-dx^3\right)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{x^8 \left(-8c+dx^3\right)^2 \sqrt{c+dx^3}}\,dx$$

```
[In] integrate(1/x**8/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(x**8*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^8 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)^2 x^8} \, dx$$

```
[In] integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^8), x)
```

Giac [F]

$$\int \frac{1}{x^8 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)^2 x^8} \, dx$$

```
[In] integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^8 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{1}{x^8 \sqrt{d\,x^3 + c}\,(8\,c - d\,x^3)^2} \, dx$$

```
[In] int(1/(x^8*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
[Out] int(1/(x^8*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.437 $\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{x^7\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{7}{3},2,\frac{1}{2},\frac{10}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{448c^2\sqrt{c+dx^3}}$$

```
[Out] 1/448*x^7*AppellF1(7/3,1/2,2,10/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c
^2/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {525, 524}

$$\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{x^7\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{7}{3},2,\frac{1}{2},\frac{10}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{448c^2\sqrt{c+dx^3}}$$

```
[In] Int[x^6/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^7*Sqrt[1 + (d*x^3)/c]*AppellF1[7/3, 2, 1/2, 10/3, (d*x^3)/(8*c), -((d*x^
3)/c)])/(448*c^2*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^6}{(8c-dx^3)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$

$$= \frac{x^7\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{7}{3};2,\frac{1}{2};\frac{10}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{448c^2\sqrt{c+dx^3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 239 vs. $2(66) = 132$.

Time = 10.33 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.62

$$\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$$

$$= \frac{x\left(-\frac{23dx^3\sqrt{1+\frac{dx^3}{c}}\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c}+\frac{256\left(c+dx^3-\frac{32c^2\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{32c\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\left(\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4\text{AppellF1}\left(\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}\right)}{8c-dx^3}\right)}{864d^2\sqrt{c+dx^3}}$$

```
[In] Integrate[x^6/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*((-23*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c),
 (d*x^3)/(8*c)])/c + (256*(c + d*x^3 - (32*c^2*AppellF1[1/3, 1/2, 1, 4/3, -
((d*x^3)/c), (d*x^3)/(8*c)])/(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c),
 (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)
/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]))))/(8*
c - d*x^3)))/(864*d^2*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.50 (sec) , antiderivative size = 723, normalized size of antiderivative = 10.95

method	result	size
elliptic	Expression too large to display	723
default	Expression too large to display	1432

```
[In] int(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 8/27*x*(d*x^3+c)^(1/2)/d^2/(-d*x^3+8*c)-46/81*I/d^3*3^(1/2)*(-c*d^2)^(1/3)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3
^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d
/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+64/243*I/d^5*2^(1/2)*sum(1/_alpha^2*
(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))
)/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1
/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c
*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alph
a*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d
-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^
2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3
*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2
)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2425 vs. 2(52) = 104.

Time = 0.72 (sec) , antiderivative size = 2425, normalized size of antiderivative = 36.74

$$\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/243*(36*sqrt(d*x^3 + c)*d*x - 63*(d*x^3 - 8*c)*sqrt(d)*weierstrassPInver
se(0, -4*c/d, x) + 2*(d^4*x^3 - 8*c*d^3 + sqrt(-3)*(d^4*x^3 - 8*c*d^3))*(1/
(c*d^14))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9
*(c*d^12*x^8 + 38*c^2*d^11*x^5 + 64*c^3*d^10*x^2 + sqrt(-3)*(c*d^12*x^8 + 3
```

8*c^2*d^11*x^5 + 64*c^3*d^10*x^2))*(1/(c*d^14))^(2/3) + 3*sqrt(d*x^3 + c)*((c*d^14*x^7 + 80*c^2*d^13*x^4 + 160*c^3*d^12*x - sqrt(-3)*(c*d^14*x^7 + 80*c^2*d^13*x^4 + 160*c^3*d^12*x))*(1/(c*d^14))^(5/6) - 2*(7*c*d^9*x^6 + 152*c^2*d^8*x^3 + 64*c^3*d^7)*sqrt(1/(c*d^14)) + 6*(5*c*d^4*x^5 + 32*c^2*d^3*x^2 + sqrt(-3)*(5*c*d^4*x^5 + 32*c^2*d^3*x^2))*(1/(c*d^14))^(1/6)) - 9*(5*c*d^7*x^7 + 64*c^2*d^6*x^4 + 32*c^3*d^5*x - sqrt(-3)*(5*c*d^7*x^7 + 64*c^2*d^6*x^4 + 32*c^3*d^5*x))*(1/(c*d^14))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*(d^4*x^3 - 8*c*d^3 + sqrt(-3)*(d^4*x^3 - 8*c*d^3))*(1/(c*d^14))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c*d^12*x^8 + 38*c^2*d^11*x^5 + 64*c^3*d^10*x^2 + sqrt(-3)*(c*d^12*x^8 + 38*c^2*d^11*x^5 + 64*c^3*d^10*x^2))*(1/(c*d^14))^(2/3) - 3*sqrt(d*x^3 + c)*((c*d^14*x^7 + 80*c^2*d^13*x^4 + 160*c^3*d^12*x - sqrt(-3)*(c*d^14*x^7 + 80*c^2*d^13*x^4 + 160*c^3*d^12*x))*(1/(c*d^14))^(5/6) - 2*(7*c*d^9*x^6 + 152*c^2*d^8*x^3 + 64*c^3*d^7)*sqrt(1/(c*d^14)) + 6*(5*c*d^4*x^5 + 32*c^2*d^3*x^2 + sqrt(-3)*(5*c*d^4*x^5 + 32*c^2*d^3*x^2))*(1/(c*d^14))^(1/6)) - 9*(5*c*d^7*x^7 + 64*c^2*d^6*x^4 + 32*c^3*d^5*x - sqrt(-3)*(5*c*d^7*x^7 + 64*c^2*d^6*x^4 + 32*c^3*d^5*x))*(1/(c*d^14))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 2*(d^4*x^3 - 8*c*d^3 - sqrt(-3)*(d^4*x^3 - 8*c*d^3))*(1/(c*d^14))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c*d^12*x^8 + 38*c^2*d^11*x^5 + 64*c^3*d^10*x^2 - sqrt(-3)*(c*d^12*x^8 + 38*c^2*d^11*x^5 + 64*c^3*d^10*x^2))*(1/(c*d^14))^(2/3) + 3*sqrt(d*x^3 + c)*((c*d^14*x^7 + 80*c^2*d^13*x^4 + 160*c^3*d^12*x + sqrt(-3)*(c*d^14*x^7 + 80*c^2*d^13*x^4 + 160*c^3*d^12*x))*(1/(c*d^14))^(5/6) - 2*(7*c*d^9*x^6 + 152*c^2*d^8*x^3 + 64*c^3*d^7)*sqrt(1/(c*d^14)) + 6*(5*c*d^4*x^5 + 32*c^2*d^3*x^2 - sqrt(-3)*(5*c*d^4*x^5 + 32*c^2*d^3*x^2))*(1/(c*d^14))^(1/6)) - 9*(5*c*d^7*x^7 + 64*c^2*d^6*x^4 + 32*c^3*d^5*x + sqrt(-3)*(5*c*d^7*x^7 + 64*c^2*d^6*x^4 + 32*c^3*d^5*x))*(1/(c*d^14))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*(d^4*x^3 - 8*c*d^3 - sqrt(-3)*(d^4*x^3 - 8*c*d^3))*(1/(c*d^14))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c*d^12*x^8 + 38*c^2*d^11*x^5 + 64*c^3*d^10*x^2 - sqrt(-3)*(c*d^12*x^8 + 38*c^2*d^11*x^5 + 64*c^3*d^10*x^2))*(1/(c*d^14))^(2/3) - 3*sqrt(d*x^3 + c)*((c*d^14*x^7 + 80*c^2*d^13*x^4 + 160*c^3*d^12*x + sqrt(-3)*(c*d^14*x^7 + 80*c^2*d^13*x^4 + 160*c^3*d^12*x))*(1/(c*d^14))^(5/6) - 2*(7*c*d^9*x^6 + 152*c^2*d^8*x^3 + 64*c^3*d^7)*sqrt(1/(c*d^14)) + 6*(5*c*d^4*x^5 + 32*c^2*d^3*x^2 - sqrt(-3)*(5*c*d^4*x^5 + 32*c^2*d^3*x^2))*(1/(c*d^14))^(1/6)) - 9*(5*c*d^7*x^7 + 64*c^2*d^6*x^4 + 32*c^3*d^5*x + sqrt(-3)*(5*c*d^7*x^7 + 64*c^2*d^6*x^4 + 32*c^3*d^5*x))*(1/(c*d^14))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 4*(d^4*x^3 - 8*c*d^3)*(1/(c*d^14))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(c*d^12*x^8 + 38*c^2*d^11*x^5 + 64*c^3*d^10*x^2)*(1/(c*d^14))^(2/3) + 6*sqrt(d*x^3 + c)*((c*d^14*x^7 + 80*c^2*d^13*x^4 + 160*c^3*d^12*x)*(1/(c*d^14))^(5/6) + (7*c*d^9*x^6 + 152*c^2*d^8*x^3 + 64*c^3*d^7)*sqrt(1/(c*d^14)) + 6*(5*c*d^4*x^5 + 32*c^2*d^3*x^2)*(1/(c*d^14))^(1/6)) + 18*(5*c*d^7*x^7 + 64*c^2*d^6*x^4 + 32*c^3*d^5*x)*(1/(c*d^14))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 4*(d^4*x^3 - 8*c*d^3)*(1/(c*d^14))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 120

```
0*c^2*d*x^3 + 640*c^3 + 18*(c*d^12*x^8 + 38*c^2*d^11*x^5 + 64*c^3*d^10*x^2)
*(1/(c*d^14))^(2/3) - 6*sqrt(d*x^3 + c)*((c*d^14*x^7 + 80*c^2*d^13*x^4 + 16
0*c^3*d^12*x)*(1/(c*d^14))^(5/6) + (7*c*d^9*x^6 + 152*c^2*d^8*x^3 + 64*c^3*
d^7)*sqrt(1/(c*d^14)) + 6*(5*c*d^4*x^5 + 32*c^2*d^3*x^2)*(1/(c*d^14))^(1/6)
) + 18*(5*c*d^7*x^7 + 64*c^2*d^6*x^4 + 32*c^3*d^5*x)*(1/(c*d^14))^(1/3))/(d
^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^4*x^3 - 8*c*d^3)
```

Sympy [F]

$$\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^6}{(-8c+dx^3)^2\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(x**6/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(x**6/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^6}{\sqrt{dx^3+c}(dx^3-8\,c)^2}\,dx$$

```
[In] integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^6/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Giac [F]

$$\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^6}{\sqrt{dx^3+c}(dx^3-8\,c)^2}\,dx$$

```
[In] integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^6/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{x^6}{\sqrt{d\,x^3+c}\,(8\,c-d\,x^3)^2}\,dx$$

```
[In] int(x^6/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
[Out] int(x^6/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.438 $\int \frac{x^3}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{x^3}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{x^4\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},2,\frac{1}{2},\frac{7}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{256c^2\sqrt{c+dx^3}}$$

```
[Out] 1/256*x^4*AppellF1(4/3,1/2,2,7/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^
2/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {525, 524}

$$\int \frac{x^3}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \frac{x^4\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{4}{3},2,\frac{1}{2},\frac{7}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{256c^2\sqrt{c+dx^3}}$$

```
[In] Int[x^3/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 2, 1/2, 7/3, (d*x^3)/(8*c), -((d*x^3
)/c)])/(256*c^2*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{(8c-dx^3)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$

$$= \frac{x^4\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{4}{3};2,\frac{1}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{256c^2\sqrt{c+dx^3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 237 vs. $2(66) = 132$.

Time = 10.22 (sec) , antiderivative size = 237, normalized size of antiderivative = 3.59

$$\int \frac{x^3}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$$

$$= \frac{x\left(x^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-\frac{64c\left(c+dx^3-\frac{32c^2\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{d}{}\right.}{32c\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\left(\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\right.\right.}\right.}{d(-8c+dx^3)}\right.}{1728c^2\sqrt{c+dx^3}}$$

```
[In] Integrate[x^3/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*(x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d*x^3
)/(8*c)] - (64*c*(c + d*x^3 - (32*c^2*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/
c), (d*x^3)/(8*c)])/(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/
(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] -
 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]))))/(d*(-8*c + d
*x^3))))/(1728*c^2*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.58 (sec) , antiderivative size = 732, normalized size of antiderivative = 11.09

method	result	size
elliptic	Expression too large to display	732
default	Expression too large to display	1151

```
[In] int(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/27*x*(d*x^3+c)^(1/2)/c/d/(-d*x^3+8*c)+1/81*I/d^2/c*3^(1/2)*(-c*d^2)^(1/3)
*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*
3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/243*I/d^4/c*2^(1/2)*sum(1/_alpha^
2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3
)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^
(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(
-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_al
pha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha
*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*
d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d
-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d
^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2548 vs. $2(52) = 104$.

Time = 0.90 (sec) , antiderivative size = 2548, normalized size of antiderivative = 38.61

$$\int \frac{x^3}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3888*(144*sqrt(d*x^3 + c)*d*x + 72*(d*x^3 - 8*c)*sqrt(d)*weierstrassPInv
erse(0, -4*c/d, x) - (c*d^3*x^3 - 8*c^2*d^2 + sqrt(-3)*(c*d^3*x^3 - 8*c^2*d
^2))*(1/(c^7*d^8))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 64
```

```
0*c^3 - 9*(c^5*d^8*x^8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x^2 + sqrt(-3)*(c^5*d^
8*x^8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x^2))*(1/(c^7*d^8))^(2/3) + 3*sqrt(d*x^
3 + c)*((c^6*d^9*x^7 + 80*c^7*d^8*x^4 + 160*c^8*d^7*x - sqrt(-3)*(c^6*d^9*x
^7 + 80*c^7*d^8*x^4 + 160*c^8*d^7*x))*(1/(c^7*d^8))^(5/6) - 2*(7*c^4*d^6*x^
6 + 152*c^5*d^5*x^3 + 64*c^6*d^4)*sqrt(1/(c^7*d^8)) + 6*(5*c^2*d^3*x^5 + 32
*c^3*d^2*x^2 + sqrt(-3)*(5*c^2*d^3*x^5 + 32*c^3*d^2*x^2))*(1/(c^7*d^8))^(1/
6)) - 9*(5*c^3*d^5*x^7 + 64*c^4*d^4*x^4 + 32*c^5*d^3*x - sqrt(-3)*(5*c^3*d^
5*x^7 + 64*c^4*d^4*x^4 + 32*c^5*d^3*x))*(1/(c^7*d^8))^(1/3))/(d^3*x^9 - 24*
c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (c*d^3*x^3 - 8*c^2*d^2 + sqrt(-3)*(
c*d^3*x^3 - 8*c^2*d^2))*(1/(c^7*d^8))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 +
1200*c^2*d*x^3 + 640*c^3 - 9*(c^5*d^8*x^8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x^2
 + sqrt(-3)*(c^5*d^8*x^8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x^2))*(1/(c^7*d^8))^
(2/3) - 3*sqrt(d*x^3 + c)*((c^6*d^9*x^7 + 80*c^7*d^8*x^4 + 160*c^8*d^7*x -
sqrt(-3)*(c^6*d^9*x^7 + 80*c^7*d^8*x^4 + 160*c^8*d^7*x))*(1/(c^7*d^8))^(5/6
) - 2*(7*c^4*d^6*x^6 + 152*c^5*d^5*x^3 + 64*c^6*d^4)*sqrt(1/(c^7*d^8)) + 6*
(5*c^2*d^3*x^5 + 32*c^3*d^2*x^2 + sqrt(-3)*(5*c^2*d^3*x^5 + 32*c^3*d^2*x^2)
)*(1/(c^7*d^8))^(1/6)) - 9*(5*c^3*d^5*x^7 + 64*c^4*d^4*x^4 + 32*c^5*d^3*x -
 sqrt(-3)*(5*c^3*d^5*x^7 + 64*c^4*d^4*x^4 + 32*c^5*d^3*x))*(1/(c^7*d^8))^(1
/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c*d^3*x^3 - 8*c
^2*d^2 - sqrt(-3)*(c*d^3*x^3 - 8*c^2*d^2))*(1/(c^7*d^8))^(1/6)*log((d^3*x^9
 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c^5*d^8*x^8 + 38*c^6*d^7*x
^5 + 64*c^7*d^6*x^2 - sqrt(-3)*(c^5*d^8*x^8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x
^2))*(1/(c^7*d^8))^(2/3) + 3*sqrt(d*x^3 + c)*((c^6*d^9*x^7 + 80*c^7*d^8*x^4
 + 160*c^8*d^7*x + sqrt(-3)*(c^6*d^9*x^7 + 80*c^7*d^8*x^4 + 160*c^8*d^7*x))
*(1/(c^7*d^8))^(5/6) - 2*(7*c^4*d^6*x^6 + 152*c^5*d^5*x^3 + 64*c^6*d^4)*sqr
t(1/(c^7*d^8)) + 6*(5*c^2*d^3*x^5 + 32*c^3*d^2*x^2 - sqrt(-3)*(5*c^2*d^3*x^
5 + 32*c^3*d^2*x^2))*(1/(c^7*d^8))^(1/6)) - 9*(5*c^3*d^5*x^7 + 64*c^4*d^4*x
^4 + 32*c^5*d^3*x + sqrt(-3)*(5*c^3*d^5*x^7 + 64*c^4*d^4*x^4 + 32*c^5*d^3*x
))*(1/(c^7*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3))
 + (c*d^3*x^3 - 8*c^2*d^2 - sqrt(-3)*(c*d^3*x^3 - 8*c^2*d^2))*(1/(c^7*d^8))
^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c^5*d^8
*x^8 + 38*c^6*d^7*x^5 + 64*c^7*d^6*x^2 - sqrt(-3)*(c^5*d^8*x^8 + 38*c^6*d^7
*x^5 + 64*c^7*d^6*x^2))*(1/(c^7*d^8))^(2/3) - 3*sqrt(d*x^3 + c)*((c^6*d^9*x
^7 + 80*c^7*d^8*x^4 + 160*c^8*d^7*x + sqrt(-3)*(c^6*d^9*x^7 + 80*c^7*d^8*x^
4 + 160*c^8*d^7*x))*(1/(c^7*d^8))^(5/6) - 2*(7*c^4*d^6*x^6 + 152*c^5*d^5*x^
3 + 64*c^6*d^4)*sqrt(1/(c^7*d^8)) + 6*(5*c^2*d^3*x^5 + 32*c^3*d^2*x^2 - sqr
t(-3)*(5*c^2*d^3*x^5 + 32*c^3*d^2*x^2))*(1/(c^7*d^8))^(1/6)) - 9*(5*c^3*d^5
*x^7 + 64*c^4*d^4*x^4 + 32*c^5*d^3*x + sqrt(-3)*(5*c^3*d^5*x^7 + 64*c^4*d^4
*x^4 + 32*c^5*d^3*x))*(1/(c^7*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^
2*d*x^3 - 512*c^3)) - 2*(c*d^3*x^3 - 8*c^2*d^2)*(1/(c^7*d^8))^(1/6)*log((d^
3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(c^5*d^8*x^8 + 38*c^6
*d^7*x^5 + 64*c^7*d^6*x^2)*(1/(c^7*d^8))^(2/3) + 6*sqrt(d*x^3 + c)*((c^6*d^
9*x^7 + 80*c^7*d^8*x^4 + 160*c^8*d^7*x)*(1/(c^7*d^8))^(5/6) + (7*c^4*d^6*x^
6 + 152*c^5*d^5*x^3 + 64*c^6*d^4)*sqrt(1/(c^7*d^8)) + 6*(5*c^2*d^3*x^5 + 32
*c^3*d^2*x^2)*(1/(c^7*d^8))^(1/6)) + 18*(5*c^3*d^5*x^7 + 64*c^4*d^4*x^4 + 3
```

```
2*c^5*d^3*x)*(1/(c^7*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 -
 512*c^3)) + 2*(c*d^3*x^3 - 8*c^2*d^2)*(1/(c^7*d^8))^(1/6)*log((d^3*x^9 + 3
18*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(c^5*d^8*x^8 + 38*c^6*d^7*x^5
+ 64*c^7*d^6*x^2)*(1/(c^7*d^8))^(2/3) - 6*sqrt(d*x^3 + c)*((c^6*d^9*x^7 + 8
0*c^7*d^8*x^4 + 160*c^8*d^7*x)*(1/(c^7*d^8))^(5/6) + (7*c^4*d^6*x^6 + 152*c
^5*d^5*x^3 + 64*c^6*d^4)*sqrt(1/(c^7*d^8)) + 6*(5*c^2*d^3*x^5 + 32*c^3*d^2*
x^2)*(1/(c^7*d^8))^(1/6)) + 18*(5*c^3*d^5*x^7 + 64*c^4*d^4*x^4 + 32*c^5*d^3
*x)*(1/(c^7*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)
))/(c*d^3*x^3 - 8*c^2*d^2)
```

Sympy [F]

$$\int \frac{x^3}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{(-8c+dx^3)^2\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(x**3/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(x**3/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^3}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{\sqrt{dx^3+c}(dx^3-8\,c)^2}\,dx$$

```
[In] integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Giac [F]

$$\int \frac{x^3}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{\sqrt{dx^3+c}(dx^3-8\,c)^2}\,dx$$

```
[In] integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^3/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\int \frac{x^3}{\sqrt{d\,x^3+c}\,(8\,c-d\,x^3)^2}\,dx$$

```
[In] int(x^3/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
[Out] int(x^3/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.439 $\int \frac{1}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\frac{x\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{1}{3},2,\frac{1}{2},\frac{4}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{64c^2\sqrt{c+dx^3}}$$

```
[Out] 1/64*x*AppellF1(1/3,1/2,2,4/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^2/(
d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {441, 440}

$$\int \frac{1}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx=\frac{x\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},2,\frac{1}{2},\frac{4}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{64c^2\sqrt{c+dx^3}}$$

```
[In] Int[1/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 2, 1/2, 4/3, (d*x^3)/(8*c), -((d*x^3)/
c)])/(64*c^2*Sqrt[c + d*x^3])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{(8c-dx^3)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$

$$= \frac{x\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{1}{3};2,\frac{1}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{64c^2\sqrt{c+dx^3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 237 vs. $2(64) = 128$.

Time = 10.20 (sec) , antiderivative size = 237, normalized size of antiderivative = 3.70

$$\int \frac{1}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$$

$$= \frac{x\left(\frac{dx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^3}+\frac{64\left(\frac{c+dx^3}{c^2}+\frac{832\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{32c\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\left(\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4\,\text{App}\right.}\right.}{8c-dx^3}\right.}{13824\sqrt{c+dx^3}}$$

```
[In] Integrate[1/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*((d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d*
x^3)/(8*c)])/c^3 + (64*((c + d*x^3)/c^2 + (832*AppellF1[1/3, 1/2, 1, 4/3, -
((d*x^3)/c), (d*x^3)/(8*c)])/(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c),
 (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)
/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]))))/(8*
c - d*x^3)))/(13824*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.48 (sec) , antiderivative size = 729, normalized size of antiderivative = 11.39

method	result	size
default	Expression too large to display	729
elliptic	Expression too large to display	729

```
[In] int(1/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/216*x*(d*x^3+c)^(1/2)/c^2/(-d*x^3+8*c)+1/648*I/c^2*3^(1/2)/d*(-c*d^2)^(1/
3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c
*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2
)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3
)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-5/972*I/c^2/d^3*2^(1/2)*sum(1/_al
pha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^
(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+
I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)
*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_a
lpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*
(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)
*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-
8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2498 vs. $2(50) = 100$.

Time = 0.87 (sec) , antiderivative size = 2498, normalized size of antiderivative = 39.03

$$\int \frac{1}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15552*(72*sqrt(d*x^3 + c)*d*x - 288*(d*x^3 - 8*c)*sqrt(d)*weierstrassPIn
verse(0, -4*c/d, x) - 5*(c^2*d^2*x^3 - 8*c^3*d + sqrt(-3)*(c^2*d^2*x^3 - 8*
c^3*d))*(1/(c^13*d^2))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3
+ 640*c^3 - 9*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2 + sqrt(-3)*(
```

c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2))*(1/(c^13*d^2))^(2/3) + 3*
sqrt(d*x^3 + c)*((c^11*d^4*x^7 + 80*c^12*d^3*x^4 + 160*c^13*d^2*x - sqrt(-3
)*(c^11*d^4*x^7 + 80*c^12*d^3*x^4 + 160*c^13*d^2*x))*(1/(c^13*d^2))^(5/6) -
2*(7*c^7*d^3*x^6 + 152*c^8*d^2*x^3 + 64*c^9*d)*sqrt(1/(c^13*d^2)) + 6*(5*c
^3*d^2*x^5 + 32*c^4*d*x^2 + sqrt(-3)*(5*c^3*d^2*x^5 + 32*c^4*d*x^2))*(1/(c^
13*d^2))^(1/6)) - 9*(5*c^5*d^3*x^7 + 64*c^6*d^2*x^4 + 32*c^7*d*x - sqrt(-3)
*(5*c^5*d^3*x^7 + 64*c^6*d^2*x^4 + 32*c^7*d*x))*(1/(c^13*d^2))^(1/3))/(d^3*
x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 5*(c^2*d^2*x^3 - 8*c^3*d +
sqrt(-3)*(c^2*d^2*x^3 - 8*c^3*d))*(1/(c^13*d^2))^(1/6)*log((d^3*x^9 + 318*
c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 6
4*c^11*d^2*x^2 + sqrt(-3)*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2)
)*(1/(c^13*d^2))^(2/3) - 3*sqrt(d*x^3 + c)*((c^11*d^4*x^7 + 80*c^12*d^3*x^4
+ 160*c^13*d^2*x - sqrt(-3)*(c^11*d^4*x^7 + 80*c^12*d^3*x^4 + 160*c^13*d^2
x))(1/(c^13*d^2))^(5/6) - 2*(7*c^7*d^3*x^6 + 152*c^8*d^2*x^3 + 64*c^9*d)*
sqrt(1/(c^13*d^2)) + 6*(5*c^3*d^2*x^5 + 32*c^4*d*x^2 + sqrt(-3)*(5*c^3*d^2*
x^5 + 32*c^4*d*x^2))*(1/(c^13*d^2))^(1/6)) - 9*(5*c^5*d^3*x^7 + 64*c^6*d^2*
x^4 + 32*c^7*d*x - sqrt(-3)*(5*c^5*d^3*x^7 + 64*c^6*d^2*x^4 + 32*c^7*d*x))*
(1/(c^13*d^2))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) -
5*(c^2*d^2*x^3 - 8*c^3*d - sqrt(-3)*(c^2*d^2*x^3 - 8*c^3*d))*(1/(c^13*d^2)
)^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c^9*d^
4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2 - sqrt(-3)*(c^9*d^4*x^8 + 38*c^10
*d^3*x^5 + 64*c^11*d^2*x^2))*(1/(c^13*d^2))^(2/3) + 3*sqrt(d*x^3 + c)*((c^1
1*d^4*x^7 + 80*c^12*d^3*x^4 + 160*c^13*d^2*x + sqrt(-3)*(c^11*d^4*x^7 + 80*
c^12*d^3*x^4 + 160*c^13*d^2*x))*(1/(c^13*d^2))^(5/6) - 2*(7*c^7*d^3*x^6 + 1
52*c^8*d^2*x^3 + 64*c^9*d)*sqrt(1/(c^13*d^2)) + 6*(5*c^3*d^2*x^5 + 32*c^4*d
x^2 - sqrt(-3)(5*c^3*d^2*x^5 + 32*c^4*d*x^2))*(1/(c^13*d^2))^(1/6)) - 9*(
5*c^5*d^3*x^7 + 64*c^6*d^2*x^4 + 32*c^7*d*x + sqrt(-3)*(5*c^5*d^3*x^7 + 64*
c^6*d^2*x^4 + 32*c^7*d*x))*(1/(c^13*d^2))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 +
192*c^2*d*x^3 - 512*c^3)) + 5*(c^2*d^2*x^3 - 8*c^3*d - sqrt(-3)*(c^2*d^2*x^
3 - 8*c^3*d))*(1/(c^13*d^2))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*
d*x^3 + 640*c^3 - 9*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2 - sqrt
(-3)*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2))*(1/(c^13*d^2))^(2/3
) - 3*sqrt(d*x^3 + c)*((c^11*d^4*x^7 + 80*c^12*d^3*x^4 + 160*c^13*d^2*x + s
qrt(-3)*(c^11*d^4*x^7 + 80*c^12*d^3*x^4 + 160*c^13*d^2*x))*(1/(c^13*d^2))^(
5/6) - 2*(7*c^7*d^3*x^6 + 152*c^8*d^2*x^3 + 64*c^9*d)*sqrt(1/(c^13*d^2)) +
6*(5*c^3*d^2*x^5 + 32*c^4*d*x^2 - sqrt(-3)*(5*c^3*d^2*x^5 + 32*c^4*d*x^2))*
(1/(c^13*d^2))^(1/6)) - 9*(5*c^5*d^3*x^7 + 64*c^6*d^2*x^4 + 32*c^7*d*x + sq
rt(-3)*(5*c^5*d^3*x^7 + 64*c^6*d^2*x^4 + 32*c^7*d*x))*(1/(c^13*d^2))^(1/3))
/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 10*(c^2*d^2*x^3 - 8*
c^3*d)*(1/(c^13*d^2))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 +
640*c^3 + 18*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2)*(1/(c^13*d^
2))^(2/3) + 6*sqrt(d*x^3 + c)*((c^11*d^4*x^7 + 80*c^12*d^3*x^4 + 160*c^13*d
^2*x)*(1/(c^13*d^2))^(5/6) + (7*c^7*d^3*x^6 + 152*c^8*d^2*x^3 + 64*c^9*d)*s
qrt(1/(c^13*d^2)) + 6*(5*c^3*d^2*x^5 + 32*c^4*d*x^2)*(1/(c^13*d^2))^(1/6))
+ 18*(5*c^5*d^3*x^7 + 64*c^6*d^2*x^4 + 32*c^7*d*x)*(1/(c^13*d^2))^(1/3))/(d

```
^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 10*(c^2*d^2*x^3 - 8*c^3
*d)*(1/(c^13*d^2))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 64
0*c^3 + 18*(c^9*d^4*x^8 + 38*c^10*d^3*x^5 + 64*c^11*d^2*x^2)*(1/(c^13*d^2))
^(2/3) - 6*sqrt(d*x^3 + c)*((c^11*d^4*x^7 + 80*c^12*d^3*x^4 + 160*c^13*d^2*
x)*(1/(c^13*d^2))^(5/6) + (7*c^7*d^3*x^6 + 152*c^8*d^2*x^3 + 64*c^9*d)*sqrt
(1/(c^13*d^2)) + 6*(5*c^3*d^2*x^5 + 32*c^4*d*x^2)*(1/(c^13*d^2))^(1/6)) + 1
8*(5*c^5*d^3*x^7 + 64*c^6*d^2*x^4 + 32*c^7*d*x)*(1/(c^13*d^2))^(1/3))/(d^3*
x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(c^2*d^2*x^3 - 8*c^3*d)
```

Sympy [F]

$$\int \frac{1}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{1}{(-8c+dx^3)^2\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(1/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(1/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)^2}\,dx$$

```
[In] integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Giac [F]

$$\int \frac{1}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)^2}\,dx$$

```
[In] integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{d\,x^3+c}\,(8\,c-d\,x^3)^2}\,dx$$

```
[In] int(1/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
[Out] int(1/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.440 $\int \frac{1}{x^3(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{1}{x^3 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = -\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{2}{3},2,\frac{1}{2},\frac{1}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{128c^2x^2\sqrt{c+dx^3}}$$

```
[Out] -1/128*AppellF1(-2/3,1/2,2,1/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^2/
x^2/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {525, 524}

$$\int \frac{1}{x^3 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = -\frac{\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{2}{3},2,\frac{1}{2},\frac{1}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{128c^2x^2\sqrt{c+dx^3}}$$

```
[In] Int[1/(x^3*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/128*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 2, 1/2, 1/3, (d*x^3)/(8*c), -((d
*x^3)/c)])/(c^2*x^2*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^3(8c-dx^3)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$

$$= -\frac{\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{2}{3};2,\frac{1}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{128c^2x^2\sqrt{c+dx^3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 266 vs. 2(66) = 132.

Time = 10.21 (sec) , antiderivative size = 266, normalized size of antiderivative = 4.03

$$\int \frac{1}{x^3\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$$
$$= \frac{-\frac{64(c+dx^3)(-216c+29dx^3)}{c^3x^2(-8c+dx^3)}+\frac{29d^2x^4\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^4}-\frac{4096dx\,\text{Appe}\cdots}{c(8c-dx^3)\left(32c\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+3dx^3\left(\cdots\right.\right.}}{221184\sqrt{c+dx^3}}$$

```
[In] Integrate[1/(x^3*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((-64*(c + d*x^3)*(-216*c + 29*d*x^3))/(c^3*x^2*(-8*c + d*x^3)) + (29*d^2*x
^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*
c)])/c^4 - (4096*d*x*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)
])/(c*(8*c - d*x^3)*(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/
(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] -
 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]))))/(221184*Sqrt
[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 5.56 (sec) , antiderivative size = 744, normalized size of antiderivative = 11.27

method	result	size
elliptic	Expression too large to display	744
default	Expression too large to display	1456
risch	Expression too large to display	1457

```
[In] int(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1728*x/c^3*d*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-1/128*(d*x^3+c)^(1/2)/c^3/x^2+2
9/10368*I/c^3*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3)
)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-
c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))
-19/15552*I/c^3/d^2*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d
*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/
d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2
*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1
/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(
2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*
3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-
c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,
(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(8c - dx^3\right)^2 \sqrt{c + dx^3}} \, dx = \text{Timed out}$$

```
[In] integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^3 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{1}{x^3 (-8c + dx^3)^2 \sqrt{c + dx^3}} \, dx$$

```
[In] integrate(1/x**3/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(x**3*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)^2 x^3} \, dx$$

```
[In] integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{1}{\sqrt{dx^3 + c}(dx^3 - 8\,c)^2 x^3} \, dx$$

```
[In] integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (8c - dx^3)^2 \sqrt{c + dx^3}} \, dx = \int \frac{1}{x^3 \sqrt{d\,x^3 + c}\,(8\,c - d\,x^3)^2} \, dx$$

```
[In] int(1/(x^3*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
[Out] int(1/(x^3*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.441 $\int \frac{1}{x^6(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{1}{x^6\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx=-\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{5}{3},2,\frac{1}{2},-\frac{2}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{320c^2x^5\sqrt{c+dx^3}}$$

```
[Out] -1/320*AppellF1(-5/3,1/2,2,-2/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^2
/x^5/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {525, 524}

$$\int \frac{1}{x^6\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx=-\frac{\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{5}{3},2,\frac{1}{2},-\frac{2}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{320c^2x^5\sqrt{c+dx^3}}$$

```
[In] Int[1/(x^6*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/320*(Sqrt[1 + (d*x^3)/c]*AppellF1[-5/3, 2, 1/2, -2/3, (d*x^3)/(8*c), -((
d*x^3)/c)])/(c^2*x^5*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^6(8c-dx^3)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$

$$= -\frac{\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{5}{3};2,\frac{1}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{320c^2x^5\sqrt{c+dx^3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 279 vs. 2(66) = 132.

Time = 10.22 (sec) , antiderivative size = 279, normalized size of antiderivative = 4.23

$$\int \frac{1}{x^6\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{\frac{64(c+dx^3)(864c^2-1080cdx^3+119d^2x^6)}{c^4x^5(-8c+dx^3)} - \frac{119d^3x^4\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^5} + \frac{140\cdots}{c^2(8c-dx^3)\left(32c\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{d\cdots}{8\cdots}\right.\right.}}{2211840\sqrt{c+dx^3}}$$

```
[In] Integrate[1/(x^6*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((64*(c + d*x^3)*(864*c^2 - 1080*c*d*x^3 + 119*d^2*x^6))/(c^4*x^5*(-8*c + d
*x^3)) - (119*d^3*x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x
^3)/c), (d*x^3)/(8*c)])/c^5 + (1404928*d^2*x*AppellF1[1/3, 1/2, 1, 4/3, -((
d*x^3)/c), (d*x^3)/(8*c)])/(c^2*(8*c - d*x^3)*(32*c*AppellF1[1/3, 1/2, 1, 4
/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d
*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^
3)/(8*c)]))))/(2211840*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 5.56 (sec) , antiderivative size = 765, normalized size of antiderivative = 11.59

method	result	size
elliptic	Expression too large to display	765
risch	Expression too large to display	1464
default	Expression too large to display	1783

```
[In] int(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/13824*d^2*x/c^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-1/320*(d*x^3+c)^(1/2)/c^3/x^
5+9/2560*d*(d*x^3+c)^(1/2)/c^4/x^2-119/103680*I*d/c^4*3^(1/2)*(-c*d^2)^(1/3
)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-7/31104*I/d/c^4*2^(1/2)*sum(1/_alp
ha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(
1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I
*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3
)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*
_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_al
pha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(
-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*
c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-
c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8
*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2561 vs. 2(52) = 104.

Time = 4.25 (sec) , antiderivative size = 2561, normalized size of antiderivative = 38.80

$$\int \frac{1}{x^6 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2488320*(11088*(d^2*x^8 - 8*c*d*x^5)*sqrt(d)*weierstrassPInverse(0, -4*c/
d, x) + 35*(c^4*d*x^8 - 8*c^5*x^5 + sqrt(-3)*(c^4*d*x^8 - 8*c^5*x^5))*(d^10
```

/c^25)^(1/6)*log(16807*(d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*
c^3*d^8 - 9*(c^17*d^4*x^8 + 38*c^18*d^3*x^5 + 64*c^19*d^2*x^2 + sqrt(-3)*(c
^17*d^4*x^8 + 38*c^18*d^3*x^5 + 64*c^19*d^2*x^2))*(d^10/c^25)^(2/3) + 3*sqr
t(d*x^3 + c)*((c^21*d^2*x^7 + 80*c^22*d*x^4 + 160*c^23*x - sqrt(-3)*(c^21*d
^2*x^7 + 80*c^22*d*x^4 + 160*c^23*x))*(d^10/c^25)^(5/6) - 2*(7*c^13*d^5*x^6
 + 152*c^14*d^4*x^3 + 64*c^15*d^3)*sqrt(d^10/c^25) + 6*(5*c^5*d^8*x^5 + 32*
c^6*d^7*x^2 + sqrt(-3)*(5*c^5*d^8*x^5 + 32*c^6*d^7*x^2))*(d^10/c^25)^(1/6))
 - 9*(5*c^9*d^7*x^7 + 64*c^10*d^6*x^4 + 32*c^11*d^5*x - sqrt(-3)*(5*c^9*d^7
*x^7 + 64*c^10*d^6*x^4 + 32*c^11*d^5*x))*(d^10/c^25)^(1/3))/(d^3*x^9 - 24*c
*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 35*(c^4*d*x^8 - 8*c^5*x^5 + sqrt(-3)
*(c^4*d*x^8 - 8*c^5*x^5))*(d^10/c^25)^(1/6)*log(16807*(d^11*x^9 + 318*c*d^1
0*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 - 9*(c^17*d^4*x^8 + 38*c^18*d^3*x^5
 + 64*c^19*d^2*x^2 + sqrt(-3)*(c^17*d^4*x^8 + 38*c^18*d^3*x^5 + 64*c^19*d^2*
x^2))*(d^10/c^25)^(2/3) - 3*sqrt(d*x^3 + c)*((c^21*d^2*x^7 + 80*c^22*d*x^4
 + 160*c^23*x - sqrt(-3)*(c^21*d^2*x^7 + 80*c^22*d*x^4 + 160*c^23*x))*(d^10/
c^25)^(5/6) - 2*(7*c^13*d^5*x^6 + 152*c^14*d^4*x^3 + 64*c^15*d^3)*sqrt(d^10
/c^25) + 6*(5*c^5*d^8*x^5 + 32*c^6*d^7*x^2 + sqrt(-3)*(5*c^5*d^8*x^5 + 32*c
^6*d^7*x^2))*(d^10/c^25)^(1/6)) - 9*(5*c^9*d^7*x^7 + 64*c^10*d^6*x^4 + 32*c
^11*d^5*x - sqrt(-3)*(5*c^9*d^7*x^7 + 64*c^10*d^6*x^4 + 32*c^11*d^5*x))*(d^
10/c^25)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 35*(c
^4*d*x^8 - 8*c^5*x^5 - sqrt(-3)*(c^4*d*x^8 - 8*c^5*x^5))*(d^10/c^25)^(1/6)*
log(16807*(d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 - 9*(
c^17*d^4*x^8 + 38*c^18*d^3*x^5 + 64*c^19*d^2*x^2 - sqrt(-3)*(c^17*d^4*x^8 +
 38*c^18*d^3*x^5 + 64*c^19*d^2*x^2))*(d^10/c^25)^(2/3) + 3*sqrt(d*x^3 + c)*
((c^21*d^2*x^7 + 80*c^22*d*x^4 + 160*c^23*x + sqrt(-3)*(c^21*d^2*x^7 + 80*c
^22*d*x^4 + 160*c^23*x))*(d^10/c^25)^(5/6) - 2*(7*c^13*d^5*x^6 + 152*c^14*d
^4*x^3 + 64*c^15*d^3)*sqrt(d^10/c^25) + 6*(5*c^5*d^8*x^5 + 32*c^6*d^7*x^2 -
 sqrt(-3)*(5*c^5*d^8*x^5 + 32*c^6*d^7*x^2))*(d^10/c^25)^(1/6)) - 9*(5*c^9*d
^7*x^7 + 64*c^10*d^6*x^4 + 32*c^11*d^5*x + sqrt(-3)*(5*c^9*d^7*x^7 + 64*c^1
0*d^6*x^4 + 32*c^11*d^5*x))*(d^10/c^25)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 19
2*c^2*d*x^3 - 512*c^3)) - 35*(c^4*d*x^8 - 8*c^5*x^5 - sqrt(-3)*(c^4*d*x^8 -
 8*c^5*x^5))*(d^10/c^25)^(1/6)*log(16807*(d^11*x^9 + 318*c*d^10*x^6 + 1200*
c^2*d^9*x^3 + 640*c^3*d^8 - 9*(c^17*d^4*x^8 + 38*c^18*d^3*x^5 + 64*c^19*d^2
x^2 - sqrt(-3)(c^17*d^4*x^8 + 38*c^18*d^3*x^5 + 64*c^19*d^2*x^2))*(d^10/c
^25)^(2/3) - 3*sqrt(d*x^3 + c)*((c^21*d^2*x^7 + 80*c^22*d*x^4 + 160*c^23*x
 + sqrt(-3)*(c^21*d^2*x^7 + 80*c^22*d*x^4 + 160*c^23*x))*(d^10/c^25)^(5/6) -
 2*(7*c^13*d^5*x^6 + 152*c^14*d^4*x^3 + 64*c^15*d^3)*sqrt(d^10/c^25) + 6*(5
*c^5*d^8*x^5 + 32*c^6*d^7*x^2 - sqrt(-3)*(5*c^5*d^8*x^5 + 32*c^6*d^7*x^2))*
(d^10/c^25)^(1/6)) - 9*(5*c^9*d^7*x^7 + 64*c^10*d^6*x^4 + 32*c^11*d^5*x + s
qrt(-3)*(5*c^9*d^7*x^7 + 64*c^10*d^6*x^4 + 32*c^11*d^5*x))*(d^10/c^25)^(1/3
))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 70*(c^4*d*x^8 - 8*
c^5*x^5)*(d^10/c^25)^(1/6)*log(16807*(d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*
d^9*x^3 + 640*c^3*d^8 + 18*(c^17*d^4*x^8 + 38*c^18*d^3*x^5 + 64*c^19*d^2*x^
2)*(d^10/c^25)^(2/3) + 6*sqrt(d*x^3 + c)*((c^21*d^2*x^7 + 80*c^22*d*x^4 + 1
60*c^23*x)*(d^10/c^25)^(5/6) + (7*c^13*d^5*x^6 + 152*c^14*d^4*x^3 + 64*c^15

```
*d^3)*sqrt(d^10/c^25) + 6*(5*c^5*d^8*x^5 + 32*c^6*d^7*x^2)*(d^10/c^25)^(1/6
)) + 18*(5*c^9*d^7*x^7 + 64*c^10*d^6*x^4 + 32*c^11*d^5*x)*(d^10/c^25)^(1/3)
)/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 70*(c^4*d*x^8 - 8*c
^5*x^5)*(d^10/c^25)^(1/6)*log(16807*(d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d
^9*x^3 + 640*c^3*d^8 + 18*(c^17*d^4*x^8 + 38*c^18*d^3*x^5 + 64*c^19*d^2*x^2
)*(d^10/c^25)^(2/3) - 6*sqrt(d*x^3 + c)*((c^21*d^2*x^7 + 80*c^22*d*x^4 + 16
0*c^23*x)*(d^10/c^25)^(5/6) + (7*c^13*d^5*x^6 + 152*c^14*d^4*x^3 + 64*c^15*
d^3)*sqrt(d^10/c^25) + 6*(5*c^5*d^8*x^5 + 32*c^6*d^7*x^2)*(d^10/c^25)^(1/6)
) + 18*(5*c^9*d^7*x^7 + 64*c^10*d^6*x^4 + 32*c^11*d^5*x)*(d^10/c^25)^(1/3))
/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 72*(119*d^2*x^6 - 10
80*c*d*x^3 + 864*c^2)*sqrt(d*x^3 + c))/(c^4*d*x^8 - 8*c^5*x^5)
```

Sympy [F]

$$\int \frac{1}{x^6 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{x^6 (-8c+dx^3)^2 \sqrt{c+dx^3}}\,dx$$

```
[In] integrate(1/x**6/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(x**6*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^6 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)^2 x^6}\,dx$$

```
[In] integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 (8c-dx^3)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{dx^3+c}(dx^3-8\,c)^2 x^6}\,dx$$

```
[In] integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (8c - dx^3)^2 \sqrt{c + dx^3}} dx = \int \frac{1}{x^6 \sqrt{d x^3 + c} (8 c - d x^3)^2} dx$$

```
[In] int(1/(x^6*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
[Out] int(1/(x^6*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

3.442 $\int \frac{x^{11}}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 95

$$\int \frac{x^{11}}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{8x^6}{27d^2(8c-dx^3)\sqrt{c+dx^3}} + \frac{2(38c+39dx^3)}{81d^4\sqrt{c+dx^3}} - \frac{640\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243d^4}$$

```
[Out] -640/243*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/d^4+8/27*x^6/d^2/(-d*
x^3+8*c)/(d*x^3+c)^(1/2)+2/81*(39*d*x^3+38*c)/d^4/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {457, 100, 151, 65, 212}

$$\int \frac{x^{11}}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = -\frac{640\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243d^4} + \frac{2(38c+39dx^3)}{81d^4\sqrt{c+dx^3}} + \frac{8x^6}{27d^2(8c-dx^3)\sqrt{c+dx^3}}$$

```
[In] Int[x^11/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (8*x^6)/(27*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (2*(38*c + 39*d*x^3))/(81*
d^4*Sqrt[c + d*x^3]) - (640*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(
243*d^4)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)
*(c + d*x)^(n + 1), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$
$$= \frac{8x^6}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{x(16c^2+13cdx)}{(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{27cd^2}$$
$$= \frac{8x^6}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2(38c+39dx^3)}{81d^4\sqrt{c+dx^3}} - \frac{(320c)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{81d^3}$$
$$= \frac{8x^6}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2(38c+39dx^3)}{81d^4\sqrt{c+dx^3}} - \frac{(640c)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{81d^4}$$
$$= \frac{8x^6}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2(38c+39dx^3)}{81d^4\sqrt{c+dx^3}} - \frac{640\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243d^4}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04

$$\int \frac{x^{11}}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = -\frac{2\left(912c^2+822cdx^3-81d^2x^6-320\sqrt{c}(8c-dx^3)\sqrt{c+dx^3}\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{243d^4\left(-8c+dx^3\right)\sqrt{c+dx^3}}$$

```
[In] Integrate[x^11/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*(912*c^2 + 822*c*d*x^3 - 81*d^2*x^6 - 320*Sqrt[c]*(8*c - d*x^3)*Sqrt[c
+ d*x^3]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(243*d^4*(-8*c + d*x^3)*Sqr
t[c + d*x^3])
```

Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{\frac{2\sqrt{d\,x^{3}+c}}{3}+\frac{128c\left(\frac{4\sqrt{d\,x^{3}+c}}{-d\,x^{3}+8c}-\frac{5\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{\sqrt{c}}\right)}{243}+\frac{2c}{243\sqrt{d\,x^{3}+c}}}{d^{4}}$
risch	$\frac{2\sqrt{d\,x^{3}+c}}{3d^{4}}+\frac{c\left(\frac{2}{243d\sqrt{d\,x^{3}+c}}-\frac{2432\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{729d\sqrt{c}}+\frac{512c\left(-\frac{\sqrt{d\,x^{3}+c}}{c\left(d\,x^{3}-8c\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{243d}\right)}{d^{3}}$
default	$\frac{d\left(\frac{2c}{3d^{2}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}+\frac{2\sqrt{d\,x^{3}+c}}{3d^{2}}\right)-\frac{32c}{3d\sqrt{d\,x^{3}+c}}}{d^{3}}+\frac{128\sqrt{c}\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\sqrt{d\,x^{3}+c}}{3}+\sqrt{c}\right)}{9d^{4}\sqrt{d\,x^{3}+c}}+\frac{512c\left(-\frac{2}{\sqrt{d\,x^{3}+c}}+\frac{\sqrt{d\,x^{3}}}{-d\,x^{3}}\right.}{24}$
elliptic	$\frac{512c\sqrt{d\,x^{3}+c}}{243d^{4}(-d\,x^{3}+8c)}+\frac{2c}{243d^{4}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}+\frac{2\sqrt{d\,x^{3}+c}}{3d^{4}}+\frac{320i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-\right.}{}\right.}{\left(-\right.}}}{}\right.}{}$

```
[In] int(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*((d*x^3+c)^(1/2)+64/81*c*(4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-5*arctanh(1/3*
(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))+1/81*c/(d*x^3+c)^(1/2))/d^4
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.45

$$\int \frac{x^{11}}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \left[\frac{2\left(160\,(d^2x^6-7\,cdx^3-8\,c^2)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+3\,(27\,d^2x^6-2\right.}{243\,(d^6x^6-7\,cd^5x^3-8\,c^2d^4)}\right.$$

```
[In] integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [2/243*(160*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3
 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 3*(27*d^2*x^6 - 274*c*d*x^3 - 304*c^
2)*sqrt(d*x^3 + c))/(d^6*x^6 - 7*c*d^5*x^3 - 8*c^2*d^4), 2/243*(320*(d^2*x^
6 - 7*c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*
(27*d^2*x^6 - 274*c*d*x^3 - 304*c^2)*sqrt(d*x^3 + c))/(d^6*x^6 - 7*c*d^5*x^
3 - 8*c^2*d^4)]
```

Sympy [F]

$$\int \frac{x^{11}}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{x^{11}}{(-8c+dx^3)^2 (c+dx^3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**11/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(x**11/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.03

$$\int \frac{x^{11}}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{2\left(160\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+81\sqrt{dx^3+c}-\frac{3\,(85\,(dx^3+c)c+3\,c^2)}{(dx^3+c)^{\frac{3}{2}}-9\sqrt{dx^3+c}c}\right)}{243\,d^4}$$

```
[In] integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] 2/243*(160*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*s
qrt(c))) + 81*sqrt(d*x^3 + c) - 3*(85*(d*x^3 + c)*c + 3*c^2)/((d*x^3 + c)^(
3/2) - 9*sqrt(d*x^3 + c)*c))/d^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int \frac{x^{11}}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{640\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{243\sqrt{-c}d^4} + \frac{2\sqrt{dx^3+c}}{3\,d^4} - \frac{2\,(85\,(dx^3+c)c+3\,c^2)}{81\left((dx^3+c)^{\frac{3}{2}}-9\sqrt{dx^3+c}c\right)d^4}$$

```
[In] integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 640/243*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) + 2/3*sqrt(d*
x^3 + c)/d^4 - 2/81*(85*(d*x^3 + c)*c + 3*c^2)/(((d*x^3 + c)^(3/2) - 9*sqrt
(d*x^3 + c)*c)*d^4)
```

Mupad [B] (verification not implemented)

Time = 8.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.17

$$\int \frac{x^{11}}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \frac{2\sqrt{d\,x^3+c}}{3\,d^4} + \frac{320\,\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{243\,d^4} + \frac{\sqrt{d\,x^3+c}\,\left(\frac{176\,c^2}{81\,d^4}+\frac{170\,c\,x^3}{81\,d^3}\right)}{8\,c^2+7\,c\,d\,x^3-d^2\,x^6}$$

```
[In] int(x^11/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*d^4) + (320*c^(1/2)*log((10*c + d*x^3 - 6*c^(1/2)*
(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(243*d^4) + ((c + d*x^3)^(1/2)*((176*c^2
)/(81*d^4) + (170*c*x^3)/(81*d^3)))/(8*c^2 - d^2*x^6 + 7*c*d*x^3)
```

3.443 $\int \frac{x^8}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 83

$$\int \frac{x^8}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=-\frac{22}{81d^3\sqrt{c+dx^3}}+\frac{64c}{27d^3\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{32\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243\sqrt{c}d^3}$$

```
[Out] -32/243*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^3/c^(1/2)-22/81/d^3/(d*x^3+c
)^(1/2)+64/27*c/d^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {457, 91, 79, 65, 212}

$$\int \frac{x^8}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=-\frac{32\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243\sqrt{c}d^3}+\frac{64c}{27d^3\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{22}{81d^3\sqrt{c+dx^3}}$$

```
[In] Int[x^8/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -22/(81*d^3*Sqrt[c + d*x^3]) + (64*c)/(27*d^3*(8*c - d*x^3)*Sqrt[c + d*x^3]
) - (32*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(243*Sqrt[c]*d^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{64c}{27d^3(8c-dx^3)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{-24c^2d+9cd^2x}{(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{27cd^3}$$

$$= -\frac{22}{81d^3\sqrt{c+dx^3}} + \frac{64c}{27d^3(8c-dx^3)\sqrt{c+dx^3}} - \frac{16\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{81d^2}$$

$$= -\frac{22}{81d^3\sqrt{c+dx^3}} + \frac{64c}{27d^3(8c-dx^3)\sqrt{c+dx^3}} - \frac{32\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{81d^3}$$

$$= -\frac{22}{81d^3\sqrt{c+dx^3}} + \frac{64c}{27d^3(8c-dx^3)\sqrt{c+dx^3}} - \frac{32\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243\sqrt{c}d^3}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.86

$$\int \frac{x^8}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{2\left(\frac{3(8c+11dx^3)}{(8c-dx^3)\sqrt{c+dx^3}} - \frac{16\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{\sqrt{c}}\right)}{243d^3}$$

```
[In] Integrate[x^8/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
[Out] (2*((3*(8*c + 11*d*x^3))/((8*c - d*x^3)*Sqrt[c + d*x^3]) - (16*ArcTanh[Sqrt
[c + d*x^3]/(3*Sqrt[c])])/Sqrt[c]))/(243*d^3)
```

Maple [A] (verified)

Time = 4.77 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$\frac{\frac{64\sqrt{d\,x^{3}+c}}{243\left(-d\,x^{3}+8c\right)}-\frac{32\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{243\sqrt{c}}-\frac{2}{243\sqrt{d\,x^{3}+c}}}{d^{3}}$
default	$-\frac{2}{3d^{3}\sqrt{d\,x^{3}+c}}+\frac{-\frac{32\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)\sqrt{d\,x^{3}+c}}{81}+\frac{32\sqrt{c}}{27}}{d^{3}\sqrt{c}\,\sqrt{d\,x^{3}+c}}+\frac{-\frac{128}{243\sqrt{d\,x^{3}+c}}+\frac{64\sqrt{d\,x^{3}+c}}{243\left(-d\,x^{3}+8c\right)}+\frac{64\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{3\sqrt{c}}\right)}{243\sqrt{c}}}{d^{3}}$
elliptic	$\frac{64\sqrt{d\,x^{3}+c}}{243d^{3}(-d\,x^{3}+8c)}-\frac{2}{243d^{3}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}+\frac{16i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(d\_Z^{3}-8c\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)}{d}\right.}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}}{}\right.}{}$

[In] int(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/243*(32*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-16*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)-1/(d*x^3+c)^(1/2))/d^3

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.69

$$\int \frac{x^8}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \left[\frac{2\left(8\left(d^2x^6-7\,cdx^3-8\,c^2\right)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-3\left(11\,cdx^3+8\,c^2\right.\right.}{243\left(cd^5x^6-7\,c^2d^4x^3-8\,c^3d^3\right)}\right.$$

[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")

[Out] [2/243*(8*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - 3*(11*c*d*x^3 + 8*c^2)*sqrt(d*x^3 + c))/(c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3), 2/243*(16*(d^2*x^6 - 7*c*d*x^3 -

```
8*c^2)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 3*(11*c*d*x^3 + 8
*c^2)*sqrt(d*x^3 + c))/(c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3)]
```

Sympy [F]

$$\int \frac{x^8}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{x^8}{(-8c+dx^3)^2 (c+dx^3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**8/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(x**8/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98

$$\int \frac{x^8}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{2\left(\frac{8\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{\sqrt{c}} - \frac{3\,(11\,dx^3+8\,c)}{(dx^3+c)^{\frac{3}{2}}-9\sqrt{dx^3+c}c}\right)}{243\,d^3}$$

```
[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] 2/243*(8*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/s
qrt(c) - 3*(11*d*x^3 + 8*c)/((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c))/d^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int \frac{x^8}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{32\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{243\sqrt{-c}d^3} - \frac{2\,(11\,dx^3+8\,c)}{81\left((dx^3+c)^{\frac{3}{2}}-9\sqrt{dx^3+c}c\right)d^3}$$

```
[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] 32/243*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/81*(11*d*x^3
 + 8*c)/(((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*d^3)
```

Mupad [B] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.13

$$\int \frac{x^8}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{16\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{243\,\sqrt{c}\,d^3}+\frac{\sqrt{d\,x^3+c}\,\left(\frac{16\,c}{81\,d^3}+\frac{22\,x^3}{81\,d^2}\right)}{8\,c^2+7\,c\,d\,x^3-d^2\,x^6}$$

```
[In] int(x^8/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] (16*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(243*c
^(1/2)*d^3) + ((c + d*x^3)^(1/2)*((16*c)/(81*d^3) + (22*x^3)/(81*d^2)))/(8*
c^2 - d^2*x^6 + 7*c*d*x^3)
```

3.444 $\int \frac{x^5}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 85

$$\int \frac{x^5}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = -\frac{2}{81cd^2\sqrt{c+dx^3}} + \frac{8}{27d^2(8c-dx^3)\sqrt{c+dx^3}} + \frac{2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{3/2}d^2}$$

```
[Out] 2/243*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)/d^2-2/81/c/d^2/(d*x^3+c)
^(1/2)+8/27/d^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {457, 79, 53, 65, 212}

$$\int \frac{x^5}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{3/2}d^2} + \frac{8}{27d^2(8c-dx^3)\sqrt{c+dx^3}} - \frac{2}{81cd^2\sqrt{c+dx^3}}$$

```
[In] Int[x^5/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -2/(81*c*d^2*Sqrt[c + d*x^3]) + 8/(27*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) +
(2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(243*c^(3/2)*d^2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)\\ &= \frac{8}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{9d}\end{aligned}$$

$$= -\frac{2}{81cd^2\sqrt{c+dx^3}} + \frac{8}{27d^2(8c-dx^3)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}dx, x, x^3\right)}{81cd}$$

$$= -\frac{2}{81cd^2\sqrt{c+dx^3}} + \frac{8}{27d^2(8c-dx^3)\sqrt{c+dx^3}} + \frac{2\text{Subst}\left(\int \frac{1}{9c-x^2}dx, x, \sqrt{c+dx^3}\right)}{81cd^2}$$

$$= -\frac{2}{81cd^2\sqrt{c+dx^3}} + \frac{8}{27d^2(8c-dx^3)\sqrt{c+dx^3}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{3/2}d^2}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{x^5}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{2\left(\frac{3\sqrt{c}(4c+dx^3)}{(8c-dx^3)\sqrt{c+dx^3}} + \operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{243c^{3/2}d^2}$$

```
[In] Integrate[x^5/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
[Out] (2*((3*Sqrt[c]*(4*c + d*x^3))/((8*c - d*x^3)*Sqrt[c + d*x^3]) + ArcTanh[Sqr
t[c + d*x^3]/(3*Sqrt[c])]))/(243*c^(3/2)*d^2)
```

Maple [A] (verified)

Time = 4.38 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$\frac{\frac{8\sqrt{d\,x^3+c}}{243\left(-d\,x^3+8c\right)}+\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{243\sqrt{c}}+\frac{2}{243\sqrt{d\,x^3+c}}}{c\,d^2}$
default	$\frac{-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)\sqrt{d\,x^3+c}}{81}+\frac{2\sqrt{c}}{27}}{d^2\sqrt{d\,x^3+c}\,c\,c^{\frac{3}{2}}}+\frac{-\frac{16}{243\sqrt{d\,x^3+c}}+\frac{8\sqrt{d\,x^3+c}}{243\left(-d\,x^3+8c\right)}+\frac{8\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{243\sqrt{c}}}{c\,d^2}$
elliptic	$\frac{8\sqrt{d\,x^3+c}}{243d^2c(-d\,x^3+8c)}+\frac{2}{243d^2c\sqrt{(x^3+\frac{c}{d})d}}-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-8c\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}}{}\right.}{}$

[In] int(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/243/c*(4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)+1/(d*x^3+c)^(1/2))/d^2

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.62

$$\int \frac{x^5}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \left[\frac{(d^2x^6-7\,cdx^3-8\,c^2)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-6\,(cdx^3+4\,c^2)\sqrt{dx^3}}{243\,(c^2d^4x^6-7\,c^3d^3x^3-8\,c^4d^2)}\right.$$
$$\left.-\frac{2\left((d^2x^6-7\,cdx^3-8\,c^2)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+3\,(cdx^3+4\,c^2)\sqrt{dx^3+c}\right)}{243\,(c^2d^4x^6-7\,c^3d^3x^3-8\,c^4d^2)}\right]$$

[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")

[Out] [1/243*((d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - 6*(c*d*x^3 + 4*c^2)*sqrt(d*x^3 + c))/(c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2), -2/243*((d^2*x^6 - 7*c*d*x^3 - 8*c^

```
2)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(c*d*x^3 + 4*c^2)*sq
rt(d*x^3 + c))/(c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2)]
```

Sympy [F]

$$\int \frac{x^5}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \int \frac{x^5}{(-8c+dx^3)^2(c+dx^3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**5/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(x**5/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int \frac{x^5}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = -\frac{\frac{6\,(dx^3+4\,c)}{(dx^3+c)^{\frac{3}{2}}c-9\,\sqrt{dx^3+c}c^2}+\frac{\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)}{c^{\frac{3}{2}}}}{243\,d^2}$$

```
[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] -1/243*(6*(d*x^3 + 4*c)/((d*x^3 + c)^(3/2)*c - 9*sqrt(d*x^3 + c)*c^2) + log
((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/c^(3/2))/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = -\frac{2\left(\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}cd}+\frac{3\,(dx^3+4\,c)}{\left((dx^3+c)^{\frac{3}{2}}-9\,\sqrt{dx^3+c}c\right)cd}\right)}{243\,d}$$

```
[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] -2/243*(arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c*d) + 3*(d*x^3 + 4*
c)/(((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*c*d))/d
```

Mupad [B] (verification not implemented)

Time = 8.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.13

$$\int \frac{x^5}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx=\frac{\left(\frac{8}{81\,d^2}+\frac{2\,x^3}{81\,c\,d}\right)\,\sqrt{d\,x^3+c}}{8\,c^2+7\,c\,d\,x^3-d^2\,x^6}+\frac{\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{243\,c^{3/2}\,d^2}$$

```
[In] int(x^5/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] ((8/(81*d^2) + (2*x^3)/(81*c*d))*(c + d*x^3)^(1/2))/(8*c^2 - d^2*x^6 + 7*c*
d*x^3) + log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(2
43*c^(3/2)*d^2)
```

3.445 $\int \frac{x^2}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 88

$$\int \frac{x^2}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = -\frac{1}{81c^2d\sqrt{c+dx^3}} + \frac{1}{27cd(8c-dx^3)\sqrt{c+dx^3}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/2}d}$$

```
[Out] 1/243*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)/d-1/81/c^2/d/(d*x^3+c)^(
1/2)+1/27/c/d/(-d*x^3+8*c)/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {455, 44, 53, 65, 212}

$$\int \frac{x^2}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/2}d} - \frac{1}{81c^2d\sqrt{c+dx^3}} + \frac{1}{27cd(8c-dx^3)\sqrt{c+dx^3}}$$

```
[In] Int[x^2/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/81*1/(c^2*d*Sqrt[c + d*x^3]) + 1/(27*c*d*(8*c - d*x^3)*Sqrt[c + d*x^3])
+ ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(243*c^(5/2)*d)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)\\ &= \frac{1}{27cd\,(8c-dx^3)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{18c}\\ &= -\frac{1}{81c^2d\sqrt{c+dx^3}} + \frac{1}{27cd\,(8c-dx^3)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{162c^2}\end{aligned}$$

$$= -\frac{1}{81c^2d\sqrt{c+dx^3}} + \frac{1}{27cd\,(8c-dx^3)\,\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{81c^2d}$$

$$= -\frac{1}{81c^2d\sqrt{c+dx^3}} + \frac{1}{27cd\,(8c-dx^3)\,\sqrt{c+dx^3}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/2}d}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \frac{\frac{3\sqrt{c}(-5c+dx^3)}{(8c-dx^3)\sqrt{c+dx^3}} + \operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/2}d}$$

```
[In] Integrate[x^2/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] ((3*Sqrt[c]*(-5*c + d*x^3))/((8*c - d*x^3)*Sqrt[c + d*x^3]) + ArcTanh[Sqrt[
c + d*x^3]/(3*Sqrt[c])])/(243*c^(5/2)*d)
```

Maple [A] (verified)

Time = 4.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.70

method	result
default	$\frac{-\frac{2}{\sqrt{d\,x^3+c}}+\frac{\sqrt{d\,x^3+c}}{-d\,x^3+8c}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{\sqrt{c}}}{243c^2d}$
pseudoelliptic	$\frac{-\frac{2}{\sqrt{d\,x^3+c}}+\frac{\sqrt{d\,x^3+c}}{-d\,x^3+8c}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{\sqrt{c}}}{243c^2d}$
elliptic	$\frac{\sqrt{d\,x^3+c}}{243c^2d(-d\,x^3+8c)} - \frac{2}{243d\,c^2\sqrt{\left(x^3+\frac{c}{d}\right)d}} - \frac{i\sqrt{2}\left(\sum_{\_\alpha=\text{RootOf}\left(d\_Z^3-8c\right)} \frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right.}{\left(-c\,d^2\right)^{\frac{1}{3}}}}}{}\right.}{}$

```
[In] int(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/243/c^2*(-2/(d*x^3+c)^(1/2)+(d*x^3+c)^(1/2)/(-d*x^3+8*c)+arctanh(1/3*(d*x
^3+c)^(1/2)/c^(1/2))/c^(1/2))/d
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.49

$$\int \frac{x^2}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \left[\frac{(d^2x^6-7\,cdx^3-8\,c^2)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-6\,(cdx^3-5\,c^2)\sqrt{dx^3}}{486\,(c^3d^3x^6-7\,c^4d^2x^3-8\,c^5d)}\right.$$
$$\left.-\frac{(d^2x^6-7\,cdx^3-8\,c^2)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+3\,(cdx^3-5\,c^2)\sqrt{dx^3+c}}{243\,(c^3d^3x^6-7\,c^4d^2x^3-8\,c^5d)}\right]$$

```
[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/486*((d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c
)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - 6*(c*d*x^3 - 5*c^2)*sqrt(d*x^3 + c))/(c^
3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d), -1/243*((d^2*x^6 - 7*c*d*x^3 - 8*c^2)
*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(c*d*x^3 - 5*c^2)*sqrt
(d*x^3 + c))/(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d)]
```

Sympy [F]

$$\int \frac{x^2}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \int \frac{x^2}{(-8c+dx^3)^2\,(c+dx^3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**2/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**2/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = -\frac{\frac{6\,(dx^3-5\,c)}{(dx^3+c)^{\frac{3}{2}}c^2-9\,\sqrt{dx^3+c}c^3}+\frac{\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)}{c^{\frac{5}{2}}}}{486\,d}$$

```
[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/486*(6*(d*x^3 - 5*c)/((d*x^3 + c)^(3/2)*c^2 - 9*sqrt(d*x^3 + c)*c^3) + l
og((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/c^(5/2))/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{243\sqrt{-c}c^2d} - \frac{dx^3-5\,c}{81\left((dx^3+c)^{\frac{3}{2}}-9\sqrt{dx^3+c}c\right)c^2d}$$

```
[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/243*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2*d) - 1/81*(d*x^3
- 5*c)/(((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*c^2*d)
```

Mupad [B] (verification not implemented)

Time = 8.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{486\,c^{5/2}\,d} - \frac{\left(\frac{5}{81\,c\,d}-\frac{x^3}{81\,c^2}\right)\sqrt{d\,x^3+c}}{8\,c^2+7\,c\,d\,x^3-d^2\,x^6}$$

```
[In] int(x^2/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(486*c^(5/2
)*d) - ((5/(81*c*d) - x^3/(81*c^2))*(c + d*x^3)^(1/2))/(8*c^2 - d^2*x^6 + 7
*c*d*x^3)
```

3.446 $\int \frac{1}{x\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 106

$$\int \frac{1}{x\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{5}{648c^3\sqrt{c+dx^3}} + \frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{7\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{7776c^{7/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}}$$

```
[Out] 7/7776*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)-1/96*arctanh((d*x^3+c)^
(1/2)/c^(1/2))/c^(7/2)+5/648/c^3/(d*x^3+c)^(1/2)+1/216/c^2/(-d*x^3+8*c)/(d*
x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {457, 105, 157, 162, 65, 214, 212}

$$\int \frac{1}{x\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{7\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{7776c^{7/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}} + \frac{5}{648c^3\sqrt{c+dx^3}} + \frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

```
[In] Int[1/(x*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] 5/(648*c^3*Sqrt[c + d*x^3]) + 1/(216*c^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (
7*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(7776*c^(7/2)) - ArcTanh[Sqrt[c + d
*x^3]/Sqrt[c]]/(96*c^(7/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)\\
&= \frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{-9cd-\frac{3d^2x}{2}}{x(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{216c^2d}\\
&= \frac{5}{648c^3\sqrt{c+dx^3}} + \frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{-\frac{81}{2}c^2d^2+\frac{15}{4}cd^3x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{972c^4d^2}\\
&= \frac{5}{648c^3\sqrt{c+dx^3}} + \frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{192c^3} + \frac{(7d)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{5184c^3}\\
&= \frac{5}{648c^3\sqrt{c+dx^3}} + \frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\\
&\quad + \frac{7\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{2592c^3} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{96c^3d}\\
&= \frac{5}{648c^3\sqrt{c+dx^3}} + \frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{7\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{7776c^{7/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{\frac{12\sqrt{c}(43c-5dx^3)}{(8c-dx^3)\sqrt{c+dx^3}} + 7\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 81\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{7776c^{7/2}}$$

```
[In] Integrate[1/(x*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] ((12*Sqrt[c]*(43*c - 5*d*x^3))/((8*c - d*x^3)*Sqrt[c + d*x^3]) + 7*ArcTanh[
Sqrt[c + d*x^3]/(3*Sqrt[c])] - 81*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(7776*c
^(7/2))
```

Maple [A] (verified)

Time = 4.55 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{96 c^{\frac{7}{2}}}+\frac{\frac{7 \operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{3 \sqrt{c}}\right)\left(d x^3-8 c\right)}{\sqrt{c}}-4 \sqrt{d x^3+c}}{7776\left(d x^3-8 c\right) c^3}+\frac{2}{243 c^3 \sqrt{d x^3+c}}$
default	$\frac{\frac{2}{3 c \sqrt{\left(x^3+\frac{c}{d}\right) d}}-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{3 c^{\frac{3}{2}}}}{64 c^2}+\frac{-\frac{2}{\sqrt{d x^3+c}}+\frac{\sqrt{d x^3+c}}{-d x^3+8 c}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{3 \sqrt{c}}\right)}{\sqrt{c}}}{1944 c^3}-\frac{-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{3 \sqrt{c}}\right) \sqrt{d x^3+c}}{3}+\sqrt{c}}{864 c^{\frac{7}{2}} \sqrt{d x^3+c}}$
elliptic	Expression too large to display

```
[In] int(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/96*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)+1/7776*(7*arctanh(1/3*(d*x^3
+c)^(1/2)/c^(1/2))/c^(1/2)*(d*x^3-8*c)-4*(d*x^3+c)^(1/2))/(d*x^3-8*c)/c^3+2
/243/c^3/(d*x^3+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.98

$$\int \frac{1}{x\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \left[\frac{7\left(d^2x^6-7\,cdx^3-8\,c^2\right)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+81\left(d^2x^6-7\,cdx^3\right.}{15552\left(c^4d^2x^6-7\,c^5\right.}\right.$$

```
[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/15552*(7*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3
 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 81*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqr
t(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 24*(5*c*d*x^3 - 4
3*c^2)*sqrt(d*x^3 + c))/(c^4*d^2*x^6 - 7*c^5*d*x^3 - 8*c^6), 1/7776*(81*(d^
2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 7*
(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/
c) + 12*(5*c*d*x^3 - 43*c^2)*sqrt(d*x^3 + c))/(c^4*d^2*x^6 - 7*c^5*d*x^3 -
8*c^6)]
```

Sympy [F]

$$\int \frac{1}{x\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{x\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(1/(x*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x}\,dx$$

```
[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}c^3}-\frac{7\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{7776\sqrt{-c}c^3}+\frac{5\,dx^3-43\,c}{648\left(\left(dx^3+c\right)^{\frac{3}{2}}-9\sqrt{dx^3+c}c\right)c^3}$$

```
[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] 1/96*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 7/7776*arctan(1/3*sq
rt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) + 1/648*(5*d*x^3 - 43*c)/(((d*x^3 +
c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*c^3)
```

Mupad [B] (verification not implemented)

Time = 8.35 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.95

$$\int \frac{1}{x\,(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx=-\frac{\frac{5\,(d\,x^3+c)}{216\,c^3}-\frac{2}{9\,c^2}}{27\,c\,\sqrt{d\,x^3+c}-3\,(d\,x^3+c)^{3/2}}+\frac{\left(\operatorname{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{\sqrt{c^7}}\right)\,1\mathrm{i}-\frac{\operatorname{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^7}}\right)\,7\mathrm{i}}{81}\right)\,1\mathrm{i}}{96\,\sqrt{c^7}}$$

[In] int(1/(x*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)

[Out] ((atanh((c^3*(c + d*x^3)^(1/2))/(c^7)^(1/2))*1i - (atanh((c^3*(c + d*x^3)^(1/2))/(3*(c^7)^(1/2)))*7i)/81)*1i)/(96*(c^7)^(1/2)) - ((5*(c + d*x^3))/(216*c^3) - 2/(9*c^2))/(27*c*(c + d*x^3)^(1/2) - 3*(c + d*x^3)^(3/2))

3.447 $\int \frac{1}{x^4(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 143

$$\int \frac{1}{x^4 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = -\frac{35d}{2592c^4\sqrt{c+dx^3}}+\frac{5d}{864c^3(8c-dx^3)\sqrt{c+dx^3}} - \frac{1}{24c^2x^3(8c-dx^3)\sqrt{c+dx^3}}+\frac{5d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{31104c^{9/2}}+\frac{5d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{9/2}}$$

```
[Out] 5/31104*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(9/2)+5/384*d*arctanh((d*x
^3+c)^(1/2)/c^(1/2))/c^(9/2)-35/2592*d/c^4/(d*x^3+c)^(1/2)+5/864*d/c^3/(-d*
x^3+8*c)/(d*x^3+c)^(1/2)-1/24/c^2/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {457, 105, 156, 157, 162, 65, 214, 212}

$$\int \frac{1}{x^4 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{5d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{31104c^{9/2}}+\frac{5d\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{9/2}} - \frac{35d}{2592c^4\sqrt{c+dx^3}}+\frac{5d}{864c^3(8c-dx^3)\sqrt{c+dx^3}}-\frac{1}{24c^2x^3(8c-dx^3)\sqrt{c+dx^3}}$$

```
[In] Int[1/(x^4*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-35*d)/(2592*c^4*Sqrt[c + d*x^3]) + (5*d)/(864*c^3*(8*c - d*x^3)*Sqrt[c +
d*x^3]) - 1/(24*c^2*x^3*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (5*d*ArcTanh[Sqrt[
```

```
c + d*x^3]/(3*Sqrt[c])])/(31104*c^(9/2)) + (5*d*ArcTanh[Sqrt[c + d*x^3]/Sqr
t[c]])/(384*c^(9/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right) \\
&= -\frac{1}{24c^2x^3\,(8c-dx^3)\,\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{10cd-\frac{5d^2x}{2}}{x(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)}{24c^2} \\
&= \frac{5d}{864c^3\,(8c-dx^3)\,\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\,(8c-dx^3)\,\sqrt{c+dx^3}} \\
&\quad + \frac{\text{Subst}\left(\int \frac{-90c^2d^2+15cd^3x}{x(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{1728c^4d} \\
&= -\frac{35d}{2592c^4\sqrt{c+dx^3}} + \frac{5d}{864c^3\,(8c-dx^3)\,\sqrt{c+dx^3}} \\
&\quad - \frac{1}{24c^2x^3\,(8c-dx^3)\,\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{-405c^3d^3+\frac{105}{2}c^2d^4x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{7776c^6d^2} \\
&= -\frac{35d}{2592c^4\sqrt{c+dx^3}} + \frac{5d}{864c^3\,(8c-dx^3)\,\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\,(8c-dx^3)\,\sqrt{c+dx^3}} \\
&\quad - \frac{(5d)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{768c^4} + \frac{(5d^2)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{20736c^4}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{35d}{2592c^4\sqrt{c+dx^3}} + \frac{5d}{864c^3\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\left(8c-dx^3\right)\sqrt{c+dx^3}} \\
&\quad - \frac{5\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{384c^4} + \frac{(5d)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{10368c^4} \\
&= -\frac{35d}{2592c^4\sqrt{c+dx^3}} + \frac{5d}{864c^3\left(8c-dx^3\right)\sqrt{c+dx^3}} \\
&\quad - \frac{1}{24c^2x^3\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{31104c^{9/2}} + \frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{9/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.76

$$
\int \frac{1}{x^4\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{-\frac{12\sqrt{c}\left(108c^2+265cdx^3-35d^2x^6\right)}{x^3\left(8c-dx^3\right)\sqrt{c+dx^3}} + 5d\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) + 405d\,\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{31104c^{9/2}}
$$

```
[In] Integrate[1/(x^4*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] ((-12*Sqrt[c]*(108*c^2 + 265*c*d*x^3 - 35*d^2*x^6))/(x^3*(8*c - d*x^3)*Sqrt
[c + d*x^3]) + 5*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] + 405*d*ArcTanh[Sqr
t[c + d*x^3]/Sqrt[c]])/(31104*c^(9/2))
```

Maple [A] (verified)

Time = 4.70 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{d\left(-\frac{-5\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)d\,x^3+2\sqrt{d\,x^3+c}\,\sqrt{c}}{128d\,x^3c^{\frac{9}{2}}}+\frac{\frac{\sqrt{d\,x^3+c}}{-d\,x^3+8c}+\frac{5\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{2\sqrt{c}}}{5184c^4}-\frac{2}{81c^4\sqrt{d\,x^3+c}}\right)}{3}$
risch	$-\frac{\sqrt{d\,x^3+c}}{192c^4x^3}-\frac{d\left(-\frac{5\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3\sqrt{c}}+\frac{256}{243\sqrt{d\,x^3+c}}-\frac{13\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{729\sqrt{c}}-\frac{2c\left(-\frac{\sqrt{d\,x^3+c}}{c\left(d\,x^3-8c\right)}+\frac{\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{3\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{243}\right)}{128c^4}$
default	$\frac{-\frac{\sqrt{d\,x^3+c}}{3c^2x^3}-\frac{2d}{3c^2\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{d\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}}{64c^2}+\frac{d\left(\frac{2}{3c\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{2\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{256c^3}+\frac{d\left(-\frac{2}{\sqrt{d\,x^3+c}}+\frac{\sqrt{d\,x^3+c}}{-d\,x^3+8c}\right.}{1555}$
elliptic	Expression too large to display

```
[In] int(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*d*(-1/128*(-5*arctanh((d*x^3+c)^(1/2)/c^(1/2))*d*x^3+2*(d*x^3+c)^(1/2)*
c^(1/2))/d/x^3/c^(9/2)+1/5184*((d*x^3+c)^(1/2)/(-d*x^3+8*c)+5/2*arctanh(1/3
*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))/c^4-2/81/c^4/(d*x^3+c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 368, normalized size of antiderivative = 2.57

$$\int \frac{1}{x^4\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \left[\frac{5\left(d^3x^9-7\,cd^2x^6-8\,c^2dx^3\right)\sqrt{c}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+405\left(d^3x^9-\right.}{62208}\right.$$

$$-\frac{405\left(d^3x^9-7\,cd^2x^6-8\,c^2dx^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+5\left(d^3x^9-7\,cd^2x^6-8\,c^2dx^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+}}{3}\right.}{31104\left(c^5d^2x^9-7\,c^6dx^6-8\,c^7x^3\right)}$$

```
[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/62208*(5*(d^3*x^9 - 7*c*d^2*x^6 - 8*c^2*d*x^3)*sqrt(c)*log((d*x^3 + 6*sq
rt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 405*(d^3*x^9 - 7*c*d^2*x^6 -
 8*c^2*d*x^3)*sqrt(c)*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) -
24*(35*c*d^2*x^6 - 265*c^2*d*x^3 - 108*c^3)*sqrt(d*x^3 + c))/(c^5*d^2*x^9 -
 7*c^6*d*x^6 - 8*c^7*x^3), -1/31104*(405*(d^3*x^9 - 7*c*d^2*x^6 - 8*c^2*d*x
^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + 5*(d^3*x^9 - 7*c*d^2*x^6
- 8*c^2*d*x^3)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*(35*c*d
^2*x^6 - 265*c^2*d*x^3 - 108*c^3)*sqrt(d*x^3 + c))/(c^5*d^2*x^9 - 7*c^6*d*x
^6 - 8*c^7*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \int \frac{1}{x^4\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**4/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x**4*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)^2 x^4}\,dx$$

```
[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^4 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = -\frac{5\,d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{384\,\sqrt{-c}c^4} - \frac{5\,d\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{31104\,\sqrt{-c}c^4} - \frac{35\,(dx^3+c)^2 d - 335\,(dx^3+c)cd + 192\,c^2 d}{2592\left((dx^3+c)^{\frac{5}{2}} - 10\,(dx^3+c)^{\frac{3}{2}}c + 9\,\sqrt{dx^3+c}c^2\right)c^4}$$

```
[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] -5/384*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 5/31104*d*arctan
(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 1/2592*(35*(d*x^3 + c)^2*d
- 335*(d*x^3 + c)*c*d + 192*c^2*d)/(((d*x^3 + c)^(5/2) - 10*(d*x^3 + c)^(3/
2)*c + 9*sqrt(d*x^3 + c)*c^2)*c^4)
```

Mupad [B] (verification not implemented)

Time = 8.48 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^4 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = -\frac{\frac{2\,d}{9\,c^2} + \frac{35\,d\,(d\,x^3+c)^2}{864\,c^4} - \frac{335\,d\,(d\,x^3+c)}{864\,c^3}}{3\,(d\,x^3+c)^{5/2} - 30\,c\,(d\,x^3+c)^{3/2} + 27\,c^2\,\sqrt{d\,x^3+c}} - \frac{d\left(\mathrm{atanh}\left(\frac{c^4\,\sqrt{d\,x^3+c}}{\sqrt{c^9}}\right)\,1\mathrm{i} + \frac{\mathrm{atanh}\left(\frac{c^4\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^9}}\right)\,1\mathrm{i}}{81}\right)\,5\mathrm{i}}{384\,\sqrt{c^9}}$$

```
[In] int(1/(x^4*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
[Out] - ((2*d)/(9*c^2) + (35*d*(c + d*x^3)^2)/(864*c^4) - (335*d*(c + d*x^3))/(86
4*c^3))/(3*(c + d*x^3)^(5/2) - 30*c*(c + d*x^3)^(3/2) + 27*c^2*(c + d*x^3)^
(1/2)) - (d*(atanh((c^4*(c + d*x^3)^(1/2))/(c^9)^(1/2))*1i + (atanh((c^4*(c
 + d*x^3)^(1/2))/(3*(c^9)^(1/2)))*1i)/81)*5i)/(384*(c^9)^(1/2))
```

3.448 $\int \frac{1}{x^7(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 185

$$\int \frac{1}{x^7 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{665d^2}{41472c^5\sqrt{c+dx^3}} - \frac{71d^2}{13824c^4(8c-dx^3)\sqrt{c+dx^3}} - \frac{1}{48c^2x^6(8c-dx^3)\sqrt{c+dx^3}} + \frac{17d}{384c^3x^3(8c-dx^3)\sqrt{c+dx^3}} + \frac{13d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{497664c^{11/2}} - \frac{33d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{11/2}}$$

```
[Out] 13/497664*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/2)-33/2048*d^2*arc
tanh((d*x^3+c)^(1/2)/c^(1/2))/c^(11/2)+665/41472*d^2/c^5/(d*x^3+c)^(1/2)-71
/13824*d^2/c^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2)-1/48/c^2/x^6/(-d*x^3+8*c)/(d*x^
3+c)^(1/2)+17/384*d/c^3/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {457, 105, 156, 157, 162, 65, 214, 212}

$$\int \frac{1}{x^7 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{13d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{497664c^{11/2}} - \frac{33d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{11/2}} + \frac{665d^2}{41472c^5\sqrt{c+dx^3}} - \frac{71d^2}{13824c^4(8c-dx^3)\sqrt{c+dx^3}} + \frac{17d}{384c^3x^3(8c-dx^3)\sqrt{c+dx^3}} - \frac{1}{48c^2x^6(8c-dx^3)\sqrt{c+dx^3}}$$

```
[In] Int[1/(x^7*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (665*d^2)/(41472*c^5*Sqrt[c + d*x^3]) - (71*d^2)/(13824*c^4*(8*c - d*x^3)*S
qrt[c + d*x^3]) - 1/(48*c^2*x^6*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (17*d)/(38
4*c^3*x^3*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (13*d^2*ArcTanh[Sqrt[c + d*x^3]/
(3*Sqrt[c])])/(497664*c^(11/2)) - (33*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])
/(2048*c^(11/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^3(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right) \\
&= -\frac{1}{48c^2x^6\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{17cd-\frac{7d^2x}{2}}{x^2(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)}{48c^2} \\
&= -\frac{1}{48c^2x^6\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{17d}{384c^3x^3\left(8c-dx^3\right)\sqrt{c+dx^3}} \\
&\quad + \frac{\text{Subst}\left(\int \frac{198c^2d^2-\frac{85}{2}cd^3x}{x(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)}{384c^4} \\
&= -\frac{71d^2}{13824c^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\left(8c-dx^3\right)\sqrt{c+dx^3}} \\
&\quad + \frac{17d}{384c^3x^3\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{-1782c^3d^3+213c^2d^4x}{x(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{27648c^6d}
\end{aligned}$$

$$= \frac{665d^2}{41472c^5\sqrt{c+dx^3}} - \frac{71d^2}{13824c^4(8c-dx^3)\sqrt{c+dx^3}} - \frac{1}{48c^2x^6(8c-dx^3)\sqrt{c+dx^3}} + \frac{17d}{384c^3x^3(8c-dx^3)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{-8019c^4d^4+\frac{1995}{2}c^3d^5x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{124416c^8d^2}$$

$$= \frac{665d^2}{41472c^5\sqrt{c+dx^3}} - \frac{71d^2}{13824c^4(8c-dx^3)\sqrt{c+dx^3}} - \frac{1}{48c^2x^6(8c-dx^3)\sqrt{c+dx^3}} + \frac{17d}{384c^3x^3(8c-dx^3)\sqrt{c+dx^3}} + \frac{(33d^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{4096c^5} + \frac{(13d^3)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{331776c^5}$$

$$= \frac{665d^2}{41472c^5\sqrt{c+dx^3}} - \frac{71d^2}{13824c^4(8c-dx^3)\sqrt{c+dx^3}} - \frac{1}{48c^2x^6(8c-dx^3)\sqrt{c+dx^3}} + \frac{17d}{384c^3x^3(8c-dx^3)\sqrt{c+dx^3}} + \frac{(33d)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{2048c^5} + \frac{(13d^2)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{165888c^5}$$

$$= \frac{665d^2}{41472c^5\sqrt{c+dx^3}} - \frac{71d^2}{13824c^4(8c-dx^3)\sqrt{c+dx^3}} - \frac{1}{48c^2x^6(8c-dx^3)\sqrt{c+dx^3}} + \frac{17d}{384c^3x^3(8c-dx^3)\sqrt{c+dx^3}} + \frac{13d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{497664c^{11/2}} - \frac{33d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{11/2}}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^7(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{\frac{12\sqrt{c}(864c^3-1836c^2dx^3-5107cd^2x^6+665d^3x^9)}{x^6(-8c+dx^3)\sqrt{c+dx^3}} + 13d^2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 8019d^2\text{arc}}{497664c^{11/2}}$$

```
[In] Integrate[1/(x^7*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] ((12*Sqrt[c]*(864*c^3 - 1836*c^2*d*x^3 - 5107*c*d^2*x^6 + 665*d^3*x^9))/(x^
6*(-8*c + d*x^3)*Sqrt[c + d*x^3]) + 13*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[
c])] - 8019*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(497664*c^(11/2))
```

Maple [A] (verified)

Time = 4.50 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{\sqrt{dx^3+c}\,(-3dx^3+c)}{384c^5x^6}+\frac{d^2\left(-\frac{33\,\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)}{8\sqrt{c}}+\frac{512}{243\sqrt{dx^3+c}}+\frac{35\,\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{5832\sqrt{c}}+\frac{c\left(-\frac{\sqrt{dx^3+c}}{c(dx^3-8c)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{}}{}\right.}{3c}\right.}{486}\right.}{256c^5}$
pseudoelliptic	$\frac{d^2\left(-\frac{99\,\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)d^2x^6-48dx^3\sqrt{dx^3+c}\,\sqrt{c}+16\sqrt{dx^3+c}\,c^{\frac{3}{2}}}{2048d^2x^6c^{\frac{11}{2}}}+\frac{\frac{\sqrt{dx^3+c}}{-dx^3+8c}+\frac{13\,\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{4\sqrt{c}}}{41472c^5}+\frac{2}{81c^5\sqrt{dx^3+c}}\right)}{3}$
default	$\frac{-\frac{\sqrt{dx^3+c}}{6c^2x^6}+\frac{7d\sqrt{dx^3+c}}{12c^3x^3}+\frac{2d^2}{3c^3\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{5d^2\,\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)}{4c^{\frac{7}{2}}}}{64c^2}+\frac{d\left(-\frac{\sqrt{dx^3+c}}{3c^2x^3}-\frac{2d}{3c^2\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{d\,\operatorname{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right)}{256c^3}$
elliptic	Expression too large to display

```
[In] int(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/384*(d*x^3+c)^(1/2)*(-3*d*x^3+c)/c^5/x^6+1/256/c^5*d^2*(-33/8*arctanh((d
*x^3+c)^(1/2)/c^(1/2))/c^(1/2)+512/243/(d*x^3+c)^(1/2)+35/5832*arctanh(1/3*
(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)+1/486*c*(-(d*x^3+c)^(1/2)/c/(d*x^3-8*c)+1/
3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.15

$$\int \frac{1}{x^7\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\left[\frac{13\left(d^4x^{12}-7\,cd^3x^9-8\,c^2d^2x^6\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+8019\left(d^4\right.}{}\right.$$

```
[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/995328*(13*(d^4*x^12 - 7*c*d^3*x^9 - 8*c^2*d^2*x^6)*sqrt(c)*log((d*x^3 +
 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 8019*(d^4*x^12 - 7*c*d^
3*x^9 - 8*c^2*d^2*x^6)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c
)/x^3) + 24*(665*c*d^3*x^9 - 5107*c^2*d^2*x^6 - 1836*c^3*d*x^3 + 864*c^4)*s
qrt(d*x^3 + c))/(c^6*d^2*x^12 - 7*c^7*d*x^9 - 8*c^8*x^6), 1/497664*(8019*(d
^4*x^12 - 7*c*d^3*x^9 - 8*c^2*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt
(-c)/c) - 13*(d^4*x^12 - 7*c*d^3*x^9 - 8*c^2*d^2*x^6)*sqrt(-c)*arctan(1/3*s
qrt(d*x^3 + c)*sqrt(-c)/c) + 12*(665*c*d^3*x^9 - 5107*c^2*d^2*x^6 - 1836*c^
```

```
3*d*x^3 + 864*c^4)*sqrt(d*x^3 + c))/(c^6*d^2*x^12 - 7*c^7*d*x^9 - 8*c^8*x^6
)]
```

Sympy [F]

$$\int \frac{1}{x^7 (8c - dx^3)^2 (c + dx^3)^{3/2}} \, dx = \int \frac{1}{x^7 (-8c + dx^3)^2 (c + dx^3)^{\frac{3}{2}}} \, dx$$

```
[In] integrate(1/x**7/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(1/(x**7*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^7 (8c - dx^3)^2 (c + dx^3)^{3/2}} \, dx = \int \frac{1}{(dx^3 + c)^{\frac{3}{2}} (dx^3 - 8\,c)^2 x^7} \, dx$$

```
[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^7), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^7 (8c - dx^3)^2 (c + dx^3)^{3/2}} \, dx = \frac{33\,d^2 \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{2048\,\sqrt{-c}c^5} - \frac{13\,d^2 \arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{497664\,\sqrt{-c}c^5} + \frac{341\,(dx^3 + c)d^2 - 3072\,cd^2}{41472\,\left((dx^3 + c)^{\frac{3}{2}} - 9\,\sqrt{dx^3 + c}c\right)c^5} + \frac{3\,(dx^3 + c)^{\frac{3}{2}} d^2 - 4\,\sqrt{dx^3 + c}cd^2}{384\,c^5 d^2 x^6}$$

```
[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] 33/2048*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^5) - 13/497664*d^2
*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^5) + 1/41472*(341*(d*x^3
+ c)*d^2 - 3072*c*d^2)/(((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*c^5) + 1/
384*(3*(d*x^3 + c)^(3/2)*d^2 - 4*sqrt(d*x^3 + c)*c*d^2)/(c^5*d^2*x^6)
```

Mupad [B] (verification not implemented)

Time = 8.82 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^7 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{\frac{2\,d^2}{9\,c^2}-\frac{10373\,d^2\,\left(d\,x^3+c\right)}{13824\,c^3}+\frac{3551\,d^2\,{\left(d\,x^3+c\right)}^2}{6912\,c^4}-\frac{665\,d^2\,{\left(d\,x^3+c\right)}^3}{13824\,c^5}}{33\,c\,{\left(d\,x^3+c\right)}^{5/2}-3\,{\left(d\,x^3+c\right)}^{7/2}+27\,c^3\,\sqrt{d\,x^3+c}-57\,c^2\,{\left(d\,x^3+c\right)}^{3/2}} + \frac{d^2\,\left(\mathrm{atanh}\left(\frac{c^5\,\sqrt{d\,x^3+c}}{\sqrt{c^{11}}}\right)\,1{}\mathrm{i}-\frac{\mathrm{atanh}\left(\frac{c^5\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^{11}}}\right)\,13{}\mathrm{i}}{8019}\right)\,33{}\mathrm{i}}{2048\,\sqrt{c^{11}}}$$

```
[In] int(1/(x^7*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] ((2*d^2)/(9*c^2) - (10373*d^2*(c + d*x^3))/(13824*c^3) + (3551*d^2*(c + d*x
^3)^2)/(6912*c^4) - (665*d^2*(c + d*x^3)^3)/(13824*c^5))/(33*c*(c + d*x^3)^
(5/2) - 3*(c + d*x^3)^(7/2) + 27*c^3*(c + d*x^3)^(1/2) - 57*c^2*(c + d*x^3)
^(3/2)) + (d^2*(atanh((c^5*(c + d*x^3)^(1/2))/(c^11)^(1/2))*1i - (atanh((c^
5*(c + d*x^3)^(1/2))/(3*(c^11)^(1/2)))*13i)/8019)*33i)/(2048*(c^11)^(1/2))
```

3.449 $\int \frac{x^7}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 668

$$\begin{aligned}
\int \frac{x^7}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx &= -\frac{2x^2}{81cd^2\sqrt{c+dx^3}}+\frac{8x^2}{27d^2(8c-dx^3)\sqrt{c+dx^3}} \\
&+\frac{2\sqrt{c+dx^3}}{81cd^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{4\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{81\sqrt{3}c^{5/6}d^{8/3}} \\
&-\frac{4\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{243c^{5/6}d^{8/3}}+\frac{4\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/6}d^{8/3}} \\
&-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{27\ 3^{3/4}c^{2/3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+\frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{81\sqrt[4]{3}c^{2/3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}$$

```
[Out] -4/243*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(5/6)/d
^(8/3)+4/243*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/6)/d^(8/3)+4/243*arc
```

```
tan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(5/6)/d^(8/3)*3^
(1/2)-2/81*x^2/c/d^2/(d*x^3+c)^(1/2)+8/27*x^2/d^2/(-d*x^3+8*c)/(d*x^3+c)^(1
/2)+2/81*(d*x^3+c)^(1/2)/c/d^(8/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+2/243*(c
^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1
/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)
*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(2/3)/d^(8/3)/(d*x
^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)
^(1/2)-1/81*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(
d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c
^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1
/2)*3^(1/4)/c^(2/3)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d
^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 668, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {481, 593, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{x^7}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)\right.}{81\sqrt[4]{3}c^{2/3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{27\ 3^{3/4}c^{2/3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{4\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{81\sqrt{3}c^{5/6}d^{8/3}} - \frac{4\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{243c^{5/6}d^{8/3}} + \frac{4\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/6}d^{8/3}} + \frac{2\sqrt{c+dx^3}}{81cd^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{8x^2}{27d^2(8c-dx^3)\sqrt{c+dx^3}} - \frac{2x^2}{81cd^2\sqrt{c+dx^3}}$$

```
[In] Int[x^7/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*x^2)/(81*c*d^2*Sqrt[c + d*x^3]) + (8*x^2)/(27*d^2*(8*c - d*x^3)*Sqrt[c
+ d*x^3]) + (2*Sqrt[c + d*x^3])/(81*c*d^(8/3)*((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)) + (4*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]
```

```
])/(81*Sqrt[3]*c^(5/6)*d^(8/3)) - (4*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(
1/6)*Sqrt[c + d*x^3])])/(243*c^(5/6)*d^(8/3)) + (4*ArcTanh[Sqrt[c + d*x^3]/
(3*Sqrt[c])])/(243*c^(5/6)*d^(8/3)) - (Sqrt[2 - Sqrt[3]]*(c^(1/3) + d^(1/3)
*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(3/4)*c^(2/3)*d^(8/
3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)
^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(
1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)], -7 - 4*Sqrt[3]])/(81*3^(1/4)*c^(2/3)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/
3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
```

```
/; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\text{integral} = \frac{8x^2}{27d^2(8c-dx^3)\sqrt{c+dx^3}} - \frac{\int \frac{x(16c^2+7cdx^3)}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx}{27cd^2}$$

$$= -\frac{2x^2}{81cd^2\sqrt{c+dx^3}} + \frac{8x^2}{27d^2(8c-dx^3)\sqrt{c+dx^3}} + \frac{2\int \frac{x\left(-72c^3d-\frac{9}{2}c^2d^2x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{729c^3d^3}$$

$$= -\frac{2x^2}{81cd^2\sqrt{c+dx^3}} + \frac{8x^2}{27d^2(8c-dx^3)\sqrt{c+dx^3}} + \frac{2\int \left(\frac{9c^2dx}{2\sqrt{c+dx^3}} - \frac{108c^3dx}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{729c^3d^3}$$

$$= -\frac{2x^2}{81cd^2\sqrt{c+dx^3}} + \frac{8x^2}{27d^2(8c-dx^3)\sqrt{c+dx^3}} - \frac{8\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{27d^2} + \frac{\int \frac{x}{\sqrt{c+dx^3}}\,dx}{81cd^2}$$

$$= -\frac{2x^2}{81cd^2\sqrt{c+dx^3}} + \frac{8x^2}{27d^2(8c-dx^3)\sqrt{c+dx^3}} + \frac{2\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}dx}{81cd^3} + \frac{\int \frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}dx}{81cd^{7/3}} - \frac{2\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}dx}{81c^{2/3}d^{7/3}} - \frac{(1-\sqrt{3})\int \frac{1}{\sqrt{c+dx^3}}dx}{81c^{2/3}d^{7/3}} + \frac{2\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}dx}{27\sqrt[3]{c}d^{5/3}}$$

$$= -\frac{2x^2}{81cd^2\sqrt{c+dx^3}} + \frac{8x^2}{27d^2(8c-dx^3)\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{81cd^{8/3}\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{(1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{27\ 3^{3/4}c^{2/3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{(1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{81\sqrt[4]{3}c^{2/3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{4\text{Subst}\left(\int \frac{1}{9-cx^2}dx, x, \frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{81\sqrt[3]{c}d^{8/3}} + \frac{2\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}dx, x, x^3\right)}{81\sqrt[3]{c}d^{5/3}} - \frac{8\text{Subst}\left(\int \frac{1}{-\frac{2d^2}{c}-6d^2x^2}dx, x, \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{81c^{4/3}d^{2/3}}$$

$$= -\frac{2x^2}{81cd^2\sqrt{c+dx^3}} + \frac{8x^2}{27d^2(8c-dx^3)\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{81cd^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{4\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{81\sqrt{3}c^{5/6}d^{8/3}} - \frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{243c^{5/6}d^{8/3}} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{27\ 3^{3/4}c^{2/3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{81\sqrt[4]{3}c^{2/3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{4\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{81\sqrt[3]{c}d^{8/3}}$$

$$= -\frac{2x^2}{81cd^2\sqrt{c+dx^3}} + \frac{8x^2}{27d^2(8c-dx^3)\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{81cd^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{4\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{81\sqrt{3}c^{5/6}d^{8/3}} - \frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{243c^{5/6}d^{8/3}} + \frac{4\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/6}d^{8/3}} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{27\ 3^{3/4}c^{2/3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{81\sqrt[4]{3}c^{2/3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.25

$$\int \frac{x^7}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{80cx^2(4c+dx^3)+40cx^2(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\right.}{3240c^2d^2\,(8c-dx^3)}$$

```
[In] Integrate[x^7/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (80*c*x^2*(4*c + d*x^3) + 40*c*x^2*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*Appel
lF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(-8*c + d*x^3)*S
qrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])
/(3240*c^2*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.59 (sec) , antiderivative size = 910, normalized size of antiderivative = 1.36

method	result	size
elliptic	Expression too large to display	910
default	Expression too large to display	2256

```
[In] int(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 8/243*x^2/c/d^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+2/243/d^2*x^2/c/((x^3+c/d)*d)^
(1/2)-2/243*I/d^3/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)
^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(
x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/
(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)
^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+8/729*I/d^5/c
*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)
^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3
*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1
/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(
I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(
```

```
-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(
-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/
2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*
_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)
^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alph
a=RootOf(_Z^3*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.60 (sec) , antiderivative size = 2681, normalized size of antiderivative = 4.01

$$\int \frac{x^7}{(8c-dx^3)^2 (c+dx^3)^{3/2}} dx = \text{Too large to display}$$

```
[In] integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/729*(18*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(d)*weierstrassZeta(0, -4*c/d,
 weierstrassPInverse(0, -4*c/d, x)) + (c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^
3 + sqrt(-3)*(c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3))*(1/(c^5*d^16))^(1/6)*
log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^4*d^13*x^7
 + 64*c^5*d^12*x^4 + 32*c^6*d^11*x + sqrt(-3)*(5*c^4*d^13*x^7 + 64*c^5*d^12
*x^4 + 32*c^6*d^11*x))*(1/(c^5*d^16))^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^5*d
^15*x^5 + 32*c^6*d^14*x^2 - sqrt(-3)*(5*c^5*d^15*x^5 + 32*c^6*d^14*x^2))*(1
/(c^5*d^16))^(5/6) - 2*(7*c^3*d^10*x^6 + 152*c^4*d^9*x^3 + 64*c^5*d^8)*sqrt
(1/(c^5*d^16)) + (c*d^5*x^7 + 80*c^2*d^4*x^4 + 160*c^3*d^3*x + sqrt(-3)*(c*
d^5*x^7 + 80*c^2*d^4*x^4 + 160*c^3*d^3*x))*(1/(c^5*d^16))^(1/6)) - 9*(c^2*d
^8*x^8 + 38*c^3*d^7*x^5 + 64*c^4*d^6*x^2 - sqrt(-3)*(c^2*d^8*x^8 + 38*c^3*d
^7*x^5 + 64*c^4*d^6*x^2))*(1/(c^5*d^16))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 1
92*c^2*d*x^3 - 512*c^3)) - (c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3 + sqrt(-3
)*(c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3))*(1/(c^5*d^16))^(1/6)*log((d^3*x^
9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^4*d^13*x^7 + 64*c^5*d
^12*x^4 + 32*c^6*d^11*x + sqrt(-3)*(5*c^4*d^13*x^7 + 64*c^5*d^12*x^4 + 32*c
^6*d^11*x))*(1/(c^5*d^16))^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^5*d^15*x^5 + 3
2*c^6*d^14*x^2 - sqrt(-3)*(5*c^5*d^15*x^5 + 32*c^6*d^14*x^2))*(1/(c^5*d^16)
)^(5/6) - 2*(7*c^3*d^10*x^6 + 152*c^4*d^9*x^3 + 64*c^5*d^8)*sqrt(1/(c^5*d^1
6)) + (c*d^5*x^7 + 80*c^2*d^4*x^4 + 160*c^3*d^3*x + sqrt(-3)*(c*d^5*x^7 + 8
0*c^2*d^4*x^4 + 160*c^3*d^3*x))*(1/(c^5*d^16))^(1/6)) - 9*(c^2*d^8*x^8 + 38
*c^3*d^7*x^5 + 64*c^4*d^6*x^2 - sqrt(-3)*(c^2*d^8*x^8 + 38*c^3*d^7*x^5 + 64
*c^4*d^6*x^2))*(1/(c^5*d^16))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^
3 - 512*c^3)) + (c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3 - sqrt(-3)*(c*d^5*x^
6 - 7*c^2*d^4*x^3 - 8*c^3*d^3))*(1/(c^5*d^16))^(1/6)*log((d^3*x^9 + 318*c*d
^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^4*d^13*x^7 + 64*c^5*d^12*x^4 + 3
2*c^6*d^11*x - sqrt(-3)*(5*c^4*d^13*x^7 + 64*c^5*d^12*x^4 + 32*c^6*d^11*x))
```

```
*(1/(c^5*d^16))^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^5*d^15*x^5 + 32*c^6*d^14*
x^2 + sqrt(-3)*(5*c^5*d^15*x^5 + 32*c^6*d^14*x^2))*(1/(c^5*d^16))^(5/6) - 2
*(7*c^3*d^10*x^6 + 152*c^4*d^9*x^3 + 64*c^5*d^8)*sqrt(1/(c^5*d^16)) + (c*d^
5*x^7 + 80*c^2*d^4*x^4 + 160*c^3*d^3*x - sqrt(-3)*(c*d^5*x^7 + 80*c^2*d^4*x
^4 + 160*c^3*d^3*x))*(1/(c^5*d^16))^(1/6)) - 9*(c^2*d^8*x^8 + 38*c^3*d^7*x^
5 + 64*c^4*d^6*x^2 + sqrt(-3)*(c^2*d^8*x^8 + 38*c^3*d^7*x^5 + 64*c^4*d^6*x^
2))*(1/(c^5*d^16))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3
)) - (c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3 - sqrt(-3)*(c*d^5*x^6 - 7*c^2*d
^4*x^3 - 8*c^3*d^3))*(1/(c^5*d^16))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 12
00*c^2*d*x^3 + 640*c^3 - 9*(5*c^4*d^13*x^7 + 64*c^5*d^12*x^4 + 32*c^6*d^11*
x - sqrt(-3)*(5*c^4*d^13*x^7 + 64*c^5*d^12*x^4 + 32*c^6*d^11*x))*(1/(c^5*d^
16))^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^5*d^15*x^5 + 32*c^6*d^14*x^2 + sqrt(
-3)*(5*c^5*d^15*x^5 + 32*c^6*d^14*x^2))*(1/(c^5*d^16))^(5/6) - 2*(7*c^3*d^1
0*x^6 + 152*c^4*d^9*x^3 + 64*c^5*d^8)*sqrt(1/(c^5*d^16)) + (c*d^5*x^7 + 80*
c^2*d^4*x^4 + 160*c^3*d^3*x - sqrt(-3)*(c*d^5*x^7 + 80*c^2*d^4*x^4 + 160*c^
3*d^3*x))*(1/(c^5*d^16))^(1/6)) - 9*(c^2*d^8*x^8 + 38*c^3*d^7*x^5 + 64*c^4*
d^6*x^2 + sqrt(-3)*(c^2*d^8*x^8 + 38*c^3*d^7*x^5 + 64*c^4*d^6*x^2))*(1/(c^5
*d^16))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 2*(c*d
^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3)*(1/(c^5*d^16))^(1/6)*log((d^3*x^9 + 318
*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(5*c^4*d^13*x^7 + 64*c^5*d^12*x^
4 + 32*c^6*d^11*x)*(1/(c^5*d^16))^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^5*d^15*
x^5 + 32*c^6*d^14*x^2)*(1/(c^5*d^16))^(5/6) + (7*c^3*d^10*x^6 + 152*c^4*d^9
*x^3 + 64*c^5*d^8)*sqrt(1/(c^5*d^16)) + (c*d^5*x^7 + 80*c^2*d^4*x^4 + 160*c
^3*d^3*x)*(1/(c^5*d^16))^(1/6)) + 18*(c^2*d^8*x^8 + 38*c^3*d^7*x^5 + 64*c^4
*d^6*x^2)*(1/(c^5*d^16))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 5
12*c^3)) - 2*(c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3)*(1/(c^5*d^16))^(1/6)*l
og((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(5*c^4*d^13*x^7
 + 64*c^5*d^12*x^4 + 32*c^6*d^11*x)*(1/(c^5*d^16))^(2/3) - 6*sqrt(d*x^3 + c
)*(6*(5*c^5*d^15*x^5 + 32*c^6*d^14*x^2)*(1/(c^5*d^16))^(5/6) + (7*c^3*d^10*
x^6 + 152*c^4*d^9*x^3 + 64*c^5*d^8)*sqrt(1/(c^5*d^16)) + (c*d^5*x^7 + 80*c^
2*d^4*x^4 + 160*c^3*d^3*x)*(1/(c^5*d^16))^(1/6)) + 18*(c^2*d^8*x^8 + 38*c^3
*d^7*x^5 + 64*c^4*d^6*x^2)*(1/(c^5*d^16))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 +
192*c^2*d*x^3 - 512*c^3)) + 18*(d^2*x^5 + 4*c*d*x^2)*sqrt(d*x^3 + c))/(c*d^
5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3)
```

Sympy [F]

$$\int \frac{x^7}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx=\int \frac{x^7}{(-8c+dx^3)^2\,(c+dx^3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**7/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(x**7/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^7}{(8c-dx^3)^2 (c+dx^3)^{3/2}} dx = \int \frac{x^7}{(dx^3+c)^{\frac{3}{2}} (dx^3-8c)^2} dx$$

```
[In] integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(x^7/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Giac [F]

$$\int \frac{x^7}{(8c-dx^3)^2 (c+dx^3)^{3/2}} dx = \int \frac{x^7}{(dx^3+c)^{\frac{3}{2}} (dx^3-8c)^2} dx$$

```
[In] integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(x^7/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(8c-dx^3)^2 (c+dx^3)^{3/2}} dx = \int \frac{x^7}{(d\,x^3+c)^{3/2} (8\,c-d\,x^3)^2} dx$$

```
[In] int(x^7/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
[Out] int(x^7/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

3.450 $\int \frac{x^4}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 671

$$\begin{aligned}
\int \frac{x^4}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx &= -\frac{x^2}{81c^2d\sqrt{c+dx^3}} \\
&+ \frac{x^2}{27cd(8c-dx^3)\sqrt{c+dx^3}} + \frac{\sqrt{c+dx^3}}{81c^2d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} \\
&- \frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{81\sqrt{3}c^{11/6}d^{5/3}} + \frac{\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{243c^{11/6}d^{5/3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{11/6}d^{5/3}} \\
&- \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{54\ 3^{3/4}c^{5/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+ \frac{\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{81\sqrt[4]{3}c^{5/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}$$

```
[Out] 1/243*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(11/6)/d
^(5/3)-1/243*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)/d^(5/3)-1/243*ar
ctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(11/6)/d^(5/3)*
```

```
3^(1/2)-1/81*x^2/c^2/d/(d*x^3+c)^(1/2)+1/27*x^2/c/d/(-d*x^3+8*c)/(d*x^3+c)^
(1/2)+1/81*(d*x^3+c)^(1/2)/c^2/d^(5/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/24
3*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+
c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(
2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(5/3)/d^(5/3)/
(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))
)^2)^(1/2)-1/162*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2
)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2)
)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^
2)^(1/2)*3^(1/4)/c^(5/3)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*
x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 671, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.481, Rules used = {482, 593, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{x^4}{(8c-dx^3)^2 (c+dx^3)^{3/2}} dx = \frac{\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right.\right.}{81\sqrt[4]{3}c^{5/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{54\ 3^{3/4}c^{5/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{81\sqrt{3}c^{11/6}d^{5/3}}+\frac{\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{243c^{11/6}d^{5/3}}-\frac{\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{11/6}d^{5/3}}$$

$$+\frac{\sqrt{c+dx^3}}{81c^2d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{x^2}{81c^2d\sqrt{c+dx^3}}+\frac{x^2}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

```
[In] Int[x^4/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/81*x^2/(c^2*d*Sqrt[c + d*x^3]) + x^2/(27*c*d*(8*c - d*x^3)*Sqrt[c + d*x^
3]) + Sqrt[c + d*x^3]/(81*c^2*d^(5/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x))
- ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]]/(81*Sqrt[
3]*c^(11/6)*d^(5/3)) + ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c +
```

```
d*x^3])]/(243*c^(11/6)*d^(5/3)) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(243
*c^(11/6)*d^(5/3)) - (Sqrt[2 - Sqrt[3]]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)
 - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(54*3^(3/4)*c^(5/3)*d^(5/3)*Sqrt[(c^(1/3)*
(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^
3]) + (Sqrt[2]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^
(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sq
rt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sq
rt[3]])/(81*3^(1/4)*c^(5/3)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\text{integral} = \frac{x^2}{27cd\,(8c-dx^3)\sqrt{c+dx^3}} - \frac{\int \frac{x\left(2c-\frac{5dx^3}{2}\right)}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx}{27cd}$$

$$= -\frac{x^2}{81c^2d\sqrt{c+dx^3}} + \frac{x^2}{27cd\,(8c-dx^3)\sqrt{c+dx^3}} + \frac{2\int \frac{x\left(45c^2d-\frac{9}{4}cd^2x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{729c^3d^2}$$

$$= -\frac{x^2}{81c^2d\sqrt{c+dx^3}} + \frac{x^2}{27cd\,(8c-dx^3)\sqrt{c+dx^3}} + \frac{2\int \left(\frac{9cdx}{4\sqrt{c+dx^3}} + \frac{27c^2dx}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{729c^3d^2}$$

$$= -\frac{x^2}{81c^2d\sqrt{c+dx^3}} + \frac{x^2}{27cd\,(8c-dx^3)\sqrt{c+dx^3}} + \frac{\int \frac{x}{\sqrt{c+dx^3}}\,dx}{162c^2d} + \frac{2\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{27cd}$$

$$= -\frac{x^2}{81c^2d\sqrt{c+dx^3}} + \frac{x^2}{27cd(8c-dx^3)\sqrt{c+dx^3}} - \frac{\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{162c^2d^2} + \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{162c^2d^{4/3}}$$

$$+ \frac{\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{162c^{5/3}d^{4/3}} - \frac{\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{162c^{5/3}d^{4/3}} - \frac{\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{54c^{4/3}d^{2/3}}$$

$$= -\frac{x^2}{81c^2d\sqrt{c+dx^3}} + \frac{x^2}{27cd(8c-dx^3)\sqrt{c+dx^3}} + \frac{\sqrt{c+dx^3}}{81c^2d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$- \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{54\ 3^{3/4}c^{5/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{81\sqrt[4]{3}c^{5/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{\text{Subst}\left(\int \frac{1}{9-cx^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{81c^{4/3}d^{5/3}} - \frac{\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{162c^{4/3}d^{2/3}}$$

$$+ \frac{\left(2\sqrt[3]{d}\right)\text{Subst}\left(\int \frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{81c^{7/3}}$$

$$= -\frac{x^2}{81c^2d\sqrt{c+dx^3}} + \frac{x^2}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\sqrt{c+dx^3}}{81c^2d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{81\sqrt{3}c^{11/6}d^{5/3}} + \frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{243c^{11/6}d^{5/3}} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{54\ 3^{3/4}c^{5/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{81\sqrt[4]{3}c^{5/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{81c^{4/3}d^{5/3}}$$

$$= -\frac{x^2}{81c^2d\sqrt{c+dx^3}} + \frac{x^2}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\sqrt{c+dx^3}}{81c^2d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{81\sqrt{3}c^{11/6}d^{5/3}} + \frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{243c^{11/6}d^{5/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{11/6}d^{5/3}} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{54\ 3^{3/4}c^{5/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{81\sqrt[4]{3}c^{5/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.25

$$\int \frac{x^4}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \frac{80cx^2(-5c+dx^3)+50cx^2(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+\cdots}{6480c^3d\,(8c-dx^3)\sqrt{c+dx^3}}$$

```
[In] Integrate[x^4/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (80*c*x^2*(-5*c + d*x^3) + 50*c*x^2*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*Appel
lF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(-8*c + d*x^3)*S
qrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])
/(6480*c^3*d*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.62 (sec) , antiderivative size = 910, normalized size of antiderivative = 1.36

method	result	size
elliptic	Expression too large to display	910
default	Expression too large to display	1789

```
[In] int(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/243*x^2/c^2/d*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-2/243/d*x^2/c^2/((x^3+c/d)*d)^
(1/2)-1/243*I/c^2/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^
2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I
*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)
^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3
)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^
2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/
(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-2/729*I/c^2
/d^4*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))
/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/
2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d
```

```
^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1
/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*
d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_
alpha=RootOf(_Z^3*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.65 (sec) , antiderivative size = 2723, normalized size of antiderivative = 4.06

$$\int \frac{x^4}{(8c-dx^3)^2 (c+dx^3)^{3/2}} dx = \text{Too large to display}$$

```
[In] integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2916*(36*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(d)*weierstrassZeta(0, -4*c/d
, weierstrassPInverse(0, -4*c/d, x)) - (c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4
*d^2 + sqrt(-3)*(c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2))*(1/(c^11*d^10))^
(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^8*d^
9*x^7 + 64*c^9*d^8*x^4 + 32*c^10*d^7*x + sqrt(-3)*(5*c^8*d^9*x^7 + 64*c^9*d
^8*x^4 + 32*c^10*d^7*x))*(1/(c^11*d^10))^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^
10*d^10*x^5 + 32*c^11*d^9*x^2 - sqrt(-3)*(5*c^10*d^10*x^5 + 32*c^11*d^9*x^2
))*(1/(c^11*d^10))^(5/6) - 2*(7*c^6*d^7*x^6 + 152*c^7*d^6*x^3 + 64*c^8*d^5)
*sqrt(1/(c^11*d^10)) + (c^2*d^4*x^7 + 80*c^3*d^3*x^4 + 160*c^4*d^2*x + sqrt
(-3)*(c^2*d^4*x^7 + 80*c^3*d^3*x^4 + 160*c^4*d^2*x))*(1/(c^11*d^10))^(1/6))
 - 9*(c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2 - sqrt(-3)*(c^4*d^6*x^8
 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2))*(1/(c^11*d^10))^(1/3))/(d^3*x^9 - 24*c
*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4
*d^2 + sqrt(-3)*(c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2))*(1/(c^11*d^10))^
(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^8*d^
9*x^7 + 64*c^9*d^8*x^4 + 32*c^10*d^7*x + sqrt(-3)*(5*c^8*d^9*x^7 + 64*c^9*d
^8*x^4 + 32*c^10*d^7*x))*(1/(c^11*d^10))^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^
10*d^10*x^5 + 32*c^11*d^9*x^2 - sqrt(-3)*(5*c^10*d^10*x^5 + 32*c^11*d^9*x^2
))*(1/(c^11*d^10))^(5/6) - 2*(7*c^6*d^7*x^6 + 152*c^7*d^6*x^3 + 64*c^8*d^5)
*sqrt(1/(c^11*d^10)) + (c^2*d^4*x^7 + 80*c^3*d^3*x^4 + 160*c^4*d^2*x + sqrt
(-3)*(c^2*d^4*x^7 + 80*c^3*d^3*x^4 + 160*c^4*d^2*x))*(1/(c^11*d^10))^(1/6))
 - 9*(c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2 - sqrt(-3)*(c^4*d^6*x^8
 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2))*(1/(c^11*d^10))^(1/3))/(d^3*x^9 - 24*c
*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4
*d^2 - sqrt(-3)*(c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2))*(1/(c^11*d^10))^
(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^8*d^
9*x^7 + 64*c^9*d^8*x^4 + 32*c^10*d^7*x - sqrt(-3)*(5*c^8*d^9*x^7 + 64*c^9*d
```

```
^8*x^4 + 32*c^10*d^7*x))*(1/(c^11*d^10))^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^
10*d^10*x^5 + 32*c^11*d^9*x^2 + sqrt(-3)*(5*c^10*d^10*x^5 + 32*c^11*d^9*x^2
))*(1/(c^11*d^10))^(5/6) - 2*(7*c^6*d^7*x^6 + 152*c^7*d^6*x^3 + 64*c^8*d^5)
*sqrt(1/(c^11*d^10)) + (c^2*d^4*x^7 + 80*c^3*d^3*x^4 + 160*c^4*d^2*x - sqrt
(-3)*(c^2*d^4*x^7 + 80*c^3*d^3*x^4 + 160*c^4*d^2*x))*(1/(c^11*d^10))^(1/6))
 - 9*(c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2 + sqrt(-3)*(c^4*d^6*x^8
 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2))*(1/(c^11*d^10))^(1/3))/(d^3*x^9 - 24*c
*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4
*d^2 - sqrt(-3)*(c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2))*(1/(c^11*d^10))^
(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^8*d^
9*x^7 + 64*c^9*d^8*x^4 + 32*c^10*d^7*x - sqrt(-3)*(5*c^8*d^9*x^7 + 64*c^9*d
^8*x^4 + 32*c^10*d^7*x))*(1/(c^11*d^10))^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^
10*d^10*x^5 + 32*c^11*d^9*x^2 + sqrt(-3)*(5*c^10*d^10*x^5 + 32*c^11*d^9*x^2
))*(1/(c^11*d^10))^(5/6) - 2*(7*c^6*d^7*x^6 + 152*c^7*d^6*x^3 + 64*c^8*d^5)
*sqrt(1/(c^11*d^10)) + (c^2*d^4*x^7 + 80*c^3*d^3*x^4 + 160*c^4*d^2*x - sqrt
(-3)*(c^2*d^4*x^7 + 80*c^3*d^3*x^4 + 160*c^4*d^2*x))*(1/(c^11*d^10))^(1/6))
 - 9*(c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2 + sqrt(-3)*(c^4*d^6*x^8
 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2))*(1/(c^11*d^10))^(1/3))/(d^3*x^9 - 24*c
*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*(c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c
^4*d^2)*(1/(c^11*d^10))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3
 + 640*c^3 + 18*(5*c^8*d^9*x^7 + 64*c^9*d^8*x^4 + 32*c^10*d^7*x)*(1/(c^11*d
^10))^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^10*d^10*x^5 + 32*c^11*d^9*x^2)*(1/(
c^11*d^10))^(5/6) + (7*c^6*d^7*x^6 + 152*c^7*d^6*x^3 + 64*c^8*d^5)*sqrt(1/(
c^11*d^10)) + (c^2*d^4*x^7 + 80*c^3*d^3*x^4 + 160*c^4*d^2*x)*(1/(c^11*d^10)
)^(1/6)) + 18*(c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c^6*d^4*x^2)*(1/(c^11*d^10
))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 2*(c^2*d^4*
x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2)*(1/(c^11*d^10))^(1/6)*log((d^3*x^9 + 318*c
*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(5*c^8*d^9*x^7 + 64*c^9*d^8*x^4 +
32*c^10*d^7*x)*(1/(c^11*d^10))^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^10*d^10*x^
5 + 32*c^11*d^9*x^2)*(1/(c^11*d^10))^(5/6) + (7*c^6*d^7*x^6 + 152*c^7*d^6*x
^3 + 64*c^8*d^5)*sqrt(1/(c^11*d^10)) + (c^2*d^4*x^7 + 80*c^3*d^3*x^4 + 160*
c^4*d^2*x)*(1/(c^11*d^10))^(1/6)) + 18*(c^4*d^6*x^8 + 38*c^5*d^5*x^5 + 64*c
^6*d^4*x^2)*(1/(c^11*d^10))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3
- 512*c^3)) + 36*(d^2*x^5 - 5*c*d*x^2)*sqrt(d*x^3 + c))/(c^2*d^4*x^6 - 7*c^
3*d^3*x^3 - 8*c^4*d^2)
```

Sympy [F]

$$\int \frac{x^4}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx=\int \frac{x^4}{(-8c+dx^3)^2(c+dx^3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**4/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(x**4/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^4}{(8c - dx^3)^2 (c + dx^3)^{3/2}} \, dx = \int \frac{x^4}{(dx^3 + c)^{\frac{3}{2}} (dx^3 - 8\,c)^2} \, dx$$

```
[In] integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(x^4/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Giac [F]

$$\int \frac{x^4}{(8c - dx^3)^2 (c + dx^3)^{3/2}} \, dx = \int \frac{x^4}{(dx^3 + c)^{\frac{3}{2}} (dx^3 - 8\,c)^2} \, dx$$

```
[In] integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(x^4/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(8c - dx^3)^2 (c + dx^3)^{3/2}} \, dx = \int \frac{x^4}{(d\,x^3 + c)^{3/2} \, (8\,c - d\,x^3)^2} \, dx$$

```
[In] int(x^4/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
[Out] int(x^4/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

3.451 $\int \frac{x}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 665

$$\int \frac{x}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{5x^2}{648c^3\sqrt{c+dx^3}}+\frac{x^2}{216c^2(8c-dx^3)\sqrt{c+dx^3}} - \frac{5\sqrt{c+dx^3}}{648c^3d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{5\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1296\sqrt{3}c^{17/6}d^{2/3}} + \frac{5\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{3888c^{17/6}d^{2/3}} - \frac{5\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3888c^{17/6}d^{2/3}} + \frac{5\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{432\ 3^{3/4}c^{8/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{5\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{324\sqrt{2}\sqrt[4]{3}c^{8/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 5/3888*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(17/6)/
d^(2/3)-5/3888*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)/d^(2/3)-5/3888
```

```
*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(17/6)/d^(2/
3)*3^(1/2)+5/648*x^2/c^3/(d*x^3+c)^(1/2)+1/216*x^2/c^2/(-d*x^3+8*c)/(d*x^3+
c)^(1/2)-5/648*(d*x^3+c)^(1/2)/c^3/d^(2/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-
5/1944*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/
3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3
)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(8/3)/d^(2/3)*2^(
1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(
1/2)))^2)^(1/2)+5/1296*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-
3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2
^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1
/2)))^2)^(1/2)*3^(1/4)/c^(8/3)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^
(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 665, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {483, 593, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$
\begin{aligned}
&\int \frac{x}{(8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx = \\
&-\frac{5\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{324\sqrt{2}\sqrt[4]{3}c^{8/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+\frac{5\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{432\ 3^{3/4}c^{8/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&-\frac{5\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1296\sqrt{3}c^{17/6}d^{2/3}}+\frac{5\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{3888c^{17/6}d^{2/3}}-\frac{5\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3888c^{17/6}d^{2/3}} \\
&-\frac{5\sqrt{c+dx^3}}{648c^3d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{5x^2}{648c^3\sqrt{c+dx^3}}+\frac{x^2}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}}
\end{aligned}
$$

```
[In] Int[x/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (5*x^2)/(648*c^3*Sqrt[c + d*x^3]) + x^2/(216*c^2*(8*c - d*x^3)*Sqrt[c + d*x
^3]) - (5*Sqrt[c + d*x^3])/(648*c^3*d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)) - (5*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])
/(1296*Sqrt[3]*c^(17/6)*d^(2/3)) + (5*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^
(1/6)*Sqrt[c + d*x^3])])/(3888*c^(17/6)*d^(2/3)) - (5*ArcTanh[Sqrt[c + d*x^
3]/(3*Sqrt[c])])/(3888*c^(17/6)*d^(2/3)) + (5*Sqrt[2 - Sqrt[3]]*(c^(1/3) +
d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*
c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x
)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(432*3^(3/4)*c^(8/
3)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*Sqrt[c + d*x^3]) - (5*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(
1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)], -7 - 4*Sqrt[3]])/(324*Sqrt[2]*3^(1/4)*c^(8/3)*d^(2/3)*Sqrt[(c^(1/
3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d
*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^2}{216c^2(8c-dx^3)\sqrt{c+dx^3}} + \frac{\int \frac{x\left(25cd+\frac{5d^2x^3}{2}\right)}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx}{216c^2d} \\
&= \frac{5x^2}{648c^3\sqrt{c+dx^3}} + \frac{x^2}{216c^2(8c-dx^3)\sqrt{c+dx^3}} - \frac{\int \frac{x\left(\frac{45c^2d^2}{2}-\frac{45}{4}cd^3x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{2916c^4d^2} \\
&= \frac{5x^2}{648c^3\sqrt{c+dx^3}} + \frac{x^2}{216c^2(8c-dx^3)\sqrt{c+dx^3}} - \frac{\int \left(\frac{45cd^2x}{4\sqrt{c+dx^3}} - \frac{135c^2d^2x}{2(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{2916c^4d^2} \\
&= \frac{5x^2}{648c^3\sqrt{c+dx^3}} + \frac{x^2}{216c^2(8c-dx^3)\sqrt{c+dx^3}} - \frac{5\int \frac{x}{\sqrt{c+dx^3}}\,dx}{1296c^3} + \frac{5\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{216c^2}
\end{aligned}
$$

$$= \frac{5x^2}{648c^3\sqrt{c+dx^3}} + \frac{x^2}{216c^2(8c-dx^3)\sqrt{c+dx^3}} - \frac{5\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}dx}{2592c^3d}$$

$$- \frac{5\int \frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}dx}{1296c^3\sqrt[3]{d}} + \frac{5\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}dx}{2592c^{8/3}\sqrt[3]{d}}$$

$$+ \frac{(5(1-\sqrt{3}))\int \frac{1}{\sqrt{c+dx^3}}dx}{1296c^{8/3}\sqrt[3]{d}} - \frac{\left(5\sqrt[3]{d}\right)\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}dx}{864c^{7/3}}$$

$$= \frac{5x^2}{648c^3\sqrt{c+dx^3}} + \frac{x^2}{216c^2(8c-dx^3)\sqrt{c+dx^3}} - \frac{5\sqrt{c+dx^3}}{648c^3d^{2/3}\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$+ \frac{5\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{(1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{432\ 3^{3/4}c^{8/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$- \frac{5\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}{(1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{324\sqrt{2}\sqrt[4]{3}c^{8/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{5\text{Subst}\left(\int \frac{1}{9-cx^2}dx, x, \frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{1296c^{7/3}d^{2/3}} - \frac{\left(5\sqrt[3]{d}\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}dx, x, x^3\right)}{2592c^{7/3}}$$

$$+ \frac{\left(5d^{4/3}\right)\text{Subst}\left(\int \frac{1}{-\frac{2d^2}{c}-6d^2x^2}dx, x, \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{648c^{10/3}}$$

$$= \frac{5x^2}{648c^3\sqrt{c+dx^3}} + \frac{x^2}{216c^2(8c-dx^3)\sqrt{c+dx^3}} - \frac{5\sqrt{c+dx^3}}{648c^3d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{5\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1296\sqrt{3}c^{17/6}d^{2/3}} + \frac{5\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{3888c^{17/6}d^{2/3}} + \frac{5\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{432\ 3^{3/4}c^{8/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{5\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{324\sqrt{2}\sqrt[4]{3}c^{8/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{5\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{1296c^{7/3}d^{2/3}}$$

$$= \frac{5x^2}{648c^3\sqrt{c+dx^3}} + \frac{x^2}{216c^2(8c-dx^3)\sqrt{c+dx^3}} - \frac{5\sqrt{c+dx^3}}{648c^3d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{5\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1296\sqrt{3}c^{17/6}d^{2/3}} + \frac{5\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{3888c^{17/6}d^{2/3}} - \frac{5\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3888c^{17/6}d^{2/3}} + \frac{5\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{432\ 3^{3/4}c^{8/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{5\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{324\sqrt{2}\sqrt[4]{3}c^{8/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.25

$$\int \frac{x}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \frac{16cx^2(43c-5dx^3)+5cx^2(-8c+dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c}\right.}{10368c^4\,(8c-dx^3)}$$

```
[In] Integrate[x/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (16*c*x^2*(43*c - 5*d*x^3) + 5*c*x^2*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*App
ellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(8*c - d*x^3)*
Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]
)/(10368*c^4*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.53 (sec) , antiderivative size = 904, normalized size of antiderivative = 1.36

method	result	size
default	Expression too large to display	904
elliptic	Expression too large to display	904

```
[In] int(x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1944/c^3*x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+2/243*x^2/c^3/((x^3+c/d)*d)^(1/
2)+5/1944*I/c^3*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(
1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+
1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-
3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(
1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-5/5832*I/c^3/d^
3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-
3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*
(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-
```

```
(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*
(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)
*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2
)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alp
ha=RootOf(_Z^3*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.76 (sec) , antiderivative size = 2684, normalized size of antiderivative = 4.04

$$\int \frac{x}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/46656*(360*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(d)*weierstrassZeta(0, -4*c/
d, weierstrassPInverse(0, -4*c/d, x)) + 5*(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*
c^5*d + sqrt(-3)*(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d))*(1/(c^17*d^4))^(1
/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^12*d^5
*x^7 + 64*c^13*d^4*x^4 + 32*c^14*d^3*x + sqrt(-3)*(5*c^12*d^5*x^7 + 64*c^13
*d^4*x^4 + 32*c^14*d^3*x))*(1/(c^17*d^4))^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c
^15*d^5*x^5 + 32*c^16*d^4*x^2 - sqrt(-3)*(5*c^15*d^5*x^5 + 32*c^16*d^4*x^2)
)*(1/(c^17*d^4))^(5/6) - 2*(7*c^9*d^4*x^6 + 152*c^10*d^3*x^3 + 64*c^11*d^2)
*sqrt(1/(c^17*d^4)) + (c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x + sqrt(-3
)*(c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x))*(1/(c^17*d^4))^(1/6)) - 9*(
c^6*d^4*x^8 + 38*c^7*d^3*x^5 + 64*c^8*d^2*x^2 - sqrt(-3)*(c^6*d^4*x^8 + 38*
c^7*d^3*x^5 + 64*c^8*d^2*x^2))*(1/(c^17*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^
6 + 192*c^2*d*x^3 - 512*c^3)) - 5*(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d +
sqrt(-3)*(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d))*(1/(c^17*d^4))^(1/6)*log(
(d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^12*d^5*x^7 + 6
4*c^13*d^4*x^4 + 32*c^14*d^3*x + sqrt(-3)*(5*c^12*d^5*x^7 + 64*c^13*d^4*x^4
 + 32*c^14*d^3*x))*(1/(c^17*d^4))^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^15*d^5*
x^5 + 32*c^16*d^4*x^2 - sqrt(-3)*(5*c^15*d^5*x^5 + 32*c^16*d^4*x^2))*(1/(c^
17*d^4))^(5/6) - 2*(7*c^9*d^4*x^6 + 152*c^10*d^3*x^3 + 64*c^11*d^2)*sqrt(1/
(c^17*d^4)) + (c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x + sqrt(-3)*(c^3*d
^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x))*(1/(c^17*d^4))^(1/6)) - 9*(c^6*d^4*
x^8 + 38*c^7*d^3*x^5 + 64*c^8*d^2*x^2 - sqrt(-3)*(c^6*d^4*x^8 + 38*c^7*d^3*
x^5 + 64*c^8*d^2*x^2))*(1/(c^17*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*
c^2*d*x^3 - 512*c^3)) + 5*(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d - sqrt(-3)
*(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d))*(1/(c^17*d^4))^(1/6)*log((d^3*x^9
 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^12*d^5*x^7 + 64*c^13*d
^4*x^4 + 32*c^14*d^3*x - sqrt(-3)*(5*c^12*d^5*x^7 + 64*c^13*d^4*x^4 + 32*c^
```

14*d^3*x))*(1/(c^17*d^4))^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^15*d^5*x^5 + 32*c^16*d^4*x^2 + sqrt(-3)*(5*c^15*d^5*x^5 + 32*c^16*d^4*x^2))*(1/(c^17*d^4))^(5/6) - 2*(7*c^9*d^4*x^6 + 152*c^10*d^3*x^3 + 64*c^11*d^2)*sqrt(1/(c^17*d^4)) + (c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x - sqrt(-3)*(c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x))*(1/(c^17*d^4))^(1/6)) - 9*(c^6*d^4*x^8 + 38*c^7*d^3*x^5 + 64*c^8*d^2*x^2 + sqrt(-3)*(c^6*d^4*x^8 + 38*c^7*d^3*x^5 + 64*c^8*d^2*x^2))*(1/(c^17*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 5*(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d - sqrt(-3)*(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d))*(1/(c^17*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(5*c^12*d^5*x^7 + 64*c^13*d^4*x^4 + 32*c^14*d^3*x - sqrt(-3)*(5*c^12*d^5*x^7 + 64*c^13*d^4*x^4 + 32*c^14*d^3*x))*(1/(c^17*d^4))^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^15*d^5*x^5 + 32*c^16*d^4*x^2 + sqrt(-3)*(5*c^15*d^5*x^5 + 32*c^16*d^4*x^2))*(1/(c^17*d^4))^(5/6) - 2*(7*c^9*d^4*x^6 + 152*c^10*d^3*x^3 + 64*c^11*d^2)*sqrt(1/(c^17*d^4)) + (c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x - sqrt(-3)*(c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x))*(1/(c^17*d^4))^(1/6)) - 9*(c^6*d^4*x^8 + 38*c^7*d^3*x^5 + 64*c^8*d^2*x^2 + sqrt(-3)*(c^6*d^4*x^8 + 38*c^7*d^3*x^5 + 64*c^8*d^2*x^2))*(1/(c^17*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 10*(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d)*(1/(c^17*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(5*c^12*d^5*x^7 + 64*c^13*d^4*x^4 + 32*c^14*d^3*x)*(1/(c^17*d^4))^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^15*d^5*x^5 + 32*c^16*d^4*x^2)*(1/(c^17*d^4))^(5/6) + (7*c^9*d^4*x^6 + 152*c^10*d^3*x^3 + 64*c^11*d^2)*sqrt(1/(c^17*d^4)) + (c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x)*(1/(c^17*d^4))^(1/6)) + 18*(c^6*d^4*x^8 + 38*c^7*d^3*x^5 + 64*c^8*d^2*x^2)*(1/(c^17*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 10*(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d)*(1/(c^17*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(5*c^12*d^5*x^7 + 64*c^13*d^4*x^4 + 32*c^14*d^3*x)*(1/(c^17*d^4))^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^15*d^5*x^5 + 32*c^16*d^4*x^2)*(1/(c^17*d^4))^(5/6) + (7*c^9*d^4*x^6 + 152*c^10*d^3*x^3 + 64*c^11*d^2)*sqrt(1/(c^17*d^4)) + (c^3*d^3*x^7 + 80*c^4*d^2*x^4 + 160*c^5*d*x)*(1/(c^17*d^4))^(1/6)) + 18*(c^6*d^4*x^8 + 38*c^7*d^3*x^5 + 64*c^8*d^2*x^2)*(1/(c^17*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 72*(5*d^2*x^5 - 43*c*d*x^2)*sqrt(d*x^3 + c))/(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d)

Sympy [F]

$$\int \frac{x}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{x}{(-8c+dx^3)^2 (c+dx^3)^{\frac{3}{2}}}\,dx$$

[In] integrate(x/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)

[Out] Integral(x/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)

Maxima [F]

$$\int \frac{x}{(8c - dx^3)^2 (c + dx^3)^{3/2}} \, dx = \int \frac{x}{(dx^3 + c)^{\frac{3}{2}} (dx^3 - 8\,c)^2} \, dx$$

```
[In] integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(x/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Giac [F]

$$\int \frac{x}{(8c - dx^3)^2 (c + dx^3)^{3/2}} \, dx = \int \frac{x}{(dx^3 + c)^{\frac{3}{2}} (dx^3 - 8\,c)^2} \, dx$$

```
[In] integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(x/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(8c - dx^3)^2 (c + dx^3)^{3/2}} \, dx = \int \frac{x}{(d\,x^3 + c)^{3/2}\,(8\,c - d\,x^3)^2} \, dx$$

```
[In] int(x/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
[Out] int(x/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

3.452 $\int \frac{1}{x^2\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 686

$$\int \frac{1}{x^2\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{5}{648c^3x\sqrt{c+dx^3}}+\frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{31\sqrt{c+dx^3}}{1296c^4x}+\frac{31\sqrt[3]{d}\sqrt{c+dx^3}}{1296c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1296\sqrt{3}c^{23/6}}+\frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{3888c^{23/6}}-\frac{\sqrt[3]{d}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3888c^{23/6}}-\frac{31\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{864\,3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{31\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{648\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 1/3888*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c
^(23/6)-1/3888*d^(1/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(23/6)-1/3888
```

```
*d^(1/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(23/
6)*3^(1/2)+5/648/c^3/x/(d*x^3+c)^(1/2)+1/216/c^2/x/(-d*x^3+8*c)/(d*x^3+c)^(
1/2)-31/1296*(d*x^3+c)^(1/2)/c^4/x+31/1296*d^(1/3)*(d*x^3+c)^(1/2)/c^4/(d^(
1/3)*x+c^(1/3)*(1+3^(1/2)))+31/3888*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((
d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*
I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))
^2)^(1/2)*3^(3/4)/c^(11/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3
)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-31/2592*d^(1/3)*(c^(1/3)+d^(1
/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1
/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+
d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(11/3)/(d*x
^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)
^(1/2)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 686, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.519$, Rules used = {483, 593, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{1}{x^2 (8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx = \frac{31\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right.}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right.}\right.\right.}{648\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{31\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{864\ 3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{\sqrt[3]{d}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1296\sqrt{3}c^{23/6}}+\frac{\sqrt[3]{d}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{3888c^{23/6}}$$

$$-\frac{\sqrt[3]{d}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3888c^{23/6}}-\frac{31\sqrt{c+dx^3}}{1296c^4x}+\frac{31\sqrt[3]{d}\sqrt{c+dx^3}}{1296c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$+\frac{5}{648c^3x\sqrt{c+dx^3}}+\frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

```
[In] Int[1/(x^2*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] 5/(648*c^3*x*Sqrt[c + d*x^3]) + 1/(216*c^2*x*(8*c - d*x^3)*Sqrt[c + d*x^3])
 - (31*Sqrt[c + d*x^3])/(1296*c^4*x) + (31*d^(1/3)*Sqrt[c + d*x^3])/(1296*c
^4*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*
(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(1296*Sqrt[3]*c^(23/6)) + (d^(1/3)
*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(3888*c^(23/
6)) - (d^(1/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3888*c^(23/6)) - (31*
Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(
1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcS
in[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(864*3^(3/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x
))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (31*d^(1/3)*(c
^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) +
d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(648*Sqrt
[2]*3^(1/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^
(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
```

```
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{216c^2x\,(8c-dx^3)\,\sqrt{c+dx^3}} + \frac{\int \frac{28cd+\frac{11d^2x^3}{2}}{x^2(8c-dx^3)(c+dx^3)^{3/2}}\,dx}{216c^2d}$$

$$= \frac{5}{648c^3x\sqrt{c+dx^3}} + \frac{1}{216c^2x\,(8c-dx^3)\,\sqrt{c+dx^3}} - \frac{\int \frac{-558c^2d^2+\frac{225}{4}cd^3x^3}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{2916c^4d^2}$$

$$= \frac{5}{648c^3x\sqrt{c+dx^3}} + \frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{31\sqrt{c+dx^3}}{1296c^4x} + \frac{\int \frac{x\left(2340c^3d^3-279c^2d^4x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{23328c^6d^2}$$

$$= \frac{5}{648c^3x\sqrt{c+dx^3}} + \frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{31\sqrt{c+dx^3}}{1296c^4x} + \frac{\int \left(\frac{279c^2d^3x}{\sqrt{c+dx^3}} + \frac{108c^3d^3x}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{23328c^6d^2}$$

$$= \frac{5}{648c^3x\sqrt{c+dx^3}} + \frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{31\sqrt{c+dx^3}}{1296c^4x} + \frac{(31d)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{2592c^4} + \frac{d\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{216c^3}$$

$$= \frac{5}{648c^3x\sqrt{c+dx^3}} + \frac{1}{216c^2x\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{31\sqrt{c+dx^3}}{1296c^4x} - \frac{\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{2592c^4}$$
$$+ \frac{\left(31d^{2/3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{2592c^4} + \frac{d^{2/3}\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{2592c^{11/3}}$$
$$- \frac{\left(31\left(1-\sqrt{3}\right)d^{2/3}\right)\int \frac{1}{\sqrt{c+dx^3}}\,dx}{2592c^{11/3}} - \frac{d^{4/3}\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{864c^{10/3}}$$

$$= \frac{5}{648c^3x\sqrt{c+dx^3}} + \frac{1}{216c^2x\,(8c-dx^3)\,\sqrt{c+dx^3}} - \frac{31\sqrt{c+dx^3}}{1296c^4x} + \frac{31\sqrt[3]{d}\sqrt{c+dx^3}}{1296c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$

$$- \frac{31\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{864\ 3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{31\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{648\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{\sqrt[3]{d}\text{Subst}\left(\int \frac{1}{9-cx^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{1296c^{10/3}} - \frac{d^{4/3}\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{2592c^{10/3}}$$

$$+ \frac{d^{7/3}\text{Subst}\left(\int \frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{648c^{13/3}}$$

$$= \frac{5}{648c^3x\sqrt{c+dx^3}} + \frac{1}{216c^2x(8c-dx^3)\sqrt{c+dx^3}} - \frac{31\sqrt{c+dx^3}}{1296c^4x} + \frac{31\sqrt[3]{d}\sqrt{c+dx^3}}{1296c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1296\sqrt{3}c^{23/6}} + \frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{3888c^{23/6}} - \frac{31\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{864\ 3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{31\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{648\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{\sqrt[3]{d}\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{1296c^{10/3}}$$

$$
\begin{aligned}
&= \frac{5}{648c^3x\sqrt{c+dx^3}} + \frac{1}{216c^2x\,(8c-dx^3)\sqrt{c+dx^3}} - \frac{31\sqrt{c+dx^3}}{1296c^4x} \\
&+ \frac{31\sqrt[3]{d}\sqrt{c+dx^3}}{1296c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1296\sqrt{3}c^{23/6}} \\
&+ \frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{3888c^{23/6}} - \frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3888c^{23/6}} \\
&- \frac{31\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{864\ 3^{3/4}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+ \frac{31\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{648\sqrt{2}\sqrt[4]{3}c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.15 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.26

$$
\int \frac{1}{x^2\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{-80c(162c^2+227cdx^3-31d^2x^6)+650cdx^3(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{Appell}}{103680}
$$

```
[In] Integrate[1/(x^2*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-80*c*(162*c^2 + 227*c*d*x^3 - 31*d^2*x^6) + 650*c*d*x^3*(8*c - d*x^3)*Sqr
t[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] +
31*d^2*x^6*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -(
(d*x^3)/c), (d*x^3)/(8*c)])/(103680*c^5*Sqrt[c + d*x^3]*(8*c*x - d*x^4))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.33 (sec) , antiderivative size = 920, normalized size of antiderivative = 1.34

method	result	size
elliptic	Expression too large to display	920
risch	Expression too large to display	2220
default	Expression too large to display	2270

```
[In] int(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15552*d*x^2/c^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-2/243*d*x^2/c^4/((x^3+c/d)*d
)^(1/2)-1/64*(d*x^3+c)^(1/2)/c^4/x-31/3888*I/c^4*3^(1/2)*(-c*d^2)^(1/3)*(I*
(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c
*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d
^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(
I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2)))-1/5832*I/c^4/d^2*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I
*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/
2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^
(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2
)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)
*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Elli
pticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alp
ha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alph
a-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.91 (sec) , antiderivative size = 2534, normalized size of antiderivative = 3.69

$$\int \frac{1}{x^2 (8c - dx^3)^2 (c + dx^3)^{3/2}} dx = \text{Too large to display}$$

```
[In] integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/46656*(1116*(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)*sqrt(d)*weierstrassZeta(0, -
4*c/d, weierstrassPInverse(0, -4*c/d, x)) - (c^4*d^2*x^7 - 7*c^5*d*x^4 - 8*
c^6*x + sqrt(-3)*(c^4*d^2*x^7 - 7*c^5*d*x^4 - 8*c^6*x))*(d^2/c^23)^(1/6)*lo
g((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d - 9*(5*c^16*d^2*x
^7 + 64*c^17*d*x^4 + 32*c^18*x + sqrt(-3)*(5*c^16*d^2*x^7 + 64*c^17*d*x^4 +
 32*c^18*x))*(d^2/c^23)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^2
1*x^2 - sqrt(-3)*(5*c^20*d*x^5 + 32*c^21*x^2))*(d^2/c^23)^(5/6) - 2*(7*c^12
*d^2*x^6 + 152*c^13*d*x^3 + 64*c^14)*sqrt(d^2/c^23) + (c^4*d^3*x^7 + 80*c^5
*d^2*x^4 + 160*c^6*d*x + sqrt(-3)*(c^4*d^3*x^7 + 80*c^5*d^2*x^4 + 160*c^6*d
*x))*(d^2/c^23)^(1/6)) - 9*(c^8*d^3*x^8 + 38*c^9*d^2*x^5 + 64*c^10*d*x^2 -
sqrt(-3)*(c^8*d^3*x^8 + 38*c^9*d^2*x^5 + 64*c^10*d*x^2))*(d^2/c^23)^(1/3))/
(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (c^4*d^2*x^7 - 7*c^5*
d*x^4 - 8*c^6*x + sqrt(-3)*(c^4*d^2*x^7 - 7*c^5*d*x^4 - 8*c^6*x))*(d^2/c^23
)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d - 9*(5*
c^16*d^2*x^7 + 64*c^17*d*x^4 + 32*c^18*x + sqrt(-3)*(5*c^16*d^2*x^7 + 64*c^
17*d*x^4 + 32*c^18*x))*(d^2/c^23)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^
5 + 32*c^21*x^2 - sqrt(-3)*(5*c^20*d*x^5 + 32*c^21*x^2))*(d^2/c^23)^(5/6) -
 2*(7*c^12*d^2*x^6 + 152*c^13*d*x^3 + 64*c^14)*sqrt(d^2/c^23) + (c^4*d^3*x^
7 + 80*c^5*d^2*x^4 + 160*c^6*d*x + sqrt(-3)*(c^4*d^3*x^7 + 80*c^5*d^2*x^4 +
 160*c^6*d*x))*(d^2/c^23)^(1/6)) - 9*(c^8*d^3*x^8 + 38*c^9*d^2*x^5 + 64*c^1
0*d*x^2 - sqrt(-3)*(c^8*d^3*x^8 + 38*c^9*d^2*x^5 + 64*c^10*d*x^2))*(d^2/c^2
3)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c^4*d^2*x^
7 - 7*c^5*d*x^4 - 8*c^6*x - sqrt(-3)*(c^4*d^2*x^7 - 7*c^5*d*x^4 - 8*c^6*x))
*(d^2/c^23)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3
*d - 9*(5*c^16*d^2*x^7 + 64*c^17*d*x^4 + 32*c^18*x - sqrt(-3)*(5*c^16*d^2*x
^7 + 64*c^17*d*x^4 + 32*c^18*x))*(d^2/c^23)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5
*c^20*d*x^5 + 32*c^21*x^2 + sqrt(-3)*(5*c^20*d*x^5 + 32*c^21*x^2))*(d^2/c^2
3)^(5/6) - 2*(7*c^12*d^2*x^6 + 152*c^13*d*x^3 + 64*c^14)*sqrt(d^2/c^23) + (
c^4*d^3*x^7 + 80*c^5*d^2*x^4 + 160*c^6*d*x - sqrt(-3)*(c^4*d^3*x^7 + 80*c^5
*d^2*x^4 + 160*c^6*d*x))*(d^2/c^23)^(1/6)) - 9*(c^8*d^3*x^8 + 38*c^9*d^2*x^
5 + 64*c^10*d*x^2 + sqrt(-3)*(c^8*d^3*x^8 + 38*c^9*d^2*x^5 + 64*c^10*d*x^2)
)*(d^2/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (
c^4*d^2*x^7 - 7*c^5*d*x^4 - 8*c^6*x - sqrt(-3)*(c^4*d^2*x^7 - 7*c^5*d*x^4 -
 8*c^6*x))*(d^2/c^23)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3
 + 640*c^3*d - 9*(5*c^16*d^2*x^7 + 64*c^17*d*x^4 + 32*c^18*x - sqrt(-3)*(5*
```

```
c^16*d^2*x^7 + 64*c^17*d*x^4 + 32*c^18*x))*(d^2/c^23)^(2/3) - 3*sqrt(d*x^3
+ c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2 + sqrt(-3)*(5*c^20*d*x^5 + 32*c^21*x^2)
)*(d^2/c^23)^(5/6) - 2*(7*c^12*d^2*x^6 + 152*c^13*d*x^3 + 64*c^14)*sqrt(d^2
/c^23) + (c^4*d^3*x^7 + 80*c^5*d^2*x^4 + 160*c^6*d*x - sqrt(-3)*(c^4*d^3*x^
7 + 80*c^5*d^2*x^4 + 160*c^6*d*x))*(d^2/c^23)^(1/6)) - 9*(c^8*d^3*x^8 + 38*
c^9*d^2*x^5 + 64*c^10*d*x^2 + sqrt(-3)*(c^8*d^3*x^8 + 38*c^9*d^2*x^5 + 64*c
^10*d*x^2))*(d^2/c^23)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512
*c^3)) - 2*(c^4*d^2*x^7 - 7*c^5*d*x^4 - 8*c^6*x)*(d^2/c^23)^(1/6)*log((d^4*
x^9 + 318*c*d^3*x^6 + 1200*c^2*d^2*x^3 + 640*c^3*d + 18*(5*c^16*d^2*x^7 + 6
4*c^17*d*x^4 + 32*c^18*x)*(d^2/c^23)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^20*d
*x^5 + 32*c^21*x^2)*(d^2/c^23)^(5/6) + (7*c^12*d^2*x^6 + 152*c^13*d*x^3 + 6
4*c^14)*sqrt(d^2/c^23) + (c^4*d^3*x^7 + 80*c^5*d^2*x^4 + 160*c^6*d*x)*(d^2/
c^23)^(1/6)) + 18*(c^8*d^3*x^8 + 38*c^9*d^2*x^5 + 64*c^10*d*x^2)*(d^2/c^23)
^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 2*(c^4*d^2*x^
7 - 7*c^5*d*x^4 - 8*c^6*x)*(d^2/c^23)^(1/6)*log((d^4*x^9 + 318*c*d^3*x^6 +
1200*c^2*d^2*x^3 + 640*c^3*d + 18*(5*c^16*d^2*x^7 + 64*c^17*d*x^4 + 32*c^18
*x)*(d^2/c^23)^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^20*d*x^5 + 32*c^21*x^2)*(d
^2/c^23)^(5/6) + (7*c^12*d^2*x^6 + 152*c^13*d*x^3 + 64*c^14)*sqrt(d^2/c^23)
 + (c^4*d^3*x^7 + 80*c^5*d^2*x^4 + 160*c^6*d*x)*(d^2/c^23)^(1/6)) + 18*(c^8
*d^3*x^8 + 38*c^9*d^2*x^5 + 64*c^10*d*x^2)*(d^2/c^23)^(1/3))/(d^3*x^9 - 24*
c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 36*(31*d^2*x^6 - 227*c*d*x^3 - 162*
c^2)*sqrt(d*x^3 + c))/(c^4*d^2*x^7 - 7*c^5*d*x^4 - 8*c^6*x)
```

Sympy [F]

$$\int \frac{1}{x^2\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{x^2\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**2/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(1/(x**2*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x^2}\,dx$$

```
[In] integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 \left(8c-dx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{{\left(dx^3+c\right)}^{\frac{3}{2}}{\left(dx^3-8\,c\right)}^2x^2}\,dx$$

[In] integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")

[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(8c-dx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{x^2\,{\left(d\,x^3+c\right)}^{3/2}\,{\left(8\,c-d\,x^3\right)}^2}\,dx$$

[In] int(1/(x^2*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)

[Out] int(1/(x^2*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)

3.453 $\int \frac{1}{x^5(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 708

$$\int \frac{1}{x^5(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{5}{648c^3x^4\sqrt{c+dx^3}} + \frac{1}{216c^2x^4(8c-dx^3)\sqrt{c+dx^3}} - \frac{253\sqrt{c+dx^3}}{20736c^4x^4} + \frac{77d\sqrt{c+dx^3}}{2592c^5x} - \frac{77d^{4/3}\sqrt{c+dx^3}}{2592c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{11d^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{82944\sqrt{3}c^{29/6}} + \frac{11d^{4/3}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{248832c^{29/6}} - \frac{11d^{4/3}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{248832c^{29/6}} + \frac{77\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{1728\,3^{3/4}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{77d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{1296\sqrt{2}\sqrt[4]{3}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 11/248832*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2)
)/c^(29/6)-11/248832*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(29/6)-
11/248832*d^(4/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2
))/c^(29/6)*3^(1/2)+5/648/c^3/x^4/(d*x^3+c)^(1/2)+1/216/c^2/x^4/(-d*x^3+8*c
)/(d*x^3+c)^(1/2)-253/20736*(d*x^3+c)^(1/2)/c^4/x^4+77/2592*d*(d*x^3+c)^(1/
2)/c^5/x-77/2592*d^(4/3)*(d*x^3+c)^(1/2)/c^5/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))
)-77/7776*d^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/
2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/
3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(14/3)
*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1
+3^(1/2)))^2)^(1/2)+77/5184*d^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*
x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*
6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^
(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(14/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1
/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 708, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.519$, Rules

used = {483, 593, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{1}{x^5 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = -\frac{77d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{1296\sqrt{2}\sqrt[4]{3}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} + \frac{77\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{1728\ 3^{3/4}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{11d^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{82944\sqrt{3}c^{29/6}} + \frac{11d^{4/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{248832c^{29/6}} - \frac{11d^{4/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{248832c^{29/6}} - \frac{77d^{4/3}\sqrt{c+dx^3}}{2592c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{77d\sqrt{c+dx^3}}{2592c^5x} - \frac{253\sqrt{c+dx^3}}{20736c^4x^4} + \frac{5}{648c^3x^4\sqrt{c+dx^3}} + \frac{1}{216c^2x^4(8c-dx^3)\sqrt{c+dx^3}}$$

```
[In] Int[1/(x^5*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] 5/(648*c^3*x^4*Sqrt[c + d*x^3]) + 1/(216*c^2*x^4*(8*c - d*x^3)*Sqrt[c + d*x
^3]) - (253*Sqrt[c + d*x^3])/(20736*c^4*x^4) + (77*d*Sqrt[c + d*x^3])/(2592
*c^5*x) - (77*d^(4/3)*Sqrt[c + d*x^3])/(2592*c^5*((1 + Sqrt[3])*c^(1/3) + d
^(1/3)*x)) - (11*d^(4/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqr
t[c + d*x^3]])/(82944*Sqrt[3]*c^(29/6)) + (11*d^(4/3)*ArcTanh[(c^(1/3) + d^
(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]])/(248832*c^(29/6)) - (11*d^(4/3)*Ar
cTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]])/(248832*c^(29/6)) + (77*Sqrt[2 - Sqrt[3
]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3
)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3
])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3
]])/(1728*3^(3/4)*c^(14/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (77*d^(4/3)*(c^(1/3) + d^(1/
3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(1296*Sqrt[2]*3^(1/4)*c
```

```
^(14/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{216c^2x^4(8c-dx^3)\sqrt{c+dx^3}}+\frac{\int \frac{31cd+\frac{17d^2x^3}{2}}{x^5(8c-dx^3)(c+dx^3)^{3/2}}\,dx}{216c^2d}\\
&= \frac{5}{648c^3x^4\sqrt{c+dx^3}}+\frac{1}{216c^2x^4(8c-dx^3)\sqrt{c+dx^3}}-\frac{\int \frac{-\frac{2277}{2}c^2d^2+\frac{495}{4}cd^3x^3}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx}{2916c^4d^2}\\
&= \frac{5}{648c^3x^4\sqrt{c+dx^3}}+\frac{1}{216c^2x^4(8c-dx^3)\sqrt{c+dx^3}}-\frac{253\sqrt{c+dx^3}}{20736c^4x^4}+\frac{\int \frac{-22176c^3d^3+\frac{11385}{4}c^2d^4x^3}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{93312c^6d^2}\\
&= \frac{5}{648c^3x^4\sqrt{c+dx^3}}+\frac{1}{216c^2x^4(8c-dx^3)\sqrt{c+dx^3}}\\
&\quad-\frac{253\sqrt{c+dx^3}}{20736c^4x^4}+\frac{77d\sqrt{c+dx^3}}{2592c^5x}-\frac{\int \frac{x(88110c^4d^4-11088c^3d^5x^3)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{746496c^8d^2}\\
&= \frac{5}{648c^3x^4\sqrt{c+dx^3}}+\frac{1}{216c^2x^4(8c-dx^3)\sqrt{c+dx^3}}-\frac{253\sqrt{c+dx^3}}{20736c^4x^4}\\
&\quad+\frac{77d\sqrt{c+dx^3}}{2592c^5x}-\frac{\int \left(\frac{11088c^3d^4x}{\sqrt{c+dx^3}}-\frac{594c^4d^4x}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{746496c^8d^2}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{5}{648c^3x^4\sqrt{c+dx^3}}+\frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{253\sqrt{c+dx^3}}{20736c^4x^4} \\
&\quad+\frac{77d\sqrt{c+dx^3}}{2592c^5x}-\frac{(77d^2)\int\frac{x}{\sqrt{c+dx^3}}\,dx}{5184c^5}+\frac{(11d^2)\int\frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{13824c^4} \\
&= \frac{5}{648c^3x^4\sqrt{c+dx^3}}+\frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{253\sqrt{c+dx^3}}{20736c^4x^4} \\
&\quad+\frac{77d\sqrt{c+dx^3}}{2592c^5x}-\frac{(11d)\int\frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{165888c^5} \\
&\quad-\frac{\left(77d^{5/3}\right)\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{5184c^5}+\frac{\left(11d^{5/3}\right)\int\frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{165888c^{14/3}} \\
&\quad+\frac{\left(77\left(1-\sqrt{3}\right)d^{5/3}\right)\int\frac{1}{\sqrt{c+dx^3}}\,dx}{5184c^{14/3}}-\frac{\left(11d^{7/3}\right)\int\frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{55296c^{13/3}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{5}{648c^3x^4\sqrt{c+dx^3}} + \frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}} \\
&- \frac{253\sqrt{c+dx^3}}{20736c^4x^4} + \frac{77d\sqrt{c+dx^3}}{2592c^5x} - \frac{77d^{4/3}\sqrt{c+dx^3}}{2592c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} \\
&+ \frac{77\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{1728\ 3^{3/4}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&- \frac{77d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{1296\sqrt{2}\sqrt[4]{3}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+ \frac{\left(11d^{4/3}\right)\text{Subst}\left(\int\frac{1}{9-cx^2}\,dx,x,\frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{82944c^{13/3}} \\
&- \frac{\left(11d^{7/3}\right)\text{Subst}\left(\int\frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{165888c^{13/3}} \\
&+ \frac{\left(11d^{10/3}\right)\text{Subst}\left(\int\frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{41472c^{16/3}}
\end{aligned}
$$

$$= \frac{5}{648c^3x^4\sqrt{c+dx^3}} + \frac{1}{216c^2x^4(8c-dx^3)\sqrt{c+dx^3}} - \frac{253\sqrt{c+dx^3}}{20736c^4x^4} + \frac{77d\sqrt{c+dx^3}}{2592c^5x} - \frac{77d^{4/3}\sqrt{c+dx^3}}{2592c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{11d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{82944\sqrt{3}c^{29/6}} + \frac{11d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{248832c^{29/6}} + \frac{77\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{1728\ 3^{3/4}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{77d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{1296\sqrt{2}\sqrt[4]{3}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} - \frac{\left(11d^{4/3}\right)\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{82944c^{13/3}}$$

$$= \frac{5}{648c^3x^4\sqrt{c+dx^3}} + \frac{1}{216c^2x^4(8c-dx^3)\sqrt{c+dx^3}} - \frac{253\sqrt{c+dx^3}}{20736c^4x^4} + \frac{77d\sqrt{c+dx^3}}{2592c^5x}$$
$$- \frac{77d^{4/3}\sqrt{c+dx^3}}{2592c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{11d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{82944\sqrt{3}c^{29/6}}$$
$$+ \frac{11d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{248832c^{29/6}} - \frac{11d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{248832c^{29/6}}$$
$$+ \frac{77\sqrt{2-\sqrt{3}}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{1728\ 3^{3/4}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$- \frac{77d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{1296\sqrt{2}\sqrt[4]{3}c^{14/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.28

$$\int \frac{1}{x^5(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{-24475cd^2x^6(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-16\left(\right.}{}$$

```
[In] Integrate[1/(x^5*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-24475*c*d^2*x^6*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5
/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 16*(10*c*(648*c^3 - 2997*c^2*d*x^3 - 456
5*c*d^2*x^6 + 616*d^3*x^9) + 77*d^3*x^9*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*
AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(3317760*c^6*x^4*
(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.31 (sec) , antiderivative size = 943, normalized size of antiderivative = 1.33

method	result	size
elliptic	Expression too large to display	943
risch	Expression too large to display	2232
default	Expression too large to display	2775

```
[In] int(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/124416*d^2*x^2/c^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+2/243*d^2*x^2/c^5/((x^3+c
/d)*d)^(1/2)-1/256*(d*x^3+c)^(1/2)/c^4/x^4+11/512*d*(d*x^3+c)^(1/2)/c^5/x+7
7/7776*I*d/c^5*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3
))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d
*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))-11/373248*I/c^5/d*
2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*
(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/
2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I
*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-
c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-
c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_
alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^
(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha
=RootOf(_Z^3*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.80 (sec) , antiderivative size = 2692, normalized size of antiderivative = 3.80

$$\int \frac{1}{x^5 (8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx = \text{Too large to display}$$

```
[In] integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2985984*(88704*(d^3*x^10 - 7*c*d^2*x^7 - 8*c^2*d*x^4)*sqrt(d)*weierstrass
Zeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) + 11*(c^5*d^2*x^10 - 7*c
^6*d*x^7 - 8*c^7*x^4 + sqrt(-3)*(c^5*d^2*x^10 - 7*c^6*d*x^7 - 8*c^7*x^4))*(
d^8/c^29)^(1/6)*log(161051*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 64
0*c^3*d^6 - 9*(5*c^20*d^3*x^7 + 64*c^21*d^2*x^4 + 32*c^22*d*x + sqrt(-3)*(5
*c^20*d^3*x^7 + 64*c^21*d^2*x^4 + 32*c^22*d*x))*(d^8/c^29)^(2/3) + 3*sqrt(d
*x^3 + c)*(6*(5*c^25*d*x^5 + 32*c^26*x^2 - sqrt(-3)*(5*c^25*d*x^5 + 32*c^26
*x^2))*(d^8/c^29)^(5/6) - 2*(7*c^15*d^4*x^6 + 152*c^16*d^3*x^3 + 64*c^17*d^
2)*sqrt(d^8/c^29) + (c^5*d^7*x^7 + 80*c^6*d^6*x^4 + 160*c^7*d^5*x + sqrt(-3
)*(c^5*d^7*x^7 + 80*c^6*d^6*x^4 + 160*c^7*d^5*x))*(d^8/c^29)^(1/6)) - 9*(c^
10*d^6*x^8 + 38*c^11*d^5*x^5 + 64*c^12*d^4*x^2 - sqrt(-3)*(c^10*d^6*x^8 + 3
8*c^11*d^5*x^5 + 64*c^12*d^4*x^2))*(d^8/c^29)^(1/3))/(d^3*x^9 - 24*c*d^2*x^
6 + 192*c^2*d*x^3 - 512*c^3)) - 11*(c^5*d^2*x^10 - 7*c^6*d*x^7 - 8*c^7*x^4
+ sqrt(-3)*(c^5*d^2*x^10 - 7*c^6*d*x^7 - 8*c^7*x^4))*(d^8/c^29)^(1/6)*log(1
61051*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^20
*d^3*x^7 + 64*c^21*d^2*x^4 + 32*c^22*d*x + sqrt(-3)*(5*c^20*d^3*x^7 + 64*c^
21*d^2*x^4 + 32*c^22*d*x))*(d^8/c^29)^(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^25*
d*x^5 + 32*c^26*x^2 - sqrt(-3)*(5*c^25*d*x^5 + 32*c^26*x^2))*(d^8/c^29)^(5/
6) - 2*(7*c^15*d^4*x^6 + 152*c^16*d^3*x^3 + 64*c^17*d^2)*sqrt(d^8/c^29) + (
c^5*d^7*x^7 + 80*c^6*d^6*x^4 + 160*c^7*d^5*x + sqrt(-3)*(c^5*d^7*x^7 + 80*c
^6*d^6*x^4 + 160*c^7*d^5*x))*(d^8/c^29)^(1/6)) - 9*(c^10*d^6*x^8 + 38*c^11*
d^5*x^5 + 64*c^12*d^4*x^2 - sqrt(-3)*(c^10*d^6*x^8 + 38*c^11*d^5*x^5 + 64*c
^12*d^4*x^2))*(d^8/c^29)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 5
12*c^3)) + 11*(c^5*d^2*x^10 - 7*c^6*d*x^7 - 8*c^7*x^4 - sqrt(-3)*(c^5*d^2*x
^10 - 7*c^6*d*x^7 - 8*c^7*x^4))*(d^8/c^29)^(1/6)*log(161051*(d^9*x^9 + 318*
c*d^8*x^6 + 1200*c^2*d^7*x^3 + 640*c^3*d^6 - 9*(5*c^20*d^3*x^7 + 64*c^21*d^
2*x^4 + 32*c^22*d*x - sqrt(-3)*(5*c^20*d^3*x^7 + 64*c^21*d^2*x^4 + 32*c^22*
d*x))*(d^8/c^29)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^25*d*x^5 + 32*c^26*x^2 +
 sqrt(-3)*(5*c^25*d*x^5 + 32*c^26*x^2))*(d^8/c^29)^(5/6) - 2*(7*c^15*d^4*x^
6 + 152*c^16*d^3*x^3 + 64*c^17*d^2)*sqrt(d^8/c^29) + (c^5*d^7*x^7 + 80*c^6*
d^6*x^4 + 160*c^7*d^5*x - sqrt(-3)*(c^5*d^7*x^7 + 80*c^6*d^6*x^4 + 160*c^7*
d^5*x))*(d^8/c^29)^(1/6)) - 9*(c^10*d^6*x^8 + 38*c^11*d^5*x^5 + 64*c^12*d^4
*x^2 + sqrt(-3)*(c^10*d^6*x^8 + 38*c^11*d^5*x^5 + 64*c^12*d^4*x^2))*(d^8/c^
29)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 11*(c^5*d^
2*x^10 - 7*c^6*d*x^7 - 8*c^7*x^4 - sqrt(-3)*(c^5*d^2*x^10 - 7*c^6*d*x^7 - 8
```

```
*c^7*x^4))*(d^8/c^29)^(1/6)*log(161051*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^2*
d^7*x^3 + 640*c^3*d^6 - 9*(5*c^20*d^3*x^7 + 64*c^21*d^2*x^4 + 32*c^22*d*x -
 sqrt(-3)*(5*c^20*d^3*x^7 + 64*c^21*d^2*x^4 + 32*c^22*d*x))*(d^8/c^29)^(2/3
) - 3*sqrt(d*x^3 + c)*(6*(5*c^25*d*x^5 + 32*c^26*x^2 + sqrt(-3)*(5*c^25*d*x
^5 + 32*c^26*x^2))*(d^8/c^29)^(5/6) - 2*(7*c^15*d^4*x^6 + 152*c^16*d^3*x^3
+ 64*c^17*d^2)*sqrt(d^8/c^29) + (c^5*d^7*x^7 + 80*c^6*d^6*x^4 + 160*c^7*d^5
*x - sqrt(-3)*(c^5*d^7*x^7 + 80*c^6*d^6*x^4 + 160*c^7*d^5*x))*(d^8/c^29)^(1
/6)) - 9*(c^10*d^6*x^8 + 38*c^11*d^5*x^5 + 64*c^12*d^4*x^2 + sqrt(-3)*(c^10
*d^6*x^8 + 38*c^11*d^5*x^5 + 64*c^12*d^4*x^2))*(d^8/c^29)^(1/3))/(d^3*x^9 -
 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 22*(c^5*d^2*x^10 - 7*c^6*d*x^7
- 8*c^7*x^4)*(d^8/c^29)^(1/6)*log(161051*(d^9*x^9 + 318*c*d^8*x^6 + 1200*c^
2*d^7*x^3 + 640*c^3*d^6 + 18*(5*c^20*d^3*x^7 + 64*c^21*d^2*x^4 + 32*c^22*d*
x)*(d^8/c^29)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^25*d*x^5 + 32*c^26*x^2)*(d^
8/c^29)^(5/6) + (7*c^15*d^4*x^6 + 152*c^16*d^3*x^3 + 64*c^17*d^2)*sqrt(d^8/
c^29) + (c^5*d^7*x^7 + 80*c^6*d^6*x^4 + 160*c^7*d^5*x)*(d^8/c^29)^(1/6)) +
18*(c^10*d^6*x^8 + 38*c^11*d^5*x^5 + 64*c^12*d^4*x^2)*(d^8/c^29)^(1/3))/(d^
3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 22*(c^5*d^2*x^10 - 7*c^6
*d*x^7 - 8*c^7*x^4)*(d^8/c^29)^(1/6)*log(161051*(d^9*x^9 + 318*c*d^8*x^6 +
1200*c^2*d^7*x^3 + 640*c^3*d^6 + 18*(5*c^20*d^3*x^7 + 64*c^21*d^2*x^4 + 32*
c^22*d*x)*(d^8/c^29)^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^25*d*x^5 + 32*c^26*x
^2)*(d^8/c^29)^(5/6) + (7*c^15*d^4*x^6 + 152*c^16*d^3*x^3 + 64*c^17*d^2)*sq
rt(d^8/c^29) + (c^5*d^7*x^7 + 80*c^6*d^6*x^4 + 160*c^7*d^5*x)*(d^8/c^29)^(1
/6)) + 18*(c^10*d^6*x^8 + 38*c^11*d^5*x^5 + 64*c^12*d^4*x^2)*(d^8/c^29)^(1/
3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 144*(616*d^3*x^9
- 4565*c*d^2*x^6 - 2997*c^2*d*x^3 + 648*c^3)*sqrt(d*x^3 + c))/(c^5*d^2*x^10
 - 7*c^6*d*x^7 - 8*c^7*x^4)
```

Sympy [F]

$$\int \frac{1}{x^5 \left(8c - dx^3\right)^2 \left(c + dx^3\right)^{3/2}}\, dx = \int \frac{1}{x^5 \left(-8c + dx^3\right)^2 \left(c + dx^3\right)^{\frac{3}{2}}}\, dx$$

```
[In] integrate(1/x**5/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x**5*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^5 (8c - dx^3)^2 (c + dx^3)^{3/2}} \, dx = \int \frac{1}{(dx^3 + c)^{\frac{3}{2}} (dx^3 - 8\,c)^2 x^5} \, dx$$

```
[In] integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5 (8c - dx^3)^2 (c + dx^3)^{3/2}} \, dx = \int \frac{1}{(dx^3 + c)^{\frac{3}{2}} (dx^3 - 8\,c)^2 x^5} \, dx$$

```
[In] integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (8c - dx^3)^2 (c + dx^3)^{3/2}} \, dx = \int \frac{1}{x^5 \, (d\,x^3 + c)^{3/2} \, (8\,c - d\,x^3)^2} \, dx$$

```
[In] int(1/(x^5*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
[Out] int(1/(x^5*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

3.454 $\int \frac{1}{x^8 (8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 732

$$
\begin{aligned}
\int \frac{1}{x^8 (8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx &= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7(8c-dx^3)\sqrt{c+dx^3}} \\
&- \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8257d\sqrt{c+dx^3}}{580608c^5x^4} - \frac{5179d^2\sqrt{c+dx^3}}{145152c^6x} \\
&+ \frac{5179d^{7/3}\sqrt{c+dx^3}}{145152c^6\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{7d^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{331776\sqrt{3}c^{35/6}} \\
&+ \frac{7d^{7/3}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{995328c^{35/6}} - \frac{7d^{7/3}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{995328c^{35/6}} \\
&- \frac{5179\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{96768\, 3^{3/4}c^{17/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+ \frac{5179d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{72576\sqrt{2}\sqrt[4]{3}c^{17/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}
\end{aligned}
$$

```
[Out] 7/995328*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))
/c^(35/6)-7/995328*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(35/6)-7/
995328*d^(7/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/
c^(35/6)*3^(1/2)+5/648/c^3/x^7/(d*x^3+c)^(1/2)+1/216/c^2/x^7/(-d*x^3+8*c)/(
d*x^3+c)^(1/2)-191/18144*(d*x^3+c)^(1/2)/c^4/x^7+8257/580608*d*(d*x^3+c)^(1
/2)/c^5/x^4-5179/145152*d^2*(d*x^3+c)^(1/2)/c^6/x+5179/145152*d^(7/3)*(d*x^
3+c)^(1/2)/c^6/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+5179/435456*d^(7/3)*(c^(1/3)
+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1
+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)
*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(17/3)*2^(1/2)/(d*x^3+c)^(1/2)/(
c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-5179/2
90304*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))
/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*(
(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^
(1/2)*3^(1/4)/c^(17/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3
)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 732, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.519$, Rules

used = {483, 593, 597, 598, 309, 224, 1891, 499, 455, 65, 212, 2163, 2170, 211}

$$\int \frac{1}{x^8 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{5179 d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right), -7-4\sqrt{3}\right)}{72576\sqrt{2}\sqrt[4]{3}c^{17/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{5179\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{96768\ 3^{3/4}c^{17/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$-\frac{7d^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{331776\sqrt{3}c^{35/6}}+\frac{7d^{7/3}\text{arctanh}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{995328c^{35/6}}$$

$$-\frac{7d^{7/3}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{995328c^{35/6}}+\frac{5179d^{7/3}\sqrt{c+dx^3}}{145152c^6\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{5179d^2\sqrt{c+dx^3}}{145152c^6x}$$

$$+\frac{8257d\sqrt{c+dx^3}}{580608c^5x^4}-\frac{191\sqrt{c+dx^3}}{18144c^4x^7}+\frac{5}{648c^3x^7\sqrt{c+dx^3}}+\frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

```
[In] Int[1/(x^8*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] 5/(648*c^3*x^7*Sqrt[c + d*x^3]) + 1/(216*c^2*x^7*(8*c - d*x^3)*Sqrt[c + d*x
^3]) - (191*Sqrt[c + d*x^3])/(18144*c^4*x^7) + (8257*d*Sqrt[c + d*x^3])/(58
0608*c^5*x^4) - (5179*d^2*Sqrt[c + d*x^3])/(145152*c^6*x) + (5179*d^(7/3)*S
qrt[c + d*x^3])/(145152*c^6*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (7*d^(7/
3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(331776
*Sqrt[3]*c^(35/6)) + (7*d^(7/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*
Sqrt[c + d*x^3])])/(995328*c^(35/6)) - (7*d^(7/3)*ArcTanh[Sqrt[c + d*x^3]/(
3*Sqrt[c])])/(995328*c^(35/6)) - (5179*Sqrt[2 - Sqrt[3]]*d^(7/3)*(c^(1/3) +
 d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*
x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(96768*3^(3/4)*c^
(17/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)^2]*Sqrt[c + d*x^3]) + (5179*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3
) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]
*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(72576*Sqrt[2]*3^(1/4)*c^(17/3)*Sqrt[(c^(
1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c +
 d*x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
```

```
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
```

```
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{216c^2x^7(8c-dx^3)\sqrt{c+dx^3}} + \frac{\int \frac{34cd+\frac{23d^2x^3}{2}}{x^8(8c-dx^3)(c+dx^3)^{3/2}}\,dx}{216c^2d}$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7(8c-dx^3)\sqrt{c+dx^3}} - \frac{\int \frac{-1719c^2d^2+\frac{765}{4}cd^3x^3}{x^8(8c-dx^3)\sqrt{c+dx^3}}\,dx}{2916c^4d^2}$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7(8c-dx^3)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{\int \frac{-74313c^3d^3+\frac{18909}{2}c^2d^4x^3}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx}{163296c^6d^2}$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7(8c-dx^3)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8257d\sqrt{c+dx^3}}{580608c^5x^4} - \frac{\int \frac{-1491552c^4d^4+\frac{371565}{2}c^3d^5x^3}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{5225472c^8d^2}$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7(8c-dx^3)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8257d\sqrt{c+dx^3}}{580608c^5x^4} - \frac{5179d^2\sqrt{c+dx^3}}{145152c^6x} + \frac{\int \frac{x(5971500c^5d^5-745776c^4d^6x^3)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{41803776c^{10}d^2}$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7(8c-dx^3)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8257d\sqrt{c+dx^3}}{580608c^5x^4} - \frac{5179d^2\sqrt{c+dx^3}}{145152c^6x} + \frac{\int \left(\frac{745776c^4d^5x}{\sqrt{c+dx^3}} + \frac{5292c^5d^5x}{(8c-dx^3)\sqrt{c+dx^3}}\right) dx}{41803776c^{10}d^2}$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7(8c-dx^3)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8257d\sqrt{c+dx^3}}{580608c^5x^4} - \frac{5179d^2\sqrt{c+dx^3}}{145152c^6x} + \frac{(5179d^3)\int \frac{x}{\sqrt{c+dx^3}}dx}{290304c^6} + \frac{(7d^3)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}dx}{55296c^5}$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7(8c-dx^3)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8257d\sqrt{c+dx^3}}{580608c^5x^4} - \frac{5179d^2\sqrt{c+dx^3}}{145152c^6x} - \frac{(7d^2)\int \frac{2\sqrt[3]{c}d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}dx}{663552c^6} + \frac{\left(5179d^{8/3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}dx}{290304c^6} + \frac{\left(7d^{8/3}\right)\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}dx}{663552c^{17/3}} - \frac{\left(5179\left(1-\sqrt{3}\right)d^{8/3}\right)\int \frac{1}{\sqrt{c+dx^3}}dx}{290304c^{17/3}} - \frac{\left(7d^{10/3}\right)\int \frac{x^2}{(8c-dx^3)\sqrt{c+dx^3}}dx}{221184c^{16/3}}$$

$$
\begin{aligned}
&= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} \\
&+ \frac{8257d\sqrt{c+dx^3}}{580608c^5x^4} - \frac{5179d^2\sqrt{c+dx^3}}{145152c^6x} + \frac{5179d^{7/3}\sqrt{c+dx^3}}{145152c^6\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} \\
&- \frac{5179\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right.}{96768\ 3^{3/4}c^{17/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+ \frac{5179d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{72576\sqrt{2}\sqrt[4]{3}c^{17/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}} \\
&+ \frac{\left(7d^{7/3}\right)\text{Subst}\left(\int\frac{1}{9-cx^2}\,dx,x,\frac{\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{331776c^{16/3}} \\
&- \frac{\left(7d^{10/3}\right)\text{Subst}\left(\int\frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{663552c^{16/3}} \\
&+ \frac{\left(7d^{13/3}\right)\text{Subst}\left(\int\frac{1}{-\frac{2d^2}{c}-6d^2x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{c+dx^3}}\right)}{165888c^{19/3}}
\end{aligned}
$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7}$$
$$+ \frac{8257d\sqrt{c+dx^3}}{580608c^5x^4} - \frac{5179d^2\sqrt{c+dx^3}}{145152c^6x} + \frac{5179d^{7/3}\sqrt{c+dx^3}}{145152c^6\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}$$
$$- \frac{7d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{331776\sqrt{3}c^{35/6}} + \frac{7d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{995328c^{35/6}}$$
$$- \frac{5179\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{96768\ 3^{3/4}c^{17/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$+ \frac{5179d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{72576\sqrt{2}\sqrt[4]{3}c^{17/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$
$$- \frac{\left(7d^{7/3}\right)\text{Subst}\left(\int\frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{331776c^{16/3}}$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7(8c-dx^3)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8257d\sqrt{c+dx^3}}{580608c^5x^4} - \frac{5179d^2\sqrt{c+dx^3}}{145152c^6x}$$

$$+ \frac{5179d^{7/3}\sqrt{c+dx^3}}{145152c^6\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{7d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{331776\sqrt{3}c^{35/6}}$$

$$+ \frac{7d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{995328c^{35/6}} - \frac{7d^{7/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{995328c^{35/6}}$$

$$- \frac{5179\sqrt{2-\sqrt{3}}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right.}{96768\ 3^{3/4}c^{17/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

$$+ \frac{5179d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right)|-7-4\sqrt{3}\right)}{72576\sqrt{2}\sqrt[4]{3}c^{17/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.19 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.29

$$\int \frac{1}{x^8(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{829375cd^3x^9(8c-dx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-8\left(2\right.}{}$$

```
[In] Integrate[1/(x^8*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (829375*c*d^3*x^9*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5
/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 8*(20*c*(10368*c^4 - 18792*c^3*d*x^3 + 1
01817*c^2*d^2*x^6 + 153269*c*d^3*x^9 - 20716*d^4*x^12) + 5179*d^4*x^12*(8*c
 - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x
^3)/(8*c)]))/(92897280*c^7*x^7*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.67 (sec) , antiderivative size = 962, normalized size of antiderivative = 1.31

method	result	size
elliptic	Expression too large to display	962
risch	Expression too large to display	2243
default	Expression too large to display	3300

```
[In] int(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/995328*d^3*x^2/c^6*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-2/243*d^3*x^2/c^6/((x^3+c
/d)*d)^(1/2)-1/448*(d*x^3+c)^(1/2)/c^4/x^7+43/7168*d*(d*x^3+c)^(1/2)/c^5/x^
4-787/28672*d^2*(d*x^3+c)^(1/2)/c^6/x-5179/435456*I/c^6*d^2*3^(1/2)*(-c*d^2
)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3)))^(1/2)))-7/1492992*I/c^6*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1
/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^
(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2
)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d
-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(
2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^
(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(
2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.57 (sec) , antiderivative size = 2725, normalized size of antiderivative = 3.72

$$\int \frac{1}{x^8 (8c-dx^3)^2 (c+dx^3)^{3/2}} \, dx = \text{Too large to display}$$

```
[In] integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/83607552*(2983104*(d^4*x^13 - 7*c*d^3*x^10 - 8*c^2*d^2*x^7)*sqrt(d)*weie
rstrassZeta(0, -4*c/d, weierstrassPInverse(0, -4*c/d, x)) - 49*(c^6*d^2*x^1
3 - 7*c^7*d*x^10 - 8*c^8*x^7 + sqrt(-3)*(c^6*d^2*x^13 - 7*c^7*d*x^10 - 8*c^
8*x^7))*(d^14/c^35)^(1/6)*log(16807*(d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d
^12*x^3 + 640*c^3*d^11 - 9*(5*c^24*d^4*x^7 + 64*c^25*d^3*x^4 + 32*c^26*d^2*
x + sqrt(-3)*(5*c^24*d^4*x^7 + 64*c^25*d^3*x^4 + 32*c^26*d^2*x))*(d^14/c^35
)^(2/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^30*d*x^5 + 32*c^31*x^2 - sqrt(-3)*(5*c^
30*d*x^5 + 32*c^31*x^2))*(d^14/c^35)^(5/6) - 2*(7*c^18*d^6*x^6 + 152*c^19*d
^5*x^3 + 64*c^20*d^4)*sqrt(d^14/c^35) + (c^6*d^11*x^7 + 80*c^7*d^10*x^4 + 1
60*c^8*d^9*x + sqrt(-3)*(c^6*d^11*x^7 + 80*c^7*d^10*x^4 + 160*c^8*d^9*x))*(
d^14/c^35)^(1/6)) - 9*(c^12*d^9*x^8 + 38*c^13*d^8*x^5 + 64*c^14*d^7*x^2 - s
qrt(-3)*(c^12*d^9*x^8 + 38*c^13*d^8*x^5 + 64*c^14*d^7*x^2))*(d^14/c^35)^(1/
3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 49*(c^6*d^2*x^13
 - 7*c^7*d*x^10 - 8*c^8*x^7 + sqrt(-3)*(c^6*d^2*x^13 - 7*c^7*d*x^10 - 8*c^8*
x^7))*(d^14/c^35)^(1/6)*log(16807*(d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^1
2*x^3 + 640*c^3*d^11 - 9*(5*c^24*d^4*x^7 + 64*c^25*d^3*x^4 + 32*c^26*d^2*x
 + sqrt(-3)*(5*c^24*d^4*x^7 + 64*c^25*d^3*x^4 + 32*c^26*d^2*x))*(d^14/c^35)^
(2/3) - 3*sqrt(d*x^3 + c)*(6*(5*c^30*d*x^5 + 32*c^31*x^2 - sqrt(-3)*(5*c^30
*d*x^5 + 32*c^31*x^2))*(d^14/c^35)^(5/6) - 2*(7*c^18*d^6*x^6 + 152*c^19*d^5
*x^3 + 64*c^20*d^4)*sqrt(d^14/c^35) + (c^6*d^11*x^7 + 80*c^7*d^10*x^4 + 160
*c^8*d^9*x + sqrt(-3)*(c^6*d^11*x^7 + 80*c^7*d^10*x^4 + 160*c^8*d^9*x))*(d^
14/c^35)^(1/6)) - 9*(c^12*d^9*x^8 + 38*c^13*d^8*x^5 + 64*c^14*d^7*x^2 - sqr
t(-3)*(c^12*d^9*x^8 + 38*c^13*d^8*x^5 + 64*c^14*d^7*x^2))*(d^14/c^35)^(1/3)
)/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 49*(c^6*d^2*x^13 -
7*c^7*d*x^10 - 8*c^8*x^7 - sqrt(-3)*(c^6*d^2*x^13 - 7*c^7*d*x^10 - 8*c^8*x^
7))*(d^14/c^35)^(1/6)*log(16807*(d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*
x^3 + 640*c^3*d^11 - 9*(5*c^24*d^4*x^7 + 64*c^25*d^3*x^4 + 32*c^26*d^2*x -
sqrt(-3)*(5*c^24*d^4*x^7 + 64*c^25*d^3*x^4 + 32*c^26*d^2*x))*(d^14/c^35)^(2
/3) + 3*sqrt(d*x^3 + c)*(6*(5*c^30*d*x^5 + 32*c^31*x^2 + sqrt(-3)*(5*c^30*d
*x^5 + 32*c^31*x^2))*(d^14/c^35)^(5/6) - 2*(7*c^18*d^6*x^6 + 152*c^19*d^5*x
^3 + 64*c^20*d^4)*sqrt(d^14/c^35) + (c^6*d^11*x^7 + 80*c^7*d^10*x^4 + 160*c
^8*d^9*x - sqrt(-3)*(c^6*d^11*x^7 + 80*c^7*d^10*x^4 + 160*c^8*d^9*x))*(d^14
/c^35)^(1/6)) - 9*(c^12*d^9*x^8 + 38*c^13*d^8*x^5 + 64*c^14*d^7*x^2 + sqrt(
-3)*(c^12*d^9*x^8 + 38*c^13*d^8*x^5 + 64*c^14*d^7*x^2))*(d^14/c^35)^(1/3))/
(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 49*(c^6*d^2*x^13 - 7*
```

```
c^7*d*x^10 - 8*c^8*x^7 - sqrt(-3)*(c^6*d^2*x^13 - 7*c^7*d*x^10 - 8*c^8*x^7)
)*(d^14/c^35)^(1/6)*log(16807*(d^14*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^
3 + 640*c^3*d^11 - 9*(5*c^24*d^4*x^7 + 64*c^25*d^3*x^4 + 32*c^26*d^2*x - sq
rt(-3)*(5*c^24*d^4*x^7 + 64*c^25*d^3*x^4 + 32*c^26*d^2*x))*(d^14/c^35)^(2/3
) - 3*sqrt(d*x^3 + c)*(6*(5*c^30*d*x^5 + 32*c^31*x^2 + sqrt(-3)*(5*c^30*d*x
^5 + 32*c^31*x^2))*(d^14/c^35)^(5/6) - 2*(7*c^18*d^6*x^6 + 152*c^19*d^5*x^3
 + 64*c^20*d^4)*sqrt(d^14/c^35) + (c^6*d^11*x^7 + 80*c^7*d^10*x^4 + 160*c^8
*d^9*x - sqrt(-3)*(c^6*d^11*x^7 + 80*c^7*d^10*x^4 + 160*c^8*d^9*x))*(d^14/c
^35)^(1/6)) - 9*(c^12*d^9*x^8 + 38*c^13*d^8*x^5 + 64*c^14*d^7*x^2 + sqrt(-3
)*(c^12*d^9*x^8 + 38*c^13*d^8*x^5 + 64*c^14*d^7*x^2))*(d^14/c^35)^(1/3))/(d
^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 98*(c^6*d^2*x^13 - 7*c^
7*d*x^10 - 8*c^8*x^7)*(d^14/c^35)^(1/6)*log(16807*(d^14*x^9 + 318*c*d^13*x^
6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 + 18*(5*c^24*d^4*x^7 + 64*c^25*d^3*x^4
 + 32*c^26*d^2*x)*(d^14/c^35)^(2/3) + 6*sqrt(d*x^3 + c)*(6*(5*c^30*d*x^5 +
32*c^31*x^2)*(d^14/c^35)^(5/6) + (7*c^18*d^6*x^6 + 152*c^19*d^5*x^3 + 64*c^
20*d^4)*sqrt(d^14/c^35) + (c^6*d^11*x^7 + 80*c^7*d^10*x^4 + 160*c^8*d^9*x)*
(d^14/c^35)^(1/6)) + 18*(c^12*d^9*x^8 + 38*c^13*d^8*x^5 + 64*c^14*d^7*x^2)*
(d^14/c^35)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 98
*(c^6*d^2*x^13 - 7*c^7*d*x^10 - 8*c^8*x^7)*(d^14/c^35)^(1/6)*log(16807*(d^1
4*x^9 + 318*c*d^13*x^6 + 1200*c^2*d^12*x^3 + 640*c^3*d^11 + 18*(5*c^24*d^4*
x^7 + 64*c^25*d^3*x^4 + 32*c^26*d^2*x)*(d^14/c^35)^(2/3) - 6*sqrt(d*x^3 + c
)*(6*(5*c^30*d*x^5 + 32*c^31*x^2)*(d^14/c^35)^(5/6) + (7*c^18*d^6*x^6 + 152
*c^19*d^5*x^3 + 64*c^20*d^4)*sqrt(d^14/c^35) + (c^6*d^11*x^7 + 80*c^7*d^10*
x^4 + 160*c^8*d^9*x)*(d^14/c^35)^(1/6)) + 18*(c^12*d^9*x^8 + 38*c^13*d^8*x^
5 + 64*c^14*d^7*x^2)*(d^14/c^35)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d
*x^3 - 512*c^3)) + 144*(20716*d^4*x^12 - 153269*c*d^3*x^9 - 101817*c^2*d^2*
x^6 + 18792*c^3*d*x^3 - 10368*c^4)*sqrt(d*x^3 + c))/(c^6*d^2*x^13 - 7*c^7*d
*x^10 - 8*c^8*x^7)
```

Sympy [F]

$$\int \frac{1}{x^8\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{x^8\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**8/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x**8*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^8 (8c - dx^3)^2 (c + dx^3)^{3/2}}\,dx = \int \frac{1}{(dx^3 + c)^{\frac{3}{2}} (dx^3 - 8\,c)^2 x^8}\,dx$$

```
[In] integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^8), x)
```

Giac [F]

$$\int \frac{1}{x^8 (8c - dx^3)^2 (c + dx^3)^{3/2}}\,dx = \int \frac{1}{(dx^3 + c)^{\frac{3}{2}} (dx^3 - 8\,c)^2 x^8}\,dx$$

```
[In] integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^8 (8c - dx^3)^2 (c + dx^3)^{3/2}}\,dx = \int \frac{1}{x^8\,(d\,x^3 + c)^{3/2}\,(8\,c - d\,x^3)^2}\,dx$$

```
[In] int(1/(x^8*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
[Out] int(1/(x^8*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

3.455 $\int \frac{x^6}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 256

$$\int \frac{x^6}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{2x(4c+dx^3)}{81cd^2(8c-dx^3)\sqrt{c+dx^3}} - \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}\right),-7-4\sqrt{3}\right)}{81\sqrt[4]{3}cd^{7/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}$$

```
[Out] 2/81*x*(d*x^3+4*c)/c/d^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2)-2/243*(c^(1/3)+d^(1/3
)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2
))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^
(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c/d^(7/3)/(d*x^
3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^
(1/2)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.26, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {525, 524}

$$\int \frac{x^6}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{x^7\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{7}{3},2,\frac{3}{2},\frac{10}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{448c^3\sqrt{c+dx^3}}$$

```
[In] Int[x^6/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^7*Sqrt[1 + (d*x^3)/c]*AppellF1[7/3, 2, 3/2, 10/3, (d*x^3)/(8*c), -((d*x^
3)/c)])/(448*c^3*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^6}{(8c-dx^3)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$
$$= \frac{x^7\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{7}{3};2,\frac{3}{2};\frac{10}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{448c^3\sqrt{c+dx^3}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.56 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.74

$$\int \frac{x^6}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx =$$
$$-\frac{6\sqrt[3]{-d}x(4c+dx^3)+2i3^{3/4}\sqrt[3]{c}\sqrt{\frac{(-1)^{5/6}\left(-\sqrt[3]{c}+\sqrt[3]{-d}x\right)}{\sqrt[3]{c}}}\sqrt{1+\frac{\sqrt[3]{-d}x}{\sqrt[3]{c}}+\frac{(-d)^{2/3}x^2}{c^{2/3}}}(-8c+dx^3)\,\text{EllipticF}\left(\text{arc}\right.}{243c(-d)^{7/3}\,(-8c+dx^3)\,\sqrt{c+dx^3}}$$

```
[In] Integrate[x^6/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/243*(6*(-d)^(1/3)*x*(4*c + d*x^3) + (2*I)*3^(3/4)*c^(1/3)*Sqrt[((-1)^(5/
6)*(-c^(1/3) + (-d)^(1/3)*x))/c^(1/3)]*Sqrt[1 + ((-d)^(1/3)*x)/c^(1/3) + ((
-d)^(2/3)*x^2)/c^(2/3)]*(-8*c + d*x^3)*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) -
(I*(-d)^(1/3)*x)/c^(1/3)]/3^(1/4)], (-1)^(1/3)])/(c*(-d)^(7/3)*(-8*c + d*x^
3)*Sqrt[c + d*x^3])
```

Maple [A] (verified)

Time = 4.40 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.32

method	result
elliptic	$\frac{8x\sqrt{d\,x^{3}+c}}{243c\,d^{2}(-d\,x^{3}+8c)}+\frac{2x}{243d^{2}c\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}+\frac{2i\sqrt{3}\,(-c\,d^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-c\,d^{2})^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,(-c\,d^{2})^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{(-c\,d^{2})^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-c\,d^{2})^{\frac{1}{3}}}{d}}{-\frac{3(-c\,d^{2})^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,(-c\,d^{2})^{\frac{1}{3}}}{2d}}}\cdots}{\cdots}$
default	Expression too large to display

```
[In] int(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 8/243*x/c/d^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+2/243/d^2*x/c/((x^3+c/d)*d)^(1/2
)+2/243*I/d^3/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/
3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*
(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2
))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.35

$$\int \frac{x^6}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = -\frac{2\left((d^2x^6-7\,cdx^3-8\,c^2)\sqrt{d}\text{weierstrassPInverse}\left(0,-\frac{4\,c}{d},x\right)+(d^2x^4+4\,cdx)\sqrt{dx^3+c}\right)}{81\,(cd^5x^6-7\,c^2d^4x^3-8\,c^3d^3)}$$

```
[In] integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/81*((d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(d)*weierstrassPInverse(0, -4*c/d,
 x) + (d^2*x^4 + 4*c*d*x)*sqrt(d*x^3 + c))/(c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c
^3*d^3)
```

Sympy [F]

$$\int \frac{x^6}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \int \frac{x^6}{(-8c+dx^3)^2(c+dx^3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**6/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(x**6/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^6}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \int \frac{x^6}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)^2}\,dx$$

```
[In] integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(x^6/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Giac [F]

$$\int \frac{x^6}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \int \frac{x^6}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)^2}\,dx$$

```
[In] integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(x^6/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \int \frac{x^6}{(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)^2}\,dx$$

```
[In] int(x^6/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
[Out] int(x^6/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

3.456 $\int \frac{x^3}{(8c-dx^3)^2(c+dx^3)^{3/2}} dx$

Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{x^3}{(8c-dx^3)^2(c+dx^3)^{3/2}} dx = \frac{x^4\sqrt{1+\frac{dx^3}{c}}\operatorname{AppellF1}\left(\frac{4}{3},2,\frac{3}{2},\frac{7}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{256c^3\sqrt{c+dx^3}}$$

```
[Out] 1/256*x^4*AppellF1(4/3,3/2,2,7/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^
3/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {525, 524}

$$\int \frac{x^3}{(8c-dx^3)^2(c+dx^3)^{3/2}} dx = \frac{x^4\sqrt{\frac{dx^3}{c}+1}\operatorname{AppellF1}\left(\frac{4}{3},2,\frac{3}{2},\frac{7}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{256c^3\sqrt{c+dx^3}}$$

```
[In] Int[x^3/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 2, 3/2, 7/3, (d*x^3)/(8*c), -((d*x^3
)/c)])/(256*c^3*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{(8c-dx^3)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$

$$= \frac{x^4\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{4}{3};2,\frac{3}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{256c^3\sqrt{c+dx^3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 242 vs. $2(66) = 132$.

Time = 10.29 (sec) , antiderivative size = 242, normalized size of antiderivative = 3.67

$$\int \frac{x^3}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \frac{x\left(\frac{3x^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^3}+\frac{192\left(\frac{-5c+dx^3}{c^2}+\frac{\cdots}{32c\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\cdots}\right.}{\cdots}\right.}{15552\sqrt{c+dx^3}}$$

```
[In] Integrate[x^3/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*((3*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d*
x^3)/(8*c)])/c^3 + (192*((-5*c + d*x^3)/c^2 + (160*AppellF1[1/3, 1/2, 1, 4/
3, -((d*x^3)/c), (d*x^3)/(8*c)])/(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)
/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*
x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]))))
/(d*(8*c - d*x^3))))/(15552*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.42 (sec) , antiderivative size = 754, normalized size of antiderivative = 11.42

method	result	size
elliptic	Expression too large to display	754
default	Expression too large to display	1479

```
[In] int(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/243*x/c^2/d*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-2/243/d*x/c^2/((x^3+c/d)*d)^(1/2
)+1/243*I/c^2/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(
1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+
1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/
d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1
/2))-1/243*I/c^2/d^4*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/
d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1
/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/
2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(
1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^
(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(
-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c
,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2713 vs. 2(52) = 104.

Time = 1.07 (sec) , antiderivative size = 2713, normalized size of antiderivative = 41.11

$$\int \frac{x^3}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3888*(24*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(d)*weierstrassPInverse(0, -4*
c/d, x) + (c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2 + sqrt(-3)*(c^2*d^4*x^6
- 7*c^3*d^3*x^3 - 8*c^4*d^2))*(1/(c^13*d^8))^(1/6)*log((d^3*x^9 + 318*c*d^2
```

*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c^9*d^8*x^8 + 38*c^10*d^7*x^5 + 64*c^1
1*d^6*x^2 + sqrt(-3)*(c^9*d^8*x^8 + 38*c^10*d^7*x^5 + 64*c^11*d^6*x^2))*(1/
(c^13*d^8))^(2/3) + 3*sqrt(d*x^3 + c)*((c^11*d^9*x^7 + 80*c^12*d^8*x^4 + 16
0*c^13*d^7*x - sqrt(-3)*(c^11*d^9*x^7 + 80*c^12*d^8*x^4 + 160*c^13*d^7*x))*
(1/(c^13*d^8))^(5/6) - 2*(7*c^7*d^6*x^6 + 152*c^8*d^5*x^3 + 64*c^9*d^4)*sqr
t(1/(c^13*d^8)) + 6*(5*c^3*d^3*x^5 + 32*c^4*d^2*x^2 + sqrt(-3)*(5*c^3*d^3*x
^5 + 32*c^4*d^2*x^2))*(1/(c^13*d^8))^(1/6)) - 9*(5*c^5*d^5*x^7 + 64*c^6*d^4
*x^4 + 32*c^7*d^3*x - sqrt(-3)*(5*c^5*d^5*x^7 + 64*c^6*d^4*x^4 + 32*c^7*d^3
x))(1/(c^13*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^
3)) - (c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2 + sqrt(-3)*(c^2*d^4*x^6 - 7*
c^3*d^3*x^3 - 8*c^4*d^2))*(1/(c^13*d^8))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6
 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c^9*d^8*x^8 + 38*c^10*d^7*x^5 + 64*c^11*d^
6*x^2 + sqrt(-3)*(c^9*d^8*x^8 + 38*c^10*d^7*x^5 + 64*c^11*d^6*x^2))*(1/(c^1
3*d^8))^(2/3) - 3*sqrt(d*x^3 + c)*((c^11*d^9*x^7 + 80*c^12*d^8*x^4 + 160*c^
13*d^7*x - sqrt(-3)*(c^11*d^9*x^7 + 80*c^12*d^8*x^4 + 160*c^13*d^7*x))*(1/(
c^13*d^8))^(5/6) - 2*(7*c^7*d^6*x^6 + 152*c^8*d^5*x^3 + 64*c^9*d^4)*sqrt(1/
(c^13*d^8)) + 6*(5*c^3*d^3*x^5 + 32*c^4*d^2*x^2 + sqrt(-3)*(5*c^3*d^3*x^5 +
 32*c^4*d^2*x^2))*(1/(c^13*d^8))^(1/6)) - 9*(5*c^5*d^5*x^7 + 64*c^6*d^4*x^4
 + 32*c^7*d^3*x - sqrt(-3)*(5*c^5*d^5*x^7 + 64*c^6*d^4*x^4 + 32*c^7*d^3*x))
*(1/(c^13*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3))
+ (c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2 - sqrt(-3)*(c^2*d^4*x^6 - 7*c^3*
d^3*x^3 - 8*c^4*d^2))*(1/(c^13*d^8))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1
200*c^2*d*x^3 + 640*c^3 - 9*(c^9*d^8*x^8 + 38*c^10*d^7*x^5 + 64*c^11*d^6*x^
2 - sqrt(-3)*(c^9*d^8*x^8 + 38*c^10*d^7*x^5 + 64*c^11*d^6*x^2))*(1/(c^13*d^
8))^(2/3) + 3*sqrt(d*x^3 + c)*((c^11*d^9*x^7 + 80*c^12*d^8*x^4 + 160*c^13*d
^7*x + sqrt(-3)*(c^11*d^9*x^7 + 80*c^12*d^8*x^4 + 160*c^13*d^7*x))*(1/(c^13
d^8))^(5/6) - 2(7*c^7*d^6*x^6 + 152*c^8*d^5*x^3 + 64*c^9*d^4)*sqrt(1/(c^1
3*d^8)) + 6*(5*c^3*d^3*x^5 + 32*c^4*d^2*x^2 - sqrt(-3)*(5*c^3*d^3*x^5 + 32*
c^4*d^2*x^2))*(1/(c^13*d^8))^(1/6)) - 9*(5*c^5*d^5*x^7 + 64*c^6*d^4*x^4 + 3
2*c^7*d^3*x + sqrt(-3)*(5*c^5*d^5*x^7 + 64*c^6*d^4*x^4 + 32*c^7*d^3*x))*(1/
(c^13*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c
^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2 - sqrt(-3)*(c^2*d^4*x^6 - 7*c^3*d^3*
x^3 - 8*c^4*d^2))*(1/(c^13*d^8))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*
c^2*d*x^3 + 640*c^3 - 9*(c^9*d^8*x^8 + 38*c^10*d^7*x^5 + 64*c^11*d^6*x^2 -
sqrt(-3)*(c^9*d^8*x^8 + 38*c^10*d^7*x^5 + 64*c^11*d^6*x^2))*(1/(c^13*d^8))^
(2/3) - 3*sqrt(d*x^3 + c)*((c^11*d^9*x^7 + 80*c^12*d^8*x^4 + 160*c^13*d^7*x
 + sqrt(-3)*(c^11*d^9*x^7 + 80*c^12*d^8*x^4 + 160*c^13*d^7*x))*(1/(c^13*d^8
))^(5/6) - 2*(7*c^7*d^6*x^6 + 152*c^8*d^5*x^3 + 64*c^9*d^4)*sqrt(1/(c^13*d^
8)) + 6*(5*c^3*d^3*x^5 + 32*c^4*d^2*x^2 - sqrt(-3)*(5*c^3*d^3*x^5 + 32*c^4*
d^2*x^2))*(1/(c^13*d^8))^(1/6)) - 9*(5*c^5*d^5*x^7 + 64*c^6*d^4*x^4 + 32*c^
7*d^3*x + sqrt(-3)*(5*c^5*d^5*x^7 + 64*c^6*d^4*x^4 + 32*c^7*d^3*x))*(1/(c^1
3*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 2*(c^2
*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2)*(1/(c^13*d^8))^(1/6)*log((d^3*x^9 + 3
18*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(c^9*d^8*x^8 + 38*c^10*d^7*x^5
 + 64*c^11*d^6*x^2)*(1/(c^13*d^8))^(2/3) + 6*sqrt(d*x^3 + c)*((c^11*d^9*x^7

```
 + 80*c^12*d^8*x^4 + 160*c^13*d^7*x)*(1/(c^13*d^8))^(5/6) + (7*c^7*d^6*x^6
+ 152*c^8*d^5*x^3 + 64*c^9*d^4)*sqrt(1/(c^13*d^8)) + 6*(5*c^3*d^3*x^5 + 32*
c^4*d^2*x^2)*(1/(c^13*d^8))^(1/6)) + 18*(5*c^5*d^5*x^7 + 64*c^6*d^4*x^4 + 3
2*c^7*d^3*x)*(1/(c^13*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3
- 512*c^3)) - 2*(c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2)*(1/(c^13*d^8))^(1
/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(c^9*d^8*x
^8 + 38*c^10*d^7*x^5 + 64*c^11*d^6*x^2)*(1/(c^13*d^8))^(2/3) - 6*sqrt(d*x^3
 + c)*((c^11*d^9*x^7 + 80*c^12*d^8*x^4 + 160*c^13*d^7*x)*(1/(c^13*d^8))^(5/
6) + (7*c^7*d^6*x^6 + 152*c^8*d^5*x^3 + 64*c^9*d^4)*sqrt(1/(c^13*d^8)) + 6*
(5*c^3*d^3*x^5 + 32*c^4*d^2*x^2)*(1/(c^13*d^8))^(1/6)) + 18*(5*c^5*d^5*x^7
+ 64*c^6*d^4*x^4 + 32*c^7*d^3*x)*(1/(c^13*d^8))^(1/3))/(d^3*x^9 - 24*c*d^2*
x^6 + 192*c^2*d*x^3 - 512*c^3)) - 48*(d^2*x^4 - 5*c*d*x)*sqrt(d*x^3 + c))/(
c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2)
```

Sympy [F]

$$\int \frac{x^3}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \int \frac{x^3}{(-8c+dx^3)^2\,(c+dx^3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**3/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(x**3/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^3}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \int \frac{x^3}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)^2}\,dx$$

```
[In] integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(x^3/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Giac [F]

$$\int \frac{x^3}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \int \frac{x^3}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)^2}\,dx$$

```
[In] integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(x^3/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx=\int \frac{x^3}{{\left(d\,x^3+c\right)}^{3/2}\,{\left(8\,c-d\,x^3\right)}^2} \,d x$$

```
[In] int(x^3/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
[Out] int(x^3/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

3.457 $\int \frac{1}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx=\frac{x\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{1}{3},2,\frac{3}{2},\frac{4}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{64c^3\sqrt{c+dx^3}}$$

```
[Out] 1/64*x*AppellF1(1/3,3/2,2,4/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^3/(
d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {441, 440}

$$\int \frac{1}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx=\frac{x\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},2,\frac{3}{2},\frac{4}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{64c^3\sqrt{c+dx^3}}$$

```
[In] Int[1/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 2, 3/2, 4/3, (d*x^3)/(8*c), -((d*x^3)/
c)])/(64*c^3*Sqrt[c + d*x^3])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{(8c-dx^3)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}\\ &= \frac{x\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{1}{3};2,\frac{3}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{64c^3\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 253 vs. $2(64) = 128$.

Time = 10.27 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.95

$$\int \frac{1}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{x\left(-15dx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+192c\left(\frac{-43c+5dx^3}{-8c+dx^3}+\right.\right.}{}$$

```
[In] Integrate[1/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*(-15*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c),
(d*x^3)/(8*c)] + 192*c*((-43*c + 5*d*x^3)/(-8*c + d*x^3) + (1216*c^2*Appell
F1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((8*c - d*x^3)*(32*c*App
ellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/
3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3,
 -((d*x^3)/c), (d*x^3)/(8*c)]))))))/(124416*c^4*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.55 (sec) , antiderivative size = 748, normalized size of antiderivative = 11.69

method	result	size
default	Expression too large to display	748
elliptic	Expression too large to display	748

```
[In] int(1/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1944/c^3*x*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+2/243*x/c^3/((x^3+c/d)*d)^(1/2)-5
/1944*I/c^3*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3)
)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-
c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))
-1/972*I/c^3/d^3*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(
-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*
d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)
/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3
)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d
^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*
3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2640 vs. $2(50) = 100$.

Time = 0.97 (sec) , antiderivative size = 2640, normalized size of antiderivative = 41.25

$$\int \frac{1}{(8c - dx^3)^2 (c + dx^3)^{3/2}} \, dx = \text{Too large to display}$$

```
[In] integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15552*(192*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(d)*weierstrassPInverse(0, -
4*c/d, x) + (c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d + sqrt(-3)*(c^3*d^3*x^6
- 7*c^4*d^2*x^3 - 8*c^5*d))*(1/(c^19*d^2))^(1/6)*log((d^3*x^9 + 318*c*d^2*x
^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c^13*d^4*x^8 + 38*c^14*d^3*x^5 + 64*c^15
*d^2*x^2 + sqrt(-3)*(c^13*d^4*x^8 + 38*c^14*d^3*x^5 + 64*c^15*d^2*x^2))*(1/
(c^19*d^2))^(2/3) + 3*sqrt(d*x^3 + c)*((c^16*d^4*x^7 + 80*c^17*d^3*x^4 + 16
0*c^18*d^2*x - sqrt(-3)*(c^16*d^4*x^7 + 80*c^17*d^3*x^4 + 160*c^18*d^2*x))*
(1/(c^19*d^2))^(5/6) - 2*(7*c^10*d^3*x^6 + 152*c^11*d^2*x^3 + 64*c^12*d)*sq
rt(1/(c^19*d^2)) + 6*(5*c^4*d^2*x^5 + 32*c^5*d*x^2 + sqrt(-3)*(5*c^4*d^2*x^
5 + 32*c^5*d*x^2))*(1/(c^19*d^2))^(1/6)) - 9*(5*c^7*d^3*x^7 + 64*c^8*d^2*x^
4 + 32*c^9*d*x - sqrt(-3)*(5*c^7*d^3*x^7 + 64*c^8*d^2*x^4 + 32*c^9*d*x))*(1
/(c^19*d^2))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (
c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d + sqrt(-3)*(c^3*d^3*x^6 - 7*c^4*d^2*x
```

^3 - 8*c^5*d))*(1/(c^19*d^2))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2
*d*x^3 + 640*c^3 - 9*(c^13*d^4*x^8 + 38*c^14*d^3*x^5 + 64*c^15*d^2*x^2 + sq
rt(-3)*(c^13*d^4*x^8 + 38*c^14*d^3*x^5 + 64*c^15*d^2*x^2))*(1/(c^19*d^2))^(
2/3) - 3*sqrt(d*x^3 + c)*((c^16*d^4*x^7 + 80*c^17*d^3*x^4 + 160*c^18*d^2*x
- sqrt(-3)*(c^16*d^4*x^7 + 80*c^17*d^3*x^4 + 160*c^18*d^2*x))*(1/(c^19*d^2)
)^(5/6) - 2*(7*c^10*d^3*x^6 + 152*c^11*d^2*x^3 + 64*c^12*d)*sqrt(1/(c^19*d^
2)) + 6*(5*c^4*d^2*x^5 + 32*c^5*d*x^2 + sqrt(-3)*(5*c^4*d^2*x^5 + 32*c^5*d*
x^2))*(1/(c^19*d^2))^(1/6)) - 9*(5*c^7*d^3*x^7 + 64*c^8*d^2*x^4 + 32*c^9*d*
x - sqrt(-3)*(5*c^7*d^3*x^7 + 64*c^8*d^2*x^4 + 32*c^9*d*x))*(1/(c^19*d^2))^
(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (c^3*d^3*x^6 -
7*c^4*d^2*x^3 - 8*c^5*d - sqrt(-3)*(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d)
)*(1/(c^19*d^2))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*
c^3 - 9*(c^13*d^4*x^8 + 38*c^14*d^3*x^5 + 64*c^15*d^2*x^2 - sqrt(-3)*(c^13*
d^4*x^8 + 38*c^14*d^3*x^5 + 64*c^15*d^2*x^2))*(1/(c^19*d^2))^(2/3) + 3*sqrt
(d*x^3 + c)*((c^16*d^4*x^7 + 80*c^17*d^3*x^4 + 160*c^18*d^2*x + sqrt(-3)*(c
^16*d^4*x^7 + 80*c^17*d^3*x^4 + 160*c^18*d^2*x))*(1/(c^19*d^2))^(5/6) - 2*(
7*c^10*d^3*x^6 + 152*c^11*d^2*x^3 + 64*c^12*d)*sqrt(1/(c^19*d^2)) + 6*(5*c^
4*d^2*x^5 + 32*c^5*d*x^2 - sqrt(-3)*(5*c^4*d^2*x^5 + 32*c^5*d*x^2))*(1/(c^1
9*d^2))^(1/6)) - 9*(5*c^7*d^3*x^7 + 64*c^8*d^2*x^4 + 32*c^9*d*x + sqrt(-3)*
(5*c^7*d^3*x^7 + 64*c^8*d^2*x^4 + 32*c^9*d*x))*(1/(c^19*d^2))^(1/3))/(d^3*x
^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - (c^3*d^3*x^6 - 7*c^4*d^2*x^
3 - 8*c^5*d - sqrt(-3)*(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d))*(1/(c^19*d^
2))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 - 9*(c^13
*d^4*x^8 + 38*c^14*d^3*x^5 + 64*c^15*d^2*x^2 - sqrt(-3)*(c^13*d^4*x^8 + 38*
c^14*d^3*x^5 + 64*c^15*d^2*x^2))*(1/(c^19*d^2))^(2/3) - 3*sqrt(d*x^3 + c)*(
(c^16*d^4*x^7 + 80*c^17*d^3*x^4 + 160*c^18*d^2*x + sqrt(-3)*(c^16*d^4*x^7 +
80*c^17*d^3*x^4 + 160*c^18*d^2*x))*(1/(c^19*d^2))^(5/6) - 2*(7*c^10*d^3*x^
6 + 152*c^11*d^2*x^3 + 64*c^12*d)*sqrt(1/(c^19*d^2)) + 6*(5*c^4*d^2*x^5 + 3
2*c^5*d*x^2 - sqrt(-3)*(5*c^4*d^2*x^5 + 32*c^5*d*x^2))*(1/(c^19*d^2))^(1/6)
) - 9*(5*c^7*d^3*x^7 + 64*c^8*d^2*x^4 + 32*c^9*d*x + sqrt(-3)*(5*c^7*d^3*x^
7 + 64*c^8*d^2*x^4 + 32*c^9*d*x))*(1/(c^19*d^2))^(1/3))/(d^3*x^9 - 24*c*d^2
*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 2*(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d
)*(1/(c^19*d^2))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*
c^3 + 18*(c^13*d^4*x^8 + 38*c^14*d^3*x^5 + 64*c^15*d^2*x^2)*(1/(c^19*d^2))^
(2/3) + 6*sqrt(d*x^3 + c)*((c^16*d^4*x^7 + 80*c^17*d^3*x^4 + 160*c^18*d^2*x
)*(1/(c^19*d^2))^(5/6) + (7*c^10*d^3*x^6 + 152*c^11*d^2*x^3 + 64*c^12*d)*sq
rt(1/(c^19*d^2)) + 6*(5*c^4*d^2*x^5 + 32*c^5*d*x^2)*(1/(c^19*d^2))^(1/6)) +
18*(5*c^7*d^3*x^7 + 64*c^8*d^2*x^4 + 32*c^9*d*x)*(1/(c^19*d^2))^(1/3))/(d^
3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 2*(c^3*d^3*x^6 - 7*c^4*d
^2*x^3 - 8*c^5*d)*(1/(c^19*d^2))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*
c^2*d*x^3 + 640*c^3 + 18*(c^13*d^4*x^8 + 38*c^14*d^3*x^5 + 64*c^15*d^2*x^2)
*(1/(c^19*d^2))^(2/3) - 6*sqrt(d*x^3 + c)*((c^16*d^4*x^7 + 80*c^17*d^3*x^4
+ 160*c^18*d^2*x)*(1/(c^19*d^2))^(5/6) + (7*c^10*d^3*x^6 + 152*c^11*d^2*x^3
+ 64*c^12*d)*sqrt(1/(c^19*d^2)) + 6*(5*c^4*d^2*x^5 + 32*c^5*d*x^2)*(1/(c^1
9*d^2))^(1/6)) + 18*(5*c^7*d^3*x^7 + 64*c^8*d^2*x^4 + 32*c^9*d*x)*(1/(c^19*

```
d^2))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 24*(5*d^
2*x^4 - 43*c*d*x)*sqrt(d*x^3 + c))/(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d)
```

Sympy [F]

$$\int \frac{1}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(-8c+dx^3)^2 (c+dx^3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(1/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)^2}\,dx$$

```
[In] integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Giac [F]

$$\int \frac{1}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(dx^3+c)^{\frac{3}{2}}(dx^3-8\,c)^2}\,dx$$

```
[In] integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(d\,x^3+c)^{3/2}\,(8\,c-d\,x^3)^2}\,dx$$

```
[In] int(1/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
[Out] int(1/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

3.458 $\int \frac{1}{x^3(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{1}{x^3 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = -\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{2}{3},2,\frac{3}{2},\frac{1}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{128c^3x^2\sqrt{c+dx^3}}$$

```
[Out] -1/128*AppellF1(-2/3,3/2,2,1/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^3/
x^2/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {525, 524}

$$\int \frac{1}{x^3 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = -\frac{\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{2}{3},2,\frac{3}{2},\frac{1}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{128c^3x^2\sqrt{c+dx^3}}$$

```
[In] Int[1/(x^3*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/128*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 2, 3/2, 1/3, (d*x^3)/(8*c), -((d
*x^3)/c)])/(c^3*x^2*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^3(8c-dx^3)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$

$$= -\frac{\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{2}{3};2,\frac{3}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{128c^3x^2\sqrt{c+dx^3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 259 vs. 2(66) = 132.

Time = 10.25 (sec) , antiderivative size = 259, normalized size of antiderivative = 3.92

$$\int \frac{1}{x^3\,(8c-dx^3)^2\,(c+dx^3)^{3/2}}\,dx = \frac{167d^2x^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+\frac{64c\left(-648c^2-1249cdx^3+16\right.}{}}{6635}$$

```
[In] Integrate[1/(x^3*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (167*d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (
d*x^3)/(8*c)] + (64*c*(-648*c^2 - 1249*c*d*x^3 + 167*d^2*x^6 - (19648*c^2*d
*x^3*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(32*c*AppellF
1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1
/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((
d*x^3)/c), (d*x^3)/(8*c)]))))/(8*c - d*x^3))/(663552*c^5*x^2*Sqrt[c + d*x^3
])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 6.04 (sec) , antiderivative size = 764, normalized size of antiderivative = 11.58

method	result	size
elliptic	Expression too large to display	764
risch	Expression too large to display	1760
default	Expression too large to display	1806

```
[In] int(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15552*d*x/c^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)-2/243*d*x/c^4/((x^3+c/d)*d)^(1
/2)-1/128*(d*x^3+c)^(1/2)/c^4/x^2+167/31104*I/c^4*3^(1/2)*(-c*d^2)^(1/3)*(I
*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)
^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/
d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(
-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2
*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/5184*I/c^4/d^2*2^(1/2)*sum(1/_alpha^
2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3
)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^
(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(
-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_al
pha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha
*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*
d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d
-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d
^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2650 vs. 2(52) = 104.

Time = 2.07 (sec) , antiderivative size = 2650, normalized size of antiderivative = 40.15

$$\int \frac{1}{x^3 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/82944*(1264*(d^2*x^8 - 7*c*d*x^5 - 8*c^2*x^2)*sqrt(d)*weierstrassPInvers
e(0, -4*c/d, x) - (c^4*d^2*x^8 - 7*c^5*d*x^5 - 8*c^6*x^2 + sqrt(-3)*(c^4*d^
```

```
2*x^8 - 7*c^5*d*x^5 - 8*c^6*x^2))*(d^4/c^25)^(1/6)*log((d^6*x^9 + 318*c*d^5
*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 - 9*(c^17*d^3*x^8 + 38*c^18*d^2*x^5 +
 64*c^19*d*x^2 + sqrt(-3)*(c^17*d^3*x^8 + 38*c^18*d^2*x^5 + 64*c^19*d*x^2))
*(d^4/c^25)^(2/3) + 3*sqrt(d*x^3 + c)*((c^21*d^2*x^7 + 80*c^22*d*x^4 + 160*
c^23*x - sqrt(-3)*(c^21*d^2*x^7 + 80*c^22*d*x^4 + 160*c^23*x))*(d^4/c^25)^(
5/6) - 2*(7*c^13*d^3*x^6 + 152*c^14*d^2*x^3 + 64*c^15*d)*sqrt(d^4/c^25) + 6
*(5*c^5*d^4*x^5 + 32*c^6*d^3*x^2 + sqrt(-3)*(5*c^5*d^4*x^5 + 32*c^6*d^3*x^2
))*(d^4/c^25)^(1/6)) - 9*(5*c^9*d^4*x^7 + 64*c^10*d^3*x^4 + 32*c^11*d^2*x -
 sqrt(-3)*(5*c^9*d^4*x^7 + 64*c^10*d^3*x^4 + 32*c^11*d^2*x))*(d^4/c^25)^(1/
3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (c^4*d^2*x^8 - 7*
c^5*d*x^5 - 8*c^6*x^2 + sqrt(-3)*(c^4*d^2*x^8 - 7*c^5*d*x^5 - 8*c^6*x^2))*(
d^4/c^25)^(1/6)*log((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d
^3 - 9*(c^17*d^3*x^8 + 38*c^18*d^2*x^5 + 64*c^19*d*x^2 + sqrt(-3)*(c^17*d^3
*x^8 + 38*c^18*d^2*x^5 + 64*c^19*d*x^2))*(d^4/c^25)^(2/3) - 3*sqrt(d*x^3 +
c)*((c^21*d^2*x^7 + 80*c^22*d*x^4 + 160*c^23*x - sqrt(-3)*(c^21*d^2*x^7 + 8
0*c^22*d*x^4 + 160*c^23*x))*(d^4/c^25)^(5/6) - 2*(7*c^13*d^3*x^6 + 152*c^14
*d^2*x^3 + 64*c^15*d)*sqrt(d^4/c^25) + 6*(5*c^5*d^4*x^5 + 32*c^6*d^3*x^2 +
sqrt(-3)*(5*c^5*d^4*x^5 + 32*c^6*d^3*x^2))*(d^4/c^25)^(1/6)) - 9*(5*c^9*d^4
*x^7 + 64*c^10*d^3*x^4 + 32*c^11*d^2*x - sqrt(-3)*(5*c^9*d^4*x^7 + 64*c^10*
d^3*x^4 + 32*c^11*d^2*x))*(d^4/c^25)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c
^2*d*x^3 - 512*c^3)) - (c^4*d^2*x^8 - 7*c^5*d*x^5 - 8*c^6*x^2 - sqrt(-3)*(c
^4*d^2*x^8 - 7*c^5*d*x^5 - 8*c^6*x^2))*(d^4/c^25)^(1/6)*log((d^6*x^9 + 318*
c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 - 9*(c^17*d^3*x^8 + 38*c^18*d^2*
x^5 + 64*c^19*d*x^2 - sqrt(-3)*(c^17*d^3*x^8 + 38*c^18*d^2*x^5 + 64*c^19*d*
x^2))*(d^4/c^25)^(2/3) + 3*sqrt(d*x^3 + c)*((c^21*d^2*x^7 + 80*c^22*d*x^4 +
 160*c^23*x + sqrt(-3)*(c^21*d^2*x^7 + 80*c^22*d*x^4 + 160*c^23*x))*(d^4/c^
25)^(5/6) - 2*(7*c^13*d^3*x^6 + 152*c^14*d^2*x^3 + 64*c^15*d)*sqrt(d^4/c^25
) + 6*(5*c^5*d^4*x^5 + 32*c^6*d^3*x^2 - sqrt(-3)*(5*c^5*d^4*x^5 + 32*c^6*d^
3*x^2))*(d^4/c^25)^(1/6)) - 9*(5*c^9*d^4*x^7 + 64*c^10*d^3*x^4 + 32*c^11*d^
2*x + sqrt(-3)*(5*c^9*d^4*x^7 + 64*c^10*d^3*x^4 + 32*c^11*d^2*x))*(d^4/c^25
)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + (c^4*d^2*x^8
 - 7*c^5*d*x^5 - 8*c^6*x^2 - sqrt(-3)*(c^4*d^2*x^8 - 7*c^5*d*x^5 - 8*c^6*x^
2))*(d^4/c^25)^(1/6)*log((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*
c^3*d^3 - 9*(c^17*d^3*x^8 + 38*c^18*d^2*x^5 + 64*c^19*d*x^2 - sqrt(-3)*(c^1
7*d^3*x^8 + 38*c^18*d^2*x^5 + 64*c^19*d*x^2))*(d^4/c^25)^(2/3) - 3*sqrt(d*x
^3 + c)*((c^21*d^2*x^7 + 80*c^22*d*x^4 + 160*c^23*x + sqrt(-3)*(c^21*d^2*x^
7 + 80*c^22*d*x^4 + 160*c^23*x))*(d^4/c^25)^(5/6) - 2*(7*c^13*d^3*x^6 + 152
*c^14*d^2*x^3 + 64*c^15*d)*sqrt(d^4/c^25) + 6*(5*c^5*d^4*x^5 + 32*c^6*d^3*x
^2 - sqrt(-3)*(5*c^5*d^4*x^5 + 32*c^6*d^3*x^2))*(d^4/c^25)^(1/6)) - 9*(5*c^
9*d^4*x^7 + 64*c^10*d^3*x^4 + 32*c^11*d^2*x + sqrt(-3)*(5*c^9*d^4*x^7 + 64*
c^10*d^3*x^4 + 32*c^11*d^2*x))*(d^4/c^25)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 +
192*c^2*d*x^3 - 512*c^3)) - 2*(c^4*d^2*x^8 - 7*c^5*d*x^5 - 8*c^6*x^2)*(d^4/
c^25)^(1/6)*log((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 +
 18*(c^17*d^3*x^8 + 38*c^18*d^2*x^5 + 64*c^19*d*x^2)*(d^4/c^25)^(2/3) + 6*s
qrt(d*x^3 + c)*((c^21*d^2*x^7 + 80*c^22*d*x^4 + 160*c^23*x)*(d^4/c^25)^(5/6
```

```
) + (7*c^13*d^3*x^6 + 152*c^14*d^2*x^3 + 64*c^15*d)*sqrt(d^4/c^25) + 6*(5*c
^5*d^4*x^5 + 32*c^6*d^3*x^2)*(d^4/c^25)^(1/6)) + 18*(5*c^9*d^4*x^7 + 64*c^1
0*d^3*x^4 + 32*c^11*d^2*x)*(d^4/c^25)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*
c^2*d*x^3 - 512*c^3)) + 2*(c^4*d^2*x^8 - 7*c^5*d*x^5 - 8*c^6*x^2)*(d^4/c^25
)^(1/6)*log((d^6*x^9 + 318*c*d^5*x^6 + 1200*c^2*d^4*x^3 + 640*c^3*d^3 + 18*
(c^17*d^3*x^8 + 38*c^18*d^2*x^5 + 64*c^19*d*x^2)*(d^4/c^25)^(2/3) - 6*sqrt(
d*x^3 + c)*((c^21*d^2*x^7 + 80*c^22*d*x^4 + 160*c^23*x)*(d^4/c^25)^(5/6) +
(7*c^13*d^3*x^6 + 152*c^14*d^2*x^3 + 64*c^15*d)*sqrt(d^4/c^25) + 6*(5*c^5*d
^4*x^5 + 32*c^6*d^3*x^2)*(d^4/c^25)^(1/6)) + 18*(5*c^9*d^4*x^7 + 64*c^10*d^
3*x^4 + 32*c^11*d^2*x)*(d^4/c^25)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*
d*x^3 - 512*c^3)) + 8*(167*d^2*x^6 - 1249*c*d*x^3 - 648*c^2)*sqrt(d*x^3 + c
))/(c^4*d^2*x^8 - 7*c^5*d*x^5 - 8*c^6*x^2)
```

Sympy [F]

$$\int \frac{1}{x^3\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{x^3\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**3/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(1/(x**3*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x^3}\,dx$$

```
[In] integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x^3}\,dx$$

```
[In] integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(8c-dx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx = \int \frac{1}{x^3\,{\left(d\,x^3+c\right)}^{3/2}\,{\left(8\,c-d\,x^3\right)}^2} \,d x$$

```
[In] int(1/(x^3*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
[Out] int(1/(x^3*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

3.459 $\int \frac{1}{x^6(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{1}{x^6\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=-\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{5}{3},2,\frac{3}{2},-\frac{2}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{320c^3x^5\sqrt{c+dx^3}}$$

```
[Out] -1/320*AppellF1(-5/3,3/2,2,-2/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^3
/x^5/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {525, 524}

$$\int \frac{1}{x^6\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=-\frac{\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{5}{3},2,\frac{3}{2},-\frac{2}{3},\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{320c^3x^5\sqrt{c+dx^3}}$$

```
[In] Int[1/(x^6*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/320*(Sqrt[1 + (d*x^3)/c]*AppellF1[-5/3, 2, 3/2, -2/3, (d*x^3)/(8*c), -((
d*x^3)/c)])/(c^3*x^5*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^6(8c-dx^3)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$

$$= -\frac{\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{5}{3};2,\frac{3}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{320c^3x^5\sqrt{c+dx^3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 283 vs. $2(66) = 132$.

Time = 10.24 (sec) , antiderivative size = 283, normalized size of antiderivative = 4.29

$$\int \frac{1}{x^6\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{\frac{64\left(2592c^3-7128c^2dx^3-15373cd^2x^6+2027d^3x^9\right)}{c^5x^5\left(-8c+dx^3\right)} - \frac{2027d^3x^4\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},\ldots\right.}{c^6}}{\ldots}$$

```
[In] Integrate[1/(x^6*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] ((64*(2592*c^3 - 7128*c^2*d*x^3 - 15373*c*d^2*x^6 + 2027*d^3*x^9))/(c^5*x^5
*(-8*c + d*x^3)) - (2027*d^3*x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1,
7/3, -((d*x^3)/c), (d*x^3)/(8*c)])/c^6 + (16789504*d^2*x*AppellF1[1/3, 1/2,
 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(c^3*(8*c - d*x^3)*(32*c*AppellF1[1/
3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2,
2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^
3)/c), (d*x^3)/(8*c)]))))/(6635520*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 6.21 (sec) , antiderivative size = 787, normalized size of antiderivative = 11.92

method	result	size
elliptic	Expression too large to display	787
risch	Expression too large to display	1772
default	Expression too large to display	2157

```
[In] int(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/124416*d^2*x/c^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+2/243*d^2*x/c^5/((x^3+c/d)*
d)^(1/2)-1/320*(d*x^3+c)^(1/2)/c^4/x^5+29/2560*d*(d*x^3+c)^(1/2)/c^5/x^2-20
27/311040*I*d/c^5*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(
1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+
1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/
d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1
/2))-1/31104*I/c^5/d*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/
d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1
/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/
2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(
1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^
(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(
-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c
,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2698 vs. 2(52) = 104.

Time = 5.32 (sec) , antiderivative size = 2698, normalized size of antiderivative = 40.88

$$\int \frac{1}{x^6 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2488320*(49008*(d^3*x^11 - 7*c*d^2*x^8 - 8*c^2*d*x^5)*sqrt(d)*weierstrass
PInverse(0, -4*c/d, x) + 5*(c^5*d^2*x^11 - 7*c^6*d*x^8 - 8*c^7*x^5 + sqrt(-
```

3)*(c^5*d^2*x^11 - 7*c^6*d*x^8 - 8*c^7*x^5))*(d^10/c^31)^(1/6)*log((d^11*x^
9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 - 9*(c^21*d^4*x^8 + 38*
c^22*d^3*x^5 + 64*c^23*d^2*x^2 + sqrt(-3)*(c^21*d^4*x^8 + 38*c^22*d^3*x^5 +
64*c^23*d^2*x^2))*(d^10/c^31)^(2/3) + 3*sqrt(d*x^3 + c)*((c^26*d^2*x^7 + 8
0*c^27*d*x^4 + 160*c^28*x - sqrt(-3)*(c^26*d^2*x^7 + 80*c^27*d*x^4 + 160*c^
28*x))*(d^10/c^31)^(5/6) - 2*(7*c^16*d^5*x^6 + 152*c^17*d^4*x^3 + 64*c^18*d
^3)*sqrt(d^10/c^31) + 6*(5*c^6*d^8*x^5 + 32*c^7*d^7*x^2 + sqrt(-3)*(5*c^6*d
^8*x^5 + 32*c^7*d^7*x^2))*(d^10/c^31)^(1/6)) - 9*(5*c^11*d^7*x^7 + 64*c^12*
d^6*x^4 + 32*c^13*d^5*x - sqrt(-3)*(5*c^11*d^7*x^7 + 64*c^12*d^6*x^4 + 32*c
^13*d^5*x))*(d^10/c^31)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 51
2*c^3)) - 5*(c^5*d^2*x^11 - 7*c^6*d*x^8 - 8*c^7*x^5 + sqrt(-3)*(c^5*d^2*x^1
1 - 7*c^6*d*x^8 - 8*c^7*x^5))*(d^10/c^31)^(1/6)*log((d^11*x^9 + 318*c*d^10*
x^6 + 1200*c^2*d^9*x^3 + 640*c^3*d^8 - 9*(c^21*d^4*x^8 + 38*c^22*d^3*x^5 +
64*c^23*d^2*x^2 + sqrt(-3)*(c^21*d^4*x^8 + 38*c^22*d^3*x^5 + 64*c^23*d^2*x^
2))*(d^10/c^31)^(2/3) - 3*sqrt(d*x^3 + c)*((c^26*d^2*x^7 + 80*c^27*d*x^4 +
160*c^28*x - sqrt(-3)*(c^26*d^2*x^7 + 80*c^27*d*x^4 + 160*c^28*x))*(d^10/c^
31)^(5/6) - 2*(7*c^16*d^5*x^6 + 152*c^17*d^4*x^3 + 64*c^18*d^3)*sqrt(d^10/c
^31) + 6*(5*c^6*d^8*x^5 + 32*c^7*d^7*x^2 + sqrt(-3)*(5*c^6*d^8*x^5 + 32*c^7
*d^7*x^2))*(d^10/c^31)^(1/6)) - 9*(5*c^11*d^7*x^7 + 64*c^12*d^6*x^4 + 32*c^
13*d^5*x - sqrt(-3)*(5*c^11*d^7*x^7 + 64*c^12*d^6*x^4 + 32*c^13*d^5*x))*(d^
10/c^31)^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 5*(c^
5*d^2*x^11 - 7*c^6*d*x^8 - 8*c^7*x^5 - sqrt(-3)*(c^5*d^2*x^11 - 7*c^6*d*x^8
- 8*c^7*x^5))*(d^10/c^31)^(1/6)*log((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*
d^9*x^3 + 640*c^3*d^8 - 9*(c^21*d^4*x^8 + 38*c^22*d^3*x^5 + 64*c^23*d^2*x^2
- sqrt(-3)*(c^21*d^4*x^8 + 38*c^22*d^3*x^5 + 64*c^23*d^2*x^2))*(d^10/c^31)
^(2/3) + 3*sqrt(d*x^3 + c)*((c^26*d^2*x^7 + 80*c^27*d*x^4 + 160*c^28*x + sq
rt(-3)*(c^26*d^2*x^7 + 80*c^27*d*x^4 + 160*c^28*x))*(d^10/c^31)^(5/6) - 2*(
7*c^16*d^5*x^6 + 152*c^17*d^4*x^3 + 64*c^18*d^3)*sqrt(d^10/c^31) + 6*(5*c^6
*d^8*x^5 + 32*c^7*d^7*x^2 - sqrt(-3)*(5*c^6*d^8*x^5 + 32*c^7*d^7*x^2))*(d^1
0/c^31)^(1/6)) - 9*(5*c^11*d^7*x^7 + 64*c^12*d^6*x^4 + 32*c^13*d^5*x + sqrt
(-3)*(5*c^11*d^7*x^7 + 64*c^12*d^6*x^4 + 32*c^13*d^5*x))*(d^10/c^31)^(1/3))
/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 5*(c^5*d^2*x^11 - 7*
c^6*d*x^8 - 8*c^7*x^5 - sqrt(-3)*(c^5*d^2*x^11 - 7*c^6*d*x^8 - 8*c^7*x^5))*
(d^10/c^31)^(1/6)*log((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^3 + 640*c
^3*d^8 - 9*(c^21*d^4*x^8 + 38*c^22*d^3*x^5 + 64*c^23*d^2*x^2 - sqrt(-3)*(c^
21*d^4*x^8 + 38*c^22*d^3*x^5 + 64*c^23*d^2*x^2))*(d^10/c^31)^(2/3) - 3*sqrt
(d*x^3 + c)*((c^26*d^2*x^7 + 80*c^27*d*x^4 + 160*c^28*x + sqrt(-3)*(c^26*d^
2*x^7 + 80*c^27*d*x^4 + 160*c^28*x))*(d^10/c^31)^(5/6) - 2*(7*c^16*d^5*x^6
+ 152*c^17*d^4*x^3 + 64*c^18*d^3)*sqrt(d^10/c^31) + 6*(5*c^6*d^8*x^5 + 32*c
^7*d^7*x^2 - sqrt(-3)*(5*c^6*d^8*x^5 + 32*c^7*d^7*x^2))*(d^10/c^31)^(1/6))
- 9*(5*c^11*d^7*x^7 + 64*c^12*d^6*x^4 + 32*c^13*d^5*x + sqrt(-3)*(5*c^11*d^
7*x^7 + 64*c^12*d^6*x^4 + 32*c^13*d^5*x))*(d^10/c^31)^(1/3))/(d^3*x^9 - 24*
c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 10*(c^5*d^2*x^11 - 7*c^6*d*x^8 - 8*
c^7*x^5)*(d^10/c^31)^(1/6)*log((d^11*x^9 + 318*c*d^10*x^6 + 1200*c^2*d^9*x^
3 + 640*c^3*d^8 + 18*(c^21*d^4*x^8 + 38*c^22*d^3*x^5 + 64*c^23*d^2*x^2)*(d^

```
10/c^31)^(2/3) + 6*sqrt(d*x^3 + c)*((c^26*d^2*x^7 + 80*c^27*d*x^4 + 160*c^2
8*x)*(d^10/c^31)^(5/6) + (7*c^16*d^5*x^6 + 152*c^17*d^4*x^3 + 64*c^18*d^3)*
sqrt(d^10/c^31) + 6*(5*c^6*d^8*x^5 + 32*c^7*d^7*x^2)*(d^10/c^31)^(1/6)) + 1
8*(5*c^11*d^7*x^7 + 64*c^12*d^6*x^4 + 32*c^13*d^5*x)*(d^10/c^31)^(1/3))/(d^
3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 10*(c^5*d^2*x^11 - 7*c^6
*d*x^8 - 8*c^7*x^5)*(d^10/c^31)^(1/6)*log((d^11*x^9 + 318*c*d^10*x^6 + 1200
*c^2*d^9*x^3 + 640*c^3*d^8 + 18*(c^21*d^4*x^8 + 38*c^22*d^3*x^5 + 64*c^23*d
^2*x^2)*(d^10/c^31)^(2/3) - 6*sqrt(d*x^3 + c)*((c^26*d^2*x^7 + 80*c^27*d*x^
4 + 160*c^28*x)*(d^10/c^31)^(5/6) + (7*c^16*d^5*x^6 + 152*c^17*d^4*x^3 + 64
*c^18*d^3)*sqrt(d^10/c^31) + 6*(5*c^6*d^8*x^5 + 32*c^7*d^7*x^2)*(d^10/c^31)
^(1/6)) + 18*(5*c^11*d^7*x^7 + 64*c^12*d^6*x^4 + 32*c^13*d^5*x)*(d^10/c^31)
^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) + 24*(2027*d^3*
x^9 - 15373*c*d^2*x^6 - 7128*c^2*d*x^3 + 2592*c^3)*sqrt(d*x^3 + c))/(c^5*d^
2*x^11 - 7*c^6*d*x^8 - 8*c^7*x^5)
```

Sympy [F]

$$\int \frac{1}{x^6 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{x^6 (-8c+dx^3)^2 (c+dx^3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**6/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(1/(x**6*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^6 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(dx^3+c)^{\frac{3}{2}} (dx^3-8\,c)^2 x^6}\,dx$$

```
[In] integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 (8c-dx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(dx^3+c)^{\frac{3}{2}} (dx^3-8\,c)^2 x^6}\,dx$$

```
[In] integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 \left(8c-dx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx = \int \frac{1}{x^6 \left(d\,x^3+c\right)^{3/2} \left(8\,c-d\,x^3\right)^2}\,dx$$

```
[In] int(1/(x^6*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
[Out] int(1/(x^6*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

3.460 $\int \frac{x^8\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 161

$$\int \frac{x^8\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx = -\frac{a(4bc-5ad)\sqrt{c+dx^3}}{3b^3(bc-ad)}+\frac{2\left(c+dx^3\right)^{3/2}}{9b^2d} - \frac{a^2\left(c+dx^3\right)^{3/2}}{3b^2(bc-ad)\left(a+bx^3\right)}+\frac{a(4bc-5ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}\sqrt{bc-ad}}$$

```
[Out] 2/9*(d*x^3+c)^(3/2)/b^2/d-1/3*a^2*(d*x^3+c)^(3/2)/b^2/(-a*d+b*c)/(b*x^3+a)+
1/3*a*(-5*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(7
/2)/(-a*d+b*c)^(1/2)-1/3*a*(-5*a*d+4*b*c)*(d*x^3+c)^(1/2)/b^3/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {457, 91, 81, 52, 65, 214}

$$\int \frac{x^8\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx = -\frac{a^2\left(c+dx^3\right)^{3/2}}{3b^2\left(a+bx^3\right)(bc-ad)}+\frac{a(4bc-5ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}\sqrt{bc-ad}} - \frac{a\sqrt{c+dx^3}(4bc-5ad)}{3b^3(bc-ad)}+\frac{2\left(c+dx^3\right)^{3/2}}{9b^2d}$$

```
[In] Int[(x^8*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] -1/3*(a*(4*b*c - 5*a*d)*Sqrt[c + d*x^3])/(b^3*(b*c - a*d)) + (2*(c + d*x^3)
^(3/2))/(9*b^2*d) - (a^2*(c + d*x^3)^(3/2))/(3*b^2*(b*c - a*d)*(a + b*x^3))
```

```
 + (a*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(
3*b^(7/2)*Sqrt[b*c - a*d])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2\sqrt{c+dx}}{(a+bx)^2}\,dx, x, x^3\right) \\
&= -\frac{a^2(c+dx^3)^{3/2}}{3b^2(bc-ad)\,(a+bx^3)} + \frac{\text{Subst}\left(\int \frac{\sqrt{c+dx}\left(-\frac{1}{2}a(2bc-3ad)+b(bc-ad)x\right)}{a+bx}\,dx, x, x^3\right)}{3b^2(bc-ad)} \\
&= \frac{2(c+dx^3)^{3/2}}{9b^2d} - \frac{a^2(c+dx^3)^{3/2}}{3b^2(bc-ad)\,(a+bx^3)} - \frac{(a(4bc-5ad))\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)}{6b^2(bc-ad)} \\
&= -\frac{a(4bc-5ad)\sqrt{c+dx^3}}{3b^3(bc-ad)} + \frac{2(c+dx^3)^{3/2}}{9b^2d} - \frac{a^2(c+dx^3)^{3/2}}{3b^2(bc-ad)\,(a+bx^3)} \\
&\quad - \frac{(a(4bc-5ad))\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{6b^3} \\
&= -\frac{a(4bc-5ad)\sqrt{c+dx^3}}{3b^3(bc-ad)} + \frac{2(c+dx^3)^{3/2}}{9b^2d} - \frac{a^2(c+dx^3)^{3/2}}{3b^2(bc-ad)\,(a+bx^3)} \\
&\quad - \frac{(a(4bc-5ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3b^3d} \\
&= -\frac{a(4bc-5ad)\sqrt{c+dx^3}}{3b^3(bc-ad)} + \frac{2(c+dx^3)^{3/2}}{9b^2d} \\
&\quad - \frac{a^2(c+dx^3)^{3/2}}{3b^2(bc-ad)\,(a+bx^3)} + \frac{a(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.78

$$\int \frac{x^8\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \frac{\sqrt{c+dx^3}(-15a^2d+2ab(c-5dx^3)+2b^2x^3(c+dx^3))}{9b^3d\,(a+bx^3)} + \frac{a(-4bc+5ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{7/2}\sqrt{-bc+ad}}$$

```
[In] Integrate[(x^8*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
[Out] (Sqrt[c + d*x^3]*(-15*a^2*d + 2*a*b*(c - 5*d*x^3) + 2*b^2*x^3*(c + d*x^3)))
/(9*b^3*d*(a + b*x^3)) + (a*(-4*b*c + 5*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3
])/Sqrt[-(b*c) + a*d]])/(3*b^(7/2)*Sqrt[-(b*c) + a*d])
```

Maple [A] (verified)

Time = 4.80 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$-\frac{5\left(-\left(ad-\frac{4bc}{5}\right)d\left(b\,x^{3}+a\right)a\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{(ad-bc)b}\left(-\frac{2x^{3}\left(d\,x^{3}+c\right)b^{2}}{15}-\frac{2a\left(-5d\,x^{3}+c\right)b}{15}+a^{2}d\right)\sqrt{d\,x^{3}+c}\right)}{3\sqrt{(ad-bc)b}\,d\,b^{3}\left(b\,x^{3}+a\right)}$
default	$\frac{2\left(d\,x^{3}+c\right)^{\frac{3}{2}}}{9b^{2}d}+\frac{a^{2}\left(-\frac{\sqrt{d\,x^{3}+c}}{b\,x^{3}+a}+\frac{d\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}\right)}{3b^{3}}-\frac{4a\left(\sqrt{d\,x^{3}+c}-\frac{(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}\right)}{3b^{3}}$
risch	$-\frac{2\left(-bd\,x^{3}+6ad-bc\right)\sqrt{d\,x^{3}+c}}{9d\,b^{3}}+\frac{a\left(\frac{2(3ad-2bc)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}-\frac{a\left(d\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)\left(b\,x^{3}+a\right)+\sqrt{d\,x^{3}+c}\,\sqrt{(ad-bc)b}\right)}{3\sqrt{(ad-bc)b}\left(b\,x^{3}+a\right)}\right)}{b^{3}}$
elliptic	$-\frac{a^{2}\sqrt{d\,x^{3}+c}}{3b^{3}\left(b\,x^{3}+a\right)}+\frac{2x^{3}\sqrt{d\,x^{3}+c}}{9b^{2}}+\frac{2\left(-\frac{2ad-bc}{b^{3}}-\frac{2c}{3b^{2}}\right)\sqrt{d\,x^{3}+c}}{3d}-\frac{ia\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{(5ad-4bc)\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id}{-}}}{}\right)}{}$

[In] int(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)

[Out] -5/3/((a*d-b*c)*b)^(1/2)*(-(a*d-4/5*b*c)*d*(b*x^3+a)*a*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(-2/15*x^3*(d*x^3+c)*b^2-2/15*a*(-5*d*x^3+c)*b+a^2*d)*(d*x^3+c)^(1/2))/d/b^3/(b*x^3+a)

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.91

$$\int \frac{x^8\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \left[-\frac{3\,(4\,a^2bcd-5\,a^3d^2+(4\,ab^2cd-5\,a^2bd^2)x^3)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)-2\,(2\,(b^4cd\cdots}{18\,(ab^5cd-a^2b^4d^2+(b^6cd\cdots}\right.$$
$$-\frac{3\,(4\,a^2bcd-5\,a^3d^2+(4\,ab^2cd-5\,a^2bd^2)x^3)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-b^2c+abd}}{bdx^3+bc}\right)-(2\,(b^4cd-ab^3d^2)\cdots}{9\,(ab^5cd-a^2b^4d^2+(b^6cd-ab\cdots}$$

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/18*(3*(4*a^2*b*c*d - 5*a^3*d^2 + (4*a*b^2*c*d - 5*a^2*b*d^2)*x^3)*sqrt(
b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c -
a*b*d))/(b*x^3 + a)) - 2*(2*(b^4*c*d - a*b^3*d^2)*x^6 + 2*a*b^3*c^2 - 17*a^
2*b^2*c*d + 15*a^3*b*d^2 + 2*(b^4*c^2 - 6*a*b^3*c*d + 5*a^2*b^2*d^2)*x^3)*s
qrt(d*x^3 + c))/(a*b^5*c*d - a^2*b^4*d^2 + (b^6*c*d - a*b^5*d^2)*x^3), -1/9
*(3*(4*a^2*b*c*d - 5*a^3*d^2 + (4*a*b^2*c*d - 5*a^2*b*d^2)*x^3)*sqrt(-b^2*c
 + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) - (2
*(b^4*c*d - a*b^3*d^2)*x^6 + 2*a*b^3*c^2 - 17*a^2*b^2*c*d + 15*a^3*b*d^2 +
2*(b^4*c^2 - 6*a*b^3*c*d + 5*a^2*b^2*d^2)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c*d
- a^2*b^4*d^2 + (b^6*c*d - a*b^5*d^2)*x^3)]
```

Sympy [F]

$$\int \frac{x^8\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{x^8\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$$

```
[In] integrate(x**8*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

```
[Out] Integral(x**8*sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.84

$$\int \frac{x^8\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = -\frac{\sqrt{dx^3+c}a^2d}{3\left((dx^3+c)b-bc+ad\right)b^3} - \frac{(4\,abc-5\,a^2d)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}b^3} + \frac{2\left((dx^3+c)^{\frac{3}{2}}b^4d^2-6\sqrt{dx^3+c}ab^3d^3\right)}{9\,b^6d^3}$$

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*sqrt(d*x^3 + c)*a^2*d/(((d*x^3 + c)*b - b*c + a*d)*b^3) - 1/3*(4*a*b*c
 - 5*a^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a
*b*d)*b^3) + 2/9*((d*x^3 + c)^(3/2)*b^4*d^2 - 6*sqrt(d*x^3 + c)*a*b^3*d^3)/
(b^6*d^3)
```

Mupad [B] (verification not implemented)

Time = 10.88 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.25

$$\int \frac{x^8\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,b^2} - \frac{\sqrt{d\,x^3+c}\,\left(\frac{4\,c}{3\,b^2}-\frac{2\,b^2\,c-2\,a\,b\,d}{b^4}+\frac{2\,a\,d}{b^3}\right)}{3\,d} + \frac{a^2\,\left(\frac{2\,a\,d}{3\,(2\,b^2\,c-2\,a\,b\,d)}-\frac{2\,b\,c}{3\,(2\,b^2\,c-2\,a\,b\,d)}\right)\,\sqrt{d\,x^3+c}}{b^2\,\left(b\,x^3+a\right)} + \frac{a\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\left(5\,a\,d-4\,b\,c\right)\,1{}\mathrm{i}}{6\,b^{7/2}\,\sqrt{a\,d-b\,c}}$$

```
[In] int((x^8*(c + d*x^3)^(1/2))/(a + b*x^3)^2,x)
```

```
[Out] (2*x^3*(c + d*x^3)^(1/2))/(9*b^2) - ((c + d*x^3)^(1/2)*((4*c)/(3*b^2) - (2*
b^2*c - 2*a*b*d)/b^4 + (2*a*d)/b^3))/(3*d) + (a*log((2*b*c - a*d + b^(1/2)*
(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(5*a*d - 4*b
*c)*1i)/(6*b^(7/2)*(a*d - b*c)^(1/2)) + (a^2*((2*a*d)/(3*(2*b^2*c - 2*a*b*d
)) - (2*b*c)/(3*(2*b^2*c - 2*a*b*d)))*(c + d*x^3)^(1/2))/(b^2*(a + b*x^3))
```

3.461 $\int \frac{x^5\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 136

$$\int \frac{x^5\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \frac{(2bc-3ad)\sqrt{c+dx^3}}{3b^2(bc-ad)} + \frac{a(c+dx^3)^{3/2}}{3b(bc-ad)(a+bx^3)} - \frac{(2bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}$$

```
[Out] 1/3*a*(d*x^3+c)^(3/2)/b/(-a*d+b*c)/(b*x^3+a)-1/3*(-3*a*d+2*b*c)*arctanh(b^(
1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(5/2)/(-a*d+b*c)^(1/2)+1/3*(-3*a*d
+2*b*c)*(d*x^3+c)^(1/2)/b^2/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 79, 52, 65, 214}

$$\int \frac{x^5\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = -\frac{(2bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}} + \frac{\sqrt{c+dx^3}(2bc-3ad)}{3b^2(bc-ad)} + \frac{a(c+dx^3)^{3/2}}{3b(a+bx^3)(bc-ad)}$$

```
[In] Int[(x^5*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] ((2*b*c - 3*a*d)*Sqrt[c + d*x^3])/(3*b^2*(b*c - a*d)) + (a*(c + d*x^3)^(3/2
))/(3*b*(b*c - a*d)*(a + b*x^3)) - ((2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2)*Sqrt[b*c - a*d])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x\sqrt{c+dx}}{(a+bx)^2}\,dx, x, x^3\right)$$

$$= \frac{a(c+dx^3)^{3/2}}{3b(bc-ad)\,(a+bx^3)} + \frac{(2bc-3ad)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)}{6b(bc-ad)}$$

$$= \frac{(2bc-3ad)\sqrt{c+dx^3}}{3b^2(bc-ad)} + \frac{a(c+dx^3)^{3/2}}{3b(bc-ad)(a+bx^3)} + \frac{(2bc-3ad)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{6b^2}$$

$$= \frac{(2bc-3ad)\sqrt{c+dx^3}}{3b^2(bc-ad)} + \frac{a(c+dx^3)^{3/2}}{3b(bc-ad)(a+bx^3)} + \frac{(2bc-3ad)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3b^2d}$$

$$= \frac{(2bc-3ad)\sqrt{c+dx^3}}{3b^2(bc-ad)} + \frac{a(c+dx^3)^{3/2}}{3b(bc-ad)(a+bx^3)} - \frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.72

$$\int \frac{x^5\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \frac{\frac{\sqrt{b}(3a+2bx^3)\sqrt{c+dx^3}}{a+bx^3} + \frac{(2bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}}{3b^{5/2}}$$

```
[In] Integrate[(x^5*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] ((Sqrt[b]*(3*a + 2*b*x^3)*Sqrt[c + d*x^3])/(a + b*x^3) + ((2*b*c - 3*a*d)*A
rcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/Sqrt[-(b*c) + a*d])/(3
*b^(5/2))
```

Maple [A] (verified)

Time = 4.67 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$\frac{-\left(ad-\frac{2bc}{3}\right)\left(b\,x^{3}+a\right)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(ad-bc\right)b}}\right)+\left(\frac{2b\,x^{3}}{3}+a\right)\sqrt{\left(ad-bc\right)b}\,\sqrt{d\,x^{3}+c}}{\sqrt{\left(ad-bc\right)b}\,b^{2}\left(b\,x^{3}+a\right)}$
default	$\frac{\frac{2\sqrt{d\,x^{3}+c}}{3}-\frac{2\left(ad-bc\right)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(ad-bc\right)b}}\right)}{3\sqrt{\left(ad-bc\right)b}}}{b^{2}}-\frac{a\left(-\frac{\sqrt{d\,x^{3}+c}}{b\,x^{3}+a}+\frac{d\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(ad-bc\right)b}}\right)}{\sqrt{\left(ad-bc\right)b}}\right)}{3b^{2}}$
risch	$\frac{2\sqrt{d\,x^{3}+c}}{3b^{2}}-\frac{\frac{2\left(2ad-bc\right)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(ad-bc\right)b}}\right)}{3\sqrt{\left(ad-bc\right)b}}-\frac{a\left(d\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{\left(ad-bc\right)b}}\right)\left(b\,x^{3}+a\right)+\sqrt{d\,x^{3}+c}\,\sqrt{\left(ad-bc\right)b}\right)}{3\sqrt{\left(ad-bc\right)b}\,\left(b\,x^{3}+a\right)}}{b^{2}}$
elliptic	$\frac{a\sqrt{d\,x^{3}+c}}{3b^{2}\left(b\,x^{3}+a\right)}+\frac{2\sqrt{d\,x^{3}+c}}{3b^{2}}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(3ad-2bc\right)\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\cdots}}{\cdots}\right.}{\cdots}$

[In] int(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)

[Out] 1/((a*d-b*c)*b)^(1/2)*(-(a*d-2/3*b*c)*(b*x^3+a)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))+(2/3*b*x^3+a)*((a*d-b*c)*b)^(1/2)*(d*x^3+c)^(1/2))/b^2/(b*x^3+a)

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.46

$$\int \frac{x^5\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$$
$$=\left[-\frac{\left((2\,b^2c-3\,abd)x^3+2\,abc-3\,a^2d\right)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)-2\left(3\,ab^2c-3\,a^2bd+\cdots\right.}{6\left(ab^4c-a^2b^3d+(b^5c-ab^4d)x^3\right)}\right.$$

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(((2*b^2*c - 3*a*b*d)*x^3 + 2*a*b*c - 3*a^2*d)*sqrt(b^2*c - a*b*d)*lo
g((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 +
a)) - 2*(3*a*b^2*c - 3*a^2*b*d + 2*(b^3*c - a*b^2*d)*x^3)*sqrt(d*x^3 + c))/
(a*b^4*c - a^2*b^3*d + (b^5*c - a*b^4*d)*x^3), 1/3*(((2*b^2*c - 3*a*b*d)*x^
3 + 2*a*b*c - 3*a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^
2*c + a*b*d)/(b*d*x^3 + b*c)) + (3*a*b^2*c - 3*a^2*b*d + 2*(b^3*c - a*b^2*d
)*x^3)*sqrt(d*x^3 + c))/(a*b^4*c - a^2*b^3*d + (b^5*c - a*b^4*d)*x^3)]
```

Sympy [F]

$$\int \frac{x^5\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{x^5\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$$

```
[In] integrate(x**5*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

```
[Out] Integral(x**5*sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.75

$$\int \frac{x^5\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \frac{\sqrt{dx^3+c}ad}{3\left((dx^3+c)b-bc+ad\right)b^2} + \frac{(2\,bc-3\,ad)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}b^2} + \frac{2\sqrt{dx^3+c}}{3\,b^2}$$

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*sqrt(d*x^3 + c)*a*d/(((d*x^3 + c)*b - b*c + a*d)*b^2) + 1/3*(2*b*c - 3*
a*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b
^2) + 2/3*sqrt(d*x^3 + c)/b^2
```

Mupad [B] (verification not implemented)

Time = 10.13 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.12

$$\int \frac{x^5\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \frac{2\sqrt{d\,x^3+c}}{3\,b^2} - \frac{a\left(\frac{2\,a\,d}{3\,(2\,b^2\,c-2\,a\,b\,d)} - \frac{2\,b\,c}{3\,(2\,b^2\,c-2\,a\,b\,d)}\right)\sqrt{d\,x^3+c}}{b\,(b\,x^3+a)} + \frac{\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)(3\,a\,d-2\,b\,c)\,1\mathrm{i}}{6\,b^{5/2}\,\sqrt{a\,d-b\,c}}$$

```
[In] int((x^5*(c + d*x^3)^(1/2))/(a + b*x^3)^2,x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*b^2) + (log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/
2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*(3*a*d - 2*b*c)*1i)/(6*b^(5
/2)*(a*d - b*c)^(1/2)) - (a*((2*a*d)/(3*(2*b^2*c - 2*a*b*d)) - (2*b*c)/(3*(
2*b^2*c - 2*a*b*d)))*(c + d*x^3)^(1/2))/(b*(a + b*x^3))
```

3.462 $\int \frac{x^2\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \frac{x^2\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = -\frac{\sqrt{c+dx^3}}{3b(a+bx^3)} - \frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}$$

```
[Out] -1/3*d*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)
^(1/2)-1/3*(d*x^3+c)^(1/2)/b/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {455, 43, 65, 214}

$$\int \frac{x^2\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = -\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}} - \frac{\sqrt{c+dx^3}}{3b(a+bx^3)}$$

```
[In] Int[(x^2*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] -1/3*Sqrt[c + d*x^3]/(b*(a + b*x^3)) - (d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])
/Sqrt[b*c - a*d]])/(3*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{(a+bx)^2}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{c+dx^3}}{3b\,(a+bx^3)} + \frac{d\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{6b}\\
&= -\frac{\sqrt{c+dx^3}}{3b\,(a+bx^3)} + \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3b}\\
&= -\frac{\sqrt{c+dx^3}}{3b\,(a+bx^3)} - \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{x^2\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = -\frac{\sqrt{c+dx^3}}{3b\,(a+bx^3)} + \frac{d\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{3/2}\sqrt{-bc+ad}}$$

```
[In] Integrate[(x^2*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
[Out] -1/3*Sqrt[c + d*x^3]/(b*(a + b*x^3)) + (d*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/
Sqrt[-(b*c) + a*d]])/(3*b^(3/2)*Sqrt[-(b*c) + a*d])
```

Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.81

method	result
default	$\frac{-\frac{\sqrt{d\,x^3+c}}{b\,x^3+a}+\frac{d \arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}}{3b}$
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^3+c}}{b\,x^3+a}+\frac{d \arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}}{3b}$
elliptic	$-\frac{\sqrt{d\,x^3+c}}{3b(b\,x^3+a)}-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}}{}\right.}{}$

[In] int(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)

[Out] 1/3/b*(-(d*x^3+c)^(1/2)/(b*x^3+a)+d/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 255, normalized size of antiderivative = 3.19

$$\int \frac{x^2\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$$
$$= \left[\frac{(bdx^3+ad)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)-2\,\sqrt{dx^3+c}(b^2c-abd)}{6\,(ab^3c-a^2b^2d+(b^4c-ab^3d)x^3)}, \frac{(bdx^3+ad)\sqrt{-b^2c+\ldots}}{3\ldots}\right.$$

[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] [1/6*((b*d*x^3 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) - 2*sqrt(d*x^3 + c)*(b^2*c

```
- a*b*d))/(a*b^3*c - a^2*b^2*d + (b^4*c - a*b^3*d)*x^3), 1/3*((b*d*x^3 + a*
d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^
3 + b*c)) - sqrt(d*x^3 + c)*(b^2*c - a*b*d))/(a*b^3*c - a^2*b^2*d + (b^4*c
- a*b^3*d)*x^3)]
```

Sympy [F]

$$\int \frac{x^2\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx = \int \frac{x^2\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$$

```
[In] integrate(x**2*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

```
[Out] Integral(x**2*sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\int \frac{x^2\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx = \frac{d\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}b} - \frac{\sqrt{dx^3+c}d}{3\left((dx^3+c)b-bc+ad\right)b}$$

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*
b) - 1/3*sqrt(d*x^3 + c)*d/(((d*x^3 + c)*b - b*c + a*d)*b)
```

Mupad [B] (verification not implemented)

Time = 9.67 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.56

$$\int \frac{x^2\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx = \frac{\left(\frac{2\,a\,d}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}-\frac{2\,b\,c}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}\right)\,\sqrt{d\,x^3+c}}{b\,x^3+a} + \frac{d\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{6\,b^{3/2}\,\sqrt{a\,d-b\,c}}$$

```
[In] int((x^2*(c + d*x^3)^(1/2))/(a + b*x^3)^2,x)
```

```
[Out] (((2*a*d)/(3*(2*b^2*c - 2*a*b*d)) - (2*b*c)/(3*(2*b^2*c - 2*a*b*d)))*(c + d
*x^3)^(1/2))/(a + b*x^3) + (d*log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*
(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*1i)/(6*b^(3/2)*(a*d - b*c)^(1/
2))
```

3.463 $\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 121

$$\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)^2}\,dx=\frac{\sqrt{c+dx^3}}{3a(a+bx^3)}-\frac{2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}\sqrt{bc-ad}}$$

```
[Out] -2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/a^2+1/3*(-a*d+2*b*c)*arctanh(
b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/b^(1/2)/(-a*d+b*c)^(1/2)+1/3*
(d*x^3+c)^(1/2)/a/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 101, 162, 65, 214}

$$\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)^2}\,dx=\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}\sqrt{bc-ad}}-\frac{2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}+\frac{\sqrt{c+dx^3}}{3a(a+bx^3)}$$

```
[In] Int[Sqrt[c + d*x^3]/(x*(a + b*x^3)^2),x]
```

```
[Out] Sqrt[c + d*x^3]/(3*a*(a + b*x^3)) - (2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt
[c]])/(3*a^2) + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c -
 a*d]])/(3*a^2*Sqrt[b]*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x(a+bx)^2}\,dx, x, x^3\right) \\
&= \frac{\sqrt{c+dx^3}}{3a\,(a+bx^3)} - \frac{\text{Subst}\left(\int \frac{-c-\frac{dx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a} \\
&= \frac{\sqrt{c+dx^3}}{3a\,(a+bx^3)} + \frac{c\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{3a^2} - \frac{(2bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{6a^2}
\end{aligned}$$

$$= \frac{\sqrt{c+dx^3}}{3a(a+bx^3)} + \frac{(2c)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^2d} - \frac{(2bc-ad)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^2d}$$

$$= \frac{\sqrt{c+dx^3}}{3a(a+bx^3)} - \frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2} + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}\sqrt{bc-ad}}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)^2}\,dx = \frac{\frac{a\sqrt{c+dx^3}}{a+bx^3} + \frac{(-2bc+ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}\sqrt{-bc+ad}} - 2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}$$

```
[In] Integrate[Sqrt[c + d*x^3]/(x*(a + b*x^3)^2),x]
```

```
[Out] ((a*Sqrt[c + d*x^3])/(a + b*x^3) + ((-2*b*c + a*d)*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^3])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*Sqrt[-(b*c) + a*d]) - 2*Sqrt[c]*ArcT
anh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2)
```

Maple [A] (verified)

Time = 4.44 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99

method	result
pseudoelliptic	$\frac{(b\,x^3+a)(ad-2bc)\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)-2\left(\sqrt{c}\,(b\,x^3+a)\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)-\frac{\sqrt{d\,x^3+c}\,a}{2}\right)\sqrt{(ad-bc)b}}{3\sqrt{(ad-bc)b}\,a^2(b\,x^3+a)}$
default	$\frac{\frac{2\sqrt{d\,x^3+c}}{3}-\frac{2\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)\sqrt{c}}{3}}{a^2} - \frac{2\left(\sqrt{d\,x^3+c}-\frac{(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}\right)}{3a^2} - \frac{-\frac{\sqrt{d\,x^3+c}}{b\,x^3+a}+\frac{d\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}}{3a}$
elliptic	Expression too large to display

```
[In] int((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/((a*d-b*c)*b)^(1/2)*((b*x^3+a)*(a*d-2*b*c)*arctan(b*(d*x^3+c)^(1/2)/((a
*d-b*c)*b)^(1/2))-2*(c^(1/2)*(b*x^3+a)*arctanh((d*x^3+c)^(1/2)/c^(1/2))-1/2
*(d*x^3+c)^(1/2)*a)*((a*d-b*c)*b)^(1/2))/a^2/(b*x^3+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(97) = 194.

Time = 0.39 (sec) , antiderivative size = 856, normalized size of antiderivative = 7.07

$$\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)^2}\,dx$$
$$= \left[-\frac{((2\,b^2c-abd)x^3+2\,abc-a^2d)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)-2\,(ab^2c-a^2bd+(b^3c-\right.}{6\,(a^3b^2c-a^4bd+(a^2b^3c-a^3b^2d)x^3)}$$
$$-\frac{((2\,b^2c-abd)x^3+2\,abc-a^2d)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-b^2c+abd}}{bdx^3+bc}\right)-(ab^2c-a^2bd+(b^3c-ab^2d)x^3)}{3\,(a^3b^2c-a^4bd+(a^2b^3c-a^3b^2d)x^3)}$$
$$-\frac{((2\,b^2c-abd)x^3+2\,abc-a^2d)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-b^2c+abd}}{bdx^3+bc}\right)-2\,(ab^2c-a^2bd+(b^3c-ab^2d)x}{3\,(a^3b^2c-a^4bd+(a^2b^3c-a^3b^2d)x^3)}$$

```
[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(((2*b^2*c - a*b*d)*x^3 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b
*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a))
- 2*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt
(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*(a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a
^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^3), -1/3*(((2*b^2*c - a*b*d)
*x^3 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b
^2*c + a*b*d)/(b*d*x^3 + b*c)) - (a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^3
)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - (a*b^2*c - a
^2*b*d)*sqrt(d*x^3 + c))/(a^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^3
), 1/6*(4*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^3)*sqrt(-c)*arctan(sqrt(
d*x^3 + c)*sqrt(-c)/c) - ((2*b^2*c - a*b*d)*x^3 + 2*a*b*c - a^2*d)*sqrt(b^2
*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b
*d))/(b*x^3 + a)) + 2*(a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a^3*b^2*c - a^4
*b*d + (a^2*b^3*c - a^3*b^2*d)*x^3), -1/3*(((2*b^2*c - a*b*d)*x^3 + 2*a*b*c
 - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/
(b*d*x^3 + b*c)) - 2*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^3)*sqrt(-c)*a
rctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - (a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a
^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^3)]
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)^2}\,dx = \int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)^2}\,dx$$

```
[In] integrate((d*x**3+c)**(1/2)/x/(b*x**3+a)**2,x)
[Out] Integral(sqrt(c + d*x**3)/(x*(a + b*x**3)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)^2}\,dx = \int \frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)^2 x}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)^2}\,dx = \frac{\sqrt{dx^3+c}d}{3\left((dx^3+c)b-bc+ad\right)a} - \frac{(2\,bc-ad)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}a^2} + \frac{2\,c\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}}$$

```
[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x, algorithm="giac")
[Out] 1/3*sqrt(d*x^3 + c)*d/(((d*x^3 + c)*b - b*c + a*d)*a) - 1/3*(2*b*c - a*d)*a
rctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2) +
2/3*c*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c))
```

Mupad [B] (verification not implemented)

Time = 12.81 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)^2}\,dx=\frac{\sqrt{c}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a^2}-\frac{\left(\frac{b\,d}{3\,\left(b^2\,c-a\,b\,d\right)}-\frac{b^2\,c}{3\,a\,\left(b^2\,c-a\,b\,d\right)}\right)\,\sqrt{d\,x^3+c}}{b\,x^3+a}+\frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{d\,x^3+c}\,\sqrt{a\,b\,d-b^2\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\left(a\,d-2\,b\,c\right)\,1{}\mathrm{i}}{6\,a^2\,\sqrt{a\,b\,d-b^2\,c}}$$

```
[In] int((c + d*x^3)^(1/2)/(x*(a + b*x^3)^2),x)
```

```
[Out] (c^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))
)/x^6))/(3*a^2) - (((b*d)/(3*(b^2*c - a*b*d)) - (b^2*c)/(3*a*(b^2*c - a*b*d
)))*(c + d*x^3)^(1/2))/(a + b*x^3) + (log((2*b*c - a*d + (c + d*x^3)^(1/2)*
(a*b*d - b^2*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(a*d - 2*b*c)*1i)/(6*a^2*(
a*b*d - b^2*c)^(1/2))
```

3.464 $\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 161

$$\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)^2}\,dx = -\frac{2b\sqrt{c+dx^3}}{3a^2(a+bx^3)} - \frac{\sqrt{c+dx^3}}{3ax^3(a+bx^3)} + \frac{(4bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3\sqrt{c}} - \frac{\sqrt{b}(4bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3\sqrt{bc-ad}}$$

```
[Out] 1/3*(-a*d+4*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^3/c^(1/2)-1/3*(-3*a*d+4
*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/a^3/(-a*d+b
*c)^(1/2)-2/3*b*(d*x^3+c)^(1/2)/a^2/(b*x^3+a)-1/3*(d*x^3+c)^(1/2)/a/x^3/(b*
x^3+a)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {457, 101, 156, 162, 65, 214}

$$\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)^2}\,dx = -\frac{\sqrt{b}(4bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3\sqrt{bc-ad}} + \frac{(4bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3\sqrt{c}} - \frac{2b\sqrt{c+dx^3}}{3a^2(a+bx^3)} - \frac{\sqrt{c+dx^3}}{3ax^3(a+bx^3)}$$

```
[In] Int[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)^2),x]
```

```
[Out] (-2*b*Sqrt[c + d*x^3])/(3*a^2*(a + b*x^3)) - Sqrt[c + d*x^3]/(3*a*x^3*(a +
b*x^3)) + ((4*b*c - a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^3*Sqrt[c])
- (Sqrt[b]*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d
]])/(3*a^3*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(a+bx)^2}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{3ax^3(a+bx^3)} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-4bc+ad)-\frac{3bdx}{2}}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{3a}$$

$$= -\frac{2b\sqrt{c+dx^3}}{3a^2(a+bx^3)} - \frac{\sqrt{c+dx^3}}{3ax^3(a+bx^3)} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(bc-ad)(4bc-ad)-bd(bc-ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a^2(bc-ad)}$$

$$= -\frac{2b\sqrt{c+dx^3}}{3a^2(a+bx^3)} - \frac{\sqrt{c+dx^3}}{3ax^3(a+bx^3)} + \frac{(b(4bc-3ad))\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{6a^3} - \frac{(4bc-ad)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{6a^3}$$

$$= -\frac{2b\sqrt{c+dx^3}}{3a^2(a+bx^3)} - \frac{\sqrt{c+dx^3}}{3ax^3(a+bx^3)} + \frac{(b(4bc-3ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^3d} - \frac{(4bc-ad)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^3d}$$

$$= -\frac{2b\sqrt{c+dx^3}}{3a^2(a+bx^3)} - \frac{\sqrt{c+dx^3}}{3ax^3(a+bx^3)} + \frac{(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3\sqrt{c}} - \frac{\sqrt{b}(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3\sqrt{bc-ad}}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{c+dx^3}}{x^4 (a+bx^3)^2} \, dx = \frac{-\frac{a(a+2bx^3)\sqrt{c+dx^3}}{x^3(a+bx^3)} + \frac{\sqrt{b}(4bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}} + \frac{(4bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a^3}$$

```
[In] Integrate[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)^2),x]
```

```
[Out] (-((a*(a + 2*b*x^3)*Sqrt[c + d*x^3])/(x^3*(a + b*x^3))) + (Sqrt[b]*(4*b*c -
 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/Sqrt[-(b*c) +
 a*d] + ((4*b*c - a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/Sqrt[c])/(3*a^3)
```

Maple [A] (verified)

Time = 4.76 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$-\frac{-4x^3 b\sqrt{c}\,(b\,x^3+a)\left(bc-\frac{3ad}{4}\right)\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{(ad-bc)b}\left(x^3(b\,x^3+a)(ad-4bc)\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)+(2b\,x^3+a)\sqrt{d}\right.}{3\sqrt{c}\,\sqrt{(ad-bc)b}\,a^3(b\,x^3+a)x^3}$
risch	$-\frac{\sqrt{d\,x^3+c}}{3a^2x^3}-\frac{-\frac{2(-ad+4bc)\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3a\sqrt{c}}+\frac{2b\left(d\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)(b\,x^3+a)+\sqrt{d\,x^3+c}\,\sqrt{(ad-bc)b}\right)}{3\sqrt{(ad-bc)b}\,(b\,x^3+a)}+\frac{4b(ad-2bc)\arctan(}{3a\sqrt{(ad-b}}}{2a^2}$
default	$\frac{-\frac{\sqrt{d\,x^3+c}}{3x^3}-\frac{d\ \text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3\sqrt{c}}}{a^2}-\frac{2b\left(\frac{2\sqrt{d\,x^3+c}}{3}-\frac{2\ \text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)\sqrt{c}}{3}\right)}{a^3}+\frac{b\left(-\frac{\sqrt{d\,x^3+c}}{b\,x^3+a}+\frac{d\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}\right)}{3a^2}+$
elliptic	Expression too large to display

```
[In] int((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/c^(1/2)*(-4*x^3*b*c^(1/2)*(b*x^3+a)*(b*c-3/4*a*d)*arctan(b*(d*x^3+c)^(
1/2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(x^3*(b*x^3+a)*(a*d-4*b*c)*ar
ctanh((d*x^3+c)^(1/2)/c^(1/2))+(2*b*x^3+a)*(d*x^3+c)^(1/2)*c^(1/2)*a))/((a*
d-b*c)*b)^(1/2)/a^3/(b*x^3+a)/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 870, normalized size of antiderivative = 5.40

$$\int \frac{\sqrt{c+dx^3}}{x^4 (a+bx^3)^2}\,dx = \left[-\frac{((4\,b^2c^2-3\,abcd)x^6+(4\,abc^2-3\,a^2cd)x^3)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)+((4\,b^2c-\ldots}{6\,(a^3bcx^6+a^4cx^3)}\right.$$

$$-\frac{2\,((4\,b^2c^2-3\,abcd)x^6+(4\,abc^2-3\,a^2cd)x^3)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right)+((4\,b^2c-abd\ldots}{6\,(a^3bcx^6+a^4cx^3)}$$

$$-\frac{2\,((4\,b^2c-abd)x^6+(4\,abc-a^2d)x^3)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+((4\,b^2c^2-3\,abcd)x^6+(4\,abc^2-3\,a^2c\ldots}{6\,(a^3bcx^6+a^4cx^3)}$$

$$-\frac{((4\,b^2c^2-3\,abcd)x^6+(4\,abc^2-3\,a^2cd)x^3)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^3+bc}\right)+((4\,b^2c-abd)x\ldots}{3\,(a^3bcx^6+a^4cx^3)}$$

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(((4*b^2*c^2 - 3*a*b*c*d)*x^6 + (4*a*b*c^2 - 3*a^2*c*d)*x^3)*sqrt(b/(
b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt
(b/(b*c - a*d)))/(b*x^3 + a)) + ((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*
x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*(2*a*b*
c*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*c*x^6 + a^4*c*x^3), -1/6*(2*((4*b^2*
c^2 - 3*a*b*c*d)*x^6 + (4*a*b*c^2 - 3*a^2*c*d)*x^3)*sqrt(-b/(b*c - a*d))*ar
ctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) + (
(4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(
d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*(2*a*b*c*x^3 + a^2*c)*sqrt(d*x^3 + c))/(
a^3*b*c*x^6 + a^4*c*x^3), -1/6*(2*((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d
)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + ((4*b^2*c^2 - 3*a*b*c*
d)*x^6 + (4*a*b*c^2 - 3*a^2*c*d)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*
b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a))
 + 2*(2*a*b*c*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*c*x^6 + a^4*c*x^3), -1/3
*(((4*b^2*c^2 - 3*a*b*c*d)*x^6 + (4*a*b*c^2 - 3*a^2*c*d)*x^3)*sqrt(-b/(b*c
- a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 +
 b*c)) + ((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*sqrt(-c)*arctan(sq
rt(d*x^3 + c)*sqrt(-c)/c) + (2*a*b*c*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*c
*x^6 + a^4*c*x^3)]
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)^2}\,dx = \int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)^2}\,dx$$

```
[In] integrate((d*x**3+c)**(1/2)/x**4/(b*x**3+a)**2,x)
[Out] Integral(sqrt(c + d*x**3)/(x**4*(a + b*x**3)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}}{(bx^3+a)^2x^4}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{c+dx^3}}{x^4(a+bx^3)^2}\,dx = \frac{(4\,b^2c-3\,abd)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}a^3} - \frac{(4\,bc-ad)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^3\sqrt{-c}} - \frac{2\,(dx^3+c)^{\frac{3}{2}}bd-2\,\sqrt{dx^3+c}bcd+\sqrt{dx^3+c}ad^2}{3\left((dx^3+c)^2b-2\,(dx^3+c)bc+bc^2+(dx^3+c)ad-acd\right)a^2}$$

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x, algorithm="giac")
[Out] 1/3*(4*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqr
t(-b^2*c + a*b*d)*a^3) - 1/3*(4*b*c - a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))
/(a^3*sqrt(-c)) - 1/3*(2*(d*x^3 + c)^(3/2)*b*d - 2*sqrt(d*x^3 + c)*b*c*d +
sqrt(d*x^3 + c)*a*d^2)/(((d*x^3 + c)^2*b - 2*(d*x^3 + c)*b*c + b*c^2 + (d*x
^3 + c)*a*d - a*c*d)*a^2)
```

Mupad [B] (verification not implemented)

Time = 14.41 (sec) , antiderivative size = 438, normalized size of antiderivative = 2.72

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)^2}\,dx$$

$$= \frac{\left(\frac{a\left(\frac{a\left(\frac{a\left(\frac{b^2d^2}{2a^3c^2}-\frac{b^2d^2(3ad-4bc)}{6a^2c^2\left(a^2d-abc\right)}+\frac{b^2d(2ad-bc)(3ad-4bc)}{6a^3c^2\left(a^2d-abc\right)}\right)}{b}-\frac{bd(2ad-bc)}{2a^3c^2}+\frac{b(3ad-4bc)\left(-a^2d^2+2abcd+2b^2c^2\right)}{6a^3c^2\left(a^2d-abc\right)}\right)}{b}-\frac{-a^2d^2+2abcd+2b^2 \cdots}{2a^3c^2}\cdots\right)}{b}\cdots \right)}{bx^3+a}$$

$$-\frac{\sqrt{dx^3+c}}{3a^2x^3}+\frac{\ln\left(\frac{\left(\sqrt{dx^3+c}-\sqrt{c}\right)^3\left(\sqrt{dx^3+c}+\sqrt{c}\right)}{x^6}\right)(ad-4bc)}{6a^3\sqrt{c}}$$
$$+\frac{\sqrt{b}\,\ln\left(\frac{ad-2bc-bdx^3+\sqrt{b}\sqrt{dx^3+c}\sqrt{ad-bc}\,2\mathrm{i}}{bx^3+a}\right)(3ad-4bc)\,1\mathrm{i}}{6a^3\sqrt{ad-bc}}$$

```
[In] int((c + d*x^3)^(1/2)/(x^4*(a + b*x^3)^2),x)
```

```
[Out] (((a*((a*((a*((b^2*d^2)/(2*a^3*c^2) - (b^2*d^2*(3*a*d - 4*b*c))/(6*a^2*c^2*
(a^2*d - a*b*c)) + (b^2*d*(2*a*d - b*c)*(3*a*d - 4*b*c))/(6*a^3*c^2*(a^2*d
- a*b*c))))/b - (b*d*(2*a*d - b*c))/(2*a^3*c^2) + (b*(3*a*d - 4*b*c)*(2*b^2
*c^2 - a^2*d^2 + 2*a*b*c*d))/(6*a^3*c^2*(a^2*d - a*b*c))))/b - (2*b^2*c^2 -
 a^2*d^2 + 2*a*b*c*d)/(2*a^3*c^2) + (b*(a*d - 4*b*c)*(3*a*d - 4*b*c))/(6*a^
2*c*(a^2*d - a*b*c))))/b - (a*d - 4*b*c)/(2*a^2*c))*(c + d*x^3)^(1/2))/(a +
 b*x^3) - (c + d*x^3)^(1/2)/(3*a^2*x^3) + (log((((c + d*x^3)^(1/2) - c^(1/2
))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)*(a*d - 4*b*c))/(6*a^3*c^(1/2)) + (
b^(1/2)*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i -
 b*d*x^3)/(a + b*x^3))*(3*a*d - 4*b*c)*1i)/(6*a^3*(a*d - b*c)^(1/2))
```

3.465 $\int \frac{x^3\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^3\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \frac{x^4\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{4}{3},2,-\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] 1/4*x^4*AppellF1(4/3,2,-1/2,7/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/(1+d
*x^3/c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{x^3\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \frac{x^4\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{4}{3},2,-\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[In] Int[(x^3*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] (x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 2, -1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)
])/(4*a^2*Sqrt[1 + (d*x^3)/c])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c+dx^3}\int \frac{x^3\sqrt{1+\frac{dx^3}{c}}}{(a+bx^3)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$

$$= \frac{x^4\sqrt{c+dx^3}F_1\left(\frac{4}{3};2,-\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 235 vs. $2(64) = 128$.

Time = 10.22 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.67

$$\int \frac{x^3\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$$

$$= \frac{x\left(\frac{5dx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a}+\frac{8\left(-c-dx^3+\frac{8ac^2\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-3x^3\left(2bc\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\cdots\right.}\right.}{a+bx^3}\right.}{24b\sqrt{c+dx^3}}$$

```
[In] Integrate[(x^3*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] (x*((5*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -
((b*x^3)/a)])/a + (8*(-c - d*x^3 + (8*a*c^2*AppellF1[1/3, 1/2, 1, 4/3, -((d
*x^3)/c), -((b*x^3)/a)])/(8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -(
(b*x^3)/a)] - 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^
3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(a
+ b*x^3)))/(24*b*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.74 (sec) , antiderivative size = 748, normalized size of antiderivative = 11.69

method	result	size
elliptic	Expression too large to display	748
default	Expression too large to display	1468

```
[In] int(x^3*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*x/b*(d*x^3+c)^(1/2)/(b*x^3+a)-5/9*I/b^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I/b^2/d^2*2^(1/2)*sum((5*a*d-2*b*c)/_a
lpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3
)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*
d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c
*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^
2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^
2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/
2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alph
a+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d
^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_a
lpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^3\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{x^3\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$$

```
[In] integrate(x**3*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
[Out] Integral(x**3*sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

Maxima [F]

$$\int \frac{x^3\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}x^3}{(bx^3+a)^2}\,dx$$

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)*x^3/(b*x^3 + a)^2, x)
```

Giac [F]

$$\int \frac{x^3\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}x^3}{(bx^3+a)^2}\,dx$$

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)*x^3/(b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{x^3\,\sqrt{d\,x^3+c}}{(b\,x^3+a)^2}\,dx$$

```
[In] int((x^3*(c + d*x^3)^(1/2))/(a + b*x^3)^2,x)
[Out] int((x^3*(c + d*x^3)^(1/2))/(a + b*x^3)^2, x)
```

3.466 $\int \frac{x\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{x\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx=\frac{x^2\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{2}{3},2,-\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] 1/2*x^2*AppellF1(2/3,2,-1/2,5/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/(1+d
*x^3/c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {525, 524}

$$\int \frac{x\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx=\frac{x^2\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{2}{3},2,-\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[In] Int[(x*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] (x^2*Sqrt[c + d*x^3]*AppellF1[2/3, 2, -1/2, 5/3, -((b*x^3)/a), -((d*x^3)/c)
])/(2*a^2*Sqrt[1 + (d*x^3)/c])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c+dx^3}\int\frac{x\sqrt{1+\frac{dx^3}{c}}}{(a+bx^3)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$

$$= \frac{x^2\sqrt{c+dx^3}F_1\left(\frac{2}{3};2,-\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 153 vs. 2(64) = 128.

Time = 10.11 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.39

$$\int\frac{x\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$$
$$= \frac{10ax^2(c+dx^3)+5cx^2(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-dx^5(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{Ap}}{30a^2\left(a+bx^3\right)\sqrt{c+dx^3}}$$

```
[In] Integrate[(x*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] (10*a*x^2*(c + d*x^3) + 5*c*x^2*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/
3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] - d*x^5*(a + b*x^3)*Sqrt[1 + (d
*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(30*a^2*(a
 + b*x^3)*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.56 (sec) , antiderivative size = 908, normalized size of antiderivative = 14.19

method	result	size
default	Expression too large to display	908
elliptic	Expression too large to display	908

```
[In] int(x*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^2/a*(d*x^3+c)^(1/2)/(b*x^3+a)+1/9*I/a/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+
1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/
3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-
c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^
2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I
*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)
^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3
^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2)))+1/18*I/a/b/d^2*2^(1/2)*sum((-a*d-2*b*c)/_alpha/(a*d-b*c)*(-c*d
^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c
*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(
-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(
1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*
d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1
/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*
d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{x\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$$

```
[In] integrate(x*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
[Out] Integral(x*sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

Maxima [F]

$$\int \frac{x\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}x}{(bx^3+a)^2}\,dx$$

```
[In] integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)*x/(b*x^3 + a)^2, x)
```

Giac [F]

$$\int \frac{x\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}x}{(bx^3+a)^2}\,dx$$

```
[In] integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)*x/(b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{x\,\sqrt{d\,x^3+c}}{{\left(b\,x^3+a\right)}^2}\,dx$$

```
[In] int((x*(c + d*x^3)^(1/2))/(a + b*x^3)^2,x)
[Out] int((x*(c + d*x^3)^(1/2))/(a + b*x^3)^2, x)
```

3.467 $\int \frac{\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \frac{x\sqrt{c+dx^3}\,\mathrm{AppellF1}\left(\frac{1}{3},2,-\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] x*AppellF1(1/3,2,-1/2,4/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/(1+d*x^3/c
)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {441, 440}

$$\int \frac{\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \frac{x\sqrt{c+dx^3}\,\mathrm{AppellF1}\left(\frac{1}{3},2,-\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[In] Int[Sqrt[c + d*x^3]/(a + b*x^3)^2,x]
```

```
[Out] (x*Sqrt[c + d*x^3]*AppellF1[1/3, 2, -1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)])
/(a^2*Sqrt[1 + (d*x^3)/c])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
```

```
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c+dx^3}\int \frac{\sqrt{1+\frac{dx^3}{c}}}{(a+bx^3)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$

$$= \frac{x\sqrt{c+dx^3}F_1\left(\frac{1}{3};2,-\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 232 vs. 2(59) = 118.

Time = 10.23 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.93

$$\int \frac{\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$$

$$= \frac{x\left(\frac{dx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a^2}+\frac{8\left(\frac{c+dx^3}{a}+\frac{16c^2\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-3x^3\left(2bc\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\cdots\right.}\right.}{a+bx^3}\right.}{24\sqrt{c+dx^3}}$$

```
[In] Integrate[Sqrt[c + d*x^3]/(a + b*x^3)^2,x]
```

```
[Out] (x*((d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((
b*x^3)/a)])/a^2 + (8*((c + d*x^3)/a + (16*c^2*AppellF1[1/3, 1/2, 1, 4/3, -(
(d*x^3)/c), -((b*x^3)/a)])/(8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c),
-((b*x^3)/a)] - 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*
x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(
a + b*x^3)))/(24*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.40 (sec) , antiderivative size = 753, normalized size of antiderivative = 12.76

method	result	size
default	Expression too large to display	753
elliptic	Expression too large to display	753

```
[In] int((d*x^3+c)^(1/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x/a*(d*x^3+c)^(1/2)/(b*x^3+a)-1/9*I/a/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/
2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(
I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2
)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I/a/b/d^2*2^(1/2)*sum((a*d-4*b*c)/_alph
a^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(
-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2
)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^
2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^
(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^
(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b
/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I
*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)
^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alph
a=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$$

```
[In] integrate((d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
[Out] Integral(sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}}{(bx^3+a)^2}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)/(b*x^3 + a)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{\sqrt{dx^3+c}}{(bx^3+a)^2}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)/(b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx = \int \frac{\sqrt{d\,x^3+c}}{(b\,x^3+a)^2}\,dx$$

```
[In] int((c + d*x^3)^(1/2)/(a + b*x^3)^2,x)
[Out] int((c + d*x^3)^(1/2)/(a + b*x^3)^2, x)
```

3.468 $\int \frac{\sqrt{c+dx^3}}{x^2(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)^2}\,dx = -\frac{\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{1}{3},2,-\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] -AppellF1(-1/3,2,-1/2,2/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/x/(1+d*x^3
/c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)^2}\,dx = -\frac{\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{1}{3},2,-\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{\frac{dx^3}{c}+1}}$$

```
[In] Int[Sqrt[c + d*x^3]/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -((Sqrt[c + d*x^3]*AppellF1[-1/3, 2, -1/2, 2/3, -((b*x^3)/a), -((d*x^3)/c)]
)/(a^2*x*Sqrt[1 + (d*x^3)/c]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c+dx^3}\int \frac{\sqrt{1+\frac{dx^3}{c}}}{x^2(a+bx^3)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$

$$= -\frac{\sqrt{c+dx^3}F_1\left(-\frac{1}{3};2,-\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(62) = 124.

Time = 10.15 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.77

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)^2}\,dx$$

$$= \frac{-20a(3a+4bx^3)\left(c+dx^3\right)+5(-8bc+9ad)x^3(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+8b}{60a^3x\left(a+bx^3\right)\sqrt{c+dx^3}}$$

```
[In] Integrate[Sqrt[c + d*x^3]/(x^2*(a + b*x^3)^2),x]
```

```
[Out] (-20*a*(3*a + 4*b*x^3)*(c + d*x^3) + 5*(-8*b*c + 9*a*d)*x^3*(a + b*x^3)*Sqr
t[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 8
*b*d*x^6*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^
3)/c), -((b*x^3)/a)])/(60*a^3*x*(a + b*x^3)*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 6.39 (sec) , antiderivative size = 920, normalized size of antiderivative = 14.84

method	result	size
elliptic	Expression too large to display	920
risch	Expression too large to display	1819
default	Expression too large to display	2227

```
[In] int((d*x^3+c)^(1/2)/x^2/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b/a^2*x^2*(d*x^3+c)^(1/2)/(b*x^3+a)-1/a^2*(d*x^3+c)^(1/2)/x-4/9*I/a^2*
3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*
d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c
)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1
/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3
*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)
*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/18*I/a^2/d^2*2^(1/2)*sum((-5*a*
d+8*b*c)/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))
/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/
2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d
^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1
/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2
)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^
(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)^2}\,dx = \int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)^2}\,dx$$

```
[In] integrate((d*x**3+c)**(1/2)/x**2/(b*x**3+a)**2,x)
[Out] Integral(sqrt(c + d*x**3)/(x**2*(a + b*x**3)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)^2}\,dx = \int \frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)^2x^2}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)^2}\,dx = \int \frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)^2x^2}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a)^2,x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)^2}\,dx=\int \frac{\sqrt{d\,x^3+c}}{x^2\,{\left(b\,x^3+a\right)}^2}\,dx$$

```
[In] int((c + d*x^3)^(1/2)/(x^2*(a + b*x^3)^2),x)
[Out] int((c + d*x^3)^(1/2)/(x^2*(a + b*x^3)^2), x)
```

3.469 $\int \frac{\sqrt{c+dx^3}}{x^3(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)^2}\,dx=-\frac{\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{2}{3},2,-\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] -1/2*AppellF1(-2/3,2,-1/2,1/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/x^2/(1
+d*x^3/c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)^2}\,dx=-\frac{\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{2}{3},2,-\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[In] Int[Sqrt[c + d*x^3]/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -1/2*(Sqrt[c + d*x^3]*AppellF1[-2/3, 2, -1/2, 1/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a^2*x^2*Sqrt[1 + (d*x^3)/c])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c+dx^3}\int \frac{\sqrt{1+\frac{dx^3}{c}}}{x^3(a+bx^3)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$

$$= -\frac{\sqrt{c+dx^3}F_1\left(-\frac{2}{3};2,-\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 338 vs. 2(64) = 128.

Time = 10.32 (sec) , antiderivative size = 338, normalized size of antiderivative = 5.28

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)^2}\,dx$$

$$= \frac{-5bdx^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{a\left(32ac\left(6ac+30bcx^3-3adx^3+10bdx^6\right)\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\cdots\right.}{(a+bx^3)\left(-8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\cdots\right.}}{48a^3x^2\sqrt{c+dx^3}}$$

```
[In] Integrate[Sqrt[c + d*x^3]/(x^3*(a + b*x^3)^2),x]
```

```
[Out] (-5*b*d*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -(
(b*x^3)/a)] + (a*(32*a*c*(6*a*c + 30*b*c*x^3 - 3*a*d*x^3 + 10*b*d*x^6)*Appe
llF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 24*x^3*(3*a + 5*b*x^3)
*(c + d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)]
+ a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3
)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2
*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[
4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(48*a^3*x^2*Sqrt[c + d*x^
3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 6.58 (sec) , antiderivative size = 766, normalized size of antiderivative = 11.97

method	result	size
elliptic	Expression too large to display	766
risch	Expression too large to display	1513
default	Expression too large to display	1768

```
[In] int((d*x^3+c)^(1/2)/x^3/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b/a^2*x*(d*x^3+c)^(1/2)/(b*x^3+a)-1/2/a^2*(d*x^3+c)^(1/2)/x^2+5/18*I/a
^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^
3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3
)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I/a^2
/d^2*2^(1/2)*sum((-7*a*d+10*b*c)/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d
*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)
*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1
/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^
(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(
-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*Ellipt
icPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))
*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha
^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-
3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)^2}\,dx = \int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)^2}\,dx$$

```
[In] integrate((d*x**3+c)**(1/2)/x**3/(b*x**3+a)**2,x)
[Out] Integral(sqrt(c + d*x**3)/(x**3*(a + b*x**3)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)^2}\,dx = \int \frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)^2x^3}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x^3), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)^2}\,dx = \int \frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)^2x^3}\,dx$$

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a)^2,x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)^2}\,dx = \int \frac{\sqrt{d\,x^3+c}}{x^3\,\left(b\,x^3+a\right)^2}\,dx$$

```
[In] int((c + d*x^3)^(1/2)/(x^3*(a + b*x^3)^2),x)
[Out] int((c + d*x^3)^(1/2)/(x^3*(a + b*x^3)^2), x)
```

3.470 $\int \frac{x^8(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 189

$$\int \frac{x^8(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = -\frac{a(4bc-7ad)\sqrt{c+dx^3}}{3b^4} - \frac{a(4bc-7ad)\,(c+dx^3)^{3/2}}{9b^3(bc-ad)} + \frac{2(c+dx^3)^{5/2}}{15b^2d} - \frac{a^2(c+dx^3)^{5/2}}{3b^2(bc-ad)\,(a+bx^3)} + \frac{a(4bc-7ad)\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{9/2}}$$

```
[Out] -1/9*a*(-7*a*d+4*b*c)*(d*x^3+c)^(3/2)/b^3/(-a*d+b*c)+2/15*(d*x^3+c)^(5/2)/b
^2/d-1/3*a^2*(d*x^3+c)^(5/2)/b^2/(-a*d+b*c)/(b*x^3+a)+1/3*a*(-7*a*d+4*b*c)*
arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(9/2)-
1/3*a*(-7*a*d+4*b*c)*(d*x^3+c)^(1/2)/b^4
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {457, 91, 81, 52, 65, 214}

$$\int \frac{x^8(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = -\frac{a^2(c+dx^3)^{5/2}}{3b^2\,(a+bx^3)\,(bc-ad)} + \frac{a(4bc-7ad)\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{9/2}} - \frac{a\sqrt{c+dx^3}(4bc-7ad)}{3b^4} - \frac{a(c+dx^3)^{3/2}\,(4bc-7ad)}{9b^3(bc-ad)} + \frac{2(c+dx^3)^{5/2}}{15b^2d}$$

```
[In] Int[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] -1/3*(a*(4*b*c - 7*a*d)*Sqrt[c + d*x^3])/b^4 - (a*(4*b*c - 7*a*d)*(c + d*x^
3)^(3/2))/(9*b^3*(b*c - a*d)) + (2*(c + d*x^3)^(5/2))/(15*b^2*d) - (a^2*(c
+ d*x^3)^(5/2))/(3*b^2*(b*c - a*d)*(a + b*x^3)) + (a*(4*b*c - 7*a*d)*Sqrt[b
*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(9/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x^2(c+dx)^{3/2}}{(a+bx)^2}\,dx, x, x^3\right)$$

$$= -\frac{a^2(c+dx^3)^{5/2}}{3b^2(bc-ad)(a+bx^3)} + \frac{\text{Subst}\left(\int \frac{(c+dx)^{3/2}\left(-\frac{1}{2}a(2bc-5ad)+b(bc-ad)x\right)}{a+bx}\,dx, x, x^3\right)}{3b^2(bc-ad)}$$

$$= \frac{2(c+dx^3)^{5/2}}{15b^2d} - \frac{a^2(c+dx^3)^{5/2}}{3b^2(bc-ad)(a+bx^3)} - \frac{(a(4bc-7ad))\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx, x, x^3\right)}{6b^2(bc-ad)}$$

$$= -\frac{a(4bc-7ad)(c+dx^3)^{3/2}}{9b^3(bc-ad)} + \frac{2(c+dx^3)^{5/2}}{15b^2d} - \frac{a^2(c+dx^3)^{5/2}}{3b^2(bc-ad)(a+bx^3)} - \frac{(a(4bc-7ad))\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)}{6b^3}$$

$$= -\frac{a(4bc-7ad)\sqrt{c+dx^3}}{3b^4} - \frac{a(4bc-7ad)(c+dx^3)^{3/2}}{9b^3(bc-ad)} + \frac{2(c+dx^3)^{5/2}}{15b^2d} - \frac{a^2(c+dx^3)^{5/2}}{3b^2(bc-ad)(a+bx^3)} - \frac{(a(4bc-7ad)(bc-ad))\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{6b^4}$$

$$= -\frac{a(4bc-7ad)\sqrt{c+dx^3}}{3b^4} - \frac{a(4bc-7ad)(c+dx^3)^{3/2}}{9b^3(bc-ad)} + \frac{2(c+dx^3)^{5/2}}{15b^2d} - \frac{a^2(c+dx^3)^{5/2}}{3b^2(bc-ad)(a+bx^3)} - \frac{(a(4bc-7ad)(bc-ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3b^4d}$$

$$= -\frac{a(4bc-7ad)\sqrt{c+dx^3}}{3b^4} - \frac{a(4bc-7ad)(c+dx^3)^{3/2}}{9b^3(bc-ad)} + \frac{2(c+dx^3)^{5/2}}{15b^2d} - \frac{a^2(c+dx^3)^{5/2}}{3b^2(bc-ad)(a+bx^3)} + \frac{a(4bc-7ad)\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{9/2}}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.86

$$\int \frac{x^8(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \frac{\sqrt{c+dx^3}\left(105a^3d^2+6b^3x^3(c+dx^3)^2+5a^2bd(-19c+14dx^3)+2ab^2(3c^2-34cdx^3-\right.}{45b^4d\,(a+bx^3)}$$
$$+\frac{a(4bc-7ad)\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{9/2}}$$

```
[In] Integrate[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (Sqrt[c + d*x^3]*(105*a^3*d^2 + 6*b^3*x^3*(c + d*x^3)^2 + 5*a^2*b*d*(-19*c
+ 14*d*x^3) + 2*a*b^2*(3*c^2 - 34*c*d*x^3 - 7*d^2*x^6)))/(45*b^4*d*(a + b*x
^3)) + (a*(4*b*c - 7*a*d)*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3
])/Sqrt[-(b*c) + a*d]])/(3*b^(9/2))
```

Maple [A] (verified)

Time = 4.65 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$-\frac{7\left(\left(ad-\frac{4bc}{7}\right)d(b\,x^{3}+a)a(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)-\sqrt{d\,x^{3}+c}\left(\frac{2x^{3}\left(d\,x^{3}+c\right)^{2}b^{3}}{35}+\frac{2\left(-\frac{7}{3}d^{2}x^{6}-\frac{34}{3}cd\,x^{3}+c^{2}\right)a\,b^{2}}{35}-\frac{19d\,a^{2}}{}\right.\right.}{3\sqrt{(ad-bc)b}\,b^{4}d(b\,x^{3}+a)}$
default	$\frac{2(d\,x^{3}+c)^{\frac{5}{2}}}{15b^{2}d}+\frac{a^{2}\left(-d(b\,x^{3}+a)(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)+\left(\frac{(2d\,x^{3}-c)b}{3}+ad\right)\sqrt{d\,x^{3}+c}\,\sqrt{(ad-bc)b}\right)}{b^{4}\sqrt{(ad-bc)b}\,(b\,x^{3}+a)}+\frac{4a\left(-(ad-b\right.}{}$
risch	$\frac{2(3b^{2}d^{2}x^{6}-10x^{3}ab\,d^{2}+6x^{3}b^{2}cd+45a^{2}d^{2}-40abcd+3b^{2}c^{2})\sqrt{d\,x^{3}+c}}{45d\,b^{4}}-\frac{a\left(\frac{2\left(4a^{2}d^{2}-6abcd+2b^{2}c^{2}\right)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}-\frac{a(a}{}\right.}{}$
elliptic	$\frac{a^{2}(ad-bc)\sqrt{d\,x^{3}+c}}{3b^{4}(b\,x^{3}+a)}+\frac{2d\,x^{6}\sqrt{d\,x^{3}+c}}{15b^{2}}+\frac{2\left(-\frac{2(ad-bc)d}{b^{3}}-\frac{4cd}{5b^{2}}\right)x^{3}\sqrt{d\,x^{3}+c}}{9d}+\frac{2\left(\frac{3a^{2}d^{2}-4abcd+b^{2}c^{2}}{b^{4}}-\frac{2\left(-\frac{2(ad-bc)d}{b^{3}}-\frac{4cd}{5b^{2}}\right.}{3d}\right.}{3d}$

```
[In] int(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -7/3*((a*d-4/7*b*c)*d*(b*x^3+a)*a*(a*d-b*c)*arctan(b*(d*x^3+c)^(1/2)/((a*d-
b*c)*b)^(1/2))-(d*x^3+c)^(1/2)*(2/35*x^3*(d*x^3+c)^2*b^3+2/35*(-7/3*d^2*x^6
-34/3*c*d*x^3+c^2)*a*b^2-19/21*d*a^2*(-14/19*d*x^3+c)*b+a^3*d^2)*((a*d-b*c)
*b)^(1/2))/((a*d-b*c)*b)^(1/2)/b^4/d/(b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 443, normalized size of antiderivative = 2.34

$$\int \frac{x^{8}(c+dx^{3})^{3/2}}{(a+bx^{3})^{2}}\,dx=\left[-\frac{15\left(4\,a^{2}bcd-7\,a^{3}d^{2}+(4\,ab^{2}cd-7\,a^{2}bd^{2})x^{3}\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^{3}+2\,bc-ad-2\,\sqrt{dx^{3}+c}b\sqrt{\frac{bc}{}}}{bx^{3}+a}\right.}{}\right.$$

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/90*(15*(4*a^2*b*c*d - 7*a^3*d^2 + (4*a*b^2*c*d - 7*a^2*b*d^2)*x^3)*sqrt
((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*b*sqrt((b*c
- a*d)/b))/(b*x^3 + a)) - 2*(6*b^3*d^2*x^9 + 2*(6*b^3*c*d - 7*a*b^2*d^2)*x^
6 + 6*a*b^2*c^2 - 95*a^2*b*c*d + 105*a^3*d^2 + 2*(3*b^3*c^2 - 34*a*b^2*c*d
+ 35*a^2*b*d^2)*x^3)*sqrt(d*x^3 + c))/(b^5*d*x^3 + a*b^4*d), 1/45*(15*(4*a^
2*b*c*d - 7*a^3*d^2 + (4*a*b^2*c*d - 7*a^2*b*d^2)*x^3)*sqrt(-(b*c - a*d)/b)
*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (6*b^3*d^2*x
^9 + 2*(6*b^3*c*d - 7*a*b^2*d^2)*x^6 + 6*a*b^2*c^2 - 95*a^2*b*c*d + 105*a^3
*d^2 + 2*(3*b^3*c^2 - 34*a*b^2*c*d + 35*a^2*b*d^2)*x^3)*sqrt(d*x^3 + c))/(b
^5*d*x^3 + a*b^4*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^8 (c+dx^3)^{3/2}}{(a+bx^3)^2}\, dx = \text{Timed out}$$

```
[In] integrate(x**8*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8 (c+dx^3)^{3/2}}{(a+bx^3)^2}\, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.12

$$\int \frac{x^8(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = -\frac{(4\,ab^2c^2-11\,a^2bcd+7\,a^3d^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}b^4} - \frac{\sqrt{dx^3+c}a^2bcd-\sqrt{dx^3+c}a^3d^2}{3\,((dx^3+c)b-bc+ad)b^4} + \frac{2\left(3\,(dx^3+c)^{\frac{5}{2}}b^8d^4-10\,(dx^3+c)^{\frac{3}{2}}ab^7d^5-30\sqrt{dx^3+c}ab^7cd^5+45\sqrt{dx^3+c}a^2b^6d^6\right)}{45\,b^{10}d^5}$$

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*(4*a*b^2*c^2 - 11*a^2*b*c*d + 7*a^3*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt
(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^4) - 1/3*(sqrt(d*x^3 + c)*a^2*b*c
*d - sqrt(d*x^3 + c)*a^3*d^2)/(((d*x^3 + c)*b - b*c + a*d)*b^4) + 2/45*(3*(
d*x^3 + c)^(5/2)*b^8*d^4 - 10*(d*x^3 + c)^(3/2)*a*b^7*d^5 - 30*sqrt(d*x^3 +
 c)*a*b^7*c*d^5 + 45*sqrt(d*x^3 + c)*a^2*b^6*d^6)/(b^10*d^5)
```

Mupad [B] (verification not implemented)

Time = 12.04 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.75

$$\int \frac{x^8(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \frac{\sqrt{d\,x^3+c}\left(\frac{2\,(a\,d-b\,c)^2}{b^4}+\frac{2\,c\left(\frac{2\,d\,(a\,d-2\,b\,c)}{b^3}+\frac{2\,a\,d^2}{b^3}+\frac{8\,c\,d}{5\,b^2}\right)}{3\,d}+\frac{2\,a\left(\frac{d\,(a\,d-2\,b\,c)}{b^3}+\frac{a\,d^2}{b^3}\right)}{b}\right)}{3\,d} + \frac{2\,d\,x^6\sqrt{d\,x^3+c}}{15\,b^2} - \frac{x^3\sqrt{d\,x^3+c}\left(\frac{2\,d\,(a\,d-2\,b\,c)}{b^3}+\frac{2\,a\,d^2}{b^3}+\frac{8\,c\,d}{5\,b^2}\right)}{9\,d} - \frac{a^2\left(\frac{2\,b\,c^2}{3\,(2\,b^2\,c-2\,a\,b\,d)}+\frac{a\left(\frac{2\,a\,d^2}{3\,(2\,b^2\,c-2\,a\,b\,d)}-\frac{4\,b\,c\,d}{3\,(2\,b^2\,c-2\,a\,b\,d)}\right)}{b}\right)\sqrt{d\,x^3+c}}{b^2\,(b\,x^3+a)} + \frac{a\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\sqrt{a\,d-b\,c}\,(7\,a\,d-4\,b\,c)\,1\mathrm{i}}{6\,b^{9/2}}$$

```
[In] int((x^8*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x)
```

```
[Out] ((c + d*x^3)^(1/2)*((2*(a*d - b*c)^2)/b^4 + (2*c*((2*d*(a*d - 2*b*c))/b^3 +
 (2*a*d^2)/b^3 + (8*c*d)/(5*b^2)))/(3*d) + (2*a*((d*(a*d - 2*b*c))/b^3 + (a
*d^2)/b^3))/b))/(3*d) + (2*d*x^6*(c + d*x^3)^(1/2))/(15*b^2) - (x^3*(c + d*
x^3)^(1/2)*((2*d*(a*d - 2*b*c))/b^3 + (2*a*d^2)/b^3 + (8*c*d)/(5*b^2)))/(9*
d) + (a*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i -
 b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*(7*a*d - 4*b*c)*1i)/(6*b^(9/2)) -
```

```
(a^2*((2*b*c^2)/(3*(2*b^2*c - 2*a*b*d)) + (a*((2*a*d^2)/(3*(2*b^2*c - 2*a*b
*d)) - (4*b*c*d)/(3*(2*b^2*c - 2*a*b*d))))/b)*(c + d*x^3)^(1/2))/(b^2*(a +
b*x^3))
```

3.471 $\int \frac{x^5(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 163

$$\int \frac{x^5(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \frac{(2bc-5ad)\sqrt{c+dx^3}}{3b^3} + \frac{(2bc-5ad)(c+dx^3)^{3/2}}{9b^2(bc-ad)} + \frac{a(c+dx^3)^{5/2}}{3b(bc-ad)(a+bx^3)} - \frac{(2bc-5ad)\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}$$

```
[Out] 1/9*(-5*a*d+2*b*c)*(d*x^3+c)^(3/2)/b^2/(-a*d+b*c)+1/3*a*(d*x^3+c)^(5/2)/b/(
-a*d+b*c)/(b*x^3+a)-1/3*(-5*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*
d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(7/2)+1/3*(-5*a*d+2*b*c)*(d*x^3+c)^(1/2)/b
^3
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 79, 52, 65, 214}

$$\int \frac{x^5(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = -\frac{(2bc-5ad)\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}} + \frac{\sqrt{c+dx^3}(2bc-5ad)}{3b^3} + \frac{(c+dx^3)^{3/2}(2bc-5ad)}{9b^2(bc-ad)} + \frac{a(c+dx^3)^{5/2}}{3b(a+bx^3)(bc-ad)}$$

```
[In] Int[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] ((2*b*c - 5*a*d)*Sqrt[c + d*x^3])/(3*b^3) + ((2*b*c - 5*a*d)*(c + d*x^3)^(3
/2))/(9*b^2*(b*c - a*d)) + (a*(c + d*x^3)^(5/2))/(3*b*(b*c - a*d)*(a + b*x^
```

```
3)) - ((2*b*c - 5*a*d)*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sq
rt[b*c - a*d]])/(3*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x(c+dx)^{3/2}}{(a+bx)^2}\,dx, x, x^3\right)$$

$$= \frac{a(c+dx^3)^{5/2}}{3b(bc-ad)(a+bx^3)} + \frac{(2bc-5ad)\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx, x, x^3\right)}{6b(bc-ad)}$$

$$= \frac{(2bc-5ad)(c+dx^3)^{3/2}}{9b^2(bc-ad)} + \frac{a(c+dx^3)^{5/2}}{3b(bc-ad)(a+bx^3)} + \frac{(2bc-5ad)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)}{6b^2}$$

$$= \frac{(2bc-5ad)\sqrt{c+dx^3}}{3b^3} + \frac{(2bc-5ad)(c+dx^3)^{3/2}}{9b^2(bc-ad)} + \frac{a(c+dx^3)^{5/2}}{3b(bc-ad)(a+bx^3)} + \frac{((2bc-5ad)(bc-ad))\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{6b^3}$$

$$= \frac{(2bc-5ad)\sqrt{c+dx^3}}{3b^3} + \frac{(2bc-5ad)(c+dx^3)^{3/2}}{9b^2(bc-ad)} + \frac{a(c+dx^3)^{5/2}}{3b(bc-ad)(a+bx^3)} + \frac{((2bc-5ad)(bc-ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3b^3d}$$

$$= \frac{(2bc-5ad)\sqrt{c+dx^3}}{3b^3} + \frac{(2bc-5ad)(c+dx^3)^{3/2}}{9b^2(bc-ad)} + \frac{a(c+dx^3)^{5/2}}{3b(bc-ad)(a+bx^3)} - \frac{(2bc-5ad)\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.77

$$\int \frac{x^5(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \frac{\sqrt{c+dx^3}(-15a^2d+ab(11c-10dx^3)+2b^2x^3(4c+dx^3))}{9b^3(a+bx^3)} - \frac{(2bc-5ad)\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{3b^{7/2}}$$

```
[In] Integrate[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (Sqrt[c + d*x^3]*(-15*a^2*d + a*b*(11*c - 10*d*x^3) + 2*b^2*x^3*(4*c + d*x^
3)))/(9*b^3*(a + b*x^3)) - ((2*b*c - 5*a*d)*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt
[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(3*b^(7/2))
```

Maple [A] (verified)

Time = 4.72 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$-\frac{5\left(-(b x^3+a)\left(ad-\frac{2bc}{5}\right)(ad-bc)\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{(ad-bc)b}\left(-\frac{8x^3\left(\frac{d x^3}{4}+c\right)b^2}{15}-\frac{11\left(-\frac{10d x^3}{11}+c\right)ab}{15}+a^2d\right)\sqrt{d x^3+\cdots}\right.}{3\sqrt{(ad-bc)b}\, b^3 (b x^3+a)}$
default	$-\frac{2\left(-(ad-bc)^2\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{d x^3+c}\left(\frac{(-d x^3-4c)b}{3}+ad\right)\sqrt{(ad-bc)b}\right)}{3b^3\sqrt{(ad-bc)b}}-\frac{a\left(-d(b x^3+a)(ad-bc)\arctan\left(\frac{b\sqrt{d\cdots}}{\sqrt{(ad\cdots}}\right.\right.}{b^3\sqrt{\cdots}}$
risch	$-\frac{2(-bd x^3+6ad-4bc)\sqrt{d x^3+c}}{9b^3}+\frac{\frac{2(3a^2d^2-4abcd+b^2c^2)\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}-\frac{a(a^2d^2-2abcd+b^2c^2)\left(d\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)(b\cdots\right.}{3\sqrt{(ad-bc)b}\,(ad-bc)(b x\cdots}}{b^3}$
elliptic	$-\frac{(ad-bc)a\sqrt{d x^3+c}}{3b^3(b x^3+a)}+\frac{2d x^3\sqrt{d x^3+c}}{9b^2}+\frac{2\left(-\frac{2(ad-bc)d}{b^3}-\frac{2cd}{3b^2}\right)\sqrt{d x^3+c}}{3d}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}(b\_Z^3+a)}\frac{(-5a^2d^2+7abcd-\cdots}{\cdots}\right.}{\cdots}$

```
[In] int(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -5/3/((a*d-b*c)*b)^(1/2)*(-(b*x^3+a)*(a*d-2/5*b*c)*(a*d-b*c)*arctan(b*(d*x^
3+c)^(1/2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(-8/15*x^3*(1/4*d*x^3+c
)*b^2-11/15*(-10/11*d*x^3+c)*a*b+a^2*d)*(d*x^3+c)^(1/2))/b^3/(b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.93

$$\int \frac{x^5 (c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \left[-\frac{3\left((2\,b^2c-5\,abd)x^3+2\,abc-5\,a^2d\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)-2}{18\,(b^4x^3+ab^3)}\right.$$

$$-\frac{3\left((2\,b^2c-5\,abd)x^3+2\,abc-5\,a^2d\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-(2\,b^2dx^6+2\,(4\,b^2c-5\,abd)x^3}{9\,(b^4x^3+ab^3)}$$

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/18*(3*((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*d)*sqrt((b*c - a*d)/b)
*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x
^3 + a)) - 2*(2*b^2*d*x^6 + 2*(4*b^2*c - 5*a*b*d)*x^3 + 11*a*b*c - 15*a^2*d
)*sqrt(d*x^3 + c))/(b^4*x^3 + a*b^3), -1/9*(3*((2*b^2*c - 5*a*b*d)*x^3 + 2*
a*b*c - 5*a^2*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c
- a*d)/b)/(b*c - a*d)) - (2*b^2*d*x^6 + 2*(4*b^2*c - 5*a*b*d)*x^3 + 11*a*b*
c - 15*a^2*d)*sqrt(d*x^3 + c))/(b^4*x^3 + a*b^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5 (c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**5*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5 (c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.06

$$\int \frac{x^5(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \frac{(2\,b^2c^2-7\,abcd+5\,a^2d^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}b^3} + \frac{\sqrt{dx^3+c}abcd-\sqrt{dx^3+c}a^2d^2}{3\,((dx^3+c)b-bc+ad)b^3} + \frac{2\left((dx^3+c)^{\frac{3}{2}}b^4+3\,\sqrt{dx^3+c}b^4c-6\,\sqrt{dx^3+c}ab^3d\right)}{9\,b^6}$$

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(2*b^2*c^2 - 7*a*b*c*d + 5*a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*
c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 1/3*(sqrt(d*x^3 + c)*a*b*c*d - sqr
t(d*x^3 + c)*a^2*d^2)/(((d*x^3 + c)*b - b*c + a*d)*b^3) + 2/9*((d*x^3 + c)^
(3/2)*b^4 + 3*sqrt(d*x^3 + c)*b^4*c - 6*sqrt(d*x^3 + c)*a*b^3*d)/b^6
```

Mupad [B] (verification not implemented)

Time = 11.58 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.40

$$\int \frac{x^5(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \frac{2\,d\,x^3\,\sqrt{d\,x^3+c}}{9\,b^2} - \frac{\sqrt{d\,x^3+c}\left(\frac{2\,d\,(a\,d-2\,b\,c)}{b^3}+\frac{2\,a\,d^2}{b^3}+\frac{4\,c\,d}{3\,b^2}\right)}{3\,d} + \frac{a\left(\frac{2\,b\,c^2}{3\,(2\,b^2\,c-2\,a\,b\,d)}+\frac{a\left(\frac{2\,a\,d^2}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}-\frac{4\,b\,c\,d}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}\right)}{b}\right)\sqrt{d\,x^3+c}}{b\,\left(b\,x^3+a\right)} + \frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\sqrt{a\,d-b\,c}\,(5\,a\,d-2\,b\,c)\,1{}\mathrm{i}}{6\,b^{7/2}}$$

```
[In] int((x^5*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x)
```

```
[Out] (2*d*x^3*(c + d*x^3)^(1/2))/(9*b^2) - ((c + d*x^3)^(1/2)*((2*d*(a*d - 2*b*c
))/b^3 + (2*a*d^2)/b^3 + (4*c*d)/(3*b^2)))/(3*d) + (log((2*b*c - a*d + b^(1
/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(a*d - b
*c)^(1/2)*(5*a*d - 2*b*c)*1i)/(6*b^(7/2)) + (a*((2*b*c^2)/(3*(2*b^2*c - 2*a
*b*d)) + (a*((2*a*d^2)/(3*(2*b^2*c - 2*a*b*d)) - (4*b*c*d)/(3*(2*b^2*c - 2*
a*b*d))))/b)*(c + d*x^3)^(1/2))/(b*(a + b*x^3))
```

3.472 $\int \frac{x^2(c+dx^3)^{3/2}}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 94

$$\int \frac{x^2(c+dx^3)^{3/2}}{(a+bx^3)^2} dx = \frac{d\sqrt{c+dx^3}}{b^2} - \frac{(c+dx^3)^{3/2}}{3b(a+bx^3)} - \frac{d\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{5/2}}$$

```
[Out] -1/3*(d*x^3+c)^(3/2)/b/(b*x^3+a)-d*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*
c)^(1/2))*(-a*d+b*c)^(1/2)/b^(5/2)+d*(d*x^3+c)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {455, 43, 52, 65, 214}

$$\int \frac{x^2(c+dx^3)^{3/2}}{(a+bx^3)^2} dx = -\frac{d\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{5/2}} - \frac{(c+dx^3)^{3/2}}{3b(a+bx^3)} + \frac{d\sqrt{c+dx^3}}{b^2}$$

```
[In] Int[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (d*Sqrt[c + d*x^3])/b^2 - (c + d*x^3)^(3/2)/(3*b*(a + b*x^3)) - (d*Sqrt[b*c
 - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/b^(5/2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{(a+bx)^2}\,dx, x, x^3\right)\\
&= -\frac{(c+dx^3)^{3/2}}{3b\,(a+bx^3)} + \frac{d\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)}{2b}\\
&= \frac{d\sqrt{c+dx^3}}{b^2} - \frac{(c+dx^3)^{3/2}}{3b\,(a+bx^3)} + \frac{(d(bc-ad))\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{2b^2}\\
&= \frac{d\sqrt{c+dx^3}}{b^2} - \frac{(c+dx^3)^{3/2}}{3b\,(a+bx^3)} + \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{b^2}
\end{aligned}$$

$$= \frac{d\sqrt{c+dx^3}}{b^2} - \frac{(c+dx^3)^{3/2}}{3b\,(a+bx^3)} - \frac{d\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{5/2}}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int \frac{x^2(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \frac{\sqrt{c+dx^3}(-bc+3ad+2bdx^3)}{3b^2\,(a+bx^3)} - \frac{d\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{b^{5/2}}$$

```
[In] Integrate[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
[Out] (Sqrt[c + d*x^3]*(-(b*c) + 3*a*d + 2*b*d*x^3))/(3*b^2*(a + b*x^3)) - (d*Sqr
t[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/b^(5/
2)
```

Maple [A] (verified)

Time = 4.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.15

method	result
default	$\frac{-d(b x^3+a)(ad-bc)\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)+\left(\frac{(2d x^3-c)b}{3}+ad\right)\sqrt{d x^3+c}\sqrt{(ad-bc)b}}{\sqrt{(ad-bc)b}\, b^2 (b x^3+a)}$
pseudoelliptic	$\frac{-d(b x^3+a)(ad-bc)\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)+\left(\frac{(2d x^3-c)b}{3}+ad\right)\sqrt{d x^3+c}\sqrt{(ad-bc)b}}{\sqrt{(ad-bc)b}\, b^2 (b x^3+a)}$
risch	$\frac{2d\sqrt{d x^3+c}}{3b^2}-\frac{\frac{4(ad-bc)d\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}+\frac{\left(-a^2d^2+2abcd-b^2c^2\right)\left(d\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)(b x^3+a)+\sqrt{d x^3+c}\sqrt{(ad-bc)b}\right)}{3\sqrt{(ad-bc)b}\,(ad-bc)(b x^3+a)}}{b^2}$
elliptic	$\frac{(ad-bc)\sqrt{d x^3+c}}{3b^2(b x^3+a)}+\frac{2d\sqrt{d x^3+c}}{3b^2}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d \cdots}{-3\left(-c \cdots\right.}}}{\cdots}\right.}{\cdots}$

[In] int(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)

[Out] 1/((a*d-b*c)*b)^(1/2)*(-d*(b*x^3+a)*(a*d-b*c)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))+(1/3*(2*d*x^3-c)*b+a*d)*(d*x^3+c)^(1/2)*((a*d-b*c)*b)^(1/2))/b^2/(b*x^3+a)

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.49

$$\int \frac{x^2(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \left[\frac{3\left(bdx^3+ad\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\left(2\,bdx^3-bc+3\,ad\right)\sqrt{dx^3+c}}{6\left(b^3x^3+ab^2\right)}\right.$$

$$\left.-\frac{3\left(bdx^3+ad\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\left(2\,bdx^3-bc+3\,ad\right)\sqrt{dx^3+c}}{3\left(b^3x^3+ab^2\right)}\right]$$

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b*d*x^3 + a*d)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2*
sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*(2*b*d*x^3 - b*c +
3*a*d)*sqrt(d*x^3 + c))/(b^3*x^3 + a*b^2), -1/3*(3*(b*d*x^3 + a*d)*sqrt(-(b
*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) -
(2*b*d*x^3 - b*c + 3*a*d)*sqrt(d*x^3 + c))/(b^3*x^3 + a*b^2)]
```

Sympy [F]

$$\int \frac{x^2(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \int \frac{x^2(c+dx^3)^{\frac{3}{2}}}{(a+bx^3)^2}\,dx$$

```
[In] integrate(x**2*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

```
[Out] Integral(x**2*(c + d*x**3)**(3/2)/(a + b*x**3)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.30

$$\int \frac{x^2(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \frac{2\sqrt{dx^3+c}d}{3b^2} + \frac{(bcd-ad^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^2} - \frac{\sqrt{dx^3+c}bcd-\sqrt{dx^3+c}ad^2}{3\left((dx^3+c)b-bc+ad\right)b^2}$$

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 2/3*sqrt(d*x^3 + c)*d/b^2 + (b*c*d - a*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-
b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^2) - 1/3*(sqrt(d*x^3 + c)*b*c*d - s
qrt(d*x^3 + c)*a*d^2)/(((d*x^3 + c)*b - b*c + a*d)*b^2)
```

Mupad [B] (verification not implemented)

Time = 11.37 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.81

$$\int \frac{x^2(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \frac{2\,d\,\sqrt{d\,x^3+c}}{3\,b^2} - \frac{\left(\frac{2\,b\,c^2}{3\,(2\,b^2\,c-2\,a\,b\,d)}+\frac{a\left(\frac{2\,a\,d^2}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}-\frac{4\,b\,c\,d}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}\right)}{b}\right)\sqrt{d\,x^3+c}}{b\,x^3+a} + \frac{d\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\sqrt{a\,d-b\,c}\,1\mathrm{i}}{2\,b^{5/2}}$$

```
[In] int((x^2*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x)
```

```
[Out] (2*d*(c + d*x^3)^(1/2))/(3*b^2) - (((2*b*c^2)/(3*(2*b^2*c - 2*a*b*d)) + (a*
((2*a*d^2)/(3*(2*b^2*c - 2*a*b*d)) - (4*b*c*d)/(3*(2*b^2*c - 2*a*b*d))))/b)
*(c + d*x^3)^(1/2))/(a + b*x^3) + (d*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)
^(1/2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*1i)/(
2*b^(5/2))
```

3.473 $\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(a+bx^3\right)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 131

$$\int \frac{(c+dx^3)^{3/2}}{x\,(a+bx^3)^2}\,dx = \frac{(bc-ad)\sqrt{c+dx^3}}{3ab\,(a+bx^3)} - \frac{2c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2} + \frac{\sqrt{bc-ad}(2bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2b^{3/2}}$$

```
[Out] -2/3*c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^2+1/3*(a*d+2*b*c)*arctanh(b
^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/a^2/b^(3/2)+1/3*(
-a*d+b*c)*(d*x^3+c)^(1/2)/a/b/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 100, 162, 65, 214}

$$\int \frac{(c+dx^3)^{3/2}}{x\,(a+bx^3)^2}\,dx = \frac{\sqrt{bc-ad}(ad+2bc)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2b^{3/2}} - \frac{2c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2} + \frac{\sqrt{c+dx^3}(bc-ad)}{3ab\,(a+bx^3)}$$

```
[In] Int[(c + d*x^3)^(3/2)/(x*(a + b*x^3)^2),x]
```

```
[Out] ((b*c - a*d)*Sqrt[c + d*x^3])/(3*a*b*(a + b*x^3)) - (2*c^(3/2)*ArcTanh[Sqrt
[c + d*x^3]/Sqrt[c]])/(3*a^2) + (Sqrt[b*c - a*d]*(2*b*c + a*d)*ArcTanh[(Sqr
t[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^2*b^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x(a+bx)^2}\,dx, x, x^3\right)$$

$$= \frac{(bc-ad)\sqrt{c+dx^3}}{3ab\,(a+bx^3)} + \frac{\text{Subst}\left(\int \frac{bc^2+\frac{1}{2}d(bc+ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3ab}$$

$$= \frac{(bc-ad)\sqrt{c+dx^3}}{3ab\,(a+bx^3)} + \frac{c^2\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{3a^2} - \frac{((bc-ad)(2bc+ad))\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{6a^2b}$$

$$= \frac{(bc-ad)\sqrt{c+dx^3}}{3ab\,(a+bx^3)} + \frac{(2c^2)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^2d} - \frac{((bc-ad)(2bc+ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^2bd}$$

$$= \frac{(bc-ad)\sqrt{c+dx^3}}{3ab\,(a+bx^3)} - \frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2} + \frac{\sqrt{bc-ad}(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2b^{3/2}}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx^3)^{3/2}}{x\,(a+bx^3)^2}\,dx = \frac{\frac{a(bc-ad)\sqrt{c+dx^3}}{b(a+bx^3)} + \frac{\sqrt{-bc+ad}(2bc+ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{b^{3/2}} - 2c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}$$

```
[In] Integrate[(c + d*x^3)^(3/2)/(x*(a + b*x^3)^2),x]
```

```
[Out] ((a*(b*c - a*d)*Sqrt[c + d*x^3])/(b*(a + b*x^3)) + (Sqrt[-(b*c) + a*d]*(2*b
*c + a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/b^(3/2) - 2
*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2)
```

Maple [A] (verified)

Time = 4.66 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$-\frac{-(b\,x^{3}+a)(ad+2bc)(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{(ad-bc)b}\left(2bc^{\frac{3}{2}}(b\,x^{3}+a)\,\text{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)+a\sqrt{d\,x^{3}+c}\,(ad-bc)\right)}{3\sqrt{(ad-bc)b}\,a^{2}b(b\,x^{3}+a)}$
default	$\frac{\frac{2d\,x^{3}\sqrt{d\,x^{3}+c}}{9}+\frac{8c\sqrt{d\,x^{3}+c}}{9}-\frac{2c^{\frac{3}{2}}\,\text{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3}}{a^{2}}+\frac{-\frac{2(ad-bc)^{2}\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{3}+\frac{2\sqrt{d\,x^{3}+c}\left(\frac{(-d\,x^{3}-4c)b}{3}+ad\right)\sqrt{(ad-}}{3}}{b\,a^{2}\sqrt{(ad-bc)b}}$
elliptic	Expression too large to display

[In] int((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x,method=_RETURNVERBOSE)

[Out] -1/3*(-(b*x^3+a)*(a*d+2*b*c)*(a*d-b*c)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(2*b*c^(3/2)*(b*x^3+a)*arctanh((d*x^3+c)^(1/2)/c^(1/2))+a*(d*x^3+c)^(1/2)*(a*d-b*c)))/((a*d-b*c)*b)^(1/2)/a^2/b/(b*x^3+a)
)

Fricas [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 686, normalized size of antiderivative = 5.24

$$\int \frac{(c+dx^3)^{3/2}}{x\,(a+bx^3)^2}\,dx = \left[\frac{((2\,b^2c+abd)x^3+2\,abc+a^2d)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\,(b^2cx^3+}{6\,(a^2b^2x^3+a^3b)}\right.$$

[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x, algorithm="fricas")

[Out] [1/6*(((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*(b^2*c*x^3 + a*b*c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt(d*x^3 + c)*(a*b*c - a^2*d))/(a^2*b^2*x^3 + a^3*b), 1/3*(((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (b^2*c*x^3 + a*b*c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + sqrt(d*x^3 + c)*(a*b*c - a^2*d))/(a^2*b^2*x^3 + a^3*b), 1/6*(4*(b^2*c*x^3 + a*b*c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + ((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*sqrt(d*x^3 + c)*(a*b*c - a^2*d))/(a^2*b^2*x^3 + a^3*b), 1/3*(((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + 2*(b^2*c*x^3 + a*b*c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + sqrt(d*x^3 + c)*(a*b*c - a^2*d))/(a^2*b^2*x^3 + a^3*b)]

Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)^2}\,dx = \int \frac{(c+dx^3)^{\frac{3}{2}}}{x(a+bx^3)^2}\,dx$$

```
[In] integrate((d*x**3+c)**(3/2)/x/(b*x**3+a)**2,x)
```

```
[Out] Integral((c + d*x**3)**(3/2)/(x*(a + b*x**3)**2), x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)^2x}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.18

$$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)^2}\,dx = \frac{2c^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3a^2\sqrt{-c}} - \frac{(2b^2c^2-abcd-a^2d^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}a^2b} + \frac{\sqrt{dx^3+c}bcd-\sqrt{dx^3+c}ad^2}{3((dx^3+c)b-bc+ad)ab}$$

```
[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 2/3*c^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)) - 1/3*(2*b^2*c^2 -
a*b*c*d - a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^
2*c + a*b*d)*a^2*b) + 1/3*(sqrt(d*x^3 + c)*b*c*d - sqrt(d*x^3 + c)*a*d^2)/(
((d*x^3 + c)*b - b*c + a*d)*a*b)
```

Mupad [B] (verification not implemented)

Time = 13.94 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.63

$$\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)^2}\,dx = \frac{c^{3/2}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a^2} + \frac{\sqrt{d\,x^3+c}\left(\frac{a\left(\frac{b\,d^2}{3\left(b^2\,c-a\,b\,d\right)}-\frac{2\,b^2\,c\,d}{3\,a\left(b^2\,c-a\,b\,d\right)}\right)}{b}+\frac{b^2\,c^2}{3\,a\,(b^2\,c-a\,b\,d)}\right)}{b\,x^3+a} + \frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,\sqrt{a\,d-b\,c}\,(a\,d+2\,b\,c)\,1\mathrm{i}}{6\,a^2\,b^{3/2}}$$

```
[In] int((c + d*x^3)^(3/2)/(x*(a + b*x^3)^2),x)
```

```
[Out] (c^(3/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))
)/x^6))/(3*a^2) + ((c + d*x^3)^(1/2)*((a*((b*d^2)/(3*(b^2*c - a*b*d)) - (2*
b^2*c*d)/(3*a*(b^2*c - a*b*d))))/b + (b^2*c^2)/(3*a*(b^2*c - a*b*d))))/(a +
 b*x^3) + (log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2
i + b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*(a*d + 2*b*c)*1i)/(6*a^2*b^(3/2
))
```

3.474 $\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 170

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)^2} dx = -\frac{(2bc-ad)\sqrt{c+dx^3}}{3a^2(a+bx^3)} - \frac{c\sqrt{c+dx^3}}{3ax^3(a+bx^3)} + \frac{\sqrt{c}(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3} - \frac{\sqrt{bc-ad}(4bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3\sqrt{b}}$$

```
[Out] 1/3*(-3*a*d+4*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/a^3-1/3*(-a*d+4
*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/a^
3/b^(1/2)-1/3*(-a*d+2*b*c)*(d*x^3+c)^(1/2)/a^2/(b*x^3+a)-1/3*c*(d*x^3+c)^(1
/2)/a/x^3/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {457, 100, 156, 162, 65, 214}

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)^2} dx = \frac{\sqrt{c}(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3} - \frac{\sqrt{bc-ad}(4bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3\sqrt{b}} - \frac{\sqrt{c+dx^3}(2bc-ad)}{3a^2(a+bx^3)} - \frac{c\sqrt{c+dx^3}}{3ax^3(a+bx^3)}$$

```
[In] Int[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] -1/3*((2*b*c - a*d)*Sqrt[c + d*x^3])/(a^2*(a + b*x^3)) - (c*Sqrt[c + d*x^3]
)/(3*a*x^3*(a + b*x^3)) + (Sqrt[c]*(4*b*c - 3*a*d)*ArcTanh[Sqrt[c + d*x^3]/
Sqrt[c]])/(3*a^3) - (Sqrt[b*c - a*d]*(4*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c
+ d*x^3])/Sqrt[b*c - a*d]])/(3*a^3*Sqrt[b])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(a+bx)^2}\,dx, x, x^3\right)$$

$$= -\frac{c\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}c(4bc-3ad)+\frac{1}{2}d(3bc-2ad)x}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{3a}$$

$$= -\frac{(2bc-ad)\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)} - \frac{c\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}c(4bc-3ad)(bc-ad)+\frac{1}{2}d(bc-ad)(2bc-ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a^2(bc-ad)}$$

$$= -\frac{(2bc-ad)\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)} - \frac{c\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} - \frac{(c(4bc-3ad))\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{6a^3} + \frac{((bc-ad)(4bc-ad))\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{6a^3}$$

$$= -\frac{(2bc-ad)\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)} - \frac{c\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} - \frac{(c(4bc-3ad))\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^3d} + \frac{((bc-ad)(4bc-ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^3d}$$

$$= -\frac{(2bc-ad)\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)} - \frac{c\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} + \frac{\sqrt{c}(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3} - \frac{\sqrt{bc-ad}(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)^2}\,dx = \frac{\frac{a\sqrt{c+dx^3}(-ac-2bcx^3+adx^3)}{x^3(a+bx^3)} + \frac{(4b^2c^2-5abcd+a^2d^2)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}\sqrt{-bc+ad}} + \sqrt{c}(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{c-}}{\sqrt{}}\right.}{3a^3}$$

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] ((a*Sqrt[c + d*x^3]*(-(a*c) - 2*b*c*x^3 + a*d*x^3))/(x^3*(a + b*x^3)) + ((4
*b^2*c^2 - 5*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c
) + a*d]])/(Sqrt[b]*Sqrt[-(b*c) + a*d]) + Sqrt[c]*(4*b*c - 3*a*d)*ArcTanh[S
qrt[c + d*x^3]/Sqrt[c]])/(3*a^3)
```

Maple [A] (verified)

Time = 4.72 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$\frac{\frac{x^3(bx^3+a)(ad-bc)(ad-4bc)\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)}{3}+\frac{4\sqrt{(ad-bc)b}\left(x^3\left(c^{\frac{3}{2}}b-\frac{3ad\sqrt{c}}{4}\right)(bx^3+a)\,\text{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)-\frac{(2bcx^3+a(-dx^3+}{4}\right.}{3}}{x^3\sqrt{(ad-bc)b}\,(bx^3+a)a^3}$
risch	$-\frac{c\sqrt{dx^3+c}}{3a^2x^3}-\frac{\frac{2\sqrt{c}\,(3ad-4bc)\,\text{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)}{3a}+\frac{8bc(ad-bc)\arctan\left(\frac{b\sqrt{dx^3+c}}{\sqrt{(ad-bc)b}}\right)}{3a\sqrt{(ad-bc)b}}+\frac{(-2a^2d^2+4abcd-2b^2c^2)\left(d\arctan\left(\frac{b\sqrt{dx^3+}}{\sqrt{(ad-bc}}\right.\right.}{3\sqrt{(ad-bc)b}\,(ad-bc}}{2a^2}$
default	$\frac{-\frac{c\sqrt{dx^3+c}}{3x^3}+\frac{2d\sqrt{dx^3+c}}{3}-\sqrt{c}\,d\,\text{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)}{a^2}-\frac{2b\left(\frac{2dx^3\sqrt{dx^3+c}}{9}+\frac{8c\sqrt{dx^3+c}}{9}-\frac{2c^{\frac{3}{2}}\,\text{arctanh}\left(\frac{\sqrt{dx^3+c}}{\sqrt{c}}\right)}{3}\right)}{a^3}+\frac{-d(bx}{}$
elliptic	Expression too large to display

```
[In] int((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4/3*(1/4*x^3*(b*x^3+a)*(a*d-b*c)*(a*d-4*b*c)*arctan(b*(d*x^3+c)^(1/2)/((a*d
-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(x^3*(c^(3/2)*b-3/4*a*d*c^(1/2))*(b*x^3
+a)*arctanh((d*x^3+c)^(1/2)/c^(1/2))-1/4*(2*b*c*x^3+a*(-d*x^3+c))*a*(d*x^3+
c)^(1/2)))/((a*d-b*c)*b)^(1/2)/x^3/(b*x^3+a)/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 838, normalized size of antiderivative = 4.93

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)^2}\,dx = \left[-\frac{((4\,b^2c-abd)x^6+(4\,abc-a^2d)x^3)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+((4\right.$$

$$-\frac{2\,((4\,b^2c-abd)x^6+(4\,abc-a^2d)x^3)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+((4\,b^2c-3\,abd)x^6+(4\,abc-}{6\,(a^3bx^6+a^4x^3)}$$

$$-\frac{2\,((4\,b^2c-3\,abd)x^6+(4\,abc-3\,a^2d)x^3)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right)+((4\,b^2c-abd)x^6+(4\,abc-a^2d)x^3)\sqrt{}}{6\,(a^3bx^6+a^4x^3)}$$

$$-\frac{((4\,b^2c-abd)x^6+(4\,abc-a^2d)x^3)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+((4\,b^2c-3\,abd)x^6+(4\,abc-3}{3\,(a^3bx^6+a^4x^3)}$$

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*sqrt((b*c - a*d)/b)*
log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^
3 + a)) + ((4*b^2*c - 3*a*b*d)*x^6 + (4*a*b*c - 3*a^2*d)*x^3)*sqrt(c)*log((
d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*((2*a*b*c - a^2*d)*x^3 +
a^2*c)*sqrt(d*x^3 + c))/(a^3*b*x^6 + a^4*x^3), -1/6*(2*((4*b^2*c - a*b*d)*x
^6 + (4*a*b*c - a^2*d)*x^3)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*
sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + ((4*b^2*c - 3*a*b*d)*x^6 + (4*a*b*c - 3
*a^2*d)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2
*((2*a*b*c - a^2*d)*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*x^6 + a^4*x^3), -1
/6*(2*((4*b^2*c - 3*a*b*d)*x^6 + (4*a*b*c - 3*a^2*d)*x^3)*sqrt(-c)*arctan(s
qrt(d*x^3 + c)*sqrt(-c)/c) + ((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3
)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt
((b*c - a*d)/b))/(b*x^3 + a)) + 2*((2*a*b*c - a^2*d)*x^3 + a^2*c)*sqrt(d*x^
3 + c))/(a^3*b*x^6 + a^4*x^3), -1/3*(((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^
2*d)*x^3)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/
b)/(b*c - a*d)) + ((4*b^2*c - 3*a*b*d)*x^6 + (4*a*b*c - 3*a^2*d)*x^3)*sqrt(
-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + ((2*a*b*c - a^2*d)*x^3 + a^2*c)*sq
rt(d*x^3 + c))/(a^3*b*x^6 + a^4*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*x**3+c)**(3/2)/x**4/(b*x**3+a)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)^2x^4}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.27

$$\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)^2}\,dx = \frac{(4\,b^2c^2-5\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}a^3} - \frac{(4\,bc^2-3\,acd)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^3\sqrt{-c}} - \frac{2\,(dx^3+c)^{\frac{3}{2}}bcd-2\,\sqrt{dx^3+c}bc^2d-(dx^3+c)^{\frac{3}{2}}ad^2+2\,\sqrt{dx^3+c}acd^2}{3\,\left((dx^3+c)^2b-2\,(dx^3+c)bc+bc^2+(dx^3+c)ad-acd\right)a^2}$$

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x, algorithm="giac")
[Out] 1/3*(4*b^2*c^2 - 5*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c
+ a*b*d))/(sqrt(-b^2*c + a*b*d)*a^3) - 1/3*(4*b*c^2 - 3*a*c*d)*arctan(sqrt(
d*x^3 + c)/sqrt(-c))/(a^3*sqrt(-c)) - 1/3*(2*(d*x^3 + c)^(3/2)*b*c*d - 2*sq
rt(d*x^3 + c)*b*c^2*d - (d*x^3 + c)^(3/2)*a*d^2 + 2*sqrt(d*x^3 + c)*a*c*d^2
)/(((d*x^3 + c)^2*b - 2*(d*x^3 + c)*b*c + b*c^2 + (d*x^3 + c)*a*d - a*c*d)*
a^2)
```

Mupad [B] (verification not implemented)

Time = 15.79 (sec) , antiderivative size = 531, normalized size of antiderivative = 3.12

$$\int \frac{(c+dx^3)^{3/2}}{x^4\,(a+bx^3)^2}\,dx = \frac{\sqrt{c}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)\,(3\,a\,d-4\,b\,c)}{6\,a^3} - \frac{c\,\sqrt{d\,x^3+c}}{3\,a^2\,x^3}$$

$$-\frac{\sqrt{d\,x^3+c}\,\left(\frac{3\,a\,d-4\,b\,c}{2\,a^2} - \frac{a\,\left(\frac{a\,\left(\frac{a\,\left(\frac{b\,d^2\,(a\,d+b\,c)}{a^3\,c^2} - \frac{a\,\left(\frac{b^2\,d^3}{2\,a^3\,c^2} - \frac{b^2\,d^3\,(3\,a\,d-4\,b\,c)}{6\,a^2\,c^2\,\left(a^2\,d-a\,b\,c\right)} + \frac{b^2\,d^2\,(a\,d+b\,c)\,(3\,a\,d-4\,b\,c)}{3\,a^3\,c^2\,\left(a^2\,d-a\,b\,c\right)}\right)}{b}\right)}{b} + \frac{b\,(3\,a\,d-4\,b\,c)\,\left(a^2\,d^3+4\,a\,b\,c\,d^2 \cdots\right.}{6\,a^3\,c^2\,\left(a^2\,d-a\,b\,c\right)} \cdots\right)}{b} \cdots\right)}{\cdots}\right)}{b\,x^3+a}$$

$$+\frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,\sqrt{a\,d-b\,c}\,(a\,d-4\,b\,c)\,1\mathrm{i}}{6\,a^3\,\sqrt{b}}$$

```
[In] int((c + d*x^3)^(3/2)/(x^4*(a + b*x^3)^2),x)
```

```
[Out] (c^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))
)/x^6)*(3*a*d - 4*b*c))/(6*a^3) - (c*(c + d*x^3)^(1/2))/(3*a^2*x^3) - ((c +
 d*x^3)^(1/2)*((3*a*d - 4*b*c)/(2*a^2) - (a*((a*((a*((b*d^2*(a*d + b*c))/(a
^3*c^2) - (a*((b^2*d^3)/(2*a^3*c^2) - (b^2*d^3*(3*a*d - 4*b*c))/(6*a^2*c^2*
(a^2*d - a*b*c)) + (b^2*d^2*(a*d + b*c)*(3*a*d - 4*b*c))/(3*a^3*c^2*(a^2*d
- a*b*c))))/b + (b*(3*a*d - 4*b*c)*(a^2*d^3 - b^2*c^2*d + 4*a*b*c*d^2))/(6*
```

```
a^3*c^2*(a^2*d - a*b*c))))/b - (a^2*d^3 - b^2*c^2*d + 4*a*b*c*d^2)/(2*a^3*c
^2) + (b*(3*a*d - 4*b*c)*(2*b^2*c^3 - 4*a^2*c*d^2 + 2*a*b*c^2*d))/(6*a^3*c^
2*(a^2*d - a*b*c))))/b - (2*b^2*c^3 - 4*a^2*c*d^2 + 2*a*b*c^2*d)/(2*a^3*c^2
) + (b*(3*a*d - 4*b*c)^2)/(6*a^2*(a^2*d - a*b*c))))/b))/(a + b*x^3) + (log(
(2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a
 + b*x^3))*(a*d - b*c)^(1/2)*(a*d - 4*b*c)*1i)/(6*a^3*b^(1/2))
```

3.475 $\int \frac{x^3(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{x^3(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \frac{cx^4\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{4}{3},2,-\frac{3}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] 1/4*c*x^4*AppellF1(4/3,2,-3/2,7/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/(1
+d*x^3/c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{x^3(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \frac{cx^4\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{4}{3},2,-\frac{3}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[In] Int[(x^3*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (c*x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 2, -3/2, 7/3, -((b*x^3)/a), -((d*x^3)/
c)])/(4*a^2*Sqrt[1 + (d*x^3)/c])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{x^3\left(1+\frac{dx^3}{c}\right)^{3/2}}{(a+bx^3)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$

$$= \frac{cx^4\sqrt{c+dx^3}F_1\left(\frac{4}{3};2,-\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 338 vs. 2(65) = 130.

Time = 10.47 (sec) , antiderivative size = 338, normalized size of antiderivative = 5.20

$$\int \frac{x^3(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \frac{x^4\left(\frac{d(43bc-55ad)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a}+\frac{8\left(-8acd(11ad+b(c+6dx^3))\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\right.\right.}{(a+bx^3)\left(-8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2}\right.\right.}\right.}{120b}$$

```
[In] Integrate[(x^3*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (x^4*((d*(43*b*c - 55*a*d)*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -
((d*x^3)/c), -((b*x^3)/a)])/a + (8*(-8*a*c*d*(11*a*d + b*(c + 6*d*x^3))*App
ellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 3*(c + d*x^3)*(5*b*c
- 11*a*d - 6*b*d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*
x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((
a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] +
 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*
AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))))/(120*b^2*Sqrt[c
 + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 6.48 (sec) , antiderivative size = 808, normalized size of antiderivative = 12.43

method	result	size
elliptic	Expression too large to display	808
risch	Expression too large to display	1564
default	Expression too large to display	1587

```
[In] int(x^3*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(a*d-b*c)*x/b^2*(d*x^3+c)^(1/2)/(b*x^3+a)+2/5*x/b^2*d*(d*x^3+c)^(1/2)-2
/3*I*(-11/6*(a*d-b*c)*d/b^3-2/5*c*d/b^2)*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)
^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I/b^3/d^2*2^(1/2)*sum((-11*a^2*d^2+13*a*
b*c*d-2*b^2*c^2)/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d
^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2
*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*
x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*
_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)
*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^
(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c)
,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^3(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx=\int \frac{x^3(c+dx^3)^{\frac{3}{2}}}{(a+bx^3)^2}\,dx$$

```
[In] integrate(x**3*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
[Out] Integral(x**3*(c + d*x**3)**(3/2)/(a + b*x**3)**2, x)
```

Maxima [F]

$$\int \frac{x^3(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx=\int \frac{(dx^3+c)^{\frac{3}{2}}x^3}{(bx^3+a)^2}\,dx$$

```
[In] integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)*x^3/(b*x^3 + a)^2, x)
```

Giac [F]

$$\int \frac{x^3(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx=\int \frac{(dx^3+c)^{\frac{3}{2}}x^3}{(bx^3+a)^2}\,dx$$

```
[In] integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
[Out] integrate((d*x^3 + c)^(3/2)*x^3/(b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx=\int \frac{x^3\,{\left(d\,x^3+c\right)}^{3/2}}{{\left(b\,x^3+a\right)}^2}\,dx$$

```
[In] int((x^3*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x)
[Out] int((x^3*(c + d*x^3)^(3/2))/(a + b*x^3)^2, x)
```

3.476 $\int \frac{x(c+dx^3)^{3/2}}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 65

$$\int \frac{x(c+dx^3)^{3/2}}{(a+bx^3)^2} dx = \frac{cx^2\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{2}{3},2,-\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] 1/2*c*x^2*AppellF1(2/3,2,-3/2,5/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/(1
+d*x^3/c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {525, 524}

$$\int \frac{x(c+dx^3)^{3/2}}{(a+bx^3)^2} dx = \frac{cx^2\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{2}{3},2,-\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[In] Int[(x*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (c*x^2*Sqrt[c + d*x^3]*AppellF1[2/3, 2, -3/2, 5/3, -((b*x^3)/a), -((d*x^3)/
c)])/(2*a^2*Sqrt[1 + (d*x^3)/c])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{x\left(1+\frac{dx^3}{c}\right)^{3/2}}{(a+bx^3)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$

$$= \frac{cx^2\sqrt{c+dx^3}F_1\left(\frac{2}{3};2,-\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 177 vs. 2(65) = 130.

Time = 10.20 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.72

$$\int \frac{x(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \frac{x^2\left(-10a(-bc+ad)(c+dx^3)+5c(bc+2ad)(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\right.\right.}{30a^2b(a+bx}$$

```
[In] Integrate[(x*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (x^2*(-10*a*(-(b*c) + a*d)*(c + d*x^3) + 5*c*(b*c + 2*a*d)*(a + b*x^3)*Sqrt
[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] - d*
(b*c - 7*a*d)*x^3*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3
, -((d*x^3)/c), -((b*x^3)/a)]))/(30*a^2*b*(a + b*x^3)*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.77 (sec) , antiderivative size = 955, normalized size of antiderivative = 14.69

method	result	size
default	Expression too large to display	955
elliptic	Expression too large to display	955

```
[In] int(x*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(a*d-b*c)/b/a*x^2*(d*x^3+c)^(1/2)/(b*x^3+a)-2/3*I*(d^2/b^2+1/6/b^2*d*(
a*d-b*c)/a)*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3)
)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/18*I/a/b^2/d^2*2^
(1/2)*sum((7*a^2*d^2-5*a*b*c*d-2*b^2*c^2)/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(
1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3)
)^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/
3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-
c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^
(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))
*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)
*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*
_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+
1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \int \frac{x(c+dx^3)^{\frac{3}{2}}}{(a+bx^3)^2}\,dx$$

```
[In] integrate(x*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
[Out] Integral(x*(c + d*x**3)**(3/2)/(a + b*x**3)**2, x)
```

Maxima [F]

$$\int \frac{x(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}x}{(bx^3+a)^2}\,dx$$

```
[In] integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)*x/(b*x^3 + a)^2, x)
```

Giac [F]

$$\int \frac{x(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}x}{(bx^3+a)^2}\,dx$$

```
[In] integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
[Out] integrate((d*x^3 + c)^(3/2)*x/(b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \int \frac{x\,(d\,x^3+c)^{3/2}}{(b\,x^3+a)^2}\,dx$$

```
[In] int((x*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x)
[Out] int((x*(c + d*x^3)^(3/2))/(a + b*x^3)^2, x)
```

3.477 $\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^2} dx = \frac{cx\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},2,-\frac{3}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] c*x*AppellF1(1/3,2,-3/2,4/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/(1+d*x^3
/c)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {441, 440}

$$\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^2} dx = \frac{cx\sqrt{c+dx^3}\,\text{AppellF1}\left(\frac{1}{3},2,-\frac{3}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[In] Int[(c + d*x^3)^(3/2)/(a + b*x^3)^2,x]
```

```
[Out] (c*x*Sqrt[c + d*x^3]*AppellF1[1/3, 2, -3/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(a^2*Sqrt[1 + (d*x^3)/c])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
```

```
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{\left(1+\frac{dx^3}{c}\right)^{3/2}}{(a+bx^3)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$

$$= \frac{cx\sqrt{c+dx^3}F_1\left(\frac{1}{3};2,-\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 339 vs. $2(60) = 120$.

Time = 10.32 (sec) , antiderivative size = 339, normalized size of antiderivative = 5.65

$$\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \frac{x\left(d(bc+5ad)x^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{a\left(-64ac\left(-ad^2x^3+bc(3c+dx^3)\right)\right.\ldots}{(a+bx^3)\left(-8ac\ldots\right.}\right.}{2\ldots}$$

```
[In] Integrate[(c + d*x^3)^(3/2)/(a + b*x^3)^2,x]
```

```
[Out] (x*(d*(b*c + 5*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d
*x^3)/c), -((b*x^3)/a)] + (a*(-64*a*c*(-(a*d^2*x^3) + b*c*(3*c + d*x^3))*Ap
pellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 24*(b*c - a*d)*x^3*(
c + d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] +
a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3)*
(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b
*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/
3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))))/(24*a^2*b*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.57 (sec) , antiderivative size = 801, normalized size of antiderivative = 13.35

method	result	size
default	Expression too large to display	801
elliptic	Expression too large to display	801

```
[In] int((d*x^3+c)^(3/2)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(a*d-b*c)/b/a*x*(d*x^3+c)^(1/2)/(b*x^3+a)-2/3*I*(d^2/b^2-1/6/b^2*d*(a*
d-b*c)/a)*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)
/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/
(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1
/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*
d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1
/18*I/a/b^2/d^2*2^(1/2)*sum((5*a^2*d^2-a*b*c*d-4*b^2*c^2)/_alpha^2/(a*d-b*c
)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3
)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^
(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(
-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_al
pha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha
*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*
3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*
d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d
-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2
/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^
3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \int \frac{(c+dx^3)^{\frac{3}{2}}}{(a+bx^3)^2}\,dx$$

```
[In] integrate((d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
[Out] Integral((c + d*x**3)**(3/2)/(a + b*x**3)**2, x)
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)^2}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)/(b*x^3 + a)^2, x)
```

Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)^2}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
[Out] integrate((d*x^3 + c)^(3/2)/(b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx = \int \frac{(d\,x^3+c)^{3/2}}{(b\,x^3+a)^2}\,dx$$

```
[In] int((c + d*x^3)^(3/2)/(a + b*x^3)^2,x)
[Out] int((c + d*x^3)^(3/2)/(a + b*x^3)^2, x)
```

3.478 $\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 63

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)^2}\,dx = -\frac{c\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{1}{3},2,-\frac{3}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] -c*AppellF1(-1/3,2,-3/2,2/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/x/(1+d*x
^3/c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)^2}\,dx = -\frac{c\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{1}{3},2,-\frac{3}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{\frac{dx^3}{c}+1}}$$

```
[In] Int[(c + d*x^3)^(3/2)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -((c*Sqrt[c + d*x^3]*AppellF1[-1/3, 2, -3/2, 2/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a^2*x*Sqrt[1 + (d*x^3)/c]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{\left(1+\frac{dx^3}{c}\right)^{3/2}}{x^2(a+bx^3)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}$$

$$= -\frac{c\sqrt{c+dx^3}F_1\left(-\frac{1}{3};2,-\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{1+\frac{dx^3}{c}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 190 vs. 2(63) = 126.

Time = 10.25 (sec) , antiderivative size = 190, normalized size of antiderivative = 3.02

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)^2}\,dx = \frac{-20a(c+dx^3)(3ac+4bcx^3-adx^3)+5c(-8bc+11ad)x^3(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF}}{60a^3x\,(a}$$

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] (-20*a*(c + d*x^3)*(3*a*c + 4*b*c*x^3 - a*d*x^3) + 5*c*(-8*b*c + 11*a*d)*x^
3*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c),
-((b*x^3)/a)] + 2*d*(4*b*c - a*d)*x^6*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*Appel
lF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(60*a^3*x*(a + b*x^3)*Sq
rt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 6.32 (sec) , antiderivative size = 970, normalized size of antiderivative = 15.40

method	result	size
elliptic	Expression too large to display	970
risch	Expression too large to display	1854
default	Expression too large to display	2364

```
[In] int((d*x^3+c)^(3/2)/x^2/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(a*d-b*c)/a^2*x^2*(d*x^3+c)^(1/2)/(b*x^3+a)-c/a^2*(d*x^3+c)^(1/2)/x-2/3
*I*(-1/6*d*(a*d-b*c)/a^2/b+1/2*d*c/a^2)*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/
d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
)*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1
/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(
1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(
1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)
))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/
3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/
2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))
^(1/2)))+1/18*I/a^2/b/d^2*2^(1/2)*sum((-a^2*d^2-7*a*b*c*d+8*b^2*c^2)/_alpha
/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*
d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(
1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^
(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/
3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/
3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*
(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^
(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1
/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=R
ootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^2\left(a+bx^3\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^2\left(a+bx^3\right)^2}\,dx=\int \frac{\left(c+dx^3\right)^{\frac{3}{2}}}{x^2\left(a+bx^3\right)^2}\,dx$$

```
[In] integrate((d*x**3+c)**(3/2)/x**2/(b*x**3+a)**2,x)
[Out] Integral((c + d*x**3)**(3/2)/(x**2*(a + b*x**3)**2), x)
```

Maxima [F]

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^2\left(a+bx^3\right)^2}\,dx=\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(bx^3+a\right)^2x^2}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x^2), x)
```

Giac [F]

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^2\left(a+bx^3\right)^2}\,dx=\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(bx^3+a\right)^2x^2}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a)^2,x, algorithm="giac")
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)^2}\,dx = \int \frac{(d\,x^3+c)^{3/2}}{x^2\,(b\,x^3+a)^2}\,dx$$

```
[In] int((c + d*x^3)^(3/2)/(x^2*(a + b*x^3)^2),x)
[Out] int((c + d*x^3)^(3/2)/(x^2*(a + b*x^3)^2), x)
```

3.479 $\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)^2}\,dx = -\frac{c\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{2}{3},2,-\frac{3}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{1+\frac{dx^3}{c}}}$$

```
[Out] -1/2*c*AppellF1(-2/3,2,-3/2,1/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/x^2/
(1+d*x^3/c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)^2}\,dx = -\frac{c\sqrt{c+dx^3}\,\text{AppellF1}\left(-\frac{2}{3},2,-\frac{3}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[In] Int[(c + d*x^3)^(3/2)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -1/2*(c*Sqrt[c + d*x^3]*AppellF1[-2/3, 2, -3/2, 1/3, -((b*x^3)/a), -((d*x^3
)/c)])/(a^2*x^2*Sqrt[1 + (d*x^3)/c])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{\left(1+\frac{dx^3}{c}\right)^{3/2}}{x^3(a+bx^3)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}} \\ &= -\frac{c\sqrt{c+dx^3}F_1\left(-\frac{2}{3};2,-\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{1+\frac{dx^3}{c}}} \end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 370 vs. $2(65) = 130$.

Time = 10.41 (sec) , antiderivative size = 370, normalized size of antiderivative = 5.69

$$\int \frac{(c+dx^3)^{3/2}}{x^3\left(a+bx^3\right)^2}\,dx = \frac{-d(5bc-2ad)x^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{8a\left(4ac\left(10bcx^3(3c+dx^3)+a(6\cdots\right.\right.}{\cdots}}{\cdots}$$

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] (-(d*(5*b*c - 2*a*d)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((
d*x^3)/c), -((b*x^3)/a)]) + (8*a*(4*a*c*(10*b*c*x^3*(3*c + d*x^3) + a*(6*c^
2 - 15*c*d*x^3 - 4*d^2*x^6))*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*
x^3)/a)] - 3*x^3*(c + d*x^3)*(3*a*c + 5*b*c*x^3 - 2*a*d*x^3)*(2*b*c*AppellF
1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1,
 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2
, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2,
7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)
/c), -((b*x^3)/a)]))))/(48*a^3*x^2*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 6.81 (sec) , antiderivative size = 815, normalized size of antiderivative = 12.54

method	result	size
elliptic	Expression too large to display	815
risch	Expression too large to display	1549
default	Expression too large to display	1902

```
[In] int((d*x^3+c)^(3/2)/x^3/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(a*d-b*c)/a^2*x*(d*x^3+c)^(1/2)/(b*x^3+a)-1/2*c/a^2*(d*x^3+c)^(1/2)/x^2
-2/3*I*(1/6*d*(a*d-b*c)/a^2/b-1/4*d*c/a^2)*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1
/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3
))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^
2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I/a^2/b/d^2*2^(1/2)*sum((a^2*d^2-11*a*
b*c*d+10*b^2*c^2)/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*
d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(
2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d
*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2
*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2
)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*
d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)
^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c
),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)^2}\,dx=\text{Timed out}$$

```
[In] integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*x**3+c)**(3/2)/x**3/(b*x**3+a)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)^2x^3}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x^3), x)
```

Giac [F]

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)^2}\,dx = \int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)^2x^3}\,dx$$

```
[In] integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a)^2,x, algorithm="giac")
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)^2}\,dx = \int \frac{(d\,x^3+c)^{3/2}}{x^3\,(b\,x^3+a)^2}\,dx$$

```
[In] int((c + d*x^3)^(3/2)/(x^3*(a + b*x^3)^2),x)
[Out] int((c + d*x^3)^(3/2)/(x^3*(a + b*x^3)^2), x)
```

3.480 $\int \frac{x^8}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 123

$$\int \frac{x^8}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{2\sqrt{c+dx^3}}{3b^2d} - \frac{a^2\sqrt{c+dx^3}}{3b^2(bc-ad)\,(a+bx^3)} + \frac{a(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}(bc-ad)^{3/2}}$$

```
[Out] 1/3*a*(-3*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(5
/2)/(-a*d+b*c)^(3/2)+2/3*(d*x^3+c)^(1/2)/b^2/d-1/3*a^2*(d*x^3+c)^(1/2)/b^2/
(-a*d+b*c)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 91, 81, 65, 214}

$$\int \frac{x^8}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = -\frac{a^2\sqrt{c+dx^3}}{3b^2\,(a+bx^3)\,(bc-ad)} + \frac{a(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}(bc-ad)^{3/2}} + \frac{2\sqrt{c+dx^3}}{3b^2d}$$

```
[In] Int[x^8/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*b^2*d) - (a^2*Sqrt[c + d*x^3])/(3*b^2*(b*c - a*d)*(a
 + b*x^3)) + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c
- a*d]])/(3*b^(5/2)*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= -\frac{a^2\sqrt{c+dx^3}}{3b^2(bc-ad)\,(a+bx^3)} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(2bc-ad)+b(bc-ad)x}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3b^2(bc-ad)}$$

$$= \frac{2\sqrt{c+dx^3}}{3b^2d} - \frac{a^2\sqrt{c+dx^3}}{3b^2(bc-ad)(a+bx^3)} - \frac{(a(4bc-3ad))\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{6b^2(bc-ad)}$$

$$= \frac{2\sqrt{c+dx^3}}{3b^2d} - \frac{a^2\sqrt{c+dx^3}}{3b^2(bc-ad)(a+bx^3)} - \frac{(a(4bc-3ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3b^2d(bc-ad)}$$

$$= \frac{2\sqrt{c+dx^3}}{3b^2d} - \frac{a^2\sqrt{c+dx^3}}{3b^2(bc-ad)(a+bx^3)} + \frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}(bc-ad)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.06

$$\int \frac{x^8}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{\frac{\sqrt{b}\sqrt{c+dx^3}(-3a^2d+2b^2cx^3+2ab(c-dx^3))}{d(bc-ad)(a+bx^3)} + \frac{a(4bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{3b^{5/2}}$$

```
[In] Integrate[x^8/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((Sqrt[b]*Sqrt[c + d*x^3]*(-3*a^2*d + 2*b^2*c*x^3 + 2*a*b*(c - d*x^3)))/(d*
(b*c - a*d)*(a + b*x^3)) + (a*(4*b*c - 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^
3])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/(3*b^(5/2))
```

Maple [A] (verified)

Time = 4.60 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$\frac{-d(b x^3+a)a\left(ad-\frac{4bc}{3}\right)\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{d x^3+c}\left(-\frac{2b^2 c x^3}{3}-\frac{2a\left(-d x^3+c\right)b}{3}+a^2 d\right)\sqrt{(ad-bc)b}}{\sqrt{(ad-bc)b}\, d\, b^2 (ad-bc)(b x^3+a)}$
risch	$\frac{2\sqrt{d x^3+c}}{3b^2 d}-\frac{a\left(\frac{4\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}-\frac{a\left(d\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)\left(b x^3+a\right)+\sqrt{d x^3+c}\sqrt{(ad-bc)b}\right)}{3\sqrt{(ad-bc)b}\,(ad-bc)\left(b x^3+a\right)}\right)}{b^2}$
default	$\frac{2\sqrt{d x^3+c}}{3b^2 d}+\frac{a^2\left(d\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)(b x^3+a)+\sqrt{d x^3+c}\sqrt{(ad-bc)b}\right)}{3b^2\sqrt{(ad-bc)b}\,(ad-bc)(b x^3+a)}-\frac{4a\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3b^2\sqrt{(ad-bc)b}}$
elliptic	$\frac{a^2\sqrt{d x^3+c}}{3(ad-bc)b^2(b x^3+a)}+\frac{2\sqrt{d x^3+c}}{3b^2 d}+\frac{ia\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(3ad-4bc)\left(-c d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right.}{d}\right.}{\left(-c d^2\right)^{\frac{1}{3}}}}}{}\right.}{}$

[In] int(x^8/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/((a*d-b*c)*b)^(1/2)*(-d*(b*x^3+a)*a*(a*d-4/3*b*c)*arctan(b*(d*x^3+c)^(1/2
)/((a*d-b*c)*b)^(1/2))+(d*x^3+c)^(1/2)*(-2/3*b^2*c*x^3-2/3*a*(-d*x^3+c)*b+a
^2*d)*((a*d-b*c)*b)^(1/2))/d/b^2/(a*d-b*c)/(b*x^3+a)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(103) = 206$.

Time = 0.29 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.86

$$\int \frac{x^8}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$

$$= \left[\frac{(4\,a^2bcd-3\,a^3d^2+(4\,ab^2cd-3\,a^2bd^2)x^3)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)+2\,(2\,ab^3c^2-5}{6\,(ab^5c^2d-2\,a^2b^4cd^2+a^3b^3d^3+(b^6c^2d-2\,ab^5cd^2+a}\right.$$

$$-\frac{(4\,a^2bcd-3\,a^3d^2+(4\,ab^2cd-3\,a^2bd^2)x^3)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-b^2c+abd}}{bdx^3+bc}\right)-(2\,ab^3c^2-5\,a^2b^2cd}{3\,(ab^5c^2d-2\,a^2b^4cd^2+a^3b^3d^3+(b^6c^2d-2\,ab^5cd^2+a^2b^4}$$

```
[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^3)*sqrt(b^2*
c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*
d))/(b*x^3 + a)) + 2*(2*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2 + 2*(b^4*c^
2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c^2*d - 2*a^2*b
^4*c*d^2 + a^3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^3), -1
/3*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^3)*sqrt(-b^2*c
 + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) - (2
*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2 + 2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b
^2*d^2)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c^2*d - 2*a^2*b^4*c*d^2 + a^3*b^3*d^3
+ (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^3)]
```

Sympy [F]

$$\int \frac{x^8}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^8}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(x**8/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**8/((a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.09

$$\int \frac{x^8}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{dx^3+c}a^2d}{3\,(b^3c-ab^2d)((dx^3+c)b-bc+ad)} - \frac{(4\,abc-3\,a^2d)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,(b^3c-ab^2d)\sqrt{-b^2c+abd}} + \frac{2\sqrt{dx^3+c}}{3\,b^2d}$$

```
[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(d*x^3 + c)*a^2*d/((b^3*c - a*b^2*d)*((d*x^3 + c)*b - b*c + a*d))
- 1/3*(4*a*b*c - 3*a^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((
b^3*c - a*b^2*d)*sqrt(-b^2*c + a*b*d)) + 2/3*sqrt(d*x^3 + c)/(b^2*d)
```

Mupad [B] (verification not implemented)

Time = 11.61 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.30

$$\int \frac{x^8}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{2\sqrt{d\,x^3+c}\,(2\,b^2\,c-2\,a\,b\,d)}{3\,d\,\left(2\,b^4\,c-2\,a\,b^3\,d\right)} - \frac{2\,a^2\sqrt{d\,x^3+c}}{3\,b\,\left(b\,x^3+a\right)\,\left(2\,b^2\,c-2\,a\,b\,d\right)} + \frac{a\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,\left(3\,a\,d-4\,b\,c\right)\,1\mathrm{i}}{6\,b^{5/2}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

```
[In] int(x^8/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

```
[Out] (2*(c + d*x^3)^(1/2)*(2*b^2*c - 2*a*b*d))/(3*d*(2*b^4*c - 2*a*b^3*d)) - (2*
a^2*(c + d*x^3)^(1/2))/(3*b*(a + b*x^3)*(2*b^2*c - 2*a*b*d)) + (a*log((a*d
- 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*
x^3))*(3*a*d - 4*b*c)*1i)/(6*b^(5/2)*(a*d - b*c)^(3/2))
```

3.481 $\int \frac{x^5}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 99

$$\int \frac{x^5}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx=\frac{a\sqrt{c+dx^3}}{3b(bc-ad)\,(a+bx^3)}-\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}$$

```
[Out] -1/3*(-a*d+2*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)
/(-a*d+b*c)^(3/2)+1/3*a*(d*x^3+c)^(1/2)/b/(-a*d+b*c)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {457, 79, 65, 214}

$$\int \frac{x^5}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx=\frac{a\sqrt{c+dx^3}}{3b\,(a+bx^3)\,(bc-ad)}-\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}$$

```
[In] Int[x^5/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (a*Sqrt[c + d*x^3])/(3*b*(b*c - a*d)*(a + b*x^3)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(3/2)*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{(a+bx)^2\sqrt{c+dx}}\,dx,x,x^3\right)\\
&= \frac{a\sqrt{c+dx^3}}{3b(bc-ad)\left(a+bx^3\right)}+\frac{(2bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{6b(bc-ad)}\\
&= \frac{a\sqrt{c+dx^3}}{3b(bc-ad)\left(a+bx^3\right)}+\frac{(2bc-ad)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3bd(bc-ad)}\\
&= \frac{a\sqrt{c+dx^3}}{3b(bc-ad)\left(a+bx^3\right)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

$$\int \frac{x^5}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{\frac{a\sqrt{b}\sqrt{c+dx^3}}{(bc-ad)(a+bx^3)}-\frac{(2bc-ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{3b^{3/2}}$$

```
[In] Integrate[x^5/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((a*Sqrt[b]*Sqrt[c + d*x^3])/((b*c - a*d)*(a + b*x^3)) - ((2*b*c - a*d)*Arc
Tan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/(3
*b^(3/2))
```

Maple [A] (verified)

Time = 4.64 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{-\frac{a\sqrt{d\,x^3+c}}{b\,x^3+a}+\frac{(ad-2bc)\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}}{3(ad-bc)b}$
default	$\frac{2\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3b\sqrt{(ad-bc)b}}-\frac{a\left(d\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)(b\,x^3+a)+\sqrt{d\,x^3+c}\,\sqrt{(ad-bc)b}\right)}{3b\sqrt{(ad-bc)b}\,(ad-bc)(b\,x^3+a)}$
elliptic	$-\frac{a\sqrt{d\,x^3+c}}{3(ad-bc)b(b\,x^3+a)}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(-ad+2bc)\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d(\cdots}{-3(-c\,d\cdots}}\cdots}{\cdots}\right.}{\cdots}$

```
[In] int(x^5/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/(a*d-b*c)/b*(-a*(d*x^3+c)^(1/2)/(b*x^3+a)+(a*d-2*b*c)/((a*d-b*c)*b)^(1/
2)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 348, normalized size of antiderivative = 3.52

$$\int \frac{x^5}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$
$$= \left[\frac{((2\,b^2c-abd)x^3+2\,abc-a^2d)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)+2\,(ab^2c-a^2bd)\sqrt{dx^3+c}}{6\,(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2+(b^5c^2-2\,ab^4cd+a^2b^3d^2)x^3)}\right.$$

```
[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(((2*b^2*c - a*b*d)*x^3 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b*
d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) +
 2*(a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^
2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^3), 1/3*(((2*b^2*c - a*b*d)
*x^3 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b
^2*c + a*b*d)/(b*d*x^3 + b*c)) + (a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a*b^
4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)
*x^3)]
```

Sympy [F]

$$\int \frac{x^5}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^5}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(x**5/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.17

$$\int \frac{x^5}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{\frac{\sqrt{dx^3+c}ad^2}{(b^2c-abd)((dx^3+c)b-bc+ad)}+\frac{(2bcd-ad^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^2c-abd)\sqrt{-b^2c+abd}}}{3d}$$

```
[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(sqrt(d*x^3 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^3 + c)*b - b*c + a*d)) +
(2*b*c*d - a*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c -
a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

Mupad [B] (verification not implemented)

Time = 11.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.12

$$\int \frac{x^5}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{2a\sqrt{d\,x^3+c}}{3\,(b\,x^3+a)\,(2\,b^2\,c-2\,a\,b\,d)} + \frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,(a\,d-2\,b\,c)\,1\mathrm{i}}{6\,b^{3/2}\,(a\,d-b\,c)^{3/2}}$$

```
[In] int(x^5/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

```
[Out] (log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^
3)/(a + b*x^3))*(a*d - 2*b*c)*1i)/(6*b^(3/2)*(a*d - b*c)^(3/2)) + (2*a*(c +
 d*x^3)^(1/2))/(3*(a + b*x^3)*(2*b^2*c - 2*a*b*d))
```

3.482 $\int \frac{x^2}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{x^2}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{c+dx^3}}{3(bc-ad)(a+bx^3)}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{3/2}}$$

```
[Out] 1/3*d*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(3/2)/b^
(1/2)-1/3*(d*x^3+c)^(1/2)/(-a*d+b*c)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {455, 44, 65, 214}

$$\int \frac{x^2}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^3}}{3(a+bx^3)(bc-ad)}$$

```
[In] Int[x^2/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/3*Sqrt[c + d*x^3]/((b*c - a*d)*(a + b*x^3)) + (d*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^3])/Sqrt[b*c - a*d]])/(3*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
```

```
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{c+dx^3}}{3(bc-ad)\left(a+bx^3\right)} - \frac{d\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{6(bc-ad)}\\
&= -\frac{\sqrt{c+dx^3}}{3(bc-ad)\left(a+bx^3\right)} - \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3(bc-ad)}\\
&= -\frac{\sqrt{c+dx^3}}{3(bc-ad)\left(a+bx^3\right)} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \frac{x^2}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{1}{3}\left(-\frac{\sqrt{c+dx^3}}{(bc-ad)(a+bx^3)}+\frac{d\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}(-bc+ad)^{3/2}}\right)$$

```
[In] Integrate[x^2/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
[Out] (-(Sqrt[c + d*x^3]/((b*c - a*d)*(a + b*x^3))) + (d*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^3])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/3
```

Maple [A] (verified)

Time = 4.44 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

method	result
default	$\frac{d\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)(b\,x^3+a)+\sqrt{d\,x^3+c}\,\sqrt{(ad-bc)b}}{3\sqrt{(ad-bc)b}\,(ad-bc)(b\,x^3+a)}$
pseudoelliptic	$\frac{d\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)(b\,x^3+a)+\sqrt{d\,x^3+c}\,\sqrt{(ad-bc)b}}{3\sqrt{(ad-bc)b}\,(ad-bc)(b\,x^3+a)}$
elliptic	$\frac{\sqrt{d\,x^3+c}}{3(ad-bc)(b\,x^3+a)}-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(\cdots\right.}}\cdots}{\cdots}\right.}{\cdots}$

```
[In] int(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3/((a*d-b*c)*b)^(1/2)*(d*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))*(b
*x^3+a)+(d*x^3+c)^(1/2)*((a*d-b*c)*b)^(1/2))/(a*d-b*c)/(b*x^3+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(71) = 142$.

Time = 0.27 (sec) , antiderivative size = 302, normalized size of antiderivative = 3.47

$$\int \frac{x^2}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \left[-\frac{(bdx^3+ad)\sqrt{b^2c-abd}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\sqrt{b^2c-abd}}{bx^3+a}\right)+2\,\sqrt{dx^3+c}(b^2c-abd)}{6\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^3)}, -\frac{(bdx^3+ad)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-b^2c+abd}}{bdx^3+bc}\right)+\sqrt{dx^3+c}(b^2c-abd)}{3\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^3)}\right]$$

```
[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/6*((b*d*x^3 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*s
qrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*sqrt(d*x^3 + c)*(b^2*c
 - a*b*d))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d
+ a^2*b^2*d^2)*x^3), -1/3*((b*d*x^3 + a*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt
(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) + sqrt(d*x^3 + c)*(b^2*c
- a*b*d))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d +
 a^2*b^2*d^2)*x^3)]
```

Sympy [F]

$$\int \frac{x^2}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^2}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(x**2/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**2/((a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = -\frac{d\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}(bc-ad)} - \frac{\sqrt{dx^3+c}d}{3\left((dx^3+c)b-bc+ad\right)(bc-ad)}$$

```
[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*(b*c - a*d)) - 1/3*sqrt(d*x^3 + c)*d/((((d*x^3 + c)*b - b*c + a*d)*(b*c - a
*d))
```

Mupad [B] (verification not implemented)

Time = 10.66 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.20

$$\int \frac{x^2}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = -\frac{2\,b\,\sqrt{d\,x^3+c}}{3\,\left(b\,x^3+a\right)\,\left(2\,b^2\,c-2\,a\,b\,d\right)} + \frac{d\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{6\,\sqrt{b}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

```
[In] int(x^2/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

```
[Out] (d*log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*
x^3)/(a + b*x^3))*1i)/(6*b^(1/2)*(a*d - b*c)^(3/2)) - (2*b*(c + d*x^3)^(1/2
))/(3*(a + b*x^3)*(2*b^2*c - 2*a*b*d))
```

3.483 $\int \frac{1}{x(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 132

$$\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\frac{b\sqrt{c+dx^3}}{3a(bc-ad)\left(a+bx^3\right)}-\frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}+\frac{\sqrt{b}(2bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}}$$

```
[Out] 1/3*(-3*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2
)/a^2/(-a*d+b*c)^(3/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^2/c^(1/2)+1/3
*b*(d*x^3+c)^(1/2)/a/(-a*d+b*c)/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 105, 162, 65, 214}

$$\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\frac{\sqrt{b}(2bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}+\frac{b\sqrt{c+dx^3}}{3a\left(a+bx^3\right)(bc-ad)}$$

```
[In] Int[1/(x*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (b*Sqrt[c + d*x^3])/(3*a*(b*c - a*d)*(a + b*x^3)) - (2*ArcTanh[Sqrt[c + d*x
^3]/Sqrt[c]])/(3*a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*S
qrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^2*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= \frac{b\sqrt{c+dx^3}}{3a(bc-ad)\,(a+bx^3)} + \frac{\text{Subst}\left(\int \frac{bc-ad+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a(bc-ad)}$$

$$= \frac{b\sqrt{c+dx^3}}{3a(bc-ad)\left(a+bx^3\right)} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{3a^2} - \frac{(b(2bc-3ad))\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{6a^2(bc-ad)}$$

$$= \frac{b\sqrt{c+dx^3}}{3a(bc-ad)\left(a+bx^3\right)} + \frac{2\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3a^2d} - \frac{(b(2bc-3ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3a^2d(bc-ad)}$$

$$= \frac{b\sqrt{c+dx^3}}{3a(bc-ad)\left(a+bx^3\right)} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} + \frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{-\frac{ab\sqrt{c+dx^3}}{(-bc+ad)(a+bx^3)} + \frac{\sqrt{b}(2bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}} - \frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a^2}$$

```
[In] Integrate[1/(x*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
[Out] (-((a*b*Sqrt[c + d*x^3])/((-(b*c) + a*d)*(a + b*x^3))) + (Sqrt[b]*(2*b*c -
3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)
^(3/2) - (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/Sqrt[c])/(3*a^2)
```

Maple [A] (verified)

Time = 4.57 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11

method	result
pseudoelliptic	$-\frac{-2b\sqrt{c}\left(bc-\frac{3ad}{2}\right)\left(b\,x^3+a\right)\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)+\left(2\left(b\,x^3+a\right)(ad-bc)\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)+\sqrt{d\,x^3+c}\,\sqrt{c}\,ab\right)\sqrt{(ad-bc)b}}{3\sqrt{c}\,\sqrt{(ad-bc)b}\,a^2(ad-bc)\left(b\,x^3+a\right)}$
default	$-\frac{2\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}-\frac{2b\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3a^2\sqrt{(ad-bc)b}}-\frac{b\left(d\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)\left(b\,x^3+a\right)+\sqrt{d\,x^3+c}\,\sqrt{(ad-bc)b}\right)}{3a\sqrt{(ad-bc)b}\,(ad-bc)\left(b\,x^3+a\right)}$
elliptic	Expression too large to display

```
[In] int(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/c^(1/2)*(-2*b*c^(1/2)*(b*c-3/2*a*d)*(b*x^3+a)*arctan(b*(d*x^3+c)^(1/2)
/((a*d-b*c)*b)^(1/2))+(2*(b*x^3+a)*(a*d-b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2
))+(d*x^3+c)^(1/2)*c^(1/2)*a*b)*((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)/a^
2/(a*d-b*c)/(b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 862, normalized size of antiderivative = 6.53

$$\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = \left[\frac{2\sqrt{dx^3+c}abc+(2abc^2-3a^2cd+(2b^2c^2-3abcd)x^3)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2bc-ad+2\sqrt{dx^3+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)-}{6\left(a^3bc^2-a^4cd+(a^2b^2c^2-a^3bcd)x^3\right)}\right.$$

```
[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*sqrt(d*x^3 + c)*a*b*c + (2*a*b*c^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b
*c*d)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 +
c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*((b^2*c - a*b*d)*x^3 +
 a*b*c - a^2*d)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))
/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^3), 1/3*(sqrt(d*x^3 + c
)*a*b*c + (2*a*b*c^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^3)*sqrt(-b/(b*
c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3
 + b*c)) + ((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*sqrt(c)*log((d*x^3 - 2*sqr
t(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3
*b*c*d)*x^3), 1/6*(2*sqrt(d*x^3 + c)*a*b*c + 4*((b^2*c - a*b*d)*x^3 + a*b*c
 - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (2*a*b*c^2 - 3*a^2*
c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c
 - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)))/(
a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^3), 1/3*(sqrt(d*x^3 + c)*
a*b*c + (2*a*b*c^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^3)*sqrt(-b/(b*c
- a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 +
 b*c)) + 2*((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^3
 + c)*sqrt(-c)/c))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^3)]
```

Sympy [F]

$$\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(1/x/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(x*(a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\int \frac{1}{\left(bx^3+a\right)^2\sqrt{dx^3+c}x}\,dx$$

```
[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\frac{\sqrt{dx^3+c}bd}{3\left(abc-a^2d\right)\left(\left(dx^3+c\right)b-bc+ad\right)}-\frac{\left(2\,b^2c-3\,abd\right)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\left(a^2bc-a^3d\right)\sqrt{-b^2c+abd}}+\frac{2\,\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}}$$

```
[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] 1/3*sqrt(d*x^3 + c)*b*d/((a*b*c - a^2*d)*((d*x^3 + c)*b - b*c + a*d)) - 1/3
*(2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b
*c - a^3*d)*sqrt(-b^2*c + a*b*d)) + 2/3*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a
^2*sqrt(-c))
```

Mupad [B] (verification not implemented)

Time = 14.69 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.23

$$\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a^2\,\sqrt{c}}+\frac{b^2\,\sqrt{d\,x^3+c}}{3\,a\,\left(b\,x^3+a\right)\,\left(b^2\,c-a\,b\,d\right)}+\frac{\sqrt{b}\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\left(3\,a\,d-2\,b\,c\right)\,1{}\mathrm{i}}{6\,a^2\,{\left(a\,d-b\,c\right)}^{3/2}}$$

```
[In] int(1/(x*(a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

```
[Out] log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(3
*a^2*c^(1/2)) + (b^2*(c + d*x^3)^(1/2))/(3*a*(a + b*x^3)*(b^2*c - a*b*d)) +
 (b^(1/2)*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i
 - b*d*x^3)/(a + b*x^3))*(3*a*d - 2*b*c)*1i)/(6*a^2*(a*d - b*c)^(3/2))
```

3.484 $\int \frac{1}{x^4(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 185

$$\int \frac{1}{x^4(a+bx^3)^2\sqrt{c+dx^3}}\,dx = -\frac{b(2bc-ad)\sqrt{c+dx^3}}{3a^2c(bc-ad)(a+bx^3)} - \frac{\sqrt{c+dx^3}}{3acx^3(a+bx^3)} + \frac{(4bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{3/2}} - \frac{b^{3/2}(4bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3(bc-ad)^{3/2}}$$

```
[Out] 1/3*(a*d+4*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^3/c^(3/2)-1/3*b^(3/2)*(-
5*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a^3/(-a*d+b*
c)^(3/2)-1/3*b*(-a*d+2*b*c)*(d*x^3+c)^(1/2)/a^2/c/(-a*d+b*c)/(b*x^3+a)-1/3*
(d*x^3+c)^(1/2)/a/c/x^3/(b*x^3+a)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {457, 105, 156, 162, 65, 214}

$$\int \frac{1}{x^4(a+bx^3)^2\sqrt{c+dx^3}}\,dx = -\frac{b^{3/2}(4bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3(bc-ad)^{3/2}} + \frac{(ad+4bc)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{3/2}} - \frac{b\sqrt{c+dx^3}(2bc-ad)}{3a^2c(a+bx^3)(bc-ad)} - \frac{\sqrt{c+dx^3}}{3acx^3(a+bx^3)}$$

```
[In] Int[1/(x^4*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/3*(b*(2*b*c - a*d)*Sqrt[c + d*x^3])/(a^2*c*(b*c - a*d)*(a + b*x^3)) - Sq
rt[c + d*x^3]/(3*a*c*x^3*(a + b*x^3)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x
^3]/Sqrt[c]])/(3*a^3*c^(3/2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*S
qrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^3*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{3acx^3(a+bx^3)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4bc+ad)+\frac{3bdx}{2}}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^3}}{3a^2c(bc-ad)(a+bx^3)} - \frac{\sqrt{c+dx^3}}{3acx^3(a+bx^3)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-ad)(4bc+ad)+\frac{1}{2}bd(2bc-ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a^2c(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^3}}{3a^2c(bc-ad)(a+bx^3)} - \frac{\sqrt{c+dx^3}}{3acx^3(a+bx^3)} + \frac{(b^2(4bc-5ad))\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{6a^3(bc-ad)} - \frac{(4bc+ad)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{6a^3c}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^3}}{3a^2c(bc-ad)(a+bx^3)} - \frac{\sqrt{c+dx^3}}{3acx^3(a+bx^3)} + \frac{(b^2(4bc-5ad))\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^3d(bc-ad)} - \frac{(4bc+ad)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^3cd}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^3}}{3a^2c(bc-ad)(a+bx^3)} - \frac{\sqrt{c+dx^3}}{3acx^3(a+bx^3)} + \frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{3/2}} - \frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3(bc-ad)^{3/2}}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^4 (a+bx^3)^2 \sqrt{c+dx^3}} \, dx = \frac{\frac{a\sqrt{c+dx^3}(-a^2d+2b^2cx^3+ab(c-dx^3))}{c(-bc+ad)x^3(a+bx^3)} - \frac{b^{3/2}(4bc-5ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}} + \frac{(4bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{c^{3/2}}}{3a^3}$$

```
[In] Integrate[1/(x^4*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((a*Sqrt[c + d*x^3]*(-(a^2*d) + 2*b^2*c*x^3 + a*b*(c - d*x^3)))/(c*(-(b*c)
+ a*d)*x^3*(a + b*x^3)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^3])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2) + ((4*b*c + a*d)*ArcTanh
[Sqrt[c + d*x^3]/Sqrt[c]])/c^(3/2))/(3*a^3)
```

Maple [A] (verified)

Time = 4.80 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{-4x^3c^{\frac{5}{2}}\left(bc-\frac{5ad}{4}\right)b^2(b\,x^3+a)\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)+\left(c\,x^3(b\,x^3+a)(ad+4bc)(ad-bc)\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)+c^{\frac{3}{2}}(2b^2c\,x^3+a(\cdots\right.}{3\sqrt{(ad-bc)b}\,a^3(ad-bc)(b\,x^3+a)c^{\frac{5}{2}}x^3}$
risch	$-\frac{\sqrt{d\,x^3+c}}{3c\,a^2x^3}-\frac{-\frac{2(ad+4bc)\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3a\sqrt{c}}-\frac{2b^2c\left(d\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)(b\,x^3+a)+\sqrt{d\,x^3+c}\,\sqrt{(ad-bc)b}\right)}{3\sqrt{(ad-bc)b}\,(ad-bc)(b\,x^3+a)}-\frac{8b^2c\arctan\left(\frac{b\sqrt{d\cdots}}{\sqrt{(a\cdots}}\right.}{3a\sqrt{(ad-bc\cdots}}}{2c\,a^2}$
default	$\frac{-\frac{\sqrt{d\,x^3+c}}{3c\,x^3}+\frac{d\ \text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}}{a^2}+\frac{4b\ \text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3a^3\sqrt{c}}+\frac{b^2\left(d\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)(b\,x^3+a)+\sqrt{d\,x^3+c}\,\sqrt{(ad-bc)b}\cdots\right.}{3a^2\sqrt{(ad-bc)b}\,(ad-bc)(b\,x^3+a)}$
elliptic	Expression too large to display

```
[In] int(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/((a*d-b*c)*b)^(1/2)*(-4*x^3*c^(5/2)*(b*c-5/4*a*d)*b^2*(b*x^3+a)*arctan(
b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))+(c*x^3*(b*x^3+a)*(a*d+4*b*c)*(a*d-b*
c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))+c^(3/2)*(2*b^2*c*x^3+a*(-d*x^3+c)*b-a^2
*d)*a*(d*x^3+c)^(1/2))*((a*d-b*c)*b)^(1/2))/a^3/(a*d-b*c)/(b*x^3+a)/c^(5/2)
/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 1236, normalized size of antiderivative = 6.68

$$\int \frac{1}{x^4 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^6 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^3)
*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*(b*c -
a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*
d^2)*x^6 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^3)*sqrt(c)*log((d*x^3 +
2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*(a^2*b*c^2 - a^3*c*d + (2*a*b^2*c
^2 - a^2*b*c*d)*x^3)*sqrt(d*x^3 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^6 + (a
^4*b*c^3 - a^5*c^2*d)*x^3), -1/6*(2*((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^6 + (4*a
*b^2*c^3 - 5*a^2*b*c^2*d)*x^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)
*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) - ((4*b^3*c^2 - 3*a*b^2*
c*d - a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^3)*sqrt(c)*l
og((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*(a^2*b*c^2 - a^3*c*d
 + (2*a*b^2*c^2 - a^2*b*c*d)*x^3)*sqrt(d*x^3 + c))/((a^3*b^2*c^3 - a^4*b*c^2
*d)*x^6 + (a^4*b*c^3 - a^5*c^2*d)*x^3), -1/6*(2*((4*b^3*c^2 - 3*a*b^2*c*d -
 a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^3)*sqrt(-c)*arcta
n(sqrt(d*x^3 + c)*sqrt(-c)/c) - ((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^6 + (4*a*b^2
*c^3 - 5*a^2*b*c^2*d)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d -
 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*(a^2*b
*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^3)*sqrt(d*x^3 + c))/((a^3*b^2*
c^3 - a^4*b*c^2*d)*x^6 + (a^4*b*c^3 - a^5*c^2*d)*x^3), -1/3*(((4*b^3*c^3 -
5*a*b^2*c^2*d)*x^6 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^3)*sqrt(-b/(b*c - a*d)
)*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c))
 + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - 3*a^2*b*c*d
- a^3*d^2)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (a^2*b*c^2 -
a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^3)*sqrt(d*x^3 + c))/((a^3*b^2*c^3 - a
^4*b*c^2*d)*x^6 + (a^4*b*c^3 - a^5*c^2*d)*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{x^4 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx$$

```
[In] integrate(1/x**4/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{(bx^3+a)^2\sqrt{dx^3+c}x^4}\,dx$$

```
[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^4 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx = \frac{(4\,b^3c-5\,ab^2d)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,(a^3bc-a^4d)\sqrt{-b^2c+abd}} - \frac{2\,(dx^3+c)^{\frac{3}{2}}b^2cd-2\,\sqrt{dx^3+c}b^2c^2d-(dx^3+c)^{\frac{3}{2}}abd^2+2\,\sqrt{dx^3+c}abcd^2-\sqrt{dx^3+c}a^2d^3}{3\,(a^2bc^2-a^3cd)\left((dx^3+c)^2b-2\,(dx^3+c)bc+bc^2+(dx^3+c)ad-acd\right)} - \frac{(4\,bc+ad)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^3\sqrt{-c}c}$$

```
[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] 1/3*(4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((
a^3*b*c - a^4*d)*sqrt(-b^2*c + a*b*d)) - 1/3*(2*(d*x^3 + c)^(3/2)*b^2*c*d -
 2*sqrt(d*x^3 + c)*b^2*c^2*d - (d*x^3 + c)^(3/2)*a*b*d^2 + 2*sqrt(d*x^3 + c
)*a*b*c*d^2 - sqrt(d*x^3 + c)*a^2*d^3)/((a^2*b*c^2 - a^3*c*d)*((d*x^3 + c)^
2*b - 2*(d*x^3 + c)*b*c + b*c^2 + (d*x^3 + c)*a*d - a*c*d)) - 1/3*(4*b*c +
a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^3*sqrt(-c)*c)
```

Mupad [B] (verification not implemented)

Time = 15.99 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.92

$$\int \frac{1}{x^4 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx$$
$$= \frac{\sqrt{d\,x^3+c}\left(\frac{d\,a^2+4\,b\,c\,a}{2\,a^3\,c^2}-\frac{a\left(\frac{2\,c\,b^2+2\,a\,d\,b}{2\,a^3\,c^2}-\frac{a\left(\frac{b^2\,d}{2\,a^3\,c^2}+\frac{b\left(2\,c\,b^2+2\,a\,d\,b\right)\,(3\,a\,d-4\,b\,c)}{6\,a^3\,c^2\left(a^2\,d-a\,b\,c\right)}-\frac{b^2\,d\,(3\,a\,d-4\,b\,c)}{6\,a^2\,c^2\left(a^2\,d-a\,b\,c\right)}\right)}{b}+\frac{b\left(d\,a^2+4\,b\,c\,a\right)\,(3\,a\,d-4\,b\,c)}{6\,a^3\,c^2\left(a^2\,d-a\,b\,c\right)}\right)}{b}\right)}{b\,x^3+a}$$
$$-\frac{\sqrt{d\,x^3+c}}{3\,a^2\,c\,x^3}+\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)^3}{x^6}\right)\,(a\,d+4\,b\,c)}{6\,a^3\,c^{3/2}}$$
$$+\frac{b^{3/2}\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,(5\,a\,d-4\,b\,c)\,1\mathrm{i}}{6\,a^3\,(a\,d-b\,c)^{3/2}}$$

```
[In] int(1/(x^4*(a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

```
[Out] ((c + d*x^3)^(1/2)*((a^2*d + 4*a*b*c)/(2*a^3*c^2) - (a*((2*b^2*c + 2*a*b*d)
/(2*a^3*c^2) - (a*((b^2*d)/(2*a^3*c^2) + (b*(2*b^2*c + 2*a*b*d)*(3*a*d - 4*
b*c))/(6*a^3*c^2*(a^2*d - a*b*c)) - (b^2*d*(3*a*d - 4*b*c))/(6*a^2*c^2*(a^2
*d - a*b*c))))/b + (b*(a^2*d + 4*a*b*c)*(3*a*d - 4*b*c))/(6*a^3*c^2*(a^2*d
- a*b*c))))/b))/(a + b*x^3) - (c + d*x^3)^(1/2)/(3*a^2*c*x^3) + (log((((c +
 d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2) + c^(1/2))^3)/x^6)*(a*d + 4*b*c
))/(6*a^3*c^(3/2)) + (b^(3/2)*log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*
(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(5*a*d - 4*b*c)*1i)/(6*a^3*(a*
d - b*c)^(3/2))
```

3.485 $\int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{x^4\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},2,\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{c+dx^3}}$$

```
[Out] 1/4*x^4*AppellF1(4/3,2,1/2,7/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/(d*
x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{x^4\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{4}{3},2,\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{c+dx^3}}$$

```
[In] Int[x^3/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)])/(4*a^2*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{(a+bx^3)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}} \\
&= \frac{x^4\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{4}{3};2,\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{c+dx^3}}
\end{aligned}
$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 238 vs. $2(64) = 128$.

Time = 10.23 (sec) , antiderivative size = 238, normalized size of antiderivative = 3.72

$$
\int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{x\left(\frac{dx^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a}+\frac{8\left(c+dx^3+\frac{8ac^2\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{-8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+3x^3\left(2bc\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},2,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+ad\,\text{AppellF1}\left(\frac{4}{3},\frac{3}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}\right)}{a+bx^3}\right)}{24(-bc+ad)\sqrt{c+dx^3}}
$$

```
[In] Integrate[x^3/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*((d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((
b*x^3)/a)])/a + (8*(c + d*x^3 + (8*a*c^2*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^
3)/c), -((b*x^3)/a)])/(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b
*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)
/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(a +
b*x^3)))/(24*(-(b*c) + a*d)*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.51 (sec) , antiderivative size = 764, normalized size of antiderivative = 11.94

method	result	size
elliptic	Expression too large to display	764
default	Expression too large to display	1207

```
[In] int(x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/(a*d-b*c)*x*(d*x^3+c)^(1/2)/(b*x^3+a)-1/9*I/(a*d-b*c)/b*3^(1/2)*(-c*d^2
)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*
d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2
)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3
^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)
^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I/b/d^2*2^(1/2)*sum((a*
d+2*b*c)/(a*d-b*c)^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*
(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1
/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d
*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)
^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha
^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x
+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1
/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*
3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^
(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx=\text{Timed out}$$

```
[In] integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(x**3/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(x**3/((a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{(bx^3+a)^2\sqrt{dx^3+c}}\,dx$$

```
[In] integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{(bx^3+a)^2\sqrt{dx^3+c}}\,dx$$

```
[In] integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^3/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{(b\,x^3+a)^2\,\sqrt{d\,x^3+c}}\,dx$$

```
[In] int(x^3/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
[Out] int(x^3/((a + b*x^3)^2*(c + d*x^3)^(1/2)), x)
```

3.486 $\int \frac{x}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{x}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{x^2\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},2,\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{c+dx^3}}$$

```
[Out] 1/2*x^2*AppellF1(2/3,2,1/2,5/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/(d*
x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {525, 524}

$$\int \frac{x}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{x^2\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{2}{3},2,\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{c+dx^3}}$$

```
[In] Int[x/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 2, 1/2, 5/3, -((b*x^3)/a), -((d*x^3)
/c)])/(2*a^2*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x}{(a+bx^3)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$= \frac{x^2\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{2}{3};2,\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{c+dx^3}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(64) = 128.

Time = 10.18 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.69

$$\int \frac{x}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$
$$= \frac{10abx^2(c+dx^3)+5(bc-3ad)x^2(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-bdx^5(a+bx^3)\sqrt{1}}{30a^2(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}}$$

```
[In] Integrate[x/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (10*a*b*x^2*(c + d*x^3) + 5*(b*c - 3*a*d)*x^2*(a + b*x^3)*Sqrt[1 + (d*x^3)/
c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] - b*d*x^5*(a + b*
x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)
/a)])/(30*a^2*(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.56 (sec) , antiderivative size = 923, normalized size of antiderivative = 14.42

method	result	size
default	Expression too large to display	923
elliptic	Expression too large to display	923

```
[In] int(x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b/(a*d-b*c)/a*x^2*(d*x^3+c)^(1/2)/(b*x^3+a)-1/9*I/(a*d-b*c)/a*3^(1/2)*
(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3
^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*
((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/
2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c
*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/18*I/a/d^2*2^(1/2)*sum((-5*a*d+2*b*c)/(
a*d-b*c)^2/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c
*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*
(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-
c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d
^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d
^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1
/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alp
ha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*
d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_
alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \text{Timed out}$$

```
[In] integrate(x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(x/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(x/((a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x}{(bx^3+a)^2\sqrt{dx^3+c}}\,dx$$

```
[In] integrate(x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x}{(bx^3+a)^2\sqrt{dx^3+c}}\,dx$$

```
[In] integrate(x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(x/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{x}{(b\,x^3+a)^2\,\sqrt{d\,x^3+c}}\,dx$$

```
[In] int(x/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
[Out] int(x/((a + b*x^3)^2*(c + d*x^3)^(1/2)), x)
```

3.487 $\int \frac{1}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{x\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{1}{3},2,\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{c+dx^3}}$$

```
[Out] x*AppellF1(1/3,2,1/2,4/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/(d*x^3+c)
^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {441, 440}

$$\int \frac{1}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \frac{x\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},2,\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{c+dx^3}}$$

```
[In] Int[1/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a^2*Sqrt[c + d*x^3])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int\frac{1}{(a+bx^3)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$= \frac{x\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{1}{3};2,\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{c+dx^3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 392 vs. 2(59) = 118.

Time = 10.28 (sec) , antiderivative size = 392, normalized size of antiderivative = 6.64

$$\int\frac{1}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$
$$= \frac{-8acx\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\left(8a(3bc-3ad+bdx^3)+bdx^3(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2}\right.\right.}{24a^2(bc-ad)(a+bx^3)\sqrt{c+dx^3}\left(-8ac\,\text{A}\right.}$$

```
[In] Integrate[1/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-8*a*c*x*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)]*(8*a*(3*b*
c - 3*a*d + b*d*x^3) + b*d*x^3*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3
, 1/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]) + 3*b*x^4*(8*a*(c + d*x^3) + d*
x^3*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c)
, -((b*x^3)/a)])*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/
a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))/(24*a^2*
(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3]*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3,
-((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*
x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*
x^3)/a)])))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.67 (sec) , antiderivative size = 769, normalized size of antiderivative = 13.03

method	result	size
default	Expression too large to display	769
elliptic	Expression too large to display	769

```
[In] int(1/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b/(a*d-b*c)/a*x*(d*x^3+c)^(1/2)/(b*x^3+a)+1/9*I/(a*d-b*c)/a*3^(1/2)*(-
c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(
1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I/a/d^2*2^(1/2)*su
m((-7*a*d+4*b*c)/(a*d-b*c)^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c
*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*
(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(
d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+
2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/
2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c
*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2
)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*
c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{1}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(1/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(1/((a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{1}{(bx^3+a)^2\sqrt{dx^3+c}}\,dx$$

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{1}{(bx^3+a)^2\sqrt{dx^3+c}}\,dx$$

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx = \int \frac{1}{(b\,x^3+a)^2\,\sqrt{d\,x^3+c}}\,dx$$

```
[In] int(1/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
[Out] int(1/((a + b*x^3)^2*(c + d*x^3)^(1/2)), x)
```

3.488 $\int \frac{1}{x^2(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{1}{x^2\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{1}{3},2,\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{c+dx^3}}$$

```
[Out] -AppellF1(-1/3,2,1/2,2/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/x/(d*x^3+
c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{1}{x^2\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{1}{3},2,\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{c+dx^3}}$$

```
[In] Int[1/(x^2*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] -((Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 2, 1/2, 2/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a^2*x*Sqrt[c + d*x^3]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^2(a+bx^3)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$= -\frac{\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{1}{3};2,\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{c+dx^3}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 226 vs. $2(62) = 124$.

Time = 10.26 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.65

$$\int \frac{1}{x^2\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$$
$$= \frac{20a(c+dx^3)\left(3a^2d-4b^2cx^3-3ab(c-dx^3)\right)-5\left(8b^2c^2-15abcd+3a^2d^2\right)x^3(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}}{60a^3c(bc-ad)x\left(a+b\right.}$$

```
[In] Integrate[1/(x^2*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (20*a*(c + d*x^3)*(3*a^2*d - 4*b^2*c*x^3 - 3*a*b*(c - d*x^3)) - 5*(8*b^2*c^
2 - 15*a*b*c*d + 3*a^2*d^2)*x^3*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/
3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*b*d*(4*b*c - 3*a*d)*x^6*(a
+ b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*
x^3)/a)])/(60*a^3*c*(b*c - a*d)*x*(a + b*x^3)*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 6.40 (sec) , antiderivative size = 963, normalized size of antiderivative = 15.53

method	result	size
elliptic	Expression too large to display	963
default	Expression too large to display	1818
risch	Expression too large to display	1819

```
[In] int(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/(a*d-b*c)/a^2*b^2*x^2*(d*x^3+c)^(1/2)/(b*x^3+a)-1/c/a^2*(d*x^3+c)^(1/2)
/x-2/3*I*(-1/6*b*d/(a*d-b*c)/a^2+1/2*d/c/a^2)*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(
1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*
d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),
(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)
^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^
2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I
*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3)))^(1/2)))+1/18*I*b/a^2/d^2*2^(1/2)*sum((11*a*d-8*b*c)/(a*d-b*c)^2/_alp
ha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/
3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3
^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+
(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_a
lpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alph
a*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c
*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*
d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z
^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^2\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = \int \frac{1}{x^2\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(1/x**2/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(x**2*(a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = \int \frac{1}{\left(bx^3+a\right)^2\sqrt{dx^3+c}x^2}\,dx$$

```
[In] integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = \int \frac{1}{\left(bx^3+a\right)^2\sqrt{dx^3+c}x^2}\,dx$$

```
[In] integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{x^2 \left(b\,x^3+a\right)^2 \sqrt{d\,x^3+c}}\,dx$$

```
[In] int(1/(x^2*(a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
[Out] int(1/(x^2*(a + b*x^3)^2*(c + d*x^3)^(1/2)), x)
```

3.489 $\int \frac{1}{x^3(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{x^3\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{2}{3},2,\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{c+dx^3}}$$

```
[Out] -1/2*AppellF1(-2/3,2,1/2,1/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/x^2/(
d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{1}{x^3\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{2}{3},2,\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{c+dx^3}}$$

```
[In] Int[1/(x^3*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/2*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 2, 1/2, 1/3, -((b*x^3)/a), -((d*x^
3)/c)])/(a^2*x^2*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^3(a+bx^3)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$

$$= -\frac{\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{2}{3};2,\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{c+dx^3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 411 vs. $2(64) = 128$.

Time = 10.63 (sec) , antiderivative size = 411, normalized size of antiderivative = 6.42

$$\int \frac{1}{x^3(a+bx^3)^2\sqrt{c+dx^3}}\,dx$$

$$= \frac{bd(5bc-3ad)x^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{a\left(32ac\left(-10b^2cx^3(3c+dx^3)+3a^2d(2c+3dx^3)+3ab(-2c^2+\right.\right.}{(a+bx^3)}}{48}$$

```
[In] Integrate[1/(x^3*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (b*d*(5*b*c - 3*a*d)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((
d*x^3)/c), -((b*x^3)/a)] + (a*(32*a*c*(-10*b^2*c*x^3*(3*c + d*x^3) + 3*a^2*
d*(2*c + 3*d*x^3) + 3*a*b*(-2*c^2 + 7*c*d*x^3 + 2*d^2*x^6))*AppellF1[1/3, 1
/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 24*x^3*(c + d*x^3)*(-3*a^2*d + 5*
b^2*c*x^3 + 3*a*b*(c - d*x^3))*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/
c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/
a)])))/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x
^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a
)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(48*a^3
*c*(-(b*c) + a*d)*x^2*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 6.31 (sec) , antiderivative size = 809, normalized size of antiderivative = 12.64

method	result	size
elliptic	Expression too large to display	809
default	Expression too large to display	1512
risch	Expression too large to display	1513

```
[In] int(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/(a*d-b*c)/a^2*b^2*x*(d*x^3+c)^(1/2)/(b*x^3+a)-1/2/c/a^2*(d*x^3+c)^(1/2)
/x^2-2/3*I*(1/6*b*d/(a*d-b*c)/a^2-1/4*d/c/a^2)*3^(1/2)/d*(-c*d^2)^(1/3)*(I*
(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d
*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1
/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-
c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I*b/a^2/d^2*2^(1/2)*sum((13*a*d-10
*b*c)/(a*d-b*c)^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c
*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3)
)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1
/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*
d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/
2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(
1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/
2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))
^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^3 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{x^3 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx$$

```
[In] integrate(1/x**3/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(x**3*(a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{(bx^3+a)^2 \sqrt{dx^3+c}x^3}\,dx$$

```
[In] integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{(bx^3+a)^2 \sqrt{dx^3+c}x^3}\,dx$$

```
[In] integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx = \int \frac{1}{x^3 (b\,x^3+a)^2 \sqrt{d\,x^3+c}}\,dx$$

```
[In] int(1/(x^3*(a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
[Out] int(1/(x^3*(a + b*x^3)^2*(c + d*x^3)^(1/2)), x)
```

3.490 $\int \frac{x^8}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 150

$$\int \frac{x^8}{(a+bx^3)^2\,(c+dx^3)^{3/2}}\,dx = \frac{-2b^2c^2-a^2d^2}{3b^2d(bc-ad)^2\sqrt{c+dx^3}} - \frac{a^2}{3b^2(bc-ad)\,(a+bx^3)\,\sqrt{c+dx^3}} + \frac{a(4bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{5/2}}$$

```
[Out] 1/3*a*(-a*d+4*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2
)/(-a*d+b*c)^(5/2)+1/3*(-a^2*d^2-2*b^2*c^2)/b^2/d/(-a*d+b*c)^2/(d*x^3+c)^(1
/2)-1/3*a^2/b^2/(-a*d+b*c)/(b*x^3+a)/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 91, 79, 65, 214}

$$\int \frac{x^8}{(a+bx^3)^2\,(c+dx^3)^{3/2}}\,dx = -\frac{a^2d^2+2b^2c^2}{3b^2d\sqrt{c+dx^3}(bc-ad)^2} - \frac{a^2}{3b^2\,(a+bx^3)\,\sqrt{c+dx^3}(bc-ad)} + \frac{a(4bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{5/2}}$$

```
[In] Int[x^8/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/3*(2*b^2*c^2 + a^2*d^2)/(b^2*d*(b*c - a*d)^2*Sqrt[c + d*x^3]) - a^2/(3*b
^2*(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3]) + (a*(4*b*c - a*d)*ArcTanh[(Sqr
t[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(3/2)*(b*c - a*d)^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= -\frac{a^2}{3b^2(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(2bc+ad)+b(bc-ad)x}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{3b^2(bc-ad)}$$

$$= -\frac{2b^2c^2+a^2d^2}{3b^2d(bc-ad)^2\sqrt{c+dx^3}} - \frac{a^2}{3b^2(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{(a(4bc-ad))\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{6b(bc-ad)^2}$$

$$= -\frac{2b^2c^2+a^2d^2}{3b^2d(bc-ad)^2\sqrt{c+dx^3}} - \frac{a^2}{3b^2(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{(a(4bc-ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3bd(bc-ad)^2}$$

$$= -\frac{2b^2c^2+a^2d^2}{3b^2d(bc-ad)^2\sqrt{c+dx^3}} - \frac{a^2}{3b^2(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.89

$$\int \frac{x^8}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{-\frac{\sqrt{b}\left(2abc^2+2b^2c^2x^3+a^2d\left(c+dx^3\right)\right)}{d(bc-ad)^2(a+bx^3)\sqrt{c+dx^3}} + \frac{a(-4bc+ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{5/2}}}{3b^{3/2}}$$

```
[In] Integrate[x^8/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-((Sqrt[b]*(2*a*b*c^2 + 2*b^2*c^2*x^3 + a^2*d*(c + d*x^3)))/(d*(b*c - a*d)
^2*(a + b*x^3)*Sqrt[c + d*x^3])) + (a*(-4*b*c + a*d)*ArcTan[(Sqrt[b]*Sqrt[c
 + d*x^3])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(5/2))/(3*b^(3/2))
```

Maple [A] (verified)

Time = 4.39 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$-\frac{-ad\sqrt{d\,x^{3}+c}\,(b\,x^{3}+a)(ad-4bc)\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{(ad-bc)b}\,(2b^{2}c^{2}x^{3}+2b\,c^{2}a+d\,a^{2}(d\,x^{3}+c))}{3\sqrt{d\,x^{3}+c}\,\sqrt{(ad-bc)b}\,db(b\,x^{3}+a)(ad-bc)^{2}}$
default	$-\frac{2}{3b^{2}d\sqrt{d\,x^{3}+c}}+\frac{a^{2}d\left(-\frac{2}{\sqrt{d\,x^{3}+c}}-\frac{b\sqrt{d\,x^{3}+c}}{d\left(b\,x^{3}+a\right)}-\frac{3\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)b}{\sqrt{(ad-bc)b}}\right)}{3b^{2}(ad-bc)^{2}}+\frac{4a\left(b\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)\sqrt{d\,x^{3}+c}+\sqrt{(ad-\cdots}\right.}{b^{2}\sqrt{(ad-bc)b}\,\sqrt{d\,x^{3}+c}\,(3ad-3bc)}$
elliptic	$-\frac{2c^{2}}{3d(ad-bc)^{2}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}-\frac{a^{2}\sqrt{d\,x^{3}+c}}{3(ad-bc)^{2}b(b\,x^{3}+a)}-\frac{ia\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{(ad-4bc)\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\,(\cdots}{\cdots}\right)}{(\cdots}}\cdots}{\cdots}\right.}{\cdots}$

[In] int(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/3*(-a*d*(d*x^3+c)^(1/2)*(b*x^3+a)*(a*d-4*b*c)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(2*b^2*c^2*x^3+2*b*c^2*a+d*a^2*(d*x^3+c)))/(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2)/d/b/(b*x^3+a)/(a*d-b*c)^2

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(129) = 258$.

Time = 0.35 (sec) , antiderivative size = 746, normalized size of antiderivative = 4.97

$$\int \frac{x^8}{(a+bx^3)^2\,(c+dx^3)^{3/2}}\,dx = \left[-\frac{((4\,ab^2cd^2-a^2bd^3)x^6+4\,a^2bc^2d-a^3cd^2+(4\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)x\cdots}{6\,(ab^5c^4d-3\,a^2b^4c^3d^2+3\,a^3b^3c^2d^3-a\cdots}\right.$$

$$-\frac{((4\,ab^2cd^2-a^2bd^3)x^6+4\,a^2bc^2d-a^3cd^2+(4\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)x^3)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-\cdots}}{bdx^3-\cdots}\right.}{3\,(ab^5c^4d-3\,a^2b^4c^3d^2+3\,a^3b^3c^2d^3-a^4b^2cd^4+(b^6c^3d^2-3\,ab^5c^2d^3+3\,a^2b^4\cdots}$$

[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")

[Out] [-1/6*(((4*a*b^2*c*d^2 - a^2*b*d^3)*x^6 + 4*a^2*b*c^2*d - a^3*c*d^2 + (4*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^3)*sqrt(b^2*c - a*b*d)*log((b*d*x^3

```
+ 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*(2*
a*b^3*c^3 - a^2*b^2*c^2*d - a^3*b*c*d^2 + (2*b^4*c^3 - 2*a*b^3*c^2*d + a^2*
b^2*c*d^2 - a^3*b*d^3)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c^4*d - 3*a^2*b^4*c^3*d
^2 + 3*a^3*b^3*c^2*d^3 - a^4*b^2*c*d^4 + (b^6*c^3*d^2 - 3*a*b^5*c^2*d^3 + 3
*a^2*b^4*c*d^4 - a^3*b^3*d^5)*x^6 + (b^6*c^4*d - 2*a*b^5*c^3*d^2 + 2*a^3*b^
3*c*d^4 - a^4*b^2*d^5)*x^3), -1/3*(((4*a*b^2*c*d^2 - a^2*b*d^3)*x^6 + 4*a^2
*b*c^2*d - a^3*c*d^2 + (4*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^3)*sqrt(
-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)
) + (2*a*b^3*c^3 - a^2*b^2*c^2*d - a^3*b*c*d^2 + (2*b^4*c^3 - 2*a*b^3*c^2*d
 + a^2*b^2*c*d^2 - a^3*b*d^3)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c^4*d - 3*a^2*b^
4*c^3*d^2 + 3*a^3*b^3*c^2*d^3 - a^4*b^2*c*d^4 + (b^6*c^3*d^2 - 3*a*b^5*c^2*
d^3 + 3*a^2*b^4*c*d^4 - a^3*b^3*d^5)*x^6 + (b^6*c^4*d - 2*a*b^5*c^3*d^2 + 2
*a^3*b^3*c*d^4 - a^4*b^2*d^5)*x^3)]
```

Sympy [F]

$$\int \frac{x^8}{(a+bx^3)^2\,(c+dx^3)^{3/2}}\,dx=\int \frac{x^8}{(a+bx^3)^2\,(c+dx^3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**8/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(x**8/((a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{(a+bx^3)^2\,(c+dx^3)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.30

$$\int \frac{x^8}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = -\frac{(4\,abc - a^2d)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\,(b^3c^2 - 2\,ab^2cd + a^2bd^2)\sqrt{-b^2c+abd}} - \frac{2\,(dx^3+c)b^2c^2 - 2\,b^2c^3 + 2\,abc^2d + (dx^3+c)a^2d^2}{3\,(b^3c^2d - 2\,ab^2cd^2 + a^2bd^3)\left((dx^3+c)^{\frac{3}{2}}b - \sqrt{dx^3+c}bc + \sqrt{dx^3+c}ad\right)}$$

```
[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*(4*a*b*c - a^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^3
*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*sqrt(-b^2*c + a*b*d)) - 1/3*(2*(d*x^3 + c)*
b^2*c^2 - 2*b^2*c^3 + 2*a*b*c^2*d + (d*x^3 + c)*a^2*d^2)/((b^3*c^2*d - 2*a*
b^2*c*d^2 + a^2*b*d^3)*((d*x^3 + c)^(3/2)*b - sqrt(d*x^3 + c)*b*c + sqrt(d*
x^3 + c)*a*d))
```

Mupad [B] (verification not implemented)

Time = 12.27 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.45

$$\int \frac{x^8}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \frac{\sqrt{d\,x^3+c}\,\left(x^3\,\left(\frac{\left(\frac{3\,b\,d\,(a\,d+b\,c)-b\,d\,(a\,d+2\,b\,c)}{3\,\left(a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d\right)}-\frac{b\,d\,(a\,d+b\,c)}{a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d}\right)\,(a\,d+b\,c)}{b\,d}+\frac{}{a^2\,b\,d^3-}\right.\right.}{b\,d\,x^6+(a\,d+b\,c)} + \frac{a\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,(a\,d-4\,b\,c)\,1{}\mathrm{i}}{6\,b^{3/2}\,{(a\,d-b\,c)}^{5/2}}$$

```
[In] int(x^8/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

```
[Out] ((c + d*x^3)^(1/2)*(x^3*((((3*b*d*(a*d + b*c) - b*d*(a*d + 2*b*c))/(3*(a^2*
b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) - (b*d*(a*d + b*c))/(a^2*b*d^3 + b^3*c^
2*d - 2*a*b^2*c*d^2))*(a*d + b*c))/(b*d) + (a*b*c*d)/(a^2*b*d^3 + b^3*c^2*d
 - 2*a*b^2*c*d^2)) + (a*c*((3*b*d*(a*d + b*c) - b*d*(a*d + 2*b*c))/(3*(a^2*
b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) - (b*d*(a*d + b*c))/(a^2*b*d^3 + b^3*c^
2*d - 2*a*b^2*c*d^2)))/(b*d)))/(a*c + x^3*(a*d + b*c) + b*d*x^6) + (a*log((
2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a
+ b*x^3))*(a*d - 4*b*c)*1i)/(6*b^(3/2)*(a*d - b*c)^(5/2))
```

3.491 $\int \frac{x^5}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 134

$$\int \frac{x^5}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx=\frac{2bc+ad}{3b(bc-ad)^2\sqrt{c+dx^3}}+\frac{a}{3b(bc-ad)(a+bx^3)\sqrt{c+dx^3}}-\frac{(2bc+ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{5/2}}$$

```
[Out] -1/3*(a*d+2*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*
c)^(5/2)/b^(1/2)+1/3*(a*d+2*b*c)/b/(-a*d+b*c)^2/(d*x^3+c)^(1/2)+1/3*a/b/(-a
*d+b*c)/(b*x^3+a)/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 79, 53, 65, 214}

$$\int \frac{x^5}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx=-\frac{(ad+2bc)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{5/2}}+\frac{a}{3b(a+bx^3)\sqrt{c+dx^3}(bc-ad)}+\frac{ad+2bc}{3b\sqrt{c+dx^3}(bc-ad)^2}$$

```
[In] Int[x^5/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*b*c + a*d)/(3*b*(b*c - a*d)^2*Sqrt[c + d*x^3]) + a/(3*b*(b*c - a*d)*(a +
 b*x^3)*Sqrt[c + d*x^3]) - ((2*b*c + a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])
/Sqrt[b*c - a*d]])/(3*Sqrt[b]*(b*c - a*d)^(5/2))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x}{(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{a}{3b(bc-ad)\,(a+bx^3)\,\sqrt{c+dx^3}} + \frac{(2bc+ad)\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{6b(bc-ad)}$$

$$= \frac{2bc+ad}{3b(bc-ad)^2\sqrt{c+dx^3}} + \frac{a}{3b(bc-ad)(a+bx^3)\sqrt{c+dx^3}} + \frac{(2bc+ad)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{6(bc-ad)^2}$$

$$= \frac{2bc+ad}{3b(bc-ad)^2\sqrt{c+dx^3}} + \frac{a}{3b(bc-ad)(a+bx^3)\sqrt{c+dx^3}} + \frac{(2bc+ad)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3d(bc-ad)^2}$$

$$= \frac{2bc+ad}{3b(bc-ad)^2\sqrt{c+dx^3}} + \frac{a}{3b(bc-ad)(a+bx^3)\sqrt{c+dx^3}} - \frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.82

$$\int \frac{x^5}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{1}{3}\left(\frac{3ac+2bcx^3+adx^3}{(bc-ad)^2(a+bx^3)\sqrt{c+dx^3}} + \frac{(2bc+ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}(-bc+ad)^{5/2}}\right)$$

```
[In] Integrate[x^5/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] ((3*a*c + 2*b*c*x^3 + a*d*x^3)/((b*c - a*d)^2*(a + b*x^3)*Sqrt[c + d*x^3])
+ ((2*b*c + a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(Sqr
t[b]*(-(b*c) + a*d)^(5/2)))/3
```

Maple [A] (verified)

Time = 4.45 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.93

method	result
pseudoelliptic	$\frac{\frac{\sqrt{d\,x^3+c}\,\left(b\,x^3+a\right)(ad+2bc)\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)}{3}+\left(\frac{2bc\,x^3}{3}+a\left(\frac{d\,x^3}{3}+c\right)\right)\sqrt{(ad-bc)b}}{\sqrt{d\,x^3+c}\,\sqrt{(ad-bc)b}\,(ad-bc)^2\left(b\,x^3+a\right)}$
default	$-\frac{2\left(b\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)\sqrt{d\,x^3+c}+\sqrt{(ad-bc)b}\right)}{b\sqrt{(ad-bc)b}\,\sqrt{d\,x^3+c}\,(3ad-3bc)}-\frac{ad\left(-\frac{2}{\sqrt{d\,x^3+c}}-\frac{b\sqrt{d\,x^3+c}}{d\left(b\,x^3+a\right)}-\frac{3\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)b}{\sqrt{(ad-bc)b}}\right)}{3b(ad-bc)^2}$
elliptic	$\frac{2c}{3(ad-bc)^2\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{a\sqrt{d\,x^3+c}}{3(ad-bc)^2\left(b\,x^3+a\right)}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{(-ad-2bc)\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right.}{}\right.}{\left(-c\,d^2\right.}}}{}\right.}{}$

[In] int(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] (1/3*(d*x^3+c)^(1/2)*(b*x^3+a)*(a*d+2*b*c)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))+(2/3*b*c*x^3+a*(1/3*d*x^3+c))*((a*d-b*c)*b)^(1/2))/(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2)/(a*d-b*c)^2/(b*x^3+a)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(114) = 228$.

Time = 0.38 (sec) , antiderivative size = 630, normalized size of antiderivative = 4.70

$$\int \frac{x^5}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \left[\frac{\left((2\,b^2cd+abd^2)x^6+2\,abc^2+a^2cd+(2\,b^2c^2+3\,abcd+a^2d^2)x^3\right)\sqrt{b^2c-ab}}{6\left(ab^4c^4-3\,a^2b^3c^3d+3\,a^3b^2c^2d^2-a^4bcd^3+(b^5c^3d-\right.}\right.$$

[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")

[Out] [1/6*(((2*b^2*c*d + a*b*d^2)*x^6 + 2*a*b*c^2 + a^2*c*d + (2*b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^3)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*(3*a*b^2*c^2 - 3*a^2*b*c*d + (2*b^3*c^2 - a*b^2*c*d - a^2*b*d^2)*x^3)*sqrt(d*x^3 + c))/(a*b^4*c^4 -

```
3*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 - a^4*b*c*d^3 + (b^5*c^3*d - 3*a*b^4*c
^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (b^5*c^4 - 2*a*b^4*c^3*d + 2*
a^3*b^2*c*d^3 - a^4*b*d^4)*x^3), 1/3*(((2*b^2*c*d + a*b*d^2)*x^6 + 2*a*b*c^
2 + a^2*c*d + (2*b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^3)*sqrt(-b^2*c + a*b*d)*a
rctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) + (3*a*b^2*c^2
- 3*a^2*b*c*d + (2*b^3*c^2 - a*b^2*c*d - a^2*b*d^2)*x^3)*sqrt(d*x^3 + c))/(
a*b^4*c^4 - 3*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 - a^4*b*c*d^3 + (b^5*c^3*d
- 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (b^5*c^4 - 2*a*b^4
*c^3*d + 2*a^3*b^2*c*d^3 - a^4*b*d^4)*x^3)]
```

Sympy [F]

$$\int \frac{x^5}{(a+bx^3)^2 (c+dx^3)^{3/2}}\, dx = \int \frac{x^5}{(a+bx^3)^2 (c+dx^3)^{\frac{3}{2}}}\, dx$$

```
[In] integrate(x**5/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(x**5/((a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^3)^2 (c+dx^3)^{3/2}}\, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.35

$$\int \frac{x^5}{(a+bx^3)^2 (c+dx^3)^{3/2}}\, dx = \frac{\frac{(2\,bcd+ad^2)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{-b^2c+abd}} + \frac{2\,(dx^3+c)bcd-2\,bc^2d+(dx^3+c)ad^2+2\,acd^2}{(b^2c^2-2\,abcd+a^2d^2)\left((dx^3+c)^{\frac{3}{2}}b-\sqrt{dx^3+c}bc+\sqrt{dx^3+c}ad\right)}}{3\,d}$$

```
[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*((2*b*c*d + a*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2
*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-b^2*c + a*b*d)) + (2*(d*x^3 + c)*b*c*d -
2*b*c^2*d + (d*x^3 + c)*a*d^2 + 2*a*c*d^2)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)
*((d*x^3 + c)^(3/2)*b - sqrt(d*x^3 + c)*b*c + sqrt(d*x^3 + c)*a*d)))/d
```

Mupad [B] (verification not implemented)

Time = 11.73 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.84

$$\int \frac{x^5}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx = -\frac{\sqrt{d\,x^3+c}\,\left(x^3\,\left(\frac{3\,b\,d\,(a\,d+b\,c)-b\,d\,(a\,d+2\,b\,c)}{3\,(a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d)}-\frac{b\,d\,(a\,d+b\,c)}{a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d}\right)-\frac{a\,b\,c\,d}{a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d}\right)}{b\,d\,x^6+(a\,d+b\,c)\,x^3+a\,c} + \frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,(a\,d+2\,b\,c)\,1\mathrm{i}}{6\,\sqrt{b}\,(a\,d-b\,c)^{5/2}}$$

```
[In] int(x^5/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

```
[Out] (log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^
3)/(a + b*x^3))*(a*d + 2*b*c)*1i)/(6*b^(1/2)*(a*d - b*c)^(5/2)) - ((c + d*x
^3)^(1/2)*(x^3*((3*b*d*(a*d + b*c) - b*d*(a*d + 2*b*c))/(3*(a^2*b*d^3 + b^3
*c^2*d - 2*a*b^2*c*d^2)) - (b*d*(a*d + b*c))/(a^2*b*d^3 + b^3*c^2*d - 2*a*b
^2*c*d^2)) - (a*b*c*d)/(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)))/(a*c + x^3
*(a*d + b*c) + b*d*x^6)
```

3.492 $\int \frac{x^2}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 108

$$\int \frac{x^2}{(a+bx^3)^2\,(c+dx^3)^{3/2}}\,dx = -\frac{d}{(bc-ad)^2\sqrt{c+dx^3}} - \frac{1}{3(bc-ad)\,(a+bx^3)\,\sqrt{c+dx^3}} + \frac{\sqrt{b}d\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

```
[Out] d*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/(-a*d+b*c)^(5/2
)-d/(-a*d+b*c)^2/(d*x^3+c)^(1/2)-1/3/(-a*d+b*c)/(b*x^3+a)/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {455, 44, 53, 65, 214}

$$\int \frac{x^2}{(a+bx^3)^2\,(c+dx^3)^{3/2}}\,dx = \frac{\sqrt{b}d\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}} - \frac{d}{\sqrt{c+dx^3}(bc-ad)^2} - \frac{1}{3\,(a+bx^3)\,\sqrt{c+dx^3}(bc-ad)}$$

```
[In] Int[x^2/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -(d/((b*c - a*d)^2*Sqrt[c + d*x^3])) - 1/(3*(b*c - a*d)*(a + b*x^3)*Sqrt[c
+ d*x^3]) + (Sqrt[b]*d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/
(b*c - a*d)^(5/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= -\frac{1}{3(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{d\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{2(bc-ad)}$$

$$= -\frac{d}{(bc-ad)^2\sqrt{c+dx^3}} - \frac{1}{3(bc-ad)(a+bx^3)\sqrt{c+dx^3}} - \frac{(bd)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{2(bc-ad)^2}$$

$$= -\frac{d}{(bc-ad)^2\sqrt{c+dx^3}} - \frac{1}{3(bc-ad)(a+bx^3)\sqrt{c+dx^3}} - \frac{b\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{(bc-ad)^2}$$

$$= -\frac{d}{(bc-ad)^2\sqrt{c+dx^3}} - \frac{1}{3(bc-ad)(a+bx^3)\sqrt{c+dx^3}} + \frac{\sqrt{b}d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{-2ad-b(c+3dx^3)}{3(bc-ad)^2(a+bx^3)\sqrt{c+dx^3}} - \frac{\sqrt{b}d\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{5/2}}$$

```
[In] Integrate[x^2/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*a*d - b*(c + 3*d*x^3))/(3*(b*c - a*d)^2*(a + b*x^3)*Sqrt[c + d*x^3]) -
(Sqrt[b]*d*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(-(b*c) +
a*d)^(5/2)
```

Maple [A] (verified)

Time = 4.48 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{-\frac{b\sqrt{d\,x^{3}+c}}{3\left(b\,x^{3}+a\right)}-\frac{db\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}-\frac{2d}{3\sqrt{d\,x^{3}+c}}}{(ad-bc)^{2}}$
default	$\frac{d\left(-\frac{2}{\sqrt{d\,x^{3}+c}}-\frac{b\sqrt{d\,x^{3}+c}}{d\left(b\,x^{3}+a\right)}-\frac{3\arctan\left(\frac{b\sqrt{d\,x^{3}+c}}{\sqrt{(ad-bc)b}}\right)b}{\sqrt{(ad-bc)b}}\right)}{3(ad-bc)^{2}}$
elliptic	$-\frac{2d}{3(ad-bc)^{2}\sqrt{\left(x^{3}+\frac{c}{d}\right)d}}-\frac{b\sqrt{d\,x^{3}+c}}{3(ad-bc)^{2}(b\,x^{3}+a)}+\frac{ib\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+(}{d}\right.}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}}{}\right.}{}$

[In] int(x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/(a*d-b*c)^2*(-1/3*b*(d*x^3+c)^(1/2)/(b*x^3+a)-d*b/((a*d-b*c)*b)^(1/2)*arc tan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))-2/3*d/(d*x^3+c)^(1/2))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(92) = 184.

Time = 0.33 (sec) , antiderivative size = 450, normalized size of antiderivative = 4.17

$$\int\frac{x^{2}}{\left(a+bx^{3}\right)^{2}\left(c+dx^{3}\right)^{3/2}}\,dx=\left[\frac{3\left(bd^{2}x^{6}+(bcd+ad^{2})x^{3}+acd\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^{3}+2\,bc-ad+2\,\sqrt{dx^{3}+c}(bc-ad)\sqrt{\frac{b}{bc-}}}{bx^{3}+a}\right.}{6\left((b^{3}c^{2}d-2\,ab^{2}cd^{2}+a^{2}bd^{3})x^{6}+ab^{2}c^{3}-2\,a^{2}bc^{2}d+a^{3}cd^{2}+(b^{3}c^{3}\right.}\right.$$

[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")

```
[Out] [1/6*(3*(b*d^2*x^6 + (b*c*d + a*d^2)*x^3 + a*c*d)*sqrt(b/(b*c - a*d))*log((
b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/
(b*x^3 + a)) - 2*(3*b*d*x^3 + b*c + 2*a*d)*sqrt(d*x^3 + c))/((b^3*c^2*d - 2
*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^
3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^3), 1/3*(3*(b*d^2*x^6 + (b*c
*d + a*d^2)*x^3 + a*c*d)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c
- a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) - (3*b*d*x^3 + b*c + 2*a*d)*sq
rt(d*x^3 + c))/((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3 - 2
*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*
x^3)]
```

Sympy [F]

$$\int \frac{x^2}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{x^2}{(a+bx^3)^2 (c+dx^3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**2/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(x**2/((a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.42

$$\int \frac{x^2}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = -\frac{bd\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{-b^2c+abd}} - \frac{3\,(dx^3+c)bd-2\,bcd+2\,ad^2}{3\,(b^2c^2-2\,abcd+a^2d^2)\left((dx^3+c)^{\frac{3}{2}}b-\sqrt{dx^3+c}bc+\sqrt{dx^3+c}ad\right)}$$

```
[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -b*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)*sqrt(-b^2*c + a*b*d)) - 1/3*(3*(d*x^3 + c)*b*d - 2*b*c*d + 2*a*d^
2)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((d*x^3 + c)^(3/2)*b - sqrt(d*x^3 + c)*
b*c + sqrt(d*x^3 + c)*a*d))
```

Mupad [B] (verification not implemented)

Time = 11.31 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.84

$$\int \frac{x^2}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = -\frac{\left(\frac{3\,b\,d\,(a\,d+b\,c)-b\,d\,(a\,d+2\,b\,c)}{3\,(a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d)}+\frac{b^2\,d^2\,x^3}{a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d}\right)\sqrt{d\,x^3+c}}{b\,d\,x^6+(a\,d+b\,c)\,x^3+a\,c} + \frac{\sqrt{b}\,d\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{2\,(a\,d-b\,c)^{5/2}}$$

```
[In] int(x^2/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

```
[Out] (b^(1/2)*d*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2
i - b*d*x^3)/(a + b*x^3))*1i)/(2*(a*d - b*c)^(5/2)) - (((3*b*d*(a*d + b*c)
- b*d*(a*d + 2*b*c))/(3*(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) + (b^2*d^2
*x^3)/(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2))*(c + d*x^3)^(1/2))/(a*c + x^
3*(a*d + b*c) + b*d*x^6)
```

3.493 $\int \frac{1}{x(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 172

$$\int \frac{1}{x\,(a+bx^3)^2\,(c+dx^3)^{3/2}}\,dx = \frac{d(bc+2ad)}{3ac(bc-ad)^2\sqrt{c+dx^3}} + \frac{b}{3a(bc-ad)\,(a+bx^3)\,\sqrt{c+dx^3}} - \frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}} + \frac{b^{3/2}(2bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{5/2}}$$

```
[Out] -2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^2/c^(3/2)+1/3*b^(3/2)*(-5*a*d+2*b*c
)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(5/2)+1/
3*d*(2*a*d+b*c)/a/c/(-a*d+b*c)^2/(d*x^3+c)^(1/2)+1/3*b/a/(-a*d+b*c)/(b*x^3+
a)/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {457, 105, 157, 162, 65, 214}

$$\int \frac{1}{x\,(a+bx^3)^2\,(c+dx^3)^{3/2}}\,dx = \frac{b^{3/2}(2bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{5/2}} - \frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}} + \frac{b}{3a\,(a+bx^3)\,\sqrt{c+dx^3}(bc-ad)} + \frac{d(2ad+bc)}{3ac\sqrt{c+dx^3}(bc-ad)^2}$$

```
[In] Int[1/(x*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (d*(b*c + 2*a*d))/(3*a*c*(b*c - a*d)^2*Sqrt[c + d*x^3]) + b/(3*a*(b*c - a*d
)*(a + b*x^3)*Sqrt[c + d*x^3]) - (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^
2*c^(3/2)) + (b^(3/2)*(2*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqr
t[b*c - a*d]])/(3*a^2*(b*c - a*d)^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(a+bx)^2(c+dx)^{3/2}}\,dx,x,x^3\right)\\
&= \frac{b}{3a(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}}+\frac{\text{Subst}\left(\int \frac{bc-ad+\frac{3bdx}{2}}{x(a+bx)(c+dx)^{3/2}}\,dx,x,x^3\right)}{3a(bc-ad)}\\
&= \frac{d(bc+2ad)}{3ac(bc-ad)^2\sqrt{c+dx^3}}+\frac{b}{3a(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}}\\
&\quad-\frac{2\text{Subst}\left(\int \frac{-\frac{1}{2}(bc-ad)^2-\frac{1}{4}bd(bc+2ad)x}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3ac(bc-ad)^2}\\
&= \frac{d(bc+2ad)}{3ac(bc-ad)^2\sqrt{c+dx^3}}+\frac{b}{3a(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}}\\
&\quad+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{3a^2c}-\frac{\left(b^2(2bc-5ad)\right)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{6a^2(bc-ad)^2}\\
&= \frac{d(bc+2ad)}{3ac(bc-ad)^2\sqrt{c+dx^3}}+\frac{b}{3a(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}}\\
&\quad+\frac{2\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3a^2cd}\\
&\quad-\frac{\left(b^2(2bc-5ad)\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3a^2d(bc-ad)^2}\\
&= \frac{d(bc+2ad)}{3ac(bc-ad)^2\sqrt{c+dx^3}}+\frac{b}{3a(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}}\\
&\quad-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}}+\frac{b^{3/2}(2bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\,(a+bx^3)^2\,(c+dx^3)^{3/2}}\,dx = \frac{\frac{a(2a^2d^2+2abd^2x^3+b^2c(c+dx^3))}{c(bc-ad)^2(a+bx^3)\sqrt{c+dx^3}} - \frac{b^{3/2}(2bc-5ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{5/2}} - \frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right.}{c^{3/2}}}{3a^2}$$

[In] Integrate[1/(x*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]

[Out] ((a*(2*a^2*d^2 + 2*a*b*d^2*x^3 + b^2*c*(c + d*x^3)))/(c*(b*c - a*d)^2*(a + b*x^3)*Sqrt[c + d*x^3]) - (b^(3/2)*(2*b*c - 5*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(5/2) - (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/c^(3/2))/(3*a^2)

Maple [A] (verified)

Time = 4.55 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.16

method	result
pseudoelliptic	$\frac{-2\left(-\frac{5ad}{2}+bc\right)c^{\frac{5}{2}}\sqrt{d\,x^3+c}\,b^2(b\,x^3+a)\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)+\left(-2c\sqrt{d\,x^3+c}\,(ad-bc)^2(b\,x^3+a)\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)+c^{\frac{3}{2}}a(c\right.}{3\sqrt{(ad-bc)b}\,\sqrt{d\,x^3+c}\,a^2(b\,x^3+a)(ad-bc)^2c^{\frac{5}{2}}}$
default	$\frac{\frac{2}{3c\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{2\,\text{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}}{a^2}+\frac{2b\left(b\arctan\left(\frac{b\sqrt{d\,x^3+c}}{\sqrt{(ad-bc)b}}\right)\sqrt{d\,x^3+c}+\sqrt{(ad-bc)b}\right)}{a^2\sqrt{(ad-bc)b}\,\sqrt{d\,x^3+c}\,(3ad-3bc)}-\frac{bd\left(-\frac{2}{\sqrt{d\,x^3+c}}-\frac{b\sqrt{d\,x^3+c}}{d(b\,x^3+a)}\right.}{3a(ad-}$
elliptic	Expression too large to display

[In] int(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/3*(-2*(-5/2*a*d+b*c)*c^(5/2)*(d*x^3+c)^(1/2)*b^2*(b*x^3+a)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))+(-2*c*(d*x^3+c)^(1/2)*(a*d-b*c)^2*(b*x^3+a)*arctanh((d*x^3+c)^(1/2)/c^(1/2))+c^(3/2)*a*(c*(d*x^3+c)*b^2+2*x^3*a*b*d^2+2*a^2*d^2))*((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)/(d*x^3+c)^(1/2)/a^2/(b*x^3+a)/(a*d-b*c)^2/c^(5/2)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(144) = 288$.

Time = 0.60 (sec) , antiderivative size = 1819, normalized size of antiderivative = 10.58

$$\int \frac{1}{x\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/6*((2*a*b^2*c^4 - 5*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*b^2*c^2*d^2)*x^6 +
 (2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2*b*c^2*d^2)*x^3)*sqrt(b/(b*c - a*d))*log
((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d))
)/(b*x^3 + a)) - 2*((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3
 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d
^3)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*(a*
b^2*c^3 + 2*a^3*c*d^2 + (a*b^2*c^2*d + 2*a^2*b*c*d^2)*x^3)*sqrt(d*x^3 + c))
/(a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^
3*d^2 + a^4*b*c^2*d^3)*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 +
 a^5*c^2*d^3)*x^3), 1/3*((2*a*b^2*c^4 - 5*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*
b^2*c^2*d^2)*x^6 + (2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2*b*c^2*d^2)*x^3)*sqrt(
-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b
*d*x^3 + b*c)) + ((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3 -
 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3
)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + (a*b^2*
c^3 + 2*a^3*c*d^2 + (a*b^2*c^2*d + 2*a^2*b*c*d^2)*x^3)*sqrt(d*x^3 + c))/(a^
3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^
2 + a^4*b*c^2*d^3)*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5
*c^2*d^3)*x^3), 1/6*(4*((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2
*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a
^3*d^3)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - (2*a*b^2*c^4 - 5
*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*b^2*c^2*d^2)*x^6 + (2*b^3*c^4 - 3*a*b^2*c
^3*d - 5*a^2*b*c^2*d^2)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d
 - 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*(a*b
^2*c^3 + 2*a^3*c*d^2 + (a*b^2*c^2*d + 2*a^2*b*c*d^2)*x^3)*sqrt(d*x^3 + c))/
(a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3
*d^2 + a^4*b*c^2*d^3)*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 +
a^5*c^2*d^3)*x^3), 1/3*((2*a*b^2*c^4 - 5*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*b
^2*c^2*d^2)*x^6 + (2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2*b*c^2*d^2)*x^3)*sqrt(-
b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*
d*x^3 + b*c)) + 2*((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3
- 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^
3)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (a*b^2*c^3 + 2*a^3*c*
d^2 + (a*b^2*c^2*d + 2*a^2*b*c*d^2)*x^3)*sqrt(d*x^3 + c))/(a^3*b^2*c^5 - 2*
a^4*b*c^4*d + a^5*c^3*d^2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*
```

```
d^3)*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*x^3)
]
```

Sympy [F]

$$\int \frac{1}{x\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{x\left(a+bx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(1/(x*(a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^3+a\right)^2\left(dx^3+c\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.31

$$\int \frac{1}{x\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=-\frac{\left(2\,b^3c-5\,ab^2d\right)\arctan\left(\frac{\sqrt{dx^3+c}b}{\sqrt{-b^2c+abd}}\right)}{3\left(a^2b^2c^2-2\,a^3bcd+a^4d^2\right)\sqrt{-b^2c+abd}}+\frac{(dx^3+c)b^2cd+2\,(dx^3+c)abd^2-2\,abcd^2+2\,a^2d^3}{3\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2\right)\left(\left(dx^3+c\right)^{\frac{3}{2}}b-\sqrt{dx^3+c}bc+\sqrt{dx^3+c}ad\right)}+\frac{2\,\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}c}$$

```
[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] -1/3*(2*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(
(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(-b^2*c + a*b*d)) + 1/3*((d*x^3 +
 c)*b^2*c*d + 2*(d*x^3 + c)*a*b*d^2 - 2*a*b*c*d^2 + 2*a^2*d^3)/((a*b^2*c^3
- 2*a^2*b*c^2*d + a^3*c*d^2)*((d*x^3 + c)^(3/2)*b - sqrt(d*x^3 + c)*b*c + s
qrt(d*x^3 + c)*a*d)) + 2/3*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)*c
)
```

Mupad [B] (verification not implemented)

Time = 16.41 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.67

$$\int \frac{1}{x(a+bx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a^2\,c^{3/2}} + \frac{\left(\frac{(2\,a\,d+b\,c)^4+(2\,a\,d+b\,c)^2\,((a\,d+2\,b\,c)\,(2\,a\,d+b\,c)-9\,a\,b\,c\,d)}{9\,a\,c\,(2\,a\,d+b\,c)^2\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}+\frac{b\,d\,x^3\,(2\,a\,d+b\,c)}{3\,a\,c\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)\,\sqrt{d\,x^3+c}}{b\,d\,x^6+(a\,d+b\,c)\,x^3+a\,c} + \frac{b^{3/2}\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,(5\,a\,d-2\,b\,c)\,1\mathrm{i}}{6\,a^2\,(a\,d-b\,c)^{5/2}}$$

```
[In] int(1/(x*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

```
[Out] log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(3
*a^2*c^(3/2)) + ((((2*a*d + b*c)^4 + (2*a*d + b*c)^2*((a*d + 2*b*c)*(2*a*d
+ b*c) - 9*a*b*c*d))/(9*a*c*(2*a*d + b*c)^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)
) + (b*d*x^3*(2*a*d + b*c))/(3*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*(c + d
*x^3)^(1/2))/(a*c + x^3*(a*d + b*c) + b*d*x^6) + (b^(3/2)*log((2*b*c - a*d
+ b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(5
*a*d - 2*b*c)*1i)/(6*a^2*(a*d - b*c)^(5/2))
```

3.494 $\int \frac{1}{x^4(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 241

$$\int \frac{1}{x^4(a+bx^3)^2(c+dx^3)^{3/2}}\,dx = -\frac{d(2b^2c^2-2abcd+3a^2d^2)}{3a^2c^2(bc-ad)^2\sqrt{c+dx^3}} - \frac{b(2bc-ad)}{3a^2c(bc-ad)(a+bx^3)\sqrt{c+dx^3}} - \frac{1}{3acx^3(a+bx^3)\sqrt{c+dx^3}} + \frac{(4bc+3ad)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{5/2}} - \frac{b^{5/2}(4bc-7ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3(bc-ad)^{5/2}}$$

```
[Out] 1/3*(3*a*d+4*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^3/c^(5/2)-1/3*b^(5/2)*
(-7*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a^3/(-a*d+
b*c)^(5/2)-1/3*d*(3*a^2*d^2-2*a*b*c*d+2*b^2*c^2)/a^2/c^2/(-a*d+b*c)^2/(d*x^
3+c)^(1/2)-1/3*b*(-a*d+2*b*c)/a^2/c/(-a*d+b*c)/(b*x^3+a)/(d*x^3+c)^(1/2)-1/
3/a/c/x^3/(b*x^3+a)/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {457, 105, 156, 157, 162, 65, 214}

$$\int \frac{1}{x^4(a+bx^3)^2(c+dx^3)^{3/2}}\,dx = -\frac{b^{5/2}(4bc-7ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3(bc-ad)^{5/2}} + \frac{(3ad+4bc)\text{arctanh}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{5/2}} - \frac{d(3a^2d^2-2abcd+2b^2c^2)}{3a^2c^2\sqrt{c+dx^3}(bc-ad)^2} - \frac{b(2bc-ad)}{3a^2c(a+bx^3)\sqrt{c+dx^3}(bc-ad)} - \frac{1}{3acx^3(a+bx^3)\sqrt{c+dx^3}}$$

```
[In] Int[1/(x^4*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/3*(d*(2*b^2*c^2 - 2*a*b*c*d + 3*a^2*d^2))/(a^2*c^2*(b*c - a*d)^2*Sqrt[c
+ d*x^3]) - (b*(2*b*c - a*d))/(3*a^2*c*(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x
^3]) - 1/(3*a*c*x^3*(a + b*x^3)*Sqrt[c + d*x^3]) + ((4*b*c + 3*a*d)*ArcTanh
[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^3*c^(5/2)) - (b^(5/2)*(4*b*c - 7*a*d)*ArcTa
nh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^3*(b*c - a*d)^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)\\
&= -\frac{1}{3acx^3\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4bc+3ad)+\frac{5bdx}{2}}{x(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)}{3ac}\\
&= -\frac{b(2bc-ad)}{3a^2c(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{1}{3acx^3\left(a+bx^3\right)\sqrt{c+dx^3}}\\
&\quad - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-ad)(4bc+3ad)+\frac{3}{2}bd(2bc-ad)x}{x(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{3a^2c(bc-ad)}\\
&= -\frac{d(2b^2c^2-2abcd+3a^2d^2)}{3a^2c^2(bc-ad)^2\sqrt{c+dx^3}} - \frac{b(2bc-ad)}{3a^2c(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}}\\
&\quad - \frac{1}{3acx^3\left(a+bx^3\right)\sqrt{c+dx^3}}\\
&\quad + \frac{2\text{Subst}\left(\int \frac{-\frac{1}{4}(bc-ad)^2(4bc+3ad)-\frac{1}{4}bd\left(2b^2c^2-2abcd+3a^2d^2\right)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a^2c^2(bc-ad)^2}\\
&= -\frac{d(2b^2c^2-2abcd+3a^2d^2)}{3a^2c^2(bc-ad)^2\sqrt{c+dx^3}} - \frac{b(2bc-ad)}{3a^2c(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{1}{3acx^3\left(a+bx^3\right)\sqrt{c+dx^3}}\\
&\quad + \frac{\left(b^3(4bc-7ad)\right)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{6a^3(bc-ad)^2} - \frac{(4bc+3ad)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{6a^3c^2}
\end{aligned}$$

$$= -\frac{d(2b^2c^2 - 2abcd + 3a^2d^2)}{3a^2c^2(bc-ad)^2\sqrt{c+dx^3}} - \frac{b(2bc-ad)}{3a^2c(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}}$$
$$- \frac{1}{3acx^3\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{\left(b^3(4bc-7ad)\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^3d(bc-ad)^2}$$
$$- \frac{(4bc+3ad)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^3c^2d}$$
$$= -\frac{d(2b^2c^2 - 2abcd + 3a^2d^2)}{3a^2c^2(bc-ad)^2\sqrt{c+dx^3}} - \frac{b(2bc-ad)}{3a^2c(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}}$$
$$- \frac{1}{3acx^3\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{(4bc+3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{5/2}}$$
$$- \frac{b^{5/2}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3(bc-ad)^{5/2}}$$

Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^4\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{-\frac{a\left(2b^3c^2x^3\left(c+dx^3\right)+a^3d^2\left(c+3dx^3\right)+ab^2c\left(c^2-cdx^3-2d^2x^6\right)+a^2bd\left(-2c^2-cdx^3+3d^2x^6\right)\right)}{c^2(bc-ad)^2x^3(a+bx^3)\sqrt{c+dx^3}} + \frac{b^{5/2}(\cdots}{\cdots}}{3a^3}$$

```
[In] Integrate[1/(x^4*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-((a*(2*b^3*c^2*x^3*(c + d*x^3) + a^3*d^2*(c + 3*d*x^3) + a*b^2*c*(c^2 - c
*d*x^3 - 2*d^2*x^6) + a^2*b*d*(-2*c^2 - c*d*x^3 + 3*d^2*x^6)))/(c^2*(b*c -
a*d)^2*x^3*(a + b*x^3)*Sqrt[c + d*x^3])) + (b^(5/2)*(4*b*c - 7*a*d)*ArcTan[
(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(5/2) + ((4*b
*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/c^(5/2))/(3*a^3)
```

Maple [A] (verified)

Time = 4.89 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.04

method	result
risch	$-\frac{\sqrt{d x^3+c}}{3c^2a^2x^3} - \frac{-\frac{2(3ad+4bc)\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{3a\sqrt{c}} + \frac{4a^2d^3}{3(ad-bc)^2\sqrt{d x^3+c}} + \frac{2b^3c^2\left(d\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)(b x^3+a)+\sqrt{d x^3+c}\sqrt{(ad-b}{3(ad-bc)^2\sqrt{(ad-bc)b}\left(b x^3+a\right)}}{2a^2c^2}$
pseudoelliptic	$-\frac{-4x^3c^{\frac{9}{2}}b^3\sqrt{d x^3+c}\left(bc-\frac{7ad}{4}\right)(b x^3+a)\arctan\left(\frac{b\sqrt{d x^3+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{(ad-bc)b}\left(-3\left(ad+\frac{4bc}{3}\right)x^3\sqrt{d x^3+c}\,(b x^3+a)c^2(ad-bc)\right.}{3\sqrt{d x^3+c}\sqrt{(ad-bc)b}}$
default	$\frac{-\frac{\sqrt{d x^3+c}}{3c^2x^3}-\frac{2d}{3c^2\sqrt{\left(x^3+\frac{c}{d}\right)d}}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}}{a^2} - \frac{2b\left(\frac{2}{3c\sqrt{\left(x^3+\frac{c}{d}\right)d}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}\right)}{a^3} + \frac{b^2d\left(-\frac{2}{\sqrt{d x^3+c}}-\frac{b\sqrt{c}}{d(b}\right.}{3}$
elliptic	Expression too large to display

[In] int(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/3/c^2/a^2*(d*x^3+c)^(1/2)/x^3-1/2/a^2/c^2*(-2/3/a*(3*a*d+4*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)+4/3*a^2*d^3/(a*d-b*c)^2/(d*x^3+c)^(1/2)+2/3*b^3*c^2/(a*d-b*c)^2*(d*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2))*(b*x^3+a)+(d*x^3+c)^(1/2)*((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)/(b*x^3+a)+4/3*b^3*c^2*(3*a*d-2*b*c)/a/(a*d-b*c)^2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^3+c)^(1/2)/((a*d-b*c)*b)^(1/2)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 585 vs. $2(209) = 418$.

Time = 0.72 (sec) , antiderivative size = 2384, normalized size of antiderivative = 9.89

$$\int \frac{1}{x^4 (a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \text{Too large to display}$$

[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")

[Out] [-1/6*(((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^9 + (4*b^4*c^5 - 3*a*b^3*c^4*d - 7*a^2*b^2*c^3*d^2)*x^6 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) - ((4*b^4*c^3*d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + 3*a^3*b*d^4)*x^9 + (4*b^4*c^4 - a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 + a^3*b*c*d^3 + 3*a^4*d^4)*x^6 + (4*a*b^3*c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^3)*sqrt(c)*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (2*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 3*a^3*b*c*d^3)*x^6 + (2*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^3)*sqrt(d*x^3 + c))/((a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3)*x^9 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x^6 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^3), -1/6*(2*((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^9 + (4*b^4*c^5 - 3*a*b^3*c^4*d - 7*a^2*

```
b^2*c^3*d^2)*x^6 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^3)*sqrt(-b/(b*c - a*d)
)*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c))
 - ((4*b^4*c^3*d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + 3*a^3*b*d^4)*x^9 + (
4*b^4*c^4 - a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 + a^3*b*c*d^3 + 3*a^4*d^4)*x^6
+ (4*a*b^3*c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^3)*sqrt
(c)*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*(a^2*b^2*c^4 - 2
*a^3*b*c^3*d + a^4*c^2*d^2 + (2*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 3*a^3*b*c
*d^3)*x^6 + (2*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^3
)*sqrt(d*x^3 + c))/((a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3)*x^9
 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x^6 + (a^4*b
^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^3), -1/6*(2*((4*b^4*c^3*d - 5*a*b^3
*c^2*d^2 - 2*a^2*b^2*c*d^3 + 3*a^3*b*d^4)*x^9 + (4*b^4*c^4 - a*b^3*c^3*d -
7*a^2*b^2*c^2*d^2 + a^3*b*c*d^3 + 3*a^4*d^4)*x^6 + (4*a*b^3*c^4 - 5*a^2*b^2
*c^3*d - 2*a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c
)*sqrt(-c)/c) + ((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^9 + (4*b^4*c^5 - 3*a*b^3
*c^4*d - 7*a^2*b^2*c^3*d^2)*x^6 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^3)*sqrt
(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*
sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^
2*d^2 + (2*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 3*a^3*b*c*d^3)*x^6 + (2*a*b^3*
c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^3)*sqrt(d*x^3 + c))/((
a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3)*x^9 + (a^3*b^3*c^6 - a^4
*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x^6 + (a^4*b^2*c^6 - 2*a^5*b*c^5*
d + a^6*c^4*d^2)*x^3), -1/3*(((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^9 + (4*b^4*
c^5 - 3*a*b^3*c^4*d - 7*a^2*b^2*c^3*d^2)*x^6 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4
*d)*x^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(
b*c - a*d))/(b*d*x^3 + b*c)) + ((4*b^4*c^3*d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*
c*d^3 + 3*a^3*b*d^4)*x^9 + (4*b^4*c^4 - a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 + a
^3*b*c*d^3 + 3*a^4*d^4)*x^6 + (4*a*b^3*c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^2*
d^2 + 3*a^4*c*d^3)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (a^2*
b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (2*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2
+ 3*a^3*b*c*d^3)*x^6 + (2*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 3*a^4
*c*d^3)*x^3)*sqrt(d*x^3 + c))/((a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c
^3*d^3)*x^9 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x
^6 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4 (a+bx^3)^2 (c+dx^3)^{3/2}} \, dx = \int \frac{1}{x^4 (a+bx^3)^2 (c+dx^3)^{\frac{3}{2}}} \, dx$$

```
[In] integrate(1/x**4/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 \left(a+b x^3\right)^2 \left(c+d x^3\right)^{3/2}}\, dx = \int \frac{1}{{\left(b x^{3} + a\right)}^{2} {\left(d x^{3} + c\right)}^{\frac{3}{2}} x^{4}}\, dx$$

```
[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.52

$$\int \frac{1}{x^4 \left(a+b x^3\right)^2 \left(c+d x^3\right)^{3/2}}\, dx = \frac{{\left(4 \, b^{4} c - 7 \, a b^{3} d\right)} \arctan\left(\frac{\sqrt{d x^{3} + c} b}{\sqrt{-b^{2} c + a b d}}\right)}{3 \, {\left(a^{3} b^{2} c^{2} - 2 \, a^{4} b c d + a^{5} d^{2}\right)} \sqrt{-b^{2} c + a b d}}$$
$$- \frac{2 \, {\left(d x^{3} + c\right)}^{2} b^{3} c^{2} d - 2 \, {\left(d x^{3} + c\right)} b^{3} c^{3} d - 2 \, {\left(d x^{3} + c\right)}^{2} a b^{2} c d^{2} + 3 \, {\left(d x^{3} + c\right)} a b^{2} c^{2} d^{2} + 3 \, {\left(d x^{3} + c\right)}^{2} a^{2} b d^{3} - 7 \, (d}{3 \, {\left(a^{2} b^{2} c^{4} - 2 \, a^{3} b c^{3} d + a^{4} c^{2} d^{2}\right)} \left({\left(d x^{3} + c\right)}^{\frac{5}{2}} b - 2 \, {\left(d x^{3} + c\right)}^{\frac{3}{2}} b c + \sqrt{d x^{3} + c} b c^{2} + (d\right.}$$
$$- \frac{{\left(4 \, b c + 3 \, a d\right)} \arctan\left(\frac{\sqrt{d x^{3} + c}}{\sqrt{-c}}\right)}{3 \, a^{3} \sqrt{-c} c^{2}}$$

```
[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] 1/3*(4*b^4*c - 7*a*b^3*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((
a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2)*sqrt(-b^2*c + a*b*d)) - 1/3*(2*(d*x^3
+ c)^2*b^3*c^2*d - 2*(d*x^3 + c)*b^3*c^3*d - 2*(d*x^3 + c)^2*a*b^2*c*d^2 +
3*(d*x^3 + c)*a*b^2*c^2*d^2 + 3*(d*x^3 + c)^2*a^2*b*d^3 - 7*(d*x^3 + c)*a^2
*b*c*d^3 + 2*a^2*b*c^2*d^3 + 3*(d*x^3 + c)*a^3*d^4 - 2*a^3*c*d^4)/((a^2*b^2
*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*((d*x^3 + c)^(5/2)*b - 2*(d*x^3 + c)^(3
/2)*b*c + sqrt(d*x^3 + c)*b*c^2 + (d*x^3 + c)^(3/2)*a*d - sqrt(d*x^3 + c)*a
*c*d)) - 1/3*(4*b*c + 3*a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^3*sqrt(-c)
*c^2)
```

Mupad [B] (verification not implemented)

Time = 23.90 (sec) , antiderivative size = 18847, normalized size of antiderivative = 78.20

$$\int \frac{1}{x^4 (a + bx^3)^2 (c + dx^3)^{3/2}} \, dx = \text{Too large to display}$$

```
[In] int(1/(x^4*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

```
[Out] (2*b*log(1/x^6))/(3*a^3*c^(3/2)) - (c + d*x^3)^(1/2)/(3*a^2*c^2*x^3) + (d*l
og(1/x^6))/(2*a^2*c^(5/2)) + (2*b*log(c^(3/2)*(c + d*x^3)^(1/2) - c^(1/2)*(
c + d*x^3)^(3/2) + d^2*x^6 + 2*c*d*x^3 + 3*c^(1/2)*d*x^3*(c + d*x^3)^(1/2))
)/(3*a^3*c^(3/2)) + (d*log(c^(3/2)*(c + d*x^3)^(1/2) - c^(1/2)*(c + d*x^3)^
(3/2) + d^2*x^6 + 2*c*d*x^3 + 3*c^(1/2)*d*x^3*(c + d*x^3)^(1/2)))/(2*a^2*c^
(5/2)) - (b^7*c^9*x^4*(c + d*x^3)^(1/2))/(2*(2*a^9*c^6*d^5*x + 2*a^9*c^5*d^
6*x^4 + a^5*b^4*c^9*d^2*x^4 + a^6*b^3*c^8*d^3*x^4 - 3*a^7*b^2*c^7*d^4*x^4 +
 a^5*b^4*c^8*d^3*x^7 - 3*a^7*b^2*c^6*d^5*x^7 - 3*a^8*b*c^7*d^4*x + a^6*b^3*
c^9*d^2*x - a^8*b*c^6*d^5*x^4 + 2*a^8*b*c^5*d^6*x^7)) - (5*a^9*d^7*x^4*(c +
 d*x^3)^(1/2))/(4*(a^6*b^5*c^9*x + a^5*b^6*c^9*x^4 - 3*a^7*b^4*c^7*d^2*x^4
- a^8*b^3*c^6*d^3*x^4 + 2*a^9*b^2*c^5*d^4*x^4 - 3*a^7*b^4*c^6*d^3*x^7 + 2*a
^8*b^3*c^5*d^4*x^7 - 3*a^8*b^3*c^7*d^2*x + 2*a^9*b^2*c^6*d^3*x + a^6*b^5*c^
8*d*x^4 + a^5*b^6*c^8*d*x^7)) + (3*a^2*d^2*x*(c + d*x^3)^(1/2))/(a^2*b^2*c^
4*x^4 + 2*a^4*c^2*d^2*x^4 + a^3*b*c^4*x + 2*a^4*c^3*d*x + 3*a^3*b*c^3*d*x^4
 + a^2*b^2*c^3*d*x^7 + 2*a^3*b*c^2*d^2*x^7) + (2*b^2*c^2*x*(c + d*x^3)^(1/2
))/(a^2*b^2*c^4*x^4 + 2*a^4*c^2*d^2*x^4 + a^3*b*c^4*x + 2*a^4*c^3*d*x + 3*a
^3*b*c^3*d*x^4 + a^2*b^2*c^3*d*x^7 + 2*a^3*b*c^2*d^2*x^7) - (b^(7/2)*c*log(
(a^6*b^(15/2)*c^10*36i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (
a^7*b^(13/2)*c^9*d*198i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) +
(a^12*b^(3/2)*c^4*d^6*18i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2))
- (a^11*b^(5/2)*c^5*d^5*126i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2
)) + (a^10*b^(7/2)*c^6*d^4*360i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(
1/2)) - (a^9*b^(9/2)*c^7*d^3*540i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)
^(1/2)) + (a^8*b^(11/2)*c^8*d^2*450i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b
*c)^(1/2)) + (a^6*b^(15/2)*c^9*d*x^3*18i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d
 - b*c)^(1/2)) - (a^11*b^(5/2)*c^4*d^6*x^3*18i)/(a*(a*d - b*c)^(1/2) + b*x^
3*(a*d - b*c)^(1/2)) + (a^10*b^(7/2)*c^5*d^5*x^3*90i)/(a*(a*d - b*c)^(1/2)
+ b*x^3*(a*d - b*c)^(1/2)) - (a^9*b^(9/2)*c^6*d^4*x^3*180i)/(a*(a*d - b*c)^
(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (a^8*b^(11/2)*c^7*d^3*x^3*180i)/(a*(a*d
- b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (a^7*b^(13/2)*c^8*d^2*x^3*90i)/(a
*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (36*a^6*b^7*c^9*(c + d*x^3)
^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) +
 (360*a^8*b^5*c^7*d^2*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(
1/2) + b*x^3*(a*d - b*c)^(1/2)) - (360*a^9*b^4*c^6*d^3*(c + d*x^3)^(1/2)*(a
*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (180*a^1
0*b^3*c^5*d^4*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b
```

x^3(a*d - b*c)^(1/2)) - (36*a^11*b^2*c^4*d^5*(c + d*x^3)^(1/2)*(a*d - b*c
)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (180*a^7*b^6*c^8
d(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d -
 b*c)^(1/2)))*2i)/(3*a^3*(a*d - b*c)^(5/2)) + (b^(5/2)*d*log((a^6*b^(15/2)*
c^10*36i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (a^7*b^(13/2)*c
^9*d*198i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (a^12*b^(3/2)*
c^4*d^6*18i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (a^11*b^(5/2
)*c^5*d^5*126i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (a^10*b^(
7/2)*c^6*d^4*360i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (a^9*b
^(9/2)*c^7*d^3*540i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (a^8
*b^(11/2)*c^8*d^2*450i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (
a^6*b^(15/2)*c^9*d*x^3*18i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2))
 - (a^11*b^(5/2)*c^4*d^6*x^3*18i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^
(1/2)) + (a^10*b^(7/2)*c^5*d^5*x^3*90i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d -
 b*c)^(1/2)) - (a^9*b^(9/2)*c^6*d^4*x^3*180i)/(a*(a*d - b*c)^(1/2) + b*x^3*
(a*d - b*c)^(1/2)) + (a^8*b^(11/2)*c^7*d^3*x^3*180i)/(a*(a*d - b*c)^(1/2) +
 b*x^3*(a*d - b*c)^(1/2)) - (a^7*b^(13/2)*c^8*d^2*x^3*90i)/(a*(a*d - b*c)^(
1/2) + b*x^3*(a*d - b*c)^(1/2)) + (36*a^6*b^7*c^9*(c + d*x^3)^(1/2)*(a*d -
b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (360*a^8*b^5*
c^7*d^2*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(
a*d - b*c)^(1/2)) - (360*a^9*b^4*c^6*d^3*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2
))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (180*a^10*b^3*c^5*d^4*
(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*
c)^(1/2)) - (36*a^11*b^2*c^4*d^5*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a
*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (180*a^7*b^6*c^8*d*(c + d*x^3)
^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)))*
7i)/(6*a^2*(a*d - b*c)^(5/2)) + (5*a^4*d^4*x^4*(c + d*x^3)^(1/2))/(2*(a^4*b
^2*c^6*x + a^3*b^3*c^6*x^4 + 2*a^5*b*c^5*d*x + 2*a^4*b^2*c^4*d^2*x^7 + 3*a^
4*b^2*c^5*d*x^4 + 2*a^5*b*c^4*d^2*x^4 + a^3*b^3*c^5*d*x^7)) - (65*a^3*d^3*x
^4*(c + d*x^3)^(1/2))/(24*(a^3*b^2*c^5*x^4 + 2*a^5*c^3*d^2*x^4 + a^4*b*c^5*
x + 2*a^5*c^4*d*x + 3*a^4*b*c^4*d*x^4 + a^3*b^2*c^4*d*x^7 + 2*a^4*b*c^3*d^2
*x^7)) - (8*b^3*c^3*x^4*(c + d*x^3)^(1/2))/(3*(a^3*b^2*c^5*x^4 + 2*a^5*c^3*
d^2*x^4 + a^4*b*c^5*x + 2*a^5*c^4*d*x + 3*a^4*b*c^4*d*x^4 + a^3*b^2*c^4*d*x
^7 + 2*a^4*b*c^3*d^2*x^7)) + (14*b^3*c^4*x^4*(c + d*x^3)^(1/2))/(a^3*b^2*c^
6*x^4 + 2*a^5*c^4*d^2*x^4 + a^4*b*c^6*x + 2*a^5*c^5*d*x + 3*a^4*b*c^5*d*x^4
 + a^3*b^2*c^5*d*x^7 + 2*a^4*b*c^4*d^2*x^7) - (5*a^7*c^2*d^5*x*(c + d*x^3)^
(1/2))/(2*(a^5*b^4*c^9*x + a^4*b^5*c^9*x^4 - 3*a^6*b^3*c^7*d^2*x^4 - a^7*b^
2*c^6*d^3*x^4 - 3*a^6*b^3*c^6*d^3*x^7 + 2*a^7*b^2*c^5*d^4*x^7 + 2*a^8*b*c^6
*d^3*x - 3*a^7*b^2*c^7*d^2*x + a^5*b^4*c^8*d*x^4 + 2*a^8*b*c^5*d^4*x^4 + a^
4*b^5*c^8*d*x^7)) - (5*a^7*c*d^6*x^4*(c + d*x^3)^(1/2))/(2*(a^5*b^4*c^9*x +
 a^4*b^5*c^9*x^4 - 3*a^6*b^3*c^7*d^2*x^4 - a^7*b^2*c^6*d^3*x^4 - 3*a^6*b^3*
c^6*d^3*x^7 + 2*a^7*b^2*c^5*d^4*x^7 + 2*a^8*b*c^6*d^3*x - 3*a^7*b^2*c^7*d^2
*x + a^5*b^4*c^8*d*x^4 + 2*a^8*b*c^5*d^4*x^4 + a^4*b^5*c^8*d*x^7)) - (3*a^8
*c^2*d^5*x*(c + d*x^3)^(1/2))/(8*(a^6*b^4*c^9*x + a^5*b^5*c^9*x^4 - 3*a^7*b
^3*c^7*d^2*x^4 - a^8*b^2*c^6*d^3*x^4 - 3*a^7*b^3*c^6*d^3*x^7 + 2*a^8*b^2*c^

5*d^4*x^7 + 2*a^9*b*c^6*d^3*x - 3*a^8*b^2*c^7*d^2*x + a^6*b^4*c^8*d*x^4 + 2
*a^9*b*c^5*d^4*x^4 + a^5*b^5*c^8*d*x^7)) - (3*a^8*c*d^6*x^4*(c + d*x^3)^(1/
2))/(8*(a^6*b^4*c^9*x + a^5*b^5*c^9*x^4 - 3*a^7*b^3*c^7*d^2*x^4 - a^8*b^2*c
^6*d^3*x^4 - 3*a^7*b^3*c^6*d^3*x^7 + 2*a^8*b^2*c^5*d^4*x^7 + 2*a^9*b*c^6*d^
3*x - 3*a^8*b^2*c^7*d^2*x + a^6*b^4*c^8*d*x^4 + 2*a^9*b*c^5*d^4*x^4 + a^5*b
^5*c^8*d*x^7)) + (23*a^9*c^3*d^5*x*(c + d*x^3)^(1/2))/(8*(a^7*b^4*c^10*x +
a^6*b^5*c^10*x^4 - 3*a^8*b^3*c^8*d^2*x^4 - a^9*b^2*c^7*d^3*x^4 - 3*a^8*b^3*
c^7*d^3*x^7 + 2*a^9*b^2*c^6*d^4*x^7 + 2*a^10*b*c^7*d^3*x - 3*a^9*b^2*c^8*d^
2*x + a^7*b^4*c^9*d*x^4 + 2*a^10*b*c^6*d^4*x^4 + a^6*b^5*c^9*d*x^7)) - (a*b
^6*c^9*x*(c + d*x^3)^(1/2))/(2*(2*a^9*c^6*d^5*x + 2*a^9*c^5*d^6*x^4 + a^5*b
^4*c^9*d^2*x^4 + a^6*b^3*c^8*d^3*x^4 - 3*a^7*b^2*c^7*d^4*x^4 + a^5*b^4*c^8*
d^3*x^7 - 3*a^7*b^2*c^6*d^5*x^7 - 3*a^8*b*c^7*d^4*x + a^6*b^3*c^9*d^2*x - a
^8*b*c^6*d^5*x^4 + 2*a^8*b*c^5*d^6*x^7)) + (3*a^2*b^6*c^9*x^4*(c + d*x^3)^(
1/2))/(4*(2*a^10*c^7*d^4*x + 2*a^10*c^6*d^5*x^4 + a^7*b^3*c^9*d^2*x^4 - 3*a
^8*b^2*c^8*d^3*x^4 + a^6*b^4*c^9*d^2*x^7 - 3*a^8*b^2*c^7*d^4*x^7 + a^7*b^3*
c^10*d*x - 3*a^9*b*c^8*d^3*x + a^6*b^4*c^10*d*x^4 - a^9*b*c^7*d^4*x^4 + 2*a
^9*b*c^6*d^5*x^7)) - (5*a^6*c^2*d^3*x*(c + d*x^3)^(1/2))/(2*(a^6*b^2*c^7*x
+ a^5*b^3*c^7*x^4 + 2*a^7*b*c^6*d*x + 2*a^6*b^2*c^5*d^2*x^7 + 3*a^6*b^2*c^6
*d*x^4 + 2*a^7*b*c^5*d^2*x^4 + a^5*b^3*c^6*d*x^7)) - (5*a^6*c*d^4*x^4*(c +
d*x^3)^(1/2))/(2*(a^6*b^2*c^7*x + a^5*b^3*c^7*x^4 + 2*a^7*b*c^6*d*x + 2*a^6
*b^2*c^5*d^2*x^7 + 3*a^6*b^2*c^6*d*x^4 + 2*a^7*b*c^5*d^2*x^4 + a^5*b^3*c^6*
d*x^7)) + (4*a^2*b^4*c^6*x*(c + d*x^3)^(1/2))/(a^5*b^3*c^8*x + 2*a^8*c^5*d^
3*x + a^4*b^4*c^8*x^4 + 2*a^8*c^4*d^4*x^4 - 3*a^6*b^2*c^6*d^2*x^4 - 3*a^6*b
^2*c^5*d^3*x^7 - 3*a^7*b*c^6*d^2*x + a^5*b^3*c^7*d*x^4 - a^7*b*c^5*d^3*x^4
+ a^4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d^4*x^7) + (8*a*b^5*c^6*x^4*(c + d*x^3)^(
1/2))/(3*(a^5*b^3*c^8*x + 2*a^8*c^5*d^3*x + a^4*b^4*c^8*x^4 + 2*a^8*c^4*d^4
*x^4 - 3*a^6*b^2*c^6*d^2*x^4 - 3*a^6*b^2*c^5*d^3*x^7 - 3*a^7*b*c^6*d^2*x +
a^5*b^3*c^7*d*x^4 - a^7*b*c^5*d^3*x^4 + a^4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d^4
*x^7)) - (14*a^2*b^4*c^7*x*(c + d*x^3)^(1/2))/(a^5*b^3*c^9*x + 2*a^8*c^6*d^
3*x + a^4*b^4*c^9*x^4 + 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c^7*d^2*x^4 - 3*a^6*b
^2*c^6*d^3*x^7 - 3*a^7*b*c^7*d^2*x + a^5*b^3*c^8*d*x^4 - a^7*b*c^6*d^3*x^4
+ a^4*b^4*c^8*d*x^7 + 2*a^7*b*c^5*d^4*x^7) - (14*a*b^5*c^7*x^4*(c + d*x^3)^
(1/2))/(a^5*b^3*c^9*x + 2*a^8*c^6*d^3*x + a^4*b^4*c^9*x^4 + 2*a^8*c^5*d^4*x
^4 - 3*a^6*b^2*c^7*d^2*x^4 - 3*a^6*b^2*c^6*d^3*x^7 - 3*a^7*b*c^7*d^2*x + a^
5*b^3*c^8*d*x^4 - a^7*b*c^6*d^3*x^4 + a^4*b^4*c^8*d*x^7 + 2*a^7*b*c^5*d^4*x
^7) + (a^3*b^4*c^7*x*(c + d*x^3)^(1/2))/(8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3*x
 + a^5*b^4*c^9*x^4 + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^2*
c^6*d^3*x^7 - 3*a^8*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a
^5*b^4*c^8*d*x^7 + 2*a^8*b*c^5*d^4*x^7)) + (269*a^4*b^4*c^8*x*(c + d*x^3)^(
1/2))/(24*(a^7*b^3*c^10*x + 2*a^10*c^7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^
6*d^4*x^4 - 3*a^8*b^2*c^8*d^2*x^4 - 3*a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2
*x + a^7*b^3*c^9*d*x^4 - a^9*b*c^7*d^3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^
6*d^4*x^7)) + (65*a^6*c^2*d^4*x*(c + d*x^3)^(1/2))/(8*(a^5*b^3*c^8*x + 2*a^
8*c^5*d^3*x + a^4*b^4*c^8*x^4 + 2*a^8*c^4*d^4*x^4 - 3*a^6*b^2*c^6*d^2*x^4 -
 3*a^6*b^2*c^5*d^3*x^7 - 3*a^7*b*c^6*d^2*x + a^5*b^3*c^7*d*x^4 - a^7*b*c^5*

d^3*x^4 + a^4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d^4*x^7)) + (65*a^6*c*d^5*x^4*(c + d*x^3)^(1/2))/(24*(a^5*b^3*c^8*x + 2*a^8*c^5*d^3*x + a^4*b^4*c^8*x^4 + 2*a^8*c^4*d^4*x^4 - 3*a^6*b^2*c^6*d^2*x^4 - 3*a^6*b^2*c^5*d^3*x^7 - 3*a^7*b*c^6*d^2*x + a^5*b^3*c^7*d*x^4 - a^7*b*c^5*d^3*x^4 + a^4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d^4*x^7)) - (41*a^6*c^3*d^4*x*(c + d*x^3)^(1/2))/(3*(a^5*b^3*c^9*x + 2*a^8*c^6*d^3*x + a^4*b^4*c^9*x^4 + 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c^7*d^2*x^4 - 3*a^6*b^2*c^6*d^3*x^7 - 3*a^7*b*c^7*d^2*x + a^5*b^3*c^8*d*x^4 - a^7*b*c^6*d^3*x^4 + a^4*b^4*c^8*d*x^7 + 2*a^7*b*c^5*d^4*x^7)) - (5*a^7*c^3*d^4*x*(c + d*x^3)^(1/2))/(8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3*x + a^5*b^4*c^9*x^4 + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^2*c^6*d^3*x^7 - 3*a^8*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a^5*b^4*c^8*d*x^7 + 2*a^8*b*c^5*d^4*x^7)) + (47*a^8*c^4*d^4*x*(c + d*x^3)^(1/2))/(6*(a^7*b^3*c^10*x + 2*a^10*c^7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 - 3*a^8*b^2*c^8*d^2*x^4 - 3*a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2*x + a^7*b^3*c^9*d*x^4 - a^9*b*c^7*d^3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7)) - (34*a^3*b^2*c^5*x*(c + d*x^3)^(1/2))/(3*(a^5*b^2*c^7*x^4 + 2*a^7*c^5*d^2*x^4 + a^6*b*c^7*x + 2*a^7*c^6*d*x + 3*a^6*b*c^6*d*x^4 + a^5*b^2*c^6*d*x^7 + 2*a^6*b*c^5*d^2*x^7)) + (47*a^3*c^2*d^2*x*(c + d*x^3)^(1/2))/(3*(a^3*b^2*c^6*x^4 + 2*a^5*c^4*d^2*x^4 + a^4*b*c^6*x + 2*a^5*c^5*d*x + 3*a^4*b*c^5*d*x^4 + a^3*b^2*c^5*d*x^7 + 2*a^4*b*c^4*d^2*x^7)) + (38*a^3*c*d^3*x^4*(c + d*x^3)^(1/2))/(3*(a^3*b^2*c^6*x^4 + 2*a^5*c^4*d^2*x^4 + a^4*b*c^6*x + 2*a^5*c^5*d*x + 3*a^4*b*c^5*d*x^4 + a^3*b^2*c^5*d*x^7 + 2*a^4*b*c^4*d^2*x^7)) - (257*a^5*c^3*d^2*x*(c + d*x^3)^(1/2))/(24*(a^5*b^2*c^7*x^4 + 2*a^7*c^5*d^2*x^4 + a^6*b*c^7*x + 2*a^7*c^6*d*x + 3*a^6*b*c^6*d*x^4 + a^5*b^2*c^6*d*x^7 + 2*a^6*b*c^5*d^2*x^7)) - (5*a^9*c*d^6*x*(c + d*x^3)^(1/2))/(4*(a^6*b^5*c^9*x + a^5*b^6*c^9*x^4 - 3*a^7*b^4*c^7*d^2*x^4 - a^8*b^3*c^6*d^3*x^4 + 2*a^9*b^2*c^5*d^4*x^4 - 3*a^7*b^4*c^6*d^3*x^7 + 2*a^8*b^3*c^5*d^4*x^7 - 3*a^8*b^3*c^7*d^2*x + 2*a^9*b^2*c^6*d^3*x + a^6*b^5*c^8*d*x^4 + a^5*b^6*c^8*d*x^7)) + (23*a^9*c^2*d^6*x^4*(c + d*x^3)^(1/2))/(8*(a^7*b^4*c^10*x + a^6*b^5*c^10*x^4 - 3*a^8*b^3*c^8*d^2*x^4 - a^9*b^2*c^7*d^3*x^4 - 3*a^8*b^3*c^7*d^3*x^7 + 2*a^9*b^2*c^6*d^4*x^7 + 2*a^10*b*c^7*d^3*x - 3*a^9*b^2*c^8*d^2*x + a^7*b^4*c^9*d*x^4 + 2*a^10*b*c^6*d^4*x^4 + a^6*b^5*c^9*d*x^7)) + (a^2*b^6*c^10*x*(c + d*x^3)^(1/2))/(2*(2*a^10*c^7*d^5*x + 2*a^10*c^6*d^6*x^4 + a^6*b^4*c^10*d^2*x^4 + a^7*b^3*c^9*d^3*x^4 - 3*a^8*b^2*c^8*d^4*x^4 + a^6*b^4*c^9*d^3*x^7 - 3*a^8*b^2*c^7*d^5*x^7 - 3*a^9*b*c^8*d^4*x + a^7*b^3*c^10*d^2*x - a^9*b*c^7*d^5*x^4 + 2*a^9*b*c^6*d^6*x^7)) + (a*b^7*c^10*x^4*(c + d*x^3)^(1/2))/(2*(2*a^10*c^7*d^5*x + 2*a^10*c^6*d^6*x^4 + a^6*b^4*c^10*d^2*x^4 + a^7*b^3*c^9*d^3*x^4 - 3*a^8*b^2*c^8*d^4*x^4 + a^6*b^4*c^9*d^3*x^7 - 3*a^8*b^2*c^7*d^5*x^7 - 3*a^9*b*c^8*d^4*x + a^7*b^3*c^10*d^2*x - a^9*b*c^7*d^5*x^4 + 2*a^9*b*c^6*d^6*x^7)) + (a^2*b^5*c^7*x^4*(c + d*x^3)^(1/2))/(8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3*x + a^5*b^4*c^9*x^4 + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^2*c^6*d^3*x^7 - 3*a^8*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a^5*b^4*c^8*d*x^7 + 2*a^8*b*c^5*d^4*x^7)) + (269*a^3*b^5*c^8*x^4*(c + d*x^3)^(1/2))/(24*(a^7*b^3*c^10*x + 2*a^10*c^7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 - 3*a^8*b^2*c^8*d^2*x^4 - 3*a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2 *x + a^7*b^3*c^9*d*x^4 - a^9*b*c^7*d^3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7)) - (26*a^6*c^2*d^5*x^4*(c + d*x^3)^(1/2))/(3*(a^5*b^3*c^9*x + 2*a^8*c^6*d^3*x + a^4*b^4*c^9*x^4 + 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c^7*d^2*x^4 - 3*a^6*b^2*c^6*d^3*x^7 - 3*a^7*b*c^7*d^2*x + a^5*b^3*c^8*d*x^4 - a^7*b*c^6*d^3*x^4 + a^4*b^4*c^8*d*x^7 + 2*a^7*b*c^5*d^4*x^7)) - (5*a^7*c^2*d^5*x^4*(c + d*x^3)^(1/2))/(8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3*x + a^5*b^4*c^9*x^4 + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^2*c^6*d^3*x^7 - 3*a^8*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a^5*b^4*c^8*d*x^7 + 2*a^8*b*c^5*d^4*x^7)) + (79*a^8*c^3*d^5*x^4*(c + d*x^3)^(1/2))/(12*(a^7*b^3*c^10*x + 2*a^10*c^7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 - 3*a^8*b^2*c^8*d^2*x^4 - 3*a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2*x + a^7*b^3*c^9*d*x^4 - a^9*b*c^7*d^3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7)) - (34*a^2*b^3*c^5*x^4*(c + d*x^3)^(1/2))/(3*(a^5*b^2*c^7*x^4 + 2*a^7*c^5*d^2*x^4 + a^6*b*c^7*x + 2*a^7*c^6*d*x + 3*a^6*b*c^6*d*x^4 + a^5*b^2*c^6*d*x^7 + 2*a^6*b*c^5*d^2*x^7)) - (239*a^5*c^2*d^3*x^4*(c + d*x^3)^(1/2))/(24*(a^5*b^2*c^7*x^4 + 2*a^7*c^5*d^2*x^4 + a^6*b*c^7*x + 2*a^7*c^6*d*x + 3*a^6*b*c^6*d*x^4 + a^5*b^2*c^6*d*x^7 + 2*a^6*b*c^5*d^2*x^7)) - (3*a^2*b^5*c^8*x*(c + d*x^3)^(1/2))/(4*(2*a^9*c^6*d^4*x + 2*a^9*c^5*d^5*x^4 + a^6*b^3*c^8*d^2*x^4 - 3*a^7*b^2*c^7*d^3*x^4 + a^5*b^4*c^8*d^2*x^7 - 3*a^7*b^2*c^6*d^4*x^7 + a^6*b^3*c^9*d*x - 3*a^8*b*c^7*d^3*x + a^5*b^4*c^9*d*x^4 - a^8*b*c^6*d^4*x^4 + 2*a^8*b*c^5*d^5*x^7)) - (3*a*b^6*c^8*x^4*(c + d*x^3)^(1/2))/(4*(2*a^9*c^6*d^4*x + 2*a^9*c^5*d^5*x^4 + a^6*b^3*c^8*d^2*x^4 - 3*a^7*b^2*c^7*d^3*x^4 + a^5*b^4*c^8*d^2*x^7 - 3*a^7*b^2*c^6*d^4*x^7 + a^6*b^3*c^9*d*x - 3*a^8*b*c^7*d^3*x + a^5*b^4*c^9*d*x^4 - a^8*b*c^6*d^4*x^4 + 2*a^8*b*c^5*d^5*x^7)) + (3*a^3*b^5*c^9*x*(c + d*x^3)^(1/2))/(4*(2*a^10*c^7*d^4*x + 2*a^10*c^6*d^5*x^4 + a^7*b^3*c^9*d^2*x^4 - 3*a^8*b^2*c^8*d^3*x^4 + a^6*b^4*c^9*d^2*x^7 - 3*a^8*b^2*c^7*d^4*x^7 + a^7*b^3*c^10*d*x - 3*a^9*b*c^8*d^3*x + a^6*b^4*c^10*d*x^4 - a^9*b*c^7*d^4*x^4 + 2*a^9*b*c^6*d^5*x^7)) + (5*a^4*c*d^3*x*(c + d*x^3)^(1/2))/(2*(a^4*b^2*c^6*x + a^3*b^3*c^6*x^4 + 2*a^5*b*c^5*d*x + 2*a^4*b^2*c^4*d^2*x^7 + 3*a^4*b^2*c^5*d*x^4 + 2*a^5*b*c^4*d^2*x^4 + a^3*b^3*c^5*d*x^7)) - (8*a*b^4*c^5*x*(c + d*x^3)^(1/2))/(3*(a^4*b^3*c^7*x + 2*a^7*c^4*d^3*x + a^3*b^4*c^7*x^4 + 2*a^7*c^3*d^4*x^4 - 3*a^5*b^2*c^5*d^2*x^4 - 3*a^5*b^2*c^4*d^3*x^7 - 3*a^6*b*c^5*d^2*x + a^4*b^3*c^6*d*x^4 - a^6*b*c^4*d^3*x^4 + a^3*b^4*c^6*d*x^7 + 2*a^6*b*c^3*d^4*x^7)) - (5*a^5*c*d^4*x*(c + d*x^3)^(1/2))/(a^4*b^3*c^7*x + 2*a^7*c^4*d^3*x + a^3*b^4*c^7*x^4 + 2*a^7*c^3*d^4*x^4 - 3*a^5*b^2*c^5*d^2*x^4 - 3*a^5*b^2*c^4*d^3*x^7 - 3*a^6*b*c^5*d^2*x + a^4*b^3*c^6*d*x^4 - a^6*b*c^4*d^3*x^4 + a^3*b^4*c^6*d*x^7 + 2*a^6*b*c^3*d^4*x^7) + (5*a^10*c^2*d^6*x*(c + d*x^3)^(1/2))/(4*(a^7*b^5*c^10*x + a^6*b^6*c^10*x^4 - 3*a^8*b^4*c^8*d^2*x^4 - a^9*b^3*c^7*d^3*x^4 + 2*a^10*b^2*c^6*d^4*x^4 - 3*a^8*b^4*c^7*d^3*x^7 + 2*a^9*b^3*c^6*d^4*x^7 - 3*a^9*b^3*c^8*d^2*x + 2*a^10*b^2*c^7*d^3*x + a^7*b^5*c^9*d*x^4 + a^6*b^6*c^9*d*x^7)) + (5*a^10*c*d^7*x^4*(c + d*x^3)^(1/2))/(4*(a^7*b^5*c^10*x + a^6*b^6*c^10*x^4 - 3*a^8*b^4*c^8*d^2*x^4 - a^9*b^3*c^7*d^3*x^4 + 2*a^10*b^2*c^6*d^4*x^4 - 3*a^8*b^4*c^7*d^3*x^7 + 2*a^9*b^3*c^6*d^4*x^7 - 3*a^9*b^3*c^8*d^2*x + 2*a^10*b^2*c^7*d^3*x + a^7*b^5*c^9*d*x^4 + a^6*b^6*c^9*d*x^7)) - (11*a*b^2*c^3*x*(c + d*x^3)^(1/2))/(3*(a^3*

b^2*c^5*x^4 + 2*a^5*c^3*d^2*x^4 + a^4*b*c^5*x + 2*a^5*c^4*d*x + 3*a^4*b*c^4
*d*x^4 + a^3*b^2*c^4*d*x^7 + 2*a^4*b*c^3*d^2*x^7)) + (14*a*b^2*c^4*x*(c + d
*x^3)^(1/2))/(a^3*b^2*c^6*x^4 + 2*a^5*c^4*d^2*x^4 + a^4*b*c^6*x + 2*a^5*c^5
*d*x + 3*a^4*b*c^5*d*x^4 + a^3*b^2*c^5*d*x^7 + 2*a^4*b*c^4*d^2*x^7) + (11*a
*b*d^2*x^4*(c + d*x^3)^(1/2))/(2*(a^2*b^2*c^4*x^4 + 2*a^4*c^2*d^2*x^4 + a^3
*b*c^4*x + 2*a^4*c^3*d*x + 3*a^3*b*c^3*d*x^4 + a^2*b^2*c^3*d*x^7 + 2*a^3*b*
c^2*d^2*x^7)) - (143*a^3*c*d^2*x*(c + d*x^3)^(1/2))/(24*(a^3*b^2*c^5*x^4 +
2*a^5*c^3*d^2*x^4 + a^4*b*c^5*x + 2*a^5*c^4*d*x + 3*a^4*b*c^4*d*x^4 + a^3*b
^2*c^4*d*x^7 + 2*a^4*b*c^3*d^2*x^7)) + (22*b^2*c*d*x^4*(c + d*x^3)^(1/2))/(
3*(a^2*b^2*c^4*x^4 + 2*a^4*c^2*d^2*x^4 + a^3*b*c^4*x + 2*a^4*c^3*d*x + 3*a^
3*b*c^3*d*x^4 + a^2*b^2*c^3*d*x^7 + 2*a^3*b*c^2*d^2*x^7)) + (11*a^3*b^2*c^3
*d^2*x*(c + d*x^3)^(1/2))/(3*(a^4*b^3*c^7*x + 2*a^7*c^4*d^3*x + a^3*b^4*c^7
*x^4 + 2*a^7*c^3*d^4*x^4 - 3*a^5*b^2*c^5*d^2*x^4 - 3*a^5*b^2*c^4*d^3*x^7 -
3*a^6*b*c^5*d^2*x + a^4*b^3*c^6*d*x^4 - a^6*b*c^4*d^3*x^4 + a^3*b^4*c^6*d*x
^7 + 2*a^6*b*c^3*d^4*x^7)) - (31*a^4*b^2*c^4*d^2*x*(c + d*x^3)^(1/2))/(2*(a
^5*b^3*c^8*x + 2*a^8*c^5*d^3*x + a^4*b^4*c^8*x^4 + 2*a^8*c^4*d^4*x^4 - 3*a^
6*b^2*c^6*d^2*x^4 - 3*a^6*b^2*c^5*d^3*x^7 - 3*a^7*b*c^6*d^2*x + a^5*b^3*c^7
*d*x^4 - a^7*b*c^5*d^3*x^4 + a^4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d^4*x^7)) + (2
8*a^2*b^4*c^5*d*x^4*(c + d*x^3)^(1/2))/(3*(a^5*b^3*c^8*x + 2*a^8*c^5*d^3*x
+ a^4*b^4*c^8*x^4 + 2*a^8*c^4*d^4*x^4 - 3*a^6*b^2*c^6*d^2*x^4 - 3*a^6*b^2*c
^5*d^3*x^7 - 3*a^7*b*c^6*d^2*x + a^5*b^3*c^7*d*x^4 - a^7*b*c^5*d^3*x^4 + a^
4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d^4*x^7)) + (29*a^5*b*c^2*d^4*x^4*(c + d*x^3)
^(1/2))/(3*(a^5*b^3*c^8*x + 2*a^8*c^5*d^3*x + a^4*b^4*c^8*x^4 + 2*a^8*c^4*d
^4*x^4 - 3*a^6*b^2*c^6*d^2*x^4 - 3*a^6*b^2*c^5*d^3*x^7 - 3*a^7*b*c^6*d^2*x
+ a^5*b^3*c^7*d*x^4 - a^7*b*c^5*d^3*x^4 + a^4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d
^4*x^7)) + (23*a^4*b^2*c^5*d^2*x*(c + d*x^3)^(1/2))/(3*(a^5*b^3*c^9*x + 2*a
^8*c^6*d^3*x + a^4*b^4*c^9*x^4 + 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c^7*d^2*x^4
- 3*a^6*b^2*c^6*d^3*x^7 - 3*a^7*b*c^7*d^2*x + a^5*b^3*c^8*d*x^4 - a^7*b*c^6
*d^3*x^4 + a^4*b^4*c^8*d*x^7 + 2*a^7*b*c^5*d^4*x^7)) + (7*a^2*b^4*c^6*d*x^4
*(c + d*x^3)^(1/2))/(3*(a^5*b^3*c^9*x + 2*a^8*c^6*d^3*x + a^4*b^4*c^9*x^4 +
 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c^7*d^2*x^4 - 3*a^6*b^2*c^6*d^3*x^7 - 3*a^7*
b*c^7*d^2*x + a^5*b^3*c^8*d*x^4 - a^7*b*c^6*d^3*x^4 + a^4*b^4*c^8*d*x^7 + 2
*a^7*b*c^5*d^4*x^7)) - (7*a^5*b*c^3*d^4*x^4*(c + d*x^3)^(1/2))/(4*(a^5*b^3*
c^9*x + 2*a^8*c^6*d^3*x + a^4*b^4*c^9*x^4 + 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c
^7*d^2*x^4 - 3*a^6*b^2*c^6*d^3*x^7 - 3*a^7*b*c^7*d^2*x + a^5*b^3*c^8*d*x^4
- a^7*b*c^6*d^3*x^4 + a^4*b^4*c^8*d*x^7 + 2*a^7*b*c^5*d^4*x^7)) - (9*a^5*b^
2*c^5*d^2*x*(c + d*x^3)^(1/2))/(8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3*x + a^5*b^
4*c^9*x^4 + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^2*c^6*d^3*x
^7 - 3*a^8*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a^5*b^4*c^
8*d*x^7 + 2*a^8*b*c^5*d^4*x^7)) - (9*a^3*b^4*c^6*d*x^4*(c + d*x^3)^(1/2))/(
8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3*x + a^5*b^4*c^9*x^4 + 2*a^9*c^5*d^4*x^4 -
3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^2*c^6*d^3*x^7 - 3*a^8*b*c^7*d^2*x + a^6*b^3
*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a^5*b^4*c^8*d*x^7 + 2*a^8*b*c^5*d^4*x^7))
- (9*a^6*b*c^3*d^4*x^4*(c + d*x^3)^(1/2))/(8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3
*x + a^5*b^4*c^9*x^4 + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^

2*c^6*d^3*x^7 - 3*a^8*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a^5*b^4*c^8*d*x^7 + 2*a^8*b*c^5*d^4*x^7)) + (23*a^6*b^2*c^6*d^2*x*(c + d*x^3)^(1/2))/(24*(a^7*b^3*c^10*x + 2*a^10*c^7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 - 3*a^8*b^2*c^8*d^2*x^4 - 3*a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2*x + a^7*b^3*c^9*d*x^4 - a^9*b*c^7*d^3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7)) - (31*a^4*b^4*c^7*d*x^4*(c + d*x^3)^(1/2))/(8*(a^7*b^3*c^10*x + 2*a^10*c^7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 - 3*a^8*b^2*c^8*d^2*x^4 - 3*a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2*x + a^7*b^3*c^9*d*x^4 - a^9*b*c^7*d^3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7)) - (21*a^7*b*c^4*d^4*x^4*(c + d*x^3)^(1/2))/(8*(a^7*b^3*c^10*x + 2*a^10*c^7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 - 3*a^8*b^2*c^8*d^2*x^4 - 3*a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2*x + a^7*b^3*c^9*d*x^4 - a^9*b*c^7*d^3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7)) + (209*a^2*b*c^2*d^2*x^4*(c + d*x^3)^(1/2))/(12*(a^3*b^2*c^6*x^4 + 2*a^5*c^4*d^2*x^4 + a^4*b*c^6*x + 2*a^5*c^5*d*x + 3*a^4*b*c^5*d*x^4 + a^3*b^2*c^5*d*x^7 + 2*a^4*b*c^4*d^2*x^7)) - (89*a^3*b^2*c^4*d*x^4*(c + d*x^3)^(1/2))/(6*(a^5*b^2*c^7*x^4 + 2*a^7*c^5*d^2*x^4 + a^6*b*c^7*x + 2*a^7*c^6*d*x + 3*a^6*b*c^6*d*x^4 + a^5*b^2*c^6*d*x^7 + 2*a^6*b*c^5*d^2*x^7)) - (109*a^4*b*c^3*d^2*x^4*(c + d*x^3)^(1/2))/(12*(a^5*b^2*c^7*x^4 + 2*a^7*c^5*d^2*x^4 + a^6*b*c^7*x + 2*a^7*c^6*d*x + 3*a^6*b*c^6*d*x^4 + a^5*b^2*c^6*d*x^7 + 2*a^6*b*c^5*d^2*x^7)) - (33*a^2*b*c^2*d*x*(c + d*x^3)^(1/2))/(2*(a^3*b^2*c^5*x^4 + 2*a^5*c^3*d^2*x^4 + a^4*b*c^5*x + 2*a^5*c^4*d*x + 3*a^4*b*c^4*d*x^4 + a^3*b^2*c^4*d*x^7 + 2*a^4*b*c^3*d^2*x^7)) + (329*a^2*b*c^3*d*x*(c + d*x^3)^(1/2))/(12*(a^3*b^2*c^6*x^4 + 2*a^5*c^4*d^2*x^4 + a^4*b*c^6*x + 2*a^5*c^5*d*x + 3*a^4*b*c^5*d*x^4 + a^3*b^2*c^5*d*x^7 + 2*a^4*b*c^4*d^2*x^7)) - (205*a^4*b*c^4*d*x*(c + d*x^3)^(1/2))/(12*(a^5*b^2*c^7*x^4 + 2*a^7*c^5*d^2*x^4 + a^6*b*c^7*x + 2*a^7*c^6*d*x + 3*a^6*b*c^6*d*x^4 + a^5*b^2*c^6*d*x^7 + 2*a^6*b*c^5*d^2*x^7)) + (6*a^2*b^3*c^3*d^2*x^4*(c + d*x^3)^(1/2))/(a^4*b^3*c^7*x + 2*a^7*c^4*d^3*x + a^3*b^4*c^7*x^4 + 2*a^7*c^3*d^4*x^4 - 3*a^5*b^2*c^5*d^2*x^4 - 3*a^5*b^2*c^4*d^3*x^7 - 3*a^6*b*c^5*d^2*x + a^4*b^3*c^6*d*x^4 - a^6*b*c^4*d^3*x^4 + a^3*b^4*c^6*d*x^7 + 2*a^6*b*c^3*d^4*x^7) - (a^3*b^2*c^2*d^3*x^4*(c + d*x^3)^(1/2))/(a^4*b^3*c^7*x + 2*a^7*c^4*d^3*x + a^3*b^4*c^7*x^4 + 2*a^7*c^3*d^4*x^4 - 3*a^5*b^2*c^5*d^2*x^4 - 3*a^5*b^2*c^4*d^3*x^7 - 3*a^6*b*c^5*d^2*x + a^4*b^3*c^6*d*x^4 - a^6*b*c^4*d^3*x^4 + a^3*b^4*c^6*d*x^7 + 2*a^6*b*c^3*d^4*x^7) - (19*a^3*b^3*c^4*d^2*x^4*(c + d*x^3)^(1/2))/(3*(a^5*b^3*c^8*x + 2*a^8*c^5*d^3*x + a^4*b^4*c^8*x^4 + 2*a^8*c^4*d^4*x^4 - 3*a^6*b^2*c^6*d^2*x^4 - 3*a^6*b^2*c^5*d^3*x^7 - 3*a^7*b*c^6*d^2*x + a^5*b^3*c^7*d*x^4 - a^7*b*c^5*d^3*x^4 + a^4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d^4*x^7)) + (10*a^4*b^2*c^3*d^3*x^4*(c + d*x^3)^(1/2))/(3*(a^5*b^3*c^8*x + 2*a^8*c^5*d^3*x + a^4*b^4*c^8*x^4 + 2*a^8*c^4*d^4*x^4 - 3*a^6*b^2*c^6*d^2*x^4 - 3*a^6*b^2*c^5*d^3*x^7 - 3*a^7*b*c^6*d^2*x + a^5*b^3*c^7*d*x^4 - a^7*b*c^5*d^3*x^4 + a^4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d^4*x^7)) - (a^3*b^3*c^5*d^2*x^4*(c + d*x^3)^(1/2))/(2*(a^5*b^3*c^9*x + 2*a^8*c^6*d^3*x + a^4*b^4*c^9*x^4 + 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c^7*d^2*x^4 - 3*a^6*b^2*c^6*d^3*x^7 - 3*a^7*b*c^7*d^2*x + a^5*b^3*c^8*d*x^4 - a^7*b*c^6*d^3*x^4 + a^4*b^4*c^8*d*x^7 + 2*a^7*b*c^5*d^4*x^7)) - (97*a^4*b^2*c^4*d^3*x^4*(c + d*x^3)^(

1/2))/(6*(a^5*b^3*c^9*x + 2*a^8*c^6*d^3*x + a^4*b^4*c^9*x^4 + 2*a^8*c^5*d^4
*x^4 - 3*a^6*b^2*c^7*d^2*x^4 - 3*a^6*b^2*c^6*d^3*x^7 - 3*a^7*b*c^7*d^2*x +
a^5*b^3*c^8*d*x^4 - a^7*b*c^6*d^3*x^4 + a^4*b^4*c^8*d*x^7 + 2*a^7*b*c^5*d^4
*x^7)) - (9*a^4*b^3*c^5*d^2*x^4*(c + d*x^3)^(1/2))/(8*(a^6*b^3*c^9*x + 2*a^
9*c^6*d^3*x + a^5*b^4*c^9*x^4 + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 -
3*a^7*b^2*c^6*d^3*x^7 - 3*a^8*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*
d^3*x^4 + a^5*b^4*c^8*d*x^7 + 2*a^8*b*c^5*d^4*x^7)) - (9*a^5*b^2*c^4*d^3*x^
4*(c + d*x^3)^(1/2))/(8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3*x + a^5*b^4*c^9*x^4
+ 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^2*c^6*d^3*x^7 - 3*a^8
*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a^5*b^4*c^8*d*x^7 +
2*a^8*b*c^5*d^4*x^7)) + (31*a^5*b^3*c^6*d^2*x^4*(c + d*x^3)^(1/2))/(24*(a^7
*b^3*c^10*x + 2*a^10*c^7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 - 3*
a^8*b^2*c^8*d^2*x^4 - 3*a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2*x + a^7*b^3*c
^9*d*x^4 - a^9*b*c^7*d^3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7)) +
(415*a^6*b^2*c^5*d^3*x^4*(c + d*x^3)^(1/2))/(24*(a^7*b^3*c^10*x + 2*a^10*c^
7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 - 3*a^8*b^2*c^8*d^2*x^4 - 3
*a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2*x + a^7*b^3*c^9*d*x^4 - a^9*b*c^7*d^
3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7)) + (11*a*b*c*d*x*(c + d*x^
3)^(1/2))/(2*(a^2*b^2*c^4*x^4 + 2*a^4*c^2*d^2*x^4 + a^3*b*c^4*x + 2*a^4*c^3
*d*x + 3*a^3*b*c^3*d*x^4 + a^2*b^2*c^3*d*x^7 + 2*a^3*b*c^2*d^2*x^7)) - (10*
a^2*b^3*c^4*d*x*(c + d*x^3)^(1/2))/(3*(a^4*b^3*c^7*x + 2*a^7*c^4*d^3*x + a^
3*b^4*c^7*x^4 + 2*a^7*c^3*d^4*x^4 - 3*a^5*b^2*c^5*d^2*x^4 - 3*a^5*b^2*c^4*d
^3*x^7 - 3*a^6*b*c^5*d^2*x + a^4*b^3*c^6*d*x^4 - a^6*b*c^4*d^3*x^4 + a^3*b^
4*c^6*d*x^7 + 2*a^6*b*c^3*d^4*x^7)) - (19*a^4*b*c^2*d^3*x*(c + d*x^3)^(1/2)
)/(6*(a^4*b^3*c^7*x + 2*a^7*c^4*d^3*x + a^3*b^4*c^7*x^4 + 2*a^7*c^3*d^4*x^4
- 3*a^5*b^2*c^5*d^2*x^4 - 3*a^5*b^2*c^4*d^3*x^7 - 3*a^6*b*c^5*d^2*x + a^4*
b^3*c^6*d*x^4 - a^6*b*c^4*d^3*x^4 + a^3*b^4*c^6*d*x^7 + 2*a^6*b*c^3*d^4*x^7
)) - (8*a*b^4*c^4*d*x^4*(c + d*x^3)^(1/2))/(a^4*b^3*c^7*x + 2*a^7*c^4*d^3*x
+ a^3*b^4*c^7*x^4 + 2*a^7*c^3*d^4*x^4 - 3*a^5*b^2*c^5*d^2*x^4 - 3*a^5*b^2*
c^4*d^3*x^7 - 3*a^6*b*c^5*d^2*x + a^4*b^3*c^6*d*x^4 - a^6*b*c^4*d^3*x^4 + a
^3*b^4*c^6*d*x^7 + 2*a^6*b*c^3*d^4*x^7) - (15*a^4*b*c*d^4*x^4*(c + d*x^3)^(
1/2))/(2*(a^4*b^3*c^7*x + 2*a^7*c^4*d^3*x + a^3*b^4*c^7*x^4 + 2*a^7*c^3*d^4
*x^4 - 3*a^5*b^2*c^5*d^2*x^4 - 3*a^5*b^2*c^4*d^3*x^7 - 3*a^6*b*c^5*d^2*x +
a^4*b^3*c^6*d*x^4 - a^6*b*c^4*d^3*x^4 + a^3*b^4*c^6*d*x^7 + 2*a^6*b*c^3*d^4
*x^7)) + (12*a^3*b^3*c^5*d*x*(c + d*x^3)^(1/2))/(a^5*b^3*c^8*x + 2*a^8*c^5*
d^3*x + a^4*b^4*c^8*x^4 + 2*a^8*c^4*d^4*x^4 - 3*a^6*b^2*c^6*d^2*x^4 - 3*a^6
*b^2*c^5*d^3*x^7 - 3*a^7*b*c^6*d^2*x + a^5*b^3*c^7*d*x^4 - a^7*b*c^5*d^3*x^
4 + a^4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d^4*x^7) + (51*a^5*b*c^3*d^3*x*(c + d*x
^3)^(1/2))/(4*(a^5*b^3*c^8*x + 2*a^8*c^5*d^3*x + a^4*b^4*c^8*x^4 + 2*a^8*c^
4*d^4*x^4 - 3*a^6*b^2*c^6*d^2*x^4 - 3*a^6*b^2*c^5*d^3*x^7 - 3*a^7*b*c^6*d^2
*x + a^5*b^3*c^7*d*x^4 - a^7*b*c^5*d^3*x^4 + a^4*b^4*c^7*d*x^7 + 2*a^7*b*c^
4*d^4*x^7)) - (14*a^3*b^3*c^6*d*x*(c + d*x^3)^(1/2))/(3*(a^5*b^3*c^9*x + 2*
a^8*c^6*d^3*x + a^4*b^4*c^9*x^4 + 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c^7*d^2*x^4
- 3*a^6*b^2*c^6*d^3*x^7 - 3*a^7*b*c^7*d^2*x + a^5*b^3*c^8*d*x^4 - a^7*b*c^
6*d^3*x^4 + a^4*b^4*c^8*d*x^7 + 2*a^7*b*c^5*d^4*x^7)) - (169*a^5*b*c^4*d^3*

```
x*(c + d*x^3)^(1/2))/(12*(a^5*b^3*c^9*x + 2*a^8*c^6*d^3*x + a^4*b^4*c^9*x^4
 + 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c^7*d^2*x^4 - 3*a^6*b^2*c^6*d^3*x^7 - 3*a^
7*b*c^7*d^2*x + a^5*b^3*c^8*d*x^4 - a^7*b*c^6*d^3*x^4 + a^4*b^4*c^8*d*x^7 +
 2*a^7*b*c^5*d^4*x^7)) - (9*a^4*b^3*c^6*d*x*(c + d*x^3)^(1/2))/(8*(a^6*b^3*
c^9*x + 2*a^9*c^6*d^3*x + a^5*b^4*c^9*x^4 + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c
^7*d^2*x^4 - 3*a^7*b^2*c^6*d^3*x^7 - 3*a^8*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4
- a^8*b*c^6*d^3*x^4 + a^5*b^4*c^8*d*x^7 + 2*a^8*b*c^5*d^4*x^7)) - (9*a^6*b*
c^4*d^3*x*(c + d*x^3)^(1/2))/(8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3*x + a^5*b^4*
c^9*x^4 + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^2*c^6*d^3*x^7
 - 3*a^8*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a^5*b^4*c^8*
d*x^7 + 2*a^8*b*c^5*d^4*x^7)) - (7*a^5*b^3*c^7*d*x*(c + d*x^3)^(1/2))/(8*(a
^7*b^3*c^10*x + 2*a^10*c^7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 -
3*a^8*b^2*c^8*d^2*x^4 - 3*a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2*x + a^7*b^3
*c^9*d*x^4 - a^9*b*c^7*d^3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7))
+ (77*a^7*b*c^5*d^3*x*(c + d*x^3)^(1/2))/(8*(a^7*b^3*c^10*x + 2*a^10*c^7*d^
3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 - 3*a^8*b^2*c^8*d^2*x^4 - 3*a^8
*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2*x + a^7*b^3*c^9*d*x^4 - a^9*b*c^7*d^3*x^
4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7)) - (41*a*b^2*c^2*d*x^4*(c + d*
x^3)^(1/2))/(3*(a^3*b^2*c^5*x^4 + 2*a^5*c^3*d^2*x^4 + a^4*b*c^5*x + 2*a^5*c
^4*d*x + 3*a^4*b*c^4*d*x^4 + a^3*b^2*c^4*d*x^7 + 2*a^4*b*c^3*d^2*x^7)) - (7
1*a^2*b*c*d^2*x^4*(c + d*x^3)^(1/2))/(6*(a^3*b^2*c^5*x^4 + 2*a^5*c^3*d^2*x^
4 + a^4*b*c^5*x + 2*a^5*c^4*d*x + 3*a^4*b*c^4*d*x^4 + a^3*b^2*c^4*d*x^7 + 2
*a^4*b*c^3*d^2*x^7)) + (133*a*b^2*c^3*d*x^4*(c + d*x^3)^(1/2))/(6*(a^3*b^2*
c^6*x^4 + 2*a^5*c^4*d^2*x^4 + a^4*b*c^6*x + 2*a^5*c^5*d*x + 3*a^4*b*c^5*d*x
^4 + a^3*b^2*c^5*d*x^7 + 2*a^4*b*c^4*d^2*x^7))
```

3.495 $\int \frac{x^3}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{x^3}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx=\frac{x^4\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},2,\frac{3}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2c\sqrt{c+dx^3}}$$

```
[Out] 1/4*x^4*AppellF1(4/3,2,3/2,7/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/c/(
d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{x^3}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx=\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{4}{3},2,\frac{3}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2c\sqrt{c+dx^3}}$$

```
[In] Int[x^3/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 2, 3/2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)])/(4*a^2*c*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{(a+bx^3)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$
$$= \frac{x^4\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{4}{3};2,\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2c\sqrt{c+dx^3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 381 vs. $2(67) = 134$.

Time = 10.32 (sec) , antiderivative size = 381, normalized size of antiderivative = 5.69

$$\int \frac{x^3}{(a+bx^3)^2\,(c+dx^3)^{3/2}}\,dx = -\frac{x^4\left(-8abcd\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\left(8a+(a+bx^3)\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right.}{8a(bc-ad)^2\,(a+bx^3)\sqrt{c+dx^3}\,\left(-8ac\,\text{Appel}\right.}$$

```
[In] Integrate[x^3/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/8*(x^4*(-8*a*b*c*d*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)
]*(8*a + (a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^
3)/c), -((b*x^3)/a)]) + (8*a*(2*a*d + b*(c + 3*d*x^3)) + 3*b*d*x^3*(a + b*x
^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/
a)])*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*Ap
pellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/(a*(b*c - a*d)^2*(a
 + b*x^3)*Sqrt[c + d*x^3]*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c),
-((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*
x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.44 (sec) , antiderivative size = 787, normalized size of antiderivative = 11.75

method	result	size
elliptic	Expression too large to display	787
default	Expression too large to display	1593

```
[In] int(x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*d*x/(a*d-b*c)^2/((x^3+c/d)*d)^(1/2)-1/3*b/(a*d-b*c)^2*x*(d*x^3+c)^(1/2
)/(b*x^3+a)+1/3*I/(a*d-b*c)^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(
1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/
d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(
1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/
(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c
*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3)))^(1/2))+1/18*I/d^2*2^(1/2)*sum((-7*a*d-2*b*c)/(a*d-b*c)^3/_alpha^2
*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(
1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alp
ha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*
d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3
^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d
^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-
3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/
d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3
*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)^2 (c+dx^3)^{3/2}}\, dx = \text{Timed out}$$

```
[In] integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^3}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{x^3}{(a+bx^3)^2 (c+dx^3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**3/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(x**3/((a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^3}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{x^3}{(bx^3+a)^2(dx^3+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(x^3/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{x^3}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{x^3}{(bx^3+a)^2(dx^3+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(x^3/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{x^3}{(b\,x^3+a)^2\,(d\,x^3+c)^{3/2}}\,dx$$

```
[In] int(x^3/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
[Out] int(x^3/((a + b*x^3)^2*(c + d*x^3)^(3/2)), x)
```

3.496 $\int \frac{x}{(a+bx^3)^2 (c+dx^3)^{3/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 67

$$\int \frac{x}{(a+bx^3)^2 (c+dx^3)^{3/2}} \, dx = \frac{x^2 \sqrt{1+\frac{dx^3}{c}} \operatorname{AppellF1}\left(\frac{2}{3},2,\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2 c\sqrt{c+dx^3}}$$

```
[Out] 1/2*x^2*AppellF1(2/3,2,3/2,5/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/c/(
d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {525, 524}

$$\int \frac{x}{(a+bx^3)^2 (c+dx^3)^{3/2}} \, dx = \frac{x^2 \sqrt{\frac{dx^3}{c}+1} \operatorname{AppellF1}\left(\frac{2}{3},2,\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2 c\sqrt{c+dx^3}}$$

```
[In] Int[x/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 2, 3/2, 5/3, -((b*x^3)/a), -((d*x^3)
/c)])/(2*a^2*c*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{1+\frac{dx^3}{c}}\int\frac{x}{(a+bx^3)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}\\ &= \frac{x^2\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{2}{3};2,\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2c\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 216 vs. $2(67) = 134$.

Time = 10.28 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.22

$$\int\frac{x}{(a+bx^3)^2\,(c+dx^3)^{3/2}}\,dx = -\frac{x^2\left(-10a(2a^2d^2+2abd^2x^3+b^2c(c+dx^3))+5(-b^2c^2+6abcd+a^2d^2)\,(a+bx^3)\,\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\right.\right.}{30a^2c(bc-ad)^2\,(a+bx^3)}$$

```
[In] Integrate[x/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/30*(x^2*(-10*a*(2*a^2*d^2 + 2*a*b*d^2*x^3 + b^2*c*(c + d*x^3)) + 5*(-(b^
2*c^2) + 6*a*b*c*d + a^2*d^2)*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3,
 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + b*d*(b*c + 2*a*d)*x^3*(a + b*x^
3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a
)]))/(a^2*c*(b*c - a*d)^2*(a + b*x^3)*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.38 (sec) , antiderivative size = 986, normalized size of antiderivative = 14.72

method	result	size
default	Expression too large to display	986
elliptic	Expression too large to display	986

```
[In] int(x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/3*d^2*x^2/c/(a*d-b*c)^2/((x^3+c/d)*d)^(1/2)+1/3*b^2/(a*d-b*c)^2/a*x^2*(d*
x^3+c)^(1/2)/(b*x^3+a)-2/3*I*(-1/3*d^2/(a*d-b*c)^2/c-1/6*b*d/(a*d-b*c)^2/a)
*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(
-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^
3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*Elliptic
E(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^
(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^
(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(
1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)))+1/18*I/a/d^2*b*2^(1/2)*sum((11
*a*d-2*b*c)/(a*d-b*c)^3/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(
1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/
d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c
)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alph
a^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(
1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)
*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3
^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)^2 (c+dx^3)^{3/2}} \, dx = \text{Timed out}$$

```
[In] integrate(x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x}{(a+bx^3)^2 (c+dx^3)^{3/2}} \, dx = \int \frac{x}{(a+bx^3)^2 (c+dx^3)^{\frac{3}{2}}} \, dx$$

```
[In] integrate(x/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(x/((a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x}{(a+bx^3)^2 (c+dx^3)^{3/2}} \, dx = \int \frac{x}{(bx^3+a)^2 (dx^3+c)^{\frac{3}{2}}} \, dx$$

```
[In] integrate(x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(x/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{x}{(a+bx^3)^2 (c+dx^3)^{3/2}} \, dx = \int \frac{x}{(bx^3+a)^2 (dx^3+c)^{\frac{3}{2}}} \, dx$$

```
[In] integrate(x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(x/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx=\int \frac{x}{(b\,x^3+a)^2\,(d\,x^3+c)^{3/2}}\,dx$$

```
[In] int(x/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
[Out] int(x/((a + b*x^3)^2*(c + d*x^3)^(3/2)), x)
```

3.497 $\int \frac{1}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{1}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx=\frac{x\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{1}{3},2,\frac{3}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c\sqrt{c+dx^3}}$$

```
[Out] x*AppellF1(1/3,2,3/2,4/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/c/(d*x^3+
c)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {441, 440}

$$\int \frac{1}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx=\frac{x\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},2,\frac{3}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c\sqrt{c+dx^3}}$$

```
[In] Int[1/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 2, 3/2, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a^2*c*Sqrt[c + d*x^3])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{(a+bx^3)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$

$$= \frac{x\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{1}{3};2,\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c\sqrt{c+dx^3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 381 vs. 2(62) = 124.

Time = 10.52 (sec) , antiderivative size = 381, normalized size of antiderivative = 6.15

$$\int \frac{1}{(a+bx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{x\left(bd(bc+2ad)x^3\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{a\left(64ac(3a^2d^2+\right.}{}\right.}{}$$

```
[In] Integrate[1/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*(b*d*(b*c + 2*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -(
(d*x^3)/c), -((b*x^3)/a)] + (a*(64*a*c*(3*a^2*d^2 + 2*a*b*d*(-3*c + d*x^3)
+ b^2*c*(3*c + d*x^3))*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a
)] - 24*x^3*(2*a^2*d^2 + 2*a*b*d^2*x^3 + b^2*c*(c + d*x^3))*(2*b*c*AppellF1
[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1,
7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3)*(8*a*c*AppellF1[1/3, 1/2,
1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/
3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c
), -((b*x^3)/a)])))))/(24*a^2*c*(b*c - a*d)^2*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 4.41 (sec) , antiderivative size = 830, normalized size of antiderivative = 13.39

method	result	size
default	Expression too large to display	830
elliptic	Expression too large to display	830

```
[In] int(1/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*d^2*x/c/(a*d-b*c)^2/((x^3+c/d)*d)^(1/2)+1/3*b^2/(a*d-b*c)^2/a*x*(d*x^3+
c)^(1/2)/(b*x^3+a)-2/3*I*(1/3*d^2/(a*d-b*c)^2/c+1/6*b*d/(a*d-b*c)^2/a)*3^(1
/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^
2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)
+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^
(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*
d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3
/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))+1/18*I/a/d^2*b*
2^(1/2)*sum((13*a*d-4*b*c)/(a*d-b*c)^3/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*
x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*
(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*
(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3
))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d
^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(
1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d
-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*
d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/
2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^2 (c+dx^3)^{3/2}}\, dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(a+bx^3)^2 (c+dx^3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(1/((a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)^2 (dx^3+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)^2 (dx^3+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(b\,x^3+a)^2\,(d\,x^3+c)^{3/2}}\,dx$$

```
[In] int(1/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
[Out] int(1/((a + b*x^3)^2*(c + d*x^3)^(3/2)), x)
```

3.498 $\int \frac{1}{x^2(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{1}{x^2\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=-\frac{\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{1}{3},2,\frac{3}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2cx\sqrt{c+dx^3}}$$

```
[Out] -AppellF1(-1/3,2,3/2,2/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/c/x/(d*x^
3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{1}{x^2\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx=-\frac{\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{1}{3},2,\frac{3}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2cx\sqrt{c+dx^3}}$$

```
[In] Int[1/(x^2*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -((Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 2, 3/2, 2/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a^2*c*x*Sqrt[c + d*x^3]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^2(a+bx^3)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}\\ &= -\frac{\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{1}{3};2,\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2cx\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 308 vs. $2(65) = 130$.

Time = 10.42 (sec) , antiderivative size = 308, normalized size of antiderivative = 4.74

$$\int \frac{1}{x^2\,(a+bx^3)^2\,(c+dx^3)^{3/2}}\,dx = \frac{-20a(4b^3c^2x^3(c+dx^3)+a^3d^2(3c+5dx^3)+3ab^2c(c^2-cdx^3-2d^2x^6)+}{}$$

```
[In] Integrate[1/(x^2*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-20*a*(4*b^3*c^2*x^3*(c + d*x^3) + a^3*d^2*(3*c + 5*d*x^3) + 3*a*b^2*c*(c^
2 - c*d*x^3 - 2*d^2*x^6) + a^2*b*d*(-6*c^2 - 3*c*d*x^3 + 5*d^2*x^6)) + 5*(-
8*b^3*c^3 + 21*a*b^2*c^2*d - 6*a^2*b*c*d^2 + 5*a^3*d^3)*x^3*(a + b*x^3)*Sqr
t[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2
*b*d*(4*b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*x^6*(a + b*x^3)*Sqrt[1 + (d*x^3)/c
]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(60*a^3*c^2*(b*c
- a*d)^2*x*(a + b*x^3)*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 7.53 (sec) , antiderivative size = 1019, normalized size of antiderivative = 15.68

method	result	size
elliptic	Expression too large to display	1019
risch	Expression too large to display	2334
default	Expression too large to display	2383

```
[In] int(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*d^3*x^2/c^2/(a*d-b*c)^2/((x^3+c/d)*d)^(1/2)-1/3/(a*d-b*c)^2/a^2*b^3*x^
2*(d*x^3+c)^(1/2)/(b*x^3+a)-1/c^2/a^2*(d*x^3+c)^(1/2)/x-2/3*I*(1/3*d^3/c^2/
(a*d-b*c)^2+1/6*b^2*d/a^2/(a*d-b*c)^2+1/2*d/c^2/a^2)*3^(1/2)/d*(-c*d^2)^(1/
3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c
*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c
*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d
*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(
1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3)))^(1/2)))-1/18*I*b^2/a^2/d^2*2^(1/2)*sum((17*a*d-8*b*c)/(a*d-b*
c)^3/_alpha*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c
*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/(-3*(-c*d^2)^
(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)*(-c*d^
2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*(I*(-c*d^2)
^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^2-(-c*d^2)^(1
/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1
/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),1/2*b/d
*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha+I*3
^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c),(I*3^(1/2)/d*(-c*d^2)^(
1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alpha=
RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^2 (a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{x^2 (a+bx^3)^2 (c+dx^3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**2/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(1/(x**2*(a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)^2(dx^3+c)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a+bx^3)^2 (c+dx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)^2(dx^3+c)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + bx^3)^2 (c + dx^3)^{3/2}} \, dx = \int \frac{1}{x^2 (b\, x^3 + a)^2 (d\, x^3 + c)^{3/2}} \, dx$$

```
[In] int(1/(x^2*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
[Out] int(1/(x^2*(a + b*x^3)^2*(c + d*x^3)^(3/2)), x)
```

3.499 $\int \frac{1}{x^3(a+bx^3)^2(c+dx^3)^{3/2}} dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{1}{x^3 (a+bx^3)^2 (c+dx^3)^{3/2}} dx = -\frac{\sqrt{1+\frac{dx^3}{c}}\, \text{AppellF1}\left(-\frac{2}{3},2,\frac{3}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2cx^2\sqrt{c+dx^3}}$$

```
[Out] -1/2*AppellF1(-2/3,2,3/2,1/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/c/x^2
/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\int \frac{1}{x^3 (a+bx^3)^2 (c+dx^3)^{3/2}} dx = -\frac{\sqrt{\frac{dx^3}{c}+1}\, \text{AppellF1}\left(-\frac{2}{3},2,\frac{3}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2cx^2\sqrt{c+dx^3}}$$

```
[In] Int[1/(x^3*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/2*(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 2, 3/2, 1/3, -((b*x^3)/a), -((d*x^
3)/c)])/(a^2*c*x^2*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^3(a+bx^3)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$
$$= -\frac{\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{2}{3};2,\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2cx^2\sqrt{c+dx^3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 515 vs. $2(67) = 134$.

Time = 10.92 (sec) , antiderivative size = 515, normalized size of antiderivative = 7.69

$$\int \frac{1}{x^3(a+bx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{-bd(5b^2c^2-6abcd+7a^2d^2)\,x^6\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{}$$

```
[In] Integrate[1/(x^3*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-(b*d*(5*b^2*c^2 - 6*a*b*c*d + 7*a^2*d^2)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1
[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]) + (a*(32*a*c*(10*b^3*c^2*x^
3*(3*c + d*x^3) + 3*a^3*d^2*(2*c + 7*d*x^3) + 3*a*b^2*c*(2*c^2 - 13*c*d*x^3
 - 4*d^2*x^6) + 2*a^2*b*d*(-6*c^2 - 6*c*d*x^3 + 7*d^2*x^6))*AppellF1[1/3, 1
/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 24*x^3*(5*b^3*c^2*x^3*(c + d*x^3)
 + a^3*d^2*(3*c + 7*d*x^3) + 3*a*b^2*c*(c^2 - c*d*x^3 - 2*d^2*x^6) + a^2*b*
d*(-6*c^2 - 3*c*d*x^3 + 7*d^2*x^6))*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*
x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*
x^3)/a)])))/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -
((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x
^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(4
8*a^3*c^2*(b*c - a*d)^2*x^2*Sqrt[c + d*x^3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 7.34 (sec) , antiderivative size = 863, normalized size of antiderivative = 12.88

method	result	size
elliptic	Expression too large to display	863
risch	Expression too large to display	1874
default	Expression too large to display	1919

```
[In] int(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*d^3*x/c^2/(a*d-b*c)^2/((x^3+c/d)*d)^(1/2)-1/3/(a*d-b*c)^2/a^2*b^3*x*(d
*x^3+c)^(1/2)/(b*x^3+a)-1/2/c^2/a^2*(d*x^3+c)^(1/2)/x^2-2/3*I*(-1/3*d^3/c^2
/(a*d-b*c)^2-1/6*b^2*d/a^2/(a*d-b*c)^2-1/4*d/c^2/a^2)*3^(1/2)/d*(-c*d^2)^(1
/3)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-
c*d^2)^(1/3))^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1
/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/
3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))-1/18*I*b^2/a^2/d^2*2^(1/2)*sum((
19*a*d-10*b*c)/(a*d-b*c)^3/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d
^2)^(1/3))/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2
*x+1/d*(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*
x^3+c)^(1/2)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*
_alpha^2*d^2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)
*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d
^2)^(1/3))^(1/2),1/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^
(2/3)*3^(1/2)*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/(a*d-b*c)
,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2
)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^3)^2 (c+dx^3)^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^3 (a+bx^3)^2 (c+dx^3)^{3/2}} dx = \int \frac{1}{x^3 (a+bx^3)^2 (c+dx^3)^{\frac{3}{2}}} dx$$

```
[In] integrate(1/x**3/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(1/(x**3*(a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (a+bx^3)^2 (c+dx^3)^{3/2}} dx = \int \frac{1}{(bx^3+a)^2 (dx^3+c)^{\frac{3}{2}} x^3} dx$$

```
[In] integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a+bx^3)^2 (c+dx^3)^{3/2}} dx = \int \frac{1}{(bx^3+a)^2 (dx^3+c)^{\frac{3}{2}} x^3} dx$$

```
[In] integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^3)^2 (c+dx^3)^{3/2}} dx = \int \frac{1}{x^3 (b\,x^3+a)^2 (d\,x^3+c)^{3/2}} dx$$

```
[In] int(1/(x^3*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
[Out] int(1/(x^3*(a + b*x^3)^2*(c + d*x^3)^(3/2)), x)
```

3.500 $\int (ex)^m \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 134

$$\int (ex)^m \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \frac{2B(ex)^{1+m} \left(a+bx^3\right)^{7/2}}{be(23+2m)} - \frac{a^2(2aB(1+m)-Ab(23+2m))(ex)^{1+m}\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{be(1+m)(23+2m)\sqrt{1+\frac{bx^3}{a}}}$$

```
[Out] 2*B*(e*x)^(1+m)*(b*x^3+a)^(7/2)/b/e/(23+2*m)-a^2*(2*a*B*(1+m)-A*b*(23+2*m))
*(e*x)^(1+m)*hypergeom([-5/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)*(b*x^3+a)^(1
/2)/b/e/(1+m)/(23+2*m)/(1+b*x^3/a)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {470, 372, 371}

$$\int (ex)^m \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \frac{2B\left(a+bx^3\right)^{7/2}(ex)^{m+1}}{be(2m+23)} - \frac{a^2\sqrt{a+bx^3}(ex)^{m+1}(2aB(m+1)-Ab(2m+23))\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{be(m+1)(2m+23)\sqrt{\frac{bx^3}{a}+1}}$$

```
[In] Int[(e*x)^m*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (2*B*(e*x)^(1 + m)*(a + b*x^3)^(7/2))/(b*e*(23 + 2*m)) - (a^2*(2*a*B*(1 + m
) - A*b*(23 + 2*m))*(e*x)^(1 + m)*Sqrt[a + b*x^3]*Hypergeometric2F1[-5/2, (
```

```
1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(b*e*(1 + m)*(23 + 2*m)*Sqrt[1 + (b*x^3
)/a])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{7/2}}{be(23+2m)} - \frac{\left(aB(1+m)-Ab\left(\frac{23}{2}+m\right)\right)\int (ex)^m\left(a+bx^3\right)^{5/2}\,dx}{b\left(\frac{23}{2}+m\right)}$$

$$= \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{7/2}}{be(23+2m)} - \frac{\left(a^2\left(aB(1+m)-Ab\left(\frac{23}{2}+m\right)\right)\sqrt{a+bx^3}\right)\int (ex)^m\left(1+\frac{bx^3}{a}\right)^{5/2}\,dx}{b\left(\frac{23}{2}+m\right)\sqrt{1+\frac{bx^3}{a}}}$$

$$= \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{7/2}}{be(23+2m)} - \frac{a^2(2aB(1+m)-Ab(23+2m))(ex)^{1+m}\sqrt{a+bx^3}\,{}_2F_1\left(-\frac{5}{2},\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{be(1+m)(23+2m)\sqrt{1+\frac{bx^3}{a}}}$$

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.84

$$\int (ex)^m \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) dx = \frac{a^2x(ex)^m\sqrt{a+bx^3}\left(A(4+m)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)+B(1+m)x^3\,\text{Hype}\right.}{(1+m)(4+m)\sqrt{1+\frac{bx^3}{a}}}$$

```
[In] Integrate[(e*x)^m*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (a^2*x*(e*x)^m*Sqrt[a + b*x^3]*(A*(4 + m)*Hypergeometric2F1[-5/2, (1 + m)/3
, (4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[-5/2, (4 + m)/
3, (7 + m)/3, -((b*x^3)/a)]))/((1 + m)*(4 + m)*Sqrt[1 + (b*x^3)/a])
```

Maple [F]

$$\int (ex)^m \left(b\,x^3+a\right)^{\frac{5}{2}} \left(x^3B+A\right) dx$$

```
[In] int((e*x)^m*(b*x^3+a)^(5/2)*(B*x^3+A),x)
```

```
[Out] int((e*x)^m*(b*x^3+a)^(5/2)*(B*x^3+A),x)
```

Fricas [F]

$$\int (ex)^m \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}(ex)^m\, dx$$

```
[In] integrate((e*x)^m*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] integral((B*b^2*x^9 + (2*B*a*b + A*b^2)*x^6 + (B*a^2 + 2*A*a*b)*x^3 + A*a^2
)*sqrt(b*x^3 + a)*(e*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 18.96 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.83

$$\int (ex)^m \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right)\,dx = \frac{Aa^{\frac{5}{2}}e^m x^{m+1}\Gamma\left(\frac{m}{3}+\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{3}+\frac{1}{3} \\ \frac{m}{3}+\frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{2Aa^{\frac{3}{2}}be^m x^{m+4}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{3}+\frac{4}{3} \\ \frac{m}{3}+\frac{7}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)} + \frac{A\sqrt{a}b^2e^m x^{m+7}\Gamma\left(\frac{m}{3}+\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{3}+\frac{7}{3} \\ \frac{m}{3}+\frac{10}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{10}{3}\right)} + \frac{Ba^{\frac{5}{2}}e^m x^{m+4}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{3}+\frac{4}{3} \\ \frac{m}{3}+\frac{7}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)} + \frac{2Ba^{\frac{3}{2}}be^m x^{m+7}\Gamma\left(\frac{m}{3}+\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{3}+\frac{7}{3} \\ \frac{m}{3}+\frac{10}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{10}{3}\right)} + \frac{B\sqrt{a}b^2e^m x^{m+10}\Gamma\left(\frac{m}{3}+\frac{10}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{3}+\frac{10}{3} \\ \frac{m}{3}+\frac{13}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{13}{3}\right)}$$

```
[In] integrate((e*x)**m*(b*x**3+a)**(5/2)*(B*x**3+A),x)
```

```
[Out] A*a**(5/2)*e**m*x**(m + 1)*gamma(m/3 + 1/3)*hyper((-1/2, m/3 + 1/3), (m/3 +
 4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 4/3)) + 2*A*a**(3/2)*b*e**
m*x**(m + 4)*gamma(m/3 + 4/3)*hyper((-1/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3
*exp_polar(I*pi)/a)/(3*gamma(m/3 + 7/3)) + A*sqrt(a)*b**2*e**m*x**(m + 7)*g
amma(m/3 + 7/3)*hyper((-1/2, m/3 + 7/3), (m/3 + 10/3,), b*x**3*exp_polar(I*
pi)/a)/(3*gamma(m/3 + 10/3)) + B*a**(5/2)*e**m*x**(m + 4)*gamma(m/3 + 4/3)*
hyper((-1/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m
/3 + 7/3)) + 2*B*a**(3/2)*b*e**m*x**(m + 7)*gamma(m/3 + 7/3)*hyper((-1/2, m
/3 + 7/3), (m/3 + 10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 10/3)) +
 B*sqrt(a)*b**2*e**m*x**(m + 10)*gamma(m/3 + 10/3)*hyper((-1/2, m/3 + 10/3)
, (m/3 + 13/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 13/3))
```

Maxima [F]

$$\int (ex)^m \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}(ex)^m \, dx$$

```
[In] integrate((e*x)^m*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}(ex)^m \, dx$$

```
[In] integrate((e*x)^m*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^m\,{\left(b\,x^3+a\right)}^{5/2} \,d x$$

```
[In] int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(5/2),x)
[Out] int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(5/2), x)
```

3.501 $\int (ex)^m \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 132

$$\int (ex)^m \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{5/2}}{be(17+2m)} - \frac{a(2aB(1+m)-Ab(17+2m))(ex)^{1+m}\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{be(1+m)(17+2m)\sqrt{1+\frac{bx^3}{a}}}$$

```
[Out] 2*B*(e*x)^(1+m)*(b*x^3+a)^(5/2)/b/e/(17+2*m)-a*(2*a*B*(1+m)-A*b*(17+2*m))*(
e*x)^(1+m)*hypergeom([-3/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)*(b*x^3+a)^(1/2
)/b/e/(1+m)/(17+2*m)/(1+b*x^3/a)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {470, 372, 371}

$$\int (ex)^m \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \frac{2B\left(a+bx^3\right)^{5/2}(ex)^{m+1}}{be(2m+17)} - \frac{a\sqrt{a+bx^3}(ex)^{m+1}(2aB(m+1)-Ab(2m+17))\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{be(m+1)(2m+17)\sqrt{\frac{bx^3}{a}+1}}$$

```
[In] Int[(e*x)^m*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*B*(e*x)^(1 + m)*(a + b*x^3)^(5/2))/(b*e*(17 + 2*m)) - (a*(2*a*B*(1 + m)
- A*b*(17 + 2*m))*(e*x)^(1 + m)*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, (1
```

```
+ m)/3, (4 + m)/3, -((b*x^3)/a)])/(b*e*(1 + m)*(17 + 2*m)*Sqrt[1 + (b*x^3)/
a])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{5/2}}{be(17+2m)} - \frac{\left(aB(1+m)-Ab\left(\frac{17}{2}+m\right)\right)\int (ex)^m\left(a+bx^3\right)^{3/2}\,dx}{b\left(\frac{17}{2}+m\right)}\\
&= \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{5/2}}{be(17+2m)} - \frac{\left(a\left(aB(1+m)-Ab\left(\frac{17}{2}+m\right)\right)\sqrt{a+bx^3}\right)\int (ex)^m\left(1+\frac{bx^3}{a}\right)^{3/2}\,dx}{b\left(\frac{17}{2}+m\right)\sqrt{1+\frac{bx^3}{a}}}\\
&= \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{5/2}}{be(17+2m)}\\
&\quad - \frac{a(2aB(1+m)-Ab(17+2m))(ex)^{1+m}\sqrt{a+bx^3}\,{}_2F_1\left(-\frac{3}{2},\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{be(1+m)(17+2m)\sqrt{1+\frac{bx^3}{a}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.84

$$\int (ex)^m \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \frac{ax(ex)^m\sqrt{a+bx^3}\left(A(4+m)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)+B(1+m)x^3\,\text{Hyperge}\right.}{(1+m)(4+m)\sqrt{1+\frac{bx^3}{a}}}$$

```
[In] Integrate[(e*x)^m*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (a*x*(e*x)^m*Sqrt[a + b*x^3]*(A*(4 + m)*Hypergeometric2F1[-3/2, (1 + m)/3,
(4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[-3/2, (4 + m)/3,
 (7 + m)/3, -((b*x^3)/a)]))/((1 + m)*(4 + m)*Sqrt[1 + (b*x^3)/a])
```

Maple [F]

$$\int (ex)^m \left(b\,x^3+a\right)^{\frac{3}{2}} \left(x^3B+A\right) dx$$

```
[In] int((e*x)^m*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

```
[Out] int((e*x)^m*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

Fricas [F]

$$\int (ex)^m \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}(ex)^m \, dx$$

```
[In] integrate((e*x)^m*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)*(e*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.89 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.86

$$\int (ex)^m \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right)\,dx = \frac{Aa^{\frac{3}{2}}e^m x^{m+1}\Gamma\left(\frac{m}{3}+\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{m}{3}+\frac{1}{3}\\ \frac{m}{3}+\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{A\sqrt{a}be^m x^{m+4}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{m}{3}+\frac{4}{3}\\ \frac{m}{3}+\frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)} + \frac{Ba^{\frac{3}{2}}e^m x^{m+4}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{m}{3}+\frac{4}{3}\\ \frac{m}{3}+\frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)} + \frac{B\sqrt{a}be^m x^{m+7}\Gamma\left(\frac{m}{3}+\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{m}{3}+\frac{7}{3}\\ \frac{m}{3}+\frac{10}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{10}{3}\right)}$$

```
[In] integrate((e*x)**m*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] A*a**(3/2)*e**m*x**(m + 1)*gamma(m/3 + 1/3)*hyper((-1/2, m/3 + 1/3), (m/3 +
 4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 4/3)) + A*sqrt(a)*b*e**m*x
**(m + 4)*gamma(m/3 + 4/3)*hyper((-1/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*gamma(m/3 + 7/3)) + B*a**(3/2)*e**m*x**(m + 4)*gamma(m/
3 + 4/3)*hyper((-1/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(m/3 + 7/3)) + B*sqrt(a)*b*e**m*x**(m + 7)*gamma(m/3 + 7/3)*hyper((-
1/2, m/3 + 7/3), (m/3 + 10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 10
/3))
```

Maxima [F]

$$\int (ex)^m \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right)\,dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}(ex)^m\,dx$$

```
[In] integrate((e*x)^m*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m (a+bx^3)^{3/2} (A+Bx^3)\ dx = \int (Bx^3+A)(bx^3+a)^{\frac{3}{2}} (ex)^m\ dx$$

```
[In] integrate((e*x)^m*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m (a+bx^3)^{3/2} (A+Bx^3)\ dx = \int (B\,x^3+A)\ (e\,x)^m\ (b\,x^3+a)^{3/2}\,dx$$

```
[In] int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(3/2),x)
[Out] int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(3/2), x)
```

3.502 $\int (ex)^m \sqrt{a+bx^3} (A+Bx^3) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 131

$$\int (ex)^m \sqrt{a+bx^3} (A+Bx^3) \, dx = \frac{2B(ex)^{1+m} (a+bx^3)^{3/2}}{be(11+2m)} - \frac{(2aB(1+m)-Ab(11+2m))(ex)^{1+m}\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{be(1+m)(11+2m)\sqrt{1+\frac{bx^3}{a}}}$$

```
[Out] 2*B*(e*x)^(1+m)*(b*x^3+a)^(3/2)/b/e/(11+2*m)-(2*a*B*(1+m)-A*b*(11+2*m))*(e*
x)^(1+m)*hypergeom([-1/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)*(b*x^3+a)^(1/2)/
b/e/(1+m)/(11+2*m)/(1+b*x^3/a)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {470, 372, 371}

$$\int (ex)^m \sqrt{a+bx^3} (A+Bx^3) \, dx = \frac{2B(a+bx^3)^{3/2} (ex)^{m+1}}{be(2m+11)} - \frac{\sqrt{a+bx^3}(ex)^{m+1}(2aB(m+1)-Ab(2m+11))\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{be(m+1)(2m+11)\sqrt{\frac{bx^3}{a}+1}}$$

```
[In] Int[(e*x)^m*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*B*(e*x)^(1 + m)*(a + b*x^3)^(3/2))/(b*e*(11 + 2*m)) - ((2*a*B*(1 + m) -
A*b*(11 + 2*m))*(e*x)^(1 + m)*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, (1 +
```

```
m)/3, (4 + m)/3, -((b*x^3)/a)])/(b*e*(1 + m)*(11 + 2*m)*Sqrt[1 + (b*x^3)/a]
)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{3/2}}{be(11+2m)} - \frac{\left(aB(1+m)-Ab\left(\frac{11}{2}+m\right)\right)\int (ex)^m\sqrt{a+bx^3}\,dx}{b\left(\frac{11}{2}+m\right)}$$

$$= \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{3/2}}{be(11+2m)} - \frac{\left(\left(aB(1+m)-Ab\left(\frac{11}{2}+m\right)\right)\sqrt{a+bx^3}\right)\int (ex)^m\sqrt{1+\frac{bx^3}{a}}\,dx}{b\left(\frac{11}{2}+m\right)\sqrt{1+\frac{bx^3}{a}}}$$

$$= \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{3/2}}{be(11+2m)} - \frac{(2aB(1+m)-Ab(11+2m))(ex)^{1+m}\sqrt{a+bx^3}\,{}_2F_1\left(-\frac{1}{2},\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{be(1+m)(11+2m)\sqrt{1+\frac{bx^3}{a}}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.84

$$\int (ex)^m\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{x(ex)^m\sqrt{a+bx^3}\left(A(4+m)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)+B(1+m)x^3\,\text{Hypergeometric2F1}\right.}{(1+m)(4+m)\sqrt{1+\frac{bx^3}{a}}}$$

```
[In] Integrate[(e*x)^m*Sqrt[a + b*x^3]*(A + B*x^3),x]
[Out] (x*(e*x)^m*Sqrt[a + b*x^3]*(A*(4 + m)*Hypergeometric2F1[-1/2, (1 + m)/3, (4
 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[-1/2, (4 + m)/3, (
7 + m)/3, -((b*x^3)/a)]))/((1 + m)*(4 + m)*Sqrt[1 + (b*x^3)/a])
```

Maple [F]

$$\int \left(e x \right)^{m} \sqrt{b\,x^{3}+a}\,\left(x^{3} B +A \right)dx$$

```
[In] int((e*x)^m*(b*x^3+a)^(1/2)*(B*x^3+A),x)
[Out] int((e*x)^m*(b*x^3+a)^(1/2)*(B*x^3+A),x)
```

Fricas [F]

$$\int (ex)^m\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \int \left(Bx^3+A\right)\sqrt{bx^3+a}(ex)^m\,dx$$

```
[In] integrate((e*x)^m*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="fricas")
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.22 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.92

$$\int (ex)^m\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{A\sqrt{a}e^mx^{m+1}\Gamma\left(\frac{m}{3}+\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{3}+\frac{1}{3}\\ \frac{m}{3}+\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{B\sqrt{a}e^mx^{m+4}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{3}+\frac{4}{3}\\ \frac{m}{3}+\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}$$

```
[In] integrate((e*x)**m*(b*x**3+a)**(1/2)*(B*x**3+A),x)
```

```
[Out] A*sqrt(a)*e**m*x**(m + 1)*gamma(m/3 + 1/3)*hyper((-1/2, m/3 + 1/3), (m/3 +
4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 4/3)) + B*sqrt(a)*e**m*x**(
m + 4)*gamma(m/3 + 4/3)*hyper((-1/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_p
olar(I*pi)/a)/(3*gamma(m/3 + 7/3))
```

Maxima [F]

$$\int (ex)^m \sqrt{a + bx^3} (A + Bx^3)\, dx = \int (Bx^3 + A)\sqrt{bx^3 + a}(ex)^m\, dx$$

```
[In] integrate((e*x)^m*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m \sqrt{a + bx^3} (A + Bx^3)\, dx = \int (Bx^3 + A)\sqrt{bx^3 + a}(ex)^m\, dx$$

```
[In] integrate((e*x)^m*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \sqrt{a + bx^3} (A + Bx^3)\, dx = \int (B\,x^3 + A)\ (e\,x)^m\ \sqrt{b\,x^3 + a}\,dx$$

```
[In] int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(1/2),x)
```

```
[Out] int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(1/2), x)
```

3.503 $\int \frac{(ex)^m(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 131

$$\int \frac{(ex)^m (A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2B(ex)^{1+m}\sqrt{a+bx^3}}{be(5+2m)} - \frac{(2aB(1+m)-Ab(5+2m))(ex)^{1+m}\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{be(1+m)(5+2m)\sqrt{a+bx^3}}$$

```
[Out] 2*B*(e*x)^(1+m)*(b*x^3+a)^(1/2)/b/e/(5+2*m)-(2*a*B*(1+m)-A*b*(5+2*m))*(e*x)
^(1+m)*hypergeom([1/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)*(1+b*x^3/a)^(1/2)/b
/e/(1+m)/(5+2*m)/(b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {470, 372, 371}

$$\int \frac{(ex)^m (A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{2B\sqrt{a+bx^3}(ex)^{m+1}}{be(2m+5)} - \frac{\sqrt{\frac{bx^3}{a}+1}(ex)^{m+1}(2aB(m+1)-Ab(2m+5))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{be(m+1)(2m+5)\sqrt{a+bx^3}}$$

```
[In] Int[((e*x)^m*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*B*(e*x)^(1 + m)*Sqrt[a + b*x^3])/(b*e*(5 + 2*m)) - ((2*a*B*(1 + m) - A*b
*(5 + 2*m))*(e*x)^(1 + m)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, (1 + m
)/3, (4 + m)/3, -((b*x^3)/a)])/(b*e*(1 + m)*(5 + 2*m)*Sqrt[a + b*x^3])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2B(ex)^{1+m}\sqrt{a+bx^3}}{be(5+2m)} - \frac{\left(aB(1+m)-Ab\left(\frac{5}{2}+m\right)\right)\int \frac{(ex)^m}{\sqrt{a+bx^3}}\,dx}{b\left(\frac{5}{2}+m\right)} \\
&= \frac{2B(ex)^{1+m}\sqrt{a+bx^3}}{be(5+2m)} - \frac{\left(\left(aB(1+m)-Ab\left(\frac{5}{2}+m\right)\right)\sqrt{1+\frac{bx^3}{a}}\right)\int \frac{(ex)^m}{\sqrt{1+\frac{bx^3}{a}}}\,dx}{b\left(\frac{5}{2}+m\right)\sqrt{a+bx^3}} \\
&= \frac{2B(ex)^{1+m}\sqrt{a+bx^3}}{be(5+2m)} \\
&\quad - \frac{(2aB(1+m)-Ab(5+2m))(ex)^{1+m}\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{be(1+m)(5+2m)\sqrt{a+bx^3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.84

$$\int \frac{(ex)^m (A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{x(ex)^m\sqrt{1+\frac{bx^3}{a}}\left(A(4+m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)+B(1+m)x^3\,\text{Hypergeometric2F1}\right.}{(1+m)(4+m)\sqrt{a+bx^3}}$$

```
[In] Integrate[((e*x)^m*(A + B*x^3))/Sqrt[a + b*x^3],x]
[Out] (x*(e*x)^m*Sqrt[1 + (b*x^3)/a]*(A*(4 + m)*Hypergeometric2F1[1/2, (1 + m)/3,
 (4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[1/2, (4 + m)/3,
 (7 + m)/3, -((b*x^3)/a)]))/((1 + m)*(4 + m)*Sqrt[a + b*x^3])
```

Maple [F]

$$\int \frac{(ex)^m (x^3B+A)}{\sqrt{b\,x^3+a}}dx$$

```
[In] int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/2),x)
[Out] int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{(ex)^m (A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \int \frac{(Bx^3+A)(ex)^m}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] integral((B*x^3 + A)*(e*x)^m/sqrt(b*x^3 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.95 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.89

$$\int \frac{(ex)^m (A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{Ae^m x^{m+1}\Gamma\left(\frac{m}{3}+\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{3}+\frac{1}{3}\\ \frac{m}{3}+\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{Be^m x^{m+4}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{3}+\frac{4}{3}\\ \frac{m}{3}+\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}$$

```
[In] integrate((e*x)**m*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*e**m*x**(m + 1)*gamma(m/3 + 1/3)*hyper((1/2, m/3 + 1/3), (m/3 + 4/3,), b*
x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(m/3 + 4/3)) + B*e**m*x**(m + 4)*ga
mma(m/3 + 4/3)*hyper((1/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_polar(I*pi)
/a)/(3*sqrt(a)*gamma(m/3 + 7/3))
```

Maxima [F]

$$\int \frac{(ex)^m (A + Bx^3)}{\sqrt{a + bx^3}} \, dx = \int \frac{(Bx^3 + A)(ex)^m}{\sqrt{bx^3 + a}} \, dx$$

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^m/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{(ex)^m (A + Bx^3)}{\sqrt{a + bx^3}} \, dx = \int \frac{(Bx^3 + A)(ex)^m}{\sqrt{bx^3 + a}} \, dx$$

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^m/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A + Bx^3)}{\sqrt{a + bx^3}} \, dx = \int \frac{(B\,x^3 + A)\,(e\,x)^m}{\sqrt{b\,x^3 + a}} \, dx$$

```
[In] int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(1/2),x)
```

```
[Out] int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(1/2), x)
```

3.504 $\int \frac{(ex)^m\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 133

$$\int \frac{(ex)^m\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx=\frac{2(Ab-aB)(ex)^{1+m}}{3abe\sqrt{a+bx^3}}+\frac{(2aB(1+m)+A(b-2bm))(ex)^{1+m}\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{3abe(1+m)\sqrt{a+bx^3}}$$

```
[Out] 2/3*(A*b-B*a)*(e*x)^(1+m)/a/b/e/(b*x^3+a)^(1/2)+1/3*(2*a*B*(1+m)+A*(-2*b*m+
b))*(e*x)^(1+m)*hypergeom([1/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)*(1+b*x^3/a
)^(1/2)/a/b/e/(1+m)/(b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {468, 372, 371}

$$\int \frac{(ex)^m\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx=\frac{\sqrt{\frac{bx^3}{a}+1}(ex)^{m+1}(2aB(m+1)+A(b-2bm))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{3},\frac{m+4}{3},\right.}{3abe(m+1)\sqrt{a+bx^3}}+\frac{2(ex)^{m+1}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

```
[In] Int[((e*x)^m*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(1 + m))/(3*a*b*e*Sqrt[a + b*x^3]) + ((2*a*B*(1 + m) +
 A*(b - 2*b*m))*(e*x)^(1 + m)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, (1
 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(3*a*b*e*(1 + m)*Sqrt[a + b*x^3])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\text{integral} = \frac{2(Ab-aB)(ex)^{1+m}}{3abe\sqrt{a+bx^3}} + \frac{(2aB(1+m)+A(b-2bm))\int \frac{(ex)^m}{\sqrt{a+bx^3}}\,dx}{3ab}$$

$$= \frac{2(Ab-aB)(ex)^{1+m}}{3abe\sqrt{a+bx^3}} + \frac{\left((2aB(1+m)+A(b-2bm))\sqrt{1+\frac{bx^3}{a}}\right)\int \frac{(ex)^m}{\sqrt{1+\frac{bx^3}{a}}}\,dx}{3ab\sqrt{a+bx^3}}$$

$$= \frac{2(Ab-aB)(ex)^{1+m}}{3abe\sqrt{a+bx^3}} + \frac{(2aB(1+m)+A(b-2bm))(ex)^{1+m}\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{3abe(1+m)\sqrt{a+bx^3}}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.85

$$\int \frac{(ex)^m\,(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{x(ex)^m\sqrt{1+\frac{bx^3}{a}}\left(A(4+m)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)+B(1+m)x\right.}{a(1+m)(4+m)\sqrt{a+bx^3}}$$

```
[In] Integrate[((e*x)^m*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (x*(e*x)^m*Sqrt[1 + (b*x^3)/a]*(A*(4 + m)*Hypergeometric2F1[3/2, (1 + m)/3,
 (4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[3/2, (4 + m)/3,
 (7 + m)/3, -((b*x^3)/a)]))/(a*(1 + m)*(4 + m)*Sqrt[a + b*x^3])
```

Maple [F]

$$\int \frac{(ex)^m (x^3 B + A)}{(b\,x^3 + a)^{\frac{3}{2}}}dx$$

```
[In] int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

```
[Out] int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^3)}{(a + bx^3)^{3/2}}\,dx = \int \frac{(Bx^3 + A)(ex)^m}{(bx^3 + a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2), x
)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 29.54 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.88

$$\int \frac{(ex)^m (A + Bx^3)}{(a + bx^3)^{3/2}}\,dx = \frac{Ae^m x^{m+1}\Gamma\left(\frac{m}{3} + \frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{3}{2}, \frac{m}{3} + \frac{1}{3} \\ \frac{m}{3} + \frac{4}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{m}{3} + \frac{4}{3}\right)} + \frac{Be^m x^{m+4}\Gamma\left(\frac{m}{3} + \frac{4}{3}\right) {}_2F_1\left(\begin{matrix} \frac{3}{2}, \frac{m}{3} + \frac{4}{3} \\ \frac{m}{3} + \frac{7}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{m}{3} + \frac{7}{3}\right)}$$

```
[In] integrate((e*x)**m*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] A*e**m*x**(m + 1)*gamma(m/3 + 1/3)*hyper((3/2, m/3 + 1/3), (m/3 + 4/3,), b*
x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(m/3 + 4/3)) + B*e**m*x**(m + 4)*g
amma(m/3 + 4/3)*hyper((3/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_polar(I*pi
)/a)/(3*a**(3/2)*gamma(m/3 + 7/3))
```

Maxima [F]

$$\int \frac{(ex)^m (A + Bx^3)}{(a + bx^3)^{3/2}}\,dx = \int \frac{(Bx^3 + A)(ex)^m}{(bx^3 + a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(e*x)^m/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(ex)^m (A + Bx^3)}{(a + bx^3)^{3/2}}\,dx = \int \frac{(Bx^3 + A)(ex)^m}{(bx^3 + a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(e*x)^m/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A + Bx^3)}{(a + bx^3)^{3/2}}\,dx = \int \frac{(B\,x^3 + A)\,(e\,x)^m}{(b\,x^3 + a)^{3/2}}\,dx$$

```
[In] int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(3/2),x)
[Out] int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(3/2), x)
```

3.505 $\int \frac{(ex)^m\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 133

$$\int \frac{(ex)^m\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx=\frac{2(Ab-aB)(ex)^{1+m}}{9abe\left(a+bx^3\right)^{3/2}}+\frac{(Ab(7-2m)+2aB(1+m))(ex)^{1+m}\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{9a^2be(1+m)\sqrt{a+bx^3}}$$

```
[Out] 2/9*(A*b-B*a)*(e*x)^(1+m)/a/b/e/(b*x^3+a)^(3/2)+1/9*(A*b*(7-2*m)+2*a*B*(1+m
))*(e*x)^(1+m)*hypergeom([3/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)*(1+b*x^3/a)
^(1/2)/a^2/b/e/(1+m)/(b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {468, 372, 371}

$$\int \frac{(ex)^m\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx=\frac{\sqrt{\frac{bx^3}{a}+1}(ex)^{m+1}(2aB(m+1)+Ab(7-2m))\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{3},\frac{m+4}{3},\right.}{9a^2be(m+1)\sqrt{a+bx^3}}+\frac{2(ex)^{m+1}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

```
[In] Int[((e*x)^m*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(1 + m))/(9*a*b*e*(a + b*x^3)^(3/2)) + ((A*b*(7 - 2*m)
 + 2*a*B*(1 + m))*(e*x)^(1 + m)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[3/2,
(1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(9*a^2*b*e*(1 + m)*Sqrt[a + b*x^3])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\text{integral} = \frac{2(Ab-aB)(ex)^{1+m}}{9abe\,(a+bx^3)^{3/2}} + \frac{\left(2\left(-Ab\left(-\frac{7}{2}+m\right)+aB(1+m)\right)\right)\int \frac{(ex)^m}{(a+bx^3)^{3/2}}\,dx}{9ab}$$

$$= \frac{2(Ab-aB)(ex)^{1+m}}{9abe\,(a+bx^3)^{3/2}} + \frac{\left(2\left(-Ab\left(-\frac{7}{2}+m\right)+aB(1+m)\right)\sqrt{1+\frac{bx^3}{a}}\right)\int \frac{(ex)^m}{\left(1+\frac{bx^3}{a}\right)^{3/2}}\,dx}{9a^2b\sqrt{a+bx^3}}$$

$$= \frac{2(Ab-aB)(ex)^{1+m}}{9abe\,(a+bx^3)^{3/2}} + \frac{(Ab(7-2m)+2aB(1+m))(ex)^{1+m}\sqrt{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{3}{2},\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{9a^2be(1+m)\sqrt{a+bx^3}}$$

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.85

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{x(ex)^m\sqrt{1+\frac{bx^3}{a}}\left(A(4+m)\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)+B(1+m\right.}{a^2(1+m)(4+m)\sqrt{a+bx^3}}$$

```
[In] Integrate[((e*x)^m*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (x*(e*x)^m*Sqrt[1 + (b*x^3)/a]*(A*(4 + m)*Hypergeometric2F1[5/2, (1 + m)/3,
 (4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[5/2, (4 + m)/3,
 (7 + m)/3, -((b*x^3)/a)]))/(a^2*(1 + m)*(4 + m)*Sqrt[a + b*x^3])
```

Maple [F]

$$\int \frac{(ex)^m (x^3B+A)}{(b\,x^3+a)^{\frac{5}{2}}}dx$$

```
[In] int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

```
[Out] int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)(ex)^m}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^m/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2
*b*x^3 + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x)**m*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ex)^m (A + Bx^3)}{(a + bx^3)^{5/2}} \, dx = \int \frac{(Bx^3 + A)(ex)^m}{(bx^3 + a)^{\frac{5}{2}}} \, dx$$

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(e*x)^m/(b*x^3 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(ex)^m (A + Bx^3)}{(a + bx^3)^{5/2}} \, dx = \int \frac{(Bx^3 + A)(ex)^m}{(bx^3 + a)^{\frac{5}{2}}} \, dx$$

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(e*x)^m/(b*x^3 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A + Bx^3)}{(a + bx^3)^{5/2}} \, dx = \int \frac{(B\,x^3 + A)\,(e\,x)^m}{(b\,x^3 + a)^{5/2}} \, dx$$

```
[In] int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(5/2),x)
[Out] int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(5/2), x)
```

3.506 $\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd}-\frac{(bc+ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3b^{3/2}d^{3/2}}$$

```
[Out] -1/3*(a*d+b*c)*arctanh(d^(1/2)*(b*x^3+a)^(1/2)/b^(1/2)/(d*x^3+c)^(1/2))/b^(
3/2)/d^(3/2)+1/3*(b*x^3+a)^(1/2)*(d*x^3+c)^(1/2)/b/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {457, 81, 65, 223, 212}

$$\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd}-\frac{(ad+bc)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3b^{3/2}d^{3/2}}$$

```
[In] Int[x^5/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (Sqrt[a + b*x^3]*Sqrt[c + d*x^3])/(3*b*d) - ((b*c + a*d)*ArcTanh[(Sqrt[d]*S
qrt[a + b*x^3])/(Sqrt[b]*Sqrt[c + d*x^3])])/(3*b^(3/2)*d^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= \frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd} - \frac{(bc+ad)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^3\right)}{6bd}\\
&= \frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd} - \frac{(bc+ad)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^3}\right)}{3b^2d}\\
&= \frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd} - \frac{(bc+ad)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^3}}{\sqrt{c+dx^3}}\right)}{3b^2d}\\
&= \frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd} - \frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3b^{3/2}d^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd}-\frac{(bc+ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{d}\sqrt{a+bx^3}}\right)}{3b^{3/2}d^{3/2}}$$

```
[In] Integrate[x^5/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (Sqrt[a + b*x^3]*Sqrt[c + d*x^3])/(3*b*d) - ((b*c + a*d)*ArcTanh[(Sqrt[b]*S
qrt[c + d*x^3])/(Sqrt[d]*Sqrt[a + b*x^3])]])/(3*b^(3/2)*d^(3/2))
```

Maple [F]

$$\int \frac{x^5}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}dx$$

```
[In] int(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.91

$$\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \left[\frac{4\sqrt{bx^3+a}\sqrt{dx^3+c}bd+(bc+ad)\sqrt{bd}\log\left(8\,b^2d^2x^6+b^2c^2+6\,abcd+a^2d^2+8\,(b^2cd+abd^2)x^3-4\,(2\,b\right.}{12\,b^2d^2}\right.$$

```
[In] integrate(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*b*d + (b*c + a*d)*sqrt(b*d)*log(8*
b^2*d^2*x^6 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^3 - 4
*(2*b*d*x^3 + b*c + a*d)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(b*d)))/(b^2*d
^2), 1/6*(2*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*b*d + (b*c + a*d)*sqrt(-b*d)*ar
ctan(1/2*(2*b*d*x^3 + b*c + a*d)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(-b*d)
/(b^2*d^2*x^6 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^3)))/(b^2*d^2)]
```

Sympy [F]

$$\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(x**5/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**5/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.18

$$\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\frac{\frac{(bc+ad)\log\left(\left|-\sqrt{bx^3+a}\sqrt{bd}+\sqrt{b^2c+(bx^3+a)bd-abd}\right|\right)}{\sqrt{bd}d}+\frac{\sqrt{bx^3+a}\sqrt{b^2c+(bx^3+a)bd-abd}}{bd}}{3\,|b|}$$

```
[In] integrate(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*((b*c + a*d)*log(abs(-sqrt(b*x^3 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^3 +
 a)*b*d - a*b*d)))/(sqrt(b*d)*d) + sqrt(b*x^3 + a)*sqrt(b^2*c + (b*x^3 + a)
*b*d - a*b*d)/(b*d))/abs(b)
```

Mupad [B] (verification not implemented)

Time = 13.12 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.22

$$\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\left(\frac{2\,a\,d}{3}+\frac{2\,b\,c}{3}\right)}{d^3\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}+\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\frac{2\,a\,d}{3}+\frac{2\,b\,c}{3}\right)}{b\,d^2\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3}-\frac{8\,\sqrt{a}\,\sqrt{c}\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^2}{3\,d^2\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^4}{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^4}+\frac{b^2}{d^2}-\frac{2\,b\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^2}{d\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^2}} - \frac{2\,\mathrm{atanh}\left(\frac{\sqrt{d}\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}{\sqrt{b}\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}\right)(a\,d+b\,c)}{3\,b^{3/2}\,d^{3/2}}$$

```
[In] int(x^5/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

```
[Out] ((((a + b*x^3)^(1/2) - a^(1/2))*((2*a*d)/3 + (2*b*c)/3))/(d^3*((c + d*x^3)^
(1/2) - c^(1/2))) + (((a + b*x^3)^(1/2) - a^(1/2))^3*((2*a*d)/3 + (2*b*c)/3
))/(b*d^2*((c + d*x^3)^(1/2) - c^(1/2))^3) - (8*a^(1/2)*c^(1/2)*((a + b*x^3
)^(1/2) - a^(1/2))^2)/(3*d^2*((c + d*x^3)^(1/2) - c^(1/2))^2))/((((a + b*x^3
)^(1/2) - a^(1/2))^4/((c + d*x^3)^(1/2) - c^(1/2))^4 + b^2/d^2 - (2*b*((a +
 b*x^3)^(1/2) - a^(1/2))^2)/(d*((c + d*x^3)^(1/2) - c^(1/2))^2)) - (2*atanh
((d^(1/2)*((a + b*x^3)^(1/2) - a^(1/2)))/(b^(1/2)*((c + d*x^3)^(1/2) - c^(1
/2))))*(a*d + b*c))/(3*b^(3/2)*d^(3/2))
```

3.507 $\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 48

$$\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3\sqrt{b}\sqrt{d}}$$

```
[Out] 2/3*arctanh(d^(1/2)*(b*x^3+a)^(1/2)/b^(1/2)/(d*x^3+c)^(1/2))/b^(1/2)/d^(1/2
)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {455, 65, 223, 212}

$$\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3\sqrt{b}\sqrt{d}}$$

```
[In] Int[x^2/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^3])/(Sqrt[b]*Sqrt[c + d*x^3])])/(3*Sqrt[b]
*Sqrt[d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^3}\right)}{3b}\\
&= \frac{2\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^3}}{\sqrt{c+dx^3}}\right)}{3b}\\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3\sqrt{b}\sqrt{d}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{d}\sqrt{a+bx^3}}\right)}{3\sqrt{b}\sqrt{d}}$$

```
[In] Integrate[x^2/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/(Sqrt[d]*Sqrt[a + b*x^3])])/(3*Sqrt[b]
*Sqrt[d])
```

Maple [F]

$$\int \frac{x^2}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}dx$$

[In] int(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)

[Out] int(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(34) = 68$.

Time = 0.28 (sec) , antiderivative size = 194, normalized size of antiderivative = 4.04

$$\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \left[\frac{\sqrt{bd}\log\left(8\,b^2d^2x^6+b^2c^2+6\,abcd+a^2d^2+8\,(b^2cd+abd^2)x^3+4\,(2\,bdx^3+bc+ad)\sqrt{bx^3+a}\sqrt{dx^3+c}\sqrt{bd}\right)}{6\,bd}, -\frac{\sqrt{-bd}\arctan\left(\frac{(2\,bdx^3+bc+ad)\sqrt{bx^3+a}\sqrt{dx^3+c}\sqrt{-bd}}{2\,(b^2d^2x^6+abcd+(b^2cd+abd^2)x^3)}\right)}{3\,bd}\right]$$

[In] integrate(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] [1/6*sqrt(b*d)*log(8*b^2*d^2*x^6 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^3 + 4*(2*b*d*x^3 + b*c + a*d)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(b*d))/(b*d), -1/3*sqrt(-b*d)*arctan(1/2*(2*b*d*x^3 + b*c + a*d)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(-b*d)/(b^2*d^2*x^6 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^3))/(b*d)]

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

[In] integrate(x**2/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)

[Out] Integral(x**2/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = -\frac{2\,b\log\left(\left|-\sqrt{bx^3+a}\sqrt{bd}+\sqrt{b^2c+(bx^3+a)bd-abd}\right|\right)}{3\sqrt{bd}|b|}$$

```
[In] integrate(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*b*log(abs(-sqrt(b*x^3 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^3 + a)*b*d -
a*b*d)))/(sqrt(b*d)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = -\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}{\sqrt{-b\,d}\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}\right)}{3\sqrt{-b\,d}}$$

```
[In] int(x^2/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

```
[Out] -(4*atan((b*((c + d*x^3)^(1/2) - c^(1/2)))/((-b*d)^(1/2)*((a + b*x^3)^(1/2)
 - a^(1/2)))))/(3*(-b*d)^(1/2))
```

3.508 $\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 48

$$\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3\sqrt{a}\sqrt{c}}$$

```
[Out] -2/3*arctanh(c^(1/2)*(b*x^3+a)^(1/2)/a^(1/2)/(d*x^3+c)^(1/2))/a^(1/2)/c^(1/
2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {457, 95, 214}

$$\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3\sqrt{a}\sqrt{c}}$$

```
[In] Int[1/(x*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^3])/(Sqrt[a]*Sqrt[c + d*x^3])])/(3*Sqrt[a
]*Sqrt[c])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^3\right)\\ &= \frac{2}{3}\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+bx^3}}{\sqrt{c+dx^3}}\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3\sqrt{a}\sqrt{c}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a}\sqrt{c+dx^3}}{\sqrt{c}\sqrt{a+bx^3}}\right)}{3\sqrt{a}\sqrt{c}}$$

```
[In] Integrate[1/(x*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
[Out] (-2*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^3])/(Sqrt[c]*Sqrt[a + b*x^3])])/(3*Sqrt[a
]*Sqrt[c])
```

Maple [F]

$$\int \frac{1}{x\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}dx$$

```
[In] int(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
[Out] int(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(34) = 68$.

Time = 0.29 (sec) , antiderivative size = 204, normalized size of antiderivative = 4.25

$$\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \left[\frac{\sqrt{ac}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^6+8\,a^2c^2+8\,(abc^2+a^2cd)x^3-4\,((bc+ad)x^3+2\,ac)\sqrt{bx^3+a}\sqrt{dx^3+c}\sqrt{ac}}{x^6}\right)}{6\,ac}, \frac{\sqrt{-ac}\arctan\left(\frac{((bc+ad)x^3\cdots}{2\,(abcdx\cdots}\right)}{3\,a\cdots}\right.$$

```
[In] integrate(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*sqrt(a*c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6 + 8*a^2*c^2 + 8*(a*
b*c^2 + a^2*c*d)*x^3 - 4*((b*c + a*d)*x^3 + 2*a*c)*sqrt(b*x^3 + a)*sqrt(d*x
^3 + c)*sqrt(a*c))/x^6)/(a*c), 1/3*sqrt(-a*c)*arctan(1/2*((b*c + a*d)*x^3 +
 2*a*c)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(-a*c)/(a*b*c*d*x^6 + a^2*c^2 +
 (a*b*c^2 + a^2*c*d)*x^3))/(a*c)]
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(1/x/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(34) = 68$.

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.85

$$\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = -\frac{2\sqrt{bd}b\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^3+a}\sqrt{bd}-\sqrt{b^2c+(bx^3+a)bd-abd}\right)^2}{2\sqrt{-abcd}b}\right)}{3\sqrt{-abcd}|b|}$$

```
[In] integrate(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(b*d)*b*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^3 + a)*sqrt(b*d) -
sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sqrt(-a*b*c*
d)*abs(b))
```

Mupad [B] (verification not implemented)

Time = 11.45 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.83

$$\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = -\frac{\ln\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{dx^3+c}-\sqrt{c}}\right)-\ln\left(\frac{\left(\sqrt{c}\sqrt{bx^3+a}-\sqrt{a}\sqrt{dx^3+c}\right)\left(b\sqrt{c}-\frac{\sqrt{a}\,d\left(\sqrt{bx^3+a}-\sqrt{a}\right)}{\sqrt{dx^3+c}-\sqrt{c}}\right)}{\sqrt{dx^3+c}-\sqrt{c}}\right)}{3\sqrt{a}\sqrt{c}}$$

```
[In] int(1/(x*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

```
[Out] -(log(((a + b*x^3)^(1/2) - a^(1/2))/((c + d*x^3)^(1/2) - c^(1/2))) - log(((
c^(1/2)*(a + b*x^3)^(1/2) - a^(1/2)*(c + d*x^3)^(1/2))*(b*c^(1/2) - (a^(1/2
)*d*((a + b*x^3)^(1/2) - a^(1/2)))/((c + d*x^3)^(1/2) - c^(1/2))))/((c + d*
x^3)^(1/2) - c^(1/2))))/(3*a^(1/2)*c^(1/2))
```

3.509 $\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 91

$$\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3acx^3} + \frac{(bc+ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3a^{3/2}c^{3/2}}$$

```
[Out] 1/3*(a*d+b*c)*arctanh(c^(1/2)*(b*x^3+a)^(1/2)/a^(1/2)/(d*x^3+c)^(1/2))/a^(3
/2)/c^(3/2)-1/3*(b*x^3+a)^(1/2)*(d*x^3+c)^(1/2)/a/c/x^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {457, 98, 95, 214}

$$\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{(ad+bc)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3a^{3/2}c^{3/2}} - \frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3acx^3}$$

```
[In] Int[1/(x^4*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/3*(Sqrt[a + b*x^3]*Sqrt[c + d*x^3])/(a*c*x^3) + ((b*c + a*d)*ArcTanh[(Sq
rt[c]*Sqrt[a + b*x^3])/(Sqrt[a]*Sqrt[c + d*x^3])])/(3*a^(3/2)*c^(3/2))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3acx^3} - \frac{(bc+ad)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^3\right)}{6ac}\\
&= -\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3acx^3} - \frac{(bc+ad)\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+bx^3}}{\sqrt{c+dx^3}}\right)}{3ac}\\
&= -\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3acx^3} + \frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3a^{3/2}c^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3acx^3} + \frac{(bc+ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3a^{3/2}c^{3/2}}$$

```
[In] Integrate[1/(x^4*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
[Out] -1/3*(Sqrt[a + b*x^3]*Sqrt[c + d*x^3])/(a*c*x^3) + ((b*c + a*d)*ArcTanh[(Sq
rt[c]*Sqrt[a + b*x^3])/(Sqrt[a]*Sqrt[c + d*x^3])])/(3*a^(3/2)*c^(3/2))
```

Maple [F]

$$\int \frac{1}{x^4\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}dx$$

```
[In] int(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 278, normalized size of antiderivative = 3.05

$$\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \left[\frac{\sqrt{ac}(bc+ad)x^3\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^6+8\,a^2c^2+8\,(abc^2+a^2cd)x^3+4\,((bc+ad)x^3+2\,ac)\sqrt{bx^3+a}\sqrt{dx^3+c}\sqrt{ac}}{x^6}\right)-4\,\sqrt{bx^3+a}}{12\,a^2c^2x^3}\right.$$
$$\left.-\frac{\sqrt{-ac}(bc+ad)x^3\arctan\left(\frac{((bc+ad)x^3+2\,ac)\sqrt{bx^3+a}\sqrt{dx^3+c}\sqrt{-ac}}{2\,(abcdx^6+a^2c^2+(abc^2+a^2cd)x^3)}\right)+2\,\sqrt{bx^3+a}\sqrt{dx^3+c}ac}{6\,a^2c^2x^3}\right]$$

```
[In] integrate(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(sqrt(a*c)*(b*c + a*d)*x^3*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6 +
 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^3 + 4*((b*c + a*d)*x^3 + 2*a*c)*sqrt(b
*x^3 + a)*sqrt(d*x^3 + c)*sqrt(a*c))/x^6) - 4*sqrt(b*x^3 + a)*sqrt(d*x^3 +
c)*a*c)/(a^2*c^2*x^3), -1/6*(sqrt(-a*c)*(b*c + a*d)*x^3*arctan(1/2*((b*c +
a*d)*x^3 + 2*a*c)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(-a*c)/(a*b*c*d*x^6 +
 a^2*c^2 + (a*b*c^2 + a^2*c*d)*x^3)) + 2*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*a*
c)/(a^2*c^2*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(1/x**4/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(71) = 142$.

Time = 0.29 (sec) , antiderivative size = 413, normalized size of antiderivative = 4.54

$$\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{\sqrt{bd}b^4d\left(\frac{(bc+ad)\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^3+a}\sqrt{bd}-\sqrt{b^2c+(bx^3+a)bd-abd}\right)^2}{2\sqrt{-abcd}b}\right)}{\sqrt{-abcd}ab^3cd} - \frac{2\left(b^3c^2-2ab^2cd+a^2bd^2-\left(\sqrt{b\cdots}\right)\right)}{\left(b^4c^2-2ab^3cd+a^2b^2d^2-2\left(\sqrt{bx^3+a}\sqrt{bd}-\sqrt{b^2c+(bx^3+\cdots}\right)\right)}\cdots\right)}{3|b|}$$

```
[In] integrate(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(b*d)*b^4*d*((b*c + a*d)*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^3 +
 a)*sqrt(b*d) - sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b
))/(sqrt(-a*b*c*d)*a*b^3*c*d) - 2*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2 - (sqr
t(b*x^3 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2*b*c - (sq
rt(b*x^3 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2*a*d)/((b
^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 - 2*(sqrt(b*x^3 + a)*sqrt(b*d) - sqrt(b^
2*c + (b*x^3 + a)*b*d - a*b*d))^2*b^2*c - 2*(sqrt(b*x^3 + a)*sqrt(b*d) - sq
rt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2*a*b*d + (sqrt(b*x^3 + a)*sqrt(b*d) -
 sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^4)*a*b^2*c*d))/abs(b)
```

Mupad [B] (verification not implemented)

Time = 15.12 (sec) , antiderivative size = 481, normalized size of antiderivative = 5.29

$$\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\left(\frac{c\,b^2}{12}+\frac{a\,d\,b}{12}\right)}{a^{3/2}\,c^{3/2}\,d\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}-\frac{b^2}{12\,a\,c\,d}+\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^2\left(\frac{a^2\,d^2}{12}-\frac{a\,b\,c\,d}{4}+\frac{b^2\,c^2}{12}\right)}{a^2\,c^2\,d\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3}{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3}+\frac{b\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}{d\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}-\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^2(a\,d+b\,c)}{\sqrt{a}\,\sqrt{c}\,d\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^2}} + \frac{\ln\left(\frac{\sqrt{b\,x^3+a}-\sqrt{a}}{\sqrt{d\,x^3+c}-\sqrt{c}}\right)\left(\sqrt{a}\,b\,c^{3/2}+a^{3/2}\,\sqrt{c}\,d\right)}{6\,a^2\,c^2} - \frac{\ln\left(\frac{\left(\sqrt{c}\,\sqrt{b\,x^3+a}-\sqrt{a}\,\sqrt{d\,x^3+c}\right)\left(b\,\sqrt{c}-\frac{\sqrt{a}\,d\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}{\sqrt{d\,x^3+c}-\sqrt{c}}\right)}{\sqrt{d\,x^3+c}-\sqrt{c}}\right)\left(\sqrt{a}\,b\,c^{3/2}+a^{3/2}\,\sqrt{c}\,d\right)}{6\,a^2\,c^2} - \frac{d\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}{12\,a\,c\,\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}$$

```
[In] int(1/(x^4*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

```
[Out] ((((a + b*x^3)^(1/2) - a^(1/2))*((b^2*c)/12 + (a*b*d)/12))/(a^(3/2)*c^(3/2)
*d*((c + d*x^3)^(1/2) - c^(1/2))) - b^2/(12*a*c*d) + (((a + b*x^3)^(1/2) -
a^(1/2))^2*((a^2*d^2)/12 + (b^2*c^2)/12 - (a*b*c*d)/4))/(a^2*c^2*d*((c + d*
x^3)^(1/2) - c^(1/2))^2))/(((a + b*x^3)^(1/2) - a^(1/2))^3/((c + d*x^3)^(1/
2) - c^(1/2))^3 + (b*((a + b*x^3)^(1/2) - a^(1/2)))/(d*((c + d*x^3)^(1/2) -
 c^(1/2))) - (((a + b*x^3)^(1/2) - a^(1/2))^2*(a*d + b*c))/(a^(1/2)*c^(1/2)
*d*((c + d*x^3)^(1/2) - c^(1/2))^2)) + (log(((a + b*x^3)^(1/2) - a^(1/2))/(
(c + d*x^3)^(1/2) - c^(1/2)))*(a^(1/2)*b*c^(3/2) + a^(3/2)*c^(1/2)*d))/(6*a
^2*c^2) - (log((((c^(1/2)*(a + b*x^3)^(1/2) - a^(1/2)*(c + d*x^3)^(1/2))*(b*
c^(1/2) - (a^(1/2)*d*((a + b*x^3)^(1/2) - a^(1/2)))/((c + d*x^3)^(1/2) - c^
(1/2))))/((c + d*x^3)^(1/2) - c^(1/2)))*(a^(1/2)*b*c^(3/2) + a^(3/2)*c^(1/2
)*d))/(6*a^2*c^2) - (d*((a + b*x^3)^(1/2) - a^(1/2)))/(12*a*c*((c + d*x^3)^
(1/2) - c^(1/2)))
```

3.510 $\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{x^5\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{2},\frac{1}{2},\frac{8}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

```
[Out] 1/5*x^5*AppellF1(5/3,1/2,1/2,8/3,-b*x^3/a,-d*x^3/c)*(1+b*x^3/a)^(1/2)*(1+d*
x^3/c)^(1/2)/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {525, 524}

$$\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{x^5\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{2},\frac{1}{2},\frac{8}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

```
[In] Int[x^4/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^5*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1/2, 8/3, -
((b*x^3)/a), -((d*x^3)/c)])/(5*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{1+\frac{bx^3}{a}}\int \frac{x^4}{\sqrt{1+\frac{bx^3}{a}}\sqrt{c+dx^3}}\,dx}{\sqrt{a+bx^3}}\\ &= \frac{\left(\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\right)\int \frac{x^4}{\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\\ &= \frac{x^5\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5\sqrt{a+bx^3}\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 2.48 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02

$$\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{x^5\sqrt{\frac{a+bx^3}{a}}\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{2},\frac{1}{2},\frac{8}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

```
[In] Integrate[x^4/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^5*Sqrt[(a + b*x^3)/a]*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2, 1/2, 8/3, -
((b*x^3)/a), -((d*x^3)/c)])/(5*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Maple [F]

$$\int \frac{x^4}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}dx$$

```
[In] int(x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x^4}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

[In] integrate(x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^4/(b*d*x^6 + (b*c + a*d)*x^3 + a*c), x)

Sympy [F]

$$\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

[In] integrate(x**4/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)

[Out] Integral(x**4/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)

Maxima [F]

$$\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x^4}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

[In] integrate(x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] integrate(x^4/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)

Giac [F]

$$\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x^4}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

[In] integrate(x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] integrate(x^4/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x^4}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

```
[In] int(x^4/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
[Out] int(x^4/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)), x)
```

3.511 $\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{x^4\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

```
[Out] 1/4*x^4*AppellF1(4/3,1/2,1/2,7/3,-b*x^3/a,-d*x^3/c)*(1+b*x^3/a)^(1/2)*(1+d*
x^3/c)^(1/2)/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {525, 524}

$$\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{x^4\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

```
[In] Int[x^3/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^4*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1/2, 7/3, -
((b*x^3)/a), -((d*x^3)/c)])/(4*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{bx^3}{a}}\int \frac{x^3}{\sqrt{1+\frac{bx^3}{a}}\sqrt{c+dx^3}}\,dx}{\sqrt{a+bx^3}}$$

$$= \frac{\left(\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\right)\int \frac{x^3}{\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

$$= \frac{x^4\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

Mathematica [A] (verified)

Time = 2.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{x^4\sqrt{\frac{a+bx^3}{a}}\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

```
[In] Integrate[x^3/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
[Out] (x^4*Sqrt[(a + b*x^3)/a]*Sqrt[(c + d*x^3)/c]*AppellF1[4/3, 1/2, 1/2, 7/3, -
((b*x^3)/a), -((d*x^3)/c)])/(4*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Maple [F]

$$\int \frac{x^3}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}dx$$

```
[In] int(x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
[Out] int(x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

```
[In] integrate(x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^3/(b*d*x^6 + (b*c + a*d)*x^3 + a
*c), x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(x**3/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
[Out] Integral(x**3/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

```
[In] integrate(x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

```
[In] integrate(x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^3/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x^3}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

```
[In] int(x^3/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
[Out] int(x^3/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)), x)
```

3.512 $\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 88

$$\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{x^2\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

```
[Out] 1/2*x^2*AppellF1(2/3,1/2,1/2,5/3,-b*x^3/a,-d*x^3/c)*(1+b*x^3/a)^(1/2)*(1+d*
x^3/c)^(1/2)/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{x^2\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

```
[In] Int[x/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1/2, 5/3, -
((b*x^3)/a), -((d*x^3)/c)])/(2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{1+\frac{bx^3}{a}}\int \frac{x}{\sqrt{1+\frac{bx^3}{a}}\sqrt{c+dx^3}}\,dx}{\sqrt{a+bx^3}} \\ &= \frac{\left(\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\right)\int \frac{x}{\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{a+bx^3}\sqrt{c+dx^3}} \\ &= \frac{x^2\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2\sqrt{a+bx^3}\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 2.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02

$$\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{x^2\sqrt{\frac{a+bx^3}{a}}\sqrt{\frac{c+dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

```
[In] Integrate[x/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
[Out] (x^2*Sqrt[(a + b*x^3)/a]*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1/2, 5/3, -
((b*x^3)/a), -((d*x^3)/c)])/(2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Maple [F]

$$\int \frac{x}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}dx$$

```
[In] int(x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
[Out] int(x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{x}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

[In] integrate(x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x/(b*d*x^6 + (b*c + a*d)*x^3 + a*c
), x)

Sympy [F]

$$\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

[In] integrate(x/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)

[Out] Integral(x/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)

Maxima [F]

$$\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{x}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

[In] integrate(x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] integrate(x/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)

Giac [F]

$$\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{x}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

[In] integrate(x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] integrate(x/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{x}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

```
[In] int(x/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
[Out] int(x/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)), x)
```

3.513 $\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}} dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}} dx = \frac{x\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

```
[Out] x*AppellF1(1/3,1/2,1/2,4/3,-b*x^3/a,-d*x^3/c)*(1+b*x^3/a)^(1/2)*(1+d*x^3/c)
^(1/2)/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {441, 440}

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}} dx = \frac{x\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

```
[In] Int[1/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1/2, 1/2, 4/3, -((
b*x^3)/a), -((d*x^3)/c)])/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{bx^3}{a}}\int\frac{1}{\sqrt{1+\frac{bx^3}{a}}\sqrt{c+dx^3}}\,dx}{\sqrt{a+bx^3}}$$

$$= \frac{\left(\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\right)\int\frac{1}{\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

$$= \frac{x\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 170 vs. 2(83) = 166.

Time = 2.40 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.05

$$\int\frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx =$$

$$-\frac{8acx\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\sqrt{c+dx^3}\left(-8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+3x^3\left(ad\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{3}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\right.\right.}$$

```
[In] Integrate[1/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]

[Out] (-8*a*c*x*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(Sqrt[a
 + b*x^3]*Sqrt[c + d*x^3]*(-8*a*c*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a)
, -((d*x^3)/c)] + 3*x^3*(a*d*AppellF1[4/3, 1/2, 3/2, 7/3, -((b*x^3)/a), -((
d*x^3)/c)] + b*c*AppellF1[4/3, 3/2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))
)
```

Maple [F]

$$\int \frac{1}{\sqrt{b\,x^{3}+a}\,\sqrt{d\,x^{3}+c}}dx$$

```
[In] int(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
[Out] int(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)/(b*d*x^6 + (b*c + a*d)*x^3 + a*c),
 x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

```
[In] integrate(1/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=\int \frac{1}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

```
[In] int(1/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
[Out] int(1/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)), x)
```

3.514 $\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 86

$$\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=-\frac{\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

```
[Out] -AppellF1(-1/3,1/2,1/2,2/3,-b*x^3/a,-d*x^3/c)*(1+b*x^3/a)^(1/2)*(1+d*x^3/c)
^(1/2)/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {525, 524}

$$\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx=-\frac{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

```
[In] Int[1/(x^2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] -((Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 1/2, 1/2, 2/3, -(
(b*x^3)/a), -((d*x^3)/c)])/(x*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1+\frac{bx^3}{a}} \int \frac{1}{x^2\sqrt{1+\frac{bx^3}{a}}\sqrt{c+dx^3}}\,dx}{\sqrt{a+bx^3}} \\
&= \frac{\left(\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\right) \int \frac{1}{x^2\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{a+bx^3}\sqrt{c+dx^3}} \\
&= -\frac{\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 189 vs. 2(86) = 172.

Time = 2.50 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.20

$$\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx \\
= \frac{-20(a+bx^3)(c+dx^3)+5(bc+ad)x^3\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+8bdx^6\sqrt{1+}}{20acx\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

```
[In] Integrate[1/(x^2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (-20*(a + b*x^3)*(c + d*x^3) + 5*(b*c + a*d)*x^3*Sqrt[1 + (b*x^3)/a]*Sqrt[1
 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1/2, 5/3, -((b*x^3)/a), -((d*x^3)/c)] + 8*
b*d*x^6*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1/2, 8/3
, -((b*x^3)/a), -((d*x^3)/c)])/(20*a*c*x*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Maple [F]

$$\int \frac{1}{x^{2} \sqrt{b\,x^{3}+a}\,\sqrt{d\,x^{3}+c}}d x$$

[In] int(1/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)

[Out] int(1/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)

Fricas [F]

$$\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+cx^2}}\,dx$$

[In] integrate(1/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)/(b*d*x^8 + (b*c + a*d)*x^5 + a*c*x^2), x)

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

[In] integrate(1/x**2/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)

[Out] Integral(1/(x**2*sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+cx^2}}\,dx$$

[In] integrate(1/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^2), x)

Giac [F]

$$\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}x^2}\,dx$$

```
[In] integrate(1/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{x^2\,\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

```
[In] int(1/(x^2*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
[Out] int(1/(x^2*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)), x)
```

3.515 $\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

```
[Out] -1/2*AppellF1(-2/3,1/2,1/2,1/3,-b*x^3/a,-d*x^3/c)*(1+b*x^3/a)^(1/2)*(1+d*x^
3/c)^(1/2)/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {525, 524}

$$\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\,\text{AppellF1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

```
[In] Int[1/(x^3*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/2*(Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1/2, 1/2, 1/3,
 -((b*x^3)/a), -((d*x^3)/c)])/(x^2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{bx^3}{a}}\int \frac{1}{x^3\sqrt{1+\frac{bx^3}{a}}\sqrt{c+dx^3}}\,dx}{\sqrt{a+bx^3}}$$
$$= \frac{\left(\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\right)\int \frac{1}{x^3\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$
$$= -\frac{\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 365 vs. $2(88) = 176$.

Time = 2.59 (sec) , antiderivative size = 365, normalized size of antiderivative = 4.15

$$\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$
$$= \frac{bdx^6\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},\frac{1}{2},\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{4\left(-4ac(2ac+3bcx^3+3adx^3+2bdx^6)\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right.\right.}{8ac\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}}{8acx^2\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

```
[In] Integrate[1/(x^3*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (b*d*x^6*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1/2, 7/
3, -((b*x^3)/a), -((d*x^3)/c)] + (4*(-4*a*c*(2*a*c + 3*b*c*x^3 + 3*a*d*x^3
+ 2*b*d*x^6)*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + 3*x
^3*(a + b*x^3)*(c + d*x^3)*(a*d*AppellF1[4/3, 1/2, 3/2, 7/3, -((b*x^3)/a),
-((d*x^3)/c)] + b*c*AppellF1[4/3, 3/2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)
])))/(8*a*c*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - 3*x^
3*(a*d*AppellF1[4/3, 1/2, 3/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + b*c*Appel
lF1[4/3, 3/2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/(8*a*c*x^2*Sqrt[a +
b*x^3]*Sqrt[c + d*x^3])
```

Maple [F]

$$\int \frac{1}{x^3\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}dx$$

[In] int(1/x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)

[Out] int(1/x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)

Fricas [F]

$$\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}x^3}\,dx$$

[In] integrate(1/x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)/(b*d*x^9 + (b*c + a*d)*x^6 + a*c*x^3), x)

Sympy [F]

$$\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$$

[In] integrate(1/x**3/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)

[Out] Integral(1/(x**3*sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)

Maxima [F]

$$\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}x^3}\,dx$$

[In] integrate(1/x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^3), x)

Giac [F]

$$\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}x^3}\,dx$$

```
[In] integrate(1/x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \int \frac{1}{x^3\,\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

```
[In] int(1/(x^3*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
[Out] int(1/(x^3*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)), x)
```

3.516 $\int (ex)^{7/2}\sqrt{a+bx^3}(A+Bx^3)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 161

$$\int (ex)^{7/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{a(2Ab-aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{24b^2}+\frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be}-\frac{a^2(2Ab-aB)e^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24b^{5/2}}$$

```
[Out] 1/9*B*(e*x)^(9/2)*(b*x^3+a)^(3/2)/b/e-1/24*a^2*(2*A*b-B*a)*e^(7/2)*arctanh(
(e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))/b^(5/2)+1/24*a*(2*A*b-B*a)*e^2
*(e*x)^(3/2)*(b*x^3+a)^(1/2)/b^2+1/12*(2*A*b-B*a)*(e*x)^(9/2)*(b*x^3+a)^(1/
2)/b/e
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {470, 285, 327, 335, 281, 223, 212}

$$\int (ex)^{7/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = -\frac{a^2e^{7/2}(2Ab-aB)\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24b^{5/2}}+\frac{ae^2(ex)^{3/2}\sqrt{a+bx^3}(2Ab-aB)}{24b^2}+\frac{(ex)^{9/2}\sqrt{a+bx^3}(2Ab-aB)}{12be}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be}$$

```
[In] Int[(e*x)^(7/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (a*(2*A*b - a*B)*e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(24*b^2) + ((2*A*b - a*B)
*(e*x)^(9/2)*Sqrt[a + b*x^3])/(12*b*e) + (B*(e*x)^(9/2)*(a + b*x^3)^(3/2))/
(9*b*e) - (a^2*(2*A*b - a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)
*Sqrt[a + b*x^3])])/(24*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be} - \frac{\left(-9Ab+\frac{9aB}{2}\right)\int (ex)^{7/2}\sqrt{a+bx^3}\,dx}{9b} \\
&= \frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be} + \frac{(a(2Ab-aB))\int \frac{(ex)^{7/2}}{\sqrt{a+bx^3}}\,dx}{8b} \\
&= \frac{a(2Ab-aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{24b^2} + \frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be} \\
&\quad + \frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be} - \frac{\left(a^2(2Ab-aB)e^3\right)\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{16b^2} \\
&= \frac{a(2Ab-aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{24b^2} + \frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be} \\
&\quad + \frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be} - \frac{\left(a^2(2Ab-aB)e^2\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{8b^2} \\
&= \frac{a(2Ab-aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{24b^2} + \frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be} \\
&\quad + \frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be} - \frac{\left(a^2(2Ab-aB)e^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx,x,(ex)^{3/2}\right)}{24b^2} \\
&= \frac{a(2Ab-aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{24b^2} + \frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be} \\
&\quad + \frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be} - \frac{\left(a^2(2Ab-aB)e^2\right)\text{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx,x,\frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{24b^2} \\
&= \frac{a(2Ab-aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{24b^2} + \frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be} \\
&\quad + \frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be} - \frac{a^2(2Ab-aB)e^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24b^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.76

$$\int (ex)^{7/2}\sqrt{a+bx^3}(A+Bx^3)\,dx=\frac{(ex)^{7/2}\sqrt{a+bx^3}(6aAb-3a^2B+12Ab^2x^3+2abBx^3+8b^2Bx^6)}{72b^2x^2}+\frac{a^2(-2Ab+aB)(ex)^{7/2}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{24b^{5/2}x^{7/2}}$$

[In] Integrate[(e*x)^(7/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]

[Out] ((e*x)^(7/2)*Sqrt[a + b*x^3]*(6*a*A*b - 3*a^2*B + 12*A*b^2*x^3 + 2*a*b*B*x^3 + 8*b^2*B*x^6))/(72*b^2*x^2) + (a^2*(-2*A*b + a*B)*(e*x)^(7/2)*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]])/(24*b^(5/2)*x^(7/2))

Maple [A] (verified)

Time = 5.65 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.86

method	result
risch	$\frac{x^2(8b^2Bx^6+12Ab^2x^3+2Babx^3+6abA-3a^2B)\sqrt{bx^3+a}\,e^4}{72b^2\sqrt{ex}}-\frac{a^2(2Ab-Ba)\operatorname{arctanh}\left(\frac{\sqrt{(bx^3+a)ex}}{x^2\sqrt{be}}\right)e^4\sqrt{(bx^3+a)ex}}{24b^2\sqrt{be}\sqrt{ex}\sqrt{bx^3+a}}$
default	$-\frac{e^3\sqrt{ex}\sqrt{bx^3+a}\left(-8B\sqrt{(bx^3+a)ex}\sqrt{be}\,b^2x^7-12A\sqrt{(bx^3+a)ex}\sqrt{be}\,b^2x^4-2B\sqrt{(bx^3+a)ex}\sqrt{be}\,abx^4+6A\operatorname{arctanh}\left(\frac{\sqrt{(bx^3+a)e}}{x^2\sqrt{be}}\right.\right.}{72\sqrt{(bx^3+a)ex}\,b^2\sqrt{be}}$
elliptic	Expression too large to display

[In] int((e*x)^(7/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/72*x^2*(8*B*b^2*x^6+12*A*b^2*x^3+2*B*a*b*x^3+6*A*a*b-3*B*a^2)*(b*x^3+a)^(1/2)/b^2*e^4/(e*x)^(1/2)-1/24*a^2*(2*A*b-B*a)/b^2/(b*e)^(1/2)*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))*e^4*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.57 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.83

$$\int (ex)^{7/2}\sqrt{a+bx^3}(A+Bx^3)\,dx = \left[-\frac{3\,(Ba^3-2\,Aa^2b)e^3\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e+4\,(2\,b^2x^4+abx)\sqrt{bx^3+a}\sqrt{ex}\sqrt{\frac{e}{b}}\right)}{288\,b^2}\right.$$

$$-\frac{3\,(Ba^3-2\,Aa^2b)e^3\sqrt{-\frac{e}{b}}\arctan\left(\frac{2\,\sqrt{bx^3+a}\sqrt{ex}bx\sqrt{-\frac{e}{b}}}{2\,bex^3+ae}\right)-2\,(8\,Bb^2e^3x^7+2\,(Bab+6\,Ab^2)e^3x^4-3\,(Ba^2-2\,A}{144\,b^2}$$

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/288*(3*(B*a^3 - 2*A*a^2*b)*e^3*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3
 - a^2*e + 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(
8*B*b^2*e^3*x^7 + 2*(B*a*b + 6*A*b^2)*e^3*x^4 - 3*(B*a^2 - 2*A*a*b)*e^3*x)*
sqrt(b*x^3 + a)*sqrt(e*x))/b^2, -1/144*(3*(B*a^3 - 2*A*a^2*b)*e^3*sqrt(-e/b
)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) - 2*
(8*B*b^2*e^3*x^7 + 2*(B*a*b + 6*A*b^2)*e^3*x^4 - 3*(B*a^2 - 2*A*a*b)*e^3*x)
*sqrt(b*x^3 + a)*sqrt(e*x))/b^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(141) = 282.

Time = 18.80 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.85

$$\int (ex)^{7/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \begin{cases} \frac{2\left(\begin{cases} \text{NaN} \\ Ae^3\left(\begin{cases} -\frac{a^2e^3\left(\begin{cases} \frac{\log\left(\frac{2b(ex)^{\frac{3}{2}}}{e^3}+2\sqrt{\frac{b}{e^3}}\sqrt{a+bx^3}\right)}{\sqrt{\frac{b}{e^3}}} & \text{for } a\neq 0 \\ \frac{(ex)^{\frac{3}{2}}\log\left((ex)^{\frac{3}{2}}\right)}{\sqrt{bx^3}} & \text{otherwise} \end{cases}\right)}{8b}+\sqrt{a+bx^3}\left(\frac{ae^3(ex)^{\frac{3}{2}}}{8b}+\frac{(ex)^{\frac{9}{2}}}{4}\right) & \text{for } \ldots \\ \frac{\sqrt{a}(ex)^{\frac{9}{2}}}{3} & \text{othe}\ldots \end{cases}\right. \end{cases}\right.}{\ldots} \\ 0 \end{cases}$$

[In] integrate((e*x)**(7/2)*(B*x**3+A)*(b*x**3+a)**(1/2),x)

[Out] Piecewise((2*Piecewise((nan, Eq(e**3, 0)), ((A*e**3*Piecewise((-a**2*e**3*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), True))/(8*b) + sqrt(a + b*x**3)*(a*e**3*(e*x)**(3/2)/(8*b) + (e*x)**(9/2)/4), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(9/2)/3, True)) + B*Piecewise((a**3*e**6*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), True))/(16*b**2) + sqrt(a + b*x**3)*(-a**2*e**6*(e*x)**(3/2)/(16*b**2) + a*e**3*(e*x)**(9/2)/(24*b) + (e*x)**(15/2)/6), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(15/2)/5, True)))/(3*e**3), True))/e, Ne(e, 0)), (0, True))

Maxima [F]

$$\int (ex)^{7/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \int \left(Bx^3+A\right)\sqrt{bx^3+a}(ex)^{\frac{7}{2}}\,dx$$

[In] integrate((e*x)^(7/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^(7/2), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(126) = 252$.

Time = 0.43 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.03

$$\int (ex)^{7/2}\sqrt{a+bx^3}(A+Bx^3)\,dx = \frac{1}{12}\sqrt{be^4x^3+ae^4}\sqrt{ex}\left(\frac{2\,x^3}{e}+\frac{a}{be}\right)Ax|e|^2 + \frac{1}{72}\sqrt{be^4x^3+ae^4}\left(2\,e^3x^3\left(\frac{4\,x^3}{e^4}+\frac{a}{be^4}\right)-\frac{3\,a^2}{b^2e}\right)\sqrt{ex}Bx|e|^2 - \frac{(B^2a^6e-4\,ABa^5be+4\,A^2a^4b^2e)^2e^5\log\left(\left|-(\sqrt{ex}Ba^3e^2x-2\,\sqrt{ex}Aa^2be^2x)\sqrt{be}+\sqrt{B^2a^7e^6-4\,ABa^6be^6}\right.\right.}{24\,\sqrt{be}b^2|B^2a^6e-4\,ABa^5be+4\,A^2a^4b^2e||-Ba^3+2\,Aa^2}$$

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*(2*x^3/e + a/(b*e))*A*x*abs(e)^2 + 1
/72*sqrt(b*e^4*x^3 + a*e^4)*(2*e^3*x^3*(4*x^3/e^4 + a/(b*e^4)) - 3*a^2/(b^2
*e))*sqrt(e*x)*B*x*abs(e)^2 - 1/24*(B^2*a^6*e - 4*A*B*a^5*b*e + 4*A^2*a^4*b
^2*e)^2*e^5*log(abs(-(sqrt(e*x)*B*a^3*e^2*x - 2*sqrt(e*x)*A*a^2*b*e^2*x)*sq
rt(b*e) + sqrt(B^2*a^7*e^6 - 4*A*B*a^6*b*e^6 + 4*A^2*a^5*b^2*e^6 + (sqrt(e*
x)*B*a^3*e^2*x - 2*sqrt(e*x)*A*a^2*b*e^2*x)^2*b*e)))/(sqrt(b*e)*b^2*abs(B^2
*a^6*e - 4*A*B*a^5*b*e + 4*A^2*a^4*b^2*e)*abs(-B*a^3 + 2*A*a^2*b)*abs(e)^2)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{7/2}\sqrt{a+bx^3}(A+Bx^3)\,dx = \int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{7/2}\,\sqrt{b\,x^3+a}\,dx$$

```
[In] int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(1/2),x)
```

```
[Out] int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(1/2), x)
```

3.517 $\int (ex)^{5/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 324

$$\int (ex)^{5/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{3a(16Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{320b^2}$$

$$+\frac{(16Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{80be}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8be}$$

$$-\frac{3^{3/4}a^{5/3}(16Ab-7aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(\right.\right.}{640b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 1/8*B*(e*x)^(7/2)*(b*x^3+a)^(3/2)/b/e+1/80*(16*A*b-7*B*a)*(e*x)^(7/2)*(b*x^
3+a)^(1/2)/b/e+3/320*a*(16*A*b-7*B*a)*e^2*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^2-1
/640*3^(3/4)*a^(5/3)*(16*A*b-7*B*a)*e^2*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/
3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/
3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(
1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+
1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+
b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(
1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {470, 285, 327, 335, 231}

$$\int (ex)^{5/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx =$$

$$-\frac{3^{3/4}a^{5/3}e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(16Ab-7aB)\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}\right.}{640b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{3ae^2\sqrt{ex}\sqrt{a+bx^3}(16Ab-7aB)}{320b^2}+\frac{(ex)^{7/2}\sqrt{a+bx^3}(16Ab-7aB)}{80be}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8be}$$

```
[In] Int[(e*x)^(5/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (3*a*(16*A*b - 7*a*B)*e^2*Sqrt[e*x]*Sqrt[a + b*x^3])/(320*b^2) + ((16*A*b -
 7*a*B)*(e*x)^(7/2)*Sqrt[a + b*x^3])/(80*b*e) + (B*(e*x)^(7/2)*(a + b*x^3)^
(3/2))/(8*b*e) - (3^(3/4)*a^(5/3)*(16*A*b - 7*a*B)*e^2*Sqrt[e*x]*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1
+ Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*
x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(640*b^2*Sqrt[(b
^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt
[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{B(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8be}-\frac{\left(-8Ab+\frac{7aB}{2}\right)\int (ex)^{5/2}\sqrt{a+bx^3}\,dx}{8b}\\
&= \frac{(16Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{80be}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8be}+\frac{(3a(16Ab-7aB))\int \frac{(ex)^{5/2}}{\sqrt{a+bx^3}}\,dx}{160b}\\
&= \frac{3a(16Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{320b^2}+\frac{(16Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{80be}\\
&\quad+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8be}-\frac{\left(3a^2(16Ab-7aB)e^3\right)\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{640b^2}\\
&= \frac{3a(16Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{320b^2}+\frac{(16Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{80be}\\
&\quad+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8be}-\frac{\left(3a^2(16Ab-7aB)e^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{320b^2}
\end{aligned}$$

$$= \frac{3a(16Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{320b^2} + \frac{(16Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{80be} + \frac{B(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8be} - \frac{3^{3/4}a^{5/3}(16Ab-7aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\right.}{640b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.35

$$\int (ex)^{5/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{e^2\sqrt{ex}\sqrt{a+bx^3}\left(-\left(\left(a+bx^3\right)\sqrt{1+\frac{bx^3}{a}}\left(-16Ab+7aB-10bBx^3\right)\right)+a(-16Ab+7aB)\,\text{Hype}\right.}{80b^2\sqrt{1+\frac{bx^3}{a}}}$$

```
[In] Integrate[(e*x)^(5/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
[Out] (e^2*Sqrt[e*x]*Sqrt[a + b*x^3]*(-((a + b*x^3)*Sqrt[1 + (b*x^3)/a]*(-16*A*b
+ 7*a*B - 10*b*B*x^3)) + a*(-16*A*b + 7*a*B)*Hypergeometric2F1[-1/2, 1/6, 7
/6, -((b*x^3)/a)]))/(80*b^2*Sqrt[1 + (b*x^3)/a])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.04 (sec) , antiderivative size = 777, normalized size of antiderivative = 2.40

method	result
risch	$\frac{\left(40b^2Bx^6+64Ab^2x^3+12Babx^3+48abA-21a^2B\right)x\sqrt{bx^3+a}\,e^3}{320b^2\sqrt{ex}} - \frac{3a^2(16Ab-7Ba)\left(\frac{\left(-ab^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\right.}{}\right.}{\left(-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{2b}+\right.}}}{}$
elliptic	Expression too large to display
default	Expression too large to display

```
[In] int((e*x)^(5/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/320*(40*B*b^2*x^6+64*A*b^2*x^3+12*B*a*b*x^3+48*A*a*b-21*B*a^2)*x*(b*x^3+a
)^(1/2)/b^2*e^3/(e*x)^(1/2)-3/320*a^2*(16*A*b-7*B*a)/b*(1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-
1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2
)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*e*x
*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*Ellipti
cF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))*e^3*
((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [F]

$$\int (ex)^{5/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\sqrt{bx^3+a}(ex)^{\frac{5}{2}}\,dx$$

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*e^2*x^5 + A*e^2*x^2)*sqrt(b*x^3 + a)*sqrt(e*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 21.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.30

$$\int (ex)^{5/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{A\sqrt{a}e^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\\frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{6}\right)}+\frac{B\sqrt{a}e^{\frac{5}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\\frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{19}{6}\right)}$$

```
[In] integrate((e*x)**(5/2)*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*e**(5/2)*x**(7/2)*gamma(7/6)*hyper((-1/2, 7/6), (13/6,), b*x**3*e
xp_polar(I*pi)/a)/(3*gamma(13/6)) + B*sqrt(a)*e**(5/2)*x**(13/2)*gamma(13/6
)*hyper((-1/2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(19/6))
```

Maxima [F]

$$\int (ex)^{5/2}\sqrt{a+bx^3}(A+Bx^3)\,dx = \int (Bx^3+A)\sqrt{bx^3+a}(ex)^{\frac{5}{2}}\,dx$$

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^(5/2), x)
```

Giac [F]

$$\int (ex)^{5/2}\sqrt{a+bx^3}(A+Bx^3)\,dx = \int (Bx^3+A)\sqrt{bx^3+a}(ex)^{\frac{5}{2}}\,dx$$

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{5/2}\sqrt{a+bx^3}(A+Bx^3)\,dx = \int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{5/2}\,\sqrt{b\,x^3+a}\,dx$$

```
[In] int((A + B*x^3)*(e*x)^(5/2)*(a + b*x^3)^(1/2),x)
```

```
[Out] int((A + B*x^3)*(e*x)^(5/2)*(a + b*x^3)^(1/2), x)
```

3.518 $\int (ex)^{3/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 581

$$\int (ex)^{3/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{(14Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{56be}$$
$$+\frac{3\left(1+\sqrt{3}\right)a(14Ab-5aB)e\sqrt{ex}\sqrt{a+bx^3}}{112b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}+\frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be}$$
$$-\frac{3\sqrt[4]{3}a^{4/3}(14Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right.}{112b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{3^{3/4}\left(1-\sqrt{3}\right)a^{4/3}(14Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{}}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{}}\right.\right.}{224b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 1/7*B*(e*x)^(5/2)*(b*x^3+a)^(3/2)/b/e+1/56*(14*A*b-5*B*a)*(e*x)^(5/2)*(b*x^
3+a)^(1/2)/b/e+3/112*a*(14*A*b-5*B*a)*e*(1+3^(1/2))*(e*x)^(1/2)*(b*x^3+a)^(
1/2)/b^(5/3)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))-3/112*3^(1/4)*a^(4/3)*(14*A*b-
5*B*a)*e*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^
(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1
/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+
b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2
/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

```
/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*
(1+3^(1/2)))^2)^(1/2)-1/224*3^(3/4)*a^(4/3)*(14*A*b-5*B*a)*e*(a^(1/3)+b^(1/
3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)
^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*Elli
pticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^
2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)
*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b
*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2))
)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {470, 285, 335, 314, 231, 1895}

$$\int (ex)^{3/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx =$$

$$-\frac{3^{3/4}\left(1-\sqrt{3}\right)a^{4/3}e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(14Ab-5aB)\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+}\right.\right.}{224b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3\sqrt[4]{3}a^{4/3}e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(14Ab-5aB)E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right.\right.}{112b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{3\left(1+\sqrt{3}\right)ae\sqrt{ex}\sqrt{a+bx^3}(14Ab-5aB)}{112b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$

$$+\frac{(ex)^{5/2}\sqrt{a+bx^3}(14Ab-5aB)}{56be}+\frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be}$$

```
[In] Int[(e*x)^(3/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] ((14*A*b - 5*a*B)*(e*x)^(5/2)*Sqrt[a + b*x^3])/(56*b*e) + (3*(1 + Sqrt[3])*
a*(14*A*b - 5*a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^3])/(112*b^(5/3)*(a^(1/3) + (1
+ Sqrt[3])*b^(1/3)*x)) + (B*(e*x)^(5/2)*(a + b*x^3)^(3/2))/(7*b*e) - (3*3^(
1/4)*a^(4/3)*(14*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2
]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt
```

```
[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(112*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) +
 b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (3^(
3/4)*(1 - Sqrt[3])*a^(4/3)*(14*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/
3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(224*b^(5/3)*Sqrt[(b^(1/3
)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a +
b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
```

```
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\text{integral} = \frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be} - \frac{\left(-7Ab+\frac{5aB}{2}\right)\int (ex)^{3/2}\sqrt{a+bx^3}\,dx}{7b}$$

$$= \frac{(14Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{56be} + \frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be} + \frac{(3a(14Ab-5aB))\int \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\,dx}{112b}$$

$$= \frac{(14Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{56be} + \frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be} + \frac{(3a(14Ab-5aB))\text{Subst}\left(\int \frac{x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{56be}$$

$$= \frac{(14Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{56be} + \frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be} - \frac{(3a(14Ab-5aB))\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}e^2-2b^{2/3}x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{112b^{5/3}e} - \frac{\left(3\left(1-\sqrt{3}\right)a^{5/3}(14Ab-5aB)e\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{112b^{5/3}}$$

$$= \frac{(14Ab - 5aB)(ex)^{5/2}\sqrt{a + bx^3}}{56be} + \frac{3(1+\sqrt{3})\, a(14Ab - 5aB)e\sqrt{ex}\sqrt{a + bx^3}}{112b^{5/3}\left(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{b}x\right)} + \frac{B(ex)^{5/2}\left(a + bx^3\right)^{3/2}}{7be}$$

$$- \frac{3\sqrt[4]{3}a^{4/3}(14Ab - 5aB)e\sqrt{ex}\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}{\left(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{b}x\right)^2}}E\left(\cos^{-1}\left(\frac{\sqrt[3]{a} + (1-\sqrt{3})\sqrt[3]{b}x}{\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{b}x}\right)|\frac{1}{4}}{112b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{\left(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{a + bx^3}}$$

$$- \frac{3^{3/4}(1-\sqrt{3})\, a^{4/3}(14Ab - 5aB)e\sqrt{ex}\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}{\left(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a} + (1-\sqrt{3})}{\sqrt[3]{a} + (1+\sqrt{3})}\right.\right.}{224b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{\left(\sqrt[3]{a} + (1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{a + bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.16

$$\int (ex)^{3/2}\sqrt{a + bx^3}\left(A + Bx^3\right)\, dx = \frac{x(ex)^{3/2}\sqrt{a + bx^3}\left(5B(a + bx^3)\sqrt{1 + \frac{bx^3}{a}} + (14Ab - 5aB)\,\text{Hypergeometric2F1}\left(-\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, -\frac{bx^3}{a}\right.\right.}{35b\sqrt{1 + \frac{bx^3}{a}}}$$

```
[In] Integrate[(e*x)^(3/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (x*(e*x)^(3/2)*Sqrt[a + b*x^3]*(5*B*(a + b*x^3)*Sqrt[1 + (b*x^3)/a] + (14*A
*b - 5*a*B)*Hypergeometric2F1[-1/2, 5/6, 11/6, -((b*x^3)/a)]))/(35*b*Sqrt[1
 + (b*x^3)/a])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.59 (sec) , antiderivative size = 1140, normalized size of antiderivative = 1.96

method	result	size
risch	Expression too large to display	1140
elliptic	Expression too large to display	1184
default	Expression too large to display	5358

```
[In] int((e*x)^(3/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/56*x^3*(8*B*b*x^3+14*A*b+3*B*a)*(b*x^3+a)^(1/2)/b*e^2/(e*x)^(1/2)+3/112*a
*(14*A*b-5*B*a)/b*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/
(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*
(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*
b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2
)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)
+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*
(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(
1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))*b/(-a*b^2)^(1/3)))/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3)))^(1/2)*e^2*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2
)
```

Fricas [F]

$$\int (ex)^{3/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\sqrt{bx^3+a}(ex)^{\frac{3}{2}}\,dx$$

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] integral((B*e*x^4 + A*e*x)*sqrt(b*x^3 + a)*sqrt(e*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.92 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.17

$$\int (ex)^{3/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{A\sqrt{a}e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{6}\right)}+\frac{B\sqrt{a}e^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{17}{6}\right)}$$

```
[In] integrate((e*x)**(3/2)*(B*x**3+A)*(b*x**3+a)**(1/2),x)
[Out] A*sqrt(a)*e**(3/2)*x**(5/2)*gamma(5/6)*hyper((-1/2, 5/6), (11/6,), b*x**3*e
xp_polar(I*pi)/a)/(3*gamma(11/6)) + B*sqrt(a)*e**(3/2)*x**(11/2)*gamma(11/6
)*hyper((-1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(17/6))
```

Maxima [F]

$$\int (ex)^{3/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\sqrt{bx^3+a}(ex)^{\frac{3}{2}}\,dx$$

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^(3/2), x)
```

Giac [F]

$$\int (ex)^{3/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \int \left(Bx^3+A\right)\sqrt{bx^3+a}(ex)^{\frac{3}{2}}\,dx$$

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{3/2}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{3/2}\,\sqrt{b\,x^3+a}\,dx$$

```
[In] int((A + B*x^3)*(e*x)^(3/2)*(a + b*x^3)^(1/2),x)
[Out] int((A + B*x^3)*(e*x)^(3/2)*(a + b*x^3)^(1/2), x)
```

3.519 $\int \sqrt{ex}\sqrt{a+bx^3}(A+Bx^3)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 121

$$\int \sqrt{ex}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{(4Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{12be}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be}+\frac{a(4Ab-aB)\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{12b^{3/2}}$$

```
[Out] 1/6*B*(e*x)^(3/2)*(b*x^3+a)^(3/2)/b/e+1/12*a*(4*A*b-B*a)*arctanh((e*x)^(3/2
)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))*e^(1/2)/b^(3/2)+1/12*(4*A*b-B*a)*(e*x)^(
3/2)*(b*x^3+a)^(1/2)/b/e
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {470, 285, 335, 281, 223, 212}

$$\int \sqrt{ex}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\frac{a\sqrt{e}(4Ab-aB)\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{12b^{3/2}}+\frac{(ex)^{3/2}\sqrt{a+bx^3}(4Ab-aB)}{12be}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be}$$

```
[In] Int[Sqrt[e*x]*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] ((4*A*b - a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(12*b*e) + (B*(e*x)^(3/2)*(a +
b*x^3)^(3/2))/(6*b*e) + (a*(4*A*b - a*B)*Sqrt[e]*ArcTanh[(Sqrt[b]*(e*x)^(3/
2))/(e^(3/2)*Sqrt[a + b*x^3])])/(12*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be} - \frac{\left(-6Ab+\frac{3aB}{2}\right)\int\sqrt{ex}\sqrt{a+bx^3}\,dx}{6b}$$

$$= \frac{(4Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be} + \frac{(a(4Ab-aB))\int\frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{8b}$$

$$= \frac{(4Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be} + \frac{(a(4Ab-aB))\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{4be}$$

$$= \frac{(4Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be} + \frac{(a(4Ab-aB))\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx, x, (ex)^{3/2}\right)}{12be}$$

$$= \frac{(4Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be} + \frac{(a(4Ab-aB))\text{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx, x, \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{12be}$$

$$= \frac{(4Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be} + \frac{a(4Ab-aB)\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{12b^{3/2}}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79

$$\int \sqrt{ex}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \frac{x\sqrt{ex}\sqrt{a+bx^3}(4Ab+aB+2bBx^3)}{12b} - \frac{a(-4Ab+aB)\sqrt{ex}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{12b^{3/2}\sqrt{x}}$$

```
[In] Integrate[Sqrt[e*x]*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (x*Sqrt[e*x]*Sqrt[a + b*x^3]*(4*A*b + a*B + 2*b*B*x^3))/(12*b) - (a*(-4*A*b
 + a*B)*Sqrt[e*x]*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]])/(12*b^(3/2)*Sqrt[
x])
```

Maple [A] (verified)

Time = 4.77 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.90

method	result
risch	$\frac{x^{2}\left(2 b B \,x^{3}+4 A b +B a \right) \sqrt{b \,x^{3}+a}\, e}{12 b \sqrt{e x}}+\frac{a \left(4 A b -B a \right) \operatorname{arctanh}\left(\frac{\sqrt{\left(b \,x^{3}+a \right) e x}}{x^{2} \sqrt{b e}}\right) e \sqrt{\left(b \,x^{3}+a \right) e x}}{12 b \sqrt{b e}\, \sqrt{e x}\, \sqrt{b \,x^{3}+a}}$
default	$\frac{\sqrt{e x}\, \sqrt{b \,x^{3}+a}\, \left(2 B \sqrt{\left(b \,x^{3}+a \right) e x}\, \sqrt{b e}\, b \,x^{4}+4 A \,\operatorname{arctanh}\left(\frac{\sqrt{\left(b \,x^{3}+a \right) e x}}{x^{2} \sqrt{b e}}\right) a b e +4 A \sqrt{\left(b \,x^{3}+a \right) e x}\, \sqrt{b e}\, b x -B \,\operatorname{arctanh}\left(\frac{\sqrt{\left(b \,x^{3}+a \right) e x}}{x^{2} \sqrt{b e}}\right) a$
elliptic	Expression too large to display

```
[In] int((B*x^3+A)*(e*x)^(1/2)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*x^2*(2*B*b*x^3+4*A*b+B*a)*(b*x^3+a)^(1/2)/b*e/(e*x)^(1/2)+1/12*a*(4*A*
b-B*a)/b/(b*e)^(1/2)*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))*e*((b*x
^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.58 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.83

$$\int \sqrt{ex}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$

$$= \left[-\frac{(Ba^2-4\,Aab)\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\,(2\,b^2x^4+abx)\sqrt{bx^3+a}\sqrt{ex}\sqrt{\frac{e}{b}}\right)-4\,(2\,Bbx^4+(\right.$$

$$48\,b$$

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*((B*a^2 - 4*A*a*b)*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e
- 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(2*B*b*x^4
 + (B*a + 4*A*b)*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b, 1/24*((B*a^2 - 4*A*a*b)*s
qrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*
e)) + 2*(2*B*b*x^4 + (B*a + 4*A*b)*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(104) = 208$.

Time = 1.55 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.09

$$\int \sqrt{ex}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$

$$= \begin{cases} \frac{2\left(\begin{cases} \text{NaN} \\ \frac{Ae^3\left(\begin{cases} \frac{a\left(\begin{cases} \frac{\log\left(\frac{2b(ex)^{\frac{3}{2}}}{e^3}+2\sqrt{\frac{b}{e^3}}\sqrt{a+bx^3}\right)}{\sqrt{\frac{b}{e^3}}} & \text{for } a \neq 0 \\ \frac{(ex)^{\frac{3}{2}}\log\left((ex)^{\frac{3}{2}}\right)}{\sqrt{bx^3}} & \text{otherwise} \end{cases}\right)}{2} + \frac{(ex)^{\frac{3}{2}}\sqrt{a+bx^3}}{2} & \text{for } \frac{b}{e^3} \neq 0 \\ \sqrt{a}(ex)^{\frac{3}{2}} & \text{otherwise} \end{cases}\right) + B\left(\begin{cases} -\frac{a^2e^3\left(\begin{cases} \frac{\log\left(\frac{2b(ex)^{\frac{3}{2}}}{e^3} \cdots\right.}{\cdots} \\ \frac{(ex)^{\frac{3}{2}}\log\left((e\cdots\right.}{\sqrt{bx^3}} \end{cases}\right.}{\cdots} \\ \frac{\sqrt{a}(ex)^{\frac{9}{2}}}{3} \end{cases}\right.}{3e^3} \end{cases}\right.}{e} \cdots \\ 0 \end{cases}$$

```
[In] integrate((B*x**3+A)*(e*x)**(1/2)*(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((2*Piecewise((nan, Eq(e**3, 0)), ((A*e**3*Piecewise((a*Piecewise(
(log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3),
 Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), True))/2 + (e*x)*
*(3/2)*sqrt(a + b*x**3)/2, Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(3/2), True)) +
B*Piecewise((-a**2*e**3*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**
3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2
))/sqrt(b*x**3), True))/(8*b) + sqrt(a + b*x**3)*(a*e**3*(e*x)**(3/2)/(8*b)
 + (e*x)**(9/2)/4), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(9/2)/3, True)))/(3*e**
3), True))/e, Ne(e, 0)), (0, True))
```

Maxima [F]

$$\int \sqrt{ex}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx = \int \left(Bx^3+A\right)\sqrt{bx^3+a}\sqrt{ex}\,dx$$

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.46

$$\int \sqrt{ex}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx$$
$$=\frac{Ba^2e\log\left(\left|-\sqrt{be}\sqrt{ex}ex+\sqrt{be^4x^3+ae^4}\right|\right)}{12\sqrt{be}b}+\frac{\sqrt{be^4x^3+ae^4}\sqrt{ex}\left(\frac{2x^3}{e}+\frac{a}{be}\right)Bx|e|^2}{12e^3}$$
$$-\frac{\left(\frac{ae^4\log\left(\left|-\sqrt{be}\sqrt{ex}ex+\sqrt{be^4x^3+ae^4}\right|\right)}{\sqrt{be}}-\sqrt{be^4x^3+ae^4}\sqrt{ex}ex\right)A|e|^2}{3e^5}$$

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*B*a^2*e*log(abs(-sqrt(b*e)*sqrt(e*x)*e*x + sqrt(b*e^4*x^3 + a*e^4)))/(
sqrt(b*e)*b) + 1/12*sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*(2*x^3/e + a/(b*e))*B
*x*abs(e)^2/e^3 - 1/3*(a*e^4*log(abs(-sqrt(b*e)*sqrt(e*x)*e*x + sqrt(b*e^4*
x^3 + a*e^4)))/sqrt(b*e) - sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*e*x)*A*abs(e)^
2/e^5
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{ex}\sqrt{a+bx^3}\left(A+Bx^3\right)\,dx=\int \left(B\,x^3+A\right)\,\sqrt{e\,x}\,\sqrt{b\,x^3+a}\,dx$$

```
[In] int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(1/2),x)
```

```
[Out] int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(1/2), x)
```

3.520 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{\sqrt{ex}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 286

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{\sqrt{ex}}\,dx=\frac{(10Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{20be}+\frac{B\sqrt{ex}\left(a+bx^3\right)^{3/2}}{5be}$$
$$+\frac{3^{3/4}a^{2/3}(10Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(2\right.}{40be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 1/5*B*(b*x^3+a)^(3/2)*(e*x)^(1/2)/b/e+1/20*(10*A*b-B*a)*(e*x)^(1/2)*(b*x^3+
a)^(1/2)/b/e+1/40*3^(3/4)*a^(2/3)*(10*A*b-B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3
)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3
)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1
/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6
^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a
^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b/e/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1
/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {470, 285, 335, 231}

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{\sqrt{ex}}\,dx$$

$$=\frac{3^{3/4}a^{2/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(10Ab-aB)\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}(2\right.}{40be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{\sqrt{ex}\sqrt{a+bx^3}(10Ab-aB)}{20be}+\frac{B\sqrt{ex}(a+bx^3)^{3/2}}{5be}$$

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/Sqrt[e*x],x]
```

```
[Out] ((10*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(20*b*e) + (B*Sqrt[e*x]*(a + b*x
^3)^(3/2))/(5*b*e) + (3^(3/4)*a^(2/3)*(10*A*b - a*B)*Sqrt[e*x]*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 +
Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)
/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(40*b*e*Sqrt[(b^(1
/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a
+ b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{B\sqrt{ex}(a+bx^3)^{3/2}}{5be} - \frac{\left(-5Ab+\frac{aB}{2}\right)\int \frac{\sqrt{a+bx^3}}{\sqrt{ex}}\,dx}{5b}$$

$$= \frac{(10Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{20be} + \frac{B\sqrt{ex}(a+bx^3)^{3/2}}{5be} + \frac{(3a(10Ab-aB))\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{40b}$$

$$= \frac{(10Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{20be} + \frac{B\sqrt{ex}(a+bx^3)^{3/2}}{5be} + \frac{(3a(10Ab-aB))\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{20be}$$

$$= \frac{(10Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{20be} + \frac{B\sqrt{ex}(a+bx^3)^{3/2}}{5be} + \frac{3^{3/4}a^{2/3}(10Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}(2\cdots}{40be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{\sqrt{ex}}\,dx = \frac{x\sqrt{a+bx^3}\left(B(a+bx^3)\sqrt{1+\frac{bx^3}{a}}+(10Ab-aB)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{6},\frac{7}{6},-\frac{bx^3}{a}\right)\right)}{5b\sqrt{ex}\sqrt{1+\frac{bx^3}{a}}}$$

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/Sqrt[e*x],x]
[Out] (x*Sqrt[a + b*x^3]*(B*(a + b*x^3)*Sqrt[1 + (b*x^3)/a] + (10*A*b - a*B)*Hype
rgeometric2F1[-1/2, 1/6, 7/6, -((b*x^3)/a)]))/(5*b*Sqrt[e*x]*Sqrt[1 + (b*x^
3)/a])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.98 (sec) , antiderivative size = 744, normalized size of antiderivative = 2.60

method	result
risch	$\frac{(4bB\,x^3+10Ab+3Ba)x\sqrt{b\,x^3+a}}{20b\sqrt{ex}}+\frac{3a(10Ab-Ba)\left(\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}\right)}}\ \left(\cdots\right.}{\cdots}$
elliptic	$\frac{\sqrt{(b\,x^3+a)ex}\left(\frac{B\,x^3\sqrt{be\,x^4+aex}}{5e}+\frac{\left(Ab+\frac{3Ba}{10}\right)\sqrt{be\,x^4+aex}}{2be}+\frac{2\left(Aa-\frac{a\left(Ab+\frac{3Ba}{10}\right)}{4b}\right)\left(\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}\cdots\right.}{\left(-\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-\cdots\right.}}}{\cdots}\right.}{\cdots}$
default	Expression too large to display

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/20*(4*B*b*x^3+10*A*b+3*B*a)*x*(b*x^3+a)^(1/2)/b/(e*x)^(1/2)+3/20*a*(10*A*
b-B*a)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3)
)^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)
^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1
/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*
(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2))*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{\sqrt{ex}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{\sqrt{ex}}\,dx$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(1/2),x, algorithm="fricas")
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(e*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.64 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{\sqrt{ex}}\,dx = \frac{A\sqrt{a}\sqrt{x}\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\Gamma\left(\frac{7}{6}\right)} + \frac{B\sqrt{a}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\Gamma\left(\frac{13}{6}\right)}$$

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/(e*x)**(1/2),x)
```

```
[Out] A*sqrt(a)*sqrt(x)*gamma(1/6)*hyper((-1/2, 1/6), (7/6,), b*x**3*exp_polar(I*
pi)/a)/(3*sqrt(e)*gamma(7/6)) + B*sqrt(a)*x**(7/2)*gamma(7/6)*hyper((-1/2,
7/6), (13/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(13/6))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{\sqrt{ex}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{\sqrt{ex}}\,dx$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/sqrt(e*x), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{\sqrt{ex}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{\sqrt{ex}}\,dx$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/sqrt(e*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{\sqrt{ex}}\,dx = \int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{\sqrt{e\,x}}\,dx$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(1/2),x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(1/2), x)
```

3.521 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{(ex)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 580

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{(ex)^{3/2}}\,dx = \frac{(8Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{4ae^4} + \frac{3\left(1+\sqrt{3}\right)(8Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{8b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)} - \frac{2A\left(a+bx^3\right)^{3/2}}{ae\sqrt{ex}}$$
$$- \frac{3\sqrt[4]{3}\sqrt[3]{a}(8Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{8b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$- \frac{3^{3/4}\left(1-\sqrt{3}\right)\sqrt[3]{a}(8Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right.\right.}{16b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -2*A*(b*x^3+a)^(3/2)/a/e/(e*x)^(1/2)+1/4*(8*A*b+B*a)*(e*x)^(5/2)*(b*x^3+a)^
(1/2)/a/e^4+3/8*(8*A*b+B*a)*(1+3^(1/2))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^(2/3)
/e^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))-3/8*3^(1/4)*a^(1/3)*(8*A*b+B*a)*(a^(1/
3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1
/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2
)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^
(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(
```

```
1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/e^2/(b
*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2))
)^2)^(1/2)-1/16*3^(3/4)*a^(1/3)*(8*A*b+B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b
^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b
^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)
+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1
/2)+1/4*2^(1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3
)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/e^2/(b*x^3+a)^(1/2)
/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {464, 285, 335, 314, 231, 1895}

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{3/2}}\,dx =$$

$$-\frac{3^{3/4}\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+8Ab)\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right.\right.}{16b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3\sqrt[4]{3}\sqrt[3]{a}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+8Ab)E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{8b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{3\left(1+\sqrt{3}\right)\sqrt{ex}\sqrt{a+bx^3}(aB+8Ab)}{8b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}+\frac{(ex)^{5/2}\sqrt{a+bx^3}(aB+8Ab)}{4ae^4}-\frac{2A\left(a+bx^3\right)^{3/2}}{ae\sqrt{ex}}$$

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(3/2),x]
```

```
[Out] ((8*A*b + a*B)*(e*x)^(5/2)*Sqrt[a + b*x^3])/(4*a*e^4) + (3*(1 + Sqrt[3])*(8
*A*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(8*b^(2/3)*e^2*(a^(1/3) + (1 + Sqrt[
3])*b^(1/3)*x)) - (2*A*(a + b*x^3)^(3/2))/(a*e*Sqrt[e*x]) - (3*3^(1/4)*a^(1
/3)*(8*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[Ar
cCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)], (2 + Sqrt[3])/4])/(8*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))
/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (3^(3/4)*(1 - Sq
```

```
rt[3])*a^(1/3)*(8*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*E
llipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x)], (2 + Sqrt[3])/4])/(16*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) +
 b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\text{integral} = -\frac{2A(a+bx^3)^{3/2}}{ae\sqrt{ex}} + \frac{(8Ab+aB)\int (ex)^{3/2}\sqrt{a+bx^3}\,dx}{ae^3}$$

$$= \frac{(8Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{4ae^4} - \frac{2A(a+bx^3)^{3/2}}{ae\sqrt{ex}} + \frac{(3(8Ab+aB))\int \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\,dx}{8e^3}$$

$$= \frac{(8Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{4ae^4} - \frac{2A(a+bx^3)^{3/2}}{ae\sqrt{ex}} + \frac{(3(8Ab+aB))\text{Subst}\left(\int \frac{x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{4e^4}$$

$$= \frac{(8Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{4ae^4} - \frac{2A(a+bx^3)^{3/2}}{ae\sqrt{ex}} - \frac{(3(8Ab+aB))\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}e^2-2b^{2/3}x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{8b^{2/3}e^4} - \frac{\left(3\left(1-\sqrt{3}\right)a^{2/3}(8Ab+aB)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{8b^{2/3}e^2}$$

$$= \frac{(8Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{4ae^4} + \frac{3\left(1+\sqrt{3}\right)(8Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{8b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)} - \frac{2A(a+bx^3)^{3/2}}{ae\sqrt{ex}} - \frac{3\sqrt[4]{3}\sqrt[3]{a}(8Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}(2 \ldots}{8b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{3^{3/4}\left(1-\sqrt{3}\right)\sqrt[3]{a}(8Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{\ldots}}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{\ldots}}\right.\right.}{16b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{3/2}}\,dx = -\frac{2Ax(a+bx^3)^{3/2}}{a(ex)^{3/2}} - \frac{4\left(-4Ab-\frac{aB}{2}\right)x^4\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{5}{6},\frac{11}{6},-\frac{bx^3}{a}\right)}{5a(ex)^{3/2}\sqrt{1+\frac{bx^3}{a}}}$$

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(3/2),x]
```

```
[Out] (-2*A*x*(a + b*x^3)^(3/2))/(a*(e*x)^(3/2)) - (4*(-4*A*b - (a*B)/2)*x^4*Sqrt
[a + b*x^3]*Hypergeometric2F1[-1/2, 5/6, 11/6, -((b*x^3)/a)])/(5*a*(e*x)^(3
/2)*Sqrt[1 + (b*x^3)/a])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.83 (sec) , antiderivative size = 1123, normalized size of antiderivative = 1.94

method	result	size
risch	Expression too large to display	1123
elliptic	Expression too large to display	1161
default	Expression too large to display	5736

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(b*x^3+a)^(1/2)*(-B*x^3+8*A)/e/(e*x)^(1/2)+(3*A*b+3/8*B*a)*(x*(x+1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)
*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF
(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/
```

```
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))*b/(-a*b^2)^(1/3)))/(b*e*
x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)/e*((b*
x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{3/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{(ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(3/2),x, algorithm="fricas")
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(e^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.52 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{3/2}}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{6}\\ \frac{5}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{5}{6}\right)} + \frac{B\sqrt{a}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)}$$

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/(e*x)**(3/2),x)
[Out] A*sqrt(a)*gamma(-1/6)*hyper((-1/2, -1/6), (5/6,), b*x**3*exp_polar(I*pi)/a)
/(3*e**(3/2)*sqrt(x)*gamma(5/6)) + B*sqrt(a)*x**(5/2)*gamma(5/6)*hyper((-1/
2, 5/6), (11/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(11/6))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{3/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{(ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(3/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/(e*x)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{3/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{(ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/(e*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{3/2}}\,dx = \int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{(e\,x)^{3/2}}\,dx$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(3/2),x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(3/2), x)
```

3.522 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{5/2}} dx$

Optimal result

Integrand size = 26, antiderivative size = 118

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{5/2}} dx = \frac{(2Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{3ae^4} - \frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB)\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}e^{5/2}}$$

```
[Out] -2/3*A*(b*x^3+a)^(3/2)/a/e/(e*x)^(3/2)+1/3*(2*A*b+B*a)*arctanh((e*x)^(3/2)*
b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))/e^(5/2)/b^(1/2)+1/3*(2*A*b+B*a)*(e*x)^(3/2
)*(b*x^3+a)^(1/2)/a/e^4
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {464, 285, 335, 281, 223, 212}

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{5/2}} dx = \frac{(aB+2Ab)\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}e^{5/2}} + \frac{(ex)^{3/2}\sqrt{a+bx^3}(aB+2Ab)}{3ae^4} - \frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}}$$

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(5/2),x]
```

```
[Out] ((2*A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(3*a*e^4) - (2*A*(a + b*x^3)^(3
/2))/(3*a*e*(e*x)^(3/2)) + ((2*A*b + a*B)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^
(3/2)*Sqrt[a + b*x^3])])/(3*Sqrt[b]*e^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB)\int\sqrt{ex}\sqrt{a+bx^3}\,dx}{ae^3}$$

$$= \frac{(2Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{3ae^4} - \frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB)\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{2e^3}$$

$$= \frac{(2Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{3ae^4} - \frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{e^4}$$

$$= \frac{(2Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{3ae^4} - \frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx, x, (ex)^{3/2}\right)}{3e^4}$$

$$= \frac{(2Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{3ae^4} - \frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB)\text{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx, x, \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{3e^4}$$

$$= \frac{(2Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{3ae^4} - \frac{2A(a+bx^3)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}e^{5/2}}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{5/2}}\,dx = \frac{x\left(\sqrt{b}\sqrt{a+bx^3}(-2A+Bx^3)+(2Ab+aB)x^{3/2}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)\right)}{3\sqrt{b}(ex)^{5/2}}$$

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(5/2),x]
[Out] (x*(Sqrt[b]*Sqrt[a + b*x^3]*(-2*A + B*x^3) + (2*A*b + a*B)*x^(3/2)*Log[Sqrt
[b]*x^(3/2) + Sqrt[a + b*x^3]]))/(3*Sqrt[b]*(e*x)^(5/2))
```

Maple [A] (verified)

Time = 4.60 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.85

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,(-x^3B+2A)}{3x\,e^2\sqrt{ex}}+\frac{2\left(Ab+\frac{Ba}{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^3+a\right)ex}}{x^2\sqrt{be}}\right)\sqrt{(b\,x^3+a)ex}}{3\sqrt{be}\,e^2\sqrt{ex}\,\sqrt{b\,x^3+a}}$
default	$\frac{\sqrt{b\,x^3+a}\left(2A\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^3+a\right)ex}}{x^2\sqrt{be}}\right)be\,x^2+B\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^3+a\right)ex}}{x^2\sqrt{be}}\right)ae\,x^2+B\sqrt{(b\,x^3+a)ex}\,\sqrt{be}\,x^3-2A\sqrt{(b\,x^3+a)ex}\,\sqrt{be}\right)}{3x\,e^2\sqrt{ex}\,\sqrt{(b\,x^3+a)ex}\,\sqrt{be}}$
elliptic	Expression too large to display

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(b*x^3+a)^(1/2)*(-B*x^3+2*A)/x/e^2/(e*x)^(1/2)+2/3*(A*b+1/2*B*a)/(b*e)
^(1/2)*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))/e^2*((b*x^3+a)*e*x)^(
1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.57 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{5/2}}\,dx = \left[\frac{(Ba+2\,Ab)\sqrt{be}x^2\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\,(2\,bx^4+ax)\sqrt{bx^3+a}\sqrt{b\right.}{12\,be^3x^2}\right.$$

$$\left.-\frac{(Ba+2\,Ab)\sqrt{-be}x^2\arctan\left(\frac{2\sqrt{bx^3+a}\sqrt{-be}\sqrt{ex}x}{2\,bex^3+ae}\right)-2\,(Bbx^3-2\,Ab)\sqrt{bx^3+a}\sqrt{ex}}{6\,be^3x^2}\right]$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*((B*a + 2*A*b)*sqrt(b*e)*x^2*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e -
 4*(2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b*e)*sqrt(e*x)) + 4*(B*b*x^3 - 2*A*
b)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2), -1/6*((B*a + 2*A*b)*sqrt(-b*e)*x
^2*arctan(2*sqrt(b*x^3 + a)*sqrt(-b*e)*sqrt(e*x)*x/(2*b*e*x^3 + a*e)) - 2*(
B*b*x^3 - 2*A*b)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2)]
```

Sympy [A] (verification not implemented)

Time = 6.39 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{5/2}}\,dx = -\frac{2A\sqrt{a}}{3e^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{2A\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3e^{\frac{5}{2}}} - \frac{2Abx^{\frac{3}{2}}}{3\sqrt{a}e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{B\sqrt{a}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}}+\frac{Ba\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3\sqrt{b}e^{\frac{5}{2}}}$$

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/(e*x)**(5/2),x)
```

```
[Out] -2*A*sqrt(a)/(3*e**(5/2)*x**(3/2)*sqrt(1 + b*x**3/a)) + 2*A*sqrt(b)*asinh(s
qrt(b)*x**(3/2)/sqrt(a))/(3*e**(5/2)) - 2*A*b*x**(3/2)/(3*sqrt(a)*e**(5/2)*
sqrt(1 + b*x**3/a)) + B*sqrt(a)*x**(3/2)*sqrt(1 + b*x**3/a)/(3*e**(5/2)) +
B*a*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(3*sqrt(b)*e**(5/2))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{5/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{(ex)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(5/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/(e*x)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{5/2}}\,dx = \text{Exception raised: NotImplementedError}$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(5/2),x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> unable to parse Giac output: Recur
sive assumption sageVARa>=(-sageVARb*sageVARe/(sageVARe^4*t_nostep^6)) igno
red2/sageVARe^3*sageVARB/6/sageVARe^3*sqrt(sageVARe*sageVARx)*sqrt(sageVARe
*sageVARx)*sqrt(
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{5/2}}\,dx = \int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{(e\,x)^{5/2}}\,dx$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(5/2),x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(5/2), x)
```

3.523 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 283

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{7/2}}\,dx = \frac{(4Ab+5aB)\sqrt{ex}\sqrt{a+bx^3}}{10ae^4} - \frac{2A(a+bx^3)^{3/2}}{5ae(ex)^{5/2}} + \frac{3^{3/4}(4Ab+5aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{20\sqrt[3]{a}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -2/5*A*(b*x^3+a)^(3/2)/a/e/(e*x)^(5/2)+1/10*(4*A*b+5*B*a)*(e*x)^(1/2)*(b*x^
3+a)^(1/2)/a/e^4+1/20*3^(3/4)*(4*A*b+5*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b
^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b
^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)
+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1
/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1
/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/a^(1/3)/e^4/(b*x^3+a)^(1/2)/(b^(1/3)*x*
(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {464, 285, 335, 231}

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{7/2}}\,dx = \frac{3^{3/4}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(5aB+4Ab)\,\text{EllipticF}\left(\arccos\left(\right.\right.}{20\sqrt[3]{a}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{\sqrt{ex}\sqrt{a+bx^3}(5aB+4Ab)}{10ae^4} - \frac{2A\left(a+bx^3\right)^{3/2}}{5ae(ex)^{5/2}}$$

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(7/2),x]
```

```
[Out] ((4*A*b + 5*a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(10*a*e^4) - (2*A*(a + b*x^3)^(
3/2))/(5*a*e*(e*x)^(5/2)) + (3^(3/4)*(4*A*b + 5*a*B)*Sqrt[e*x]*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 +
Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)
/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(20*a^(1/3)*e^4*Sq
rt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]
*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{2A(a+bx^3)^{3/2}}{5ae(ex)^{5/2}} + \frac{(4Ab+5aB)\int \frac{\sqrt{a+bx^3}}{\sqrt{ex}}\,dx}{5ae^3}$$

$$= \frac{(4Ab+5aB)\sqrt{ex}\sqrt{a+bx^3}}{10ae^4} - \frac{2A(a+bx^3)^{3/2}}{5ae(ex)^{5/2}} + \frac{(3(4Ab+5aB))\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{20e^3}$$

$$= \frac{(4Ab+5aB)\sqrt{ex}\sqrt{a+bx^3}}{10ae^4} - \frac{2A(a+bx^3)^{3/2}}{5ae(ex)^{5/2}} + \frac{(3(4Ab+5aB))\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{10e^4}$$

$$= \frac{(4Ab+5aB)\sqrt{ex}\sqrt{a+bx^3}}{10ae^4} - \frac{2A(a+bx^3)^{3/2}}{5ae(ex)^{5/2}} + \frac{3^{3/4}(4Ab+5aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}(2+\sqrt{}\right.}{20\sqrt[3]{a}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{7/2}}\,dx = \frac{2x\sqrt{a+bx^3}\left(-A(a+bx^3)\sqrt{1+\frac{bx^3}{a}}+(4Ab+5aB)x^3\,\text{Hypergeometric2F1}\left(-\right.\right.}{5a(ex)^{7/2}\sqrt{1+\frac{bx^3}{a}}}$$

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(7/2),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*(-(A*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]) + (4*A*b + 5*a*B
)*x^3*Hypergeometric2F1[-1/2, 1/6, 7/6, -((b*x^3)/a)]))/(5*a*(e*x)^(7/2)*Sq
rt[1 + (b*x^3)/a])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.70 (sec) , antiderivative size = 743, normalized size of antiderivative = 2.63

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,(-5x^3B+4A)}{10x^2e^3\sqrt{ex}}+\frac{2\left(\frac{3Ab}{5}+\frac{3Ba}{4}\right)\left(\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{(\cdots}{}\right.}{(\cdots}$
elliptic	Expression too large to display
default	Expression too large to display

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*(b*x^3+a)^(1/2)*(-5*B*x^3+4*A)/x^2/e^3/(e*x)^(1/2)+2*(3/5*A*b+3/4*B*a
)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(
1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/
(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3
)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-
a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*b/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a
*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2))/e^3*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{7/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{(ex)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(7/2),x, algorithm="fricas")
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(e^4*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.58 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.35

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{7/2}}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{5}{6}\right)\,{}_2F_1\left(\begin{matrix}-\frac{5}{6},-\frac{1}{2}\\ \frac{1}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)} + \frac{B\sqrt{a}\sqrt{x}\Gamma\left(\frac{1}{6}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)}$$

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/(e*x)**(7/2),x)
[Out] A*sqrt(a)*gamma(-5/6)*hyper((-5/6, -1/2), (1/6,), b*x**3*exp_polar(I*pi)/a)
/(3*e**(7/2)*x**(5/2)*gamma(1/6)) + B*sqrt(a)*sqrt(x)*gamma(1/6)*hyper((-1/
2, 1/6), (7/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(7/2)*gamma(7/6))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{7/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{(ex)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(7/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/(e*x)^(7/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{7/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{(ex)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(7/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/(e*x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{(ex)^{7/2}}\,dx = \int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{(e\,x)^{7/2}}\,dx$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(7/2),x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(7/2), x)
```

3.524 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{9/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 564

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{9/2}}\,dx = -\frac{2(2Ab+7aB)\sqrt{a+bx^3}}{7a\sqrt{x}}$$

$$+\frac{3(1+\sqrt{3})\sqrt[3]{b}(2Ab+7aB)\sqrt{x}\sqrt{a+bx^3}}{7a\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)}-\frac{2A(a+bx^3)^{3/2}}{7ax^{7/2}}$$

$$-\frac{3\sqrt[4]{3}\sqrt[3]{b}(2Ab+7aB)\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+(1-\sqrt{3})\sqrt[3]{b}x}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{7a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3^{3/4}(1-\sqrt{3})\sqrt[3]{b}(2Ab+7aB)\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+(1-\sqrt{3})\sqrt[3]{b}x}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x}\right)\right.}{14a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -2/7*A*(b*x^3+a)^(3/2)/a/x^(7/2)-2/7*(2*A*b+7*B*a)*(b*x^3+a)^(1/2)/a/x^(1/2
)+3/7*b^(1/3)*(2*A*b+7*B*a)*(1+3^(1/2))*x^(1/2)*(b*x^3+a)^(1/2)/a/(a^(1/3)+
b^(1/3)*x*(1+3^(1/2)))-3/7*3^(1/4)*b^(1/3)*(2*A*b+7*B*a)*(a^(1/3)+b^(1/3)*x
)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/
2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*Elliptic
E((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(
1/2),1/4*6^(1/2)+1/4*2^(1/2))*x^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(b^(1/
```

```
3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)-1/14*3^(3
/4)*b^(1/3)*(2*A*b+7*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2
)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2
)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1
/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(
1-3^(1/2))*x^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3
)*x*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/
3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {464, 283, 335, 314, 231, 1895}

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{9/2}}\,dx =$$

$$-\frac{3^{3/4}\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(7aB+2Ab)\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)\right.}{14a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(7aB+2Ab)E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{7a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2\sqrt{a+bx^3}(7aB+2Ab)}{7a\sqrt{x}}+\frac{3\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt{x}\sqrt{a+bx^3}(7aB+2Ab)}{7a\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}-\frac{2A\left(a+bx^3\right)^{3/2}}{7ax^{7/2}}$$

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(9/2),x]

[Out] (-2*(2*A*b + 7*a*B)*Sqrt[a + b*x^3])/(7*a*Sqrt[x]) + (3*(1 + Sqrt[3])*b^(1/
3)*(2*A*b + 7*a*B)*Sqrt[x]*Sqrt[a + b*x^3])/(7*a*(a^(1/3) + (1 + Sqrt[3])*b
^(1/3)*x)) - (2*A*(a + b*x^3)^(3/2))/(7*a*x^(7/2)) - (3*3^(1/4)*b^(1/3)*(2*
A*b + 7*a*B)*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a
^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2
+ Sqrt[3])/4])/(7*a^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) +
 (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (3^(3/4)*(1 - Sqrt[3])*b^(1
/3)*(2*A*b + 7*a*B)*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
```

```
^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[Ar
cCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)], (2 + Sqrt[3])/4])/(14*a^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a
^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\text{integral} = -\frac{2A(a+bx^3)^{3/2}}{7ax^{7/2}} - \frac{\left(2\left(-Ab-\frac{7aB}{2}\right)\right)\int \frac{\sqrt{a+bx^3}}{x^{3/2}}\,dx}{7a}$$

$$= -\frac{2(2Ab+7aB)\sqrt{a+bx^3}}{7a\sqrt{x}} - \frac{2A(a+bx^3)^{3/2}}{7ax^{7/2}} + \frac{(3b(2Ab+7aB))\int \frac{x^{3/2}}{\sqrt{a+bx^3}}\,dx}{7a}$$

$$= -\frac{2(2Ab+7aB)\sqrt{a+bx^3}}{7a\sqrt{x}} - \frac{2A(a+bx^3)^{3/2}}{7ax^{7/2}} + \frac{(6b(2Ab+7aB))\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{7a}$$

$$= -\frac{2(2Ab+7aB)\sqrt{a+bx^3}}{7a\sqrt{x}} - \frac{2A(a+bx^3)^{3/2}}{7ax^{7/2}} - \frac{\left(3\sqrt[3]{b}(2Ab+7aB)\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{7a} - \frac{\left(3\left(1-\sqrt{3}\right)\sqrt[3]{b}(2Ab+7aB)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{7\sqrt[3]{a}}$$

$$= -\frac{2(2Ab+7aB)\sqrt{a+bx^3}}{7a\sqrt{x}} + \frac{3\left(1+\sqrt{3}\right)\sqrt[3]{b}(2Ab+7aB)\sqrt{x}\sqrt{a+bx^3}}{7a\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)} - \frac{2A(a+bx^3)^{3/2}}{7ax^{7/2}} - \frac{3\sqrt[4]{3}\sqrt[3]{b}(2Ab+7aB)\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}(2 \cdots}{7a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{3^{3/4}\left(1-\sqrt{3}\right)\sqrt[3]{b}(2Ab+7aB)\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}}\right.\right.}{14a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.14

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{9/2}}\,dx = \frac{2\sqrt{a+bx^3}\left(-A(a+bx^3)-\frac{(2Ab+7aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{6},\frac{5}{6},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{7ax^{7/2}}$$

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(9/2),x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(-(A*(a + b*x^3)) - ((2*A*b + 7*a*B)*x^3*Hypergeometric2
F1[-1/2, -1/6, 5/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(7*a*x^(7/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.98 (sec) , antiderivative size = 1127, normalized size of antiderivative = 2.00

method	result	size
risch	Expression too large to display	1127
elliptic	Expression too large to display	1177
default	Expression too large to display	5911

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/7*(b*x^3+a)^(1/2)*(3*A*b*x^3+7*B*a*x^3+A*a)/x^(7/2)/a+3/7*b*(2*A*b+7*B*a
)/a*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2
)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2
)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^
(1/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(
-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)
))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
)^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((
```

```
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))*b/(-a*b^2
)^(1/3)))/(b*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2)*(x*(b*x^3+a))^(1/2)/x^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{9/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^{\frac{9}{2}}}\,dx$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(9/2),x, algorithm="fricas")
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/x^(9/2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.85 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{9/2}}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{7}{6},-\frac{1}{2}\\-\frac{1}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{\frac{7}{2}}\Gamma\left(-\frac{1}{6}\right)} + \frac{B\sqrt{a}\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{6}\\\frac{5}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{x}\Gamma\left(\frac{5}{6}\right)}$$

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**(9/2),x)
[Out] A*sqrt(a)*gamma(-7/6)*hyper((-7/6, -1/2), (-1/6,), b*x**3*exp_polar(I*pi)/a
)/(3*x**(7/2)*gamma(-1/6)) + B*sqrt(a)*gamma(-1/6)*hyper((-1/2, -1/6), (5/6
,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(x)*gamma(5/6))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{9/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^{\frac{9}{2}}}\,dx$$

[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(9/2),x, algorithm="maxima")

[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^(9/2), x)

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{9/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^{\frac{9}{2}}}\,dx$$

[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(9/2),x, algorithm="giac")

[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^(9/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{9/2}}\,dx = \int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{x^{9/2}}\,dx$$

[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(9/2),x)

[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(9/2), x)

3.525 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 79

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11/2}}\,dx = -\frac{2B\sqrt{a+bx^3}}{3x^{3/2}} - \frac{2A(a+bx^3)^{3/2}}{9ax^{9/2}} + \frac{2}{3}\sqrt{b}B\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a+bx^3}}\right)$$

```
[Out] -2/9*A*(b*x^3+a)^(3/2)/a/x^(9/2)+2/3*B*arctanh(x^(3/2)*b^(1/2)/(b*x^3+a)^(1
/2))*b^(1/2)-2/3*B*(b*x^3+a)^(1/2)/x^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {462, 283, 335, 281, 223, 212}

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11/2}}\,dx = -\frac{2A(a+bx^3)^{3/2}}{9ax^{9/2}} + \frac{2}{3}\sqrt{b}B\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a+bx^3}}\right) - \frac{2B\sqrt{a+bx^3}}{3x^{3/2}}$$

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(11/2),x]
```

```
[Out] (-2*B*Sqrt[a + b*x^3])/(3*x^(3/2)) - (2*A*(a + b*x^3)^(3/2))/(9*a*x^(9/2))
+ (2*Sqrt[b]*B*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a + b*x^3]])/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2A(a+bx^3)^{3/2}}{9ax^{9/2}} + B\int \frac{\sqrt{a+bx^3}}{x^{5/2}}\,dx \\
&= -\frac{2B\sqrt{a+bx^3}}{3x^{3/2}} - \frac{2A(a+bx^3)^{3/2}}{9ax^{9/2}} + (bB)\int \frac{\sqrt{x}}{\sqrt{a+bx^3}}\,dx \\
&= -\frac{2B\sqrt{a+bx^3}}{3x^{3/2}} - \frac{2A(a+bx^3)^{3/2}}{9ax^{9/2}} + (2bB)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right) \\
&= -\frac{2B\sqrt{a+bx^3}}{3x^{3/2}} - \frac{2A(a+bx^3)^{3/2}}{9ax^{9/2}} + \frac{1}{3}(2bB)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, x^{3/2}\right)
\end{aligned}$$

$$= -\frac{2B\sqrt{a+bx^3}}{3x^{3/2}} - \frac{2A(a+bx^3)^{3/2}}{9ax^{9/2}} + \frac{1}{3}(2bB)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{3/2}}{\sqrt{a+bx^3}}\right)$$

$$= -\frac{2B\sqrt{a+bx^3}}{3x^{3/2}} - \frac{2A(a+bx^3)^{3/2}}{9ax^{9/2}} + \frac{2}{3}\sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a+bx^3}}\right)$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11/2}}\,dx = -\frac{2\sqrt{a+bx^3}(aA+Abx^3+3aBx^3)}{9ax^{9/2}} + \frac{2}{3}\sqrt{b}B\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)$$

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(11/2),x]
```

```
[Out] (-2*Sqrt[a + b*x^3]*(a*A + A*b*x^3 + 3*a*B*x^3))/(9*a*x^(9/2)) + (2*Sqrt[b]
*B*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]])/3
```

Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.06

method	result	size
risch	$-\frac{2\sqrt{b\,x^3+a}\,(A b\,x^3+3 B a\,x^3+A a)}{9 x^{\frac{9}{2}}\,a}+\frac{2 B\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{x\left(b\,x^3+a\right)}}{x^2\sqrt{b}}\right)\sqrt{x(b\,x^3+a)}}{3\sqrt{x}\,\sqrt{b\,x^3+a}}$	84
default	$-\frac{2\sqrt{b\,x^3+a}\left(-3B\,\operatorname{arctanh}\left(\frac{\sqrt{x\left(b\,x^3+a\right)}}{x^2\sqrt{b}}\right)\sqrt{b}\,a\,x^5+A\sqrt{x(b\,x^3+a)}\,b\,x^3+3B\sqrt{x(b\,x^3+a)}\,a\,x^3+A\sqrt{x(b\,x^3+a)}\,a\right)}{9x^{\frac{9}{2}}\sqrt{x(b\,x^3+a)}\,a}$	108
elliptic	Expression too large to display	1051

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/9*(b*x^3+a)^(1/2)*(A*b*x^3+3*B*a*x^3+A*a)/x^(9/2)/a+2/3*B*b^(1/2)*arctan
h(1/x^2*(x*(b*x^3+a))^(1/2)/b^(1/2))*(x*(b*x^3+a))^(1/2)/x^(1/2)/(b*x^3+a)^
(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.28

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11/2}}\,dx = \left[\frac{3\,Ba\sqrt{b}x^5\log\left(-8\,b^2x^6-8\,abx^3-4\,(2\,bx^4+ax)\sqrt{bx^3+a}\sqrt{b}\sqrt{x}-a^2\right)-4\,((3\right.$$

$$\left.-\frac{3\,Ba\sqrt{-b}x^5\arctan\left(\frac{2\,\sqrt{bx^3+a}\sqrt{-b}x^{\frac{3}{2}}}{2\,bx^3+a}\right)+2\,((3\,Ba+Ab)x^3+Aa)\sqrt{bx^3+a}\sqrt{x}}{9\,ax^5}\right]$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] [1/18*(3*B*a*sqrt(b)*x^5*log(-8*b^2*x^6 - 8*a*b*x^3 - 4*(2*b*x^4 + a*x)*sqr
t(b*x^3 + a)*sqrt(b)*sqrt(x) - a^2) - 4*((3*B*a + A*b)*x^3 + A*a)*sqrt(b*x^
3 + a)*sqrt(x))/(a*x^5), -1/9*(3*B*a*sqrt(-b)*x^5*arctan(2*sqrt(b*x^3 + a)*
sqrt(-b)*x^(3/2)/(2*b*x^3 + a)) + 2*((3*B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 +
a)*sqrt(x))/(a*x^5)]
```

Sympy [A] (verification not implemented)

Time = 26.87 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.66

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11/2}}\,dx = -\frac{2A\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{9x^3}-\frac{2Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}{9a} - \frac{2B\sqrt{a}}{3x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{2B\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3}-\frac{2Bbx^{\frac{3}{2}}}{3\sqrt{a}\sqrt{1+\frac{bx^3}{a}}}$$

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**(11/2),x)
```

```
[Out] -2*A*sqrt(b)*sqrt(a/(b*x**3) + 1)/(9*x**3) - 2*A*b**(3/2)*sqrt(a/(b*x**3) +
 1)/(9*a) - 2*B*sqrt(a)/(3*x**(3/2)*sqrt(1 + b*x**3/a)) + 2*B*sqrt(b)*asinh
(sqrt(b)*x**(3/2)/sqrt(a))/3 - 2*B*b*x**(3/2)/(3*sqrt(a)*sqrt(1 + b*x**3/a)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11/2}}\,dx = -\frac{1}{3}\left(\sqrt{b}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}\right)+\frac{2\sqrt{bx^3+a}}{x^{\frac{3}{2}}}\right)B-\frac{2\,(bx^3+a)^{\frac{3}{2}}A}{9\,ax^{\frac{9}{2}}}$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] -1/3*(sqrt(b)*log(-(sqrt(b) - sqrt(b*x^3 + a)/x^(3/2))/(sqrt(b) + sqrt(b*x^
3 + a)/x^(3/2))) + 2*sqrt(b*x^3 + a)/x^(3/2))*B - 2/9*(b*x^3 + a)^(3/2)*A/(
a*x^(9/2))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11/2}}\,dx = -\frac{2\,Bb\arctan\left(\frac{\sqrt{b+\frac{a}{x^3}}}{\sqrt{-b}}\right)}{3\sqrt{-b}} + \frac{2\left(3\,Bab\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+3\,Ba\sqrt{-b}\sqrt{b}+A\sqrt{-b}b^{\frac{3}{2}}\right)}{9\,a\sqrt{-b}} - \frac{2\left(3\,Ba^3\sqrt{b+\frac{a}{x^3}}+Aa^2\left(b+\frac{a}{x^3}\right)^{\frac{3}{2}}\right)}{9\,a^3}$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(11/2),x, algorithm="giac")
```

```
[Out] -2/3*B*b*arctan(sqrt(b + a/x^3)/sqrt(-b))/sqrt(-b) + 2/9*(3*B*a*b*arctan(sq
rt(b)/sqrt(-b)) + 3*B*a*sqrt(-b)*sqrt(b) + A*sqrt(-b)*b^(3/2))/(a*sqrt(-b))
 - 2/9*(3*B*a^3*sqrt(b + a/x^3) + A*a^2*(b + a/x^3)^(3/2))/a^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{11/2}}\,dx = \int \frac{\left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}}{x^{11/2}}\,dx$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(11/2),x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(11/2), x)
```

3.526 $\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{13/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 269

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{13/2}}\,dx = \frac{2(2Ab-11aB)\sqrt{a+bx^3}}{55ax^{5/2}} - \frac{2A(a+bx^3)^{3/2}}{11ax^{11/2}} - \frac{3^{3/4}b(2Ab-11aB)\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\mathrm{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{55a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -2/11*A*(b*x^3+a)^(3/2)/a/x^(11/2)+2/55*(2*A*b-11*B*a)*(b*x^3+a)^(1/2)/a/x^
(5/2)-1/55*3^(3/4)*b*(2*A*b-11*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x
*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x
*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)
*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*
2^(1/2))*x^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*
x*(1+3^(1/2)))^2)^(1/2)/a^(4/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)
*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {464, 283, 335, 231}

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{13/2}}\,dx = -\frac{3^{3/4}b\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(2Ab-11aB)\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}(2+\sqrt{3})\right)}{55a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{2\sqrt{a+bx^3}(2Ab-11aB)}{55ax^{5/2}} - \frac{2A(a+bx^3)^{3/2}}{11ax^{11/2}}$$

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(13/2),x]
```

```
[Out] (2*(2*A*b - 11*a*B)*Sqrt[a + b*x^3])/(55*a*x^(5/2)) - (2*A*(a + b*x^3)^(3/2
))/(11*a*x^(11/2)) - (3^(3/4)*b*(2*A*b - 11*a*B)*Sqrt[x]*(a^(1/3) + b^(1/3)
*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3
])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1
/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(55*a^(4/3)*Sqrt[(b^(1/3
)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a +
b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{2A(a+bx^3)^{3/2}}{11ax^{11/2}} - \frac{\left(2\left(Ab-\frac{11aB}{2}\right)\right)\int\frac{\sqrt{a+bx^3}}{x^{7/2}}\,dx}{11a}$$

$$= \frac{2(2Ab-11aB)\sqrt{a+bx^3}}{55ax^{5/2}} - \frac{2A(a+bx^3)^{3/2}}{11ax^{11/2}} - \frac{(3b(2Ab-11aB))\int\frac{1}{\sqrt{x}\sqrt{a+bx^3}}\,dx}{55a}$$

$$= \frac{2(2Ab-11aB)\sqrt{a+bx^3}}{55ax^{5/2}} - \frac{2A(a+bx^3)^{3/2}}{11ax^{11/2}} - \frac{(6b(2Ab-11aB))\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{55a}$$

$$= \frac{2(2Ab-11aB)\sqrt{a+bx^3}}{55ax^{5/2}} - \frac{2A(a+bx^3)^{3/2}}{11ax^{11/2}} - \frac{3^{3/4}b(2Ab-11aB)\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{55a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.30

$$\int\frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{13/2}}\,dx = \frac{2\sqrt{a+bx^3}\left(-5A(a+bx^3)+\frac{(2Ab-11aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{2},\frac{1}{6},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{55ax^{11/2}}$$

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(13/2),x]
[Out] (2*Sqrt[a + b*x^3]*(-5*A*(a + b*x^3) + ((2*A*b - 11*a*B)*x^3*Hypergeometric
2F1[-5/6, -1/2, 1/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(55*a*x^(11/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.52 (sec) , antiderivative size = 745, normalized size of antiderivative = 2.77

method	result
risch	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(3Ab\,x^{3}+11Ba\,x^{3}+5Aa\right)}{55x^{\frac{11}{2}}a}-\frac{6b^{2}(2Ab-11Ba)\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b\right.}{b}\right.}}}{}$
elliptic	$\frac{\sqrt{x\left(b\,x^{3}+a\right)}\left(-\frac{2A\sqrt{b\,x^{4}+ax}}{11x^{6}}-\frac{2(3Ab+11Ba)\sqrt{b\,x^{4}+ax}}{55a\,x^{3}}+\frac{2\left(Bb-\frac{2b(3Ab+11Ba)}{55a}\right)\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\right.}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\right.}{2b}\right.}}}{}\right.}{}$
default	Expression too large to display

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/55*(b*x^3+a)^(1/2)*(3*A*b*x^3+11*B*a*x^3+5*A*a)/x^(11/2)/a-6/55*b^2*(2*A
*b-11*B*a)/a*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)
^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-
a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))/(-a*b^2)^(1/3)/(b*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-
1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2))*(x*(b*x^3+a))^(1/2)/x^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.28

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{13/2}}\,dx = -\frac{2\left(3\,(11\,Bab-2\,Ab^2)\sqrt{a}x^6\text{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)+((11\,Ba^2+3\,Aab)x^3+5\,Aa^2)\sqrt{bx^3+a}\sqrt{x}\right)}{55\,a^2x^6}$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(13/2),x, algorithm="fricas")
```

```
[Out] -2/55*(3*(11*B*a*b - 2*A*b^2)*sqrt(a)*x^6*weierstrassPInverse(0, -4*b/a, 1/
x) + ((11*B*a^2 + 3*A*a*b)*x^3 + 5*A*a^2)*sqrt(b*x^3 + a)*sqrt(x))/(a^2*x^6
)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 67.78 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{13/2}}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{11}{6}\right){}_2F_1\left(\begin{matrix}-\frac{11}{6},-\frac{1}{2}\\ -\frac{5}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{\frac{11}{2}}\Gamma\left(-\frac{5}{6}\right)} + \frac{B\sqrt{a}\Gamma\left(-\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{5}{6},-\frac{1}{2}\\ \frac{1}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)}$$

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**(13/2),x)
```

```
[Out] A*sqrt(a)*gamma(-11/6)*hyper((-11/6, -1/2), (-5/6,), b*x**3*exp_polar(I*pi)
/a)/(3*x**(11/2)*gamma(-5/6)) + B*sqrt(a)*gamma(-5/6)*hyper((-5/6, -1/2), (
1/6,), b*x**3*exp_polar(I*pi)/a)/(3*x**(5/2)*gamma(1/6))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{13/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^{\frac{13}{2}}}\,dx$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(13/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^(13/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{13/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{bx^3+a}}{x^{\frac{13}{2}}}\,dx$$

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(13/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^(13/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}(A+Bx^3)}{x^{13/2}}\,dx = \int \frac{(B\,x^3+A)\,\sqrt{b\,x^3+a}}{x^{13/2}}\,dx$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(13/2),x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(13/2), x)
```

3.527 $\int (ex)^{7/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx$

Optimal result

Integrand size = 26, antiderivative size = 201

$$\int (ex)^{7/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \frac{a^2(8Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{192b^2} + \frac{a(8Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{96be} + \frac{(8Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{72be} + \frac{B(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12be} - \frac{a^3(8Ab-3aB)e^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{192b^{5/2}}$$

```
[Out] 1/72*(8*A*b-3*B*a)*(e*x)^(9/2)*(b*x^3+a)^(3/2)/b/e+1/12*B*(e*x)^(9/2)*(b*x^
3+a)^(5/2)/b/e-1/192*a^3*(8*A*b-3*B*a)*e^(7/2)*arctanh((e*x)^(3/2)*b^(1/2)/
e^(3/2)/(b*x^3+a)^(1/2))/b^(5/2)+1/192*a^2*(8*A*b-3*B*a)*e^2*(e*x)^(3/2)*(b
*x^3+a)^(1/2)/b^2+1/96*a*(8*A*b-3*B*a)*(e*x)^(9/2)*(b*x^3+a)^(1/2)/b/e
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {470, 285, 327, 335, 281, 223, 212}

$$\int (ex)^{7/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = -\frac{a^3e^{7/2}(8Ab-3aB)\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{192b^{5/2}} + \frac{a^2e^2(ex)^{3/2}\sqrt{a+bx^3}(8Ab-3aB)}{192b^2} + \frac{(ex)^{9/2}\left(a+bx^3\right)^{3/2}(8Ab-3aB)}{72be} + \frac{a(ex)^{9/2}\sqrt{a+bx^3}(8Ab-3aB)}{96be} + \frac{B(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12be}$$

```
[In] Int[(e*x)^(7/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (a^2*(8*A*b - 3*a*B)*e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(192*b^2) + (a*(8*A*b
 - 3*a*B)*(e*x)^(9/2)*Sqrt[a + b*x^3])/(96*b*e) + ((8*A*b - 3*a*B)*(e*x)^(9
/2)*(a + b*x^3)^(3/2))/(72*b*e) + (B*(e*x)^(9/2)*(a + b*x^3)^(5/2))/(12*b*e
) - (a^3*(8*A*b - 3*a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqr
t[a + b*x^3])])/(192*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{B(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12be} - \frac{\left(-12Ab+\frac{9aB}{2}\right)\int (ex)^{7/2}\left(a+bx^3\right)^{3/2}\,dx}{12b} \\
&= \frac{(8Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{72be} + \frac{B(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12be} \\
&\quad + \frac{(a(8Ab-3aB))\int (ex)^{7/2}\sqrt{a+bx^3}\,dx}{16b} \\
&= \frac{a(8Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{96be} + \frac{(8Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{72be} \\
&\quad + \frac{B(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12be} + \frac{\left(a^2(8Ab-3aB)\right)\int \frac{(ex)^{7/2}}{\sqrt{a+bx^3}}\,dx}{64b} \\
&= \frac{a^2(8Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{192b^2} + \frac{a(8Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{96be} \\
&\quad + \frac{(8Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{72be} \\
&\quad + \frac{B(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12be} - \frac{\left(a^3(8Ab-3aB)e^3\right)\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{128b^2} \\
&= \frac{a^2(8Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{192b^2} + \frac{a(8Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{96be} \\
&\quad + \frac{(8Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{72be} + \frac{B(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12be} \\
&\quad - \frac{\left(a^3(8Ab-3aB)e^2\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{64b^2} \\
&= \frac{a^2(8Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{192b^2} + \frac{a(8Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{96be} \\
&\quad + \frac{(8Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{72be} + \frac{B(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12be} \\
&\quad - \frac{\left(a^3(8Ab-3aB)e^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx, x, (ex)^{3/2}\right)}{192b^2}
\end{aligned}$$

$$= \frac{a^2(8Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{192b^2} + \frac{a(8Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{96be} + \frac{(8Ab-3aB)(ex)^{9/2}(a+bx^3)^{3/2}}{72be} + \frac{B(ex)^{9/2}(a+bx^3)^{5/2}}{12be} - \frac{(a^3(8Ab-3aB)e^2)\,\text{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx, x, \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{192b^2}$$

$$= \frac{a^2(8Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{192b^2} + \frac{a(8Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{96be} + \frac{(8Ab-3aB)(ex)^{9/2}(a+bx^3)^{3/2}}{72be} + \frac{B(ex)^{9/2}(a+bx^3)^{5/2}}{12be} - \frac{a^3(8Ab-3aB)e^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{192b^{5/2}}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.72

$$\int (ex)^{7/2} (a+bx^3)^{3/2} (A+Bx^3)\, dx = \frac{e^3\sqrt{ex}\left(\sqrt{b}x^{3/2}\sqrt{a+bx^3}(-9a^3B+6a^2b(4A+Bx^3)+16b^3x^6(4A+3Bx^3)+8ab^2x\right.}{576b^{5/2}\sqrt{x}}$$

```
[In] Integrate[(e*x)^(7/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (e^3*Sqrt[e*x]*(Sqrt[b]*x^(3/2)*Sqrt[a + b*x^3]*(-9*a^3*B + 6*a^2*b*(4*A +
B*x^3) + 16*b^3*x^6*(4*A + 3*B*x^3) + 8*a*b^2*x^3*(14*A + 9*B*x^3)) + 3*a^3
*(-8*A*b + 3*a*B)*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]]))/(576*b^(5/2)*Sqr
t[x])
```

Maple [A] (verified)

Time = 4.62 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.81

method	result
risch	$\frac{x^2(48b^3Bx^9+64x^6b^3A+72Bx^6ab^2+112aAb^2x^3+6Ba^2bx^3+24a^2bA-9a^3B)\sqrt{bx^3+a}\,e^4}{576b^2\sqrt{ex}} - \frac{a^3(8Ab-3Ba)\,\text{arctanh}\left(\frac{\sqrt{(bx^3+a)e}}{x^2\sqrt{be}}\right.}{192b^2\sqrt{be}\sqrt{ex}\sqrt{bx^3}}$
default	$-\frac{e^3\sqrt{ex}\sqrt{bx^3+a}\left(-48B\sqrt{(bx^3+a)ex}\sqrt{be}\,b^3x^{10}-64A\sqrt{(bx^3+a)ex}\sqrt{be}\,b^3x^7-72B\sqrt{(bx^3+a)ex}\sqrt{be}\,ab^2x^7-112A\sqrt{(bx^3+a)ex}\right.}{}$
elliptic	Expression too large to display

```
[In] int((e*x)^(7/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/576/b^2*x^2*(48*B*b^3*x^9+64*A*b^3*x^6+72*B*a*b^2*x^6+112*A*a*b^2*x^3+6*B
*a^2*b*x^3+24*A*a^2*b-9*B*a^3)*(b*x^3+a)^(1/2)*e^4/(e*x)^(1/2)-1/192*a^3/b^
2*(8*A*b-3*B*a)/(b*e)^(1/2)*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))*
e^4*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.57 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.77

$$\int (ex)^{7/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \left[-\frac{3 \left(3 Ba^4 - 8 Aa^3 b\right)e^3 \sqrt{\frac{e}{b}} \log\left(-8 b^2 ex^6 - 8 abex^3 - a^2 e + 4 \left(2 b^2 x^4 + abx\right)\sqrt{bx^3 +$$

```
[In] integrate((e*x)^(7/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] [-1/2304*(3*(3*B*a^4 - 8*A*a^3*b)*e^3*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*
x^3 - a^2*e + 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) -
4*(48*B*b^3*e^3*x^10 + 8*(9*B*a*b^2 + 8*A*b^3)*e^3*x^7 + 2*(3*B*a^2*b + 56*
A*a*b^2)*e^3*x^4 - 3*(3*B*a^3 - 8*A*a^2*b)*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x)
)/b^2, -1/1152*(3*(3*B*a^4 - 8*A*a^3*b)*e^3*sqrt(-e/b)*arctan(2*sqrt(b*x^3
+ a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) - 2*(48*B*b^3*e^3*x^10 + 8
*(9*B*a*b^2 + 8*A*b^3)*e^3*x^7 + 2*(3*B*a^2*b + 56*A*a*b^2)*e^3*x^4 - 3*(3*
B*a^3 - 8*A*a^2*b)*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 634 vs. 2(180) = 360.

Time = 31.07 (sec) , antiderivative size = 634, normalized size of antiderivative = 3.15

$$\int (ex)^{7/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \text{Too large to display}$$

```
[In] integrate((e*x)**(7/2)*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] Piecewise((2*Piecewise((nan, Eq(e**3, 0)), ((A*a*e**3*Piecewise((-a**2*e**3
*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/sq
rt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), True))
/(8*b) + sqrt(a + b*x**3)*(a*e**3*(e*x)**(3/2)/(8*b) + (e*x)**(9/2)/4), Ne(
```

```
b/e**3, 0)), (sqrt(a)*(e*x)**(9/2)/3, True)) + A*b*Piecewise((a**3*e**6*Pie
cewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b
/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), True))/(16
*b**2) + sqrt(a + b*x**3)*(-a**2*e**6*(e*x)**(3/2)/(16*b**2) + a*e**3*(e*x)
**(9/2)/(24*b) + (e*x)**(15/2)/6), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(15/2)/5
, True)) + B*a*Piecewise((a**3*e**6*Piecewise((log(2*b*(e*x)**(3/2)/e**3 +
2*sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log
((e*x)**(3/2))/sqrt(b*x**3), True))/(16*b**2) + sqrt(a + b*x**3)*(-a**2*e**
6*(e*x)**(3/2)/(16*b**2) + a*e**3*(e*x)**(9/2)/(24*b) + (e*x)**(15/2)/6), N
e(b/e**3, 0)), (sqrt(a)*(e*x)**(15/2)/5, True)) + B*b*Piecewise((-5*a**4*e*
*9*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/
sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), True
))/(128*b**3) + sqrt(a + b*x**3)*(5*a**3*e**9*(e*x)**(3/2)/(128*b**3) - 5*a
**2*e**6*(e*x)**(9/2)/(192*b**2) + a*e**3*(e*x)**(15/2)/(48*b) + (e*x)**(21
/2)/8), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(21/2)/7, True))/e**3)/(3*e**3), Tr
ue))/e, Ne(e, 0)), (0, True))
```

Maxima [F]

$$\int (ex)^{7/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{7}{2}}\,dx$$

```
[In] integrate((e*x)^(7/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^(7/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(163) = 326$.

Time = 0.51 (sec) , antiderivative size = 488, normalized size of antiderivative = 2.43

$$\int (ex)^{7/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{1}{12}\sqrt{be^4x^3+ae^4}\sqrt{ex}\left(\frac{2\,x^3}{e}+\frac{a}{be}\right)Aax|e|^2$$
$$+\frac{1}{72}\sqrt{be^4x^3+ae^4}\left(2\,e^3x^3\left(\frac{4\,x^3}{e^4}+\frac{a}{be^4}\right)-\frac{3\,a^2}{b^2e}\right)\sqrt{ex}Bax|e|^2$$
$$+\frac{1}{72}\sqrt{be^4x^3+ae^4}\left(2\,e^3x^3\left(\frac{4\,x^3}{e^4}+\frac{a}{be^4}\right)-\frac{3\,a^2}{b^2e}\right)\sqrt{ex}Abx|e|^2$$
$$+\frac{1}{576}\sqrt{be^4x^3+ae^4}\left(2\left(4\,e^3x^3\left(\frac{6\,x^3}{e^7}+\frac{a}{be^7}\right)-\frac{5\,a^2}{b^2e^4}\right)e^3x^3+\frac{15\,a^3}{b^3e}\right)\sqrt{ex}Bbx|e|^2$$
$$-\frac{\left(9\,B^2a^8e-48\,ABa^7be+64\,A^2a^6b^2e\right)^2e^5\log\left(\left|-\left(3\,\sqrt{ex}Ba^4e^2x-8\,\sqrt{ex}Aa^3be^2x\right)\sqrt{be}+\sqrt{9\,B^2a^9e^6-48\right.\right.}{192\,\sqrt{be}b^2|9\,B^2a^8e-48\,ABa^7be+64\,A^2a^6b^2e||-3\,Ba^4}$$

```
[In] integrate((e*x)^(7/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*(2*x^3/e + a/(b*e))*A*a*x*abs(e)^2 +
 1/72*sqrt(b*e^4*x^3 + a*e^4)*(2*e^3*x^3*(4*x^3/e^4 + a/(b*e^4)) - 3*a^2/(b
^2*e))*sqrt(e*x)*B*a*x*abs(e)^2 + 1/72*sqrt(b*e^4*x^3 + a*e^4)*(2*e^3*x^3*(
4*x^3/e^4 + a/(b*e^4)) - 3*a^2/(b^2*e))*sqrt(e*x)*A*b*x*abs(e)^2 + 1/576*sq
rt(b*e^4*x^3 + a*e^4)*(2*(4*e^3*x^3*(6*x^3/e^7 + a/(b*e^7)) - 5*a^2/(b^2*e^
4))*e^3*x^3 + 15*a^3/(b^3*e))*sqrt(e*x)*B*b*x*abs(e)^2 - 1/192*(9*B^2*a^8*e
 - 48*A*B*a^7*b*e + 64*A^2*a^6*b^2*e)^2*e^5*log(abs(-(3*sqrt(e*x)*B*a^4*e^2
*x - 8*sqrt(e*x)*A*a^3*b*e^2*x)*sqrt(b*e) + sqrt(9*B^2*a^9*e^6 - 48*A*B*a^8
*b*e^6 + 64*A^2*a^7*b^2*e^6 + (3*sqrt(e*x)*B*a^4*e^2*x - 8*sqrt(e*x)*A*a^3*
b*e^2*x)^2*b*e)))/(sqrt(b*e)*b^2*abs(9*B^2*a^8*e - 48*A*B*a^7*b*e + 64*A^2*
a^6*b^2*e)*abs(-3*B*a^4 + 8*A*a^3*b)*abs(e)^2)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{7/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{7/2}\,{\left(b\,x^3+a\right)}^{3/2} \,d x$$

```
[In] int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(3/2),x)
```

```
[Out] int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(3/2), x)
```

3.528 $\int (ex)^{5/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx$

Optimal result

Integrand size = 26, antiderivative size = 364

$$\int (ex)^{5/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \frac{27a^2(22Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{7040b^2} + \frac{9a(22Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{1760be} + \frac{(22Ab-7aB)(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{176be} + \frac{B(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11be} - \frac{9\,3^{3/4}a^{8/3}(22Ab-7aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{14080b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 1/176*(22*A*b-7*B*a)*(e*x)^(7/2)*(b*x^3+a)^(3/2)/b/e+1/11*B*(e*x)^(7/2)*(b*
x^3+a)^(5/2)/b/e+9/1760*a*(22*A*b-7*B*a)*(e*x)^(7/2)*(b*x^3+a)^(1/2)/b/e+27
/7040*a^2*(22*A*b-7*B*a)*e^2*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^2-9/14080*3^(3/4
)*a^(8/3)*(22*A*b-7*B*a)*e^2*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(
1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(
1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3
^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2)
)*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(
1+3^(1/2)))^2)^(1/2)/b^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^
(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {470, 285, 327, 335, 231}

$$\int (ex)^{5/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx =$$

$$-\frac{9\,3^{3/4}a^{8/3}e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(22Ab-7aB)\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\right.}{14080b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{27a^2e^2\sqrt{ex}\sqrt{a+bx^3}(22Ab-7aB)}{7040b^2}+\frac{(ex)^{7/2}\left(a+bx^3\right)^{3/2}(22Ab-7aB)}{176be}$$

$$+\frac{9a(ex)^{7/2}\sqrt{a+bx^3}(22Ab-7aB)}{1760be}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11be}$$

```
[In] Int[(e*x)^(5/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (27*a^2*(22*A*b - 7*a*B)*e^2*Sqrt[e*x]*Sqrt[a + b*x^3])/(7040*b^2) + (9*a*(
22*A*b - 7*a*B)*(e*x)^(7/2)*Sqrt[a + b*x^3])/(1760*b*e) + ((22*A*b - 7*a*B)
*(e*x)^(7/2)*(a + b*x^3)^(3/2))/(176*b*e) + (B*(e*x)^(7/2)*(a + b*x^3)^(5/2
))/(11*b*e) - (9*3^(3/4)*a^(8/3)*(22*A*b - 7*a*B)*e^2*Sqrt[e*x]*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 +
 Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x
)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(14080*b^2*Sqrt[(
b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqr
t[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{B(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11be}-\frac{\left(-11Ab+\frac{7aB}{2}\right)\int (ex)^{5/2}\left(a+bx^3\right)^{3/2}\,dx}{11b}\\
&= \frac{(22Ab-7aB)(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{176be}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11be}\\
&\quad+\frac{(9a(22Ab-7aB))\int (ex)^{5/2}\sqrt{a+bx^3}\,dx}{352b}\\
&= \frac{9a(22Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{1760be}+\frac{(22Ab-7aB)(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{176be}\\
&\quad+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11be}+\frac{\left(27a^2(22Ab-7aB)\right)\int \frac{(ex)^{5/2}}{\sqrt{a+bx^3}}\,dx}{3520b}\\
&= \frac{27a^2(22Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{7040b^2}+\frac{9a(22Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{1760be}\\
&\quad+\frac{(22Ab-7aB)(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{176be}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11be}\\
&\quad-\frac{\left(27a^3(22Ab-7aB)e^3\right)\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{14080b^2}
\end{aligned}$$

$$= \frac{27a^2(22Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{7040b^2} + \frac{9a(22Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{1760be} + \frac{(22Ab-7aB)(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{176be} + \frac{B(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11be} - \frac{(27a^3(22Ab-7aB)e^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{7040b^2}$$

$$= \frac{27a^2(22Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{7040b^2} + \frac{9a(22Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{1760be} + \frac{(22Ab-7aB)(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{176be} + \frac{B(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11be} - \frac{9\,3^{3/4}a^{8/3}(22Ab-7aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)\right.}{14080b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.19 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.32

$$\int (ex)^{5/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx = \frac{e^2\sqrt{ex}\sqrt{a+bx^3}\left(-\left(a+bx^3\right)^2\sqrt{1+\frac{bx^3}{a}}(-22Ab+7aB-16bBx^3)+a^2(-22Ab+7\right.}{176b^2\sqrt{1+\frac{bx^3}{a}}}$$

```
[In] Integrate[(e*x)^(5/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (e^2*Sqrt[e*x]*Sqrt[a + b*x^3]*(-((a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*(-22*A*
b + 7*a*B - 16*b*B*x^3)) + a^2*(-22*A*b + 7*a*B)*Hypergeometric2F1[-3/2, 1/
6, 7/6, -((b*x^3)/a)]))/(176*b^2*Sqrt[1 + (b*x^3)/a])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.53 (sec) , antiderivative size = 801, normalized size of antiderivative = 2.20

method	result	size
risch	Expression too large to display	801
elliptic	Expression too large to display	976
default	Expression too large to display	4619

```
[In] int((e*x)^(5/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7040/b^2*(640*B*b^3*x^9+880*A*b^3*x^6+1000*B*a*b^2*x^6+1672*A*a*b^2*x^3+1
08*B*a^2*b*x^3+594*A*a^2*b-189*B*a^3)*x*(b*x^3+a)^(1/2)*e^3/(e*x)^(1/2)-27/
7040*a^3/b*(22*A*b-7*B*a)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x
-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))*e^3*((b*x^3+a)*e*x)^(1/2)/(e*x)^(
1/2)/(b*x^3+a)^(1/2)
```

Fricas [F]

$$\int (ex)^{5/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right) \left(bx^3+a\right)^{\frac{3}{2}} (ex)^{\frac{5}{2}} \, dx$$

```
[In] integrate((e*x)^(5/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] integral((B*b*e^2*x^8 + (B*a + A*b)*e^2*x^5 + A*a*e^2*x^2)*sqrt(b*x^3 + a)*
sqrt(e*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 63.41 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.55

$$\int (ex)^{5/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \frac{Aa^{\frac{3}{2}}e^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{7}{6} \\ \frac{13}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{6}\right)} + \frac{A\sqrt{a}be^{\frac{5}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{13}{6} \\ \frac{19}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{19}{6}\right)} + \frac{Ba^{\frac{3}{2}}e^{\frac{5}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{13}{6} \\ \frac{19}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{19}{6}\right)} + \frac{B\sqrt{a}be^{\frac{5}{2}}x^{\frac{19}{2}}\Gamma\left(\frac{19}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{19}{6} \\ \frac{25}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{25}{6}\right)}$$

```
[In] integrate((e*x)**(5/2)*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] A*a**(3/2)*e**(5/2)*x**(7/2)*gamma(7/6)*hyper((-1/2, 7/6), (13/6,), b*x**3*
exp_polar(I*pi)/a)/(3*gamma(13/6)) + A*sqrt(a)*b*e**(5/2)*x**(13/2)*gamma(1
3/6)*hyper((-1/2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(19/6))
 + B*a**(3/2)*e**(5/2)*x**(13/2)*gamma(13/6)*hyper((-1/2, 13/6), (19/6,), b
*x**3*exp_polar(I*pi)/a)/(3*gamma(19/6)) + B*sqrt(a)*b*e**(5/2)*x**(19/2)*g
amma(19/6)*hyper((-1/2, 19/6), (25/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(
25/6))
```

Maxima [F]

$$\int (ex)^{5/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{5}{2}} \, dx$$

```
[In] integrate((e*x)^(5/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^(5/2), x)
```

Giac [F]

$$\int (ex)^{5/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{5}{2}} \, dx$$

```
[In] integrate((e*x)^(5/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{5/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{5/2}\,{\left(b\,x^3+a\right)}^{3/2} \,d x$$

```
[In] int((A + B*x^3)*(e*x)^(5/2)*(a + b*x^3)^(3/2),x)
[Out] int((A + B*x^3)*(e*x)^(5/2)*(a + b*x^3)^(3/2), x)
```

3.529 $\int (ex)^{3/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 621

$$\int (ex)^{3/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{9a(4Ab-aB)(ex)^{5/2}\sqrt{a+bx^3}}{224be}$$
$$+\frac{27\left(1+\sqrt{3}\right)a^2(4Ab-aB)e\sqrt{ex}\sqrt{a+bx^3}}{448b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$
$$+\frac{(4Ab-aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28be}+\frac{B(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10be}$$
$$-\frac{27\sqrt[4]{3}a^{7/3}(4Ab-aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{448b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{9\,3^{3/4}\left(1-\sqrt{3}\right)a^{7/3}(4Ab-aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{}}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{}}\right.\right.}{896b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 1/28*(4*A*b-B*a)*(e*x)^(5/2)*(b*x^3+a)^(3/2)/b/e+1/10*B*(e*x)^(5/2)*(b*x^3+
a)^(5/2)/b/e+9/224*a*(4*A*b-B*a)*(e*x)^(5/2)*(b*x^3+a)^(1/2)/b/e+27/448*a^2
*(4*A*b-B*a)*e*(1+3^(1/2))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^(5/3)/(a^(1/3)+b^(
1/3)*x*(1+3^(1/2)))-27/448*3^(1/4)*a^(7/3)*(4*A*b-B*a)*e*(a^(1/3)+b^(1/3)*x
)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/
2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*Elliptic
```

```
E((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(
1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/
3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(b
^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)-9/896
*3^(3/4)*a^(7/3)*(4*A*b-B*a)*e*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3
^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3
^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1
-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/
2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1
/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 621, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {470, 285, 335, 314, 231, 1895}

$$\int (ex)^{3/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx =$$

$$-\frac{9\,3^{3/4}\left(1-\sqrt{3}\right)a^{7/3}e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(4Ab-aB)\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+}\right.\right.}{896b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{27\sqrt[4]{3}a^{7/3}e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(4Ab-aB)E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right.}{448b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{27\left(1+\sqrt{3}\right)a^2e\sqrt{ex}\sqrt{a+bx^3}(4Ab-aB)}{448b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}+\frac{(ex)^{5/2}\left(a+bx^3\right)^{3/2}(4Ab-aB)}{28be}$$

$$+\frac{9a(ex)^{5/2}\sqrt{a+bx^3}(4Ab-aB)}{224be}+\frac{B(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10be}$$

```
[In] Int[(e*x)^(3/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (9*a*(4*A*b - a*B)*(e*x)^(5/2)*Sqrt[a + b*x^3])/(224*b*e) + (27*(1 + Sqrt[3
])*a^2*(4*A*b - a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^3])/(448*b^(5/3)*(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x)) + ((4*A*b - a*B)*(e*x)^(5/2)*(a + b*x^3)^(3/2))/(2
8*b*e) + (B*(e*x)^(5/2)*(a + b*x^3)^(5/2))/(10*b*e) - (27*3^(1/4)*a^(7/3)*(
```

```
4*A*b - a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCo
s[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)],
 (2 + Sqrt[3])/4])/(448*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(
1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (9*3^(3/4)*(1 - Sqrt[
3])*a^(7/3)*(4*A*b - a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*El
lipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])
*b^(1/3)*x)], (2 + Sqrt[3])/4])/(896*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(
1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
```

```
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{B(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10be} - \frac{\left(-10Ab+\frac{5aB}{2}\right)\int (ex)^{3/2}\left(a+bx^3\right)^{3/2}\,dx}{10b} \\
&= \frac{(4Ab-aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28be} + \frac{B(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10be} \\
&\quad + \frac{(9a(4Ab-aB))\int (ex)^{3/2}\sqrt{a+bx^3}\,dx}{56b} \\
&= \frac{9a(4Ab-aB)(ex)^{5/2}\sqrt{a+bx^3}}{224be} + \frac{(4Ab-aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28be} \\
&\quad + \frac{B(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10be} + \frac{\left(27a^2(4Ab-aB)\right)\int \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\,dx}{448b} \\
&= \frac{9a(4Ab-aB)(ex)^{5/2}\sqrt{a+bx^3}}{224be} + \frac{(4Ab-aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28be} \\
&\quad + \frac{B(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10be} + \frac{\left(27a^2(4Ab-aB)\right)\operatorname{Subst}\left(\int \frac{x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{224be} \\
&= \frac{9a(4Ab-aB)(ex)^{5/2}\sqrt{a+bx^3}}{224be} \\
&\quad + \frac{(4Ab-aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28be} + \frac{B(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10be} \\
&\quad - \frac{\left(27a^2(4Ab-aB)\right)\operatorname{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}e^2-2b^{2/3}x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{448b^{5/3}e} \\
&\quad - \frac{\left(27\left(1-\sqrt{3}\right)a^{8/3}(4Ab-aB)e\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{448b^{5/3}}
\end{aligned}
$$

$$= \frac{9a(4Ab-aB)(ex)^{5/2}\sqrt{a+bx^3}}{224be} + \frac{27\left(1+\sqrt{3}\right)a^2(4Ab-aB)e\sqrt{ex}\sqrt{a+bx^3}}{448b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$

$$+ \frac{(4Ab-aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28be} + \frac{B(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10be}$$

$$- \frac{27\sqrt[4]{3}a^{7/3}(4Ab-aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}(2}{448b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$- \frac{9\ 3^{3/4}\left(1-\sqrt{3}\right)a^{7/3}(4Ab-aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)}\right.\right.}{896b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.15

$$\int (ex)^{3/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx = \frac{x(ex)^{3/2}\sqrt{a+bx^3}\left(B\left(a+bx^3\right)^2\sqrt{1+\frac{bx^3}{a}}+a(4Ab-aB)\,\text{Hypergeometric2F1}\left(-\frac{3}{2}\right.\right.}{10b\sqrt{1+\frac{bx^3}{a}}}$$

```
[In] Integrate[(e*x)^(3/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (x*(e*x)^(3/2)*Sqrt[a + b*x^3]*(B*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a] + a*(4*
A*b - a*B)*Hypergeometric2F1[-3/2, 5/6, 11/6, -((b*x^3)/a)]))/(10*b*Sqrt[1
+ (b*x^3)/a])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.99 (sec) , antiderivative size = 1164, normalized size of antiderivative = 1.87

method	result	size
risch	Expression too large to display	1164
elliptic	Expression too large to display	1279
default	Expression too large to display	5790

```
[In] int((e*x)^(3/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1120/b*x^3*(112*B*b^2*x^6+160*A*b^2*x^3+184*B*a*b*x^3+340*A*a*b+27*B*a^2)
*(b*x^3+a)^(1/2)*e^2/(e*x)^(1/2)+27/448*a^2/b*(4*A*b-B*a)*(x*(x+1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/
b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))*b/(-a*b^2)^(1/3)))/(b*e*x*(x-1
/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*e^2*((b*x^3+
a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [F]

$$\int (ex)^{3/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}} \, dx$$

```
[In] integrate((e*x)^(3/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
[Out] integral((B*b*e*x^7 + (B*a + A*b)*e*x^4 + A*a*e*x)*sqrt(b*x^3 + a)*sqrt(e*x
), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 20.76 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.32

$$\int (ex)^{3/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx = \frac{Aa^{\frac{3}{2}}e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{6} \\ \frac{11}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{6}\right)} + \frac{A\sqrt{a}be^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{11}{6} \\ \frac{17}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{17}{6}\right)} + \frac{Ba^{\frac{3}{2}}e^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{11}{6} \\ \frac{17}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{17}{6}\right)} + \frac{B\sqrt{a}be^{\frac{3}{2}}x^{\frac{17}{2}}\Gamma\left(\frac{17}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{17}{6} \\ \frac{23}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{23}{6}\right)}$$

```
[In] integrate((e*x)**(3/2)*(b*x**3+a)**(3/2)*(B*x**3+A),x)
[Out] A*a**(3/2)*e**(3/2)*x**(5/2)*gamma(5/6)*hyper((-1/2, 5/6), (11/6,), b*x**3*
exp_polar(I*pi)/a)/(3*gamma(11/6)) + A*sqrt(a)*b*e**(3/2)*x**(11/2)*gamma(1
1/6)*hyper((-1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(17/6))
 + B*a**(3/2)*e**(3/2)*x**(11/2)*gamma(11/6)*hyper((-1/2, 11/6), (17/6,), b
*x**3*exp_polar(I*pi)/a)/(3*gamma(17/6)) + B*sqrt(a)*b*e**(3/2)*x**(17/2)*g
amma(17/6)*hyper((-1/2, 17/6), (23/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(
23/6))
```

Maxima [F]

$$\int (ex)^{3/2} \left(a + bx^3\right)^{3/2} \left(A + Bx^3\right) \, dx = \int \left(Bx^3 + A\right) \left(bx^3 + a\right)^{\frac{3}{2}} (ex)^{\frac{3}{2}} \, dx$$

```
[In] integrate((e*x)^(3/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^(3/2), x)
```

Giac [F]

$$\int (ex)^{3/2} \left(a + bx^3\right)^{3/2} \left(A + Bx^3\right) \, dx = \int \left(Bx^3 + A\right) \left(bx^3 + a\right)^{\frac{3}{2}} (ex)^{\frac{3}{2}} \, dx$$

```
[In] integrate((e*x)^(3/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{3/2} \left(a + bx^3\right)^{3/2} \left(A + Bx^3\right) \, dx = \int \left(B\,x^3 + A\right)\,{\left(e\,x\right)}^{3/2}\,{\left(b\,x^3 + a\right)}^{3/2} \,d x$$

```
[In] int((A + B*x^3)*(e*x)^(3/2)*(a + b*x^3)^(3/2),x)
[Out] int((A + B*x^3)*(e*x)^(3/2)*(a + b*x^3)^(3/2), x)
```

3.530 $\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 161

$$\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{a(6Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{24be}+\frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be}+\frac{a^2(6Ab-aB)\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24b^{3/2}}$$

```
[Out] 1/36*(6*A*b-B*a)*(e*x)^(3/2)*(b*x^3+a)^(3/2)/b/e+1/9*B*(e*x)^(3/2)*(b*x^3+a
)^(5/2)/b/e+1/24*a^2*(6*A*b-B*a)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3
+a)^(1/2))*e^(1/2)/b^(3/2)+1/24*a*(6*A*b-B*a)*(e*x)^(3/2)*(b*x^3+a)^(1/2)/b
/e
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {470, 285, 335, 281, 223, 212}

$$\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{a^2\sqrt{e}(6Ab-aB)\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24b^{3/2}}+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}(6Ab-aB)}{36be}+\frac{a(ex)^{3/2}\sqrt{a+bx^3}(6Ab-aB)}{24be}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be}$$

```
[In] Int[Sqrt[e*x]*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (a*(6*A*b - a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(24*b*e) + ((6*A*b - a*B)*(e*
x)^(3/2)*(a + b*x^3)^(3/2))/(36*b*e) + (B*(e*x)^(3/2)*(a + b*x^3)^(5/2))/(9
*b*e) + (a^2*(6*A*b - a*B)*Sqrt[e]*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*S
qrt[a + b*x^3])])/(24*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be} - \frac{\left(-9Ab+\frac{3aB}{2}\right)\int\sqrt{ex}\left(a+bx^3\right)^{3/2}\,dx}{9b}$$

$$= \frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be} + \frac{(a(6Ab-aB))\int\sqrt{ex}\sqrt{a+bx^3}\,dx}{8b}$$

$$= \frac{a(6Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{24be} + \frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be} + \frac{\left(a^2(6Ab-aB)\right)\int\frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{16b}$$

$$= \frac{a(6Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{24be} + \frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be} + \frac{\left(a^2(6Ab-aB)\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{8be}$$

$$= \frac{a(6Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{24be} + \frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be} + \frac{\left(a^2(6Ab-aB)\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx,x,(ex)^{3/2}\right)}{24be}$$

$$= \frac{a(6Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{24be} + \frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be} + \frac{\left(a^2(6Ab-aB)\right)\text{Subst}\left(\int\frac{1}{1-\frac{bx^2}{e^3}}\,dx,x,\frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{24be}$$

$$= \frac{a(6Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{24be} + \frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be} + \frac{a^2(6Ab-aB)\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24b^{3/2}}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.75

$$\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\frac{x\sqrt{ex}\sqrt{a+bx^3}(30aAb+3a^2B+12Ab^2x^3+14abBx^3+8b^2Bx^6)}{72b}-\frac{a^2(-6Ab+aB)\sqrt{ex}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{24b^{3/2}\sqrt{x}}$$

```
[In] Integrate[Sqrt[e*x]*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (x*Sqrt[e*x]*Sqrt[a + b*x^3]*(30*a*A*b + 3*a^2*B + 12*A*b^2*x^3 + 14*a*b*B*
x^3 + 8*b^2*B*x^6))/(72*b) - (a^2*(-6*A*b + a*B)*Sqrt[e*x]*Log[Sqrt[b]*x^(3
/2) + Sqrt[a + b*x^3]])/(24*b^(3/2)*Sqrt[x])
```

Maple [A] (verified)

Time = 4.63 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.83

method	result
risch	$\frac{x^{2}\left(8b^{2}B\,x^{6}+12A\,b^{2}x^{3}+14Bab\,x^{3}+30abA+3a^{2}B\right)\sqrt{b\,x^{3}+a}\,e}{72b\sqrt{ex}}+\frac{a^{2}(6Ab-Ba)\,\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right)ex}}{x^{2}\sqrt{be}}\right)e\sqrt{(b\,x^{3}+a)ex}}{24b\sqrt{be}\,\sqrt{ex}\,\sqrt{b\,x^{3}+a}}$
default	$\frac{\sqrt{b\,x^{3}+a}\,\sqrt{ex}\,\left(8B\sqrt{(b\,x^{3}+a)ex}\,\sqrt{be}\,b^{2}x^{7}+12A\sqrt{(b\,x^{3}+a)ex}\,\sqrt{be}\,b^{2}x^{4}+14B\sqrt{(b\,x^{3}+a)ex}\,\sqrt{be}\,ab\,x^{4}+30A\sqrt{(b\,x^{3}+a)ex}\,\sqrt{be}\,abx+1\right.}{72\sqrt{(b\,x^{3}+a)ex}\,b\sqrt{be}}$
elliptic	Expression too large to display

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)*(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/72/b*x^2*(8*B*b^2*x^6+12*A*b^2*x^3+14*B*a*b*x^3+30*A*a*b+3*B*a^2)*(b*x^3+
a)^(1/2)*e/(e*x)^(1/2)+1/24*a^2/b*(6*A*b-B*a)/(b*e)^(1/2)*arctanh(((b*x^3+a
)*e*x)^(1/2)/x^2/(b*e)^(1/2))*e*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)
^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.69 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.70

$$\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx = \left[-\frac{3\left(Ba^3-6\,Aa^2b\right)\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\left(2\,b^2x^4+abx\right)\sqrt{bx^3+a}\sqrt{ex}\sqrt{\frac{e}{b}}\right)-4}{288\,b}\right.$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="fricas")
[Out] [-1/288*(3*(B*a^3 - 6*A*a^2*b)*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a
^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(8*B*
b^2*x^7 + 2*(7*B*a*b + 6*A*b^2)*x^4 + 3*(B*a^2 + 10*A*a*b)*x)*sqrt(b*x^3 +
a)*sqrt(e*x))/b, 1/144*(3*(B*a^3 - 6*A*a^2*b)*sqrt(-e/b)*arctan(2*sqrt(b*x^
3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) + 2*(8*B*b^2*x^7 + 2*(7*
B*a*b + 6*A*b^2)*x^4 + 3*(B*a^2 + 10*A*a*b)*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b
]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 546 vs. $2(138) = 276$.

Time = 3.44 (sec) , antiderivative size = 546, normalized size of antiderivative = 3.39

$$\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx = \begin{cases} \frac{2\left(\begin{cases}\text{NaN} \\ \frac{Aae^3\left(\begin{cases}\frac{a\left(\begin{cases}\frac{\log\left(\frac{2b(ex)^{\frac{3}{2}}}{e^3}+2\sqrt{\frac{b}{e^3}}\sqrt{a+bx^3}\right)}{\sqrt{\frac{b}{e^3}}} & \text{for } a\neq 0\\ \frac{(ex)^{\frac{3}{2}}\log\left((ex)^{\frac{3}{2}}\right)}{\sqrt{bx^3}} & \text{otherwise}\end{cases}\right)}{2}+\frac{(ex)^{\frac{3}{2}}\sqrt{a+bx^3}}{2} & \text{for } \frac{b}{e^3}\neq 0\\ \sqrt{a}(ex)^{\frac{3}{2}} & \text{otherwise}\end{cases}\right)+Ab\left(\left(\begin{cases} a^2e^3 \cdots \\ -\frac{\cdots}{\cdots} \\ \frac{\sqrt{a}(ex)^{\frac{9}{2}}}{3} \cdots\end{cases}\right.\right.}{\cdots}\end{cases}\right.}{\cdots} \\ 0 \end{cases}$$

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)*(e*x)**(1/2),x)
```

```
[Out] Piecewise((2*Piecewise((nan, Eq(e**3, 0)), ((A*a*e**3*Piecewise((a*Piecewis
e((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3
), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), True))/2 + (e*x
)**(3/2)*sqrt(a + b*x**3)/2, Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(3/2), True))
+ A*b*Piecewise((-a**2*e**3*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b
/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**
(3/2))/sqrt(b*x**3), True))/(8*b) + sqrt(a + b*x**3)*(a*e**3*(e*x)**(3/2)/(
8*b) + (e*x)**(9/2)/4), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(9/2)/3, True)) + B
*a*Piecewise((-a**2*e**3*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e*
*3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/
2))/sqrt(b*x**3), True))/(8*b) + sqrt(a + b*x**3)*(a*e**3*(e*x)**(3/2)/(8*b
) + (e*x)**(9/2)/4), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(9/2)/3, True)) + B*b*
Piecewise((a**3*e**6*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*
sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/
sqrt(b*x**3), True))/(16*b**2) + sqrt(a + b*x**3)*(-a**2*e**6*(e*x)**(3/2)/
(16*b**2) + a*e**3*(e*x)**(9/2)/(24*b) + (e*x)**(15/2)/6), Ne(b/e**3, 0)),
(sqrt(a)*(e*x)**(15/2)/5, True))/e**3)/(3*e**3), True))/e, Ne(e, 0)), (0, T
rue))
```

Maxima [F]

$$\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}\sqrt{ex}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*sqrt(e*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(126) = 252$.

Time = 0.45 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.58

$$\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx = \frac{\sqrt{be^4x^3+ae^4}\sqrt{ex}\left(\frac{2\,x^3}{e}+\frac{a}{be}\right)Bax|e|^2}{12\,e^3}$$
$$+\frac{\sqrt{be^4x^3+ae^4}\sqrt{ex}\left(\frac{2\,x^3}{e}+\frac{a}{be}\right)Abx|e|^2}{12\,e^3}$$
$$+\frac{\sqrt{be^4x^3+ae^4}\left(2\,e^3x^3\left(\frac{4\,x^3}{e^4}+\frac{a}{be^4}\right)-\frac{3\,a^2}{b^2e}\right)\sqrt{ex}Bbx|e|^2}{72\,e^3}$$
$$-\frac{(B^2a^6+4\,ABa^5b+4\,A^2a^4b^2)e^4\log\left(\left|\left(\sqrt{ex}Ba^3x+2\,\sqrt{ex}Aa^2bx\right)\sqrt{be}+\sqrt{B^2a^7e^2+4\,ABa^6be^2+4\,A^2a^5b^2\cdots}\right.\right.}{24\,\sqrt{be}b|Ba^3e+2\,Aa^2be||e|^2}$$
$$-\frac{\left(\frac{ae^4\log\left(\left|-\sqrt{be}\sqrt{ex}ex+\sqrt{be^4x^3+ae^4}\right|\right)}{\sqrt{be}}-\sqrt{be^4x^3+ae^4}\sqrt{ex}ex\right)Aa|e|^2}{3\,e^5}$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*(2*x^3/e + a/(b*e))*B*a*x*abs(e)^2/e
^3 + 1/12*sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*(2*x^3/e + a/(b*e))*A*b*x*abs(e
)^2/e^3 + 1/72*sqrt(b*e^4*x^3 + a*e^4)*(2*e^3*x^3*(4*x^3/e^4 + a/(b*e^4)) -
 3*a^2/(b^2*e))*sqrt(e*x)*B*b*x*abs(e)^2/e^3 - 1/24*(B^2*a^6 + 4*A*B*a^5*b
+ 4*A^2*a^4*b^2)*e^4*log(abs((sqrt(e*x)*B*a^3*x + 2*sqrt(e*x)*A*a^2*b*x)*sq
rt(b*e) + sqrt(B^2*a^7*e^2 + 4*A*B*a^6*b*e^2 + 4*A^2*a^5*b^2*e^2 + (sqrt(e*
x)*B*a^3*x + 2*sqrt(e*x)*A*a^2*b*x)^2*b*e)))/(sqrt(b*e)*b*abs(B*a^3*e + 2*A
*a^2*b*e)*abs(e)^2) - 1/3*(a*e^4*log(abs(-sqrt(b*e)*sqrt(e*x)*e*x + sqrt(b*
e^4*x^3 + a*e^4)))/sqrt(b*e) - sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*e*x)*A*a*a
bs(e)^2/e^5
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx = \int \left(B\,x^3+A\right)\,\sqrt{e\,x}\,{\left(b\,x^3+a\right)}^{3/2} \,d x$$

```
[In] int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(3/2),x)
```

```
[Out] int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(3/2), x)
```

3.531 $\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{\sqrt{ex}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 324

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{\sqrt{ex}}\,dx = \frac{9a(16Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{320be} + \frac{(16Ab-aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{80be} + \frac{B\sqrt{ex}\left(a+bx^3\right)^{5/2}}{8be} + \frac{9\,3^{3/4}a^{5/3}(16Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(2\right.}{640be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 1/80*(16*A*b-B*a)*(b*x^3+a)^(3/2)*(e*x)^(1/2)/b/e+1/8*B*(b*x^3+a)^(5/2)*(e*
x)^(1/2)/b/e+9/320*a*(16*A*b-B*a)*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b/e+9/640*3^(
3/4)*a^(5/3)*(16*A*b-B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2
)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2
)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1
/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(
e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3
^(1/2)))^2)^(1/2)/b/e/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {470, 285, 335, 231}

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \frac{9\,3^{3/4}a^{5/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(16Ab-aB)\,\text{EllipticF}\left(\text{ar}\right.}{640be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{\sqrt{ex}(a+bx^3)^{3/2}(16Ab-aB)}{80be}+\frac{9a\sqrt{ex}\sqrt{a+bx^3}(16Ab-aB)}{320be}+\frac{B\sqrt{ex}(a+bx^3)^{5/2}}{8be}$$

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/Sqrt[e*x],x]
```

```
[Out] (9*a*(16*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(320*b*e) + ((16*A*b - a*B)*
Sqrt[e*x]*(a + b*x^3)^(3/2))/(80*b*e) + (B*Sqrt[e*x]*(a + b*x^3)^(5/2))/(8*
b*e) + (9*3^(3/4)*a^(5/3)*(16*A*b - a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(
1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) +
(1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(640*b*e*Sqrt[(b^(1/3)*x*(a^(1
/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{B\sqrt{ex}\left(a+bx^3\right)^{5/2}}{8be} - \frac{\left(-8Ab+\frac{aB}{2}\right)\int \frac{\left(a+bx^3\right)^{3/2}}{\sqrt{ex}}\,dx}{8b}$$

$$= \frac{(16Ab-aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{80be} + \frac{B\sqrt{ex}\left(a+bx^3\right)^{5/2}}{8be} + \frac{(9a(16Ab-aB))\int \frac{\sqrt{a+bx^3}}{\sqrt{ex}}\,dx}{160b}$$

$$= \frac{9a(16Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{320be} + \frac{(16Ab-aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{80be} + \frac{B\sqrt{ex}\left(a+bx^3\right)^{5/2}}{8be} + \frac{\left(27a^2(16Ab-aB)\right)\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{640b}$$

$$= \frac{9a(16Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{320be} + \frac{(16Ab-aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{80be} + \frac{B\sqrt{ex}\left(a+bx^3\right)^{5/2}}{8be} + \frac{\left(27a^2(16Ab-aB)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{320be}$$

$$= \frac{9a(16Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{320be} + \frac{(16Ab-aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{80be} + \frac{B\sqrt{ex}\left(a+bx^3\right)^{5/2}}{8be} + \frac{9\ 3^{3/4}a^{5/3}(16Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}}{640be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.30

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \frac{x\sqrt{a+bx^3}\left(B(a+bx^3)^2\sqrt{1+\frac{bx^3}{a}}+a(16Ab-aB)\,\text{Hypergeometric2F1}\left(-\frac{3}{2}\right.\right.}{8b\sqrt{ex}\sqrt{1+\frac{bx^3}{a}}}$$

[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/Sqrt[e*x],x]

[Out] (x*Sqrt[a + b*x^3]*(B*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a] + a*(16*A*b - a*B)*
Hypergeometric2F1[-3/2, 1/6, 7/6, -((b*x^3)/a)]))/(8*b*Sqrt[e*x]*Sqrt[1 + (
b*x^3)/a])

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.80 (sec) , antiderivative size = 768, normalized size of antiderivative = 2.37

method	result
risch	$\frac{(40b^2Bx^6+64Ab^2x^3+76Babx^3+208abA+27a^2B)x\sqrt{bx^3+a}}{320b\sqrt{ex}}+\frac{27a^2(16Ab-Ba)\left(\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3(-ab^2)}{2b}\right.}{\left(-\frac{(-ab^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}}{}\right.}}}{}$
elliptic	Expression too large to display
default	Expression too large to display

[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/320/b*(40*B*b^2*x^6+64*A*b^2*x^3+76*B*a*b*x^3+208*A*a*b+27*B*a^2)*x*(b*x^
3+a)^(1/2)/(e*x)^(1/2)+27/320*a^2*(16*A*b-B*a)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*
b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*

```
(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))*((b*x^3+a)*e
*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{\sqrt{ex}}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)*sqrt(e*x)/(e*x),
 x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.28 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.61

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \frac{Aa^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\Gamma\left(\frac{7}{6}\right)} + \frac{A\sqrt{a}bx^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\Gamma\left(\frac{13}{6}\right)} + \frac{Ba^{\frac{3}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\Gamma\left(\frac{13}{6}\right)} + \frac{B\sqrt{a}bx^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\Gamma\left(\frac{19}{6}\right)}$$

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/(e*x)**(1/2),x)
```

```
[Out] A*a**(3/2)*sqrt(x)*gamma(1/6)*hyper((-1/2, 1/6), (7/6,), b*x**3*exp_polar(I
*pi)/a)/(3*sqrt(e)*gamma(7/6)) + A*sqrt(a)*b*x**(7/2)*gamma(7/6)*hyper((-1/
2, 7/6), (13/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(13/6)) + B*a**
```

```
(3/2)*x**(7/2)*gamma(7/6)*hyper((-1/2, 7/6), (13/6,), b*x**3*exp_polar(I*pi
)/a)/(3*sqrt(e)*gamma(13/6)) + B*sqrt(a)*b*x**(13/2)*gamma(13/6)*hyper((-1/
2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(19/6))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{\sqrt{ex}}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/sqrt(e*x), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{\sqrt{ex}}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/sqrt(e*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{3/2}}{\sqrt{e\,x}}\,dx$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(1/2),x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(1/2), x)
```

3.532 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 614

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = \frac{9(14Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{56e^4}$$
$$+\frac{27(1+\sqrt{3})\,a(14Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{112b^{2/3}e^2\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)}$$
$$+\frac{(14Ab+aB)(ex)^{5/2}(a+bx^3)^{3/2}}{7ae^4}-\frac{2A(a+bx^3)^{5/2}}{ae\sqrt{ex}}$$
$$-\frac{27\sqrt[4]{3}a^{4/3}(14Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+(1-\sqrt{3})\sqrt[3]{b}x}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x}\right)|\frac{1}{4}(2+\sqrt{3})\right.}{112b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{9\,3^{3/4}(1-\sqrt{3})\,a^{4/3}(14Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+(1-\sqrt{3})}{\sqrt[3]{a}+(1+\sqrt{3})}\right.\right.}{224b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 1/7*(14*A*b+B*a)*(e*x)^(5/2)*(b*x^3+a)^(3/2)/a/e^4-2*A*(b*x^3+a)^(5/2)/a/e/
(e*x)^(1/2)+9/56*(14*A*b+B*a)*(e*x)^(5/2)*(b*x^3+a)^(1/2)/e^4+27/112*a*(14*
A*b+B*a)*(1+3^(1/2))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^(2/3)/e^2/(a^(1/3)+b^(1/
3)*x*(1+3^(1/2)))-27/112*3^(1/4)*a^(4/3)*(14*A*b+B*a)*(a^(1/3)+b^(1/3)*x)*(
(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/
```

```
(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE((
1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2
),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*
x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/e^2/(b*x^3+a)^(1/2)/(
b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)-9/22
4*3^(3/4)*a^(4/3)*(14*A*b+B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3
^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3
^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1
-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/
2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/e^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(
a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 614, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {464, 285, 335, 314, 231, 1895}

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{3/2}}\,dx =$$

$$-\frac{9\, 3^{3/4}(1-\sqrt{3})\, a^{4/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}(aB+14Ab)\,\text{EllipticF}\left(\arccos\left(\frac{(1-\sqrt{3})\sqrt[3]{b}x+\sqrt[3]{a}}{(1+\sqrt{3})\sqrt[3]{b}x+\sqrt[3]{a}}\right)\right.}{224b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{27\sqrt[4]{3}a^{4/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}(aB+14Ab)E\left(\arccos\left(\frac{(1-\sqrt{3})\sqrt[3]{b}x+\sqrt[3]{a}}{(1+\sqrt{3})\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{112b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{27(1+\sqrt{3})\,a\sqrt{ex}\sqrt{a+bx^3}(aB+14Ab)}{112b^{2/3}e^2\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)}+\frac{(ex)^{5/2}(a+bx^3)^{3/2}(aB+14Ab)}{7ae^4}$$

$$+\frac{9(ex)^{5/2}\sqrt{a+bx^3}(aB+14Ab)}{56e^4}-\frac{2A(a+bx^3)^{5/2}}{ae\sqrt{ex}}$$

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(3/2),x]
```

```
[Out] (9*(14*A*b + a*B)*(e*x)^(5/2)*Sqrt[a + b*x^3])/(56*e^4) + (27*(1 + Sqrt[3])
*a*(14*A*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(112*b^(2/3)*e^2*(a^(1/3) + (1
```

```
 + Sqrt[3])*b^(1/3)*x)) + ((14*A*b + a*B)*(e*x)^(5/2)*(a + b*x^3)^(3/2))/(7
*a*e^4) - (2*A*(a + b*x^3)^(5/2))/(a*e*Sqrt[e*x]) - (27*3^(1/4)*a^(4/3)*(14
*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(
a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2
 + Sqrt[3])/4])/(112*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^
(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (9*3^(3/4)*(1 - Sqrt
[3])*a^(4/3)*(14*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*El
lipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])
*b^(1/3)*x)], (2 + Sqrt[3])/4])/(224*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) +
 b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
```

```
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2A(a+bx^3)^{5/2}}{ae\sqrt{ex}}+\frac{(14Ab+aB)\int (ex)^{3/2}\,(a+bx^3)^{3/2}\,dx}{ae^3}\\
&= \frac{(14Ab+aB)(ex)^{5/2}\,(a+bx^3)^{3/2}}{7ae^4}-\frac{2A(a+bx^3)^{5/2}}{ae\sqrt{ex}}+\frac{(9(14Ab+aB))\int (ex)^{3/2}\sqrt{a+bx^3}\,dx}{14e^3}\\
&= \frac{9(14Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{56e^4}+\frac{(14Ab+aB)(ex)^{5/2}\,(a+bx^3)^{3/2}}{7ae^4}\\
&\quad-\frac{2A(a+bx^3)^{5/2}}{ae\sqrt{ex}}+\frac{(27a(14Ab+aB))\int \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\,dx}{112e^3}\\
&= \frac{9(14Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{56e^4}+\frac{(14Ab+aB)(ex)^{5/2}\,(a+bx^3)^{3/2}}{7ae^4}\\
&\quad-\frac{2A(a+bx^3)^{5/2}}{ae\sqrt{ex}}+\frac{(27a(14Ab+aB))\text{Subst}\left(\int \frac{x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{56e^4}\\
&= \frac{9(14Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{56e^4}+\frac{(14Ab+aB)(ex)^{5/2}\,(a+bx^3)^{3/2}}{7ae^4}\\
&\quad-\frac{2A(a+bx^3)^{5/2}}{ae\sqrt{ex}}-\frac{(27a(14Ab+aB))\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}e^2-2b^{2/3}x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{112b^{2/3}e^4}\\
&\quad-\frac{\left(27\left(1-\sqrt{3}\right)a^{5/3}(14Ab+aB)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{112b^{2/3}e^2}
\end{aligned}$$

$$= \frac{9(14Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{56e^4} + \frac{27\left(1+\sqrt{3}\right)a(14Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{112b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$

$$+ \frac{(14Ab+aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7ae^4} - \frac{2A\left(a+bx^3\right)^{5/2}}{ae\sqrt{ex}}$$

$$- \frac{27\sqrt[4]{3}a^{4/3}(14Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}}{112b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$- \frac{9\ 3^{3/4}\left(1-\sqrt{3}\right)a^{4/3}(14Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right.}{\sqrt[3]{a}+\left(1+\sqrt{3}\right.}\right.\right.}{224b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.14

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{(ex)^{3/2}}\,dx = \frac{2x\sqrt{a+bx^3}\left(-\frac{5A\left(a+bx^3\right)^2}{a}+\frac{(14Ab+aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{6},\frac{11}{6},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{5(ex)^{3/2}}$$

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(3/2),x]
[Out] (2*x*Sqrt[a + b*x^3]*((-5*A*(a + b*x^3)^2)/a + ((14*A*b + a*B)*x^3*Hypergeo
metric2F1[-3/2, 5/6, 11/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(5*(e*x)^(3
/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.05 (sec) , antiderivative size = 1140, normalized size of antiderivative = 1.86

method	result	size
risch	Expression too large to display	1140
elliptic	Expression too large to display	1233
default	Expression too large to display	6142

[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/56*(b*x^3+a)^(1/2)*(-8*B*b*x^6-14*A*b*x^3-17*B*a*x^3+112*A*a)/e/(e*x)^(1/2)+27/112*a*(14*A*b+B*a)*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))*b/(-a*b^2)^(1/3)))/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)/e*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)

Fricas [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{3/2}}\,dx=\int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="fricas")

[Out] integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)*sqrt(e*x)/(e^2*x^2), x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.87 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.33

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = \frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{6}\\ \frac{5}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{5}{6}\right)} + \frac{A\sqrt{a}bx^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)} + \frac{Ba^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)} + \frac{B\sqrt{a}bx^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{17}{6}\right)}$$

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/(e*x)**(3/2),x)
```

```
[Out] A*a**(3/2)*gamma(-1/6)*hyper((-1/2, -1/6), (5/6,), b*x**3*exp_polar(I*pi)/a
)/(3*e**(3/2)*sqrt(x)*gamma(5/6)) + A*sqrt(a)*b*x**(5/2)*gamma(5/6)*hyper((
-1/2, 5/6), (11/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(11/6)) + B
*a**(3/2)*x**(5/2)*gamma(5/6)*hyper((-1/2, 5/6), (11/6,), b*x**3*exp_polar(
I*pi)/a)/(3*e**(3/2)*gamma(11/6)) + B*sqrt(a)*b*x**(11/2)*gamma(11/6)*hyper
((-1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(17/6))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/(e*x)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/(e*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{3/2}}{(e\,x)^{3/2}}\,dx$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(3/2),x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(3/2), x)
```

3.533 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 152

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = \frac{(4Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{4e^4} + \frac{(4Ab+aB)(ex)^{3/2}(a+bx^3)^{3/2}}{6ae^4} - \frac{2A(a+bx^3)^{5/2}}{3ae(ex)^{3/2}} + \frac{a(4Ab+aB)\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{4\sqrt{b}e^{5/2}}$$

```
[Out] 1/6*(4*A*b+B*a)*(e*x)^(3/2)*(b*x^3+a)^(3/2)/a/e^4-2/3*A*(b*x^3+a)^(5/2)/a/e
/(e*x)^(3/2)+1/4*a*(4*A*b+B*a)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a
)^(1/2))/e^(5/2)/b^(1/2)+1/4*(4*A*b+B*a)*(e*x)^(3/2)*(b*x^3+a)^(1/2)/e^4
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {464, 285, 335, 281, 223, 212}

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = \frac{a(aB+4Ab)\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{4\sqrt{b}e^{5/2}} + \frac{(ex)^{3/2}(a+bx^3)^{3/2}(aB+4Ab)}{6ae^4} + \frac{(ex)^{3/2}\sqrt{a+bx^3}(aB+4Ab)}{4e^4} - \frac{2A(a+bx^3)^{5/2}}{3ae(ex)^{3/2}}$$

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(5/2),x]
```

```
[Out] ((4*A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(4*e^4) + ((4*A*b + a*B)*(e*x)^
(3/2)*(a + b*x^3)^(3/2))/(6*a*e^4) - (2*A*(a + b*x^3)^(5/2))/(3*a*e*(e*x)^(
```

```
3/2)) + (a*(4*A*b + a*B)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*
x^3])])/(4*Sqrt[b]*e^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{2A(a+bx^3)^{5/2}}{3ae(ex)^{3/2}} + \frac{(4Ab+aB)\int \sqrt{ex}(a+bx^3)^{3/2}\,dx}{ae^3}$$

$$= \frac{(4Ab+aB)(ex)^{3/2}(a+bx^3)^{3/2}}{6ae^4} - \frac{2A(a+bx^3)^{5/2}}{3ae(ex)^{3/2}} + \frac{(3(4Ab+aB))\int \sqrt{ex}\sqrt{a+bx^3}\,dx}{4e^3}$$

$$= \frac{(4Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{4e^4} + \frac{(4Ab+aB)(ex)^{3/2}(a+bx^3)^{3/2}}{6ae^4} - \frac{2A(a+bx^3)^{5/2}}{3ae(ex)^{3/2}} + \frac{(3a(4Ab+aB))\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{8e^3}$$

$$= \frac{(4Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{4e^4} + \frac{(4Ab+aB)(ex)^{3/2}(a+bx^3)^{3/2}}{6ae^4} - \frac{2A(a+bx^3)^{5/2}}{3ae(ex)^{3/2}} + \frac{(3a(4Ab+aB))\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{4e^4}$$

$$= \frac{(4Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{4e^4} + \frac{(4Ab+aB)(ex)^{3/2}(a+bx^3)^{3/2}}{6ae^4} - \frac{2A(a+bx^3)^{5/2}}{3ae(ex)^{3/2}} + \frac{(a(4Ab+aB))\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx, x, (ex)^{3/2}\right)}{4e^4}$$

$$= \frac{(4Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{4e^4} + \frac{(4Ab+aB)(ex)^{3/2}(a+bx^3)^{3/2}}{6ae^4} - \frac{2A(a+bx^3)^{5/2}}{3ae(ex)^{3/2}} + \frac{(a(4Ab+aB))\text{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx, x, \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{4e^4}$$

$$= \frac{(4Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{4e^4} + \frac{(4Ab+aB)(ex)^{3/2}(a+bx^3)^{3/2}}{6ae^4} - \frac{2A(a+bx^3)^{5/2}}{3ae(ex)^{3/2}} + \frac{a(4Ab+aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{4\sqrt{b}e^{5/2}}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.66

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = \frac{x\left(\sqrt{b}\sqrt{a+bx^3}(-8aA+4Abx^3+5aBx^3+2bBx^6)+3a(4Ab+aB)x^{3/2}\log\left(\right.\right.}{12\sqrt{b}(ex)^{5/2}}$$

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(5/2),x]
```

```
[Out] (x*(Sqrt[b]*Sqrt[a + b*x^3]*(-8*a*A + 4*A*b*x^3 + 5*a*B*x^3 + 2*b*B*x^6) +
3*a*(4*A*b + a*B)*x^(3/2)*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]]))/(12*Sqrt
[b]*(e*x)^(5/2))
```

Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,(-2bB\,x^6-4Ab\,x^3-5Ba\,x^3+8Aa)}{12x\,e^2\sqrt{ex}}+\frac{a(4Ab+Ba)\operatorname{arctanh}\left(\frac{\sqrt{(b\,x^3+a)ex}}{x^2\sqrt{be}}\right)\sqrt{(b\,x^3+a)ex}}{4\sqrt{be}\,e^2\sqrt{ex}\,\sqrt{b\,x^3+a}}$
default	$\frac{\sqrt{b\,x^3+a}\left(2B\sqrt{(b\,x^3+a)ex}\,\sqrt{be}\,b\,x^6+12A\operatorname{arctanh}\left(\frac{\sqrt{(b\,x^3+a)ex}}{x^2\sqrt{be}}\right)abe\,x^2+4A\sqrt{(b\,x^3+a)ex}\,\sqrt{be}\,b\,x^3+3B\operatorname{arctanh}\left(\frac{\sqrt{(b\,x^3+a)ex}}{x^2\sqrt{be}}\right)a\right.}{12x\,e^2\sqrt{ex}\,\sqrt{(b\,x^3+a)ex}\,\sqrt{be}}$
elliptic	Expression too large to display

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(b*x^3+a)^(1/2)*(-2*B*b*x^6-4*A*b*x^3-5*B*a*x^3+8*A*a)/x/e^2/(e*x)^(1
/2)+1/4*a*(4*A*b+B*a)/(b*e)^(1/2)*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(
1/2))/e^2*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.56 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.68

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = \left[\frac{3\,(Ba^2+4\,Aab)\sqrt{be}x^2\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\,(2\,bx^4+ax)\sqrt{bx^3}\right.}{48\,be^3}\right.$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(B*a^2 + 4*A*a*b)*sqrt(b*e)*x^2*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a
^2*e - 4*(2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b*e)*sqrt(e*x)) + 4*(2*B*b^2*
x^6 + (5*B*a*b + 4*A*b^2)*x^3 - 8*A*a*b)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*
x^2), -1/24*(3*(B*a^2 + 4*A*a*b)*sqrt(-b*e)*x^2*arctan(2*sqrt(b*x^3 + a)*sq
rt(-b*e)*sqrt(e*x)*x/(2*b*e*x^3 + a*e)) - 2*(2*B*b^2*x^6 + (5*B*a*b + 4*A*b
^2)*x^3 - 8*A*a*b)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(138) = 276$.

Time = 17.82 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.90

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = -\frac{2Aa^{\frac{3}{2}}}{3e^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{A\sqrt{a}bx^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}} - \frac{2A\sqrt{a}bx^{\frac{3}{2}}}{3e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{Aa\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{e^{\frac{5}{2}}} + \frac{Ba^{\frac{3}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}} + \frac{Ba^{\frac{3}{2}}x^{\frac{3}{2}}}{12e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{B\sqrt{a}bx^{\frac{9}{2}}}{4e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{Ba^2\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{4\sqrt{b}e^{\frac{5}{2}}} + \frac{Bb^2x^{\frac{15}{2}}}{6\sqrt{a}e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}$$

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/(e*x)**(5/2),x)
```

```
[Out] -2*A*a**(3/2)/(3*e**(5/2)*x**(3/2)*sqrt(1 + b*x**3/a)) + A*sqrt(a)*b*x**(3/
2)*sqrt(1 + b*x**3/a)/(3*e**(5/2)) - 2*A*sqrt(a)*b*x**(3/2)/(3*e**(5/2)*sqr
t(1 + b*x**3/a)) + A*a*sqrt(b)*asinh(sqrt(b)*x**(3/2)/sqrt(a))/e**(5/2) + B
*a**(3/2)*x**(3/2)*sqrt(1 + b*x**3/a)/(3*e**(5/2)) + B*a**(3/2)*x**(3/2)/(1
2*e**(5/2)*sqrt(1 + b*x**3/a)) + B*sqrt(a)*b*x**(9/2)/(4*e**(5/2)*sqrt(1 +
b*x**3/a)) + B*a**2*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(4*sqrt(b)*e**(5/2)) +
B*b**2*x**(15/2)/(6*sqrt(a)*e**(5/2)*sqrt(1 + b*x**3/a))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{(ex)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/(e*x)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = \text{Exception raised: NotImplementedError}$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> unable to parse Giac output: Recur
sive assumption sageVARa>=(-sageVARb*sageVARe/(sageVARe^4*t_nostep^6)) igno
red2/sageVARe^3*(120*sageVARb^5*sageVARe^3*sageVARB/1440/sageVARb^4/sageVAR
e^9*sqrt(sageVAR
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = \int \frac{(B\,x^3+A)\,\left(b\,x^3+a\right)^{3/2}}{(e\,x)^{5/2}}\,dx$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(5/2),x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(5/2), x)
```

3.534 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 314

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{7/2}}\,dx = \frac{9(2Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{20e^4} + \frac{(2Ab+aB)\sqrt{ex}(a+bx^3)^{3/2}}{5ae^4} - \frac{2A(a+bx^3)^{5/2}}{5ae(ex)^{5/2}} + \frac{9\,3^{3/4}a^{2/3}(2Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(2}{40e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -2/5*A*(b*x^3+a)^(5/2)/a/e/(e*x)^(5/2)+1/5*(2*A*b+B*a)*(b*x^3+a)^(3/2)*(e*x
)^(1/2)/a/e^4+9/20*(2*A*b+B*a)*(e*x)^(1/2)*(b*x^3+a)^(1/2)/e^4+9/40*3^(3/4)
*a^(2/3)*(2*A*b+B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2
/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(
a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))
^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^
(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2
)))^2)^(1/2)/e^4/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^
(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {464, 285, 335, 231}

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{7/2}}\,dx = \frac{9\,3^{3/4}a^{2/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+2Ab)\,\text{EllipticF}\left(\text{arc}\right.}{40e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{\sqrt{ex}(a+bx^3)^{3/2}(aB+2Ab)}{5ae^4} + \frac{9\sqrt{ex}\sqrt{a+bx^3}(aB+2Ab)}{20e^4} - \frac{2A(a+bx^3)^{5/2}}{5ae(ex)^{5/2}}$$

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(7/2),x]
```

```
[Out] (9*(2*A*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(20*e^4) + ((2*A*b + a*B)*Sqrt[
e*x]*(a + b*x^3)^(3/2))/(5*a*e^4) - (2*A*(a + b*x^3)^(5/2))/(5*a*e*(e*x)^(5
/2)) + (9*3^(3/4)*a^(2/3)*(2*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1
/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(40*e^4*Sqrt[(b^(1/3)*x*(a^(1/3
) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2A(a+bx^3)^{5/2}}{5ae(ex)^{5/2}}+\frac{(2Ab+aB)\int\frac{(a+bx^3)^{3/2}}{\sqrt{ex}}\,dx}{ae^3}\\
&=\frac{(2Ab+aB)\sqrt{ex}(a+bx^3)^{3/2}}{5ae^4}-\frac{2A(a+bx^3)^{5/2}}{5ae(ex)^{5/2}}+\frac{(9(2Ab+aB))\int\frac{\sqrt{a+bx^3}}{\sqrt{ex}}\,dx}{10e^3}\\
&=\frac{9(2Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{20e^4}+\frac{(2Ab+aB)\sqrt{ex}(a+bx^3)^{3/2}}{5ae^4}\\
&\quad-\frac{2A(a+bx^3)^{5/2}}{5ae(ex)^{5/2}}+\frac{(27a(2Ab+aB))\int\frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{40e^3}\\
&=\frac{9(2Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{20e^4}+\frac{(2Ab+aB)\sqrt{ex}(a+bx^3)^{3/2}}{5ae^4}\\
&\quad-\frac{2A(a+bx^3)^{5/2}}{5ae(ex)^{5/2}}+\frac{(27a(2Ab+aB))\text{Subst}\left(\int\frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{20e^4}\\
&=\frac{9(2Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{20e^4}+\frac{(2Ab+aB)\sqrt{ex}(a+bx^3)^{3/2}}{5ae^4}-\frac{2A(a+bx^3)^{5/2}}{5ae(ex)^{5/2}}\\
&\quad+\frac{9\,3^{3/4}a^{2/3}(2Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}(\right.}{40e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.27

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{7/2}}\,dx=\frac{2x\sqrt{a+bx^3}\left(-\frac{A(a+bx^3)^2}{a}+\frac{5(2Ab+aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{6},\frac{7}{6},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{5(ex)^{7/2}}$$

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(7/2),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*(-((A*(a + b*x^3)^2)/a) + (5*(2*A*b + a*B)*x^3*Hyperge
ometric2F1[-3/2, 1/6, 7/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(5*(e*x)^(7
/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.77 (sec) , antiderivative size = 759, normalized size of antiderivative = 2.42

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,(-4bB\,x^6-10Ab\,x^3-13Ba\,x^3+8Aa)}{20x^2e^3\sqrt{ex}}+\frac{27a(2Ab+Ba)\left(\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)}{2b}\right.}{\left(-\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\left(x-\right.}}}{}$
elliptic	Expression too large to display
default	Expression too large to display

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*(b*x^3+a)^(1/2)*(-4*B*b*x^6-10*A*b*x^3-13*B*a*x^3+8*A*a)/x^2/e^3/(e*x
)^(1/2)+27/20*a*(2*A*b+B*a)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*
(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(
x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1
```

```
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))/e^3*((b*x^3+a)*e*x)^(1/2)/(e*
x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [F]

$$\int \frac{(a+bx^3)^{3/2}\,(A+Bx^3)}{(ex)^{7/2}}\,dx=\int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="fricas")
[Out] integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)*sqrt(e*x)/(e^4*x
^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 23.15 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx^3)^{3/2}\,(A+Bx^3)}{(ex)^{7/2}}\,dx=\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{5}{6}\right)\,{}_2F_1\left(\begin{matrix}-\frac{5}{6},-\frac{1}{2}\\ \frac{1}{6}\end{matrix}\middle|\,\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)}+\frac{A\sqrt{a}b\sqrt{x}\Gamma\left(\frac{1}{6}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\,\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)}+\frac{Ba^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{1}{6}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\,\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)}+\frac{B\sqrt{a}bx^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\,\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{13}{6}\right)}$$

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/(e*x)**(7/2),x)
[Out] A*a**(3/2)*gamma(-5/6)*hyper((-5/6, -1/2), (1/6,), b*x**3*exp_polar(I*pi)/a
)/(3*e**(7/2)*x**(5/2)*gamma(1/6)) + A*sqrt(a)*b*sqrt(x)*gamma(1/6)*hyper((
-1/2, 1/6), (7/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(7/2)*gamma(7/6)) + B*a
**(3/2)*sqrt(x)*gamma(1/6)*hyper((-1/2, 1/6), (7/6,), b*x**3*exp_polar(I*pi
)/a)/(3*e**(7/2)*gamma(7/6)) + B*sqrt(a)*b*x**(7/2)*gamma(7/6)*hyper((-1/2,
 7/6), (13/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(7/2)*gamma(13/6))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{7/2}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/(e*x)^(7/2), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{7/2}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/(e*x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{7/2}}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{3/2}}{(e\,x)^{7/2}}\,dx$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(7/2),x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(7/2), x)
```

3.535 $\int (ex)^{7/2} (a+bx^3)^{5/2} (A+Bx^3) \, dx$

Optimal result

Integrand size = 26, antiderivative size = 241

$$\int (ex)^{7/2} (a+bx^3)^{5/2} (A+Bx^3) \, dx = \frac{a^3(10Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{384b^2} + \frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{192be} + \frac{a(10Ab-3aB)(ex)^{9/2}(a+bx^3)^{3/2}}{144be} + \frac{(10Ab-3aB)(ex)^{9/2}(a+bx^3)^{5/2}}{120be} + \frac{B(ex)^{9/2}(a+bx^3)^{7/2}}{15be} - \frac{a^4(10Ab-3aB)e^{7/2}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{384b^{5/2}}$$

```
[Out] 1/144*a*(10*A*b-3*B*a)*(e*x)^(9/2)*(b*x^3+a)^(3/2)/b/e+1/120*(10*A*b-3*B*a)
*(e*x)^(9/2)*(b*x^3+a)^(5/2)/b/e+1/15*B*(e*x)^(9/2)*(b*x^3+a)^(7/2)/b/e-1/3
84*a^4*(10*A*b-3*B*a)*e^(7/2)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)
^(1/2))/b^(5/2)+1/384*a^3*(10*A*b-3*B*a)*e^2*(e*x)^(3/2)*(b*x^3+a)^(1/2)/b^
2+1/192*a^2*(10*A*b-3*B*a)*(e*x)^(9/2)*(b*x^3+a)^(1/2)/b/e
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used

$= \{470, 285, 327, 335, 281, 223, 212\}$

$$\int (ex)^{7/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx = -\frac{a^4e^{7/2}(10Ab-3aB)\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{384b^{5/2}} + \frac{a^3e^2(ex)^{3/2}\sqrt{a+bx^3}(10Ab-3aB)}{384b^2} + \frac{a^2(ex)^{9/2}\sqrt{a+bx^3}(10Ab-3aB)}{192be} + \frac{(ex)^{9/2}\left(a+bx^3\right)^{5/2}(10Ab-3aB)}{120be} + \frac{a(ex)^{9/2}\left(a+bx^3\right)^{3/2}(10Ab-3aB)}{144be} + \frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be}$$

```
[In] Int[(e*x)^(7/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (a^3*(10*A*b - 3*a*B)*e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(384*b^2) + (a^2*(10
*A*b - 3*a*B)*(e*x)^(9/2)*Sqrt[a + b*x^3])/(192*b*e) + (a*(10*A*b - 3*a*B)*
(e*x)^(9/2)*(a + b*x^3)^(3/2))/(144*b*e) + ((10*A*b - 3*a*B)*(e*x)^(9/2)*(a
 + b*x^3)^(5/2))/(120*b*e) + (B*(e*x)^(9/2)*(a + b*x^3)^(7/2))/(15*b*e) - (
a^4*(10*A*b - 3*a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a
+ b*x^3])])/(384*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be}-\frac{\left(-15Ab+\frac{9aB}{2}\right)\int (ex)^{7/2}\left(a+bx^3\right)^{5/2}\,dx}{15b}\\
&= \frac{(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{120be}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be}\\
&\quad+\frac{(a(10Ab-3aB))\int (ex)^{7/2}\left(a+bx^3\right)^{3/2}\,dx}{16b}\\
&= \frac{a(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{144be}+\frac{(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{120be}\\
&\quad+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be}+\frac{\left(a^2(10Ab-3aB)\right)\int (ex)^{7/2}\sqrt{a+bx^3}\,dx}{32b}\\
&= \frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{192be}+\frac{a(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{144be}\\
&\quad+\frac{(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{120be}\\
&\quad+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be}+\frac{\left(a^3(10Ab-3aB)\right)\int \frac{(ex)^{7/2}}{\sqrt{a+bx^3}}\,dx}{128b}
\end{aligned}
$$

$$= \frac{a^3(10Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{384b^2}+\frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{192be}+\frac{a(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{144be}+\frac{(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{120be}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be}-\frac{\left(a^4(10Ab-3aB)e^3\right)\int\frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{256b^2}$$

$$= \frac{a^3(10Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{384b^2}+\frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{192be}+\frac{a(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{144be}+\frac{(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{120be}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be}-\frac{\left(a^4(10Ab-3aB)e^2\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{128b^2}$$

$$= \frac{a^3(10Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{384b^2}+\frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{192be}+\frac{a(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{144be}+\frac{(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{120be}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be}-\frac{\left(a^4(10Ab-3aB)e^2\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx,x,(ex)^{3/2}\right)}{384b^2}$$

$$= \frac{a^3(10Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{384b^2}+\frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{192be}+\frac{a(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{144be}+\frac{(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{120be}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be}-\frac{\left(a^4(10Ab-3aB)e^2\right)\text{Subst}\left(\int\frac{1}{1-\frac{bx^2}{e^3}}\,dx,x,\frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{384b^2}$$

$$= \frac{a^3(10Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{384b^2}+\frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{192be}+\frac{a(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{144be}+\frac{(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{120be}+\frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be}-\frac{a^4(10Ab-3aB)e^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{384b^{5/2}}$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.68

$$\int (ex)^{7/2} (a + bx^3)^{5/2} (A+Bx^3) \, dx = \frac{e^3\sqrt{ex}\Big(\sqrt{b}x^{3/2}\sqrt{a+bx^3}(-45a^4B+30a^3b(5A+Bx^3)+96b^4x^9(5A+4Bx^3)+16}{}$$

[In] Integrate[(e*x)^(7/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]

[Out] (e^3*Sqrt[e*x]*(Sqrt[b]*x^(3/2)*Sqrt[a + b*x^3]*(-45*a^4*B + 30*a^3*b*(5*A + B*x^3) + 96*b^4*x^9*(5*A + 4*B*x^3) + 16*a*b^3*x^6*(85*A + 63*B*x^3) + 4*a^2*b^2*x^3*(295*A + 186*B*x^3)) + 15*a^4*(-10*A*b + 3*a*B)*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]]))/(5760*b^(5/2)*Sqrt[x])

Maple [A] (verified)

Time = 4.98 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.77

method	result
risch	$\frac{x^2(384B\,b^4x^{12}+480A\,b^4x^9+1008Ba\,b^3x^9+1360Aa\,b^3x^6+744B\,a^2b^2x^6+1180A\,a^2b^2x^3+30B\,a^3b\,x^3+150A\,a^3b-45B\,a^4)\sqrt{b\,x^3+a}\,e^4}{5760b^2\sqrt{ex}}$
elliptic	Expression too large to display
default	Expression too large to display

[In] int((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x,method=_RETURNVERBOSE)

[Out] 1/5760/b^2*x^2*(384*B*b^4*x^12+480*A*b^4*x^9+1008*B*a*b^3*x^9+1360*A*a*b^3*x^6+744*B*a^2*b^2*x^6+1180*A*a^2*b^2*x^3+30*B*a^3*b*x^3+150*A*a^3*b-45*B*a^4)*(b*x^3+a)^(1/2)*e^4/(e*x)^(1/2)-1/384*a^4/b^2*(10*A*b-3*B*a)/(b*e)^(1/2)*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))*e^4*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.59 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.70

$$\int (ex)^{7/2} (a +bx^3)^{5/2} (A+Bx^3) \, dx = \left[-\frac{15 (3 Ba^5 - 10 Aa^4 b)e^3 \sqrt{\frac{e}{b}} \log \left(-8 b^2 ex^6 - 8 abex^3 - a^2 e + 4 (2 b^2 x^4 + abx)\sqrt{bx\right.$$

```
[In] integrate((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] [-1/23040*(15*(3*B*a^5 - 10*A*a^4*b)*e^3*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b
*e*x^3 - a^2*e + 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b))
 - 4*(384*B*b^4*e^3*x^13 + 48*(21*B*a*b^3 + 10*A*b^4)*e^3*x^10 + 8*(93*B*a^
2*b^2 + 170*A*a*b^3)*e^3*x^7 + 10*(3*B*a^3*b + 118*A*a^2*b^2)*e^3*x^4 - 15*
(3*B*a^4 - 10*A*a^3*b)*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b^2, -1/11520*(15*
(3*B*a^5 - 10*A*a^4*b)*e^3*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*
x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) - 2*(384*B*b^4*e^3*x^13 + 48*(21*B*a*b^3 +
10*A*b^4)*e^3*x^10 + 8*(93*B*a^2*b^2 + 170*A*a*b^3)*e^3*x^7 + 10*(3*B*a^3*b
 + 118*A*a^2*b^2)*e^3*x^4 - 15*(3*B*a^4 - 10*A*a^3*b)*e^3*x)*sqrt(b*x^3 + a
)*sqrt(e*x))/b^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1028 vs. 2(216) = 432.

Time = 49.13 (sec) , antiderivative size = 1028, normalized size of antiderivative = 4.27

$$\int (ex)^{7/2} \left(a + bx^3\right)^{5/2} \left(A + Bx^3\right) \, dx = \text{Too large to display}$$

```
[In] integrate((e*x)**(7/2)*(b*x**3+a)**(5/2)*(B*x**3+A),x)
```

```
[Out] Piecewise((2*Piecewise((nan, Eq(e**3, 0)), ((A*a**2*e**3*Piecewise((-a**2*e
**3*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))
/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), Tru
e))/(8*b) + sqrt(a + b*x**3)*(a*e**3*(e*x)**(3/2)/(8*b) + (e*x)**(9/2)/4),
Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(9/2)/3, True)) + 2*A*a*b*Piecewise((a**3*e
**6*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))
/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), Tru
e))/(16*b**2) + sqrt(a + b*x**3)*(-a**2*e**6*(e*x)**(3/2)/(16*b**2) + a*e**
3*(e*x)**(9/2)/(24*b) + (e*x)**(15/2)/6), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(
```

```
15/2)/5, True)) + A*b**2*Piecewise((-5*a**4*e**9*Piecewise((log(2*b*(e*x)**
(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*
x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), True))/(128*b**3) + sqrt(a + b*x*
*3)*(5*a**3*e**9*(e*x)**(3/2)/(128*b**3) - 5*a**2*e**6*(e*x)**(9/2)/(192*b*
*2) + a*e**3*(e*x)**(15/2)/(48*b) + (e*x)**(21/2)/8), Ne(b/e**3, 0)), (sqrt
(a)*(e*x)**(21/2)/7, True))/e**3 + B*a**2*Piecewise((a**3*e**6*Piecewise((l
og(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3), N
e(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), True))/(16*b**2) +
sqrt(a + b*x**3)*(-a**2*e**6*(e*x)**(3/2)/(16*b**2) + a*e**3*(e*x)**(9/2)/(
24*b) + (e*x)**(15/2)/6), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(15/2)/5, True))
+ 2*B*a*b*Piecewise((-5*a**4*e**9*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*
sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((
e*x)**(3/2))/sqrt(b*x**3), True))/(128*b**3) + sqrt(a + b*x**3)*(5*a**3*e**
9*(e*x)**(3/2)/(128*b**3) - 5*a**2*e**6*(e*x)**(9/2)/(192*b**2) + a*e**3*(e
*x)**(15/2)/(48*b) + (e*x)**(21/2)/8), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(21/
2)/7, True))/e**3 + B*b**2*Piecewise((7*a**5*e**12*Piecewise((log(2*b*(e*x)
**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((
e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), True))/(256*b**4) + sqrt(a + b*
x**3)*(-7*a**4*e**12*(e*x)**(3/2)/(256*b**4) + 7*a**3*e**9*(e*x)**(9/2)/(38
4*b**3) - 7*a**2*e**6*(e*x)**(15/2)/(480*b**2) + a*e**3*(e*x)**(21/2)/(80*b
) + (e*x)**(27/2)/10), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(27/2)/9, True))/e**
6)/(3*e**3), True))/e, Ne(e, 0)), (0, True))
```

Maxima [F]

$$\int (ex)^{7/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right)\, dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{7}{2}}\, dx$$

```
[In] integrate((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^(7/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 692 vs. 2(197) = 394.

Time = 0.60 (sec) , antiderivative size = 692, normalized size of antiderivative = 2.87

$$\int (ex)^{7/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx = \frac{1}{12}\sqrt{be^4x^3+ae^4}\sqrt{ex}\left(\frac{2x^3}{e}+\frac{a}{be}\right)Aa^2x|e|^2$$
$$+\frac{1}{72}\sqrt{be^4x^3+ae^4}\left(2e^3x^3\left(\frac{4x^3}{e^4}+\frac{a}{be^4}\right)-\frac{3a^2}{b^2e}\right)\sqrt{ex}Ba^2x|e|^2$$
$$+\frac{1}{36}\sqrt{be^4x^3+ae^4}\left(2e^3x^3\left(\frac{4x^3}{e^4}+\frac{a}{be^4}\right)-\frac{3a^2}{b^2e}\right)\sqrt{ex}Aabx|e|^2$$
$$+\frac{1}{288}\sqrt{be^4x^3+ae^4}\left(2\left(4e^3x^3\left(\frac{6x^3}{e^7}+\frac{a}{be^7}\right)-\frac{5a^2}{b^2e^4}\right)e^3x^3+\frac{15a^3}{b^3e}\right)\sqrt{ex}Babx|e|^2$$
$$+\frac{1}{576}\sqrt{be^4x^3+ae^4}\left(2\left(4e^3x^3\left(\frac{6x^3}{e^7}+\frac{a}{be^7}\right)-\frac{5a^2}{b^2e^4}\right)e^3x^3+\frac{15a^3}{b^3e}\right)\sqrt{ex}Ab^2x|e|^2$$
$$+\frac{1}{5760}\sqrt{be^4x^3+ae^4}\left(2\left(4\left(6e^3x^3\left(\frac{8x^3}{e^{10}}+\frac{a}{be^{10}}\right)-\frac{7a^2}{b^2e^7}\right)e^3x^3+\frac{35a^3}{b^3e^4}\right)e^3x^3-\frac{105a^4}{b^4e}\right)\sqrt{ex}Bb^2x|e|^2$$
$$-\frac{\left(9B^2a^{10}e-60ABa^9be+100A^2a^8b^2e\right)^2e^5\log\left(\left|-\left(3\sqrt{ex}Ba^5e^2x-10\sqrt{ex}Aa^4be^2x\right)\sqrt{be}+\sqrt{9B^2a^{11}e^6-\cdots}\right.\right.}{384\sqrt{be}b^2|9B^2a^{10}e-60ABa^9be+100A^2a^8b^2e||-3\cdots}$$

```
[In] integrate((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*(2*x^3/e + a/(b*e))*A*a^2*x*abs(e)^2
 + 1/72*sqrt(b*e^4*x^3 + a*e^4)*(2*e^3*x^3*(4*x^3/e^4 + a/(b*e^4)) - 3*a^2/
(b^2*e))*sqrt(e*x)*B*a^2*x*abs(e)^2 + 1/36*sqrt(b*e^4*x^3 + a*e^4)*(2*e^3*x
^3*(4*x^3/e^4 + a/(b*e^4)) - 3*a^2/(b^2*e))*sqrt(e*x)*A*a*b*x*abs(e)^2 + 1/
288*sqrt(b*e^4*x^3 + a*e^4)*(2*(4*e^3*x^3*(6*x^3/e^7 + a/(b*e^7)) - 5*a^2/(
b^2*e^4))*e^3*x^3 + 15*a^3/(b^3*e))*sqrt(e*x)*B*a*b*x*abs(e)^2 + 1/576*sqrt
(b*e^4*x^3 + a*e^4)*(2*(4*e^3*x^3*(6*x^3/e^7 + a/(b*e^7)) - 5*a^2/(b^2*e^4)
)*e^3*x^3 + 15*a^3/(b^3*e))*sqrt(e*x)*A*b^2*x*abs(e)^2 + 1/5760*sqrt(b*e^4*
x^3 + a*e^4)*(2*(4*(6*e^3*x^3*(8*x^3/e^10 + a/(b*e^10)) - 7*a^2/(b^2*e^7))*
e^3*x^3 + 35*a^3/(b^3*e^4))*e^3*x^3 - 105*a^4/(b^4*e))*sqrt(e*x)*B*b^2*x*ab
s(e)^2 - 1/384*(9*B^2*a^10*e - 60*A*B*a^9*b*e + 100*A^2*a^8*b^2*e)^2*e^5*lo
g(abs(-(3*sqrt(e*x)*B*a^5*e^2*x - 10*sqrt(e*x)*A*a^4*b*e^2*x)*sqrt(b*e) + s
qrt(9*B^2*a^11*e^6 - 60*A*B*a^10*b*e^6 + 100*A^2*a^9*b^2*e^6 + (3*sqrt(e*x)
*B*a^5*e^2*x - 10*sqrt(e*x)*A*a^4*b*e^2*x)^2*b*e)))/(sqrt(b*e)*b^2*abs(9*B^
2*a^10*e - 60*A*B*a^9*b*e + 100*A^2*a^8*b^2*e)*abs(-3*B*a^5 + 10*A*a^4*b)*a
bs(e)^2)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{7/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \int \left(B\,x^3+A\right) \,\left(e\,x\right)^{7/2} \left(b\,x^3+a\right)^{5/2} \, dx$$

```
[In] int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(5/2),x)
[Out] int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(5/2), x)
```

3.536 $\int (ex)^{5/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx$

Optimal result

Integrand size = 26, antiderivative size = 404

$$\int (ex)^{5/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \frac{81a^3(4Ab-aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{5632b^2} + \frac{27a^2(4Ab-aB)(ex)^{7/2}\sqrt{a+bx^3}}{1408be} + \frac{15a(4Ab-aB)(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{704be} + \frac{(4Ab-aB)(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{44be} + \frac{B(ex)^{7/2}\left(a+bx^3\right)^{7/2}}{14be} - \frac{27\, 3^{3/4}a^{11/3}(4Ab-aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\ldots\right)}{11264b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 15/704*a*(4*A*b-B*a)*(e*x)^(7/2)*(b*x^3+a)^(3/2)/b/e+1/44*(4*A*b-B*a)*(e*x)
^(7/2)*(b*x^3+a)^(5/2)/b/e+1/14*B*(e*x)^(7/2)*(b*x^3+a)^(7/2)/b/e+27/1408*a
^2*(4*A*b-B*a)*(e*x)^(7/2)*(b*x^3+a)^(1/2)/b/e+81/5632*a^3*(4*A*b-B*a)*e^2*
(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^2-27/11264*3^(3/4)*a^(11/3)*(4*A*b-B*a)*e^2*(
a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1
+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3
^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*
(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^2/(b*x^
3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2
)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {470, 285, 327, 335, 231}

$$\int (ex)^{5/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx = -\frac{27\,3^{3/4}a^{11/3}e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(4Ab-aB)\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\right.}{11264b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{81a^3e^2\sqrt{ex}\sqrt{a+bx^3}(4Ab-aB)}{5632b^2} + \frac{27a^2(ex)^{7/2}\sqrt{a+bx^3}(4Ab-aB)}{1408be} + \frac{15a(ex)^{7/2}\left(a+bx^3\right)^{3/2}(4Ab-aB)}{704be} + \frac{(ex)^{7/2}\left(a+bx^3\right)^{5/2}(4Ab-aB)}{44be} + \frac{B(ex)^{7/2}\left(a+bx^3\right)^{7/2}}{14be}$$

```
[In] Int[(e*x)^(5/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (81*a^3*(4*A*b - a*B)*e^2*Sqrt[e*x]*Sqrt[a + b*x^3])/(5632*b^2) + (27*a^2*(
4*A*b - a*B)*(e*x)^(7/2)*Sqrt[a + b*x^3])/(1408*b*e) + (15*a*(4*A*b - a*B)*
(e*x)^(7/2)*(a + b*x^3)^(3/2))/(704*b*e) + ((4*A*b - a*B)*(e*x)^(7/2)*(a +
b*x^3)^(5/2))/(44*b*e) + (B*(e*x)^(7/2)*(a + b*x^3)^(7/2))/(14*b*e) - (27*3
^(3/4)*a^(11/3)*(4*A*b - a*B)*e^2*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sq
rt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(11264*b^2*Sqrt[(b^(1/3)*x*(a^(1/3) +
 b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
```

```
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{B(ex)^{7/2}\left(a+bx^3\right)^{7/2}}{14be}-\frac{\left(-14Ab+\frac{7aB}{2}\right)\int (ex)^{5/2}\left(a+bx^3\right)^{5/2}\,dx}{14b}\\
&= \frac{(4Ab-aB)(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{44be}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{7/2}}{14be}\\
&\quad+\frac{(15a(4Ab-aB))\int (ex)^{5/2}\left(a+bx^3\right)^{3/2}\,dx}{88b}\\
&= \frac{15a(4Ab-aB)(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{704be}+\frac{(4Ab-aB)(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{44be}\\
&\quad+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{7/2}}{14be}+\frac{\left(135a^2(4Ab-aB)\right)\int (ex)^{5/2}\sqrt{a+bx^3}\,dx}{1408b}
\end{aligned}$$

$$= \frac{27a^2(4Ab-aB)(ex)^{7/2}\sqrt{a+bx^3}}{1408be} + \frac{15a(4Ab-aB)(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{704be} + \frac{(4Ab-aB)(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{44be} + \frac{B(ex)^{7/2}\left(a+bx^3\right)^{7/2}}{14be} + \frac{(81a^3(4Ab-aB))\int \frac{(ex)^{5/2}}{\sqrt{a+bx^3}}\,dx}{2816b}$$

$$= \frac{81a^3(4Ab-aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{5632b^2} + \frac{27a^2(4Ab-aB)(ex)^{7/2}\sqrt{a+bx^3}}{1408be} + \frac{15a(4Ab-aB)(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{704be} + \frac{(4Ab-aB)(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{44be} + \frac{B(ex)^{7/2}\left(a+bx^3\right)^{7/2}}{14be} - \frac{(81a^4(4Ab-aB)e^3)\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{11264b^2}$$

$$= \frac{81a^3(4Ab-aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{5632b^2} + \frac{27a^2(4Ab-aB)(ex)^{7/2}\sqrt{a+bx^3}}{1408be} + \frac{15a(4Ab-aB)(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{704be} + \frac{(4Ab-aB)(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{44be} + \frac{B(ex)^{7/2}\left(a+bx^3\right)^{7/2}}{14be} - \frac{(81a^4(4Ab-aB)e^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{5632b^2}$$

$$= \frac{81a^3(4Ab-aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{5632b^2} + \frac{27a^2(4Ab-aB)(ex)^{7/2}\sqrt{a+bx^3}}{1408be} + \frac{15a(4Ab-aB)(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{704be} + \frac{(4Ab-aB)(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{44be} + \frac{B(ex)^{7/2}\left(a+bx^3\right)^{7/2}}{14be} - \frac{27\ 3^{3/4}a^{11/3}(4Ab-aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)\right.}{11264b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.23 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.29

$$\int (ex)^{5/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \frac{e^2\sqrt{ex}\sqrt{a+bx^3}\left(-\left(a+bx^3\right)^3 \sqrt{1+\frac{bx^3}{a}}\left(-28Ab+7aB-22bBx^3\right)+7a^3(-4Ab+a\right.}{308b^2\sqrt{1+\frac{bx^3}{a}}}$$

[In] Integrate[(e*x)^(5/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]

[Out] (e^2*Sqrt[e*x]*Sqrt[a + b*x^3]*(-((a + b*x^3)^3*Sqrt[1 + (b*x^3)/a]*(-28*A*b + 7*a*B - 22*b*B*x^3)) + 7*a^3*(-4*A*b + a*B)*Hypergeometric2F1[-5/2, 1/6, 7/6, -((b*x^3)/a)]))/(308*b^2*Sqrt[1 + (b*x^3)/a])

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.89 (sec) , antiderivative size = 825, normalized size of antiderivative = 2.04

method	result	size
risch	Expression too large to display	825
elliptic	Expression too large to display	1134
default	Expression too large to display	5063

[In] int((e*x)^(5/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x,method=_RETURNVERBOSE)

[Out] 1/39424/b^2*(2816*B*b^4*x^12+3584*A*b^4*x^9+7552*B*a*b^3*x^9+10528*A*a*b^3*x^6+5816*B*a^2*b^2*x^6+9968*A*a^2*b^2*x^3+324*B*a^3*b*x^3+2268*A*a^3*b-567*B*a^4)*x*(b*x^3+a)^(1/2)*e^3/(e*x)^(1/2)-81/5632*a^4/b*(4*A*b-B*a)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-

```
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3))
)^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2))*e^3*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [F]

$$\int (ex)^{5/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right)\, dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{5}{2}}\, dx$$

```
[In] integrate((e*x)^(5/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] integral((B*b^2*e^2*x^11 + (2*B*a*b + A*b^2)*e^2*x^8 + (B*a^2 + 2*A*a*b)*e^
2*x^5 + A*a^2*e^2*x^2)*sqrt(b*x^3 + a)*sqrt(e*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 150.12 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.76

$$\int (ex)^{5/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right)\, dx = \frac{Aa^{\frac{5}{2}}e^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{6}\right)} + \frac{2Aa^{\frac{3}{2}}be^{\frac{5}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{19}{6}\right)} + \frac{A\sqrt{a}b^2e^{\frac{5}{2}}x^{\frac{19}{2}}\Gamma\left(\frac{19}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{19}{6}\\ \frac{25}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{25}{6}\right)} + \frac{Ba^{\frac{5}{2}}e^{\frac{5}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{19}{6}\right)} + \frac{2Ba^{\frac{3}{2}}be^{\frac{5}{2}}x^{\frac{19}{2}}\Gamma\left(\frac{19}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{19}{6}\\ \frac{25}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{25}{6}\right)} + \frac{B\sqrt{a}b^2e^{\frac{5}{2}}x^{\frac{25}{2}}\Gamma\left(\frac{25}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{25}{6}\\ \frac{31}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{31}{6}\right)}$$

```
[In] integrate((e*x)**(5/2)*(b*x**3+a)**(5/2)*(B*x**3+A),x)
```

```
[Out] A*a**(5/2)*e**(5/2)*x**(7/2)*gamma(7/6)*hyper((-1/2, 7/6), (13/6,), b*x**3*
exp_polar(I*pi)/a)/(3*gamma(13/6)) + 2*A*a**(3/2)*b*e**(5/2)*x**(13/2)*gamm
```

```
a(13/6)*hyper((-1/2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(19/
6)) + A*sqrt(a)*b**2*e**(5/2)*x**(19/2)*gamma(19/6)*hyper((-1/2, 19/6), (25
/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(25/6)) + B*a**(5/2)*e**(5/2)*x**(1
3/2)*gamma(13/6)*hyper((-1/2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*
gamma(19/6)) + 2*B*a**(3/2)*b*e**(5/2)*x**(19/2)*gamma(19/6)*hyper((-1/2, 1
9/6), (25/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(25/6)) + B*sqrt(a)*b**2*e
**(5/2)*x**(25/2)*gamma(25/6)*hyper((-1/2, 25/6), (31/6,), b*x**3*exp_polar
(I*pi)/a)/(3*gamma(31/6))
```

Maxima [F]

$$\int (ex)^{5/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{5}{2}} \, dx$$

```
[In] integrate((e*x)^(5/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^(5/2), x)
```

Giac [F]

$$\int (ex)^{5/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{5}{2}} \, dx$$

```
[In] integrate((e*x)^(5/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{5/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{5/2}\,{\left(b\,x^3+a\right)}^{5/2} \,d x$$

```
[In] int((A + B*x^3)*(e*x)^(5/2)*(a + b*x^3)^(5/2),x)
[Out] int((A + B*x^3)*(e*x)^(5/2)*(a + b*x^3)^(5/2), x)
```

3.537 $\int (ex)^{3/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx$

Optimal result

Integrand size = 26, antiderivative size = 661

$$\int (ex)^{3/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \frac{27a^2(26Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{5824be}$$
$$+\frac{81\left(1+\sqrt{3}\right)a^3(26Ab-5aB)e\sqrt{ex}\sqrt{a+bx^3}}{11648b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}+\frac{3a(26Ab-5aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{728be}$$
$$+\frac{(26Ab-5aB)(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{260be}+\frac{B(ex)^{5/2}\left(a+bx^3\right)^{7/2}}{13be}$$
$$-\frac{81\sqrt[4]{3}a^{10/3}(26Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}(2+\sqrt{\ldots}}{11648b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{27\ 3^{3/4}\left(1-\sqrt{3}\right)a^{10/3}(26Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+(1-\ldots}{\sqrt[3]{a}+(1-\ldots}\right.\right.}{23296b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 3/728*a*(26*A*b-5*B*a)*(e*x)^(5/2)*(b*x^3+a)^(3/2)/b/e+1/260*(26*A*b-5*B*a)
*(e*x)^(5/2)*(b*x^3+a)^(5/2)/b/e+1/13*B*(e*x)^(5/2)*(b*x^3+a)^(7/2)/b/e+27/
5824*a^2*(26*A*b-5*B*a)*(e*x)^(5/2)*(b*x^3+a)^(1/2)/b/e+81/11648*a^3*(26*A*
b-5*B*a)*e*(1+3^(1/2))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^(5/3)/(a^(1/3)+b^(1/3)
*x*(1+3^(1/2)))-81/11648*3^(1/4)*a^(10/3)*(26*A*b-5*B*a)*e*(a^(1/3)+b^(1/3)
*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(
```

```
1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*Ellipt
icE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)
^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/
(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)-27/
23296*3^(3/4)*a^(10/3)*(26*A*b-5*B*a)*e*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/
3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/
3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(
1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+
1/4*2^(1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3
)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 661, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {470, 285, 335, 314, 231, 1895}

$$\int (ex)^{3/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) dx =$$

$$-\frac{27\ 3^{3/4}\left(1-\sqrt{3}\right) a^{10/3} e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(26Ab-5aB)\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{}}{\left(1+\sqrt{3}\right)\sqrt[3]{}}\right.\right.}{23296b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{81\sqrt[4]{3}a^{10/3}e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(26Ab-5aB)E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}(2+\sqrt{3}\right.}{11648b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{81\left(1+\sqrt{3}\right)a^3e\sqrt{ex}\sqrt{a+bx^3}(26Ab-5aB)}{11648b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$

$$+\frac{27a^2(ex)^{5/2}\sqrt{a+bx^3}(26Ab-5aB)}{5824be}+\frac{(ex)^{5/2}\left(a+bx^3\right)^{5/2}(26Ab-5aB)}{260be}$$

$$+\frac{3a(ex)^{5/2}\left(a+bx^3\right)^{3/2}(26Ab-5aB)}{728be}+\frac{B(ex)^{5/2}\left(a+bx^3\right)^{7/2}}{13be}$$

```
[In] Int[(e*x)^(3/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (27*a^2*(26*A*b - 5*a*B)*(e*x)^(5/2)*Sqrt[a + b*x^3])/(5824*b*e) + (81*(1 +
 Sqrt[3])*a^3*(26*A*b - 5*a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^3])/(11648*b^(5/3)*
(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)) + (3*a*(26*A*b - 5*a*B)*(e*x)^(5/2)*(a
 + b*x^3)^(3/2))/(728*b*e) + ((26*A*b - 5*a*B)*(e*x)^(5/2)*(a + b*x^3)^(5/2
))/(260*b*e) + (B*(e*x)^(5/2)*(a + b*x^3)^(7/2))/(13*b*e) - (81*3^(1/4)*a^(
10/3)*(26*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Ellip
ticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^
(1/3)*x)], (2 + Sqrt[3])/4])/(11648*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1
/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (27*3^(3/
4)*(1 - Sqrt[3])*a^(10/3)*(26*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x
)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])
*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3
) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(23296*b^(5/3)*Sqrt[(b^(1/
3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a +
 b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{B(ex)^{5/2}\left(a+bx^3\right)^{7/2}}{13be} - \frac{\left(-13Ab+\frac{5aB}{2}\right)\int (ex)^{3/2}\left(a+bx^3\right)^{5/2}\,dx}{13b} \\
&= \frac{(26Ab-5aB)(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{260be} + \frac{B(ex)^{5/2}\left(a+bx^3\right)^{7/2}}{13be} \\
&\quad + \frac{(3a(26Ab-5aB))\int (ex)^{3/2}\left(a+bx^3\right)^{3/2}\,dx}{104b} \\
&= \frac{3a(26Ab-5aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{728be} + \frac{(26Ab-5aB)(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{260be} \\
&\quad + \frac{B(ex)^{5/2}\left(a+bx^3\right)^{7/2}}{13be} + \frac{\left(27a^2(26Ab-5aB)\right)\int (ex)^{3/2}\sqrt{a+bx^3}\,dx}{1456b} \\
&= \frac{27a^2(26Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{5824be} + \frac{3a(26Ab-5aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{728be} \\
&\quad + \frac{(26Ab-5aB)(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{260be} \\
&\quad + \frac{B(ex)^{5/2}\left(a+bx^3\right)^{7/2}}{13be} + \frac{\left(81a^3(26Ab-5aB)\right)\int \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\,dx}{11648b}
\end{aligned}$$

$$= \frac{27a^2(26Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{5824be} + \frac{3a(26Ab-5aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{728be} + \frac{(26Ab-5aB)(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{260be} + \frac{B(ex)^{5/2}\left(a+bx^3\right)^{7/2}}{13be} + \frac{(81a^3(26Ab-5aB))\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{5824be}$$

$$= \frac{27a^2(26Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{5824be} + \frac{3a(26Ab-5aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{728be} + \frac{(26Ab-5aB)(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{260be} + \frac{B(ex)^{5/2}\left(a+bx^3\right)^{7/2}}{13be} - \frac{(81a^3(26Ab-5aB))\,\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}e^2-2b^{2/3}x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{11648b^{5/3}e} - \frac{\left(81\left(1-\sqrt{3}\right)a^{11/3}(26Ab-5aB)e\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{11648b^{5/3}}$$

$$= \frac{27a^2(26Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{5824be} + \frac{81\left(1+\sqrt{3}\right)a^3(26Ab-5aB)e\sqrt{ex}\sqrt{a+bx^3}}{11648b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)} + \frac{3a(26Ab-5aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{728be} + \frac{(26Ab-5aB)(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{260be} + \frac{B(ex)^{5/2}\left(a+bx^3\right)^{7/2}}{13be} - \frac{81\sqrt[4]{3}a^{10/3}(26Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)\right.}{11648b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{27\ 3^{3/4}\left(1-\sqrt{3}\right)a^{10/3}(26Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+(}{\sqrt[3]{a}+(}\right.\right.}{23296b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.15

$$\int (ex)^{3/2} (a+bx^3)^{5/2} (A+Bx^3) \, dx = \frac{x(ex)^{3/2}\sqrt{a+bx^3}\left(5B(a+bx^3)^3 \sqrt{1+\frac{bx^3}{a}}+a^2(26Ab-5aB) \text{Hypergeometric2F1}\right.}{65b\sqrt{1+\frac{bx^3}{a}}}$$

```
[In] Integrate[(e*x)^(3/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (x*(e*x)^(3/2)*Sqrt[a + b*x^3]*(5*B*(a + b*x^3)^3*Sqrt[1 + (b*x^3)/a] + a^2
*(26*A*b - 5*a*B)*Hypergeometric2F1[-5/2, 5/6, 11/6, -((b*x^3)/a)]))/(65*b*
Sqrt[1 + (b*x^3)/a])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.82 (sec) , antiderivative size = 1188, normalized size of antiderivative = 1.80

method	result	size
risch	Expression too large to display	1188
elliptic	Expression too large to display	1410
default	Expression too large to display	6202

```
[In] int((e*x)^(3/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/29120/b*x^3*(2240*B*b^3*x^9+2912*A*b^3*x^6+6160*B*a*b^2*x^6+8944*A*a*b^2*
x^3+5000*B*a^2*b*x^3+9542*A*a^2*b+405*B*a^3)*(b*x^3+a)^(1/2)*e^2/(e*x)^(1/2
)+81/11648*a^3/b*(26*A*b-5*B*a)*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(
-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(
1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2
)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*
(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
```

```
^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*
b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2))*b/(-a*b^2)^(1/3)))/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*e^2*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/
(b*x^3+a)^(1/2)
```

Fricas [F]

$$\int (ex)^{3/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{3}{2}}\,dx$$

```
[In] integrate((e*x)^(3/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="fricas")
[Out] integral((B*b^2*e*x^10 + (2*B*a*b + A*b^2)*e*x^7 + (B*a^2 + 2*A*a*b)*e*x^4
+ A*a^2*e*x)*sqrt(b*x^3 + a)*sqrt(e*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 52.73 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.47

$$\int (ex)^{3/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \frac{Aa^{\frac{5}{2}}e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{6} \\ \frac{11}{6} \end{matrix} \middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{6}\right)} + \frac{2Aa^{\frac{3}{2}}be^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{11}{6} \\ \frac{17}{6} \end{matrix} \middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{17}{6}\right)} + \frac{A\sqrt{a}b^2e^{\frac{3}{2}}x^{\frac{17}{2}}\Gamma\left(\frac{17}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{17}{6} \\ \frac{23}{6} \end{matrix} \middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{23}{6}\right)} + \frac{Ba^{\frac{5}{2}}e^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{11}{6} \\ \frac{17}{6} \end{matrix} \middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{17}{6}\right)} + \frac{2Ba^{\frac{3}{2}}be^{\frac{3}{2}}x^{\frac{17}{2}}\Gamma\left(\frac{17}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{17}{6} \\ \frac{23}{6} \end{matrix} \middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{23}{6}\right)} + \frac{B\sqrt{a}b^2e^{\frac{3}{2}}x^{\frac{23}{2}}\Gamma\left(\frac{23}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{23}{6} \\ \frac{29}{6} \end{matrix} \middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{29}{6}\right)}$$

```
[In] integrate((e*x)**(3/2)*(b*x**3+a)**(5/2)*(B*x**3+A),x)
```

```
[Out] A*a**(5/2)*e**(3/2)*x**(5/2)*gamma(5/6)*hyper((-1/2, 5/6), (11/6,), b*x**3*
exp_polar(I*pi)/a)/(3*gamma(11/6)) + 2*A*a**(3/2)*b*e**(3/2)*x**(11/2)*gamm
a(11/6)*hyper((-1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(17/
6)) + A*sqrt(a)*b**2*e**(3/2)*x**(17/2)*gamma(17/6)*hyper((-1/2, 17/6), (23
/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(23/6)) + B*a**(5/2)*e**(3/2)*x**(1
1/2)*gamma(11/6)*hyper((-1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*
gamma(17/6)) + 2*B*a**(3/2)*b*e**(3/2)*x**(17/2)*gamma(17/6)*hyper((-1/2, 1
7/6), (23/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(23/6)) + B*sqrt(a)*b**2*e
**(3/2)*x**(23/2)*gamma(23/6)*hyper((-1/2, 23/6), (29/6,), b*x**3*exp_polar
(I*pi)/a)/(3*gamma(29/6))
```

Maxima [F]

$$\int (ex)^{3/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{3}{2}} \, dx$$

```
[In] integrate((e*x)^(3/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^(3/2), x)
```

Giac [F]

$$\int (ex)^{3/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \int \left(Bx^3+A\right) \left(bx^3+a\right)^{\frac{5}{2}} (ex)^{\frac{3}{2}} \, dx$$

```
[In] integrate((e*x)^(3/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{3/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) \, dx = \int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{3/2}\,{\left(b\,x^3+a\right)}^{5/2} \,d x$$

```
[In] int((A + B*x^3)*(e*x)^(3/2)*(a + b*x^3)^(5/2),x)
[Out] int((A + B*x^3)*(e*x)^(3/2)*(a + b*x^3)^(5/2), x)
```

3.538 $\int \sqrt{ex}(a+bx^3)^{5/2}\left(A+Bx^3\right)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 201

$$\int \sqrt{ex}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx=\frac{5a^2(8Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{192be}+\frac{5a(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{288be}+\frac{(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{72be}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{7/2}}{12be}+\frac{5a^3(8Ab-aB)\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{192b^{3/2}}$$

```
[Out] 5/288*a*(8*A*b-B*a)*(e*x)^(3/2)*(b*x^3+a)^(3/2)/b/e+1/72*(8*A*b-B*a)*(e*x)^
(3/2)*(b*x^3+a)^(5/2)/b/e+1/12*B*(e*x)^(3/2)*(b*x^3+a)^(7/2)/b/e+5/192*a^3*
(8*A*b-B*a)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))*e^(1/2)/b^
(3/2)+5/192*a^2*(8*A*b-B*a)*(e*x)^(3/2)*(b*x^3+a)^(1/2)/b/e
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {470, 285, 335, 281, 223, 212}

$$\int \sqrt{ex}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx=\frac{5a^3\sqrt{e}(8Ab-aB)\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{192b^{3/2}}+\frac{5a^2(ex)^{3/2}\sqrt{a+bx^3}(8Ab-aB)}{192be}+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}(8Ab-aB)}{72be}+\frac{5a(ex)^{3/2}\left(a+bx^3\right)^{3/2}(8Ab-aB)}{288be}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{7/2}}{12be}$$

```
[In] Int[Sqrt[e*x]*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (5*a^2*(8*A*b - a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(192*b*e) + (5*a*(8*A*b -
 a*B)*(e*x)^(3/2)*(a + b*x^3)^(3/2))/(288*b*e) + ((8*A*b - a*B)*(e*x)^(3/2)
*(a + b*x^3)^(5/2))/(72*b*e) + (B*(e*x)^(3/2)*(a + b*x^3)^(7/2))/(12*b*e) +
 (5*a^3*(8*A*b - a*B)*Sqrt[e]*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a
 + b*x^3])])/(192*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{B(ex)^{3/2}(a+bx^3)^{7/2}}{12be} - \frac{\left(-12Ab+\frac{3aB}{2}\right)\int \sqrt{ex}(a+bx^3)^{5/2}\,dx}{12b}$$

$$= \frac{(8Ab-aB)(ex)^{3/2}(a+bx^3)^{5/2}}{72be} + \frac{B(ex)^{3/2}(a+bx^3)^{7/2}}{12be} + \frac{(5a(8Ab-aB))\int \sqrt{ex}(a+bx^3)^{3/2}\,dx}{48b}$$

$$= \frac{5a(8Ab-aB)(ex)^{3/2}(a+bx^3)^{3/2}}{288be} + \frac{(8Ab-aB)(ex)^{3/2}(a+bx^3)^{5/2}}{72be} + \frac{B(ex)^{3/2}(a+bx^3)^{7/2}}{12be} + \frac{(5a^2(8Ab-aB))\int \sqrt{ex}\sqrt{a+bx^3}\,dx}{64b}$$

$$= \frac{5a^2(8Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{192be} + \frac{5a(8Ab-aB)(ex)^{3/2}(a+bx^3)^{3/2}}{288be} + \frac{(8Ab-aB)(ex)^{3/2}(a+bx^3)^{5/2}}{72be} + \frac{B(ex)^{3/2}(a+bx^3)^{7/2}}{12be} + \frac{(5a^3(8Ab-aB))\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{128b}$$

$$= \frac{5a^2(8Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{192be} + \frac{5a(8Ab-aB)(ex)^{3/2}(a+bx^3)^{3/2}}{288be} + \frac{(8Ab-aB)(ex)^{3/2}(a+bx^3)^{5/2}}{72be} + \frac{B(ex)^{3/2}(a+bx^3)^{7/2}}{12be} + \frac{(5a^3(8Ab-aB))\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{64be}$$

$$= \frac{5a^2(8Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{192be} + \frac{5a(8Ab-aB)(ex)^{3/2}(a+bx^3)^{3/2}}{288be} + \frac{(8Ab-aB)(ex)^{3/2}(a+bx^3)^{5/2}}{72be} + \frac{B(ex)^{3/2}(a+bx^3)^{7/2}}{12be} + \frac{(5a^3(8Ab-aB))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx, x, (ex)^{3/2}\right)}{192be}$$

$$= \frac{5a^2(8Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{192be} + \frac{5a(8Ab-aB)(ex)^{3/2}(a+bx^3)^{3/2}}{288be} + \frac{(8Ab-aB)(ex)^{3/2}(a+bx^3)^{5/2}}{72be} + \frac{B(ex)^{3/2}(a+bx^3)^{7/2}}{12be} + \frac{(5a^3(8Ab-aB))\,\text{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx, x, \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{192be}$$

$$=\frac{5a^2(8Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{192be}+\frac{5a(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{288be}+\frac{(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{72be}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{7/2}}{12be}+\frac{5a^3(8Ab-aB)\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{192b^{3/2}}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.70

$$\int \sqrt{ex}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx=\frac{\sqrt{ex}\left(\sqrt{b}x^{3/2}\sqrt{a+bx^3}(15a^3B+16b^3x^6(4A+3Bx^3)+8ab^2x^3(26A+17Bx^3)+2a^2b(132A+5\cdots\right.}{576b^{3/2}\sqrt{x}}$$

```
[In] Integrate[Sqrt[e*x]*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (Sqrt[e*x]*(Sqrt[b]*x^(3/2)*Sqrt[a + b*x^3]*(15*a^3*B + 16*b^3*x^6*(4*A + 3
*B*x^3) + 8*a*b^2*x^3*(26*A + 17*B*x^3) + 2*a^2*b*(132*A + 59*B*x^3)) - 15*
a^3*(-8*A*b + a*B)*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]]))/(576*b^(3/2)*Sq
rt[x])
```

Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.79

method	result
risch	$\frac{x^2\left(48b^3Bx^9+64x^6b^3A+136Bx^6ab^2+208aAb^2x^3+118Ba^2bx^3+264a^2bA+15a^3B\right)\sqrt{bx^3+a}\,e}{576b\sqrt{ex}}+\frac{5a^3(8Ab-Ba)\operatorname{arctanh}\left(\frac{\sqrt{(bx^3\cdots}}{x^2\sqrt{\cdots}}\right)}{192b\sqrt{be}\sqrt{ex}\sqrt{\cdots}}$
default	$\frac{\sqrt{bx^3+a}\sqrt{ex}\left(48B\sqrt{(bx^3+a)ex}\sqrt{be}\,b^3x^{10}+64A\sqrt{(bx^3+a)ex}\sqrt{be}\,b^3x^7+136B\sqrt{(bx^3+a)ex}\sqrt{be}\,ab^2x^7+208A\sqrt{(bx^3+a)ex}\sqrt{be}\,a\cdots\right.}{\cdots}$
elliptic	Expression too large to display

```
[In] int((b*x^3+a)^(5/2)*(B*x^3+A)*(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/576/b*x^2*(48*B*b^3*x^9+64*A*b^3*x^6+136*B*a*b^2*x^6+208*A*a*b^2*x^3+118*
B*a^2*b*x^3+264*A*a^2*b+15*B*a^3)*(b*x^3+a)^(1/2)*e/(e*x)^(1/2)+5/192*a^3/b
*(8*A*b-B*a)/(b*e)^(1/2)*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))*e*(
(b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.57 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.61

$$\int \sqrt{ex}(a+bx^3)^{5/2}(A+Bx^3)\,dx = \left[-\frac{15\,(Ba^4-8\,Aa^3b)\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\,(2\,b^2x^4+abx)\sqrt{bx^3+a}\sqrt{ex}\sqrt{\frac{e}{b}}\right)-\right.$$

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2304*(15*(B*a^4 - 8*A*a^3*b)*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 -
 a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(48
*B*b^3*x^10 + 8*(17*B*a*b^2 + 8*A*b^3)*x^7 + 2*(59*B*a^2*b + 104*A*a*b^2)*x
^4 + 3*(5*B*a^3 + 88*A*a^2*b)*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b, 1/1152*(15*(
B*a^4 - 8*A*a^3*b)*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-
e/b)/(2*b*e*x^3 + a*e)) + 2*(48*B*b^3*x^10 + 8*(17*B*a*b^2 + 8*A*b^3)*x^7 +
 2*(59*B*a^2*b + 104*A*a*b^2)*x^4 + 3*(5*B*a^3 + 88*A*a^2*b)*x)*sqrt(b*x^3
+ a)*sqrt(e*x))/b]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 896 vs. 2(177) = 354.

Time = 7.75 (sec) , antiderivative size = 896, normalized size of antiderivative = 4.46

$$\int \sqrt{ex}(a+bx^3)^{5/2}(A+Bx^3)\,dx = \text{Too large to display}$$

```
[In] integrate((b*x**3+a)**(5/2)*(B*x**3+A)*(e*x)**(1/2),x)
```

```
[Out] Piecewise((2*Piecewise((nan, Eq(e**3, 0)), ((A*a**2*e**3*Piecewise((a*Piece
wise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e
**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), True))/2 + (
e*x)**(3/2)*sqrt(a + b*x**3)/2, Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(3/2), True
)) + 2*A*a*b*Piecewise((-a**2*e**3*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2
*sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log(
(e*x)**(3/2))/sqrt(b*x**3), True))/(8*b) + sqrt(a + b*x**3)*(a*e**3*(e*x)**
(3/2)/(8*b) + (e*x)**(9/2)/4), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(9/2)/3, Tru
e)) + A*b**2*Piecewise((a**3*e**6*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*
sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((
e*x)**(3/2))/sqrt(b*x**3), True))/(16*b**2) + sqrt(a + b*x**3)*(-a**2*e**6*
```

```
(e*x)**(3/2)/(16*b**2) + a*e**3*(e*x)**(9/2)/(24*b) + (e*x)**(15/2)/6), Ne(
b/e**3, 0)), (sqrt(a)*(e*x)**(15/2)/5, True))/e**3 + B*a**2*Piecewise((-a**
2*e**3*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**
3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3),
True))/(8*b) + sqrt(a + b*x**3)*(a*e**3*(e*x)**(3/2)/(8*b) + (e*x)**(9/2)/4
), Ne(b/e**3, 0)), (sqrt(a)*(e*x)**(9/2)/3, True)) + 2*B*a*b*Piecewise((a**
3*e**6*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**
3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3),
True))/(16*b**2) + sqrt(a + b*x**3)*(-a**2*e**6*(e*x)**(3/2)/(16*b**2) + a*
e**3*(e*x)**(9/2)/(24*b) + (e*x)**(15/2)/6), Ne(b/e**3, 0)), (sqrt(a)*(e*x)
**(15/2)/5, True))/e**3 + B*b**2*Piecewise((-5*a**4*e**9*Piecewise((log(2*b
*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0
)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), True))/(128*b**3) + sqrt(
a + b*x**3)*(5*a**3*e**9*(e*x)**(3/2)/(128*b**3) - 5*a**2*e**6*(e*x)**(9/2)
/(192*b**2) + a*e**3*(e*x)**(15/2)/(48*b) + (e*x)**(21/2)/8), Ne(b/e**3, 0)
), (sqrt(a)*(e*x)**(21/2)/7, True))/e**6)/(3*e**3), True))/e, Ne(e, 0)), (0
, True))
```

Maxima [F]

$$\int \sqrt{ex}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx=\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}\sqrt{ex}\,dx$$

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*sqrt(e*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 590 vs. $2(159) = 318$.

Time = 0.52 (sec) , antiderivative size = 590, normalized size of antiderivative = 2.94

$$\int \sqrt{ex}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx = \frac{\sqrt{be^4x^3+ae^4}\sqrt{ex}\left(\frac{2\,x^3}{e}+\frac{a}{be}\right)Ba^2x|e|^2}{12\,e^3}$$
$$+\frac{\sqrt{be^4x^3+ae^4}\sqrt{ex}\left(\frac{2\,x^3}{e}+\frac{a}{be}\right)Aabx|e|^2}{6\,e^3}$$
$$+\frac{\sqrt{be^4x^3+ae^4}\left(2\,e^3x^3\left(\frac{4\,x^3}{e^4}+\frac{a}{be^4}\right)-\frac{3\,a^2}{b^2e}\right)\sqrt{ex}Babx|e|^2}{36\,e^3}$$
$$+\frac{\sqrt{be^4x^3+ae^4}\left(2\,e^3x^3\left(\frac{4\,x^3}{e^4}+\frac{a}{be^4}\right)-\frac{3\,a^2}{b^2e}\right)\sqrt{ex}Ab^2x|e|^2}{72\,e^3}$$
$$+\frac{\sqrt{be^4x^3+ae^4}\left(2\left(4\,e^3x^3\left(\frac{6\,x^3}{e^7}+\frac{a}{be^7}\right)-\frac{5\,a^2}{b^2e^4}\right)e^3x^3+\frac{15\,a^3}{b^3e}\right)\sqrt{ex}Bb^2x|e|^2}{576\,e^3}$$
$$-\frac{(25\,B^2a^8+240\,ABa^7b+576\,A^2a^6b^2)e^4\log\left(\left|(5\,\sqrt{ex}Ba^4x+24\,\sqrt{ex}Aa^3bx)\sqrt{be}+\sqrt{25\,B^2a^9e^2+240\,ABa\right.\right.}{192\,\sqrt{be}b|5\,Ba^4e+24\,Aa^3be||e|^2}$$
$$-\frac{\left(\frac{ae^4\log\left(\left|-\sqrt{be}\sqrt{ex}ex+\sqrt{be^4x^3+ae^4}\right|\right)}{\sqrt{be}}-\sqrt{be^4x^3+ae^4}\sqrt{ex}ex\right)Aa^2|e|^2}{3\,e^5}$$

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*(2*x^3/e + a/(b*e))*B*a^2*x*abs(e)^2
/e^3 + 1/6*sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*(2*x^3/e + a/(b*e))*A*a*b*x*ab
s(e)^2/e^3 + 1/36*sqrt(b*e^4*x^3 + a*e^4)*(2*e^3*x^3*(4*x^3/e^4 + a/(b*e^4)
) - 3*a^2/(b^2*e))*sqrt(e*x)*B*a*b*x*abs(e)^2/e^3 + 1/72*sqrt(b*e^4*x^3 + a
*e^4)*(2*e^3*x^3*(4*x^3/e^4 + a/(b*e^4)) - 3*a^2/(b^2*e))*sqrt(e*x)*A*b^2*x
*abs(e)^2/e^3 + 1/576*sqrt(b*e^4*x^3 + a*e^4)*(2*(4*e^3*x^3*(6*x^3/e^7 + a/
(b*e^7)) - 5*a^2/(b^2*e^4))*e^3*x^3 + 15*a^3/(b^3*e))*sqrt(e*x)*B*b^2*x*abs
(e)^2/e^3 - 1/192*(25*B^2*a^8 + 240*A*B*a^7*b + 576*A^2*a^6*b^2)*e^4*log(ab
s((5*sqrt(e*x)*B*a^4*x + 24*sqrt(e*x)*A*a^3*b*x)*sqrt(b*e) + sqrt(25*B^2*a^
9*e^2 + 240*A*B*a^8*b*e^2 + 576*A^2*a^7*b^2*e^2 + (5*sqrt(e*x)*B*a^4*x + 24
*sqrt(e*x)*A*a^3*b*x)^2*b*e)))/(sqrt(b*e)*b*abs(5*B*a^4*e + 24*A*a^3*b*e)*a
bs(e)^2) - 1/3*(a*e^4*log(abs(-sqrt(b*e)*sqrt(e*x)*e*x + sqrt(b*e^4*x^3 + a
*e^4)))/sqrt(b*e) - sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*e*x)*A*a^2*abs(e)^2/e
^5
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{ex}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)\,dx=\int \left(B\,x^3+A\right)\,\sqrt{e\,x}\,\left(b\,x^3+a\right)^{5/2}\,dx$$

```
[In] int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(5/2),x)
[Out] int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(5/2), x)
```

3.539 $\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{\sqrt{ex}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 364

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \frac{27a^2(22Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{1408be} + \frac{3a(22Ab-aB)\sqrt{ex}(a+bx^3)^{3/2}}{352be} + \frac{(22Ab-aB)\sqrt{ex}(a+bx^3)^{5/2}}{176be} + \frac{B\sqrt{ex}(a+bx^3)^{7/2}}{11be} + \frac{27\ 3^{3/4}a^{8/3}(22Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\ \text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(2}{2816be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 3/352*a*(22*A*b-B*a)*(b*x^3+a)^(3/2)*(e*x)^(1/2)/b/e+1/176*(22*A*b-B*a)*(b*
x^3+a)^(5/2)*(e*x)^(1/2)/b/e+1/11*B*(b*x^3+a)^(7/2)*(e*x)^(1/2)/b/e+27/1408
*a^2*(22*A*b-B*a)*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b/e+27/2816*3^(3/4)*a^(8/3)*(
22*A*b-B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)
+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b
^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1
/2)/b/e/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(
1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {470, 285, 335, 231}

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \frac{27\,3^{3/4}a^{8/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(22Ab-aB)\,\text{EllipticF}\left(\right.}{2816be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+b}} + \frac{27a^2\sqrt{ex}\sqrt{a+bx^3}(22Ab-aB)}{1408be} + \frac{\sqrt{ex}(a+bx^3)^{5/2}(22Ab-aB)}{176be} + \frac{3a\sqrt{ex}(a+bx^3)^{3/2}(22Ab-aB)}{352be} + \frac{B\sqrt{ex}(a+bx^3)^{7/2}}{11be}$$

```
[In] Int[((a + b*x^3)^(5/2)*(A + B*x^3))/Sqrt[e*x],x]
```

```
[Out] (27*a^2*(22*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(1408*b*e) + (3*a*(22*A*b
 - a*B)*Sqrt[e*x]*(a + b*x^3)^(3/2))/(352*b*e) + ((22*A*b - a*B)*Sqrt[e*x]*
(a + b*x^3)^(5/2))/(176*b*e) + (B*Sqrt[e*x]*(a + b*x^3)^(7/2))/(11*b*e) + (
27*3^(3/4)*a^(8/3)*(22*A*b - a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sq
rt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(2816*b*e*Sqrt[(b^(1/3)*x*(a^(1/3) +
b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{B\sqrt{ex}(a+bx^3)^{7/2}}{11be} - \frac{\left(-11Ab+\frac{aB}{2}\right)\int \frac{(a+bx^3)^{5/2}}{\sqrt{ex}}\,dx}{11b}$$

$$= \frac{(22Ab-aB)\sqrt{ex}(a+bx^3)^{5/2}}{176be} + \frac{B\sqrt{ex}(a+bx^3)^{7/2}}{11be} + \frac{(15a(22Ab-aB))\int \frac{(a+bx^3)^{3/2}}{\sqrt{ex}}\,dx}{352b}$$

$$= \frac{3a(22Ab-aB)\sqrt{ex}(a+bx^3)^{3/2}}{352be} + \frac{(22Ab-aB)\sqrt{ex}(a+bx^3)^{5/2}}{176be} + \frac{B\sqrt{ex}(a+bx^3)^{7/2}}{11be} + \frac{(27a^2(22Ab-aB))\int \frac{\sqrt{a+bx^3}}{\sqrt{ex}}\,dx}{704b}$$

$$= \frac{27a^2(22Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{1408be} + \frac{3a(22Ab-aB)\sqrt{ex}(a+bx^3)^{3/2}}{352be} + \frac{(22Ab-aB)\sqrt{ex}(a+bx^3)^{5/2}}{176be} + \frac{B\sqrt{ex}(a+bx^3)^{7/2}}{11be} + \frac{(81a^3(22Ab-aB))\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{2816b}$$

$$= \frac{27a^2(22Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{1408be} + \frac{3a(22Ab-aB)\sqrt{ex}(a+bx^3)^{3/2}}{352be} + \frac{(22Ab-aB)\sqrt{ex}(a+bx^3)^{5/2}}{176be} + \frac{B\sqrt{ex}(a+bx^3)^{7/2}}{11be} + \frac{(81a^3(22Ab-aB))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{1408be}$$

$$= \frac{27a^2(22Ab - aB)\sqrt{ex}\sqrt{a+bx^3}}{1408be} + \frac{3a(22Ab - aB)\sqrt{ex}(a+bx^3)^{3/2}}{352be}$$
$$+ \frac{(22Ab - aB)\sqrt{ex}(a+bx^3)^{5/2}}{176be} + \frac{B\sqrt{ex}(a+bx^3)^{7/2}}{11be}$$
$$+ \frac{27\ 3^{3/4}a^{8/3}(22Ab - aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)\right|}{2816be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.23

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \frac{x\sqrt{a+bx^3}\left(B(a+bx^3)^3 - \frac{a^2(-22Ab+aB)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{6},\frac{7}{6},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{11b\sqrt{ex}}$$

```
[In] Integrate[((a + b*x^3)^(5/2)*(A + B*x^3))/Sqrt[e*x],x]
[Out] (x*Sqrt[a + b*x^3]*(B*(a + b*x^3)^3 - (a^2*(-22*A*b + a*B)*Hypergeometric2F
1[-5/2, 1/6, 7/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(11*b*Sqrt[e*x])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.84 (sec) , antiderivative size = 792, normalized size of antiderivative = 2.18

method	result
risch	$\frac{(128b^3Bx^9+176x^6b^3A+376Bx^6ab^2+616aAb^2x^3+356Ba^2bx^3+1034a^2bA+81a^3B)x\sqrt{bx^3+a}}{1408b\sqrt{ex}} + \frac{81a^3(22Ab-Ba)\left(\frac{\left(-ab^2\right)^{\frac{1}{3}}}{2b}-\right.}{}$
elliptic	Expression too large to display
default	Expression too large to display

```
[In] int((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

[Out] 1/1408/b*(128*B*b^3*x^9+176*A*b^3*x^6+376*B*a*b^2*x^6+616*A*a*b^2*x^3+356*B
*a^2*b*x^3+1034*A*a^2*b+81*B*a^3)*x*(b*x^3+a)^(1/2)/(e*x)^(1/2)+81/1408*a^3
*(22*A*b-B*a)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2
)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(
-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/
(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2))*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(
1/2)

Fricas [F]

$$\int \frac{\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)}{\sqrt{ex}}\,dx=\int \frac{(Bx^3+A)(bx^3+a)^{\frac{5}{2}}}{\sqrt{ex}}\,dx$$

[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="fricas")

[Out] integral((B*b^2*x^9 + (2*B*a*b + A*b^2)*x^6 + (B*a^2 + 2*A*a*b)*x^3 + A*a^2
)*sqrt(b*x^3 + a)*sqrt(e*x)/(e*x), x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 17.85 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \frac{Aa^{\frac{5}{2}}\sqrt{x}\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\Gamma\left(\frac{7}{6}\right)} + \frac{2Aa^{\frac{3}{2}}bx^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\Gamma\left(\frac{13}{6}\right)} + \frac{A\sqrt{a}b^2x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\Gamma\left(\frac{19}{6}\right)} + \frac{Ba^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\Gamma\left(\frac{13}{6}\right)} + \frac{2Ba^{\frac{3}{2}}bx^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\Gamma\left(\frac{19}{6}\right)} + \frac{B\sqrt{a}b^2x^{\frac{19}{2}}\Gamma\left(\frac{19}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{19}{6}\\ \frac{25}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\Gamma\left(\frac{25}{6}\right)}$$

```
[In] integrate((b*x**3+a)**(5/2)*(B*x**3+A)/(e*x)**(1/2),x)
```

```
[Out] A*a**(5/2)*sqrt(x)*gamma(1/6)*hyper((-1/2, 1/6), (7/6,), b*x**3*exp_polar(I
*pi)/a)/(3*sqrt(e)*gamma(7/6)) + 2*A*a**(3/2)*b*x**(7/2)*gamma(7/6)*hyper((
-1/2, 7/6), (13/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(13/6)) + A*
sqrt(a)*b**2*x**(13/2)*gamma(13/6)*hyper((-1/2, 13/6), (19/6,), b*x**3*exp_
polar(I*pi)/a)/(3*sqrt(e)*gamma(19/6)) + B*a**(5/2)*x**(7/2)*gamma(7/6)*hyp
er((-1/2, 7/6), (13/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(13/6))
+ 2*B*a**(3/2)*b*x**(13/2)*gamma(13/6)*hyper((-1/2, 13/6), (19/6,), b*x**3*
exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(19/6)) + B*sqrt(a)*b**2*x**(19/2)*gamma
(19/6)*hyper((-1/2, 19/6), (25/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*ga
mma(25/6))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{5}{2}}}{\sqrt{ex}}\,dx$$

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/sqrt(e*x), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{5}{2}}}{\sqrt{ex}}\,dx$$

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/sqrt(e*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{\sqrt{ex}}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{5/2}}{\sqrt{e\,x}}\,dx$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(1/2),x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(1/2), x)
```

3.540 $\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 650

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = \frac{27a(20Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{224e^4}$$
$$+\frac{81\left(1+\sqrt{3}\right)a^2(20Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{448b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}+\frac{3(20Ab+aB)(ex)^{5/2}(a+bx^3)^{3/2}}{28e^4}$$
$$+\frac{(20Ab+aB)(ex)^{5/2}(a+bx^3)^{5/2}}{10ae^4}-\frac{2A(a+bx^3)^{7/2}}{ae\sqrt{ex}}$$
$$-\frac{81\sqrt[4]{3}a^{7/3}(20Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right.}{448b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{27\ 3^{3/4}\left(1-\sqrt{3}\right)a^{7/3}(20Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right.}{\sqrt[3]{a}+\left(1+\sqrt{3}\right.}\right.\right.}{896b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 3/28*(20*A*b+B*a)*(e*x)^(5/2)*(b*x^3+a)^(3/2)/e^4+1/10*(20*A*b+B*a)*(e*x)^(
5/2)*(b*x^3+a)^(5/2)/a/e^4-2*A*(b*x^3+a)^(7/2)/a/e/(e*x)^(1/2)+27/224*a*(20
*A*b+B*a)*(e*x)^(5/2)*(b*x^3+a)^(1/2)/e^4+81/448*a^2*(20*A*b+B*a)*(1+3^(1/2
))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^(2/3)/e^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))-
81/448*3^(1/4)*a^(7/3)*(20*A*b+B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x
```

```
*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x
*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)
*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*
2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1
/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/e^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)
+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)-27/896*3^(3/4)*a^(7/3)
*(20*A*b+B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)
+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(
1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(
e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3
^(1/2)))^2)^(1/2)/b^(2/3)/e^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x
)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 650, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {464, 285, 335, 314, 231, 1895}

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{3/2}}\,dx =$$

$$-\frac{27\ 3^{3/4}\left(1-\sqrt{3}\right)a^{7/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+20Ab)\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+}\right.\right.}{896b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{81\sqrt[4]{3}a^{7/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+20Ab)E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{448b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{81\left(1+\sqrt{3}\right)a^2\sqrt{ex}\sqrt{a+bx^3}(aB+20Ab)}{448b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}+\frac{(ex)^{5/2}\left(a+bx^3\right)^{5/2}(aB+20Ab)}{10ae^4}$$

$$+\frac{3(ex)^{5/2}\left(a+bx^3\right)^{3/2}(aB+20Ab)}{28e^4}+\frac{27a(ex)^{5/2}\sqrt{a+bx^3}(aB+20Ab)}{224e^4}-\frac{2A\left(a+bx^3\right)^{7/2}}{ae\sqrt{ex}}$$

```
[In] Int[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(3/2),x]
```

```
[Out] (27*a*(20*A*b + a*B)*(e*x)^(5/2)*Sqrt[a + b*x^3])/(224*e^4) + (81*(1 + Sqrt
[3])*a^2*(20*A*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(448*b^(2/3)*e^2*(a^(1/3
```

```
) + (1 + Sqrt[3])*b^(1/3)*x)) + (3*(20*A*b + a*B)*(e*x)^(5/2)*(a + b*x^3)^(
3/2))/(28*e^4) + ((20*A*b + a*B)*(e*x)^(5/2)*(a + b*x^3)^(5/2))/(10*a*e^4)
- (2*A*(a + b*x^3)^(7/2))/(a*e*Sqrt[e*x]) - (81*3^(1/4)*a^(7/3)*(20*A*b + a
*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3)
+ (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[
3])/4])/(448*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) +
(1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (27*3^(3/4)*(1 - Sqrt[3])*a^
(7/3)*(20*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF
[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3
)*x)], (2 + Sqrt[3])/4])/(896*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3
)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2A(a+bx^3)^{7/2}}{ae\sqrt{ex}}+\frac{(20Ab+aB)\int (ex)^{3/2}\left(a+bx^3\right)^{5/2}\,dx}{ae^3}\\
&= \frac{(20Ab+aB)(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10ae^4}-\frac{2A(a+bx^3)^{7/2}}{ae\sqrt{ex}}\\
&\quad+\frac{(3(20Ab+aB))\int (ex)^{3/2}\left(a+bx^3\right)^{3/2}\,dx}{4e^3}\\
&= \frac{3(20Ab+aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28e^4}+\frac{(20Ab+aB)(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10ae^4}\\
&\quad-\frac{2A(a+bx^3)^{7/2}}{ae\sqrt{ex}}+\frac{(27a(20Ab+aB))\int (ex)^{3/2}\sqrt{a+bx^3}\,dx}{56e^3}\\
&= \frac{27a(20Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{224e^4}+\frac{3(20Ab+aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28e^4}\\
&\quad+\frac{(20Ab+aB)(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10ae^4}\\
&\quad-\frac{2A(a+bx^3)^{7/2}}{ae\sqrt{ex}}+\frac{(81a^2(20Ab+aB))\int \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\,dx}{448e^3}
\end{aligned}$$

$$= \frac{27a(20Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{224e^4} + \frac{3(20Ab+aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28e^4} + \frac{(20Ab+aB)(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10ae^4} - \frac{2A\left(a+bx^3\right)^{7/2}}{ae\sqrt{ex}} + \frac{\left(81a^2(20Ab+aB)\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{224e^4}$$

$$= \frac{27a(20Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{224e^4} + \frac{3(20Ab+aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28e^4} + \frac{(20Ab+aB)(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10ae^4} - \frac{2A\left(a+bx^3\right)^{7/2}}{ae\sqrt{ex}} - \frac{\left(81a^2(20Ab+aB)\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}e^2-2b^{2/3}x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{448b^{2/3}e^4} - \frac{\left(81\left(1-\sqrt{3}\right)a^{8/3}(20Ab+aB)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{448b^{2/3}e^2}$$

$$= \frac{27a(20Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{224e^4} + \frac{81\left(1+\sqrt{3}\right)a^2(20Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{448b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)} + \frac{3(20Ab+aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28e^4} + \frac{(20Ab+aB)(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10ae^4} - \frac{2A\left(a+bx^3\right)^{7/2}}{ae\sqrt{ex}} - \frac{81\sqrt[4]{3}a^{7/3}(20Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\cos^{-1}\left(\frac{\sqrt[3]{a}\ldots}{\sqrt[3]{a}\ldots}\right.\right.}{448b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{27\ 3^{3/4}\left(1-\sqrt{3}\right)a^{7/3}(20Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\ldots\right.}{\sqrt[3]{a}+\left(1+\ldots\right.}\right.\right.}{896b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.13

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = \frac{2x\sqrt{a+bx^3}\left(-5A(a+bx^3)^3+\frac{a^2(20Ab+aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{5}{6},\frac{11}{6},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{5a(ex)^{3/2}}$$

```
[In] Integrate[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(3/2),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*(-5*A*(a + b*x^3)^3 + (a^2*(20*A*b + a*B)*x^3*Hypergeo
metric2F1[-5/2, 5/6, 11/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(5*a*(e*x)^
(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.94 (sec) , antiderivative size = 1166, normalized size of antiderivative = 1.79

method	result	size
risch	Expression too large to display	1166
elliptic	Expression too large to display	1341
default	Expression too large to display	6530

```
[In] int((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/1120*(b*x^3+a)^(1/2)*(-112*B*b^2*x^9-160*A*b^2*x^6-344*B*a*b*x^6-620*A*a
*b*x^3-367*B*a^2*x^3+2240*A*a^2)/e/(e*x)^(1/2)+81/448*a^2*(20*A*b+B*a)*(x*(
x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3))
)^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*El
lipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)
,((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
```

```
a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))*b/(-a*b^2)^(1/3))
)/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
/e*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [F]

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{5}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="fricas")
[Out] integral((B*b^2*x^9 + (2*B*a*b + A*b^2)*x^6 + (B*a^2 + 2*A*a*b)*x^3 + A*a^2
)*sqrt(b*x^3 + a)*sqrt(e*x)/(e^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 21.62 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.48

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = \frac{Aa^{\frac{5}{2}}\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{6}\\ \frac{5}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{5}{6}\right)} + \frac{2Aa^{\frac{3}{2}}bx^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)} + \frac{A\sqrt{a}b^2x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{17}{6}\right)} + \frac{Ba^{\frac{5}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)} + \frac{2Ba^{\frac{3}{2}}bx^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{17}{6}\right)} + \frac{B\sqrt{a}b^2x^{\frac{17}{2}}\Gamma\left(\frac{17}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{17}{6}\\ \frac{23}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{23}{6}\right)}$$

```
[In] integrate((b*x**3+a)**(5/2)*(B*x**3+A)/(e*x)**(3/2),x)
```

```
[Out] A*a**(5/2)*gamma(-1/6)*hyper((-1/2, -1/6), (5/6,), b*x**3*exp_polar(I*pi)/a
)/(3*e**(3/2)*sqrt(x)*gamma(5/6)) + 2*A*a**(3/2)*b*x**(5/2)*gamma(5/6)*hype
r((-1/2, 5/6), (11/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(11/6))
+ A*sqrt(a)*b**2*x**(11/2)*gamma(11/6)*hyper((-1/2, 11/6), (17/6,), b*x**3*
exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(17/6)) + B*a**(5/2)*x**(5/2)*gamma(5/6
)*hyper((-1/2, 5/6), (11/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(1
1/6)) + 2*B*a**(3/2)*b*x**(11/2)*gamma(11/6)*hyper((-1/2, 11/6), (17/6,), b
*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(17/6)) + B*sqrt(a)*b**2*x**(17/2
)*gamma(17/6)*hyper((-1/2, 17/6), (23/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**
(3/2)*gamma(23/6))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{5}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/(e*x)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{5}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/(e*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{5/2}}{(e\,x)^{3/2}}\,dx$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(3/2),x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(3/2), x)
```

3.541 $\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{5/2}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 188

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{5/2}} \, dx = \frac{5a(6Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{24e^4} + \frac{5(6Ab+aB)(ex)^{3/2}(a+bx^3)^{3/2}}{36e^4} + \frac{(6Ab+aB)(ex)^{3/2}(a+bx^3)^{5/2}}{9ae^4} - \frac{2A(a+bx^3)^{7/2}}{3ae(ex)^{3/2}} + \frac{5a^2(6Ab+aB)\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24\sqrt{b}e^{5/2}}$$

```
[Out] 5/36*(6*A*b+B*a)*(e*x)^(3/2)*(b*x^3+a)^(3/2)/e^4+1/9*(6*A*b+B*a)*(e*x)^(3/2
)*(b*x^3+a)^(5/2)/a/e^4-2/3*A*(b*x^3+a)^(7/2)/a/e/(e*x)^(3/2)+5/24*a^2*(6*A
*b+B*a)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))/e^(5/2)/b^(1/2
)+5/24*a*(6*A*b+B*a)*(e*x)^(3/2)*(b*x^3+a)^(1/2)/e^4
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {464, 285, 335, 281, 223, 212}

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{5/2}} \, dx = \frac{5a^2(aB+6Ab)\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24\sqrt{b}e^{5/2}} + \frac{(ex)^{3/2}(a+bx^3)^{5/2}(aB+6Ab)}{9ae^4} + \frac{5(ex)^{3/2}(a+bx^3)^{3/2}(aB+6Ab)}{36e^4} + \frac{5a(ex)^{3/2}\sqrt{a+bx^3}(aB+6Ab)}{24e^4} - \frac{2A(a+bx^3)^{7/2}}{3ae(ex)^{3/2}}$$

```
[In] Int[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(5/2),x]
```

```
[Out] (5*a*(6*A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(24*e^4) + (5*(6*A*b + a*B)
*(e*x)^(3/2)*(a + b*x^3)^(3/2))/(36*e^4) + ((6*A*b + a*B)*(e*x)^(3/2)*(a +
b*x^3)^(5/2))/(9*a*e^4) - (2*A*(a + b*x^3)^(7/2))/(3*a*e*(e*x)^(3/2)) + (5*
a^2*(6*A*b + a*B)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])
/(24*Sqrt[b]*e^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{2A(a+bx^3)^{7/2}}{3ae(ex)^{3/2}} + \frac{(6Ab+aB)\int \sqrt{ex}(a+bx^3)^{5/2}\,dx}{ae^3}$$

$$= \frac{(6Ab+aB)(ex)^{3/2}(a+bx^3)^{5/2}}{9ae^4} - \frac{2A(a+bx^3)^{7/2}}{3ae(ex)^{3/2}} + \frac{(5(6Ab+aB))\int \sqrt{ex}(a+bx^3)^{3/2}\,dx}{6e^3}$$

$$= \frac{5(6Ab+aB)(ex)^{3/2}(a+bx^3)^{3/2}}{36e^4} + \frac{(6Ab+aB)(ex)^{3/2}(a+bx^3)^{5/2}}{9ae^4} - \frac{2A(a+bx^3)^{7/2}}{3ae(ex)^{3/2}} + \frac{(5a(6Ab+aB))\int \sqrt{ex}\sqrt{a+bx^3}\,dx}{8e^3}$$

$$= \frac{5a(6Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{24e^4} + \frac{5(6Ab+aB)(ex)^{3/2}(a+bx^3)^{3/2}}{36e^4} + \frac{(6Ab+aB)(ex)^{3/2}(a+bx^3)^{5/2}}{9ae^4} - \frac{2A(a+bx^3)^{7/2}}{3ae(ex)^{3/2}} + \frac{(5a^2(6Ab+aB))\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{16e^3}$$

$$= \frac{5a(6Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{24e^4} + \frac{5(6Ab+aB)(ex)^{3/2}(a+bx^3)^{3/2}}{36e^4} + \frac{(6Ab+aB)(ex)^{3/2}(a+bx^3)^{5/2}}{9ae^4} - \frac{2A(a+bx^3)^{7/2}}{3ae(ex)^{3/2}} + \frac{(5a^2(6Ab+aB))\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{8e^4}$$

$$= \frac{5a(6Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{24e^4} + \frac{5(6Ab+aB)(ex)^{3/2}(a+bx^3)^{3/2}}{36e^4} + \frac{(6Ab+aB)(ex)^{3/2}(a+bx^3)^{5/2}}{9ae^4} - \frac{2A(a+bx^3)^{7/2}}{3ae(ex)^{3/2}} + \frac{(5a^2(6Ab+aB))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx, x, (ex)^{3/2}\right)}{24e^4}$$

$$= \frac{5a(6Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{24e^4} + \frac{5(6Ab+aB)(ex)^{3/2}(a+bx^3)^{3/2}}{36e^4} + \frac{(6Ab+aB)(ex)^{3/2}(a+bx^3)^{5/2}}{9ae^4} - \frac{2A(a+bx^3)^{7/2}}{3ae(ex)^{3/2}} + \frac{(5a^2(6Ab+aB))\,\text{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx, x, \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{24e^4}$$

$$= \frac{5a(6Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{24e^4} + \frac{5(6Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36e^4} + \frac{(6Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9ae^4} - \frac{2A\left(a+bx^3\right)^{7/2}}{3ae(ex)^{3/2}} + \frac{5a^2(6Ab+aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24\sqrt{b}e^{5/2}}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.68

$$\int \frac{\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)}{(ex)^{5/2}}\,dx = \frac{x\left(\sqrt{b}\sqrt{a+bx^3}(4b^2x^6(3A+2Bx^3)+a^2(-48A+33Bx^3)+a(54Abx^3+26bB\right.}{72\sqrt{b}(ex)^{5/2}}$$

```
[In] Integrate[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(5/2),x]
```

```
[Out] (x*(Sqrt[b]*Sqrt[a + b*x^3]*(4*b^2*x^6*(3*A + 2*B*x^3) + a^2*(-48*A + 33*B*
x^3) + a*(54*A*b*x^3 + 26*b*B*x^6)) + 15*a^2*(6*A*b + a*B)*x^(3/2)*Log[Sqrt
[b]*x^(3/2) + Sqrt[a + b*x^3]]))/(72*Sqrt[b]*(e*x)^(5/2))
```

Maple [A] (verified)

Time = 4.59 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,\left(-8b^2B\,x^9-12A\,b^2x^6-26B\,x^6ab-54aAb\,x^3-33a^2B\,x^3+48a^2A\right)}{72x\,e^2\sqrt{ex}}+\frac{5a^2(6Ab+Ba)\,\mathrm{arctanh}\left(\frac{\sqrt{\left(b\,x^3+a\right)ex}}{x^2\sqrt{be}}\right)\sqrt{(b\,x^3+a)e}}{24\sqrt{be}\,e^2\sqrt{ex}\,\sqrt{b\,x^3+a}}$
default	$\frac{\sqrt{b\,x^3+a}\left(8B\sqrt{(b\,x^3+a)ex}\,\sqrt{be}\,b^2x^9+12A\sqrt{(b\,x^3+a)ex}\,\sqrt{be}\,b^2x^6+26B\sqrt{(b\,x^3+a)ex}\,\sqrt{be}\,ab\,x^6+90A\,\mathrm{arctanh}\left(\frac{\sqrt{\left(b\,x^3+a\right)ex}}{x^2\sqrt{be}}\right)a^2be\,x\right.}{72x\,e^2\sqrt{ex}\,\sqrt{(b\,x^3}}$
elliptic	Expression too large to display

```
[In] int((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/72*(b*x^3+a)^(1/2)*(-8*B*b^2*x^9-12*A*b^2*x^6-26*B*a*b*x^6-54*A*a*b*x^3-
33*B*a^2*x^3+48*A*a^2)/x/e^2/(e*x)^(1/2)+5/24*a^2*(6*A*b+B*a)/(b*e)^(1/2)*a
rctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))/e^2*((b*x^3+a)*e*x)^(1/2)/(e*
x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.58 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.64

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = \left[\frac{15\,(Ba^3+6\,Aa^2b)\sqrt{be}x^2\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\,(2\,bx^4+ax)\sqrt{}\right.}{}\right.$$

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/288*(15*(B*a^3 + 6*A*a^2*b)*sqrt(b*e)*x^2*log(-8*b^2*e*x^6 - 8*a*b*e*x^3
 - a^2*e - 4*(2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b*e)*sqrt(e*x)) + 4*(8*B*
b^3*x^9 + 2*(13*B*a*b^2 + 6*A*b^3)*x^6 - 48*A*a^2*b + 3*(11*B*a^2*b + 18*A*
a*b^2)*x^3)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2), -1/144*(15*(B*a^3 + 6*A
*a^2*b)*sqrt(-b*e)*x^2*arctan(2*sqrt(b*x^3 + a)*sqrt(-b*e)*sqrt(e*x)*x/(2*b
*e*x^3 + a*e)) - 2*(8*B*b^3*x^9 + 2*(13*B*a*b^2 + 6*A*b^3)*x^6 - 48*A*a^2*b
 + 3*(11*B*a^2*b + 18*A*a*b^2)*x^3)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. 2(180) = 360.

Time = 40.09 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.14

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = -\frac{2Aa^{\frac{5}{2}}}{3e^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{2Aa^{\frac{3}{2}}bx^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}} - \frac{7Aa^{\frac{3}{2}}bx^{\frac{3}{2}}}{12e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{A\sqrt{a}b^2x^{\frac{9}{2}}}{4e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{5Aa^2\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{4e^{\frac{5}{2}}} + \frac{Ab^3x^{\frac{15}{2}}}{6\sqrt{a}e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{Ba^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}} + \frac{Ba^{\frac{5}{2}}x^{\frac{3}{2}}}{8e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{35Ba^{\frac{3}{2}}bx^{\frac{9}{2}}}{72e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{17B\sqrt{a}b^2x^{\frac{15}{2}}}{36e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{5Ba^3\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{24\sqrt{b}e^{\frac{5}{2}}} + \frac{Bb^3x^{\frac{21}{2}}}{9\sqrt{a}e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}$$

```
[In] integrate((b*x**3+a)**(5/2)*(B*x**3+A)/(e*x)**(5/2),x)
```

```
[Out] -2*A*a**(5/2)/(3*e**(5/2)*x**(3/2)*sqrt(1 + b*x**3/a)) + 2*A*a**(3/2)*b*x**
(3/2)*sqrt(1 + b*x**3/a)/(3*e**(5/2)) - 7*A*a**(3/2)*b*x**(3/2)/(12*e**(5/2
)*sqrt(1 + b*x**3/a)) + A*sqrt(a)*b**2*x**(9/2)/(4*e**(5/2)*sqrt(1 + b*x**3
```

```
/a)) + 5*A*a**2*sqrt(b)*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(4*e**(5/2)) + A*b*
*3*x**(15/2)/(6*sqrt(a)*e**(5/2)*sqrt(1 + b*x**3/a)) + B*a**(5/2)*x**(3/2)*
sqrt(1 + b*x**3/a)/(3*e**(5/2)) + B*a**(5/2)*x**(3/2)/(8*e**(5/2)*sqrt(1 +
b*x**3/a)) + 35*B*a**(3/2)*b*x**(9/2)/(72*e**(5/2)*sqrt(1 + b*x**3/a)) + 17
*B*sqrt(a)*b**2*x**(15/2)/(36*e**(5/2)*sqrt(1 + b*x**3/a)) + 5*B*a**3*asinh
(sqrt(b)*x**(3/2)/sqrt(a))/(24*sqrt(b)*e**(5/2)) + B*b**3*x**(21/2)/(9*sqrt
(a)*e**(5/2)*sqrt(1 + b*x**3/a))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{5}{2}}}{(ex)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/(e*x)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = \text{Exception raised: NotImplementedError}$$

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> unable to parse Giac output: Recur
sive assumption sageVARa>=(-sageVARb*sageVARe/(sageVARe^4*t_nostep^6)) igno
red2/sageVARe^3*((8870400*sageVARb^12*sageVARe^9*sageVARB/159667200/sageVAR
b^10/sageVARe^18
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{5/2}}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{5/2}}{(e\,x)^{5/2}}\,dx$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(5/2),x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(5/2), x)
```

3.542 $\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 352

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{7/2}}\,dx = \frac{27a(16Ab+5aB)\sqrt{ex}\sqrt{a+bx^3}}{320e^4} + \frac{3(16Ab+5aB)\sqrt{ex}(a+bx^3)^{3/2}}{80e^4} + \frac{(16Ab+5aB)\sqrt{ex}(a+bx^3)^{5/2}}{40ae^4} - \frac{2A(a+bx^3)^{7/2}}{5ae(ex)^{5/2}} + \frac{27\,3^{3/4}a^{5/3}(16Ab+5aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\ldots}{640e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -2/5*A*(b*x^3+a)^(7/2)/a/e/(e*x)^(5/2)+3/80*(16*A*b+5*B*a)*(b*x^3+a)^(3/2)*
(e*x)^(1/2)/e^4+1/40*(16*A*b+5*B*a)*(b*x^3+a)^(5/2)*(e*x)^(1/2)/a/e^4+27/32
0*a*(16*A*b+5*B*a)*(e*x)^(1/2)*(b*x^3+a)^(1/2)/e^4+27/640*3^(3/4)*a^(5/3)*(
16*A*b+5*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)
+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(
1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*(
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^
(1/2)/e^4/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x
*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {464, 285, 335, 231}

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{7/2}}\,dx = \frac{27\ 3^{3/4}a^{5/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(5aB+16Ab)\,\text{EllipticF}\left(\right.}{640e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx}}$$

$$+\frac{\sqrt{ex}(a+bx^3)^{5/2}(5aB+16Ab)}{40ae^4}+\frac{3\sqrt{ex}(a+bx^3)^{3/2}(5aB+16Ab)}{80e^4}$$
$$+\frac{27a\sqrt{ex}\sqrt{a+bx^3}(5aB+16Ab)}{320e^4}-\frac{2A(a+bx^3)^{7/2}}{5ae(ex)^{5/2}}$$

```
[In] Int[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(7/2),x]
```

```
[Out] (27*a*(16*A*b + 5*a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(320*e^4) + (3*(16*A*b +
5*a*B)*Sqrt[e*x]*(a + b*x^3)^(3/2))/(80*e^4) + ((16*A*b + 5*a*B)*Sqrt[e*x]*
(a + b*x^3)^(5/2))/(40*a*e^4) - (2*A*(a + b*x^3)^(7/2))/(5*a*e*(e*x)^(5/2))
 + (27*3^(3/4)*a^(5/3)*(16*A*b + 5*a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1
/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(640*e^4*Sqrt[(b^(1/3)*x*(a^(1/
3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{2A(a+bx^3)^{7/2}}{5ae(ex)^{5/2}} + \frac{(16Ab+5aB)\int \frac{(a+bx^3)^{5/2}}{\sqrt{ex}}\,dx}{5ae^3}$$

$$= \frac{(16Ab+5aB)\sqrt{ex}(a+bx^3)^{5/2}}{40ae^4} - \frac{2A(a+bx^3)^{7/2}}{5ae(ex)^{5/2}} + \frac{(3(16Ab+5aB))\int \frac{(a+bx^3)^{3/2}}{\sqrt{ex}}\,dx}{16e^3}$$

$$= \frac{3(16Ab+5aB)\sqrt{ex}(a+bx^3)^{3/2}}{80e^4} + \frac{(16Ab+5aB)\sqrt{ex}(a+bx^3)^{5/2}}{40ae^4} - \frac{2A(a+bx^3)^{7/2}}{5ae(ex)^{5/2}} + \frac{(27a(16Ab+5aB))\int \frac{\sqrt{a+bx^3}}{\sqrt{ex}}\,dx}{160e^3}$$

$$= \frac{27a(16Ab+5aB)\sqrt{ex}\sqrt{a+bx^3}}{320e^4} + \frac{3(16Ab+5aB)\sqrt{ex}(a+bx^3)^{3/2}}{80e^4} + \frac{(16Ab+5aB)\sqrt{ex}(a+bx^3)^{5/2}}{40ae^4} - \frac{2A(a+bx^3)^{7/2}}{5ae(ex)^{5/2}} + \frac{(81a^2(16Ab+5aB))\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{640e^3}$$

$$= \frac{27a(16Ab+5aB)\sqrt{ex}\sqrt{a+bx^3}}{320e^4} + \frac{3(16Ab+5aB)\sqrt{ex}(a+bx^3)^{3/2}}{80e^4} + \frac{(16Ab+5aB)\sqrt{ex}(a+bx^3)^{5/2}}{40ae^4} - \frac{2A(a+bx^3)^{7/2}}{5ae(ex)^{5/2}} + \frac{(81a^2(16Ab+5aB))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{320e^4}$$

$$= \frac{27a(16Ab+5aB)\sqrt{ex}\sqrt{a+bx^3}}{320e^4} + \frac{3(16Ab+5aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{80e^4}$$
$$+ \frac{(16Ab+5aB)\sqrt{ex}\left(a+bx^3\right)^{5/2}}{40ae^4} - \frac{2A\left(a+bx^3\right)^{7/2}}{5ae(ex)^{5/2}}$$
$$+ \frac{27\ 3^{3/4}a^{5/3}(16Ab+5aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\right.}{640e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.25

$$\int \frac{\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)}{(ex)^{7/2}}\,dx = \frac{2x\sqrt{a+bx^3}\left(-A\left(a+bx^3\right)^3+\frac{a^2(16Ab+5aB)x^3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{6},\frac{7}{6},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{5a(ex)^{7/2}}$$

```
[In] Integrate[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(7/2),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*(-(A*(a + b*x^3)^3) + (a^2*(16*A*b + 5*a*B)*x^3*Hyperg
eometric2F1[-5/2, 1/6, 7/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(5*a*(e*x)
^(7/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.86 (sec) , antiderivative size = 786, normalized size of antiderivative = 2.23

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,\left(-40b^2B\,x^9-64A\,b^2x^6-140B\,x^6ab-368aAb\,x^3-235a^2B\,x^3+128a^2A\right)}{320x^2e^3\sqrt{ex}}+\frac{81a^2(16Ab+5Ba)\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right.}{}$
elliptic	Expression too large to display
default	Expression too large to display

```
[In] int((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/320*(b*x^3+a)^(1/2)*(-40*B*b^2*x^9-64*A*b^2*x^6-140*B*a*b*x^6-368*A*a*b*
x^3-235*B*a^2*x^3+128*A*a^2)/x^2/e^3/(e*x)^(1/2)+81/320*a^2*(16*A*b+5*B*a)*
(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/
b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3))
)^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*
b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b
/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b
^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2))/e^3*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [F]

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{7/2}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{5}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((B*b^2*x^9 + (2*B*a*b + A*b^2)*x^6 + (B*a^2 + 2*A*a*b)*x^3 + A*a^2
)*sqrt(b*x^3 + a)*sqrt(e*x)/(e^4*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 44.17 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{7/2}}\,dx = \frac{Aa^{\frac{5}{2}}\Gamma\left(-\frac{5}{6}\right){}_2F_1\left(\begin{matrix} -\frac{5}{6}, -\frac{1}{2} \\ \frac{1}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)} + \frac{2Aa^{\frac{3}{2}}b\sqrt{x}\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{6} \\ \frac{7}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)} + \frac{A\sqrt{a}b^2x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{7}{6} \\ \frac{13}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{13}{6}\right)} + \frac{Ba^{\frac{5}{2}}\sqrt{x}\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{6} \\ \frac{7}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)} + \frac{2Ba^{\frac{3}{2}}bx^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{7}{6} \\ \frac{13}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{13}{6}\right)} + \frac{B\sqrt{a}b^2x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{13}{6} \\ \frac{19}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{19}{6}\right)}$$

```
[In] integrate((b*x**3+a)**(5/2)*(B*x**3+A)/(e*x)**(7/2),x)
```

```
[Out] A*a**(5/2)*gamma(-5/6)*hyper((-5/6, -1/2), (1/6,), b*x**3*exp_polar(I*pi)/a
)/(3*e**(7/2)*x**(5/2)*gamma(1/6)) + 2*A*a**(3/2)*b*sqrt(x)*gamma(1/6)*hype
r((-1/2, 1/6), (7/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(7/2)*gamma(7/6)) +
A*sqrt(a)*b**2*x**(7/2)*gamma(7/6)*hyper((-1/2, 7/6), (13/6,), b*x**3*exp_p
olar(I*pi)/a)/(3*e**(7/2)*gamma(13/6)) + B*a**(5/2)*sqrt(x)*gamma(1/6)*hype
r((-1/2, 1/6), (7/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(7/2)*gamma(7/6)) +
2*B*a**(3/2)*b*x**(7/2)*gamma(7/6)*hyper((-1/2, 7/6), (13/6,), b*x**3*exp_p
olar(I*pi)/a)/(3*e**(7/2)*gamma(13/6)) + B*sqrt(a)*b**2*x**(13/2)*gamma(13/
6)*hyper((-1/2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(7/2)*gamma
(19/6))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{7/2}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{5}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/(e*x)^(7/2), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{7/2}}\,dx = \int \frac{(Bx^3+A)(bx^3+a)^{\frac{5}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/(e*x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{7/2}}\,dx = \int \frac{(B\,x^3+A)\,(b\,x^3+a)^{5/2}}{(e\,x)^{7/2}}\,dx$$

```
[In] int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(7/2),x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(7/2), x)
```

3.543 $\int \frac{(ex)^{7/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 121

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx=\frac{(4Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{12b^2}+\frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be}-\frac{a(4Ab-3aB)e^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{12b^{5/2}}$$

```
[Out] -1/12*a*(4*A*b-3*B*a)*e^(7/2)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)
^(1/2))/b^(5/2)+1/12*(4*A*b-3*B*a)*e^2*(e*x)^(3/2)*(b*x^3+a)^(1/2)/b^2+1/6*
B*(e*x)^(9/2)*(b*x^3+a)^(1/2)/b/e
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {470, 327, 335, 281, 223, 212}

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx=-\frac{ae^{7/2}(4Ab-3aB)\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{12b^{5/2}}+\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}(4Ab-3aB)}{12b^2}+\frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be}$$

```
[In] Int[((e*x)^(7/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] ((4*A*b - 3*a*B)*e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(12*b^2) + (B*(e*x)^(9/2)
*Sqrt[a + b*x^3])/(6*b*e) - (a*(4*A*b - 3*a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*
x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(12*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be} - \frac{\left(-6Ab+\frac{9aB}{2}\right)\int \frac{(ex)^{7/2}}{\sqrt{a+bx^3}}\,dx}{6b}$$

$$= \frac{(4Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{12b^2} + \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be} - \frac{(a(4Ab-3aB)e^3)\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{8b^2}$$

$$= \frac{(4Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{12b^2} + \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be} - \frac{(a(4Ab-3aB)e^2)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{4b^2}$$

$$= \frac{(4Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{12b^2} + \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be} - \frac{(a(4Ab-3aB)e^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx, x, (ex)^{3/2}\right)}{12b^2}$$

$$= \frac{(4Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{12b^2} + \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be} - \frac{(a(4Ab-3aB)e^2)\,\text{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx, x, \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{12b^2}$$

$$= \frac{(4Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{12b^2} + \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be} - \frac{a(4Ab-3aB)e^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{12b^{5/2}}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.83

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx = \frac{(ex)^{7/2}\sqrt{a+bx^3}(4Ab-3aB+2bBx^3)}{12b^2x^2} + \frac{a(-4Ab+3aB)(ex)^{7/2}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{12b^{5/2}x^{7/2}}$$

```
[In] Integrate[((e*x)^(7/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] ((e*x)^(7/2)*Sqrt[a + b*x^3]*(4*A*b - 3*a*B + 2*b*B*x^3))/(12*b^2*x^2) + (a
*(-4*A*b + 3*a*B)*(e*x)^(7/2)*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]])/(12*b
^(5/2)*x^(7/2))
```

Maple [A] (verified)

Time = 4.68 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.94

method	result
risch	$\frac{x^{2}\left(2bB\,x^{3}+4Ab-3Ba\right)\sqrt{b\,x^{3}+a}\,e^{4}}{12b^{2}\sqrt{ex}}-\frac{a\left(4Ab-3Ba\right)\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right)ex}}{x^{2}\sqrt{be}}\right)e^{4}\sqrt{\left(b\,x^{3}+a\right)ex}}{12b^{2}\sqrt{be}\,\sqrt{ex}\,\sqrt{b\,x^{3}+a}}$
default	$-\frac{e^{3}\sqrt{ex}\,\sqrt{b\,x^{3}+a}\,\left(-2B\sqrt{\left(b\,x^{3}+a\right)ex}\,\sqrt{be}\,b\,x^{4}+4A\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+a\right)ex}}{x^{2}\sqrt{be}}\right)abe-4A\sqrt{\left(b\,x^{3}+a\right)ex}\,\sqrt{be}\,bx-3B\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^{3}+\right.}}{x^{2}\sqrt{b}}\right.}{12\sqrt{\left(b\,x^{3}+a\right)ex}\,b^{2}\sqrt{be}}$
elliptic	Expression too large to display

```
[In] int((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*x^2*(2*B*b*x^3+4*A*b-3*B*a)*(b*x^3+a)^(1/2)/b^2*e^4/(e*x)^(1/2)-1/12*a
*(4*A*b-3*B*a)/b^2/(b*e)^(1/2)*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2
))*e^4*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.55 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.02

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx=\left[-\frac{(3\,Ba^2-4\,Aab)e^3\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e+4\,(2\,b^2x^4+abx)\sqrt{bx^3+\cdots}\right.}{48\,b^2}\right.$$

$$\left.-\frac{(3\,Ba^2-4\,Aab)e^3\sqrt{-\frac{e}{b}}\arctan\left(\frac{2\sqrt{bx^3+a}\sqrt{ex}bx\sqrt{-\frac{e}{b}}}{2\,bex^3+ae}\right)-2\,(2\,Bbe^3x^4-(3\,Ba-4\,Ab)e^3x)\sqrt{bx^3+a}\sqrt{ex}}{24\,b^2}\right]$$

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*((3*B*a^2 - 4*A*a*b)*e^3*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 -
a^2*e + 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(2*B
*b*e^3*x^4 - (3*B*a - 4*A*b)*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b^2, -1/24*(
(3*B*a^2 - 4*A*a*b)*e^3*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*s
qrt(-e/b)/(2*b*e*x^3 + a*e)) - 2*(2*B*b*e^3*x^4 - (3*B*a - 4*A*b)*e^3*x)*sq
rt(b*x^3 + a)*sqrt(e*x))/b^2]
```

Sympy [A] (verification not implemented)

Time = 13.37 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.60

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx=\begin{cases}\frac{2\left(\begin{cases}\text{NaN} \\ \frac{\begin{cases}-\frac{ae^3\left(Ae^3-\frac{3Bae^3}{4b}\right)\left(\begin{cases}\frac{\log\left(\frac{2b(ex)^{\frac{3}{2}}}{e^3}+2\sqrt{\frac{b}{e^3}}\sqrt{a+bx^3}\right)}{\sqrt{\frac{b}{e^3}}} & \text{for } a\neq 0\\ \frac{(ex)^{\frac{3}{2}}\log\left((ex)^{\frac{3}{2}}\right)}{\sqrt{bx^3}} & \text{otherwise}\end{cases}\right)}{2b}+\sqrt{a+bx^3}\left(\frac{Be^3}{\cdots}\right. \\ \frac{\frac{Ae^3(ex)^{\frac{9}{2}}}{3}+\frac{B(ex)^{\frac{15}{2}}}{5}}{\sqrt{a}}\end{cases}}{3e^3}\end{cases}\right)}{e} \\ 0\end{cases}$$

```
[In] integrate((e*x)**(7/2)*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((2*Piecewise((nan, Eq(e**3, 0)), (Piecewise((-a*e**3*(A*e**3 - 3*
B*a*e**3/(4*b))*Piecewise((log(2*b*(e*x)**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(
a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((e*x)**(3/2)*log((e*x)**(3/2))/sqrt(
b*x**3), True))/(2*b) + sqrt(a + b*x**3)*(B*e**3*(e*x)**(9/2)/(4*b) + e**3*
(e*x)**(3/2)*(A*e**3 - 3*B*a*e**3/(4*b))/(2*b)), Ne(b/e**3, 0)), ((A*e**3*(
e*x)**(9/2)/3 + B*(e*x)**(15/2)/5)/sqrt(a), True))/(3*e**3), True))/e, Ne(e
, 0)), (0, True))
```

Maxima [F]

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx=\int \frac{(Bx^3+A)(ex)^{\frac{7}{2}}}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(7/2)/sqrt(b*x^3 + a), x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.21

$$\int \frac{(ex)^{7/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{\sqrt{be^4x^3+ae^4}\sqrt{ex}e^5x\left(\frac{2\,Bx^3}{be^2}-\frac{3\,Bab^3e^5-4\,Ab^4e^5}{b^5e^7}\right)}{12\,|e|^2} - \frac{(3\,Ba^2b^3e^9-4\,Aab^4e^9)\log\left(\left|-\sqrt{be}\sqrt{ex}ex+\sqrt{be^4x^3+ae^4}\right|\right)}{12\,\sqrt{be}b^5e|e|^4}$$

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*e^5*x*(2*B*x^3/(b*e^2) - (3*B*a*b^3*
e^5 - 4*A*b^4*e^5)/(b^5*e^7))/abs(e)^2 - 1/12*(3*B*a^2*b^3*e^9 - 4*A*a*b^4*
e^9)*log(abs(-sqrt(b*e)*sqrt(e*x)*e*x + sqrt(b*e^4*x^3 + a*e^4)))/(sqrt(b*e
)*b^5*e*abs(e)^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \int \frac{(B\,x^3+A)\,(e\,x)^{7/2}}{\sqrt{b\,x^3+a}}\,dx$$

```
[In] int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(1/2),x)
```

```
[Out] int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(1/2), x)
```

3.544 $\int \frac{(ex)^{5/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 286

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx = \frac{(10Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{20b^2}+\frac{B(ex)^{7/2}\sqrt{a+bx^3}}{5be} - \frac{a^{2/3}(10Ab-7aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(2+}{40\sqrt[4]{3}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 1/5*B*(e*x)^(7/2)*(b*x^3+a)^(1/2)/b/e+1/20*(10*A*b-7*B*a)*e^2*(e*x)^(1/2)*(
b*x^3+a)^(1/2)/b^2-1/120*a^(2/3)*(10*A*b-7*B*a)*e^2*(a^(1/3)+b^(1/3)*x)*((a
^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a
^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-
(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),
1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^2/(b*x^3+a)^(1/2)/(b^
(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {470, 327, 335, 231}

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx =$$

$$-\frac{a^{2/3}e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(10Ab-7aB)\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}(2-\right.}{40\sqrt[4]{3}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{e^2\sqrt{ex}\sqrt{a+bx^3}(10Ab-7aB)}{20b^2}+\frac{B(ex)^{7/2}\sqrt{a+bx^3}}{5be}$$

```
[In] Int[((e*x)^(5/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] ((10*A*b - 7*a*B)*e^2*Sqrt[e*x]*Sqrt[a + b*x^3])/(20*b^2) + (B*(e*x)^(7/2)*
Sqrt[a + b*x^3])/(5*b*e) - (a^(2/3)*(10*A*b - 7*a*B)*e^2*Sqrt[e*x]*(a^(1/3)
 + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3
)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(40*3^(1/4)*b^
2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{B(ex)^{7/2}\sqrt{a+bx^3}}{5be} - \frac{\left(-5Ab+\frac{7aB}{2}\right)\int \frac{(ex)^{5/2}}{\sqrt{a+bx^3}}\,dx}{5b}$$

$$= \frac{(10Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{20b^2} + \frac{B(ex)^{7/2}\sqrt{a+bx^3}}{5be} - \frac{\left(a(10Ab-7aB)e^3\right)\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{40b^2}$$

$$= \frac{(10Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{20b^2} + \frac{B(ex)^{7/2}\sqrt{a+bx^3}}{5be} - \frac{\left(a(10Ab-7aB)e^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{20b^2}$$

$$= \frac{(10Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{20b^2} + \frac{B(ex)^{7/2}\sqrt{a+bx^3}}{5be} - \frac{a^{2/3}(10Ab-7aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\right.\right.}{40\sqrt[4]{3}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.34

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx=\frac{e^2\sqrt{ex}\left(-\left(\left(a+bx^3\right)\left(-10Ab+7aB-4bBx^3\right)\right)+a(-10Ab+7aB)\sqrt{1+\frac{bx^3}{a}}\,\text{Hy}\right.}{20b^2\sqrt{a+bx^3}}$$

```
[In] Integrate[((e*x)^(5/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (e^2*Sqrt[e*x]*(-((a + b*x^3)*(-10*A*b + 7*a*B - 4*b*B*x^3)) + a*(-10*A*b +
 7*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)])
)/(20*b^2*Sqrt[a + b*x^3])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.90 (sec) , antiderivative size = 753, normalized size of antiderivative = 2.63

method	result
risch	$\frac{\left(4bB\,x^3+10Ab-7Ba\right)x\sqrt{b\,x^3+a}\,e^3}{20b^2\sqrt{ex}}-\frac{a(10Ab-7Ba)\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right.}}}{\cdots}$
elliptic	$\frac{\sqrt{ex}\,\sqrt{\left(b\,x^3+a\right)ex}\left(\frac{B\,e^2x^3\sqrt{be\,x^4+aex}}{5b}+\frac{\left(A\,e^3-\frac{7B\,e^3a}{10b}\right)\sqrt{be\,x^4+aex}}{2be}-\frac{\left(A\,e^3-\frac{7B\,e^3a}{10b}\right)a\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-\cdots\right.}{\cdots}\right.}{\left(-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\cdots\right.}}}{\cdots}\right.}{\cdots}$
default	Expression too large to display

```
[In] int((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/20*(4*B*b*x^3+10*A*b-7*B*a)*x*(b*x^3+a)^(1/2)/b^2*e^3/(e*x)^(1/2)-1/20*a*
(10*A*b-7*B*a)/b*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3
```

```
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*
b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/
b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))*e^3*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x
^3+a)^(1/2)
```

Fricas [F]

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx=\int \frac{(Bx^3+A)(ex)^{\frac{5}{2}}}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*e^2*x^5 + A*e^2*x^2)*sqrt(e*x)/sqrt(b*x^3 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.70 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.33

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx=\frac{Ae^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{13}{6}\right)}+\frac{Be^{\frac{5}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{19}{6}\right)}$$

```
[In] integrate((e*x)**(5/2)*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*e**(5/2)*x**(7/2)*gamma(7/6)*hyper((1/2, 7/6), (13/6,), b*x**3*exp_polar(
I*pi)/a)/(3*sqrt(a)*gamma(13/6)) + B*e**(5/2)*x**(13/2)*gamma(13/6)*hyper((
1/2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(19/6))
```

Maxima [F]

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \int \frac{(Bx^3+A)(ex)^{\frac{5}{2}}}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(e*x)^(5/2)/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \int \frac{(Bx^3+A)(ex)^{\frac{5}{2}}}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(e*x)^(5/2)/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \int \frac{(B\,x^3+A)\,(e\,x)^{5/2}}{\sqrt{b\,x^3+a}}\,dx$$

```
[In] int(((A + B*x^3)*(e*x)^(5/2))/(a + b*x^3)^(1/2),x)
[Out] int(((A + B*x^3)*(e*x)^(5/2))/(a + b*x^3)^(1/2), x)
```

3.545 $\int \frac{(ex)^{3/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 543

$$\int \frac{(ex)^{3/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be}+\frac{(1+\sqrt{3})(8Ab-5aB)e\sqrt{ex}\sqrt{a+bx^3}}{8b^{5/3}\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)}$$
$$-\frac{\sqrt[4]{3}\sqrt[3]{a}(8Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+(1-\sqrt{3})\sqrt[3]{b}x}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{8b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{(1-\sqrt{3})\sqrt[3]{a}(8Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+(1-\sqrt{3})\sqrt[3]{b}x}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x}\right)\right.}{16\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 1/4*B*(e*x)^(5/2)*(b*x^3+a)^(1/2)/b/e+1/8*(8*A*b-5*B*a)*e*(1+3^(1/2))*(e*x)
^(1/2)*(b*x^3+a)^(1/2)/b^(5/3)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))-1/8*3^(1/4)*
a^(1/3)*(8*A*b-5*B*a)*e*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2))
)^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2))
)*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2
)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*
x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(
1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(
1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)-1/48*a^(1/3)*(8*A*b-5*B*a)*e*(a^(1/3)+
b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)
```

```
))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))
*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/
2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/
4)/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*
x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {470, 335, 314, 231, 1895}

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx = -\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(8Ab-5aB)\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)\right.}{16\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{\sqrt[4]{3}\sqrt[3]{a}e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(8Ab-5aB)E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{8b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{\left(1+\sqrt{3}\right)e\sqrt{ex}\sqrt{a+bx^3}(8Ab-5aB)}{8b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)} + \frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be}$$

```
[In] Int[((e*x)^(3/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (B*(e*x)^(5/2)*Sqrt[a + b*x^3])/(4*b*e) + ((1 + Sqrt[3])*(8*A*b - 5*a*B)*e*
Sqrt[e*x]*Sqrt[a + b*x^3])/(8*b^(5/3)*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x))
- (3^(1/4)*a^(1/3)*(8*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)
*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 +
 Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(8*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3
) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (
(1 - Sqrt[3])*a^(1/3)*(8*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1
/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(16*3^(1/4)*b^(5/3)*Sqrt[(b^(1/
```

```
3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a +
 b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\text{integral} = \frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be} - \frac{\left(-4Ab+\frac{5aB}{2}\right)\int \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\,dx}{4b}$$

$$= \frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be} + \frac{(8Ab-5aB)\text{Subst}\left(\int \frac{x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{4be}$$

$$= \frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be} - \frac{(8Ab-5aB)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}e^2-2b^{2/3}x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{8b^{5/3}e} - \frac{\left(\left(1-\sqrt{3}\right)a^{2/3}(8Ab-5aB)e\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{8b^{5/3}}$$

$$= \frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be} + \frac{\left(1+\sqrt{3}\right)(8Ab-5aB)e\sqrt{ex}\sqrt{a+bx^3}}{8b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt[3]{a}(8Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}(2\right.}{8b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}(8Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right.\right.}{16\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.15

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx = \frac{x(ex)^{3/2}\left(5B\left(a+bx^3\right)+(8Ab-5aB)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-\right.\right.}{20b\sqrt{a+bx^3}}$$

```
[In] Integrate[((e*x)^(3/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (x*(e*x)^(3/2)*(5*B*(a + b*x^3) + (8*A*b - 5*a*B)*Sqrt[1 + (b*x^3)/a]*Hyper
geometric2F1[1/2, 5/6, 11/6, -((b*x^3)/a)]))/(20*b*Sqrt[a + b*x^3])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.81 (sec) , antiderivative size = 1124, normalized size of antiderivative = 2.07

method	result	size
risch	Expression too large to display	1124
elliptic	Expression too large to display	1128
default	Expression too large to display	4914

```
[In] int((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*B*x^3/b*(b*x^3+a)^(1/2)*e^2/(e*x)^(1/2)+1/8*(8*A*b-5*B*a)/b*(x*(x+1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)
*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF
(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))*b/(-a*b^2)^(1/3)))/(b*e*
x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*e^2*((
b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [F]

$$\int \frac{(ex)^{3/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\int \frac{(Bx^3+A)(ex)^{\frac{3}{2}}}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] integral((B*e*x^4 + A*e*x)*sqrt(e*x)/sqrt(b*x^3 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.78 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.17

$$\int \frac{(ex)^{3/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\frac{Ae^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{11}{6}\right)}+\frac{Be^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{17}{6}\right)}$$

```
[In] integrate((e*x)**(3/2)*(B*x**3+A)/(b*x**3+a)**(1/2),x)
[Out] A*e**(3/2)*x**(5/2)*gamma(5/6)*hyper((1/2, 5/6), (11/6,), b*x**3*exp_polar(
I*pi)/a)/(3*sqrt(a)*gamma(11/6)) + B*e**(3/2)*x**(11/2)*gamma(11/6)*hyper((
1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(17/6))
```

Maxima [F]

$$\int \frac{(ex)^{3/2}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\int \frac{(Bx^3+A)(ex)^{\frac{3}{2}}}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(e*x)^(3/2)/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{(ex)^{3/2} (A + Bx^3)}{\sqrt{a + bx^3}} \, dx = \int \frac{(Bx^3 + A)(ex)^{\frac{3}{2}}}{\sqrt{bx^3 + a}} \, dx$$

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(e*x)^(3/2)/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2} (A + Bx^3)}{\sqrt{a + bx^3}} \, dx = \int \frac{(B\,x^3 + A)\,(e\,x)^{3/2}}{\sqrt{b\,x^3 + a}} \, dx$$

```
[In] int(((A + B*x^3)*(e*x)^(3/2))/(a + b*x^3)^(1/2),x)
[Out] int(((A + B*x^3)*(e*x)^(3/2))/(a + b*x^3)^(1/2), x)
```

3.546 $\int \frac{\sqrt{ex}(A+Bx^3)}{\sqrt{a+bx^3}} dx$

Optimal result

Integrand size = 26, antiderivative size = 83

$$\int \frac{\sqrt{ex}(A+Bx^3)}{\sqrt{a+bx^3}} dx = \frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be} + \frac{(2Ab-aB)\sqrt{e}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}$$

```
[Out] 1/3*(2*A*b-B*a)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))*e^(1/2
)/b^(3/2)+1/3*B*(e*x)^(3/2)*(b*x^3+a)^(1/2)/b/e
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {470, 335, 281, 223, 212}

$$\int \frac{\sqrt{ex}(A+Bx^3)}{\sqrt{a+bx^3}} dx = \frac{\sqrt{e}(2Ab-aB)\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}} + \frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}$$

```
[In] Int[(Sqrt[e*x]*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (B*(e*x)^(3/2)*Sqrt[a + b*x^3])/(3*b*e) + ((2*A*b - a*B)*Sqrt[e]*ArcTanh[(S
qrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(3*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be} - \frac{\left(-3Ab+\frac{3aB}{2}\right)\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{3b} \\
&= \frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be} + \frac{(2Ab-aB)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{be} \\
&= \frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be} + \frac{(2Ab-aB)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx, x, (ex)^{3/2}\right)}{3be} \\
&= \frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be} + \frac{(2Ab-aB)\text{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx, x, \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{3be} \\
&= \frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be} + \frac{(2Ab-aB)\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{ex}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx=\frac{\sqrt{ex}\left(\sqrt{b}Bx^{3/2}\sqrt{a+bx^3}+(2Ab-aB)\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)\right)}{3b^{3/2}\sqrt{x}}$$

```
[In] Integrate[(Sqrt[e*x]*(A + B*x^3))/Sqrt[a + b*x^3],x]
[Out] (Sqrt[e*x]*(Sqrt[b]*B*x^(3/2)*Sqrt[a + b*x^3] + (2*A*b - a*B)*Log[Sqrt[b]*x
^(3/2) + Sqrt[a + b*x^3]]))/(3*b^(3/2)*Sqrt[x])
```

Maple [A] (verified)

Time = 4.62 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.13

method	result	size
risch	$\frac{B\,x^2\sqrt{b\,x^3+a}\,e}{3b\sqrt{ex}}+\frac{(2Ab-Ba)\operatorname{arctanh}\left(\frac{\sqrt{(b\,x^3+a)ex}}{x^2\sqrt{be}}\right)e\sqrt{(b\,x^3+a)ex}}{3b\sqrt{be}\,\sqrt{ex}\,\sqrt{b\,x^3+a}}$	94
default	$\frac{\sqrt{ex}\,\sqrt{b\,x^3+a}\left(2A\operatorname{arctanh}\left(\frac{\sqrt{(b\,x^3+a)ex}}{x^2\sqrt{be}}\right)be+B\sqrt{(b\,x^3+a)ex}\,\sqrt{be}\,x-B\operatorname{arctanh}\left(\frac{\sqrt{(b\,x^3+a)ex}}{x^2\sqrt{be}}\right)ae\right)}{3\sqrt{(b\,x^3+a)ex}\,b\sqrt{be}}$	112
elliptic	Expression too large to display	1046

```
[In] int((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*B*x^2/b*(b*x^3+a)^(1/2)*e/(e*x)^(1/2)+1/3*(2*A*b-B*a)/b/(b*e)^(1/2)*arc
tanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))*e*((b*x^3+a)*e*x)^(1/2)/(e*x)^(
1/2)/(b*x^3+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.54 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.22

$$\int \frac{\sqrt{ex}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$$

$$=\left[\frac{4\sqrt{bx^3+a}\sqrt{ex}Bx-(Ba-2\,Ab)\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\,(2\,b^2x^4+abx)\sqrt{bx^3+a}\sqrt{ex}\sqrt{\frac{e}{b}}\right.}{12\,b}\right.$$

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*sqrt(b*x^3 + a)*sqrt(e*x)*B*x - (B*a - 2*A*b)*sqrt(e/b)*log(-8*b^2
*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e
*x)*sqrt(e/b)))/b, 1/6*(2*sqrt(b*x^3 + a)*sqrt(e*x)*B*x + (B*a - 2*A*b)*sqr
t(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)
))/b]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(73) = 146$.

Time = 1.51 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.82

$$\int \frac{\sqrt{ex}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \begin{cases} \frac{2\left(\begin{cases} \text{NaN} & \text{for } e^3 = 0 \\ \frac{\begin{cases} \frac{Be^3(ex)^{\frac{3}{2}}\sqrt{a+bx^3}}{2b} + \left(Ae^3 - \frac{Bae^3}{2b}\right)\left(\begin{cases} \frac{\log\left(\frac{2b(ex)^{\frac{3}{2}}}{e^3} + 2\sqrt{\frac{b}{e^3}}\sqrt{a+bx^3}\right)}{\sqrt{\frac{b}{e^3}}} & \text{for } a \neq 0 \\ \frac{(ex)^{\frac{3}{2}}\log\left((ex)^{\frac{3}{2}}\right)}{\sqrt{bx^3}} & \text{otherwise} \end{cases}\right) & \text{for } \frac{b}{e^3} \neq 0 \\ \frac{Ae^3(ex)^{\frac{3}{2}} + \frac{B(ex)^{\frac{9}{2}}}{3}}{\sqrt{a}} & \text{otherwise} \end{cases}}{3e^3} & \text{otherwise} \end{cases}\right)}{e} \\ 0 \end{cases}$$

```
[In] integrate((B*x**3+A)*(e*x)**(1/2)/(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((2*Piecewise((nan, Eq(e**3, 0)), (Piecewise((B*e**3*(e*x)**(3/2)*
sqrt(a + b*x**3)/(2*b) + (A*e**3 - B*a*e**3/(2*b))*Piecewise((log(2*b*(e*x)
**(3/2)/e**3 + 2*sqrt(b/e**3)*sqrt(a + b*x**3))/sqrt(b/e**3), Ne(a, 0)), ((
e*x)**(3/2)*log((e*x)**(3/2))/sqrt(b*x**3), True)), Ne(b/e**3, 0)), ((A*e**
3*(e*x)**(3/2) + B*(e*x)**(9/2)/3)/sqrt(a), True))/(3*e**3), True))/e, Ne(e
, 0)), (0, True))
```

Maxima [F]

$$\int \frac{\sqrt{ex}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \int \frac{(Bx^3+A)\sqrt{ex}}{\sqrt{bx^3+a}}\,dx$$

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*sqrt(e*x)/sqrt(b*x^3 + a), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{ex}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \frac{\sqrt{be^4x^3+ae^4}\sqrt{ex}Bx}{3\,b|e|^2} + \frac{(Bae^5-2\,Abe^5)\log\left(\left|-\sqrt{be}\sqrt{ex}ex+\sqrt{be^4x^3+ae^4}\right|\right)}{3\,\sqrt{be}b|e|^4}$$

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] 1/3*sqrt(b*e^4*x^3 + a*e^4)*sqrt(e*x)*B*x/(b*abs(e)^2) + 1/3*(B*a*e^5 - 2*A
*b*e^5)*log(abs(-sqrt(b*e)*sqrt(e*x)*e*x + sqrt(b*e^4*x^3 + a*e^4)))/(sqrt(
b*e)*b*abs(e)^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(A+Bx^3)}{\sqrt{a+bx^3}}\,dx = \int \frac{(B\,x^3+A)\,\sqrt{e\,x}}{\sqrt{b\,x^3+a}}\,dx$$

```
[In] int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(1/2),x)
[Out] int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(1/2), x)
```

3.547 $\int \frac{A+Bx^3}{\sqrt{ex}\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 249

$$\int \frac{A+Bx^3}{\sqrt{ex}\sqrt{a+bx^3}}\,dx=\frac{B\sqrt{ex}\sqrt{a+bx^3}}{2be}+\frac{(4Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}\sqrt[3]{a}be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 1/2*B*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b/e+1/12*(4*A*b-B*a)*(a^(1/3)+b^(1/3)*x)*
((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF(
(1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/
2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)
*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(1/3)/b/e/(b*x^3+a
)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(
1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {470, 335, 231}

$$\int \frac{A+Bx^3}{\sqrt{ex}\sqrt{a+bx^3}}\,dx$$
$$=\frac{\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(4Ab-aB)\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}\sqrt[3]{a}be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{B\sqrt{ex}\sqrt{a+bx^3}}{2be}$$

```
[In] Int[(A + B*x^3)/(Sqrt[e*x]*Sqrt[a + b*x^3]),x]
```

```
[Out] (B*Sqrt[e*x]*Sqrt[a + b*x^3])/(2*b*e) + ((4*A*b - a*B)*Sqrt[e*x]*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1
+ Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*
x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*a^(1/
3)*b*e*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1
/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
```

```
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{B\sqrt{ex}\sqrt{a+bx^3}}{2be} - \frac{\left(-2Ab+\frac{aB}{2}\right)\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{2b}\\ &= \frac{B\sqrt{ex}\sqrt{a+bx^3}}{2be} + \frac{(4Ab-aB)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{2be}\\ &= \frac{B\sqrt{ex}\sqrt{a+bx^3}}{2be}\\ &\quad + \frac{(4Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}\sqrt[3]{a}be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.32

$$\int \frac{A+Bx^3}{\sqrt{ex}\sqrt{a+bx^3}}\,dx = \frac{Bx(a+bx^3)+(4Ab-aB)x\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^3}{a}\right)}{2b\sqrt{ex}\sqrt{a+bx^3}}$$

```
[In] Integrate[(A + B*x^3)/(Sqrt[e*x]*Sqrt[a + b*x^3]),x]
[Out] (B*x*(a + b*x^3) + (4*A*b - a*B)*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/
6, 1/2, 7/6, -((b*x^3)/a)])/(2*b*Sqrt[e*x]*Sqrt[a + b*x^3])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.70 (sec) , antiderivative size = 728, normalized size of antiderivative = 2.92

method	result
risch	$\frac{Bx\sqrt{b\,x^3+a}}{2b\sqrt{ex}}+\frac{(4Ab-Ba)\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)^2\sqrt{\frac{\cdots}{b\left(-\cdots\right.}}}{2\left(-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\cdots\right.}$
elliptic	$\frac{\sqrt{(b\,x^3+a)ex}\left(\frac{B\sqrt{be\,x^4+aex}}{2be}+\frac{2\left(A-\frac{aB}{4b}\right)\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a\,b^2\right)\cdots}{b}\right.}{\left(\cdots\right.}\right.}{\cdots}$
default	Expression too large to display

```
[In] int((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*B/b*x*(b*x^3+a)^(1/2)/(e*x)^(1/2)+1/2*(4*A*b-B*a)*(1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1
/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)
^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*e*x*
(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*Elliptic
F(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))*((b*x
^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [F]

$$\int \frac{A+Bx^3}{\sqrt{ex}\sqrt{a+bx^3}}\,dx=\int \frac{Bx^3+A}{\sqrt{bx^3+a}\sqrt{ex}}\,dx$$

[In] integrate((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(b*e*x^4 + a*e*x), x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.38

$$\int \frac{A+Bx^3}{\sqrt{ex}\sqrt{a+bx^3}}\,dx=\frac{A\sqrt{x}\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{6},\frac{1}{2}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\sqrt{e}\Gamma\left(\frac{7}{6}\right)}+\frac{Bx^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\sqrt{e}\Gamma\left(\frac{13}{6}\right)}$$

[In] integrate((B*x**3+A)/(e*x)**(1/2)/(b*x**3+a)**(1/2),x)

[Out] A*sqrt(x)*gamma(1/6)*hyper((1/6, 1/2), (7/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*sqrt(e)*gamma(7/6)) + B*x**(7/2)*gamma(7/6)*hyper((1/2, 7/6), (13/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*sqrt(e)*gamma(13/6))

Maxima [F]

$$\int \frac{A+Bx^3}{\sqrt{ex}\sqrt{a+bx^3}}\,dx=\int \frac{Bx^3+A}{\sqrt{bx^3+a}\sqrt{ex}}\,dx$$

[In] integrate((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*sqrt(e*x)), x)

Giac [F]

$$\int \frac{A+Bx^3}{\sqrt{ex}\sqrt{a+bx^3}}\,dx=\int \frac{Bx^3+A}{\sqrt{bx^3+a}\sqrt{ex}}\,dx$$

```
[In] integrate((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*sqrt(e*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{\sqrt{ex}\sqrt{a+bx^3}}\,dx=\int \frac{B\,x^3+A}{\sqrt{e\,x}\,\sqrt{b\,x^3+a}}\,dx$$

```
[In] int((A + B*x^3)/((e*x)^(1/2)*(a + b*x^3)^(1/2)),x)
[Out] int((A + B*x^3)/((e*x)^(1/2)*(a + b*x^3)^(1/2)), x)
```

3.548 $\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 542

$$\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx=-\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}}+\frac{\left(1+\sqrt{3}\right)(2Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{ab^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$
$$-\frac{\sqrt[4]{3}(2Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{a^{2/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{\left(1-\sqrt{3}\right)(2Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(2-\right.}{2\sqrt[4]{3}a^{2/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -2*A*(b*x^3+a)^(1/2)/a/e/(e*x)^(1/2)+(2*A*b+B*a)*(1+3^(1/2))*(e*x)^(1/2)*(b
*x^3+a)^(1/2)/a/b^(2/3)/e^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))-3^(1/4)*(2*A*b+
B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3
)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*
x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1
/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/a^(
2/3)/b^(2/3)/e^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^
(1/3)*x*(1+3^(1/2)))^2)^(1/2)-1/6*(2*A*b+B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)
+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)
```

```
+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/
3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^
(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2
/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(2/3)/b^(2/3)/e
^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(
1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {464, 335, 314, 231, 1895}

$$\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx =$$

$$-\frac{\left(1-\sqrt{3}\right)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+2Ab)\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}(2}{2\sqrt[4]{3}a^{2/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{\sqrt[4]{3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+2Ab)E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{a^{2/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{\left(1+\sqrt{3}\right)\sqrt{ex}\sqrt{a+bx^3}(aB+2Ab)}{ab^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}-\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}}$$

```
[In] Int[(A + B*x^3)/((e*x)^(3/2)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*A*Sqrt[a + b*x^3])/(a*e*Sqrt[e*x]) + ((1 + Sqrt[3])*(2*A*b + a*B)*Sqrt[
e*x]*Sqrt[a + b*x^3])/(a*b^(2/3)*e^2*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)) -
 (3^(1/4)*(2*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Ellipt
icE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(
1/3)*x)], (2 + Sqrt[3])/4])/(a^(2/3)*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) +
 b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - ((1
- Sqrt[3])*(2*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Ellip
ticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^
(1/3)*x)], (2 + Sqrt[3])/4])/(2*3^(1/4)*a^(2/3)*b^(2/3)*e^2*Sqrt[(b^(1/3)*x
```

```
*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x
^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\text{integral} = -\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}} + \frac{(2Ab+aB)\int \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\,dx}{ae^3}$$

$$= -\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}} + \frac{(2(2Ab+aB))\text{Subst}\left(\int \frac{x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{ae^4}$$

$$= -\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}} - \frac{(2Ab+aB)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}e^2-2b^{2/3}x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{ab^{2/3}e^4} - \frac{\left(\left(1-\sqrt{3}\right)(2Ab+aB)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{\sqrt[3]{a}b^{2/3}e^2}$$

$$= -\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}} + \frac{\left(1+\sqrt{3}\right)(2Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{ab^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}(2Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right.\right.}{a^{2/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{\left(1-\sqrt{3}\right)(2Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(\right.\right.}{2\sqrt[4]{3}a^{2/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.15

$$\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx = \frac{x\left(-10A(a+bx^3)+2(2Ab+aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-\frac{bx^3}{a}\right.\right.}{5a(ex)^{3/2}\sqrt{a+bx^3}}$$

```
[In] Integrate[(A + B*x^3)/((e*x)^(3/2)*Sqrt[a + b*x^3]),x]
[Out] (x*(-10*A*(a + b*x^3) + 2*(2*A*b + a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeomet
ric2F1[1/2, 5/6, 11/6, -((b*x^3)/a)]))/(5*a*(e*x)^(3/2)*Sqrt[a + b*x^3])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.96 (sec) , antiderivative size = 1119, normalized size of antiderivative = 2.06

method	result	size
risch	Expression too large to display	1119
elliptic	Expression too large to display	1132
default	Expression too large to display	5385

```
[In] int((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*A*(b*x^3+a)^(1/2)/a/e/(e*x)^(1/2)+(2*A*b+B*a)/a*(x*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b
^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b
*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))*b/(-a*b^2)^(1/3)))/(b*e*x*(x-1/b*(-a*
b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)/e*((b*x^3+a)*e*x)^(
1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [F]

$$\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx = \int \frac{Bx^3+A}{\sqrt{bx^3+a}\,(ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(b*e^2*x^5 + a*e^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.41 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.18

$$\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx = \frac{A\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{6},\frac{1}{2}\\ \frac{5}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}e^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{5}{6}\right)} + \frac{Bx^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)}$$

```
[In] integrate((B*x**3+A)/(e*x)**(3/2)/(b*x**3+a)**(1/2),x)
[Out] A*gamma(-1/6)*hyper((-1/6, 1/2), (5/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*e**(3/2)*sqrt(x)*gamma(5/6)) + B*x**(5/2)*gamma(5/6)*hyper((1/2, 5/6), (
11/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*e**(3/2)*gamma(11/6))
```

Maxima [F]

$$\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx = \int \frac{Bx^3+A}{\sqrt{bx^3+a}\,(ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*(e*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx=\int \frac{Bx^3+A}{\sqrt{bx^3+a}\,(ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*(e*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx=\int \frac{B\,x^3+A}{{(e\,x)}^{3/2}\,\sqrt{b\,x^3+a}}\,dx$$

```
[In] int((A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(1/2)),x)
[Out] int((A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(1/2)), x)
```

3.549 $\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 75

$$\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx=-\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}+\frac{2B\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}e^{5/2}}$$

```
[Out] 2/3*B*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))/e^(5/2)/b^(1/2)-
2/3*A*(b*x^3+a)^(1/2)/a/e/(e*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {462, 335, 281, 223, 212}

$$\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx=\frac{2B\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}e^{5/2}}-\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}$$

```
[In] Int[(A + B*x^3)/((e*x)^(5/2)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*A*Sqrt[a + b*x^3])/(3*a*e*(e*x)^(3/2)) + (2*B*ArcTanh[(Sqrt[b]*(e*x)^(3
/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(3*Sqrt[b]*e^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}+\frac{B\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{e^3}\\
&= -\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}+\frac{(2B)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{e^4}\\
&= -\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}+\frac{(2B)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx,x,(ex)^{3/2}\right)}{3e^4}\\
&= -\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}+\frac{(2B)\mathrm{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx,x,\frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{3e^4}\\
&= -\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}+\frac{2B\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}e^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx=\frac{2x\left(-\frac{A\sqrt{a+bx^3}}{a}+\frac{Bx^{3/2}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{\sqrt{b}}\right)}{3(ex)^{5/2}}$$

```
[In] Integrate[(A + B*x^3)/((e*x)^(5/2)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*x*(-((A*Sqrt[a + b*x^3])/a) + (B*x^(3/2)*Log[Sqrt[b]*x^(3/2) + Sqrt[a +
b*x^3]])/Sqrt[b]))/(3*(e*x)^(5/2))
```

Maple [A] (verified)

Time = 4.62 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.16

method	result	size
risch	$-\frac{2A\sqrt{b\,x^3+a}}{3ax\,e^2\sqrt{ex}}+\frac{2B\,\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^3+a\right)ex}}{x^2\sqrt{be}}\right)\sqrt{(b\,x^3+a)ex}}{3\sqrt{be}\,e^2\sqrt{ex}\,\sqrt{b\,x^3+a}}$	87
default	$-\frac{2\sqrt{b\,x^3+a}\left(-B\,\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^3+a\right)ex}}{x^2\sqrt{be}}\right)ae\,x^2+A\sqrt{(b\,x^3+a)ex}\,\sqrt{be}\right)}{3x\,e^2\sqrt{ex}\,\sqrt{(b\,x^3+a)ex}\,a\sqrt{be}}$	93
elliptic	Expression too large to display	1037

```
[In] int((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/a*A*(b*x^3+a)^(1/2)/x/e^2/(e*x)^(1/2)+2/3*B/(b*e)^(1/2)*arctanh(((b*x^
3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))/e^2*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x
^3+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.44

$$\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx=\left[\frac{\sqrt{be}Bax^2\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\,(2\,bx^4+ax)\sqrt{bx^3+a}\sqrt{be}\sqrt{ex}\right)-4}{6\,abe^3x^2}\right.$$
$$\left.-\frac{\sqrt{-be}Bax^2\arctan\left(\frac{2\sqrt{bx^3+a}\sqrt{-be}\sqrt{ex}x}{2\,bex^3+ae}\right)+2\,\sqrt{bx^3+a}\sqrt{ex}Ab}{3\,abe^3x^2}\right]$$

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(b*e)*B*a*x^2*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b*x^4
 + a*x)*sqrt(b*x^3 + a)*sqrt(b*e)*sqrt(e*x)) - 4*sqrt(b*x^3 + a)*sqrt(e*x)*
A*b)/(a*b*e^3*x^2), -1/3*(sqrt(-b*e)*B*a*x^2*arctan(2*sqrt(b*x^3 + a)*sqrt(
-b*e)*sqrt(e*x)*x/(2*b*e*x^3 + a*e)) + 2*sqrt(b*x^3 + a)*sqrt(e*x)*A*b)/(a*
b*e^3*x^2)]
```

Sympy [A] (verification not implemented)

Time = 5.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx = -\frac{2A\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3ae^{\frac{5}{2}}} + \frac{2B\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3\sqrt{b}e^{\frac{5}{2}}}$$

```
[In] integrate((B*x**3+A)/(e*x)**(5/2)/(b*x**3+a)**(1/2),x)
```

```
[Out] -2*A*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*a*e**(5/2)) + 2*B*asinh(sqrt(b)*x**(3/
2)/sqrt(a))/(3*sqrt(b)*e**(5/2))
```

Maxima [F]

$$\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx = \int \frac{Bx^3+A}{\sqrt{bx^3+a}\,(ex)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] B*integrate(sqrt(x)/sqrt(b*x^3 + a), x)/e^(5/2) - 2/3*(b*sqrt(e)*x^4 + a*sq
rt(e)*x)*A/(sqrt(b*x^3 + a)*a*e^3*x^(5/2))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.41

$$\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx = -\frac{2\,e\left(\frac{\frac{B\arctan\left(\frac{\sqrt{be+\frac{ae}{x^3}}}{\sqrt{-be}}\right)}{\sqrt{-be}}+\frac{\sqrt{be+\frac{ae}{x^3}}A}{ae}}{e} - \frac{Bae\arctan\left(\frac{\sqrt{be}}{\sqrt{-be}}\right)+\sqrt{be}\sqrt{-be}A}{\sqrt{-be}ae^2}\right)}{3\,|e|^2}$$

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*e*((B*arctan(sqrt(b*e + a*e/x^3)/sqrt(-b*e))/sqrt(-b*e) + sqrt(b*e + a
*e/x^3)*A/(a*e))/e - (B*a*e*arctan(sqrt(b*e)/sqrt(-b*e)) + sqrt(b*e)*sqrt(-
b*e)*A)/(sqrt(-b*e)*a*e^2))/abs(e)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx=\int \frac{B\,x^3+A}{{\left(e\,x\right)}^{5/2}\,\sqrt{b\,x^3+a}}\,dx$$

```
[In] int((A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(1/2)),x)
[Out] int((A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(1/2)), x)
```

3.550 $\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 246

$$\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx = -\frac{2A\sqrt{a+bx^3}}{5ae(ex)^{5/2}} - \frac{(2Ab-5aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -2/5*A*(b*x^3+a)^(1/2)/a/e/(e*x)^(5/2)-1/15*(2*A*b-5*B*a)*(a^(1/3)+b^(1/3)*
x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1
/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*Ellipti
cF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^
(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2
/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(4/3)/e^4/(b*x^
3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2
)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {464, 335, 231}

$$\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx =$$
$$-\frac{\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(2Ab-5aB)\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}(2+\sqrt{3})\right)}{5\sqrt[4]{3}a^{4/3}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{2A\sqrt{a+bx^3}}{5ae(ex)^{5/2}}$$

```
[In] Int[(A + B*x^3)/((e*x)^(7/2)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*A*Sqrt[a + b*x^3])/(5*a*e*(e*x)^(5/2)) - ((2*A*b - 5*a*B)*Sqrt[e*x]*(a^
(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3
) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b
^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(5*3^(1/4
)*a^(4/3)*e^4*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3
])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
```

```
x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{2A\sqrt{a+bx^3}}{5ae(ex)^{5/2}} - \frac{(2Ab-5aB)\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{5ae^3}$$

$$= -\frac{2A\sqrt{a+bx^3}}{5ae(ex)^{5/2}} - \frac{(2(2Ab-5aB))\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{5ae^4}$$

$$= -\frac{2A\sqrt{a+bx^3}}{5ae(ex)^{5/2}} - \frac{(2Ab-5aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.33

$$\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx = -\frac{2x\left(A(a+bx^3)+(2Ab-5aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^3}{a}\right)\right)}{5a(ex)^{7/2}\sqrt{a+bx^3}}$$

```
[In] Integrate[(A + B*x^3)/((e*x)^(7/2)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*x*(A*(a + b*x^3) + (2*A*b - 5*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometr
ic2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(5*a*(e*x)^(7/2)*Sqrt[a + b*x^3])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.94 (sec) , antiderivative size = 740, normalized size of antiderivative = 3.01

method	result
risch	$-\frac{2A\sqrt{b\,x^3+a}}{5a\,x^2e^3\sqrt{ex}}-\frac{2(2Ab-5Ba)\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)^2\sqrt{}}{5a\left(-\frac{3\left(-a\,b^2\right)}{2b}\right.}$
elliptic	$\frac{\sqrt{(b\,x^3+a)ex}\left(-\frac{2A\sqrt{be\,x^4+aex}}{5e^4a\,x^3}+\frac{2\left(\frac{B}{e^3}-\frac{2bA}{5a\,e^3}\right)\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\right.}{}\right.}{}$
default	Expression too large to display

```
[In] int((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/a*A*(b*x^3+a)^(1/2)/x^2/e^3/(e*x)^(1/2)-2/5*(2*A*b-5*B*a)/a*(1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)
^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1
/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(
-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)
))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^
(1/3)/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3))
)^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2))/e^3*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.24

$$\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx = -\frac{2\left((5\,Ba-2\,Ab)\sqrt{ae}x^3\text{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)+\sqrt{bx^3+a}\sqrt{ex}Aa\right)}{5\,a^2e^4x^3}$$

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*((5*B*a - 2*A*b)*sqrt(a*e)*x^3*weierstrassPInverse(0, -4*b/a, 1/x) + s
qrt(b*x^3 + a)*sqrt(e*x)*A*a)/(a^2*e^4*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.39

$$\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx = \frac{A\Gamma\left(-\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{5}{6},\frac{1}{2}\\ \frac{1}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)}+\frac{B\sqrt{x}\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{6},\frac{1}{2}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)}$$

```
[In] integrate((B*x**3+A)/(e*x)**(7/2)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*gamma(-5/6)*hyper((-5/6, 1/2), (1/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*e**(7/2)*x**(5/2)*gamma(1/6)) + B*sqrt(x)*gamma(1/6)*hyper((1/6, 1/2), (
7/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*e**(7/2)*gamma(7/6))
```

Maxima [F]

$$\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx = \int \frac{Bx^3+A}{\sqrt{bx^3+a}\,(ex)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*(e*x)^(7/2)), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx = \int \frac{Bx^3+A}{\sqrt{bx^3+a}\,(ex)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*(e*x)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx = \int \frac{B\,x^3+A}{{(e\,x)}^{7/2}\,\sqrt{b\,x^3+a}}\,dx$$

```
[In] int((A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(1/2)),x)
[Out] int((A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(1/2)), x)
```

3.551 $\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 120

$$\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = -\frac{(2Ab-3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}} + \frac{(2Ab-3aB)e^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}$$

```
[Out] 1/3*(2*A*b-3*B*a)*e^(7/2)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/
2))/b^(5/2)-1/3*(2*A*b-3*B*a)*e^2*(e*x)^(3/2)/b^2/(b*x^3+a)^(1/2)+1/3*B*(e*
x)^(9/2)/b/e/(b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {470, 294, 335, 281, 223, 212}

$$\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{e^{7/2}(2Ab-3aB)\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}} - \frac{e^2(ex)^{3/2}(2Ab-3aB)}{3b^2\sqrt{a+bx^3}} + \frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}}$$

```
[In] Int[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] -1/3*((2*A*b - 3*a*B)*e^2*(e*x)^(3/2))/(b^2*Sqrt[a + b*x^3]) + (B*(e*x)^(9/
2))/(3*b*e*Sqrt[a + b*x^3]) + ((2*A*b - 3*a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*
x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(3*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}} - \frac{\left(-3Ab+\frac{9aB}{2}\right)\int\frac{(ex)^{7/2}}{(a+bx^3)^{3/2}}\,dx}{3b}$$

$$= -\frac{(2Ab-3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}} + \frac{((2Ab-3aB)e^3)\int\frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{2b^2}$$

$$=-\frac{(2Ab-3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}}+\frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}}+\frac{((2Ab-3aB)e^2)\operatorname{Subst}\left(\int\frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{b^2}$$

$$=-\frac{(2Ab-3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}}+\frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}}+\frac{((2Ab-3aB)e^2)\operatorname{Subst}\left(\int\frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx,x,(ex)^{3/2}\right)}{3b^2}$$

$$=-\frac{(2Ab-3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}}+\frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}}+\frac{((2Ab-3aB)e^2)\operatorname{Subst}\left(\int\frac{1}{1-\frac{bx^2}{e^3}}\,dx,x,\frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{3b^2}$$

$$=-\frac{(2Ab-3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}}+\frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}}+\frac{(2Ab-3aB)e^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}$$

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.78

$$\int\frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx=\frac{(ex)^{7/2}\left(\frac{\sqrt{b}x^{3/2}(-2Ab+3aB+bBx^3)}{\sqrt{a+bx^3}}+(2Ab-3aB)\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)\right)}{3b^{5/2}x^{7/2}}$$

```
[In] Integrate[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
[Out] ((e*x)^(7/2)*((Sqrt[b]*x^(3/2)*(-2*A*b + 3*a*B + b*B*x^3))/Sqrt[a + b*x^3]
+ (2*A*b - 3*a*B)*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]]))/(3*b^(5/2)*x^(7/
2))
```

Maple [A] (verified)

Time = 4.69 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.08

method	result
risch	$\frac{B\,x^2\sqrt{b\,x^3+a}\,e^4}{3b^2\sqrt{ex}}+\frac{\left(\frac{2(2Ab-3Ba)\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^3+a\right)ex}}{x^2\sqrt{be}}\right)}{3\sqrt{be}}-\frac{4(Ab-Ba)x^2}{3\sqrt{\left(x^3+\frac{a}{b}\right)bex}}\right)e^4\sqrt{(b\,x^3+a)ex}}{2b^2\sqrt{ex}\,\sqrt{b\,x^3+a}}$
default	$\frac{e^3\sqrt{ex}\left(B\sqrt{be}\,b\,x^5-2A\sqrt{be}\,b\,x^2+3B\sqrt{be}\,a\,x^2+2A\sqrt{(b\,x^3+a)ex}\,\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^3+a\right)ex}}{x^2\sqrt{be}}\right)b-3B\sqrt{(b\,x^3+a)ex}\,\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^3+a\right)ex}}{x^2\sqrt{be}}\right.\right.}{3x\sqrt{b\,x^3+a}\,b^2\sqrt{be}}$
elliptic	Expression too large to display

```
[In] int((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*B*x^2/b^2*(b*x^3+a)^(1/2)*e^4/(e*x)^(1/2)+1/2/b^2*(2/3*(2*A*b-3*B*a)/(b
*e)^(1/2)*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))-4/3*(A*b-B*a)*x^2/
((x^3+a/b)*b*e*x)^(1/2))*e^4*((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1
/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.56 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.56

$$\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \left[-\frac{((3\,Bab-2\,Ab^2)e^3x^3+(3\,Ba^2-2\,Aab)e^3)\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e\right.}{12(}\right.$$

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(((3*B*a*b - 2*A*b^2)*e^3*x^3 + (3*B*a^2 - 2*A*a*b)*e^3)*sqrt(e/b)*l
og(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 +
a)*sqrt(e*x)*sqrt(e/b)) - 4*(B*b*e^3*x^4 + (3*B*a - 2*A*b)*e^3*x)*sqrt(b*x^
3 + a)*sqrt(e*x))/(b^3*x^3 + a*b^2), 1/6*(((3*B*a*b - 2*A*b^2)*e^3*x^3 + (3
*B*a^2 - 2*A*a*b)*e^3)*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sq
rt(-e/b)/(2*b*e*x^3 + a*e)) + 2*(B*b*e^3*x^4 + (3*B*a - 2*A*b)*e^3*x)*sqrt(
b*x^3 + a)*sqrt(e*x))/(b^3*x^3 + a*b^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x)**(7/2)*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)(ex)^{\frac{7}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")

[Out] integrate((B*x^3 + A)*(e*x)^(7/2)/(b*x^3 + a)^(3/2), x)

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.10

$$\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{\left(\frac{Be^4x^3}{b} + \frac{3\,Bab^3e^4-2\,Ab^4e^4}{b^5}\right)\sqrt{ex}ex}{3\,\sqrt{be^4x^3+ae^4}} + \frac{(3\,Bab^3e^4-2\,Ab^4e^4)e^2\log\left(\left|-\sqrt{be}\sqrt{ex}ex+\sqrt{be^4x^3+ae^4}\right|\right)}{3\,\sqrt{be}b^5|e|^2}$$

[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")

[Out] 1/3*(B*e^4*x^3/b + (3*B*a*b^3*e^4 - 2*A*b^4*e^4)/b^5)*sqrt(e*x)*e*x/sqrt(b*e^4*x^3 + a*e^4) + 1/3*(3*B*a*b^3*e^4 - 2*A*b^4*e^4)*e^2*log(abs(-sqrt(b*e)*sqrt(e*x)*e*x + sqrt(b*e^4*x^3 + a*e^4)))/(sqrt(b*e)*b^5*abs(e)^2)

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(B\,x^3+A)\,(e\,x)^{7/2}}{(b\,x^3+a)^{3/2}}\,dx$$

[In] int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(3/2),x)

[Out] int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(3/2), x)

3.552 $\int \frac{(ex)^{5/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 286

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx = -\frac{(4Ab-7aB)e^2\sqrt{ex}}{6b^2\sqrt{a+bx^3}} + \frac{B(ex)^{7/2}}{2be\sqrt{a+bx^3}} + \frac{(4Ab-7aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{12\sqrt[4]{3}\sqrt[3]{a}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 1/2*B*(e*x)^(7/2)/b/e/(b*x^3+a)^(1/2)-1/6*(4*A*b-7*B*a)*e^2*(e*x)^(1/2)/b^2
/(b*x^3+a)^(1/2)+1/36*(4*A*b-7*B*a)*e^2*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/
3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/
3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(
1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+
1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+
b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(1/3)/b^2/(b*x^3+a)^(1/2)/(b^(1/3
)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {470, 294, 335, 231}

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(4Ab-7aB)\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right.}{\left(1+\sqrt{3}\right.}\right.\right.}{12\sqrt[4]{3}\sqrt[3]{a}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{e^2\sqrt{ex}(4Ab-7aB)}{6b^2\sqrt{a+bx^3}}+\frac{B(ex)^{7/2}}{2be\sqrt{a+bx^3}}$$

```
[In] Int[((e*x)^(5/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] -1/6*((4*A*b - 7*a*B)*e^2*Sqrt[e*x])/(b^2*Sqrt[a + b*x^3]) + (B*(e*x)^(7/2)
)/(2*b*e*Sqrt[a + b*x^3]) + ((4*A*b - 7*a*B)*e^2*Sqrt[e*x]*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt
[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^
(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(12*3^(1/4)*a^(1/3)*b^
2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{B(ex)^{7/2}}{2be\sqrt{a+bx^3}} - \frac{\left(-2Ab+\frac{7aB}{2}\right)\int \frac{(ex)^{5/2}}{(a+bx^3)^{3/2}}\,dx}{2b}$$

$$= -\frac{(4Ab-7aB)e^2\sqrt{ex}}{6b^2\sqrt{a+bx^3}} + \frac{B(ex)^{7/2}}{2be\sqrt{a+bx^3}} + \frac{((4Ab-7aB)e^3)\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{12b^2}$$

$$= -\frac{(4Ab-7aB)e^2\sqrt{ex}}{6b^2\sqrt{a+bx^3}} + \frac{B(ex)^{7/2}}{2be\sqrt{a+bx^3}} + \frac{((4Ab-7aB)e^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{6b^2}$$

$$= -\frac{(4Ab-7aB)e^2\sqrt{ex}}{6b^2\sqrt{a+bx^3}} + \frac{B(ex)^{7/2}}{2be\sqrt{a+bx^3}} + \frac{(4Ab-7aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}(2+\sqrt{3}}{12\sqrt[4]{3}\sqrt[3]{a}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.30

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{e^2\sqrt{ex}\left(-4Ab+7aB+3bBx^3+(4Ab-7aB)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6}\right.\right.}{6b^2\sqrt{a+bx^3}}$$

```
[In] Integrate[((e*x)^(5/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (e^2*Sqrt[e*x]*(-4*A*b + 7*a*B + 3*b*B*x^3 + (4*A*b - 7*a*B)*Sqrt[1 + (b*x^
3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(6*b^2*Sqrt[a + b*x^
3])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.91 (sec) , antiderivative size = 787, normalized size of antiderivative = 2.75

method	result	size
elliptic	Expression too large to display	787
risch	Expression too large to display	2115
default	Expression too large to display	3760

```
[In] int((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e/x*(e*x)^(1/2)/(b*x^3+a)^(1/2)*((b*x^3+a)*e*x)^(1/2)*(-2/3/b^2*e^3*x*(A*
b-B*a)/((x^3+a/b)*b*e*x)^(1/2)+1/2*B*e^2/b^2*(b*e*x^4+a*e*x)^(1/2)+2*(1/3*(
A*b-B*a)*e^3/b^2-1/4*B/b^2*e^3*a)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^
(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2)^(1
/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [F]

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{(Bx^3+A)(ex)^{\frac{5}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*e^2*x^5 + A*e^2*x^2)*sqrt(b*x^3 + a)*sqrt(e*x)/(b^2*x^6 + 2*a*b
*x^3 + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x)**(5/2)*(B*x**3+A)/(b*x**3+a)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)(ex)^{\frac{5}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(e*x)^(5/2)/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)(ex)^{\frac{5}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(e*x)^(5/2)/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(B\,x^3+A)\,{(e\,x)}^{5/2}}{{(b\,x^3+a)}^{3/2}}\,dx$$

```
[In] int(((A + B*x^3)*(e*x)^(5/2))/(a + b*x^3)^(3/2),x)
[Out] int(((A + B*x^3)*(e*x)^(5/2))/(a + b*x^3)^(3/2), x)
```

3.553 $\int \frac{(ex)^{3/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 553

$$\int \frac{(ex)^{3/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2(Ab-aB)(ex)^{5/2}}{3abe\sqrt{a+bx^3}} - \frac{(1+\sqrt{3})(2Ab-5aB)e\sqrt{ex}\sqrt{a+bx^3}}{3ab^{5/3}\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)}$$
$$+\frac{(2Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+(1-\sqrt{3})\sqrt[3]{b}x}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{3^{3/4}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{(1-\sqrt{3})(2Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+(1-\sqrt{3})\sqrt[3]{b}x}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x}\right),\frac{1}{4}(2\right.}{6\sqrt[4]{3}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/3*(A*b-B*a)*(e*x)^(5/2)/a/b/e/(b*x^3+a)^(1/2)-1/3*(2*A*b-5*B*a)*e*(1+3^(1
/2))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/a/b^(5/3)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))+
1/3*(2*A*b-5*B*a)*e*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/
(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a
^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^
2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(
1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)
))^2)^(1/2)*3^(1/4)/a^(2/3)/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(
1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)+1/18*(2*A*b-5*B*a)*e*(a^(1
```

```
/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(
1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/
2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3
^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3
)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3
^(3/4)/a^(2/3)/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1
/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {468, 335, 314, 231, 1895}

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{\left(1-\sqrt{3}\right)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(2Ab-5aB)\,\text{EllipticF}\left(\text{arcc}\right.}{6\sqrt[4]{3}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(2Ab-5aB)E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3^{3/4}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{\left(1+\sqrt{3}\right)e\sqrt{ex}\sqrt{a+bx^3}(2Ab-5aB)}{3ab^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}+\frac{2(ex)^{5/2}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

```
[In] Int[((e*x)^(3/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(5/2))/(3*a*b*e*Sqrt[a + b*x^3]) - ((1 + Sqrt[3])*(2*A
*b - 5*a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^3])/(3*a*b^(5/3)*(a^(1/3) + (1 + Sqrt[
3])*b^(1/3)*x)) + ((2*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)
*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 +
 Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(3^(3/4)*a^(2/3)*b^(5/3)*Sqrt[(b^(
1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a
 + b*x^3]) + ((1 - Sqrt[3])*(2*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/
3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(6*3^(1/4)*a^(2/3)*b^(5/3
)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\text{integral} = \frac{2(Ab-aB)(ex)^{5/2}}{3abe\sqrt{a+bx^3}} + \frac{\left(2\left(-Ab+\frac{5aB}{2}\right)\right)\int\frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\,dx}{3ab}$$

$$= \frac{2(Ab-aB)(ex)^{5/2}}{3abe\sqrt{a+bx^3}} - \frac{(2(2Ab-5aB))\text{Subst}\left(\int\frac{x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{3abe}$$

$$= \frac{2(Ab-aB)(ex)^{5/2}}{3abe\sqrt{a+bx^3}} + \frac{(2Ab-5aB)\text{Subst}\left(\int\frac{\left(-1+\sqrt{3}\right)a^{2/3}e^2-2b^{2/3}x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{3ab^{5/3}e} + \frac{\left(\left(1-\sqrt{3}\right)(2Ab-5aB)e\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{3\sqrt[3]{a}b^{5/3}}$$

$$= \frac{2(Ab-aB)(ex)^{5/2}}{3abe\sqrt{a+bx^3}} - \frac{\left(1+\sqrt{3}\right)(2Ab-5aB)e\sqrt{ex}\sqrt{a+bx^3}}{3ab^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)} + \frac{(2Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}(2+\sqrt{3}\right.}{3^{3/4}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{\left(1-\sqrt{3}\right)(2Ab-5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\right.}{6\sqrt[4]{3}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.14

$$\int\frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{x(ex)^{3/2}\left(5aB+(2Ab-5aB)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{3}{2},\frac{11}{6},-\frac{bx^3}{a}\right)\right)}{5ab\sqrt{a+bx^3}}$$

```
[In] Integrate[((e*x)^(3/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
[Out] (x*(e*x)^(3/2)*(5*a*B + (2*A*b - 5*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2
F1[5/6, 3/2, 11/6, -((b*x^3)/a)]))/(5*a*b*Sqrt[a + b*x^3])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.59 (sec) , antiderivative size = 1154, normalized size of antiderivative = 2.09

method	result	size
elliptic	Expression too large to display	1154
default	Expression too large to display	5392

```
[In] int((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e/x*(e*x)^(1/2)/(b*x^3+a)^(1/2)*((b*x^3+a)*e*x)^(1/2)*(2/3/b*e^2*x^3/a*(A
*b-B*a)/((x^3+a/b)*b*e*x)^(1/2)+(B*e^2/b-2/3*(A*b-B*a)/a/b*e^2)*(x*(x+1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)
*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF
(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))*b/(-a*b^2)^(1/3)))/(b*e*
x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [F]

$$\int \frac{(ex)^{3/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)(ex)^{\frac{3}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] integral((B*e*x^4 + A*e*x)*sqrt(b*x^3 + a)*sqrt(e*x)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 77.73 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.17

$$\int \frac{(ex)^{3/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{Ae^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right) {}_2F_1\left(\begin{matrix}\frac{5}{6},\frac{3}{2}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)} + \frac{Be^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right) {}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{17}{6}\right)}$$

[In] integrate((e*x)**(3/2)*(B*x**3+A)/(b*x**3+a)**(3/2),x)

[Out] A*e**(3/2)*x**(5/2)*gamma(5/6)*hyper((5/6, 3/2), (11/6,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(11/6)) + B*e**(3/2)*x**(11/2)*gamma(11/6)*hyper((3/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(17/6))

Maxima [F]

$$\int \frac{(ex)^{3/2}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)(ex)^{\frac{3}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")

[Out] integrate((B*x^3 + A)*(e*x)^(3/2)/(b*x^3 + a)^(3/2), x)

Giac [F]

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{(Bx^3+A)(ex)^{\frac{3}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(e*x)^(3/2)/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{\left(B\,x^3+A\right)\,{\left(e\,x\right)}^{3/2}}{{\left(b\,x^3+a\right)}^{3/2}}\,dx$$

```
[In] int(((A + B*x^3)*(e*x)^(3/2))/(a + b*x^3)^(3/2),x)
[Out] int(((A + B*x^3)*(e*x)^(3/2))/(a + b*x^3)^(3/2), x)
```

3.554 $\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{3/2}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 85

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{3/2}} \, dx = \frac{2(Ab-aB)(ex)^{3/2}}{3abe\sqrt{a+bx^3}} + \frac{2B\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}$$

```
[Out] 2/3*B*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))*e^(1/2)/b^(3/2)+
2/3*(A*b-B*a)*(e*x)^(3/2)/a/b/e/(b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {463, 335, 281, 223, 212}

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{3/2}} \, dx = \frac{2(ex)^{3/2}(Ab-aB)}{3abe\sqrt{a+bx^3}} + \frac{2B\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}$$

```
[In] Int[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(3/2))/(3*a*b*e*Sqrt[a + b*x^3]) + (2*B*Sqrt[e]*ArcTan
h[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(3*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2(Ab-aB)(ex)^{3/2}}{3abe\sqrt{a+bx^3}}+\frac{B\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{b}\\
&= \frac{2(Ab-aB)(ex)^{3/2}}{3abe\sqrt{a+bx^3}}+\frac{(2B)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{be}\\
&= \frac{2(Ab-aB)(ex)^{3/2}}{3abe\sqrt{a+bx^3}}+\frac{(2B)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx,x,(ex)^{3/2}\right)}{3be}\\
&= \frac{2(Ab-aB)(ex)^{3/2}}{3abe\sqrt{a+bx^3}}+\frac{(2B)\text{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx,x,\frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{3be}\\
&= \frac{2(Ab-aB)(ex)^{3/2}}{3abe\sqrt{a+bx^3}}+\frac{2B\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2\sqrt{ex}\left(\frac{\sqrt{b}(Ab-aB)x^{3/2}}{a\sqrt{a+bx^3}}+B\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)\right)}{3b^{3/2}\sqrt{x}}$$

```
[In] Integrate[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(3/2),x]
[Out] (2*Sqrt[e*x]*((Sqrt[b]*(A*b - a*B)*x^(3/2))/(a*Sqrt[a + b*x^3]) + B*Log[Sqr
t[b]*x^(3/2) + Sqrt[a + b*x^3]]))/(3*b^(3/2)*Sqrt[x])
```

Maple [A] (verified)

Time = 4.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{2\sqrt{ex}\left(A\sqrt{be}\,b\,x^2-B\sqrt{be}\,a\,x^2+B\sqrt{(b\,x^3+a)ex}\,\operatorname{arctanh}\left(\frac{\sqrt{(b\,x^3+a)\,ex}}{x^2\sqrt{be}}\right)a\right)}{3\sqrt{b\,x^3+a}\,xb\sqrt{be}\,a}$	92
elliptic	Expression too large to display	1050

```
[In] int((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(e*x)^(1/2)/(b*x^3+a)^(1/2)*(A*(b*e)^(1/2)*b*x^2-B*(b*e)^(1/2)*a*x^2+B*
((b*x^3+a)*e*x)^(1/2)*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))*a)/x/b
/(b*e)^(1/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.75

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \left[-\frac{4\sqrt{bx^3+a}(Ba-Ab)\sqrt{ex}x-(Babx^3+Ba^2)\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e\right.}{6\,(ab^2x^3+a^2b)}\right.$$

$$\left.-\frac{2\sqrt{bx^3+a}(Ba-Ab)\sqrt{ex}x+(Babx^3+Ba^2)\sqrt{-\frac{e}{b}}\arctan\left(\frac{2\sqrt{bx^3+a}\sqrt{ex}bx\sqrt{-\frac{e}{b}}}{2\,bex^3+ae}\right)}{3\,(ab^2x^3+a^2b)}\right]$$

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(4*sqrt(b*x^3 + a)*(B*a - A*b)*sqrt(e*x)*x - (B*a*b*x^3 + B*a^2)*sqrt
(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b
*x^3 + a)*sqrt(e*x)*sqrt(e/b)))/(a*b^2*x^3 + a^2*b), -1/3*(2*sqrt(b*x^3 + a
)*(B*a - A*b)*sqrt(e*x)*x + (B*a*b*x^3 + B*a^2)*sqrt(-e/b)*arctan(2*sqrt(b*
x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)))/(a*b^2*x^3 + a^2*b)]
```

Sympy [A] (verification not implemented)

Time = 19.57 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2A\sqrt{e}x^{\frac{3}{2}}}{3a^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}} + B\left(\frac{2\sqrt{e}\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3b^{\frac{3}{2}}} - \frac{2\sqrt{e}x^{\frac{3}{2}}}{3\sqrt{a}b\sqrt{1+\frac{bx^3}{a}}}\right)$$

```
[In] integrate((B*x**3+A)*(e*x)**(1/2)/(b*x**3+a)**(3/2),x)
```

```
[Out] 2*A*sqrt(e)*x**(3/2)/(3*a**(3/2)*sqrt(1 + b*x**3/a)) + B*(2*sqrt(e)*asinh(s
qrt(b)*x**(3/2)/sqrt(a))/(3*b**(3/2)) - 2*sqrt(e)*x**(3/2)/(3*sqrt(a)*b*sqr
t(1 + b*x**3/a)))
```

Maxima [F]

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{ex}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(e*x)/(b*x^3 + a)^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = -\frac{2\,Be^3\log\left(\left|-\sqrt{be}\sqrt{ex}ex+\sqrt{be^4x^3+ae^4}\right|\right)}{3\,\sqrt{be}b|e|^2} - \frac{2\,(Bae-Abe)\sqrt{ex}ex}{3\,\sqrt{be^4x^3+ae^4}ab}$$

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*B*e^3*log(abs(-sqrt(b*e)*sqrt(e*x)*e*x + sqrt(b*e^4*x^3 + a*e^4)))/(sq
rt(b*e)*b*abs(e)^2) - 2/3*(B*a*e - A*b*e)*sqrt(e*x)*e*x/(sqrt(b*e^4*x^3 + a
*e^4)*a*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \int \frac{(B\,x^3+A)\,\sqrt{e\,x}}{(b\,x^3+a)^{3/2}}\,dx$$

```
[In] int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(3/2),x)
[Out] int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(3/2), x)
```

3.555 $\int \frac{A+Bx^3}{\sqrt{ex}(a+bx^3)^{3/2}} dx$

Optimal result

Integrand size = 26, antiderivative size = 258

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{3/2}}\,dx = \frac{2(Ab-aB)\sqrt{ex}}{3abe\sqrt{a+bx^3}} + \frac{(2Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3\sqrt[4]{3}a^{4/3}be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/3*(A*b-B*a)*(e*x)^(1/2)/a/b/e/(b*x^3+a)^(1/2)+1/9*(2*A*b+B*a)*(a^(1/3)+b^
(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))
^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*E
llipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)
))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(4/3)/b/e
/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/
2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {468, 335, 231}

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{3/2}}\,dx = \frac{\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+2Ab)\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{}}{\left(1+\sqrt{3}\right)\sqrt[3]{}}\right.\right.}{3\sqrt[4]{3}a^{4/3}be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2\sqrt{ex}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

```
[In] Int[(A + B*x^3)/(Sqrt[e*x]*(a + b*x^3)^(3/2)),x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[e*x])/(3*a*b*e*Sqrt[a + b*x^3]) + ((2*A*b + a*B)*Sqrt[e
*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/
(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqr
t[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(3
*3^(1/4)*a^(4/3)*b*e*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 +
 Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
```

```
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\text{integral} = \frac{2(Ab-aB)\sqrt{ex}}{3abe\sqrt{a+bx^3}} + \frac{\left(2\left(Ab+\frac{aB}{2}\right)\right)\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{3ab}$$

$$= \frac{2(Ab-aB)\sqrt{ex}}{3abe\sqrt{a+bx^3}} + \frac{(2(2Ab+aB))\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{3abe}$$

$$= \frac{2(Ab-aB)\sqrt{ex}}{3abe\sqrt{a+bx^3}} + \frac{(2Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3\sqrt[4]{3}a^{4/3}be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.31

$$\int \frac{A+Bx^3}{\sqrt{ex}\left(a+bx^3\right)^{3/2}}\,dx = \frac{2x\left(Ab-aB+(2Ab+aB)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^3}{a}\right)\right)}{3ab\sqrt{ex}\sqrt{a+bx^3}}$$

```
[In] Integrate[(A + B*x^3)/(Sqrt[e*x]*(a + b*x^3)^(3/2)),x]
[Out] (2*x*(A*b - a*B + (2*A*b + a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6,
1/2, 7/6, -((b*x^3)/a)]))/(3*a*b*Sqrt[e*x]*Sqrt[a + b*x^3])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.79 (sec) , antiderivative size = 754, normalized size of antiderivative = 2.92

method	result
elliptic	$\frac{\sqrt{(b x^3+a)ex}\left(\frac{2x(Ab-Ba)}{3ba\sqrt{\left(x^3+\frac{a}{b}\right)bex}}+\frac{2\left(\frac{B}{b}+\frac{\frac{2Ab}{3}-\frac{2Ba}{3}}{ab}\right)\left(\frac{\left(-a b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a b^2\right)^{\frac{1}{3}}}{b}\right)}}\left(\right.}{}\right.}{}$
default	Expression too large to display

```
[In] int((B*x^3+A)/(b*x^3+a)^(3/2)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)*(2/3/b*x/a*(A*b-B*a)/((x^
3+a/b)*b*e*x)^(1/2)+2*(B/b+2/3*(A*b-B*a)/a/b)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b
^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(
x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)/(b*e*x*(x-1/b
*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.36

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{3/2}}\,dx = -\frac{2\left(((Bab+2\,Ab^2)x^3+Ba^2+2\,Aab)\sqrt{ae}\text{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)+\sqrt{bx^3+a}(Ba^2-Aab)\sqrt{ex}\right)}{3\,(a^2b^2ex^3+a^3be)}$$

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(3/2)/(e*x)^(1/2),x, algorithm="fricas")
[Out] -2/3*(((B*a*b + 2*A*b^2)*x^3 + B*a^2 + 2*A*a*b)*sqrt(a*e)*weierstrassPInver
se(0, -4*b/a, 1/x) + sqrt(b*x^3 + a)*(B*a^2 - A*a*b)*sqrt(e*x))/(a^2*b^2*e*
x^3 + a^3*b*e)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 22.61 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.36

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{3/2}}\,dx = \frac{A\sqrt{x}\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{6},\frac{3}{2}\\\frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\sqrt{e}\Gamma\left(\frac{7}{6}\right)}+\frac{Bx^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}\frac{7}{6},\frac{3}{2}\\\frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\sqrt{e}\Gamma\left(\frac{13}{6}\right)}$$

```
[In] integrate((B*x**3+A)/(b*x**3+a)**(3/2)/(e*x)**(1/2),x)
[Out] A*sqrt(x)*gamma(1/6)*hyper((1/6, 3/2), (7/6,), b*x**3*exp_polar(I*pi)/a)/(3
*a**(3/2)*sqrt(e)*gamma(7/6)) + B*x**(7/2)*gamma(7/6)*hyper((7/6, 3/2), (13
/6,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*sqrt(e)*gamma(13/6))
```

Maxima [F]

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{3/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(3/2)/(e*x)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*sqrt(e*x)), x)
```

Giac [F]

$$\int \frac{A + Bx^3}{\sqrt{ex}\,(a + bx^3)^{3/2}}\,dx = \int \frac{Bx^3 + A}{(bx^3 + a)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(3/2)/(e*x)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*sqrt(e*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^3}{\sqrt{ex}\,(a + bx^3)^{3/2}}\,dx = \int \frac{B\,x^3 + A}{\sqrt{e\,x}\,(b\,x^3 + a)^{3/2}}\,dx$$

```
[In] int((A + B*x^3)/((e*x)^(1/2)*(a + b*x^3)^(3/2)),x)
[Out] int((A + B*x^3)/((e*x)^(1/2)*(a + b*x^3)^(3/2)), x)
```

3.556 $\int \frac{A+Bx^3}{(ex)^{3/2}(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 585

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}\,dx = -\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}} - \frac{2(4Ab-aB)(ex)^{5/2}}{3a^2e^4\sqrt{a+bx^3}} + \frac{2\left(1+\sqrt{3}\right)(4Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{3a^2b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)} - \frac{2(4Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3^{3/4}a^{5/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{\left(1-\sqrt{3}\right)(4Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(2 -}{3\sqrt[4]{3}a^{5/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -2/3*(4*A*b-B*a)*(e*x)^(5/2)/a^2/e^4/(b*x^3+a)^(1/2)-2*A/a/e/(e*x)^(1/2)/(b
*x^3+a)^(1/2)+2/3*(4*A*b-B*a)*(1+3^(1/2))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/a^2/b
^(2/3)/e^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))-2/3*(4*A*b-B*a)*(a^(1/3)+b^(1/3)
*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(
1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*Ellipt
icE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)
^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
```

```
2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(5/3)/b^(2/3)/
e^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^
(1/2)))^2)^(1/2)-1/9*(4*A*b-B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1
-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1
-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*
(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(
1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(
1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(5/3)/b^(2/3)/e^2/(b*x^3+a)^
(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/
2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {464, 296, 335, 314, 231, 1895}

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}\,dx =$$

$$-\frac{\left(1-\sqrt{3}\right)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(4Ab-aB)\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}(2\right.}{3\sqrt[4]{3}a^{5/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(4Ab-aB)E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3^{3/4}a^{5/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2\left(1+\sqrt{3}\right)\sqrt{ex}\sqrt{a+bx^3}(4Ab-aB)}{3a^2b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}-\frac{2(ex)^{5/2}(4Ab-aB)}{3a^2e^4\sqrt{a+bx^3}}-\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}}$$

```
[In] Int[(A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-2*A)/(a*e*Sqrt[e*x]*Sqrt[a + b*x^3]) - (2*(4*A*b - a*B)*(e*x)^(5/2))/(3*a
^2*e^4*Sqrt[a + b*x^3]) + (2*(1 + Sqrt[3])*(4*A*b - a*B)*Sqrt[e*x]*Sqrt[a +
 b*x^3])/(3*a^2*b^(2/3)*e^2*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)) - (2*(4*A*
b - a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(
1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 +
Sqrt[3])/4])/(3^(3/4)*a^(5/3)*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3
```

```
)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - ((1 - Sqrt[
3])*(4*A*b - a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[Ar
cCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)], (2 + Sqrt[3])/4])/(3*3^(1/4)*a^(5/3)*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/
3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\text{integral} = -\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}} - \frac{(4Ab-aB)\int \frac{(ex)^{3/2}}{(a+bx^3)^{3/2}}\,dx}{ae^3}$$

$$= -\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}} - \frac{2(4Ab-aB)(ex)^{5/2}}{3a^2e^4\sqrt{a+bx^3}} + \frac{(2(4Ab-aB))\int \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\,dx}{3a^2e^3}$$

$$= -\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}} - \frac{2(4Ab-aB)(ex)^{5/2}}{3a^2e^4\sqrt{a+bx^3}} + \frac{(4(4Ab-aB))\text{Subst}\left(\int \frac{x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{3a^2e^4}$$

$$= -\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}} - \frac{2(4Ab-aB)(ex)^{5/2}}{3a^2e^4\sqrt{a+bx^3}} - \frac{(2(4Ab-aB))\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}e^2-2b^{2/3}x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{3a^2b^{2/3}e^4} - \frac{\left(2\left(1-\sqrt{3}\right)(4Ab-aB)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{3a^{4/3}b^{2/3}e^2}$$

$$= -\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^3}} - \frac{2(4Ab-aB)(ex)^{5/2}}{3a^2e^4\sqrt{a+bx^3}} + \frac{2\left(1+\sqrt{3}\right)(4Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{3a^2b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$

$$-\frac{2(4Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3^{3/4}a^{5/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{\left(1-\sqrt{3}\right)(4Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}(2\right.\right)}{3\sqrt[4]{3}a^{5/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.13

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}\,dx = \frac{x\left(-10aA+2(-4Ab+aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{3}{2},\frac{11}{6},-\frac{bx^3}{a}\right)\right)}{5a^2(ex)^{3/2}\sqrt{a+bx^3}}$$

```
[In] Integrate[(A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(3/2)),x]
[Out] (x*(-10*a*A + 2*(-4*A*b + a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[5/
6, 3/2, 11/6, -((b*x^3)/a)]))/(5*a^2*(e*x)^(3/2)*Sqrt[a + b*x^3])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.20 (sec) , antiderivative size = 1177, normalized size of antiderivative = 2.01

method	result	size
elliptic	Expression too large to display	1177
risch	Expression too large to display	2209
default	Expression too large to display	5563

```
[In] int((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)*(-2*(b*e*x^3+a*e)/e^2/a^2
*A/(x*(b*e*x^3+a*e))^(1/2)-2/3/e*x^3/a^2*(A*b-B*a)/((x^3+a/b)*b*e*x)^(1/2)+
(2*b/a^2/e*A+2/3/a^2*(A*b-B*a)/e)*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b
*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))
^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b
^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/
b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-
a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2))*b/(-a*b^2)^(1/3)))/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.18

$$\int \frac{A+Bx^3}{(ex)^{3/2}\,(a+bx^3)^{3/2}}\,dx =$$
$$-\frac{2\left(\left((Bab-4\,Ab^2)x^4+(Ba^2-4\,Aab)x\right)\sqrt{ae}\,\mathrm{weierstrassZeta}\left(0,-\frac{4\,b}{a},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)\right)+\sqrt{}\right.}{3\,(a^2b^2e^2x^4+a^3be^2x)}$$

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(((B*a*b - 4*A*b^2)*x^4 + (B*a^2 - 4*A*a*b)*x)*sqrt(a*e)*weierstrassZe
ta(0, -4*b/a, weierstrassPInverse(0, -4*b/a, 1/x)) + sqrt(b*x^3 + a)*(B*a^2
 - A*a*b)*sqrt(e*x))/(a^2*b^2*e^2*x^4 + a^3*b*e^2*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 42.00 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.17

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}\,dx = \frac{A\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{6},\frac{3}{2}\\ \frac{5}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}e^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{5}{6}\right)} + \frac{Bx^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}\frac{5}{6},\frac{3}{2}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)}$$

```
[In] integrate((B*x**3+A)/(e*x)**(3/2)/(b*x**3+a)**(3/2),x)
```

```
[Out] A*gamma(-1/6)*hyper((-1/6, 3/2), (5/6,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3
/2)*e**(3/2)*sqrt(x)*gamma(5/6)) + B*x**(5/2)*gamma(5/6)*hyper((5/6, 3/2),
(11/6,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*e**(3/2)*gamma(11/6))
```

Maxima [F]

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}\,dx = \int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*(e*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}\,dx = \int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*(e*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{B\,x^3+A}{\left(e\,x\right)^{3/2}\,\left(b\,x^3+a\right)^{3/2}}\,dx$$

```
[In] int((A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(3/2)),x)
[Out] int((A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(3/2)), x)
```

3.557 $\int \frac{A+Bx^3}{(ex)^{5/2}(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 67

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}\,dx=-\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^3}}-\frac{2(2Ab-aB)(ex)^{3/2}}{3a^2e^4\sqrt{a+bx^3}}$$

```
[Out] -2/3*A/a/e/(e*x)^(3/2)/(b*x^3+a)^(1/2)-2/3*(2*A*b-B*a)*(e*x)^(3/2)/a^2/e^4/
(b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {464, 270}

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}\,dx=-\frac{2(ex)^{3/2}(2Ab-aB)}{3a^2e^4\sqrt{a+bx^3}}-\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^3}}$$

```
[In] Int[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-2*A)/(3*a*e*(e*x)^(3/2)*Sqrt[a + b*x^3]) - (2*(2*A*b - a*B)*(e*x)^(3/2))/
(3*a^2*e^4*Sqrt[a + b*x^3])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^3}} - \frac{(2Ab-aB)\int \frac{\sqrt{ex}}{(a+bx^3)^{3/2}}\,dx}{ae^3}\\ &= -\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^3}} - \frac{2(2Ab-aB)(ex)^{3/2}}{3a^2e^4\sqrt{a+bx^3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx^3}{(ex)^{5/2}\,(a+bx^3)^{3/2}}\,dx = \frac{2x(-aA-2Abx^3+aBx^3)}{3a^2(ex)^{5/2}\sqrt{a+bx^3}}$$

```
[In] Integrate[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(3/2)),x]
[Out] (2*x*(-(a*A) - 2*A*b*x^3 + a*B*x^3))/(3*a^2*(e*x)^(5/2)*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 4.48 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

method	result	size
gosper	$-\frac{2x(2Ab\,x^3-Ba\,x^3+Aa)}{3\sqrt{b\,x^3+a}\,a^2(ex)^{\frac{5}{2}}}$	39
default	$-\frac{2(2Ab\,x^3-Ba\,x^3+Aa)}{3x\sqrt{b\,x^3+a}\,a^2e^2\sqrt{ex}}$	44
risch	$-\frac{2A\sqrt{b\,x^3+a}}{3a^2x\,e^2\sqrt{ex}}-\frac{2(Ab-Ba)x^2}{3a^2e^2\sqrt{ex}\,\sqrt{b\,x^3+a}}$	61
elliptic	$\frac{\sqrt{(b\,x^3+a)ex}\left(-\frac{2x^2(Ab-Ba)}{3e^2a^2\sqrt{\left(x^3+\frac{a}{b}\right)bex}}-\frac{2A\sqrt{be\,x^4+aex}}{3e^3a^2x^2}\right)}{\sqrt{ex}\,\sqrt{b\,x^3+a}}$	88

```
[In] int((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/3*x*(2*A*b*x^3-B*a*x^3+A*a)/(b*x^3+a)^(1/2)/a^2/(e*x)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^3}{(ex)^{5/2}\,(a+bx^3)^{3/2}}\,dx = \frac{2\,((Ba-2\,Ab)x^3-Aa)\sqrt{bx^3+a}\sqrt{ex}}{3\,(a^2be^3x^5+a^3e^3x^2)}$$

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*((B*a - 2*A*b)*x^3 - A*a)*sqrt(b*x^3 + a)*sqrt(e*x)/(a^2*b*e^3*x^5 + a^
3*e^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 83.59 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.34

$$\int \frac{A+Bx^3}{(ex)^{5/2}\,(a+bx^3)^{3/2}}\,dx = A\left(-\frac{2}{3a\sqrt{b}e^{\frac{5}{2}}x^3\sqrt{\frac{a}{bx^3}+1}}-\frac{4\sqrt{b}}{3a^2e^{\frac{5}{2}}\sqrt{\frac{a}{bx^3}+1}}\right)+\frac{2B}{3a\sqrt{b}e^{\frac{5}{2}}\sqrt{\frac{a}{bx^3}+1}}$$

```
[In] integrate((B*x**3+A)/(e*x)**(5/2)/(b*x**3+a)**(3/2),x)
```

```
[Out] A*(-2/(3*a*sqrt(b)*e**(5/2)*x**3*sqrt(a/(b*x**3) + 1)) - 4*sqrt(b)/(3*a**2*
e**(5/2)*sqrt(a/(b*x**3) + 1))) + 2*B/(3*a*sqrt(b)*e**(5/2)*sqrt(a/(b*x**3)
 + 1))
```

Maxima [F]

$$\int \frac{A+Bx^3}{(ex)^{5/2}\,(a+bx^3)^{3/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{3}{2}}\,(ex)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*(e*x)^(5/2)), x)
```

Giac [B] (verification not implemented)

Error detected during grading. Assigning place holder grade for now.

Time = 0.36 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.46

$$\int \frac{A+Bx^3}{(ex)^{5/2}\,(a+bx^3)^{3/2}}\,dx = Recursiveaassumption \geq -\frac{2\,Ae\left(\frac{\sqrt{be+\frac{ae}{x^3}}}{ae^2}-\frac{\sqrt{be}}{ae^2}\right)}{3\,a|e|^2}+\frac{2\,(Ba-Ab)\sqrt{ex}x}{3\,\sqrt{be^4x^3+ae^4}a^2e}-\frac{bignored}{e^3t_{nostep}^6}$$

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] Recursive*a*assumption >= -2/3*A*e*(sqrt(b*e + a*e/x^3)/(a*e^2) - sqrt(b*e)
/(a*e^2))/(a*abs(e)^2) + 2/3*(B*a - A*b)*sqrt(e*x)*x/(sqrt(b*e^4*x^3 + a*e^
4)*a^2*e) - b*ignored/(e^3*t_nostep^6)
```

Mupad [B] (verification not implemented)

Time = 8.86 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^3}{(ex)^{5/2}\,(a+bx^3)^{3/2}}\,dx = -\frac{\left(\frac{2\,A}{3\,a\,b\,e^2}+\frac{x^3\,(4\,A\,b-2\,B\,a)}{3\,a^2\,b\,e^2}\right)\,\sqrt{b\,x^3+a}}{x^4\,\sqrt{e\,x}+\frac{a\,x\,\sqrt{e\,x}}{b}}$$

```
[In] int((A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(3/2)),x)
```

```
[Out] -(((2*A)/(3*a*b*e^2) + (x^3*(4*A*b - 2*B*a))/(3*a^2*b*e^2))*(a + b*x^3)^(1/
2))/(x^4*(e*x)^(1/2) + (a*x*(e*x)^(1/2))/b)
```

3.558 $\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 283

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}\,dx=-\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^3}}-\frac{2(8Ab-5aB)\sqrt{ex}}{15a^2e^4\sqrt{a+bx^3}}-\frac{2(8Ab-5aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{15\sqrt[4]{3}a^{7/3}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -2/5*A/a/e/(e*x)^(5/2)/(b*x^3+a)^(1/2)-2/15*(8*A*b-5*B*a)*(e*x)^(1/2)/a^2/e
^4/(b*x^3+a)^(1/2)-2/45*(8*A*b-5*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)
*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)
*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/
3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/
4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^
(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(7/3)/e^4/(b*x^3+a)^(1/2)/(b^(1/3)*
x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {464, 296, 335, 231}

$$\int \frac{A+Bx^3}{(ex)^{7/2}(a+bx^3)^{3/2}}\,dx = -\frac{2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(8Ab-5aB)\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}(2+\sqrt{3})\right)}{15\sqrt[4]{3}a^{7/3}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{2\sqrt{ex}(8Ab-5aB)}{15a^2e^4\sqrt{a+bx^3}} - \frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^3}}$$

```
[In] Int[(A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-2*A)/(5*a*e*(e*x)^(5/2)*Sqrt[a + b*x^3]) - (2*(8*A*b - 5*a*B)*Sqrt[e*x])/
(15*a^2*e^4*Sqrt[a + b*x^3]) - (2*(8*A*b - 5*a*B)*Sqrt[e*x]*(a^(1/3) + b^(1
/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqr
t[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a
^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(15*3^(1/4)*a^(7/3)*e
^4*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*
x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^3}} - \frac{(8Ab-5aB)\int \frac{1}{\sqrt{ex}(a+bx^3)^{3/2}}\,dx}{5ae^3}$$

$$= -\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^3}} - \frac{2(8Ab-5aB)\sqrt{ex}}{15a^2e^4\sqrt{a+bx^3}} - \frac{(2(8Ab-5aB))\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{15a^2e^3}$$

$$= -\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^3}} - \frac{2(8Ab-5aB)\sqrt{ex}}{15a^2e^4\sqrt{a+bx^3}} - \frac{(4(8Ab-5aB))\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{15a^2e^4}$$

$$= -\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^3}} - \frac{2(8Ab-5aB)\sqrt{ex}}{15a^2e^4\sqrt{a+bx^3}} - \frac{2(8Ab-5aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right.}{15\sqrt[4]{3}a^{7/3}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.34

$$\int \frac{A+Bx^3}{(ex)^{7/2}\,(a+bx^3)^{3/2}}\,dx = \frac{x\left(-2(3aA+8Abx^3-5aBx^3)+4(-8Ab+5aB)x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F}\right.}{15a^2(ex)^{7/2}\sqrt{a+bx^3}}$$

```
[In] Integrate[(A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(3/2)),x]
```

```
[Out] (x*(-2*(3*a*A + 8*A*b*x^3 - 5*a*B*x^3) + 4*(-8*A*b + 5*a*B)*x^3*Sqrt[1 + (b
*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(15*a^2*(e*x)^(7/
2)*Sqrt[a + b*x^3])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.02 (sec) , antiderivative size = 784, normalized size of antiderivative = 2.77

method	result	size
elliptic	Expression too large to display	784
risch	Expression too large to display	1444
default	Expression too large to display	3783

```
[In] int((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)*(-2/3/e^3*x/a^2*(A*b-B*a)
/((x^3+a/b)*b*e*x)^(1/2)-2/5/e^4/a^2*A*(b*e*x^4+a*e*x)^(1/2)/x^3+2*(-2/3/a^
2*(A*b-B*a)/e^3-2/5*b/a^2/e^3*A)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(
1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2)^(1/
3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.41

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}\,dx = -\frac{2\left(2\left((5\,Bab-8\,Ab^2)x^6+(5\,Ba^2-8\,Aab)x^3\right)\sqrt{ae}\,\text{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)-\left((5\,Ba^2-8\,Aab)x^3-3\right.\right.}{15\left(a^3be^4x^6+a^4e^4x^3\right)}$$

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/15*(2*((5*B*a*b - 8*A*b^2)*x^6 + (5*B*a^2 - 8*A*a*b)*x^3)*sqrt(a*e)*weie
rstrassPInverse(0, -4*b/a, 1/x) - ((5*B*a^2 - 8*A*a*b)*x^3 - 3*A*a^2)*sqrt(
b*x^3 + a)*sqrt(e*x))/(a^3*b*e^4*x^6 + a^4*e^4*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 152.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.34

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}\,dx = \frac{A\Gamma\left(-\frac{5}{6}\right)\,{}_2F_1\left(\begin{matrix}-\frac{5}{6},\frac{3}{2}\\ \frac{1}{6}\end{matrix}\middle|\,\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)}+\frac{B\sqrt{x}\Gamma\left(\frac{1}{6}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{6},\frac{3}{2}\\ \frac{7}{6}\end{matrix}\middle|\,\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)}$$

```
[In] integrate((B*x**3+A)/(e*x)**(7/2)/(b*x**3+a)**(3/2),x)
```

```
[Out] A*gamma(-5/6)*hyper((-5/6, 3/2), (1/6,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3
/2)*e**(7/2)*x**(5/2)*gamma(1/6)) + B*sqrt(x)*gamma(1/6)*hyper((1/6, 3/2),
(7/6,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*e**(7/2)*gamma(7/6))
```

Maxima [F]

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}\,dx = \int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*(e*x)^(7/2)), x)
```

Giac [F]

$$\int \frac{A + Bx^3}{(ex)^{7/2} (a + bx^3)^{3/2}} \, dx = \int \frac{Bx^3 + A}{(bx^3 + a)^{\frac{3}{2}} (ex)^{\frac{7}{2}}} \, dx$$

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*(e*x)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^3}{(ex)^{7/2} (a + bx^3)^{3/2}} \, dx = \int \frac{B\,x^3 + A}{(e\,x)^{7/2}\,(b\,x^3 + a)^{3/2}} \, dx$$

```
[In] int((A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(3/2)),x)
[Out] int((A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(3/2)), x)
```

3.559 $\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 114

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx=\frac{2(Ab-aB)(ex)^{9/2}}{9abe\left(a+bx^3\right)^{3/2}}-\frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}}+\frac{2Be^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}$$

```
[Out] 2/9*(A*b-B*a)*(e*x)^(9/2)/a/b/e/(b*x^3+a)^(3/2)+2/3*B*e^(7/2)*arctanh((e*x)
^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))/b^(5/2)-2/3*B*e^2*(e*x)^(3/2)/b^2/(
b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {463, 294, 335, 281, 223, 212}

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx=\frac{2(ex)^{9/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}+\frac{2Be^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}-\frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}}$$

```
[In] Int[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(9/2))/(9*a*b*e*(a + b*x^3)^(3/2)) - (2*B*e^2*(e*x)^(3
/2))/(3*b^2*Sqrt[a + b*x^3]) + (2*B*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(
e^(3/2)*Sqrt[a + b*x^3])])/(3*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{2(Ab-aB)(ex)^{9/2}}{9abe\,(a+bx^3)^{3/2}} + \frac{B\int \frac{(ex)^{7/2}}{(a+bx^3)^{3/2}}\,dx}{b}$$

$$= \frac{2(Ab-aB)(ex)^{9/2}}{9abe\,(a+bx^3)^{3/2}} - \frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{(Be^3)\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{b^2}$$

$$= \frac{2(Ab-aB)(ex)^{9/2}}{9abe\left(a+bx^3\right)^{3/2}} - \frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{(2Be^2)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{b^2}$$

$$= \frac{2(Ab-aB)(ex)^{9/2}}{9abe\left(a+bx^3\right)^{3/2}} - \frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{(2Be^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx, x, (ex)^{3/2}\right)}{3b^2}$$

$$= \frac{2(Ab-aB)(ex)^{9/2}}{9abe\left(a+bx^3\right)^{3/2}} - \frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{(2Be^2)\,\text{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx, x, \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{3b^2}$$

$$= \frac{2(Ab-aB)(ex)^{9/2}}{9abe\left(a+bx^3\right)^{3/2}} - \frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{2Be^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{2e^3\sqrt{ex}\left(\frac{\sqrt{b}x^{3/2}\left(-3a^2B+Ab^2x^3-4abBx^3\right)}{a\left(a+bx^3\right)^{3/2}} + 3B\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)\right)}{9b^{5/2}\sqrt{x}}$$

```
[In] Integrate[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*e^3*Sqrt[e*x]*((Sqrt[b]*x^(3/2)*(-3*a^2*B + A*b^2*x^3 - 4*a*b*B*x^3))/(a
*(a + b*x^3)^(3/2)) + 3*B*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]]))/(9*b^(5/
2)*Sqrt[x])
```

Maple [A] (verified)

Time = 4.42 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.38

method	result
default	$\frac{2\left(A\,b^2x^5\sqrt{be}-4Bab\,x^5\sqrt{be}+3B\,\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^3+a\right)ex}}{x^2\sqrt{be}}\right)ab\,x^3\sqrt{(b\,x^3+a)ex}-3B\,a^2x^2\sqrt{be}+3B\,\operatorname{arctanh}\left(\frac{\sqrt{\left(b\,x^3+a\right)ex}}{x^2\sqrt{be}}\right)a^2\sqrt{(b\,x^3+}\right.}{9\sqrt{be}\,b^2a(b\,x^3+a)^{\frac{3}{2}}\,x}$
elliptic	Expression too large to display

```
[In] int((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*(A*b^2*x^5*(b*e)^(1/2)-4*B*a*b*x^5*(b*e)^(1/2)+3*B*arctanh(((b*x^3+a)*e
*x)^(1/2)/x^2/(b*e)^(1/2))*a*b*x^3*((b*x^3+a)*e*x)^(1/2)-3*B*a^2*x^2*(b*e)^
```

```
(1/2)+3*B*arctanh(((b*x^3+a)*e*x)^(1/2)/x^2/(b*e)^(1/2))*a^2*((b*x^3+a)*e*x
)^(1/2))*(e*x)^(1/2)*e^3/(b*e)^(1/2)/b^2/a/(b*x^3+a)^(3/2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 345, normalized size of antiderivative = 3.03

$$\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \left[\frac{3\left(Bab^2e^3x^6+2\,Ba^2be^3x^3+Ba^3e^3\right)\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\,(2\,b^2\right.}{18\,(ab^4x^6+2}\right.$$

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/18*(3*(B*a*b^2*e^3*x^6 + 2*B*a^2*b*e^3*x^3 + B*a^3*e^3)*sqrt(e/b)*log(-8
*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sq
rt(e*x)*sqrt(e/b)) - 4*((4*B*a*b - A*b^2)*e^3*x^4 + 3*B*a^2*e^3*x)*sqrt(b*x
^3 + a)*sqrt(e*x))/(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2), -1/9*(3*(B*a*b^2*
e^3*x^6 + 2*B*a^2*b*e^3*x^3 + B*a^3*e^3)*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a
)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) + 2*((4*B*a*b - A*b^2)*e^3*x^
4 + 3*B*a^2*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/(a*b^4*x^6 + 2*a^2*b^3*x^3 +
a^3*b^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x)**(7/2)*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)(ex)^{\frac{7}{2}}}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(7/2)/(b*x^3 + a)^(5/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.11

$$\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2\,Be^6\log\left(\left|-\sqrt{be}\sqrt{ex}ex+\sqrt{be^4x^3+ae^4}\right|\right)}{3\,\sqrt{be}b^2|e|^2} - \frac{2\left(\frac{3\,Bae^8}{b^2}+\frac{(4\,Ba^5b^6e^{24}-Aa^4b^7e^{24})x^3}{a^5b^7e^{16}}\right)\sqrt{ex}ex}{9\,(be^4x^3+ae^4)^{\frac{3}{2}}}$$

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*B*e^6*log(abs(-sqrt(b*e)*sqrt(e*x)*e*x + sqrt(b*e^4*x^3 + a*e^4)))/(sq
rt(b*e)*b^2*abs(e)^2) - 2/9*(3*B*a*e^8/b^2 + (4*B*a^5*b^6*e^24 - A*a^4*b^7*
e^24)*x^3/(a^5*b^7*e^16))*sqrt(e*x)*e*x/(b*e^4*x^3 + a*e^4)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(B\,x^3+A)\,(e\,x)^{7/2}}{(b\,x^3+a)^{5/2}}\,dx$$

```
[In] int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(5/2),x)
```

```
[Out] int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(5/2), x)
```

3.560 $\int \frac{(ex)^{5/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 299

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx=\frac{2(Ab-aB)(ex)^{7/2}}{9abe\left(a+bx^3\right)^{3/2}}-\frac{2(2Ab+7aB)e^2\sqrt{ex}}{27ab^2\sqrt{a+bx^3}}+\frac{(2Ab+7aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{27\sqrt[4]{3}a^{4/3}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/9*(A*b-B*a)*(e*x)^(7/2)/a/b/e/(b*x^3+a)^(3/2)-2/27*(2*A*b+7*B*a)*e^2*(e*x
)^(1/2)/a/b^2/(b*x^3+a)^(1/2)+1/81*(2*A*b+7*B*a)*e^2*(a^(1/3)+b^(1/3)*x)*((
a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(
a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1
-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
,1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(4/3)/b^2/(b*x^3+a)^
(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/
2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {468, 294, 335, 231}

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(7aB+2Ab)\operatorname{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right.}{\left(1+\sqrt{3}\right.}\right.\right.}{27\sqrt[4]{3}a^{4/3}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{2e^2\sqrt{ex}(7aB+2Ab)}{27ab^2\sqrt{a+bx^3}}+\frac{2(ex)^{7/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

```
[In] Int[((e*x)^(5/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(7/2))/(9*a*b*e*(a + b*x^3)^(3/2)) - (2*(2*A*b + 7*a*B
)*e^2*Sqrt[e*x])/(27*a*b^2*Sqrt[a + b*x^3]) + ((2*A*b + 7*a*B)*e^2*Sqrt[e*x
]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a
^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[
3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(27*
3^(1/4)*a^(4/3)*b^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 +
Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\text{integral} = \frac{2(Ab-aB)(ex)^{7/2}}{9abe\left(a+bx^3\right)^{3/2}} + \frac{\left(2\left(Ab+\frac{7aB}{2}\right)\right)\int\frac{(ex)^{5/2}}{(a+bx^3)^{3/2}}\,dx}{9ab}$$

$$= \frac{2(Ab-aB)(ex)^{7/2}}{9abe\left(a+bx^3\right)^{3/2}} - \frac{2(2Ab+7aB)e^2\sqrt{ex}}{27ab^2\sqrt{a+bx^3}} + \frac{\left((2Ab+7aB)e^3\right)\int\frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{27ab^2}$$

$$= \frac{2(Ab-aB)(ex)^{7/2}}{9abe\left(a+bx^3\right)^{3/2}} - \frac{2(2Ab+7aB)e^2\sqrt{ex}}{27ab^2\sqrt{a+bx^3}} + \frac{\left(2(2Ab+7aB)e^2\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{27ab^2}$$

$$= \frac{2(Ab-aB)(ex)^{7/2}}{9abe\left(a+bx^3\right)^{3/2}} - \frac{2(2Ab+7aB)e^2\sqrt{ex}}{27ab^2\sqrt{a+bx^3}} + \frac{(2Ab+7aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right.\right.}{27\sqrt[4]{3}a^{4/3}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.36

$$\int\frac{(ex)^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{2e^2\sqrt{ex}\left(-7a^2B+Ab^2x^3-2ab(A+5Bx^3)+(2Ab+7aB)\left(a+bx^3\right)\sqrt{1+\frac{bx^3}{a}}\right.}{27ab^2\left(a+bx^3\right)^{3/2}}$$

```
[In] Integrate[((e*x)^(5/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*e^2*Sqrt[e*x]*(-7*a^2*B + A*b^2*x^3 - 2*a*b*(A + 5*B*x^3) + (2*A*b + 7*a
*B)*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x
^3)/a)]))/(27*a*b^2*(a + b*x^3)^(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.60 (sec) , antiderivative size = 809, normalized size of antiderivative = 2.71

method	result	size
elliptic	Expression too large to display	809
default	Expression too large to display	7083

```
[In] int((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e/x*(e*x)^(1/2)/(b*x^3+a)^(1/2)*((b*x^3+a)*e*x)^(1/2)*(-2/9*e^2/b^4*(A*b-
B*a)*(b*e*x^4+a*e*x)^(1/2)/(x^3+a/b)^2+2/27/b^2*e^3*x/a*(A*b-10*B*a)/((x^3+
a/b)*b*e*x)^(1/2)+2*(B*e^3/b^2+2/27/b^2/a*e^3*(A*b-10*B*a))*(1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3
)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-
a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1
/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)
/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*
EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/
2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.57

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx = -\frac{2\left(\left(\left(7\,Bab^2+2\,Ab^3\right)e^2x^6+2\left(7\,Ba^2b+2\,Aab^2\right)e^2x^3+\left(7\,Ba^3+2\,Aa^2b\right)e^2\right)\sqrt{ae}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4}{a}\right.\right.}{27\left(a^2b^4x^6+2\,a^3b^3x^3+a^4b^2\right)}$$

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/27*(((7*B*a*b^2 + 2*A*b^3)*e^2*x^6 + 2*(7*B*a^2*b + 2*A*a*b^2)*e^2*x^3 +
 (7*B*a^3 + 2*A*a^2*b)*e^2)*sqrt(a*e)*weierstrassPInverse(0, -4*b/a, 1/x) +
 ((10*B*a^2*b - A*a*b^2)*e^2*x^3 + (7*B*a^3 + 2*A*a^2*b)*e^2)*sqrt(b*x^3 +
a)*sqrt(e*x))/(a^2*b^4*x^6 + 2*a^3*b^3*x^3 + a^4*b^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x)**(5/2)*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx = \int \frac{(Bx^3+A)(ex)^{\frac{5}{2}}}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(5/2)/(b*x^3 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)(ex)^{\frac{5}{2}}}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")

[Out] integrate((B*x^3 + A)*(e*x)^(5/2)/(b*x^3 + a)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(B\,x^3+A)\,(e\,x)^{5/2}}{(b\,x^3+a)^{5/2}}\,dx$$

[In] int(((A + B*x^3)*(e*x)^(5/2))/(a + b*x^3)^(5/2),x)

[Out] int(((A + B*x^3)*(e*x)^(5/2))/(a + b*x^3)^(5/2), x)

3.561 $\int \frac{(ex)^{3/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 596

$$\int \frac{(ex)^{3/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2(Ab-aB)(ex)^{5/2}}{9abe(a+bx^3)^{3/2}} + \frac{2(4Ab+5aB)(ex)^{5/2}}{27a^2be\sqrt{a+bx^3}} - \frac{2(1+\sqrt{3})(4Ab+5aB)e\sqrt{ex}\sqrt{a+bx^3}}{27a^2b^{5/3}\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)} + \frac{2(4Ab+5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+(1-\sqrt{3})\sqrt[3]{b}x}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{9\,3^{3/4}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{(1-\sqrt{3})(4Ab+5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+(1-\sqrt{3})\sqrt[3]{b}x}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x}\right),\frac{1}{4}\right.}{27\sqrt[4]{3}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/9*(A*b-B*a)*(e*x)^(5/2)/a/b/e/(b*x^3+a)^(3/2)+2/27*(4*A*b+5*B*a)*(e*x)^(5
/2)/a^2/b/e/(b*x^3+a)^(1/2)-2/27*(4*A*b+5*B*a)*e*(1+3^(1/2))*(e*x)^(1/2)*(b
*x^3+a)^(1/2)/a^2/b^(5/3)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))+2/27*(4*A*b+5*B*a
)*e*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)
*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x
*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/
3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a
```

```
^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(1
/4)/a^(5/3)/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)
+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)+1/81*(4*A*b+5*B*a)*e*(a^(1/3)+b^(1/3)*x)*(
(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/
(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((
1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2
),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3
)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(5/3)/b
^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1
+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {468, 296, 335, 314, 231, 1895}

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx=\frac{\left(1-\sqrt{3}\right)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(5aB+4Ab)\,\text{EllipticF}\left(\arccos\right.}{27\sqrt[4]{3}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(5aB+4Ab)E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{9\ 3^{3/4}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{2\left(1+\sqrt{3}\right)e\sqrt{ex}\sqrt{a+bx^3}(5aB+4Ab)}{27a^2b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}+\frac{2(ex)^{5/2}(5aB+4Ab)}{27a^2be\sqrt{a+bx^3}}+\frac{2(ex)^{5/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

```
[In] Int[((e*x)^(3/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(5/2))/(9*a*b*e*(a + b*x^3)^(3/2)) + (2*(4*A*b + 5*a*B
)*(e*x)^(5/2))/(27*a^2*b*e*Sqrt[a + b*x^3]) - (2*(1 + Sqrt[3])*(4*A*b + 5*a
*B)*e*Sqrt[e*x]*Sqrt[a + b*x^3])/(27*a^2*b^(5/3)*(a^(1/3) + (1 + Sqrt[3])*b
^(1/3)*x)) + (2*(4*A*b + 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sq
rt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(9*3^(3/4)*a^(5/3)*b^(5/3)*Sqrt[(b^(1
/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a
+ b*x^3]) + ((1 - Sqrt[3])*(4*A*b + 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x
```

```
)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])
*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3
) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(27*3^(1/4)*a^(5/3)*b^(5/3
)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\text{integral} = \frac{2(Ab-aB)(ex)^{5/2}}{9abe\left(a+bx^3\right)^{3/2}} + \frac{\left(2\left(2Ab+\frac{5aB}{2}\right)\right)\int\frac{(ex)^{3/2}}{\left(a+bx^3\right)^{3/2}}\,dx}{9ab}$$

$$= \frac{2(Ab-aB)(ex)^{5/2}}{9abe\left(a+bx^3\right)^{3/2}} + \frac{2(4Ab+5aB)(ex)^{5/2}}{27a^2be\sqrt{a+bx^3}} - \frac{(2(4Ab+5aB))\int\frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\,dx}{27a^2b}$$

$$= \frac{2(Ab-aB)(ex)^{5/2}}{9abe\left(a+bx^3\right)^{3/2}} + \frac{2(4Ab+5aB)(ex)^{5/2}}{27a^2be\sqrt{a+bx^3}} - \frac{(4(4Ab+5aB))\text{Subst}\left(\int\frac{x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{27a^2be}$$

$$= \frac{2(Ab-aB)(ex)^{5/2}}{9abe\left(a+bx^3\right)^{3/2}} + \frac{2(4Ab+5aB)(ex)^{5/2}}{27a^2be\sqrt{a+bx^3}} + \frac{(2(4Ab+5aB))\text{Subst}\left(\int\frac{\left(-1+\sqrt{3}\right)a^{2/3}e^2-2b^{2/3}x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{27a^2b^{5/3}e} + \frac{\left(2\left(1-\sqrt{3}\right)(4Ab+5aB)e\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{27a^{4/3}b^{5/3}}$$

$$= \frac{2(Ab-aB)(ex)^{5/2}}{9abe\,(a+bx^3)^{3/2}} + \frac{2(4Ab+5aB)(ex)^{5/2}}{27a^2be\sqrt{a+bx^3}} - \frac{2(1+\sqrt{3})\,(4Ab+5aB)e\sqrt{ex}\sqrt{a+bx^3}}{27a^2b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$

$$+ \frac{2(4Ab+5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,E\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}(2+\sqrt{\cdots}\right.}{9\,3^{3/4}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{\left(1-\sqrt{3}\right)(4Ab+5aB)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\cdots\right.}{27\sqrt[4]{3}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.14

$$\int \frac{(ex)^{3/2}\,(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{x(ex)^{3/2}\left(-5a^2B+(4Ab+5aB)\,(a+bx^3)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{5}{2},\cdots\right.\right.}{10a^2b\,(a+bx^3)^{3/2}}$$

```
[In] Integrate[((e*x)^(3/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
[Out] (x*(e*x)^(3/2)*(-5*a^2*B + (4*A*b + 5*a*B)*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*
Hypergeometric2F1[5/6, 5/2, 11/6, -((b*x^3)/a)]))/(10*a^2*b*(a + b*x^3)^(3/
2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.63 (sec) , antiderivative size = 1190, normalized size of antiderivative = 2.00

method	result	size
elliptic	Expression too large to display	1190
default	Expression too large to display	10786

```
[In] int((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e/x*(e*x)^(1/2)/(b*x^3+a)^(1/2)*((b*x^3+a)*e*x)^(1/2)*(2/9*e/a/b^3*x^2*(A
*b-B*a)*(b*e*x^4+a*e*x)^(1/2)/(x^3+a/b)^2+2/27/b*e^2*x^3/a^2*(4*A*b+5*B*a)/
((x^3+a/b)*b*e*x)^(1/2)-2/27/b/a^2*e^2*(4*A*b+5*B*a)*(x*(x+1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a
*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1
/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))*b/(-a*b^2)^(1/3)))/(b*e*x*(x-1/b*(-
a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.28

$$\int \frac{(ex)^{3/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = -\frac{2\left(\left((5\,Bab^2+4\,Ab^3)ex^7+2\,(5\,Ba^2b+4\,Aab^2)ex^4+(5\,Ba^3+4\,Aa^2b)ex\right)\sqrt{ae}\mathrm{weierstrassZeta}\left(0,-\frac{4\,b}{a},\mathrm{weier}\right.\right.}{27\,(a^2b^4x^7+2\,a^3b^3x^4+a}$$

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/27*(((5*B*a*b^2 + 4*A*b^3)*e*x^7 + 2*(5*B*a^2*b + 4*A*a*b^2)*e*x^4 + (5*
B*a^3 + 4*A*a^2*b)*e*x)*sqrt(a*e)*weierstrassZeta(0, -4*b/a, weierstrassPIn
verse(0, -4*b/a, 1/x)) + ((8*B*a^2*b + A*a*b^2)*e*x^3 + (5*B*a^3 + 4*A*a^2*
b)*e)*sqrt(b*x^3 + a)*sqrt(e*x))/(a^2*b^4*x^7 + 2*a^3*b^3*x^4 + a^4*b^2*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate((e*x)**(3/2)*(B*x**3+A)/(b*x**3+a)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{(Bx^3+A)(ex)^{\frac{3}{2}}}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(e*x)^(3/2)/(b*x^3 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{(Bx^3+A)(ex)^{\frac{3}{2}}}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(e*x)^(3/2)/(b*x^3 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx=\int \frac{\left(B\,x^3+A\right)\,{\left(e\,x\right)}^{3/2}}{{\left(b\,x^3+a\right)}^{5/2}}\,dx$$

```
[In] int(((A + B*x^3)*(e*x)^(3/2))/(a + b*x^3)^(5/2),x)
[Out] int(((A + B*x^3)*(e*x)^(3/2))/(a + b*x^3)^(5/2), x)
```

3.562 $\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{5/2}} dx$

Optimal result

Integrand size = 26, antiderivative size = 79

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{5/2}} dx = \frac{2(Ab-aB)(ex)^{3/2}}{9abe(a+bx^3)^{3/2}} + \frac{2(2Ab+aB)(ex)^{3/2}}{9a^2be\sqrt{a+bx^3}}$$

```
[Out] 2/9*(A*b-B*a)*(e*x)^(3/2)/a/b/e/(b*x^3+a)^(3/2)+2/9*(2*A*b+B*a)*(e*x)^(3/2)
/a^2/b/e/(b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {468, 270}

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{5/2}} dx = \frac{2(ex)^{3/2}(aB+2Ab)}{9a^2be\sqrt{a+bx^3}} + \frac{2(ex)^{3/2}(Ab-aB)}{9abe(a+bx^3)^{3/2}}$$

```
[In] Int[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(3/2))/(9*a*b*e*(a + b*x^3)^(3/2)) + (2*(2*A*b + a*B)*
(e*x)^(3/2))/(9*a^2*b*e*Sqrt[a + b*x^3])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2(Ab-aB)(ex)^{3/2}}{9abe\,(a+bx^3)^{3/2}}+\frac{\left(2\left(3Ab+\frac{3aB}{2}\right)\right)\int\frac{\sqrt{ex}}{(a+bx^3)^{3/2}}\,dx}{9ab}\\ &= \frac{2(Ab-aB)(ex)^{3/2}}{9abe\,(a+bx^3)^{3/2}}+\frac{2(2Ab+aB)(ex)^{3/2}}{9a^2be\sqrt{a+bx^3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.56

$$\int\frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx=\frac{2x\sqrt{ex}(3aA+2Abx^3+aBx^3)}{9a^2\,(a+bx^3)^{3/2}}$$

```
[In] Integrate[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(5/2),x]
[Out] (2*x*Sqrt[e*x]*(3*a*A + 2*A*b*x^3 + a*B*x^3))/(9*a^2*(a + b*x^3)^(3/2))
```

Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

method	result	size
gosper	$\frac{2x\left(2Ab\,x^3+Ba\,x^3+3Aa\right)\sqrt{ex}}{9\left(b\,x^3+a\right)^{\frac{3}{2}}a^2}$	39
default	$\frac{2x\left(2Ab\,x^3+Ba\,x^3+3Aa\right)\sqrt{ex}}{9\left(b\,x^3+a\right)^{\frac{3}{2}}a^2}$	39
elliptic	$\frac{\sqrt{ex}\,\sqrt{(b\,x^3+a)ex}\left(\frac{2x(Ab-Ba)\sqrt{be\,x^4+aex}}{9a\,b^3\left(x^3+\frac{a}{b}\right)^2}+\frac{2e\,x^2(2Ab+Ba)}{9b\,a^2\sqrt{\left(x^3+\frac{a}{b}\right)bex}}\right)}{ex\sqrt{b\,x^3+a}}$	111

```
[In] int((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/9*x*(2*A*b*x^3+B*a*x^3+3*A*a)*(e*x)^(1/2)/(b*x^3+a)^(3/2)/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2\left((Ba+2\,Ab)x^4+3\,Aax\right)\sqrt{bx^3+a}\sqrt{ex}}{9\left(a^2b^2x^6+2\,a^3bx^3+a^4\right)}$$

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(5/2),x, algorithm="fricas")
[Out] 2/9*((B*a + 2*A*b)*x^4 + 3*A*a*x)*sqrt(b*x^3 + a)*sqrt(e*x)/(a^2*b^2*x^6 +
2*a^3*b*x^3 + a^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((B*x**3+A)*(e*x)**(1/2)/(b*x**3+a)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \int \frac{(Bx^3+A)\sqrt{ex}}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(5/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*sqrt(e*x)/(b*x^3 + a)^(5/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2\left(\frac{3\,Ae^5}{a}+\frac{(Ba^5b^5e^{21}+2\,Aa^4b^6e^{21})x^3}{a^6b^5e^{16}}\right)\sqrt{ex}ex}{9\left(be^4x^3+ae^4\right)^{\frac{3}{2}}}$$

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(5/2),x, algorithm="giac")
[Out] 2/9*(3*A*e^5/a + (B*a^5*b^5*e^21 + 2*A*a^4*b^6*e^21)*x^3/(a^6*b^5*e^16))*sq
rt(e*x)*e*x/(b*e^4*x^3 + a*e^4)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 8.58 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{ex}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{\left(\frac{2\,A\,x\,\sqrt{e\,x}}{3\,a\,b^2}+\frac{x^4\,\sqrt{e\,x}\,(4\,A\,b+2\,B\,a)}{9\,a^2\,b^2}\right)\,\sqrt{b\,x^3+a}}{x^6+\frac{a^2}{b^2}+\frac{2\,a\,x^3}{b}}$$

```
[In] int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(5/2),x)
```

```
[Out] (((2*A*x*(e*x)^(1/2))/(3*a*b^2) + (x^4*(e*x)^(1/2)*(4*A*b + 2*B*a))/(9*a^2*
b^2))*(a + b*x^3)^(1/2))/(x^6 + a^2/b^2 + (2*a*x^3)/b)
```

3.563 $\int \frac{A+Bx^3}{\sqrt{ex}\left(a+bx^3\right)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 297

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{5/2}}\,dx=\frac{2(Ab-aB)\sqrt{ex}}{9abe\,(a+bx^3)^{3/2}}+\frac{2(8Ab+aB)\sqrt{ex}}{27a^2be\sqrt{a+bx^3}}+\frac{2(8Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(2+\sqrt{3})\right)}{27\sqrt[4]{3}a^{7/3}be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/9*(A*b-B*a)*(e*x)^(1/2)/a/b/e/(b*x^3+a)^(3/2)+2/27*(8*A*b+B*a)*(e*x)^(1/2
)/a^2/b/e/(b*x^3+a)^(1/2)+2/81*(8*A*b+B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^
(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^
(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+
b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/
2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(7/3)/b/e/(b*x^3+a)^(1/2)/(b^(
1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {468, 296, 335, 231}

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{5/2}}\,dx = \frac{2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+8Ab)\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{\ }}{\left(1+\sqrt{3}\right)\sqrt[3]{\ }}\right.\right.}{27\sqrt[4]{3}a^{7/3}be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2\sqrt{ex}(aB+8Ab)}{27a^2be\sqrt{a+bx^3}}+\frac{2\sqrt{ex}(Ab-aB)}{9abe\,(a+bx^3)^{3/2}}$$

```
[In] Int[(A + B*x^3)/(Sqrt[e*x]*(a + b*x^3)^(5/2)),x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[e*x])/(9*a*b*e*(a + b*x^3)^(3/2)) + (2*(8*A*b + a*B)*Sq
rt[e*x])/(27*a^2*b*e*Sqrt[a + b*x^3]) + (2*(8*A*b + a*B)*Sqrt[e*x]*(a^(1/3)
 + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3
)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(27*3^(1/4)*a^
(7/3)*b*e*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b
^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(Ab-aB)\sqrt{ex}}{9abe\,(a+bx^3)^{3/2}} + \frac{\left(2\left(4Ab+\frac{aB}{2}\right)\right)\int \frac{1}{\sqrt{ex}(a+bx^3)^{3/2}}\,dx}{9ab} \\
&= \frac{2(Ab-aB)\sqrt{ex}}{9abe\,(a+bx^3)^{3/2}} + \frac{2(8Ab+aB)\sqrt{ex}}{27a^2be\sqrt{a+bx^3}} + \frac{(2(8Ab+aB))\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{27a^2b} \\
&= \frac{2(Ab-aB)\sqrt{ex}}{9abe\,(a+bx^3)^{3/2}} + \frac{2(8Ab+aB)\sqrt{ex}}{27a^2be\sqrt{a+bx^3}} + \frac{(4(8Ab+aB))\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{27a^2be} \\
&= \frac{2(Ab-aB)\sqrt{ex}}{9abe\,(a+bx^3)^{3/2}} + \frac{2(8Ab+aB)\sqrt{ex}}{27a^2be\sqrt{a+bx^3}} \\
&\quad + \frac{2(8Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{27\sqrt[4]{3}a^{7/3}be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.36

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{5/2}}\,dx = \frac{2x\left(-2a^2B+8Ab^2x^3+ab(11A+Bx^3)+2(8Ab+aB)\,(a+bx^3)\,\sqrt{1+\frac{bx^3}{a}}\,\text{Hyperg}\right.}{27a^2b\sqrt{ex}\,(a+bx^3)^{3/2}}$$

```
[In] Integrate[(A + B*x^3)/(Sqrt[e*x]*(a + b*x^3)^(5/2)),x]
```

```
[Out] (2*x*(-2*a^2*B + 8*A*b^2*x^3 + a*b*(11*A + B*x^3) + 2*(8*A*b + a*B)*(a + b*
x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(
27*a^2*b*Sqrt[e*x]*(a + b*x^3)^(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.54 (sec) , antiderivative size = 785, normalized size of antiderivative = 2.64

method	result	size
elliptic	Expression too large to display	785
default	Expression too large to display	7077

```
[In] int((B*x^3+A)/(b*x^3+a)^(5/2)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)*(2/9/e/a/b^3*(A*b-B*a)*(b
*e*x^4+a*e*x)^(1/2)/(x^3+a/b)^2+2/27/b*x/a^2*(8*A*b+B*a)/((x^3+a/b)*b*e*x)^
(1/2)+4/27/a^2*(8*A*b+B*a)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(
x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.51

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{5/2}}\,dx =$$
$$-\frac{2\left(2\left(\left(Bab^2+8\,Ab^3\right)x^6+Ba^3+8\,Aa^2b+2\left(Ba^2b+8\,Aab^2\right)x^3\right)\sqrt{ae}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)+(2\,Ba\cdots}{27\left(a^3b^3ex^6+2\,a^4b^2ex^3+a^5be\right)}$$

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(5/2)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/27*(2*((B*a*b^2 + 8*A*b^3)*x^6 + B*a^3 + 8*A*a^2*b + 2*(B*a^2*b + 8*A*a*
b^2)*x^3)*sqrt(a*e)*weierstrassPInverse(0, -4*b/a, 1/x) + (2*B*a^3 - 11*A*a
^2*b - (B*a^2*b + 8*A*a*b^2)*x^3)*sqrt(b*x^3 + a)*sqrt(e*x))/(a^3*b^3*e*x^6
 + 2*a^4*b^2*e*x^3 + a^5*b*e)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((B*x**3+A)/(b*x**3+a)**(5/2)/(e*x)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{5/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{5}{2}}\sqrt{ex}}\,dx$$

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(5/2)/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*sqrt(e*x)), x)
```

Giac [F]

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{5/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{5}{2}}\sqrt{ex}}\,dx$$

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(5/2)/(e*x)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*sqrt(e*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{\sqrt{ex}\,(a+bx^3)^{5/2}}\,dx = \int \frac{B\,x^3+A}{\sqrt{e\,x}\,(b\,x^3+a)^{5/2}}\,dx$$

```
[In] int((A + B*x^3)/((e*x)^(1/2)*(a + b*x^3)^(5/2)),x)
[Out] int((A + B*x^3)/((e*x)^(1/2)*(a + b*x^3)^(5/2)), x)
```

3.564 $\int \frac{A+Bx^3}{(ex)^{3/2}(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 624

$$\int \frac{A+Bx^3}{(ex)^{3/2}(a+bx^3)^{5/2}}\,dx = -\frac{2A}{ae\sqrt{ex}\,(a+bx^3)^{3/2}} - \frac{2(10Ab-aB)(ex)^{5/2}}{9a^2e^4(a+bx^3)^{3/2}} - \frac{8(10Ab-aB)(ex)^{5/2}}{27a^3e^4\sqrt{a+bx^3}} + \frac{8(1+\sqrt{3})(10Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{27a^3b^{2/3}e^2\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)} - \frac{8(10Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\,E\left(\arccos\left(\frac{\sqrt[3]{a}+(1-\sqrt{3})\sqrt[3]{b}x}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{9\,3^{3/4}a^{8/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{4(1-\sqrt{3})(10Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+(1-\sqrt{3})\sqrt[3]{b}x}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x}\right),\frac{1}{4}(\right.}{27\sqrt[4]{3}a^{8/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -2/9*(10*A*b-B*a)*(e*x)^(5/2)/a^2/e^4/(b*x^3+a)^(3/2)-2*A/a/e/(b*x^3+a)^(3/
2)/(e*x)^(1/2)-8/27*(10*A*b-B*a)*(e*x)^(5/2)/a^3/e^4/(b*x^3+a)^(1/2)+8/27*(
10*A*b-B*a)*(1+3^(1/2))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/a^3/b^(2/3)/e^2/(a^(1/3
)+b^(1/3)*x*(1+3^(1/2)))-8/27*(10*A*b-B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^
(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^
(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+
b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/
```

```
2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(8/3)/b^(2/3)/e^2/(b*x^3+a)^(1
/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
-4/81*(10*A*b-B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(
a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^
(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2
/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/
2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x
*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(8/3)/b^(2/3)/e^2/(b*x^3+a)^(1/2)/(b^(1/3)
*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 624, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {464, 296, 335, 314, 231, 1895}

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}\,dx =$$

$$-\frac{4(1-\sqrt{3})\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(10Ab-aB)\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}\right.}{27\sqrt[4]{3}a^{8/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{8\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(10Ab-aB)E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{9\ 3^{3/4}a^{8/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{8\left(1+\sqrt{3}\right)\sqrt{ex}\sqrt{a+bx^3}(10Ab-aB)}{27a^3b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}-\frac{8(ex)^{5/2}(10Ab-aB)}{27a^3e^4\sqrt{a+bx^3}}$$

$$-\frac{2(ex)^{5/2}(10Ab-aB)}{9a^2e^4\left(a+bx^3\right)^{3/2}}-\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}}$$

```
[In] Int[(A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(5/2)),x]
```

```
[Out] (-2*A)/(a*e*Sqrt[e*x]*(a + b*x^3)^(3/2)) - (2*(10*A*b - a*B)*(e*x)^(5/2))/(
9*a^2*e^4*(a + b*x^3)^(3/2)) - (8*(10*A*b - a*B)*(e*x)^(5/2))/(27*a^3*e^4*S
qrt[a + b*x^3]) + (8*(1 + Sqrt[3])*(10*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^3]
)/(27*a^3*b^(2/3)*e^2*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)) - (8*(10*A*b - a
```

```
*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3)
+ (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[
3])/4])/(9*3^(3/4)*a^(8/3)*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x
))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (4*(1 - Sqrt[3
])*(10*A*b - a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[Ar
cCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)], (2 + Sqrt[3])/4])/(27*3^(1/4)*a^(8/3)*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1
/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
```

```
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\text{integral} = -\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}} - \frac{(10Ab-aB)\int \frac{(ex)^{3/2}}{(a+bx^3)^{5/2}}\,dx}{ae^3}$$

$$= -\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}} - \frac{2(10Ab-aB)(ex)^{5/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{(4(10Ab-aB))\int \frac{(ex)^{3/2}}{(a+bx^3)^{3/2}}\,dx}{9a^2e^3}$$

$$= -\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}} - \frac{2(10Ab-aB)(ex)^{5/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{8(10Ab-aB)(ex)^{5/2}}{27a^3e^4\sqrt{a+bx^3}} + \frac{(8(10Ab-aB))\int \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\,dx}{27a^3e^3}$$

$$= -\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}} - \frac{2(10Ab-aB)(ex)^{5/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{8(10Ab-aB)(ex)^{5/2}}{27a^3e^4\sqrt{a+bx^3}} + \frac{(16(10Ab-aB))\text{Subst}\left(\int \frac{x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{27a^3e^4}$$

$$= -\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}} - \frac{2(10Ab-aB)(ex)^{5/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{8(10Ab-aB)(ex)^{5/2}}{27a^3e^4\sqrt{a+bx^3}} - \frac{(8(10Ab-aB))\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}e^2-2b^{2/3}x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{27a^3b^{2/3}e^4} - \frac{\left(8\left(1-\sqrt{3}\right)(10Ab-aB)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{27a^{7/3}b^{2/3}e^2}$$

$$= -\frac{2A}{ae\sqrt{ex}\,(a+bx^3)^{3/2}} - \frac{2(10Ab-aB)(ex)^{5/2}}{9a^2e^4\,(a+bx^3)^{3/2}} - \frac{8(10Ab-aB)(ex)^{5/2}}{27a^3e^4\sqrt{a+bx^3}} + \frac{8\left(1+\sqrt{3}\right)(10Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{27a^3b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)} - \frac{8(10Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right.}{9\,3^{3/4}a^{8/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{4\left(1-\sqrt{3}\right)(10Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\right.}{27\sqrt[4]{3}a^{8/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.14

$$\int \frac{A+Bx^3}{(ex)^{3/2}\,(a+bx^3)^{5/2}}\,dx = \frac{2x\left(-5a^2A+(-10Ab+aB)x^3(a+bx^3)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{5}{2},\frac{11}{6},\right.\right.}{5a^3(ex)^{3/2}\,(a+bx^3)^{3/2}}$$

```
[In] Integrate[(A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(5/2)),x]
```

```
[Out] (2*x*(-5*a^2*A + (-10*A*b + a*B)*x^3*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hyperg
eometric2F1[5/6, 5/2, 11/6, -((b*x^3)/a)]))/(5*a^3*(e*x)^(3/2)*(a + b*x^3)^
(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.03 (sec) , antiderivative size = 1225, normalized size of antiderivative = 1.96

method	result	size
elliptic	Expression too large to display	1225
risch	Expression too large to display	3336
default	Expression too large to display	10961

```
[In] int((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)*(-2*(b*e*x^3+a*e)/e^2/a^3
*A/(x*(b*e*x^3+a*e))^(1/2)-2/9/e^2/a^2/b^2*x^2*(A*b-B*a)*(b*e*x^4+a*e*x)^(1
/2)/(x^3+a/b)^2-2/27/e*x^3/a^3*(13*A*b-4*B*a)/((x^3+a/b)*b*e*x)^(1/2)+(2*b/
a^3/e*A+2/27/a^3*(13*A*b-4*B*a)/e)*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+
(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/
b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3))
)^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*
b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(
-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2))*b/(-a*b^2)^(1/3)))/(b*e*x*(x-1/b*(-a*b^2)^(1/3))*(x+1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.27

$$\int \frac{A+Bx^3}{(ex)^{3/2}\,(a+bx^3)^{5/2}}\,dx = -\frac{2\,\left(4\,\left((Bab^2-10\,Ab^3)x^7+2\,(Ba^2b-10\,Aab^2)x^4+(Ba^3-10\,Aa^2b)x\right)\sqrt{ae}\,\mathrm{weierstrassZeta}\left(0,-\frac{4\,b}{a},\mathrm{weierst}\right.}{27\,(a^3b^3e^2x^7+2\,a^4b^2e^2x^4+}$$

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/27*(4*((B*a*b^2 - 10*A*b^3)*x^7 + 2*(B*a^2*b - 10*A*a*b^2)*x^4 + (B*a^3
- 10*A*a^2*b)*x)*sqrt(a*e)*weierstrassZeta(0, -4*b/a, weierstrassPInverse(0
```

```
, -4*b/a, 1/x)) + (4*B*a^3 - 13*A*a^2*b + (B*a^2*b - 10*A*a*b^2)*x^3)*sqrt(
b*x^3 + a)*sqrt(e*x))/(a^3*b^3*e^2*x^7 + 2*a^4*b^2*e^2*x^4 + a^5*b*e^2*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx^3}{(ex)^{3/2} (a + bx^3)^{5/2}} \, dx = \text{Timed out}$$

```
[In] integrate((B*x**3+A)/(e*x)**(3/2)/(b*x**3+a)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{A + Bx^3}{(ex)^{3/2} (a + bx^3)^{5/2}} \, dx = \int \frac{Bx^3 + A}{(bx^3 + a)^{\frac{5}{2}} (ex)^{\frac{3}{2}}} \, dx$$

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(5/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*(e*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{A + Bx^3}{(ex)^{3/2} (a + bx^3)^{5/2}} \, dx = \int \frac{Bx^3 + A}{(bx^3 + a)^{\frac{5}{2}} (ex)^{\frac{3}{2}}} \, dx$$

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(5/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*(e*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^3}{(ex)^{3/2} (a + bx^3)^{5/2}} \, dx = \int \frac{B\,x^3 + A}{(e\,x)^{3/2}\,(b\,x^3 + a)^{5/2}} \, dx$$

```
[In] int((A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(5/2)),x)
[Out] int((A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(5/2)), x)
```

3.565 $\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 104

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}\,dx = -\frac{2A}{3ae(ex)^{3/2}\left(a+bx^3\right)^{3/2}} - \frac{2(4Ab-aB)(ex)^{3/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{4(4Ab-aB)(ex)^{3/2}}{9a^3e^4\sqrt{a+bx^3}}$$

```
[Out] -2/3*A/a/e/(e*x)^(3/2)/(b*x^3+a)^(3/2)-2/9*(4*A*b-B*a)*(e*x)^(3/2)/a^2/e^4/
(b*x^3+a)^(3/2)-4/9*(4*A*b-B*a)*(e*x)^(3/2)/a^3/e^4/(b*x^3+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {464, 279, 270}

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}\,dx = -\frac{4(ex)^{3/2}(4Ab-aB)}{9a^3e^4\sqrt{a+bx^3}} - \frac{2(ex)^{3/2}(4Ab-aB)}{9a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{2A}{3ae(ex)^{3/2}\left(a+bx^3\right)^{3/2}}$$

```
[In] Int[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(5/2)),x]
```

```
[Out] (-2*A)/(3*a*e*(e*x)^(3/2)*(a + b*x^3)^(3/2)) - (2*(4*A*b - a*B)*(e*x)^(3/2)
)/(9*a^2*e^4*(a + b*x^3)^(3/2)) - (4*(4*A*b - a*B)*(e*x)^(3/2))/(9*a^3*e^4*
Sqrt[a + b*x^3])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2A}{3ae(ex)^{3/2}\left(a+bx^3\right)^{3/2}}-\frac{(4Ab-aB)\int\frac{\sqrt{ex}}{\left(a+bx^3\right)^{5/2}}\,dx}{ae^3}\\
&= -\frac{2A}{3ae(ex)^{3/2}\left(a+bx^3\right)^{3/2}}-\frac{2(4Ab-aB)(ex)^{3/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}}-\frac{(2(4Ab-aB))\int\frac{\sqrt{ex}}{\left(a+bx^3\right)^{3/2}}\,dx}{3a^2e^3}\\
&= -\frac{2A}{3ae(ex)^{3/2}\left(a+bx^3\right)^{3/2}}-\frac{2(4Ab-aB)(ex)^{3/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}}-\frac{4(4Ab-aB)(ex)^{3/2}}{9a^3e^4\sqrt{a+bx^3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.64

$$\int\frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}\,dx=\frac{2x\left(-3a^2A-12aAbx^3+3a^2Bx^3-8Ab^2x^6+2abBx^6\right)}{9a^3(ex)^{5/2}\left(a+bx^3\right)^{3/2}}$$

```
[In] Integrate[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(5/2)),x]
[Out] (2*x*(-3*a^2*A - 12*a*A*b*x^3 + 3*a^2*B*x^3 - 8*A*b^2*x^6 + 2*a*b*B*x^6))/(
9*a^3*(e*x)^(5/2)*(a + b*x^3)^(3/2))
```

Maple [A] (verified)

Time = 3.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{2x\left(8A\,b^{2}x^{6}-2B\,x^{6}ab+12aAb\,x^{3}-3a^{2}B\,x^{3}+3a^{2}A\right)}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}}a^{3}\left(ex\right)^{\frac{5}{2}}}$	62
default	$-\frac{2\left(8A\,b^{2}x^{6}-2B\,x^{6}ab+12aAb\,x^{3}-3a^{2}B\,x^{3}+3a^{2}A\right)}{9a^{3}\sqrt{ex}\,e^{2}\left(b\,x^{3}+a\right)^{\frac{3}{2}}x}$	67
risch	$-\frac{2A\sqrt{b\,x^{3}+a}}{3a^{3}x\,e^{2}\sqrt{ex}}-\frac{2\left(5A\,b^{2}x^{3}-2Bab\,x^{3}+6abA-3a^{2}B\right)x^{2}}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}}a^{3}e^{2}\sqrt{ex}}$	82
elliptic	$\frac{\sqrt{\left(b\,x^{3}+a\right)ex}\left(-\frac{2x\left(Ab-Ba\right)\sqrt{be\,x^{4}+aex}}{9e^{3}a^{2}b^{2}\left(x^{3}+\frac{a}{b}\right)^{2}}-\frac{2x^{2}\left(5Ab-2Ba\right)}{9e^{2}a^{3}\sqrt{\left(x^{3}+\frac{a}{b}\right)bex}}-\frac{2A\sqrt{be\,x^{4}+aex}}{3e^{3}a^{3}x^{2}}\right)}{\sqrt{ex}\,\sqrt{b\,x^{3}+a}}$	133

```
[In] int((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/9*x*(8*A*b^2*x^6-2*B*a*b*x^6+12*A*a*b*x^3-3*B*a^2*x^3+3*A*a^2)/(b*x^3+a)
^(3/2)/a^3/(e*x)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^3}{(ex)^{5/2}\,(a+bx^3)^{5/2}}\,dx = \frac{2\,(2\,(Bab-4\,Ab^2)x^6+3\,(Ba^2-4\,Aab)x^3-3\,Aa^2)\sqrt{bx^3+a}\sqrt{ex}}{9\,(a^3b^2e^3x^8+2\,a^4be^3x^5+a^5e^3x^2)}$$

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x, algorithm="fricas")
[Out] 2/9*(2*(B*a*b - 4*A*b^2)*x^6 + 3*(B*a^2 - 4*A*a*b)*x^3 - 3*A*a^2)*sqrt(b*x^
3 + a)*sqrt(e*x)/(a^3*b^2*e^3*x^8 + 2*a^4*b*e^3*x^5 + a^5*e^3*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{(ex)^{5/2}\,(a+bx^3)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((B*x**3+A)/(e*x)**(5/2)/(b*x**3+a)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+Bx^3}{(ex)^{5/2}\,(a+bx^3)^{5/2}}\,dx = \int \frac{Bx^3+A}{(bx^3+a)^{\frac{5}{2}}\,(ex)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*(e*x)^(5/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx^3}{(ex)^{5/2}\,(a+bx^3)^{5/2}}\,dx = \text{Exception raised: NotImplementedError}$$

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> unable to parse Giac output: Recur
sive assumption sageVARa>=(-sageVARb*sageVARe/(sageVARe^4*t_nostep^6)) igno
red2*(-(23914845*sageVARb^7*sageVARe^18*sageVARa^6*sageVARA-9565938*sageVAR
b^6*sageVARe^18*
```

Mupad [B] (verification not implemented)

Time = 8.76 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx^3}{(ex)^{5/2}\,(a+bx^3)^{5/2}}\,dx = -\frac{\sqrt{b\,x^3+a}\,\left(\frac{2\,A}{3\,a\,b^2\,e^2}-\frac{x^3\,\left(6\,B\,a^2-24\,A\,a\,b\right)}{9\,a^3\,b^2\,e^2}+\frac{x^6\,\left(16\,A\,b^2-4\,B\,a\,b\right)}{9\,a^3\,b^2\,e^2}\right)}{x^7\,\sqrt{e\,x}+\frac{a^2\,x\,\sqrt{e\,x}}{b^2}+\frac{2\,a\,x^4\,\sqrt{e\,x}}{b}}$$

```
[In] int((A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(5/2)),x)
[Out] -((a + b*x^3)^(1/2)*((2*A)/(3*a*b^2*e^2) - (x^3*(6*B*a^2 - 24*A*a*b))/(9*a^
3*b^2*e^2) + (x^6*(16*A*b^2 - 4*B*a*b))/(9*a^3*b^2*e^2)))/(x^7*(e*x)^(1/2)
+ (a^2*x*(e*x)^(1/2))/b^2 + (2*a*x^4*(e*x)^(1/2))/b)
```

3.566 $\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 320

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{5/2}}\,dx = -\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}} - \frac{2(14Ab-5aB)\sqrt{ex}}{45a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{16(14Ab-5aB)\sqrt{ex}}{135a^3e^4\sqrt{a+bx^3}} - \frac{16(14Ab-5aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{\right.\right.}{135\sqrt[4]{3}a^{10/3}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -2/5*A/a/e/(e*x)^(5/2)/(b*x^3+a)^(3/2)-2/45*(14*A*b-5*B*a)*(e*x)^(1/2)/a^2/
e^4/(b*x^3+a)^(3/2)-16/135*(14*A*b-5*B*a)*(e*x)^(1/2)/a^3/e^4/(b*x^3+a)^(1/
2)-16/405*(14*A*b-5*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)
))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)
))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/
2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e
*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^
(1/2)))^2)^(1/2)*3^(3/4)/a^(10/3)/e^4/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b
^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {464, 296, 335, 231}

$$\int \frac{A+Bx^3}{(ex)^{7/2}(a+bx^3)^{5/2}}\,dx = -\frac{16\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(14Ab-5aB)\operatorname{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{135\sqrt[4]{3}a^{10/3}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{16\sqrt{ex}(14Ab-5aB)}{135a^3e^4\sqrt{a+bx^3}} - \frac{2\sqrt{ex}(14Ab-5aB)}{45a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}}$$

```
[In] Int[(A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(5/2)),x]
```

```
[Out] (-2*A)/(5*a*e*(e*x)^(5/2)*(a + b*x^3)^(3/2)) - (2*(14*A*b - 5*a*B)*Sqrt[e*x
])/(45*a^2*e^4*(a + b*x^3)^(3/2)) - (16*(14*A*b - 5*a*B)*Sqrt[e*x])/(135*a^
3*e^4*Sqrt[a + b*x^3]) - (16*(14*A*b - 5*a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/
3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(135*3^(1/4)*a^(10/3)*e^4
*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*Sqrt[a + b*x^3])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}} - \frac{(14Ab-5aB)\int \frac{1}{\sqrt{ex}\left(a+bx^3\right)^{5/2}}\,dx}{5ae^3}$$

$$= -\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}} - \frac{2(14Ab-5aB)\sqrt{ex}}{45a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{(8(14Ab-5aB))\int \frac{1}{\sqrt{ex}\left(a+bx^3\right)^{3/2}}\,dx}{45a^2e^3}$$

$$= -\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}} - \frac{2(14Ab-5aB)\sqrt{ex}}{45a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{16(14Ab-5aB)\sqrt{ex}}{135a^3e^4\sqrt{a+bx^3}} - \frac{(16(14Ab-5aB))\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{135a^3e^3}$$

$$= -\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}} - \frac{2(14Ab-5aB)\sqrt{ex}}{45a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{16(14Ab-5aB)\sqrt{ex}}{135a^3e^4\sqrt{a+bx^3}} - \frac{(32(14Ab-5aB))\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{135a^3e^4}$$

$$= -\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}} - \frac{2(14Ab-5aB)\sqrt{ex}}{45a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{16(14Ab-5aB)\sqrt{ex}}{135a^3e^4\sqrt{a+bx^3}} - \frac{16(14Ab-5aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}(2+\sqrt{}\right.}{135\sqrt[4]{3}a^{10/3}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.38

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{5/2}}\,dx = \frac{x\Big(-224Ab^2x^6+a^2(-54A+110Bx^3)+a(-308Abx^3+80bBx^6)+32(-14Ab+\cdots}{135a^3(ex)^{7/2}\left(a+bx\cdots\right.}$$

```
[In] Integrate[(A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(5/2)),x]
```

```
[Out] (x*(-224*A*b^2*x^6 + a^2*(-54*A + 110*B*x^3) + a*(-308*A*b*x^3 + 80*b*B*x^6
) + 32*(-14*A*b + 5*a*B)*x^3*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric
2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(135*a^3*(e*x)^(7/2)*(a + b*x^3)^(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.98 (sec) , antiderivative size = 829, normalized size of antiderivative = 2.59

method	result	size
elliptic	Expression too large to display	829
risch	Expression too large to display	2182
default	Expression too large to display	7299

```
[In] int((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x^3+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^3+a)^(1/2)*(-2/9/e^4/a^2/b^2*(A*b-B*
a)*(b*e*x^4+a*e*x)^(1/2)/(x^3+a/b)^2-2/27/e^3*x/a^3*(17*A*b-8*B*a)/((x^3+a/
b)*b*e*x)^(1/2)-2/5/e^4/a^3*A*(b*e*x^4+a*e*x)^(1/2)/x^3+2*(-2/27/a^3*(17*A*
b-8*B*a)/e^3-2/5*b/a^3/e^3*A)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2
)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)/(b*e*x*(x-1/b*(-a*b^2)^(1/3))
*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
```

```
(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.56

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{5/2}}\,dx = -\frac{2\left(16\left((5\,Bab^2-14\,Ab^3)x^9+2\,(5\,Ba^2b-14\,Aab^2)x^6+(5\,Ba^3-14\,Aa^2b)x^3\right)\sqrt{ae}\,\mathrm{weierstrassPInverse}(0,-}{135\left(a^4b^2e^4x^9+2\,a^5be^4x^6+\right.}$$

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/135*(16*((5*B*a*b^2 - 14*A*b^3)*x^9 + 2*(5*B*a^2*b - 14*A*a*b^2)*x^6 + (
5*B*a^3 - 14*A*a^2*b)*x^3)*sqrt(a*e)*weierstrassPInverse(0, -4*b/a, 1/x) -
(8*(5*B*a^2*b - 14*A*a*b^2)*x^6 - 27*A*a^3 + 11*(5*B*a^3 - 14*A*a^2*b)*x^3)
*sqrt(b*x^3 + a)*sqrt(e*x))/(a^4*b^2*e^4*x^9 + 2*a^5*b*e^4*x^6 + a^6*e^4*x^
3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((B*x**3+A)/(e*x)**(7/2)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{5/2}}\,dx = \int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*(e*x)^(7/2)), x)
```

Giac [F]

$$\int \frac{A + Bx^3}{(ex)^{7/2} (a + bx^3)^{5/2}} \, dx = \int \frac{Bx^3 + A}{(bx^3 + a)^{\frac{5}{2}} (ex)^{\frac{7}{2}}} \, dx$$

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(5/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*(e*x)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^3}{(ex)^{7/2} (a + bx^3)^{5/2}} \, dx = \int \frac{B\,x^3 + A}{(e\,x)^{7/2} (b\,x^3 + a)^{5/2}} \, dx$$

```
[In] int((A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(5/2)),x)
[Out] int((A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(5/2)), x)
```

3.567 $\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 220

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{a^3\sqrt[3]{a+bx^3}}{b^4d}-\frac{a^2(a+bx^3)^{4/3}}{4b^4d}+\frac{a(a+bx^3)^{7/3}}{7b^4d} -\frac{(a+bx^3)^{10/3}}{10b^4d}+\frac{\sqrt[3]{2}a^{10/3}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^4d} +\frac{a^{10/3}\log\left(a-bx^3\right)}{3\ 2^{2/3}b^4d}-\frac{a^{10/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^4d}$$

```
[Out] -a^3*(b*x^3+a)^(1/3)/b^4/d-1/4*a^2*(b*x^3+a)^(4/3)/b^4/d+1/7*a*(b*x^3+a)^(7
/3)/b^4/d-1/10*(b*x^3+a)^(10/3)/b^4/d+1/6*a^(10/3)*ln(-b*x^3+a)*2^(1/3)/b^4
/d-1/2*a^(10/3)*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(1/3)/b^4/d+1/3*2^(1/
3)*a^(10/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/b
^4/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

$= \{457, 90, 52, 59, 631, 210, 31\}$

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{\sqrt[3]{2}a^{10/3}\arctan\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^4d} + \frac{a^{10/3}\log\left(a-bx^3\right)}{3\ 2^{2/3}b^4d} - \frac{a^{10/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^4d} - \frac{a^3\sqrt[3]{a+bx^3}}{b^4d} - \frac{a^2\left(a+bx^3\right)^{4/3}}{4b^4d} + \frac{a\left(a+bx^3\right)^{7/3}}{7b^4d} - \frac{\left(a+bx^3\right)^{10/3}}{10b^4d}$$

```
[In] Int[(x^11*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -((a^3*(a + b*x^3)^(1/3))/(b^4*d)) - (a^2*(a + b*x^3)^(4/3))/(4*b^4*d) + (a
*(a + b*x^3)^(7/3))/(7*b^4*d) - (a + b*x^3)^(10/3)/(10*b^4*d) + (2^(1/3)*a^
(10/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*b^4*d) + (a^(10/3)*Log[a - b*x^3])/(3*2^(2/3)*b^4*d) - (a^(10/3)*Log[
2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*b^4*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^3\sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{a^2\sqrt[3]{a+bx}}{b^3d}+\frac{a(a+bx)^{4/3}}{b^3d}-\frac{(a+bx)^{7/3}}{b^3d}+\frac{a^3\sqrt[3]{a+bx}}{b^3(ad-bdx)}\right)\,dx, x, x^3\right)\\
&= -\frac{a^2(a+bx^3)^{4/3}}{4b^4d}+\frac{a(a+bx^3)^{7/3}}{7b^4d}-\frac{(a+bx^3)^{10/3}}{10b^4d}+\frac{a^3\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)}{3b^3}\\
&= -\frac{a^3\sqrt[3]{a+bx^3}}{b^4d}-\frac{a^2(a+bx^3)^{4/3}}{4b^4d}+\frac{a(a+bx^3)^{7/3}}{7b^4d}\\
&\quad -\frac{(a+bx^3)^{10/3}}{10b^4d}+\frac{(2a^4)\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)}\,dx, x, x^3\right)}{3b^3}\\
&= -\frac{a^3\sqrt[3]{a+bx^3}}{b^4d}-\frac{a^2(a+bx^3)^{4/3}}{4b^4d}+\frac{a(a+bx^3)^{7/3}}{7b^4d}-\frac{(a+bx^3)^{10/3}}{10b^4d}\\
&\quad +\frac{a^{10/3}\log\left(a-bx^3\right)}{3\ 2^{2/3}b^4d}+\frac{a^{10/3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2^{2/3}b^4d}\\
&\quad +\frac{a^{11/3}\text{Subst}\left(\int \frac{1}{2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^4d}
\end{aligned}$$

$$= -\frac{a^3\sqrt[3]{a+bx^3}}{b^4d} - \frac{a^2(a+bx^3)^{4/3}}{4b^4d} + \frac{a(a+bx^3)^{7/3}}{7b^4d} - \frac{(a+bx^3)^{10/3}}{10b^4d} + \frac{a^{10/3}\log(a-bx^3)}{3\ 2^{2/3}b^4d} - \frac{a^{10/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^4d} - \frac{\left(\sqrt[3]{2}a^{10/3}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{b^4d}$$

$$= -\frac{a^3\sqrt[3]{a+bx^3}}{b^4d} - \frac{a^2(a+bx^3)^{4/3}}{4b^4d} + \frac{a(a+bx^3)^{7/3}}{7b^4d} - \frac{(a+bx^3)^{10/3}}{10b^4d} + \frac{\sqrt[3]{2}a^{10/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}b^4d} + \frac{a^{10/3}\log(a-bx^3)}{3\ 2^{2/3}b^4d} - \frac{a^{10/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^4d}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.92

$$\int\frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{-3\sqrt[3]{a+bx^3}(169a^3+37a^2bx^3+22ab^2x^6+14b^3x^9)+140\sqrt[3]{2}\sqrt{3}a^{10/3}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-140\sqrt[3]{2}a}{420b^4d}$$

```
[In] Integrate[(x^11*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-3*(a + b*x^3)^(1/3)*(169*a^3 + 37*a^2*b*x^3 + 22*a*b^2*x^6 + 14*b^3*x^9)
+ 140*2^(1/3)*Sqrt[3]*a^(10/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1
/3))/Sqrt[3]] - 140*2^(1/3)*a^(10/3)*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(
1/3)] + 70*2^(1/3)*a^(10/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/
3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(420*b^4*d)
```

Maple [A] (verified)

Time = 6.64 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$\frac{\left(-42b^3x^9-66ab^2x^6-111a^2bx^3-507a^3\right)\left(bx^3+a\right)^{\frac{1}{3}}+70\,2^{\frac{1}{3}}a^{\frac{10}{3}}\left(2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(bx^3+a\right)^{\frac{2}{3}}+2\right.\right.}{420b^4d}$

```
[In] int(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/420*((-42*b^3*x^9-66*a*b^2*x^6-111*a^2*b*x^3-507*a^3)*(b*x^3+a)^(1/3)+70*
2^(1/3)*a^(10/3)*(2*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^
(1/2))*3^(1/2)+ln((b*x^3+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*a
^(2/3))-2*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3))))/b^4/d
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.85

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{140\sqrt{3}2^{\frac{1}{3}}(-a)^{\frac{1}{3}}a^3\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+70\cdot 2^{\frac{1}{3}}(-a)^{\frac{1}{3}}a^3\log\left(2^{\frac{2}{3}}(-a)^{\frac{2}{3}}-2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}(-\right.}{}$$

```
[In] integrate(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/420*(140*sqrt(3)*2^(1/3)*(-a)^(1/3)*a^3*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x
^3 + a)^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + 70*2^(1/3)*(-a)^(1/3)*a^3*log(2^
(2/3)*(-a)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(2/3)
) - 140*2^(1/3)*(-a)^(1/3)*a^3*log(2^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(1/3))
+ 3*(14*b^3*x^9 + 22*a*b^2*x^6 + 37*a^2*b*x^3 + 169*a^3)*(b*x^3 + a)^(1/3))
/(b^4*d)
```

Sympy [F]

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^{11}\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

```
[In] integrate(x**11*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**11*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.83

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$$
$$=\frac{\frac{140\sqrt{3}2^{\frac{1}{3}}a^{\frac{10}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{d}+\frac{70\cdot 2^{\frac{1}{3}}a^{\frac{10}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{d}-\frac{140\cdot 2^{\frac{1}{3}}a^{\frac{10}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3\right.}{d}}{420\,b^4}$$

```
[In] integrate(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] 1/420*(140*sqrt(3)*2^(1/3)*a^(10/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(
1/3) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d + 70*2^(1/3)*a^(10/3)*log(2^(2/3)*a^
(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d - 140*2^(1
/3)*a^(10/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d - 3*(14*(b*x^3 + a
)^(10/3) - 20*(b*x^3 + a)^(7/3)*a + 35*(b*x^3 + a)^(4/3)*a^2 + 140*(b*x^3 +
 a)^(1/3)*a^3)/d)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.76 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.98

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=\frac{\sqrt{3}2^{\frac{1}{3}}a^{\frac{10}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{3\,b^4d}$$
$$+\frac{2^{\frac{1}{3}}a^{\frac{10}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,b^4d}-\frac{2^{\frac{1}{3}}a^{\frac{10}{3}}\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,b^4d}$$
$$-\frac{14\,(bx^3+a)^{\frac{10}{3}}b^{36}d^9-20\,(bx^3+a)^{\frac{7}{3}}ab^{36}d^9+35\,(bx^3+a)^{\frac{4}{3}}a^2b^{36}d^9+140\,(bx^3+a)^{\frac{1}{3}}a^3b^{36}d^9}{140\,b^{40}d^{10}}$$

```
[In] integrate(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*2^(1/3)*a^(10/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) +
2*(b*x^3 + a)^(1/3))/a^(1/3))/(b^4*d) + 1/6*2^(1/3)*a^(10/3)*log(2^(2/3)*a^
(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(b^4*d) - 1/
3*2^(1/3)*a^(10/3)*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(b^4*d) -
 1/140*(14*(b*x^3 + a)^(10/3)*b^36*d^9 - 20*(b*x^3 + a)^(7/3)*a*b^36*d^9 +
35*(b*x^3 + a)^(4/3)*a^2*b^36*d^9 + 140*(b*x^3 + a)^(1/3)*a^3*b^36*d^9)/(b^
40*d^10)
```

Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.09

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{a\,{\left(b\,x^3+a\right)}^{7/3}}{7\,b^4\,d}-\frac{a^3\,{\left(b\,x^3+a\right)}^{1/3}}{b^4\,d}-\frac{a^2\,{\left(b\,x^3+a\right)}^{4/3}}{4\,b^4\,d}-\frac{{\left(b\,x^3+a\right)}^{10/3}}{10\,b^4\,d}-\frac{2^{1/3}\,a^{10/3}\,\ln\left({\left(b\,x^3+a\right)}^{1/3}-2^{1/3}\,a^{1/3}\right)}{3\,b^4\,d}-\frac{2^{1/3}\,a^{10/3}\,\ln\left(\frac{6\,a^4\,{\left(b\,x^3+a\right)}^{1/3}}{b^4\,d}-\frac{6\,2^{1/3}\,a^{13/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^4\,d}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b^4\,d}+\frac{2^{1/3}\,a^{10/3}\,\ln\left(\frac{6\,a^4\,{\left(b\,x^3+a\right)}^{1/3}}{b^4\,d}+\frac{18\,2^{1/3}\,a^{13/3}\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{b^4\,d}\right)\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{b^4\,d}$$

```
[In] int((x^11*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

```
[Out] (a*(a + b*x^3)^(7/3))/(7*b^4*d) - (a^3*(a + b*x^3)^(1/3))/(b^4*d) - (a^2*(a
 + b*x^3)^(4/3))/(4*b^4*d) - (a + b*x^3)^(10/3)/(10*b^4*d) - (2^(1/3)*a^(10
/3)*log((a + b*x^3)^(1/3) - 2^(1/3)*a^(1/3)))/(3*b^4*d) - (2^(1/3)*a^(10/3)
*log((6*a^4*(a + b*x^3)^(1/3))/(b^4*d) - (6*2^(1/3)*a^(13/3)*((3^(1/2)*1i)/
2 - 1/2))/(b^4*d))*((3^(1/2)*1i)/2 - 1/2))/(3*b^4*d) + (2^(1/3)*a^(10/3)*lo
g((6*a^4*(a + b*x^3)^(1/3))/(b^4*d) + (18*2^(1/3)*a^(13/3)*((3^(1/2)*1i)/6
+ 1/6))/(b^4*d))*((3^(1/2)*1i)/6 + 1/6))/(b^4*d)
```

3.568 $\int \frac{x^8 \sqrt[3]{a+bx^3}}{ad-bdx^3} dx$

Optimal result

Integrand size = 28, antiderivative size = 174

$$\int \frac{x^8 \sqrt[3]{a+bx^3}}{ad-bdx^3} dx = -\frac{a^2 \sqrt[3]{a+bx^3}}{b^3 d} - \frac{(a+bx^3)^{7/3}}{7b^3 d} + \frac{\sqrt[3]{2}a^{7/3} \arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^3 d} + \frac{a^{7/3}\log(a-bx^3)}{3\ 2^{2/3}b^3 d} - \frac{a^{7/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^3 d}$$

```
[Out] -a^2*(b*x^3+a)^(1/3)/b^3/d-1/7*(b*x^3+a)^(7/3)/b^3/d+1/6*a^(7/3)*ln(-b*x^3+
a)*2^(1/3)/b^3/d-1/2*a^(7/3)*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(1/3)/b^
3/d+1/3*2^(1/3)*a^(7/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3
)*3^(1/2))/b^3/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {457, 90, 52, 59, 631, 210, 31}

$$\int \frac{x^8 \sqrt[3]{a+bx^3}}{ad-bdx^3} dx = \frac{\sqrt[3]{2}a^{7/3} \arctan\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^3 d} + \frac{a^{7/3}\log(a-bx^3)}{3\ 2^{2/3}b^3 d} - \frac{a^{7/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^3 d} - \frac{a^2\sqrt[3]{a+bx^3}}{b^3 d} - \frac{(a+bx^3)^{7/3}}{7b^3 d}$$

```
[In] Int[(x^8*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -((a^2*(a + b*x^3)^(1/3))/(b^3*d)) - (a + b*x^3)^(7/3)/(7*b^3*d) + (2^(1/3)
*a^(7/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*b^3*d) + (a^(7/3)*Log[a - b*x^3])/(3*2^(2/3)*b^3*d) - (a^(7/3)*Log[
2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*b^3*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x^2\sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{(a+bx)^{4/3}}{b^2d}+\frac{a^2\sqrt[3]{a+bx}}{b^2(ad-bdx)}\right)\,dx, x, x^3\right)$$

$$= -\frac{(a+bx^3)^{7/3}}{7b^3d}+\frac{a^2\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)}{3b^2}$$

$$= -\frac{a^2\sqrt[3]{a+bx^3}}{b^3d}-\frac{(a+bx^3)^{7/3}}{7b^3d}+\frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)}\,dx, x, x^3\right)}{3b^2}$$

$$= -\frac{a^2\sqrt[3]{a+bx^3}}{b^3d}-\frac{(a+bx^3)^{7/3}}{7b^3d}+\frac{a^{7/3}\log(a-bx^3)}{3\ 2^{2/3}b^3d}+\frac{a^{7/3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2^{2/3}b^3d}+\frac{a^{8/3}\text{Subst}\left(\int \frac{1}{2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^3d}$$

$$= -\frac{a^2\sqrt[3]{a+bx^3}}{b^3d}-\frac{(a+bx^3)^{7/3}}{7b^3d}+\frac{a^{7/3}\log(a-bx^3)}{3\ 2^{2/3}b^3d}-\frac{a^{7/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^3d}-\frac{\left(\sqrt[3]{2}a^{7/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{b^3d}$$

$$= -\frac{a^2\sqrt[3]{a+bx^3}}{b^3d}-\frac{(a+bx^3)^{7/3}}{7b^3d}+\frac{\sqrt[3]{2}a^{7/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}b^3d}+\frac{a^{7/3}\log(a-bx^3)}{3\ 2^{2/3}b^3d}-\frac{a^{7/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^3d}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.21

$$\int \frac{x^8\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{48a^2\sqrt[3]{a+bx^3}+12abx^3\sqrt[3]{a+bx^3}+6b^2x^6\sqrt[3]{a+bx^3}-14\sqrt[3]{2}\sqrt{3}a^{7/3}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+14\sqrt[3]{2}\cdots}{42b^3d}$$

```
[In] Integrate[(x^8*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/42*(48*a^2*(a + b*x^3)^(1/3) + 12*a*b*x^3*(a + b*x^3)^(1/3) + 6*b^2*x^6*
(a + b*x^3)^(1/3) - 14*2^(1/3)*Sqrt[3]*a^(7/3)*ArcTan[(1 + (2^(2/3)*(a + b*
x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 14*2^(1/3)*a^(7/3)*Log[-2*a^(1/3) + 2^(2/3)
*(a + b*x^3)^(1/3)] - 7*2^(1/3)*a^(7/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a
+ b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(b^3*d)
```

Maple [A] (verified)

Time = 5.05 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$\frac{\left(-6b^2x^6-12ab\,x^3-48a^2\right)\left(b\,x^3+a\right)^{\frac{1}{3}}+7\,2^{\frac{1}{3}}a^{\frac{7}{3}}\left(2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^3+a\right)^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}\cdots\right.\right.}{42b^3d}$

```
[In] int(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/42*((-6*b^2*x^6-12*a*b*x^3-48*a^2)*(b*x^3+a)^(1/3)+7*2^(1/3)*a^(7/3)*(2*a
rctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)+ln((b*
x^3+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*a^(2/3))-2*ln((b*x^3+a
)^(1/3)-2^(1/3)*a^(1/3))))/b^3/d
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00

$$\int \frac{x^8\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{14\sqrt{3}2^{\frac{1}{3}}(-a)^{\frac{1}{3}}a^2\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+7\cdot2^{\frac{1}{3}}(-a)^{\frac{1}{3}}a^2\log\left(2^{\frac{2}{3}}(-a)^{\frac{2}{3}}-2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}\right.}{42\,b^3d}$$

```
[In] integrate(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
[Out] -1/42*(14*sqrt(3)*2^(1/3)*(-a)^(1/3)*a^2*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x^3
 + a)^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + 7*2^(1/3)*(-a)^(1/3)*a^2*log(2^(2/
3)*(-a)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(2/3)) -
 14*2^(1/3)*(-a)^(1/3)*a^2*log(2^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(1/3)) + 6*
(b^2*x^6 + 2*a*b*x^3 + 8*a^2)*(b*x^3 + a)^(1/3))/(b^3*d)
```

Sympy [F]

$$\int \frac{x^8\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^8\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

```
[In] integrate(x**8*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
[Out] -Integral(x**8*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.89

$$\int \frac{x^8\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{\frac{14\sqrt{3}2^{\frac{1}{3}}a^{\frac{7}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{d}+\frac{7\cdot2^{\frac{1}{3}}a^{\frac{7}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{d}-\frac{14\cdot2^{\frac{1}{3}}a^{\frac{7}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right.}{d}}{42\,b^3}$$

```
[In] integrate(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] 1/42*(14*sqrt(3)*2^(1/3)*a^(7/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3
) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d + 7*2^(1/3)*a^(7/3)*log(2^(2/3)*a^(2/3)
 + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d - 14*2^(1/3)*a^
(7/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d - 6*((b*x^3 + a)^(7/3) +
7*(b*x^3 + a)^(1/3)*a^2)/d)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.74 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.01

$$\int \frac{x^8 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{\sqrt{3}2^{\frac{1}{3}}a^{\frac{7}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(bx^3+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{3\,b^3d} + \frac{2^{\frac{1}{3}}a^{\frac{7}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,b^3d} - \frac{2^{\frac{1}{3}}a^{\frac{7}{3}}\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,b^3d} - \frac{(bx^3+a)^{\frac{7}{3}}b^{18}d^6+7\,(bx^3+a)^{\frac{1}{3}}a^2b^{18}d^6}{7\,b^{21}d^7}$$

```
[In] integrate(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*2^(1/3)*a^(7/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2
*(b*x^3 + a)^(1/3))/a^(1/3))/(b^3*d) + 1/6*2^(1/3)*a^(7/3)*log(2^(2/3)*a^(2
/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(b^3*d) - 1/3*
2^(1/3)*a^(7/3)*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(b^3*d) - 1/
7*((b*x^3 + a)^(7/3)*b^18*d^6 + 7*(b*x^3 + a)^(1/3)*a^2*b^18*d^6)/(b^21*d^7
)
```

Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.26

$$\int \frac{x^8 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{2^{1/3}\,(-a)^{7/3}\,\ln\left(6\,a^3\,\left(b\,x^3+a\right)^{1/3}-6\,2^{1/3}\,(-a)^{10/3}\right)}{3\,b^3\,d}-\frac{a^2\,\left(b\,x^3+a\right)^{1/3}}{b^3\,d}-\frac{\left(b\,x^3+a\right)^{7/3}}{7\,b^3\,d} - \frac{2^{1/3}\,(-a)^{7/3}\,\ln\left(\frac{6\,a^3\,\left(b\,x^3+a\right)^{1/3}}{b^3\,d}+\frac{6\,2^{1/3}\,(-a)^{10/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b^3\,d}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b^3\,d} + \frac{2^{1/3}\,(-a)^{7/3}\,\ln\left(\frac{6\,a^3\,\left(b\,x^3+a\right)^{1/3}}{b^3\,d}-\frac{18\,2^{1/3}\,(-a)^{10/3}\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{b^3\,d}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{b^3\,d}$$

```
[In] int((x^8*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

```
[Out] (2^(1/3)*(-a)^(7/3)*log(6*a^3*(a + b*x^3)^(1/3) - 6*2^(1/3)*(-a)^(10/3)))/(
3*b^3*d) - (a^2*(a + b*x^3)^(1/3))/(b^3*d) - (a + b*x^3)^(7/3)/(7*b^3*d) -
(2^(1/3)*(-a)^(7/3)*log((6*a^3*(a + b*x^3)^(1/3))/(b^3*d) + (6*2^(1/3)*(-a)
^(10/3)*((3^(1/2)*1i)/2 + 1/2))/(b^3*d))*((3^(1/2)*1i)/2 + 1/2))/(3*b^3*d)
+ (2^(1/3)*(-a)^(7/3)*log((6*a^3*(a + b*x^3)^(1/3))/(b^3*d) - (18*2^(1/3)*(
-a)^(10/3)*((3^(1/2)*1i)/6 - 1/6))/(b^3*d))*((3^(1/2)*1i)/6 - 1/6))/(b^3*d)
```

3.569 $\int \frac{x^5 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 172

$$\int \frac{x^5 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx = -\frac{a\sqrt[3]{a+bx^3}}{b^2d} - \frac{(a+bx^3)^{4/3}}{4b^2d} + \frac{\sqrt[3]{2}a^{4/3}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^2d} + \frac{a^{4/3}\log(a-bx^3)}{3\ 2^{2/3}b^2d} - \frac{a^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^2d}$$

```
[Out] -a*(b*x^3+a)^(1/3)/b^2/d-1/4*(b*x^3+a)^(4/3)/b^2/d+1/6*a^(4/3)*ln(-b*x^3+a)
*2^(1/3)/b^2/d-1/2*a^(4/3)*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(1/3)/b^2/
d+1/3*2^(1/3)*a^(4/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*
3^(1/2))/b^2/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {457, 81, 52, 59, 631, 210, 31}

$$\int \frac{x^5 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx = \frac{\sqrt[3]{2}a^{4/3}\arctan\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^2d} + \frac{a^{4/3}\log(a-bx^3)}{3\ 2^{2/3}b^2d} - \frac{a^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^2d} - \frac{a\sqrt[3]{a+bx^3}}{b^2d} - \frac{(a+bx^3)^{4/3}}{4b^2d}$$

```
[In] Int[(x^5*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -((a*(a + b*x^3)^(1/3))/(b^2*d)) - (a + b*x^3)^(4/3)/(4*b^2*d) + (2^(1/3)*a
^(4/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*b^2*d) + (a^(4/3)*Log[a - b*x^3])/(3*2^(2/3)*b^2*d) - (a^(4/3)*Log[2^
(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*b^2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x\sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)$$

$$= -\frac{(a+bx^3)^{4/3}}{4b^2d} + \frac{a\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)}{3b}$$

$$= -\frac{a\sqrt[3]{a+bx^3}}{b^2d} - \frac{(a+bx^3)^{4/3}}{4b^2d} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)}\,dx, x, x^3\right)}{3b}$$

$$= -\frac{a\sqrt[3]{a+bx^3}}{b^2d} - \frac{(a+bx^3)^{4/3}}{4b^2d} + \frac{a^{4/3}\log(a-bx^3)}{3\,2^{2/3}b^2d} + \frac{a^{4/3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2^{2/3}b^2d} + \frac{a^{5/3}\text{Subst}\left(\int \frac{1}{2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^2d}$$

$$= -\frac{a\sqrt[3]{a+bx^3}}{b^2d} - \frac{(a+bx^3)^{4/3}}{4b^2d} + \frac{a^{4/3}\log(a-bx^3)}{3\,2^{2/3}b^2d} - \frac{a^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^2d} - \frac{\left(\sqrt[3]{2}a^{4/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{b^2d}$$

$$= -\frac{a\sqrt[3]{a+bx^3}}{b^2d} - \frac{(a+bx^3)^{4/3}}{4b^2d} + \frac{\sqrt[3]{2}a^{4/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}b^2d} + \frac{a^{4/3}\log(a-bx^3)}{3\,2^{2/3}b^2d} - \frac{a^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^2d}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.09

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{15a\sqrt[3]{a+bx^3}+3bx^3\sqrt[3]{a+bx^3}-4\sqrt[3]{2}\sqrt{3}a^{4/3}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+4\sqrt[3]{2}a^{4/3}\log\left(-2\sqrt[3]{a}+2^{2/3}\sqrt[3]{\right.}{12b^2d}$$

```
[In] Integrate[(x^5*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/12*(15*a*(a + b*x^3)^(1/3) + 3*b*x^3*(a + b*x^3)^(1/3) - 4*2^(1/3)*Sqrt[
3]*a^(4/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 4*2^
(1/3)*a^(4/3)*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)] - 2*2^(1/3)*a^(4/
3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^
(2/3)])/(b^2*d)
```

Maple [A] (verified)

Time = 4.60 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$\frac{\left(-3b\,x^3-15a\right)\left(b\,x^3+a\right)^{\frac{1}{3}}+2\,2^{\frac{1}{3}}a^{\frac{4}{3}}\left(2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^3+a\right)^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)-2\,\mathrm{l}}{12b^2d}$

```
[In] int(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*((-3*b*x^3-15*a)*(b*x^3+a)^(1/3)+2*2^(1/3)*a^(4/3)*(2*arctan(1/3*(a^(1
/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)+ln((b*x^3+a)^(2/3)+2^
(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*a^(2/3))-2*ln((b*x^3+a)^(1/3)-2^(1/3)
*a^(1/3))))/b^2/d
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.91

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{4\sqrt{3}2^{\frac{1}{3}}(-a)^{\frac{1}{3}}\,a\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+\sqrt{3}a}{3\,a}\right)+2\cdot 2^{\frac{1}{3}}(-a)^{\frac{1}{3}}\,a\log\left(2^{\frac{2}{3}}(-a)^{\frac{2}{3}}-2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+\right.}{12\,b^2d}$$

[In] integrate(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")

[Out] -1/12*(4*sqrt(3)*2^(1/3)*(-a)^(1/3)*a*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x^3 + a)^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + 2*2^(1/3)*(-a)^(1/3)*a*log(2^(2/3)*(-a)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(2/3)) - 4*2^(1/3)*(-a)^(1/3)*a*log(2^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(1/3)) + 3*(b*x^3 + 5*a)*(b*x^3 + a)^(1/3))/(b^2*d)

Sympy [F]

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^5\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

[In] integrate(x**5*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)

[Out] -Integral(x**5*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.89

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{\frac{4\sqrt{3}2^{\frac{1}{3}}a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{d}+\frac{2\cdot 2^{\frac{1}{3}}a^{\frac{4}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d}-\frac{4\cdot 2^{\frac{1}{3}}a^{\frac{4}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right)}{d}}{12\,b^2}$$

[In] integrate(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")

```
[Out] 1/12*(4*sqrt(3)*2^(1/3)*a^(4/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3)
 + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d + 2*2^(1/3)*a^(4/3)*log(2^(2/3)*a^(2/3)
+ 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d - 4*2^(1/3)*a^(4
/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d - 3*((b*x^3 + a)^(4/3) + 4*
(b*x^3 + a)^(1/3)*a)/d)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.75 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.01

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{\sqrt{3}2^{\frac{1}{3}}a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(bx^3+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{3\,b^2d} + \frac{2^{\frac{1}{3}}a^{\frac{4}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,b^2d} - \frac{2^{\frac{1}{3}}a^{\frac{4}{3}}\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,b^2d} - \frac{(bx^3+a)^{\frac{4}{3}}b^6d^3+4\,(bx^3+a)^{\frac{1}{3}}ab^6d^3}{4\,b^8d^4}$$

```
[In] integrate(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*2^(1/3)*a^(4/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2
*(b*x^3 + a)^(1/3))/a^(1/3))/(b^2*d) + 1/6*2^(1/3)*a^(4/3)*log(2^(2/3)*a^(2
/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(b^2*d) - 1/3*
2^(1/3)*a^(4/3)*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(b^2*d) - 1/
4*((b*x^3 + a)^(4/3)*b^6*d^3 + 4*(b*x^3 + a)^(1/3)*a*b^6*d^3)/(b^8*d^4)
```

Mupad [B] (verification not implemented)

Time = 8.90 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.16

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{(b\,x^3+a)^{4/3}}{4\,b^2\,d} - \frac{a\,(b\,x^3+a)^{1/3}}{b^2\,d} - \frac{2^{1/3}\,a^{4/3}\,\ln\left((b\,x^3+a)^{1/3}-2^{1/3}\,a^{1/3}\right)}{3\,b^2\,d} - \frac{2^{1/3}\,a^{4/3}\,\ln\left(\frac{6\,a^2\,(b\,x^3+a)^{1/3}}{b^2\,d}-\frac{6\,2^{1/3}\,a^{7/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b^2\,d}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b^2\,d} + \frac{2^{1/3}\,a^{4/3}\,\ln\left(\frac{6\,a^2\,(b\,x^3+a)^{1/3}}{b^2\,d}+\frac{18\,2^{1/3}\,a^{7/3}\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{b^2\,d}\right)\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{b^2\,d}$$

```
[In] int((x^5*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

```
[Out] (2^(1/3)*a^(4/3)*log((6*a^2*(a + b*x^3)^(1/3))/(b^2*d) + (18*2^(1/3)*a^(7/3
)*((3^(1/2)*1i)/6 + 1/6))/(b^2*d))*((3^(1/2)*1i)/6 + 1/6))/(b^2*d) - (a*(a
+ b*x^3)^(1/3))/(b^2*d) - (2^(1/3)*a^(4/3)*log((a + b*x^3)^(1/3) - 2^(1/3)*
a^(1/3)))/(3*b^2*d) - (2^(1/3)*a^(4/3)*log((6*a^2*(a + b*x^3)^(1/3))/(b^2*d
) - (6*2^(1/3)*a^(7/3)*((3^(1/2)*1i)/2 - 1/2))/(b^2*d))*((3^(1/2)*1i)/2 - 1
/2))/(3*b^2*d) - (a + b*x^3)^(4/3)/(4*b^2*d)
```

3.570 $\int \frac{x^2 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 150

$$\int \frac{x^2 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\sqrt[3]{a+bx^3}}{bd}+\frac{\sqrt[3]{2}\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}bd}+\frac{\sqrt[3]{a}\log\left(a-bx^3\right)}{3\ 2^{2/3}bd}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}bd}$$

```
[Out] -(b*x^3+a)^(1/3)/b/d+1/6*a^(1/3)*ln(-b*x^3+a)*2^(1/3)/b/d-1/2*a^(1/3)*ln(2^
(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(1/3)/b/d+1/3*2^(1/3)*a^(1/3)*arctan(1/3*(
a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/b/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {455, 52, 59, 631, 210, 31}

$$\int \frac{x^2 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{\sqrt[3]{2}\sqrt[3]{a}\arctan\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}bd}-\frac{\sqrt[3]{a+bx^3}}{bd}+\frac{\sqrt[3]{a}\log\left(a-bx^3\right)}{3\ 2^{2/3}bd}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}bd}$$

```
[In] Int[(x^2*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -((a + b*x^3)^(1/3)/(b*d)) + (2^(1/3)*a^(1/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a
+ b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b*d) + (a^(1/3)*Log[a - b*x^3]
```

```
)/(3*2^(2/3)*b*d) - (a^(1/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(
2/3)*b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt[3]{a+bx^3}}{bd} + \frac{1}{3}(2a)\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt[3]{a+bx^3}}{bd} + \frac{\sqrt[3]{a}\log(a-bx^3)}{3\ 2^{2/3}bd} + \frac{\sqrt[3]{a}\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2^{2/3}bd} + \frac{a^{2/3}\text{Subst}\left(\int \frac{1}{2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}bd}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{bd} + \frac{\sqrt[3]{a}\log(a-bx^3)}{3\ 2^{2/3}bd} - \frac{\sqrt[3]{a}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}bd} - \frac{\left(\sqrt[3]{2}\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{bd}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{bd} + \frac{\sqrt[3]{2}\sqrt[3]{a}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}bd} + \frac{\sqrt[3]{a}\log(a-bx^3)}{3\ 2^{2/3}bd} - \frac{\sqrt[3]{a}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}bd}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.13

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{-6\sqrt[3]{a+bx^3}+2\sqrt[3]{2}\sqrt{3}\sqrt[3]{a}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\sqrt[3]{2}\sqrt[3]{a}\log\left(-2\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}\right)+\sqrt[3]{2}\sqrt[3]{a}\log}{6bd}$$

```
[In] Integrate[(x^2*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-6*(a + b*x^3)^(1/3) + 2*2^(1/3)*Sqrt[3]*a^(1/3)*ArcTan[(1 + (2^(2/3)*(a +
 b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*2^(1/3)*a^(1/3)*Log[-2*a^(1/3) + 2^(2/
3)*(a + b*x^3)^(1/3)] + 2^(1/3)*a^(1/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a
+ b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(6*b*d)
```

Maple [A] (verified)

Time = 4.73 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{2a^{\frac{1}{3}}2^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)-2a^{\frac{1}{3}}2^{\frac{1}{3}}\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)+a^{\frac{1}{3}}2^{\frac{1}{3}}\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\right.}{6bd}$

[In] int(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)

[Out] 1/6*(2*a^(1/3)*2^(1/3)*3^(1/2)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))-2*a^(1/3)*2^(1/3)*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3))+a^(1/3)*2^(1/3)*ln((b*x^3+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*a^(2/3))-6*(b*x^3+a)^(1/3))/b/d

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.96

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{2\sqrt{3}2^{\frac{1}{3}}(-a)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+\sqrt{3}a}{3\,a}\right)+2^{\frac{1}{3}}(-a)^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}(-a)^{\frac{2}{3}}-2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(bx^3\right.}{6\,bd}$$

[In] integrate(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")

[Out] -1/6*(2*sqrt(3)*2^(1/3)*(-a)^(1/3)*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x^3 + a)^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + 2^(1/3)*(-a)^(1/3)*log(2^(2/3)*(-a)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(2/3)) - 2*2^(1/3)*(-a)^(1/3)*log(2^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(1/3)) + 6*(b*x^3 + a)^(1/3))/(b*d)

Sympy [F]

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^2\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

[In] integrate(x**2*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)

[Out] -Integral(x**2*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.93

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{\frac{2\sqrt{3}2^{\frac{1}{3}}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{d}+\frac{2^{\frac{1}{3}}a^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d}-\frac{2\cdot 2^{\frac{1}{3}}a^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right)}{d}-}{6b}$$

[In] integrate(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")

[Out] 1/6*(2*sqrt(3)*2^(1/3)*a^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d + 2^(1/3)*a^(1/3)*log(2^(2/3)*a^(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d - 2*2^(1/3)*a^(1/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d - 6*(b*x^3 + a)^(1/3)/d)/b

Giac [A] (verification not implemented)

none

Time = 0.74 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{\sqrt{3}2^{\frac{1}{3}}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{3bd} + \frac{2^{\frac{1}{3}}a^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6bd} - \frac{2^{\frac{1}{3}}a^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3bd} - \frac{(bx^3+a)^{\frac{1}{3}}}{bd}$$

[In] integrate(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")

[Out] 1/3*sqrt(3)*2^(1/3)*a^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/(b*d) + 1/6*2^(1/3)*a^(1/3)*log(2^(2/3)*a^(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(b*d) - 1/3*2^(1/3)*a^(1/3)*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(b*d) - (b*x^3 + a)^(1/3)/(b*d)

Mupad [B] (verification not implemented)

Time = 8.66 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.29

$$\int \frac{x^2 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{2^{1/3}\,(-a)^{1/3}\,\ln\left(6\,a\,(b\,x^3+a)^{1/3}-6\,2^{1/3}\,(-a)^{4/3}\right)}{3\,b\,d}-\frac{(b\,x^3+a)^{1/3}}{b\,d}+\frac{2^{1/3}\,(-a)^{1/3}\,\ln\left(\frac{6\,a\,(b\,x^3+a)^{1/3}}{b\,d}-\frac{6\,2^{1/3}\,(-a)^{4/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b\,d}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b\,d}-\frac{2^{1/3}\,(-a)^{1/3}\,\ln\left(\frac{6\,a\,(b\,x^3+a)^{1/3}}{b\,d}+\frac{6\,2^{1/3}\,(-a)^{4/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b\,d}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b\,d}$$

```
[In] int((x^2*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

```
[Out] (2^(1/3)*(-a)^(1/3)*log(6*a*(a + b*x^3)^(1/3) - 6*2^(1/3)*(-a)^(4/3)))/(3*b
*d) - (a + b*x^3)^(1/3)/(b*d) + (2^(1/3)*(-a)^(1/3)*log((6*a*(a + b*x^3)^(1
/3))/(b*d) - (6*2^(1/3)*(-a)^(4/3)*((3^(1/2)*1i)/2 - 1/2))/(b*d))*((3^(1/2)
*1i)/2 - 1/2))/(3*b*d) - (2^(1/3)*(-a)^(1/3)*log((6*a*(a + b*x^3)^(1/3))/(b
*d) + (6*2^(1/3)*(-a)^(4/3)*((3^(1/2)*1i)/2 + 1/2))/(b*d))*((3^(1/2)*1i)/2
+ 1/2))/(3*b*d)
```

3.571 $\int \frac{\sqrt[3]{a+bx^3}}{x\left(ad-bdx^3\right)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 214

$$\int \frac{\sqrt[3]{a+bx^3}}{x\left(ad-bdx^3\right)}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}d} + \frac{\sqrt[3]{2}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}d} - \frac{\log(x)}{2a^{2/3}d} + \frac{\log\left(a-bx^3\right)}{3\ 2^{2/3}a^{2/3}d} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}d} - \frac{\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{2/3}d}$$

```
[Out] -1/2*ln(x)/a^(2/3)/d+1/6*ln(-b*x^3+a)*2^(1/3)/a^(2/3)/d+1/2*ln(a^(1/3)-(b*x
^3+a)^(1/3))/a^(2/3)/d-1/2*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(1/3)/a^(2
/3)/d-1/3*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(2/3)/d
*3^(1/2)+1/3*2^(1/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3
^(1/2))/a^(2/3)/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used

= {457, 85, 59, 631, 210, 31}

$$\int \frac{\sqrt[3]{a+bx^3}}{x\,(ad-bdx^3)}\,dx = -\frac{\arctan\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}d} + \frac{\sqrt[3]{2}\arctan\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}d} + \frac{\log\left(a-bx^3\right)}{3\ 2^{2/3}a^{2/3}d} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}d} - \frac{\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{2/3}d} - \frac{\log(x)}{2a^{2/3}d}$$

```
[In] Int[(a + b*x^3)^(1/3)/(x*(a*d - b*d*x^3)),x]
```

```
[Out] -(ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3
)*d)) + (2^(1/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1
/3))])/(Sqrt[3]*a^(2/3)*d) - Log[x]/(2*a^(2/3)*d) + Log[a - b*x^3]/(3*2^(2/
3)*a^(2/3)*d) + Log[a^(1/3) - (a + b*x^3)^(1/3)]/(2*a^(2/3)*d) - Log[2^(1/3
)*a^(1/3) - (a + b*x^3)^(1/3)]/(2^(2/3)*a^(2/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x(ad-bdx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}(2b)\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)}\,dx, x, x^3\right) + \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^3\right)}{3d}$$

$$= -\frac{\log(x)}{2a^{2/3}d} + \frac{\log\left(a-bx^3\right)}{3\ 2^{2/3}a^{2/3}d} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2a^{2/3}d} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{2/3}d} - \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}d} + \frac{\text{Subst}\left(\int \frac{1}{2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\sqrt[3]{a}d}$$

$$= -\frac{\log(x)}{2a^{2/3}d} + \frac{\log\left(a-bx^3\right)}{3\ 2^{2/3}a^{2/3}d} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}d} - \frac{\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{2/3}d} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{a^{2/3}d} - \frac{\sqrt[3]{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{a^{2/3}d}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}d} + \frac{\sqrt[3]{2}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}d} - \frac{\log(x)}{2a^{2/3}d} + \frac{\log\left(a-bx^3\right)}{3\ 2^{2/3}a^{2/3}d} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}d} - \frac{\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{2/3}d}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt[3]{a+bx^3}}{x\,(ad-bdx^3)}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)+2\sqrt[3]{}$$

[In] Integrate[(a + b*x^3)^(1/3)/(x*(a*d - b*d*x^3)),x]

[Out] -1/6*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*2^(1/3)*Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[-a^(1/3) + (a + b*x^3)^(1/3)] + 2*2^(1/3)*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] - 2^(1/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(a^(2/3)*d)

Maple [A] (verified)

Time = 4.94 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.85

method	result
pseudoelliptic	$\frac{2\,2^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)-2\,2^{\frac{1}{3}}\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)+2^{\frac{1}{3}}\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{6d\,a^{\frac{2}{3}}}$

[In] int((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)

[Out] 1/6*(2*2^(1/3)*3^(1/2)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))-2*2^(1/3)*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3))+2^(1/3)*ln((b*x^3+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*a^(2/3))-2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)+2*ln((b*x^3+a)^(1/3)-a^(1/3))-ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3)))/d/a^(2/3)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 634 vs. $2(161) = 322$.

Time = 0.26 (sec) , antiderivative size = 634, normalized size of antiderivative = 2.96

$$\int \frac{\sqrt[3]{a+bx^3}}{x\,(ad-bdx^3)}\,dx =$$

$$-\frac{1}{6}\left(\frac{1}{2}\right)^{\frac{1}{3}}(\sqrt{-3}+1)\left(-\frac{3\,a^2d^3\sqrt{\frac{1}{a^4d^6}}+1}{a^2d^3}\right)^{\frac{1}{3}}\log\left(-\left(\frac{1}{2}\right)^{\frac{1}{3}}(\sqrt{-3}a^3d^4+a^3d^4)\left(-\frac{3\,a^2d^3\sqrt{\frac{1}{a^4d^6}}+1}{a^2d^3}\right)^{\frac{1}{3}}\right.$$

$$\left.+2\left(bx^3+a\right)^{\frac{1}{3}}\right)$$

$$+\frac{1}{6}\left(\frac{1}{2}\right)^{\frac{1}{3}}(\sqrt{-3}-1)\left(-\frac{3\,a^2d^3\sqrt{\frac{1}{a^4d^6}}+1}{a^2d^3}\right)^{\frac{1}{3}}\log\left(\left(\frac{1}{2}\right)^{\frac{1}{3}}(\sqrt{-3}a^3d^4-a^3d^4)\left(-\frac{3\,a^2d^3\sqrt{\frac{1}{a^4d^6}}+1}{a^2d^3}\right)^{\frac{1}{3}}\sqrt{\right.$$

$$\left.+2\left(bx^3+a\right)^{\frac{1}{3}}\right)$$

$$-\frac{1}{6}\left(\frac{1}{2}\right)^{\frac{1}{3}}(\sqrt{-3}+1)\left(\frac{3\,a^2d^3\sqrt{\frac{1}{a^4d^6}}-1}{a^2d^3}\right)^{\frac{1}{3}}\log\left(\left(\frac{1}{2}\right)^{\frac{1}{3}}(\sqrt{-3}a^3d^4+a^3d^4)\left(\frac{3\,a^2d^3\sqrt{\frac{1}{a^4d^6}}-1}{a^2d^3}\right)^{\frac{1}{3}}\sqrt{\frac{1}{a^4d}}\right.$$

$$\left.+2\left(bx^3+a\right)^{\frac{1}{3}}\right)$$

$$+\frac{1}{6}\left(\frac{1}{2}\right)^{\frac{1}{3}}(\sqrt{-3}-1)\left(\frac{3\,a^2d^3\sqrt{\frac{1}{a^4d^6}}-1}{a^2d^3}\right)^{\frac{1}{3}}\log\left(-\left(\frac{1}{2}\right)^{\frac{1}{3}}(\sqrt{-3}a^3d^4-a^3d^4)\left(\frac{3\,a^2d^3\sqrt{\frac{1}{a^4d^6}}-1}{a^2d^3}\right)^{\frac{1}{3}}\sqrt{\frac{}{a}}\right.$$

$$\left.+2\left(bx^3+a\right)^{\frac{1}{3}}\right)$$

$$+\frac{1}{3}\left(\frac{1}{2}\right)^{\frac{1}{3}}\left(-\frac{3\,a^2d^3\sqrt{\frac{1}{a^4d^6}}+1}{a^2d^3}\right)^{\frac{1}{3}}\log\left(\left(\frac{1}{2}\right)^{\frac{1}{3}}a^3d^4\left(-\frac{3\,a^2d^3\sqrt{\frac{1}{a^4d^6}}+1}{a^2d^3}\right)^{\frac{1}{3}}\sqrt{\frac{1}{a^4d^6}}\right.$$

$$\left.+\left(bx^3+a\right)^{\frac{1}{3}}\right)$$

$$+\frac{1}{3}\left(\frac{1}{2}\right)^{\frac{1}{3}}\left(\frac{3\,a^2d^3\sqrt{\frac{1}{a^4d^6}}-1}{a^2d^3}\right)^{\frac{1}{3}}\log\left(-\left(\frac{1}{2}\right)^{\frac{1}{3}}a^3d^4\left(\frac{3\,a^2d^3\sqrt{\frac{1}{a^4d^6}}-1}{a^2d^3}\right)^{\frac{1}{3}}\sqrt{\frac{1}{a^4d^6}}\right.$$

[In] integrate((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x, algorithm="fricas")

[Out] -1/6*(1/2)^(1/3)*(sqrt(-3) + 1)*(-(3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^(1/3)*log(-(1/2)^(1/3)*(sqrt(-3)*a^3*d^4 + a^3*d^4)*(-(3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^(1/3)*sqrt(1/(a^4*d^6)) + 2*(b*x^3 + a)^(1/3)) + 1/6*(1/2)^(1/3)*(sqrt(-3) - 1)*(-(3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^(1/3)*log((1/2)^(1/3)*(sqrt(-3)*a^3*d^4 - a^3*d^4)*(-(3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^(1/3)*sqrt(1/(a^4*d^6)) + 2*(b*x^3 + a)^(1/3)) - 1/6*(1/2)^(1/3)*(sqrt(-3) + 1)*((3*a^2*d^3*sqrt(1/(a^4*d^6)) - 1)/(a^2*d^3))^(1/3)*log((1/2)^(1/3)*(sqrt(-3)*a^3*d^4 + a^3*d^4)*((3*a^2*d^3*sqrt(1/(a^4*d^6)) - 1)/(a^2*d^3))^(1/3)*sqrt(1/(a^4*d^6)) + 2*(b*x^3 + a)^(1/3)) + 1/6*(1/2)^(1/3)*(sqrt(-3) - 1)*((3*a^2*d^3*sqrt(1/(a^4*d^6)) - 1)/(a^2*d^3))^(1/3)*log(-(1/2)^(1/3)*(sqrt(-3)*a^3*d^4 - a^3*d^4)*((3*a^2*d^3*sqrt(1/(a^4*d^6)) - 1)/(a^2*d^3))^(1/3)*sqrt(1/(a^4*d^6)) + 2*(b*x^3 + a)^(1/3)) + 1/3*(1/2)^(1/3)*(-(3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^(1/3)*log((1/2)^(1/3)*a^3*d^4*(-(3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^(1/3)*sqrt(1/(a^4*d^6)) + (b*x^3 + a)^(1/3)) + 1/3*(1/2)^(1/3)*((3*a^2*d^3*sqrt(1/(a^4*d^6)) - 1)/(a^2*d^3))^(1/3)*log(-(1/2)^(1/3)*a^3*d^4*((3*a^2*d^3*sqrt(1/(a^4*d^6)) - 1)/(a^2*d^3))^(1/3)*sqrt(1/(a^4*d^6)) + (b*x^3 + a)^(1/3))

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x\left(ad-bdx^3\right)}\,dx=-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax+bx^4}\,dx}{d}$$

[In] integrate((b*x**3+a)**(1/3)/x/(-b*d*x**3+a*d),x)

[Out] -Integral((a + b*x**3)**(1/3)/(-a*x + b*x**4), x)/d

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x\left(ad-bdx^3\right)}\,dx=\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x}\,dx$$

[In] integrate((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x, algorithm="maxima")

[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x), x)

Giac [A] (verification not implemented)

none

Time = 0.99 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt[3]{a+bx^3}}{x(ad-bdx^3)}\,dx = \frac{\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(bx^3+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{2}{3}}d} - \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{2}{3}}d} + \frac{2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,a^{\frac{2}{3}}d} - \frac{2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{2}{3}}d} - \frac{\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{2}{3}}d} + \frac{\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{2}{3}}d}$$

```
[In] integrate((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(b*x^3
+ a)^(1/3))/a^(1/3))/(a^(2/3)*d) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^3
 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(2/3)*d) + 1/6*2^(1/3)*log(2^(2/3)*a^(2/
3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(a^(2/3)*d) - 1
/3*2^(1/3)*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(a^(2/3)*d) - 1/6
*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(2/3)*d) +
 1/3*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(2/3)*d)
```

Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.61

$$\int \frac{\sqrt[3]{a+bx^3}}{x(ad-bdx^3)}\,dx = \ln\left(\left(b\,x^3+a\right)^{1/3}-a\,d\left(\frac{1}{a^2\,d^3}\right)^{1/3}\right)\left(\frac{1}{27\,a^2\,d^3}\right)^{1/3} + \ln\left(\left(b\,x^3+a\right)^{1/3}+2^{1/3}\,a\,d\left(-\frac{1}{a^2\,d^3}\right)^{1/3}\right)\left(-\frac{2}{27\,a^2\,d^3}\right)^{1/3} - \ln\left(2^{1/3}\,a\,d\left(-\frac{1}{a^2\,d^3}\right)^{1/3}-2\left(b\,x^3+a\right)^{1/3}\right.$$

```
[In] int((a + b*x^3)^(1/3)/(x*(a*d - b*d*x^3)),x)
```

```
[Out] log((a + b*x^3)^(1/3) - a*d*(1/(a^2*d^3))^(1/3))*(1/(27*a^2*d^3))^(1/3) + l
og((a + b*x^3)^(1/3) + 2^(1/3)*a*d*(-1/(a^2*d^3))^(1/3))*(-2/(27*a^2*d^3))^
(1/3) - log(2^(1/3)*a*d*(-1/(a^2*d^3))^(1/3) - 2*(a + b*x^3)^(1/3) + 2^(1/3
)*3^(1/2)*a*d*(-1/(a^2*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(-2/(27*a^2*d
^3))^(1/3) + log(2*(a + b*x^3)^(1/3) - 2^(1/3)*a*d*(-1/(a^2*d^3))^(1/3) + 2
^(1/3)*3^(1/2)*a*d*(-1/(a^2*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(-2/(27*
a^2*d^3))^(1/3) + log(2*(a + b*x^3)^(1/3) + a*d*(1/(a^2*d^3))^(1/3) - 3^(1/
2)*a*d*(1/(a^2*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(1/(27*a^2*d^3))^(1/3
) - log(2*(a + b*x^3)^(1/3) + a*d*(1/(a^2*d^3))^(1/3) + 3^(1/2)*a*d*(1/(a^2
*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(1/(27*a^2*d^3))^(1/3)
```

3.572 $\int \frac{\sqrt[3]{a+bx^3}}{x^4(ad-bdx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 268

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4(ad-bdx^3)}\,dx = \frac{b\sqrt[3]{a+bx^3}}{3a^2d}-\frac{(a+bx^3)^{4/3}}{3a^2dx^3}-\frac{4b\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}d}+\frac{\sqrt[3]{2}b\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}d}-\frac{2b\log(x)}{3a^{5/3}d}+\frac{b\log(a-bx^3)}{3\,2^{2/3}a^{5/3}d}+\frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{3a^{5/3}d}-\frac{b\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{5/3}d}$$

```
[Out] 1/3*b*(b*x^3+a)^(1/3)/a^2/d-1/3*(b*x^3+a)^(4/3)/a^2/d/x^3-2/3*b*ln(x)/a^(5/
3)/d+1/6*b*ln(-b*x^3+a)*2^(1/3)/a^(5/3)/d+2/3*b*ln(a^(1/3)-(b*x^3+a)^(1/3))
/a^(5/3)/d-1/2*b*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(1/3)/a^(5/3)/d-4/9*
b*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(5/3)/d*3^(1/2)
+1/3*2^(1/3)*b*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2)
)/a^(5/3)/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used

$= \{457, 105, 162, 52, 59, 631, 210, 31\}$

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4(ad-bdx^3)}\,dx = -\frac{4b\arctan\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}d} + \frac{\sqrt[3]{2}b\arctan\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}d} + \frac{b\log(a-bx^3)}{3\,2^{2/3}a^{5/3}d} + \frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{3a^{5/3}d} - \frac{b\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{5/3}d} - \frac{2b\log(x)}{3a^{5/3}d} - \frac{(a+bx^3)^{4/3}}{3a^2dx^3} + \frac{b\sqrt[3]{a+bx^3}}{3a^2d}$$

```
[In] Int[(a + b*x^3)^(1/3)/(x^4*(a*d - b*d*x^3)),x]
```

```
[Out] (b*(a + b*x^3)^(1/3))/(3*a^2*d) - (a + b*x^3)^(4/3)/(3*a^2*d*x^3) - (4*b*Ar
cTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)
*d) + (2^(1/3)*b*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1
/3))])/(Sqrt[3]*a^(5/3)*d) - (2*b*Log[x])/(3*a^(5/3)*d) + (b*Log[a - b*x^3]
)/(3*2^(2/3)*a^(5/3)*d) + (2*b*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(3*a^(5/3)
*d) - (b*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*a^(5/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
```

```
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^2(ad-bdx)}\,dx, x, x^3\right)$$

$$= -\frac{(a+bx^3)^{4/3}}{3a^2dx^3} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}\left(-\frac{4}{3}abd+\frac{1}{3}b^2dx\right)}{x(ad-bdx)}\,dx, x, x^3\right)}{3a^2d}$$

$$= -\frac{(a+bx^3)^{4/3}}{3a^2dx^3} + \frac{b^2\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)}{3a^2} + \frac{(4b)\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x}\,dx, x, x^3\right)}{9a^2d}$$

$$= \frac{b\sqrt[3]{a+bx^3}}{3a^2d} - \frac{(a+bx^3)^{4/3}}{3a^2dx^3} + \frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)}\,dx, x, x^3\right)}{3a} + \frac{(4b)\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^3\right)}{9ad}$$

$$= \frac{b\sqrt[3]{a+bx^3}}{3a^2d} - \frac{(a+bx^3)^{4/3}}{3a^2dx^3} - \frac{2b\log(x)}{3a^{5/3}d} + \frac{b\log(a-bx^3)}{3\ 2^{2/3}a^{5/3}d} - \frac{(2b)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{3a^{5/3}d} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{5/3}d} - \frac{(2b)\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{3a^{4/3}d} + \frac{b\text{Subst}\left(\int \frac{1}{2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^{4/3}d}$$

$$= \frac{b\sqrt[3]{a+bx^3}}{3a^2d} - \frac{(a+bx^3)^{4/3}}{3a^2dx^3} - \frac{2b\log(x)}{3a^{5/3}d} + \frac{b\log(a-bx^3)}{3\ 2^{2/3}a^{5/3}d} + \frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{3a^{5/3}d} - \frac{b\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{5/3}d} + \frac{(4b)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{3a^{5/3}d} - \frac{\left(\sqrt[3]{2}b\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{a^{5/3}d}$$

$$= \frac{b\sqrt[3]{a+bx^3}}{3a^2d} - \frac{(a+bx^3)^{4/3}}{3a^2dx^3} - \frac{4b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{5/3}d} + \frac{\sqrt[3]{2}b\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{5/3}d} - \frac{2b\log(x)}{3a^{5/3}d} + \frac{b\log(a-bx^3)}{3\ 2^{2/3}a^{5/3}d} + \frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{3a^{5/3}d} - \frac{b\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{5/3}d}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4(ad-bdx^3)}\,dx =$$

$$-\frac{6a^{2/3}\sqrt[3]{a+bx^3} + 8\sqrt{3}bx^3\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 6\sqrt[3]{2}\sqrt{3}bx^3\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 8bx^3\log\left(\right.}{}$$

[In] Integrate[(a + b*x^3)^(1/3)/(x^4*(a*d - b*d*x^3)),x]

[Out] -1/18*(6*a^(2/3)*(a + b*x^3)^(1/3) + 8*Sqrt[3]*b*x^3*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 6*2^(1/3)*Sqrt[3]*b*x^3*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 8*b*x^3*Log[-a^(1/3) + (a + b*x^3)^(1/3)] + 6*2^(1/3)*b*x^3*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)] + 4*b*x^3*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] - 3*2^(1/3)*b*x^3*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(a^(5/3)*d*x^3)

Maple [A] (verified)

Time = 4.86 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$-\frac{-2^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)b\,x^{3}+2^{\frac{1}{3}}\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)b\,x^{3}+\frac{4\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}\,b\,x}{3}}{}$

[In] int((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)

[Out] -1/3*(-2^(1/3)*3^(1/2)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*b*x^3+2^(1/3)*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3))*b*x^3+4/3*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)*b*x^3-1/2*2^(1/3)*ln((b*x^3+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*a^(2/3))*b*x^3-4/3*ln((b*x^3+a)^(1/3)-a^(1/3))*b*x^3+2/3*ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3))*b*x^3+(b*x^3+a)^(1/3)*a^(2/3))/a^(5/3)/x^3/d

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(ad-bdx^3\right)}\,dx =$$

$$-\frac{6\sqrt{3}2^{\frac{1}{3}}a^2bx^3\left(-\frac{1}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a\left(-\frac{1}{a^2}\right)^{\frac{2}{3}}+\frac{1}{3}\sqrt{3}\right)+3\cdot 2^{\frac{1}{3}}a^2bx^3\left(-\frac{1}{a^2}\right)^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^2\left(-\frac{1}{a^2}\right)\right.}{}$$

[In] integrate((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x, algorithm="fricas")

[Out] -1/18*(6*sqrt(3)*2^(1/3)*a^2*b*x^3*(-1/a^2)^(1/3)*arctan(1/3*sqrt(3)*2^(2/3)*(b*x^3 + a)^(1/3)*a*(-1/a^2)^(2/3) + 1/3*sqrt(3)) + 3*2^(1/3)*a^2*b*x^3*(

```
-1/a^2)^(1/3)*log(2^(2/3)*a^2*(-1/a^2)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*a*
(-1/a^2)^(1/3) + (b*x^3 + a)^(2/3)) - 6*2^(1/3)*a^2*b*x^3*(-1/a^2)^(1/3)*lo
g(2^(1/3)*a*(-1/a^2)^(1/3) + (b*x^3 + a)^(1/3)) + 8*sqrt(3)*(a^2)^(1/6)*a*b
*x^3*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^(1/3)*a + 2*sqrt(3)*(b*x^3 + a)^
(1/3)*(a^2)^(2/3))/a^2) + 4*(a^2)^(2/3)*b*x^3*log((b*x^3 + a)^(2/3)*a + (a^
2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) - 8*(a^2)^(2/3)*b*x^3*log((b*x^
3 + a)^(1/3)*a - (a^2)^(2/3)) + 6*(b*x^3 + a)^(1/3)*a^2)/(a^3*d*x^3)
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(ad-bdx^3\right)}\,dx = -\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^4+bx^7}\,dx}{d}$$

```
[In] integrate((b*x**3+a)**(1/3)/x**4/(-b*d*x**3+a*d),x)
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**4 + b*x**7), x)/d
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(ad-bdx^3\right)}\,dx = \int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^4}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.98 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4(ad-bdx^3)}\,dx = \frac{\sqrt{3}2^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(bx^3+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{5}{3}}d} - \frac{4\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{5}{3}}d} + \frac{2^{\frac{1}{3}}b\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,a^{\frac{5}{3}}d} - \frac{2^{\frac{1}{3}}b\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{5}{3}}d} - \frac{2\,b\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9\,a^{\frac{5}{3}}d} + \frac{4\,b\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{5}{3}}d} - \frac{(bx^3+a)^{\frac{1}{3}}}{3\,adx^3}$$

```
[In] integrate((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*2^(1/3)*b*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(b*x^
3 + a)^(1/3))/a^(1/3))/(a^(5/3)*d) - 4/9*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b
*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(5/3)*d) + 1/6*2^(1/3)*b*log(2^(2/3)
*a^(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(a^(5/3)*
d) - 1/3*2^(1/3)*b*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(a^(5/3)*
d) - 2/9*b*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^
(5/3)*d) + 4/9*b*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(5/3)*d) - 1/3*(b
*x^3 + a)^(1/3)/(a*d*x^3)
```

Mupad [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.70

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4(ad-bdx^3)}\,dx = \frac{4\ln\left(b\,(b\,x^3+a)^{1/3}-a^2\,d\left(\frac{b^3}{a^5\,d^3}\right)^{1/3}\right)\left(\frac{b^3}{a^5\,d^3}\right)^{1/3}}{9} + \ln\left(b\,(b\,x^3+a)^{1/3}+2^{1/3}\,a^2\,d\left(-\frac{b^3}{a^5\,d^3}\right)^{1/3}\right)\left(-\frac{2\,b^3}{27\,a^5\,d^3}\right)^{1/3} + \ln\left(2\,b\,(b\,x^3+a)^{1/3}+a^2\,d\left(\frac{b^3}{a^5\,d^3}\right)^{1/3} -\right.$$

```
[In] int((a + b*x^3)^(1/3)/(x^4*(a*d - b*d*x^3)),x)
```

```
[Out] (4*log(b*(a + b*x^3)^(1/3) - a^2*d*(b^3/(a^5*d^3))^(1/3))*(b^3/(a^5*d^3))^(
1/3))/9 + log(b*(a + b*x^3)^(1/3) + 2^(1/3)*a^2*d*(-b^3/(a^5*d^3))^(1/3))*(
-(2*b^3)/(27*a^5*d^3))^(1/3) + log(2*b*(a + b*x^3)^(1/3) + a^2*d*(b^3/(a^5*
d^3))^(1/3) - 3^(1/2)*a^2*d*(b^3/(a^5*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 - 1/2
)*((64*b^3)/(729*a^5*d^3))^(1/3) - log(2*b*(a + b*x^3)^(1/3) + a^2*d*(b^3/(
a^5*d^3))^(1/3) + 3^(1/2)*a^2*d*(b^3/(a^5*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 +
 1/2)*((64*b^3)/(729*a^5*d^3))^(1/3) - log(2^(1/3)*a^2*d*(-b^3/(a^5*d^3))^(
1/3) - 2*b*(a + b*x^3)^(1/3) + 2^(1/3)*3^(1/2)*a^2*d*(-b^3/(a^5*d^3))^(1/3)
*1i)*((3^(1/2)*1i)/2 + 1/2)*(-(2*b^3)/(27*a^5*d^3))^(1/3) + log(2*b*(a + b*
x^3)^(1/3) - 2^(1/3)*a^2*d*(-b^3/(a^5*d^3))^(1/3) + 2^(1/3)*3^(1/2)*a^2*d*(
-b^3/(a^5*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(-(2*b^3)/(27*a^5*d^3))^(1
/3) - (b*(a + b*x^3)^(1/3))/(3*a*(d*(a + b*x^3) - a*d))
```

3.573 $\int \frac{\sqrt[3]{a+bx^3}}{x^7(ad-bdx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 283

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7(ad-bdx^3)}\,dx = -\frac{2b\sqrt[3]{a+bx^3}}{9a^2dx^3}-\frac{(a+bx^3)^{4/3}}{6a^2dx^6}-\frac{11b^2\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}d}+\frac{\sqrt[3]{2}b^2\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}d}-\frac{11b^2\log(x)}{18a^{8/3}d}+\frac{b^2\log(a-bx^3)}{3\ 2^{2/3}a^{8/3}d}+\frac{11b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{8/3}d}-\frac{b^2\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{8/3}d}$$

```
[Out] -2/9*b*(b*x^3+a)^(1/3)/a^2/d/x^3-1/6*(b*x^3+a)^(4/3)/a^2/d/x^6-11/18*b^2*ln
(x)/a^(8/3)/d+1/6*b^2*ln(-b*x^3+a)*2^(1/3)/a^(8/3)/d+11/18*b^2*ln(a^(1/3)-(
b*x^3+a)^(1/3))/a^(8/3)/d-1/2*b^2*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(1/
3)/a^(8/3)/d-11/27*b^2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/
2))/a^(8/3)/d*3^(1/2)+1/3*2^(1/3)*b^2*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)
^(1/3))/a^(1/3)*3^(1/2))/a^(8/3)/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used

$= \{457, 105, 154, 162, 59, 631, 210, 31\}$

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(ad-bdx^3\right)}\,dx = -\frac{11b^2\arctan\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}d} + \frac{\sqrt[3]{2}b^2\arctan\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}d} + \frac{b^2\log\left(a-bx^3\right)}{3\ 2^{2/3}a^{8/3}d} + \frac{11b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{8/3}d} - \frac{b^2\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{8/3}d} - \frac{11b^2\log(x)}{18a^{8/3}d} - \frac{2b\sqrt[3]{a+bx^3}}{9a^2dx^3} - \frac{\left(a+bx^3\right)^{4/3}}{6a^2dx^6}$$

```
[In] Int[(a + b*x^3)^(1/3)/(x^7*(a*d - b*d*x^3)),x]
```

```
[Out] (-2*b*(a + b*x^3)^(1/3))/(9*a^2*d*x^3) - (a + b*x^3)^(4/3)/(6*a^2*d*x^6) -
(11*b^2*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[
3]*a^(8/3)*d) + (2^(1/3)*b^2*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(
Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(8/3)*d) - (11*b^2*Log[x])/(18*a^(8/3)*d) + (
b^2*Log[a - b*x^3])/(3*2^(2/3)*a^(8/3)*d) + (11*b^2*Log[a^(1/3) - (a + b*x^
3)^(1/3)])/(18*a^(8/3)*d) - (b^2*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/
(2^(2/3)*a^(8/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^3(ad-bdx)}\,dx, x, x^3\right)$$

$$= -\frac{(a+bx^3)^{4/3}}{6a^2dx^6} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}\left(-\frac{4}{3}abd-\frac{2}{3}b^2dx\right)}{x^2(ad-bdx)}\,dx, x, x^3\right)}{6a^2d}$$

$$= -\frac{2b\sqrt[3]{a+bx^3}}{9a^2dx^3} - \frac{(a+bx^3)^{4/3}}{6a^2dx^6} - \frac{\text{Subst}\left(\int \frac{-\frac{22}{9}a^2b^2d^2-\frac{14}{9}ab^3d^2x}{x(a+bx)^{2/3}(ad-bdx)}\,dx, x, x^3\right)}{6a^3d^2}$$

$$= -\frac{2b\sqrt[3]{a+bx^3}}{9a^2dx^3} - \frac{(a+bx^3)^{4/3}}{6a^2dx^6} + \frac{(2b^3)\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)}\,dx, x, x^3\right)}{3a^2} + \frac{(11b^2)\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^3\right)}{27a^2d}$$

$$= -\frac{2b\sqrt[3]{a+bx^3}}{9a^2dx^3} - \frac{(a+bx^3)^{4/3}}{6a^2dx^6} - \frac{11b^2\log(x)}{18a^{8/3}d} + \frac{b^2\log\left(a-bx^3\right)}{3\ 2^{2/3}a^{8/3}d} - \frac{(11b^2)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{18a^{8/3}d} + \frac{b^2\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{8/3}d} - \frac{(11b^2)\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{18a^{7/3}d} + \frac{b^2\text{Subst}\left(\int \frac{1}{2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^{7/3}d}$$

$$= -\frac{2b\sqrt[3]{a+bx^3}}{9a^2dx^3} - \frac{(a+bx^3)^{4/3}}{6a^2dx^6} - \frac{11b^2\log(x)}{18a^{8/3}d} + \frac{b^2\log\left(a-bx^3\right)}{3\ 2^{2/3}a^{8/3}d} + \frac{11b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{8/3}d} - \frac{b^2\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{8/3}d} + \frac{(11b^2)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{9a^{8/3}d} - \frac{\left(\sqrt[3]{2}b^2\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{a^{8/3}d}$$

$$= -\frac{2b\sqrt[3]{a+bx^3}}{9a^2dx^3} - \frac{(a+bx^3)^{4/3}}{6a^2dx^6} - \frac{11b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}a^{8/3}d} + \frac{\sqrt[3]{2}b^2\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{8/3}d} - \frac{11b^2\log(x)}{18a^{8/3}d} + \frac{b^2\log(a-bx^3)}{3\ 2^{2/3}a^{8/3}d} + \frac{11b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{8/3}d} - \frac{b^2\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{8/3}d}$$

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7(ad-bdx^3)}\,dx = -\frac{9a^{5/3}\sqrt[3]{a+bx^3} + 21a^{2/3}bx^3\sqrt[3]{a+bx^3} + 22\sqrt{3}b^2x^6\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 18\sqrt[3]{2}\sqrt{3}b^2x^6\arctan\left(\frac{1+}{}\right.}{}$$

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^7*(a*d - b*d*x^3)),x]
```

```
[Out] -1/54*(9*a^(5/3)*(a + b*x^3)^(1/3) + 21*a^(2/3)*b*x^3*(a + b*x^3)^(1/3) + 2
2*Sqrt[3]*b^2*x^6*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 18*
2^(1/3)*Sqrt[3]*b^2*x^6*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sq
rt[3]] - 22*b^2*x^6*Log[-a^(1/3) + (a + b*x^3)^(1/3)] + 18*2^(1/3)*b^2*x^6*
Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)] + 11*b^2*x^6*Log[a^(2/3) + a^(1
/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] - 9*2^(1/3)*b^2*x^6*Log[2*a^(2/3
) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(a^(8/3
)*d*x^6)
```

Maple [A] (verified)

Time = 4.70 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$\frac{18\,2^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)b^{2}x^{6}-18\,2^{\frac{1}{3}}\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)b^{2}x^{6}+9\,2^{\frac{1}{3}}\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right.}{}$

```
[In] int((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/54/a^(8/3)*(18*2^(1/3)*3^(1/2)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3
))/a^(1/3)*3^(1/2))*b^2*x^6-18*2^(1/3)*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3))*
b^2*x^6+9*2^(1/3)*ln((b*x^3+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3
)*a^(2/3))*b^2*x^6-22*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2
))*3^(1/2)*b^2*x^6+22*ln((b*x^3+a)^(1/3)-a^(1/3))*b^2*x^6-11*ln((b*x^3+a)^(
2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3))*b^2*x^6-21*b*x^3*a^(2/3)*(b*x^3+a)^(1
/3)-9*(b*x^3+a)^(1/3)*a^(5/3))/x^6/d
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(ad-bdx^3\right)}\,dx = -\frac{18\sqrt{3}2^{\frac{1}{3}}a^2b^2x^6\left(-\frac{1}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}2^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}a\left(-\frac{1}{a^2}\right)^{\frac{2}{3}}+\frac{1}{3}\sqrt{3}\right)+9\cdot 2^{\frac{1}{3}}a^2b^2x^6\left(-\frac{1}{a^2}\right)^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^2\left(-\frac{1}{a}\right.\right.}{}$$

```
[In] integrate((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/54*(18*sqrt(3)*2^(1/3)*a^2*b^2*x^6*(-1/a^2)^(1/3)*arctan(1/3*sqrt(3)*2^(
2/3)*(b*x^3 + a)^(1/3)*a*(-1/a^2)^(2/3) + 1/3*sqrt(3)) + 9*2^(1/3)*a^2*b^2*
x^6*(-1/a^2)^(1/3)*log(2^(2/3)*a^2*(-1/a^2)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/
3)*a*(-1/a^2)^(1/3) + (b*x^3 + a)^(2/3)) - 18*2^(1/3)*a^2*b^2*x^6*(-1/a^2)^
(1/3)*log(2^(1/3)*a*(-1/a^2)^(1/3) + (b*x^3 + a)^(1/3)) + 22*sqrt(3)*(a^2)^
(1/6)*a*b^2*x^6*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^(1/3)*a + 2*sqrt(3)*(
b*x^3 + a)^(1/3)*(a^2)^(2/3))/a^2) + 11*(a^2)^(2/3)*b^2*x^6*log((b*x^3 + a)
^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) - 22*(a^2)^(2/3)*
b^2*x^6*log((b*x^3 + a)^(1/3)*a - (a^2)^(2/3)) + 3*(7*a^2*b*x^3 + 3*a^3)*(b
*x^3 + a)^(1/3))/(a^4*d*x^6)
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7 (ad-bdx^3)}\,dx = -\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^7+bx^{10}}\,dx}{d}$$

```
[In] integrate((b*x**3+a)**(1/3)/x**7/(-b*d*x**3+a*d),x)
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**7 + b*x**10), x)/d
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7 (ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}}{(bdx^3-ad)x^7}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^7), x)
```

Giac [A] (verification not implemented)

none

Time = 0.97 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{x^7 (ad-bdx^3)}\,dx = {} & \frac{\sqrt{3}2^{\frac{1}{3}}b^2 \arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(bx^3+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{8}{3}}d} \\ & - \frac{11\,\sqrt{3}b^2 \arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{27\,a^{\frac{8}{3}}d} \\ & + \frac{2^{\frac{1}{3}}b^2 \log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,a^{\frac{8}{3}}d} \\ & - \frac{2^{\frac{1}{3}}b^2 \log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{8}{3}}d} \\ & - \frac{11\,b^2 \log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{54\,a^{\frac{8}{3}}d} \\ & + \frac{11\,b^2 \log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{27\,a^{\frac{8}{3}}d} - \frac{7\,(bx^3+a)^{\frac{4}{3}}b^2-4\,(bx^3+a)^{\frac{1}{3}}ab^2}{18\,a^2b^2dx^6}\end{aligned}$$

```
[In] integrate((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*2^(1/3)*b^2*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(b*
x^3 + a)^(1/3))/a^(1/3))/(a^(8/3)*d) - 11/27*sqrt(3)*b^2*arctan(1/3*sqrt(3)
*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(8/3)*d) + 1/6*2^(1/3)*b^2*log
(2^(2/3)*a^(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(
a^(8/3)*d) - 1/3*2^(1/3)*b^2*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))
/(a^(8/3)*d) - 11/54*b^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3)
+ a^(2/3))/(a^(8/3)*d) + 11/27*b^2*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a
^(8/3)*d) - 1/18*(7*(b*x^3 + a)^(4/3)*b^2 - 4*(b*x^3 + a)^(1/3)*a*b^2)/(a^2
*b^2*d*x^6)
```

Mupad [B] (verification not implemented)

Time = 9.45 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.73

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(ad-bdx^3\right)}\,dx=\frac{\frac{2\,b^2\,{\left(b\,x^3+a\right)}^{1/3}}{9\,a}-\frac{7\,b^2\,{\left(b\,x^3+a\right)}^{4/3}}{18\,a^2}}{d\,{\left(b\,x^3+a\right)}^2+a^2\,d-2\,a\,d\,\left(b\,x^3+a\right)}+\frac{11\,\ln\left(b^2\,{\left(b\,x^3+a\right)}^{1/3}-a^3\,d\,{\left(\frac{b^6}{a^8\,d^3}\right)}^{1/3}\right)\,{\left(\frac{b^6}{a^8\,d^3}\right)}^{1/3}}{27}+\ln\left(b^2\,{\left(b\,x^3+a\right)}^{1/3}+2^{1/3}\,a^3\,d\,{\left(-\frac{b^6}{a^8\,d^3}\right)}^{1/3}\right)\,{\left(-\frac{2\,b^6}{27\,a^8\,d^3}\right)}^{1/3}-\ln\left(2^{1/3}\,a^3\,d\,{\left(-\frac{b^6}{a^8\,d^3}\right)}^{1/3}-2\,b^2\,\left(b\,x^3+\right.\right.$$

```
[In] int((a + b*x^3)^(1/3)/(x^7*(a*d - b*d*x^3)),x)
```

```
[Out] ((2*b^2*(a + b*x^3)^(1/3))/(9*a) - (7*b^2*(a + b*x^3)^(4/3))/(18*a^2))/(d*(
a + b*x^3)^2 + a^2*d - 2*a*d*(a + b*x^3)) + (11*log(b^2*(a + b*x^3)^(1/3) -
 a^3*d*(b^6/(a^8*d^3))^(1/3))*(b^6/(a^8*d^3))^(1/3))/27 + log(b^2*(a + b*x^
3)^(1/3) + 2^(1/3)*a^3*d*(-b^6/(a^8*d^3))^(1/3))*(-(2*b^6)/(27*a^8*d^3))^(1
/3) - log(2^(1/3)*a^3*d*(-b^6/(a^8*d^3))^(1/3) - 2*b^2*(a + b*x^3)^(1/3) +
2^(1/3)*3^(1/2)*a^3*d*(-b^6/(a^8*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(-(
2*b^6)/(27*a^8*d^3))^(1/3) + log(2*b^2*(a + b*x^3)^(1/3) - 2^(1/3)*a^3*d*(-
b^6/(a^8*d^3))^(1/3) + 2^(1/3)*3^(1/2)*a^3*d*(-b^6/(a^8*d^3))^(1/3)*1i)*((3
^(1/2)*1i)/2 - 1/2)*(-(2*b^6)/(27*a^8*d^3))^(1/3) + (11*log(2*b^2*(a + b*x^
3)^(1/3) + a^3*d*(b^6/(a^8*d^3))^(1/3) - 3^(1/2)*a^3*d*(b^6/(a^8*d^3))^(1/3
)*1i)*(3^(1/2)*1i - 1)*(b^6/(a^8*d^3))^(1/3))/54 - (11*log(2*b^2*(a + b*x^3
)^(1/3) + a^3*d*(b^6/(a^8*d^3))^(1/3) + 3^(1/2)*a^3*d*(b^6/(a^8*d^3))^(1/3)
*1i)*(3^(1/2)*1i + 1)*(b^6/(a^8*d^3))^(1/3))/54
```

3.574 $\int \frac{x^7\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 268

$$\int \frac{x^7\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=-\frac{7ax^2\sqrt[3]{a+bx^3}}{18b^2d}-\frac{x^5\sqrt[3]{a+bx^3}}{6bd}+\frac{11a^2\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{8/3}d}-\frac{\sqrt[3]{2}a^2\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{8/3}d}+\frac{a^2\log\left(ad-bdx^3\right)}{3\ 2^{2/3}b^{8/3}d}+\frac{11a^2\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{18b^{8/3}d}-\frac{a^2\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^{8/3}d}$$

```
[Out] -7/18*a*x^2*(b*x^3+a)^(1/3)/b^2/d-1/6*x^5*(b*x^3+a)^(1/3)/b/d+1/6*a^2*ln(-b
*d*x^3+a*d)*2^(1/3)/b^(8/3)/d+11/18*a^2*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(8/
3)/d-1/2*a^2*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(1/3)/b^(8/3)/d+11/27*
a^2*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(8/3)/d*3^(1/2)-1
/3*2^(1/3)*a^2*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/
b^(8/3)/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {489, 596, 598, 337, 503}

$$\int \frac{x^7\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{11a^2\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}b^{8/3}d} - \frac{\sqrt[3]{2}a^2\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{8/3}d} + \frac{a^2\log\left(ad-bdx^3\right)}{3\ 2^{2/3}b^{8/3}d} + \frac{11a^2\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{18b^{8/3}d} - \frac{a^2\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^{8/3}d} - \frac{7ax^2\sqrt[3]{a+bx^3}}{18b^2d} - \frac{x^5\sqrt[3]{a+bx^3}}{6bd}$$

```
[In] Int[(x^7*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-7*a*x^2*(a + b*x^3)^(1/3))/(18*b^2*d) - (x^5*(a + b*x^3)^(1/3))/(6*b*d) +
 (11*a^2*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*
b^(8/3)*d) - (2^(1/3)*a^2*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x^3)^(1/
3))/Sqrt[3]])/(Sqrt[3]*b^(8/3)*d) + (a^2*Log[a*d - b*d*x^3])/(3*2^(2/3)*b^(
8/3)*d) + (11*a^2*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(18*b^(8/3)*d) - (a^2
*Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)])/(2^(2/3)*b^(8/3)*d)
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
```

```
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^5\sqrt[3]{a+bx^3}}{6bd}+\frac{\int \frac{x^4(5a^2d+7abdx^3)}{(a+bx^3)^{2/3}(ad-bdx^3)}\,dx}{6bd}\\
&= -\frac{7ax^2\sqrt[3]{a+bx^3}}{18b^2d}-\frac{x^5\sqrt[3]{a+bx^3}}{6bd}+\frac{\int \frac{x(14a^3bd^2+22a^2b^2d^2x^3)}{(a+bx^3)^{2/3}(ad-bdx^3)}\,dx}{18b^3d^2}\\
&= -\frac{7ax^2\sqrt[3]{a+bx^3}}{18b^2d}-\frac{x^5\sqrt[3]{a+bx^3}}{6bd}+\frac{\int \left(-\frac{22a^2bdx}{(a+bx^3)^{2/3}}+\frac{36a^3bd^2x}{(a+bx^3)^{2/3}(ad-bdx^3)}\right)dx}{18b^3d^2}\\
&= -\frac{7ax^2\sqrt[3]{a+bx^3}}{18b^2d}-\frac{x^5\sqrt[3]{a+bx^3}}{6bd}+\frac{(2a^3)\int \frac{x}{(a+bx^3)^{2/3}(ad-bdx^3)}\,dx}{b^2}-\frac{(11a^2)\int \frac{x}{(a+bx^3)^{2/3}}\,dx}{9b^2d}\\
&= -\frac{7ax^2\sqrt[3]{a+bx^3}}{18b^2d}-\frac{x^5\sqrt[3]{a+bx^3}}{6bd}+\frac{11a^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{8/3}d}\\
&\quad -\frac{\sqrt[3]{2}a^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{8/3}d}+\frac{a^2\log(ad-bdx^3)}{3\ 2^{2/3}b^{8/3}d}\\
&\quad +\frac{11a^2\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{18b^{8/3}d}-\frac{a^2\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^{8/3}d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.22

$$\int \frac{x^7 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx =$$

$$-\frac{21ab^{2/3}x^2\sqrt[3]{a+bx^3}+9b^{5/3}x^5\sqrt[3]{a+bx^3}-22\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)+18\sqrt[3]{2}\sqrt{3}a^2\arctan\left(\frac{}{\sqrt[3]{b}x}\right.}{}$$

[In] Integrate[(x^7*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]

[Out] -1/54*(21*a*b^(2/3)*x^2*(a + b*x^3)^(1/3) + 9*b^(5/3)*x^5*(a + b*x^3)^(1/3) - 22*Sqrt[3]*a^2*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] + 18*2^(1/3)*Sqrt[3]*a^2*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))] - 22*a^2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + 18*2^(1/3)*a^2*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)] + 11*a^2*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] - 9*2^(1/3)*a^2*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(b^(8/3)*d)

Maple [A] (verified)

Time = 7.07 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$\frac{-9x^5\left(b\,x^3+a\right)^{\frac{1}{3}}b^{\frac{5}{3}}-21a\,x^2\left(b\,x^3+a\right)^{\frac{1}{3}}b^{\frac{2}{3}}+18\,2^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{3b^{\frac{1}{3}}x}\right)a^2-18\,2^{\frac{1}{3}}\ln\left(\frac{-2^{\frac{1}{3}}b^{\frac{1}{3}}x+\left(b\,x^3+a\right)^{\frac{1}{3}}}{x}\right.}{}$

[In] int(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)

[Out] 1/54/b^(8/3)*(-9*x^5*(b*x^3+a)^(1/3)*b^(5/3)-21*a*x^2*(b*x^3+a)^(1/3)*b^(2/3)+18*2^(1/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)+b^(1/3)*x)/b^(1/3)/x)*a^2-18*2^(1/3)*ln((-2^(1/3)*b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a^2+9*2^(1/3)*ln((2^(2/3)*b^(2/3)*x^2+2^(1/3)*b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a^2-22*a^2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+22*a^2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-11*a^2*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))/d

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.35

$$\int \frac{x^7 \sqrt[3]{a+bx^3}}{ad-bdx^3} dx = -\frac{18 \sqrt{3} 2^{\frac{1}{3}} a^2 b^2 \left(-\frac{1}{b^2}\right)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3} 2^{\frac{2}{3}} (bx^3+a)^{\frac{1}{3}} b \left(-\frac{1}{b^2}\right)^{\frac{2}{3}} + \sqrt{3}x}{3x}\right) - 18 \cdot 2^{\frac{1}{3}} a^2 b^2 \left(-\frac{1}{b^2}\right)^{\frac{1}{3}} \log\left(\frac{2^{\frac{1}{3}} bx \left(-\frac{1}{b^2}\right)^{\frac{1}{3}} + (bx^3+a)^{\frac{1}{3}}}{x}\right.}{}$$

```
[In] integrate(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/54*(18*sqrt(3)*2^(1/3)*a^2*b^2*(-1/b^2)^(1/3)*arctan(1/3*(sqrt(3)*2^(2/3
)*(b*x^3 + a)^(1/3)*b*(-1/b^2)^(2/3) + sqrt(3)*x)/x) - 18*2^(1/3)*a^2*b^2*(
-1/b^2)^(1/3)*log((2^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(1/3))/x) + 9*2
^(1/3)*a^2*b^2*(-1/b^2)^(1/3)*log((2^(2/3)*b^2*x^2*(-1/b^2)^(2/3) - 2^(1/3)
*(b*x^3 + a)^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(2/3))/x^2) + 22*sqrt(3
)*a^2*(b^2)^(1/6)*b*arctan(1/3*(sqrt(3)*(b^2)^(1/3)*b*x + 2*sqrt(3)*(b*x^3
+ a)^(1/3)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x)) - 22*a^2*(b^2)^(2/3)*log(-((b^
2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + 11*a^2*(b^2)^(2/3)*log(((b^2)^(1/3)*
b*x^2 + (b*x^3 + a)^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)/x^2) + 3*(3*
b^3*x^5 + 7*a*b^2*x^2)*(b*x^3 + a)^(1/3))/(b^4*d)
```

Sympy [F]

$$\int \frac{x^7 \sqrt[3]{a+bx^3}}{ad-bdx^3} dx = -\frac{\int \frac{x^7 \sqrt[3]{a+bx^3}}{-a+bx^3} dx}{d}$$

```
[In] integrate(x**7*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**7*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Maxima [F]

$$\int \frac{x^7 \sqrt[3]{a+bx^3}}{ad-bdx^3} dx = \int -\frac{(bx^3+a)^{\frac{1}{3}} x^7}{bdx^3-ad} dx$$

```
[In] integrate(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)*x^7/(b*d*x^3 - a*d), x)
```

Giac [F]

$$\int \frac{x^7 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx = \int -\frac{(bx^3+a)^{\frac{1}{3}} x^7}{bdx^3-ad} \, dx$$

```
[In] integrate(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(1/3)*x^7/(b*d*x^3 - a*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx = \int \frac{x^7 \left(b\,x^3+a\right)^{1/3}}{a\,d-b\,d\,x^3} \, dx$$

```
[In] int((x^7*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
[Out] int((x^7*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)
```

3.575 $\int \frac{x^4\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 233

$$\int \frac{x^4\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=-\frac{x^2\sqrt[3]{a+bx^3}}{3bd}+\frac{4a\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{5/3}d}-\frac{\sqrt[3]{2}a\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{5/3}d}+\frac{a\log\left(ad-bdx^3\right)}{3\ 2^{2/3}b^{5/3}d}+\frac{2a\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{3b^{5/3}d}-\frac{a\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^{5/3}d}$$

```
[Out] -1/3*x^2*(b*x^3+a)^(1/3)/b/d+1/6*a*ln(-b*d*x^3+a*d)*2^(1/3)/b^(5/3)/d+2/3*a
*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(5/3)/d-1/2*a*ln(2^(1/3)*b^(1/3)*x-(b*x^3+
a)^(1/3))*2^(1/3)/b^(5/3)/d+4/9*a*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3)
)*3^(1/2))/b^(5/3)/d*3^(1/2)-1/3*2^(1/3)*a*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*
x/(b*x^3+a)^(1/3))*3^(1/2))/b^(5/3)/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {489, 598, 337, 503}

$$\int \frac{x^4\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{4a\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{5/3}d}-\frac{\sqrt[3]{2}a\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{5/3}d}+\frac{a\log\left(ad-bdx^3\right)}{3\ 2^{2/3}b^{5/3}d}+\frac{2a\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{3b^{5/3}d}-\frac{a\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^{5/3}d}-\frac{x^2\sqrt[3]{a+bx^3}}{3bd}$$

```
[In] Int[(x^4*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/3*(x^2*(a + b*x^3)^(1/3))/(b*d) + (4*a*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*
x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(5/3)*d) - (2^(1/3)*a*ArcTan[(1 + (2*2^(
1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(5/3)*d) + (a*Log[a
*d - b*d*x^3])/(3*2^(2/3)*b^(5/3)*d) + (2*a*Log[b^(1/3)*x - (a + b*x^3)^(1/
3)])/(3*b^(5/3)*d) - (a*Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)])/(2^(2/3
)*b^(5/3)*d)
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
```

```
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x^2\sqrt[3]{a+bx^3}}{3bd}+\frac{\int \frac{x(2a^2d+4abdx^3)}{(a+bx^3)^{2/3}(ad-bdx^3)}\,dx}{3bd}\\ &= -\frac{x^2\sqrt[3]{a+bx^3}}{3bd}+\frac{\int \left(-\frac{4ax}{(a+bx^3)^{2/3}}+\frac{6a^2dx}{(a+bx^3)^{2/3}(ad-bdx^3)}\right)dx}{3bd}\\ &= -\frac{x^2\sqrt[3]{a+bx^3}}{3bd}+\frac{(2a^2)\int \frac{x}{(a+bx^3)^{2/3}(ad-bdx^3)}\,dx}{b}-\frac{(4a)\int \frac{x}{(a+bx^3)^{2/3}}\,dx}{3bd}\\ &= -\frac{x^2\sqrt[3]{a+bx^3}}{3bd}+\frac{4a\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{5/3}d}-\frac{\sqrt[3]{2}a\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{5/3}d}\\ &\quad+\frac{a\log(ad-bdx^3)}{3\ 2^{2/3}b^{5/3}d}+\frac{2a\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{3b^{5/3}d}-\frac{a\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^{5/3}d}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.26

$$\int \frac{x^4\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx =$$

$$-\frac{6b^{2/3}x^2\sqrt[3]{a+bx^3}-8\sqrt{3}a\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)+6\sqrt[3]{2}\sqrt{3}a\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right)-8a\log}{}$$

```
[In] Integrate[(x^4*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/18*(6*b^(2/3)*x^2*(a + b*x^3)^(1/3) - 8*Sqrt[3]*a*ArcTan[(Sqrt[3]*b^(1/3
)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] + 6*2^(1/3)*Sqrt[3]*a*ArcTan[(Sqrt[
3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))] - 8*a*Log[-(b^(1/3)*
```

```
x) + (a + b*x^3)^(1/3)] + 6*2^(1/3)*a*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3
)^(1/3)] + 4*a*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^
(2/3)] - 3*2^(1/3)*a*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3
) + 2^(1/3)*(a + b*x^3)^(2/3)])/(b^(5/3)*d)
```

Maple [A] (verified)

Time = 4.75 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$\frac{-6\left(b\,x^{3}+a\right)^{\frac{1}{3}} x^{2} b^{\frac{2}{3}}+6\,2^{\frac{1}{3}} \sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}} x\right)}{3 b^{\frac{1}{3}} x}\right) a-6\,2^{\frac{1}{3}} \ln\left(\frac{-2^{\frac{1}{3}} b^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right) a+3\,2^{\frac{1}{3}} \ln\left(\frac{2^{\frac{2}{3}} b^{\frac{2}{3}} x^{2}+2^{\frac{1}{3}}}{}\right.}{}$

```
[In] int(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/18/b^(5/3)*(-6*(b*x^3+a)^(1/3)*x^2*b^(2/3)+6*2^(1/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)+b^(1/3)*x)/b^(1/3)/x)*a-6*2^(1/3)*ln((-2^(1
/3)*b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a+3*2^(1/3)*ln((2^(2/3)*b^(2/3)*x^2+2^(1/
3)*b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a-8*3^(1/2)*arctan(1/3*3
^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)*a+8*ln((-b^(1/3)*x+(b*x^3+a
)^(1/3))/x)*a-4*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/
x^2)*a)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.45

$$\int \frac{x^4\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{6\,\sqrt{3}2^{\frac{1}{3}}ab^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}b\left(-\frac{1}{b^2}\right)^{\frac{2}{3}}+\sqrt{3}x}{3\,x}\right)-6\cdot 2^{\frac{1}{3}}ab^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{1}{3}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)+3}{}$$

```
[In] integrate(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/18*(6*sqrt(3)*2^(1/3)*a*b^2*(-1/b^2)^(1/3)*arctan(1/3*(sqrt(3)*2^(2/3)*(
b*x^3 + a)^(1/3)*b*(-1/b^2)^(2/3) + sqrt(3)*x)/x) - 6*2^(1/3)*a*b^2*(-1/b^2
)^(1/3)*log((2^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(1/3))/x) + 3*2^(1/3)
*a*b^2*(-1/b^2)^(1/3)*log((2^(2/3)*b^2*x^2*(-1/b^2)^(2/3) - 2^(1/3)*(b*x^3
+ a)^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(2/3))/x^2) + 6*(b*x^3 + a)^(1/
3)*b^2*x^2 + 8*sqrt(3)*a*(b^2)^(1/6)*b*arctan(1/3*(sqrt(3)*(b^2)^(1/3)*b*x
```

```
+ 2*sqrt(3)*(b*x^3 + a)^(1/3)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x)) - 8*a*(b^2)
^(2/3)*log(-((b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + 4*a*(b^2)^(2/3)*log(
((b^2)^(1/3)*b*x^2 + (b*x^3 + a)^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)
/x^2))/(b^3*d)
```

Sympy [F]

$$\int \frac{x^4 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^4 \sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

```
[In] integrate(x**4*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
[Out] -Integral(x**4*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Maxima [F]

$$\int \frac{x^4 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}x^4}{bdx^3-ad}\,dx$$

```
[In] integrate(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(1/3)*x^4/(b*d*x^3 - a*d), x)
```

Giac [F]

$$\int \frac{x^4 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}x^4}{bdx^3-ad}\,dx$$

```
[In] integrate(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(1/3)*x^4/(b*d*x^3 - a*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int \frac{x^4\,{\left(b\,x^3+a\right)}^{1/3}}{a\,d-b\,d\,x^3}\,dx$$

```
[In] int((x^4*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
[Out] int((x^4*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)
```

3.576 $\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 201

$$\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}d}-\frac{\sqrt[3]{2}\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}d}+\frac{\log\left(ad-bdx^3\right)}{3\ 2^{2/3}b^{2/3}d}+\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}d}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^{2/3}d}$$

```
[Out] 1/6*ln(-b*d*x^3+a*d)*2^(1/3)/b^(2/3)/d+1/2*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^
(2/3)/d-1/2*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(1/3)/b^(2/3)/d+1/3*arc
tan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)/d*3^(1/2)-1/3*2^(1
/3)*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)/d*3
^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {495, 337, 503}

$$\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}d}-\frac{\sqrt[3]{2}\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}d}+\frac{\log\left(ad-bdx^3\right)}{3\ 2^{2/3}b^{2/3}d}+\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}d}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^{2/3}d}$$

```
[In] Int[(x*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/3)*d) -
 (2^(1/3)*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(S
qrt[3]*b^(2/3)*d) + Log[a*d - b*d*x^3]/(3*2^(2/3)*b^(2/3)*d) + Log[b^(1/3)*
x - (a + b*x^3)^(1/3)]/(2*b^(2/3)*d) - Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^
(1/3)]/(2^(2/3)*b^(2/3)*d)
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rubi steps

$$\text{integral} = (2a)\int \frac{x}{(a+bx^3)^{2/3}(ad-bdx^3)}\,dx - \frac{\int \frac{x}{(a+bx^3)^{2/3}}\,dx}{d}$$

$$= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}d} - \frac{\sqrt[3]{2}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}d} + \frac{\log\left(ad-bdx^3\right)}{3\ 2^{2/3}b^{2/3}d} + \frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}d} - \frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^{2/3}d}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.32

$$\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-2\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right)+2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)-}{}$$

```
[In] Integrate[(x*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] -
2*2^(1/3)*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^
3)^(1/3))] + 2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] - 2*2^(1/3)*Log[-2*b^(
1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)] - Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^
3)^(1/3) + (a + b*x^3)^(2/3)] + 2^(1/3)*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)
*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(6*b^(2/3)*d)
```

Maple [A] (verified)

Time = 4.68 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.09

method	result
pseudoelliptic	$\frac{2\,2^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{3b^{\frac{1}{3}}x}\right)-2\,2^{\frac{1}{3}}\ln\left(\frac{-2^{\frac{1}{3}}b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)+2^{\frac{1}{3}}\ln\left(\frac{2^{\frac{2}{3}}b^{\frac{2}{3}}x^{2}+2^{\frac{1}{3}}b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right.}{x^{2}}\right.}{6d\,b^{\frac{2}{3}}}$

```
[In] int(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(2*2^(1/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)+b^(1/3)*
x)/b^(1/3)/x)-2*2^(1/3)*ln((-2^(1/3)*b^(1/3)*x+(b*x^3+a)^(1/3))/x)+2^(1/3)*
ln((2^(2/3)*b^(2/3)*x^2+2^(1/3)*b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/
x^2)-2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+
2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3
)*x+(b*x^3+a)^(2/3))/x^2))/d/b^(2/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(156) = 312$.

Time = 0.27 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.56

$$\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{2\sqrt{3}2^{\frac{1}{3}}b^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}b\left(-\frac{1}{b^2}\right)^{\frac{2}{3}}+\sqrt{3}x}{3x}\right)-2\cdot 2^{\frac{1}{3}}b^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{1}{3}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}}{x}\right)+2^{\frac{1}{3}}b}{}$$

```
[In] integrate(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*2^(1/3)*b^2*(-1/b^2)^(1/3)*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x
^3 + a)^(1/3)*b*(-1/b^2)^(2/3) + sqrt(3)*x)/x) - 2*2^(1/3)*b^2*(-1/b^2)^(1/
3)*log((2^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(1/3))/x) + 2^(1/3)*b^2*(-
1/b^2)^(1/3)*log((2^(2/3)*b^2*x^2*(-1/b^2)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3
)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(2/3))/x^2) + 2*sqrt(3)*(b^2)^(1/6)*b*ar
ctan(1/3*(sqrt(3)*(b^2)^(1/3)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(b^2)^(2/3)
)*(b^2)^(1/6)/(b^2*x)) - 2*(b^2)^(2/3)*log(-((b^2)^(2/3)*x - (b*x^3 + a)^(1
/3)*b)/x) + (b^2)^(2/3)*log(((b^2)^(1/3)*b*x^2 + (b*x^3 + a)^(1/3)*(b^2)^(2
/3)*x + (b*x^3 + a)^(2/3)*b)/x^2))/(b^2*d)
```

Sympy [F]

$$\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

```
[In] integrate(x*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Maxima [F]

$$\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}x}{bdx^3-ad}\,dx$$

```
[In] integrate(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)*x/(b*d*x^3 - a*d), x)
```

Giac [F]

$$\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}x}{bdx^3-ad}\,dx$$

```
[In] integrate(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(1/3)*x/(b*d*x^3 - a*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int \frac{x\,(b\,x^3+a)^{1/3}}{a\,d-b\,d\,x^3}\,dx$$

```
[In] int((x*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
[Out] int((x*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)
```

3.577 $\int \frac{\sqrt[3]{a+bx^3}}{x^2(ad-bdx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 156

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2(ad-bdx^3)}\,dx = -\frac{\sqrt[3]{a+bx^3}}{adx} - \frac{\sqrt[3]{2}\sqrt[3]{b}\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}ad} + \frac{\sqrt[3]{b}\log(ad-bdx^3)}{3\ 2^{2/3}ad} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}ad}$$

```
[Out] -(b*x^3+a)^(1/3)/a/d/x+1/6*b^(1/3)*ln(-b*d*x^3+a*d)*2^(1/3)/a/d-1/2*b^(1/3)
*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(1/3)/a/d-1/3*2^(1/3)*b^(1/3)*arct
an(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/a/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {486, 12, 503}

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2(ad-bdx^3)}\,dx = -\frac{\sqrt[3]{2}\sqrt[3]{b}\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}ad} - \frac{\sqrt[3]{a+bx^3}}{adx} + \frac{\sqrt[3]{b}\log(ad-bdx^3)}{3\ 2^{2/3}ad} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}ad}$$

```
[In] Int[(a + b*x^3)^(1/3)/(x^2*(a*d - b*d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(1/3)/(a*d*x)) - (2^(1/3)*b^(1/3)*ArcTan[(1 + (2*2^(1/3)*b^(1
/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*a*d) + (b^(1/3)*Log[a*d - b*d*
x^3])/(3*2^(2/3)*a*d) - (b^(1/3)*Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)]
)/(2^(2/3)*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt[3]{a+bx^3}}{adx}+\frac{\int \frac{2abdx}{(a+bx^3)^{2/3}(ad-bdx^3)}\,dx}{ad}\\
&= -\frac{\sqrt[3]{a+bx^3}}{adx}+(2b)\int \frac{x}{(a+bx^3)^{2/3}\,(ad-bdx^3)}\,dx\\
&= -\frac{\sqrt[3]{a+bx^3}}{adx}-\frac{\sqrt[3]{2}\sqrt[3]{b}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}ad}\\
&\quad+\frac{\sqrt[3]{b}\log\left(ad-bdx^3\right)}{3\ 2^{2/3}ad}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}ad}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2(ad-bdx^3)}\,dx = -\frac{6\sqrt[3]{a+bx^3}+2\sqrt[3]{2}\sqrt{3}\sqrt[3]{b}x\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right)+2\sqrt[3]{2}\sqrt[3]{b}x\log\left(-2\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}\right)-\sqrt[3]{2}}{6adx}$$

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^2*(a*d - b*d*x^3)),x]
```

```
[Out] -1/6*(6*(a + b*x^3)^(1/3) + 2*2^(1/3)*Sqrt[3]*b^(1/3)*x*ArcTan[(Sqrt[3]*b^(
1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))] + 2*2^(1/3)*b^(1/3)*x*Log[
-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)] - 2^(1/3)*b^(1/3)*x*Log[2*b^(2/3)
*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)]])/(a
*d*x)
```

Maple [A] (verified)

Time = 4.80 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$\frac{2b^{\frac{1}{3}}2^{\frac{1}{3}}\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{3b^{\frac{1}{3}}x}\right)x-2b^{\frac{1}{3}}2^{\frac{1}{3}}\ln\left(\frac{-2^{\frac{1}{3}}b^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)x+b^{\frac{1}{3}}2^{\frac{1}{3}}\ln\left(\frac{2^{\frac{2}{3}}b^{\frac{2}{3}}x^2+2^{\frac{1}{3}}b^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}x}{x^2}\right.}{6adx}$

```
[In] int((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(2*b^(1/3)*2^(1/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)+
b^(1/3)*x)/b^(1/3)/x)*x-2*b^(1/3)*2^(1/3)*ln((-2^(1/3)*b^(1/3)*x+(b*x^3+a)^
(1/3))/x)*x+b^(1/3)*2^(1/3)*ln((2^(2/3)*b^(2/3)*x^2+2^(1/3)*b^(1/3)*(b*x^3+
a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*x-6*(b*x^3+a)^(1/3))/a/d/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(125) = 250.

Time = 84.46 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.53

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2(ad-bdx^3)}\,dx = -\frac{2\sqrt{3}2^{\frac{1}{3}}(-b)^{\frac{1}{3}}x\arctan\left(\frac{6\sqrt{3}2^{\frac{2}{3}}(19b^2x^8+16abx^5+a^2x^2)(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}+6\sqrt{3}2^{\frac{1}{3}}(5b^2x^7-4abx^4-a^2x)(bx^3+a)^{\frac{2}{3}}(-b)^{\frac{1}{3}}+\sqrt{3}(71b^3}{3(109b^3x^9+105ab^2x^6+3a^2bx^3-a^3)}\right.}{}$$

```
[In] integrate((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/18*(2*sqrt(3)*2^(1/3)*(-b)^(1/3)*x*arctan(1/3*(6*sqrt(3)*2^(2/3)*(19*b^2
*x^8 + 16*a*b*x^5 + a^2*x^2)*(b*x^3 + a)^(1/3)*(-b)^(2/3) + 6*sqrt(3)*2^(1/
3)*(5*b^2*x^7 - 4*a*b*x^4 - a^2*x)*(b*x^3 + a)^(2/3)*(-b)^(1/3) + sqrt(3)*(
71*b^3*x^9 + 111*a*b^2*x^6 + 33*a^2*b*x^3 + a^3))/(109*b^3*x^9 + 105*a*b^2*
x^6 + 3*a^2*b*x^3 - a^3)) - 2*2^(1/3)*(-b)^(1/3)*x*log(-(6*2^(1/3)*(b*x^3 +
 a)^(1/3)*(-b)^(1/3)*b*x^2 + 6*(b*x^3 + a)^(2/3)*b*x + 2^(2/3)*(b*x^3 - a)*
(-b)^(2/3))/(b*x^3 - a)) + 2^(1/3)*(-b)^(1/3)*x*log((3*2^(2/3)*(5*b*x^4 + a
*x)*(b*x^3 + a)^(2/3)*(-b)^(2/3) - 2^(1/3)*(19*b^2*x^6 + 16*a*b*x^3 + a^2)*
(-b)^(1/3) + 12*(2*b^2*x^5 + a*b*x^2)*(b*x^3 + a)^(1/3))/(b^2*x^6 - 2*a*b*x
^3 + a^2)) + 18*(b*x^3 + a)^(1/3))/(a*d*x)
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(ad-bdx^3\right)}\,dx = -\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^2+bx^5}\,dx}{d}$$

```
[In] integrate((b*x**3+a)**(1/3)/x**2/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**2 + b*x**5), x)/d
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(ad-bdx^3\right)}\,dx = \int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{(bdx^3-ad)x^2}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(ad-bdx^3\right)}\,dx = \int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{(bdx^3-ad)x^2}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\,(ad-bdx^3)}\,dx = \int \frac{\left(b\,x^3+a\right)^{1/3}}{x^2\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

```
[In] int((a + b*x^3)^(1/3)/(x^2*(a*d - b*d*x^3)),x)
[Out] int((a + b*x^3)^(1/3)/(x^2*(a*d - b*d*x^3)), x)
```

3.578 $\int \frac{\sqrt[3]{a+bx^3}}{x^5(ad-bdx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 183

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5(ad-bdx^3)}\,dx = -\frac{\sqrt[3]{a+bx^3}}{4adx^4}-\frac{5b\sqrt[3]{a+bx^3}}{4a^2dx}-\frac{\sqrt[3]{2}b^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^2d}+\frac{b^{4/3}\log(ad-bdx^3)}{3\ 2^{2/3}a^2d}-\frac{b^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^2d}$$

```
[Out] -1/4*(b*x^3+a)^(1/3)/a/d/x^4-5/4*b*(b*x^3+a)^(1/3)/a^2/d/x+1/6*b^(4/3)*ln(-
b*d*x^3+a*d)*2^(1/3)/a^2/d-1/2*b^(4/3)*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3)
)*2^(1/3)/a^2/d-1/3*2^(1/3)*b^(4/3)*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^
3+a)^(1/3))*3^(1/2))/a^2/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {486, 597, 12, 503}

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5(ad-bdx^3)}\,dx = -\frac{\sqrt[3]{2}b^{4/3}\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}a^2d}+\frac{b^{4/3}\log(ad-bdx^3)}{3\ 2^{2/3}a^2d}-\frac{b^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^2d}-\frac{5b\sqrt[3]{a+bx^3}}{4a^2dx}-\frac{\sqrt[3]{a+bx^3}}{4adx^4}$$

```
[In] Int[(a + b*x^3)^(1/3)/(x^5*(a*d - b*d*x^3)),x]
```

```
[Out] -1/4*(a + b*x^3)^(1/3)/(a*d*x^4) - (5*b*(a + b*x^3)^(1/3))/(4*a^2*d*x) - (2
^(1/3)*b^(4/3)*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]
])/(Sqrt[3]*a^2*d) + (b^(4/3)*Log[a*d - b*d*x^3])/(3*2^(2/3)*a^2*d) - (b^(4
/3)*Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)])/(2^(2/3)*a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\sqrt[3]{a+bx^3}}{4adx^4}+\frac{\int \frac{5abd+3b^2dx^3}{x^2(a+bx^3)^{2/3}(ad-bdx^3)}\,dx}{4ad}\\ &= -\frac{\sqrt[3]{a+bx^3}}{4adx^4}-\frac{5b\sqrt[3]{a+bx^3}}{4a^2dx}-\frac{\int -\frac{8a^2b^2d^2x}{(a+bx^3)^{2/3}(ad-bdx^3)}\,dx}{4a^3d^2} \end{aligned}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{4adx^4} - \frac{5b\sqrt[3]{a+bx^3}}{4a^2dx} + \frac{(2b^2)\int \frac{x}{(a+bx^3)^{2/3}(ad-bdx^3)}\,dx}{a}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{4adx^4} - \frac{5b\sqrt[3]{a+bx^3}}{4a^2dx} - \frac{\sqrt[3]{2}b^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^2d} + \frac{b^{4/3}\log\left(ad-bdx^3\right)}{3\ 2^{2/3}a^2d} - \frac{b^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x - \sqrt[3]{a+bx^3}\right)}{2^{2/3}a^2d}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(ad-bdx^3\right)}\,dx = -\frac{\sqrt[3]{a+bx^3}(a+5bx^3)}{4a^2dx^4} - \frac{\sqrt[3]{2}b^{4/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}a^2d} - \frac{\sqrt[3]{2}b^{4/3}\log\left(-2\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}\right)}{3a^2d} + \frac{b^{4/3}\log\left(2b^{2/3}x^2+2^{2/3}\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\sqrt[3]{2}(a+bx^3)^{2/3}\right)}{3\ 2^{2/3}a^2d}$$

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^5*(a*d - b*d*x^3)),x]
```

```
[Out] -1/4*((a + b*x^3)^(1/3)*(a + 5*b*x^3))/(a^2*d*x^4) - (2^(1/3)*b^(4/3)*ArcTa
n[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))])/(Sqrt[3]*a^
2*d) - (2^(1/3)*b^(4/3)*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)])/(3*a
^2*d) + (b^(4/3)*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) +
2^(1/3)*(a + b*x^3)^(2/3)])/(3*2^(2/3)*a^2*d)
```

Maple [A] (verified)

Time = 4.69 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.81

method	result
pseudoelliptic	$\frac{\left(-15b\,x^{3}-3a\right)\left(b\,x^{3}+a\right)^{\frac{1}{3}}-4\,2^{\frac{1}{3}}b^{\frac{4}{3}}x^{4}\left(-\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{3b^{\frac{1}{3}}x}\right)\sqrt{3}+\ln\left(\frac{-2^{\frac{1}{3}}b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)-\frac{\ln\left(\frac{2^{\frac{2}{3}}b^{\frac{2}{3}}x^{2}}{}\right.}{}\right.}{12x^{4}a^{2}d}$

```
[In] int((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*((-15*b*x^3-3*a)*(b*x^3+a)^(1/3)-4*2^(1/3)*b^(4/3)*x^4*(-arctan(1/3*3^
(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)+b^(1/3)*x)/b^(1/3)/x)*3^(1/2)+ln((-2^(1/3)*b
^(1/3)*x+(b*x^3+a)^(1/3))/x)-1/2*ln((2^(2/3)*b^(2/3)*x^2+2^(1/3)*b^(1/3)*(b
*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)))/x^4/a^2/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(ad-bdx^3\right)}\,dx=\text{Timed out}$$

```
[In] integrate((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(ad-bdx^3\right)}\,dx=-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^5+bx^8}\,dx}{d}$$

```
[In] integrate((b*x**3+a)**(1/3)/x**5/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**5 + b*x**8), x)/d
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(ad-bdx^3\right)}\,dx=\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^5}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^5), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(ad-bdx^3\right)}\,dx = \int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^5}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(ad-bdx^3\right)}\,dx = \int \frac{\left(b\,x^3+a\right)^{1/3}}{x^5\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

```
[In] int((a + b*x^3)^(1/3)/(x^5*(a*d - b*d*x^3)),x)
[Out] int((a + b*x^3)^(1/3)/(x^5*(a*d - b*d*x^3)), x)
```

3.579 $\int \frac{\sqrt[3]{a+bx^3}}{x^8(ad-bdx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 210

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8(ad-bdx^3)}\,dx = -\frac{\sqrt[3]{a+bx^3}}{7adx^7}-\frac{2b\sqrt[3]{a+bx^3}}{7a^2dx^4}-\frac{8b^2\sqrt[3]{a+bx^3}}{7a^3dx} - \frac{\sqrt[3]{2}b^{7/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^3d} + \frac{b^{7/3}\log(ad-bdx^3)}{3\,2^{2/3}a^3d}-\frac{b^{7/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^3d}$$

```
[Out] -1/7*(b*x^3+a)^(1/3)/a/d/x^7-2/7*b*(b*x^3+a)^(1/3)/a^2/d/x^4-8/7*b^2*(b*x^3
+a)^(1/3)/a^3/d/x+1/6*b^(7/3)*ln(-b*d*x^3+a*d)*2^(1/3)/a^3/d-1/2*b^(7/3)*ln
(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(1/3)/a^3/d-1/3*2^(1/3)*b^(7/3)*arcta
n(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/a^3/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used

$= \{486, 597, 12, 503\}$

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8(ad-bdx^3)}\,dx = -\frac{\sqrt[3]{2}b^{7/3}\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}a^3d} + \frac{b^{7/3}\log\left(ad-bdx^3\right)}{3\ 2^{2/3}a^3d} - \frac{b^{7/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^3d} - \frac{8b^2\sqrt[3]{a+bx^3}}{7a^3dx} - \frac{2b\sqrt[3]{a+bx^3}}{7a^2dx^4} - \frac{\sqrt[3]{a+bx^3}}{7adx^7}$$

```
[In] Int[(a + b*x^3)^(1/3)/(x^8*(a*d - b*d*x^3)),x]
```

```
[Out] -1/7*(a + b*x^3)^(1/3)/(a*d*x^7) - (2*b*(a + b*x^3)^(1/3))/(7*a^2*d*x^4) -
(8*b^2*(a + b*x^3)^(1/3))/(7*a^3*d*x) - (2^(1/3)*b^(7/3)*ArcTan[(1 + (2*2^(
1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*a^3*d) + (b^(7/3)*Log
[a*d - b*d*x^3])/(3*2^(2/3)*a^3*d) - (b^(7/3)*Log[2^(1/3)*b^(1/3)*x - (a +
b*x^3)^(1/3)])/(2^(2/3)*a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
```

```
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt[3]{a+bx^3}}{7adx^7}+\frac{\int \frac{8abd+6b^2dx^3}{x^5(a+bx^3)^{2/3}(ad-bdx^3)}\,dx}{7ad}\\
&= -\frac{\sqrt[3]{a+bx^3}}{7adx^7}-\frac{2b\sqrt[3]{a+bx^3}}{7a^2dx^4}-\frac{\int \frac{-32a^2b^2d^2-24ab^3d^2x^3}{x^2(a+bx^3)^{2/3}(ad-bdx^3)}\,dx}{28a^3d^2}\\
&= -\frac{\sqrt[3]{a+bx^3}}{7adx^7}-\frac{2b\sqrt[3]{a+bx^3}}{7a^2dx^4}-\frac{8b^2\sqrt[3]{a+bx^3}}{7a^3dx}+\frac{\int \frac{56a^3b^3d^3x}{(a+bx^3)^{2/3}(ad-bdx^3)}\,dx}{28a^5d^3}\\
&= -\frac{\sqrt[3]{a+bx^3}}{7adx^7}-\frac{2b\sqrt[3]{a+bx^3}}{7a^2dx^4}-\frac{8b^2\sqrt[3]{a+bx^3}}{7a^3dx}+\frac{(2b^3)\int \frac{x}{(a+bx^3)^{2/3}(ad-bdx^3)}\,dx}{a^2}\\
&= -\frac{\sqrt[3]{a+bx^3}}{7adx^7}-\frac{2b\sqrt[3]{a+bx^3}}{7a^2dx^4}-\frac{8b^2\sqrt[3]{a+bx^3}}{7a^3dx}-\frac{\sqrt[3]{2}b^{7/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^3d}\\
&\quad+\frac{b^{7/3}\log\left(ad-bdx^3\right)}{3\ 2^{2/3}a^3d}-\frac{b^{7/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(ad-bdx^3\right)}\,dx = -\frac{\frac{6\sqrt[3]{a+bx^3}\left(a^2+2abx^3+8b^2x^6\right)}{x^7}+14\sqrt[3]{2}\sqrt{3}b^{7/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right)+14\sqrt[3]{2}b^{7/3}\log\left(-2\sqrt[3]{b}x+2^{2/3}\sqrt[3]{\cdots}\right.}{42a^3d}$$

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^8*(a*d - b*d*x^3)),x]
```

```
[Out] -1/42*((6*(a + b*x^3)^(1/3)*(a^2 + 2*a*b*x^3 + 8*b^2*x^6))/x^7 + 14*2^(1/3)
*Sqrt[3]*b^(7/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3
)^(1/3))] + 14*2^(1/3)*b^(7/3)*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)
] - 7*2^(1/3)*b^(7/3)*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/
3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(a^3*d)
```

Maple [A] (verified)

Time = 4.88 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{\left(-48b^{2}x^{6}-12ab\,x^{3}-6a^{2}\right)\left(b\,x^{3}+a\right)^{\frac{1}{3}}+7\,2^{\frac{1}{3}}b^{\frac{7}{3}}x^{7}\left(2\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{3b^{\frac{1}{3}}x}\right)\sqrt{3}+\ln\left(\frac{2^{\frac{2}{3}}b^{\frac{2}{3}}x^{2}+2^{\frac{1}{3}}b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x^{2}}\right.\right.}{42x^{7}a^{3}d}$

[In] int((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)

[Out] 1/42*((-48*b^2*x^6-12*a*b*x^3-6*a^2)*(b*x^3+a)^(1/3)+7*2^(1/3)*b^(7/3)*x^7*(2*arctan(1/3*3^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)+b^(1/3)*x)/b^(1/3)/x)*3^(1/2)+ln((2^(2/3)*b^(2/3)*x^2+2^(1/3)*b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((-2^(1/3)*b^(1/3)*x+(b*x^3+a)^(1/3))/x)))/x^7/a^3/d

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(ad-bdx^3\right)}\,dx=\text{Timed out}$$

[In] integrate((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(ad-bdx^3\right)}\,dx=-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^8+bx^{11}}\,dx}{d}$$

[In] integrate((b*x**3+a)**(1/3)/x**8/(-b*d*x**3+a*d),x)

[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**8 + b*x**11), x)/d

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\,(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}}{(bdx^3-ad)x^8}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^8), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\,(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}}{(bdx^3-ad)x^8}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\,(ad-bdx^3)}\,dx = \int \frac{(b\,x^3+a)^{1/3}}{x^8\,(a\,d-b\,d\,x^3)}\,dx$$

```
[In] int((a + b*x^3)^(1/3)/(x^8*(a*d - b*d*x^3)),x)
[Out] int((a + b*x^3)^(1/3)/(x^8*(a*d - b*d*x^3)), x)
```

3.580 $\int \frac{\sqrt[3]{a+bx^3}}{x^{11}(ad-bdx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 237

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(ad-bdx^3\right)}\,dx = -\frac{\sqrt[3]{a+bx^3}}{10adx^{10}}-\frac{11b\sqrt[3]{a+bx^3}}{70a^2dx^7}-\frac{37b^2\sqrt[3]{a+bx^3}}{140a^3dx^4}-\frac{169b^3\sqrt[3]{a+bx^3}}{140a^4dx}-\frac{\sqrt[3]{2}b^{10/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^4d}+\frac{b^{10/3}\log\left(ad-bdx^3\right)}{3\ 2^{2/3}a^4d}-\frac{b^{10/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^4d}$$

```
[Out] -1/10*(b*x^3+a)^(1/3)/a/d/x^10-11/70*b*(b*x^3+a)^(1/3)/a^2/d/x^7-37/140*b^2
*(b*x^3+a)^(1/3)/a^3/d/x^4-169/140*b^3*(b*x^3+a)^(1/3)/a^4/d/x+1/6*b^(10/3)
*ln(-b*d*x^3+a*d)*2^(1/3)/a^4/d-1/2*b^(10/3)*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)
^(1/3))*2^(1/3)/a^4/d-1/3*2^(1/3)*b^(10/3)*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*
x/(b*x^3+a)^(1/3))*3^(1/2))/a^4/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used

$= \{486, 597, 12, 503\}$

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(ad-bdx^3\right)}\,dx = -\frac{\sqrt[3]{2}b^{10/3}\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}a^4d} + \frac{b^{10/3}\log\left(ad-bdx^3\right)}{3\ 2^{2/3}a^4d} - \frac{b^{10/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^4d} - \frac{169b^3\sqrt[3]{a+bx^3}}{140a^4dx} - \frac{37b^2\sqrt[3]{a+bx^3}}{140a^3dx^4} - \frac{11b\sqrt[3]{a+bx^3}}{70a^2dx^7} - \frac{\sqrt[3]{a+bx^3}}{10adx^{10}}$$

```
[In] Int[(a + b*x^3)^(1/3)/(x^11*(a*d - b*d*x^3)),x]
```

```
[Out] -1/10*(a + b*x^3)^(1/3)/(a*d*x^10) - (11*b*(a + b*x^3)^(1/3))/(70*a^2*d*x^7
) - (37*b^2*(a + b*x^3)^(1/3))/(140*a^3*d*x^4) - (169*b^3*(a + b*x^3)^(1/3)
)/(140*a^4*d*x) - (2^(1/3)*b^(10/3)*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a +
b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*a^4*d) + (b^(10/3)*Log[a*d - b*d*x^3])/(3*
2^(2/3)*a^4*d) - (b^(10/3)*Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)])/(2^(
2/3)*a^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt[3]{a+bx^3}}{10adx^{10}}+\frac{\int \frac{11abd+9b^2dx^3}{x^8(a+bx^3)^{2/3}(ad-bdx^3)}\,dx}{10ad}\\
&= -\frac{\sqrt[3]{a+bx^3}}{10adx^{10}}-\frac{11b\sqrt[3]{a+bx^3}}{70a^2dx^7}-\frac{\int \frac{-74a^2b^2d^2-66ab^3d^2x^3}{x^5(a+bx^3)^{2/3}(ad-bdx^3)}\,dx}{70a^3d^2}\\
&= -\frac{\sqrt[3]{a+bx^3}}{10adx^{10}}-\frac{11b\sqrt[3]{a+bx^3}}{70a^2dx^7}-\frac{37b^2\sqrt[3]{a+bx^3}}{140a^3dx^4}+\frac{\int \frac{338a^3b^3d^3+222a^2b^4d^3x^3}{x^2(a+bx^3)^{2/3}(ad-bdx^3)}\,dx}{280a^5d^3}\\
&= -\frac{\sqrt[3]{a+bx^3}}{10adx^{10}}-\frac{11b\sqrt[3]{a+bx^3}}{70a^2dx^7}-\frac{37b^2\sqrt[3]{a+bx^3}}{140a^3dx^4}-\frac{169b^3\sqrt[3]{a+bx^3}}{140a^4dx}-\frac{\int -\frac{560a^4b^4d^4x}{(a+bx^3)^{2/3}(ad-bdx^3)}\,dx}{280a^7d^4}\\
&= -\frac{\sqrt[3]{a+bx^3}}{10adx^{10}}-\frac{11b\sqrt[3]{a+bx^3}}{70a^2dx^7}-\frac{37b^2\sqrt[3]{a+bx^3}}{140a^3dx^4}-\frac{169b^3\sqrt[3]{a+bx^3}}{140a^4dx}+\frac{(2b^4)\int \frac{x}{(a+bx^3)^{2/3}(ad-bdx^3)}\,dx}{a^3}\\
&= -\frac{\sqrt[3]{a+bx^3}}{10adx^{10}}-\frac{11b\sqrt[3]{a+bx^3}}{70a^2dx^7}-\frac{37b^2\sqrt[3]{a+bx^3}}{140a^3dx^4}\\
&\quad -\frac{169b^3\sqrt[3]{a+bx^3}}{140a^4dx}-\frac{\sqrt[3]{2}b^{10/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^4d}\\
&\quad +\frac{b^{10/3}\log\left(ad-bdx^3\right)}{3\ 2^{2/3}a^4d}-\frac{b^{10/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(ad-bdx^3\right)}\,dx =$$
$$-\frac{\frac{3\sqrt[3]{a+bx^3}(14a^3+22a^2bx^3+37ab^2x^6+169b^3x^9)}{x^{10}}+140\sqrt[3]{2}\sqrt{3}b^{10/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right)+140\sqrt[3]{2}b^{10/3}\log}{420a^4d}$$

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^11*(a*d - b*d*x^3)),x]
```

```
[Out] -1/420*((3*(a + b*x^3)^(1/3)*(14*a^3 + 22*a^2*b*x^3 + 37*a*b^2*x^6 + 169*b^
3*x^9))/x^10 + 140*2^(1/3)*Sqrt[3]*b^(10/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(
1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))] + 140*2^(1/3)*b^(10/3)*Log[-2*b^(1/3)*
x + 2^(2/3)*(a + b*x^3)^(1/3)] - 70*2^(1/3)*b^(10/3)*Log[2*b^(2/3)*x^2 + 2^
(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(a^4*d)
```

Maple [A] (verified)

Time = 4.93 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$\frac{\left(-507 b^{3} x^{9}-111 a\,b^{2} x^{6}-66 a^{2} b\,x^{3}-42 a^{3}\right) \left(b\,x^{3}+a\right)^{\frac{1}{3}}+70\,2^{\frac{1}{3}} b^{\frac{10}{3}} x^{10} \left(2 \arctan\left(\frac{\sqrt{3}\,\left(2^{\frac{2}{3}} \left(b\,x^{3}+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}} x\right)}{3 b^{\frac{1}{3}} x}\right) \sqrt{3}+\ln\left(\frac{2^{\frac{2}{3}} b^{\frac{2}{3}} x^{2}+}{}\right.\right.}{420 x^{10} a^{4} d}$

```
[In] int((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/420*((-507*b^3*x^9-111*a*b^2*x^6-66*a^2*b*x^3-42*a^3)*(b*x^3+a)^(1/3)+70*
2^(1/3)*b^(10/3)*x^10*(2*arctan(1/3*3^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)+b^(1/3
)*x)/b^(1/3)/x)*3^(1/2)+ln((2^(2/3)*b^(2/3)*x^2+2^(1/3)*b^(1/3)*(b*x^3+a)^(
1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((-2^(1/3)*b^(1/3)*x+(b*x^3+a)^(1/3))/x)))
/x^10/a^4/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(ad-bdx^3\right)}\,dx=\text{Timed out}$$

```
[In] integrate((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(ad-bdx^3\right)}\,dx=-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^{11}+bx^{14}}\,dx}{d}$$

```
[In] integrate((b*x**3+a)**(1/3)/x**11/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**11 + b*x**14), x)/d
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(ad-bdx^3\right)}\,dx = \int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^{11}}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^11), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(ad-bdx^3\right)}\,dx = \int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^{11}}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^11), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(ad-bdx^3\right)}\,dx = \int \frac{\left(b\,x^3+a\right)^{1/3}}{x^{11}\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

```
[In] int((a + b*x^3)^(1/3)/(x^11*(a*d - b*d*x^3)),x)
[Out] int((a + b*x^3)^(1/3)/(x^11*(a*d - b*d*x^3)), x)
```

3.581 $\int \frac{x^6 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 521

$$\int \frac{x^6\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{3ax\sqrt[3]{a+bx^3}}{5b^2d}-\frac{x^4\sqrt[3]{a+bx^3}}{5bd}-\frac{\sqrt[3]{2}a^{5/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{7/3}d}$$
$$-\frac{a^{5/3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}b^{7/3}d}$$
$$-\frac{2a^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5b^2d\left(a+bx^3\right)^{2/3}}$$
$$-\frac{a^{5/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{7/3}d}$$
$$+\frac{a^{5/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{7/3}d}$$
$$-\frac{\sqrt[3]{2}a^{5/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3b^{7/3}d}$$
$$+\frac{a^{5/3}\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}b^{7/3}d}$$

```
[Out] -3/5*a*x*(b*x^3+a)^(1/3)/b^2/d-1/5*x^4*(b*x^3+a)^(1/3)/b/d-2/5*a^2*x*(1+b*x
^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/b^2/d/(b*x^3+a)^(2/3)-1/6*
a^(5/3)*ln(2^(2/3)+(-a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/b^(7/3)/d+
1/6*a^(5/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(
1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/b^(7/3)/d-1/3*2^(1/3)*a^(5/3)*ln(1
+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/b^(7/3)/d+1/12*a^(5/3)*ln(2*2
^(1/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b
*x^3+a)^(1/3))*2^(1/3)/b^(7/3)/d-1/3*2^(1/3)*a^(5/3)*arctan(1/3*(1-2*2^(1/3
)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/b^(7/3)/d*3^(1/2)-1/6*a^(5/
```

```
3)*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1
/3)/b^(7/3)/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {489, 596, 544, 252, 251, 421, 420, 493, 298, 31, 648, 631, 210, 642}

$$\int \frac{x^6\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\sqrt[3]{2}a^{5/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{7/3}d} - \frac{a^{5/3}\arctan\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}b^{7/3}d} - \frac{a^{5/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{7/3}d} + \frac{a^{5/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}b^{7/3}d} - \frac{\sqrt[3]{2}a^{5/3}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3b^{7/3}d} + \frac{a^{5/3}\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+2\sqrt[3]{2}\right)}{6\ 2^{2/3}b^{7/3}d} - \frac{2a^2x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5b^2d\left(a+bx^3\right)^{2/3}} - \frac{3ax\sqrt[3]{a+bx^3}}{5b^2d} - \frac{x^4\sqrt[3]{a+bx^3}}{5bd}$$

```
[In] Int[(x^6*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-3*a*x*(a + b*x^3)^(1/3))/(5*b^2*d) - (x^4*(a + b*x^3)^(1/3))/(5*b*d) - (2
^(1/3)*a^(5/3)*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1
/3))/Sqrt[3]])/(Sqrt[3]*b^(7/3)*d) - (a^(5/3)*ArcTan[(1 + (2^(1/3)*(a^(1/3)
```

```
 + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*b^(7/3)*d) - (
2*a^2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)
])/(5*b^2*d*(a + b*x^3)^(2/3)) - (a^(5/3)*Log[2^(2/3) - (a^(1/3) + b^(1/3)*
x)/(a + b*x^3)^(1/3)])/(3*2^(2/3)*b^(7/3)*d) + (a^(5/3)*Log[1 + (2^(2/3)*(a
^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/
(a + b*x^3)^(1/3)])/(3*2^(2/3)*b^(7/3)*d) - (2^(1/3)*a^(5/3)*Log[1 + (2^(1/
3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*b^(7/3)*d) + (a^(5/3)*Log[
2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x^3)^(2/3) + (2^(2/3)*(a^(1/3) +
 b^(1/3)*x))/(a + b*x^3)^(1/3)])/(6*2^(2/3)*b^(7/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
```

```
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 421

```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^3)^(2/3), x], x] - Dist[d/(b*c - a*d), Int[
(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^4\sqrt[3]{a+bx^3}}{5bd}+\frac{\int \frac{x^3(4a^2d+6abdx^3)}{(a+bx^3)^{2/3}(ad-bdx^3)}\,dx}{5bd}\\
&= -\frac{3ax\sqrt[3]{a+bx^3}}{5b^2d}-\frac{x^4\sqrt[3]{a+bx^3}}{5bd}+\frac{\int \frac{6a^3bd^2+14a^2b^2d^2x^3}{(a+bx^3)^{2/3}(ad-bdx^3)}\,dx}{10b^3d^2}\\
&= -\frac{3ax\sqrt[3]{a+bx^3}}{5b^2d}-\frac{x^4\sqrt[3]{a+bx^3}}{5bd}+\frac{(2a^3)\int \frac{1}{(a+bx^3)^{2/3}(ad-bdx^3)}\,dx}{b^2}-\frac{(7a^2)\int \frac{1}{(a+bx^3)^{2/3}}\,dx}{5b^2d}\\
&= -\frac{3ax\sqrt[3]{a+bx^3}}{5b^2d}-\frac{x^4\sqrt[3]{a+bx^3}}{5bd}+\frac{a^2\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx}{b^2}\\
&\quad+\frac{a^2\int \frac{1}{(a+bx^3)^{2/3}}\,dx}{b^2d}-\frac{\left(7a^2\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{5b^2d\,(a+bx^3)^{2/3}}\\
&= -\frac{3ax\sqrt[3]{a+bx^3}}{5b^2d}-\frac{x^4\sqrt[3]{a+bx^3}}{5bd}-\frac{7a^2x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5b^2d\,(a+bx^3)^{2/3}}\\
&\quad+\frac{(9a^{7/3})\,\text{Subst}\left(\int \frac{x}{(4-ax^3)(1+2ax^3)}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{b^{7/3}d}\\
&\quad+\frac{\left(a^2\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{b^2d\,(a+bx^3)^{2/3}}
\end{aligned}$$

$$= -\frac{3ax\sqrt[3]{a+bx^3}}{5b^2d} - \frac{x^4\sqrt[3]{a+bx^3}}{5bd} - \frac{2a^2x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5b^2d\,(a+bx^3)^{2/3}} + \frac{a^{7/3}\text{Subst}\left(\int \frac{x}{4-ax^3}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{b^{7/3}d} + \frac{\left(2a^{7/3}\right)\text{Subst}\left(\int \frac{x}{1+2ax^3}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{b^{7/3}d}$$

$$= -\frac{3ax\sqrt[3]{a+bx^3}}{5b^2d} - \frac{x^4\sqrt[3]{a+bx^3}}{5bd} - \frac{2a^2x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5b^2d\,(a+bx^3)^{2/3}} + \frac{a^2\text{Subst}\left(\int \frac{1}{2^{2/3}-\sqrt[3]{a}x}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{7/3}d} - \frac{a^2\text{Subst}\left(\int \frac{2^{2/3}-\sqrt[3]{a}x}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{7/3}d} - \frac{\left(2^{2/3}a^2\right)\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}\sqrt[3]{a}x}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3b^{7/3}d} + \frac{\left(2^{2/3}a^2\right)\text{Subst}\left(\int \frac{1+\sqrt[3]{2}\sqrt[3]{a}x}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3b^{7/3}d}$$

$$= -\frac{3ax\sqrt[3]{a+bx^3}}{5b^2d} - \frac{x^4\sqrt[3]{a+bx^3}}{5bd} - \frac{2a^2x\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5b^2d\left(a+bx^3\right)^{2/3}}$$

$$- \frac{a^{5/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{7/3}d} - \frac{\sqrt[3]{2}a^{5/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3b^{7/3}d}$$

$$+ \frac{a^{5/3}\text{Subst}\left(\int \frac{2^{2/3}\sqrt[3]{a}+2a^{2/3}x}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}b^{7/3}d}$$

$$+ \frac{a^{5/3}\text{Subst}\left(\int \frac{-\sqrt[3]{2}\sqrt[3]{a}+2\ 2^{2/3}a^{2/3}x}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{7/3}d}$$

$$- \frac{a^2\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{2b^{7/3}d}$$

$$+ \frac{a^2\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{2}b^{7/3}d}$$

$$= -\frac{3ax\sqrt[3]{a+bx^3}}{5b^2d} - \frac{x^4\sqrt[3]{a+bx^3}}{5bd} - \frac{2a^2x\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5b^2d\,(a+bx^3)^{2/3}}$$

$$- \frac{a^{5/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{7/3}d}$$

$$+ \frac{a^{5/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{7/3}d}$$

$$- \frac{\sqrt[3]{2}a^{5/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3b^{7/3}d}$$

$$+ \frac{a^{5/3}\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}b^{7/3}d}$$

$$+ \frac{a^{5/3}\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{2^{2/3}b^{7/3}d}$$

$$+ \frac{\left(\sqrt[3]{2}a^{5/3}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{b^{7/3}d}$$

$$= -\frac{3ax\sqrt[3]{a+bx^3}}{5b^2d} - \frac{x^4\sqrt[3]{a+bx^3}}{5bd} - \frac{\sqrt[3]{2}a^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{7/3}d}$$

$$- \frac{a^{5/3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}b^{7/3}d}$$

$$- \frac{2a^2x\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5b^2d\left(a+bx^3\right)^{2/3}} - \frac{a^{5/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{7/3}d}$$

$$+ \frac{a^{5/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{7/3}d}$$

$$- \frac{\sqrt[3]{2}a^{5/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3b^{7/3}d}$$

$$+ \frac{a^{5/3}\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}b^{7/3}d}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 7.81 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.45

$$\int \frac{x^6\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$$

$$= \frac{-4(a+bx^3)\left(3ax+bx^4\right)+7abx^4\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+\frac{}{(a-bx^3)\left(4a\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\right.\right.}}{20b^2d\left(a+bx^3\right)^{2/3}}$$

```
[In] Integrate[(x^6*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-4*(a + b*x^3)*(3*a*x + b*x^4) + 7*a*b*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[
4/3, 2/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a] + (48*a^4*x*AppellF1[1/3, 2/3, 1
```

```
, 4/3, -((b*x^3)/a), (b*x^3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1, 4/
3, -((b*x^3)/a), (b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)
/a), (b*x^3)/a] - 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))
/(20*b^2*d*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{x^{6} \left(b\,x^{3}+a \right)^{\frac{1}{3}}}{-bd\,x^{3}+ad}dx$$

```
[In] int(x^6*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
[Out] int(x^6*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=\text{Timed out}$$

```
[In] integrate(x^6*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=-\frac{\int \frac{x^{6} \sqrt[3]{a+bx^{3}}}{-a+bx^{3}}\,dx}{d}$$

```
[In] integrate(x**6*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
[Out] -Integral(x**6*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Maxima [F]

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=\int -\frac{(bx^{3}+a)^{\frac{1}{3}}x^{6}}{bdx^{3}-ad}\,dx$$

```
[In] integrate(x^6*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(1/3)*x^6/(b*d*x^3 - a*d), x)
```

Giac [F]

$$\int \frac{x^6\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}x^6}{bdx^3-ad}\,dx$$

```
[In] integrate(x^6*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(1/3)*x^6/(b*d*x^3 - a*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int \frac{x^6\,(b\,x^3+a)^{1/3}}{a\,d-b\,d\,x^3}\,dx$$

```
[In] int((x^6*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
[Out] int((x^6*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)
```

3.582 $\int \frac{x^3 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 494

$$\int \frac{x^3\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{x\sqrt[3]{a+bx^3}}{2bd} - \frac{\sqrt[3]{2}a^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}d} - \frac{a^{2/3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}b^{4/3}d} - \frac{ax\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2bd\left(a+bx^3\right)^{2/3}} - \frac{a^{2/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{4/3}d} + \frac{a^{2/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{4/3}d} - \frac{\sqrt[3]{2}a^{2/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3b^{4/3}d} + \frac{a^{2/3}\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}b^{4/3}d}$$

```
[Out] -1/2*x*(b*x^3+a)^(1/3)/b/d-1/2*a*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[
4/3],-b*x^3/a)/b/d/(b*x^3+a)^(2/3)-1/6*a^(2/3)*ln(2^(2/3)+(-a^(1/3)-b^(1/3)
*x)/(b*x^3+a)^(1/3))*2^(1/3)/b^(4/3)/d+1/6*a^(2/3)*ln(1+2^(2/3)*(a^(1/3)+b^
(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(
1/3)/b^(4/3)/d-1/3*2^(1/3)*a^(2/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+
a)^(1/3))/b^(4/3)/d+1/12*a^(2/3)*ln(2*2^(1/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+
a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/b^(4/3)/d-1/3
*2^(1/3)*a^(2/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3
))*3^(1/2))/b^(4/3)/d*3^(1/2)-1/6*a^(2/3)*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^
(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)/b^(4/3)/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {489, 544, 252, 251, 421, 420, 493, 298, 31, 648, 631, 210, 642}

$$\int \frac{x^3\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\sqrt[3]{2}a^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}d} - \frac{a^{2/3}\arctan\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}b^{4/3}d} - \frac{a^{2/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{4/3}d} + \frac{a^{2/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}b^{4/3}d} - \frac{\sqrt[3]{2}a^{2/3}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3b^{4/3}d} + \frac{a^{2/3}\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+2\sqrt[3]{2}\right)}{6\ 2^{2/3}b^{4/3}d} - \frac{ax\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2bd\,(a+bx^3)^{2/3}} - \frac{x\sqrt[3]{a+bx^3}}{2bd}$$

```
[In] Int[(x^3*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/2*(x*(a + b*x^3)^(1/3))/(b*d) - (2^(1/3)*a^(2/3)*ArcTan[(1 - (2*2^(1/3)*
(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(4/3)*d) - (
a^(2/3)*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt
[3]])/(2^(2/3)*Sqrt[3]*b^(4/3)*d) - (a*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometr
ic2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*b*d*(a + b*x^3)^(2/3)) - (a^(2/3)*Lo
g[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*2^(2/3)*b^(4/3)*d)
 + (a^(2/3)*Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (
2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*2^(2/3)*b^(4/3)*d) -
(2^(1/3)*a^(2/3)*Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]
```

```
)/(3*b^(4/3)*d) + (a^(2/3)*Log[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x
^3)^(2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(6*2^(2/3)*
b^(4/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 421

```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^3)^(2/3), x], x] - Dist[d/(b*c - a*d), Int[
```

```
(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = -\frac{x\sqrt[3]{a+bx^3}}{2bd} + \frac{\int \frac{a^2d+3abdx^3}{(a+bx^3)^{2/3}(ad-bdx^3)}\,dx}{2bd}$$

$$= -\frac{x\sqrt[3]{a+bx^3}}{2bd} + \frac{(2a^2)\int \frac{1}{(a+bx^3)^{2/3}(ad-bdx^3)}\,dx}{b} - \frac{(3a)\int \frac{1}{(a+bx^3)^{2/3}}\,dx}{2bd}$$

$$= -\frac{x\sqrt[3]{a+bx^3}}{2bd} + \frac{a\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx}{b} + \frac{a\int \frac{1}{(a+bx^3)^{2/3}}\,dx}{bd} - \frac{\left(3a\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{2bd\,(a+bx^3)^{2/3}}$$

$$= -\frac{x\sqrt[3]{a+bx^3}}{2bd} - \frac{3ax\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2bd\,(a+bx^3)^{2/3}} + \frac{(9a^{4/3})\,\text{Subst}\left(\int \frac{x}{(4-ax^3)(1+2ax^3)}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{b^{4/3}d} + \frac{\left(a\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{bd\,(a+bx^3)^{2/3}}$$

$$= -\frac{x\sqrt[3]{a+bx^3}}{2bd} - \frac{ax\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2bd\,(a+bx^3)^{2/3}} + \frac{a^{4/3}\text{Subst}\left(\int \frac{x}{4-ax^3}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{b^{4/3}d} + \frac{(2a^{4/3})\,\text{Subst}\left(\int \frac{x}{1+2ax^3}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{b^{4/3}d}$$

$$= -\frac{x\sqrt[3]{a+bx^3}}{2bd} - \frac{ax\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2bd\left(a+bx^3\right)^{2/3}}$$

$$+ \frac{a\text{Subst}\left(\int \frac{1}{2^{2/3}-\sqrt[3]{a}x}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{4/3}d}$$

$$- \frac{a\text{Subst}\left(\int \frac{2^{2/3}-\sqrt[3]{a}x}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{4/3}d}$$

$$- \frac{\left(2^{2/3}a\right)\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}\sqrt[3]{a}x}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3b^{4/3}d}$$

$$+ \frac{\left(2^{2/3}a\right)\text{Subst}\left(\int \frac{1+\sqrt[3]{2}\sqrt[3]{a}x}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3b^{4/3}d}$$

$$= -\frac{x\sqrt[3]{a+bx^3}}{2bd} - \frac{ax\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2bd\left(a+bx^3\right)^{2/3}} - \frac{a^{2/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{4/3}d} - \frac{\sqrt[3]{2}a^{2/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3b^{4/3}d} + \frac{a^{2/3}\text{Subst}\left(\int\frac{2^{2/3}\sqrt[3]{a}+2a^{2/3}x}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}b^{4/3}d} + \frac{a^{2/3}\text{Subst}\left(\int\frac{-\sqrt[3]{2}\sqrt[3]{a}+2\ 2^{2/3}a^{2/3}x}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{4/3}d} - \frac{a\text{Subst}\left(\int\frac{1}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{2b^{4/3}d} + \frac{a\text{Subst}\left(\int\frac{1}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{2}b^{4/3}d}$$

$$=-\frac{x\sqrt[3]{a+bx^3}}{2bd}-\frac{ax\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2bd\left(a+bx^3\right)^{2/3}}-\frac{a^{2/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{4/3}d}$$

$$+\frac{a^{2/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{4/3}d}$$

$$-\frac{\sqrt[3]{2}a^{2/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3b^{4/3}d}$$

$$+\frac{a^{2/3}\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}b^{4/3}d}$$

$$+\frac{a^{2/3}\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{2^{2/3}b^{4/3}d}$$

$$+\frac{\left(\sqrt[3]{2}a^{2/3}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{b^{4/3}d}$$

$$= -\frac{x\sqrt[3]{a+bx^3}}{2bd} - \frac{\sqrt[3]{2}a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}d} - \frac{a^{2/3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}b^{4/3}d} - \frac{ax\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2bd(a+bx^3)^{2/3}} - \frac{a^{2/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{4/3}d} + \frac{a^{2/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}b^{4/3}d} - \frac{\sqrt[3]{2}a^{2/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3b^{4/3}d} + \frac{a^{2/3}\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}b^{4/3}d}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 7.19 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.46

$$\int \frac{x^3\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{x\left(3x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+\frac{4\left(-a-bx^3+\frac{4a^3\text{AppellF1}\left(\frac{1}{3},\right.}{\left(a-bx^3\right)\left(4a\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+bx^3\left(3\text{AppellF}\right.\right.}\right.}{b}\right.}{8d\left(a+bx^3\right)^{2/3}}$$

```
[In] Integrate[(x^3*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x*(3*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), (b
*x^3)/a] + (4*(-a - b*x^3 + (4*a^3*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a),
 (b*x^3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*
x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] - 2*
AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))))/b))/(8*d*(a + b*x^
3)^(2/3))
```

Maple [F]

$$\int \frac{x^{3} \left(b\,x^{3}+a\right)^{\frac{1}{3}}}{-bd\,x^{3}+ad}dx$$

```
[In] int(x^3*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^3*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=\text{Timed out}$$

```
[In] integrate(x^3*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^3\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=-\frac{\int \frac{x^3\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

```
[In] integrate(x**3*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**3*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Maxima [F]

$$\int \frac{x^3 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}x^3}{bdx^3-ad}\,dx$$

```
[In] integrate(x^3*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(1/3)*x^3/(b*d*x^3 - a*d), x)
```

Giac [F]

$$\int \frac{x^3 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}x^3}{bdx^3-ad}\,dx$$

```
[In] integrate(x^3*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(1/3)*x^3/(b*d*x^3 - a*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int \frac{x^3\,(b\,x^3+a)^{1/3}}{a\,d-b\,d\,x^3}\,dx$$

```
[In] int((x^3*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
[Out] int((x^3*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)
```

3.583 $\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 416

$$\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}d} - \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}d} - \frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}d} + \frac{\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}d} - \frac{\sqrt[3]{2}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d} + \frac{\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}d}$$

```
[Out] -1/6*ln(2^(2/3)+(-a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(1/3)/b^(1/
3)/d+1/6*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3
)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(1/3)/b^(1/3)/d-1/3*2^(1/3)*ln(1+2^
(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/a^(1/3)/b^(1/3)/d+1/12*ln(2*2^(1
/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^
3+a)^(1/3))*2^(1/3)/a^(1/3)/b^(1/3)/d-1/3*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*(
a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/a^(1/3)/b^(1/3)/d*3^(1/2)-1/6*
arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)
/a^(1/3)/b^(1/3)/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {420, 493, 298, 31, 648, 631, 210, 642}

$$\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}d} - \frac{\arctan\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}d} - \frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}d} + \frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}d} - \frac{\sqrt[3]{2}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{a}\sqrt[3]{b}d} + \frac{\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+2\sqrt[3]{2}\right)}{6\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}d}$$

```
[In] Int[(a + b*x^3)^(1/3)/(a*d - b*d*x^3),x]
```

```
[Out] -((2^(1/3)*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))
/Sqrt[3]])/(Sqrt[3]*a^(1/3)*b^(1/3)*d)) - ArcTan[(1 + (2^(1/3)*(a^(1/3) + b
^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]*a^(1/3)*b^(1/3)*d)
- Log[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(1/3)
*b^(1/3)*d) + Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) -
 (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(1/3)*b^(1
/3)*d) - (2^(1/3)*Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)
])/(3*a^(1/3)*b^(1/3)*d) + Log[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x
^3)^(2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(6*2^(2/3)*a
^(1/3)*b^(1/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{\left(9\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{x}{(4-ax^3)(1+2ax^3)}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}d}$$

$$= \frac{\sqrt[3]{a}\text{Subst}\left(\int \frac{x}{4-ax^3}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}d} + \frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{x}{1+2ax^3}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}d}$$

$$=\frac{\text{Subst}\left(\int \frac{1}{2^{2/3}-\sqrt[3]{a}x}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{b}d}-\frac{\text{Subst}\left(\int \frac{2^{2/3}-\sqrt[3]{a}x}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{b}d}-\frac{2^{2/3}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}\sqrt[3]{a}x}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}d}+\frac{2^{2/3}\text{Subst}\left(\int \frac{1+\sqrt[3]{2}\sqrt[3]{a}x}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}d}$$

$$=-\frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}d}-\frac{\sqrt[3]{2}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d}-\frac{\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{2\sqrt[3]{b}d}+\frac{\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{2}\sqrt[3]{b}d}+\frac{\text{Subst}\left(\int \frac{2^{2/3}\sqrt[3]{a}+2a^{2/3}x}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}d}+\frac{\text{Subst}\left(\int \frac{-\sqrt[3]{2}\sqrt[3]{a}+2\ 2^{2/3}a^{2/3}x}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}d}$$

$$= -\frac{\log\left(2^{2/3} - \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}d} + \frac{\log\left(1 + \frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}} - \frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}d}$$
$$-\frac{\sqrt[3]{2}\log\left(1 + \frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d} + \frac{\log\left(2\sqrt[3]{2} + \frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}} + \frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}d}$$
$$+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 + \frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{2^{2/3}\sqrt[3]{a}\sqrt[3]{b}d}$$
$$+\frac{\sqrt[3]{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{a}\sqrt[3]{b}d}$$

$$= -\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}d} - \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}d}$$
$$-\frac{\log\left(2^{2/3} - \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}d} + \frac{\log\left(1 + \frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}} - \frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}d}$$
$$-\frac{\sqrt[3]{2}\log\left(1 + \frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d} + \frac{\log\left(2\sqrt[3]{2} + \frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}} + \frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}d}$$

Mathematica [A] (verified)

Time = 3.33 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$$
$$= \frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx^3}}{-2\sqrt[3]{2}\sqrt[3]{a}-2\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a+bx^3}}\right) + 2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{a+bx^3}}{\sqrt[3]{2}\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a+bx^3}}\right) - 4\log\left(\sqrt[3]{2}\right.}{}$$

```
[In] Integrate[(a + b*x^3)^(1/3)/(a*d - b*d*x^3),x]
```

```
[Out] (4*Sqrt[3]*ArcTan[(Sqrt[3]*(a + b*x^3)^(1/3))/(-2*2^(1/3)*a^(1/3) - 2*2^(1/
3)*b^(1/3)*x + (a + b*x^3)^(1/3))] + 2*Sqrt[3]*ArcTan[(Sqrt[3]*(a + b*x^3)^
(1/3))/(2^(1/3)*a^(1/3) + 2^(1/3)*b^(1/3)*x + (a + b*x^3)^(1/3))] - 4*Log[2
^(1/3)*a^(1/3) + 2^(1/3)*b^(1/3)*x + (a + b*x^3)^(1/3)] - 2*Log[-(2^(1/3)*a
^(1/3)) - 2^(1/3)*b^(1/3)*x + 2*(a + b*x^3)^(1/3)] + Log[2^(2/3)*a^(2/3) +
2^(2/3)*b^(2/3)*x^2 + 2*2^(1/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 4*(a + b*x^3)
^(2/3) + 2*2^(1/3)*a^(1/3)*(2^(1/3)*b^(1/3)*x + (a + b*x^3)^(1/3))] + 2*Log
[2^(2/3)*a^(2/3) + 2^(2/3)*b^(2/3)*x^2 - 2^(1/3)*b^(1/3)*x*(a + b*x^3)^(1/3
) + (a + b*x^3)^(2/3) + a^(1/3)*(2*2^(2/3)*b^(1/3)*x - 2^(1/3)*(a + b*x^3)^
(1/3))])/(6*2^(2/3)*a^(1/3)*b^(1/3)*d)
```

Maple [F]

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{-bd\,x^{3}+ad}dx$$

```
[In] int((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = -\frac{\int \frac{\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

```
[In] integrate((b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}}{bdx^3-ad}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(1/3)/(b*d*x^3 - a*d), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}}{bdx^3-ad}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(1/3)/(b*d*x^3 - a*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \int \frac{(b\,x^3+a)^{1/3}}{a\,d-b\,d\,x^3}\,dx$$

```
[In] int((a + b*x^3)^(1/3)/(a*d - b*d*x^3),x)
[Out] int((a + b*x^3)^(1/3)/(a*d - b*d*x^3), x)
```

3.584 $\int \frac{\sqrt[3]{a+bx^3}}{x^3(ad-bdx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 496

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(ad-bdx^3\right)}\,dx = -\frac{\sqrt[3]{a+bx^3}}{2adx^2} - \frac{\sqrt[3]{2}b^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^{4/3}d} - \frac{b^{2/3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a^{4/3}d} + \frac{bx\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2ad\left(a+bx^3\right)^{2/3}} - \frac{b^{2/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^{4/3}d} + \frac{b^{2/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^{4/3}d} - \frac{\sqrt[3]{2}b^{2/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3a^{4/3}d} + \frac{b^{2/3}\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}a^{4/3}d}$$

```
[Out] -1/2*(b*x^3+a)^(1/3)/a/d/x^2+1/2*b*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3]
,[4/3],-b*x^3/a)/a/d/(b*x^3+a)^(2/3)-1/6*b^(2/3)*ln(2^(2/3)+(-a^(1/3)-b^(1/
3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(4/3)/d+1/6*b^(2/3)*ln(1+2^(2/3)*(a^(1/3)+
b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2
^(1/3)/a^(4/3)/d-1/3*2^(1/3)*b^(2/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^
3+a)^(1/3))/a^(4/3)/d+1/12*b^(2/3)*ln(2*2^(1/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^
3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(4/3)/d-1
/3*2^(1/3)*b^(2/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1
/3))*3^(1/2))/a^(4/3)/d*3^(1/2)-1/6*b^(2/3)*arctan(1/3*(1+2^(1/3)*(a^(1/3)+
b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)/a^(4/3)/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {486, 544, 252, 251, 421, 420, 493, 298, 31, 648, 631, 210, 642}

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(ad-bdx^3\right)}\,dx = -\frac{\sqrt[3]{2}b^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^{4/3}d} - \frac{b^{2/3}\arctan\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a^{4/3}d} - \frac{b^{2/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^{4/3}d} + \frac{b^{2/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{4/3}d} - \frac{\sqrt[3]{2}b^{2/3}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3a^{4/3}d} + \frac{b^{2/3}\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+2\sqrt[3]{2}\right)}{6\ 2^{2/3}a^{4/3}d} + \frac{bx\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2ad\left(a+bx^3\right)^{2/3}} - \frac{\sqrt[3]{a+bx^3}}{2adx^2}$$

```
[In] Int[(a + b*x^3)^(1/3)/(x^3*(a*d - b*d*x^3)),x]
```

```
[Out] -1/2*(a + b*x^3)^(1/3)/(a*d*x^2) - (2^(1/3)*b^(2/3)*ArcTan[(1 - (2*2^(1/3)*
(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(4/3)*d) - (
b^(2/3)*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt
[3]])/(2^(2/3)*Sqrt[3]*a^(4/3)*d) + (b*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometr
ic2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*d*(a + b*x^3)^(2/3)) - (b^(2/3)*Lo
g[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*2^(2/3)*a^(4/3)*d)
 + (b^(2/3)*Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (
2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*2^(2/3)*a^(4/3)*d) -
(2^(1/3)*b^(2/3)*Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]
```

```
)/(3*a^(4/3)*d) + (b^(2/3)*Log[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x
^3)^(2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(6*2^(2/3)*
a^(4/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 421

```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^3)^(2/3), x], x] - Dist[d/(b*c - a*d), Int[
```

```
(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt[3]{a+bx^3}}{2adx^2} + \frac{\int \frac{3abd+b^2dx^3}{(a+bx^3)^{2/3}(ad-bdx^3)}\,dx}{2ad}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{2adx^2} + (2b)\int \frac{1}{(a+bx^3)^{2/3}(ad-bdx^3)}\,dx - \frac{b\int \frac{1}{(a+bx^3)^{2/3}}\,dx}{2ad}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{2adx^2} + \frac{b\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx}{a} + \frac{b\int \frac{1}{(a+bx^3)^{2/3}}\,dx}{ad} - \frac{\left(b\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{2ad(a+bx^3)^{2/3}}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{2adx^2} - \frac{bx\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2ad(a+bx^3)^{2/3}} + \frac{(9b^{2/3})\,\text{Subst}\left(\int \frac{x}{(4-ax^3)(1+2ax^3)}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{a^{2/3}d} + \frac{\left(b\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{ad(a+bx^3)^{2/3}}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{2adx^2} + \frac{bx\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2ad(a+bx^3)^{2/3}} + \frac{b^{2/3}\text{Subst}\left(\int \frac{x}{4-ax^3}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{a^{2/3}d} + \frac{(2b^{2/3})\,\text{Subst}\left(\int \frac{x}{1+2ax^3}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{a^{2/3}d}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{2adx^2} + \frac{bx\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2ad\left(a+bx^3\right)^{2/3}}$$

$$+ \frac{b^{2/3}\text{Subst}\left(\int \frac{1}{2^{2/3}-\sqrt[3]{a}x}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}ad}$$

$$- \frac{b^{2/3}\text{Subst}\left(\int \frac{2^{2/3}-\sqrt[3]{a}x}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}ad}$$

$$- \frac{\left(2^{2/3}b^{2/3}\right)\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}\sqrt[3]{a}x}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3ad}$$

$$+ \frac{\left(2^{2/3}b^{2/3}\right)\text{Subst}\left(\int \frac{1+\sqrt[3]{2}\sqrt[3]{a}x}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3ad}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{2adx^2} + \frac{bx\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2ad\left(a+bx^3\right)^{2/3}}$$

$$-\frac{b^{2/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^{4/3}d} - \frac{\sqrt[3]{2}b^{2/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3a^{4/3}d}$$

$$+\frac{b^{2/3}\text{Subst}\left(\int\frac{2^{2/3}\sqrt[3]{a}+2a^{2/3}x}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}a^{4/3}d}$$

$$+\frac{b^{2/3}\text{Subst}\left(\int\frac{-\sqrt[3]{2}\sqrt[3]{a}+2\ 2^{2/3}a^{2/3}x}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^{4/3}d}$$

$$-\frac{b^{2/3}\text{Subst}\left(\int\frac{1}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{2ad}$$

$$+\frac{b^{2/3}\text{Subst}\left(\int\frac{1}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{2}ad}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{2adx^2} + \frac{bx\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2ad\left(a+bx^3\right)^{2/3}} - \frac{b^{2/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^{4/3}d}$$

$$+ \frac{b^{2/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^{4/3}d}$$

$$- \frac{\sqrt[3]{2}b^{2/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3a^{4/3}d}$$

$$+ \frac{b^{2/3}\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}a^{4/3}d}$$

$$+ \frac{b^{2/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{2^{2/3}a^{4/3}d}$$

$$+ \frac{\left(\sqrt[3]{2}b^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{a^{4/3}d}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{2adx^2} - \frac{\sqrt[3]{2}b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^{4/3}d}$$
$$- \frac{b^{2/3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a^{4/3}d}$$
$$+ \frac{bx\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2ad\left(a+bx^3\right)^{2/3}} - \frac{b^{2/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^{4/3}d}$$
$$+ \frac{b^{2/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^{4/3}d}$$
$$- \frac{\sqrt[3]{2}b^{2/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3a^{4/3}d}$$
$$+ \frac{b^{2/3}\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}a^{4/3}d}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 11.16 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(ad-bdx^3\right)}\,dx$$
$$= \frac{-4a(a+bx^3)+b^2x^6\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+\frac{48a^3bx^3\,\text{A}}{(a-bx^3)\left(4a\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+bx^3\left(\right.\right.}}{8a^2dx^2\left(a+bx^3\right)^{2/3}}$$

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^3*(a*d - b*d*x^3)),x]
```

```
[Out] (-4*a*(a + b*x^3) + b^2*x^6*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3
, -((b*x^3)/a), (b*x^3)/a] + (48*a^3*b*x^3*AppellF1[1/3, 2/3, 1, 4/3, -((b*
```

```
x^3)/a), (b*x^3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)
/a), (b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)
/a] - 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a]))))/(8*a^2*d*x^
2*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{1}{3}}}{x^{3}\left(-bd\,x^{3}+ad\right)}dx$$

```
[In] int((b*x^3+a)^(1/3)/x^3/(-b*d*x^3+a*d),x)
[Out] int((b*x^3+a)^(1/3)/x^3/(-b*d*x^3+a*d),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\,(ad-bdx^3)}\,dx=\text{Timed out}$$

```
[In] integrate((b*x^3+a)^(1/3)/x^3/(-b*d*x^3+a*d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\,(ad-bdx^3)}\,dx=-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^3+bx^6}\,dx}{d}$$

```
[In] integrate((b*x**3+a)**(1/3)/x**3/(-b*d*x**3+a*d),x)
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**3 + b*x**6), x)/d
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\,(ad-bdx^3)}\,dx=\int -\frac{(bx^3+a)^{\frac{1}{3}}}{(bdx^3-ad)x^3}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/x^3/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(ad-bdx^3\right)}\,dx = \int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{(bdx^3-ad)x^3}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/x^3/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(ad-bdx^3\right)}\,dx = \int \frac{\left(b\,x^3+a\right)^{1/3}}{x^3\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

```
[In] int((a + b*x^3)^(1/3)/(x^3*(a*d - b*d*x^3)),x)
[Out] int((a + b*x^3)^(1/3)/(x^3*(a*d - b*d*x^3)), x)
```

3.585 $\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(ad-bdx^3\right)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 523

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6(ad-bdx^3)}\,dx = -\frac{\sqrt[3]{a+bx^3}}{5adx^5}-\frac{3b\sqrt[3]{a+bx^3}}{5a^2dx^2}-\frac{\sqrt[3]{2}b^{5/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(\sqrt[3]{a}+\sqrt[3]{b}x)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^{7/3}d}$$
$$-\frac{b^{5/3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}(\sqrt[3]{a}+\sqrt[3]{b}x)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a^{7/3}d}$$
$$+\frac{2b^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5a^2d(a+bx^3)^{2/3}}$$
$$-\frac{b^{5/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^{7/3}d}$$
$$+\frac{b^{5/3}\log\left(1+\frac{2^{2/3}(\sqrt[3]{a}+\sqrt[3]{b}x)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}(\sqrt[3]{a}+\sqrt[3]{b}x)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^{7/3}d}$$
$$-\frac{\sqrt[3]{2}b^{5/3}\log\left(1+\frac{\sqrt[3]{2}(\sqrt[3]{a}+\sqrt[3]{b}x)}{\sqrt[3]{a+bx^3}}\right)}{3a^{7/3}d}$$
$$+\frac{b^{5/3}\log\left(2\sqrt[3]{2}+\frac{(\sqrt[3]{a}+\sqrt[3]{b}x)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}(\sqrt[3]{a}+\sqrt[3]{b}x)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}a^{7/3}d}$$

```
[Out] -1/5*(b*x^3+a)^(1/3)/a/d/x^5-3/5*b*(b*x^3+a)^(1/3)/a^2/d/x^2+2/5*b^2*x*(1+b
*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/a^2/d/(b*x^3+a)^(2/3)-1/
6*b^(5/3)*ln(2^(2/3)+(-a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(7/3)/
d+1/6*b^(5/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a
^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(7/3)/d-1/3*2^(1/3)*b^(5/3)*ln
(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/a^(7/3)/d+1/12*b^(5/3)*ln(2
*2^(1/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/
(b*x^3+a)^(1/3))*2^(1/3)/a^(7/3)/d-1/3*2^(1/3)*b^(5/3)*arctan(1/3*(1-2*2^(1
/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/a^(7/3)/d*3^(1/2)-1/6*b^(
```

```
5/3)*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^
(1/3)/a^(7/3)/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {486, 597, 544, 252, 251, 421, 420, 493, 298, 31, 648, 631, 210, 642}

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(ad-bdx^3\right)}\,dx = -\frac{\sqrt[3]{2}b^{5/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^{7/3}d} - \frac{b^{5/3}\arctan\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a^{7/3}d} - \frac{b^{5/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^{7/3}d} + \frac{b^{5/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{7/3}d} - \frac{\sqrt[3]{2}b^{5/3}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3a^{7/3}d} + \frac{b^{5/3}\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+2\sqrt[3]{2}\right)}{6\ 2^{2/3}a^{7/3}d} + \frac{2b^2x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{5a^2d\left(a+bx^3\right)^{2/3}} - \frac{3b\sqrt[3]{a+bx^3}}{5a^2dx^2} - \frac{\sqrt[3]{a+bx^3}}{5adx^5}$$

```
[In] Int[(a + b*x^3)^(1/3)/(x^6*(a*d - b*d*x^3)),x]
```

```
[Out] -1/5*(a + b*x^3)^(1/3)/(a*d*x^5) - (3*b*(a + b*x^3)^(1/3))/(5*a^2*d*x^2) -
(2^(1/3)*b^(5/3)*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^
(1/3))/Sqrt[3]])/(Sqrt[3]*a^(7/3)*d) - (b^(5/3)*ArcTan[(1 + (2^(1/3)*(a^(1/
```

```
3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*a^(7/3)*d) +
 (2*b^2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/
a)])/(5*a^2*d*(a + b*x^3)^(2/3)) - (b^(5/3)*Log[2^(2/3) - (a^(1/3) + b^(1/3
)*x)/(a + b*x^3)^(1/3)])/(3*2^(2/3)*a^(7/3)*d) + (b^(5/3)*Log[1 + (2^(2/3)*
(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x)
)/(a + b*x^3)^(1/3)])/(3*2^(2/3)*a^(7/3)*d) - (2^(1/3)*b^(5/3)*Log[1 + (2^(
1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*a^(7/3)*d) + (b^(5/3)*Lo
g[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x^3)^(2/3) + (2^(2/3)*(a^(1/3)
 + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(6*2^(2/3)*a^(7/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
```

```
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 421

```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^3)^(2/3), x], x] - Dist[d/(b*c - a*d), Int[
(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt[3]{a+bx^3}}{5adx^5}+\frac{\int \frac{6abd+4b^2dx^3}{x^3(a+bx^3)^{2/3}(ad-bdx^3)}\,dx}{5ad}\\
&= -\frac{\sqrt[3]{a+bx^3}}{5adx^5}-\frac{3b\sqrt[3]{a+bx^3}}{5a^2dx^2}-\frac{\int \frac{-14a^2b^2d^2-6ab^3d^2x^3}{(a+bx^3)^{2/3}(ad-bdx^3)}\,dx}{10a^3d^2}\\
&= -\frac{\sqrt[3]{a+bx^3}}{5adx^5}-\frac{3b\sqrt[3]{a+bx^3}}{5a^2dx^2}+\frac{(2b^2)\int \frac{1}{(a+bx^3)^{2/3}(ad-bdx^3)}\,dx}{a}-\frac{(3b^2)\int \frac{1}{(a+bx^3)^{2/3}}\,dx}{5a^2d}\\
&= -\frac{\sqrt[3]{a+bx^3}}{5adx^5}-\frac{3b\sqrt[3]{a+bx^3}}{5a^2dx^2}+\frac{b^2\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx}{a^2}\\
&\quad+\frac{b^2\int \frac{1}{(a+bx^3)^{2/3}}\,dx}{a^2d}-\frac{\left(3b^2\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{5a^2d\,(a+bx^3)^{2/3}}\\
&= -\frac{\sqrt[3]{a+bx^3}}{5adx^5}-\frac{3b\sqrt[3]{a+bx^3}}{5a^2dx^2}-\frac{3b^2x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5a^2d\,(a+bx^3)^{2/3}}\\
&\quad+\frac{(9b^{5/3})\,\text{Subst}\left(\int \frac{x}{(4-ax^3)(1+2ax^3)}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{a^{5/3}d}\\
&\quad+\frac{\left(b^2\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{a^2d\,(a+bx^3)^{2/3}}
\end{aligned}
$$

$$= -\frac{\sqrt[3]{a+bx^3}}{5adx^5} - \frac{3b\sqrt[3]{a+bx^3}}{5a^2dx^2} + \frac{2b^2x\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5a^2d\left(a+bx^3\right)^{2/3}} + \frac{b^{5/3}\text{Subst}\left(\int \frac{x}{4-ax^3}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{a^{5/3}d} + \frac{\left(2b^{5/3}\right)\text{Subst}\left(\int \frac{x}{1+2ax^3}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{a^{5/3}d}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{5adx^5} - \frac{3b\sqrt[3]{a+bx^3}}{5a^2dx^2} + \frac{2b^2x\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5a^2d\left(a+bx^3\right)^{2/3}} + \frac{b^{5/3}\text{Subst}\left(\int \frac{1}{2^{2/3}-\sqrt[3]{a}x}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^2d} - \frac{b^{5/3}\text{Subst}\left(\int \frac{2^{2/3}-\sqrt[3]{a}x}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^2d} - \frac{\left(2^{2/3}b^{5/3}\right)\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}\sqrt[3]{a}x}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3a^2d} + \frac{\left(2^{2/3}b^{5/3}\right)\text{Subst}\left(\int \frac{1+\sqrt[3]{2}\sqrt[3]{a}x}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3a^2d}$$

$$=-\frac{\sqrt[3]{a+bx^3}}{5adx^5}-\frac{3b\sqrt[3]{a+bx^3}}{5a^2dx^2}+\frac{2b^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\ {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5a^2d\,(a+bx^3)^{2/3}}$$

$$-\frac{b^{5/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^{7/3}d}-\frac{\sqrt[3]{2}b^{5/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3a^{7/3}d}$$

$$+\frac{b^{5/3}\text{Subst}\left(\int\frac{2^{2/3}\sqrt[3]{a}+2a^{2/3}x}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}a^{7/3}d}$$

$$+\frac{b^{5/3}\text{Subst}\left(\int\frac{-\sqrt[3]{2}\sqrt[3]{a}+2\ 2^{2/3}a^{2/3}x}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^{7/3}d}$$

$$-\frac{b^{5/3}\text{Subst}\left(\int\frac{1}{2\sqrt[3]{2}+2^{2/3}\sqrt[3]{a}x+a^{2/3}x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{2a^2d}$$

$$+\frac{b^{5/3}\text{Subst}\left(\int\frac{1}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{2}a^2d}$$

$$
\begin{aligned}
&= -\frac{\sqrt[3]{a+bx^3}}{5adx^5} - \frac{3b\sqrt[3]{a+bx^3}}{5a^2dx^2} + \frac{2b^2x\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5a^2d\left(a+bx^3\right)^{2/3}} \\
&- \frac{b^{5/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^{7/3}d} \\
&+ \frac{b^{5/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^{7/3}d} \\
&- \frac{\sqrt[3]{2}b^{5/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3a^{7/3}d} \\
&+ \frac{b^{5/3}\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}a^{7/3}d} \\
&+ \frac{b^{5/3}\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{2^{2/3}a^{7/3}d} \\
&+ \frac{\left(\sqrt[3]{2}b^{5/3}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{a^{7/3}d}
\end{aligned}
$$

$$= -\frac{\sqrt[3]{a+bx^3}}{5adx^5} - \frac{3b\sqrt[3]{a+bx^3}}{5a^2dx^2} - \frac{\sqrt[3]{2}b^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^{7/3}d}$$

$$- \frac{b^{5/3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a^{7/3}d}$$

$$+ \frac{2b^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\ {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5a^2d\left(a+bx^3\right)^{2/3}} - \frac{b^{5/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^{7/3}d}$$

$$+ \frac{b^{5/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}a^{7/3}d}$$

$$- \frac{\sqrt[3]{2}b^{5/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3a^{7/3}d}$$

$$+ \frac{b^{5/3}\log\left(2\sqrt[3]{2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}a^{7/3}d}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 11.19 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(ad-bdx^3\right)}\,dx$$

$$= \frac{-\frac{4\left(a^2+4abx^3+3b^2x^6\right)}{a^2x^5}+\frac{3b^3x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\mathrm{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{a^3}+\frac{112b^2x\,\mathrm{AppellF1}\left(\frac{1}{3}\right.}{(a-bx^3)\left(4a\,\mathrm{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+bx^3\left(3\,\mathrm{AppellF}\right.\right.}}{20d\left(a+bx^3\right)^{2/3}}$$

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^6*(a*d - b*d*x^3)),x]
```

```
[Out] ((-4*(a^2 + 4*a*b*x^3 + 3*b^2*x^6))/(a^2*x^5) + (3*b^3*x^4*(1 + (b*x^3)/a)^
(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])/a^3 + (112*b^2*x
```

```
*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/((a - b*x^3)*(4*a*App
ellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2
/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] - 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^
3)/a), (b*x^3)/a]))))/(20*d*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{x^6\,(-bd\,x^3+ad)}dx$$

```
[In] int((b*x^3+a)^(1/3)/x^6/(-b*d*x^3+a*d),x)
[Out] int((b*x^3+a)^(1/3)/x^6/(-b*d*x^3+a*d),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\,(ad-bdx^3)}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^3+a)^(1/3)/x^6/(-b*d*x^3+a*d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\,(ad-bdx^3)}\,dx = -\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^6+bx^9}\,dx}{d}$$

```
[In] integrate((b*x**3+a)**(1/3)/x**6/(-b*d*x**3+a*d),x)
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**6 + b*x**9), x)/d
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\,(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}}{(bdx^3-ad)x^6}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/x^6/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^6), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{1}{3}}}{(bdx^3-ad)x^6}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/x^6/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6(ad-bdx^3)}\,dx = \int \frac{(b\,x^3+a)^{1/3}}{x^6\,(a\,d-b\,d\,x^3)}\,dx$$

```
[In] int((a + b*x^3)^(1/3)/(x^6*(a*d - b*d*x^3)),x)
[Out] int((a + b*x^3)^(1/3)/(x^6*(a*d - b*d*x^3)), x)
```

3.586 $\int \frac{x^{11}(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 223

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{a^3(a+bx^3)^{2/3}}{2b^4d}-\frac{a^2(a+bx^3)^{5/3}}{5b^4d}+\frac{a(a+bx^3)^{8/3}}{8b^4d}-\frac{(a+bx^3)^{11/3}}{11b^4d}-\frac{2^{2/3}a^{11/3}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^4d}+\frac{a^{11/3}\log(a-bx^3)}{3\sqrt[3]{2}b^4d}-\frac{a^{11/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^4d}$$

```
[Out] -1/2*a^3*(b*x^3+a)^(2/3)/b^4/d-1/5*a^2*(b*x^3+a)^(5/3)/b^4/d+1/8*a*(b*x^3+a
)^(8/3)/b^4/d-1/11*(b*x^3+a)^(11/3)/b^4/d+1/6*a^(11/3)*ln(-b*x^3+a)*2^(2/3)
/b^4/d-1/2*a^(11/3)*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(2/3)/b^4/d-1/3*2
^(2/3)*a^(11/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2
))/b^4/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used

$= \{457, 90, 52, 57, 631, 210, 31\}$

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{2^{2/3}a^{11/3}\arctan\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}}{}\right)}{\sqrt{3}b^4d} + \frac{a^{11/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}b^4d} - \frac{a^{11/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^4d} - \frac{a^3(a+bx^3)^{2/3}}{2b^4d} - \frac{a^2(a+bx^3)^{5/3}}{5b^4d} + \frac{a(a+bx^3)^{8/3}}{8b^4d} - \frac{(a+bx^3)^{11/3}}{11b^4d}$$

```
[In] Int[(x^11*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/2*(a^3*(a + b*x^3)^(2/3))/(b^4*d) - (a^2*(a + b*x^3)^(5/3))/(5*b^4*d) +
(a*(a + b*x^3)^(8/3))/(8*b^4*d) - (a + b*x^3)^(11/3)/(11*b^4*d) - (2^(2/3)*
a^(11/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*b^4*d) + (a^(11/3)*Log[a - b*x^3])/(3*2^(1/3)*b^4*d) - (a^(11/3)*Lo
g[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*b^4*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^3(a+bx)^{2/3}}{ad-bdx}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{a^2(a+bx)^{2/3}}{b^3d}+\frac{a(a+bx)^{5/3}}{b^3d}-\frac{(a+bx)^{8/3}}{b^3d}+\frac{a^3(a+bx)^{2/3}}{b^3(ad-bdx)}\right)\,dx, x, x^3\right)\\
&= -\frac{a^2(a+bx^3)^{5/3}}{5b^4d}+\frac{a(a+bx^3)^{8/3}}{8b^4d}-\frac{(a+bx^3)^{11/3}}{11b^4d}+\frac{a^3\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{ad-bdx}\,dx, x, x^3\right)}{3b^3}\\
&= -\frac{a^3(a+bx^3)^{2/3}}{2b^4d}-\frac{a^2(a+bx^3)^{5/3}}{5b^4d}+\frac{a(a+bx^3)^{8/3}}{8b^4d}\\
&\quad -\frac{(a+bx^3)^{11/3}}{11b^4d}+\frac{(2a^4)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(ad-bdx)}\,dx, x, x^3\right)}{3b^3}\\
&= -\frac{a^3(a+bx^3)^{2/3}}{2b^4d}-\frac{a^2(a+bx^3)^{5/3}}{5b^4d}+\frac{a(a+bx^3)^{8/3}}{8b^4d}-\frac{(a+bx^3)^{11/3}}{11b^4d}\\
&\quad +\frac{a^{11/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}b^4d}+\frac{a^{11/3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^4d}\\
&\quad -\frac{a^4\text{Subst}\left(\int \frac{1}{2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{b^4d}
\end{aligned}$$

$$= -\frac{a^3(a+bx^3)^{2/3}}{2b^4d} - \frac{a^2(a+bx^3)^{5/3}}{5b^4d} + \frac{a(a+bx^3)^{8/3}}{8b^4d} - \frac{(a+bx^3)^{11/3}}{11b^4d} + \frac{a^{11/3}\log(a-bx^3)}{3\sqrt[3]{2}b^4d}$$
$$- \frac{a^{11/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^4d} + \frac{\left(2^{2/3}a^{11/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{b^4d}$$
$$= -\frac{a^3(a+bx^3)^{2/3}}{2b^4d} - \frac{a^2(a+bx^3)^{5/3}}{5b^4d} + \frac{a(a+bx^3)^{8/3}}{8b^4d}$$
$$- \frac{(a+bx^3)^{11/3}}{11b^4d} - \frac{2^{2/3}a^{11/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}b^4d}$$
$$+ \frac{a^{11/3}\log(a-bx^3)}{3\sqrt[3]{2}b^4d} - \frac{a^{11/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^4d}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.91

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{ad-bdx^3}\,dx =$$
$$-\frac{3(a+bx^3)^{2/3}\left(293a^3+98a^2bx^3+65ab^2x^6+40b^3x^9\right)+440\ 2^{2/3}\sqrt{3}a^{11/3}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+440}{1320}$$

```
[In] Integrate[(x^11*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/1320*(3*(a + b*x^3)^(2/3)*(293*a^3 + 98*a^2*b*x^3 + 65*a*b^2*x^6 + 40*b^
3*x^9) + 440*2^(2/3)*Sqrt[3]*a^(11/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3
))/a^(1/3))/Sqrt[3]] + 440*2^(2/3)*a^(11/3)*Log[-2*a^(1/3) + 2^(2/3)*(a + b
*x^3)^(1/3)] - 220*2^(2/3)*a^(11/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*
x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(b^4*d)
```

Maple [A] (verified)

Time = 4.64 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{220\,2^{\frac{2}{3}}\left(-2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)-2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)\right)}{1320b^{4}d}$

```
[In] int(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1320*(220*2^(2/3)*(-2*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3
)*3^(1/2))*3^(1/2)+ln((b*x^3+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/
3)*a^(2/3))-2*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3)))*a^(11/3)-3*(b*x^3+a)^(2/
3)*(40*b^3*x^9+65*a*b^2*x^6+98*a^2*b*x^3+293*a^3))/b^4/d
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.94

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{ad-bdx^3}\,dx =$$

$$-\frac{440\cdot 4^{\frac{1}{3}}\sqrt{3}(-a^2)^{\frac{1}{3}}\,a^3\arctan\left(\frac{4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}(-a^2)^{\frac{1}{3}}-\sqrt{3}a}{3\,a}\right)+220\cdot 4^{\frac{1}{3}}(-a^2)^{\frac{1}{3}}\,a^3\log\left(4^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}(-a^2)^{\frac{2}{3}}+2\,(b\right.}{}$$

```
[In] integrate(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/1320*(440*4^(1/3)*sqrt(3)*(-a^2)^(1/3)*a^3*arctan(1/3*(4^(1/3)*sqrt(3)*(
b*x^3 + a)^(1/3)*(-a^2)^(1/3) - sqrt(3)*a)/a) + 220*4^(1/3)*(-a^2)^(1/3)*a^
3*log(4^(2/3)*(b*x^3 + a)^(1/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(2/3)*a - 2*4^
(1/3)*(-a^2)^(1/3)*a) - 440*4^(1/3)*(-a^2)^(1/3)*a^3*log(-4^(2/3)*(-a^2)^(2
/3) + 2*(b*x^3 + a)^(1/3)*a) + 3*(40*b^3*x^9 + 65*a*b^2*x^6 + 98*a^2*b*x^3
+ 293*a^3)*(b*x^3 + a)^(2/3))/(b^4*d)
```

Sympy [F]

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^{11}(a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

[In] integrate(x**11*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)

[Out] -Integral(x**11*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.82

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\frac{440\sqrt{3}2^{\frac{2}{3}}a^{\frac{11}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{d} - \frac{220\cdot 2^{\frac{2}{3}}a^{\frac{11}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d} + \frac{440\cdot 2^{\frac{2}{3}}a^{\frac{11}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(\right.\right.}{d}}{1320\,b^4}$$

[In] integrate(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")

```
[Out] -1/1320*(440*sqrt(3)*2^(2/3)*a^(11/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a
^(1/3) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d - 220*2^(2/3)*a^(11/3)*log(2^(2/3)
*a^(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d + 440*2
^(2/3)*a^(11/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d + 3*(40*(b*x^3
+ a)^(11/3) - 55*(b*x^3 + a)^(8/3)*a + 88*(b*x^3 + a)^(5/3)*a^2 + 220*(b*x^
3 + a)^(2/3)*a^3)/d)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.75 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.96

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\sqrt{3}2^{\frac{2}{3}}a^{\frac{11}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{3\,b^4d} + \frac{2^{\frac{2}{3}}a^{\frac{11}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,b^4d} - \frac{2^{\frac{2}{3}}a^{\frac{11}{3}}\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,b^4d} - \frac{40\,(bx^3+a)^{\frac{11}{3}}b^{40}d^{10} - 55\,(bx^3+a)^{\frac{8}{3}}ab^{40}d^{10} + 88\,(bx^3+a)^{\frac{5}{3}}a^2b^{40}d^{10} + 220\,(bx^3+a)^{\frac{2}{3}}a^3b^{40}d^{10}}{440\,b^{44}d^{11}}$$

```
[In] integrate(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*2^(2/3)*a^(11/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) +
 2*(b*x^3 + a)^(1/3))/a^(1/3))/(b^4*d) + 1/6*2^(2/3)*a^(11/3)*log(2^(2/3)*a
^(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(b^4*d) - 1
/3*2^(2/3)*a^(11/3)*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(b^4*d)
- 1/440*(40*(b*x^3 + a)^(11/3)*b^40*d^10 - 55*(b*x^3 + a)^(8/3)*a*b^40*d^10
 + 88*(b*x^3 + a)^(5/3)*a^2*b^40*d^10 + 220*(b*x^3 + a)^(2/3)*a^3*b^40*d^10
)/(b^44*d^11)
```

Mupad [B] (verification not implemented)

Time = 8.72 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.17

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \frac{a\,(b\,x^3+a)^{8/3}}{8\,b^4\,d} - \frac{a^3\,(b\,x^3+a)^{2/3}}{2\,b^4\,d} - \frac{a^2\,(b\,x^3+a)^{5/3}}{5\,b^4\,d} - \frac{(b\,x^3+a)^{11/3}}{11\,b^4\,d} + \frac{4^{1/3}\,(-a)^{11/3}\,\ln\left(4\,a^8\,(b\,x^3+a)^{1/3}+4\,2^{1/3}\,(-a)^{25/3}\right)}{3\,b^4\,d} - \frac{4^{1/3}\,(-a)^{11/3}\,\ln\left(\frac{4\,a^8\,(b\,x^3+a)^{1/3}}{b^8\,d^2}+\frac{2\,4^{2/3}\,(-a)^{25/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{b^8\,d^2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b^4\,d} + \frac{4^{1/3}\,(-a)^{11/3}\,\ln\left(\frac{4\,a^8\,(b\,x^3+a)^{1/3}}{b^8\,d^2}+\frac{18\,4^{2/3}\,(-a)^{25/3}\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2}{b^8\,d^2}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{b^4\,d}$$

```
[In] int((x^11*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

```
[Out] (a*(a + b*x^3)^(8/3))/(8*b^4*d) - (a^3*(a + b*x^3)^(2/3))/(2*b^4*d) - (a^2*
(a + b*x^3)^(5/3))/(5*b^4*d) - (a + b*x^3)^(11/3)/(11*b^4*d) + (4^(1/3)*(-a
)^(11/3)*log(4*a^8*(a + b*x^3)^(1/3) + 4*2^(1/3)*(-a)^(25/3)))/(3*b^4*d) -
(4^(1/3)*(-a)^(11/3)*log((4*a^8*(a + b*x^3)^(1/3))/(b^8*d^2) + (2*4^(2/3)*(
-a)^(25/3)*((3^(1/2)*1i)/2 + 1/2)^2)/(b^8*d^2))*((3^(1/2)*1i)/2 + 1/2))/(3*
b^4*d) + (4^(1/3)*(-a)^(11/3)*log((4*a^8*(a + b*x^3)^(1/3))/(b^8*d^2) + (18
*4^(2/3)*(-a)^(25/3)*((3^(1/2)*1i)/6 - 1/6)^2)/(b^8*d^2))*((3^(1/2)*1i)/6 -
 1/6))/(b^4*d)
```

3.587 $\int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3} dx$

Optimal result

Integrand size = 28, antiderivative size = 177

$$\int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3} dx = -\frac{a^2(a+bx^3)^{2/3}}{2b^3d} - \frac{(a+bx^3)^{8/3}}{8b^3d} - \frac{2^{2/3}a^{8/3}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^3d} + \frac{a^{8/3}\log(a-bx^3)}{3\sqrt[3]{2}b^3d} - \frac{a^{8/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^3d}$$

```
[Out] -1/2*a^2*(b*x^3+a)^(2/3)/b^3/d-1/8*(b*x^3+a)^(8/3)/b^3/d+1/6*a^(8/3)*ln(-b*
x^3+a)*2^(2/3)/b^3/d-1/2*a^(8/3)*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(2/3
)/b^3/d-1/3*2^(2/3)*a^(8/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^
(1/3)*3^(1/2))/b^3/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {457, 90, 52, 57, 631, 210, 31}

$$\int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3} dx = -\frac{2^{2/3}a^{8/3}\arctan\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^3d} + \frac{a^{8/3}\log(a-bx^3)}{3\sqrt[3]{2}b^3d} - \frac{a^{8/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^3d} - \frac{a^2(a+bx^3)^{2/3}}{2b^3d} - \frac{(a+bx^3)^{8/3}}{8b^3d}$$

```
[In] Int[(x^8*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/2*(a^2*(a + b*x^3)^(2/3))/(b^3*d) - (a + b*x^3)^(8/3)/(8*b^3*d) - (2^(2/
3)*a^(8/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])
/(Sqrt[3]*b^3*d) + (a^(8/3)*Log[a - b*x^3])/(3*2^(1/3)*b^3*d) - (a^(8/3)*Lo
g[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*b^3*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x^2(a+bx)^{2/3}}{ad-bdx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{(a+bx)^{5/3}}{b^2d} + \frac{a^2(a+bx)^{2/3}}{b^2(ad-bdx)}\right)dx, x, x^3\right)$$

$$= -\frac{(a+bx^3)^{8/3}}{8b^3d} + \frac{a^2\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{ad-bdx}\,dx, x, x^3\right)}{3b^2}$$

$$= -\frac{a^2(a+bx^3)^{2/3}}{2b^3d} - \frac{(a+bx^3)^{8/3}}{8b^3d} + \frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(ad-bdx)}\,dx, x, x^3\right)}{3b^2}$$

$$= -\frac{a^2(a+bx^3)^{2/3}}{2b^3d} - \frac{(a+bx^3)^{8/3}}{8b^3d} + \frac{a^{8/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}b^3d} + \frac{a^{8/3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^3d} - \frac{a^3\text{Subst}\left(\int \frac{1}{2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{b^3d}$$

$$= -\frac{a^2(a+bx^3)^{2/3}}{2b^3d} - \frac{(a+bx^3)^{8/3}}{8b^3d} + \frac{a^{8/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}b^3d} - \frac{a^{8/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^3d} + \frac{\left(2^{2/3}a^{8/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{b^3d}$$

$$= -\frac{a^2(a+bx^3)^{2/3}}{2b^3d} - \frac{(a+bx^3)^{8/3}}{8b^3d} - \frac{2^{2/3}a^{8/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}b^3d} + \frac{a^{8/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}b^3d} - \frac{a^{8/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^3d}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.19

$$\int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3}\,dx =$$

$$-\frac{15a^2(a+bx^3)^{2/3}+6abx^3(a+bx^3)^{2/3}+3b^2x^6(a+bx^3)^{2/3}+8\ 2^{2/3}\sqrt{3}a^{8/3}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+8\ 2}{24b^3d}$$

[In] Integrate[(x^8*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]

[Out] -1/24*(15*a^2*(a + b*x^3)^(2/3) + 6*a*b*x^3*(a + b*x^3)^(2/3) + 3*b^2*x^6*(a + b*x^3)^(2/3) + 8*2^(2/3)*Sqrt[3]*a^(8/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 8*2^(2/3)*a^(8/3)*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)] - 4*2^(2/3)*a^(8/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(b^3*d)

Maple [A] (verified)

Time = 4.79 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.77

method	result
pseudoelliptic	$\frac{4\,2^{\frac{2}{3}}\left(-2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)-2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)\right)a^{\frac{8}{3}}}{24b^3d}$

[In] int(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)

[Out] 1/24*(4*2^(2/3)*(-2*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)+ln((b*x^3+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*a^(2/3))-2*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3)))*a^(8/3)-3*(b*x^3+a)^(2/3)*(b^2*x^6+2*a*b*x^3+5*a^2))/b^3/d

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.11

$$\int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{8\cdot 4^{\frac{1}{3}}\sqrt{3}(-a^2)^{\frac{1}{3}}\,a^2\arctan\left(\frac{4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}(-a^2)^{\frac{1}{3}}-\sqrt{3}a}{3a}\right)+4\cdot 4^{\frac{1}{3}}(-a^2)^{\frac{1}{3}}\,a^2\log\left(4^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}(-a^2)^{\frac{2}{3}}+2\,(bx^3\right.}{}$$

```
[In] integrate(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/24*(8*4^(1/3)*sqrt(3)*(-a^2)^(1/3)*a^2*arctan(1/3*(4^(1/3)*sqrt(3)*(b*x^
3 + a)^(1/3)*(-a^2)^(1/3) - sqrt(3)*a)/a) + 4*4^(1/3)*(-a^2)^(1/3)*a^2*log(
4^(2/3)*(b*x^3 + a)^(1/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(2/3)*a - 2*4^(1/3)*
(-a^2)^(1/3)*a) - 8*4^(1/3)*(-a^2)^(1/3)*a^2*log(-4^(2/3)*(-a^2)^(2/3) + 2*
(b*x^3 + a)^(1/3)*a) + 3*(b^2*x^6 + 2*a*b*x^3 + 5*a^2)*(b*x^3 + a)^(2/3))/(
b^3*d)
```

Sympy [F]

$$\int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^8(a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

```
[In] integrate(x**8*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**8*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.88

$$\int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\frac{8\sqrt{3}2^{\frac{2}{3}}a^{\frac{8}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{d}-\frac{4\cdot 2^{\frac{2}{3}}a^{\frac{8}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d}+\frac{8\cdot 2^{\frac{2}{3}}a^{\frac{8}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right.}{d}}{24\,b^3}$$

```
[In] integrate(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -1/24*(8*sqrt(3)*2^(2/3)*a^(8/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3
) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d - 4*2^(2/3)*a^(8/3)*log(2^(2/3)*a^(2/3)
 + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d + 8*2^(2/3)*a^(
8/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d + 3*((b*x^3 + a)^(8/3) + 4
*(b*x^3 + a)^(2/3)*a^2)/d)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.75 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.99

$$\int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\sqrt{3}2^{\frac{2}{3}}a^{\frac{8}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(bx^3+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{3\,b^3d} + \frac{2^{\frac{2}{3}}a^{\frac{8}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,b^3d} - \frac{2^{\frac{2}{3}}a^{\frac{8}{3}}\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,b^3d} - \frac{(bx^3+a)^{\frac{8}{3}}b^{21}d^7+4\,(bx^3+a)^{\frac{2}{3}}a^2b^{21}d^7}{8\,b^{24}d^8}$$

```
[In] integrate(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*2^(2/3)*a^(8/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) +
2*(b*x^3 + a)^(1/3))/a^(1/3))/(b^3*d) + 1/6*2^(2/3)*a^(8/3)*log(2^(2/3)*a^(
2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(b^3*d) - 1/3
*2^(2/3)*a^(8/3)*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(b^3*d) - 1
/8*((b*x^3 + a)^(8/3)*b^21*d^7 + 4*(b*x^3 + a)^(2/3)*a^2*b^21*d^7)/(b^24*d^
8)
```

Mupad [B] (verification not implemented)

Time = 8.76 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.16

$$\int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{(b\,x^3+a)^{8/3}}{8\,b^3\,d} - \frac{a^2\,(b\,x^3+a)^{2/3}}{2\,b^3\,d} - \frac{4^{1/3}\,a^{8/3}\,\ln\left((b\,x^3+a)^{1/3}-2^{1/3}\,a^{1/3}\right)}{3\,b^3\,d} - \frac{4^{1/3}\,a^{8/3}\,\ln\left(\frac{4\,a^6\,(b\,x^3+a)^{1/3}}{b^6\,d^2}-\frac{2\,4^{2/3}\,a^{19/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{b^6\,d^2}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b^3\,d} + \frac{4^{1/3}\,a^{8/3}\,\ln\left(\frac{4\,a^6\,(b\,x^3+a)^{1/3}}{b^6\,d^2}-\frac{18\,4^{2/3}\,a^{19/3}\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2}{b^6\,d^2}\right)\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{b^3\,d}$$

```
[In] int((x^8*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

```
[Out] (4^(1/3)*a^(8/3)*log((4*a^6*(a + b*x^3)^(1/3))/(b^6*d^2) - (18*4^(2/3)*a^(1
9/3)*((3^(1/2)*1i)/6 + 1/6)^2)/(b^6*d^2))*((3^(1/2)*1i)/6 + 1/6))/(b^3*d) -
 (a^2*(a + b*x^3)^(2/3))/(2*b^3*d) - (4^(1/3)*a^(8/3)*log((a + b*x^3)^(1/3)
 - 2^(1/3)*a^(1/3)))/(3*b^3*d) - (4^(1/3)*a^(8/3)*log((4*a^6*(a + b*x^3)^(1
/3))/(b^6*d^2) - (2*4^(2/3)*a^(19/3)*((3^(1/2)*1i)/2 - 1/2)^2)/(b^6*d^2))*(
(3^(1/2)*1i)/2 - 1/2))/(3*b^3*d) - (a + b*x^3)^(8/3)/(8*b^3*d)
```

3.588 $\int \frac{x^5(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 175

$$\int \frac{x^5(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{a(a+bx^3)^{2/3}}{2b^2d} - \frac{(a+bx^3)^{5/3}}{5b^2d} - \frac{2^{2/3}a^{5/3}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^2d} + \frac{a^{5/3}\log(a-bx^3)}{3\sqrt[3]{2}b^2d} - \frac{a^{5/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^2d}$$

```
[Out] -1/2*a*(b*x^3+a)^(2/3)/b^2/d-1/5*(b*x^3+a)^(5/3)/b^2/d+1/6*a^(5/3)*ln(-b*x^
3+a)*2^(2/3)/b^2/d-1/2*a^(5/3)*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(2/3)/
b^2/d-1/3*2^(2/3)*a^(5/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1
/3)*3^(1/2))/b^2/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {457, 81, 52, 57, 631, 210, 31}

$$\int \frac{x^5(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{2^{2/3}a^{5/3}\arctan\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^2d} + \frac{a^{5/3}\log(a-bx^3)}{3\sqrt[3]{2}b^2d} - \frac{a^{5/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^2d} - \frac{a(a+bx^3)^{2/3}}{2b^2d} - \frac{(a+bx^3)^{5/3}}{5b^2d}$$

```
[In] Int[(x^5*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/2*(a*(a + b*x^3)^(2/3))/(b^2*d) - (a + b*x^3)^(5/3)/(5*b^2*d) - (2^(2/3)
*a^(5/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*b^2*d) + (a^(5/3)*Log[a - b*x^3])/(3*2^(1/3)*b^2*d) - (a^(5/3)*Log[
2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*b^2*d)
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

<u>Rule 52</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

<u>Rule 57</u>

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

<u>Rule 81</u>

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

<u>Rule 210</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

<u>Rule 457</u>

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x(a+bx)^{2/3}}{ad-bdx}\,dx, x, x^3\right) \\
&= -\frac{(a+bx^3)^{5/3}}{5b^2d} + \frac{a\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{ad-bdx}\,dx, x, x^3\right)}{3b} \\
&= -\frac{a(a+bx^3)^{2/3}}{2b^2d} - \frac{(a+bx^3)^{5/3}}{5b^2d} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(ad-bdx)}\,dx, x, x^3\right)}{3b} \\
&= -\frac{a(a+bx^3)^{2/3}}{2b^2d} - \frac{(a+bx^3)^{5/3}}{5b^2d} + \frac{a^{5/3}\log(a-bx^3)}{3\sqrt[3]{2}b^2d} \\
&\quad + \frac{a^{5/3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^2d} \\
&\quad - \frac{a^2\text{Subst}\left(\int \frac{1}{2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{b^2d} \\
&= -\frac{a(a+bx^3)^{2/3}}{2b^2d} - \frac{(a+bx^3)^{5/3}}{5b^2d} + \frac{a^{5/3}\log(a-bx^3)}{3\sqrt[3]{2}b^2d} - \frac{a^{5/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^2d} \\
&\quad + \frac{(2^{2/3}a^{5/3})\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{b^2d} \\
&= -\frac{a(a+bx^3)^{2/3}}{2b^2d} - \frac{(a+bx^3)^{5/3}}{5b^2d} - \frac{2^{2/3}a^{5/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}b^2d} \\
&\quad + \frac{a^{5/3}\log(a-bx^3)}{3\sqrt[3]{2}b^2d} - \frac{a^{5/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.07

$$\int \frac{x^5(a+bx^3)^{2/3}}{ad-bdx^3}\,dx =$$

$$-\frac{21a(a+bx^3)^{2/3}+6bx^3(a+bx^3)^{2/3}+10\ 2^{2/3}\sqrt{3}a^{5/3}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+10\ 2^{2/3}a^{5/3}\log\left(-2\sqrt[3]{a}\right.}{30b^2d}$$

```
[In] Integrate[(x^5*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/30*(21*a*(a + b*x^3)^(2/3) + 6*b*x^3*(a + b*x^3)^(2/3) + 10*2^(2/3)*Sqrt
[3]*a^(5/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 10*
2^(2/3)*a^(5/3)*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)] - 5*2^(2/3)*a^(
5/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3
)^(2/3)])/(b^2*d)
```

Maple [A] (verified)

Time = 5.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$\frac{-5\,2^{\frac{2}{3}}\left(2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)-\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)\right)a}{30b^2d}$

```
[In] int(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30*(-5*2^(2/3)*(2*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^
(1/2))*3^(1/2)+2*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3))-ln((b*x^3+a)^(2/3)+2^(
1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*a^(2/3)))*a^(5/3)-3*(b*x^3+a)^(2/3)*(2
*b*x^3+7*a))/b^2/d
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.03

$$\int \frac{x^5(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{10\cdot 4^{\frac{1}{3}}\sqrt{3}(-a^2)^{\frac{1}{3}}\,a\arctan\left(\frac{4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}(-a^2)^{\frac{1}{3}}-\sqrt{3}a}{3\,a}\right)+5\cdot 4^{\frac{1}{3}}(-a^2)^{\frac{1}{3}}\,a\log\left(4^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}(-a^2)^{\frac{2}{3}}+2\,(bx^3+}{30\,b^2}$$

```
[In] integrate(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/30*(10*4^(1/3)*sqrt(3)*(-a^2)^(1/3)*a*arctan(1/3*(4^(1/3)*sqrt(3)*(b*x^3
 + a)^(1/3)*(-a^2)^(1/3) - sqrt(3)*a)/a) + 5*4^(1/3)*(-a^2)^(1/3)*a*log(4^(
2/3)*(b*x^3 + a)^(1/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(2/3)*a - 2*4^(1/3)*(-a
^2)^(1/3)*a) - 10*4^(1/3)*(-a^2)^(1/3)*a*log(-4^(2/3)*(-a^2)^(2/3) + 2*(b*x
^3 + a)^(1/3)*a) + 3*(2*b*x^3 + 7*a)*(b*x^3 + a)^(2/3))/(b^2*d)
```

Sympy [F]

$$\int \frac{x^5(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^5(a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

```
[In] integrate(x**5*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**5*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.89

$$\int \frac{x^5(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\frac{10\sqrt{3}2^{\frac{2}{3}}a^{\frac{5}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{d}-\frac{5\cdot 2^{\frac{2}{3}}a^{\frac{5}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d}+\frac{10\cdot 2^{\frac{2}{3}}a^{\frac{5}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}}{d}}{30\,b^2}$$

```
[In] integrate(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -1/30*(10*sqrt(3)*2^(2/3)*a^(5/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/
3) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d - 5*2^(2/3)*a^(5/3)*log(2^(2/3)*a^(2/3
) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d + 10*2^(2/3)*a
^(5/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d + 3*(2*(b*x^3 + a)^(5/3)
 + 5*(b*x^3 + a)^(2/3)*a)/d)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.73 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00

$$\int \frac{x^5(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\sqrt{3}2^{\frac{2}{3}}a^{\frac{5}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(bx^3+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{3\,b^2d} + \frac{2^{\frac{2}{3}}a^{\frac{5}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,b^2d} - \frac{2^{\frac{2}{3}}a^{\frac{5}{3}}\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,b^2d} - \frac{2\,(bx^3+a)^{\frac{5}{3}}b^8d^4+5\,(bx^3+a)^{\frac{2}{3}}ab^8d^4}{10\,b^{10}d^5}$$

```
[In] integrate(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*2^(2/3)*a^(5/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) +
2*(b*x^3 + a)^(1/3))/a^(1/3))/(b^2*d) + 1/6*2^(2/3)*a^(5/3)*log(2^(2/3)*a^(
2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(b^2*d) - 1/3
*2^(2/3)*a^(5/3)*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(b^2*d) - 1
/10*(2*(b*x^3 + a)^(5/3)*b^8*d^4 + 5*(b*x^3 + a)^(2/3)*a*b^8*d^4)/(b^10*d^5
)
```

Mupad [B] (verification not implemented)

Time = 8.66 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.26

$$\int \frac{x^5(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \frac{4^{1/3}\,{(-a)}^{5/3}\,\ln\left(4\,a^4\,{\left(b\,x^3+a\right)}^{1/3}+4\,2^{1/3}\,{(-a)}^{13/3}\right)}{3\,b^2\,d} - \frac{a\,{\left(b\,x^3+a\right)}^{2/3}}{2\,b^2\,d} - \frac{{\left(b\,x^3+a\right)}^{5/3}}{5\,b^2\,d} - \frac{4^{1/3}\,{(-a)}^{5/3}\,\ln\left(\frac{4\,a^4\,{\left(b\,x^3+a\right)}^{1/3}}{b^4\,d^2}+\frac{2\,4^{2/3}\,{(-a)}^{13/3}\,{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2}{b^4\,d^2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b^2\,d} + \frac{4^{1/3}\,{(-a)}^{5/3}\,\ln\left(\frac{4\,a^4\,{\left(b\,x^3+a\right)}^{1/3}}{b^4\,d^2}+\frac{18\,4^{2/3}\,{(-a)}^{13/3}\,{\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}^2}{b^4\,d^2}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{b^2\,d}$$

```
[In] int((x^5*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

```
[Out] (4^(1/3)*(-a)^(5/3)*log(4*a^4*(a + b*x^3)^(1/3) + 4*2^(1/3)*(-a)^(13/3)))/(
3*b^2*d) - (a*(a + b*x^3)^(2/3))/(2*b^2*d) - (a + b*x^3)^(5/3)/(5*b^2*d) -
(4^(1/3)*(-a)^(5/3)*log((4*a^4*(a + b*x^3)^(1/3))/(b^4*d^2) + (2*4^(2/3)*(-
a)^(13/3)*((3^(1/2)*1i)/2 + 1/2)^2)/(b^4*d^2))*((3^(1/2)*1i)/2 + 1/2))/(3*b
^2*d) + (4^(1/3)*(-a)^(5/3)*log((4*a^4*(a + b*x^3)^(1/3))/(b^4*d^2) + (18*4
^(2/3)*(-a)^(13/3)*((3^(1/2)*1i)/6 - 1/6)^2)/(b^4*d^2))*((3^(1/2)*1i)/6 - 1
/6))/(b^2*d)
```

3.589 $\int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 153

$$\int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{(a+bx^3)^{2/3}}{2bd} - \frac{2^{2/3}a^{2/3}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}bd} + \frac{a^{2/3}\log(a-bx^3)}{3\sqrt[3]{2}bd} - \frac{a^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}bd}$$

```
[Out] -1/2*(b*x^3+a)^(2/3)/b/d+1/6*a^(2/3)*ln(-b*x^3+a)*2^(2/3)/b/d-1/2*a^(2/3)*l
n(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(2/3)/b/d-1/3*2^(2/3)*a^(2/3)*arctan(1
/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/b/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {455, 52, 57, 631, 210, 31}

$$\int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{2^{2/3}a^{2/3}\arctan\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}bd} + \frac{a^{2/3}\log(a-bx^3)}{3\sqrt[3]{2}bd} - \frac{a^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}bd} - \frac{(a+bx^3)^{2/3}}{2bd}$$

```
[In] Int[(x^2*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/2*(a + b*x^3)^(2/3)/(b*d) - (2^(2/3)*a^(2/3)*ArcTan[(a^(1/3) + 2^(2/3)*(
a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b*d) + (a^(2/3)*Log[a - b*x^
3])/(3*2^(1/3)*b*d) - (a^(2/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2
^(1/3)*b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{ad-bdx}\,dx, x, x^3\right)\\
&= -\frac{(a+bx^3)^{2/3}}{2bd} + \frac{1}{3}(2a)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(ad-bdx)}\,dx, x, x^3\right)\\
&= -\frac{(a+bx^3)^{2/3}}{2bd} + \frac{a^{2/3}\log(a-bx^3)}{3\sqrt[3]{2}bd} + \frac{a^{2/3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}bd}\\
&\quad - \frac{a\text{Subst}\left(\int \frac{1}{2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{bd}\\
&= -\frac{(a+bx^3)^{2/3}}{2bd} + \frac{a^{2/3}\log(a-bx^3)}{3\sqrt[3]{2}bd} - \frac{a^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}bd}\\
&\quad + \frac{\left(2^{2/3}a^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{bd}\\
&= -\frac{(a+bx^3)^{2/3}}{2bd} - \frac{2^{2/3}a^{2/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}bd}\\
&\quad + \frac{a^{2/3}\log(a-bx^3)}{3\sqrt[3]{2}bd} - \frac{a^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}bd}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.11

$$
\int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{3(a+bx^3)^{2/3} + 2\,2^{2/3}\sqrt{3}a^{2/3}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2\,2^{2/3}a^{2/3}\log\left(-2\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}\right) - 2^{2/3}\cdots}{6bd}
$$

```
[In] Integrate[(x^2*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/6*(3*(a + b*x^3)^(2/3) + 2*2^(2/3)*Sqrt[3]*a^(2/3)*ArcTan[(1 + (2^(2/3)*
(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 2*2^(2/3)*a^(2/3)*Log[-2*a^(1/3) + 2
^(2/3)*(a + b*x^3)^(1/3)] - 2^(2/3)*a^(2/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)
*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(b*d)
```

Maple [A] (verified)

Time = 4.59 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$\frac{-2a^{\frac{2}{3}}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)-2a^{\frac{2}{3}}2^{\frac{2}{3}}\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)+a^{\frac{2}{3}}2^{\frac{2}{3}}\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\right.}{6bd}$

```
[In] int(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-2*a^(2/3)*3^(1/2)*2^(2/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3)
)/a^(1/3)*3^(1/2))-2*a^(2/3)*2^(2/3)*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3))+a^
(2/3)*2^(2/3)*ln((b*x^3+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*a^
(2/3))-3*(b*x^3+a)^(2/3))/b/d
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.09

$$\int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{2\cdot 4^{\frac{1}{3}}\sqrt{3}(-a^2)^{\frac{1}{3}}\arctan\left(\frac{4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}(-a^2)^{\frac{1}{3}}-\sqrt{3}a}{3\,a}\right)+4^{\frac{1}{3}}(-a^2)^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}(-a^2)^{\frac{2}{3}}+2\,(bx^3+a)^{\frac{2}{3}}a-\right.}{6\,bd}$$

```
[In] integrate(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/6*(2*4^(1/3)*sqrt(3)*(-a^2)^(1/3)*arctan(1/3*(4^(1/3)*sqrt(3)*(b*x^3 + a
)^(1/3)*(-a^2)^(1/3) - sqrt(3)*a)/a) + 4^(1/3)*(-a^2)^(1/3)*log(4^(2/3)*(b*
x^3 + a)^(1/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(2/3)*a - 2*4^(1/3)*(-a^2)^(1/3
)*a) - 2*4^(1/3)*(-a^2)^(1/3)*log(-4^(2/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(1/
3)*a) + 3*(b*x^3 + a)^(2/3))/(b*d)
```

Sympy [F]

$$\int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^2\left(a+bx^3\right)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

```
[In] integrate(x**2*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**2*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.92

$$\int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\frac{2\sqrt{3}2^{\frac{2}{3}}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{d}-\frac{2^{\frac{2}{3}}a^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d}+\frac{2\cdot 2^{\frac{2}{3}}a^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right)}{d}}{6b}$$

```
[In] integrate(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(3)*2^(2/3)*a^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3)
 + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d - 2^(2/3)*a^(2/3)*log(2^(2/3)*a^(2/3) +
2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d + 2*2^(2/3)*a^(2/3
)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d + 3*(b*x^3 + a)^(2/3)/d)/b
```

Giac [A] (verification not implemented)

none

Time = 0.76 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.97

$$\int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\sqrt{3}2^{\frac{2}{3}}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{3\,bd} + \frac{2^{\frac{2}{3}}a^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,bd} - \frac{2^{\frac{2}{3}}a^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,bd} - \frac{(bx^3+a)^{\frac{2}{3}}}{2\,bd}$$

```
[In] integrate(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*2^(2/3)*a^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) +
2*(b*x^3 + a)^(1/3))/a^(1/3))/(b*d) + 1/6*2^(2/3)*a^(2/3)*log(2^(2/3)*a^(2/
3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(b*d) - 1/3*2^(
2/3)*a^(2/3)*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(b*d) - 1/2*(b*
x^3 + a)^(2/3)/(b*d)
```

Mupad [B] (verification not implemented)

Time = 8.66 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.22

$$\int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{(b\,x^3+a)^{2/3}}{2\,b\,d} - \frac{4^{1/3}\,a^{2/3}\,\ln\left((b\,x^3+a)^{1/3}-2^{1/3}\,a^{1/3}\right)}{3\,b\,d} - \frac{4^{1/3}\,a^{2/3}\,\ln\left(\frac{4\,a^2\,(b\,x^3+a)^{1/3}}{b^2\,d^2}-\frac{2\,4^{2/3}\,a^{7/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{b^2\,d^2}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b\,d} + \frac{4^{1/3}\,a^{2/3}\,\ln\left(\frac{4\,a^2\,(b\,x^3+a)^{1/3}}{b^2\,d^2}-\frac{18\,4^{2/3}\,a^{7/3}\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2}{b^2\,d^2}\right)\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{b\,d}$$

```
[In] int((x^2*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

```
[Out] (4^(1/3)*a^(2/3)*log((4*a^2*(a + b*x^3)^(1/3))/(b^2*d^2) - (18*4^(2/3)*a^(7
/3)*((3^(1/2)*1i)/6 + 1/6)^2)/(b^2*d^2))*((3^(1/2)*1i)/6 + 1/6))/(b*d) - (4
^(1/3)*a^(2/3)*log((a + b*x^3)^(1/3) - 2^(1/3)*a^(1/3)))/(3*b*d) - (4^(1/3)
*a^(2/3)*log((4*a^2*(a + b*x^3)^(1/3))/(b^2*d^2) - (2*4^(2/3)*a^(7/3)*((3^(
1/2)*1i)/2 - 1/2)^2)/(b^2*d^2))*((3^(1/2)*1i)/2 - 1/2))/(3*b*d) - (a + b*x^
3)^(2/3)/(2*b*d)
```

3.590 $\int \frac{(a+bx^3)^{2/3}}{x(ad-bdx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 214

$$\int \frac{(a+bx^3)^{2/3}}{x(ad-bdx^3)}\,dx = \frac{\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}d}-\frac{2^{2/3}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}d}-\frac{\log(x)}{2\sqrt[3]{a}d}+\frac{\log(a-bx^3)}{3\sqrt[3]{2}\sqrt[3]{a}d}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}d}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\sqrt[3]{a}d}$$

```
[Out] -1/2*ln(x)/a^(1/3)/d+1/6*ln(-b*x^3+a)*2^(2/3)/a^(1/3)/d+1/2*ln(a^(1/3)-(b*x
^3+a)^(1/3))/a^(1/3)/d-1/2*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(2/3)/a^(1
/3)/d+1/3*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(1/3)/d
*3^(1/2)-1/3*2^(2/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3
^(1/2))/a^(1/3)/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {457, 85, 57, 631, 210, 31}

$$\int \frac{(a+bx^3)^{2/3}}{x(ad-bdx^3)}\,dx = \frac{\arctan\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}d}-\frac{2^{2/3}\arctan\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}d}+\frac{\log(a-bx^3)}{3\sqrt[3]{2}\sqrt[3]{a}d}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}d}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\sqrt[3]{a}d}-\frac{\log(x)}{2\sqrt[3]{a}d}$$

```
[In] Int[(a + b*x^3)^(2/3)/(x*(a*d - b*d*x^3)),x]
```

```
[Out] ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3)*
d) - (2^(2/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3)
)])/(Sqrt[3]*a^(1/3)*d) - Log[x]/(2*a^(1/3)*d) + Log[a - b*x^3]/(3*2^(1/3)*
a^(1/3)*d) + Log[a^(1/3) - (a + b*x^3)^(1/3)]/(2*a^(1/3)*d) - Log[2^(1/3)*a
^(1/3) - (a + b*x^3)^(1/3)]/(2^(1/3)*a^(1/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x(ad-bdx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}(2b)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(ad-bdx)}\,dx, x, x^3\right) + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^3\right)}{3d}$$

$$= -\frac{\log(x)}{2\sqrt[3]{a}d} + \frac{\log(a-bx^3)}{3\sqrt[3]{2}\sqrt[3]{a}d} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d} - \frac{\text{Subst}\left(\int \frac{1}{2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{d} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}d} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\sqrt[3]{a}d}$$

$$= -\frac{\log(x)}{2\sqrt[3]{a}d} + \frac{\log(a-bx^3)}{3\sqrt[3]{2}\sqrt[3]{a}d} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}d} - \frac{\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\sqrt[3]{a}d} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}d} + \frac{2^{2/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}d}$$

$$= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}d} - \frac{2^{2/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}d} - \frac{\log(x)}{2\sqrt[3]{a}d} + \frac{\log(a-bx^3)}{3\sqrt[3]{2}\sqrt[3]{a}d} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}d} - \frac{\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\sqrt[3]{a}d}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^{2/3}}{x(ad-bdx^3)}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2\log\left(-\sqrt[3]{a}\right.}{}$$

```
[In] Integrate[(a + b*x^3)^(2/3)/(x*(a*d - b*d*x^3)),x]
```

[Out] (2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*2^(2/3)*Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 2*Log[-a^(1/3) + (a + b*x^3)^(1/3)] - 2*2^(2/3)*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)] - Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + 2^(2/3)*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)]])/(6*a^(1/3)*d)

Maple [A] (verified)

Time = 4.67 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.85

method	result
pseudoelliptic	$\frac{-2\sqrt{3}\,2^{\frac{2}{3}} \arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)-2\,2^{\frac{2}{3}} \ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)+2^{\frac{2}{3}} \ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)+}{6d\,a^{\frac{1}{3}}}$

[In] int((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)

[Out] 1/6*(-2*3^(1/2)*2^(2/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))-2*2^(2/3)*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3))+2^(2/3)*ln((b*x^3+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*a^(2/3))+2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)+2*ln((b*x^3+a)^(1/3)-a^(1/3))-ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3)))/d/a^(1/3)

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 530, normalized size of antiderivative = 2.48

$$\int \frac{(a+bx^3)^{2/3}}{x\,(ad-bdx^3)}\,dx = \left[-\frac{2\cdot 4^{\frac{1}{3}}\sqrt{3}a\left(-\frac{1}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\cdot 4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(-\frac{1}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-3\sqrt{\frac{1}{3}}a\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\right.}{}\right.$$

$$-\frac{2\cdot 4^{\frac{1}{3}}\sqrt{3}a\left(-\frac{1}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\cdot 4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(-\frac{1}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+4^{\frac{1}{3}}a\left(-\frac{1}{a}\right)^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}a\left(-\frac{1}{a}\right)^{\frac{2}{3}}-2\cdot\right.}{}$$

[In] integrate((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x, algorithm="fricas")

```
[Out] [-1/6*(2*4^(1/3)*sqrt(3)*a*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*sqrt(3)*(b*x^3 +
 a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) - 3*sqrt(1/3)*a*sqrt(-1/a^(2/3))*log(
(2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a -
 a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 + a)^(1/3)*a^(2/3) + 3*a)/x^3) + 4^(1
/3)*a*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 + a)^(1/3)*a*(-1/a)^(2/3) - 2*4^(1/3)
*a*(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) - 2*4^(1/3)*a*(-1/a)^(1/3)*log(-4^(2
/3)*a*(-1/a)^(2/3) + 2*(b*x^3 + a)^(1/3)) + a^(2/3)*log((b*x^3 + a)^(2/3) +
 (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) - 2*a^(2/3)*log((b*x^3 + a)^(1/3) - a
^(1/3)))/(a*d), -1/6*(2*4^(1/3)*sqrt(3)*a*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*s
qrt(3)*(b*x^3 + a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) + 4^(1/3)*a*(-1/a)^(1/
3)*log(4^(2/3)*(b*x^3 + a)^(1/3)*a*(-1/a)^(2/3) - 2*4^(1/3)*a*(-1/a)^(1/3)
+ 2*(b*x^3 + a)^(2/3)) - 2*4^(1/3)*a*(-1/a)^(1/3)*log(-4^(2/3)*a*(-1/a)^(2/
3) + 2*(b*x^3 + a)^(1/3)) - 6*sqrt(1/3)*a^(2/3)*arctan(sqrt(1/3)*(2*(b*x^3
+ a)^(1/3) + a^(1/3))/a^(1/3)) + a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a
)^(1/3)*a^(1/3) + a^(2/3)) - 2*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)))/(a
*d)]
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x\,(ad-bdx^3)}\,dx = -\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-ax+bx^4}\,dx}{d}$$

```
[In] integrate((b*x**3+a)**(2/3)/x/(-b*d*x**3+a*d),x)
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x + b*x**4), x)/d
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x\,(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.97 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^3)^{2/3}}{x(ad-bdx^3)}\,dx = -\frac{\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(bx^3+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{1}{3}}d} + \frac{2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,a^{\frac{1}{3}}d} - \frac{2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{1}{3}}d} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{1}{3}}d} - \frac{\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{1}{3}}d} + \frac{\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{1}{3}}d}$$

```
[In] integrate((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(b*x^3
 + a)^(1/3))/a^(1/3))/(a^(1/3)*d) + 1/6*2^(2/3)*log(2^(2/3)*a^(2/3) + 2^(1/
3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(a^(1/3)*d) - 1/3*2^(2/3)
*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(a^(1/3)*d) + 1/3*sqrt(3)*a
rctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(1/3)*d) - 1/
6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(1/3)*d)
+ 1/3*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(1/3)*d)
```

Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.72

$$\int \frac{(a+bx^3)^{2/3}}{x(ad-bdx^3)}\,dx = \ln\left(2\,(b\,x^3+a)^{1/3} - 2\,2^{1/3}\,a\,d^2\left(-\frac{1}{a\,d^3}\right)^{2/3}\right)\left(-\frac{4}{27\,a\,d^3}\right)^{1/3} + \ln\left((b\,x^3+a)^{1/3} - a\,d^2\left(\frac{1}{a\,d^3}\right)^{2/3}\right)\left(\frac{1}{27\,a\,d^3}\right)^{1/3} - \ln\left(4\,(b\,x^3+\right.$$

```
[In] int((a + b*x^3)^(2/3)/(x*(a*d - b*d*x^3)),x)
```

```
[Out] log(2*(a + b*x^3)^(1/3) - 2*2^(1/3)*a*d^2*(-1/(a*d^3))^(2/3))*(-4/(27*a*d^3
))^(1/3) + log((a + b*x^3)^(1/3) - a*d^2*(1/(a*d^3))^(2/3))*(1/(27*a*d^3))^
(1/3) - log(4*(a + b*x^3)^(1/3) + 2*2^(1/3)*a*d^2*(-1/(a*d^3))^(2/3) - 2^(1
```

```
/3)*3^(1/2)*a*d^2*(-1/(a*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 + 1/2)*(-4/(27*a*d
^3))^(1/3) + log(4*(a + b*x^3)^(1/3) + 2*2^(1/3)*a*d^2*(-1/(a*d^3))^(2/3) +
 2^(1/3)*3^(1/2)*a*d^2*(-1/(a*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 - 1/2)*(-4/(2
7*a*d^3))^(1/3) - log(2*(a + b*x^3)^(1/3) + a*d^2*(1/(a*d^3))^(2/3) - 3^(1/
2)*a*d^2*(1/(a*d^3))^(2/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(1/(27*a*d^3))^(1/3)
+ log(2*(a + b*x^3)^(1/3) + a*d^2*(1/(a*d^3))^(2/3) + 3^(1/2)*a*d^2*(1/(a*d
^3))^(2/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(1/(27*a*d^3))^(1/3)
```

3.591 $\int \frac{(a+bx^3)^{2/3}}{x^4(ad-bdx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 269

$$\int \frac{(a+bx^3)^{2/3}}{x^4(ad-bdx^3)}\,dx = \frac{b(a+bx^3)^{2/3}}{3a^2d} - \frac{(a+bx^3)^{5/3}}{3a^2dx^3} + \frac{5b\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}d} - \frac{2^{2/3}b\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}d} - \frac{5b\log(x)}{6a^{4/3}d} + \frac{b\log(a-bx^3)}{3\sqrt[3]{2}a^{4/3}d} + \frac{5b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}d} - \frac{b\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^{4/3}d}$$

```
[Out] 1/3*b*(b*x^3+a)^(2/3)/a^2/d-1/3*(b*x^3+a)^(5/3)/a^2/d/x^3-5/6*b*ln(x)/a^(4/
3)/d+1/6*b*ln(-b*x^3+a)*2^(2/3)/a^(4/3)/d+5/6*b*ln(a^(1/3)-(b*x^3+a)^(1/3))
/a^(4/3)/d-1/2*b*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(2/3)/a^(4/3)/d+5/9*
b*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(4/3)/d*3^(1/2)
-1/3*2^(2/3)*b*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2)
)/a^(4/3)/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {457, 105, 162, 52, 57, 631, 210, 31}

$$\int \frac{(a+bx^3)^{2/3}}{x^4(ad-bdx^3)}\,dx = \frac{5b\arctan\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}d} - \frac{2^{2/3}b\arctan\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}d} + \frac{b\log(a-bx^3)}{3\sqrt[3]{2}a^{4/3}d} + \frac{5b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}d} - \frac{b\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^{4/3}d} - \frac{5b\log(x)}{6a^{4/3}d} - \frac{(a+bx^3)^{5/3}}{3a^2dx^3} + \frac{b(a+bx^3)^{2/3}}{3a^2d}$$

```
[In] Int[(a + b*x^3)^(2/3)/(x^4*(a*d - b*d*x^3)),x]
```

```
[Out] (b*(a + b*x^3)^(2/3))/(3*a^2*d) - (a + b*x^3)^(5/3)/(3*a^2*d*x^3) + (5*b*Ar
cTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)
*d) - (2^(2/3)*b*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1
/3))])/(Sqrt[3]*a^(4/3)*d) - (5*b*Log[x])/(6*a^(4/3)*d) + (b*Log[a - b*x^3]
)/(3*2^(1/3)*a^(4/3)*d) + (5*b*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(6*a^(4/3)
*d) - (b*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*a^(4/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x^2(ad-bdx)}\,dx, x, x^3\right)\\
&= -\frac{(a+bx^3)^{5/3}}{3a^2dx^3} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{2/3}\left(-\frac{5}{3}abd+\frac{2}{3}b^2dx\right)}{x(ad-bdx)}\,dx, x, x^3\right)}{3a^2d}\\
&= -\frac{(a+bx^3)^{5/3}}{3a^2dx^3} + \frac{b^2\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{ad-bdx}\,dx, x, x^3\right)}{3a^2} + \frac{(5b)\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x}\,dx, x, x^3\right)}{9a^2d}
\end{aligned}
$$

$$= \frac{b(a+bx^3)^{2/3}}{3a^2d} - \frac{(a+bx^3)^{5/3}}{3a^2dx^3} + \frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(ad-bdx)}\,dx, x, x^3\right)}{3a} + \frac{(5b)\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^3\right)}{9ad}$$

$$= \frac{b(a+bx^3)^{2/3}}{3a^2d} - \frac{(a+bx^3)^{5/3}}{3a^2dx^3} - \frac{5b\log(x)}{6a^{4/3}d} + \frac{b\log(a-bx^3)}{3\sqrt[3]{2}a^{4/3}d} - \frac{(5b)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{6a^{4/3}d} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^{4/3}d} + \frac{(5b)\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{6ad} - \frac{b\text{Subst}\left(\int \frac{1}{2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{ad}$$

$$= \frac{b(a+bx^3)^{2/3}}{3a^2d} - \frac{(a+bx^3)^{5/3}}{3a^2dx^3} - \frac{5b\log(x)}{6a^{4/3}d} + \frac{b\log(a-bx^3)}{3\sqrt[3]{2}a^{4/3}d} + \frac{5b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}d} - \frac{b\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^{4/3}d} - \frac{(5b)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{3a^{4/3}d} + \frac{(2^{2/3}b)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{a^{4/3}d}$$

$$= \frac{b(a+bx^3)^{2/3}}{3a^2d} - \frac{(a+bx^3)^{5/3}}{3a^2dx^3} + \frac{5b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{4/3}d} - \frac{2^{2/3}b\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{4/3}d} - \frac{5b\log(x)}{6a^{4/3}d} + \frac{b\log(a-bx^3)}{3\sqrt[3]{2}a^{4/3}d} + \frac{5b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}d} - \frac{b\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^{4/3}d}$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^{2/3}}{x^4(ad-bdx^3)}\,dx = \frac{-6\sqrt[3]{a}(a+bx^3)^{2/3}+10\sqrt{3}bx^3\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-6\,2^{2/3}\sqrt{3}bx^3\arctan\left(\frac{1+\frac{2^{2/3}\cdots}{}}{}\right)\cdots}{}$$

[In] Integrate[(a + b*x^3)^(2/3)/(x^4*(a*d - b*d*x^3)),x]

[Out] (-6*a^(1/3)*(a + b*x^3)^(2/3) + 10*Sqrt[3]*b*x^3*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 6*2^(2/3)*Sqrt[3]*b*x^3*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 10*b*x^3*Log[-a^(1/3) + (a + b*x^3)^(1/3)] - 6*2^(2/3)*b*x^3*Log[-2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)] - 5*b*x^3*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + 3*2^(2/3)*b*x^3*Log[2*a^(2/3) + 2^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(18*a^(4/3)*d*x^3)

Maple [A] (verified)

Time = 4.69 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$-\frac{2^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)bx^3+2^{\frac{2}{3}}\ln\left(\left(bx^3+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)bx^3-\frac{5\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}\,bx^3}{3}-\cdots}{}$

[In] int((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)

[Out] -1/3*(2^(2/3)*3^(1/2)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*b*x^3+2^(2/3)*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3))*b*x^3-5/3*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)*b*x^3-1/2*2^(2/3)*ln((b*x^3+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*a^(2/3))*b*x^3-5/3*ln((b*x^3+a)^(1/3)-a^(1/3))*b*x^3+5/6*ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3))*b*x^3+(b*x^3+a)^(2/3)*a^(1/3))/a^(4/3)/x^3/d

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 612, normalized size of antiderivative = 2.28

$$\int \frac{(a+bx^3)^{2/3}}{x^4(ad-bdx^3)}\,dx = \left[-\frac{6\cdot 4^{\frac{1}{3}}\sqrt{3}abx^3\left(-\frac{1}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\cdot 4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(-\frac{1}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-15\sqrt{\frac{1}{3}}abx^3\sqrt{\cdots}}{\cdots}\right.$$

$$-\frac{6\cdot 4^{\frac{1}{3}}\sqrt{3}abx^3\left(-\frac{1}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\cdot 4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(-\frac{1}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+3\cdot 4^{\frac{1}{3}}abx^3\left(-\frac{1}{a}\right)^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}a(-\cdots\right.}{\cdots}$$

```
[In] integrate((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] [-1/18*(6*4^(1/3)*sqrt(3)*a*b*x^3*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*sqrt(3)*(
b*x^3 + a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) - 15*sqrt(1/3)*a*b*x^3*sqrt(-1
/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*a^(2/3) - (b*x^3
+ a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 + a)^(1/3)*a^(2/3) + 3*
a)/x^3) + 3*4^(1/3)*a*b*x^3*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 + a)^(1/3)*a*(-
1/a)^(2/3) - 2*4^(1/3)*a*(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) - 6*4^(1/3)*a*
b*x^3*(-1/a)^(1/3)*log(-4^(2/3)*a*(-1/a)^(2/3) + 2*(b*x^3 + a)^(1/3)) + 5*a
^(2/3)*b*x^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) -
 10*a^(2/3)*b*x^3*log((b*x^3 + a)^(1/3) - a^(1/3)) + 6*(b*x^3 + a)^(2/3)*a)
/(a^2*d*x^3), -1/18*(6*4^(1/3)*sqrt(3)*a*b*x^3*(-1/a)^(1/3)*arctan(1/3*4^(1
/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) + 3*4^(1/3)*a*b*x
^3*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 + a)^(1/3)*a*(-1/a)^(2/3) - 2*4^(1/3)*a*
(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) - 6*4^(1/3)*a*b*x^3*(-1/a)^(1/3)*log(-4
^(2/3)*a*(-1/a)^(2/3) + 2*(b*x^3 + a)^(1/3)) - 30*sqrt(1/3)*a^(2/3)*b*x^3*a
rctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3)) + 5*a^(2/3)*b*x^3*
log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) - 10*a^(2/3)*b
*x^3*log((b*x^3 + a)^(1/3) - a^(1/3)) + 6*(b*x^3 + a)^(2/3)*a)/(a^2*d*x^3)]
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^4(ad-bdx^3)}\,dx = -\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-ax^4+bx^7}\,dx}{d}$$

```
[In] integrate((b*x**3+a)**(2/3)/x**4/(-b*d*x**3+a*d),x)
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**4 + b*x**7), x)/d
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^4(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^4}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.98 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.90

$$\begin{aligned}\int \frac{(a+bx^3)^{2/3}}{x^4(ad-bdx^3)}\,dx &= -\frac{\sqrt{3}2^{\frac{2}{3}}b\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(bx^3+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{4}{3}}d} \\ &+ \frac{2^{\frac{2}{3}}b\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,a^{\frac{4}{3}}d} - \frac{2^{\frac{2}{3}}b\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{4}{3}}d} \\ &+ \frac{5\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{4}{3}}d} - \frac{5\,b\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{4}{3}}d} \\ &+ \frac{5\,b\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{4}{3}}d} - \frac{(bx^3+a)^{\frac{2}{3}}}{3\,adx^3}\end{aligned}$$

```
[In] integrate((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] -1/3*sqrt(3)*2^(2/3)*b*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(b*x
^3 + a)^(1/3))/a^(1/3))/(a^(4/3)*d) + 1/6*2^(2/3)*b*log(2^(2/3)*a^(2/3) + 2
^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(a^(4/3)*d) - 1/3*2^(
```

```
2/3)*b*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(a^(4/3)*d) + 5/9*sqr
t(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(4/3)
*d) - 5/18*b*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(
a^(4/3)*d) + 5/9*b*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(4/3)*d) - 1/3*
(b*x^3 + a)^(2/3)/(a*d*x^3)
```

Mupad [B] (verification not implemented)

Time = 9.49 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.82

$$\int \frac{(a+bx^3)^{2/3}}{x^4(ad-bdx^3)}\,dx = \ln\left(2\,b^2\,\left(b\,x^3+a\right)^{1/3} -2\,2^{1/3}\,a^3\,d^2\left(-\frac{b^3}{a^4\,d^3}\right)^{2/3}\right)\left(-\frac{4\,b^3}{27\,a^4\,d^3}\right)^{1/3}+\frac{5\,\ln\left(b^2\,\left(b\,x^3+a\right)^{1/3}-a^3\,d^2\left(\frac{b^3}{a^4\,d^3}\right)^{2/3}\right)\left(\frac{b^3}{a^4\,d^3}\right)^{1/3}}{9}-\ln\left(4\,b\right.$$

```
[In] int((a + b*x^3)^(2/3)/(x^4*(a*d - b*d*x^3)),x)
```

```
[Out] log(2*b^2*(a + b*x^3)^(1/3) - 2*2^(1/3)*a^3*d^2*(-b^3/(a^4*d^3))^(2/3))*(-(
4*b^3)/(27*a^4*d^3))^(1/3) + (5*log(b^2*(a + b*x^3)^(1/3) - a^3*d^2*(b^3/(a
^4*d^3))^(2/3))*(b^3/(a^4*d^3))^(1/3))/9 - log(4*b^2*(a + b*x^3)^(1/3) + 2*
2^(1/3)*a^3*d^2*(-b^3/(a^4*d^3))^(2/3) - 2^(1/3)*3^(1/2)*a^3*d^2*(-b^3/(a^4
*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 + 1/2)*(-(4*b^3)/(27*a^4*d^3))^(1/3) + log
(4*b^2*(a + b*x^3)^(1/3) + 2*2^(1/3)*a^3*d^2*(-b^3/(a^4*d^3))^(2/3) + 2^(1/
3)*3^(1/2)*a^3*d^2*(-b^3/(a^4*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 - 1/2)*(-(4*b
^3)/(27*a^4*d^3))^(1/3) - log(2*b^2*(a + b*x^3)^(1/3) + a^3*d^2*(b^3/(a^4*d
^3))^(2/3) - 3^(1/2)*a^3*d^2*(b^3/(a^4*d^3))^(2/3)*1i)*((3^(1/2)*1i)/2 + 1/
2)*((125*b^3)/(729*a^4*d^3))^(1/3) + log(2*b^2*(a + b*x^3)^(1/3) + a^3*d^2*
(b^3/(a^4*d^3))^(2/3) + 3^(1/2)*a^3*d^2*(b^3/(a^4*d^3))^(2/3)*1i)*((3^(1/2)
*1i)/2 - 1/2)*((125*b^3)/(729*a^4*d^3))^(1/3) - (b*(a + b*x^3)^(2/3))/(3*a*
(d*(a + b*x^3) - a*d))
```

3.592 $\int \frac{(a+bx^3)^{2/3}}{x^7(ad-bdx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 284

$$\int \frac{(a+bx^3)^{2/3}}{x^7(ad-bdx^3)}\,dx = -\frac{5b(a+bx^3)^{2/3}}{18a^2dx^3} - \frac{(a+bx^3)^{5/3}}{6a^2dx^6} + \frac{14b^2\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}d} - \frac{2^{2/3}b^2\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}d} - \frac{7b^2\log(x)}{9a^{7/3}d} + \frac{b^2\log(a-bx^3)}{3\sqrt[3]{2}a^{7/3}d} + \frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{9a^{7/3}d} - \frac{b^2\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^{7/3}d}$$

```
[Out] -5/18*b*(b*x^3+a)^(2/3)/a^2/d/x^3-1/6*(b*x^3+a)^(5/3)/a^2/d/x^6-7/9*b^2*ln(
x)/a^(7/3)/d+1/6*b^2*ln(-b*x^3+a)*2^(2/3)/a^(7/3)/d+7/9*b^2*ln(a^(1/3)-(b*x
^3+a)^(1/3))/a^(7/3)/d-1/2*b^2*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(2/3)/
a^(7/3)/d+14/27*b^2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))
/a^(7/3)/d*3^(1/2)-1/3*2^(2/3)*b^2*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1
/3))/a^(1/3)*3^(1/2))/a^(7/3)/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used

$= \{457, 105, 154, 162, 57, 631, 210, 31\}$

$$\int \frac{(a+bx^3)^{2/3}}{x^7(ad-bdx^3)}\,dx = \frac{14b^2\arctan\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}d} - \frac{2^{2/3}b^2\arctan\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}d} + \frac{b^2\log(a-bx^3)}{3\sqrt[3]{2}a^{7/3}d} + \frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{9a^{7/3}d} - \frac{b^2\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^{7/3}d} - \frac{7b^2\log(x)}{9a^{7/3}d} - \frac{5b(a+bx^3)^{2/3}}{18a^2dx^3} - \frac{(a+bx^3)^{5/3}}{6a^2dx^6}$$

```
[In] Int[(a + b*x^3)^(2/3)/(x^7*(a*d - b*d*x^3)),x]
```

```
[Out] (-5*b*(a + b*x^3)^(2/3))/(18*a^2*d*x^3) - (a + b*x^3)^(5/3)/(6*a^2*d*x^6) +
 (14*b^2*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt
[3]*a^(7/3)*d) - (2^(2/3)*b^2*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/
(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)*d) - (7*b^2*Log[x])/(9*a^(7/3)*d) + (b
^2*Log[a - b*x^3])/(3*2^(1/3)*a^(7/3)*d) + (7*b^2*Log[a^(1/3) - (a + b*x^3)
^(1/3)])/(9*a^(7/3)*d) - (b^2*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^
(1/3)*a^(7/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x^3(ad-bdx)}\,dx, x, x^3\right)$$

$$= -\frac{(a+bx^3)^{5/3}}{6a^2dx^6} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{2/3}\left(-\frac{5}{3}abd-\frac{1}{3}b^2dx\right)}{x^2(ad-bdx)}\,dx, x, x^3\right)}{6a^2d}$$

$$= -\frac{5b(a+bx^3)^{2/3}}{18a^2dx^3} - \frac{(a+bx^3)^{5/3}}{6a^2dx^6} - \frac{\text{Subst}\left(\int \frac{-\frac{28}{9}a^2b^2d^2-\frac{8}{9}ab^3d^2x}{x\sqrt[3]{a+bx}(ad-bdx)}\,dx, x, x^3\right)}{6a^3d^2}$$

$$= -\frac{5b(a+bx^3)^{2/3}}{18a^2dx^3} - \frac{(a+bx^3)^{5/3}}{6a^2dx^6} + \frac{(2b^3)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(ad-bdx)}\,dx, x, x^3\right)}{3a^2} + \frac{(14b^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^3\right)}{27a^2d}$$

$$= -\frac{5b(a+bx^3)^{2/3}}{18a^2dx^3} - \frac{(a+bx^3)^{5/3}}{6a^2dx^6} - \frac{7b^2\log(x)}{9a^{7/3}d} + \frac{b^2\log(a-bx^3)}{3\sqrt[3]{2}a^{7/3}d} - \frac{(7b^2)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{9a^{7/3}d} + \frac{b^2\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^{7/3}d} + \frac{(7b^2)\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{9a^2d} - \frac{b^2\text{Subst}\left(\int \frac{1}{2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{a^2d}$$

$$= -\frac{5b(a+bx^3)^{2/3}}{18a^2dx^3} - \frac{(a+bx^3)^{5/3}}{6a^2dx^6} - \frac{7b^2\log(x)}{9a^{7/3}d} + \frac{b^2\log(a-bx^3)}{3\sqrt[3]{2}a^{7/3}d} + \frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{9a^{7/3}d} - \frac{b^2\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^{7/3}d} - \frac{(14b^2)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{9a^{7/3}d} + \frac{(2^{2/3}b^2)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{a^{7/3}d}$$

$$= -\frac{5b(a+bx^3)^{2/3}}{18a^2dx^3} - \frac{(a+bx^3)^{5/3}}{6a^2dx^6} + \frac{14b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}a^{7/3}d} - \frac{2^{2/3}b^2\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{7/3}d} - \frac{7b^2\log(x)}{9a^{7/3}d} + \frac{b^2\log(a-bx^3)}{3\sqrt[3]{2}a^{7/3}d} + \frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{9a^{7/3}d} - \frac{b^2\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^{7/3}d}$$

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3)^{2/3}}{x^7(ad-bdx^3)}\,dx = \frac{-9a^{4/3}(a+bx^3)^{2/3} - 24\sqrt[3]{a}bx^3(a+bx^3)^{2/3} + 28\sqrt{3}b^2x^6\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 18}{}$$

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^7*(a*d - b*d*x^3)),x]
```

```
[Out] (-9*a^(4/3)*(a + b*x^3)^(2/3) - 24*a^(1/3)*b*x^3*(a + b*x^3)^(2/3) + 28*Sqr
t[3]*b^2*x^6*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 18*2^(2/
3)*Sqrt[3]*b^2*x^6*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]
] + 28*b^2*x^6*Log[-a^(1/3) + (a + b*x^3)^(1/3)] - 18*2^(2/3)*b^2*x^6*Log[-
2*a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3)] - 14*b^2*x^6*Log[a^(2/3) + a^(1/3)*(
a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + 9*2^(2/3)*b^2*x^6*Log[2*a^(2/3) + 2
^(2/3)*a^(1/3)*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(54*a^(7/3)*
d*x^6)
```

Maple [A] (verified)

Time = 4.74 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$\frac{-18\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\left(a^{\frac{1}{3}}+2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)b^{2}x^{6}-18\,2^{\frac{2}{3}}\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)b^{2}x^{6}+9\,2^{\frac{2}{3}}\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}\left(b\,x^{3}+a\right)\right.}{}$

```
[In] int((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/54*(-18*3^(1/2)*2^(2/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1
/3)*3^(1/2))*b^2*x^6-18*2^(2/3)*ln((b*x^3+a)^(1/3)-2^(1/3)*a^(1/3))*b^2*x^6
+9*2^(2/3)*ln((b*x^3+a)^(2/3)+2^(1/3)*a^(1/3)*(b*x^3+a)^(1/3)+2^(2/3)*a^(2/
3))*b^2*x^6+28*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1
/2)*b^2*x^6+28*ln((b*x^3+a)^(1/3)-a^(1/3))*b^2*x^6-14*ln((b*x^3+a)^(2/3)+a^
(1/3)*(b*x^3+a)^(1/3)+a^(2/3))*b^2*x^6-24*b*x^3*a^(1/3)*(b*x^3+a)^(2/3)-9*(
b*x^3+a)^(2/3)*a^(4/3))/a^(7/3)/x^6/d
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 660, normalized size of antiderivative = 2.32

$$\int \frac{(a+bx^3)^{2/3}}{x^7(ad-bdx^3)}\,dx = \left[-\frac{18\cdot 4^{\frac{1}{3}}\sqrt{3}ab^2x^6\left(-\frac{1}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\cdot 4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(-\frac{1}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-42\sqrt{\frac{1}{3}}ab^2x}{}\right.$$

$$-\frac{18\cdot 4^{\frac{1}{3}}\sqrt{3}ab^2x^6\left(-\frac{1}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\cdot 4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(-\frac{1}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+9\cdot 4^{\frac{1}{3}}ab^2x^6\left(-\frac{1}{a}\right)^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}\right.}{}$$

```
[In] integrate((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] [-1/54*(18*4^(1/3)*sqrt(3)*a*b^2*x^6*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*sqrt(3
)*(b*x^3 + a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) - 42*sqrt(1/3)*a*b^2*x^6*sq
rt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*a^(2/3) - (b
*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 + a)^(1/3)*a^(2/3)
 + 3*a)/x^3) + 9*4^(1/3)*a*b^2*x^6*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 + a)^(1/
3)*a*(-1/a)^(2/3) - 2*4^(1/3)*a*(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) - 18*4^
(1/3)*a*b^2*x^6*(-1/a)^(1/3)*log(-4^(2/3)*a*(-1/a)^(2/3) + 2*(b*x^3 + a)^(1
/3)) + 14*a^(2/3)*b^2*x^6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3)
 + a^(2/3)) - 28*a^(2/3)*b^2*x^6*log((b*x^3 + a)^(1/3) - a^(1/3)) + 3*(8*a*
b*x^3 + 3*a^2)*(b*x^3 + a)^(2/3))/(a^3*d*x^6), -1/54*(18*4^(1/3)*sqrt(3)*a*
b^2*x^6*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/a)^(1
/3) - 1/3*sqrt(3)) + 9*4^(1/3)*a*b^2*x^6*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 +
a)^(1/3)*a*(-1/a)^(2/3) - 2*4^(1/3)*a*(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) -
 18*4^(1/3)*a*b^2*x^6*(-1/a)^(1/3)*log(-4^(2/3)*a*(-1/a)^(2/3) + 2*(b*x^3 +
 a)^(1/3)) - 84*sqrt(1/3)*a^(2/3)*b^2*x^6*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(
```

```
1/3) + a^(1/3))/a^(1/3)) + 14*a^(2/3)*b^2*x^6*log((b*x^3 + a)^(2/3) + (b*x^
3 + a)^(1/3)*a^(1/3) + a^(2/3)) - 28*a^(2/3)*b^2*x^6*log((b*x^3 + a)^(1/3)
- a^(1/3)) + 3*(8*a*b*x^3 + 3*a^2)*(b*x^3 + a)^(2/3))/(a^3*d*x^6)]
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^7(ad-bdx^3)}\,dx = -\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-ax^7+bx^{10}}\,dx}{d}$$

```
[In] integrate((b*x**3+a)**(2/3)/x**7/(-b*d*x**3+a*d),x)
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**7 + b*x**10), x)/d
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^7(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^7}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^7), x)
```

Giac [A] (verification not implemented)

none

Time = 0.99 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.98

$$\begin{aligned}\int \frac{(a+bx^3)^{2/3}}{x^7(ad-bdx^3)}\,dx = &-\frac{\sqrt{3}2^{\frac{2}{3}}b^2\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(bx^3+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{7}{3}}d} \\ &+\frac{2^{\frac{2}{3}}b^2\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{2}{3}}\right)}{6\,a^{\frac{7}{3}}d} \\ &-\frac{2^{\frac{2}{3}}b^2\log\left(\left|-2^{\frac{1}{3}}a^{\frac{1}{3}}+(bx^3+a)^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{7}{3}}d}+\frac{14\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{27\,a^{\frac{7}{3}}d} \\ &-\frac{7\,b^2\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{27\,a^{\frac{7}{3}}d} \\ &+\frac{14\,b^2\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{27\,a^{\frac{7}{3}}d}-\frac{8\,(bx^3+a)^{\frac{5}{3}}b^2-5\,(bx^3+a)^{\frac{2}{3}}ab^2}{18\,a^2b^2dx^6}\end{aligned}$$

```
[In] integrate((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*2^(2/3)*b^2*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(b
*x^3 + a)^(1/3))/a^(1/3))/(a^(7/3)*d) + 1/6*2^(2/3)*b^2*log(2^(2/3)*a^(2/3)
 + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/(a^(7/3)*d) - 1/3
*2^(2/3)*b^2*log(abs(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3)))/(a^(7/3)*d) + 1
4/27*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3)
)/(a^(7/3)*d) - 7/27*b^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3)
+ a^(2/3))/(a^(7/3)*d) + 14/27*b^2*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a
^(7/3)*d) - 1/18*(8*(b*x^3 + a)^(5/3)*b^2 - 5*(b*x^3 + a)^(2/3)*a*b^2)/(a^2
*b^2*d*x^6)
```

Mupad [B] (verification not implemented)

Time = 9.40 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.81

$$\int \frac{(a+bx^3)^{2/3}}{x^7(ad-bdx^3)}\,dx = \frac{\frac{5\,b^2\,\left(b\,x^3+a\right)^{2/3}}{18\,a}-\frac{4\,b^2\,\left(b\,x^3+a\right)^{5/3}}{9\,a^2}}{d\,\left(b\,x^3+a\right)^2+a^2\,d-2\,a\,d\,\left(b\,x^3+a\right)}$$

$$+\ln\left(2\,b^4\,\left(b\,x^3+a\right)^{1/3}-2\,2^{1/3}\,a^5\,d^2\,\left(-\frac{b^6}{a^7\,d^3}\right)^{2/3}\right)\,\left(-\frac{4\,b^6}{27\,a^7\,d^3}\right)^{1/3}+\frac{14\,\ln\left(b^4\,\left(b\,x^3+a\right)^{1/3}-a^5\,d^2\,\left(\frac{b^6}{a^7\,d^3}\right)^{2}\right.}{27}$$

```
[In] int((a + b*x^3)^(2/3)/(x^7*(a*d - b*d*x^3)),x)
```

```
[Out] ((5*b^2*(a + b*x^3)^(2/3))/(18*a) - (4*b^2*(a + b*x^3)^(5/3))/(9*a^2))/(d*(
a + b*x^3)^2 + a^2*d - 2*a*d*(a + b*x^3)) + log(2*b^4*(a + b*x^3)^(1/3) - 2
*2^(1/3)*a^5*d^2*(-b^6/(a^7*d^3))^(2/3))*(-(4*b^6)/(27*a^7*d^3))^(1/3) + (1
4*log(b^4*(a + b*x^3)^(1/3) - a^5*d^2*(b^6/(a^7*d^3))^(2/3))*(b^6/(a^7*d^3)
)^(1/3))/27 - log(4*b^4*(a + b*x^3)^(1/3) + 2*2^(1/3)*a^5*d^2*(-b^6/(a^7*d^
3))^(2/3) - 2^(1/3)*3^(1/2)*a^5*d^2*(-b^6/(a^7*d^3))^(2/3)*2i)*((3^(1/2)*1i
)/2 + 1/2)*(-(4*b^6)/(27*a^7*d^3))^(1/3) + log(4*b^4*(a + b*x^3)^(1/3) + 2*
2^(1/3)*a^5*d^2*(-b^6/(a^7*d^3))^(2/3) + 2^(1/3)*3^(1/2)*a^5*d^2*(-b^6/(a^7
*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 - 1/2)*(-(4*b^6)/(27*a^7*d^3))^(1/3) - (7*
log(2*b^4*(a + b*x^3)^(1/3) + a^5*d^2*(b^6/(a^7*d^3))^(2/3) - 3^(1/2)*a^5*d
^2*(b^6/(a^7*d^3))^(2/3)*1i)*(3^(1/2)*1i + 1)*(b^6/(a^7*d^3))^(1/3))/27 + (
7*log(2*b^4*(a + b*x^3)^(1/3) + a^5*d^2*(b^6/(a^7*d^3))^(2/3) + 3^(1/2)*a^5
*d^2*(b^6/(a^7*d^3))^(2/3)*1i)*(3^(1/2)*1i - 1)*(b^6/(a^7*d^3))^(1/3))/27
```

3.593 $\int \frac{x^6(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 264

$$\int \frac{x^6(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{4ax(a+bx^3)^{2/3}}{9b^2d}-\frac{x^4(a+bx^3)^{2/3}}{6bd}-\frac{14a^2\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{7/3}d}+\frac{2^{2/3}a^2\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{7/3}d}+\frac{a^2\log(ad-bdx^3)}{3\sqrt[3]{2}b^{7/3}d}-\frac{a^2\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^{7/3}d}+\frac{7a^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{9b^{7/3}d}$$

```
[Out] -4/9*a*x*(b*x^3+a)^(2/3)/b^2/d-1/6*x^4*(b*x^3+a)^(2/3)/b/d+1/6*a^2*ln(-b*d*
x^3+a*d)*2^(2/3)/b^(7/3)/d-1/2*a^2*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^
(2/3)/b^(7/3)/d+7/9*a^2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(7/3)/d-14/27*a^2*
arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(7/3)/d*3^(1/2)+1/3*2
^(2/3)*a^2*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(7
/3)/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {489, 596, 544, 245, 384}

$$\int \frac{x^6(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{14a^2\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}b^{7/3}d} + \frac{2^{2/3}a^2\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{7/3}d} + \frac{a^2\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}b^{7/3}d} - \frac{a^2\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^{7/3}d} + \frac{7a^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{9b^{7/3}d} - \frac{4ax(a+bx^3)^{2/3}}{9b^2d} - \frac{x^4(a+bx^3)^{2/3}}{6bd}$$

```
[In] Int[(x^6*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-4*a*x*(a + b*x^3)^(2/3))/(9*b^2*d) - (x^4*(a + b*x^3)^(2/3))/(6*b*d) - (1
4*a^2*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(
7/3)*d) + (2^(2/3)*a^2*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))
/Sqrt[3]])/(Sqrt[3]*b^(7/3)*d) + (a^2*Log[a*d - b*d*x^3])/(3*2^(1/3)*b^(7/3
)*d) - (a^2*Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)])/(2^(1/3)*b^(7/3)*d)
 + (7*a^2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(9*b^(7/3)*d)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
```

```
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^4(a+bx^3)^{2/3}}{6bd} + \frac{\int \frac{x^3(4a^2d+8abdx^3)}{\sqrt[3]{a+bx^3}(ad-bdx^3)}\,dx}{6bd} \\
&= -\frac{4ax(a+bx^3)^{2/3}}{9b^2d} - \frac{x^4(a+bx^3)^{2/3}}{6bd} + \frac{\int \frac{8a^3bd^2+28a^2b^2d^2x^3}{\sqrt[3]{a+bx^3}(ad-bdx^3)}\,dx}{18b^3d^2} \\
&= -\frac{4ax(a+bx^3)^{2/3}}{9b^2d} - \frac{x^4(a+bx^3)^{2/3}}{6bd} + \frac{(2a^3)\int \frac{1}{\sqrt[3]{a+bx^3}(ad-bdx^3)}\,dx}{b^2} - \frac{(14a^2)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{9b^2d} \\
&= -\frac{4ax(a+bx^3)^{2/3}}{9b^2d} - \frac{x^4(a+bx^3)^{2/3}}{6bd} - \frac{14a^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{7/3}d} \\
&\quad + \frac{2^{2/3}a^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{7/3}d} + \frac{a^2\log(ad-bdx^3)}{3\sqrt[3]{2}b^{7/3}d} \\
&\quad - \frac{a^2\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^{7/3}d} + \frac{7a^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{9b^{7/3}d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.23

$$\int \frac{x^6(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{24a\sqrt[3]{b}x(a+bx^3)^{2/3}+9b^{4/3}x^4(a+bx^3)^{2/3}+28\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-18\,2^{2/3}\sqrt{3}a^2\arctan\left(\cdots\right.}{}$$

```
[In] Integrate[(x^6*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/54*(24*a*b^(1/3)*x*(a + b*x^3)^(2/3) + 9*b^(4/3)*x^4*(a + b*x^3)^(2/3) +
 28*Sqrt[3]*a^2*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3)
)] - 18*2^(2/3)*Sqrt[3]*a^2*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)
*(a + b*x^3)^(1/3))] - 28*a^2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + 18*2^
(2/3)*a^2*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)] + 14*a^2*Log[b^(2/3
)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] - 9*2^(2/3)*a^2*Lo
g[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)
^(2/3)])/(b^(7/3)*d)
```

Maple [A] (verified)

Time = 4.96 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{-9x^4(b\,x^3+a)^{\frac{2}{3}}b^{\frac{4}{3}}-18\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}(b\,x^3+a)^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{3b^{\frac{1}{3}}x}\right)a^2-18\,2^{\frac{2}{3}}\ln\left(\frac{-2^{\frac{1}{3}}b^{\frac{1}{3}}x+(b\,x^3+a)^{\frac{1}{3}}}{x}\right)a^2+9\,2^{\frac{2}{3}}\ln\left(\frac{2^{\frac{2}{3}}b^{\frac{2}{3}}\cdots}{}\right.}{}$

```
[In] int(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/54*(-9*x^4*(b*x^3+a)^(2/3)*b^(4/3)-18*3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*
(2^(2/3)*(b*x^3+a)^(1/3)+b^(1/3)*x)/b^(1/3)/x)*a^2-18*2^(2/3)*ln((-2^(1/3)*
b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a^2+9*2^(2/3)*ln((2^(2/3)*b^(2/3)*x^2+2^(1/3)
*b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a^2-24*a*x*(b*x^3+a)^(2/3)
*b^(1/3)+28*a^2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^
(1/3)/x)+28*a^2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-14*a^2*ln((b^(2/3)*x^2+b
^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))/d/b^(7/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 701, normalized size of antiderivative = 2.66

$$\int \frac{x^6(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \left[-\frac{18\cdot 4^{\frac{1}{3}}\sqrt{3}a^2b\left(-\frac{1}{b}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}x-4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(-\frac{1}{b}\right)^{\frac{1}{3}}}{3x}\right)-42\sqrt{\frac{1}{3}}a^2b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\right.}{}\right.$$

$$-\frac{18\cdot 4^{\frac{1}{3}}\sqrt{3}a^2b\left(-\frac{1}{b}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}x-4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(-\frac{1}{b}\right)^{\frac{1}{3}}}{3x}\right)-18\cdot 4^{\frac{1}{3}}a^2b\left(-\frac{1}{b}\right)^{\frac{1}{3}}\log\left(-\frac{4^{\frac{2}{3}}bx\left(-\frac{1}{b}\right)^{\frac{2}{3}}-2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)+}{}$$

```
[In] integrate(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] [-1/54*(18*4^(1/3)*sqrt(3)*a^2*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1
/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3))/x) - 42*sqrt(1/3)*a^2*b*sqrt(-1
/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4
/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/
b^(2/3)) + 2*a) - 18*4^(1/3)*a^2*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*(-1/b)^(2
/3) - 2*(b*x^3 + a)^(1/3))/x) + 9*4^(1/3)*a^2*b*(-1/b)^(1/3)*log(-(2*4^(1/3
)*b*x^2*(-1/b)^(1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b*x^
3 + a)^(2/3))/x^2) - 28*a^2*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x)
 + 14*a^2*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 +
 a)^(2/3))/x^2) + 3*(3*b^2*x^4 + 8*a*b*x)*(b*x^3 + a)^(2/3))/(b^3*d), -1/54
*(18*4^(1/3)*sqrt(3)*a^2*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sq
rt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3))/x) - 18*4^(1/3)*a^2*b*(-1/b)^(1/3)*lo
g(-(4^(2/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(1/3))/x) + 9*4^(1/3)*a^2*b*(-
1/b)^(1/3)*log(-(2*4^(1/3)*b*x^2*(-1/b)^(1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b
*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(2/3))/x^2) - 84*sqrt(1/3)*a^2*b^(2/3)*arct
an(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x)) - 28*a^2*b^(2/3
)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) + 14*a^2*b^(2/3)*log((b^(2/3)*x^2
 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(3*b^2*x^4 + 8
*a*b*x)*(b*x^3 + a)^(2/3))/(b^3*d)]
```

Sympy [F]

$$\int \frac{x^6(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^6(a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

```
[In] integrate(x**6*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
[Out] -Integral(x**6*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Maxima [F]

$$\int \frac{x^6(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}x^6}{bdx^3-ad}\,dx$$

```
[In] integrate(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)*x^6/(b*d*x^3 - a*d), x)
```

Giac [F]

$$\int \frac{x^6(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}x^6}{bdx^3-ad}\,dx$$

```
[In] integrate(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(2/3)*x^6/(b*d*x^3 - a*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int \frac{x^6\,{\left(b\,x^3+a\right)}^{2/3}}{a\,d-b\,d\,x^3}\,dx$$

```
[In] int((x^6*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
[Out] int((x^6*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)
```

3.594 $\int \frac{x^3(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 229

$$\int \frac{x^3(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{x(a+bx^3)^{2/3}}{3bd} - \frac{5a\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}d} + \frac{2^{2/3}a\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}d} + \frac{a\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}b^{4/3}d} - \frac{a\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^{4/3}d} + \frac{5a\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6b^{4/3}d}$$

```
[Out] -1/3*x*(b*x^3+a)^(2/3)/b/d+1/6*a*ln(-b*d*x^3+a*d)*2^(2/3)/b^(4/3)/d-1/2*a*l
n(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(2/3)/b^(4/3)/d+5/6*a*ln(-b^(1/3)*x+
(b*x^3+a)^(1/3))/b^(4/3)/d-5/9*a*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))
*3^(1/2))/b^(4/3)/d*3^(1/2)+1/3*2^(2/3)*a*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x
/(b*x^3+a)^(1/3))*3^(1/2))/b^(4/3)/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {489, 544, 245, 384}

$$\int \frac{x^3(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{5a\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}d} + \frac{2^{2/3}a\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}d} + \frac{a\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}b^{4/3}d} - \frac{a\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^{4/3}d} + \frac{5a\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{6b^{4/3}d} - \frac{x(a+bx^3)^{2/3}}{3bd}$$

```
[In] Int[(x^3*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/3*(x*(a + b*x^3)^(2/3))/(b*d) - (5*a*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^
3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(4/3)*d) + (2^(2/3)*a*ArcTan[(1 + (2*2^(1/
3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(4/3)*d) + (a*Log[a*d
 - b*d*x^3])/(3*2^(1/3)*b^(4/3)*d) - (a*Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)
^(1/3)])/(2^(1/3)*b^(4/3)*d) + (5*a*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/
(6*b^(4/3)*d)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
```

```
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x(a+bx^3)^{2/3}}{3bd}+\frac{\int \frac{a^2d+5abdx^3}{\sqrt[3]{a+bx^3}(ad-bdx^3)}\,dx}{3bd}\\
&= -\frac{x(a+bx^3)^{2/3}}{3bd}+\frac{(2a^2)\int \frac{1}{\sqrt[3]{a+bx^3}(ad-bdx^3)}\,dx}{b}-\frac{(5a)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{3bd}\\
&= -\frac{x(a+bx^3)^{2/3}}{3bd}-\frac{5a\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}d}+\frac{2^{2/3}a\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}d}\\
&\quad+\frac{a\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}b^{4/3}d}-\frac{a\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}b^{4/3}d}+\frac{5a\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6b^{4/3}d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.27

$$\int \frac{x^3(a+bx^3)^{2/3}}{ad-bdx^3}\,dx =$$
$$-\frac{6\sqrt[3]{b}x(a+bx^3)^{2/3}+10\sqrt{3}a\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-6\,2^{2/3}\sqrt{3}a\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right)-10a\log}{}$$

```
[In] Integrate[(x^3*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/18*(6*b^(1/3)*x*(a + b*x^3)^(2/3) + 10*Sqrt[3]*a*ArcTan[(Sqrt[3]*b^(1/3)
*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 6*2^(2/3)*Sqrt[3]*a*ArcTan[(Sqrt[3
]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))] - 10*a*Log[-(b^(1/3)*
```

```
x) + (a + b*x^3)^(1/3)] + 6*2^(2/3)*a*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3
)^(1/3)] + 5*a*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^
(2/3)] - 3*2^(2/3)*a*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3
) + 2^(1/3)*(a + b*x^3)^(2/3)])/(b^(4/3)*d)
```

Maple [A] (verified)

Time = 4.88 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$\frac{-6\,2^{\frac{2}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{3b^{\frac{1}{3}}x}\right)a-6\,2^{\frac{2}{3}}\ln\left(\frac{-2^{\frac{1}{3}}b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)a+3\,2^{\frac{2}{3}}\ln\left(\frac{2^{\frac{2}{3}}b^{\frac{2}{3}}x^{2}+2^{\frac{1}{3}}b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+(}{x^{2}}\right.}{}$

```
[In] int(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*(-6*2^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)+b^(1/3
)*x)/b^(1/3)/x)*a-6*2^(2/3)*ln((-2^(1/3)*b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a+3*
2^(2/3)*ln((2^(2/3)*b^(2/3)*x^2+2^(1/3)*b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)
^(2/3))/x^2)*a-6*(b*x^3+a)^(2/3)*x*b^(1/3)+10*3^(1/2)*arctan(1/3*3^(1/2)*(b
^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)*a+10*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))
/x)*a-5*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a)/
d/b^(4/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 653, normalized size of antiderivative = 2.85

$$\int \frac{x^3(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \left[-\frac{6\cdot 4^{\frac{1}{3}}\sqrt{3}ab\left(-\frac{1}{b}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}x-4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(-\frac{1}{b}\right)^{\frac{1}{3}}}{3x}\right)-15\sqrt{\frac{1}{3}}ab\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\,bx\right.}{}\right.$$

$$-\frac{6\cdot 4^{\frac{1}{3}}\sqrt{3}ab\left(-\frac{1}{b}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}x-4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(-\frac{1}{b}\right)^{\frac{1}{3}}}{3x}\right)-6\cdot 4^{\frac{1}{3}}ab\left(-\frac{1}{b}\right)^{\frac{1}{3}}\log\left(-\frac{4^{\frac{2}{3}}bx\left(-\frac{1}{b}\right)^{\frac{2}{3}}-2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)+3\cdot}{}$$

```
[In] integrate(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] [-1/18*(6*4^(1/3)*sqrt(3)*a*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)
*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3))/x) - 15*sqrt(1/3)*a*b*sqrt(-1/b^(2
```

```
/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x
^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/
3)) + 2*a) - 6*4^(1/3)*a*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*(-1/b)^(2/3) - 2*
(b*x^3 + a)^(1/3))/x) + 3*4^(1/3)*a*b*(-1/b)^(1/3)*log(-(2*4^(1/3)*b*x^2*(-
1/b)^(1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(2/
3))/x^2) + 6*(b*x^3 + a)^(2/3)*b*x - 10*a*b^(2/3)*log(-(b^(1/3)*x - (b*x^3
+ a)^(1/3))/x) + 5*a*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x
 + (b*x^3 + a)^(2/3))/x^2))/(b^2*d), -1/18*(6*4^(1/3)*sqrt(3)*a*b*(-1/b)^(1
/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3)
)/x) - 6*4^(1/3)*a*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3
 + a)^(1/3))/x) + 3*4^(1/3)*a*b*(-1/b)^(1/3)*log(-(2*4^(1/3)*b*x^2*(-1/b)^(
1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(2/3))/x^
2) - 30*sqrt(1/3)*a*b^(2/3)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/
3))/(b^(1/3)*x)) + 6*(b*x^3 + a)^(2/3)*b*x - 10*a*b^(2/3)*log(-(b^(1/3)*x -
 (b*x^3 + a)^(1/3))/x) + 5*a*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b
^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b^2*d)]
```

Sympy [F]

$$\int \frac{x^3(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^3(a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

```
[In] integrate(x**3*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
[Out] -Integral(x**3*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Maxima [F]

$$\int \frac{x^3(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}x^3}{bdx^3-ad}\,dx$$

```
[In] integrate(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)*x^3/(b*d*x^3 - a*d), x)
```

Giac [F]

$$\int \frac{x^3(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}x^3}{bdx^3-ad}\,dx$$

```
[In] integrate(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(2/3)*x^3/(b*d*x^3 - a*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int \frac{x^3\,(b\,x^3+a)^{2/3}}{a\,d-b\,d\,x^3}\,dx$$

```
[In] int((x^3*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
[Out] int((x^3*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)
```

3.595 $\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 200

$$\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}d} + \frac{2^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}d} + \frac{\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}\sqrt[3]{b}d} - \frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\sqrt[3]{b}d} + \frac{\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}d}$$

```
[Out] 1/6*ln(-b*d*x^3+a*d)*2^(2/3)/b^(1/3)/d-1/2*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(
1/3))*2^(2/3)/b^(1/3)/d+1/2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)/d-1/3*ar
ctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)/d*3^(1/2)+1/3*2^(
2/3)*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)/d*
3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {399, 245, 384}

$$\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}d}+\frac{2^{2/3}\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}d}$$
$$+\frac{\log(ad-bdx^3)}{3\sqrt[3]{2}\sqrt[3]{b}d}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\sqrt[3]{b}d}+\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}d}$$

```
[In] Int[(a + b*x^3)^(2/3)/(a*d - b*d*x^3),x]
```

```
[Out] -(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)*d)
) + (2^(2/3)*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])
/(Sqrt[3]*b^(1/3)*d) + Log[a*d - b*d*x^3]/(3*2^(1/3)*b^(1/3)*d) - Log[2^(1/
3)*b^(1/3)*x - (a + b*x^3)^(1/3)]/(2^(1/3)*b^(1/3)*d) + Log[-(b^(1/3)*x) +
(a + b*x^3)^(1/3)]/(2*b^(1/3)*d)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = (2a)\int \frac{1}{\sqrt[3]{a+bx^3}\,(ad-bdx^3)}\,dx - \frac{\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{d}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}d} + \frac{2^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}d} + \frac{\log(ad-bdx^3)}{3\sqrt[3]{2}\sqrt[3]{b}d} - \frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\sqrt[3]{b}d} + \frac{\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}d}$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) - 2\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right) - 2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right) + 2}{}$$

```
[In] Integrate[(a + b*x^3)^(2/3)/(a*d - b*d*x^3),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3)
)] - 2*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a +
 b*x^3)^(1/3))] - 2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + 2*2^(2/3)*Log[-
2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)] + Log[b^(2/3)*x^2 + b^(1/3)*x*(a +
 b*x^3)^(1/3) + (a + b*x^3)^(2/3)] - 2^(2/3)*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^
(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(b^(1/3)*d)
```

Maple [A] (verified)

Time = 4.53 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.10

method	result
pseudoelliptic	$\frac{-2\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{3b^{\frac{1}{3}}x}\right)+2^{\frac{2}{3}}\ln\left(\frac{2^{\frac{2}{3}}b^{\frac{2}{3}}x^2+2^{\frac{1}{3}}b^{\frac{1}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}x+\left(b\,x^3+a\right)^{\frac{2}{3}}}{x^2}\right)-2\,2^{\frac{2}{3}}\ln\left(\frac{-2^{\frac{1}{3}}b^{\frac{1}{3}}x+\left(b\,x^3+a\right.}{x}\right.}{6d\,b^{\frac{1}{3}}}$

```
[In] int((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-2*3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)+b^(1/3)
*x)/b^(1/3)/x)+2^(2/3)*ln((2^(2/3)*b^(2/3)*x^2+2^(1/3)*b^(1/3)*(b*x^3+a)^(1
/3)*x+(b*x^3+a)^(2/3))/x^2)-2*2^(2/3)*ln((-2^(1/3)*b^(1/3)*x+(b*x^3+a)^(1/3
))/x)+2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)
+2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/
3)*x+(b*x^3+a)^(2/3))/x^2))/d/b^(1/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 611, normalized size of antiderivative = 3.06

$$\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \left[-\frac{2\cdot 4^{\frac{1}{3}}\sqrt{3}b\left(-\frac{1}{b}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}x-4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(-\frac{1}{b}\right)^{\frac{1}{3}}}{3x}\right)-3\sqrt{\frac{1}{3}}b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3bx^3-3\right.}{}\right.$$

$$-\frac{2\cdot 4^{\frac{1}{3}}\sqrt{3}b\left(-\frac{1}{b}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}x-4^{\frac{1}{3}}\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(-\frac{1}{b}\right)^{\frac{1}{3}}}{3x}\right)-2\cdot 4^{\frac{1}{3}}b\left(-\frac{1}{b}\right)^{\frac{1}{3}}\log\left(-\frac{4^{\frac{2}{3}}bx\left(-\frac{1}{b}\right)^{\frac{2}{3}}-2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)+4^{\frac{1}{3}}b(-}{}$$

```
[In] integrate((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*4^(1/3)*sqrt(3)*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sq
rt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3))/x) - 3*sqrt(1/3)*b*sqrt(-1/b^(2/3))*l
og(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (
b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) +
2*a) - 2*4^(1/3)*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 +
 a)^(1/3))/x) + 4^(1/3)*b*(-1/b)^(1/3)*log(-(2*4^(1/3)*b*x^2*(-1/b)^(1/3) -
 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(2/3))/x^2) - 2
*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) + b^(2/3)*log((b^(2/3)*x^2
 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b*d), -1/6*(2*4^
(1/3)*sqrt(3)*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sqrt(3)*(b*x^
3 + a)^(1/3)*(-1/b)^(1/3))/x) - 2*4^(1/3)*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*
(-1/b)^(2/3) - 2*(b*x^3 + a)^(1/3))/x) + 4^(1/3)*b*(-1/b)^(1/3)*log(-(2*4^(
1/3)*b*x^2*(-1/b)^(1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b
*x^3 + a)^(2/3))/x^2) - 6*sqrt(1/3)*b^(2/3)*arctan(sqrt(1/3)*(b^(1/3)*x + 2
*(b*x^3 + a)^(1/3))/(b^(1/3)*x)) - 2*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^
(1/3))/x) + b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3
 + a)^(2/3))/x^2))/(b*d)]
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

```
[In] integrate((b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
[Out] -Integral((a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{bdx^3-ad}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)/(b*d*x^3 - a*d), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{bdx^3-ad}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(2/3)/(b*d*x^3 - a*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{a\,d-b\,d\,x^3}\,dx$$

```
[In] int((a + b*x^3)^(2/3)/(a*d - b*d*x^3),x)
[Out] int((a + b*x^3)^(2/3)/(a*d - b*d*x^3), x)
```

3.596 $\int \frac{(a+bx^3)^{2/3}}{x^3(ad-bdx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 157

$$\int \frac{(a+bx^3)^{2/3}}{x^3(ad-bdx^3)}\,dx = -\frac{(a+bx^3)^{2/3}}{2adx^2}+\frac{2^{2/3}b^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}ad}+\frac{b^{2/3}\log(ad-bdx^3)}{3\sqrt[3]{2}ad}-\frac{b^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}ad}$$

```
[Out] -1/2*(b*x^3+a)^(2/3)/a/d/x^2+1/6*b^(2/3)*ln(-b*d*x^3+a*d)*2^(2/3)/a/d-1/2*b
^(2/3)*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(2/3)/a/d+1/3*2^(2/3)*b^(2/3
)*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/a/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {486, 12, 384}

$$\int \frac{(a+bx^3)^{2/3}}{x^3(ad-bdx^3)}\,dx = \frac{2^{2/3}b^{2/3}\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}ad}+\frac{b^{2/3}\log(ad-bdx^3)}{3\sqrt[3]{2}ad}-\frac{b^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}ad}-\frac{(a+bx^3)^{2/3}}{2adx^2}$$

```
[In] Int[(a + b*x^3)^(2/3)/(x^3*(a*d - b*d*x^3)),x]
```

```
[Out] -1/2*(a + b*x^3)^(2/3)/(a*d*x^2) + (2^(2/3)*b^(2/3)*ArcTan[(1 + (2*2^(1/3)*
b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*a*d) + (b^(2/3)*Log[a*d -
b*d*x^3])/(3*2^(1/3)*a*d) - (b^(2/3)*Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1
/3)])/(2^(1/3)*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(a+bx^3\right)^{2/3}}{2adx^2}+\frac{\int \frac{4abd}{\sqrt[3]{a+bx^3}(ad-bdx^3)}\,dx}{2ad}\\
&= -\frac{\left(a+bx^3\right)^{2/3}}{2adx^2}+(2b)\int \frac{1}{\sqrt[3]{a+bx^3}\,(ad-bdx^3)}\,dx\\
&= -\frac{\left(a+bx^3\right)^{2/3}}{2adx^2}+\frac{2^{2/3}b^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}ad}\\
&\quad+\frac{b^{2/3}\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}ad}-\frac{b^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}ad}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx^3)^{2/3}}{x^3(ad-bdx^3)}\,dx = \frac{-3(a+bx^3)^{2/3} + 2\,2^{2/3}\sqrt{3}b^{2/3}x^2\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right) - 2\,2^{2/3}b^{2/3}x^2\log\left(-\right.}{}$$

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^3*(a*d - b*d*x^3)),x]
```

```
[Out] (-3*(a + b*x^3)^(2/3) + 2*2^(2/3)*Sqrt[3]*b^(2/3)*x^2*ArcTan[(Sqrt[3]*b^(1/
3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))] - 2*2^(2/3)*b^(2/3)*x^2*Log[
-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)] + 2^(2/3)*b^(2/3)*x^2*Log[2*b^(2/
3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/
(6*a*d*x^2)
```

Maple [A] (verified)

Time = 4.75 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.99

method	result
pseudoelliptic	$\frac{-2b^{\frac{2}{3}}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{3b^{\frac{1}{3}}x}\right)x^2+b^{\frac{2}{3}}2^{\frac{2}{3}}\ln\left(\frac{2^{\frac{2}{3}}b^{\frac{2}{3}}x^2+2^{\frac{1}{3}}b^{\frac{1}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}x+\left(b\,x^3+a\right)^{\frac{2}{3}}}{x^2}\right)x^2-2b^{\frac{2}{3}}2^{\frac{2}{3}}\ln\left(\frac{-2}{}\right.}{6ad\,x^2}$

```
[In] int((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-2*b^(2/3)*3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)
+b^(1/3)*x)/b^(1/3)/x)*x^2+b^(2/3)*2^(2/3)*ln((2^(2/3)*b^(2/3)*x^2+2^(1/3)*
b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*x^2-2*b^(2/3)*2^(2/3)*ln((-
2^(1/3)*b^(1/3)*x+(b*x^3+a)^(1/3))/x)*x^2-3*(b*x^3+a)^(2/3))/a/d/x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(125) = 250.

Time = 81.74 (sec) , antiderivative size = 434, normalized size of antiderivative = 2.76

$$\int \frac{(a+bx^3)^{2/3}}{x^3(ad-bdx^3)}\,dx =$$

$$-\frac{2\cdot 4^{\frac{1}{3}}\sqrt{3}(-b^2)^{\frac{1}{3}}x^2\arctan\left(\frac{3\cdot 4^{\frac{2}{3}}\sqrt{3}(5\,b^2x^7-4\,abx^4-a^2x)(bx^3+a)^{\frac{2}{3}}(-b^2)^{\frac{2}{3}}+6\cdot 4^{\frac{1}{3}}\sqrt{3}(19\,b^3x^8+16\,ab^2x^5+a^2bx^2)(bx^3+a)^{\frac{1}{3}}(-b^2)^{\frac{1}{3}}-}{3(109\,b^4x^9+105\,ab^3x^6+3\,a^2b^2x^3-a^3b)}\right.}{}$$

```
[In] integrate((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/18*(2*4^(1/3)*sqrt(3)*(-b^2)^(1/3)*x^2*arctan(1/3*(3*4^(2/3)*sqrt(3)*(5*
b^2*x^7 - 4*a*b*x^4 - a^2*x)*(b*x^3 + a)^(2/3)*(-b^2)^(2/3) + 6*4^(1/3)*sqr
t(3)*(19*b^3*x^8 + 16*a*b^2*x^5 + a^2*b*x^2)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3)
 - sqrt(3)*(71*b^4*x^9 + 111*a*b^3*x^6 + 33*a^2*b^2*x^3 + a^3*b))/(109*b^4*
x^9 + 105*a*b^3*x^6 + 3*a^2*b^2*x^3 - a^3*b)) - 2*4^(1/3)*(-b^2)^(1/3)*x^2*
log((3*4^(2/3)*(b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x^2 - 6*(b*x^3 + a)^(2/3)*b*x
 + 4^(1/3)*(b*x^3 - a)*(-b^2)^(1/3))/(b*x^3 - a)) + 4^(1/3)*(-b^2)^(1/3)*x^
2*log(-(6*4^(1/3)*(5*b^2*x^4 + a*b*x)*(b*x^3 + a)^(2/3)*(-b^2)^(1/3) - 4^(2
/3)*(19*b^2*x^6 + 16*a*b*x^3 + a^2)*(-b^2)^(2/3) - 24*(2*b^3*x^5 + a*b^2*x^
2)*(b*x^3 + a)^(1/3))/(b^2*x^6 - 2*a*b*x^3 + a^2)) + 9*(b*x^3 + a)^(2/3))/(
a*d*x^2)
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^3(ad-bdx^3)}\,dx = -\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-ax^3+bx^6}\,dx}{d}$$

```
[In] integrate((b*x**3+a)**(2/3)/x**3/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**3 + b*x**6), x)/d
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^3(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^3}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^3), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^3(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^3}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^3\,(ad-bdx^3)}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{x^3\,(a\,d-b\,d\,x^3)}\,dx$$

```
[In] int((a + b*x^3)^(2/3)/(x^3*(a*d - b*d*x^3)),x)
[Out] int((a + b*x^3)^(2/3)/(x^3*(a*d - b*d*x^3)), x)
```

3.597 $\int \frac{(a+bx^3)^{2/3}}{x^6(ad-bdx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 182

$$\int \frac{(a+bx^3)^{2/3}}{x^6(ad-bdx^3)}\,dx = -\frac{(a+bx^3)^{2/3}}{5adx^5} - \frac{7b(a+bx^3)^{2/3}}{10a^2dx^2} + \frac{2^{2/3}b^{5/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^2d} + \frac{b^{5/3}\log(ad-bdx^3)}{3\sqrt[3]{2}a^2d} - \frac{b^{5/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x - \sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^2d}$$

```
[Out] -1/5*(b*x^3+a)^(2/3)/a/d/x^5-7/10*b*(b*x^3+a)^(2/3)/a^2/d/x^2+1/6*b^(5/3)*l
n(-b*d*x^3+a*d)*2^(2/3)/a^2/d-1/2*b^(5/3)*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1
/3))*2^(2/3)/a^2/d+1/3*2^(2/3)*b^(5/3)*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b
*x^3+a)^(1/3))*3^(1/2))/a^2/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {486, 597, 12, 384}

$$\int \frac{(a+bx^3)^{2/3}}{x^6(ad-bdx^3)}\,dx = \frac{2^{2/3}b^{5/3}\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}a^2d} + \frac{b^{5/3}\log(ad-bdx^3)}{3\sqrt[3]{2}a^2d} - \frac{b^{5/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x - \sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^2d} - \frac{7b(a+bx^3)^{2/3}}{10a^2dx^2} - \frac{(a+bx^3)^{2/3}}{5adx^5}$$

```
[In] Int[(a + b*x^3)^(2/3)/(x^6*(a*d - b*d*x^3)),x]
```

```
[Out] -1/5*(a + b*x^3)^(2/3)/(a*d*x^5) - (7*b*(a + b*x^3)^(2/3))/(10*a^2*d*x^2) +
 (2^(2/3)*b^(5/3)*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt
[3]])/(Sqrt[3]*a^2*d) + (b^(5/3)*Log[a*d - b*d*x^3])/(3*2^(1/3)*a^2*d) - (b
^(5/3)*Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)])/(2^(1/3)*a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{(a+bx^3)^{2/3}}{5adx^5} + \frac{\int \frac{7abd+3b^2dx^3}{x^3\sqrt[3]{a+bx^3}(ad-bdx^3)}\,dx}{5ad}$$

$$= -\frac{(a+bx^3)^{2/3}}{5adx^5} - \frac{7b(a+bx^3)^{2/3}}{10a^2dx^2} - \frac{\int -\frac{20a^2b^2d^2}{\sqrt[3]{a+bx^3}(ad-bdx^3)}\,dx}{10a^3d^2}$$

$$= -\frac{(a+bx^3)^{2/3}}{5adx^5} - \frac{7b(a+bx^3)^{2/3}}{10a^2dx^2} + \frac{(2b^2)\int \frac{1}{\sqrt[3]{a+bx^3}(ad-bdx^3)}dx}{a}$$

$$= -\frac{(a+bx^3)^{2/3}}{5adx^5} - \frac{7b(a+bx^3)^{2/3}}{10a^2dx^2} + \frac{2^{2/3}b^{5/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^2d} + \frac{b^{5/3}\log(ad-bdx^3)}{3\sqrt[3]{2}a^2d} - \frac{b^{5/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^2d}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^3)^{2/3}}{x^6(ad-bdx^3)}dx = -\frac{(a+bx^3)^{2/3}(2a+7bx^3)}{10a^2dx^5} + \frac{2^{2/3}b^{5/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}a^2d} - \frac{2^{2/3}b^{5/3}\log\left(-2\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}\right)}{3a^2d} + \frac{b^{5/3}\log\left(2b^{2/3}x^2+2^{2/3}\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\sqrt[3]{2}(a+bx^3)^{2/3}\right)}{3\sqrt[3]{2}a^2d}$$

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^6*(a*d - b*d*x^3)),x]
```

```
[Out] -1/10*((a + b*x^3)^(2/3)*(2*a + 7*b*x^3))/(a^2*d*x^5) + (2^(2/3)*b^(5/3)*Ar
cTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3))])/(Sqrt[3]
*a^2*d) - (2^(2/3)*b^(5/3)*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)])/(
3*a^2*d) + (b^(5/3)*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3)
 + 2^(1/3)*(a + b*x^3)^(2/3)])/(3*2^(1/3)*a^2*d)
```

Maple [A] (verified)

Time = 4.88 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$\frac{5x^5 2^{\frac{2}{3}}\left(-2\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{3b^{\frac{1}{3}}x}\right)\sqrt{3}+\ln\left(\frac{2^{\frac{2}{3}}b^{\frac{2}{3}}x^2+2^{\frac{1}{3}}b^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)-2\ln\left(\frac{-2^{\frac{1}{3}}b^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right.\right.}{30x^5a^2d}$

```
[In] int((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30*(5*x^5*2^(2/3)*(-2*arctan(1/3*3^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)+b^(1/3)
*x)/b^(1/3)/x)*3^(1/2)+ln((2^(2/3)*b^(2/3)*x^2+2^(1/3)*b^(1/3)*(b*x^3+a)^(1
/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((-2^(1/3)*b^(1/3)*x+(b*x^3+a)^(1/3))/x))*b
^(5/3)-3*(b*x^3+a)^(2/3)*(7*b*x^3+2*a))/x^5/a^2/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^6\,(ad-bdx^3)}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^6\,(ad-bdx^3)}\,dx = -\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-ax^6+bx^9}\,dx}{d}$$

```
[In] integrate((b*x**3+a)**(2/3)/x**6/(-b*d*x**3+a*d),x)
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**6 + b*x**9), x)/d
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^6\,(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^6}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^6), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^6(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^6}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^6(ad-bdx^3)}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{x^6\,(a\,d-b\,d\,x^3)}\,dx$$

```
[In] int((a + b*x^3)^(2/3)/(x^6*(a*d - b*d*x^3)),x)
[Out] int((a + b*x^3)^(2/3)/(x^6*(a*d - b*d*x^3)), x)
```

3.598 $\int \frac{(a+bx^3)^{2/3}}{x^9(ad-bdx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 209

$$\int \frac{(a+bx^3)^{2/3}}{x^9(ad-bdx^3)}\,dx = -\frac{(a+bx^3)^{2/3}}{8adx^8} - \frac{b(a+bx^3)^{2/3}}{4a^2dx^5} - \frac{5b^2(a+bx^3)^{2/3}}{8a^3dx^2} + \frac{2^{2/3}b^{8/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^3d} + \frac{b^{8/3}\log(ad-bdx^3)}{3\sqrt[3]{2}a^3d} - \frac{b^{8/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^3d}$$

```
[Out] -1/8*(b*x^3+a)^(2/3)/a/d/x^8-1/4*b*(b*x^3+a)^(2/3)/a^2/d/x^5-5/8*b^2*(b*x^3
+a)^(2/3)/a^3/d/x^2+1/6*b^(8/3)*ln(-b*d*x^3+a*d)*2^(2/3)/a^3/d-1/2*b^(8/3)*
ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(2/3)/a^3/d+1/3*2^(2/3)*b^(8/3)*arc
tan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/a^3/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {486, 597, 12, 384}

$$\int \frac{(a+bx^3)^{2/3}}{x^9(ad-bdx^3)}\,dx = \frac{2^{2/3}b^{8/3}\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}a^3d} + \frac{b^{8/3}\log(ad-bdx^3)}{3\sqrt[3]{2}a^3d} - \frac{b^{8/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^3d} - \frac{5b^2(a+bx^3)^{2/3}}{8a^3dx^2} - \frac{b(a+bx^3)^{2/3}}{4a^2dx^5} - \frac{(a+bx^3)^{2/3}}{8adx^8}$$

```
[In] Int[(a + b*x^3)^(2/3)/(x^9*(a*d - b*d*x^3)),x]
```

```
[Out] -1/8*(a + b*x^3)^(2/3)/(a*d*x^8) - (b*(a + b*x^3)^(2/3))/(4*a^2*d*x^5) - (5
*b^2*(a + b*x^3)^(2/3))/(8*a^3*d*x^2) + (2^(2/3)*b^(8/3)*ArcTan[(1 + (2*2^(
1/3)*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*a^3*d) + (b^(8/3)*Log
[a*d - b*d*x^3])/(3*2^(1/3)*a^3*d) - (b^(8/3)*Log[2^(1/3)*b^(1/3)*x - (a +
b*x^3)^(1/3)])/(2^(1/3)*a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{(a+bx^3)^{2/3}}{8adx^8}+\frac{\int \frac{10abd+6b^2dx^3}{x^6\sqrt[3]{a+bx^3}(ad-bdx^3)}\,dx}{8ad}$$

$$= -\frac{(a+bx^3)^{2/3}}{8adx^8} - \frac{b(a+bx^3)^{2/3}}{4a^2dx^5} - \frac{\int \frac{-50a^2b^2d^2-30ab^3d^2x^3}{x^3\sqrt[3]{a+bx^3}(ad-bdx^3)}dx}{40a^3d^2}$$

$$= -\frac{(a+bx^3)^{2/3}}{8adx^8} - \frac{b(a+bx^3)^{2/3}}{4a^2dx^5} - \frac{5b^2(a+bx^3)^{2/3}}{8a^3dx^2} + \frac{\int \frac{160a^3b^3d^3}{\sqrt[3]{a+bx^3}(ad-bdx^3)}dx}{80a^5d^3}$$

$$= -\frac{(a+bx^3)^{2/3}}{8adx^8} - \frac{b(a+bx^3)^{2/3}}{4a^2dx^5} - \frac{5b^2(a+bx^3)^{2/3}}{8a^3dx^2} + \frac{(2b^3)\int \frac{1}{\sqrt[3]{a+bx^3}(ad-bdx^3)}dx}{a^2}$$

$$= -\frac{(a+bx^3)^{2/3}}{8adx^8} - \frac{b(a+bx^3)^{2/3}}{4a^2dx^5} - \frac{5b^2(a+bx^3)^{2/3}}{8a^3dx^2} + \frac{2^{2/3}b^{8/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^3d} + \frac{b^{8/3}\log(ad-bdx^3)}{3\sqrt[3]{2}a^3d} - \frac{b^{8/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^3d}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3)^{2/3}}{x^9(ad-bdx^3)}dx = \frac{-\frac{3(a+bx^3)^{2/3}(a^2+2abx^3+5b^2x^6)}{x^8} + 8\,2^{2/3}\sqrt{3}b^{8/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+bx^3}}\right) - 8\,2^{2/3}b^{8/3}}{}$$

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^9*(a*d - b*d*x^3)),x]
```

```
[Out] ((-3*(a + b*x^3)^(2/3)*(a^2 + 2*a*b*x^3 + 5*b^2*x^6))/x^8 + 8*2^(2/3)*Sqrt[
3]*b^(8/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3
))] - 8*2^(2/3)*b^(8/3)*Log[-2*b^(1/3)*x + 2^(2/3)*(a + b*x^3)^(1/3)] + 4*2
^(2/3)*b^(8/3)*Log[2*b^(2/3)*x^2 + 2^(2/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^
(1/3)*(a + b*x^3)^(2/3)])/(24*a^3*d)
```

Maple [A] (verified)

Time = 4.83 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{4x^8 2^{\frac{2}{3}}\left(-2\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{3b^{\frac{1}{3}}x}\right)\sqrt{3}+\ln\left(\frac{2^{\frac{2}{3}}b^{\frac{2}{3}}x^2+2^{\frac{1}{3}}b^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)-2\ln\left(\frac{-2^{\frac{1}{3}}b^{\frac{1}{3}}x+\left(bx^3+a\right.}{x}\right.}{24x^8a^3d}$

```
[In] int((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(4*x^8*2^(2/3)*(-2*arctan(1/3*3^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)+b^(1/3)
*x)/b^(1/3)/x)*3^(1/2)+ln((2^(2/3)*b^(2/3)*x^2+2^(1/3)*b^(1/3)*(b*x^3+a)^(1
/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((-2^(1/3)*b^(1/3)*x+(b*x^3+a)^(1/3))/x))*b
^(8/3)-3*(b*x^3+a)^(2/3)*(5*b^2*x^6+2*a*b*x^3+a^2))/x^8/a^3/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^9\,(ad-bdx^3)}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^9\,(ad-bdx^3)}\,dx = -\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-ax^9+bx^{12}}\,dx}{d}$$

```
[In] integrate((b*x**3+a)**(2/3)/x**9/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**9 + b*x**12), x)/d
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^9\,(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^9}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^9), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^9(ad-bdx^3)}\,dx=\int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^9}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^9), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^9(ad-bdx^3)}\,dx=\int \frac{(b\,x^3+a)^{2/3}}{x^9\,(a\,d-b\,d\,x^3)}\,dx$$

```
[In] int((a + b*x^3)^(2/3)/(x^9*(a*d - b*d*x^3)),x)
[Out] int((a + b*x^3)^(2/3)/(x^9*(a*d - b*d*x^3)), x)
```

3.599 $\int \frac{(a+bx^3)^{2/3}}{x^{12}(ad-bdx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 236

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(ad-bdx^3)}\,dx = -\frac{(a+bx^3)^{2/3}}{11adx^{11}}-\frac{13b(a+bx^3)^{2/3}}{88a^2dx^8}-\frac{49b^2(a+bx^3)^{2/3}}{220a^3dx^5}-\frac{293b^3(a+bx^3)^{2/3}}{440a^4dx^2}+\frac{2^{2/3}b^{11/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^4d}+\frac{b^{11/3}\log(ad-bdx^3)}{3\sqrt[3]{2}a^4d}-\frac{b^{11/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^4d}$$

```
[Out] -1/11*(b*x^3+a)^(2/3)/a/d/x^11-13/88*b*(b*x^3+a)^(2/3)/a^2/d/x^8-49/220*b^2
*(b*x^3+a)^(2/3)/a^3/d/x^5-293/440*b^3*(b*x^3+a)^(2/3)/a^4/d/x^2+1/6*b^(11/
3)*ln(-b*d*x^3+a*d)*2^(2/3)/a^4/d-1/2*b^(11/3)*ln(2^(1/3)*b^(1/3)*x-(b*x^3+
a)^(1/3))*2^(2/3)/a^4/d+1/3*2^(2/3)*b^(11/3)*arctan(1/3*(1+2*2^(1/3)*b^(1/3
)*x/(b*x^3+a)^(1/3))*3^(1/2))/a^4/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used

$= \{486, 597, 12, 384\}$

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(ad-bdx^3)}\,dx = \frac{2^{2/3}b^{11/3}\arctan\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}a^4d} + \frac{b^{11/3}\log(ad-bdx^3)}{3\sqrt[3]{2}a^4d} - \frac{b^{11/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^4d} - \frac{293b^3(a+bx^3)^{2/3}}{440a^4dx^2} - \frac{49b^2(a+bx^3)^{2/3}}{220a^3dx^5} - \frac{13b(a+bx^3)^{2/3}}{88a^2dx^8} - \frac{(a+bx^3)^{2/3}}{11adx^{11}}$$

```
[In] Int[(a + b*x^3)^(2/3)/(x^12*(a*d - b*d*x^3)),x]
```

```
[Out] -1/11*(a + b*x^3)^(2/3)/(a*d*x^11) - (13*b*(a + b*x^3)^(2/3))/(88*a^2*d*x^8
) - (49*b^2*(a + b*x^3)^(2/3))/(220*a^3*d*x^5) - (293*b^3*(a + b*x^3)^(2/3)
)/(440*a^4*d*x^2) + (2^(2/3)*b^(11/3)*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(a
+ b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*a^4*d) + (b^(11/3)*Log[a*d - b*d*x^3])/(
3*2^(1/3)*a^4*d) - (b^(11/3)*Log[2^(1/3)*b^(1/3)*x - (a + b*x^3)^(1/3)])/(2
^(1/3)*a^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+bx^3)^{2/3}}{11adx^{11}}+\frac{\int \frac{13abd+9b^2dx^3}{x^9\sqrt[3]{a+bx^3}(ad-bdx^3)}\,dx}{11ad}\\
&= -\frac{(a+bx^3)^{2/3}}{11adx^{11}}-\frac{13b(a+bx^3)^{2/3}}{88a^2dx^8}-\frac{\int \frac{-98a^2b^2d^2-78ab^3d^2x^3}{x^6\sqrt[3]{a+bx^3}(ad-bdx^3)}\,dx}{88a^3d^2}\\
&= -\frac{(a+bx^3)^{2/3}}{11adx^{11}}-\frac{13b(a+bx^3)^{2/3}}{88a^2dx^8}-\frac{49b^2(a+bx^3)^{2/3}}{220a^3dx^5}+\frac{\int \frac{586a^3b^3d^3+294a^2b^4d^3x^3}{x^3\sqrt[3]{a+bx^3}(ad-bdx^3)}\,dx}{440a^5d^3}\\
&= -\frac{(a+bx^3)^{2/3}}{11adx^{11}}-\frac{13b(a+bx^3)^{2/3}}{88a^2dx^8}-\frac{49b^2(a+bx^3)^{2/3}}{220a^3dx^5}\\
&\quad-\frac{293b^3(a+bx^3)^{2/3}}{440a^4dx^2}-\frac{\int -\frac{1760a^4b^4d^4}{\sqrt[3]{a+bx^3}(ad-bdx^3)}\,dx}{880a^7d^4}\\
&= -\frac{(a+bx^3)^{2/3}}{11adx^{11}}-\frac{13b(a+bx^3)^{2/3}}{88a^2dx^8}-\frac{49b^2(a+bx^3)^{2/3}}{220a^3dx^5}\\
&\quad-\frac{293b^3(a+bx^3)^{2/3}}{440a^4dx^2}+\frac{(2b^4)\int \frac{1}{\sqrt[3]{a+bx^3}(ad-bdx^3)}\,dx}{a^3}\\
&= -\frac{(a+bx^3)^{2/3}}{11adx^{11}}-\frac{13b(a+bx^3)^{2/3}}{88a^2dx^8}-\frac{49b^2(a+bx^3)^{2/3}}{220a^3dx^5}\\
&\quad-\frac{293b^3(a+bx^3)^{2/3}}{440a^4dx^2}+\frac{2^{2/3}b^{11/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^4d}\\
&\quad+\frac{b^{11/3}\log(ad-bdx^3)}{3\sqrt[3]{2}a^4d}-\frac{b^{11/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}a^4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(ad-bdx^3)}\,dx = \frac{-\frac{3(a+bx^3)^{2/3}(40a^3+65a^2bx^3+98ab^2x^6+293b^3x^9)}{x^{11}} + 440\,2^{2/3}\sqrt{3}b^{11/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a+b}}\right.}{}$$

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^12*(a*d - b*d*x^3)),x]
```

```
[Out] ((-3*(a + b*x^3)^(2/3)*(40*a^3 + 65*a^2*b*x^3 + 98*a*b^2*x^6 + 293*b^3*x^9)
)/x^11 + 440*2^(2/3)*Sqrt[3]*b^(11/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x
 + 2^(2/3)*(a + b*x^3)^(1/3))] - 440*2^(2/3)*b^(11/3)*Log[-2*b^(1/3)*x + 2^
(2/3)*(a + b*x^3)^(1/3)] + 220*2^(2/3)*b^(11/3)*Log[2*b^(2/3)*x^2 + 2^(2/3)
*b^(1/3)*x*(a + b*x^3)^(1/3) + 2^(1/3)*(a + b*x^3)^(2/3)])/(1320*a^4*d)
```

Maple [A] (verified)

Time = 5.07 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.74

method	result
pseudoelliptic	$\frac{-220x^{11}2^{\frac{2}{3}}\left(2\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{3b^{\frac{1}{3}}x}\right)\sqrt{3}+2\ln\left(\frac{-2^{\frac{1}{3}}b^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)-\ln\left(\frac{2^{\frac{2}{3}}b^{\frac{2}{3}}x^2+2^{\frac{1}{3}}b^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}x+\left(bx^3\right.}{x^2}\right.\right.}{1320x^{11}a^4d}$

```
[In] int((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1320*(-220*x^11*2^(2/3)*(2*arctan(1/3*3^(1/2)*(2^(2/3)*(b*x^3+a)^(1/3)+b^
(1/3)*x)/b^(1/3)/x)*3^(1/2)+2*ln((-2^(1/3)*b^(1/3)*x+(b*x^3+a)^(1/3))/x)-ln
((2^(2/3)*b^(2/3)*x^2+2^(1/3)*b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^
2))*b^(11/3)-3*(b*x^3+a)^(2/3)*(293*b^3*x^9+98*a*b^2*x^6+65*a^2*b*x^3+40*a^
3))/x^11/a^4/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(ad-bdx^3)}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(ad-bdx^3)}\,dx = -\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-ax^{12}+bx^{15}}\,dx}{d}$$

```
[In] integrate((b*x**3+a)**(2/3)/x**12/(-b*d*x**3+a*d),x)
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**12 + b*x**15), x)/d
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^{12}}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^12), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^{12}}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^12), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(ad-bdx^3)}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{x^{12}\,(a\,d-b\,d\,x^3)}\,dx$$

```
[In] int((a + b*x^3)^(2/3)/(x^12*(a*d - b*d*x^3)),x)
[Out] int((a + b*x^3)^(2/3)/(x^12*(a*d - b*d*x^3)), x)
```

3.600 $\int \frac{x^7 (a+bx^3)^{2/3}}{ad-bdx^3} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 512

$$\int \frac{x^7(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{9ax^2(a+bx^3)^{2/3}}{28b^2d} - \frac{x^5(a+bx^3)^{2/3}}{7bd} + \frac{2^{2/3}a^{7/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{8/3}d} + \frac{a^{7/3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}b^{8/3}d} - \frac{19a^2x^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{28b^2d\sqrt[3]{a+bx^3}} + \frac{a^{7/3}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}b^{8/3}d} + \frac{a^{7/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{2}b^{8/3}d} - \frac{2^{2/3}a^{7/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3b^{8/3}d} - \frac{a^{7/3}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}b^{8/3}d}$$

```
[Out] -9/28*a*x^2*(b*x^3+a)^(2/3)/b^2/d-1/7*x^5*(b*x^3+a)^(2/3)/b/d-19/28*a^2*x^2
*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/b^2/d/(b*x^3+a)^(1/
3)+1/12*a^(7/3)*ln((a^(1/3)-b^(1/3)*x)^2*(a^(1/3)+b^(1/3)*x)/a)*2^(2/3)/b^(
8/3)/d+1/6*a^(7/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/
3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(2/3)/b^(8/3)/d-1/3*2^(2/3)*a^(7/
3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/b^(8/3)/d-1/4*a^(7/3)*
ln(b^(1/3)*(a^(1/3)+b^(1/3)*x)/a^(1/3)-2^(2/3)*b^(1/3)*(b*x^3+a)^(1/3)/a^(1
/3))*2^(2/3)/b^(8/3)/d+1/3*2^(2/3)*a^(7/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)
+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/b^(8/3)/d*3^(1/2)+1/6*a^(7/3)*arctan(
1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(2/3)/b^(8/3
)/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {489, 596, 598, 372, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$
\begin{aligned}
\int \frac{x^7(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = {} & \frac{2^{2/3}a^{7/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{8/3}d} \\
& + \frac{a^{7/3}\arctan\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}b^{8/3}d} \\
& + \frac{a^{7/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{2}b^{8/3}d} \\
& - \frac{2^{2/3}a^{7/3}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3b^{8/3}d} \\
& - \frac{a^{7/3}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}b^{8/3}d} \\
& + \frac{a^{7/3}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}b^{8/3}d} \\
& - \frac{19a^2x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{28b^2d\sqrt[3]{a+bx^3}} \\
& - \frac{9ax^2(a+bx^3)^{2/3}}{28b^2d}-\frac{x^5(a+bx^3)^{2/3}}{7bd}
\end{aligned}
$$

```
[In] Int[(x^7*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-9*a*x^2*(a + b*x^3)^(2/3))/(28*b^2*d) - (x^5*(a + b*x^3)^(2/3))/(7*b*d) +
 (2^(2/3)*a^(7/3)*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)
```

```
^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(8/3)*d) + (a^(7/3)*ArcTan[(1 + (2^(1/3)*(a^(1
/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*b^(8/3)*d)
- (19*a^2*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x
^3)/a)])/(28*b^2*d*(a + b*x^3)^(1/3)) + (a^(7/3)*Log[((a^(1/3) - b^(1/3)*x)
^2*(a^(1/3) + b^(1/3)*x))/a])/(6*2^(1/3)*b^(8/3)*d) + (a^(7/3)*Log[1 + (2^(
2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/
3)*x))/(a + b*x^3)^(1/3)])/(3*2^(1/3)*b^(8/3)*d) - (2^(2/3)*a^(7/3)*Log[1 +
 (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*b^(8/3)*d) - (a^(7/
3)*Log[(b^(1/3)*(a^(1/3) + b^(1/3)*x))/a^(1/3) - (2^(2/3)*b^(1/3)*(a + b*x^
3)^(1/3))/a^(1/3)])/(2*2^(1/3)*b^(8/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
```

```
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{x^5(a+bx^3)^{2/3}}{7bd} + \frac{\int \frac{x^4(5a^2d+9abdx^3)}{\sqrt[3]{a+bx^3}(ad-bdx^3)}\,dx}{7bd}$$

$$= -\frac{9ax^2(a+bx^3)^{2/3}}{28b^2d} - \frac{x^5(a+bx^3)^{2/3}}{7bd} + \frac{\int \frac{x(18a^3bd^2+38a^2b^2d^2x^3)}{\sqrt[3]{a+bx^3}(ad-bdx^3)}\,dx}{28b^3d^2}$$

$$= -\frac{9ax^2(a+bx^3)^{2/3}}{28b^2d} - \frac{x^5(a+bx^3)^{2/3}}{7bd} + \frac{\int \left(-\frac{38a^2bdx}{\sqrt[3]{a+bx^3}} + \frac{56a^3bd^2x}{\sqrt[3]{a+bx^3}(ad-bdx^3)}\right)dx}{28b^3d^2}$$

$$= -\frac{9ax^2(a+bx^3)^{2/3}}{28b^2d} - \frac{x^5(a+bx^3)^{2/3}}{7bd} + \frac{(2a^3)\int \frac{x}{\sqrt[3]{a+bx^3}(ad-bdx^3)}\,dx}{b^2} - \frac{(19a^2)\int \frac{x}{\sqrt[3]{a+bx^3}}\,dx}{14b^2d}$$

$$= -\frac{9ax^2(a+bx^3)^{2/3}}{28b^2d} - \frac{x^5(a+bx^3)^{2/3}}{7bd} - \frac{(2a^{8/3})\,\text{Subst}\left(\int \frac{1}{1+2ax^3}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{b^{8/3}d} + \frac{(2a^{7/3})\int \frac{1}{\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)\sqrt[3]{a+bx^3}}\,dx}{3b^{7/3}d} - \frac{\left(19a^2\sqrt[3]{1+\frac{bx^3}{a}}\right)\int \frac{x}{\sqrt[3]{1+\frac{bx^3}{a}}}\,dx}{14b^2d\sqrt[3]{a+bx^3}}$$

$$= -\frac{9ax^2(a+bx^3)^{2/3}}{28b^2d} - \frac{x^5(a+bx^3)^{2/3}}{7bd} + \frac{a^{7/3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}b^{8/3}d}$$

$$-\frac{19a^2x^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{28b^2d\sqrt[3]{a+bx^3}} + \frac{a^{7/3}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}b^{8/3}d}$$

$$-\frac{a^{7/3}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}} - \frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}b^{8/3}d}$$

$$-\frac{\left(2a^{8/3}\right)\operatorname{Subst}\left(\int\frac{1}{1+\sqrt[3]{2}\sqrt[3]{a}x}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3b^{8/3}d}$$

$$-\frac{\left(2a^{8/3}\right)\operatorname{Subst}\left(\int\frac{2-\sqrt[3]{2}\sqrt[3]{a}x}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3b^{8/3}d}$$

$$= -\frac{9ax^2(a+bx^3)^{2/3}}{28b^2d} - \frac{x^5(a+bx^3)^{2/3}}{7bd} + \frac{a^{7/3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}b^{8/3}d}$$

$$- \frac{19a^2x^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{28b^2d\sqrt[3]{a+bx^3}}$$

$$+ \frac{a^{7/3}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}b^{8/3}d} - \frac{2^{2/3}a^{7/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3b^{8/3}d}$$

$$- \frac{a^{7/3}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}} - \frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}b^{8/3}d}$$

$$+ \frac{a^{7/3}\text{Subst}\left(\int \frac{-\sqrt[3]{2}\sqrt[3]{a}+2\ 2^{2/3}a^{2/3}x}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{2}b^{8/3}d}$$

$$- \frac{a^{8/3}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{b^{8/3}d}$$

$$= -\frac{9ax^2(a+bx^3)^{2/3}}{28b^2d} - \frac{x^5(a+bx^3)^{2/3}}{7bd} + \frac{a^{7/3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}b^{8/3}d}$$

$$- \frac{19a^2x^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{28b^2d\sqrt[3]{a+bx^3}} + \frac{a^{7/3}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}b^{8/3}d}$$

$$+ \frac{a^{7/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}} - \frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{2}b^{8/3}d}$$

$$- \frac{2^{2/3}a^{7/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3b^{8/3}d}$$

$$- \frac{a^{7/3}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}} - \frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}b^{8/3}d}$$

$$- \frac{\left(2^{2/3}a^{7/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{b^{8/3}d}$$

$$= -\frac{9ax^2(a+bx^3)^{2/3}}{28b^2d} - \frac{x^5(a+bx^3)^{2/3}}{7bd} + \frac{2^{2/3}a^{7/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{8/3}d}$$

$$+ \frac{a^{7/3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}b^{8/3}d} - \frac{19a^2x^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{28b^2d\sqrt[3]{a+bx^3}}$$

$$+ \frac{a^{7/3}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}b^{8/3}d}$$

$$+ \frac{a^{7/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{2}b^{8/3}d}$$

$$- \frac{2^{2/3}a^{7/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3b^{8/3}d}$$

$$- \frac{a^{7/3}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}b^{8/3}d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 8.54 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.29

$$\int \frac{x^7(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \frac{-5(9a^2x^2+13abx^5+4b^2x^8)+45a^2x^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+38}{140b^2d\sqrt[3]{a+bx^3}}$$

```
[In] Integrate[(x^7*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-5*(9*a^2*x^2 + 13*a*b*x^5 + 4*b^2*x^8) + 45*a^2*x^2*(1 + (b*x^3)/a)^(1/3)
*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), (b*x^3)/a] + 38*a*b*x^5*(1 + (b*x
^3)/a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), (b*x^3)/a])/(140*b^2*
d*(a + b*x^3)^(1/3))
```

Maple [F]

$$\int \frac{x^{7} \left(b\,x^{3}+a \right)^{\frac{2}{3}}}{-bd\,x^{3}+ad}d x$$

```
[In] int(x^7*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
[Out] int(x^7*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^7(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \text{Timed out}$$

```
[In] integrate(x^7*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^7(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x^7(a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

```
[In] integrate(x**7*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
[Out] -Integral(x**7*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Maxima [F]

$$\int \frac{x^7(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}x^7}{bdx^3-ad}\,dx$$

```
[In] integrate(x^7*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)*x^7/(b*d*x^3 - a*d), x)
```

Giac [F]

$$\int \frac{x^7(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}x^7}{bdx^3-ad}\,dx$$

```
[In] integrate(x^7*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(2/3)*x^7/(b*d*x^3 - a*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int \frac{x^7\,(b\,x^3+a)^{2/3}}{a\,d-b\,d\,x^3}\,dx$$

```
[In] int((x^7*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
[Out] int((x^7*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)
```

3.601 $\int \frac{x^4 (a+bx^3)^{2/3}}{ad-bdx^3} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 485

$$\int \frac{x^4(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{x^2(a+bx^3)^{2/3}}{4bd} + \frac{2^{2/3}a^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{5/3}d} + \frac{a^{4/3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}b^{5/3}d} - \frac{3ax^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{4bd\sqrt[3]{a+bx^3}} + \frac{a^{4/3}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}b^{5/3}d} + \frac{a^{4/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{2}b^{5/3}d} - \frac{2^{2/3}a^{4/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3b^{5/3}d} - \frac{a^{4/3}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}b^{5/3}d}$$

```
[Out] -1/4*x^2*(b*x^3+a)^(2/3)/b/d-3/4*a*x^2*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 2/
3],[5/3],-b*x^3/a)/b/d/(b*x^3+a)^(1/3)+1/12*a^(4/3)*ln((a^(1/3)-b^(1/3)*x)^
2*(a^(1/3)+b^(1/3)*x)/a)*2^(2/3)/b^(5/3)/d+1/6*a^(4/3)*ln(1+2^(2/3)*(a^(1/3
)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))
*2^(2/3)/b^(5/3)/d-1/3*2^(2/3)*a^(4/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*
x^3+a)^(1/3))/b^(5/3)/d-1/4*a^(4/3)*ln(b^(1/3)*(a^(1/3)+b^(1/3)*x)/a^(1/3)-
2^(2/3)*b^(1/3)*(b*x^3+a)^(1/3)/a^(1/3))*2^(2/3)/b^(5/3)/d+1/3*2^(2/3)*a^(4
/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/b
^(5/3)/d*3^(1/2)+1/6*a^(4/3)*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x
^3+a)^(1/3))*3^(1/2))*2^(2/3)/b^(5/3)/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {489, 598, 372, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^4(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \frac{2^{2/3}a^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(\sqrt[3]{a}+\sqrt[3]{b}x)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{5/3}d} + \frac{a^{4/3}\arctan\left(\frac{\frac{\sqrt[3]{2}(\sqrt[3]{a}+\sqrt[3]{b}x)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}b^{5/3}d} + \frac{a^{4/3}\log\left(\frac{2^{2/3}(\sqrt[3]{a}+\sqrt[3]{b}x)^2}{(a+bx^3)^{2/3}} - \frac{\sqrt[3]{2}(\sqrt[3]{a}+\sqrt[3]{b}x)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{2}b^{5/3}d} - \frac{2^{2/3}a^{4/3}\log\left(\frac{\sqrt[3]{2}(\sqrt[3]{a}+\sqrt[3]{b}x)}{\sqrt[3]{a+bx^3}}+1\right)}{3b^{5/3}d} - \frac{a^{4/3}\log\left(\frac{\sqrt[3]{b}(\sqrt[3]{a}+\sqrt[3]{b}x)}{\sqrt[3]{a}} - \frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}b^{5/3}d} + \frac{a^{4/3}\log\left(\frac{(\sqrt[3]{a}-\sqrt[3]{b}x)^2(\sqrt[3]{a}+\sqrt[3]{b}x)}{a}\right)}{6\sqrt[3]{2}b^{5/3}d} - \frac{3ax^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{4bd\sqrt[3]{a+bx^3}} - \frac{x^2(a+bx^3)^{2/3}}{4bd}$$

```
[In] Int[(x^4*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/4*(x^2*(a + b*x^3)^(2/3))/(b*d) + (2^(2/3)*a^(4/3)*ArcTan[(1 - (2*2^(1/3
)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(5/3)*d) +
 (a^(4/3)*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sq
rt[3]])/(2^(1/3)*Sqrt[3]*b^(5/3)*d) - (3*a*x^2*(1 + (b*x^3)/a)^(1/3)*Hyperg
```

```
eometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/(4*b*d*(a + b*x^3)^(1/3)) + (a^(4
/3)*Log[((a^(1/3) - b^(1/3)*x)^2*(a^(1/3) + b^(1/3)*x))/a])/(6*2^(1/3)*b^(5
/3)*d) + (a^(4/3)*Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/
3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*2^(1/3)*b^(5/3)
*d) - (2^(2/3)*a^(4/3)*Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^
(1/3)])/(3*b^(5/3)*d) - (a^(4/3)*Log[(b^(1/3)*(a^(1/3) + b^(1/3)*x))/a^(1/3
) - (2^(2/3)*b^(1/3)*(a + b*x^3)^(1/3))/a^(1/3)])/(2*2^(1/3)*b^(5/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
```

```
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{x^2(a+bx^3)^{2/3}}{4bd} + \frac{\int \frac{x(2a^2d+6abdx^3)}{\sqrt[3]{a+bx^3}(ad-bdx^3)}\,dx}{4bd}$$

$$= -\frac{x^2(a+bx^3)^{2/3}}{4bd} + \frac{\int \left(-\frac{6ax}{\sqrt[3]{a+bx^3}} + \frac{8a^2dx}{\sqrt[3]{a+bx^3}(ad-bdx^3)}\right)dx}{4bd}$$

$$= -\frac{x^2(a+bx^3)^{2/3}}{4bd} + \frac{(2a^2)\int \frac{x}{\sqrt[3]{a+bx^3}(ad-bdx^3)}\,dx}{b} - \frac{(3a)\int \frac{x}{\sqrt[3]{a+bx^3}}\,dx}{2bd}$$

$$= -\frac{x^2(a+bx^3)^{2/3}}{4bd} - \frac{(2a^{5/3})\,\text{Subst}\left(\int \frac{1}{1+2ax^3}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{b^{5/3}d}$$

$$+ \frac{(2a^{4/3})\int \frac{1}{\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)\sqrt[3]{a+bx^3}}\,dx}{3b^{4/3}d} - \frac{\left(3a\sqrt[3]{1+\frac{bx^3}{a}}\right)\int \frac{x}{\sqrt[3]{1+\frac{bx^3}{a}}}\,dx}{2bd\sqrt[3]{a+bx^3}}$$

$$= -\frac{x^2(a+bx^3)^{2/3}}{4bd} + \frac{a^{4/3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}b^{5/3}d}$$

$$- \frac{3ax^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{4bd\sqrt[3]{a+bx^3}} + \frac{a^{4/3}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}b^{5/3}d}$$

$$- \frac{a^{4/3}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}} - \frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}b^{5/3}d}$$

$$- \frac{(2a^{5/3})\,\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}\sqrt[3]{a}x}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3b^{5/3}d}$$

$$- \frac{(2a^{5/3})\,\text{Subst}\left(\int \frac{2-\sqrt[3]{2}\sqrt[3]{a}x}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3b^{5/3}d}$$

$$= -\frac{x^2(a+bx^3)^{2/3}}{4bd} + \frac{a^{4/3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}b^{5/3}d}$$

$$- \frac{3ax^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{4bd\sqrt[3]{a+bx^3}} + \frac{a^{4/3}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}b^{5/3}d}$$

$$- \frac{2^{2/3}a^{4/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3b^{5/3}d}$$

$$- \frac{a^{4/3}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}} - \frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}b^{5/3}d}$$

$$+ \frac{a^{4/3}\text{Subst}\left(\int \frac{-\sqrt[3]{2}\sqrt[3]{a}+2\ 2^{2/3}a^{2/3}x}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{2}b^{5/3}d}$$

$$- \frac{a^{5/3}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{b^{5/3}d}$$

$$= -\frac{x^2(a+bx^3)^{2/3}}{4bd} + \frac{a^{4/3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}b^{5/3}d}$$

$$- \frac{3ax^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{4bd\sqrt[3]{a+bx^3}} + \frac{a^{4/3}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}b^{5/3}d}$$

$$+ \frac{a^{4/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{2}b^{5/3}d}$$

$$- \frac{2^{2/3}a^{4/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3b^{5/3}d}$$

$$- \frac{a^{4/3}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}b^{5/3}d}$$

$$- \frac{\left(2^{2/3}a^{4/3}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{b^{5/3}d}$$

$$= -\frac{x^2(a+bx^3)^{2/3}}{4bd} + \frac{2^{2/3}a^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{5/3}d}$$
$$+ \frac{a^{4/3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}b^{5/3}d} - \frac{3ax^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{4bd\sqrt[3]{a+bx^3}}$$
$$+ \frac{a^{4/3}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}b^{5/3}d}$$
$$+ \frac{a^{4/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{2}b^{5/3}d}$$
$$- \frac{2^{2/3}a^{4/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3b^{5/3}d}$$
$$- \frac{a^{4/3}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}b^{5/3}d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 8.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.26

$$\int \frac{x^4(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \frac{x^2\left(-5(a+bx^3)+5a\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)+6bx^3\sqrt[3]{1+\frac{bx^3}{a}}\,\text{A}\right.}{20bd\sqrt[3]{a+bx^3}}$$

```
[In] Integrate[(x^4*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^2*(-5*(a + b*x^3) + 5*a*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3,
 -((b*x^3)/a), (b*x^3)/a] + 6*b*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3
, 1, 8/3, -((b*x^3)/a), (b*x^3)/a]))/(20*b*d*(a + b*x^3)^(1/3))
```

Maple [F]

$$\int \frac{x^{4} \left(b\,x^{3}+a \right)^{\frac{2}{3}}}{-b d\,x^{3}+a d}d x$$

```
[In] int(x^4*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
[Out] int(x^4*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^3)^{2/3}}{ad-bdx^3}\,dx=\text{Timed out}$$

```
[In] integrate(x^4*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^4(a+bx^3)^{2/3}}{ad-bdx^3}\,dx=-\frac{\int \frac{x^4(a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

```
[In] integrate(x**4*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
[Out] -Integral(x**4*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Maxima [F]

$$\int \frac{x^4(a+bx^3)^{2/3}}{ad-bdx^3}\,dx=\int -\frac{(bx^3+a)^{\frac{2}{3}}x^4}{bdx^3-ad}\,dx$$

```
[In] integrate(x^4*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)*x^4/(b*d*x^3 - a*d), x)
```

Giac [F]

$$\int \frac{x^4(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}x^4}{bdx^3-ad}\,dx$$

```
[In] integrate(x^4*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(2/3)*x^4/(b*d*x^3 - a*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int \frac{x^4\,(b\,x^3+a)^{2/3}}{a\,d-b\,d\,x^3}\,dx$$

```
[In] int((x^4*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
[Out] int((x^4*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)
```

3.602 $\int \frac{x\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 457

$$\int \frac{x(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \frac{2^{2/3}\sqrt[3]{a}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}d} + \frac{\sqrt[3]{a}\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}b^{2/3}d} - \frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}} + \frac{\sqrt[3]{a}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}b^{2/3}d} + \frac{\sqrt[3]{a}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{2}b^{2/3}d} - \frac{2^{2/3}\sqrt[3]{a}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}d} - \frac{\sqrt[3]{a}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}b^{2/3}d}$$

```
[Out] -1/2*x^2*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/d/(b*x^3+a)
^(1/3)+1/12*a^(1/3)*ln((a^(1/3)-b^(1/3)*x)^2*(a^(1/3)+b^(1/3)*x)/a)*2^(2/3)
/b^(2/3)/d+1/6*a^(1/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2
^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(2/3)/b^(2/3)/d-1/3*2^(2/3)*a
^(1/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/b^(2/3)/d-1/4*a^(1
/3)*ln(b^(1/3)*(a^(1/3)+b^(1/3)*x)/a^(1/3)-2^(2/3)*b^(1/3)*(b*x^3+a)^(1/3)/
a^(1/3))*2^(2/3)/b^(2/3)/d+1/3*2^(2/3)*a^(1/3)*arctan(1/3*(1-2*2^(1/3)*(a^(
```

```
1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)/d*3^(1/2)+1/6*a^(1/3)*arc
tan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(2/3)/b^
(2/3)/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {495, 372, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$\int \frac{x(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \frac{2^{2/3}\sqrt[3]{a}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}d} + \frac{\sqrt[3]{a}\arctan\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}b^{2/3}d} + \frac{\sqrt[3]{a}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{2}b^{2/3}d} - \frac{2^{2/3}\sqrt[3]{a}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3b^{2/3}d} - \frac{\sqrt[3]{a}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}b^{2/3}d} + \frac{\sqrt[3]{a}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}b^{2/3}d} - \frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

```
[In] Int[(x*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (2^(2/3)*a^(1/3)*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^
(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)*d) + (a^(1/3)*ArcTan[(1 + (2^(1/3)*(a^(1/
3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*b^(2/3)*d) -
 (x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])
/(2*d*(a + b*x^3)^(1/3)) + (a^(1/3)*Log[((a^(1/3) - b^(1/3)*x)^2*(a^(1/3) +
 b^(1/3)*x))/a])/(6*2^(1/3)*b^(2/3)*d) + (a^(1/3)*Log[1 + (2^(2/3)*(a^(1/3)
 + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b
*x^3)^(1/3)])/(3*2^(1/3)*b^(2/3)*d) - (2^(2/3)*a^(1/3)*Log[1 + (2^(1/3)*(a^
(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*b^(2/3)*d) - (a^(1/3)*Log[(b^(1/
3)*(a^(1/3) + b^(1/3)*x))/a^(1/3) - (2^(2/3)*b^(1/3)*(a + b*x^3)^(1/3))/a^(
1/3)])/(2*2^(1/3)*b^(2/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
```

```
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\text{integral} = (2a)\int \frac{x}{\sqrt[3]{a+bx^3}\,(ad-bdx^3)}\,dx - \frac{\int \frac{x}{\sqrt[3]{a+bx^3}}\,dx}{d}$$

$$= -\frac{\left(2a^{2/3}\right)\text{Subst}\left(\int \frac{1}{1+2ax^3}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{b^{2/3}d} + \frac{\left(2\sqrt[3]{a}\right)\int \frac{1}{\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)\sqrt[3]{a+bx^3}}\,dx}{3\sqrt[3]{b}d} - \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x}{\sqrt[3]{1+\frac{bx^3}{a}}}\,dx}{d\sqrt[3]{a+bx^3}}$$

$$= \frac{\sqrt[3]{a}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}b^{2/3}d} - \frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}} + \frac{\sqrt[3]{a}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}b^{2/3}d} - \frac{\sqrt[3]{a}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}} - \frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}b^{2/3}d} - \frac{\left(2a^{2/3}\right)\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}\sqrt[3]{a}x}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}d} - \frac{\left(2a^{2/3}\right)\text{Subst}\left(\int \frac{2-\sqrt[3]{2}\sqrt[3]{a}x}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}d}$$

$$= \frac{\sqrt[3]{a}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}b^{2/3}d} - \frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

$$+ \frac{\sqrt[3]{a}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}b^{2/3}d} - \frac{2^{2/3}\sqrt[3]{a}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}d}$$

$$- \frac{\sqrt[3]{a}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}} - \frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}b^{2/3}d}$$

$$+ \frac{\sqrt[3]{a}\text{Subst}\left(\int \frac{-\sqrt[3]{2}\sqrt[3]{a}+2\ 2^{2/3}a^{2/3}x}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{2}b^{2/3}d}$$

$$- \frac{a^{2/3}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{b^{2/3}d}$$

$$= \frac{\sqrt[3]{a}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}b^{2/3}d} - \frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

$$+ \frac{\sqrt[3]{a}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}b^{2/3}d}$$

$$+ \frac{\sqrt[3]{a}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}} - \frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{2}b^{2/3}d}$$

$$- \frac{2^{2/3}\sqrt[3]{a}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}d}$$

$$- \frac{\sqrt[3]{a}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}} - \frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}b^{2/3}d}$$

$$- \frac{\left(2^{2/3}\sqrt[3]{a}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{b^{2/3}d}$$

$$= \frac{2^{2/3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}d} + \frac{\sqrt[3]{a}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}b^{2/3}d}$$

$$- \frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}} + \frac{\sqrt[3]{a}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}b^{2/3}d}$$

$$+ \frac{\sqrt[3]{a}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{2}b^{2/3}d}$$

$$- \frac{2^{2/3}\sqrt[3]{a}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}d}$$

$$- \frac{\sqrt[3]{a}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}b^{2/3}d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.14

$$\int \frac{x(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

```
[In] Integrate[(x*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
[Out] (x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, -2/3, 1, 5/3, -((b*x^3)/a), (b*x^3
)/a])/(2*d*(a + b*x^3)^(1/3))
```

Maple [F]

$$\int \frac{x(b\,x^{3}+a)^{\frac{2}{3}}}{-bd\,x^{3}+ad}dx$$

```
[In] int(x*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
[Out] int(x*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \text{Timed out}$$

```
[In] integrate(x*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = -\frac{\int \frac{x(a+bx^3)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

```
[In] integrate(x*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
[Out] -Integral(x*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

Maxima [F]

$$\int \frac{x(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}x}{bdx^3-ad}\,dx$$

```
[In] integrate(x*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)*x/(b*d*x^3 - a*d), x)
```

Giac [F]

$$\int \frac{x(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}x}{bdx^3-ad}\,dx$$

```
[In] integrate(x*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(2/3)*x/(b*d*x^3 - a*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx^3)^{2/3}}{ad-bdx^3}\,dx = \int \frac{x\,(b\,x^3+a)^{2/3}}{a\,d-b\,d\,x^3}\,dx$$

```
[In] int((x*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
[Out] int((x*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)
```

3.603 $\int \frac{(a+bx^3)^{2/3}}{x^2(ad-bdx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 483

$$\int \frac{(a+bx^3)^{2/3}}{x^2(ad-bdx^3)}\,dx = -\frac{(a+bx^3)^{2/3}}{adx} + \frac{2^{2/3}\sqrt[3]{b}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}d} + \frac{\sqrt[3]{b}\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a^{2/3}d} + \frac{bx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2ad\sqrt[3]{a+bx^3}} + \frac{\sqrt[3]{b}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}a^{2/3}d} + \frac{\sqrt[3]{b}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{2}a^{2/3}d} - \frac{2^{2/3}\sqrt[3]{b}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3a^{2/3}d} - \frac{\sqrt[3]{b}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}a^{2/3}d}$$

```
[Out] -(b*x^3+a)^(2/3)/a/d/x+1/2*b*x^2*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/
3],-b*x^3/a)/a/d/(b*x^3+a)^(1/3)+1/12*b^(1/3)*ln((a^(1/3)-b^(1/3)*x)^2*(a^(
1/3)+b^(1/3)*x)/a)*2^(2/3)/a^(2/3)/d+1/6*b^(1/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1
/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(2/
3)/a^(2/3)/d-1/3*2^(2/3)*b^(1/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)
^(1/3))/a^(2/3)/d-1/4*b^(1/3)*ln(b^(1/3)*(a^(1/3)+b^(1/3)*x)/a^(1/3)-2^(2/3
)*b^(1/3)*(b*x^3+a)^(1/3)/a^(1/3))*2^(2/3)/a^(2/3)/d+1/3*2^(2/3)*b^(1/3)*ar
ctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/a^(2/3)
/d*3^(1/2)+1/6*b^(1/3)*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^
(1/3))*3^(1/2))*2^(2/3)/a^(2/3)/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {486, 598, 372, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$\int \frac{(a+bx^3)^{2/3}}{x^2(ad-bdx^3)}\,dx = \frac{2^{2/3}\sqrt[3]{b}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}d} + \frac{\sqrt[3]{b}\arctan\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a^{2/3}d} + \frac{\sqrt[3]{b}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{2}a^{2/3}d} - \frac{2^{2/3}\sqrt[3]{b}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3a^{2/3}d} - \frac{\sqrt[3]{b}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}a^{2/3}d} + \frac{\sqrt[3]{b}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}a^{2/3}d} + \frac{bx^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2ad\sqrt[3]{a+bx^3}} - \frac{(a+bx^3)^{2/3}}{adx}$$

```
[In] Int[(a + b*x^3)^(2/3)/(x^2*(a*d - b*d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(2/3)/(a*d*x)) + (2^(2/3)*b^(1/3)*ArcTan[(1 - (2*2^(1/3)*(a^(
1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(2/3)*d) + (b^(1
/3)*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]]
)/(2^(1/3)*Sqrt[3]*a^(2/3)*d) + (b*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric
```

```
2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/(2*a*d*(a + b*x^3)^(1/3)) + (b^(1/3)*Log[
((a^(1/3) - b^(1/3)*x)^2*(a^(1/3) + b^(1/3)*x))/a])/(6*2^(1/3)*a^(2/3)*d) +
 (b^(1/3)*Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^
(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*2^(1/3)*a^(2/3)*d) - (2
^(2/3)*b^(1/3)*Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/
(3*a^(2/3)*d) - (b^(1/3)*Log[(b^(1/3)*(a^(1/3) + b^(1/3)*x))/a^(1/3) - (2^(
2/3)*b^(1/3)*(a + b*x^3)^(1/3))/a^(1/3)])/(2*2^(1/3)*a^(2/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
```

```
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{(a+bx^3)^{2/3}}{adx} + \frac{\int \frac{x(3abd-b^2dx^3)}{\sqrt[3]{a+bx^3}(ad-bdx^3)}\,dx}{ad}$$

$$= -\frac{(a+bx^3)^{2/3}}{adx} + \frac{\int \left(\frac{bx}{\sqrt[3]{a+bx^3}} + \frac{2abdx}{\sqrt[3]{a+bx^3}(ad-bdx^3)}\right)dx}{ad}$$

$$= -\frac{(a+bx^3)^{2/3}}{adx} + (2b)\int \frac{x}{\sqrt[3]{a+bx^3}\,(ad-bdx^3)}\,dx + \frac{b\int \frac{x}{\sqrt[3]{a+bx^3}}\,dx}{ad}$$

$$= -\frac{(a+bx^3)^{2/3}}{adx} - \frac{\left(2\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{1}{1+2ax^3}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{a}d}$$

$$+ \frac{\left(2b^{2/3}\right)\int \frac{1}{\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)\sqrt[3]{a+bx^3}}\,dx}{3a^{2/3}d} + \frac{\left(b\sqrt[3]{1+\frac{bx^3}{a}}\right)\int \frac{x}{\sqrt[3]{1+\frac{bx^3}{a}}}\,dx}{ad\sqrt[3]{a+bx^3}}$$

$$= -\frac{(a+bx^3)^{2/3}}{adx} + \frac{\sqrt[3]{b}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a^{2/3}d}$$

$$+ \frac{bx^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2ad\sqrt[3]{a+bx^3}} + \frac{\sqrt[3]{b}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}a^{2/3}d}$$

$$- \frac{\sqrt[3]{b}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}} - \frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}a^{2/3}d}$$

$$- \frac{\left(2\sqrt[3]{b}\right)\text{Subst}\left(\int\frac{1}{1+\sqrt[3]{2}\sqrt[3]{a}x}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a}d}$$

$$- \frac{\left(2\sqrt[3]{b}\right)\text{Subst}\left(\int\frac{2-\sqrt[3]{2}\sqrt[3]{a}x}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a}d}$$

$$= -\frac{(a+bx^3)^{2/3}}{adx} + \frac{\sqrt[3]{b}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a^{2/3}d} + \frac{bx^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2ad\sqrt[3]{a+bx^3}}$$

$$+ \frac{\sqrt[3]{b}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}a^{2/3}d} - \frac{2^{2/3}\sqrt[3]{b}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3a^{2/3}d}$$

$$- \frac{\sqrt[3]{b}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}} - \frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}a^{2/3}d}$$

$$+ \frac{\sqrt[3]{b}\text{Subst}\left(\int \frac{-\sqrt[3]{2}\sqrt[3]{a}+2\ 2^{2/3}a^{2/3}x}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{2}a^{2/3}d}$$

$$- \frac{\sqrt[3]{b}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{a}d}$$

$$= -\frac{(a+bx^3)^{2/3}}{adx} + \frac{\sqrt[3]{b}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a^{2/3}d}$$

$$+ \frac{bx^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2ad\sqrt[3]{a+bx^3}} + \frac{\sqrt[3]{b}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}a^{2/3}d}$$

$$+ \frac{\sqrt[3]{b}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{2}a^{2/3}d}$$

$$- \frac{2^{2/3}\sqrt[3]{b}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3a^{2/3}d}$$

$$- \frac{\sqrt[3]{b}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}a^{2/3}d}$$

$$- \frac{\left(2^{2/3}\sqrt[3]{b}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{a^{2/3}d}$$

$$= -\frac{(a+bx^3)^{2/3}}{adx} + \frac{2^{2/3}\sqrt[3]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}d}$$

$$+ \frac{\sqrt[3]{b}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a^{2/3}d} + \frac{bx^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2ad\sqrt[3]{a+bx^3}}$$

$$+ \frac{\sqrt[3]{b}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}a^{2/3}d}$$

$$+ \frac{\sqrt[3]{b}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{2}a^{2/3}d}$$

$$- \frac{2^{2/3}\sqrt[3]{b}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3a^{2/3}d}$$

$$- \frac{\sqrt[3]{b}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}a^{2/3}d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 11.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.28

$$\int \frac{(a+bx^3)^{2/3}}{x^2(ad-bdx^3)}\,dx = \frac{15abx^3\sqrt[3]{1+\frac{bx^3}{a}}\,\mathrm{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2\left(5a(a+bx^3)+b^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\,A\right.}{10a^2dx\sqrt[3]{a+bx^3}}$$

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^2*(a*d - b*d*x^3)),x]
```

```
[Out] (15*a*b*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a),
(b*x^3)/a] - 2*(5*a*(a + b*x^3) + b^2*x^6*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/
3, 1/3, 1, 8/3, -((b*x^3)/a), (b*x^3)/a]))/(10*a^2*d*x*(a + b*x^3)^(1/3))
```

Maple [F]

$$\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{x^2\,(-bd\,x^3+ad)}dx$$

```
[In] int((b*x^3+a)^(2/3)/x^2/(-b*d*x^3+a*d),x)
[Out] int((b*x^3+a)^(2/3)/x^2/(-b*d*x^3+a*d),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^2\,(ad-bdx^3)}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^3+a)^(2/3)/x^2/(-b*d*x^3+a*d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^2\,(ad-bdx^3)}\,dx = -\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-ax^2+bx^5}\,dx}{d}$$

```
[In] integrate((b*x**3+a)**(2/3)/x**2/(-b*d*x**3+a*d),x)
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**2 + b*x**5), x)/d
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^2\,(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^2}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/x^2/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^2), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^2(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^2}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/x^2/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^2(ad-bdx^3)}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{x^2\,(a\,d-b\,d\,x^3)}\,dx$$

```
[In] int((a + b*x^3)^(2/3)/(x^2*(a*d - b*d*x^3)),x)
[Out] int((a + b*x^3)^(2/3)/(x^2*(a*d - b*d*x^3)), x)
```

3.604 $\int \frac{(a+bx^3)^{2/3}}{x^5(ad-bdx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 512

$$\int \frac{(a+bx^3)^{2/3}}{x^5(ad-bdx^3)}\,dx = -\frac{(a+bx^3)^{2/3}}{4adx^4} - \frac{3b(a+bx^3)^{2/3}}{2a^2dx} + \frac{2^{2/3}b^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^{5/3}d} + \frac{b^{4/3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a^{5/3}d} + \frac{3b^2x^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{4a^2d\sqrt[3]{a+bx^3}} + \frac{b^{4/3}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}a^{5/3}d} + \frac{b^{4/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{2}a^{5/3}d} - \frac{2^{2/3}b^{4/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3a^{5/3}d} - \frac{b^{4/3}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}a^{5/3}d}$$

```
[Out] -1/4*(b*x^3+a)^(2/3)/a/d/x^4-3/2*b*(b*x^3+a)^(2/3)/a^2/d/x+3/4*b^2*x^2*(1+b
*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/a^2/d/(b*x^3+a)^(1/3)+1/
12*b^(4/3)*ln((a^(1/3)-b^(1/3)*x)^2*(a^(1/3)+b^(1/3)*x)/a)*2^(2/3)/a^(5/3)/
d+1/6*b^(4/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a
^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(2/3)/a^(5/3)/d-1/3*2^(2/3)*b^(4/3)*ln
(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/a^(5/3)/d-1/4*b^(4/3)*ln(b^
(1/3)*(a^(1/3)+b^(1/3)*x)/a^(1/3)-2^(2/3)*b^(1/3)*(b*x^3+a)^(1/3)/a^(1/3))*
2^(2/3)/a^(5/3)/d+1/3*2^(2/3)*b^(4/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1
/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/a^(5/3)/d*3^(1/2)+1/6*b^(4/3)*arctan(1/3*(
1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(2/3)/a^(5/3)/d*3
^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {486, 597, 598, 372, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$\begin{aligned}
\int \frac{(a+bx^3)^{2/3}}{x^5(ad-bdx^3)}dx &= \frac{2^{2/3}b^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^{5/3}d} \\
&+ \frac{b^{4/3}\arctan\left(\frac{\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a^{5/3}d} \\
&+ \frac{b^{4/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{2}a^{5/3}d} \\
&- \frac{2^{2/3}b^{4/3}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3a^{5/3}d} \\
&- \frac{b^{4/3}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}a^{5/3}d} \\
&+ \frac{b^{4/3}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}a^{5/3}d} \\
&+ \frac{3b^2x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{4a^2d\sqrt[3]{a+bx^3}} \\
&- \frac{3b(a+bx^3)^{2/3}}{2a^2dx} - \frac{(a+bx^3)^{2/3}}{4adx^4}
\end{aligned}$$

```
[In] Int[(a + b*x^3)^(2/3)/(x^5*(a*d - b*d*x^3)),x]
```

```
[Out] -1/4*(a + b*x^3)^(2/3)/(a*d*x^4) - (3*b*(a + b*x^3)^(2/3))/(2*a^2*d*x) + (2
^(2/3)*b^(4/3)*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1
```

```
/3))/Sqrt[3]])/(Sqrt[3]*a^(5/3)*d) + (b^(4/3)*ArcTan[(1 + (2^(1/3)*(a^(1/3)
 + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*a^(5/3)*d) + (
3*b^2*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/
a)])/(4*a^2*d*(a + b*x^3)^(1/3)) + (b^(4/3)*Log[((a^(1/3) - b^(1/3)*x)^2*(a
^(1/3) + b^(1/3)*x))/a])/(6*2^(1/3)*a^(5/3)*d) + (b^(4/3)*Log[1 + (2^(2/3)*
(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x)
)/(a + b*x^3)^(1/3)])/(3*2^(1/3)*a^(5/3)*d) - (2^(2/3)*b^(4/3)*Log[1 + (2^(
1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*a^(5/3)*d) - (b^(4/3)*Lo
g[(b^(1/3)*(a^(1/3) + b^(1/3)*x))/a^(1/3) - (2^(2/3)*b^(1/3)*(a + b*x^3)^(1
/3))/a^(1/3)])/(2*2^(1/3)*a^(5/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
```

```
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{(a+bx^3)^{2/3}}{4adx^4} + \frac{\int \frac{6abd+2b^2dx^3}{x^2\sqrt[3]{a+bx^3}(ad-bdx^3)}\,dx}{4ad}$$

$$= -\frac{(a+bx^3)^{2/3}}{4adx^4} - \frac{3b(a+bx^3)^{2/3}}{2a^2dx} - \frac{\int \frac{x(-14a^2b^2d^2+6ab^3d^2x^3)}{\sqrt[3]{a+bx^3}(ad-bdx^3)}\,dx}{4a^3d^2}$$

$$= -\frac{(a+bx^3)^{2/3}}{4adx^4} - \frac{3b(a+bx^3)^{2/3}}{2a^2dx} - \frac{\int \left(-\frac{6ab^2dx}{\sqrt[3]{a+bx^3}} - \frac{8a^2b^2d^2x}{\sqrt[3]{a+bx^3}(ad-bdx^3)}\right)dx}{4a^3d^2}$$

$$= -\frac{(a+bx^3)^{2/3}}{4adx^4} - \frac{3b(a+bx^3)^{2/3}}{2a^2dx} + \frac{(2b^2)\int \frac{x}{\sqrt[3]{a+bx^3}(ad-bdx^3)}\,dx}{a} + \frac{(3b^2)\int \frac{x}{\sqrt[3]{a+bx^3}}\,dx}{2a^2d}$$

$$= -\frac{(a+bx^3)^{2/3}}{4adx^4} - \frac{3b(a+bx^3)^{2/3}}{2a^2dx} - \frac{(2b^{4/3})\,\text{Subst}\left(\int \frac{1}{1+2ax^3}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{a^{4/3}d}$$

$$+ \frac{(2b^{5/3})\int \frac{1}{\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)\sqrt[3]{a+bx^3}}\,dx}{3a^{5/3}d} + \frac{\left(3b^2\sqrt[3]{1+\frac{bx^3}{a}}\right)\int \frac{x}{\sqrt[3]{1+\frac{bx^3}{a}}}\,dx}{2a^2d\sqrt[3]{a+bx^3}}$$

$$= -\frac{(a+bx^3)^{2/3}}{4adx^4} - \frac{3b(a+bx^3)^{2/3}}{2a^2dx} + \frac{b^{4/3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a^{5/3}d}$$

$$+ \frac{3b^2x^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{4a^2d\sqrt[3]{a+bx^3}} + \frac{b^{4/3}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}a^{5/3}d}$$

$$- \frac{b^{4/3}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}} - \frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}a^{5/3}d}$$

$$- \frac{\left(2b^{4/3}\right)\text{Subst}\left(\int\frac{1}{1+\sqrt[3]{2}\sqrt[3]{a}x}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3a^{4/3}d}$$

$$- \frac{\left(2b^{4/3}\right)\text{Subst}\left(\int\frac{2-\sqrt[3]{2}\sqrt[3]{a}x}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3a^{4/3}d}$$

$$=-\frac{(a+bx^3)^{2/3}}{4adx^4}-\frac{3b(a+bx^3)^{2/3}}{2a^2dx}+\frac{b^{4/3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a^{5/3}d}$$

$$+\frac{3b^2x^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{4a^2d\sqrt[3]{a+bx^3}}+\frac{b^{4/3}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}a^{5/3}d}$$

$$-\frac{2^{2/3}b^{4/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3a^{5/3}d}$$

$$-\frac{b^{4/3}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}a^{5/3}d}$$

$$+\frac{b^{4/3}\text{Subst}\left(\int\frac{-\sqrt[3]{2}\sqrt[3]{a}+2\ 2^{2/3}a^{2/3}x}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{2}a^{5/3}d}$$

$$-\frac{b^{4/3}\text{Subst}\left(\int\frac{1}{1-\sqrt[3]{2}\sqrt[3]{a}x+2^{2/3}a^{2/3}x^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt[3]{a+bx^3}}\right)}{a^{4/3}d}$$

$$= -\frac{(a+bx^3)^{2/3}}{4adx^4} - \frac{3b(a+bx^3)^{2/3}}{2a^2dx} + \frac{b^{4/3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a^{5/3}d}$$

$$+ \frac{3b^2x^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{4a^2d\sqrt[3]{a+bx^3}} + \frac{b^{4/3}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}a^{5/3}d}$$

$$+ \frac{b^{4/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}} - \frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{2}a^{5/3}d}$$

$$- \frac{2^{2/3}b^{4/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3a^{5/3}d}$$

$$- \frac{b^{4/3}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}} - \frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}a^{5/3}d}$$

$$- \frac{\left(2^{2/3}b^{4/3}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{a^{5/3}d}$$

$$
\begin{aligned}
&= -\frac{\left(a+bx^3\right)^{2/3}}{4adx^4} - \frac{3b\left(a+bx^3\right)^{2/3}}{2a^2dx} + \frac{2^{2/3}b^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}a^{5/3}d} \\
&+ \frac{b^{4/3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a^{5/3}d} + \frac{3b^2x^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{4a^2d\sqrt[3]{a+bx^3}} \\
&+ \frac{b^{4/3}\log\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a}\right)}{6\sqrt[3]{2}a^{5/3}d} \\
&+ \frac{b^{4/3}\log\left(1+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{2}a^{5/3}d} \\
&- \frac{2^{2/3}b^{4/3}\log\left(1+\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right)}{3a^{5/3}d} \\
&- \frac{b^{4/3}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{2\sqrt[3]{2}a^{5/3}d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 11.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.29

$$
\int \frac{\left(a+bx^3\right)^{2/3}}{x^5\left(ad-bdx^3\right)}\,dx = \frac{-5a\left(a^2+7abx^3+6b^2x^6\right)+35ab^2x^6\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},\frac{bx^3}{a}\right)-6b^3x}{20a^3dx^4\sqrt[3]{a+bx^3}}
$$

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^5*(a*d - b*d*x^3)),x]
```

```
[Out] (-5*a*(a^2 + 7*a*b*x^3 + 6*b^2*x^6) + 35*a*b^2*x^6*(1 + (b*x^3)/a)^(1/3)*Ap
pellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), (b*x^3)/a] - 6*b^3*x^9*(1 + (b*x^3)/
a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), (b*x^3)/a])/(20*a^3*d*x^4
*(a + b*x^3)^(1/3))
```

Maple [F]

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{5}\left(-bd\,x^{3}+ad\right)}dx$$

```
[In] int((b*x^3+a)^(2/3)/x^5/(-b*d*x^3+a*d),x)
[Out] int((b*x^3+a)^(2/3)/x^5/(-b*d*x^3+a*d),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^5\,(ad-bdx^3)}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^3+a)^(2/3)/x^5/(-b*d*x^3+a*d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^5\,(ad-bdx^3)}\,dx = -\frac{\int \frac{(a+bx^3)^{\frac{2}{3}}}{-ax^5+bx^8}\,dx}{d}$$

```
[In] integrate((b*x**3+a)**(2/3)/x**5/(-b*d*x**3+a*d),x)
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**5 + b*x**8), x)/d
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^5\,(ad-bdx^3)}\,dx = \int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^5}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/x^5/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^5), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^5\,(ad-bdx^3)}\,dx=\int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^5}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/x^5/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^5\,(ad-bdx^3)}\,dx=\int \frac{(b\,x^3+a)^{2/3}}{x^5\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

```
[In] int((a + b*x^3)^(2/3)/(x^5*(a*d - b*d*x^3)),x)
[Out] int((a + b*x^3)^(2/3)/(x^5*(a*d - b*d*x^3)), x)
```

3.605 $\int \frac{x^{14}}{\sqrt[3]{1-x^3}(1+x^3)} dx$

Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{x^{14}}{\sqrt[3]{1-x^3}\,(1+x^3)} dx = \frac{2}{5}\left(1-x^3\right)^{5/3} - \frac{1}{4}\left(1-x^3\right)^{8/3} + \frac{1}{11}\left(1-x^3\right)^{11/3} + \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[Out] 2/5*(-x^3+1)^(5/3)-1/4*(-x^3+1)^(8/3)+1/11*(-x^3+1)^(11/3)-1/12*ln(x^3+1)*2
^(2/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(1/3*(1+2^(2/3)*(-x
^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {457, 90, 57, 631, 210, 31}

$$\int \frac{x^{14}}{\sqrt[3]{1-x^3}\,(1+x^3)} dx = \frac{\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{1}{11}\left(1-x^3\right)^{11/3} - \frac{1}{4}\left(1-x^3\right)^{8/3} + \frac{2}{5}\left(1-x^3\right)^{5/3} - \frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[In] Int[x^14/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (2*(1 - x^3)^(5/3))/5 - (1 - x^3)^(8/3)/4 + (1 - x^3)^(11/3)/11 + ArcTan[(1
 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[1 + x^3]/(6*2^
(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{\sqrt[3]{1-x}(1+x)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-2(1-x)^{2/3}+2(1-x)^{5/3}-(1-x)^{8/3}+\frac{1}{\sqrt[3]{1-x}(1+x)}\right)\,dx, x, x^3\right)\\
&= \frac{2}{5}\left(1-x^3\right)^{5/3}-\frac{1}{4}\left(1-x^3\right)^{8/3}+\frac{1}{11}\left(1-x^3\right)^{11/3}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,dx, x, x^3\right)\\
&= \frac{2}{5}\left(1-x^3\right)^{5/3}-\frac{1}{4}\left(1-x^3\right)^{8/3}+\frac{1}{11}\left(1-x^3\right)^{11/3}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}\\
&\quad+\frac{1}{2}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right)-\frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\\
&= \frac{2}{5}\left(1-x^3\right)^{5/3}-\frac{1}{4}\left(1-x^3\right)^{8/3}+\frac{1}{11}\left(1-x^3\right)^{11/3}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}\\
&\quad+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}-\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}}\\
&= \frac{2}{5}\left(1-x^3\right)^{5/3}-\frac{1}{4}\left(1-x^3\right)^{8/3}\\
&\quad+\frac{1}{11}\left(1-x^3\right)^{11/3}+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.14

$$
\begin{aligned}
\int \frac{x^{14}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx &= \frac{1}{220}\left(1-x^3\right)^{2/3}\left(53-38x^3+5x^6-20x^9\right)\\
&\quad+\frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)}{3\sqrt[3]{2}}\\
&\quad-\frac{\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(1-x^3)^{2/3}\right)}{6\sqrt[3]{2}}
\end{aligned}
$$

```
[In] Integrate[x^14/((1 - x^3)^(1/3)*(1 + x^3)),x]
[Out] ((1 - x^3)^(2/3)*(53 - 38*x^3 + 5*x^6 - 20*x^9))/220 + ArcTan[(1 + 2^(2/3)*
(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[-2 + 2^(2/3)*(1 - x^3)^(1
/3)]/(3*2^(1/3)) - Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3
)]/(6*2^(1/3))
```

Maple [A] (verified)

Time = 9.74 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$\frac{\left(2\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}-\ln\left(\left(-x^3+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)+2\ln\left(\left(-x^3+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)\right)2^{\frac{2}{3}}}{12}-\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{}$
trager	$\left(-\frac{1}{11}x^9+\frac{1}{44}x^6-\frac{19}{110}x^3+\frac{53}{220}\right)\left(-x^3+1\right)^{\frac{2}{3}}+\mathrm{RootOf}\left(\mathrm{RootOf}\left(\_Z^3-4\right)^2+6\_Z\mathrm{RootOf}\left(\right.\right.$
risch	$\frac{\left(20x^9-5x^6+38x^3-53\right)\left(x^3-1\right)}{220\left(-x^3+1\right)^{\frac{1}{3}}}+\mathrm{RootOf}\left(\mathrm{RootOf}\left(\_Z^3-4\right)^2+6\_Z\mathrm{RootOf}\left(\_Z^3-4\right)+36\_Z^2\right.$

```
[In] int(x^14/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(2*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-ln((-x^3+1)^
(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))+2*ln((-x^3+1)^(1/3)-2^(1/3)))*2^(2/3)
-1/220*(-x^3+1)^(2/3)*(20*x^9-5*x^6+38*x^3-53)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.93

$$\begin{aligned}\int \frac{x^{14}}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx &= -\frac{1}{220}\left(20\,x^9-5\,x^6+38\,x^3-53\right)\left(-x^3+1\right)^{\frac{2}{3}} \\ &\quad+\frac{1}{6}\sqrt{6}2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}+2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) \\ &\quad-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) \\ &\quad+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)\end{aligned}$$

```
[In] integrate(x^14/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/220*(20*x^9 - 5*x^6 + 38*x^3 - 53)*(-x^3 + 1)^(2/3) + 1/6*sqrt(6)*2^(1/6
)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*sqrt(6)*(-x^3 + 1)^(1/3))) - 1/12
*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2
^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Sympy [F]

$$\int \frac{x^{14}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^{14}}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

```
[In] integrate(x**14/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**14/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.94

$$\begin{aligned}\int \frac{x^{14}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx &= \frac{1}{11}\left(-x^3+1\right)^{\frac{11}{3}} - \frac{1}{4}\left(-x^3+1\right)^{\frac{8}{3}} \\ &\quad + \frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) \\ &\quad + \frac{2}{5}\left(-x^3+1\right)^{\frac{5}{3}} - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) \\ &\quad + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)\end{aligned}$$

```
[In] integrate(x^14/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/11*(-x^3 + 1)^(11/3) - 1/4*(-x^3 + 1)^(8/3) + 1/6*sqrt(3)*2^(2/3)*arctan(
1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) + 2/5*(-x^3 + 1)^(5/3)
- 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) +
 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06

$$\begin{aligned}\int \frac{x^{14}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx &= -\frac{1}{11}\left(x^3-1\right)^3\left(-x^3+1\right)^{\frac{2}{3}} - \frac{1}{4}\left(x^3-1\right)^2\left(-x^3+1\right)^{\frac{2}{3}} \\ &\quad + \frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) \\ &\quad + \frac{2}{5}\left(-x^3+1\right)^{\frac{5}{3}} - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) \\ &\quad + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)\end{aligned}$$

```
[In] integrate(x^14/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/11*(x^3 - 1)^3*(-x^3 + 1)^(2/3) - 1/4*(x^3 - 1)^2*(-x^3 + 1)^(2/3) + 1/6
*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3)))
 + 2/5*(-x^3 + 1)^(5/3) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/
3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3)))
```

Mupad [B] (verification not implemented)

Time = 8.43 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.05

$$\int \frac{x^{14}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{2^{2/3}\ln\left((1-x^3)^{1/3}-2^{1/3}\right)}{6}+\frac{2\,(1-x^3)^{5/3}}{5} - \frac{(1-x^3)^{8/3}}{4}+\frac{(1-x^3)^{11/3}}{11} + \frac{2^{2/3}\ln\left((1-x^3)^{1/3}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12} - \frac{2^{2/3}\ln\left((1-x^3)^{1/3}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

```
[In] int(x^14/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3) - 2^(1/3)))/6 + (2*(1 - x^3)^(5/3))/5 - (1 - x
^3)^(8/3)/4 + (1 - x^3)^(11/3)/11 + (2^(2/3)*log((1 - x^3)^(1/3) - (2^(1/3)
*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log((1 - x^3)^(1/3)
 - (2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/12
```

3.606 $\int \frac{x^{11}}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 128

$$\int \frac{x^{11}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{1}{8}\left(1-x^3\right)^{8/3} - \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[Out] -1/2*(-x^3+1)^(2/3)+1/5*(-x^3+1)^(5/3)-1/8*(-x^3+1)^(8/3)+1/12*ln(x^3+1)*2^
(2/3)-1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)-1/6*arctan(1/3*(1+2^(2/3)*(-x^
3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {457, 90, 57, 631, 210, 31}

$$\int \frac{x^{11}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{1}{8}\left(1-x^3\right)^{8/3}+\frac{1}{5}\left(1-x^3\right)^{5/3} -\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[In] Int[x^11/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/2*(1 - x^3)^(2/3) + (1 - x^3)^(5/3)/5 - (1 - x^3)^(8/3)/8 - ArcTan[(1 +
2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(1/
3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{\sqrt[3]{1-x}(1+x)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{\sqrt[3]{1-x}} - (1-x)^{2/3} + (1-x)^{5/3} - \frac{1}{\sqrt[3]{1-x}(1+x)}\right)\,dx, x, x^3\right)\\
&= -\frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{1}{5}\left(1-x^3\right)^{5/3} - \frac{1}{8}\left(1-x^3\right)^{8/3} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,dx, x, x^3\right)\\
&= -\frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{1}{5}\left(1-x^3\right)^{5/3} - \frac{1}{8}\left(1-x^3\right)^{8/3} + \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}\\
&\quad -\frac{1}{2}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right) + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\\
&= -\frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{1}{5}\left(1-x^3\right)^{5/3} - \frac{1}{8}\left(1-x^3\right)^{8/3} + \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}\\
&\quad - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}}\\
&= -\frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{1}{5}\left(1-x^3\right)^{5/3}\\
&\quad - \frac{1}{8}\left(1-x^3\right)^{8/3} - \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.09

$$
\int \frac{x^{11}}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx = \frac{1}{120}\left(-3\left(1-x^3\right)^{2/3}\left(17-2x^3+5x^6\right)\right.
$$
$$
-20\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) - 20\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) + 10\,2^{2/3}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\right.
$$

```
[In] Integrate[x^11/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (-3*(1 - x^3)^(2/3)*(17 - 2*x^3 + 5*x^6) - 20*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2
^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 20*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1
/3)] + 10*2^(2/3)*Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)
])/120
```

Maple [A] (verified)

Time = 10.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{\left(-2\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}+\ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)-2\ln\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)\right)2^{\frac{2}{3}}}{12}-\frac{\left(-x^{3}+1\right)}{}$
risch	Expression too large to display
trager	Expression too large to display

```
[In] int(x^11/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(-2*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+ln((-x^3+1)
^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))-2*ln((-x^3+1)^(1/3)-2^(1/3)))*2^(2/3
)-1/40*(-x^3+1)^(2/3)*(5*x^6-2*x^3+17)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.07

$$\int \frac{x^{11}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{1}{6}\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(2\sqrt{6}(-1)^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}-\sqrt{6}2^{\frac{1}{3}}\right)\right) - \frac{1}{12} \cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}(-1)^{\frac{2}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}\right) - \frac{1}{40}\left(5\,x^{6}-2\,x^{3}+17\right)\left(-x^{3}+1\right)^{\frac{2}{3}}$$

```
[In] integrate(x^11/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(6)*2^(1/6)*(-1)^(1/3)*arctan(1/6*2^(1/6)*(2*sqrt(6)*(-1)^(1/3)*(-
x^3 + 1)^(1/3) - sqrt(6)*2^(1/3))) - 1/12*2^(2/3)*(-1)^(1/3)*log(2^(1/3)*(-
1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^
(2/3)*(-1)^(1/3)*log(-2^(1/3)*(-1)^(2/3) + (-x^3 + 1)^(1/3)) - 1/40*(5*x^6
- 2*x^3 + 17)*(-x^3 + 1)^(2/3)
```

Sympy [F]

$$\int \frac{x^{11}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^{11}}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

```
[In] integrate(x**11/(-x**3+1)**(1/3)/(x**3+1),x)
[Out] Integral(x**11/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Maxima [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.93

$$\begin{aligned}\int \frac{x^{11}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = &-\frac{1}{8}\left(-x^3+1\right)^{\frac{8}{3}} - \frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)\\ &+\frac{1}{5}\left(-x^3+1\right)^{\frac{5}{3}} + \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)\\ &-\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right) - \frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}\end{aligned}$$

```
[In] integrate(x^11/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] -1/8*(-x^3 + 1)^(8/3) - 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(
1/3) + 2*(-x^3 + 1)^(1/3))) + 1/5*(-x^3 + 1)^(5/3) + 1/12*2^(2/3)*log(2^(2/
3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(2/3)*log(-2^(1/3
) + (-x^3 + 1)^(1/3)) - 1/2*(-x^3 + 1)^(2/3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \frac{x^{11}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = &-\frac{1}{8}\left(x^3-1\right)^2\left(-x^3+1\right)^{\frac{2}{3}}\\ &-\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)\\ &+\frac{1}{5}\left(-x^3+1\right)^{\frac{5}{3}} + \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)\\ &-\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right) - \frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}\end{aligned}$$

```
[In] integrate(x^11/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/8*(x^3 - 1)^2*(-x^3 + 1)^(2/3) - 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*
2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) + 1/5*(-x^3 + 1)^(5/3) + 1/12*2^(2/
3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(2/3)
*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - 1/2*(-x^3 + 1)^(2/3)
```

Mupad [B] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.04

$$\int \frac{x^{11}}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{(1-x^3)^{5/3}}{5} - \frac{(1-x^3)^{2/3}}{2} - \frac{2^{2/3}\,\ln\left((1-x^3)^{1/3}-2^{1/3}\right)}{6} - \frac{(1-x^3)^{8/3}}{8} - \frac{2^{2/3}\,\ln\left((1-x^3)^{1/3}-\frac{2^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12} + \frac{2^{2/3}\,\ln\left((1-x^3)^{1/3}-\frac{2^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

```
[In] int(x^11/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] (1 - x^3)^(5/3)/5 - (1 - x^3)^(2/3)/2 - (2^(2/3)*log((1 - x^3)^(1/3) - 2^(1
/3)))/6 - (1 - x^3)^(8/3)/8 - (2^(2/3)*log((1 - x^3)^(1/3) - (2^(1/3)*(3^(1
/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 + (2^(2/3)*log((1 - x^3)^(1/3) - (2^
(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/12
```

3.607 $\int \frac{x^8}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 97

$$\int \frac{x^8}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{5}\left(1-x^3\right)^{5/3} + \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[Out] 1/5*(-x^3+1)^(5/3)-1/12*ln(x^3+1)*2^(2/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^
(2/3)+1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {457, 90, 57, 631, 210, 31}

$$\int \frac{x^8}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{1}{5}\left(1-x^3\right)^{5/3} - \frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[In] Int[x^8/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (1 - x^3)^(5/3)/5 + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*
Sqrt[3]) - Log[1 + x^3]/(6*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(
1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{\sqrt[3]{1-x}(1+x)}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(-(1-x)^{2/3} + \frac{1}{\sqrt[3]{1-x}(1+x)}\right)\,dx, x, x^3\right)$$
$$= \frac{1}{5}\left(1-x^3\right)^{5/3} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,dx, x, x^3\right)$$
$$= \frac{1}{5}\left(1-x^3\right)^{5/3} - \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right) - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$
$$= \frac{1}{5}\left(1-x^3\right)^{5/3} - \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}}$$
$$= \frac{1}{5}\left(1-x^3\right)^{5/3} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.31

$$\int \frac{x^8}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx = \frac{1}{60}\left(12\left(1-x^3\right)^{5/3} + 10\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) + 10\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) - 5\,2^{2/3}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{\cdots}\right.\right.$$

```
[In] Integrate[x^8/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (12*(1 - x^3)^(5/3) + 10*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3
))/Sqrt[3]] + 10*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] - 5*2^(2/3)*Log[
2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/60
```

Maple [A] (verified)

Time = 9.16 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$-\frac{\left(-x^{3}+1\right)^{\frac{2}{3}} x^{3}}{5}+\frac{\left(-x^{3}+1\right)^{\frac{2}{3}}}{5}+\frac{2^{\frac{2}{3}} \ln \left(\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)}{6}-\frac{2^{\frac{2}{3}} \ln \left(\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}} \left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)}{12}+\frac{\arctan\left(\frac{\left(1+\right.}{}\right.}{}$
trager	Expression too large to display
risch	Expression too large to display

```
[In] int(x^8/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*(-x^3+1)^(2/3)*x^3+1/5*(-x^3+1)^(2/3)+1/6*2^(2/3)*ln((-x^3+1)^(1/3)-2^
(1/3))-1/12*2^(2/3)*ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))+1/6*a
rctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.09

$$\int \frac{x^8}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{1}{5}\,\left(x^3-1\right)\left(-x^3+1\right)^{\frac{2}{3}} + \frac{1}{6}\,\sqrt{6}2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}+2\,\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)$$

```
[In] integrate(x^8/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/5*(x^3 - 1)*(-x^3 + 1)^(2/3) + 1/6*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(s
qrt(6)*2^(1/3) + 2*sqrt(6)*(-x^3 + 1)^(1/3))) - 1/12*2^(2/3)*log(2^(2/3) +
2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (
-x^3 + 1)^(1/3))
```

Sympy [F]

$$\int \frac{x^8}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^8}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

```
[In] integrate(x**8/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**8/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{x^8}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx &= \frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{5}\left(-x^3+1\right)^{\frac{5}{3}}\\ &\quad-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)\\ &\quad+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)\end{aligned}$$

```
[In] integrate(x^8/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) + 1/5*(-x^3 + 1)^(5/3) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^
(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{x^8}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx &= \frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{5}\left(-x^3+1\right)^{\frac{5}{3}}\\ &\quad-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)\\ &\quad+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)\end{aligned}$$

```
[In] integrate(x^8/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) + 1/5*(-x^3 + 1)^(5/3) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^
(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3)
))
```

Mupad [B] (verification not implemented)

Time = 8.33 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.14

$$\int \frac{x^8}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-2^{1/3}\right)}{6}+\frac{\left(1-x^3\right)^{5/3}}{5} + \frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12} - \frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

```
[In] int(x^8/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3) - 2^(1/3)))/6 + (1 - x^3)^(5/3)/5 + (2^(2/3)*l
og((1 - x^3)^(1/3) - (2^(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 -
 (2^(2/3)*log((1 - x^3)^(1/3) - (2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i
 + 1))/12
```

3.608 $\int \frac{x^5}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \frac{x^5}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[Out] -1/2*(-x^3+1)^(2/3)+1/12*ln(x^3+1)*2^(2/3)-1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2
^(2/3)-1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {457, 81, 57, 631, 210, 31}

$$\int \frac{x^5}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[In] Int[x^5/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/2*(1 - x^3)^(2/3) - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/
3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(1/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*
2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x}{\sqrt[3]{1-x}(1+x)}\,dx, x, x^3\right)$$
$$= -\frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,dx, x, x^3\right)$$
$$= -\frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right) + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$
$$= -\frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}}$$
$$= -\frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.29

$$\int \frac{x^5}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx = \frac{1}{12}\left(-6\left(1-x^3\right)^{2/3} - 2\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) - 2\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) + 2^{2/3}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(1\right.\right.$$

```
[In] Integrate[x^5/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (-6*(1 - x^3)^(2/3) - 2*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3)
)/Sqrt[3]] - 2*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] + 2^(2/3)*Log[2 +
2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/12
```

Maple [A] (verified)

Time = 9.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$-\frac{\left(-x^{3}+1\right)^{\frac{2}{3}}}{2}-\frac{2^{\frac{2}{3}} \ln\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)}{6}+\frac{2^{\frac{2}{3}} \ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}} \left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)}{12}-\frac{\arctan\left(\frac{\left(1+2^{\frac{2}{3}} \left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right.}{6}$
trager	Expression too large to display
risch	Expression too large to display

[In] int(x^5/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)

[Out] -1/2*(-x^3+1)^(2/3)-1/6*2^(2/3)*ln((-x^3+1)^(1/3)-2^(1/3))+1/12*2^(2/3)*ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))-1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.28

$$\int \frac{x^5}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{1}{6}\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(2\sqrt{6}(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}-\sqrt{6}2^{\frac{1}{3}}\right)\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}-2^{\frac{2}{3}}(-1)^{\frac{1}{3}} + \left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}(-1)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right) - \frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}$$

[In] integrate(x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")

[Out] -1/6*sqrt(6)*2^(1/6)*(-1)^(1/3)*arctan(1/6*2^(1/6)*(2*sqrt(6)*(-1)^(1/3)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3))) - 1/12*2^(2/3)*(-1)^(1/3)*log(2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*(-1)^(1/3)*log(-2^(1/3)*(-1)^(2/3) + (-x^3 + 1)^(1/3)) - 1/2*(-x^3 + 1)^(2/3)

Sympy [F]

$$\int \frac{x^5}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^5}{\sqrt[3]{-(x-1)(x^2+x+1)}\,(x+1)(x^2-x+1)}\,dx$$

```
[In] integrate(x**5/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**5/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \frac{x^5}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = & -\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) \\ & +\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) \\ & -\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)-\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}\end{aligned}$$

```
[In] integrate(x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) - 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) - 1/2*(-x^3 + 1)^(2/3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{x^5}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = & -\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) \\ & +\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) \\ & -\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)-\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}\end{aligned}$$

```
[In] integrate(x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) - 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - 1/2*(-x^3 + 1)^(2
/3)
```

Mupad [B] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.13

$$\int \frac{x^5}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-2^{1/3}\right)}{6}-\frac{\left(1-x^3\right)^{2/3}}{2} - \frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12} + \frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

```
[In] int(x^5/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3) - (2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i
+ 1))/12 - (1 - x^3)^(2/3)/2 - (2^(2/3)*log((1 - x^3)^(1/3) - (2^(1/3)*(3^(
1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log((1 - x^3)^(1/3) - 2^
(1/3)))/6
```

3.609 $\int \frac{x^2}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 82

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=\frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log(1+x^3)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[Out] -1/12*ln(x^3+1)*2^(2/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(1
/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {455, 57, 631, 210, 31}

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=\frac{\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log(x^3+1)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[In] Int[x^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[1 + x
^3]/(6*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,dx, x, x^3\right)\\
&= -\frac{\log(1+x^3)}{6\sqrt[3]{2}}\\
&\quad +\frac{1}{2}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right)-\frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\\
&= -\frac{\log(1+x^3)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}-\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}}\\
&= \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log(1+x^3)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.27

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)+2\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)-\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(1-x^3)^{2/3}\right)}{6\sqrt[3]{2}}$$

```
[In] Integrate[x^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 2*Log[-2 + 2^(2/
3)*(1 - x^3)^(1/3)] - Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(
2/3)])/(6*2^(1/3))
```

Maple [A] (verified)

Time = 4.97 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

method	result	size
pseudoelliptic	$\frac{\left(2\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}-\ln\left(\left(-x^3+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)+2\ln\left(\left(-x^3+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)\right)2^{\frac{2}{3}}}{12}$	80
trager	Expression too large to display	759

```
[In] int(x^2/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(2*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-ln((-x^3+1)^
(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))+2*ln((-x^3+1)^(1/3)-2^(1/3)))*2^(2/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.10

$$\begin{aligned}\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx &= \frac{1}{6}\sqrt{6}2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}+2\sqrt{6}(-x^3+1)^{\frac{1}{3}}\right)\right)\\ &\quad-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right)\\ &\quad+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right)\end{aligned}$$

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*sqrt(6)*(-x^3 +
 1)^(1/3))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 +
 1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Sympy [F]

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^2}{\sqrt[3]{-(x-1)(x^2+x+1)}\,(x+1)(x^2-x+1)}\,dx$$

```
[In] integrate(x**2/(-x**3+1)**(1/3)/(x**3+1),x)
[Out] Integral(x**2/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)$$

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.06

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)$$

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) + 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3)))
```

Mupad [B] (verification not implemented)

Time = 8.52 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{2^{2/3}\,\ln\left(\left(1-x^3\right)^{1/3}-2^{1/3}\right)}{6} + \frac{2^{2/3}\,\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12} - \frac{2^{2/3}\,\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

```
[In] int(x^2/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3) - 2^(1/3)))/6 + (2^(2/3)*log((1 - x^3)^(1/3) -
 (2^(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log((1 - x
^3)^(1/3) - (2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/12
```

3.610 $\int \frac{1}{x\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 137

$$\int \frac{1}{x\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log(x)}{2} + \frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[Out] -1/2*ln(x)+1/12*ln(x^3+1)*2^(2/3)+1/2*ln(1-(-x^3+1)^(1/3))-1/4*ln(2^(1/3)-(
-x^3+1)^(1/3))*2^(2/3)+1/3*arctan(1/3*(1+2*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)
-1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {457, 88, 57, 632, 210, 31, 631}

$$\int \frac{1}{x\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{\arctan\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log(x^3+1)}{6\sqrt[3]{2}} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} - \frac{\log(x)}{2}$$

```
[In] Int[1/(x*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2^(2/3)*(1 -
x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[x]/2 + Log[1 + x^3]/(6*2^(1/3)
) + Log[1 - (1 - x^3)^(1/3)]/2 - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x(1+x)}\,dx, x, x^3\right) \\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x}\,dx, x, x^3\right) - \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,dx, x, x^3\right) \\
&= -\frac{\log(x)}{2} + \frac{\log(1+x^3)}{6\sqrt[3]{2}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right) \\
&\quad + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right) \\
&\quad - \frac{1}{2}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right) + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} \\
&= -\frac{\log(x)}{2} + \frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) \\
&\quad - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}} \\
&\quad - \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^3}\right) \\
&= \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log(x)}{2} \\
&\quad + \frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.35

$$\int \frac{1}{x\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{12}\left(4\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)\right.$$
$$-2\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)+4\log\left(-1+\sqrt[3]{1-x^3}\right)-2\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)-2\log\left(1+\right.$$

```
[In] Integrate[1/(x*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (4*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] - 2*2^(2/3)*Sqrt[3]*ArcT
an[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 4*Log[-1 + (1 - x^3)^(1/3)] - 2
*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] - 2*Log[1 + (1 - x^3)^(1/3) + (1
 - x^3)^(2/3)] + 2^(2/3)*Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3
)^(2/3)])/12
```

Maple [A] (verified)

Time = 4.93 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.06

method	result
pseudoelliptic	$-\frac{\ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}+1\right)}{6}+\frac{\arctan\left(\frac{\left(1+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln\left(-1+\left(-x^{3}+1\right)^{\frac{1}{3}}\right)}{3}-\frac{2^{\frac{2}{3}}\ln\left(\left(-x^{3}+1\right)^{\frac{1}{3}}\cdots\right.}{6}$

```
[In] int(1/x/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*ln((-x^3+1)^(2/3)+(-x^3+1)^(1/3)+1)+1/3*arctan(1/3*(1+2*(-x^3+1)^(1/3)
)*3^(1/2))*3^(1/2)+1/3*ln(-1+(-x^3+1)^(1/3))-1/6*2^(2/3)*ln((-x^3+1)^(1/3)-
2^(1/3))+1/12*2^(2/3)*ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))-1/6
*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 410, normalized size of antiderivative = 2.99

$$\int \frac{1}{x\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$$
$$= \frac{1}{12}\cdot 2^{\frac{2}{3}}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)\log\left(\frac{1}{8}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^3-\frac{3}{4}\cdot 2^{\frac{1}{3}}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^2+3\left(-x^3+1\right)^{\frac{1}{3}}+1\right)$$
$$-\frac{1}{24}\left(2^{\frac{2}{3}}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)-2\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^2}\right)\log\left(\frac{3}{8}\cdot 2^{\frac{2}{3}}\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^2}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)+\frac{3}{8}\cdot 2^{\frac{1}{3}}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^2+3\left(-x^3+1\right)^{\frac{1}{3}}\right)$$
$$-\frac{1}{24}\left(2^{\frac{2}{3}}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)+2\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^2}\right)\log\left(-\frac{3}{8}\cdot 2^{\frac{2}{3}}\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^2}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)+\frac{3}{8}\cdot 2^{\frac{1}{3}}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^2+3\left(-x^3+1\right)^{\frac{1}{3}}\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}(-x^3+1)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)$$
$$+\frac{1}{3}\log\left(-\frac{1}{24}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^3+\left(-x^3+1\right)^{\frac{1}{3}}-\frac{4}{3}\right)-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)$$

```
[In] integrate(1/x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/12*2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))*log(1/8*(I*sqrt(3)*(-1)^(1
/3) - (-1)^(1/3))^3 - 3/4*2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2 + 3
*(-x^3 + 1)^(1/3) + 1) - 1/24*(2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))
- 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2))*log(3/8
*2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2)*(I*
sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) + 3/8*2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)
^(1/3))^2 + 3*(-x^3 + 1)^(1/3)) - 1/24*(2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-1
)^(1/3)) + 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2)
)*log(-3/8*2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/
3))^2)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) + 3/8*2^(1/3)*(I*sqrt(3)*(-1)^(1
/3) - (-1)^(1/3))^2 + 3*(-x^3 + 1)^(1/3)) + 1/3*sqrt(3)*arctan(2/3*sqrt(3)*
(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) + 1/3*log(-1/24*(I*sqrt(3)*(-1)^(1/3) - (-1
)^(1/3))^3 + (-x^3 + 1)^(1/3) - 4/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1
)^(1/3) + 1)
```

Sympy [F]

$$\int \frac{1}{x\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x\sqrt[3]{-(x-1)(x^2+x+1)}\,(x+1)(x^2-x+1)}\,dx$$

```
[In] integrate(1/x/(-x**3+1)**(1/3)/(x**3+1),x)
[Out] Integral(1/(x*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x}\,dx$$

```
[In] integrate(1/x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.09

$$\begin{aligned}\int \frac{1}{x\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = &-\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)\\ &+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)\\ &-\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)\\ &+\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)\\ &-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\\ &+\frac{1}{3}\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)\end{aligned}$$

```
[In] integrate(1/x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
[Out] -1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) - 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/3*sqrt(3)*arcta
n(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3
+ 1)^(1/3) + 1) + 1/3*log(abs((-x^3 + 1)^(1/3) - 1))
```

Mupad [B] (verification not implemented)

Time = 8.41 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.87

$$\int \frac{1}{x\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{\ln\left(6-6\,(1-x^3)^{1/3}\right)}{3}$$

$$+\ln\left(\left(\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^3\left(1458\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2-135\,(1-x^3)^{1/3}\right)-(1-x^3)^{1/3}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(\right.\right.$$

```
[In] int(1/(x*(1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] log(6 - 6*(1 - x^3)^(1/3))/3 + log(((3^(1/2)*1i)/6 - 1/6)^3*(1458*((3^(1/2)
*1i)/6 - 1/6)^2 - 135*(1 - x^3)^(1/3)) - (1 - x^3)^(1/3))*((3^(1/2)*1i)/6 -
 1/6) - log(- ((3^(1/2)*1i)/6 + 1/6)^3*(1458*((3^(1/2)*1i)/6 + 1/6)^2 - 135
*(1 - x^3)^(1/3)) - (1 - x^3)^(1/3))*((3^(1/2)*1i)/6 + 1/6) - (2^(2/3)*log(
(3*(1 - x^3)^(1/3))/2 - (3*2^(1/3))/2))/6 + ((-1)^(1/3)*2^(2/3)*log((3*(1 -
 x^3)^(1/3))/2 - (3*(-1)^(2/3)*2^(1/3))/2))/6 - ((-1)^(1/3)*2^(2/3)*log(- (
(3^(1/2)*1i + 1)^3*(135*(1 - x^3)^(1/3) - (81*(-1)^(2/3)*2^(1/3)*(3^(1/2)*1
i + 1)^2)/4))/432 - (1 - x^3)^(1/3))*(3^(1/2)*1i + 1))/12
```

3.611 $\int \frac{1}{x^4 \sqrt[3]{1-x^3}(1+x^3)} dx$

Optimal result

Integrand size = 22, antiderivative size = 157

$$\int \frac{1}{x^4 \sqrt[3]{1-x^3}\,(1+x^3)} dx = -\frac{(1-x^3)^{2/3}}{3x^3} - \frac{2\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log(x)}{3} - \frac{\log(1+x^3)}{6\sqrt[3]{2}} - \frac{1}{3}\log\left(1-\sqrt[3]{1-x^3}\right) + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[Out] -1/3*(-x^3+1)^(2/3)/x^3+1/3*ln(x)-1/12*ln(x^3+1)*2^(2/3)-1/3*ln(1-(-x^3+1)^
(1/3))+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)-2/9*arctan(1/3*(1+2*(-x^3+1)^
(1/3))*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*
2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used

$= \{457, 105, 162, 57, 632, 210, 31, 631\}$

$$\int \frac{1}{x^4\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{2\arctan\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{(1-x^3)^{2/3}}{3x^3} - \frac{\log(x^3+1)}{6\sqrt[3]{2}} - \frac{1}{3}\log\left(1-\sqrt[3]{1-x^3}\right) + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} + \frac{\log(x)}{3}$$

```
[In] Int[1/(x^4*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/3*(1 - x^3)^(2/3)/x^3 - (2*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]])/(3*S
qrt[3]) + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) +
 Log[x]/3 - Log[1 + x^3]/(6*2^(1/3)) - Log[1 - (1 - x^3)^(1/3)]/3 + Log[2^(
1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x^2(1+x)}\,dx, x, x^3\right) \\
&= -\frac{(1-x^3)^{2/3}}{3x^3} - \frac{1}{3}\text{Subst}\left(\int \frac{\frac{2}{3}-\frac{x}{3}}{\sqrt[3]{1-x}x(1+x)}\,dx, x, x^3\right) \\
&= -\frac{(1-x^3)^{2/3}}{3x^3} - \frac{2}{9}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x}\,dx, x, x^3\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,dx, x, x^3\right) \\
&= -\frac{(1-x^3)^{2/3}}{3x^3} + \frac{\log(x)}{3} - \frac{\log(1+x^3)}{6\sqrt[3]{2}} \\
&\quad + \frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right) - \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right) \\
&\quad + \frac{1}{2}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right) - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}
\end{aligned}
$$

$$= -\frac{(1-x^3)^{2/3}}{3x^3} + \frac{\log(x)}{3} - \frac{\log(1+x^3)}{6\sqrt[3]{2}} - \frac{1}{3}\log\left(1-\sqrt[3]{1-x^3}\right) + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$
$$+ \frac{2}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^3}\right) - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}}$$

$$= -\frac{(1-x^3)^{2/3}}{3x^3} - \frac{2\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$
$$+ \frac{\log(x)}{3} - \frac{\log(1+x^3)}{6\sqrt[3]{2}} - \frac{1}{3}\log\left(1-\sqrt[3]{1-x^3}\right) + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^4\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{36}\left(-\frac{12(1-x^3)^{2/3}}{x^3} - 8\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)\right.$$
$$+6\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) - 8\log\left(-1+\sqrt[3]{1-x^3}\right) + 6\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) + 4\log\left(1+\right.$$

```
[In] Integrate[1/(x^4*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ((-12*(1 - x^3)^(2/3))/x^3 - 8*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[
3]] + 6*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 8*L
og[-1 + (1 - x^3)^(1/3)] + 6*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] + 4*
Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)] - 3*2^(2/3)*Log[2 + 2^(2/3)*(1 -
 x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/36
```

Maple [A] (verified)

Time = 6.52 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.34

method	result
pseudoelliptic	$\frac{-6\,2^{\frac{2}{3}}\sqrt{3}\,\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)x^3-6\,2^{\frac{2}{3}}\ln\left(\left(-x^3+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)x^3+3\,2^{\frac{2}{3}}\ln\left(\left(-x^3+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)x^3+}{36\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right.}$

```
[In] int(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/36*(-6*2^(2/3)*3^(1/2)*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*x^3
-6*2^(2/3)*ln((-x^3+1)^(1/3)-2^(1/3))*x^3+3*2^(2/3)*ln((-x^3+1)^(2/3)+2^(1/
```

3)*(-x^3+1)^(1/3)+2^(2/3))*x^3+8*3^(1/2)*arctan(1/3*(1+2*(-x^3+1)^(1/3))*3^(1/2))*x^3-4*ln((-x^3+1)^(2/3)+(-x^3+1)^(1/3)+1)*x^3+8*ln(-1+(-x^3+1)^(1/3))*x^3+12*(-x^3+1)^(2/3))/((-x^3+1)^(2/3)+(-x^3+1)^(1/3)+1)/(-1+(-x^3+1)^(1/3))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^4\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{6\sqrt{6}2^{\frac{1}{6}}x^3\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}+2\sqrt{6}(-x^3+1)^{\frac{1}{3}}\right)\right)-3\cdot 2^{\frac{2}{3}}x^3\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right)$$

[In] integrate(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")

[Out] 1/36*(6*sqrt(6)*2^(1/6)*x^3*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*sqrt(6)*(-x^3 + 1)^(1/3))) - 3*2^(2/3)*x^3*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 6*2^(2/3)*x^3*log(-2^(1/3) + (-x^3 + 1)^(1/3)) - 8*sqrt(3)*x^3*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) + 4*x^3*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) - 8*x^3*log((-x^3 + 1)^(1/3) - 1) - 12*(-x^3 + 1)^(2/3))/x^3

Sympy [F]

$$\int \frac{1}{x^4\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x^4\sqrt[3]{-(x-1)(x^2+x+1)}\,(x+1)(x^2-x+1)}\,dx$$

[In] integrate(1/x**4/(-x**3+1)**(1/3)/(x**3+1),x)

[Out] Integral(1/(x**4*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)

Maxima [F]

$$\int \frac{1}{x^4\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^4}\,dx$$

[In] integrate(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")

[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^4), x)

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.04

$$\begin{aligned}\int \frac{1}{x^4\sqrt[3]{1-x^3}\,(1+x^3)}\,dx &= \frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\,\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)\\ &\quad-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)\\ &\quad+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)\\ &\quad-\frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{3\,x^3}\\ &\quad+\frac{1}{9}\,\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\\ &\quad-\frac{2}{9}\,\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)\end{aligned}$$

[In] integrate(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")

[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) + 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - 2/9*sqrt(3)*arctan
(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/3*(-x^3 + 1)^(2/3)/x^3 + 1/9*log
((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) - 2/9*log(abs((-x^3 + 1)^(1/3) -
1))

Mupad [B] (verification not implemented)

Time = 8.48 (sec) , antiderivative size = 382, normalized size of antiderivative = 2.43

$$\int \frac{1}{x^4\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{2^{2/3}\ln\left(\frac{2^{1/3}\left(\frac{2^{2/3}\left(81\,2^{1/3}-75\,(1-x^3)^{1/3}\right)}{6}-\frac{38}{3}\right)}{18}+\frac{16\,(1-x^3)^{1/3}}{27}\right)}{6} - \frac{(1-x^3)^{2/3}}{3\,x^3} - \frac{2\ln\left(\frac{344\,(1-x^3)^{1/3}}{243}-\frac{344}{243}\right)}{9}$$

$$+\ln\left(\left(\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)^2\left(\left(\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)\left(1458\left(\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)^2-75\,(1-x^3)^{1/3}\right)-\frac{38}{3}\right)+\frac{16\,(1-x^3)^{1/3}}{27}\right)$$

```
[In] int(1/(x^4*(1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((2^(1/3)*((2^(2/3)*(81*2^(1/3) - 75*(1 - x^3)^(1/3)))/6 - 38/3
))/18 + (16*(1 - x^3)^(1/3))/27))/6 - (1 - x^3)^(2/3)/(3*x^3) - (2*log((344
*(1 - x^3)^(1/3))/243 - 344/243))/9 + log(((3^(1/2)*1i)/9 + 1/9)^2*(((3^(1/
2)*1i)/9 + 1/9)*(1458*((3^(1/2)*1i)/9 + 1/9)^2 - 75*(1 - x^3)^(1/3)) - 38/3
) + (16*(1 - x^3)^(1/3))/27)*((3^(1/2)*1i)/9 + 1/9) - log((16*(1 - x^3)^(1/
3))/27 - ((3^(1/2)*1i)/9 - 1/9)^2*(((3^(1/2)*1i)/9 - 1/9)*(1458*((3^(1/2)*1
i)/9 - 1/9)^2 - 75*(1 - x^3)^(1/3)) + 38/3))*((3^(1/2)*1i)/9 - 1/9) + (2^(2
/3)*log((16*(1 - x^3)^(1/3))/27 + (2^(1/3)*(3^(1/2)*1i - 1)^2*((2^(2/3)*(3^
(1/2)*1i - 1)*((81*2^(1/3)*(3^(1/2)*1i - 1)^2)/4 - 75*(1 - x^3)^(1/3)))/12
- 38/3))/72)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log((16*(1 - x^3)^(1/3))/27 -
(2^(1/3)*(3^(1/2)*1i + 1)^2*((2^(2/3)*(3^(1/2)*1i + 1)*((81*2^(1/3)*(3^(1/2
)*1i + 1)^2)/4 - 75*(1 - x^3)^(1/3)))/12 + 38/3))/72)*(3^(1/2)*1i + 1))/12
```

3.612 $\int \frac{x^6}{\sqrt[3]{1-x^3}(1+x^3)} dx$

Optimal result

Integrand size = 22, antiderivative size = 154

$$\int \frac{x^6}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{1}{3}x\left(1-x^3\right)^{2/3} + \frac{2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} - \frac{1}{3}\log\left(x+\sqrt[3]{1-x^3}\right)$$

```
[Out] -1/3*x*(-x^3+1)^(2/3)-1/12*ln(x^3+1)*2^(2/3)+1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/
3))*2^(2/3)-1/3*ln(x+(-x^3+1)^(1/3))+2/9*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*
3^(1/2))*3^(1/2)-1/6*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(
2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {490, 544, 245, 384}

$$\int \frac{x^6}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=\frac{2\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{1}{3}\left(1-x^3\right)^{2/3}x-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}-\frac{1}{3}\log\left(\sqrt[3]{1-x^3}+x\right)$$

```
[In] Int[x^6/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/3*(x*(1 - x^3)^(2/3)) + (2*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/
(3*Sqrt[3]) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*
Sqrt[3]) - Log[1 + x^3]/(6*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(
2*2^(1/3)) - Log[x + (1 - x^3)^(1/3)]/3
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{3}x\left(1-x^3\right)^{2/3}+\frac{1}{3}\int\frac{1-2x^3}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx\\ &= -\frac{1}{3}x\left(1-x^3\right)^{2/3}-\frac{2}{3}\int\frac{1}{\sqrt[3]{1-x^3}}\,dx+\int\frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx\\ &= -\frac{1}{3}x\left(1-x^3\right)^{2/3}+\frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}\\ &\quad-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}-\frac{1}{3}\log\left(x+\sqrt[3]{1-x^3}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.43

$$\int\frac{x^6}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=\frac{1}{36}\left(-12x\left(1-x^3\right)^{2/3}+8\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2\sqrt[3]{1-x^3}}\right)\right.$$
$$-6\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-8\log\left(x+\sqrt[3]{1-x^3}\right)+6\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+4\log\left(x^2-x\right.$$

```
[In] Integrate[x^6/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (-12*x*(1 - x^3)^(2/3) + 8*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2*(1 - x^3)^(1/3
))] - 6*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] -
 8*Log[x + (1 - x^3)^(1/3)] + 6*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)]
+ 4*Log[x^2 - x*(1 - x^3)^(1/3) + (1 - x^3)^(2/3)] - 3*2^(2/3)*Log[-2*x^2 +
 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/36
```

Maple [A] (verified)

Time = 6.40 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.51

method	result
pseudoelliptic	$\frac{6\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)+6\,2^{\frac{2}{3}}\ln\left(\frac{2^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)-3\,2^{\frac{2}{3}}\ln\left(\frac{2^{\frac{2}{3}}x^2-2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}\right)-1}{36\left(\left(-x^3+1\right)^{\frac{2}{3}}-\left(-x^3+1\right)^{\frac{1}{3}}x+\right.}$

```
[In] int(x^6/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/36*(6*3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)+6
*2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)-3*2^(2/3)*ln((2^(2/3)*x^2-2^(1/3)
*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))/x^2)-12*x*(-x^3+1)^(2/3)-8*3^(1/2)*arctan
(1/3*(-2*(-x^3+1)^(1/3)+x)*3^(1/2)/x)+4*ln(((-x^3+1)^(2/3)-(-x^3+1)^(1/3)*x
+x^2)/x^2)-8*ln((x+(-x^3+1)^(1/3))/x))/((-x^3+1)^(2/3)-(-x^3+1)^(1/3)*x+x^2
)/(x+(-x^3+1)^(1/3))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.31

$$\begin{aligned}\int \frac{x^6}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = & -\frac{1}{3}\left(-x^3+1\right)^{\frac{2}{3}}x \\ & -\frac{1}{6}\sqrt{6}2^{\frac{1}{6}}\arctan\left(-\frac{2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}x-2\sqrt{6}(-x^3+1)^{\frac{1}{3}}\right)}{6\,x}\right) \\ & +\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\frac{2^{\frac{1}{3}}x+(-x^3+1)^{\frac{1}{3}}}{x}\right)-\frac{1}{12} \\ & \cdot 2^{\frac{2}{3}}\log\left(\frac{2^{\frac{2}{3}}x^2-2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}x+(-x^3+1)^{\frac{2}{3}}}{x^2}\right) \\ & +\frac{2}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}(-x^3+1)^{\frac{1}{3}}}{3\,x}\right) \\ & -\frac{2}{9}\log\left(\frac{x+(-x^3+1)^{\frac{1}{3}}}{x}\right) \\ & +\frac{1}{9}\log\left(\frac{x^2-(-x^3+1)^{\frac{1}{3}}x+(-x^3+1)^{\frac{2}{3}}}{x^2}\right)\end{aligned}$$

```
[In] integrate(x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*(-x^3 + 1)^(2/3)*x - 1/6*sqrt(6)*2^(1/6)*arctan(-1/6*2^(1/6)*(sqrt(6)*
2^(1/3)*x - 2*sqrt(6)*(-x^3 + 1)^(1/3))/x) + 1/6*2^(2/3)*log((2^(1/3)*x + (
-x^3 + 1)^(1/3))/x) - 1/12*2^(2/3)*log((2^(2/3)*x^2 - 2^(1/3)*(-x^3 + 1)^(1
/3)*x + (-x^3 + 1)^(2/3))/x^2) + 2/9*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqr
t(3)*(-x^3 + 1)^(1/3))/x) - 2/9*log((x + (-x^3 + 1)^(1/3))/x) + 1/9*log((x^
2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

Sympy [F]

$$\int \frac{x^6}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^6}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

```
[In] integrate(x**6/(-x**3+1)**(1/3)/(x**3+1),x)
[Out] Integral(x**6/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{x^6}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^6}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^6}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^6}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

```
[In] int(x^6/((1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int(x^6/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.613 $\int \frac{x^3}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 135

$$\int \frac{x^3}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log(1+x^3)}{6\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right)$$

```
[Out] 1/12*ln(x^3+1)*2^(2/3)-1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)+1/2*ln(x+(
-x^3+1)^(1/3))-1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+1/6*a
rctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {494, 245, 384}

$$\int \frac{x^3}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log(x^3+1)}{6\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}+\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)$$

```
[In] Int[x^3/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 - (2*2^(
1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(1/
3)) - Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3)) + Log[x + (1 - x^3)^(
1/3)]/2
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\sqrt[3]{1-x^3}}\,dx - \int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log(1+x^3)}{6\sqrt[3]{2}} - \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} + \frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right)$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.52

$$\int \frac{x^3}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{12}\left(-4\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2\sqrt[3]{1-x^3}}\right) +2\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)+4\log\left(x+\sqrt[3]{1-x^3}\right)-2\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)-2\log\left(x^2-x\right.\right.$$

```
[In] Integrate[x^3/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (-4*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2*(1 - x^3)^(1/3))] + 2*2^(2/3)*Sqrt[3]
*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] + 4*Log[x + (1 - x^3)^(1
/3)] - 2*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)] - 2*Log[x^2 - x*(1 - x^
3)^(1/3) + (1 - x^3)^(2/3)] + 2^(2/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3
) - 2^(1/3)*(1 - x^3)^(2/3)]])/12
```

Maple [A] (verified)

Time = 4.67 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.33

method	result
pseudoelliptic	$-\frac{2^{\frac{2}{3}}\ln\left(\frac{2^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)}{6}+\frac{2^{\frac{2}{3}}\ln\left(\frac{2^{\frac{2}{3}}x^2-2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}\right)}{12}-\frac{\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)}{6}$

```
[In] int(x^3/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)+1/12*2^(2/3)*ln((2^(2/3)*x^2-
2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))/x^2)-1/6*3^(1/2)*2^(2/3)*arctan(1/
3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)-1/6*ln(((-x^3+1)^(2/3)-(-x^3+1)^(1
/3)*x+x^2)/x^2)+1/3*3^(1/2)*arctan(1/3*(-2*(-x^3+1)^(1/3)+x)*3^(1/2)/x)+1/3
*ln((x+(-x^3+1)^(1/3))/x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 452, normalized size of antiderivative = 3.35

$$\int \frac{x^3}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{12}$$

$$\cdot 2^{\frac{2}{3}}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)\log\left(-\frac{x\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^3-6\cdot 2^{\frac{1}{3}}x\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^2+8\,x-24\,(\ldots}{8\,x}\right.$$

$$-\frac{1}{24}\left(2^{\frac{2}{3}}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)-2\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^2}\right)\log\left(-\frac{3\left(2^{\frac{2}{3}}\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}\left(i\sqrt{3}\,(\ldots\right.}}{\ldots}\right.$$

$$-\frac{1}{24}\left(2^{\frac{2}{3}}\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)+2\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^2}\right)\log\left(\frac{3\left(2^{\frac{2}{3}}\sqrt{\frac{3}{2}}\sqrt{-2^{\frac{1}{3}}\left(i\sqrt{3}\,(-\ldots\right.}}{\ldots}\right.$$

$$-\frac{1}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}(-x^3+1)^{\frac{1}{3}}}{3\,x}\right)$$

$$+\frac{1}{3}\log\left(\frac{x\left(i\sqrt{3}(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^3+32\,x+24\,(-x^3+1)^{\frac{1}{3}}}{24\,x}\right)$$

$$-\frac{1}{6}\log\left(\frac{x^2-(-x^3+1)^{\frac{1}{3}}x+(-x^3+1)^{\frac{2}{3}}}{x^2}\right)$$

```
[In] integrate(x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/12*2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))*log(-1/8*(x*(I*sqrt(3)*(-1
)^(1/3) - (-1)^(1/3))^3 - 6*2^(1/3)*x*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2
 + 8*x - 24*(-x^3 + 1)^(1/3))/x) - 1/24*(2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-
1)^(1/3)) - 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2
))*log(-3/8*(2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(
1/3))^2)*x*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) + 2^(1/3)*x*(I*sqrt(3)*(-1)^
(1/3) - (-1)^(1/3))^2 - 8*(-x^3 + 1)^(1/3))/x) - 1/24*(2^(2/3)*(I*sqrt(3)*(
-1)^(1/3) - (-1)^(1/3)) + 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) -
 (-1)^(1/3))^2))*log(3/8*(2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(
1/3) - (-1)^(1/3))^2)*x*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) - 2^(1/3)*x*(I*
sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2 + 8*(-x^3 + 1)^(1/3))/x) - 1/3*sqrt(3)*a
```

```
rctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) + 1/3*log(1/24*(x*(I
*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^3 + 32*x + 24*(-x^3 + 1)^(1/3))/x) - 1/6*
log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^3}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

```
[In] integrate(x**3/(-x**3+1)**(1/3)/(x**3+1),x)
[Out] Integral(x**3/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^3}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^3}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^3}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

```
[In] int(x^3/((1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int(x^3/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.614 $\int \frac{1}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 88

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log(1+x^3)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[Out] -1/12*ln(x^3+1)*2^(2/3)+1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/6*arcta
n(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {384}

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log(x^3+1)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}$$

```
[In] Int[1/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])) -
Log[1 + x^3]/(6*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
```

```
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\text{integral} = -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right) - 2\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right) + \log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}(1-x^3)^{2/3}\right)}{6\sqrt[3]{2}}$$

```
[In] Integrate[1/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] - 2*Log[2
*x + 2^(2/3)*(1 - x^3)^(1/3)] + Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^
(1/3)*(1 - x^3)^(2/3)])/2^(1/3)
```

Maple [A] (verified)

Time = 4.63 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08

method	result	size
pseudoelliptic	$\frac{\left(\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)+\ln\left(\frac{2^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)-\frac{\ln\left(\frac{2^{\frac{2}{3}}x^2-2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}\right)}{2}\right)2^{\frac{2}{3}}}{6}$	95
trager	Expression too large to display	825

```
[In] int(1/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(3^(1/2)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)+ln((2^(1/3)*
x+(-x^3+1)^(1/3))/x)-1/2*ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^
(2/3))/x^2))*2^(2/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(67) = 134$.

Time = 1.71 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.88

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx =$$

$$-\frac{1}{18}\sqrt{6}2^{\frac{1}{6}}\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}2^{\frac{2}{3}}(5x^7+4x^4-x)(-x^3+1)^{\frac{2}{3}}-\sqrt{6}2^{\frac{1}{3}}(71x^9-111x^6+33x^3-1)+12\sqrt{6}\right.}{6\,(109x^9-105x^6+3x^3+1)}\right.$$

$$+\frac{1}{18}\cdot 2^{\frac{2}{3}}\log\left(\frac{6\cdot 2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}x^2+2^{\frac{2}{3}}(x^3+1)+6\,(-x^3+1)^{\frac{2}{3}}x}{x^3+1}\right)-\frac{1}{36}$$

$$\cdot 2^{\frac{2}{3}}\log\left(\frac{3\cdot 2^{\frac{2}{3}}(5x^4-x)(-x^3+1)^{\frac{2}{3}}+2^{\frac{1}{3}}(19x^6-16x^3+1)-12\,(2x^5-x^2)(-x^3+1)^{\frac{1}{3}}}{x^6+2x^3+1}\right)$$

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)*(5*x^7 + 4*x^4
- x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6 + 33*x^3 - 1) + 1
2*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 - 105*x^6 + 3*
x^3 + 1)) + 1/18*2^(2/3)*log((6*2^(1/3)*(-x^3 + 1)^(1/3)*x^2 + 2^(2/3)*(x^3
 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) - 1/36*2^(2/3)*log((3*2^(2/3)*(5*x
^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x^3 + 1) - 12*(2*x^5 - x^2)
*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1))
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

```
[In] integrate(1/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

```
[In] int(1/((1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int(1/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.615 $\int \frac{1}{x^3 \sqrt[3]{1-x^3}(1+x^3)} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 105

$$\int \frac{1}{x^3 \sqrt[3]{1-x^3} (1+x^3)} \, dx = -\frac{(1-x^3)^{2/3}}{2x^2} + \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log(1+x^3)}{6\sqrt[3]{2}} - \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[Out] -1/2*(-x^3+1)^(2/3)/x^2+1/12*ln(x^3+1)*2^(2/3)-1/4*ln(-2^(1/3)*x-(-x^3+1)^(
1/3))*2^(2/3)+1/6*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3
)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {491, 12, 384}

$$\int \frac{1}{x^3 \sqrt[3]{1-x^3} (1+x^3)} \, dx = \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log(x^3+1)}{6\sqrt[3]{2}} - \frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}} - \frac{(1-x^3)^{2/3}}{2x^2}$$

```
[In] Int[1/(x^3*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/2*(1 - x^3)^(2/3)/x^2 + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[
3]]/(2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(1/3)) - Log[-(2^(1/3)*x) - (1 -
x^3)^(1/3)]/(2*2^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(1-x^3)^{2/3}}{2x^2}+\frac{1}{2}\int -\frac{2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx\\
&= -\frac{(1-x^3)^{2/3}}{2x^2}-\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx\\
&= -\frac{(1-x^3)^{2/3}}{2x^2}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log(1+x^3)}{6\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.34

$$\int \frac{1}{x^3\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{12}\left(-\frac{6(1-x^3)^{2/3}}{x^2} + 2\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right) - 2\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right) + 2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3} - \right.\right.$$

```
[In] Integrate[1/(x^3*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ((-6*(1 - x^3)^(2/3))/x^2 + 2*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/
3)*(1 - x^3)^(1/3))] - 2*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)] + 2^(2/
3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/12
```

Maple [A] (verified)

Time = 22.00 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.20

method	result
pseudoelliptic	$\frac{-2\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)x^2-2\,2^{\frac{2}{3}}\ln\left(\frac{2^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)x^2+2^{\frac{2}{3}}\ln\left(\frac{2^{\frac{2}{3}}x^2-2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}\right.}{12x^2}$
trager	Expression too large to display
risch	Expression too large to display

```
[In] int(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(-2*3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)*
x^2-2*2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)*x^2+2^(2/3)*ln((2^(2/3)*x^2-
2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))/x^2)*x^2-6*(-x^3+1)^(2/3))/x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. 2(81) = 162.

Time = 1.69 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.92

$$\int \frac{1}{x^3\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{2\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}x^2\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}2^{\frac{2}{3}}(-1)^{\frac{2}{3}}(5x^7+4x^4-x)(-x^3+1)^{\frac{2}{3}}-12\sqrt{6}(-1)^{\frac{1}{3}}(19x^8-16x^5+x^2)(-x^3+1)^{\frac{1}{3}}-\sqrt{6}2^{\frac{1}{3}}(71x^9-\right.}{6(109x^9-105x^6+3x^3+1)}\right.}{}$$

```
[In] integrate(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/36*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*x^2*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/
3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 12*sqrt(6)*(-1)^(1/3)*
(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^
6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 2*2^(2/3)*(-1)^(1/3)*x^
2*log((6*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*x^2 - 2^(2/3)*(-1)^(1/3)*(x^3
+ 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 2^(2/3)*(-1)^(1/3)*x^2*log(-(3*2^
(2/3)*(-1)^(1/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(-1)^(2/3)*(19*x^6
- 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) + 18*
(-x^3 + 1)^(2/3))/x^2
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x^3\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

```
[In] integrate(1/x**3/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**3*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^3}\,dx$$

```
[In] integrate(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^3}\,dx$$

```
[In] integrate(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x^3\,(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

```
[In] int(1/(x^3*(1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int(1/(x^3*(1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.616 $\int \frac{1}{x^6 \sqrt[3]{1-x^3}(1+x^3)} dx$

Optimal result

Integrand size = 22, antiderivative size = 124

$$\int \frac{1}{x^6 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{(1-x^3)^{2/3}}{5x^5} + \frac{(1-x^3)^{2/3}}{5x^2} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[Out] -1/5*(-x^3+1)^(2/3)/x^5+1/5*(-x^3+1)^(2/3)/x^2-1/12*ln(x^3+1)*2^(2/3)+1/4*l
n(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/6*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)
^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {491, 597, 12, 384}

$$\int \frac{1}{x^6 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log(x^3+1)}{6\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}} - \frac{(1-x^3)^{2/3}}{5x^5} + \frac{(1-x^3)^{2/3}}{5x^2}$$

```
[In] Int[1/(x^6*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/5*(1 - x^3)^(2/3)/x^5 + (1 - x^3)^(2/3)/(5*x^2) - ArcTan[(1 - (2*2^(1/3)
*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[1 + x^3]/(6*2^(1/3))
+ Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{(1-x^3)^{2/3}}{5x^5} + \frac{1}{5}\int \frac{-2+3x^3}{x^3\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$$

$$= -\frac{(1-x^3)^{2/3}}{5x^5} + \frac{(1-x^3)^{2/3}}{5x^2} - \frac{1}{10}\int -\frac{10}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$$

$$= -\frac{(1-x^3)^{2/3}}{5x^5} + \frac{(1-x^3)^{2/3}}{5x^2} + \int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$$

$$= -\frac{(1-x^3)^{2/3}}{5x^5} + \frac{(1-x^3)^{2/3}}{5x^2} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^6\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{60}\left(-\frac{12(1-x^3)^{5/3}}{x^5} - 10\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right) + 10\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right) - 5\,2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3} \cdots\right.\right.$$

```
[In] Integrate[1/(x^6*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ((-12*(1 - x^3)^(5/3))/x^5 - 10*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(
2/3)*(1 - x^3)^(1/3))] + 10*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)] - 5*
2^(2/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/
60
```

Maple [A] (verified)

Time = 22.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$\frac{10\,2^{\frac{2}{3}}\ln\left(\frac{2^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)x^5+12\left(x^3-1\right)\left(-x^3+1\right)^{\frac{2}{3}}+5\,2^{\frac{2}{3}}x^5\left(2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)-\ln\left(\frac{2^{\frac{2}{3}}x^2-2^{\frac{1}{3}}\left(-x \cdots\right.}{}\right)\right)}{60x^5}$
risch	Expression too large to display
trager	Expression too large to display

```
[In] int(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/60*(10*2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)*x^5+12*(x^3-1)*(-x^3+1)^(
2/3)+5*2^(2/3)*x^5*(2*3^(1/2)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x
)/x)-ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))/x^2)))/x^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(95) = 190.

Time = 1.76 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.28

$$\int \frac{1}{x^6\sqrt[3]{1-x^3}\,(1+x^3)}\,dx =$$

$$-\frac{10\,\sqrt{6}2^{\frac{1}{6}}x^5\arctan\left(\frac{2^{\frac{1}{6}}\left(6\,\sqrt{6}2^{\frac{2}{3}}\,(5\,x^7+4\,x^4-x)\,(-x^3+1)^{\frac{2}{3}}-\sqrt{6}2^{\frac{1}{3}}\,(71\,x^9-111\,x^6+33\,x^3-1)+12\,\sqrt{6}(19\,x^8-16\,x^5+x^2)\,(-x^3+1)^{\frac{1}{3}}\right)}{6\,(109\,x^9-105\,x^6+3\,x^3+1)}\right.}{}$$

```
[In] integrate(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/180*(10*sqrt(6)*2^(1/6)*x^5*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)*(5*x^7
 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6 + 33*x^3
 - 1) + 12*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 - 105
*x^6 + 3*x^3 + 1)) - 10*2^(2/3)*x^5*log((6*2^(1/3)*(-x^3 + 1)^(1/3)*x^2 + 2
^(2/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 5*2^(2/3)*x^5*log((3*
2^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x^3 + 1) - 12*(
2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) - 36*(x^3 - 1)*(-x^3 + 1)
^(2/3))/x^5
```

Sympy [F]

$$\int \frac{1}{x^6\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x^6\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

```
[In] integrate(1/x**6/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**6*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Maxima [F]

$$\int \frac{1}{x^6\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^6}\,dx$$

```
[In] integrate(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^6}\,dx$$

```
[In] integrate(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x^6\,(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

```
[In] int(1/(x^6*(1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int(1/(x^6*(1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.617 $\int \frac{1}{x^9 \sqrt[3]{1-x^3}(1+x^3)} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 141

$$\int \frac{1}{x^9 \sqrt[3]{1-x^3}(1+x^3)} \, dx = -\frac{(1-x^3)^{2/3}}{8x^8} + \frac{(1-x^3)^{2/3}}{20x^5} - \frac{17(1-x^3)^{2/3}}{40x^2} + \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log(1+x^3)}{6\sqrt[3]{2}} - \frac{\log\left(-\sqrt[3]{2}x - \sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[Out] -1/8*(-x^3+1)^(2/3)/x^8+1/20*(-x^3+1)^(2/3)/x^5-17/40*(-x^3+1)^(2/3)/x^2+1/
12*ln(x^3+1)*2^(2/3)-1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(1
/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used

$= \{491, 597, 12, 384\}$

$$\int \frac{1}{x^9 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log(x^3+1)}{6\sqrt[3]{2}} - \frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}} - \frac{(1-x^3)^{2/3}}{8x^8} + \frac{(1-x^3)^{2/3}}{20x^5} - \frac{17(1-x^3)^{2/3}}{40x^2}$$

```
[In] Int[1/(x^9*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/8*(1 - x^3)^(2/3)/x^8 + (1 - x^3)^(2/3)/(20*x^5) - (17*(1 - x^3)^(2/3))/
(40*x^2) + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqr
t[3]) + Log[1 + x^3]/(6*2^(1/3)) - Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2
^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
```

```
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(1-x^3)^{2/3}}{8x^8}+\frac{1}{8}\int\frac{-2+6x^3}{x^6\sqrt[3]{1-x^3}\,(1+x^3)}\,dx\\ &= -\frac{(1-x^3)^{2/3}}{8x^8}+\frac{(1-x^3)^{2/3}}{20x^5}-\frac{1}{40}\int\frac{-34+6x^3}{x^3\sqrt[3]{1-x^3}\,(1+x^3)}\,dx\\ &= -\frac{(1-x^3)^{2/3}}{8x^8}+\frac{(1-x^3)^{2/3}}{20x^5}-\frac{17(1-x^3)^{2/3}}{40x^2}+\frac{1}{80}\int-\frac{80}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx\\ &= -\frac{(1-x^3)^{2/3}}{8x^8}+\frac{(1-x^3)^{2/3}}{20x^5}-\frac{17(1-x^3)^{2/3}}{40x^2}-\int\frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx\\ &= -\frac{(1-x^3)^{2/3}}{8x^8}+\frac{(1-x^3)^{2/3}}{20x^5}-\frac{17(1-x^3)^{2/3}}{40x^2}\\ &\quad+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log(1+x^3)}{6\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.09

$$\int\frac{1}{x^9\sqrt[3]{1-x^3}\,(1+x^3)}\,dx=\frac{1}{120}\left(-\frac{3(1-x^3)^{2/3}(5-2x^3+17x^6)}{x^8}\right.$$
$$+20\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-20\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+10\,2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-\right.$$

```
[In] Integrate[1/(x^9*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ((-3*(1 - x^3)^(2/3)*(5 - 2*x^3 + 17*x^6))/x^8 + 20*2^(2/3)*Sqrt[3]*ArcTan[
(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] - 20*2^(2/3)*Log[2*x + 2^(2/3)*(
1 - x^3)^(1/3)] + 10*2^(2/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/
3)*(1 - x^3)^(2/3)])/120
```

Maple [A] (verified)

Time = 22.18 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$\frac{-20\,2^{\frac{2}{3}}\ln\left(\frac{2^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)x^{8}+\left(-51x^{6}+6x^{3}-15\right)\left(-x^{3}+1\right)^{\frac{2}{3}}-10\,2^{\frac{2}{3}}x^{8}\left(2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)-\ln\left(\frac{2^{\frac{2}{3}}}{}\right.\right.}{120x^{8}}$
risch	Expression too large to display
trager	Expression too large to display

```
[In] int(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*(-20*2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)*x^8+(-51*x^6+6*x^3-15)*
(-x^3+1)^(2/3)-10*2^(2/3)*x^8*(2*3^(1/2)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3
+1)^(1/3)+x)/x)-ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))/x^
2)))/x^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(109) = 218$.

Time = 1.70 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.27

$$\int \frac{1}{x^9\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{20\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}x^{8}\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}2^{\frac{2}{3}}(-1)^{\frac{2}{3}}(5x^{7}+4x^{4}-x)\left(-x^{3}+1\right)^{\frac{2}{3}}-12\sqrt{6}(-1)^{\frac{1}{3}}(19x^{8}-16x^{5}+x^{2})\left(-x^{3}+1\right)^{\frac{1}{3}}-\sqrt{6}2^{\frac{1}{3}}(71x^{9}-\right.}{6\left(109x^{9}-105x^{6}+3x^{3}+1\right)}\right.}{}$$

```
[In] integrate(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/360*(20*sqrt(6)*2^(1/6)*(-1)^(1/3)*x^8*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(
2/3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 12*sqrt(6)*(-1)^(1/3
)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*
x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 20*2^(2/3)*(-1)^(1/3)
*x^8*log((6*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*x^2 - 2^(2/3)*(-1)^(1/3)*(x
^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 10*2^(2/3)*(-1)^(1/3)*x^8*log(
-(3*2^(2/3)*(-1)^(1/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(-1)^(2/3)*(1
9*x^6 - 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1))
 + 9*(17*x^6 - 2*x^3 + 5)*(-x^3 + 1)^(2/3))/x^8
```

Sympy [F]

$$\int \frac{1}{x^9 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x^9 \sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

```
[In] integrate(1/x**9/(-x**3+1)**(1/3)/(x**3+1),x)
[Out] Integral(1/(x**9*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Maxima [F]

$$\int \frac{1}{x^9 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^9}\,dx$$

```
[In] integrate(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^9), x)
```

Giac [F]

$$\int \frac{1}{x^9 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^9}\,dx$$

```
[In] integrate(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^9), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^9 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x^9\,(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

```
[In] int(1/(x^9*(1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int(1/(x^9*(1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.618 $\int \frac{x^7}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 271

$$
\begin{aligned}
\int \frac{x^7}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx &= -\frac{1}{4}x^2\left(1-x^3\right)^{2/3}+\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} \\
&\quad+\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} \\
&\quad-\frac{1}{4}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) \\
&\quad+\frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}}+\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} \\
&\quad-\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}-\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}
\end{aligned}
$$

```
[Out] -1/4*x^2*(-x^3+1)^(2/3)-1/4*x^2*hypergeom([1/3, 2/3],[5/3],x^3)+1/24*ln((1-
x)*(1+x)^2)*2^(2/3)+1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/
(-x^3+1)^(1/3))*2^(2/3)-1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/8*
ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(1/3*(1-2*2^(1/3)*(1-x)/(
-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/12*arctan(1/3*(1+2^(1/3)*(1-x)/(-
x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {490, 21, 495, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^7}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{1}{4}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) + \frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}} - \frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}} - \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}} - \frac{1}{4}\left(1-x^3\right)^{2/3}x^2 + \frac{\log\left((1-x)(x+1)^2\right)}{12\sqrt[3]{2}}$$

```
[In] Int[x^7/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/4*(x^2*(1 - x^3)^(2/3)) + ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3
))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/
3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - (x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^
3])/4 + Log[(1 - x)*(1 + x)^2]/(12*2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(
1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[1 + (
2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) - Log[-1 + x + 2^(2/3)*(1 - x
^3)^(1/3)]/(4*2^(1/3))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{4}x^2\left(1-x^3\right)^{2/3}+\frac{1}{4}\int \frac{x(2-2x^3)}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx \\
&= -\frac{1}{4}x^2\left(1-x^3\right)^{2/3}+\frac{1}{2}\int \frac{x(1-x^3)^{2/3}}{1+x^3}\,dx \\
&= -\frac{1}{4}x^2\left(1-x^3\right)^{2/3}-\frac{1}{2}\int \frac{x}{\sqrt[3]{1-x^3}}\,dx+\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx \\
&= -\frac{1}{4}x^2\left(1-x^3\right)^{2/3}-\frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) \\
&\quad -\frac{1}{3}\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx-\text{Subst}\left(\int \frac{1}{1+2x^3}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right)
\end{aligned}
$$

$$= -\frac{1}{4}x^2\left(1-x^3\right)^{2/3} + \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) + \frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}} - \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} - \frac{1}{3}\text{Subst}\left(\int\frac{1}{1+\sqrt[3]{2}x}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{3}\text{Subst}\left(\int\frac{2-\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{1}{4}x^2\left(1-x^3\right)^{2/3} + \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) + \frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}} - \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} - \frac{1}{2}\text{Subst}\left(\int\frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right) + \frac{\text{Subst}\left(\int\frac{-\sqrt[3]{2}+2\ 2^{2/3}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}$$

$$= -\frac{1}{4}x^2\left(1-x^3\right)^{2/3} + \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) + \frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} - \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} - \frac{\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}}$$

$$= -\frac{1}{4}x^2(1-x^3)^{2/3} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) + \frac{\log((1-x)(1+x)^2)}{12\sqrt[3]{2}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} - \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.15

$$\int \frac{x^7}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{4}x^2\left(-(1-x^3)^{2/3} + \text{AppellF1}\left(\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},x^3,-x^3\right)\right)$$

```
[In] Integrate[x^7/((1 - x^3)^(1/3)*(1 + x^3)),x]
[Out] (x^2*(-(1 - x^3)^(2/3) + AppellF1[2/3, -2/3, 1, 5/3, x^3, -x^3]))/4
```

Maple [F]

$$\int \frac{x^7}{(-x^3+1)^{\frac{1}{3}}\,(x^3+1)}dx$$

```
[In] int(x^7/(-x^3+1)^(1/3)/(x^3+1),x)
[Out] int(x^7/(-x^3+1)^(1/3)/(x^3+1),x)
```

Fricas [F]

$$\int \frac{x^7}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^7}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(x^7/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
[Out] integral(-(-x^3 + 1)^(2/3)*x^7/(x^6 - 1), x)
```

Sympy [F]

$$\int \frac{x^7}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^7}{\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

```
[In] integrate(x**7/(-x**3+1)**(1/3)/(x**3+1),x)
[Out] Integral(x**7/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{x^7}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^7}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(x^7/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Giac [F]

$$\int \frac{x^7}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^7}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(x^7/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^7}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

```
[In] int(x^7/((1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int(x^7/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.619 $\int \frac{x^4}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 254

$$\begin{aligned}\int \frac{x^4}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx &= -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} \\ &\quad + \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) \\ &\quad - \frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}} - \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} \\ &\quad + \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}\end{aligned}$$

```
[Out] 1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/24*ln((1-x)*(1+x)^2)*2^(2/3)-1/12
*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+
1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/8*ln(-1+x+2^(2/3)*(-x^3+1)
^(1/3))*2^(2/3)-1/6*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*
2^(2/3)*3^(1/2)-1/12*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2
^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {494, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^4}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) - \frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}} + \frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}} + \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}} - \frac{\log\left((1-x)(x+1)^2\right)}{12\sqrt[3]{2}}$$

```
[In] Int[x^4/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3
])) - ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sq
rt[3]) + (x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/2 - Log[(1 - x)*(1 + x
)^2]/(12*2^(1/3)) - Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*
(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) + Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)
^(1/3)]/(3*2^(1/3)) + Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
```

```
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\text{integral} = \int \frac{x}{\sqrt[3]{1-x^3}}\,dx - \int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$$

$$= \frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) + \frac{1}{3}\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx + \text{Subst}\left(\int \frac{1}{1+2x^3}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) - \frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}} + \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{2-\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) - \frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{2}+2\ 2^{2/3}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{1}{2}x^2 {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) - \frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}} - \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{1}{2}x^2 {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) - \frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}} - \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.10

$$\int \frac{x^4}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{5}x^5\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right)$$

```
[In] Integrate[x^4/((1 - x^3)^(1/3)*(1 + x^3)),x]
[Out] (x^5*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/5
```

Maple [F]

$$\int \frac{x^4}{\left(-x^3+1\right)^{\frac{1}{3}}\left(x^3+1\right)}dx$$

```
[In] int(x^4/(-x^3+1)^(1/3)/(x^3+1),x)
[Out] int(x^4/(-x^3+1)^(1/3)/(x^3+1),x)
```

Fricas [F]

$$\int \frac{x^4}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\, dx = \int \frac{x^4}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\, dx$$

```
[In] integrate(x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
[Out] integral(-(-x^3 + 1)^(2/3)*x^4/(x^6 - 1), x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\, dx = \int \frac{x^4}{\sqrt[3]{-\left(x-1\right)\left(x^2+x+1\right)}\left(x+1\right)\left(x^2-x+1\right)}\, dx$$

```
[In] integrate(x**4/(-x**3+1)**(1/3)/(x**3+1),x)
[Out] Integral(x**4/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\, dx = \int \frac{x^4}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\, dx$$

```
[In] integrate(x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^4}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x^4}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

```
[In] int(x^4/((1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int(x^4/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.620 $\int \frac{x}{\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 233

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} - \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[Out] 1/24*ln((1-x)*(1+x)^2)*2^(2/3)+1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(
1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2
^(2/3)-1/8*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(1/3*(1-2*2^(1
/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/12*arctan(1/3*(1+2^(1/
3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {502, 2174, 206, 31, 648, 631, 210, 642}

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}} - \frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}} - \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}} + \frac{\log\left((1-x)(x+1)^2\right)}{12\sqrt[3]{2}}$$

```
[In] Int[x/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])
 + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[
3]) + Log[(1 - x)*(1 + x)^2]/(12*2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(1
- x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[1 + (2^
(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) - Log[-1 + x + 2^(2/3)*(1 - x^3
)^(1/3)]/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{3}\int\frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx\right)-\text{Subst}\left(\int\frac{1}{1+2x^3}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right)$$

$$=\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}}-\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}-\frac{1}{3}\text{Subst}\left(\int\frac{1}{1+\sqrt[3]{2}x}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right)-\frac{1}{3}\text{Subst}\left(\int\frac{2-\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right)$$

$$=\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}}-\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}-\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}-\frac{1}{2}\text{Subst}\left(\int\frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right)+\frac{\text{Subst}\left(\int\frac{-\sqrt[3]{2}+2\ 2^{2/3}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}$$

$$=\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}}+\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}-\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}-\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}-\frac{\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}}$$

$$=\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}}+\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}-\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}-\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.21

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$$
$$= \frac{-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{\sqrt[3]{2}-\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right)-4\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{-2\sqrt[3]{2}+2\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right)-4\log\left(-\sqrt[3]{2}+\sqrt[3]{2}x-\sqrt[3]{1}\right.}{}$$

```
[In] Integrate[x/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(2^(1/3) - 2^(1/3)*x + (1 - x^
3)^(1/3))] - 4*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(-2*2^(1/3) + 2*2^(
1/3)*x + (1 - x^3)^(1/3))] - 4*Log[-2^(1/3) + 2^(1/3)*x - (1 - x^3)^(1/3)]
- 2*Log[-2^(1/3) + 2^(1/3)*x + 2*(1 - x^3)^(1/3)] + 2*Log[2^(2/3) - 2*2^(2/
3)*x + 2^(2/3)*x^2 + (-1 + x)*(2 - 2*x^3)^(1/3) + (1 - x^3)^(2/3)] + Log[2^
(2/3) - 2*2^(2/3)*x + 2^(2/3)*x^2 - 2*(-1 + x)*(2 - 2*x^3)^(1/3) + 4*(1 - x
^3)^(2/3)])/(12*2^(1/3))
```

Maple [F]

$$\int \frac{x}{\left(-x^3+1\right)^{\frac{1}{3}}\left(x^3+1\right)}dx$$

```
[In] int(x/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(x/(-x^3+1)^(1/3)/(x^3+1),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(171) = 342$.

Time = 1.70 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.60

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{1}{36}\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\arctan\left(\frac{2^{\frac{1}{6}}\left(24\sqrt{6}2^{\frac{2}{3}}(-1)^{\frac{2}{3}}(x^{14}-2x^{11}-6x^8-2x^5+x^2)(-x^3+1)^{\frac{2}{3}}+12\sqrt{6}(-1)^{\frac{1}{3}}(\ldots\right.}{6(x^{18}-\ldots}\right) - \frac{1}{72}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{12\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}(x^8-4x^5+x^2)(-x^3+1)^{\frac{2}{3}}-2^{\frac{1}{3}}(-1)^{\frac{2}{3}}(x^{12}-32x^9+78x^6-32x^3+1)-\ldots}{x^{12}+4x^9+6x^6+4x^3+1}\right) + \frac{1}{36}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{12(-x^3+1)^{\frac{2}{3}}x^2-6\cdot 2^{\frac{1}{3}}(-1)^{\frac{2}{3}}(x^4-x)(-x^3+1)^{\frac{1}{3}}-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}(x^6+2x^3+1)}{x^6+2x^3+1}\right)$$

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(6)*2^(1/6)*(-1)^(1/3)*arctan(1/6*2^(1/6)*(24*sqrt(6)*2^(2/3)*(-1
)^(2/3)*(x^14 - 2*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^(2/3) + 12*sqrt(6)
*(-1)^(1/3)*(x^16 - 33*x^13 + 110*x^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1)^(
1/3) + sqrt(6)*2^(1/3)*(x^18 + 42*x^15 - 417*x^12 + 812*x^9 - 417*x^6 + 42*
x^3 + 1))/(x^18 - 102*x^15 + 447*x^12 - 628*x^9 + 447*x^6 - 102*x^3 + 1)) -
 1/72*2^(2/3)*(-1)^(1/3)*log(-(12*2^(2/3)*(-1)^(1/3)*(x^8 - 4*x^5 + x^2)*(-
x^3 + 1)^(2/3) - 2^(1/3)*(-1)^(2/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 + 1) -
 6*(x^10 - 11*x^7 + 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x^6 + 4
*x^3 + 1)) + 1/36*2^(2/3)*(-1)^(1/3)*log(-(12*(-x^3 + 1)^(2/3)*x^2 - 6*2^(1
/3)*(-1)^(2/3)*(x^4 - x)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3)*(x^6 + 2*x^3
 + 1))/(x^6 + 2*x^3 + 1))
```

Sympy [F]

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x}{\sqrt[3]{-(x-1)(x^2+x+1)}\,(x+1)(x^2-x+1)}\,dx$$

```
[In] integrate(x/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{x}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

```
[In] int(x/((1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int(x/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.621 $\int \frac{1}{x^2 \sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 270

$$\int \frac{1}{x^2 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{(1-x^3)^{2/3}}{x} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{1}{2}x^2 \operatorname{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) - \frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}} - \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[Out] -(-x^3+1)^(2/3)/x-1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/24*ln((1-x)*(1+
x)^2)*2^(2/3)-1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+
1)^(1/3))*2^(2/3)+1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/8*ln(-1+
x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)-1/6*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1
)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/12*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)
^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {491, 598, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{x^2\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)-\frac{(1-x^3)^{2/3}}{x}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}-\frac{\log\left((1-x)(x+1)^2\right)}{12\sqrt[3]{2}}$$

```
[In] Int[1/(x^2*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -((1 - x^3)^(2/3)/x) - ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqr
t[3]]/(2^(1/3)*Sqrt[3]) - ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sq
rt[3]]/(2*2^(1/3)*Sqrt[3]) - (x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/2
- Log[(1 - x)*(1 + x)^2]/(12*2^(1/3)) - Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^
3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) + Log[1 + (2^(1/3
)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) + Log[-1 + x + 2^(2/3)*(1 - x^3)^(1
/3)]/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(1-x^3)^{2/3}}{x}+\int \frac{x(-2-x^3)}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx\\
&= -\frac{(1-x^3)^{2/3}}{x}+\int \left(-\frac{x}{\sqrt[3]{1-x^3}}-\frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\right)dx\\
&= -\frac{(1-x^3)^{2/3}}{x}-\int \frac{x}{\sqrt[3]{1-x^3}}\,dx-\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx\\
&= -\frac{(1-x^3)^{2/3}}{x}-\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{1}{3}\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx\\
&\quad+\text{Subst}\left(\int \frac{1}{1+2x^3}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right)\\
&= -\frac{(1-x^3)^{2/3}}{x}-\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}\\
&\quad-\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}}\\
&\quad+\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right)\\
&\quad+\frac{1}{3}\text{Subst}\left(\int \frac{2-\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right)
\end{aligned}
$$

$$= -\frac{(1-x^3)^{2/3}}{x} - \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) - \frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{2}+2\ 2^{2/3}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}$$

$$= -\frac{(1-x^3)^{2/3}}{x} - \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) - \frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}} - \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}}$$

$$= -\frac{(1-x^3)^{2/3}}{x} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) - \frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}} - \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 11.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.25

$$\int \frac{1}{x^2\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = -\frac{(1-x^3)^{2/3}}{x} - x^2\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right) - \frac{1}{5}x^5\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right)$$

```
[In] Integrate[1/(x^2*(1 - x^3)^(1/3)*(1 + x^3)),x]
[Out] -((1 - x^3)^(2/3)/x) - x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3] - (x^5*App
ellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/5
```

Maple [F]

$$\int \frac{1}{x^2\,(-x^3+1)^{\frac{1}{3}}\,(x^3+1)}dx$$

```
[In] int(1/x^2/(-x^3+1)^(1/3)/(x^3+1),x)
[Out] int(1/x^2/(-x^3+1)^(1/3)/(x^3+1),x)
```

Fricas [F]

$$\int \frac{1}{x^2\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^2}\,dx$$

```
[In] integrate(1/x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
[Out] integral(-(-x^3 + 1)^(2/3)/(x^8 - x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x^2\sqrt[3]{-(x-1)\,(x^2+x+1)}\,(x+1)\,(x^2-x+1)}\,dx$$

```
[In] integrate(1/x**2/(-x**3+1)**(1/3)/(x**3+1),x)
[Out] Integral(1/(x**2*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Maxima [F]

$$\int \frac{1}{x^2 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^2}\,dx$$

```
[In] integrate(1/x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^2}\,dx$$

```
[In] integrate(1/x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x^2\,(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

```
[In] int(1/(x^2*(1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int(1/(x^2*(1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.622 $\int \frac{1}{x^5\sqrt[3]{1-x^3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 289

$$\begin{aligned}\int \frac{1}{x^5\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = &-\frac{(1-x^3)^{2/3}}{4x^4}+\frac{(1-x^3)^{2/3}}{2x}\\ &+\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}\\ &+\frac{1}{4}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)\\ &+\frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}}+\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}\\ &-\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}-\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}\end{aligned}$$

```
[Out] -1/4*(-x^3+1)^(2/3)/x^4+1/2*(-x^3+1)^(2/3)/x+1/4*x^2*hypergeom([1/3, 2/3],[
5/3],x^3)+1/24*ln((1-x)*(1+x)^2)*2^(2/3)+1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)
^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1
)^(1/3))*2^(2/3)-1/8*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(1/3
*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/12*arctan(1/
3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {491, 21, 486, 598, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{x^5\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{1}{4}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) + \frac{(1-x^3)^{2/3}}{2x} + \frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} + 1\right)}{6\sqrt[3]{2}} - \frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} + 1\right)}{3\sqrt[3]{2}} - \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}} - \frac{(1-x^3)^{2/3}}{4x^4} + \frac{\log\left((1-x)(x+1)^2\right)}{12\sqrt[3]{2}}$$

```
[In] Int[1/(x^5*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/4*(1 - x^3)^(2/3)/x^4 + (1 - x^3)^(2/3)/(2*x) + ArcTan[(1 - (2*2^(1/3)*(
1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + ArcTan[(1 + (2^(1/3)*
(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) + (x^2*Hypergeometri
c2F1[1/3, 2/3, 5/3, x^3])/4 + Log[(1 - x)*(1 + x)^2]/(12*2^(1/3)) + Log[1 +
 (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(
6*2^(1/3)) - Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) - Log[-
1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(1-x^3)^{2/3}}{4x^4}+\frac{1}{4}\int \frac{-2+2x^3}{x^2\sqrt[3]{1-x^3}\,(1+x^3)}\,dx \\
&= -\frac{(1-x^3)^{2/3}}{4x^4}-\frac{1}{2}\int \frac{(1-x^3)^{2/3}}{x^2\,(1+x^3)}\,dx \\
&= -\frac{(1-x^3)^{2/3}}{4x^4}+\frac{(1-x^3)^{2/3}}{2x}-\frac{1}{2}\int \frac{x(-3-x^3)}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx
\end{aligned}$$

$$=-\frac{(1-x^3)^{2/3}}{4x^4}+\frac{(1-x^3)^{2/3}}{2x}-\frac{1}{2}\int\left(-\frac{x}{\sqrt[3]{1-x^3}}-\frac{2x}{\sqrt[3]{1-x^3}\,(1+x^3)}\right)dx$$

$$=-\frac{(1-x^3)^{2/3}}{4x^4}+\frac{(1-x^3)^{2/3}}{2x}+\frac{1}{2}\int\frac{x}{\sqrt[3]{1-x^3}}\,dx+\int\frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$$

$$=-\frac{(1-x^3)^{2/3}}{4x^4}+\frac{(1-x^3)^{2/3}}{2x}+\frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)$$
$$-\frac{1}{3}\int\frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx-\text{Subst}\left(\int\frac{1}{1+2x^3}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right)$$

$$=-\frac{(1-x^3)^{2/3}}{4x^4}+\frac{(1-x^3)^{2/3}}{2x}+\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$
$$+\frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}}-\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$
$$-\frac{1}{3}\text{Subst}\left(\int\frac{1}{1+\sqrt[3]{2}x}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right)-\frac{1}{3}\text{Subst}\left(\int\frac{2-\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right)$$

$$=-\frac{(1-x^3)^{2/3}}{4x^4}+\frac{(1-x^3)^{2/3}}{2x}+\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)$$
$$+\frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}}-\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}-\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$
$$-\frac{1}{2}\text{Subst}\left(\int\frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right)+\frac{\text{Subst}\left(\int\frac{-\sqrt[3]{2}+2\;2^{2/3}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}$$

$$=-\frac{(1-x^3)^{2/3}}{4x^4}+\frac{(1-x^3)^{2/3}}{2x}+\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)$$
$$+\frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}}+\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}-\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}$$
$$-\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}-\frac{\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}}$$

$$= -\frac{(1-x^3)^{2/3}}{4x^4} + \frac{(1-x^3)^{2/3}}{2x} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) + \frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} - \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 11.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^5\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{5(1-x^3)^{2/3}\left(-1+2x^3\right)+15x^6\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right)+2x^9\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right)}{20x^4}$$

```
[In] Integrate[1/(x^5*(1 - x^3)^(1/3)*(1 + x^3)),x]
[Out] (5*(1 - x^3)^(2/3)*(-1 + 2*x^3) + 15*x^6*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x
^3] + 2*x^9*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/(20*x^4)
```

Maple [F]

$$\int \frac{1}{x^5\left(-x^3+1\right)^{\frac{1}{3}}\left(x^3+1\right)}dx$$

```
[In] int(1/x^5/(-x^3+1)^(1/3)/(x^3+1),x)
[Out] int(1/x^5/(-x^3+1)^(1/3)/(x^3+1),x)
```

Fricas [F]

$$\int \frac{1}{x^5 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^5}\,dx$$

[In] integrate(1/x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")

[Out] integral(-(-x^3 + 1)^(2/3)/(x^11 - x^5), x)

Sympy [F]

$$\int \frac{1}{x^5 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x^5 \sqrt[3]{-(x-1)(x^2+x+1)}\,(x+1)(x^2-x+1)}\,dx$$

[In] integrate(1/x**5/(-x**3+1)**(1/3)/(x**3+1),x)

[Out] Integral(1/(x**5*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)

Maxima [F]

$$\int \frac{1}{x^5 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^5}\,dx$$

[In] integrate(1/x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")

[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^5), x)

Giac [F]

$$\int \frac{1}{x^5 \sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^5}\,dx$$

[In] integrate(1/x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")

[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^5), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1}{x^5\,(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

```
[In] int(1/(x^5*(1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int(1/(x^5*(1 - x^3)^(1/3)*(x^3 + 1)), x)
```

3.623 $\int \frac{x^{11}}{(1-x^3)^{2/3}(1+x^3)} dx$

Optimal result

Integrand size = 22, antiderivative size = 125

$$\int \frac{x^{11}}{(1-x^3)^{2/3}(1+x^3)} dx = -\sqrt[3]{1-x^3}+\frac{1}{4}(1-x^3)^{4/3} - \frac{1}{7}(1-x^3)^{7/3}+\frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log(1+x^3)}{6\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

```
[Out] -(-x^3+1)^(1/3)+1/4*(-x^3+1)^(4/3)-1/7*(-x^3+1)^(7/3)+1/12*ln(x^3+1)*2^(1/3
)-1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(1/3)+1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)
^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {457, 90, 59, 631, 210, 31}

$$\int \frac{x^{11}}{(1-x^3)^{2/3}(1+x^3)} dx = \frac{\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{1}{7}(1-x^3)^{7/3}+\frac{1}{4}(1-x^3)^{4/3}-\sqrt[3]{1-x^3}+\frac{\log(x^3+1)}{6\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

```
[In] Int[x^11/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(1/3) + (1 - x^3)^(4/3)/4 - (1 - x^3)^(7/3)/7 + ArcTan[(1 + 2^(2
/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(2/3))
- Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{(1-x)^{2/3}}-\sqrt[3]{1-x}+(1-x)^{4/3}-\frac{1}{(1-x)^{2/3}(1+x)}\right)dx, x, x^3\right)\\
&= -\sqrt[3]{1-x^3}+\frac{1}{4}\left(1-x^3\right)^{4/3}-\frac{1}{7}\left(1-x^3\right)^{7/3}-\frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)\\
&= -\sqrt[3]{1-x^3}+\frac{1}{4}\left(1-x^3\right)^{4/3}-\frac{1}{7}\left(1-x^3\right)^{7/3}+\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}\\
&\quad+\frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}+\frac{\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\\
&= -\sqrt[3]{1-x^3}+\frac{1}{4}\left(1-x^3\right)^{4/3}-\frac{1}{7}\left(1-x^3\right)^{7/3}+\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}\\
&\quad-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}-\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{2^{2/3}}\\
&= -\sqrt[3]{1-x^3}+\frac{1}{4}\left(1-x^3\right)^{4/3}\\
&\quad-\frac{1}{7}\left(1-x^3\right)^{7/3}+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.10

$$
\int \frac{x^{11}}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \frac{1}{84}\left(3\sqrt[3]{1-x^3}\left(-25+x^3-4x^6\right)\right.
$$

$$
+14\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)-14\sqrt[3]{2}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)+7\sqrt[3]{2}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(1-x
$$

```
[In] Integrate[x^11/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (3*(1 - x^3)^(1/3)*(-25 + x^3 - 4*x^6) + 14*2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(
2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 14*2^(1/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3
)] + 7*2^(1/3)*Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/
84
```

Maple [A] (verified)

Time = 9.50 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{\left(-4x^{6}+x^{3}-25\right)\left(-x^{3}+1\right)^{\frac{1}{3}}}{28}+\frac{2^{\frac{1}{3}}\left(2\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}+\ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)-2\ln\left(\left(-\right.\right.}{12}$
trager	Expression too large to display
risch	Expression too large to display

```
[In] int(x^11/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/28*(-4*x^6+x^3-25)*(-x^3+1)^(1/3)+1/12*2^(1/3)*(2*arctan(1/3*(1+2^(2/3)*(
-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^
(2/3))-2*ln((-x^3+1)^(1/3)-2^(1/3)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.14

$$\int \frac{x^{11}}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}(-1)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+2\cdot 4^{\frac{1}{3}}(-1)^{\frac{2}{3}}+2\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}(-1)^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)-\frac{1}{28}\left(4x^6-x^3+25\right)\left(-x^3+1\right)^{\frac{1}{3}}$$

```
[In] integrate(x^11/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-1)^(2
/3)*(-x^3 + 1)^(1/3) + 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*(-1)^(1/3)*log(-4^(
2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3) + 2*4^(1/3)*(-1)^(2/3) + 2*(-x^3 + 1)^(2/3
)) + 1/12*4^(2/3)*(-1)^(1/3)*log(4^(2/3)*(-1)^(1/3) + 2*(-x^3 + 1)^(1/3)) -
 1/28*(4*x^6 - x^3 + 25)*(-x^3 + 1)^(1/3)
```

Sympy [F]

$$\int \frac{x^{11}}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^{11}}{\left(-(x-1)(x^2+x+1)\right)^{\frac{2}{3}}(x+1)(x^2-x+1)}\,dx$$

```
[In] integrate(x**11/(-x**3+1)**(2/3)/(x**3+1),x)
[Out] Integral(x**11/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.95

$$\begin{aligned}\int \frac{x^{11}}{(1-x^3)^{2/3}(1+x^3)}\,dx = & -\frac{1}{7}\left(-x^3+1\right)^{\frac{7}{3}} \\ & + \frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) + \frac{1}{4}\left(-x^3+1\right)^{\frac{4}{3}} \\ & + \frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) \\ & - \frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right) - \left(-x^3+1\right)^{\frac{1}{3}}\end{aligned}$$

```
[In] integrate(x^11/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
[Out] -1/7*(-x^3 + 1)^(7/3) + 1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(
1/3) + 2*(-x^3 + 1)^(1/3))) + 1/4*(-x^3 + 1)^(4/3) + 1/12*2^(1/3)*log(2^(2/
3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(1/3)*log(-2^(1/3
) + (-x^3 + 1)^(1/3)) - (-x^3 + 1)^(1/3)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \frac{x^{11}}{(1-x^3)^{2/3}(1+x^3)}\,dx = & -\frac{1}{7}\left(x^3-1\right)^2\left(-x^3+1\right)^{\frac{1}{3}} \\ & + \frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) + \frac{1}{4}\left(-x^3+1\right)^{\frac{4}{3}} \\ & + \frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) \\ & - \frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right) - \left(-x^3+1\right)^{\frac{1}{3}}\end{aligned}$$

```
[In] integrate(x^11/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/7*(x^3 - 1)^2*(-x^3 + 1)^(1/3) + 1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*
2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) + 1/4*(-x^3 + 1)^(4/3) + 1/12*2^(1/
3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(1/3)
*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - (-x^3 + 1)^(1/3)
```

Mupad [B] (verification not implemented)

Time = 8.40 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.08

$$\int \frac{x^{11}}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{(1-x^3)^{4/3}}{4} - (1-x^3)^{1/3} - \frac{2^{1/3}\ln\left(3\,2^{1/3}-3\,(1-x^3)^{1/3}\right)}{6} - \frac{(1-x^3)^{7/3}}{7} - \frac{2^{1/3}\ln\left(3\,(1-x^3)^{1/3}-\frac{3\,2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12} + \frac{2^{1/3}\ln\left(\frac{3\,2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}+3\,(1-x^3)^{1/3}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

```
[In] int(x^11/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] (1 - x^3)^(4/3)/4 - (1 - x^3)^(1/3) - (2^(1/3)*log(3*2^(1/3) - 3*(1 - x^3)^
(1/3)))/6 - (1 - x^3)^(7/3)/7 - (2^(1/3)*log(3*(1 - x^3)^(1/3) - (3*2^(1/3)
*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/12 + (2^(1/3)*log((3*2^(1/3)*(3^(1/
2)*1i + 1))/2 + 3*(1 - x^3)^(1/3))*(3^(1/2)*1i + 1))/12
```

3.624 $\int \frac{x^8}{(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \frac{x^8}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{4}\left(1-x^3\right)^{4/3} - \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log(1+x^3)}{6\ 2^{2/3}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

```
[Out] 1/4*(-x^3+1)^(4/3)-1/12*ln(x^3+1)*2^(1/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^
(1/3)-1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {457, 90, 59, 631, 210, 31}

$$\int \frac{x^8}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{1}{4}\left(1-x^3\right)^{4/3} - \frac{\log(x^3+1)}{6\ 2^{2/3}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

```
[In] Int[x^8/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (1 - x^3)^(4/3)/4 - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*
Sqrt[3]) - Log[1 + x^3]/(6*2^(2/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(
2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(-\sqrt[3]{1-x}+\frac{1}{(1-x)^{2/3}(1+x)}\right)\,dx, x, x^3\right)\end{aligned}$$

$$= \frac{1}{4}\left(1-x^3\right)^{4/3} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)$$

$$= \frac{1}{4}\left(1-x^3\right)^{4/3} - \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

$$= \frac{1}{4}\left(1-x^3\right)^{4/3} - \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{2^{2/3}}$$

$$= \frac{1}{4}\left(1-x^3\right)^{4/3} - \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.30

$$\int \frac{x^8}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \frac{1}{12}\left(3\left(1-x^3\right)^{4/3} - 2\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) + 2\sqrt[3]{2}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) - \sqrt[3]{2}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}\left(1-x^3\right)^{2/3}\right)\right)$$

```
[In] Integrate[x^8/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (3*(1 - x^3)^(4/3) - 2*2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))
/Sqrt[3]] + 2*2^(1/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] - 2^(1/3)*Log[2 + 2
^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/12
```

Maple [A] (verified)

Time = 9.51 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.11

method	result
pseudoelliptic	$-\frac{(-x^3+1)^{\frac{1}{3}}x^3}{4}+\frac{(-x^3+1)^{\frac{1}{3}}}{4}+\frac{2^{\frac{1}{3}}\ln\left((-x^3+1)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)}{6}-\frac{2^{\frac{1}{3}}\ln\left((-x^3+1)^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)}{12}-\frac{\arctan\left(\left(\right.\right.}{}$
trager	Expression too large to display
risch	Expression too large to display

```
[In] int(x^8/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-x^3+1)^(1/3)*x^3+1/4*(-x^3+1)^(1/3)+1/6*2^(1/3)*ln((-x^3+1)^(1/3)-2^
(1/3))-1/12*2^(1/3)*ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))-1/6*a
rctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.16

$$\int \frac{x^8}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}(-x^3+1)^{\frac{1}{3}}+4^{\frac{1}{3}}\sqrt{3}\right)\right) - \frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}(-x^3+1)^{\frac{1}{3}}+2(-x^3+1)^{\frac{2}{3}}+2\cdot 4^{\frac{1}{3}}\right)+\frac{1}{12} \cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{2}{3}}+2(-x^3+1)^{\frac{1}{3}}\right)-\frac{1}{4}(x^3-1)(-x^3+1)^{\frac{1}{3}}$$

```
[In] integrate(x^8/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-x^3 + 1)^(1/3) +
 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*log(4^(2/3)*(-x^3 + 1)^(1/3) + 2*(-x^3 +
1)^(2/3) + 2*4^(1/3)) + 1/12*4^(2/3)*log(-4^(2/3) + 2*(-x^3 + 1)^(1/3)) - 1
/4*(x^3 - 1)*(-x^3 + 1)^(1/3)
```

Sympy [F]

$$\int \frac{x^8}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^8}{\left(-(x-1)(x^2+x+1)\right)^{\frac{2}{3}}(x+1)(x^2-x+1)}\,dx$$

```
[In] integrate(x**8/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x**8/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.99

$$\int \frac{x^8}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{4}\left(-x^3+1\right)^{\frac{4}{3}}-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)$$

```
[In] integrate(x^8/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/4*(-x^3 + 1)^(4/3) - 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)
^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(1/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \frac{x^8}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{4}\left(-x^3+1\right)^{\frac{4}{3}}-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)$$

```
[In] integrate(x^8/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/4*(-x^3 + 1)^(4/3) - 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)
^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(1/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3
)))
```

Mupad [B] (verification not implemented)

Time = 8.44 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.15

$$\int \frac{x^8}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{2^{1/3}\ln\left(\frac{(1-x^3)^{1/3}}{2}-\frac{2^{1/3}}{2}\right)}{6}+\frac{(1-x^3)^{4/3}}{4} + \frac{2^{1/3}\ln\left(3(1-x^3)^{1/3}-\frac{3\,2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12} - \frac{2^{1/3}\ln\left(\frac{3\,2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}+3(1-x^3)^{1/3}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

```
[In] int(x^8/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] (2^(1/3)*log((1 - x^3)^(1/3)/2 - 2^(1/3)/2))/6 + (1 - x^3)^(4/3)/4 + (2^(1/
3)*log(3*(1 - x^3)^(1/3) - (3*2^(1/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1)
)/12 - (2^(1/3)*log((3*2^(1/3)*(3^(1/2)*1i + 1))/2 + 3*(1 - x^3)^(1/3))*(3^
(1/2)*1i + 1))/12
```

3.625 $\int \frac{x^5}{(1-x^3)^{2/3}(1+x^3)} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 95

$$\int \frac{x^5}{(1-x^3)^{2/3}(1+x^3)} \, dx = -\sqrt[3]{1-x^3}+\frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log(1+x^3)}{6\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

```
[Out] -(-x^3+1)^(1/3)+1/12*ln(x^3+1)*2^(1/3)-1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(1/
3)+1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {457, 81, 59, 631, 210, 31}

$$\int \frac{x^5}{(1-x^3)^{2/3}(1+x^3)} \, dx = \frac{\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \sqrt[3]{1-x^3}+\frac{\log(x^3+1)}{6\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

```
[In] Int[x^5/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(1/3) + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*S
qrt[3]) + Log[1 + x^3]/(6*2^(2/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2
/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)$$

$$= -\sqrt[3]{1-x^3} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)$$

$$= -\sqrt[3]{1-x^3} + \frac{\log(1+x^3)}{6\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

$$= -\sqrt[3]{1-x^3} + \frac{\log(1+x^3)}{6\ 2^{2/3}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{2^{2/3}}$$

$$= -\sqrt[3]{1-x^3} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log(1+x^3)}{6\ 2^{2/3}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.33

$$\int \frac{x^5}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{12}\left(-12\sqrt[3]{1-x^3} + 2\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) - 2\sqrt[3]{2}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) + \sqrt[3]{2}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(1-x^3)^{2/\cdots}\right.\right.$$

```
[In] Integrate[x^5/((1 - x^3)^(2/3)*(1 + x^3)),x]
[Out] (-12*(1 - x^3)^(1/3) + 2*2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3
))/Sqrt[3]] - 2*2^(1/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] + 2^(1/3)*Log[2 +
 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/12
```

Maple [A] (verified)

Time = 8.61 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$-(-x^3+1)^{\frac{1}{3}} - \frac{2^{\frac{1}{3}}\ln\left((-x^3+1)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)}{6} + \frac{2^{\frac{1}{3}}\ln\left((-x^3+1)^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)}{12} + \frac{\arctan\left(\frac{\left(1+2^{\frac{2}{3}}(-x^3+1)^{\frac{1}{3}}\right)\cdots}{3}\right.}{6}$
trager	Expression too large to display
risch	Expression too large to display

```
[In] int(x^5/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -(-x^3+1)^(1/3)-1/6*2^(1/3)*ln((-x^3+1)^(1/3)-2^(1/3))+1/12*2^(1/3)*ln((-x^
3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))+1/6*arctan(1/3*(1+2^(2/3)*(-x^3+
1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.37

$$\int \frac{x^5}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}(-1)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+2\cdot 4^{\frac{1}{3}}(-1)^{\frac{2}{3}}+2\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}(-1)^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)-\left(-x^3+1\right)^{\frac{1}{3}}$$

```
[In] integrate(x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-1)^(2
/3)*(-x^3 + 1)^(1/3) + 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*(-1)^(1/3)*log(-4^(
2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3) + 2*4^(1/3)*(-1)^(2/3) + 2*(-x^3 + 1)^(2/3
)) + 1/12*4^(2/3)*(-1)^(1/3)*log(4^(2/3)*(-1)^(1/3) + 2*(-x^3 + 1)^(1/3)) -
 (-x^3 + 1)^(1/3)
```

Sympy [F]

$$\int \frac{x^5}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^5}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

```
[In] integrate(x**5/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x**5/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.02

$$\int \frac{x^5}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)-\left(-x^3+1\right)^{\frac{1}{3}}$$

```
[In] integrate(x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) + 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) - 1/6*2^(1/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) - (-x^3 + 1)^(1/3)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.03

$$\int \frac{x^5}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)-\left(-x^3+1\right)^{\frac{1}{3}}$$

```
[In] integrate(x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) + 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) - 1/6*2^(1/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - (-x^3 + 1)^(1/3)
```

Mupad [B] (verification not implemented)

Time = 8.42 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.19

$$\int \frac{x^5}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=-\frac{2^{1/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{2}-\frac{2^{1/3}}{2}\right)}{6}-\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\ln\left(3\left(1-x^3\right)^{1/3}-\frac{3\,2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12}+\frac{2^{1/3}\ln\left(\frac{3\,2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}+3\left(1-x^3\right)^{1/3}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

```
[In] int(x^5/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] (2^(1/3)*log((3*2^(1/3)*(3^(1/2)*1i + 1))/2 + 3*(1 - x^3)^(1/3))*(3^(1/2)*1
i + 1))/12 - (1 - x^3)^(1/3) - (2^(1/3)*log(3*(1 - x^3)^(1/3) - (3*2^(1/3)*
(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/12 - (2^(1/3)*log((1 - x^3)^(1/3)/2
- 2^(1/3)/2))/6
```

3.626 $\int \frac{x^2}{(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{x^2}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log(1+x^3)}{6\ 2^{2/3}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

```
[Out] -1/12*ln(x^3+1)*2^(1/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(1/3)-1/6*arctan(1
/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {455, 59, 631, 210, 31}

$$\int \frac{x^2}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log(x^3+1)}{6\ 2^{2/3}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

```
[In] Int[x^2/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3])) - Log[1
+ x^3]/(6*2^(2/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)\\
&= -\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\\
&= -\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{2^{2/3}}\\
&= -\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.23

$$\int \frac{x^2}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)-2\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)+\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(1-x^3)^{2/3}\right)}{6\ 2^{2/3}}$$

```
[In] Integrate[x^2/((1 - x^3)^(2/3)*(1 + x^3)),x]
[Out] -1/6*(2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 2*Log[-2 +
2^(2/3)*(1 - x^3)^(1/3)] + Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x
^3)^(2/3)])/2^(2/3)
```

Maple [A] (verified)

Time = 4.62 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$-\frac{2^{\frac{1}{3}}\left(2\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}+\ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)-2\ln\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)\right)}{12}$
trager	$\text{RootOf}\left(\_Z^{3}-2\right)\ln\left(\frac{-6\,\text{RootOf}\left(\text{RootOf}\left(\_Z^{3}-2\right)^{2}+6\_Z\,\text{RootOf}\left(\_Z^{3}-2\right)+36\_Z^{2}\right)\text{RootOf}\left(\_Z^{3}-2\right)^{4}x^{3}-180\text{RootOf}\left(\ldots\right.}{\ldots}\right.$

```
[In] int(x^2/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
[Out] -1/12*2^(1/3)*(2*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+ln(
(-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))-2*ln((-x^3+1)^(1/3)-2^(1/3))
)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.18

$$\int \frac{x^2}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}(-x^3+1)^{\frac{1}{3}}+4^{\frac{1}{3}}\sqrt{3}\right)\right) - \frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}(-x^3+1)^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{2}{3}}+2\cdot 4^{\frac{1}{3}}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{2}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)$$

```
[In] integrate(x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-x^3 + 1)^(1/3) +
 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*log(4^(2/3)*(-x^3 + 1)^(1/3) + 2*(-x^3 +
1)^(2/3) + 2*4^(1/3)) + 1/12*4^(2/3)*log(-4^(2/3) + 2*(-x^3 + 1)^(1/3))
```

Sympy [F]

$$\int \frac{x^2}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^2}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

```
[In] integrate(x**2/(-x**3+1)**(2/3)/(x**3+1),x)
[Out] Integral(x**2/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)$$

```
[In] integrate(x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
[Out] -1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) + 1/6*2^(1/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)$$

```
[In] integrate(x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) + 1/6*2^(1/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3)))
```

Mupad [B] (verification not implemented)

Time = 8.56 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.23

$$\int \frac{x^2}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{2^{1/3}\ln\left(3\,2^{1/3}-3\left(1-x^3\right)^{1/3}\right)}{6}+\frac{2^{1/3}\ln\left(3\left(1-x^3\right)^{1/3}-\frac{3\,2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12}-\frac{2^{1/3}\ln\left(\frac{3\,2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}+3\left(1-x^3\right)^{1/3}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

```
[In] int(x^2/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] (2^(1/3)*log(3*2^(1/3) - 3*(1 - x^3)^(1/3)))/6 + (2^(1/3)*log(3*(1 - x^3)^(
1/3) - (3*2^(1/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/12 - (2^(1/3)*log(
(3*2^(1/3)*(3^(1/2)*1i + 1))/2 + 3*(1 - x^3)^(1/3))*(3^(1/2)*1i + 1))/12
```

3.627 $\int \frac{1}{x(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 137

$$\int \frac{1}{x\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = -\frac{\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} -\frac{\log(x)}{2}+\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

```
[Out] -1/2*ln(x)+1/12*ln(x^3+1)*2^(1/3)+1/2*ln(1-(-x^3+1)^(1/3))-1/4*ln(2^(1/3)-(
-x^3+1)^(1/3))*2^(1/3)-1/3*arctan(1/3*(1+2*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)
+1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {457, 88, 59, 632, 210, 31, 631}

$$\int \frac{1}{x\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = -\frac{\arctan\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} +\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}-\frac{\log(x)}{2}$$

```
[In] Int[1/(x*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 + 2^(2/3)*(1
 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) - Log[x]/2 + Log[1 + x^3]/(6*2^(2
```

```
/3)) + Log[1 - (1 - x^3)^(1/3)]/2 - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2/
3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}x(1+x)}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}x}\,dx, x, x^3\right) - \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)$$
$$= -\frac{\log(x)}{2} + \frac{\log(1+x^3)}{6\ 2^{2/3}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right) + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$
$$= -\frac{\log(x)}{2} + \frac{\log(1+x^3)}{6\ 2^{2/3}} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{2^{2/3}} + \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^3}\right)$$
$$= -\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log(x)}{2} + \frac{\log(1+x^3)}{6\ 2^{2/3}} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.35

$$\int \frac{1}{x\left(1-x^3\right)^{2/3}(1+x^3)}\,dx = \frac{1}{12}\left(-4\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right) + 2\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) + 4\log\left(-1+\sqrt[3]{1-x^3}\right) - 2\sqrt[3]{2}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) - 2\log\left(1+\sqrt[3]{1-\right.\right.$$

```
[In] Integrate[1/(x*(1 - x^3)^(2/3)*(1 + x^3)),x]
[Out] (-4*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] + 2*2^(1/3)*Sqrt[3]*Arc
Tan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 4*Log[-1 + (1 - x^3)^(1/3)] -
```

```
2*2^(1/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] - 2*Log[1 + (1 - x^3)^(1/3) + (
1 - x^3)^(2/3)] + 2^(1/3)*Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^
3)^(2/3)])/12
```

Maple [A] (verified)

Time = 4.51 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.06

method	result
pseudoelliptic	$-\frac{\ln\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)}{6}-\frac{\arctan\left(\frac{\left(1+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln\left(-1+\left(-x^3+1\right)^{\frac{1}{3}}\right)}{3}-\frac{2^{\frac{1}{3}}\ln\left(\left(-x^3+1\right)^{\frac{1}{3}}-\right.}{6}$

```
[In] int(1/x/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*ln((-x^3+1)^(2/3)+(-x^3+1)^(1/3)+1)-1/3*arctan(1/3*(1+2*(-x^3+1)^(1/3)
)*3^(1/2))*3^(1/2)+1/3*ln(-1+(-x^3+1)^(1/3))-1/6*2^(1/3)*ln((-x^3+1)^(1/3)-
2^(1/3))+1/12*2^(1/3)*ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))+1/6
*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.33

$$\int \frac{1}{x\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=-\frac{1}{6}$$
$$\cdot 4^{\frac{1}{6}}\sqrt{3}(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}(-1)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{24}$$
$$\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+2\cdot 4^{\frac{1}{3}}(-1)^{\frac{2}{3}}+2\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{12}$$
$$\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}(-1)^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)$$
$$-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)$$

```
[In] integrate(1/x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-1)^(2
/3)*(-x^3 + 1)^(1/3) + 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*(-1)^(1/3)*log(-4^(
2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3) + 2*4^(1/3)*(-1)^(2/3) + 2*(-x^3 + 1)^(2/3
)) + 1/12*4^(2/3)*(-1)^(1/3)*log(4^(2/3)*(-1)^(1/3) + 2*(-x^3 + 1)^(1/3)) -
 1/3*sqrt(3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) - 1/6*log((
-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)^(1/3) - 1)
```

Sympy [F]

$$\int \frac{1}{x(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{1}{x\left(-(x-1)(x^2+x+1)\right)^{\frac{2}{3}}(x+1)(x^2-x+1)}\,dx$$

```
[In] integrate(1/x/(-x**3+1)**(2/3)/(x**3+1),x)
[Out] Integral(1/(x*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}}x}\,dx$$

```
[In] integrate(1/x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.09

$$\begin{aligned}\int \frac{1}{x(1-x^3)^{2/3}(1+x^3)}\,dx &= \frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)\\ &\quad-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{12}\\ &\quad\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)\\ &\quad-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)\end{aligned}$$

```
[In] integrate(1/x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
[Out] 1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + 1/12*2^(1/
3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(1/3)
*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 +
 1)^(1/3) + 1) + 1/3*log(abs((-x^3 + 1)^(1/3) - 1))
```

Mupad [B] (verification not implemented)

Time = 8.49 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.51

$$\int \frac{1}{x(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{\ln\left(5-5(1-x^3)^{1/3}\right)}{3} - \frac{2^{1/3}\ln\left(6(1-x^3)^{1/3}-\frac{2^{1/3}\left(\frac{2^{2/3}\left(243\,2^{1/3}+243\,(1-x^3)^{1/3}\right)}{36}+9\right)}{6}\right)}{6}$$

$$+\ln\left(\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\left(\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2\left(243\,(1-x^3)^{1/3}+243-\sqrt{3}\,243\mathrm{i}\right)+9\right)+6\,(1-x^3)^{1/3}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}}{6}\right.$$

[In] int(1/(x*(1 - x^3)^(2/3)*(x^3 + 1)),x)

[Out] log(5 - 5*(1 - x^3)^(1/3))/3 - (2^(1/3)*log(6*(1 - x^3)^(1/3) - (2^(1/3)*((2^(2/3)*(243*2^(1/3) + 243*(1 - x^3)^(1/3)))/36 + 9))/6))/6 + log(((3^(1/2)*1i)/6 - 1/6)*(((3^(1/2)*1i)/6 - 1/6)^2*(243*(1 - x^3)^(1/3) - 3^(1/2)*243i + 243) + 9) + 6*(1 - x^3)^(1/3))*((3^(1/2)*1i)/6 - 1/6) - log(6*(1 - x^3)^(1/3) - ((3^(1/2)*1i)/6 + 1/6)*(((3^(1/2)*1i)/6 + 1/6)^2*(3^(1/2)*243i + 243*(1 - x^3)^(1/3) + 243) + 9))*((3^(1/2)*1i)/6 + 1/6) + ((-1)^(1/3)*2^(1/3)*log(6*(1 - x^3)^(1/3) - ((-1)^(1/3)*2^(1/3)*(((-1)^(2/3)*2^(2/3)*(243*(-1)^(1/3)*2^(1/3) - 243*(1 - x^3)^(1/3)))/36 - 9))/6))/6 - ((-1)^(1/3)*2^(1/3)*log(6*(1 - x^3)^(1/3) - ((-1)^(1/3)*2^(1/3)*(3^(1/2)*1i + 1)*(((-1)^(2/3)*2^(2/3)*(3^(1/2)*1i + 1)^2*(243*(1 - x^3)^(1/3) + (243*(-1)^(1/3)*2^(1/3)*(3^(1/2)*1i + 1))/2))/144 + 9))/12)*(3^(1/2)*1i + 1))/12

3.628 $\int \frac{1}{x^4(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 158

$$\int \frac{1}{x^4(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{\sqrt[3]{1-x^3}}{3x^3}+\frac{\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log(x)}{6}-\frac{\log(1+x^3)}{6\ 2^{2/3}} - \frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

```
[Out] -1/3*(-x^3+1)^(1/3)/x^3+1/6*ln(x)-1/12*ln(x^3+1)*2^(1/3)-1/6*ln(1-(-x^3+1)^
(1/3))+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(1/3)+1/9*arctan(1/3*(1+2*(-x^3+1)^
(1/3))*3^(1/2))*3^(1/2)-1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*
2^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {457, 105, 162, 59, 632, 210, 31, 631}

$$\int \frac{1}{x^4(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{\arctan\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\sqrt[3]{1-x^3}}{3x^3}-\frac{\log(x^3+1)}{6\ 2^{2/3}}-\frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}+\frac{\log(x)}{6}$$

```
[In] Int[1/(x^4*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -1/3*(1 - x^3)^(1/3)/x^3 + ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/(3*Sqrt[
3]) - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + Log
[x]/6 - Log[1 + x^3]/(6*2^(2/3)) - Log[1 - (1 - x^3)^(1/3)]/6 + Log[2^(1/3)
 - (1 - x^3)^(1/3)]/(2*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}x^2(1+x)}\,dx, x, x^3\right)\\
&= -\frac{\sqrt[3]{1-x^3}}{3x^3} - \frac{1}{3}\text{Subst}\left(\int \frac{\frac{1}{3}-\frac{2x}{3}}{(1-x)^{2/3}x(1+x)}\,dx, x, x^3\right)\\
&= -\frac{\sqrt[3]{1-x^3}}{3x^3} - \frac{1}{9}\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}x}\,dx, x, x^3\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)\\
&= -\frac{\sqrt[3]{1-x^3}}{3x^3} + \frac{\log(x)}{6} - \frac{\log(1+x^3)}{6\ 2^{2/3}}\\
&\quad + \frac{1}{6}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right) + \frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right)\\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\\
&= -\frac{\sqrt[3]{1-x^3}}{3x^3} + \frac{\log(x)}{6} - \frac{\log(1+x^3)}{6\ 2^{2/3}} - \frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right) + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}\\
&\quad - \frac{1}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^3}\right) + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{2^{2/3}}\\
&= -\frac{\sqrt[3]{1-x^3}}{3x^3} + \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}\\
&\quad + \frac{\log(x)}{6} - \frac{\log(1+x^3)}{6\ 2^{2/3}} - \frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right) + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^4 (1-x^3)^{2/3} (1+x^3)} \, dx = \frac{1}{36}\left(-\frac{12\sqrt[3]{1-x^3}}{x^3} + 4\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right) - 6\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) - 4\log\left(-1+\sqrt[3]{1-x^3}\right) + 6\sqrt[3]{2}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) + 2\log\left(1+\sqrt[3]{1-\right.\right.$$

```
[In] Integrate[1/(x^4*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] ((-12*(1 - x^3)^(1/3))/x^3 + 4*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[
3]] - 6*2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 4*L
og[-1 + (1 - x^3)^(1/3)] + 6*2^(1/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] + 2*
Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)] - 3*2^(1/3)*Log[2 + 2^(2/3)*(1 -
 x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/36
```

Maple [A] (verified)

Time = 7.28 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.34

method	result
pseudoelliptic	$\frac{6\,2^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)x^{3}-6\,2^{\frac{1}{3}}\ln\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)x^{3}+3\,2^{\frac{1}{3}}\ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)x^{3}-4}{36\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}+\right.}$

```
[In] int(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/36*(6*2^(1/3)*3^(1/2)*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*x^3-
6*2^(1/3)*ln((-x^3+1)^(1/3)-2^(1/3))*x^3+3*2^(1/3)*ln((-x^3+1)^(2/3)+2^(1/3
)*(-x^3+1)^(1/3)+2^(2/3))*x^3-4*3^(1/2)*arctan(1/3*(1+2*(-x^3+1)^(1/3))*3^(
1/2))*x^3-2*ln((-x^3+1)^(2/3)+(-x^3+1)^(1/3)+1)*x^3+4*ln(-1+(-x^3+1)^(1/3))
*x^3+12*(-x^3+1)^(1/3))/((-x^3+1)^(2/3)+(-x^3+1)^(1/3)+1)/(-1+(-x^3+1)^(1/3
))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^4 (1-x^3)^{2/3} (1+x^3)} dx =$$
$$-\frac{12 \cdot 4^{\frac{1}{6}} \sqrt{3} x^3 \arctan\left(\frac{1}{6} \cdot 4^{\frac{1}{6}} \left(4^{\frac{2}{3}} \sqrt{3}(-x^3+1)^{\frac{1}{3}} + 4^{\frac{1}{3}} \sqrt{3}\right)\right) + 3 \cdot 4^{\frac{2}{3}} x^3 \log\left(4^{\frac{2}{3}}(-x^3+1)^{\frac{1}{3}} + 2(-x^3+1)^{\frac{2}{3}} + \cdots}{\cdots}$$

```
[In] integrate(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/72*(12*4^(1/6)*sqrt(3)*x^3*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-x^3 + 1
)^(1/3) + 4^(1/3)*sqrt(3))) + 3*4^(2/3)*x^3*log(4^(2/3)*(-x^3 + 1)^(1/3) +
2*(-x^3 + 1)^(2/3) + 2*4^(1/3)) - 6*4^(2/3)*x^3*log(-4^(2/3) + 2*(-x^3 + 1)
^(1/3)) - 8*sqrt(3)*x^3*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3))
- 4*x^3*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 8*x^3*log((-x^3 + 1)
^(1/3) - 1) + 24*(-x^3 + 1)^(1/3))/x^3
```

Sympy [F]

$$\int \frac{1}{x^4 (1-x^3)^{2/3} (1+x^3)} dx = \int \frac{1}{x^4 (-(x-1)(x^2+x+1))^{\frac{2}{3}} (x+1)(x^2-x+1)} dx$$

```
[In] integrate(1/x**4/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**4*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Maxima [F]

$$\int \frac{1}{x^4 (1-x^3)^{2/3} (1+x^3)} dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}} x^4} dx$$

```
[In] integrate(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^4 (1-x^3)^{2/3} (1+x^3)} dx = -\frac{1}{6}\sqrt{3}2^{\frac{1}{3}} \arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) + \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right) - \frac{1}{12} \cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right) - \frac{\left(-x^3+1\right)^{\frac{1}{3}}}{3x^3} + \frac{1}{18}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right) - \frac{1}{9}\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

```
[In] integrate(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/12*2^(1
/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(1/3
)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - 1/3*(-x^3 + 1)^(1/3)/x^3 + 1/18*l
og((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) - 1/9*log(abs((-x^3 + 1)^(1/3)
- 1))
```

Mupad [B] (verification not implemented)

Time = 8.56 (sec) , antiderivative size = 368, normalized size of antiderivative = 2.33

$$\int \frac{1}{x^4 (1-x^3)^{2/3} (1+x^3)} dx = \frac{2^{1/3}\ln\left(\frac{10\,(1-x^3)^{1/3}}{9}-\frac{2^{1/3}\left(\frac{2^{2/3}\left(243\,2^{1/3}+27\,(1-x^3)^{1/3}\right)}{36}-\frac{25}{3}\right)}{6}\right)}{6} - \frac{(1-x^3)^{1/3}}{3\,x^3} - \frac{\ln\left(\frac{31\,(1-x^3)^{1/3}}{243}-\frac{31}{243}\right)}{9}$$

$$-\ln\left(\left(-\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)\left(\left(-\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)^2\left(27\,(1-x^3)^{1/3}+81-\sqrt{3}\,81\mathrm{i}\right)-\frac{25}{3}\right)+\frac{10\,(1-x^3)^{1/3}}{9}\right)\left(-\frac{1}{18}+\right.$$

```
[In] int(1/(x^4*(1 - x^3)^(2/3)*(x^3 + 1)),x)

[Out] (2^(1/3)*log((10*(1 - x^3)^(1/3))/9 - (2^(1/3)*((2^(2/3)*(243*2^(1/3) + 27*
(1 - x^3)^(1/3)))/36 - 25/3))/6))/6 - (1 - x^3)^(1/3)/(3*x^3) - log((31*(1
- x^3)^(1/3))/243 - 31/243)/9 - log((((3^(1/2)*1i)/18 - 1/18)*(((3^(1/2)*1i)
/18 - 1/18)^2*(27*(1 - x^3)^(1/3) - 3^(1/2)*81i + 81) - 25/3) + (10*(1 - x^
3)^(1/3))/9)*((3^(1/2)*1i)/18 - 1/18) + log((10*(1 - x^3)^(1/3))/9 - ((3^(1
/2)*1i)/18 + 1/18)*(((3^(1/2)*1i)/18 + 1/18)^2*(3^(1/2)*81i + 27*(1 - x^3)^
(1/3) + 81) - 25/3))*((3^(1/2)*1i)/18 + 1/18) + (2^(1/3)*log((10*(1 - x^3)^
(1/3))/9 - (2^(1/3)*(3^(1/2)*1i - 1)*((2^(2/3)*(3^(1/2)*1i - 1)^2*((243*2^(
1/3)*(3^(1/2)*1i - 1))/2 + 27*(1 - x^3)^(1/3)))/144 - 25/3))/12)*(3^(1/2)*1
i - 1))/12 - (2^(1/3)*log((10*(1 - x^3)^(1/3))/9 - (2^(1/3)*(3^(1/2)*1i + 1
)*((2^(2/3)*(3^(1/2)*1i + 1)^2*((243*2^(1/3)*(3^(1/2)*1i + 1))/2 - 27*(1 -
x^3)^(1/3)))/144 + 25/3))/12)*(3^(1/2)*1i + 1))/12
```

3.629 $\int \frac{x^7}{(1-x^3)^{2/3}(1+x^3)} dx$

Optimal result

Integrand size = 22, antiderivative size = 160

$$\int \frac{x^7}{(1-x^3)^{2/3}(1+x^3)} dx = -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log(1+x^3)}{6\ 2^{2/3}} + \frac{1}{6}\log\left(-x-\sqrt[3]{1-x^3}\right) - \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

```
[Out] -1/3*x^2*(-x^3+1)^(1/3)+1/12*ln(x^3+1)*2^(1/3)+1/6*ln(-x-(-x^3+1)^(1/3))-1/
4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(1/3)+1/9*arctan(1/3*(1-2*x/(-x^3+1)^(1/3
))*3^(1/2))*3^(1/2)-1/6*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*
2^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used

$= \{490, 598, 337, 503\}$

$$\int \frac{x^7}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log(x^3+1)}{6\ 2^{2/3}} + \frac{1}{6}\log\left(-\sqrt[3]{1-x^3}-x\right) - \frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\ 2^{2/3}} - \frac{1}{3}\sqrt[3]{1-x^3}x^2$$

```
[In] Int[x^7/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -1/3*(x^2*(1 - x^3)^(1/3)) + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(3
*Sqrt[3]) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sq
rt[3]) + Log[1 + x^3]/(6*2^(2/3)) + Log[-x - (1 - x^3)^(1/3)]/6 - Log[-(2^(
1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(2/3))
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{3}x^2\sqrt[3]{1-x^3}+\frac{1}{3}\int \frac{x(2-x^3)}{(1-x^3)^{2/3}(1+x^3)}\,dx \\
&= -\frac{1}{3}x^2\sqrt[3]{1-x^3}+\frac{1}{3}\int \left(-\frac{x}{(1-x^3)^{2/3}}+\frac{3x}{(1-x^3)^{2/3}(1+x^3)}\right)dx \\
&= -\frac{1}{3}x^2\sqrt[3]{1-x^3}-\frac{1}{3}\int \frac{x}{(1-x^3)^{2/3}}\,dx+\int \frac{x}{(1-x^3)^{2/3}(1+x^3)}\,dx \\
&= -\frac{1}{3}x^2\sqrt[3]{1-x^3}+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} \\
&\quad+\frac{\log(1+x^3)}{6\,2^{2/3}}+\frac{1}{6}\log\left(-x-\sqrt[3]{1-x^3}\right)-\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\,2^{2/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.39

$$\int \frac{x^7}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{36}\left(-12x^2\sqrt[3]{1-x^3}+4\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2\sqrt[3]{1-x^3}}\right)\right.$$
$$-6\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)+4\log\left(x+\sqrt[3]{1-x^3}\right)-6\sqrt[3]{2}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)-2\log\left(x^2-x\sqrt[3]{1-\right.$$

```
[In] Integrate[x^7/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (-12*x^2*(1 - x^3)^(1/3) + 4*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2*(1 - x^3)^(1
/3))] - 6*2^(1/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))]
 + 4*Log[x + (1 - x^3)^(1/3)] - 6*2^(1/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)
] - 2*Log[x^2 - x*(1 - x^3)^(1/3) + (1 - x^3)^(2/3)] + 3*2^(1/3)*Log[-2*x^2
 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/36
```

Maple [A] (verified)

Time = 6.54 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.46

method	result
pseudoelliptic	$\frac{3\,2^{\frac{1}{3}} \ln\left(\frac{2^{\frac{2}{3}} x^{2}-2^{\frac{1}{3}} \left(-x^{3}+1\right)^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)-2 \ln\left(\frac{\left(-x^{3}+1\right)^{\frac{2}{3}}-\left(-x^{3}+1\right)^{\frac{1}{3}} x+x^{2}}{x^{2}}\right)-6\,2^{\frac{1}{3}} \ln\left(\frac{2^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)+4 \ln\left(\frac{x+}{}\right.}{36\left(x+\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\left(\left(-x^{3}+1\right)\right.}$

[In] int(x^7/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)

[Out] 1/36*(3*2^(1/3)*ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))/x^2)-2*ln(((-x^3+1)^(2/3)-(-x^3+1)^(1/3)*x+x^2)/x^2)-6*2^(1/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)+4*ln((x+(-x^3+1)^(1/3))/x)-12*x^2*(-x^3+1)^(1/3)+(6*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)*2^(1/3)-4*arctan(1/3*(-2*(-x^3+1)^(1/3)+x)*3^(1/2)/x))*3^(1/2))/(x+(-x^3+1)^(1/3))/((-x^3+1)^(2/3)+x*(x-(-x^3+1)^(1/3)))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.45

$$\begin{aligned}\int \frac{x^7}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx &= -\frac{1}{3}\left(-x^{3}+1\right)^{\frac{1}{3}} x^{2}+\frac{1}{6} \\ &\cdot 4^{\frac{1}{6}} \sqrt{3}(-1)^{\frac{1}{3}} \arctan\left(\frac{4^{\frac{1}{6}}\left(4^{\frac{2}{3}} \sqrt{3}(-1)^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}-4^{\frac{1}{3}} \sqrt{3}x\right)}{6\,x}\right) \\ &+\frac{1}{12}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}} \log\left(-\frac{4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,x-2\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{24} \\ &\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}} \log\left(\frac{2\cdot 4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\,x^{2}+4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}} x+2\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right) \\ &+\frac{1}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}\left(-x^{3}+1\right)^{\frac{1}{3}}}{3\,x}\right)+\frac{1}{9}\log\left(\frac{x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right) \\ &-\frac{1}{18}\log\left(\frac{x^{2}-\left(-x^{3}+1\right)^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)\end{aligned}$$

[In] integrate(x^7/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")

[Out] -1/3*(-x^3 + 1)^(1/3)*x^2 + 1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-1)^(2/3)*(-x^3 + 1)^(1/3) - 4^(1/3)*sqrt(3)*x)/x) + 1

```
/12*4^(2/3)*(-1)^(1/3)*log(-(4^(2/3)*(-1)^(1/3)*x - 2*(-x^3 + 1)^(1/3))/x)
- 1/24*4^(2/3)*(-1)^(1/3)*log((2*4^(1/3)*(-1)^(2/3)*x^2 + 4^(2/3)*(-1)^(1/3
)*(-x^3 + 1)^(1/3)*x + 2*(-x^3 + 1)^(2/3))/x^2) + 1/9*sqrt(3)*arctan(-1/3*(
sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) + 1/9*log((x + (-x^3 + 1)^(1/3))
/x) - 1/18*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

Sympy [F]

$$\int \frac{x^7}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^7}{\left(-(x-1)(x^2+x+1)\right)^{\frac{2}{3}}(x+1)(x^2-x+1)}\,dx$$

```
[In] integrate(x**7/(-x**3+1)**(2/3)/(x**3+1),x)
[Out] Integral(x**7/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{x^7}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^7}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(x^7/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Giac [F]

$$\int \frac{x^7}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^7}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(x^7/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^7}{(1-x^3)^{2/3}\,(x^3+1)}\,dx$$

```
[In] int(x^7/((1 - x^3)^(2/3)*(x^3 + 1)),x)
[Out] int(x^7/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

3.630 $\int \frac{x^4}{(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 139

$$\int \frac{x^4}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(1+x^3)}{6\ 2^{2/3}}-\frac{1}{2}\log\left(-x-\sqrt[3]{1-x^3}\right)+\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

```
[Out] -1/12*ln(x^3+1)*2^(1/3)-1/2*ln(-x-(-x^3+1)^(1/3))+1/4*ln(-2^(1/3)*x-(-x^3+1
)^(1/3))*2^(1/3)-1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+1/6
*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {494, 337, 503}

$$\int \frac{x^4}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(x^3+1)}{6\ 2^{2/3}}-\frac{1}{2}\log\left(-\sqrt[3]{1-x^3}-x\right)+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\ 2^{2/3}}$$

```
[In] Int[x^4/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 - (2*2^(
1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) - Log[1 + x^3]/(6*2^(2/
3)) - Log[-x - (1 - x^3)^(1/3)]/2 + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*
2^(2/3))
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x}{(1-x^3)^{2/3}}\,dx - \int \frac{x}{(1-x^3)^{2/3}(1+x^3)}\,dx\\ &= -\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log(1+x^3)}{6\ 2^{2/3}}\\ &\quad -\frac{1}{2}\log\left(-x-\sqrt[3]{1-x^3}\right) + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.48

$$\int \frac{x^4}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{12}\left(-4\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2\sqrt[3]{1-x^3}}\right)\right.$$
$$+2\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-4\log\left(x+\sqrt[3]{1-x^3}\right)+2\sqrt[3]{2}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+2\log\left(x^2-x\sqrt[3]{1-\right.$$

```
[In] Integrate[x^4/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (-4*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2*(1 - x^3)^(1/3))] + 2*2^(1/3)*Sqrt[3]
*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] - 4*Log[x + (1 - x^3)^(1
/3)] + 2*2^(1/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)] + 2*Log[x^2 - x*(1 - x^
3)^(1/3) + (1 - x^3)^(2/3)] - 2^(1/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3
) - 2^(1/3)*(1 - x^3)^(2/3)])/12
```

Maple [A] (verified)

Time = 4.67 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.29

method	result
pseudoelliptic	$-\frac{2^{\frac{1}{3}}\ln\left(\frac{2^{\frac{2}{3}}x^2-2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}\right)}{12}+\frac{\ln\left(\frac{\left(-x^3+1\right)^{\frac{2}{3}}-\left(-x^3+1\right)^{\frac{1}{3}}x+x^2}{x^2}\right)}{6}+\frac{2^{\frac{1}{3}}\ln\left(\frac{2^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)}{6}-\frac{\ln\left(\right.}{}$

```
[In] int(x^4/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*2^(1/3)*ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))/x^2)
+1/6*ln(((-x^3+1)^(2/3)-(-x^3+1)^(1/3)*x+x^2)/x^2)+1/6*2^(1/3)*ln((2^(1/3)*
x+(-x^3+1)^(1/3))/x)-1/3*ln((x+(-x^3+1)^(1/3))/x)+1/6*(-arctan(1/3*3^(1/2)*
(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)*2^(1/3)+2*arctan(1/3*(-2*(-x^3+1)^(1/3)+x)*3
^(1/2)/x))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.42

$$\int \frac{x^4}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(-\frac{4^{\frac{1}{6}}\left(4^{\frac{1}{3}}\sqrt{3}x-4^{\frac{2}{3}}\sqrt{3}(-x^3+1)^{\frac{1}{3}}\right)}{6\,x}\right) + \frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(\frac{4^{\frac{2}{3}}x+2\,(-x^3+1)^{\frac{1}{3}}}{x}\right) - \frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(\frac{2\cdot 4^{\frac{1}{3}}x^2-4^{\frac{2}{3}}(-x^3+1)^{\frac{1}{3}}x+2\,(-x^3+1)^{\frac{2}{3}}}{x^2}\right) - \frac{1}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}x-2\sqrt{3}(-x^3+1)^{\frac{1}{3}}}{3\,x}\right) - \frac{1}{3}\log\left(\frac{x+(-x^3+1)^{\frac{1}{3}}}{x}\right) + \frac{1}{6}\log\left(\frac{x^2-(-x^3+1)^{\frac{1}{3}}x+(-x^3+1)^{\frac{2}{3}}}{x^2}\right)$$

```
[In] integrate(x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/6*4^(1/6)*sqrt(3)*arctan(-1/6*4^(1/6)*(4^(1/3)*sqrt(3)*x - 4^(2/3)*sqrt(3
)*(-x^3 + 1)^(1/3))/x) + 1/12*4^(2/3)*log((4^(2/3)*x + 2*(-x^3 + 1)^(1/3))/
x) - 1/24*4^(2/3)*log((2*4^(1/3)*x^2 - 4^(2/3)*(-x^3 + 1)^(1/3)*x + 2*(-x^3
 + 1)^(2/3))/x^2) - 1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 +
1)^(1/3))/x) - 1/3*log((x + (-x^3 + 1)^(1/3))/x) + 1/6*log((x^2 - (-x^3 + 1
)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

Sympy [F]

$$\int \frac{x^4}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^4}{\left(-(x-1)(x^2+x+1)\right)^{\frac{2}{3}}(x+1)(x^2-x+1)}\,dx$$

```
[In] integrate(x**4/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x**4/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{x^4}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^4}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Giac [F]

$$\int \frac{x^4}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^4}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^4}{(1-x^3)^{2/3}\,(x^3+1)}\,dx$$

```
[In] int(x^4/((1 - x^3)^(2/3)*(x^3 + 1)),x)
[Out] int(x^4/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

3.631 $\int \frac{x}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 88

$$\int \frac{x}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)} \, dx=-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(1+x^3\right)}{6\ 2^{2/3}}-\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

```
[Out] 1/12*ln(x^3+1)*2^(1/3)-1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(1/3)-1/6*arctan
(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {503}

$$\int \frac{x}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)} \, dx=-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\ 2^{2/3}}$$

```
[In] Int[x/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3])) +
Log[1 + x^3]/(6*2^(2/3)) - Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(2/3))
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
```

```
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rubi steps

$$\text{integral} = -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log(1+x^3)}{6\ 2^{2/3}} - \frac{\log\left(-\sqrt[3]{2}x - \sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.30

$$\int \frac{x}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{-2\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right) - 2\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right) + \log\left(-2x^2+2^{2/3}\right.}{6\ 2^{2/3}}$$

```
[In] Integrate[x/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] - 2*Log[2*x +
 2^(2/3)*(1 - x^3)^(1/3)] + Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3
)*(1 - x^3)^(2/3)])/(6*2^(2/3))
```

Maple [A] (verified)

Time = 4.89 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.09

method	result	size
pseudoelliptic	$\frac{2^{\frac{1}{3}}\left(2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)-2\ln\left(\frac{2^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)+\ln\left(\frac{2^{\frac{2}{3}}x^{2}-2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)\right)}{12}$	96
trager	Expression too large to display	939

```
[In] int(x/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*2^(1/3)*(2*3^(1/2)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)-2
*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)+ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+
(-x^3+1)^(2/3))/x^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(67) = 134$.

Time = 1.47 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.22

$$\int \frac{x}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{1}{18} \cdot 4^{\frac{1}{6}}\sqrt{3}(-1)^{\frac{1}{3}}\arctan\left(-\frac{4^{\frac{1}{6}}\left(6\cdot 4^{\frac{2}{3}}\sqrt{3}(-1)^{\frac{2}{3}}\left(19\,x^8-16\,x^5+x^2\right)(-x^3+1)^{\frac{1}{3}}-12\,\sqrt{3}(-1)^{\frac{1}{3}}\left(5\,x^7+4\,x^4-x\right)\right.}{6\left(109\,x^9-105\,x^6+3\,x^3+1\right)}\right. + \frac{1}{36}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{3\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}x^2-4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^3+1\right)-6\left(-x^3+1\right)^{\frac{2}{3}}x}{x^3+1}\right)-\frac{1}{72} \cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(\frac{6\cdot 4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(5\,x^4-x\right)(-x^3+1)^{\frac{2}{3}}-4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(19\,x^6-16\,x^3+1\right)-24\left(2\,x^5-x^2\right)(-x^3+1)^{\frac{1}{3}}}{x^6+2\,x^3+1}\right.$$

```
[In] integrate(x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/18*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(-1/6*4^(1/6)*(6*4^(2/3)*sqrt(3)*(-1
)^(2/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - 12*sqrt(3)*(-1)^(1/3)*(5
*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 4^(1/3)*sqrt(3)*(71*x^9 - 111*x^6 + 33
*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) + 1/36*4^(2/3)*(-1)^(1/3)*log(-
(3*4^(2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3)*x^2 - 4^(1/3)*(-1)^(2/3)*(x^3 + 1) -
 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) - 1/72*4^(2/3)*(-1)^(1/3)*log((6*4^(1/3)*
(-1)^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 4^(2/3)*(-1)^(1/3)*(19*x^6 - 16*x
^3 + 1) - 24*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1))
```

Sympy [F]

$$\int \frac{x}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

```
[In] integrate(x/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{x}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Giac [F]

$$\int \frac{x}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x}{(1-x^3)^{2/3}(x^3+1)}\,dx$$

```
[In] int(x/((1 - x^3)^(2/3)*(x^3 + 1)),x)
[Out] int(x/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

3.632 $\int \frac{1}{x^2(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int \frac{1}{x^2(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{\sqrt[3]{1-x^3}}{x}+\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log(1+x^3)}{6\ 2^{2/3}}+\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

```
[Out] -(-x^3+1)^(1/3)/x-1/12*ln(x^3+1)*2^(1/3)+1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*
2^(1/3)+1/6*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1
/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {491, 503}

$$\int \frac{1}{x^2(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\sqrt[3]{1-x^3}}{x}-\frac{\log(x^3+1)}{6\ 2^{2/3}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\ 2^{2/3}}$$

```
[In] Int[1/(x^2*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -((1 - x^3)^(1/3)/x) + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/
(2^(2/3)*Sqrt[3]) - Log[1 + x^3]/(6*2^(2/3)) + Log[-(2^(1/3)*x) - (1 - x^3)
^(1/3)]/(2*2^(2/3))
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt[3]{1-x^3}}{x} - \int \frac{x}{(1-x^3)^{2/3}(1+x^3)}\,dx$$

$$= -\frac{\sqrt[3]{1-x^3}}{x} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log(1+x^3)}{6\ 2^{2/3}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^2(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{\sqrt[3]{1-x^3}}{x} - \frac{\arctan\left(\frac{\sqrt{3}x}{-x+2^{2/3}\sqrt[3]{1-x^3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)}{3\ 2^{2/3}} - \frac{\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}(1-x^3)^{2/3}\right)}{6\ 2^{2/3}}$$

```
[In] Integrate[1/(x^2*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -((1 - x^3)^(1/3)/x) - ArcTan[(Sqrt[3]*x)/(-x + 2^(2/3)*(1 - x^3)^(1/3))]/(
2^(2/3)*Sqrt[3]) + Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)]/(3*2^(2/3)) - Log[-2*
x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)]/(6*2^(2/3))
```

Maple [A] (verified)

Time = 22.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.17

method	result
pseudoelliptic	$\frac{-2\,2^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)x+2\,2^{\frac{1}{3}}\ln\left(\frac{2^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)x-2^{\frac{1}{3}}\ln\left(\frac{2^{\frac{2}{3}}x^{2}-2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)}{12x}$
trager	Expression too large to display
risch	Expression too large to display

```
[In] int(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(-2*2^(1/3)*3^(1/2)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)*
x+2*2^(1/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)*x-2^(1/3)*ln((2^(2/3)*x^2-2^(1
/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))/x^2)*x-12*(-x^3+1)^(1/3))/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(81) = 162.

Time = 1.42 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.64

$$\int \frac{1}{x^2\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \frac{4\cdot 4^{\frac{1}{6}}\sqrt{3}x\arctan\left(\frac{4^{\frac{1}{6}}\left(6\cdot 4^{\frac{2}{3}}\sqrt{3}\left(19\,x^{8}-16\,x^{5}+x^{2}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}+12\,\sqrt{3}\left(5\,x^{7}+4\,x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{2}{3}}-}{6\left(109\,x^{9}-105\,x^{6}+3\,x^{3}+1\right)}\right.}{}$$

```
[In] integrate(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/72*(4*4^(1/6)*sqrt(3)*x*arctan(1/6*4^(1/6)*(6*4^(2/3)*sqrt(3)*(19*x^8 - 1
6*x^5 + x^2)*(-x^3 + 1)^(1/3) + 12*sqrt(3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(
2/3) - 4^(1/3)*sqrt(3)*(71*x^9 - 111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6
+ 3*x^3 + 1)) + 2*4^(2/3)*x*log((3*4^(2/3)*(-x^3 + 1)^(1/3)*x^2 + 6*(-x^3 +
 1)^(2/3)*x + 4^(1/3)*(x^3 + 1))/(x^3 + 1)) - 4^(2/3)*x*log((6*4^(1/3)*(5*x
^4 - x)*(-x^3 + 1)^(2/3) + 4^(2/3)*(19*x^6 - 16*x^3 + 1) - 24*(2*x^5 - x^2)
*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) - 72*(-x^3 + 1)^(1/3))/x
```

Sympy [F]

$$\int \frac{1}{x^2 (1-x^3)^{2/3} (1+x^3)} dx = \int \frac{1}{x^2 \left(-(x-1)(x^2+x+1)\right)^{\frac{2}{3}} (x+1)(x^2-x+1)} dx$$

```
[In] integrate(1/x**2/(-x**3+1)**(2/3)/(x**3+1),x)
[Out] Integral(1/(x**2*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Maxima [F]

$$\int \frac{1}{x^2 (1-x^3)^{2/3} (1+x^3)} dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}} x^2} dx$$

```
[In] integrate(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (1-x^3)^{2/3} (1+x^3)} dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}} x^2} dx$$

```
[In] integrate(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (1-x^3)^{2/3} (1+x^3)} dx = \int \frac{1}{x^2 (1-x^3)^{2/3} (x^3+1)} dx$$

```
[In] int(1/(x^2*(1 - x^3)^(2/3)*(x^3 + 1)),x)
[Out] int(1/(x^2*(1 - x^3)^(2/3)*(x^3 + 1)), x)
```

3.633 $\int \frac{1}{x^5(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 124

$$\int \frac{1}{x^5(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{\sqrt[3]{1-x^3}}{4x^4}+\frac{\sqrt[3]{1-x^3}}{4x} -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log(1+x^3)}{6\ 2^{2/3}}-\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

```
[Out] -1/4*(-x^3+1)^(1/3)/x^4+1/4*(-x^3+1)^(1/3)/x+1/12*ln(x^3+1)*2^(1/3)-1/4*ln(
-2^(1/3)*x-(-x^3+1)^(1/3))*2^(1/3)-1/6*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(
1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {491, 597, 12, 503}

$$\int \frac{1}{x^5(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} +\frac{\sqrt[3]{1-x^3}}{4x}+\frac{\log(x^3+1)}{6\ 2^{2/3}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\ 2^{2/3}}-\frac{\sqrt[3]{1-x^3}}{4x^4}$$

```
[In] Int[1/(x^5*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -1/4*(1 - x^3)^(1/3)/x^4 + (1 - x^3)^(1/3)/(4*x) - ArcTan[(1 - (2*2^(1/3)*x
)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(2/3)) -
Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(2/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\sqrt[3]{1-x^3}}{4x^4} + \frac{1}{4}\int \frac{-1+3x^3}{x^2\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx \\ &= -\frac{\sqrt[3]{1-x^3}}{4x^4} + \frac{\sqrt[3]{1-x^3}}{4x} - \frac{1}{4}\int -\frac{4x}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx \end{aligned}$$

$$= -\frac{\sqrt[3]{1-x^3}}{4x^4} + \frac{\sqrt[3]{1-x^3}}{4x} + \int \frac{x}{(1-x^3)^{2/3}(1+x^3)}\,dx$$

$$= -\frac{\sqrt[3]{1-x^3}}{4x^4} + \frac{\sqrt[3]{1-x^3}}{4x} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log(1+x^3)}{6\ 2^{2/3}} - \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^5(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{12}\left(-\frac{3(1-x^3)^{4/3}}{x^4}\right.$$
$$-2\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right) - 2\sqrt[3]{2}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right) + \sqrt[3]{2}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}(1\right.$$

```
[In] Integrate[1/(x^5*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] ((-3*(1 - x^3)^(4/3))/x^4 - 2*2^(1/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/
3)*(1 - x^3)^(1/3))] - 2*2^(1/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)] + 2^(1/
3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/12
```

Maple [A] (verified)

Time = 22.48 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$\frac{-2\,2^{\frac{1}{3}}\ln\left(\frac{2^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)x^4+\left(3x^3-3\right)\left(-x^3+1\right)^{\frac{1}{3}}+2^{\frac{1}{3}}x^4\left(2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)+\ln\left(\frac{2^{\frac{2}{3}}x^2-2^{\frac{1}{3}}\left(-x\right.}{}\right.\right.}{12x^4}$
trager	Expression too large to display
risch	Expression too large to display

```
[In] int(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(-2*2^(1/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)*x^4+(3*x^3-3)*(-x^3+1)^(1
/3)+2^(1/3)*x^4*(2*3^(1/2)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x
)+ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))/x^2)))/x^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 312 vs. $2(95) = 190$.

Time = 1.41 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.52

$$\int \frac{1}{x^5 (1-x^3)^{2/3} (1+x^3)} \, dx = -\frac{4 \cdot 4^{\frac{1}{6}} \sqrt{3} (-1)^{\frac{1}{3}} x^4 \arctan\left(-\frac{4^{\frac{1}{6}} \left(6 \cdot 4^{\frac{2}{3}} \sqrt{3} (-1)^{\frac{2}{3}} (19 x^8 - 16 x^5 + x^2) (-x^3+1)^{\frac{1}{3}} - 12 \sqrt{3} (-1)^{\frac{1}{3}} (5 x^7 + 4 x^4 - x) (-x^3+1)^{\frac{2}{3}} - 4^{\frac{1}{3}} \sqrt{3} (71 x^9 - 1}{6 (109 x^9 - 105 x^6 + 3 x^3 + 1)}\right.}{}$$

```
[In] integrate(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/72*(4*4^(1/6)*sqrt(3)*(-1)^(1/3)*x^4*arctan(-1/6*4^(1/6)*(6*4^(2/3)*sqrt
(3)*(-1)^(2/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - 12*sqrt(3)*(-1)^(
1/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 4^(1/3)*sqrt(3)*(71*x^9 - 111*x
^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 2*4^(2/3)*(-1)^(1/3)*x
^4*log(-(3*4^(2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3)*x^2 - 4^(1/3)*(-1)^(2/3)*(x^
3 + 1) - 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 4^(2/3)*(-1)^(1/3)*x^4*log((6*4
^(1/3)*(-1)^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 4^(2/3)*(-1)^(1/3)*(19*x^6
 - 16*x^3 + 1) - 24*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) - 18
*(x^3 - 1)*(-x^3 + 1)^(1/3))/x^4
```

Sympy [F]

$$\int \frac{1}{x^5 (1-x^3)^{2/3} (1+x^3)} \, dx = \int \frac{1}{x^5 \left(-(x-1)(x^2+x+1)\right)^{\frac{2}{3}} (x+1)(x^2-x+1)} \, dx$$

```
[In] integrate(1/x**5/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**5*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Maxima [F]

$$\int \frac{1}{x^5 (1-x^3)^{2/3} (1+x^3)} \, dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}} x^5} \, dx$$

```
[In] integrate(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}}x^5}\,dx$$

```
[In] integrate(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \int \frac{1}{x^5\left(1-x^3\right)^{2/3}\left(x^3+1\right)}\,dx$$

```
[In] int(1/(x^5*(1 - x^3)^(2/3)*(x^3 + 1)),x)
[Out] int(1/(x^5*(1 - x^3)^(2/3)*(x^3 + 1)), x)
```

3.634 $\int \frac{x^6}{(1-x^3)^{2/3}(1+x^3)} dx$

Optimal result

Integrand size = 22, antiderivative size = 291

$$\int \frac{x^6}{(1-x^3)^{2/3}(1+x^3)} dx = -\frac{1}{2}x\sqrt[3]{1-x^3} + \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} - \frac{\log\left(2\sqrt[3]{2}+\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\ 2^{2/3}}$$

```
[Out] -1/2*x*(-x^3+1)^(1/3)+1/12*ln(2^(2/3)+(-1+x)/(-x^3+1)^(1/3))*2^(1/3)-1/12*l
n(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/
6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)-1/24*ln(2*2^(1/3)+(1-x)^2/(-x^
3+1)^(2/3)+2^(2/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/6*arctan(1/3*(1-2*2^(1/3
)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)+1/12*arctan(1/3*(1+2^(1/3)
*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {490, 420, 493, 298, 31, 648, 631, 210, 642}

$$\int \frac{x^6}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{1}{2}\sqrt[3]{1-x^3}x + \frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\ 2^{2/3}} + \frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}} - \frac{\log\left(\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}\right)}{12\ 2^{2/3}}$$

```
[In] Int[x^6/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -1/2*(x*(1 - x^3)^(1/3)) + ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))
/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)
)/Sqrt[3]]/(2*2^(2/3)*Sqrt[3]) + Log[2^(2/3) - (1 - x)/(1 - x^3)^(1/3)]/(6*
2^(2/3)) - Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/
(1 - x^3)^(1/3)]/(6*2^(2/3)) + Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(
3*2^(2/3)) - Log[2*2^(1/3) + (1 - x)^2/(1 - x^3)^(2/3) + (2^(2/3)*(1 - x))/
(1 - x^3)^(1/3)]/(12*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = -\frac{1}{2}x\sqrt[3]{1-x^3}+\frac{1}{2}\int\frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx$$

$$= -\frac{1}{2}x\sqrt[3]{1-x^3} - \frac{9}{2}\text{Subst}\left(\int \frac{x}{(4-x^3)(1+2x^3)}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{1}{2}x\sqrt[3]{1-x^3} - \frac{1}{2}\text{Subst}\left(\int \frac{x}{4-x^3}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) - \text{Subst}\left(\int \frac{x}{1+2x^3}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{1}{2}x\sqrt[3]{1-x^3} - \frac{\text{Subst}\left(\int \frac{1}{2^{2/3}-x}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{2^{2/3}-x}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{\text{Subst}\left(\int \frac{1+\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}$$

$$= -\frac{1}{2}x\sqrt[3]{1-x^3} + \frac{\log\left(2^{2/3} - \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\log\left(1 + \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) - \frac{\text{Subst}\left(\int \frac{2^{2/3}+2x}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{12\ 2^{2/3}} - \text{Su}$$

$$= -\frac{1}{2}x\sqrt[3]{1-x^3} + \frac{\log\left(2^{2/3} - \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\log\left(1 + \frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\log\left(1 + \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} - \frac{\log\left(2\sqrt[3]{2} + \frac{(1-x)^2}{(1-x^3)^{2/3}} + \frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\ 2^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 + \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2\ 2^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2^{2/3}}$$

$$= -\frac{1}{2}x\sqrt[3]{1-x^3} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{\log\left(2^{2/3} - \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\log\left(1 + \frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\log\left(1 + \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} - \frac{\log\left(2\sqrt[3]{2} + \frac{(1-x)^2}{(1-x^3)^{2/3}} + \frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\ 2^{2/3}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.13 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.40

$$\int \frac{x^6}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{2}x\sqrt[3]{1-x^3}\Bigg(-1 - \frac{4\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},x^3,-x^3\right)}{(1+x^3)\left(-4\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},x^3,-x^3\right)+x^3\left(3\,\text{AppellF1}\left(\frac{4}{3},-\frac{1}{3},2,\frac{7}{3},x^3,-x^3\right)+\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},\right.\right.\right.}$$

```
[In] Integrate[x^6/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (x*(1 - x^3)^(1/3)*(-1 - (4*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3])/((1 + x
^3)*(-4*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, -1/3,
 2, 7/3, x^3, -x^3] + AppellF1[4/3, 2/3, 1, 7/3, x^3, -x^3])))))/2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 18.21 (sec) , antiderivative size = 694, normalized size of antiderivative = 2.38

method	result	size
trager	Expression too large to display	694
risch	Expression too large to display	988

```
[In] int(x^6/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-x^3+1)^(1/3)*x+1/4*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^
2)*ln(-(18*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)^2*RootOf(_Z^
3-2)^3*x^3+12*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z
^3-2)^4*x^3+3*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z
^3-2)*x^6+2*RootOf(_Z^3-2)^2*x^6-18*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+
3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)^2*x^2+18*(-x^3+1)^(1/3)*RootOf(R
ootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*x^4-6*RootOf(RootOf(_Z^3-2)^2+3
*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)*x^3-4*RootOf(_Z^3-2)^2*x^3-12*x^2
*(-x^3+1)^(2/3)-18*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-
2)+9*_Z^2)*x+3*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_
Z^3-2)+2*RootOf(_Z^3-2)^2)/(1+x)^2/(x^2-x+1)^2)+1/12*RootOf(_Z^3-2)*ln(-(18
*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)^2*RootOf(_Z^3-2)^2*x^3
+12*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)^3*x^
3-3*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*x^6-2*RootOf(_Z^3-2
)*x^6+18*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)
*RootOf(_Z^3-2)*x^2+6*(-x^3+1)^(1/3)*x^4+18*RootOf(RootOf(_Z^3-2)^2+3*_Z*Ro
```

```
otOf(_Z^3-2)+9*_Z^2)*x^3+12*RootOf(_Z^3-2)*x^3-6*(-x^3+1)^(1/3)*x-3*RootOf(
RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)-2*RootOf(_Z^3-2))/(1+x)^2/(x^2
-x+1)^2)
```

Fricas [A] (verification not implemented)

none

Time = 1.72 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.22

$$\int \frac{x^6}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{36} \cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(-\frac{4^{\frac{1}{6}}\left(6\cdot 4^{\frac{2}{3}}\sqrt{3}(x^{16}-33\,x^{13}+110\,x^{10}-110\,x^7+33\,x^4-x)(-x^3+1)^{\frac{1}{3}}-48\sqrt{3}(x^{14}-2\,x^{11}\right.}{6\,(x^{18}-102\,x^{15}+447\,x^{12}}\right) + \frac{1}{72}\cdot 4^{\frac{2}{3}}\log\left(-\frac{12\,(-x^3+1)^{\frac{2}{3}}x^2-3\cdot 4^{\frac{2}{3}}(x^4-x)(-x^3+1)^{\frac{1}{3}}+4^{\frac{1}{3}}(x^6+2\,x^3+1)}{x^6+2\,x^3+1}\right) - \frac{1}{144} \cdot 4^{\frac{2}{3}}\log\left(\frac{24\cdot 4^{\frac{1}{3}}(x^8-4\,x^5+x^2)(-x^3+1)^{\frac{2}{3}}+4^{\frac{2}{3}}(x^{12}-32\,x^9+78\,x^6-32\,x^3+1)+12\,(x^{10}-11\,x^7+11\,x^4}{x^{12}+4\,x^9+6\,x^6+4\,x^3+1}\right) - \frac{1}{2}\left(-x^3+1\right)^{\frac{1}{3}}x$$

```
[In] integrate(x^6/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/36*4^(1/6)*sqrt(3)*arctan(-1/6*4^(1/6)*(6*4^(2/3)*sqrt(3)*(x^16 - 33*x^13
 + 110*x^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1)^(1/3) - 48*sqrt(3)*(x^14 - 2
*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^(2/3) - 4^(1/3)*sqrt(3)*(x^18 + 42*
x^15 - 417*x^12 + 812*x^9 - 417*x^6 + 42*x^3 + 1))/(x^18 - 102*x^15 + 447*x
^12 - 628*x^9 + 447*x^6 - 102*x^3 + 1)) + 1/72*4^(2/3)*log(-(12*(-x^3 + 1)^
(2/3)*x^2 - 3*4^(2/3)*(x^4 - x)*(-x^3 + 1)^(1/3) + 4^(1/3)*(x^6 + 2*x^3 + 1
))/(x^6 + 2*x^3 + 1)) - 1/144*4^(2/3)*log((24*4^(1/3)*(x^8 - 4*x^5 + x^2)*(
-x^3 + 1)^(2/3) + 4^(2/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 + 1) + 12*(x^10
- 11*x^7 + 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x^6 + 4*x^3 + 1)
) - 1/2*(-x^3 + 1)^(1/3)*x
```

Sympy [F]

$$\int \frac{x^6}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^6}{\left(-(x-1)(x^2+x+1)\right)^{\frac{2}{3}}(x+1)(x^2-x+1)}\,dx$$

```
[In] integrate(x**6/(-x**3+1)**(2/3)/(x**3+1),x)
[Out] Integral(x**6/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{x^6}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^6}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(x^6/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Giac [F]

$$\int \frac{x^6}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^6}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(x^6/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^6}{(1-x^3)^{2/3}(x^3+1)}\,dx$$

```
[In] int(x^6/((1 - x^3)^(2/3)*(x^3 + 1)),x)
[Out] int(x^6/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

3.635 $\int \frac{x^3}{(1-x^3)^{2/3}(1+x^3)} dx$

Optimal result

Integrand size = 22, antiderivative size = 294

$$\int \frac{x^3}{(1-x^3)^{2/3}(1+x^3)} dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right) - \frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\log\left(\right.}{}$$

```
[Out] 1/2*x*hypergeom([1/3, 2/3],[4/3],x^3)-1/12*ln(2^(2/3)+(-1+x)/(-x^3+1)^(1/3)
)*2^(1/3)+1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(
1/3))*2^(1/3)-1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/24*ln(2*2^(1
/3)+(1-x)^2/(-x^3+1)^(2/3)+2^(2/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)-1/6*arctan
(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)-1/12*arcta
n(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules

used = {494, 251, 421, 420, 493, 298, 31, 648, 631, 210, 642}

$$\int \frac{x^3}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\,2^{2/3}\sqrt{3}} + \frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right) - \frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\,2^{2/3}} + \frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\,2^{2/3}} - \frac{\log\left(\cdots\right.}{\cdots}$$

```
[In] Int[x^3/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3
])) - ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(2/3)*Sq
rt[3]) + (x*Hypergeometric2F1[1/3, 2/3, 4/3, x^3])/2 - Log[2^(2/3) - (1 - x
)/(1 - x^3)^(1/3)]/(6*2^(2/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3
) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(2/3)) - Log[1 + (2^(1/3)*(1 -
x))/(1 - x^3)^(1/3)]/(3*2^(2/3)) + Log[2*2^(1/3) + (1 - x)^2/(1 - x^3)^(2/3
) + (2^(2/3)*(1 - x))/(1 - x^3)^(1/3)]/(12*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 421

```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^3)^(2/3), x], x] - Dist[d/(b*c - a*d), Int[
(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(1-x^3)^{2/3}}\,dx - \int \frac{1}{(1-x^3)^{2/3}(1+x^3)}\,dx$$

$$= x\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^3\right) - \frac{1}{2}\int \frac{1}{(1-x^3)^{2/3}}\,dx - \frac{1}{2}\int \frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx$$

$$= \frac{1}{2}x\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^3\right) + \frac{9}{2}\text{Subst}\left(\int \frac{x}{(4-x^3)(1+2x^3)}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)$$

$$= \frac{1}{2}x\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^3\right) + \frac{1}{2}\text{Subst}\left(\int \frac{x}{4-x^3}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) + \text{Subst}\left(\int \frac{x}{1+2x^3}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)$$

$$= \frac{1}{2}x\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^3\right) + \frac{\text{Subst}\left(\int \frac{1}{2^{2/3}-x}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\text{Subst}\left(\int \frac{2^{2/3}-x}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1+\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}$$

$$= \frac{1}{2}x\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^3\right) - \frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) + \frac{\text{Subst}\left(\int \frac{2^{2/3}+2x}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{12\ 2^{2/3}} + \text{Sub}$$

$$= \frac{1}{2}x\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^3\right) - \frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{\log\left(2\sqrt[3]{2}+\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2^{2/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{1}{2}x\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^3\right)$$
$$-\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}}$$
$$-\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{\log\left(2\sqrt[3]{2}+\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\ 2^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.09

$$\int \frac{x^3}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{1}{4}x^4\,\mathrm{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},x^3,-x^3\right)$$

```
[In] Integrate[x^3/((1 - x^3)^(2/3)*(1 + x^3)),x]
[Out] (x^4*AppellF1[4/3, 2/3, 1, 7/3, x^3, -x^3])/4
```

Maple [F]

$$\int \frac{x^3}{(-x^3+1)^{\frac{2}{3}}(x^3+1)}dx$$

```
[In] int(x^3/(-x^3+1)^(2/3)/(x^3+1),x)
[Out] int(x^3/(-x^3+1)^(2/3)/(x^3+1),x)
```

Fricas [F]

$$\int \frac{x^3}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^3}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
[Out] integral(-(-x^3 + 1)^(1/3)*x^3/(x^6 - 1), x)
```

Sympy [F]

$$\int \frac{x^3}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^3}{(-(x-1)(x^2+x+1))^{\frac{2}{3}}(x+1)(x^2-x+1)}\,dx$$

```
[In] integrate(x**3/(-x**3+1)**(2/3)/(x**3+1),x)
[Out] Integral(x**3/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{x^3}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^3}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Giac [F]

$$\int \frac{x^3}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^3}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{x^3}{(1-x^3)^{2/3}(x^3+1)}\,dx$$

```
[In] int(x^3/((1 - x^3)^(2/3)*(x^3 + 1)),x)
[Out] int(x^3/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

3.636 $\int \frac{1}{(1-x^3)^{2/3}(1+x^3)} dx$

Optimal result

Integrand size = 19, antiderivative size = 293

$$\int \frac{1}{(1-x^3)^{2/3}(1+x^3)} dx = \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{1}{2}x \,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right) + \frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\log\left(\right.}{}$$

```
[Out] 1/2*x*hypergeom([1/3, 2/3],[4/3],x^3)+1/12*ln(2^(2/3)+(-1+x)/(-x^3+1)^(1/3)
)*2^(1/3)-1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(
1/3))*2^(1/3)+1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)-1/24*ln(2*2^(1
/3)+(1-x)^2/(-x^3+1)^(2/3)+2^(2/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/6*arctan
(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)+1/12*arcta
n(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules

used = {421, 251, 420, 493, 298, 31, 648, 631, 210, 642}

$$\int \frac{1}{(1-x^3)^{2/3}(1+x^3)}\,dx = \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\,2^{2/3}\sqrt{3}} + \frac{1}{2}x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},x^3\right) + \frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\,2^{2/3}} - \frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\,2^{2/3}} + \frac{\log\left(\ldots\right.}{}$$

```
[In] Int[1/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3])
 + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(2/3)*Sqrt[
3]) + (x*Hypergeometric2F1[1/3, 2/3, 4/3, x^3])/2 + Log[2^(2/3) - (1 - x)/(
1 - x^3)^(1/3)]/(6*2^(2/3)) - Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) -
 (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(2/3)) + Log[1 + (2^(1/3)*(1 - x))
/(1 - x^3)^(1/3)]/(3*2^(2/3)) - Log[2*2^(1/3) + (1 - x)^2/(1 - x^3)^(2/3) +
 (2^(2/3)*(1 - x))/(1 - x^3)^(1/3)]/(12*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 421

```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^3)^(2/3), x], x] - Dist[d/(b*c - a*d), Int[
(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\int \frac{1}{(1-x^3)^{2/3}}\,dx + \frac{1}{2}\int \frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx$$

$$= \frac{1}{2}x\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^3\right) - \frac{9}{2}\text{Subst}\left(\int \frac{x}{(4-x^3)\,(1+2x^3)}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)$$

$$= \frac{1}{2} x\, {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; x^3\right) - \frac{1}{2}\text{Subst}\left(\int \frac{x}{4-x^3}\, dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) - \text{Subst}\left(\int \frac{x}{1+2x^3}\, dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)$$

$$= \frac{1}{2} x\, {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; x^3\right) - \frac{\text{Subst}\left(\int \frac{1}{2^{2/3}-x}\, dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{2^{2/3}-x}{2\sqrt[3]{2}+2^{2/3}x+x^2}\, dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\, dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{\text{Subst}\left(\int \frac{1+\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\, dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}$$

$$= \frac{1}{2} x\, {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; x^3\right) + \frac{\log\left(2^{2/3} - \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\log\left(1 + \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}x+x^2}\, dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) - \frac{\text{Subst}\left(\int \frac{2^{2/3}+2x}{2\sqrt[3]{2}+2^{2/3}x+x^2}\, dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{12\ 2^{2/3}} - \text{Sub}$$

$$= \frac{1}{2} x\, {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; x^3\right) + \frac{\log\left(2^{2/3} - \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\log\left(1 + \frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\log\left(1 + \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} - \frac{\log\left(2\sqrt[3]{2} + \frac{(1-x)^2}{(1-x^3)^{2/3}} + \frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\ 2^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\, dx, x, 1 + \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2\ 2^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\, dx, x, 1 - \frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2^{2/3}}$$

$$= \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{1}{2} x\, {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; x^3\right) + \frac{\log\left(2^{2/3} - \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\log\left(1 + \frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\log\left(1 + \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} - \frac{\log\left(2\sqrt[3]{2} + \frac{(1-x)^2}{(1-x^3)^{2/3}} + \frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\ 2^{2/3}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.38

$$\int \frac{1}{(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{4x\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},x^3,-x^3\right)}{(1-x^3)^{2/3}(1+x^3)\left(-4\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},x^3,-x^3\right)+x^3\left(3\,\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},2,\frac{7}{3},x^3,-x^3\right)-2\,\text{AppellF1}\right.\right.}$$

```
[In] Integrate[1/((1 - x^3)^(2/3)*(1 + x^3)),x]
[Out] (-4*x*AppellF1[1/3, 2/3, 1, 4/3, x^3, -x^3])/((1 - x^3)^(2/3)*(1 + x^3)*(-4
*AppellF1[1/3, 2/3, 1, 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, 2/3, 2, 7/3,
x^3, -x^3] - 2*AppellF1[4/3, 5/3, 1, 7/3, x^3, -x^3])))
```

Maple [F]

$$\int \frac{1}{(-x^3+1)^{\frac{2}{3}}(x^3+1)}dx$$

```
[In] int(1/(-x^3+1)^(2/3)/(x^3+1),x)
[Out] int(1/(-x^3+1)^(2/3)/(x^3+1),x)
```

Fricas [F]

$$\int \frac{1}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
[Out] integral(-(-x^3 + 1)^(1/3)/(x^6 - 1), x)
```

Sympy [F]

$$\int \frac{1}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{1}{(-(x-1)(x^2+x+1))^{\frac{2}{3}}(x+1)(x^2-x+1)}\,dx$$

```
[In] integrate(1/(-x**3+1)**(2/3)/(x**3+1),x)
[Out] Integral(1/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Giac [F]

$$\int \frac{1}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-x^3)^{2/3}(1+x^3)}\,dx = \int \frac{1}{(1-x^3)^{2/3}\,(x^3+1)}\,dx$$

```
[In] int(1/((1 - x^3)^(2/3)*(x^3 + 1)),x)
[Out] int(1/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

3.637 $\int \frac{1}{x^3(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 294

$$\int \frac{1}{x^3(1-x^3)^{2/3}(1+x^3)}\,dx = -\frac{\sqrt[3]{1-x^3}}{2x^2} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{\log\left(2\sqrt[3]{2}+\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\ 2^{2/3}}$$

```
[Out] -1/2*(-x^3+1)^(1/3)/x^2-1/12*ln(2^(2/3)+(-1+x)/(-x^3+1)^(1/3))*2^(1/3)+1/12
*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)-
1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/24*ln(2*2^(1/3)+(1-x)^2/(-
x^3+1)^(2/3)+2^(2/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)-1/6*arctan(1/3*(1-2*2^(1
/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)-1/12*arctan(1/3*(1+2^(1/
3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {491, 21, 420, 493, 298, 31, 648, 631, 210, 642}

$$\int \frac{1}{x^3 (1-x^3)^{2/3} (1+x^3)} dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\ 2^{2/3}} - \frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}} + \frac{\log\left(\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}\right)}{12\ 2^{2/3}} - \frac{\sqrt[3]{1-x^3}}{2x^2}$$

```
[In] Int[1/(x^3*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -1/2*(1 - x^3)^(1/3)/x^2 - ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))
/Sqrt[3]]/(2^(2/3)*Sqrt[3]) - ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)
)/Sqrt[3]]/(2*2^(2/3)*Sqrt[3]) - Log[2^(2/3) - (1 - x)/(1 - x^3)^(1/3)]/(6*
2^(2/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/
(1 - x^3)^(1/3)]/(6*2^(2/3)) - Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(
3*2^(2/3)) + Log[2*2^(1/3) + (1 - x)^2/(1 - x^3)^(2/3) + (2^(2/3)*(1 - x))/
(1 - x^3)^(1/3)]/(12*2^(2/3))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt[3]{1-x^3}}{2x^2} + \frac{1}{2}\int \frac{-1+x^3}{(1-x^3)^{2/3}(1+x^3)}\,dx$$

$$= -\frac{\sqrt[3]{1-x^3}}{2x^2} - \frac{1}{2}\int \frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx$$

$$= -\frac{\sqrt[3]{1-x^3}}{2x^2} + \frac{9}{2}\text{Subst}\left(\int \frac{x}{(4-x^3)(1+2x^3)}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{\sqrt[3]{1-x^3}}{2x^2} + \frac{1}{2}\text{Subst}\left(\int \frac{x}{4-x^3}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) + \text{Subst}\left(\int \frac{x}{1+2x^3}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{\sqrt[3]{1-x^3}}{2x^2} + \frac{\text{Subst}\left(\int \frac{1}{2^{2/3}-x}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\text{Subst}\left(\int \frac{2^{2/3}-x}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1+\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}$$

$$= -\frac{\sqrt[3]{1-x^3}}{2x^2} - \frac{\log\left(2^{2/3} - \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\log\left(1 + \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) + \frac{\text{Subst}\left(\int \frac{2^{2/3}+2x}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{12\ 2^{2/3}} + \text{Sub}$$

$$= -\frac{\sqrt[3]{1-x^3}}{2x^2} - \frac{\log\left(2^{2/3} - \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\log\left(1 + \frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\log\left(1 + \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{\log\left(2\sqrt[3]{2} + \frac{(1-x)^2}{(1-x^3)^{2/3}} + \frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 + \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2^{2/3}}$$

$$= -\frac{\sqrt[3]{1-x^3}}{2x^2} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}}$$

$$- \frac{\log\left(2^{2/3} - \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\log\left(1 + \frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}}$$

$$- \frac{\log\left(1 + \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{\log\left(2\sqrt[3]{2} + \frac{(1-x)^2}{(1-x^3)^{2/3}} + \frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\ 2^{2/3}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 11.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^3 (1-x^3)^{2/3} (1+x^3)} dx = \frac{\sqrt[3]{1-x^3}\left(-1 + \frac{4x^3 \operatorname{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},x^3,-x^3\right)}{(1+x^3)\left(-4 \operatorname{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},x^3,-x^3\right)+x^3\left(3 \operatorname{AppellF1}\left(\frac{4}{3},-\frac{1}{3},2,\frac{7}{3},x^3,-x^3\right)+\operatorname{Ap}\right.\right.}\right.}{2x^2}$$

```
[In] Integrate[1/(x^3*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] ((1 - x^3)^(1/3)*(-1 + (4*x^3*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3])/((1 +
 x^3)*(-4*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, -1/
3, 2, 7/3, x^3, -x^3] + AppellF1[4/3, 2/3, 1, 7/3, x^3, -x^3])))))/(2*x^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 47.56 (sec) , antiderivative size = 695, normalized size of antiderivative = 2.36

method	result	size
risch	Expression too large to display	695
trager	Expression too large to display	1729

```
[In] int(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(x^3-1)/x^2/(-x^3+1)^(2/3)+(1/4*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^
3+2)+9*_Z^2)*ln(-(18*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)^2*
RootOf(_Z^3+2)^2*x^3+12*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)
*RootOf(_Z^3+2)^3*x^3-3*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)
*x^6-2*RootOf(_Z^3+2)*x^6+9*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+3*_Z*R
ootOf(_Z^3+2)+9*_Z^2)*(x^6-2*x^3+1)^(2/3)*x-18*RootOf(_Z^3+2)*(x^6-2*x^3+1)
```

^(1/3)*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*x^2-6*RootOf(_Z^
3+2)^2*(x^6-2*x^3+1)^(1/3)*x^2+6*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2
)+9*_Z^2)*x^3+4*RootOf(_Z^3+2)*x^3-3*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z
^3+2)+9*_Z^2)-2*RootOf(_Z^3+2))/(1+x)^2/(x^2-x+1)^2)+1/12*RootOf(_Z^3+2)*ln
((36*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)^2*RootOf(_Z^3+2)^2
*x^3+6*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*RootOf(_Z^3+2)^3
*x^3+6*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*x^6+RootOf(_Z^3+
2)*x^6+9*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^
2)*(x^6-2*x^3+1)^(2/3)*x-6*RootOf(_Z^3+2)^2*(x^6-2*x^3+1)^(1/3)*x^2-36*Root
Of(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*x^3-6*RootOf(_Z^3+2)*x^3-6*
(x^6-2*x^3+1)^(2/3)*x+6*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)
+RootOf(_Z^3+2))/(1+x)^2/(x^2-x+1)^2))/(-x^3+1)^(2/3)*((x^3-1)^2)^(1/3)

Fricas [A] (verification not implemented)

none

Time = 1.73 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.35

$$\int \frac{1}{x^3 (1-x^3)^{2/3} (1+x^3)}\,dx = -\frac{4\cdot 4^{\frac{1}{6}}\sqrt{3}(-1)^{\frac{1}{3}}\,x^2 \arctan\left(\frac{4^{\frac{1}{6}}\left(6\cdot 4^{\frac{2}{3}}\sqrt{3}(-1)^{\frac{2}{3}}\left(x^{16}-33\,x^{13}+110\,x^{10}-110\,x^7+33\,x^4-x\right)\left(-x^3+1\right)^{\frac{1}{3}}+48\sqrt{3}(-1)^{\frac{1}{3}}\left(x^{14}-2\,x^{11}-6\,x^8-2\,x^5 \cdots}{6\left(x^{18}-102\,x^{15}+447\,x^{12}-628\,x^9+447\,x^6-1 \cdots}\right.}{}$$

[In] integrate(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")

[Out] -1/144*(4*4^(1/6)*sqrt(3)*(-1)^(1/3)*x^2*arctan(1/6*4^(1/6)*(6*4^(2/3)*sqrt
(3)*(-1)^(2/3)*(x^16 - 33*x^13 + 110*x^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1
)^(1/3) + 48*sqrt(3)*(-1)^(1/3)*(x^14 - 2*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3
 + 1)^(2/3) - 4^(1/3)*sqrt(3)*(x^18 + 42*x^15 - 417*x^12 + 812*x^9 - 417*x^
6 + 42*x^3 + 1))/(x^18 - 102*x^15 + 447*x^12 - 628*x^9 + 447*x^6 - 102*x^3
+ 1)) + 4^(2/3)*(-1)^(1/3)*x^2*log((24*4^(1/3)*(-1)^(2/3)*(x^8 - 4*x^5 + x^
2)*(-x^3 + 1)^(2/3) - 4^(2/3)*(-1)^(1/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 +
 1) + 12*(x^10 - 11*x^7 + 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x
^6 + 4*x^3 + 1)) - 2*4^(2/3)*(-1)^(1/3)*x^2*log(-(12*(-x^3 + 1)^(2/3)*x^2 +
 3*4^(2/3)*(-1)^(1/3)*(x^4 - x)*(-x^3 + 1)^(1/3) + 4^(1/3)*(-1)^(2/3)*(x^6
+ 2*x^3 + 1))/(x^6 + 2*x^3 + 1)) + 72*(-x^3 + 1)^(1/3))/x^2

Sympy [F]

$$\int \frac{1}{x^3 (1-x^3)^{2/3} (1+x^3)} dx = \int \frac{1}{x^3 \left(-(x-1)(x^2+x+1)\right)^{\frac{2}{3}} (x+1)(x^2-x+1)} dx$$

```
[In] integrate(1/x**3/(-x**3+1)**(2/3)/(x**3+1),x)
[Out] Integral(1/(x**3*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Maxima [F]

$$\int \frac{1}{x^3 (1-x^3)^{2/3} (1+x^3)} dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}} x^3} dx$$

```
[In] integrate(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (1-x^3)^{2/3} (1+x^3)} dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}} x^3} dx$$

```
[In] integrate(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (1-x^3)^{2/3} (1+x^3)} dx = \int \frac{1}{x^3 (1-x^3)^{2/3} (x^3+1)} dx$$

```
[In] int(1/(x^3*(1 - x^3)^(2/3)*(x^3 + 1)),x)
[Out] int(1/(x^3*(1 - x^3)^(2/3)*(x^3 + 1)), x)
```

3.638 $\int \frac{x^{14}}{(1-x^3)^{4/3}(1+x^3)} dx$

Optimal result

Integrand size = 22, antiderivative size = 141

$$\int \frac{x^{14}}{(1-x^3)^{4/3}(1+x^3)} dx = \frac{1}{2\sqrt[3]{1-x^3}} + (1-x^3)^{2/3} - \frac{2}{5}(1-x^3)^{5/3} + \frac{1}{8}(1-x^3)^{8/3} + \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[Out] 1/2/(-x^3+1)^(1/3)+(-x^3+1)^(2/3)-2/5*(-x^3+1)^(5/3)+1/8*(-x^3+1)^(8/3)-1/2
4*ln(x^3+1)*2^(2/3)+1/8*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)+1/12*arctan(1/3*
(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 89, 45, 641, 53, 57, 631, 210, 31}

$$\int \frac{x^{14}}{(1-x^3)^{4/3}(1+x^3)} dx = \frac{\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{1}{8}(1-x^3)^{8/3} - \frac{2}{5}(1-x^3)^{5/3} + (1-x^3)^{2/3} + \frac{1}{2\sqrt[3]{1-x^3}} - \frac{\log(x^3+1)}{12\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[In] Int[x^14/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) + (1 - x^3)^(2/3) - (2*(1 - x^3)^(5/3))/5 + (1 - x^3)
^(8/3)/8 + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]
) - Log[1 + x^3]/(12*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{(1-x)^{4/3}(1+x)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{1}{\sqrt[3]{1-x}}-\frac{x^2}{\sqrt[3]{1-x}}+\frac{1}{\sqrt[3]{1-x}\,(1-x^2)}\right)dx, x, x^3\right)\\
&= \frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{1}{3}\text{Subst}\left(\int \frac{x^2}{\sqrt[3]{1-x}}\,dx, x, x^3\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1-x^2)}\,dx, x, x^3\right)\\
&= \frac{1}{2}\left(1-x^3\right)^{2/3}\\
&\quad -\frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{\sqrt[3]{1-x}}-2(1-x)^{2/3}+(1-x)^{5/3}\right)dx, x, x^3\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{4/3}(1+x)}\,dx, x, \right.\\
&= \frac{1}{2\sqrt[3]{1-x^3}}+\left(1-x^3\right)^{2/3}-\frac{2}{5}\left(1-x^3\right)^{5/3}+\frac{1}{8}\left(1-x^3\right)^{8/3}+\frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,dx, x, x^3\right)\\
&= \frac{1}{2\sqrt[3]{1-x^3}}+\left(1-x^3\right)^{2/3}-\frac{2}{5}\left(1-x^3\right)^{5/3}+\frac{1}{8}\left(1-x^3\right)^{8/3}-\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}\\
&\quad +\frac{1}{4}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right)-\frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}\\
&= \frac{1}{2\sqrt[3]{1-x^3}}+\left(1-x^3\right)^{2/3}-\frac{2}{5}\left(1-x^3\right)^{5/3}+\frac{1}{8}\left(1-x^3\right)^{8/3}-\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}\\
&\quad +\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}-\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}
\end{aligned}
$$

$$= \frac{1}{2\sqrt[3]{1-x^3}} + \left(1-x^3\right)^{2/3} - \frac{2}{5}\left(1-x^3\right)^{5/3} + \frac{1}{8}\left(1-x^3\right)^{8/3} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.01

$$\int \frac{x^{14}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{120}\left(-\frac{3(-49+23x^3+x^6+5x^9)}{\sqrt[3]{1-x^3}} + 10\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) + 10\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) - 5\,2^{2/3}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(\right.\right.$$

```
[In] Integrate[x^14/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((-3*(-49 + 23*x^3 + x^6 + 5*x^9))/(1 - x^3)^(1/3) + 10*2^(2/3)*Sqrt[3]*Arc
Tan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 10*2^(2/3)*Log[-2 + 2^(2/3)*(1
 - x^3)^(1/3)] - 5*2^(2/3)*Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x
^3)^(2/3)])/120
```

Maple [A] (verified)

Time = 8.93 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.98

method	result
pseudoelliptic	$\frac{-15x^9-3x^6+10\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)2^{\frac{2}{3}}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+10\,2^{\frac{2}{3}}\ln\left(\left(-x^3+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)\left(-x^3+1\right)^{\frac{1}{3}}-5\,2^{\frac{2}{3}}\ln\left(\left(-x\right.\right.}{120\left(-x^3+1\right)^{\frac{1}{3}}}$
trager	$\frac{(5x^9+x^6+23x^3-49)\left(-x^3+1\right)^{\frac{2}{3}}}{40x^3-40} + \frac{\mathrm{RootOf}\left(\mathrm{RootOf}\left(\_Z^3-4\right)^2+6\_Z\,\mathrm{RootOf}\left(\_Z^3-4\right)+36\_Z^2\right)\ln\left(\frac{15\,\mathrm{RootOf}\left(\mathrm{Ro}\right.}{}\right.}{}$
risch	Expression too large to display

```
[In] int(x^14/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*(-15*x^9-3*x^6+10*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2
/3)*3^(1/2)*(-x^3+1)^(1/3)+10*2^(2/3)*ln((-x^3+1)^(1/3)-2^(1/3))*(-x^3+1)^(
1/3)-5*2^(2/3)*ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))*(-x^3+1)^(
1/3)-69*x^3+147)/(-x^3+1)^(1/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.99

$$\int \frac{x^{14}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{10\sqrt{6}2^{\frac{1}{6}}(x^3-1)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}+2\sqrt{6}(-x^3+1)^{\frac{1}{3}}\right)\right)-5\cdot 2^{\frac{2}{3}}(x^3-1)\log}{}$$

```
[In] integrate(x^14/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/120*(10*sqrt(6)*2^(1/6)*(x^3 - 1)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2
*sqrt(6)*(-x^3 + 1)^(1/3))) - 5*2^(2/3)*(x^3 - 1)*log(2^(2/3) + 2^(1/3)*(-x
^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 10*2^(2/3)*(x^3 - 1)*log(-2^(1/3) + (-x
^3 + 1)^(1/3)) + 3*(5*x^9 + x^6 + 23*x^3 - 49)*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

Sympy [F]

$$\int \frac{x^{14}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^{14}}{\left(-(x-1)(x^2+x+1)\right)^{\frac{4}{3}}(x+1)(x^2-x+1)}\,dx$$

```
[In] integrate(x**14/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**14/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.91

$$\begin{aligned}\int \frac{x^{14}}{(1-x^3)^{4/3}(1+x^3)}\,dx &= \frac{1}{8}(-x^3+1)^{\frac{8}{3}} \\ &+ \frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2(-x^3+1)^{\frac{1}{3}}\right)\right) - \frac{2}{5}(-x^3+1)^{\frac{5}{3}} \\ &- \frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}+(-x^3+1)^{\frac{2}{3}}\right) + \frac{1}{12} \\ &\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+(-x^3+1)^{\frac{1}{3}}\right) + (-x^3+1)^{\frac{2}{3}} + \frac{1}{2(-x^3+1)^{\frac{1}{3}}}\end{aligned}$$

```
[In] integrate(x^14/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/8*(-x^3 + 1)^(8/3) + 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(
1/3) + 2*(-x^3 + 1)^(1/3))) - 2/5*(-x^3 + 1)^(5/3) - 1/24*2^(2/3)*log(2^(2/
3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/12*2^(2/3)*log(-2^(1/
3) + (-x^3 + 1)^(1/3)) + (-x^3 + 1)^(2/3) + 1/2/(-x^3 + 1)^(1/3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.96

$$\int \frac{x^{14}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{8}\left(x^3-1\right)^2\left(-x^3+1\right)^{\frac{2}{3}} + \frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) - \frac{2}{5}\left(-x^3+1\right)^{\frac{5}{3}} - \frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right) + \left(-x^3+1\right)^{\frac{2}{3}} + \frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

```
[In] integrate(x^14/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/8*(x^3 - 1)^2*(-x^3 + 1)^(2/3) + 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*
2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) - 2/5*(-x^3 + 1)^(5/3) - 1/24*2^(2/
3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/12*2^(2/3
)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + (-x^3 + 1)^(2/3) + 1/2/(-x^3 + 1)
^(1/3)
```

Mupad [B] (verification not implemented)

Time = 8.54 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.05

$$\int \frac{x^{14}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}}{4}\right)}{12} + \frac{1}{2\left(1-x^3\right)^{1/3}} + \left(1-x^3\right)^{2/3} - \frac{2\left(1-x^3\right)^{5/3}}{5} + \frac{\left(1-x^3\right)^{8/3}}{8} + \frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{24} - \frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{24}$$

```
[In] int(x^14/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3)/4 - 2^(1/3)/4))/12 + 1/(2*(1 - x^3)^(1/3)) + (
1 - x^3)^(2/3) - (2*(1 - x^3)^(5/3))/5 + (1 - x^3)^(8/3)/8 + (2^(2/3)*log((
1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1i - 1))/24 -
(2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i + 1)^2)/16)*(3^(1/2)*
1i + 1))/24
```

3.639 $\int \frac{x^{11}}{(1-x^3)^{4/3}(1+x^3)} dx$

Optimal result

Integrand size = 22, antiderivative size = 130

$$\int \frac{x^{11}}{(1-x^3)^{4/3}(1+x^3)} dx = \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{2}(1-x^3)^{2/3} - \frac{1}{5}(1-x^3)^{5/3} - \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\log(1+x^3)}{12\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[Out] 1/2/(-x^3+1)^(1/3)+1/2*(-x^3+1)^(2/3)-1/5*(-x^3+1)^(5/3)+1/24*ln(x^3+1)*2^(
2/3)-1/8*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)-1/12*arctan(1/3*(1+2^(2/3)*(-x^
3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 89, 45, 797, 79, 57, 631, 210, 31}

$$\int \frac{x^{11}}{(1-x^3)^{4/3}(1+x^3)} dx = -\frac{\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{1}{5}(1-x^3)^{5/3} + \frac{1}{2}(1-x^3)^{2/3} + \frac{1}{2\sqrt[3]{1-x^3}} + \frac{\log(x^3+1)}{12\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[In] Int[x^11/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) + (1 - x^3)^(2/3)/2 - (1 - x^3)^(5/3)/5 - ArcTan[(1 +
 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(12*2
^(1/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 797

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)*(a/d + (c/e)*x)^p, x] /; F
reeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{(1-x)^{4/3}(1+x)}\,dx, x, x^3\right) \\
&= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{x}{\sqrt[3]{1-x}} - \frac{x}{\sqrt[3]{1-x}\,(-1+x^2)}\right)\,dx, x, x^3\right) \\
&= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{x}{\sqrt[3]{1-x}}\,dx, x, x^3\right)\right) - \frac{1}{3}\text{Subst}\left(\int \frac{x}{\sqrt[3]{1-x}\,(-1+x^2)}\,dx, x, x^3\right) \\
&= -\left(\frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{\sqrt[3]{1-x}} - (1-x)^{2/3}\right)\,dx, x, x^3\right)\right) \\
&\quad - \frac{1}{3}\text{Subst}\left(\int \frac{x}{(-1-x)(1-x)^{4/3}}\,dx, x, x^3\right) \\
&= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{1}{5}\left(1-x^3\right)^{5/3} + \frac{1}{6}\text{Subst}\left(\int \frac{1}{(-1-x)\sqrt[3]{1-x}}\,dx, x, x^3\right) \\
&= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{1}{5}\left(1-x^3\right)^{5/3} + \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} \\
&\quad - \frac{1}{4}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right) + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}
\end{aligned}
$$

$$= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{1}{5}\left(1-x^3\right)^{5/3} + \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

$$= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{1}{5}\left(1-x^3\right)^{5/3} - \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05

$$\int \frac{x^{11}}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{1}{120}\left(-\frac{12(-8+x^3+2x^6)}{\sqrt[3]{1-x^3}} - 10\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) - 10\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) + 5\,2^{2/3}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(\right.\right.$$

```
[In] Integrate[x^11/((1 - x^3)^(4/3)*(1 + x^3)),x]
[Out] ((-12*(-8 + x^3 + 2*x^6))/(1 - x^3)^(1/3) - 10*2^(2/3)*Sqrt[3]*ArcTan[(1 +
2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 10*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(
1/3)] + 5*2^(2/3)*Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)
])/120
```

Maple [A] (verified)

Time = 8.62 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$\frac{-24x^6-10\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)2^{\frac{2}{3}}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}-10\,2^{\frac{2}{3}}\ln\left(\left(-x^3+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)\left(-x^3+1\right)^{\frac{1}{3}}+5\,2^{\frac{2}{3}}\ln\left(\left(-x^3+1\right)\right.}{120\left(-x^3+1\right)^{\frac{1}{3}}}$
trager	$\frac{\left(2x^6+x^3-8\right)\left(-x^3+1\right)^{\frac{2}{3}}}{10x^3-10}+\frac{\operatorname{RootOf}\left(\operatorname{RootOf}\left(\_Z^3+4\right)^2+6\_Z\operatorname{RootOf}\left(\_Z^3+4\right)+36\_Z^2\right)\ln\left(\frac{15\operatorname{RootOf}\left(\operatorname{RootOf}\left(\_\right.\right.}{}\right.}{}$
risch	Expression too large to display

```
[In] int(x^11/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*(-24*x^6-10*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^
(1/2)*(-x^3+1)^(1/3)-10*2^(2/3)*ln((-x^3+1)^(1/3)-2^(1/3))*(-x^3+1)^(1/3)+5
*2^(2/3)*ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))*(-x^3+1)^(1/3)-1
2*x^3+96)/(-x^3+1)^(1/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.22

$$\int \frac{x^{11}}{(1-x^3)^{4/3}(1+x^3)}\,dx =$$
$$-\frac{10\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}(x^3-1)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(2\sqrt{6}(-1)^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}-\sqrt{6}2^{\frac{1}{3}}\right)\right)+5\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}(x^3-1)\log\left(2^{\frac{1}{3}}(-\right.}{}$$

```
[In] integrate(x^11/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/120*(10*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^3 - 1)*arctan(1/6*2^(1/6)*(2*sqrt(
6)*(-1)^(1/3)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3))) + 5*2^(2/3)*(-1)^(1/3)*(
x^3 - 1)*log(2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (-x
^3 + 1)^(2/3)) - 10*2^(2/3)*(-1)^(1/3)*(x^3 - 1)*log(-2^(1/3)*(-1)^(2/3) +
(-x^3 + 1)^(1/3)) - 12*(2*x^6 + x^3 - 8)*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

Sympy [F]

$$\int \frac{x^{11}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^{11}}{\left(-(x-1)(x^2+x+1)\right)^{\frac{4}{3}}(x+1)(x^2-x+1)}\,dx$$

```
[In] integrate(x**11/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**11/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.92

$$\int \frac{x^{11}}{(1-x^3)^{4/3}(1+x^3)}\,dx = -\frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{5}\left(-x^3+1\right)^{\frac{5}{3}} + \frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right) + \frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}} + \frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

```
[In] integrate(x^11/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) - 1/5*(-x^3 + 1)^(5/3) + 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1
)^(1/3) + (-x^3 + 1)^(2/3)) - 1/12*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
 + 1/2*(-x^3 + 1)^(2/3) + 1/2/(-x^3 + 1)^(1/3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.92

$$\int \frac{x^{11}}{(1-x^3)^{4/3}(1+x^3)}\,dx = -\frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{5}\left(-x^3+1\right)^{\frac{5}{3}} + \frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right) + \frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}} + \frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

```
[In] integrate(x^11/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) - 1/5*(-x^3 + 1)^(5/3) + 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1
)^(1/3) + (-x^3 + 1)^(2/3)) - 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1
/3))) + 1/2*(-x^3 + 1)^(2/3) + 1/2/(-x^3 + 1)^(1/3)
```

Mupad [B] (verification not implemented)

Time = 8.49 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.07

$$\int \frac{x^{11}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{2(1-x^3)^{1/3}} - \frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}}{4}\right)}{12} + \frac{(1-x^3)^{2/3}}{2} - \frac{(1-x^3)^{5/3}}{5} - \frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{24} + \frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{24}$$

```
[In] int(x^11/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) - (2^(2/3)*log((1 - x^3)^(1/3)/4 - 2^(1/3)/4))/12 + (
1 - x^3)^(2/3)/2 - (1 - x^3)^(5/3)/5 - (2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^
(1/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1i - 1))/24 + (2^(2/3)*log((1 - x^3)
^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i + 1)^2)/16)*(3^(1/2)*1i + 1))/24
```

3.640 $\int \frac{x^8}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 115

$$\int \frac{x^8}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)} \, dx = \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[Out] 1/2/(-x^3+1)^(1/3)+1/2*(-x^3+1)^(2/3)-1/24*ln(x^3+1)*2^(2/3)+1/8*ln(2^(1/3)
-(-x^3+1)^(1/3))*2^(2/3)+1/12*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2)
)*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {457, 89, 641, 53, 57, 631, 210, 31}

$$\int \frac{x^8}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)} \, dx = \frac{\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{1}{2\sqrt[3]{1-x^3}} - \frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[In] Int[x^8/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) + (1 - x^3)^(2/3)/2 + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(
1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - Log[1 + x^3]/(12*2^(1/3)) + Log[2^(1/3
) - (1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(1-x)^{4/3}(1+x)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{1}{\sqrt[3]{1-x}}+\frac{1}{\sqrt[3]{1-x}\,(1-x^2)}\right)\,dx, x, x^3\right)\\
&= \frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1-x^2)}\,dx, x, x^3\right)\\
&= \frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{4/3}(1+x)}\,dx, x, x^3\right)\\
&= \frac{1}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,dx, x, x^3\right)\\
&= \frac{1}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}\\
&\quad+\frac{1}{4}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right)-\frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}\\
&= \frac{1}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}\\
&\quad-\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\\
&= \frac{1}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.15

$$\int \frac{x^8}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{24}\left(-\frac{12(-2+x^3)}{\sqrt[3]{1-x^3}} + 2\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) + 2\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) - 2^{2/3}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(1-x^3\right.\right.$$

```
[In] Integrate[x^8/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((-12*(-2 + x^3))/(1 - x^3)^(1/3) + 2*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(
1 - x^3)^(1/3))/Sqrt[3]] + 2*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] - 2^
(2/3)*Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/24
```

Maple [A] (verified)

Time = 8.37 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.10

method	result
pseudoelliptic	$-\frac{-2\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)2^{\frac{2}{3}}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+12x^3+2^{\frac{2}{3}}\ln\left(\left(-x^3+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)\left(-x^3+1\right)^{\frac{1}{3}}-2\,2^{\frac{2}{3}}\ln}{24\left(-x^3+1\right)^{\frac{1}{3}}}$
risch	Expression too large to display
trager	Expression too large to display

```
[In] int(x^8/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(-2*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)*(-
x^3+1)^(1/3)+12*x^3+2^(2/3)*ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3
))*(-x^3+1)^(1/3)-2*2^(2/3)*ln((-x^3+1)^(1/3)-2^(1/3))*(-x^3+1)^(1/3)-24)/(
-x^3+1)^(1/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.13

$$\int \frac{x^8}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{2\sqrt{6}2^{\frac{1}{6}}(x^3-1)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}+2\sqrt{6}(-x^3+1)^{\frac{1}{3}}\right)\right)-2^{\frac{2}{3}}(x^3-1)\log\left(2\right.}{}$$

```
[In] integrate(x^8/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/24*(2*sqrt(6)*2^(1/6)*(x^3 - 1)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*s
qrt(6)*(-x^3 + 1)^(1/3))) - 2^(2/3)*(x^3 - 1)*log(2^(2/3) + 2^(1/3)*(-x^3 +
 1)^(1/3) + (-x^3 + 1)^(2/3)) + 2*2^(2/3)*(x^3 - 1)*log(-2^(1/3) + (-x^3 +
1)^(1/3)) + 12*(x^3 - 2)*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

Sympy [F]

$$\int \frac{x^8}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^8}{\left(-(x-1)(x^2+x+1)\right)^{\frac{4}{3}}(x+1)(x^2-x+1)}\,dx$$

```
[In] integrate(x**8/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**8/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.94

$$\int \frac{x^8}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)+\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}+\frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

```
[In] integrate(x^8/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) + 1/12*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) + 1/2*(-x^3 + 1)^(2/3)
+ 1/2/(-x^3 + 1)^(1/3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95

$$\int \frac{x^8}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{12} \cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)+\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}+\frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

```
[In] integrate(x^8/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) + 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/2*(-x^3 + 1)^(
2/3) + 1/2/(-x^3 + 1)^(1/3)
```

Mupad [B] (verification not implemented)

Time = 8.48 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.11

$$\int \frac{x^8}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}}{4}\right)}{12}+\frac{1}{2\,(1-x^3)^{1/3}} + \frac{(1-x^3)^{2/3}}{2}+\frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{24} - \frac{2^{2/3}\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{24}$$

```
[In] int(x^8/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3)/4 - 2^(1/3)/4))/12 + 1/(2*(1 - x^3)^(1/3)) + (
1 - x^3)^(2/3)/2 + (2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i -
1)^2)/16)*(3^(1/2)*1i - 1))/24 - (2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*
(3^(1/2)*1i + 1)^2)/16)*(3^(1/2)*1i + 1))/24
```

3.641 $\int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 100

$$\int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{2\sqrt[3]{1-x^3}} - \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\log(1+x^3)}{12\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[Out] 1/2/(-x^3+1)^(1/3)+1/24*ln(x^3+1)*2^(2/3)-1/8*ln(2^(1/3)-(-x^3+1)^(1/3))*2^
(2/3)-1/12*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {457, 79, 57, 631, 210, 31}

$$\int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)}\,dx = -\frac{\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{1}{2\sqrt[3]{1-x^3}} + \frac{\log(x^3+1)}{12\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[In] Int[x^5/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^
(1/3)*Sqrt[3]) + Log[1 + x^3]/(12*2^(1/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]
/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x}{(1-x)^{4/3}(1+x)}\,dx, x, x^3\right)$$

$$= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,dx, x, x^3\right)$$

$$= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{\log(1+x^3)}{12\sqrt[3]{2}} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right) + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

$$= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{\log(1+x^3)}{12\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

$$= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\log(1+x^3)}{12\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.26

$$\int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{24}\left(\frac{12}{\sqrt[3]{1-x^3}} - 2\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) - 2\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) + 2^{2/3}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(1-\right.\right.$$

```
[In] Integrate[x^5/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (12/(1 - x^3)^(1/3) - 2*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3)
)/Sqrt[3]] - 2*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] + 2^(2/3)*Log[2 +
2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/24
```

Maple [A] (verified)

Time = 8.47 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.22

method	result
pseudoelliptic	$\frac{-2\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)2^{\frac{2}{3}}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}-2\,2^{\frac{2}{3}}\ln\left(\left(-x^3+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)\left(-x^3+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\ln\left(\left(-x^3+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3\right.\right.}{24\left(-x^3+1\right)^{\frac{1}{3}}}$
trager	$-\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{2(x^3-1)}+\frac{\mathrm{RootOf}\left(\_Z^3+4\right)\ln\left(-\frac{6\,\mathrm{RootOf}\left(\mathrm{RootOf}\left(\_Z^3+4\right)^2+6\_Z\,\mathrm{RootOf}\left(\_Z^3+4\right)+36\_Z^2\right)\mathrm{RootOf}\left(\_Z^3+4\right.}{}\right.}{}$
risch	Expression too large to display

[In] int(x^5/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)

[Out] 1/24*(-2*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)*(-x
^3+1)^(1/3)-2*2^(2/3)*ln((-x^3+1)^(1/3)-2^(1/3))*(-x^3+1)^(1/3)+2^(2/3)*ln(
(-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))*(-x^3+1)^(1/3)+12)/(-x^3+1)^
(1/3)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(73) = 146.

Time = 0.33 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.48

$$\int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)}\,dx = -\frac{2\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}(x^3-1)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(2\sqrt{6}(-1)^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}-\sqrt{6}2^{\frac{1}{3}}\right)\right)+2^{\frac{2}{3}}(-1)^{\frac{1}{3}}(x^3-1)\log\left(2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\right.}{24\,(}$$

[In] integrate(x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")

[Out] -1/24*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^3 - 1)*arctan(1/6*2^(1/6)*(2*sqrt(6)
(-1)^(1/3)(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3))) + 2^(2/3)*(-1)^(1/3)*(x^3
- 1)*log(2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (-x^3 +
1)^(2/3)) - 2*2^(2/3)*(-1)^(1/3)*(x^3 - 1)*log(-2^(1/3)*(-1)^(2/3) + (-x^3
+ 1)^(1/3)) + 12*(-x^3 + 1)^(2/3))/(x^3 - 1)

Sympy [F]

$$\int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^5}{\left(-(x-1)(x^2+x+1)\right)^{\frac{4}{3}}(x+1)(x^2-x+1)}\,dx$$

```
[In] integrate(x**5/(-x**3+1)**(4/3)/(x**3+1),x)
[Out] Integral(x**5/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.97

$$\int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)}\,dx = -\frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) + \frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right) + \frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

```
[In] integrate(x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
[Out] -1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) + 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2
/3)) - 1/12*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) + 1/2/(-x^3 + 1)^(1/3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98

$$\int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)}\,dx = -\frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) + \frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right) + \frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

```
[In] integrate(x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
[Out] -1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) + 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2
/3)) - 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/2/(-x^3 + 1)^
(1/3)
```

Mupad [B] (verification not implemented)

Time = 8.43 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.17

$$\int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{2\,(1-x^3)^{1/3}} - \frac{2^{2/3}\,\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}}{4}\right)}{12} - \frac{2^{2/3}\,\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{24} + \frac{2^{2/3}\,\ln\left(\frac{(1-x^3)^{1/3}}{4}-\frac{2^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{24}$$

```
[In] int(x^5/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) - (2^(2/3)*log((1 - x^3)^(1/3)/4 - 2^(1/3)/4))/12 - (
2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1
i - 1))/24 + (2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i + 1)^2)/
16)*(3^(1/2)*1i + 1))/24
```

3.642 $\int \frac{x^2}{(1-x^3)^{4/3}(1+x^3)} dx$

Optimal result

Integrand size = 22, antiderivative size = 100

$$\int \frac{x^2}{(1-x^3)^{4/3}(1+x^3)} dx = \frac{1}{2\sqrt[3]{1-x^3}} + \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[Out] 1/2/(-x^3+1)^(1/3)-1/24*ln(x^3+1)*2^(2/3)+1/8*ln(2^(1/3)-(-x^3+1)^(1/3))*2^
(2/3)+1/12*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {455, 53, 57, 631, 210, 31}

$$\int \frac{x^2}{(1-x^3)^{4/3}(1+x^3)} dx = \frac{\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{1}{2\sqrt[3]{1-x^3}} - \frac{\log(x^3+1)}{12\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[In] Int[x^2/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^
(1/3)*Sqrt[3]) - Log[1 + x^3]/(12*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]
/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{4/3}(1+x)}\,dx, x, x^3\right)$$

$$
\begin{aligned}
&= \frac{1}{2\sqrt[3]{1-x^3}}+\frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,dx,x,x^3\right)\\
&= \frac{1}{2\sqrt[3]{1-x^3}}-\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}\\
&\quad +\frac{1}{4}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx,x,\sqrt[3]{1-x^3}\right)-\frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx,x,\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}\\
&= \frac{1}{2\sqrt[3]{1-x^3}}-\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}-\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2^{2/3}\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\\
&= \frac{1}{2\sqrt[3]{1-x^3}}+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.27

$$
\int \frac{x^2}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{1}{24}\left(\frac{12}{\sqrt[3]{1-x^3}}\right.
$$
$$
+2\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)+2\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)-2^{2/3}\log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}(1-x
$$

```
[In] Integrate[x^2/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (12/(1 - x^3)^(1/3) + 2*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3)
)/Sqrt[3]] + 2*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] - 2^(2/3)*Log[2 +
2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/24
```

Maple [A] (verified)

Time = 8.42 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.22

method	result
pseudoelliptic	$-\frac{-2\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)2^{\frac{2}{3}}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\ln\left(\left(-x^3+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)\left(-x^3+1\right)^{\frac{1}{3}}-2\,2^{\frac{2}{3}}\ln\left(\left(-x\right.\right.}{24\left(-x^3+1\right)^{\frac{1}{3}}}$
trager	$-\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{2\left(x^3-1\right)}+\frac{\operatorname{RootOf}\left(\_Z^3-4\right)\ln\left(\frac{-6\operatorname{RootOf}\left(\operatorname{RootOf}\left(\_Z^3-4\right)^2+6\_Z\operatorname{RootOf}\left(\_Z^3-4\right)+36\_Z^2\right)\operatorname{RootOf}\left(\_Z^3-4\right.}{}\right.}{}$
risch	Expression too large to display

[In] int(x^2/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)

[Out] -1/24*(-2*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)*(-x^3+1)^(1/3)+2^(2/3)*ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))*(-x^3+1)^(1/3)-2*2^(2/3)*ln((-x^3+1)^(1/3)-2^(1/3))*(-x^3+1)^(1/3)-12)/(-x^3+1)^(1/3)

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.25

$$\int \frac{x^2}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=\frac{2\sqrt{6}2^{\frac{1}{6}}\left(x^3-1\right)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}+2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-2^{\frac{2}{3}}\left(x^3-1\right)\log\left(2^{\frac{2}{3}}\right.}{}$$

[In] integrate(x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")

[Out] 1/24*(2*sqrt(6)*2^(1/6)*(x^3 - 1)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*sqrt(6)*(-x^3 + 1)^(1/3))) - 2^(2/3)*(x^3 - 1)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 2*2^(2/3)*(x^3 - 1)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) - 12*(-x^3 + 1)^(2/3))/(x^3 - 1)

Sympy [F]

$$\int \frac{x^2}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=\int \frac{x^2}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

[In] integrate(x**2/(-x**3+1)**(4/3)/(x**3+1),x)

[Out] Integral(x**2/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right) + \frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

```
[In] integrate(x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) + 1/12*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) + 1/2/(-x^3 + 1)^(1/3)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98

$$\int \frac{x^2}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right) + \frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}}$$

```
[In] integrate(x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) + 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/2/(-x^3 + 1)^(
1/3)
```

Mupad [B] (verification not implemented)

Time = 8.45 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.17

$$\int \frac{x^2}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{2^{2/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/3}}{4}\right)}{12}+\frac{1}{2\left(1-x^3\right)^{1/3}} + \frac{2^{2/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{24} - \frac{2^{2/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{24}$$

```
[In] int(x^2/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3)/4 - 2^(1/3)/4))/12 + 1/(2*(1 - x^3)^(1/3)) + (
2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1
i - 1))/24 - (2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i + 1)^2)/
16)*(3^(1/2)*1i + 1))/24
```

3.643 $\int \frac{1}{x(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 154

$$\int \frac{1}{x(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{2\sqrt[3]{1-x^3}}+\frac{\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\log(x)}{2}+\frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[Out] 1/2/(-x^3+1)^(1/3)-1/2*ln(x)+1/24*ln(x^3+1)*2^(2/3)+1/2*ln(1-(-x^3+1)^(1/3)
)-1/8*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)+1/3*arctan(1/3*(1+2*(-x^3+1)^(1/3)
)*3^(1/2))*3^(1/2)-1/12*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2
/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {457, 87, 162, 57, 632, 210, 31, 631}

$$\int \frac{1}{x(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{\arctan\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{1}{2\sqrt[3]{1-x^3}}+\frac{\log(x^3+1)}{12\sqrt[3]{2}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}-\frac{\log(x)}{2}$$

```
[In] Int[1/(x*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) + ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - A
rcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - Log[x]/2
 + Log[1 + x^3]/(12*2^(1/3)) + Log[1 - (1 - x^3)^(1/3)]/2 - Log[2^(1/3) - (
1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 87

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x
)*((e + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{4/3}x(1+x)}\,dx, x, x^3\right)\\
&= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{6}\text{Subst}\left(\int \frac{2+x}{\sqrt[3]{1-x}x(1+x)}\,dx, x, x^3\right)\\
&= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,dx, x, x^3\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x}\,dx, x, x^3\right)\\
&= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{\log(x)}{2} + \frac{\log(1+x^3)}{12\sqrt[3]{2}} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right)\\
&\quad - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right)\\
&\quad + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right) + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}\\
&= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{\log(x)}{2} + \frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}\\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} - \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^3}\right)\\
&= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log(x)}{2}\\
&\quad + \frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.29

$$\int \frac{1}{x(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{24}\left(\frac{12}{\sqrt[3]{1-x^3}}+8\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)\right.$$
$$-2\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)+8\log\left(-1+\sqrt[3]{1-x^3}\right)-2\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)-4\log\left(1+\sqrt[3]{1}\right.$$

```
[In] Integrate[1/(x*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (12/(1 - x^3)^(1/3) + 8*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] - 2
*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 8*Log[-1 +
 (1 - x^3)^(1/3)] - 2*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^3)^(1/3)] - 4*Log[1 +
 (1 - x^3)^(1/3) + (1 - x^3)^(2/3)] + 2^(2/3)*Log[2 + 2^(2/3)*(1 - x^3)^(1/
3) + 2^(1/3)*(1 - x^3)^(2/3)])/24
```

Maple [A] (verified)

Time = 6.86 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.36

method	result
pseudoelliptic	$\frac{-2\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)2^{\frac{2}{3}}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}-2\,2^{\frac{2}{3}}\ln\left(\left(-x^3+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)\left(-x^3+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\ln\left(\left(-x^3+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3\right.\right.}{}$

```
[In] int(1/x/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(-2*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)*(-x
^3+1)^(1/3)-2*2^(2/3)*ln((-x^3+1)^(1/3)-2^(1/3))*(-x^3+1)^(1/3)+2^(2/3)*ln(
(-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/3)+2^(2/3))*(-x^3+1)^(1/3)+8*arctan(1/3*
(1+2*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*(-x^3+1)^(1/3)-4*ln((-x^3+1)^(2/3)+(-
x^3+1)^(1/3)+1)*(-x^3+1)^(1/3)+8*ln(-1+(-x^3+1)^(1/3))*(-x^3+1)^(1/3)+12)/(
-x^3+1)^(1/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.47

$$\int \frac{1}{x(1-x^3)^{4/3}(1+x^3)}\,dx = -\frac{2\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}(x^3-1)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(2\sqrt{6}(-1)^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}-\sqrt{6}2^{\frac{1}{3}}\right)\right)+2^{\frac{2}{3}}(-1)^{\frac{1}{3}}(x^3-1)\log\left(2^{\frac{1}{3}}(-1)\right.}{}$$

```
[In] integrate(1/x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^3 - 1)*arctan(1/6*2^(1/6)*(2*sqrt(6)
*(-1)^(1/3)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3))) + 2^(2/3)*(-1)^(1/3)*(x^3
- 1)*log(2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (-x^3 +
 1)^(2/3)) - 2*2^(2/3)*(-1)^(1/3)*(x^3 - 1)*log(-2^(1/3)*(-1)^(2/3) + (-x^3
 + 1)^(1/3)) - 8*sqrt(3)*(x^3 - 1)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/
3*sqrt(3)) + 4*(x^3 - 1)*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) - 8*(
x^3 - 1)*log((-x^3 + 1)^(1/3) - 1) + 12*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

Sympy [F]

$$\int \frac{1}{x(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{1}{x\left(-(x-1)(x^2+x+1)\right)^{\frac{4}{3}}(x+1)(x^2-x+1)}\,dx$$

```
[In] integrate(1/x/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{x(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}x}\,dx$$

```
[In] integrate(1/x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = -\frac{1}{12}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) + \frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right) + \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right) + \frac{1}{2\left(-x^3+1\right)^{\frac{1}{3}}} - \frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right) + \frac{1}{3}\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

```
[In] integrate(1/x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) + 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2
/3)) - 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/3*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + 1/2/(-x^3 + 1)^(1/3) - 1/6*log(
(-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log(abs((-x^3 + 1)^(1/3) - 1
))
```

Mupad [B] (verification not implemented)

Time = 8.56 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.64

$$\int \frac{1}{x\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{\ln\left(\frac{17}{4}-\frac{17\left(1-x^3\right)^{1/3}}{4}\right)}{3} + \ln\left(\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)\left(1458\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)^2-\frac{459\left(1-x^3\right)^{1/3}}{4}\right)-\frac{63}{4}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)-\ln\left(\left(\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)\right.$$

```
[In] int(1/(x*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] log(17/4 - (17*(1 - x^3)^(1/3))/4)/3 + log(((3^(1/2)*1i)/6 - 1/6)*(1458*((3
^(1/2)*1i)/6 - 1/6)^2 - (459*(1 - x^3)^(1/3))/4) - 63/4)*((3^(1/2)*1i)/6 -
1/6) - log(((3^(1/2)*1i)/6 + 1/6)*(1458*((3^(1/2)*1i)/6 + 1/6)^2 - (459*(1
- x^3)^(1/3))/4) + 63/4)*((3^(1/2)*1i)/6 + 1/6) - (2^(2/3)*log((2^(2/3)*((8
1*2^(1/3))/4 - (459*(1 - x^3)^(1/3))/4))/12 + 63/4))/12 + 1/(2*(1 - x^3)^(1
```

```
/3)) + ((-1)^(1/3)*2^(2/3)*log(((-1)^(1/3)*2^(2/3)*((81*(-1)^(2/3)*2^(1/3))
/4 - (459*(1 - x^3)^(1/3))/4))/12 - 63/4))/12 - ((-1)^(1/3)*2^(2/3)*log(((-
1)^(1/3)*2^(2/3)*(3^(1/2)*1i + 1)*((459*(1 - x^3)^(1/3))/4 - (81*(-1)^(2/3)
*2^(1/3)*(3^(1/2)*1i + 1)^2)/16))/24 - 63/4)*(3^(1/2)*1i + 1))/24
```

3.644 $\int \frac{1}{x^4(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 175

$$\int \frac{1}{x^4(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{5}{6\sqrt[3]{1-x^3}} - \frac{1}{3x^3\sqrt[3]{1-x^3}} + \frac{\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log(x)}{6} - \frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right) + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[Out] 5/6/(-x^3+1)^(1/3)-1/3/x^3/(-x^3+1)^(1/3)-1/6*ln(x)-1/24*ln(x^3+1)*2^(2/3)+
1/6*ln(1-(-x^3+1)^(1/3))+1/8*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)+1/9*arctan(
1/3*(1+2*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+1/12*arctan(1/3*(1+2^(2/3)*(-x^3+
1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 105, 162, 53, 57, 632, 210, 31, 631}

$$\int \frac{1}{x^4 (1-x^3)^{4/3} (1+x^3)} \, dx = \frac{\arctan\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{1}{3x^3\sqrt[3]{1-x^3}} + \frac{5}{6\sqrt[3]{1-x^3}} - \frac{\log(x^3+1)}{12\sqrt[3]{2}} + \frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right) + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} - \frac{\log(x)}{6}$$

```
[In] Int[1/(x^4*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 5/(6*(1 - x^3)^(1/3)) - 1/(3*x^3*(1 - x^3)^(1/3)) + ArcTan[(1 + 2*(1 - x^3)
^(1/3))/Sqrt[3]]/(3*Sqrt[3]) + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]
]/(2*2^(1/3)*Sqrt[3]) - Log[x]/6 - Log[1 + x^3]/(12*2^(1/3)) + Log[1 - (1 -
 x^3)^(1/3)]/6 + Log[2^(1/3) - (1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{4/3}x^2(1+x)}\,dx, x, x^3\right)$$

$$
\begin{aligned}
&= -\frac{1}{3x^3\sqrt[3]{1-x^3}} - \frac{1}{3}\text{Subst}\left(\int \frac{-\frac{1}{3}-\frac{4x}{3}}{(1-x)^{4/3}x(1+x)}\,dx, x, x^3\right) \\
&= -\frac{1}{3x^3\sqrt[3]{1-x^3}} + \frac{1}{9}\text{Subst}\left(\int \frac{1}{(1-x)^{4/3}x}\,dx, x, x^3\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{4/3}(1+x)}\,dx, x, x^3\right) \\
&= \frac{5}{6\sqrt[3]{1-x^3}} - \frac{1}{3x^3\sqrt[3]{1-x^3}} + \frac{1}{9}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}x}\,dx, x, x^3\right) \\
&\quad + \frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}(1+x)}\,dx, x, x^3\right) \\
&= \frac{5}{6\sqrt[3]{1-x^3}} - \frac{1}{3x^3\sqrt[3]{1-x^3}} - \frac{\log(x)}{6} - \frac{\log(1+x^3)}{12\sqrt[3]{2}} \\
&\quad - \frac{1}{6}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right) + \frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right) \\
&\quad + \frac{1}{4}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right) - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} \\
&= \frac{5}{6\sqrt[3]{1-x^3}} - \frac{1}{3x^3\sqrt[3]{1-x^3}} - \frac{\log(x)}{6} - \frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right) \\
&\quad + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^3}\right) \\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} \\
&= \frac{5}{6\sqrt[3]{1-x^3}} - \frac{1}{3x^3\sqrt[3]{1-x^3}} + \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} \\
&\quad - \frac{\log(x)}{6} - \frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right) + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.19

$$
\int \frac{1}{x^4(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{72}\left(\frac{12(-2+5x^3)}{x^3\sqrt[3]{1-x^3}} + 8\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)\right.
$$
$$
+6\,2^{2/3}\sqrt{3}\arctan\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)+8\log\left(-1+\sqrt[3]{1-x^3}\right)+6\,2^{2/3}\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right)-4\log\left(1+\sqrt[3]{\right.
$$

```
[In] Integrate[1/(x^4*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((12*(-2 + 5*x^3))/(x^3*(1 - x^3)^(1/3)) + 8*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3
)^(1/3))/Sqrt[3]] + 6*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/
Sqrt[3]] + 8*Log[-1 + (1 - x^3)^(1/3)] + 6*2^(2/3)*Log[-2 + 2^(2/3)*(1 - x^
3)^(1/3)] - 4*Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)] - 3*2^(2/3)*Log[2
+ 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(2/3)])/72
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(131) = 262$.

Time = 6.55 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.53

method	result
pseudoelliptic	$-\frac{\frac{3\,2^{\frac{2}{3}}\sqrt{3}\,\arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)x^{3}\left(-x^{3}+1\right)^{\frac{1}{3}}}{4}+\sqrt{3}\,\arctan\left(\frac{\left(1+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)x^{3}\left(-x^{3}+1\right)^{\frac{1}{3}}-\frac{3\,2^{\frac{2}{3}}\ln\left(\left(-x^{3}+1\right)^{\frac{2}{3}}\right.}{}}{9(-x^{3}+}$

```
[In] int(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9/(-x^3+1)^(1/3)*(3/4*2^(2/3)*3^(1/2)*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/
3))*3^(1/2))*x^3*(-x^3+1)^(1/3)+3^(1/2)*arctan(1/3*(1+2*(-x^3+1)^(1/3))*3^(
1/2))*x^3*(-x^3+1)^(1/3)-3/8*2^(2/3)*ln((-x^3+1)^(2/3)+2^(1/3)*(-x^3+1)^(1/
3)+2^(2/3))*x^3*(-x^3+1)^(1/3)+3/4*2^(2/3)*ln((-x^3+1)^(1/3)-2^(1/3))*x^3*(
-x^3+1)^(1/3)-1/2*ln((-x^3+1)^(2/3)+(-x^3+1)^(1/3)+1)*x^3*(-x^3+1)^(1/3)+ln
(-1+(-x^3+1)^(1/3))*x^3*(-x^3+1)^(1/3)+15/2*x^3-3)/((-x^3+1)^(2/3)+(-x^3+1)
^(1/3)+1)/(-1+(-x^3+1)^(1/3))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.36

$$\int \frac{1}{x^4 \left(1-x^3\right)^{4/3} (1+x^3)}\,dx = \frac{6\sqrt{6}2^{\frac{1}{6}}(x^6-x^3)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}+2\sqrt{6}(-x^3+1)^{\frac{1}{3}}\right)\right)-3\cdot 2^{\frac{2}{3}}(x^6-x^3)}{}$$

```
[In] integrate(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/72*(6*sqrt(6)*2^(1/6)*(x^6 - x^3)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2
*sqrt(6)*(-x^3 + 1)^(1/3))) - 3*2^(2/3)*(x^6 - x^3)*log(2^(2/3) + 2^(1/3)*(
-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 6*2^(2/3)*(x^6 - x^3)*log(-2^(1/3) +
(-x^3 + 1)^(1/3)) + 8*sqrt(3)*(x^6 - x^3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/
3) + 1/3*sqrt(3)) - 4*(x^6 - x^3)*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) +
 1) + 8*(x^6 - x^3)*log((-x^3 + 1)^(1/3) - 1) - 12*(5*x^3 - 2)*(-x^3 + 1)^(
2/3))/(x^6 - x^3)
```

Sympy [F]

$$\int \frac{1}{x^4 (1-x^3)^{4/3} (1+x^3)} \, dx = \int \frac{1}{x^4 (-(x-1)(x^2+x+1))^{\frac{4}{3}} (x+1)(x^2-x+1)} \, dx$$

```
[In] integrate(1/x**4/(-x**3+1)**(4/3)/(x**3+1),x)
[Out] Integral(1/(x**4*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Maxima [F]

$$\int \frac{1}{x^4 (1-x^3)^{4/3} (1+x^3)} \, dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}} x^4} \, dx$$

```
[In] integrate(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.03

$$\begin{aligned}\int \frac{1}{x^4 (1-x^3)^{4/3} (1+x^3)} \, dx &= \frac{1}{12} \sqrt{3} 2^{\frac{2}{3}} \arctan\left(\frac{1}{6} \sqrt{3} 2^{\frac{2}{3}} \left(2^{\frac{1}{3}} + 2 (-x^3+1)^{\frac{1}{3}}\right)\right) \\ &\quad - \frac{1}{24} \cdot 2^{\frac{2}{3}} \log\left(2^{\frac{2}{3}} + 2^{\frac{1}{3}} (-x^3+1)^{\frac{1}{3}} + (-x^3+1)^{\frac{2}{3}}\right) + \frac{1}{12} \cdot 2^{\frac{2}{3}} \log\left(\left|-2^{\frac{1}{3}} + (-x^3+1)^{\frac{1}{3}}\right|\right) \\ &\quad + \frac{1}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \left(2 (-x^3+1)^{\frac{1}{3}} + 1\right)\right) - \frac{5 x^3 - 2}{6 \left((-x^3+1)^{\frac{4}{3}} - (-x^3+1)^{\frac{1}{3}}\right)} \\ &\quad - \frac{1}{18} \log\left((-x^3+1)^{\frac{2}{3}} + (-x^3+1)^{\frac{1}{3}} + 1\right) + \frac{1}{9} \log\left(\left|(-x^3+1)^{\frac{1}{3}} - 1\right|\right)\end{aligned}$$

```
[In] integrate(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
[Out] 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) + 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/9*sqrt(3)*arct
an(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/6*(5*x^3 - 2)/((-x^3 + 1)^(4/3
) - (-x^3 + 1)^(1/3)) - 1/18*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) +
 1/9*log(abs((-x^3 + 1)^(1/3) - 1))
```

Mupad [B] (verification not implemented)

Time = 8.60 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.28

$$\int \frac{1}{x^4 (1-x^3)^{4/3} (1+x^3)} dx = \frac{\ln \left(\frac{11 (1-x^3)^{1/3}}{972} - \frac{11}{972}\right)}{9} + \frac{2^{2/3} \ln \left(\frac{2^{1/3} \left(\frac{2^{2/3} \left(\frac{81\, 2^{1/3}}{4} - \frac{75 (1-x^3)^{1/3}}{4}\right)}{12} - \frac{35}{12}\right)}{72} + \frac{(1-x^3)^{1/3}}{27}\right)}{12}$$

$$+\ln \left(\left(-\frac{1}{18} + \frac{\sqrt{3}\, 1\mathrm{i}}{18}\right)^2 \left(\left(-\frac{1}{18}+\frac{\sqrt{3}\, 1\mathrm{i}}{18}\right) \left(1458 \left(-\frac{1}{18} + \frac{\sqrt{3}\, 1\mathrm{i}}{18}\right)^2 - \frac{75 (1-x^3)^{1/3}}{4}\right) - \frac{35}{12}\right) + \frac{(1-x^3)^{1/3}}{27}\right.$$

```
[In] int(1/(x^4*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] log((11*(1 - x^3)^(1/3))/972 - 11/972)/9 + (2^(2/3)*log((2^(1/3)*((2^(2/3)*
((81*2^(1/3))/4 - (75*(1 - x^3)^(1/3))/4))/12 - 35/12))/72 + (1 - x^3)^(1/3
)/27))/12 + log(((3^(1/2)*1i)/18 - 1/18)^2*(((3^(1/2)*1i)/18 - 1/18)*(1458*
((3^(1/2)*1i)/18 - 1/18)^2 - (75*(1 - x^3)^(1/3))/4) - 35/12) + (1 - x^3)^(
1/3)/27)*((3^(1/2)*1i)/18 - 1/18) - log((1 - x^3)^(1/3)/27 - ((3^(1/2)*1i)/
18 + 1/18)^2*(((3^(1/2)*1i)/18 + 1/18)*(1458*((3^(1/2)*1i)/18 + 1/18)^2 - (
75*(1 - x^3)^(1/3))/4) + 35/12))*((3^(1/2)*1i)/18 + 1/18) + ((5*x^3)/6 - 1/
3)/((1 - x^3)^(1/3) - (1 - x^3)^(4/3)) + (2^(2/3)*log((1 - x^3)^(1/3)/27 +
(2^(1/3)*(3^(1/2)*1i - 1)^2*((2^(2/3)*(3^(1/2)*1i - 1)*((81*2^(1/3)*(3^(1/2
)*1i - 1)^2)/16 - (75*(1 - x^3)^(1/3))/4))/24 - 35/12))/288)*(3^(1/2)*1i -
1))/24 - (2^(2/3)*log((1 - x^3)^(1/3)/27 - (2^(1/3)*(3^(1/2)*1i + 1)^2*((2^
(2/3)*(3^(1/2)*1i + 1)*((81*2^(1/3)*(3^(1/2)*1i + 1)^2)/16 - (75*(1 - x^3)^
(1/3))/4))/24 + 35/12))/288)*(3^(1/2)*1i + 1))/24
```

3.645 $\int \frac{x^9}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 174

$$\int \frac{x^9}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{x^4}{2\sqrt[3]{1-x^3}}+\frac{5}{6}x\left(1-x^3\right)^{2/3}+\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}-\frac{1}{6}\log\left(x+\sqrt[3]{1-x^3}\right)$$

```
[Out] 1/2*x^4/(-x^3+1)^(1/3)+5/6*x*(-x^3+1)^(2/3)+1/24*ln(x^3+1)*2^(2/3)-1/8*ln(-
2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/6*ln(x+(-x^3+1)^(1/3))+1/9*arctan(1/3*(
1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+1/12*arctan(1/3*(1-2*2^(1/3)*x/(-x^3
+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used

$= \{481, 596, 544, 245, 384\}$

$$\int \frac{x^9}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{5}{6}\left(1-x^3\right)^{2/3}x + \frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}} - \frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{4\sqrt[3]{2}} - \frac{1}{6}\log\left(\sqrt[3]{1-x^3}+x\right) + \frac{x^4}{2\sqrt[3]{1-x^3}}$$

```
[In] Int[x^9/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] x^4/(2*(1 - x^3)^(1/3)) + (5*x*(1 - x^3)^(2/3))/6 + ArcTan[(1 - (2*x)/(1 -
x^3)^(1/3))/Sqrt[3]]/(3*Sqrt[3]) + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3
))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(12*2^(1/3)) - Log[-(2^(1/3)
*x) - (1 - x^3)^(1/3)]/(4*2^(1/3)) - Log[x + (1 - x^3)^(1/3)]/6
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^4}{2\sqrt[3]{1-x^3}} - \frac{1}{2}\int \frac{x^3(4+5x^3)}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx \\
&= \frac{x^4}{2\sqrt[3]{1-x^3}} + \frac{5}{6}x\left(1-x^3\right)^{2/3} - \frac{1}{6}\int \frac{5+2x^3}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx \\
&= \frac{x^4}{2\sqrt[3]{1-x^3}} + \frac{5}{6}x\left(1-x^3\right)^{2/3} - \frac{1}{3}\int \frac{1}{\sqrt[3]{1-x^3}}\,dx - \frac{1}{2}\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx \\
&= \frac{x^4}{2\sqrt[3]{1-x^3}} + \frac{5}{6}x\left(1-x^3\right)^{2/3} + \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} \\
&\quad + \frac{\log(1+x^3)}{12\sqrt[3]{2}} - \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} - \frac{1}{6}\log\left(x+\sqrt[3]{1-x^3}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.30

$$\int \frac{x^9}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{72}\left(-\frac{12x(-5+2x^3)}{\sqrt[3]{1-x^3}} + 8\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2\sqrt[3]{1-x^3}}\right)\right.$$
$$+6\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right) - 8\log\left(x+\sqrt[3]{1-x^3}\right) - 6\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right) + 4\log\left(x^2-x\sqrt[3]{\right.$$

```
[In] Integrate[x^9/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((-12*x*(-5 + 2*x^3))/(1 - x^3)^(1/3) + 8*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2
*(1 - x^3)^(1/3))] + 6*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 -
 x^3)^(1/3))] - 8*Log[x + (1 - x^3)^(1/3)] - 6*2^(2/3)*Log[2*x + 2^(2/3)*(1
 - x^3)^(1/3)] + 4*Log[x^2 - x*(1 - x^3)^(1/3) + (1 - x^3)^(2/3)] + 3*2^(2/
3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/72
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(132) = 264$.

Time = 6.65 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.68

method	result
pseudoelliptic	$\frac{-6\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)\left(-x^3+1\right)^{\frac{1}{3}}-6\,2^{\frac{2}{3}}\ln\left(\frac{2^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)\left(-x^3+1\right)^{\frac{1}{3}}+3\,2^{\frac{2}{3}}\ln\left(\frac{2^{\frac{2}{3}}x^2-2^{\frac{1}{3}}\left(-x^3+1\right)\cdots}{x\cdots}\right.}{72\left(\left(\cdots\right.\right.}$

```
[In] int(x^9/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/72*(-6*3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x)*
(-x^3+1)^(1/3)-6*2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)*(-x^3+1)^(1/3)+3*
2^(2/3)*ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))/x^2)*(-x^3
+1)^(1/3)-24*x^4-8*3^(1/2)*arctan(1/3*(-2*(-x^3+1)^(1/3)+x)*3^(1/2)/x)*(-x^
3+1)^(1/3)+4*ln(((-x^3+1)^(2/3)-(-x^3+1)^(1/3)*x+x^2)/x^2)*(-x^3+1)^(1/3)-8
*ln((x+(-x^3+1)^(1/3))/x)*(-x^3+1)^(1/3)+60*x)/((-x^3+1)^(2/3)-(-x^3+1)^(1/
3)*x+x^2)/(x+(-x^3+1)^(1/3))/(-x^3+1)^(1/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(132) = 264$.

Time = 0.32 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.56

$$\int \frac{x^9}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{6\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\left(x^3-1\right)\arctan\left(\frac{2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}x+2\sqrt{6}(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}\right)}{6x}\right)+6\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^3\cdots\right.}{\cdots}$$

```
[In] integrate(x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/72*(6*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^3 - 1)*arctan(1/6*2^(1/6)*(sqrt(6)*2^
(1/3)*x + 2*sqrt(6)*(-1)^(1/3)*(-x^3 + 1)^(1/3))/x) + 6*2^(2/3)*(-1)^(1/3)*
(x^3 - 1)*log((2^(1/3)*(-1)^(2/3)*x + (-x^3 + 1)^(1/3))/x) - 3*2^(2/3)*(-1)
^(1/3)*(x^3 - 1)*log(-(2^(2/3)*(-1)^(1/3)*x^2 + 2^(1/3)*(-1)^(2/3)*(-x^3 +
```

```
1)^(1/3)*x - (-x^3 + 1)^(2/3))/x^2) + 8*sqrt(3)*(x^3 - 1)*arctan(-1/3*(sqrt
(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) - 8*(x^3 - 1)*log((x + (-x^3 + 1)^(1
/3))/x) + 4*(x^3 - 1)*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2
) + 12*(2*x^4 - 5*x)*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

Sympy [F]

$$\int \frac{x^9}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^9}{\left(-(x-1)(x^2+x+1)\right)^{\frac{4}{3}}(x+1)(x^2-x+1)}\,dx$$

```
[In] integrate(x**9/(-x**3+1)**(4/3)/(x**3+1),x)
[Out] Integral(x**9/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{x^9}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^9}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x^9/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Giac [F]

$$\int \frac{x^9}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^9}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x^9/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^9}{(1-x^3)^{4/3}\,(x^3+1)}\,dx$$

```
[In] int(x^9/((1 - x^3)^(4/3)*(x^3 + 1)),x)
[Out] int(x^9/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.646 $\int \frac{x^6}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 153

$$\int \frac{x^6}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{x}{2\sqrt[3]{1-x^3}}+\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} - \frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right)$$

```
[Out] 1/2*x/(-x^3+1)^(1/3)-1/24*ln(x^3+1)*2^(2/3)+1/8*ln(-2^(1/3)*x-(-x^3+1)^(1/3
))*2^(2/3)-1/2*ln(x+(-x^3+1)^(1/3))+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3
^(1/2))*3^(1/2)-1/12*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(
2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {481, 544, 245, 384}

$$\int \frac{x^6}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{x}{2\sqrt[3]{1-x^3}} - \frac{\log(x^3+1)}{12\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{4\sqrt[3]{2}} - \frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)$$

```
[In] Int[x^6/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] x/(2*(1 - x^3)^(1/3)) + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]
 - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3])
- Log[1 + x^3]/(12*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(4*2^(1/3
)) - Log[x + (1 - x^3)^(1/3)]/2
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x}{2\sqrt[3]{1-x^3}} - \frac{1}{2}\int \frac{1+2x^3}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx \\
&= \frac{x}{2\sqrt[3]{1-x^3}} + \frac{1}{2}\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx - \int \frac{1}{\sqrt[3]{1-x^3}}\,dx \\
&= \frac{x}{2\sqrt[3]{1-x^3}} + \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} \\
&\quad - \frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} - \frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.44

$$\int \frac{x^6}{(1-x^3)^{4/3}\,(1+x^3)}\,dx = \frac{1}{24}\left(\frac{12x}{\sqrt[3]{1-x^3}} + 8\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2\sqrt[3]{1-x^3}}\right)\right.$$
$$-2\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right) - 8\log\left(x+\sqrt[3]{1-x^3}\right) + 2\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right) + 4\log\left(x^2-x\sqrt[3]{\right.$$

```
[In] Integrate[x^6/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((12*x)/(1 - x^3)^(1/3) + 8*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2*(1 - x^3)^(1/
3))] - 2*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))]
- 8*Log[x + (1 - x^3)^(1/3)] + 2*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)]
 + 4*Log[x^2 - x*(1 - x^3)^(1/3) + (1 - x^3)^(2/3)] - 2^(2/3)*Log[-2*x^2 +
2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/24
```

Maple [A] (verified)

Time = 5.54 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.43

method	result
pseudoelliptic	$\frac{2^{\frac{2}{3}} \ln\left(\frac{2^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right) \left(-x^{3}+1\right)^{\frac{1}{3}}-4 \ln\left(\frac{x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right) \left(-x^{3}+1\right)^{\frac{1}{3}}+\left(\left(-\frac{\ln\left(\frac{2^{\frac{2}{3}} x^{2}-2^{\frac{1}{3}} \left(-x^{3}+1\right)^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)}{2}+\sqrt{3}\right.\right.}{1}$

```
[In] int(x^6/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/(-x^3+1)^(1/3)*(2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)*(-x^3+1)^(1/3
)-4*ln((x+(-x^3+1)^(1/3))/x)*(-x^3+1)^(1/3)+((-1/2*ln((2^(2/3)*x^2-2^(1/3)*
(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))/x^2)+3^(1/2)*arctan(1/3*3^(1/2)*(-2^(2/3)*
(-x^3+1)^(1/3)+x)/x))*2^(2/3)-4*3^(1/2)*arctan(1/3*(-2*(-x^3+1)^(1/3)+x)*3^
(1/2)/x)+2*ln(((-x^3+1)^(2/3)-(-x^3+1)^(1/3)*x+x^2)/x^2))*(-x^3+1)^(1/3)+6*
x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(118) = 236.

Time = 0.33 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.56

$$\int \frac{x^6}{(1-x^3)^{4/3}(1+x^3)}\,dx = -\frac{2\sqrt{6}2^{\frac{1}{6}}(x^3-1)\arctan\left(-\frac{2^{\frac{1}{6}}\left(\sqrt{6}2^{\frac{1}{3}}x-2\sqrt{6}(-x^3+1)^{\frac{1}{3}}\right)}{6x}\right)-2\cdot 2^{\frac{2}{3}}(x^3-1)\log\left(\frac{2^{\frac{1}{3}}x+(-x^3+1)^{\frac{1}{3}}}{x}\right)+2^{\frac{2}{3}}(x^3-1)\log}{}$$

```
[In] integrate(x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(6)*2^(1/6)*(x^3 - 1)*arctan(-1/6*2^(1/6)*(sqrt(6)*2^(1/3)*x -
 2*sqrt(6)*(-x^3 + 1)^(1/3))/x) - 2*2^(2/3)*(x^3 - 1)*log((2^(1/3)*x + (-x^
3 + 1)^(1/3))/x) + 2^(2/3)*(x^3 - 1)*log((2^(2/3)*x^2 - 2^(1/3)*(-x^3 + 1)^
(1/3)*x + (-x^3 + 1)^(2/3))/x^2) - 8*sqrt(3)*(x^3 - 1)*arctan(-1/3*(sqrt(3)
*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) + 8*(x^3 - 1)*log((x + (-x^3 + 1)^(1/3)
)/x) - 4*(x^3 - 1)*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2) +
 12*(-x^3 + 1)^(2/3)*x)/(x^3 - 1)
```

Sympy [F]

$$\int \frac{x^6}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \int \frac{x^6}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

```
[In] integrate(x**6/(-x**3+1)**(4/3)/(x**3+1),x)
[Out] Integral(x**6/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{x^6}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \int \frac{x^6}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Giac [F]

$$\int \frac{x^6}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \int \frac{x^6}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \int \frac{x^6}{\left(1-x^3\right)^{4/3}\left(x^3+1\right)}\,dx$$

```
[In] int(x^6/((1 - x^3)^(4/3)*(x^3 + 1)),x)
[Out] int(x^6/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.647 $\int \frac{x^3}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 106

$$\int \frac{x^3}{(1-x^3)^{4/3}(1+x^3)}\,dx=\frac{x}{2\sqrt[3]{1-x^3}}+\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\log(1+x^3)}{12\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[Out] 1/2*x/(-x^3+1)^(1/3)+1/24*ln(x^3+1)*2^(2/3)-1/8*ln(-2^(1/3)*x-(-x^3+1)^(1/3
))*2^(2/3)+1/12*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*
3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {482, 384}

$$\int \frac{x^3}{(1-x^3)^{4/3}(1+x^3)}\,dx=\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{x}{2\sqrt[3]{1-x^3}}+\frac{\log(x^3+1)}{12\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{4\sqrt[3]{2}}$$

```
[In] Int[x^3/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] x/(2*(1 - x^3)^(1/3)) + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]
/(2*2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(12*2^(1/3)) - Log[-(2^(1/3)*x) - (1 -
x^3)^(1/3)]/(4*2^(1/3))
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x}{2\sqrt[3]{1-x^3}}-\frac{1}{2}\int\frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx\\ &= \frac{x}{2\sqrt[3]{1-x^3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\log(1+x^3)}{12\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.31

$$\int\frac{x^3}{(1-x^3)^{4/3}\,(1+x^3)}\,dx=\frac{1}{24}\left(\frac{12x}{\sqrt[3]{1-x^3}}+2\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-2\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}\right.\right.$$

```
[In] Integrate[x^3/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((12*x)/(1 - x^3)^(1/3) + 2*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)
*(1 - x^3)^(1/3))] - 2*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)] + 2^(2/3)
*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/24
```

Maple [A] (verified)

Time = 7.63 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.33

method	result
pseudoelliptic	$-\frac{\sqrt{3}\,2^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}-\frac{2^{\frac{2}{3}} \ln\left(\frac{2^{\frac{2}{3}} x^{2}-2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}}{2}+2^{\frac{2}{3}} \ln\left(\frac{2^{\frac{1}{3}} x}{}\right.}{12\left(-x^{3}+1\right)^{\frac{1}{3}}}$
risch	Expression too large to display
trager	Expression too large to display

```
[In] int(x^3/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12/(-x^3+1)^(1/3)*(3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)
^(1/3)+x)/x)*(-x^3+1)^(1/3)-1/2*2^(2/3)*ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1
/3)*x+(-x^3+1)^(2/3))/x^2)*(-x^3+1)^(1/3)+2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^(1
/3))/x)*(-x^3+1)^(1/3)-6*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(79) = 158.

Time = 1.91 (sec) , antiderivative size = 318, normalized size of antiderivative = 3.00

$$\int \frac{x^{3}}{(1-x^{3})^{4/3}(1+x^{3})}\,dx =$$

$$-\frac{2\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\left(x^{3}-1\right)\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}2^{\frac{2}{3}}(-1)^{\frac{2}{3}}(5x^{7}+4x^{4}-x)(-x^{3}+1)^{\frac{2}{3}}-12\sqrt{6}(-1)^{\frac{1}{3}}(19x^{8}-16x^{5}+x^{2})(-x^{3}+1)^{\frac{1}{3}}-\sqrt{6}2^{\frac{1}{3}}(71x^{9}\cdots}{6(109x^{9}-105x^{6}+3x^{3}+1)}\right.}{}$$

```
[In] integrate(x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/72*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^3 - 1)*arctan(1/6*2^(1/6)*(6*sqrt(6)
*2^(2/3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 12*sqrt(6)*(-1)^
(1/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)*(71*x^9 -
111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 2*2^(2/3)*(-1)^(1
/3)*(x^3 - 1)*log((6*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*x^2 - 2^(2/3)*(-1)
^(1/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 2^(2/3)*(-1)^(1/3)*(x
^3 - 1)*log(-(3*2^(2/3)*(-1)^(1/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(
-1)^(2/3)*(19*x^6 - 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 +
 2*x^3 + 1)) + 36*(-x^3 + 1)^(2/3)*x)/(x^3 - 1)
```

Sympy [F]

$$\int \frac{x^3}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^3}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

```
[In] integrate(x**3/(-x**3+1)**(4/3)/(x**3+1),x)
[Out] Integral(x**3/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{x^3}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^3}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Giac [F]

$$\int \frac{x^3}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^3}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^3}{(1-x^3)^{4/3}\,(x^3+1)}\,dx$$

```
[In] int(x^3/((1 - x^3)^(4/3)*(x^3 + 1)),x)
[Out] int(x^3/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.648 $\int \frac{1}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 106

$$\int \frac{1}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{x}{2\sqrt[3]{1-x^3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[Out] 1/2*x/(-x^3+1)^(1/3)-1/24*ln(x^3+1)*2^(2/3)+1/8*ln(-2^(1/3)*x-(-x^3+1)^(1/3
))*2^(2/3)-1/12*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*
3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {390, 384}

$$\int \frac{1}{(1-x^3)^{4/3}(1+x^3)}\,dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{x}{2\sqrt[3]{1-x^3}} - \frac{\log(x^3+1)}{12\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{4\sqrt[3]{2}}$$

```
[In] Int[1/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] x/(2*(1 - x^3)^(1/3)) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]
/(2*2^(1/3)*Sqrt[3]) - Log[1 + x^3]/(12*2^(1/3)) + Log[-(2^(1/3)*x) - (1 -
x^3)^(1/3)]/(4*2^(1/3))
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx\\ &= \frac{x}{2\sqrt[3]{1-x^3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.32

$$\int \frac{1}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=\frac{1}{24}\left(\frac{12x}{\sqrt[3]{1-x^3}}\right.$$
$$-2\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)+2\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)-2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}\right.$$

```
[In] Integrate[1/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((12*x)/(1 - x^3)^(1/3) - 2*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)
*(1 - x^3)^(1/3))] + 2*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)] - 2^(2/3)
*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3)^(2/3)])/24
```

Maple [A] (verified)

Time = 6.83 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.33

method	result
pseudoelliptic	$\frac{\sqrt{3}\,2^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}-\frac{2^{\frac{2}{3}} \ln\left(\frac{2^{\frac{2}{3}} x^{2}-2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}}{2}+2^{\frac{2}{3}} \ln\left(\frac{2^{\frac{1}{3}} x+(\cdots}{\cdots}\right.}{12(-x^{3}+1)^{\frac{1}{3}}}$
trager	Expression too large to display
risch	Expression too large to display

```
[In] int(1/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/(-x^3+1)^(1/3)*(3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^
(1/3)+x)/x)*(-x^3+1)^(1/3)-1/2*2^(2/3)*ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/
3)*x+(-x^3+1)^(2/3))/x^2)*(-x^3+1)^(1/3)+2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^(1/
3))/x)*(-x^3+1)^(1/3)+6*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(79) = 158.

Time = 1.77 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.72

$$\int \frac{1}{(1-x^3)^{4/3}\,(1+x^3)}\,dx =$$

$$-\frac{2\sqrt{6}2^{\frac{1}{6}}(x^3-1)\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}2^{\frac{2}{3}}(5x^7+4x^4-x)(-x^3+1)^{\frac{2}{3}}-\sqrt{6}2^{\frac{1}{3}}(71x^9-111x^6+33x^3-1)+12\sqrt{6}(19x^8-16x^5+x^2)(-x^3+1)^{\frac{1}{3}}\right)}{6(109x^9-105x^6+3x^3+1)}\right.\cdots}{\cdots}$$

```
[In] integrate(1/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/72*(2*sqrt(6)*2^(1/6)*(x^3 - 1)*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)*(5
*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6 + 33
*x^3 - 1) + 12*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 -
 105*x^6 + 3*x^3 + 1)) - 2*2^(2/3)*(x^3 - 1)*log((6*2^(1/3)*(-x^3 + 1)^(1/3
)*x^2 + 2^(2/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 2^(2/3)*(x^3
 - 1)*log((3*2^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x^
3 + 1) - 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) + 36*(-x^3 +
 1)^(2/3)*x)/(x^3 - 1)
```

Sympy [F]

$$\int \frac{1}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{1}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

```
[In] integrate(1/(-x**3+1)**(4/3)/(x**3+1),x)
[Out] Integral(1/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Giac [F]

$$\int \frac{1}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{1}{(1-x^3)^{4/3}(x^3+1)}\,dx$$

```
[In] int(1/((1 - x^3)^(4/3)*(x^3 + 1)),x)
[Out] int(1/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.649 $\int \frac{1}{x^3(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 124

$$\int \frac{1}{x^3(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{2x^2\sqrt[3]{1-x^3}} - \frac{(1-x^3)^{2/3}}{x^2} + \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\log(1+x^3)}{12\sqrt[3]{2}} - \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[Out] 1/2/x^2/(-x^3+1)^(1/3)-(-x^3+1)^(2/3)/x^2+1/24*ln(x^3+1)*2^(2/3)-1/8*ln(-2^
(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)+1/12*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/
3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {483, 597, 12, 384}

$$\int \frac{1}{x^3(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\log(x^3+1)}{12\sqrt[3]{2}} - \frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{4\sqrt[3]{2}} - \frac{(1-x^3)^{2/3}}{x^2} + \frac{1}{2x^2\sqrt[3]{1-x^3}}$$

```
[In] Int[1/(x^3*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*x^2*(1 - x^3)^(1/3)) - (1 - x^3)^(2/3)/x^2 + ArcTan[(1 - (2*2^(1/3)*x)
/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(12*2^(1/3))
- Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2x^2\sqrt[3]{1-x^3}}+\frac{1}{2}\int\frac{4+3x^3}{x^3\sqrt[3]{1-x^3}\,(1+x^3)}\,dx\\ &= \frac{1}{2x^2\sqrt[3]{1-x^3}}-\frac{(1-x^3)^{2/3}}{x^2}-\frac{1}{4}\int\frac{2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx\end{aligned}$$

$$= \frac{1}{2x^2\sqrt[3]{1-x^3}} - \frac{(1-x^3)^{2/3}}{x^2} - \frac{1}{2}\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$$

$$= \frac{1}{2x^2\sqrt[3]{1-x^3}} - \frac{(1-x^3)^{2/3}}{x^2} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\log(1+x^3)}{12\sqrt[3]{2}} - \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^3\,(1-x^3)^{4/3}\,(1+x^3)}\,dx = \frac{1}{24}\left(\frac{12(-1+2x^3)}{x^2\sqrt[3]{1-x^3}} + 2\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right) - 2\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right) + 2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}(\right.\right.$$

```
[In] Integrate[1/(x^3*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((12*(-1 + 2*x^3))/(x^2*(1 - x^3)^(1/3)) + 2*2^(2/3)*Sqrt[3]*ArcTan[(Sqrt[3
]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] - 2*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)
^(1/3)] + 2^(2/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3)*(1 - x^3
)^(2/3)])/24
```

Maple [A] (verified)

Time = 21.60 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.26

method	result
pseudoelliptic	$-\frac{\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)x^2\left(-x^3+1\right)^{\frac{1}{3}} - \frac{2^{\frac{2}{3}}\ln\left(\frac{2^{\frac{2}{3}}x^2-2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}\right)x^2\left(-x^3+1\right)^{\frac{1}{3}}}{2} + 2^{\frac{2}{3}}\ln\left(\right.}{12\left(-x^3+1\right)^{\frac{1}{3}}x^2}$
risch	Expression too large to display
trager	Expression too large to display

```
[In] int(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12/(-x^3+1)^(1/3)*(3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)
^(1/3)+x)/x)*x^2*(-x^3+1)^(1/3)-1/2*2^(2/3)*ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1
)^(1/3)*x+(-x^3+1)^(2/3))/x^2)*x^2*(-x^3+1)^(1/3)+2^(2/3)*ln((2^(1/3)*x+(-x
^3+1)^(1/3))/x)*x^2*(-x^3+1)^(1/3)-12*x^3+6)/x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(95) = 190$.

Time = 1.68 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.74

$$\int \frac{1}{x^3 (1-x^3)^{4/3} (1+x^3)} \, dx =$$
$$-\frac{2\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}(x^5-x^2)\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}2^{\frac{2}{3}}(-1)^{\frac{2}{3}}(5x^7+4x^4-x)(-x^3+1)^{\frac{2}{3}}-12\sqrt{6}(-1)^{\frac{1}{3}}(19x^8-16x^5+x^2)(-x^3+1)^{\frac{1}{3}}-\sqrt{6}2^{\frac{1}{3}}(7\right.}{6(109x^9-105x^6+3x^3+1)}\right.}{}$$

```
[In] integrate(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/72*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^5 - x^2)*arctan(1/6*2^(1/6)*(6*sqrt(
6)*2^(2/3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 12*sqrt(6)*(-1
)^(1/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)*(71*x^9
- 111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 2*2^(2/3)*(-1)^
(1/3)*(x^5 - x^2)*log((6*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*x^2 - 2^(2/3)*
(-1)^(1/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 2^(2/3)*(-1)^(1/3
)*(x^5 - x^2)*log(-(3*2^(2/3)*(-1)^(1/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 2^(
1/3)*(-1)^(2/3)*(19*x^6 - 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/
(x^6 + 2*x^3 + 1)) + 36*(2*x^3 - 1)*(-x^3 + 1)^(2/3))/(x^5 - x^2)
```

Sympy [F]

$$\int \frac{1}{x^3 (1-x^3)^{4/3} (1+x^3)} \, dx = \int \frac{1}{x^3 \left(-(x-1)(x^2+x+1)\right)^{\frac{4}{3}} (x+1)(x^2-x+1)} \, dx$$

```
[In] integrate(1/x**3/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**3*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Maxima [F]

$$\int \frac{1}{x^3 (1-x^3)^{4/3} (1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}x^3}\,dx$$

```
[In] integrate(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (1-x^3)^{4/3} (1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}x^3}\,dx$$

```
[In] integrate(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (1-x^3)^{4/3} (1+x^3)}\,dx = \int \frac{1}{x^3 (1-x^3)^{4/3} (x^3+1)}\,dx$$

```
[In] int(1/(x^3*(1 - x^3)^(4/3)*(x^3 + 1)),x)
[Out] int(1/(x^3*(1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.650 $\int \frac{1}{x^6(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 144

$$\int \frac{1}{x^6(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{2x^5\sqrt[3]{1-x^3}} - \frac{7(1-x^3)^{2/3}}{10x^5} - \frac{4(1-x^3)^{2/3}}{5x^2} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[Out] 1/2/x^5/(-x^3+1)^(1/3)-7/10*(-x^3+1)^(2/3)/x^5-4/5*(-x^3+1)^(2/3)/x^2-1/24*
ln(x^3+1)*2^(2/3)+1/8*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/12*arctan(1/3
*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {483, 597, 12, 384}

$$\int \frac{1}{x^6(1-x^3)^{4/3}(1+x^3)}\,dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log(x^3+1)}{12\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{4\sqrt[3]{2}} - \frac{7(1-x^3)^{2/3}}{10x^5} + \frac{1}{2x^5\sqrt[3]{1-x^3}} - \frac{4(1-x^3)^{2/3}}{5x^2}$$

```
[In] Int[1/(x^6*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*x^5*(1 - x^3)^(1/3)) - (7*(1 - x^3)^(2/3))/(10*x^5) - (4*(1 - x^3)^(2/
3))/(5*x^2) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3
)*Sqrt[3]) - Log[1 + x^3]/(12*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)
]/(4*2^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2x^5\sqrt[3]{1-x^3}}+\frac{1}{2}\int \frac{7+6x^3}{x^6\sqrt[3]{1-x^3}\,(1+x^3)}\,dx\\
&= \frac{1}{2x^5\sqrt[3]{1-x^3}}-\frac{7(1-x^3)^{2/3}}{10x^5}-\frac{1}{10}\int \frac{-16-21x^3}{x^3\sqrt[3]{1-x^3}\,(1+x^3)}\,dx
\end{aligned}
$$

$$= \frac{1}{2x^5\sqrt[3]{1-x^3}} - \frac{7(1-x^3)^{2/3}}{10x^5} - \frac{4(1-x^3)^{2/3}}{5x^2} + \frac{1}{20}\int \frac{10}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$$

$$= \frac{1}{2x^5\sqrt[3]{1-x^3}} - \frac{7(1-x^3)^{2/3}}{10x^5} - \frac{4(1-x^3)^{2/3}}{5x^2} + \frac{1}{2}\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$$

$$= \frac{1}{2x^5\sqrt[3]{1-x^3}} - \frac{7(1-x^3)^{2/3}}{10x^5} - \frac{4(1-x^3)^{2/3}}{5x^2} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^6(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{120}\left(-\frac{12(2+x^3-8x^6)}{x^5\sqrt[3]{1-x^3}} - 10\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right) + 10\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right) - 5\,2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3} - \right.\right.$$

```
[In] Integrate[1/(x^6*(1 - x^3)^(4/3)*(1 + x^3)),x]
[Out] ((-12*(2 + x^3 - 8*x^6))/(x^5*(1 - x^3)^(1/3)) - 10*2^(2/3)*Sqrt[3]*ArcTan[
(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] + 10*2^(2/3)*Log[2*x + 2^(2/3)*(
1 - x^3)^(1/3)] - 5*2^(2/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^(1/3
)*(1 - x^3)^(2/3)])/120
```

Maple [A] (verified)

Time = 22.17 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$\frac{\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}(-x^3+1)^{\frac{1}{3}}+x\right)}{3x}\right)x^5(-x^3+1)^{\frac{1}{3}} - \frac{2^{\frac{2}{3}}\ln\left(\frac{2^{\frac{2}{3}}x^2-2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}x+(-x^3+1)^{\frac{2}{3}}}{x^2}\right)x^5(-x^3+1)^{\frac{1}{3}}}{2} + 2^{\frac{2}{3}}\ln\left(\right.}{12(-x^3+1)^{\frac{1}{3}}x^5}$
trager	Expression too large to display
risch	Expression too large to display

```
[In] int(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/(-x^3+1)^(1/3)*(3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^
(1/3)+x)/x)*x^5*(-x^3+1)^(1/3)-1/2*2^(2/3)*ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)
^(1/3)*x+(-x^3+1)^(2/3))/x^2)*x^5*(-x^3+1)^(1/3)+2^(2/3)*ln((2^(1/3)*x+(-x^
3+1)^(1/3))/x)*x^5*(-x^3+1)^(1/3)+48/5*x^6-6/5*x^3-12/5)/x^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(109) = 218$.

Time = 1.69 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.19

$$\int \frac{1}{x^6 (1-x^3)^{4/3} (1+x^3)} \, dx = -\frac{10 \sqrt{6} 2^{\frac{1}{6}} (x^8 - x^5) \arctan \left(\frac{2^{\frac{1}{6}} \left(6 \sqrt{6} 2^{\frac{2}{3}} (5 x^7 + 4 x^4 - x) (-x^3+1)^{\frac{2}{3}} - \sqrt{6} 2^{\frac{1}{3}} (71 x^9 - 111 x^6 + 33 x^3 - 1) + 12 \sqrt{6} (19 x^8 - 16 x^5 + x^2) (-x^3+1)^{\frac{1}{3}}\right.}{6 (109 x^9 - 105 x^6 + 3 x^3 + 1)} \right.}{}$$

```
[In] integrate(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/360*(10*sqrt(6)*2^(1/6)*(x^8 - x^5)*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3
)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6
+ 33*x^3 - 1) + 12*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x
^9 - 105*x^6 + 3*x^3 + 1)) - 10*2^(2/3)*(x^8 - x^5)*log((6*2^(1/3)*(-x^3 +
1)^(1/3)*x^2 + 2^(2/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 5*2^(
2/3)*(x^8 - x^5)*log((3*2^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*
x^6 - 16*x^3 + 1) - 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) +
 36*(8*x^6 - x^3 - 2)*(-x^3 + 1)^(2/3))/(x^8 - x^5)
```

Sympy [F]

$$\int \frac{1}{x^6 (1-x^3)^{4/3} (1+x^3)} \, dx = \int \frac{1}{x^6 \left(-(x-1)(x^2+x+1)\right)^{\frac{4}{3}} (x+1)(x^2-x+1)} \, dx$$

```
[In] integrate(1/x**6/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**6*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Maxima [F]

$$\int \frac{1}{x^6 (1-x^3)^{4/3} (1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}x^6}\,dx$$

```
[In] integrate(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 (1-x^3)^{4/3} (1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}x^6}\,dx$$

```
[In] integrate(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (1-x^3)^{4/3} (1+x^3)}\,dx = \int \frac{1}{x^6 (1-x^3)^{4/3} (x^3+1)}\,dx$$

```
[In] int(1/(x^6*(1 - x^3)^(4/3)*(x^3 + 1)),x)
[Out] int(1/(x^6*(1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.651 $\int \frac{1}{x^9(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 162

$$\int \frac{1}{x^9 (1-x^3)^{4/3} (1+x^3)}\,dx = \frac{1}{2x^8\sqrt[3]{1-x^3}} - \frac{5(1-x^3)^{2/3}}{8x^8} - \frac{13(1-x^3)^{2/3}}{20x^5} - \frac{49(1-x^3)^{2/3}}{40x^2} + \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\log(1+x^3)}{12\sqrt[3]{2}} - \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[Out] 1/2/x^8/(-x^3+1)^(1/3)-5/8*(-x^3+1)^(2/3)/x^8-13/20*(-x^3+1)^(2/3)/x^5-49/4
0*(-x^3+1)^(2/3)/x^2+1/24*ln(x^3+1)*2^(2/3)-1/8*ln(-2^(1/3)*x-(-x^3+1)^(1/3
))*2^(2/3)+1/12*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*
3^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {483, 597, 12, 384}

$$\int \frac{1}{x^9 (1-x^3)^{4/3} (1+x^3)}\,dx = \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\log(x^3+1)}{12\sqrt[3]{2}} - \frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{4\sqrt[3]{2}} - \frac{5(1-x^3)^{2/3}}{8x^8} + \frac{1}{2x^8\sqrt[3]{1-x^3}} - \frac{13(1-x^3)^{2/3}}{20x^5} - \frac{49(1-x^3)^{2/3}}{40x^2}$$

```
[In] Int[1/(x^9*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*x^8*(1 - x^3)^(1/3)) - (5*(1 - x^3)^(2/3))/(8*x^8) - (13*(1 - x^3)^(2/
3))/(20*x^5) - (49*(1 - x^3)^(2/3))/(40*x^2) + ArcTan[(1 - (2*2^(1/3)*x)/(1
 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(12*2^(1/3)) - L
og[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2x^8\sqrt[3]{1-x^3}}+\frac{1}{2}\int\frac{10+9x^3}{x^9\sqrt[3]{1-x^3}\,(1+x^3)}\,dx\\
&= \frac{1}{2x^8\sqrt[3]{1-x^3}}-\frac{5(1-x^3)^{2/3}}{8x^8}-\frac{1}{16}\int\frac{-52-60x^3}{x^6\sqrt[3]{1-x^3}\,(1+x^3)}\,dx
\end{aligned}$$

$$= \frac{1}{2x^8\sqrt[3]{1-x^3}} - \frac{5(1-x^3)^{2/3}}{8x^8} - \frac{13(1-x^3)^{2/3}}{20x^5} + \frac{1}{80}\int \frac{196+156x^3}{x^3\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$$

$$= \frac{1}{2x^8\sqrt[3]{1-x^3}} - \frac{5(1-x^3)^{2/3}}{8x^8} - \frac{13(1-x^3)^{2/3}}{20x^5} - \frac{49(1-x^3)^{2/3}}{40x^2} - \frac{1}{160}\int \frac{80}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$$

$$= \frac{1}{2x^8\sqrt[3]{1-x^3}} - \frac{5(1-x^3)^{2/3}}{8x^8} - \frac{13(1-x^3)^{2/3}}{20x^5} - \frac{49(1-x^3)^{2/3}}{40x^2} - \frac{1}{2}\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$$

$$= \frac{1}{2x^8\sqrt[3]{1-x^3}} - \frac{5(1-x^3)^{2/3}}{8x^8} - \frac{13(1-x^3)^{2/3}}{20x^5} - \frac{49(1-x^3)^{2/3}}{40x^2} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\log(1+x^3)}{12\sqrt[3]{2}} - \frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^9(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{120}\left(-\frac{3(5+x^3+23x^6-49x^9)}{x^8\sqrt[3]{1-x^3}} + 10\,2^{2/3}\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right) - 10\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right) + 5\,2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\right.\right.$$

```
[In] Integrate[1/(x^9*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((-3*(5 + x^3 + 23*x^6 - 49*x^9))/(x^8*(1 - x^3)^(1/3)) + 10*2^(2/3)*Sqrt[3
]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] - 10*2^(2/3)*Log[2*x +
2^(2/3)*(1 - x^3)^(1/3)] + 5*2^(2/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3)
 - 2^(1/3)*(1 - x^3)^(2/3)])/120
```

Maple [A] (verified)

Time = 21.04 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$\frac{-10\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(-2^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+x\right)}{3x}\right)x^8\left(-x^3+1\right)^{\frac{1}{3}}-10\,2^{\frac{2}{3}}\ln\left(\frac{2^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)x^8\left(-x^3+1\right)^{\frac{1}{3}}+5\,2^{\frac{2}{3}}\ln\left(\frac{2^{\frac{2}{3}}x^2-2}{}\right.}{120x^8\left(-x^3+1\right)^{\frac{1}{3}}}$
risch	Expression too large to display
trager	Expression too large to display

```
[In] int(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*(-10*3^(1/2)*2^(2/3)*arctan(1/3*3^(1/2)*(-2^(2/3)*(-x^3+1)^(1/3)+x)/x
)*x^8*(-x^3+1)^(1/3)-10*2^(2/3)*ln((2^(1/3)*x+(-x^3+1)^(1/3))/x)*x^8*(-x^3+
1)^(1/3)+5*2^(2/3)*ln((2^(2/3)*x^2-2^(1/3)*(-x^3+1)^(1/3)*x+(-x^3+1)^(2/3))
/x^2)*x^8*(-x^3+1)^(1/3)+147*x^9-69*x^6-3*x^3-15)/x^8/(-x^3+1)^(1/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(123) = 246$.

Time = 1.71 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.17

$$\int \frac{1}{x^9\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx =$$

$$-\frac{10\sqrt{6}2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\left(x^{11}-x^8\right)\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}2^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(5x^7+4x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}-12\sqrt{6}(-1)^{\frac{1}{3}}\left(19x^8-16x^5+x^2\right)\left(-x^3+1\right)^{\frac{1}{3}}-\sqrt{6}2^{\frac{1}{3}}\cdots\right.}{6\left(109x^9-105x^6+3x^3+1\right)}\right.}{}$$

```
[In] integrate(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/360*(10*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^11 - x^8)*arctan(1/6*2^(1/6)*(6*sq
rt(6)*2^(2/3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 12*sqrt(6)*
(-1)^(1/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)*(71*x
^9 - 111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 10*2^(2/3)*(
-1)^(1/3)*(x^11 - x^8)*log((6*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*x^2 - 2^(
2/3)*(-1)^(1/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 5*2^(2/3)*(-
1)^(1/3)*(x^11 - x^8)*log(-(3*2^(2/3)*(-1)^(1/3)*(5*x^4 - x)*(-x^3 + 1)^(2/
3) - 2^(1/3)*(-1)^(2/3)*(19*x^6 - 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)
^(1/3))/(x^6 + 2*x^3 + 1)) + 9*(49*x^9 - 23*x^6 - x^3 - 5)*(-x^3 + 1)^(2/3)
)/(x^11 - x^8)
```

Sympy [F]

$$\int \frac{1}{x^9 (1-x^3)^{4/3} (1+x^3)} dx = \int \frac{1}{x^9 \left(-(x-1)(x^2+x+1)\right)^{\frac{4}{3}} (x+1)(x^2-x+1)} dx$$

```
[In] integrate(1/x**9/(-x**3+1)**(4/3)/(x**3+1),x)
[Out] Integral(1/(x**9*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Maxima [F]

$$\int \frac{1}{x^9 (1-x^3)^{4/3} (1+x^3)} dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}} x^9} dx$$

```
[In] integrate(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^9), x)
```

Giac [F]

$$\int \frac{1}{x^9 (1-x^3)^{4/3} (1+x^3)} dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}} x^9} dx$$

```
[In] integrate(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^9), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^9 (1-x^3)^{4/3} (1+x^3)} dx = \int \frac{1}{x^9 (1-x^3)^{4/3} (x^3+1)} dx$$

```
[In] int(1/(x^9*(1 - x^3)^(4/3)*(x^3 + 1)),x)
[Out] int(1/(x^9*(1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.652 $\int \frac{x^{10}}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 292

$$\int \frac{x^{10}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{x^5}{2\sqrt[3]{1-x^3}}+\frac{3}{4}x^2\left(1-x^3\right)^{2/3}-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}}-\frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right)-\frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}}-\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}}+\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}+\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}}$$

```
[Out] 1/2*x^5/(-x^3+1)^(1/3)+3/4*x^2*(-x^3+1)^(2/3)-1/2*x^2*hypergeom([1/3, 2/3],
[5/3],x^3)-1/48*ln((1-x)*(1+x)^2)*2^(2/3)-1/24*ln(1+2^(2/3)*(1-x)^2/(-x^3+1
)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/12*ln(1+2^(1/3)*(1-x)/(-x^3
+1)^(1/3))*2^(2/3)+1/16*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)-1/12*arctan
(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/24*arcta
n(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {481, 596, 598, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^{10}}{(1-x^3)^{4/3}(1+x^3)}\,dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}} - \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) - \frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{12\sqrt[3]{2}} + \frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}} + \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{8\sqrt[3]{2}} + \frac{x^5}{2\sqrt[3]{1-x^3}} + \frac{3}{4}\left(1-x^3\right)^{2/3}x^2 - \frac{\log\left((1-x)(x+1)^2\right)}{24\sqrt[3]{2}}$$

```
[In] Int[x^10/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] x^5/(2*(1 - x^3)^(1/3)) + (3*x^2*(1 - x^3)^(2/3))/4 - ArcTan[(1 - (2*2^(1/3
)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - ArcTan[(1 + (2^(
1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(4*2^(1/3)*Sqrt[3]) - (x^2*Hypergeo
metric2F1[1/3, 2/3, 5/3, x^3])/2 - Log[(1 - x)*(1 + x)^2]/(24*2^(1/3)) - Lo
g[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/
3)]/(12*2^(1/3)) + Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) +
 Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(8*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^5}{2\sqrt[3]{1-x^3}}-\frac{1}{2}\int \frac{x^4(5+6x^3)}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx \\
&= \frac{x^5}{2\sqrt[3]{1-x^3}}+\frac{3}{4}x^2\left(1-x^3\right)^{2/3}-\frac{1}{8}\int \frac{x(12+8x^3)}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx \\
&= \frac{x^5}{2\sqrt[3]{1-x^3}}+\frac{3}{4}x^2\left(1-x^3\right)^{2/3}-\frac{1}{8}\int \left(\frac{8x}{\sqrt[3]{1-x^3}}+\frac{4x}{\sqrt[3]{1-x^3}\,(1+x^3)}\right)dx \\
&= \frac{x^5}{2\sqrt[3]{1-x^3}}+\frac{3}{4}x^2\left(1-x^3\right)^{2/3}-\frac{1}{2}\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx-\int \frac{x}{\sqrt[3]{1-x^3}}\,dx
\end{aligned}$$

$$= \frac{x^5}{2\sqrt[3]{1-x^3}} + \frac{3}{4}x^2\left(1-x^3\right)^{2/3} - \frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) + \frac{1}{6}\int\frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx + \frac{1}{2}\text{Subst}\left(\int\frac{1}{1+2x^3}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right)$$

$$= \frac{x^5}{2\sqrt[3]{1-x^3}} + \frac{3}{4}x^2\left(1-x^3\right)^{2/3} - \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}} - \frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) - \frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}} + \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}} + \frac{1}{6}\text{Subst}\left(\int\frac{1}{1+\sqrt[3]{2}x}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{6}\text{Subst}\left(\int\frac{2-\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right)$$

$$= \frac{x^5}{2\sqrt[3]{1-x^3}} + \frac{3}{4}x^2\left(1-x^3\right)^{2/3} - \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}} - \frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) - \frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}} + \frac{1}{4}\text{Subst}\left(\int\frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right) - \frac{\text{Subst}\left(\int\frac{-\sqrt[3]{2}+2\ 2^{2/3}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}}$$

$$= \frac{x^5}{2\sqrt[3]{1-x^3}} + \frac{3}{4}x^2\left(1-x^3\right)^{2/3} - \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}} - \frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) - \frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}} - \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}} + \frac{\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2\sqrt[3]{2}}$$

$$= \frac{x^5}{2\sqrt[3]{1-x^3}} + \frac{3}{4}x^2(1-x^3)^{2/3} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}} - \frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) - \frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}} - \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.24

$$\int \frac{x^{10}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{20}x^2\left(-\frac{5(-3+x^3)}{\sqrt[3]{1-x^3}} - 15\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right) - 4x^3\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right)\right)$$

```
[In] Integrate[x^10/((1 - x^3)^(4/3)*(1 + x^3)),x]
[Out] (x^2*((-5*(-3 + x^3))/(1 - x^3)^(1/3) - 15*AppellF1[2/3, 1/3, 1, 5/3, x^3,
-x^3] - 4*x^3*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3]))/20
```

Maple [F]

$$\int \frac{x^{10}}{(-x^3+1)^{\frac{4}{3}}(x^3+1)}dx$$

```
[In] int(x^10/(-x^3+1)^(4/3)/(x^3+1),x)
[Out] int(x^10/(-x^3+1)^(4/3)/(x^3+1),x)
```

Fricas [F]

$$\int \frac{x^{10}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^{10}}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(x^10/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)*x^10/(x^9 - x^6 - x^3 + 1), x)
```

Sympy [F]

$$\int \frac{x^{10}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^{10}}{\left(-(x-1)(x^2+x+1)\right)^{\frac{4}{3}}(x+1)(x^2-x+1)}\,dx$$

```
[In] integrate(x**10/(-x**3+1)**(4/3)/(x**3+1),x)
[Out] Integral(x**10/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x
)
```

Maxima [F]

$$\int \frac{x^{10}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^{10}}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(x^10/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x^10/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Giac [F]

$$\int \frac{x^{10}}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^{10}}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(x^10/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x^10/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}}{(1-x^3)^{4/3}\,(1+x^3)}\,dx = \int \frac{x^{10}}{(1-x^3)^{4/3}\,(x^3+1)}\,dx$$

```
[In] int(x^10/((1 - x^3)^(4/3)*(x^3 + 1)),x)
[Out] int(x^10/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.653 $\int \frac{x^7}{(1-x^3)^{4/3}(1+x^3)} dx$

Optimal result

Integrand size = 22, antiderivative size = 274

$$\int \frac{x^7}{(1-x^3)^{4/3}(1+x^3)} dx = \frac{x^2}{2\sqrt[3]{1-x^3}} + \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}} - \frac{3}{4}x^2 \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) + \frac{\log((1-x)(1+x)^2)}{24\sqrt[3]{2}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}} - \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} - \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}}$$

```
[Out] 1/2*x^2/(-x^3+1)^(1/3)-3/4*x^2*hypergeom([1/3, 2/3],[5/3],x^3)+1/48*ln((1-x
)*(1+x)^2)*2^(2/3)+1/24*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(
-x^3+1)^(1/3))*2^(2/3)-1/12*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/16
*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)+1/12*arctan(1/3*(1-2*2^(1/3)*(1-x)
/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/24*arctan(1/3*(1+2^(1/3)*(1-x)/
(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {481, 598, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^7}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}} - \frac{3}{4}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) + \frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{12\sqrt[3]{2}} - \frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}} - \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{8\sqrt[3]{2}} + \frac{x^2}{2\sqrt[3]{1-x^3}} + \frac{\log((1-x)(x+1)^2)}{24\sqrt[3]{2}}$$

```
[In] Int[x^7/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] x^2/(2*(1 - x^3)^(1/3)) + ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/
Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3
))/Sqrt[3]]/(4*2^(1/3)*Sqrt[3]) - (3*x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x
^3])/4 + Log[(1 - x)*(1 + x)^2]/(24*2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/
(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(12*2^(1/3)) - Log[1 +
 (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[-1 + x + 2^(2/3)*(1 -
 x^3)^(1/3)]/(8*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^2}{2\sqrt[3]{1-x^3}} - \frac{1}{2}\int \frac{x(2+3x^3)}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx \\
&= \frac{x^2}{2\sqrt[3]{1-x^3}} - \frac{1}{2}\int \left(\frac{3x}{\sqrt[3]{1-x^3}} - \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\right)dx \\
&= \frac{x^2}{2\sqrt[3]{1-x^3}} + \frac{1}{2}\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx - \frac{3}{2}\int \frac{x}{\sqrt[3]{1-x^3}}\,dx \\
&= \frac{x^2}{2\sqrt[3]{1-x^3}} - \frac{3}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) - \frac{1}{6}\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx \\
&\quad - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+2x^3}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) \\
&= \frac{x^2}{2\sqrt[3]{1-x^3}} + \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}} - \frac{3}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) + \frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}} \\
&\quad - \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}} - \frac{1}{6}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) \\
&\quad - \frac{1}{6}\text{Subst}\left(\int \frac{2-\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)
\end{aligned}
$$

$$
= \frac{x^2}{2\sqrt[3]{1-x^3}} + \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}} - \frac{3}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)
+ \frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}} - \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} - \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}}
- \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)
+ \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{2}+2\ 2^{2/3}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}}
$$

$$
= \frac{x^2}{2\sqrt[3]{1-x^3}} + \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}} - \frac{3}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)
+ \frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}} - \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}
- \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2\sqrt[3]{2}}
$$

$$
= \frac{x^2}{2\sqrt[3]{1-x^3}} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}}
- \frac{3}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) + \frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}}
- \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} - \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.24

$$\int \frac{x^7}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{10}x^2\left(\frac{5}{\sqrt[3]{1-x^3}} - 5\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right) - 3x^3\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right)\right)$$

```
[In] Integrate[x^7/((1 - x^3)^(4/3)*(1 + x^3)),x]
[Out] (x^2*(5/(1 - x^3)^(1/3) - 5*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3] - 3*x^3*A
ppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3]))/10
```

Maple [F]

$$\int \frac{x^7}{(-x^3+1)^{\frac{4}{3}}(x^3+1)}dx$$

```
[In] int(x^7/(-x^3+1)^(4/3)/(x^3+1),x)
[Out] int(x^7/(-x^3+1)^(4/3)/(x^3+1),x)
```

Fricas [F]

$$\int \frac{x^7}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^7}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(x^7/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)*x^7/(x^9 - x^6 - x^3 + 1), x)
```

Sympy [F]

$$\int \frac{x^7}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^7}{(-(x-1)(x^2+x+1))^{\frac{4}{3}}(x+1)(x^2-x+1)}\,dx$$

```
[In] integrate(x**7/(-x**3+1)**(4/3)/(x**3+1),x)
[Out] Integral(x**7/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{x^7}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^7}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(x^7/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Giac [F]

$$\int \frac{x^7}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^7}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(x^7/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^7}{(1-x^3)^{4/3}(x^3+1)}\,dx$$

```
[In] int(x^7/((1 - x^3)^(4/3)*(x^3 + 1)),x)
[Out] int(x^7/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.654 $\int \frac{x^4}{(1-x^3)^{4/3}(1+x^3)} dx$

Optimal result

Integrand size = 22, antiderivative size = 274

$$\int \frac{x^4}{(1-x^3)^{4/3}(1+x^3)} dx = \frac{x^2}{2\sqrt[3]{1-x^3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}} - \frac{1}{4}x^2 \,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) - \frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}} - \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}}$$

```
[Out] 1/2*x^2/(-x^3+1)^(1/3)-1/4*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/48*ln((1-x
)*(1+x)^2)*2^(2/3)-1/24*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(
-x^3+1)^(1/3))*2^(2/3)+1/12*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/16
*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)-1/12*arctan(1/3*(1-2*2^(1/3)*(1-x)
/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/24*arctan(1/3*(1+2^(1/3)*(1-x)/
(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {482, 598, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^4}{(1-x^3)^{4/3}(1+x^3)}dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}} - \frac{1}{4}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) - \frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{12\sqrt[3]{2}} + \frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}} + \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{8\sqrt[3]{2}} + \frac{x^2}{2\sqrt[3]{1-x^3}} - \frac{\log\left((1-x)(x+1)^2\right)}{24\sqrt[3]{2}}$$

```
[In] Int[x^4/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] x^2/(2*(1 - x^3)^(1/3)) - ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/
Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3
))/Sqrt[3]]/(4*2^(1/3)*Sqrt[3]) - (x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3
])/4 - Log[(1 - x)*(1 + x)^2]/(24*2^(1/3)) - Log[1 + (2^(2/3)*(1 - x)^2)/(1
 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(12*2^(1/3)) + Log[1 + (
2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) + Log[-1 + x + 2^(2/3)*(1 - x
^3)^(1/3)]/(8*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^2}{2\sqrt[3]{1-x^3}}-\frac{1}{2}\int\frac{x(2+x^3)}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx\\
&= \frac{x^2}{2\sqrt[3]{1-x^3}}-\frac{1}{2}\int\left(\frac{x}{\sqrt[3]{1-x^3}}+\frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\right)dx\\
&= \frac{x^2}{2\sqrt[3]{1-x^3}}-\frac{1}{2}\int\frac{x}{\sqrt[3]{1-x^3}}\,dx-\frac{1}{2}\int\frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx\\
&= \frac{x^2}{2\sqrt[3]{1-x^3}}-\frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{1}{6}\int\frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx\\
&\quad+\frac{1}{2}\text{Subst}\left(\int\frac{1}{1+2x^3}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right)\\
&= \frac{x^2}{2\sqrt[3]{1-x^3}}-\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}}-\frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}}\\
&\quad+\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}}+\frac{1}{6}\text{Subst}\left(\int\frac{1}{1+\sqrt[3]{2}x}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right)\\
&\quad+\frac{1}{6}\text{Subst}\left(\int\frac{2-\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right)
\end{aligned}
$$

$$= \frac{x^2}{2\sqrt[3]{1-x^3}} - \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}} - \frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) - \frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{2}+2\ 2^{2/3}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}}$$

$$= \frac{x^2}{2\sqrt[3]{1-x^3}} - \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}} - \frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) - \frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}} - \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2\sqrt[3]{2}}$$

$$= \frac{x^2}{2\sqrt[3]{1-x^3}} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}} - \frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) - \frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}} - \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.24

$$\int \frac{x^4}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{10}x^2\left(\frac{5}{\sqrt[3]{1-x^3}} - 5\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right) - x^3\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right)\right)$$

```
[In] Integrate[x^4/((1 - x^3)^(4/3)*(1 + x^3)),x]
[Out] (x^2*(5/(1 - x^3)^(1/3) - 5*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3] - x^3*App
ellF1[5/3, 1/3, 1, 8/3, x^3, -x^3]))/10
```

Maple [F]

$$\int \frac{x^4}{(-x^3+1)^{\frac{4}{3}}(x^3+1)}dx$$

```
[In] int(x^4/(-x^3+1)^(4/3)/(x^3+1),x)
[Out] int(x^4/(-x^3+1)^(4/3)/(x^3+1),x)
```

Fricas [F]

$$\int \frac{x^4}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^4}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)*x^4/(x^9 - x^6 - x^3 + 1), x)
```

Sympy [F]

$$\int \frac{x^4}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^4}{(-(x-1)(x^2+x+1))^{\frac{4}{3}}(x+1)(x^2-x+1)}\,dx$$

```
[In] integrate(x**4/(-x**3+1)**(4/3)/(x**3+1),x)
[Out] Integral(x**4/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{x^4}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^4}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Giac [F]

$$\int \frac{x^4}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^4}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x^4}{(1-x^3)^{4/3}(x^3+1)}\,dx$$

```
[In] int(x^4/((1 - x^3)^(4/3)*(x^3 + 1)),x)
[Out] int(x^4/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.655 $\int \frac{x}{(1-x^3)^{4/3}(1+x^3)} dx$

Optimal result

Integrand size = 20, antiderivative size = 274

$$\int \frac{x}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{x^2}{2\sqrt[3]{1-x^3}} + \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}} - \frac{1}{4}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) + \frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}} - \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} - \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}}$$

```
[Out] 1/2*x^2/(-x^3+1)^(1/3)-1/4*x^2*hypergeom([1/3, 2/3],[5/3],x^3)+1/48*ln((1-x
)*(1+x)^2)*2^(2/3)+1/24*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(
-x^3+1)^(1/3))*2^(2/3)-1/12*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/16
*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)+1/12*arctan(1/3*(1-2*2^(1/3)*(1-x)
/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/24*arctan(1/3*(1+2^(1/3)*(1-x)/
(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {483, 494, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$\int \frac{x}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}} - \frac{1}{4}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) + \frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{12\sqrt[3]{2}} - \frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}} - \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{8\sqrt[3]{2}} + \frac{x^2}{2\sqrt[3]{1-x^3}} + \frac{\log\left((1-x)(x+1)^2\right)}{24\sqrt[3]{2}}$$

```
[In] Int[x/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] x^2/(2*(1 - x^3)^(1/3)) + ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/
Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3
))/Sqrt[3]]/(4*2^(1/3)*Sqrt[3]) - (x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3
])/4 + Log[(1 - x)*(1 + x)^2]/(24*2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(1
 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(12*2^(1/3)) - Log[1 + (
2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[-1 + x + 2^(2/3)*(1 - x
^3)^(1/3)]/(8*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^2}{2\sqrt[3]{1-x^3}}-\frac{1}{2}\int \frac{x^4}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx\\
&= \frac{x^2}{2\sqrt[3]{1-x^3}}-\frac{1}{2}\int \frac{x}{\sqrt[3]{1-x^3}}\,dx+\frac{1}{2}\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx\\
&= \frac{x^2}{2\sqrt[3]{1-x^3}}-\frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{1}{6}\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx\\
&\quad -\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+2x^3}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right)\\
&= \frac{x^2}{2\sqrt[3]{1-x^3}}+\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}}-\frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}}\\
&\quad -\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}}-\frac{1}{6}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right)\\
&\quad -\frac{1}{6}\text{Subst}\left(\int \frac{2-\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right)
\end{aligned}
$$

$$=\frac{x^2}{2\sqrt[3]{1-x^3}}+\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}}-\frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}}-\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}-\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}}-\frac{1}{4}\text{Subst}\left(\int\frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right)+\frac{\text{Subst}\left(\int\frac{-\sqrt[3]{2}+2\ 2^{2/3}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx,x,\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}}$$

$$=\frac{x^2}{2\sqrt[3]{1-x^3}}+\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}}-\frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}}+\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}}-\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}-\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}}-\frac{\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2\sqrt[3]{2}}$$

$$=\frac{x^2}{2\sqrt[3]{1-x^3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}}-\frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}}+\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}}-\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}-\frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 10.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.16

$$\int \frac{x}{(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{x^2}{2\sqrt[3]{1-x^3}} - \frac{1}{10}x^5\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right)$$

```
[In] Integrate[x/((1 - x^3)^(4/3)*(1 + x^3)),x]
[Out] x^2/(2*(1 - x^3)^(1/3)) - (x^5*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/10
```

Maple [F]

$$\int \frac{x}{\left(-x^3+1\right)^{\frac{4}{3}}\left(x^3+1\right)}dx$$

```
[In] int(x/(-x^3+1)^(4/3)/(x^3+1),x)
[Out] int(x/(-x^3+1)^(4/3)/(x^3+1),x)
```

Fricas [F]

$$\int \frac{x}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)*x/(x^9 - x^6 - x^3 + 1), x)
```

Sympy [F]

$$\int \frac{x}{(1-x^3)^{4/3}(1+x^3)}\,dx = \int \frac{x}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

```
[In] integrate(x/(-x**3+1)**(4/3)/(x**3+1),x)
[Out] Integral(x/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

Maxima [F]

$$\int \frac{x}{(1-x^3)^{4/3}\,(1+x^3)}\,dx = \int \frac{x}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Giac [F]

$$\int \frac{x}{(1-x^3)^{4/3}\,(1+x^3)}\,dx = \int \frac{x}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(1-x^3)^{4/3}\,(1+x^3)}\,dx = \int \frac{x}{(1-x^3)^{4/3}\,(x^3+1)}\,dx$$

```
[In] int(x/((1 - x^3)^(4/3)*(x^3 + 1)),x)
[Out] int(x/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.656 $\int \frac{1}{x^2(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 292

$$\int \frac{1}{x^2(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{2x\sqrt[3]{1-x^3}} - \frac{3(1-x^3)^{2/3}}{2x} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}} - \frac{3}{4}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) - \frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}} - \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}}$$

```
[Out] 1/2/x/(-x^3+1)^(1/3)-3/2*(-x^3+1)^(2/3)/x-3/4*x^2*hypergeom([1/3, 2/3],[5/3
],x^3)-1/48*ln((1-x)*(1+x)^2)*2^(2/3)-1/24*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2
/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/12*ln(1+2^(1/3)*(1-x)/(-x^3+1)^
(1/3))*2^(2/3)+1/16*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)-1/12*arctan(1/3
*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/24*arctan(1/
3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {483, 597, 598, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{x^2 (1-x^3)^{4/3} (1+x^3)} \, dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}} - \frac{3}{4}x^2 \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) - \frac{3(1-x^3)^{2/3}}{2x} + \frac{1}{2\sqrt[3]{1-x^3}x} - \frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} + 1\right)}{12\sqrt[3]{2}} + \frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} + 1\right)}{6\sqrt[3]{2}} + \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3} + x - 1\right)}{8\sqrt[3]{2}} - \frac{\log((1-x)(x+1)^2)}{24\sqrt[3]{2}}$$

```
[In] Int[1/(x^2*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*x*(1 - x^3)^(1/3)) - (3*(1 - x^3)^(2/3))/(2*x) - ArcTan[(1 - (2*2^(1/3
)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - ArcTan[(1 + (2^(
1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(4*2^(1/3)*Sqrt[3]) - (3*x^2*Hyperg
eometric2F1[1/3, 2/3, 5/3, x^3])/4 - Log[(1 - x)*(1 + x)^2]/(24*2^(1/3)) -
Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(
1/3)]/(12*2^(1/3)) + Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3))
 + Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(8*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2x\sqrt[3]{1-x^3}} + \frac{1}{2}\int \frac{3+2x^3}{x^2\sqrt[3]{1-x^3}\,(1+x^3)}\,dx \\
&= \frac{1}{2x\sqrt[3]{1-x^3}} - \frac{3(1-x^3)^{2/3}}{2x} - \frac{1}{2}\int \frac{x(4+3x^3)}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx \\
&= \frac{1}{2x\sqrt[3]{1-x^3}} - \frac{3(1-x^3)^{2/3}}{2x} - \frac{1}{2}\int \left(\frac{3x}{\sqrt[3]{1-x^3}} + \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\right)dx \\
&= \frac{1}{2x\sqrt[3]{1-x^3}} - \frac{3(1-x^3)^{2/3}}{2x} - \frac{1}{2}\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx - \frac{3}{2}\int \frac{x}{\sqrt[3]{1-x^3}}\,dx \\
&= \frac{1}{2x\sqrt[3]{1-x^3}} - \frac{3(1-x^3)^{2/3}}{2x} - \frac{3}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) \\
&\quad + \frac{1}{6}\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+2x^3}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)
\end{aligned}$$

$$= \frac{1}{2x\sqrt[3]{1-x^3}} - \frac{3(1-x^3)^{2/3}}{2x} - \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}} - \frac{3}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) - \frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}} + \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}} + \frac{1}{6}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{6}\text{Subst}\left(\int \frac{2-\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)$$

$$= \frac{1}{2x\sqrt[3]{1-x^3}} - \frac{3(1-x^3)^{2/3}}{2x} - \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}} - \frac{3}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) - \frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{2}+2\ 2^{2/3}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}}$$

$$= \frac{1}{2x\sqrt[3]{1-x^3}} - \frac{3(1-x^3)^{2/3}}{2x} - \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}} - \frac{3}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) - \frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}} - \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2\sqrt[3]{2}}$$

$$= \frac{1}{2x\sqrt[3]{1-x^3}} - \frac{3(1-x^3)^{2/3}}{2x} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}}$$
$$- \frac{3}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) - \frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}} - \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}}$$
$$+ \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 11.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^2(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{-2+3x^3}{2x\sqrt[3]{1-x^3}}$$
$$- x^2\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right) - \frac{3}{10}x^5\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right)$$

```
[In] Integrate[1/(x^2*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (-2 + 3*x^3)/(2*x*(1 - x^3)^(1/3)) - x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x
^3] - (3*x^5*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/10
```

Maple [F]

$$\int \frac{1}{x^2(-x^3+1)^{\frac{4}{3}}(x^3+1)}\,dx$$

```
[In] int(1/x^2/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(1/x^2/(-x^3+1)^(4/3)/(x^3+1),x)
```

Fricas [F]

$$\int \frac{1}{x^2 (1-x^3)^{4/3} (1+x^3)} \, dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}} x^2} \, dx$$

```
[In] integrate(1/x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)/(x^11 - x^8 - x^5 + x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2 (1-x^3)^{4/3} (1+x^3)} \, dx = \int \frac{1}{x^2 \left(-(x-1)(x^2+x+1)\right)^{\frac{4}{3}} (x+1)(x^2-x+1)} \, dx$$

```
[In] integrate(1/x**2/(-x**3+1)**(4/3)/(x**3+1),x)
[Out] Integral(1/(x**2*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Maxima [F]

$$\int \frac{1}{x^2 (1-x^3)^{4/3} (1+x^3)} \, dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}} x^2} \, dx$$

```
[In] integrate(1/x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (1-x^3)^{4/3} (1+x^3)} \, dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}} x^2} \, dx$$

```
[In] integrate(1/x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (1 - x^3)^{4/3} (1 + x^3)} \, dx = \int \frac{1}{x^2 (1 - x^3)^{4/3} (x^3 + 1)} \, dx$$

```
[In] int(1/(x^2*(1 - x^3)^(4/3)*(x^3 + 1)),x)
[Out] int(1/(x^2*(1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.657 $\int \frac{1}{x^5(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 308

$$\int \frac{1}{x^5(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{2x^4\sqrt[3]{1-x^3}} - \frac{3(1-x^3)^{2/3}}{4x^4} - \frac{(1-x^3)^{2/3}}{x} + \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}} - \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) + \frac{\log((1-x)(1+x)^2)}{24\sqrt[3]{2}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}} - \frac{\log\left(1 \right.}{}$$

```
[Out] 1/2/x^4/(-x^3+1)^(1/3)-3/4*(-x^3+1)^(2/3)/x^4-(-x^3+1)^(2/3)/x-1/2*x^2*hype
rgeom([1/3, 2/3],[5/3],x^3)+1/48*ln((1-x)*(1+x)^2)*2^(2/3)+1/24*ln(1+2^(2/3
)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/12*ln(1+2^
(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/16*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^
(2/3)+1/12*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3
^(1/2)+1/24*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^
(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {483, 597, 598, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{x^5 (1-x^3)^{4/3} (1+x^3)} dx = \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}} - \frac{1}{2}x^2 \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},x^3\right) - \frac{(1-x^3)^{2/3}}{x} + \frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} + 1\right)}{12\sqrt[3]{2}} - \frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}} + 1\right)}{6\sqrt[3]{2}} - \frac{\log\left(2^{2/3}\sqrt[3]{1-x^3} + x - 1\right)}{8\sqrt[3]{2}} - \frac{3(1-x^3)^{2/3}}{4x^4} + \frac{1}{2\sqrt[3]{1-x^3}x^4} + \frac{\log((1-x)(x+1)^2)}{24\sqrt[3]{2}}$$

```
[In] Int[1/(x^5*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*x^4*(1 - x^3)^(1/3)) - (3*(1 - x^3)^(2/3))/(4*x^4) - (1 - x^3)^(2/3)/x
 + ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqr
t[3]) + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(4*2^(1/3)*
Sqrt[3]) - (x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/2 + Log[(1 - x)*(1 +
 x)^2]/(24*2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3
)*(1 - x))/(1 - x^3)^(1/3)]/(12*2^(1/3)) - Log[1 + (2^(1/3)*(1 - x))/(1 - x
^3)^(1/3)]/(6*2^(1/3)) - Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(8*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2x^4\sqrt[3]{1-x^3}}+\frac{1}{2}\int \frac{6+5x^3}{x^5\sqrt[3]{1-x^3}\,(1+x^3)}\,dx\\
&= \frac{1}{2x^4\sqrt[3]{1-x^3}}-\frac{3(1-x^3)^{2/3}}{4x^4}-\frac{1}{8}\int \frac{-8-12x^3}{x^2\sqrt[3]{1-x^3}\,(1+x^3)}\,dx\\
&= \frac{1}{2x^4\sqrt[3]{1-x^3}}-\frac{3(1-x^3)^{2/3}}{4x^4}-\frac{(1-x^3)^{2/3}}{x}+\frac{1}{8}\int \frac{x(-4-8x^3)}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx\\
&= \frac{1}{2x^4\sqrt[3]{1-x^3}}-\frac{3(1-x^3)^{2/3}}{4x^4}-\frac{(1-x^3)^{2/3}}{x}+\frac{1}{8}\int \left(-\frac{8x}{\sqrt[3]{1-x^3}}+\frac{4x}{\sqrt[3]{1-x^3}\,(1+x^3)}\right)dx\\
&= \frac{1}{2x^4\sqrt[3]{1-x^3}}-\frac{3(1-x^3)^{2/3}}{4x^4}-\frac{(1-x^3)^{2/3}}{x}+\frac{1}{2}\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx-\int \frac{x}{\sqrt[3]{1-x^3}}\,dx
\end{aligned}
$$

$$= \frac{1}{2x^4\sqrt[3]{1-x^3}} - \frac{3(1-x^3)^{2/3}}{4x^4} - \frac{(1-x^3)^{2/3}}{x} - \frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) - \frac{1}{6}\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+2x^3}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)$$

$$= \frac{1}{2x^4\sqrt[3]{1-x^3}} - \frac{3(1-x^3)^{2/3}}{4x^4} - \frac{(1-x^3)^{2/3}}{x} + \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}} - \frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) + \frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}} - \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}} - \frac{1}{6}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{6}\text{Subst}\left(\int \frac{2-\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)$$

$$= \frac{1}{2x^4\sqrt[3]{1-x^3}} - \frac{3(1-x^3)^{2/3}}{4x^4} - \frac{(1-x^3)^{2/3}}{x} + \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}} - \frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) + \frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}} - \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} - \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right) + \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{2}+2\ 2^{2/3}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{1-x}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}}$$

$$= \frac{1}{2x^4\sqrt[3]{1-x^3}} - \frac{3(1-x^3)^{2/3}}{4x^4} - \frac{(1-x^3)^{2/3}}{x} + \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}} - \frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) + \frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}} - \frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} - \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2\sqrt[3]{2}}$$

$$= \frac{1}{2x^4\sqrt[3]{1-x^3}} - \frac{3(1-x^3)^{2/3}}{4x^4} - \frac{(1-x^3)^{2/3}}{x} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt{3}}$$

$$-\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) + \frac{\log\left((1-x)(1+x)^2\right)}{24\sqrt[3]{2}} + \frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}} - \frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}}$$

$$-\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} - \frac{\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{8\sqrt[3]{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 11.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^5\,(1-x^3)^{4/3}\,(1+x^3)}\,dx = -\frac{\frac{5(1+x^3-4x^6)}{\sqrt[3]{1-x^3}} + 5x^6\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},x^3,-x^3\right) + 4x^9\operatorname{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},x^3,-x^3\right)}{20x^4}$$

```
[In] Integrate[1/(x^5*(1 - x^3)^(4/3)*(1 + x^3)),x]
[Out] -1/20*((5*(1 + x^3 - 4*x^6))/(1 - x^3)^(1/3) + 5*x^6*AppellF1[2/3, 1/3, 1,
5/3, x^3, -x^3] + 4*x^9*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/x^4
```

Maple [F]

$$\int \frac{1}{x^5\,(-x^3+1)^{\frac{4}{3}}\,(x^3+1)}dx$$

```
[In] int(1/x^5/(-x^3+1)^(4/3)/(x^3+1),x)
[Out] int(1/x^5/(-x^3+1)^(4/3)/(x^3+1),x)
```

Fricas [F]

$$\int \frac{1}{x^5 (1-x^3)^{4/3} (1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}x^5}\,dx$$

```
[In] integrate(1/x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)/(x^14 - x^11 - x^8 + x^5), x)
```

Sympy [F]

$$\int \frac{1}{x^5 (1-x^3)^{4/3} (1+x^3)}\,dx = \int \frac{1}{x^5 \left(-(x-1)(x^2+x+1)\right)^{\frac{4}{3}} (x+1)(x^2-x+1)}\,dx$$

```
[In] integrate(1/x**5/(-x**3+1)**(4/3)/(x**3+1),x)
[Out] Integral(1/(x**5*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

Maxima [F]

$$\int \frac{1}{x^5 (1-x^3)^{4/3} (1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}x^5}\,dx$$

```
[In] integrate(1/x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5 (1-x^3)^{4/3} (1+x^3)}\,dx = \int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}x^5}\,dx$$

```
[In] integrate(1/x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (1-x^3)^{4/3} (1+x^3)} \, dx = \int \frac{1}{x^5 (1-x^3)^{4/3} (x^3+1)} \, dx$$

```
[In] int(1/(x^5*(1 - x^3)^(4/3)*(x^3 + 1)),x)
[Out] int(1/(x^5*(1 - x^3)^(4/3)*(x^3 + 1)), x)
```

3.658 $\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 264

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=-\frac{c^3\sqrt[3]{a+bx^3}}{d^4}+\frac{(b^2c^2+abcd+a^2d^2)\,(a+bx^3)^{4/3}}{4b^3d^3}-\frac{(bc+2ad)\,(a+bx^3)^{7/3}}{7b^3d^2}+\frac{(a+bx^3)^{10/3}}{10b^3d}-\frac{c^3\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{13/3}}-\frac{c^3\sqrt[3]{bc-ad}\log(c+dx^3)}{6d^{13/3}}+\frac{c^3\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{13/3}}$$

```
[Out] -c^3*(b*x^3+a)^(1/3)/d^4+1/4*(a^2*d^2+a*b*c*d+b^2*c^2)*(b*x^3+a)^(4/3)/b^3/
d^3-1/7*(2*a*d+b*c)*(b*x^3+a)^(7/3)/b^3/d^2+1/10*(b*x^3+a)^(10/3)/b^3/d-1/6
*c^3*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/d^(13/3)+1/2*c^3*(-a*d+b*c)^(1/3)*ln((-a*
d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(13/3)-1/3*c^3*(-a*d+b*c)^(1/3)*arc
tan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(13/3)*3^
(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {457, 90, 52, 60, 631, 210, 31}

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{(a+bx^3)^{4/3}(a^2d^2+abcd+b^2c^2)}{4b^3d^3} - \frac{c^3\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{13/3}} - \frac{(a+bx^3)^{7/3}(2ad+bc)}{7b^3d^2} + \frac{(a+bx^3)^{10/3}}{10b^3d} - \frac{c^3\sqrt[3]{bc-ad}\log(c+dx^3)}{6d^{13/3}} + \frac{c^3\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{13/3}} - \frac{c^3\sqrt[3]{a+bx^3}}{d^4}$$

```
[In] Int[(x^11*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] -((c^3*(a + b*x^3)^(1/3))/d^4) + ((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)
^(4/3))/(4*b^3*d^3) - ((b*c + 2*a*d)*(a + b*x^3)^(7/3))/(7*b^3*d^2) + (a +
b*x^3)^(10/3)/(10*b^3*d) - (c^3*(b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a
 + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(13/3)) - (c^3*(b*
c - a*d)^(1/3)*Log[c + d*x^3])/(6*d^(13/3)) + (c^3*(b*c - a*d)^(1/3)*Log[(b
*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
```

```
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^3\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right) \\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{(b^2c^2+abcd+a^2d^2)\sqrt[3]{a+bx}}{b^2d^3} + \frac{(-bc-2ad)(a+bx)^{4/3}}{b^2d^2} \right.\right. \\
&\qquad \left.\left. + \frac{(a+bx)^{7/3}}{b^2d} - \frac{c^3\sqrt[3]{a+bx}}{d^3(c+dx)}\right)dx, x, x^3\right) \\
&= \frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{4/3}}{4b^3d^3} - \frac{(bc+2ad)(a+bx^3)^{7/3}}{7b^3d^2} \\
&\quad + \frac{(a+bx^3)^{10/3}}{10b^3d} - \frac{c^3\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)}{3d^3}
\end{aligned}
$$

$$=-\frac{c^3\sqrt[3]{a+bx^3}}{d^4}+\frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{4/3}}{4b^3d^3}-\frac{(bc+2ad)(a+bx^3)^{7/3}}{7b^3d^2}+\frac{(a+bx^3)^{10/3}}{10b^3d}+\frac{(c^3(bc-ad))\,\text{Subst}\left(\int\frac{1}{(a+bx)^{2/3}(c+dx)}\,dx,x,x^3\right)}{3d^4}$$

$$=-\frac{c^3\sqrt[3]{a+bx^3}}{d^4}+\frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{4/3}}{4b^3d^3}-\frac{(bc+2ad)(a+bx^3)^{7/3}}{7b^3d^2}+\frac{(a+bx^3)^{10/3}}{10b^3d}-\frac{c^3\sqrt[3]{bc-ad}\log(c+dx^3)}{6d^{13/3}}+\frac{\left(c^3\sqrt[3]{bc-ad}\right)\text{Subst}\left(\int\frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx,x,\sqrt[3]{a+bx^3}\right)}{2d^{13/3}}+\frac{\left(c^3(bc-ad)^{2/3}\right)\text{Subst}\left(\int\frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx,x,\sqrt[3]{a+bx^3}\right)}{2d^{14/3}}$$

$$=-\frac{c^3\sqrt[3]{a+bx^3}}{d^4}+\frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{4/3}}{4b^3d^3}-\frac{(bc+2ad)(a+bx^3)^{7/3}}{7b^3d^2}+\frac{(a+bx^3)^{10/3}}{10b^3d}-\frac{c^3\sqrt[3]{bc-ad}\log(c+dx^3)}{6d^{13/3}}+\frac{c^3\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{13/3}}+\frac{\left(c^3\sqrt[3]{bc-ad}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{d^{13/3}}$$

$$=-\frac{c^3\sqrt[3]{a+bx^3}}{d^4}+\frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{4/3}}{4b^3d^3}-\frac{(bc+2ad)(a+bx^3)^{7/3}}{7b^3d^2}+\frac{(a+bx^3)^{10/3}}{10b^3d}-\frac{c^3\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{13/3}}-\frac{c^3\sqrt[3]{bc-ad}\log(c+dx^3)}{6d^{13/3}}+\frac{c^3\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{13/3}}$$

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.17

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

$$= \frac{\frac{3\sqrt[3]{d}\sqrt[3]{a+bx^3}(9a^3d^3-3a^2bd^2(-5c+dx^3)+ab^2d(35c^2-5cdx^3+2d^2x^6)+b^3(-140c^3+35c^2dx^3-20cd^2x^6+14d^3x^9))}{b^3}-140\sqrt{3}c^3\sqrt[3]{bc-}}{}$$

[In] Integrate[(x^11*(a + b*x^3)^(1/3))/(c + d*x^3),x]

[Out] ((3*d^(1/3)*(a + b*x^3)^(1/3)*(9*a^3*d^3 - 3*a^2*b*d^2*(-5*c + d*x^3) + a*b^2*d*(35*c^2 - 5*c*d*x^3 + 2*d^2*x^6) + b^3*(-140*c^3 + 35*c^2*d*x^3 - 20*c*d^2*x^6 + 14*d^3*x^9)))/b^3 - 140*Sqrt[3]*c^3*(b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 140*c^3*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - 70*c^3*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(420*d^(13/3))

Maple [A] (verified)

Time = 6.04 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$\frac{\frac{27\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}d\left(\frac{\left(14d^3x^9-20c\,d^2x^6+35c^2d\,x^3-140c^3\right)b^3}{9}+\frac{35\left(\frac{2}{35}d^2x^6-\frac{1}{7}cd\,x^3+c^2\right)da\,b^2}{9}+\frac{5\left(-\frac{d\,x^3}{5}+c\right)d^2a^2b}{3}+a^3d^3\right)}{70}+b^3}{}$

[In] int(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] 1/6*(27/70*(1/d*(a*d-b*c))^(2/3)*(b*x^3+a)^(1/3)*d*(1/9*(14*d^3*x^9-20*c*d^2*x^6+35*c^2*d*x^3-140*c^3)*b^3+35/9*(2/35*d^2*x^6-1/7*c*d*x^3+c^2)*d*a*b^2+5/3*(-1/5*d*x^3+c)*d^2*a^2*b+a^3*d^3)+b^3*c^3*(a*d-b*c)*(2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))))/(1/d*(a*d-b*c))^(2/3)/b^3/d^5

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.23

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = -\frac{140\sqrt{3}b^3c^3\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+70\,b^3c^3\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}-(bx\right.}{}$$

```
[In] integrate(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/420*(140*sqrt(3)*b^3*c^3*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b
*x^3 + a)^(1/3)*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d))
 + 70*b^3*c^3*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/
3)*((b*c - a*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3)) - 140*b^3*c^3*((b*c - a*d
)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b*c - a*d)/d)^(1/3)) - 3*(14*b^3*d^3*x
^9 - 2*(10*b^3*c*d^2 - a*b^2*d^3)*x^6 - 140*b^3*c^3 + 35*a*b^2*c^2*d + 15*a
^2*b*c*d^2 + 9*a^3*d^3 + (35*b^3*c^2*d - 5*a*b^2*c*d^2 - 3*a^2*b*d^3)*x^3)*
(b*x^3 + a)^(1/3))/(b^3*d^4)
```

Sympy [F]

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

```
[In] integrate(x**11*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**11*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.44

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = -\frac{(b^{34}c^4d^6 - ab^{33}c^3d^7)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^{34}cd^{10} - ab^{33}d^{11})} + \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}c^3\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^5} + \frac{(-bcd^2+ad^3)^{\frac{1}{3}}c^3\log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^5} - \frac{140\,(bx^3+a)^{\frac{1}{3}}b^{30}c^3d^6 - 35\,(bx^3+a)^{\frac{4}{3}}b^{29}c^2d^7 + 20\,(bx^3+a)^{\frac{7}{3}}b^{28}cd^8 - 35\,(bx^3+a)^{\frac{4}{3}}ab^{28}cd^8 - 14\,(bx^3+a}{140\,b^{30}d^{10}}$$

```
[In] integrate(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^34*c^4*d^6 - a*b^33*c^3*d^7)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3
+ a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^34*c*d^10 - a*b^33*d^11) + 1/3*sqr
t(3)*(-b*c*d^2 + a*d^3)^(1/3)*c^3*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) +
 (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^5 + 1/6*(-b*c*d^2 + a*d^
3)^(1/3)*c^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/
3) + (-(b*c - a*d)/d)^(2/3))/d^5 - 1/140*(140*(b*x^3 + a)^(1/3)*b^30*c^3*d^
6 - 35*(b*x^3 + a)^(4/3)*b^29*c^2*d^7 + 20*(b*x^3 + a)^(7/3)*b^28*c*d^8 - 3
5*(b*x^3 + a)^(4/3)*a*b^28*c*d^8 - 14*(b*x^3 + a)^(10/3)*b^27*d^9 + 40*(b*x
^3 + a)^(7/3)*a*b^27*d^9 - 35*(b*x^3 + a)^(4/3)*a^2*b^27*d^9)/(b^30*d^10)
```

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.67

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \left(\frac{3\,a^2}{4\,b^3\,d} + \frac{\left(\frac{3\,a}{b^3\,d} + \frac{b^4\,c-a\,b^3\,d}{b^6\,d^2}\right)\,(b^4\,c-a\,b^3\,d)}{4\,b^3\,d}\right)\,{\left(b\,x^3+a\right)}^{4/3} - \left(\frac{3\,a}{7\,b^3\,d} + \frac{b^4\,c-a\,b^3\,d}{7\,b^6\,d^2}\right)\,{\left(b\,x^3+a\right)}^{7/3} - {\left(b\,x^3+a\right)}^{1/3}\left(\frac{a^3}{b^3\,d} + \frac{\left(\frac{3\,a^2}{b^3\,d} + \frac{\left(\frac{3\,a}{b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{b^6\,d^2}\right)\,(b^4\,c-a\,b^3\,d)}{b^3\,d}\right)\,(b^4\,c-a\,b^3\,d)}{b^3\,d}\right) + \frac{{\left(b\,x^3+a\right)}^{10/3}}{10\,b^3\,d} - \frac{c^3\,\ln\left((a\,d-b\right.}{}$$

```
[In] int((x^11*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

```
[Out] ((3*a^2)/(4*b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c
- a*b^3*d))/(4*b^3*d))*(a + b*x^3)^(4/3) - ((3*a)/(7*b^3*d) + (b^4*c - a*b^
3*d)/(7*b^6*d^2))*(a + b*x^3)^(7/3) - (a + b*x^3)^(1/3)*(a^3/(b^3*d) + (((3
*a^2)/(b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c - a*b
^3*d))/(b^3*d))*(b^4*c - a*b^3*d))/(b^3*d)) + (a + b*x^3)^(10/3)/(10*b^3*d)
 - (c^3*log((a*d - b*c)^(1/3) - d^(1/3)*(a + b*x^3)^(1/3))*(a*d - b*c)^(1/3
))/(3*d^(13/3)) - (c^3*log((3*(a + b*x^3)^(1/3)*(b*c^4 - a*c^3*d))/d^2 + (3
*c^3*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^(4/3))/d^(7/3))*((3^(1/2)*1i)/2 - 1
/2)*(a*d - b*c)^(1/3))/(3*d^(13/3)) + (c^3*log((3*(a + b*x^3)^(1/3)*(b*c^4
- a*c^3*d))/d^2 - (9*c^3*((3^(1/2)*1i)/6 + 1/6)*(a*d - b*c)^(4/3))/d^(7/3))
*((3^(1/2)*1i)/6 + 1/6)*(a*d - b*c)^(1/3))/d^(13/3)
```

3.659 $\int \frac{x^8 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 220

$$\int \frac{x^8 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx = \frac{c^2 \sqrt[3]{a+bx^3}}{d^3} - \frac{(bc+ad)\left(a+bx^3\right)^{4/3}}{4b^2d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7b^2d} + \frac{c^2 \sqrt[3]{bc-ad} \arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{10/3}} + \frac{c^2 \sqrt[3]{bc-ad} \log\left(c+dx^3\right)}{6d^{10/3}} - \frac{c^2 \sqrt[3]{bc-ad} \log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}}$$

```
[Out] c^2*(b*x^3+a)^(1/3)/d^3-1/4*(a*d+b*c)*(b*x^3+a)^(4/3)/b^2/d^2+1/7*(b*x^3+a)
^(7/3)/b^2/d+1/6*c^2*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/d^(10/3)-1/2*c^2*(-a*d+b*
c)^(1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(10/3)+1/3*c^2*(-a*
d+b*c)^(1/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1
/2))/d^(10/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used

$= \{457, 90, 52, 60, 631, 210, 31\}$

$$\int \frac{x^8 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{c^2 \sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{10/3}} - \frac{(a+bx^3)^{4/3}(ad+bc)}{4b^2d^2} + \frac{(a+bx^3)^{7/3}}{7b^2d} + \frac{c^2\sqrt[3]{bc-ad}\log(c+dx^3)}{6d^{10/3}} - \frac{c^2\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}} + \frac{c^2\sqrt[3]{a+bx^3}}{d^3}$$

```
[In] Int[(x^8*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (c^2*(a + b*x^3)^(1/3))/d^3 - ((b*c + a*d)*(a + b*x^3)^(4/3))/(4*b^2*d^2) +
 (a + b*x^3)^(7/3)/(7*b^2*d) + (c^2*(b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3
)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(10/3)) + (c^2
*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(6*d^(10/3)) - (c^2*(b*c - a*d)^(1/3)*Lo
g[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{(-bc-ad)\sqrt[3]{a+bx}}{bd^2}+\frac{(a+bx)^{4/3}}{bd}+\frac{c^2\sqrt[3]{a+bx}}{d^2(c+dx)}\right)\,dx, x, x^3\right)\\
&= -\frac{(bc+ad)\left(a+bx^3\right)^{4/3}}{4b^2d^2}+\frac{\left(a+bx^3\right)^{7/3}}{7b^2d}+\frac{c^2\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)}{3d^2}\\
&= \frac{c^2\sqrt[3]{a+bx^3}}{d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{4/3}}{4b^2d^2}+\frac{\left(a+bx^3\right)^{7/3}}{7b^2d}\\
&\quad -\frac{\left(c^2(bc-ad)\right)\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3d^3}
\end{aligned}
$$

$$=\frac{c^2\sqrt[3]{a+bx^3}}{d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{4/3}}{4b^2d^2}+\frac{\left(a+bx^3\right)^{7/3}}{7b^2d}+\frac{c^2\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{10/3}}-\frac{\left(c^2\sqrt[3]{bc-ad}\right)\text{Subst}\left(\int\frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx,x,\sqrt[3]{a+bx^3}\right)}{2d^{10/3}}-\frac{\left(c^2(bc-ad)^{2/3}\right)\text{Subst}\left(\int\frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx,x,\sqrt[3]{a+bx^3}\right)}{2d^{11/3}}$$

$$=\frac{c^2\sqrt[3]{a+bx^3}}{d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{4/3}}{4b^2d^2}+\frac{\left(a+bx^3\right)^{7/3}}{7b^2d}+\frac{c^2\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{10/3}}-\frac{c^2\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}}-\frac{\left(c^2\sqrt[3]{bc-ad}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{d^{10/3}}$$

$$=\frac{c^2\sqrt[3]{a+bx^3}}{d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{4/3}}{4b^2d^2}+\frac{\left(a+bx^3\right)^{7/3}}{7b^2d}+\frac{c^2\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{10/3}}+\frac{c^2\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{10/3}}-\frac{c^2\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.20

$$\int\frac{x^8\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

$$=\frac{\frac{3\sqrt[3]{d}\sqrt[3]{a+bx^3}\left(-3a^2d^2+abd\left(-7c+dx^3\right)+b^2\left(28c^2-7cdx^3+4d^2x^6\right)\right)}{b^2}+28\sqrt{3}c^2\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)-\cdots}{\cdots}$$

```
[In] Integrate[(x^8*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] ((3*d^(1/3)*(a + b*x^3)^(1/3)*(-3*a^2*d^2 + a*b*d*(-7*c + d*x^3) + b^2*(28*
c^2 - 7*c*d*x^3 + 4*d^2*x^6)))/b^2 + 28*Sqrt[3]*c^2*(b*c - a*d)^(1/3)*ArcTa
n[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 28*c^2*(
b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + 14*c^
2*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a +
b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]])/(84*d^(10/3))
```

Maple [A] (verified)

Time = 4.73 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$-\frac{\frac{9\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}} d\left(\left(-\frac{4b x^3}{3}+a\right)\left(b x^3+a\right)d^2+\frac{7\left(b x^3+a\right)bcd}{3}-\frac{28b^2c^2}{3}\right)\left(b x^3+a\right)^{\frac{1}{3}}}{14}+b^2c^2(ad-bc)\left(2\arctan\left(\frac{\sqrt{3}\left(2\left(b x^3+a\right)^{\frac{1}{3}}+\left(\frac{ad-}{d}\right.\right.}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right.\right.}{6\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}} b^2d^4}$

```
[In] int(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/(1/d*(a*d-b*c))^(2/3)*(9/14*(1/d*(a*d-b*c))^(2/3)*d*((-4/3*b*x^3+a)*(b
*x^3+a)*d^2+7/3*(b*x^3+a)*b*c*d-28/3*b^2*c^2)*(b*x^3+a)^(1/3)+b^2*c^2*(a*d-
b*c)*(2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(
a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)
^(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3)))
/b^2/d^4
```

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.28

$$\int \frac{x^8\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = -\frac{28\sqrt{3}b^2c^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+14\,b^2c^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(\right.\right.}{}$$

```
[In] integrate(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/84*(28*sqrt(3)*b^2*c^2*(-(b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*
x^3 + a)^(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d))
 + 14*b^2*c^2*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1
/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3)) - 28*b^2*c^2*(-(b*c -
a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)) - 3*(4*b^2*d^
```

```
2*x^6 + 28*b^2*c^2 - 7*a*b*c*d - 3*a^2*d^2 - (7*b^2*c*d - a*b*d^2)*x^3)*(b*
x^3 + a)^(1/3))/(b^2*d^3)
```

Sympy [F]

$$\int \frac{x^8\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{x^8\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

```
[In] integrate(x**8*(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(x**8*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.45

$$\begin{aligned}\int \frac{x^8\sqrt[3]{a+bx^3}}{c+dx^3}\,dx &= \frac{(b^{17}c^3d^4 - ab^{16}c^2d^5)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^{17}cd^7 - ab^{16}d^8)} \\ &\quad - \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}c^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^4} \\ &\quad - \frac{(-bcd^2+ad^3)^{\frac{1}{3}}c^2\log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^4} \\ &\quad + \frac{28\,(bx^3+a)^{\frac{1}{3}}b^{14}c^2d^4 - 7\,(bx^3+a)^{\frac{4}{3}}b^{13}cd^5 + 4\,(bx^3+a)^{\frac{7}{3}}b^{12}d^6 - 7\,(bx^3+a)^{\frac{4}{3}}ab^{12}d^6}{28\,b^{14}d^7}\end{aligned}$$

```
[In] integrate(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b^17*c^3*d^4 - a*b^16*c^2*d^5)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 +
 a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^17*c*d^7 - a*b^16*d^8) - 1/3*sqrt(3
)*(-b*c*d^2 + a*d^3)^(1/3)*c^2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-
(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^4 - 1/6*(-b*c*d^2 + a*d^3)^
(1/3)*c^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3)
+ (-(b*c - a*d)/d)^(2/3))/d^4 + 1/28*(28*(b*x^3 + a)^(1/3)*b^14*c^2*d^4 - 7
*(b*x^3 + a)^(4/3)*b^13*c*d^5 + 4*(b*x^3 + a)^(7/3)*b^12*d^6 - 7*(b*x^3 + a
)^(4/3)*a*b^12*d^6)/(b^14*d^7)
```

Mupad [B] (verification not implemented)

Time = 8.89 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.53

$$\int \frac{x^8\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \left(\frac{a^2}{b^2\,d}+\frac{\left(\frac{2\,a}{b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{b^4\,d^2}\right)\,\left(b^3\,c-a\,b^2\,d\right)}{b^2\,d}\right)\,{\left(b\,x^3+a\right)}^{1/3} - \left(\frac{a}{2\,b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{4\,b^4\,d^2}\right)\,{\left(b\,x^3+a\right)}^{4/3}+\frac{{\left(b\,x^3+a\right)}^{7/3}}{7\,b^2\,d} + \frac{c^2\,\ln\left({\left(a\,d-b\,c\right)}^{1/3}-d^{1/3}\,{\left(b\,x^3+a\right)}^{1/3}\right)\,{\left(a\,d-b\,c\right)}^{1/3}}{3\,d^{10/3}} - \frac{c^2\,\ln\left(\frac{3\,{\left(b\,x^3+a\right)}^{1/3}\,\left(b\,c^3-a\,c^2\,d\right)}{d}-\frac{3\,c^2\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{4/3}}{d^{4/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{1/3}}{3\,d^{10/3}} + \frac{c^2\,\ln\left(\frac{3\,{\left(b\,x^3+a\right)}^{1/3}\,\left(b\,c^3-a\,c^2\,d\right)}{d}+\frac{9\,c^2\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,{\left(a\,d-b\,c\right)}^{4/3}}{d^{4/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,{\left(a\,d-b\,c\right)}^{1/3}}{d^{10/3}}$$

```
[In] int((x^8*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

```
[Out] (a^2/(b^2*d) + (((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2))*(b^3*c - a*b^
2*d))/(b^2*d))*(a + b*x^3)^(1/3) - (a/(2*b^2*d) + (b^3*c - a*b^2*d)/(4*b^4*
d^2))*(a + b*x^3)^(4/3) + (a + b*x^3)^(7/3)/(7*b^2*d) + (c^2*log((a*d - b*c
)^(1/3) - d^(1/3)*(a + b*x^3)^(1/3))*(a*d - b*c)^(1/3))/(3*d^(10/3)) - (c^2
*log((3*(a + b*x^3)^(1/3)*(b*c^3 - a*c^2*d))/d - (3*c^2*((3^(1/2)*1i)/2 + 1
/2)*(a*d - b*c)^(4/3))/d^(4/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(1/3))/(
3*d^(10/3)) + (c^2*log((3*(a + b*x^3)^(1/3)*(b*c^3 - a*c^2*d))/d + (9*c^2*(
(3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(4/3))/d^(4/3))*((3^(1/2)*1i)/6 - 1/6)*(a
*d - b*c)^(1/3))/d^(10/3)
```

3.660 $\int \frac{x^5 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 186

$$\int \frac{x^5 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx = -\frac{c\sqrt[3]{a+bx^3}}{d^2} + \frac{(a+bx^3)^{4/3}}{4bd} - \frac{c\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{7/3}} - \frac{c\sqrt[3]{bc-ad}\log(c+dx^3)}{6d^{7/3}} + \frac{c\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}}$$

```
[Out] -c*(b*x^3+a)^(1/3)/d^2+1/4*(b*x^3+a)^(4/3)/b/d-1/6*c*(-a*d+b*c)^(1/3)*ln(d*
x^3+c)/d^(7/3)+1/2*c*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)
^(1/3))/d^(7/3)-1/3*c*(-a*d+b*c)^(1/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1
/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(7/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {457, 81, 52, 60, 631, 210, 31}

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = -\frac{c\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{7/3}}-\frac{c\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{7/3}}+\frac{c\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}}-\frac{c\sqrt[3]{a+bx^3}}{d^2}+\frac{\left(a+bx^3\right)^{4/3}}{4bd}$$

```
[In] Int[(x^5*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] -((c*(a + b*x^3)^(1/3))/d^2) + (a + b*x^3)^(4/3)/(4*b*d) - (c*(b*c - a*d)^(
1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])
/(Sqrt[3]*d^(7/3)) - (c*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(6*d^(7/3)) + (c*
(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^
(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)\\
&= \frac{(a+bx^3)^{4/3}}{4bd} - \frac{c\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)}{3d}\\
&= -\frac{c\sqrt[3]{a+bx^3}}{d^2} + \frac{(a+bx^3)^{4/3}}{4bd} + \frac{(c(bc-ad))\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3d^2}
\end{aligned}
$$

$$= -\frac{c\sqrt[3]{a+bx^3}}{d^2} + \frac{(a+bx^3)^{4/3}}{4bd} - \frac{c\sqrt[3]{bc-ad}\log(c+dx^3)}{6d^{7/3}} + \frac{\left(c\sqrt[3]{bc-ad}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{7/3}} + \frac{\left(c(bc-ad)^{2/3}\right)\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{8/3}}$$

$$= -\frac{c\sqrt[3]{a+bx^3}}{d^2} + \frac{(a+bx^3)^{4/3}}{4bd} - \frac{c\sqrt[3]{bc-ad}\log(c+dx^3)}{6d^{7/3}} + \frac{c\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}} + \frac{\left(c\sqrt[3]{bc-ad}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{d^{7/3}}$$

$$= -\frac{c\sqrt[3]{a+bx^3}}{d^2} + \frac{(a+bx^3)^{4/3}}{4bd} - \frac{c\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{7/3}} - \frac{c\sqrt[3]{bc-ad}\log(c+dx^3)}{6d^{7/3}} + \frac{c\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.22

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{c+dx^3}dx = \frac{\frac{3\sqrt[3]{d}\sqrt[3]{a+bx^3}(-4bc+ad+bdx^3)}{b} - 4\sqrt{3}c\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) + 4c\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\right.}{12d^{7/3}}$$

```
[In] Integrate[(x^5*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] ((3*d^(1/3)*(a + b*x^3)^(1/3)*(-4*b*c + a*d + b*d*x^3))/b - 4*Sqrt[3]*c*(b*
c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))
/Sqrt[3]] + 4*c*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^
```

3)^(1/3)] - 2*c*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(12*d^(7/3))

Maple [A] (verified)

Time = 4.82 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.10

method	result
pseudoelliptic	$\frac{\frac{3d\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\left(d\left(b\,x^3+a\right)-4bc\right)\left(b\,x^3+a\right)^{\frac{1}{3}}}{2}+bc(ad-bc)\left(2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^3+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^3+a\right)^{\frac{2}{3}}+\left(\frac{ad-}{d}\right.\right.\right.}{6\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}b\,d^3}$

[In] int(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] 1/6/(1/d*(a*d-b*c))^(2/3)*(3/2*d*(1/d*(a*d-b*c))^(2/3)*(d*(b*x^3+a)-4*b*c)*(b*x^3+a)^(1/3)+b*c*(a*d-b*c)*(2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))))/b/d^3

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.19

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = -\frac{4\sqrt{3}bc\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+2\,bc\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}\right.}{12\,ba}$$

[In] integrate(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")

[Out] -1/12*(4*sqrt(3)*b*c*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 2*b*c*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*((b*c - a*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3)) - 4*b*c*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b*c - a*d)/d)^(1/3)) - 3*(b*d*x^3 - 4*b*c + a*d)*(b*x^3 + a)^(1/3))/(b*d^2)

Sympy [F]

$$\int \frac{x^5 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx = \int \frac{x^5 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$$

```
[In] integrate(x**5*(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(x**5*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.48

$$\begin{aligned}
\int \frac{x^5 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx &= -\frac{\left(b^6c^2d^2 - ab^5cd^3\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} \log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^6cd^4 - ab^5d^5\right)} \\
&\quad + \frac{\sqrt{3}\left(-bcd^2 + ad^3\right)^{\frac{1}{3}} c \arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^3} \\
&\quad + \frac{\left(-bcd^2 + ad^3\right)^{\frac{1}{3}} c \log\left(\left(bx^3+a\right)^{\frac{2}{3}} + \left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^3} \\
&\quad - \frac{4\left(bx^3+a\right)^{\frac{1}{3}} b^4cd^2 - \left(bx^3+a\right)^{\frac{4}{3}} b^3d^3}{4\,b^4d^4}
\end{aligned}$$

```
[In] integrate(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^6*c^2*d^2 - a*b^5*c*d^3)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)
^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^6*c*d^4 - a*b^5*d^5) + 1/3*sqrt(3)*(-b
*c*d^2 + a*d^3)^(1/3)*c*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c -
a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^3 + 1/6*(-b*c*d^2 + a*d^3)^(1/3)*c
*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c
- a*d)/d)^(2/3))/d^3 - 1/4*(4*(b*x^3 + a)^(1/3)*b^4*c*d^2 - (b*x^3 + a)^(4/
3)*b^3*d^3)/(b^4*d^4)
```

Mupad [B] (verification not implemented)

Time = 8.54 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.60

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{\left(b\,x^3+a\right)^{4/3}}{4\,b\,d}-\left(b\,x^3+a\right)^{1/3}\left(\frac{a}{b\,d}+\frac{b^2\,c-a\,b\,d}{b^2\,d^2}\right)$$
$$-\frac{c\,\ln\left(\left(b\,x^3+a\right)^{1/3}\left(3\,b\,c^2-3\,a\,c\,d\right)+\frac{c\,\left(a\,d-b\,c\right)^{1/3}\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{7/3}}\right)\left(a\,d-b\,c\right)^{1/3}}{3\,d^{7/3}}$$
$$-\frac{c\,\ln\left(\left(b\,x^3+a\right)^{1/3}\left(3\,b\,c^2-3\,a\,c\,d\right)+\frac{c\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(a\,d-b\,c\right)^{1/3}\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{7/3}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(a\,d-b\,c\right)^{1/3}}{3\,d^{7/3}}$$
$$+\frac{c\,\ln\left(\left(b\,x^3+a\right)^{1/3}\left(3\,b\,c^2-3\,a\,c\,d\right)-\frac{c\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(a\,d-b\,c\right)^{1/3}\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{7/3}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(a\,d-b\,c\right)^{1/3}}{3\,d^{7/3}}$$

```
[In] int((x^5*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

```
[Out] (a + b*x^3)^(4/3)/(4*b*d) - (a + b*x^3)^(1/3)*(a/(b*d) + (b^2*c - a*b*d)/(b
^2*d^2)) - (c*log((a + b*x^3)^(1/3)*(3*b*c^2 - 3*a*c*d) + (c*(a*d - b*c)^(1
/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(7/3)))*(a*d - b*c)^(1/3))/(3*d^(7/3)) - (c
*log((a + b*x^3)^(1/3)*(3*b*c^2 - 3*a*c*d) + (c*((3^(1/2)*1i)/2 - 1/2)*(a*d
 - b*c)^(1/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(7/3)))*((3^(1/2)*1i)/2 - 1/2)*(a
*d - b*c)^(1/3))/(3*d^(7/3)) + (c*log((a + b*x^3)^(1/3)*(3*b*c^2 - 3*a*c*d)
 - (c*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(1/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^
(7/3)))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(1/3))/(3*d^(7/3))
```

3.661 $\int \frac{x^2 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 159

$$\int \frac{x^2 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx = \frac{\sqrt[3]{a+bx^3}}{d} + \frac{\sqrt[3]{bc-ad} \arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{4/3}} + \frac{\sqrt[3]{bc-ad} \log(c+dx^3)}{6d^{4/3}} - \frac{\sqrt[3]{bc-ad} \log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}}$$

```
[Out] (b*x^3+a)^(1/3)/d+1/6*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/d^(4/3)-1/2*(-a*d+b*c)^(
1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(4/3)+1/3*(-a*d+b*c)^(1
/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(4
/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {455, 52, 60, 631, 210, 31}

$$\int \frac{x^2 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx = \frac{\sqrt[3]{bc-ad} \arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{4/3}} + \frac{\sqrt[3]{bc-ad} \log(c+dx^3)}{6d^{4/3}} - \frac{\sqrt[3]{bc-ad} \log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}} + \frac{\sqrt[3]{a+bx^3}}{d}$$

```
[In] Int[(x^2*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (a + b*x^3)^(1/3)/d + ((b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)
^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(4/3)) + ((b*c - a*d)^(1/3)
*Log[c + d*x^3])/(6*d^(4/3)) - ((b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d
^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{\sqrt[3]{a+bx^3}}{d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3d}$$

$$= \frac{\sqrt[3]{a+bx^3}}{d} + \frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{4/3}} - \frac{\sqrt[3]{bc-ad}\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{4/3}} - \frac{(bc-ad)^{2/3}\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{5/3}}$$

$$= \frac{\sqrt[3]{a+bx^3}}{d} + \frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{4/3}} - \frac{\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}} - \frac{\sqrt[3]{bc-ad}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{d^{4/3}}$$

$$= \frac{\sqrt[3]{a+bx^3}}{d} + \frac{\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{4/3}} + \frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{4/3}} - \frac{\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.29

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$
$$= \frac{6\sqrt[3]{d}\sqrt[3]{a+bx^3}+2\sqrt{3}\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)-2\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{6d^{4/3}}$$

```
[In] Integrate[(x^2*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (6*d^(1/3)*(a + b*x^3)^(1/3) + 2*Sqrt[3]*(b*c - a*d)^(1/3)*ArcTan[(1 - (2*d
^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*(b*c - a*d)^(1/3)
*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + (b*c - a*d)^(1/3)*Log
[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*
(a + b*x^3)^(2/3)])/(6*d^(4/3))
```

Maple [A] (verified)

Time = 4.71 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.39

method	result
pseudoelliptic	$\frac{(b\,x^3+a)^{\frac{1}{3}}}{d}+\frac{\ln\left((b\,x^3+a)^{\frac{1}{3}}-\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)(ad-bc)}{3d^2\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left((b\,x^3+a)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}(b\,x^3+a)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\right)(ad-bc)}{6d^2\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}}-$

```
[In] int(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x^3+a)^(1/3)/d+1/3/d^2/(1/d*(a*d-b*c))^(2/3)*ln((b*x^3+a)^(1/3)-(1/d*(a*
d-b*c))^(1/3))*(a*d-b*c)-1/6/d^2/(1/d*(a*d-b*c))^(2/3)*ln((b*x^3+a)^(2/3)+(
1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))*(a*d-b*c)-1/3/d
^2/(1/d*(a*d-b*c))^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d
*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*(a*d-b*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.30

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = -\frac{2\sqrt{3}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\right.\right.}{6\,d}$$

```
[In] integrate(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(-(b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(
1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) + (-(b*c
- a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^
(1/3) + (-(b*c - a*d)/d)^(2/3)) - 2*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^
(1/3) - (-(b*c - a*d)/d)^(1/3)) - 6*(b*x^3 + a)^(1/3))/d
```

Sympy [F]

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{x^2\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

```
[In] integrate(x**2*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**2*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.40

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{(bc-ad)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bcd-ad^2)} - \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^2} + \frac{(bx^3+a)^{\frac{1}{3}}}{d} - \frac{(-bcd^2+ad^3)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^2}$$

```
[In] integrate(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b*c - a*d)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c -
 a*d)/d)^(1/3)))/(b*c*d - a*d^2) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*arc
tan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d
)/d)^(1/3))/d^2 + (b*x^3 + a)^(1/3)/d - 1/6*(-b*c*d^2 + a*d^3)^(1/3)*log((b
*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/
d)^(2/3))/d^2
```

Mupad [B] (verification not implemented)

Time = 8.46 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.57

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{(b\,x^3+a)^{1/3}}{d} + \frac{\ln\left((b\,x^3+a)^{1/3}\,(3\,a\,d^2-3\,b\,c\,d)-\frac{(a\,d-b\,c)^{1/3}\,(9\,a\,d^3-9\,b\,c\,d^2)}{3\,d^{4/3}}\right)\,(a\,d-b\,c)^{1/3}}{3\,d^{4/3}} - \frac{\ln\left((b\,x^3+a)^{1/3}\,(3\,a\,d^2-3\,b\,c\,d)+\frac{\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^{1/3}\,(9\,a\,d^3-9\,b\,c\,d^2)}{3\,d^{4/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(a\,d-b\,c)^{1/3}}{3\,d^{4/3}} + \frac{\ln\left((b\,x^3+a)^{1/3}\,(3\,a\,d^2-3\,b\,c\,d)-\frac{\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\,(a\,d-b\,c)^{1/3}\,(9\,a\,d^3-9\,b\,c\,d^2)}{d^{4/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\,(a\,d-b\,c)^{1/3}}{d^{4/3}}$$

```
[In] int((x^2*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

```
[Out] (a + b*x^3)^(1/3)/d + (log((a + b*x^3)^(1/3)*(3*a*d^2 - 3*b*c*d) - ((a*d -
b*c)^(1/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(4/3)))*(a*d - b*c)^(1/3))/(3*d^(4/3
)) - (log((a + b*x^3)^(1/3)*(3*a*d^2 - 3*b*c*d) + (((3^(1/2)*1i)/2 + 1/2)*(
a*d - b*c)^(1/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(4/3)))*((3^(1/2)*1i)/2 + 1/2)
*(a*d - b*c)^(1/3))/(3*d^(4/3)) + (log((a + b*x^3)^(1/3)*(3*a*d^2 - 3*b*c*d
) - (((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(1/3)*(9*a*d^3 - 9*b*c*d^2))/d^(4/3
))*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(1/3))/d^(4/3)
```

3.662 $\int \frac{\sqrt[3]{a+bx^3}}{x(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 246

$$\int \frac{\sqrt[3]{a+bx^3}}{x(c+dx^3)}\,dx = -\frac{\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}c} - \frac{\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c\sqrt[3]{d}} - \frac{\sqrt[3]{a}\log(x)}{2c} - \frac{\sqrt[3]{bc-ad}\log(c+dx^3)}{6c\sqrt[3]{d}} + \frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c} + \frac{\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c\sqrt[3]{d}}$$

```
[Out] -1/2*a^(1/3)*ln(x)/c-1/6*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/c/d^(1/3)+1/2*a^(1/3)
*ln(a^(1/3)-(b*x^3+a)^(1/3))/c+1/2*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3)+d^(
1/3)*(b*x^3+a)^(1/3))/c/d^(1/3)-1/3*a^(1/3)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)
^(1/3))/a^(1/3)*3^(1/2))/c*3^(1/2)-1/3*(-a*d+b*c)^(1/3)*arctan(1/3*(1-2*d^(
1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c/d^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {457, 85, 59, 631, 210, 31, 60}

$$\int \frac{\sqrt[3]{a+bx^3}}{x(c+dx^3)}\,dx = -\frac{\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c\sqrt[3]{d}} - \frac{\sqrt[3]{a}\arctan\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}c} - \frac{\sqrt[3]{bc-ad}\log(c+dx^3)}{6c\sqrt[3]{d}} + \frac{\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c\sqrt[3]{d}} + \frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c} - \frac{\sqrt[3]{a}\log(x)}{2c}$$

```
[In] Int[(a + b*x^3)^(1/3)/(x*(c + d*x^3)),x]
```

```
[Out] -((a^(1/3)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt
[3]*c)) - ((b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c
 - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c*d^(1/3)) - (a^(1/3)*Log[x])/(2*c) - ((b
*c - a*d)^(1/3)*Log[c + d*x^3])/(6*c*d^(1/3)) + (a^(1/3)*Log[a^(1/3) - (a +
 b*x^3)^(1/3)])/(2*c) + ((b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*
(a + b*x^3)^(1/3)])/(2*c*d^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
```

```
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{a\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^3\right)}{3c} + \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3c}$$

$$= -\frac{\sqrt[3]{a}\log(x)}{2c} - \frac{\sqrt[3]{bc-ad}\log(c+dx^3)}{6c\sqrt[3]{d}} - \frac{\sqrt[3]{a}\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c} - \frac{a^{2/3}\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c} + \frac{\sqrt[3]{bc-ad}\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c\sqrt[3]{d}} + \frac{(bc-ad)^{2/3}\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2cd^{2/3}}$$

$$= -\frac{\sqrt[3]{a}\log(x)}{2c} - \frac{\sqrt[3]{bc-ad}\log(c+dx^3)}{6c\sqrt[3]{d}} + \frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c} + \frac{\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c\sqrt[3]{d}} + \frac{\sqrt[3]{a}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{c} + \frac{\sqrt[3]{bc-ad}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{c\sqrt[3]{d}}$$

$$= -\frac{\sqrt[3]{a}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}c} - \frac{\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c\sqrt[3]{d}} - \frac{\sqrt[3]{a}\log(x)}{2c} - \frac{\sqrt[3]{bc-ad}\log(c+dx^3)}{6c\sqrt[3]{d}} + \frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c} + \frac{\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c\sqrt[3]{d}}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt[3]{a+bx^3}}{x\,(c+dx^3)}\,dx = -\frac{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{d}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\sqrt{3}\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)-2\sqrt[3]{a}\sqrt[3]{d}\log\left(-\sqrt[3]{a}\right.}{}$$

```
[In] Integrate[(a + b*x^3)^(1/3)/(x*(c + d*x^3)),x]
```

```
[Out] -1/6*(2*Sqrt[3]*a^(1/3)*d^(1/3)*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/
Sqrt[3]] + 2*Sqrt[3]*(b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(
1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*a^(1/3)*d^(1/3)*Log[-a^(1/3) + (a + b
*x^3)^(1/3)] - 2*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x
^3)^(1/3)] + a^(1/3)*d^(1/3)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a +
 b*x^3)^(2/3)] + (b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a
*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(c*d^(1/3))
```

Maple [A] (verified)

Time = 4.83 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$\frac{-\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}} d\left(2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)\right)a^{\frac{1}{3}}-}{}$

```
[In] int((b*x^3+a)^(1/3)/x/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/(1/d*(a*d-b*c))^(2/3)*(-(1/d*(a*d-b*c))^(2/3)*d*(2*arctan(1/3*(a^(1/3)+
2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)+ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x
^3+a)^(1/3)+a^(2/3))-2*ln((b*x^3+a)^(1/3)-a^(1/3)))*a^(1/3)+(a*d-b*c)*(2*ar
ctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^
(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/
d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))))/c/d
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt[3]{a+bx^3}}{x\,(c+dx^3)}\,dx =$$
$$-\frac{2\sqrt{3}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+2\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}a}{3\,a}\right)+a^{\frac{1}{3}}\log}{}$$

```
[In] integrate((b*x^3+a)^(1/3)/x/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1
/3)*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 2*sqrt(3)
*a^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*a^(2/3) + sqrt(3)*a)/a) +
a^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + ((b*
c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*((b*c - a*d)/d)
^(1/3) + ((b*c - a*d)/d)^(2/3)) - 2*a^(1/3)*log((b*x^3 + a)^(1/3) - a^(1/3)
) - 2*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b*c - a*d)/d)^(1/3)))
/c
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x\,(c+dx^3)}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{x\,(c+dx^3)}\,dx$$

```
[In] integrate((b*x**3+a)**(1/3)/x/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(x*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x\,(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx^3+c)x}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/x/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.55 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt[3]{a+bx^3}}{x(c+dx^3)}\,dx = -\frac{(bc-ad)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2-acd)} - \frac{\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,c} - \frac{a^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,c} + \frac{a^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,c} + \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,cd} + \frac{(-bcd^2+ad^3)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,cd}$$

```
[In] integrate((b*x^3+a)^(1/3)/x/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b*c - a*d)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
- a*d)/d)^(1/3)))/(b*c^2 - a*c*d) - 1/3*sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*
(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/c - 1/6*a^(1/3)*log((b*x^3 + a)^(2
/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/c + 1/3*a^(1/3)*log(abs((b*x^3 +
 a)^(1/3) - a^(1/3)))/c + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*arctan(1/3*s
qrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3
))/(c*d) + 1/6*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)
^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(c*d)
```

Mupad [B] (verification not implemented)

Time = 8.82 (sec) , antiderivative size = 1607, normalized size of antiderivative = 6.53

$$\int \frac{\sqrt[3]{a+bx^3}}{x(c+dx^3)}\,dx = \text{Too large to display}$$

```
[In] int((a + b*x^3)^(1/3)/(x*(c + d*x^3)),x)
```

```
[Out] log((a + b*x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^7*c^3*d^2 - 12*a^3*b^5*c*d^4 +
 9*a^2*b^6*c^2*d^3) - (a/(27*c^3))^(1/3)*(((243*a*b^6*c^6*d^3 - 729*a^2*b^5
*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(a/(27*c^3))^(1/3) - (a + b*x^3)^(1/3)*(81*
a*b^6*c^5*d^3 - 81*a^2*b^5*c^4*d^4))*(a/(27*c^3))^(2/3) - 9*a*b^7*c^4*d^2 +
 27*a^2*b^6*c^3*d^3 - 18*a^3*b^5*c^2*d^4))*(a/(27*c^3))^(1/3) + log((a + b*
x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^7*c^3*d^2 - 12*a^3*b^5*c*d^4 + 9*a^2*b^6*
c^2*d^3) - (((243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5
)*(-(a*d - b*c)/(27*c^3*d))^(1/3) - (a + b*x^3)^(1/3)*(81*a*b^6*c^5*d^3 - 8
1*a^2*b^5*c^4*d^4))*(-(a*d - b*c)/(27*c^3*d))^(2/3) - 9*a*b^7*c^4*d^2 + 27*
a^2*b^6*c^3*d^3 - 18*a^3*b^5*c^2*d^4)*(-(a*d - b*c)/(27*c^3*d))^(1/3))*(-(a
*d - b*c)/(27*c^3*d))^(1/3) + log((a + b*x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^
7*c^3*d^2 - 12*a^3*b^5*c*d^4 + 9*a^2*b^6*c^2*d^3) + ((3^(1/2)*1i)/2 - 1/2)*
(-(a*d - b*c)/(27*c^3*d))^(1/3)*(((3^(1/2)*1i)/2 - 1/2)^2*((a + b*x^3)^(1/3
)*(81*a*b^6*c^5*d^3 - 81*a^2*b^5*c^4*d^4) - ((3^(1/2)*1i)/2 - 1/2)*(243*a*b
^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(-(a*d - b*c)/(27*c
^3*d))^(1/3))*(-(a*d - b*c)/(27*c^3*d))^(2/3) + 9*a*b^7*c^4*d^2 - 27*a^2*b^
6*c^3*d^3 + 18*a^3*b^5*c^2*d^4))*((3^(1/2)*1i)/2 - 1/2)*(-(a*d - b*c)/(27*c
^3*d))^(1/3) - log((a + b*x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^7*c^3*d^2 - 12*
a^3*b^5*c*d^4 + 9*a^2*b^6*c^2*d^3) - ((3^(1/2)*1i)/2 + 1/2)*(-(a*d - b*c)/(
27*c^3*d))^(1/3)*(((3^(1/2)*1i)/2 + 1/2)^2*((a + b*x^3)^(1/3)*(81*a*b^6*c^5
*d^3 - 81*a^2*b^5*c^4*d^4) + ((3^(1/2)*1i)/2 + 1/2)*(243*a*b^6*c^6*d^3 - 72
9*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(-(a*d - b*c)/(27*c^3*d))^(1/3))*(
-(a*d - b*c)/(27*c^3*d))^(2/3) + 9*a*b^7*c^4*d^2 - 27*a^2*b^6*c^3*d^3 + 18*
a^3*b^5*c^2*d^4))*((3^(1/2)*1i)/2 + 1/2)*(-(a*d - b*c)/(27*c^3*d))^(1/3) +
log((a + b*x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^7*c^3*d^2 - 12*a^3*b^5*c*d^4 +
 9*a^2*b^6*c^2*d^3) + ((3^(1/2)*1i)/2 - 1/2)*(a/(27*c^3))^(1/3)*(((3^(1/2)*
1i)/2 - 1/2)^2*((a + b*x^3)^(1/3)*(81*a*b^6*c^5*d^3 - 81*a^2*b^5*c^4*d^4) -
 ((3^(1/2)*1i)/2 - 1/2)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*
b^4*c^4*d^5)*(a/(27*c^3))^(1/3))*(a/(27*c^3))^(2/3) + 9*a*b^7*c^4*d^2 - 27*
a^2*b^6*c^3*d^3 + 18*a^3*b^5*c^2*d^4))*((3^(1/2)*1i)/2 - 1/2)*(a/(27*c^3))^
(1/3) - log((a + b*x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^7*c^3*d^2 - 12*a^3*b^5
*c*d^4 + 9*a^2*b^6*c^2*d^3) - ((3^(1/2)*1i)/2 + 1/2)*(a/(27*c^3))^(1/3)*(((
3^(1/2)*1i)/2 + 1/2)^2*((a + b*x^3)^(1/3)*(81*a*b^6*c^5*d^3 - 81*a^2*b^5*c^
4*d^4) + ((3^(1/2)*1i)/2 + 1/2)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 +
486*a^3*b^4*c^4*d^5)*(a/(27*c^3))^(1/3))*(a/(27*c^3))^(2/3) + 9*a*b^7*c^4*d
^2 - 27*a^2*b^6*c^3*d^3 + 18*a^3*b^5*c^2*d^4))*((3^(1/2)*1i)/2 + 1/2)*(a/(2
7*c^3))^(1/3)
```

3.663 $\int \frac{\sqrt[3]{a+bx^3}}{x^4(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 340

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(c+dx^3\right)}\,dx = \frac{d\sqrt[3]{a+bx^3}}{c^2}+\frac{(bc-3ad)\sqrt[3]{a+bx^3}}{3ac^2}-\frac{\left(a+bx^3\right)^{4/3}}{3acx^3}-\frac{(bc-3ad)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}c^2}+\frac{d^{2/3}\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2}-\frac{(bc-3ad)\log(x)}{6a^{2/3}c^2}+\frac{d^{2/3}\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c^2}+\frac{(bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{2/3}c^2}-\frac{d^{2/3}\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2}$$

```
[Out] d*(b*x^3+a)^(1/3)/c^2+1/3*(-3*a*d+b*c)*(b*x^3+a)^(1/3)/a/c^2-1/3*(b*x^3+a)^
(4/3)/a/c/x^3-1/6*(-3*a*d+b*c)*ln(x)/a^(2/3)/c^2+1/6*d^(2/3)*(-a*d+b*c)^(1/
3)*ln(d*x^3+c)/c^2+1/6*(-3*a*d+b*c)*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(2/3)/c^2
-1/2*d^(2/3)*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/
c^2-1/9*(-3*a*d+b*c)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2)
)/a^(2/3)/c^2*3^(1/2)+1/3*d^(2/3)*(-a*d+b*c)^(1/3)*arctan(1/3*(1-2*d^(1/3)*
(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c^2*3^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {457, 105, 162, 52, 59, 631, 210, 31, 60}

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(c+dx^3\right)}dx = -\frac{\arctan\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)(bc-3ad)}{3\sqrt{3}a^{2/3}c^2} + \frac{(bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{2/3}c^2} - \frac{\log(x)(bc-3ad)}{6a^{2/3}c^2} + \frac{d^{2/3}\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2} + \frac{d^{2/3}\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c^2} - \frac{d^{2/3}\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2} + \frac{d\sqrt[3]{a+bx^3}}{c^2} + \frac{\sqrt[3]{a+bx^3}(bc-3ad)}{3ac^2} - \frac{\left(a+bx^3\right)^{4/3}}{3acx^3}$$

```
[In] Int[(a + b*x^3)^(1/3)/(x^4*(c + d*x^3)),x]
```

```
[Out] (d*(a + b*x^3)^(1/3))/c^2 + ((b*c - 3*a*d)*(a + b*x^3)^(1/3))/(3*a*c^2) - (
a + b*x^3)^(4/3)/(3*a*c*x^3) - ((b*c - 3*a*d)*ArcTan[(a^(1/3) + 2*(a + b*x^
3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)*c^2) + (d^(2/3)*(b*c - a*d
)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3
]])/(Sqrt[3]*c^2) - ((b*c - 3*a*d)*Log[x])/(6*a^(2/3)*c^2) + (d^(2/3)*(b*c
- a*d)^(1/3)*Log[c + d*x^3])/(6*c^2) + ((b*c - 3*a*d)*Log[a^(1/3) - (a + b*
x^3)^(1/3)])/(6*a^(2/3)*c^2) - (d^(2/3)*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(
1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^2(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{(a+bx^3)^{4/3}}{3acx^3} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}\left(\frac{1}{3}(-bc+3ad)-\frac{bdx}{3}\right)}{x(c+dx)}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{(a+bx^3)^{4/3}}{3acx^3} + \frac{d^2\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)}{3c^2} + \frac{(bc-3ad)\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x}\,dx, x, x^3\right)}{9ac^2}$$

$$= \frac{d\sqrt[3]{a+bx^3}}{c^2} + \frac{(bc-3ad)\sqrt[3]{a+bx^3}}{3ac^2} - \frac{(a+bx^3)^{4/3}}{3acx^3} + \frac{(bc-3ad)\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^3\right)}{9c^2} - \frac{(d(bc-ad))\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3c^2}$$

$$= \frac{d\sqrt[3]{a+bx^3}}{c^2} + \frac{(bc-3ad)\sqrt[3]{a+bx^3}}{3ac^2} - \frac{(a+bx^3)^{4/3}}{3acx^3} - \frac{(bc-3ad)\log(x)}{6a^{2/3}c^2} + \frac{d^{2/3}\sqrt[3]{bc-ad}\log(c+dx^3)}{6c^2} - \frac{(bc-3ad)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{6a^{2/3}c^2} - \frac{(bc-3ad)\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{6\sqrt[3]{a}c^2} - \frac{\left(d^{2/3}\sqrt[3]{bc-ad}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c^2} - \frac{\left(\sqrt[3]{d}(bc-ad)^{2/3}\right)\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c^2}$$

$$= \frac{d\sqrt[3]{a+bx^3}}{c^2} + \frac{(bc-3ad)\sqrt[3]{a+bx^3}}{3ac^2} - \frac{(a+bx^3)^{4/3}}{3acx^3} - \frac{(bc-3ad)\log(x)}{6a^{2/3}c^2} + \frac{d^{2/3}\sqrt[3]{bc-ad}\log(c+dx^3)}{6c^2} + \frac{(bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{2/3}c^2} - \frac{d^{2/3}\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2} + \frac{(bc-3ad)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{3a^{2/3}c^2} - \frac{\left(d^{2/3}\sqrt[3]{bc-ad}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{c^2}$$

$$= \frac{d\sqrt[3]{a+bx^3}}{c^2} + \frac{(bc-3ad)\sqrt[3]{a+bx^3}}{3ac^2} - \frac{(a+bx^3)^{4/3}}{3acx^3} - \frac{(bc-3ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{2/3}c^2} + \frac{d^{2/3}\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2} - \frac{(bc-3ad)\log(x)}{6a^{2/3}c^2} + \frac{d^{2/3}\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c^2} + \frac{(bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{2/3}c^2} - \frac{d^{2/3}\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2}$$

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(c+dx^3\right)}\,dx$$

$$= \frac{-\frac{6c\sqrt[3]{a+bx^3}}{x^3} + \frac{2\sqrt{3}(-bc+3ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} + 6\sqrt{3}d^{2/3}\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) + \frac{2(bc-}{}}{}$$

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^4*(c + d*x^3)),x]
```

```
[Out] ((-6*c*(a + b*x^3)^(1/3))/x^3 + (2*Sqrt[3]*(-(b*c) + 3*a*d)*ArcTan[(1 + (2*
(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3) + 6*Sqrt[3]*d^(2/3)*(b*c - a*
d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[
3]] + (2*(b*c - 3*a*d)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(2/3) - 6*d^(2/
3)*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + (
(-(b*c) + 3*a*d)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3
)])/a^(2/3) + 3*d^(2/3)*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(
b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(18*c^2)
```

Maple [A] (verified)

Time = 5.23 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$-\frac{-\frac{x^3\left(a^{\frac{2}{3}}bc-a^{\frac{5}{3}}d\right)\ln\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\right)}{2}-x^3\sqrt{3}\left(a^{\frac{2}{3}}bc-a^{\frac{5}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(\frac{ad}{}\right.\right.}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right.}{}$

```
[In] int((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-1/2*x^3*(a^(2/3)*b*c-a^(5/3)*d)*ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(
1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-x^3*3^(1/2)*(a^(2/3)*b*c-a^(5/3
)*d)*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d
-b*c))^(1/3))-1/2*(1/d*(a*d-b*c))^(2/3)*x^3*(a*d-1/3*b*c)*ln((b*x^3+a)^(2/3
)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3))+x^3*(a^(2/3)*b*c-a^(5/3)*d)*ln((b*x^3+a)
^(1/3)-(1/d*(a*d-b*c))^(1/3))+(1/d*(a*d-b*c))^(2/3)*(-x^3*3^(1/2)*(a*d-1/3*
b*c)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))+x^3*(a*d-1/3*b
*c)*ln((b*x^3+a)^(1/3)-a^(1/3))+(b*x^3+a)^(1/3)*a^(2/3)*c))/a^(2/3)/(1/d*(a
*d-b*c))^(2/3)/c^2/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(c+dx^3\right)}\,dx =$$

$$-\frac{6\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}a^2x^3\arctan\left(-\frac{2\sqrt{3}(-bcd^2+ad^3)^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}-\sqrt{3}(bcd-ad^2)}{3\,(bcd-ad^2)}\right)+3\,(-bcd^2+ad^3)^{\frac{1}{3}}a^2x^3\log\left(\left(\right.\right.}{}$$

```
[In] integrate((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/18*(6*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*a^2*x^3*arctan(-1/3*(2*sqrt(3)*(-
b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(1/3) - sqrt(3)*(b*c*d - a*d^2))/(b*c*d
- a*d^2)) + 3*(-b*c*d^2 + a*d^3)^(1/3)*a^2*x^3*log((b*x^3 + a)^(2/3)*d^2 +
(-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) -
6*(-b*c*d^2 + a*d^3)^(1/3)*a^2*x^3*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*
d^3)^(1/3)) + 2*sqrt(3)*(a*b*c - 3*a^2*d)*x^3*sqrt(-(-a^2)^(1/3))*arctan(-1
/3*(sqrt(3)*(-a^2)^(1/3)*a - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-a^2)^(2/3))*sqrt
(-(-a^2)^(1/3))/a^2) + (-a^2)^(2/3)*(b*c - 3*a*d)*x^3*log((b*x^3 + a)^(2/3)
*a - (-a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(-a^2)^(2/3)) - 2*(-a^2)^(2/3)*(b*c
```

```
 - 3*a*d)*x^3*log((b*x^3 + a)^(1/3)*a - (-a^2)^(2/3)) + 6*(b*x^3 + a)^(1/3)
*a^2*c)/(a^2*c^2*x^3)
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(c+dx^3\right)}\,dx=\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(c+dx^3\right)}\,dx$$

```
[In] integrate((b*x**3+a)**(1/3)/x**4/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(1/3)/(x**4*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(c+dx^3\right)}\,dx=\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^4}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4 (c+dx^3)} \, dx = \frac{(bcd-ad^2)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^3-ac^2d)} - \frac{\sqrt{3}(bc-3\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{2}{3}}c^2} - \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,c^2} - \frac{(bc-3\,ad)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{2}{3}}c^2} - \frac{(-bcd^2+ad^3)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,c^2} + \frac{(bc-3\,ad)\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{2}{3}}c^2} - \frac{(bx^3+a)^{\frac{1}{3}}}{3\,cx^3}$$

```
[In] integrate((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b*c*d - a*d^2)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b
*c - a*d)/d)^(1/3)))/(b*c^3 - a*c^2*d) - 1/9*sqrt(3)*(b*c - 3*a*d)*arctan(1
/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(2/3)*c^2) - 1/3*sqr
t(3)*(-b*c*d^2 + a*d^3)^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(
b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/c^2 - 1/18*(b*c - 3*a*d)*log((
b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(2/3)*c^2) - 1/6
*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c
- a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/c^2 + 1/9*(b*c - 3*a*d)*log(abs((
b*x^3 + a)^(1/3) - a^(1/3)))/(a^(2/3)*c^2) - 1/3*(b*x^3 + a)^(1/3)/(c*x^3)
```

Mupad [B] (verification not implemented)

Time = 14.13 (sec) , antiderivative size = 1917, normalized size of antiderivative = 5.64

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\,(c+dx^3)}\,dx = \text{Too large to display}$$

```
[In] int((a + b*x^3)^(1/3)/(x^4*(c + d*x^3)),x)
```

```
[Out] log(- ((((27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d
) - 27*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d - b*c)^3/(a
^2*c^6))^(1/3))*(-(3*a*d - b*c)^3/(a^2*c^6))^(2/3))/81 - (b^5*d^4*(27*a^3*d
^3 + b^3*c^3 + 17*a*b^2*c^2*d - 45*a^2*b*c*d^2))/(3*c))*(-(3*a*d - b*c)^3/(
a^2*c^6))^(1/3))/9 - (2*b^4*d^5*(a + b*x^3)^(1/3)*(27*a^4*d^4 + 5*b^4*c^4 +
 72*a^2*b^2*c^2*d^2 - 32*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*c^4))*(-(27*a^3*
d^3 - b^3*c^3 + 9*a*b^2*c^2*d - 27*a^2*b*c*d^2)/(729*a^2*c^6))^(1/3) + log(
- ((((27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d) -
81*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((d^2*(a*d - b*c))/c^6)^
(1/3))*((d^2*(a*d - b*c))/c^6)^(2/3))/9 - (b^5*d^4*(27*a^3*d^3 + b^3*c^3 +
17*a*b^2*c^2*d - 45*a^2*b*c*d^2))/(3*c))*((d^2*(a*d - b*c))/c^6)^(1/3))/3 -
 (2*b^4*d^5*(a + b*x^3)^(1/3)*(27*a^4*d^4 + 5*b^4*c^4 + 72*a^2*b^2*c^2*d^2
- 32*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*c^4))*((a*d^3 - b*c*d^2)/(27*c^6))^(
1/3) + log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*(27*b^5*c^3*d^3
*(a + b*x^3)^(1/3)*(4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d) - 81*a*b^4*c^4*d^3*((3
^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((d^2*(a*d - b*c))/c^
6)^(1/3))*((d^2*(a*d - b*c))/c^6)^(2/3))/9 + (b^5*d^4*(27*a^3*d^3 + b^3*c^3
 + 17*a*b^2*c^2*d - 45*a^2*b*c*d^2))/(3*c))*((d^2*(a*d - b*c))/c^6)^(1/3))/
3 - (2*b^4*d^5*(a + b*x^3)^(1/3)*(27*a^4*d^4 + 5*b^4*c^4 + 72*a^2*b^2*c^2*d
^2 - 32*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*c^4))*((3^(1/2)*1i)/2 - 1/2)*((a*
d^3 - b*c*d^2)/(27*c^6))^(1/3) - log((2*b^4*d^5*(a + b*x^3)^(1/3)*(27*a^4*d
^4 + 5*b^4*c^4 + 72*a^2*b^2*c^2*d^2 - 32*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*
c^4) - (((3^(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*(27*b^5*c^3*d^3*(a
+ b*x^3)^(1/3)*(4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d) + 81*a*b^4*c^4*d^3*((3^(1/
2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((d^2*(a*d - b*c))/c^6)^(
1/3))*((d^2*(a*d - b*c))/c^6)^(2/3))/9 - (b^5*d^4*(27*a^3*d^3 + b^3*c^3 + 1
7*a*b^2*c^2*d - 45*a^2*b*c*d^2))/(3*c))*((d^2*(a*d - b*c))/c^6)^(1/3))/3)*(
(3^(1/2)*1i)/2 + 1/2)*((a*d^3 - b*c*d^2)/(27*c^6))^(1/3) + log((((3^(1/2)*1
i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*(27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a
^2*d^2 + b^2*c^2 - 5*a*b*c*d) - 27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*
a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d - b*c)^3/(a^2*c^6))^(1/3))*(-(3*a*d
 - b*c)^3/(a^2*c^6))^(2/3))/81 + (b^5*d^4*(27*a^3*d^3 + b^3*c^3 + 17*a*b^2*
c^2*d - 45*a^2*b*c*d^2))/(3*c))*(-(3*a*d - b*c)^3/(a^2*c^6))^(1/3))/9 - (2*
b^4*d^5*(a + b*x^3)^(1/3)*(27*a^4*d^4 + 5*b^4*c^4 + 72*a^2*b^2*c^2*d^2 - 32
*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*c^4))*((3^(1/2)*1i)/2 - 1/2)*(-(27*a^3*d
^3 - b^3*c^3 + 9*a*b^2*c^2*d - 27*a^2*b*c*d^2)/(729*a^2*c^6))^(1/3) - log((
```

```
2*b^4*d^5*(a + b*x^3)^(1/3)*(27*a^4*d^4 + 5*b^4*c^4 + 72*a^2*b^2*c^2*d^2 -
32*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*c^4) - (((3^(1/2)*1i)/2 + 1/2)*((((3^(
1/2)*1i)/2 - 1/2)*(27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a^2*d^2 + b^2*c^2 -
5*a*b*c*d) + 27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 -
 3*a*b*c*d)*(-(3*a*d - b*c)^3/(a^2*c^6))^(1/3))*(-(3*a*d - b*c)^3/(a^2*c^6)
)^(2/3))/81 - (b^5*d^4*(27*a^3*d^3 + b^3*c^3 + 17*a*b^2*c^2*d - 45*a^2*b*c*
d^2))/(3*c))*(-(3*a*d - b*c)^3/(a^2*c^6))^(1/3))/9)*((3^(1/2)*1i)/2 + 1/2)*
(-(27*a^3*d^3 - b^3*c^3 + 9*a*b^2*c^2*d - 27*a^2*b*c*d^2)/(729*a^2*c^6))^(1
/3) - (a + b*x^3)^(1/3)/(3*c*x^3)
```

3.664 $\int \frac{\sqrt[3]{a+bx^3}}{x^7(c+dx^3)} dx$

Optimal result

Integrand size = 24, antiderivative size = 370

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7(c+dx^3)} dx = \frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3} - \frac{(a+bx^3)^{4/3}}{6acx^6} + \frac{(b^2c^2+3abcd-9a^2d^2)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}c^3} - \frac{d^{5/3}\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^3} + \frac{(b^2c^2+3abcd-9a^2d^2)\log(x)}{18a^{5/3}c^3} - \frac{d^{5/3}\sqrt[3]{bc-ad}\log(c+dx^3)}{6c^3} - \frac{(b^2c^2+3abcd-9a^2d^2)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{5/3}c^3} + \frac{d^{5/3}\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^3}$$

```
[Out] 1/9*(3*a*d+b*c)*(b*x^3+a)^(1/3)/a/c^2/x^3-1/6*(b*x^3+a)^(4/3)/a/c/x^6+1/18*
(-9*a^2*d^2+3*a*b*c*d+b^2*c^2)*ln(x)/a^(5/3)/c^3-1/6*d^(5/3)*(-a*d+b*c)^(1/
3)*ln(d*x^3+c)/c^3-1/18*(-9*a^2*d^2+3*a*b*c*d+b^2*c^2)*ln(a^(1/3)-(b*x^3+a)
^(1/3))/a^(5/3)/c^3+1/2*d^(5/3)*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3
)*(b*x^3+a)^(1/3))/c^3+1/27*(-9*a^2*d^2+3*a*b*c*d+b^2*c^2)*arctan(1/3*(a^(1
/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(5/3)/c^3*3^(1/2)-1/3*d^(5/3)*(-a
*d+b*c)^(1/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(
1/2))/c^3*3^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {457, 105, 154, 162, 59, 631, 210, 31, 60}

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7(c+dx^3)}\,dx = \frac{\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)(-9a^2d^2+3abcd+b^2c^2)}{9\sqrt{3}a^{5/3}c^3} - \frac{(-9a^2d^2+3abcd+b^2c^2)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{5/3}c^3} + \frac{\log(x)(-9a^2d^2+3abcd+b^2c^2)}{18a^{5/3}c^3} - \frac{d^{5/3}\sqrt[3]{bc-ad}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^3} - \frac{d^{5/3}\sqrt[3]{bc-ad}\log(c+dx^3)}{6c^3} + \frac{d^{5/3}\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^3} + \frac{\sqrt[3]{a+bx^3}(3ad+bc)}{9ac^2x^3} - \frac{(a+bx^3)^{4/3}}{6acx^6}$$

```
[In] Int[(a + b*x^3)^(1/3)/(x^7*(c + d*x^3)),x]
```

```
[Out] ((b*c + 3*a*d)*(a + b*x^3)^(1/3))/(9*a*c^2*x^3) - (a + b*x^3)^(4/3)/(6*a*c*
x^6) + ((b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(
1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*c^3) - (d^(5/3)*(b*c - a*d)^(1
/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/
(Sqrt[3]*c^3) + ((b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*Log[x])/(18*a^(5/3)*c^3)
 - (d^(5/3)*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(6*c^3) - ((b^2*c^2 + 3*a*b*c
*d - 9*a^2*d^2)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(18*a^(5/3)*c^3) + (d^(5/
3)*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2
*c^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
```

```
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^3(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{(a+bx^3)^{4/3}}{6acx^6} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}\left(\frac{2}{3}(bc+3ad)+\frac{2bdx}{3}\right)}{x^2(c+dx)}\,dx, x, x^3\right)}{6ac}$$

$$= \frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3} - \frac{(a+bx^3)^{4/3}}{6acx^6} - \frac{\text{Subst}\left(\int \frac{\frac{2}{9}(b^2c^2+3abcd-9a^2d^2)+\frac{2}{9}bd(bc-6ad)x}{x(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{6ac^2}$$

$$= \frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3} - \frac{(a+bx^3)^{4/3}}{6acx^6} + \frac{(d^2(bc-ad))\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3c^3} - \frac{(b^2c^2+3abcd-9a^2d^2)\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^3\right)}{27ac^3}$$

$$= \frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3} - \frac{(a+bx^3)^{4/3}}{6acx^6} + \frac{(b^2c^2+3abcd-9a^2d^2)\log(x)}{18a^{5/3}c^3} - \frac{d^{5/3}\sqrt[3]{bc-ad}\log(c+dx^3)}{6c^3} + \frac{\left(d^{5/3}\sqrt[3]{bc-ad}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c^3} + \frac{\left(d^{4/3}(bc-ad)^{2/3}\right)\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c^3} + \frac{(b^2c^2+3abcd-9a^2d^2)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{18a^{5/3}c^3} + \frac{(b^2c^2+3abcd-9a^2d^2)\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{18a^{4/3}c^3}$$

$$= \frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3} - \frac{(a+bx^3)^{4/3}}{6acx^6} + \frac{(b^2c^2+3abcd-9a^2d^2)\log(x)}{18a^{5/3}c^3} - \frac{d^{5/3}\sqrt[3]{bc-ad}\log(c+dx^3)}{6c^3} - \frac{(b^2c^2+3abcd-9a^2d^2)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{5/3}c^3} + \frac{d^{5/3}\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^3} + \frac{\left(d^{5/3}\sqrt[3]{bc-ad}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{c^3} - \frac{(b^2c^2+3abcd-9a^2d^2)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{9a^{5/3}c^3}$$

$$= \frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3} - \frac{(a+bx^3)^{4/3}}{6acx^6} + \frac{(b^2c^2+3abcd-9a^2d^2)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}a^{5/3}c^3} - \frac{d^{5/3}\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^3} + \frac{(b^2c^2+3abcd-9a^2d^2)\log(x)}{18a^{5/3}c^3} - \frac{d^{5/3}\sqrt[3]{bc-ad}\log(c+dx^3)}{6c^3} - \frac{(b^2c^2+3abcd-9a^2d^2)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{5/3}c^3} + \frac{d^{5/3}\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^3}$$

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7(c+dx^3)}\,dx$$

$$= \frac{\frac{3c\sqrt[3]{a+bx^3}(-3ac-bcx^3+6adx^3)}{ax^6} + \frac{2\sqrt{3}(b^2c^2+3abcd-9a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}} - 18\sqrt{3}d^{5/3}\sqrt[3]{bc-ad}\arctan\left(\frac{1-}{}\right.}{}$$

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^7*(c + d*x^3)),x]
```

```
[Out] ((3*c*(a + b*x^3)^(1/3)*(-3*a*c - b*c*x^3 + 6*a*d*x^3))/(a*x^6) + (2*Sqrt[3
]*(b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/
3))/Sqrt[3]])/a^(5/3) - 18*Sqrt[3]*d^(5/3)*(b*c - a*d)^(1/3)*ArcTan[(1 - (2
*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - (2*(b^2*c^2 + 3*a
*b*c*d - 9*a^2*d^2)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(5/3) + 18*d^(5/3)
*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + ((b
^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (
a + b*x^3)^(2/3)])/a^(5/3) - 9*d^(5/3)*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2
/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/
3)])/(54*c^3)
```

Maple [A] (verified)

Time = 5.11 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.16

method	result
pseudoelliptic	$-\frac{-\frac{\left(a^{\frac{11}{3}} d-a^{\frac{8}{3}} b c\right) x^{6} d \ln\left(\left(b x^{3}+a\right)^{\frac{2}{3}}+\left(\frac{a d-b c}{d}\right)^{\frac{1}{3}}\left(b x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{a d-b c}{d}\right)^{\frac{2}{3}}\right)}{2}-\left(a^{\frac{11}{3}} d-a^{\frac{8}{3}} b c\right) x^{6} \sqrt{3}\, d \arctan\left(\frac{\sqrt{3}\left(2\left(b x^{3}+a\right)^{\frac{1}{3}}+\left(\right.\right.}{3\left(\frac{a d-b c}{d}\right)}\right.}{}$

```
[In] int((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/(1/d*(a*d-b*c))^(2/3)*(-1/2*(a^(11/3)*d-a^(8/3)*b*c)*x^6*d*ln((b*x^3+a
)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-(a^(11
/3)*d-a^(8/3)*b*c)*x^6*3^(1/2)*d*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d
*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))+1/2*(1/d*(a*d-b*c))^(2/3)*x^6*(a^
2*d^2-1/3*a*b*c*d-1/9*b^2*c^2)*a*ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)
+a^(2/3))+(a^(11/3)*d-a^(8/3)*b*c)*x^6*d*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))
^(1/3))-(1/d*(a*d-b*c))^(2/3)*(-x^6*3^(1/2)*(a^2*d^2-1/3*a*b*c*d-1/9*b^2*c^
2)*a*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))+x^6*(a^2*d^2-1
/3*a*b*c*d-1/9*b^2*c^2)*a*ln((b*x^3+a)^(1/3)-a^(1/3))-1/6*c*(b*x^3+a)^(1/3)
*((-6*d*x^3+3*c)*a^(8/3)+a^(5/3)*b*c*x^3)))/a^(8/3)/c^3/x^6
```

Fricas [A] (verification not implemented)

none

Time = 1.12 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(c+dx^3\right)}\,dx =$$

$$-\frac{18\sqrt{3}(bcd^2-ad^3)^{\frac{1}{3}}a^3dx^6\arctan\left(-\frac{2\sqrt{3}(bcd^2-ad^3)^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}-\sqrt{3}(bcd-ad^2)}{3(bcd-ad^2)}\right)+9(bcd^2-ad^3)^{\frac{1}{3}}a^3dx^6\log\left((bx^3\right.}{}$$

```
[In] integrate((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/54*(18*sqrt(3)*(b*c*d^2 - a*d^3)^(1/3)*a^3*d*x^6*arctan(-1/3*(2*sqrt(3)*
(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(1/3) - sqrt(3)*(b*c*d - a*d^2))/(b*c*d
 - a*d^2)) + 9*(b*c*d^2 - a*d^3)^(1/3)*a^3*d*x^6*log((b*x^3 + a)^(2/3)*d^2
- (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(2/3)) -
18*(b*c*d^2 - a*d^3)^(1/3)*a^3*d*x^6*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a
*d^3)^(1/3)) - 2*sqrt(3)*(a*b^2*c^2 + 3*a^2*b*c*d - 9*a^3*d^2)*(a^2)^(1/6)*
x^6*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^(1/3)*a + 2*sqrt(3)*(b*x^3 + a)^(
1/3)*(a^2)^(2/3))/a^2) - (b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*(a^2)^(2/3)*x^6*
```

```
log((b*x^3 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) +
2*(b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*(a^2)^(2/3)*x^6*log((b*x^3 + a)^(1/3)*a
 - (a^2)^(2/3)) + 3*(3*a^3*c^2 + (a^2*b*c^2 - 6*a^3*c*d)*x^3)*(b*x^3 + a)^(
1/3))/(a^3*c^3*x^6)
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(c+dx^3\right)}\,dx=\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(c+dx^3\right)}\,dx$$

```
[In] integrate((b*x**3+a)**(1/3)/x**7/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(1/3)/(x**7*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(c+dx^3\right)}\,dx=\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^7}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^7), x)
```

Giac [A] (verification not implemented)

none

Time = 0.56 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(c+dx^3\right)}\,dx$$
$$= -\frac{(bcd^2-ad^3)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^4-ac^3d\right)}$$
$$+\frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}d\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,c^3}$$
$$+\frac{(-bcd^2+ad^3)^{\frac{1}{3}}d\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,c^3}$$
$$+\frac{\sqrt{3}(b^2c^2+3\,abcd-9\,a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{27\,a^{\frac{5}{3}}c^3}$$
$$+\frac{(b^2c^2+3\,abcd-9\,a^2d^2)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{54\,a^{\frac{5}{3}}c^3}$$
$$-\frac{(b^2c^2+3\,abcd-9\,a^2d^2)\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{27\,a^{\frac{5}{3}}c^3}$$
$$-\frac{(bx^3+a)^{\frac{4}{3}}b^2c+2\,(bx^3+a)^{\frac{1}{3}}ab^2c-6\,(bx^3+a)^{\frac{4}{3}}abd+6\,(bx^3+a)^{\frac{1}{3}}a^2bd}{18\,ab^2c^2x^6}$$

```
[In] integrate((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b*c*d^2 - a*d^3)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (
-(b*c - a*d)/d)^(1/3)))/(b*c^4 - a*c^3*d) + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^
(1/3)*d*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(
-(b*c - a*d)/d)^(1/3))/c^3 + 1/6*(-b*c*d^2 + a*d^3)^(1/3)*d*log((b*x^3 + a)
^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))
/c^3 + 1/27*sqrt(3)*(b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*arctan(1/3*sqrt(3)*(2
*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(5/3)*c^3) + 1/54*(b^2*c^2 + 3*a*
b*c*d - 9*a^2*d^2)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2
/3))/(a^(5/3)*c^3) - 1/27*(b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*log(abs((b*x^3
+ a)^(1/3) - a^(1/3)))/(a^(5/3)*c^3) - 1/18*((b*x^3 + a)^(4/3)*b^2*c + 2*(b
*x^3 + a)^(1/3)*a*b^2*c - 6*(b*x^3 + a)^(4/3)*a*b*d + 6*(b*x^3 + a)^(1/3)*a
^2*b*d)/(a*b^2*c^2*x^6)
```

Mupad [B] (verification not implemented)

Time = 16.50 (sec) , antiderivative size = 2767, normalized size of antiderivative = 7.48

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(c+dx^3\right)}\,dx = \text{Too large to display}$$

[In] int((a + b*x^3)^(1/3)/(x^7*(c + d*x^3)),x)

[Out] log((((81*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^5*(a*d - b*c))/c^9)^(1/3) + (9*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(12*a^3*d^3 + b^3*c^3 + a*b^2*c^2*d - 14*a^2*b*c*d^2))/a)*(-(d^5*(a*d - b*c))/c^9)^(2/3))/9 - (b^5*d^4*(729*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 135*a^3*b^3*c^3*d^3 + 864*a^4*b^2*c^2*d^4 + 8*a*b^5*c^5*d - 1458*a^5*b*c*d^5))/(81*a^3*c^4))*(-(d^5*(a*d - b*c))/c^9)^(1/3))/3 - (b^4*d^6*(a + b*x^3)^(1/3)*(1458*a^7*d^7 + b^7*c^7 + 72*a^2*b^5*c^5*d^2 - 135*a^3*b^4*c^4*d^3 - 1080*a^4*b^3*c^3*d^4 + 3564*a^5*b^2*c^2*d^5 + 8*a*b^6*c^6*d - 3888*a^6*b*c*d^6))/(243*a^3*c^8))*(-(a*d^6 - b*c*d^5)/(27*c^9))^(1/3) + log((((((9*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(12*a^3*d^3 + b^3*c^3 + a*b^2*c^2*d - 14*a^2*b*c*d^2))/a + 9*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*c^9))^(1/3))*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*c^9))^(2/3))/729 - (b^5*d^4*(729*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 135*a^3*b^3*c^3*d^3 + 864*a^4*b^2*c^2*d^4 + 8*a*b^5*c^5*d - 1458*a^5*b*c*d^5))/(81*a^3*c^4))*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*c^9))^(1/3))/27 - (b^4*d^6*(a + b*x^3)^(1/3)*(1458*a^7*d^7 + b^7*c^7 + 72*a^2*b^5*c^5*d^2 - 135*a^3*b^4*c^4*d^3 - 1080*a^4*b^3*c^3*d^4 + 3564*a^5*b^2*c^2*d^5 + 8*a*b^6*c^6*d - 3888*a^6*b*c*d^6))/(243*a^3*c^8))*(-(b^6*c^6 - 729*a^6*d^6 - 135*a^3*b^3*c^3*d^3 + 9*a*b^5*c^5*d + 729*a^5*b*c*d^5)/(19683*a^5*c^9))^(1/3) - (((a + b*x^3)^(1/3)*(b^2*c + 3*a*b*d))/(9*c^2) - (b*(a + b*x^3)^(4/3)*(6*a*d - b*c))/(18*a*c^2))/((a + b*x^3)^2 - 2*a*(a + b*x^3) + a^2) + log(- (((3^(1/2)*1i)/2 - 1/2)*((((9*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(12*a^3*d^3 + b^3*c^3 + a*b^2*c^2*d - 14*a^2*b*c*d^2))/a + 81*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^5*(a*d - b*c))/c^9)^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(-(d^5*(a*d - b*c))/c^9)^(2/3))/9 + (b^5*d^4*(729*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 135*a^3*b^3*c^3*d^3 + 864*a^4*b^2*c^2*d^4 + 8*a*b^5*c^5*d - 1458*a^5*b*c*d^5))/(81*a^3*c^4))*(-(d^5*(a*d - b*c))/c^9)^(1/3))/3 - (b^4*d^6*(a + b*x^3)^(1/3)*(1458*a^7*d^7 + b^7*c^7 + 72*a^2*b^5*c^5*d^2 - 135*a^3*b^4*c^4*d^3 - 1080*a^4*b^3*c^3*d^4 + 3564*a^5*b^2*c^2*d^5 + 8*a*b^6*c^6*d - 3888*a^6*b*c*d^6))/(243*a^3*c^8))*((3^(1/2)*1i)/2 - 1/2)*(-(a*d^6 - b*c*d^5)/(27*c^9))^(1/3) - log((((3^(1/2)*1i)/2 + 1/2)*((((9*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(12*a^3*d^3 + b^3*c^3 + a*b^2*c^2*d - 14*a^2*b*c*d^2))/a - 81*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^5*(a*d - b*c))/c^9)^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(-(d^5*(a*d - b*c))/c^9)^(2/3))/9 - (b^5*d^4*(729*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 135*a^3*b^3*c^3*d^3 + 864*a^4*b^2*c^2*d^4 + 8*a*b^5*c^5*d - 1458*a^5*b*c*d^5))/(81*a^3

```
*c^4))*(-(d^5*(a*d - b*c))/c^9)^(1/3))/3 + (b^4*d^6*(a + b*x^3)^(1/3)*(1458
*a^7*d^7 + b^7*c^7 + 72*a^2*b^5*c^5*d^2 - 135*a^3*b^4*c^4*d^3 - 1080*a^4*b^
3*c^3*d^4 + 3564*a^5*b^2*c^2*d^5 + 8*a*b^6*c^6*d - 3888*a^6*b*c*d^6))/(243*
a^3*c^8))*((3^(1/2)*1i)/2 + 1/2)*(-(a*d^6 - b*c*d^5)/(27*c^9))^(1/3) + log(
- (((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*((9*b^5*c^2*d^3*(a + b*x
^3)^(1/3)*(12*a^3*d^3 + b^3*c^3 + a*b^2*c^2*d - 14*a^2*b*c*d^2))/a + 9*a*b^
4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(b^2*c
^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*c^9))^(1/3))*(-(b^2*c^2 - 9*a^2*d^2 + 3*
a*b*c*d)^3/(a^5*c^9))^(2/3))/729 + (b^5*d^4*(729*a^6*d^6 + b^6*c^6 - 9*a^2*
b^4*c^4*d^2 - 135*a^3*b^3*c^3*d^3 + 864*a^4*b^2*c^2*d^4 + 8*a*b^5*c^5*d - 1
458*a^5*b*c*d^5))/(81*a^3*c^4))*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*
c^9))^(1/3))/27 - (b^4*d^6*(a + b*x^3)^(1/3)*(1458*a^7*d^7 + b^7*c^7 + 72*a
^2*b^5*c^5*d^2 - 135*a^3*b^4*c^4*d^3 - 1080*a^4*b^3*c^3*d^4 + 3564*a^5*b^2*
c^2*d^5 + 8*a*b^6*c^6*d - 3888*a^6*b*c*d^6))/(243*a^3*c^8))*((3^(1/2)*1i)/2
 - 1/2)*(-(b^6*c^6 - 729*a^6*d^6 - 135*a^3*b^3*c^3*d^3 + 9*a*b^5*c^5*d + 72
9*a^5*b*c*d^5)/(19683*a^5*c^9))^(1/3) - log((((3^(1/2)*1i)/2 + 1/2)*((((3^(
1/2)*1i)/2 - 1/2)*((9*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(12*a^3*d^3 + b^3*c^3 +
 a*b^2*c^2*d - 14*a^2*b*c*d^2))/a - 9*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*
(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^
5*c^9))^(1/3))*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*c^9))^(2/3))/729
- (b^5*d^4*(729*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 135*a^3*b^3*c^3*d^3
 + 864*a^4*b^2*c^2*d^4 + 8*a*b^5*c^5*d - 1458*a^5*b*c*d^5))/(81*a^3*c^4))*(
-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*c^9))^(1/3))/27 + (b^4*d^6*(a + b
*x^3)^(1/3)*(1458*a^7*d^7 + b^7*c^7 + 72*a^2*b^5*c^5*d^2 - 135*a^3*b^4*c^4*
d^3 - 1080*a^4*b^3*c^3*d^4 + 3564*a^5*b^2*c^2*d^5 + 8*a*b^6*c^6*d - 3888*a^
6*b*c*d^6))/(243*a^3*c^8))*((3^(1/2)*1i)/2 + 1/2)*(-(b^6*c^6 - 729*a^6*d^6
- 135*a^3*b^3*c^3*d^3 + 9*a*b^5*c^5*d + 729*a^5*b*c*d^5)/(19683*a^5*c^9))^(
1/3)
```

3.665 $\int \frac{x^7 \sqrt[3]{a+bx^3}}{c+dx^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 336

$$\int \frac{x^7 \sqrt[3]{a+bx^3}}{c+dx^3} dx = -\frac{(6bc-ad)x^2\sqrt[3]{a+bx^3}}{18bd^2} + \frac{x^5\sqrt[3]{a+bx^3}}{6d} - \frac{(9b^2c^2-3abcd-a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{5/3}d^3} + \frac{c^{5/3}\sqrt[3]{bc-ad}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^3} - \frac{c^{5/3}\sqrt[3]{bc-ad}\log(c+dx^3)}{6d^3} - \frac{(9b^2c^2-3abcd-a^2d^2)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{18b^{5/3}d^3} + \frac{c^{5/3}\sqrt[3]{bc-ad}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^3}$$

```
[Out] -1/18*(-a*d+6*b*c)*x^2*(b*x^3+a)^(1/3)/b/d^2+1/6*x^5*(b*x^3+a)^(1/3)/d-1/6*
c^(5/3)*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/d^3-1/18*(-a^2*d^2-3*a*b*c*d+9*b^2*c^2
)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(5/3)/d^3+1/2*c^(5/3)*(-a*d+b*c)^(1/3)*ln
((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^3-1/27*(-a^2*d^2-3*a*b*c*d+9
*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(5/3)/d^3*3
```

```
^(1/2)+1/3*c^(5/3)*(-a*d+b*c)^(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1
/3)/(b*x^3+a)^(1/3))*3^(1/2))/d^3*3^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {489, 596, 598, 337, 503}

$$\int \frac{x^7\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = -\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)\left(-a^2d^2-3abcd+9b^2c^2\right)}{9\sqrt{3}b^{5/3}d^3} - \frac{\left(-a^2d^2-3abcd+9b^2c^2\right)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{18b^{5/3}d^3} + \frac{c^{5/3}\sqrt[3]{bc-ad}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d^3} - \frac{c^{5/3}\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^3} + \frac{c^{5/3}\sqrt[3]{bc-ad}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^3} - \frac{x^2\sqrt[3]{a+bx^3}(6bc-ad)}{18bd^2} + \frac{x^5\sqrt[3]{a+bx^3}}{6d}$$

```
[In] Int[(x^7*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] -1/18*((6*b*c - a*d)*x^2*(a + b*x^3)^(1/3))/(b*d^2) + (x^5*(a + b*x^3)^(1/3
))/(6*d) - ((9*b^2*c^2 - 3*a*b*c*d - a^2*d^2)*ArcTan[(1 + (2*b^(1/3)*x)/(a
+ b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(5/3)*d^3) + (c^(5/3)*(b*c - a*d)^(1
/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3
]])/(Sqrt[3]*d^3) - (c^(5/3)*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(6*d^3) - ((
9*b^2*c^2 - 3*a*b*c*d - a^2*d^2)*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(18*b^
(5/3)*d^3) + (c^(5/3)*(b*c - a*d)^(1/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) -
 (a + b*x^3)^(1/3)])/(2*d^3)
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^5\sqrt[3]{a+bx^3}}{6d} - \frac{\int \frac{x^4(5ac+(6bc-ad)x^3)}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{6d} \\
&= -\frac{(6bc-ad)x^2\sqrt[3]{a+bx^3}}{18bd^2} + \frac{x^5\sqrt[3]{a+bx^3}}{6d} + \frac{\int \frac{x(2ac(6bc-ad)+2(9b^2c^2-3abcd-a^2d^2)x^3)}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{18bd^2} \\
&= -\frac{(6bc-ad)x^2\sqrt[3]{a+bx^3}}{18bd^2} + \frac{x^5\sqrt[3]{a+bx^3}}{6d} + \frac{\int \left(\frac{2(9b^2c^2-3abcd-a^2d^2)x}{d(a+bx^3)^{2/3}} + \frac{18(-b^2c^3+abc^2d)x}{d(a+bx^3)^{2/3}(c+dx^3)}\right)dx}{18bd^2}
\end{aligned}
$$

$$= -\frac{(6bc-ad)x^2\sqrt[3]{a+bx^3}}{18bd^2} + \frac{x^5\sqrt[3]{a+bx^3}}{6d} - \frac{(c^2(bc-ad))\int\frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{d^3} + \frac{(9b^2c^2-3abcd-a^2d^2)\int\frac{x}{(a+bx^3)^{2/3}}\,dx}{9bd^3}$$

$$= -\frac{(6bc-ad)x^2\sqrt[3]{a+bx^3}}{18bd^2} + \frac{x^5\sqrt[3]{a+bx^3}}{6d} - \frac{(9b^2c^2-3abcd-a^2d^2)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{5/3}d^3} + \frac{c^{5/3}\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^3} - \frac{c^{5/3}\sqrt[3]{bc-ad}\log(c+dx^3)}{6d^3} - \frac{(9b^2c^2-3abcd-a^2d^2)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{18b^{5/3}d^3} + \frac{c^{5/3}\sqrt[3]{bc-ad}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^3}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.45 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.57

$$\int \frac{x^7\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

$$= \frac{\frac{6dx^2\sqrt[3]{a+bx^3}(-6bc+ad+3bdx^3)}{b} - \frac{4\sqrt{3}(9b^2c^2-3abcd-a^2d^2)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{b^{5/3}} - 18\sqrt{-6-6i\sqrt{3}}c^{5/3}\sqrt[3]{bc-ad}}{}$$

```
[In] Integrate[(x^7*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] ((6*d*x^2*(a + b*x^3)^(1/3)*(-6*b*c + a*d + 3*b*d*x^3))/b - (4*Sqrt[3]*(9*b
^2*c^2 - 3*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a
+ b*x^3)^(1/3))])/b^(5/3) - 18*Sqrt[-6 - (6*I)*Sqrt[3]]*c^(5/3)*(b*c - a*d)
^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I +
 Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (4*(-9*b^2*c^2 + 3*a*b*c*d + a^2*d^
2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(5/3) + (18*I)*(I + Sqrt[3])*c^
```

```
(5/3)*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)
*(a + b*x^3)^(1/3)] + (2*(9*b^2*c^2 - 3*a*b*c*d - a^2*d^2)*Log[b^(2/3)*x^2
+ b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(5/3) + 9*(1 - I*Sqrt
[3])*c^(5/3)*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3
])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*
(a + b*x^3)^(2/3)])/(108*d^3)
```

Maple [A] (verified)

Time = 6.38 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.32

method	result
pseudoelliptic	$\frac{-\frac{\left(b^{\frac{11}{3}}c-b^{\frac{8}{3}}ad\right)c\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^2-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{2}-\left(b^{\frac{11}{3}}c-b^{\frac{8}{3}}ad\right)\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2(b}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}\right.}{}$

```
[In] int(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(-1/2*(b^(11/3)*c-b^(8/3)*a*d)*c*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)
/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-(b^(11/3)*c-b^(8/3)*a*d)*
3^(1/2)*c*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*
d-b*c)/c)^(1/3)/x)-1/18*((a*d-b*c)/c)^(2/3)*b*(a^2*d^2+3*a*b*c*d-9*b^2*c^2)
*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)+(b^(11/3)*
c-b^(8/3)*a*d)*c*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)+1/9*(-3^(1/2
)*b*(a^2*d^2+3*a*b*c*d-9*b^2*c^2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)
^(1/3))/b^(1/3)/x)+b*(a^2*d^2+3*a*b*c*d-9*b^2*c^2)*ln((-b^(1/3)*x+(b*x^3+a)
^(1/3))/x)+3/2*x^2*((3*d*x^3-6*c)*b^(8/3)+a*b^(5/3)*d)*d*(b*x^3+a)^(1/3))*(
(a*d-b*c)/c)^(2/3))/((a*d-b*c)/c)^(2/3)/b^(8/3)/d^3
```

Fricas [A] (verification not implemented)

none

Time = 1.15 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.47

$$\int \frac{x^7\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$
$$=\frac{18\sqrt{3}(bc^3-ac^2d)^{\frac{1}{3}}b^3c\arctan\left(-\frac{\sqrt{3}(bc^2-acd)x+2\sqrt{3}(bc^3-ac^2d)^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}}{3\,(bc^2-acd)x}\right)+18\,(bc^3-ac^2d)^{\frac{1}{3}}b^3c\log\left(\frac{(bx^3+a)^{\frac{1}{3}}c}{}\right.}{}$$

```
[In] integrate(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/54*(18*sqrt(3)*(b*c^3 - a*c^2*d)^(1/3)*b^3*c*arctan(-1/3*(sqrt(3)*(b*c^2
- a*c*d)*x + 2*sqrt(3)*(b*c^3 - a*c^2*d)^(2/3)*(b*x^3 + a)^(1/3))/((b*c^2 -
 a*c*d)*x)) + 18*(b*c^3 - a*c^2*d)^(1/3)*b^3*c*log(((b*x^3 + a)^(1/3)*c - (
b*c^3 - a*c^2*d)^(1/3)*x)/x) - 9*(b*c^3 - a*c^2*d)^(1/3)*b^3*c*log(((b*x^3
+ a)^(2/3)*c^2 + (b*c^3 - a*c^2*d)^(1/3)*(b*x^3 + a)^(1/3)*c*x + (b*c^3 - a
*c^2*d)^(2/3)*x^2)/x^2) + 2*sqrt(3)*(9*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*(
b^2)^(1/6)*arctan(1/3*(sqrt(3)*(b^2)^(1/3)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3
)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x)) - 2*(9*b^2*c^2 - 3*a*b*c*d - a^2*d^2)*(
b^2)^(2/3)*log(-((b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (9*b^2*c^2 - 3*a
*b*c*d - a^2*d^2)*(b^2)^(2/3)*log(((b^2)^(1/3)*b*x^2 + (b*x^3 + a)^(1/3)*(b
^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)/x^2) + 3*(3*b^3*d^2*x^5 - (6*b^3*c*d - a
*b^2*d^2)*x^2)*(b*x^3 + a)^(1/3))/(b^3*d^3)
```

Sympy [F]

$$\int \frac{x^7\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{x^7\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

```
[In] integrate(x**7*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**7*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^7\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}x^7}{dx^3+c}\,dx$$

```
[In] integrate(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x^7/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{x^7\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}x^7}{dx^3+c}\,dx$$

```
[In] integrate(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x^7/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7 \sqrt[3]{a + bx^3}}{c + dx^3} \, dx = \int \frac{x^7 \, (b \, x^3 + a)^{1/3}}{d \, x^3 + c} \, dx$$

```
[In] int((x^7*(a + b*x^3)^(1/3))/(c + d*x^3),x)
[Out] int((x^7*(a + b*x^3)^(1/3))/(c + d*x^3), x)
```

3.666 $\int \frac{x^4 \sqrt[3]{a+bx^3}}{c+dx^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 276

$$\int \frac{x^4 \sqrt[3]{a+bx^3}}{c+dx^3} dx = \frac{x^2\sqrt[3]{a+bx^3}}{3d} + \frac{(3bc-ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}d^2} - \frac{c^{2/3}\sqrt[3]{bc-ad}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2} + \frac{c^{2/3}\sqrt[3]{bc-ad}\log(c+dx^3)}{6d^2} + \frac{(3bc-ad)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6b^{2/3}d^2} - \frac{c^{2/3}\sqrt[3]{bc-ad}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^2}$$

```
[Out] 1/3*x^2*(b*x^3+a)^(1/3)/d+1/6*c^(2/3)*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/d^2+1/6*
(-a*d+3*b*c)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)/d^2-1/2*c^(2/3)*(-a*d+b*
c)^(1/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^2+1/9*(-a*d+3*b*c
)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)/d^2*3^(1/2)-1
/3*c^(2/3)*(-a*d+b*c)^(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x
^3+a)^(1/3))*3^(1/2))/d^2*3^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {489, 598, 337, 503}

$$\int \frac{x^4 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx = \frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(3bc-ad)}{3\sqrt{3}b^{2/3}d^2} - \frac{c^{2/3}\sqrt[3]{bc-ad}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d^2} + \frac{(3bc-ad)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6b^{2/3}d^2} + \frac{c^{2/3}\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^2} - \frac{c^{2/3}\sqrt[3]{bc-ad}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^2} + \frac{x^2\sqrt[3]{a+bx^3}}{3d}$$

```
[In] Int[(x^4*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (x^2*(a + b*x^3)^(1/3))/(3*d) + ((3*b*c - a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a
 + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(2/3)*d^2) - (c^(2/3)*(b*c - a*d)^(
1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[
3]])/(Sqrt[3]*d^2) + (c^(2/3)*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(6*d^2) + (
(3*b*c - a*d)*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(6*b^(2/3)*d^2) - (c^(2/3
)*(b*c - a*d)^(1/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])
/(2*d^2)
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
```

```
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^2\sqrt[3]{a+bx^3}}{3d} - \frac{\int \frac{x(2ac+(3bc-ad)x^3)}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{3d} \\
&= \frac{x^2\sqrt[3]{a+bx^3}}{3d} - \frac{\int \left(\frac{(3bc-ad)x}{d(a+bx^3)^{2/3}} + \frac{3(-bc^2+acd)x}{d(a+bx^3)^{2/3}(c+dx^3)}\right)dx}{3d} \\
&= \frac{x^2\sqrt[3]{a+bx^3}}{3d} + \frac{(c(bc-ad))\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{d^2} - \frac{(3bc-ad)\int \frac{x}{(a+bx^3)^{2/3}}\,dx}{3d^2} \\
&= \frac{x^2\sqrt[3]{a+bx^3}}{3d} + \frac{(3bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}d^2} \\
&\quad - \frac{c^{2/3}\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2} \\
&\quad + \frac{c^{2/3}\sqrt[3]{bc-ad}\log(c+dx^3)}{6d^2} + \frac{(3bc-ad)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6b^{2/3}d^2} \\
&\quad - \frac{c^{2/3}\sqrt[3]{bc-ad}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^2}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.47 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.69

$$\int \frac{x^4\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$
$$= \frac{12dx^2\sqrt[3]{a+bx^3}+\frac{4\sqrt{3}(3bc-ad)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{b^{2/3}}+6\sqrt{-6-6i\sqrt{3}}c^{2/3}\sqrt[3]{bc-ad}\arctan\left(\frac{3}{\sqrt{3}\sqrt[3]{bc-ad}}\right.}{}$$

```
[In] Integrate[(x^4*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (12*d*x^2*(a + b*x^3)^(1/3) + (4*Sqrt[3]*(3*b*c - a*d)*ArcTan[(Sqrt[3]*b^(1
/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/b^(2/3) + 6*Sqrt[-6 - (6*I)*Sqrt
[3]]*c^(2/3)*(b*c - a*d)^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c
 - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (4*(3*b*c -
 a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(2/3) + 6*(1 - I*Sqrt[3])*c^
(2/3)*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)
*(a + b*x^3)^(1/3)] + (2*(-3*b*c + a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*
x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(2/3) + (3*I)*(I + Sqrt[3])*c^(2/3)*(b*c
 - a*d)^(1/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c -
 a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)
])/(36*d^2)
```

Maple [A] (verified)

Time = 4.86 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.42

method	result
pseudoelliptic	$-\frac{-\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}(ad-3bc)\ln\left(\frac{b^{\frac{2}{3}}x^2+b^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{6}+\left(-ad\,b^{\frac{2}{3}}+b^{\frac{5}{3}}c\right)\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)-\frac{\sqrt{3}\left(\frac{ad-bc}{c}\right.}{}}{}$

```
[In] int(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b^(2/3)/((a*d-b*c)/c)^(2/3)*(-1/6*((a*d-b*c)/c)^(2/3)*(a*d-3*b*c)*ln((
b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)+(-a*d*b^(2/3)+b
^(5/3)*c)*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)-1/3*3^(1/2)*((a*d-b
*c)/c)^(2/3)*(a*d-3*b*c)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b
```

```
^(1/3)/x)+1/3*((a*d-b*c)/c)^(2/3)*(a*d-3*b*c)*ln((-b^(1/3)*x+(b*x^3+a)^(1/3
))/x)-(b*x^3+a)^(1/3)*x^2*((a*d-b*c)/c)^(2/3)*d*b^(2/3)+(a*d*b^(2/3)-b^(5/3
)*c)*(arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*
c)/c)^(1/3)/x)*3^(1/2)+1/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*
(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)))/d^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(222) = 444$.

Time = 0.34 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.64

$$\int \frac{x^4\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

$$= \frac{6\left(bx^3+a\right)^{\frac{1}{3}}b^2dx^2 + 6\sqrt{3}(-bc^3+ac^2d)^{\frac{1}{3}}b^2\arctan\left(-\frac{\sqrt{3}(bc^2-acd)x+2\sqrt{3}(-bc^3+ac^2d)^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}}{3(bc^2-acd)x}\right) + 6\left(-bc^3+ac^2\right.}{}$$

```
[In] integrate(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/18*(6*(b*x^3 + a)^(1/3)*b^2*d*x^2 + 6*sqrt(3)*(-b*c^3 + a*c^2*d)^(1/3)*b^
2*arctan(-1/3*(sqrt(3)*(b*c^2 - a*c*d)*x + 2*sqrt(3)*(-b*c^3 + a*c^2*d)^(2/
3)*(b*x^3 + a)^(1/3))/((b*c^2 - a*c*d)*x)) + 6*(-b*c^3 + a*c^2*d)^(1/3)*b^2
*log(((b*x^3 + a)^(1/3)*c + (-b*c^3 + a*c^2*d)^(1/3)*x)/x) - 3*(-b*c^3 + a*
c^2*d)^(1/3)*b^2*log(((b*x^3 + a)^(2/3)*c^2 - (-b*c^3 + a*c^2*d)^(1/3)*(b*x
^3 + a)^(1/3)*c*x + (-b*c^3 + a*c^2*d)^(2/3)*x^2)/x^2) - 2*sqrt(3)*(3*b^2*c
 - a*b*d)*sqrt(-(-b^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-b^2)^(1/3)*b*x - 2*sqr
t(3)*(b*x^3 + a)^(1/3)*(-b^2)^(2/3))*sqrt(-(-b^2)^(1/3))/(b^2*x)) + 2*(-b^2
)^(2/3)*(3*b*c - a*d)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) - (-b^
2)^(2/3)*(3*b*c - a*d)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^
(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2))/(b^2*d^2)
```

Sympy [F]

$$\int \frac{x^4\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{x^4\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

```
[In] integrate(x**4*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**4*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^4 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx = \int \frac{(bx^3+a)^{\frac{1}{3}}x^4}{dx^3+c} \, dx$$

```
[In] integrate(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)*x^4/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{x^4 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx = \int \frac{(bx^3+a)^{\frac{1}{3}}x^4}{dx^3+c} \, dx$$

```
[In] integrate(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)*x^4/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx = \int \frac{x^4\,(b\,x^3+a)^{1/3}}{d\,x^3+c} \, dx$$

```
[In] int((x^4*(a + b*x^3)^(1/3))/(c + d*x^3),x)
[Out] int((x^4*(a + b*x^3)^(1/3))/(c + d*x^3), x)
```

3.667 $\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 234

$$\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = -\frac{\sqrt[3]{b}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d}+\frac{\sqrt[3]{bc-ad}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}d} - \frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6\sqrt[3]{c}d}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2d} + \frac{\sqrt[3]{bc-ad}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}d}$$

```
[Out] -1/6*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/c^(1/3)/d-1/2*b^(1/3)*ln(b^(1/3)*x-(b*x^3
+a)^(1/3))/d+1/2*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(
1/3))/c^(1/3)/d-1/3*b^(1/3)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1
/2))/d*3^(1/2)+1/3*(-a*d+b*c)^(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1
/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(1/3)/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {495, 337, 503}

$$\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\frac{\sqrt[3]{bc-ad}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}d}-\frac{\sqrt[3]{b}\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d}-\frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6\sqrt[3]{c}d}+\frac{\sqrt[3]{bc-ad}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}d}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2d}$$

```
[In] Int[(x*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] -((b^(1/3)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*
d)) + ((b*c - a*d)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a +
b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(1/3)*d) - ((b*c - a*d)^(1/3)*Log[c + d
*x^3])/(6*c^(1/3)*d) - (b^(1/3)*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(2*d) +
 ((b*c - a*d)^(1/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])
/(2*c^(1/3)*d)
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[b*c - a*d, 0]
```

Rubi steps

$$\text{integral} = \frac{b \int \frac{x}{(a+bx^3)^{2/3}}\,dx}{d} - \frac{(bc-ad)\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{d}$$

$$= -\frac{\sqrt[3]{b}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d} + \frac{\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}d} - \frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6\sqrt[3]{c}d} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2d} + \frac{\sqrt[3]{bc-ad}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}d}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.79 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.81

$$\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

$$= \frac{-4\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) - \frac{2\sqrt{-6-6i\sqrt{3}}\sqrt[3]{bc-ad}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{c}} - 4\sqrt[3]{\cdots}}{\cdots}$$

```
[In] Integrate[(x*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (-4*Sqrt[3]*b^(1/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(
1/3))] - (2*Sqrt[-6 - (6*I)*Sqrt[3]]*(b*c - a*d)^(1/3)*ArcTan[(3*(b*c - a*d
)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^
3)^(1/3))])/c^(1/3) - 4*b^(1/3)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + ((2
*I)*(I + Sqrt[3])*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt
[3])*c^(1/3)*(a + b*x^3)^(1/3)])/c^(1/3) + 2*b^(1/3)*Log[b^(2/3)*x^2 + b^(1
/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + ((1 - I*Sqrt[3])*(b*c - a*d)
^(1/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(
1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/c^(1
/3))/(12*d)
```

Maple [A] (verified)

Time = 4.63 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.44

method	result
pseudoelliptic	$\frac{b^{\frac{1}{3}} \ln\left(\frac{b^{\frac{2}{3}} x^2+b^{\frac{1}{3}} \left(b\,x^3+a\right)^{\frac{1}{3}} x+\left(b\,x^3+a\right)^{\frac{2}{3}}}{x^2}\right) c\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}+(-2ad+2bc)\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x+\left(b\,x^3+a\right)^{\frac{1}{3}}}{x}\right)+2b^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(\cdots}{\cdots}\right.}{\cdots}$

```
[In] int(x*(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(b^(1/3)*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2
)*c*((a*d-b*c)/c)^(2/3)+(-2*a*d+2*b*c)*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^
(1/3))/x)+2*b^(1/3)*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3)
)/b^(1/3)/x)*c*((a*d-b*c)/c)^(2/3)-2*b^(1/3)*ln((-b^(1/3)*x+(b*x^3+a)^(1/3)
)/x)*c*((a*d-b*c)/c)^(2/3)+(2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(
b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-
((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*(a*d-b*c))/((a
*d-b*c)/c)^(2/3)/c/d
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.41

$$\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$
$$= \frac{2\sqrt{3}\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}(bx^3+a)^{\frac{1}{3}}c\left(\frac{bc-ad}{c}\right)^{\frac{2}{3}}}{3\,(bc-ad)x}\right)-2\sqrt{3}(-b)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}bx+2\sqrt{3}(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}}{3\,bx}\right)+\cdots}{\cdots}$$

```
[In] integrate(x*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*((b*c - a*d)/c)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2
*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b*c - a*d)/c)^(2/3))/((b*c - a*d)*x)) - 2*sq
rt(3)*(-b)^(1/3)*arctan(1/3*(sqrt(3)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b)
^(2/3))/(b*x)) + 2*(-b)^(1/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 2
*((b*c - a*d)/c)^(1/3)*log(-(x*((b*c - a*d)/c)^(1/3) - (b*x^3 + a)^(1/3))/x
) - (-b)^(1/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^
3 + a)^(2/3))/x^2) - ((b*c - a*d)/c)^(1/3)*log((x^2*((b*c - a*d)/c)^(2/3) +
 (b*x^3 + a)^(1/3)*x*((b*c - a*d)/c)^(1/3) + (b*x^3 + a)^(2/3))/x^2))/d
```

Sympy [F]

$$\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

```
[In] integrate(x*(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(x*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}x}{dx^3+c}\,dx$$

```
[In] integrate(x*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)*x/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}x}{dx^3+c}\,dx$$

```
[In] integrate(x*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)*x/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{x\,(b\,x^3+a)^{1/3}}{d\,x^3+c}\,dx$$

```
[In] int((x*(a + b*x^3)^(1/3))/(c + d*x^3),x)
[Out] int((x*(a + b*x^3)^(1/3))/(c + d*x^3), x)
```

3.668 $\int \frac{\sqrt[3]{a+bx^3}}{x^2(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 168

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2(c+dx^3)}\,dx = -\frac{\sqrt[3]{a+bx^3}}{cx} - \frac{\sqrt[3]{bc-ad}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{4/3}} + \frac{\sqrt[3]{bc-ad}\log(c+dx^3)}{6c^{4/3}} - \frac{\sqrt[3]{bc-ad}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{4/3}}$$

```
[Out] -(b*x^3+a)^(1/3)/c/x+1/6*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/c^(4/3)-1/2*(-a*d+b*c
)^(1/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(4/3)-1/3*(-a*d+b*
c)^(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2
))/c^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {486, 12, 503}

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2(c+dx^3)}\,dx = -\frac{\sqrt[3]{bc-ad}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{4/3}}+\frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c^{4/3}}-\frac{\sqrt[3]{bc-ad}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{4/3}}-\frac{\sqrt[3]{a+bx^3}}{cx}$$

```
[In] Int[(a + b*x^3)^(1/3)/(x^2*(c + d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(1/3)/(c*x)) - ((b*c - a*d)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^
(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(4/3)) + ((b*c -
 a*d)^(1/3)*Log[c + d*x^3])/(6*c^(4/3)) - ((b*c - a*d)^(1/3)*Log[((b*c - a*
d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(4/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt[3]{a+bx^3}}{cx} + \frac{\int \frac{(bc-ad)x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{c} \\
&= -\frac{\sqrt[3]{a+bx^3}}{cx} + \frac{(bc-ad)\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{c} \\
&= -\frac{\sqrt[3]{a+bx^3}}{cx} - \frac{\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{4/3}} \\
&\quad + \frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c^{4/3}} - \frac{\sqrt[3]{bc-ad}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{4/3}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.41 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.84

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(c+dx^3\right)}\,dx$$

$$= \frac{-\frac{12\sqrt[3]{c}\sqrt[3]{a+bx^3}}{x} + 2\sqrt{-6-6i\sqrt{3}}\sqrt[3]{bc-ad}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right) + 2\left(1-i\sqrt{3}\right)}{}$$

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^2*(c + d*x^3)),x]
```

```
[Out] ((-12*c^(1/3)*(a + b*x^3)^(1/3))/x + 2*Sqrt[-6 - (6*I)*Sqrt[3]]*(b*c - a*d)
^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I +
 Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + 2*(1 - I*Sqrt[3])*(b*c - a*d)^(1/3)
*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)] + I
*(I + Sqrt[3])*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt
[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3
)*(a + b*x^3)^(2/3)])/(12*c^(4/3))
```

Maple [A] (verified)

Time = 4.76 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.30

method	result
pseudoelliptic	$-\frac{-\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)(ad-bc)x+3\left(b\,x^{3}+a\right)^{\frac{1}{3}}c\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}+x\left(\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)\sqrt{3}+\frac{\ln}{}\right.}{3\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x\,c^{2}}$

```
[In] int((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/((a*d-b*c)/c)^(2/3)*(-ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)*(a
*d-b*c)*x+3*(b*x^3+a)^(1/3)*c*((a*d-b*c)/c)^(2/3)+x*(arctan(1/3*3^(1/2)*(((
a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+1/2*l
n(((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^
(2/3))/x^2))*(a*d-b*c))/x/c^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(c+dx^3\right)}\,dx=\text{Timed out}$$

```
[In] integrate((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(c+dx^3\right)}\,dx=\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(c+dx^3\right)}\,dx$$

```
[In] integrate((b*x**3+a)**(1/3)/x**2/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(x**2*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(c+dx^3\right)}\,dx=\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^2}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(c+dx^3\right)}\,dx=\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^2}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(c+dx^3\right)}\,dx=\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^2\,\left(d\,x^3+c\right)}\,dx$$

```
[In] int((a + b*x^3)^(1/3)/(x^2*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(1/3)/(x^2*(c + d*x^3)), x)
```

3.669 $\int \frac{\sqrt[3]{a+bx^3}}{x^5(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 204

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5(c+dx^3)}\,dx = -\frac{\sqrt[3]{a+bx^3}}{4cx^4} - \frac{(bc-4ad)\sqrt[3]{a+bx^3}}{4ac^2x} + \frac{d\sqrt[3]{bc-ad}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{7/3}} - \frac{d\sqrt[3]{bc-ad}\log(c+dx^3)}{6c^{7/3}} + \frac{d\sqrt[3]{bc-ad}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{7/3}}$$

```
[Out] -1/4*(b*x^3+a)^(1/3)/c/x^4-1/4*(-4*a*d+b*c)*(b*x^3+a)^(1/3)/a/c^2/x-1/6*d*(
-a*d+b*c)^(1/3)*ln(d*x^3+c)/c^(7/3)+1/2*d*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1
/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(7/3)+1/3*d*(-a*d+b*c)^(1/3)*arctan(1/3*(1
+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(7/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used

$= \{486, 597, 12, 503\}$

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5(c+dx^3)}\,dx = \frac{d\sqrt[3]{bc-ad}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{7/3}} - \frac{d\sqrt[3]{bc-ad}\log(c+dx^3)}{6c^{7/3}} + \frac{d\sqrt[3]{bc-ad}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{7/3}} - \frac{\sqrt[3]{a+bx^3}(bc-4ad)}{4ac^2x} - \frac{\sqrt[3]{a+bx^3}}{4cx^4}$$

```
[In] Int[(a + b*x^3)^(1/3)/(x^5*(c + d*x^3)),x]
```

```
[Out] -1/4*(a + b*x^3)^(1/3)/(c*x^4) - ((b*c - 4*a*d)*(a + b*x^3)^(1/3))/(4*a*c^2
*x) + (d*(b*c - a*d)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a
+ b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(7/3)) - (d*(b*c - a*d)^(1/3)*Log[c +
 d*x^3])/(6*c^(7/3)) + (d*(b*c - a*d)^(1/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/
3) - (a + b*x^3)^(1/3)])/(2*c^(7/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
```

```
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt[3]{a+bx^3}}{4cx^4}+\frac{\int \frac{bc-4ad-3bdx^3}{x^2(a+bx^3)^{2/3}(c+dx^3)}\,dx}{4c}\\
&= -\frac{\sqrt[3]{a+bx^3}}{4cx^4}-\frac{(bc-4ad)\sqrt[3]{a+bx^3}}{4ac^2x}-\frac{\int \frac{4ad(bc-ad)x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{4ac^2}\\
&= -\frac{\sqrt[3]{a+bx^3}}{4cx^4}-\frac{(bc-4ad)\sqrt[3]{a+bx^3}}{4ac^2x}-\frac{(d(bc-ad))\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{c^2}\\
&= -\frac{\sqrt[3]{a+bx^3}}{4cx^4}-\frac{(bc-4ad)\sqrt[3]{a+bx^3}}{4ac^2x}+\frac{d\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{7/3}}\\
&\quad -\frac{d\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c^{7/3}}+\frac{d\sqrt[3]{bc-ad}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{7/3}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.62 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.63

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(c+dx^3\right)}\,dx = \frac{\frac{3\sqrt[3]{c}\sqrt[3]{a+bx^3}\left(-ac-bcx^3+4adx^3\right)}{ax^4}-2\sqrt{-6-6i\sqrt{3}}d\sqrt[3]{bc-ad}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)+\cdots}{}$$

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^5*(c + d*x^3)),x]
```

```
[Out] ((3*c^(1/3)*(a + b*x^3)^(1/3)*(-(a*c) - b*c*x^3 + 4*a*d*x^3))/(a*x^4) - 2*S
qrt[-6 - (6*I)*Sqrt[3]]*d*(b*c - a*d)^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/
(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))]
+ (2*I)*(I + Sqrt[3])*d*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 +
```

I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)] + (1 - I*Sqrt[3])*d*(b*c - a*d)^(1/3)
Log[2(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x
*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(12*c^(7/3
))

Maple [A] (verified)

Time = 4.81 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.20

method	result
pseudoelliptic	$\frac{-2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)a(ad-bc)d\,x^{4}-\frac{3\left((-4ad+bc)x^{3}+ac\right)\left(b\,x^{3}+a\right)^{\frac{1}{3}}c\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}}{2}+x^{4}\left(2\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x\right.}{3\left(\frac{ad-b}{c}\right.}\right.\right.}{6\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^{4}c^{3}a}$

```
[In] int((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/((a*d-b*c)/c)^(2/3)*(-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)*a
*(a*d-b*c)*d*x^4-3/2*((-4*a*d+b*c)*x^3+a*c)*(b*x^3+a)^(1/3)*c*((a*d-b*c)/c)
^(2/3)+x^4*(2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/
((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1
/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*d*a*(a*d-b*c))/x^4/c^3/a
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(c+dx^3\right)}\,dx=\text{Timed out}$$

```
[In] integrate((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(c+dx^3\right)}\,dx=\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(c+dx^3\right)}\,dx$$

```
[In] integrate((b*x**3+a)**(1/3)/x**5/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(x**5*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(c+dx^3\right)}\,dx=\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^5}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^5), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(c+dx^3\right)}\,dx=\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^5}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(c+dx^3\right)}\,dx=\int \frac{{\left(b\,x^3+a\right)}^{1/3}}{x^5\,\left(d\,x^3+c\right)}\,dx$$

```
[In] int((a + b*x^3)^(1/3)/(x^5*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(1/3)/(x^5*(c + d*x^3)), x)
```

3.670 $\int \frac{\sqrt[3]{a+bx^3}}{x^8(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 258

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8(c+dx^3)}\,dx = -\frac{\sqrt[3]{a+bx^3}}{7cx^7} - \frac{(bc-7ad)\sqrt[3]{a+bx^3}}{28ac^2x^4} + \frac{(3b^2c^2+7abcd-28a^2d^2)\sqrt[3]{a+bx^3}}{28a^2c^3x} - \frac{d^2\sqrt[3]{bc-ad}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{10/3}} + \frac{d^2\sqrt[3]{bc-ad}\log(c+dx^3)}{6c^{10/3}} - \frac{d^2\sqrt[3]{bc-ad}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{10/3}}$$

```
[Out] -1/7*(b*x^3+a)^(1/3)/c/x^7-1/28*(-7*a*d+b*c)*(b*x^3+a)^(1/3)/a/c^2/x^4+1/28
*(-28*a^2*d^2+7*a*b*c*d+3*b^2*c^2)*(b*x^3+a)^(1/3)/a^2/c^3/x+1/6*d^2*(-a*d+
b*c)^(1/3)*ln(d*x^3+c)/c^(10/3)-1/2*d^2*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3
)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(10/3)-1/3*d^2*(-a*d+b*c)^(1/3)*arctan(1/3*(
1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(10/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {486, 597, 12, 503}

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8(c+dx^3)}\,dx = \frac{\sqrt[3]{a+bx^3}(-28a^2d^2+7abcd+3b^2c^2)}{28a^2c^3x} - \frac{d^2\sqrt[3]{bc-ad}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{10/3}} + \frac{d^2\sqrt[3]{bc-ad}\log(c+dx^3)}{6c^{10/3}} - \frac{d^2\sqrt[3]{bc-ad}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{10/3}} - \frac{\sqrt[3]{a+bx^3}(bc-7ad)}{28ac^2x^4} - \frac{\sqrt[3]{a+bx^3}}{7cx^7}$$

```
[In] Int[(a + b*x^3)^(1/3)/(x^8*(c + d*x^3)),x]
```

```
[Out] -1/7*(a + b*x^3)^(1/3)/(c*x^7) - ((b*c - 7*a*d)*(a + b*x^3)^(1/3))/(28*a*c^
2*x^4) + ((3*b^2*c^2 + 7*a*b*c*d - 28*a^2*d^2)*(a + b*x^3)^(1/3))/(28*a^2*c
^3*x) - (d^2*(b*c - a*d)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)
*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(10/3)) + (d^2*(b*c - a*d)^(1/3)*
Log[c + d*x^3])/(6*c^(10/3)) - (d^2*(b*c - a*d)^(1/3)*Log[((b*c - a*d)^(1/3
)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(10/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
```

```
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt[3]{a+bx^3}}{7cx^7}+\frac{\int \frac{bc-7ad-6bdx^3}{x^5(a+bx^3)^{2/3}(c+dx^3)}\,dx}{7c}\\
&= -\frac{\sqrt[3]{a+bx^3}}{7cx^7}-\frac{(bc-7ad)\sqrt[3]{a+bx^3}}{28ac^2x^4}-\frac{\int \frac{3b^2c^2+7abcd-28a^2d^2+3bd(bc-7ad)x^3}{x^2(a+bx^3)^{2/3}(c+dx^3)}\,dx}{28ac^2}\\
&= -\frac{\sqrt[3]{a+bx^3}}{7cx^7}-\frac{(bc-7ad)\sqrt[3]{a+bx^3}}{28ac^2x^4}\\
&\quad+\frac{(3b^2c^2+7abcd-28a^2d^2)\sqrt[3]{a+bx^3}}{28a^2c^3x}+\frac{\int \frac{28a^2d^2(bc-ad)x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{28a^2c^3}\\
&= -\frac{\sqrt[3]{a+bx^3}}{7cx^7}-\frac{(bc-7ad)\sqrt[3]{a+bx^3}}{28ac^2x^4}+\frac{(3b^2c^2+7abcd-28a^2d^2)\sqrt[3]{a+bx^3}}{28a^2c^3x}\\
&\quad+\frac{(d^2(bc-ad))\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{c^3}\\
&= -\frac{\sqrt[3]{a+bx^3}}{7cx^7}-\frac{(bc-7ad)\sqrt[3]{a+bx^3}}{28ac^2x^4}+\frac{(3b^2c^2+7abcd-28a^2d^2)\sqrt[3]{a+bx^3}}{28a^2c^3x}\\
&\quad-\frac{d^2\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{10/3}}+\frac{d^2\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c^{10/3}}\\
&\quad-\frac{d^2\sqrt[3]{bc-ad}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{10/3}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.04 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(c+dx^3\right)}\,dx$$
$$= \frac{-\frac{3\sqrt[3]{c}\sqrt[3]{a+bx^3}\left(-3b^2c^2x^6+abcx^3\left(c-7dx^3\right)+a^2\left(4c^2-7cdx^3+28d^2x^6\right)\right)}{a^2x^7}+14\sqrt{-6-6i\sqrt{3}}d^2\sqrt[3]{bc-ad}\arctan\left(\frac{}{\sqrt{3}\sqrt[3]{bc-}}\right.}{}$$

[In] Integrate[(a + b*x^3)^(1/3)/(x^8*(c + d*x^3)),x]

[Out] ((-3*c^(1/3)*(a + b*x^3)^(1/3)*(-3*b^2*c^2*x^6 + a*b*c*x^3*(c - 7*d*x^3) + a^2*(4*c^2 - 7*c*d*x^3 + 28*d^2*x^6)))/(a^2*x^7) + 14*Sqrt[-6 - (6*I)*Sqrt[3]]*d^2*(b*c - a*d)^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + 14*(1 - I*Sqrt[3])*d^2*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)] + (7*I)*(I + Sqrt[3])*d^2*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(84*c^(10/3))

Maple [A] (verified)

Time = 4.84 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$-\frac{\frac{6\left(\left(-\frac{3bx^3}{4}+a\right)\left(bx^3+a\right)c^2-\frac{7\left(bx^3+a\right)acdx^3}{4}+7a^2d^2x^6\right)c\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}}{7}+a^2d^2x^7(ad-bc)\left(2\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x\right.}{3\left(\frac{ad-b}{c}\right.}\right.\right.}{6\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^7}$

[In] int((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] -1/6/((a*d-b*c)/c)^(2/3)*(6/7*((-3/4*b*x^3+a)*(b*x^3+a)*c^2-7/4*(b*x^3+a)*a*c*d*x^3+7*a^2*d^2*x^6)*c*((a*d-b*c)/c)^(2/3)*(b*x^3+a)^(1/3)+a^2*d^2*x^7*(a*d-b*c)*(2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/(((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)))/x^7/c^4/a^2

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(c+dx^3\right)}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(c+dx^3\right)}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{x^8\left(c+dx^3\right)}\,dx$$

```
[In] integrate((b*x**3+a)**(1/3)/x**8/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(1/3)/(x**8*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(c+dx^3\right)}\,dx = \int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^8}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^8), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(c+dx^3\right)}\,dx = \int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^8}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\,(c+dx^3)}\,dx=\int \frac{{\left(b\,x^3+a\right)}^{1/3}}{x^8\,\left(d\,x^3+c\right)}\,dx$$

```
[In] int((a + b*x^3)^(1/3)/(x^8*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(1/3)/(x^8*(c + d*x^3)), x)
```

3.671 $\int \frac{\sqrt[3]{a+bx^3}}{x^{11}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 318

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}(c+dx^3)}\,dx = -\frac{\sqrt[3]{a+bx^3}}{10cx^{10}} - \frac{(bc-10ad)\sqrt[3]{a+bx^3}}{70ac^2x^7} + \frac{(3b^2c^2+5abcd-35a^2d^2)\sqrt[3]{a+bx^3}}{140a^2c^3x^4} - \frac{(9b^3c^3+15ab^2c^2d+35a^2bcd^2-140a^3d^3)\sqrt[3]{a+bx^3}}{140a^3c^4x} + \frac{d^3\sqrt[3]{bc-ad}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{13/3}} - \frac{d^3\sqrt[3]{bc-ad}\log(c+dx^3)}{6c^{13/3}} + \frac{d^3\sqrt[3]{bc-ad}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{13/3}}$$

```
[Out] -1/10*(b*x^3+a)^(1/3)/c/x^10-1/70*(-10*a*d+b*c)*(b*x^3+a)^(1/3)/a/c^2/x^7+1
/140*(-35*a^2*d^2+5*a*b*c*d+3*b^2*c^2)*(b*x^3+a)^(1/3)/a^2/c^3/x^4-1/140*(-
140*a^3*d^3+35*a^2*b*c*d^2+15*a*b^2*c^2*d+9*b^3*c^3)*(b*x^3+a)^(1/3)/a^3/c^
4/x-1/6*d^3*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/c^(13/3)+1/2*d^3*(-a*d+b*c)^(1/3)*
ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(13/3)+1/3*d^3*(-a*d+b*c)^
(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/
c^(13/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {486, 597, 12, 503}

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}(c+dx^3)}\,dx = \frac{\sqrt[3]{a+bx^3}(-35a^2d^2+5abcd+3b^2c^2)}{140a^2c^3x^4} - \frac{\sqrt[3]{a+bx^3}(-140a^3d^3+35a^2bcd^2+15ab^2c^2d+9b^3c^3)}{140a^3c^4x} + \frac{d^3\sqrt[3]{bc-ad}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{13/3}} - \frac{d^3\sqrt[3]{bc-ad}\log(c+dx^3)}{6c^{13/3}} + \frac{d^3\sqrt[3]{bc-ad}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{13/3}} - \frac{\sqrt[3]{a+bx^3}(bc-10ad)}{70ac^2x^7} - \frac{\sqrt[3]{a+bx^3}}{10cx^{10}}$$

```
[In] Int[(a + b*x^3)^(1/3)/(x^11*(c + d*x^3)),x]
```

```
[Out] -1/10*(a + b*x^3)^(1/3)/(c*x^10) - ((b*c - 10*a*d)*(a + b*x^3)^(1/3))/(70*a
*c^2*x^7) + ((3*b^2*c^2 + 5*a*b*c*d - 35*a^2*d^2)*(a + b*x^3)^(1/3))/(140*a
^2*c^3*x^4) - ((9*b^3*c^3 + 15*a*b^2*c^2*d + 35*a^2*b*c*d^2 - 140*a^3*d^3)*
(a + b*x^3)^(1/3))/(140*a^3*c^4*x) + (d^3*(b*c - a*d)^(1/3)*ArcTan[(1 + (2*
(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(13/
3)) - (d^3*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(6*c^(13/3)) + (d^3*(b*c - a*d
)^(1/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(13/3)
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
```

```
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt[3]{a+bx^3}}{10cx^{10}}+\frac{\int \frac{bc-10ad-9bdx^3}{x^8(a+bx^3)^{2/3}(c+dx^3)}\,dx}{10c}\\
&= -\frac{\sqrt[3]{a+bx^3}}{10cx^{10}}-\frac{(bc-10ad)\sqrt[3]{a+bx^3}}{70ac^2x^7}-\frac{\int \frac{2(3b^2c^2+5abcd-35a^2d^2)+6bd(bc-10ad)x^3}{x^5(a+bx^3)^{2/3}(c+dx^3)}\,dx}{70ac^2}\\
&= -\frac{\sqrt[3]{a+bx^3}}{10cx^{10}}-\frac{(bc-10ad)\sqrt[3]{a+bx^3}}{70ac^2x^7}+\frac{(3b^2c^2+5abcd-35a^2d^2)\sqrt[3]{a+bx^3}}{140a^2c^3x^4}\\
&\quad+\frac{\int \frac{2(9b^3c^3+15ab^2c^2d+35a^2bcd^2-140a^3d^3)+6bd(3b^2c^2+5abcd-35a^2d^2)x^3}{x^2(a+bx^3)^{2/3}(c+dx^3)}\,dx}{280a^2c^3}\\
&= -\frac{\sqrt[3]{a+bx^3}}{10cx^{10}}-\frac{(bc-10ad)\sqrt[3]{a+bx^3}}{70ac^2x^7}+\frac{(3b^2c^2+5abcd-35a^2d^2)\sqrt[3]{a+bx^3}}{140a^2c^3x^4}\\
&\quad-\frac{(9b^3c^3+15ab^2c^2d+35a^2bcd^2-140a^3d^3)\sqrt[3]{a+bx^3}}{140a^3c^4x}-\frac{\int \frac{280a^3d^3(bc-ad)x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{280a^3c^4}\\
&= -\frac{\sqrt[3]{a+bx^3}}{10cx^{10}}-\frac{(bc-10ad)\sqrt[3]{a+bx^3}}{70ac^2x^7}+\frac{(3b^2c^2+5abcd-35a^2d^2)\sqrt[3]{a+bx^3}}{140a^2c^3x^4}\\
&\quad-\frac{(9b^3c^3+15ab^2c^2d+35a^2bcd^2-140a^3d^3)\sqrt[3]{a+bx^3}}{140a^3c^4x}\\
&\quad-\frac{(d^3(bc-ad))\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{c^4}
\end{aligned}$$

$$=-\frac{\sqrt[3]{a+bx^3}}{10cx^{10}}-\frac{(bc-10ad)\sqrt[3]{a+bx^3}}{70ac^2x^7}+\frac{(3b^2c^2+5abcd-35a^2d^2)\sqrt[3]{a+bx^3}}{140a^2c^3x^4}-\frac{(9b^3c^3+15ab^2c^2d+35a^2bcd^2-140a^3d^3)\sqrt[3]{a+bx^3}}{140a^3c^4x}+\frac{d^3\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{13/3}}-\frac{d^3\sqrt[3]{bc-ad}\log(c+dx^3)}{6c^{13/3}}+\frac{d^3\sqrt[3]{bc-ad}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{13/3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.44 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.32

$$\int\frac{\sqrt[3]{a+bx^3}}{x^{11}(c+dx^3)}\,dx=\frac{\frac{3\sqrt[3]{c}\sqrt[3]{a+bx^3}(-9b^3c^3x^9+3ab^2c^2x^6(c-5dx^3)+a^2bcx^3(-2c^2+5cdx^3-35d^2x^6)+a^3(-14c^3+20c^2dx^3-35cd^2x^6+140d^3x^9))}{a^3x^{10}}-70\sqrt{-6-}}{}$$

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^11*(c + d*x^3)),x]
```

```
[Out] ((3*c^(1/3)*(a + b*x^3)^(1/3)*(-9*b^3*c^3*x^9 + 3*a*b^2*c^2*x^6*(c - 5*d*x^
3) + a^2*b*c*x^3*(-2*c^2 + 5*c*d*x^3 - 35*d^2*x^6) + a^3*(-14*c^3 + 20*c^2*
d*x^3 - 35*c*d^2*x^6 + 140*d^3*x^9)))/(a^3*x^10) - 70*Sqrt[-6 - (6*I)*Sqrt[
3]]*d^3*(b*c - a*d)^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*
d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (70*I)*(I + Sqrt
[3])*d^3*(b*c - a*d)^(1/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1
/3)*(a + b*x^3)^(1/3)] + 35*(1 - I*Sqrt[3])*d^3*(b*c - a*d)^(1/3)*Log[2*(b*
c - a*d)^(1/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^
3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(420*c^(13/3))
```

Maple [A] (verified)

Time = 4.92 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$\frac{-\frac{3\left(\left(\frac{9}{14}b^{2}x^{6}-\frac{6}{7}ab\,x^{3}+a^{2}\right)\left(b\,x^{3}+a\right)c^{3}-\frac{10x^{3}\left(-\frac{3b\,x^{3}}{4}+a\right)d\left(b\,x^{3}+a\right)a\,c^{2}}{7}+\frac{5\left(b\,x^{3}+a\right)a^{2}c\,d^{2}x^{6}}{2}-10a^{3}d^{3}x^{9}\right)\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}c\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{5}}{}$

[In] int((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] 1/6/((a*d-b*c)/c)^(2/3)*(-3/5*((9/14*b^2*x^6-6/7*a*b*x^3+a^2)*(b*x^3+a)*c^3 -10/7*x^3*(-3/4*b*x^3+a)*d*(b*x^3+a)*a*c^2+5/2*(b*x^3+a)*a^2*c*d^2*x^6-10*a ^3*d^3*x^9)*((a*d-b*c)/c)^(2/3)*c*(b*x^3+a)^(1/3)+a^3*d^3*x^10*(a*d-b*c)*(2 *arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c) ^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a) ^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3)) /x)))/x^10/c^5/a^3

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(c+dx^3\right)}\,dx = \text{Timed out}$$

[In] integrate((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(c+dx^3\right)}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(c+dx^3\right)}\,dx$$

[In] integrate((b*x**3+a)**(1/3)/x**11/(d*x**3+c),x)

[Out] Integral((a + b*x**3)**(1/3)/(x**11*(c + d*x**3)), x)

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(c+dx^3\right)}\,dx=\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^{11}}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^11), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(c+dx^3\right)}\,dx=\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^{11}}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^11), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(c+dx^3\right)}\,dx=\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^{11}\,\left(d\,x^3+c\right)}\,dx$$

```
[In] int((a + b*x^3)^(1/3)/(x^11*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(1/3)/(x^11*(c + d*x^3)), x)
```

3.672 $\int \frac{x^6 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{x^7 \sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{7}{3},-\frac{1}{3},1,\frac{10}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] 1/7*x^7*(b*x^3+a)^(1/3)*AppellF1(7/3,-1/3,1,10/3,-b*x^3/a,-d*x^3/c)/c/(1+b*
x^3/a)^(1/3)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{x^7 \sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{7}{3},-\frac{1}{3},1,\frac{10}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[In] Int[(x^6*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (x^7*(a + b*x^3)^(1/3)*AppellF1[7/3, -1/3, 1, 10/3, -((b*x^3)/a), -((d*x^3)
/c)])/(7*c*(1 + (b*x^3)/a)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{a+bx^3}\int \frac{x^6\sqrt[3]{1+\frac{bx^3}{a}}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{x^7\sqrt[3]{a+bx^3}F_1\left(\frac{7}{3};-\frac{1}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 281 vs. 2(64) = 128.

Time = 7.70 (sec) , antiderivative size = 281, normalized size of antiderivative = 4.39

$$\int \frac{x^6\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{x\left(4(a+bx^3)\left(-5bc+ad+2bdx^3\right)-\frac{(-10b^2c^2+5abcd+a^2d^2)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}+\frac{\cdots}{(c+dx^3)\left(-4ac\cdots\right.}\right.}{40bd^2\left(a+bx^3\right)^{2/3}}$$

```
[In] Integrate[(x^6*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (x*(4*(a + b*x^3)*(-5*b*c + a*d + 2*b*d*x^3) - ((-10*b^2*c^2 + 5*a*b*c*d +
a^2*d^2)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a),
 -((d*x^3)/c)])/c + (16*a^2*c^2*(-5*b*c + a*d)*AppellF1[1/3, 2/3, 1, 4/3, -
((b*x^3)/a), -((d*x^3)/c)])/((c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3,
 -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x
^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d
*x^3)/c)])))))/(40*b*d^2*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{x^6 (b\,x^3+a)^{\frac{1}{3}}}{d\,x^3+c}dx$$

```
[In] int(x^6*(b*x^3+a)^(1/3)/(d*x^3+c),x)
[Out] int(x^6*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \text{Timed out}$$

```
[In] integrate(x^6*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{x^6 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

```
[In] integrate(x**6*(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(x**6*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}x^6}{dx^3+c}\,dx$$

```
[In] integrate(x^6*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)*x^6/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}x^6}{dx^3+c}\,dx$$

```
[In] integrate(x^6*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)*x^6/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{x^6\,(b\,x^3+a)^{1/3}}{d\,x^3+c}\,dx$$

```
[In] int((x^6*(a + b*x^3)^(1/3))/(c + d*x^3),x)
[Out] int((x^6*(a + b*x^3)^(1/3))/(c + d*x^3), x)
```

3.673 $\int \frac{x^3 \sqrt[3]{a+bx^3}}{c+dx^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^3 \sqrt[3]{a+bx^3}}{c+dx^3} dx = \frac{x^4 \sqrt[3]{a+bx^3} \operatorname{AppellF1}\left(\frac{4}{3},-\frac{1}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] 1/4*x^4*(b*x^3+a)^(1/3)*AppellF1(4/3,-1/3,1,7/3,-b*x^3/a,-d*x^3/c)/c/(1+b*x
^3/a)^(1/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{x^3 \sqrt[3]{a+bx^3}}{c+dx^3} dx = \frac{x^4 \sqrt[3]{a+bx^3} \operatorname{AppellF1}\left(\frac{4}{3},-\frac{1}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[In] Int[(x^3*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (x^4*(a + b*x^3)^(1/3)*AppellF1[4/3, -1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/
c)])/(4*c*(1 + (b*x^3)/a)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{a+bx^3}\int \frac{x^3\sqrt[3]{1+\frac{bx^3}{a}}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{x^4\sqrt[3]{a+bx^3}F_1\left(\frac{4}{3};-\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 240 vs. $2(64) = 128$.

Time = 7.11 (sec) , antiderivative size = 240, normalized size of antiderivative = 3.75

$$\int \frac{x^3\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$
$$= \frac{x\left(\frac{(-2bc+ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}+4\left(a+bx^3+\frac{4a^2c^2\,\text{Appe}}{(c+dx^3)\left(-4ac\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(3ad\right.\right.}\right.\right.}{8d\,(a+bx^3)^{2/3}}$$

```
[In] Integrate[(x^3*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (x*(((-2*b*c + a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -(
(b*x^3)/a), -((d*x^3)/c)])/c + 4*(a + b*x^3 + (4*a^2*c^2*AppellF1[1/3, 2/3,
 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/((c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/
3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7
/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3
)/a), -((d*x^3)/c)]))))))/(8*d*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{x^{3}(b\,x^{3}+a)^{\frac{1}{3}}}{d\,x^{3}+c}dx$$

```
[In] int(x^3*(b*x^3+a)^(1/3)/(d*x^3+c),x)
[Out] int(x^3*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \text{Timed out}$$

```
[In] integrate(x^3*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^3\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{x^3\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

```
[In] integrate(x**3*(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(x**3*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^3\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}x^3}{dx^3+c}\,dx$$

```
[In] integrate(x^3*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)*x^3/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{x^3\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}x^3}{dx^3+c}\,dx$$

```
[In] integrate(x^3*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)*x^3/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{x^3\,(b\,x^3+a)^{1/3}}{d\,x^3+c}\,dx$$

```
[In] int((x^3*(a + b*x^3)^(1/3))/(c + d*x^3),x)
[Out] int((x^3*(a + b*x^3)^(1/3))/(c + d*x^3), x)
```

3.674 $\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{x\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] x*(b*x^3+a)^(1/3)*AppellF1(1/3,-1/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(1+b*x^3/a)^
(1/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {441, 440}

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{x\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[In] Int[(a + b*x^3)^(1/3)/(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(c*(1 + (b*x^3)/a)^(1/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
```

```
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{x\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] (warning: unable to verify)

Leave count is larger than twice the leaf count of optimal. 160 vs. $2(59) = 118$.

Time = 0.04 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.71

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

$$= \frac{4acx\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(c+dx^3)\left(4ac\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(-3ad\,\text{AppellF1}\left(\frac{4}{3},-\frac{1}{3},2,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+bc\,\text{Appe}\right.\right.}$$

```
[In] Integrate[(a + b*x^3)^(1/3)/(c + d*x^3),x]
```

```
[Out] (4*a*c*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d*x
^3)/c)])/((c + d*x^3)*(4*a*c*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d
*x^3)/c)] + x^3*(-3*a*d*AppellF1[4/3, -1/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)] + b*c*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Maple [F]

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{d\,x^{3}+c}dx$$

```
[In] int((b*x^3+a)^(1/3)/(d*x^3+c),x)
[Out] int((b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

```
[In] integrate((b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{dx^3+c}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{dx^3+c}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \int \frac{(b\,x^3+a)^{1/3}}{d\,x^3+c}\,dx$$

```
[In] int((a + b*x^3)^(1/3)/(c + d*x^3),x)
[Out] int((a + b*x^3)^(1/3)/(c + d*x^3), x)
```

3.675 $\int \frac{\sqrt[3]{a+bx^3}}{x^3(c+dx^3)} dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3 (c+dx^3)} dx = -\frac{\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(-\frac{2}{3},-\frac{1}{3},1,\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] -1/2*(b*x^3+a)^(1/3)*AppellF1(-2/3,-1/3,1,1/3,-b*x^3/a,-d*x^3/c)/c/x^2/(1+b
*x^3/a)^(1/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3 (c+dx^3)} dx = -\frac{\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(-\frac{2}{3},-\frac{1}{3},1,\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[In] Int[(a + b*x^3)^(1/3)/(x^3*(c + d*x^3)),x]
```

```
[Out] -1/2*((a + b*x^3)^(1/3)*AppellF1[-2/3, -1/3, 1, 1/3, -((b*x^3)/a), -((d*x^3
)/c)])/(c*x^2*(1 + (b*x^3)/a)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^3(c+dx^3)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{\sqrt[3]{a+bx^3}F_1\left(-\frac{2}{3};-\frac{1}{3},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 327 vs. $2(64) = 128$.

Time = 10.27 (sec) , antiderivative size = 327, normalized size of antiderivative = 5.11

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(c+dx^3\right)}\,dx = \frac{-bdx^6\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{c\left(16ac\left(bdx^6+a\left(c+3dx^3\right)\right)\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-4x^3(a+}{(c+dx^3)\left(-4ac\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(\right.\right.}}{8c^2x^2\left(a+bx^3\right)^{2/3}}$$

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^3*(c + d*x^3)),x]
```

```
[Out] (-(b*d*x^6*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -
((d*x^3)/c)]) + (c*(16*a*c*(b*d*x^6 + a*(c + 3*d*x^3))*AppellF1[1/3, 2/3, 1
, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - 4*x^3*(a + b*x^3)*(c + d*x^3)*(3*a*d*A
ppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3,
 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)*(-4*a*c*AppellF1[
1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/
3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -
((b*x^3)/a), -((d*x^3)/c)]))))/(8*c^2*x^2*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{x^3\,(d\,x^3+c)}dx$$

```
[In] int((b*x^3+a)^(1/3)/x^3/(d*x^3+c),x)
[Out] int((b*x^3+a)^(1/3)/x^3/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\,(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^3+a)^(1/3)/x^3/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\,(c+dx^3)}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{x^3\,(c+dx^3)}\,dx$$

```
[In] integrate((b*x**3+a)**(1/3)/x**3/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(1/3)/(x**3*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\,(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx^3+c)x^3}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/x^3/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(c+dx^3\right)}\,dx=\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{(dx^3+c)x^3}\,dx$$

[In] integrate((b*x^3+a)^(1/3)/x^3/(d*x^3+c),x, algorithm="giac")

[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(c+dx^3\right)}\,dx=\int \frac{{\left(b\,x^3+a\right)}^{1/3}}{x^3\,\left(d\,x^3+c\right)}\,dx$$

[In] int((a + b*x^3)^(1/3)/(x^3*(c + d*x^3)),x)

[Out] int((a + b*x^3)^(1/3)/(x^3*(c + d*x^3)), x)

3.676 $\int \frac{\sqrt[3]{a+bx^3}}{x^6(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6(c+dx^3)}\,dx = -\frac{\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(-\frac{5}{3},-\frac{1}{3},1,-\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5cx^5\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] -1/5*(b*x^3+a)^(1/3)*AppellF1(-5/3,-1/3,1,-2/3,-b*x^3/a,-d*x^3/c)/c/x^5/(1+
b*x^3/a)^(1/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6(c+dx^3)}\,dx = -\frac{\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(-\frac{5}{3},-\frac{1}{3},1,-\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5cx^5\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[In] Int[(a + b*x^3)^(1/3)/(x^6*(c + d*x^3)),x]
```

```
[Out] -1/5*((a + b*x^3)^(1/3)*AppellF1[-5/3, -1/3, 1, -2/3, -((b*x^3)/a), -((d*x^
3)/c)])/(c*x^5*(1 + (b*x^3)/a)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^6(c+dx^3)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= -\frac{\sqrt[3]{a+bx^3}F_1\left(-\frac{5}{3};-\frac{1}{3},1;-\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5cx^5\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 289 vs. 2(64) = 128.

Time = 10.38 (sec) , antiderivative size = 289, normalized size of antiderivative = 4.52

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6(c+dx^3)}\,dx$$
$$= \frac{-\frac{4(a+bx^3)(2ac+bcx^3-5adx^3)}{ac^2x^5}+\frac{bd(-bc+5ad)x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^3}+\frac{16(b^2\cdots}{c(c+dx^3)\left(-4ac\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a}\cdots\right.\right.}}{40\left(a+bx^3\right)^{2/3}}$$

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^6*(c + d*x^3)),x]
```

```
[Out] ((-4*(a + b*x^3)*(2*a*c + b*c*x^3 - 5*a*d*x^3))/(a*c^2*x^5) + (b*d*(-(b*c)
+ 5*a*d)*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a),
 -((d*x^3)/c)])/(a*c^3) + (16*(b^2*c^2 + 5*a*b*c*d - 10*a^2*d^2)*x*AppellF1
[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*(c + d*x^3)*(-4*a*c*Appe
llF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/
3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7
/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(40*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{x^6\,(d\,x^3+c)}dx$$

```
[In] int((b*x^3+a)^(1/3)/x^6/(d*x^3+c),x)
[Out] int((b*x^3+a)^(1/3)/x^6/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\,(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^3+a)^(1/3)/x^6/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\,(c+dx^3)}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{x^6\,(c+dx^3)}\,dx$$

```
[In] integrate((b*x**3+a)**(1/3)/x**6/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(1/3)/(x**6*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\,(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx^3+c)x^6}\,dx$$

```
[In] integrate((b*x^3+a)^(1/3)/x^6/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^6), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6 (c+dx^3)} \, dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx^3+c)x^6} \, dx$$

```
[In] integrate((b*x^3+a)^(1/3)/x^6/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6 (c+dx^3)} \, dx = \int \frac{(b\,x^3+a)^{1/3}}{x^6\,\left(d\,x^3+c\right)} \, dx$$

```
[In] int((a + b*x^3)^(1/3)/(x^6*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(1/3)/(x^6*(c + d*x^3)), x)
```

3.677 $\int \frac{x^{11}\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 266

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{c+dx^3}\,dx = -\frac{c^3(a+bx^3)^{2/3}}{2d^4} + \frac{(b^2c^2+abcd+a^2d^2)\,(a+bx^3)^{5/3}}{5b^3d^3} - \frac{(bc+2ad)\,(a+bx^3)^{8/3}}{8b^3d^2} + \frac{(a+bx^3)^{11/3}}{11b^3d} - \frac{c^3(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{14/3}} + \frac{c^3(bc-ad)^{2/3}\log(c+dx^3)}{6d^{14/3}} - \frac{c^3(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{14/3}}$$

```
[Out] -1/2*c^3*(b*x^3+a)^(2/3)/d^4+1/5*(a^2*d^2+a*b*c*d+b^2*c^2)*(b*x^3+a)^(5/3)/
b^3/d^3-1/8*(2*a*d+b*c)*(b*x^3+a)^(8/3)/b^3/d^2+1/11*(b*x^3+a)^(11/3)/b^3/d
+1/6*c^3*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/d^(14/3)-1/2*c^3*(-a*d+b*c)^(2/3)*ln(
(-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(14/3)-1/3*c^3*(-a*d+b*c)^(2/3)
*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(14/3
)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {457, 90, 52, 58, 631, 210, 31}

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{(a+bx^3)^{5/3}\,(a^2d^2+abcd+b^2c^2)}{5b^3d^3} - \frac{c^3(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{14/3}} - \frac{(a+bx^3)^{8/3}\,(2ad+bc)}{8b^3d^2} + \frac{(a+bx^3)^{11/3}}{11b^3d} + \frac{c^3(bc-ad)^{2/3}\log(c+dx^3)}{6d^{14/3}} - \frac{c^3(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{14/3}} - \frac{c^3(a+bx^3)^{2/3}}{2d^4}$$

```
[In] Int[(x^11*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] -1/2*(c^3*(a + b*x^3)^(2/3))/d^4 + ((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^
3)^(5/3))/(5*b^3*d^3) - ((b*c + 2*a*d)*(a + b*x^3)^(8/3))/(8*b^3*d^2) + (a
+ b*x^3)^(11/3)/(11*b^3*d) - (c^3*(b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*
(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(14/3)) + (c^3*(
b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*d^(14/3)) - (c^3*(b*c - a*d)^(2/3)*Log[
(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(14/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
```

```
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^3(a+bx)^{2/3}}{c+dx}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{\left(b^2c^2+abcd+a^2d^2\right)(a+bx)^{2/3}}{b^2d^3}+\frac{(-bc-2ad)(a+bx)^{5/3}}{b^2d^2}\right.\right.\\
&\qquad\left.\left.+\frac{(a+bx)^{8/3}}{b^2d}-\frac{c^3(a+bx)^{2/3}}{d^3(c+dx)}\right)dx, x, x^3\right)\\
&= \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^3d^3}-\frac{(bc+2ad)\left(a+bx^3\right)^{8/3}}{8b^3d^2}\\
&\quad+\frac{\left(a+bx^3\right)^{11/3}}{11b^3d}-\frac{c^3\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{c+dx}\,dx, x, x^3\right)}{3d^3}
\end{aligned}$$

$$= -\frac{c^3(a+bx^3)^{2/3}}{2d^4} + \frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{5/3}}{5b^3d^3} - \frac{(bc+2ad)(a+bx^3)^{8/3}}{8b^3d^2} + \frac{(a+bx^3)^{11/3}}{11b^3d} + \frac{(c^3(bc-ad))\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)}{3d^4}$$

$$= -\frac{c^3(a+bx^3)^{2/3}}{2d^4} + \frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{5/3}}{5b^3d^3} - \frac{(bc+2ad)(a+bx^3)^{8/3}}{8b^3d^2} + \frac{(a+bx^3)^{11/3}}{11b^3d} + \frac{c^3(bc-ad)^{2/3}\log(c+dx^3)}{6d^{14/3}} - \frac{(c^3(bc-ad)^{2/3})\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{14/3}} + \frac{(c^3(bc-ad))\,\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^5}$$

$$= -\frac{c^3(a+bx^3)^{2/3}}{2d^4} + \frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{5/3}}{5b^3d^3} - \frac{(bc+2ad)(a+bx^3)^{8/3}}{8b^3d^2} + \frac{(a+bx^3)^{11/3}}{11b^3d} + \frac{c^3(bc-ad)^{2/3}\log(c+dx^3)}{6d^{14/3}} - \frac{c^3(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{14/3}} + \frac{(c^3(bc-ad)^{2/3})\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{d^{14/3}}$$

$$= -\frac{c^3(a+bx^3)^{2/3}}{2d^4} + \frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{5/3}}{5b^3d^3} - \frac{(bc+2ad)(a+bx^3)^{8/3}}{8b^3d^2} + \frac{(a+bx^3)^{11/3}}{11b^3d} - \frac{c^3(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{14/3}} + \frac{c^3(bc-ad)^{2/3}\log(c+dx^3)}{6d^{14/3}} - \frac{c^3(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{14/3}}$$

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.17

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{\frac{3d^{2/3}(a+bx^3)^{2/3}(18a^3d^3+3a^2bd^2(11c-4dx^3)+2ab^2d(44c^2-11cdx^3+5d^2x^6)+b^3(-220c^3+88c^2dx^3-55cd^2x^6+40}{b^3}}{}$$

[In] Integrate[(x^11*(a + b*x^3)^(2/3))/(c + d*x^3),x]

[Out] ((3*d^(2/3)*(a + b*x^3)^(2/3)*(18*a^3*d^3 + 3*a^2*b*d^2*(11*c - 4*d*x^3) + 2*a*b^2*d*(44*c^2 - 11*c*d*x^3 + 5*d^2*x^6) + b^3*(-220*c^3 + 88*c^2*d*x^3 - 55*c*d^2*x^6 + 40*d^3*x^9)))/b^3 - 440*Sqrt[3]*c^3*(b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 440*c^3*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + 220*c^3*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(1320*d^(14/3))

Maple [A] (verified)

Time = 4.73 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$\frac{\frac{27\left(\left(bx^3+a\right)\left(\frac{20}{9}b^2x^6-\frac{5}{3}abx^3+a^2\right)d^3+\frac{11\left(-\frac{5bx^3}{3}+a\right)b\left(bx^3+a\right)cd^2}{6}+\frac{44b^2c^2\left(bx^3+a\right)d}{9}-\frac{110b^3c^3}{9}\right)d\left(bx^3+a\right)^{\frac{2}{3}}\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}{110}+b^3c^3}{}$

[In] int(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] 1/6*(27/110*((b*x^3+a)*(20/9*b^2*x^6-5/3*a*b*x^3+a^2)*d^3+11/6*(-5/3*b*x^3+a)*b*(b*x^3+a)*c*d^2+44/9*b^2*c^2*(b*x^3+a)*d-110/9*b^3*c^3)*d*(b*x^3+a)^(2/3)*(1/d*(a*d-b*c))^(1/3)+b^3*c^3*(a*d-b*c)*(-2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))))/(1/d*(a*d-b*c))^(1/3)/b^3/d^5

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 455 vs. 2(219) = 438.

Time = 0.52 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.71

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{c+dx^3}\,dx =$$

$$-\frac{440\sqrt{3}b^3c^3\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}+\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+220\,b^3c^3\left(-\frac{b^2c^2-2\,abcd}{d^2}\right.}{}$$

```
[In] integrate(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/1320*(440*sqrt(3)*b^3*c^3*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*a
rctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)
/d^2)^(1/3) + sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 220*b^3*c^3*(-(b^2*c^2 -
2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*log((b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*
c*d + a^2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(2/3)*(b*c - a*d) + (b*c - a*d)*(-(
b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)) - 440*b^3*c^3*(-(b^2*c^2 - 2*a*b
*c*d + a^2*d^2)/d^2)^(1/3)*log(-d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2
/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)) - 3*(40*b^3*d^3*x^9 - 5*(11*b^3*c*d^2
- 2*a*b^2*d^3)*x^6 - 220*b^3*c^3 + 88*a*b^2*c^2*d + 33*a^2*b*c*d^2 + 18*a^3
*d^3 + 2*(44*b^3*c^2*d - 11*a*b^2*c*d^2 - 6*a^2*b*d^3)*x^3)*(b*x^3 + a)^(2/
3))/(b^3*d^4)
```

Sympy [F]

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{x^{11}(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

```
[In] integrate(x**11*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**11*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{c+dx^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.54

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{c+dx^3}\,dx = -\frac{\left(b^{37}c^4d^7\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}-ab^{36}c^3d^8\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{37}cd^{11}-ab^{36}d^{12}\right)} - \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{2}{3}}c^3\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^6} + \frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^3\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^6} - \frac{220\,(bx^3+a)^{\frac{2}{3}}b^{33}c^3d^7-88\,(bx^3+a)^{\frac{5}{3}}b^{32}c^2d^8+55\,(bx^3+a)^{\frac{8}{3}}b^{31}cd^9-88\,(bx^3+a)^{\frac{5}{3}}ab^{31}cd^9-40\,(bx^3+a)}{440\,b^{33}d^{11}}$$

```
[In] integrate(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^37*c^4*d^7*(-(b*c - a*d)/d)^(1/3) - a*b^36*c^3*d^8*(-(b*c - a*d)/d)
^(1/3))*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)
^(1/3)))/(b^37*c*d^11 - a*b^36*d^12) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)
*c^3*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b
*c - a*d)/d)^(1/3))/d^6 + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^3*log((b*x^3 + a)^
(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/
d^6 - 1/440*(220*(b*x^3 + a)^(2/3)*b^33*c^3*d^7 - 88*(b*x^3 + a)^(5/3)*b^32
*c^2*d^8 + 55*(b*x^3 + a)^(8/3)*b^31*c*d^9 - 88*(b*x^3 + a)^(5/3)*a*b^31*c*
d^9 - 40*(b*x^3 + a)^(11/3)*b^30*d^10 + 110*(b*x^3 + a)^(8/3)*a*b^30*d^10 -
 88*(b*x^3 + a)^(5/3)*a^2*b^30*d^10)/(b^33*d^11)
```

Mupad [B] (verification not implemented)

Time = 9.47 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.84

$$\int \frac{x^{11}(a+bx^3)^{2/3}}{c+dx^3}\,dx = \left(\frac{3\,a^2}{5\,b^3\,d}+\frac{\left(\frac{3\,a}{b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{b^6\,d^2}\right)\,\left(b^4\,c-a\,b^3\,d\right)}{5\,b^3\,d}\right)\,{\left(b\,x^3+a\right)}^{5/3}$$

$$-\left(\frac{3\,a}{8\,b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{8\,b^6\,d^2}\right)\,{\left(b\,x^3+a\right)}^{8/3}$$

$$-{\left(b\,x^3+a\right)}^{2/3}\,\left(\frac{a^3}{2\,b^3\,d}+\frac{\left(\frac{3\,a^2}{b^3\,d}+\frac{\left(\frac{3\,a}{b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{b^6\,d^2}\right)\,\left(b^4\,c-a\,b^3\,d\right)}{b^3\,d}\right)\,\left(b^4\,c-a\,b^3\,d\right)}{2\,b^3\,d}\right)+\frac{{\left(b\,x^3+a\right)}^{11/3}}{11\,b^3\,d}-\frac{c^3\,\ln\left(\frac{{\left(b\,x^3+a\right)}^{1/}\cdots}{\cdots}\right.}{\cdots}$$

```
[In] int((x^11*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

```
[Out] ((3*a^2)/(5*b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c
- a*b^3*d))/(5*b^3*d))*(a + b*x^3)^(5/3) - ((3*a)/(8*b^3*d) + (b^4*c - a*b^
3*d)/(8*b^6*d^2))*(a + b*x^3)^(8/3) - (a + b*x^3)^(2/3)*(a^3/(2*b^3*d) + ((
(3*a^2)/(b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c - a
*b^3*d))/(b^3*d))*(b^4*c - a*b^3*d))/(2*b^3*d)) + (a + b*x^3)^(11/3)/(11*b^
3*d) - (c^3*log(((a + b*x^3)^(1/3)*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d))/d
^7 - (c^6*(a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(9*d^(28/3)))*(a*d - b*c
)^(2/3))/(3*d^(14/3)) - (c^3*log((c^6*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(7
/3))/d^(22/3) + (c^6*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d^7)*((3^(1/2)*1i)/2
- 1/2)*(a*d - b*c)^(2/3))/(3*d^(14/3)) + (c^3*log((c^6*(a + b*x^3)^(1/3)*(a
*d - b*c)^2)/d^7 - (c^6*(3^(1/2)*1i + 1)^2*(a*d - b*c)^(7/3))/(4*d^(22/3)))
*((3^(1/2)*1i)/6 + 1/6)*(a*d - b*c)^(2/3))/d^(14/3)
```

3.678 $\int \frac{x^8(a+bx^3)^{2/3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 223

$$\int \frac{x^8(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{c^2(a+bx^3)^{2/3}}{2d^3} - \frac{(bc+ad)(a+bx^3)^{5/3}}{5b^2d^2} + \frac{(a+bx^3)^{8/3}}{8b^2d} + \frac{c^2(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{11/3}} - \frac{c^2(bc-ad)^{2/3}\log(c+dx^3)}{6d^{11/3}} + \frac{c^2(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{11/3}}$$

```
[Out] 1/2*c^2*(b*x^3+a)^(2/3)/d^3-1/5*(a*d+b*c)*(b*x^3+a)^(5/3)/b^2/d^2+1/8*(b*x^
3+a)^(8/3)/b^2/d-1/6*c^2*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/d^(11/3)+1/2*c^2*(-a*
d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(11/3)+1/3*c^2*
(-a*d+b*c)^(2/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*
3^(1/2))/d^(11/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used

$= \{457, 90, 52, 58, 631, 210, 31\}$

$$\int \frac{x^8(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{c^2(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{11/3}} - \frac{(a+bx^3)^{5/3}(ad+bc)}{5b^2d^2} + \frac{(a+bx^3)^{8/3}}{8b^2d} - \frac{c^2(bc-ad)^{2/3}\log(c+dx^3)}{6d^{11/3}} + \frac{c^2(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{11/3}} + \frac{c^2(a+bx^3)^{2/3}}{2d^3}$$

```
[In] Int[(x^8*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (c^2*(a + b*x^3)^(2/3))/(2*d^3) - ((b*c + a*d)*(a + b*x^3)^(5/3))/(5*b^2*d^
2) + (a + b*x^3)^(8/3)/(8*b^2*d) + (c^2*(b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^
(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(11/3)) -
(c^2*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*d^(11/3)) + (c^2*(b*c - a*d)^(2/3
)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(11/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2(a+bx)^{2/3}}{c+dx}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{(-bc-ad)(a+bx)^{2/3}}{bd^2} + \frac{(a+bx)^{5/3}}{bd} + \frac{c^2(a+bx)^{2/3}}{d^2(c+dx)}\right)\,dx, x, x^3\right)\\
&= -\frac{(bc+ad)\left(a+bx^3\right)^{5/3}}{5b^2d^2} + \frac{\left(a+bx^3\right)^{8/3}}{8b^2d} + \frac{c^2\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{c+dx}\,dx, x, x^3\right)}{3d^2}\\
&= \frac{c^2\left(a+bx^3\right)^{2/3}}{2d^3} - \frac{(bc+ad)\left(a+bx^3\right)^{5/3}}{5b^2d^2} + \frac{\left(a+bx^3\right)^{8/3}}{8b^2d}\\
&\quad - \frac{\left(c^2(bc-ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)}{3d^3}
\end{aligned}$$

$$= \frac{c^2(a+bx^3)^{2/3}}{2d^3} - \frac{(bc+ad)(a+bx^3)^{5/3}}{5b^2d^2} + \frac{(a+bx^3)^{8/3}}{8b^2d} - \frac{c^2(bc-ad)^{2/3}\log(c+dx^3)}{6d^{11/3}} + \frac{\left(c^2(bc-ad)^{2/3}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{11/3}} - \frac{\left(c^2(bc-ad)\right)\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^4}$$

$$= \frac{c^2(a+bx^3)^{2/3}}{2d^3} - \frac{(bc+ad)(a+bx^3)^{5/3}}{5b^2d^2} + \frac{(a+bx^3)^{8/3}}{8b^2d} - \frac{c^2(bc-ad)^{2/3}\log(c+dx^3)}{6d^{11/3}} + \frac{c^2(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{11/3}} - \frac{\left(c^2(bc-ad)^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{d^{11/3}}$$

$$= \frac{c^2(a+bx^3)^{2/3}}{2d^3} - \frac{(bc+ad)(a+bx^3)^{5/3}}{5b^2d^2} + \frac{(a+bx^3)^{8/3}}{8b^2d} + \frac{c^2(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{11/3}} - \frac{c^2(bc-ad)^{2/3}\log(c+dx^3)}{6d^{11/3}} + \frac{c^2(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{11/3}}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.19

$$\int \frac{x^8(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{\frac{3d^{2/3}(a+bx^3)^{2/3}\left(-3a^2d^2+2abd(-4c+dx^3)+b^2(20c^2-8cdx^3+5d^2x^6)\right)}{b^2} + 40\sqrt{3}c^2(bc-ad)^{2/3}\arctan\left(\frac{1}{-}\right.}{}$$

```
[In] Integrate[(x^8*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] ((3*d^(2/3)*(a + b*x^3)^(2/3)*(-3*a^2*d^2 + 2*a*b*d*(-4*c + d*x^3) + b^2*(2
0*c^2 - 8*c*d*x^3 + 5*d^2*x^6)))/b^2 + 40*Sqrt[3]*c^2*(b*c - a*d)^(2/3)*Arc
Tan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 40*c^2
```

```
*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - 20*
c^2*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a
+ b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(120*d^(11/3))
```

Maple [A] (verified)

Time = 4.76 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$-\frac{\frac{9d\left(\left(-\frac{5b\,x^{3}}{3}+a\right)\left(b\,x^{3}+a\right)d^{2}+\frac{8\left(b\,x^{3}+a\right)bcd}{3}-\frac{20b^{2}c^{2}}{3}\right)\left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}{20}+b^{2}c^{2}(ad-bc)\Bigg(-2\arctan\Bigg(\frac{\sqrt{3}\,\Big(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\Big(}{3\left(\frac{ad-bc}{d}\right)}}{6\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}d^{4}b}$

```
[In] int(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/(1/d*(a*d-b*c))^(1/3)*(9/20*d*((-5/3*b*x^3+a)*(b*x^3+a)*d^2+8/3*(b*x^3
+a)*b*c*d-20/3*b^2*c^2)*(b*x^3+a)^(2/3)*(1/d*(a*d-b*c))^(1/3)+b^2*c^2*(a*d-
b*c)*(-2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*
(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a
)^(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3)))
)/d^4/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(181) = 362$.

Time = 0.49 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.78

$$\int \frac{x^8(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{40\sqrt{3}b^2c^2\left(\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)-20}{}$$

```
[In] integrate(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/120*(40*sqrt(3)*b^2*c^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*arcta
n(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)
^(1/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) - 20*b^2*c^2*((b^2*c^2 - 2*a*b*c
*d + a^2*d^2)/d^2)^(1/3)*log((b*x^3 + a)^(1/3)*d*((b^2*c^2 - 2*a*b*c*d + a^
2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(2/3)*(b*c - a*d) - (b*c - a*d)*((b^2*c^2 -
 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)) + 40*b^2*c^2*((b^2*c^2 - 2*a*b*c*d + a^2*
d^2)/d^2)^(1/3)*log(-d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) - (b*x^3
 + a)^(1/3)*(b*c - a*d)) + 3*(5*b^2*d^2*x^6 + 20*b^2*c^2 - 8*a*b*c*d - 3*a^
2*d^2 - 2*(4*b^2*c*d - a*b*d^2)*x^3)*(b*x^3 + a)^(2/3))/(b^2*d^3)
```

Sympy [F]

$$\int \frac{x^8(a+bx^3)^{2/3}}{c+dx^3}\,dx=\int \frac{x^8(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

```
[In] integrate(x**8*(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(x**8*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8(a+bx^3)^{2/3}}{c+dx^3}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.57

$$\int \frac{x^8(a+bx^3)^{2/3}}{c+dx^3}\,dx=\frac{\left(b^{19}c^3d^5\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}-ab^{18}c^2d^6\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^{19}cd^8-ab^{18}d^9)}$$
$$+\frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{2}{3}}c^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^5}$$
$$-\frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^2\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^5}$$
$$+\frac{20\,(bx^3+a)^{\frac{2}{3}}b^{16}c^2d^5-8\,(bx^3+a)^{\frac{5}{3}}b^{15}cd^6+5\,(bx^3+a)^{\frac{8}{3}}b^{14}d^7-8\,(bx^3+a)^{\frac{5}{3}}ab^{14}d^7}{40\,b^{16}d^8}$$

```
[In] integrate(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b^19*c^3*d^5*(-(b*c - a*d)/d)^(1/3) - a*b^18*c^2*d^6*(-(b*c - a*d)/d)^
(1/3))*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^
(1/3)))/(b^19*c*d^8 - a*b^18*d^9) + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*c^
2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c
- a*d)/d)^(1/3))/d^5 - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^2*log((b*x^3 + a)^(2/
3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^5
 + 1/40*(20*(b*x^3 + a)^(2/3)*b^16*c^2*d^5 - 8*(b*x^3 + a)^(5/3)*b^15*c*d^6
 + 5*(b*x^3 + a)^(8/3)*b^14*d^7 - 8*(b*x^3 + a)^(5/3)*a*b^14*d^7)/(b^16*d^8
)
```

Mupad [B] (verification not implemented)

Time = 9.30 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.73

$$\int \frac{x^8(a+bx^3)^{2/3}}{c+dx^3}\,dx = \left(\frac{a^2}{2\,b^2\,d}+\frac{\left(\frac{2\,a}{b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{b^4\,d^2}\right)\,\left(b^3\,c-a\,b^2\,d\right)}{2\,b^2\,d}\right)\,{\left(b\,x^3+a\right)}^{2/3}$$
$$-\left(\frac{2\,a}{5\,b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{5\,b^4\,d^2}\right)\,{\left(b\,x^3+a\right)}^{5/3}+\frac{{\left(b\,x^3+a\right)}^{8/3}}{8\,b^2\,d}$$
$$+\frac{c^2\,\ln\left(\frac{{\left(b\,x^3+a\right)}^{1/3}\,\left(a^2\,c^4\,d^2-2\,a\,b\,c^5\,d+b^2\,c^6\right)}{d^5}-\frac{c^4\,{\left(a\,d-b\,c\right)}^{4/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{9\,d^{22/3}}\right)\,{\left(a\,d-b\,c\right)}^{2/3}}{3\,d^{11/3}}$$
$$-\frac{c^2\,\ln\left(\frac{c^4\,{\left(b\,x^3+a\right)}^{1/3}\,{\left(a\,d-b\,c\right)}^2}{d^5}-\frac{c^4\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{7/3}}{d^{16/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{2/3}}{3\,d^{11/3}}$$
$$+\frac{c^2\,\ln\left(\frac{c^4\,{\left(b\,x^3+a\right)}^{1/3}\,{\left(a\,d-b\,c\right)}^2}{d^5}-\frac{c^4\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,{\left(a\,d-b\,c\right)}^{7/3}}{4\,d^{16/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,{\left(a\,d-b\,c\right)}^{2/3}}{d^{11/3}}$$

```
[In] int((x^8*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

```
[Out] (a^2/(2*b^2*d) + (((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2))*(b^3*c - a*
b^2*d))/(2*b^2*d))*(a + b*x^3)^(2/3) - ((2*a)/(5*b^2*d) + (b^3*c - a*b^2*d)
/(5*b^4*d^2))*(a + b*x^3)^(5/3) + (a + b*x^3)^(8/3)/(8*b^2*d) + (c^2*log(((
a + b*x^3)^(1/3)*(b^2*c^6 + a^2*c^4*d^2 - 2*a*b*c^5*d))/d^5 - (c^4*(a*d - b
*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(9*d^(22/3)))*(a*d - b*c)^(2/3))/(3*d^(11/
3)) - (c^2*log((c^4*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d^5 - (c^4*((3^(1/2)*1
i)/2 - 1/2)*(a*d - b*c)^(7/3))/d^(16/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)
^(2/3))/(3*d^(11/3)) + (c^2*log((c^4*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d^5 -
 (c^4*(3^(1/2)*1i - 1)^2*(a*d - b*c)^(7/3))/(4*d^(16/3)))*((3^(1/2)*1i)/6 -
 1/6)*(a*d - b*c)^(2/3))/d^(11/3)
```

3.679 $\int \frac{x^5 (a+bx^3)^{2/3}}{c+dx^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 188

$$\int \frac{x^5 (a+bx^3)^{2/3}}{c+dx^3} dx = -\frac{c(a+bx^3)^{2/3}}{2d^2} + \frac{(a+bx^3)^{5/3}}{5bd} - \frac{c(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{8/3}} + \frac{c(bc-ad)^{2/3}\log(c+dx^3)}{6d^{8/3}} - \frac{c(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}}$$

```
[Out] -1/2*c*(b*x^3+a)^(2/3)/d^2+1/5*(b*x^3+a)^(5/3)/b/d+1/6*c*(-a*d+b*c)^(2/3)*l
n(d*x^3+c)/d^(8/3)-1/2*c*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^
3+a)^(1/3))/d^(8/3)-1/3*c*(-a*d+b*c)^(2/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a
)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(8/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used

$= \{457, 81, 52, 58, 631, 210, 31\}$

$$\int \frac{x^5(a+bx^3)^{2/3}}{c+dx^3}\,dx = -\frac{c(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{8/3}} + \frac{c(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{8/3}} - \frac{c(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}} - \frac{c(a+bx^3)^{2/3}}{2d^2} + \frac{(a+bx^3)^{5/3}}{5bd}$$

```
[In] Int[(x^5*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] -1/2*(c*(a + b*x^3)^(2/3))/d^2 + (a + b*x^3)^(5/3)/(5*b*d) - (c*(b*c - a*d)
^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]
])/(Sqrt[3]*d^(8/3)) + (c*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*d^(8/3)) - (
c*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*
d^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x(a+bx)^{2/3}}{c+dx}\,dx,x,x^3\right)\\
&= \frac{(a+bx^3)^{5/3}}{5bd} - \frac{c\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{c+dx}\,dx,x,x^3\right)}{3d}\\
&= -\frac{c(a+bx^3)^{2/3}}{2d^2} + \frac{(a+bx^3)^{5/3}}{5bd} + \frac{(c(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx,x,x^3\right)}{3d^2}\\
&= -\frac{c(a+bx^3)^{2/3}}{2d^2} + \frac{(a+bx^3)^{5/3}}{5bd} + \frac{c(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{8/3}}\\
&\quad - \frac{\left(c(bc-ad)^{2/3}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx,x,\sqrt[3]{a+bx^3}\right)}{2d^{8/3}}\\
&\quad + \frac{(c(bc-ad))\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx,x,\sqrt[3]{a+bx^3}\right)}{2d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{c(a+bx^3)^{2/3}}{2d^2} + \frac{(a+bx^3)^{5/3}}{5bd} + \frac{c(bc-ad)^{2/3}\log(c+dx^3)}{6d^{8/3}} \\
&\quad - \frac{c(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}} \\
&\quad + \frac{\left(c(bc-ad)^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{d^{8/3}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{c(a+bx^3)^{2/3}}{2d^2} + \frac{(a+bx^3)^{5/3}}{5bd} - \frac{c(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{8/3}} \\
&\quad + \frac{c(bc-ad)^{2/3}\log(c+dx^3)}{6d^{8/3}} - \frac{c(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.21

$$
\int \frac{x^5(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{\frac{3d^{2/3}(a+bx^3)^{2/3}(-5bc+2ad+2bdx^3)}{b} - 10\sqrt{3}c(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) - 10c
$$

```
[In] Integrate[(x^5*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] ((3*d^(2/3)*(a + b*x^3)^(2/3)*(-5*b*c + 2*a*d + 2*b*d*x^3))/b - 10*Sqrt[3]*
c*(b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(
1/3))/Sqrt[3]] - 10*c*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a
+ b*x^3)^(1/3)] + 5*c*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*
c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(30*d^(8/3))
```

Maple [A] (verified)

Time = 4.72 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.10

method	result
pseudoelliptic	$\frac{\frac{6\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}} d\left(\left(d\,x^3-\frac{5c}{2}\right)b+ad\right)\left(b\,x^3+a\right)^{\frac{2}{3}}}{5}+bc(ad-bc)\left(-2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^3+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^3+a\right)^{\frac{2}{3}}+\left(a\right.\right.\right.}{6\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}} b\,d^3}$

```
[In] int(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/(1/d*(a*d-b*c))^(1/3)*(6/5*(1/d*(a*d-b*c))^(1/3)*d*((d*x^3-5/2*c)*b+a*d
)*(b*x^3+a)^(2/3)+b*c*(a*d-b*c)*(-2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(
1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+(1/
d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a)^(1
/3)-(1/d*(a*d-b*c))^(1/3))))/b/d^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(149) = 298$.

Time = 0.45 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.88

$$\int \frac{x^5(a+bx^3)^{2/3}}{c+dx^3}\,dx =$$

$$-\frac{10\sqrt{3}bc\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}+\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+5\,bc\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}}{}$$

```
[In] integrate(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/30*(10*sqrt(3)*b*c*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*arctan(-
1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(
1/3) + sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 5*b*c*(-(b^2*c^2 - 2*a*b*c*d + a
^2*d^2)/d^2)^(1/3)*log((b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2
)/d^2)^(2/3) - (b*x^3 + a)^(2/3)*(b*c - a*d) + (b*c - a*d)*(-(b^2*c^2 - 2*a
*b*c*d + a^2*d^2)/d^2)^(1/3)) - 10*b*c*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^
2)^(1/3)*log(-d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) - (b*x^3 + a)^
(1/3)*(b*c - a*d)) - 3*(2*b*d*x^3 - 5*b*c + 2*a*d)*(b*x^3 + a)^(2/3))/(b*d^
2)
```

Sympy [F]

$$\int \frac{x^5(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{x^5(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

```
[In] integrate(x**5*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**5*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5 (a + bx^3)^{2/3}}{c + dx^3} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(149) = 298$.

Time = 0.30 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.63

$$\int \frac{x^5 (a + bx^3)^{2/3}}{c + dx^3} \, dx = -\frac{\left(b^7c^2d^3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} - ab^6cd^4\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^7cd^5 - ab^6d^6\right)} - \frac{\sqrt{3}(-bcd^2 + ad^3)^{\frac{2}{3}}c\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^4} + \frac{(-bcd^2 + ad^3)^{\frac{2}{3}}c\log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^4} - \frac{5\,(bx^3+a)^{\frac{2}{3}}b^5cd^3 - 2\,(bx^3+a)^{\frac{5}{3}}b^4d^4}{10\,b^5d^5}$$

```
[In] integrate(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^7*c^2*d^3*(-(b*c - a*d)/d)^(1/3) - a*b^6*c*d^4*(-(b*c - a*d)/d)^(1/
3))*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/
3)))/(b^7*c*d^5 - a*b^6*d^6) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*c*arcta
n(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/
d)^(1/3))/d^4 + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c*log((b*x^3 + a)^(2/3) + (b*x
^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^4 - 1/10*(
5*(b*x^3 + a)^(2/3)*b^5*c*d^3 - 2*(b*x^3 + a)^(5/3)*b^4*d^4)/(b^5*d^5)
```

Mupad [B] (verification not implemented)

Time = 9.49 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.61

$$\int \frac{x^5(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{(b\,x^3+a)^{5/3}}{5\,b\,d} - (b\,x^3+a)^{2/3}\left(\frac{a}{2\,b\,d}+\frac{b^2\,c-a\,b\,d}{2\,b^2\,d^2}\right) - \frac{c\,\ln\left(\frac{(b\,x^3+a)^{1/3}\,(a^2\,c^2\,d^2-2\,a\,b\,c^3\,d+b^2\,c^4)}{d^3}-\frac{c^2\,(a\,d-b\,c)^{4/3}\,(9\,a\,d^3-9\,b\,c\,d^2)}{9\,d^{16/3}}\right)\,(a\,d- [illegible]}{3\,d^{8/3}}$$

```
[In] int((x^5*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

```
[Out] (a + b*x^3)^(5/3)/(5*b*d) - (a + b*x^3)^(2/3)*(a/(2*b*d) + (b^2*c - a*b*d)/
(2*b^2*d^2)) - (c*log(((a + b*x^3)^(1/3)*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3
*d))/d^3 - (c^2*(a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(9*d^(16/3)))*(a*d
 - b*c)^(2/3))/(3*d^(8/3)) - (c*log((c^2*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)
^(7/3))/d^(10/3) + (c^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d^3)*((3^(1/2)*1i)
/2 - 1/2)*(a*d - b*c)^(2/3))/(3*d^(8/3)) + (c*log((c^2*(a + b*x^3)^(1/3)*(a
*d - b*c)^2)/d^3 - (c^2*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^(7/3))/d^(10/3))
*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(2/3))/(3*d^(8/3))
```

3.680 $\int \frac{x^2\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 162

$$\int \frac{x^2(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{(a+bx^3)^{2/3}}{2d} + \frac{(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{5/3}} - \frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{5/3}} + \frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}}$$

```
[Out] 1/2*(b*x^3+a)^(2/3)/d-1/6*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/d^(5/3)+1/2*(-a*d+b*
c)^(2/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(5/3)+1/3*(-a*d+b*c
)^(2/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/
d^(5/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {455, 52, 58, 631, 210, 31}

$$\int \frac{x^2(a+bx^3)^{2/3}}{c+dx^3}\,dx=\frac{(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{5/3}}-\frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{5/3}}+\frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}}+\frac{(a+bx^3)^{2/3}}{2d}$$

```
[In] Int[(x^2*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (a + b*x^3)^(2/3)/(2*d) + ((b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*
x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(5/3)) - ((b*c - a*d)^(
2/3)*Log[c + d*x^3])/(6*d^(5/3)) + ((b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3)
 + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{c+dx}\,dx, x, x^3\right)\\
&= \frac{(a+bx^3)^{2/3}}{2d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)}{3d}\\
&= \frac{(a+bx^3)^{2/3}}{2d} - \frac{(bc-ad)^{2/3}\log(c+dx^3)}{6d^{5/3}}\\
&\quad+ \frac{(bc-ad)^{2/3}\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{5/3}}\\
&\quad- \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^2}\\
&= \frac{(a+bx^3)^{2/3}}{2d} - \frac{(bc-ad)^{2/3}\log(c+dx^3)}{6d^{5/3}}\\
&\quad+ \frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}}\\
&\quad- \frac{(bc-ad)^{2/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{d^{5/3}}
\end{aligned}$$

$$= \frac{(a+bx^3)^{2/3}}{2d} + \frac{(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{5/3}} - \frac{(bc-ad)^{2/3}\log(c+dx^3)}{6d^{5/3}} + \frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.27

$$\int \frac{x^2(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{3d^{2/3}(a+bx^3)^{2/3} + 2\sqrt{3}(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) + 2(bc-ad)^{2/3}\log(}{}$$

```
[In] Integrate[(x^2*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (3*d^(2/3)*(a + b*x^3)^(2/3) + 2*Sqrt[3]*(b*c - a*d)^(2/3)*ArcTan[(1 - (2*d
^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 2*(b*c - a*d)^(2/3)
*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - (b*c - a*d)^(2/3)*Log
[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*
(a + b*x^3)^(2/3)])/(6*d^(5/3))
```

Maple [A] (verified)

Time = 4.72 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.37

method	result
pseudoelliptic	$\frac{(b\,x^3+a)^{\frac{2}{3}}}{2d} + \frac{\ln\left((b\,x^3+a)^{\frac{1}{3}} - \left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)(ad-bc)}{3d^2\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}} - \frac{\ln\left((b\,x^3+a)^{\frac{2}{3}} + \left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}(b\,x^3+a)^{\frac{1}{3}} + \left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\right)(ad-bc)}{6d^2\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}} + -$

```
[In] int(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(b*x^3+a)^(2/3)/d+1/3/d^2/(1/d*(a*d-b*c))^(1/3)*ln((b*x^3+a)^(1/3)-(1/d
*(a*d-b*c))^(1/3))*(a*d-b*c)-1/6/d^2/(1/d*(a*d-b*c))^(1/3)*ln((b*x^3+a)^(2/
3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))*(a*d-b*c)+1
/3*3^(1/2)/d^2/(1/d*(a*d-b*c))^(1/3)*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+
(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*(a*d-b*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(128) = 256$.

Time = 0.52 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.99

$$\int \frac{x^2(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{2\sqrt{3}\left(\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)-\left(\frac{b^2c^2-2}{}\right.}{}$$

```
[In] integrate(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*arctan(-1/3*(2*s
qrt(3)*(b*x^3 + a)^(1/3)*d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3) - sq
rt(3)*(b*c - a*d))/(b*c - a*d)) - ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/
3)*log((b*x^3 + a)^(1/3)*d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) - (b
*x^3 + a)^(2/3)*(b*c - a*d) - (b*c - a*d)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/
d^2)^(1/3)) + 2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*log(-d*((b^2*c^
2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)) + 3*(b
*x^3 + a)^(2/3))/d
```

Sympy [F]

$$\int \frac{x^2(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{x^2(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

```
[In] integrate(x**2*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**2*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+bx^3)^{2/3}}{c+dx^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(128) = 256.

Time = 0.31 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.60

$$\int \frac{x^2(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{\left(bcd\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} - ad^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bcd^2-ad^3)} + \frac{(bx^3+a)^{\frac{2}{3}}}{2\,d} + \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^3} - \frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^3}$$

```
[In] integrate(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b*c*d*(-(b*c - a*d)/d)^(1/3) - a*d^2*(-(b*c - a*d)/d)^(1/3))*(-(b*c -
a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c*d^2
 - a*d^3) + 1/2*(b*x^3 + a)^(2/3)/d + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*
arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
a*d)/d)^(1/3))/d^3 - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (
b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^3
```

Mupad [B] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.47

$$\int \frac{x^2(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{(b\,x^3+a)^{2/3}}{2\,d} + \frac{\ln\left(\frac{(b\,x^3+a)^{1/3}\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{d} - \frac{(a\,d-b\,c)^{4/3}\,(9\,a\,d^3-9\,b\,c\,d^2)}{9\,d^{10/3}}\right)(a\,d-b\,c)^{2/3}}{3\,d^{5/3}} - \frac{\ln\left(\frac{(b\,x^3+a)^{1/3}\,(a\,d-b\,c)^2}{d} - \frac{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(a\,d-b\,c)^{7/3}}{d^{4/3}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(a\,d-b\,c)^{2/3}}{3\,d^{5/3}} + \frac{\ln\left(\frac{(b\,x^3+a)^{1/3}\,(a\,d-b\,c)^2}{d} - \frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2(a\,d-b\,c)^{7/3}}{4\,d^{4/3}}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)(a\,d-b\,c)^{2/3}}{d^{5/3}}$$

```
[In] int((x^2*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

```
[Out] (a + b*x^3)^(2/3)/(2*d) + (log(((a + b*x^3)^(1/3)*(a^2*d^2 + b^2*c^2 - 2*a*
b*c*d))/d - ((a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(9*d^(10/3)))*(a*d -
```

```
b*c)^(2/3))/(3*d^(5/3)) - (log(((a + b*x^3)^(1/3)*(a*d - b*c)^2)/d - (((3^(
1/2)*1i)/2 - 1/2)*(a*d - b*c)^(7/3))/d^(4/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d -
 b*c)^(2/3))/(3*d^(5/3)) + (log(((a + b*x^3)^(1/3)*(a*d - b*c)^2)/d - ((3^(
1/2)*1i - 1)^2*(a*d - b*c)^(7/3))/(4*d^(4/3)))*((3^(1/2)*1i)/6 - 1/6)*(a*d
- b*c)^(2/3))/d^(5/3)
```

3.681 $\int \frac{(a+bx^3)^{2/3}}{x(c+dx^3)} dx$

Optimal result

Integrand size = 24, antiderivative size = 245

$$\int \frac{(a+bx^3)^{2/3}}{x(c+dx^3)} dx = \frac{a^{2/3}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}c} - \frac{(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}cd^{2/3}} - \frac{a^{2/3}\log(x)}{2c} + \frac{(bc-ad)^{2/3}\log(c+dx^3)}{6cd^{2/3}} + \frac{a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c} - \frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2cd^{2/3}}$$

```
[Out] -1/2*a^(2/3)*ln(x)/c+1/6*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/c/d^(2/3)+1/2*a^(2/3)
*ln(a^(1/3)-(b*x^3+a)^(1/3))/c-1/2*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)+d^(
1/3)*(b*x^3+a)^(1/3))/c/d^(2/3)+1/3*a^(2/3)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)
^(1/3))/a^(1/3)*3^(1/2))/c*3^(1/2)-1/3*(-a*d+b*c)^(2/3)*arctan(1/3*(1-2*d^(
1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c/d^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {457, 85, 57, 631, 210, 31, 58}

$$\int \frac{(a+bx^3)^{2/3}}{x(c+dx^3)}\,dx = \frac{a^{2/3}\arctan\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}c} + \frac{a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c} - \frac{a^{2/3}\log(x)}{2c} - \frac{(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}cd^{2/3}} + \frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6cd^{2/3}} - \frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2cd^{2/3}}$$

```
[In] Int[(a + b*x^3)^(2/3)/(x*(c + d*x^3)),x]
```

```
[Out] (a^(2/3)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3
]*c) - ((b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c -
a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c*d^(2/3)) - (a^(2/3)*Log[x])/(2*c) + ((b*c
- a*d)^(2/3)*Log[c + d*x^3])/(6*c*d^(2/3)) + (a^(2/3)*Log[a^(1/3) - (a + b*
x^3)^(1/3)])/(2*c) - ((b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a
+ b*x^3)^(1/3)])/(2*c*d^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^3\right)}{3c} + \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)}{3c}$$

$$= -\frac{a^{2/3}\log(x)}{2c} + \frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6cd^{2/3}} - \frac{a^{2/3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c} + \frac{a\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c} - \frac{(bc-ad)^{2/3}\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2cd^{2/3}} + \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2cd}$$

$$= -\frac{a^{2/3}\log(x)}{2c} + \frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6cd^{2/3}} + \frac{a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c} - \frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2cd^{2/3}} - \frac{a^{2/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{c} + \frac{(bc-ad)^{2/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{cd^{2/3}}$$

$$= \frac{a^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}c} - \frac{(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}cd^{2/3}} - \frac{a^{2/3}\log(x)}{2c} + \frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6cd^{2/3}} + \frac{a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c} - \frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2cd^{2/3}}$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx^3)^{2/3}}{x(c+dx^3)}\,dx = \frac{2\sqrt{3}a^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\frac{-2\sqrt{3}(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)+2a^{2/3}d^{2/3}\log\left(-\right.}{}}{}$$

[In] Integrate[(a + b*x^3)^(2/3)/(x*(c + d*x^3)),x]

[Out] (2*Sqrt[3]*a^(2/3)*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + (-2*Sqrt[3]*(b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 2*a^(2/3)*d^(2/3)*Log[-a^(1/3) + (a + b*x^3)^(1/3)] - 2*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - a^(2/3)*d^(2/3)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + (b*c - a*d)^(2/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/d^(2/3))/(6*c)

Maple [A] (verified)

Time = 4.63 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$\frac{-a^{\frac{2}{3}}\left(-2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)\right)d\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}+}{}$

[In] int((b*x^3+a)^(2/3)/x/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] 1/6/(1/d*(a*d-b*c))^(1/3)*(-a^(2/3)*(-2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)+ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3))-2*ln((b*x^3+a)^(1/3)-a^(1/3)))*d*(1/d*(a*d-b*c))^(1/3)+(a*d-b*c)*(-2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))))/c/d

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(192) = 384$.

Time = 0.54 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.73

$$\int \frac{(a+bx^3)^{2/3}}{x(c+dx^3)}\,dx =$$
$$-\frac{2\sqrt{3}\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}+\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)-2\sqrt{3}(a^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}a+2}{}\right.}{}$$

```
[In] integrate((b*x^3+a)^(2/3)/x/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*arctan(-1/3*(2
*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3) +
 sqrt(3)*(b*c - a*d))/(b*c - a*d)) - 2*sqrt(3)*(a^2)^(1/3)*arctan(1/3*(sqrt
(3)*a + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(a^2)^(1/3))/a) + (-(b^2*c^2 - 2*a*b*c*
d + a^2*d^2)/d^2)^(1/3)*log((b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^
2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(2/3)*(b*c - a*d) + (b*c - a*d)*(-(b^2*c^2
- 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)) + (a^2)^(1/3)*log((b*x^3 + a)^(2/3)*a +
(a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) - 2*(-(b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)/d^2)^(1/3)*log(-d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) -
(b*x^3 + a)^(1/3)*(b*c - a*d)) - 2*(a^2)^(1/3)*log((b*x^3 + a)^(1/3)*a - (a
^2)^(2/3)))/c
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{2}{3}}}{x(c+dx^3)}\,dx$$

```
[In] integrate((b*x**3+a)**(2/3)/x/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(2/3)/(x*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x\,(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/x/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.63 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.39

$$\begin{aligned}\int \frac{(a+bx^3)^{2/3}}{x\,(c+dx^3)}\,dx =& -\frac{\left(bc\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} - ad\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2-acd)} \\ &+ \frac{\sqrt{3}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,c} \\ &- \frac{a^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{6\,c} + \frac{a^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}} - a^{\frac{1}{3}}\right|\right)}{3\,c} \\ &- \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,cd^2} \\ &+ \frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,cd^2}\end{aligned}$$

```
[In] integrate((b*x^3+a)^(2/3)/x/(d*x^3+c),x, algorithm="giac")
[Out] -1/3*(b*c*(-(b*c - a*d)/d)^(1/3) - a*d*(-(b*c - a*d)/d)^(1/3))*(-(b*c - a*d
)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c^2 - a*
c*d) + 1/3*sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3
))/a^(1/3))/c - 1/6*a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/
3) + a^(2/3))/c + 1/3*a^(2/3)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/c - 1/3
*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) +
 (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(c*d^2) + 1/6*(-b*c*d^2 +
a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/
3) + (-(b*c - a*d)/d)^(2/3))/(c*d^2)
```

Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 1963, normalized size of antiderivative = 8.01

$$\int \frac{(a+bx^3)^{2/3}}{x\,(c+dx^3)}\,dx = \text{Too large to display}$$

```
[In] int((a + b*x^3)^(2/3)/(x*(c + d*x^3)),x)
```

```
[Out] log((a + b*x^3)^(1/3)*(2*a^5*b^5*d^4 - a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4*
a^3*b^7*c^2*d^2) - (a^2/(27*c^3))^(2/3)*(((a + b*x^3)^(1/3)*(54*a^2*b^6*c^4
*d^3 - 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^5) - (243*a*b^6*c^6*d^3 - 729
*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(a^2/(27*c^3))^(2/3))*(a^2/(27*c^3)
)^(1/3) + 36*a^2*b^7*c^4*d^2 - 54*a^3*b^6*c^3*d^3 + 27*a^4*b^5*c^2*d^4 - 9*
a*b^8*c^5*d))*(a^2/(27*c^3))^(1/3) + log((a + b*x^3)^(1/3)*(2*a^5*b^5*d^4 -
 a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4*a^3*b^7*c^2*d^2) - (-(a^2*d^2 + b^2*c^
2 - 2*a*b*c*d)/(27*c^3*d^2))^(2/3)*(((a + b*x^3)^(1/3)*(54*a^2*b^6*c^4*d^3
- 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^5) - (243*a*b^6*c^6*d^3 - 729*a^2*
b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^
3*d^2))^(2/3))*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(1/3) + 36*a
^2*b^7*c^4*d^2 - 54*a^3*b^6*c^3*d^3 + 27*a^4*b^5*c^2*d^4 - 9*a*b^8*c^5*d))*
(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(1/3) - log((a + b*x^3)^(1/
3)*(2*a^5*b^5*d^4 - a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4*a^3*b^7*c^2*d^2) +
((3^(1/2)*1i)/2 + 1/2)^2*(a^2/(27*c^3))^(2/3)*(((3^(1/2)*1i)/2 + 1/2)*((a +
 b*x^3)^(1/3)*(54*a^2*b^6*c^4*d^3 - 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^
5) - ((3^(1/2)*1i)/2 + 1/2)^2*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 48
6*a^3*b^4*c^4*d^5)*(a^2/(27*c^3))^(2/3))*(a^2/(27*c^3))^(1/3) - 36*a^2*b^7*
c^4*d^2 + 54*a^3*b^6*c^3*d^3 - 27*a^4*b^5*c^2*d^4 + 9*a*b^8*c^5*d))*((3^(1/
2)*1i)/2 + 1/2)*(a^2/(27*c^3))^(1/3) + log((a + b*x^3)^(1/3)*(2*a^5*b^5*d^4
 - a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4*a^3*b^7*c^2*d^2) - ((3^(1/2)*1i)/2 -
 1/2)^2*(a^2/(27*c^3))^(2/3)*(((3^(1/2)*1i)/2 - 1/2)*((a + b*x^3)^(1/3)*(54
*a^2*b^6*c^4*d^3 - 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^5) - ((3^(1/2)*1i
)/2 - 1/2)^2*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5
)*(a^2/(27*c^3))^(2/3))*(a^2/(27*c^3))^(1/3) + 36*a^2*b^7*c^4*d^2 - 54*a^3*
b^6*c^3*d^3 + 27*a^4*b^5*c^2*d^4 - 9*a*b^8*c^5*d))*((3^(1/2)*1i)/2 - 1/2)*(
a^2/(27*c^3))^(1/3) - log((a + b*x^3)^(1/3)*(2*a^5*b^5*d^4 - a^2*b^8*c^3*d
- 5*a^4*b^6*c*d^3 + 4*a^3*b^7*c^2*d^2) + ((3^(1/2)*1i)/2 + 1/2)^2*(-(a^2*d^
2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(2/3)*(((3^(1/2)*1i)/2 + 1/2)*((a +
b*x^3)^(1/3)*(54*a^2*b^6*c^4*d^3 - 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^5
) - ((3^(1/2)*1i)/2 + 1/2)^2*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486
*a^3*b^4*c^4*d^5)*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(2/3))*(-
(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(1/3) - 36*a^2*b^7*c^4*d^2 +
54*a^3*b^6*c^3*d^3 - 27*a^4*b^5*c^2*d^4 + 9*a*b^8*c^5*d))*((3^(1/2)*1i)/2 +
 1/2)*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(1/3) + log((a + b*x^
3)^(1/3)*(2*a^5*b^5*d^4 - a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4*a^3*b^7*c^2*d
```

^2) - ((3^(1/2)*1i)/2 - 1/2)^2*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(2/3)*(((3^(1/2)*1i)/2 - 1/2)*((a + b*x^3)^(1/3)*(54*a^2*b^6*c^4*d^3 - 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^5) - ((3^(1/2)*1i)/2 - 1/2)^2*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(2/3))*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(1/3) + 36*a^2*b^7*c^4*d^2 - 54*a^3*b^6*c^3*d^3 + 27*a^4*b^5*c^2*d^4 - 9*a*b^8*c^5*d))*((3^(1/2)*1i)/2 - 1/2)*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(1/3)

3.682 $\int \frac{(a+bx^3)^{2/3}}{x^4(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 347

$$\int \frac{(a+bx^3)^{2/3}}{x^4(c+dx^3)}\,dx = \frac{d(a+bx^3)^{2/3}}{2c^2} + \frac{(2bc-3ad)(a+bx^3)^{2/3}}{6ac^2} - \frac{(a+bx^3)^{5/3}}{3acx^3} + \frac{(2bc-3ad)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}c^2} + \frac{\sqrt[3]{d}(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2} - \frac{(2bc-3ad)\log(x)}{6\sqrt[3]{a}c^2} - \frac{\sqrt[3]{d}(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^2} + \frac{(2bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6\sqrt[3]{a}c^2} + \frac{\sqrt[3]{d}(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2}$$

```
[Out] 1/2*d*(b*x^3+a)^(2/3)/c^2+1/6*(-3*a*d+2*b*c)*(b*x^3+a)^(2/3)/a/c^2-1/3*(b*x
^3+a)^(5/3)/a/c/x^3-1/6*(-3*a*d+2*b*c)*ln(x)/a^(1/3)/c^2-1/6*d^(1/3)*(-a*d+
b*c)^(2/3)*ln(d*x^3+c)/c^2+1/6*(-3*a*d+2*b*c)*ln(a^(1/3)-(b*x^3+a)^(1/3))/a
^(1/3)/c^2+1/2*d^(1/3)*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+
a)^(1/3))/c^2+1/9*(-3*a*d+2*b*c)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(
1/3)*3^(1/2))/a^(1/3)/c^2*3^(1/2)+1/3*d^(1/3)*(-a*d+b*c)^(2/3)*arctan(1/3*(
1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c^2*3^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {457, 105, 162, 52, 57, 631, 210, 31, 58}

$$\int \frac{(a+bx^3)^{2/3}}{x^4(c+dx^3)}\,dx = \frac{\arctan\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)(2bc-3ad)}{3\sqrt{3}\sqrt[3]{a}c^2} + \frac{\sqrt[3]{d}(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2} + \frac{d(a+bx^3)^{2/3}}{2c^2} + \frac{(a+bx^3)^{2/3}(2bc-3ad)}{6ac^2} - \frac{\sqrt[3]{d}(bc-ad)^{2/3}\log(c+dx^3)}{6c^2} + \frac{(2bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6\sqrt[3]{a}c^2} + \frac{\sqrt[3]{d}(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2} - \frac{\log(x)(2bc-3ad)}{6\sqrt[3]{a}c^2} - \frac{(a+bx^3)^{5/3}}{3acx^3}$$

```
[In] Int[(a + b*x^3)^(2/3)/(x^4*(c + d*x^3)),x]
```

```
[Out] (d*(a + b*x^3)^(2/3))/(2*c^2) + ((2*b*c - 3*a*d)*(a + b*x^3)^(2/3))/(6*a*c^
2) - (a + b*x^3)^(5/3)/(3*a*c*x^3) + ((2*b*c - 3*a*d)*ArcTan[(a^(1/3) + 2*(
a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*c^2) + (d^(1/3)*(b
*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3)
)/Sqrt[3]])/(Sqrt[3]*c^2) - ((2*b*c - 3*a*d)*Log[x])/(6*a^(1/3)*c^2) - (d^(
1/3)*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*c^2) + ((2*b*c - 3*a*d)*Log[a^(1/
3) - (a + b*x^3)^(1/3)])/(6*a^(1/3)*c^2) + (d^(1/3)*(b*c - a*d)^(2/3)*Log[(
b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x^2(c+dx)}\,dx, x, x^3\right) \\
&= -\frac{\left(a+bx^3\right)^{5/3}}{3acx^3} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{2/3}\left(\frac{1}{3}(-2bc+3ad)-\frac{2bdx}{3}\right)}{x(c+dx)}\,dx, x, x^3\right)}{3ac} \\
&= -\frac{\left(a+bx^3\right)^{5/3}}{3acx^3} + \frac{d^2\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{c+dx}\,dx, x, x^3\right)}{3c^2} + \frac{(2bc-3ad)\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x}\,dx, x, x^3\right)}{9ac^2} \\
&= \frac{d\left(a+bx^3\right)^{2/3}}{2c^2} + \frac{(2bc-3ad)\left(a+bx^3\right)^{2/3}}{6ac^2} - \frac{\left(a+bx^3\right)^{5/3}}{3acx^3} \\
&\quad + \frac{(2bc-3ad)\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^3\right)}{9c^2} \\
&\quad - \frac{(d(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)}{3c^2}
\end{aligned}
$$

$$= \frac{d(a+bx^3)^{2/3}}{2c^2} + \frac{(2bc-3ad)(a+bx^3)^{2/3}}{6ac^2} - \frac{(a+bx^3)^{5/3}}{3acx^3} - \frac{(2bc-3ad)\log(x)}{6\sqrt[3]{a}c^2} - \frac{\sqrt[3]{d}(bc-ad)^{2/3}\log(c+dx^3)}{6c^2} + \frac{(2bc-3ad)\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{6c^2} - \frac{(2bc-3ad)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{6\sqrt[3]{a}c^2} + \frac{\left(\sqrt[3]{d}(bc-ad)^{2/3}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c^2} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c^2}$$

$$= \frac{d(a+bx^3)^{2/3}}{2c^2} + \frac{(2bc-3ad)(a+bx^3)^{2/3}}{6ac^2} - \frac{(a+bx^3)^{5/3}}{3acx^3} - \frac{(2bc-3ad)\log(x)}{6\sqrt[3]{a}c^2} - \frac{\sqrt[3]{d}(bc-ad)^{2/3}\log(c+dx^3)}{6c^2} + \frac{(2bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6\sqrt[3]{a}c^2} + \frac{\sqrt[3]{d}(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2} - \frac{(2bc-3ad)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{3\sqrt[3]{a}c^2} - \frac{\left(\sqrt[3]{d}(bc-ad)^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{c^2}$$

$$= \frac{d(a+bx^3)^{2/3}}{2c^2} + \frac{(2bc-3ad)(a+bx^3)^{2/3}}{6ac^2} - \frac{(a+bx^3)^{5/3}}{3acx^3} + \frac{(2bc-3ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{a}c^2} + \frac{\sqrt[3]{d}(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2} - \frac{(2bc-3ad)\log(x)}{6\sqrt[3]{a}c^2} - \frac{\sqrt[3]{d}(bc-ad)^{2/3}\log(c+dx^3)}{6c^2} + \frac{(2bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6\sqrt[3]{a}c^2} + \frac{\sqrt[3]{d}(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2}$$

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^3)^{2/3}}{x^4(c+dx^3)}\,dx = \frac{-\frac{6c(a+bx^3)^{2/3}}{x^3} + \frac{2\sqrt{3}(2bc-3ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} + 6\sqrt{3}\sqrt[3]{d}(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) \cdots}{\cdots}$$

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^4*(c + d*x^3)),x]
```

```
[Out] ((-6*c*(a + b*x^3)^(2/3))/x^3 + (2*Sqrt[3]*(2*b*c - 3*a*d)*ArcTan[(1 + (2*(
a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) + 6*Sqrt[3]*d^(1/3)*(b*c - a*d
)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3
]] + (2*(2*b*c - 3*a*d)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(1/3) + 6*d^(1
/3)*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] +
((-2*b*c + 3*a*d)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/
3)])/a^(1/3) - 3*d^(1/3)*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*
(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(18*c^2)
```

Maple [A] (verified)

Time = 4.90 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{\left(-2\left(b\,x^{3}+a\right)^{\frac{2}{3}} a^{\frac{1}{3}} c+x^{3}\left(-2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right.\right.\right.}{}$

[In] int((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] 1/6/a^(1/3)*((-2*(b*x^3+a)^(2/3)*a^(1/3)*c+x^3*(-2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)+ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3))-2*ln((b*x^3+a)^(1/3)-a^(1/3)))*(a*d-2/3*b*c))*(1/d*(a*d-b*c))^(1/3)+2*x^3*(arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))-1/2*ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3)))*(d*a^(4/3)-a^(1/3)*b*c))/(1/d*(a*d-b*c))^(1/3)/c^2/x^3

Fricas [A] (verification not implemented)

none

Time = 0.63 (sec) , antiderivative size = 1030, normalized size of antiderivative = 2.97

$$\int \frac{(a+bx^3)^{2/3}}{x^4\,(c+dx^3)}\,dx = \text{Too large to display}$$

[In] integrate((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x, algorithm="fricas")

[Out] [-1/18*(3*sqrt(1/3)*(2*a*b*c - 3*a^2*d)*x^3*sqrt((-a)^(1/3)/a)*log((2*b*x^3 - 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(-a)^(2/3) - (b*x^3 + a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)^(1/3)/a) - 3*(b*x^3 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^3) - 6*sqrt(3)*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*a*x^3*arctan(1/3*(sqrt(3)*(b*c - a*d) - 2*sqrt(3)*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3))/(b*c - a*d)) + (2*b*c - 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(2*b*c - 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*a*x^3*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*a*x^3*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)) + 6*(b*x^3 + a)^(2/3)*a*c)/(a*c^2*x^3), 1/18*(6*sqrt(1/3)*(2*a*b*c - 3*a^2*d)*x^3*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + 6*sqrt(3)*(b^2*c^2*d - 2*a*b*

```
c*d^2 + a^2*d^3)^(1/3)*a*x^3*arctan(1/3*(sqrt(3)*(b*c - a*d) - 2*sqrt(3)*(b
^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3))/(b*c - a*d)) - (
2*b*c - 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a
)^(1/3) + (-a)^(2/3)) + 2*(2*b*c - 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(1
/3) + (-a)^(1/3)) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*a*x^3*log(-
(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/
3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3
)) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*a*x^3*log(-(b*x^3 + a)^(1/
3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)) - 6*(b*x^3
+ a)^(2/3)*a*c)/(a*c^2*x^3)]
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^4\,(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{2}{3}}}{x^4\,(c+dx^3)}\,dx$$

```
[In] integrate((b*x**3+a)**(2/3)/x**4/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(2/3)/(x**4*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^4\,(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^4}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.56 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^3)^{2/3}}{x^4(c+dx^3)}\,dx = \frac{\left(bcd\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} - ad^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^3-ac^2d)}$$
$$+ \frac{\sqrt{3}(2\,bc-3\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{1}{3}}c^2}$$
$$- \frac{(2\,bc-3\,ad)\log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{18\,a^{\frac{1}{3}}c^2}$$
$$+ \frac{\left(2\,a^{\frac{1}{3}}bc - 3\,a^{\frac{4}{3}}d\right)\log\left(\left|(bx^3+a)^{\frac{1}{3}} - a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{2}{3}}c^2}$$
$$+ \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,c^2d}$$
$$- \frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,c^2d} - \frac{(bx^3+a)^{\frac{2}{3}}}{3\,cx^3}$$

```
[In] integrate((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b*c*d*(-(b*c - a*d)/d)^(1/3) - a*d^2*(-(b*c - a*d)/d)^(1/3))*(-(b*c -
a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c^3 -
 a*c^2*d) + 1/9*sqrt(3)*(2*b*c - 3*a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(
1/3) + a^(1/3))/a^(1/3))/(a^(1/3)*c^2) - 1/18*(2*b*c - 3*a*d)*log((b*x^3 +
a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(1/3)*c^2) + 1/9*(2*a^(1
/3)*b*c - 3*a^(4/3)*d)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(2/3)*c^2)
+ 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1
/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(c^2*d) - 1/6*(-b*c*d
^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d
)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(c^2*d) - 1/3*(b*x^3 + a)^(2/3)/(c*x^3)
```

Mupad [B] (verification not implemented)

Time = 14.44 (sec) , antiderivative size = 1908, normalized size of antiderivative = 5.50

$$\int \frac{(a+bx^3)^{2/3}}{x^4(c+dx^3)}\,dx = \text{Too large to display}$$

```
[In] int((a + b*x^3)^(2/3)/(x^4*(c + d*x^3)),x)
```

```
[Out] log(- ((((6*b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(9*a^2*d^2 + 2*b^2*c^2
- 6*a*b*c*d) - 27*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((d*(a*d
- b*c)^2)/c^6)^(2/3))*((d*(a*d - b*c)^2)/c^6)^(1/3))/3 - (a*b^5*d^4*(27*a^3
*d^3 - 19*b^3*c^3 + 64*a*b^2*c^2*d - 72*a^2*b*c*d^2))/(3*c))*((d*(a*d - b*c
)^2)/c^6)^(2/3))/9 - (b^5*d^4*(a + b*x^3)^(1/3)*(4*a*d - 3*b*c)*(3*a^2*d^2
+ 2*b^2*c^2 - 5*a*b*c*d)^2)/(27*c^5))*((a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)/
(27*c^6))^(1/3) + log(- ((((6*b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(9*a^
2*d^2 + 2*b^2*c^2 - 6*a*b*c*d) - 3*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a
*b*c*d)*(-(3*a*d - 2*b*c)^3/(a*c^6))^(2/3))*(-(3*a*d - 2*b*c)^3/(a*c^6))^(1
/3))/9 - (a*b^5*d^4*(27*a^3*d^3 - 19*b^3*c^3 + 64*a*b^2*c^2*d - 72*a^2*b*c*
d^2))/(3*c))*(-(3*a*d - 2*b*c)^3/(a*c^6))^(2/3))/81 - (b^5*d^4*(a + b*x^3)^
(1/3)*(4*a*d - 3*b*c)*(3*a^2*d^2 + 2*b^2*c^2 - 5*a*b*c*d)^2)/(27*c^5))*(-(2
7*a^3*d^3 - 8*b^3*c^3 + 36*a*b^2*c^2*d - 54*a^2*b*c*d^2)/(729*a*c^6))^(1/3)
 - log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*(6*b^4*d^3*(a + b*x
^3)^(1/3)*(a*d - b*c)^2*(9*a^2*d^2 + 2*b^2*c^2 - 6*a*b*c*d) - 3*a*b^4*c^4*d
^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d - 2*b*
c)^3/(a*c^6))^(2/3))*(-(3*a*d - 2*b*c)^3/(a*c^6))^(1/3))/9 + (a*b^5*d^4*(27
*a^3*d^3 - 19*b^3*c^3 + 64*a*b^2*c^2*d - 72*a^2*b*c*d^2))/(3*c))*(-(3*a*d -
 2*b*c)^3/(a*c^6))^(2/3))/81 - (b^5*d^4*(a + b*x^3)^(1/3)*(4*a*d - 3*b*c)*(
3*a^2*d^2 + 2*b^2*c^2 - 5*a*b*c*d)^2)/(27*c^5))*((3^(1/2)*1i)/2 + 1/2)*(-(2
7*a^3*d^3 - 8*b^3*c^3 + 36*a*b^2*c^2*d - 54*a^2*b*c*d^2)/(729*a*c^6))^(1/3)
 + log((((3^(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*(6*b^4*d^3*(a + b*x
^3)^(1/3)*(a*d - b*c)^2*(9*a^2*d^2 + 2*b^2*c^2 - 6*a*b*c*d) + 3*a*b^4*c^4*d
^3*((3^(1/2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d - 2*b*
c)^3/(a*c^6))^(2/3))*(-(3*a*d - 2*b*c)^3/(a*c^6))^(1/3))/9 - (a*b^5*d^4*(27
*a^3*d^3 - 19*b^3*c^3 + 64*a*b^2*c^2*d - 72*a^2*b*c*d^2))/(3*c))*(-(3*a*d -
 2*b*c)^3/(a*c^6))^(2/3))/81 - (b^5*d^4*(a + b*x^3)^(1/3)*(4*a*d - 3*b*c)*(
3*a^2*d^2 + 2*b^2*c^2 - 5*a*b*c*d)^2)/(27*c^5))*((3^(1/2)*1i)/2 - 1/2)*(-(2
7*a^3*d^3 - 8*b^3*c^3 + 36*a*b^2*c^2*d - 54*a^2*b*c*d^2)/(729*a*c^6))^(1/3)
 - (a + b*x^3)^(2/3)/(3*c*x^3) - log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i
)/2 + 1/2)*(6*b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(9*a^2*d^2 + 2*b^2*c^
2 - 6*a*b*c*d) - 27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c
^2 - 3*a*b*c*d)*((d*(a*d - b*c)^2)/c^6)^(2/3))*((d*(a*d - b*c)^2)/c^6)^(1/3
))/3 + (a*b^5*d^4*(27*a^3*d^3 - 19*b^3*c^3 + 64*a*b^2*c^2*d - 72*a^2*b*c*d^
2))/(3*c))*((d*(a*d - b*c)^2)/c^6)^(2/3))/9 - (b^5*d^4*(a + b*x^3)^(1/3)*(4
*a*d - 3*b*c)*(3*a^2*d^2 + 2*b^2*c^2 - 5*a*b*c*d)^2)/(27*c^5))*((3^(1/2)*1i
)/2 + 1/2)*((a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)/(27*c^6))^(1/3) + log((((3^
(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*(6*b^4*d^3*(a + b*x^3)^(1/3)*(a
*d - b*c)^2*(9*a^2*d^2 + 2*b^2*c^2 - 6*a*b*c*d) + 27*a*b^4*c^4*d^3*((3^(1/2
)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((d*(a*d - b*c)^2)/c^6)^(2
/3))*((d*(a*d - b*c)^2)/c^6)^(1/3))/3 - (a*b^5*d^4*(27*a^3*d^3 - 19*b^3*c^3
 + 64*a*b^2*c^2*d - 72*a^2*b*c*d^2))/(3*c))*((d*(a*d - b*c)^2)/c^6)^(2/3))/
9 - (b^5*d^4*(a + b*x^3)^(1/3)*(4*a*d - 3*b*c)*(3*a^2*d^2 + 2*b^2*c^2 - 5*a
*b*c*d)^2)/(27*c^5))*((3^(1/2)*1i)/2 - 1/2)*((a^2*d^3 + b^2*c^2*d - 2*a*b*c
*d^2)/(27*c^6))^(1/3)
```

3.683 $\int \frac{(a+bx^3)^{2/3}}{x^7(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 370

$$\begin{aligned}\int \frac{(a+bx^3)^{2/3}}{x^7(c+dx^3)}\,dx &= \frac{(bc+6ad)(a+bx^3)^{2/3}}{18ac^2x^3}-\frac{(a+bx^3)^{5/3}}{6acx^6} \\ &-\frac{(b^2c^2+6abcd-9a^2d^2)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}c^3} \\ &-\frac{d^{4/3}(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^3}+\frac{(b^2c^2+6abcd-9a^2d^2)\log(x)}{18a^{4/3}c^3} \\ &+\frac{d^{4/3}(bc-ad)^{2/3}\log(c+dx^3)}{6c^3}-\frac{(b^2c^2+6abcd-9a^2d^2)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{4/3}c^3} \\ &-\frac{d^{4/3}(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^3}\end{aligned}$$

```
[Out] 1/18*(6*a*d+b*c)*(b*x^3+a)^(2/3)/a/c^2/x^3-1/6*(b*x^3+a)^(5/3)/a/c/x^6+1/18
*(-9*a^2*d^2+6*a*b*c*d+b^2*c^2)*ln(x)/a^(4/3)/c^3+1/6*d^(4/3)*(-a*d+b*c)^(2
/3)*ln(d*x^3+c)/c^3-1/18*(-9*a^2*d^2+6*a*b*c*d+b^2*c^2)*ln(a^(1/3)-(b*x^3+a
)^(1/3))/a^(4/3)/c^3-1/2*d^(4/3)*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)+d^(1/
3)*(b*x^3+a)^(1/3))/c^3-1/27*(-9*a^2*d^2+6*a*b*c*d+b^2*c^2)*arctan(1/3*(a^(
1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(4/3)/c^3*3^(1/2)-1/3*d^(4/3)*(-
a*d+b*c)^(2/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^
(1/2))/c^3*3^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {457, 105, 154, 162, 57, 631, 210, 31, 58}

$$\int \frac{(a+bx^3)^{2/3}}{x^7(c+dx^3)}\,dx = -\frac{\arctan\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)(-9a^2d^2+6abcd+b^2c^2)}{9\sqrt{3}a^{4/3}c^3} - \frac{(-9a^2d^2+6abcd+b^2c^2)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{4/3}c^3} + \frac{\log(x)(-9a^2d^2+6abcd+b^2c^2)}{18a^{4/3}c^3} - \frac{d^{4/3}(bc-ad)^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^3} + \frac{d^{4/3}(bc-ad)^{2/3}\log(c+dx^3)}{6c^3} - \frac{d^{4/3}(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^3} + \frac{(a+bx^3)^{2/3}(6ad+bc)}{18ac^2x^3} - \frac{(a+bx^3)^{5/3}}{6acx^6}$$

```
[In] Int[(a + b*x^3)^(2/3)/(x^7*(c + d*x^3)),x]
```

```
[Out] ((b*c + 6*a*d)*(a + b*x^3)^(2/3))/(18*a*c^2*x^3) - (a + b*x^3)^(5/3)/(6*a*c
*x^6) - ((b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^
(1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(4/3)*c^3) - (d^(4/3)*(b*c - a*d)^(
2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])
/(Sqrt[3]*c^3) + ((b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*Log[x])/(18*a^(4/3)*c^3
) + (d^(4/3)*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*c^3) - ((b^2*c^2 + 6*a*b*
c*d - 9*a^2*d^2)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(18*a^(4/3)*c^3) - (d^(4
/3)*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(
2*c^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x^3(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{(a+bx^3)^{5/3}}{6acx^6} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{2/3}\left(\frac{1}{3}(bc+6ad)+\frac{bdx}{3}\right)}{x^2(c+dx)}\,dx, x, x^3\right)}{6ac}$$

$$= \frac{(bc+6ad)\,(a+bx^3)^{2/3}}{18ac^2x^3} - \frac{(a+bx^3)^{5/3}}{6acx^6} - \frac{\text{Subst}\left(\int \frac{\frac{2}{9}(b^2c^2+6abcd-9a^2d^2)+\frac{2}{9}bd(bc-3ad)x}{x\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)}{6ac^2}$$

$$= \frac{(bc+6ad)\,(a+bx^3)^{2/3}}{18ac^2x^3} - \frac{(a+bx^3)^{5/3}}{6acx^6} + \frac{(d^2(bc-ad))\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)}{3c^3} - \frac{(b^2c^2+6abcd-9a^2d^2)\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^3\right)}{27ac^3}$$

$$= \frac{(bc+6ad)\left(a+bx^3\right)^{2/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{5/3}}{6acx^6} + \frac{\left(b^2c^2+6abcd-9a^2d^2\right)\log(x)}{18a^{4/3}c^3} + \frac{d^{4/3}(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^3} - \frac{\left(d^{4/3}(bc-ad)^{2/3}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c^3} + \frac{(d(bc-ad))\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c^3} + \frac{\left(b^2c^2+6abcd-9a^2d^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{18a^{4/3}c^3} - \frac{\left(b^2c^2+6abcd-9a^2d^2\right)\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{18ac^3}$$

$$= \frac{(bc+6ad)\left(a+bx^3\right)^{2/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{5/3}}{6acx^6} + \frac{\left(b^2c^2+6abcd-9a^2d^2\right)\log(x)}{18a^{4/3}c^3} + \frac{d^{4/3}(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^3} - \frac{\left(b^2c^2+6abcd-9a^2d^2\right)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{4/3}c^3} - \frac{d^{4/3}(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^3} + \frac{\left(d^{4/3}(bc-ad)^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{c^3} + \frac{\left(b^2c^2+6abcd-9a^2d^2\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{9a^{4/3}c^3}$$

$$= \frac{(bc+6ad)\left(a+bx^3\right)^{2/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{5/3}}{6acx^6} - \frac{\left(b^2c^2+6abcd-9a^2d^2\right)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}a^{4/3}c^3} - \frac{d^{4/3}(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^3} + \frac{\left(b^2c^2+6abcd-9a^2d^2\right)\log(x)}{18a^{4/3}c^3} + \frac{d^{4/3}(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^3} - \frac{\left(b^2c^2+6abcd-9a^2d^2\right)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{4/3}c^3} - \frac{d^{4/3}(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^3}$$

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.12

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^7\left(c+dx^3\right)}\,dx = \frac{\frac{3c\left(a+bx^3\right)^{2/3}\left(-3ac-2bcx^3+6adx^3\right)}{ax^6} - \frac{2\sqrt{3}\left(b^2c^2+6abcd-9a^2d^2\right)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}} - 18\sqrt{3}d^{4/3}(bc-}{}$$

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^7*(c + d*x^3)),x]
```

```
[Out] ((3*c*(a + b*x^3)^(2/3)*(-3*a*c - 2*b*c*x^3 + 6*a*d*x^3))/(a*x^6) - (2*Sqrt
[3]*(b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(
1/3))/Sqrt[3]])/a^(4/3) - 18*Sqrt[3]*d^(4/3)*(b*c - a*d)^(2/3)*ArcTan[(1 -
(2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - (2*(b^2*c^2 + 6
*a*b*c*d - 9*a^2*d^2)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(4/3) - 18*d^(4/
3)*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + (
(b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) +
 (a + b*x^3)^(2/3)])/a^(4/3) + 9*d^(4/3)*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^
(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(
2/3)])/(54*c^3)
```

Maple [A] (verified)

Time = 5.03 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$\frac{\frac{-\left(b x^3+a\right)^{\frac{2}{3}} a^{\frac{4}{3}} c\left(-6 a d x^3+2 b c x^3+3 a c\right)+\frac{x^6\left(9 a^2 d^2-6 a b c d-b^2 c^2\right)\left(2 \arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b x^3+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3 a^{\frac{1}{3}}}\right)\sqrt{3}+2 \ln\left(\left(b x^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)\cdots\right)}{3}\cdots}{3}}{\cdots}$

[In] int((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] 1/6/(1/d*(a*d-b*c))^(1/3)*(1/3*(-(b*x^3+a)^(2/3)*a^(4/3)*c*(-6*a*d*x^3+2*b*c*x^3+3*a*c)+1/3*x^6*(9*a^2*d^2-6*a*b*c*d-b^2*c^2)*(2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)+2*ln((b*x^3+a)^(1/3)-a^(1/3))-ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3)))*a)*(1/d*(a*d-b*c))^(1/3)-x^6*(2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*3^(1/2)+2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))-ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3)))*a^(7/3)*(a*d-b*c)*d)/c^3/x^6/a^(7/3)

Fricas [A] (verification not implemented)

none

Time = 1.36 (sec) , antiderivative size = 1151, normalized size of antiderivative = 3.11

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^7\left(c+dx^3\right)}\,dx = \text{Too large to display}$$

[In] integrate((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x, algorithm="fricas")

[Out] [-1/54*(18*sqrt(3)*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*a^2*d*x^6*arctan(-1/3*(sqrt(3)*(b*c - a*d) + 2*sqrt(3)*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3))/(b*c - a*d)) + 9*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*a^2*d*x^6*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 18*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*a^2*d*x^6*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)) + 3*sqrt(1/3)*(a*b^2*c^2 + 6*a^2*b*c*d - 9*a^3*d^2)*x^6*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 + a)^(1/

```
3)*a^(2/3) + 3*a)/x^3) - (b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*a^(2/3)*x^6*log(
(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*(b^2*c^2 + 6*a
*b*c*d - 9*a^2*d^2)*a^(2/3)*x^6*log((b*x^3 + a)^(1/3) - a^(1/3)) + 3*(3*a^2
*c^2 + 2*(a*b*c^2 - 3*a^2*c*d)*x^3)*(b*x^3 + a)^(2/3))/(a^2*c^3*x^6), -1/54
*(18*sqrt(3)*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*a^2*d*x^6*arctan(-1
/3*(sqrt(3)*(b*c - a*d) + 2*sqrt(3)*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1
/3)*(b*x^3 + a)^(1/3))/(b*c - a*d)) + 9*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3
)^(1/3)*a^2*d*x^6*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*
a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^
3)^(2/3)*(b*x^3 + a)^(1/3)) - 18*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)
*a^2*d*x^6*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (-b^2*c^2*d + 2*a*b*c*d
^2 - a^2*d^3)^(2/3)) - (b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*a^(2/3)*x^6*log((b
*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*(b^2*c^2 + 6*a*b
*c*d - 9*a^2*d^2)*a^(2/3)*x^6*log((b*x^3 + a)^(1/3) - a^(1/3)) + 6*sqrt(1/3
)*(a*b^2*c^2 + 6*a^2*b*c*d - 9*a^3*d^2)*x^6*arctan(sqrt(1/3)*(2*(b*x^3 + a)
^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) + 3*(3*a^2*c^2 + 2*(a*b*c^2 - 3*a^2*c*d)
*x^3)*(b*x^3 + a)^(2/3))/(a^2*c^3*x^6)]
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^7\,(c+dx^3)}\,dx=\int \frac{(a+bx^3)^{\frac{2}{3}}}{x^7\,(c+dx^3)}\,dx$$

```
[In] integrate((b*x**3+a)**(2/3)/x**7/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(2/3)/(x**7*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^7\,(c+dx^3)}\,dx=\int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^7}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^7), x)
```

Giac [A] (verification not implemented)

none

Time = 0.59 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^3)^{2/3}}{x^7(c+dx^3)}\,dx = -\frac{\left(bcd^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} - ad^3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^4-ac^3d)} - \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,c^3} + \frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,c^3} - \frac{\sqrt{3}(b^2c^2+6\,abcd-9\,a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{27\,a^{\frac{4}{3}}c^3} + \frac{(b^2c^2+6\,abcd-9\,a^2d^2)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{54\,a^{\frac{4}{3}}c^3} - \frac{\left(a^{\frac{1}{3}}b^2c^2+6\,a^{\frac{4}{3}}bcd-9\,a^{\frac{7}{3}}d^2\right)\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{27\,a^{\frac{5}{3}}c^3} - \frac{2\,(bx^3+a)^{\frac{5}{3}}b^2c+(bx^3+a)^{\frac{2}{3}}ab^2c-6\,(bx^3+a)^{\frac{5}{3}}abd+6\,(bx^3+a)^{\frac{2}{3}}a^2bd}{18\,ab^2c^2x^6}$$

```
[In] integrate((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b*c*d^2*(-(b*c - a*d)/d)^(1/3) - a*d^3*(-(b*c - a*d)/d)^(1/3))*(-(b*c
 - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c^
4 - a*c^3*d) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(
b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/c^3 + 1/
6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c
 - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/c^3 - 1/27*sqrt(3)*(b^2*c^2 + 6*
a*b*c*d - 9*a^2*d^2)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(
1/3))/(a^(4/3)*c^3) + 1/54*(b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*log((b*x^3 + a
)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(4/3)*c^3) - 1/27*(a^(1/3
)*b^2*c^2 + 6*a^(4/3)*b*c*d - 9*a^(7/3)*d^2)*log(abs((b*x^3 + a)^(1/3) - a^
(1/3)))/(a^(5/3)*c^3) - 1/18*(2*(b*x^3 + a)^(5/3)*b^2*c + (b*x^3 + a)^(2/3)
*a*b^2*c - 6*(b*x^3 + a)^(5/3)*a*b*d + 6*(b*x^3 + a)^(2/3)*a^2*b*d)/(a*b^2*
c^2*x^6)
```

Mupad [B] (verification not implemented)

Time = 19.14 (sec) , antiderivative size = 2788, normalized size of antiderivative = 7.54

$$\int \frac{(a+bx^3)^{2/3}}{x^7(c+dx^3)}\,dx = \text{Too large to display}$$

```
[In] int((a + b*x^3)^(2/3)/(x^7*(c + d*x^3)),x)
```

```
[Out] log(((((27*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^4*(a*d - b*
c)^2)/c^9)^(2/3) - (b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(162*a^4*d^4 +
b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 108*a^3*b*c*d^3))/(3*a^2*c^
2))*(-(d^4*(a*d - b*c)^2)/c^9)^(1/3))/3 - (b^5*d^4*(729*a^6*d^6 + b^6*c^6 +
 63*a^2*b^4*c^4*d^2 - 918*a^3*b^3*c^3*d^3 + 2295*a^4*b^2*c^2*d^4 + 17*a*b^5
*c^5*d - 2187*a^5*b*c*d^5))/(81*a^2*c^4))*(-(d^4*(a*d - b*c)^2)/c^9)^(2/3))
/9 + (2*b^5*d^7*(a + b*x^3)^(1/3)*(6*a*d - 5*b*c)*(9*a^3*d^3 + b^3*c^3 + 5*
a*b^2*c^2*d - 15*a^2*b*c*d^2)^2)/(729*a^2*c^10))*(-(a^2*d^6 + b^2*c^2*d^4 -
 2*a*b*c*d^5)/(27*c^9))^(1/3) + log((((((a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2
 - 3*a*b*c*d)*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(2/3))/3 - (
b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(162*a^4*d^4 + b^4*c^4 + 18*a^2*b^2
*c^2*d^2 + 12*a*b^3*c^3*d - 108*a^3*b*c*d^3))/(3*a^2*c^2))*(-(b^2*c^2 - 9*a
^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(1/3))/27 - (b^5*d^4*(729*a^6*d^6 + b^6*c^
6 + 63*a^2*b^4*c^4*d^2 - 918*a^3*b^3*c^3*d^3 + 2295*a^4*b^2*c^2*d^4 + 17*a*
b^5*c^5*d - 2187*a^5*b*c*d^5))/(81*a^2*c^4))*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b
*c*d)^3/(a^4*c^9))^(2/3))/729 + (2*b^5*d^7*(a + b*x^3)^(1/3)*(6*a*d - 5*b*c
)*(9*a^3*d^3 + b^3*c^3 + 5*a*b^2*c^2*d - 15*a^2*b*c*d^2)^2)/(729*a^2*c^10))
*(-(b^6*c^6 - 729*a^6*d^6 + 81*a^2*b^4*c^4*d^2 - 108*a^3*b^3*c^3*d^3 - 729*
a^4*b^2*c^2*d^4 + 18*a*b^5*c^5*d + 1458*a^5*b*c*d^5)/(19683*a^4*c^9))^(1/3)
 - (((a + b*x^3)^(2/3)*(b^2*c + 6*a*b*d))/(18*c^2) - (b*(a + b*x^3)^(5/3)*(
3*a*d - b*c))/(9*a*c^2))/((a + b*x^3)^2 - 2*a*(a + b*x^3) + a^2) - log((((3
^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*((b^4*d^3*(a + b*x^3)^(1/3)*(a
*d - b*c)^2*(162*a^4*d^4 + b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d -
108*a^3*b*c*d^3))/(3*a^2*c^2) - 27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*
a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^4*(a*d - b*c)^2)/c^9)^(2/3))*(-(d^4*(a*
d - b*c)^2)/c^9)^(1/3))/3 - (b^5*d^4*(729*a^6*d^6 + b^6*c^6 + 63*a^2*b^4*c^
4*d^2 - 918*a^3*b^3*c^3*d^3 + 2295*a^4*b^2*c^2*d^4 + 17*a*b^5*c^5*d - 2187*
a^5*b*c*d^5))/(81*a^2*c^4))*(-(d^4*(a*d - b*c)^2)/c^9)^(2/3))/9 + (2*b^5*d^
7*(a + b*x^3)^(1/3)*(6*a*d - 5*b*c)*(9*a^3*d^3 + b^3*c^3 + 5*a*b^2*c^2*d -
15*a^2*b*c*d^2)^2)/(729*a^2*c^10))*((3^(1/2)*1i)/2 + 1/2)*(-(a^2*d^6 + b^2*
c^2*d^4 - 2*a*b*c*d^5)/(27*c^9))^(1/3) + log((((3^(1/2)*1i)/2 + 1/2)*((((3^
(1/2)*1i)/2 - 1/2)*((b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(162*a^4*d^4 +
 b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 108*a^3*b*c*d^3))/(3*a^2*c
^2) + 27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*
c*d)*(-(d^4*(a*d - b*c)^2)/c^9)^(2/3))*(-(d^4*(a*d - b*c)^2)/c^9)^(1/3))/3
+ (b^5*d^4*(729*a^6*d^6 + b^6*c^6 + 63*a^2*b^4*c^4*d^2 - 918*a^3*b^3*c^3*d^
```

3 + 2295*a^4*b^2*c^2*d^4 + 17*a*b^5*c^5*d - 2187*a^5*b*c*d^5))/(81*a^2*c^4))*(-(d^4*(a*d - b*c)^2)/c^9)^(2/3))/9 + (2*b^5*d^7*(a + b*x^3)^(1/3)*(6*a*d - 5*b*c)*(9*a^3*d^3 + b^3*c^3 + 5*a*b^2*c^2*d - 15*a^2*b*c*d^2)^2)/(729*a^2*c^10))*((3^(1/2)*1i)/2 - 1/2)*(-(a^2*d^6 + b^2*c^2*d^4 - 2*a*b*c*d^5)/(27*c^9))^(1/3) - log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*((b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(162*a^4*d^4 + b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 108*a^3*b*c*d^3))/(3*a^2*c^2) - (a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(2/3))/3)*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(1/3))/27 - (b^5*d^4*(729*a^6*d^6 + b^6*c^6 + 63*a^2*b^4*c^4*d^2 - 918*a^3*b^3*c^3*d^3 + 2295*a^4*b^2*c^2*d^4 + 17*a*b^5*c^5*d - 2187*a^5*b*c*d^5))/(81*a^2*c^4))*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(2/3))/729 + (2*b^5*d^7*(a + b*x^3)^(1/3)*(6*a*d - 5*b*c)*(9*a^3*d^3 + b^3*c^3 + 5*a*b^2*c^2*d - 15*a^2*b*c*d^2)^2)/(729*a^2*c^10))*((3^(1/2)*1i)/2 + 1/2)*(-(b^6*c^6 - 729*a^6*d^6 + 81*a^2*b^4*c^4*d^2 - 108*a^3*b^3*c^3*d^3 - 729*a^4*b^2*c^2*d^4 + 18*a*b^5*c^5*d + 1458*a^5*b*c*d^5)/(19683*a^4*c^9))^(1/3) + log((((3^(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*((b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(162*a^4*d^4 + b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 108*a^3*b*c*d^3))/(3*a^2*c^2) + (a*b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(2/3))/3)*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(1/3))/27 + (b^5*d^4*(729*a^6*d^6 + b^6*c^6 + 63*a^2*b^4*c^4*d^2 - 918*a^3*b^3*c^3*d^3 + 2295*a^4*b^2*c^2*d^4 + 17*a*b^5*c^5*d - 2187*a^5*b*c*d^5))/(81*a^2*c^4))*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(2/3))/729 + (2*b^5*d^7*(a + b*x^3)^(1/3)*(6*a*d - 5*b*c)*(9*a^3*d^3 + b^3*c^3 + 5*a*b^2*c^2*d - 15*a^2*b*c*d^2)^2)/(729*a^2*c^10))*((3^(1/2)*1i)/2 - 1/2)*(-(b^6*c^6 - 729*a^6*d^6 + 81*a^2*b^4*c^4*d^2 - 108*a^3*b^3*c^3*d^3 - 729*a^4*b^2*c^2*d^4 + 18*a*b^5*c^5*d + 1458*a^5*b*c*d^5)/(19683*a^4*c^9))^(1/3)

3.684 $\int \frac{x^6(a+bx^3)^{2/3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 334

$$\int \frac{x^6(a+bx^3)^{2/3}}{c+dx^3}\,dx = -\frac{(3bc-ad)x(a+bx^3)^{2/3}}{9bd^2} + \frac{x^4(a+bx^3)^{2/3}}{6d} + \frac{(9b^2c^2-6abcd-a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{4/3}d^3} - \frac{c^{4/3}(bc-ad)^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^3} - \frac{c^{4/3}(bc-ad)^{2/3}\log(c+dx^3)}{6d^3} + \frac{c^{4/3}(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^3} - \frac{(9b^2c^2-6abcd-a^2d^2)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18b^{4/3}d^3}$$

```
[Out] -1/9*(-a*d+3*b*c)*x*(b*x^3+a)^(2/3)/b/d^2+1/6*x^4*(b*x^3+a)^(2/3)/d-1/6*c^(
4/3)*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/d^3+1/2*c^(4/3)*(-a*d+b*c)^(2/3)*ln((-a*d
+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^3-1/18*(-a^2*d^2-6*a*b*c*d+9*b^2*c
^2)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4/3)/d^3+1/27*(-a^2*d^2-6*a*b*c*d+9*b
^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(4/3)/d^3*3^(
1/2)-1/3*c^(4/3)*(-a*d+b*c)^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3
)/(b*x^3+a)^(1/3))*3^(1/2))/d^3*3^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {489, 596, 544, 245, 384}

$$\int \frac{x^6(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(-a^2d^2-6abcd+9b^2c^2)}{9\sqrt{3}b^{4/3}d^3} - \frac{(-a^2d^2-6abcd+9b^2c^2)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{18b^{4/3}d^3} - \frac{c^{4/3}(bc-ad)^{2/3}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d^3} - \frac{c^{4/3}(bc-ad)^{2/3}\log(c+dx^3)}{6d^3} + \frac{c^{4/3}(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^3} - \frac{x(a+bx^3)^{2/3}(3bc-ad)}{9bd^2} + \frac{x^4(a+bx^3)^{2/3}}{6d}$$

```
[In] Int[(x^6*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] -1/9*((3*b*c - a*d)*x*(a + b*x^3)^(2/3))/(b*d^2) + (x^4*(a + b*x^3)^(2/3))/
(6*d) + ((9*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b
*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(4/3)*d^3) - (c^(4/3)*(b*c - a*d)^(2/3)
*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])
/(Sqrt[3]*d^3) - (c^(4/3)*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*d^3) + (c^(4
/3)*(b*c - a*d)^(2/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)
])/(2*d^3) - ((9*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x
^3)^(1/3)])/(18*b^(4/3)*d^3)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
```

```
+ Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^4(a+bx^3)^{2/3}}{6d} - \frac{\int \frac{x^3(4ac+2(3bc-ad)x^3)}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{6d} \\
&= -\frac{(3bc-ad)x(a+bx^3)^{2/3}}{9bd^2} + \frac{x^4(a+bx^3)^{2/3}}{6d} + \frac{\int \frac{2ac(3bc-ad)+2(9b^2c^2-6abcd-a^2d^2)x^3}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{18bd^2} \\
&= -\frac{(3bc-ad)x(a+bx^3)^{2/3}}{9bd^2} + \frac{x^4(a+bx^3)^{2/3}}{6d} \\
&\quad - \frac{(c^2(bc-ad))\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{d^3} + \frac{(9b^2c^2-6abcd-a^2d^2)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{9bd^3}
\end{aligned}$$

$$= -\frac{(3bc-ad)x(a+bx^3)^{2/3}}{9bd^2} + \frac{x^4(a+bx^3)^{2/3}}{6d} + \frac{(9b^2c^2-6abcd-a^2d^2)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{4/3}d^3} - \frac{c^{4/3}(bc-ad)^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^3} - \frac{c^{4/3}(bc-ad)^{2/3}\log(c+dx^3)}{6d^3} + \frac{c^{4/3}(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2d^3} - \frac{(9b^2c^2-6abcd-a^2d^2)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18b^{4/3}d^3}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.29 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.58

$$\int \frac{x^6(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{\frac{6d(a+bx^3)^{2/3}(-6bcx+2adx+3bdx^4)}{b} + \frac{4\sqrt{3}(9b^2c^2-6abcd-a^2d^2)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{b^{4/3}} + 18\sqrt{-6}$$

```
[In] Integrate[(x^6*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] ((6*d*(a + b*x^3)^(2/3)*(-6*b*c*x + 2*a*d*x + 3*b*d*x^4))/b + (4*Sqrt[3]*(9
*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(
a + b*x^3)^(1/3))])/b^(4/3) + 18*Sqrt[-6 + (6*I)*Sqrt[3]]*c^(4/3)*(b*c - a*
d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I
 + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (4*(-9*b^2*c^2 + 6*a*b*c*d + a^2*
d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(4/3) - (18*I)*(-I + Sqrt[3])
*c^(4/3)*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1
/3)*(a + b*x^3)^(1/3)] + (2*(9*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*Log[b^(2/3)*x
^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(4/3) + 9*(1 + I*S
qrt[3])*c^(4/3)*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqr
t[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/
3)*(a + b*x^3)^(2/3)])/(108*d^3)
```

Maple [A] (verified)

Time = 5.32 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$-\frac{\left(-6x\left(\left(\frac{3d\,x^3}{2}-3c\right)b^{\frac{7}{3}}+b^{\frac{4}{3}}ad\right)d\left(b\,x^3+a\right)^{\frac{2}{3}}+b\left(-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^3+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)+\ln\left(\frac{b^{\frac{2}{3}}x^2+b^{\frac{1}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}x+\left(b\,x^3\right.}{x^2}\right.\right.\right.}{}$

```
[In] int(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/54/((a*d-b*c)/c)^(1/3)/b^(7/3)*((-6*x*((3/2*d*x^3-3*c)*b^(7/3)+b^(4/3)*a
*d)*d*(b*x^3+a)^(2/3)+b*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+
a)^(1/3))/b^(1/3)/x)+ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2
/3))/x^2)-2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x))*(a^2*d^2+6*a*b*c*d-9*b^2*c^
2))*((a*d-b*c)/c)^(1/3)+18*(b^(10/3)*c-b^(7/3)*a*d)*(arctan(1/3*3^(1/2)*(((
a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln(((
(a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)-1/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((
a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*c)/d^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 555 vs. 2(276) = 552.

Time = 1.59 (sec) , antiderivative size = 1164, normalized size of antiderivative = 3.49

$$\int \frac{x^6(a+bx^3)^{2/3}}{c+dx^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/54*(18*sqrt(3)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(1/3)*b^2*c*arctan(-
1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^
(1/3)*(b*x^3 + a)^(1/3))/((b*c - a*d)*x)) - 18*(b^2*c^3 - 2*a*b*c^2*d + a^2
*c*d^2)^(1/3)*b^2*c*log(((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(2/3)*x - (b*x
^3 + a)^(1/3)*(b*c^2 - a*c*d))/x) + 9*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(
1/3)*b^2*c*log(-((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(1/3)*(b*c - a*d)*x^2
+ (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(2/3)*(b*x^3 + a)^(1/3)*x + (b*x^3 +
a)^(2/3)*(b*c^2 - a*c*d))/x^2) + 3*sqrt(1/3)*(9*b^3*c^2 - 6*a*b^2*c*d - a^2
*b*d^2)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*
sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2
/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 2*(9*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*b^(2/3
)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (9*b^2*c^2 - 6*a*b*c*d - a^2*d^
```

```
2)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/
3))/x^2) - 3*(3*b^2*d^2*x^4 - 2*(3*b^2*c*d - a*b*d^2)*x)*(b*x^3 + a)^(2/3))
/(b^2*d^3), -1/54*(18*sqrt(3)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(1/3)*b^2
*c*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b^2*c^3 - 2*a*b*c^2*d +
a^2*c*d^2)^(1/3)*(b*x^3 + a)^(1/3))/((b*c - a*d)*x)) - 18*(b^2*c^3 - 2*a*b*
c^2*d + a^2*c*d^2)^(1/3)*b^2*c*log(((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(2/
3)*x - (b*x^3 + a)^(1/3)*(b*c^2 - a*c*d))/x) + 9*(b^2*c^3 - 2*a*b*c^2*d + a
^2*c*d^2)^(1/3)*b^2*c*log(-((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(1/3)*(b*c
- a*d)*x^2 + (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(2/3)*(b*x^3 + a)^(1/3)*x
+ (b*x^3 + a)^(2/3)*(b*c^2 - a*c*d))/x^2) + 2*(9*b^2*c^2 - 6*a*b*c*d - a^2*
d^2)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (9*b^2*c^2 - 6*a*b*c
*d - a^2*d^2)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x
^3 + a)^(2/3))/x^2) + 6*sqrt(1/3)*(9*b^3*c^2 - 6*a*b^2*c*d - a^2*b*d^2)*arc
tan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 3*(3
*b^2*d^2*x^4 - 2*(3*b^2*c*d - a*b*d^2)*x)*(b*x^3 + a)^(2/3))/(b^2*d^3)]
```

Sympy [F]

$$\int \frac{x^6(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{x^6(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

```
[In] integrate(x**6*(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(x**6*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^6(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}x^6}{dx^3+c}\,dx$$

```
[In] integrate(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)*x^6/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{x^6(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}x^6}{dx^3+c}\,dx$$

```
[In] integrate(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)*x^6/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{x^6\,(b\,x^3+a)^{2/3}}{d\,x^3+c}\,dx$$

```
[In] int((x^6*(a + b*x^3)^(2/3))/(c + d*x^3),x)
[Out] int((x^6*(a + b*x^3)^(2/3))/(c + d*x^3), x)
```

3.685 $\int \frac{x^3(a+bx^3)^{2/3}}{c+dx^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 272

$$\begin{aligned}\int \frac{x^3(a+bx^3)^{2/3}}{c+dx^3} dx &= \frac{x(a+bx^3)^{2/3}}{3d} - \frac{(3bc-2ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}d^2} \\ &+ \frac{\sqrt[3]{c}(bc-ad)^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2} + \frac{\sqrt[3]{c}(bc-ad)^{2/3}\log(c+dx^3)}{6d^2} \\ &- \frac{\sqrt[3]{c}(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2d^2} \\ &+ \frac{(3bc-2ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6\sqrt[3]{b}d^2}\end{aligned}$$

```
[Out] 1/3*x*(b*x^3+a)^(2/3)/d+1/6*c^(1/3)*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/d^2-1/2*c^
(1/3)*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^2+1
/6*(-2*a*d+3*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)/d^2-1/9*(-2*a*d+3*
b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)/d^2*3^(1/2
)+1/3*c^(1/3)*(-a*d+b*c)^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(
b*x^3+a)^(1/3))*3^(1/2))/d^2*3^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {489, 544, 245, 384}

$$\int \frac{x^3(a+bx^3)^{2/3}}{c+dx^3}\,dx = -\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(3bc-2ad)}{3\sqrt{3}\sqrt[3]{b}d^2} + \frac{\sqrt[3]{c}(bc-ad)^{2/3}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d^2} + \frac{\sqrt[3]{c}(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^2} - \frac{\sqrt[3]{c}(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^2} + \frac{(3bc-2ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{6\sqrt[3]{b}d^2} + \frac{x(a+bx^3)^{2/3}}{3d}$$

```
[In] Int[(x^3*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(2/3))/(3*d) - ((3*b*c - 2*a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a
 + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(1/3)*d^2) + (c^(1/3)*(b*c - a*d)^(
2/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[
3]])/(Sqrt[3]*d^2) + (c^(1/3)*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*d^2) - (
c^(1/3)*(b*c - a*d)^(2/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(
1/3)])/(2*d^2) + ((3*b*c - 2*a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(6
*b^(1/3)*d^2)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\text{integral} = \frac{x(a+bx^3)^{2/3}}{3d} - \frac{\int \frac{ac+(3bc-2ad)x^3}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{3d}$$

$$= \frac{x(a+bx^3)^{2/3}}{3d} - \frac{(3bc-2ad)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{3d^2} + \frac{(c(bc-ad))\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{d^2}$$

$$= \frac{x(a+bx^3)^{2/3}}{3d} - \frac{(3bc-2ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}d^2} + \frac{\sqrt[3]{c}(bc-ad)^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2} + \frac{\sqrt[3]{c}(bc-ad)^{2/3}\log(c+dx^3)}{6d^2} - \frac{\sqrt[3]{c}(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2d^2} + \frac{(3bc-2ad)\log\left(-\sqrt[3]{b}x + \sqrt[3]{a+bx^3}\right)}{6\sqrt[3]{b}d^2}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.05 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.71

$$\int \frac{x^3(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{12dx(a+bx^3)^{2/3} - \frac{4\sqrt{3}(3bc-2ad)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}} - 6\sqrt{-6+6i\sqrt{3}}\sqrt[3]{c}(bc-ad)^{\cdots}}{\cdots}$$

```
[In] Integrate[(x^3*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (12*d*x*(a + b*x^3)^(2/3) - (4*Sqrt[3]*(3*b*c - 2*a*d)*ArcTan[(Sqrt[3]*b^(1
/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))]])/b^(1/3) - 6*Sqrt[-6 + (6*I)*Sqrt
[3]]*c^(1/3)*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c
 - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (4*(3*b*c -
 2*a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(1/3) + 6*(1 + I*Sqrt[3])*
c^(1/3)*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/
3)*(a + b*x^3)^(1/3)] + (2*(-3*b*c + 2*a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a
+ b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(1/3) - (3*I)*(-I + Sqrt[3])*c^(1/3)
*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(
b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^
(2/3)])/(36*d^2)
```

Maple [A] (verified)

Time = 4.82 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.44

method	result
pseudoelliptic	$-\frac{-\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(ad-\frac{3bc}{2}\right)\ln\left(\frac{b^{\frac{2}{3}}x^2+b^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{3}+\left(adb^{\frac{1}{3}}-b^{\frac{4}{3}}c\right)\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)+\frac{2\sqrt{3}\left(\frac{ad-bc}{c}\right)\cdots}{\cdots}}{\cdots}$

```
[In] int(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-1/3*((a*d-b*c)/c)^(1/3)*(a*d-3/2*b*c)*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3
+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)+(a*d*b^(1/3)-b^(4/3)*c)*ln((((a*d-b*c)/c)
^(1/3)*x+(b*x^3+a)^(1/3))/x)+2/3*3^(1/2)*((a*d-b*c)/c)^(1/3)*(a*d-3/2*b*c)*
arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+2/3*((a*d-b*c)/
c)^(1/3)*(a*d-3/2*b*c)*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-(b*x^3+a)^(2/3)*x
*((a*d-b*c)/c)^(1/3)*d*b^(1/3)+(a*d*b^(1/3)-b^(4/3)*c)*(arctan(1/3*3^(1/2)*
```

```
(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)-1/
2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+
a)^(2/3))/x^2)))/b^(1/3)/((a*d-b*c)/c)^(1/3)/d^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(219) = 438$.

Time = 0.60 (sec) , antiderivative size = 1091, normalized size of antiderivative = 4.01

$$\int \frac{x^3(a+bx^3)^{2/3}}{c+dx^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/18*(6*(b*x^3 + a)^(2/3)*b*d*x - 3*sqrt(1/3)*(3*b^2*c - 2*a*b*d)*sqrt((-b
)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*(
(-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)
*x)*sqrt((-b)^(1/3)/b) + 2*a) + 6*sqrt(3)*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d
^2)^(1/3)*b*arctan(1/3*(sqrt(3)*(b*c - a*d)*x - 2*sqrt(3)*(-b^2*c^3 + 2*a*b
*c^2*d - a^2*c*d^2)^(1/3)*(b*x^3 + a)^(1/3))/((b*c - a*d)*x)) + 2*(3*b*c -
2*a*d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (3*b*c - 2*a*
d)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3
 + a)^(2/3))/x^2) + 6*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(1/3)*b*log(((-b
^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(2/3)*x - (b*x^3 + a)^(1/3)*(b*c^2 - a*c*
d))/x) - 3*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(1/3)*b*log(((-b^2*c^3 + 2*
a*b*c^2*d - a^2*c*d^2)^(1/3)*(b*c - a*d)*x^2 - (-b^2*c^3 + 2*a*b*c^2*d - a^
2*c*d^2)^(2/3)*(b*x^3 + a)^(1/3)*x - (b*x^3 + a)^(2/3)*(b*c^2 - a*c*d))/x^2
))/(b*d^2), 1/18*(6*(b*x^3 + a)^(2/3)*b*d*x + 6*sqrt(1/3)*(3*b^2*c - 2*a*b*
d)*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3
))*sqrt(-(-b)^(1/3)/b)/x) + 6*sqrt(3)*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^
(1/3)*b*arctan(1/3*(sqrt(3)*(b*c - a*d)*x - 2*sqrt(3)*(-b^2*c^3 + 2*a*b*c^2
*d - a^2*c*d^2)^(1/3)*(b*x^3 + a)^(1/3))/((b*c - a*d)*x)) + 2*(3*b*c - 2*a*
d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (3*b*c - 2*a*d)*(
-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a
)^(2/3))/x^2) + 6*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(1/3)*b*log(((-b^2*c
^3 + 2*a*b*c^2*d - a^2*c*d^2)^(2/3)*x - (b*x^3 + a)^(1/3)*(b*c^2 - a*c*d))/
x) - 3*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(1/3)*b*log(((-b^2*c^3 + 2*a*b*
c^2*d - a^2*c*d^2)^(1/3)*(b*c - a*d)*x^2 - (-b^2*c^3 + 2*a*b*c^2*d - a^2*c*
d^2)^(2/3)*(b*x^3 + a)^(1/3)*x - (b*x^3 + a)^(2/3)*(b*c^2 - a*c*d))/x^2))/(
b*d^2)]
```

Sympy [F]

$$\int \frac{x^3(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{x^3(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

```
[In] integrate(x**3*(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(x**3*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^3(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}x^3}{dx^3+c}\,dx$$

```
[In] integrate(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)*x^3/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{x^3(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}x^3}{dx^3+c}\,dx$$

```
[In] integrate(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)*x^3/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{x^3\,(b\,x^3+a)^{2/3}}{d\,x^3+c}\,dx$$

```
[In] int((x^3*(a + b*x^3)^(2/3))/(c + d*x^3),x)
[Out] int((x^3*(a + b*x^3)^(2/3))/(c + d*x^3), x)
```

3.686 $\int \frac{(a+bx^3)^{2/3}}{c+dx^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 233

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3} dx = \frac{b^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d} - \frac{(bc-ad)^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}d} - \frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^{2/3}d} + \frac{(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}d} - \frac{b^{2/3}\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2d}$$

```
[Out] -1/6*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/c^(2/3)/d+1/2*(-a*d+b*c)^(2/3)*ln((-a*d+b
*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/d-1/2*b^(2/3)*ln(-b^(1/3)*x+(b
*x^3+a)^(1/3))/d+1/3*b^(2/3)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(
1/2))/d*3^(1/2)-1/3*(-a*d+b*c)^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(
1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(2/3)/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {399, 245, 384}

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{b^{2/3}\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d} - \frac{(bc-ad)^{2/3}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}d} - \frac{b^{2/3}\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2d} - \frac{(bc-ad)^{2/3}\log(c+dx^3)}{6c^{2/3}d} + \frac{(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}d}$$

```
[In] Int[(a + b*x^3)^(2/3)/(c + d*x^3),x]
```

```
[Out] (b^(2/3)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d)
 - ((b*c - a*d)^(2/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x
^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(2/3)*d) - ((b*c - a*d)^(2/3)*Log[c + d*x^
3])/(6*c^(2/3)*d) + ((b*c - a*d)^(2/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) -
(a + b*x^3)^(1/3)])/(2*c^(2/3)*d) - (b^(2/3)*Log[-(b^(1/3)*x) + (a + b*x^3)
^(1/3)])/(2*d)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
```

```
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{b\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{d} - \frac{(bc-ad)\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{d}$$

$$= \frac{b^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d} - \frac{(bc-ad)^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}d}$$
$$- \frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^{2/3}d} + \frac{(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}d}$$
$$- \frac{b^{2/3}\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2d}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.82

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{4\sqrt{3}b^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) + \frac{2\sqrt{-6+6i\sqrt{3}}(bc-ad)^{2/3}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\right.}{c^{2/3}}$$

```
[In] Integrate[(a + b*x^3)^(2/3)/(c + d*x^3),x]
```

```
[Out] (4*Sqrt[3]*b^(2/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1
/3))] + (2*Sqrt[-6 + (6*I)*Sqrt[3]]*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d)
^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3
)^(1/3))])/c^(2/3) - 4*b^(2/3)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] - ((2*
I)*(-I + Sqrt[3])*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt
[3])*c^(1/3)*(a + b*x^3)^(1/3)])/c^(2/3) + 2*b^(2/3)*Log[b^(2/3)*x^2 + b^(1
/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + ((1 + I*Sqrt[3])*(b*c - a*d)
^(2/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(
1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/c^(2
/3))/(12*d)
```

Maple [A] (verified)

Time = 4.78 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.45

method	result
pseudoelliptic	$\frac{\frac{b^{\frac{2}{3}} \ln\left(\frac{b^{\frac{2}{3}} x^{2}+b^{\frac{1}{3}} \left(b\,x^{3}+a\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right) c \left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}{2}+\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right) (ad-bc)-\sqrt{3}\, b^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} x+2\left(b\right.\right.}{3 b^{\frac{1}{3}} x}\right.}{}$

[In] int((b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] 1/3/((a*d-b*c)/c)^(1/3)*(1/2*b^(2/3)*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3
)*x+(b*x^3+a)^(2/3))/x^2)*c*((a*d-b*c)/c)^(1/3)+ln((((a*d-b*c)/c)^(1/3)*x+(
b*x^3+a)^(1/3))/x)*(a*d-b*c)-3^(1/2)*b^(2/3)*arctan(1/3*3^(1/2)*(b^(1/3)*x+
2*(b*x^3+a)^(1/3))/b^(1/3)/x)*c*((a*d-b*c)/c)^(1/3)-b^(2/3)*ln((-b^(1/3)*x+
(b*x^3+a)^(1/3))/x)*c*((a*d-b*c)/c)^(1/3)+(arctan(1/3*3^(1/2)*(((a*d-b*c)/c
)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)-1/2*ln((((a*d-b
*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2
))*(a*d-b*c))/c/d

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. 2(186) = 372.

Time = 0.50 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.01

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx =$$

$$-\frac{2\sqrt{3}\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}(bx^3+a)^{\frac{1}{3}}c\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}}{3\,(bc-ad)x}\right)+2\sqrt{3}(-b^2)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}bx-\ldots}{}\right.}{}$$

[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")

[Out] -1/6*(2*sqrt(3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sq
rt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*x)) + 2*sqrt(3)*(-b^2)^(1/3)*arctan(-1/3
*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 2*((b^2*
c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^2*
d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 2*(-b^2)^(1/3)*log(-(
(-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (-b^2)^(1/3)*log(-((-b^2)^(1/3)*
b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + ((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2

```
*a*b*c*d + a^2*d^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*
d + a^2*d^2)/c^2)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2))/d
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

```
[In] integrate((b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{dx^3+c}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{dx^3+c}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{d\,x^3+c}\,dx$$

```
[In] int((a + b*x^3)^(2/3)/(c + d*x^3),x)
[Out] int((a + b*x^3)^(2/3)/(c + d*x^3), x)
```

3.687 $\int \frac{(a+bx^3)^{2/3}}{x^3(c+dx^3)} dx$

Optimal result

Integrand size = 24, antiderivative size = 169

$$\int \frac{(a+bx^3)^{2/3}}{x^3(c+dx^3)} dx = -\frac{(a+bx^3)^{2/3}}{2cx^2} + \frac{(bc-ad)^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{5/3}} + \frac{(bc-ad)^{2/3}\log(c+dx^3)}{6c^{5/3}} - \frac{(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{5/3}}$$

```
[Out] -1/2*(b*x^3+a)^(2/3)/c/x^2+1/6*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/c^(5/3)-1/2*(-a
*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(5/3)+1/3*(-
a*d+b*c)^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*
3^(1/2))/c^(5/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {486, 12, 384}

$$\int \frac{(a+bx^3)^{2/3}}{x^3(c+dx^3)}\,dx = \frac{(bc-ad)^{2/3}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{5/3}} + \frac{(bc-ad)^{2/3}\log(c+dx^3)}{6c^{5/3}} - \frac{(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{5/3}} - \frac{(a+bx^3)^{2/3}}{2cx^2}$$

```
[In] Int[(a + b*x^3)^(2/3)/(x^3*(c + d*x^3)),x]
```

```
[Out] -1/2*(a + b*x^3)^(2/3)/(c*x^2) + ((b*c - a*d)^(2/3)*ArcTan[(1 + (2*(b*c - a
*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(5/3)) + ((b
*c - a*d)^(2/3)*Log[c + d*x^3])/(6*c^(5/3)) - ((b*c - a*d)^(2/3)*Log[((b*c
- a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(5/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\text{integral} = -\frac{(a+bx^3)^{2/3}}{2cx^2} + \frac{\int \frac{2bc-2ad}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{2c}$$

$$= -\frac{(a+bx^3)^{2/3}}{2cx^2} + \frac{(bc-ad)\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{c}$$

$$= -\frac{(a+bx^3)^{2/3}}{2cx^2} + \frac{(bc-ad)^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{5/3}}$$

$$+ \frac{(bc-ad)^{2/3}\log(c+dx^3)}{6c^{5/3}} - \frac{(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{5/3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.59 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.83

$$\int \frac{(a+bx^3)^{2/3}}{x^3(c+dx^3)}\,dx = \frac{-\frac{6c^{2/3}(a+bx^3)^{2/3}}{x^2} - 2\sqrt{-6+6i\sqrt{3}}(bc-ad)^{2/3}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{}$$

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^3*(c + d*x^3)),x]
```

```
[Out] ((-6*c^(2/3)*(a + b*x^3)^(2/3))/x^2 - 2*Sqrt[-6 + (6*I)*Sqrt[3]]*(b*c - a*d
)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I
+ Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + 2*(1 + I*Sqrt[3])*(b*c - a*d)^(2/3
)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)] -
I*(-I + Sqrt[3])*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sq
rt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2
/3)*(a + b*x^3)^(2/3)])/(12*c^(5/3))
```

Maple [A] (verified)

Time = 5.08 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.31

method	result
pseudoelliptic	$\frac{-2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)(ad-bc)x^{2}-3\left(b\,x^{3}+a\right)^{\frac{2}{3}}c\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}+x^{2}\left(-2\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)\right.}{6\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}c^{2}x^{2}}$

```
[In] int((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/((a*d-b*c)/c)^(1/3)*(-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)*(
a*d-b*c)*x^2-3*(b*x^3+a)^(2/3)*c*((a*d-b*c)/c)^(1/3)+x^2*(-2*arctan(1/3*3^(
1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/
2)+ln(((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3
+a)^(2/3))/x^2))*(a*d-b*c))/c^2/x^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^3\left(c+dx^3\right)}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^3\left(c+dx^3\right)}\,dx = \int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{x^3\left(c+dx^3\right)}\,dx$$

```
[In] integrate((b*x**3+a)**(2/3)/x**3/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(2/3)/(x**3*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^3(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^3}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^3), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^3(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^3}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^3(c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{x^3\,(d\,x^3+c)}\,dx$$

```
[In] int((a + b*x^3)^(2/3)/(x^3*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(2/3)/(x^3*(c + d*x^3)), x)
```

3.688 $\int \frac{(a+bx^3)^{2/3}}{x^6(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 206

$$\int \frac{(a+bx^3)^{2/3}}{x^6(c+dx^3)}\,dx = -\frac{(a+bx^3)^{2/3}}{5cx^5}-\frac{(2bc-5ad)(a+bx^3)^{2/3}}{10ac^2x^2} - \frac{d(bc-ad)^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{8/3}} - \frac{d(bc-ad)^{2/3}\log(c+dx^3)}{6c^{8/3}} + \frac{d(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{8/3}}$$

```
[Out] -1/5*(b*x^3+a)^(2/3)/c/x^5-1/10*(-5*a*d+2*b*c)*(b*x^3+a)^(2/3)/a/c^2/x^2-1/
6*d*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/c^(8/3)+1/2*d*(-a*d+b*c)^(2/3)*ln((-a*d+b*
c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)-1/3*d*(-a*d+b*c)^(2/3)*arctan(1
/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)*3^(1/2
)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {486, 597, 12, 384}

$$\int \frac{(a+bx^3)^{2/3}}{x^6(c+dx^3)}\,dx = -\frac{d(bc-ad)^{2/3}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{8/3}} - \frac{d(bc-ad)^{2/3}\log(c+dx^3)}{6c^{8/3}} + \frac{d(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{8/3}} - \frac{(a+bx^3)^{2/3}(2bc-5ad)}{10ac^2x^2} - \frac{(a+bx^3)^{2/3}}{5cx^5}$$

```
[In] Int[(a + b*x^3)^(2/3)/(x^6*(c + d*x^3)),x]
```

```
[Out] -1/5*(a + b*x^3)^(2/3)/(c*x^5) - ((2*b*c - 5*a*d)*(a + b*x^3)^(2/3))/(10*a*
c^2*x^2) - (d*(b*c - a*d)^(2/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3
)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(8/3)) - (d*(b*c - a*d)^(2/3)*Lo
g[c + d*x^3])/(6*c^(8/3)) + (d*(b*c - a*d)^(2/3)*Log[((b*c - a*d)^(1/3)*x)/
c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(8/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
```

```
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{(a+bx^3)^{2/3}}{5cx^5} + \frac{\int \frac{2bc-5ad-3bdx^3}{x^3\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{5c}$$

$$= -\frac{(a+bx^3)^{2/3}}{5cx^5} - \frac{(2bc-5ad)(a+bx^3)^{2/3}}{10ac^2x^2} - \frac{\int \frac{10ad(bc-ad)}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{10ac^2}$$

$$= -\frac{(a+bx^3)^{2/3}}{5cx^5} - \frac{(2bc-5ad)(a+bx^3)^{2/3}}{10ac^2x^2} - \frac{(d(bc-ad))\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{c^2}$$

$$= -\frac{(a+bx^3)^{2/3}}{5cx^5} - \frac{(2bc-5ad)(a+bx^3)^{2/3}}{10ac^2x^2} - \frac{d(bc-ad)^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{8/3}}$$

$$- \frac{d(bc-ad)^{2/3}\log(c+dx^3)}{6c^{8/3}} + \frac{d(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{8/3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.86 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.62

$$\int \frac{(a+bx^3)^{2/3}}{x^6(c+dx^3)}\,dx = \frac{\frac{6c^{2/3}(a+bx^3)^{2/3}(-2ac-2bcx^3+5adx^3)}{ax^5} + 10\sqrt{-6+6i\sqrt{3}}d(bc-ad)^{2/3}\arctan\left(\frac{3\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{bc-ad}x-(}\right.}{}$$

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^6*(c + d*x^3)),x]
```

[Out] ((6*c^(2/3)*(a + b*x^3)^(2/3)*(-2*a*c - 2*b*c*x^3 + 5*a*d*x^3))/(a*x^5) + 1
0*Sqrt[-6 + (6*I)*Sqrt[3]]*d*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*
x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)
)] - (10*I)*(-I + Sqrt[3])*d*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(1/3)*x +
(1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)] + 5*(1 + I*Sqrt[3])*d*(b*c - a*d
)^(2/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^
(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(60
*c^(8/3))

Maple [A] (verified)

Time = 4.85 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.19

method	result
pseudoelliptic	$-\frac{-2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)a(ad-bc)d\,x^{5}+\frac{6\left(\left(-\frac{5ad}{2}+bc\right)x^{3}+ac\right)c\left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}{5}+x^{5}da\left(-2\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-}{c}\right.\right.}{3}\right.\right.}{6\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x^{5}c^{3}a}$

[In] int((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] -1/6/((a*d-b*c)/c)^(1/3)*(-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)*
a*(a*d-b*c)*d*x^5+6/5*((-5/2*a*d+b*c)*x^3+a*c)*c*(b*x^3+a)^(2/3)*((a*d-b*c)
/c)^(1/3)+x^5*d*a*(-2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)
^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln(((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*
c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*(a*d-b*c))/x^5/c^3/a

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^6(c+dx^3)}\,dx=\text{Timed out}$$

[In] integrate((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^6\,(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{2}{3}}}{x^6\,(c+dx^3)}\,dx$$

```
[In] integrate((b*x**3+a)**(2/3)/x**6/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(2/3)/(x**6*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^6\,(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^6}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^6), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^6\,(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^6}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^6\,(c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{x^6\,(d\,x^3+c)}\,dx$$

```
[In] int((a + b*x^3)^(2/3)/(x^6*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(2/3)/(x^6*(c + d*x^3)), x)
```

3.689 $\int \frac{(a+bx^3)^{2/3}}{x^9(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 257

$$\int \frac{(a+bx^3)^{2/3}}{x^9(c+dx^3)}\,dx = -\frac{(a+bx^3)^{2/3}}{8cx^8} - \frac{(bc-4ad)(a+bx^3)^{2/3}}{20ac^2x^5} + \frac{(3b^2c^2+8abcd-20a^2d^2)(a+bx^3)^{2/3}}{40a^2c^3x^2} + \frac{d^2(bc-ad)^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{11/3}} + \frac{d^2(bc-ad)^{2/3}\log(c+dx^3)}{6c^{11/3}} - \frac{d^2(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{11/3}}$$

```
[Out] -1/8*(b*x^3+a)^(2/3)/c/x^8-1/20*(-4*a*d+b*c)*(b*x^3+a)^(2/3)/a/c^2/x^5+1/40
*(-20*a^2*d^2+8*a*b*c*d+3*b^2*c^2)*(b*x^3+a)^(2/3)/a^2/c^3/x^2+1/6*d^2*(-a*
d+b*c)^(2/3)*ln(d*x^3+c)/c^(11/3)-1/2*d^2*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1
/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(11/3)+1/3*d^2*(-a*d+b*c)^(2/3)*arctan(1/3
*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(11/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {486, 597, 12, 384}

$$\int \frac{(a+bx^3)^{2/3}}{x^9(c+dx^3)}\,dx = \frac{(a+bx^3)^{2/3}(-20a^2d^2+8abcd+3b^2c^2)}{40a^2c^3x^2} + \frac{d^2(bc-ad)^{2/3}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{11/3}} + \frac{d^2(bc-ad)^{2/3}\log(c+dx^3)}{6c^{11/3}} - \frac{d^2(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{11/3}} - \frac{(a+bx^3)^{2/3}(bc-4ad)}{20ac^2x^5} - \frac{(a+bx^3)^{2/3}}{8cx^8}$$

```
[In] Int[(a + b*x^3)^(2/3)/(x^9*(c + d*x^3)),x]
```

```
[Out] -1/8*(a + b*x^3)^(2/3)/(c*x^8) - ((b*c - 4*a*d)*(a + b*x^3)^(2/3))/(20*a*c^
2*x^5) + ((3*b^2*c^2 + 8*a*b*c*d - 20*a^2*d^2)*(a + b*x^3)^(2/3))/(40*a^2*c
^3*x^2) + (d^2*(b*c - a*d)^(2/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/
3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(11/3)) + (d^2*(b*c - a*d)^(2/3
)*Log[c + d*x^3])/(6*c^(11/3)) - (d^2*(b*c - a*d)^(2/3)*Log[((b*c - a*d)^(1
/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(11/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
```

```
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{(a+bx^3)^{2/3}}{8cx^8} + \frac{\int \frac{2(bc-4ad)-6bdx^3}{x^6\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{8c}$$

$$= -\frac{(a+bx^3)^{2/3}}{8cx^8} - \frac{(bc-4ad)(a+bx^3)^{2/3}}{20ac^2x^5} - \frac{\int \frac{2(3b^2c^2+8abcd-20a^2d^2)+6bd(bc-4ad)x^3}{x^3\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{40ac^2}$$

$$= -\frac{(a+bx^3)^{2/3}}{8cx^8} - \frac{(bc-4ad)(a+bx^3)^{2/3}}{20ac^2x^5} + \frac{(3b^2c^2+8abcd-20a^2d^2)(a+bx^3)^{2/3}}{40a^2c^3x^2} + \frac{\int \frac{80a^2d^2(bc-ad)}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{80a^2c^3}$$

$$= -\frac{(a+bx^3)^{2/3}}{8cx^8} - \frac{(bc-4ad)(a+bx^3)^{2/3}}{20ac^2x^5} + \frac{(3b^2c^2+8abcd-20a^2d^2)(a+bx^3)^{2/3}}{40a^2c^3x^2} + \frac{(d^2(bc-ad))\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{c^3}$$

$$= -\frac{(a+bx^3)^{2/3}}{8cx^8} - \frac{(bc-4ad)(a+bx^3)^{2/3}}{20ac^2x^5} + \frac{(3b^2c^2+8abcd-20a^2d^2)(a+bx^3)^{2/3}}{40a^2c^3x^2} + \frac{d^2(bc-ad)^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{11/3}} + \frac{d^2(bc-ad)^{2/3}\log(c+dx^3)}{6c^{11/3}} - \frac{d^2(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{11/3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.25 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.46

$$\int \frac{(a+bx^3)^{2/3}}{x^9(c+dx^3)}\,dx = \frac{-\frac{3c^{2/3}(a+bx^3)^{2/3}(-3b^2c^2x^6+2abcx^3(c-4dx^3)+a^2(5c^2-8cdx^3+20d^2x^6))}{a^2x^8} - 20\sqrt{-6+6i\sqrt{3}}d^2(bc-ad)}{}$$

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^9*(c + d*x^3)),x]
```

```
[Out] ((-3*c^(2/3)*(a + b*x^3)^(2/3)*(-3*b^2*c^2*x^6 + 2*a*b*c*x^3*(c - 4*d*x^3)
+ a^2*(5*c^2 - 8*c*d*x^3 + 20*d^2*x^6)))/(a^2*x^8) - 20*Sqrt[-6 + (6*I)*Sqr
t[3]]*d^2*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c -
a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + 20*(1 + I*Sqrt
[3])*d^2*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1
/3)*(a + b*x^3)^(1/3)] - (10*I)*(-I + Sqrt[3])*d^2*(b*c - a*d)^(2/3)*Log[2*
(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b
*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(120*c^(11/3))
```

Maple [A] (verified)

Time = 4.86 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$\frac{-3c\left(\left(4a^2d^2-\frac{8}{5}abcd-\frac{3}{5}b^2c^2\right)x^6+\frac{2\left(-4a^2cd+b\,c^2a\right)x^3}{5}+a^2c^2\right)\left(b\,x^3+a\right)^{\frac{2}{3}}\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}+4a^2d^2x^8(ad-bc)\left(-2\arctan\left(\frac{\sqrt{3}\left(\left(\frac{a}{}\right.\right.}{}\right.\right.}{24\left(\frac{ad-bc}{c}\right)}$

```
[In] int((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/((a*d-b*c)/c)^(1/3)*(-3*c*((4*a^2*d^2-8/5*a*b*c*d-3/5*b^2*c^2)*x^6+2/5
*(-4*a^2*c*d+a*b*c^2)*x^3+a^2*c^2)*(b*x^3+a)^(2/3)*((a*d-b*c)/c)^(1/3)+4*a^
2*d^2*x^8*(a*d-b*c)*(-2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+
a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-
b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((((a*d-b*c)/c)^(
1/3)*x+(b*x^3+a)^(1/3))/x)))/x^8/c^4/a^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^9\,(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^9\,(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{2}{3}}}{x^9\,(c+dx^3)}\,dx$$

```
[In] integrate((b*x**3+a)**(2/3)/x**9/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(2/3)/(x**9*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^9\,(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^9}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^9), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^9\,(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^9}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^9), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^9(c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{x^9\,(d\,x^3+c)}\,dx$$

```
[In] int((a + b*x^3)^(2/3)/(x^9*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(2/3)/(x^9*(c + d*x^3)), x)
```

3.690 $\int \frac{(a+bx^3)^{2/3}}{x^{12}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 320

$$\begin{aligned}\int \frac{(a+bx^3)^{2/3}}{x^{12}(c+dx^3)}\,dx = &-\frac{(a+bx^3)^{2/3}}{11cx^{11}}-\frac{(2bc-11ad)(a+bx^3)^{2/3}}{88ac^2x^8}\\&+\frac{(6b^2c^2+11abcd-44a^2d^2)(a+bx^3)^{2/3}}{220a^2c^3x^5}\\&-\frac{(18b^3c^3+33ab^2c^2d+88a^2bcd^2-220a^3d^3)(a+bx^3)^{2/3}}{440a^3c^4x^2}\\&-\frac{d^3(bc-ad)^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{14/3}}-\frac{d^3(bc-ad)^{2/3}\log(c+dx^3)}{6c^{14/3}}\\&+\frac{d^3(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{14/3}}\end{aligned}$$

```
[Out] -1/11*(b*x^3+a)^(2/3)/c/x^11-1/88*(-11*a*d+2*b*c)*(b*x^3+a)^(2/3)/a/c^2/x^8
+1/220*(-44*a^2*d^2+11*a*b*c*d+6*b^2*c^2)*(b*x^3+a)^(2/3)/a^2/c^3/x^5-1/440
*(-220*a^3*d^3+88*a^2*b*c*d^2+33*a*b^2*c^2*d+18*b^3*c^3)*(b*x^3+a)^(2/3)/a^
3/c^4/x^2-1/6*d^3*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/c^(14/3)+1/2*d^3*(-a*d+b*c)^
(2/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(14/3)-1/3*d^3*(-a*d
+b*c)^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(
1/2))/c^(14/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {486, 597, 12, 384}

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(c+dx^3)}\,dx = \frac{(a+bx^3)^{2/3}(-44a^2d^2+11abcd+6b^2c^2)}{220a^2c^3x^5} - \frac{(a+bx^3)^{2/3}(-220a^3d^3+88a^2bcd^2+33ab^2c^2d+18b^3c^3)}{440a^3c^4x^2} - \frac{d^3(bc-ad)^{2/3}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{14/3}} - \frac{d^3(bc-ad)^{2/3}\log(c+dx^3)}{6c^{14/3}} + \frac{d^3(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{14/3}} - \frac{(a+bx^3)^{2/3}(2bc-11ad)}{88ac^2x^8} - \frac{(a+bx^3)^{2/3}}{11cx^{11}}$$

```
[In] Int[(a + b*x^3)^(2/3)/(x^12*(c + d*x^3)),x]
```

```
[Out] -1/11*(a + b*x^3)^(2/3)/(c*x^11) - ((2*b*c - 11*a*d)*(a + b*x^3)^(2/3))/(88
*a*c^2*x^8) + ((6*b^2*c^2 + 11*a*b*c*d - 44*a^2*d^2)*(a + b*x^3)^(2/3))/(22
0*a^2*c^3*x^5) - ((18*b^3*c^3 + 33*a*b^2*c^2*d + 88*a^2*b*c*d^2 - 220*a^3*d
^3)*(a + b*x^3)^(2/3))/(440*a^3*c^4*x^2) - (d^3*(b*c - a*d)^(2/3)*ArcTan[(1
 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*
c^(14/3)) - (d^3*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*c^(14/3)) + (d^3*(b*c
 - a*d)^(2/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^
(14/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+bx^3)^{2/3}}{11cx^{11}}+\frac{\int \frac{2bc-11ad-9bdx^3}{x^9\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{11c}\\
&= -\frac{(a+bx^3)^{2/3}}{11cx^{11}}-\frac{(2bc-11ad)\,(a+bx^3)^{2/3}}{88ac^2x^8}-\frac{\int \frac{2(6b^2c^2+11abcd-44a^2d^2)+6bd(2bc-11ad)x^3}{x^6\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{88ac^2}\\
&= -\frac{(a+bx^3)^{2/3}}{11cx^{11}}-\frac{(2bc-11ad)\,(a+bx^3)^{2/3}}{88ac^2x^8}+\frac{(6b^2c^2+11abcd-44a^2d^2)\,(a+bx^3)^{2/3}}{220a^2c^3x^5}\\
&\quad+\frac{\int \frac{2(18b^3c^3+33ab^2c^2d+88a^2bcd^2-220a^3d^3)+6bd(6b^2c^2+11abcd-44a^2d^2)x^3}{x^3\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{440a^2c^3}\\
&= -\frac{(a+bx^3)^{2/3}}{11cx^{11}}-\frac{(2bc-11ad)\,(a+bx^3)^{2/3}}{88ac^2x^8}+\frac{(6b^2c^2+11abcd-44a^2d^2)\,(a+bx^3)^{2/3}}{220a^2c^3x^5}\\
&\quad-\frac{(18b^3c^3+33ab^2c^2d+88a^2bcd^2-220a^3d^3)\,(a+bx^3)^{2/3}}{440a^3c^4x^2}-\frac{\int \frac{880a^3d^3(bc-ad)}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{880a^3c^4}\\
&= -\frac{(a+bx^3)^{2/3}}{11cx^{11}}-\frac{(2bc-11ad)\,(a+bx^3)^{2/3}}{88ac^2x^8}+\frac{(6b^2c^2+11abcd-44a^2d^2)\,(a+bx^3)^{2/3}}{220a^2c^3x^5}\\
&\quad-\frac{(18b^3c^3+33ab^2c^2d+88a^2bcd^2-220a^3d^3)\,(a+bx^3)^{2/3}}{440a^3c^4x^2}\\
&\quad-\frac{(d^3(bc-ad))\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{c^4}
\end{aligned}$$

$$= -\frac{(a+bx^3)^{2/3}}{11cx^{11}} - \frac{(2bc-11ad)(a+bx^3)^{2/3}}{88ac^2x^8} + \frac{(6b^2c^2+11abcd-44a^2d^2)(a+bx^3)^{2/3}}{220a^2c^3x^5} - \frac{(18b^3c^3+33ab^2c^2d+88a^2bcd^2-220a^3d^3)(a+bx^3)^{2/3}}{440a^3c^4x^2} - \frac{d^3(bc-ad)^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{14/3}} - \frac{d^3(bc-ad)^{2/3}\log(c+dx^3)}{6c^{14/3}} + \frac{d^3(bc-ad)^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{14/3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.92 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(c+dx^3)}\,dx = \frac{\frac{3c^{2/3}(a+bx^3)^{2/3}(-18b^3c^3x^9+3ab^2c^2x^6(4c-11dx^3)-2a^2bcx^3(5c^2-11cdx^3+44d^2x^6)+a^3(-40c^3+55c^2dx^3-88cd^2x^6}{a^3x^{11}}}{}$$

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^12*(c + d*x^3)),x]
```

```
[Out] ((3*c^(2/3)*(a + b*x^3)^(2/3)*(-18*b^3*c^3*x^9 + 3*a*b^2*c^2*x^6*(4*c - 11*
d*x^3) - 2*a^2*b*c*x^3*(5*c^2 - 11*c*d*x^3 + 44*d^2*x^6) + a^3*(-40*c^3 + 5
5*c^2*d*x^3 - 88*c*d^2*x^6 + 220*d^3*x^9)))/(a^3*x^11) + 220*Sqrt[-6 + (6*I
)*Sqrt[3]]*d^3*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b
*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] - (220*I)*(
-I + Sqrt[3])*d^3*(b*c - a*d)^(2/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt
[3])*c^(1/3)*(a + b*x^3)^(1/3)] + 110*(1 + I*Sqrt[3])*d^3*(b*c - a*d)^(2/3)
*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x
*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(1320*c^(1
4/3))
```

Maple [A] (verified)

Time = 4.92 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$-\frac{6\left(\left(b x^{3}+a\right)\left(\frac{9}{20} b^{2} x^{6}-\frac{3}{4} a b x^{3}+a^{2}\right) c^{3}-\frac{11 x^{3}\left(-\frac{3 b x^{3}}{5}+a\right) d\left(b x^{3}+a\right) a c^{2}}{8}+\frac{11\left(b x^{3}+a\right) a^{2} c d^{2} x^{6}}{5}-\frac{11 a^{3} d^{3} x^{9}}{2}\right) c\left(b x^{3}+a\right)^{\frac{2}{3}}\left(\frac{ad-b\ldots}{c}\right.}{}$

[In] int((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] -1/66/((a*d-b*c)/c)^(1/3)*(6*((b*x^3+a)*(9/20*b^2*x^6-3/4*a*b*x^3+a^2)*c^3-11/8*x^3*(-3/5*b*x^3+a)*d*(b*x^3+a)*a*c^2+11/5*(b*x^3+a)*a^2*c*d^2*x^6-11/2*a^3*d^3*x^9)*c*(b*x^3+a)^(2/3)*((a*d-b*c)/c)^(1/3)+11*a^3*d^3*x^11*(a*d-b*c)*(-2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)))/x^11/c^5/a^3

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(c+dx^3)}\,dx = \text{Timed out}$$

[In] integrate((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{2}{3}}}{x^{12}(c+dx^3)}\,dx$$

[In] integrate((b*x**3+a)**(2/3)/x**12/(d*x**3+c),x)

[Out] Integral((a + b*x**3)**(2/3)/(x**12*(c + d*x**3)), x)

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^{12}}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^12), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^{12}}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^12), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{x^{12}\,(d\,x^3+c)}\,dx$$

```
[In] int((a + b*x^3)^(2/3)/(x^12*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(2/3)/(x^12*(c + d*x^3)), x)
```

3.691 $\int \frac{x^7(a+bx^3)^{2/3}}{c+dx^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^7(a+bx^3)^{2/3}}{c+dx^3} dx = \frac{x^8(a+bx^3)^{2/3} \operatorname{AppellF1}\left(\frac{8}{3},-\frac{2}{3},1,\frac{11}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

```
[Out] 1/8*x^8*(b*x^3+a)^(2/3)*AppellF1(8/3,-2/3,1,11/3,-b*x^3/a,-d*x^3/c)/c/(1+b*
x^3/a)^(2/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{x^7(a+bx^3)^{2/3}}{c+dx^3} dx = \frac{x^8(a+bx^3)^{2/3} \operatorname{AppellF1}\left(\frac{8}{3},-\frac{2}{3},1,\frac{11}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

```
[In] Int[(x^7*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (x^8*(a + b*x^3)^(2/3)*AppellF1[8/3, -2/3, 1, 11/3, -((b*x^3)/a), -((d*x^3)
/c)])/(8*c*(1 + (b*x^3)/a)^(2/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{(a+bx^3)^{2/3} \int \frac{x^7\left(1+\frac{bx^3}{a}\right)^{2/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= \frac{x^8(a+bx^3)^{2/3}\,F_1\left(\frac{8}{3};-\frac{2}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 181 vs. 2(64) = 128.

Time = 8.42 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.83

$$\int \frac{x^7(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{x^2\left(5c(a+bx^3)\left(-7bc+2ad+4bdx^3\right)+5ac(7bc-2ad)\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1\right.\right.}{140}$$

```
[In] Integrate[(x^7*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (x^2*(5*c*(a + b*x^3)*(-7*b*c + 2*a*d + 4*b*d*x^3) + 5*a*c*(7*b*c - 2*a*d)*
(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/c)
] - 2*(-14*b^2*c^2 + 7*a*b*c*d + 2*a^2*d^2)*x^3*(1 + (b*x^3)/a)^(1/3)*Appel
lF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)]))/(140*b*c*d^2*(a + b*x^3
)^(1/3))
```

Maple [F]

$$\int \frac{x^{7} \left(b\,x^{3}+a \right)^{\frac{2}{3}}}{d\,x^{3}+c}d x$$

[In] int(x^7*(b*x^3+a)^(2/3)/(d*x^3+c),x)

[Out] int(x^7*(b*x^3+a)^(2/3)/(d*x^3+c),x)

Fricas [F(-1)]

Timed out.

$$\int \frac{x^7(a+bx^3)^{2/3}}{c+dx^3}\,dx = \text{Timed out}$$

[In] integrate(x^7*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{x^7(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{x^{7} \left(a + b x^{3}\right)^{\frac{2}{3}}}{c + d x^{3}}\, dx$$

[In] integrate(x**7*(b*x**3+a)**(2/3)/(d*x**3+c),x)

[Out] Integral(x**7*(a + b*x**3)**(2/3)/(c + d*x**3), x)

Maxima [F]

$$\int \frac{x^7(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int { \frac {{\left (b x^{3} + a\right)}^{\frac {2}{3}} x^{7}}{d x^{3} + c} \,d x }$$

[In] integrate(x^7*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(2/3)*x^7/(d*x^3 + c), x)

Giac [F]

$$\int \frac{x^7(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}x^7}{dx^3+c}\,dx$$

```
[In] integrate(x^7*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)*x^7/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{x^7\,(b\,x^3+a)^{2/3}}{d\,x^3+c}\,dx$$

```
[In] int((x^7*(a + b*x^3)^(2/3))/(c + d*x^3),x)
[Out] int((x^7*(a + b*x^3)^(2/3))/(c + d*x^3), x)
```

3.692 $\int \frac{x^4(a+bx^3)^{2/3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^4(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{x^5(a+bx^3)^{2/3}\operatorname{AppellF1}\left(\frac{5}{3},-\frac{2}{3},1,\frac{8}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

```
[Out] 1/5*x^5*(b*x^3+a)^(2/3)*AppellF1(5/3,-2/3,1,8/3,-b*x^3/a,-d*x^3/c)/c/(1+b*x
^3/a)^(2/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{x^4(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{x^5(a+bx^3)^{2/3}\operatorname{AppellF1}\left(\frac{5}{3},-\frac{2}{3},1,\frac{8}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

```
[In] Int[(x^4*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (x^5*(a + b*x^3)^(2/3)*AppellF1[5/3, -2/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/
c)])/(5*c*(1 + (b*x^3)/a)^(2/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{(a+bx^3)^{2/3}\int \frac{x^4\left(1+\frac{bx^3}{a}\right)^{2/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= \frac{x^5(a+bx^3)^{2/3}\,F_1\left(\frac{5}{3};-\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(64) = 128.

Time = 8.03 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.20

$$\int \frac{x^4(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{5cx^2(a+bx^3)-5acx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2(-2bc+ad)x}{20cd\sqrt[3]{a+bx^3}}$$

```
[In] Integrate[(x^4*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (5*c*x^2*(a + b*x^3) - 5*a*c*x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1
, 5/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*(-2*b*c + a*d)*x^5*(1 + (b*x^3)/a)^(
1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(20*c*d*(a + b
*x^3)^(1/3))
```

Maple [F]

$$\int \frac{x^4(b\,x^3+a)^{\frac{2}{3}}}{d\,x^3+c}dx$$

```
[In] int(x^4*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^4*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^3)^{2/3}}{c+dx^3}\,dx = \text{Timed out}$$

```
[In] integrate(x^4*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^4(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{x^4(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

```
[In] integrate(x**4*(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(x**4*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^4(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}x^4}{dx^3+c}\,dx$$

```
[In] integrate(x^4*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)*x^4/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{x^4(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}x^4}{dx^3+c}\,dx$$

```
[In] integrate(x^4*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)*x^4/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{x^4\,(b\,x^3+a)^{2/3}}{d\,x^3+c}\,dx$$

```
[In] int((x^4*(a + b*x^3)^(2/3))/(c + d*x^3),x)
[Out] int((x^4*(a + b*x^3)^(2/3))/(c + d*x^3), x)
```

3.693 $\int \frac{x(a+bx^3)^{2/3}}{c+dx^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{x(a+bx^3)^{2/3}}{c+dx^3} dx = \frac{x^2(a+bx^3)^{2/3} \operatorname{AppellF1}\left(\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

```
[Out] 1/2*x^2*(b*x^3+a)^(2/3)*AppellF1(2/3,-2/3,1,5/3,-b*x^3/a,-d*x^3/c)/c/(1+b*x
^3/a)^(2/3)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {525, 524}

$$\int \frac{x(a+bx^3)^{2/3}}{c+dx^3} dx = \frac{x^2(a+bx^3)^{2/3} \operatorname{AppellF1}\left(\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

```
[In] Int[(x*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (x^2*(a + b*x^3)^(2/3)*AppellF1[2/3, -2/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/
c)])/(2*c*(1 + (b*x^3)/a)^(2/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(a+bx^3)^{2/3}\int\frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= \frac{x^2(a+bx^3)^{2/3}\,F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int\frac{x(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{x^2(a+bx^3)^{2/3}\,\text{AppellF1}\left(\frac{2}{3},-\frac{2}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\left(\frac{a+bx^3}{a}\right)^{2/3}}$$

```
[In] Integrate[(x*(a + b*x^3)^(2/3))/(c + d*x^3),x]
[Out] (x^2*(a + b*x^3)^(2/3)*AppellF1[2/3, -2/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/
c)])/(2*c*((a + b*x^3)/a)^(2/3))
```

Maple [F]

$$\int\frac{x(b\,x^3+a)^{\frac{2}{3}}}{d\,x^3+c}dx$$

```
[In] int(x*(b*x^3+a)^(2/3)/(d*x^3+c),x)
[Out] int(x*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x(a+bx^3)^{2/3}}{c+dx^3}\,dx = \text{Timed out}$$

```
[In] integrate(x*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{x(a+bx^3)^{\frac{2}{3}}}{c+dx^3}\,dx$$

```
[In] integrate(x*(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(x*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}x}{dx^3+c}\,dx$$

```
[In] integrate(x*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)*x/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{x(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}x}{dx^3+c}\,dx$$

```
[In] integrate(x*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)*x/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx^3)^{2/3}}{c+dx^3}\,dx = \int \frac{x\,(b\,x^3+a)^{2/3}}{d\,x^3+c}\,dx$$

```
[In] int((x*(a + b*x^3)^(2/3))/(c + d*x^3),x)
[Out] int((x*(a + b*x^3)^(2/3))/(c + d*x^3), x)
```

3.694 $\int \frac{(a+bx^3)^{2/3}}{x^2(c+dx^3)} dx$

Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{(a+bx^3)^{2/3}}{x^2(c+dx^3)} dx = -\frac{(a+bx^3)^{2/3} \operatorname{AppellF1}\left(-\frac{1}{3},-\frac{2}{3},1,\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

```
[Out] -(b*x^3+a)^(2/3)*AppellF1(-1/3,-2/3,1,2/3,-b*x^3/a,-d*x^3/c)/c/x/(1+b*x^3/a
)^(2/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{(a+bx^3)^{2/3}}{x^2(c+dx^3)} dx = -\frac{(a+bx^3)^{2/3} \operatorname{AppellF1}\left(-\frac{1}{3},-\frac{2}{3},1,\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

```
[In] Int[(a + b*x^3)^(2/3)/(x^2*(c + d*x^3)),x]
```

```
[Out] -(((a + b*x^3)^(2/3)*AppellF1[-1/3, -2/3, 1, 2/3, -((b*x^3)/a), -((d*x^3)/c
)])/(c*x*(1 + (b*x^3)/a)^(2/3)))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{(a+bx^3)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^2(c+dx^3)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= -\frac{(a+bx^3)^{2/3}\,F_1\left(-\frac{1}{3};-\frac{2}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 138 vs. 2(62) = 124.

Time = 10.11 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.23

$$\int \frac{(a+bx^3)^{2/3}}{x^2(c+dx^3)}\,dx = \frac{-10c(a+bx^3)-5(-2bc+ad)x^3\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bdx^6}{10c^2x\sqrt[3]{a+bx^3}}$$

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^2*(c + d*x^3)),x]
```

```
[Out] (-10*c*(a + b*x^3) - 5*(-2*b*c + a*d)*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/
3, 1/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*d*x^6*(1 + (b*x^3)/a)^(1/
3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(10*c^2*x*(a + b
*x^3)^(1/3))
```

Maple [F]

$$\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{x^2\,(d\,x^3+c)}dx$$

```
[In] int((b*x^3+a)^(2/3)/x^2/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^2/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^2(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^3+a)^(2/3)/x^2/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^2(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{2}{3}}}{x^2(c+dx^3)}\,dx$$

```
[In] integrate((b*x**3+a)**(2/3)/x**2/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(2/3)/(x**2*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^2(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^2}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/x^2/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^2), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^2(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^2}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/x^2/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^2(c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{x^2\,(d\,x^3+c)}\,dx$$

```
[In] int((a + b*x^3)^(2/3)/(x^2*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(2/3)/(x^2*(c + d*x^3)), x)
```

3.695 $\int \frac{(a+bx^3)^{2/3}}{x^5(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{(a+bx^3)^{2/3}}{x^5(c+dx^3)}\,dx=-\frac{(a+bx^3)^{2/3}\operatorname{AppellF1}\left(-\frac{4}{3},-\frac{2}{3},1,-\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4cx^4\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

```
[Out] -1/4*(b*x^3+a)^(2/3)*AppellF1(-4/3,-2/3,1,-1/3,-b*x^3/a,-d*x^3/c)/c/x^4/(1+
b*x^3/a)^(2/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{(a+bx^3)^{2/3}}{x^5(c+dx^3)}\,dx=-\frac{(a+bx^3)^{2/3}\operatorname{AppellF1}\left(-\frac{4}{3},-\frac{2}{3},1,-\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4cx^4\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

```
[In] Int[(a + b*x^3)^(2/3)/(x^5*(c + d*x^3)),x]
```

```
[Out] -1/4*((a + b*x^3)^(2/3)*AppellF1[-4/3, -2/3, 1, -1/3, -((b*x^3)/a), -((d*x^
3)/c)])/(c*x^4*(1 + (b*x^3)/a)^(2/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{(a+bx^3)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^5(c+dx^3)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= -\frac{(a+bx^3)^{2/3}\,F_1\left(-\frac{4}{3};-\frac{2}{3},1;-\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4cx^4\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 181 vs. 2(64) = 128.

Time = 10.16 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.83

$$\int \frac{(a+bx^3)^{2/3}}{x^5(c+dx^3)}\,dx = \frac{-5c(a+bx^3)\left(2bcx^3+a(c-4dx^3)\right)+5(b^2c^2-4abcd+2a^2d^2)\,x^6\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}}{20ac^3x^4\sqrt[3]{}}$$

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^5*(c + d*x^3)),x]
```

```
[Out] (-5*c*(a + b*x^3)*(2*b*c*x^3 + a*(c - 4*d*x^3)) + 5*(b^2*c^2 - 4*a*b*c*d +
2*a^2*d^2)*x^6*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a
), -((d*x^3)/c)] + 2*b*d*(b*c - 2*a*d)*x^9*(1 + (b*x^3)/a)^(1/3)*AppellF1[5
/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(20*a*c^3*x^4*(a + b*x^3)^(1/
3))
```

Maple [F]

$$\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{x^5\,(d\,x^3+c)}dx$$

```
[In] int((b*x^3+a)^(2/3)/x^5/(d*x^3+c),x)
[Out] int((b*x^3+a)^(2/3)/x^5/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^5\,(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^3+a)^(2/3)/x^5/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^5\,(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{2}{3}}}{x^5\,(c+dx^3)}\,dx$$

```
[In] integrate((b*x**3+a)**(2/3)/x**5/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(2/3)/(x**5*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^5\,(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^5}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/x^5/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^5), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^5\,(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^5}\,dx$$

```
[In] integrate((b*x^3+a)^(2/3)/x^5/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^5\,(c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{x^5\,(d\,x^3+c)}\,dx$$

```
[In] int((a + b*x^3)^(2/3)/(x^5*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(2/3)/(x^5*(c + d*x^3)), x)
```

3.696 $\int \frac{x^8(a+bx^3)^{4/3}}{c+dx^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 251

$$\int \frac{x^8(a+bx^3)^{4/3}}{c+dx^3} dx = -\frac{c^2(bc-ad)\sqrt[3]{a+bx^3}}{d^4} + \frac{c^2(a+bx^3)^{4/3}}{4d^3} - \frac{(bc+ad)(a+bx^3)^{7/3}}{7b^2d^2} + \frac{(a+bx^3)^{10/3}}{10b^2d} - \frac{c^2(bc-ad)^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{13/3}} - \frac{c^2(bc-ad)^{4/3}\log(c+dx^3)}{6d^{13/3}} + \frac{c^2(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{13/3}}$$

```
[Out] -c^2*(-a*d+b*c)*(b*x^3+a)^(1/3)/d^4+1/4*c^2*(b*x^3+a)^(4/3)/d^3-1/7*(a*d+b*
c)*(b*x^3+a)^(7/3)/b^2/d^2+1/10*(b*x^3+a)^(10/3)/b^2/d-1/6*c^2*(-a*d+b*c)^(
4/3)*ln(d*x^3+c)/d^(13/3)+1/2*c^2*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)+d^(1
/3)*(b*x^3+a)^(1/3))/d^(13/3)-1/3*c^2*(-a*d+b*c)^(4/3)*arctan(1/3*(1-2*d^(1
/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(13/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used

$= \{457, 90, 52, 60, 631, 210, 31\}$

$$\int \frac{x^8(a+bx^3)^{4/3}}{c+dx^3}\,dx = -\frac{c^2(bc-ad)^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{13/3}} - \frac{(a+bx^3)^{7/3}(ad+bc)}{7b^2d^2} + \frac{(a+bx^3)^{10/3}}{10b^2d} - \frac{c^2(bc-ad)^{4/3}\log(c+dx^3)}{6d^{13/3}} + \frac{c^2(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{13/3}} - \frac{c^2\sqrt[3]{a+bx^3}(bc-ad)}{d^4} + \frac{c^2(a+bx^3)^{4/3}}{4d^3}$$

```
[In] Int[(x^8*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] -((c^2*(b*c - a*d)*(a + b*x^3)^(1/3))/d^4) + (c^2*(a + b*x^3)^(4/3))/(4*d^3
) - ((b*c + a*d)*(a + b*x^3)^(7/3))/(7*b^2*d^2) + (a + b*x^3)^(10/3)/(10*b^
2*d) - (c^2*(b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*
c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(13/3)) - (c^2*(b*c - a*d)^(4/3)*Log[c
 + d*x^3])/(6*d^(13/3)) + (c^2*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^
(1/3)*(a + b*x^3)^(1/3)])/(2*d^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2(a+bx)^{4/3}}{c+dx}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{(-bc-ad)(a+bx)^{4/3}}{bd^2}+\frac{(a+bx)^{7/3}}{bd}+\frac{c^2(a+bx)^{4/3}}{d^2(c+dx)}\right)\,dx,x,x^3\right)\\
&= -\frac{(bc+ad)\left(a+bx^3\right)^{7/3}}{7b^2d^2}+\frac{\left(a+bx^3\right)^{10/3}}{10b^2d}+\frac{c^2\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{c+dx}\,dx,x,x^3\right)}{3d^2}\\
&= \frac{c^2\left(a+bx^3\right)^{4/3}}{4d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{7/3}}{7b^2d^2}+\frac{\left(a+bx^3\right)^{10/3}}{10b^2d}\\
&\quad-\frac{\left(c^2(bc-ad)\right)\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx,x,x^3\right)}{3d^3}\\
&= -\frac{c^2(bc-ad)\sqrt[3]{a+bx^3}}{d^4}+\frac{c^2\left(a+bx^3\right)^{4/3}}{4d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{7/3}}{7b^2d^2}\\
&\quad+\frac{\left(a+bx^3\right)^{10/3}}{10b^2d}+\frac{\left(c^2(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx,x,x^3\right)}{3d^4}
\end{aligned}$$

$$= -\frac{c^2(bc-ad)\sqrt[3]{a+bx^3}}{d^4} + \frac{c^2(a+bx^3)^{4/3}}{4d^3} - \frac{(bc+ad)\left(a+bx^3\right)^{7/3}}{7b^2d^2} + \frac{\left(a+bx^3\right)^{10/3}}{10b^2d} - \frac{c^2(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{13/3}} + \frac{\left(c^2(bc-ad)^{4/3}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{13/3}} + \frac{\left(c^2(bc-ad)^{5/3}\right)\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{14/3}}$$

$$= -\frac{c^2(bc-ad)\sqrt[3]{a+bx^3}}{d^4} + \frac{c^2(a+bx^3)^{4/3}}{4d^3} - \frac{(bc+ad)\left(a+bx^3\right)^{7/3}}{7b^2d^2} + \frac{\left(a+bx^3\right)^{10/3}}{10b^2d} - \frac{c^2(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{13/3}} + \frac{c^2(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{13/3}} + \frac{\left(c^2(bc-ad)^{4/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{d^{13/3}}$$

$$= -\frac{c^2(bc-ad)\sqrt[3]{a+bx^3}}{d^4} + \frac{c^2(a+bx^3)^{4/3}}{4d^3} - \frac{(bc+ad)\left(a+bx^3\right)^{7/3}}{7b^2d^2} + \frac{\left(a+bx^3\right)^{10/3}}{10b^2d} - \frac{c^2(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{13/3}} - \frac{c^2(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{13/3}} + \frac{c^2(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{13/3}}$$

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.23

$$\int \frac{x^8\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx = \frac{\frac{3\sqrt[3]{d}\sqrt[3]{a+bx^3}\left(-6a^3d^3+2a^2bd^2\left(-10c+dx^3\right)+ab^2d\left(175c^2-40cdx^3+22d^2x^6\right)+b^3\left(-140c^3+35c^2dx^3-20cd^2x^6-\right.\right.}{b^2}}{}$$

[In] Integrate[(x^8*(a + b*x^3)^(4/3))/(c + d*x^3),x]

```
[Out] ((3*d^(1/3)*(a + b*x^3)^(1/3)*(-6*a^3*d^3 + 2*a^2*b*d^2*(-10*c + d*x^3) + a
*b^2*d*(175*c^2 - 40*c*d*x^3 + 22*d^2*x^6) + b^3*(-140*c^3 + 35*c^2*d*x^3 -
 20*c*d^2*x^6 + 14*d^3*x^9)))/b^2 - 140*Sqrt[3]*c^2*(b*c - a*d)^(4/3)*ArcTa
n[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 140*c^2*
(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - 70*c
^2*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a +
 b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(420*d^(13/3))
```

Maple [A] (verified)

Time = 4.89 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$-\frac{\frac{9\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}} d\left(\left(-\frac{7b\,x^{3}}{3}+a\right)\left(b\,x^{3}+a\right)^{2} d^{3}+\frac{10bc\left(b\,x^{3}+a\right)^{2} d^{2}}{3}-\frac{175b^{2}\left(\frac{b\,x^{3}}{5}+a\right) c^{2} d}{6}+\frac{70b^{3} c^{3}}{3}\right)\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{35}+b^{2} c^{2} (ad-bc)^{2}\left(2\arctan\right.}{6\left(\frac{ad-}{}\right.}$

```
[In] int(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/(1/d*(a*d-b*c))^(2/3)*(9/35*(1/d*(a*d-b*c))^(2/3)*d*((-7/3*b*x^3+a)*(b
*x^3+a)^2*d^3+10/3*b*c*(b*x^3+a)^2*d^2-175/6*b^2*(1/5*b*x^3+a)*c^2*d+70/3*b
^3*c^3)*(b*x^3+a)^(1/3)+b^2*c^2*(a*d-b*c)^2*(2*arctan(1/3*3^(1/2)*(2*(b*x^3
+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a
)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((
b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))))/b^2/d^5
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.47

$$\int \frac{x^8(a+bx^3)^{4/3}}{c+dx^3}\,dx=\frac{140\,\sqrt{3}(b^3c^3-ab^2c^2d)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\,\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+70\,(b^3}{}$$

```
[In] integrate(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/420*(140*sqrt(3)*(b^3*c^3 - a*b^2*c^2*d)*(-(b*c - a*d)/d)^(1/3)*arctan(-1
/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a
*d))/(b*c - a*d)) + 70*(b^3*c^3 - a*b^2*c^2*d)*(-(b*c - a*d)/d)^(1/3)*log((
b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)
/d)^(2/3)) - 140*(b^3*c^3 - a*b^2*c^2*d)*(-(b*c - a*d)/d)^(1/3)*log((b*x^3
```

```
+ a)^(1/3) - (-(b*c - a*d)/d)^(1/3)) + 3*(14*b^3*d^3*x^9 - 2*(10*b^3*c*d^2
- 11*a*b^2*d^3)*x^6 - 140*b^3*c^3 + 175*a*b^2*c^2*d - 20*a^2*b*c*d^2 - 6*a^
3*d^3 + (35*b^3*c^2*d - 40*a*b^2*c*d^2 + 2*a^2*b*d^3)*x^3)*(b*x^3 + a)^(1/3
))/(b^2*d^4)
```

Sympy [F]

$$\int \frac{x^8(a+bx^3)^{4/3}}{c+dx^3}\,dx=\int \frac{x^8(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

```
[In] integrate(x**8*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**8*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8(a+bx^3)^{4/3}}{c+dx^3}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.57

$$\int \frac{x^8(a+bx^3)^{4/3}}{c+dx^3}\,dx =$$
$$-\frac{(b^{24}c^4d^6-2\,ab^{23}c^3d^7+a^2b^{22}c^2d^8)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^{23}cd^{10}-ab^{22}d^{11})}$$
$$+\frac{\sqrt{3}(bc^3-ac^2d)(-bcd^2+ad^3)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^5}$$
$$+\frac{(bc^3-ac^2d)(-bcd^2+ad^3)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^5}$$
$$-\frac{140\,(bx^3+a)^{\frac{1}{3}}b^{21}c^3d^6-35\,(bx^3+a)^{\frac{4}{3}}b^{20}c^2d^7-140\,(bx^3+a)^{\frac{1}{3}}ab^{20}c^2d^7+20\,(bx^3+a)^{\frac{7}{3}}b^{19}cd^8-14\,(bx^3+a}{140\,b^{20}d^{10}}$$

```
[In] integrate(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^24*c^4*d^6 - 2*a*b^23*c^3*d^7 + a^2*b^22*c^2*d^8)*(-(b*c - a*d)/d)^
(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^23*c*d^10 - a
*b^22*d^11) + 1/3*sqrt(3)*(b*c^3 - a*c^2*d)*(-b*c*d^2 + a*d^3)^(1/3)*arctan
(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d
)^(1/3))/d^5 + 1/6*(b*c^3 - a*c^2*d)*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 +
a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3
))/d^5 - 1/140*(140*(b*x^3 + a)^(1/3)*b^21*c^3*d^6 - 35*(b*x^3 + a)^(4/3)*b
^20*c^2*d^7 - 140*(b*x^3 + a)^(1/3)*a*b^20*c^2*d^7 + 20*(b*x^3 + a)^(7/3)*b
^19*c*d^8 - 14*(b*x^3 + a)^(10/3)*b^18*d^9 + 20*(b*x^3 + a)^(7/3)*a*b^18*d^
9)/(b^20*d^10)
```

Mupad [B] (verification not implemented)

Time = 8.91 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.90

$$\int \frac{x^8(a+bx^3)^{4/3}}{c+dx^3}\,dx = \left(\frac{a^2}{4\,b^2\,d}+\frac{\left(\frac{2\,a}{b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{b^4\,d^2}\right)\,\left(b^3\,c-a\,b^2\,d\right)}{4\,b^2\,d}\right)\,{\left(b\,x^3+a\right)}^{4/3} - \left(\frac{2\,a}{7\,b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{7\,b^4\,d^2}\right)\,{\left(b\,x^3+a\right)}^{7/3}+\frac{{\left(b\,x^3+a\right)}^{10/3}}{10\,b^2\,d} + \frac{c^2\,\ln\left(\frac{3\,{\left(b\,x^3+a\right)}^{1/3}\,\left(a^2\,c^2\,d^2-2\,a\,b\,c^3\,d+b^2\,c^4\right)}{d^2}-\frac{c^2\,{\left(a\,d-b\,c\right)}^{4/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{13/3}}\right)\,{\left(a\,d-b\,c\right)}^{4/3}}{3\,d^{13/3}} - \frac{\left(\frac{a^2}{b^2\,d}+\frac{\left(\frac{2\,a}{b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{b^4\,d^2}\right)\,\left(b^3\,c-a\,b^2\,d\right)}{b^2\,d}\right)\,{\left(b\,x^3+a\right)}^{1/3}\,\left(b^3\,c-a\,b^2\,d\right)}{b^2\,d} - \frac{c^2\,\ln\left(\frac{3\,c^2\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{7/3}}{d^{7/3}}+\frac{3\,c^2\,{\left(b\,x^3+a\right)}^{1/3}\,{\left(a\,d-b\,c\right)}^2}{d^2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{4/3}}{3\,d^{13/3}} + \frac{c^2\,\ln\left(\frac{3\,c^2\,{\left(b\,x^3+a\right)}^{1/3}\,{\left(a\,d-b\,c\right)}^2}{d^2}-\frac{9\,c^2\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,{\left(a\,d-b\,c\right)}^{7/3}}{d^{7/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,{\left(a\,d-b\,c\right)}^{4/3}}{d^{13/3}}$$

```
[In] int((x^8*(a + b*x^3)^(4/3))/(c + d*x^3),x)
```

```
[Out] (a^2/(4*b^2*d) + (((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2))*(b^3*c - a*
b^2*d))/(4*b^2*d))*(a + b*x^3)^(4/3) - ((2*a)/(7*b^2*d) + (b^3*c - a*b^2*d)
/(7*b^4*d^2))*(a + b*x^3)^(7/3) + (a + b*x^3)^(10/3)/(10*b^2*d) + (c^2*log(
(3*(a + b*x^3)^(1/3)*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d))/d^2 - (c^2*(a*d
 - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(13/3)))*(a*d - b*c)^(4/3))/(3*d^
(13/3)) - ((a^2/(b^2*d) + (((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2))*(b
^3*c - a*b^2*d))/(b^2*d))*(a + b*x^3)^(1/3)*(b^3*c - a*b^2*d))/(b^2*d) - (c
^2*log((3*c^2*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(7/3))/d^(7/3) + (3*c^2*(a
 + b*x^3)^(1/3)*(a*d - b*c)^2)/d^2)*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(4/3
))/(3*d^(13/3)) + (c^2*log((3*c^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d^2 - (9
*c^2*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(7/3))/d^(7/3))*((3^(1/2)*1i)/6 - 1
/6)*(a*d - b*c)^(4/3))/d^(13/3)
```

3.697 $\int \frac{x^5(a+bx^3)^{4/3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 211

$$\int \frac{x^5(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{c(bc-ad)\sqrt[3]{a+bx^3}}{d^3} - \frac{c(a+bx^3)^{4/3}}{4d^2} + \frac{(a+bx^3)^{7/3}}{7bd} + \frac{c(bc-ad)^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{10/3}} + \frac{c(bc-ad)^{4/3}\log(c+dx^3)}{6d^{10/3}} - \frac{c(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}}$$

```
[Out] c*(-a*d+b*c)*(b*x^3+a)^(1/3)/d^3-1/4*c*(b*x^3+a)^(4/3)/d^2+1/7*(b*x^3+a)^(7
/3)/b/d+1/6*c*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/d^(10/3)-1/2*c*(-a*d+b*c)^(4/3)*
ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(10/3)+1/3*c*(-a*d+b*c)^(4/3
)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(10/
3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used

$= \{457, 81, 52, 60, 631, 210, 31\}$

$$\int \frac{x^5(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{c(bc-ad)^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{10/3}} + \frac{c(bc-ad)^{4/3}\log(c+dx^3)}{6d^{10/3}} - \frac{c(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}} + \frac{c\sqrt[3]{a+bx^3}(bc-ad)}{d^3} - \frac{c(a+bx^3)^{4/3}}{4d^2} + \frac{(a+bx^3)^{7/3}}{7bd}$$

```
[In] Int[(x^5*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (c*(b*c - a*d)*(a + b*x^3)^(1/3))/d^3 - (c*(a + b*x^3)^(4/3))/(4*d^2) + (a
+ b*x^3)^(7/3)/(7*b*d) + (c*(b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b
*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(10/3)) + (c*(b*c - a*
d)^(4/3)*Log[c + d*x^3])/(6*d^(10/3)) - (c*(b*c - a*d)^(4/3)*Log[(b*c - a*d
)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x(a+bx)^{4/3}}{c+dx}\,dx,x,x^3\right)\\
&= \frac{(a+bx^3)^{7/3}}{7bd}-\frac{c\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{c+dx}\,dx,x,x^3\right)}{3d}\\
&= -\frac{c(a+bx^3)^{4/3}}{4d^2}+\frac{(a+bx^3)^{7/3}}{7bd}+\frac{(c(bc-ad))\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx,x,x^3\right)}{3d^2}\\
&= \frac{c(bc-ad)\sqrt[3]{a+bx^3}}{d^3}-\frac{c(a+bx^3)^{4/3}}{4d^2}+\frac{(a+bx^3)^{7/3}}{7bd}\\
&\quad -\frac{(c(bc-ad)^2)\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx,x,x^3\right)}{3d^3}
\end{aligned}
$$

$$= \frac{c(bc-ad)\sqrt[3]{a+bx^3}}{d^3} - \frac{c(a+bx^3)^{4/3}}{4d^2} + \frac{(a+bx^3)^{7/3}}{7bd} + \frac{c(bc-ad)^{4/3}\log(c+dx^3)}{6d^{10/3}} - \frac{\left(c(bc-ad)^{4/3}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{10/3}} - \frac{\left(c(bc-ad)^{5/3}\right)\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{11/3}}$$

$$= \frac{c(bc-ad)\sqrt[3]{a+bx^3}}{d^3} - \frac{c(a+bx^3)^{4/3}}{4d^2} + \frac{(a+bx^3)^{7/3}}{7bd} + \frac{c(bc-ad)^{4/3}\log(c+dx^3)}{6d^{10/3}} - \frac{c(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}} - \frac{\left(c(bc-ad)^{4/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{d^{10/3}}$$

$$= \frac{c(bc-ad)\sqrt[3]{a+bx^3}}{d^3} - \frac{c(a+bx^3)^{4/3}}{4d^2} + \frac{(a+bx^3)^{7/3}}{7bd} + \frac{c(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{10/3}} + \frac{c(bc-ad)^{4/3}\log(c+dx^3)}{6d^{10/3}} - \frac{c(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.23

$$\int \frac{x^5(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{\frac{3\sqrt[3]{d}\sqrt[3]{a+bx^3}(4a^2d^2+abd(-35c+8dx^3)+b^2(28c^2-7cdx^3+4d^2x^6))}{b} + 28\sqrt{3}c(bc-ad)^{4/3}\arctan\left(\right.}{}$$

```
[In] Integrate[(x^5*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] ((3*d^(1/3)*(a + b*x^3)^(1/3)*(4*a^2*d^2 + a*b*d*(-35*c + 8*d*x^3) + b^2*(2
8*c^2 - 7*c*d*x^3 + 4*d^2*x^6)))/b + 28*Sqrt[3]*c*(b*c - a*d)^(4/3)*ArcTan[
(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 28*c*(b*c
```

```
- a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + 14*c*(b*c
 - a*d)^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)
^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(84*d^(10/3))
```

Maple [A] (verified)

Time = 4.90 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.08

method	result
pseudoelliptic	$\frac{\frac{6\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}} d\left(\left(b\,x^{3}+a\right)^{2} d^{2}-\frac{35b\left(\frac{b\,x^{3}}{5}+a\right) cd}{4}+7b^{2}c^{2}\right)\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{7}+bc(ad-bc)^{2}\left(2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)\sqrt{3}+\cdots\right.}{6\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}} d^{4}b}$

```
[In] int(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/(1/d*(a*d-b*c))^(2/3)*(6/7*(1/d*(a*d-b*c))^(2/3)*d*((b*x^3+a)^2*d^2-35/
4*b*(1/5*b*x^3+a)*c*d+7*b^2*c^2)*(b*x^3+a)^(1/3)+b*c*(a*d-b*c)^2*(2*arctan(
1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3)
)*3^(1/2)+ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*
d-b*c))^(2/3))-2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))))/d^4/b
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.41

$$\int \frac{x^5(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{28\sqrt{3}(b^2c^2-abcd)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+14\,(b^2c^2-a\cdots}{\cdots}$$

```
[In] integrate(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/84*(28*sqrt(3)*(b^2*c^2 - a*b*c*d)*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*s
qrt(3)*(b*x^3 + a)^(1/3)*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*
c - a*d)) + 14*(b^2*c^2 - a*b*c*d)*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2
/3) - (b*x^3 + a)^(1/3)*((b*c - a*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3)) - 28
*(b^2*c^2 - a*b*c*d)*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b*c -
a*d)/d)^(1/3)) + 3*(4*b^2*d^2*x^6 + 28*b^2*c^2 - 35*a*b*c*d + 4*a^2*d^2 - (
7*b^2*c*d - 8*a*b*d^2)*x^3)*(b*x^3 + a)^(1/3))/(b*d^3)
```

Sympy [F]

$$\int \frac{x^5(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{x^5(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

```
[In] integrate(x**5*(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(x**5*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(a+bx^3)^{4/3}}{c+dx^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(171) = 342$.

Time = 0.30 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.65

$$\int \frac{x^5(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{(b^{10}c^3d^4 - 2\,ab^9c^2d^5 + a^2b^8cd^6)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^9cd^7 - ab^8d^8)}$$
$$- \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}(bc^2-acd)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^4}$$
$$- \frac{(-bcd^2+ad^3)^{\frac{1}{3}}(bc^2-acd)\log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,d^4}$$
$$+ \frac{28\,(bx^3+a)^{\frac{1}{3}}b^8c^2d^4 - 7\,(bx^3+a)^{\frac{4}{3}}b^7cd^5 - 28\,(bx^3+a)^{\frac{1}{3}}ab^7cd^5 + 4\,(bx^3+a)^{\frac{7}{3}}b^6d^6}{28\,b^7d^7}$$

```
[In] integrate(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b^10*c^3*d^4 - 2*a*b^9*c^2*d^5 + a^2*b^8*c*d^6)*(-(b*c - a*d)/d)^(1/3)
*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^9*c*d^7 - a*b^8*d^
8) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*(b*c^2 - a*c*d)*arctan(1/3*sqrt(3
)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^
4 - 1/6*(-b*c*d^2 + a*d^3)^(1/3)*(b*c^2 - a*c*d)*log((b*x^3 + a)^(2/3) + (b
*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^4 + 1/28
*(28*(b*x^3 + a)^(1/3)*b^8*c^2*d^4 - 7*(b*x^3 + a)^(4/3)*b^7*c*d^5 - 28*(b*
x^3 + a)^(1/3)*a*b^7*c*d^5 + 4*(b*x^3 + a)^(7/3)*b^6*d^6)/(b^7*d^7)
```

Mupad [B] (verification not implemented)

Time = 8.91 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.65

$$\int \frac{x^5(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{(b\,x^3+a)^{7/3}}{7\,b\,d} - (b\,x^3+a)^{4/3}\left(\frac{a}{4\,b\,d}+\frac{b^2\,c-a\,b\,d}{4\,b^2\,d^2}\right) - \frac{c\,\ln\left(\frac{3\,(b\,x^3+a)^{1/3}\,(a^2\,c\,d^2-2\,a\,b\,c^2\,d+b^2\,c^3)}{d}-\frac{c\,(a\,d-b\,c)^{4/3}\,(9\,a\,d^3-9\,b\,c\,d^2)}{3\,d^{10/3}}\right)\,(a\,d-}{3\,d^{10/3}}$$

```
[In] int((x^5*(a + b*x^3)^(4/3))/(c + d*x^3),x)
```

```
[Out] (a + b*x^3)^(7/3)/(7*b*d) - (a + b*x^3)^(4/3)*(a/(4*b*d) + (b^2*c - a*b*d)/
(4*b^2*d^2)) - (c*log((3*(a + b*x^3)^(1/3)*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2
*d))/d - (c*(a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(10/3)))*(a*d - b
*c)^(4/3))/(3*d^(10/3)) - (c*log((3*c*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d -
(3*c*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^(7/3))/d^(4/3))*((3^(1/2)*1i)/2 - 1
/2)*(a*d - b*c)^(4/3))/(3*d^(10/3)) + (c*log((3*c*(a + b*x^3)^(1/3)*(a*d -
b*c)^2)/d + (3*c*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(7/3))/d^(4/3))*((3^(1/
2)*1i)/2 + 1/2)*(a*d - b*c)^(4/3))/(3*d^(10/3)) + ((a + b*x^3)^(1/3)*(b^2*c
 - a*b*d)*(a/(b*d) + (b^2*c - a*b*d)/(b^2*d^2)))/(b*d)
```

3.698 $\int \frac{x^2(a+bx^3)^{4/3}}{c+dx^3} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 187

$$\int \frac{x^2(a+bx^3)^{4/3}}{c+dx^3} \, dx = -\frac{(bc-ad)\sqrt[3]{a+bx^3}}{d^2} + \frac{(a+bx^3)^{4/3}}{4d} - \frac{(bc-ad)^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{7/3}} - \frac{(bc-ad)^{4/3}\log(c+dx^3)}{6d^{7/3}} + \frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}}$$

```
[Out] -(-a*d+b*c)*(b*x^3+a)^(1/3)/d^2+1/4*(b*x^3+a)^(4/3)/d-1/6*(-a*d+b*c)^(4/3)*
ln(d*x^3+c)/d^(7/3)+1/2*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3
+a)^(1/3))/d^(7/3)-1/3*(-a*d+b*c)^(4/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(
1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(7/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

$= \{455, 52, 60, 631, 210, 31\}$

$$\int \frac{x^2(a+bx^3)^{4/3}}{c+dx^3}\,dx = -\frac{(bc-ad)^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{7/3}} - \frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{7/3}} + \frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}} - \frac{\sqrt[3]{a+bx^3}(bc-ad)}{d^2} + \frac{(a+bx^3)^{4/3}}{4d}$$

```
[In] Int[(x^2*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] -(((b*c - a*d)*(a + b*x^3)^(1/3))/d^2) + (a + b*x^3)^(4/3)/(4*d) - ((b*c -
a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqr
t[3]])/(Sqrt[3]*d^(7/3)) - ((b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*d^(7/3)) +
 ((b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*
d^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{(a+bx^3)^{4/3}}{4d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)}{3d}$$

$$= -\frac{(bc-ad)\sqrt[3]{a+bx^3}}{d^2} + \frac{(a+bx^3)^{4/3}}{4d} + \frac{(bc-ad)^2\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3d^2}$$

$$= -\frac{(bc-ad)\sqrt[3]{a+bx^3}}{d^2} + \frac{(a+bx^3)^{4/3}}{4d} - \frac{(bc-ad)^{4/3}\log(c+dx^3)}{6d^{7/3}} + \frac{(bc-ad)^{4/3}\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{7/3}} + \frac{(bc-ad)^{5/3}\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{8/3}}$$

$$= -\frac{(bc-ad)\sqrt[3]{a+bx^3}}{d^2} + \frac{(a+bx^3)^{4/3}}{4d} - \frac{(bc-ad)^{4/3}\log(c+dx^3)}{6d^{7/3}} + \frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}} + \frac{(bc-ad)^{4/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{d^{7/3}}$$

$$= -\frac{(bc-ad)\sqrt[3]{a+bx^3}}{d^2} + \frac{(a+bx^3)^{4/3}}{4d} - \frac{(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{7/3}} - \frac{(bc-ad)^{4/3}\log(c+dx^3)}{6d^{7/3}} + \frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.18

$$\int \frac{x^2(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{3\sqrt[3]{d}\sqrt[3]{a+bx^3}(-4bc+5ad+bdx^3) - 4\sqrt{3}(bc-ad)^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) + }{}$$

```
[In] Integrate[(x^2*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (3*d^(1/3)*(a + b*x^3)^(1/3)*(-4*b*c + 5*a*d + b*d*x^3) - 4*Sqrt[3]*(b*c -
a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqr
t[3]] + 4*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/
3)] - 2*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)
*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(12*d^(7/3))
```

Maple [A] (verified)

Time = 5.01 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.09

method	result
pseudoelliptic	$\frac{\frac{5\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}d\left(\left(\frac{b\,x^3}{5}+a\right)d-\frac{4bc}{5}\right)}{4}-\frac{(ad-bc)^2\left(2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^3+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^3+a\right)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right.}{6}}{d^3\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}}$

```
[In] int(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 5/4/(1/d*(a*d-b*c))^(2/3)*((1/d*(a*d-b*c))^(2/3)*(b*x^3+a)^(1/3)*d*((1/5*b*
x^3+a)*d-4/5*b*c)-2/15*(a*d-b*c)^2*(2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)
+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+(
1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a)^
(1/3)-(1/d*(a*d-b*c))^(1/3))))/d^3
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.32

$$\int \frac{x^2(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{4\sqrt{3}(bc-ad)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+2\,(bc-ad)\left(\cdots\right.}{}$$

```
[In] integrate(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/12*(4*sqrt(3)*(b*c - a*d)*(-(b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(
b*x^3 + a)^(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d
)) + 2*(b*c - a*d)*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 +
a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3)) - 4*(b*c - a*d)*(
-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)) + 3*(
b*d*x^3 - 4*b*c + 5*a*d)*(b*x^3 + a)^(1/3))/d^2
```

Sympy [F]

$$\int \frac{x^2(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{x^2(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

```
[In] integrate(x**2*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**2*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+bx^3)^{4/3}}{c+dx^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.59

$$\int \frac{x^2(a+bx^3)^{4/3}}{c+dx^3}\,dx = -\frac{(b^2c^2d^2-2\,abcd^3+a^2d^4)\left(-\frac{bc-ad}{d}\right)^{\frac13}\log\left(\left|(bx^3+a)^{\frac13}-\left(-\frac{bc-ad}{d}\right)^{\frac13}\right|\right)}{3\,(bcd^4-ad^5)} + \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac13}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac13}+\left(-\frac{bc-ad}{d}\right)^{\frac13}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac13}}\right)}{3\,d^3} + \frac{(-bcd^2+ad^3)^{\frac13}(bc-ad)\log\left((bx^3+a)^{\frac23}+(bx^3+a)^{\frac13}\left(-\frac{bc-ad}{d}\right)^{\frac13}+\left(-\frac{bc-ad}{d}\right)^{\frac23}\right)}{6\,d^3} - \frac{4\,(bx^3+a)^{\frac13}bcd^2-(bx^3+a)^{\frac43}d^3-4\,(bx^3+a)^{\frac13}ad^3}{4\,d^4}$$

```
[In] integrate(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(-(b*c - a*d)/d)^(1/3)*log(abs((
b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c*d^4 - a*d^5) + 1/3*sqrt(3)
*(-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/
3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^3 + 1/6*(-b*c*d^2 +
a*d^3)^(1/3)*(b*c - a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c -
 a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^3 - 1/4*(4*(b*x^3 + a)^(1/3)*b*c
*d^2 - (b*x^3 + a)^(4/3)*d^3 - 4*(b*x^3 + a)^(1/3)*a*d^3)/d^4
```

Mupad [B] (verification not implemented)

Time = 8.55 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.63

$$\int \frac{x^2(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{(b\,x^3+a)^{4/3}}{4\,d} + \frac{\ln\left((b\,x^3+a)^{1/3}\,(3\,a^2\,d^2-6\,a\,b\,c\,d+3\,b^2\,c^2)-\frac{(a\,d-b\,c)^{4/3}\,(9\,a\,d^3-9\,b\,c\,d^2)}{3\,d^{7/3}}\right)\,(a\,d-b\,c)^{4/3}}{3\,d^{7/3}} + \frac{(b\,x^3+a)^{1/3}\,(a\,d-b\,c)}{d^2} - \frac{\ln\left((b\,x^3+a)^{1/3}\,(3\,a^2\,d^2-6\,a\,b\,c\,d+3\,b^2\,c^2)+\frac{\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(a\,d-b\,c)^{4/3}\,(9\,a\,d^3-9\,b\,c\,d^2)}{3\,d^{7/3}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(a\,d-b\,c)^{4/3}}{3\,d^{7/3}} + \frac{\ln\left((b\,x^3+a)^{1/3}\,(3\,a^2\,d^2-6\,a\,b\,c\,d+3\,b^2\,c^2)-\frac{\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)(a\,d-b\,c)^{4/3}\,(9\,a\,d^3-9\,b\,c\,d^2)}{d^{7/3}}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)(a\,d-b\,c)^{4}}{d^{7/3}}$$

```
[In] int((x^2*(a + b*x^3)^(4/3))/(c + d*x^3),x)
```

```
[Out] (a + b*x^3)^(4/3)/(4*d) + (log((a + b*x^3)^(1/3)*(3*a^2*d^2 + 3*b^2*c^2 - 6
*a*b*c*d) - ((a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(7/3)))*(a*d - b
*c)^(4/3))/(3*d^(7/3)) + ((a + b*x^3)^(1/3)*(a*d - b*c))/d^2 - (log((a + b*
x^3)^(1/3)*(3*a^2*d^2 + 3*b^2*c^2 - 6*a*b*c*d) + (((3^(1/2)*1i)/2 + 1/2)*(a
*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(7/3)))*((3^(1/2)*1i)/2 + 1/2)*
(a*d - b*c)^(4/3))/(3*d^(7/3)) + (log((a + b*x^3)^(1/3)*(3*a^2*d^2 + 3*b^2*
c^2 - 6*a*b*c*d) - (((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(4/3)*(9*a*d^3 - 9*b
*c*d^2))/d^(7/3))*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(4/3))/d^(7/3)
```

3.699 $\int \frac{(a+bx^3)^{4/3}}{x(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 261

$$\int \frac{(a+bx^3)^{4/3}}{x(c+dx^3)}\,dx = \frac{b\sqrt[3]{a+bx^3}}{d} - \frac{a^{4/3}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}c} + \frac{(bc-ad)^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}cd^{4/3}} - \frac{a^{4/3}\log(x)}{2c} + \frac{(bc-ad)^{4/3}\log(c+dx^3)}{6cd^{4/3}} + \frac{a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c} - \frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2cd^{4/3}}$$

```
[Out] b*(b*x^3+a)^(1/3)/d-1/2*a^(4/3)*ln(x)/c+1/6*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/c/
d^(4/3)+1/2*a^(4/3)*ln(a^(1/3)-(b*x^3+a)^(1/3))/c-1/2*(-a*d+b*c)^(4/3)*ln((
-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/c/d^(4/3)-1/3*a^(4/3)*arctan(1/3*(
a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/c*3^(1/2)+1/3*(-a*d+b*c)^(4/3)*
arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c/d^(4/3
)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {457, 86, 162, 59, 631, 210, 31, 60}

$$\int \frac{(a+bx^3)^{4/3}}{x(c+dx^3)}\,dx = -\frac{a^{4/3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt{3}c}+\frac{a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c}-\frac{a^{4/3}\log(x)}{2c}+\frac{(bc-ad)^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}cd^{4/3}}+\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6cd^{4/3}}-\frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2cd^{4/3}}+\frac{b\sqrt[3]{a+bx^3}}{d}$$

```
[In] Int[(a + b*x^3)^(4/3)/(x*(c + d*x^3)),x]
```

```
[Out] (b*(a + b*x^3)^(1/3))/d - (a^(4/3)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(
Sqrt[3]*a^(1/3))])/(Sqrt[3]*c) + ((b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*
(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c*d^(4/3)) - (a^(4
/3)*Log[x])/(2*c) + ((b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*c*d^(4/3)) + (a^(
4/3)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*c) - ((b*c - a*d)^(4/3)*Log[(b*c
- a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c*d^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Dist[1/(b*d), I
nt[(b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/(
(a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{x(c+dx)}\,dx, x, x^3\right) \\
&= \frac{b\sqrt[3]{a+bx^3}}{d} + \frac{\text{Subst}\left(\int \frac{a^2d+b(-bc+2ad)x}{x(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3d} \\
&= \frac{b\sqrt[3]{a+bx^3}}{d} + \frac{a^2\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^3\right)}{3c} - \frac{(bc-ad)^2\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3cd}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b\sqrt[3]{a+bx^3}}{d} - \frac{a^{4/3}\log(x)}{2c} + \frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6cd^{4/3}} \\
&- \frac{a^{4/3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c} \\
&- \frac{a^{5/3}\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c} \\
&- \frac{(bc-ad)^{4/3}\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2cd^{4/3}} \\
&- \frac{(bc-ad)^{5/3}\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2cd^{5/3}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b\sqrt[3]{a+bx^3}}{d} - \frac{a^{4/3}\log(x)}{2c} + \frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6cd^{4/3}} \\
&+ \frac{a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c} - \frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2cd^{4/3}} \\
&+ \frac{a^{4/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{c} \\
&- \frac{(bc-ad)^{4/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{cd^{4/3}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b\sqrt[3]{a+bx^3}}{d} - \frac{a^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}c} \\
&+ \frac{(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}cd^{4/3}} - \frac{a^{4/3}\log(x)}{2c} \\
&+ \frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6cd^{4/3}} + \frac{a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c} \\
&- \frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2cd^{4/3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^3)^{4/3}}{x(c+dx^3)}\,dx = \frac{6bc\sqrt[3]{d}\sqrt[3]{a+bx^3}-2\sqrt{3}a^{4/3}d^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\sqrt{3}(bc-ad)^{4/3}\arctan\left(\frac{1-\cdots}{\cdots}\right.}{}$$

[In] Integrate[(a + b*x^3)^(4/3)/(x*(c + d*x^3)),x]

[Out] (6*b*c*d^(1/3)*(a + b*x^3)^(1/3) - 2*Sqrt[3]*a^(4/3)*d^(4/3)*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 2*Sqrt[3]*(b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 2*a^(4/3)*d^(4/3)*Log[-a^(1/3) + (a + b*x^3)^(1/3)] - 2*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - a^(4/3)*d^(4/3)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + (b*c - a*d)^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*c*d^(4/3))

Maple [A] (verified)

Time = 4.77 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.11

method	result
pseudoelliptic	$-\frac{-3(bx^3+a)^{\frac{1}{3}}\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}bcd+\frac{d^2\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\left(2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2(bx^3+a)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left((bx^3+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\ln\left(\cdots\right.\right.}{2}}{}$

[In] int((b*x^3+a)^(4/3)/x/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] -1/3/(1/d*(a*d-b*c))^(2/3)*(-3*(b*x^3+a)^(1/3)*(1/d*(a*d-b*c))^(2/3)*b*c*d+1/2*d^2*(1/d*(a*d-b*c))^(2/3)*(2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)+ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3))-2*ln((b*x^3+a)^(1/3)-a^(1/3)))*a^(4/3)-1/2*(a*d-b*c)^2*(2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))))/c/d^2

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.23

$$\int \frac{(a+bx^3)^{4/3}}{x(c+dx^3)}\,dx =$$

$$-\frac{2\sqrt{3}a^{\frac{4}{3}}d\arctan\left(\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+a^{\frac{4}{3}}d\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2a^{\frac{4}{3}}d\log\left((bx^3+a\right.}{}$$

```
[In] integrate((b*x^3+a)^(4/3)/x/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*a^(4/3)*d*arctan(1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*a^(2/3) +
 sqrt(3)*a)/a) + a^(4/3)*d*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3
) + a^(2/3)) - 2*a^(4/3)*d*log((b*x^3 + a)^(1/3) - a^(1/3)) - 2*sqrt(3)*(b*
c - a*d)*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(
(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) - 6*(b*x^3 + a)^(1
/3)*b*c - (b*c - a*d)*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) - (b*x^3
+ a)^(1/3)*((b*c - a*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3)) + 2*(b*c - a*d)*(
(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b*c - a*d)/d)^(1/3)))/(c*d)
```

Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{x(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{4}{3}}}{x(c+dx^3)}\,dx$$

```
[In] integrate((b*x**3+a)**(4/3)/x/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(x*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{x(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x}\,dx$$

```
[In] integrate((b*x^3+a)^(4/3)/x/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.55 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx^3)^{4/3}}{x(c+dx^3)}\,dx = -\frac{\sqrt{3}a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,c} - \frac{a^{\frac{4}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,c} + \frac{a^{\frac{4}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,c} + \frac{(b^2c^2-2\,abcd+a^2d^2)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2d-acd^2)} + \frac{(bx^3+a)^{\frac{1}{3}}b}{d} - \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,cd^2} - \frac{(-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,cd^2}$$

```
[In] integrate((b*x^3+a)^(4/3)/x/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*a^(4/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(
1/3))/c - 1/6*a^(4/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a
^(2/3))/c + 1/3*a^(4/3)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/c + 1/3*(b^2*
c^2 - 2*a*b*c*d + a^2*d^2)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3)
 - (-(b*c - a*d)/d)^(1/3)))/(b*c^2*d - a*c*d^2) + (b*x^3 + a)^(1/3)*b/d - 1
/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*(b*x^
3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(c*d^2) - 1/
6*(-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^
(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(c*d^2)
```

Mupad [B] (verification not implemented)

Time = 9.92 (sec) , antiderivative size = 796, normalized size of antiderivative = 3.05

$$\int \frac{(a+bx^3)^{4/3}}{x(c+dx^3)}\,dx = \ln\left(c\,d\left(-\frac{(a\,d-b\,c)^4}{c^3\,d^4}\right)^{1/3} + a\,d\,(b\,x^3+a)^{1/3} - b\,c\,(b\,x^3+a)^{1/3}\right)\left(-\frac{a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4}{27\,c^3\,d^4}\right)^{1/3} + \ln\left(c\left(\frac{a^4}{c^3}\right)^{1/3} - a\,(b\,x^3+a\right.$$

```
[In] int((a + b*x^3)^(4/3)/(x*(c + d*x^3)),x)
```

```
[Out] log(c*d*(-(a*d - b*c)^4/(c^3*d^4))^(1/3) + a*d*(a + b*x^3)^(1/3) - b*c*(a +
 b*x^3)^(1/3))*(-(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4
*a^3*b*c*d^3)/(27*c^3*d^4))^(1/3) + log(c*(a^4/c^3)^(1/3) - a*(a + b*x^3)^(
1/3))*(a^4/(27*c^3))^(1/3) + (b*(a + b*x^3)^(1/3))/d - log(c*(a^4/c^3)^(1/3
) + 2*a*(a + b*x^3)^(1/3) + 3^(1/2)*c*(a^4/c^3)^(1/3)*1i)*((3^(1/2)*1i)/2 +
 1/2)*(a^4/(27*c^3))^(1/3) + log(c*(a^4/c^3)^(1/3)*1i + a*(a + b*x^3)^(1/3)
*2i + 3^(1/2)*c*(a^4/c^3)^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a^4/(27*c^3))^(1/3
) + log((3*a^2*b^4*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(2*a^4*d^4 + b^4*c^4 + 6
*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d + 3*a^2*b^4*c*((3^(1/2
)*1i)/2 - 1/2)*(-(a*d - b*c)^4/(c^3*d^4))^(1/3)*(2*a^5*d^5 - b^5*c^5 - 10*a
^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 6*a^4*b*c*d^4))*((3^(
1/2)*1i)/2 - 1/2)*(-(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d
- 4*a^3*b*c*d^3)/(27*c^3*d^4))^(1/3) - log((3*a^2*b^4*(a + b*x^3)^(1/3)*(a*
d - b*c)^2*(2*a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3
*b*c*d^3))/d - 3*a^2*b^4*c*((3^(1/2)*1i)/2 + 1/2)*(-(a*d - b*c)^4/(c^3*d^4)
)^(1/3)*(2*a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 5*
a*b^4*c^4*d - 6*a^4*b*c*d^4))*((3^(1/2)*1i)/2 + 1/2)*(-(a^4*d^4 + b^4*c^4 +
 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)/(27*c^3*d^4))^(1/3)
```

3.700 $\int \frac{(a+bx^3)^{4/3}}{x^4(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 399

$$\begin{aligned}\int \frac{(a+bx^3)^{4/3}}{x^4(c+dx^3)}\,dx &= \frac{(4bc-3ad)\sqrt[3]{a+bx^3}}{3c^2}-\frac{(bc-ad)\sqrt[3]{a+bx^3}}{c^2}+\frac{d(a+bx^3)^{4/3}}{4c^2}\\ &+\frac{(4bc-3ad)(a+bx^3)^{4/3}}{12ac^2}-\frac{(a+bx^3)^{7/3}}{3acx^3}-\frac{\sqrt[3]{a}(4bc-3ad)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}c^2}\\ &-\frac{(bc-ad)^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2\sqrt[3]{d}}-\frac{\sqrt[3]{a}(4bc-3ad)\log(x)}{6c^2}\\ &-\frac{(bc-ad)^{4/3}\log(c+dx^3)}{6c^2\sqrt[3]{d}}+\frac{\sqrt[3]{a}(4bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6c^2}\\ &+\frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2\sqrt[3]{d}}\end{aligned}$$

```
[Out] 1/3*(-3*a*d+4*b*c)*(b*x^3+a)^(1/3)/c^2-(-a*d+b*c)*(b*x^3+a)^(1/3)/c^2+1/4*d
*(b*x^3+a)^(4/3)/c^2+1/12*(-3*a*d+4*b*c)*(b*x^3+a)^(4/3)/a/c^2-1/3*(b*x^3+a
)^(7/3)/a/c/x^3-1/6*a^(1/3)*(-3*a*d+4*b*c)*ln(x)/c^2-1/6*(-a*d+b*c)^(4/3)*l
n(d*x^3+c)/c^2/d^(1/3)+1/6*a^(1/3)*(-3*a*d+4*b*c)*ln(a^(1/3)-(b*x^3+a)^(1/3
))/c^2+1/2*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/c^
2/d^(1/3)-1/9*a^(1/3)*(-3*a*d+4*b*c)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))
/a^(1/3)*3^(1/2))/c^2*3^(1/2)-1/3*(-a*d+b*c)^(4/3)*arctan(1/3*(1-2*d^(1/3)*
(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c^2/d^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {457, 105, 162, 52, 59, 631, 210, 31, 60}

$$\int \frac{(a+bx^3)^{4/3}}{x^4(c+dx^3)}\,dx = -\frac{\sqrt[3]{a}\arctan\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)(4bc-3ad)}{3\sqrt{3}c^2} - \frac{(bc-ad)^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2\sqrt[3]{d}} + \frac{d(a+bx^3)^{4/3}}{4c^2} + \frac{(a+bx^3)^{4/3}(4bc-3ad)}{12ac^2} + \frac{\sqrt[3]{a+bx^3}(4bc-3ad)}{3c^2} - \frac{\sqrt[3]{a+bx^3}(bc-ad)}{c^2} - \frac{(bc-ad)^{4/3}\log(c+dx^3)}{6c^2\sqrt[3]{d}} + \frac{\sqrt[3]{a}(4bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6c^2} + \frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2\sqrt[3]{d}} - \frac{\sqrt[3]{a}\log(x)(4bc-3ad)}{6c^2} - \frac{(a+bx^3)^{7/3}}{3acx^3}$$

```
[In] Int[(a + b*x^3)^(4/3)/(x^4*(c + d*x^3)),x]
```

```
[Out] ((4*b*c - 3*a*d)*(a + b*x^3)^(1/3))/(3*c^2) - ((b*c - a*d)*(a + b*x^3)^(1/3
))/c^2 + (d*(a + b*x^3)^(4/3))/(4*c^2) + ((4*b*c - 3*a*d)*(a + b*x^3)^(4/3)
)/(12*a*c^2) - (a + b*x^3)^(7/3)/(3*a*c*x^3) - (a^(1/3)*(4*b*c - 3*a*d)*Arc
Tan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*c^2) - (
(b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/
3))/Sqrt[3]])/(Sqrt[3]*c^2*d^(1/3)) - (a^(1/3)*(4*b*c - 3*a*d)*Log[x])/(6*c
^2) - ((b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*c^2*d^(1/3)) + (a^(1/3)*(4*b*c
- 3*a*d)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(6*c^2) + ((b*c - a*d)^(4/3)*Log
[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^2*d^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{x^2(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{(a+bx^3)^{7/3}}{3acx^3} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{4/3}\left(\frac{1}{3}(-4bc+3ad)-\frac{4bdx}{3}\right)}{x(c+dx)}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{(a+bx^3)^{7/3}}{3acx^3} + \frac{d^2\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{c+dx}\,dx, x, x^3\right)}{3c^2} + \frac{(4bc-3ad)\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{x}\,dx, x, x^3\right)}{9ac^2}$$

$$= \frac{d(a+bx^3)^{4/3}}{4c^2} + \frac{(4bc-3ad)\,(a+bx^3)^{4/3}}{12ac^2} - \frac{(a+bx^3)^{7/3}}{3acx^3} + \frac{(4bc-3ad)\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x}\,dx, x, x^3\right)}{9c^2} - \frac{(d(bc-ad))\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)}{3c^2}$$

$$= \frac{(4bc-3ad)\sqrt[3]{a+bx^3}}{3c^2} - \frac{(bc-ad)\sqrt[3]{a+bx^3}}{c^2} + \frac{d(a+bx^3)^{4/3}}{4c^2} + \frac{(4bc-3ad)\,(a+bx^3)^{4/3}}{12ac^2} - \frac{(a+bx^3)^{7/3}}{3acx^3} + \frac{(a(4bc-3ad))\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^3\right)}{9c^2} + \frac{(bc-ad)^2\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3c^2}$$

$$= \frac{(4bc-3ad)\sqrt[3]{a+bx^3}}{3c^2} - \frac{(bc-ad)\sqrt[3]{a+bx^3}}{c^2} + \frac{d(a+bx^3)^{4/3}}{4c^2} + \frac{(4bc-3ad)\left(a+bx^3\right)^{4/3}}{12ac^2} - \frac{\left(a+bx^3\right)^{7/3}}{3acx^3} - \frac{\sqrt[3]{a}(4bc-3ad)\log(x)}{6c^2} - \frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6c^2\sqrt[3]{d}} - \frac{\left(\sqrt[3]{a}(4bc-3ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{6c^2} - \frac{\left(a^{2/3}(4bc-3ad)\right)\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{6c^2} + \frac{(bc-ad)^{4/3}\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c^2\sqrt[3]{d}} + \frac{(bc-ad)^{5/3}\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c^2d^{2/3}}$$

$$= \frac{(4bc-3ad)\sqrt[3]{a+bx^3}}{3c^2} - \frac{(bc-ad)\sqrt[3]{a+bx^3}}{c^2} + \frac{d(a+bx^3)^{4/3}}{4c^2} + \frac{(4bc-3ad)\left(a+bx^3\right)^{4/3}}{12ac^2} - \frac{\left(a+bx^3\right)^{7/3}}{3acx^3} - \frac{\sqrt[3]{a}(4bc-3ad)\log(x)}{6c^2} - \frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6c^2\sqrt[3]{d}} + \frac{\sqrt[3]{a}(4bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6c^2} + \frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2\sqrt[3]{d}} + \frac{\left(\sqrt[3]{a}(4bc-3ad)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{3c^2} + \frac{(bc-ad)^{4/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{c^2\sqrt[3]{d}}$$

$$= \frac{(4bc-3ad)\sqrt[3]{a+bx^3}}{3c^2} - \frac{(bc-ad)\sqrt[3]{a+bx^3}}{c^2} + \frac{d(a+bx^3)^{4/3}}{4c^2} + \frac{(4bc-3ad)\left(a+bx^3\right)^{4/3}}{12ac^2} - \frac{(a+bx^3)^{7/3}}{3acx^3} - \frac{\sqrt[3]{a}(4bc-3ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}c^2} - \frac{(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2\sqrt[3]{d}} - \frac{\sqrt[3]{a}(4bc-3ad)\log(x)}{6c^2} - \frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6c^2\sqrt[3]{d}} + \frac{\sqrt[3]{a}(4bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6c^2} + \frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2\sqrt[3]{d}}$$

Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 353, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^{4/3}}{x^4\left(c+dx^3\right)}\,dx = \frac{-\frac{6ac\sqrt[3]{a+bx^3}}{x^3} + 2\sqrt{3}\sqrt[3]{a}(-4bc+3ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - \frac{6\sqrt{3}(bc-ad)^{4/3}\arctan\left(\frac{1-\cdots}{\cdots}\right)}{\sqrt[3]{\cdots}}\cdots}{\cdots}$$

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^4*(c + d*x^3)),x]
```

```
[Out] ((-6*a*c*(a + b*x^3)^(1/3))/x^3 + 2*Sqrt[3]*a^(1/3)*(-4*b*c + 3*a*d)*ArcTan
[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - (6*Sqrt[3]*(b*c - a*d)^(4/3
)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/d^
(1/3) - 2*a^(1/3)*(-4*b*c + 3*a*d)*Log[-a^(1/3) + (a + b*x^3)^(1/3)] + (6*(
b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/d^(1/3
) + a^(1/3)*(-4*b*c + 3*a*d)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a +
 b*x^3)^(2/3)] - (3*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c
- a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/d^(1/3))/(18*c
^2)
```

Maple [A] (verified)

Time = 5.04 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.91

method	result
pseudoelliptic	$-\frac{x^{3}(ad-bc)^{2}\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\right)}{2}-x^{3}\sqrt{3}\,(ad-bc)^{2}\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)+-\frac{x^{3}}{}$

[In] int((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] 1/3*(-1/2*x^3*(a*d-b*c)^2*ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-x^3*3^(1/2)*(a*d-b*c)^2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))+1/2*x^3*(-4/3*a^(1/3)*b*c+d*a^(4/3))*d*(1/d*(a*d-b*c))^(2/3)*ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3))+x^3*(a*d-b*c)^2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))-d*(1/d*(a*d-b*c))^(2/3)*(-x^3*(-4/3*a^(1/3)*b*c+d*a^(4/3))*3^(1/2)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))+x^3*(-4/3*a^(1/3)*b*c+d*a^(4/3))*ln((b*x^3+a)^(1/3)-a^(1/3))+(b*x^3+a)^(1/3)*a*c))/(1/d*(a*d-b*c))^(2/3)/c^2/d/x^3

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^4\left(c+dx^3\right)}\,dx=\frac{6\sqrt{3}(bc-ad)x^3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3\,(bc-ad)}\right)+2\sqrt{3}(4\,bc-3}{}$$

[In] integrate((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x, algorithm="fricas")

[Out] 1/18*(6*sqrt(3)*(b*c - a*d)*x^3*(-(b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 2*sqrt(3)*(4*b*c - 3*a*d)*(-a)^(1/3)*x^3*arctan(1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + (4*b*c - 3*a*d)*(-a)^(1/3)*x^3*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 3*(b*c - a*d)*x^3*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3)) - 2*(4*b*c - 3*a*d)*(-a)^(1/3)*x^3*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) - 6*(b*c - a*d)*x^3*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)) - 6*(b*x^3 + a)^(1/3)*a*c)/(c^2*x^3)

Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^4(c+dx^3)}\,dx=\int \frac{(a+bx^3)^{\frac{4}{3}}}{x^4(c+dx^3)}\,dx$$

```
[In] integrate((b*x**3+a)**(4/3)/x**4/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(4/3)/(x**4*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^4(c+dx^3)}\,dx=\int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^4}\,dx$$

```
[In] integrate((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.58 (sec) , antiderivative size = 394, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \frac{(a+bx^3)^{4/3}}{x^4(c+dx^3)}\,dx=&-\frac{(b^2c^2-2\,abcd+a^2d^2)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^3-ac^2d)}\\&-\frac{\sqrt{3}\left(4\,a^{\frac{1}{3}}bc-3\,a^{\frac{4}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,c^2}\\&-\frac{\left(4\,a^{\frac{1}{3}}bc-3\,a^{\frac{4}{3}}d\right)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,c^2}\\&+\frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,c^2d}\\&+\frac{(-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,c^2d}\\&+\frac{(4\,abc-3\,a^2d)\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{2}{3}}c^2}-\frac{(bx^3+a)^{\frac{1}{3}}a}{3\,cx^3}\end{aligned}$$

```
[In] integrate((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3
+ a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c^3 - a*c^2*d) - 1/9*sqrt(3)*(4*a^
(1/3)*b*c - 3*a^(4/3)*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))
/a^(1/3))/c^2 - 1/18*(4*a^(1/3)*b*c - 3*a^(4/3)*d)*log((b*x^3 + a)^(2/3) +
(b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/c^2 + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(
1/3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)
^(1/3))/(-(b*c - a*d)/d)^(1/3))/(c^2*d) + 1/6*(-b*c*d^2 + a*d^3)^(1/3)*(b*c
 - a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) +
(-(b*c - a*d)/d)^(2/3))/(c^2*d) + 1/9*(4*a*b*c - 3*a^2*d)*log(abs((b*x^3 +
a)^(1/3) - a^(1/3)))/(a^(2/3)*c^2) - 1/3*(b*x^3 + a)^(1/3)*a/(c*x^3)
```

Mupad [B] (verification not implemented)

Time = 14.72 (sec) , antiderivative size = 2047, normalized size of antiderivative = 5.13

$$\int \frac{(a+bx^3)^{4/3}}{x^4(c+dx^3)}\,dx = \text{Too large to display}$$

```
[In] int((a + b*x^3)^(4/3)/(x^4*(c + d*x^3)),x)
```

```
[Out] log(c^2*(-(a*(3*a*d - 4*b*c)^3)/c^6)^(1/3) + 3*a*d*(a + b*x^3)^(1/3) - 4*b*
c*(a + b*x^3)^(1/3))*(-(27*a^4*d^3 - 64*a*b^3*c^3 + 144*a^2*b^2*c^2*d - 108
*a^3*b*c*d^2)/(729*c^6))^(1/3) + log((((81*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*
c^2 - 3*a*b*c*d)*((a*d - b*c)^4/(c^6*d))^(1/3) - 108*a*b^5*c^3*d^3*(a + b*x
^3)^(1/3)*(a*d - b*c)^2)*((a*d - b*c)^4/(c^6*d))^(2/3))/9 + (a*b^5*d^2*(27*
a^5*d^5 - 27*b^5*c^5 - 341*a^2*b^3*c^3*d^2 + 332*a^3*b^2*c^2*d^3 + 162*a*b^
4*c^4*d - 153*a^4*b*c*d^4))/(3*c))*((a*d - b*c)^4/(c^6*d))^(1/3))/3 - (a*b^
4*d^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(54*a^5*d^5 - 36*b^5*c^5 - 388*a^2*b^
3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 171*a*b^4*c^4*d - 252*a^4*b*c*d^4))/(9*c^
4))*((a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3
)/(27*c^6*d))^(1/3) + log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*
(108*a*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2 - 81*a*b^4*c^4*d^3*((3^(
1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((a*d - b*c)^4/(c^6*d))
^(1/3))*((a*d - b*c)^4/(c^6*d))^(2/3))/9 + (a*b^5*d^2*(27*a^5*d^5 - 27*b^5*
c^5 - 341*a^2*b^3*c^3*d^2 + 332*a^3*b^2*c^2*d^3 + 162*a*b^4*c^4*d - 153*a^4
*b*c*d^4))/(3*c))*((a*d - b*c)^4/(c^6*d))^(1/3))/3 - (a*b^4*d^2*(a + b*x^3)
^(1/3)*(a*d - b*c)^2*(54*a^5*d^5 - 36*b^5*c^5 - 388*a^2*b^3*c^3*d^2 + 450*a
^3*b^2*c^2*d^3 + 171*a*b^4*c^4*d - 252*a^4*b*c*d^4))/(9*c^4))*((3^(1/2)*1i)
/2 - 1/2)*((a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b
*c*d^3)/(27*c^6*d))^(1/3) - log((a*b^4*d^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2*
(54*a^5*d^5 - 36*b^5*c^5 - 388*a^2*b^3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 171*
a*b^4*c^4*d - 252*a^4*b*c*d^4))/(9*c^4) - (((3^(1/2)*1i)/2 + 1/2)*((((3^(1/
2)*1i)/2 - 1/2)*(108*a*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2 + 81*a*b
```

```
^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((a*d -
 b*c)^4/(c^6*d))^(1/3))*((a*d - b*c)^4/(c^6*d))^(2/3))/9 - (a*b^5*d^2*(27*a
^5*d^5 - 27*b^5*c^5 - 341*a^2*b^3*c^3*d^2 + 332*a^3*b^2*c^2*d^3 + 162*a*b^4
*c^4*d - 153*a^4*b*c*d^4))/(3*c))*((a*d - b*c)^4/(c^6*d))^(1/3))/3)*((3^(1/
2)*1i)/2 + 1/2)*((a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4
*a^3*b*c*d^3)/(27*c^6*d))^(1/3) + log((a*b^4*d^2*(a + b*x^3)^(1/3)*(a*d - b
*c)^2*(54*a^5*d^5 - 36*b^5*c^5 - 388*a^2*b^3*c^3*d^2 + 450*a^3*b^2*c^2*d^3
+ 171*a*b^4*c^4*d - 252*a^4*b*c*d^4))/(9*c^4) - (((3^(1/2)*1i)/2 - 1/2)*(((
(3^(1/2)*1i)/2 + 1/2)*(108*a*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2 -
27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(
-(a*(3*a*d - 4*b*c)^3)/c^6)^(1/3))*(-(a*(3*a*d - 4*b*c)^3)/c^6)^(2/3))/81 +
 (a*b^5*d^2*(27*a^5*d^5 - 27*b^5*c^5 - 341*a^2*b^3*c^3*d^2 + 332*a^3*b^2*c^
2*d^3 + 162*a*b^4*c^4*d - 153*a^4*b*c*d^4))/(3*c))*(-(a*(3*a*d - 4*b*c)^3)/
c^6)^(1/3))/9)*((3^(1/2)*1i)/2 - 1/2)*(-(27*a^4*d^3 - 64*a*b^3*c^3 + 144*a^
2*b^2*c^2*d - 108*a^3*b*c*d^2)/(729*c^6))^(1/3) - log((a*b^4*d^2*(a + b*x^3
)^(1/3)*(a*d - b*c)^2*(54*a^5*d^5 - 36*b^5*c^5 - 388*a^2*b^3*c^3*d^2 + 450*
a^3*b^2*c^2*d^3 + 171*a*b^4*c^4*d - 252*a^4*b*c*d^4))/(9*c^4) - (((3^(1/2)*
1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*(108*a*b^5*c^3*d^3*(a + b*x^3)^(1/3)*
(a*d - b*c)^2 + 27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^
2 - 3*a*b*c*d)*(-(a*(3*a*d - 4*b*c)^3)/c^6)^(1/3))*(-(a*(3*a*d - 4*b*c)^3)/
c^6)^(2/3))/81 - (a*b^5*d^2*(27*a^5*d^5 - 27*b^5*c^5 - 341*a^2*b^3*c^3*d^2
+ 332*a^3*b^2*c^2*d^3 + 162*a*b^4*c^4*d - 153*a^4*b*c*d^4))/(3*c))*(-(a*(3*
a*d - 4*b*c)^3)/c^6)^(1/3))/9)*((3^(1/2)*1i)/2 + 1/2)*(-(27*a^4*d^3 - 64*a*
b^3*c^3 + 144*a^2*b^2*c^2*d - 108*a^3*b*c*d^2)/(729*c^6))^(1/3) - (a*(a + b
*x^3)^(1/3))/(3*c*x^3)
```

3.701 $\int \frac{(a+bx^3)^{4/3}}{x^7(c+dx^3)} dx$

Optimal result

Integrand size = 24, antiderivative size = 440

$$\begin{aligned}
\int \frac{(a+bx^3)^{4/3}}{x^7(c+dx^3)}\,dx &= \frac{d(bc-ad)\sqrt[3]{a+bx^3}}{c^3} \\
&+ \frac{(2b^2c^2-12abcd+9a^2d^2)\sqrt[3]{a+bx^3}}{9ac^3} - \frac{(bc-6ad)(a+bx^3)^{4/3}}{18ac^2x^3} \\
&- \frac{(a+bx^3)^{7/3}}{6acx^6} - \frac{(2b^2c^2-12abcd+9a^2d^2)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{2/3}c^3} \\
&+ \frac{d^{2/3}(bc-ad)^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^3} - \frac{(2b^2c^2-12abcd+9a^2d^2)\log(x)}{18a^{2/3}c^3} \\
&+ \frac{d^{2/3}(bc-ad)^{4/3}\log(c+dx^3)}{6c^3} + \frac{(2b^2c^2-12abcd+9a^2d^2)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{2/3}c^3} \\
&- \frac{d^{2/3}(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^3}
\end{aligned}$$

```
[Out] d*(-a*d+b*c)*(b*x^3+a)^(1/3)/c^3+1/9*(9*a^2*d^2-12*a*b*c*d+2*b^2*c^2)*(b*x^
3+a)^(1/3)/a/c^3-1/18*(-6*a*d+b*c)*(b*x^3+a)^(4/3)/a/c^2/x^3-1/6*(b*x^3+a)^
(7/3)/a/c/x^6-1/18*(9*a^2*d^2-12*a*b*c*d+2*b^2*c^2)*ln(x)/a^(2/3)/c^3+1/6*d
^(2/3)*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/c^3+1/18*(9*a^2*d^2-12*a*b*c*d+2*b^2*c^
2)*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(2/3)/c^3-1/2*d^(2/3)*(-a*d+b*c)^(4/3)*ln(
(-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/c^3-1/27*(9*a^2*d^2-12*a*b*c*d+2*
b^2*c^2)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(2/3)/c^
```

```
3*3^(1/2)+1/3*d^(2/3)*(-a*d+b*c)^(4/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1
/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c^3*3^(1/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {457, 105, 154, 162, 52, 59, 631, 210, 31, 60}

$$\int \frac{(a+bx^3)^{4/3}}{x^7(c+dx^3)}\,dx = \frac{\sqrt[3]{a+bx^3}(9a^2d^2-12abcd+2b^2c^2)}{9ac^3} - \frac{\arctan\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)(9a^2d^2-12abcd+2b^2c^2)}{9\sqrt{3}a^{2/3}c^3} + \frac{(9a^2d^2-12abcd+2b^2c^2)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{2/3}c^3} - \frac{\log(x)(9a^2d^2-12abcd+2b^2c^2)}{18a^{2/3}c^3} + \frac{d^{2/3}(bc-ad)^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^3} + \frac{d^{2/3}(bc-ad)^{4/3}\log(c+dx^3)}{6c^3} - \frac{d^{2/3}(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^3} + \frac{d\sqrt[3]{a+bx^3}(bc-ad)}{c^3} - \frac{(a+bx^3)^{4/3}(bc-6ad)}{18ac^2x^3} - \frac{(a+bx^3)^{7/3}}{6acx^6}$$

```
[In] Int[(a + b*x^3)^(4/3)/(x^7*(c + d*x^3)),x]
```

```
[Out] (d*(b*c - a*d)*(a + b*x^3)^(1/3))/c^3 + ((2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^
2)*(a + b*x^3)^(1/3))/(9*a*c^3) - ((b*c - 6*a*d)*(a + b*x^3)^(4/3))/(18*a*c
^2*x^3) - (a + b*x^3)^(7/3)/(6*a*c*x^6) - ((2*b^2*c^2 - 12*a*b*c*d + 9*a^2*
d^2)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*
a^(2/3)*c^3) + (d^(2/3)*(b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3
)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c^3) - ((2*b^2*c^2 - 12*a*b*
c*d + 9*a^2*d^2)*Log[x])/(18*a^(2/3)*c^3) + (d^(2/3)*(b*c - a*d)^(4/3)*Log[
c + d*x^3])/(6*c^3) + ((2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*Log[a^(1/3) - (
a + b*x^3)^(1/3)])/(18*a^(2/3)*c^3) - (d^(2/3)*(b*c - a*d)^(4/3)*Log[(b*c -
 a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{x^3(c+dx)}\,dx, x, x^3\right)\\
&= -\frac{\left(a+bx^3\right)^{7/3}}{6acx^6} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{4/3}\left(\frac{1}{3}(-bc+6ad)-\frac{bdx}{3}\right)}{x^2(c+dx)}\,dx, x, x^3\right)}{6ac}\\
&= -\frac{(bc-6ad)\left(a+bx^3\right)^{4/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{7/3}}{6acx^6}\\
&\quad - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}\left(-\frac{2}{9}\left(2b^2c^2-12abcd+9a^2d^2\right)-\frac{2}{9}bd(2bc-3ad)x\right)}{x(c+dx)}\,dx, x, x^3\right)}{6ac^2}
\end{aligned}$$

$$= -\frac{(bc-6ad)(a+bx^3)^{4/3}}{18ac^2x^3} - \frac{(a+bx^3)^{7/3}}{6acx^6} + \frac{(d^2(bc-ad))\operatorname{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)}{3c^3} + \frac{(2b^2c^2-12abcd+9a^2d^2)\operatorname{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x}\,dx, x, x^3\right)}{27ac^3}$$

$$= \frac{d(bc-ad)\sqrt[3]{a+bx^3}}{c^3} + \frac{(2b^2c^2-12abcd+9a^2d^2)\sqrt[3]{a+bx^3}}{9ac^3} - \frac{(bc-6ad)(a+bx^3)^{4/3}}{18ac^2x^3} - \frac{(a+bx^3)^{7/3}}{6acx^6} - \frac{(d(bc-ad)^2)\operatorname{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3c^3} + \frac{(2b^2c^2-12abcd+9a^2d^2)\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^3\right)}{27c^3}$$

$$= \frac{d(bc-ad)\sqrt[3]{a+bx^3}}{c^3} + \frac{(2b^2c^2-12abcd+9a^2d^2)\sqrt[3]{a+bx^3}}{9ac^3} - \frac{(bc-6ad)(a+bx^3)^{4/3}}{18ac^2x^3} - \frac{(a+bx^3)^{7/3}}{6acx^6} - \frac{(2b^2c^2-12abcd+9a^2d^2)\log(x)}{18a^{2/3}c^3} + \frac{d^{2/3}(bc-ad)^{4/3}\log(c+dx^3)}{6c^3} - \frac{\left(d^{2/3}(bc-ad)^{4/3}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c^3} - \frac{\left(\sqrt[3]{d}(bc-ad)^{5/3}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c^3} - \frac{(2b^2c^2-12abcd+9a^2d^2)\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{18a^{2/3}c^3} - \frac{(2b^2c^2-12abcd+9a^2d^2)\operatorname{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{18\sqrt[3]{a}c^3}$$

$$= \frac{d(bc-ad)\sqrt[3]{a+bx^3}}{c^3} + \frac{(2b^2c^2-12abcd+9a^2d^2)\sqrt[3]{a+bx^3}}{9ac^3} - \frac{(bc-6ad)(a+bx^3)^{4/3}}{18ac^2x^3} - \frac{(a+bx^3)^{7/3}}{6acx^6} - \frac{(2b^2c^2-12abcd+9a^2d^2)\log(x)}{18a^{2/3}c^3} + \frac{d^{2/3}(bc-ad)^{4/3}\log(c+dx^3)}{6c^3} + \frac{(2b^2c^2-12abcd+9a^2d^2)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{2/3}c^3} - \frac{d^{2/3}(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^3} - \frac{\left(d^{2/3}(bc-ad)^{4/3}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{c^3} + \frac{(2b^2c^2-12abcd+9a^2d^2)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{9a^{2/3}c^3}$$

$$= \frac{d(bc-ad)\sqrt[3]{a+bx^3}}{c^3} + \frac{(2b^2c^2-12abcd+9a^2d^2)\sqrt[3]{a+bx^3}}{9ac^3} - \frac{(bc-6ad)(a+bx^3)^{4/3}}{18ac^2x^3} - \frac{(a+bx^3)^{7/3}}{6acx^6} - \frac{(2b^2c^2-12abcd+9a^2d^2)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}a^{2/3}c^3} + \frac{d^{2/3}(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^3} - \frac{(2b^2c^2-12abcd+9a^2d^2)\log(x)}{18a^{2/3}c^3} + \frac{d^{2/3}(bc-ad)^{4/3}\log(c+dx^3)}{6c^3} + \frac{(2b^2c^2-12abcd+9a^2d^2)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{2/3}c^3} - \frac{d^{2/3}(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^3}$$

Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 413, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)^{4/3}}{x^7(c+dx^3)}\,dx = \frac{\frac{3c\sqrt[3]{a+bx^3}(-3ac-7bcx^3+6adx^3)}{x^6} - \frac{2\sqrt{3}(2b^2c^2-12abcd+9a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} + 18\sqrt{3}d^{2/3}}{}$$

[In] Integrate[(a + b*x^3)^(4/3)/(x^7*(c + d*x^3)),x]

[Out] ((3*c*(a + b*x^3)^(1/3)*(-3*a*c - 7*b*c*x^3 + 6*a*d*x^3))/x^6 - (2*Sqrt[3]*(2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3) + 18*Sqrt[3]*d^(2/3)*(b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + (2*(2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(2/3) - 18*d^(2/3)*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + ((-2*b^2*c^2 + 12*a*b*c*d - 9*a^2*d^2)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/a^(2/3) + 9*d^(2/3)*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(54*c^3)

Maple [A] (verified)

Time = 5.02 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$-\frac{-\frac{x^{6}\left(b^{2}c^{2}a^{\frac{2}{3}}+a^{\frac{5}{3}}(ad-2bc)d\right)\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\right)}{2}-x^{6}\sqrt{3}\left(b^{2}c^{2}a^{\frac{2}{3}}+a^{\frac{5}{3}}(ad-2bc)d\right)\arctan\left(\cdots\right.}{\cdots}$

[In] int((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] -1/3*(-1/2*x^6*(b^2*c^2*a^(2/3)+a^(5/3)*(a*d-2*b*c)*d)*ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-x^6*3^(1/2)*(b^2*c^2*a^(2/3)+a^(5/3)*(a*d-2*b*c)*d)*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))+1/2*(1/d*(a*d-b*c))^(2/3)*x^6*(a^2*d^2-4/3*a*b*c*d+2/9*b^2*c^2)*ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3))+x^6*(b^2*c^2*a^(2/3)+a^(5/3)*(a*d-2*b*c)*d)*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))+1/2*(1/d*(a*d-b*c))^(2/3)*(2*x^6*(a^2*d^2-4/3*a*b*c*d+2/9*b^2*c^2)*3^(1/2)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))-2*x^6*(a^2*d^2-4/3*a*b*c*d+2/9*b^2*c^2)*ln((b*x^3+a)^(1/3)-a^(1/3))+(b*x^3+a)^(1/3)*(7/3*b*c*x^3*a^(2/3)+a^(5/3)*(-2*d*x^3+c))*c))/a^(2/3)/(1/d*(a*d-b*c))^(2/3)/c^3/x^6

Fricas [A] (verification not implemented)

none

Time = 1.17 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^3)^{4/3}}{x^7(c+dx^3)}\,dx = \frac{18\sqrt{3}(a^2bc-a^3d)(bcd^2-ad^3)^{\frac{1}{3}}x^6\arctan\left(-\frac{2\sqrt{3}(bcd^2-ad^3)^{\frac{2}{3}}(bx^3+a)^{\frac{1}{3}}-\sqrt{3}(bcd-ad^2)}{3(bcd-ad^2)}\right)-2\sqrt{}}{}$$

```
[In] integrate((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/54*(18*sqrt(3)*(a^2*b*c - a^3*d)*(b*c*d^2 - a*d^3)^(1/3)*x^6*arctan(-1/3*
(2*sqrt(3)*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(1/3) - sqrt(3)*(b*c*d - a*d
^2))/(b*c*d - a*d^2)) - 2*sqrt(3)*(2*a*b^2*c^2 - 12*a^2*b*c*d + 9*a^3*d^2)*
(a^2)^(1/6)*x^6*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^(1/3)*a + 2*sqrt(3)*(
b*x^3 + a)^(1/3)*(a^2)^(2/3))/a^2) - (2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*(
a^2)^(2/3)*x^6*log((b*x^3 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*
(a^2)^(2/3)) + 2*(2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*(a^2)^(2/3)*x^6*log((
b*x^3 + a)^(1/3)*a - (a^2)^(2/3)) + 9*(a^2*b*c - a^3*d)*(b*c*d^2 - a*d^3)^(
1/3)*x^6*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1
/3)*d + (b*c*d^2 - a*d^3)^(2/3)) - 18*(a^2*b*c - a^3*d)*(b*c*d^2 - a*d^3)^(
1/3)*x^6*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) - 3*(3*a^3*c^2
+ (7*a^2*b*c^2 - 6*a^3*c*d)*x^3)*(b*x^3 + a)^(1/3))/(a^2*c^3*x^6)
```

Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^7(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{4}{3}}}{x^7(c+dx^3)}\,dx$$

```
[In] integrate((b*x**3+a)**(4/3)/x**7/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(x**7*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^7(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^7}\,dx$$

```
[In] integrate((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^7), x)
```

Giac [A] (verification not implemented)

none

Time = 0.56 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^3)^{4/3}}{x^7(c+dx^3)}\,dx = \frac{(b^2c^2d-2\,abcd^2+a^2d^3)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^4-ac^3d)} - \frac{\sqrt{3}(-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,c^3} - \frac{(-bcd^2+ad^3)^{\frac{1}{3}}(bc-ad)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,c^3} - \frac{\sqrt{3}(2\,b^2c^2-12\,abcd+9\,a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{27\,a^{\frac{2}{3}}c^3} - \frac{(2\,b^2c^2-12\,abcd+9\,a^2d^2)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{54\,a^{\frac{2}{3}}c^3} + \frac{(2\,b^2c^2-12\,abcd+9\,a^2d^2)\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{27\,a^{\frac{2}{3}}c^3} - \frac{7\,(bx^3+a)^{\frac{4}{3}}b^2c-4\,(bx^3+a)^{\frac{1}{3}}ab^2c-6\,(bx^3+a)^{\frac{4}{3}}abd+6\,(bx^3+a)^{\frac{1}{3}}a^2bd}{18\,b^2c^2x^6}$$

```
[In] integrate((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x
^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c^4 - a*c^3*d) - 1/3*sqrt(3)*(-
b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3)
+ (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/c^3 - 1/6*(-b*c*d^2 + a*d
^3)^(1/3)*(b*c - a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*
d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/c^3 - 1/27*sqrt(3)*(2*b^2*c^2 - 12*a*
b*c*d + 9*a^2*d^2)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/
3))/(a^(2/3)*c^3) - 1/54*(2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*log((b*x^3 +
a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(2/3)*c^3) + 1/27*(2*b^2
*c^2 - 12*a*b*c*d + 9*a^2*d^2)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(2/
3)*c^3) - 1/18*(7*(b*x^3 + a)^(4/3)*b^2*c - 4*(b*x^3 + a)^(1/3)*a*b^2*c - 6
*(b*x^3 + a)^(4/3)*a*b*d + 6*(b*x^3 + a)^(1/3)*a^2*b*d)/(b^2*c^2*x^6)
```

Mupad [B] (verification not implemented)

Time = 16.70 (sec) , antiderivative size = 2841, normalized size of antiderivative = 6.46

$$\int \frac{(a+bx^3)^{4/3}}{x^7(c+dx^3)}\,dx = \text{Too large to display}$$

```
[In] int((a + b*x^3)^(4/3)/(x^7*(c + d*x^3)),x)
```

```
[Out] log(((((18*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(6*a*d - b*c) + 9*a*
b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((9*a^2*d^2 + 2*b^2*c^2 - 12*
a*b*c*d)^3/(a^2*c^9))^(1/3))*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c
^9))^(2/3))/729 + (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 - 3258*a^2*b^4*c^4*d^2
+ 6939*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a*b^5*c^5*d + 3645*a^5*
b*c*d^5))/(81*c^4))*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9))^(1/3
))/27 - (b^4*d^5*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(1458*a^6*d^6 + 170*b^6*c^
6 + 6561*a^2*b^4*c^4*d^2 - 12420*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^2*d^4 -
1764*a*b^5*c^5*d - 6804*a^5*b*c*d^5))/(243*c^8))*((729*a^6*d^6 + 8*b^6*c^6
+ 972*a^2*b^4*c^4*d^2 - 3024*a^3*b^3*c^3*d^3 + 4374*a^4*b^2*c^2*d^4 - 144*a
*b^5*c^5*d - 2916*a^5*b*c*d^5)/(19683*a^2*c^9))^(1/3) + log(((((18*b^5*c^2*
d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(6*a*d - b*c) + 81*a*b^4*c^4*d^3*(2*a^2
*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^2*(a*d - b*c)^4)/c^9)^(1/3))*(-(d^2*(a*d -
 b*c)^4)/c^9)^(2/3))/9 + (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 - 3258*a^2*b^4*c
^4*d^2 + 6939*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a*b^5*c^5*d + 36
45*a^5*b*c*d^5))/(81*c^4))*(-(d^2*(a*d - b*c)^4)/c^9)^(1/3))/3 - (b^4*d^5*(
a + b*x^3)^(1/3)*(a*d - b*c)^2*(1458*a^6*d^6 + 170*b^6*c^6 + 6561*a^2*b^4*c
^4*d^2 - 12420*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^2*d^4 - 1764*a*b^5*c^5*d -
 6804*a^5*b*c*d^5))/(243*c^8))*(-(a^4*d^6 + b^4*c^4*d^2 - 4*a*b^3*c^3*d^3 +
 6*a^2*b^2*c^2*d^4 - 4*a^3*b*c*d^5)/(27*c^9))^(1/3) + (((2*a*b^2*c - 3*a^2*
b*d)*(a + b*x^3)^(1/3))/(9*c^2) + (b*(a + b*x^3)^(4/3)*(6*a*d - 7*b*c))/(18
*c^2))/((a + b*x^3)^2 - 2*a*(a + b*x^3) + a^2) + log((((3^(1/2)*1i)/2 - 1/2
)*((((3^(1/2)*1i)/2 + 1/2)*(18*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*
(6*a*d - b*c) + 81*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^
2 - 3*a*b*c*d)*(-(d^2*(a*d - b*c)^4)/c^9)^(1/3))*(-(d^2*(a*d - b*c)^4)/c^9)
^(2/3))/9 - (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 - 3258*a^2*b^4*c^4*d^2 + 6939
*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a*b^5*c^5*d + 3645*a^5*b*c*d^
5))/(81*c^4))*(-(d^2*(a*d - b*c)^4)/c^9)^(1/3))/3 + (b^4*d^5*(a + b*x^3)^(1
/3)*(a*d - b*c)^2*(1458*a^6*d^6 + 170*b^6*c^6 + 6561*a^2*b^4*c^4*d^2 - 1242
0*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^2*d^4 - 1764*a*b^5*c^5*d - 6804*a^5*b*c
*d^5))/(243*c^8))*((3^(1/2)*1i)/2 - 1/2)*(-(a^4*d^6 + b^4*c^4*d^2 - 4*a*b^3
*c^3*d^3 + 6*a^2*b^2*c^2*d^4 - 4*a^3*b*c*d^5)/(27*c^9))^(1/3) - log((((3^(1
/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*(18*b^5*c^2*d^3*(a + b*x^3)^(1/3)
*(a*d - b*c)^2*(6*a*d - b*c) - 81*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*(2*a
^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^2*(a*d - b*c)^4)/c^9)^(1/3))*(-(d^2*(a*d
 - b*c)^4)/c^9)^(2/3))/9 + (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 - 3258*a^2*b^4
```

```
*c^4*d^2 + 6939*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a*b^5*c^5*d +
3645*a^5*b*c*d^5))/(81*c^4))*(-(d^2*(a*d - b*c)^4)/c^9)^(1/3))/3 + (b^4*d^5
*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(1458*a^6*d^6 + 170*b^6*c^6 + 6561*a^2*b^4
*c^4*d^2 - 12420*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^2*d^4 - 1764*a*b^5*c^5*d
 - 6804*a^5*b*c*d^5))/(243*c^8))*((3^(1/2)*1i)/2 + 1/2)*(-(a^4*d^6 + b^4*c^
4*d^2 - 4*a*b^3*c^3*d^3 + 6*a^2*b^2*c^2*d^4 - 4*a^3*b*c*d^5)/(27*c^9))^(1/3
) + log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*(18*b^5*c^2*d^3*(a
 + b*x^3)^(1/3)*(a*d - b*c)^2*(6*a*d - b*c) + 9*a*b^4*c^4*d^3*((3^(1/2)*1i)
/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*
b*c*d)^3/(a^2*c^9))^(1/3))*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9
))^(2/3))/729 - (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 - 3258*a^2*b^4*c^4*d^2 +
6939*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a*b^5*c^5*d + 3645*a^5*b*
c*d^5))/(81*c^4))*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9))^(1/3))
/27 + (b^4*d^5*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(1458*a^6*d^6 + 170*b^6*c^6
+ 6561*a^2*b^4*c^4*d^2 - 12420*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^2*d^4 - 17
64*a*b^5*c^5*d - 6804*a^5*b*c*d^5))/(243*c^8))*((3^(1/2)*1i)/2 - 1/2)*((729
*a^6*d^6 + 8*b^6*c^6 + 972*a^2*b^4*c^4*d^2 - 3024*a^3*b^3*c^3*d^3 + 4374*a^
4*b^2*c^2*d^4 - 144*a*b^5*c^5*d - 2916*a^5*b*c*d^5)/(19683*a^2*c^9))^(1/3)
- log((((3^(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*(18*b^5*c^2*d^3*(a +
 b*x^3)^(1/3)*(a*d - b*c)^2*(6*a*d - b*c) - 9*a*b^4*c^4*d^3*((3^(1/2)*1i)/2
 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*
c*d)^3/(a^2*c^9))^(1/3))*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9))
^(2/3))/729 + (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 - 3258*a^2*b^4*c^4*d^2 + 69
39*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a*b^5*c^5*d + 3645*a^5*b*c*
d^5))/(81*c^4))*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9))^(1/3))/2
7 + (b^4*d^5*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(1458*a^6*d^6 + 170*b^6*c^6 +
6561*a^2*b^4*c^4*d^2 - 12420*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^2*d^4 - 1764
*a*b^5*c^5*d - 6804*a^5*b*c*d^5))/(243*c^8))*((3^(1/2)*1i)/2 + 1/2)*((729*a
^6*d^6 + 8*b^6*c^6 + 972*a^2*b^4*c^4*d^2 - 3024*a^3*b^3*c^3*d^3 + 4374*a^4*
b^2*c^2*d^4 - 144*a*b^5*c^5*d - 2916*a^5*b*c*d^5)/(19683*a^2*c^9))^(1/3)
```

3.702 $\int \frac{x^4\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 334

$$\int \frac{x^4(a+bx^3)^{4/3}}{c+dx^3}\,dx = -\frac{(6bc-7ad)x^2\sqrt[3]{a+bx^3}}{18d^2}+\frac{bx^5\sqrt[3]{a+bx^3}}{6d} - \frac{(9b^2c^2-12abcd+2a^2d^2)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{2/3}d^3} + \frac{c^{2/3}(bc-ad)^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^3} - \frac{c^{2/3}(bc-ad)^{4/3}\log(c+dx^3)}{6d^3} - \frac{(9b^2c^2-12abcd+2a^2d^2)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{18b^{2/3}d^3} + \frac{c^{2/3}(bc-ad)^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^3}$$

```
[Out] -1/18*(-7*a*d+6*b*c)*x^2*(b*x^3+a)^(1/3)/d^2+1/6*b*x^5*(b*x^3+a)^(1/3)/d-1/
6*c^(2/3)*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/d^3-1/18*(2*a^2*d^2-12*a*b*c*d+9*b^2
*c^2)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)/d^3+1/2*c^(2/3)*(-a*d+b*c)^(4/3
)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^3-1/27*(2*a^2*d^2-12*a*b
*c*d+9*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)
/d^3*3^(1/2)+1/3*c^(2/3)*(-a*d+b*c)^(4/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*
x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/d^3*3^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {488, 596, 598, 337, 503}

$$\int \frac{x^4(a+bx^3)^{4/3}}{c+dx^3}\,dx = -\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(2a^2d^2-12abcd+9b^2c^2)}{9\sqrt{3}b^{2/3}d^3} - \frac{(2a^2d^2-12abcd+9b^2c^2)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{18b^{2/3}d^3} + \frac{c^{2/3}(bc-ad)^{4/3}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d^3} - \frac{c^{2/3}(bc-ad)^{4/3}\log(c+dx^3)}{6d^3} + \frac{c^{2/3}(bc-ad)^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^3} - \frac{x^2\sqrt[3]{a+bx^3}(6bc-7ad)}{18d^2} + \frac{bx^5\sqrt[3]{a+bx^3}}{6d}$$

```
[In] Int[(x^4*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] -1/18*((6*b*c - 7*a*d)*x^2*(a + b*x^3)^(1/3))/d^2 + (b*x^5*(a + b*x^3)^(1/3
))/(6*d) - ((9*b^2*c^2 - 12*a*b*c*d + 2*a^2*d^2)*ArcTan[(1 + (2*b^(1/3)*x)/
(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(2/3)*d^3) + (c^(2/3)*(b*c - a*d)
^(4/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqr
t[3]])/(Sqrt[3]*d^3) - (c^(2/3)*(b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*d^3) -
 ((9*b^2*c^2 - 12*a*b*c*d + 2*a^2*d^2)*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/
(18*b^(2/3)*d^3) + (c^(2/3)*(b*c - a*d)^(4/3)*Log[((b*c - a*d)^(1/3)*x)/c^(
1/3) - (a + b*x^3)^(1/3)])/(2*d^3)
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
```

```
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx^5\sqrt[3]{a+bx^3}}{6d}+\frac{\int \frac{x^4\left(-a(5bc-6ad)-b(6bc-7ad)x^3\right)}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx}{6d}\\
&=-\frac{(6bc-7ad)x^2\sqrt[3]{a+bx^3}}{18d^2}+\frac{bx^5\sqrt[3]{a+bx^3}}{6d}-\frac{\int \frac{x\left(-2abc(6bc-7ad)-2b\left(9b^2c^2-12abcd+2a^2d^2\right)x^3\right)}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx}{18bd^2}\\
&=-\frac{(6bc-7ad)x^2\sqrt[3]{a+bx^3}}{18d^2}+\frac{bx^5\sqrt[3]{a+bx^3}}{6d}\\
&\quad-\frac{\int \left(-\frac{2b\left(9b^2c^2-12abcd+2a^2d^2\right)x}{d\left(a+bx^3\right)^{2/3}}+\frac{18\left(b^3c^3-2ab^2c^2d+a^2bcd^2\right)x}{d\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\right)\,dx}{18bd^2}
\end{aligned}$$

$$= -\frac{(6bc-7ad)x^2\sqrt[3]{a+bx^3}}{18d^2} + \frac{bx^5\sqrt[3]{a+bx^3}}{6d} - \frac{(c(bc-ad)^2)\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{d^3} + \frac{(9b^2c^2-12abcd+2a^2d^2)\int \frac{x}{(a+bx^3)^{2/3}}\,dx}{9d^3}$$

$$= -\frac{(6bc-7ad)x^2\sqrt[3]{a+bx^3}}{18d^2} + \frac{bx^5\sqrt[3]{a+bx^3}}{6d} - \frac{(9b^2c^2-12abcd+2a^2d^2)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{2/3}d^3} + \frac{c^{2/3}(bc-ad)^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^3} - \frac{c^{2/3}(bc-ad)^{4/3}\log(c+dx^3)}{6d^3} - \frac{(9b^2c^2-12abcd+2a^2d^2)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{18b^{2/3}d^3} + \frac{c^{2/3}(bc-ad)^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^3}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.38 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.57

$$\int \frac{x^4(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{6dx^2\sqrt[3]{a+bx^3}(-6bc+7ad+3bdx^3) - \frac{4\sqrt{3}(9b^2c^2-12abcd+2a^2d^2)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{b^{2/3}} \cdots}{\cdots}$$

```
[In] Integrate[(x^4*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (6*d*x^2*(a + b*x^3)^(1/3)*(-6*b*c + 7*a*d + 3*b*d*x^3) - (4*Sqrt[3]*(9*b^2
*c^2 - 12*a*b*c*d + 2*a^2*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a
 + b*x^3)^(1/3))])/b^(2/3) - 18*Sqrt[-6 - (6*I)*Sqrt[3]]*c^(2/3)*(b*c - a*d
)^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I
+ Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] - (4*(9*b^2*c^2 - 12*a*b*c*d + 2*a^2
*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(2/3) + (18*I)*(I + Sqrt[3])
*c^(2/3)*(b*c - a*d)^(4/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1
/3)*(a + b*x^3)^(1/3)] + (2*(9*b^2*c^2 - 12*a*b*c*d + 2*a^2*d^2)*Log[b^(2/3
```

```
)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(2/3) + 9*(1 -
I*Sqrt[3])*c^(2/3)*(b*c - a*d)^(4/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*
Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^
(2/3)*(a + b*x^3)^(2/3)])/(108*d^3)
```

Maple [A] (verified)

Time = 5.11 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.41

method	result
pseudoelliptic	$\frac{\frac{\left(-a^{2}d^{2}b^{\frac{2}{3}}-(-2ad+bc)c\,b^{\frac{5}{3}}\right)\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{2}-\left(a^{2}d^{2}b^{\frac{2}{3}}+(-2ad+bc)c\,b^{\frac{5}{3}}\right)\sqrt{3}\,\arctan\left(\right.}{}$

```
[In] int(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(1/2*(-a^2*d^2*b^(2/3)-(-2*a*d+b*c)*c*b^(5/3))*ln((((a*d-b*c)/c)^(2/3)*
x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-(a^2*d^2*b^
(2/3)+(-2*a*d+b*c)*c*b^(5/3))*3^(1/2)*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/
3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)+1/9*((a*d-b*c)/c)^(2/3)*(a^2
*d^2-6*a*b*c*d+9/2*b^2*c^2)*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^
3+a)^(2/3))/x^2)+(a^2*d^2*b^(2/3)+(-2*a*d+b*c)*c*b^(5/3))*ln((((a*d-b*c)/c)
^(1/3)*x+(b*x^3+a)^(1/3))/x)-((a*d-b*c)/c)^(2/3)*(-2/9*3^(1/2)*(a^2*d^2-6*a
*b*c*d+9/2*b^2*c^2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3
)/x)+ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*(b^2*c^2-4/3*a*b*c*d+2/9*a^2*d^2)+x
^2*(b*x^3+a)^(1/3)*(-7/6*a*d*b^(2/3)+(-1/2*d*x^3+c)*b^(5/3))*d))/b^(2/3)/((
a*d-b*c)/c)^(2/3)/d^3
```

Fricas [A] (verification not implemented)

none

Time = 1.18 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.65

$$\int \frac{x^4(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{2\sqrt{3}(9\,b^3c^2-12\,ab^2cd+2\,a^2bd^2)\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\left(\sqrt{3}\left(-b^2\right)^{\frac{1}{3}}bx-2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-b^2\right)\right.}{3\,b^2x}\right.}{}$$

```
[In] integrate(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/54*(2*sqrt(3)*(9*b^3*c^2 - 12*a*b^2*c*d + 2*a^2*b*d^2)*sqrt(-(-b^2)^(1/3)
)*arctan(-1/3*(sqrt(3)*(-b^2)^(1/3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2
```

```
)^(2/3))*sqrt(-(-b^2)^(1/3))/(b^2*x)) - 18*sqrt(3)*(b^3*c - a*b^2*d)*(-b*c^
3 + a*c^2*d)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c^2 - a*c*d)*x + 2*sqrt(3)*(-b*c
^3 + a*c^2*d)^(2/3)*(b*x^3 + a)^(1/3))/((b*c^2 - a*c*d)*x)) - 2*(9*b^2*c^2
- 12*a*b*c*d + 2*a^2*d^2)*(-b^2)^(2/3)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(
1/3)*b)/x) + (9*b^2*c^2 - 12*a*b*c*d + 2*a^2*d^2)*(-b^2)^(2/3)*log(-((-b^2)
^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2)
 - 18*(b^3*c - a*b^2*d)*(-b*c^3 + a*c^2*d)^(1/3)*log(((b*x^3 + a)^(1/3)*c +
 (-b*c^3 + a*c^2*d)^(1/3)*x)/x) + 9*(b^3*c - a*b^2*d)*(-b*c^3 + a*c^2*d)^(1
/3)*log(((b*x^3 + a)^(2/3)*c^2 - (-b*c^3 + a*c^2*d)^(1/3)*(b*x^3 + a)^(1/3)
*c*x + (-b*c^3 + a*c^2*d)^(2/3)*x^2)/x^2) + 3*(3*b^3*d^2*x^5 - (6*b^3*c*d -
 7*a*b^2*d^2)*x^2)*(b*x^3 + a)^(1/3))/(b^2*d^3)
```

Sympy [F]

$$\int \frac{x^4(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{x^4(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

```
[In] integrate(x**4*(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(x**4*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^4(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}x^4}{dx^3+c}\,dx$$

```
[In] integrate(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)*x^4/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{x^4(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}x^4}{dx^3+c}\,dx$$

```
[In] integrate(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(4/3)*x^4/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{x^4\,(b\,x^3+a)^{4/3}}{d\,x^3+c}\,dx$$

```
[In] int((x^4*(a + b*x^3)^(4/3))/(c + d*x^3),x)
[Out] int((x^4*(a + b*x^3)^(4/3))/(c + d*x^3), x)
```

3.703 $\int \frac{x(a+bx^3)^{4/3}}{c+dx^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 277

$$\int \frac{x(a+bx^3)^{4/3}}{c+dx^3} dx = \frac{bx^2\sqrt[3]{a+bx^3}}{3d} + \frac{\sqrt[3]{b}(3bc-4ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}d^2} - \frac{(bc-ad)^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}d^2} + \frac{(bc-ad)^{4/3}\log(c+dx^3)}{6\sqrt[3]{c}d^2} + \frac{\sqrt[3]{b}(3bc-4ad)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6d^2} - \frac{(bc-ad)^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}d^2}$$

```
[Out] 1/3*b*x^2*(b*x^3+a)^(1/3)/d+1/6*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/c^(1/3)/d^2+1/
6*b^(1/3)*(-4*a*d+3*b*c)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/d^2-1/2*(-a*d+b*c)^(
4/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(1/3)/d^2+1/9*b^(1/3)
*(-4*a*d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/d^2*3^(
1/2)-1/3*(-a*d+b*c)^(4/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3
+a)^(1/3))*3^(1/2))/c^(1/3)/d^2*3^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {488, 598, 337, 503}

$$\int \frac{x(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{\sqrt[3]{b}\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(3bc-4ad)}{3\sqrt{3}d^2} - \frac{(bc-ad)^{4/3}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}d^2} + \frac{(bc-ad)^{4/3}\log(c+dx^3)}{6\sqrt[3]{c}d^2} + \frac{\sqrt[3]{b}(3bc-4ad)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6d^2} - \frac{(bc-ad)^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}d^2} + \frac{bx^2\sqrt[3]{a+bx^3}}{3d}$$

```
[In] Int[(x*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (b*x^2*(a + b*x^3)^(1/3))/(3*d) + (b^(1/3)*(3*b*c - 4*a*d)*ArcTan[(1 + (2*b
^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*d^2) - ((b*c - a*d)^(4/3)
*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])
/(Sqrt[3]*c^(1/3)*d^2) + ((b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*c^(1/3)*d^2)
 + (b^(1/3)*(3*b*c - 4*a*d)*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(6*d^2) - (
(b*c - a*d)^(4/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(
2*c^(1/3)*d^2)
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 488

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
```

```
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx^2\sqrt[3]{a+bx^3}}{3d} + \frac{\int \frac{x(-a(2bc-3ad)-b(3bc-4ad)x^3)}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{3d} \\
&= \frac{bx^2\sqrt[3]{a+bx^3}}{3d} + \frac{\int \left(-\frac{b(3bc-4ad)x}{d(a+bx^3)^{2/3}} + \frac{3(b^2c^2-2abcd+a^2d^2)x}{d(a+bx^3)^{2/3}(c+dx^3)}\right)dx}{3d} \\
&= \frac{bx^2\sqrt[3]{a+bx^3}}{3d} - \frac{(b(3bc-4ad))\int \frac{x}{(a+bx^3)^{2/3}}\,dx}{3d^2} + \frac{(bc-ad)^2\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{d^2} \\
&= \frac{bx^2\sqrt[3]{a+bx^3}}{3d} + \frac{\sqrt[3]{b}(3bc-4ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}d^2} \\
&\quad - \frac{(bc-ad)^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}d^2} \\
&\quad + \frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6\sqrt[3]{c}d^2} + \frac{\sqrt[3]{b}(3bc-4ad)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6d^2} \\
&\quad - \frac{(bc-ad)^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}d^2}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.54 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.69

$$\int \frac{x(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{12bdx^2\sqrt[3]{a+bx^3}+4\sqrt{3}\sqrt[3]{b}(3bc-4ad)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)+\frac{6\sqrt{-6-6i\sqrt{3}}(bc-ad)^{4/3}\cdots}{\cdots}}{\cdots}$$

```
[In] Integrate[(x*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (12*b*d*x^2*(a + b*x^3)^(1/3) + 4*Sqrt[3]*b^(1/3)*(3*b*c - 4*a*d)*ArcTan[(S
qrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] + (6*Sqrt[-6 - (6*I)*S
qrt[3]]*(b*c - a*d)^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*
d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))]])/c^(1/3) + 4*b^(1/
3)*(3*b*c - 4*a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + (6*(1 - I*Sqrt[3
])*(b*c - a*d)^(4/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a
 + b*x^3)^(1/3)])/c^(1/3) - 2*b^(1/3)*(3*b*c - 4*a*d)*Log[b^(2/3)*x^2 + b^(
1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] + ((3*I)*(I + Sqrt[3])*(b*c -
 a*d)^(4/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a
*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])
/c^(1/3))/(36*d^2)
```

Maple [A] (verified)

Time = 5.19 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.39

method	result
pseudoelliptic	$-\frac{-\frac{(ad-bc)^2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^2-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{2}-\sqrt{3}\,(ad-bc)^2\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)\cdots}{\cdots}$

```
[In] int(x*(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/((a*d-b*c)/c)^(2/3)*(-1/2*(a*d-b*c)^2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*
d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-3^(1/2)*(a*d-b*c)^2
*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)
^(1/3)/x)-2/3*((a*d-b*c)/c)^(2/3)*c*(a*d*b^(1/3)-3/4*b^(4/3)*c)*ln((b^(2/3)
*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)+(a*d-b*c)^2*ln((((a*d-
b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)+4/3*(-3^(1/2)*(a*d*b^(1/3)-3/4*b^(4/3)*
c)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+(a*d*b^(1/3)
```

```
-3/4*b^(4/3)*c)*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-3/4*b*x^2*(b*x^3+a)^(1/3
)*d)*((a*d-b*c)/c)^(2/3)*c)/c/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.43

$$\int \frac{x(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{6\left(bx^3+a\right)^{\frac{1}{3}}bdx^2 - 6\sqrt{3}(bc-ad)\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}c\left(\frac{bc-ad}{c}\right)^{\frac{2}{3}}}{3\,(bc-ad)x}\right)}{}$$

```
[In] integrate(x*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/18*(6*(b*x^3 + a)^(1/3)*b*d*x^2 - 6*sqrt(3)*(b*c - a*d)*((b*c - a*d)/c)^(
1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b
*c - a*d)/c)^(2/3))/((b*c - a*d)*x)) + 2*sqrt(3)*(3*b*c - 4*a*d)*(-b)^(1/3)
*arctan(1/3*(sqrt(3)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b)^(2/3))/(b*x)) -
 2*(3*b*c - 4*a*d)*(-b)^(1/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - 6
*(b*c - a*d)*((b*c - a*d)/c)^(1/3)*log(-(x*((b*c - a*d)/c)^(1/3) - (b*x^3 +
 a)^(1/3))/x) + (3*b*c - 4*a*d)*(-b)^(1/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a
)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(b*c - a*d)*((b*c - a*d)
/c)^(1/3)*log((x^2*((b*c - a*d)/c)^(2/3) + (b*x^3 + a)^(1/3)*x*((b*c - a*d)
/c)^(1/3) + (b*x^3 + a)^(2/3))/x^2))/d^2
```

Sympy [F]

$$\int \frac{x(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{x\left(a+bx^3\right)^{\frac{4}{3}}}{c+dx^3}\,dx$$

```
[In] integrate(x*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}x}{dx^3+c}\,dx$$

```
[In] integrate(x*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)*x/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{x(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}x}{dx^3+c}\,dx$$

```
[In] integrate(x*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(4/3)*x/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{x\,(b\,x^3+a)^{4/3}}{d\,x^3+c}\,dx$$

```
[In] int((x*(a + b*x^3)^(4/3))/(c + d*x^3),x)
[Out] int((x*(a + b*x^3)^(4/3))/(c + d*x^3), x)
```

3.704 $\int \frac{(a+bx^3)^{4/3}}{x^2(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 254

$$\int \frac{(a+bx^3)^{4/3}}{x^2(c+dx^3)}\,dx = -\frac{a\sqrt[3]{a+bx^3}}{cx} - \frac{b^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d} + \frac{(bc-ad)^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{4/3}d} - \frac{(bc-ad)^{4/3}\log(c+dx^3)}{6c^{4/3}d} - \frac{b^{4/3}\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2d} + \frac{(bc-ad)^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{4/3}d}$$

```
[Out] -a*(b*x^3+a)^(1/3)/c/x-1/6*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/c^(4/3)/d-1/2*b^(4/
3)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/d+1/2*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)
*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(4/3)/d-1/3*b^(4/3)*arctan(1/3*(1+2*b^(1/3)*x
/(b*x^3+a)^(1/3))*3^(1/2))/d*3^(1/2)+1/3*(-a*d+b*c)^(4/3)*arctan(1/3*(1+2*(
-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(4/3)/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {485, 598, 337, 503}

$$\int \frac{(a+bx^3)^{4/3}}{x^2(c+dx^3)}\,dx = -\frac{b^{4/3}\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d} + \frac{(bc-ad)^{4/3}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{4/3}d} - \frac{b^{4/3}\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2d} - \frac{(bc-ad)^{4/3}\log(c+dx^3)}{6c^{4/3}d} + \frac{(bc-ad)^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{4/3}d} - \frac{a\sqrt[3]{a+bx^3}}{cx}$$

```
[In] Int[(a + b*x^3)^(4/3)/(x^2*(c + d*x^3)),x]
```

```
[Out] -((a*(a + b*x^3)^(1/3))/(c*x)) - (b^(4/3)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*
x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d) + ((b*c - a*d)^(4/3)*ArcTan[(1 + (2*(b*c
- a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(4/3)*d)
- ((b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*c^(4/3)*d) - (b^(4/3)*Log[b^(1/3)*x
 - (a + b*x^3)^(1/3)])/(2*d) + ((b*c - a*d)^(4/3)*Log[((b*c - a*d)^(1/3)*x)
/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(4/3)*d)
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a\sqrt[3]{a+bx^3}}{cx}+\frac{\int \frac{x\left(a(2bc-ad)+b^2cx^3\right)}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{c}\\
&= -\frac{a\sqrt[3]{a+bx^3}}{cx}+\frac{\int \left(\frac{b^2cx}{d(a+bx^3)^{2/3}}-\frac{(b^2c^2-2abcd+a^2d^2)x}{d(a+bx^3)^{2/3}(c+dx^3)}\right)dx}{c}\\
&= -\frac{a\sqrt[3]{a+bx^3}}{cx}+\frac{b^2\int \frac{x}{(a+bx^3)^{2/3}}\,dx}{d}-\frac{(bc-ad)^2\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{cd}\\
&= -\frac{a\sqrt[3]{a+bx^3}}{cx}-\frac{b^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d}\\
&\quad+\frac{(bc-ad)^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{4/3}d}-\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6c^{4/3}d}\\
&\quad-\frac{b^{4/3}\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2d}+\frac{(bc-ad)^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{4/3}d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.25 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.80

$$\int \frac{(a+bx^3)^{4/3}}{x^2\,(c+dx^3)}\,dx = \frac{-12a\sqrt[3]{c}d\sqrt[3]{a+bx^3} - 4\sqrt{3}b^{4/3}c^{4/3}x\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) - 2\sqrt{-6-6i\sqrt{3}}(bc -}{}$$

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^2*(c + d*x^3)),x]
```

```
[Out] (-12*a*c^(1/3)*d*(a + b*x^3)^(1/3) - 4*Sqrt[3]*b^(4/3)*c^(4/3)*x*ArcTan[(Sq
rt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 2*Sqrt[-6 - (6*I)*Sqr
t[3]]*(b*c - a*d)^(4/3)*x*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*
d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] - 4*b^(4/3)*c^(4/3
)*x*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + (2*I)*(I + Sqrt[3])*(b*c - a*d)
^(4/3)*x*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1
/3)] + 2*b^(4/3)*c^(4/3)*x*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) +
(a + b*x^3)^(2/3)] + (1 - I*Sqrt[3])*(b*c - a*d)^(4/3)*x*Log[2*(b*c - a*d)^
(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3)
+ I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(12*c^(4/3)*d*x)
```

Maple [A] (verified)

Time = 4.95 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.31

method	result
pseudoelliptic	$-\frac{6\left(b\,x^3+a\right)^{\frac{1}{3}}c\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}ad+x\left(-\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}c^2\left(2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^3+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)+\ln\left(\frac{b^{\frac{2}{3}}x^2+b^{\frac{1}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}x+\left(b\,x^3\right.}{x^2}\right.\right.\right.}{}$

```
[In] int((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(6*(b*x^3+a)^(1/3)*c*((a*d-b*c)/c)^(2/3)*a*d+x*(-((a*d-b*c)/c)^(2/3)*c
^2*(2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+l
n((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((-b^(1/
3)*x+(b*x^3+a)^(1/3))/x))*b^(4/3)+(a*d-b*c)^2*(2*arctan(1/3*3^(1/2)*(((a*d-
b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d
-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x
^2)-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x))))/((a*d-b*c)/c)^(2/3)/
c^2/x/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^2\,(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^2\,(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{4}{3}}}{x^2\,(c+dx^3)}\,dx$$

```
[In] integrate((b*x**3+a)**(4/3)/x**2/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(4/3)/(x**2*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^2\,(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^2}\,dx$$

```
[In] integrate((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^2), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^2\,(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^2}\,dx$$

```
[In] integrate((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^2\,(c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{4/3}}{x^2\,(d\,x^3+c)}\,dx$$

```
[In] int((a + b*x^3)^(4/3)/(x^2*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(4/3)/(x^2*(c + d*x^3)), x)
```

3.705 $\int \frac{(a+bx^3)^{4/3}}{x^5(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 201

$$\int \frac{(a+bx^3)^{4/3}}{x^5(c+dx^3)}\,dx = -\frac{a\sqrt[3]{a+bx^3}}{4cx^4} - \frac{(5bc-4ad)\sqrt[3]{a+bx^3}}{4c^2x} - \frac{(bc-ad)^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{7/3}} + \frac{(bc-ad)^{4/3}\log(c+dx^3)}{6c^{7/3}} - \frac{(bc-ad)^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{7/3}}$$

```
[Out] -1/4*a*(b*x^3+a)^(1/3)/c/x^4-1/4*(-4*a*d+5*b*c)*(b*x^3+a)^(1/3)/c^2/x+1/6*(
-a*d+b*c)^(4/3)*ln(d*x^3+c)/c^(7/3)-1/2*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3
)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(7/3)-1/3*(-a*d+b*c)^(4/3)*arctan(1/3*(1+2*(
-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(7/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used

$= \{485, 597, 12, 503\}$

$$\int \frac{(a+bx^3)^{4/3}}{x^5(c+dx^3)}\,dx = -\frac{(bc-ad)^{4/3}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{7/3}} + \frac{(bc-ad)^{4/3}\log(c+dx^3)}{6c^{7/3}} - \frac{(bc-ad)^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{7/3}} - \frac{\sqrt[3]{a+bx^3}(5bc-4ad)}{4c^2x} - \frac{a\sqrt[3]{a+bx^3}}{4cx^4}$$

```
[In] Int[(a + b*x^3)^(4/3)/(x^5*(c + d*x^3)),x]
```

```
[Out] -1/4*(a*(a + b*x^3)^(1/3))/(c*x^4) - ((5*b*c - 4*a*d)*(a + b*x^3)^(1/3))/(4
*c^2*x) - ((b*c - a*d)^(4/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(
a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(7/3)) + ((b*c - a*d)^(4/3)*Log[c +
 d*x^3])/(6*c^(7/3)) - ((b*c - a*d)^(4/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3)
 - (a + b*x^3)^(1/3)])/(2*c^(7/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
```

```
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a\sqrt[3]{a+bx^3}}{4cx^4}+\frac{\int \frac{a(5bc-4ad)+b(4bc-3ad)x^3}{x^2(a+bx^3)^{2/3}(c+dx^3)}\,dx}{4c}\\
&= -\frac{a\sqrt[3]{a+bx^3}}{4cx^4}-\frac{(5bc-4ad)\sqrt[3]{a+bx^3}}{4c^2x}-\frac{\int -\frac{4a(bc-ad)^2x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{4ac^2}\\
&= -\frac{a\sqrt[3]{a+bx^3}}{4cx^4}-\frac{(5bc-4ad)\sqrt[3]{a+bx^3}}{4c^2x}+\frac{(bc-ad)^2\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{c^2}\\
&= -\frac{a\sqrt[3]{a+bx^3}}{4cx^4}-\frac{(5bc-4ad)\sqrt[3]{a+bx^3}}{4c^2x}-\frac{(bc-ad)^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{7/3}}\\
&\quad+\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6c^{7/3}}-\frac{(bc-ad)^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{7/3}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.06 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.63

$$\int \frac{(a+bx^3)^{4/3}}{x^5(c+dx^3)}\,dx = \frac{\frac{3\sqrt[3]{c}\sqrt[3]{a+bx^3}(-ac-5bcx^3+4adx^3)}{x^4}+2\sqrt{-6-6i\sqrt{3}}(bc-ad)^{4/3}\arctan\left(\frac{3\sqrt[3]{bc-}}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+}\right.}{}$$

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^5*(c + d*x^3)),x]
```

```
[Out] ((3*c^(1/3)*(a + b*x^3)^(1/3)*(-(a*c) - 5*b*c*x^3 + 4*a*d*x^3))/x^4 + 2*Sqr
t[-6 - (6*I)*Sqrt[3]]*(b*c - a*d)^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqr
t[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + 2*
(1 - I*Sqrt[3])*(b*c - a*d)^(4/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3
])*c^(1/3)*(a + b*x^3)^(1/3)] + I*(I + Sqrt[3])*(b*c - a*d)^(4/3)*Log[2*(b*
c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^
3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(12*c^(7/3))
```

Maple [A] (verified)

Time = 5.02 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.21

method	result
pseudoelliptic	$\frac{-2x^4(ad-bc)^2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^3+a\right)^{\frac{1}{3}}}{x}\right)-\frac{3\left(b\,x^3+a\right)^{\frac{1}{3}}\left((-4ad+5bc)x^3+ac\right)c\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}}{2}+x^4\left(2\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x\cdots\right.}{3\left(\frac{ad-bc}{c}\right.\cdots}\right.\right.}{6\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^4c^3}$

```
[In] int((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/((a*d-b*c)/c)^(2/3)*(-2*x^4*(a*d-b*c)^2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^
3+a)^(1/3))/x)-3/2*(b*x^3+a)^(1/3)*((-4*a*d+5*b*c)*x^3+a*c)*c*((a*d-b*c)/c)
^(2/3)+x^4*(2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/
((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1
/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*(a*d-b*c)^2)/x^4/c^3
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^5(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^5(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{4}{3}}}{x^5(c+dx^3)}\,dx$$

```
[In] integrate((b*x**3+a)**(4/3)/x**5/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(x**5*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^5(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^5}\,dx$$

```
[In] integrate((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^5), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^5(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^5}\,dx$$

```
[In] integrate((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^5(c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{4/3}}{x^5\,(d\,x^3+c)}\,dx$$

```
[In] int((a + b*x^3)^(4/3)/(x^5*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(4/3)/(x^5*(c + d*x^3)), x)
```

3.706 $\int \frac{(a+bx^3)^{4/3}}{x^8(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 250

$$\int \frac{(a+bx^3)^{4/3}}{x^8(c+dx^3)}\,dx = -\frac{a\sqrt[3]{a+bx^3}}{7cx^7} - \frac{(8bc-7ad)\sqrt[3]{a+bx^3}}{28c^2x^4} - \frac{(4b^2c^2-35abcd+28a^2d^2)\sqrt[3]{a+bx^3}}{28ac^3x} + \frac{d(bc-ad)^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{10/3}} - \frac{d(bc-ad)^{4/3}\log(c+dx^3)}{6c^{10/3}} + \frac{d(bc-ad)^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{10/3}}$$

```
[Out] -1/7*a*(b*x^3+a)^(1/3)/c/x^7-1/28*(-7*a*d+8*b*c)*(b*x^3+a)^(1/3)/c^2/x^4-1/
28*(28*a^2*d^2-35*a*b*c*d+4*b^2*c^2)*(b*x^3+a)^(1/3)/a/c^3/x-1/6*d*(-a*d+b*
c)^(4/3)*ln(d*x^3+c)/c^(10/3)+1/2*d*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)*x/
c^(1/3)-(b*x^3+a)^(1/3))/c^(10/3)+1/3*d*(-a*d+b*c)^(4/3)*arctan(1/3*(1+2*(-
a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(10/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {485, 597, 12, 503}

$$\int \frac{(a+bx^3)^{4/3}}{x^8(c+dx^3)}\,dx = -\frac{\sqrt[3]{a+bx^3}(28a^2d^2-35abcd+4b^2c^2)}{28ac^3x} + \frac{d(bc-ad)^{4/3}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{10/3}} - \frac{d(bc-ad)^{4/3}\log(c+dx^3)}{6c^{10/3}} + \frac{d(bc-ad)^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{10/3}} - \frac{\sqrt[3]{a+bx^3}(8bc-7ad)}{28c^2x^4} - \frac{a\sqrt[3]{a+bx^3}}{7cx^7}$$

```
[In] Int[(a + b*x^3)^(4/3)/(x^8*(c + d*x^3)),x]
```

```
[Out] -1/7*(a*(a + b*x^3)^(1/3))/(c*x^7) - ((8*b*c - 7*a*d)*(a + b*x^3)^(1/3))/(2
8*c^2*x^4) - ((4*b^2*c^2 - 35*a*b*c*d + 28*a^2*d^2)*(a + b*x^3)^(1/3))/(28*
a*c^3*x) + (d*(b*c - a*d)^(4/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3
)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(10/3)) - (d*(b*c - a*d)^(4/3)*L
og[c + d*x^3])/(6*c^(10/3)) + (d*(b*c - a*d)^(4/3)*Log[((b*c - a*d)^(1/3)*x
)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(10/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
```

```
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a\sqrt[3]{a+bx^3}}{7cx^7}+\frac{\int \frac{a(8bc-7ad)+b(7bc-6ad)x^3}{x^5(a+bx^3)^{2/3}(c+dx^3)}\,dx}{7c}\\
&= -\frac{a\sqrt[3]{a+bx^3}}{7cx^7}-\frac{(8bc-7ad)\sqrt[3]{a+bx^3}}{28c^2x^4}-\frac{\int \frac{-a(4b^2c^2-35abcd+28a^2d^2)+3abd(8bc-7ad)x^3}{x^2(a+bx^3)^{2/3}(c+dx^3)}\,dx}{28ac^2}\\
&= -\frac{a\sqrt[3]{a+bx^3}}{7cx^7}-\frac{(8bc-7ad)\sqrt[3]{a+bx^3}}{28c^2x^4}\\
&\quad-\frac{(4b^2c^2-35abcd+28a^2d^2)\sqrt[3]{a+bx^3}}{28ac^3x}+\frac{\int -\frac{28a^2d(bc-ad)^2x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{28a^2c^3}\\
&= -\frac{a\sqrt[3]{a+bx^3}}{7cx^7}-\frac{(8bc-7ad)\sqrt[3]{a+bx^3}}{28c^2x^4}\\
&\quad-\frac{(4b^2c^2-35abcd+28a^2d^2)\sqrt[3]{a+bx^3}}{28ac^3x}-\frac{(d(bc-ad)^2)\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{c^3}\\
&= -\frac{a\sqrt[3]{a+bx^3}}{7cx^7}-\frac{(8bc-7ad)\sqrt[3]{a+bx^3}}{28c^2x^4}-\frac{(4b^2c^2-35abcd+28a^2d^2)\sqrt[3]{a+bx^3}}{28ac^3x}\\
&\quad+\frac{d(bc-ad)^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{10/3}}-\frac{d(bc-ad)^{4/3}\log(c+dx^3)}{6c^{10/3}}\\
&\quad+\frac{d(bc-ad)^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{10/3}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.41 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^3)^{4/3}}{x^8(c+dx^3)}\,dx = \frac{-\frac{3\sqrt[3]{c}\sqrt[3]{a+bx^3}(4b^2c^2x^6+abcx^3(8c-35dx^3)+a^2(4c^2-7cdx^3+28d^2x^6))}{ax^7} - 14\sqrt{-6-6i\sqrt{3}}d(bc-ad)^{4/}}{}$$

[In] Integrate[(a + b*x^3)^(4/3)/(x^8*(c + d*x^3)),x]

[Out] ((-3*c^(1/3)*(a + b*x^3)^(1/3)*(4*b^2*c^2*x^6 + a*b*c*x^3*(8*c - 35*d*x^3) + a^2*(4*c^2 - 7*c*d*x^3 + 28*d^2*x^6)))/(a*x^7) - 14*Sqrt[-6 - (6*I)*Sqrt[3]]*d*(b*c - a*d)^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (14*I)*(I + Sqrt[3])*d*(b*c - a*d)^(4/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)] + 7*(1 - I*Sqrt[3])*d*(b*c - a*d)^(4/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(84*c^(10/3))

Maple [A] (verified)

Time = 4.84 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$-\frac{\frac{6\left(\left(b\,x^3+a\right)^2c^2-\frac{7ad\,x^3\left(5b\,x^3+a\right)c}{4}+7a^2d^2x^6\right)c\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}}{7}+ad\,x^7(ad-bc)^2\left(2\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(b\,x^3+a\right)\right.}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right.\right.}{6\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^7c^4a}$

[In] int((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] -1/6/((a*d-b*c)/c)^(2/3)*(6/7*((b*x^3+a)^2*c^2-7/4*a*d*x^3*(5*b*x^3+a)*c+7*a^2*d^2*x^6)*c*((a*d-b*c)/c)^(2/3)*(b*x^3+a)^(1/3)+a*d*x^7*(a*d-b*c)^2*(2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)))/x^7/c^4/a

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^8\,(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^8\,(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{4}{3}}}{x^8\,(c+dx^3)}\,dx$$

```
[In] integrate((b*x**3+a)**(4/3)/x**8/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(4/3)/(x**8*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^8\,(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^8}\,dx$$

```
[In] integrate((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^8), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^8\,(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^8}\,dx$$

```
[In] integrate((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^8\,(c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{4/3}}{x^8\,\left(d\,x^3+c\right)}\,dx$$

```
[In] int((a + b*x^3)^(4/3)/(x^8*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(4/3)/(x^8*(c + d*x^3)), x)
```

3.707 $\int \frac{(a+bx^3)^{4/3}}{x^{11}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 318

$$\int \frac{(a+bx^3)^{4/3}}{x^{11}(c+dx^3)}\,dx = -\frac{a\sqrt[3]{a+bx^3}}{10cx^{10}}-\frac{(11bc-10ad)\sqrt[3]{a+bx^3}}{70c^2x^7} -\frac{(2b^2c^2-40abcd+35a^2d^2)\sqrt[3]{a+bx^3}}{140ac^3x^4} +\frac{(6b^3c^3+20ab^2c^2d-175a^2bcd^2+140a^3d^3)\sqrt[3]{a+bx^3}}{140a^2c^4x} -\frac{d^2(bc-ad)^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{13/3}} +\frac{d^2(bc-ad)^{4/3}\log(c+dx^3)}{6c^{13/3}} -\frac{d^2(bc-ad)^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{13/3}}$$

```
[Out] -1/10*a*(b*x^3+a)^(1/3)/c/x^10-1/70*(-10*a*d+11*b*c)*(b*x^3+a)^(1/3)/c^2/x^
7-1/140*(35*a^2*d^2-40*a*b*c*d+2*b^2*c^2)*(b*x^3+a)^(1/3)/a/c^3/x^4+1/140*(
140*a^3*d^3-175*a^2*b*c*d^2+20*a*b^2*c^2*d+6*b^3*c^3)*(b*x^3+a)^(1/3)/a^2/c
^4/x+1/6*d^2*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/c^(13/3)-1/2*d^2*(-a*d+b*c)^(4/3)
*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(13/3)-1/3*d^2*(-a*d+b*c)
^(4/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))
/c^(13/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {485, 597, 12, 503}

$$\int \frac{(a+bx^3)^{4/3}}{x^{11}(c+dx^3)}\,dx = -\frac{\sqrt[3]{a+bx^3}(35a^2d^2-40abcd+2b^2c^2)}{140ac^3x^4} + \frac{\sqrt[3]{a+bx^3}(140a^3d^3-175a^2bcd^2+20ab^2c^2d+6b^3c^3)}{140a^2c^4x} - \frac{d^2(bc-ad)^{4/3}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{13/3}} + \frac{d^2(bc-ad)^{4/3}\log(c+dx^3)}{6c^{13/3}} - \frac{d^2(bc-ad)^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{13/3}} - \frac{\sqrt[3]{a+bx^3}(11bc-10ad)}{70c^2x^7} - \frac{a\sqrt[3]{a+bx^3}}{10cx^{10}}$$

```
[In] Int[(a + b*x^3)^(4/3)/(x^11*(c + d*x^3)),x]
```

```
[Out] -1/10*(a*(a + b*x^3)^(1/3))/(c*x^10) - ((11*b*c - 10*a*d)*(a + b*x^3)^(1/3)
)/(70*c^2*x^7) - ((2*b^2*c^2 - 40*a*b*c*d + 35*a^2*d^2)*(a + b*x^3)^(1/3))/
(140*a*c^3*x^4) + ((6*b^3*c^3 + 20*a*b^2*c^2*d - 175*a^2*b*c*d^2 + 140*a^3*
d^3)*(a + b*x^3)^(1/3))/(140*a^2*c^4*x) - (d^2*(b*c - a*d)^(4/3)*ArcTan[(1
+ (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c
^(13/3)) + (d^2*(b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*c^(13/3)) - (d^2*(b*c
- a*d)^(4/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(
13/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
```

```
1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a\sqrt[3]{a+bx^3}}{10cx^{10}}+\frac{\int \frac{a(11bc-10ad)+b(10bc-9ad)x^3}{x^8(a+bx^3)^{2/3}(c+dx^3)}\,dx}{10c} \\
&= -\frac{a\sqrt[3]{a+bx^3}}{10cx^{10}}-\frac{(11bc-10ad)\sqrt[3]{a+bx^3}}{70c^2x^7}-\frac{\int \frac{-2a(2b^2c^2-40abcd+35a^2d^2)+6abd(11bc-10ad)x^3}{x^5(a+bx^3)^{2/3}(c+dx^3)}\,dx}{70ac^2} \\
&= -\frac{a\sqrt[3]{a+bx^3}}{10cx^{10}}-\frac{(11bc-10ad)\sqrt[3]{a+bx^3}}{70c^2x^7}-\frac{(2b^2c^2-40abcd+35a^2d^2)\sqrt[3]{a+bx^3}}{140ac^3x^4} \\
&\quad+\frac{\int \frac{-2a(6b^3c^3+20ab^2c^2d-175a^2bcd^2+140a^3d^3)-6abd(2b^2c^2-40abcd+35a^2d^2)x^3}{x^2(a+bx^3)^{2/3}(c+dx^3)}\,dx}{280a^2c^3} \\
&= -\frac{a\sqrt[3]{a+bx^3}}{10cx^{10}}-\frac{(11bc-10ad)\sqrt[3]{a+bx^3}}{70c^2x^7}-\frac{(2b^2c^2-40abcd+35a^2d^2)\sqrt[3]{a+bx^3}}{140ac^3x^4} \\
&\quad+\frac{(6b^3c^3+20ab^2c^2d-175a^2bcd^2+140a^3d^3)\sqrt[3]{a+bx^3}}{140a^2c^4x}-\frac{\int -\frac{280a^3d^2(bc-ad)^2x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{280a^3c^4} \\
&= -\frac{a\sqrt[3]{a+bx^3}}{10cx^{10}}-\frac{(11bc-10ad)\sqrt[3]{a+bx^3}}{70c^2x^7}-\frac{(2b^2c^2-40abcd+35a^2d^2)\sqrt[3]{a+bx^3}}{140ac^3x^4} \\
&\quad+\frac{(6b^3c^3+20ab^2c^2d-175a^2bcd^2+140a^3d^3)\sqrt[3]{a+bx^3}}{140a^2c^4x} \\
&\quad+\frac{(d^2(bc-ad)^2)\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{c^4}
\end{aligned}
$$

$$
= -\frac{a\sqrt[3]{a+bx^3}}{10cx^{10}} - \frac{(11bc-10ad)\sqrt[3]{a+bx^3}}{70c^2x^7} - \frac{(2b^2c^2-40abcd+35a^2d^2)\sqrt[3]{a+bx^3}}{140ac^3x^4}
+ \frac{(6b^3c^3+20ab^2c^2d-175a^2bcd^2+140a^3d^3)\sqrt[3]{a+bx^3}}{140a^2c^4x}
- \frac{d^2(bc-ad)^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{13/3}} + \frac{d^2(bc-ad)^{4/3}\log(c+dx^3)}{6c^{13/3}}
- \frac{d^2(bc-ad)^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{13/3}}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.73 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.32

$$
\int \frac{(a+bx^3)^{4/3}}{x^{11}(c+dx^3)}\,dx = \frac{\frac{3\sqrt[3]{c}\sqrt[3]{a+bx^3}(6b^3c^3x^9-2ab^2c^2x^6(c-10dx^3)+a^2bcx^3(-22c^2+40cdx^3-175d^2x^6)+a^3(-14c^3+20c^2dx^3-35cd^2x^6+}{a^2x^{10}}}{}
$$

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^11*(c + d*x^3)),x]
```

```
[Out] ((3*c^(1/3)*(a + b*x^3)^(1/3)*(6*b^3*c^3*x^9 - 2*a*b^2*c^2*x^6*(c - 10*d*x^
3) + a^2*b*c*x^3*(-22*c^2 + 40*c*d*x^3 - 175*d^2*x^6) + a^3*(-14*c^3 + 20*c
^2*d*x^3 - 35*c*d^2*x^6 + 140*d^3*x^9)))/(a^2*x^10) + 70*Sqrt[-6 - (6*I)*Sq
rt[3]]*d^2*(b*c - a*d)^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c -
 a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + 70*(1 - I*Sqr
t[3])*d^2*(b*c - a*d)^(4/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(
1/3)*(a + b*x^3)^(1/3)] + (35*I)*(I + Sqrt[3])*d^2*(b*c - a*d)^(4/3)*Log[2*
(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b
*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(420*c^(13/3))
```

Maple [A] (verified)

Time = 5.05 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.92

method	result
pseudoelliptic	$\frac{-\frac{3\left(\left(-\frac{3b\,x^{3}}{7}+a\right)\left(b\,x^{3}+a\right)^{2}c^{3}-\frac{10ad\,x^{3}\left(b\,x^{3}+a\right)^{2}c^{2}}{7}+\frac{5a^{2}d^{2}x^{6}\left(5b\,x^{3}+a\right)c}{2}-10a^{3}d^{3}x^{9}\right)c\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{5}+a^{2}d^{2}x^{10}(ad-bc)^{2}}{6}$

```
[In] int((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/((a*d-b*c)/c)^(2/3)*(-3/5*((-3/7*b*x^3+a)*(b*x^3+a)^2*c^3-10/7*a*d*x^3*
(b*x^3+a)^2*c^2+5/2*a^2*d^2*x^6*(5*b*x^3+a)*c-10*a^3*d^3*x^9)*c*((a*d-b*c)/
c)^(2/3)*(b*x^3+a)^(1/3)+a^2*d^2*x^10*(a*d-b*c)^2*(2*arctan(1/3*3^(1/2)*(((
a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln(((
(a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3
))/x^2)-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)))/x^10/c^5/a^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^{11}(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^{11}(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{4}{3}}}{x^{11}(c+dx^3)}\,dx$$

```
[In] integrate((b*x**3+a)**(4/3)/x**11/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(x**11*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^{11}(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^{11}}\,dx$$

```
[In] integrate((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^11), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^{11}(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^{11}}\,dx$$

```
[In] integrate((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^11), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^{11}(c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{4/3}}{x^{11}\,(d\,x^3+c)}\,dx$$

```
[In] int((a + b*x^3)^(4/3)/(x^11*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(4/3)/(x^11*(c + d*x^3)), x)
```

3.708 $\int \frac{(a+bx^3)^{4/3}}{x^{14}(c+dx^3)} dx$

Optimal result

Integrand size = 24, antiderivative size = 392

$$\int \frac{(a+bx^3)^{4/3}}{x^{14}(c+dx^3)} dx = -\frac{a\sqrt[3]{a+bx^3}}{13cx^{13}} - \frac{(14bc-13ad)\sqrt[3]{a+bx^3}}{130c^2x^{10}} - \frac{(4b^2c^2-143abcd+130a^2d^2)\sqrt[3]{a+bx^3}}{910ac^3x^7} + \frac{(12b^3c^3+26ab^2c^2d-520a^2bcd^2+455a^3d^3)\sqrt[3]{a+bx^3}}{1820a^2c^4x^4} - \frac{(36b^4c^4+78ab^3c^3d+260a^2b^2c^2d^2-2275a^3bcd^3+1820a^4d^4)\sqrt[3]{a+bx^3}}{1820a^3c^5x} + \frac{d^3(bc-ad)^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{16/3}} - \frac{d^3(bc-ad)^{4/3}\log(c+dx^3)}{6c^{16/3}} + \frac{d^3(bc-ad)^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{16/3}}$$

```
[Out] -1/13*a*(b*x^3+a)^(1/3)/c/x^13-1/130*(-13*a*d+14*b*c)*(b*x^3+a)^(1/3)/c^2/x
^10-1/910*(130*a^2*d^2-143*a*b*c*d+4*b^2*c^2)*(b*x^3+a)^(1/3)/a/c^3/x^7+1/1
820*(455*a^3*d^3-520*a^2*b*c*d^2+26*a*b^2*c^2*d+12*b^3*c^3)*(b*x^3+a)^(1/3)
/a^2/c^4/x^4-1/1820*(1820*a^4*d^4-2275*a^3*b*c*d^3+260*a^2*b^2*c^2*d^2+78*a
*b^3*c^3*d+36*b^4*c^4)*(b*x^3+a)^(1/3)/a^3/c^5/x-1/6*d^3*(-a*d+b*c)^(4/3)*l
n(d*x^3+c)/c^(16/3)+1/2*d^3*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-
(b*x^3+a)^(1/3))/c^(16/3)+1/3*d^3*(-a*d+b*c)^(4/3)*arctan(1/3*(1+2*(-a*d+b*
c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(16/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {485, 597, 12, 503}

$$\int \frac{(a+bx^3)^{4/3}}{x^{14}(c+dx^3)}\,dx = -\frac{\sqrt[3]{a+bx^3}(130a^2d^2-143abcd+4b^2c^2)}{910ac^3x^7} + \frac{\sqrt[3]{a+bx^3}(455a^3d^3-520a^2bcd^2+26ab^2c^2d+12b^3c^3)}{1820a^2c^4x^4} - \frac{\sqrt[3]{a+bx^3}(1820a^4d^4-2275a^3bcd^3+260a^2b^2c^2d^2+78ab^3c^3d+36b^4c^4)}{1820a^3c^5x} + \frac{d^3(bc-ad)^{4/3}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{16/3}} - \frac{d^3(bc-ad)^{4/3}\log(c+dx^3)}{6c^{16/3}} + \frac{d^3(bc-ad)^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{16/3}} - \frac{\sqrt[3]{a+bx^3}(14bc-13ad)}{130c^2x^{10}} - \frac{a\sqrt[3]{a+bx^3}}{13cx^{13}}$$

```
[In] Int[(a + b*x^3)^(4/3)/(x^14*(c + d*x^3)),x]
```

```
[Out] -1/13*(a*(a + b*x^3)^(1/3))/(c*x^13) - ((14*b*c - 13*a*d)*(a + b*x^3)^(1/3)
)/(130*c^2*x^10) - ((4*b^2*c^2 - 143*a*b*c*d + 130*a^2*d^2)*(a + b*x^3)^(1/
3))/(910*a*c^3*x^7) + ((12*b^3*c^3 + 26*a*b^2*c^2*d - 520*a^2*b*c*d^2 + 455
*a^3*d^3)*(a + b*x^3)^(1/3))/(1820*a^2*c^4*x^4) - ((36*b^4*c^4 + 78*a*b^3*c
^3*d + 260*a^2*b^2*c^2*d^2 - 2275*a^3*b*c*d^3 + 1820*a^4*d^4)*(a + b*x^3)^(
1/3))/(1820*a^3*c^5*x) + (d^3*(b*c - a*d)^(4/3)*ArcTan[(1 + (2*(b*c - a*d)^
(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(16/3)) - (d^3*(
b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*c^(16/3)) + (d^3*(b*c - a*d)^(4/3)*Log[
((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(16/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
```

```
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a\sqrt[3]{a+bx^3}}{13cx^{13}}+\frac{\int \frac{a(14bc-13ad)+b(13bc-12ad)x^3}{x^{11}(a+bx^3)^{2/3}(c+dx^3)}\,dx}{13c}\\
&= -\frac{a\sqrt[3]{a+bx^3}}{13cx^{13}}-\frac{(14bc-13ad)\sqrt[3]{a+bx^3}}{130c^2x^{10}}-\frac{\int \frac{-a(4b^2c^2-143abcd+130a^2d^2)+9abd(14bc-13ad)x^3}{x^8(a+bx^3)^{2/3}(c+dx^3)}\,dx}{130ac^2}\\
&= -\frac{a\sqrt[3]{a+bx^3}}{13cx^{13}}-\frac{(14bc-13ad)\sqrt[3]{a+bx^3}}{130c^2x^{10}}-\frac{(4b^2c^2-143abcd+130a^2d^2)\sqrt[3]{a+bx^3}}{910ac^3x^7}\\
&\quad+\frac{\int \frac{-2a(12b^3c^3+26ab^2c^2d-520a^2bcd^2+455a^3d^3)-6abd(4b^2c^2-143abcd+130a^2d^2)x^3}{x^5(a+bx^3)^{2/3}(c+dx^3)}\,dx}{910a^2c^3}\\
&= -\frac{a\sqrt[3]{a+bx^3}}{13cx^{13}}-\frac{(14bc-13ad)\sqrt[3]{a+bx^3}}{130c^2x^{10}}-\frac{(4b^2c^2-143abcd+130a^2d^2)\sqrt[3]{a+bx^3}}{910ac^3x^7}\\
&\quad+\frac{(12b^3c^3+26ab^2c^2d-520a^2bcd^2+455a^3d^3)\sqrt[3]{a+bx^3}}{1820a^2c^4x^4}\\
&\quad-\frac{\int \frac{-2a(36b^4c^4+78ab^3c^3d+260a^2b^2c^2d^2-2275a^3bcd^3+1820a^4d^4)-6abd(12b^3c^3+26ab^2c^2d-520a^2bcd^2+455a^3d^3)x^3}{x^2(a+bx^3)^{2/3}(c+dx^3)}\,dx}{3640a^3c^4}
\end{aligned}$$

$$= -\frac{a\sqrt[3]{a+bx^3}}{13cx^{13}} - \frac{(14bc-13ad)\sqrt[3]{a+bx^3}}{130c^2x^{10}} - \frac{(4b^2c^2-143abcd+130a^2d^2)\sqrt[3]{a+bx^3}}{910ac^3x^7} + \frac{(12b^3c^3+26ab^2c^2d-520a^2bcd^2+455a^3d^3)\sqrt[3]{a+bx^3}}{1820a^2c^4x^4} - \frac{(36b^4c^4+78ab^3c^3d+260a^2b^2c^2d^2-2275a^3bcd^3+1820a^4d^4)\sqrt[3]{a+bx^3}}{1820a^3c^5x} + \frac{\int -\frac{3640a^4d^3(bc-ad)^2x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{3640a^4c^5}$$

$$= -\frac{a\sqrt[3]{a+bx^3}}{13cx^{13}} - \frac{(14bc-13ad)\sqrt[3]{a+bx^3}}{130c^2x^{10}} - \frac{(4b^2c^2-143abcd+130a^2d^2)\sqrt[3]{a+bx^3}}{910ac^3x^7} + \frac{(12b^3c^3+26ab^2c^2d-520a^2bcd^2+455a^3d^3)\sqrt[3]{a+bx^3}}{1820a^2c^4x^4} - \frac{(36b^4c^4+78ab^3c^3d+260a^2b^2c^2d^2-2275a^3bcd^3+1820a^4d^4)\sqrt[3]{a+bx^3}}{1820a^3c^5x} - \frac{(d^3(bc-ad)^2)\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{c^5}$$

$$= -\frac{a\sqrt[3]{a+bx^3}}{13cx^{13}} - \frac{(14bc-13ad)\sqrt[3]{a+bx^3}}{130c^2x^{10}} - \frac{(4b^2c^2-143abcd+130a^2d^2)\sqrt[3]{a+bx^3}}{910ac^3x^7} + \frac{(12b^3c^3+26ab^2c^2d-520a^2bcd^2+455a^3d^3)\sqrt[3]{a+bx^3}}{1820a^2c^4x^4} - \frac{(36b^4c^4+78ab^3c^3d+260a^2b^2c^2d^2-2275a^3bcd^3+1820a^4d^4)\sqrt[3]{a+bx^3}}{1820a^3c^5x} + \frac{d^3(bc-ad)^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{16/3}} - \frac{d^3(bc-ad)^{4/3}\log(c+dx^3)}{6c^{16/3}} + \frac{d^3(bc-ad)^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{16/3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.75 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx^3)^{4/3}}{x^{14}(c+dx^3)}\,dx = \frac{-\frac{3\sqrt[3]{c}\sqrt[3]{a+bx^3}(36b^4c^4x^{12}+6ab^3c^3x^9(-2c+13dx^3)+2a^2b^2c^2x^6(4c^2-13cdx^3+130d^2x^6)+a^3bcx^3(196c^3-286c^2d\ldots}{a^3x^{13}}}{\ldots}$$

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^14*(c + d*x^3)),x]
```

```
[Out] ((-3*c^(1/3)*(a + b*x^3)^(1/3)*(36*b^4*c^4*x^12 + 6*a*b^3*c^3*x^9*(-2*c + 1
3*d*x^3) + 2*a^2*b^2*c^2*x^6*(4*c^2 - 13*c*d*x^3 + 130*d^2*x^6) + a^3*b*c*x
^3*(196*c^3 - 286*c^2*d*x^3 + 520*c*d^2*x^6 - 2275*d^3*x^9) + a^4*(140*c^4
- 182*c^3*d*x^3 + 260*c^2*d^2*x^6 - 455*c*d^3*x^9 + 1820*d^4*x^12)))/(a^3*x
^13) - 910*Sqrt[-6 - (6*I)*Sqrt[3]]*d^3*(b*c - a*d)^(4/3)*ArcTan[(3*(b*c -
a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b
*x^3)^(1/3))] + (910*I)*(I + Sqrt[3])*d^3*(b*c - a*d)^(4/3)*Log[2*(b*c - a*
d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)] + 455*(1 - I*Sqrt[3
])*d^3*(b*c - a*d)^(4/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(
1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b
*x^3)^(2/3)])/(5460*c^(16/3))
```

Maple [A] (verified)

Time = 5.06 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$-\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}\left(\left(\frac{9}{35}b^2x^6-\frac{3}{5}ab\,x^3+a^2\right)\left(b\,x^3+a\right)^2c^4-\frac{13x^3\left(-\frac{3b\,x^3}{7}+a\right)d\left(b\,x^3+a\right)^2a\,c^3}{10}+\frac{13\left(b\,x^3+a\right)^2a^2c^2d^2x^6}{7}-\frac{13a^3d^3x^9\left(5b\,x^3\right.}{4}\right.}{}$

```
[In] int((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/13/((a*d-b*c)/c)^(2/3)*(((a*d-b*c)/c)^(2/3)*((9/35*b^2*x^6-3/5*a*b*x^3+a
^2)*(b*x^3+a)^2*c^4-13/10*x^3*(-3/7*b*x^3+a)*d*(b*x^3+a)^2*a*c^3+13/7*(b*x^
3+a)^2*a^2*c^2*d^2*x^6-13/4*a^3*d^3*x^9*(5*b*x^3+a)*c+13*a^4*d^4*x^12)*c*(b
*x^3+a)^(1/3)+13/6*a^3*d^3*x^13*(a*d-b*c)^2*(2*arctan(1/3*3^(1/2)*(((a*d-b*
c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b
*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2
)-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)))/x^13/c^6/a^3
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^{14}\,(c+dx^3)}\,dx=\text{Timed out}$$

```
[In] integrate((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^{14}\left(c+dx^3\right)}\,dx=\text{Timed out}$$

```
[In] integrate((b*x**3+a)**(4/3)/x**14/(d*x**3+c),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^{14}\left(c+dx^3\right)}\,dx=\int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^{14}}\,dx$$

```
[In] integrate((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^14), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^{14}\left(c+dx^3\right)}\,dx=\int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^{14}}\,dx$$

```
[In] integrate((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^14), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^{14}\left(c+dx^3\right)}\,dx=\int \frac{(b\,x^3+a)^{4/3}}{x^{14}\,\left(d\,x^3+c\right)}\,dx$$

```
[In] int((a + b*x^3)^(4/3)/(x^14*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(4/3)/(x^14*(c + d*x^3)), x)
```

3.709 $\int \frac{x^6(a+bx^3)^{4/3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{x^6(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{ax^7\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{7}{3},-\frac{4}{3},1,\frac{10}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] 1/7*a*x^7*(b*x^3+a)^(1/3)*AppellF1(7/3,-4/3,1,10/3,-b*x^3/a,-d*x^3/c)/c/(1+
b*x^3/a)^(1/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{x^6(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{ax^7\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{7}{3},-\frac{4}{3},1,\frac{10}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[In] Int[(x^6*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (a*x^7*(a + b*x^3)^(1/3)*AppellF1[7/3, -4/3, 1, 10/3, -((b*x^3)/a), -((d*x^
3)/c)])/(7*c*(1 + (b*x^3)/a)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{x^6\left(1+\frac{bx^3}{a}\right)^{4/3}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{ax^7\sqrt[3]{a+bx^3}F_1\left(\frac{7}{3};-\frac{4}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 343 vs. 2(65) = 130.

Time = 9.84 (sec) , antiderivative size = 343, normalized size of antiderivative = 5.28

$$\int \frac{x^6(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{x\left(2(a+bx^3)\left(2a^2d^2+3abd(-8c+3dx^3)+b^2(20c^2-8cdx^3+5d^2x^6)\right)-\frac{(20b^3c^3-30ab^2c\cdots}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[(x^6*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (x*(2*(a + b*x^3)*(2*a^2*d^2 + 3*a*b*d*(-8*c + 3*d*x^3) + b^2*(20*c^2 - 8*c
*d*x^3 + 5*d^2*x^6)) - ((20*b^3*c^3 - 30*a*b^2*c^2*d + 8*a^2*b*c*d^2 + a^3*
d^3)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((
d*x^3)/c)])/c + (16*a^2*c^2*(10*b^2*c^2 - 12*a*b*c*d + a^2*d^2)*AppellF1[1/
3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/((c + d*x^3)*(-4*a*c*AppellF1[
1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/
3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -
((b*x^3)/a), -((d*x^3)/c)])))))/(80*b*d^3*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{x^{6} \left(b \,x^{3}+a \right)^{\frac{4}{3}}}{d \,x^{3}+c}d x$$

```
[In] int(x^6*(b*x^3+a)^(4/3)/(d*x^3+c),x)
[Out] int(x^6*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6(a+bx^3)^{4/3}}{c+dx^3}\,dx = \text{Timed out}$$

```
[In] integrate(x^6*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^6(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{x^{6} \left(a + b x^{3}\right)^{\frac{4}{3}}}{c + d x^{3}}\, dx$$

```
[In] integrate(x**6*(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(x**6*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^6(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int { \frac {{\left (b x^{3} + a\right)}^{\frac {4}{3}} x^{6}}{d x^{3} + c} \,d x }$$

```
[In] integrate(x^6*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)*x^6/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{x^6(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}x^6}{dx^3+c}\,dx$$

```
[In] integrate(x^6*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(4/3)*x^6/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{x^6\,(b\,x^3+a)^{4/3}}{d\,x^3+c}\,dx$$

```
[In] int((x^6*(a + b*x^3)^(4/3))/(c + d*x^3),x)
[Out] int((x^6*(a + b*x^3)^(4/3))/(c + d*x^3), x)
```

3.710 $\int \frac{x^3(a+bx^3)^{4/3}}{c+dx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{x^3(a+bx^3)^{4/3}}{c+dx^3}\,dx=\frac{ax^4\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{4}{3},-\frac{4}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] 1/4*a*x^4*(b*x^3+a)^(1/3)*AppellF1(4/3,-4/3,1,7/3,-b*x^3/a,-d*x^3/c)/c/(1+b
*x^3/a)^(1/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{x^3(a+bx^3)^{4/3}}{c+dx^3}\,dx=\frac{ax^4\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{4}{3},-\frac{4}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[In] Int[(x^3*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (a*x^4*(a + b*x^3)^(1/3)*AppellF1[4/3, -4/3, 1, 7/3, -((b*x^3)/a), -((d*x^3
)/c)])/(4*c*(1 + (b*x^3)/a)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int\frac{x^3\left(1+\frac{bx^3}{a}\right)^{4/3}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{ax^4\sqrt[3]{a+bx^3}F_1\left(\frac{4}{3};-\frac{4}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 280 vs. 2(65) = 130.

Time = 9.29 (sec) , antiderivative size = 280, normalized size of antiderivative = 4.31

$$\int\frac{x^3(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{x\left(4(a+bx^3)\left(-5bc+6ad+2bdx^3\right)+\frac{(10b^2c^2-15abcd+4a^2d^2)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{b}{}\right.}{c}\right.}{}$$

```
[In] Integrate[(x^3*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (x*(4*(a + b*x^3)*(-5*b*c + 6*a*d + 2*b*d*x^3) + ((10*b^2*c^2 - 15*a*b*c*d
+ 4*a^2*d^2)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)
/a), -((d*x^3)/c)])/c + (16*a^2*c^2*(-5*b*c + 6*a*d)*AppellF1[1/3, 2/3, 1,
4/3, -((b*x^3)/a), -((d*x^3)/c)])/((c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1
, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3,
-((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a)
, -((d*x^3)/c)])))))/(40*d^2*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{x^3(b\,x^3+a)^{\frac{4}{3}}}{d\,x^3+c}dx$$

```
[In] int(x^3*(b*x^3+a)^(4/3)/(d*x^3+c),x)
[Out] int(x^3*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^3)^{4/3}}{c+dx^3}\,dx = \text{Timed out}$$

```
[In] integrate(x^3*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^3(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{x^3(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

```
[In] integrate(x**3*(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(x**3*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{x^3(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}x^3}{dx^3+c}\,dx$$

```
[In] integrate(x^3*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)*x^3/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{x^3(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}x^3}{dx^3+c}\,dx$$

```
[In] integrate(x^3*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(4/3)*x^3/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{x^3\,(b\,x^3+a)^{4/3}}{d\,x^3+c}\,dx$$

```
[In] int((x^3*(a + b*x^3)^(4/3))/(c + d*x^3),x)
[Out] int((x^3*(a + b*x^3)^(4/3))/(c + d*x^3), x)
```

3.711 $\int \frac{(a+bx^3)^{4/3}}{c+dx^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3} dx = \frac{ax\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{4}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] a*x*(b*x^3+a)^(1/3)*AppellF1(1/3,-4/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(1+b*x^3/a
)^(1/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {441, 440}

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3} dx = \frac{ax\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{4}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[In] Int[(a + b*x^3)^(4/3)/(c + d*x^3),x]
```

```
[Out] (a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -4/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(c*(1 + (b*x^3)/a)^(1/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
```

```
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{ax\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{4}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 346 vs. $2(60) = 120$.

Time = 0.30 (sec) , antiderivative size = 346, normalized size of antiderivative = 5.77

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx = \frac{x\left(\frac{b(-2bc+3ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}+\frac{4\left(-4ac\left(2a^2d+abdx^3+b^2x^3(c+dx^3)\right)\text{AppellF1}\left(\right.\right.}{(c+dx^3)\left(-4ac\,\text{AppellF1}\left(\frac{1}{3},\right.\right.}\right.}{8d\,(a}$$

```
[In] Integrate[(a + b*x^3)^(4/3)/(c + d*x^3),x]
```

```
[Out] (x*((b*(-2*b*c + 3*a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3
, -((b*x^3)/a), -((d*x^3)/c)])/c + (4*(-4*a*c*(2*a^2*d + a*b*d*x^3 + b^2*x^
3*(c + d*x^3))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + b*x
^3*(a + b*x^3)*(c + d*x^3)*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a),
-((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)
])))/((c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] +
 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))))/(8*d*(a
+ b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}}{d\,x^{3}+c}dx$$

```
[In] int((b*x^3+a)^(4/3)/(d*x^3+c),x)
[Out] int((b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

```
[In] integrate((b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{dx^3+c}\,dx$$

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)/(d*x^3 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{dx^3+c}\,dx$$

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(4/3)/(d*x^3 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx = \int \frac{(b\,x^3+a)^{4/3}}{d\,x^3+c}\,dx$$

```
[In] int((a + b*x^3)^(4/3)/(c + d*x^3),x)
[Out] int((a + b*x^3)^(4/3)/(c + d*x^3), x)
```

3.712 $\int \frac{(a+bx^3)^{4/3}}{x^3(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{(a+bx^3)^{4/3}}{x^3(c+dx^3)}\,dx = -\frac{a\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(-\frac{2}{3},-\frac{4}{3},1,\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] -1/2*a*(b*x^3+a)^(1/3)*AppellF1(-2/3,-4/3,1,1/3,-b*x^3/a,-d*x^3/c)/c/x^2/(1
+b*x^3/a)^(1/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {525, 524}

$$\int \frac{(a+bx^3)^{4/3}}{x^3(c+dx^3)}\,dx = -\frac{a\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(-\frac{2}{3},-\frac{4}{3},1,\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[In] Int[(a + b*x^3)^(4/3)/(x^3*(c + d*x^3)),x]
```

```
[Out] -1/2*(a*(a + b*x^3)^(1/3)*AppellF1[-2/3, -4/3, 1, 1/3, -((b*x^3)/a), -((d*x
^3)/c)])/(c*x^2*(1 + (b*x^3)/a)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int\frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{x^3(c+dx^3)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= -\frac{a\sqrt[3]{a+bx^3}F_1\left(-\frac{2}{3};-\frac{4}{3},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 341 vs. 2(65) = 130.

Time = 10.36 (sec) , antiderivative size = 341, normalized size of antiderivative = 5.25

$$\int\frac{(a+bx^3)^{4/3}}{x^3(c+dx^3)}\,dx =$$
$$-\frac{b(-2bc+ad)x^6\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{4ac\left(-4ac(ac-2bcx^3+3adx^3+bdx^6)\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\right.\right.}{(c+dx^3)\left(-4ac\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{b}{}\right.\right.}}{8c^2x^2(a+bx^3)^{2/3}}$$

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^3*(c + d*x^3)),x]
```

```
[Out] -1/8*(b*(-2*b*c + a*d)*x^6*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3,
 -((b*x^3)/a), -((d*x^3)/c)] + (4*a*c*(-4*a*c*(a*c - 2*b*c*x^3 + 3*a*d*x^3
+ b*d*x^6)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(a
+ b*x^3)*(c + d*x^3)*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x
^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/(
(c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)]
+ x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c
*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(c^2*x^2*(a + b
*x^3)^(2/3))
```

Maple [F]

$$\int \frac{(b\,x^3+a)^{\frac{4}{3}}}{x^3\,(d\,x^3+c)}dx$$

```
[In] int((b*x^3+a)^(4/3)/x^3/(d*x^3+c),x)
[Out] int((b*x^3+a)^(4/3)/x^3/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^3\,(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^3+a)^(4/3)/x^3/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^3\,(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{4}{3}}}{x^3\,(c+dx^3)}\,dx$$

```
[In] integrate((b*x**3+a)**(4/3)/x**3/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(4/3)/(x**3*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^3\,(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^3}\,dx$$

```
[In] integrate((b*x^3+a)^(4/3)/x^3/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^3), x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^3(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^3}\,dx$$

```
[In] integrate((b*x^3+a)^(4/3)/x^3/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^3(c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{4/3}}{x^3\,(d\,x^3+c)}\,dx$$

```
[In] int((a + b*x^3)^(4/3)/(x^3*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(4/3)/(x^3*(c + d*x^3)), x)
```

3.713 $\int \frac{(a+bx^3)^{4/3}}{x^6(c+dx^3)} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{(a+bx^3)^{4/3}}{x^6(c+dx^3)} \, dx = -\frac{a\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(-\frac{5}{3},-\frac{4}{3},1,-\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5cx^5\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] -1/5*a*(b*x^3+a)^(1/3)*AppellF1(-5/3,-4/3,1,-2/3,-b*x^3/a,-d*x^3/c)/c/x^5/(
1+b*x^3/a)^(1/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{(a+bx^3)^{4/3}}{x^6(c+dx^3)} \, dx = -\frac{a\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(-\frac{5}{3},-\frac{4}{3},1,-\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5cx^5\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[In] Int[(a + b*x^3)^(4/3)/(x^6*(c + d*x^3)),x]
```

```
[Out] -1/5*(a*(a + b*x^3)^(1/3)*AppellF1[-5/3, -4/3, 1, -2/3, -((b*x^3)/a), -((d*
x^3)/c)])/(c*x^5*(1 + (b*x^3)/a)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{x^6(c+dx^3)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{a\sqrt[3]{a+bx^3}F_1\left(-\frac{5}{3};-\frac{4}{3},1;-\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5cx^5\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 286 vs. 2(65) = 130.

Time = 10.38 (sec) , antiderivative size = 286, normalized size of antiderivative = 4.40

$$\int \frac{(a+bx^3)^{4/3}}{x^6(c+dx^3)}\,dx = \frac{-\frac{4(a+bx^3)(2ac+6bcx^3-5adx^3)}{c^2x^5} + \frac{bd(-6bc+5ad)x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3} - \frac{\cdots}{c(c+dx^3)\left(-4\cdots\right.}}{40\,(a+\cdots}$$

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^6*(c + d*x^3)),x]
```

```
[Out] ((-4*(a + b*x^3)*(2*a*c + 6*b*c*x^3 - 5*a*d*x^3))/(c^2*x^5) + (b*d*(-6*b*c
+ 5*a*d)*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a),
 -((d*x^3)/c)])/c^3 - (16*a*(4*b^2*c^2 - 15*a*b*c*d + 10*a^2*d^2)*x*AppellF
1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*(c + d*x^3)*(-4*a*c*App
ellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4
/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1,
7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(40*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{(b\,x^3+a)^{\frac{4}{3}}}{x^6\,(d\,x^3+c)}dx$$

[In] int((b*x^3+a)^(4/3)/x^6/(d*x^3+c),x)

[Out] int((b*x^3+a)^(4/3)/x^6/(d*x^3+c),x)

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^6\,(c+dx^3)}\,dx = \text{Timed out}$$

[In] integrate((b*x^3+a)^(4/3)/x^6/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^6\,(c+dx^3)}\,dx = \int \frac{(a+bx^3)^{\frac{4}{3}}}{x^6\,(c+dx^3)}\,dx$$

[In] integrate((b*x**3+a)**(4/3)/x**6/(d*x**3+c),x)

[Out] Integral((a + b*x**3)**(4/3)/(x**6*(c + d*x**3)), x)

Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^6\,(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^6}\,dx$$

[In] integrate((b*x^3+a)^(4/3)/x^6/(d*x^3+c),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^6), x)

Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^6(c+dx^3)}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^6}\,dx$$

```
[In] integrate((b*x^3+a)^(4/3)/x^6/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^6(c+dx^3)}\,dx = \int \frac{(b\,x^3+a)^{4/3}}{x^6\,(d\,x^3+c)}\,dx$$

```
[In] int((a + b*x^3)^(4/3)/(x^6*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(4/3)/(x^6*(c + d*x^3)), x)
```

3.714 $\int \frac{x^{14}}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 290

$$\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = -\frac{(bc+ad)\,(b^2c^2+a^2d^2)\,(a+bx^3)^{2/3}}{2b^4d^4} + \frac{(b^2c^2+2abcd+3a^2d^2)\,(a+bx^3)^{5/3}}{5b^4d^3} - \frac{(bc+3ad)\,(a+bx^3)^{8/3}}{8b^4d^2} + \frac{(a+bx^3)^{11/3}}{11b^4d} - \frac{c^4\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{14/3}\sqrt[3]{bc-ad}} + \frac{c^4\log\left(c+dx^3\right)}{6d^{14/3}\sqrt[3]{bc-ad}} - \frac{c^4\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{14/3}\sqrt[3]{bc-ad}}$$

```
[Out] -1/2*(a*d+b*c)*(a^2*d^2+b^2*c^2)*(b*x^3+a)^(2/3)/b^4/d^4+1/5*(3*a^2*d^2+2*a
*b*c*d+b^2*c^2)*(b*x^3+a)^(5/3)/b^4/d^3-1/8*(3*a*d+b*c)*(b*x^3+a)^(8/3)/b^4
/d^2+1/11*(b*x^3+a)^(11/3)/b^4/d+1/6*c^4*ln(d*x^3+c)/d^(14/3)/(-a*d+b*c)^(1
/3)-1/2*c^4*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(14/3)/(-a*d+b*c
)^(1/3)-1/3*c^4*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3
^(1/2))/d^(14/3)/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {457, 90, 58, 631, 210, 31}

$$\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = -\frac{(a+bx^3)^{2/3}\,(ad+bc)\,(a^2d^2+b^2c^2)}{2b^4d^4} + \frac{(a+bx^3)^{5/3}\,(3a^2d^2+2abcd+b^2c^2)}{5b^4d^3} - \frac{c^4\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{14/3}\sqrt[3]{bc-ad}} - \frac{(a+bx^3)^{8/3}\,(3ad+bc)}{8b^4d^2} + \frac{(a+bx^3)^{11/3}}{11b^4d} + \frac{c^4\log(c+dx^3)}{6d^{14/3}\sqrt[3]{bc-ad}} - \frac{c^4\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{14/3}\sqrt[3]{bc-ad}}$$

```
[In] Int[x^14/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -1/2*((b*c + a*d)*(b^2*c^2 + a^2*d^2)*(a + b*x^3)^(2/3))/(b^4*d^4) + ((b^2*
c^2 + 2*a*b*c*d + 3*a^2*d^2)*(a + b*x^3)^(5/3))/(5*b^4*d^3) - ((b*c + 3*a*d
)*(a + b*x^3)^(8/3))/(8*b^4*d^2) + (a + b*x^3)^(11/3)/(11*b^4*d) - (c^4*Arc
Tan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3
]*d^(14/3)*(b*c - a*d)^(1/3)) + (c^4*Log[c + d*x^3])/(6*d^(14/3)*(b*c - a*d
)^(1/3)) - (c^4*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(1
4/3)*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{(bc+ad)\left(-b^2c^2-a^2d^2\right)}{b^3d^4\sqrt[3]{a+bx}} + \frac{\left(b^2c^2+2abcd+3a^2d^2\right)(a+bx)^{2/3}}{b^3d^3}\right.\right.\\
&\qquad\left.\left. + \frac{(-bc-3ad)(a+bx)^{5/3}}{b^3d^2} + \frac{(a+bx)^{8/3}}{b^3d} + \frac{c^4}{d^4\sqrt[3]{a+bx}(c+dx)}\right)dx, x, x^3\right)\\
&= -\frac{(bc+ad)\left(b^2c^2+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^4d^4} + \frac{\left(b^2c^2+2abcd+3a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^4d^3}\\
&\quad - \frac{(bc+3ad)\left(a+bx^3\right)^{8/3}}{8b^4d^2} + \frac{\left(a+bx^3\right)^{11/3}}{11b^4d} + \frac{c^4\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)}{3d^4}
\end{aligned}$$

$$= -\frac{(bc+ad)\left(b^2c^2+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^4d^4} + \frac{\left(b^2c^2+2abcd+3a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^4d^3} - \frac{(bc+3ad)\left(a+bx^3\right)^{8/3}}{8b^4d^2} + \frac{\left(a+bx^3\right)^{11/3}}{11b^4d} + \frac{c^4\log\left(c+dx^3\right)}{6d^{14/3}\sqrt[3]{bc-ad}} + \frac{c^4\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}} + x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^5} - \frac{c^4\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}} + x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{14/3}\sqrt[3]{bc-ad}}$$

$$= -\frac{(bc+ad)\left(b^2c^2+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^4d^4} + \frac{\left(b^2c^2+2abcd+3a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^4d^3} - \frac{(bc+3ad)\left(a+bx^3\right)^{8/3}}{8b^4d^2} + \frac{\left(a+bx^3\right)^{11/3}}{11b^4d} + \frac{c^4\log\left(c+dx^3\right)}{6d^{14/3}\sqrt[3]{bc-ad}} - \frac{c^4\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{14/3}\sqrt[3]{bc-ad}} + \frac{c^4\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{d^{14/3}\sqrt[3]{bc-ad}}$$

$$= -\frac{(bc+ad)\left(b^2c^2+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^4d^4} + \frac{\left(b^2c^2+2abcd+3a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^4d^3} - \frac{(bc+3ad)\left(a+bx^3\right)^{8/3}}{8b^4d^2} + \frac{\left(a+bx^3\right)^{11/3}}{11b^4d} - \frac{c^4\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{14/3}\sqrt[3]{bc-ad}} + \frac{c^4\log\left(c+dx^3\right)}{6d^{14/3}\sqrt[3]{bc-ad}} - \frac{c^4\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{14/3}\sqrt[3]{bc-ad}}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.06

$$\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$= \frac{-3d^{2/3}\sqrt[3]{bc-ad}(a+bx^3)^{2/3}\left(81a^3d^3+9a^2bd^2(11c-6dx^3)+3ab^2d(44c^2-22cdx^3+15d^2x^6)+b^3(220c^3\right.}{}$$

```
[In] Integrate[x^14/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (-3*d^(2/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(2/3)*(81*a^3*d^3 + 9*a^2*b*d^2*(
11*c - 6*d*x^3) + 3*a*b^2*d*(44*c^2 - 22*c*d*x^3 + 15*d^2*x^6) + b^3*(220*c
^3 - 88*c^2*d*x^3 + 55*c*d^2*x^6 - 40*d^3*x^9)) - 440*Sqrt[3]*b^4*c^4*ArcTa
n[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 440*b^4*
c^4*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + 220*b^4*c^4*Log[(b
*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a
+ b*x^3)^(2/3)])/(1320*b^4*d^(14/3)*(b*c - a*d)^(1/3))
```

Maple [A] (verified)

Time = 4.80 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.95

method	result
pseudoelliptic	$-\frac{\frac{243\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\left(\frac{\left(-40d^3x^9+55c\,d^2x^6-88c^2d\,x^3+220c^3\right)b^3}{81}+\frac{44da\left(\frac{15}{44}d^2x^6-\frac{1}{2}cd\,x^3+c^2\right)b^2}{27}+\frac{11\left(-\frac{6d\,x^3}{11}+c\right)d^2a^2b}{9}+a^3d^3\right)d\left(b\,x^3+\right.}{220}}{}$

```
[In] int(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/(1/d*(a*d-b*c))^(1/3)*(243/220*(1/d*(a*d-b*c))^(1/3)*(1/81*(-40*d^3*x^
9+55*c*d^2*x^6-88*c^2*d*x^3+220*c^3)*b^3+44/27*d*a*(15/44*d^2*x^6-1/2*c*d*x
^3+c^2)*b^2+11/9*(-6/11*d*x^3+c)*d^2*a^2*b+a^3*d^3)*d*(b*x^3+a)^(2/3)+b^4*c
^4*(-2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a
*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^
(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))))/
d^5/b^4
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 1004, normalized size of antiderivative = 3.46

$$\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/1320*(220*(-b*c*d^2 + a*d^3)^(2/3)*b^4*c^4*log((b*x^3 + a)^(2/3)*d^2 + (
-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 4
40*(-b*c*d^2 + a*d^3)^(2/3)*b^4*c^4*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a
*d^3)^(1/3)) + 660*sqrt(1/3)*(b^5*c^5*d - a*b^4*c^4*d^2)*sqrt((-b*c*d^2 + a
*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 + 3*sqrt(1/3)*(
2*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a
*d^2) + (-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d))*sqrt((-b*c*d^2 + a*d^3)^(1/3)
/(b*c - a*d)) - 3*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c))
- 3*(220*b^4*c^4*d^2 - 88*a*b^3*c^3*d^3 - 33*a^2*b^2*c^2*d^4 - 18*a^3*b*c*d
^5 - 81*a^4*d^6 - 40*(b^4*c*d^5 - a*b^3*d^6)*x^9 + 5*(11*b^4*c^2*d^4 - 2*a*
b^3*c*d^5 - 9*a^2*b^2*d^6)*x^6 - 2*(44*b^4*c^3*d^3 - 11*a*b^3*c^2*d^4 - 6*a
^2*b^2*c*d^5 - 27*a^3*b*d^6)*x^3)*(b*x^3 + a)^(2/3))/(b^5*c*d^6 - a*b^4*d^7
), 1/1320*(220*(-b*c*d^2 + a*d^3)^(2/3)*b^4*c^4*log((b*x^3 + a)^(2/3)*d^2 +
 (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) -
 440*(-b*c*d^2 + a*d^3)^(2/3)*b^4*c^4*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 +
 a*d^3)^(1/3)) + 1320*sqrt(1/3)*(b^5*c^5*d - a*b^4*c^4*d^2)*sqrt(-(-b*c*d^2
 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d + (-b*
c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))/d) - 3*(2
20*b^4*c^4*d^2 - 88*a*b^3*c^3*d^3 - 33*a^2*b^2*c^2*d^4 - 18*a^3*b*c*d^5 - 8
1*a^4*d^6 - 40*(b^4*c*d^5 - a*b^3*d^6)*x^9 + 5*(11*b^4*c^2*d^4 - 2*a*b^3*c*
d^5 - 9*a^2*b^2*d^6)*x^6 - 2*(44*b^4*c^3*d^3 - 11*a*b^3*c^2*d^4 - 6*a^2*b^2
*c*d^5 - 27*a^3*b*d^6)*x^3)*(b*x^3 + a)^(2/3))/(b^5*c*d^6 - a*b^4*d^7)]
```

Sympy [F]

$$\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^{14}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

```
[In] integrate(x**14/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**14/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.57

$$\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = -\frac{b^{48}c^4d^7\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^{49}cd^{11}-ab^{48}d^{12})} - \frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^4\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^6-\sqrt{3}ad^7} + \frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^4\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(bcd^6-ad^7)} - \frac{220\,(bx^3+a)^{\frac{2}{3}}b^{43}c^3d^7-88\,(bx^3+a)^{\frac{5}{3}}b^{42}c^2d^8+220\,(bx^3+a)^{\frac{2}{3}}ab^{42}c^2d^8+55\,(bx^3+a)^{\frac{8}{3}}b^{41}cd^9-176\,(bx}{}$$

```
[In] integrate(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b^48*c^4*d^7*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
 - a*d)/d)^(1/3)))/(b^49*c*d^11 - a*b^48*d^12) - (-b*c*d^2 + a*d^3)^(2/3)*c
^4*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c
 - a*d)/d)^(1/3))/(sqrt(3)*b*c*d^6 - sqrt(3)*a*d^7) + 1/6*(-b*c*d^2 + a*d^3
)^(2/3)*c^4*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3
) + (-(b*c - a*d)/d)^(2/3))/(b*c*d^6 - a*d^7) - 1/440*(220*(b*x^3 + a)^(2/3
)*b^43*c^3*d^7 - 88*(b*x^3 + a)^(5/3)*b^42*c^2*d^8 + 220*(b*x^3 + a)^(2/3)*
a*b^42*c^2*d^8 + 55*(b*x^3 + a)^(8/3)*b^41*c*d^9 - 176*(b*x^3 + a)^(5/3)*a*
b^41*c*d^9 + 220*(b*x^3 + a)^(2/3)*a^2*b^41*c*d^9 - 40*(b*x^3 + a)^(11/3)*b
^40*d^10 + 165*(b*x^3 + a)^(8/3)*a*b^40*d^10 - 264*(b*x^3 + a)^(5/3)*a^2*b^
40*d^10 + 220*(b*x^3 + a)^(2/3)*a^3*b^40*d^10)/(b^44*d^11)
```

Mupad [B] (verification not implemented)

Time = 8.95 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.51

$$\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \left(\frac{6\,a^2}{5\,b^4\,d}+\frac{\left(\frac{4\,a}{b^4\,d}+\frac{b^5\,c-a\,b^4\,d}{b^8\,d^2}\right)\,\left(b^5\,c-a\,b^4\,d\right)}{5\,b^4\,d}\right)\,{\left(b\,x^3+a\right)}^{5/3} - \left(\frac{a}{2\,b^4\,d}+\frac{b^5\,c-a\,b^4\,d}{8\,b^8\,d^2}\right)\,{\left(b\,x^3+a\right)}^{8/3} - {\left(b\,x^3+a\right)}^{2/3}\,\left(\frac{2\,a^3}{b^4\,d}+\frac{\left(\frac{6\,a^2}{b^4\,d}+\frac{\left(\frac{4\,a}{b^4\,d}+\frac{b^5\,c-a\,b^4\,d}{b^8\,d^2}\right)\,\left(b^5\,c-a\,b^4\,d\right)}{b^4\,d}\right)\,\left(b^5\,c-a\,b^4\,d\right)}{2\,b^4\,d}\right)+\frac{{\left(b\,x^3+a\right)}^{11/3}}{11\,b^4\,d}+\frac{c^4\,\ln\left(\frac{c^8\,\left(b\,x^3+ \right.}{d^7}\right.}{3\,d^{14/3}}$$

```
[In] int(x^14/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] ((6*a^2)/(5*b^4*d) + (((4*a)/(b^4*d) + (b^5*c - a*b^4*d)/(b^8*d^2))*(b^5*c
- a*b^4*d))/(5*b^4*d))*(a + b*x^3)^(5/3) - (a/(2*b^4*d) + (b^5*c - a*b^4*d)
/(8*b^8*d^2))*(a + b*x^3)^(8/3) - (a + b*x^3)^(2/3)*((2*a^3)/(b^4*d) + (((6
*a^2)/(b^4*d) + (((4*a)/(b^4*d) + (b^5*c - a*b^4*d)/(b^8*d^2))*(b^5*c - a*b
^4*d))/(b^4*d))*(b^5*c - a*b^4*d))/(2*b^4*d)) + (a + b*x^3)^(11/3)/(11*b^4*
d) + (c^4*log((c^8*(a + b*x^3)^(1/3))/d^7 - (c^8*(a*d - b*c)^(1/3))/d^(22/3
)))/(3*d^(14/3)*(a*d - b*c)^(1/3)) - (log((c^8*(a + b*x^3)^(1/3))/d^7 - (c^
8*(3^(1/2)*1i + 1)^2*(a*d - b*c)^(1/3))/(4*d^(22/3)))*(3^(1/2)*c^4*1i + c^4
))/(6*d^(14/3)*(a*d - b*c)^(1/3)) + (c^4*log((c^8*(a + b*x^3)^(1/3))/d^7 -
(c^8*(3^(1/2)*1i - 1)^2*(a*d - b*c)^(1/3))/(4*d^(22/3)))*((3^(1/2)*1i)/6 -
1/6))/(d^(14/3)*(a*d - b*c)^(1/3))
```

3.715 $\int \frac{x^{11}}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 244

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{(b^2c^2+abcd+a^2d^2)\,(a+bx^3)^{2/3}}{2b^3d^3} - \frac{(bc+2ad)\,(a+bx^3)^{5/3}}{5b^3d^2} + \frac{(a+bx^3)^{8/3}}{8b^3d} + \frac{c^3\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{11/3}\sqrt[3]{bc-ad}} - \frac{c^3\log(c+dx^3)}{6d^{11/3}\sqrt[3]{bc-ad}} + \frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{11/3}\sqrt[3]{bc-ad}}$$

```
[Out] 1/2*(a^2*d^2+a*b*c*d+b^2*c^2)*(b*x^3+a)^(2/3)/b^3/d^3-1/5*(2*a*d+b*c)*(b*x^
3+a)^(5/3)/b^3/d^2+1/8*(b*x^3+a)^(8/3)/b^3/d-1/6*c^3*ln(d*x^3+c)/d^(11/3)/(
-a*d+b*c)^(1/3)+1/2*c^3*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(11/
3)/(-a*d+b*c)^(1/3)+1/3*c^3*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b
*c)^(1/3))*3^(1/2))/d^(11/3)/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

$= \{457, 90, 58, 631, 210, 31\}$

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{(a+bx^3)^{2/3}\,(a^2d^2+abcd+b^2c^2)}{2b^3d^3} + \frac{c^3\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{11/3}\sqrt[3]{bc-ad}} - \frac{(a+bx^3)^{5/3}\,(2ad+bc)}{5b^3d^2} + \frac{(a+bx^3)^{8/3}}{8b^3d} - \frac{c^3\log(c+dx^3)}{6d^{11/3}\sqrt[3]{bc-ad}} + \frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{11/3}\sqrt[3]{bc-ad}}$$

```
[In] Int[x^11/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)^(2/3))/(2*b^3*d^3) - ((b*c + 2*a
*d)*(a + b*x^3)^(5/3))/(5*b^3*d^2) + (a + b*x^3)^(8/3)/(8*b^3*d) + (c^3*Arc
Tan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3
]*d^(11/3)*(b*c - a*d)^(1/3)) - (c^3*Log[c + d*x^3])/(6*d^(11/3)*(b*c - a*d
)^(1/3)) + (c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(1
1/3)*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right) \\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{b^2c^2+abcd+a^2d^2}{b^2d^3\sqrt[3]{a+bx}} + \frac{(-bc-2ad)(a+bx)^{2/3}}{b^2d^2} + \frac{(a+bx)^{5/3}}{b^2d} - \frac{c^3}{d^3\sqrt[3]{a+bx}(c+dx)}\right) dx, x, x^3\right) \\
&= \frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{2/3}}{2b^3d^3} - \frac{(bc+2ad)(a+bx^3)^{5/3}}{5b^3d^2} + \frac{(a+bx^3)^{8/3}}{8b^3d} - \frac{c^3\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)}{3d^3} \\
&= \frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{2/3}}{2b^3d^3} - \frac{(bc+2ad)(a+bx^3)^{5/3}}{5b^3d^2} + \frac{(a+bx^3)^{8/3}}{8b^3d} \\
&\quad - \frac{c^3\log(c+dx^3)}{6d^{11/3}\sqrt[3]{bc-ad}} - \frac{c^3\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}} + x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^4} \\
&\quad + \frac{c^3\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}} + x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{11/3}\sqrt[3]{bc-ad}}
\end{aligned}$$

$$= \frac{(b^2c^2+abcd+a^2d^2)\,(a+bx^3)^{2/3}}{2b^3d^3} - \frac{(bc+2ad)\,(a+bx^3)^{5/3}}{5b^3d^2} + \frac{(a+bx^3)^{8/3}}{8b^3d} - \frac{c^3\log(c+dx^3)}{6d^{11/3}\sqrt[3]{bc-ad}} + \frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{11/3}\sqrt[3]{bc-ad}} - \frac{c^3\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{d^{11/3}\sqrt[3]{bc-ad}}$$

$$= \frac{(b^2c^2+abcd+a^2d^2)\,(a+bx^3)^{2/3}}{2b^3d^3} - \frac{(bc+2ad)\,(a+bx^3)^{5/3}}{5b^3d^2} + \frac{(a+bx^3)^{8/3}}{8b^3d} + \frac{c^3\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{11/3}\sqrt[3]{bc-ad}} - \frac{c^3\log(c+dx^3)}{6d^{11/3}\sqrt[3]{bc-ad}} + \frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{11/3}\sqrt[3]{bc-ad}}$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.08

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$= \frac{3d^{2/3}\sqrt[3]{bc-ad}(a+bx^3)^{2/3}\left(9a^2d^2-6abd(-2c+dx^3)+b^2(20c^2-8cdx^3+5d^2x^6)\right)+40\sqrt{3}b^3c^3\arctan\left(\frac{1-}{—}\right.}{}$$

```
[In] Integrate[x^11/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (3*d^(2/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(2/3)*(9*a^2*d^2 - 6*a*b*d*(-2*c +
 d*x^3) + b^2*(20*c^2 - 8*c*d*x^3 + 5*d^2*x^6)) + 40*Sqrt[3]*b^3*c^3*ArcTan
[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 40*b^3*c^
3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - 20*b^3*c^3*Log[(b*c
- a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b
*x^3)^(2/3)])/(120*b^3*d^(11/3)*(b*c - a*d)^(1/3))
```

Maple [A] (verified)

Time = 4.73 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$\frac{\frac{27\left(\frac{\left(5d^2x^6-8cd\,x^3+20c^2\right)b^2}{9}+\frac{4\left(-\frac{d\,x^3}{2}+c\right)dab}{3}+a^2d^2\right)\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}d\left(b\,x^3+a\right)^{\frac{2}{3}}}{20}+b^3c^3\left(-2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^3+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right.}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right.\right.}{6\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}b^3d^4}$

```
[In] int(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/(1/d*(a*d-b*c))^(1/3)*(27/20*(1/9*(5*d^2*x^6-8*c*d*x^3+20*c^2)*b^2+4/3*
(-1/2*d*x^3+c)*d*a*b+a^2*d^2)*(1/d*(a*d-b*c))^(1/3)*d*(b*x^3+a)^(2/3)+b^3*c
^3*(-2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a
*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^
(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))))/
b^3/d^4
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 873, normalized size of antiderivative = 3.58

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \left[-\frac{20\left(bcd^2-ad^3\right)^{\frac{2}{3}}b^3c^3\log\left(\left(bx^3+a\right)^{\frac{2}{3}}d^2-\left(bcd^2-ad^3\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}d+\left(bcd^2-ad^3\right)^{\frac{2}{3}}\right)-40\left(bcd^2-ad^3\right.}{}\right.$$

$$-\frac{20\left(bcd^2-ad^3\right)^{\frac{2}{3}}b^3c^3\log\left(\left(bx^3+a\right)^{\frac{2}{3}}d^2-\left(bcd^2-ad^3\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}d+\left(bcd^2-ad^3\right)^{\frac{2}{3}}\right)-40\left(bcd^2-ad^3\right.}{}$$

```
[In] integrate(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/120*(20*(b*c*d^2 - a*d^3)^(2/3)*b^3*c^3*log((b*x^3 + a)^(2/3)*d^2 - (b*
c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(2/3)) - 40*(b
*c*d^2 - a*d^3)^(2/3)*b^3*c^3*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(
1/3)) - 60*sqrt(1/3)*(b^4*c^4*d - a*b^3*c^3*d^2)*sqrt(-(b*c*d^2 - a*d^3)^(1
/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 - 3*sqrt(1/3)*(2*(b*c*d
^2 - a*d^3)^(2/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (
b*c*d^2 - a*d^3)^(1/3)*(b*c - a*d))*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*
d)) - 3*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)) - 3*(20*b^3
*c^3*d^2 - 8*a*b^2*c^2*d^3 - 3*a^2*b*c*d^4 - 9*a^3*d^5 + 5*(b^3*c*d^4 - a*b
^2*d^5)*x^6 - 2*(4*b^3*c^2*d^3 - a*b^2*c*d^4 - 3*a^2*b*d^5)*x^3)*(b*x^3 + a
)^(2/3))/(b^4*c*d^5 - a*b^3*d^6), -1/120*(20*(b*c*d^2 - a*d^3)^(2/3)*b^3*c^
3*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d +
 (b*c*d^2 - a*d^3)^(2/3)) - 40*(b*c*d^2 - a*d^3)^(2/3)*b^3*c^3*log((b*x^3 +
 a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) + 120*sqrt(1/3)*(b^4*c^4*d - a*b^3*c
^3*d^2)*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^
3 + a)^(1/3)*d - (b*c*d^2 - a*d^3)^(1/3))*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c
 - a*d))/d) - 3*(20*b^3*c^3*d^2 - 8*a*b^2*c^2*d^3 - 3*a^2*b*c*d^4 - 9*a^3*d
^5 + 5*(b^3*c*d^4 - a*b^2*d^5)*x^6 - 2*(4*b^3*c^2*d^3 - a*b^2*c*d^4 - 3*a^2
*b*d^5)*x^3)*(b*x^3 + a)^(2/3))/(b^4*c*d^5 - a*b^3*d^6)]
```

Sympy [F]

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

```
[In] integrate(x**11/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**11/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.52

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{b^{27}c^3d^5\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^{28}cd^8-ab^{27}d^9)} + \frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^3\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^5-\sqrt{3}ad^6} - \frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^3\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(bcd^5-ad^6)} + \frac{20\,(bx^3+a)^{\frac{2}{3}}b^{23}c^2d^5-8\,(bx^3+a)^{\frac{5}{3}}b^{22}cd^6+20\,(bx^3+a)^{\frac{2}{3}}ab^{22}cd^6+5\,(bx^3+a)^{\frac{8}{3}}b^{21}d^7-16\,(bx^3+a)^{\frac{5}{3}}a}{40\,b^{24}d^8}$$

```
[In] integrate(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^27*c^3*d^5*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
- a*d)/d)^(1/3)))/(b^28*c*d^8 - a*b^27*d^9) + (-b*c*d^2 + a*d^3)^(2/3)*c^3*
arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
a*d)/d)^(1/3))/(sqrt(3)*b*c*d^5 - sqrt(3)*a*d^6) - 1/6*(-b*c*d^2 + a*d^3)^(
2/3)*c^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) +
 (-(b*c - a*d)/d)^(2/3))/(b*c*d^5 - a*d^6) + 1/40*(20*(b*x^3 + a)^(2/3)*b^2
3*c^2*d^5 - 8*(b*x^3 + a)^(5/3)*b^22*c*d^6 + 20*(b*x^3 + a)^(2/3)*a*b^22*c*
d^6 + 5*(b*x^3 + a)^(8/3)*b^21*d^7 - 16*(b*x^3 + a)^(5/3)*a*b^21*d^7 + 20*(
b*x^3 + a)^(2/3)*a^2*b^21*d^7)/(b^24*d^8)
```

Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.39

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \left(\frac{3\,a^2}{2\,b^3\,d}+\frac{\left(\frac{3\,a}{b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{b^6\,d^2}\right)\,\left(b^4\,c-a\,b^3\,d\right)}{2\,b^3\,d}\right)\,{\left(b\,x^3+a\right)}^{2/3} - \left(\frac{3\,a}{5\,b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{5\,b^6\,d^2}\right)\,{\left(b\,x^3+a\right)}^{5/3} + \frac{{\left(b\,x^3+a\right)}^{8/3}}{8\,b^3\,d} - \frac{c^3\,\ln\left(\frac{c^6\,{\left(b\,x^3+a\right)}^{1/3}}{d^5}+\frac{b\,c^7-a\,c^6\,d}{d^{16/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)}{3\,d^{11/3}\,{\left(a\,d-b\,c\right)}^{1/3}} + \frac{\ln\left(\frac{c^6\,{\left(b\,x^3+a\right)}^{1/3}}{d^5}-\frac{c^6\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,{\left(a\,d-b\,c\right)}^{1/3}}{4\,d^{16/3}}\right)\,\left(c^3+\sqrt{3}\,c^3\,1{}\mathrm{i}\right)}{6\,d^{11/3}\,{\left(a\,d-b\,c\right)}^{1/3}} - \frac{c^3\,\ln\left(\frac{c^6\,{\left(b\,x^3+a\right)}^{1/3}}{d^5}+\frac{c^6\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^{1/3}}{d^{16/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,d^{11/3}\,{\left(a\,d-b\,c\right)}^{1/3}}$$

```
[In] int(x^11/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] ((3*a^2)/(2*b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c
- a*b^3*d))/(2*b^3*d))*(a + b*x^3)^(2/3) - ((3*a)/(5*b^3*d) + (b^4*c - a*b^
3*d)/(5*b^6*d^2))*(a + b*x^3)^(5/3) + (a + b*x^3)^(8/3)/(8*b^3*d) - (c^3*lo
g((c^6*(a + b*x^3)^(1/3))/d^5 + (b*c^7 - a*c^6*d)/(d^(16/3)*(a*d - b*c)^(2/
3))))/(3*d^(11/3)*(a*d - b*c)^(1/3)) + (log((c^6*(a + b*x^3)^(1/3))/d^5 - (
c^6*(3^(1/2)*1i + 1)^2*(a*d - b*c)^(1/3))/(4*d^(16/3)))*(3^(1/2)*c^3*1i + c
^3))/(6*d^(11/3)*(a*d - b*c)^(1/3)) - (c^3*log((c^6*(a + b*x^3)^(1/3))/d^5
+ (c^6*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(1/3))/d^(16/3))*((3^(1/2)*1i)/2
- 1/2))/(3*d^(11/3)*(a*d - b*c)^(1/3))
```

3.716 $\int \frac{x^8}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 203

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = -\frac{(bc+ad)\,(a+bx^3)^{2/3}}{2b^2d^2}+\frac{(a+bx^3)^{5/3}}{5b^2d}-\frac{c^2\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{8/3}\sqrt[3]{bc-ad}}+\frac{c^2\log\left(c+dx^3\right)}{6d^{8/3}\sqrt[3]{bc-ad}}-\frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}\sqrt[3]{bc-ad}}$$

```
[Out] -1/2*(a*d+b*c)*(b*x^3+a)^(2/3)/b^2/d^2+1/5*(b*x^3+a)^(5/3)/b^2/d+1/6*c^2*ln
(d*x^3+c)/d^(8/3)/(-a*d+b*c)^(1/3)-1/2*c^2*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x
^3+a)^(1/3))/d^(8/3)/(-a*d+b*c)^(1/3)-1/3*c^2*arctan(1/3*(1-2*d^(1/3)*(b*x^
3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(8/3)/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

$= \{457, 90, 58, 631, 210, 31\}$

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = -\frac{c^2\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{8/3}\sqrt[3]{bc-ad}} - \frac{(a+bx^3)^{2/3}\,(ad+bc)}{2b^2d^2} + \frac{(a+bx^3)^{5/3}}{5b^2d} + \frac{c^2\log\left(c+dx^3\right)}{6d^{8/3}\sqrt[3]{bc-ad}} - \frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}\sqrt[3]{bc-ad}}$$

```
[In] Int[x^8/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -1/2*((b*c + a*d)*(a + b*x^3)^(2/3))/(b^2*d^2) + (a + b*x^3)^(5/3)/(5*b^2*d
) - (c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[
3]])/(Sqrt[3]*d^(8/3)*(b*c - a*d)^(1/3)) + (c^2*Log[c + d*x^3])/(6*d^(8/3)*
(b*c - a*d)^(1/3)) - (c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)
])/(2*d^(8/3)*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{-bc-ad}{bd^2\sqrt[3]{a+bx}} + \frac{(a+bx)^{2/3}}{bd} + \frac{c^2}{d^2\sqrt[3]{a+bx}(c+dx)}\right)\,dx, x, x^3\right)$$

$$= -\frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^2d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5b^2d} + \frac{c^2\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)}{3d^2}$$

$$= -\frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^2d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5b^2d} + \frac{c^2\log\left(c+dx^3\right)}{6d^{8/3}\sqrt[3]{bc-ad}} + \frac{c^2\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}} + x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^3} - \frac{c^2\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}} + x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{8/3}\sqrt[3]{bc-ad}}$$

$$= -\frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^2d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5b^2d} + \frac{c^2\log\left(c+dx^3\right)}{6d^{8/3}\sqrt[3]{bc-ad}} - \frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}\sqrt[3]{bc-ad}} + \frac{c^2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{d^{8/3}\sqrt[3]{bc-ad}}$$

$$= -\frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^2d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5b^2d} - \frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{8/3}\sqrt[3]{bc-ad}} + \frac{c^2\log\left(c+dx^3\right)}{6d^{8/3}\sqrt[3]{bc-ad}} - \frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}\sqrt[3]{bc-ad}}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.14

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$= \frac{-3d^{2/3}\sqrt[3]{bc-ad}\left(a+bx^3\right)^{2/3}\left(5bc+3ad-2bdx^3\right) - 10\sqrt{3}b^2c^2\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) - 10b^2c^2\log\left(\sqrt[3]{\right.}}{30b^2d^{8/3}\sqrt[3]{bc-}}$$

```
[In] Integrate[x^8/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (-3*d^(2/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(2/3)*(5*b*c + 3*a*d - 2*b*d*x^3)
 - 10*Sqrt[3]*b^2*c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)
^(1/3))/Sqrt[3]] - 10*b^2*c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(
1/3)] + 5*b^2*c^2*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*
x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(30*b^2*d^(8/3)*(b*c - a*d)^(1/3))
```

Maple [A] (verified)

Time = 5.06 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$-\frac{\frac{9\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}d\left(\frac{\left(-2d\,x^3+5c\right)b}{3}+ad\right)\left(b\,x^3+a\right)^{\frac{2}{3}}}{5}+b^2c^2\left(-2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^3+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^3+a\right)^{\frac{2}{3}}+\left(\frac{ad}{}\right.\right.\right.}{6\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}b^2d^3}$

```
[In] int(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(9/5*(1/d*(a*d-b*c))^(1/3)*d*(1/3*(-2*d*x^3+5*c)*b+a*d)*(b*x^3+a)^(2/3
)+b^2*c^2*(-2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/
(1/d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*
```

```
x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1
/3))))/(1/d*(a*d-b*c))^(1/3)/b^2/d^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(164) = 328$.

Time = 0.32 (sec) , antiderivative size = 768, normalized size of antiderivative = 3.78

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$
$$= \left[\frac{5\left(-bcd^2+ad^3\right)^{\frac{2}{3}}b^2c^2\log\left(\left(bx^3+a\right)^{\frac{2}{3}}d^2+\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}d+\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\right)-10\left(-bcd^2\right.}{}\right.$$

```
[In] integrate(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/30*(5*(-b*c*d^2 + a*d^3)^(2/3)*b^2*c^2*log((b*x^3 + a)^(2/3)*d^2 + (-b*c
*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 10*(-
b*c*d^2 + a*d^3)^(2/3)*b^2*c^2*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)
^(1/3)) + 15*sqrt(1/3)*(b^3*c^3*d - a*b^2*c^2*d^2)*sqrt((-b*c*d^2 + a*d^3)^
(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 + 3*sqrt(1/3)*(2*(-b*
c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2)
+ (-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d))*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c
- a*d)) - 3*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)) - 3*(5
*b^2*c^2*d^2 - 2*a*b*c*d^3 - 3*a^2*d^4 - 2*(b^2*c*d^3 - a*b*d^4)*x^3)*(b*x^
3 + a)^(2/3))/(b^3*c*d^4 - a*b^2*d^5), 1/30*(5*(-b*c*d^2 + a*d^3)^(2/3)*b^2
*c^2*log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)
*d + (-b*c*d^2 + a*d^3)^(2/3)) - 10*(-b*c*d^2 + a*d^3)^(2/3)*b^2*c^2*log((b
*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)) + 30*sqrt(1/3)*(b^3*c^3*d - a
*b^2*c^2*d^2)*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*
(2*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^2 + a*d^3)
^(1/3)/(b*c - a*d))/d) - 3*(5*b^2*c^2*d^2 - 2*a*b*c*d^3 - 3*a^2*d^4 - 2*(b^
2*c*d^3 - a*b*d^4)*x^3)*(b*x^3 + a)^(2/3))/(b^3*c*d^4 - a*b^2*d^5)]
```

Sympy [F]

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

```
[In] integrate(x**8/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**8/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.54

$$\begin{aligned}
&\int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx \\
&= -\frac{b^{12}c^2d^3\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^{13}cd^5-ab^{12}d^6)} \\
&\quad -\frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^4-\sqrt{3}ad^5} \\
&\quad +\frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^2\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(bcd^4-ad^5)} \\
&\quad -\frac{5\,(bx^3+a)^{\frac{2}{3}}b^9cd^3-2\,(bx^3+a)^{\frac{5}{3}}b^8d^4+5\,(bx^3+a)^{\frac{2}{3}}ab^8d^4}{10\,b^{10}d^5}
\end{aligned}$$

```
[In] integrate(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b^12*c^2*d^3*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
 - a*d)/d)^(1/3)))/(b^13*c*d^5 - a*b^12*d^6) - (-b*c*d^2 + a*d^3)^(2/3)*c^2
*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
 a*d)/d)^(1/3))/(sqrt(3)*b*c*d^4 - sqrt(3)*a*d^5) + 1/6*(-b*c*d^2 + a*d^3)^
(2/3)*c^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3)
+ (-(b*c - a*d)/d)^(2/3))/(b*c*d^4 - a*d^5) - 1/10*(5*(b*x^3 + a)^(2/3)*b^9
*c*d^3 - 2*(b*x^3 + a)^(5/3)*b^8*d^4 + 5*(b*x^3 + a)^(2/3)*a*b^8*d^4)/(b^10
*d^5)
```

Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.32

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{\left(b\,x^3+a\right)^{5/3}}{5\,b^2\,d} - \left(\frac{a}{b^2\,d} + \frac{b^3\,c-a\,b^2\,d}{2\,b^4\,d^2}\right)\,\left(b\,x^3+a\right)^{2/3} + \frac{c^2\,\ln\left(\frac{c^4\,\left(b\,x^3+a\right)^{1/3}}{d^3} + \frac{b\,c^5-a\,c^4\,d}{d^{10/3}\,\left(a\,d-b\,c\right)^{2/3}}\right)}{3\,d^{8/3}\,\left(a\,d-b\,c\right)^{1/3}} - \frac{\ln\left(\frac{c^4\,\left(b\,x^3+a\right)^{1/3}}{d^3} - \frac{c^4\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)^2\,\left(a\,d-b\,c\right)^{1/3}}{4\,d^{10/3}}\right)\,\left(c^2+\sqrt{3}\,c^2\,1{}\mathrm{i}\right)}{6\,d^{8/3}\,\left(a\,d-b\,c\right)^{1/3}} + \frac{c^2\,\ln\left(\frac{c^4\,\left(b\,x^3+a\right)^{1/3}}{d^3} - \frac{c^4\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)^2\,\left(a\,d-b\,c\right)^{1/3}}{4\,d^{10/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{d^{8/3}\,\left(a\,d-b\,c\right)^{1/3}}$$

```
[In] int(x^8/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] (a + b*x^3)^(5/3)/(5*b^2*d) - (a/(b^2*d) + (b^3*c - a*b^2*d)/(2*b^4*d^2))*(
a + b*x^3)^(2/3) + (c^2*log((c^4*(a + b*x^3)^(1/3))/d^3 + (b*c^5 - a*c^4*d)
/(d^(10/3)*(a*d - b*c)^(2/3))))/(3*d^(8/3)*(a*d - b*c)^(1/3)) - (log((c^4*(
a + b*x^3)^(1/3))/d^3 - (c^4*(3^(1/2)*1i + 1)^2*(a*d - b*c)^(1/3))/(4*d^(10
/3)))*(3^(1/2)*c^2*1i + c^2))/(6*d^(8/3)*(a*d - b*c)^(1/3)) + (c^2*log((c^4
*(a + b*x^3)^(1/3))/d^3 - (c^4*(3^(1/2)*1i - 1)^2*(a*d - b*c)^(1/3))/(4*d^(
10/3)))*((3^(1/2)*1i)/6 - 1/6))/(d^(8/3)*(a*d - b*c)^(1/3))
```

3.717 $\int \frac{x^5}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 168

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\frac{(a+bx^3)^{2/3}}{2bd}+\frac{c\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{5/3}\sqrt[3]{bc-ad}}-\frac{c\log(c+dx^3)}{6d^{5/3}\sqrt[3]{bc-ad}}+\frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}\sqrt[3]{bc-ad}}$$

```
[Out] 1/2*(b*x^3+a)^(2/3)/b/d-1/6*c*ln(d*x^3+c)/d^(5/3)/(-a*d+b*c)^(1/3)+1/2*c*ln
((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(5/3)/(-a*d+b*c)^(1/3)+1/3*c*a
rctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(5/3)/(
-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {457, 81, 58, 631, 210, 31}

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\frac{c\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{5/3}\sqrt[3]{bc-ad}}-\frac{c\log(c+dx^3)}{6d^{5/3}\sqrt[3]{bc-ad}}+\frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}\sqrt[3]{bc-ad}}+\frac{(a+bx^3)^{2/3}}{2bd}$$

```
[In] Int[x^5/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (a + b*x^3)^(2/3)/(2*b*d) + (c*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b
*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(5/3)*(b*c - a*d)^(1/3)) - (c*Log[c +
 d*x^3])/(6*d^(5/3)*(b*c - a*d)^(1/3)) + (c*Log[(b*c - a*d)^(1/3) + d^(1/3)
*(a + b*x^3)^(1/3)])/(2*d^(5/3)*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{(a+bx^3)^{2/3}}{2bd} - \frac{c\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)}{3d}$$

$$= \frac{(a+bx^3)^{2/3}}{2bd} - \frac{c\log(c+dx^3)}{6d^{5/3}\sqrt[3]{bc-ad}} - \frac{c\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}} + x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^2} + \frac{c\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}} + x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{5/3}\sqrt[3]{bc-ad}}$$

$$= \frac{(a+bx^3)^{2/3}}{2bd} - \frac{c\log(c+dx^3)}{6d^{5/3}\sqrt[3]{bc-ad}} + \frac{c\log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}\sqrt[3]{bc-ad}} - \frac{c\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{d^{5/3}\sqrt[3]{bc-ad}}$$

$$= \frac{(a+bx^3)^{2/3}}{2bd} + \frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{5/3}\sqrt[3]{bc-ad}} - \frac{c\log(c+dx^3)}{6d^{5/3}\sqrt[3]{bc-ad}} + \frac{c\log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}\sqrt[3]{bc-ad}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.21

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$
$$= \frac{3d^{2/3}\sqrt[3]{bc-ad}(a+bx^3)^{2/3}+2\sqrt{3}bc\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)+2bc\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)-}{6bd^{5/3}\sqrt[3]{bc-ad}}$$

[In] Integrate[x^5/((a + b*x^3)^(1/3)*(c + d*x^3)),x]

[Out] (3*d^(2/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(2/3) + 2*Sqrt[3]*b*c*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 2*b*c*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - b*c*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*b*d^(5/3)*(b*c - a*d)^(1/3))

Maple [A] (verified)

Time = 4.85 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.11

method	result
pseudoelliptic	$\frac{-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)bc+3\left(b\,x^{3}+a\right)^{\frac{2}{3}}d\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}-2c\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)b+c\ln\left(\left(b\,x^{3}\right.\right.}{6b\,d^{2}\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}$

[In] int(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] 1/6*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*b*c+3*(b*x^3+a)^(2/3)*d*(1/d*(a*d-b*c))^(1/3)-2*c*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))*b+c*ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))*b)/b/d^2/(1/d*(a*d-b*c))^(1/3)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(134) = 268.

Time = 0.27 (sec) , antiderivative size = 667, normalized size of antiderivative = 3.97

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$= \left[-\frac{(bcd^2-ad^3)^{\frac{2}{3}}bc\log\left((bx^3+a)^{\frac{2}{3}}d^2-(bcd^2-ad^3)^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}d+(bcd^2-ad^3)^{\frac{2}{3}}\right)-2\,(bcd^2-ad^3)^{\frac{2}{3}}bc\log}{} \right.$$

$$-\frac{(bcd^2-ad^3)^{\frac{2}{3}}bc\log\left((bx^3+a)^{\frac{2}{3}}d^2-(bcd^2-ad^3)^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}d+(bcd^2-ad^3)^{\frac{2}{3}}\right)-2\,(bcd^2-ad^3)^{\frac{2}{3}}bc\log}{}$$

```
[In] integrate(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/6*((b*c*d^2 - a*d^3)^(2/3)*b*c*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a
*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(2/3)) - 2*(b*c*d^2 - a
*d^3)^(2/3)*b*c*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) - 3*sqrt
(1/3)*(b^2*c^2*d - a*b*c*d^2)*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*lo
g((2*b*d^2*x^3 - b*c*d + 3*a*d^2 - 3*sqrt(1/3)*(2*(b*c*d^2 - a*d^3)^(2/3)*(
b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (b*c*d^2 - a*d^3)^(1
/3)*(b*c - a*d))*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d)) - 3*(b*c*d^2 -
a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)) - 3*(b*c*d^2 - a*d^3)*(b*x^3 +
 a)^(2/3))/(b^2*c*d^3 - a*b*d^4), -1/6*((b*c*d^2 - a*d^3)^(2/3)*b*c*log((b*
x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2
 - a*d^3)^(2/3)) - 2*(b*c*d^2 - a*d^3)^(2/3)*b*c*log((b*x^3 + a)^(1/3)*d +
(b*c*d^2 - a*d^3)^(1/3)) + 6*sqrt(1/3)*(b^2*c^2*d - a*b*c*d^2)*sqrt((b*c*d^
2 - a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d - (b*
c*d^2 - a*d^3)^(1/3))*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))/d) - 3*(b*c
*d^2 - a*d^3)*(b*x^3 + a)^(2/3))/(b^2*c*d^3 - a*b*d^4)]
```

Sympy [F]

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

```
[In] integrate(x**5/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**5/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.53

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{\frac{2\,bcd\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{bcd^2-ad^3}+\frac{6\left(-bcd^2+ad^3\right)^{\frac{2}{3}}bc\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^3-\sqrt{3}ad^4}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}bc\log\left(\right.}{}}{6\,b}$$

```
[In] integrate(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*c*d*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*
d)/d)^(1/3)))/(b*c*d^2 - a*d^3) + 6*(-b*c*d^2 + a*d^3)^(2/3)*b*c*arctan(1/3
*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1
/3))/(sqrt(3)*b*c*d^3 - sqrt(3)*a*d^4) - (-b*c*d^2 + a*d^3)^(2/3)*b*c*log((
b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)
/d)^(2/3))/(b*c*d^3 - a*d^4) + 3*(b*x^3 + a)^(2/3)/d)/b
```

Mupad [B] (verification not implemented)

Time = 8.93 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.30

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{(b\,x^3+a)^{2/3}}{2\,b\,d} + \frac{\ln\left(\frac{c^2\,(b\,x^3+a)^{1/3}}{d}-\frac{c^2\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2\,(a\,d-b\,c)^{1/3}}{4\,d^{4/3}}\right)\,(c-\sqrt{3}\,c\,1\mathrm{i})}{6\,d^{5/3}\,(a\,d-b\,c)^{1/3}} + \frac{\ln\left(\frac{c^2\,(b\,x^3+a)^{1/3}}{d}-\frac{c^2\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2\,(a\,d-b\,c)^{1/3}}{4\,d^{4/3}}\right)\,(c+\sqrt{3}\,c\,1\mathrm{i})}{6\,d^{5/3}\,(a\,d-b\,c)^{1/3}} - \frac{c\,\ln\left(\frac{c^2\,(b\,x^3+a)^{1/3}}{d}+\frac{b\,c^3-a\,c^2\,d}{d^{4/3}\,(a\,d-b\,c)^{2/3}}\right)}{3\,d^{5/3}\,(a\,d-b\,c)^{1/3}}$$

```
[In] int(x^5/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] (a + b*x^3)^(2/3)/(2*b*d) + (log((c^2*(a + b*x^3)^(1/3))/d - (c^2*(3^(1/2)*
1i - 1)^2*(a*d - b*c)^(1/3))/(4*d^(4/3)))*(c - 3^(1/2)*c*1i))/(6*d^(5/3)*(a
*d - b*c)^(1/3)) + (log((c^2*(a + b*x^3)^(1/3))/d - (c^2*(3^(1/2)*1i + 1)^2
*(a*d - b*c)^(1/3))/(4*d^(4/3)))*(c + 3^(1/2)*c*1i))/(6*d^(5/3)*(a*d - b*c)
^(1/3)) - (c*log((c^2*(a + b*x^3)^(1/3))/d + (b*c^3 - a*c^2*d)/(d^(4/3)*(a*
d - b*c)^(2/3))))/(3*d^(5/3)*(a*d - b*c)^(1/3))
```

3.718 $\int \frac{x^2}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 145

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6d^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}$$

```
[Out] 1/6*ln(d*x^3+c)/d^(2/3)/(-a*d+b*c)^(1/3)-1/2*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b
*x^3+a)^(1/3))/d^(2/3)/(-a*d+b*c)^(1/3)-1/3*arctan(1/3*(1-2*d^(1/3)*(b*x^3+
a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(2/3)/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {455, 58, 631, 210, 31}

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(c+dx^3\right)}{6d^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}$$

```
[In] Int[x^2/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]]/(Sq
rt[3]*d^(2/3)*(b*c - a*d)^(1/3))) + Log[c + d*x^3]/(6*d^(2/3)*(b*c - a*d)^(
1/3)) - Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(2*d^(2/3)*(b*c
- a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{\log(c+dx^3)}{6d^{2/3}\sqrt[3]{bc-ad}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}} + x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}$$

$$= \frac{\log(c+dx^3)}{6d^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{d^{2/3}\sqrt[3]{bc-ad}}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{2/3}\sqrt[3]{bc-ad}} + \frac{\log(c+dx^3)}{6d^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$= \frac{-2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) - 2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right) + \log\left((bc-ad)^{2/3} - \sqrt[3]{d}\sqrt[3]{bc-a\ldots}\right.}{6d^{2/3}\sqrt[3]{bc-ad}}$$

```
[In] Integrate[x^2/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sq
rt[3]] - 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + Log[(b*c -
a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x
^3)^(2/3)])/(6*d^(2/3)*(b*c - a*d)^(1/3))
```

Maple [A] (verified)

Time = 4.50 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$\frac{2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)\sqrt{3}+2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)-\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-}{d}\right.\right.}{6d\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}$

```
[In] int(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a
*d-b*c))^(1/3))*3^(1/2)+2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))-ln((b*x
^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3)))/d
/(1/d*(a*d-b*c))^(1/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(114) = 228.

Time = 0.26 (sec) , antiderivative size = 592, normalized size of antiderivative = 4.08

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$= \left[\frac{3\sqrt{\frac{1}{3}}(bcd-ad^2)\sqrt{\frac{(-bcd^2+ad^3)^{\frac{1}{3}}}{bc-ad}}\log\left(\frac{2\,bd^2x^3-bcd+3\,ad^2+3\,\sqrt{\frac{1}{3}}\left(2\,(-bcd^2+ad^3)^{\frac{2}{3}}\,(bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\,(bcd-ad^2)+(-bcd^2+ad^3\right.}{dx^3+c}\right.}{}\right.$$

```
[In] integrate(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*(b*c*d - a*d^2)*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d)
)*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 + 3*sqrt(1/3)*(2*(-b*c*d^2 + a*d^3)^(2
/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) + (-b*c*d^2 + a*d
^3)^(1/3)*(b*c - a*d))*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d)) - 3*(-b*c
*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)) + (-b*c*d^2 + a*d^3)^(2
/3)*log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*
d + (-b*c*d^2 + a*d^3)^(2/3)) - 2*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^
(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)))/(b*c*d^2 - a*d^3), 1/6*(6*sqrt(1/3)*(b
*c*d - a*d^2)*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*
(2*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^2 + a*d^3)
```

```
^(1/3)/(b*c - a*d))/d) + (-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3)*d^2
 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3))
 - 2*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^
(1/3)))/(b*c*d^2 - a*d^3)]
```

Sympy [F]

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

```
[In] integrate(x**2/(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(x**2/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.56

$$\begin{aligned}\int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx &= -\frac{(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^2-\sqrt{3}ad^3}\\ &\quad+\frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(bcd^2-ad^3)}\\ &\quad-\frac{\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc-ad)}\end{aligned}$$

```
[In] integrate(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -(-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c
- a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b*c*d^2 - sqrt(3)*a*d^3)
+ 1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-
(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d^2 - a*d^3) - 1/3*(-(b
*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*
c - a*d)
```

Mupad [B] (verification not implemented)

Time = 8.85 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.43

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{\ln\left(d\,(b\,x^3+a)^{1/3}-\frac{9\,a\,d^3-9\,b\,c\,d^2}{9\,d^{4/3}\,(a\,d-b\,c)^{2/3}}\right)}{3\,d^{2/3}\,(a\,d-b\,c)^{1/3}} + \frac{\ln\left(d\,(b\,x^3+a)^{1/3}-\frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2\,(9\,a\,d^3-9\,b\,c\,d^2)}{36\,d^{4/3}\,(a\,d-b\,c)^{2/3}}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,d^{2/3}\,(a\,d-b\,c)^{1/3}} - \frac{\ln\left(d\,(b\,x^3+a)^{1/3}-\frac{\left(1+\sqrt{3}\,1\mathrm{i}\right)^2\,(9\,a\,d^3-9\,b\,c\,d^2)}{36\,d^{4/3}\,(a\,d-b\,c)^{2/3}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,d^{2/3}\,(a\,d-b\,c)^{1/3}}$$

```
[In] int(x^2/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] log(d*(a + b*x^3)^(1/3) - (9*a*d^3 - 9*b*c*d^2)/(9*d^(4/3)*(a*d - b*c)^(2/3
)))/(3*d^(2/3)*(a*d - b*c)^(1/3)) + (log(d*(a + b*x^3)^(1/3) - ((3^(1/2)*1i
 - 1)^2*(9*a*d^3 - 9*b*c*d^2))/(36*d^(4/3)*(a*d - b*c)^(2/3)))*(3^(1/2)*1i
- 1))/(6*d^(2/3)*(a*d - b*c)^(1/3)) - (log(d*(a + b*x^3)^(1/3) - ((3^(1/2)*
1i + 1)^2*(9*a*d^3 - 9*b*c*d^2))/(36*d^(4/3)*(a*d - b*c)^(2/3)))*(3^(1/2)*1
i + 1))/(6*d^(2/3)*(a*d - b*c)^(1/3))
```

3.719 $\int \frac{1}{x\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 244

$$\int \frac{1}{x\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx=\frac{\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}c}+\frac{\sqrt[3]{d}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c\sqrt[3]{bc-ad}}-\frac{\log(x)}{2\sqrt[3]{a}c}-\frac{\sqrt[3]{d}\log\left(c+dx^3\right)}{6c\sqrt[3]{bc-ad}}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}c}+\frac{\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c\sqrt[3]{bc-ad}}$$

```
[Out] -1/2*ln(x)/a^(1/3)/c-1/6*d^(1/3)*ln(d*x^3+c)/c/(-a*d+b*c)^(1/3)+1/2*ln(a^(1
/3)-(b*x^3+a)^(1/3))/a^(1/3)/c+1/2*d^(1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x
^3+a)^(1/3))/c/(-a*d+b*c)^(1/3)+1/3*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/
a^(1/3)*3^(1/2))/a^(1/3)/c*3^(1/2)+1/3*d^(1/3)*arctan(1/3*(1-2*d^(1/3)*(b*x
^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used

$= \{457, 88, 57, 631, 210, 31, 58\}$

$$\int \frac{1}{x\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{\sqrt[3]{d}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c\sqrt[3]{bc-ad}} + \frac{\arctan\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}c} - \frac{\sqrt[3]{d}\log\left(c+dx^3\right)}{6c\sqrt[3]{bc-ad}} + \frac{\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c\sqrt[3]{bc-ad}} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}c} - \frac{\log(x)}{2\sqrt[3]{a}c}$$

```
[In] Int[1/(x*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3)*
c) + (d^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/
Sqrt[3]])/(Sqrt[3]*c*(b*c - a*d)^(1/3)) - Log[x]/(2*a^(1/3)*c) - (d^(1/3)*L
og[c + d*x^3])/(6*c*(b*c - a*d)^(1/3)) + Log[a^(1/3) - (a + b*x^3)^(1/3)]/(
2*a^(1/3)*c) + (d^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])
/(2*c*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
```

```
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^3\right)}{3c} - \frac{d\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)}{3c}\end{aligned}$$

$$= -\frac{\log(x)}{2\sqrt[3]{a}c} - \frac{\sqrt[3]{d}\log(c+dx^3)}{6c\sqrt[3]{bc-ad}} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c} - \frac{\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}c} + \frac{\sqrt[3]{d}\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c\sqrt[3]{bc-ad}}$$

$$= -\frac{\log(x)}{2\sqrt[3]{a}c} - \frac{\sqrt[3]{d}\log(c+dx^3)}{6c\sqrt[3]{bc-ad}} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}c} + \frac{\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c\sqrt[3]{bc-ad}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}c} - \frac{\sqrt[3]{d}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{c\sqrt[3]{bc-ad}}$$

$$= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}c} + \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c\sqrt[3]{bc-ad}} - \frac{\log(x)}{2\sqrt[3]{a}c} - \frac{\sqrt[3]{d}\log(c+dx^3)}{6c\sqrt[3]{bc-ad}} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}c} + \frac{\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c\sqrt[3]{bc-ad}}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.27

$$\int \frac{1}{x\sqrt[3]{a+bx^3}(c+dx^3)}\,dx = \frac{\frac{2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} + \frac{2\sqrt{3}\sqrt[3]{d}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{bc-ad}} + \frac{2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{a}} + \frac{2\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad-}\right.}{\sqrt[3]{bc-}}}{}$$

```
[In] Integrate[1/(x*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

[Out] ((2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) + (2*Sqrt[3]*d^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(b*c - a*d)^(1/3) + (2*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(1/3) + (2*d^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(1/3) - Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]/a^(1/3) - (d^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(1/3))/(6*c)

Maple [A] (verified)

Time = 4.62 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$-\frac{\left(-2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)\right)\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}+2}{6}$

[In] int(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] -1/6/(1/d*(a*d-b*c))^(1/3)/a^(1/3)*((-2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)+ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3))-2*ln((b*x^3+a)^(1/3)-a^(1/3)))*(1/d*(a*d-b*c))^(1/3)+2*(arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))-1/2*ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3)))*a^(1/3))/c

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 628, normalized size of antiderivative = 2.57

$$\int \frac{1}{x\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$= \left[\frac{3\sqrt{\frac{1}{3}}a\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^3+3\sqrt{\frac{1}{3}}\left(2\,(bx^3+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx^3+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x^3}\right)-2\sqrt{3}a\left(\frac{d}{bc-ad}\right)^{\frac{1}{3}}\arctan}{} \right.$$

$$-\frac{2\sqrt{3}a\left(\frac{d}{bc-ad}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(\frac{d}{bc-ad}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+a\left(\frac{d}{bc-ad}\right)^{\frac{1}{3}}\log\left(-(bx^3+a)^{\frac{1}{3}}(bc-ad)\left(\frac{d}{bc-ad}\right)\right.}{}$$

```
[In] integrate(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 +
 a)^(2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*
x^3 + a)^(1/3)*a^(2/3) + 3*a)/x^3) - 2*sqrt(3)*a*(d/(b*c - a*d))^(1/3)*arct
an(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(d/(b*c - a*d))^(1/3) - 1/3*sqrt(3)) - a*(
d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(d/(b*c - a*d))^(2/
3) + (b*x^3 + a)^(2/3)*d + (b*c - a*d)*(d/(b*c - a*d))^(1/3)) + 2*a*(d/(b*c
 - a*d))^(1/3)*log((b*c - a*d)*(d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1/3)*d)
 - a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2
*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)))/(a*c), -1/6*(2*sqrt(3)*a*(d/(b*c
 - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(d/(b*c - a*d))^(1/3) -
 1/3*sqrt(3)) + a*(d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*
(d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*d + (b*c - a*d)*(d/(b*c - a*d))^(
1/3)) - 2*a*(d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(d/(b*c - a*d))^(2/3) + (
b*x^3 + a)^(1/3)*d) - 6*sqrt(1/3)*a^(2/3)*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(
1/3) + a^(1/3))/a^(1/3)) + a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3
)*a^(1/3) + a^(2/3)) - 2*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)))/(a*c)]
```

Sympy [F]

$$\int \frac{1}{x\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{1}{x\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

```
[In] integrate(1/x/(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(1/(x*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x}\,dx$$

```
[In] integrate(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.34

$$\begin{aligned}\int \frac{1}{x\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx &= \frac{d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2-acd)} \\ &\quad+\frac{(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^2d-\sqrt{3}acd^2} \\ &\quad-\frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(bc^2d-acd^2)} \\ &\quad+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{1}{3}}c} \\ &\quad-\frac{\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{1}{3}}c}+\frac{\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{1}{3}}c}\end{aligned}$$

[In] integrate(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")

[Out] 1/3*d*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c^2 - a*c*d) + (-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b*c^2*d - sqrt(3)*a*c*d^2) - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c^2*d - a*c*d^2) + 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(1/3)*c) - 1/6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(1/3)*c) + 1/3*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(1/3)*c)

Mupad [B] (verification not implemented)

Time = 10.23 (sec) , antiderivative size = 702, normalized size of antiderivative = 2.88

$$\int \frac{1}{x\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \ln\left(b^5\,d^4\,\left(b\,x^3+a\right)^{1/3}\right.$$

$$-\frac{d\left(27\,b^4\,c^2\,d^3\,\left(b\,x^3+a\right)^{1/3}\left(2\,a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)-243\,a\,b^4\,c^4\,d^3\,\left(\frac{d}{27\,b\,c^4-27\,a\,c^3\,d}\right)^{2/3}\left(2\,a^2\,d^2-3\,a\,b\,c\,d\right.\right.}{27\,b\,c^4-27\,a\,c^3\,d}$$

[In] int(1/(x*(a + b*x^3)^(1/3)*(c + d*x^3)),x)

[Out] log(b^5*d^4*(a + b*x^3)^(1/3) - (d*(27*b^4*c^2*d^3*(a + b*x^3)^(1/3)*(2*a^2*d^2 + b^2*c^2 - 2*a*b*c*d) - 243*a*b^4*c^4*d^3*(d/(27*b*c^4 - 27*a*c^3*d))^(2/3)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)))/(27*b*c^4 - 27*a*c^3*d))*(d/(27*b*c^4 - 27*a*c^3*d))^(1/3) + log((a + b*x^3)^(1/3) - a*c^2*(1/(a*c^3))^(2/3))*(1/(27*a*c^3))^(1/3) + (log(b^5*d^4*(a + b*x^3)^(1/3) - (d*(3^(1/2)*1i - 1)^3*(27*b^4*c^2*d^3*(a + b*x^3)^(1/3)*(2*a^2*d^2 + b^2*c^2 - 2*a*b*c*d) - (243*a*b^4*c^4*d^3*(3^(1/2)*1i - 1)^2*(d/(27*b*c^4 - 27*a*c^3*d))^(2/3)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/4))/(8*(27*b*c^4 - 27*a*c^3*d)))*(3^(1/2)*1i - 1)*(d/(27*b*c^4 - 27*a*c^3*d))^(1/3))/2 - (log(b^5*d^4*(a + b*x^3)^(1/3) + (d*(3^(1/2)*1i + 1)^3*(27*b^4*c^2*d^3*(a + b*x^3)^(1/3)*(2*a^2*d^2 + b^2*c^2 - 2*a*b*c*d) - (243*a*b^4*c^4*d^3*(3^(1/2)*1i + 1)^2*(d/(27*b*c^4 - 27*a*c^3*d))^(2/3)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/4))/(8*(27*b*c^4 - 27*a*c^3*d)))*(3^(1/2)*1i + 1)*(d/(27*b*c^4 - 27*a*c^3*d))^(1/3))/2 - log((a + b*x^3)^(1/3)*2i + a*c^2*(1/(a*c^3))^(2/3)*1i + 3^(1/2)*a*c^2*(1/(a*c^3))^(2/3))*((3^(1/2)*1i)/2 + 1/2)*(1/(27*a*c^3))^(1/3) + log((a + b*x^3)^(1/3)*2i + a*c^2*(1/(a*c^3))^(2/3)*1i - 3^(1/2)*a*c^2*(1/(a*c^3))^(2/3))*((3^(1/2)*1i)/2 - 1/2)*(1/(27*a*c^3))^(1/3)

3.720 $\int \frac{1}{x^4\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 296

$$\int \frac{1}{x^4\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = -\frac{(a+bx^3)^{2/3}}{3acx^3} - \frac{(bc+3ad)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}c^2} - \frac{d^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2\sqrt[3]{bc-ad}} + \frac{(bc+3ad)\log(x)}{6a^{4/3}c^2} + \frac{d^{4/3}\log(c+dx^3)}{6c^2\sqrt[3]{bc-ad}} - \frac{(bc+3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}c^2} - \frac{d^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2\sqrt[3]{bc-ad}}$$

```
[Out] -1/3*(b*x^3+a)^(2/3)/a/c/x^3+1/6*(3*a*d+b*c)*ln(x)/a^(4/3)/c^2+1/6*d^(4/3)*
ln(d*x^3+c)/c^2/(-a*d+b*c)^(1/3)-1/6*(3*a*d+b*c)*ln(a^(1/3)-(b*x^3+a)^(1/3)
)/a^(4/3)/c^2-1/2*d^(4/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/c^2/
(-a*d+b*c)^(1/3)-1/9*(3*a*d+b*c)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(
1/3)*3^(1/2))/a^(4/3)/c^2*3^(1/2)-1/3*d^(4/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^
3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c^2/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {457, 105, 162, 57, 631, 210, 31, 58}

$$\int \frac{1}{x^4\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = -\frac{\arctan\left(\frac{2\frac{\sqrt[3]{a+bx^3}}{}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)(3ad+bc)}{3\sqrt{3}a^{4/3}c^2} - \frac{(3ad+bc)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}c^2} + \frac{\log(x)(3ad+bc)}{6a^{4/3}c^2} - \frac{d^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2\sqrt[3]{bc-ad}} + \frac{d^{4/3}\log\left(c+dx^3\right)}{6c^2\sqrt[3]{bc-ad}} - \frac{d^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2\sqrt[3]{bc-ad}} - \frac{\left(a+bx^3\right)^{2/3}}{3acx^3}$$

```
[In] Int[1/(x^4*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -1/3*(a + b*x^3)^(2/3)/(a*c*x^3) - ((b*c + 3*a*d)*ArcTan[(a^(1/3) + 2*(a +
b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)*c^2) - (d^(4/3)*ArcTan
[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c
^2*(b*c - a*d)^(1/3)) + ((b*c + 3*a*d)*Log[x])/(6*a^(4/3)*c^2) + (d^(4/3)*L
og[c + d*x^3])/(6*c^2*(b*c - a*d)^(1/3)) - ((b*c + 3*a*d)*Log[a^(1/3) - (a
+ b*x^3)^(1/3)])/(6*a^(4/3)*c^2) - (d^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)
*(a + b*x^3)^(1/3)])/(2*c^2*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
```

```
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)$$

$$=-\frac{\left(a+bx^3\right)^{2/3}}{3acx^3}-\frac{\text{Subst}\left(\int\frac{\frac{1}{3}(bc+3ad)+\frac{bdx}{3}}{x\sqrt[3]{a+bx}(c+dx)}\,dx,x,x^3\right)}{3ac}$$

$$=-\frac{\left(a+bx^3\right)^{2/3}}{3acx^3}+\frac{d^2\text{Subst}\left(\int\frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx,x,x^3\right)}{3c^2}-\frac{(bc+3ad)\text{Subst}\left(\int\frac{1}{x\sqrt[3]{a+bx}}\,dx,x,x^3\right)}{9ac^2}$$

$$=-\frac{\left(a+bx^3\right)^{2/3}}{3acx^3}+\frac{(bc+3ad)\log(x)}{6a^{4/3}c^2}+\frac{d^{4/3}\log\left(c+dx^3\right)}{6c^2\sqrt[3]{bc-ad}}+\frac{d\text{Subst}\left(\int\frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx,x,\sqrt[3]{a+bx^3}\right)}{2c^2}-\frac{d^{4/3}\text{Subst}\left(\int\frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx,x,\sqrt[3]{a+bx^3}\right)}{2c^2\sqrt[3]{bc-ad}}+\frac{(bc+3ad)\text{Subst}\left(\int\frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx^3}\right)}{6a^{4/3}c^2}-\frac{(bc+3ad)\text{Subst}\left(\int\frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx,x,\sqrt[3]{a+bx^3}\right)}{6ac^2}$$

$$=-\frac{\left(a+bx^3\right)^{2/3}}{3acx^3}+\frac{(bc+3ad)\log(x)}{6a^{4/3}c^2}+\frac{d^{4/3}\log\left(c+dx^3\right)}{6c^2\sqrt[3]{bc-ad}}-\frac{(bc+3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}c^2}-\frac{d^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2\sqrt[3]{bc-ad}}+\frac{d^{4/3}\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{c^2\sqrt[3]{bc-ad}}+\frac{(bc+3ad)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{3a^{4/3}c^2}$$

$$= -\frac{(a+bx^3)^{2/3}}{3acx^3} - \frac{(bc+3ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{4/3}c^2}$$

$$- \frac{d^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2\sqrt[3]{bc-ad}} + \frac{(bc+3ad)\log(x)}{6a^{4/3}c^2} + \frac{d^{4/3}\log(c+dx^3)}{6c^2\sqrt[3]{bc-ad}}$$

$$- \frac{(bc+3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}c^2} - \frac{d^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2\sqrt[3]{bc-ad}}$$

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^4\sqrt[3]{a+bx^3}(c+dx^3)}\,dx =$$

$$-\frac{\frac{6c(a+bx^3)^{2/3}}{ax^3} + \frac{2\sqrt{3}(bc+3ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}} + \frac{6\sqrt{3}d^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{bc-ad}} + \frac{2(bc+3ad)\log\left(-\sqrt[3]{a}+\sqrt[3]{\right.}}{a^{4/3}}}{}$$

```
[In] Integrate[1/(x^4*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -1/18*((6*c*(a + b*x^3)^(2/3))/(a*x^3) + (2*Sqrt[3]*(b*c + 3*a*d)*ArcTan[(1
 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(4/3) + (6*Sqrt[3]*d^(4/3)*Ar
cTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(b*c -
 a*d)^(1/3) + (2*(b*c + 3*a*d)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(4/3) +
 (6*d^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)
^(1/3) - ((b*c + 3*a*d)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^
3)^(2/3)])/a^(4/3) - (3*d^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)
^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(1/3))/c
^2
```

Maple [A] (verified)

Time = 4.84 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$-\frac{\left(2\left(b\,x^{3}+a\right)^{\frac{2}{3}}a^{\frac{1}{3}}c-x^{3}\left(-2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\right.\right.\right.}{}$

```
[In] int(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/a^(4/3)*((2*(b*x^3+a)^(2/3)*a^(1/3)*c-x^3*(-2*arctan(1/3*(a^(1/3)+2*(b
*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)+ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a
)^(1/3)+a^(2/3))-2*ln((b*x^3+a)^(1/3)-a^(1/3)))*(1/3*b*c+a*d))*(1/d*(a*d-b*
c))^(1/3)+(-2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/
(1/d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*
x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1
/3)))*x^3*a^(4/3)*d)/(1/d*(a*d-b*c))^(1/3)/c^2/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 837, normalized size of antiderivative = 2.83

$$\int \frac{1}{x^4\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/18*(6*sqrt(3)*a^2*d*x^3*(-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3
 + a)^(1/3)*(-d/(b*c - a*d))^(1/3) + 1/3*sqrt(3)) - 3*a^2*d*x^3*(-d/(b*c -
a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*
x^3 + a)^(2/3)*d - (b*c - a*d)*(-d/(b*c - a*d))^(1/3)) + 6*a^2*d*x^3*(-d/(b
*c - a*d))^(1/3)*log((b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1/3)
*d) + 3*sqrt(1/3)*(a*b*c + 3*a^2*d)*x^3*sqrt((-a)^(1/3)/a)*log((2*b*x^3 - 3
*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(-a)^(2/3) - (b*x^3 + a)^(1/3)*a + (-a)^(1/
3)*a)*sqrt((-a)^(1/3)/a) - 3*(b*x^3 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^3) + (b*
c + 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1
/3) + (-a)^(2/3)) - 2*(b*c + 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(1/3) +
(-a)^(1/3)) - 6*(b*x^3 + a)^(2/3)*a*c)/(a^2*c^2*x^3), 1/18*(6*sqrt(3)*a^2*d
*x^3*(-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(-d/(b*c -
 a*d))^(1/3) + 1/3*sqrt(3)) - 3*a^2*d*x^3*(-d/(b*c - a*d))^(1/3)*log(-(b*x^
3 + a)^(1/3)*(b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*d - (b*
c - a*d)*(-d/(b*c - a*d))^(1/3)) + 6*a^2*d*x^3*(-d/(b*c - a*d))^(1/3)*log((
```

```
b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1/3)*d) - 6*sqrt(1/3)*(a*b
*c + 3*a^2*d)*x^3*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)
 - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + (b*c + 3*a*d)*(-a)^(2/3)*x^3*log((b*x
^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(b*c + 3*a*d
)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) - 6*(b*x^3 + a)^(2/3)*
a*c)/(a^2*c^2*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{1}{x^4\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

```
[In] integrate(1/x**4/(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(1/(x**4*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x^4}\,dx$$

```
[In] integrate(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.53 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^4\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = -\frac{d^2\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^3-ac^2d)} - \frac{(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^3-\sqrt{3}ac^2d} + \frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(bc^3-ac^2d)} - \frac{\sqrt{3}(bc+3\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{4}{3}}c^2} + \frac{(bc+3\,ad)\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{4}{3}}c^2} - \frac{\left(a^{\frac{1}{3}}bc+3\,a^{\frac{4}{3}}d\right)\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{5}{3}}c^2} - \frac{(bx^3+a)^{\frac{2}{3}}}{3\,acx^3}$$

```
[In] integrate(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*d^2*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d
)^(1/3)))/(b*c^3 - a*c^2*d) - (-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(
2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt
(3)*b*c^3 - sqrt(3)*a*c^2*d) + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)
^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))
/(b*c^3 - a*c^2*d) - 1/9*sqrt(3)*(b*c + 3*a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3
 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(4/3)*c^2) + 1/18*(b*c + 3*a*d)*log((b*x
^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(4/3)*c^2) - 1/9*(a
^(1/3)*b*c + 3*a^(4/3)*d)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(5/3)*c^
2) - 1/3*(b*x^3 + a)^(2/3)/(a*c*x^3)
```

Mupad [B] (verification not implemented)

Time = 15.39 (sec) , antiderivative size = 1929, normalized size of antiderivative = 6.52

$$\int \frac{1}{x^4\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Too large to display}$$

[In] int(1/(x^4*(a + b*x^3)^(1/3)*(c + d*x^3)),x)

[Out] log(- (((((3*b^4*d^3*(a + b*x^3)^(1/3)*(18*a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d - 12*a^3*b*c*d^3))/a^2 - 3*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d + b*c)^3/(a^4*c^6))^(2/3))*(-(3*a*d + b*c)^3 /(a^4*c^6))^(1/3))/9 + (b^5*d^4*(b^3*c^3 - 27*a^3*d^3 + 8*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^2*c))*(-(3*a*d + b*c)^3/(a^4*c^6))^(2/3))/81 - (4*b^5*d^7*(a + b*x^3)^(1/3)*(3*a*d + b*c)^2)/(27*a^2*c^5))*(-(27*a^3*d^3 + b^3*c^3 + 9*a*b^2*c^2*d + 27*a^2*b*c*d^2)/(729*a^4*c^6))^(1/3) + log(- (-d^4/(27*b*c^7 - 27*a*c^6*d))^(2/3)*((-d^4/(27*b*c^7 - 27*a*c^6*d))^(1/3)*((3*b^4*d^3*(a + b*x^3)^(1/3)*(18*a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d - 12*a^3*b*c*d^3))/a^2 - 243*a*b^4*c^4*d^3*(-d^4/(27*b*c^7 - 27*a*c^6*d))^(2/3)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)) + (b^5*d^4*(b^3*c^3 - 27*a^3*d^3 + 8*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^2*c)) - (4*b^5*d^7*(a + b*x^3)^(1/3) *(3*a*d + b*c)^2)/(27*a^2*c^5))*(-d^4/(27*b*c^7 - 27*a*c^6*d))^(1/3) - log((((((3*b^4*d^3*(a + b*x^3)^(1/3)*(18*a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d - 12*a^3*b*c*d^3))/a^2 - 3*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d + b*c)^3/(a^4*c^6))^(2/3))* ((3^(1/2)*1i)/2 + 1/2)*(-(3*a*d + b*c)^3/(a^4*c^6))^(1/3))/9 - (b^5*d^4*(b^3*c^3 - 27*a^3*d^3 + 8*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^2*c))*((3^(1/2)*1i)/2 - 1/2)*(-(3*a*d + b*c)^3/(a^4*c^6))^(2/3))/81 - (4*b^5*d^7*(a + b*x^3)^(1/3)*(3*a*d + b*c)^2)/(27*a^2*c^5))*((3^(1/2)*1i)/2 + 1/2)*(-(27*a^3*d^3 + b^3*c^3 + 9*a*b^2*c^2*d + 27*a^2*b*c*d^2)/(729*a^4*c^6))^(1/3) + log((((((3*b^4*d^3*(a + b*x^3)^(1/3)*(18*a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2 + 4 *a*b^3*c^3*d - 12*a^3*b*c*d^3))/a^2 + 3*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d + b*c)^3/(a^4*c^6))^(2/3))*((3 ^(1/2)*1i)/2 - 1/2)*(-(3*a*d + b*c)^3/(a^4*c^6))^(1/3))/9 + (b^5*d^4*(b^3*c ^3 - 27*a^3*d^3 + 8*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^2*c))*((3^(1/2)*1i) /2 + 1/2)*(-(3*a*d + b*c)^3/(a^4*c^6))^(2/3))/81 - (4*b^5*d^7*(a + b*x^3)^(1/3)*(3*a*d + b*c)^2)/(27*a^2*c^5))*((3^(1/2)*1i)/2 - 1/2)*(-(27*a^3*d^3 + b^3*c^3 + 9*a*b^2*c^2*d + 27*a^2*b*c*d^2)/(729*a^4*c^6))^(1/3) + (log(- ((3 ^(1/2)*1i - 1)^2*(((3^(1/2)*1i - 1)*(-d^4/(27*b*c^7 - 27*a*c^6*d))^(1/3)*((3*b^4*d^3*(a + b*x^3)^(1/3)*(18*a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2 + 4*a *b^3*c^3*d - 12*a^3*b*c*d^3))/a^2 - (243*a*b^4*c^4*d^3*(3^(1/2)*1i - 1)^2*(-d^4/(27*b*c^7 - 27*a*c^6*d))^(2/3)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/4))/ 2 + (b^5*d^4*(b^3*c^3 - 27*a^3*d^3 + 8*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^ 2*c))*(-d^4/(27*b*c^7 - 27*a*c^6*d))^(2/3))/4 - (4*b^5*d^7*(a + b*x^3)^(1/3)*(3*a*d + b*c)^2)/(27*a^2*c^5))*(3^(1/2)*1i - 1)*(-d^4/(27*b*c^7 - 27*a*c^

```
6*d))^(1/3))/2 - (log(((3^(1/2)*1i + 1)^2*(((3^(1/2)*1i + 1)*(-d^4/(27*b*c^
7 - 27*a*c^6*d))^(1/3)*((3*b^4*d^3*(a + b*x^3)^(1/3)*(18*a^4*d^4 + b^4*c^4
- 2*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d - 12*a^3*b*c*d^3))/a^2 - (243*a*b^4*c^4
*d^3*(3^(1/2)*1i + 1)^2*(-d^4/(27*b*c^7 - 27*a*c^6*d))^(2/3)*(2*a^2*d^2 + b
^2*c^2 - 3*a*b*c*d))/4))/2 - (b^5*d^4*(b^3*c^3 - 27*a^3*d^3 + 8*a*b^2*c^2*d
 + 18*a^2*b*c*d^2))/(3*a^2*c))*(-d^4/(27*b*c^7 - 27*a*c^6*d))^(2/3))/4 - (4
*b^5*d^7*(a + b*x^3)^(1/3)*(3*a*d + b*c)^2)/(27*a^2*c^5))*(3^(1/2)*1i + 1)*
(-d^4/(27*b*c^7 - 27*a*c^6*d))^(1/3))/2 - (a + b*x^3)^(2/3)/(3*a*c*x^3)
```

3.721 $\int \frac{x^6}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 273

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{x(a+bx^3)^{2/3}}{3bd} - \frac{(3bc+ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}d^2} + \frac{c^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2\sqrt[3]{bc-ad}} + \frac{c^{4/3}\log(c+dx^3)}{6d^2\sqrt[3]{bc-ad}} - \frac{c^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2d^2\sqrt[3]{bc-ad}} + \frac{(3bc+ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6b^{4/3}d^2}$$

```
[Out] 1/3*x*(b*x^3+a)^(2/3)/b/d+1/6*c^(4/3)*ln(d*x^3+c)/d^2/(-a*d+b*c)^(1/3)-1/2*
c^(4/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^2/(-a*d+b*c)^(1/3)
+1/6*(a*d+3*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4/3)/d^2-1/9*(a*d+3*b*c)
*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(4/3)/d^2*3^(1/2)+1/
3*c^(4/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/
2))/d^2/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {490, 544, 245, 384}

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = -\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(ad+3bc)}{3\sqrt{3}b^{4/3}d^2} + \frac{c^{4/3}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d^2\sqrt[3]{bc-ad}} + \frac{(ad+3bc)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{6b^{4/3}d^2} + \frac{c^{4/3}\log\left(c+dx^3\right)}{6d^2\sqrt[3]{bc-ad}} - \frac{c^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^2\sqrt[3]{bc-ad}} + \frac{x(a+bx^3)^{2/3}}{3bd}$$

```
[In] Int[x^6/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (x*(a + b*x^3)^(2/3))/(3*b*d) - ((3*b*c + a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a
 + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(4/3)*d^2) + (c^(4/3)*ArcTan[(1 + (
2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*d^2*
(b*c - a*d)^(1/3)) + (c^(4/3)*Log[c + d*x^3])/(6*d^2*(b*c - a*d)^(1/3)) - (
c^(4/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*d^2*(b*c
 - a*d)^(1/3)) + ((3*b*c + a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(6*b
^(4/3)*d^2)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(a+bx^3)^{2/3}}{3bd} - \frac{\int \frac{ac+(3bc+ad)x^3}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{3bd} \\
&= \frac{x(a+bx^3)^{2/3}}{3bd} + \frac{c^2\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{d^2} - \frac{(3bc+ad)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{3bd^2} \\
&= \frac{x(a+bx^3)^{2/3}}{3bd} - \frac{(3bc+ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}d^2} \\
&\quad + \frac{c^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2\sqrt[3]{bc-ad}} + \frac{c^{4/3}\log\left(c+dx^3\right)}{6d^2\sqrt[3]{bc-ad}} \\
&\quad - \frac{c^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2d^2\sqrt[3]{bc-ad}} + \frac{(3bc+ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6b^{4/3}d^2}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.82 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.71

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$= \frac{\frac{12dx(a+bx^3)^{2/3}}{b} - \frac{4\sqrt{3}(3bc+ad)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{b^{4/3}} - \frac{6\sqrt{-6+6i\sqrt{3}}c^{4/3}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{bc-ad}}}{}$$

```
[In] Integrate[x^6/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((12*d*x*(a + b*x^3)^(2/3))/b - (4*Sqrt[3]*(3*b*c + a*d)*ArcTan[(Sqrt[3]*b^
(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))]])/b^(4/3) - (6*Sqrt[-6 + (6*I)*S
qrt[3]]*c^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x
 - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))]])/(b*c - a*d)^(1/3) + (4*(3*b
*c + a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]])/b^(4/3) + (6*(1 + I*Sqrt[3
])*c^(4/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^
(1/3)]])/(b*c - a*d)^(1/3) - (2*(3*b*c + a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a
 + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]])/b^(4/3) - ((3*I)*(-I + Sqrt[3])*c^(4/
3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)
*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]])/(b*c - a
*d)^(1/3))/(36*d^2)
```

Maple [A] (verified)

Time = 4.97 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.14

method	result
pseudoelliptic	$\frac{\left((b\,x^3+a)^{\frac{2}{3}}\,dx\,b^{\frac{1}{3}}+\frac{\left(\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^3+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)+\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^3+a\right)^{\frac{1}{3}}}{x}\right)-\frac{\ln\left(\frac{b^{\frac{2}{3}}x^2+b^{\frac{1}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}x+\left(b\,x^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{2}\right)}{3}\right.}{}$

```
[In] int(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/b^(4/3)/((a*d-b*c)/c)^(1/3)*(((b*x^3+a)^(2/3)*d*x*b^(1/3)+1/3*(3^(1/2)*
arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+ln((-b^(1/3)*x+
(b*x^3+a)^(1/3))/x)-1/2*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)
^(2/3))/x^2))*(a*d+3*b*c))*((a*d-b*c)/c)^(1/3)+b^(4/3)*(arctan(1/3*3^(1/2)*
(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln
((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)-1/2*ln((((a*d-b*c)/c)^(2/3)*x^2
-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*c)/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 826, normalized size of antiderivative = 3.03

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/18*(6*sqrt(3)*b^2*c*(-c/(b*c - a*d))^(1/3)*arctan(-1/3*(sqrt(3)*x - 2*s
qrt(3)*(b*x^3 + a)^(1/3)*(-c/(b*c - a*d))^(1/3))/x) - 6*b^2*c*(-c/(b*c - a*
d))^(1/3)*log(-((b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*c)
/x) + 3*b^2*c*(-c/(b*c - a*d))^(1/3)*log(-((b*c - a*d)*x^2*(-c/(b*c - a*d))
^(1/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 +
a)^(2/3)*c)/x^2) - 6*(b*x^3 + a)^(2/3)*b*d*x - 3*sqrt(1/3)*(3*b^2*c + a*b*d
)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1
/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)
*sqrt(-1/b^(2/3)) + 2*a) - 2*(3*b*c + a*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3
 + a)^(1/3))/x) + (3*b*c + a*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3
)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b^2*d^2), -1/18*(6*sqrt(3)*b^2*c*(-
c/(b*c - a*d))^(1/3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(
-c/(b*c - a*d))^(1/3))/x) - 6*b^2*c*(-c/(b*c - a*d))^(1/3)*log(-((b*c - a*d
)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*c)/x) + 3*b^2*c*(-c/(b*c - a
*d))^(1/3)*log(-((b*c - a*d)*x^2*(-c/(b*c - a*d))^(1/3) - (b*x^3 + a)^(1/3)
*(b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(2/3)*c)/x^2) - 6*(b*x^
3 + a)^(2/3)*b*d*x - 2*(3*b*c + a*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^
(1/3))/x) + (3*b*c + a*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1
/3)*x + (b*x^3 + a)^(2/3))/x^2) - 6*sqrt(1/3)*(3*b^2*c + a*b*d)*arctan(sqrt
(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3))/(b^2*d^2)]
```

Sympy [F]

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

```
[In] integrate(x**6/(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(x**6/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^6}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(x^6/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^6}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^6/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^6}{(b\,x^3+a)^{1/3}\,(d\,x^3+c)}\,dx$$

```
[In] int(x^6/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
[Out] int(x^6/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.722 $\int \frac{x^3}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 233

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}d} - \frac{\sqrt[3]{c}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d\sqrt[3]{bc-ad}} - \frac{\sqrt[3]{c}\log(c+dx^3)}{6d\sqrt[3]{bc-ad}} + \frac{\sqrt[3]{c}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d\sqrt[3]{bc-ad}} - \frac{\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}d}$$

```
[Out] -1/6*c^(1/3)*ln(d*x^3+c)/d/(-a*d+b*c)^(1/3)+1/2*c^(1/3)*ln((-a*d+b*c)^(1/3)
*x/c^(1/3)-(b*x^3+a)^(1/3))/d/(-a*d+b*c)^(1/3)-1/2*ln(-b^(1/3)*x+(b*x^3+a)^
(1/3))/b^(1/3)/d+1/3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^
(1/3)/d*3^(1/2)-1/3*c^(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x
^3+a)^(1/3))*3^(1/2))/d/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {494, 245, 384}

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = -\frac{\sqrt[3]{c}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d\sqrt[3]{bc-ad}} + \frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}d} - \frac{\sqrt[3]{c}\log(c+dx^3)}{6d\sqrt[3]{bc-ad}} + \frac{\sqrt[3]{c}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d\sqrt[3]{bc-ad}} - \frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}d}$$

```
[In] Int[x^3/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)*d) -
 (c^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/
Sqrt[3]])/(Sqrt[3]*d*(b*c - a*d)^(1/3)) - (c^(1/3)*Log[c + d*x^3])/(6*d*(b*
c - a*d)^(1/3)) + (c^(1/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^
(1/3)])/(2*d*(b*c - a*d)^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*
b^(1/3)*d)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
```

```
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{d} - \frac{c\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{d}$$

$$= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}d} - \frac{\sqrt[3]{c}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d\sqrt[3]{bc-ad}} - \frac{\sqrt[3]{c}\log\left(c+dx^3\right)}{6d\sqrt[3]{bc-ad}} + \frac{\sqrt[3]{c}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d\sqrt[3]{bc-ad}} - \frac{\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}d}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.10 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.82

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$= \frac{\frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}} + \frac{2\sqrt{-6+6i\sqrt{3}}\sqrt[3]{c}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{bc-ad}} - \frac{4\log\left(-\sqrt[3]{b}x+\sqrt[3]{a-\cdots}\right.}{\sqrt[3]{b}}}{}$$

```
[In] Integrate[x^3/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((4*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/
b^(1/3) + (2*Sqrt[-6 + (6*I)*Sqrt[3]]*c^(1/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x
)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))
]])/(b*c - a*d)^(1/3) - (4*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(1/3) -
((2*I)*(-I + Sqrt[3])*c^(1/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c
^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(1/3) + (2*Log[b^(2/3)*x^2 + b^(1/3)
*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(1/3) + ((1 + I*Sqrt[3])*c^(1/
3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)
*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a
*d)^(1/3))/(12*d)
```

Maple [A] (verified)

Time = 4.64 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.23

method	result
pseudoelliptic	$\frac{\left(-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)+\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)-2\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)\right)\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}}{}$

[In] int(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] 1/6/((a*d-b*c)/c)^(1/3)*((-2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x))*((a*d-b*c)/c)^(1/3)+(-2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x))*b^(1/3))/b^(1/3)/d

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 761, normalized size of antiderivative = 3.27

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$= \left[\frac{3\sqrt{\frac{1}{3}}b\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}\,x^2-3\,\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}\,bx^3-(bx^3+a)^{\frac{1}{3}}bx^2+2\,(bx^3+a)^{\frac{2}{3}}(-b)^{\frac{1}{3}}\right.\right.}{}\right.$$

$$-\frac{6\sqrt{\frac{1}{3}}b\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)-2\,\sqrt{3}b\left(\frac{c}{bc-ad}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}x+2\,\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(\frac{c}{bc}\right.}{3\,x}\right.}{}$$

[In] integrate(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")

[Out] [1/6*(3*sqrt(1/3)*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*

```
(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) + 2*sqrt(3)*b*(c/
(b*c - a*d))^(1/3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(c/(
b*c - a*d))^(1/3))/x) + 2*b*(c/(b*c - a*d))^(1/3)*log(-((b*c - a*d)*x*(c/(b
*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*c)/x) - b*(c/(b*c - a*d))^(1/3)*log(((
b*c - a*d)*x^2*(c/(b*c - a*d))^(1/3) + (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(c/(
b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*c)/x^2) - 2*(-b)^(2/3)*log(((-b)^(1/3
)*x + (b*x^3 + a)^(1/3))/x) + (-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^
(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b*d), -1/6*(6*sqrt(1/3)*b*sq
rt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sq
rt(-(-b)^(1/3)/b)/x) - 2*sqrt(3)*b*(c/(b*c - a*d))^(1/3)*arctan(1/3*(sqrt(3
)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(c/(b*c - a*d))^(1/3))/x) - 2*b*(c/(b*c -
 a*d))^(1/3)*log(-((b*c - a*d)*x*(c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*
c)/x) + b*(c/(b*c - a*d))^(1/3)*log(((b*c - a*d)*x^2*(c/(b*c - a*d))^(1/3)
+ (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(c/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)
*c)/x^2) + 2*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (-b)^(2
/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3
))/x^2))/(b*d)]
```

Sympy [F]

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

```
[In] integrate(x**3/(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(x**3/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^3}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(x^3/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^3}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^3/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^3}{(b\,x^3+a)^{1/3}\,(d\,x^3+c)}\,dx$$

```
[In] int(x^3/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
[Out] int(x^3/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.723 $\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 148

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}$$

```
[Out] 1/6*ln(d*x^3+c)/c^(2/3)/(-a*d+b*c)^(1/3)-1/2*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-
(b*x^3+a)^(1/3))/c^(2/3)/(-a*d+b*c)^(1/3)+1/3*arctan(1/3*(1+2*(-a*d+b*c)^(1
/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(2/3)/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {384}

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}$$

```
[In] Int[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(
Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^
(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*
(b*c - a*d)^(1/3))
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\text{integral} = \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}} + \frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.72

$$\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx = \frac{-2\sqrt{-6+6i\sqrt{3}}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right) + \left(1+i\sqrt{3}\right)\left(2\log\left(2\sqrt[3]{bc-ad}x + \left(1+i\sqrt{3}\right)\right.\right.}{12c^{2/3}}$$

```
[In] Integrate[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (-2*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c -
 a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (1 + I*Sqrt[3
])*(2*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)
] - Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3
)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]))/(12*c^
(2/3)*(b*c - a*d)^(1/3))
```

Maple [A] (verified)

Time = 4.54 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.16

method	result
pseudoelliptic	$\frac{2\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)\sqrt{3}+2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)-\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x}{x^{2}}\right.}{6\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}c}$

[In] int(1/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] 1/6*(2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)-ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))/((a*d-b*c)/c)^(1/3)/c

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Timed out}$$

[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

[In] integrate(1/(b*x**3+a)**(1/3)/(d*x**3+c),x)

[Out] Integral(1/((a + b*x**3)**(1/3)*(c + d*x**3)), x)

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{1/3}\,(d\,x^3+c)}\,dx$$

```
[In] int(1/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
[Out] int(1/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.724 $\int \frac{1}{x^3 \sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 176

$$\int \frac{1}{x^3 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = -\frac{(a+bx^3)^{2/3}}{2acx^2} - \frac{d\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{5/3}\sqrt[3]{bc-ad}} - \frac{d\log(c+dx^3)}{6c^{5/3}\sqrt[3]{bc-ad}} + \frac{d\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{5/3}\sqrt[3]{bc-ad}}$$

```
[Out] -1/2*(b*x^3+a)^(2/3)/a/c/x^2-1/6*d*ln(d*x^3+c)/c^(5/3)/(-a*d+b*c)^(1/3)+1/2
*d*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(5/3)/(-a*d+b*c)^(1/3)-
1/3*d*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/
c^(5/3)/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {491, 12, 384}

$$\int \frac{1}{x^3\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = -\frac{d\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{5/3}\sqrt[3]{bc-ad}} - \frac{d\log(c+dx^3)}{6c^{5/3}\sqrt[3]{bc-ad}} + \frac{d\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{5/3}\sqrt[3]{bc-ad}} - \frac{(a+bx^3)^{2/3}}{2acx^2}$$

```
[In] Int[1/(x^3*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -1/2*(a + b*x^3)^(2/3)/(a*c*x^2) - (d*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(
c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(5/3)*(b*c - a*d)^(1/3)) -
 (d*Log[c + d*x^3])/(6*c^(5/3)*(b*c - a*d)^(1/3)) + (d*Log[((b*c - a*d)^(1/
3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(5/3)*(b*c - a*d)^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\text{integral} = -\frac{(a+bx^3)^{2/3}}{2acx^2} + \frac{\int -\frac{2ad}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{2ac}$$
$$= -\frac{(a+bx^3)^{2/3}}{2acx^2} - \frac{d\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{c}$$
$$= -\frac{(a+bx^3)^{2/3}}{2acx^2} - \frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{5/3}\sqrt[3]{bc-ad}}$$
$$-\frac{d\log\left(c+dx^3\right)}{6c^{5/3}\sqrt[3]{bc-ad}} + \frac{d\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{5/3}\sqrt[3]{bc-ad}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.14 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.78

$$\int \frac{1}{x^3\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$
$$= \frac{-6c^{2/3}\sqrt[3]{bc-ad}(a+bx^3)^{2/3} + 2\sqrt{-6+6i\sqrt{3}}adx^2\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right) - 2i(-i\ldots}{\ldots}$$

```
[In] Integrate[1/(x^3*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (-6*c^(2/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(2/3) + 2*Sqrt[-6 + (6*I)*Sqrt[3]
]*a*d*x^2*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*
I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] - (2*I)*(-I + Sqrt[3])*a*d*x^2*Log
[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)] + a*(d
+ I*Sqrt[3]*d)*x^2*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(
b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^
(2/3)])/(12*a*c^(5/3)*(b*c - a*d)^(1/3)*x^2)
```

Maple [A] (verified)

Time = 4.86 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.23

method	result
pseudoelliptic	$-\frac{\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)\sqrt{3}\,a d\,x^{2}+a\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)d\,x^{2}-\frac{a\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\right.}{x^{2}}\right.}{2}}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}a\,c^{2}x^{2}}$

[In] int(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] -1/3/((a*d-b*c)/c)^(1/3)*(arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)*a*d*x^2+a*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)*d*x^2-1/2*a*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*d*x^2+3/2*(b*x^3+a)^(2/3)*c*((a*d-b*c)/c)^(1/3))/a/c^2/x^2

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\text{Timed out}$$

[In] integrate(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{1}{x^3\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{1}{x^3\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

[In] integrate(1/x**3/(b*x**3+a)**(1/3)/(d*x**3+c),x)

[Out] Integral(1/(x**3*(a + b*x**3)**(1/3)*(c + d*x**3)), x)

Maxima [F]

$$\int \frac{1}{x^3\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x^3}\,dx$$

```
[In] integrate(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x^3}\,dx$$

```
[In] integrate(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{1}{x^3\,{\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

```
[In] int(1/(x^3*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
[Out] int(1/(x^3*(a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.725 $\int \frac{1}{x^6 \sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 214

$$\int \frac{1}{x^6 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = -\frac{(a+bx^3)^{2/3}}{5acx^5}+\frac{(3bc+5ad)\,(a+bx^3)^{2/3}}{10a^2c^2x^2} + \frac{d^2\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{8/3}\sqrt[3]{bc-ad}}+\frac{d^2\log(c+dx^3)}{6c^{8/3}\sqrt[3]{bc-ad}} - \frac{d^2\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{8/3}\sqrt[3]{bc-ad}}$$

```
[Out] -1/5*(b*x^3+a)^(2/3)/a/c/x^5+1/10*(5*a*d+3*b*c)*(b*x^3+a)^(2/3)/a^2/c^2/x^2
+1/6*d^2*ln(d*x^3+c)/c^(8/3)/(-a*d+b*c)^(1/3)-1/2*d^2*ln((-a*d+b*c)^(1/3)*x
/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)/(-a*d+b*c)^(1/3)+1/3*d^2*arctan(1/3*(1+2*
(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/(-a*d+b*c)^(1/
3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {491, 597, 12, 384}

$$\int \frac{1}{x^6\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{(a+bx^3)^{2/3}\,(5ad+3bc)}{10a^2c^2x^2} + \frac{d^2\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{8/3}\sqrt[3]{bc-ad}} + \frac{d^2\log(c+dx^3)}{6c^{8/3}\sqrt[3]{bc-ad}} - \frac{d^2\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{8/3}\sqrt[3]{bc-ad}} - \frac{(a+bx^3)^{2/3}}{5acx^5}$$

```
[In] Int[1/(x^6*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -1/5*(a + b*x^3)^(2/3)/(a*c*x^5) + ((3*b*c + 5*a*d)*(a + b*x^3)^(2/3))/(10*
a^2*c^2*x^2) + (d^2*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3
)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(8/3)*(b*c - a*d)^(1/3)) + (d^2*Log[c + d*x^
3])/(6*c^(8/3)*(b*c - a*d)^(1/3)) - (d^2*Log[((b*c - a*d)^(1/3)*x)/c^(1/3)
- (a + b*x^3)^(1/3)])/(2*c^(8/3)*(b*c - a*d)^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
```

```
, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+bx^3)^{2/3}}{5acx^5}+\frac{\int \frac{-3bc-5ad-3bdx^3}{x^3\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{5ac}\\
&= -\frac{(a+bx^3)^{2/3}}{5acx^5}+\frac{(3bc+5ad)\,(a+bx^3)^{2/3}}{10a^2c^2x^2}-\frac{\int -\frac{10a^2d^2}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{10a^2c^2}\\
&= -\frac{(a+bx^3)^{2/3}}{5acx^5}+\frac{(3bc+5ad)\,(a+bx^3)^{2/3}}{10a^2c^2x^2}+\frac{d^2\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{c^2}\\
&= -\frac{(a+bx^3)^{2/3}}{5acx^5}+\frac{(3bc+5ad)\,(a+bx^3)^{2/3}}{10a^2c^2x^2}+\frac{d^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{8/3}\sqrt[3]{bc-ad}}\\
&\quad+\frac{d^2\log\left(c+dx^3\right)}{6c^{8/3}\sqrt[3]{bc-ad}}-\frac{d^2\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{8/3}\sqrt[3]{bc-ad}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.62 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.59

$$\int \frac{1}{x^6\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$=\frac{\frac{6c^{2/3}(a+bx^3)^{2/3}(-2ac+3bcx^3+5adx^3)}{a^2x^5}-\frac{10\sqrt{-6+6i\sqrt{3}}d^2\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{bc-ad}}+\frac{10\left(1+i\sqrt{3}\right)d^2\log\left(2\sqrt[3]{\ldots}\right.}{\ldots}}{\ldots}$$

```
[In] Integrate[1/(x^6*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((6*c^(2/3)*(a + b*x^3)^(2/3)*(-2*a*c + 3*b*c*x^3 + 5*a*d*x^3))/(a^2*x^5) -
 (10*Sqrt[-6 + (6*I)*Sqrt[3]]*d^2*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(
b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))]])/(b*c - a*
d)^(1/3) + (10*(1 + I*Sqrt[3])*d^2*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[
3])*c^(1/3)*(a + b*x^3)^(1/3)]])/(b*c - a*d)^(1/3) - ((5*I)*(-I + Sqrt[3])*d
^2*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)
*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]])/(b*c - a
*d)^(1/3))/(60*c^(8/3))
```

Maple [A] (verified)

Time = 4.99 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.08

method	result
pseudoelliptic	$-\frac{6c\left(\frac{(-5ad-3bc)x^3}{2}+ac\right)(bx^3+a)^{\frac{2}{3}}\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}+5a^2d^2x^5\left(-2\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2(bx^3+a)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)\sqrt{3}+\ln\left(\frac{\left(\frac{ad-bc}{c}\right)\cdots}{}\right.\right.}{30\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}a^2c^3x^5}$

```
[In] int(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/30*(6*c*(1/2*(-5*a*d-3*b*c)*x^3+a*c)*(b*x^3+a)^(2/3)*((a*d-b*c)/c)^(1/3)
+5*a^2*d^2*x^5*(-2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1
/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/
c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln(((((a*d-b*c)/c)^(1/3)*
x+(b*x^3+a)^(1/3))/x)))/((a*d-b*c)/c)^(1/3)/a^2/c^3/x^5
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^6\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{1}{x^6\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

```
[In] integrate(1/x**6/(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(1/(x**6*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^6\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x^6}\,dx$$

```
[In] integrate(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x^6}\,dx$$

```
[In] integrate(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=\int \frac{1}{x^6\,(b\,x^3+a)^{1/3}\,(d\,x^3+c)}\,dx$$

```
[In] int(1/(x^6*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
[Out] int(1/(x^6*(a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.726 $\int \frac{1}{x^9 \sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 262

$$\int \frac{1}{x^9 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = -\frac{(a+bx^3)^{2/3}}{8acx^8}+\frac{(3bc+4ad)\,(a+bx^3)^{2/3}}{20a^2c^2x^5} - \frac{(9b^2c^2+12abcd+20a^2d^2)\,(a+bx^3)^{2/3}}{40a^3c^3x^2} - \frac{d^3\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{11/3}\sqrt[3]{bc-ad}} - \frac{d^3\log(c+dx^3)}{6c^{11/3}\sqrt[3]{bc-ad}} + \frac{d^3\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{11/3}\sqrt[3]{bc-ad}}$$

```
[Out] -1/8*(b*x^3+a)^(2/3)/a/c/x^8+1/20*(4*a*d+3*b*c)*(b*x^3+a)^(2/3)/a^2/c^2/x^5
-1/40*(20*a^2*d^2+12*a*b*c*d+9*b^2*c^2)*(b*x^3+a)^(2/3)/a^3/c^3/x^2-1/6*d^3
*ln(d*x^3+c)/c^(11/3)/(-a*d+b*c)^(1/3)+1/2*d^3*ln((-a*d+b*c)^(1/3)*x/c^(1/3
)-(b*x^3+a)^(1/3))/c^(11/3)/(-a*d+b*c)^(1/3)-1/3*d^3*arctan(1/3*(1+2*(-a*d+
b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(11/3)/(-a*d+b*c)^(1/3)*3^
(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {491, 597, 12, 384}

$$\int \frac{1}{x^9\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{(a+bx^3)^{2/3}(4ad+3bc)}{20a^2c^2x^5} - \frac{(a+bx^3)^{2/3}(20a^2d^2+12abcd+9b^2c^2)}{40a^3c^3x^2} - \frac{d^3\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{11/3}\sqrt[3]{bc-ad}} - \frac{d^3\log(c+dx^3)}{6c^{11/3}\sqrt[3]{bc-ad}} + \frac{d^3\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{11/3}\sqrt[3]{bc-ad}} - \frac{(a+bx^3)^{2/3}}{8acx^8}$$

```
[In] Int[1/(x^9*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -1/8*(a + b*x^3)^(2/3)/(a*c*x^8) + ((3*b*c + 4*a*d)*(a + b*x^3)^(2/3))/(20*
a^2*c^2*x^5) - ((9*b^2*c^2 + 12*a*b*c*d + 20*a^2*d^2)*(a + b*x^3)^(2/3))/(4
0*a^3*c^3*x^2) - (d^3*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x
^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(11/3)*(b*c - a*d)^(1/3)) - (d^3*Log[c + d
*x^3])/(6*c^(11/3)*(b*c - a*d)^(1/3)) + (d^3*Log[((b*c - a*d)^(1/3)*x)/c^(1
/3) - (a + b*x^3)^(1/3)])/(2*c^(11/3)*(b*c - a*d)^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
```

```
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{(a+bx^3)^{2/3}}{8acx^8} + \frac{\int \frac{-2(3bc+4ad)-6bdx^3}{x^6\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{8ac}$$

$$= -\frac{(a+bx^3)^{2/3}}{8acx^8} + \frac{(3bc+4ad)(a+bx^3)^{2/3}}{20a^2c^2x^5} - \frac{\int \frac{-2(9b^2c^2+12abcd+20a^2d^2)-6bd(3bc+4ad)x^3}{x^3\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{40a^2c^2}$$

$$= -\frac{(a+bx^3)^{2/3}}{8acx^8} + \frac{(3bc+4ad)(a+bx^3)^{2/3}}{20a^2c^2x^5} - \frac{(9b^2c^2+12abcd+20a^2d^2)(a+bx^3)^{2/3}}{40a^3c^3x^2} + \frac{\int -\frac{80a^3d^3}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{80a^3c^3}$$

$$= -\frac{(a+bx^3)^{2/3}}{8acx^8} + \frac{(3bc+4ad)(a+bx^3)^{2/3}}{20a^2c^2x^5} - \frac{(9b^2c^2+12abcd+20a^2d^2)(a+bx^3)^{2/3}}{40a^3c^3x^2} - \frac{d^3\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{c^3}$$

$$= -\frac{(a+bx^3)^{2/3}}{8acx^8} + \frac{(3bc+4ad)(a+bx^3)^{2/3}}{20a^2c^2x^5} - \frac{(9b^2c^2+12abcd+20a^2d^2)(a+bx^3)^{2/3}}{40a^3c^3x^2} - \frac{d^3\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{11/3}\sqrt[3]{bc-ad}} - \frac{d^3\log(c+dx^3)}{6c^{11/3}\sqrt[3]{bc-ad}} + \frac{d^3\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{11/3}\sqrt[3]{bc-ad}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.59 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.43

$$\int \frac{1}{x^9\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

$$= \frac{-\frac{3c^{2/3}(a+bx^3)^{2/3}(9b^2c^2x^6-6abcx^3(c-2dx^3)+a^2(5c^2-8cdx^3+20d^2x^6))}{a^3x^8}+\frac{20\sqrt{-6+6i\sqrt{3}}d^3\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+}}\right.}{\sqrt[3]{bc-ad}}}{}$$

```
[In] Integrate[1/(x^9*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((-3*c^(2/3)*(a + b*x^3)^(2/3)*(9*b^2*c^2*x^6 - 6*a*b*c*x^3*(c - 2*d*x^3) +
 a^2*(5*c^2 - 8*c*d*x^3 + 20*d^2*x^6)))/(a^3*x^8) + (20*Sqrt[-6 + (6*I)*Sqr
t[3]]*d^3*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*
I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))])/(b*c - a*d)^(1/3) - ((20*I)*(-I +
 Sqrt[3])*d^3*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^
3)^(1/3)])/(b*c - a*d)^(1/3) + (10*(1 + I*Sqrt[3])*d^3*Log[2*(b*c - a*d)^(2
/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) +
I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(1/3))/(120*c^(11/3
))
```

Maple [A] (verified)

Time = 4.98 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$-\frac{3\left(\left(4a^2d^2+\frac{12}{5}abcd+\frac{9}{5}b^2c^2\right)x^6+\frac{2\left(-4a^2cd-3b\,c^2a\right)x^3}{5}+a^2c^2\right)c\left(b\,x^3+a\right)^{\frac{2}{3}}\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}+4a^3d^3x^8\left(2\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x\right.}{3\left(\frac{ad-b}{c}\right.}\right.\right.}{24\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x^8}$

```
[In] int(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24/((a*d-b*c)/c)^(1/3)*(3*((4*a^2*d^2+12/5*a*b*c*d+9/5*b^2*c^2)*x^6+2/5*
(-4*a^2*c*d-3*a*b*c^2)*x^3+a^2*c^2)*c*(b*x^3+a)^(2/3)*((a*d-b*c)/c)^(1/3)+4
*a^3*d^3*x^8*(2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3)
)/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3
))/x)-ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*
x^3+a)^(2/3))/x^2)))/x^8/c^4/a^3
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^9\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^9\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{x^9\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

```
[In] integrate(1/x**9/(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(1/(x**9*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^9\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x^9}\,dx$$

```
[In] integrate(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^9), x)
```

Giac [F]

$$\int \frac{1}{x^9\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x^9}\,dx$$

```
[In] integrate(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^9), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^9\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{x^9\,\left(b\,x^3+a\right)^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

```
[In] int(1/(x^9*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
[Out] int(1/(x^9*(a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.727 $\int \frac{x^7}{\sqrt[3]{a+bx^3}(c+dx^3)} dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{x^8\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{8}{3},\frac{1}{3},1,\frac{11}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\sqrt[3]{a+bx^3}}$$

```
[Out] 1/8*x^8*(1+b*x^3/a)^(1/3)*AppellF1(8/3,1/3,1,11/3,-b*x^3/a,-d*x^3/c)/c/(b*x
^3+a)^(1/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{x^8\sqrt[3]{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{8}{3},\frac{1}{3},1,\frac{11}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\sqrt[3]{a+bx^3}}$$

```
[In] Int[x^7/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (x^8*(1 + (b*x^3)/a)^(1/3)*AppellF1[8/3, 1/3, 1, 11/3, -((b*x^3)/a), -((d*x
^3)/c)])/(8*c*(a + b*x^3)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^7}{\sqrt[3]{1+\frac{bx^3}{a}}(c+dx^3)}\,dx}{\sqrt[3]{a+bx^3}}$$

$$= \frac{x^8\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(\frac{8}{3};\frac{1}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\sqrt[3]{a+bx^3}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 144 vs. 2(64) = 128.

Time = 8.46 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.25

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$
$$= \frac{5cx^2(a+bx^3)-5acx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-2(2bc+ad)x^5\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{5}{3}\right.}{20bcd\sqrt[3]{a+bx^3}}$$

```
[In] Integrate[x^7/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (5*c*x^2*(a + b*x^3) - 5*a*c*x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1
, 5/3, -((b*x^3)/a), -((d*x^3)/c)] - 2*(2*b*c + a*d)*x^5*(1 + (b*x^3)/a)^(1
/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(20*b*c*d*(a +
b*x^3)^(1/3))
```

Maple [F]

$$\int \frac{x^7}{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(d\,x^3+c\right)}dx$$

```
[In] int(x^7/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^7/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate(x^7/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

```
[In] integrate(x**7/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**7/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^7}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(x^7/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^7/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^7}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(x^7/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^7/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^7}{(b\,x^3+a)^{1/3}\,(d\,x^3+c)}\,dx$$

```
[In] int(x^7/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
[Out] int(x^7/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.728 $\int \frac{x^4}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{x^5\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{a+bx^3}}$$

```
[Out] 1/5*x^5*(1+b*x^3/a)^(1/3)*AppellF1(5/3,1/3,1,8/3,-b*x^3/a,-d*x^3/c)/c/(b*x^
3+a)^(1/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{x^5\sqrt[3]{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{a+bx^3}}$$

```
[In] Int[x^4/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (x^5*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^
3)/c)])/(5*c*(a + b*x^3)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^4}{\sqrt[3]{1+\frac{bx^3}{a}}(c+dx^3)}\,dx}{\sqrt[3]{a+bx^3}}\\ &= \frac{x^5\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{a+bx^3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 7.95 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{x^5\sqrt[3]{\frac{a+bx^3}{a}}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},1,\frac{8}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{a+bx^3}}$$

```
[In] Integrate[x^4/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
[Out] (x^5*((a + b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^
3)/c)])/(5*c*(a + b*x^3)^(1/3))
```

Maple [F]

$$\int \frac{x^4}{(b\,x^3+a)^{\frac{1}{3}}\,(d\,x^3+c)}dx$$

```
[In] int(x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x)
[Out] int(x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate(x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^4}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

```
[In] integrate(x**4/(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(x**4/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^4}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(x^4/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^4}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^4/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x^4}{(b\,x^3+a)^{1/3}\,(d\,x^3+c)}\,dx$$

```
[In] int(x^4/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
[Out] int(x^4/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.729 $\int \frac{x}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{x}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{a+bx^3}}$$

```
[Out] 1/2*x^2*(1+b*x^3/a)^(1/3)*AppellF1(2/3,1/3,1,5/3,-b*x^3/a,-d*x^3/c)/c/(b*x^
3+a)^(1/3)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {525, 524}

$$\int \frac{x}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{a+bx^3}}$$

```
[In] Int[x/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), -((d*x^
3)/c)])/(2*c*(a + b*x^3)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x}{\sqrt[3]{1+\frac{bx^3}{a}}(c+dx^3)}\,dx}{\sqrt[3]{a+bx^3}}$$

$$= \frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{a+bx^3}}$$

Mathematica [A] (verified)

Time = 10.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{x^2\sqrt[3]{\frac{a+bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{a+bx^3}}$$

```
[In] Integrate[x/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
[Out] (x^2*((a + b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), -((d*x^
3)/c)])/(2*c*(a + b*x^3)^(1/3))
```

Maple [F]

$$\int \frac{x}{(b\,x^3+a)^{\frac{1}{3}}\,(d\,x^3+c)}dx$$

```
[In] int(x/(b*x^3+a)^(1/3)/(d*x^3+c),x)
[Out] int(x/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate(x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$

```
[In] integrate(x/(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(x/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(x/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{x}{\left(b\,x^3+a\right)^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

```
[In] int(x/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
[Out] int(x/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.730 $\int \frac{1}{x^2\sqrt[3]{a+bx^3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{1}{x^2\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=-\frac{\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(-\frac{1}{3},\frac{1}{3},1,\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\sqrt[3]{a+bx^3}}$$

```
[Out] -(1+b*x^3/a)^(1/3)*AppellF1(-1/3,1/3,1,2/3,-b*x^3/a,-d*x^3/c)/c/x/(b*x^3+a)
^(1/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{1}{x^2\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=-\frac{\sqrt[3]{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(-\frac{1}{3},\frac{1}{3},1,\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\sqrt[3]{a+bx^3}}$$

```
[In] Int[1/(x^2*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -(((1 + (b*x^3)/a)^(1/3)*AppellF1[-1/3, 1/3, 1, 2/3, -((b*x^3)/a), -((d*x^3
)/c)])/(c*x*(a + b*x^3)^(1/3)))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{x^2\sqrt[3]{1+\frac{bx^3}{a}}(c+dx^3)}\,dx}{\sqrt[3]{a+bx^3}}$$
$$= -\frac{\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(-\frac{1}{3};\frac{1}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\sqrt[3]{a+bx^3}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(62) = 124.

Time = 10.11 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.27

$$\int \frac{1}{x^2\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$
$$= \frac{-10c(a+bx^3)+5(bc-ad)x^3\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bdx^6\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{5}{3}\right.}{10ac^2x\sqrt[3]{a+bx^3}}$$

```
[In] Integrate[1/(x^2*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (-10*c*(a + b*x^3) + 5*(b*c - a*d)*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3,
1/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*d*x^6*(1 + (b*x^3)/a)^(1/3)*
AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(10*a*c^2*x*(a + b*
x^3)^(1/3))
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b\,x^{3}+a \right)^{\frac{1}{3}} \left(d\,x^{3}+c \right)}d x$$

```
[In] int(1/x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x)
[Out] int(1/x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \sqrt[3]{a+bx^3} (c+dx^3)} \, dx = \text{Timed out}$$

```
[In] integrate(1/x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^2 \sqrt[3]{a+bx^3} (c+dx^3)} \, dx = \int \frac{1}{x^2 \sqrt[3]{a+bx^3} (c+dx^3)} \, dx$$

```
[In] integrate(1/x**2/(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(1/(x**2*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 \sqrt[3]{a+bx^3} (c+dx^3)} \, dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x^2} \, dx$$

```
[In] integrate(1/x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x^2}\,dx$$

```
[In] integrate(1/x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{x^2\,(b\,x^3+a)^{1/3}\,(d\,x^3+c)}\,dx$$

```
[In] int(1/(x^2*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
[Out] int(1/(x^2*(a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.731 $\int \frac{1}{x^5 \sqrt[3]{a+bx^3}(c+dx^3)} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{x^5 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=-\frac{\sqrt[3]{1+\frac{bx^3}{a}}\,\mathrm{AppellF1}\left(-\frac{4}{3},\frac{1}{3},1,-\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4cx^4\sqrt[3]{a+bx^3}}$$

```
[Out] -1/4*(1+b*x^3/a)^(1/3)*AppellF1(-4/3,1/3,1,-1/3,-b*x^3/a,-d*x^3/c)/c/x^4/(b
*x^3+a)^(1/3)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{1}{x^5 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx=-\frac{\sqrt[3]{\frac{bx^3}{a}+1}\,\mathrm{AppellF1}\left(-\frac{4}{3},\frac{1}{3},1,-\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4cx^4\sqrt[3]{a+bx^3}}$$

```
[In] Int[1/(x^5*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -1/4*((1 + (b*x^3)/a)^(1/3)*AppellF1[-4/3, 1/3, 1, -1/3, -((b*x^3)/a), -((d
*x^3)/c)])/(c*x^4*(a + b*x^3)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{x^5\sqrt[3]{1+\frac{bx^3}{a}}(c+dx^3)}\,dx}{\sqrt[3]{a+bx^3}}$$
$$= -\frac{\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(-\frac{4}{3};\frac{1}{3},1;-\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4cx^4\sqrt[3]{a+bx^3}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 183 vs. $2(64) = 128$.

Time = 10.18 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.86

$$\int \frac{1}{x^5\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$$
$$= \frac{5c(a+bx^3)\,(-ac+2bcx^3+4adx^3)+5(-b^2c^2-2abcd+2a^2d^2)\,x^6\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\right.}{20a^2c^3x^4\sqrt[3]{a+bx^3}}$$

```
[In] Integrate[1/(x^5*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (5*c*(a + b*x^3)*(-(a*c) + 2*b*c*x^3 + 4*a*d*x^3) + 5*(-(b^2*c^2) - 2*a*b*c
*d + 2*a^2*d^2)*x^6*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x
^3)/a), -((d*x^3)/c)] - 2*b*d*(b*c + 2*a*d)*x^9*(1 + (b*x^3)/a)^(1/3)*Appel
lF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(20*a^2*c^3*x^4*(a + b*x
^3)^(1/3))
```

Maple [F]

$$\int \frac{1}{x^{5} \left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(d\,x^{3}+c\right)}d x$$

```
[In] int(1/x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x)
[Out] int(1/x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^5 \sqrt[3]{a + bx^3} \, (c + dx^3)} \, dx = \text{Timed out}$$

```
[In] integrate(1/x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^5 \sqrt[3]{a + bx^3} \, (c + dx^3)} \, dx = \int \frac{1}{x^5 \sqrt[3]{a + bx^3} \, (c + dx^3)} \, dx$$

```
[In] integrate(1/x**5/(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(1/(x**5*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^5 \sqrt[3]{a + bx^3} \, (c + dx^3)} \, dx = \int \frac{1}{(bx^3 + a)^{\frac{1}{3}} (dx^3 + c) x^5} \, dx$$

```
[In] integrate(1/x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)x^5}\,dx$$

```
[In] integrate(1/x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 \sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \int \frac{1}{x^5\,(b\,x^3+a)^{1/3}\,(d\,x^3+c)}\,dx$$

```
[In] int(1/(x^5*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
[Out] int(1/(x^5*(a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

3.732 $\int \frac{x^{11}}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 241

$$\int \frac{x^{11}}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{(b^2c^2+abcd+a^2d^2)\sqrt[3]{a+bx^3}}{b^3d^3} - \frac{(bc+2ad)(a+bx^3)^{4/3}}{4b^3d^2} + \frac{(a+bx^3)^{7/3}}{7b^3d} + \frac{c^3\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{10/3}(bc-ad)^{2/3}} + \frac{c^3\log(c+dx^3)}{6d^{10/3}(bc-ad)^{2/3}} - \frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}(bc-ad)^{2/3}}$$

```
[Out] (a^2*d^2+a*b*c*d+b^2*c^2)*(b*x^3+a)^(1/3)/b^3/d^3-1/4*(2*a*d+b*c)*(b*x^3+a)
^(4/3)/b^3/d^2+1/7*(b*x^3+a)^(7/3)/b^3/d+1/6*c^3*ln(d*x^3+c)/d^(10/3)/(-a*d
+b*c)^(2/3)-1/2*c^3*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(10/3)/(
-a*d+b*c)^(2/3)+1/3*c^3*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^
(1/3))*3^(1/2))/d^(10/3)/(-a*d+b*c)^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used

= {457, 90, 60, 631, 210, 31}

$$\int \frac{x^{11}}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{\sqrt[3]{a+bx^3}(a^2d^2+abcd+b^2c^2)}{b^3d^3} + \frac{c^3\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{10/3}(bc-ad)^{2/3}} - \frac{(a+bx^3)^{4/3}(2ad+bc)}{4b^3d^2} + \frac{(a+bx^3)^{7/3}}{7b^3d} + \frac{c^3\log(c+dx^3)}{6d^{10/3}(bc-ad)^{2/3}} - \frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}(bc-ad)^{2/3}}$$

```
[In] Int[x^11/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] ((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)^(1/3))/(b^3*d^3) - ((b*c + 2*a*d
)*(a + b*x^3)^(4/3))/(4*b^3*d^2) + (a + b*x^3)^(7/3)/(7*b^3*d) + (c^3*ArcTa
n[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*
d^(10/3)*(b*c - a*d)^(2/3)) + (c^3*Log[c + d*x^3])/(6*d^(10/3)*(b*c - a*d)^
(2/3)) - (c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(10/
3)*(b*c - a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{b^2c^2+abcd+a^2d^2}{b^2d^3(a+bx)^{2/3}}+\frac{(-bc-2ad)\sqrt[3]{a+bx}}{b^2d^2}+\frac{(a+bx)^{4/3}}{b^2d}-\frac{c^3}{d^3(a+bx)^{2/3}(c+dx)}\right)dx, x, x^3\right)\\
&= \frac{(b^2c^2+abcd+a^2d^2)\sqrt[3]{a+bx^3}}{b^3d^3}-\frac{(bc+2ad)\left(a+bx^3\right)^{4/3}}{4b^3d^2}+\frac{\left(a+bx^3\right)^{7/3}}{7b^3d}-\frac{c^3\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3d^3}\\
&= \frac{(b^2c^2+abcd+a^2d^2)\sqrt[3]{a+bx^3}}{b^3d^3}-\frac{(bc+2ad)\left(a+bx^3\right)^{4/3}}{4b^3d^2}+\frac{\left(a+bx^3\right)^{7/3}}{7b^3d}\\
&\quad+\frac{c^3\log\left(c+dx^3\right)}{6d^{10/3}(bc-ad)^{2/3}}-\frac{c^3\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{10/3}(bc-ad)^{2/3}}\\
&\quad-\frac{c^3\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{11/3}\sqrt[3]{bc-ad}}
\end{aligned}$$

$$= \frac{(b^2c^2+abcd+a^2d^2)\sqrt[3]{a+bx^3}}{b^3d^3} - \frac{(bc+2ad)(a+bx^3)^{4/3}}{4b^3d^2} + \frac{(a+bx^3)^{7/3}}{7b^3d} + \frac{c^3\log(c+dx^3)}{6d^{10/3}(bc-ad)^{2/3}} - \frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}(bc-ad)^{2/3}} - \frac{c^3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{d^{10/3}(bc-ad)^{2/3}}$$

$$= \frac{(b^2c^2+abcd+a^2d^2)\sqrt[3]{a+bx^3}}{b^3d^3} - \frac{(bc+2ad)(a+bx^3)^{4/3}}{4b^3d^2} + \frac{(a+bx^3)^{7/3}}{7b^3d} + \frac{c^3\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{10/3}(bc-ad)^{2/3}} + \frac{c^3\log(c+dx^3)}{6d^{10/3}(bc-ad)^{2/3}} - \frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}(bc-ad)^{2/3}}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.10

$$\int \frac{x^{11}}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{3\sqrt[3]{d}(bc-ad)^{2/3}\sqrt[3]{a+bx^3}(18a^2d^2+3abd(7c-2dx^3)+b^2(28c^2-7cdx^3+4d^2}{}$$

```
[In] Integrate[x^11/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (3*d^(1/3)*(b*c - a*d)^(2/3)*(a + b*x^3)^(1/3)*(18*a^2*d^2 + 3*a*b*d*(7*c -
 2*d*x^3) + b^2*(28*c^2 - 7*c*d*x^3 + 4*d^2*x^6)) + 28*Sqrt[3]*b^3*c^3*ArcT
an[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 28*b^3*
c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + 14*b^3*c^3*Log[(b*
c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a +
 b*x^3)^(2/3)])/(84*b^3*d^(10/3)*(b*c - a*d)^(2/3))
```

Maple [A] (verified)

Time = 4.77 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$\frac{9\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}} d\left(\frac{\left(2d^{2}x^{6}-\frac{7}{2}cd\,x^{3}+14c^{2}\right)b^{2}}{9}+\frac{7\left(-\frac{2d\,x^{3}}{7}+c\right)dab}{6}+a^{2}d^{2}\right)}{14}+\frac{b^{3}c^{3}\left(2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right.\right.}{b^{3}d^{4}\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}}$

```
[In] int(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 9/14*((1/d*(a*d-b*c))^(2/3)*(b*x^3+a)^(1/3)*d*(1/9*(2*d^2*x^6-7/2*c*d*x^3+1
4*c^2)*b^2+7/6*(-2/7*d*x^3+c)*d*a*b+a^2*d^2)+7/27*b^3*c^3*(2*arctan(1/3*3^(
1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*3^(1/
2)+ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))
^(2/3))-2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))))/(1/d*(a*d-b*c))^(2/3)
/b^3/d^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(201) = 402$.

Time = 0.35 (sec) , antiderivative size = 1322, normalized size of antiderivative = 5.49

$$\int \frac{x^{11}}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/84*(14*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b^3*c^3*log(-(b*x^3 +
a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c
- a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 28
*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b^3*c^3*log(-(b*x^3 + a)^(1/3)*
(b*c*d - a*d^2) - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)) - 42*sqrt(1/3
)*(b^4*c^4*d - a*b^3*c^3*d^2)*sqrt((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/
3)/d)*log((b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x^3 - 3*
sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2
- a^2*d^3)^(1/3)*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(
b*x^3 + a)^(1/3))*sqrt((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d) - 3*(-
b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3)*(b*c - a*d))/(d*
x^3 + c)) + 3*(28*b^4*c^4*d - 35*a*b^3*c^3*d^2 + 4*a^2*b^2*c^2*d^3 - 15*a^3
*b*c*d^4 + 18*a^4*d^5 + 4*(b^4*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^6 -
 (7*b^4*c^3*d^2 - 8*a*b^3*c^2*d^3 - 5*a^2*b^2*c*d^4 + 6*a^3*b*d^5)*x^3)*(b*
x^3 + a)^(1/3))/(b^5*c^2*d^4 - 2*a*b^4*c*d^5 + a^2*b^3*d^6), 1/84*(14*(-b^2
*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b^3*c^3*log(-(b*x^3 + a)^(2/3)*(b*c*d
 - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) + (-b^2*
c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 28*(-b^2*c^2*d +
2*a*b*c*d^2 - a^2*d^3)^(2/3)*b^3*c^3*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2)
```

```
 - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)) - 84*sqrt(1/3)*(b^4*c^4*d -
a*b^3*c^3*d^2)*sqrt(-(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d)*arctan(s
qrt(1/3)*((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) + 2*(-b^2*
c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3))*sqrt(-(-b^2*c^2*d +
 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2)) + 3*(28*b
^4*c^4*d - 35*a*b^3*c^3*d^2 + 4*a^2*b^2*c^2*d^3 - 15*a^3*b*c*d^4 + 18*a^4*d
^5 + 4*(b^4*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^6 - (7*b^4*c^3*d^2 - 8
*a*b^3*c^2*d^3 - 5*a^2*b^2*c*d^4 + 6*a^3*b*d^5)*x^3)*(b*x^3 + a)^(1/3))/(b^
5*c^2*d^4 - 2*a*b^4*c*d^5 + a^2*b^3*d^6)]
```

Sympy [F]

$$\int \frac{x^{11}}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\int \frac{x^{11}}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

```
[In] integrate(x**11/(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(x**11/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.54

$$\int \frac{x^{11}}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{b^{24}c^3d^4\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{25}cd^7-ab^{24}d^8\right)} - \frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}c^3\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^4-\sqrt{3}ad^5} - \frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}c^3\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(bcd^4-ad^5\right)} + \frac{28\left(bx^3+a\right)^{\frac{1}{3}}b^{20}c^2d^4-7\left(bx^3+a\right)^{\frac{4}{3}}b^{19}cd^5+28\left(bx^3+a\right)^{\frac{1}{3}}ab^{19}cd^5+4\left(bx^3+a\right)^{\frac{7}{3}}b^{18}d^6-14\left(bx^3+a\right)^{\frac{4}{3}}ab^{18}d^6 \cdots}{28\,b^{21}d^7}$$

```
[In] integrate(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^24*c^3*d^4*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
- a*d)/d)^(1/3)))/(b^25*c*d^7 - a*b^24*d^8) - (-b*c*d^2 + a*d^3)^(1/3)*c^3*
arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
a*d)/d)^(1/3))/(sqrt(3)*b*c*d^4 - sqrt(3)*a*d^5) - 1/6*(-b*c*d^2 + a*d^3)^(
1/3)*c^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) +
 (-(b*c - a*d)/d)^(2/3))/(b*c*d^4 - a*d^5) + 1/28*(28*(b*x^3 + a)^(1/3)*b^2
0*c^2*d^4 - 7*(b*x^3 + a)^(4/3)*b^19*c*d^5 + 28*(b*x^3 + a)^(1/3)*a*b^19*c*
d^5 + 4*(b*x^3 + a)^(7/3)*b^18*d^6 - 14*(b*x^3 + a)^(4/3)*a*b^18*d^6 + 28*(
b*x^3 + a)^(1/3)*a^2*b^18*d^6)/(b^21*d^7)
```

Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.37

$$\int \frac{x^{11}}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \left(\frac{3\,a^2}{b^3\,d} + \frac{\left(\frac{3\,a}{b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{b^6\,d^2}\right)\,\left(b^4\,c-a\,b^3\,d\right)}{b^3\,d}\right)\,\left(b\,x^3+a\right)^{1/3} - \left(\frac{3\,a}{4\,b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{4\,b^6\,d^2}\right)\,\left(b\,x^3+a\right)^{4/3} + \frac{\left(b\,x^3+a\right)^{7/3}}{7\,b^3\,d} + \frac{\ln\left(\frac{3\,c^3\,\left(b\,x^3+a\right)^{1/3}}{d}+\frac{3\,c^3\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,\left(a\,d-b\,c\right)^{1/3}}{2\,d^{4/3}}\right)\,\left(c^3+\sqrt{3}\,c^3\,1\mathrm{i}\right)}{6\,d^{10/3}\,\left(a\,d-b\,c\right)^{2/3}} - \frac{c^3\,\ln\left(\frac{3\,c^3\,\left(b\,x^3+a\right)^{1/3}}{d}-\frac{3\,c^3\,\left(a\,d-b\,c\right)^{1/3}}{d^{4/3}}\right)}{3\,d^{10/3}\,\left(a\,d-b\,c\right)^{2/3}} - \frac{c^3\,\ln\left(\frac{3\,c^3\,\left(b\,x^3+a\right)^{1/3}}{d}-\frac{3\,c^3\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)^{1/3}}{d^{4/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,d^{10/3}\,\left(a\,d-b\,c\right)^{2/3}}$$

```
[In] int(x^11/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

```
[Out] ((3*a^2)/(b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c -
a*b^3*d))/(b^3*d))*(a + b*x^3)^(1/3) - ((3*a)/(4*b^3*d) + (b^4*c - a*b^3*d)
/(4*b^6*d^2))*(a + b*x^3)^(4/3) + (a + b*x^3)^(7/3)/(7*b^3*d) + (log((3*c^3
*(a + b*x^3)^(1/3))/d + (3*c^3*(3^(1/2)*1i + 1)*(a*d - b*c)^(1/3))/(2*d^(4/
3)))*(3^(1/2)*c^3*1i + c^3))/(6*d^(10/3)*(a*d - b*c)^(2/3)) - (c^3*log((3*c
^3*(a + b*x^3)^(1/3))/d - (3*c^3*(a*d - b*c)^(1/3))/d^(4/3)))/(3*d^(10/3)*(
a*d - b*c)^(2/3)) - (c^3*log((3*c^3*(a + b*x^3)^(1/3))/d - (3*c^3*((3^(1/2)
*1i)/2 - 1/2)*(a*d - b*c)^(1/3))/d^(4/3))*((3^(1/2)*1i)/2 - 1/2))/(3*d^(10/
3)*(a*d - b*c)^(2/3))
```

3.733 $\int \frac{x^8}{(a+bx^3)^{2/3}(c+dx^3)} dx$

Optimal result

Integrand size = 24, antiderivative size = 201

$$\int \frac{x^8}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = -\frac{(bc+ad)\sqrt[3]{a+bx^3}}{b^2d^2} + \frac{(a+bx^3)^{4/3}}{4b^2d} - \frac{c^2\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{7/3}(bc-ad)^{2/3}} - \frac{c^2\log(c+dx^3)}{6d^{7/3}(bc-ad)^{2/3}} + \frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}(bc-ad)^{2/3}}$$

```
[Out] -(a*d+b*c)*(b*x^3+a)^(1/3)/b^2/d^2+1/4*(b*x^3+a)^(4/3)/b^2/d-1/6*c^2*ln(d*x
^3+c)/d^(7/3)/(-a*d+b*c)^(2/3)+1/2*c^2*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a
)^(1/3))/d^(7/3)/(-a*d+b*c)^(2/3)-1/3*c^2*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)
^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(7/3)/(-a*d+b*c)^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

$= \{457, 90, 60, 631, 210, 31\}$

$$\int \frac{x^8}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = -\frac{c^2 \arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{7/3}(bc-ad)^{2/3}} - \frac{\sqrt[3]{a+bx^3}(ad+bc)}{b^2d^2} + \frac{(a+bx^3)^{4/3}}{4b^2d} - \frac{c^2\log(c+dx^3)}{6d^{7/3}(bc-ad)^{2/3}} + \frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}(bc-ad)^{2/3}}$$

```
[In] Int[x^8/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -(((b*c + a*d)*(a + b*x^3)^(1/3))/(b^2*d^2)) + (a + b*x^3)^(4/3)/(4*b^2*d)
- (c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]
])/(Sqrt[3]*d^(7/3)*(b*c - a*d)^(2/3)) - (c^2*Log[c + d*x^3])/(6*d^(7/3)*(b
*c - a*d)^(2/3)) + (c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])
/(2*d^(7/3)*(b*c - a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{-bc-ad}{bd^2(a+bx)^{2/3}} + \frac{\sqrt[3]{a+bx}}{bd} + \frac{c^2}{d^2(a+bx)^{2/3}(c+dx)}\right)dx, x, x^3\right)$$

$$= -\frac{(bc+ad)\sqrt[3]{a+bx^3}}{b^2d^2} + \frac{(a+bx^3)^{4/3}}{4b^2d} + \frac{c^2\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3d^2}$$

$$= -\frac{(bc+ad)\sqrt[3]{a+bx^3}}{b^2d^2} + \frac{(a+bx^3)^{4/3}}{4b^2d} - \frac{c^2\log(c+dx^3)}{6d^{7/3}(bc-ad)^{2/3}} + \frac{c^2\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{7/3}(bc-ad)^{2/3}} + \frac{c^2\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{8/3}\sqrt[3]{bc-ad}}$$

$$= -\frac{(bc+ad)\sqrt[3]{a+bx^3}}{b^2d^2} + \frac{(a+bx^3)^{4/3}}{4b^2d} - \frac{c^2\log(c+dx^3)}{6d^{7/3}(bc-ad)^{2/3}} + \frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}(bc-ad)^{2/3}} + \frac{c^2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{d^{7/3}(bc-ad)^{2/3}}$$

$$= -\frac{(bc+ad)\sqrt[3]{a+bx^3}}{b^2d^2} + \frac{(a+bx^3)^{4/3}}{4b^2d} - \frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{7/3}(bc-ad)^{2/3}} - \frac{c^2\log(c+dx^3)}{6d^{7/3}(bc-ad)^{2/3}} + \frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}(bc-ad)^{2/3}}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.15

$$\int \frac{x^8}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{-3\sqrt[3]{d}(bc-ad)^{2/3}\sqrt[3]{a+bx^3}(4bc+3ad-bdx^3) - 4\sqrt{3}b^2c^2\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a}}{\sqrt[3]{bc-}}}{\sqrt{3}}\right.}{}$$

```
[In] Integrate[x^8/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (-3*d^(1/3)*(b*c - a*d)^(2/3)*(a + b*x^3)^(1/3)*(4*b*c + 3*a*d - b*d*x^3) -
 4*Sqrt[3]*b^2*c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1
/3))/Sqrt[3]] + 4*b^2*c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)
] - 2*b^2*c^2*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)
^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(12*b^2*d^(7/3)*(b*c - a*d)^(2/3))
```

Maple [A] (verified)

Time = 4.94 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$-\frac{\frac{9d\left(\left(-\frac{bx^3}{3}+a\right)d+\frac{4bc}{3}\right)\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}}{2}+b^2c^2\left(2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(\frac{ad-b}{d}\right.\right.\right.}{6\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}b^2d^3}$

```
[In] int(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/(1/d*(a*d-b*c))^(2/3)*(9/2*d*((-1/3*b*x^3+a)*d+4/3*b*c)*(1/d*(a*d-b*c)
)^(2/3)*(b*x^3+a)^(1/3)+b^2*c^2*(2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1
/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+(1/d
*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a)^(1/
3)-(1/d*(a*d-b*c))^(1/3))))/b^2/d^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. 2(164) = 328.

Time = 0.32 (sec) , antiderivative size = 1156, normalized size of antiderivative = 5.75

$$\int \frac{x^8}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/12*(2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*b^2*c^2*log(-(b*x^3 + a
)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*c -
a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 4*(b^
2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*b^2*c^2*log(-(b*x^3 + a)^(1/3)*(b*c*
d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)) + 6*sqrt(1/3)*(b^3*
c^3*d - a*b^2*c^2*d^2)*sqrt(-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d)*l
og((b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x^3 + 3*sqrt(1/
3)*(2*(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^
3)^(1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a
)^(1/3))*sqrt(-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d) + 3*(b^2*c^2*d
- 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3)*(b*c - a*d))/(d*x^3 + c))
+ 3*(4*b^3*c^3*d - 5*a*b^2*c^2*d^2 - 2*a^2*b*c*d^3 + 3*a^3*d^4 - (b^3*c^2*d
^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3)*(b*x^3 + a)^(1/3))/(b^4*c^2*d^3 - 2*a*
b^3*c*d^4 + a^2*b^2*d^5), -1/12*(2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3
)*b^2*c^2*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2
 + a^2*d^3)^(1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(
b*x^3 + a)^(1/3)) - 4*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*b^2*c^2*log
(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(
2/3)) - 12*sqrt(1/3)*(b^3*c^3*d - a*b^2*c^2*d^2)*sqrt((b^2*c^2*d - 2*a*b*c*
d^2 + a^2*d^3)^(1/3)/d)*arctan(-sqrt(1/3)*((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d
^3)^(1/3)*(b*c - a*d) - 2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3
+ a)^(1/3))*sqrt((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d)/(b^2*c^2 - 2*
a*b*c*d + a^2*d^2)) + 3*(4*b^3*c^3*d - 5*a*b^2*c^2*d^2 - 2*a^2*b*c*d^3 + 3*
a^3*d^4 - (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3)*(b*x^3 + a)^(1/3))
/(b^4*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)]
```

Sympy [F]

$$\int \frac{x^8}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^8}{(a+bx^3)^{\frac{2}{3}}(c+dx^3)}\,dx$$

```
[In] integrate(x**8/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**8/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.55

$$\int \frac{x^8}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = -\frac{b^{10}c^2d^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^{11}cd^4-ab^{10}d^5)} + \frac{(-bcd^2+ad^3)^{\frac{1}{3}}c^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^3-\sqrt{3}ad^4} + \frac{(-bcd^2+ad^3)^{\frac{1}{3}}c^2\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(bcd^3-ad^4)} - \frac{4\,(bx^3+a)^{\frac{1}{3}}b^7cd^2-(bx^3+a)^{\frac{4}{3}}b^6d^3+4\,(bx^3+a)^{\frac{1}{3}}ab^6d^3}{4\,b^8d^4}$$

```
[In] integrate(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b^10*c^2*d^2*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
 - a*d)/d)^(1/3)))/(b^11*c*d^4 - a*b^10*d^5) + (-b*c*d^2 + a*d^3)^(1/3)*c^2
*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
 a*d)/d)^(1/3))/(sqrt(3)*b*c*d^3 - sqrt(3)*a*d^4) + 1/6*(-b*c*d^2 + a*d^3)^
(1/3)*c^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3)
+ (-(b*c - a*d)/d)^(2/3))/(b*c*d^3 - a*d^4) - 1/4*(4*(b*x^3 + a)^(1/3)*b^7*
c*d^2 - (b*x^3 + a)^(4/3)*b^6*d^3 + 4*(b*x^3 + a)^(1/3)*a*b^6*d^3)/(b^8*d^4
)
```

Mupad [B] (verification not implemented)

Time = 8.67 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.45

$$\int \frac{x^8}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \frac{(b\,x^3+a)^{4/3}}{4\,b^2\,d} - \left(\frac{2\,a}{b^2\,d} + \frac{b^3\,c-a\,b^2\,d}{b^4\,d^2}\right)\,(b\,x^3+a)^{1/3} - \frac{\ln\left(3\,c^2\,(b\,x^3+a)^{1/3} + \frac{\left(c^2+\sqrt{3}\,c^2\,1\mathrm{i}\right)\,(9\,a\,d^3-9\,b\,c\,d^2)}{6\,d^{7/3}\,(a\,d-b\,c)^{2/3}}\right)\,\left(c^2+\sqrt{3}\,c^2\,1\mathrm{i}\right)}{6\,d^{7/3}\,(a\,d-b\,c)^{2/3}} + \frac{c^2\,\ln\left(3\,c^2\,(b\,x^3+a)^{1/3} - \frac{c^2\,(9\,a\,d^3-9\,b\,c\,d^2)}{3\,d^{7/3}\,(a\,d-b\,c)^{2/3}}\right)}{3\,d^{7/3}\,(a\,d-b\,c)^{2/3}} + \frac{c^2\,\ln\left(3\,c^2\,(b\,x^3+a)^{1/3} - \frac{c^2\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\,(9\,a\,d^3-9\,b\,c\,d^2)}{d^{7/3}\,(a\,d-b\,c)^{2/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{d^{7/3}\,(a\,d-b\,c)^{2/3}}$$

```
[In] int(x^8/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

```
[Out] (a + b*x^3)^(4/3)/(4*b^2*d) - ((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2))
*(a + b*x^3)^(1/3) - (log(3*c^2*(a + b*x^3)^(1/3) + ((3^(1/2)*c^2*1i + c^2)
*(9*a*d^3 - 9*b*c*d^2))/(6*d^(7/3)*(a*d - b*c)^(2/3)))*(3^(1/2)*c^2*1i + c^
2))/(6*d^(7/3)*(a*d - b*c)^(2/3)) + (c^2*log(3*c^2*(a + b*x^3)^(1/3) - (c^2
*(9*a*d^3 - 9*b*c*d^2))/(3*d^(7/3)*(a*d - b*c)^(2/3))))/(3*d^(7/3)*(a*d - b
*c)^(2/3)) + (c^2*log(3*c^2*(a + b*x^3)^(1/3) - (c^2*((3^(1/2)*1i)/6 - 1/6)
*(9*a*d^3 - 9*b*c*d^2))/(d^(7/3)*(a*d - b*c)^(2/3)))*((3^(1/2)*1i)/6 - 1/6)
)/(d^(7/3)*(a*d - b*c)^(2/3))
```

3.734 $\int \frac{x^5}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 165

$$\int \frac{x^5}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{\sqrt[3]{a+bx^3}}{bd} + \frac{c\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{4/3}(bc-ad)^{2/3}} + \frac{c\log(c+dx^3)}{6d^{4/3}(bc-ad)^{2/3}} - \frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}(bc-ad)^{2/3}}$$

```
[Out] (b*x^3+a)^(1/3)/b/d+1/6*c*ln(d*x^3+c)/d^(4/3)/(-a*d+b*c)^(2/3)-1/2*c*ln((-a
*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(4/3)/(-a*d+b*c)^(2/3)+1/3*c*arcta
n(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(4/3)/(-a*d
+b*c)^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {457, 81, 60, 631, 210, 31}

$$\int \frac{x^5}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{c\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{4/3}(bc-ad)^{2/3}} + \frac{c\log(c+dx^3)}{6d^{4/3}(bc-ad)^{2/3}} - \frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}(bc-ad)^{2/3}} + \frac{\sqrt[3]{a+bx^3}}{bd}$$

```
[In] Int[x^5/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (a + b*x^3)^(1/3)/(b*d) + (c*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c
 - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(4/3)*(b*c - a*d)^(2/3)) + (c*Log[c + d
*x^3])/(6*d^(4/3)*(b*c - a*d)^(2/3)) - (c*Log[(b*c - a*d)^(1/3) + d^(1/3)*(
a + b*x^3)^(1/3)])/(2*d^(4/3)*(b*c - a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{\sqrt[3]{a+bx^3}}{bd} - \frac{c\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3d}$$

$$= \frac{\sqrt[3]{a+bx^3}}{bd} + \frac{c\log\left(c+dx^3\right)}{6d^{4/3}(bc-ad)^{2/3}} - \frac{c\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{4/3}(bc-ad)^{2/3}}$$
$$- \frac{c\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{5/3}\sqrt[3]{bc-ad}}$$

$$= \frac{\sqrt[3]{a+bx^3}}{bd} + \frac{c\log\left(c+dx^3\right)}{6d^{4/3}(bc-ad)^{2/3}} - \frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}(bc-ad)^{2/3}}$$
$$- \frac{c\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{d^{4/3}(bc-ad)^{2/3}}$$

$$= \frac{\sqrt[3]{a+bx^3}}{bd} + \frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{4/3}(bc-ad)^{2/3}}$$
$$+ \frac{c\log\left(c+dx^3\right)}{6d^{4/3}(bc-ad)^{2/3}} - \frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}(bc-ad)^{2/3}}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.22

$$\int \frac{x^5}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{6\sqrt[3]{d}(bc-ad)^{2/3}\sqrt[3]{a+bx^3} + 2\sqrt{3}bc\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) - 2bc\log\left(\sqrt[3]{b\right.}{6b}$$

```
[In] Integrate[x^5/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (6*d^(1/3)*(b*c - a*d)^(2/3)*(a + b*x^3)^(1/3) + 2*Sqrt[3]*b*c*ArcTan[(1 -
(2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*b*c*Log[(b*c
- a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + b*c*Log[(b*c - a*d)^(2/3) - d^(
1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*b
*d^(4/3)*(b*c - a*d)^(2/3))
```

Maple [A] (verified)

Time = 5.04 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.13

method	result
pseudoelliptic	$\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)bc+6\left(b\,x^{3}+a\right)^{\frac{1}{3}}d\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}-2c\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)b+c\ln\left(\left(b\,x^{3}+c\right.\right.}{6b\,d^{2}\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}}$

```
[In] int(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(2*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))
/(1/d*(a*d-b*c))^(1/3))*b*c+6*(b*x^3+a)^(1/3)*d*(1/d*(a*d-b*c))^(2/3)-2*c*l
n((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))*b+c*ln((b*x^3+a)^(2/3)+(1/d*(a*d-b
*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))*b)/b/d^2/(1/d*(a*d-b*c))^
(2/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 482 vs. $2(133) = 266$.

Time = 0.32 (sec) , antiderivative size = 1060, normalized size of antiderivative = 6.42

$$\int \frac{x^5}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b*c*log(-(b*x^3 + a)^(2/3)
*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) +
 (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 2*(-b^2*c^
2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b*c*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^
2) - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)) - 3*sqrt(1/3)*(b^2*c^2*d -
 a*b*c*d^2)*sqrt((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d)*log((b^2*c^2
 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x^3 - 3*sqrt(1/3)*(2*(b*x^
3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(
b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3))*
sqrt((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d) - 3*(-b^2*c^2*d + 2*a*b*
```

```
c*d^2 - a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3)*(b*c - a*d))/(d*x^3 + c)) + 6*(b^2
*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x^3 + a)^(1/3))/(b^3*c^2*d^2 - 2*a*b^2*c
*d^3 + a^2*b*d^4), 1/6*((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b*c*log(
-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(
1/3)*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(
1/3)) - 2*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b*c*log(-(b*x^3 + a)^(
1/3)*(b*c*d - a*d^2) - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)) - 6*sqrt
(1/3)*(b^2*c^2*d - a*b*c*d^2)*sqrt(-(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1
/3)/d)*arctan(sqrt(1/3)*((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c -
a*d) + 2*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3))*sqrt
(-(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d)/(b^2*c^2 - 2*a*b*c*d + a^2*
d^2)) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x^3 + a)^(1/3))/(b^3*c^2*d
^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)]
```

Sympy [F]

$$\int \frac{x^5}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \int \frac{x^5}{(a+bx^3)^{\frac{2}{3}}\,(c+dx^3)}\,dx$$

```
[In] integrate(x**5/(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(x**5/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.53

$$\int \frac{x^5}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = -\frac{\frac{6\,(-bcd^2+ad^3)^{\frac{1}{3}}bc\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^2-\sqrt{3}ad^3}+\frac{(-bcd^2+ad^3)^{\frac{1}{3}}bc\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{bcd^2-ad^3}-}{6\,b}$$

```
[In] integrate(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/6*(6*(-b*c*d^2 + a*d^3)^(1/3)*b*c*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3
) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b*c*d^2 - sqrt
(3)*a*d^3) + (-b*c*d^2 + a*d^3)^(1/3)*b*c*log((b*x^3 + a)^(2/3) + (b*x^3 +
a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d^2 - a*d^3)
 - 2*b*c*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d
)^(1/3)))/(b*c*d - a*d^2) - 6*(b*x^3 + a)^(1/3)/d)/b
```

Mupad [B] (verification not implemented)

Time = 8.80 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.41

$$\int \frac{x^5}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{(b\,x^3+a)^{1/3}}{b\,d} - \frac{c\,\ln\left(3\,c\,d\,(b\,x^3+a)^{1/3}-\frac{c\,(9\,a\,d^3-9\,b\,c\,d^2)}{3\,d^{4/3}\,(a\,d-b\,c)^{2/3}}\right)}{3\,d^{4/3}\,(a\,d-b\,c)^{2/3}} + \frac{\ln\left(3\,c\,d\,(b\,x^3+a)^{1/3}+\frac{(9\,a\,d^3-9\,b\,c\,d^2)\,(c-\sqrt{3}\,c\,1\mathrm{i})}{6\,d^{4/3}\,(a\,d-b\,c)^{2/3}}\right)\,(c-\sqrt{3}\,c\,1\mathrm{i})}{6\,d^{4/3}\,(a\,d-b\,c)^{2/3}} + \frac{\ln\left(3\,c\,d\,(b\,x^3+a)^{1/3}+\frac{(9\,a\,d^3-9\,b\,c\,d^2)\,(c+\sqrt{3}\,c\,1\mathrm{i})}{6\,d^{4/3}\,(a\,d-b\,c)^{2/3}}\right)\,(c+\sqrt{3}\,c\,1\mathrm{i})}{6\,d^{4/3}\,(a\,d-b\,c)^{2/3}}$$

```
[In] int(x^5/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

```
[Out] (a + b*x^3)^(1/3)/(b*d) - (c*log(3*c*d*(a + b*x^3)^(1/3) - (c*(9*a*d^3 - 9*
b*c*d^2))/(3*d^(4/3)*(a*d - b*c)^(2/3))))/(3*d^(4/3)*(a*d - b*c)^(2/3)) + (
log(3*c*d*(a + b*x^3)^(1/3) + ((9*a*d^3 - 9*b*c*d^2)*(c - 3^(1/2)*c*1i))/(6
*d^(4/3)*(a*d - b*c)^(2/3)))*(c - 3^(1/2)*c*1i))/(6*d^(4/3)*(a*d - b*c)^(2/
```

```
3)) + (log(3*c*d*(a + b*x^3)^(1/3) + ((9*a*d^3 - 9*b*c*d^2)*(c + 3^(1/2)*c*
1i))/(6*d^(4/3)*(a*d - b*c)^(2/3)))*(c + 3^(1/2)*c*1i))/(6*d^(4/3)*(a*d - b
*c)^(2/3))
```

3.735 $\int \frac{x^2}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 145

$$\int \frac{x^2}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{d}(bc-ad)^{2/3}} - \frac{\log(c+dx^3)}{6\sqrt[3]{d}(bc-ad)^{2/3}} + \frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}$$

```
[Out] -1/6*ln(d*x^3+c)/d^(1/3)/(-a*d+b*c)^(2/3)+1/2*ln((-a*d+b*c)^(1/3)+d^(1/3)*(
b*x^3+a)^(1/3))/d^(1/3)/(-a*d+b*c)^(2/3)-1/3*arctan(1/3*(1-2*d^(1/3)*(b*x^3
+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(1/3)/(-a*d+b*c)^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {455, 60, 631, 210, 31}

$$\int \frac{x^2}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{d}(bc-ad)^{2/3}} - \frac{\log(c+dx^3)}{6\sqrt[3]{d}(bc-ad)^{2/3}} + \frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}$$

```
[In] Int[x^2/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]]/(Sq
rt[3]*d^(1/3)*(b*c - a*d)^(2/3))) - Log[c + d*x^3]/(6*d^(1/3)*(b*c - a*d)^(
2/3)) + Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(2*d^(1/3)*(b*c
- a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{\log(c+dx^3)}{6\sqrt[3]{d}(bc-ad)^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}$$

$$= -\frac{\log(c+dx^3)}{6\sqrt[3]{d}(bc-ad)^{2/3}} + \frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{\sqrt[3]{d}(bc-ad)^{2/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{d}(bc-ad)^{2/3}} - \frac{\log(c+dx^3)}{6\sqrt[3]{d}(bc-ad)^{2/3}} + \frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}(bc-ad)^{2/3}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right) - 2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right) + \log\left((bc-ad)^{2/3} - \sqrt[3]{d}\sqrt[3]{bc-ad}\right.}{6\sqrt[3]{d}(bc-ad)^{2/3}}$$

```
[In] Integrate[x^2/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3)
)/Sqrt[3]] - 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + Log[(b*
c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a +
 b*x^3)^(2/3)])/(d^(1/3)*(b*c - a*d)^(2/3))
```

Maple [A] (verified)

Time = 4.50 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$\frac{-2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)\sqrt{3}+2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)-\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{a}{}\right.\right.}{6d\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}}$

```
[In] int(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(
a*d-b*c))^(1/3))*3^(1/2)+2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))-ln((b*
x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3)))/
d/(1/d*(a*d-b*c))^(2/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. 2(114) = 228.

Time = 0.33 (sec) , antiderivative size = 927, normalized size of antiderivative = 6.39

$$\int \frac{x^2}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \left[-\frac{3\sqrt{\frac{1}{3}}(bcd-ad^2)\sqrt{-\frac{\left(b^2c^2d-2\,abcd^2+a^2d^3\right)^{\frac{1}{3}}}{d}}\log\left(\frac{b^2c^2-4\,abcd+3\,a^2d^2-2\,\left(b^2cd-abd^2\right)x^3 \cdots}{\cdots}\right.}{\cdots}\right.$$

```
[In] integrate(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(1/3)*(b*c*d - a*d^2)*sqrt(-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3
)^(1/3)/d)*log((b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x^3
 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*
d^2 + a^2*d^3)^(1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3
)*(b*x^3 + a)^(1/3))*sqrt(-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d) + 3
*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3)*(b*c - a*d))/(
d*x^3 + c)) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*log(-(b*x^3 + a)^(2
/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*c - a*d)
 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 2*(b^2*c^
2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) -
 (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2
*d^3), 1/6*(6*sqrt(1/3)*(b*c*d - a*d^2)*sqrt((b^2*c^2*d - 2*a*b*c*d^2 + a^2
```

```
*d^3)^(1/3)/d)*arctan(-sqrt(1/3)*((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)
*(b*c - a*d) - 2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3
))*sqrt((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d)/(b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*log(-(b*x^3 + a)^(2/
3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*c - a*d)
+ (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) + 2*(b^2*c^2
*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) -
(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*
d^3)]
```

Sympy [F]

$$\int \frac{x^2}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \int \frac{x^2}{(a+bx^3)^{\frac{2}{3}}\,(c+dx^3)}\,dx$$

```
[In] integrate(x**2/(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(x**2/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.52

$$\int \frac{x^2}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{(-bcd^2+ad^3)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd-\sqrt{3}ad^2} + \frac{(-bcd^2+ad^3)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(bcd-ad^2)} - \frac{\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc-ad)}$$

```
[In] integrate(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] (-b*c*d^2 + a*d^3)^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c -
 a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b*c*d - sqrt(3)*a*d^2) + 1
/6*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*
c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d - a*d^2) - 1/3*(-(b*c -
a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c - a
*d)
```

Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.47

$$\int \frac{x^2}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{\ln\left(3\,d^2\,(b\,x^3+a)^{1/3}-\frac{9\,a\,d^3-9\,b\,c\,d^2}{3\,d^{1/3}\,(a\,d-b\,c)^{2/3}}\right)}{3\,d^{1/3}\,(a\,d-b\,c)^{2/3}} + \frac{\ln\left(3\,d^2\,(b\,x^3+a)^{1/3}-\frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,(9\,a\,d^3-9\,b\,c\,d^2)}{6\,d^{1/3}\,(a\,d-b\,c)^{2/3}}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,d^{1/3}\,(a\,d-b\,c)^{2/3}} - \frac{\ln\left(3\,d^2\,(b\,x^3+a)^{1/3}+\frac{\left(1+\sqrt{3}\,1\mathrm{i}\right)\,(9\,a\,d^3-9\,b\,c\,d^2)}{6\,d^{1/3}\,(a\,d-b\,c)^{2/3}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,d^{1/3}\,(a\,d-b\,c)^{2/3}}$$

```
[In] int(x^2/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

```
[Out] log(3*d^2*(a + b*x^3)^(1/3) - (9*a*d^3 - 9*b*c*d^2)/(3*d^(1/3)*(a*d - b*c)^
(2/3)))/(3*d^(1/3)*(a*d - b*c)^(2/3)) + (log(3*d^2*(a + b*x^3)^(1/3) - ((3^
(1/2)*1i - 1)*(9*a*d^3 - 9*b*c*d^2))/(6*d^(1/3)*(a*d - b*c)^(2/3)))*(3^(1/2
)*1i - 1))/(6*d^(1/3)*(a*d - b*c)^(2/3)) - (log(3*d^2*(a + b*x^3)^(1/3) + (
(3^(1/2)*1i + 1)*(9*a*d^3 - 9*b*c*d^2))/(6*d^(1/3)*(a*d - b*c)^(2/3)))*(3^(
1/2)*1i + 1))/(6*d^(1/3)*(a*d - b*c)^(2/3))
```

3.736 $\int \frac{1}{x(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 245

$$\int \frac{1}{x(a+bx^3)^{2/3}(c+dx^3)}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}c} + \frac{d^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c(bc-ad)^{2/3}} - \frac{\log(x)}{2a^{2/3}c} + \frac{d^{2/3}\log(c+dx^3)}{6c(bc-ad)^{2/3}} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}c} - \frac{d^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{2/3}}$$

```
[Out] -1/2*ln(x)/a^(2/3)/c+1/6*d^(2/3)*ln(d*x^3+c)/c/(-a*d+b*c)^(2/3)+1/2*ln(a^(1
/3)-(b*x^3+a)^(1/3))/a^(2/3)/c-1/2*d^(2/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x
^3+a)^(1/3))/c/(-a*d+b*c)^(2/3)-1/3*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/
a^(1/3)*3^(1/2))/a^(2/3)/c*3^(1/2)+1/3*d^(2/3)*arctan(1/3*(1-2*d^(1/3)*(b*x
^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c/(-a*d+b*c)^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {457, 88, 59, 631, 210, 31, 60}

$$\int \frac{1}{x\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=-\frac{\arctan\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}c}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}c}-\frac{\log(x)}{2a^{2/3}c}+\frac{d^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c(bc-ad)^{2/3}}+\frac{d^{2/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{2/3}}-\frac{d^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{2/3}}$$

```
[In] Int[1/(x*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -(ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3
)*c)) + (d^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3
))/Sqrt[3]])/(Sqrt[3]*c*(b*c - a*d)^(2/3)) - Log[x]/(2*a^(2/3)*c) + (d^(2/3
)*Log[c + d*x^3])/(6*c*(b*c - a*d)^(2/3)) + Log[a^(1/3) - (a + b*x^3)^(1/3)
]/(2*a^(2/3)*c) - (d^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3
)])/(2*c*(b*c - a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}(c+dx)}\,dx,x,x^3\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx,x,x^3\right)}{3c}-\frac{d\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx,x,x^3\right)}{3c}\end{aligned}$$

$$= -\frac{\log(x)}{2a^{2/3}c} + \frac{d^{2/3}\log(c+dx^3)}{6c(bc-ad)^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2a^{2/3}c} - \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}c} - \frac{d^{2/3}\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{2/3}} - \frac{\sqrt[3]{d}\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c\sqrt[3]{bc-ad}}$$

$$= -\frac{\log(x)}{2a^{2/3}c} + \frac{d^{2/3}\log(c+dx^3)}{6c(bc-ad)^{2/3}} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}c} - \frac{d^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{a^{2/3}c} - \frac{d^{2/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{c(bc-ad)^{2/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}c} + \frac{d^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c(bc-ad)^{2/3}} - \frac{\log(x)}{2a^{2/3}c} + \frac{d^{2/3}\log(c+dx^3)}{6c(bc-ad)^{2/3}} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}c} - \frac{d^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{2/3}}$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.26

$$\int \frac{1}{x(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{-\frac{2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} + \frac{2\sqrt{3}d^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{(bc-ad)^{2/3}} + \frac{2\log\left(-\sqrt[3]{a}+\sqrt[3]{\ldots}\right.}{a^{2/\ldots}}}{}$$

```
[In] Integrate[1/(x*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] ((-2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3) +
 (2*Sqrt[3]*d^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(
1/3))/Sqrt[3]])/(b*c - a*d)^(2/3) + (2*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a
^(2/3) - (2*d^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(b*
c - a*d)^(2/3) - Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3
)]/a^(2/3) + (d^(2/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a
+ b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(2/3))/(6*c)
```

Maple [A] (verified)

Time = 4.65 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$\frac{\left(2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}\right)-2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\left(\right.\right.}{6a}$

```
[In] int(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*((2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(
a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)
^(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3)))*
a^(2/3)-(2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)+
ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3))-2*ln((b*x^3+a)^(1/3)-a^
(1/3)))*(1/d*(a*d-b*c))^(2/3))/a^(2/3)/(1/d*(a*d-b*c))^(2/3)/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(192) = 384$.

Time = 0.32 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.93

$$\int \frac{1}{x\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = -\frac{2\sqrt{3}a^2\left(-\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}(bc-ad)\left(-\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{2}{3}}-\sqrt{3}d}{3\,d}\right)+a^2\left(-\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}}}{}$$

```
[In] integrate(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*a^2*(-d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*arctan(-1/
3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*(b*c - a*d)*(-d^2/(b^2*c^2 - 2*a*b*c*d + a^2
*d^2))^(2/3) - sqrt(3)*d)/d) + a^2*(-d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(
1/3)*log((b*x^3 + a)^(2/3)*d^2 + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2)*(-d^2/(b
```

```
^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-d^
2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(2/3)) - 2*a^2*(-d^2/(b^2*c^2 - 2*a*b*c*
d + a^2*d^2))^(1/3)*log((b*x^3 + a)^(1/3)*d - (b*c - a*d)*(-d^2/(b^2*c^2 -
2*a*b*c*d + a^2*d^2))^(1/3)) + 2*sqrt(3)*(a^2)^(1/6)*a*arctan(1/3*(a^2)^(1/
6)*(sqrt(3)*(a^2)^(1/3)*a + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(a^2)^(2/3))/a^2) +
 (a^2)^(2/3)*log((b*x^3 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a
^2)^(2/3)) - 2*(a^2)^(2/3)*log((b*x^3 + a)^(1/3)*a - (a^2)^(2/3)))/(a^2*c)
```

Sympy [F]

$$\int \frac{1}{x\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\int \frac{1}{x\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

```
[In] integrate(1/x/(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(1/(x*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)x}\,dx$$

```
[In] integrate(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.31

$$\int \frac{1}{x(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{d\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(bc^2-acd)} - \frac{(-bcd^2+ad^3)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^2-\sqrt{3}acd} - \frac{(-bcd^2+ad^3)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(bc^2-acd)} - \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{2}{3}}c} - \frac{\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{2}{3}}c} + \frac{\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{2}{3}}c}$$

```
[In] integrate(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*d*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(
1/3)))/(b*c^2 - a*c*d) - (-b*c*d^2 + a*d^3)^(1/3)*arctan(1/3*sqrt(3)*(2*(b*
x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b
*c^2 - sqrt(3)*a*c*d) - 1/6*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 + a)^(2/3)
+ (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c^2
 - a*c*d) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/
a^(1/3))/(a^(2/3)*c) - 1/6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3
) + a^(2/3))/(a^(2/3)*c) + 1/3*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(2/
3)*c)
```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 1413, normalized size of antiderivative = 5.77

$$\int \frac{1}{x(a+bx^3)^{2/3}(c+dx^3)}\,dx = \text{Too large to display}$$

```
[In] int(1/(x*(a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

```
[Out] log((((81*b^6*c^5*d^3 - 162*a*b^5*c^4*d^4)*(a + b*x^3)^(1/3) - (243*a*b^6*c
^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(1/(27*a^2*c^3))^(1/3))
*(1/(27*a^2*c^3))^(2/3) - 9*b^5*c^2*d^4)*(1/(27*a^2*c^3))^(1/3) + 6*b^4*d^5
*(a + b*x^3)^(1/3))*(1/(27*a^2*c^3))^(1/3) + log(- (((81*b^6*c^5*d^3 - 162*
a*b^5*c^4*d^4)*(a + b*x^3)^(1/3) - (-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*
a*b*c^4*d))^(1/3)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^
4*d^5))*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(2/3) - 9*b^5*c
^2*d^4)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(1/3) - 6*b^4*d
^5*(a + b*x^3)^(1/3))*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(
1/3) + log(((3^(1/2)*1i)/2 - 1/2)*(((3^(1/2)*1i)/2 - 1/2)^2*((81*b^6*c^5*d^
3 - 162*a*b^5*c^4*d^4)*(a + b*x^3)^(1/3) - ((3^(1/2)*1i)/2 - 1/2)*(243*a*b^
6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(1/(27*a^2*c^3))^(1/
3))*(1/(27*a^2*c^3))^(2/3) - 9*b^5*c^2*d^4)*(1/(27*a^2*c^3))^(1/3) + 6*b^4*
d^5*(a + b*x^3)^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(1/(27*a^2*c^3))^(1/3) - log(
6*b^4*d^5*(a + b*x^3)^(1/3) - ((3^(1/2)*1i)/2 + 1/2)*(((3^(1/2)*1i)/2 + 1/2
)^2*((81*b^6*c^5*d^3 - 162*a*b^5*c^4*d^4)*(a + b*x^3)^(1/3) + ((3^(1/2)*1i)
/2 + 1/2)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(
1/(27*a^2*c^3))^(1/3))*(1/(27*a^2*c^3))^(2/3) - 9*b^5*c^2*d^4)*(1/(27*a^2*c
^3))^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(1/(27*a^2*c^3))^(1/3) + (log(6*b^4*d^5*
(a + b*x^3)^(1/3) + ((3^(1/2)*1i - 1)*(((3^(1/2)*1i - 1)^2*((81*b^6*c^5*d^3
 - 162*a*b^5*c^4*d^4)*(a + b*x^3)^(1/3) - ((3^(1/2)*1i - 1)*(-d^2/(27*b^2*c
^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(1/3)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5
*c^5*d^4 + 486*a^3*b^4*c^4*d^5))/2)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54
*a*b*c^4*d))^(2/3))/4 - 9*b^5*c^2*d^4)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 -
 54*a*b*c^4*d))^(1/3))/2)*(3^(1/2)*1i - 1)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d
^2 - 54*a*b*c^4*d))^(1/3))/2 - (log(6*b^4*d^5*(a + b*x^3)^(1/3) - ((3^(1/2)
*1i + 1)*(((3^(1/2)*1i + 1)^2*((81*b^6*c^5*d^3 - 162*a*b^5*c^4*d^4)*(a + b*
x^3)^(1/3) + ((3^(1/2)*1i + 1)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*
c^4*d))^(1/3)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^
5))/2)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(2/3))/4 - 9*b^5
*c^2*d^4)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(1/3))/2)*(3^
(1/2)*1i + 1)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(1/3))/2
```

3.737 $\int \frac{1}{x^4(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 299

$$\int \frac{1}{x^4(a+bx^3)^{2/3}(c+dx^3)}\,dx = -\frac{\sqrt[3]{a+bx^3}}{3acx^3}+\frac{(2bc+3ad)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}c^2} - \frac{d^{5/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2(bc-ad)^{2/3}}+\frac{(2bc+3ad)\log(x)}{6a^{5/3}c^2}-\frac{d^{5/3}\log(c+dx^3)}{6c^2(bc-ad)^{2/3}} - \frac{(2bc+3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{5/3}c^2}+\frac{d^{5/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2(bc-ad)^{2/3}}$$

```
[Out] -1/3*(b*x^3+a)^(1/3)/a/c/x^3+1/6*(3*a*d+2*b*c)*ln(x)/a^(5/3)/c^2-1/6*d^(5/3
)*ln(d*x^3+c)/c^2/(-a*d+b*c)^(2/3)-1/6*(3*a*d+2*b*c)*ln(a^(1/3)-(b*x^3+a)^(
1/3))/a^(5/3)/c^2+1/2*d^(5/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/
c^2/(-a*d+b*c)^(2/3)+1/9*(3*a*d+2*b*c)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3
))/a^(1/3)*3^(1/2))/a^(5/3)/c^2*3^(1/2)-1/3*d^(5/3)*arctan(1/3*(1-2*d^(1/3)
*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c^2/(-a*d+b*c)^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {457, 105, 162, 59, 631, 210, 31, 60}

$$\int \frac{1}{x^4 (a+bx^3)^{2/3} (c+dx^3)} \, dx = \frac{\arctan\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)(3ad+2bc)}{3\sqrt{3}a^{5/3}c^2} - \frac{(3ad+2bc)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{5/3}c^2} + \frac{\log(x)(3ad+2bc)}{6a^{5/3}c^2} - \frac{d^{5/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2(bc-ad)^{2/3}} - \frac{d^{5/3}\log(c+dx^3)}{6c^2(bc-ad)^{2/3}} + \frac{d^{5/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2(bc-ad)^{2/3}} - \frac{\sqrt[3]{a+bx^3}}{3acx^3}$$

```
[In] Int[1/(x^4*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -1/3*(a + b*x^3)^(1/3)/(a*c*x^3) + ((2*b*c + 3*a*d)*ArcTan[(a^(1/3) + 2*(a
+ b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*c^2) - (d^(5/3)*ArcT
an[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]
*c^2*(b*c - a*d)^(2/3)) + ((2*b*c + 3*a*d)*Log[x])/(6*a^(5/3)*c^2) - (d^(5/
3)*Log[c + d*x^3])/(6*c^2*(b*c - a*d)^(2/3)) - ((2*b*c + 3*a*d)*Log[a^(1/3)
 - (a + b*x^3)^(1/3)])/(6*a^(5/3)*c^2) + (d^(5/3)*Log[(b*c - a*d)^(1/3) + d
^(1/3)*(a + b*x^3)^(1/3)])/(2*c^2*(b*c - a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
```

```
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt[3]{a+bx^3}}{3acx^3} - \frac{\text{Subst}\left(\int \frac{\frac{1}{3}(2bc+3ad)+\frac{2bdx}{3}}{x(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{3acx^3} + \frac{d^2\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3c^2} - \frac{(2bc+3ad)\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^3\right)}{9ac^2}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{3acx^3} + \frac{(2bc+3ad)\log(x)}{6a^{5/3}c^2} - \frac{d^{5/3}\log\left(c+dx^3\right)}{6c^2(bc-ad)^{2/3}} + \frac{d^{5/3}\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c^2(bc-ad)^{2/3}} + \frac{d^{4/3}\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c^2\sqrt[3]{bc-ad}} + \frac{(2bc+3ad)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{6a^{5/3}c^2} + \frac{(2bc+3ad)\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{6a^{4/3}c^2}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{3acx^3} + \frac{(2bc+3ad)\log(x)}{6a^{5/3}c^2} - \frac{d^{5/3}\log\left(c+dx^3\right)}{6c^2(bc-ad)^{2/3}} - \frac{(2bc+3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{5/3}c^2} + \frac{d^{5/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2(bc-ad)^{2/3}} + \frac{d^{5/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{c^2(bc-ad)^{2/3}} - \frac{(2bc+3ad)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{3a^{5/3}c^2}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{3acx^3} + \frac{(2bc+3ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{5/3}c^2} - \frac{d^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2(bc-ad)^{2/3}} + \frac{(2bc+3ad)\log(x)}{6a^{5/3}c^2} - \frac{d^{5/3}\log\left(c+dx^3\right)}{6c^2(bc-ad)^{2/3}} - \frac{(2bc+3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{5/3}c^2} + \frac{d^{5/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2(bc-ad)^{2/3}}$$

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^4\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{-\frac{6c\sqrt[3]{a+bx^3}}{ax^3} + \frac{2\sqrt{3}(2bc+3ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}} - \frac{6\sqrt{3}d^{5/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{\ldots}}{\sqrt[3]{bc\ldots}}}{\sqrt{\ldots}}\right)}{(bc-ad)^{2/3}} \ldots}{\ldots}$$

```
[In] Integrate[1/(x^4*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] ((-6*c*(a + b*x^3)^(1/3))/(a*x^3) + (2*Sqrt[3]*(2*b*c + 3*a*d)*ArcTan[(1 +
(2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(5/3) - (6*Sqrt[3]*d^(5/3)*ArcTa
n[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(b*c - a*
d)^(2/3) - (2*(2*b*c + 3*a*d)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(5/3) +
(6*d^(5/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^
(2/3) + ((2*b*c + 3*a*d)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x
^3)^(2/3)])/a^(5/3) - (3*d^(5/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d
)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(2/3))/
(18*c^2)
```

Maple [A] (verified)

Time = 4.90 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.99

method	result
pseudoelliptic	$\frac{-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(\frac{ad-bc}{d}\right)^{\frac{2}{3}}c\,a^{\frac{2}{3}}+x^{3}\left(-d\left(2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)\right.\right.\right.}{}$

```
[In] int(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-2*(b*x^3+a)^(1/3)*(1/d*(a*d-b*c))^(2/3)*c*a^(2/3)+x^3*(-d*(2*arctan(1
/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))
*3^(1/2)+ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d
-b*c))^(2/3))-2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3)))*a^(5/3)+1/3*(1/d
*(a*d-b*c))^(2/3)*(3*a*d+2*b*c)*(2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a
^(1/3)*3^(1/2))*3^(1/2)+ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3))
-2*ln((b*x^3+a)^(1/3)-a^(1/3)))))/a^(5/3)/(1/d*(a*d-b*c))^(2/3)/c^2/x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 562 vs. $2(239) = 478$.

Time = 0.79 (sec) , antiderivative size = 562, normalized size of antiderivative = 1.88

$$\int \frac{1}{x^4 (a+bx^3)^{2/3} (c+dx^3)}\,dx = -\frac{6\sqrt{3}a^3d\left(\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}}x^3\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}(bc-ad)\left(\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{2}{3}}-\sqrt{3}d}{3\,d}\right)+3\,a^3d\left(\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}}}{}$$

```
[In] integrate(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/18*(6*sqrt(3)*a^3*d*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*x^3*arct
an(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*(b*c - a*d)*(d^2/(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2))^(2/3) - sqrt(3)*d)/d) + 3*a^3*d*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2
*d^2))^(1/3)*x^3*log((b*x^3 + a)^(2/3)*d^2 - (b*x^3 + a)^(1/3)*(b*c*d - a*d
^2)*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3) + (b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(2/3)) - 6*a^3*d*(d^2/(b^2*c^2
 - 2*a*b*c*d + a^2*d^2))^(1/3)*x^3*log((b*x^3 + a)^(1/3)*d + (b*c - a*d)*(d
^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)) - 2*sqrt(3)*(2*a*b*c + 3*a^2*d)*
x^3*sqrt(-(-a^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-a^2)^(1/3)*a - 2*sqrt(3)*(b*
x^3 + a)^(1/3)*(-a^2)^(2/3))*sqrt(-(-a^2)^(1/3))/a^2) - (-a^2)^(2/3)*(2*b*c
 + 3*a*d)*x^3*log((b*x^3 + a)^(2/3)*a - (-a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*
(-a^2)^(2/3)) + 2*(-a^2)^(2/3)*(2*b*c + 3*a*d)*x^3*log((b*x^3 + a)^(1/3)*a
- (-a^2)^(2/3)) + 6*(b*x^3 + a)^(1/3)*a^2*c)/(a^3*c^2*x^3)
```

Sympy [F]

$$\int \frac{1}{x^4 (a+bx^3)^{2/3} (c+dx^3)} \, dx = \int \frac{1}{x^4 (a+bx^3)^{\frac{2}{3}} (c+dx^3)} \, dx$$

```
[In] integrate(1/x**4/(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(1/(x**4*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (a+bx^3)^{2/3} (c+dx^3)} \, dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)x^4} \, dx$$

```
[In] integrate(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.54 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.26

$$\begin{aligned} \int \frac{1}{x^4 (a+bx^3)^{2/3} (c+dx^3)} \, dx = & -\frac{d^2 \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} \log \left(\left| (bx^3+a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} \right| \right)}{3\,(bc^3-ac^2d)} \\ & + \frac{(-bcd^2+ad^3)^{\frac{1}{3}} d \arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}} \right)}{\sqrt{3}bc^3 - \sqrt{3}ac^2d} \\ & + \frac{(-bcd^2+ad^3)^{\frac{1}{3}} d \log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}} \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}} \right)}{6\,(bc^3-ac^2d)} \\ & + \frac{\sqrt{3}(2\,bc+3\,ad) \arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}} + a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}} \right)}{9\,a^{\frac{5}{3}} c^2} \\ & + \frac{(2\,bc+3\,ad) \log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}} a^{\frac{1}{3}} + a^{\frac{2}{3}} \right)}{18\,a^{\frac{5}{3}} c^2} \\ & - \frac{(2\,bc+3\,ad) \log\left(\left| (bx^3+a)^{\frac{1}{3}} - a^{\frac{1}{3}} \right| \right)}{9\,a^{\frac{5}{3}} c^2} - \frac{(bx^3+a)^{\frac{1}{3}}}{3\,acx^3} \end{aligned}$$

```
[In] integrate(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*d^2*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d
)^(1/3)))/(b*c^3 - a*c^2*d) + (-b*c*d^2 + a*d^3)^(1/3)*d*arctan(1/3*sqrt(3)
*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sq
rt(3)*b*c^3 - sqrt(3)*a*c^2*d) + 1/6*(-b*c*d^2 + a*d^3)^(1/3)*d*log((b*x^3
+ a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2
/3))/(b*c^3 - a*c^2*d) + 1/9*sqrt(3)*(2*b*c + 3*a*d)*arctan(1/3*sqrt(3)*(2*
(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(5/3)*c^2) + 1/18*(2*b*c + 3*a*d)*
log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(5/3)*c^2)
- 1/9*(2*b*c + 3*a*d)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(5/3)*c^2) -
 1/3*(b*x^3 + a)^(1/3)/(a*c*x^3)
```

Mupad [B] (verification not implemented)

Time = 15.75 (sec) , antiderivative size = 1959, normalized size of antiderivative = 6.55

$$\int \frac{1}{x^4 (a+bx^3)^{2/3} (c+dx^3)} \, dx = \text{Too large to display}$$

```
[In] int(1/(x^4*(a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

```
[Out] log(- (((((27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a^2*d^2 - 2*b^2*c^2 + a*b*c*
d))/a - 81*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(d^5/(c^6*(a*d -
 b*c)^2))^(1/3))*(d^5/(c^6*(a*d - b*c)^2))^(2/3))/9 + (b^5*d^4*(8*b^3*c^3 -
 27*a^3*d^3 + 28*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^3*c))*(d^5/(c^6*(a*d -
 b*c)^2))^(1/3))/3 - (2*b^4*d^6*(a + b*x^3)^(1/3)*(27*a^3*d^3 + 4*b^3*c^3 +
 18*a*b^2*c^2*d + 36*a^2*b*c*d^2))/(9*a^3*c^4))*(d^5/(27*b^2*c^8 + 27*a^2*c
^6*d^2 - 54*a*b*c^7*d))^(1/3) + log(- (((((27*b^5*c^3*d^3*(a + b*x^3)^(1/3)
*(4*a^2*d^2 - 2*b^2*c^2 + a*b*c*d))/a - 27*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c
^2 - 3*a*b*c*d)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^(1/3))*(-(3*a*d + 2*b*c)^3/(
a^5*c^6))^(2/3))/81 + (b^5*d^4*(8*b^3*c^3 - 27*a^3*d^3 + 28*a*b^2*c^2*d + 1
8*a^2*b*c*d^2))/(3*a^3*c))*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^(1/3))/9 - (2*b^4
*d^6*(a + b*x^3)^(1/3)*(27*a^3*d^3 + 4*b^3*c^3 + 18*a*b^2*c^2*d + 36*a^2*b*
c*d^2))/(9*a^3*c^4))*(-(27*a^3*d^3 + 8*b^3*c^3 + 36*a*b^2*c^2*d + 54*a^2*b*
c*d^2)/(729*a^5*c^6))^(1/3) + (log(((3^(1/2)*1i - 1)*((((27*b^5*c^3*d^3*(a
+ b*x^3)^(1/3)*(4*a^2*d^2 - 2*b^2*c^2 + a*b*c*d))/a - (81*a*b^4*c^4*d^3*(3^
(1/2)*1i - 1)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(d^5/(c^6*(a*d - b*c)^2))^(
1/3))/2)*(3^(1/2)*1i - 1)^2*(d^5/(c^6*(a*d - b*c)^2))^(2/3))/36 + (b^5*d^4*
(8*b^3*c^3 - 27*a^3*d^3 + 28*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^3*c))*(d^5
/(c^6*(a*d - b*c)^2))^(1/3))/6 + (2*b^4*d^6*(a + b*x^3)^(1/3)*(27*a^3*d^3 +
 4*b^3*c^3 + 18*a*b^2*c^2*d + 36*a^2*b*c*d^2))/(9*a^3*c^4))*(3^(1/2)*1i - 1
)*(d^5/(27*b^2*c^8 + 27*a^2*c^6*d^2 - 54*a*b*c^7*d))^(1/3))/2 - (log(((3^(1
/2)*1i + 1)*((((27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a^2*d^2 - 2*b^2*c^2 + a
*b*c*d))/a + (81*a*b^4*c^4*d^3*(3^(1/2)*1i + 1)*(2*a^2*d^2 + b^2*c^2 - 3*a*
```

```
b*c*d)*(d^5/(c^6*(a*d - b*c)^2))^(1/3))/2)*(3^(1/2)*1i + 1)^2*(d^5/(c^6*(a*
d - b*c)^2))^(2/3))/36 + (b^5*d^4*(8*b^3*c^3 - 27*a^3*d^3 + 28*a*b^2*c^2*d
+ 18*a^2*b*c*d^2))/(3*a^3*c))*(d^5/(c^6*(a*d - b*c)^2))^(1/3))/6 - (2*b^4*d
^6*(a + b*x^3)^(1/3)*(27*a^3*d^3 + 4*b^3*c^3 + 18*a*b^2*c^2*d + 36*a^2*b*c*
d^2))/(9*a^3*c^4))*(3^(1/2)*1i + 1)*(d^5/(27*b^2*c^8 + 27*a^2*c^6*d^2 - 54*
a*b*c^7*d))^(1/3))/2 + log((2*b^4*d^6*(a + b*x^3)^(1/3)*(27*a^3*d^3 + 4*b^3
*c^3 + 18*a*b^2*c^2*d + 36*a^2*b*c*d^2))/(9*a^3*c^4) - (((((27*b^5*c^3*d^3*
(a + b*x^3)^(1/3)*(4*a^2*d^2 - 2*b^2*c^2 + a*b*c*d))/a - 27*a*b^4*c^4*d^3*(
(3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d + 2*b*c)^3
/(a^5*c^6))^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^(2
/3))/81 - (b^5*d^4*(8*b^3*c^3 - 27*a^3*d^3 + 28*a*b^2*c^2*d + 18*a^2*b*c*d^
2))/(3*a^3*c))*((3^(1/2)*1i)/2 - 1/2)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^(1/3))
/9)*((3^(1/2)*1i)/2 - 1/2)*(-(27*a^3*d^3 + 8*b^3*c^3 + 36*a*b^2*c^2*d + 54*
a^2*b*c*d^2)/(729*a^5*c^6))^(1/3) - log((((((27*b^5*c^3*d^3*(a + b*x^3)^(1/
3)*(4*a^2*d^2 - 2*b^2*c^2 + a*b*c*d))/a + 27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2
+ 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^(1/
3))*((3^(1/2)*1i)/2 - 1/2)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^(2/3))/81 + (b^5*
d^4*(8*b^3*c^3 - 27*a^3*d^3 + 28*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^3*c))*
((3^(1/2)*1i)/2 + 1/2)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^(1/3))/9 - (2*b^4*d^6
*(a + b*x^3)^(1/3)*(27*a^3*d^3 + 4*b^3*c^3 + 18*a*b^2*c^2*d + 36*a^2*b*c*d^
2))/(9*a^3*c^4))*((3^(1/2)*1i)/2 + 1/2)*(-(27*a^3*d^3 + 8*b^3*c^3 + 36*a*b^
2*c^2*d + 54*a^2*b*c*d^2)/(729*a^5*c^6))^(1/3) - (a + b*x^3)^(1/3)/(3*a*c*x
^3)
```

3.738 $\int \frac{x^7}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 279

$$\int \frac{x^7}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{x^2\sqrt[3]{a+bx^3}}{3bd} + \frac{(3bc+2ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{5/3}d^2} - \frac{c^{5/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2(bc-ad)^{2/3}} + \frac{c^{5/3}\log(c+dx^3)}{6d^2(bc-ad)^{2/3}} + \frac{(3bc+2ad)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6b^{5/3}d^2} - \frac{c^{5/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^2(bc-ad)^{2/3}}$$

```
[Out] 1/3*x^2*(b*x^3+a)^(1/3)/b/d+1/6*c^(5/3)*ln(d*x^3+c)/d^2/(-a*d+b*c)^(2/3)+1/
6*(2*a*d+3*b*c)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(5/3)/d^2-1/2*c^(5/3)*ln((-
a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^2/(-a*d+b*c)^(2/3)+1/9*(2*a*d+3
*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(5/3)/d^2*3^(1/
2)-1/3*c^(5/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*
3^(1/2))/d^2/(-a*d+b*c)^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {490, 598, 337, 503}

$$\int \frac{x^7}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(2ad+3bc)}{3\sqrt{3}b^{5/3}d^2} - \frac{c^{5/3}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d^2(bc-ad)^{2/3}} + \frac{(2ad+3bc)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6b^{5/3}d^2} + \frac{c^{5/3}\log(c+dx^3)}{6d^2(bc-ad)^{2/3}} - \frac{c^{5/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^2(bc-ad)^{2/3}} + \frac{x^2\sqrt[3]{a+bx^3}}{3bd}$$

```
[In] Int[x^7/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (x^2*(a + b*x^3)^(1/3))/(3*b*d) + ((3*b*c + 2*a*d)*ArcTan[(1 + (2*b^(1/3)*x
)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(5/3)*d^2) - (c^(5/3)*ArcTan[(1
 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*
d^2*(b*c - a*d)^(2/3)) + (c^(5/3)*Log[c + d*x^3])/(6*d^2*(b*c - a*d)^(2/3))
 + ((3*b*c + 2*a*d)*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(6*b^(5/3)*d^2) - (
c^(5/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*d^2*(b*c
 - a*d)^(2/3))
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^2\sqrt[3]{a+bx^3}}{3bd} - \frac{\int \frac{x(2ac+(3bc+2ad)x^3)}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{3bd} \\
&= \frac{x^2\sqrt[3]{a+bx^3}}{3bd} - \frac{\int \left(\frac{(3bc+2ad)x}{d(a+bx^3)^{2/3}} - \frac{3bc^2x}{d(a+bx^3)^{2/3}(c+dx^3)}\right)dx}{3bd} \\
&= \frac{x^2\sqrt[3]{a+bx^3}}{3bd} + \frac{c^2\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{d^2} - \frac{(3bc+2ad)\int \frac{x}{(a+bx^3)^{2/3}}\,dx}{3bd^2} \\
&= \frac{x^2\sqrt[3]{a+bx^3}}{3bd} + \frac{(3bc+2ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{5/3}d^2} \\
&\quad - \frac{c^{5/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2(bc-ad)^{2/3}} + \frac{c^{5/3}\log\left(c+dx^3\right)}{6d^2(bc-ad)^{2/3}} \\
&\quad + \frac{(3bc+2ad)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6b^{5/3}d^2} - \frac{c^{5/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^2(bc-ad)^{2/3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.26 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.69

$$\int \frac{x^7}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{\frac{12dx^2\sqrt[3]{a+bx^3}}{b}+\frac{4\sqrt{3}(3bc+2ad)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{b^{5/3}}+\frac{6\sqrt{-6-6i\sqrt{3}}c^{5/3}\arctan\left(\cdots\right.}{\cdots}}{\cdots}$$

```
[In] Integrate[x^7/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] ((12*d*x^2*(a + b*x^3)^(1/3))/b + (4*Sqrt[3]*(3*b*c + 2*a*d)*ArcTan[(Sqrt[3
]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/b^(5/3) + (6*Sqrt[-6 - (6*
I)*Sqrt[3]]*c^(5/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/
3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))]])/(b*c - a*d)^(2/3) + (4*
(3*b*c + 2*a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(5/3) + (6*(1 - I*
Sqrt[3])*c^(5/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b
*x^3)^(1/3)])/(b*c - a*d)^(2/3) - (2*(3*b*c + 2*a*d)*Log[b^(2/3)*x^2 + b^(1
/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(5/3) + ((3*I)*(I + Sqrt[3]
)*c^(5/3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d
)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(
b*c - a*d)^(2/3))/(36*d^2)
```

Maple [A] (verified)

Time = 4.96 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.18

method	result
pseudoelliptic	$\frac{(b\,x^3+a)^{\frac{1}{3}} x^2 \left(\frac{ad-bc}{c}\right)^{\frac{2}{3}} d\, b^{\frac{2}{3}}+\frac{c\left(2\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(b\,x^3+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)\sqrt{3}+\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^2-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}x+(b\cdots}{x^2}\right.\cdots}{2}}{\cdots}$

```
[In] int(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((b*x^3+a)^(1/3)*x^2*((a*d-b*c)/c)^(2/3)*d*b^(2/3)+1/2*c*(2*arctan(1/3*
3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^
(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*
x^3+a)^(2/3))/x^2)-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x))*b^(5/3)
-1/6*((a*d-b*c)/c)^(2/3)*(2*a*d+3*b*c)*(2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/
3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*
x+(b*x^3+a)^(2/3))/x^2)-2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)))/b^(5/3)/((a*
d-b*c)/c)^(2/3)/d^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(225) = 450$.

Time = 0.65 (sec) , antiderivative size = 558, normalized size of antiderivative = 2.00

$$\int \frac{x^7}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{6\sqrt{3}b^3c\left(-\frac{c^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}(bc-ad)\left(-\frac{c^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{2}{3}}+\sqrt{3}c}{3\,cx}\right.}{}$$

```
[In] integrate(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/18*(6*sqrt(3)*b^3*c*(-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*arctan(-
1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*(b*c - a*d)*(-c^2/(b^2*c^2 - 2*a*b*c*d + a
^2*d^2))^(2/3) + sqrt(3)*c*x)/(c*x)) + 6*(b*x^3 + a)^(1/3)*b^2*d*x^2 + 6*b^
3*c*(-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*log(((b*c - a*d)*(-c^2/(b^
2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*x + (b*x^3 + a)^(1/3)*c)/x) - 3*b^3*c*(
-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*log(((b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*(-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(2/3)*x^2 + (b*x^3 + a)^(2/3)*
c^2 - (b*x^3 + a)^(1/3)*(b*c^2 - a*c*d)*(-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^
2))^(1/3)*x)/x^2) - 2*sqrt(3)*(3*b^2*c + 2*a*b*d)*(b^2)^(1/6)*arctan(1/3*(s
qrt(3)*(b^2)^(1/3)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(b^2)^(2/3))*(b^2)^(1/
6)/(b^2*x)) + 2*(b^2)^(2/3)*(3*b*c + 2*a*d)*log(-((b^2)^(2/3)*x - (b*x^3 +
a)^(1/3)*b)/x) - (b^2)^(2/3)*(3*b*c + 2*a*d)*log(((b^2)^(1/3)*b*x^2 + (b*x^
3 + a)^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)/x^2))/(b^3*d^2)
```

Sympy [F]

$$\int \frac{x^7}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^7}{(a+bx^3)^{\frac{2}{3}}(c+dx^3)}\,dx$$

```
[In] integrate(x**7/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**7/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^7}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^7}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^7/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^7}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^7}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^7/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^7}{(b\,x^3+a)^{2/3}\,(d\,x^3+c)}\,dx$$

```
[In] int(x^7/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
[Out] int(x^7/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

3.739 $\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 234

$$\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = -\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}d} + \frac{c^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d(bc-ad)^{2/3}} - \frac{c^{2/3}\log(c+dx^3)}{6d(bc-ad)^{2/3}} - \frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}d} + \frac{c^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d(bc-ad)^{2/3}}$$

```
[Out] -1/6*c^(2/3)*ln(d*x^3+c)/d/(-a*d+b*c)^(2/3)-1/2*ln(b^(1/3)*x-(b*x^3+a)^(1/3
))/b^(2/3)/d+1/2*c^(2/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d/(
-a*d+b*c)^(2/3)-1/3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(
2/3)/d*3^(1/2)+1/3*c^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^
3+a)^(1/3))*3^(1/2))/d/(-a*d+b*c)^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {494, 337, 503}

$$\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = -\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}d} + \frac{c^{2/3}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d(bc-ad)^{2/3}} - \frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}d} - \frac{c^{2/3}\log\left(c+dx^3\right)}{6d(bc-ad)^{2/3}} + \frac{c^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d(bc-ad)^{2/3}}$$

```
[In] Int[x^4/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/3)*d)
) + (c^(2/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)
))/Sqrt[3]])/(Sqrt[3]*d*(b*c - a*d)^(2/3)) - (c^(2/3)*Log[c + d*x^3])/(6*d*
(b*c - a*d)^(2/3)) - Log[b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*b^(2/3)*d) + (c^
(2/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*d*(b*c - a
*d)^(2/3))
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
```

```
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{x}{(a+bx^3)^{2/3}}\,dx}{d} - \frac{c\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{d}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}d} + \frac{c^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d(bc-ad)^{2/3}} - \frac{c^{2/3}\log\left(c+dx^3\right)}{6d(bc-ad)^{2/3}}$$
$$- \frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}d} + \frac{c^{2/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d(bc-ad)^{2/3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.47 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.81

$$\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{-\frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{b^{2/3}} - \frac{2\sqrt{-6-6i\sqrt{3}}c^{2/3}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{\cdots}}\right)}{(bc-ad)^{2/3}}\cdots}{\cdots}$$

```
[In] Integrate[x^4/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] ((-4*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])
/b^(2/3) - (2*Sqrt[-6 - (6*I)*Sqrt[3]]*c^(2/3)*ArcTan[(3*(b*c - a*d)^(1/3)*
x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)
)])/(b*c - a*d)^(2/3) - (4*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(2/3) +
 ((2*I)*(I + Sqrt[3])*c^(2/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c
^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(2/3) + (2*Log[b^(2/3)*x^2 + b^(1/3)
*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/b^(2/3) + ((1 - I*Sqrt[3])*c^(2/
3)*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)
*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a
*d)^(2/3))/(12*d)
```

Maple [A] (verified)

Time = 4.66 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.23

method	result
pseudoelliptic	$\frac{\left(-\frac{\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^2-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}x+\left(b\,x^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{2}-\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(b\,x^3+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)\sqrt{3}+\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}{}\right.\right.}{}$

[In] int(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] 1/3/b^(2/3)*((-1/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x))*b^(2/3)+1/2*(2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x))*((a*d-b*c)/c)^(2/3))/((a*d-b*c)/c)^(2/3)/d

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(187) = 374$.

Time = 0.28 (sec) , antiderivative size = 530, normalized size of antiderivative = 2.26

$$\int \frac{x^4}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{2\sqrt{3}b^2\left(\frac{c^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(bc-ad\right)\left(\frac{c^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{2}{3}}+\sqrt{3}cx}{3\,cx}\right)}{}$$

[In] integrate(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")

[Out] 1/6*(2*sqrt(3)*b^2*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*(b*c - a*d)*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(2/3) + sqrt(3)*c*x)/(c*x)) + 2*b^2*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*log(-((b*c - a*d)*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*x - (b*x^3 + a)^(1/3)*c)/x) - b^2*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*log(((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(2/3)*x^2 + (b*x^3 + a)^(2/3)*c^2 + (b*x^3 + a)^(1/3)*(b*c^2 - a*c*d)*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*x)/x^2) + 2*sqrt(3)*b*sqrt(-(-b^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-b^2)^(1/3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(2/3))*sqrt(-(-b^2)^(1/3))/(b^2*x)) - 2*(-b^2)^(2/3)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (-b^2)^(2/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2))/(b^2*d)

Sympy [F]

$$\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^4}{(a+bx^3)^{\frac{2}{3}}(c+dx^3)}\,dx$$

```
[In] integrate(x**4/(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(x**4/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^4}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(x^4/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^4}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^4/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^4}{(b\,x^3+a)^{2/3}\,(d\,x^3+c)}\,dx$$

```
[In] int(x^4/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
[Out] int(x^4/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

3.740 $\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 149

$$\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = -\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}} + \frac{\log(c+dx^3)}{6\sqrt[3]{c}(bc-ad)^{2/3}} - \frac{\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}(bc-ad)^{2/3}}$$

```
[Out] 1/6*ln(d*x^3+c)/c^(1/3)/(-a*d+b*c)^(2/3)-1/2*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-
(b*x^3+a)^(1/3))/c^(1/3)/(-a*d+b*c)^(2/3)-1/3*arctan(1/3*(1+2*(-a*d+b*c)^(1
/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(1/3)/(-a*d+b*c)^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {503}

$$\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = -\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}} + \frac{\log(c+dx^3)}{6\sqrt[3]{c}(bc-ad)^{2/3}} - \frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}(bc-ad)^{2/3}}$$

```
[In] Int[x/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -(ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]
/(Sqrt[3]*c^(1/3)*(b*c - a*d)^(2/3))) + Log[c + d*x^3]/(6*c^(1/3)*(b*c - a*
d)^(2/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(1/
3)*(b*c - a*d)^(2/3))
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rubi steps

$$\text{integral} = -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}}+\frac{\log\left(c+dx^3\right)}{6\sqrt[3]{c}(bc-ad)^{2/3}}-\frac{\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}(bc-ad)^{2/3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.89 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.71

$$\int \frac{x}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{2\sqrt{-6-6i\sqrt{3}}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)+\left(1-i\sqrt{3}\right)\left(21\right.}{}$$

```
[In] Integrate[x/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (2*Sqrt[-6 - (6*I)*Sqrt[3]]*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c -
a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (1 - I*Sqrt[3]
)*(2*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]
 - Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)
*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]))/(12*c^(
1/3)*(b*c - a*d)^(2/3))
```

Maple [A] (verified)

Time = 4.54 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.13

method	result
pseudoelliptic	$\frac{2\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)\sqrt{3}+\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)-2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x\cdots}{x}\right.}{6\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}c}$

```
[In] int(x/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
[Out] 1/6*(2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b
*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*
x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^
(1/3))/x))/((a*d-b*c)/c)^(2/3)/c
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate(x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \int \frac{x}{(a+bx^3)^{\frac{2}{3}}\,(c+dx^3)}\,dx$$

```
[In] integrate(x/(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(x/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx=\int \frac{x}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(x/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx=\int \frac{x}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx=\int \frac{x}{(b\,x^3+a)^{2/3}\,(d\,x^3+c)}\,dx$$

```
[In] int(x/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
[Out] int(x/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

3.741 $\int \frac{1}{x^2(a+bx^3)^{2/3}(c+dx^3)} dx$

Optimal result

Integrand size = 24, antiderivative size = 173

$$\int \frac{1}{x^2 (a+bx^3)^{2/3} (c+dx^3)} dx = -\frac{\sqrt[3]{a+bx^3}}{acx} + \frac{d \arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{4/3}(bc-ad)^{2/3}} - \frac{d\log(c+dx^3)}{6c^{4/3}(bc-ad)^{2/3}} + \frac{d\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{4/3}(bc-ad)^{2/3}}$$

```
[Out] -(b*x^3+a)^(1/3)/a/c/x-1/6*d*ln(d*x^3+c)/c^(4/3)/(-a*d+b*c)^(2/3)+1/2*d*ln(
(-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(4/3)/(-a*d+b*c)^(2/3)+1/3*d*
arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(4/3
)/(-a*d+b*c)^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {491, 12, 503}

$$\int \frac{1}{x^2\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\frac{d\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{4/3}(bc-ad)^{2/3}}-\frac{d\log\left(c+dx^3\right)}{6c^{4/3}(bc-ad)^{2/3}}+\frac{d\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{4/3}(bc-ad)^{2/3}}-\frac{\sqrt[3]{a+bx^3}}{acx}$$

```
[In] Int[1/(x^2*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(1/3)/(a*c*x)) + (d*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1
/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(4/3)*(b*c - a*d)^(2/3)) - (d*
Log[c + d*x^3])/(6*c^(4/3)*(b*c - a*d)^(2/3)) + (d*Log[((b*c - a*d)^(1/3)*x
)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(4/3)*(b*c - a*d)^(2/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt[3]{a+bx^3}}{acx} - \frac{\int \frac{adx}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{ac} \\
&= -\frac{\sqrt[3]{a+bx^3}}{acx} - \frac{d\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{c} \\
&= -\frac{\sqrt[3]{a+bx^3}}{acx} + \frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{4/3}(bc-ad)^{2/3}} \\
&\quad - \frac{d\log\left(c+dx^3\right)}{6c^{4/3}(bc-ad)^{2/3}} + \frac{d\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{4/3}(bc-ad)^{2/3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.19 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.78

$$
\int \frac{1}{x^2\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{-12\sqrt[3]{c}(bc-ad)^{2/3}\sqrt[3]{a+bx^3} - 2\sqrt{-6-6i\sqrt{3}}adx\arctan\left(\frac{3\sqrt[3]{\cdots}}{\sqrt{3}\sqrt[3]{bc-ad}x-\cdots}\right.\cdots}{\cdots}
$$

```
[In] Integrate[1/(x^2*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (-12*c^(1/3)*(b*c - a*d)^(2/3)*(a + b*x^3)^(1/3) - 2*Sqrt[-6 - (6*I)*Sqrt[3
]]*a*d*x*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I
 + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))] + (2*I)*(I + Sqrt[3])*a*d*x*Log[2*(
b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)] + a*(d - I*
Sqrt[3]*d)*x*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c -
a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]
)/(12*a*c^(4/3)*(b*c - a*d)^(2/3)*x)
```

Maple [A] (verified)

Time = 4.76 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.22

method	result
pseudoelliptic	$\frac{-\sqrt{3}\, \arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x}\right) a d x+\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right) a d x-\frac{\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}} x^{2}-\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right.}{x^{2}}\right.}{2}}{3\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}} a\,c^{2} x}$

```
[In] int(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/((a*d-b*c)/c)^(2/3)*(-3^(1/2)*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x
-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*a*d*x+ln(((((a*d-b*c)/c)^(1/3)*x+
(b*x^3+a)^(1/3))/x)*a*d*x-1/2*ln(((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/
3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a*d*x-3*(b*x^3+a)^(1/3)*c*((a*d-
b*c)/c)^(2/3))/a/c^2/x
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\text{Timed out}$$

```
[In] integrate(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^2\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\int \frac{1}{x^2\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

```
[In] integrate(1/x**2/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 \left(a + bx^3\right)^{2/3} \left(c + dx^3\right)} \, dx = \int \frac{1}{\left(bx^3 + a\right)^{\frac{2}{3}} \left(dx^3 + c\right) x^2} \, dx$$

```
[In] integrate(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 \left(a + bx^3\right)^{2/3} \left(c + dx^3\right)} \, dx = \int \frac{1}{\left(bx^3 + a\right)^{\frac{2}{3}} \left(dx^3 + c\right) x^2} \, dx$$

```
[In] integrate(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(a + bx^3\right)^{2/3} \left(c + dx^3\right)} \, dx = \int \frac{1}{x^2 \left(b\,x^3 + a\right)^{2/3} \left(d\,x^3 + c\right)} \, dx$$

```
[In] int(1/(x^2*(a + b*x^3)^(2/3)*(c + d*x^3)),x)
[Out] int(1/(x^2*(a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

3.742 $\int \frac{1}{x^5(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 215

$$\int \frac{1}{x^5(a+bx^3)^{2/3}(c+dx^3)}\,dx=-\frac{\sqrt[3]{a+bx^3}}{4acx^4}+\frac{(3bc+4ad)\sqrt[3]{a+bx^3}}{4a^2c^2x}-\frac{d^2\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{7/3}(bc-ad)^{2/3}}+\frac{d^2\log(c+dx^3)}{6c^{7/3}(bc-ad)^{2/3}}-\frac{d^2\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{7/3}(bc-ad)^{2/3}}$$

```
[Out] -1/4*(b*x^3+a)^(1/3)/a/c/x^4+1/4*(4*a*d+3*b*c)*(b*x^3+a)^(1/3)/a^2/c^2/x+1/
6*d^2*ln(d*x^3+c)/c^(7/3)/(-a*d+b*c)^(2/3)-1/2*d^2*ln((-a*d+b*c)^(1/3)*x/c^
(1/3)-(b*x^3+a)^(1/3))/c^(7/3)/(-a*d+b*c)^(2/3)-1/3*d^2*arctan(1/3*(1+2*(-a
*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(7/3)/(-a*d+b*c)^(2/3)*
3^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {491, 597, 12, 503}

$$\int \frac{1}{x^5 (a+bx^3)^{2/3} (c+dx^3)} dx = \frac{\sqrt[3]{a+bx^3}(4ad+3bc)}{4a^2c^2x} - \frac{d^2 \arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{7/3}(bc-ad)^{2/3}} + \frac{d^2 \log(c+dx^3)}{6c^{7/3}(bc-ad)^{2/3}} - \frac{d^2 \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{7/3}(bc-ad)^{2/3}} - \frac{\sqrt[3]{a+bx^3}}{4acx^4}$$

```
[In] Int[1/(x^5*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -1/4*(a + b*x^3)^(1/3)/(a*c*x^4) + ((3*b*c + 4*a*d)*(a + b*x^3)^(1/3))/(4*a
^2*c^2*x) - (d^2*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(
1/3)))/Sqrt[3]])/(Sqrt[3]*c^(7/3)*(b*c - a*d)^(2/3)) + (d^2*Log[c + d*x^3])
/(6*c^(7/3)*(b*c - a*d)^(2/3)) - (d^2*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (
a + b*x^3)^(1/3)])/(2*c^(7/3)*(b*c - a*d)^(2/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt[3]{a+bx^3}}{4acx^4}+\frac{\int \frac{-3bc-4ad-3bdx^3}{x^2(a+bx^3)^{2/3}(c+dx^3)}\,dx}{4ac}\\
&= -\frac{\sqrt[3]{a+bx^3}}{4acx^4}+\frac{(3bc+4ad)\sqrt[3]{a+bx^3}}{4a^2c^2x}-\frac{\int -\frac{4a^2d^2x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{4a^2c^2}\\
&= -\frac{\sqrt[3]{a+bx^3}}{4acx^4}+\frac{(3bc+4ad)\sqrt[3]{a+bx^3}}{4a^2c^2x}+\frac{d^2\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx}{c^2}\\
&= -\frac{\sqrt[3]{a+bx^3}}{4acx^4}+\frac{(3bc+4ad)\sqrt[3]{a+bx^3}}{4a^2c^2x}-\frac{d^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{7/3}(bc-ad)^{2/3}}\\
&\quad+\frac{d^2\log\left(c+dx^3\right)}{6c^{7/3}(bc-ad)^{2/3}}-\frac{d^2\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{7/3}(bc-ad)^{2/3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.81 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.58

$$
\int \frac{1}{x^5\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\frac{\frac{3\sqrt[3]{c}\sqrt[3]{a+bx^3}(-ac+3bcx^3+4adx^3)}{a^2x^4}+\frac{2\sqrt{-6-6i\sqrt{3}}d^2\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{\cdots}}\right.}{(bc-ad)^{2/3}}}{\cdots}
$$

```
[In] Integrate[1/(x^5*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] ((3*c^(1/3)*(a + b*x^3)^(1/3)*(-(a*c) + 3*b*c*x^3 + 4*a*d*x^3))/(a^2*x^4) +
 (2*Sqrt[-6 - (6*I)*Sqrt[3]]*d^2*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b
```

```
*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))])/(b*c - a*d
)^(2/3) + (2*(1 - I*Sqrt[3])*d^2*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3]
)*c^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(2/3) + (I*(I + Sqrt[3])*d^2*Log[
2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a +
 b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(2/
3))/(12*c^(7/3))
```

Maple [A] (verified)

Time = 4.88 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$-\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}} \left(\left(-4ad-3bc\right)x^{3}+ac\right) c \left(b\,x^{3}+a\right)^{\frac{1}{3}}+\frac{2a^{2}d^{2}x^{4}\left(-2\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)\sqrt{3}+2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+}{x}\right.\right.}{3}}{4\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}a^{2}c^{3}x^{4}}$

```
[In] int(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/((a*d-b*c)/c)^(2/3)*(((a*d-b*c)/c)^(2/3)*((-4*a*d-3*b*c)*x^3+a*c)*c*(b
*x^3+a)^(1/3)+2/3*a^2*d^2*x^4*(-2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x
-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+2*ln((((a*d-b*c)/c)^(1/3
)*x+(b*x^3+a)^(1/3))/x)-ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*
x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)))/a^2/c^3/x^4
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^5 \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^5 (a+bx^3)^{2/3} (c+dx^3)} dx = \int \frac{1}{x^5 (a+bx^3)^{\frac{2}{3}} (c+dx^3)} dx$$

```
[In] integrate(1/x**5/(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(1/(x**5*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^5 (a+bx^3)^{2/3} (c+dx^3)} dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}} (dx^3+c)x^5} dx$$

```
[In] integrate(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5 (a+bx^3)^{2/3} (c+dx^3)} dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}} (dx^3+c)x^5} dx$$

```
[In] integrate(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (a+bx^3)^{2/3} (c+dx^3)} dx = \int \frac{1}{x^5 (b\,x^3+a)^{2/3} (d\,x^3+c)} dx$$

```
[In] int(1/(x^5*(a + b*x^3)^(2/3)*(c + d*x^3)),x)
[Out] int(1/(x^5*(a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

3.743 $\int \frac{x^6}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^6}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{x^7\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{7}{3},\frac{2}{3},1,\frac{10}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c(a+bx^3)^{2/3}}$$

```
[Out] 1/7*x^7*(1+b*x^3/a)^(2/3)*AppellF1(7/3,2/3,1,10/3,-b*x^3/a,-d*x^3/c)/c/(b*x
^3+a)^(2/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{x^6}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{x^7\left(\frac{bx^3}{a}+1\right)^{2/3}\operatorname{AppellF1}\left(\frac{7}{3},\frac{2}{3},1,\frac{10}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c(a+bx^3)^{2/3}}$$

```
[In] Int[x^6/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (x^7*(1 + (b*x^3)/a)^(2/3)*AppellF1[7/3, 2/3, 1, 10/3, -((b*x^3)/a), -((d*x
^3)/c)])/(7*c*(a + b*x^3)^(2/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{x^6}{\left(1+\frac{bx^3}{a}\right)^{2/3}(c+dx^3)}\,dx}{(a+bx^3)^{2/3}}$$

$$= \frac{x^7\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{7}{3};\frac{2}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\,(a+bx^3)^{2/3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 249 vs. $2(64) = 128$.

Time = 9.49 (sec) , antiderivative size = 249, normalized size of antiderivative = 3.89

$$\int \frac{x^6}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \frac{x\left(-\frac{(2bc+ad)x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{bc}+4\left(\frac{a}{b}+x^3+\frac{}{b(c+dx^3)\left(-4ac\,\text{App}\right.}\right.\right.}{8d}$$

```
[In] Integrate[x^6/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (x*(-(((2*b*c + a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -
((b*x^3)/a), -((d*x^3)/c)])/(b*c)) + 4*(a/b + x^3 + (4*a^2*c^2*AppellF1[1/3
, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(b*(c + d*x^3)*(-4*a*c*AppellF1
[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2
/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3,
-((b*x^3)/a), -((d*x^3)/c)]))))))/(8*d*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{x^6}{\left(b\,x^3+a\right)^{\frac{2}{3}}\left(d\,x^3+c\right)}dx$$

```
[In] int(x^6/(b*x^3+a)^(2/3)/(d*x^3+c),x)
[Out] int(x^6/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate(x^6/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^6}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \int \frac{x^6}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

```
[In] integrate(x**6/(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(x**6/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^6}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \int \frac{x^6}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

```
[In] integrate(x^6/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(x^6/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^6}{(a+bx^3)^{2/3}(c+dx^3)}\,dx=\int \frac{x^6}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(x^6/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^6/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^3)^{2/3}(c+dx^3)}\,dx=\int \frac{x^6}{(b\,x^3+a)^{2/3}\,(d\,x^3+c)}\,dx$$

```
[In] int(x^6/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
[Out] int(x^6/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

3.744 $\int \frac{x^3}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^3}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\,(a+bx^3)^{2/3}}$$

```
[Out] 1/4*x^4*(1+b*x^3/a)^(2/3)*AppellF1(4/3,2/3,1,7/3,-b*x^3/a,-d*x^3/c)/c/(b*x^
3+a)^(2/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{x^3}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \frac{x^4\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\,(a+bx^3)^{2/3}}$$

```
[In] Int[x^3/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^
3)/c)])/(4*c*(a + b*x^3)^(2/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{x^3}{\left(1+\frac{bx^3}{a}\right)^{2/3}(c+dx^3)}\,dx}{(a+bx^3)^{2/3}}$$

$$= \frac{x^4\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\,(a+bx^3)^{2/3}}$$

Mathematica [A] (verified)

Time = 8.96 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \frac{x^4\left(\frac{a+bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\,(a+bx^3)^{2/3}}$$

```
[In] Integrate[x^3/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
[Out] (x^4*((a + b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^
3)/c)])/(4*c*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{x^3}{(b\,x^3+a)^{\frac{2}{3}}\,(d\,x^3+c)}dx$$

```
[In] int(x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x)
[Out] int(x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate(x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^3}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^3}{(a+bx^3)^{\frac{2}{3}}(c+dx^3)}\,dx$$

```
[In] integrate(x**3/(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(x**3/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^3}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^3}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(x^3/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^3}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{x^3}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^3/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx=\int \frac{x^3}{\left(b\,x^3+a\right)^{2/3}\,\left(d\,x^3+c\right)}\,dx$$

```
[In] int(x^3/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
[Out] int(x^3/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

3.745 $\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)} dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)} dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} \text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c(a+bx^3)^{2/3}}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(b*x^3+a)^(
2/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {441, 440}

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)} dx = \frac{x\left(\frac{bx^3}{a}+1\right)^{2/3} \text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c(a+bx^3)^{2/3}}$$

```
[In] Int[1/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(c*(a + b*x^3)^(2/3))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int\frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}(c+dx^3)}\,dx}{\left(a+bx^3\right)^{2/3}}$$
$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(a+bx^3\right)^{2/3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 161 vs. 2(59) = 118.

Time = 0.06 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.73

$$\int\frac{1}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx =$$
$$-\frac{4acx\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)\left(-4ac\,\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+x^3\left(3ad\,\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\right.\right.}$$

```
[In] Integrate[1/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (-4*a*c*x*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/((a + b*x
^3)^(2/3)*(c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d
*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c
)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Maple [F]

$$\int\frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}}\left(d\,x^3+c\right)}dx$$

```
[In] int(1/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(1/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{1}{(a+bx^3)^{\frac{2}{3}}(c+dx^3)}\,dx$$

```
[In] integrate(1/(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(1/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{2/3}\,\left(d\,x^3+c\right)}\,dx$$

```
[In] int(1/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
[Out] int(1/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

3.746 $\int \frac{1}{x^3(a+bx^3)^{2/3}(c+dx^3)} dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{x^3 (a+bx^3)^{2/3} (c+dx^3)} dx = -\frac{\left(1+\frac{bx^3}{a}\right)^{2/3} \text{AppellF1}\left(-\frac{2}{3},\frac{2}{3},1,\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2 (a+bx^3)^{2/3}}$$

```
[Out] -1/2*(1+b*x^3/a)^(2/3)*AppellF1(-2/3,2/3,1,1/3,-b*x^3/a,-d*x^3/c)/c/x^2/(b*
x^3+a)^(2/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{1}{x^3 (a+bx^3)^{2/3} (c+dx^3)} dx = -\frac{\left(\frac{bx^3}{a}+1\right)^{2/3} \text{AppellF1}\left(-\frac{2}{3},\frac{2}{3},1,\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2 (a+bx^3)^{2/3}}$$

```
[In] Int[1/(x^3*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -1/2*((1 + (b*x^3)/a)^(2/3)*AppellF1[-2/3, 2/3, 1, 1/3, -((b*x^3)/a), -((d*
x^3)/c)])/(c*x^2*(a + b*x^3)^(2/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{x^3\left(1+\frac{bx^3}{a}\right)^{2/3}(c+dx^3)}\,dx}{(a+bx^3)^{2/3}}$$

$$= -\frac{\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(-\frac{2}{3};\frac{2}{3},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\,(a+bx^3)^{2/3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 338 vs. $2(64) = 128$.

Time = 10.28 (sec) , antiderivative size = 338, normalized size of antiderivative = 5.28

$$\int \frac{1}{x^3\,(a+bx^3)^{2/3}\,(c+dx^3)}\,dx = \frac{-bdx^6\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{4c\left(-4ac\left(ac+2bcx^3+3adx^3\right.\right.}{(c+dx^3}}{}$$

```
[In] Integrate[1/(x^3*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (-(b*d*x^6*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -
((d*x^3)/c)]) + (4*c*(-4*a*c*(a*c + 2*b*c*x^3 + 3*a*d*x^3 + b*d*x^6)*Appell
F1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(a + b*x^3)*(c + d*x
^3)*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*A
ppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)*(4*a*
c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - x^3*(3*a*d*Appel
lF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3
, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(8*a*c^2*x^2*(a + b*x^3)^(2/3))
```

Maple [F]

$$\int \frac{1}{x^{3} \left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(d\,x^{3}+c\right)}d x$$

```
[In] int(1/x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x)
[Out] int(1/x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^3)^{2/3} (c+dx^3)} \, dx = \text{Timed out}$$

```
[In] integrate(1/x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^3 (a+bx^3)^{2/3} (c+dx^3)} \, dx = \int \frac{1}{x^{3} \left(a + b x^{3}\right)^{\frac{2}{3}} \left(c + d x^{3}\right)}\, dx$$

```
[In] integrate(1/x**3/(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(1/(x**3*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (a+bx^3)^{2/3} (c+dx^3)} \, dx = \int { \frac{1}{{\left(b x^{3} + a\right)}^{\frac{2}{3}} {\left(d x^{3} + c\right)} x^{3}} \,d x }$$

```
[In] integrate(1/x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a+bx^3)^{2/3} (c+dx^3)} \, dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}} (dx^3+c)x^3} \, dx$$

```
[In] integrate(1/x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^3)^{2/3} (c+dx^3)} \, dx = \int \frac{1}{x^3 (b\,x^3+a)^{2/3} \,(d\,x^3+c)} \, dx$$

```
[In] int(1/(x^3*(a + b*x^3)^(2/3)*(c + d*x^3)),x)
[Out] int(1/(x^3*(a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

3.747 $\int \frac{x^{14}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 347

$$\begin{aligned}\int \frac{x^{14}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx &= -\frac{a^4}{b^4(bc-ad)\sqrt[3]{a+bx^3}}+\frac{a^2(a+bx^3)^{2/3}}{2b^4d} \\ &+\frac{a(bc+ad)(a+bx^3)^{2/3}}{2b^4d^2}+\frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{2/3}}{2b^4d^3}-\frac{2a(a+bx^3)^{5/3}}{5b^4d} \\ &-\frac{(bc+ad)(a+bx^3)^{5/3}}{5b^4d^2}+\frac{(a+bx^3)^{8/3}}{8b^4d}+\frac{c^4\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{11/3}(bc-ad)^{4/3}} \\ &-\frac{c^4\log(c+dx^3)}{6d^{11/3}(bc-ad)^{4/3}}+\frac{c^4\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{11/3}(bc-ad)^{4/3}}\end{aligned}$$

```
[Out] -a^4/b^4/(-a*d+b*c)/(b*x^3+a)^(1/3)+1/2*a^2*(b*x^3+a)^(2/3)/b^4/d+1/2*a*(a*
d+b*c)*(b*x^3+a)^(2/3)/b^4/d^2+1/2*(a^2*d^2+a*b*c*d+b^2*c^2)*(b*x^3+a)^(2/3
)/b^4/d^3-2/5*a*(b*x^3+a)^(5/3)/b^4/d-1/5*(a*d+b*c)*(b*x^3+a)^(5/3)/b^4/d^2
+1/8*(b*x^3+a)^(8/3)/b^4/d-1/6*c^4*ln(d*x^3+c)/d^(11/3)/(-a*d+b*c)^(4/3)+1/
2*c^4*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(11/3)/(-a*d+b*c)^(4/3
)+1/3*c^4*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2)
)/d^(11/3)/(-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {457, 89, 45, 58, 631, 210, 31}

$$\int \frac{x^{14}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = -\frac{a^4}{b^4\sqrt[3]{a+bx^3}(bc-ad)}+\frac{a^2(a+bx^3)^{2/3}}{2b^4d}+\frac{(a+bx^3)^{2/3}(a^2d^2+abcd+b^2c^2)}{2b^4d^3}+\frac{c^4\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{11/3}(bc-ad)^{4/3}}+\frac{a(a+bx^3)^{2/3}(ad+bc)}{2b^4d^2}-\frac{(a+bx^3)^{5/3}(ad+bc)}{5b^4d^2}-\frac{2a(a+bx^3)^{5/3}}{5b^4d}+\frac{(a+bx^3)^{8/3}}{8b^4d}-\frac{c^4\log(c+dx^3)}{6d^{11/3}(bc-ad)^{4/3}}+\frac{c^4\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{11/3}(bc-ad)^{4/3}}$$

```
[In] Int[x^14/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(a^4/(b^4*(b*c - a*d)*(a + b*x^3)^(1/3))) + (a^2*(a + b*x^3)^(2/3))/(2*b^4
*d) + (a*(b*c + a*d)*(a + b*x^3)^(2/3))/(2*b^4*d^2) + ((b^2*c^2 + a*b*c*d +
 a^2*d^2)*(a + b*x^3)^(2/3))/(2*b^4*d^3) - (2*a*(a + b*x^3)^(5/3))/(5*b^4*d
) - ((b*c + a*d)*(a + b*x^3)^(5/3))/(5*b^4*d^2) + (a + b*x^3)^(8/3)/(8*b^4*
d) + (c^4*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt
[3]])/(Sqrt[3]*d^(11/3)*(b*c - a*d)^(4/3)) - (c^4*Log[c + d*x^3])/(6*d^(11/
3)*(b*c - a*d)^(4/3)) + (c^4*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1
/3)])/(2*d^(11/3)*(b*c - a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
```

```
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{(a+bx)^{4/3}(c+dx)}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a^4}{b^3(bc-ad)(a+bx)^{4/3}}+\frac{b^2c^2+abcd+a^2d^2}{b^3d^3\sqrt[3]{a+bx}}-\frac{(bc+ad)x}{b^2d^2\sqrt[3]{a+bx}}\right.\right.\\
&\qquad\left.\left.+\frac{x^2}{bd\sqrt[3]{a+bx}}+\frac{c^4}{d^3(-bc+ad)\sqrt[3]{a+bx}(c+dx)}\right)dx,x,x^3\right)\\
&= -\frac{a^4}{b^4(bc-ad)\sqrt[3]{a+bx^3}}+\frac{(b^2c^2+abcd+a^2d^2)\left(a+bx^3\right)^{2/3}}{2b^4d^3}+\frac{\text{Subst}\left(\int \frac{x^2}{\sqrt[3]{a+bx}}\,dx,x,x^3\right)}{3bd}\\
&\quad-\frac{c^4\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx,x,x^3\right)}{3d^3(bc-ad)}-\frac{(bc+ad)\text{Subst}\left(\int \frac{x}{\sqrt[3]{a+bx}}\,dx,x,x^3\right)}{3b^2d^2}
\end{aligned}$$

$$= -\frac{a^4}{b^4(bc-ad)\sqrt[3]{a+bx^3}} + \frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{2/3}}{2b^4d^3} - \frac{c^4\log(c+dx^3)}{6d^{11/3}(bc-ad)^{4/3}}$$
$$+ \frac{\text{Subst}\left(\int \left(\frac{a^2}{b^2\sqrt[3]{a+bx}} - \frac{2a(a+bx)^{2/3}}{b^2} + \frac{(a+bx)^{5/3}}{b^2}\right) dx, x, x^3\right)}{3bd}$$
$$+ \frac{c^4\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x} dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{11/3}(bc-ad)^{4/3}}$$
$$- \frac{c^4\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2} dx, x, \sqrt[3]{a+bx^3}\right)}{2d^4(bc-ad)}$$
$$- \frac{(bc+ad)\text{Subst}\left(\int \left(-\frac{a}{b\sqrt[3]{a+bx}} + \frac{(a+bx)^{2/3}}{b}\right) dx, x, x^3\right)}{3b^2d^2}$$

$$= -\frac{a^4}{b^4(bc-ad)\sqrt[3]{a+bx^3}} + \frac{a^2(a+bx^3)^{2/3}}{2b^4d} + \frac{a(bc+ad)(a+bx^3)^{2/3}}{2b^4d^2}$$
$$+ \frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{2/3}}{2b^4d^3} - \frac{2a(a+bx^3)^{5/3}}{5b^4d} - \frac{(bc+ad)(a+bx^3)^{5/3}}{5b^4d^2}$$
$$+ \frac{(a+bx^3)^{8/3}}{8b^4d} - \frac{c^4\log(c+dx^3)}{6d^{11/3}(bc-ad)^{4/3}} + \frac{c^4\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{11/3}(bc-ad)^{4/3}}$$
$$- \frac{c^4\text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1 - \frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{d^{11/3}(bc-ad)^{4/3}}$$

$$= -\frac{a^4}{b^4(bc-ad)\sqrt[3]{a+bx^3}} + \frac{a^2(a+bx^3)^{2/3}}{2b^4d} + \frac{a(bc+ad)(a+bx^3)^{2/3}}{2b^4d^2}$$
$$+ \frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{2/3}}{2b^4d^3} - \frac{2a(a+bx^3)^{5/3}}{5b^4d}$$
$$- \frac{(bc+ad)(a+bx^3)^{5/3}}{5b^4d^2} + \frac{(a+bx^3)^{8/3}}{8b^4d} + \frac{c^4\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{11/3}(bc-ad)^{4/3}}$$
$$- \frac{c^4\log(c+dx^3)}{6d^{11/3}(bc-ad)^{4/3}} + \frac{c^4\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{11/3}(bc-ad)^{4/3}}$$

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.01

$$\int \frac{x^{14}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{\frac{3d^{2/3}(-81a^4d^3+9a^3bd^2(c-3dx^3)+3a^2b^2d(4c^2+cdx^3+3d^2x^6)+b^4cx^3(20c^2-8cdx^3+5d^2x^6)+ab^3(20c^3+}{b^4(bc-ad)\sqrt[3]{a+bx^3}}}{}$$

```
[In] Integrate[x^14/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] ((3*d^(2/3)*(-81*a^4*d^3 + 9*a^3*b*d^2*(c - 3*d*x^3) + 3*a^2*b^2*d*(4*c^2 +
 c*d*x^3 + 3*d^2*x^6) + b^4*c*x^3*(20*c^2 - 8*c*d*x^3 + 5*d^2*x^6) + a*b^3*
(20*c^3 + 4*c^2*d*x^3 - c*d^2*x^6 - 5*d^3*x^9)))/(b^4*(b*c - a*d)*(a + b*x^
3)^(1/3)) + (40*Sqrt[3]*c^4*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c
- a*d)^(1/3))/Sqrt[3]])/(b*c - a*d)^(4/3) + (40*c^4*Log[(b*c - a*d)^(1/3) +
 d^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(4/3) - (20*c^4*Log[(b*c - a*d)^(2
/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/
3)])/(b*c - a*d)^(4/3))/(120*d^(11/3))
```

Maple [A] (verified)

Time = 4.80 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$-\frac{-\frac{243\left(a\left(\frac{5}{81}b^3x^9-\frac{1}{9}ab^2x^6+\frac{1}{3}a^2bx^3+a^3\right)d^3-\frac{b\left(\frac{5}{9}b^2x^6-\frac{2}{3}abx^3+a^2\right)\left(bx^3+a\right)cd^2}{9}-\frac{4b^2\left(-\frac{2bx^3}{3}+a\right)\left(bx^3+a\right)c^2d}{27}-\frac{20b^3c^3\left(bx^3+\right.}{81}}{20}}{}$

```
[In] int(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/(1/d*(a*d-b*c))^(1/3)*(-243/20*(a*(5/81*b^3*x^9-1/9*a*b^2*x^6+1/3*a^2*
b*x^3+a^3)*d^3-1/9*b*(5/9*b^2*x^6-2/3*a*b*x^3+a^2)*(b*x^3+a)*c*d^2-4/27*b^2
*(-2/3*b*x^3+a)*(b*x^3+a)*c^2*d-20/81*b^3*c^3*(b*x^3+a))*d*(1/d*(a*d-b*c))^
(1/3)+b^4*c^4*(b*x^3+a)^(1/3)*(-2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/
d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+(1/d*
(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a)^(1/3
)-(1/d*(a*d-b*c))^(1/3))))/(b*x^3+a)^(1/3)/d^4/(a*d-b*c)/b^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. 2(291) = 582.

Time = 0.36 (sec) , antiderivative size = 1300, normalized size of antiderivative = 3.75

$$\int \frac{x^{14}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/120*(60*sqrt(1/3)*(a*b^5*c^5*d - a^2*b^4*c^4*d^2 + (b^6*c^5*d - a*b^5*c
^4*d^2)*x^3)*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 -
b*c*d + 3*a*d^2 + 3*sqrt(1/3)*(2*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(2/3)
 + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) + (-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)
)*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d)) - 3*(-b*c*d^2 + a*d^3)^(2/3)*(
b*x^3 + a)^(1/3))/(d*x^3 + c)) + 20*(b^5*c^4*x^3 + a*b^4*c^4)*(-b*c*d^2 + a
*d^3)^(2/3)*log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a
)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 40*(b^5*c^4*x^3 + a*b^4*c^4)*(-b*c*
d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)) - 3*
(20*a*b^4*c^4*d^2 - 8*a^2*b^3*c^3*d^3 - 3*a^3*b^2*c^2*d^4 - 90*a^4*b*c*d^5
+ 81*a^5*d^6 + 5*(b^5*c^2*d^4 - 2*a*b^4*c*d^5 + a^2*b^3*d^6)*x^9 - (8*b^5*c
^3*d^3 - 7*a*b^4*c^2*d^4 - 10*a^2*b^3*c*d^5 + 9*a^3*b^2*d^6)*x^6 + (20*b^5*
c^4*d^2 - 16*a*b^4*c^3*d^3 - a^2*b^3*c^2*d^4 - 30*a^3*b^2*c*d^5 + 27*a^4*b*
d^6)*x^3)*(b*x^3 + a)^(2/3))/(a*b^6*c^2*d^5 - 2*a^2*b^5*c*d^6 + a^3*b^4*d^7
 + (b^7*c^2*d^5 - 2*a*b^6*c*d^6 + a^2*b^5*d^7)*x^3), -1/120*(120*sqrt(1/3)*
(a*b^5*c^5*d - a^2*b^4*c^4*d^2 + (b^6*c^5*d - a*b^5*c^4*d^2)*x^3)*sqrt(-(-b
*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d
+ (-b*c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))/d)
+ 20*(b^5*c^4*x^3 + a*b^4*c^4)*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/
3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^
(2/3)) - 40*(b^5*c^4*x^3 + a*b^4*c^4)*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 +
 a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)) - 3*(20*a*b^4*c^4*d^2 - 8*a^2*b^3*c
^3*d^3 - 3*a^3*b^2*c^2*d^4 - 90*a^4*b*c*d^5 + 81*a^5*d^6 + 5*(b^5*c^2*d^4 -
 2*a*b^4*c*d^5 + a^2*b^3*d^6)*x^9 - (8*b^5*c^3*d^3 - 7*a*b^4*c^2*d^4 - 10*a
^2*b^3*c*d^5 + 9*a^3*b^2*d^6)*x^6 + (20*b^5*c^4*d^2 - 16*a*b^4*c^3*d^3 - a^
2*b^3*c^2*d^4 - 30*a^3*b^2*c*d^5 + 27*a^4*b*d^6)*x^3)*(b*x^3 + a)^(2/3))/(a
*b^6*c^2*d^5 - 2*a^2*b^5*c*d^6 + a^3*b^4*d^7 + (b^7*c^2*d^5 - 2*a*b^6*c*d^6
 + a^2*b^5*d^7)*x^3)]
```

Sympy [F]

$$\int \frac{x^{14}}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx = \int \frac{x^{14}}{(a+bx^3)^{\frac{4}{3}}\,(c+dx^3)}\,dx$$

```
[In] integrate(x**14/(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(x**14/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{14}}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.24

$$\int \frac{x^{14}}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx = \frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^4\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^2d^5-2\sqrt{3}abcd^6+\sqrt{3}a^2d^7} - \frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^4\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(b^2c^2d^5-2\,abcd^6+a^2d^7)} + \frac{c^4\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^2c^2d^3-2\,abcd^4+a^2d^5)} - \frac{a^4}{(b^5c-ab^4d)(bx^3+a)^{\frac{1}{3}}} + \frac{20\,(bx^3+a)^{\frac{2}{3}}b^{30}c^2d^5-8\,(bx^3+a)^{\frac{5}{3}}b^{29}cd^6+40\,(bx^3+a)^{\frac{2}{3}}ab^{29}cd^6+5\,(bx^3+a)^{\frac{8}{3}}b^{28}d^7-24\,(bx^3+a)^{\frac{5}{3}}ab^{28}}{40\,b^{32}d^8}$$

```
[In] integrate(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] (-b*c*d^2 + a*d^3)^(2/3)*c^4*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b
*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b^2*c^2*d^5 - 2*sqrt(3
)*a*b*c*d^6 + sqrt(3)*a^2*d^7) - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^4*log((b*x^
3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^
(2/3))/(b^2*c^2*d^5 - 2*a*b*c*d^6 + a^2*d^7) + 1/3*c^4*(-(b*c - a*d)/d)^(2/
3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^2*c^2*d^3 - 2*a*
b*c*d^4 + a^2*d^5) - a^4/((b^5*c - a*b^4*d)*(b*x^3 + a)^(1/3)) + 1/40*(20*(
b*x^3 + a)^(2/3)*b^30*c^2*d^5 - 8*(b*x^3 + a)^(5/3)*b^29*c*d^6 + 40*(b*x^3
+ a)^(2/3)*a*b^29*c*d^6 + 5*(b*x^3 + a)^(8/3)*b^28*d^7 - 24*(b*x^3 + a)^(5/
3)*a*b^28*d^7 + 60*(b*x^3 + a)^(2/3)*a^2*b^28*d^7)/(b^32*d^8)
```

Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.63

$$\int \frac{x^{14}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \left(\frac{3\,a^2}{b^4\,d}+\frac{\left(\frac{4\,a}{b^4\,d}+\frac{b^5\,c-a\,b^4\,d}{b^8\,d^2}\right)\,\left(b^5\,c-a\,b^4\,d\right)}{2\,b^4\,d}\right)\,{\left(b\,x^3+a\right)}^{2/3} - \left(\frac{4\,a}{5\,b^4\,d}+\frac{b^5\,c-a\,b^4\,d}{5\,b^8\,d^2}\right)\,{\left(b\,x^3+a\right)}^{5/3}+\frac{{\left(b\,x^3+a\right)}^{8/3}}{8\,b^4\,d}+\frac{a^4}{b^4\,{\left(b\,x^3+a\right)}^{1/3}\,\left(a\,d-b\,c\right)} + \frac{c^4\,\ln\left({\left(b\,x^3+a\right)}^{1/3}\,\left(a\,c^8\,d^5-b\,c^9\,d^4\right)-\frac{c^8\,\left(9\,a^4\,d^{15}-36\,a^3\,b\,c\,d^{14}+54\,a^2\,b^2\,c^2\,d^{13}-36\,a\,b^3\,c^3\,d^{12}+9\,b^4\,c^4\,d^{11}\right)}{9\,d^{22/3}\,{\left(a\,d-b\,c\right)}^{8/3}}\right)}{3\,d^{11/3}\,{\left(a\,d-b\,c\right)}^{4/3}} - \frac{\ln\left({\left(b\,x^3+a\right)}^{1/3}\,\left(a\,c^8\,d^5-b\,c^9\,d^4\right)-\frac{{\left(c^4+\sqrt{3}\,c^4\,1{}\mathrm{i}\right)}^2\,\left(9\,a^4\,d^{15}-36\,a^3\,b\,c\,d^{14}+54\,a^2\,b^2\,c^2\,d^{13}-36\,a\,b^3\,c^3\,d^{12}+9\,b^4\,c^4\,d^{11}\right)}{36\,d^{22/3}\,{\left(a\,d-b\,c\right)}^{8/3}}\right)\,\left(c^4+\sqrt{\ }\right.}{6\,d^{11/3}\,{\left(a\,d-b\,c\right)}^{4/3}} + \frac{c^4\,\ln\left({\left(b\,x^3+a\right)}^{1/3}\,\left(a\,c^8\,d^5-b\,c^9\,d^4\right)-\frac{c^8\,{\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}^2\,\left(9\,a^4\,d^{15}-36\,a^3\,b\,c\,d^{14}+54\,a^2\,b^2\,c^2\,d^{13}-36\,a\,b^3\,c^3\,d^{12}+9\,b^4\,c^4\,d^{11}\right)}{d^{22/3}\,{\left(a\,d-b\,c\right)}^{8/3}}\right)\,\left(-\right.}{d^{11/3}\,{\left(a\,d-b\,c\right)}^{4/3}}$$

```
[In] int(x^14/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] ((3*a^2)/(b^4*d) + (((4*a)/(b^4*d) + (b^5*c - a*b^4*d)/(b^8*d^2))*(b^5*c -
a*b^4*d))/(2*b^4*d))*(a + b*x^3)^(2/3) - ((4*a)/(5*b^4*d) + (b^5*c - a*b^4*
d)/(5*b^8*d^2))*(a + b*x^3)^(5/3) + (a + b*x^3)^(8/3)/(8*b^4*d) + a^4/(b^4*
(a + b*x^3)^(1/3)*(a*d - b*c)) + (c^4*log((a + b*x^3)^(1/3)*(a*c^8*d^5 - b*
c^9*d^4) - (c^8*(9*a^4*d^15 + 9*b^4*c^4*d^11 - 36*a*b^3*c^3*d^12 + 54*a^2*b
^2*c^2*d^13 - 36*a^3*b*c*d^14))/(9*d^(22/3)*(a*d - b*c)^(8/3))))/(3*d^(11/3
)*(a*d - b*c)^(4/3)) - (log((a + b*x^3)^(1/3)*(a*c^8*d^5 - b*c^9*d^4) - ((3
^(1/2)*c^4*1i + c^4)^2*(9*a^4*d^15 + 9*b^4*c^4*d^11 - 36*a*b^3*c^3*d^12 + 5
4*a^2*b^2*c^2*d^13 - 36*a^3*b*c*d^14))/(36*d^(22/3)*(a*d - b*c)^(8/3)))*(3^
(1/2)*c^4*1i + c^4))/(6*d^(11/3)*(a*d - b*c)^(4/3)) + (c^4*log((a + b*x^3)^
```

```
(1/3)*(a*c^8*d^5 - b*c^9*d^4) - (c^8*((3^(1/2)*1i)/6 - 1/6)^2*(9*a^4*d^15 +
 9*b^4*c^4*d^11 - 36*a*b^3*c^3*d^12 + 54*a^2*b^2*c^2*d^13 - 36*a^3*b*c*d^14
))/(d^(22/3)*(a*d - b*c)^(8/3)))*((3^(1/2)*1i)/6 - 1/6))/(d^(11/3)*(a*d - b
*c)^(4/3))
```

3.748 $\int \frac{x^{11}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 253

$$\int \frac{x^{11}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{a^3}{b^3(bc-ad)\sqrt[3]{a+bx^3}} - \frac{a(a+bx^3)^{2/3}}{2b^3d} - \frac{(bc+ad)(a+bx^3)^{2/3}}{2b^3d^2} + \frac{(a+bx^3)^{5/3}}{5b^3d} - \frac{c^3\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{8/3}(bc-ad)^{4/3}} + \frac{c^3\log(c+dx^3)}{6d^{8/3}(bc-ad)^{4/3}} - \frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}(bc-ad)^{4/3}}$$

```
[Out] a^3/b^3/(-a*d+b*c)/(b*x^3+a)^(1/3)-1/2*a*(b*x^3+a)^(2/3)/b^3/d-1/2*(a*d+b*c
)*(b*x^3+a)^(2/3)/b^3/d^2+1/5*(b*x^3+a)^(5/3)/b^3/d+1/6*c^3*ln(d*x^3+c)/d^(
8/3)/(-a*d+b*c)^(4/3)-1/2*c^3*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/
d^(8/3)/(-a*d+b*c)^(4/3)-1/3*c^3*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-
a*d+b*c)^(1/3))*3^(1/2))/d^(8/3)/(-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used

$= \{457, 89, 45, 58, 631, 210, 31\}$

$$\int \frac{x^{11}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{a^3}{b^3\sqrt[3]{a+bx^3}(bc-ad)} - \frac{c^3\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{8/3}(bc-ad)^{4/3}} - \frac{(a+bx^3)^{2/3}(ad+bc)}{2b^3d^2} - \frac{a(a+bx^3)^{2/3}}{2b^3d} + \frac{(a+bx^3)^{5/3}}{5b^3d} + \frac{c^3\log(c+dx^3)}{6d^{8/3}(bc-ad)^{4/3}} - \frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}(bc-ad)^{4/3}}$$

```
[In] Int[x^11/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] a^3/(b^3*(b*c - a*d)*(a + b*x^3)^(1/3)) - (a*(a + b*x^3)^(2/3))/(2*b^3*d) -
 ((b*c + a*d)*(a + b*x^3)^(2/3))/(2*b^3*d^2) + (a + b*x^3)^(5/3)/(5*b^3*d)
- (c^3*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]
])/(Sqrt[3]*d^(8/3)*(b*c - a*d)^(4/3)) + (c^3*Log[c + d*x^3])/(6*d^(8/3)*(b
*c - a*d)^(4/3)) - (c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])
/(2*d^(8/3)*(b*c - a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
```

```
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{a^3}{b^2(bc-ad)(a+bx)^{4/3}} + \frac{-bc-ad}{b^2d^2\sqrt[3]{a+bx}} + \frac{x}{bd\sqrt[3]{a+bx}} - \frac{c^3}{d^2(-bc+ad)\sqrt[3]{a+bx}(c+dx)}\right) dx, x, x^3\right)\\
&= \frac{a^3}{b^3(bc-ad)\sqrt[3]{a+bx^3}} - \frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^3d^2} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt[3]{a+bx}}\,dx, x, x^3\right)}{3bd} + \frac{c^3\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)}{3d^2(bc-ad)}
\end{aligned}$$

$$= \frac{a^3}{b^3(bc-ad)\sqrt[3]{a+bx^3}} - \frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^3d^2} + \frac{c^3\log\left(c+dx^3\right)}{6d^{8/3}(bc-ad)^{4/3}} + \frac{\text{Subst}\left(\int\left(-\frac{a}{b\sqrt[3]{a+bx}}+\frac{(a+bx)^{2/3}}{b}\right)dx,x,x^3\right)}{3bd} - \frac{c^3\text{Subst}\left(\int\frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}dx,x,\sqrt[3]{a+bx^3}\right)}{2d^{8/3}(bc-ad)^{4/3}} + \frac{c^3\text{Subst}\left(\int\frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}dx,x,\sqrt[3]{a+bx^3}\right)}{2d^3(bc-ad)}$$

$$= \frac{a^3}{b^3(bc-ad)\sqrt[3]{a+bx^3}} - \frac{a\left(a+bx^3\right)^{2/3}}{2b^3d} - \frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^3d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5b^3d} + \frac{c^3\log\left(c+dx^3\right)}{6d^{8/3}(bc-ad)^{4/3}} - \frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}(bc-ad)^{4/3}} + \frac{c^3\text{Subst}\left(\int\frac{1}{-3-x^2}dx,x,1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{d^{8/3}(bc-ad)^{4/3}}$$

$$= \frac{a^3}{b^3(bc-ad)\sqrt[3]{a+bx^3}} - \frac{a\left(a+bx^3\right)^{2/3}}{2b^3d} - \frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^3d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5b^3d} - \frac{c^3\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{8/3}(bc-ad)^{4/3}} + \frac{c^3\log\left(c+dx^3\right)}{6d^{8/3}(bc-ad)^{4/3}} - \frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}(bc-ad)^{4/3}}$$

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.18

$$\int \frac{x^{11}}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}dx = \frac{\frac{3d^{2/3}\left(18a^3d^2+b^3cx^3\left(-5c+2dx^3\right)+3a^2bd\left(-c+2dx^3\right)-ab^2\left(5c^2+cdx^3+2d^2x^6\right)\right)}{b^3(bc-ad)\sqrt[3]{a+bx^3}} - \frac{10\sqrt{3}c^3\arctan\left(\frac{1-2\cdots}{\cdots}\right)}{(bc-a\cdots}}{\cdots}$$

```
[In] Integrate[x^11/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] ((3*d^(2/3)*(18*a^3*d^2 + b^3*c*x^3*(-5*c + 2*d*x^3) + 3*a^2*b*d*(-c + 2*d*
x^3) - a*b^2*(5*c^2 + c*d*x^3 + 2*d^2*x^6)))/(b^3*(b*c - a*d)*(a + b*x^3)^(
1/3)) - (10*Sqrt[3]*c^3*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*
d)^(1/3))/Sqrt[3]])/(b*c - a*d)^(4/3) - (10*c^3*Log[(b*c - a*d)^(1/3) + d^(
1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(4/3) + (5*c^3*Log[(b*c - a*d)^(2/3) -
 d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/
(b*c - a*d)^(4/3))/(30*d^(8/3))
```

Maple [A] (verified)

Time = 4.72 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$\frac{-\frac{54d\left(a\left(-\frac{1}{9}b^2x^6+\frac{1}{3}ab\,x^3+a^2\right)d^2-\frac{b\left(-\frac{2b\,x^3}{3}+a\right)\left(b\,x^3+a\right)cd}{6}-\frac{5b^2c^2\left(b\,x^3+a\right)}{18}\right)\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}{5}+b^3c^3\left(b\,x^3+a\right)^{\frac{1}{3}}\left(-2\arctan\left(\frac{\sqrt{3}\left(2(b \cdots\right.}{}\right.\right.}{6\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\left(b\,x^3+a\right)^{\cdots}}$

```
[In] int(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/(1/d*(a*d-b*c))^(1/3)/(b*x^3+a)^(1/3)*(-54/5*d*(a*(-1/9*b^2*x^6+1/3*a*b
*x^3+a^2)*d^2-1/6*b*(-2/3*b*x^3+a)*(b*x^3+a)*c*d-5/18*b^2*c^2*(b*x^3+a))*(1
/d*(a*d-b*c))^(1/3)+b^3*c^3*(b*x^3+a)^(1/3)*(-2*arctan(1/3*3^(1/2)*(2*(b*x^
3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+
a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln(
(b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3)))))/d^3/(a*d-b*c)/b^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. 2(208) = 416.

Time = 0.36 (sec) , antiderivative size = 1141, normalized size of antiderivative = 4.51

$$\int \frac{x^{11}}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/30*(15*sqrt(1/3)*(a*b^4*c^4*d - a^2*b^3*c^3*d^2 + (b^5*c^4*d - a*b^4*c^
3*d^2)*x^3)*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b
*c*d + 3*a*d^2 - 3*sqrt(1/3)*(2*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(2/3) +
 (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (b*c*d^2 - a*d^3)^(1/3)*(b*c - a*d))*s
qrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d)) - 3*(b*c*d^2 - a*d^3)^(2/3)*(b*x^
3 + a)^(1/3))/(d*x^3 + c)) - 5*(b^4*c^3*x^3 + a*b^3*c^3)*(b*c*d^2 - a*d^3)^
(2/3)*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)
```

```
*d + (b*c*d^2 - a*d^3)^(2/3)) + 10*(b^4*c^3*x^3 + a*b^3*c^3)*(b*c*d^2 - a*d
^3)^(2/3)*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) + 3*(5*a*b^3*c
^3*d^2 - 2*a^2*b^2*c^2*d^3 - 21*a^3*b*c*d^4 + 18*a^4*d^5 - 2*(b^4*c^2*d^3 -
 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^6 + (5*b^4*c^3*d^2 - 4*a*b^3*c^2*d^3 - 7*a^
2*b^2*c*d^4 + 6*a^3*b*d^5)*x^3)*(b*x^3 + a)^(2/3))/(a*b^5*c^2*d^4 - 2*a^2*b
^4*c*d^5 + a^3*b^3*d^6 + (b^6*c^2*d^4 - 2*a*b^5*c*d^5 + a^2*b^4*d^6)*x^3),
1/30*(30*sqrt(1/3)*(a*b^4*c^4*d - a^2*b^3*c^3*d^2 + (b^5*c^4*d - a*b^4*c^3*
d^2)*x^3)*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*
x^3 + a)^(1/3)*d - (b*c*d^2 - a*d^3)^(1/3))*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b
*c - a*d))/d) + 5*(b^4*c^3*x^3 + a*b^3*c^3)*(b*c*d^2 - a*d^3)^(2/3)*log((b*
x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2
 - a*d^3)^(2/3)) - 10*(b^4*c^3*x^3 + a*b^3*c^3)*(b*c*d^2 - a*d^3)^(2/3)*log
((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) - 3*(5*a*b^3*c^3*d^2 - 2*a^
2*b^2*c^2*d^3 - 21*a^3*b*c*d^4 + 18*a^4*d^5 - 2*(b^4*c^2*d^3 - 2*a*b^3*c*d^
4 + a^2*b^2*d^5)*x^6 + (5*b^4*c^3*d^2 - 4*a*b^3*c^2*d^3 - 7*a^2*b^2*c*d^4 +
 6*a^3*b*d^5)*x^3)*(b*x^3 + a)^(2/3))/(a*b^5*c^2*d^4 - 2*a^2*b^4*c*d^5 + a^
3*b^3*d^6 + (b^6*c^2*d^4 - 2*a*b^5*c*d^5 + a^2*b^4*d^6)*x^3)]
```

Sympy [F]

$$\int \frac{x^{11}}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx = \int \frac{x^{11}}{(a+bx^3)^{\frac{4}{3}}\,(c+dx^3)}\,dx$$

```
[In] integrate(x**11/(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(x**11/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.47

$$\int \frac{x^{11}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = -\frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^3\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^2d^4-2\sqrt{3}abcd^5+\sqrt{3}a^2d^6} + \frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^3\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(b^2c^2d^4-2\,abcd^5+a^2d^6)} - \frac{c^3\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^2c^2d^2-2\,abcd^3+a^2d^4)} + \frac{a^3}{(b^4c-ab^3d)(bx^3+a)^{\frac{1}{3}}} - \frac{5\,(bx^3+a)^{\frac{2}{3}}b^{13}cd^3-2\,(bx^3+a)^{\frac{5}{3}}b^{12}d^4+10\,(bx^3+a)^{\frac{2}{3}}ab^{12}d^4}{10\,b^{15}d^5}$$

```
[In] integrate(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -(-b*c*d^2 + a*d^3)^(2/3)*c^3*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(
b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b^2*c^2*d^4 - 2*sqrt(
3)*a*b*c*d^5 + sqrt(3)*a^2*d^6) + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^3*log((b*x
^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)
^(2/3))/(b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6) - 1/3*c^3*(-(b*c - a*d)/d)^(2
/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^2*c^2*d^2 - 2*a
*b*c*d^3 + a^2*d^4) + a^3/((b^4*c - a*b^3*d)*(b*x^3 + a)^(1/3)) - 1/10*(5*(
b*x^3 + a)^(2/3)*b^13*c*d^3 - 2*(b*x^3 + a)^(5/3)*b^12*d^4 + 10*(b*x^3 + a)
^(2/3)*a*b^12*d^4)/(b^15*d^5)
```

Mupad [B] (verification not implemented)

Time = 9.48 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.95

$$\int \frac{x^{11}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{(b\,x^3+a)^{5/3}}{5\,b^3\,d} - \left(\frac{3\,a}{2\,b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{2\,b^6\,d^2}\right)(b\,x^3+a)^{2/3} - \frac{a^3}{b^3\,(b\,x^3+a)^{1/3}\,(a\,d-b\,c)} - \frac{c^3\,\ln\left((b\,x^3+a)^{1/3}\,(a\,c^6\,d^4-b\,c^7\,d^3)-\frac{c^6\,(9\,a^4\,d^{12}-36\,a^3\,b\,c\,d^{11}+54\,a^2\,b^2\,c^2\,d^{10}-36\,a\,b^3\,c^3\,d^9+9\,b^4\,c^4\,d^8)}{9\,d^{16/3}\,(a\,d-b\,c)^{8/3}}\right)}{3\,d^{8/3}\,(a\,d-b\,c)^{4/3}} + \frac{\ln\left((b\,x^3+a)^{1/3}\,(a\,c^6\,d^4-b\,c^7\,d^3)-\frac{\left(c^3+\sqrt{3}\,c^3\,1\mathrm{i}\right)^2\,(9\,a^4\,d^{12}-36\,a^3\,b\,c\,d^{11}+54\,a^2\,b^2\,c^2\,d^{10}-36\,a\,b^3\,c^3\,d^9+9\,b^4\,c^4\,d^8)}{36\,d^{16/3}\,(a\,d-b\,c)^{8/3}}\right)\,(c^3+\sqrt{}}{6\,d^{8/3}\,(a\,d-b\,c)^{4/3}} - \frac{c^3\,\ln\left((b\,x^3+a)^{1/3}\,(a\,c^6\,d^4-b\,c^7\,d^3)-\frac{c^6\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2\,(9\,a^4\,d^{12}-36\,a^3\,b\,c\,d^{11}+54\,a^2\,b^2\,c^2\,d^{10}-36\,a\,b^3\,c^3\,d^9+9\,b^4\,c^4\,d^8)}{9\,d^{16/3}\,(a\,d-b\,c)^{8/3}}\right)\,(-}{3\,d^{8/3}\,(a\,d-b\,c)^{4/3}}$$

```
[In] int(x^11/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] (a + b*x^3)^(5/3)/(5*b^3*d) - ((3*a)/(2*b^3*d) + (b^4*c - a*b^3*d)/(2*b^6*d
^2))*(a + b*x^3)^(2/3) - a^3/(b^3*(a + b*x^3)^(1/3)*(a*d - b*c)) - (c^3*log
((a + b*x^3)^(1/3)*(a*c^6*d^4 - b*c^7*d^3) - (c^6*(9*a^4*d^12 + 9*b^4*c^4*d
^8 - 36*a*b^3*c^3*d^9 + 54*a^2*b^2*c^2*d^10 - 36*a^3*b*c*d^11))/(9*d^(16/3)
*(a*d - b*c)^(8/3))))/(3*d^(8/3)*(a*d - b*c)^(4/3)) + (log((a + b*x^3)^(1/3
)*(a*c^6*d^4 - b*c^7*d^3) - ((3^(1/2)*c^3*1i + c^3)^2*(9*a^4*d^12 + 9*b^4*c
^4*d^8 - 36*a*b^3*c^3*d^9 + 54*a^2*b^2*c^2*d^10 - 36*a^3*b*c*d^11))/(36*d^(
16/3)*(a*d - b*c)^(8/3)))*(3^(1/2)*c^3*1i + c^3))/(6*d^(8/3)*(a*d - b*c)^(4
/3)) - (c^3*log((a + b*x^3)^(1/3)*(a*c^6*d^4 - b*c^7*d^3) - (c^6*((3^(1/2)*
1i)/2 - 1/2)^2*(9*a^4*d^12 + 9*b^4*c^4*d^8 - 36*a*b^3*c^3*d^9 + 54*a^2*b^2*
c^2*d^10 - 36*a^3*b*c*d^11))/(9*d^(16/3)*(a*d - b*c)^(8/3)))*((3^(1/2)*1i)/
2 - 1/2))/(3*d^(8/3)*(a*d - b*c)^(4/3))
```

3.749 $\int \frac{x^8}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 203

$$\int \frac{x^8}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = -\frac{a^2}{b^2(bc-ad)\sqrt[3]{a+bx^3}}+\frac{(a+bx^3)^{2/3}}{2b^2d}+\frac{c^2\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{5/3}(bc-ad)^{4/3}}-\frac{c^2\log(c+dx^3)}{6d^{5/3}(bc-ad)^{4/3}}+\frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}(bc-ad)^{4/3}}$$

```
[Out] -a^2/b^2/(-a*d+b*c)/(b*x^3+a)^(1/3)+1/2*(b*x^3+a)^(2/3)/b^2/d-1/6*c^2*ln(d*
x^3+c)/d^(5/3)/(-a*d+b*c)^(4/3)+1/2*c^2*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+
a)^(1/3))/d^(5/3)/(-a*d+b*c)^(4/3)+1/3*c^2*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a
)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(5/3)/(-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 89, 58, 631, 210, 31}

$$\int \frac{x^8}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = -\frac{a^2}{b^2\sqrt[3]{a+bx^3}(bc-ad)}+\frac{c^2\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{5/3}(bc-ad)^{4/3}}+\frac{(a+bx^3)^{2/3}}{2b^2d}-\frac{c^2\log(c+dx^3)}{6d^{5/3}(bc-ad)^{4/3}}+\frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}(bc-ad)^{4/3}}$$

```
[In] Int[x^8/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(a^2/(b^2*(b*c - a*d)*(a + b*x^3)^(1/3))) + (a + b*x^3)^(2/3)/(2*b^2*d) +
(c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])
/(Sqrt[3]*d^(5/3)*(b*c - a*d)^(4/3)) - (c^2*Log[c + d*x^3])/(6*d^(5/3)*(b*c
 - a*d)^(4/3)) + (c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(
2*d^(5/3)*(b*c - a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 89

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*
x)^n*((e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a^2}{b(bc-ad)(a+bx)^{4/3}} + \frac{1}{bd\sqrt[3]{a+bx}} + \frac{c^2}{d(-bc+ad)\sqrt[3]{a+bx}(c+dx)}\right)\,dx, x, x^3\right)$$

$$= -\frac{a^2}{b^2(bc-ad)\sqrt[3]{a+bx^3}} + \frac{(a+bx^3)^{2/3}}{2b^2d} - \frac{c^2\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)}{3d(bc-ad)}$$

$$= -\frac{a^2}{b^2(bc-ad)\sqrt[3]{a+bx^3}} + \frac{(a+bx^3)^{2/3}}{2b^2d} - \frac{c^2\log(c+dx^3)}{6d^{5/3}(bc-ad)^{4/3}} + \frac{c^2\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{5/3}(bc-ad)^{4/3}} - \frac{c^2\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^2(bc-ad)}$$

$$= -\frac{a^2}{b^2(bc-ad)\sqrt[3]{a+bx^3}} + \frac{(a+bx^3)^{2/3}}{2b^2d} - \frac{c^2\log(c+dx^3)}{6d^{5/3}(bc-ad)^{4/3}} + \frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}(bc-ad)^{4/3}} - \frac{c^2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{d^{5/3}(bc-ad)^{4/3}}$$

$$= -\frac{a^2}{b^2(bc-ad)\sqrt[3]{a+bx^3}} + \frac{(a+bx^3)^{2/3}}{2b^2d} + \frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{5/3}(bc-ad)^{4/3}} - \frac{c^2\log(c+dx^3)}{6d^{5/3}(bc-ad)^{4/3}} + \frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}(bc-ad)^{4/3}}$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.25

$$\int \frac{x^8}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{\frac{3d^{2/3}(-3a^2d+b^2cx^3+ab(c-dx^3))}{b^2(bc-ad)\sqrt[3]{a+bx^3}} + \frac{2\sqrt{3}c^2\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{(bc-ad)^{4/3}} + \frac{2c^2\log\left(\sqrt[3]{bc-a\right.}}{(bc-}}{6d^{5/3}}$$

[In] Integrate[x^8/((a + b*x^3)^(4/3)*(c + d*x^3)),x]

[Out] ((3*d^(2/3)*(-3*a^2*d + b^2*c*x^3 + a*b*(c - d*x^3)))/(b^2*(b*c - a*d)*(a + b*x^3)^(1/3)) + (2*Sqrt[3]*c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(b*c - a*d)^(4/3) + (2*c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(4/3) - (c^2*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(4/3))/(6*d^(5/3))

Maple [A] (verified)

Time = 4.69 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.15

method	result
pseudoelliptic	$-\frac{-9\left(-\frac{b^2cx^3}{3}-\frac{a(-dx^3+c)b}{3}+a^2d\right)\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}d+b^2c^2(bx^3+a)^{\frac{1}{3}}\left(-2\arctan\left(\frac{\sqrt{3}\left(2(bx^3+a)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(\right.\right.\right.}{6\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}(bx^3+a)^{\frac{1}{3}}d^2(ad-bc)b^2}$

[In] int(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] -1/6*(-9*(-1/3*b^2*c*x^3-1/3*a*(-d*x^3+c)*b+a^2*d)*(1/d*(a*d-b*c))^(1/3)*d+b^2*c^2*(b*x^3+a)^(1/3)*(-2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))))/(1/d*(a*d-b*c))^(1/3)/(b*x^3+a)^(1/3)/d^2/(a*d-b*c)/b^2

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 453 vs. 2(167) = 334.

Time = 0.34 (sec) , antiderivative size = 1004, normalized size of antiderivative = 4.95

$$\int \frac{x^8}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(1/3)*(a*b^3*c^3*d - a^2*b^2*c^2*d^2 + (b^4*c^3*d - a*b^3*c^2*
d^2)*x^3)*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c
*d + 3*a*d^2 + 3*sqrt(1/3)*(2*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(2/3) +
(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) + (-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d))*s
qrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d)) - 3*(-b*c*d^2 + a*d^3)^(2/3)*(b*x
^3 + a)^(1/3))/(d*x^3 + c)) + (b^3*c^2*x^3 + a*b^2*c^2)*(-b*c*d^2 + a*d^3)^
(2/3)*log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3
)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 2*(b^3*c^2*x^3 + a*b^2*c^2)*(-b*c*d^2 + a
*d^3)^(2/3)*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)) - 3*(a*b^2*
c^2*d^2 - 4*a^2*b*c*d^3 + 3*a^3*d^4 + (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*
d^4)*x^3)*(b*x^3 + a)^(2/3))/(a*b^4*c^2*d^3 - 2*a^2*b^3*c*d^4 + a^3*b^2*d^5
 + (b^5*c^2*d^3 - 2*a*b^4*c*d^4 + a^2*b^3*d^5)*x^3), -1/6*(6*sqrt(1/3)*(a*b
^3*c^3*d - a^2*b^2*c^2*d^2 + (b^4*c^3*d - a*b^3*c^2*d^2)*x^3)*sqrt(-(-b*c*d
^2 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d + (-
b*c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))/d) + (b
^3*c^2*x^3 + a*b^2*c^2)*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3)*d^2
+ (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3))
- 2*(b^3*c^2*x^3 + a*b^2*c^2)*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(1/3
)*d - (-b*c*d^2 + a*d^3)^(1/3)) - 3*(a*b^2*c^2*d^2 - 4*a^2*b*c*d^3 + 3*a^3*
d^4 + (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3)*(b*x^3 + a)^(2/3))/(a*
b^4*c^2*d^3 - 2*a^2*b^3*c*d^4 + a^3*b^2*d^5 + (b^5*c^2*d^3 - 2*a*b^4*c*d^4
+ a^2*b^3*d^5)*x^3)]
```

Sympy [F]

$$\int \frac{x^8}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^8}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)}\,dx$$

```
[In] integrate(x**8/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**8/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.60

$$\int \frac{x^8}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^2d^3-2\sqrt{3}abcd^4+\sqrt{3}a^2d^5} - \frac{(-bcd^2+ad^3)^{\frac{2}{3}}c^2\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(b^2c^2d^3-2\,abcd^4+a^2d^5)} + \frac{c^2\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^2c^2d-2\,abcd^2+a^2d^3)} - \frac{a^2}{(b^3c-ab^2d)(bx^3+a)^{\frac{1}{3}}} + \frac{(bx^3+a)^{\frac{2}{3}}}{2\,b^2d}$$

```
[In] integrate(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] (-b*c*d^2 + a*d^3)^(2/3)*c^2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b
*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b^2*c^2*d^3 - 2*sqrt(3
)*a*b*c*d^4 + sqrt(3)*a^2*d^5) - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^2*log((b*x^
3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^
(2/3))/(b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5) + 1/3*c^2*(-(b*c - a*d)/d)^(2/
3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^2*c^2*d - 2*a*b*
c*d^2 + a^2*d^3) - a^2/((b^3*c - a*b^2*d)*(b*x^3 + a)^(1/3)) + 1/2*(b*x^3 +
 a)^(2/3)/(b^2*d)
```

Mupad [B] (verification not implemented)

Time = 9.62 (sec) , antiderivative size = 449, normalized size of antiderivative = 2.21

$$\int \frac{x^8}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{(b\,x^3+a)^{2/3}}{2\,b^2\,d} + \frac{a^2}{b^2\,(b\,x^3+a)^{1/3}\,(a\,d-b\,c)} + \frac{c^2\,\ln\left((b\,x^3+a)^{1/3}\,(a\,c^4\,d^3-b\,c^5\,d^2)-\frac{c^4\,(9\,a^4\,d^9-36\,a^3\,b\,c\,d^8+54\,a^2\,b^2\,c^2\,d^7-36\,a\,b^3\,c^3\,d^6+9\,b^4\,c^4\,d^5)}{9\,d^{10/3}\,(a\,d-b\,c)^{8/3}}\right)}{3\,d^{5/3}\,(a\,d-b\,c)^{4/3}} - \frac{\ln\left((b\,x^3+a)^{1/3}\,(a\,c^4\,d^3-b\,c^5\,d^2)-\frac{\left(c^2+\sqrt{3}\,c^2\,1\mathrm{i}\right)^2\,(9\,a^4\,d^9-36\,a^3\,b\,c\,d^8+54\,a^2\,b^2\,c^2\,d^7-36\,a\,b^3\,c^3\,d^6+9\,b^4\,c^4\,d^5)}{36\,d^{10/3}\,(a\,d-b\,c)^{8/3}}\right)\,(c^2+\sqrt{3}\,c}{6\,d^{5/3}\,(a\,d-b\,c)^{4/3}} + \frac{c^2\,\ln\left((b\,x^3+a)^{1/3}\,(a\,c^4\,d^3-b\,c^5\,d^2)-\frac{c^4\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2\,(9\,a^4\,d^9-36\,a^3\,b\,c\,d^8+54\,a^2\,b^2\,c^2\,d^7-36\,a\,b^3\,c^3\,d^6+9\,b^4\,c^4\,d^5)}{d^{10/3}\,(a\,d-b\,c)^{8/3}}\right)\,\left(-\frac{1}{6}+\right.}{d^{5/3}\,(a\,d-b\,c)^{4/3}}$$

```
[In] int(x^8/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] (a + b*x^3)^(2/3)/(2*b^2*d) + a^2/(b^2*(a + b*x^3)^(1/3)*(a*d - b*c)) + (c^
2*log((a + b*x^3)^(1/3)*(a*c^4*d^3 - b*c^5*d^2) - (c^4*(9*a^4*d^9 + 9*b^4*c
^4*d^5 - 36*a*b^3*c^3*d^6 + 54*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8))/(9*d^(10/
3)*(a*d - b*c)^(8/3))))/(3*d^(5/3)*(a*d - b*c)^(4/3)) - (log((a + b*x^3)^(1
/3)*(a*c^4*d^3 - b*c^5*d^2) - ((3^(1/2)*c^2*1i + c^2)^2*(9*a^4*d^9 + 9*b^4*
c^4*d^5 - 36*a*b^3*c^3*d^6 + 54*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8))/(36*d^(1
0/3)*(a*d - b*c)^(8/3)))*(3^(1/2)*c^2*1i + c^2))/(6*d^(5/3)*(a*d - b*c)^(4/
3)) + (c^2*log((a + b*x^3)^(1/3)*(a*c^4*d^3 - b*c^5*d^2) - (c^4*((3^(1/2)*1
i)/6 - 1/6)^2*(9*a^4*d^9 + 9*b^4*c^4*d^5 - 36*a*b^3*c^3*d^6 + 54*a^2*b^2*c^
2*d^7 - 36*a^3*b*c*d^8))/(d^(10/3)*(a*d - b*c)^(8/3)))*((3^(1/2)*1i)/6 - 1/
6))/(d^(5/3)*(a*d - b*c)^(4/3))
```

3.750 $\int \frac{x^5}{(a+bx^3)^{4/3}(c+dx^3)} dx$

Optimal result

Integrand size = 24, antiderivative size = 174

$$\int \frac{x^5}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{a}{b(bc-ad)\sqrt[3]{a+bx^3}} - \frac{c\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{2/3}(bc-ad)^{4/3}} + \frac{c\log(c+dx^3)}{6d^{2/3}(bc-ad)^{4/3}} - \frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}(bc-ad)^{4/3}}$$

```
[Out] a/b/(-a*d+b*c)/(b*x^3+a)^(1/3)+1/6*c*ln(d*x^3+c)/d^(2/3)/(-a*d+b*c)^(4/3)-1
/2*c*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(2/3)/(-a*d+b*c)^(4/3)-
1/3*c*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^
(2/3)/(-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {457, 79, 58, 631, 210, 31}

$$\int \frac{x^5}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = -\frac{c\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{2/3}(bc-ad)^{4/3}} + \frac{c\log(c+dx^3)}{6d^{2/3}(bc-ad)^{4/3}} - \frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}(bc-ad)^{4/3}} + \frac{a}{b\sqrt[3]{a+bx^3}(bc-ad)}$$

```
[In] Int[x^5/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] a/(b*(b*c - a*d)*(a + b*x^3)^(1/3)) - (c*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)
^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(2/3)*(b*c - a*d)^(4/3)) +
(c*Log[c + d*x^3])/(6*d^(2/3)*(b*c - a*d)^(4/3)) - (c*Log[(b*c - a*d)^(1/3)
 + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(2/3)*(b*c - a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x}{(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{a}{b(bc-ad)\sqrt[3]{a+bx^3}} + \frac{c\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)}{3(bc-ad)}$$

$$= \frac{a}{b(bc-ad)\sqrt[3]{a+bx^3}} + \frac{c\log\left(c+dx^3\right)}{6d^{2/3}(bc-ad)^{4/3}} - \frac{c\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{2/3}(bc-ad)^{4/3}} + \frac{c\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d(bc-ad)}$$

$$= \frac{a}{b(bc-ad)\sqrt[3]{a+bx^3}} + \frac{c\log\left(c+dx^3\right)}{6d^{2/3}(bc-ad)^{4/3}} - \frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}(bc-ad)^{4/3}} + \frac{c\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{d^{2/3}(bc-ad)^{4/3}}$$

$$= \frac{a}{b(bc-ad)\sqrt[3]{a+bx^3}} - \frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}d^{2/3}(bc-ad)^{4/3}} + \frac{c\log\left(c+dx^3\right)}{6d^{2/3}(bc-ad)^{4/3}} - \frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}(bc-ad)^{4/3}}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.30

$$\int \frac{x^5}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\frac{1}{6}\left(\frac{6a}{\left(b^2c-abd\right)\sqrt[3]{a+bx^3}}-\frac{2\sqrt{3}c\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{d^{2/3}(bc-ad)^{4/3}}-\frac{2c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{d^{2/3}(bc-ad)^{4/3}}+\frac{c\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{d^{2/3}(bc-ad)^{4/3}}\right)$$

```
[In] Integrate[x^5/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] ((6*a)/((b^2*c - a*b*d)*(a + b*x^3)^(1/3)) - (2*Sqrt[3]*c*ArcTan[(1 - (2*d^
(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(d^(2/3)*(b*c - a*d)^
(4/3)) - (2*c*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(d^(2/3)*
(b*c - a*d)^(4/3)) + (c*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(
a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(d^(2/3)*(b*c - a*d)^(4/3)))
/6
```

Maple [A] (verified)

Time = 4.50 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.29

method	result
pseudoelliptic	$-\frac{\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^3+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)bc\left(b\,x^3+a\right)^{\frac{1}{3}}}{3}+\frac{\ln\left(\left(b\,x^3+a\right)^{\frac{1}{3}}-\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)bc\left(b\,x^3+a\right)^{\frac{1}{3}}}{3}-\frac{\ln\left(\left(b\,x^3+a\right)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)\cdots\right.}{\cdots}}{\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}(ad-bc)db}$

```
[In] int(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(1/d*(a*d-b*c))^(1/3)*(1/3*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)
+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*b*c*(b*x^3+a)^(1/3)+1/3*ln((
b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))*b*c*(b*x^3+a)^(1/3)-1/6*ln((b*x^3+a)^
(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))*b*c*(b*x
^3+a)^(1/3)+a*d*(1/d*(a*d-b*c))^(1/3))/(b*x^3+a)^(1/3)/(a*d-b*c)/d/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(141) = 282$.

Time = 0.34 (sec) , antiderivative size = 872, normalized size of antiderivative = 5.01

$$\int \frac{x^5}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \left[-\frac{3\sqrt{\frac{1}{3}}(ab^2c^2d - a^2bcd^2 + (b^3c^2d - ab^2cd^2)x^3)\sqrt{-\frac{(bcd^2-ad^3)^{\frac{1}{3}}}{bc-ad}}\log\left(\frac{2\,bd^2x^3-bc\cdots}{\cdots}\right.}{\cdots}\right.$$

```
[In] integrate(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(1/3)*(a*b^2*c^2*d - a^2*b*c*d^2 + (b^3*c^2*d - a*b^2*c*d^2)*x
^3)*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c*d + 3
*a*d^2 - 3*sqrt(1/3)*(2*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(2/3) + (b*x^3
+ a)^(1/3)*(b*c*d - a*d^2) - (b*c*d^2 - a*d^3)^(1/3)*(b*c - a*d))*sqrt(-(b*
c*d^2 - a*d^3)^(1/3)/(b*c - a*d)) - 3*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(
1/3))/(d*x^3 + c)) - (b^2*c*x^3 + a*b*c)*(b*c*d^2 - a*d^3)^(2/3)*log((b*x^3
 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2 -
a*d^3)^(2/3)) + 2*(b^2*c*x^3 + a*b*c)*(b*c*d^2 - a*d^3)^(2/3)*log((b*x^3 +
a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) - 6*(a*b*c*d^2 - a^2*d^3)*(b*x^3 + a)
^(2/3))/(a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + a^3*b*d^4 + (b^4*c^2*d^2 - 2*a*b
^3*c*d^3 + a^2*b^2*d^4)*x^3), 1/6*(6*sqrt(1/3)*(a*b^2*c^2*d - a^2*b*c*d^2 +
 (b^3*c^2*d - a*b^2*c*d^2)*x^3)*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*a
rctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d - (b*c*d^2 - a*d^3)^(1/3))*sqrt((b*c
*d^2 - a*d^3)^(1/3)/(b*c - a*d))/d) + (b^2*c*x^3 + a*b*c)*(b*c*d^2 - a*d^3)
^(2/3)*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3
)*d + (b*c*d^2 - a*d^3)^(2/3)) - 2*(b^2*c*x^3 + a*b*c)*(b*c*d^2 - a*d^3)^(2
/3)*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) + 6*(a*b*c*d^2 - a^2
*d^3)*(b*x^3 + a)^(2/3))/(a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + a^3*b*d^4 + (b^
4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*x^3)]
```

Sympy [F]

$$\int \frac{x^5}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \int \frac{x^5}{\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

```
[In] integrate(x**5/(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(x**5/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(141) = 282$.

Time = 0.30 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.73

$$\int \frac{x^5}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = -\frac{\frac{6\left(-bcd^2+ad^3\right)^{\frac{2}{3}}bc\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^2d^2-2\sqrt{3}abcd^3+\sqrt{3}a^2d^4} - \frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}bc\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{b^2c^2d^2-2\,abcd^3+a^2d^4} + \cdots}{6\,b}$$

```
[In] integrate(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] -1/6*(6*(-b*c*d^2 + a*d^3)^(2/3)*b*c*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3
) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b^2*c^2*d^2 -
2*sqrt(3)*a*b*c*d^3 + sqrt(3)*a^2*d^4) - (-b*c*d^2 + a*d^3)^(2/3)*b*c*log((
b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)
/d)^(2/3))/(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4) + 2*b*c*(-(b*c - a*d)/d)^(
2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^2*c^2 - 2*a*b*
c*d + a^2*d^2) - 6*a/((b*x^3 + a)^(1/3)*(b*c - a*d)))/b
```

Mupad [B] (verification not implemented)

Time = 9.52 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.37

$$\int \frac{x^5}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = -\frac{a}{b\,(b\,x^3+a)^{1/3}\,(a\,d-b\,c)} - \frac{c\,\ln\left((b\,x^3+a)^{1/3}\,(a\,c^2\,d^2-b\,c^3\,d)-\frac{c^2\,(9\,a^4\,d^6-36\,a^3\,b\,c\,d^5+54\,a^2\,b^2\,c^2\,d^4-36\,a\,b^3\,c^3\,d^3+9\,b^4\,c^4\,d^2)}{9\,d^{4/3}\,(a\,d-b\,c)^{8/3}}\right)}{3\,d^{2/3}\,(a\,d-b\,c)^{4/3}} + \frac{\ln\left((b\,x^3+a)^{1/3}\,(a\,c^2\,d^2-b\,c^3\,d)-\frac{\left(c-\sqrt{3}\,c\,1\mathrm{i}\right)^2\,(9\,a^4\,d^6-36\,a^3\,b\,c\,d^5+54\,a^2\,b^2\,c^2\,d^4-36\,a\,b^3\,c^3\,d^3+9\,b^4\,c^4\,d^2)}{36\,d^{4/3}\,(a\,d-b\,c)^{8/3}}\right)\,(c-\sqrt{3}\,c\,1\mathrm{i}}{6\,d^{2/3}\,(a\,d-b\,c)^{4/3}} + \frac{\ln\left((b\,x^3+a)^{1/3}\,(a\,c^2\,d^2-b\,c^3\,d)-\frac{\left(c+\sqrt{3}\,c\,1\mathrm{i}\right)^2\,(9\,a^4\,d^6-36\,a^3\,b\,c\,d^5+54\,a^2\,b^2\,c^2\,d^4-36\,a\,b^3\,c^3\,d^3+9\,b^4\,c^4\,d^2)}{36\,d^{4/3}\,(a\,d-b\,c)^{8/3}}\right)\,(c+\sqrt{3}\,c\,1\mathrm{i}}{6\,d^{2/3}\,(a\,d-b\,c)^{4/3}}$$

```
[In] int(x^5/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] (log((a + b*x^3)^(1/3)*(a*c^2*d^2 - b*c^3*d) - ((c - 3^(1/2)*c*1i)^2*(9*a^4
*d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*d^3 + 54*a^2*b^2*c^2*d^4 - 36*a^3*b*c*d
^5))/(36*d^(4/3)*(a*d - b*c)^(8/3)))*(c - 3^(1/2)*c*1i))/(6*d^(2/3)*(a*d -
b*c)^(4/3)) - (c*log((a + b*x^3)^(1/3)*(a*c^2*d^2 - b*c^3*d) - (c^2*(9*a^4*
d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*d^3 + 54*a^2*b^2*c^2*d^4 - 36*a^3*b*c*d^
5))/(9*d^(4/3)*(a*d - b*c)^(8/3))))/(3*d^(2/3)*(a*d - b*c)^(4/3)) - a/(b*(a
 + b*x^3)^(1/3)*(a*d - b*c)) + (log((a + b*x^3)^(1/3)*(a*c^2*d^2 - b*c^3*d)
 - ((c + 3^(1/2)*c*1i)^2*(9*a^4*d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*d^3 + 54
*a^2*b^2*c^2*d^4 - 36*a^3*b*c*d^5))/(36*d^(4/3)*(a*d - b*c)^(8/3)))*(c + 3^
(1/2)*c*1i))/(6*d^(2/3)*(a*d - b*c)^(4/3))
```

3.751 $\int \frac{x^2}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 167

$$\int \frac{x^2}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = -\frac{1}{(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\sqrt[3]{d}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}(bc-ad)^{4/3}} - \frac{\sqrt[3]{d}\log\left(c+dx^3\right)}{6(bc-ad)^{4/3}} + \frac{\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2(bc-ad)^{4/3}}$$

```
[Out] -1/(-a*d+b*c)/(b*x^3+a)^(1/3)-1/6*d^(1/3)*ln(d*x^3+c)/(-a*d+b*c)^(4/3)+1/2*
d^(1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/(-a*d+b*c)^(4/3)+1/3*d
^(1/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/(
-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {455, 53, 58, 631, 210, 31}

$$\int \frac{x^2}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{\sqrt[3]{d}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}(bc-ad)^{4/3}} - \frac{1}{\sqrt[3]{a+bx^3}(bc-ad)} - \frac{\sqrt[3]{d}\log\left(c+dx^3\right)}{6(bc-ad)^{4/3}} + \frac{\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2(bc-ad)^{4/3}}$$

```
[In] Int[x^2/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(1/((b*c - a*d)*(a + b*x^3)^(1/3))) + (d^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a +
 b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*(b*c - a*d)^(4/3)) - (
d^(1/3)*Log[c + d*x^3])/(6*(b*c - a*d)^(4/3)) + (d^(1/3)*Log[(b*c - a*d)^(1
/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*(b*c - a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{1}{(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)}{3(bc-ad)}$$

$$= -\frac{1}{(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\sqrt[3]{d}\log(c+dx^3)}{6(bc-ad)^{4/3}} + \frac{\sqrt[3]{d}\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2(bc-ad)^{4/3}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2(bc-ad)}$$

$$= -\frac{1}{(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\sqrt[3]{d}\log(c+dx^3)}{6(bc-ad)^{4/3}} + \frac{\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2(bc-ad)^{4/3}} - \frac{\sqrt[3]{d}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{(bc-ad)^{4/3}}$$

$$= -\frac{1}{(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}(bc-ad)^{4/3}} - \frac{\sqrt[3]{d}\log(c+dx^3)}{6(bc-ad)^{4/3}} + \frac{\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2(bc-ad)^{4/3}}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.32

$$\int \frac{x^2}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = -\frac{1}{(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\sqrt[3]{d}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}(bc-ad)^{4/3}} + \frac{\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{3(bc-ad)^{4/3}} - \frac{\sqrt[3]{d}\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}(a+bx^3)^{2/3}\right)}{6(bc-ad)^{4/3}}$$

```
[In] Integrate[x^2/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(1/((b*c - a*d)*(a + b*x^3)^(1/3))) + (d^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a +
 b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*(b*c - a*d)^(4/3)) + (
d^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(3*(b*c - a*d)^
(4/3)) - (d^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*
x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*(b*c - a*d)^(4/3))
```

Maple [A] (verified)

Time = 4.64 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.18

method	result
pseudoelliptic	$\frac{6\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}+\left(2\arctan\left(\frac{\sqrt{3}\left(2\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}}\right)\sqrt{3}+2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right)-\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\right.}{\left(\frac{ad-bc}{d}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}(6ad-6bc)}$

```
[In] int(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(3*(1/d*(a*d-b*c))^(1/3)+1/2*(2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/
d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*3^(1/2)+2*ln((b*x^3+a)^(1/3)-(1/
d*(a*d-b*c))^(1/3))-ln((b*x^3+a)^(2/3)+(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3
)+(1/d*(a*d-b*c))^(2/3)))*(b*x^3+a)^(1/3))/(1/d*(a*d-b*c))^(1/3)/(b*x^3+a)^
(1/3)/(6*a*d-6*b*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.57

$$\int \frac{x^2}{(a+bx^3)^{4/3}(c+dx^3)}\,dx =$$
$$-\frac{2\sqrt{3}(bx^3+a)\left(-\frac{d}{bc-ad}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}(bx^3+a)^{\frac{1}{3}}\left(-\frac{d}{bc-ad}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-(bx^3+a)\left(-\frac{d}{bc-ad}\right)^{\frac{1}{3}}\log\left(-(bx^3+a\right.}{}$$

```
[In] integrate(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(b*x^3 + a)*(-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^
3 + a)^(1/3)*(-d/(b*c - a*d))^(1/3) + 1/3*sqrt(3)) - (b*x^3 + a)*(-d/(b*c -
 a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b
*x^3 + a)^(2/3)*d - (b*c - a*d)*(-d/(b*c - a*d))^(1/3)) + 2*(b*x^3 + a)*(-d
/(b*c - a*d))^(1/3)*log((b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1
/3)*d) + 6*(b*x^3 + a)^(2/3))/((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)
```

Sympy [F]

$$\int \frac{x^2}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^2}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)}\,dx$$

```
[In] integrate(x**2/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**2/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(135) = 270$.

Time = 0.30 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.71

$$\int \frac{x^2}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,(b^2c^2-2\,abcd+a^2d^2)} + \frac{(-bcd^2+ad^3)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^2d-2\,\sqrt{3}abcd^2+\sqrt{3}a^2d^3} - \frac{(-bcd^2+ad^3)^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(b^2c^2d-2\,abcd^2+a^2d^3)} - \frac{1}{(bx^3+a)^{\frac{1}{3}}(bc-ad)}$$

```
[In] integrate(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*d*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(
1/3)))/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + (-b*c*d^2 + a*d^3)^(2/3)*arctan(1/
3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(
1/3))/(sqrt(3)*b^2*c^2*d - 2*sqrt(3)*a*b*c*d^2 + sqrt(3)*a^2*d^3) - 1/6*(-b
*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*
d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3) -
 1/((b*x^3 + a)^(1/3)*(b*c - a*d))
```

Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.33

$$\int \frac{x^2}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{1}{(b\,x^3+a)^{1/3}\,(a\,d-b\,c)} + \frac{d^{1/3}\,\ln\left((b\,x^3+a)^{1/3}\,(a\,d^4-b\,c\,d^3)-\frac{d^{2/3}\,(9\,a^4\,d^6-36\,a^3\,b\,c\,d^5+54\,a^2\,b^2\,c^2\,d^4-36\,a\,b^3\,c^3\,d^3+9\,b^4\,c^4\,d^2)}{9\,(a\,d-b\,c)^{8/3}}\right)}{3\,(a\,d-b\,c)^{4/3}} - \frac{d^{1/3}\,\ln\left((b\,x^3+a)^{1/3}\,(a\,d^4-b\,c\,d^3)-\frac{d^{2/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2\,(9\,a^4\,d^6-36\,a^3\,b\,c\,d^5+54\,a^2\,b^2\,c^2\,d^4-36\,a\,b^3\,c^3\,d^3+9\,b^4\,c^4\,d^2)}{9\,(a\,d-b\,c)^{8/3}}\right)\,\left(\frac{1}{2}+\right.}{3\,(a\,d-b\,c)^{4/3}} + \frac{d^{1/3}\,\ln\left((b\,x^3+a)^{1/3}\,(a\,d^4-b\,c\,d^3)-\frac{d^{2/3}\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2\,(9\,a^4\,d^6-36\,a^3\,b\,c\,d^5+54\,a^2\,b^2\,c^2\,d^4-36\,a\,b^3\,c^3\,d^3+9\,b^4\,c^4\,d^2)}{(a\,d-b\,c)^{8/3}}\right)\,\left(-\frac{1}{6}\right.}{(a\,d-b\,c)^{4/3}}$$

```
[In] int(x^2/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] 1/((a + b*x^3)^(1/3)*(a*d - b*c)) + (d^(1/3)*log((a + b*x^3)^(1/3)*(a*d^4 -
 b*c*d^3) - (d^(2/3)*(9*a^4*d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*d^3 + 54*a^2
*b^2*c^2*d^4 - 36*a^3*b*c*d^5))/(9*(a*d - b*c)^(8/3))))/(3*(a*d - b*c)^(4/3
)) - (d^(1/3)*log((a + b*x^3)^(1/3)*(a*d^4 - b*c*d^3) - (d^(2/3)*((3^(1/2)*
1i)/2 + 1/2)^2*(9*a^4*d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*d^3 + 54*a^2*b^2*c
^2*d^4 - 36*a^3*b*c*d^5))/(9*(a*d - b*c)^(8/3)))*((3^(1/2)*1i)/2 + 1/2))/(3
*(a*d - b*c)^(4/3)) + (d^(1/3)*log((a + b*x^3)^(1/3)*(a*d^4 - b*c*d^3) - (d
^(2/3)*((3^(1/2)*1i)/6 - 1/6)^2*(9*a^4*d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*d
^3 + 54*a^2*b^2*c^2*d^4 - 36*a^3*b*c*d^5))/(a*d - b*c)^(8/3))*((3^(1/2)*1i)
/6 - 1/6))/(a*d - b*c)^(4/3)
```

3.752 $\int \frac{1}{x(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 271

$$\int \frac{1}{x(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}c} - \frac{d^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c(bc-ad)^{4/3}} - \frac{\log(x)}{2a^{4/3}c} + \frac{d^{4/3}\log(c+dx^3)}{6c(bc-ad)^{4/3}} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{4/3}c} - \frac{d^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{4/3}}$$

```
[Out] b/a/(-a*d+b*c)/(b*x^3+a)^(1/3)-1/2*ln(x)/a^(4/3)/c+1/6*d^(4/3)*ln(d*x^3+c)/
c/(-a*d+b*c)^(4/3)+1/2*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(4/3)/c-1/2*d^(4/3)*ln
((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/c/(-a*d+b*c)^(4/3)+1/3*arctan(1/
3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(4/3)/c*3^(1/2)-1/3*d^(4/3
)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c/(-a*
d+b*c)^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {457, 87, 162, 57, 631, 210, 31, 58}

$$\int \frac{1}{x(a+bx^3)^{4/3}(c+dx^3)}dx = \frac{\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt{3}a^{4/3}c}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{4/3}c}-\frac{\log(x)}{2a^{4/3}c}-\frac{d^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c(bc-ad)^{4/3}}+\frac{d^{4/3}\log(c+dx^3)}{6c(bc-ad)^{4/3}}-\frac{d^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{4/3}}+\frac{b}{a\sqrt[3]{a+bx^3}(bc-ad)}$$

```
[In] Int[1/(x*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] b/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) + ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3)
)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(4/3)*c) - (d^(4/3)*ArcTan[(1 - (2*d^(1/3)*
(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c*(b*c - a*d)^(4/3
)) - Log[x]/(2*a^(4/3)*c) + (d^(4/3)*Log[c + d*x^3])/(6*c*(b*c - a*d)^(4/3)
) + Log[a^(1/3) - (a + b*x^3)^(1/3)]/(2*a^(4/3)*c) - (d^(4/3)*Log[(b*c - a*
d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c*(b*c - a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 87

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x
)*((e + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\text{Subst}\left(\int \frac{-bc+ad-bdx}{x\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)}{3a(bc-ad)}$$

$$= \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^3\right)}{3ac} + \frac{d^2\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)}{3c(bc-ad)}$$

$$= \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\log(x)}{2a^{4/3}c} + \frac{d^{4/3}\log(c+dx^3)}{6c(bc-ad)^{4/3}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2a^{4/3}c} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2ac} - \frac{d^{4/3}\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{4/3}} + \frac{d\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c(bc-ad)}$$

$$= \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\log(x)}{2a^{4/3}c} + \frac{d^{4/3}\log(c+dx^3)}{6c(bc-ad)^{4/3}} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{4/3}c} - \frac{d^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{4/3}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{a^{4/3}c} + \frac{d^{4/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{c(bc-ad)^{4/3}}$$

$$= \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{4/3}c} - \frac{d^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c(bc-ad)^{4/3}} - \frac{\log(x)}{2a^{4/3}c} + \frac{d^{4/3}\log(c+dx^3)}{6c(bc-ad)^{4/3}} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{4/3}c} - \frac{d^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{4/3}}$$

Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.29

$$\int \frac{1}{x\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{1}{6}\left(\frac{6b}{(abc-a^2d)\sqrt[3]{a+bx^3}} + \frac{2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}c} - \frac{2\sqrt{3}d^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{c(bc-ad)^{4/3}} + \frac{2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)}{a^{4/3}c} - \frac{2d^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{c(bc-ad)^{4/3}} - \frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{a^{4/3}c} + \frac{d^{4/3}\log\left((bc-ad)^{2/3}-\sqrt[3]{d}\sqrt[3]{bc-ad}\sqrt[3]{a+bx^3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)}{c(bc-ad)^{4/3}}\right)$$

```
[In] Integrate[1/(x*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] ((6*b)/((a*b*c - a^2*d)*(a + b*x^3)^(1/3)) + (2*Sqrt[3]*ArcTan[(1 + (2*(a +
 b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/(a^(4/3)*c) - (2*Sqrt[3]*d^(4/3)*ArcTan[(
1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(c*(b*c - a*
d)^(4/3)) + (2*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/(a^(4/3)*c) - (2*d^(4/3)*
Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(c*(b*c - a*d)^(4/3)) -
 Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]/(a^(4/3)*c) +
 (d^(4/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/
3) + d^(2/3)*(a + b*x^3)^(2/3)])/(c*(b*c - a*d)^(4/3)))/6
```

Maple [A] (verified)

Time = 4.74 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$\frac{\left(-\left(-2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)\right)(ad-bc)\left(b\,x^{3}+\right.}{}$

```
[In] int(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/(b*x^3+a)^(1/3)/(1/d*(a*d-b*c))^(1/3)*((-(-2*arctan(1/3*(a^(1/3)+2*(b*x
^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)+ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^
(1/3)+a^(2/3))-2*ln((b*x^3+a)^(1/3)-a^(1/3)))*(a*d-b*c)*(b*x^3+a)^(1/3)-6*a
^(1/3)*b*c)*(1/d*(a*d-b*c))^(1/3)+(-2*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)
+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3))*3^(1/2)+ln((b*x^3+a)^(2/3)+(
1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3))-2*ln((b*x^3+a)^
(1/3)-(1/d*(a*d-b*c))^(1/3)))*a^(4/3)*d*(b*x^3+a)^(1/3))/a^(4/3)/(a*d-b*c)/
c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 460 vs. 2(216) = 432.

Time = 0.34 (sec) , antiderivative size = 975, normalized size of antiderivative = 3.60

$$\int \frac{1}{x\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*(6*(b*x^3 + a)^(2/3)*a*b*c + 3*sqrt(1/3)*(a^2*b*c - a^3*d + (a*b^2*c -
 a^2*b*d)*x^3)*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(
2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 +
 a)^(1/3)*a^(2/3) + 3*a)/x^3) + 2*sqrt(3)*(a^2*b*d*x^3 + a^3*d)*(d/(b*c - a
*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(d/(b*c - a*d))^(1/3) - 1/3
*sqrt(3)) - ((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*a^(2/3)*log((b*x^3 + a)^(
2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*((b^2*c - a*b*d)*x^3 + a*b*
c - a^2*d)*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)) + (a^2*b*d*x^3 + a^3*d)
*(d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(d/(b*c - a*d))^(
2/3) + (b*x^3 + a)^(2/3)*d + (b*c - a*d)*(d/(b*c - a*d))^(1/3)) - 2*(a^2*b*
d*x^3 + a^3*d)*(d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(d/(b*c - a*d))^(2/3)
+ (b*x^3 + a)^(1/3)*d))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^
3), 1/6*(6*(b*x^3 + a)^(2/3)*a*b*c + 2*sqrt(3)*(a^2*b*d*x^3 + a^3*d)*(d/(b*
c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(d/(b*c - a*d))^(1/3)
```

```
- 1/3*sqrt(3)) - ((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*a^(2/3)*log((b*x^3 +
 a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*((b^2*c - a*b*d)*x^3 +
 a*b*c - a^2*d)*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)) + (a^2*b*d*x^3 + a
^3*d)*(d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(d/(b*c - a*
d))^(2/3) + (b*x^3 + a)^(2/3)*d + (b*c - a*d)*(d/(b*c - a*d))^(1/3)) - 2*(a
^2*b*d*x^3 + a^3*d)*(d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(d/(b*c - a*d))^(
2/3) + (b*x^3 + a)^(1/3)*d) + 6*sqrt(1/3)*(a^2*b*c - a^3*d + (a*b^2*c - a^2
*b*d)*x^3)*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3
))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^3)]
```

Sympy [F]

$$\int \frac{1}{x\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\int \frac{1}{x\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

```
[In] integrate(1/x/(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(1/(x*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)x}\,dx$$

```
[In] integrate(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.44

$$\int \frac{1}{x\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=-\frac{d^2\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^2c^3-2\,abc^2d+a^2cd^2\right)}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^3-2\sqrt{3}abc^2d+\sqrt{3}a^2cd^2}+\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\left(b^2c^3-2\,abc^2d+a^2cd^2\right)}+\frac{b}{\left(bx^3+a\right)^{\frac{1}{3}}\left(abc-a^2d\right)}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{4}{3}}c}-\frac{\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{4}{3}}c}+\frac{\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{4}{3}}c}$$

```
[In] integrate(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*d^2*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d
)^(1/3)))/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2) - (-b*c*d^2 + a*d^3)^(2/3)*ar
ctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*
d)/d)^(1/3))/(sqrt(3)*b^2*c^3 - 2*sqrt(3)*a*b*c^2*d + sqrt(3)*a^2*c*d^2) +
1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b
*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b^2*c^3 - 2*a*b*c^2*d + a^2*c
*d^2) + b/((b*x^3 + a)^(1/3)*(a*b*c - a^2*d)) + 1/3*sqrt(3)*arctan(1/3*sqrt
(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(4/3)*c) - 1/6*log((b*x^3 +
 a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(4/3)*c) + 1/3*log(abs(
(b*x^3 + a)^(1/3) - a^(1/3)))/(a^(4/3)*c)
```

Mupad [B] (verification not implemented)

Time = 9.71 (sec) , antiderivative size = 3804, normalized size of antiderivative = 14.04

$$\int \frac{1}{x\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\text{Too large to display}$$

```
[In] int(1/(x*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] log(9*a^7*b^14*c^11*d^4 - ((a + b*x^3)^(1/3)*(27*a^7*b^15*c^13*d^3 - 297*a^
8*b^14*c^12*d^4 + 1485*a^9*b^13*c^11*d^5 - 4455*a^10*b^12*c^10*d^6 + 8937*a
```

^11*b^11*c^9*d^7 - 12663*a^12*b^10*c^8*d^8 + 13041*a^13*b^9*c^7*d^9 - 9855*a^14*b^8*c^6*d^10 + 5400*a^15*b^7*c^5*d^11 - 2052*a^16*b^6*c^4*d^12 + 486*a^17*b^5*c^3*d^13 - 54*a^18*b^4*c^2*d^14) - (-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(2/3)*(243*a^10*b^15*c^15*d^3 - 2916*a^11*b^14*c^14*d^4 + 15795*a^12*b^13*c^13*d^5 - 51030*a^13*b^12*c^12*d^6 + 109350*a^14*b^11*c^11*d^7 - 163296*a^15*b^10*c^10*d^8 + 173502*a^16*b^9*c^9*d^9 - 131220*a^17*b^8*c^8*d^10 + 69255*a^18*b^7*c^7*d^11 - 24300*a^19*b^6*c^6*d^12 + 5103*a^20*b^5*c^5*d^13 - 486*a^21*b^4*c^4*d^14))*(-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(1/3) - 90*a^8*b^13*c^10*d^5 + 405*a^9*b^12*c^9*d^6 - 1071*a^10*b^11*c^8*d^7 + 1827*a^11*b^10*c^7*d^8 - 2079*a^12*b^9*c^6*d^9 + 1575*a^13*b^8*c^5*d^10 - 765*a^14*b^7*c^4*d^11 + 216*a^15*b^6*c^3*d^12 - 27*a^16*b^5*c^2*d^13)*(-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(1/3) + log(9*a^7*b^14*c^11*d^4 - ((a + b*x^3)^(1/3)*(27*a^7*b^15*c^13*d^3 - 297*a^8*b^14*c^12*d^4 + 1485*a^9*b^13*c^11*d^5 - 4455*a^10*b^12*c^10*d^6 + 8937*a^11*b^11*c^9*d^7 - 12663*a^12*b^10*c^8*d^8 + 13041*a^13*b^9*c^7*d^9 - 9855*a^14*b^8*c^6*d^10 + 5400*a^15*b^7*c^5*d^11 - 2052*a^16*b^6*c^4*d^12 + 486*a^17*b^5*c^3*d^13 - 54*a^18*b^4*c^2*d^14) - (1/(27*a^4*c^3))^(2/3)*(243*a^10*b^15*c^15*d^3 - 2916*a^11*b^14*c^14*d^4 + 15795*a^12*b^13*c^13*d^5 - 51030*a^13*b^12*c^12*d^6 + 109350*a^14*b^11*c^11*d^7 - 163296*a^15*b^10*c^10*d^8 + 173502*a^16*b^9*c^9*d^9 - 131220*a^17*b^8*c^8*d^10 + 69255*a^18*b^7*c^7*d^11 - 24300*a^19*b^6*c^6*d^12 + 5103*a^20*b^5*c^5*d^13 - 486*a^21*b^4*c^4*d^14))*(1/(27*a^4*c^3))^(1/3) - 90*a^8*b^13*c^10*d^5 + 405*a^9*b^12*c^9*d^6 - 1071*a^10*b^11*c^8*d^7 + 1827*a^11*b^10*c^7*d^8 - 2079*a^12*b^9*c^6*d^9 + 1575*a^13*b^8*c^5*d^10 - 765*a^14*b^7*c^4*d^11 + 216*a^15*b^6*c^3*d^12 - 27*a^16*b^5*c^2*d^13)*(1/(27*a^4*c^3))^(1/3) + (log(((3^(1/2)*1i - 1)*(-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(1/3)*((a + b*x^3)^(1/3)*(27*a^7*b^15*c^13*d^3 - 297*a^8*b^14*c^12*d^4 + 1485*a^9*b^13*c^11*d^5 - 4455*a^10*b^12*c^10*d^6 + 8937*a^11*b^11*c^9*d^7 - 12663*a^12*b^10*c^8*d^8 + 13041*a^13*b^9*c^7*d^9 - 9855*a^14*b^8*c^6*d^10 + 5400*a^15*b^7*c^5*d^11 - 2052*a^16*b^6*c^4*d^12 + 486*a^17*b^5*c^3*d^13 - 54*a^18*b^4*c^2*d^14) - ((3^(1/2)*1i - 1)^2*(-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(2/3)*(243*a^10*b^15*c^15*d^3 - 2916*a^11*b^14*c^14*d^4 + 15795*a^12*b^13*c^13*d^5 - 51030*a^13*b^12*c^12*d^6 + 109350*a^14*b^11*c^11*d^7 - 163296*a^15*b^10*c^10*d^8 + 173502*a^16*b^9*c^9*d^9 - 131220*a^17*b^8*c^8*d^10 + 69255*a^18*b^7*c^7*d^11 - 24300*a^19*b^6*c^6*d^12 + 5103*a^20*b^5*c^5*d^13 - 486*a^21*b^4*c^4*d^14))/4))/2 - 9*a^7*b^14*c^11*d^4 + 90*a^8*b^13*c^10*d^5 - 405*a^9*b^12*c^9*d^6 + 1071*a^10*b^11*c^8*d^7 - 1827*a^11*b^10*c^7*d^8 + 2079*a^12*b^9*c^6*d^9 - 1575*a^13*b^8*c^5*d^10 + 765*a^14*b^7*c^4*d^11 - 216*a^15*b^6*c^3*d^12 + 27*a^16*b^5*c^2*d^13)*(3^(1/2)*1i - 1)*(-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(1/3))/2 - (log(((3^(1/2)*1i + 1)*(-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(1/3)*((a + b*x^3)^

(1/3)*(27*a^7*b^15*c^13*d^3 - 297*a^8*b^14*c^12*d^4 + 1485*a^9*b^13*c^11*d^5 - 4455*a^10*b^12*c^10*d^6 + 8937*a^11*b^11*c^9*d^7 - 12663*a^12*b^10*c^8*d^8 + 13041*a^13*b^9*c^7*d^9 - 9855*a^14*b^8*c^6*d^10 + 5400*a^15*b^7*c^5*d^11 - 2052*a^16*b^6*c^4*d^12 + 486*a^17*b^5*c^3*d^13 - 54*a^18*b^4*c^2*d^14) - ((3^(1/2)*1i + 1)^2*(-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(2/3)*(243*a^10*b^15*c^15*d^3 - 2916*a^11*b^14*c^14*d^4 + 15795*a^12*b^13*c^13*d^5 - 51030*a^13*b^12*c^12*d^6 + 109350*a^14*b^11*c^11*d^7 - 163296*a^15*b^10*c^10*d^8 + 173502*a^16*b^9*c^9*d^9 - 131220*a^17*b^8*c^8*d^10 + 69255*a^18*b^7*c^7*d^11 - 24300*a^19*b^6*c^6*d^12 + 5103*a^20*b^5*c^5*d^13 - 486*a^21*b^4*c^4*d^14))/4))/2 + 9*a^7*b^14*c^11*d^4 - 90*a^8*b^13*c^10*d^5 + 405*a^9*b^12*c^9*d^6 - 1071*a^10*b^11*c^8*d^7 + 1827*a^11*b^10*c^7*d^8 - 2079*a^12*b^9*c^6*d^9 + 1575*a^13*b^8*c^5*d^10 - 765*a^14*b^7*c^4*d^11 + 216*a^15*b^6*c^3*d^12 - 27*a^16*b^5*c^2*d^13)*(3^(1/2)*1i + 1)*(-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(1/3))/2 - b/((a + b*x^3)^(1/3)*(a^2*d - a*b*c)) + log(((a + b*x^3)^(1/3)*(27*a^7*b^15*c^13*d^3 - 297*a^8*b^14*c^12*d^4 + 1485*a^9*b^13*c^11*d^5 - 4455*a^10*b^12*c^10*d^6 + 8937*a^11*b^11*c^9*d^7 - 12663*a^12*b^10*c^8*d^8 + 13041*a^13*b^9*c^7*d^9 - 9855*a^14*b^8*c^6*d^10 + 5400*a^15*b^7*c^5*d^11 - 2052*a^16*b^6*c^4*d^12 + 486*a^17*b^5*c^3*d^13 - 54*a^18*b^4*c^2*d^14) - ((3^(1/2)*1i)/2 - 1/2)^2*(1/(27*a^4*c^3))^(2/3)*(243*a^10*b^15*c^15*d^3 - 2916*a^11*b^14*c^14*d^4 + 15795*a^12*b^13*c^13*d^5 - 51030*a^13*b^12*c^12*d^6 + 109350*a^14*b^11*c^11*d^7 - 163296*a^15*b^10*c^10*d^8 + 173502*a^16*b^9*c^9*d^9 - 131220*a^17*b^8*c^8*d^10 + 69255*a^18*b^7*c^7*d^11 - 24300*a^19*b^6*c^6*d^12 + 5103*a^20*b^5*c^5*d^13 - 486*a^21*b^4*c^4*d^14))*((3^(1/2)*1i)/2 - 1/2)*(1/(27*a^4*c^3))^(1/3) - 9*a^7*b^14*c^11*d^4 + 90*a^8*b^13*c^10*d^5 - 405*a^9*b^12*c^9*d^6 + 1071*a^10*b^11*c^8*d^7 - 1827*a^11*b^10*c^7*d^8 + 2079*a^12*b^9*c^6*d^9 - 1575*a^13*b^8*c^5*d^10 + 765*a^14*b^7*c^4*d^11 - 216*a^15*b^6*c^3*d^12 + 27*a^16*b^5*c^2*d^13)*((3^(1/2)*1i)/2 - 1/2)*(1/(27*a^4*c^3))^(1/3) - log(((a + b*x^3)^(1/3)*(27*a^7*b^15*c^13*d^3 - 297*a^8*b^14*c^12*d^4 + 1485*a^9*b^13*c^11*d^5 - 4455*a^10*b^12*c^10*d^6 + 8937*a^11*b^11*c^9*d^7 - 12663*a^12*b^10*c^8*d^8 + 13041*a^13*b^9*c^7*d^9 - 9855*a^14*b^8*c^6*d^10 + 5400*a^15*b^7*c^5*d^11 - 2052*a^16*b^6*c^4*d^12 + 486*a^17*b^5*c^3*d^13 - 54*a^18*b^4*c^2*d^14) - ((3^(1/2)*1i)/2 + 1/2)^2*(1/(27*a^4*c^3))^(2/3)*(243*a^10*b^15*c^15*d^3 - 2916*a^11*b^14*c^14*d^4 + 15795*a^12*b^13*c^13*d^5 - 51030*a^13*b^12*c^12*d^6 + 109350*a^14*b^11*c^11*d^7 - 163296*a^15*b^10*c^10*d^8 + 173502*a^16*b^9*c^9*d^9 - 131220*a^17*b^8*c^8*d^10 + 69255*a^18*b^7*c^7*d^11 - 24300*a^19*b^6*c^6*d^12 + 5103*a^20*b^5*c^5*d^13 - 486*a^21*b^4*c^4*d^14))*((3^(1/2)*1i)/2 + 1/2)*(1/(27*a^4*c^3))^(1/3) + 9*a^7*b^14*c^11*d^4 - 90*a^8*b^13*c^10*d^5 + 405*a^9*b^12*c^9*d^6 - 1071*a^10*b^11*c^8*d^7 + 1827*a^11*b^10*c^7*d^8 - 2079*a^12*b^9*c^6*d^9 + 1575*a^13*b^8*c^5*d^10 - 765*a^14*b^7*c^4*d^11 + 216*a^15*b^6*c^3*d^12 - 27*a^16*b^5*c^2*d^13)*((3^(1/2)*1i)/2 + 1/2)*(1/(27*a^4*c^3))^(1/3)

3.753 $\int \frac{1}{x^4(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 357

$$\int \frac{1}{x^4(a+bx^3)^{4/3}(c+dx^3)}\,dx = -\frac{d^2}{c^2(bc-ad)\sqrt[3]{a+bx^3}} - \frac{4bc+3ad}{3a^2c^2\sqrt[3]{a+bx^3}} - \frac{1}{3acx^3\sqrt[3]{a+bx^3}} - \frac{(4bc+3ad)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}c^2} + \frac{d^{7/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2(bc-ad)^{4/3}} + \frac{(4bc+3ad)\log(x)}{6a^{7/3}c^2} - \frac{d^{7/3}\log(c+dx^3)}{6c^2(bc-ad)^{4/3}} - \frac{(4bc+3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{7/3}c^2} + \frac{d^{7/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2(bc-ad)^{4/3}}$$

```
[Out] -d^2/c^2/(-a*d+b*c)/(b*x^3+a)^(1/3)+1/3*(-3*a*d-4*b*c)/a^2/c^2/(b*x^3+a)^(1
/3)-1/3/a/c/x^3/(b*x^3+a)^(1/3)+1/6*(3*a*d+4*b*c)*ln(x)/a^(7/3)/c^2-1/6*d^(
7/3)*ln(d*x^3+c)/c^2/(-a*d+b*c)^(4/3)-1/6*(3*a*d+4*b*c)*ln(a^(1/3)-(b*x^3+a
)^(1/3))/a^(7/3)/c^2+1/2*d^(7/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3
))/c^2/(-a*d+b*c)^(4/3)-1/9*(3*a*d+4*b*c)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(
1/3))/a^(1/3)*3^(1/2))/a^(7/3)/c^2*3^(1/2)+1/3*d^(7/3)*arctan(1/3*(1-2*d^(1
/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c^2/(-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {457, 105, 162, 53, 57, 631, 210, 31, 58}

$$\int \frac{1}{x^4 \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)}\,dx = -\frac{\arctan\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)(3ad+4bc)}{3\sqrt{3}a^{7/3}c^2} - \frac{(3ad+4bc)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{7/3}c^2} + \frac{\log(x)(3ad+4bc)}{6a^{7/3}c^2} - \frac{3ad+4bc}{3a^2c^2\sqrt[3]{a+bx^3}} + \frac{d^{7/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2(bc-ad)^{4/3}} - \frac{d^{7/3}\log\left(c+dx^3\right)}{6c^2(bc-ad)^{4/3}} + \frac{d^{7/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2(bc-ad)^{4/3}} - \frac{d^2}{c^2\sqrt[3]{a+bx^3}(bc-ad)} - \frac{1}{3acx^3\sqrt[3]{a+bx^3}}$$

```
[In] Int[1/(x^4*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(d^2/(c^2*(b*c - a*d)*(a + b*x^3)^(1/3))) - (4*b*c + 3*a*d)/(3*a^2*c^2*(a
+ b*x^3)^(1/3)) - 1/(3*a*c*x^3*(a + b*x^3)^(1/3)) - ((4*b*c + 3*a*d)*ArcTan
[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)*c^2
) + (d^(7/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/S
qrt[3]])/(Sqrt[3]*c^2*(b*c - a*d)^(4/3)) + ((4*b*c + 3*a*d)*Log[x])/(6*a^(7
/3)*c^2) - (d^(7/3)*Log[c + d*x^3])/(6*c^2*(b*c - a*d)^(4/3)) - ((4*b*c + 3
*a*d)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(6*a^(7/3)*c^2) + (d^(7/3)*Log[(b*c
 - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^2*(b*c - a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{1}{3acx^3\sqrt[3]{a+bx^3}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{3}(4bc+3ad)+\frac{4bdx}{3}}{x(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{1}{3acx^3\sqrt[3]{a+bx^3}} + \frac{d^2\text{Subst}\left(\int \frac{1}{(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)}{3c^2} - \frac{(4bc+3ad)\text{Subst}\left(\int \frac{1}{x(a+bx)^{4/3}}\,dx, x, x^3\right)}{9ac^2}$$

$$= -\frac{d^2}{c^2(bc-ad)\sqrt[3]{a+bx^3}} - \frac{4bc+3ad}{3a^2c^2\sqrt[3]{a+bx^3}} - \frac{1}{3acx^3\sqrt[3]{a+bx^3}} - \frac{d^3\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}(c+dx)}\,dx, x, x^3\right)}{3c^2(bc-ad)} - \frac{(4bc+3ad)\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^3\right)}{9a^2c^2}$$

$$= -\frac{d^2}{c^2(bc-ad)\sqrt[3]{a+bx^3}} - \frac{4bc+3ad}{3a^2c^2\sqrt[3]{a+bx^3}} - \frac{1}{3acx^3\sqrt[3]{a+bx^3}} + \frac{(4bc+3ad)\log(x)}{6a^{7/3}c^2} - \frac{d^{7/3}\log(c+dx^3)}{6c^2(bc-ad)^{4/3}} + \frac{d^{7/3}\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c^2(bc-ad)^{4/3}} - \frac{d^2\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{d}}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c^2(bc-ad)} + \frac{(4bc+3ad)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{6a^{7/3}c^2} - \frac{(4bc+3ad)\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{6a^2c^2}$$

$$= -\frac{d^2}{c^2(bc-ad)\sqrt[3]{a+bx^3}} - \frac{4bc+3ad}{3a^2c^2\sqrt[3]{a+bx^3}} - \frac{1}{3acx^3\sqrt[3]{a+bx^3}} + \frac{(4bc+3ad)\log(x)}{6a^{7/3}c^2} - \frac{d^{7/3}\log(c+dx^3)}{6c^2(bc-ad)^{4/3}} - \frac{(4bc+3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{7/3}c^2} + \frac{d^{7/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2(bc-ad)^{4/3}} - \frac{d^{7/3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}\right)}{c^2(bc-ad)^{4/3}} + \frac{(4bc+3ad)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right)}{3a^{7/3}c^2}$$

$$= -\frac{d^2}{c^2(bc-ad)\sqrt[3]{a+bx^3}} - \frac{4bc+3ad}{3a^2c^2\sqrt[3]{a+bx^3}} - \frac{1}{3acx^3\sqrt[3]{a+bx^3}} - \frac{(4bc+3ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}a^{7/3}c^2} + \frac{d^{7/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c^2(bc-ad)^{4/3}} + \frac{(4bc+3ad)\log(x)}{6a^{7/3}c^2} - \frac{d^{7/3}\log(c+dx^3)}{6c^2(bc-ad)^{4/3}} - \frac{(4bc+3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{7/3}c^2} + \frac{d^{7/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c^2(bc-ad)^{4/3}}$$

Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^4 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \frac{\frac{6c(-a^2d+4b^2cx^3+ab(c-dx^3))}{a^2(-bc+ad)x^3\sqrt[3]{a+bx^3}} - \frac{2\sqrt{3}(4bc+3ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{7/3}} + \frac{6\sqrt{3}d^{7/3}\arctan\left(\frac{1 \ldots}{\ldots}\right)}{(bc \ldots}}{\ldots}$$

```
[In] Integrate[1/(x^4*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] ((6*c*(-(a^2*d) + 4*b^2*c*x^3 + a*b*(c - d*x^3)))/(a^2*(-(b*c) + a*d)*x^3*(
a + b*x^3)^(1/3)) - (2*Sqrt[3]*(4*b*c + 3*a*d)*ArcTan[(1 + (2*(a + b*x^3)^(
1/3))/a^(1/3))/Sqrt[3]])/a^(7/3) + (6*Sqrt[3]*d^(7/3)*ArcTan[(1 - (2*d^(1/3
)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(b*c - a*d)^(4/3) - (2*(4
*b*c + 3*a*d)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/a^(7/3) + (6*d^(7/3)*Log[(
b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(4/3) + ((4*b*c
+ 3*a*d)*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/a^(7
/3) - (3*d^(7/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x
^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(4/3))/(18*c^2)
```

Maple [A] (verified)

Time = 5.00 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.98

method	result
pseudoelliptic	$-\frac{\left(x^3\left(\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^3+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+\ln\left(\left(b\,x^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-\frac{\ln\left(\left(b\,x^3+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2}\right)\left(ad+\frac{4bc}{3}\right)\left(ad-bc \ldots\right.}{\ldots}$

```
[In] int(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*((x^3*(arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)
+ln((b*x^3+a)^(1/3)-a^(1/3))-1/2*ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)
+a^(2/3)))*(a*d+4/3*b*c)*(a*d-b*c)*(b*x^3+a)^(1/3)-(-a^(7/3)*d+b*((-d*x^3+c
)*a^(4/3)+4*b*c*x^3*a^(1/3)))*c)*(1/d*(a*d-b*c))^(1/3)-a^(7/3)*x^3*(arctan(
1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(1/3))/(1/d*(a*d-b*c))^(1/3)
)*3^(1/2)+ln((b*x^3+a)^(1/3)-(1/d*(a*d-b*c))^(1/3))-1/2*ln((b*x^3+a)^(2/3)+
(1/d*(a*d-b*c))^(1/3)*(b*x^3+a)^(1/3)+(1/d*(a*d-b*c))^(2/3)))*d^2*(b*x^3+a)
^(1/3))/a^(7/3)/(1/d*(a*d-b*c))^(1/3)/(b*x^3+a)^(1/3)/(a*d-b*c)/c^2/x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 670 vs. $2(292) = 584$.

Time = 0.82 (sec) , antiderivative size = 1386, normalized size of antiderivative = 3.88

$$\int \frac{1}{x^4 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \text{Too large to display}$$

```
[In] integrate(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*((4*a*b^3*c^2 - a^2*b^2*c*d - 3*a^3*b*d^2)*x^6 + (4*a^2*
b^2*c^2 - a^3*b*c*d - 3*a^4*d^2)*x^3)*sqrt((-a)^(1/3)/a)*log((2*b*x^3 - 3*s
qrt(1/3)*(2*(b*x^3 + a)^(2/3)*(-a)^(2/3) - (b*x^3 + a)^(1/3)*a + (-a)^(1/3)
*a)*sqrt((-a)^(1/3)/a) - 3*(b*x^3 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^3) - 6*sqr
t(3)*(a^3*b*d^2*x^6 + a^4*d^2*x^3)*(-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3
)*(b*x^3 + a)^(1/3)*(-d/(b*c - a*d))^(1/3) + 1/3*sqrt(3)) + ((4*b^3*c^2 - a
*b^2*c*d - 3*a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - a^2*b*c*d - 3*a^3*d^2)*x^3)*(-
a)^(2/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3))
 - 2*((4*b^3*c^2 - a*b^2*c*d - 3*a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - a^2*b*c*d
- 3*a^3*d^2)*x^3)*(-a)^(2/3)*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) + 3*(a^3*b
*d^2*x^6 + a^4*d^2*x^3)*(-d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c
- a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*d - (b*c - a*d)*(-d/(b*c
- a*d))^(1/3)) - 6*(a^3*b*d^2*x^6 + a^4*d^2*x^3)*(-d/(b*c - a*d))^(1/3)*log
((b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1/3)*d) - 6*(a^2*b*c^2 -
 a^3*c*d + (4*a*b^2*c^2 - a^2*b*c*d)*x^3)*(b*x^3 + a)^(2/3))/((a^3*b^2*c^3
- a^4*b*c^2*d)*x^6 + (a^4*b*c^3 - a^5*c^2*d)*x^3), -1/18*(6*sqrt(1/3)*((4*a
*b^3*c^2 - a^2*b^2*c*d - 3*a^3*b*d^2)*x^6 + (4*a^2*b^2*c^2 - a^3*b*c*d - 3*
a^4*d^2)*x^3)*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) - (
-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + 6*sqrt(3)*(a^3*b*d^2*x^6 + a^4*d^2*x^3)*(
-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(-d/(b*c - a*d))
^(1/3) + 1/3*sqrt(3)) - ((4*b^3*c^2 - a*b^2*c*d - 3*a^2*b*d^2)*x^6 + (4*a*b
^2*c^2 - a^2*b*c*d - 3*a^3*d^2)*x^3)*(-a)^(2/3)*log((b*x^3 + a)^(2/3) - (b*
x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 2*((4*b^3*c^2 - a*b^2*c*d - 3*a^2
*b*d^2)*x^6 + (4*a*b^2*c^2 - a^2*b*c*d - 3*a^3*d^2)*x^3)*(-a)^(2/3)*log((b*
x^3 + a)^(1/3) + (-a)^(1/3)) - 3*(a^3*b*d^2*x^6 + a^4*d^2*x^3)*(-d/(b*c - a
*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x
^3 + a)^(2/3)*d - (b*c - a*d)*(-d/(b*c - a*d))^(1/3)) + 6*(a^3*b*d^2*x^6 +
a^4*d^2*x^3)*(-d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(-d/(b*c - a*d))^(2/3)
+ (b*x^3 + a)^(1/3)*d) + 6*(a^2*b*c^2 - a^3*c*d + (4*a*b^2*c^2 - a^2*b*c*d)
*x^3)*(b*x^3 + a)^(2/3))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^6 + (a^4*b*c^3 - a^
5*c^2*d)*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \int \frac{1}{x^4 (a+bx^3)^{\frac{4}{3}} (c+dx^3)} \, dx$$

[In] integrate(1/x**4/(b*x**3+a)**(4/3)/(d*x**3+c),x)

[Out] Integral(1/(x**4*(a + b*x**3)**(4/3)*(c + d*x**3)), x)

Maxima [F]

$$\int \frac{1}{x^4 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}} (dx^3+c) x^4} \, dx$$

[In] integrate(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")

[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^4), x)

Giac [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.35

$$\begin{aligned}\int \frac{1}{x^4 (a+bx^3)^{4/3} (c+dx^3)} \, dx &= \frac{d^3 \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}} \log \left(\left|(bx^3+a)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3 \, (b^2c^4 - 2\,abc^3d + a^2c^2d^2)} \\ &+ \frac{(-bcd^2+ad^3)^{\frac{2}{3}} d \arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^4 - 2\sqrt{3}abc^3d + \sqrt{3}a^2c^2d^2} \\ &- \frac{(-bcd^2+ad^3)^{\frac{2}{3}} d \log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}} + \left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{6\,(b^2c^4 - 2\,abc^3d + a^2c^2d^2)} \\ &- \frac{4\,(bx^3+a)b^2c - 3\,ab^2c - (bx^3+a)abd}{3\,(a^2bc^2 - a^3cd)\left((bx^3+a)^{\frac{4}{3}} - (bx^3+a)^{\frac{1}{3}}a\right)} \\ &- \frac{\sqrt{3}(4\,bc+3\,ad) \arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}} + a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{7}{3}}c^2} \\ &+ \frac{(4\,bc+3\,ad) \log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{18\,a^{\frac{7}{3}}c^2} \\ &- \frac{\left(4\,a^{\frac{1}{3}}bc + 3\,a^{\frac{4}{3}}d\right) \log\left(\left|(bx^3+a)^{\frac{1}{3}} - a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{8}{3}}c^2}\end{aligned}$$

```
[In] integrate(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*d^3*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)
^(1/3)))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2) + (-b*c*d^2 + a*d^3)^(2/3)*d
*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
 a*d)/d)^(1/3))/(sqrt(3)*b^2*c^4 - 2*sqrt(3)*a*b*c^3*d + sqrt(3)*a^2*c^2*d^
2) - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*d*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/
3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b^2*c^4 - 2*a*b*c^3*d
+ a^2*c^2*d^2) - 1/3*(4*(b*x^3 + a)*b^2*c - 3*a*b^2*c - (b*x^3 + a)*a*b*d)/
((a^2*b*c^2 - a^3*c*d)*((b*x^3 + a)^(4/3) - (b*x^3 + a)^(1/3)*a)) - 1/9*sqr
t(3)*(4*b*c + 3*a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(
1/3))/(a^(7/3)*c^2) + 1/18*(4*b*c + 3*a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 +
 a)^(1/3)*a^(1/3) + a^(2/3))/(a^(7/3)*c^2) - 1/9*(4*a^(1/3)*b*c + 3*a^(4/3)
*d)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(8/3)*c^2)
```

Mupad [B] (verification not implemented)

Time = 11.00 (sec) , antiderivative size = 5875, normalized size of antiderivative = 16.46

$$\int \frac{1}{x^4 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \text{Too large to display}$$

```
[In] int(1/(x^4*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] log((d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^
8*d^2 - 108*a*b^3*c^9*d))^(2/3)*(419904*a^13*b^17*c^20*d^4 - ((a + b*x^3)^(
1/3)*(8975448*a^15*b^16*c^21*d^4 - 944784*a^14*b^17*c^22*d^3 - 36905625*a^1
6*b^15*c^20*d^5 + 83790531*a^17*b^14*c^19*d^6 - 107173935*a^18*b^13*c^18*d^
7 + 56509893*a^19*b^12*c^17*d^8 + 42338133*a^20*b^11*c^16*d^9 - 93710763*a^
21*b^10*c^15*d^10 + 55092717*a^22*b^9*c^14*d^11 + 12105045*a^23*b^8*c^13*d^
12 - 38736144*a^24*b^7*c^12*d^13 + 25745364*a^25*b^6*c^11*d^14 - 8148762*a^
26*b^5*c^10*d^15 + 1062882*a^27*b^4*c^9*d^16) + (d^7/(27*b^4*c^10 + 27*a^4*
c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(2/3)
*(4782969*a^19*b^15*c^24*d^3 - 57395628*a^20*b^14*c^23*d^4 + 310892985*a^21
*b^13*c^22*d^5 - 1004423490*a^22*b^12*c^21*d^6 + 2152336050*a^23*b^11*c^20*
d^7 - 3214155168*a^24*b^10*c^19*d^8 + 3415039866*a^25*b^9*c^18*d^9 - 258280
3260*a^26*b^8*c^17*d^10 + 1363146165*a^27*b^7*c^16*d^11 - 478296900*a^28*b^
6*c^15*d^12 + 100442349*a^29*b^5*c^14*d^13 - 9565938*a^30*b^4*c^13*d^14))*(
d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2
 - 108*a*b^3*c^9*d))^(1/3) - 3254256*a^14*b^16*c^19*d^5 + 10156428*a^15*b^1
5*c^18*d^6 - 14781933*a^16*b^14*c^17*d^7 + 4920750*a^17*b^13*c^16*d^8 + 155
29887*a^18*b^12*c^15*d^9 - 22182741*a^19*b^11*c^14*d^10 + 5412825*a^20*b^10
*c^13*d^11 + 13404123*a^21*b^9*c^12*d^12 - 15713595*a^22*b^8*c^11*d^13 + 78
01029*a^23*b^7*c^10*d^14 - 1889568*a^24*b^6*c^9*d^15 + 177147*a^25*b^5*c^8*
d^16) - (a + b*x^3)^(1/3)*(256608*a^14*b^13*c^12*d^10 - 46656*a^13*b^14*c^1
```

3*d^9 - 516132*a^15*b^12*c^11*d^11 + 347004*a^16*b^11*c^10*d^12 + 265356*a^17*b^10*c^9*d^13 - 551124*a^18*b^9*c^8*d^14 + 224532*a^19*b^8*c^7*d^15 + 107892*a^20*b^7*c^6*d^16 - 113724*a^21*b^6*c^5*d^17 + 26244*a^22*b^5*c^4*d^18))*(d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(1/3) + (b^2/(a^2*d - a*b*c) + (b*(a + b*x^3)*(a*d - 4*b*c))/(3*a^2*c*(a*d - b*c)))/(a*(a + b*x^3)^(1/3) - (a + b*x^3)^(4/3)) + log((-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^(2/3)*(419904*a^13*b^17*c^20*d^4 - ((a + b*x^3)^(1/3)*(8975448*a^15*b^16*c^21*d^4 - 944784*a^14*b^17*c^22*d^3 - 36905625*a^16*b^15*c^20*d^5 + 83790531*a^17*b^14*c^19*d^6 - 107173935*a^18*b^13*c^18*d^7 + 56509893*a^19*b^12*c^17*d^8 + 42338133*a^20*b^11*c^16*d^9 - 93710763*a^21*b^10*c^15*d^10 + 55092717*a^22*b^9*c^14*d^11 + 12105045*a^23*b^8*c^13*d^12 - 38736144*a^24*b^7*c^12*d^13 + 25745364*a^25*b^6*c^11*d^14 - 8148762*a^26*b^5*c^10*d^15 + 1062882*a^27*b^4*c^9*d^16) + (-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^(2/3)*(4782969*a^19*b^15*c^24*d^3 - 57395628*a^20*b^14*c^23*d^4 + 310892985*a^21*b^13*c^22*d^5 - 1004423490*a^22*b^12*c^21*d^6 + 2152336050*a^23*b^11*c^20*d^7 - 3214155168*a^24*b^10*c^19*d^8 + 3415039866*a^25*b^9*c^18*d^9 - 2582803260*a^26*b^8*c^17*d^10 + 1363146165*a^27*b^7*c^16*d^11 - 478296900*a^28*b^6*c^15*d^12 + 100442349*a^29*b^5*c^14*d^13 - 9565938*a^30*b^4*c^13*d^14))*(-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^(1/3) - 3254256*a^14*b^16*c^19*d^5 + 10156428*a^15*b^15*c^18*d^6 - 14781933*a^16*b^14*c^17*d^7 + 4920750*a^17*b^13*c^16*d^8 + 15529887*a^18*b^12*c^15*d^9 - 22182741*a^19*b^11*c^14*d^10 + 5412825*a^20*b^10*c^13*d^11 + 13404123*a^21*b^9*c^12*d^12 - 15713595*a^22*b^8*c^11*d^13 + 7801029*a^23*b^7*c^10*d^14 - 1889568*a^24*b^6*c^9*d^15 + 177147*a^25*b^5*c^8*d^16) - (a + b*x^3)^(1/3)*(256608*a^14*b^13*c^12*d^10 - 46656*a^13*b^14*c^13*d^9 - 516132*a^15*b^12*c^11*d^11 + 347004*a^16*b^11*c^10*d^12 + 265356*a^17*b^10*c^9*d^13 - 551124*a^18*b^9*c^8*d^14 + 224532*a^19*b^8*c^7*d^15 + 107892*a^20*b^7*c^6*d^16 - 113724*a^21*b^6*c^5*d^17 + 26244*a^22*b^5*c^4*d^18))*(-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^(1/3) + (log(((3^(1/2)*1i - 1)^2*(d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(2/3)*(419904*a^13*b^17*c^20*d^4 - ((3^(1/2)*1i - 1)*(d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(1/3)*((a + b*x^3)^(1/3)*(8975448*a^15*b^16*c^21*d^4 - 944784*a^14*b^17*c^22*d^3 - 36905625*a^16*b^15*c^20*d^5 + 83790531*a^17*b^14*c^19*d^6 - 107173935*a^18*b^13*c^18*d^7 + 56509893*a^19*b^12*c^17*d^8 + 42338133*a^20*b^11*c^16*d^9 - 93710763*a^21*b^10*c^15*d^10 + 55092717*a^22*b^9*c^14*d^11 + 12105045*a^23*b^8*c^13*d^12 - 38736144*a^24*b^7*c^12*d^13 + 25745364*a^25*b^6*c^11*d^14 - 8148762*a^26*b^5*c^10*d^15 + 1062882*a^27*b^4*c^9*d^16) + ((3^(1/2)*1i - 1)^2*(d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(2/3)*(4782969*a^19*b^15*c^24*d^3 - 57395628*a^20*b^14*c^23*d^4 + 310892985*a^21*b^13*c^22*d^5 - 1004423490*a^22*b^12*c^21*d^6 + 2152336050*a^23*b^11*c^20*d^7 - 3214155168*a^24*b^10*c^19*d^8 + 3415039866*a^25*b^9*c^18*d^9 - 2582803260*a^26*b^8*c^17*d^10

+ 1363146165*a^27*b^7*c^16*d^11 - 478296900*a^28*b^6*c^15*d^12 + 100442349*a^29*b^5*c^14*d^13 - 9565938*a^30*b^4*c^13*d^14))/4))/2 - 3254256*a^14*b^16*c^19*d^5 + 10156428*a^15*b^15*c^18*d^6 - 14781933*a^16*b^14*c^17*d^7 + 4920750*a^17*b^13*c^16*d^8 + 15529887*a^18*b^12*c^15*d^9 - 22182741*a^19*b^11*c^14*d^10 + 5412825*a^20*b^10*c^13*d^11 + 13404123*a^21*b^9*c^12*d^12 - 15713595*a^22*b^8*c^11*d^13 + 7801029*a^23*b^7*c^10*d^14 - 1889568*a^24*b^6*c^9*d^15 + 177147*a^25*b^5*c^8*d^16))/4 - (a + b*x^3)^(1/3)*(256608*a^14*b^13*c^12*d^10 - 46656*a^13*b^14*c^13*d^9 - 516132*a^15*b^12*c^11*d^11 + 347004*a^16*b^11*c^10*d^12 + 265356*a^17*b^10*c^9*d^13 - 551124*a^18*b^9*c^8*d^14 + 224532*a^19*b^8*c^7*d^15 + 107892*a^20*b^7*c^6*d^16 - 113724*a^21*b^6*c^5*d^17 + 26244*a^22*b^5*c^4*d^18))*(3^(1/2)*1i - 1)*(d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(1/3))/2 - (log(((3^(1/2)*1i + 1)^2*(d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(2/3)*(((3^(1/2)*1i + 1)*(d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(1/3)*((a + b*x^3)^(1/3)*(8975448*a^15*b^16*c^21*d^4 - 944784*a^14*b^17*c^22*d^3 - 36905625*a^16*b^15*c^20*d^5 + 83790531*a^17*b^14*c^19*d^6 - 107173935*a^18*b^13*c^18*d^7 + 56509893*a^19*b^12*c^17*d^8 + 42338133*a^20*b^11*c^16*d^9 - 93710763*a^21*b^10*c^15*d^10 + 55092717*a^22*b^9*c^14*d^11 + 12105045*a^23*b^8*c^13*d^12 - 38736144*a^24*b^7*c^12*d^13 + 25745364*a^25*b^6*c^11*d^14 - 8148762*a^26*b^5*c^10*d^15 + 1062882*a^27*b^4*c^9*d^16) + ((3^(1/2)*1i + 1)^2*(d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(2/3)*(4782969*a^19*b^15*c^24*d^3 - 57395628*a^20*b^14*c^23*d^4 + 310892985*a^21*b^13*c^22*d^5 - 1004423490*a^22*b^12*c^21*d^6 + 2152336050*a^23*b^11*c^20*d^7 - 3214155168*a^24*b^10*c^19*d^8 + 3415039866*a^25*b^9*c^18*d^9 - 2582803260*a^26*b^8*c^17*d^10 + 1363146165*a^27*b^7*c^16*d^11 - 478296900*a^28*b^6*c^15*d^12 + 100442349*a^29*b^5*c^14*d^13 - 9565938*a^30*b^4*c^13*d^14))/4))/2 + 419904*a^13*b^17*c^20*d^4 - 3254256*a^14*b^16*c^19*d^5 + 10156428*a^15*b^15*c^18*d^6 - 14781933*a^16*b^14*c^17*d^7 + 4920750*a^17*b^13*c^16*d^8 + 15529887*a^18*b^12*c^15*d^9 - 22182741*a^19*b^11*c^14*d^10 + 5412825*a^20*b^10*c^13*d^11 + 13404123*a^21*b^9*c^12*d^12 - 15713595*a^22*b^8*c^11*d^13 + 7801029*a^23*b^7*c^10*d^14 - 1889568*a^24*b^6*c^9*d^15 + 177147*a^25*b^5*c^8*d^16))/4 - (a + b*x^3)^(1/3)*(256608*a^14*b^13*c^12*d^10 - 46656*a^13*b^14*c^13*d^9 - 516132*a^15*b^12*c^11*d^11 + 347004*a^16*b^11*c^10*d^12 + 265356*a^17*b^10*c^9*d^13 - 551124*a^18*b^9*c^8*d^14 + 224532*a^19*b^8*c^7*d^15 + 107892*a^20*b^7*c^6*d^16 - 113724*a^21*b^6*c^5*d^17 + 26244*a^22*b^5*c^4*d^18))*(3^(1/2)*1i + 1)*(d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(1/3))/2 - log(((3^(1/2)*1i)/2 + 1/2)^2*(-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^(2/3)*(((3^(1/2)*1i)/2 + 1/2)*((a + b*x^3)^(1/3)*(8975448*a^15*b^16*c^21*d^4 - 944784*a^14*b^17*c^22*d^3 - 36905625*a^16*b^15*c^20*d^5 + 83790531*a^17*b^14*c^19*d^6 - 107173935*a^18*b^13*c^18*d^7 + 56509893*a^19*b^12*c^17*d^8 + 42338133*a^20*b^11*c^16*d^9 - 93710763*a^21*b^10*c^15*d^10 + 55092717*a^22*b^9*c^14*d^11 + 12105045*a^23*b^8*c^13*d^12 - 38736144*a^2

4*b^7*c^12*d^13 + 25745364*a^25*b^6*c^11*d^14 - 8148762*a^26*b^5*c^10*d^15 + 1062882*a^27*b^4*c^9*d^16) + ((3^(1/2)*1i)/2 + 1/2)^2*(-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^(2/3)*(4782969*a^19*b^15*c^24*d^3 - 57395628*a^20*b^14*c^23*d^4 + 310892985*a^21*b^13*c^22*d^5 - 1004423490*a^22*b^12*c^21*d^6 + 2152336050*a^23*b^11*c^20*d^7 - 3214155168*a^24*b^10*c^19*d^8 + 3415039866*a^25*b^9*c^18*d^9 - 2582803260*a^26*b^8*c^17*d^10 + 1363146165*a^27*b^7*c^16*d^11 - 478296900*a^28*b^6*c^15*d^12 + 100442349*a^29*b^5*c^14*d^13 - 9565938*a^30*b^4*c^13*d^14))*(-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^(1/3) + 419904*a^13*b^17*c^20*d^4 - 3254256*a^14*b^16*c^19*d^5 + 10156428*a^15*b^15*c^18*d^6 - 14781933*a^16*b^14*c^17*d^7 + 4920750*a^17*b^13*c^16*d^8 + 15529887*a^18*b^12*c^15*d^9 - 22182741*a^19*b^11*c^14*d^10 + 5412825*a^20*b^10*c^13*d^11 + 13404123*a^21*b^9*c^12*d^12 - 15713595*a^22*b^8*c^11*d^13 + 7801029*a^23*b^7*c^10*d^14 - 1889568*a^24*b^6*c^9*d^15 + 177147*a^25*b^5*c^8*d^16) - (a + b*x^3)^(1/3)*(256608*a^14*b^13*c^12*d^10 - 46656*a^13*b^14*c^13*d^9 - 516132*a^15*b^12*c^11*d^11 + 347004*a^16*b^11*c^10*d^12 + 265356*a^17*b^10*c^9*d^13 - 551124*a^18*b^9*c^8*d^14 + 224532*a^19*b^8*c^7*d^15 + 107892*a^20*b^7*c^6*d^16 - 113724*a^21*b^6*c^5*d^17 + 26244*a^22*b^5*c^4*d^18))*((3^(1/2)*1i)/2 + 1/2)*(-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^(1/3) + log(((3^(1/2)*1i)/2 - 1/2)^2*(-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^(2/3)*(419904*a^13*b^17*c^20*d^4 - ((3^(1/2)*1i)/2 - 1/2)*((a + b*x^3)^(1/3)*(8975448*a^15*b^16*c^21*d^4 - 944784*a^14*b^17*c^22*d^3 - 36905625*a^16*b^15*c^20*d^5 + 83790531*a^17*b^14*c^19*d^6 - 107173935*a^18*b^13*c^18*d^7 + 56509893*a^19*b^12*c^17*d^8 + 42338133*a^20*b^11*c^16*d^9 - 93710763*a^21*b^10*c^15*d^10 + 55092717*a^22*b^9*c^14*d^11 + 12105045*a^23*b^8*c^13*d^12 - 38736144*a^24*b^7*c^12*d^13 + 25745364*a^25*b^6*c^11*d^14 - 8148762*a^26*b^5*c^10*d^15 + 1062882*a^27*b^4*c^9*d^16) + ((3^(1/2)*1i)/2 - 1/2)^2*(-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^(2/3)*(4782969*a^19*b^15*c^24*d^3 - 57395628*a^20*b^14*c^23*d^4 + 310892985*a^21*b^13*c^22*d^5 - 1004423490*a^22*b^12*c^21*d^6 + 2152336050*a^23*b^11*c^20*d^7 - 3214155168*a^24*b^10*c^19*d^8 + 3415039866*a^25*b^9*c^18*d^9 - 2582803260*a^26*b^8*c^17*d^10 + 1363146165*a^27*b^7*c^16*d^11 - 478296900*a^28*b^6*c^15*d^12 + 100442349*a^29*b^5*c^14*d^13 - 9565938*a^30*b^4*c^13*d^14))*(-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^(1/3) - 3254256*a^14*b^16*c^19*d^5 + 10156428*a^15*b^15*c^18*d^6 - 14781933*a^16*b^14*c^17*d^7 + 4920750*a^17*b^13*c^16*d^8 + 15529887*a^18*b^12*c^15*d^9 - 22182741*a^19*b^11*c^14*d^10 + 5412825*a^20*b^10*c^13*d^11 + 13404123*a^21*b^9*c^12*d^12 - 15713595*a^22*b^8*c^11*d^13 + 7801029*a^23*b^7*c^10*d^14 - 1889568*a^24*b^6*c^9*d^15 + 177147*a^25*b^5*c^8*d^16) - (a + b*x^3)^(1/3)*(256608*a^14*b^13*c^12*d^10 - 46656*a^13*b^14*c^13*d^9 - 516132*a^15*b^12*c^11*d^11 + 347004*a^16*b^11*c^10*d^12 + 265356*a^17*b^10*c^9*d^13 - 551124*a^18*b^9*c^8*d^14 + 224532*a^19*b^8*c^7*d^15 + 107892*a^20*b^7*c^6*d^16 - 113724*a^21*b^6*c^5*d^17 + 26244*a^22*b^5*c^4*d^18))*((3^(1/2)*1i)/2 - 1/2)*(-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(72

```
9*a^7*c^6))^(1/3)
```

3.754 $\int \frac{x^9}{(a+bx^3)^{4/3}(c+dx^3)} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 322

$$\int \frac{x^9}{(a+bx^3)^{4/3}(c+dx^3)} \, dx = \frac{ax^4}{b(bc-ad)\sqrt[3]{a+bx^3}} + \frac{(bc-4ad)x(a+bx^3)^{2/3}}{3b^2d(bc-ad)} - \frac{(3bc+4ad)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{7/3}d^2} + \frac{c^{7/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2(bc-ad)^{4/3}} + \frac{c^{7/3}\log(c+dx^3)}{6d^2(bc-ad)^{4/3}} - \frac{c^{7/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2d^2(bc-ad)^{4/3}} + \frac{(3bc+4ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6b^{7/3}d^2}$$

```
[Out] a*x^4/b/(-a*d+b*c)/(b*x^3+a)^(1/3)+1/3*(-4*a*d+b*c)*x*(b*x^3+a)^(2/3)/b^2/d
/(-a*d+b*c)+1/6*c^(7/3)*ln(d*x^3+c)/d^2/(-a*d+b*c)^(4/3)-1/2*c^(7/3)*ln((-a
*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^2/(-a*d+b*c)^(4/3)+1/6*(4*a*d+3*
b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(7/3)/d^2-1/9*(4*a*d+3*b*c)*arctan(1/
3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(7/3)/d^2*3^(1/2)+1/3*c^(7/3)*
arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/d^2/(-
a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {481, 596, 544, 245, 384}

$$\int \frac{x^9}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = -\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)(4ad+3bc)}{3\sqrt{3}b^{7/3}d^2} + \frac{c^{7/3}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d^2(bc-ad)^{4/3}} + \frac{(4ad+3bc)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{6b^{7/3}d^2} + \frac{x(a+bx^3)^{2/3}(bc-4ad)}{3b^2d(bc-ad)} + \frac{c^{7/3}\log(c+dx^3)}{6d^2(bc-ad)^{4/3}} - \frac{c^{7/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^2(bc-ad)^{4/3}} + \frac{ax^4}{b\sqrt[3]{a+bx^3}(bc-ad)}$$

```
[In] Int[x^9/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (a*x^4)/(b*(b*c - a*d)*(a + b*x^3)^(1/3)) + ((b*c - 4*a*d)*x*(a + b*x^3)^(2
/3))/(3*b^2*d*(b*c - a*d)) - ((3*b*c + 4*a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a
+ b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(7/3)*d^2) + (c^(7/3)*ArcTan[(1 + (2
*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*d^2*(
b*c - a*d)^(4/3)) + (c^(7/3)*Log[c + d*x^3])/(6*d^2*(b*c - a*d)^(4/3)) - (c
^(7/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*d^2*(b*c
- a*d)^(4/3)) + ((3*b*c + 4*a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(6*
b^(7/3)*d^2)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
```

```
a*d, 0]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{ax^4}{b(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\int \frac{x^3(4ac+(-bc+4ad)x^3)}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{b(bc-ad)} \\
&= \frac{ax^4}{b(bc-ad)\sqrt[3]{a+bx^3}} + \frac{(bc-4ad)x(a+bx^3)^{2/3}}{3b^2d(bc-ad)} + \frac{\int \frac{-ac(bc-4ad)-(bc-ad)(3bc+4ad)x^3}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{3b^2d(bc-ad)} \\
&= \frac{ax^4}{b(bc-ad)\sqrt[3]{a+bx^3}} + \frac{(bc-4ad)x(a+bx^3)^{2/3}}{3b^2d(bc-ad)} \\
&\quad + \frac{c^3\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{d^2(bc-ad)} - \frac{(3bc+4ad)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{3b^2d^2}
\end{aligned}$$

$$= \frac{ax^4}{b(bc-ad)\sqrt[3]{a+bx^3}} + \frac{(bc-4ad)x(a+bx^3)^{2/3}}{3b^2d(bc-ad)}$$
$$- \frac{(3bc+4ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{7/3}d^2} + \frac{c^{7/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2(bc-ad)^{4/3}}$$
$$+ \frac{c^{7/3}\log(c+dx^3)}{6d^2(bc-ad)^{4/3}} - \frac{c^{7/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2d^2(bc-ad)^{4/3}}$$
$$+ \frac{(3bc+4ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6b^{7/3}d^2}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.81 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.57

$$\int \frac{x^9}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{\frac{12d(-4a^2dx+b^2cx^4+abx(c-dx^3))}{b^2(bc-ad)\sqrt[3]{a+bx^3}} - \frac{4\sqrt{3}(3bc+4ad)\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{b^{7/3}} - \frac{6\sqrt{-6+6i\sqrt{}}}{}}{}$$

```
[In] Integrate[x^9/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] ((12*d*(-4*a^2*d*x + b^2*c*x^4 + a*b*x*(c - d*x^3)))/(b^2*(b*c - a*d)*(a +
b*x^3)^(1/3)) - (4*Sqrt[3]*(3*b*c + 4*a*d)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1
/3)*x + 2*(a + b*x^3)^(1/3))])/b^(7/3) - (6*Sqrt[-6 + (6*I)*Sqrt[3]]*c^(7/3
)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt
[3])*c^(1/3)*(a + b*x^3)^(1/3))])/(b*c - a*d)^(4/3) + (4*(3*b*c + 4*a*d)*Lo
g[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/b^(7/3) + (6*(1 + I*Sqrt[3])*c^(7/3)*L
og[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*c
 - a*d)^(4/3) - (2*(3*b*c + 4*a*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^
(1/3) + (a + b*x^3)^(2/3)])/b^(7/3) - ((3*I)*(-I + Sqrt[3])*c^(7/3)*Log[2*(
b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*
x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(4/3))
/(36*d^2)
```

Maple [A] (verified)

Time = 5.41 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.53

method	result
pseudoelliptic	$\frac{-\frac{4\left(ad+\frac{3bc}{4}\right)b^{2}\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(ad-bc\right)\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{3}-2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)c^{2}b^{\frac{13}{3}}\left(b\,x\right.}{}$

```
[In] int(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-4/3*(a*d+3/4*b*c)*b^2*((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*(a*d-b*c)*l
n((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln((((a*d-
b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)*c^2*b^(13/3)*(b*x^3+a)^(1/3)+8/3*(a*d+3
/4*b*c)*b^2*3^(1/2)*((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*(a*d-b*c)*arctan(1/
3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+8/3*(a*d+3/4*b*c)*b^2*((
a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*(a*d-b*c)*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))
/x)+(8*x*d*(-1/4*b^2*c*x^3-1/4*a*(-d*x^3+c)*b+a^2*d)*((a*d-b*c)/c)^(1/3)+(-
2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c
)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a
)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*b^2*(b*x^3+a)^(1/3)*c^2)*b^(7/3))/(b*x^3+a
)^(1/3)/((a*d-b*c)/c)^(1/3)/(a*d-b*c)/d^2/b^(13/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 637 vs. 2(267) = 534.

Time = 0.80 (sec) , antiderivative size = 1329, normalized size of antiderivative = 4.13

$$\int \frac{x^9}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(3*a*b^3*c^2 + a^2*b^2*c*d - 4*a^3*b*d^2 + (3*b^4*c^2 +
a*b^3*c*d - 4*a^2*b^2*d^2)*x^3)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a
)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 -
2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) - 6*sqrt(3)*(b^4*c^2
*x^3 + a*b^3*c^2)*(c/(b*c - a*d))^(1/3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(
b*x^3 + a)^(1/3)*(c/(b*c - a*d))^(1/3))/x) + 2*(3*a*b^2*c^2 + a^2*b*c*d - 4
*a^3*d^2 + (3*b^3*c^2 + a*b^2*c*d - 4*a^2*b*d^2)*x^3)*b^(2/3)*log(-(b^(1/3)
*x - (b*x^3 + a)^(1/3))/x) - (3*a*b^2*c^2 + a^2*b*c*d - 4*a^3*d^2 + (3*b^3*
c^2 + a*b^2*c*d - 4*a^2*b*d^2)*x^3)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^
```

```
(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 6*(b^4*c^2*x^3 + a*b^3*c^2)*(c/
(b*c - a*d))^(1/3)*log(-((b*c - a*d)*x*(c/(b*c - a*d))^(2/3) - (b*x^3 + a)^
(1/3)*c)/x) + 3*(b^4*c^2*x^3 + a*b^3*c^2)*(c/(b*c - a*d))^(1/3)*log(((b*c -
 a*d)*x^2*(c/(b*c - a*d))^(1/3) + (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(c/(b*c -
 a*d))^(2/3) + (b*x^3 + a)^(2/3)*c)/x^2) + 6*((b^3*c*d - a*b^2*d^2)*x^4 + (
a*b^2*c*d - 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))/(a*b^4*c*d^2 - a^2*b^3*d^3 +
 (b^5*c*d^2 - a*b^4*d^3)*x^3), -1/18*(6*sqrt(3)*(b^4*c^2*x^3 + a*b^3*c^2)*(
c/(b*c - a*d))^(1/3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(c
/(b*c - a*d))^(1/3))/x) - 2*(3*a*b^2*c^2 + a^2*b*c*d - 4*a^3*d^2 + (3*b^3*c
^2 + a*b^2*c*d - 4*a^2*b*d^2)*x^3)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1
/3))/x) + (3*a*b^2*c^2 + a^2*b*c*d - 4*a^3*d^2 + (3*b^3*c^2 + a*b^2*c*d - 4
*a^2*b*d^2)*x^3)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (
b*x^3 + a)^(2/3))/x^2) + 6*(b^4*c^2*x^3 + a*b^3*c^2)*(c/(b*c - a*d))^(1/3)*
log(-((b*c - a*d)*x*(c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*c)/x) - 3*(b^
4*c^2*x^3 + a*b^3*c^2)*(c/(b*c - a*d))^(1/3)*log(((b*c - a*d)*x^2*(c/(b*c -
 a*d))^(1/3) + (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(c/(b*c - a*d))^(2/3) + (b*x
^3 + a)^(2/3)*c)/x^2) - 6*sqrt(1/3)*(3*a*b^3*c^2 + a^2*b^2*c*d - 4*a^3*b*d^
2 + (3*b^4*c^2 + a*b^3*c*d - 4*a^2*b^2*d^2)*x^3)*arctan(sqrt(1/3)*(b^(1/3)*
x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 6*((b^3*c*d - a*b^2*d^2)*x^
4 + (a*b^2*c*d - 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))/(a*b^4*c*d^2 - a^2*b^3*
d^3 + (b^5*c*d^2 - a*b^4*d^3)*x^3)]
```

Sympy [F]

$$\int \frac{x^9}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx = \int \frac{x^9}{(a+bx^3)^{\frac{4}{3}}\,(c+dx^3)}\,dx$$

```
[In] integrate(x**9/(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(x**9/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^9}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx = \int \frac{x^9}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(x^9/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^9}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^9}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^9/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^9}{(b\,x^3+a)^{4/3}\,(d\,x^3+c)}\,dx$$

```
[In] int(x^9/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(x^9/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.755 $\int \frac{x^6}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 260

$$\int \frac{x^6}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{ax}{b(bc-ad)\sqrt[3]{a+bx^3}}+\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}d} - \frac{c^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d(bc-ad)^{4/3}} - \frac{c^{4/3}\log(c+dx^3)}{6d(bc-ad)^{4/3}} + \frac{c^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d(bc-ad)^{4/3}} - \frac{\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2b^{4/3}d}$$

```
[Out] a*x/b/(-a*d+b*c)/(b*x^3+a)^(1/3)-1/6*c^(4/3)*ln(d*x^3+c)/d/(-a*d+b*c)^(4/3)
+1/2*c^(4/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d/(-a*d+b*c)^(4
/3)-1/2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4/3)/d+1/3*arctan(1/3*(1+2*b^(1/3
)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(4/3)/d*3^(1/2)-1/3*c^(4/3)*arctan(1/3*(1+2
*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/d/(-a*d+b*c)^(4/3)*3^
(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {481, 544, 245, 384}

$$\int \frac{x^6}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}d} - \frac{c^{4/3}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d(bc-ad)^{4/3}} - \frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2b^{4/3}d} - \frac{c^{4/3}\log\left(c+dx^3\right)}{6d(bc-ad)^{4/3}} + \frac{c^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d(bc-ad)^{4/3}} + \frac{ax}{b\sqrt[3]{a+bx^3}(bc-ad)}$$

```
[In] Int[x^6/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (a*x)/(b*(b*c - a*d)*(a + b*x^3)^(1/3)) + ArcTan[(1 + (2*b^(1/3)*x)/(a + b*
x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(4/3)*d) - (c^(4/3)*ArcTan[(1 + (2*(b*c - a
*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*d*(b*c - a*d)^
(4/3)) - (c^(4/3)*Log[c + d*x^3])/(6*d*(b*c - a*d)^(4/3)) + (c^(4/3)*Log[((
b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*d*(b*c - a*d)^(4/3)) -
 Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(4/3)*d)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
```

```
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{ax}{b(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\int \frac{ac+(-bc+ad)x^3}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{b(bc-ad)} \\
&= \frac{ax}{b(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{bd} - \frac{c^2\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{d(bc-ad)} \\
&= \frac{ax}{b(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}d} - \frac{c^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d(bc-ad)^{4/3}} \\
&\quad - \frac{c^{4/3}\log(c+dx^3)}{6d(bc-ad)^{4/3}} + \frac{c^{4/3}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2d(bc-ad)^{4/3}} - \frac{\log\left(-\sqrt[3]{b}x + \sqrt[3]{a+bx^3}\right)}{2b^{4/3}d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.68 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.79

$$\int \frac{x^6}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{1}{12}\left(\frac{12ax}{(b^2c-abd)\sqrt[3]{a+bx^3}} + \frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{b^{4/3}d} + \frac{2\sqrt{-6+6i\sqrt{3}}c^{4/3}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{d(bc-ad)^{4/3}} - \frac{4\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{b^{4/3}d} - \frac{2i(-i+\sqrt{3})c^{4/3}\log\left(2\sqrt[3]{bc-ad}x+(1+i\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{d(bc-ad)^{4/3}} + \frac{2\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{b^{4/3}d} + \frac{(1+i\sqrt{3})c^{4/3}\log\left(2(bc-ad)^{2/3}x^2+(-1-i\sqrt{3})\sqrt[3]{c}\sqrt[3]{bc-ad}x\sqrt[3]{a+bx^3}+i(i+\sqrt{3})c^{2/3}(a+bx^3)^{2/3}\right)}{d(bc-ad)^{4/3}}\right)$$

```
[In] Integrate[x^6/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] ((12*a*x)/((b^2*c - a*b*d)*(a + b*x^3)^(1/3)) + (4*Sqrt[3]*ArcTan[(Sqrt[3]*
b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(b^(4/3)*d) + (2*Sqrt[-6 + (
6*I)*Sqrt[3]]*c^(4/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(
1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))])/(d*(b*c - a*d)^(4/3))
 - (4*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(b^(4/3)*d) - ((2*I)*(-I + Sqr
t[3])*c^(4/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^
3)^(1/3)])/(d*(b*c - a*d)^(4/3)) + (2*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^
3)^(1/3) + (a + b*x^3)^(2/3)])/(b^(4/3)*d) + ((1 + I*Sqrt[3])*c^(4/3)*Log[2
*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a +
b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(d*(b*c - a*d)^(
4/3)))/12
```

Maple [A] (verified)

Time = 4.77 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.32

method	result
pseudoelliptic	$-\frac{\left(-\left(-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)+\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)-2\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)\right)(a}{}$

```
[In] int(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/(b*x^3+a)^(1/3)*((-(-2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+
a)^(1/3))/b^(1/3)/x)+ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2
/3))/x^2)-2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x))*(a*d-b*c)*(b*x^3+a)^(1/3)+6
*a*d*x*b^(1/3))*((a*d-b*c)/c)^(1/3)+(-2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(
1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln((((a*d-b*c)/c)^
(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln(
(((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x))*b^(4/3)*c*(b*x^3+a)^(1/3))/((a*
d-b*c)/c)^(1/3)/b^(4/3)/(a*d-b*c)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(211) = 422$.

Time = 0.31 (sec) , antiderivative size = 1127, normalized size of antiderivative = 4.33

$$\int \frac{x^6}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*(6*(b*x^3 + a)^(2/3)*a*b*d*x + 3*sqrt(1/3)*(a*b^2*c - a^2*b*d + (b^3*c
 - a*b^2*d)*x^3)*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^
(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*
x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) + 2*sqrt(3)*(b^3*c*x
^3 + a*b^2*c)*(-c/(b*c - a*d))^(1/3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(b*
x^3 + a)^(1/3)*(-c/(b*c - a*d))^(1/3))/x) - 2*((b^2*c - a*b*d)*x^3 + a*b*c
- a^2*d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + ((b^2*c - a
*b*d)*x^3 + a*b*c - a^2*d)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/
3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 2*(b^3*c*x^3 + a*b^2*c)*(-c/(b*
c - a*d))^(1/3)*log(-((b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1
/3)*c)/x) + (b^3*c*x^3 + a*b^2*c)*(-c/(b*c - a*d))^(1/3)*log(-((b*c - a*d)*
x^2*(-c/(b*c - a*d))^(1/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(-c/(b*c - a*d
))^(2/3) - (b*x^3 + a)^(2/3)*c)/x^2))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d -
 a*b^3*d^2)*x^3), 1/6*(6*(b*x^3 + a)^(2/3)*a*b*d*x - 6*sqrt(1/3)*(a*b^2*c -
```

```
a^2*b*d + (b^3*c - a*b^2*d)*x^3)*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-
b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 2*sqrt(3)*(b^3*c
*x^3 + a*b^2*c)*(-c/(b*c - a*d))^(1/3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(
b*x^3 + a)^(1/3)*(-c/(b*c - a*d))^(1/3))/x) - 2*((b^2*c - a*b*d)*x^3 + a*b*
c - a^2*d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + ((b^2*c -
 a*b*d)*x^3 + a*b*c - a^2*d)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(
1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 2*(b^3*c*x^3 + a*b^2*c)*(-c/(
b*c - a*d))^(1/3)*log(-((b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^
(1/3)*c)/x) + (b^3*c*x^3 + a*b^2*c)*(-c/(b*c - a*d))^(1/3)*log(-((b*c - a*d
)*x^2*(-c/(b*c - a*d))^(1/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(-c/(b*c - a
*d))^(2/3) - (b*x^3 + a)^(2/3)*c)/x^2))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d
 - a*b^3*d^2)*x^3)]
```

Sympy [F]

$$\int \frac{x^6}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^6}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)}\,dx$$

```
[In] integrate(x**6/(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(x**6/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^6}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^6}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(x^6/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^6}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^6}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^6/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^6}{(b\,x^3+a)^{4/3}\,(d\,x^3+c)}\,dx$$

```
[In] int(x^6/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(x^6/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.756 $\int \frac{x^3}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 172

$$\int \frac{x^3}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = -\frac{x}{(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\sqrt[3]{c}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}(bc-ad)^{4/3}} + \frac{\sqrt[3]{c}\log(c+dx^3)}{6(bc-ad)^{4/3}} - \frac{\sqrt[3]{c}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2(bc-ad)^{4/3}}$$

```
[Out] -x/(-a*d+b*c)/(b*x^3+a)^(1/3)+1/6*c^(1/3)*ln(d*x^3+c)/(-a*d+b*c)^(4/3)-1/2*
c^(1/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/(-a*d+b*c)^(4/3)+1/3
*c^(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2
))/(-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {482, 12, 384}

$$\int \frac{x^3}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{\sqrt[3]{c}\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}(bc-ad)^{4/3}} - \frac{x}{\sqrt[3]{a+bx^3}(bc-ad)} + \frac{\sqrt[3]{c}\log\left(c+dx^3\right)}{6(bc-ad)^{4/3}} - \frac{\sqrt[3]{c}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2(bc-ad)^{4/3}}$$

```
[In] Int[x^3/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(x/((b*c - a*d)*(a + b*x^3)^(1/3))) + (c^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^
(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*(b*c - a*d)^(4/3))
 + (c^(1/3)*Log[c + d*x^3])/(6*(b*c - a*d)^(4/3)) - (c^(1/3)*Log[((b*c - a*
d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*(b*c - a*d)^(4/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\text{integral} = -\frac{x}{(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\int \frac{c}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{-bc+ad}$$

$$= -\frac{x}{(bc-ad)\sqrt[3]{a+bx^3}} + \frac{c\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{bc-ad}$$

$$= -\frac{x}{(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\sqrt[3]{c}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}(bc-ad)^{4/3}} + \frac{\sqrt[3]{c}\log\left(c+dx^3\right)}{6(bc-ad)^{4/3}} - \frac{\sqrt[3]{c}\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2(bc-ad)^{4/3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.84 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.87

$$\int \frac{x^3}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{1}{12}\left(-\frac{12x}{(bc-ad)\sqrt[3]{a+bx^3}} - \frac{2\sqrt{-6+6i\sqrt{3}}\sqrt[3]{c}\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{(bc-ad)^{4/3}} + \frac{2\left(1+i\sqrt{3}\right)\sqrt[3]{c}\log\left(2\sqrt[3]{bc-ad}x+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{(bc-ad)^{4/3}} - \frac{i\left(-i+\sqrt{3}\right)\sqrt[3]{c}\log\left(2(bc-ad)^{2/3}x^2+\left(-1-i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{bc-ad}x\sqrt[3]{a+bx^3}+i\left(i+\sqrt{3}\right)c^{2/3}(a+bx^3)^{2/3}\right)}{(bc-ad)^{4/3}}\right)$$

```
[In] Integrate[x^3/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] ((-12*x)/((b*c - a*d)*(a + b*x^3)^(1/3)) - (2*Sqrt[-6 + (6*I)*Sqrt[3]]*c^(1
/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sq
```

```
rt[3])*c^(1/3)*(a + b*x^3)^(1/3))])/(b*c - a*d)^(4/3) + (2*(1 + I*Sqrt[3])*
c^(1/3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/
3)])/(b*c - a*d)^(4/3) - (I*(-I + Sqrt[3])*c^(1/3)*Log[2*(b*c - a*d)^(2/3)*
x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I
 + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(4/3))/12
```

Maple [A] (verified)

Time = 4.57 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.26

method	result
pseudoelliptic	$-\frac{2\left(-3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\frac{\left(2\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)\sqrt{3}+2\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)-\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^{2}-\left(\right.}{}\right.}{2}\right.}{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(6ad-6bc\right)}$

```
[In] int(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-3*((a*d-b*c)/c)^(1/3)*x+1/2*(2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)
*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+2*ln((((a*d-b*c)/c)^(1
/3)*x+(b*x^3+a)^(1/3))/x)-ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(
b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2))*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3
)/(b*x^3+a)^(1/3)/(6*a*d-6*b*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^3}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^3}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)}\,dx$$

```
[In] integrate(x**3/(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(x**3/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^3}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^3}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(x^3/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^3}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^3}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^3/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^3}{(b\,x^3+a)^{4/3}\,(d\,x^3+c)}\,dx$$

```
[In] int(x^3/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(x^3/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.757 $\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 179

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{4/3}} - \frac{d\log(c+dx^3)}{6c^{2/3}(bc-ad)^{4/3}} + \frac{d\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{4/3}}$$

```
[Out] b*x/a/(-a*d+b*c)/(b*x^3+a)^(1/3)-1/6*d*ln(d*x^3+c)/c^(2/3)/(-a*d+b*c)^(4/3)
+1/2*d*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/(-a*d+b*c)^(4
/3)-1/3*d*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/
2))/c^(2/3)/(-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {390, 384}

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = -\frac{d\arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{4/3}} - \frac{d\log(c+dx^3)}{6c^{2/3}(bc-ad)^{4/3}} + \frac{d\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{4/3}} + \frac{bx}{a\sqrt[3]{a+bx^3}(bc-ad)}$$

```
[In] Int[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) - (d*ArcTan[(1 + (2*(b*c - a*d)^(1/
3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(
4/3)) - (d*Log[c + d*x^3])/(6*c^(2/3)*(b*c - a*d)^(4/3)) + (d*Log[((b*c - a
*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(2/3)*(b*c - a*d)^(4/3))
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{bc-ad}$$

$$= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{4/3}} - \frac{d\log(c+dx^3)}{6c^{2/3}(bc-ad)^{4/3}} + \frac{d\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{4/3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.83

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{1}{12}\left(\frac{12bx}{(abc-a^2d)\sqrt[3]{a+bx^3}} + \frac{2\sqrt{-6+6i\sqrt{3}}d\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-(3i+\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{2/3}(bc-ad)^{4/3}} - \frac{2i(-i+\sqrt{3})d\log\left(2\sqrt[3]{bc-ad}x+(1+i\sqrt{3})\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{c^{2/3}(bc-ad)^{4/3}} + \frac{(d+i\sqrt{3}d)\log\left(2(bc-ad)^{2/3}x^2+(-1-i\sqrt{3})\sqrt[3]{c}\sqrt[3]{bc-ad}x\sqrt[3]{a+bx^3}+i(i+\sqrt{3})c^{2/3}(a+bx^3)^{2/3}\right)}{c^{2/3}(bc-ad)^{4/3}}\right)$$

```
[In] Integrate[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] ((12*b*x)/((a*b*c - a^2*d)*(a + b*x^3)^(1/3)) + (2*Sqrt[-6 + (6*I)*Sqrt[3]]
*d*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqr
t[3])*c^(1/3)*(a + b*x^3)^(1/3))])/(c^(2/3)*(b*c - a*d)^(4/3)) - ((2*I)*(-I
 + Sqrt[3])*d*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^
3)^(1/3)])/(c^(2/3)*(b*c - a*d)^(4/3)) + ((d + I*Sqrt[3]*d)*Log[2*(b*c - a*
d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/
3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(c^(2/3)*(b*c - a*d)^(4/3)
))/12
```

Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.36

method	result
pseudoelliptic	$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)ad\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\ln\left(\frac{\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)ad\left(b\,x^{3}+a\right)^{\frac{1}{3}}-\frac{\ln\left(\left(\frac{ad-bc}{c}\right)^{\frac{2}{3}}x^{2}-\right.}{}}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(ad-bc\right)ca}$

[In] int(1/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] 1/3/((a*d-b*c)/c)^(1/3)/(b*x^3+a)^(1/3)*(3^(1/2)*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*a*d*(b*x^3+a)^(1/3)+ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)*a*d*(b*x^3+a)^(1/3)-1/2*ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a*d*(b*x^3+a)^(1/3)-3*b*x*c*((a*d-b*c)/c)^(1/3))/(a*d-b*c)/c/a

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\text{Timed out}$$

[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\int \frac{1}{\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

[In] integrate(1/(b*x**3+a)**(4/3)/(d*x**3+c),x)

[Out] Integral(1/((a + b*x**3)**(4/3)*(c + d*x**3)), x)

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{1}{(b\,x^3+a)^{4/3}\,(d\,x^3+c)}\,dx$$

```
[In] int(1/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(1/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.758 $\int \frac{1}{x^3(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 229

$$\int \frac{1}{x^3(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{b}{a(bc-ad)x^2\sqrt[3]{a+bx^3}} - \frac{(3bc-ad)(a+bx^3)^{2/3}}{2a^2c(bc-ad)x^2} + \frac{d^2\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{5/3}(bc-ad)^{4/3}} + \frac{d^2\log(c+dx^3)}{6c^{5/3}(bc-ad)^{4/3}} - \frac{d^2\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{5/3}(bc-ad)^{4/3}}$$

```
[Out] b/a/(-a*d+b*c)/x^2/(b*x^3+a)^(1/3)-1/2*(-a*d+3*b*c)*(b*x^3+a)^(2/3)/a^2/c/(
-a*d+b*c)/x^2+1/6*d^2*ln(d*x^3+c)/c^(5/3)/(-a*d+b*c)^(4/3)-1/2*d^2*ln((-a*d
+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(5/3)/(-a*d+b*c)^(4/3)+1/3*d^2*arc
tan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(5/3)/(
-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {483, 597, 12, 384}

$$\int \frac{1}{x^3 (a+bx^3)^{4/3} (c+dx^3)} dx = -\frac{(a+bx^3)^{2/3} (3bc-ad)}{2a^2cx^2(bc-ad)} + \frac{d^2 \arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{5/3}(bc-ad)^{4/3}} + \frac{d^2 \log(c+dx^3)}{6c^{5/3}(bc-ad)^{4/3}} - \frac{d^2 \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{5/3}(bc-ad)^{4/3}} + \frac{b}{ax^2\sqrt[3]{a+bx^3}(bc-ad)}$$

```
[In] Int[1/(x^3*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] b/(a*(b*c - a*d)*x^2*(a + b*x^3)^(1/3)) - ((3*b*c - a*d)*(a + b*x^3)^(2/3))
/(2*a^2*c*(b*c - a*d)*x^2) + (d^2*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1
/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(5/3)*(b*c - a*d)^(4/3)) + (d^
2*Log[c + d*x^3])/(6*c^(5/3)*(b*c - a*d)^(4/3)) - (d^2*Log[((b*c - a*d)^(1/
3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(5/3)*(b*c - a*d)^(4/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
```

```
IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{b}{a(bc-ad)x^2\sqrt[3]{a+bx^3}} - \frac{\int \frac{-3bc+ad-3bdx^3}{x^3\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{a(bc-ad)}$$

$$= \frac{b}{a(bc-ad)x^2\sqrt[3]{a+bx^3}} - \frac{(3bc-ad)\left(a+bx^3\right)^{2/3}}{2a^2c(bc-ad)x^2} + \frac{\int \frac{2a^2d^2}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{2a^2c(bc-ad)}$$

$$= \frac{b}{a(bc-ad)x^2\sqrt[3]{a+bx^3}} - \frac{(3bc-ad)\left(a+bx^3\right)^{2/3}}{2a^2c(bc-ad)x^2} + \frac{d^2\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{c(bc-ad)}$$

$$= \frac{b}{a(bc-ad)x^2\sqrt[3]{a+bx^3}} - \frac{(3bc-ad)\left(a+bx^3\right)^{2/3}}{2a^2c(bc-ad)x^2} + \frac{d^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{5/3}(bc-ad)^{4/3}}$$
$$+ \frac{d^2\log\left(c+dx^3\right)}{6c^{5/3}(bc-ad)^{4/3}} - \frac{d^2\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{5/3}(bc-ad)^{4/3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.74 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.56

$$\int \frac{1}{x^3\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{\frac{6c^{2/3}\left(-a^2d+3b^2cx^3+ab\left(c-dx^3\right)\right)}{a^2(-bc+ad)x^2\sqrt[3]{a+bx^3}} - \frac{2\sqrt{-6+6i\sqrt{3}}d^2\arctan\left(\frac{3\sqrt[3]{bc-ad}x}{\sqrt{3}\sqrt[3]{bc-ad}x-\left(3i+\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+}\cdots}\right.}{(bc-ad)^{4/3}}}{\cdots}$$

```
[In] Integrate[1/(x^3*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] ((6*c^(2/3)*(-(a^2*d) + 3*b^2*c*x^3 + a*b*(c - d*x^3)))/(a^2*(-(b*c) + a*d)
*x^2*(a + b*x^3)^(1/3)) - (2*Sqrt[-6 + (6*I)*Sqrt[3]]*d^2*ArcTan[(3*(b*c -
a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b
*x^3)^(1/3))]])/(b*c - a*d)^(4/3) + (2*(1 + I*Sqrt[3])*d^2*Log[2*(b*c - a*d)
^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(4/3) -
(I*(-I + Sqrt[3])*d^2*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3
)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^
3)^(2/3)])/(b*c - a*d)^(4/3))/(12*c^(5/3))
```

Maple [A] (verified)

Time = 4.76 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$\frac{-3\left(\left(abd-3b^{2}c\right)x^{3}+a^{2}d-abc\right)c\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}+a^{2}d^{2}x^{2}\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(-2\arctan\left(\frac{\sqrt{3}\left(\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x-2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}x}\right)\sqrt{3}+\ln\left(\frac{\left(ad-\cdots\right.}{c}\right.\right.}{6\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}c^{2}x^{2}\left(ad-bc\right)a^{2}}$

```
[In] int(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-3*((a*b*d-3*b^2*c)*x^3+a^2*d-a*b*c)*c*((a*d-b*c)/c)^(1/3)+a^2*d^2*x^2
*(b*x^3+a)^(1/3)*(-2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^
(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+ln(((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c
)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-2*ln(((((a*d-b*c)/c)^(1/3
)*x+(b*x^3+a)^(1/3))/x)))/((a*d-b*c)/c)^(1/3)/(b*x^3+a)^(1/3)/c^2/x^2/(a*d-
b*c)/a^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\text{Timed out}$$

```
[In] integrate(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^3 (a+bx^3)^{4/3} (c+dx^3)}\,dx = \int \frac{1}{x^3 (a+bx^3)^{\frac{4}{3}} (c+dx^3)}\,dx$$

```
[In] integrate(1/x**3/(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(1/(x**3*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (a+bx^3)^{4/3} (c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)x^3}\,dx$$

```
[In] integrate(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a+bx^3)^{4/3} (c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)x^3}\,dx$$

```
[In] integrate(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^3)^{4/3} (c+dx^3)}\,dx = \int \frac{1}{x^3 (b\,x^3+a)^{4/3}\,(d\,x^3+c)}\,dx$$

```
[In] int(1/(x^3*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(1/(x^3*(a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.759 $\int \frac{1}{x^6(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 287

$$\int \frac{1}{x^6(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{b}{a(bc-ad)x^5\sqrt[3]{a+bx^3}} - \frac{(6bc-ad)(a+bx^3)^{2/3}}{5a^2c(bc-ad)x^5} + \frac{(18b^2c^2-3abcd-5a^2d^2)(a+bx^3)^{2/3}}{10a^3c^2(bc-ad)x^2} - \frac{d^3\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{8/3}(bc-ad)^{4/3}} - \frac{d^3\log(c+dx^3)}{6c^{8/3}(bc-ad)^{4/3}} + \frac{d^3\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{8/3}(bc-ad)^{4/3}}$$

```
[Out] b/a/(-a*d+b*c)/x^5/(b*x^3+a)^(1/3)-1/5*(-a*d+6*b*c)*(b*x^3+a)^(2/3)/a^2/c/(
-a*d+b*c)/x^5+1/10*(-5*a^2*d^2-3*a*b*c*d+18*b^2*c^2)*(b*x^3+a)^(2/3)/a^3/c^
2/(-a*d+b*c)/x^2-1/6*d^3*ln(d*x^3+c)/c^(8/3)/(-a*d+b*c)^(4/3)+1/2*d^3*ln((-
a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)/(-a*d+b*c)^(4/3)-1/3*d^3*
arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3
)/(-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {483, 597, 12, 384}

$$\int \frac{1}{x^6 (a+bx^3)^{4/3} (c+dx^3)} \, dx = -\frac{(a+bx^3)^{2/3} (6bc-ad)}{5a^2cx^5(bc-ad)} + \frac{(a+bx^3)^{2/3} (-5a^2d^2-3abcd+18b^2c^2)}{10a^3c^2x^2(bc-ad)} - \frac{d^3 \arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{8/3}(bc-ad)^{4/3}} - \frac{d^3 \log(c+dx^3)}{6c^{8/3}(bc-ad)^{4/3}} + \frac{d^3 \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{8/3}(bc-ad)^{4/3}} + \frac{b}{ax^5\sqrt[3]{a+bx^3}(bc-ad)}$$

```
[In] Int[1/(x^6*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] b/(a*(b*c - a*d)*x^5*(a + b*x^3)^(1/3)) - ((6*b*c - a*d)*(a + b*x^3)^(2/3))
/(5*a^2*c*(b*c - a*d)*x^5) + ((18*b^2*c^2 - 3*a*b*c*d - 5*a^2*d^2)*(a + b*x
^3)^(2/3))/(10*a^3*c^2*(b*c - a*d)*x^2) - (d^3*ArcTan[(1 + (2*(b*c - a*d)^(
1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(8/3)*(b*c - a*d)
^(4/3)) - (d^3*Log[c + d*x^3])/(6*c^(8/3)*(b*c - a*d)^(4/3)) + (d^3*Log[((b
*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(8/3)*(b*c - a*d)^(4/
3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
```

```
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{b}{a(bc-ad)x^5\sqrt[3]{a+bx^3}} - \frac{\int \frac{-6bc+ad-6bdx^3}{x^6\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{a(bc-ad)}$$

$$= \frac{b}{a(bc-ad)x^5\sqrt[3]{a+bx^3}} - \frac{(6bc-ad)\left(a+bx^3\right)^{2/3}}{5a^2c(bc-ad)x^5} + \frac{\int \frac{-18b^2c^2+3abcd+5a^2d^2-3bd(6bc-ad)x^3}{x^3\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{5a^2c(bc-ad)}$$

$$= \frac{b}{a(bc-ad)x^5\sqrt[3]{a+bx^3}} - \frac{(6bc-ad)\left(a+bx^3\right)^{2/3}}{5a^2c(bc-ad)x^5} + \frac{\left(18b^2c^2-3abcd-5a^2d^2\right)\left(a+bx^3\right)^{2/3}}{10a^3c^2(bc-ad)x^2} - \frac{\int \frac{10a^3d^3}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{10a^3c^2(bc-ad)}$$

$$= \frac{b}{a(bc-ad)x^5\sqrt[3]{a+bx^3}} - \frac{(6bc-ad)\left(a+bx^3\right)^{2/3}}{5a^2c(bc-ad)x^5} + \frac{\left(18b^2c^2-3abcd-5a^2d^2\right)\left(a+bx^3\right)^{2/3}}{10a^3c^2(bc-ad)x^2} - \frac{d^3\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{c^2(bc-ad)}$$

$$= \frac{b}{a(bc-ad)x^5\sqrt[3]{a+bx^3}} - \frac{(6bc-ad)\left(a+bx^3\right)^{2/3}}{5a^2c(bc-ad)x^5} + \frac{\left(18b^2c^2-3abcd-5a^2d^2\right)\left(a+bx^3\right)^{2/3}}{10a^3c^2(bc-ad)x^2} - \frac{d^3\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{8/3}(bc-ad)^{4/3}} - \frac{d^3\log\left(c+dx^3\right)}{6c^{8/3}(bc-ad)^{4/3}} + \frac{d^3\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{8/3}(bc-ad)^{4/3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.70 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.40

$$\int \frac{1}{x^6 (a+bx^3)^{4/3} (c+dx^3)}\,dx = \frac{\frac{6c^{2/3}(-18b^3c^2x^6+3ab^2cx^3(-2c+dx^3)+a^3d(-2c+5dx^3)+a^2b(2c^2+cdx^3+5d^2x^6))}{a^3(-bc+ad)x^5\sqrt[3]{a+bx^3}} + \frac{10\sqrt{-6+6i\sqrt{3}}d \cdots}{\cdots}}{\cdots}$$

[In] Integrate[1/(x^6*(a + b*x^3)^(4/3)*(c + d*x^3)),x]

[Out] ((6*c^(2/3)*(-18*b^3*c^2*x^6 + 3*a*b^2*c*x^3*(-2*c + d*x^3) + a^3*d*(-2*c + 5*d*x^3) + a^2*b*(2*c^2 + c*d*x^3 + 5*d^2*x^6)))/(a^3*(-(b*c) + a*d)*x^5*(a + b*x^3)^(1/3)) + (10*Sqrt[-6 + (6*I)*Sqrt[3]]*d^3*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))]])/(b*c - a*d)^(4/3) - ((10*I)*(-I + Sqrt[3])*d^3*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]])/(b*c - a*d)^(4/3) + (5*(1 + I*Sqrt[3])*d^3*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)]])/(b*c - a*d)^(4/3))/(60*c^(8/3))

Maple [A] (verified)

Time = 4.82 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$-\frac{\frac{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}} c\left(d\left(-\frac{5d x^3}{2}+c\right)a^3-\left(\frac{5}{2}d^2x^6+\frac{1}{2}cd x^3+c^2\right)b a^2+3x^3\left(-\frac{d x^3}{2}+c\right)b^2ca+9b^3c^2x^6\right)}{5} + \frac{a^3d^3x^5\left(b x^3+a\right)^{\frac{1}{3}}\left(2\arctan\left(\frac{\sqrt{3}\left(\left(\frac{a}{\cdots}\right.\right.}{\cdots}\right)\cdots\right)}{\cdots}}{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}(b x^3+a)^{\frac{1}{3}}\cdots}$

[In] int(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)

[Out] 1/3*(-3/5*((a*d-b*c)/c)^(1/3)*c*(d*(-5/2*d*x^3+c)*a^3-(5/2*d^2*x^6+1/2*c*d*x^3+c^2)*b*a^2+3*x^3*(-1/2*d*x^3+c)*b^2*c*a+9*b^3*c^2*x^6)+1/2*a^3*d^3*x^5*(b*x^3+a)^(1/3)*(2*arctan(1/3*3^(1/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2)+2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)-ln((((a*d-b*c)/c)^(2/3)*x^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)))/((a*d-b*c)/c)^(1/3)/(b*x^3+a)^(1/3)/x^5/c^3/(a*d-b*c)/a^3

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \text{Timed out}$$

```
[In] integrate(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^6 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \int \frac{1}{x^6 (a+bx^3)^{\frac{4}{3}} (c+dx^3)} \, dx$$

```
[In] integrate(1/x**6/(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(1/(x**6*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^6 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)x^6} \, dx$$

```
[In] integrate(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)x^6} \, dx$$

```
[In] integrate(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (a+bx^3)^{4/3} (c+dx^3)}\,dx = \int \frac{1}{x^6 \left(b\,x^3+a\right)^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

```
[In] int(1/(x^6*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(1/(x^6*(a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.760 $\int \frac{1}{x^9(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 351

$$\int \frac{1}{x^9(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{b}{a(bc-ad)x^8\sqrt[3]{a+bx^3}} - \frac{(9bc-ad)(a+bx^3)^{2/3}}{8a^2c(bc-ad)x^8} + \frac{(9bc-4ad)(3bc+ad)(a+bx^3)^{2/3}}{20a^3c^2(bc-ad)x^5} - \frac{(81b^3c^3-9ab^2c^2d-12a^2bcd^2-20a^3d^3)(a+bx^3)^{2/3}}{40a^4c^3(bc-ad)x^2} + \frac{d^4\arctan\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{11/3}(bc-ad)^{4/3}} + \frac{d^4\log(c+dx^3)}{6c^{11/3}(bc-ad)^{4/3}} - \frac{d^4\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{11/3}(bc-ad)^{4/3}}$$

```
[Out] b/a/(-a*d+b*c)/x^8/(b*x^3+a)^(1/3)-1/8*(-a*d+9*b*c)*(b*x^3+a)^(2/3)/a^2/c/(
-a*d+b*c)/x^8+1/20*(-4*a*d+9*b*c)*(a*d+3*b*c)*(b*x^3+a)^(2/3)/a^3/c^2/(-a*d
+b*c)/x^5-1/40*(-20*a^3*d^3-12*a^2*b*c*d^2-9*a*b^2*c^2*d+81*b^3*c^3)*(b*x^3
+a)^(2/3)/a^4/c^3/(-a*d+b*c)/x^2+1/6*d^4*ln(d*x^3+c)/c^(11/3)/(-a*d+b*c)^(4
/3)-1/2*d^4*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(11/3)/(-a*d+b
*c)^(4/3)+1/3*d^4*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3
))*3^(1/2))/c^(11/3)/(-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {483, 597, 12, 384}

$$\int \frac{1}{x^9 (a+bx^3)^{4/3} (c+dx^3)} dx = \frac{(a+bx^3)^{2/3} (9bc-4ad)(ad+3bc)}{20a^3c^2x^5(bc-ad)} - \frac{(a+bx^3)^{2/3} (9bc-ad)}{8a^2cx^8(bc-ad)} - \frac{(a+bx^3)^{2/3} (-20a^3d^3-12a^2bcd^2-9ab^2c^2d+81b^3c^3)}{40a^4c^3x^2(bc-ad)} + \frac{d^4 \arctan\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{11/3}(bc-ad)^{4/3}} + \frac{d^4 \log(c+dx^3)}{6c^{11/3}(bc-ad)^{4/3}} - \frac{d^4 \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{11/3}(bc-ad)^{4/3}} + \frac{b}{ax^8\sqrt[3]{a+bx^3}(bc-ad)}$$

```
[In] Int[1/(x^9*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] b/(a*(b*c - a*d)*x^8*(a + b*x^3)^(1/3)) - ((9*b*c - a*d)*(a + b*x^3)^(2/3))
/(8*a^2*c*(b*c - a*d)*x^8) + ((9*b*c - 4*a*d)*(3*b*c + a*d)*(a + b*x^3)^(2/
3))/(20*a^3*c^2*(b*c - a*d)*x^5) - ((81*b^3*c^3 - 9*a*b^2*c^2*d - 12*a^2*b*
c*d^2 - 20*a^3*d^3)*(a + b*x^3)^(2/3))/(40*a^4*c^3*(b*c - a*d)*x^2) + (d^4*
ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/
(Sqrt[3]*c^(11/3)*(b*c - a*d)^(4/3)) + (d^4*Log[c + d*x^3])/(6*c^(11/3)*(b*
c - a*d)^(4/3)) - (d^4*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3
)])/(2*c^(11/3)*(b*c - a*d)^(4/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b}{a(bc-ad)x^8\sqrt[3]{a+bx^3}} - \frac{\int \frac{-9bc+ad-9bdx^3}{x^9\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{a(bc-ad)} \\
&= \frac{b}{a(bc-ad)x^8\sqrt[3]{a+bx^3}} - \frac{(9bc-ad)\left(a+bx^3\right)^{2/3}}{8a^2c(bc-ad)x^8} + \frac{\int \frac{-2(9bc-4ad)(3bc+ad)-6bd(9bc-ad)x^3}{x^6\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{8a^2c(bc-ad)} \\
&= \frac{b}{a(bc-ad)x^8\sqrt[3]{a+bx^3}} - \frac{(9bc-ad)\left(a+bx^3\right)^{2/3}}{8a^2c(bc-ad)x^8} \\
&\quad + \frac{(9bc-4ad)(3bc+ad)\left(a+bx^3\right)^{2/3}}{20a^3c^2(bc-ad)x^5} \\
&\quad - \frac{\int \frac{-2\left(81b^3c^3-9ab^2c^2d-12a^2bcd^2-20a^3d^3\right)-6bd(9bc-4ad)(3bc+ad)x^3}{x^3\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{40a^3c^2(bc-ad)} \\
&= \frac{b}{a(bc-ad)x^8\sqrt[3]{a+bx^3}} - \frac{(9bc-ad)\left(a+bx^3\right)^{2/3}}{8a^2c(bc-ad)x^8} + \frac{(9bc-4ad)(3bc+ad)\left(a+bx^3\right)^{2/3}}{20a^3c^2(bc-ad)x^5} \\
&\quad - \frac{\left(81b^3c^3-9ab^2c^2d-12a^2bcd^2-20a^3d^3\right)\left(a+bx^3\right)^{2/3}}{40a^4c^3(bc-ad)x^2} + \frac{\int \frac{80a^4d^4}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{80a^4c^3(bc-ad)} \\
&= \frac{b}{a(bc-ad)x^8\sqrt[3]{a+bx^3}} - \frac{(9bc-ad)\left(a+bx^3\right)^{2/3}}{8a^2c(bc-ad)x^8} + \frac{(9bc-4ad)(3bc+ad)\left(a+bx^3\right)^{2/3}}{20a^3c^2(bc-ad)x^5} \\
&\quad - \frac{\left(81b^3c^3-9ab^2c^2d-12a^2bcd^2-20a^3d^3\right)\left(a+bx^3\right)^{2/3}}{40a^4c^3(bc-ad)x^2} + \frac{d^4\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{c^3(bc-ad)}
\end{aligned}$$

$$= \frac{b}{a(bc-ad)x^8\sqrt[3]{a+bx^3}} - \frac{(9bc-ad)\left(a+bx^3\right)^{2/3}}{8a^2c(bc-ad)x^8} + \frac{(9bc-4ad)(3bc+ad)\left(a+bx^3\right)^{2/3}}{20a^3c^2(bc-ad)x^5} - \frac{(81b^3c^3-9ab^2c^2d-12a^2bcd^2-20a^3d^3)\left(a+bx^3\right)^{2/3}}{40a^4c^3(bc-ad)x^2} + \frac{d^4\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{11/3}(bc-ad)^{4/3}} + \frac{d^4\log\left(c+dx^3\right)}{6c^{11/3}(bc-ad)^{4/3}} - \frac{d^4\log\left(\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{11/3}(bc-ad)^{4/3}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.48 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.31

$$\int \frac{1}{x^9\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = -\frac{\frac{3c^{2/3}\left(-81b^4c^3x^9+9ab^3c^2x^6\left(-3c+dx^3\right)+3a^2b^2cx^3\left(3c^2+cdx^3+4d^2x^6\right)+a^4d\left(5c^2-8cdx^3+20d^2x^6\right)+\cdots\right)}{a^4(-bc+ad)x^8\sqrt[3]{a+bx^3}}}{}$$

```
[In] Integrate[1/(x^9*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] ((-3*c^(2/3)*(-81*b^4*c^3*x^9 + 9*a*b^3*c^2*x^6*(-3*c + d*x^3) + 3*a^2*b^2*
c*x^3*(3*c^2 + c*d*x^3 + 4*d^2*x^6) + a^4*d*(5*c^2 - 8*c*d*x^3 + 20*d^2*x^6
) + a^3*b*(-5*c^3 - c^2*d*x^3 + 4*c*d^2*x^6 + 20*d^3*x^9)))/(a^4*(-(b*c) +
a*d)*x^8*(a + b*x^3)^(1/3)) - (20*Sqrt[-6 + (6*I)*Sqrt[3]]*d^4*ArcTan[(3*(b
*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I + Sqrt[3])*c^(1/3)*(
a + b*x^3)^(1/3))])/(b*c - a*d)^(4/3) + (20*(1 + I*Sqrt[3])*d^4*Log[2*(b*c
- a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(b*c - a*d)^(4
/3) - ((10*I)*(-I + Sqrt[3])*d^4*Log[2*(b*c - a*d)^(2/3)*x^2 + (-1 - I*Sqrt
[3])*c^(1/3)*(b*c - a*d)^(1/3)*x*(a + b*x^3)^(1/3) + I*(I + Sqrt[3])*c^(2/3
)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(4/3))/(120*c^(11/3))
```

Maple [A] (verified)

Time = 4.89 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.01

method	result
pseudoelliptic	$-\frac{\frac{3\left(\frac{ad-bc}{c}\right)^{\frac{1}{3}}\left(d\left(4d^{2}x^{6}-\frac{8}{5}cd\,x^{3}+c^{2}\right)a^{4}-b\left(-4d^{3}x^{9}-\frac{4}{5}c\,d^{2}x^{6}+\frac{1}{5}c^{2}d\,x^{3}+c^{3}\right)a^{3}+\frac{9x^{3}\left(\frac{4}{3}d^{2}x^{6}+\frac{1}{3}cd\,x^{3}+c^{2}\right)b^{2}c\,a^{2}}{5}-\frac{27x^{6}\left(-\frac{d\,x^{3}}{3}+c\right)}{5}}{4}}{}$

```
[In] int(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/((a*d-b*c)/c)^(1/3)/(b*x^3+a)^(1/3)*(3/4*((a*d-b*c)/c)^(1/3)*(d*(4*d^2
*x^6-8/5*c*d*x^3+c^2)*a^4-b*(-4*d^3*x^9-4/5*c*d^2*x^6+1/5*c^2*d*x^3+c^3)*a^
3+9/5*x^3*(4/3*d^2*x^6+1/3*c*d*x^3+c^2)*b^2*c*a^2-27/5*x^6*(-1/3*d*x^3+c)*b
^3*c^2*a-81/5*b^4*c^3*x^9)*c+a^4*d^4*x^8*(b*x^3+a)^(1/3)*(2*arctan(1/3*3^(1
/2)*(((a*d-b*c)/c)^(1/3)*x-2*(b*x^3+a)^(1/3))/((a*d-b*c)/c)^(1/3)/x)*3^(1/2
)+2*ln((((a*d-b*c)/c)^(1/3)*x+(b*x^3+a)^(1/3))/x)-ln(((((a*d-b*c)/c)^(2/3)*x
^2-((a*d-b*c)/c)^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)))/x^8/c^4/(a
*d-b*c)/a^4
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^9\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\text{Timed out}$$

```
[In] integrate(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^9\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=\int \frac{1}{x^9\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

```
[In] integrate(1/x**9/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**9*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^9 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)x^9} \, dx$$

```
[In] integrate(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^9), x)
```

Giac [F]

$$\int \frac{1}{x^9 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)x^9} \, dx$$

```
[In] integrate(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^9), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^9 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \int \frac{1}{x^9 \left(b\,x^3+a\right)^{4/3} \left(d\,x^3+c\right)} \, dx$$

```
[In] int(1/(x^9*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(1/(x^9*(a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.761 $\int \frac{x^{10}}{(a+bx^3)^{4/3}(c+dx^3)} dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{x^{10}}{(a+bx^3)^{4/3}(c+dx^3)} dx = \frac{x^{11}\sqrt[3]{1+\frac{bx^3}{a}}\operatorname{AppellF1}\left(\frac{11}{3},\frac{4}{3},1,\frac{14}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{11ac\sqrt[3]{a+bx^3}}$$

```
[Out] 1/11*x^11*(1+b*x^3/a)^(1/3)*AppellF1(11/3,4/3,1,14/3,-b*x^3/a,-d*x^3/c)/a/c
/(b*x^3+a)^(1/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{x^{10}}{(a+bx^3)^{4/3}(c+dx^3)} dx = \frac{x^{11}\sqrt[3]{\frac{bx^3}{a}+1}\operatorname{AppellF1}\left(\frac{11}{3},\frac{4}{3},1,\frac{14}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{11ac\sqrt[3]{a+bx^3}}$$

```
[In] Int[x^10/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^11*(1 + (b*x^3)/a)^(1/3)*AppellF1[11/3, 4/3, 1, 14/3, -((b*x^3)/a), -((d
*x^3)/c)])/(11*a*c*(a + b*x^3)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^{10}}{\left(1+\frac{bx^3}{a}\right)^{4/3}(c+dx^3)}\,dx}{a\sqrt[3]{a+bx^3}}$$

$$= \frac{x^{11}\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(\frac{11}{3};\frac{4}{3},1;\frac{14}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{11ac\sqrt[3]{a+bx^3}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 194 vs. $2(67) = 134$.

Time = 10.23 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.90

$$\int \frac{x^{10}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{x^2\left(5c(-5a^2d+b^2cx^3+ab(c-dx^3))+5ac(-bc+5ad)\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\right.\right.}{20}$$

```
[In] Integrate[x^10/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^2*(5*c*(-5*a^2*d + b^2*c*x^3 + a*b*(c - d*x^3)) + 5*a*c*(-(b*c) + 5*a*d)
*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/c
)] + 2*(-2*b^2*c^2 - a*b*c*d + 5*a^2*d^2)*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF
1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)]))/(20*b^2*c*d*(b*c - a*d)*(
a + b*x^3)^(1/3))
```

Maple [F]

$$\int \frac{x^{10}}{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right)}dx$$

```
[In] int(x^10/(b*x^3+a)^(4/3)/(d*x^3+c),x)
[Out] int(x^10/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^{10}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate(x^10/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^{10}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^{10}}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)}\,dx$$

```
[In] integrate(x**10/(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(x**10/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^{10}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^{10}}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(x^10/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(x^10/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^{10}}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx = \int \frac{x^{10}}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(x^10/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^10/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx = \int \frac{x^{10}}{(b\,x^3+a)^{4/3}\,(d\,x^3+c)}\,dx$$

```
[In] int(x^10/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(x^10/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.762 $\int \frac{x^7}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{x^7}{(a+bx^3)^{4/3}(c+dx^3)}\,dx=\frac{x^8\sqrt[3]{1+\frac{bx^3}{a}}\operatorname{AppellF1}\left(\frac{8}{3},\frac{4}{3},1,\frac{11}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8ac\sqrt[3]{a+bx^3}}$$

```
[Out] 1/8*x^8*(1+b*x^3/a)^(1/3)*AppellF1(8/3,4/3,1,11/3,-b*x^3/a,-d*x^3/c)/a/c/(b
*x^3+a)^(1/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{x^7}{(a+bx^3)^{4/3}(c+dx^3)}\,dx=\frac{x^8\sqrt[3]{\frac{bx^3}{a}+1}\operatorname{AppellF1}\left(\frac{8}{3},\frac{4}{3},1,\frac{11}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8ac\sqrt[3]{a+bx^3}}$$

```
[In] Int[x^7/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^8*(1 + (b*x^3)/a)^(1/3)*AppellF1[8/3, 4/3, 1, 11/3, -((b*x^3)/a), -((d*x
^3)/c)])/(8*a*c*(a + b*x^3)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int\frac{x^7}{\left(1+\frac{bx^3}{a}\right)^{4/3}(c+dx^3)}\,dx}{a\sqrt[3]{a+bx^3}}$$
$$= \frac{x^8\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(\frac{8}{3};\frac{4}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8ac\sqrt[3]{a+bx^3}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 144 vs. $2(67) = 134$.

Time = 10.13 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.15

$$\int\frac{x^7}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{5acx^2-5acx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+(bc-2ad)x^5\sqrt[3]{1}}{5bc(bc-ad)\sqrt[3]{a+bx^3}}$$

```
[In] Integrate[x^7/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (5*a*c*x^2 - 5*a*c*x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((
b*x^3)/a), -((d*x^3)/c)] + (b*c - 2*a*d)*x^5*(1 + (b*x^3)/a)^(1/3)*AppellF1
[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(5*b*c*(b*c - a*d)*(a + b*x
^3)^(1/3))
```

Maple [F]

$$\int \frac{x^{7}}{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right)}d x$$

```
[In] int(x^7/(b*x^3+a)^(4/3)/(d*x^3+c),x)
[Out] int(x^7/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^7}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate(x^7/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^7}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx = \int \frac{x^7}{(a+bx^3)^{\frac{4}{3}}\,(c+dx^3)}\,dx$$

```
[In] integrate(x**7/(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(x**7/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^7}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx = \int \frac{x^7}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(x^7/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(x^7/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^7}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^7}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(x^7/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^7/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^7}{(b\,x^3+a)^{4/3}\,(d\,x^3+c)}\,dx$$

```
[In] int(x^7/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(x^7/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.763 $\int \frac{x^4}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{x^4}{(a+bx^3)^{4/3}(c+dx^3)}\,dx=\frac{x^5\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{5}{3},\frac{4}{3},1,\frac{8}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5ac\sqrt[3]{a+bx^3}}$$

```
[Out] 1/5*x^5*(1+b*x^3/a)^(1/3)*AppellF1(5/3,4/3,1,8/3,-b*x^3/a,-d*x^3/c)/a/c/(b*
x^3+a)^(1/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{x^4}{(a+bx^3)^{4/3}(c+dx^3)}\,dx=\frac{x^5\sqrt[3]{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{5}{3},\frac{4}{3},1,\frac{8}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5ac\sqrt[3]{a+bx^3}}$$

```
[In] Int[x^4/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^5*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 4/3, 1, 8/3, -((b*x^3)/a), -((d*x^
3)/c)])/(5*a*c*(a + b*x^3)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^4}{\left(1+\frac{bx^3}{a}\right)^{4/3}(c+dx^3)}\,dx}{a\sqrt[3]{a+bx^3}}$$
$$= \frac{x^5\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(\frac{5}{3};\frac{4}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5ac\sqrt[3]{a+bx^3}}$$

Mathematica [A] (verified)

Time = 10.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.93

$$\int \frac{x^4}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{x^2\left(-5c+5c\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+dx^3\sqrt[3]{1+\frac{bx^3}{a}}\,\text{App}\right.}{5c(bc-ad)\sqrt[3]{a+bx^3}}$$

```
[In] Integrate[x^4/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^2*(-5*c + 5*c*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)
/a), -((d*x^3)/c)] + d*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3,
 -((b*x^3)/a), -((d*x^3)/c)]))/(5*c*(b*c - a*d)*(a + b*x^3)^(1/3))
```

Maple [F]

$$\int \frac{x^4}{(b\,x^3+a)^{\frac{4}{3}}\,(d\,x^3+c)}dx$$

```
[In] int(x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate(x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^4}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^4}{(a+bx^3)^{\frac{4}{3}}(c+dx^3)}\,dx$$

```
[In] integrate(x**4/(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(x**4/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^4}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(x^4/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \int \frac{x^4}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^4/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^3)^{4/3}\,(c+dx^3)}\,dx=\int \frac{x^4}{\left(b\,x^3+a\right)^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

```
[In] int(x^4/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(x^4/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.764 $\int \frac{x}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 67

$$\int \frac{x}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}\,\operatorname{AppellF1}\left(\frac{2}{3},\frac{4}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ac\sqrt[3]{a+bx^3}}$$

```
[Out] 1/2*x^2*(1+b*x^3/a)^(1/3)*AppellF1(2/3,4/3,1,5/3,-b*x^3/a,-d*x^3/c)/a/c/(b*
x^3+a)^(1/3)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {525, 524}

$$\int \frac{x}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\operatorname{AppellF1}\left(\frac{2}{3},\frac{4}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ac\sqrt[3]{a+bx^3}}$$

```
[In] Int[x/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 4/3, 1, 5/3, -((b*x^3)/a), -((d*x^
3)/c)])/(2*a*c*(a + b*x^3)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x}{\left(1+\frac{bx^3}{a}\right)^{4/3}(c+dx^3)}\,dx}{a\sqrt[3]{a+bx^3}}$$

$$= \frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{4}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ac\sqrt[3]{a+bx^3}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(67) = 134.

Time = 10.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.10

$$\int \frac{x}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{x^2\left(-10bc+5(bc+ad)\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bdx^3\sqrt[3]{}\right.}{10ac(-bc+ad)\sqrt[3]{a+bx^3}}$$

```
[In] Integrate[x/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^2*(-10*b*c + 5*(b*c + a*d)*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5
/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*d*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[
5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)]))/(10*a*c*(-(b*c) + a*d)*(a +
 b*x^3)^(1/3))
```

Maple [F]

$$\int \frac{x}{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right)}dx$$

```
[In] int(x/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\left(a+bx^3\right)^{4/3} (c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate(x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x}{\left(a+bx^3\right)^{4/3} (c+dx^3)}\,dx = \int \frac{x}{\left(a+bx^{3}\right)^{\frac{4}{3}} \left(c+dx^{3}\right)}\,dx$$

```
[In] integrate(x/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\left(a+bx^3\right)^{4/3} (c+dx^3)}\,dx = \int \frac{x}{{\left(bx^{3}+a\right)}^{\frac{4}{3}}{\left(dx^{3}+c\right)}}\,dx$$

```
[In] integrate(x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Giac [F]

$$\int \frac{x}{(a+bx^3)^{4/3}(c+dx^3)}\,dx=\int \frac{x}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

```
[In] integrate(x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)^{4/3}(c+dx^3)}\,dx=\int \frac{x}{(b\,x^3+a)^{4/3}\,(d\,x^3+c)}\,dx$$

```
[In] int(x/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(x/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.765 $\int \frac{1}{x^2(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{1}{x^2(a+bx^3)^{4/3}(c+dx^3)}\,dx = -\frac{\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(-\frac{1}{3},\frac{4}{3},1,\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{acx\sqrt[3]{a+bx^3}}$$

```
[Out] -(1+b*x^3/a)^(1/3)*AppellF1(-1/3,4/3,1,2/3,-b*x^3/a,-d*x^3/c)/a/c/x/(b*x^3+
a)^(1/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{1}{x^2(a+bx^3)^{4/3}(c+dx^3)}\,dx = -\frac{\sqrt[3]{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(-\frac{1}{3},\frac{4}{3},1,\frac{2}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{acx\sqrt[3]{a+bx^3}}$$

```
[In] Int[1/(x^2*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(((1 + (b*x^3)/a)^(1/3)*AppellF1[-1/3, 4/3, 1, 2/3, -((b*x^3)/a), -((d*x^3
)/c)])/(a*c*x*(a + b*x^3)^(1/3)))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{x^2\left(1+\frac{bx^3}{a}\right)^{4/3}(c+dx^3)}\,dx}{a\sqrt[3]{a+bx^3}}$$

$$= -\frac{\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(-\frac{1}{3};\frac{4}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{acx\sqrt[3]{a+bx^3}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 193 vs. $2(65) = 130$.

Time = 10.21 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.97

$$\int \frac{1}{x^2\,(a+bx^3)^{4/3}\,(c+dx^3)}\,dx = \frac{10c(-a^2d+2b^2cx^3+ab(c-dx^3))-5(2b^2c^2-abcd+a^2d^2)\,x^3\sqrt[3]{1+\frac{bx^3}{a}}\,\mathrm{A}\ldots}{10a\ldots}$$

```
[In] Integrate[1/(x^2*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (10*c*(-(a^2*d) + 2*b^2*c*x^3 + a*b*(c - d*x^3)) - 5*(2*b^2*c^2 - a*b*c*d +
 a^2*d^2)*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a)
, -((d*x^3)/c)] + 2*b*d*(-2*b*c + a*d)*x^6*(1 + (b*x^3)/a)^(1/3)*AppellF1[5
/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(10*a^2*c^2*(-(b*c) + a*d)*x*
(a + b*x^3)^(1/3))
```

Maple [F]

$$\int \frac{1}{x^2\,(b\,x^3+a)^{\frac{4}{3}}\,(d\,x^3+c)}dx$$

```
[In] int(1/x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x)
[Out] int(1/x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\,(a+bx^3)^{4/3}\,(c+dx^3)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^2\,(a+bx^3)^{4/3}\,(c+dx^3)}\,dx = \int \frac{1}{x^2\,(a+bx^3)^{\frac{4}{3}}\,(c+dx^3)}\,dx$$

```
[In] integrate(1/x**2/(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(1/(x**2*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\,(a+bx^3)^{4/3}\,(c+dx^3)}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)x^2}\,dx$$

```
[In] integrate(1/x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)x^2} \, dx$$

```
[In] integrate(1/x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^3)^{4/3} (c+dx^3)} \, dx = \int \frac{1}{x^2 (b\,x^3+a)^{4/3} \, (d\,x^3+c)} \, dx$$

```
[In] int(1/(x^2*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(1/(x^2*(a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.766 $\int \frac{1}{x^5\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{1}{x^5\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=-\frac{\sqrt[3]{1+\frac{bx^3}{a}}\,\mathrm{AppellF1}\left(-\frac{4}{3},\frac{4}{3},1,-\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4acx^4\sqrt[3]{a+bx^3}}$$

```
[Out] -1/4*(1+b*x^3/a)^(1/3)*AppellF1(-4/3,4/3,1,-1/3,-b*x^3/a,-d*x^3/c)/a/c/x^4/
(b*x^3+a)^(1/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{1}{x^5\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx=-\frac{\sqrt[3]{\frac{bx^3}{a}+1}\,\mathrm{AppellF1}\left(-\frac{4}{3},\frac{4}{3},1,-\frac{1}{3},-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4acx^4\sqrt[3]{a+bx^3}}$$

```
[In] Int[1/(x^5*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -1/4*((1 + (b*x^3)/a)^(1/3)*AppellF1[-4/3, 4/3, 1, -1/3, -((b*x^3)/a), -((d
*x^3)/c)])/(a*c*x^4*(a + b*x^3)^(1/3))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{x^5\left(1+\frac{bx^3}{a}\right)^{4/3}(c+dx^3)}\,dx}{a\sqrt[3]{a+bx^3}}$$
$$= -\frac{\sqrt[3]{1+\frac{bx^3}{a}}F_1\left(-\frac{4}{3};\frac{4}{3},1;-\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4acx^4\sqrt[3]{a+bx^3}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 264 vs. 2(67) = 134.

Time = 10.32 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.94

$$\int \frac{1}{x^5\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{5c(-10b^3c^2x^6+ab^2cx^3(-5c+2dx^3)+a^3d(-c+4dx^3)+a^2b(c^2+cdx^3+4}{}$$

```
[In] Integrate[1/(x^5*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (5*c*(-10*b^3*c^2*x^6 + a*b^2*c*x^3*(-5*c + 2*d*x^3) + a^3*d*(-c + 4*d*x^3)
 + a^2*b*(c^2 + c*d*x^3 + 4*d^2*x^6)) + 5*(5*b^3*c^3 - a*b^2*c^2*d - 2*a^2*
b*c*d^2 + 2*a^3*d^3)*x^6*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -
((b*x^3)/a), -((d*x^3)/c)] - 2*b*d*(-5*b^2*c^2 + a*b*c*d + 2*a^2*d^2)*x^9*(
1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)]
)/(20*a^3*c^3*(-(b*c) + a*d)*x^4*(a + b*x^3)^(1/3))
```

Maple [F]

$$\int \frac{1}{x^5 \left(b\,x^3+a\right)^{\frac{4}{3}} \left(d\,x^3+c\right)}dx$$

[In] int(1/x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x)

[Out] int(1/x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x)

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (a+bx^3)^{4/3} (c+dx^3)}\,dx = \text{Timed out}$$

[In] integrate(1/x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{1}{x^5 (a+bx^3)^{4/3} (c+dx^3)}\,dx = \int \frac{1}{x^5 \left(a+bx^3\right)^{\frac{4}{3}} \left(c+dx^3\right)}\,dx$$

[In] integrate(1/x**5/(b*x**3+a)**(4/3)/(d*x**3+c),x)

[Out] Integral(1/(x**5*(a + b*x**3)**(4/3)*(c + d*x**3)), x)

Maxima [F]

$$\int \frac{1}{x^5 (a+bx^3)^{4/3} (c+dx^3)}\,dx = \int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)x^5}\,dx$$

[In] integrate(1/x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")

[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^5), x)

Giac [F]

$$\int \frac{1}{x^5 \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)}\,dx=\int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)x^5}\,dx$$

```
[In] integrate(1/x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)}\,dx=\int \frac{1}{x^5\,{\left(b\,x^3+a\right)}^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

```
[In] int(1/(x^5*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(1/(x^5*(a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

3.767 $\int \frac{x^{15}}{(a+bx^4)(c+dx^4)} dx$

Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \frac{x^{15}}{(a+bx^4)(c+dx^4)} dx = -\frac{(bc+ad)x^4}{4b^2d^2} + \frac{x^8}{8bd} - \frac{a^3\log(a+bx^4)}{4b^3(bc-ad)} + \frac{c^3\log(c+dx^4)}{4d^3(bc-ad)}$$

```
[Out] -1/4*(a*d+b*c)*x^4/b^2/d^2+1/8*x^8/b/d-1/4*a^3*ln(b*x^4+a)/b^3/(-a*d+b*c)+1
/4*c^3*ln(d*x^4+c)/d^3/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 84}

$$\int \frac{x^{15}}{(a+bx^4)(c+dx^4)} dx = -\frac{a^3\log(a+bx^4)}{4b^3(bc-ad)} - \frac{x^4(ad+bc)}{4b^2d^2} + \frac{c^3\log(c+dx^4)}{4d^3(bc-ad)} + \frac{x^8}{8bd}$$

```
[In] Int[x^15/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/4*((b*c + a*d)*x^4)/(b^2*d^2) + x^8/(8*b*d) - (a^3*Log[a + b*x^4])/(4*b^
3*(b*c - a*d)) + (c^3*Log[c + d*x^4])/(4*d^3*(b*c - a*d))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{x^3}{(a+bx)(c+dx)}\,dx, x, x^4\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \left(\frac{-bc-ad}{b^2d^2}+\frac{x}{bd}-\frac{a^3}{b^2(bc-ad)(a+bx)}-\frac{c^3}{d^2(-bc+ad)(c+dx)}\right)\,dx, x, x^4\right)\\ &= -\frac{(bc+ad)x^4}{4b^2d^2}+\frac{x^8}{8bd}-\frac{a^3\log(a+bx^4)}{4b^3(bc-ad)}+\frac{c^3\log(c+dx^4)}{4d^3(bc-ad)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int \frac{x^{15}}{(a+bx^4)(c+dx^4)}\,dx = \frac{bd(bc-ad)x^4(-2bc-2ad+bdx^4)-2a^3d^3\log(a+bx^4)+2b^3c^3\log(c+dx^4)}{8b^3d^3(bc-ad)}$$

```
[In] Integrate[x^15/((a + b*x^4)*(c + d*x^4)),x]
[Out] (b*d*(b*c - a*d)*x^4*(-2*b*c - 2*a*d + b*d*x^4) - 2*a^3*d^3*Log[a + b*x^4]
+ 2*b^3*c^3*Log[c + d*x^4])/(8*b^3*d^3*(b*c - a*d))
```

Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\left(-bd\,x^4+ad+bc\right)^2}{8b^3d^3}+\frac{a^3\ln\left(b\,x^4+a\right)}{4b^3(ad-bc)}-\frac{c^3\ln\left(d\,x^4+c\right)}{4d^3(ad-bc)}$	78
norman	$\frac{x^8}{8bd}-\frac{(ad+bc)x^4}{4b^2d^2}+\frac{a^3\ln\left(b\,x^4+a\right)}{4b^3(ad-bc)}-\frac{c^3\ln\left(d\,x^4+c\right)}{4d^3(ad-bc)}$	83
parallelrisch	$\frac{a\,b^2d^3x^8-x^8b^3c\,d^2-2a^2b\,d^3x^4+2b^3c^2d\,x^4+2a^3\ln\left(b\,x^4+a\right)d^3-2c^3\ln\left(d\,x^4+c\right)b^3}{8b^3d^3(ad-bc)}$	99
risch	$\frac{x^8}{8bd}-\frac{a\,x^4}{4b^2d}-\frac{c\,x^4}{4b\,d^2}+\frac{a^2}{8b^3d}+\frac{ac}{4b^2d^2}+\frac{c^2}{8b\,d^3}-\frac{c^3\ln\left(d\,x^4+c\right)}{4d^3(ad-bc)}+\frac{a^3\ln\left(-b\,x^4-a\right)}{4b^3(ad-bc)}$	124

```
[In] int(x^15/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(-b*d*x^4+a*d+b*c)^2/b^3/d^3+1/4*a^3/b^3/(a*d-b*c)*ln(b*x^4+a)-1/4*c^3/
d^3/(a*d-b*c)*ln(d*x^4+c)
```

Fricas [A] (verification not implemented)

none

Time = 2.69 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11

$$\int \frac{x^{15}}{(a+bx^4)(c+dx^4)}\,dx = \frac{(b^3cd^2-ab^2d^3)x^8-2\,a^3d^3\log(bx^4+a)+2\,b^3c^3\log(dx^4+c)-2\,(b^3c^2d-a^2bd^3)x^4}{8\,(b^4cd^3-ab^3d^4)}$$

```
[In] integrate(x^15/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
[Out] 1/8*((b^3*c*d^2 - a*b^2*d^3)*x^8 - 2*a^3*d^3*log(b*x^4 + a) + 2*b^3*c^3*log
(d*x^4 + c) - 2*(b^3*c^2*d - a^2*b*d^3)*x^4)/(b^4*c*d^3 - a*b^3*d^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{15}}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(x**15/(b*x**4+a)/(d*x**4+c),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.93

$$\int \frac{x^{15}}{(a+bx^4)(c+dx^4)}\,dx = -\frac{a^3\log(bx^4+a)}{4\,(b^4c-ab^3d)}+\frac{c^3\log(dx^4+c)}{4\,(bcd^3-ad^4)}+\frac{bdx^8-2\,(bc+ad)x^4}{8\,b^2d^2}$$

```
[In] integrate(x^15/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
[Out] -1/4*a^3*log(b*x^4 + a)/(b^4*c - a*b^3*d) + 1/4*c^3*log(d*x^4 + c)/(b*c*d^3
 - a*d^4) + 1/8*(b*d*x^8 - 2*(b*c + a*d)*x^4)/(b^2*d^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int \frac{x^{15}}{(a+bx^4)(c+dx^4)}\,dx = -\frac{a^3\log(|bx^4+a|)}{4(b^4c-ab^3d)} + \frac{c^3\log(|dx^4+c|)}{4(bcd^3-ad^4)} + \frac{bdx^8-2\,bcx^4-2\,adx^4}{8\,b^2d^2}$$

```
[In] integrate(x^15/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/4*a^3*log(abs(b*x^4 + a))/(b^4*c - a*b^3*d) + 1/4*c^3*log(abs(d*x^4 + c)
)/(b*c*d^3 - a*d^4) + 1/8*(b*d*x^8 - 2*b*c*x^4 - 2*a*d*x^4)/(b^2*d^2)
```

Mupad [B] (verification not implemented)

Time = 10.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int \frac{x^{15}}{(a+bx^4)(c+dx^4)}\,dx = \frac{x^8}{8\,b\,d} - \frac{c^3\,\ln(d\,x^4+c)}{4\,(a\,d^4-b\,c\,d^3)} - \frac{a^3\,\ln(b\,x^4+a)}{4\,(b^4\,c-a\,b^3\,d)} - \frac{x^4\,(a\,d+b\,c)}{4\,b^2\,d^2}$$

```
[In] int(x^15/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] x^8/(8*b*d) - (c^3*log(c + d*x^4))/(4*(a*d^4 - b*c*d^3)) - (a^3*log(a + b*x
^4))/(4*(b^4*c - a*b^3*d)) - (x^4*(a*d + b*c))/(4*b^2*d^2)
```

3.768 $\int \frac{x^{11}}{(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 70

$$\int \frac{x^{11}}{(a+bx^4)(c+dx^4)}\,dx=\frac{x^4}{4bd}+\frac{a^2\log(a+bx^4)}{4b^2(bc-ad)}-\frac{c^2\log(c+dx^4)}{4d^2(bc-ad)}$$

```
[Out] 1/4*x^4/b/d+1/4*a^2*ln(b*x^4+a)/b^2/(-a*d+b*c)-1/4*c^2*ln(d*x^4+c)/d^2/(-a*
d+b*c)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 84}

$$\int \frac{x^{11}}{(a+bx^4)(c+dx^4)}\,dx=\frac{a^2\log(a+bx^4)}{4b^2(bc-ad)}-\frac{c^2\log(c+dx^4)}{4d^2(bc-ad)}+\frac{x^4}{4bd}$$

```
[In] Int[x^11/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] x^4/(4*b*d) + (a^2*Log[a + b*x^4])/(4*b^2*(b*c - a*d)) - (c^2*Log[c + d*x^4
])/(4*d^2*(b*c - a*d))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{(a+bx)(c+dx)}\,dx, x, x^4\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{bd} + \frac{a^2}{b(bc-ad)(a+bx)} + \frac{c^2}{d(-bc+ad)(c+dx)}\right)\,dx, x, x^4\right)\\
&= \frac{x^4}{4bd} + \frac{a^2\log\left(a+bx^4\right)}{4b^2(bc-ad)} - \frac{c^2\log\left(c+dx^4\right)}{4d^2(bc-ad)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{x^{11}}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \frac{a^2d^2\log\left(a+bx^4\right) - b\left(d(-bc+ad)x^4 + bc^2\log\left(c+dx^4\right)\right)}{4b^2d^2(bc-ad)}$$

```
[In] Integrate[x^11/((a + b*x^4)*(c + d*x^4)),x]
[Out] (a^2*d^2*Log[a + b*x^4] - b*(d*(-(b*c) + a*d)*x^4 + b*c^2*Log[c + d*x^4]))/
(4*b^2*d^2*(b*c - a*d))
```

Maple [A] (verified)

Time = 4.53 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{x^4}{4bd} - \frac{a^2\ln\left(b\,x^4+a\right)}{4(ad-bc)b^2} + \frac{c^2\ln\left(d\,x^4+c\right)}{4(ad-bc)d^2}$	65
norman	$\frac{x^4}{4bd} - \frac{a^2\ln\left(b\,x^4+a\right)}{4(ad-bc)b^2} + \frac{c^2\ln\left(d\,x^4+c\right)}{4(ad-bc)d^2}$	65
risch	$\frac{x^4}{4bd} + \frac{c^2\ln\left(d\,x^4+c\right)}{4(ad-bc)d^2} - \frac{a^2\ln\left(-b\,x^4-a\right)}{4b^2(ad-bc)}$	68
parallelrisch	$-\frac{-ab\,d^2x^4+b^2cd\,x^4+a^2\ln\left(b\,x^4+a\right)d^2-c^2\ln\left(d\,x^4+c\right)b^2}{4b^2d^2(ad-bc)}$	70

```
[In] int(x^11/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
[Out] 1/4*x^4/b/d-1/4*a^2/(a*d-b*c)/b^2*ln(b*x^4+a)+1/4*c^2/(a*d-b*c)/d^2*ln(d*x^
4+c)
```

Fricas [A] (verification not implemented)

none

Time = 1.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03

$$\int \frac{x^{11}}{(a+bx^4)(c+dx^4)}\,dx = \frac{(b^2cd-abd^2)x^4+a^2d^2\log(bx^4+a)-b^2c^2\log(dx^4+c)}{4(b^3cd^2-ab^2d^3)}$$

```
[In] integrate(x^11/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
[Out] 1/4*((b^2*c*d - a*b*d^2)*x^4 + a^2*d^2*log(b*x^4 + a) - b^2*c^2*log(d*x^4 +
 c))/(b^3*c*d^2 - a*b^2*d^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(x**11/(b*x**4+a)/(d*x**4+c),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int \frac{x^{11}}{(a+bx^4)(c+dx^4)}\,dx = \frac{x^4}{4\,bd}+\frac{a^2\log(bx^4+a)}{4(b^3c-ab^2d)}-\frac{c^2\log(dx^4+c)}{4(bcd^2-ad^3)}$$

```
[In] integrate(x^11/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
[Out] 1/4*x^4/(b*d) + 1/4*a^2*log(b*x^4 + a)/(b^3*c - a*b^2*d) - 1/4*c^2*log(d*x^
4 + c)/(b*c*d^2 - a*d^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}}{(a+bx^4)(c+dx^4)}\,dx = \frac{x^4}{4\,bd} + \frac{a^2\log\left(|bx^4+a|\right)}{4\,(b^3c-ab^2d)} - \frac{c^2\log\left(|dx^4+c|\right)}{4\,(bcd^2-ad^3)}$$

```
[In] integrate(x^11/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
[Out] 1/4*x^4/(b*d) + 1/4*a^2*log(abs(b*x^4 + a))/(b^3*c - a*b^2*d) - 1/4*c^2*log
(abs(d*x^4 + c))/(b*c*d^2 - a*d^3)
```

Mupad [B] (verification not implemented)

Time = 9.91 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int \frac{x^{11}}{(a+bx^4)(c+dx^4)}\,dx = \frac{a^2\,\ln\left(b\,x^4+a\right)}{4\,b^3\,c-4\,a\,b^2\,d} + \frac{c^2\,\ln\left(d\,x^4+c\right)}{4\,a\,d^3-4\,b\,c\,d^2} + \frac{x^4}{4\,b\,d}$$

```
[In] int(x^11/((a + b*x^4)*(c + d*x^4)),x)
[Out] (a^2*log(a + b*x^4))/(4*b^3*c - 4*a*b^2*d) + (c^2*log(c + d*x^4))/(4*a*d^3
- 4*b*c*d^2) + x^4/(4*b*d)
```

3.769 $\int \frac{x^7}{(a+bx^4)(c+dx^4)} dx$

Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{x^7}{(a+bx^4)(c+dx^4)} dx = -\frac{a\log(a+bx^4)}{4b(bc-ad)} + \frac{c\log(c+dx^4)}{4d(bc-ad)}$$

```
[Out] -1/4*a*ln(b*x^4+a)/b/(-a*d+b*c)+1/4*c*ln(d*x^4+c)/d/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$\int \frac{x^7}{(a+bx^4)(c+dx^4)} dx = \frac{c\log(c+dx^4)}{4d(bc-ad)} - \frac{a\log(a+bx^4)}{4b(bc-ad)}$$

```
[In] Int[x^7/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/4*(a*Log[a + b*x^4])/(b*(b*c - a*d)) + (c*Log[c + d*x^4])/(4*d*(b*c - a*
d))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{x}{(a+bx)(c+dx)}\,dx, x, x^4\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)}+\frac{c}{(bc-ad)(c+dx)}\right)\,dx, x, x^4\right)\\ &= -\frac{a\log\left(a+bx^4\right)}{4b(bc-ad)}+\frac{c\log\left(c+dx^4\right)}{4d(bc-ad)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int \frac{x^7}{(a+bx^4)\left(c+dx^4\right)}\,dx = -\frac{ad\log\left(a+bx^4\right)-bc\log\left(c+dx^4\right)}{4b^2cd-4abd^2}$$

```
[In] Integrate[x^7/((a + b*x^4)*(c + d*x^4)),x]
[Out] -((a*d*Log[a + b*x^4] - b*c*Log[c + d*x^4])/(4*b^2*c*d - 4*a*b*d^2))
```

Maple [A] (verified)

Time = 4.52 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

method	result	size
parallelrisch	$\frac{a\ln\left(b\,x^{4}+a\right)d-c\ln\left(d\,x^{4}+c\right)b}{4(ad-bc)bd}$	43
default	$\frac{a\ln\left(b\,x^{4}+a\right)}{4(ad-bc)b}-\frac{c\ln\left(d\,x^{4}+c\right)}{4(ad-bc)d}$	50
norman	$\frac{a\ln\left(b\,x^{4}+a\right)}{4(ad-bc)b}-\frac{c\ln\left(d\,x^{4}+c\right)}{4(ad-bc)d}$	50
risch	$-\frac{c\ln\left(-d\,x^{4}-c\right)}{4(ad-bc)d}+\frac{a\ln\left(b\,x^{4}+a\right)}{4(ad-bc)b}$	53

```
[In] int(x^7/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
[Out] 1/4*(a*ln(b*x^4+a)*d-c*ln(d*x^4+c)*b)/(a*d-b*c)/b/d
```

Fricas [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \frac{x^7}{(a+bx^4)(c+dx^4)}\,dx = -\frac{ad\log(bx^4+a) - bc\log(dx^4+c)}{4(b^2cd - abd^2)}$$

```
[In] integrate(x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
[Out] -1/4*(a*d*log(b*x^4 + a) - b*c*log(d*x^4 + c))/(b^2*c*d - a*b*d^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(x**7/(b*x**4+a)/(d*x**4+c),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{x^7}{(a+bx^4)(c+dx^4)}\,dx = -\frac{a\log(bx^4+a)}{4(b^2c-abd)} + \frac{c\log(dx^4+c)}{4(bcd-ad^2)}$$

```
[In] integrate(x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
[Out] -1/4*a*log(b*x^4 + a)/(b^2*c - a*b*d) + 1/4*c*log(d*x^4 + c)/(b*c*d - a*d^2
)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{(a+bx^4)(c+dx^4)}\,dx = -\frac{a\log(|bx^4+a|)}{4(b^2c-abd)} + \frac{c\log(|dx^4+c|)}{4(bcd-ad^2)}$$

```
[In] integrate(x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
[Out] -1/4*a*log(abs(b*x^4 + a))/(b^2*c - a*b*d) + 1/4*c*log(abs(d*x^4 + c))/(b*c
*d - a*d^2)
```

Mupad [B] (verification not implemented)

Time = 9.50 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{(a+bx^4)(c+dx^4)}\,dx = -\frac{a\,\ln\left(b\,x^4+a\right)}{4\,b^2\,c-4\,a\,b\,d}-\frac{c\,\ln\left(d\,x^4+c\right)}{4\,a\,d^2-4\,b\,c\,d}$$

```
[In] int(x^7/((a + b*x^4)*(c + d*x^4)),x)
[Out] - (a*log(a + b*x^4))/(4*b^2*c - 4*a*b*d) - (c*log(c + d*x^4))/(4*a*d^2 - 4*
b*c*d)
```

3.770 $\int \frac{x^3}{(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 45

$$\int \frac{x^3}{(a+bx^4)(c+dx^4)}\,dx = \frac{\log(a+bx^4)}{4(bc-ad)} - \frac{\log(c+dx^4)}{4(bc-ad)}$$

```
[Out] 1/4*ln(b*x^4+a)/(-a*d+b*c)-1/4*ln(d*x^4+c)/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {455, 36, 31}

$$\int \frac{x^3}{(a+bx^4)(c+dx^4)}\,dx = \frac{\log(a+bx^4)}{4(bc-ad)} - \frac{\log(c+dx^4)}{4(bc-ad)}$$

```
[In] Int[x^3/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] Log[a + b*x^4]/(4*(b*c - a*d)) - Log[c + d*x^4]/(4*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)}\,dx, x, x^4\right)\\ &= \frac{b\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, x^4\right)}{4(bc-ad)} - \frac{d\text{Subst}\left(\int \frac{1}{c+dx}\,dx, x, x^4\right)}{4(bc-ad)}\\ &= \frac{\log\left(a+bx^4\right)}{4(bc-ad)} - \frac{\log\left(c+dx^4\right)}{4(bc-ad)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{(a+bx^4)\,(c+dx^4)}\,dx = \frac{\log\left(a+bx^4\right) - \log\left(c+dx^4\right)}{4bc-4ad}$$

```
[In] Integrate[x^3/((a + b*x^4)*(c + d*x^4)),x]
[Out] (Log[a + b*x^4] - Log[c + d*x^4])/(4*b*c - 4*a*d)
```

Maple [A] (verified)

Time = 4.52 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.71

method	result	size
parallelrisch	$-\frac{\ln(b\,x^4+a)-\ln(d\,x^4+c)}{4(ad-bc)}$	32
default	$-\frac{\ln(b\,x^4+a)}{4(ad-bc)}+\frac{\ln(d\,x^4+c)}{4ad-4bc}$	42
norman	$-\frac{\ln(b\,x^4+a)}{4(ad-bc)}+\frac{\ln(d\,x^4+c)}{4ad-4bc}$	42
risch	$-\frac{\ln(-b\,x^4-a)}{4(ad-bc)}+\frac{\ln(d\,x^4+c)}{4ad-4bc}$	45

```
[In] int(x^3/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
[Out] -1/4*(ln(b*x^4+a)-ln(d*x^4+c))/(a*d-b*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{(a+bx^4)(c+dx^4)}\,dx = \frac{\log(bx^4+a)-\log(dx^4+c)}{4(bc-ad)}$$

```
[In] integrate(x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
[Out] 1/4*(log(b*x^4 + a) - log(d*x^4 + c))/(b*c - a*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(36) = 72.

Time = 0.91 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.07

$$\int \frac{x^3}{(a+bx^4)(c+dx^4)}\,dx = \frac{\log\left(x^4+\frac{-\frac{a^2d^2}{ad-bc}+\frac{2abcd}{ad-bc}+ad-\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{4(ad-bc)} - \frac{\log\left(x^4+\frac{\frac{a^2d^2}{ad-bc}-\frac{2abcd}{ad-bc}+ad+\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{4(ad-bc)}$$

```
[In] integrate(x**3/(b*x**4+a)/(d*x**4+c),x)
[Out] log(x**4 + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2*c**
2/(a*d - b*c) + b*c)/(2*b*d))/(4*(a*d - b*c)) - log(x**4 + (a**2*d**2/(a*d
- b*c) - 2*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2*b*d)
)/(4*(a*d - b*c))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{(a+bx^4)(c+dx^4)}\,dx = \frac{\log(bx^4+a)}{4(bc-ad)} - \frac{\log(dx^4+c)}{4(bc-ad)}$$

```
[In] integrate(x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
[Out] 1/4*log(b*x^4 + a)/(b*c - a*d) - 1/4*log(d*x^4 + c)/(b*c - a*d)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

$$\int \frac{x^3}{(a+bx^4)(c+dx^4)}\,dx = \frac{b\log(|bx^4+a|)}{4(b^2c-abd)} - \frac{d\log(|dx^4+c|)}{4(bcd-ad^2)}$$

```
[In] integrate(x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/4*b*log(abs(b*x^4 + a))/(b^2*c - a*b*d) - 1/4*d*log(abs(d*x^4 + c))/(b*c*
d - a*d^2)
```

Mupad [B] (verification not implemented)

Time = 9.51 (sec) , antiderivative size = 1012, normalized size of antiderivative = 22.49

$$\int \frac{x^3}{(a+bx^4)(c+dx^4)}\,dx = -\frac{\operatorname{atan}\left(\frac{\left(\frac{x^4\left(96\,c\,b^5\,d^4+96\,a\,b^4\,d^5\right)+\frac{\frac{x^4\left(512\,a^3\,b^4\,d^7+1536\,a^2\,b^5\,c\,d^6+1536\,a\,b^6\,c^2\,d^5+512\,b^7\,c^3\,d^4\right)+1024\,a\,b^6\,c^3\,d^4+1024\,a^3\,b^4\,c\,d^6+2048\,a^2\,b^5\,c^2\,d^5}{4\,a\,d-4\,b\,c}+x^4}{4\,a\,d-4\,b\,c}}{4\,a\,d-4\,b\,c}\right.}{4\,a\,d-4\,b\,c}}{\frac{x^4\left(96\,c\,b^5\,d^4+96\,a\,b^4\,d^5\right)+\frac{\frac{x^4\left(512\,a^3\,b^4\,d^7+1536\,a^2\,b^5\,c\,d^6+1536\,a\,b^6\,c^2\,d^5+512\,b^7\,c^3\,d^4\right)+1024\,a\,b^6\,c^3\,d^4+1024\,a^3\,b^4\,c\,d^6+2048\,a^2\,b^5\,c^2\,d^5}{4\,a\,d-4\,b\,c}}{4\,a\,d-4\,b\,c}}{\frac{4\,a\,d-4\,b\,c}{4\,a\,d-4\,b\,c}}}\right)}{}$$

```
[In] int(x^3/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] -(atan(((((x^4*(96*a*b^4*d^5 + 96*b^5*c*d^4) + ((x^4*(512*a^3*b^4*d^7 + 512
*b^7*c^3*d^4 + 1536*a*b^6*c^2*d^5 + 1536*a^2*b^5*c*d^6) + 1024*a*b^6*c^3*d^
4 + 1024*a^3*b^4*c*d^6 + 2048*a^2*b^5*c^2*d^5)/(4*a*d - 4*b*c) + x^4*(384*a
^2*b^4*d^6 + 384*b^6*c^2*d^4 + 768*a*b^5*c*d^5) + 512*a*b^5*c^2*d^4 + 512*a
^2*b^4*c*d^5)/(4*a*d - 4*b*c) + 64*a*b^4*c*d^4)/(4*a*d - 4*b*c) + 8*b^4*d^4
*x^4)*1i)/(4*a*d - 4*b*c) - (((x^4*(96*a*b^4*d^5 + 96*b^5*c*d^4) - (x^4*(38
4*a^2*b^4*d^6 + 384*b^6*c^2*d^4 + 768*a*b^5*c*d^5) - (x^4*(512*a^3*b^4*d^7
+ 512*b^7*c^3*d^4 + 1536*a*b^6*c^2*d^5 + 1536*a^2*b^5*c*d^6) + 1024*a*b^6*c
^3*d^4 + 1024*a^3*b^4*c*d^6 + 2048*a^2*b^5*c^2*d^5)/(4*a*d - 4*b*c) + 512*a
*b^5*c^2*d^4 + 512*a^2*b^4*c*d^5)/(4*a*d - 4*b*c) + 64*a*b^4*c*d^4)/(4*a*d
- 4*b*c) - 8*b^4*d^4*x^4)*1i)/(4*a*d - 4*b*c))/(((x^4*(96*a*b^4*d^5 + 96*b^
5*c*d^4) + ((x^4*(512*a^3*b^4*d^7 + 512*b^7*c^3*d^4 + 1536*a*b^6*c^2*d^5 +
1536*a^2*b^5*c*d^6) + 1024*a*b^6*c^3*d^4 + 1024*a^3*b^4*c*d^6 + 2048*a^2*b^
5*c^2*d^5)/(4*a*d - 4*b*c) + x^4*(384*a^2*b^4*d^6 + 384*b^6*c^2*d^4 + 768*a
*b^5*c*d^5) + 512*a*b^5*c^2*d^4 + 512*a^2*b^4*c*d^5)/(4*a*d - 4*b*c) + 64*a
```

```
*b^4*c*d^4)/(4*a*d - 4*b*c) + 8*b^4*d^4*x^4)/(4*a*d - 4*b*c) + ((x^4*(96*a*
b^4*d^5 + 96*b^5*c*d^4) - (x^4*(384*a^2*b^4*d^6 + 384*b^6*c^2*d^4 + 768*a*b
^5*c*d^5) - (x^4*(512*a^3*b^4*d^7 + 512*b^7*c^3*d^4 + 1536*a*b^6*c^2*d^5 +
1536*a^2*b^5*c*d^6) + 1024*a*b^6*c^3*d^4 + 1024*a^3*b^4*c*d^6 + 2048*a^2*b^
5*c^2*d^5)/(4*a*d - 4*b*c) + 512*a*b^5*c^2*d^4 + 512*a^2*b^4*c*d^5)/(4*a*d
- 4*b*c) + 64*a*b^4*c*d^4)/(4*a*d - 4*b*c) - 8*b^4*d^4*x^4)/(4*a*d - 4*b*c)
))*2i)/(4*a*d - 4*b*c)
```

3.771 $\int \frac{1}{x(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 62

$$\int \frac{1}{x\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \frac{\log(x)}{ac} - \frac{b\log\left(a+bx^4\right)}{4a(bc-ad)} + \frac{d\log\left(c+dx^4\right)}{4c(bc-ad)}$$

```
[Out] ln(x)/a/c-1/4*b*ln(b*x^4+a)/a/(-a*d+b*c)+1/4*d*ln(d*x^4+c)/c/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 84}

$$\int \frac{1}{x\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = -\frac{b\log\left(a+bx^4\right)}{4a(bc-ad)} + \frac{d\log\left(c+dx^4\right)}{4c(bc-ad)} + \frac{\log(x)}{ac}$$

```
[In] Int[1/(x*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] Log[x]/(a*c) - (b*Log[a + b*x^4])/(4*a*(b*c - a*d)) + (d*Log[c + d*x^4])/(4
*c*(b*c - a*d))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)}\,dx, x, x^4\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{acx}+\frac{b^2}{a(-bc+ad)(a+bx)}+\frac{d^2}{c(bc-ad)(c+dx)}\right)\,dx, x, x^4\right)\\ &= \frac{\log(x)}{ac}-\frac{b\log(a+bx^4)}{4a(bc-ad)}+\frac{d\log(c+dx^4)}{4c(bc-ad)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \frac{4bc\log(x)-4ad\log(x)-bc\log\left(a+bx^4\right)+ad\log\left(c+dx^4\right)}{4abc^2-4a^2cd}$$

```
[In] Integrate[1/(x*(a + b*x^4)*(c + d*x^4)),x]
[Out] (4*b*c*Log[x] - 4*a*d*Log[x] - b*c*Log[a + b*x^4] + a*d*Log[c + d*x^4])/(4*
a*b*c^2 - 4*a^2*c*d)
```

Maple [A] (verified)

Time = 4.53 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

method	result	size
parallelrisch	$\frac{4\ln(x)ad-4\ln(x)bc+b\ln(b\,x^4+a)c-d\ln(d\,x^4+c)a}{4ac(ad-bc)}$	55
default	$\frac{\ln(x)}{ac}+\frac{b\ln(b\,x^4+a)}{4(ad-bc)a}-\frac{d\ln(d\,x^4+c)}{4(ad-bc)c}$	59
norman	$\frac{\ln(x)}{ac}+\frac{b\ln(b\,x^4+a)}{4(ad-bc)a}-\frac{d\ln(d\,x^4+c)}{4(ad-bc)c}$	59
risch	$\frac{\ln(x)}{ac}+\frac{b\ln(b\,x^4+a)}{4(ad-bc)a}-\frac{d\ln(d\,x^4+c)}{4(ad-bc)c}$	59

```
[In] int(1/x/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
[Out] 1/4*(4*ln(x)*a*d-4*ln(x)*b*c+b*ln(b*x^4+a)*c-d*ln(d*x^4+c)*a)/a/c/(a*d-b*c)
```

Fricas [A] (verification not implemented)

none

Time = 1.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = -\frac{bc\log\left(bx^4+a\right)-ad\log\left(dx^4+c\right)-4\left(bc-ad\right)\log\left(x\right)}{4\left(abc^2-a^2cd\right)}$$

```
[In] integrate(1/x/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
[Out] -1/4*(b*c*log(b*x^4 + a) - a*d*log(d*x^4 + c) - 4*(b*c - a*d)*log(x))/(a*b*
c^2 - a^2*c*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/(b*x**4+a)/(d*x**4+c),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{1}{x\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = -\frac{b\log\left(bx^4+a\right)}{4\left(abc-a^2d\right)}+\frac{d\log\left(dx^4+c\right)}{4\left(bc^2-acd\right)}+\frac{\log\left(x^4\right)}{4\,ac}$$

```
[In] integrate(1/x/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
[Out] -1/4*b*log(b*x^4 + a)/(a*b*c - a^2*d) + 1/4*d*log(d*x^4 + c)/(b*c^2 - a*c*d
) + 1/4*log(x^4)/(a*c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.18

$$\int \frac{1}{x(a+bx^4)(c+dx^4)}\,dx = -\frac{b^2\log(|bx^4+a|)}{4(ab^2c-a^2bd)} + \frac{d^2\log(|dx^4+c|)}{4(bc^2d-acd^2)} + \frac{\log(x^4)}{4\,ac}$$

```
[In] integrate(1/x/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/4*b^2*log(abs(b*x^4 + a))/(a*b^2*c - a^2*b*d) + 1/4*d^2*log(abs(d*x^4 +
c))/(b*c^2*d - a*c*d^2) + 1/4*log(x^4)/(a*c)
```

Mupad [B] (verification not implemented)

Time = 10.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{1}{x(a+bx^4)(c+dx^4)}\,dx = \frac{b\,\ln(b\,x^4+a)}{4\,a^2\,d-4\,a\,b\,c} + \frac{d\,\ln(d\,x^4+c)}{4\,b\,c^2-4\,a\,c\,d} + \frac{\ln(x)}{a\,c}$$

```
[In] int(1/(x*(a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] (b*log(a + b*x^4))/(4*a^2*d - 4*a*b*c) + (d*log(c + d*x^4))/(4*b*c^2 - 4*a*
c*d) + log(x)/(a*c)
```

3.772 $\int \frac{1}{x^5(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 87

$$\int \frac{1}{x^5(a+bx^4)(c+dx^4)}\,dx = -\frac{1}{4acx^4} - \frac{(bc+ad)\log(x)}{a^2c^2} + \frac{b^2\log(a+bx^4)}{4a^2(bc-ad)} - \frac{d^2\log(c+dx^4)}{4c^2(bc-ad)}$$

```
[Out] -1/4/a/c/x^4-(a*d+b*c)*ln(x)/a^2/c^2+1/4*b^2*ln(b*x^4+a)/a^2/(-a*d+b*c)-1/4
*d^2*ln(d*x^4+c)/c^2/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 84}

$$\int \frac{1}{x^5(a+bx^4)(c+dx^4)}\,dx = \frac{b^2\log(a+bx^4)}{4a^2(bc-ad)} - \frac{\log(x)(ad+bc)}{a^2c^2} - \frac{d^2\log(c+dx^4)}{4c^2(bc-ad)} - \frac{1}{4acx^4}$$

```
[In] Int[1/(x^5*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/4*1/(a*c*x^4) - ((b*c + a*d)*Log[x])/(a^2*c^2) + (b^2*Log[a + b*x^4])/(4
*a^2*(b*c - a*d)) - (d^2*Log[c + d*x^4])/(4*c^2*(b*c - a*d))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^2(a+bx)(c+dx)}\,dx, x, x^4\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{acx^2}+\frac{-bc-ad}{a^2c^2x}-\frac{b^3}{a^2(-bc+ad)(a+bx)}-\frac{d^3}{c^2(bc-ad)(c+dx)}\right)dx, x, x^4\right)\\ &= -\frac{1}{4acx^4}-\frac{(bc+ad)\log(x)}{a^2c^2}+\frac{b^2\log(a+bx^4)}{4a^2(bc-ad)}-\frac{d^2\log(c+dx^4)}{4c^2(bc-ad)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^5(a+bx^4)(c+dx^4)}\,dx = -\frac{1}{4acx^4}+\frac{(-bc-ad)\log(x)}{a^2c^2} - \frac{b^2\log(a+bx^4)}{4a^2(-bc+ad)}-\frac{d^2\log(c+dx^4)}{4c^2(bc-ad)}$$

```
[In] Integrate[1/(x^5*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/4*1/(a*c*x^4) + ((-(b*c) - a*d)*Log[x])/(a^2*c^2) - (b^2*Log[a + b*x^4])
/(4*a^2*(-(b*c) + a*d)) - (d^2*Log[c + d*x^4])/(4*c^2*(b*c - a*d))
```

Maple [A] (verified)

Time = 4.82 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

method	result	size
norman	$-\frac{1}{4acx^4}-\frac{b^2\ln(bx^4+a)}{4a^2(ad-bc)}+\frac{d^2\ln(dx^4+c)}{4c^2(ad-bc)}-\frac{(ad+bc)\ln(x)}{a^2c^2}$	82
default	$-\frac{1}{4acx^4}+\frac{(-ad-bc)\ln(x)}{a^2c^2}-\frac{b^2\ln(bx^4+a)}{4a^2(ad-bc)}+\frac{d^2\ln(dx^4+c)}{4c^2(ad-bc)}$	83
risch	$-\frac{1}{4acx^4}-\frac{\ln(x)d}{ac^2}-\frac{\ln(x)b}{a^2c}+\frac{d^2\ln(-dx^4-c)}{4c^2(ad-bc)}-\frac{b^2\ln(bx^4+a)}{4a^2(ad-bc)}$	90
parallelrisch	$-\frac{4\ln(x)x^4a^2d^2-4\ln(x)x^4b^2c^2+b^2\ln(bx^4+a)c^2x^4-d^2\ln(dx^4+c)a^2x^4+a^2cd-bc^2a}{4a^2c^2x^4(ad-bc)}$	99

```
[In] int(1/x^5/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a/c/x^4-1/4*b^2/a^2/(a*d-b*c)*ln(b*x^4+a)+1/4*d^2/c^2/(a*d-b*c)*ln(d*x
^4+c)-(a*d+b*c)*ln(x)/a^2/c^2
```

Fricas [A] (verification not implemented)

none

Time = 4.73 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^5 (a + bx^4) (c + dx^4)} dx = \frac{b^2c^2x^4 \log (bx^4 + a) - a^2d^2x^4 \log (dx^4 + c) - 4 (b^2c^2 - a^2d^2)x^4 \log (x) - abc^2 + a^2cd}{4 (a^2bc^3 - a^3c^2d)x^4}$$

```
[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*c^2*x^4*log(b*x^4 + a) - a^2*d^2*x^4*log(d*x^4 + c) - 4*(b^2*c^2 -
 a^2*d^2)*x^4*log(x) - a*b*c^2 + a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d)*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (a + bx^4) (c + dx^4)} dx = \text{Timed out}$$

```
[In] integrate(1/x**5/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5 (a + bx^4) (c + dx^4)} dx = \frac{b^2 \log (bx^4 + a)}{4 (a^2bc - a^3d)} - \frac{d^2 \log (dx^4 + c)}{4 (bc^3 - ac^2d)} - \frac{(bc + ad) \log (x^4)}{4 a^2c^2} - \frac{1}{4 acx^4}$$

```
[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*log(b*x^4 + a)/(a^2*b*c - a^3*d) - 1/4*d^2*log(d*x^4 + c)/(b*c^3 -
a*c^2*d) - 1/4*(b*c + a*d)*log(x^4)/(a^2*c^2) - 1/4/(a*c*x^4)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^5 (a + bx^4) (c + dx^4)} dx = \frac{b^3 \log (|bx^4 + a|)}{4 (a^2 b^2 c - a^3 bd)} - \frac{d^3 \log (|dx^4 + c|)}{4 (bc^3 d - ac^2 d^2)} - \frac{(bc + ad) \log (x^4)}{4 a^2 c^2} + \frac{bcx^4 + adx^4 - ac}{4 a^2 c^2 x^4}$$

```
[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/4*b^3*log(abs(b*x^4 + a))/(a^2*b^2*c - a^3*b*d) - 1/4*d^3*log(abs(d*x^4 +
 c))/(b*c^3*d - a*c^2*d^2) - 1/4*(b*c + a*d)*log(x^4)/(a^2*c^2) + 1/4*(b*c*
x^4 + a*d*x^4 - a*c)/(a^2*c^2*x^4)
```

Mupad [B] (verification not implemented)

Time = 11.38 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5 (a + bx^4) (c + dx^4)} dx = -\frac{b^2 \ln (b x^4 + a)}{4 (a^3 d - a^2 b c)} - \frac{d^2 \ln (d x^4 + c)}{4 (b c^3 - a c^2 d)} - \frac{1}{4 a c x^4} - \frac{\ln (x) (a d + b c)}{a^2 c^2}$$

```
[In] int(1/(x^5*(a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] - (b^2*log(a + b*x^4))/(4*(a^3*d - a^2*b*c)) - (d^2*log(c + d*x^4))/(4*(b*c
^3 - a*c^2*d)) - 1/(4*a*c*x^4) - (log(x)*(a*d + b*c))/(a^2*c^2)
```

3.773 $\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 112

$$\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx=-\frac{(bc+ad)x^2}{2b^2d^2}+\frac{x^6}{6bd}-\frac{a^{5/2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{5/2}(bc-ad)}+\frac{c^{5/2}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2d^{5/2}(bc-ad)}$$

```
[Out] -1/2*(a*d+b*c)*x^2/b^2/d^2+1/6*x^6/b/d-1/2*a^(5/2)*arctan(x^2*b^(1/2)/a^(1/
2))/b^(5/2)/(-a*d+b*c)+1/2*c^(5/2)*arctan(x^2*d^(1/2)/c^(1/2))/d^(5/2)/(-a*
d+b*c)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {476, 490, 596, 536, 211}

$$\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx=-\frac{a^{5/2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{5/2}(bc-ad)}+\frac{c^{5/2}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2d^{5/2}(bc-ad)}-\frac{x^2(ad+bc)}{2b^2d^2}+\frac{x^6}{6bd}$$

```
[In] Int[x^13/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/2*((b*c + a*d)*x^2)/(b^2*d^2) + x^6/(6*b*d) - (a^(5/2)*ArcTan[(Sqrt[b]*x
^2)/Sqrt[a]])/(2*b^(5/2)*(b*c - a*d)) + (c^(5/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[
c]])/(2*d^(5/2)*(b*c - a*d))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^6}{(a+bx^2)\,(c+dx^2)}\,dx, x, x^2\right)\\
&= \frac{x^6}{6bd} - \frac{\text{Subst}\left(\int \frac{x^2(3ac+3(bc+ad)x^2)}{(a+bx^2)(c+dx^2)}\,dx, x, x^2\right)}{6bd}\\
&= -\frac{(bc+ad)x^2}{2b^2d^2} + \frac{x^6}{6bd} + \frac{\text{Subst}\left(\int \frac{3ac(bc+ad)+3(b^2c^2+ad(bc+ad))x^2}{(a+bx^2)(c+dx^2)}\,dx, x, x^2\right)}{6b^2d^2}\\
&= -\frac{(bc+ad)x^2}{2b^2d^2} + \frac{x^6}{6bd} - \frac{a^3\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{2b^2(bc-ad)} + \frac{c^3\text{Subst}\left(\int \frac{1}{c+dx^2}\,dx, x, x^2\right)}{2d^2(bc-ad)}
\end{aligned}$$

$$= -\frac{(bc+ad)x^2}{2b^2d^2} + \frac{x^6}{6bd} - \frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{5/2}(bc-ad)} + \frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2d^{5/2}(bc-ad)}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.93

$$\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx = \frac{1}{6}\left(\frac{x^2(-3bc-3ad+bdx^4)}{b^2d^2} + \frac{3a^{5/2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{b^{5/2}(-bc+ad)} + \frac{3c^{5/2}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{d^{5/2}(bc-ad)}\right)$$

```
[In] Integrate[x^13/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((x^2*(-3*b*c - 3*a*d + b*d*x^4))/(b^2*d^2) + (3*a^(5/2)*ArcTan[(Sqrt[b]*x^
2)/Sqrt[a]])/(b^(5/2)*(-(b*c) + a*d)) + (3*c^(5/2)*ArcTan[(Sqrt[d]*x^2)/Sqr
t[c]])/(d^(5/2)*(b*c - a*d)))/6
```

Maple [A] (verified)

Time = 4.59 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.88

method	result
default	$-\frac{-\frac{bd\,x^6}{6}+\frac{(ad+bc)x^2}{2}}{b^2d^2}+\frac{a^3\arctan\left(\frac{b\,x^2}{\sqrt{ab}}\right)}{2b^2(ad-bc)\sqrt{ab}}-\frac{c^3\arctan\left(\frac{d\,x^2}{\sqrt{cd}}\right)}{2d^2(ad-bc)\sqrt{cd}}$
risch	$\frac{x^6}{6bd}-\frac{a\,x^2}{2b^2d}-\frac{c\,x^2}{2b\,d^2}+\frac{\sqrt{-cd}\,c^2\ln\left(\left(-a^6d^8+a\,d^3c^5b^5\right)x^2+(-cd)^{\frac{3}{2}}a\,b^5c^4d+(-cd)^{\frac{3}{2}}b^6c^5+a^6d^7\sqrt{-cd}+b^6c^6\sqrt{-cd}\,d\right)}{4d^3(ad-bc)}-\frac{\sqrt{-cd}\,c^2\ln}{}$

```
[In] int(x^13/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^2/d^2*(-1/6*b*d*x^6+1/2*(a*d+b*c)*x^2)+1/2*a^3/b^2/(a*d-b*c)/(a*b)^(1/
2)*arctan(b*x^2/(a*b)^(1/2))-1/2*c^3/d^2/(a*d-b*c)/(c*d)^(1/2)*arctan(d*x^2
/(c*d)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 1.49 (sec) , antiderivative size = 576, normalized size of antiderivative = 5.14

$$\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx = \left[\frac{2\,(b^2cd-abd^2)x^6 - 3\,a^2d^2\sqrt{-\frac{a}{b}}\log\left(\frac{bx^4+2\,bx^2\sqrt{-\frac{a}{b}}-a}{bx^4+a}\right) - 3\,b^2c^2\sqrt{-\frac{c}{d}}\log\left(\frac{dx^4-2\,dx^2\sqrt{-\frac{c}{d}}-c}{dx^4+c}\right) - 6\,(b^2c^2-a}{12\,(b^3cd^2-ab^2d^3)}\right.$$

```
[In] integrate(x^13/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] [1/12*(2*(b^2*c*d - a*b*d^2)*x^6 - 3*a^2*d^2*sqrt(-a/b)*log((b*x^4 + 2*b*x^
2*sqrt(-a/b) - a)/(b*x^4 + a)) - 3*b^2*c^2*sqrt(-c/d)*log((d*x^4 - 2*d*x^2*
sqrt(-c/d) - c)/(d*x^4 + c)) - 6*(b^2*c^2 - a^2*d^2)*x^2)/(b^3*c*d^2 - a*b^
2*d^3), 1/12*(2*(b^2*c*d - a*b*d^2)*x^6 - 6*a^2*d^2*sqrt(a/b)*arctan(b*x^2*
sqrt(a/b)/a) - 3*b^2*c^2*sqrt(-c/d)*log((d*x^4 - 2*d*x^2*sqrt(-c/d) - c)/(d
*x^4 + c)) - 6*(b^2*c^2 - a^2*d^2)*x^2)/(b^3*c*d^2 - a*b^2*d^3), 1/12*(2*(b
^2*c*d - a*b*d^2)*x^6 + 6*b^2*c^2*sqrt(c/d)*arctan(d*x^2*sqrt(c/d)/c) - 3*a
^2*d^2*sqrt(-a/b)*log((b*x^4 + 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)) - 6*(b^
2*c^2 - a^2*d^2)*x^2)/(b^3*c*d^2 - a*b^2*d^3), 1/6*((b^2*c*d - a*b*d^2)*x^6
 - 3*a^2*d^2*sqrt(a/b)*arctan(b*x^2*sqrt(a/b)/a) + 3*b^2*c^2*sqrt(c/d)*arct
an(d*x^2*sqrt(c/d)/c) - 3*(b^2*c^2 - a^2*d^2)*x^2)/(b^3*c*d^2 - a*b^2*d^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(x**13/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.89

$$\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx = -\frac{a^3 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\,(b^3c-ab^2d)\sqrt{ab}} + \frac{c^3 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\,(bcd^2-ad^3)\sqrt{cd}} + \frac{bdx^6-3\,(bc+ad)x^2}{6\,b^2d^2}$$

```
[In] integrate(x^13/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/2*a^3*arctan(b*x^2/sqrt(a*b))/((b^3*c - a*b^2*d)*sqrt(a*b)) + 1/2*c^3*ar
ctan(d*x^2/sqrt(c*d))/((b*c*d^2 - a*d^3)*sqrt(c*d)) + 1/6*(b*d*x^6 - 3*(b*c
 + a*d)*x^2)/(b^2*d^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx = -\frac{a^3 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\,(b^3c-ab^2d)\sqrt{ab}} + \frac{c^3 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\,(bcd^2-ad^3)\sqrt{cd}} + \frac{b^2d^2x^6-3\,b^2cdx^2-3\,abd^2x^2}{6\,b^3d^3}$$

```
[In] integrate(x^13/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/2*a^3*arctan(b*x^2/sqrt(a*b))/((b^3*c - a*b^2*d)*sqrt(a*b)) + 1/2*c^3*ar
ctan(d*x^2/sqrt(c*d))/((b*c*d^2 - a*d^3)*sqrt(c*d)) + 1/6*(b^2*d^2*x^6 - 3*
b^2*c*d*x^2 - 3*a*b*d^2*x^2)/(b^3*d^3)
```

Mupad [B] (verification not implemented)

Time = 10.77 (sec) , antiderivative size = 532, normalized size of antiderivative = 4.75

$$\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx$$
$$= \frac{\ln\left(d^{10}\left(-a^5b^5\right)^{5/2}+b^{20}c^{10}\sqrt{-a^5b^5}-a^2b^{23}c^{10}x^2-a^{12}b^{13}d^{10}x^2+2b^{10}c^5d^5\left(-a^5b^5\right)^{3/2}+2a^7b^{18}c^5d^5x\right.}{4b^6c-4ab^5d}$$
$$-\frac{\ln\left(d^{10}\left(-a^5b^5\right)^{5/2}+b^{20}c^{10}\sqrt{-a^5b^5}+a^2b^{23}c^{10}x^2+a^{12}b^{13}d^{10}x^2+2b^{10}c^5d^5\left(-a^5b^5\right)^{3/2}-2a^7b^{18}c^5d\right.}{4\left(b^6c-ab^5d\right)}$$
$$-\frac{\ln\left(b^{10}\left(-c^5d^5\right)^{5/2}+a^{10}d^{20}\sqrt{-c^5d^5}+a^{10}c^2d^{23}x^2+b^{10}c^{12}d^{13}x^2+2a^5b^5d^{10}\left(-c^5d^5\right)^{3/2}-2a^5b^5c^7d\right.}{4\left(ad^6-bcd^5\right)}$$
$$+\frac{\ln\left(b^{10}\left(-c^5d^5\right)^{5/2}+a^{10}d^{20}\sqrt{-c^5d^5}-a^{10}c^2d^{23}x^2-b^{10}c^{12}d^{13}x^2+2a^5b^5d^{10}\left(-c^5d^5\right)^{3/2}+2a^5b^5c^7d\right.}{4ad^6-4bcd^5}$$
$$+\frac{x^6}{6bd}-\frac{x^2(ad+bc)}{2b^2d^2}$$

```
[In] int(x^13/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] (log(d^10*(-a^5*b^5)^(5/2) + b^20*c^10*(-a^5*b^5)^(1/2) - a^2*b^23*c^10*x^2
 - a^12*b^13*d^10*x^2 + 2*b^10*c^5*d^5*(-a^5*b^5)^(3/2) + 2*a^7*b^18*c^5*d^
5*x^2)*(-a^5*b^5)^(1/2))/(4*b^6*c - 4*a*b^5*d) - (log(d^10*(-a^5*b^5)^(5/2)
 + b^20*c^10*(-a^5*b^5)^(1/2) + a^2*b^23*c^10*x^2 + a^12*b^13*d^10*x^2 + 2*
b^10*c^5*d^5*(-a^5*b^5)^(3/2) - 2*a^7*b^18*c^5*d^5*x^2)*(-a^5*b^5)^(1/2))/(
4*(b^6*c - a*b^5*d)) - (log(b^10*(-c^5*d^5)^(5/2) + a^10*d^20*(-c^5*d^5)^(1
/2) + a^10*c^2*d^23*x^2 + b^10*c^12*d^13*x^2 + 2*a^5*b^5*d^10*(-c^5*d^5)^(3
/2) - 2*a^5*b^5*c^7*d^18*x^2)*(-c^5*d^5)^(1/2))/(4*(a*d^6 - b*c*d^5)) + (lo
g(b^10*(-c^5*d^5)^(5/2) + a^10*d^20*(-c^5*d^5)^(1/2) - a^10*c^2*d^23*x^2 -
b^10*c^12*d^13*x^2 + 2*a^5*b^5*d^10*(-c^5*d^5)^(3/2) + 2*a^5*b^5*c^7*d^18*x
^2)*(-c^5*d^5)^(1/2))/(4*a*d^6 - 4*b*c*d^5) + x^6/(6*b*d) - (x^2*(a*d + b*c
))/(2*b^2*d^2)
```

3.774 $\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 92

$$\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx=\frac{x^2}{2bd}+\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}(bc-ad)}-\frac{c^{3/2}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)}$$

```
[Out] 1/2*x^2/b/d+1/2*a^(3/2)*arctan(x^2*b^(1/2)/a^(1/2))/b^(3/2)/(-a*d+b*c)-1/2*
c^(3/2)*arctan(x^2*d^(1/2)/c^(1/2))/d^(3/2)/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {476, 490, 536, 211}

$$\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx=\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}(bc-ad)}-\frac{c^{3/2}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)}+\frac{x^2}{2bd}$$

```
[In] Int[x^9/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] x^2/(2*b*d) + (a^(3/2)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*b^(3/2)*(b*c - a*d
)) - (c^(3/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(2*d^(3/2)*(b*c - a*d))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(a+bx^2)\,(c+dx^2)}\,dx, x, x^2\right)\\
&= \frac{x^2}{2bd} - \frac{\text{Subst}\left(\int \frac{ac+(bc+ad)x^2}{(a+bx^2)(c+dx^2)}\,dx, x, x^2\right)}{2bd}\\
&= \frac{x^2}{2bd} + \frac{a^2\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{2b(bc-ad)} - \frac{c^2\text{Subst}\left(\int \frac{1}{c+dx^2}\,dx, x, x^2\right)}{2d(bc-ad)}\\
&= \frac{x^2}{2bd} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}(bc-ad)} - \frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

$$\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx = \frac{\left(-\frac{a}{b}+\frac{c}{d}\right)x^2+\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{b^{3/2}}-\frac{c^{3/2}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{d^{3/2}}}{2bc-2ad}$$

```
[In] Integrate[x^9/((a + b*x^4)*(c + d*x^4)),x]
[Out] ((-(a/b) + c/d)*x^2 + (a^(3/2)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/b^(3/2) - (c^
(3/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/d^(3/2))/(2*b*c - 2*a*d)
```

Maple [A] (verified)

Time = 4.70 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

method	result
default	$\frac{x^2}{2bd}-\frac{a^2\arctan\left(\frac{b\,x^2}{\sqrt{ab}}\right)}{2(ad-bc)b\sqrt{ab}}+\frac{c^2\arctan\left(\frac{d\,x^2}{\sqrt{cd}}\right)}{2(ad-bc)d\sqrt{cd}}$
risch	$\frac{x^2}{2bd}+\frac{\sqrt{-ab}\,a\ln\left(\left(b^3c\,d^3a^3-b^6c^4\right)x^2+(-ab)^{\frac{3}{2}}a^3d^4+(-ab)^{\frac{3}{2}}a^2bc\,d^3+a^4\sqrt{-ab}\,d^4b+b^5c^4\sqrt{-ab}\right)}{4b^2(ad-bc)}-\frac{\sqrt{-ab}\,a\ln\left(\left(b^3c\,d^3a^3-b^6c^4\right)x^2-\right.}{}$

```
[In] int(x^9/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2/b/d-1/2*a^2/(a*d-b*c)/b/(a*b)^(1/2)*arctan(b*x^2/(a*b)^(1/2))+1/2*c
^2/(a*d-b*c)/d/(c*d)^(1/2)*arctan(d*x^2/(c*d)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 416, normalized size of antiderivative = 4.52

$$\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx$$

$$= \left[-\frac{ad\sqrt{-\frac{a}{b}}\log\left(\frac{bx^4-2\,bx^2\sqrt{-\frac{a}{b}}-a}{bx^4+a}\right)+bc\sqrt{-\frac{c}{d}}\log\left(\frac{dx^4+2\,dx^2\sqrt{-\frac{c}{d}}-c}{dx^4+c}\right)-2\,(bc-ad)x^2}{4\,(b^2cd-abd^2)}, \frac{2\,ad\sqrt{\frac{a}{b}}\arctan\left(\frac{bx^2\sqrt{\frac{a}{b}}}{a}\right.}{}\right.$$

$$-\frac{2\,bc\sqrt{\frac{c}{d}}\arctan\left(\frac{dx^2\sqrt{\frac{c}{d}}}{c}\right)+ad\sqrt{-\frac{a}{b}}\log\left(\frac{bx^4-2\,bx^2\sqrt{-\frac{a}{b}}-a}{bx^4+a}\right)-2\,(bc-ad)x^2}{4\,(b^2cd-abd^2)}, \frac{ad\sqrt{\frac{a}{b}}\arctan\left(\frac{bx^2\sqrt{\frac{a}{b}}}{a}\right)-bc}{2\,(b^2}$$

```
[In] integrate(x^9/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] [-1/4*(a*d*sqrt(-a/b)*log((b*x^4 - 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)) + b
*c*sqrt(-c/d)*log((d*x^4 + 2*d*x^2*sqrt(-c/d) - c)/(d*x^4 + c)) - 2*(b*c -
a*d)*x^2)/(b^2*c*d - a*b*d^2), 1/4*(2*a*d*sqrt(a/b)*arctan(b*x^2*sqrt(a/b)/
a) - b*c*sqrt(-c/d)*log((d*x^4 + 2*d*x^2*sqrt(-c/d) - c)/(d*x^4 + c)) + 2*(
b*c - a*d)*x^2)/(b^2*c*d - a*b*d^2), -1/4*(2*b*c*sqrt(c/d)*arctan(d*x^2*sqr
t(c/d)/c) + a*d*sqrt(-a/b)*log((b*x^4 - 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)
) - 2*(b*c - a*d)*x^2)/(b^2*c*d - a*b*d^2), 1/2*(a*d*sqrt(a/b)*arctan(b*x^2
*sqrt(a/b)/a) - b*c*sqrt(c/d)*arctan(d*x^2*sqrt(c/d)/c) + (b*c - a*d)*x^2)/
(b^2*c*d - a*b*d^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(x**9/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.87

$$\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx = \frac{a^2 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\,(b^2c-abd)\sqrt{ab}} - \frac{c^2 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\,(bcd-ad^2)\sqrt{cd}} + \frac{x^2}{2\,bd}$$

```
[In] integrate(x^9/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*arctan(b*x^2/sqrt(a*b))/((b^2*c - a*b*d)*sqrt(a*b)) - 1/2*c^2*arcta
n(d*x^2/sqrt(c*d))/((b*c*d - a*d^2)*sqrt(c*d)) + 1/2*x^2/(b*d)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.87

$$\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx = \frac{a^2 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\,(b^2c-abd)\sqrt{ab}} - \frac{c^2 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\,(bcd-ad^2)\sqrt{cd}} + \frac{x^2}{2\,bd}$$

```
[In] integrate(x^9/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*a^2*arctan(b*x^2/sqrt(a*b))/((b^2*c - a*b*d)*sqrt(a*b)) - 1/2*c^2*arcta
n(d*x^2/sqrt(c*d))/((b*c*d - a*d^2)*sqrt(c*d)) + 1/2*x^2/(b*d)
```

Mupad [B] (verification not implemented)

Time = 10.54 (sec) , antiderivative size = 518, normalized size of antiderivative = 5.63

$$\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx = \frac{\ln\left(b^9\,c^6\,\sqrt{-a^3\,b^3}-a^3\,d^6\,{\left(-a^3\,b^3\right)}^{3/2}+a\,b^{11}\,c^6\,x^2+a^7\,b^5\,d^6\,x^2+2\,b^3\,c^3\,d^3\,{\left(-a^3\,b^3\right)}^{3/2}-2\,a^4\,b^8\,c^3\,d^3\,x^2\right)\,\sqrt{-}}{4\,b^4\,c-4\,a\,b^3\,d} - \frac{\ln\left(a^3\,d^6\,{\left(-a^3\,b^3\right)}^{3/2}-b^9\,c^6\,\sqrt{-a^3\,b^3}+a\,b^{11}\,c^6\,x^2+a^7\,b^5\,d^6\,x^2-2\,b^3\,c^3\,d^3\,{\left(-a^3\,b^3\right)}^{3/2}-2\,a^4\,b^8\,c^3\,d^3\,x^2\right)\,\sqrt{}}{4\,\left(b^4\,c-a\,b^3\,d\right)} - \frac{\ln\left(b^6\,c^3\,{\left(-c^3\,d^3\right)}^{3/2}-a^6\,d^9\,\sqrt{-c^3\,d^3}+a^6\,c\,d^{11}\,x^2+b^6\,c^7\,d^5\,x^2-2\,a^3\,b^3\,d^3\,{\left(-c^3\,d^3\right)}^{3/2}-2\,a^3\,b^3\,c^4\,d^8\,x^2\right)\,\sqrt{}}{4\,\left(a\,d^4-b\,c\,d^3\right)} + \frac{\ln\left(a^6\,d^9\,\sqrt{-c^3\,d^3}-b^6\,c^3\,{\left(-c^3\,d^3\right)}^{3/2}+a^6\,c\,d^{11}\,x^2+b^6\,c^7\,d^5\,x^2+2\,a^3\,b^3\,d^3\,{\left(-c^3\,d^3\right)}^{3/2}-2\,a^3\,b^3\,c^4\,d^8\,x^2\right)\,\sqrt{}}{4\,a\,d^4-4\,b\,c\,d^3} + \frac{x^2}{2\,b\,d}$$

```
[In] int(x^9/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] (log(b^9*c^6*(-a^3*b^3)^(1/2) - a^3*d^6*(-a^3*b^3)^(3/2) + a*b^11*c^6*x^2 +
 a^7*b^5*d^6*x^2 + 2*b^3*c^3*d^3*(-a^3*b^3)^(3/2) - 2*a^4*b^8*c^3*d^3*x^2)*
(-a^3*b^3)^(1/2))/(4*b^4*c - 4*a*b^3*d) - (log(a^3*d^6*(-a^3*b^3)^(3/2) - b
^9*c^6*(-a^3*b^3)^(1/2) + a*b^11*c^6*x^2 + a^7*b^5*d^6*x^2 - 2*b^3*c^3*d^3*
(-a^3*b^3)^(3/2) - 2*a^4*b^8*c^3*d^3*x^2)*(-a^3*b^3)^(1/2))/(4*(b^4*c - a*b
^3*d)) - (log(b^6*c^3*(-c^3*d^3)^(3/2) - a^6*d^9*(-c^3*d^3)^(1/2) + a^6*c*d
^11*x^2 + b^6*c^7*d^5*x^2 - 2*a^3*b^3*d^3*(-c^3*d^3)^(3/2) - 2*a^3*b^3*c^4*
d^8*x^2)*(-c^3*d^3)^(1/2))/(4*(a*d^4 - b*c*d^3)) + (log(a^6*d^9*(-c^3*d^3)^
(1/2) - b^6*c^3*(-c^3*d^3)^(3/2) + a^6*c*d^11*x^2 + b^6*c^7*d^5*x^2 + 2*a^3
*b^3*d^3*(-c^3*d^3)^(3/2) - 2*a^3*b^3*c^4*d^8*x^2)*(-c^3*d^3)^(1/2))/(4*a*d
^4 - 4*b*c*d^3) + x^2/(2*b*d)
```

3.775 $\int \frac{x^5}{(a+bx^4)(c+dx^4)} dx$

Optimal result

Integrand size = 22, antiderivative size = 79

$$\int \frac{x^5}{(a+bx^4)(c+dx^4)} dx = -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}(bc-ad)} + \frac{\sqrt{c}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2\sqrt{d}(bc-ad)}$$

```
[Out] -1/2*arctan(x^2*b^(1/2)/a^(1/2))*a^(1/2)/(-a*d+b*c)/b^(1/2)+1/2*arctan(x^2*
d^(1/2)/c^(1/2))*c^(1/2)/(-a*d+b*c)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {476, 492, 211}

$$\int \frac{x^5}{(a+bx^4)(c+dx^4)} dx = \frac{\sqrt{c}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2\sqrt{d}(bc-ad)} - \frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}(bc-ad)}$$

```
[In] Int[x^5/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/2*(Sqrt[a]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[b]*(b*c - a*d)) + (Sqrt[
c]*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(2*Sqrt[d]*(b*c - a*d))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx^2)(c+dx^2)}\,dx, x, x^2\right)\\ &= -\frac{a\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{2(bc-ad)} + \frac{c\text{Subst}\left(\int \frac{1}{c+dx^2}\,dx, x, x^2\right)}{2(bc-ad)}\\ &= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}(bc-ad)} + \frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2\sqrt{d}(bc-ad)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int \frac{x^5}{(a+bx^4)(c+dx^4)}\,dx = \frac{-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{\sqrt{c}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{\sqrt{d}}}{2bc-2ad}$$

```
[In] Integrate[x^5/((a + b*x^4)*(c + d*x^4)),x]
[Out] (-((Sqrt[a]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[b]) + (Sqrt[c]*ArcTan[(Sqrt
[d]*x^2)/Sqrt[c]])/Sqrt[d])/(2*b*c - 2*a*d)
```

Maple [A] (verified)

Time = 4.80 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.76

method	result
default	$\frac{a\arctan\left(\frac{b\,x^{2}}{\sqrt{a b}}\right)}{2(a d-b c)\sqrt{a b}}-\frac{c\arctan\left(\frac{d\,x^{2}}{\sqrt{c d}}\right)}{2(a d-b c)\sqrt{c d}}$
risch	$\frac{\sqrt{-a b}\,\ln\left(\left(-a\,b^{3} c d+b^{4} c^{2}\right)x^{2}+\left(-a b\right)^{\frac{3}{2}} a\,d^{2}+\left(-a b\right)^{\frac{3}{2}} b c d+a^{2}\sqrt{-a b}\,d^{2} b+b^{3} c^{2}\sqrt{-a b}\right)}{4 b(a d-b c)}-\frac{\sqrt{-a b}\,\ln\left(\left(-a\,b^{3} c d+b^{4} c^{2}\right)x^{2}-\left(-a b\right)^{\frac{3}{2}} a\,d^{2}-\right.}{4b(ad-}$

[In] int(x^5/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)

[Out] 1/2*a/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x^2/(a*b)^(1/2))-1/2*c/(a*d-b*c)/(c*d)^(1/2)*arctan(d*x^2/(c*d)^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 325, normalized size of antiderivative = 4.11

$$\int \frac{x^5}{(a+bx^4)(c+dx^4)}\,dx = \left[-\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx^4+2\,bx^2\sqrt{-\frac{a}{b}}-a}{bx^4+a}\right)+\sqrt{-\frac{c}{d}}\log\left(\frac{dx^4-2\,dx^2\sqrt{-\frac{c}{d}}-c}{dx^4+c}\right)}{4\,(bc-ad)}, -\frac{2\,\sqrt{\frac{a}{b}}\arctan\left(\frac{bx^2\sqrt{\frac{a}{b}}}{a}\right)+\sqrt{-\frac{c}{d}}\log\left(\frac{dx^4-2\,dx^2\sqrt{-\frac{c}{d}}-c}{dx^4+c}\right)}{4\,(bc-ad)}, \frac{2\,\sqrt{\frac{c}{d}}\arctan\left(\frac{dx^2\sqrt{\frac{c}{d}}}{c}\right)-\sqrt{-\frac{a}{b}}\log\left(\frac{bx^4+2\,bx^2\sqrt{-\cdots}}{bx^4+a}\right.}{4\,(bc-ad)} -\frac{\sqrt{\frac{a}{b}}\arctan\left(\frac{bx^2\sqrt{\frac{a}{b}}}{a}\right)-\sqrt{\frac{c}{d}}\arctan\left(\frac{dx^2\sqrt{\frac{c}{d}}}{c}\right)}{2\,(bc-ad)}\right]$$

[In] integrate(x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")

[Out] [-1/4*(sqrt(-a/b)*log((b*x^4 + 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)) + sqrt(-c/d)*log((d*x^4 - 2*d*x^2*sqrt(-c/d) - c)/(d*x^4 + c)))/(b*c - a*d), -1/4*(2*sqrt(a/b)*arctan(b*x^2*sqrt(a/b)/a) + sqrt(-c/d)*log((d*x^4 - 2*d*x^2*sqrt(-c/d) - c)/(d*x^4 + c)))/(b*c - a*d), 1/4*(2*sqrt(c/d)*arctan(d*x^2*sqrt(c/d)/c) - sqrt(-a/b)*log((b*x^4 + 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)))/(b*c - a*d), -1/2*(sqrt(a/b)*arctan(b*x^2*sqrt(a/b)/a) - sqrt(c/d)*arctan(d*x^2*sqrt(c/d)/c))/(b*c - a*d)]

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(x**5/(b*x**4+a)/(d*x**4+c),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{(a+bx^4)(c+dx^4)}\,dx = -\frac{a\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\sqrt{ab}(bc-ad)} + \frac{c\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(bc-ad)\sqrt{cd}}$$

```
[In] integrate(x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
[Out] -1/2*a*arctan(b*x^2/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) + 1/2*c*arctan(d*x^2
/sqrt(c*d))/((b*c - a*d)*sqrt(c*d))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{(a+bx^4)(c+dx^4)}\,dx = -\frac{a\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\sqrt{ab}(bc-ad)} + \frac{c\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(bc-ad)\sqrt{cd}}$$

```
[In] integrate(x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
[Out] -1/2*a*arctan(b*x^2/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) + 1/2*c*arctan(d*x^2
/sqrt(c*d))/((b*c - a*d)*sqrt(c*d))
```

Mupad [B] (verification not implemented)

Time = 10.43 (sec) , antiderivative size = 379, normalized size of antiderivative = 4.80

$$\int \frac{x^5}{(a+bx^4)(c+dx^4)}\,dx = \frac{\ln\left(d^2(-ab)^{5/2}+b^4c^2\sqrt{-ab}-b^5c^2x^2+2b^2cd(-ab)^{3/2}-a^2b^3d^2x^2+2ab^4cdx^2\right)\sqrt{-ab}}{4b^2c-4abd} - \frac{\ln\left(d^2(-ab)^{5/2}+b^4c^2\sqrt{-ab}+b^5c^2x^2+2b^2cd(-ab)^{3/2}+a^2b^3d^2x^2-2ab^4cdx^2\right)\sqrt{-ab}}{4(b^2c-abd)} - \frac{\ln\left(b^2(-cd)^{5/2}+a^2d^4\sqrt{-cd}+a^2d^5x^2+2abd^2(-cd)^{3/2}+b^2c^2d^3x^2-2abcd^4x^2\right)\sqrt{-cd}}{4(ad^2-bcd)} + \frac{\ln\left(b^2(-cd)^{5/2}+a^2d^4\sqrt{-cd}-a^2d^5x^2+2abd^2(-cd)^{3/2}-b^2c^2d^3x^2+2abcd^4x^2\right)\sqrt{-cd}}{4ad^2-4bcd}$$

```
[In] int(x^5/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] (log(d^2*(-a*b)^(5/2) + b^4*c^2*(-a*b)^(1/2) - b^5*c^2*x^2 + 2*b^2*c*d*(-a*
b)^(3/2) - a^2*b^3*d^2*x^2 + 2*a*b^4*c*d*x^2)*(-a*b)^(1/2))/(4*b^2*c - 4*a*
b*d) - (log(d^2*(-a*b)^(5/2) + b^4*c^2*(-a*b)^(1/2) + b^5*c^2*x^2 + 2*b^2*c
*d*(-a*b)^(3/2) + a^2*b^3*d^2*x^2 - 2*a*b^4*c*d*x^2)*(-a*b)^(1/2))/(4*(b^2*
c - a*b*d)) - (log(b^2*(-c*d)^(5/2) + a^2*d^4*(-c*d)^(1/2) + a^2*d^5*x^2 +
2*a*b*d^2*(-c*d)^(3/2) + b^2*c^2*d^3*x^2 - 2*a*b*c*d^4*x^2)*(-c*d)^(1/2))/(
4*(a*d^2 - b*c*d)) + (log(b^2*(-c*d)^(5/2) + a^2*d^4*(-c*d)^(1/2) - a^2*d^5
*x^2 + 2*a*b*d^2*(-c*d)^(3/2) - b^2*c^2*d^3*x^2 + 2*a*b*c*d^4*x^2)*(-c*d)^(
1/2))/(4*a*d^2 - 4*b*c*d)
```

3.776 $\int \frac{x}{(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 79

$$\int \frac{x}{(a+bx^4)(c+dx^4)}\,dx = \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(bc-ad)} - \frac{\sqrt{d}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2\sqrt{c}(bc-ad)}$$

```
[Out] 1/2*arctan(x^2*b^(1/2)/a^(1/2))*b^(1/2)/(-a*d+b*c)/a^(1/2)-1/2*arctan(x^2*d
^(1/2)/c^(1/2))*d^(1/2)/(-a*d+b*c)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {476, 400, 211}

$$\int \frac{x}{(a+bx^4)(c+dx^4)}\,dx = \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(bc-ad)} - \frac{\sqrt{d}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2\sqrt{c}(bc-ad)}$$

```
[In] Int[x/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(b*c - a*d)) - (Sqrt[d]*
ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(2*Sqrt[c]*(b*c - a*d))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx, x, x^2\right) \\
&= \frac{b\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{2(bc-ad)} - \frac{d\text{Subst}\left(\int \frac{1}{c+dx^2}\,dx, x, x^2\right)}{2(bc-ad)} \\
&= \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(bc-ad)} - \frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2\sqrt{c}(bc-ad)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$
\int \frac{x}{(a+bx^4)(c+dx^4)}\,dx = \frac{\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{d}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{\sqrt{c}}}{2bc-2ad}
$$

```
[In] Integrate[x/((a + b*x^4)*(c + d*x^4)),x]
[Out] ((Sqrt[b]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[a] - (Sqrt[d]*ArcTan[(Sqrt[d]
*x^2)/Sqrt[c]])/Sqrt[c])/(2*b*c - 2*a*d)
```

Maple [A] (verified)

Time = 4.53 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.76

method	result
default	$-\frac{b\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2(ad-bc)\sqrt{ab}}+\frac{d\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(ad-bc)\sqrt{cd}}$
risch	$\frac{\sqrt{-cd}\,\ln\left(\left(-acd^3+bc^2d^2\right)x^2+(-cd)^{\frac{3}{2}}ad+(-cd)^{\frac{3}{2}}bc+2\sqrt{-cd}\,bc^2d\right)}{4c(ad-bc)}-\frac{\sqrt{-cd}\,\ln\left(\left(-acd^3+bc^2d^2\right)x^2-(-cd)^{\frac{3}{2}}ad-(-cd)^{\frac{3}{2}}bc-2\sqrt{-}\right.}{4c(ad-bc)}$

```
[In] int(x/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x^2/(a*b)^(1/2))+1/2*d/(a*d-b*c)/(c*d
)^(1/2)*arctan(d*x^2/(c*d)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 325, normalized size of antiderivative = 4.11

$$\int \frac{x}{(a+bx^4)(c+dx^4)}\,dx = \left[-\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4-2\,ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right)+\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4+2\,cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right)}{4\,(bc-ad)}, \frac{2\,\sqrt{\frac{d}{c}}\arctan\left(\frac{c\sqrt{\frac{d}{c}}}{dx^2}\right)-\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4-2\,ax^2 \ldots}{bx^4+ \ldots}\right)}{4\,(bc-ad)}, -\frac{2\,\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{bx^2}\right)+\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4+2\,cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right)}{4\,(bc-ad)}, -\frac{\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{bx^2}\right)-\sqrt{\frac{d}{c}}\arctan\left(\frac{c\sqrt{\frac{d}{c}}}{dx^2}\right)}{2\,(bc-ad)}\right]$$

```
[In] integrate(x/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-b/a)*log((b*x^4 - 2*a*x^2*sqrt(-b/a) - a)/(b*x^4 + a)) + sqrt(
-d/c)*log((d*x^4 + 2*c*x^2*sqrt(-d/c) - c)/(d*x^4 + c)))/(b*c - a*d), 1/4*(
2*sqrt(d/c)*arctan(c*sqrt(d/c)/(d*x^2)) - sqrt(-b/a)*log((b*x^4 - 2*a*x^2*s
qrt(-b/a) - a)/(b*x^4 + a)))/(b*c - a*d), -1/4*(2*sqrt(b/a)*arctan(a*sqrt(b
/a)/(b*x^2)) + sqrt(-d/c)*log((d*x^4 + 2*c*x^2*sqrt(-d/c) - c)/(d*x^4 + c))
)/(b*c - a*d), -1/2*(sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^2)) - sqrt(d/c)*arct
an(c*sqrt(d/c)/(d*x^2)))/(b*c - a*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(x/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int \frac{x}{(a+bx^4)(c+dx^4)}\,dx = \frac{b\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\sqrt{ab}(bc-ad)} - \frac{d\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(bc-ad)\sqrt{cd}}$$

```
[In] integrate(x/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/2*b*arctan(b*x^2/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) - 1/2*d*arctan(d*x^2/
sqrt(c*d))/((b*c - a*d)*sqrt(c*d))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int \frac{x}{(a+bx^4)(c+dx^4)}\,dx = \frac{b\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\sqrt{ab}(bc-ad)} - \frac{d\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(bc-ad)\sqrt{cd}}$$

```
[In] integrate(x/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*b*arctan(b*x^2/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) - 1/2*d*arctan(d*x^2/
sqrt(c*d))/((b*c - a*d)*sqrt(c*d))
```

Mupad [B] (verification not implemented)

Time = 10.41 (sec) , antiderivative size = 399, normalized size of antiderivative = 5.05

$$\begin{aligned}\int \frac{x}{(a+bx^4)(c+dx^4)}\,dx &= \frac{\ln\left(a^2\,d^2\,{\left(-a\,b\right)}^{5/2}+b^2\,c^2\,{\left(-a\,b\right)}^{5/2}+2\,c\,d\,{\left(-a\,b\right)}^{7/2}-a^2\,b^5\,c^2\,x^2-a^4\,b^3\,d^2\,x^2+2\,a^3\,b^4\,c\,d\,x^2\right)\,\sqrt{-a\,b}}{4\,a^2\,d-4\,a\,b\,c}\\ &\quad-\frac{\ln\left(a^2\,d^2\,{\left(-a\,b\right)}^{5/2}+b^2\,c^2\,{\left(-a\,b\right)}^{5/2}+2\,c\,d\,{\left(-a\,b\right)}^{7/2}+a^2\,b^5\,c^2\,x^2+a^4\,b^3\,d^2\,x^2-2\,a^3\,b^4\,c\,d\,x^2\right)\,\sqrt{-a\,b}}{4\,\left(a^2\,d-a\,b\,c\right)}\\ &\quad-\frac{\ln\left(a^2\,d^2\,{\left(-c\,d\right)}^{5/2}+b^2\,c^2\,{\left(-c\,d\right)}^{5/2}+2\,a\,b\,{\left(-c\,d\right)}^{7/2}+a^2\,c^2\,d^5\,x^2+b^2\,c^4\,d^3\,x^2-2\,a\,b\,c^3\,d^4\,x^2\right)\,\sqrt{-c\,d}}{4\,\left(b\,c^2-a\,c\,d\right)}\\ &\quad+\frac{\ln\left(a^2\,d^2\,{\left(-c\,d\right)}^{5/2}+b^2\,c^2\,{\left(-c\,d\right)}^{5/2}+2\,a\,b\,{\left(-c\,d\right)}^{7/2}-a^2\,c^2\,d^5\,x^2-b^2\,c^4\,d^3\,x^2+2\,a\,b\,c^3\,d^4\,x^2\right)\,\sqrt{-c\,d}}{4\,b\,c^2-4\,a\,c\,d}\end{aligned}$$

[In] int(x/((a + b*x^4)*(c + d*x^4)),x)

[Out] (log(a^2*d^2*(-a*b)^(5/2) + b^2*c^2*(-a*b)^(5/2) + 2*c*d*(-a*b)^(7/2) - a^2*b^5*c^2*x^2 - a^4*b^3*d^2*x^2 + 2*a^3*b^4*c*d*x^2)*(-a*b)^(1/2))/(4*a^2*d - 4*a*b*c) - (log(a^2*d^2*(-a*b)^(5/2) + b^2*c^2*(-a*b)^(5/2) + 2*c*d*(-a*b)^(7/2) + a^2*b^5*c^2*x^2 + a^4*b^3*d^2*x^2 - 2*a^3*b^4*c*d*x^2)*(-a*b)^(1/2))/(4*(a^2*d - a*b*c)) - (log(a^2*d^2*(-c*d)^(5/2) + b^2*c^2*(-c*d)^(5/2) + 2*a*b*(-c*d)^(7/2) + a^2*c^2*d^5*x^2 + b^2*c^4*d^3*x^2 - 2*a*b*c^3*d^4*x^2)*(-c*d)^(1/2))/(4*(b*c^2 - a*c*d)) + (log(a^2*d^2*(-c*d)^(5/2) + b^2*c^2*(-c*d)^(5/2) + 2*a*b*(-c*d)^(7/2) - a^2*c^2*d^5*x^2 - b^2*c^4*d^3*x^2 + 2*a*b*c^3*d^4*x^2)*(-c*d)^(1/2))/(4*b*c^2 - 4*a*c*d)

3.777 $\int \frac{1}{x^3(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 92

$$\int \frac{1}{x^3(a+bx^4)(c+dx^4)}\,dx = -\frac{1}{2acx^2} - \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)} + \frac{d^{3/2}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)}$$

```
[Out] -1/2/a/c/x^2-1/2*b^(3/2)*arctan(x^2*b^(1/2)/a^(1/2))/a^(3/2)/(-a*d+b*c)+1/2
*d^(3/2)*arctan(x^2*d^(1/2)/c^(1/2))/c^(3/2)/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {476, 491, 536, 211}

$$\int \frac{1}{x^3(a+bx^4)(c+dx^4)}\,dx = -\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)} + \frac{d^{3/2}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)} - \frac{1}{2acx^2}$$

```
[In] Int[1/(x^3*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/2*1/(a*c*x^2) - (b^(3/2)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*a^(3/2)*(b*c
- a*d)) + (d^(3/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(2*c^(3/2)*(b*c - a*d))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)\left(c+dx^2\right)}\,dx,x,x^2\right)\\
&= -\frac{1}{2acx^2}+\frac{\text{Subst}\left(\int \frac{-bc-ad-bdx^2}{(a+bx^2)(c+dx^2)}\,dx,x,x^2\right)}{2ac}\\
&= -\frac{1}{2acx^2}-\frac{b^2\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)}{2a(bc-ad)}+\frac{d^2\text{Subst}\left(\int \frac{1}{c+dx^2}\,dx,x,x^2\right)}{2c(bc-ad)}\\
&= -\frac{1}{2acx^2}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.84

$$\int \frac{1}{x^3(a+bx^4)(c+dx^4)}\,dx = \frac{\frac{b}{a}-\frac{d}{c}-\frac{b^{3/2}x^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{3/2}}-\frac{b^{3/2}x^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{3/2}}+\frac{d^{3/2}x^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{3/2}}+\frac{d^{3/2}x^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{3/2}}}{2(-bc+ad)x^2}$$

```
[In] Integrate[1/(x^3*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (b/a - d/c - (b^(3/2)*x^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/2)
- (b^(3/2)*x^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/2) + (d^(3/2)*
x^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(3/2) + (d^(3/2)*x^2*ArcTan[
1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(3/2))/(2*(-(b*c) + a*d)*x^2)
```

Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

method	result
default	$\frac{b^2\arctan\left(\frac{b\,x^2}{\sqrt{ab}}\right)}{2(ad-bc)a\sqrt{ab}}-\frac{1}{2ac\,x^2}-\frac{d^2\arctan\left(\frac{d\,x^2}{\sqrt{cd}}\right)}{2(ad-bc)c\sqrt{cd}}$
risch	$-\frac{1}{2ac\,x^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(d^2a^5-2a^4cbd+a^3b^2c^2\right)\_Z^2+b^3\right)}\_R\ln\left(\left(\left(-5c^3a^7d^4+18c^4a^6b\,d^3-26a^5c^5b^2d^2+18c^6a^4b^3d-5c^7a^3b^4\right)\_\right.\right.\right.}{}$

```
[In] int(1/x^3/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^2/(a*d-b*c)/a/(a*b)^(1/2)*arctan(b*x^2/(a*b)^(1/2))-1/2/a/c/x^2-1/2*d
^2/(a*d-b*c)/c/(c*d)^(1/2)*arctan(d*x^2/(c*d)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.53 (sec) , antiderivative size = 432, normalized size of antiderivative = 4.70

$$\int \frac{1}{x^3 (a + bx^4) (c + dx^4)} dx$$
$$= \left[-\frac{bcx^2\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4+2\,ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right) + adx^2\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4-2\,cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right) + 2\,bc - 2\,ad}{4\,(abc^2 - a^2cd)x^2}, -\frac{2\,adx^2\sqrt{\frac{d}{c}}\arctan\left(\frac{c\sqrt{\frac{d}{c}}}{dx^2}\right) + bcx^2\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4+2\,ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right) + 2\,bc - 2\,ad}{4\,(abc^2 - a^2cd)x^2}, \frac{2\,bcx^2\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{bx^2}\right) - \cdots}{\cdots}\right.$$

```
[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] [-1/4*(b*c*x^2*sqrt(-b/a)*log((b*x^4 + 2*a*x^2*sqrt(-b/a) - a)/(b*x^4 + a))
 + a*d*x^2*sqrt(-d/c)*log((d*x^4 - 2*c*x^2*sqrt(-d/c) - c)/(d*x^4 + c)) + 2
*b*c - 2*a*d)/((a*b*c^2 - a^2*c*d)*x^2), -1/4*(2*a*d*x^2*sqrt(d/c)*arctan(c
*sqrt(d/c)/(d*x^2)) + b*c*x^2*sqrt(-b/a)*log((b*x^4 + 2*a*x^2*sqrt(-b/a) -
a)/(b*x^4 + a)) + 2*b*c - 2*a*d)/((a*b*c^2 - a^2*c*d)*x^2), 1/4*(2*b*c*x^2*
sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^2)) - a*d*x^2*sqrt(-d/c)*log((d*x^4 - 2*c
*x^2*sqrt(-d/c) - c)/(d*x^4 + c)) - 2*b*c + 2*a*d)/((a*b*c^2 - a^2*c*d)*x^2
), 1/2*(b*c*x^2*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^2)) - a*d*x^2*sqrt(d/c)*a
rctan(c*sqrt(d/c)/(d*x^2)) - b*c + a*d)/((a*b*c^2 - a^2*c*d)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a + bx^4) (c + dx^4)} dx = \text{Timed out}$$

```
[In] integrate(1/x**3/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3 (a+bx^4) (c+dx^4)} \, dx = -\frac{b^2 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2 (abc-a^2d)\sqrt{ab}} + \frac{d^2 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2 (bc^2-acd)\sqrt{cd}} - \frac{1}{2\,acx^2}$$

```
[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
[Out] -1/2*b^2*arctan(b*x^2/sqrt(a*b))/((a*b*c - a^2*d)*sqrt(a*b)) + 1/2*d^2*arct
an(d*x^2/sqrt(c*d))/((b*c^2 - a*c*d)*sqrt(c*d)) - 1/2/(a*c*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3 (a+bx^4) (c+dx^4)} \, dx = -\frac{b^2 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2 (abc-a^2d)\sqrt{ab}} + \frac{d^2 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2 (bc^2-acd)\sqrt{cd}} - \frac{1}{2\,acx^2}$$

```
[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
[Out] -1/2*b^2*arctan(b*x^2/sqrt(a*b))/((a*b*c - a^2*d)*sqrt(a*b)) + 1/2*d^2*arct
an(d*x^2/sqrt(c*d))/((b*c^2 - a*c*d)*sqrt(c*d)) - 1/2/(a*c*x^2)
```

Mupad [B] (verification not implemented)

Time = 10.19 (sec) , antiderivative size = 354, normalized size of antiderivative = 3.85

$$\begin{aligned}\int \frac{1}{x^3 (a+bx^4) (c+dx^4)} \, dx &= \frac{\ln\left(c^3\,x^2\,{\left(-a^3\,b^3\right)}^{3/2}-a^8\,b\,d^3+a^5\,b^4\,c^3+a^6\,d^3\,x^2\,\sqrt{-a^3\,b^3}\right)\,\sqrt{-a^3\,b^3}}{4\,a^4\,d-4\,a^3\,b\,c} \\ &\quad - \frac{\ln\left(c^3\,x^2\,{\left(-a^3\,b^3\right)}^{3/2}+a^8\,b\,d^3-a^5\,b^4\,c^3+a^6\,d^3\,x^2\,\sqrt{-a^3\,b^3}\right)\,\sqrt{-a^3\,b^3}}{4\,\left(a^4\,d-a^3\,b\,c\right)} - \frac{1}{2\,a\,c\,x^2} \\ &\quad - \frac{\ln\left(a^3\,x^2\,{\left(-c^3\,d^3\right)}^{3/2}+b^3\,c^8\,d-a^3\,c^5\,d^4+b^3\,c^6\,x^2\,\sqrt{-c^3\,d^3}\right)\,\sqrt{-c^3\,d^3}}{4\,\left(b\,c^4-a\,c^3\,d\right)} \\ &\quad + \frac{\ln\left(a^3\,x^2\,{\left(-c^3\,d^3\right)}^{3/2}-b^3\,c^8\,d+a^3\,c^5\,d^4+b^3\,c^6\,x^2\,\sqrt{-c^3\,d^3}\right)\,\sqrt{-c^3\,d^3}}{4\,b\,c^4-4\,a\,c^3\,d}\end{aligned}$$

```
[In] int(1/(x^3*(a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] (log(c^3*x^2*(-a^3*b^3)^(3/2) - a^8*b*d^3 + a^5*b^4*c^3 + a^6*d^3*x^2*(-a^3
*b^3)^(1/2))*(-a^3*b^3)^(1/2))/(4*a^4*d - 4*a^3*b*c) - (log(c^3*x^2*(-a^3*b
^3)^(3/2) + a^8*b*d^3 - a^5*b^4*c^3 + a^6*d^3*x^2*(-a^3*b^3)^(1/2))*(-a^3*b
^3)^(1/2))/(4*(a^4*d - a^3*b*c)) - 1/(2*a*c*x^2) - (log(a^3*x^2*(-c^3*d^3)^
(3/2) + b^3*c^8*d - a^3*c^5*d^4 + b^3*c^6*x^2*(-c^3*d^3)^(1/2))*(-c^3*d^3)^
(1/2))/(4*(b*c^4 - a*c^3*d)) + (log(a^3*x^2*(-c^3*d^3)^(3/2) - b^3*c^8*d +
a^3*c^5*d^4 + b^3*c^6*x^2*(-c^3*d^3)^(1/2))*(-c^3*d^3)^(1/2))/(4*b*c^4 - 4*
a*c^3*d)
```

3.778 $\int \frac{1}{x^7(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 112

$$\int \frac{1}{x^7(a+bx^4)(c+dx^4)}\,dx = -\frac{1}{6acx^6}+\frac{bc+ad}{2a^2c^2x^2}+\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)}-\frac{d^{5/2}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)}$$

```
[Out] -1/6/a/c/x^6+1/2*(a*d+b*c)/a^2/c^2/x^2+1/2*b^(5/2)*arctan(x^2*b^(1/2)/a^(1/
2))/a^(5/2)/(-a*d+b*c)-1/2*d^(5/2)*arctan(x^2*d^(1/2)/c^(1/2))/c^(5/2)/(-a*
d+b*c)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {476, 491, 597, 536, 211}

$$\int \frac{1}{x^7(a+bx^4)(c+dx^4)}\,dx = \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)}+\frac{ad+bc}{2a^2c^2x^2}-\frac{d^{5/2}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)}-\frac{1}{6acx^6}$$

```
[In] Int[1/(x^7*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/6*1/(a*c*x^6) + (b*c + a*d)/(2*a^2*c^2*x^2) + (b^(5/2)*ArcTan[(Sqrt[b]*x
^2)/Sqrt[a]])/(2*a^(5/2)*(b*c - a*d)) - (d^(5/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[
c]])/(2*c^(5/2)*(b*c - a*d))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4(a+bx^2)(c+dx^2)}\,dx,x,x^2\right)\\
&= -\frac{1}{6acx^6}+\frac{\text{Subst}\left(\int \frac{-3(bc+ad)-3bdx^2}{x^2(a+bx^2)(c+dx^2)}\,dx,x,x^2\right)}{6ac}\\
&= -\frac{1}{6acx^6}+\frac{bc+ad}{2a^2c^2x^2}-\frac{\text{Subst}\left(\int \frac{-3(b^2c^2+abcd+a^2d^2)-3bd(bc+ad)x^2}{(a+bx^2)(c+dx^2)}\,dx,x,x^2\right)}{6a^2c^2}\\
&= -\frac{1}{6acx^6}+\frac{bc+ad}{2a^2c^2x^2}+\frac{b^3\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)}{2a^2(bc-ad)}-\frac{d^3\text{Subst}\left(\int \frac{1}{c+dx^2}\,dx,x,x^2\right)}{2c^2(bc-ad)}
\end{aligned}
$$

$$= -\frac{1}{6acx^6} + \frac{bc+ad}{2a^2c^2x^2} + \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)} - \frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.72

$$\int \frac{1}{x^7 (a+bx^4)(c+dx^4)} dx$$

$$= \frac{\frac{b}{a} - \frac{d}{c} - \frac{3b^2x^4}{a^2} + \frac{3d^2x^4}{c^2} + \frac{3b^{5/2}x^6 \arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{5/2}} + \frac{3b^{5/2}x^6 \arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{5/2}} - \frac{3d^{5/2}x^6 \arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{5/2}} - \frac{3d^{5/2}}{}}{6(-bc+ad)x^6}$$

```
[In] Integrate[1/(x^7*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (b/a - d/c - (3*b^2*x^4)/a^2 + (3*d^2*x^4)/c^2 + (3*b^(5/2)*x^6*ArcTan[1 -
(Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(5/2) + (3*b^(5/2)*x^6*ArcTan[1 + (Sqrt[2]*
b^(1/4)*x)/a^(1/4)])/a^(5/2) - (3*d^(5/2)*x^6*ArcTan[1 - (Sqrt[2]*d^(1/4)*x
)/c^(1/4)])/c^(5/2) - (3*d^(5/2)*x^6*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)
])/c^(5/2))/(6*(-(b*c) + a*d)*x^6)
```

Maple [A] (verified)

Time = 4.59 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.90

method	result
default	$-\frac{1}{6ac\,x^6} - \frac{-ad-bc}{2x^2a^2c^2} - \frac{b^3 \arctan\left(\frac{b\,x^2}{\sqrt{ab}}\right)}{2a^2(ad-bc)\sqrt{ab}} + \frac{d^3 \arctan\left(\frac{d\,x^2}{\sqrt{cd}}\right)}{2c^2(ad-bc)\sqrt{cd}}$
risch	$\frac{\frac{(ad+bc)x^4}{2a^2c^2} - \frac{1}{6ac}}{x^6} + \frac{\left(\sum_{\_R=\text{RootOf}\left(\left(d^2c^5a^2-2ab\,c^6d+b^2c^7\right)\_Z^2+d^5\right)} \_R \ln\left(\left(\left(5c^5a^9d^4-18c^6a^8b\,d^3+26c^7a^7b^2d^2-18c^8a^6b^3d+5c^9a^5\right.\right.\right.\right.}{}$

```
[In] int(1/x^7/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/a/c/x^6-1/2*(-a*d-b*c)/x^2/a^2/c^2-1/2*b^3/a^2/(a*d-b*c)/(a*b)^(1/2)*a
rctan(b*x^2/(a*b)^(1/2))+1/2*d^3/c^2/(a*d-b*c)/(c*d)^(1/2)*arctan(d*x^2/(c*
d)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 1.99 (sec) , antiderivative size = 592, normalized size of antiderivative = 5.29

$$\int \frac{1}{x^7 (a+bx^4) (c+dx^4)} dx = \left[-\frac{3 b^2c^2x^6\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4-2ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right) + 3a^2d^2x^6\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4+2cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right) - 6(b^2c^2-a^2d^2)x^4 + 2abc^2 \ldots}{12(a^2bc^3-a^3c^2d)x^6}, -\frac{6b^2c^2x^6\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{bx^2}\right) + 3a^2d^2x^6\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4+2cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right) - 6(b^2c^2-a^2d^2)x^4 + 2abc^2 - 2a^2cd}{12(a^2bc^3-a^3c^2d)x^6}, -\frac{3b^2c^2x^6\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{bx^2}\right) - 3a^2d^2x^6\sqrt{\frac{d}{c}}\arctan\left(\frac{c\sqrt{\frac{d}{c}}}{dx^2}\right) - 3(b^2c^2-a^2d^2)x^4 + abc^2 - a^2cd}{6(a^2bc^3-a^3c^2d)x^6} \right]$$

```
[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*b^2*c^2*x^6*sqrt(-b/a)*log((b*x^4 - 2*a*x^2*sqrt(-b/a) - a)/(b*x^
4 + a)) + 3*a^2*d^2*x^6*sqrt(-d/c)*log((d*x^4 + 2*c*x^2*sqrt(-d/c) - c)/(d*
x^4 + c)) - 6*(b^2*c^2 - a^2*d^2)*x^4 + 2*a*b*c^2 - 2*a^2*c*d)/((a^2*b*c^3
- a^3*c^2*d)*x^6), 1/12*(6*a^2*d^2*x^6*sqrt(d/c)*arctan(c*sqrt(d/c)/(d*x^2)
) - 3*b^2*c^2*x^6*sqrt(-b/a)*log((b*x^4 - 2*a*x^2*sqrt(-b/a) - a)/(b*x^4 +
a)) + 6*(b^2*c^2 - a^2*d^2)*x^4 - 2*a*b*c^2 + 2*a^2*c*d)/((a^2*b*c^3 - a^3*
c^2*d)*x^6), -1/12*(6*b^2*c^2*x^6*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^2)) + 3
*a^2*d^2*x^6*sqrt(-d/c)*log((d*x^4 + 2*c*x^2*sqrt(-d/c) - c)/(d*x^4 + c)) -
 6*(b^2*c^2 - a^2*d^2)*x^4 + 2*a*b*c^2 - 2*a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d
)*x^6), -1/6*(3*b^2*c^2*x^6*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^2)) - 3*a^2*d
^2*x^6*sqrt(d/c)*arctan(c*sqrt(d/c)/(d*x^2)) - 3*(b^2*c^2 - a^2*d^2)*x^4 +
a*b*c^2 - a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d)*x^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^7 (a+bx^4) (c+dx^4)} dx = \text{Timed out}$$

```
[In] integrate(1/x**7/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^7 (a + bx^4) (c + dx^4)} dx = \frac{b^3 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2 (a^2bc - a^3d)\sqrt{ab}} - \frac{d^3 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2 (bc^3 - ac^2d)\sqrt{cd}} + \frac{3 (bc + ad)x^4 - ac}{6 a^2c^2x^6}$$

```
[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/2*b^3*arctan(b*x^2/sqrt(a*b))/((a^2*b*c - a^3*d)*sqrt(a*b)) - 1/2*d^3*arc
tan(d*x^2/sqrt(c*d))/((b*c^3 - a*c^2*d)*sqrt(c*d)) + 1/6*(3*(b*c + a*d)*x^4
 - a*c)/(a^2*c^2*x^6)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^7 (a + bx^4) (c + dx^4)} dx = \frac{b^3 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2 (a^2bc - a^3d)\sqrt{ab}} - \frac{d^3 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2 (bc^3 - ac^2d)\sqrt{cd}} + \frac{3 bcx^4 + 3 adx^4 - ac}{6 a^2c^2x^6}$$

```
[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*b^3*arctan(b*x^2/sqrt(a*b))/((a^2*b*c - a^3*d)*sqrt(a*b)) - 1/2*d^3*arc
tan(d*x^2/sqrt(c*d))/((b*c^3 - a*c^2*d)*sqrt(c*d)) + 1/6*(3*b*c*x^4 + 3*a*d
*x^4 - a*c)/(a^2*c^2*x^6)
```

Mupad [B] (verification not implemented)

Time = 9.93 (sec) , antiderivative size = 535, normalized size of antiderivative = 4.78

$$\int \frac{1}{x^7 (a+bx^4) (c+dx^4)} \, dx$$
$$= \frac{\ln \left(c^{10} \left(-a^5 b^5\right)^{5/2} + a^{20} d^{10} \sqrt{-a^5 b^5} - a^{12} b^{13} c^{10} x^2 - a^{22} b^3 d^{10} x^2 + 2 a^{10} c^5 d^5 \left(-a^5 b^5\right)^{3/2} + 2 a^{17} b^8 c^5 d^5 x^2\right.}{4 a^6 d - 4 a^5 b c}$$
$$- \frac{\ln \left(c^{10} \left(-a^5 b^5\right)^{5/2} + a^{20} d^{10} \sqrt{-a^5 b^5} + a^{12} b^{13} c^{10} x^2 + a^{22} b^3 d^{10} x^2 + 2 a^{10} c^5 d^5 \left(-a^5 b^5\right)^{3/2} - 2 a^{17} b^8 c^5 d^5\right.}{4 \left(a^6 d - a^5 b c\right)}$$
$$- \frac{\frac{1}{6 a c} - \frac{x^4 (a d + b c)}{2 a^2 c^2}}{x^6}$$
$$- \frac{\ln \left(a^{10} \left(-c^5 d^5\right)^{5/2} + b^{10} c^{20} \sqrt{-c^5 d^5} + a^{10} c^{12} d^{13} x^2 + b^{10} c^{22} d^3 x^2 + 2 a^5 b^5 c^{10} \left(-c^5 d^5\right)^{3/2} - 2 a^5 b^5 c^{17} d^8\right.}{4 \left(b c^6 - a c^5 d\right)}$$
$$+ \frac{\ln \left(a^{10} \left(-c^5 d^5\right)^{5/2} + b^{10} c^{20} \sqrt{-c^5 d^5} - a^{10} c^{12} d^{13} x^2 - b^{10} c^{22} d^3 x^2 + 2 a^5 b^5 c^{10} \left(-c^5 d^5\right)^{3/2} + 2 a^5 b^5 c^{17} d^8\right.}{4 b c^6 - 4 a c^5 d}$$

```
[In] int(1/(x^7*(a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] (log(c^10*(-a^5*b^5)^(5/2) + a^20*d^10*(-a^5*b^5)^(1/2) - a^12*b^13*c^10*x^
2 - a^22*b^3*d^10*x^2 + 2*a^10*c^5*d^5*(-a^5*b^5)^(3/2) + 2*a^17*b^8*c^5*d^
5*x^2)*(-a^5*b^5)^(1/2))/(4*a^6*d - 4*a^5*b*c) - (log(c^10*(-a^5*b^5)^(5/2)
 + a^20*d^10*(-a^5*b^5)^(1/2) + a^12*b^13*c^10*x^2 + a^22*b^3*d^10*x^2 + 2*
a^10*c^5*d^5*(-a^5*b^5)^(3/2) - 2*a^17*b^8*c^5*d^5*x^2)*(-a^5*b^5)^(1/2))/(
4*(a^6*d - a^5*b*c)) - (1/(6*a*c) - (x^4*(a*d + b*c))/(2*a^2*c^2))/x^6 - (l
og(a^10*(-c^5*d^5)^(5/2) + b^10*c^20*(-c^5*d^5)^(1/2) + a^10*c^12*d^13*x^2
+ b^10*c^22*d^3*x^2 + 2*a^5*b^5*c^10*(-c^5*d^5)^(3/2) - 2*a^5*b^5*c^17*d^8*
x^2)*(-c^5*d^5)^(1/2))/(4*(b*c^6 - a*c^5*d)) + (log(a^10*(-c^5*d^5)^(5/2) +
 b^10*c^20*(-c^5*d^5)^(1/2) - a^10*c^12*d^13*x^2 - b^10*c^22*d^3*x^2 + 2*a^
5*b^5*c^10*(-c^5*d^5)^(3/2) + 2*a^5*b^5*c^17*d^8*x^2)*(-c^5*d^5)^(1/2))/(4*
b*c^6 - 4*a*c^5*d)
```

3.779 $\int \frac{x^8}{(a+bx^4)(c+dx^4)} dx$

Optimal result

Integrand size = 22, antiderivative size = 457

$$\begin{aligned}\int \frac{x^8}{(a+bx^4)(c+dx^4)} dx &= \frac{x}{bd} - \frac{a^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{5/4}(bc-ad)} + \frac{a^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{5/4}(bc-ad)} \\ &+ \frac{c^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}d^{5/4}(bc-ad)} - \frac{c^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}d^{5/4}(bc-ad)} \\ &- \frac{a^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}b^{5/4}(bc-ad)} \\ &+ \frac{a^{5/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}b^{5/4}(bc-ad)} \\ &+ \frac{c^{5/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}d^{5/4}(bc-ad)} \\ &- \frac{c^{5/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}d^{5/4}(bc-ad)}\end{aligned}$$

```
[Out] x/b/d+1/4*a^(5/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/b^(5/4)/(-a*d+b*c)*2
^(1/2)+1/4*a^(5/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/b^(5/4)/(-a*d+b*c)*2
^(1/2)-1/4*c^(5/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/d^(5/4)/(-a*d+b*c)*
2^(1/2)-1/4*c^(5/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/d^(5/4)/(-a*d+b*c)*
2^(1/2)-1/8*a^(5/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/b^(5
/4)/(-a*d+b*c)*2^(1/2)+1/8*a^(5/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2
*b^(1/2))/b^(5/4)/(-a*d+b*c)*2^(1/2)+1/8*c^(5/4)*ln(-c^(1/4)*d^(1/4)*x*2^(1
```

```
/2)+c^(1/2)+x^2*d^(1/2))/d^(5/4)/(-a*d+b*c)*2^(1/2)-1/8*c^(5/4)*ln(c^(1/4)*
d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/d^(5/4)/(-a*d+b*c)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {490, 536, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx = -\frac{a^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{5/4}(bc-ad)}+\frac{a^{5/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}b^{5/4}(bc-ad)} - \frac{a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}b^{5/4}(bc-ad)} + \frac{a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}b^{5/4}(bc-ad)} + \frac{c^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}d^{5/4}(bc-ad)} - \frac{c^{5/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}d^{5/4}(bc-ad)} + \frac{c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}d^{5/4}(bc-ad)} - \frac{c^{5/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}d^{5/4}(bc-ad)} + \frac{x}{bd}$$

```
[In] Int[x^8/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] x/(b*d) - (a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(5
/4)*(b*c - a*d)) + (a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqr
t[2]*b^(5/4)*(b*c - a*d)) + (c^(5/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)
])/(2*Sqrt[2]*d^(5/4)*(b*c - a*d)) - (c^(5/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x
)/c^(1/4)])/(2*Sqrt[2]*d^(5/4)*(b*c - a*d)) - (a^(5/4)*Log[Sqrt[a] - Sqrt[2
]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(5/4)*(b*c - a*d)) + (a^(5
/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(5
/4)*(b*c - a*d)) + (c^(5/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[
d]*x^2])/(4*Sqrt[2]*d^(5/4)*(b*c - a*d)) - (c^(5/4)*Log[Sqrt[c] + Sqrt[2]*c
^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*d^(5/4)*(b*c - a*d))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\text{integral} = \frac{x}{bd} - \frac{\int \frac{ac+(bc+ad)x^4}{(a+bx^4)(c+dx^4)}\,dx}{bd}$$

$$= \frac{x}{bd} + \frac{a^2\int \frac{1}{a+bx^4}\,dx}{b(bc-ad)} - \frac{c^2\int \frac{1}{c+dx^4}\,dx}{d(bc-ad)}$$

$$= \frac{x}{bd} + \frac{a^{3/2}\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2b(bc-ad)} + \frac{a^{3/2}\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2b(bc-ad)} - \frac{c^{3/2}\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{2d(bc-ad)} - \frac{c^{3/2}\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{2d(bc-ad)}$$

$$= \frac{x}{bd} + \frac{a^{3/2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b^{3/2}(bc-ad)} + \frac{a^{3/2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b^{3/2}(bc-ad)}$$

$$- \frac{a^{5/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}b^{5/4}(bc-ad)} - \frac{a^{5/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}b^{5/4}(bc-ad)} - \frac{c^{3/2}\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{4d^{3/2}(bc-ad)}$$

$$- \frac{c^{3/2}\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{4d^{3/2}(bc-ad)} + \frac{c^{5/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}+2x}{-\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}-x^2}\,dx}{4\sqrt{2}d^{5/4}(bc-ad)} + \frac{c^{5/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}-2x}{-\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}-x^2}\,dx}{4\sqrt{2}d^{5/4}(bc-ad)}$$

$$= \frac{x}{bd} - \frac{a^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}b^{5/4}(bc-ad)} + \frac{a^{5/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}b^{5/4}(bc-ad)}$$

$$+ \frac{c^{5/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}d^{5/4}(bc-ad)} - \frac{c^{5/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}d^{5/4}(bc-ad)}$$

$$+ \frac{a^{5/4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{5/4}(bc-ad)} - \frac{a^{5/4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{5/4}(bc-ad)}$$

$$- \frac{c^{5/4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}d^{5/4}(bc-ad)} + \frac{c^{5/4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}d^{5/4}(bc-ad)}$$

$$= \frac{x}{bd} - \frac{a^{5/4}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{5/4}(bc-ad)} + \frac{a^{5/4}\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{5/4}(bc-ad)} + \frac{c^{5/4}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}d^{5/4}(bc-ad)} - \frac{c^{5/4}\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}d^{5/4}(bc-ad)} - \frac{a^{5/4}\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{b}x^2\right)}{4\sqrt{2}b^{5/4}(bc-ad)} + \frac{a^{5/4}\log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{b}x^2\right)}{4\sqrt{2}b^{5/4}(bc-ad)} + \frac{c^{5/4}\log\left(\sqrt{c} - \sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x + \sqrt{d}x^2\right)}{4\sqrt{2}d^{5/4}(bc-ad)} - \frac{c^{5/4}\log\left(\sqrt{c} + \sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x + \sqrt{d}x^2\right)}{4\sqrt{2}d^{5/4}(bc-ad)}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 377, normalized size of antiderivative = 0.82

$$\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx = \frac{-\frac{8ax}{b} + \frac{8cx}{d} - \frac{2\sqrt{2}a^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{5/4}} + \frac{2\sqrt{2}a^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{5/4}} + \frac{2\sqrt{2}c^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{d^{5/4}} - \frac{2\sqrt{2}c^{5/4}\arctan\left(\right.}{d^{5/}}}{}$$

```
[In] Integrate[x^8/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((-8*a*x)/b + (8*c*x)/d - (2*Sqrt[2]*a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)
/a^(1/4)])/b^(5/4) + (2*Sqrt[2]*a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1
/4)])/b^(5/4) + (2*Sqrt[2]*c^(5/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])
/d^(5/4) - (2*Sqrt[2]*c^(5/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/d^(5
/4) - (Sqrt[2]*a^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^
2])/b^(5/4) + (Sqrt[2]*a^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2])/b^(5/4) + (Sqrt[2]*c^(5/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4
)*x + Sqrt[d]*x^2])/d^(5/4) - (Sqrt[2]*c^(5/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4
)*d^(1/4)*x + Sqrt[d]*x^2])/d^(5/4))/(8*b*c - 8*a*d)
```

Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.51

method	result
default	$\frac{x}{bd}-\frac{a\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8b(ad-bc)}+\frac{c\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)\right.}{}$
risch	$\frac{x}{bd}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\left(a^{4}d^{5}-4a^{3}bc\,d^{4}+6a^{2}b^{2}c^{2}d^{3}-4a\,b^{3}c^{3}d^{2}+b^{4}c^{4}d\right)\_Z^{4}+b^{4}c^{5}\right)}\_R\ln\left(\left(\left(-a^{5}b\,d^{6}+3a^{4}b^{2}c\,d^{5}-2a^{3}b^{3}c^{2}d^{4}-2a^{2}b^{4}c^{3}d^{3}+3a\right.\right.\right.}{4bd}$

```
[In] int(x^8/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] x/b/d-1/8/b*a/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+
(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b
)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/8/d*c/(a*d-b*c)*(c/d)^(1/
4)*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2
^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/
d)^(1/4)*x-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 1196, normalized size of antiderivative = 2.62

$$\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^8/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/4*((-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 +
 a^4*b^5*d^4))^(1/4)*b*d*log(a*x + (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b
^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*(b^2*c - a*b*d)) - (-a^5
/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d
^4))^(1/4)*b*d*log(a*x - (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2
 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*(b^2*c - a*b*d)) - I*(-a^5/(b^9*c^
4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/
4)*b*d*log(a*x - (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3
*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*(I*b^2*c - I*a*b*d)) + I*(-a^5/(b^9*c^4 -
4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*b
*d*log(a*x - (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6
*c*d^3 + a^4*b^5*d^4))^(1/4)*(-I*b^2*c + I*a*b*d)) - (-c^5/(b^4*c^4*d^5 - 4
*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*b*d*lo
g(c*x + (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*
c*d^8 + a^4*d^9))^(1/4)*(b*c*d - a*d^2)) + (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3
*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*b*d*log(c*x - (-
c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^
4*d^9))^(1/4)*(b*c*d - a*d^2)) + I*(-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6
```

```
*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*b*d*log(c*x - (-c^5/(b^4
*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^
(1/4)*(I*b*c*d - I*a*d^2)) - I*(-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2
*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*b*d*log(c*x - (-c^5/(b^4*c^4
*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4
)*(-I*b*c*d + I*a*d^2)) + 4*x)/(b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(x**8/(b*x**4+a)/(d*x**4+c),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 375, normalized size of antiderivative = 0.82

$$\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx = \frac{\frac{2\sqrt{2}a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}a^{\frac{5}{4}}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}a^{\frac{5}{4}}\log\left(\sqrt{b}x^2\right.}{}}{8\,(b^2c-abd)} - \frac{\frac{2\sqrt{2}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}c^{\frac{5}{4}}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{d^{\frac{1}{4}}}-\frac{\sqrt{2}c^{\frac{5}{4}}\log\left(\right.}{}}{8\,(bcd-ad^2)} + \frac{x}{bd}$$

```
[In] integrate(x^8/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
[Out] 1/8*(2*sqrt(2)*a^(3/2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^
(1/4))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*a^(3/2)*arc
tan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b
)))/sqrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(5/4)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/
4)*b^(1/4)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(5/4)*log(sqrt(b)*x^2 - sqrt(2)
*a^(1/4)*b^(1/4)*x + sqrt(a))/b^(1/4))/(b^2*c - a*b*d) - 1/8*(2*sqrt(2)*c^(
```

```
3/2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c
)*sqrt(d)))/sqrt(sqrt(c)*sqrt(d)) + 2*sqrt(2)*c^(3/2)*arctan(1/2*sqrt(2)*(2
*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/sqrt(sqrt(c)*s
qrt(d)) + sqrt(2)*c^(5/4)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqr
t(c))/d^(1/4) - sqrt(2)*c^(5/4)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x
 + sqrt(c))/d^(1/4))/(b*c*d - a*d^2) + x/(b*d)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.03

$$\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx = \frac{(ab^3)^{\frac{1}{4}}\,a\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}b^3c-\sqrt{2}ab^2d\right)} + \frac{(ab^3)^{\frac{1}{4}}\,a\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}b^3c-\sqrt{2}ab^2d\right)} - \frac{(cd^3)^{\frac{1}{4}}\,c\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bcd^2-\sqrt{2}ad^3\right)} - \frac{(cd^3)^{\frac{1}{4}}\,c\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bcd^2-\sqrt{2}ad^3\right)} + \frac{(ab^3)^{\frac{1}{4}}\,a\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}b^3c-\sqrt{2}ab^2d\right)} - \frac{(ab^3)^{\frac{1}{4}}\,a\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}b^3c-\sqrt{2}ab^2d\right)} - \frac{(cd^3)^{\frac{1}{4}}\,c\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bcd^2-\sqrt{2}ad^3\right)} + \frac{(cd^3)^{\frac{1}{4}}\,c\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bcd^2-\sqrt{2}ad^3\right)} + \frac{x}{bd}$$

```
[In] integrate(x^8/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*(a*b^3)^(1/4)*a*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1
/4))/(sqrt(2)*b^3*c - sqrt(2)*a*b^2*d) + 1/2*(a*b^3)^(1/4)*a*arctan(1/2*sqr
```

```
t(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*b^3*c - sqrt(2)*a*b^
2*d) - 1/2*(c*d^3)^(1/4)*c*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(
c/d)^(1/4))/(sqrt(2)*b*c*d^2 - sqrt(2)*a*d^3) - 1/2*(c*d^3)^(1/4)*c*arctan(
1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c*d^2 - sqr
t(2)*a*d^3) + 1/4*(a*b^3)^(1/4)*a*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/
b))/(sqrt(2)*b^3*c - sqrt(2)*a*b^2*d) - 1/4*(a*b^3)^(1/4)*a*log(x^2 - sqrt(
2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*b^3*c - sqrt(2)*a*b^2*d) - 1/4*(c*d^
3)^(1/4)*c*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c*d^2 -
sqrt(2)*a*d^3) + 1/4*(c*d^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt
(c/d))/(sqrt(2)*b*c*d^2 - sqrt(2)*a*d^3) + x/(b*d)
```

Mupad [B] (verification not implemented)

Time = 9.94 (sec) , antiderivative size = 6361, normalized size of antiderivative = 13.92

$$\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

```
[In] int(x^8/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] atan(((-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*
b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a
^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) - (4*x*(-a^5/(256*b^9*c^4 + 256*a^4*b^
5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(3/4
)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^
6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9))/(b*d))*(-a^5/(2
56*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 -
1024*a*b^8*c^3*d))^(1/4) - (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*1i - (-
a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*
d^2 - 1024*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c
^8*d - a^8*b*c^4*d^5))/(b*d) + (4*x*(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 -
1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(3/4)*(256*a
^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^
5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9))/(b*d))*(-a^5/(256*b^9*c
^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b
^8*c^3*d))^(1/4) + (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*1i)/((-a^5/(256
*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 10
24*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a
^8*b*c^4*d^5))/(b*d) - (4*x*(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3
*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(3/4)*(256*a^3*b^9*c
^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 -
768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9))/(b*d))*(-a^5/(256*b^9*c^4 + 256
*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d
))^(1/4) - (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d)) + (-a^5/(256*b^9*c^4 +
256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^
```

```
3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^
5))/(b*d) + (4*x*(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3
+ 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(3/4)*(256*a^3*b^9*c^8*d^4 - 76
8*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5
*c^4*d^8 + 256*a^8*b^4*c^3*d^9))/(b*d))*(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^
4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(1/4) +
(4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))))*(-a^5/(256*b^9*c^4 + 256*a^4*b^5
*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(1/4)
*2i - 2*atan(((-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1
536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3
*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) - (x*(-a^5/(256*b^9*c^4 + 256*
a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d)
)^(3/4)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 +
512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9)*4i)/(b*d))
*(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c
^2*d^2 - 1024*a*b^8*c^3*d))^(1/4)*1i + (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b
*d)) - (-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2
*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 -
a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) + (x*(-a^5/(256*b^9*c^4 + 256*a^4*b^5
*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(3/4)
*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6
*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9)*4i)/(b*d))*(-a^5/
(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2
- 1024*a*b^8*c^3*d))^(1/4)*1i - (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d)))/(
(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^
2*d^2 - 1024*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5
*c^8*d - a^8*b*c^4*d^5))/(b*d) - (x*(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 -
1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(3/4)*(256*a
^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^
5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9)*4i)/(b*d))*(-a^5/(256*b^
9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*
a*b^8*c^3*d))^(1/4)*1i + (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*1i + (-a^
5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^
2 - 1024*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8
*d - a^8*b*c^4*d^5))/(b*d) + (x*(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024
*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(3/4)*(256*a^3*b
^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^5*d^
7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9)*4i)/(b*d))*(-a^5/(256*b^9*c^
4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^
8*c^3*d))^(1/4)*1i - (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*1i))*(-a^5/(2
56*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 -
1024*a*b^8*c^3*d))^(1/4) + atan(((-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 102
4*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*(((16*(a^
3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) - (4*x*(-c^
5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^
```

7 - 1024*a^3*b*c*d^8))^(3/4)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 5
12*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^
4*c^3*d^9))/(b*d))*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^
6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4) - (4*x*(a^4*b^4*c^8 + a
^8*c^4*d^4))/(b*d))*1i - (-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*
c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*(((16*(a^3*b^6*c^
9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) + (4*x*(-c^5/(256*a
^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024
*a^3*b*c*d^8))^(3/4)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b
^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^
9))/(b*d))*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536
*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4) + (4*x*(a^4*b^4*c^8 + a^8*c^4*d
^4))/(b*d))*1i)/((-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6
+ 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*
c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) - (4*x*(-c^5/(256*a^4*d^9 +
256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c
d^8))^(3/4)(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d
^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9))/(b*d
))*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2
*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4) - (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*
d)) + (-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*
b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a
^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) + (4*x*(-c^5/(256*a^4*d^9 + 256*b^4*c^
4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(3/4
)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^
6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9))/(b*d))*(-c^5/(2
56*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 -
1024*a^3*b*c*d^8))^(1/4) + (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))))*(-c^5
/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7
 - 1024*a^3*b*c*d^8))^(1/4)*2i - 2*atan(((-c^5/(256*a^4*d^9 + 256*b^4*c^4*d
^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*(
((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) - (
x*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*
c^2*d^7 - 1024*a^3*b*c*d^8))^(3/4)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d
^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*
a^8*b^4*c^3*d^9)*4i)/(b*d))*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b
^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*1i + (4*x*(a^4
*b^4*c^8 + a^8*c^4*d^4))/(b*d)) - (-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 10
24*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*(((16*(a
^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) + (x*(-c^5
/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7
 - 1024*a^3*b*c*d^8))^(3/4)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 51
2*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4
*c^3*d^9)*4i)/(b*d))*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*
d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*1i - (4*x*(a^4*b^4*c^

```
8 + a^8*c^4*d^4))/(b*d)))/((-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^
3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*(((16*(a^3*b^6*
c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) - (x*(-c^5/(256*a
^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024
*a^3*b*c*d^8))^(3/4)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b
^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^
9)*4i)/(b*d))*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1
536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*1i + (4*x*(a^4*b^4*c^8 + a^8
*c^4*d^4))/(b*d))*1i + (-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^
3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*(((16*(a^3*b^6*c^9
+ a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) + (x*(-c^5/(256*a^4*d
^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3
*b*c*d^8))^(3/4)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c
^6*d^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9)*4
i)/(b*d))*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*
a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*1i - (4*x*(a^4*b^4*c^8 + a^8*c^4
*d^4))/(b*d))*1i))*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^
6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4) + x/(b*d)
```

3.780 $\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 449

$$\begin{aligned}\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx = & \frac{a^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{3/4}(bc-ad)} - \frac{a^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{3/4}(bc-ad)} \\ & - \frac{c^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}d^{3/4}(bc-ad)} + \frac{c^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}d^{3/4}(bc-ad)} \\ & - \frac{a^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}b^{3/4}(bc-ad)} \\ & + \frac{a^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}b^{3/4}(bc-ad)} \\ & + \frac{c^{3/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}d^{3/4}(bc-ad)} \\ & - \frac{c^{3/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}d^{3/4}(bc-ad)}\end{aligned}$$

```
[Out] -1/4*a^(3/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/b^(3/4)/(-a*d+b*c)*2^(1/2
)-1/4*a^(3/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/b^(3/4)/(-a*d+b*c)*2^(1/2
)+1/4*c^(3/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/d^(3/4)/(-a*d+b*c)*2^(1/
2)+1/4*c^(3/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/d^(3/4)/(-a*d+b*c)*2^(1/
2)-1/8*a^(3/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/b^(3/4)/(
-a*d+b*c)*2^(1/2)+1/8*a^(3/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1
/2))/b^(3/4)/(-a*d+b*c)*2^(1/2)+1/8*c^(3/4)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c
```

```
^(1/2)+x^2*d^(1/2))/d^(3/4)/(-a*d+b*c)*2^(1/2)-1/8*c^(3/4)*ln(c^(1/4)*d^(1/
4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/d^(3/4)/(-a*d+b*c)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {492, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx = \frac{a^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{3/4}(bc-ad)} - \frac{a^{3/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}b^{3/4}(bc-ad)} - \frac{a^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}b^{3/4}(bc-ad)} + \frac{a^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}b^{3/4}(bc-ad)} - \frac{c^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}d^{3/4}(bc-ad)} + \frac{c^{3/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}d^{3/4}(bc-ad)} + \frac{c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}d^{3/4}(bc-ad)} - \frac{c^{3/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}d^{3/4}(bc-ad)}$$

```
[In] Int[x^6/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (a^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(3/4)*(b*c -
 a*d)) - (a^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(3/
4)*(b*c - a*d)) - (c^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt
[2]*d^(3/4)*(b*c - a*d)) + (c^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]
)/(2*Sqrt[2]*d^(3/4)*(b*c - a*d)) - (a^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(3/4)*(b*c - a*d)) + (a^(3/4)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(3/4)*(b*c -
 a*d)) + (c^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(
4*Sqrt[2]*d^(3/4)*(b*c - a*d)) - (c^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(
1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*d^(3/4)*(b*c - a*d))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\text{integral} = -\frac{a\int \frac{x^2}{a+bx^4}\,dx}{bc-ad}+\frac{c\int \frac{x^2}{c+dx^4}\,dx}{bc-ad}$$

$$=\frac{a\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{b}(bc-ad)}-\frac{a\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{b}(bc-ad)}-\frac{c\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{2\sqrt{d}(bc-ad)}+\frac{c\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{2\sqrt{d}(bc-ad)}$$

$$=-\frac{a\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b(bc-ad)}-\frac{a\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b(bc-ad)}-\frac{a^{3/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}b^{3/4}(bc-ad)}$$

$$-\frac{a^{3/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}b^{3/4}(bc-ad)}+\frac{c\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{4d(bc-ad)}+\frac{c\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{4d(bc-ad)}$$

$$+\frac{c^{3/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}+2x}{-\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}-x^2}\,dx}{4\sqrt{2}d^{3/4}(bc-ad)}+\frac{c^{3/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}-2x}{-\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}-x^2}\,dx}{4\sqrt{2}d^{3/4}(bc-ad)}$$

$$=-\frac{a^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}b^{3/4}(bc-ad)}+\frac{a^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}b^{3/4}(bc-ad)}$$

$$+\frac{c^{3/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}d^{3/4}(bc-ad)}-\frac{c^{3/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}d^{3/4}(bc-ad)}$$

$$-\frac{a^{3/4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{3/4}(bc-ad)}+\frac{a^{3/4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{3/4}(bc-ad)}$$

$$+\frac{c^{3/4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}d^{3/4}(bc-ad)}-\frac{c^{3/4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}d^{3/4}(bc-ad)}$$

$$= \frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{3/4}(bc-ad)} - \frac{a^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{3/4}(bc-ad)} - \frac{c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}d^{3/4}(bc-ad)} + \frac{c^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}d^{3/4}(bc-ad)} - \frac{a^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}b^{3/4}(bc-ad)} + \frac{a^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}b^{3/4}(bc-ad)} + \frac{c^{3/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}d^{3/4}(bc-ad)} - \frac{c^{3/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}d^{3/4}(bc-ad)}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.76

$$\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx$$
$$= \frac{2a^{3/4}d^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) - 2a^{3/4}d^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) - 2b^{3/4}c^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right) + 2b^{3/}}{}$$

```
[In] Integrate[x^6/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (2*a^(3/4)*d^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*a^(3/4)*d^(3
/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*b^(3/4)*c^(3/4)*ArcTan[1 -
(Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*b^(3/4)*c^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4
)*x)/c^(1/4)] - a^(3/4)*d^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2] + a^(3/4)*d^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqr
t[b]*x^2] + b^(3/4)*c^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[
d]*x^2] - b^(3/4)*c^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]
*x^2])/(4*Sqrt[2]*b^(3/4)*d^(3/4)*(b*c - a*d))
```

Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.50

method	result
default	$\frac{a\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8(ad-bc)b\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{c\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}}{\left(\frac{c}{d}\right.}\right.\right.}{8(ad-bc)d\left(\frac{c}{d}\right)}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^4d^7-4a^3bc\,d^6+6a^2b^2c^2d^5-4a\,b^3c^3d^4+b^4c^4d^3\right)\_Z^4+c^3\right)}\_R\ln\left(\left(\left(2a^4b^3d^7-8a^3b^4c\,d^6+12a^2b^5c^2d^5-8a\,b^6c^3d^4+2b^7c^4d\right.\right.\right.\right.}{4}$

```
[In] int(x^6/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*a/(a*d-b*c)/b/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^
(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4
)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))-1/8*c/(a*d-b*c)/d/(c/d)^(1/4)*2^(
1/2)*(ln((x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2+(c/d)^(1/4)*x*2^(1/2)
+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/d)^(1/
4)*x-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 1427, normalized size of antiderivative = 3.18

$$\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -1/4*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 +
 a^4*b^3*d^4))^(1/4)*log(a^2*x + (b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2
 - a^3*b^2*d^3)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*
b^4*c*d^3 + a^4*b^3*d^4))^(3/4)) + 1/4*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a
^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(1/4)*log(a^2*x - (b^5*c^3
 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(-a^3/(b^7*c^4 - 4*a*b^6*
c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(3/4)) - 1/4*I*
(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*
b^3*d^4))^(1/4)*log(a^2*x - (I*b^5*c^3 - 3*I*a*b^4*c^2*d + 3*I*a^2*b^3*c*d^
2 - I*a^3*b^2*d^3)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a
^3*b^4*c*d^3 + a^4*b^3*d^4))^(3/4)) + 1/4*I*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d
+ 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(1/4)*log(a^2*x - (-I
*b^5*c^3 + 3*I*a*b^4*c^2*d - 3*I*a^2*b^3*c*d^2 + I*a^3*b^2*d^3)*(-a^3/(b^7*
c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(
3/4)) + 1/4*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^
3*b*c*d^6 + a^4*d^7))^(1/4)*log(c^2*x + (b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*
a^2*b*c*d^4 - a^3*d^5)*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2
```

```
*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(3/4)) - 1/4*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*
c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(1/4)*log(c^2*x - (
b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*(-c^3/(b^4*c^4*d^3
 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(3/4)) +
 1/4*I*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c
*d^6 + a^4*d^7))^(1/4)*log(c^2*x - (I*b^3*c^3*d^2 - 3*I*a*b^2*c^2*d^3 + 3*I
*a^2*b*c*d^4 - I*a^3*d^5)*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*
c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(3/4)) - 1/4*I*(-c^3/(b^4*c^4*d^3 - 4*a
*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(1/4)*log(c^2*
x - (-I*b^3*c^3*d^2 + 3*I*a*b^2*c^2*d^3 - 3*I*a^2*b*c*d^4 + I*a^3*d^5)*(-c^
3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*
d^7))^(3/4))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(x**6/(b*x**4+a)/(d*x**4+c),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 363, normalized size of antiderivative = 0.81

$$\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx = -\frac{a\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}x^2-\sqrt{\right.}{a^{\frac{1}{4}}}\right)}{8(bc-ad)}+\frac{c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{d}x^2-\sqrt{\right.}{c^{\frac{1}{4}}}\right)}{8(bc-ad)}$$

```
[In] integrate(x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/8*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan
(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))
/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4
)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*x^2 - sqrt(2
)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(b*c - a*d) + 1/8*c*(2*sq
rt(2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(
c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(1/2*sqrt(2)
*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(
c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x
+ sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^
(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b*c - a*d)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.01

$$\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx = -\frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}b^4c-\sqrt{2}ab^3d\right)} - \frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}b^4c-\sqrt{2}ab^3d\right)} + \frac{(cd^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bcd^3-\sqrt{2}ad^4\right)} + \frac{(cd^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bcd^3-\sqrt{2}ad^4\right)} + \frac{(ab^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}b^4c-\sqrt{2}ab^3d\right)} - \frac{(ab^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}b^4c-\sqrt{2}ab^3d\right)} - \frac{(cd^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bcd^3-\sqrt{2}ad^4\right)} + \frac{(cd^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bcd^3-\sqrt{2}ad^4\right)}$$

```
[In] integrate(x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(sqrt(2)*b^4*c - sqrt(2)*a*b^3*d) - 1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(2
)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*b^4*c - sqrt(2)*a*b^3*d
) + 1/2*(c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^
(1/4))/(sqrt(2)*b*c*d^3 - sqrt(2)*a*d^4) + 1/2*(c*d^3)^(3/4)*arctan(1/2*sqr
t(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c*d^3 - sqrt(2)*a*
d^4) + 1/4*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt
(2)*b^4*c - sqrt(2)*a*b^3*d) - 1/4*(a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/b)^
(1/4) + sqrt(a/b))/(sqrt(2)*b^4*c - sqrt(2)*a*b^3*d) - 1/4*(c*d^3)^(3/4)*lo
g(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c*d^3 - sqrt(2)*a*d^4
) + 1/4*(c*d^3)^(3/4)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)
*b*c*d^3 - sqrt(2)*a*d^4)
```

Mupad [B] (verification not implemented)

Time = 10.09 (sec) , antiderivative size = 2553, normalized size of antiderivative = 5.69

$$\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

```
[In] int(x^6/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] - 2*atan((4*b^4*c^3*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c
*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(1/4) + 4*a^3*b*d^3*x*(-a^
3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^
2 - 1024*a*b^6*c^3*d))^(1/4) + 2048*a^4*b^4*d^7*x*(-a^3/(256*b^7*c^4 + 256*
a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d)
)^(5/4) + 2048*b^8*c^4*d^3*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^
3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(5/4) - 8192*a*b^7*
c^3*d^4*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*
a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(5/4) - 8192*a^3*b^5*c*d^6*x*(-a^3/(25
6*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1
024*a*b^6*c^3*d))^(5/4) + 12288*a^2*b^6*c^2*d^5*x*(-a^3/(256*b^7*c^4 + 256*
a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d)
)^(5/4))/(a^3*d^2 + a*b^2*c^2 + a^2*b*c*d))*(-a^3/(256*b^7*c^4 + 256*a^4*b^
3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(1/4
) - atan((b^4*c^3*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d
^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(1/4)*4i + a^3*b*d^3*x*(-a^3
/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2
 - 1024*a*b^6*c^3*d))^(1/4)*4i + a^4*b^4*d^7*x*(-a^3/(256*b^7*c^4 + 256*a^4
*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(
5/4)*2048i + b^8*c^4*d^3*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*
b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(5/4)*2048i - a*b^7*c
^3*d^4*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a
^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(5/4)*8192i - a^3*b^5*c*d^6*x*(-a^3/(25
6*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1
024*a*b^6*c^3*d))^(5/4)*8192i + a^2*b^6*c^2*d^5*x*(-a^3/(256*b^7*c^4 + 256*
a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d)
)^(5/4)*12288i)/(a^3*d^2 + a*b^2*c^2 + a^2*b*c*d))*(-a^3/(256*b^7*c^4 + 256
*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d
))^(1/4)*2i - 2*atan((4*a^3*d^4*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 10
24*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^(1/4) + 2048*b
^7*c^4*d^4*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 15
36*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^(5/4) + 4*b^3*c^3*d*x*(-c^3/(256*a^
4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*
a^3*b*c*d^6))^(1/4) + 2048*a^4*b^3*d^8*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d
^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^(5/4) -
 8192*a*b^6*c^3*d^5*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3
```

*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^(5/4) - 8192*a^3*b^4*c*d^7
x(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2
*c^2*d^5 - 1024*a^3*b*c*d^6))^(5/4) + 12288*a^2*b^5*c^2*d^6*x*(-c^3/(256*a^
4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*
a^3*b*c*d^6))^(5/4))/(b^2*c^3 + a^2*c*d^2 + a*b*c^2*d))*(-c^3/(256*a^4*d^7
+ 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*
c*d^6))^(1/4) - atan((a^3*d^4*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024
*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^(1/4)*4i + b^7*c
^4*d^4*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a
^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^(5/4)*2048i + b^3*c^3*d*x*(-c^3/(256*a^
4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*
a^3*b*c*d^6))^(1/4)*4i + a^4*b^3*d^8*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3
 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^(5/4)*204
8i - a*b^6*c^3*d^5*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*
d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^(5/4)*8192i - a^3*b^4*c*d^7
x(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2
*c^2*d^5 - 1024*a^3*b*c*d^6))^(5/4)*8192i + a^2*b^5*c^2*d^6*x*(-c^3/(256*a^
4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*
a^3*b*c*d^6))^(5/4)*12288i)/(b^2*c^3 + a^2*c*d^2 + a*b*c^2*d))*(-c^3/(256*a
^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024
*a^3*b*c*d^6))^(1/4)*2i

3.781 $\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 449

$$\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx = \frac{\sqrt[4]{a}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{a}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)} - \frac{\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{d}(bc-ad)}+\frac{\sqrt[4]{c}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{d}(bc-ad)} + \frac{\sqrt[4]{a}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{b}(bc-ad)} - \frac{\sqrt[4]{a}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{b}(bc-ad)} - \frac{\sqrt[4]{c}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{d}(bc-ad)} + \frac{\sqrt[4]{c}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{d}(bc-ad)}$$

```
[Out] -1/4*a^(1/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/b^(1/4)/(-a*d+b*c)*2^(1/2
)-1/4*a^(1/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/b^(1/4)/(-a*d+b*c)*2^(1/2
)+1/4*c^(1/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/d^(1/4)/(-a*d+b*c)*2^(1/
2)+1/4*c^(1/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/d^(1/4)/(-a*d+b*c)*2^(1/
2)+1/8*a^(1/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/b^(1/4)/(
-a*d+b*c)*2^(1/2)-1/8*a^(1/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1
/2))/b^(1/4)/(-a*d+b*c)*2^(1/2)-1/8*c^(1/4)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c
```

```
^(1/2)+x^2*d^(1/2))/d^(1/4)/(-a*d+b*c)*2^(1/2)+1/8*c^(1/4)*ln(c^(1/4)*d^(1/
4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/d^(1/4)/(-a*d+b*c)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {492, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx = \frac{\sqrt[4]{a}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)} - \frac{\sqrt[4]{a}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)} - \frac{\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{d}(bc-ad)} + \frac{\sqrt[4]{c}\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\sqrt[4]{d}(bc-ad)} + \frac{\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{b}(bc-ad)} - \frac{\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{b}(bc-ad)} - \frac{\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{d}(bc-ad)} + \frac{\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{d}(bc-ad)}$$

```
[In] Int[x^4/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(1/4)*(b*c -
 a*d)) - (a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(1/
4)*(b*c - a*d)) - (c^(1/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt
[2]*d^(1/4)*(b*c - a*d)) + (c^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]
)/(2*Sqrt[2]*d^(1/4)*(b*c - a*d)) + (a^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(1/4)*(b*c - a*d)) - (a^(1/4)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(1/4)*(b*c -
 a*d)) - (c^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(
4*Sqrt[2]*d^(1/4)*(b*c - a*d)) + (c^(1/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(
1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*d^(1/4)*(b*c - a*d))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\text{integral} = -\frac{a\int \frac{1}{a+bx^4}\,dx}{bc-ad}+\frac{c\int \frac{1}{c+dx^4}\,dx}{bc-ad}$$

$$= -\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2(bc-ad)}-\frac{\sqrt{a}\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2(bc-ad)}+\frac{\sqrt{c}\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{2(bc-ad)}+\frac{\sqrt{c}\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{2(bc-ad)}$$

$$= -\frac{\sqrt{a}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{b}(bc-ad)}-\frac{\sqrt{a}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{b}(bc-ad)}+\frac{\sqrt[4]{a}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{b}(bc-ad)}$$
$$+\frac{\sqrt[4]{a}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{b}(bc-ad)}+\frac{\sqrt{c}\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{4\sqrt{d}(bc-ad)}+\frac{\sqrt{c}\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{4\sqrt{d}(bc-ad)}$$
$$-\frac{\sqrt[4]{c}\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}+2x}{-\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{d}(bc-ad)}-\frac{\sqrt[4]{c}\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}-2x}{-\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{d}(bc-ad)}$$

$$= \frac{\sqrt[4]{a}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{a}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{b}(bc-ad)}$$
$$-\frac{\sqrt[4]{c}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{d}(bc-ad)}+\frac{\sqrt[4]{c}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{d}(bc-ad)}$$
$$-\frac{\sqrt[4]{a}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)}+\frac{\sqrt[4]{a}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)}$$
$$+\frac{\sqrt[4]{c}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{d}(bc-ad)}-\frac{\sqrt[4]{c}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{d}(bc-ad)}$$

$$= \frac{\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)} - \frac{\sqrt[4]{a}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)} - \frac{\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{d}(bc-ad)} + \frac{\sqrt[4]{c}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{d}(bc-ad)} + \frac{\sqrt[4]{a}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{b}(bc-ad)} - \frac{\sqrt[4]{a}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{b}(bc-ad)} - \frac{\sqrt[4]{c}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{d}(bc-ad)} + \frac{\sqrt[4]{c}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{d}(bc-ad)}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.76

$$\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx = \frac{2\sqrt[4]{a}\sqrt[4]{d}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) - 2\sqrt[4]{a}\sqrt[4]{d}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) - 2\sqrt[4]{b}\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right) + 2\sqrt[4]{b}\sqrt[4]{c}\,a}{}$$

```
[In] Integrate[x^4/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (2*a^(1/4)*d^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*a^(1/4)*d^(1
/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*b^(1/4)*c^(1/4)*ArcTan[1 -
(Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*b^(1/4)*c^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/4
)*x)/c^(1/4)] + a^(1/4)*d^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2] - a^(1/4)*d^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqr
t[b]*x^2] - b^(1/4)*c^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[
d]*x^2] + b^(1/4)*c^(1/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]
*x^2])/(4*Sqrt[2]*b^(1/4)*d^(1/4)*(b*c - a*d))
```

Maple [A] (verified)

Time = 4.55 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.49

method	result
default	$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8ad-8bc}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\,a}{8(a}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^{4}b\,d^{4}-4a^{3}b^{2}c\,d^{3}+6a^{2}b^{3}c^{2}d^{2}-4a\,b^{4}c^{3}d+b^{5}c^{4}\right)\_Z^{4}+a\right)}\_R\ln\left(\left(\left(-a^{5}b\,d^{6}+3a^{4}b^{2}c\,d^{5}-2a^{3}b^{3}c^{2}d^{4}-2a^{2}b^{4}c^{3}d^{3}+3a\,b^{5}c^{4}\right.\right.\right.\right.}{}$

```
[In] int(x^4/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2
))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+
1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))-1/8/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*(ln
((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(
1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 1067, normalized size of antiderivative = 2.38

$$\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx =$$

$$-\frac{1}{4}\left(-\frac{a}{b^5c^4-4ab^4c^3d+6a^2b^3c^2d^2-4a^3b^2cd^3+a^4bd^4}\right)^{\frac{1}{4}}\log\left((bc-ad)\left(-\frac{a}{b^5c^4-4ab^4c^3d+6a^2b^3c^2d^2}\right.\right.$$
$$\left.\left.+x\right)\right)$$
$$+\frac{1}{4}\left(-\frac{a}{b^5c^4-4ab^4c^3d+6a^2b^3c^2d^2-4a^3b^2cd^3+a^4bd^4}\right)^{\frac{1}{4}}\log\left(-(bc-ad)\left(-\frac{a}{b^5c^4-4ab^4c^3d+6a^2b^3c^2d}\right.\right.$$
$$\left.\left.+x\right)\right)$$
$$+\frac{1}{4}i\left(-\frac{a}{b^5c^4-4ab^4c^3d+6a^2b^3c^2d^2-4a^3b^2cd^3+a^4bd^4}\right)^{\frac{1}{4}}\log\left(-(ibc-iad)\left(-\frac{}{b^5c^4-4ab^4c^3d+6a^2b^3}\right.\right.$$
$$\left.\left.+x\right)\right)$$
$$-\frac{1}{4}i\left(-\frac{a}{b^5c^4-4ab^4c^3d+6a^2b^3c^2d^2-4a^3b^2cd^3+a^4bd^4}\right)^{\frac{1}{4}}\log\left(-(-ibc+iad)\left(-\frac{}{b^5c^4-4ab^4c^3d+6a^2}\right.\right.$$
$$\left.\left.+x\right)\right)$$
$$+\frac{1}{4}\left(-\frac{c}{b^4c^4d-4ab^3c^3d^2+6a^2b^2c^2d^3-4a^3bcd^4+a^4d^5}\right)^{\frac{1}{4}}\log\left((bc-ad)\left(-\frac{c}{b^4c^4d-4ab^3c^3d^2+6a^2b^2c^2}\right.\right.$$
$$\left.\left.+x\right)\right)$$
$$-\frac{1}{4}\left(-\frac{c}{b^4c^4d-4ab^3c^3d^2+6a^2b^2c^2d^3-4a^3bcd^4+a^4d^5}\right)^{\frac{1}{4}}\log\left(-(bc-ad)\left(-\frac{c}{b^4c^4d-4ab^3c^3d^2+6a^2b^2}\right.\right.$$
$$\left.\left.+x\right)\right)$$
$$-\frac{1}{4}i\left(-\frac{c}{b^4c^4d-4ab^3c^3d^2+6a^2b^2c^2d^3-4a^3bcd^4+a^4d^5}\right)^{\frac{1}{4}}\log\left(-(ibc-iad)\left(-\frac{}{b^4c^4d-4ab^3c^3d^2+6a}\right.\right.$$
$$\left.\left.+x\right)\right)$$
$$+\frac{1}{4}i\left(-\frac{c}{b^4c^4d-4ab^3c^3d^2+6a^2b^2c^2d^3-4a^3bcd^4+a^4d^5}\right)^{\frac{1}{4}}\log\left(-(-ibc+iad)\left(-\frac{}{b^4c^4d-4ab^3c^3d^2+6}\right.\right.$$
$$\left.\left.+x\right)\right)$$

```
[In] integrate(x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -1/4*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a
^4*b*d^4))^(1/4)*log((b*c - a*d)*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c
^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4) + x) + 1/4*(-a/(b^5*c^4 - 4*a*
b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4)*log(-(b
*c - a*d)*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^
3 + a^4*b*d^4))^(1/4) + x) + 1/4*I*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3
*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4)*log(-(I*b*c - I*a*d)*(-a/(b^
5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(
1/4) + x) - 1/4*I*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*
b^2*c*d^3 + a^4*b*d^4))^(1/4)*log(-(-I*b*c + I*a*d)*(-a/(b^5*c^4 - 4*a*b^4*
c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4) + x) + 1/4*
(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*
d^5))^(1/4)*log((b*c - a*d)*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^
2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/4) + x) - 1/4*(-c/(b^4*c^4*d - 4*a*b^3
*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/4)*log(-(b*c -
a*d)*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 +
 a^4*d^5))^(1/4) + x) - 1/4*I*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*
c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/4)*log(-(I*b*c - I*a*d)*(-c/(b^4*c^4
*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/4)
+ x) + 1/4*I*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b
*c*d^4 + a^4*d^5))^(1/4)*log(-(-I*b*c + I*a*d)*(-c/(b^4*c^4*d - 4*a*b^3*c^3
*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/4) + x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(x**4/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 361, normalized size of antiderivative = 0.80

$$\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx = -\frac{\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}a^{\frac{1}{4}}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}a^{\frac{1}{4}}\log\left(\sqrt{\right.}}{}}{8(bc-ad)} + \frac{\frac{2\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}c^{\frac{1}{4}}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{d^{\frac{1}{4}}}-\frac{\sqrt{2}c^{\frac{1}{4}}\log\left(\sqrt{d}\right.}{}}{8(bc-ad)}$$

```
[In] integrate(x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/8*(2*sqrt(2)*sqrt(a)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b
^(1/4))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*sqrt(a)*ar
ctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(
b)))/sqrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(1/4)*log(sqrt(b)*x^2 + sqrt(2)*a^(1
/4)*b^(1/4)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(1/4)*log(sqrt(b)*x^2 - sqrt(2
)*a^(1/4)*b^(1/4)*x + sqrt(a))/b^(1/4))/(b*c - a*d) + 1/8*(2*sqrt(2)*sqrt(c
)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*s
qrt(d)))/sqrt(sqrt(c)*sqrt(d)) + 2*sqrt(2)*sqrt(c)*arctan(1/2*sqrt(2)*(2*sq
rt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/sqrt(sqrt(c)*sqrt
(d)) + sqrt(2)*c^(1/4)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c
))/d^(1/4) - sqrt(2)*c^(1/4)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x +
sqrt(c))/d^(1/4))/(b*c - a*d)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 437, normalized size of antiderivative = 0.97

$$\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx = -\frac{(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}b^2c-\sqrt{2}abd\right)} - \frac{(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}b^2c-\sqrt{2}abd\right)} + \frac{(cd^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bcd-\sqrt{2}ad^2\right)} + \frac{(cd^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bcd-\sqrt{2}ad^2\right)} - \frac{(ab^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}b^2c-\sqrt{2}abd\right)} + \frac{(ab^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}b^2c-\sqrt{2}abd\right)} + \frac{(cd^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bcd-\sqrt{2}ad^2\right)} - \frac{(cd^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bcd-\sqrt{2}ad^2\right)}$$

```
[In] integrate(x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(sqrt(2)*b^2*c - sqrt(2)*a*b*d) - 1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*
(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*b^2*c - sqrt(2)*a*b*d) +
1/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4
))/(sqrt(2)*b*c*d - sqrt(2)*a*d^2) + 1/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(
2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c*d - sqrt(2)*a*d^2) - 1
/4*(a*b^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*b^2*
c - sqrt(2)*a*b*d) + 1/4*(a*b^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sq
rt(a/b))/(sqrt(2)*b^2*c - sqrt(2)*a*b*d) + 1/4*(c*d^3)^(1/4)*log(x^2 + sqrt
```

```
(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c*d - sqrt(2)*a*d^2) - 1/4*(c*d^3
)^(1/4)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c*d - sqrt(
2)*a*d^2)
```

Mupad [B] (verification not implemented)

Time = 10.27 (sec) , antiderivative size = 5889, normalized size of antiderivative = 13.12

$$\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

```
[In] int(x^4/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] - atan((a^2*d^2*x*1i + b^2*c^2*x*1i - (a^6*b*d^6*x*256i)/(256*b^5*c^4 + 256
*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d)
- (a*b^6*c^5*d*x*256i)/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 +
1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d) + (a^5*b^2*c*d^5*x*768i)/(256*b^5*
c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^
4*c^3*d) + (a^2*b^5*c^4*d^2*x*768i)/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3
*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d) - (a^3*b^4*c^3*d^3*x*
512i)/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*
d^2 - 1024*a*b^4*c^3*d) - (a^4*b^3*c^2*d^4*x*512i)/(256*b^5*c^4 + 256*a^4*b
*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))/((-a/
(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 -
1024*a*b^4*c^3*d))^(1/4)*((a*(1024*a^6*b*d^7 + 1024*b^7*c^6*d - 6144*a*b^6*
c^5*d^2 - 6144*a^5*b^2*c*d^6 + 15360*a^2*b^5*c^4*d^3 - 20480*a^3*b^4*c^3*d^
4 + 15360*a^4*b^3*c^2*d^5))/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d
^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d) - 4*b^3*c^3 - 4*a^3*d^3 + 4*a
*b^2*c^2*d + 4*a^2*b*c*d^2)))*(-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b
^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*2i - atan((a^2*d
^2*x*1i + b^2*c^2*x*1i - (b^6*c^6*d*x*256i)/(256*a^4*d^5 + 256*b^4*c^4*d -
1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4) - (a^5*b*c*d^
6*x*256i)/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*
c^2*d^3 - 1024*a^3*b*c*d^4) + (a*b^5*c^5*d^2*x*768i)/(256*a^4*d^5 + 256*b^4
*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4) - (a
^2*b^4*c^4*d^3*x*512i)/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 +
1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4) - (a^3*b^3*c^3*d^4*x*512i)/(256*a^
4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^
3*b*c*d^4) + (a^4*b^2*c^2*d^5*x*768i)/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a
*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))/((-c/(256*a^4*d^5
+ 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*
d^4))^(1/4)*((c*(1024*a^6*b*d^7 + 1024*b^7*c^6*d - 6144*a*b^6*c^5*d^2 - 614
4*a^5*b^2*c*d^6 + 15360*a^2*b^5*c^4*d^3 - 20480*a^3*b^4*c^3*d^4 + 15360*a^4
*b^3*c^2*d^5))/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2
*b^2*c^2*d^3 - 1024*a^3*b*c*d^4) - 4*b^3*c^3 - 4*a^3*d^3 + 4*a*b^2*c^2*d +
```

4*a^2*b*c*d^2)))*(-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 15
36*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)*2i - 2*atan(((x*(4*a^2*b^5*c^
4*d^3 + 4*a^4*b^3*c^2*d^5) - (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^
2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*((x*(1024*a^2*b^9
*c^7*d^4 - 3072*a^3*b^8*c^6*d^5 + 2048*a^4*b^7*c^5*d^6 + 2048*a^5*b^6*c^4*d
^7 - 3072*a^6*b^5*c^3*d^8 + 1024*a^7*b^4*c^2*d^9) - (-a/(256*b^5*c^4 + 256*
a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^
(1/4)*(4096*a^2*b^10*c^8*d^4 - 24576*a^3*b^9*c^7*d^5 + 61440*a^4*b^8*c^6*d^
6 - 81920*a^5*b^7*c^5*d^7 + 61440*a^6*b^6*c^4*d^8 - 24576*a^7*b^5*c^3*d^9 +
 4096*a^8*b^4*c^2*d^10)*1i)*(-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2
*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(3/4)*1i + 16*a^2*b^6*c^
5*d^3 - 16*a^3*b^5*c^4*d^4 - 16*a^4*b^4*c^3*d^5 + 16*a^5*b^3*c^2*d^6)*1i)*(
-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2
 - 1024*a*b^4*c^3*d))^(1/4) + (x*(4*a^2*b^5*c^4*d^3 + 4*a^4*b^3*c^2*d^5) -
(-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^
2 - 1024*a*b^4*c^3*d))^(1/4)*((x*(1024*a^2*b^9*c^7*d^4 - 3072*a^3*b^8*c^6*d
^5 + 2048*a^4*b^7*c^5*d^6 + 2048*a^5*b^6*c^4*d^7 - 3072*a^6*b^5*c^3*d^8 + 1
024*a^7*b^4*c^2*d^9) + (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^
3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*(4096*a^2*b^10*c^8*d^4
- 24576*a^3*b^9*c^7*d^5 + 61440*a^4*b^8*c^6*d^6 - 81920*a^5*b^7*c^5*d^7 + 6
1440*a^6*b^6*c^4*d^8 - 24576*a^7*b^5*c^3*d^9 + 4096*a^8*b^4*c^2*d^10)*1i)*(
-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2
 - 1024*a*b^4*c^3*d))^(3/4)*1i - 16*a^2*b^6*c^5*d^3 + 16*a^3*b^5*c^4*d^4 +
16*a^4*b^4*c^3*d^5 - 16*a^5*b^3*c^2*d^6)*1i)*(-a/(256*b^5*c^4 + 256*a^4*b*d
^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4))/
((x*(4*a^2*b^5*c^4*d^3 + 4*a^4*b^3*c^2*d^5) - (-a/(256*b^5*c^4 + 256*a^4*b*
d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*
((x*(1024*a^2*b^9*c^7*d^4 - 3072*a^3*b^8*c^6*d^5 + 2048*a^4*b^7*c^5*d^6 + 2
048*a^5*b^6*c^4*d^7 - 3072*a^6*b^5*c^3*d^8 + 1024*a^7*b^4*c^2*d^9) - (-a/(2
56*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 10
24*a*b^4*c^3*d))^(1/4)*(4096*a^2*b^10*c^8*d^4 - 24576*a^3*b^9*c^7*d^5 + 614
40*a^4*b^8*c^6*d^6 - 81920*a^5*b^7*c^5*d^7 + 61440*a^6*b^6*c^4*d^8 - 24576*
a^7*b^5*c^3*d^9 + 4096*a^8*b^4*c^2*d^10)*1i)*(-a/(256*b^5*c^4 + 256*a^4*b*d
^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(3/4)*1
i + 16*a^2*b^6*c^5*d^3 - 16*a^3*b^5*c^4*d^4 - 16*a^4*b^4*c^3*d^5 + 16*a^5*b
^3*c^2*d^6)*1i)*(-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 153
6*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*1i - (x*(4*a^2*b^5*c^4*d^3 + 4
*a^4*b^3*c^2*d^5) - (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 +
 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*((x*(1024*a^2*b^9*c^7*d^4
- 3072*a^3*b^8*c^6*d^5 + 2048*a^4*b^7*c^5*d^6 + 2048*a^5*b^6*c^4*d^7 - 3072
*a^6*b^5*c^3*d^8 + 1024*a^7*b^4*c^2*d^9) + (-a/(256*b^5*c^4 + 256*a^4*b*d^4
 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*(40
96*a^2*b^10*c^8*d^4 - 24576*a^3*b^9*c^7*d^5 + 61440*a^4*b^8*c^6*d^6 - 81920
*a^5*b^7*c^5*d^7 + 61440*a^6*b^6*c^4*d^8 - 24576*a^7*b^5*c^3*d^9 + 4096*a^8
*b^4*c^2*d^10)*1i)*(-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 +

1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(3/4)*1i - 16*a^2*b^6*c^5*d^3 + 16*a^3*b^5*c^4*d^4 + 16*a^4*b^4*c^3*d^5 - 16*a^5*b^3*c^2*d^6)*1i)*(-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*1i))*(-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4) - 2*atan(((x*(4*a^2*b^5*c^4*d^3 + 4*a^4*b^3*c^2*d^5) - (-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)*((x*(1024*a^2*b^9*c^7*d^4 - 3072*a^3*b^8*c^6*d^5 + 2048*a^4*b^7*c^5*d^6 + 2048*a^5*b^6*c^4*d^7 - 3072*a^6*b^5*c^3*d^8 + 1024*a^7*b^4*c^2*d^9) - (-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)*(4096*a^2*b^10*c^8*d^4 - 24576*a^3*b^9*c^7*d^5 + 61440*a^4*b^8*c^6*d^6 - 81920*a^5*b^7*c^5*d^7 + 61440*a^6*b^6*c^4*d^8 - 24576*a^7*b^5*c^3*d^9 + 4096*a^8*b^4*c^2*d^10)*1i)*(-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(3/4)*1i + 16*a^2*b^6*c^5*d^3 - 16*a^3*b^5*c^4*d^4 - 16*a^4*b^4*c^3*d^5 + 16*a^5*b^3*c^2*d^6)*1i)*(-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4) + (x*(4*a^2*b^5*c^4*d^3 + 4*a^4*b^3*c^2*d^5) - (-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)*((x*(1024*a^2*b^9*c^7*d^4 - 3072*a^3*b^8*c^6*d^5 + 2048*a^4*b^7*c^5*d^6 + 2048*a^5*b^6*c^4*d^7 - 3072*a^6*b^5*c^3*d^8 + 1024*a^7*b^4*c^2*d^9) + (-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)*(4096*a^2*b^10*c^8*d^4 - 24576*a^3*b^9*c^7*d^5 + 61440*a^4*b^8*c^6*d^6 - 81920*a^5*b^7*c^5*d^7 + 61440*a^6*b^6*c^4*d^8 - 24576*a^7*b^5*c^3*d^9 + 4096*a^8*b^4*c^2*d^10)*1i)*(-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(3/4)*1i - 16*a^2*b^6*c^5*d^3 + 16*a^3*b^5*c^4*d^4 + 16*a^4*b^4*c^3*d^5 - 16*a^5*b^3*c^2*d^6)*1i)*(-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4))/((x*(4*a^2*b^5*c^4*d^3 + 4*a^4*b^3*c^2*d^5) - (-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)*((x*(1024*a^2*b^9*c^7*d^4 - 3072*a^3*b^8*c^6*d^5 + 2048*a^4*b^7*c^5*d^6 + 2048*a^5*b^6*c^4*d^7 - 3072*a^6*b^5*c^3*d^8 + 1024*a^7*b^4*c^2*d^9) - (-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)*(4096*a^2*b^10*c^8*d^4 - 24576*a^3*b^9*c^7*d^5 + 61440*a^4*b^8*c^6*d^6 - 81920*a^5*b^7*c^5*d^7 + 61440*a^6*b^6*c^4*d^8 - 24576*a^7*b^5*c^3*d^9 + 4096*a^8*b^4*c^2*d^10)*1i)*(-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(3/4)*1i + 16*a^2*b^6*c^5*d^3 - 16*a^3*b^5*c^4*d^4 - 16*a^4*b^4*c^3*d^5 + 16*a^5*b^3*c^2*d^6)*1i)*(-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)*1i - (x*(4*a^2*b^5*c^4*d^3 + 4*a^4*b^3*c^2*d^5) - (-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)*((x*(1024*a^2*b^9*c^7*d^4 - 3072*a^3*b^8*c^6*d^5 + 2048*a^4*b^7*c^5*d^6 + 2048*a^5*b^6*c^4*d^7 - 3072*a^6*b^5*c^3*d^8 + 1024*a^7*b^4*c^2*d^9) + (-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)

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*(4096*a^2*b^10*c^8*d^4 - 24576*a^3*b^9*c^7*d^5 + 61440*a^4*b^8*c^6*d^6 - 8
1920*a^5*b^7*c^5*d^7 + 61440*a^6*b^6*c^4*d^8 - 24576*a^7*b^5*c^3*d^9 + 4096
*a^8*b^4*c^2*d^10)*1i)*(-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^
2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(3/4)*1i - 16*a^2*b^6*c^5*d^3
 + 16*a^3*b^5*c^4*d^4 + 16*a^4*b^4*c^3*d^5 - 16*a^5*b^3*c^2*d^6)*1i)*(-c/(2
56*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 10
24*a^3*b*c*d^4))^(1/4)*1i))*(-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c
^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)
```

3.782 $\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 449

$$\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx = -\frac{\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{\sqrt[4]{b}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{\sqrt[4]{d}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{\sqrt[4]{d}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}+\frac{\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{b}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{d}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{c}(bc-ad)}+\frac{\sqrt[4]{d}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{c}(bc-ad)}$$

```
[Out] 1/4*b^(1/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(1/4)/(-a*d+b*c)*2^(1/2)
+1/4*b^(1/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(1/4)/(-a*d+b*c)*2^(1/2)
-1/4*d^(1/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(1/4)/(-a*d+b*c)*2^(1/2
)-1/4*d^(1/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(1/4)/(-a*d+b*c)*2^(1/2
)+1/8*b^(1/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(1/4)/(-
a*d+b*c)*2^(1/2)-1/8*b^(1/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/
2))/a^(1/4)/(-a*d+b*c)*2^(1/2)-1/8*d^(1/4)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^
```

```
(1/2)+x^2*d^(1/2))/c^(1/4)/(-a*d+b*c)*2^(1/2)+1/8*d^(1/4)*ln(c^(1/4)*d^(1/4
)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(1/4)/(-a*d+b*c)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {493, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx = -\frac{\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{\sqrt[4]{d}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{\sqrt[4]{d}\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}+\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{d}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{c}(bc-ad)}+\frac{\sqrt[4]{d}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{c}(bc-ad)}$$

```
[In] Int[x^2/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/2*(b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*
c - a*d)) + (b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^
(1/4)*(b*c - a*d)) + (d^(1/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*S
qrt[2]*c^(1/4)*(b*c - a*d)) - (d^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/
4)])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)) - (b^(1/4)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*(b*
c - a*d)) - (d^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]
)/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (d^(1/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*
d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\text{integral} = \frac{b\int \frac{x^2}{a+bx^4}\,dx}{bc-ad} - \frac{d\int \frac{x^2}{c+dx^4}\,dx}{bc-ad}$$

$$= -\frac{\sqrt{b}\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2(bc-ad)} + \frac{\sqrt{b}\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2(bc-ad)} + \frac{\sqrt{d}\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{2(bc-ad)} - \frac{\sqrt{d}\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{2(bc-ad)}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4(bc-ad)} + \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4(bc-ad)} - \frac{\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{4(bc-ad)} - \frac{\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{4(bc-ad)} + \frac{\sqrt[4]{b}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{a}(bc-ad)} + \frac{\sqrt[4]{b}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{a}(bc-ad)} - \frac{\sqrt[4]{d}\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}+2x}{-\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{c}(bc-ad)} - \frac{\sqrt[4]{d}\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}-2x}{-\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{c}(bc-ad)}$$

$$= \frac{\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)} - \frac{\sqrt[4]{b}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)} - \frac{\sqrt[4]{d}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{c}(bc-ad)} + \frac{\sqrt[4]{d}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{c}(bc-ad)} + \frac{\sqrt[4]{b}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)} - \frac{\sqrt[4]{b}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)} - \frac{\sqrt[4]{d}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)} + \frac{\sqrt[4]{d}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}$$

$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)} + \frac{\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)} + \frac{\sqrt[4]{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)} - \frac{\sqrt[4]{d}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)} + \frac{\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)} - \frac{\sqrt[4]{b}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)} - \frac{\sqrt[4]{d}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{c}(bc-ad)} + \frac{\sqrt[4]{d}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{c}(bc-ad)}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx = \frac{-2\sqrt[4]{b}\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\sqrt[4]{b}\sqrt[4]{c}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\sqrt[4]{a}\sqrt[4]{d}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)-2\sqrt[4]{a}\sqrt[4]{c}}{}$$

```
[In] Integrate[x^2/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (-2*b^(1/4)*c^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*b^(1/4)*c^(
1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*d^(1/4)*ArcTan[1 -
 (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 2*a^(1/4)*d^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/
4)*x)/c^(1/4)] + b^(1/4)*c^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2] - b^(1/4)*c^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2] - a^(1/4)*d^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt
[d]*x^2] + a^(1/4)*d^(1/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d
]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(1/4)*(b*c - a*d))
```

Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.49

method	result
default	$-\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\right.}{8(ad-bc)\left(\right.}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^4d^4c-4a^3b\,d^3c^2+6a^2b^2d^2c^3-4a\,b^3d\,c^4+b^4c^5\right)\_Z^4+d\right)}\_R\ln\left(\left(\left(a^6d^6-4a^5bc\,d^5+7a^4b^2c^2d^4-8a^3b^3c^3d^3+7a^2b^4c^4d^2-\right.\right.\right.\right.}{4}$

[In] int(x^2/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)

[Out] -1/8/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/8/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*(ln((x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x-1))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 1331, normalized size of antiderivative = 2.96

$$\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

[In] integrate(x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")

[Out] 1/4*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(1/4)*log(b*x + (a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(3/4)) - 1/4*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(1/4)*log(b*x - (a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(3/4)) + 1/4*I*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(1/4)*log(b*x - (I*a*b^3*c^3 - 3*I*a^2*b^2*c^2*d + 3*I*a^3*b*c*d^2 - I*a^4*d^3)*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(3/4)) - 1/4*I*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(1/4)*log(b*x - (-I*a*b^3*c^3 + 3*I*a^2*b^2*c^2*d - 3*I*a^3*b*c*d^2 + I*a^4*d^3)*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(3/4)) - 1/4*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(1/4)*log(d*x + (b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(3/4)) + 1/4*(-d/(b^4*c^5 -

```
4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(1/4)*log
(d*x - (b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*(-d/(b^4*c^5
 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(3/4))
 - 1/4*I*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3
 + a^4*c*d^4))^(1/4)*log(d*x - (I*b^3*c^4 - 3*I*a*b^2*c^3*d + 3*I*a^2*b*c^2
*d^2 - I*a^3*c*d^3)*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^
3*b*c^2*d^3 + a^4*c*d^4))^(3/4)) + 1/4*I*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a
^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(1/4)*log(d*x - (-I*b^3*c^4
+ 3*I*a*b^2*c^3*d - 3*I*a^2*b*c^2*d^2 + I*a^3*c*d^3)*(-d/(b^4*c^5 - 4*a*b^3
*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(3/4))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(x**2/(b*x**4+a)/(d*x**4+c),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 363, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx = \frac{b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}\right.}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right.}{8(bc-ad)} - \frac{d\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{d}x^2-\sqrt{2}\right.}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right.}{8(bc-ad)}$$

```
[In] integrate(x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/8*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))
/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(
1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/
(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)
*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*x^2 - sqrt(2)
*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(b*c - a*d) - 1/8*d*(2*sqr
t(2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c
)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*
(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c
)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x +
 sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(
1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b*c - a*d)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.06

$$\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx = \frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ab^3c-\sqrt{2}a^2b^2d\right)} + \frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ab^3c-\sqrt{2}a^2b^2d\right)} - \frac{(cd^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^2d^2-\sqrt{2}acd^3\right)} - \frac{(cd^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^2d^2-\sqrt{2}acd^3\right)} - \frac{(ab^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}ab^3c-\sqrt{2}a^2b^2d\right)} + \frac{(ab^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}ab^3c-\sqrt{2}a^2b^2d\right)} + \frac{(cd^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^2d^2-\sqrt{2}acd^3\right)} - \frac{(cd^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^2d^2-\sqrt{2}acd^3\right)}$$

```
[In] integrate(x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4
))/(sqrt(2)*a*b^3*c - sqrt(2)*a^2*b^2*d) + 1/2*(a*b^3)^(3/4)*arctan(1/2*sqr
t(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a*b^3*c - sqrt(2)*a^
2*b^2*d) - 1/2*(c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))
/(c/d)^(1/4))/(sqrt(2)*b*c^2*d^2 - sqrt(2)*a*c*d^3) - 1/2*(c*d^3)^(3/4)*arc
tan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^2*d^2
 - sqrt(2)*a*c*d^3) - 1/4*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + s
qrt(a/b))/(sqrt(2)*a*b^3*c - sqrt(2)*a^2*b^2*d) + 1/4*(a*b^3)^(3/4)*log(x^2
 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^3*c - sqrt(2)*a^2*b^2*d)
 + 1/4*(c*d^3)^(3/4)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*
b*c^2*d^2 - sqrt(2)*a*c*d^3) - 1/4*(c*d^3)^(3/4)*log(x^2 - sqrt(2)*x*(c/d)^
(1/4) + sqrt(c/d))/(sqrt(2)*b*c^2*d^2 - sqrt(2)*a*c*d^3)
```

Mupad [B] (verification not implemented)

Time = 9.98 (sec) , antiderivative size = 6633, normalized size of antiderivative = 14.77

$$\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

```
[In] int(x^2/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] atan(((x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) + (-b/(256*a^5*d^4 + 256*a*b^4
*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(3/4)
*(x*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^
2*d^2 - 1024*a^4*b*c*d^3))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10
 - 4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 716
8*a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) + 256*a*b^9*c^6*d^4 + 256*a^6*b^4
*c*d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 -
768*a^5*b^5*c^2*d^8))*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d
 + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4)*1i + (x*(4*a*b^6*c^2*d^5
 + 4*a^2*b^5*c*d^6) - (-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d
 + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(3/4)*(256*a*b^9*c^6*d^4 - x*(
-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2
 - 1024*a^4*b*c*d^3))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 - 40
96*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*a^5
*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) + 256*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*
d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2*d^8))*(-b
/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 -
 1024*a^4*b*c*d^3))^(1/4)*1i)/((x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) + (-b
/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 -
 1024*a^4*b*c*d^3))^(3/4)*(x*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^
3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4)*(1024*a*b^10*c^7*
d^4 + 1024*a^7*b^4*c*d^10 - 4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8
192*a^4*b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) + 256*a*
b^9*c^6*d^4 + 256*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6
 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2*d^8))*(-b/(256*a^5*d^4 + 256*a*b^4
*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4)
 - (x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) - (-b/(256*a^5*d^4 + 256*a*b^4*c^
4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(3/4)*(2
56*a*b^9*c^6*d^4 - x*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d
+ 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4)*(1024*a*b^10*c^7*d^4 + 10
24*a^7*b^4*c*d^10 - 4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*
b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) + 256*a^6*b^4*c*
d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 768
*a^5*b^5*c^2*d^8))*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d +
1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4) + 2*a*b^5*c*d^5))*(-b/(256*
a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*
```

a^4*b*c*d^3))^(1/4)*2i + 2*atan(((x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) + (
-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2
 - 1024*a^4*b*c*d^3))^(3/4)*(256*a*b^9*c^6*d^4 - x*(-b/(256*a^5*d^4 + 256*a
*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(
1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 - 4096*a^2*b^9*c^6*d^5 + 71
68*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8 - 4096*a^6
*b^5*c^2*d^9)*1i + 256*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^
4*d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2*d^8)*1i)*(-b/(256*a^5*d^4 + 2
56*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3
))^(1/4) + (x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) - (-b/(256*a^5*d^4 + 256*
a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^
(3/4)*(x*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b
^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c
*d^10 - 4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7
 + 7168*a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9)*1i + 256*a*b^9*c^6*d^4 + 256
*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^
3*d^7 - 768*a^5*b^5*c^2*d^8)*1i)*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^
2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4))/((x*(4*a*b^6
*c^2*d^5 + 4*a^2*b^5*c*d^6) - (-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b
^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(3/4)*(x*(-b/(256*a^5*
d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*
b*c*d^3))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 - 4096*a^2*b^9*c
^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8
 - 4096*a^6*b^5*c^2*d^9)*1i + 256*a*b^9*c^6*d^4 + 256*a^6*b^4*c*d^9 - 768*a
^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^
2*d^8)*1i)*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3
*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4)*1i - (x*(4*a*b^6*c^2*d^5 + 4*a^2*b^
5*c*d^6) + (-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3
*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(3/4)*(256*a*b^9*c^6*d^4 - x*(-b/(256*a^5
*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4
*b*c*d^3))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 - 4096*a^2*b^9*
c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^
8 - 4096*a^6*b^5*c^2*d^9)*1i + 256*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^5 + 51
2*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2*d^8)*1i)*(-b/(256
*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024
*a^4*b*c*d^3))^(1/4)*1i + 2*a*b^5*c*d^5))*(-b/(256*a^5*d^4 + 256*a*b^4*c^4
 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4) + at
an(((x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) + (-d/(256*b^4*c^5 + 256*a^4*c*d
^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(3/4)*(
x*(-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*
d^2 - 1024*a*b^3*c^4*d))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 -
 4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*
a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) + 256*a*b^9*c^6*d^4 + 256*a^6*b^4*c
*d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 76
8*a^5*b^5*c^2*d^8))*(-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 +

1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(1/4)*1i + (x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) - (-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(3/4)*(256*a*b^9*c^6*d^4 - x*(-d /(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 - 4096 *a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*a^5*b ^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) + 256*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^ 5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2*d^8))*(-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1 024*a*b^3*c^4*d))^(1/4)*1i)/((x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) + (-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1 024*a*b^3*c^4*d))^(3/4)*(x*(-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^ 2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(1/4)*(1024*a*b^10*c^7*d^ 4 + 1024*a^7*b^4*c*d^10 - 4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 819 2*a^4*b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) + 256*a*b^ 9*c^6*d^4 + 256*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2*d^8))*(-d/(256*b^4*c^5 + 256*a^4*c*d ^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(1/4) - (x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) - (-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(3/4)*(256 *a*b^9*c^6*d^4 - x*(-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024 *a^7*b^4*c*d^10 - 4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^ 7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) + 256*a^6*b^4*c*d^ 9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 768*a ^5*b^5*c^2*d^8))*(-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 15 36*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(1/4) + 2*a*b^5*c*d^5))*(-d/(256*b^ 4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a* b^3*c^4*d))^(1/4)*2i + 2*atan(((x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) + (-d /(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(3/4)*(256*a*b^9*c^6*d^4 - x*(-d/(256*b^4*c^5 + 256*a^4 *c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(1/ 4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 - 4096*a^2*b^9*c^6*d^5 + 7168 *a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8 - 4096*a^6*b ^5*c^2*d^9)*1i + 256*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4* d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2*d^8)*1i)*(-d/(256*b^4*c^5 + 256 *a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d)) ^(1/4) + (x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) - (-d/(256*b^4*c^5 + 256*a^ 4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(3 /4)*(x*(-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2 *c^3*d^2 - 1024*a*b^3*c^4*d))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d ^10 - 4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9)*1i + 256*a*b^9*c^6*d^4 + 256*a ^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3* d^7 - 768*a^5*b^5*c^2*d^8)*1i)*(-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*

```
b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(1/4))/((x*(4*a*b^6*c
^2*d^5 + 4*a^2*b^5*c*d^6) - (-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c
^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(3/4)*(x*(-d/(256*b^4*c^
5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*
c^4*d))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 - 4096*a^2*b^9*c^6
*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8 -
 4096*a^6*b^5*c^2*d^9)*1i + 256*a*b^9*c^6*d^4 + 256*a^6*b^4*c*d^9 - 768*a^2
*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2*
d^8)*1i)*(-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b
^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(1/4)*1i - (x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*
c*d^6) + (-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b
^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(3/4)*(256*a*b^9*c^6*d^4 - x*(-d/(256*b^4*c
^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3
*c^4*d))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 - 4096*a^2*b^9*c^
6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8
- 4096*a^6*b^5*c^2*d^9)*1i + 256*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^5 + 512*
a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2*d^8)*1i)*(-d/(256*b
^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a
*b^3*c^4*d))^(1/4)*1i + 2*a*b^5*c*d^5))*(-d/(256*b^4*c^5 + 256*a^4*c*d^4 -
1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(1/4)
```

3.783 $\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 449

$$\begin{aligned}\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = &-\frac{b^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)}\\ &+\frac{d^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)}-\frac{d^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)}\\ &-\frac{b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)}\\ &+\frac{b^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)}\\ &+\frac{d^{3/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)}\\ &-\frac{d^{3/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)}\end{aligned}$$

```
[Out] 1/4*b^(3/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/(-a*d+b*c)*2^(1/2)
+1/4*b^(3/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/(-a*d+b*c)*2^(1/2)
-1/4*d^(3/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(3/4)/(-a*d+b*c)*2^(1/2
)-1/4*d^(3/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(3/4)/(-a*d+b*c)*2^(1/2
)-1/8*b^(3/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(3/4)/(-
a*d+b*c)*2^(1/2)+1/8*b^(3/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/
2))/a^(3/4)/(-a*d+b*c)*2^(1/2)+1/8*d^(3/4)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^
(1/2)+x^2*d^(1/2))/c^(3/4)/(-a*d+b*c)*2^(1/2)-1/8*d^(3/4)*ln(c^(1/4)*d^(1/4
)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(3/4)/(-a*d+b*c)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {400, 217, 1179, 642, 1176, 631, 210}

$$\begin{aligned}\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = &-\frac{b^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}(bc-ad)}\\&-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)}\\&+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)}\\&+\frac{d^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)}-\frac{d^{3/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}c^{3/4}(bc-ad)}\\&+\frac{d^{3/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)}\\&-\frac{d^{3/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)}\end{aligned}$$

```
[In] Int[1/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/2*(b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(Sqrt[2]*a^(3/4)*(b*
c - a*d)) + (b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^
(3/4)*(b*c - a*d)) + (d^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*S
qrt[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/
4)])/(2*Sqrt[2]*c^(3/4)*(b*c - a*d)) - (b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)) + (b^(3/4)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*
c - a*d)) + (d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]
)/(4*Sqrt[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*
d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*(b*c - a*d))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\text{integral} = \frac{b\int \frac{1}{a+bx^4}\,dx}{bc-ad} - \frac{d\int \frac{1}{c+dx^4}\,dx}{bc-ad}$$

$$= \frac{b\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}(bc-ad)} + \frac{b\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}(bc-ad)} - \frac{d\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{2\sqrt{c}(bc-ad)} - \frac{d\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{2\sqrt{c}(bc-ad)}$$

$$= \frac{\sqrt{b}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}dx}{4\sqrt{a}(bc-ad)} + \frac{\sqrt{b}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}dx}{4\sqrt{a}(bc-ad)} - \frac{b^{3/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}dx}{4\sqrt{2}a^{3/4}(bc-ad)}$$

$$- \frac{b^{3/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}dx}{4\sqrt{2}a^{3/4}(bc-ad)} - \frac{\sqrt{d}\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}dx}{4\sqrt{c}(bc-ad)} - \frac{\sqrt{d}\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}dx}{4\sqrt{c}(bc-ad)}$$

$$+ \frac{d^{3/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}+2x}{-\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}-x^2}dx}{4\sqrt{2}c^{3/4}(bc-ad)} + \frac{d^{3/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}-2x}{-\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}-x^2}dx}{4\sqrt{2}c^{3/4}(bc-ad)}$$

$$= -\frac{b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)} + \frac{b^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)}$$

$$+ \frac{d^{3/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)} - \frac{d^{3/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)}$$

$$+ \frac{b^{3/4}\text{Subst}\left(\int \frac{1}{-1-x^2}dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)} - \frac{b^{3/4}\text{Subst}\left(\int \frac{1}{-1-x^2}dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)}$$

$$- \frac{d^{3/4}\text{Subst}\left(\int \frac{1}{-1-x^2}dx, x, 1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)} + \frac{d^{3/4}\text{Subst}\left(\int \frac{1}{-1-x^2}dx, x, 1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)}$$

$$= -\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)} + \frac{b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)}$$

$$+ \frac{d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)} - \frac{d^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)}$$

$$- \frac{b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)} + \frac{b^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)}$$

$$+ \frac{d^{3/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)} - \frac{d^{3/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.76

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$$
$$= \frac{-2b^{3/4}c^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2b^{3/4}c^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2a^{3/4}d^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)-2a}{}$$

[In] Integrate[1/((a + b*x^4)*(c + d*x^4)),x]

[Out] (-2*b^(3/4)*c^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*b^(3/4)*c^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(3/4)*d^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 2*a^(3/4)*d^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - b^(3/4)*c^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + b^(3/4)*c^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + a^(3/4)*d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] - a^(3/4)*d^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)*(b*c - a*d))

Maple [A] (verified)

Time = 4.44 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.50

method	result
default	$-\frac{b\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8(ad-bc)a}+\frac{d\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)\right.}{}$
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(a^4c^3d^4-4a^3bc^4d^3+6a^2b^2c^5d^2-4ab^3c^6d+b^4c^7\right)\_Z^4+d^3\right)}\_R\ln\left(\left(\left(-a^7d^7+4ca^6bd^6-6c^2a^5b^2d^5+3a^4b^3c^3d^4+3a^3b^4c^4\right.\right.\right.\right.}{}$

[In] int(1/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)

[Out] -1/8*b/(a*d-b*c)*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/8*d/(a*d-b*c)*(c/d)^(1/4)/c*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x-1))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 1171, normalized size of antiderivative = 2.61

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/4*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^
3 + a^7*d^4))^(1/4)*log(b*x + (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^
3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*(-b^3/
(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^
4))^(1/4)*log(b*x - (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d +
6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*I*(-b^3/(a^3*b^4
*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4
)*log(b*x - (I*a*b*c - I*a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^
5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) + 1/4*I*(-b^3/(a^3*b^4*c^4
 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)*lo
g(b*x - (-I*a*b*c + I*a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b
^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*(-d^3/(b^4*c^7 - 4*a*b^
3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*x
 + (b*c^2 - a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a
^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)) + 1/4*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d +
6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*x - (b*c^2
- a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d
^3 + a^4*c^3*d^4))^(1/4)) + 1/4*I*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^
2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*x - (I*b*c^2 - I*a*
c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 +
 a^4*c^3*d^4))^(1/4)) - 1/4*I*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^
5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*x - (-I*b*c^2 + I*a*c*d
)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^
4*c^3*d^4))^(1/4))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.81

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$$
$$= \frac{\frac{2\sqrt{2}b\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}b\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{b}x^2-\cdots\right.}{a^{\frac{3}{4}}}}{8\,(bc-ad)}$$
$$-\frac{\frac{2\sqrt{2}d\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}d\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}d^{\frac{3}{4}}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}}-\frac{\sqrt{2}d^{\frac{3}{4}}\log\left(\sqrt{d}x\cdots\right.}{\cdots}}{8\,(bc-ad)}$$

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/8*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))
/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b*arcta
n(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b))
)/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)*log(sqrt(b)*x^2 + sqrt(
2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(3/4)*log(sqrt(b)*x^2 -
 sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4))/(b*c - a*d) - 1/8*(2*sqrt(2)
*d*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*
sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*d*arctan(1/2*sqrt(2)*
(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqr
t(sqrt(c)*sqrt(d))) + sqrt(2)*d^(3/4)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(
1/4)*x + sqrt(c))/c^(3/4) - sqrt(2)*d^(3/4)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/
4)*d^(1/4)*x + sqrt(c))/c^(3/4))/(b*c - a*d)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 437, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = \frac{(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}abc-\sqrt{2}a^2d\right)} + \frac{(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}abc-\sqrt{2}a^2d\right)} - \frac{(cd^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^2-\sqrt{2}acd\right)} - \frac{(cd^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^2-\sqrt{2}acd\right)} + \frac{(ab^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}abc-\sqrt{2}a^2d\right)} - \frac{(ab^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}abc-\sqrt{2}a^2d\right)} - \frac{(cd^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^2-\sqrt{2}acd\right)} + \frac{(cd^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^2-\sqrt{2}acd\right)}$$

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4
))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) + 1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(
2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) - 1
/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4)
)/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) - 1/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(2
*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) + 1/
4*(a*b^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b*c
 - sqrt(2)*a^2*d) - 1/4*(a*b^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqr
t(a/b))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) - 1/4*(c*d^3)^(1/4)*log(x^2 + sqrt(
```

```
2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) + 1/4*(c*d^3)
^(1/4)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^2 - sqrt(2
)*a*c*d)
```

Mupad [B] (verification not implemented)

Time = 10.39 (sec) , antiderivative size = 6153, normalized size of antiderivative = 13.70

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

```
[In] int(1/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] - atan((((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a
^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3
*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(3/4)
*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*
c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^
11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7
 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10)
+ x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6
*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^
3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7
*c*d^7) + 8*b^7*d^7*x)*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^
4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*1i - ((-d^3/(256*b^
4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*
a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*
d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(3/4)*((-d^3/(256*b^4*c^7 +
 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c
^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^
7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d
^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10) - x*(1024*a^7*b^4*d^11
 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2
*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c
^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) - 8*b^7*d^7*x)
*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c
^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*1i)/(((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^
4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((
-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5
*d^2 - 1024*a*b^3*c^6*d))^(3/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 102
4*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b
^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*
c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3
*d^9 - 20480*a^7*b^5*c^2*d^10) + x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 -
 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^
```

3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*
d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) + 8*b^7*d^7*x)*(-d^3/(256*b^4*c^7 +
256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^
6*d))^(1/4) + ((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 +
1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a
^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))
^(3/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^
2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^
4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c
^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*
d^10) - x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 40
96*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*
b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32
*a*b^7*c*d^7) - 8*b^7*d^7*x)*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^
3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)))*(-d^3/(256*
b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 102
4*a*b^3*c^6*d))^(1/4)*2i - atan((((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 10
24*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*((-b^3/(
256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 -
 1024*a^6*b*c*d^3))^(3/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*
b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*(4096*a*b^11*c^
8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^
6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 -
 20480*a^7*b^5*c^2*d^10) + x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*
a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*
c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 -
16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) + 8*b^7*d^7*x)*(-b^3/(256*a^7*d^4 + 256*a^
3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^
(1/4)*1i - ((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 153
6*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*((-b^3/(256*a^7*d^4 + 256*a^3*
b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(3
/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b
^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c
*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*
d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^1
0) - x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*
a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7
*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*
b^7*c*d^7) - 8*b^7*d^7*x)*(-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b
^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*1i)/(((-b^3/(256
*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 10
24*a^6*b*c*d^3))^(1/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3
*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(3/4)*((-b^3/(256*a^7*d^
4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*
b*c*d^3))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10

*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10) + x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) + 8*b^7*d^7*x)*(-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4) + ((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(3/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10) - x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) - 8*b^7*d^7*x)*(-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)))*(-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*2i - 2*atan((b^3*d^3*x - (128*b^10*c^7*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) - (128*a^7*b^3*d^7*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) - (768*a^2*b^8*c^5*d^2*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) + (384*a^3*b^7*c^4*d^3*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) + (384*a^4*b^6*c^3*d^4*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) - (768*a^5*b^5*c^2*d^5*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) + (512*a*b^9*c^6*d*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) + (512*a^6*b^4*c*d^6*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))/((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*((b^3*(512*a*b^7*c^8 + 512*a^8*c*d^7 - 2560*a^2*b^6*c^7*d - 2560*a^7*b*c^2*d^6 + 4608*a^3*b^5*c^6*d^2 - 2560*a^4*b^4*c^5*d^3 - 2560*a^5*b^3*c^4*d^4 + 4608*a^6*b^2*c^3*d^5))/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) + 2*a^2*b^2*d^4 + 2*b^4*c^2*d^2 - 4*a*b^3*c*d^3)))*(-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4) - 2*atan((b^3*d^3*x - (128*a^7*d^10*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) - (128*b^7*c^7*d^3*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) - (768*a^2*b^5*c^5*d^5*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) + (384*a^3*b^4*c^4*d

^6*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) + (384*a^4*b^3*c^3*d^7*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) - (768*a^5*b^2*c^2*d^8*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) + (512*a^6*b*c*d^9*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) + (512*a*b^6*c^6*d^4*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))/((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((d^3*(512*a*b^7*c^8 + 512*a^8*c*d^7 - 2560*a^2*b^6*c^7*d - 2560*a^7*b*c^2*d^6 + 4608*a^3*b^5*c^6*d^2 - 2560*a^4*b^4*c^5*d^3 - 2560*a^5*b^3*c^4*d^4 + 4608*a^6*b^2*c^3*d^5))/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) + 2*a^2*b^2*d^4 + 2*b^4*c^2*d^2 - 4*a*b^3*c*d^3)))*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)

3.784 $\int \frac{1}{x^2(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 460

$$\begin{aligned}\int \frac{1}{x^2(a+bx^4)(c+dx^4)}\,dx = & -\frac{1}{acx}+\frac{b^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}(bc-ad)} \\ & -\frac{b^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}(bc-ad)} \\ & -\frac{d^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{5/4}(bc-ad)}+\frac{d^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{5/4}(bc-ad)} \\ & -\frac{b^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{5/4}(bc-ad)} \\ & +\frac{b^{5/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{5/4}(bc-ad)} \\ & +\frac{d^{5/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{5/4}(bc-ad)} \\ & -\frac{d^{5/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{5/4}(bc-ad)}\end{aligned}$$

```
[Out] -1/a/c/x-1/4*b^(5/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(5/4)/(-a*d+b*c
)*2^(1/2)-1/4*b^(5/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(5/4)/(-a*d+b*c
)*2^(1/2)+1/4*d^(5/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(5/4)/(-a*d+b*
c)*2^(1/2)+1/4*d^(5/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(5/4)/(-a*d+b*
```

```
c)*2^(1/2)-1/8*b^(5/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a
^(5/4)/(-a*d+b*c)*2^(1/2)+1/8*b^(5/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+
x^2*b^(1/2))/a^(5/4)/(-a*d+b*c)*2^(1/2)+1/8*d^(5/4)*ln(-c^(1/4)*d^(1/4)*x*2
^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(5/4)/(-a*d+b*c)*2^(1/2)-1/8*d^(5/4)*ln(c^(1/
4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(5/4)/(-a*d+b*c)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {491, 598, 303, 1176, 631, 210, 1179, 642}

$$\begin{aligned}\int \frac{1}{x^2 (a+bx^4)(c+dx^4)}\,dx = {} & \frac{b^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}(bc-ad)} - \frac{b^{5/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{5/4}(bc-ad)} \\ & - \frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{5/4}(bc-ad)} \\ & + \frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{5/4}(bc-ad)} \\ & - \frac{d^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{5/4}(bc-ad)} + \frac{d^{5/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}c^{5/4}(bc-ad)} \\ & + \frac{d^{5/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{5/4}(bc-ad)} \\ & - \frac{d^{5/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{5/4}(bc-ad)} - \frac{1}{acx}\end{aligned}$$

```
[In] Int[1/(x^2*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -(1/(a*c*x)) + (b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]
*a^(5/4)*(b*c - a*d)) - (b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(
2*Sqrt[2]*a^(5/4)*(b*c - a*d)) - (d^(5/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^
(1/4)])/(2*Sqrt[2]*c^(5/4)*(b*c - a*d)) + (d^(5/4)*ArcTan[1 + (Sqrt[2]*d^(1
/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(5/4)*(b*c - a*d)) - (b^(5/4)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(5/4)*(b*c - a*d)) +
(b^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]
*a^(5/4)*(b*c - a*d)) + (d^(5/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x +
Sqrt[d]*x^2])/(4*Sqrt[2]*c^(5/4)*(b*c - a*d)) - (d^(5/4)*Log[Sqrt[c] + Sqrt
[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(5/4)*(b*c - a*d))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\text{integral} = -\frac{1}{acx} + \frac{\int \frac{x^2(-bc-ad-bdx^4)}{(a+bx^4)(c+dx^4)}\,dx}{ac}$$

$$= -\frac{1}{acx} + \frac{\int \left(-\frac{b^2cx^2}{(bc-ad)(a+bx^4)} - \frac{ad^2x^2}{(-bc+ad)(c+dx^4)}\right)dx}{ac}$$

$$= -\frac{1}{acx} - \frac{b^2 \int \frac{x^2}{a+bx^4}\,dx}{a(bc-ad)} + \frac{d^2 \int \frac{x^2}{c+dx^4}\,dx}{c(bc-ad)}$$

$$= -\frac{1}{acx} + \frac{b^{3/2} \int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2a(bc-ad)} - \frac{b^{3/2} \int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2a(bc-ad)} - \frac{d^{3/2} \int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{2c(bc-ad)} + \frac{d^{3/2} \int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{2c(bc-ad)}$$

$$= -\frac{1}{acx} - \frac{b \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2}\,dx}{4a(bc-ad)} - \frac{b \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2}\,dx}{4a(bc-ad)} - \frac{b^{5/4} \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} - x^2}\,dx}{4\sqrt{2}a^{5/4}(bc-ad)}$$

$$- \frac{b^{5/4} \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} - x^2}\,dx}{4\sqrt{2}a^{5/4}(bc-ad)} + \frac{d \int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}} - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}} + x^2}\,dx}{4c(bc-ad)} + \frac{d \int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}} + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}} + x^2}\,dx}{4c(bc-ad)}$$

$$+ \frac{d^{5/4} \int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}+2x}{-\frac{\sqrt{c}}{\sqrt{d}} - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}} - x^2}\,dx}{4\sqrt{2}c^{5/4}(bc-ad)} + \frac{d^{5/4} \int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}-2x}{-\frac{\sqrt{c}}{\sqrt{d}} + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}} - x^2}\,dx}{4\sqrt{2}c^{5/4}(bc-ad)}$$

$$= -\frac{1}{acx} - \frac{b^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{5/4}(bc-ad)} + \frac{b^{5/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{5/4}(bc-ad)} + \frac{d^{5/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{5/4}(bc-ad)} - \frac{d^{5/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{5/4}(bc-ad)} - \frac{b^{5/4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}(bc-ad)} + \frac{b^{5/4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}(bc-ad)} + \frac{d^{5/4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{5/4}(bc-ad)} - \frac{d^{5/4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{5/4}(bc-ad)}$$

$$= -\frac{1}{acx} + \frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}(bc-ad)} - \frac{b^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}(bc-ad)} - \frac{d^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{5/4}(bc-ad)} + \frac{d^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{5/4}(bc-ad)} - \frac{b^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{5/4}(bc-ad)} + \frac{b^{5/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{5/4}(bc-ad)} + \frac{d^{5/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{5/4}(bc-ad)} - \frac{d^{5/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{5/4}(bc-ad)}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 385, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^2(a+bx^4)(c+dx^4)}\,dx$$

$$= \frac{\frac{8b}{a} - \frac{8d}{c} - \frac{2\sqrt{2}b^{5/4}x\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{5/4}} + \frac{2\sqrt{2}b^{5/4}x\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{5/4}} + \frac{2\sqrt{2}d^{5/4}x\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{5/4}} - \frac{2\sqrt{2}d^{5/4}x\arctan\left(\right.}{c^{5/4}}}{}$$

```
[In] Integrate[1/(x^2*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((8*b)/a - (8*d)/c - (2*Sqrt[2]*b^(5/4)*x*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^
(1/4)])/a^(5/4) + (2*Sqrt[2]*b^(5/4)*x*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/
4)])/a^(5/4) + (2*Sqrt[2]*d^(5/4)*x*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]
)/c^(5/4) - (2*Sqrt[2]*d^(5/4)*x*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c
^(5/4) + (Sqrt[2]*b^(5/4)*x*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2])/a^(5/4) - (Sqrt[2]*b^(5/4)*x*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
```

```
*x + Sqrt[b]*x^2])/a^(5/4) - (Sqrt[2]*d^(5/4)*x*Log[Sqrt[c] - Sqrt[2]*c^(1/
4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(5/4) + (Sqrt[2]*d^(5/4)*x*Log[Sqrt[c] + Sqr
t[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(5/4))/(-8*b*c*x + 8*a*d*x)
```

Maple [A] (verified)

Time = 4.68 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.52

method	result
default	$\frac{b\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8(ad-bc)a\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{1}{acx}-\frac{d\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\,\mathrm{arct}}{8(ad-}$
risch	Expression too large to display

```
[In] int(1/x^2/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*b/(a*d-b*c)/a/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^
(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4
)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))-1/a/c/x-1/8*d/(a*d-b*c)/c/(c/d)^(
1/4)*2^(1/2)*(ln((x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2+(c/d)^(1/4)*x
*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/(
c/d)^(1/4)*x-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 1461, normalized size of antiderivative = 3.18

$$\int \frac{1}{x^2\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -1/4*((-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*
d^3 + a^9*d^4))^(1/4)*a*c*x*log(b^4*x + (a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*
a^6*b*c*d^2 - a^7*d^3)*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2
*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(3/4)) - (-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^
3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(1/4)*a*c*x*log(b^4*x -
 (a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^7*d^3)*(-b^5/(a^5*b^4*c
^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(3/4))
 + I*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d
^3 + a^9*d^4))^(1/4)*a*c*x*log(b^4*x - (I*a^4*b^3*c^3 - 3*I*a^5*b^2*c^2*d +
 3*I*a^6*b*c*d^2 - I*a^7*d^3)*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*
b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(3/4)) - I*(-b^5/(a^5*b^4*c^4 - 4*a
```

```
^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(1/4)*a*c*x*lo
g(b^4*x - (-I*a^4*b^3*c^3 + 3*I*a^5*b^2*c^2*d - 3*I*a^6*b*c*d^2 + I*a^7*d^3
)*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3
+ a^9*d^4))^(3/4)) - (-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4
*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1/4)*a*c*x*log(d^4*x + (b^3*c^7 - 3*a*b^2*c
^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3)*(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^
2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(3/4)) + (-d^5/(b^4*c^9 - 4
*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1/4)*a*
c*x*log(d^4*x - (b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3)*(
-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c
^5*d^4))^(3/4)) - I*(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*
a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1/4)*a*c*x*log(d^4*x - (I*b^3*c^7 - 3*I*a*b^
2*c^6*d + 3*I*a^2*b*c^5*d^2 - I*a^3*c^4*d^3)*(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d
 + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(3/4)) + I*(-d^5/(b^
4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))
^(1/4)*a*c*x*log(d^4*x - (-I*b^3*c^7 + 3*I*a*b^2*c^6*d - 3*I*a^2*b*c^5*d^2
+ I*a^3*c^4*d^3)*(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3
*b*c^6*d^3 + a^4*c^5*d^4))^(3/4)) + 4)/(a*c*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+bx^4\right)\left(c+dx^4\right)}\,dx=\text{Timed out}$$

```
[In] integrate(1/x**2/(b*x**4+a)/(d*x**4+c),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = -\frac{b^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}x^2-\sqrt{\right.}}{a^{\frac{1}{4}}b}\right)}{8\left(abc-a^2d\right)} + \frac{d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{d}x^2-\sqrt{\right.}}{c^{\frac{1}{4}}d}\right)}{8\left(bc^2-acd\right)} - \frac{1}{acx}$$

```
[In] integrate(1/x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/8*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/
4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arct
an(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)
))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1
/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*x^2 - sqrt
(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a*b*c - a^2*d) + 1/8*d
^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sq
rt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(1/2
*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sq
rt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^
(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(sqrt(d)*x^2 - sqrt(2)*c^
(1/4)*d^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b*c^2 - a*c*d) - 1/(a*c*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2 (a+bx^4)(c+dx^4)} dx = -\frac{(ab^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2b^2c-\sqrt{2}a^3bd\right)} - \frac{(ab^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2b^2c-\sqrt{2}a^3bd\right)} + \frac{(cd^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^3d-\sqrt{2}ac^2d^2\right)} + \frac{(cd^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^3d-\sqrt{2}ac^2d^2\right)} + \frac{(ab^3)^{\frac{3}{4}} \log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}a^2b^2c-\sqrt{2}a^3bd\right)} - \frac{(ab^3)^{\frac{3}{4}} \log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}a^2b^2c-\sqrt{2}a^3bd\right)} - \frac{(cd^3)^{\frac{3}{4}} \log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^3d-\sqrt{2}ac^2d^2\right)} + \frac{(cd^3)^{\frac{3}{4}} \log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^3d-\sqrt{2}ac^2d^2\right)} - \frac{1}{acx}$$

```
[In] integrate(1/x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(sqrt(2)*a^2*b^2*c - sqrt(2)*a^3*b*d) - 1/2*(a*b^3)^(3/4)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b^2*c - sqrt(2)
*a^3*b*d) + 1/2*(c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4)
)/(c/d)^(1/4))/(sqrt(2)*b*c^3*d - sqrt(2)*a*c^2*d^2) + 1/2*(c*d^3)^(3/4)*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^3*d
- sqrt(2)*a*c^2*d^2) + 1/4*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) +
sqrt(a/b))/(sqrt(2)*a^2*b^2*c - sqrt(2)*a^3*b*d) - 1/4*(a*b^3)^(3/4)*log(x^
2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a^2*b^2*c - sqrt(2)*a^3*b*d
```

```
) - 1/4*(c*d^3)^(3/4)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)
*b*c^3*d - sqrt(2)*a*c^2*d^2) + 1/4*(c*d^3)^(3/4)*log(x^2 - sqrt(2)*x*(c/d)
^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^3*d - sqrt(2)*a*c^2*d^2) - 1/(a*c*x)
```

Mupad [B] (verification not implemented)

Time = 10.36 (sec) , antiderivative size = 5962, normalized size of antiderivative = 12.96

$$\int \frac{1}{x^2 (a+bx^4) (c+dx^4)} dx = \text{Too large to display}$$

```
[In] int(1/(x^2*(a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] 2*atan(((-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^
2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*(x*(4*a^11*b^9*c^12*d^8 + 4*a^12*b
^8*c^11*d^9) - (-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 +
1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(3/4)*(x*(-d^5/(256*b^4*c^9 + 256
*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d
))^(1/4)*(1024*a^12*b^12*c^20*d^4 - 4096*a^13*b^11*c^19*d^5 + 6144*a^14*b^1
0*c^18*d^6 - 4096*a^15*b^9*c^17*d^7 + 2048*a^16*b^8*c^16*d^8 - 4096*a^17*b^
7*c^15*d^9 + 6144*a^18*b^6*c^14*d^10 - 4096*a^19*b^5*c^13*d^11 + 1024*a^20*
b^4*c^12*d^12)*1i - 256*a^11*b^12*c^19*d^4 + 768*a^12*b^11*c^18*d^5 - 768*a
^13*b^10*c^17*d^6 + 256*a^14*b^9*c^16*d^7 + 256*a^16*b^7*c^14*d^9 - 768*a^1
7*b^6*c^13*d^10 + 768*a^18*b^5*c^12*d^11 - 256*a^19*b^4*c^11*d^12)*1i) + (-
d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*
d^2 - 1024*a*b^3*c^8*d))^(1/4)*(x*(4*a^11*b^9*c^12*d^8 + 4*a^12*b^8*c^11*d^
9) - (-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b
^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(3/4)*(x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d
^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*(
1024*a^12*b^12*c^20*d^4 - 4096*a^13*b^11*c^19*d^5 + 6144*a^14*b^10*c^18*d^6
 - 4096*a^15*b^9*c^17*d^7 + 2048*a^16*b^8*c^16*d^8 - 4096*a^17*b^7*c^15*d^9
 + 6144*a^18*b^6*c^14*d^10 - 4096*a^19*b^5*c^13*d^11 + 1024*a^20*b^4*c^12*d
^12)*1i + 256*a^11*b^12*c^19*d^4 - 768*a^12*b^11*c^18*d^5 + 768*a^13*b^10*c
^17*d^6 - 256*a^14*b^9*c^16*d^7 - 256*a^16*b^7*c^14*d^9 + 768*a^17*b^6*c^13
*d^10 - 768*a^18*b^5*c^12*d^11 + 256*a^19*b^4*c^11*d^12)*1i))/((-d^5/(256*b
^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024
*a*b^3*c^8*d))^(1/4)*(x*(4*a^11*b^9*c^12*d^8 + 4*a^12*b^8*c^11*d^9) - (-d^5
/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2
 - 1024*a*b^3*c^8*d))^(3/4)*(x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*
a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*(1024*a^12*
b^12*c^20*d^4 - 4096*a^13*b^11*c^19*d^5 + 6144*a^14*b^10*c^18*d^6 - 4096*a^
15*b^9*c^17*d^7 + 2048*a^16*b^8*c^16*d^8 - 4096*a^17*b^7*c^15*d^9 + 6144*a^
18*b^6*c^14*d^10 - 4096*a^19*b^5*c^13*d^11 + 1024*a^20*b^4*c^12*d^12)*1i -
256*a^11*b^12*c^19*d^4 + 768*a^12*b^11*c^18*d^5 - 768*a^13*b^10*c^17*d^6 +
256*a^14*b^9*c^16*d^7 + 256*a^16*b^7*c^14*d^9 - 768*a^17*b^6*c^13*d^10 + 76
```

8*a^18*b^5*c^12*d^11 - 256*a^19*b^4*c^11*d^12)*1i)*1i - (-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*(x*(4*a^11*b^9*c^12*d^8 + 4*a^12*b^8*c^11*d^9) - (-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(3/4)*(x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*(1024*a^12*b^12*c^20*d^4 - 4096*a^13*b^11*c^19*d^5 + 6144*a^14*b^10*c^18*d^6 - 4096*a^15*b^9*c^17*d^7 + 2048*a^16*b^8*c^16*d^8 - 4096*a^17*b^7*c^15*d^9 + 6144*a^18*b^6*c^14*d^10 - 4096*a^19*b^5*c^13*d^11 + 1024*a^20*b^4*c^12*d^12)*1i + 256*a^11*b^12*c^19*d^4 - 768*a^12*b^11*c^18*d^5 + 768*a^13*b^10*c^17*d^6 - 256*a^14*b^9*c^16*d^7 - 256*a^16*b^7*c^14*d^9 + 768*a^17*b^6*c^13*d^10 - 768*a^18*b^5*c^12*d^11 + 256*a^19*b^4*c^11*d^12)*1i)*1i))*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4) + atan((a^14*c*d^8*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*1024i + a^6*b^8*c^9*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*1024i + a^6*b^4*d^5*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(1/4)*4i + a^5*b^5*c*d^4*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(1/4)*4i - a^7*b^7*c^8*d*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*4096i - a^13*b*c^2*d^7*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*4096i + a^8*b^6*c^7*d^2*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*6144i - a^9*b^5*c^6*d^3*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*4096i + a^10*b^4*c^5*d^4*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*2048i - a^11*b^3*c^4*d^5*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*4096i + a^12*b^2*c^3*d^6*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*6144i)/(b^9*c^4 + a^4*b^5*d^4 + a^3*b^6*c*d^3 + a^2*b^7*c^2*d^2 + a*b^8*c^3*d))*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(1/4)*2i + atan((b^5*c^6*d^4*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*4i + a*b^8*c^14*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(5/4)*1024i + a^9*c^6*d^8*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(5/4)*1024i + a*b^4*c^5*d^5*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*4i - a^2*b^7*c^13*d*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(5/4)*4096i - a^8*b*c^7*d^7*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7* d^2 - 1024*a*b^3*c^8*d))^(5/4)*4096i + a^3*b^6*c^12*d^2*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(5/4)*6144i - a^4*b^5*c^11*d^3*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(5/4)*4096i + a^5*b^4*c^10*d^4*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(5/4)*2048i - a^6*b^3*c^9*d^5*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(5/4)*4096i + a^7*b^2*c^8*d^6*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(5/4)*6144i)/(a^4*d^9 + b^4*c^4*d^5 + a*b^3*c^3*d^6 + a^2*b^2*c^2*d^7 + a^3*b*c*d^8))*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*2i + 2*atan(((-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(1/4)*(x*(4*a^11*b^9*c^12*d^8 + 4*a^12*b^8*c^11*d^9) - (-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(3/4)*(x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(1/4)*(1024*a^12*b^12*c^20*d^4 - 4096*a^13*b^11*c^19*d^5 + 6144*a^14*b^10*c^18*d^6 - 4096*a^15*b^9*c^17*d^7 + 2048*a^16*b^8*c^16*d^8 - 4096*a^17*b^7*c^15*d^9 + 6144*a^18*b^6*c^14*d^10 - 4096*a^19*b^5*c^13*d^11 + 1024*a^20*b^4*c^12*d^12)*1i - 256*a^11*b^12*c^19*d^4 + 768*a^12*b^11*c^18*d^5 - 768*a^13*b^10*c^17*d^6 + 256*a^14*b^9*c^16*d^7 + 256*a^16*b^7*c^14*d^9 - 768*a^17*b^6*c^13*d^10 + 768*a^18*b^5*c^12*d^11 - 256*a^19*b^4*c^11*d^12)*1i) + (-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(1/4)*(x*(4*a^11*b^9*c^12*d^8 + 4*a^12*b^8*c^11*d^9) - (-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(3/4)*(x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(1/4)*(1024*a^12*b^12*c^20*d^4 - 4096*a^13*b^11*c^19*d^5 + 6144*a^14*b^10*c^18*d^6 - 4096*a^15*b^9*c^17*d^7 + 2048*a^16*b^8*c^16*d^8 - 4096*a^17*b^7*c^15*d^9 + 6144*a^18*b^6*c^14*d^10 - 4096*a^19*b^5*c^13*d^11 + 1024*a^20*b^4*c^12*d^12)*1i + 256*a^11*b^12*c^19*d^4 - 768*a^12*b^11*c^18*d^5 + 768*a^13*b^10*c^17*d^6 - 256*a^14*b^9*c^16*d^7 - 256*a^16*b^7*c^14*d^9 + 768*a^17*b^6*c^13*d^10 - 768*a^18*b^5*c^12*d^11 + 256*a^19*b^4*c^11*d^12)*1i))/((-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(1/4)*(x*(4*a^11*b^9*c^12*d^8 + 4*a^12*b^8*c^11*d^9) - (-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(3/4)*(x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(1/4)*(1024*a^12*b^12*c^20*d^4 - 4096*a^13*b^11*c^19*d^5 + 6144*a^14*b^10*c^18*d^6 - 4096*a^15*b^9*c^17*d^7 + 2048*a^16*b^8*c^16*d^8 - 4096*a^17*b^7*c^15*d^9 + 6144*a^18*b^6*c^14*d^10 - 4096*a^19*b^5*c^13*d^11 + 1024*a^20*b^4*c^12*d^12)*1i - 256*a^11*b^12*c^19*d^4 + 768*a^12*b^11*c^18*d^5 - 768*a^13*b^10*c^17*d^6 + 256*a^14*b^9*c^16*d^7 + 256*a^16*b^7*c^14*d^9 - 768*a^17*b^6*c^13*d^10 + 768*a^18*b^5*c^12*d^11 - 256*a^19*b^4*c^11*d^12)*1i)*1i - (-b^5/(256*a^9*d^4 + 2

```
56*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d
^3))^(1/4)*(x*(4*a^11*b^9*c^12*d^8 + 4*a^12*b^8*c^11*d^9) - (-b^5/(256*a^9*
d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^
8*b*c*d^3))^(3/4)*(x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^
3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(1/4)*(1024*a^12*b^12*c^20*
d^4 - 4096*a^13*b^11*c^19*d^5 + 6144*a^14*b^10*c^18*d^6 - 4096*a^15*b^9*c^1
7*d^7 + 2048*a^16*b^8*c^16*d^8 - 4096*a^17*b^7*c^15*d^9 + 6144*a^18*b^6*c^1
4*d^10 - 4096*a^19*b^5*c^13*d^11 + 1024*a^20*b^4*c^12*d^12)*1i + 256*a^11*b
^12*c^19*d^4 - 768*a^12*b^11*c^18*d^5 + 768*a^13*b^10*c^17*d^6 - 256*a^14*b
^9*c^16*d^7 - 256*a^16*b^7*c^14*d^9 + 768*a^17*b^6*c^13*d^10 - 768*a^18*b^5
*c^12*d^11 + 256*a^19*b^4*c^11*d^12)*1i)*1i))*(-b^5/(256*a^9*d^4 + 256*a^5*
b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(1
/4) - 1/(a*c*x)
```

3.785 $\int \frac{1}{x^4(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 462

$$\int \frac{1}{x^4(a+bx^4)(c+dx^4)}\,dx = -\frac{1}{3acx^3}+\frac{b^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}(bc-ad)} - \frac{b^{7/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}(bc-ad)} - \frac{d^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{7/4}(bc-ad)} + \frac{d^{7/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{7/4}(bc-ad)} + \frac{b^{7/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{7/4}(bc-ad)} - \frac{b^{7/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{7/4}(bc-ad)} - \frac{d^{7/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{7/4}(bc-ad)} + \frac{d^{7/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{7/4}(bc-ad)}$$

```
[Out] -1/3/a/c/x^3-1/4*b^(7/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/(-a*d
+b*c)*2^(1/2)-1/4*b^(7/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/(-a*d
+b*c)*2^(1/2)+1/4*d^(7/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(7/4)/(-a*
d+b*c)*2^(1/2)+1/4*d^(7/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(7/4)/(-a*
```

```
d+b*c)*2^(1/2)+1/8*b^(7/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2
))/a^(7/4)/(-a*d+b*c)*2^(1/2)-1/8*b^(7/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1
/2)+x^2*b^(1/2))/a^(7/4)/(-a*d+b*c)*2^(1/2)-1/8*d^(7/4)*ln(-c^(1/4)*d^(1/4)
*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(7/4)/(-a*d+b*c)*2^(1/2)+1/8*d^(7/4)*ln(c
^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(7/4)/(-a*d+b*c)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {491, 536, 217, 1179, 642, 1176, 631, 210}

$$\begin{aligned}\int \frac{1}{x^4 (a+bx^4)(c+dx^4)}\,dx &= \frac{b^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}(bc-ad)}-\frac{b^{7/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{7/4}(bc-ad)}\\ &+\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{7/4}(bc-ad)}\\ &-\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{7/4}(bc-ad)}\\ &-\frac{d^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{7/4}(bc-ad)}+\frac{d^{7/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}c^{7/4}(bc-ad)}\\ &-\frac{d^{7/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{7/4}(bc-ad)}\\ &+\frac{d^{7/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{7/4}(bc-ad)}-\frac{1}{3acx^3}\end{aligned}$$

```
[In] Int[1/(x^4*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/3*1/(a*c*x^3) + (b^(7/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqr
t[2]*a^(7/4)*(b*c - a*d)) - (b^(7/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)
])/(2*Sqrt[2]*a^(7/4)*(b*c - a*d)) - (d^(7/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x
)/c^(1/4)])/(2*Sqrt[2]*c^(7/4)*(b*c - a*d)) + (d^(7/4)*ArcTan[1 + (Sqrt[2]*
d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(7/4)*(b*c - a*d)) + (b^(7/4)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(7/4)*(b*c - a*d)
) - (b^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqr
t[2]*a^(7/4)*(b*c - a*d)) - (d^(7/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*
x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(7/4)*(b*c - a*d)) + (d^(7/4)*Log[Sqrt[c] +
Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(7/4)*(b*c - a*d))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\text{integral} = -\frac{1}{3acx^3} + \frac{\int \frac{-3(bc+ad)-3bdx^4}{(a+bx^4)(c+dx^4)}\,dx}{3ac}$$

$$= -\frac{1}{3acx^3} - \frac{b^2\int \frac{1}{a+bx^4}\,dx}{a(bc-ad)} + \frac{d^2\int \frac{1}{c+dx^4}\,dx}{c(bc-ad)}$$

$$= -\frac{1}{3acx^3} - \frac{b^2\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2a^{3/2}(bc-ad)} - \frac{b^2\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2a^{3/2}(bc-ad)} + \frac{d^2\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{2c^{3/2}(bc-ad)} + \frac{d^2\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{2c^{3/2}(bc-ad)}$$

$$= -\frac{1}{3acx^3} - \frac{b^{3/2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4a^{3/2}(bc-ad)} - \frac{b^{3/2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4a^{3/2}(bc-ad)}$$
$$+ \frac{b^{7/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{7/4}(bc-ad)} + \frac{b^{7/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{7/4}(bc-ad)} + \frac{d^{3/2}\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{4c^{3/2}(bc-ad)}$$
$$+ \frac{d^{3/2}\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{4c^{3/2}(bc-ad)} - \frac{d^{7/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}+2x}{-\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}-x^2}\,dx}{4\sqrt{2}c^{7/4}(bc-ad)} - \frac{d^{7/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}-2x}{-\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}-x^2}\,dx}{4\sqrt{2}c^{7/4}(bc-ad)}$$

$$
\begin{aligned}
&= -\frac{1}{3acx^3}+\frac{b^{7/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{7/4}(bc-ad)}-\frac{b^{7/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{7/4}(bc-ad)} \\
&\quad -\frac{d^{7/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{7/4}(bc-ad)}+\frac{d^{7/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{7/4}(bc-ad)} \\
&\quad -\frac{b^{7/4}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}(bc-ad)}+\frac{b^{7/4}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}(bc-ad)} \\
&\quad +\frac{d^{7/4}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{7/4}(bc-ad)}-\frac{d^{7/4}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{7/4}(bc-ad)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1}{3acx^3}+\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}(bc-ad)}-\frac{b^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}(bc-ad)} \\
&\quad -\frac{d^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{7/4}(bc-ad)}+\frac{d^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{7/4}(bc-ad)} \\
&\quad +\frac{b^{7/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{7/4}(bc-ad)}-\frac{b^{7/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{7/4}(bc-ad)} \\
&\quad -\frac{d^{7/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{7/4}(bc-ad)}+\frac{d^{7/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{7/4}(bc-ad)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 406, normalized size of antiderivative = 0.88

$$
\int\frac{1}{x^4\left(a+bx^4\right)\left(c+dx^4\right)}\,dx
$$

$$
=\frac{\frac{8b}{a}-\frac{8d}{c}-\frac{6\sqrt{2}b^{7/4}x^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{6\sqrt{2}b^{7/4}x^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{6\sqrt{2}d^{7/4}x^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{7/4}}-\frac{6\sqrt{2}d^{7/4}x^3\,\text{arcta}\cdots}{c^{\cdots}}}{\cdots}
$$

```
[In] Integrate[1/(x^4*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((8*b)/a - (8*d)/c - (6*Sqrt[2]*b^(7/4)*x^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/
a^(1/4)])/a^(7/4) + (6*Sqrt[2]*b^(7/4)*x^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a
^(1/4)])/a^(7/4) + (6*Sqrt[2]*d^(7/4)*x^3*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^
(1/4)])/c^(7/4) - (6*Sqrt[2]*d^(7/4)*x^3*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(
1/4)])/c^(7/4) - (3*Sqrt[2]*b^(7/4)*x^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*x + Sqrt[b]*x^2])/a^(7/4) + (3*Sqrt[2]*b^(7/4)*x^3*Log[Sqrt[a] + Sqrt[2]
```

```
*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4) + (3*Sqrt[2]*d^(7/4)*x^3*Log[Sqr
t[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(7/4) - (3*Sqrt[2]*d^(7/
4)*x^3*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(7/4))/(24
*(-(b*c) + a*d)*x^3)
```

Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.52

method	result
default	$\frac{b^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a^{2}(ad-bc)}-\frac{1}{3ac\,x^{3}}-\frac{d^{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}}{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right.\right.}{}$
risch	$-\frac{1}{3ac\,x^{3}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(d^{4}a^{11}-4a^{10}bc\,d^{3}+6a^{9}b^{2}c^{2}d^{2}-4a^{8}b^{3}c^{3}d+a^{7}b^{4}c^{4}\right)\_Z^{4}+b^{7}\right)}\_R\ln\left(\left(\left(-5a^{15}c^{7}d^{8}+38a^{14}b\,c^{8}d^{7}-128a^{13}b^{2}\right.\right.\right.\right.}{}$

```
[In] int(1/x^4/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/a^2*b^2/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a
/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^
(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))-1/3/a/c/x^3-1/8/c^2*d^2/(a*d-
b*c)*(c/d)^(1/4)*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(
c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arct
an(2^(1/2)/(c/d)^(1/4)*x-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.94 (sec) , antiderivative size = 1255, normalized size of antiderivative = 2.72

$$\int \frac{1}{x^4\left(a+bx^4\right)\left(c+dx^4\right)}\,dx=\text{Too large to display}$$

```
[In] integrate(1/x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -1/12*(3*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*
b*c*d^3 + a^11*d^4))^(1/4)*a*c*x^3*log(b^2*x + (-b^7/(a^7*b^4*c^4 - 4*a^8*b
^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*(a^2*b*c -
 a^3*d)) - 3*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a
^10*b*c*d^3 + a^11*d^4))^(1/4)*a*c*x^3*log(b^2*x - (-b^7/(a^7*b^4*c^4 - 4*a
^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*(a^2*b
*c - a^3*d)) - 3*I*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2
 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*a*c*x^3*log(b^2*x - (-b^7/(a^7*b^4*c^4
```

```
 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*
(I*a^2*b*c - I*a^3*d)) + 3*I*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b
^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*a*c*x^3*log(b^2*x - (-b^7/(a
^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^
4))^(1/4)*(-I*a^2*b*c + I*a^3*d)) - 3*(-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*
a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4)*a*c*x^3*log(d^2*x +
 (-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a
^4*c^7*d^4))^(1/4)*(b*c^3 - a*c^2*d)) + 3*(-d^7/(b^4*c^11 - 4*a*b^3*c^10*d
+ 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4)*a*c*x^3*log(d^2
*x - (-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3
 + a^4*c^7*d^4))^(1/4)*(b*c^3 - a*c^2*d)) + 3*I*(-d^7/(b^4*c^11 - 4*a*b^3*c
^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4)*a*c*x^3*l
og(d^2*x - (-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c
^8*d^3 + a^4*c^7*d^4))^(1/4)*(I*b*c^3 - I*a*c^2*d)) - 3*I*(-d^7/(b^4*c^11 -
 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4)
*a*c*x^3*log(d^2*x - (-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 -
 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4)*(-I*b*c^3 + I*a*c^2*d)) + 4)/(a*c*x^
3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a + bx^4) (c + dx^4)} \, dx = \text{Timed out}$$

```
[In] integrate(1/x**4/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^4\,(a+bx^4)\,(c+dx^4)}\,dx =$$

$$-\frac{\frac{2\sqrt{2}b^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}b^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}b^{\frac{7}{4}}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}b^{\frac{7}{4}}\log\left(\sqrt{b}\right.}{}}{8\,(abc-a^2d)}$$

$$+\frac{\frac{2\sqrt{2}d^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}d^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}d^{\frac{7}{4}}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}}-\frac{\sqrt{2}d^{\frac{7}{4}}\log\left(\sqrt{\right.}{}}{8\,(bc^2-acd)}$$

$$-\frac{1}{3\,acx^3}$$

```
[In] integrate(1/x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/8*(2*sqrt(2)*b^2*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/
4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b^2*
arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqr
t(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(7/4)*log(sqrt(b)*x^2 +
sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(7/4)*log(sqrt(b)*
x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4))/(a*b*c - a^2*d) + 1/8*(
2*sqrt(2)*d^2*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sq
rt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*d^2*arctan
(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))
/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*d^(7/4)*log(sqrt(d)*x^2 + sqrt(2
)*c^(1/4)*d^(1/4)*x + sqrt(c))/c^(3/4) - sqrt(2)*d^(7/4)*log(sqrt(d)*x^2 -
sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/c^(3/4))/(b*c^2 - a*c*d) - 1/3/(a*c*x^
3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^4(a+bx^4)(c+dx^4)}\,dx = -\frac{(ab^3)^{\frac{1}{4}}\,b\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2bc-\sqrt{2}a^3d\right)} - \frac{(ab^3)^{\frac{1}{4}}\,b\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2bc-\sqrt{2}a^3d\right)} + \frac{(cd^3)^{\frac{1}{4}}\,d\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^3-\sqrt{2}ac^2d\right)} + \frac{(cd^3)^{\frac{1}{4}}\,d\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^3-\sqrt{2}ac^2d\right)} - \frac{(ab^3)^{\frac{1}{4}}\,b\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}a^2bc-\sqrt{2}a^3d\right)} + \frac{(ab^3)^{\frac{1}{4}}\,b\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}a^2bc-\sqrt{2}a^3d\right)} + \frac{(cd^3)^{\frac{1}{4}}\,d\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^3-\sqrt{2}ac^2d\right)} - \frac{(cd^3)^{\frac{1}{4}}\,d\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^3-\sqrt{2}ac^2d\right)} - \frac{1}{3\,acx^3}$$

```
[In] integrate(1/x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/2*(a*b^3)^(1/4)*b*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(
1/4))/(sqrt(2)*a^2*b*c - sqrt(2)*a^3*d) - 1/2*(a*b^3)^(1/4)*b*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b*c - sqrt(2)*a
^3*d) + 1/2*(c*d^3)^(1/4)*d*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/
(c/d)^(1/4))/(sqrt(2)*b*c^3 - sqrt(2)*a*c^2*d) + 1/2*(c*d^3)^(1/4)*d*arctan
(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^3 - sqrt
(2)*a*c^2*d) - 1/4*(a*b^3)^(1/4)*b*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a
/b))/(sqrt(2)*a^2*b*c - sqrt(2)*a^3*d) + 1/4*(a*b^3)^(1/4)*b*log(x^2 - sqrt
(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a^2*b*c - sqrt(2)*a^3*d) + 1/4*(c*d
^3)^(1/4)*d*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^3 - s
qrt(2)*a*c^2*d) - 1/4*(c*d^3)^(1/4)*d*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqr
t(c/d))/(sqrt(2)*b*c^3 - sqrt(2)*a*c^2*d) - 1/3/(a*c*x^3)
```

Mupad [B] (verification not implemented)

Time = 10.65 (sec) , antiderivative size = 7459, normalized size of antiderivative = 16.15

$$\int \frac{1}{x^4 (a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

[In] int(1/(x^4*(a + b*x^4)*(c + d*x^4)),x)

[Out] - atan((a^2*b^5*d^7*x*1i + b^7*c^2*d^5*x*1i - (a^2*b^16*c^11*x*256i)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) - (a^4*b^14*c^9*d^2*x*1536i)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) + (a^5*b^13*c^8*d^3*x*1024i)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) - (a^6*b^12*c^7*d^4*x*256i)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) - (a^7*b^11*c^6*d^5*x*256i)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) + (a^8*b^10*c^5*d^6*x*1024i)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) - (a^9*b^9*c^4*d^7*x*1536i)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) + (a^10*b^8*c^3*d^8*x*1024i)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) - (a^11*b^7*c^2*d^9*x*256i)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) + (a^3*b^15*c^10*d*x*1024i)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))/((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*((b^7*(1024*a^4*b^8*c^12 + 1024*a^12*c^4*d^8 - 5120*a^5*b^7*c^11*d - 5120*a^11*b*c^5*d^7 + 10240*a^6*b^6*c^10*d^2 - 11264*a^7*b^5*c^9*d^3 + 10240*a^8*b^4*c^8*d^4 - 11264*a^9*b^3*c^7*d^5 + 10240*a^10*b^2*c^6*d^6))/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) + 4*a^5*b^3*d^8 + 4*b^8*c^5*d^3 - 4*a*b^7*c^4*d^4 - 4*a^4*b^4*c*d^7)))*(-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*2i - atan((a^2*b^5*d^7*x*1i + b^7*c^2*d^5*x*1i - (a^11*c^2*d^16*x*256i)/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) - (a^2*b^9*c^11*d^7*x*256i)/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) + (a^3*b^8*c^10*d^8*x*1024i)/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) - (a^4*b^7*c^9*d^9*x*1536i)/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) + (a^5*b^6*c^8*d^10*x*1024i)/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) - (a^6*b^5*c^7*d^11*x*256i)/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) - (a^7 *b^4*c^6*d^12*x*256i)/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) + (a^8*b^3*c^5*d^13*x*1024i)/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) - (a^9*b^2*c^4*d^14*x*1536i)/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) + (a^10*b*c^3*d^15*x*1024i)/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))/((-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*((d^7*(1024*a^4*b^8*c^12 + 1024*a^12*c^4*d^8 - 5120*a^5*b^7*c^11*d - 5120*a^11*b*c^5*d^7 + 10240*a^6*b^6*c^10*d^2 - 11264*a^7*b^5*c^9*d^3 + 10240*a^8*b^4*c^8*d^4 - 11264*a^9*b^3*c^7*d^5 + 10240*a^10*b^2*c^6*d^6))/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) + 4*a^5*b^3*d^8 + 4*b^8*c^5*d^3 - 4*a*b^7*c^4*d^4 - 4*a^4*b^4*c*d^7)))*(-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*2i - 2*atan(((-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*(x*(4*a^9*b^11*c^11*d^9 + 4*a^11*b^9*c^9*d^11) - (-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*((-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(3/4)*(x*(1024*a^11*b^13*c^20*d^4 - 4096*a^12*b^12*c^19*d^5 + 6144*a^13*b^11*c^18*d^6 - 4096*a^14*b^10*c^17*d^7 + 1024*a^15*b^9*c^16*d^8 + 1024*a^16*b^8*c^15*d^9 - 4096*a^17*b^7*c^14*d^10 + 6144*a^18*b^6*c^13*d^11 - 4096*a^19*b^5*c^12*d^12 + 1024*a^20*b^4*c^11*d^13) - (-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*(4096*a^13*b^12*c^21*d^4 - 20480*a^14*b^11*c^20*d^5 + 40960*a^15*b^10*c^19*d^6 - 45056*a^16*b^9*c^18*d^7 + 40960*a^17*b^8*c^17*d^8 - 45056*a^18*b^7*c^16*d^9 + 40960*a^19*b^6*c^15*d^10 - 20480*a^20*b^5*c^14*d^11 + 4096*a^21*b^4*c^13*d^12)*1i)*1i - 16*a^9*b^12*c^14*d^7 + 16*a^10*b^11*c^13*d^8 + 16*a^13*b^8*c^10*d^11 - 16*a^14*b^7*c^9*d^12)*1i) + (-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*(x*(4*a^9*b^11*c^11*d^9 + 4*a^11*b^9*c^9*d^11) - (-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*((-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(3/4)*(x*(1024*a^11*b^13*c^20*d^4 - 4096*a^12*b^12*c^19*d^5 + 6144*a^13*b^11*c^18*d^6 - 4096*a^14*b^10*c^17*d^7 + 1024*a^15*b^9*c^16*d^8 + 1024*a^16*b^8*c^15*d^9 - 4096*a^17*b^7*c^14*d^10 + 6144*a^18*b^6*c^13*d^11 - 4096*a^19*b^5*c^12*d^12 + 1024*a^20*b^4*c^11*d^13) + (-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*(4096*a^13*b^12*c^21*d^4 - 20480*a^14*b^11*c^20*d^5 + 40960*a^15*b^10*c^19*d^6 - 45056*a^16*b^9*c^18*d^7 + 40960*a^17*b^8*c^17*d^8 - 45056*a^18*b^7*c^16*d^9 + 40960*a^19*b^6*c^15*d^10 - 20480*a^20*b^5*c^14*d^11 + 4096*a^21*b^4*c^13*d^12)*1i)*1i + 16*a^9*b^12*c^14*d^7 - 16*a^10*b^11*c^13*d^8 - 16*a^13*b^8*c^10*d^11 + 16*a^14*b^7*c^9*d^12)*1i))/((-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536 *a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*(x*(4*a^9*b^11*c^11*d^9 + 4*a^11*b^9*c^9*d^11) - (-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*((-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(3/4)*(x*(1024*a^11*b^13*c^20*d^4 - 4096*a^12*b^12*c^19*d^5 + 6144*a^13*b^11*c^18*d^6 - 4096*a^14*b^10*c^17*d^7 + 1024*a^15*b^9*c^16*d^8 + 1024*a^16*b^8*c^15*d^9 - 4096*a^17*b^7*c^14*d^10 + 6144*a^18*b^6*c^13*d^11 - 4096*a^19*b^5*c^12*d^12 + 1024*a^20*b^4*c^11*d^13) - (-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*(4096*a^13*b^12*c^21*d^4 - 20480*a^14*b^11*c^20*d^5 + 40960*a^15*b^10*c^19*d^6 - 45056*a^16*b^9*c^18*d^7 + 40960*a^17*b^8*c^17*d^8 - 45056*a^18*b^7*c^16*d^9 + 40960*a^19*b^6*c^15*d^10 - 20480*a^20*b^5*c^14*d^11 + 4096*a^21*b^4*c^13*d^12)*1i)*1i - 16*a^9*b^12*c^14*d^7 + 16*a^10*b^11*c^13*d^8 + 16*a^13*b^8*c^10*d^11 - 16*a^14*b^7*c^9*d^12)*1i)*1i - (-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*(x*(4*a^9*b^11*c^11*d^9 + 4*a^11*b^9*c^9*d^11) - (-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*((-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(3/4)*(x*(1024*a^11*b^13*c^20*d^4 - 4096*a^12*b^12*c^19*d^5 + 6144*a^13*b^11*c^18*d^6 - 4096*a^14*b^10*c^17*d^7 + 1024*a^15*b^9*c^16*d^8 + 1024*a^16*b^8*c^15*d^9 - 4096*a^17*b^7*c^14*d^10 + 6144*a^18*b^6*c^13*d^11 - 4096*a^19*b^5*c^12*d^12 + 1024*a^20*b^4*c^11*d^13) + (-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*(4096*a^13*b^12*c^21*d^4 - 20480*a^14*b^11*c^20*d^5 + 40960*a^15*b^10*c^19*d^6 - 45056*a^16*b^9*c^18*d^7 + 40960*a^17*b^8*c^17*d^8 - 45056*a^18*b^7*c^16*d^9 + 40960*a^19*b^6*c^15*d^10 - 20480*a^20*b^5*c^14*d^11 + 4096*a^21*b^4*c^13*d^12)*1i)*1i + 16*a^9*b^12*c^14*d^7 - 16*a^10*b^11*c^13*d^8 - 16*a^13*b^8*c^10*d^11 + 16*a^14*b^7*c^9*d^12)*1i)*1i))*(-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4) - 2*atan(-((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*(x*(4*a^9*b^11*c^11*d^9 + 4*a^11*b^9*c^9*d^11) - (-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(3/4)*((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*(4096*a^13*b^12*c^21*d^4 - 20480*a^14*b^11*c^20*d^5 + 40960*a^15*b^10*c^19*d^6 - 45056*a^16*b^9*c^18*d^7 + 40960*a^17*b^8*c^17*d^8 - 45056*a^18*b^7*c^16*d^9 + 40960*a^19*b^6*c^15*d^10 - 20480*a^20*b^5*c^14*d^11 + 4096*a^21*b^4*c^13*d^12)*1i + x*(1024*a^11*b^13*c^20*d^4 - 4096*a^12*b^12*c^19*d^5 + 6144*a^13*b^11*c^18*d^6 - 4096*a^14*b^10*c^17*d^7 + 1024*a^15*b^9*c^16*d^8 + 1024*a^16*b^8*c^15*d^9 - 4096*a^17*b^7*c^14*d^10 + 6144*a^18*b^6*c^13*d^11 - 4096*a^19*b^5*c^12*d^12 + 1024*a^20*b^4*c^11*d^13))*1i + 16*a^9*b^12*c^14*d^7 - 16*a^10*b^11*c^13*d^8 - 16*a^13*b^8*c^10*d^11 + 16*a^14*b^7*c^9*d^12)*1i) + (-b^7/(256*a^11* d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*(x*(4*a^9*b^11*c^11*d^9 + 4*a^11*b^9*c^9*d^11) + (-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(3/4)*((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*(4096*a^13*b^12*c^21*d^4 - 20480*a^14*b^11*c^20*d^5 + 40960*a^15*b^10*c^19*d^6 - 45056*a^16*b^9*c^18*d^7 + 40960*a^17*b^8*c^17*d^8 - 45056*a^18*b^7*c^16*d^9 + 40960*a^19*b^6*c^15*d^10 - 20480*a^20*b^5*c^14*d^11 + 4096*a^21*b^4*c^13*d^12)*1i - x*(1024*a^11*b^13*c^20*d^4 - 4096*a^12*b^12*c^19*d^5 + 6144*a^13*b^11*c^18*d^6 - 4096*a^14*b^10*c^17*d^7 + 1024*a^15*b^9*c^16*d^8 + 1024*a^16*b^8*c^15*d^9 - 4096*a^17*b^7*c^14*d^10 + 6144*a^18*b^6*c^13*d^11 - 4096*a^19*b^5*c^12*d^12 + 1024*a^20*b^4*c^11*d^13))*1i + 16*a^9*b^12*c^14*d^7 - 16*a^10*b^11*c^13*d^8 - 16*a^13*b^8*c^10*d^11 + 16*a^14*b^7*c^9*d^12)*1i))/((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*(x*(4*a^9*b^11*c^11*d^9 + 4*a^11*b^9*c^9*d^11) - (-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(3/4)*((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*(4096*a^13*b^12*c^21*d^4 - 20480*a^14*b^11*c^20*d^5 + 40960*a^15*b^10*c^19*d^6 - 45056*a^16*b^9*c^18*d^7 + 40960*a^17*b^8*c^17*d^8 - 45056*a^18*b^7*c^16*d^9 + 40960*a^19*b^6*c^15*d^10 - 20480*a^20*b^5*c^14*d^11 + 4096*a^21*b^4*c^13*d^12)*1i + x*(1024*a^11*b^13*c^20*d^4 - 4096*a^12*b^12*c^19*d^5 + 6144*a^13*b^11*c^18*d^6 - 4096*a^14*b^10*c^17*d^7 + 1024*a^15*b^9*c^16*d^8 + 1024*a^16*b^8*c^15*d^9 - 4096*a^17*b^7*c^14*d^10 + 6144*a^18*b^6*c^13*d^11 - 4096*a^19*b^5*c^12*d^12 + 1024*a^20*b^4*c^11*d^13))*1i + 16*a^9*b^12*c^14*d^7 - 16*a^10*b^11*c^13*d^8 - 16*a^13*b^8*c^10*d^11 + 16*a^14*b^7*c^9*d^12)*1i)*1i - (-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*(x*(4*a^9*b^11*c^11*d^9 + 4*a^11*b^9*c^9*d^11) + (-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(3/4)*((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*(4096*a^13*b^12*c^21*d^4 - 20480*a^14*b^11*c^20*d^5 + 40960*a^15*b^10*c^19*d^6 - 45056*a^16*b^9*c^18*d^7 + 40960*a^17*b^8*c^17*d^8 - 45056*a^18*b^7*c^16*d^9 + 40960*a^19*b^6*c^15*d^10 - 20480*a^20*b^5*c^14*d^11 + 4096*a^21*b^4*c^13*d^12)*1i - x*(1024*a^11*b^13*c^20*d^4 - 4096*a^12*b^12*c^19*d^5 + 6144*a^13*b^11*c^18*d^6 - 4096*a^14*b^10*c^17*d^7 + 1024*a^15*b^9*c^16*d^8 + 1024*a^16*b^8*c^15*d^9 - 4096*a^17*b^7*c^14*d^10 + 6144*a^18*b^6*c^13*d^11 - 4096*a^19*b^5*c^12*d^12 + 1024*a^20*b^4*c^11*d^13))*1i + 16*a^9*b^12*c^14*d^7 - 16*a^10*b^11*c^13*d^8 - 16*a^13*b^8*c^10*d^11 + 16*a^14*b^7*c^9*d^12)*1i)*1i))*(-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024

```
*a^10*b*c*d^3))^(1/4) - 1/(3*a*c*x^3)
```

3.786 $\int \frac{1}{x^6(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 479

$$\begin{aligned}\int \frac{1}{x^6\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = & -\frac{1}{5acx^5}+\frac{bc+ad}{a^2c^2x}-\frac{b^{9/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}(bc-ad)} \\ & +\frac{b^{9/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}(bc-ad)}+\frac{d^{9/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{9/4}(bc-ad)} \\ & -\frac{d^{9/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{9/4}(bc-ad)} \\ & +\frac{b^{9/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{9/4}(bc-ad)} \\ & -\frac{b^{9/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{9/4}(bc-ad)} \\ & -\frac{d^{9/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{9/4}(bc-ad)} \\ & +\frac{d^{9/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{9/4}(bc-ad)}\end{aligned}$$

```
[Out] -1/5/a/c/x^5+(a*d+b*c)/a^2/c^2/x+1/4*b^(9/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^
(1/4))/a^(9/4)/(-a*d+b*c)*2^(1/2)+1/4*b^(9/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^
(1/4))/a^(9/4)/(-a*d+b*c)*2^(1/2)-1/4*d^(9/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c
^(1/4))/c^(9/4)/(-a*d+b*c)*2^(1/2)-1/4*d^(9/4)*arctan(1+d^(1/4)*x*2^(1/2)/c
```

```
^(1/4))/c^(9/4)/(-a*d+b*c)*2^(1/2)+1/8*b^(9/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2
)+a^(1/2)+x^2*b^(1/2))/a^(9/4)/(-a*d+b*c)*2^(1/2)-1/8*b^(9/4)*ln(a^(1/4)*b^
(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(9/4)/(-a*d+b*c)*2^(1/2)-1/8*d^(9/4)
*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(9/4)/(-a*d+b*c)*2^(1
/2)+1/8*d^(9/4)*ln(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(9/4)/(
-a*d+b*c)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {491, 597, 598, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{1}{x^6 (a+bx^4)(c+dx^4)} dx = -\frac{b^{9/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}(bc-ad)} + \frac{b^{9/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{9/4}(bc-ad)} + \frac{b^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{9/4}(bc-ad)} - \frac{b^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{9/4}(bc-ad)} + \frac{ad+bc}{a^2c^2x} + \frac{d^{9/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{9/4}(bc-ad)} - \frac{d^{9/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}c^{9/4}(bc-ad)} - \frac{d^{9/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{9/4}(bc-ad)} + \frac{d^{9/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{9/4}(bc-ad)} - \frac{1}{5acx^5}$$

```
[In] Int[1/(x^6*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/5*1/(a*c*x^5) + (b*c + a*d)/(a^2*c^2*x) - (b^(9/4)*ArcTan[1 - (Sqrt[2]*b
^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)) + (b^(9/4)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)) + (d^(9/4)*Arc
Tan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(9/4)*(b*c - a*d)) - (d^
(9/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(9/4)*(b*c - a*
d)) + (b^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*S
qrt[2]*a^(9/4)*(b*c - a*d)) - (b^(9/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(9/4)*(b*c - a*d)) - (d^(9/4)*Log[Sqrt[c]
- Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(9/4)*(b*c - a*d))
 + (d^(9/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt
[2]*c^(9/4)*(b*c - a*d))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{5acx^5}+\frac{\int \frac{-5(bc+ad)-5bdx^4}{x^2(a+bx^4)(c+dx^4)}\,dx}{5ac}\\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{a^2c^2x}-\frac{\int \frac{x^2\left(-5\left(b^2c^2+abcd+a^2d^2\right)-5bd(bc+ad)x^4\right)}{(a+bx^4)(c+dx^4)}\,dx}{5a^2c^2}\\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{a^2c^2x}-\frac{\int \left(-\frac{5b^3c^2x^2}{(bc-ad)(a+bx^4)}-\frac{5a^2d^3x^2}{(-bc+ad)(c+dx^4)}\right)dx}{5a^2c^2}\\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{a^2c^2x}+\frac{b^3\int \frac{x^2}{a+bx^4}\,dx}{a^2(bc-ad)}-\frac{d^3\int \frac{x^2}{c+dx^4}\,dx}{c^2(bc-ad)}\\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{a^2c^2x}-\frac{b^{5/2}\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2a^2(bc-ad)}+\frac{b^{5/2}\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2a^2(bc-ad)}\\
&\quad+\frac{d^{5/2}\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{2c^2(bc-ad)}-\frac{d^{5/2}\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{2c^2(bc-ad)}
\end{aligned}$$

$$=-\frac{1}{5acx^5}+\frac{bc+ad}{a^2c^2x}+\frac{b^2\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}dx}{4a^2(bc-ad)}+\frac{b^2\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}dx}{4a^2(bc-ad)}+\frac{b^{9/4}\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}dx}{4\sqrt{2}a^{9/4}(bc-ad)}+\frac{b^{9/4}\int\frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}dx}{4\sqrt{2}a^{9/4}(bc-ad)}-\frac{d^2\int\frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}dx}{4c^2(bc-ad)}-\frac{d^2\int\frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}dx}{4c^2(bc-ad)}-\frac{d^{9/4}\int\frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}+2x}{-\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}-x^2}dx}{4\sqrt{2}c^{9/4}(bc-ad)}-\frac{d^{9/4}\int\frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}-2x}{-\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}-x^2}dx}{4\sqrt{2}c^{9/4}(bc-ad)}$$

$$=-\frac{1}{5acx^5}+\frac{bc+ad}{a^2c^2x}+\frac{b^{9/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{9/4}(bc-ad)}-\frac{b^{9/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{9/4}(bc-ad)}-\frac{d^{9/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{9/4}(bc-ad)}+\frac{d^{9/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{9/4}(bc-ad)}+\frac{b^{9/4}\text{Subst}\left(\int\frac{1}{-1-x^2}dx,x,1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}(bc-ad)}-\frac{b^{9/4}\text{Subst}\left(\int\frac{1}{-1-x^2}dx,x,1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}(bc-ad)}-\frac{d^{9/4}\text{Subst}\left(\int\frac{1}{-1-x^2}dx,x,1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{9/4}(bc-ad)}+\frac{d^{9/4}\text{Subst}\left(\int\frac{1}{-1-x^2}dx,x,1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{9/4}(bc-ad)}$$

$$=-\frac{1}{5acx^5}+\frac{bc+ad}{a^2c^2x}-\frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}(bc-ad)}+\frac{b^{9/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}(bc-ad)}+\frac{d^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{9/4}(bc-ad)}-\frac{d^{9/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{9/4}(bc-ad)}+\frac{b^{9/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{9/4}(bc-ad)}-\frac{b^{9/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{9/4}(bc-ad)}-\frac{d^{9/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{9/4}(bc-ad)}+\frac{d^{9/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{9/4}(bc-ad)}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 428, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^6 (a+bx^4)(c+dx^4)}\,dx$$
$$= \frac{\frac{8b}{a}-\frac{8d}{c}-\frac{40b^2x^4}{a^2}+\frac{40d^2x^4}{c^2}+\frac{10\sqrt{2}b^{9/4}x^5\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{9/4}}-\frac{10\sqrt{2}b^{9/4}x^5\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{9/4}}-\frac{10\sqrt{2}d^{9/4}x^5\arctan\left(1-\frac{\sqrt{2}}{}\right.}{c^{9/4}}}{}$$

```
[In] Integrate[1/(x^6*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((8*b)/a - (8*d)/c - (40*b^2*x^4)/a^2 + (40*d^2*x^4)/c^2 + (10*Sqrt[2]*b^(9
/4)*x^5*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(9/4) - (10*Sqrt[2]*b^(9
/4)*x^5*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(9/4) - (10*Sqrt[2]*d^(9
/4)*x^5*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(9/4) + (10*Sqrt[2]*d^(9
/4)*x^5*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(9/4) - (5*Sqrt[2]*b^(9/
4)*x^5*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(9/4) + (5
*Sqrt[2]*b^(9/4)*x^5*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]
)/a^(9/4) + (5*Sqrt[2]*d^(9/4)*x^5*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x
+ Sqrt[d]*x^2])/c^(9/4) - (5*Sqrt[2]*d^(9/4)*x^5*Log[Sqrt[c] + Sqrt[2]*c^(1
/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(9/4))/(40*(-(b*c) + a*d)*x^5)
```

Maple [A] (verified)

Time = 5.00 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.54

method	result
default	$-\frac{1}{5acx^5}-\frac{-ad-bc}{a^2c^2x}-\frac{b^2\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a^2(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{d^2\sqrt{2}\left(\ln\left(\frac{x^2-(}{x^2+(}\right.\right.}{}$
risch	Expression too large to display

```
[In] int(1/x^6/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/a/c/x^5-1/a^2/c^2*(-a*d-b*c)/x-1/8*b^2/a^2/(a*d-b*c)/(a/b)^(1/4)*2^(1/
2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(
a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)
*x-1))+1/8*d^2/c^2/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*(ln((x^2-(c/d)^(1/4)*x*2^(
1/2)+(c/d)^(1/2))/(x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)
/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.97 (sec) , antiderivative size = 1526, normalized size of antiderivative = 3.19

$$\int \frac{1}{x^6 (a + bx^4) (c + dx^4)} dx = \text{Too large to display}$$

```
[In] integrate(1/x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/20*(5*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12
*b*c*d^3 + a^13*d^4))^(1/4)*a^2*c^2*x^5*log(b^7*x + (a^7*b^3*c^3 - 3*a^8*b^
2*c^2*d + 3*a^9*b*c*d^2 - a^10*d^3)*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d +
 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(3/4)) - 5*(-b^9/(a^9*b^4
*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^
(1/4)*a^2*c^2*x^5*log(b^7*x - (a^7*b^3*c^3 - 3*a^8*b^2*c^2*d + 3*a^9*b*c*d^
2 - a^10*d^3)*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 -
4*a^12*b*c*d^3 + a^13*d^4))^(3/4)) + 5*I*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^
3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(1/4)*a^2*c^2*x^5*lo
g(b^7*x - (I*a^7*b^3*c^3 - 3*I*a^8*b^2*c^2*d + 3*I*a^9*b*c*d^2 - I*a^10*d^3
)*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d
^3 + a^13*d^4))^(3/4)) - 5*I*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11
*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(1/4)*a^2*c^2*x^5*log(b^7*x - (-
I*a^7*b^3*c^3 + 3*I*a^8*b^2*c^2*d - 3*I*a^9*b*c*d^2 + I*a^10*d^3)*(-b^9/(a^
9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d
^4))^(3/4)) - 5*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a
^3*b*c^10*d^3 + a^4*c^9*d^4))^(1/4)*a^2*c^2*x^5*log(d^7*x + (b^3*c^10 - 3*a
*b^2*c^9*d + 3*a^2*b*c^8*d^2 - a^3*c^7*d^3)*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*
d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(3/4)) + 5*(-d^9/
(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^
9*d^4))^(1/4)*a^2*c^2*x^5*log(d^7*x - (b^3*c^10 - 3*a*b^2*c^9*d + 3*a^2*b*c
^8*d^2 - a^3*c^7*d^3)*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2
 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(3/4)) - 5*I*(-d^9/(b^4*c^13 - 4*a*b^3*
c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(1/4)*a^2*c^
2*x^5*log(d^7*x - (I*b^3*c^10 - 3*I*a*b^2*c^9*d + 3*I*a^2*b*c^8*d^2 - I*a^3
*c^7*d^3)*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c
^10*d^3 + a^4*c^9*d^4))^(3/4)) + 5*I*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a
^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(1/4)*a^2*c^2*x^5*log(d^
7*x - (-I*b^3*c^10 + 3*I*a*b^2*c^9*d - 3*I*a^2*b*c^8*d^2 + I*a^3*c^7*d^3)*(
-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a
^4*c^9*d^4))^(3/4)) + 20*(b*c + a*d)*x^4 - 4*a*c)/(a^2*c^2*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (a + bx^4) (c + dx^4)} \, dx = \text{Timed out}$$

```
[In] integrate(1/x**6/(b*x**4+a)/(d*x**4+c),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 405, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^6 (a + bx^4) (c + dx^4)} \, dx = \frac{b^3 \left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(\sqrt{b}x^2-\sqrt{2}\right.}{a^{\frac{1}{4}}b^{\frac{3}{4}}} \right.}{8(a^2bc - a^3d)} - \frac{d^3 \left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}} - \frac{\sqrt{2}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(\sqrt{d}x^2-\right.}{c} \right.}{8(bc^3 - ac^2d)} + \frac{5(bc + ad)x^4 - ac}{5a^2c^2x^5}$$

```
[In] integrate(1/x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
[Out] 1/8*b^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4
))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arcta
n(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b))
)/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/
4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*x^2 - sqrt(
2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a^2*b*c - a^3*d) - 1/8*
d^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/s
qrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(1/
2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(s
qrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d
```

```
^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(sqrt(d)*x^2 - sqrt(2)*c
^(1/4)*d^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b*c^3 - a*c^2*d) + 1/5*(5*(
b*c + a*d)*x^4 - a*c)/(a^2*c^2*x^5)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^6 (a+bx^4)(c+dx^4)} dx = \frac{(ab^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^3bc-\sqrt{2}a^4d\right)} + \frac{(ab^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^3bc-\sqrt{2}a^4d\right)} - \frac{(cd^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^4-\sqrt{2}ac^3d\right)} - \frac{(cd^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^4-\sqrt{2}ac^3d\right)} - \frac{(ab^3)^{\frac{3}{4}} \log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}a^3bc-\sqrt{2}a^4d\right)} + \frac{(ab^3)^{\frac{3}{4}} \log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}a^3bc-\sqrt{2}a^4d\right)} + \frac{(cd^3)^{\frac{3}{4}} \log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^4-\sqrt{2}ac^3d\right)} - \frac{(cd^3)^{\frac{3}{4}} \log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^4-\sqrt{2}ac^3d\right)} + \frac{5\,bcx^4+5\,adx^4-ac}{5\,a^2c^2x^5}$$

```
[In] integrate(1/x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4
))/(sqrt(2)*a^3*b*c - sqrt(2)*a^4*d) + 1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)
*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^3*b*c - sqrt(2)*a^4*d)
 - 1/2*(c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(
```

```
1/4))/(sqrt(2)*b*c^4 - sqrt(2)*a*c^3*d) - 1/2*(c*d^3)^(3/4)*arctan(1/2*sqrt
(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^4 - sqrt(2)*a*c^3
*d) - 1/4*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(
2)*a^3*b*c - sqrt(2)*a^4*d) + 1/4*(a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/b)^(
1/4) + sqrt(a/b))/(sqrt(2)*a^3*b*c - sqrt(2)*a^4*d) + 1/4*(c*d^3)^(3/4)*log
(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^4 - sqrt(2)*a*c^3*d)
 - 1/4*(c*d^3)^(3/4)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*
b*c^4 - sqrt(2)*a*c^3*d) + 1/5*(5*b*c*x^4 + 5*a*d*x^4 - a*c)/(a^2*c^2*x^5)
```

Mupad [B] (verification not implemented)

Time = 10.33 (sec) , antiderivative size = 4547, normalized size of antiderivative = 9.49

$$\int \frac{1}{x^6 (a + bx^4) (c + dx^4)} dx = \text{Too large to display}$$

```
[In] int(1/(x^6*(a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] - 2*atan((1024*a^11*b^10*c^13*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 102
4*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) + 4*a^
11*b^6*d^9*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d +
1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(1/4) + 1024*a^21*c^3*d^10*x*(-
b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c
^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) - 4096*a^12*b^9*c^12*d*x*(-b^9/(256*a^13
*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024
*a^12*b*c*d^3))^(5/4) - 4096*a^20*b*c^4*d^9*x*(-b^9/(256*a^13*d^4 + 256*a^9
*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3)
)^(5/4) + 4*a^8*b^9*c^3*d^6*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*
a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(1/4) + 6144*a
^13*b^8*c^11*d^2*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^
3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) - 4096*a^14*b^7*c^1
0*d^3*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*
a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) + 1024*a^15*b^6*c^9*d^4*x*(-b^
9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2
*d^2 - 1024*a^12*b*c*d^3))^(5/4) + 1024*a^17*b^4*c^7*d^6*x*(-b^9/(256*a^13*
d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*
a^12*b*c*d^3))^(5/4) - 4096*a^18*b^3*c^6*d^7*x*(-b^9/(256*a^13*d^4 + 256*a^
9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3
))^(5/4) + 6144*a^19*b^2*c^5*d^8*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 -
1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4))/(b
^16*c^8 + a^8*b^8*d^8 + a^7*b^9*c*d^7 + a^2*b^14*c^6*d^2 + a^3*b^13*c^5*d^3
 + a^4*b^12*c^4*d^4 + a^5*b^11*c^3*d^5 + a^6*b^10*c^2*d^6 + a*b^15*c^7*d))*
(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2
*c^2*d^2 - 1024*a^12*b*c*d^3))^(1/4) - atan((a^11*b^10*c^13*x*(-b^9/(256*a^
13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 10
```

24*a^12*b*c*d^3))^(5/4)*1024i + a^11*b^6*d^9*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(1/4)*4i + a^21*c^3*d^10*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*1024i - a^12*b^9*c^12*d*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*4096i - a^20*b*c^4*d^9*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*4096i + a^8*b^9*c^3*d^6*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(1/4)*4i + a^13*b^8*c^11*d^2*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*6144i - a^14*b^7*c^10*d^3*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*4096i + a^15*b^6*c^9*d^4*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*1024i + a^17*b^4*c^7*d^6*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*1024i - a^18*b^3*c^6*d^7*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*4096i + a^19*b^2*c^5*d^8*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*6144i)/(b^16*c^8 + a^8*b^8*d^8 + a^7*b^9*c*d^7 + a^2*b^14*c^6*d^2 + a^3*b^13*c^5*d^3 + a^4*b^12*c^4*d^4 + a^5*b^11*c^3*d^5 + a^6*b^10*c^2*d^6 + a*b^15*c^7*d))*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(1/4)*2i - 2*atan((4*b^9*c^11*d^6*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(1/4) + 1024*a^3*b^10*c^21*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4) + 1024*a^13*c^11*d^10*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4) - 4096*a^4*b^9*c^20*d*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4) - 4096*a^12*b*c^12*d^9*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4) + 4*a^3*b^6*c^8*d^9*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(1/4) + 6144*a^5*b^8*c^19*d^2*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4) - 4096*a^6*b^7*c^18*d^3*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4) + 1024*a^7*b^6*c^17*d^4*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4) + 1024*a^9*b^4*c^15*d^6*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4) - 4096*a^10*b^3*c^14*d^7*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4) + 6144*a^11*b^2*c^13*d^8*x*(-d^9/(256*b^4*c^13 + 256*a^

```
4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d
))^(5/4))/(a^8*d^16 + b^8*c^8*d^8 + a*b^7*c^7*d^9 + a^2*b^6*c^6*d^10 + a^3*
b^5*c^5*d^11 + a^4*b^4*c^4*d^12 + a^5*b^3*c^3*d^13 + a^6*b^2*c^2*d^14 + a^7
*b*c*d^15))*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1
536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(1/4) - atan((b^9*c^11*d^6*x*(-d
^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^1
1*d^2 - 1024*a*b^3*c^12*d))^(1/4)*4i + a^3*b^10*c^21*x*(-d^9/(256*b^4*c^13
+ 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^
3*c^12*d))^(5/4)*1024i + a^13*c^11*d^10*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9
*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5
/4)*1024i - a^4*b^9*c^20*d*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a
^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4)*4096i - a
^12*b*c^12*d^9*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^
3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4)*4096i + a^3*b^6*c^8*d
^9*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2
*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(1/4)*4i + a^5*b^8*c^19*d^2*x*(-d^9/(25
6*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2
- 1024*a*b^3*c^12*d))^(5/4)*6144i - a^6*b^7*c^18*d^3*x*(-d^9/(256*b^4*c^13
+ 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^
3*c^12*d))^(5/4)*4096i + a^7*b^6*c^17*d^4*x*(-d^9/(256*b^4*c^13 + 256*a^4*c
^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^
(5/4)*1024i + a^9*b^4*c^15*d^6*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 10
24*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4)*1024i
 - a^10*b^3*c^14*d^7*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c
^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4)*4096i + a^11*b^
2*c^13*d^8*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 +
1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4)*6144i)/(a^8*d^16 + b^8*c^
8*d^8 + a*b^7*c^7*d^9 + a^2*b^6*c^6*d^10 + a^3*b^5*c^5*d^11 + a^4*b^4*c^4*d
^12 + a^5*b^3*c^3*d^13 + a^6*b^2*c^2*d^14 + a^7*b*c*d^15))*(-d^9/(256*b^4*c
^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*
a*b^3*c^12*d))^(1/4)*2i - (1/(5*a*c) - (x^4*(a*d + b*c))/(a^2*c^2))/x^5
```

3.787 $\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx = -\frac{a\sqrt{c+dx^4}}{2b^2}+\frac{(c+dx^4)^{3/2}}{6bd}+\frac{a\sqrt{bc-ad}\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}}$$

```
[Out] 1/6*(d*x^4+c)^(3/2)/b/d+1/2*a*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1
/2))*(-a*d+b*c)^(1/2)/b^(5/2)-1/2*a*(d*x^4+c)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 81, 52, 65, 214}

$$\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{a\sqrt{bc-ad}\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}}-\frac{a\sqrt{c+dx^4}}{2b^2}+\frac{(c+dx^4)^{3/2}}{6bd}$$

```
[In] Int[(x^7*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] -1/2*(a*Sqrt[c + d*x^4])/b^2 + (c + d*x^4)^(3/2)/(6*b*d) + (a*Sqrt[b*c - a*
d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*b^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{x\sqrt{c+dx}}{a+bx}\,dx, x, x^4\right) \\
&= \frac{(c+dx^4)^{3/2}}{6bd} - \frac{a\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^4\right)}{4b} \\
&= -\frac{a\sqrt{c+dx^4}}{2b^2} + \frac{(c+dx^4)^{3/2}}{6bd} - \frac{(a(bc-ad))\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4b^2} \\
&= -\frac{a\sqrt{c+dx^4}}{2b^2} + \frac{(c+dx^4)^{3/2}}{6bd} - \frac{(a(bc-ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2b^2d} \\
&= -\frac{a\sqrt{c+dx^4}}{2b^2} + \frac{(c+dx^4)^{3/2}}{6bd} + \frac{a\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

$$\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{\sqrt{c+dx^4}(-3ad+b(c+dx^4))}{6b^2d} + \frac{a\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{2b^{5/2}}$$

```
[In] Integrate[(x^7*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (Sqrt[c + d*x^4]*(-3*a*d + b*(c + d*x^4)))/(6*b^2*d) + (a*Sqrt[-(b*c) + a*d
]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[-(b*c) + a*d]])/(2*b^(5/2))
```

Maple [A] (verified)

Time = 6.01 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{4}+c}\,\left(-b d\,x^{4}+3 a d-b c\right)}{3}+\frac{a d (a d-b c) \arctan\left(\frac{b \sqrt{d\,x^{4}+c}}{\sqrt{(a d-b c) b}}\right)}{\sqrt{(a d-b c) b}}}{2 b^{2} d}$
risch	$-\frac{\left(-b d\,x^{4}+3 a d-b c\right) \sqrt{d\,x^{4}+c}}{6 d\,b^{2}}+\frac{(a d-b c) a \left(-\frac{\ln\left(\frac{-\frac{2(a d-b c)}{b}-\frac{2 d \sqrt{-a b}\,\left(x^{2}+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}}\,\sqrt{d \left(x^{2}+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a\cdots}}{\cdots}}}{x^{2}+\frac{\sqrt{-a b}}{b}}\right)}{4 b \sqrt{-\frac{a d-b c}{b}}}\cdots\right)}{\cdots}$
elliptic	$\frac{\left(d\,x^{4}+c\right)^{\frac{3}{2}}}{6 b d}-\frac{a \left(\sqrt{d \left(x^{2}+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\,\left(x^{2}+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}-\frac{\sqrt{d}\,\sqrt{-a b}\,\ln\left(\frac{-\frac{d \sqrt{-a b}}{b}+d \left(x^{2}+\frac{\sqrt{-a b}}{b}\right)}{\sqrt{d}}+\sqrt{d \left(x^{2}+\frac{\sqrt{-a b}}{b}\right)\cdots}\right)}{b}\cdots\right)}{\cdots}$
default	$\frac{\left(d\,x^{4}+c\right)^{\frac{3}{2}}}{6 b d}-\frac{a \left(\frac{\sqrt{d \left(x^{2}+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\,\left(x^{2}+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}-\frac{\sqrt{d}\,\sqrt{-a b}\,\ln\left(\frac{-\frac{d \sqrt{-a b}}{b}+d \left(x^{2}+\frac{\sqrt{-a b}}{b}\right)}{\sqrt{d}}+\sqrt{d \left(x^{2}+\frac{\sqrt{-a b}}{b}\right)^{2}\cdots}\right)}{b}\cdots}{\cdots}\cdots\right)}{\cdots}$

```
[In] int(x^7*(d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b^2*(-1/3*(d*x^4+c)^(1/2)*(-b*d*x^4+3*a*d-b*c)+a*d*(a*d-b*c)/((a*d-b*c)
*b)^(1/2)*arctan(b*(d*x^4+c)^(1/2)/((a*d-b*c)*b)^(1/2)))/d
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.10

$$\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx = \left[\frac{3\,ad\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^4+2\,bc-ad+2\,\sqrt{dx^4+c}b\sqrt{\frac{bc-ad}{b}}}{bx^4+a}\right)+2\left(bdx^4+bc-3\,ad\right)\sqrt{dx^4+c}}{12\,b^2d}, \frac{3\,ad\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{}}{}\right.}{}\right.$$

```
[In] integrate(x^7*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a*d*sqrt((b*c - a*d)/b)*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4
+ c)*b*sqrt((b*c - a*d)/b))/(b*x^4 + a)) + 2*(b*d*x^4 + b*c - 3*a*d)*sqrt(d
*x^4 + c))/(b^2*d), 1/6*(3*a*d*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^4 + c)
*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (b*d*x^4 + b*c - 3*a*d)*sqrt(d*x^4 +
 c))/(b^2*d)]
```

Sympy [A] (verification not implemented)

Time = 5.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.31

$$\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx = \begin{cases} \frac{2\left(-\frac{ad^2\sqrt{c+dx^4}}{4b^2}+\frac{ad^2(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4b^3\sqrt{\frac{ad-bc}{b}}}+\frac{d\left(c+dx^4\right)^{\frac{3}{2}}}{12b}\right)}{d^2} & \text{for } d\neq 0 \\ \sqrt{c}\left(-\frac{a\left(\begin{cases}\frac{x^4}{a} & \text{for } b=0\\ \frac{\log\left(a+bx^4\right)}{b} & \text{otherwise}\end{cases}\right)}{4b}+\frac{x^4}{4b}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(x**7*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

```
[Out] Piecewise((2*(-a*d**2*sqrt(c + d*x**4)/(4*b**2) + a*d**2*(a*d - b*c)*atan(s
qrt(c + d*x**4)/sqrt((a*d - b*c)/b))/(4*b**3*sqrt((a*d - b*c)/b)) + d*(c +
d*x**4)**(3/2)/(12*b))/d**2, Ne(d, 0)), (sqrt(c)*(-a*Piecewise((x**4/a, Eq(
b, 0)), (log(a + b*x**4)/b, True))/(4*b) + x**4/(4*b)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^7*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.03

$$\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx = -\frac{(abc-a^2d)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}b^2} + \frac{(dx^4+c)^{\frac{3}{2}}b^2d^2 - 3\sqrt{dx^4+c}abd^3}{6b^3d^3}$$

```
[In] integrate(x^7*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*(a*b*c - a^2*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-
b^2*c + a*b*d)*b^2) + 1/6*((d*x^4 + c)^(3/2)*b^2*d^2 - 3*sqrt(d*x^4 + c)*a*
b*d^3)/(b^3*d^3)
```

Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.94

$$\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{(d\,x^4+c)^{3/2}}{6\,b\,d} - \frac{a\,\sqrt{d\,x^4+c}}{2\,b^2} + \frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^4+c}\,\sqrt{a\,d-b\,c}}{a^2\,d-a\,b\,c}\right)\,\sqrt{a\,d-b\,c}}{2\,b^{5/2}}$$

```
[In] int((x^7*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

```
[Out] (c + d*x^4)^(3/2)/(6*b*d) - (a*(c + d*x^4)^(1/2))/(2*b^2) + (a*atan((a*b^(1
/2)*(c + d*x^4)^(1/2)*(a*d - b*c)^(1/2))/(a^2*d - a*b*c))*(a*d - b*c)^(1/2)
)/(2*b^(5/2))
```

3.788 $\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 120

$$\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{x^2\sqrt{c+dx^4}}{4b} - \frac{\sqrt{a}\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b^2} + \frac{(bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^2\sqrt{d}}$$

```
[Out] 1/4*(-2*a*d+b*c)*arctanh(x^2*d^(1/2)/(d*x^4+c)^(1/2))/b^2/d^(1/2)-1/2*arcta
n(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))*a^(1/2)*(-a*d+b*c)^(1/2)/b^
2+1/4*x^2*(d*x^4+c)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {476, 489, 537, 223, 212, 385, 211}

$$\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx = -\frac{\sqrt{a}\sqrt{bc-ad}\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b^2} + \frac{(bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^2\sqrt{d}} + \frac{x^2\sqrt{c+dx^4}}{4b}$$

```
[In] Int[(x^5*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (x^2*Sqrt[c + d*x^4])/(4*b) - (Sqrt[a]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a
*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(2*b^2) + ((b*c - 2*a*d)*ArcTanh[(Sqrt
[d]*x^2)/Sqrt[c + d*x^4]])/(4*b^2*Sqrt[d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x^2\sqrt{c+dx^2}}{a+bx^2}\,dx, x, x^2\right)$$

$$= \frac{x^2\sqrt{c+dx^4}}{4b} - \frac{\text{Subst}\left(\int \frac{ac+(-bc+2ad)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4b}$$

$$= \frac{x^2\sqrt{c+dx^4}}{4b} + \frac{(bc-2ad)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4b^2} - \frac{(a(bc-ad))\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{2b^2}$$

$$= \frac{x^2\sqrt{c+dx^4}}{4b} + \frac{(bc-2ad)\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{4b^2} - \frac{(a(bc-ad))\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{2b^2}$$

$$= \frac{x^2\sqrt{c+dx^4}}{4b} - \frac{\sqrt{a}\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b^2} + \frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^2\sqrt{d}}$$

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.18

$$\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx$$
$$= \frac{b\sqrt{d}x^2\sqrt{c+dx^4} - 2\sqrt{a}\sqrt{d}\sqrt{bc-ad}\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right) + (bc-2ad)\log\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{4b^2\sqrt{d}}$$

```
[In] Integrate[(x^5*Sqrt[c + d*x^4])/(a + b*x^4),x]
[Out] (b*Sqrt[d]*x^2*Sqrt[c + d*x^4] - 2*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]*ArcTan[(
a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b*c - a*d]
)] + (b*c - 2*a*d)*Log[Sqrt[d]*x^2 + Sqrt[c + d*x^4]])/(4*b^2*Sqrt[d])
```

Maple [A] (verified)

Time = 5.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$-\frac{\left(-d^{\frac{3}{2}} a^{2}+\sqrt{d}\, a b c\right) \operatorname{arctanh}\left(\frac{\sqrt{d x^{4}+c}\, a}{x^{2} \sqrt{(a d-b c) a}}\right)+\sqrt{(a d-b c) a}\, \left(\left(a d-\frac{b c}{2}\right) \operatorname{arctanh}\left(\frac{\sqrt{d x^{4}+c}}{x^{2} \sqrt{d}}\right)-\frac{\sqrt{d x^{4}+c}\, b x^{2} \sqrt{d}}{2}\right)}{2 \sqrt{(a d-b c) a}\, \sqrt{d}\, b^{2}}$
risch	$\frac{x^{2} \sqrt{d x^{4}+c}}{4 b}-\frac{\frac{(2 a d-b c) \ln\left(x^{2} \sqrt{d}+\sqrt{d x^{4}+c}\right)}{2 b \sqrt{d}}-\frac{2 (a d-b c) a \left(\frac{\ln\left(\frac{-\frac{2 (a d-b c)}{b}-\frac{2 d \sqrt{-a b}\, \left(x^{2}+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}}\, \sqrt{d \left(x^{2}+\frac{\sqrt{-a b}}{b}\right)^{2} \cdots}}{x^{2}+\frac{\sqrt{-a b}}{b}}\right)}{4 \sqrt{-a b}\, \sqrt{-\frac{a d-b c}{b}}} \cdots\right)}{\cdots}}{\cdots}$
default	$\frac{\frac{x^{2} \sqrt{d x^{4}+c}}{4}+\frac{c \ln\left(x^{2} \sqrt{d}+\sqrt{d x^{4}+c}\right)}{4 \sqrt{d}}}{b}-\frac{a \left(-\frac{\sqrt{d \left(x^{2}+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\, \left(x^{2}+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}-\frac{\sqrt{d}\, \sqrt{-a b}\, \ln\left(\frac{-\frac{d \sqrt{-a b}}{b}+d \left(x^{2}+\cdots\right)}{\sqrt{d}} \cdots\right)}{\cdots}}{\cdots} \cdots\right)}{\cdots}$
elliptic	$\frac{\frac{x^{2} \sqrt{d x^{4}+c}}{2}+\frac{c \ln\left(x^{2} \sqrt{d}+\sqrt{d x^{4}+c}\right)}{2 \sqrt{d}}}{2 b}+\frac{a \left(\sqrt{d \left(x^{2}+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\, \left(x^{2}+\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}-\frac{\sqrt{d}\, \sqrt{-a b}\, \ln\left(\frac{-\frac{d \sqrt{-a b}}{b}+d \left(x^{2}+\cdots\right)}{\sqrt{d}} \cdots\right)}{\cdots} \cdots\right)}{\cdots}$

```
[In] int(x^5*(d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/((a*d-b*c)*a)^(1/2)/d^(1/2)*((-d^(3/2)*a^2+d^(1/2)*a*b*c)*arctanh((d*x
^4+c)^(1/2)/x^2*a/((a*d-b*c)*a)^(1/2))+((a*d-b*c)*a)^(1/2)*((a*d-1/2*b*c)*a
rctanh((d*x^4+c)^(1/2)/x^2/d^(1/2))-1/2*(d*x^4+c)^(1/2)*b*x^2*d^(1/2)))/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 714, normalized size of antiderivative = 5.95

$$\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx$$
$$= \left[\frac{2\sqrt{dx^4+c}bdx^2-(bc-2\,ad)\sqrt{d}\log\left(-2\,dx^4+2\,\sqrt{dx^4+c}\sqrt{d}x^2-c\right)+\sqrt{-abc+a^2d}d\log\left(\frac{(b^2c^2-8\,abcd+8}{}\right.}{8\,b^2d}\right.$$

```
[In] integrate(x^5*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(d*x^4 + c)*b*d*x^2 - (b*c - 2*a*d)*sqrt(d)*log(-2*d*x^4 + 2*sq
rt(d*x^4 + c)*sqrt(d)*x^2 - c) + sqrt(-a*b*c + a^2*d)*d*log(((b^2*c^2 - 8*a
*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c
 - 2*a*d)*x^6 - a*c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2
*a*b*x^4 + a^2)))/(b^2*d), 1/8*(2*sqrt(d*x^4 + c)*b*d*x^2 - 2*(b*c - 2*a*d)
*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) + sqrt(-a*b*c + a^2*d)*d*log
(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a
^2*c^2 - 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*
d))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/(b^2*d), 1/8*(2*sqrt(d*x^4 + c)*b*d*x^2 -
 2*sqrt(a*b*c - a^2*d)*d*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 +
c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2))
 - (b*c - 2*a*d)*sqrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c))
/(b^2*d), 1/4*(sqrt(d*x^4 + c)*b*d*x^2 - (b*c - 2*a*d)*sqrt(-d)*arctan(sqrt
(-d)*x^2/sqrt(d*x^4 + c)) - sqrt(a*b*c - a^2*d)*d*arctan(1/2*((b*c - 2*a*d)
*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 +
(a*b*c^2 - a^2*c*d)*x^2)))/(b^2*d)]
```

Sympy [F]

$$\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx=\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx$$

```
[In] integrate(x**5*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

```
[Out] Integral(x**5*sqrt(c + d*x**4)/(a + b*x**4), x)
```

Maxima [F]

$$\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}\,x^5}{bx^4+a}\,dx$$

```
[In] integrate(x^5*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^4 + c)*x^5/(b*x^4 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^5*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{x^5\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

```
[In] int((x^5*(c + d*x^4)^(1/2))/(a + b*x^4),x)
[Out] int((x^5*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

3.789 $\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 70

$$\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx=\frac{\sqrt{c+dx^4}}{2b}-\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}}$$

```
[Out] -1/2*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(
3/2)+1/2*(d*x^4+c)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {455, 52, 65, 214}

$$\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx=\frac{\sqrt{c+dx^4}}{2b}-\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}}$$

```
[In] Int[(x^3*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] Sqrt[c + d*x^4]/(2*b) - (Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/
Sqrt[b*c - a*d]])/(2*b^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^4\right)\\
&= \frac{\sqrt{c+dx^4}}{2b} + \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4b}\\
&= \frac{\sqrt{c+dx^4}}{2b} + \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2bd}\\
&= \frac{\sqrt{c+dx^4}}{2b} - \frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.99

$$\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{1}{2}\left(\frac{\sqrt{c+dx^4}}{b} - \frac{\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{b^{3/2}}\right)$$

```
[In] Integrate[(x^3*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (Sqrt[c + d*x^4]/b - (Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^4])/S
qrt[-(b*c) + a*d]])/b^(3/2))/2
```

Maple [A] (verified)

Time = 4.86 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$\frac{\sqrt{d\,x^4+c}-\frac{(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^4+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}}{2b}$
risch	$\frac{\sqrt{d\,x^4+c}}{2b}-\frac{(ad-bc)\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4b\sqrt{-\frac{ad-bc}{b}}} \cdots\right)}{b}$
default	$\frac{\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}-\frac{\sqrt{d}\sqrt{-ab}\ln\left(\frac{-\frac{d\sqrt{-ab}}{b}+d\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{d}}+\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}} \cdots\right)}{b} \cdots}{4b}$
elliptic	$\frac{\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}-\frac{\sqrt{d}\sqrt{-ab}\ln\left(\frac{-\frac{d\sqrt{-ab}}{b}+d\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{d}}+\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}} \cdots\right)}{b} \cdots}{4b}$

```
[In] int(x^3*(d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*((d*x^4+c)^(1/2)-(a*d-b*c)*arctan(b*(d*x^4+c)^(1/2)/((a*d-b*c)*b)^(1/
2))/((a*d-b*c)*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.23

$$\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx = \left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}b\sqrt{\frac{bc-ad}{b}}}{bx^4+a}\right)+2\,\sqrt{dx^4+c}}{4\,b}, -\frac{\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^4+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\sqrt{dx^4+c}}{2\,b}\right]$$

```
[In] integrate(x^3*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt((b*c - a*d)/b)*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*b*
sqrt((b*c - a*d)/b))/(b*x^4 + a)) + 2*sqrt(d*x^4 + c))/b, -1/2*(sqrt(-(b*c
- a*d)/b)*arctan(-sqrt(d*x^4 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - sqr
t(d*x^4 + c))/b]
```

Sympy [A] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.31

$$\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx = \begin{cases} \frac{2\left(\frac{d\sqrt{c+dx^4}}{4b}-\frac{d(ad-bc)\,\text{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4b^2\sqrt{\frac{ad-bc}{b}}}\right)}{d} & \text{for}\ d\neq 0 \\ \sqrt{c}\left(\begin{cases} \frac{x^4}{4a} & \text{for}\ b=0 \\ \frac{\log\left(4a+4bx^4\right)}{4b} & \text{otherwise} \end{cases}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(x**3*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

```
[Out] Piecewise((2*(d*sqrt(c + d*x**4)/(4*b) - d*(a*d - b*c)*atan(sqrt(c + d*x**4
)/sqrt((a*d - b*c)/b))/(4*b**2*sqrt((a*d - b*c)/b)))/d, Ne(d, 0)), (sqrt(c)
*Piecewise((x**4/(4*a), Eq(b, 0)), (log(4*a + 4*b*x**4)/(4*b), True)), True
))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{(bc-ad)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}b} + \frac{\sqrt{dx^4+c}}{2\,b}$$

```
[In] integrate(x^3*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*(b*c - a*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c
 + a*b*d)*b) + 1/2*sqrt(d*x^4 + c)/b
```

Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{\sqrt{d\,x^4+c}}{2\,b} - \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^4+c}}{\sqrt{a\,d-b\,c}}\right)\,\sqrt{a\,d-b\,c}}{2\,b^{3/2}}$$

```
[In] int((x^3*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

```
[Out] (c + d*x^4)^(1/2)/(2*b) - (atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c)^(1/
2))*(a*d - b*c)^(1/2))/(2*b^(3/2))
```

3.790 $\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 91

$$\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx=\frac{\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}b}+\frac{\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{2b}$$

```
[Out] 1/2*arctanh(x^2*d^(1/2)/(d*x^4+c)^(1/2))*d^(1/2)/b+1/2*arctan(x^2*(-a*d+b*c
)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))*(-a*d+b*c)^(1/2)/b/a^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {476, 399, 223, 212, 385, 211}

$$\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx=\frac{\sqrt{bc-ad}\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}b}+\frac{\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{2b}$$

```
[In] Int[(x*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(
2*Sqrt[a]*b) + (Sqrt[d]*ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]])/(2*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{c+dx^2}}{a+bx^2}\,dx,x,x^2\right)\\
&= \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,x,x^2\right)}{2b}+\frac{(bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{2b}\\
&= \frac{d\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x^2}{\sqrt{c+dx^4}}\right)}{2b}+\frac{(bc-ad)\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^2}{\sqrt{c+dx^4}}\right)}{2b}\\
&= \frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}b}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{2b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.18

$$\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx=\frac{\frac{\sqrt{bc-ad}\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}}+\sqrt{d}\log\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{2b}$$

```
[In] Integrate[(x*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] ((Sqrt[b*c - a*d]*ArcTan[(a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4])
)/(Sqrt[a]*Sqrt[b*c - a*d])])/Sqrt[a] + Sqrt[d]*Log[Sqrt[d]*x^2 + Sqrt[c +
d*x^4]])/(2*b)
```

Maple [A] (verified)

Time = 5.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.85

method	result
pseudoelliptic	$-\frac{\frac{(ad-bc)\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^4+c}\,a}{x^2\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}-\sqrt{d}\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^4+c}}{x^2\sqrt{d}}\right)}{2b}$
default	$-\frac{\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}-\frac{\sqrt{d}\sqrt{-ab}\,\ln\left(\frac{-\frac{d\sqrt{-ab}}{b}+d\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{d}}+\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\cdots\right.}{b}}\right.}{b}}{\cdots}$
elliptic	$-\frac{\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}-\frac{\sqrt{d}\sqrt{-ab}\,\ln\left(\frac{-\frac{d\sqrt{-ab}}{b}+d\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{d}}+\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\cdots\right.}{b}}\right.}{b}}{\cdots}$

```
[In] int(x*(d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b*((a*d-b*c)*arctanh((d*x^4+c)^(1/2)/x^2*a/((a*d-b*c)*a)^(1/2))/((a*d-
b*c)*a)^(1/2)-d^(1/2)*arctanh((d*x^4+c)^(1/2)/x^2/d^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 612, normalized size of antiderivative = 6.73

$$\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx$$
$$= \left[\frac{2\sqrt{d}\log\left(-2\,dx^4-2\sqrt{dx^4+c}\sqrt{d}x^2-c\right)+\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,a^2cd)x^4+a^2c^2+4\,((ab\right.}{b^2x^8+2\,abx^4+a^2}}{8\,b}\right.$$
$$-\frac{4\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x^2}{\sqrt{dx^4+c}}\right)-\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,a^2cd)x^4+a^2c^2+4\,((abc-2\,a^2d)x^6-a^2cx^2)\sqrt{dx}\right.}{b^2x^8+2\,abx^4+a^2}}{8\,b}$$
$$\left.-\frac{2\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x^2}{\sqrt{dx^4+c}}\right)-\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{((bc-2\,ad)x^4-ac)\sqrt{dx^4+c}\sqrt{\frac{bc-ad}{a}}}{2\,((bcd-ad^2)x^6+(bc^2-acd)x^2)}\right)}{4\,b}\right]$$

```
[In] integrate(x*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c) + sqrt(-(
b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4
*a^2*c*d)*x^4 + a^2*c^2 + 4*((a*b*c - 2*a^2*d)*x^6 - a^2*c*x^2)*sqrt(d*x^4
+ c)*sqrt(-(b*c - a*d)/a))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/b, -1/8*(4*sqrt(-d
)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) - sqrt(-(b*c - a*d)/a)*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4
*((a*b*c - 2*a^2*d)*x^6 - a^2*c*x^2)*sqrt(d*x^4 + c)*sqrt(-(b*c - a*d)/a))/
(b^2*x^8 + 2*a*b*x^4 + a^2)))/b, 1/4*(sqrt((b*c - a*d)/a)*arctan(1/2*((b*c
- 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^
6 + (b*c^2 - a*c*d)*x^2)) + sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d
)*x^2 - c))/b, -1/4*(2*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) - sqrt
((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt((
b*c - a*d)/a)/((b*c*d - a*d^2)*x^6 + (b*c^2 - a*c*d)*x^2)))/b]
```

Sympy [F]

$$\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx$$

```
[In] integrate(x*(d*x**4+c)**(1/2)/(b*x**4+a),x)
[Out] Integral(x*sqrt(c + d*x**4)/(a + b*x**4), x)
```

Maxima [F]

$$\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+cx}}{bx^4+a}\,dx$$

```
[In] integrate(x*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^4 + c)*x/(b*x^4 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{x\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

```
[In] int((x*(c + d*x^4)^(1/2))/(a + b*x^4),x)
[Out] int((x*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

3.791 $\int \frac{\sqrt{c+dx^4}}{x(a+bx^4)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 85

$$\int \frac{\sqrt{c+dx^4}}{x\,(a+bx^4)}\,dx=-\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a}+\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{b}}$$

```
[Out] -1/2*arctanh((d*x^4+c)^(1/2)/c^(1/2))*c^(1/2)/a+1/2*arctanh(b^(1/2)*(d*x^4+
c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/a/b^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {457, 85, 65, 214}

$$\int \frac{\sqrt{c+dx^4}}{x\,(a+bx^4)}\,dx=\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{b}}-\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a}$$

```
[In] Int[Sqrt[c + d*x^4]/(x*(a + b*x^4)),x]
```

```
[Out] -1/2*(Sqrt[c]*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/a + (Sqrt[b*c - a*d]*ArcTan
h[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*a*Sqrt[b])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x(a+bx)}\,dx, x, x^4\right)\\ &= \frac{c\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^4\right)}{4a} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4a}\\ &= \frac{c\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2ad} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2ad}\\ &= -\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a} + \frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{b}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+dx^4}}{x\,(a+bx^4)}\,dx = \frac{\frac{\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}} - \sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a}$$

```
[In] Integrate[Sqrt[c + d*x^4]/(x*(a + b*x^4)),x]
[Out] ((Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[-(b*c) + a*d]])/
Sqrt[b] - Sqrt[c]*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/(2*a)
```

Maple [A] (verified)

Time = 4.87 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.24

method	result
pseudoelliptic	$\frac{\arctan\left(\frac{b\sqrt{d\,x^4+c}}{\sqrt{(ad-bc)b}}\right)ad-\arctan\left(\frac{b\sqrt{d\,x^4+c}}{\sqrt{(ad-bc)b}}\right)bc-\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}}{\sqrt{c}}\right)\sqrt{(ad-bc)b}}{2a\sqrt{(ad-bc)b}}$
elliptic	$\frac{\sqrt{d\,x^4+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)}{2a}-\frac{\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}-\frac{\sqrt{d}\,\sqrt{-ab}\,\ln\left(\frac{-\frac{d\sqrt{-ab}}{b}+d\left(x^2+\frac{\sqrt{-c}}{b}\right.}{\sqrt{d}}\right.}{}}{}$
default	$\frac{\frac{\sqrt{d\,x^4+c}}{2}-\frac{\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)}{2}}{a}-\frac{b\left(\frac{\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}-\frac{\sqrt{d}\,\sqrt{-ab}\,\ln\left(\frac{-\frac{d\sqrt{-ab}}{b}+d\left(x^2+\frac{\sqrt{-}}{b}\right.}{\sqrt{d}}\right.}{}}{}\right.}{}$

```
[In] int((d*x^4+c)^(1/2)/x/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(arctan(b*(d*x^4+c)^(1/2)/((a*d-b*c)*b)^(1/2))*a*d-arctan(b*(d*x^4+c)^(
1/2)/((a*d-b*c)*b)^(1/2))*b*c-c^(1/2)*arctanh((d*x^4+c)^(1/2)/c^(1/2))*((a*
d-b*c)*b)^(1/2))/a/((a*d-b*c)*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 383, normalized size of antiderivative = 4.51

$$\int \frac{\sqrt{c+dx^4}}{x\,(a+bx^4)}\,dx = \left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^4+2\,bc-ad+2\,\sqrt{dx^4+c}b\sqrt{\frac{bc-ad}{b}}}{bx^4+a}\right)+\sqrt{c}\log\left(\frac{dx^4-2\,\sqrt{dx^4+c}\sqrt{c}+2\,c}{x^4}\right)}{4\,a}, \frac{2\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^4+c}b\sqrt{-}}{bc-ad}\right.}{}\right.$$

```
[In] integrate((d*x^4+c)^(1/2)/x/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt((b*c - a*d)/b)*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*b*
sqrt((b*c - a*d)/b))/(b*x^4 + a)) + sqrt(c)*log((d*x^4 - 2*sqrt(d*x^4 + c)*
```

```
sqrt(c) + 2*c)/x^4))/a, 1/4*(2*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^4 + c)
*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + sqrt(c)*log((d*x^4 - 2*sqrt(d*x^4 +
c)*sqrt(c) + 2*c)/x^4))/a, 1/4*(2*sqrt(-c)*arctan(sqrt(d*x^4 + c)*sqrt(-c)/
c) + sqrt((b*c - a*d)/b)*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*b*s
qrt((b*c - a*d)/b))/(b*x^4 + a)))/a, 1/2*(sqrt(-(b*c - a*d)/b)*arctan(-sqrt
(d*x^4 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + sqrt(-c)*arctan(sqrt(d*x^
4 + c)*sqrt(-c)/c))/a]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(70) = 140$.

Time = 4.43 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.86

$$\int \frac{\sqrt{c+dx^4}}{x(a+bx^4)}\,dx = \begin{cases} \frac{2\left(\frac{cd\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{-c}}\right)}{4a\sqrt{-c}} + \frac{d(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4ab\sqrt{\frac{ad-bc}{b}}}\right)}{d} & \text{for } d \neq 0 \\ \sqrt{c}\left(-\frac{b\left(\begin{cases} \frac{\frac{a}{2b}+x^4}{a} & \text{for } b=0 \\ -\frac{\log\left(a-2b\left(\frac{a}{2b}+x^4\right)\right)}{2b} & \text{otherwise} \end{cases}\right)}{2a} - \frac{b\left(\begin{cases} \frac{\frac{a}{2b}+x^4}{a} & \text{for } b=0 \\ \frac{\log\left(a+2b\left(\frac{a}{2b}+x^4\right)\right)}{2b} & \text{otherwise} \end{cases}\right)}{2a}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((d*x**4+c)**(1/2)/x/(b*x**4+a),x)
```

```
[Out] Piecewise((2*(c*d*atan(sqrt(c + d*x**4)/sqrt(-c))/(4*a*sqrt(-c)) + d*(a*d -
 b*c)*atan(sqrt(c + d*x**4)/sqrt((a*d - b*c)/b))/(4*a*b*sqrt((a*d - b*c)/b)
))/d, Ne(d, 0)), (sqrt(c)*(-b*Piecewise(((a/(2*b) + x**4)/a, Eq(b, 0)), (-l
og(a - 2*b*(a/(2*b) + x**4))/(2*b), True))/(2*a) - b*Piecewise(((a/(2*b) +
x**4)/a, Eq(b, 0)), (log(a + 2*b*(a/(2*b) + x**4))/(2*b), True))/(2*a)), Tr
ue))
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^4}}{x(a+bx^4)}\,dx = \int \frac{\sqrt{dx^4+c}}{(bx^4+a)x}\,dx$$

```
[In] integrate((d*x^4+c)^(1/2)/x/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c+dx^4}}{x(a+bx^4)}\,dx = -\frac{(bc-ad)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}a} + \frac{c\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{2a\sqrt{-c}}$$

```
[In] integrate((d*x^4+c)^(1/2)/x/(b*x^4+a),x, algorithm="giac")
[Out] -1/2*(b*c - a*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*
c + a*b*d)*a) + 1/2*c*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a*sqrt(-c))
```

Mupad [B] (verification not implemented)

Time = 9.24 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.34

$$\int \frac{\sqrt{c+dx^4}}{x(a+bx^4)}\,dx = \frac{\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c}\left(\sqrt{d\,x^4+c}\left(\frac{a^2\,b\,d^4}{2}-a\,b^2\,c\,d^3+b^3\,c^2\,d^2\right)+\frac{c\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\sqrt{d\,x^4+c}}{16\,a^2}\right)}{2\,a\left(\frac{b^2\,c^2\,d^3}{4}-\frac{a\,b\,c\,d^4}{4}\right)}\right)}{2\,a} + \frac{\mathrm{atanh}\left(\frac{a\,b^2\,c\,d^3\,\sqrt{d\,x^4+c}\,\sqrt{b^2\,c-a\,b\,d}}{4\left(\frac{a\,b^3\,c^2\,d^3}{4}-\frac{a^2\,b^2\,c\,d^4}{4}\right)}\right)\sqrt{b^2\,c-a\,b\,d}}{2\,a\,b}$$

```
[In] int((c + d*x^4)^(1/2)/(x*(a + b*x^4)),x)
[Out] (c^(1/2)*atanh((c^(1/2)*((c + d*x^4)^(1/2)*((a^2*b*d^4)/2 + b^3*c^2*d^2 - a
*b^2*c*d^3) + (c*(8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^4)^(1/2))/(16*
a^2)))/(2*a*((b^2*c^2*d^3)/4 - (a*b*c*d^4)/4))))/(2*a) + (atanh((a*b^2*c*d^
3*(c + d*x^4)^(1/2)*(b^2*c - a*b*d)^(1/2))/(4*((a*b^3*c^2*d^3)/4 - (a^2*b^2
*c*d^4)/4)))*(b^2*c - a*b*d)^(1/2))/(2*a*b)
```

3.792 $\int \frac{\sqrt{c+dx^4}}{x^3(a+bx^4)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 76

$$\int \frac{\sqrt{c+dx^4}}{x^3\left(a+bx^4\right)}\,dx=-\frac{\sqrt{c+dx^4}}{2ax^2}-\frac{\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}}$$

```
[Out] -1/2*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))*(-a*d+b*c)^(1/2)/
a^(3/2)-1/2*(d*x^4+c)^(1/2)/a/x^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {476, 486, 12, 385, 211}

$$\int \frac{\sqrt{c+dx^4}}{x^3\left(a+bx^4\right)}\,dx=-\frac{\sqrt{bc-ad}\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}}-\frac{\sqrt{c+dx^4}}{2ax^2}$$

```
[In] Int[Sqrt[c + d*x^4]/(x^3*(a + b*x^4)),x]
```

```
[Out] -1/2*Sqrt[c + d*x^4]/(a*x^2) - (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2
)/(Sqrt[a]*Sqrt[c + d*x^4])])/(2*a^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{c+dx^2}}{x^2(a+bx^2)}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{c+dx^4}}{2ax^2}+\frac{\text{Subst}\left(\int \frac{-bc+ad}{(a+bx^2)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{2a}\\
&= -\frac{\sqrt{c+dx^4}}{2ax^2}-\frac{(bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{2a}\\
&= -\frac{\sqrt{c+dx^4}}{2ax^2}-\frac{(bc-ad)\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^2}{\sqrt{c+dx^4}}\right)}{2a}\\
&= -\frac{\sqrt{c+dx^4}}{2ax^2}-\frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt{c+dx^4}}{x^3\,(a+bx^4)}\,dx = -\frac{\sqrt{c+dx^4}}{2ax^2} - \frac{\sqrt{bc-ad}\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{3/2}}$$

```
[In] Integrate[Sqrt[c + d*x^4]/(x^3*(a + b*x^4)),x]
[Out] -1/2*Sqrt[c + d*x^4]/(a*x^2) - (Sqrt[b*c - a*d]*ArcTan[(a*Sqrt[d] + b*x^2*(
Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(3/2))
```

Maple [A] (verified)

Time = 5.45 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.91

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^4+c}}{x^2}+\frac{(ad-bc)\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}\,a}{x^2\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}}{2a}$
risch	$-\frac{\sqrt{d\,x^4+c}}{2a\,x^2}+\frac{(ad-bc)\left(\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}}\cdots\right)}{a}$
default	$\frac{-\frac{\left(d\,x^4+c\right)^{\frac{3}{2}}}{2c\,x^2}+\frac{d\,x^2\sqrt{d\,x^4+c}}{2c}+\frac{\sqrt{d}\,\ln\left(x^2\sqrt{d}+\sqrt{d\,x^4+c}\right)}{2}}{a}-\frac{b\left(-\frac{\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}-\frac{\sqrt{d}\,\sqrt{-ab}\,\ln\left(\cdots\right)}{\cdots}}{\cdots}\cdots\right)}{\cdots}$
elliptic	$\frac{-\frac{\left(d\,x^4+c\right)^{\frac{3}{2}}}{c\,x^2}+\frac{2d\left(\frac{x^2\sqrt{d\,x^4+c}}{2}+\frac{c\ln\left(x^2\sqrt{d}+\sqrt{d\,x^4+c}\right)}{2\sqrt{d}}\right)}{c}}{2a}+\frac{b\left(\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}-\frac{\sqrt{d}\,\sqrt{-ab}\,\ln\left(\cdots\right)}{\cdots}\cdots\right)}{\cdots}$

```
[In] int((d*x^4+c)^(1/2)/x^3/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a*(-(d*x^4+c)^(1/2)/x^2+(a*d-b*c)*arctanh((d*x^4+c)^(1/2)/x^2*a/((a*d-b
*c)*a)^(1/2))/((a*d-b*c)*a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 281, normalized size of antiderivative = 3.70

$$\int \frac{\sqrt{c+dx^4}}{x^3\left(a+bx^4\right)}\,dx = \left[\frac{x^2\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^8-2\left(3\,abc^2-4\,a^2cd\right)x^4+a^2c^2-4\left(\left(abc-2\,a^2d\right)x^6-a^2cx^2\right)\sqrt{dx^4+c}\sqrt{-\frac{bc-ad}{a}}}{b^2x^8+2\,abx^4+a^2}\right)-4\sqrt{dx^4+c}}{8\,ax^2}, -\frac{x^2\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{\left((bc-2\,ad)x^4-ac\right)\sqrt{dx^4+c}\sqrt{\frac{bc-ad}{a}}}{2\left(\left(bcd-ad^2\right)x^6+\left(bc^2-acd\right)x^2\right)}\right)+2\sqrt{dx^4+c}}{4\,ax^2}\right]$$

```
[In] integrate((d*x^4+c)^(1/2)/x^3/(b*x^4+a),x, algorithm="fricas")
[Out] [1/8*(x^2*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 -
 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((a*b*c - 2*a^2*d)*x^6 - a^2*c
*x^2)*sqrt(d*x^4 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^8 + 2*a*b*x^4 + a^2)) -
4*sqrt(d*x^4 + c))/(a*x^2), -1/4*(x^2*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c
- 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^
6 + (b*c^2 - a*c*d)*x^2)) + 2*sqrt(d*x^4 + c))/(a*x^2)]
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^4}}{x^3\left(a+bx^4\right)}\,dx = \int \frac{\sqrt{c+dx^4}}{x^3\left(a+bx^4\right)}\,dx$$

```
[In] integrate((d*x**4+c)**(1/2)/x**3/(b*x**4+a),x)
[Out] Integral(sqrt(c + d*x**4)/(x**3*(a + b*x**4)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^4}}{x^3\left(a+bx^4\right)}\,dx=\int \frac{\sqrt{dx^4+c}}{(bx^4+a)x^3}\,dx$$

```
[In] integrate((d*x^4+c)^(1/2)/x^3/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(60) = 120$.

Time = 0.77 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.59

$$\int \frac{\sqrt{c+dx^4}}{x^3\left(a+bx^4\right)}\,dx=\frac{\left(bc\sqrt{d}-ad^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}a}+\frac{c\sqrt{d}}{\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2-c\right)a}$$

```
[In] integrate((d*x^4+c)^(1/2)/x^3/(b*x^4+a),x, algorithm="giac")
[Out] 1/2*(b*c*sqrt(d) - a*d^(3/2))*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2
*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a) + c*
sqrt(d)/(((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2 - c)*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{x^3\left(a+bx^4\right)}\,dx=\int \frac{\sqrt{d\,x^4+c}}{x^3\,\left(b\,x^4+a\right)}\,dx$$

```
[In] int((c + d*x^4)^(1/2)/(x^3*(a + b*x^4)),x)
[Out] int((c + d*x^4)^(1/2)/(x^3*(a + b*x^4)), x)
```

3.793 $\int \frac{\sqrt{c+dx^4}}{x^5(a+bx^4)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 115

$$\int \frac{\sqrt{c+dx^4}}{x^5\left(a+bx^4\right)}\,dx=-\frac{\sqrt{c+dx^4}}{4ax^4}+\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2\sqrt{c}}-\frac{\sqrt{b}\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2}$$

```
[Out] 1/4*(-a*d+2*b*c)*arctanh((d*x^4+c)^(1/2)/c^(1/2))/a^2/c^(1/2)-1/2*arctanh(b
^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)*(-a*d+b*c)^(1/2)/a^2-1/4*(
d*x^4+c)^(1/2)/a/x^4
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 101, 162, 65, 214}

$$\int \frac{\sqrt{c+dx^4}}{x^5\left(a+bx^4\right)}\,dx=\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2\sqrt{c}}-\frac{\sqrt{b}\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2}-\frac{\sqrt{c+dx^4}}{4ax^4}$$

```
[In] Int[Sqrt[c + d*x^4]/(x^5*(a + b*x^4)),x]
```

```
[Out] -1/4*Sqrt[c + d*x^4]/(a*x^4) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^4]/Sqrt[
c]])/(4*a^2*Sqrt[c]) - (Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^4])/Sqrt[b*c - a*d]])/(2*a^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{4}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(a+bx)}\,dx, x, x^4\right)$$

$$= -\frac{\sqrt{c+dx^4}}{4ax^4} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-2bc+ad)-\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4a}$$

$$= -\frac{\sqrt{c+dx^4}}{4ax^4} + \frac{(b(bc-ad))\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4a^2} - \frac{(2bc-ad)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^4\right)}{8a^2}$$

$$= -\frac{\sqrt{c+dx^4}}{4ax^4} + \frac{(b(bc-ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2a^2d} - \frac{(2bc-ad)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{4a^2d}$$

$$= -\frac{\sqrt{c+dx^4}}{4ax^4} + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2\sqrt{c}} - \frac{\sqrt{b}\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c+dx^4}}{x^5(a+bx^4)}\,dx = \frac{-\frac{a\sqrt{c+dx^4}}{x^4} - 2\sqrt{b}\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right) + \frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{\sqrt{c}}}{4a^2}$$

```
[In] Integrate[Sqrt[c + d*x^4]/(x^5*(a + b*x^4)),x]
[Out] (-((a*Sqrt[c + d*x^4])/x^4) - 2*Sqrt[b]*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*
Sqrt[c + d*x^4])/Sqrt[-(b*c) + a*d]] + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^
4]/Sqrt[c]])/Sqrt[c])/(4*a^2)
```

Maple [A] (verified)

Time = 4.96 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$\frac{-\frac{2(ad-bc)b\arctan\left(\frac{b\sqrt{d\,x^4+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}-\frac{a\sqrt{d\,x^4+c}}{x^4}-\frac{(ad-2bc)\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^4+c}}{\sqrt{c}}\right)}{\sqrt{c}}}{4a^2}$
risch	$-\frac{\sqrt{d\,x^4+c}}{4a\,x^4}-\frac{-\frac{(-ad+2bc)\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)}{2a\sqrt{c}}+\frac{2(ad-bc)b\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right.\cdots}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4b\sqrt{-\frac{ad-bc}{b}}}\cdots\right)}{\cdots}}{\cdots}$
elliptic	$\frac{-\frac{\left(d\,x^4+c\right)^{\frac{3}{2}}}{2c\,x^4}+\frac{d\left(\sqrt{d\,x^4+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)\right)}{2c}}{2a}-\frac{b\left(\sqrt{d\,x^4+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)\right)}{2a^2}+\frac{b\left(\sqrt{d\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2+\cdots}\cdots\right)}{\cdots}$
default	$\frac{-\frac{\left(d\,x^4+c\right)^{\frac{3}{2}}}{4c\,x^4}-\frac{d\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)}{4\sqrt{c}}+\frac{d\sqrt{d\,x^4+c}}{4c}}{a}-\frac{b\left(\frac{\sqrt{d\,x^4+c}}{2}-\frac{\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)}{2}\right)}{a^2}+\frac{b^2\left(\frac{\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\cdots}{\cdots}}}{\cdots}\cdots\right)}{\cdots}$

```
[In] int((d*x^4+c)^(1/2)/x^5/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a^2*(-2*(a*d-b*c)*b/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^4+c)^(1/2)/((a*d-
b*c)*b)^(1/2))-a*(d*x^4+c)^(1/2)/x^4-(a*d-2*b*c)/c^(1/2)*arctanh((d*x^4+c)^
(1/2)/c^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 513, normalized size of antiderivative = 4.46

$$\int \frac{\sqrt{c+dx^4}}{x^5\left(a+bx^4\right)}\,dx$$
$$= \left[\frac{2\sqrt{b^2c-abd}cx^4\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)-(2\,bc-ad)\sqrt{c}x^4\log\left(\frac{dx^4-2\,\sqrt{dx^4+c}\sqrt{c}+2\,c}{x^4}\right)-2\,\sqrt{dx^4}}{8\,a^2cx^4}\right.$$
$$-\frac{(2\,bc-ad)\sqrt{-c}x^4\arctan\left(\frac{\sqrt{dx^4+c}\sqrt{-c}}{c}\right)-\sqrt{b^2c-abd}cx^4\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)+\sqrt{dx^4+c}}{4\,a^2cx^4}$$

```
[In] integrate((d*x^4+c)^(1/2)/x^5/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(b^2*c - a*b*d)*c*x^4*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4
 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) - (2*b*c - a*d)*sqrt(c)*x^4*log((d*
x^4 - 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) - 2*sqrt(d*x^4 + c)*a*c)/(a^2*c
*x^4), 1/8*(4*sqrt(-b^2*c + a*b*d)*c*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-b^2*c
 + a*b*d)/(b*d*x^4 + b*c)) - (2*b*c - a*d)*sqrt(c)*x^4*log((d*x^4 - 2*sqrt(
d*x^4 + c)*sqrt(c) + 2*c)/x^4) - 2*sqrt(d*x^4 + c)*a*c)/(a^2*c*x^4), -1/4*(
(2*b*c - a*d)*sqrt(-c)*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) - sqrt(b^2*c
- a*b*d)*c*x^4*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*sqrt(b^2*c -
a*b*d))/(b*x^4 + a)) + sqrt(d*x^4 + c)*a*c)/(a^2*c*x^4), 1/4*(2*sqrt(-b^2*c
 + a*b*d)*c*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)
) - (2*b*c - a*d)*sqrt(-c)*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) - sqrt(d*
x^4 + c)*a*c)/(a^2*c*x^4)]
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^4}}{x^5\left(a+bx^4\right)}\,dx=\int \frac{\sqrt{c+dx^4}}{x^5\left(a+bx^4\right)}\,dx$$

```
[In] integrate((d*x**4+c)**(1/2)/x**5/(b*x**4+a),x)
```

```
[Out] Integral(sqrt(c + d*x**4)/(x**5*(a + b*x**4)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^4}}{x^5(a+bx^4)}\,dx = \int \frac{\sqrt{dx^4+c}}{(bx^4+a)x^5}\,dx$$

```
[In] integrate((d*x^4+c)^(1/2)/x^5/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^5), x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c+dx^4}}{x^5(a+bx^4)}\,dx = \frac{(b^2c-abd)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}a^2} - \frac{(2bc-ad)\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{4a^2\sqrt{-c}} - \frac{\sqrt{dx^4+c}}{4ax^4}$$

```
[In] integrate((d*x^4+c)^(1/2)/x^5/(b*x^4+a),x, algorithm="giac")
[Out] 1/2*(b^2*c - a*b*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b
^2*c + a*b*d)*a^2) - 1/4*(2*b*c - a*d)*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a^
2*sqrt(-c)) - 1/4*sqrt(d*x^4 + c)/(a*x^4)
```

Mupad [B] (verification not implemented)

Time = 9.58 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.34

$$\int \frac{\sqrt{c+dx^4}}{x^5(a+bx^4)}\,dx = \frac{\operatorname{atanh}\left(\frac{b^3\,d^4\,\sqrt{d\,x^4+c}\,\sqrt{b^2\,c-a\,b\,d}}{16\,\left(\frac{a\,b^3\,d^5}{16}-\frac{b^4\,c\,d^4}{16}\right)}\right)\,\sqrt{b^2\,c-a\,b\,d}}{2\,a^2} - \frac{\sqrt{d\,x^4+c}}{4\,a\,x^4} - \frac{\operatorname{atanh}\left(\frac{b^4\,\sqrt{c}\,d^4\,\sqrt{d\,x^4+c}}{16\,\left(\frac{b^4\,c\,d^4}{16}-\frac{3\,a\,b^3\,d^5}{32}+\frac{a^2\,b^2\,d^6}{32\,c}\right)} - \frac{3\,b^3\,d^5\,\sqrt{d\,x^4+c}}{32\,\sqrt{c}\,\left(\frac{a\,b^2\,d^6}{32\,c}-\frac{3\,b^3\,d^5}{32}+\frac{b^4\,c\,d^4}{16\,a}\right)} + \frac{b^2\,d^6\,\sqrt{d\,x^4+c}}{32\,c^{3/2}\,\left(\frac{b^2\,d^6}{32\,c}-\frac{3\,b^3\,d^5}{32\,a}+\frac{b^4\,c\,d^4}{16\,a^2}\right)}\right)\,(a\,d-2\,b\,c)}{4\,a^2\,\sqrt{c}}$$

```
[In] int((c + d*x^4)^(1/2)/(x^5*(a + b*x^4)),x)
[Out] (atanh((b^3*d^4*(c + d*x^4)^(1/2)*(b^2*c - a*b*d)^(1/2))/(16*((a*b^3*d^5)/1
6 - (b^4*c*d^4)/16)))*(b^2*c - a*b*d)^(1/2))/(2*a^2) - (c + d*x^4)^(1/2)/(4
*a*x^4) - (atanh((b^4*c^(1/2)*d^4*(c + d*x^4)^(1/2))/(16*((b^4*c*d^4)/16 -
(3*a*b^3*d^5)/32 + (a^2*b^2*d^6)/(32*c))) - (3*b^3*d^5*(c + d*x^4)^(1/2))/(
32*c^(1/2)*((a*b^2*d^6)/(32*c) - (3*b^3*d^5)/32 + (b^4*c*d^4)/(16*a))) + (b
^2*d^6*(c + d*x^4)^(1/2))/(32*c^(3/2)*((b^2*d^6)/(32*c) - (3*b^3*d^5)/(32*a
) + (b^4*c*d^4)/(16*a^2))))*(a*d - 2*b*c))/(4*a^2*c^(1/2))
```

3.794 $\int \frac{\sqrt{c+dx^4}}{x^7(a+bx^4)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 110

$$\int \frac{\sqrt{c+dx^4}}{x^7(a+bx^4)}\,dx = -\frac{\sqrt{c+dx^4}}{6ax^6} + \frac{(3bc-ad)\sqrt{c+dx^4}}{6a^2cx^2} + \frac{b\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{5/2}}$$

```
[Out] 1/2*b*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))*(-a*d+b*c)^(1/2)
/a^(5/2)-1/6*(d*x^4+c)^(1/2)/a/x^6+1/6*(-a*d+3*b*c)*(d*x^4+c)^(1/2)/a^2/c/x
^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {476, 486, 597, 12, 385, 211}

$$\int \frac{\sqrt{c+dx^4}}{x^7(a+bx^4)}\,dx = \frac{b\sqrt{bc-ad}\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{5/2}} + \frac{\sqrt{c+dx^4}(3bc-ad)}{6a^2cx^2} - \frac{\sqrt{c+dx^4}}{6ax^6}$$

```
[In] Int[Sqrt[c + d*x^4]/(x^7*(a + b*x^4)),x]
```

```
[Out] -1/6*Sqrt[c + d*x^4]/(a*x^6) + ((3*b*c - a*d)*Sqrt[c + d*x^4])/(6*a^2*c*x^2
) + (b*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4
])])/(2*a^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{c+dx^2}}{x^4\left(a+bx^2\right)}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt{c+dx^4}}{6ax^6} + \frac{\text{Subst}\left(\int \frac{-3bc+ad-2bdx^2}{x^2(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{6a}$$

$$= -\frac{\sqrt{c+dx^4}}{6ax^6} + \frac{(3bc-ad)\sqrt{c+dx^4}}{6a^2cx^2} - \frac{\text{Subst}\left(\int -\frac{3bc(bc-ad)}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{6a^2c}$$

$$= -\frac{\sqrt{c+dx^4}}{6ax^6} + \frac{(3bc-ad)\sqrt{c+dx^4}}{6a^2cx^2} + \frac{(b(bc-ad))\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{2a^2}$$

$$= -\frac{\sqrt{c+dx^4}}{6ax^6} + \frac{(3bc-ad)\sqrt{c+dx^4}}{6a^2cx^2} + \frac{(b(bc-ad))\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{2a^2}$$

$$= -\frac{\sqrt{c+dx^4}}{6ax^6} + \frac{(3bc-ad)\sqrt{c+dx^4}}{6a^2cx^2} + \frac{b\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{5/2}}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{c+dx^4}}{x^7(a+bx^4)}\,dx = \frac{\sqrt{c+dx^4}(3bcx^4 - a(c+dx^4))}{6a^2cx^6} + \frac{b\sqrt{bc-ad}\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{5/2}}$$

```
[In] Integrate[Sqrt[c + d*x^4]/(x^7*(a + b*x^4)),x]
```

```
[Out] (Sqrt[c + d*x^4]*(3*b*c*x^4 - a*(c + d*x^4)))/(6*a^2*c*x^6) + (b*Sqrt[b*c -
 a*d]*ArcTan[(a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*S
qrt[b*c - a*d])])/(2*a^(5/2))
```

Maple [A] (verified)

Time = 5.68 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{4}+c}\,\left(a d\,x^{4}-3 b c\,x^{4}+a c\right)}{3 x^{6}}-\frac{b c (a d-b c) \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{4}+c}\,a}{x^{2} \sqrt{(a d-b c) a}}\right)}{\sqrt{(a d-b c) a}}}{2 a^{2} c}$
risch	$-\frac{\sqrt{d\,x^{4}+c}\,(a d\,x^{4}-3 b c\,x^{4}+a c)}{6 c\,a^{2} x^{6}}-\frac{(a d-b c) b \left(\frac{\ln\left(\frac{-\frac{2(a d-b c)}{b}-\frac{2 d\sqrt{-a b}\,\left(x^{2}+\frac{\sqrt{-a b}}{b}\right)}{b}+2\sqrt{-\frac{a d-b c}{b}}\,\sqrt{d\left(x^{2}+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d\sqrt{-a b}\,(}{}}}{x^{2}+\frac{\sqrt{-a b}}{b}}\right)}{4\sqrt{-a b}\,\sqrt{-\frac{a d-b c}{b}}}\right.}{}$
default	$-\frac{(d\,x^{4}+c)^{\frac{3}{2}}}{6 a\,x^{6} c}-\frac{b\left(-\frac{(d\,x^{4}+c)^{\frac{3}{2}}}{2 c\,x^{2}}+\frac{d\,x^{2}\sqrt{d\,x^{4}+c}}{2 c}+\frac{\sqrt{d}\,\ln\left(x^{2}\sqrt{d}+\sqrt{d\,x^{4}+c}\right)}{2}\right)}{a^{2}}+\frac{b^{2}\left(-\frac{\sqrt{d\left(x^{2}+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d\sqrt{-a b}\,\left(x^{2}+\frac{\sqrt{-a b}}{b}\right)}{b}}}{}\right.}{}$
elliptic	$-\frac{(d\,x^{4}+c)^{\frac{3}{2}}}{6 a\,x^{6} c}-\frac{b\left(-\frac{(d\,x^{4}+c)^{\frac{3}{2}}}{c\,x^{2}}+\frac{2 d\left(\frac{x^{2}\sqrt{d\,x^{4}+c}}{2}+\frac{c\ln\left(x^{2}\sqrt{d}+\sqrt{d\,x^{4}+c}\right)}{2\sqrt{d}}\right)}{c}\right)}{2 a^{2}}-\frac{b^{2}\left(\sqrt{d\left(x^{2}+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d\sqrt{-a b}\,\left(x^{2}+\frac{\sqrt{-a b}}{b}\right.}{b}}\right.}{}$

```
[In] int((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^2*(-1/3*(d*x^4+c)^(1/2)*(a*d*x^4-3*b*c*x^4+a*c)/x^6-b*c*(a*d-b*c)/((a
*d-b*c)*a)^(1/2)*arctanh((d*x^4+c)^(1/2)/x^2*a/((a*d-b*c)*a)^(1/2)))/c
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.99

$$\int \frac{\sqrt{c+dx^4}}{x^7\left(a+bx^4\right)}\,dx = \left[\frac{3\,bcx^6\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^8-2\,\left(3\,abc^2-4\,a^2cd\right)x^4+a^2c^2+4\,\left(\left(abc-2\,a^2d\right)x^6-a^2cx^2\right)\sqrt{dx^4+c}\sqrt{-\frac{bc-ad}{a}}}{b^2x^8+2\,abx^4+a^2}\right)+4\,((3}{24\,a^2cx^6}\right.$$

[In] integrate((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x, algorithm="fricas")

[Out] [1/24*(3*b*c*x^6*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2
)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((a*b*c - 2*a^2*d)*x^6
- a^2*c*x^2)*sqrt(d*x^4 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^8 + 2*a*b*x^4 + a
^2)) + 4*((3*b*c - a*d)*x^4 - a*c)*sqrt(d*x^4 + c))/(a^2*c*x^6), 1/12*(3*b*
c*x^6*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 +
 c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^6 + (b*c^2 - a*c*d)*x^2)) + 2*((
3*b*c - a*d)*x^4 - a*c)*sqrt(d*x^4 + c))/(a^2*c*x^6)]

Sympy [F]

$$\int \frac{\sqrt{c+dx^4}}{x^7\left(a+bx^4\right)}\,dx = \int \frac{\sqrt{c+dx^4}}{x^7\left(a+bx^4\right)}\,dx$$

[In] integrate((d*x**4+c)**(1/2)/x**7/(b*x**4+a),x)

[Out] Integral(sqrt(c + d*x**4)/(x**7*(a + b*x**4)), x)

Maxima [F]

$$\int \frac{\sqrt{c+dx^4}}{x^7\left(a+bx^4\right)}\,dx = \int \frac{\sqrt{dx^4+c}}{(bx^4+a)x^7}\,dx$$

[In] integrate((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x, algorithm="maxima")

[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^7), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(90) = 180$.

Time = 1.11 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.05

$$\int \frac{\sqrt{c+dx^4}}{x^7\left(a+bx^4\right)}\,dx = -\frac{\left(b^2c\sqrt{d}-abd^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{2\sqrt{abcd-a^2d^2}a^2} - \frac{3\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4bc\sqrt{d}-3\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4ad^{\frac{3}{2}}-6\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2bc^2\sqrt{d}+3\,bc^3\sqrt{d}-}{3\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2-c\right)^3a^2}$$

```
[In] integrate((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*(b^2*c*sqrt(d) - a*b*d^(3/2))*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 +
c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^
2) - 1/3*(3*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b*c*sqrt(d) - 3*(sqrt(d)*x^2
- sqrt(d*x^4 + c))^4*a*d^(3/2) - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c^2*
sqrt(d) + 3*b*c^3*sqrt(d) - a*c^2*d^(3/2))/(((sqrt(d)*x^2 - sqrt(d*x^4 + c)
)^2 - c)^3*a^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{x^7\left(a+bx^4\right)}\,dx = \int \frac{\sqrt{d\,x^4+c}}{x^7\,\left(b\,x^4+a\right)}\,dx$$

```
[In] int((c + d*x^4)^(1/2)/(x^7*(a + b*x^4)),x)
```

```
[Out] int((c + d*x^4)^(1/2)/(x^7*(a + b*x^4)), x)
```

3.795 $\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 857

$$
\begin{aligned}
\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx &= \frac{x^3\sqrt{c+dx^4}}{5b} + \frac{(2bc-5ad)x\sqrt{c+dx^4}}{5b^2\sqrt{d}\left(\sqrt{c}+\sqrt{d}x^2\right)} \\
&- \frac{a\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4b^2} - \frac{a\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4b^2} \\
&- \frac{\sqrt[4]{c}(2bc-5ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{5b^2d^{3/4}\sqrt{c+dx^4}} \\
&+ \frac{\sqrt[4]{c}(b^2c^2+abcd-5a^2d^2)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{5b^2d^{3/4}(bc+ad)\sqrt{c+dx^4}} \\
&+ \frac{a\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\mathrm{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{8b^{5/2}\sqrt[4]{c}\left(\sqrt{-a}\sqrt{b}\sqrt{c}-a\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}} \\
&- \frac{a\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\mathrm{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{8b^{5/2}\sqrt[4]{c}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}
\end{aligned}
$$

```
[Out] 1/5*x^3*(d*x^4+c)^(1/2)/b+1/5*(-5*a*d+2*b*c)*x*(d*x^4+c)^(1/2)/b^2/d^(1/2)/
(c^(1/2)+x^2*d^(1/2))-1/4*a*arctan(x*((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(
d*x^4+c)^(1/2))*((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/b^2-1/4*a*arctan(x*((-
a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*((-a*d+b*c)/(-a)^(1/2)/
```

```
b^(1/2))^(1/2)/b^2-1/5*c^(1/4)*(-5*a*d+2*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/
4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticE(sin(2*arctan(d^(1/
4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d
^(1/2))^2)^(1/2)/b^2/d^(3/4)/(d*x^4+c)^(1/2)+1/5*c^(1/4)*(-5*a^2*d^2+a*b*c*
d+b^2*c^2)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*
x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2
)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^2/d^(3/4)/(a*d+b
*c)/(d*x^4+c)^(1/2)+1/8*a*(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(
1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(
1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2
)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^
(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^(5/2)/c^(1/4)/d^(1/4)/((
-a)^(1/2)*b^(1/2)*c^(1/2)-a*d^(1/2))/(d*x^4+c)^(1/2)-1/8*a*(-a*d+b*c)*(cos(
2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*Elli
pticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*c^(1/2)*(b^(1/2)-(-a)^(1/2)*d^
(1/2)/c^(1/2))^2/(-a)^(1/2)/b^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/
2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2
)^(1/2)/b^(5/2)/c^(1/4)/d^(1/4)/((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))/(d*x
^4+c)^(1/2)
```

Rubi [A] (warning: unable to verify)

Time = 1.45 (sec) , antiderivative size = 1067, normalized size of antiderivative = 1.25, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{489, 598, 311, 226, 1210, 504, 1231, 1721\}$

$$\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx$$

$$= \frac{\sqrt{dx^4+c}\,x^3}{5b} + \frac{(2bc-5ad)\sqrt{dx^4+c}\,x}{5b^2\sqrt{d}\left(\sqrt{d}x^2+\sqrt{c}\right)} + \frac{(-a)^{3/4}\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{4b^{9/4}}$$
$$+ \frac{(-a)^{3/4}\sqrt{ad-bc}\arctan\left(\frac{\sqrt{ad-bc}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{4b^{9/4}}$$
$$- \frac{\sqrt[4]{c}(2bc-5ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{5b^2d^{3/4}\sqrt{dx^4+c}}$$
$$+ \frac{\sqrt[4]{c}(2bc-5ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{10b^2d^{3/4}\sqrt{dx^4+c}}$$
$$+ \frac{a\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4b^2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}$$
$$+ \frac{a\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4b^2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}$$
$$+ \frac{\sqrt{-a}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\right.}{8b^{5/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}$$
$$- \frac{\sqrt{-a}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\right.}{8b^{5/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}$$

```
[In] Int[(x^6*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (x^3*Sqrt[c + d*x^4])/(5*b) + ((2*b*c - 5*a*d)*x*Sqrt[c + d*x^4])/(5*b^2*Sq
rt[d]*(Sqrt[c] + Sqrt[d]*x^2)) + ((-a)^(3/4)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b
*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*b^(9/4)) + ((-a)^(3/
4)*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqr
t[c + d*x^4])])/(4*b^(9/4)) - (c^(1/4)*(2*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x
^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)
*x)/c^(1/4)], 1/2])/(5*b^2*d^(3/4)*Sqrt[c + d*x^4]) + (c^(1/4)*(2*b*c - 5*a
*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ell
ipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(10*b^2*d^(3/4)*Sqrt[c + d*x^4]
```

```
) + (a*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - a*d)*(Sqrt[c]
+ Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcT
an[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b^2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
+ (a*(Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - a*d)*(Sqrt[c] +
Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan
[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b^2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) +
(Sqrt[-a]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqr
t[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt
[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*Arc
Tan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b^(5/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt
[c + d*x^4]) - (Sqrt[-a]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)
*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ellipt
icPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqr
t[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b^(5/2)*c^(1/4)*d^(1/4)*(b*c
 + a*d)*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^3\sqrt{c+dx^4}}{5b} - \frac{\int \frac{x^2(3ac+(-2bc+5ad)x^4)}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{5b} \\
&= \frac{x^3\sqrt{c+dx^4}}{5b} - \frac{\int \left(-\frac{(2bc-5ad)x^2}{b\sqrt{c+dx^4}} - \frac{5(-abc+a^2d)x^2}{b(a+bx^4)\sqrt{c+dx^4}}\right)dx}{5b} \\
&= \frac{x^3\sqrt{c+dx^4}}{5b} + \frac{(2bc-5ad)\int \frac{x^2}{\sqrt{c+dx^4}}\,dx}{5b^2} - \frac{(a(bc-ad))\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{b^2}
\end{aligned}$$

$$= \frac{x^3\sqrt{c+dx^4}}{5b} + \frac{\left(\sqrt{c}(2bc-5ad)\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{5b^2\sqrt{d}} - \frac{\left(\sqrt{c}(2bc-5ad)\right)\int \frac{1-\frac{\sqrt{d}x^2}{\sqrt{c}}}{\sqrt{c+dx^4}}\,dx}{5b^2\sqrt{d}} + \frac{(a(bc-ad))\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{2b^{5/2}} - \frac{(a(bc-ad))\int \frac{1}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{2b^{5/2}}$$

$$= \frac{x^3\sqrt{c+dx^4}}{5b} + \frac{(2bc-5ad)x\sqrt{c+dx^4}}{5b^2\sqrt{d}\left(\sqrt{c}+\sqrt{d}x^2\right)} - \frac{\sqrt[4]{c}(2bc-5ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{5b^2d^{3/4}\sqrt{c+dx^4}} + \frac{\sqrt[4]{c}(2bc-5ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{10b^2d^{3/4}\sqrt{c+dx^4}} + \frac{\left(a\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(bc-ad)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{2b^2(bc+ad)} - \frac{\left(a\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(bc-ad)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{2b^2(bc+ad)} + \frac{\left(a\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt{d}(bc-ad)\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{2b^{5/2}(bc+ad)} + \frac{\left(a\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt{d}(bc-ad)\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{2b^{5/2}(bc+ad)}$$

$$= \frac{x^3\sqrt{c+dx^4}}{5b} + \frac{(2bc-5ad)x\sqrt{c+dx^4}}{5b^2\sqrt{d}\left(\sqrt{c}+\sqrt{d}x^2\right)} + \frac{(-a)^{3/4}\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4b^{9/4}}$$

$$+ \frac{(-a)^{3/4}\sqrt{-bc+ad}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4b^{9/4}}$$

$$- \frac{\sqrt[4]{c}(2bc-5ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{5b^2d^{3/4}\sqrt{c+dx^4}}$$

$$+ \frac{\sqrt[4]{c}(2bc-5ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{10b^2d^{3/4}\sqrt{c+dx^4}}$$

$$+ \frac{a\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4b^{5/2}\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}}$$

$$+ \frac{a\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4b^{5/2}\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}}$$

$$+ \frac{\sqrt{-a}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\right.\right.}{8b^{5/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}}$$

$$- \frac{\sqrt{-a}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\right.\right.}{8b^{5/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.16

$$\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx$$

$$= \frac{7ax^3(c+dx^4) - 7acx^3\sqrt{1+\frac{dx^4}{c}}\operatorname{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right) + (2bc-5ad)x^7\sqrt{1+\frac{dx^4}{c}}\operatorname{AppellF1}\left(\right.}{35ab\sqrt{c+dx^4}}$$

```
[In] Integrate[(x^6*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (7*a*x^3*(c + d*x^4) - 7*a*c*x^3*Sqrt[1 + (d*x^4)/c]*AppellF1[3/4, 1/2, 1,
7/4, -((d*x^4)/c), -((b*x^4)/a)] + (2*b*c - 5*a*d)*x^7*Sqrt[1 + (d*x^4)/c]*
```

AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x^4)/a)])/(35*a*b*Sqrt[c + d
*x^4])

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.13 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.39

method	result
risch	$\frac{x^3\sqrt{d\,x^4+c}}{5b}-\frac{\frac{i(5ad-2bc)\sqrt{c}\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\sqrt{d\,x^4+c}\sqrt{d}}-\frac{5(ad-bc)a\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}-\frac{\operatorname{arctanh}\left(\frac{2\dots}{2\sqrt{\dots}}\right)}{\sqrt{\dots}}\dots\right)}{\dots}}{5b}$
elliptic	$\frac{x^3\sqrt{d\,x^4+c}}{5b}+\frac{i\left(-\frac{ad-bc}{b^2}-\frac{3c}{5b}\right)\sqrt{c}\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\sqrt{d\,x^4+c}\sqrt{d}}+\frac{a\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{(ad-bc)\left(\dots\right)}{\dots}\right)}{\dots}$
default	$\frac{\frac{x^3\sqrt{d\,x^4+c}}{5}+\frac{2ic^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{5\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\sqrt{d\,x^4+c}\sqrt{d}}}{b}-\frac{a\left(\frac{i\sqrt{d}\sqrt{c}\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\dots}\right)\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\sqrt{d\,x^4+c}}\dots\right)}{\dots}$

[In] int(x^6*(d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)

[Out] 1/5*x^3*(d*x^4+c)^(1/2)/b-1/5/b*(I*(5*a*d-2*b*c)/b*c^(1/2)/(I/c^(1/2)*d^(1/
2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(
d*x^4+c)^(1/2)/d^(1/2)*(EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-EllipticE(

```
x*(I/c^(1/2)*d^(1/2))^(1/2),I))-5/8*(a*d-b*c)*a/b^2*sum(1/_alpha*(-1/((-a*d
+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x
^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*
x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/
c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^
(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx = \text{Timed out}$$

```
[In] integrate(x^6*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx$$

```
[In] integrate(x**6*(d*x**4+c)**(1/2)/(b*x**4+a),x)
[Out] Integral(x**6*sqrt(c + d*x**4)/(a + b*x**4), x)
```

Maxima [F]

$$\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}x^6}{bx^4+a}\,dx$$

```
[In] integrate(x^6*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^4 + c)*x^6/(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}x^6}{bx^4+a}\,dx$$

```
[In] integrate(x^6*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^4 + c)*x^6/(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{x^6\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

```
[In] int((x^6*(c + d*x^4)^(1/2))/(a + b*x^4),x)
[Out] int((x^6*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

3.796 $\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 700

$$\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{x\sqrt{c+dx^4}}{3b} - \frac{(bc-ad)\arctan\left(\frac{\sqrt{\frac{\sqrt{-a}\left(\frac{bc}{a}-d\right)}{\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4b^2\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}} - \frac{(bc-ad)\arctan\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4b^2\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}$$
$$+ \frac{c^{3/4}(bc-2ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{3b\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}}$$
$$- \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{8b^2\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$
$$- \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{8b^2\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$

```
[Out] 1/3*x*(d*x^4+c)^(1/2)/b-1/4*(-a*d+b*c)*arctan(x*((b*c/a-d)*(-a)^(1/2)/b^(1/
2))^(1/2)/(d*x^4+c)^(1/2))/b^2/((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)-1/4*(-a
*d+b*c)*arctan(x*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))/b^2
/((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)+1/3*c^(3/4)*(-2*a*d+b*c)*(cos(2*arct
an(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(
sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4
```

```
+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b/d^(1/4)/(a*d+b*c)/(d*x^4+c)^(1/2)-1/8*
(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*
x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2
)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^
(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)
+x^2*d^(1/2))^2)^(1/2)/b^2/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1
/2))/(d*x^4+c)^(1/2)-1/8*(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1
/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1
/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2
)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^
(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^2/c^(1/4)/d^(1/4)/(b^(1/
2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/(d*x^4+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 904, normalized size of antiderivative = 1.29, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {489, 537, 226, 418, 1231, 1721}

$$\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx =$$

$$-\frac{(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\right.}{8b^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}$$

$$-\frac{\sqrt{-a}\sqrt[4]{d}(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)}{4b^2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}$$

$$-\frac{\sqrt[4]{-a}\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{4b^{7/4}}+\frac{\sqrt[4]{-a}\sqrt{ad-bc}\arctan\left(\frac{\sqrt{ad-bc}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{4b^{7/4}}$$

$$+\frac{(2bc-3ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{6b^2\sqrt[4]{c}\sqrt[4]{d}\sqrt{dx^4+c}}$$

$$-\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4b^2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}$$

$$-\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}\right.\right.}{\sqrt[4]{c}\right.}}{8b^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}$$

$$+\frac{x\sqrt{dx^4+c}}{3b}$$

```
[In] Int[(x^4*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (x*Sqrt[c + d*x^4])/(3*b) - ((-a)^(1/4)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c -
a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*b^(7/4)) + ((-a)^(1/4)*Sq
rt[-(b*c) + a*d]*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c +
 d*x^4])])/(4*b^(7/4)) + ((2*b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c +
 d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)],
 1/2])/(6*b^2*c^(1/4)*d^(1/4)*Sqrt[c + d*x^4]) - (a*((Sqrt[b]*Sqrt[c])/Sqrt
[-a] + Sqrt[d])*d^(1/4)*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4
)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])
/(4*b^2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - (Sqrt[-a]*(Sqrt[b]*Sqrt[c] -
 Sqrt[-a]*Sqrt[d])*d^(1/4)*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*
x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/
2])/(4*b^2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] + Sqrt[
-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c
] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/
(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8
*b^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqr
t[-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt
[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*
Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*
b^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x\sqrt{c+dx^4}}{3b} - \frac{\int \frac{ac+(-2bc+3ad)x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{3b} \\
&= \frac{x\sqrt{c+dx^4}}{3b} + \frac{(2bc-3ad)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{3b^2} - \frac{(a(bc-ad))\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{b^2} \\
&= \frac{x\sqrt{c+dx^4}}{3b} + \frac{(2bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{6b^2\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}} \\
&\quad - \frac{(bc-ad)\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2b^2} - \frac{(bc-ad)\int \frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2b^2}
\end{aligned}
$$

$$= \frac{x\sqrt{c+dx^4}}{3b} + \frac{(2bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{6b^2\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}$$

$$- \frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(bc-ad)\right)\int\frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2b^{3/2}(bc+ad)}$$

$$- \frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(bc-ad)\right)\int\frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2b^{3/2}(bc+ad)}$$

$$- \frac{\left(a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt{d}(bc-ad)\right)\int\frac{1}{\sqrt{c+dx^4}}\,dx}{2b^2(bc+ad)}$$

$$- \frac{\left(\sqrt{-a}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt{d}(bc-ad)\right)\int\frac{1}{\sqrt{c+dx^4}}\,dx}{2b^2(bc+ad)}$$

$$= \frac{x\sqrt{c+dx^4}}{3b} - \frac{\sqrt[4]{-a}\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4b^{7/4}}$$

$$+ \frac{\sqrt[4]{-a}\sqrt{-bc+ad}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4b^{7/4}}$$

$$+ \frac{(2bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{6b^2\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}$$

$$- \frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4b^2\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}}$$

$$- \frac{\sqrt{-a}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4b^2\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}}$$

$$- \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{8b^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}}$$

$$- \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)\right.}{8b^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.44 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.34

$$\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{x\left(\frac{(2bc-3ad)x^4\sqrt{1+\frac{dx^4}{c}}\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{a}+5\left(c+dx^4+\frac{5a^2c^2\operatorname{AppellF1}}{(a+bx^4)\left(-5ac\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2x^4\left(2bc\operatorname{App}\right.\right.}\right.\right.}{15b\sqrt{c+dx^4}}$$

```
[In] Integrate[(x^4*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (x*(((2*b*c - 3*a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((
d*x^4)/c), -((b*x^4)/a)])/a + 5*(c + d*x^4 + (5*a^2*c^2*AppellF1[1/4, 1/2,
1, 5/4, -((d*x^4)/c), -((b*x^4)/a)])/((a + b*x^4)*(-5*a*c*AppellF1[1/4, 1/2
, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2,
9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)
/c), -((b*x^4)/a)]))))))/(15*b*Sqrt[c + d*x^4])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.05 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.43

method	result
risch	$\frac{x\sqrt{d\,x^4+c}}{3b}-\frac{\frac{(3ad-2bc)\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{3(ad-bc)a\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2}{}}{}\right)}{8b^2}}{3b}$
elliptic	$\frac{x\sqrt{d\,x^4+c}}{3b}+\frac{\left(-\frac{ad-bc}{b^2}-\frac{c}{3b}\right)\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}+\frac{a\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{(ad-bc)\left(-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha^2}{2\sqrt{\frac{-ad+bc}{b}}}\right)}{\sqrt{\frac{-ad+bc}{b}}}\right)}{}\right)}{}$
default	$\frac{\frac{x\sqrt{d\,x^4+c}}{3}+\frac{2c\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{3\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}}{b}-\frac{a\left(\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{(ad-bc)\left(-\right)}{}\right)}{}$

[In] int(x^4*(d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)

[Out] 1/3*x*(d*x^4+c)^(1/2)/b-1/3/b*((3*a*d-2*b*c)/b/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-3/8*(a*d-b*c)*a/b^2*sum(1/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a))

Fricas [F]

$$\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}\,x^4}{bx^4+a}\,dx$$

```
[In] integrate(x^4*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
[Out] integral(sqrt(d*x^4 + c)*x^4/(b*x^4 + a), x)
```

Sympy [F]

$$\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx$$

```
[In] integrate(x**4*(d*x**4+c)**(1/2)/(b*x**4+a),x)
[Out] Integral(x**4*sqrt(c + d*x**4)/(a + b*x**4), x)
```

Maxima [F]

$$\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}\,x^4}{bx^4+a}\,dx$$

```
[In] integrate(x^4*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^4 + c)*x^4/(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}\,x^4}{bx^4+a}\,dx$$

```
[In] integrate(x^4*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^4 + c)*x^4/(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx=\int \frac{x^4\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

```
[In] int((x^4*(c + d*x^4)^(1/2))/(a + b*x^4),x)
[Out] int((x^4*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

3.797 $\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 786

$$\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{\sqrt{d}x\sqrt{c+dx^4}}{b\left(\sqrt{c}+\sqrt{d}x^2\right)}+\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4b}+\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4b}$$
$$-\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{b\sqrt{c+dx^4}}$$
$$+\frac{a\sqrt[4]{c}d^{5/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{b(bc+ad)\sqrt{c+dx^4}}$$
$$-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)\right.}{8b^{3/2}\sqrt[4]{c}\left(\sqrt{-a}\sqrt{b}\sqrt{c}-a\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$
$$+\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{8b^{3/2}\sqrt[4]{c}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$

```
[Out] x*d^(1/2)*(d*x^4+c)^(1/2)/b/(c^(1/2)+x^2*d^(1/2))+1/4*arctan(x*((a*d-b*c)/(
-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1
/2)/b+1/4*arctan(x*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*(
(-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/b-c^(1/4)*d^(1/4)*(cos(2*arctan(d^(1/4
```

```
)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticE(sin(2*arc
tan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1
/2)+x^2*d^(1/2))^2)^(1/2)/b/(d*x^4+c)^(1/2)+a*c^(1/4)*d^(5/4)*(cos(2*arctan
(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(si
n(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c
)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b/(a*d+b*c)/(d*x^4+c)^(1/2)-1/8*(-a*d+b*c)
*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4))
)*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/
2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*
d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/
2))^2)^(1/2)/b^(3/2)/c^(1/4)/d^(1/4)/((-a)^(1/2)*b^(1/2)*c^(1/2)-a*d^(1/2))
/(d*x^4+c)^(1/2)+1/8*(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/
cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))
),-1/4*c^(1/2)*(b^(1/2)-(-a)^(1/2)*d^(1/2)/c^(1/2))^2/(-a)^(1/2)/b^(1/2)/d^
(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2
))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^(3/2)/c^(1/4)/d^(1/4)/((-a)^
(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))/(d*x^4+c)^(1/2)
```

Rubi [A] (warning: unable to verify)

Time = 0.85 (sec) , antiderivative size = 1012, normalized size of antiderivative = 1.29, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used

$= \{505, 311, 226, 1210, 504, 1231, 1721\}$

$$\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx = -\frac{(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\right.}{8\sqrt{-a}b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}$$
$$+\frac{\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{4\sqrt[4]{-a}b^{5/4}}+\frac{\sqrt{ad-bc}\arctan\left(\frac{\sqrt{ad-bc}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{4\sqrt[4]{-a}b^{5/4}}$$
$$-\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{b\sqrt{dx^4+c}}$$
$$+\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt{dx^4+c}}$$
$$-\frac{\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4b\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}$$
$$-\frac{\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4b\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}$$
$$+\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{8\sqrt{-a}b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}$$
$$+\frac{\sqrt{d}x\sqrt{dx^4+c}}{b\left(\sqrt{d}x^2+\sqrt{c}\right)}$$

```
[In] Int[(x^2*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^4])/(b*(Sqrt[c] + Sqrt[d]*x^2)) + (Sqrt[b*c - a*d]*
ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(
1/4)*b^(5/4)) + (Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/
4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(1/4)*b^(5/4)) - (c^(1/4)*d^(1/4)*(Sq
rt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticE[
2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(b*Sqrt[c + d*x^4]) + (c^(1/4)*d^(1/4)
*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ellipt
icF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*b*Sqrt[c + d*x^4]) - ((Sqrt[c]
- (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*S
qrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c
```

```
^(1/4)], 1/2])/(4*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[c] + (Sqr
t[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c
 + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)
], 1/2])/(4*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] + Sq
rt[-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqr
t[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])
^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])
/(8*Sqrt[-a]*b^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[
b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(
c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a
]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(
1/4)], 1/2])/(8*Sqrt[-a]*b^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4
])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 505

```
Int[((x_)^2*Sqrt[(c_) + (d_.)*(x_)^4])/((a_) + (b_.)*(x_)^4), x_Symbol] :>
Dist[d/b, Int[x^2/Sqrt[c + d*x^4], x], x] + Dist[(b*c - a*d)/b, Int[x^2/((a
 + b*x^4)*Sqrt[c + d*x^4]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a
*d, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
```

```
}, x] && PosQ[c/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d\int \frac{x^2}{\sqrt{c+dx^4}}\,dx}{b} + \frac{(bc-ad)\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{b} \\
&= \frac{\left(\sqrt{c}\sqrt{d}\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{b} - \frac{\left(\sqrt{c}\sqrt{d}\right)\int \frac{1-\frac{\sqrt{d}x^2}{\sqrt{c}}}{\sqrt{c+dx^4}}\,dx}{b} \\
&\quad - \frac{(bc-ad)\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{2b^{3/2}} + \frac{(bc-ad)\int \frac{1}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{2b^{3/2}}
\end{aligned}$$

$$= \frac{\sqrt{d}x\sqrt{c+dx^4}}{b\left(\sqrt{c}+\sqrt{d}x^2\right)} - \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{b\sqrt{c+dx^4}}$$

$$+ \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2b\sqrt{c+dx^4}}$$

$$- \frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(bc-ad)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{2b(bc+ad)}$$

$$+ \frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(bc-ad)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{2b(bc+ad)}$$

$$- \frac{\left(\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt{d}(bc-ad)\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{2b^{3/2}(bc+ad)}$$

$$- \frac{\left(\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt{d}(bc-ad)\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{2b^{3/2}(bc+ad)}$$

$$
\begin{aligned}
&= \frac{\sqrt{d}x\sqrt{c+dx^4}}{b\left(\sqrt{c}+\sqrt{d}x^2\right)} + \frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4\sqrt[4]{-a}b^{5/4}} \\
&+ \frac{\sqrt{-bc+ad}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4\sqrt[4]{-a}b^{5/4}} \\
&- \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{b\sqrt{c+dx^4}} \\
&+ \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2b\sqrt{c+dx^4}} \\
&- \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4b^{3/2}\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}} \\
&- \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4b^{3/2}\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}} \\
&+ \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{8\sqrt{-a}b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}} \\
&- \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\right.}{8\sqrt{-a}b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.08

$$
\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{x^3\sqrt{c+dx^4}\,\mathrm{AppellF1}\left(\frac{3}{4},-\frac{1}{2},1,\frac{7}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{3a\sqrt{\frac{c+dx^4}{c}}}
$$

```
[In] Integrate[(x^2*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (x^3*Sqrt[c + d*x^4]*AppellF1[3/4, -1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)
])/(3*a*Sqrt[(c + d*x^4)/c])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.68 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.38

method	result
default	$\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{(ad-bc)\left(-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+}}\right.}{\sqrt{\frac{-ad+bc}{b}}}\right.}{}$
elliptic	$\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{(ad-bc)\left(-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+}}\right.}{\sqrt{\frac{-ad+bc}{b}}}\right.}{}$

```
[In] int(x^2*(d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] I*d^(1/2)/b*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/
2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*(EllipticF(x*(I/c^(1/2)*
d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(1/2))^(1/2),I))-1/8/b^2*sum((a*
d-b*c)/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/(
(-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b
/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c
)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a
*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^
4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx = \text{Timed out}$$

```
[In] integrate(x^2*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx$$

```
[In] integrate(x**2*(d*x**4+c)**(1/2)/(b*x**4+a),x)
[Out] Integral(x**2*sqrt(c + d*x**4)/(a + b*x**4), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}\,x^2}{bx^4+a}\,dx$$

```
[In] integrate(x^2*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^4 + c)*x^2/(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}\,x^2}{bx^4+a}\,dx$$

```
[In] integrate(x^2*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^4 + c)*x^2/(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{x^2\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

```
[In] int((x^2*(c + d*x^4)^(1/2))/(a + b*x^4),x)
[Out] int((x^2*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

3.798 $\int \frac{\sqrt{c+dx^4}}{a+bx^4} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 679

$$\int \frac{\sqrt{c+dx^4}}{a+bx^4} \, dx = \frac{(bc-ad)\arctan\left(\frac{\sqrt{\frac{\sqrt{-a}\left(\frac{bc}{a}-d\right)}{\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4ab\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}} + \frac{(bc-ad)\arctan\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4ab\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}$$
$$+ \frac{c^{3/4}d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{(bc+ad)\sqrt{c+dx^4}}$$
$$+ \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8ab\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$
$$+ \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8ab\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$

```
[Out] 1/4*(-a*d+b*c)*arctan(x*((b*c/a-d)*(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2
))/a/b/((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)+1/4*(-a*d+b*c)*arctan(x*((-a*d+
b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))/a/b/((-a*d+b*c)/(-a)^(1/2)/
b^(1/2))^(1/2)+c^(3/4)*d^(3/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/c
os(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),
1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2
)/(a*d+b*c)/(d*x^4+c)^(1/2)+1/8*(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4))
)^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)
*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/
```

```
c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1
/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/b/c^(1/4)/d^(1/4)/
(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))/(d*x^4+c)^(1/2)+1/8*(-a*d+b*c)*(cos(2*
arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*Ellipt
icPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1
/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2)
)*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^
(1/2)/a/b/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/(d*x^4+c)^(1
/2)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 881, normalized size of antiderivative = 1.30, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {415, 226, 418, 1231, 1721}

$$\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx$$

$$= \frac{(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{c\right.}{8ab\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}$$

$$-\frac{\sqrt[4]{d}(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)}{4\sqrt{-a}b\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}$$

$$-\frac{\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{4(-a)^{3/4}b^{3/4}}+\frac{\sqrt{ad-bc}\arctan\left(\frac{\sqrt{ad-bc}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{4(-a)^{3/4}b^{3/4}}$$

$$+\frac{d^{3/4}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt{dx^4+c}}$$

$$+\frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4b\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}$$

$$+\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}\right.}{\sqrt[4]{c}}\right.}{8ab\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}$$

```
[In] Int[Sqrt[c + d*x^4]/(a + b*x^4),x]
```

```
[Out] -1/4*(Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c
 + d*x^4])])/((-a)^(3/4)*b^(3/4)) + (Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[-(b*c)
```

```
 + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(3/4)*b^(3/4)) +
(d^(3/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2
]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*b*c^(1/4)*Sqrt[c + d*x^
4]) + (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(b*c - a*d)*(Sqrt[c]
+ Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcT
an[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) -
((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*d^(1/4)*(b*c - a*d)*(Sqrt[c] + Sqrt[d
]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1
/4)*x)/c^(1/4)], 1/2])/(4*Sqrt[-a]*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) +
 ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2
)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt
[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(
1/4)*x)/c^(1/4)], 1/2])/(8*a*b*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
 + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x
^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c]
 + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1
/4)*x)/c^(1/4)], 1/2])/(8*a*b*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 415

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = \frac{d\int \frac{1}{\sqrt{c+dx^4}}\,dx}{b} - \frac{(-bc+ad)\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{b}$$

$$= \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt{c+dx^4}} - \frac{(-bc+ad)\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2ab} - \frac{(-bc+ad)\int \frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2ab}$$

$$= \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt{c+dx^4}} + \frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(bc-ad)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2a\sqrt{b}(bc+ad)} + \frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(bc-ad)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2a\sqrt{b}(bc+ad)} + \frac{\left(\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt{d}(bc-ad)\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{2b(bc+ad)} - \frac{\left(\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt{d}(bc-ad)\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{2\sqrt{-a}b(bc+ad)}$$

$$= -\frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{3/4}b^{3/4}} + \frac{\sqrt{-bc+ad}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{3/4}b^{3/4}}$$
$$+ \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt{c+dx^4}}$$
$$+ \frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4b\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}}$$
$$- \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4\sqrt{-a}b\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}}$$
$$+ \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{8ab\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}}$$
$$+ \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)\right.}{8ab\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.24

$$\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx$$
$$= \frac{5acx\sqrt{c+dx^4}\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{(a+bx^4)\left(5ac\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2x^4\left(-2bc\,\text{AppellF1}\left(\frac{5}{4},-\frac{1}{2},2,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+ad\,\text{Ap}\right.\right.}$$

```
[In] Integrate[Sqrt[c + d*x^4]/(a + b*x^4),x]
```

```
[Out] (5*a*c*x*Sqrt[c + d*x^4]*AppellF1[1/4, -1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4
)/a)])/((a + b*x^4)*(5*a*c*AppellF1[1/4, -1/2, 1, 5/4, -((d*x^4)/c), -((b*x
^4)/a)] + 2*x^4*(-2*b*c*AppellF1[5/4, -1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)
/a)] + a*d*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.93 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.40

method	result
default	$\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}} - \frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{(ad-bc)\left(-\frac{\mathrm{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^3b\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+}}{}\right.}{\_\alpha^3}}{8b^2}$
elliptic	$\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}} - \frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{(ad-bc)\left(-\frac{\mathrm{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^3b\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+}}{}\right.}{\_\alpha^3}}{8b^2}$

```
[In] int((d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] d/b/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*
d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)
-1/8/b^2*sum((a*d-b*c)/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_al
pha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2)
)^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x
^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/
d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)))
,_alpha=RootOf(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx = \text{Timed out}$$

```
[In] integrate((d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx$$

```
[In] integrate((d*x**4+c)**(1/2)/(b*x**4+a),x)
[Out] Integral(sqrt(c + d*x**4)/(a + b*x**4), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}}{bx^4+a}\,dx$$

```
[In] integrate((d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^4 + c)/(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}}{bx^4+a}\,dx$$

```
[In] integrate((d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^4 + c)/(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

```
[In] int((c + d*x^4)^(1/2)/(a + b*x^4),x)
[Out] int((c + d*x^4)^(1/2)/(a + b*x^4), x)
```

3.799 $\int \frac{\sqrt{c+dx^4}}{x^2(a+bx^4)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 809

$$\int \frac{\sqrt{c+dx^4}}{x^2(a+bx^4)}\,dx = -\frac{\sqrt{c+dx^4}}{ax}+\frac{\sqrt{d}x\sqrt{c+dx^4}}{a\left(\sqrt{c}+\sqrt{d}x^2\right)} - \frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4a} - \frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4a} - \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{a\sqrt{c+dx^4}} + \frac{bc^{5/4}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{a(bc+ad)\sqrt{c+dx^4}} - \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)\right.}{8\sqrt{b}\sqrt[4]{c}\left((-a)^{3/2}\sqrt{b}\sqrt{c}+a^2\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}} - \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{8a\sqrt{b}\sqrt[4]{c}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$

```
[Out] -(d*x^4+c)^(1/2)/a/x+x*d^(1/2)*(d*x^4+c)^(1/2)/a/(c^(1/2)+x^2*d^(1/2))-1/4*
arctan(x*((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*((a*d-b*c)/(
-a)^(1/2)/b^(1/2))^(1/2)/a-1/4*arctan(x*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/
2)/(d*x^4+c)^(1/2))*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/a-c^(1/4)*d^(1/4)
```

```
*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4))
)*EllipticE(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1
/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/(d*x^4+c)^(1/2)+b*c^(5/4)*
d^(1/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c
^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x
^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/(a*d+b*c)/(d*x^4+c)
^(1/2)-1/8*(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arct
an(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(
1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2
^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^4+
c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/c^(1/4)/d^(1/4)/b^(1/2)/((-a)^(3/2)*b^(1/
2)*c^(1/2)+a^2*d^(1/2))/(d*x^4+c)^(1/2)-1/8*(-a*d+b*c)*(cos(2*arctan(d^(1/4
)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*ar
ctan(d^(1/4)*x/c^(1/4))),-1/4*c^(1/2)*(b^(1/2)-(-a)^(1/2)*d^(1/2)/c^(1/2))^
2/(-a)^(1/2)/b^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^
(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/c^(1/
4)/d^(1/4)/b^(1/2)/((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))/(d*x^4+c)^(1/2)
```

Rubi [A] (warning: unable to verify)

Time = 1.24 (sec) , antiderivative size = 1031, normalized size of antiderivative = 1.27, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{486, 598, 311, 226, 1210, 504, 1231, 1721\}$

$$\int \frac{\sqrt{c+dx^4}}{x^2\left(a+bx^4\right)}\,dx = -\frac{(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\mathrm{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\right.}{8(-a)^{3/2}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}$$

$$+\frac{\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{4(-a)^{5/4}\sqrt[4]{b}}+\frac{\sqrt{ad-bc}\arctan\left(\frac{\sqrt{ad-bc}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{4(-a)^{5/4}\sqrt[4]{b}}$$

$$-\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{a\sqrt{dx^4+c}}$$

$$+\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2a\sqrt{dx^4+c}}$$

$$+\frac{\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4a\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}$$

$$+\frac{\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4a\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}$$

$$+\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\mathrm{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{8(-a)^{3/2}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}$$

$$-\frac{\sqrt{dx^4+c}}{ax}+\frac{\sqrt{d}x\sqrt{dx^4+c}}{a\left(\sqrt{d}x^2+\sqrt{c}\right)}$$

```
[In] Int[Sqrt[c + d*x^4]/(x^2*(a + b*x^4)),x]
```

```
[Out] -(Sqrt[c + d*x^4]/(a*x)) + (Sqrt[d]*x*Sqrt[c + d*x^4])/(a*(Sqrt[c] + Sqrt[d
]*x^2)) + (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*S
qrt[c + d*x^4])])/(4*(-a)^(5/4)*b^(1/4)) + (Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt
[-(b*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(5/4)*b^(1
/4)) - (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] +
 Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(a*Sqrt[c +
 d*x^4]) + (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[
c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*a*Sq
rt[c + d*x^4]) + ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - a*d
```

```
)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ellip
ticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a*c^(1/4)*(b*c + a*d)*Sqrt[c +
 d*x^4]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - a*d)*(Sqr
t[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2
*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4
]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqrt[d]
*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*
Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[
(d^(1/4)*x)/c^(1/4)], 1/2])/(8*(-a)^(3/2)*Sqrt[b]*c^(1/4)*d^(1/4)*(b*c + a*
d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)*(
Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Elliptic
Pi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[
d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*(-a)^(3/2)*Sqrt[b]*c^(1/4)*d^(
1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{c+dx^4}}{ax} + \frac{\int \frac{x^2(-bc+2ad+bdx^4)}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{a}$$

$$= -\frac{\sqrt{c+dx^4}}{ax} + \frac{\int \left(\frac{dx^2}{\sqrt{c+dx^4}} + \frac{(-bc+ad)x^2}{(a+bx^4)\sqrt{c+dx^4}}\right)dx}{a}$$

$$= -\frac{\sqrt{c+dx^4}}{ax} + \frac{d\int \frac{x^2}{\sqrt{c+dx^4}}\,dx}{a} + \frac{(-bc+ad)\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{a}$$

$$= -\frac{\sqrt{c+dx^4}}{ax} + \frac{\left(\sqrt{c}\sqrt{d}\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{a} - \frac{\left(\sqrt{c}\sqrt{d}\right)\int \frac{1-\frac{\sqrt{d}x^2}{\sqrt{c}}}{\sqrt{c+dx^4}}\,dx}{a}$$
$$+ \frac{(bc-ad)\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{2a\sqrt{b}} - \frac{(bc-ad)\int \frac{1}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{2a\sqrt{b}}$$

$$= -\frac{\sqrt{c+dx^4}}{ax} + \frac{\sqrt{d}x\sqrt{c+dx^4}}{a\left(\sqrt{c}+\sqrt{d}x^2\right)}$$
$$- \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{a\sqrt{c+dx^4}}$$
$$+ \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2a\sqrt{c+dx^4}}$$
$$+ \frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(bc-ad)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{2a(bc+ad)}$$
$$- \frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(bc-ad)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{2a(bc+ad)}$$
$$+ \frac{\left(\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt{d}(bc-ad)\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{2a\sqrt{b}(bc+ad)}$$
$$+ \frac{\left(\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt{d}(bc-ad)\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{2a\sqrt{b}(bc+ad)}$$

$$= -\frac{\sqrt{c+dx^4}}{ax} + \frac{\sqrt{d}x\sqrt{c+dx^4}}{a\left(\sqrt{c}+\sqrt{d}x^2\right)} + \frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{5/4}\sqrt[4]{b}}$$

$$+ \frac{\sqrt{-bc+ad}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{5/4}\sqrt[4]{b}}$$

$$- \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{a\sqrt{c+dx^4}}$$

$$+ \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2a\sqrt{c+dx^4}}$$

$$+ \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4a\sqrt{b}\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}}$$

$$+ \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4a\sqrt{b}\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}}$$

$$+ \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{8(-a)^{3/2}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}}$$

$$- \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\right.}{8(-a)^{3/2}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{c+dx^4}}{x^2\left(a+bx^4\right)}\,dx$$

$$= \frac{-21a(c+dx^4)-7(bc-2ad)x^4\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+3bdx^8\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{7}{4}\right.}{21a^2x\sqrt{c+dx^4}}$$

```
[In] Integrate[Sqrt[c + d*x^4]/(x^2*(a + b*x^4)),x]
[Out] (-21*a*(c + d*x^4) - 7*(b*c - 2*a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[3/4,
1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)] + 3*b*d*x^8*Sqrt[1 + (d*x^4)/c]*Ap
```

```
pellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x^4)/a)])/(21*a^2*x*Sqrt[c + d
*x^4])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.40 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.39

method	result
risch	$-\frac{\sqrt{d\,x^4+c}}{a x}+\frac{\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}+\frac{(ad-bc)\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha}{2\sqrt{\frac{-ad+bc}{b}}}\right.}{\sqrt{\frac{-ad+bc}{b}}}}{}\right.}{}}{a}$
elliptic	$-\frac{\sqrt{d\,x^4+c}}{a x}+\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{a\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{(-ad+bc)\left(-\frac{\operatorname{arctanh}\left(\right.}{}\right.}{}$
default	$\frac{-\frac{\sqrt{d\,x^4+c}}{x}+\frac{2i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}}{a}-\frac{b\left(\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(\right.\right.}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}\right.}{}$

```
[In] int((d*x^4+c)^(1/2)/x^2/(b*x^4+a),x,method=_RETURNVERBOSE)

[Out] -(d*x^4+c)^(1/2)/a/x+1/a*(I*d^(1/2)*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/
c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*
(EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(1/2))^(
1/2),I))+1/8*(a*d-b*c)/b*sum(1/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*
(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d
^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(
```

```
1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^
(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(
1/2))),_alpha=RootOf(_Z^4*b+a)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{x^2\left(a+bx^4\right)}\,dx = \text{Timed out}$$

```
[In] integrate((d*x^4+c)^(1/2)/x^2/(b*x^4+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^4}}{x^2\left(a+bx^4\right)}\,dx = \int \frac{\sqrt{c+dx^4}}{x^2\left(a+bx^4\right)}\,dx$$

```
[In] integrate((d*x**4+c)**(1/2)/x**2/(b*x**4+a),x)
[Out] Integral(sqrt(c + d*x**4)/(x**2*(a + b*x**4)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^4}}{x^2\left(a+bx^4\right)}\,dx = \int \frac{\sqrt{dx^4+c}}{(bx^4+a)x^2}\,dx$$

```
[In] integrate((d*x^4+c)^(1/2)/x^2/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^4}}{x^2\left(a+bx^4\right)}\,dx = \int \frac{\sqrt{dx^4+c}}{(bx^4+a)x^2}\,dx$$

```
[In] integrate((d*x^4+c)^(1/2)/x^2/(b*x^4+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{x^2\left(a+bx^4\right)}\,dx=\int \frac{\sqrt{d\,x^4+c}}{x^2\,\left(b\,x^4+a\right)}\,dx$$

```
[In] int((c + d*x^4)^(1/2)/(x^2*(a + b*x^4)),x)
[Out] int((c + d*x^4)^(1/2)/(x^2*(a + b*x^4)), x)
```

3.800 $\int \frac{\sqrt{c+dx^4}}{x^4(a+bx^4)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 703

$$\int \frac{\sqrt{c+dx^4}}{x^4(a+bx^4)}\,dx = -\frac{\sqrt{c+dx^4}}{3ax^3} - \frac{(bc-ad)\arctan\left(\frac{\sqrt{\frac{\sqrt{-a}\left(\frac{bc}{a}-d\right)}{\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4a^2\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}} - \frac{(bc-ad)\arctan\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4a^2\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}$$
$$- \frac{d^{3/4}(2bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{3a\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}}$$
$$- \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{8a^2\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$
$$- \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)\right.}{8a^2\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$

```
[Out] -1/3*(d*x^4+c)^(1/2)/a/x^3-1/4*(-a*d+b*c)*arctan(x*((b*c/a-d)*(-a)^(1/2)/b^
(1/2))^(1/2)/(d*x^4+c)^(1/2))/a^2/((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)-1/4*
(-a*d+b*c)*arctan(x*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))/
a^2/((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)-1/3*d^(3/4)*(-a*d+2*b*c)*(cos(2*a
rctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*Ellipti
cF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*
```

```
x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/c^(1/4)/(a*d+b*c)/(d*x^4+c)^(1/2)-1
/8*(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/
4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(
1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*
(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1
/2)+x^2*d^(1/2))^2)^(1/2)/a^2/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d
^(1/2))/(d*x^4+c)^(1/2)-1/8*(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)
^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c
^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(
1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)
*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a^2/c^(1/4)/d^(1/4)/(b^
(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/(d*x^4+c)^(1/2)
```

Rubi [A] (warning: unable to verify)

Time = 0.95 (sec) , antiderivative size = 893, normalized size of antiderivative = 1.27, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {486, 537, 226, 418, 1231, 1721}

$$\int \frac{\sqrt{c+dx^4}}{x^4(a+bx^4)}\,dx =$$

$$-\frac{(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-\right.}{8a^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}$$

$$-\frac{\sqrt[4]{d}(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)}{4(-a)^{3/2}\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}$$

$$-\frac{\sqrt[4]{b}\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{4(-a)^{7/4}}+\frac{\sqrt[4]{b}\sqrt{ad-bc}\arctan\left(\frac{\sqrt{ad-bc}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{4(-a)^{7/4}}$$

$$-\frac{d^{3/4}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{6a\sqrt[4]{c}\sqrt{dx^4+c}}$$

$$-\frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4a\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}$$

$$-\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{8a^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}$$

$$-\frac{\sqrt{dx^4+c}}{3ax^3}$$

```
[In] Int[Sqrt[c + d*x^4]/(x^4*(a + b*x^4)),x]
```

```
[Out] -1/3*Sqrt[c + d*x^4]/(a*x^3) - (b^(1/4)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c -
a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(7/4)) + (b^(1/4)*Sq
rt[-(b*c) + a*d]*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c +
 d*x^4])])/(4*(-a)^(7/4)) - (d^(3/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^
4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2]
)/(6*a*c^(1/4)*Sqrt[c + d*x^4]) - (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d
^(1/4)*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt
[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a*c^(1/4)*(b*
c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*d^(1/4)*(
b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)
^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*(-a)^(3/2)*c^(1/4)*(b
*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(b*c -
 a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*E
llipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqr
t[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*a^2*c^(1/4)*d^(1/4)*
(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c
 - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]
*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt
[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*a^2*c^(1/4)*d^(1/4)*(
b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{c+dx^4}}{3ax^3} + \frac{\int \frac{-3bc+2ad-bdx^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{3a}$$

$$= -\frac{\sqrt{c+dx^4}}{3ax^3} - \frac{d\int \frac{1}{\sqrt{c+dx^4}}\,dx}{3a} - \frac{(bc-ad)\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{a}$$

$$= -\frac{\sqrt{c+dx^4}}{3ax^3} - \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{6a\sqrt[4]{c}\sqrt{c+dx^4}} - \frac{(bc-ad)\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2a^2} - \frac{(bc-ad)\int \frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2a^2}$$

$$= -\frac{\sqrt{c+dx^4}}{3ax^3} - \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{6a\sqrt[4]{c}\sqrt{c+dx^4}}$$

$$- \frac{\left(\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(bc-ad)\right)\int\frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2a^2(bc+ad)}$$

$$- \frac{\left(\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(bc-ad)\right)\int\frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2a^2(bc+ad)}$$

$$- \frac{\left(\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt{d}(bc-ad)\right)\int\frac{1}{\sqrt{c+dx^4}}\,dx}{2a(bc+ad)}$$

$$- \frac{\left(\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt{d}(bc-ad)\right)\int\frac{1}{\sqrt{c+dx^4}}\,dx}{2(-a)^{3/2}(bc+ad)}$$

$$= -\frac{\sqrt{c+dx^4}}{3ax^3} - \frac{\sqrt[4]{b}\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{7/4}}$$

$$+ \frac{\sqrt[4]{b}\sqrt{-bc+ad}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{7/4}}$$

$$- \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{6a\sqrt[4]{c}\sqrt{c+dx^4}}$$

$$- \frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4a\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}}$$

$$- \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4(-a)^{3/2}\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}}$$

$$- \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{8a^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}}$$

$$- \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\right.}{8a^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.29 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{c+dx^4}}{x^4\left(a+bx^4\right)}\,dx$$
$$=\frac{-bdx^8\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+\frac{a\left(25ac\left(ac+4bcx^4-adx^4+bdx^8\right)\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-10x}{\left(a+bx^4\right)\left(-5ac\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2x^4}}{15a^2x^3\sqrt{c+dx^4}}$$

```
[In] Integrate[Sqrt[c + d*x^4]/(x^4*(a + b*x^4)),x]
```

```
[Out] (-(b*d*x^8*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c), -((
b*x^4)/a)]) + (a*(25*a*c*(a*c + 4*b*c*x^4 - a*d*x^4 + b*d*x^8)*AppellF1[1/4
, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] - 10*x^4*(a + b*x^4)*(c + d*x^4)
*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*Appell
F1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))/((a + b*x^4)*(-5*a*c*Ap
pellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF
1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1,
 9/4, -((d*x^4)/c), -((b*x^4)/a)]))))/(15*a^2*x^3*Sqrt[c + d*x^4])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.80 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.42

method	result
risch	$-\frac{\sqrt{d\,x^4+c}}{3a\,x^3}-\frac{\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}+\frac{(-3ad+3bc)\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^3b\sqrt{1\cdots}}{\cdots}}{\cdots}\right)}{8b}}{3a}$
elliptic	$-\frac{\sqrt{d\,x^4+c}}{3a\,x^3}-\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{3a\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{(-ad+bc)\left(-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^3\cdots}{\cdots}\right)}{\cdots}}{8ba\cdots}$
default	$\frac{-\frac{\sqrt{d\,x^4+c}}{3x^3}+\frac{2d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{3\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}}{a}-\frac{b\left(\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{(ad-bc)\left(-\frac{\mathrm{ar}\cdots}{\cdots}\right)}{\cdots}}{\cdots}\right)}{\cdots}$

[In] int((d*x^4+c)^(1/2)/x^4/(b*x^4+a),x,method=_RETURNVERBOSE)

[Out] -1/3*(d*x^4+c)^(1/2)/a/x^3-1/3/a*(d/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)+1/8*(-3*a*d+3*b*c)/b*sum(1/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a)))

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{x^4\left(a+bx^4\right)}\,dx=\text{Timed out}$$

```
[In] integrate((d*x^4+c)^(1/2)/x^4/(b*x^4+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^4}}{x^4\left(a+bx^4\right)}\,dx=\int \frac{\sqrt{c+dx^4}}{x^4\left(a+bx^4\right)}\,dx$$

```
[In] integrate((d*x**4+c)**(1/2)/x**4/(b*x**4+a),x)
[Out] Integral(sqrt(c + d*x**4)/(x**4*(a + b*x**4)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^4}}{x^4\left(a+bx^4\right)}\,dx=\int \frac{\sqrt{dx^4+c}}{(bx^4+a)x^4}\,dx$$

```
[In] integrate((d*x^4+c)^(1/2)/x^4/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^4), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^4}}{x^4\left(a+bx^4\right)}\,dx=\int \frac{\sqrt{dx^4+c}}{(bx^4+a)x^4}\,dx$$

```
[In] integrate((d*x^4+c)^(1/2)/x^4/(b*x^4+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{x^4\,(a+bx^4)}\,dx=\int \frac{\sqrt{d\,x^4+c}}{x^4\,\left(b\,x^4+a\right)}\,dx$$

```
[In] int((c + d*x^4)^(1/2)/(x^4*(a + b*x^4)),x)
[Out] int((c + d*x^4)^(1/2)/(x^4*(a + b*x^4)), x)
```

3.801 $\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4} dx$

Optimal result

Integrand size = 28, antiderivative size = 71

$$\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4} dx = \frac{2(ex)^{5/2}\sqrt{c+dx^4}\,\text{AppellF1}\left(\frac{5}{8},1,-\frac{1}{2},\frac{13}{8},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{5ae\sqrt{1+\frac{dx^4}{c}}}$$

```
[Out] 2/5*(e*x)^(5/2)*AppellF1(5/8,1,-1/2,13/8,-b*x^4/a,-d*x^4/c)*(d*x^4+c)^(1/2)
/a/e/(1+d*x^4/c)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {477, 525, 524}

$$\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4} dx = \frac{2(ex)^{5/2}\sqrt{c+dx^4}\,\text{AppellF1}\left(\frac{5}{8},1,-\frac{1}{2},\frac{13}{8},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{5ae\sqrt{\frac{dx^4}{c}+1}}$$

```
[In] Int[((e*x)^(3/2)*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (2*(e*x)^(5/2)*Sqrt[c + d*x^4]*AppellF1[5/8, 1, -1/2, 13/8, -((b*x^4)/a), -
((d*x^4)/c)])/(5*a*e*Sqrt[1 + (d*x^4)/c])
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
```

```
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\text{Subst}\left(\int \frac{x^4\sqrt{c+\frac{dx^8}{e^4}}}{a+\frac{bx^8}{e^4}}\,dx, x, \sqrt{ex}\right)}{e}\\
&= \frac{\left(2\sqrt{c+dx^4}\right)\text{Subst}\left(\int \frac{x^4\sqrt{1+\frac{dx^8}{ce^4}}}{a+\frac{bx^8}{e^4}}\,dx, x, \sqrt{ex}\right)}{e\sqrt{1+\frac{dx^4}{c}}}\\
&= \frac{2(ex)^{5/2}\sqrt{c+dx^4}F_1\left(\frac{5}{8};1,-\frac{1}{2};\frac{13}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{5ae\sqrt{1+\frac{dx^4}{c}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 11.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{2x(ex)^{3/2}\sqrt{c+dx^4}\,\text{AppellF1}\left(\frac{5}{8},-\frac{1}{2},1,\frac{13}{8},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{5a\sqrt{\frac{c+dx^4}{c}}}$$

```
[In] Integrate[((e*x)^(3/2)*Sqrt[c + d*x^4])/(a + b*x^4),x]
[Out] (2*x*(e*x)^(3/2)*Sqrt[c + d*x^4]*AppellF1[5/8, -1/2, 1, 13/8, -((d*x^4)/c),
 -((b*x^4)/a)])/(5*a*Sqrt[(c + d*x^4)/c])
```

Maple [F]

$$\int \frac{(ex)^{\frac{3}{2}} \sqrt{d\,x^4 + c}}{b\,x^4 + a}dx$$

[In] int((e*x)^(3/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x)

[Out] int((e*x)^(3/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x)

Fricas [F]

$$\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}(ex)^{\frac{3}{2}}}{bx^4+a}\,dx$$

[In] integrate((e*x)^(3/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")

[Out] integral(sqrt(d*x^4 + c)*sqrt(e*x)*e*x/(b*x^4 + a), x)

Sympy [F]

$$\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{(ex)^{\frac{3}{2}}\sqrt{c+dx^4}}{a+bx^4}\,dx$$

[In] integrate((e*x)**(3/2)*(d*x**4+c)**(1/2)/(b*x**4+a),x)

[Out] Integral((e*x)**(3/2)*sqrt(c + d*x**4)/(a + b*x**4), x)

Maxima [F]

$$\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}(ex)^{\frac{3}{2}}}{bx^4+a}\,dx$$

[In] integrate((e*x)^(3/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")

[Out] integrate(sqrt(d*x^4 + c)*(e*x)^(3/2)/(b*x^4 + a), x)

Giac [F]

$$\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}(ex)^{\frac{3}{2}}}{bx^4+a}\,dx$$

```
[In] integrate((e*x)^(3/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^4 + c)*(e*x)^(3/2)/(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{(e\,x)^{3/2}\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

```
[In] int(((e*x)^(3/2)*(c + d*x^4)^(1/2))/(a + b*x^4),x)
[Out] int(((e*x)^(3/2)*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

3.802 $\int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4} dx$

Optimal result

Integrand size = 28, antiderivative size = 71

$$\int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4} dx = \frac{2(ex)^{3/2}\sqrt{c+dx^4}\,\text{AppellF1}\left(\frac{3}{8},1,-\frac{1}{2},\frac{11}{8},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{3ae\sqrt{1+\frac{dx^4}{c}}}$$

```
[Out] 2/3*(e*x)^(3/2)*AppellF1(3/8,1,-1/2,11/8,-b*x^4/a,-d*x^4/c)*(d*x^4+c)^(1/2)
/a/e/(1+d*x^4/c)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {477, 525, 524}

$$\int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4} dx = \frac{2(ex)^{3/2}\sqrt{c+dx^4}\,\text{AppellF1}\left(\frac{3}{8},1,-\frac{1}{2},\frac{11}{8},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{3ae\sqrt{\frac{dx^4}{c}+1}}$$

```
[In] Int[(Sqrt[e*x]*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (2*(e*x)^(3/2)*Sqrt[c + d*x^4]*AppellF1[3/8, 1, -1/2, 11/8, -((b*x^4)/a), -
((d*x^4)/c)])/(3*a*e*Sqrt[1 + (d*x^4)/c])
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
```

```
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\text{Subst}\left(\int \frac{x^2\sqrt{c+\frac{dx^8}{e^4}}}{a+\frac{bx^8}{e^4}}\,dx, x, \sqrt{ex}\right)}{e}\\
&= \frac{\left(2\sqrt{c+dx^4}\right)\text{Subst}\left(\int \frac{x^2\sqrt{1+\frac{dx^8}{ce^4}}}{a+\frac{bx^8}{e^4}}\,dx, x, \sqrt{ex}\right)}{e\sqrt{1+\frac{dx^4}{c}}}\\
&= \frac{2(ex)^{3/2}\sqrt{c+dx^4}F_1\left(\frac{3}{8};1,-\frac{1}{2};\frac{11}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{3ae\sqrt{1+\frac{dx^4}{c}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 11.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{2x\sqrt{ex}\sqrt{c+dx^4}\,\text{AppellF1}\left(\frac{3}{8},-\frac{1}{2},1,\frac{11}{8},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{3a\sqrt{\frac{c+dx^4}{c}}}$$

```
[In] Integrate[(Sqrt[e*x]*Sqrt[c + d*x^4])/(a + b*x^4),x]
[Out] (2*x*Sqrt[e*x]*Sqrt[c + d*x^4]*AppellF1[3/8, -1/2, 1, 11/8, -((d*x^4)/c), -
((b*x^4)/a)])/(3*a*Sqrt[(c + d*x^4)/c])
```

Maple [F]

$$\int \frac{\sqrt{ex}\,\sqrt{d\,x^{4}+c}}{b\,x^{4}+a}dx$$

```
[In] int((e*x)^(1/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x)
[Out] int((e*x)^(1/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4}\,dx = \text{Timed out}$$

```
[In] integrate((e*x)^(1/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4}\,dx$$

```
[In] integrate((e*x)**(1/2)*(d*x**4+c)**(1/2)/(b*x**4+a),x)
[Out] Integral(sqrt(e*x)*sqrt(c + d*x**4)/(a + b*x**4), x)
```

Maxima [F]

$$\int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}\sqrt{ex}}{bx^4+a}\,dx$$

```
[In] integrate((e*x)^(1/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^4 + c)*sqrt(e*x)/(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}\sqrt{ex}}{bx^4+a}\,dx$$

```
[In] integrate((e*x)^(1/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^4 + c)*sqrt(e*x)/(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{e\,x}\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

```
[In] int(((e*x)^(1/2)*(c + d*x^4)^(1/2))/(a + b*x^4),x)
[Out] int(((e*x)^(1/2)*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

3.803 $\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}(a+bx^4)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 69

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\,(a+bx^4)}\,dx = \frac{2\sqrt{ex}\sqrt{c+dx^4}\,\text{AppellF1}\left(\frac{1}{8},1,-\frac{1}{2},\frac{9}{8},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae\sqrt{1+\frac{dx^4}{c}}}$$

```
[Out] 2*AppellF1(1/8,1,-1/2,9/8,-b*x^4/a,-d*x^4/c)*(e*x)^(1/2)*(d*x^4+c)^(1/2)/a/
e/(1+d*x^4/c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {477, 441, 440}

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\,(a+bx^4)}\,dx = \frac{2\sqrt{ex}\sqrt{c+dx^4}\,\text{AppellF1}\left(\frac{1}{8},1,-\frac{1}{2},\frac{9}{8},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae\sqrt{\frac{dx^4}{c}+1}}$$

```
[In] Int[Sqrt[c + d*x^4]/(Sqrt[e*x]*(a + b*x^4)),x]
```

```
[Out] (2*Sqrt[e*x]*Sqrt[c + d*x^4]*AppellF1[1/8, 1, -1/2, 9/8, -((b*x^4)/a), -((d
*x^4)/c)])/(a*e*Sqrt[1 + (d*x^4)/c])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
```

```
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\text{Subst}\left(\int \frac{\sqrt{c+\frac{dx^8}{e^4}}}{a+\frac{bx^8}{e^4}} \, dx, x, \sqrt{ex}\right)}{e} \\
&= \frac{\left(2\sqrt{c+dx^4}\right) \text{Subst}\left(\int \frac{\sqrt{1+\frac{dx^8}{ce^4}}}{a+\frac{bx^8}{e^4}} \, dx, x, \sqrt{ex}\right)}{e\sqrt{1+\frac{dx^4}{c}}} \\
&= \frac{2\sqrt{ex}\sqrt{c+dx^4}F_1\left(\frac{1}{8};1,-\frac{1}{2};\frac{9}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae\sqrt{1+\frac{dx^4}{c}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 11.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\,(a+bx^4)} \, dx = \frac{2x\sqrt{c+dx^4}\,\text{AppellF1}\left(\frac{1}{8},-\frac{1}{2},1,\frac{9}{8},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{a\sqrt{ex}\sqrt{\frac{c+dx^4}{c}}}$$

```
[In] Integrate[Sqrt[c + d*x^4]/(Sqrt[e*x]*(a + b*x^4)),x]
[Out] (2*x*Sqrt[c + d*x^4]*AppellF1[1/8, -1/2, 1, 9/8, -((d*x^4)/c), -((b*x^4)/a)
])/(a*Sqrt[e*x]*Sqrt[(c + d*x^4)/c])
```

Maple [F]

$$\int \frac{\sqrt{d\,x^{4}+c}}{\sqrt{e x}\,\left(b\,x^{4}+a\right)}dx$$

```
[In] int((d*x^4+c)^(1/2)/(e*x)^(1/2)/(b*x^4+a),x)
[Out] int((d*x^4+c)^(1/2)/(e*x)^(1/2)/(b*x^4+a),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\,(a+bx^4)}\,dx = \text{Timed out}$$

```
[In] integrate((d*x^4+c)^(1/2)/(e*x)^(1/2)/(b*x^4+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\,(a+bx^4)}\,dx = \int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\,(a+bx^4)}\,dx$$

```
[In] integrate((d*x**4+c)**(1/2)/(e*x)**(1/2)/(b*x**4+a),x)
[Out] Integral(sqrt(c + d*x**4)/(sqrt(e*x)*(a + b*x**4)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\,(a+bx^4)}\,dx = \int \frac{\sqrt{dx^4+c}}{(bx^4+a)\sqrt{ex}}\,dx$$

```
[In] integrate((d*x^4+c)^(1/2)/(e*x)^(1/2)/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*sqrt(e*x)), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\,(a+bx^4)}\,dx = \int \frac{\sqrt{dx^4+c}}{(bx^4+a)\sqrt{ex}}\,dx$$

```
[In] integrate((d*x^4+c)^(1/2)/(e*x)^(1/2)/(b*x^4+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*sqrt(e*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\,(a+bx^4)}\,dx = \int \frac{\sqrt{d\,x^4+c}}{\sqrt{e\,x}\,\left(b\,x^4+a\right)}\,dx$$

```
[In] int((c + d*x^4)^(1/2)/((e*x)^(1/2)*(a + b*x^4)),x)
[Out] int((c + d*x^4)^(1/2)/((e*x)^(1/2)*(a + b*x^4)), x)
```

3.804 $\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}(a+bx^4)} dx$

Optimal result

Integrand size = 28, antiderivative size = 69

$$\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}\,(a+bx^4)}\,dx=-\frac{2\sqrt{c+dx^4}\,\text{AppellF1}\left(-\frac{1}{8},1,-\frac{1}{2},\frac{7}{8},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae\sqrt{ex}\sqrt{1+\frac{dx^4}{c}}}$$

```
[Out] -2*AppellF1(-1/8,1,-1/2,7/8,-b*x^4/a,-d*x^4/c)*(d*x^4+c)^(1/2)/a/e/(e*x)^(1
/2)/(1+d*x^4/c)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {477, 525, 524}

$$\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}\,(a+bx^4)}\,dx=-\frac{2\sqrt{c+dx^4}\,\text{AppellF1}\left(-\frac{1}{8},1,-\frac{1}{2},\frac{7}{8},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae\sqrt{ex}\sqrt{\frac{dx^4}{c}+1}}$$

```
[In] Int[Sqrt[c + d*x^4]/((e*x)^(3/2)*(a + b*x^4)),x]
```

```
[Out] (-2*Sqrt[c + d*x^4]*AppellF1[-1/8, 1, -1/2, 7/8, -((b*x^4)/a), -((d*x^4)/c)
])/(a*e*Sqrt[e*x]*Sqrt[1 + (d*x^4)/c])
```

Rule 477

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(1
```

```
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\text{Subst}\left(\int \frac{\sqrt{c+\frac{dx^8}{e^4}}}{x^2\left(a+\frac{bx^8}{e^4}\right)}\,dx, x, \sqrt{ex}\right)}{e}\\
&= \frac{\left(2\sqrt{c+dx^4}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{dx^8}{ce^4}}}{x^2\left(a+\frac{bx^8}{e^4}\right)}\,dx, x, \sqrt{ex}\right)}{e\sqrt{1+\frac{dx^4}{c}}}\\
&= -\frac{2\sqrt{c+dx^4}F_1\left(-\frac{1}{8};1,-\frac{1}{2};\frac{7}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae\sqrt{ex}\sqrt{1+\frac{dx^4}{c}}}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 143 vs. $2(69) = 138$.

Time = 11.15 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.07

$$\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}\,(a+bx^4)}\,dx = \frac{x\left(-70a(c+dx^4)-10(bc-4ad)x^4\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{7}{8},\frac{1}{2},1,\frac{15}{8},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+\right.}{35a^2(ex)^{3/2}\sqrt{c+dx^4}}$$

```
[In] Integrate[Sqrt[c + d*x^4]/((e*x)^(3/2)*(a + b*x^4)),x]
```

```
[Out] (x*(-70*a*(c + d*x^4) - 10*(b*c - 4*a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[7
/8, 1/2, 1, 15/8, -((d*x^4)/c), -((b*x^4)/a)] + 14*b*d*x^8*Sqrt[1 + (d*x^4)
/c]*AppellF1[15/8, 1/2, 1, 23/8, -((d*x^4)/c), -((b*x^4)/a)]))/(35*a^2*(e*x
)^(3/2)*Sqrt[c + d*x^4])
```

Maple [F]

$$\int \frac{\sqrt{d\,x^{4}+c}}{\left(ex\right)^{\frac{3}{2}}\left(b\,x^{4}+a\right)}dx$$

```
[In] int((d*x^4+c)^(1/2)/(e*x)^(3/2)/(b*x^4+a),x)
[Out] int((d*x^4+c)^(1/2)/(e*x)^(3/2)/(b*x^4+a),x)
```

Fricas [F]

$$\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}(a+bx^4)}\,dx=\int \frac{\sqrt{dx^4+c}}{(bx^4+a)(ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x^4+c)^(1/2)/(e*x)^(3/2)/(b*x^4+a),x, algorithm="fricas")
[Out] integral(sqrt(d*x^4 + c)*sqrt(e*x)/(b*e^2*x^6 + a*e^2*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}(a+bx^4)}\,dx=\int \frac{\sqrt{c+dx^4}}{(ex)^{\frac{3}{2}}(a+bx^4)}\,dx$$

```
[In] integrate((d*x**4+c)**(1/2)/(e*x)**(3/2)/(b*x**4+a),x)
[Out] Integral(sqrt(c + d*x**4)/((e*x)**(3/2)*(a + b*x**4)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}(a+bx^4)}\,dx=\int \frac{\sqrt{dx^4+c}}{(bx^4+a)(ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x^4+c)^(1/2)/(e*x)^(3/2)/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*(e*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}\,(a+bx^4)}\,dx = \int \frac{\sqrt{dx^4+c}}{(bx^4+a)\,(ex)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x^4+c)^(1/2)/(e*x)^(3/2)/(b*x^4+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*(e*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}\,(a+bx^4)}\,dx = \int \frac{\sqrt{d\,x^4+c}}{{(e\,x)}^{3/2}\,\left(b\,x^4+a\right)}\,dx$$

```
[In] int((c + d*x^4)^(1/2)/((e*x)^(3/2)*(a + b*x^4)),x)
[Out] int((c + d*x^4)^(1/2)/((e*x)^(3/2)*(a + b*x^4)), x)
```

3.805 $\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx = -\frac{(bc+ad)\sqrt{c+dx^4}}{2b^2d^2} + \frac{(c+dx^4)^{3/2}}{6bd^2} - \frac{a^2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}}$$

```
[Out] 1/6*(d*x^4+c)^(3/2)/b/d^2-1/2*a^2*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c
)^(1/2))/b^(5/2)/(-a*d+b*c)^(1/2)-1/2*(a*d+b*c)*(d*x^4+c)^(1/2)/b^2/d^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {457, 90, 65, 214}

$$\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx = -\frac{a^2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}} - \frac{\sqrt{c+dx^4}(ad+bc)}{2b^2d^2} + \frac{(c+dx^4)^{3/2}}{6bd^2}$$

```
[In] Int[x^11/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/2*((b*c + a*d)*Sqrt[c + d*x^4])/(b^2*d^2) + (c + d*x^4)^(3/2)/(6*b*d^2)
- (a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*b^(5/2)*Sqrt[
b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \left(\frac{-bc-ad}{b^2d\sqrt{c+dx}}+\frac{a^2}{b^2(a+bx)\sqrt{c+dx}}+\frac{\sqrt{c+dx}}{bd}\right)\,dx, x, x^4\right)\\
&= -\frac{(bc+ad)\sqrt{c+dx^4}}{2b^2d^2}+\frac{\left(c+dx^4\right)^{3/2}}{6bd^2}+\frac{a^2\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4b^2}\\
&= -\frac{(bc+ad)\sqrt{c+dx^4}}{2b^2d^2}+\frac{\left(c+dx^4\right)^{3/2}}{6bd^2}+\frac{a^2\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2b^2d}\\
&= -\frac{(bc+ad)\sqrt{c+dx^4}}{2b^2d^2}+\frac{\left(c+dx^4\right)^{3/2}}{6bd^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx=\frac{\sqrt{c+dx^4}(-2bc-3ad+bdx^4)}{6b^2d^2}+\frac{a^2\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{2b^{5/2}\sqrt{-bc+ad}}$$

```
[In] Integrate[x^11/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (Sqrt[c + d*x^4]*(-2*b*c - 3*a*d + b*d*x^4))/(6*b^2*d^2) + (a^2*ArcTan[(Sqr
t[b]*Sqrt[c + d*x^4])/Sqrt[-(b*c) + a*d]])/(2*b^(5/2)*Sqrt[-(b*c) + a*d])
```

Maple [A] (verified)

Time = 5.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$\frac{\arctan\left(\frac{b\sqrt{d\,x^4+c}}{\sqrt{(ad-bc)b}}\right)a^2d^2-\left(\left(-\frac{b\,x^4}{3}+a\right)d+\frac{2bc}{3}\right)\sqrt{d\,x^4+c}\,\sqrt{(ad-bc)b}}{2\sqrt{(ad-bc)b}\,b^2d^2}$
risch	$-\frac{(-bd\,x^4+3ad+2bc)\sqrt{d\,x^4+c}}{6d^2b^2}-\frac{a^2\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\sqrt{-}\right.}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right.}{4b^3\sqrt{-\frac{ad-bc}{b}}}$
default	$-\frac{\sqrt{d\,x^4+c}\,(-d\,x^4+2c)}{6b\,d^2}-\frac{a\sqrt{d\,x^4+c}}{2b^2d}+\frac{a^2\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-}}{}}}{x^2+\frac{\sqrt{-ab}}{b}}\right.}{4b\sqrt{-\frac{ad-bc}{b}}}\right.}{}$
elliptic	$\frac{x^4\sqrt{d\,x^4+c}}{6bd}-\frac{c\sqrt{d\,x^4+c}}{3b\,d^2}-\frac{a\sqrt{d\,x^4+c}}{2b^2d}-\frac{a^2\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-}}{}}}{x^2-\frac{\sqrt{-ab}}{b}}\right.}{4b^3\sqrt{-\frac{ad-bc}{b}}}$

```
[In] int(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(arctan(b*(d*x^4+c)^(1/2)/((a*d-b*c)*b)^(1/2))*a^2*d^2-((-1/3*b*x^4+a)*
d+2/3*b*c)*(d*x^4+c)^(1/2)*((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)/b^2/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.78

$$\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \left[\frac{3\sqrt{b^2c-abd}a^2d^2\log\left(\frac{bdx^4+2bc-ad-2\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)-2\left(2b^3c^2+ab^2cd-3a^2bd^2-(b^3cd-ab^2d^2)x^4\right)\sqrt{dx^4+c}}{12\,(b^4cd^2-ab^3d^3)}\right.$$

```
[In] integrate(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*
x^4 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) - 2*(2*b^3*c^2 + a*b^2*c*d - 3*a
^2*b*d^2 - (b^3*c*d - a*b^2*d^2)*x^4)*sqrt(d*x^4 + c))/(b^4*c*d^2 - a*b^3*d
^3), 1/6*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(sqrt(d*x^4 + c)*sqrt(-b^2*c
 + a*b*d)/(b*d*x^4 + b*c)) - (2*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2 - (b^3*c*
d - a*b^2*d^2)*x^4)*sqrt(d*x^4 + c))/(b^4*c*d^2 - a*b^3*d^3)]
```

Sympy [F]

$$\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

```
[In] integrate(x**11/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**11/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.02

$$\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{a^2\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}b^2} + \frac{(dx^4+c)^{\frac{3}{2}}b^2d^4 - 3\sqrt{dx^4+c}b^2cd^4 - 3\sqrt{dx^4+c}abd^5}{6\,b^3d^6}$$

```
[In] integrate(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*a^2*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*b^2) + 1/6*((d*x^4 + c)^(3/2)*b^2*d^4 - 3*sqrt(d*x^4 + c)*b^2*c*d^4 - 3*s
qrt(d*x^4 + c)*a*b*d^5)/(b^3*d^6)
```

Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{(d\,x^4+c)^{3/2}}{6\,b\,d^2} - \left(\frac{c}{b\,d^2} + \frac{2\,a\,d^3 - 2\,b\,c\,d^2}{4\,b^2\,d^4}\right)\sqrt{d\,x^4+c} + \frac{a^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^4+c}}{\sqrt{a\,d-b\,c}}\right)}{2\,b^{5/2}\,\sqrt{a\,d-b\,c}}$$

```
[In] int(x^11/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] (c + d*x^4)^(3/2)/(6*b*d^2) - (c/(b*d^2) + (2*a*d^3 - 2*b*c*d^2)/(4*b^2*d^4
))*(c + d*x^4)^(1/2) + (a^2*atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c)^(1
/2)))/(2*b^(5/2)*(a*d - b*c)^(1/2))
```

3.806 $\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\sqrt{c+dx^4}}{2bd} + \frac{a\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}\sqrt{bc-ad}}$$

```
[Out] 1/2*a*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)^
(1/2)+1/2*(d*x^4+c)^(1/2)/b/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {457, 81, 65, 214}

$$\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{a\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}\sqrt{bc-ad}} + \frac{\sqrt{c+dx^4}}{2bd}$$

```
[In] Int[x^7/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] Sqrt[c + d*x^4]/(2*b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a
*d]])/(2*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{x}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)\\
&= \frac{\sqrt{c+dx^4}}{2bd} - \frac{a\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4b}\\
&= \frac{\sqrt{c+dx^4}}{2bd} - \frac{a\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2bd}\\
&= \frac{\sqrt{c+dx^4}}{2bd} + \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx=\frac{1}{2}\left(\frac{\sqrt{c+dx^4}}{bd}-\frac{a\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{b^{3/2}\sqrt{-bc+ad}}\right)$$

```
[In] Integrate[x^7/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (Sqrt[c + d*x^4]/(b*d) - (a*ArcTan[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[-(b*c) +
a*d]])/(b^(3/2)*Sqrt[-(b*c) + a*d]))/2
```

Maple [A] (verified)

Time = 4.92 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$\frac{-\arctan\left(\frac{b\sqrt{d\,x^4+c}}{\sqrt{(ad-bc)b}}\right)ad+\sqrt{d\,x^4+c}\,\sqrt{(ad-bc)b}}{2bd\sqrt{(ad-bc)b}}$
risch	$\frac{\sqrt{d\,x^4+c}}{2bd}+\frac{a\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4b^2\sqrt{-\frac{ad-bc}{b}}}+\frac{a\ln\left(\cdots\right.}{\cdots}$
elliptic	$\frac{\sqrt{d\,x^4+c}}{2bd}+\frac{a\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4b^2\sqrt{-\frac{ad-bc}{b}}}+\frac{a\ln\left(\cdots\right.}{\cdots}$
default	$\frac{\sqrt{d\,x^4+c}}{2bd}-\frac{a\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4b\sqrt{-\frac{ad-bc}{b}}}-\frac{\ln\left(\cdots\right.}{\cdots}\right.}{b}$

```
[In] int(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-arctan(b*(d*x^4+c)^(1/2)/((a*d-b*c)*b)^(1/2))*a*d+(d*x^4+c)^(1/2)*((a
*d-b*c)*b)^(1/2))/b/d/((a*d-b*c)*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.77

$$\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \left[\frac{\sqrt{b^2c-abd}ad\log\left(\frac{bdx^4+2\,bc-ad+2\,\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)+2\,\sqrt{dx^4+c}(b^2c-abd)}{4\,(b^3cd-ab^2d^2)}, -\frac{\sqrt{-b^2c+abd}ad\arctan\left(\frac{\sqrt{dx^4+c}\sqrt{-b^2c+abd}}{bdx^4+bc}\right)-\sqrt{dx^4+c}(b^2c-abd)}{2\,(b^3cd-ab^2d^2)}\right]$$

```
[In] integrate(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2*c - a*b*d)*a*d*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c
)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) + 2*sqrt(d*x^4 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2), -1/2*(sqrt(-b^2*c + a*b*d)*a*d*arctan(sqrt(d*x^4 + c)*
sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) - sqrt(d*x^4 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2)]
```

Sympy [F]

$$\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

```
[In] integrate(x**7/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**7/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx = -\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b} - \frac{\sqrt{dx^4+c}}{b}}{2\,d}$$

```
[In] integrate(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(a*d*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b
*d)*b) - sqrt(d*x^4 + c)/b)/d
```

Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\sqrt{d\,x^4+c}}{2\,b\,d} - \frac{a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^4+c}}{\sqrt{a\,d-b\,c}}\right)}{2\,b^{3/2}\,\sqrt{a\,d-b\,c}}$$

```
[In] int(x^7/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] (c + d*x^4)^(1/2)/(2*b*d) - (a*atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c)
^(1/2)))/(2*b^(3/2)*(a*d - b*c)^(1/2))
```

3.807 $\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\sqrt{bc-ad}}$$

```
[Out] -1/2*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(1/2)/(-a*d+b*c)^(
1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {455, 65, 214}

$$\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\sqrt{bc-ad}}$$

```
[In] Int[x^3/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]]/(Sqrt[b]*Sqrt[b*c -
 a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2d}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{2\sqrt{b}\sqrt{-bc+ad}}$$

```
[In] Integrate[x^3/((a + b*x^4)*Sqrt[c + d*x^4]),x]
[Out] ArcTan[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[-(b*c) + a*d]]/(2*Sqrt[b]*Sqrt[-(b*c)
 + a*d])
```

Maple [A] (verified)

Time = 4.96 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{\arctan\left(\frac{b\sqrt{d\,x^4+c}}{\sqrt{(ad-bc)b}}\right)}{2\sqrt{(ad-bc)b}}$
default	$-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4b\sqrt{-\frac{ad-bc}{b}}} - \frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+}{}\right.}{}$
elliptic	$-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4b\sqrt{-\frac{ad-bc}{b}}} - \frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+}{}\right.}{}$

```
[In] int(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^4+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.55

$$\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \left[\frac{\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)}{4\,\sqrt{b^2c-abd}}, \frac{\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^4+c}\sqrt{-b^2c+abd}}{bdx^4+bc}\right)}{2\,(b^2c-abd)}\right]$$

```
[In] integrate(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
[Out] [1/4*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*sqrt(b^2*c - a*b*d))/(b
*x^4 + a))/sqrt(b^2*c - a*b*d), 1/2*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^4
+ c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c))/(b^2*c - a*b*d)]
```

Sympy [A] (verification not implemented)

Time = 5.88 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.71

$$\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \begin{cases} \frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2b\sqrt{\frac{ad-bc}{b}}} & \text{for } d \neq 0 \\ \begin{cases} \frac{x^4}{4a\sqrt{c}} & \text{for } b = 0 \\ \tilde{\infty}x^4 & \text{for } \sqrt{c} = 0 \\ \frac{\log\left(4a\sqrt{c}+4b\sqrt{c}x^4\right)}{4b\sqrt{c}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**3/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Piecewise((atan(sqrt(c + d*x**4)/sqrt((a*d - b*c)/b))/(2*b*sqrt((a*d - b*c)
/b)), Ne(d, 0)), (Piecewise((x**4/(4*a*sqrt(c)), Eq(b, 0)), (zoo*x**4, Eq(s
qrt(c), 0)), (log(4*a*sqrt(c) + 4*b*sqrt(c)*x**4)/(4*b*sqrt(c)), True)), Tr
ue))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}}$$

```
[In] integrate(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\operatorname{atan}\left(\frac{b\sqrt{d\,x^4+c}}{\sqrt{a\,b\,d-b^2\,c}}\right)}{2\sqrt{a\,b\,d-b^2\,c}}$$

```
[In] int(x^3/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
[Out] atan((b*(c + d*x^4)^(1/2))/(a*b*d - b^2*c)^(1/2))/(2*(a*b*d - b^2*c)^(1/2))
```

3.808 $\int \frac{1}{x(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 85

$$\int \frac{1}{x\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a\sqrt{c}}+\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{bc-ad}}$$

```
[Out] -1/2*arctanh((d*x^4+c)^(1/2)/c^(1/2))/a/c^(1/2)+1/2*arctanh(b^(1/2)*(d*x^4+
c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/a/(-a*d+b*c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {457, 88, 65, 214}

$$\int \frac{1}{x\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx=\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{bc-ad}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a\sqrt{c}}$$

```
[In] Int[1/(x*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/2*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]]/(a*Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[
b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*a*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^4\right)}{4a} - \frac{b\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4a} \\
&= \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2ad} - \frac{b\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2ad} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a\sqrt{c}} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\,(a+bx^4)\sqrt{c+dx^4}}\,dx = -\frac{\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{\sqrt{c}}}{2a}$$

```
[In] Integrate[1/(x*(a + b*x^4)*Sqrt[c + d*x^4]),x]
[Out] -1/2*((Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[-(b*c) + a*d]])/Sqrt[-
(b*c) + a*d] + ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]]/Sqrt[c])/a
```

Maple [A] (verified)

Time = 4.90 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

method	result
pseudoelliptic	$-\frac{b \arctan\left(\frac{b\sqrt{d\,x^4+c}}{\sqrt{(ad-bc)b}}\right)\sqrt{c}+\operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}}{\sqrt{c}}\right)\sqrt{(ad-bc)b}}{2a\sqrt{(ad-bc)b}\,\sqrt{c}}$
elliptic	$-\frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)}{2a\sqrt{c}}+\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-b}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4a\sqrt{-\frac{ad-bc}{b}}}$
default	$-\frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)}{2a\sqrt{c}}-\frac{b\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4b\sqrt{-\frac{ad-bc}{b}}}\right.}{}$

```
[In] int(1/x/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(b*arctan(b*(d*x^4+c)^(1/2)/((a*d-b*c)*b)^(1/2))*c^(1/2)+arctanh((d*x^
4+c)^(1/2)/c^(1/2))*((a*d-b*c)*b)^(1/2))/a/((a*d-b*c)*b)^(1/2)/c^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 431, normalized size of antiderivative = 5.07

$$\int \frac{1}{x\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$
$$=\left[\frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^4+2\,bc-ad+2\,\sqrt{dx^4+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^4+a}\right)+\sqrt{c}\log\left(\frac{dx^4-2\,\sqrt{dx^4+c}\sqrt{c}+2\,c}{x^4}\right)}{4\,ac},\ \frac{2\,c\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{d}}{}\right.}{}\right.$$

```
[In] integrate(1/x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(c*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*
(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + sqrt(c)*log((d*x^4 - 2*sqrt
(d*x^4 + c)*sqrt(c) + 2*c)/x^4))/(a*c), 1/4*(2*c*sqrt(-b/(b*c - a*d))*arcta
n(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c)) + sqrt
```

```
(c)*log((d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4))/(a*c), 1/4*(c*sqrt(
b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*(b*c - a*d)*s
qrt(b/(b*c - a*d)))/(b*x^4 + a)) + 2*sqrt(-c)*arctan(sqrt(d*x^4 + c)*sqrt(-
c)/c))/(a*c), 1/2*(c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^4 + c)*(b*c - a*
d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c)) + sqrt(-c)*arctan(sqrt(d*x^4 + c)*
sqrt(-c)/c))/(a*c)]
```

Sympy [A] (verification not implemented)

Time = 5.31 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{1}{x\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx = \begin{cases} \frac{2\left(-\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4a\sqrt{\frac{ad-bc}{b}}}+\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{-c}}\right)}{4a\sqrt{-c}}\right)}{d} & \text{for}\ d\neq 0 \\ \frac{\operatorname{atan}\left(\frac{2\left(\frac{a}{2b}+x^4\right)}{\sqrt{-\frac{a^2}{b^2}}}\right)}{2b\sqrt{c}\sqrt{-\frac{a^2}{b^2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/x/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Piecewise((2*(-d*atan(sqrt(c + d*x**4)/sqrt((a*d - b*c)/b))/(4*a*sqrt((a*d
- b*c)/b)) + d*atan(sqrt(c + d*x**4)/sqrt(-c))/(4*a*sqrt(-c)))/d, Ne(d, 0))
, (atan(2*(a/(2*b) + x**4)/sqrt(-a**2/b**2))/(2*b*sqrt(c)*sqrt(-a**2/b**2))
, True))
```

Maxima [F]

$$\int \frac{1}{x\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx = \int \frac{1}{(bx^4+a)\sqrt{dx^4+c}x}\,dx$$

```
[In] integrate(1/x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int \frac{1}{x\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx = -\frac{b\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}a} + \frac{\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{2a\sqrt{-c}}$$

```
[In] integrate(1/x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] -1/2*b*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*a) + 1/2*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a*sqrt(-c))
```

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 652, normalized size of antiderivative = 7.67

$$\int \frac{1}{x\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx = -\frac{\operatorname{atanh}\left(\frac{\sqrt{d\,x^4+c}}{\sqrt{c}}\right)}{2\,a\,\sqrt{c}} - \frac{\operatorname{atan}\left(\frac{\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(b^3\,d^2\,\sqrt{d\,x^4+c}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,a^2\,b^2\,d^3-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^4+c}\,\sqrt{b^2\,c-a\,b\,d}}{4\,\left(a^2\,d-a\,b\,c\right)}\right)}{4\,\left(a^2\,d-a\,b\,c\right)}\right)\,1{}\mathrm{i}}{4\,\left(a^2\,d-a\,b\,c\right)}+\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(b^3\,d^2\,\sqrt{d\,x^4+c}+\cdots\right.}{\cdots}}{\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(b^3\,d^2\,\sqrt{d\,x^4+c}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,a^2\,b^2\,d^3-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^4+c}\,\sqrt{b^2\,c-a\,b\,d}}{4\,\left(a^2\,d-a\,b\,c\right)}\right)}{4\,\left(a^2\,d-a\,b\,c\right)}\right)}{4\,\left(a^2\,d-a\,b\,c\right)}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(b^3\,d^2\,\sqrt{d\,x^4+c}+\cdots\right.}{\cdots}}\right)\cdots}{2\,\left(a^2\,d-a\,b\,c\right)}$$

```
[In] int(1/(x*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
[Out] - atanh((c + d*x^4)^(1/2)/c^(1/2))/(2*a*c^(1/2)) - (atan((((b^2*c - a*b*d)^
(1/2)*(b^3*d^2*(c + d*x^4)^(1/2) - ((b^2*c - a*b*d)^(1/2)*(2*a^2*b^2*d^3 -
((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^4)^(1/2)*(b^2*c - a*b*d)^(1/2)
)/(4*(a^2*d - a*b*c))))/(4*(a^2*d - a*b*c)))*1i)/(4*(a^2*d - a*b*c)) + ((b^
2*c - a*b*d)^(1/2)*(b^3*d^2*(c + d*x^4)^(1/2) + ((b^2*c - a*b*d)^(1/2)*(2*a
^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^4)^(1/2)*(b^2*c -
 a*b*d)^(1/2))/(4*(a^2*d - a*b*c))))/(4*(a^2*d - a*b*c)))*1i)/(4*(a^2*d - a
*b*c)))/(((b^2*c - a*b*d)^(1/2)*(b^3*d^2*(c + d*x^4)^(1/2) - ((b^2*c - a*b*
d)^(1/2)*(2*a^2*b^2*d^3 - ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^4)^(
1/2)*(b^2*c - a*b*d)^(1/2))/(4*(a^2*d - a*b*c))))/(4*(a^2*d - a*b*c))))/(4*
(a^2*d - a*b*c)) - ((b^2*c - a*b*d)^(1/2)*(b^3*d^2*(c + d*x^4)^(1/2) + ((b^
2*c - a*b*d)^(1/2)*(2*a^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c
+ d*x^4)^(1/2)*(b^2*c - a*b*d)^(1/2))/(4*(a^2*d - a*b*c))))/(4*(a^2*d - a*b
*c))))/(4*(a^2*d - a*b*c))))*(b^2*c - a*b*d)^(1/2)*1i)/(2*(a^2*d - a*b*c))
```

3.809 $\int \frac{1}{x^5(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 117

$$\int \frac{1}{x^5\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx=-\frac{\sqrt{c+dx^4}}{4acx^4}+\frac{(2bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2c^{3/2}}-\frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2\sqrt{bc-ad}}$$

```
[Out] 1/4*(a*d+2*b*c)*arctanh((d*x^4+c)^(1/2)/c^(1/2))/a^2/c^(3/2)-1/2*b^(3/2)*ar
ctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(1/2)-1/4*(d
*x^4+c)^(1/2)/a/c/x^4
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 105, 162, 65, 214}

$$\int \frac{1}{x^5\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx=-\frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2c^{3/2}}-\frac{\sqrt{c+dx^4}}{4acx^4}$$

```
[In] Int[1/(x^5*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/4*Sqrt[c + d*x^4]/(a*c*x^4) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^4]/Sqr
t[c]])/(4*a^2*c^(3/2)) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*
c - a*d]])/(2*a^2*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^2(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)$$

$$= -\frac{\sqrt{c+dx^4}}{4acx^4} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2bc+ad)+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4ac}$$

$$= -\frac{\sqrt{c+dx^4}}{4acx^4} + \frac{b^2\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4a^2} - \frac{(2bc+ad)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^4\right)}{8a^2c}$$

$$= -\frac{\sqrt{c+dx^4}}{4acx^4} + \frac{b^2\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2a^2d} - \frac{(2bc+ad)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{4a^2cd}$$

$$= -\frac{\sqrt{c+dx^4}}{4acx^4} + \frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2c^{3/2}} - \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2\sqrt{bc-ad}}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^5(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{-\frac{a\sqrt{c+dx^4}}{cx^4} + \frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}} + \frac{(2bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{c^{3/2}}}{4a^2}$$

```
[In] Integrate[1/(x^5*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (-((a*Sqrt[c + d*x^4])/(c*x^4)) + (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^4
])/Sqrt[-(b*c) + a*d]])/Sqrt[-(b*c) + a*d] + ((2*b*c + a*d)*ArcTanh[Sqrt[c
+ d*x^4]/Sqrt[c]])/c^(3/2))/(4*a^2)
```

Maple [A] (verified)

Time = 4.98 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{\frac{2b^2\arctan\left(\frac{b\sqrt{dx^4+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}-\frac{a\sqrt{dx^4+c}}{cx^4}+\frac{(ad+2bc)\operatorname{arctanh}\left(\frac{\sqrt{dx^4+c}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}}{4a^2}$
risch	$-\frac{\sqrt{dx^4+c}}{4acx^4}-\frac{-\frac{(ad+2bc)\ln\left(\frac{2c+2\sqrt{c}\sqrt{dx^4+c}}{x^2}\right)}{2a\sqrt{c}}-\frac{2b^2c\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\cdots}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4b\sqrt{-\frac{ad-bc}{b}}}\cdots\right)}{\cdots}}{\cdots}$
elliptic	$-\frac{\sqrt{dx^4+c}}{4acx^4}+\frac{d\ln\left(\frac{2c+2\sqrt{c}\sqrt{dx^4+c}}{x^2}\right)}{4ac^{\frac{3}{2}}}+\frac{b\ln\left(\frac{2c+2\sqrt{c}\sqrt{dx^4+c}}{x^2}\right)}{2a^2\sqrt{c}}-\frac{b\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\cdots}{x^2+\cdots}\right)}{4a^2\sqrt{-\cdots}}\cdots$
default	$\frac{-\frac{\sqrt{dx^4+c}}{4cx^4}+\frac{d\ln\left(\frac{2c+2\sqrt{c}\sqrt{dx^4+c}}{x^2}\right)}{4c^{\frac{3}{2}}}}{a}+\frac{b\ln\left(\frac{2c+2\sqrt{c}\sqrt{dx^4+c}}{x^2}\right)}{2a^2\sqrt{c}}+\frac{b^2\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\cdots}{x^2+\cdots}\right)}{4b\sqrt{-\cdots}}\cdots\right)}{\cdots}$

```
[In] int(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a^2*(2*b^2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^4+c)^(1/2)/((a*d-b*c)*b)^(
1/2))-a/c*(d*x^4+c)^(1/2)/x^4+(a*d+2*b*c)/c^(3/2)*arctanh((d*x^4+c)^(1/2)/c
^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 565, normalized size of antiderivative = 4.83

$$\int \frac{1}{x^5 (a+bx^4) \sqrt{c+dx^4}}\,dx$$
$$= \left[\frac{2\,bc^2x^4\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^4+a}\right)+(2\,bc+ad)\sqrt{c}x^4\log\left(\frac{dx^4+2\,\sqrt{dx^4+c}\sqrt{c}+2\,c}{x^4}\right)-2\,\sqrt{c}}{8\,a^2c^2x^4}\right.$$
$$-\frac{4\,bc^2x^4\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^4+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^4+bc}\right)-(2\,bc+ad)\sqrt{c}x^4\log\left(\frac{dx^4+2\,\sqrt{dx^4+c}\sqrt{c}+2\,c}{x^4}\right)+2\,\sqrt{dx^4}}{8\,a^2c^2x^4}$$
$$-\frac{2\,bc^2x^4\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^4+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^4+bc}\right)+(2\,bc+ad)\sqrt{-c}x^4\arctan\left(\frac{\sqrt{dx^4+c}\sqrt{-c}}{c}\right)+\sqrt{dx^4+c}}{4\,a^2c^2x^4}$$

```
[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
[Out] [1/8*(2*b*c^2*x^4*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d
*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + (2*b*c + a*d)*sqr
t(c)*x^4*log((d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) - 2*sqrt(d*x^4
+ c)*a*c)/(a^2*c^2*x^4), -1/8*(4*b*c^2*x^4*sqrt(-b/(b*c - a*d))*arctan(-sqr
t(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c)) - (2*b*c + a
*d)*sqrt(c)*x^4*log((d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) + 2*sqrt
(d*x^4 + c)*a*c)/(a^2*c^2*x^4), 1/4*(b*c^2*x^4*sqrt(b/(b*c - a*d))*log((b*d
*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*
x^4 + a)) - (2*b*c + a*d)*sqrt(-c)*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) -
 sqrt(d*x^4 + c)*a*c)/(a^2*c^2*x^4), -1/4*(2*b*c^2*x^4*sqrt(-b/(b*c - a*d))
*arctan(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c))
+ (2*b*c + a*d)*sqrt(-c)*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) + sqrt(d*x^
4 + c)*a*c)/(a^2*c^2*x^4)]
```

Sympy [F]

$$\int \frac{1}{x^5 (a+bx^4) \sqrt{c+dx^4}}\,dx = \int \frac{1}{x^5 (a+bx^4) \sqrt{c+dx^4}}\,dx$$

```
[In] integrate(1/x**5/(b*x**4+a)/(d*x**4+c)**(1/2),x)
[Out] Integral(1/(x**5*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^5 (a+bx^4) \sqrt{c+dx^4}}\,dx = \int \frac{1}{(bx^4+a)\sqrt{dx^4+c}x^5}\,dx$$

```
[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^5), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^5 (a+bx^4) \sqrt{c+dx^4}}\,dx = \frac{b^2 \arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}a^2} - \frac{(2\,bc+ad)\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{4\,a^2\sqrt{-c}c} - \frac{\sqrt{dx^4+c}}{4\,acx^4}$$

```
[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] 1/2*b^2*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*a^2) - 1/4*(2*b*c + a*d)*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a^2*sqrt(-c)*c
) - 1/4*sqrt(d*x^4 + c)/(a*c*x^4)
```

Mupad [B] (verification not implemented)

Time = 9.65 (sec) , antiderivative size = 396, normalized size of antiderivative = 3.38

$$\int \frac{1}{x^5 (a+bx^4) \sqrt{c+dx^4}}\,dx = \frac{\ln\left(\sqrt{d\,x^4+c}\,\left(b^4\,c-a\,b^3\,d\right)^{3/2}+b^6\,c^2+a^2\,b^4\,d^2-2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{4\,a^3\,d-4\,a^2\,b\,c} - \frac{\ln\left(\sqrt{d\,x^4+c}\,\left(b^4\,c-a\,b^3\,d\right)^{3/2}-b^6\,c^2-a^2\,b^4\,d^2+2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{4\,\left(a^3\,d-a^2\,b\,c\right)} - \frac{\sqrt{d\,x^4+c}}{4\,a\,c\,x^4} - \frac{\mathrm{atan}\left(\frac{b^4\,d^4\,\sqrt{d\,x^4+c}\,3{}\mathrm{i}}{16\,\sqrt{c^3}\,\left(\frac{3\,b^4\,d^4}{16\,c}+\frac{5\,a\,b^3\,d^5}{32\,c^2}+\frac{a^2\,b^2\,d^6}{32\,c^3}\right)}+\frac{b^2\,d^6\,\sqrt{d\,x^4+c}\,1{}\mathrm{i}}{32\,\sqrt{c^3}\,\left(\frac{5\,b^3\,d^5}{32\,a}+\frac{b^2\,d^6}{32\,c}+\frac{3\,b^4\,c\,d^4}{16\,a^2}\right)}+\frac{b^3\,d^5\,\sqrt{d\,x^4+c}\,5{}\mathrm{i}}{32\,\sqrt{c^3}\,\left(\frac{3\,b^4\,d^4}{16\,a}+\frac{5\,b^3\,d^5}{32\,c}+\frac{a\,b^2\,d^6}{32\,c^2}\right)}\right)\,\left(a\,d+2\,b\,c\right)}{4\,a^2\,\sqrt{c^3}}$$

```
[In] int(1/(x^5*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] (log((c + d*x^4)^(1/2)*(b^4*c - a*b^3*d)^(3/2) + b^6*c^2 + a^2*b^4*d^2 - 2*
a*b^5*c*d)*(b^4*c - a*b^3*d)^(1/2))/(4*a^3*d - 4*a^2*b*c) - (log((c + d*x^4
)^(1/2)*(b^4*c - a*b^3*d)^(3/2) - b^6*c^2 - a^2*b^4*d^2 + 2*a*b^5*c*d)*(b^4
*c - a*b^3*d)^(1/2))/(4*(a^3*d - a^2*b*c)) - (c + d*x^4)^(1/2)/(4*a*c*x^4)
- (atan((b^4*d^4*(c + d*x^4)^(1/2)*3i)/(16*(c^3)^(1/2)*((3*b^4*d^4)/(16*c)
+ (5*a*b^3*d^5)/(32*c^2) + (a^2*b^2*d^6)/(32*c^3))) + (b^2*d^6*(c + d*x^4)^
(1/2)*1i)/(32*(c^3)^(1/2)*((5*b^3*d^5)/(32*a) + (b^2*d^6)/(32*c) + (3*b^4*c
*d^4)/(16*a^2))) + (b^3*d^5*(c + d*x^4)^(1/2)*5i)/(32*(c^3)^(1/2)*((3*b^4*d
^4)/(16*a) + (5*b^3*d^5)/(32*c) + (a*b^2*d^6)/(32*c^2))))*(a*d + 2*b*c)*1i)
/(4*a^2*(c^3)^(1/2))
```

3.810 $\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 123

$$\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{x^2\sqrt{c+dx^4}}{4bd}+\frac{a^{3/2}\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b^2\sqrt{bc-ad}} - \frac{(bc+2ad)\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^2d^{3/2}}$$

```
[Out] -1/4*(2*a*d+b*c)*arctanh(x^2*d^(1/2)/(d*x^4+c)^(1/2))/b^2/d^(3/2)+1/2*a^(3/
2)*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))/b^2/(-a*d+b*c)^(1/2
)+1/4*x^2*(d*x^4+c)^(1/2)/b/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {476, 490, 537, 223, 212, 385, 211}

$$\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{a^{3/2}\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b^2\sqrt{bc-ad}} - \frac{(2ad+bc)\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^2d^{3/2}}+\frac{x^2\sqrt{c+dx^4}}{4bd}$$

```
[In] Int[x^9/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (x^2*Sqrt[c + d*x^4])/(4*b*d) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt
[a]*Sqrt[c + d*x^4])])/(2*b^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTanh[(Sq
rt[d]*x^2)/Sqrt[c + d*x^4]])/(4*b^2*d^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)\\
&= \frac{x^2\sqrt{c+dx^4}}{4bd} - \frac{\text{Subst}\left(\int \frac{ac+(bc+2ad)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4bd}\\
&= \frac{x^2\sqrt{c+dx^4}}{4bd} + \frac{a^2\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{2b^2} - \frac{(bc+2ad)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4b^2d}\\
&= \frac{x^2\sqrt{c+dx^4}}{4bd} + \frac{a^2\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{2b^2}\\
&\quad - \frac{(bc+2ad)\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{4b^2d}\\
&= \frac{x^2\sqrt{c+dx^4}}{4bd} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b^2\sqrt{bc-ad}} - \frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^2d^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$
\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\frac{bx^2\sqrt{c+dx^4}}{d} + \frac{2a^{3/2}\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}} - \frac{(bc+2ad)\log\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{d^{3/2}}}{4b^2}
$$

```
[In] Integrate[x^9/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] ((b*x^2*Sqrt[c + d*x^4])/d + (2*a^(3/2)*ArcTan[(a*Sqrt[d] + b*x^2*(Sqrt[d]*
x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b*c - a*d])])/Sqrt[b*c - a*d] - ((b*c
 + 2*a*d)*Log[Sqrt[d]*x^2 + Sqrt[c + d*x^4]])/d^(3/2))/(4*b^2)
```

Maple [A] (verified)

Time = 5.62 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.24

method	result
pseudoelliptic	$\frac{\sqrt{d\,x^4+c}\,\sqrt{d}\,\sqrt{(ad-bc)a}\,b\,x^2+2d^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}\,a}{x^2\sqrt{(ad-bc)a}}\right)a^2-2\operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}}{x^2\sqrt{d}}\right)\sqrt{(ad-bc)a}\,ad-\operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}}{x^2\sqrt{d}}\right)}{4b^2\sqrt{(ad-bc)a}\,d^{\frac{3}{2}}}$
risch	$\frac{x^2\sqrt{d\,x^4+c}}{4bd}-\frac{(2ad+bc)\ln\left(x^2\sqrt{d}+\sqrt{d\,x^4+c}\right)}{2b\sqrt{d}}-\frac{2a^2d\left(\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d}{}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}}\right.}{}$
default	$\frac{\frac{x^2\sqrt{d\,x^4+c}}{4d}-\frac{c\ln\left(x^2\sqrt{d}+\sqrt{d\,x^4+c}\right)}{4d^{\frac{3}{2}}}}{b}-\frac{a\ln\left(x^2\sqrt{d}+\sqrt{d\,x^4+c}\right)}{2b^2\sqrt{d}}+\frac{a^2\left(\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d}}{x^2+\frac{\sqrt{-}}{b}}\right)}{4\sqrt{-ab}\sqrt{-\frac{a}{}}}\right.}{}$
elliptic	$-\frac{a\ln\left(x^2\sqrt{d}+\sqrt{d\,x^4+c}\right)}{2b^2\sqrt{d}}+\frac{x^2\sqrt{d\,x^4+c}}{4bd}-\frac{c\ln\left(x^2\sqrt{d}+\sqrt{d\,x^4+c}\right)}{4b\,d^{\frac{3}{2}}}+\frac{a^2\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-}{b}}}{x}\right.}{4b^2\sqrt{-a}}$

```
[In] int(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*((d*x^4+c)^(1/2)*d^(1/2)*((a*d-b*c)*a)^(1/2)*b*x^2+2*d^(3/2)*arctanh((d
*x^4+c)^(1/2)/x^2*a/((a*d-b*c)*a)^(1/2))*a^2-2*arctanh((d*x^4+c)^(1/2)/x^2/
d^(1/2))*((a*d-b*c)*a)^(1/2)*a*d-arctanh((d*x^4+c)^(1/2)/x^2/d^(1/2))*((a*d
-b*c)*a)^(1/2)*b*c)/b^2/((a*d-b*c)*a)^(1/2)/d^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 739, normalized size of antiderivative = 6.01

$$\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$
$$= \left[\frac{2\sqrt{dx^4+c}bdx^2+ad^2\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,a^2cd)x^4+a^2c^2+4\,((b^2c^2-3\,abcd+2\,a^2d^2)x^6-(abc^2-a}{b^2x^8+2\,abx^4+a^2}\right.}{8\,b^2d^2}\right.$$

```
[In] integrate(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(d*x^4 + c)*b*d*x^2 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2
- 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*
((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x^
4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + (b*c + 2*a*d)*s
qrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c))/(b^2*d^2), 1/8*(2
*sqrt(d*x^4 + c)*b*d*x^2 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b
*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((b^2*c
^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x^4 + c)*
sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 2*(b*c + 2*a*d)*sqrt(-
d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)))/(b^2*d^2), 1/8*(2*sqrt(d*x^4 + c)*
b*d*x^2 - 2*a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)
*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)) + (b*c + 2*a*d)*s
qrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c))/(b^2*d^2), 1/4*(s
qrt(d*x^4 + c)*b*d*x^2 - a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*
d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)) + (b
*c + 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)))/(b^2*d^2)]
```

Sympy [F]

$$\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

```
[In] integrate(x**9/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**9/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^9}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

```
[In] integrate(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^9/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^9}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}\,dx$$

```
[In] int(x^9/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(x^9/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.811 $\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 91

$$\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx = -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b\sqrt{bc-ad}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{2b\sqrt{d}}$$

```
[Out] 1/2*arctanh(x^2*d^(1/2)/(d*x^4+c)^(1/2))/b/d^(1/2)-1/2*arctan(x^2*(-a*d+b*c
)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))*a^(1/2)/b/(-a*d+b*c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {476, 494, 223, 212, 385, 211}

$$\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{2b\sqrt{d}}-\frac{\sqrt{a}\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b\sqrt{bc-ad}}$$

```
[In] Int[x^5/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/2*(Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(b*S
qrt[b*c - a*d]) + ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]]/(2*b*Sqrt[d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^2\right)}{2b} - \frac{a\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{2b} - \frac{a\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{2b}\\
&= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b\sqrt{bc-ad}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{2b\sqrt{d}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.19

$$\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{-\frac{\sqrt{a}\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}+\frac{\log\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{d}}}{2b}$$

```
[In] Integrate[x^5/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (-((Sqrt[a]*ArcTan[(a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqr
t[a]*Sqrt[b*c - a*d])])/Sqrt[b*c - a*d]) + Log[Sqrt[d]*x^2 + Sqrt[c + d*x^4
]]/Sqrt[d])/(2*b)
```

Maple [A] (verified)

Time = 5.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.93

method	result
pseudoelliptic	$\frac{-a\ \operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}\,a}{x^2\sqrt{(ad-bc)a}}\right)\sqrt{d}+\operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}}{x^2\sqrt{d}}\right)\sqrt{(ad-bc)a}}{2b\sqrt{(ad-bc)a}\,\sqrt{d}}$
default	$\frac{\ln\left(x^2\sqrt{d}+\sqrt{d\,x^4+c}\right)}{2b\sqrt{d}} - \quad a\Bigg(\frac{\ln\Big(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\ \sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-b}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}}$
elliptic	$\frac{\ln\left(x^2\sqrt{d}+\sqrt{d\,x^4+c}\right)}{2b\sqrt{d}} + \frac{a\ln\Big(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\ \sqrt{d\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}}{4\sqrt{-ab}\,b\sqrt{-\frac{ad-bc}{b}}}$

```
[In] int(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-a*arctanh((d*x^4+c)^(1/2)/x^2*a/((a*d-b*c)*a)^(1/2))*d^(1/2)+arctanh(
(d*x^4+c)^(1/2)/x^2/d^(1/2))*((a*d-b*c)*a)^(1/2))/b/((a*d-b*c)*a)^(1/2)/d^(
1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 632, normalized size of antiderivative = 6.95

$$\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$
$$= \left[\frac{d\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,a^2cd)x^4+a^2c^2-4\,((b^2c^2-3\,abcd+2\,a^2d^2)x^6-(abc^2-a^2cd)x^2)\sqrt{dx^4+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^8+2\,abx^4+a^2}\right.}{8\,bd}\right.$$

```
[In] integrate(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2
*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^
2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^
2*x^8 + 2*a*b*x^4 + a^2)) + 2*sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt
(d)*x^2 - c))/(b*d), 1/8*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d
+ 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b^2*c^2 -
3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x^4 + c)*sqrt(
-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)) - 4*sqrt(-d)*arctan(sqrt(-d)*
x^2/sqrt(d*x^4 + c)))/(b*d), 1/4*(d*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c -
 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2))
 + sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c))/(b*d), 1/4*(d
*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*
sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)) - 2*sqrt(-d)*arctan(sqrt(-d)*x^2/s
qrt(d*x^4 + c)))/(b*d)]
```

Sympy [F]

$$\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

```
[In] integrate(x**5/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^5}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

```
[In] integrate(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^5/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^5}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}\,dx$$

```
[In] int(x^5/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
[Out] int(x^5/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.812 $\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}} dx$

Optimal result

Integrand size = 22, antiderivative size = 54

$$\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx=\frac{\arctan\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}\sqrt{bc-ad}}$$

```
[Out] 1/2*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))/a^(1/2)/(-a*d+b*c)
^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {476, 385, 211}

$$\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx=\frac{\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}\sqrt{bc-ad}}$$

```
[In] Int[x/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])]/(2*Sqrt[a]*Sqrt[b*c
 - a*d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.37

$$\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2\sqrt{a}\sqrt{bc-ad}}$$

```
[In] Integrate[x/((a + b*x^4)*Sqrt[c + d*x^4]),x]
[Out] ArcTan[(a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b*
c - a*d])]/(2*Sqrt[a]*Sqrt[b*c - a*d])
```

Maple [A] (verified)

Time = 5.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}\,a}{x^2\sqrt{(ad-bc)a}}\right)}{2\sqrt{(ad-bc)a}}$
default	$\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}}-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2a}{}}{}\right)}{}$
elliptic	$\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}}-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2a}{}}{}\right)}{}$

```
[In] int(x/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/((a*d-b*c)*a)^(1/2)*arctanh((d*x^4+c)^(1/2)/x^2*a/((a*d-b*c)*a)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(42) = 84$.

Time = 0.30 (sec) , antiderivative size = 245, normalized size of antiderivative = 4.54

$$\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \left[-\frac{\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,a^2cd)x^4+a^2c^2-4\,((bc-2\,ad)x^6-acx^2)\sqrt{dx^4+c}\sqrt{-abc+a^2d}}{b^2x^8+2\,abx^4+a^2}\right)}{8\,(abc-a^2d)}, \frac{\arctan\left(\right.}{}\right.$$

```
[In] integrate(x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(
3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sqrt
(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2))/(a*b*c - a^2
*d), 1/4*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c -
a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2))/sqrt(a*b*c - a^
2*d)]
```

Sympy [F]

$$\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

```
[In] integrate(x/(b*x**4+a)/(d*x**4+c)**(1/2),x)
[Out] Integral(x/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

```
[In] integrate(x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.33

$$\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx = -\frac{\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}}$$

```
[In] integrate(x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(d)*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b - b*c + 2*a*d)
/sqrt(a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}\,dx$$

```
[In] int(x/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
[Out] int(x/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.813 $\int \frac{1}{x^3(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \frac{1}{x^3\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx=-\frac{\sqrt{c+dx^4}}{2acx^2}-\frac{b\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}\sqrt{bc-ad}}$$

```
[Out] -1/2*b*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))/a^(3/2)/(-a*d+b
*c)^(1/2)-1/2*(d*x^4+c)^(1/2)/a/c/x^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {476, 491, 12, 385, 211}

$$\int \frac{1}{x^3\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx=-\frac{b\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^4}}{2acx^2}$$

```
[In] Int[1/(x^3*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/2*Sqrt[c + d*x^4]/(a*c*x^2) - (b*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*S
qrt[c + d*x^4])])/(2*a^(3/2)*Sqrt[b*c - a*d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{c+dx^4}}{2acx^2}-\frac{\text{Subst}\left(\int \frac{bc}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{2ac}\\
&= -\frac{\sqrt{c+dx^4}}{2acx^2}-\frac{b\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{2a}\\
&= -\frac{\sqrt{c+dx^4}}{2acx^2}-\frac{b\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^2}{\sqrt{c+dx^4}}\right)}{2a}\\
&= -\frac{\sqrt{c+dx^4}}{2acx^2}-\frac{b\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^3 (a+bx^4) \sqrt{c+dx^4}}\,dx = -\frac{\sqrt{c+dx^4}}{2acx^2} - \frac{b\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{3/2}\sqrt{bc-ad}}$$

```
[In] Integrate[1/(x^3*(a + b*x^4)*Sqrt[c + d*x^4]),x]
[Out] -1/2*Sqrt[c + d*x^4]/(a*c*x^2) - (b*ArcTan[(a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2
+ Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(3/2)*Sqrt[b*c - a*d])
```

Maple [A] (verified)

Time = 5.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}\,a}{x^2\sqrt{(ad-bc)a}}\right) b c\,x^2+\sqrt{d\,x^4+c}\,\sqrt{(ad-bc)a}}{2a\,x^2\sqrt{(ad-bc)a}\,c}$
default	$-\frac{\sqrt{d\,x^4+c}}{2ac\,x^2}-\frac{b\left(\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}}-\cdots\right)}{a}$
risch	$-\frac{\sqrt{d\,x^4+c}}{2ac\,x^2}-\frac{b\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4a\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}}+\cdots$
elliptic	$-\frac{\sqrt{d\,x^4+c}}{2ac\,x^2}-\frac{b\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4a\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}}+\cdots$

```
[In] int(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(arctanh((d*x^4+c)^(1/2)/x^2*a/((a*d-b*c)*a)^(1/2))*b*c*x^2+(d*x^4+c)^
(1/2)*((a*d-b*c)*a)^(1/2))/a/x^2/((a*d-b*c)*a)^(1/2)/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(64) = 128$.

Time = 0.30 (sec) , antiderivative size = 332, normalized size of antiderivative = 4.15

$$\int \frac{1}{x^3\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$
$$= \left[-\frac{\sqrt{-abc+a^2d}\,bcx^2\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,a^2cd)x^4+a^2c^2+4\,((bc-2\,ad)x^6-acx^2)\sqrt{dx^4+c}\sqrt{-abc+a^2d}}{b^2x^8+2\,abx^4+a^2}\right)+4\sqrt{}}{8\,(a^2bc^2-a^3cd)x^2}\right.$$
$$\left.-\frac{\sqrt{abc-a^2d}\,bcx^2\arctan\left(\frac{((bc-2\,ad)x^4-ac)\sqrt{dx^4+c}\sqrt{abc-a^2d}}{2\,((abcd-a^2d^2)x^6+(abc^2-a^2cd)x^2)}\right)+2\sqrt{dx^4+c}(abc-a^2d)}{4\,(a^2bc^2-a^3cd)x^2}\right]$$

```
[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a*b*c + a^2*d)*b*c*x^2*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*
x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((b*c - 2*a*d)*x^6 - a*c*
x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 4
*sqrt(d*x^4 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^2), -1/4*(sqrt(a
*b*c - a^2*d)*b*c*x^2*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*
sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)) +
2*sqrt(d*x^4 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^2)]
```

Sympy [F]

$$\int \frac{1}{x^3\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx = \int \frac{1}{x^3\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$

```
[In] integrate(1/x**3/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^3\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx = \int \frac{1}{(bx^4+a)\sqrt{dx^4+c}\,x^3}\,dx$$

```
[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.45

$$\int \frac{1}{x^3 (a+bx^4) \sqrt{c+dx^4}} \, dx = \frac{1}{2} d^{\frac{3}{2}} \left(\frac{b \arctan \left(\frac{\left(\sqrt{d}x^2 - \sqrt{dx^4+c}\right)^2 b - bc + 2\,ad}{2\,\sqrt{abcd - a^2d^2}} \right)}{\sqrt{abcd - a^2d^2}ad} + \frac{2}{\left(\left(\sqrt{d}x^2 - \sqrt{dx^4+c}\right)^2 - c\right)ad} \right)$$

```
[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*d^(3/2)*(b*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b - b*c + 2*a*
d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a*d) + 2/(((sqrt(d)*x^
2 - sqrt(d*x^4 + c))^2 - c)*a*d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^4) \sqrt{c+dx^4}} \, dx = \int \frac{1}{x^3 \left(b\,x^4+a\right) \sqrt{d\,x^4+c}} \, dx$$

```
[In] int(1/(x^3*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^3*(a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.814 $\int \frac{1}{x^7(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 115

$$\int \frac{1}{x^7\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx=-\frac{\sqrt{c+dx^4}}{6acx^6}+\frac{(3bc+2ad)\sqrt{c+dx^4}}{6a^2c^2x^2}+\frac{b^2\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{5/2}\sqrt{bc-ad}}$$

```
[Out] 1/2*b^2*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))/a^(5/2)/(-a*d+
b*c)^(1/2)-1/6*(d*x^4+c)^(1/2)/a/c/x^6+1/6*(2*a*d+3*b*c)*(d*x^4+c)^(1/2)/a^
2/c^2/x^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {476, 491, 597, 12, 385, 211}

$$\int \frac{1}{x^7\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx=\frac{b^2\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^4}(2ad+3bc)}{6a^2c^2x^2}-\frac{\sqrt{c+dx^4}}{6acx^6}$$

```
[In] Int[1/(x^7*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/6*Sqrt[c + d*x^4]/(a*c*x^6) + ((3*b*c + 2*a*d)*Sqrt[c + d*x^4])/(6*a^2*c
^2*x^2) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(2*
a^(5/2)*Sqrt[b*c - a*d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{c+dx^4}}{6acx^6}+\frac{\text{Subst}\left(\int \frac{-3bc-2ad-2bdx^2}{x^2(a+bx^2)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{6ac}\end{aligned}$$

$$= -\frac{\sqrt{c+dx^4}}{6acx^6} + \frac{(3bc+2ad)\sqrt{c+dx^4}}{6a^2c^2x^2} - \frac{\text{Subst}\left(\int -\frac{3b^2c^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{6a^2c^2}$$

$$= -\frac{\sqrt{c+dx^4}}{6acx^6} + \frac{(3bc+2ad)\sqrt{c+dx^4}}{6a^2c^2x^2} + \frac{b^2\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{2a^2}$$

$$= -\frac{\sqrt{c+dx^4}}{6acx^6} + \frac{(3bc+2ad)\sqrt{c+dx^4}}{6a^2c^2x^2} + \frac{b^2\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{2a^2}$$

$$= -\frac{\sqrt{c+dx^4}}{6acx^6} + \frac{(3bc+2ad)\sqrt{c+dx^4}}{6a^2c^2x^2} + \frac{b^2\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{5/2}\sqrt{bc-ad}}$$

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^7(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\sqrt{c+dx^4}(-ac+3bcx^4+2adx^4)}{6a^2c^2x^6} + \frac{b^2\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{5/2}\sqrt{bc-ad}}$$

```
[In] Integrate[1/(x^7*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (Sqrt[c + d*x^4]*(-(a*c) + 3*b*c*x^4 + 2*a*d*x^4))/(6*a^2*c^2*x^6) + (b^2*A
rcTan[(a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b*c
 - a*d])])/(2*a^(5/2)*Sqrt[b*c - a*d])
```

Maple [A] (verified)

Time = 6.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$-\frac{-3 \operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}\,a}{x^2\sqrt{(ad-bc)a}}\right) b^2 c^2 x^6+\left(\left(-3b\,x^4+a\right)c-2ad\,x^4\right)\sqrt{d\,x^4+c}\,\sqrt{(ad-bc)a}}{6\sqrt{(ad-bc)a}\,a^2x^6c^2}$
risch	$-\frac{\sqrt{d\,x^4+c}\,\left(-2ad\,x^4-3bc\,x^4+ac\right)}{6c^2a^2x^6}+\frac{b^2\left(\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\,\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\,\left(x^2\right.}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}}\right.}{}$
default	$-\frac{\sqrt{d\,x^4+c}\,\left(-2d\,x^4+c\right)}{6a\,c^2x^6}+\frac{b\sqrt{d\,x^4+c}}{2a^2c\,x^2}+\frac{b^2\left(\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\,\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}}{}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}}\right.}{}$
elliptic	$-\frac{\sqrt{d\,x^4+c}}{6ac\,x^6}+\frac{d\sqrt{d\,x^4+c}}{3a\,c^2x^2}+\frac{b\sqrt{d\,x^4+c}}{2a^2c\,x^2}+\frac{b^2\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\,\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-a}}{}}}{x^2+\frac{\sqrt{-ab}}{b}}\right.}{4a^2\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}}$

```
[In] int(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(-3*arctanh((d*x^4+c)^(1/2)/x^2*a/((a*d-b*c)*a)^(1/2))*b^2*c^2*x^6+((-
3*b*x^4+a)*c-2*a*d*x^4)*(d*x^4+c)^(1/2)*((a*d-b*c)*a)^(1/2))/((a*d-b*c)*a)^
(1/2)/a^2/x^6/c^2
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 418, normalized size of antiderivative = 3.63

$$\int \frac{1}{x^7\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$
$$=\left[-\frac{3\sqrt{-abc+a^2d}b^2c^2x^6\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\left(3\,abc^2-4\,a^2cd\right)x^4+a^2c^2-4\left((bc-2\,ad)x^6-acx^2\right)\sqrt{dx^4+c}\sqrt{-abc+a^2d}}{b^2x^8+2\,abx^4+a^2}\right)-}{24\left(a^3bc^3-a^4c^2d\right)x^6}\right.$$

```
[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*sqrt(-a*b*c + a^2*d)*b^2*c^2*x^6*log(((b^2*c^2 - 8*a*b*c*d + 8*a^
2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x^6
 - a*c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^
2)) + 4*(a^2*b*c^2 - a^3*c*d - (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^4)*s
qrt(d*x^4 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^6), 1/12*(3*sqrt(a*b*c - a^2*d)*
b^2*c^2*x^6*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c
 - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)) - 2*(a^2*b*c
^2 - a^3*c*d - (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^4)*sqrt(d*x^4 + c))/
((a^3*b*c^3 - a^4*c^2*d)*x^6)]
```

Sympy [F]

$$\int \frac{1}{x^7 (a+bx^4) \sqrt{c+dx^4}} \, dx = \int \frac{1}{x^7 (a+bx^4) \sqrt{c+dx^4}} \, dx$$

```
[In] integrate(1/x**7/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/(x**7*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^7 (a+bx^4) \sqrt{c+dx^4}} \, dx = \int \frac{1}{(bx^4+a)\sqrt{dx^4+c}x^7} \, dx$$

```
[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^7), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(95) = 190$.

Time = 1.16 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.78

$$\int \frac{1}{x^7 (a+bx^4) \sqrt{c+dx^4}} \, dx = -\frac{1}{6} d^{\frac{5}{2}} \left(\frac{3 b^2 \arctan\left(\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2 b-bc+2 ad}{2 \sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}a^2d^2} + \frac{2 \left(3 \left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4 b - 6 \left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2 bc\right.}{\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)\right.}\right.$$

```
[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*d^(5/2)*(3*b^2*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b - b*c +
 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2*d^2) + 2*(3*(
sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*
c - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + 3*b*c^2 + 2*a*c*d)/(((sqrt(d)
*x^2 - sqrt(d*x^4 + c))^2 - c)^3*a^2*d^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^7 (a+bx^4) \sqrt{c+dx^4}} \, dx = \int \frac{1}{x^7 \, (b\, x^4+a) \, \sqrt{d\, x^4+c}} \, dx$$

```
[In] int(1/(x^7*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^7*(a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.815 $\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 872

$$\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{x\sqrt{c+dx^4}}{3bd}-\frac{(-a)^{5/4}\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4b^{7/4}\sqrt{bc-ad}}-\frac{(-a)^{5/4}\arctan\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4b^{7/4}\sqrt{-bc+ad}}$$
$$+\frac{a^2\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4b^2\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}}$$
$$+\frac{a\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4b^2\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}}$$
$$-\frac{(bc+3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{6b^2\sqrt[4]{c}d^{5/4}\sqrt{c+dx^4}}$$
$$+\frac{a\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}}$$
$$+\frac{a\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}}$$

```
[Out] -1/4*(-a)^(5/4)*arctan(x*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2
))/b^(7/4)/(-a*d+b*c)^(1/2)-1/4*(-a)^(5/4)*arctan(x*(a*d-b*c)^(1/2)/(-a)^(1
```

```
/4)/b^(1/4)/(d*x^4+c)^(1/2))/b^(7/4)/(a*d-b*c)^(1/2)+1/3*x*(d*x^4+c)^(1/2)/
b/d-1/6*(3*a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan
(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2)
)*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^2/c^(1/
4)/d^(5/4)/(d*x^4+c)^(1/2)+1/4*a^2*d^(1/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4))
)^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*
x/c^(1/4))),1/2*2^(1/2))*(b^(1/2)*c^(1/2)/(-a)^(1/2)+d^(1/2))*(c^(1/2)+x^2*
d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^2/c^(1/4)/(a*d+b*c)/(d
*x^4+c)^(1/2)+1/4*a*d^(1/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(
2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2
*2^(1/2))*((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*
x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^2/c^(1/4)/(a*d+b*c)/(d*x^4+c)^(1/2)
+1/8*a*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^
(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-
a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2
)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x
^2*d^(1/2))^2)^(1/2)/b^2/c^(1/4)/d^(1/4)/(a*d+b*c)/(d*x^4+c)^(1/2)+1/8*a*(c
os(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*E
llipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)
*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^
(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/
2))^2)^(1/2)/b^2/c^(1/4)/d^(1/4)/(a*d+b*c)/(d*x^4+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 872, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used

$= \{490, 537, 226, 418, 1231, 1721\}$

$$\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$
$$= \frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)a^2}{4b^2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}$$
$$+\frac{\left(\sqrt{d}a+\sqrt{-a}\sqrt{b}\sqrt{c}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)a}{4b^2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}$$
$$+\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)a}{8b^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}$$
$$+\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)a}{8b^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}$$
$$-\frac{(-a)^{5/4}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{4b^{7/4}\sqrt{bc-ad}}-\frac{(-a)^{5/4}\arctan\left(\frac{\sqrt{ad-bc}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{4b^{7/4}\sqrt{ad-bc}}$$
$$-\frac{(bc+3ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{6b^2\sqrt[4]{c}d^{5/4}\sqrt{dx^4+c}}+\frac{x\sqrt{dx^4+c}}{3bd}$$

```
[In] Int[x^8/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (x*Sqrt[c + d*x^4])/(3*b*d) - ((-a)^(5/4)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^
(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*b^(7/4)*Sqrt[b*c - a*d]) - ((-a)^(5/4)*
ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*b^(
7/4)*Sqrt[-(b*c) + a*d]) + (a^2*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1
/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ell
ipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b^2*c^(1/4)*(b*c + a*d)*Sqrt
[c + d*x^4]) + (a*(Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] +
 Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTa
n[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b^2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) -
 ((b*c + 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]
*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(6*b^2*c^(1/4)*d^(5
/4)*Sqrt[c + d*x^4]) + (a*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] +
 Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(
Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2
*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt
[c + d*x^4]) + (a*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]
```

```
*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[
c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^
(1/4)*x)/c^(1/4)], 1/2])/(8*b^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]
)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
```

```
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = \frac{x\sqrt{c+dx^4}}{3bd} - \frac{\int \frac{ac+(bc+3ad)x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{3bd}$$

$$= \frac{x\sqrt{c+dx^4}}{3bd} + \frac{a^2\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{b^2} - \frac{(bc+3ad)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{3b^2d}$$

$$= \frac{x\sqrt{c+dx^4}}{3bd} - \frac{(bc+3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{6b^2\sqrt[4]{c}d^{5/4}\sqrt{c+dx^4}} + \frac{a\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2b^2} + \frac{a\int \frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2b^2}$$

$$= \frac{x\sqrt{c+dx^4}}{3bd} - \frac{(bc+3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{6b^2\sqrt[4]{c}d^{5/4}\sqrt{c+dx^4}} + \frac{\left(a\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2b^{3/2}(bc+ad)} + \frac{\left(a\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2b^{3/2}(bc+ad)} + \frac{\left(a^2\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt{d}\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{2b^2(bc+ad)} + \frac{\left(a\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt{d}\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{2b^2(bc+ad)}$$

$$= \frac{x\sqrt{c+dx^4}}{3bd} - \frac{(-a)^{5/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4b^{7/4}\sqrt{bc-ad}} - \frac{(-a)^{5/4}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4b^{7/4}\sqrt{-bc+ad}}$$
$$+ \frac{a^2\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4b^2\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}}$$
$$+ \frac{a\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4b^2\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}}$$
$$- \frac{(bc+3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{6b^2\sqrt[4]{c}d^{5/4}\sqrt{c+dx^4}}$$
$$+ \frac{a\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8b^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}}$$
$$+ \frac{a\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8b^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.35 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.29

$$\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$
$$= \frac{x\left(-\frac{(bc+3ad)x^4\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{ad}+5\left(\frac{c}{d}+x^4+\frac{5a^2c^2\,\text{Appell}}{d(a+bx^4)\left(-5ac\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2x^4\left(2bc\right.\right.}\right.\right.}{15b\sqrt{c+dx^4}}$$

```
[In] Integrate[x^8/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (x*(-(((b*c + 3*a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((
d*x^4)/c), -((b*x^4)/a)])/(a*d)) + 5*(c/d + x^4 + (5*a^2*c^2*AppellF1[1/4,
1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)])/(d*(a + b*x^4)*(-5*a*c*AppellF1[1
/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1
/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -(
(d*x^4)/c), -((b*x^4)/a)]))))))/(15*b*Sqrt[c + d*x^4])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.11 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.34

method	result
elliptic	$\frac{x\sqrt{d x^4+c}}{3bd}+\frac{\left(-\frac{a}{b^2}-\frac{c}{3db}\right)\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d x^4+c}}+\frac{a^2\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2 \cdots}{\cdots}}{\cdots}\right)}{8 \cdots}$
risch	$\frac{x\sqrt{d x^4+c}}{3bd}-\frac{\frac{(3ad+bc)\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d x^4+c}}-\frac{3a^2 d\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^3 b\sqrt{1 \cdots}}{\cdots}}{\cdots}\right)}{8b^2 \cdots}}{3bd}$
default	$\frac{\frac{x\sqrt{d x^4+c}}{3d}-\frac{c\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{3d\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d x^4+c}}}{b}-\frac{a\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b^2\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d x^4+c}}+\frac{a^2\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{-\frac{\operatorname{arctan}\cdots}{\cdots}}{\cdots}\right)}{\cdots}$

```
[In] int(x^8/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x*(d*x^4+c)^(1/2)/b/d+(-a/b^2-1/3*c/d/b)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I
/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)
*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)+1/8*a^2/b^3*sum(1/_alpha^3*(-1/((
-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/
(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1
/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x
*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/
2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \text{Timed out}$$

```
[In] integrate(x^8/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

```
[In] integrate(x**8/(b*x**4+a)/(d*x**4+c)**(1/2),x)
[Out] Integral(x**8/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^8}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

```
[In] integrate(x^8/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^8/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^8}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

```
[In] integrate(x^8/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^8/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^8}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}\,dx$$

```
[In] int(x^8/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
[Out] int(x^8/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.816 $\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 638

$$\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx = -\frac{\arctan\left(\frac{\sqrt{\frac{\sqrt{-a}\left(\frac{bc}{a}-d\right)}{\sqrt{b}}}\,x}{\sqrt{c+dx^4}}\right)}{4b\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}} - \frac{\arctan\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\,x}{\sqrt{c+dx^4}}\right)}{4b\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}$$
$$+\frac{c^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}}$$
$$-\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$
$$-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$

```
[Out] -1/4*arctan(x*((b*c/a-d)*(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))/b/((a*d
-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)-1/4*arctan(x*((-a*d+b*c)/(-a)^(1/2)/b^(1/2)
)^(1/2)/(d*x^4+c)^(1/2))/b/((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)+1/2*c^(3/4
)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)
))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(
1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/d^(1/4)/(a*d+b*c)/(d*x^4+c)
^(1/2)-1/8*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*
x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2
```

```
)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^
(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)
+x^2*d^(1/2))^2)^(1/2)/b/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2
))/(d*x^4+c)^(1/2)-1/8*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arc
tan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b
^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2
*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^
4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)-(-a)
^(1/2)*d^(1/2))/(d*x^4+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 837, normalized size of antiderivative = 1.31, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {494, 226, 418, 1231, 1721}

$$\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx =$$

$$-\frac{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8b\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}$$
$$-\frac{\sqrt[4]{-a}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{4b^{3/4}\sqrt{bc-ad}}-\frac{\sqrt[4]{-a}\arctan\left(\frac{\sqrt{ad-bc}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{4b^{3/4}\sqrt{ad-bc}}$$
$$+\frac{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{dx^4+c}}$$
$$-\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4b\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}$$
$$-\frac{\left(\sqrt{d}a+\sqrt{-a}\sqrt{b}\sqrt{c}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4b\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}$$
$$-\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}$$

```
[In] Int[x^4/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/4*((-a)^(1/4)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*
x^4])])/(b^(3/4)*Sqrt[b*c - a*d]) - ((-a)^(1/4)*ArcTan[(Sqrt[-(b*c) + a*d]*
```

```
x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*b^(3/4)*Sqrt[-(b*c) + a*d]) +
((Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ellipt
icF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*b*c^(1/4)*d^(1/4)*Sqrt[c + d*x^
4]) - (a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*
x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4
)*x)/c^(1/4)], 1/2])/(4*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[-a]
*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x
^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2
])/(4*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] + Sqrt[-a]
*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2
)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[
b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b*c^(1/4)*d^(1
/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*
(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ellipti
cPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt
[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b*c^(1/4)*d^(1/4)*(b*c + a*d)
*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{1}{\sqrt{c+dx^4}}\,dx}{b} - \frac{a\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{b}$$

$$= \frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}} - \frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2b} - \frac{\int \frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2b}$$

$$= \frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}} - \frac{\left(\sqrt{c}\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2(bc+ad)} - \frac{\left(\sqrt{c}\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2(bc+ad)} - \frac{\left(a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt{d}\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{2b(bc+ad)} - \frac{\left(\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt{d}\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{2b(bc+ad)}$$

$$= -\frac{\sqrt[4]{-a}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4b^{3/4}\sqrt{bc-ad}} - \frac{\sqrt[4]{-a}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4b^{3/4}\sqrt{-bc+ad}}$$
$$+ \frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}$$
$$- \frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4b\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}}$$
$$- \frac{\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4b\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}}$$
$$- \frac{\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)^2\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}}$$
$$- \frac{\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)^2\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.10

$$\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{x^5\sqrt{\frac{c+dx^4}{c}}\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{5a\sqrt{c+dx^4}}$$

```
[In] Integrate[x^4/((a + b*x^4)*Sqrt[c + d*x^4]),x]
[Out] (x^5*Sqrt[(c + d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)
/a)])/(5*a*Sqrt[c + d*x^4])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.72 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.42

method	result
default	$\frac{\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{a\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^3 b\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,a}}{\_\alpha^3}\right)}{8b^2}$
elliptic	$\frac{\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{a\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^3 b\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,a}}{\_\alpha^3}\right)}{8b^2}$

```
[In] int(x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*
d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)
-1/8*a/b^2*sum(1/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*
d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2
)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1
/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2
)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alph
a=RootOf(_Z^4*b+a))
```

Fricas [F]

$$\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx=\int \frac{x^4}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

```
[In] integrate(x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^4 + c)*x^4/(b*d*x^8 + (b*c + a*d)*x^4 + a*c), x)
```

Sympy [F]

$$\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

```
[In] integrate(x**4/(b*x**4+a)/(d*x**4+c)**(1/2),x)
[Out] Integral(x**4/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^4}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

```
[In] integrate(x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^4/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^4}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

```
[In] integrate(x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^4}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}\,dx$$

```
[In] int(x^4/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
[Out] int(x^4/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.817 $\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 638

$$\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\arctan\left(\frac{\sqrt{\frac{\sqrt{-a}\left(\frac{bc}{a}-d\right)}{\sqrt{b}}}\,x}{\sqrt{c+dx^4}}\right)}{4a\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}+\frac{\arctan\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\,x}{\sqrt{c+dx^4}}\right)}{4a\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}$$
$$+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}}$$
$$+\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$
$$+\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$

```
[Out] 1/4*arctan(x*((b*c/a-d)*(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))/a/((a*d-
b*c)/(-a)^(1/2)/b^(1/2))^(1/2)+1/4*arctan(x*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))
^(1/2)/(d*x^4+c)^(1/2))/a/((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)+1/2*d^(3/4)
*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4))
)*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1
/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/c^(1/4)/(a*d+b*c)/(d*x^4+c)^
(1/2)+1/8*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x
/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)
+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(
```

```
1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+
x^2*d^(1/2))^2)^(1/2)/a/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2)
)/(d*x^4+c)^(1/2)+1/8*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arct
an(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^
(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*
2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^4
+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)-(-a)^
(1/2)*d^(1/2))/(d*x^4+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 742, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {418, 1231, 226, 1721}

$$\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx = -\frac{\sqrt[4]{b}\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{3/4}\sqrt{bc-ad}} - \frac{\sqrt[4]{b}\arctan\left(\frac{x\sqrt{ad-bc}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{3/4}\sqrt{ad-bc}}$$
$$+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}$$
$$+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4a\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}$$
$$+\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}(ad+bc)}$$
$$+\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)^2\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}(ad+bc)}$$

```
[In] Int[1/((a + b*x^4)*Sqrt[c + d*x^4]),x]

[Out] -1/4*(b^(1/4)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4
])])/((-a)^(3/4)*Sqrt[b*c - a*d]) - (b^(1/4)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/
((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(3/4)*Sqrt[-(b*c) + a*d]) +
(((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqr
t[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(
1/4)], 1/2])/(4*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + ((Sqrt[-a]*Sqrt[b]*S
qrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[
c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a*c^
```

```
(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^
2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ellip
ticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]
*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*a*c^(1/4)*d^(1/4)*(b*c +
 a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] +
 Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[
b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*Ar
cTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*a*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c +
d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2a} + \frac{\int \frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2a}$$

$$= \frac{\left(\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}dx}{2a(bc+ad)}$$

$$+ \frac{\left(\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}dx}{2a(bc+ad)}$$

$$+ \frac{\left(\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt{d}\right)\int \frac{1}{\sqrt{c+dx^4}}dx}{2(bc+ad)} + \frac{\left(\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt{d}\right)\int \frac{1}{\sqrt{c+dx^4}}dx}{2a(bc+ad)}$$

$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{3/4}\sqrt{bc-ad}} - \frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{3/4}\sqrt{-bc+ad}}$$

$$+ \frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}}$$

$$+ \frac{\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4a\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}}$$

$$+ \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}}$$

$$+ \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.25

$$\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}dx =$$

$$-\frac{5acx\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{(a+bx^4)\sqrt{c+dx^4}\left(-5ac\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2x^4\left(2bc\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right.\right.}$$

```
[In] Integrate[1/((a + b*x^4)*Sqrt[c + d*x^4]),x]
[Out] (-5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)])/((a + b*x
^4)*Sqrt[c + d*x^4]*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x
^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a
)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.69 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.30

method	result	size
default	$\frac{\sum_{\_\alpha=\mathrm{RootOf}(\_Z^4 b+a)} \frac{-\frac{\operatorname{arctanh}\left(\frac{2 d x^2 \_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\sqrt{d x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^3 b\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\Pi\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},\frac{i\sqrt{c}\,\_\alpha^2 b}{\sqrt{d}\,a},\frac{\sqrt{-\frac{i\sqrt{d}}{\sqrt{c}}}}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}}\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,a\sqrt{d x^4+c}}}{\_\alpha^3}}{8b}$	191
elliptic	$\frac{\sum_{\_\alpha=\mathrm{RootOf}(\_Z^4 b+a)} \frac{-\frac{\operatorname{arctanh}\left(\frac{2 d x^2 \_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\sqrt{d x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^3 b\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\Pi\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},\frac{i\sqrt{c}\,\_\alpha^2 b}{\sqrt{d}\,a},\frac{\sqrt{-\frac{i\sqrt{d}}{\sqrt{c}}}}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}}\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,a\sqrt{d x^4+c}}}{\_\alpha^3}}{8b}$	191

```
[In] int(1/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/b*sum(1/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2
+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_al
pha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(
d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_al
pha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=Roo
tOf(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

```
[In] integrate(1/(b*x**4+a)/(d*x**4+c)**(1/2),x)
[Out] Integral(1/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{1}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}\,dx$$

```
[In] int(1/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
[Out] int(1/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.818 $\int \frac{1}{x^4(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 677

$$\int \frac{1}{x^4(a+bx^4)\sqrt{c+dx^4}}\,dx = -\frac{\sqrt{c+dx^4}}{3acx^3} - \frac{b\arctan\left(\frac{\sqrt{\frac{\sqrt{-a}\left(\frac{bc}{a}-d\right)}{\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4a^2\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}} - \frac{b\arctan\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4a^2\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}} - \frac{d^{3/4}(4bc+ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{6ac^{5/4}(bc+ad)\sqrt{c+dx^4}} - \frac{b\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a^2\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}} - \frac{b\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a^2\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$

```
[Out] -1/3*(d*x^4+c)^(1/2)/a/c/x^3-1/4*b*arctan(x*((b*c/a-d)*(-a)^(1/2)/b^(1/2))^
(1/2)/(d*x^4+c)^(1/2))/a^2/((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)-1/4*b*arcta
n(x*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))/a^2/((-a*d+b*c)/
(-a)^(1/2)/b^(1/2))^(1/2)-1/6*d^(3/4)*(a*d+4*b*c)*(cos(2*arctan(d^(1/4)*x/c
^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d
^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x
^2*d^(1/2))^2)^(1/2)/a/c^(5/4)/(a*d+b*c)/(d*x^4+c)^(1/2)-1/8*b*(cos(2*arcta
n(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(
```

```
sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2
/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(
1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/
a^2/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))/(d*x^4+c)^(1/2)-1/
8*b*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/
4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)
^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+
x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d
^(1/2))^2)^(1/2)/a^2/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/(
d*x^4+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 864, normalized size of antiderivative = 1.28, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {491, 537, 226, 418, 1231, 1721}

$$\int \frac{1}{x^4 (a+bx^4)\sqrt{c+dx^4}}\,dx =$$

$$-\frac{b\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8a^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}$$
$$-\frac{b^{5/4}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{4(-a)^{7/4}\sqrt{bc-ad}}-\frac{b^{5/4}\arctan\left(\frac{\sqrt{ad-bc}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{4(-a)^{7/4}\sqrt{ad-bc}}$$
$$-\frac{d^{3/4}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{6ac^{5/4}\sqrt{dx^4+c}}$$
$$-\frac{b\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4a\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}$$
$$-\frac{b\left(\sqrt{d}a+\sqrt{-a}\sqrt{b}\sqrt{c}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4a^2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}$$
$$-\frac{b\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}$$
$$-\frac{\sqrt{dx^4+c}}{3acx^3}$$

```
[In] Int[1/(x^4*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/3*Sqrt[c + d*x^4]/(a*c*x^3) - (b^(5/4)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^
(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(7/4)*Sqrt[b*c - a*d]) - (b^(5/4)*
ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a
)^(7/4)*Sqrt[-(b*c) + a*d]) - (d^(3/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*
x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/
2])/(6*a*c^(5/4)*Sqrt[c + d*x^4]) - (b*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d
])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)
^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a*c^(1/4)*(b*c + a*d)
*Sqrt[c + d*x^4]) - (b*(Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt
[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*
ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a^2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^
4]) - (b*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqr
t[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] -
 Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*
x)/c^(1/4)], 1/2])/(8*a^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - (b
*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d
*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqr
t[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)]
, 1/2])/(8*a^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
```

```
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{c+dx^4}}{3acx^3}+\frac{\int \frac{-3bc-ad-bdx^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{3ac}\\
&= -\frac{\sqrt{c+dx^4}}{3acx^3}-\frac{b\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{a}-\frac{d\int \frac{1}{\sqrt{c+dx^4}}\,dx}{3ac}\\
&= -\frac{\sqrt{c+dx^4}}{3acx^3}-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{6ac^{5/4}\sqrt{c+dx^4}}\\
&\quad -\frac{b\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2a^2}-\frac{b\int \frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2a^2}
\end{aligned}$$

$$= -\frac{\sqrt{c+dx^4}}{3acx^3} - \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{6ac^{5/4}\sqrt{c+dx^4}}$$
$$-\frac{\left(b^{3/2}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\right)\int\frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2a^2(bc+ad)}$$
$$-\frac{\left(b^{3/2}\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\right)\int\frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2a^2(bc+ad)}$$
$$-\frac{\left(b\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt{d}\right)\int\frac{1}{\sqrt{c+dx^4}}\,dx}{2a(bc+ad)} - \frac{\left(b\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt{d}\right)\int\frac{1}{\sqrt{c+dx^4}}\,dx}{2a^2(bc+ad)}$$

$$= -\frac{\sqrt{c+dx^4}}{3acx^3} - \frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{7/4}\sqrt{bc-ad}} - \frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{7/4}\sqrt{-bc+ad}}$$
$$-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{6ac^{5/4}\sqrt{c+dx^4}}$$
$$-\frac{b\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4a\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}}$$
$$-\frac{b\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4a^2\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}}$$
$$-\frac{b\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}}$$
$$-\frac{b\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.29 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^4 (a + bx^4) \sqrt{c + dx^4}} \, dx$$
$$= \frac{-bdx^8 \sqrt{1 + \frac{dx^4}{c}} \text{AppellF1}\left(\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, -\frac{dx^4}{c}, -\frac{bx^4}{a}\right) + \frac{5a\left(-5ac(ac+4bcx^4+2adx^4+bdx^8)\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2\right.}{(a+bx^4)\left(5ac\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-2x^4\right.}}{15a^2cx^3\sqrt{c + dx^4}}$$

```
[In] Integrate[1/(x^4*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (-(b*d*x^8*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c), -((
b*x^4)/a)]) + (5*a*(-5*a*c*(a*c + 4*b*c*x^4 + 2*a*d*x^4 + b*d*x^8)*AppellF1
[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(a + b*x^4)*(c + d*x
^4)*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*App
ellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))/((a + b*x^4)*(5*a*c*
AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] - 2*x^4*(2*b*c*Appel
lF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2,
1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]))))/(15*a^2*c*x^3*Sqrt[c + d*x^4])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.32 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.43

method	result
default	$\frac{-\frac{\sqrt{d\,x^4+c}}{3c\,x^3}-\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{3c\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}}{a}-\frac{\sum_{\_\alpha=\mathrm{RootOf}(\_Z^4b+a)}\frac{-\frac{\mathrm{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^3b\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+}}{}}{\_\alpha^3}}{8a}$
risch	$-\frac{\sqrt{d\,x^4+c}}{3ac\,x^3}-\frac{\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}+\frac{3c\left(\sum_{\_\alpha=\mathrm{RootOf}(\_Z^4b+a)}\frac{-\frac{\mathrm{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^3b\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}}{}}{\_\alpha^3}\right.}{8}}{3ac}$
elliptic	$-\frac{\sqrt{d\,x^4+c}}{3ac\,x^3}-\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{3ac\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}(\_Z^4b+a)}\frac{-\frac{\mathrm{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^3b\sqrt{1-\frac{i\sqrt{d}\,x}{\sqrt{c}}}}{}}{\_\alpha^3}}{8a}$

```
[In] int(1/x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/3/c*(d*x^4+c)^(1/2)/x^3-1/3*d/c/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1
/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*Elli
pticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I))-1/8/a*sum(1/_alpha^3*(-1/((-a*d+b*c)/
b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^
(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(
1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2
)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/
(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^4\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx=\text{Timed out}$$

```
[In] integrate(1/x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^4 (a + bx^4) \sqrt{c + dx^4}} \, dx = \int \frac{1}{x^4 (a + bx^4) \sqrt{c + dx^4}} \, dx$$

```
[In] integrate(1/x**4/(b*x**4+a)/(d*x**4+c)**(1/2),x)
[Out] Integral(1/(x**4*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (a + bx^4) \sqrt{c + dx^4}} \, dx = \int \frac{1}{(bx^4 + a)\sqrt{dx^4 + c}x^4} \, dx$$

```
[In] integrate(1/x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a + bx^4) \sqrt{c + dx^4}} \, dx = \int \frac{1}{(bx^4 + a)\sqrt{dx^4 + c}x^4} \, dx$$

```
[In] integrate(1/x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a + bx^4) \sqrt{c + dx^4}} \, dx = \int \frac{1}{x^4 \, (b\,x^4 + a) \, \sqrt{d\,x^4 + c}} \, dx$$

```
[In] int(1/(x^4*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
[Out] int(1/(x^4*(a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.819 $\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 804

$$\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{x\sqrt{c+dx^4}}{b\sqrt{d}\left(\sqrt{c}+\sqrt{d}x^2\right)} - \frac{a\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4b(bc-ad)} - \frac{a\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4b(bc-ad)} - \frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{bd^{3/4}\sqrt{c+dx^4}} + \frac{\sqrt[4]{c}(bc+2ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2bd^{3/4}(bc+ad)\sqrt{c+dx^4}} + \frac{a\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b^{3/2}\sqrt[4]{c}\left(\sqrt{-a}\sqrt{b}\sqrt{c}-a\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}} - \frac{a\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b^{3/2}\sqrt[4]{c}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$

```
[Out] x*(d*x^4+c)^(1/2)/b/d^(1/2)/(c^(1/2)+x^2*d^(1/2))-1/4*a*arctan(x*((a*d-b*c)
/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*((a*d-b*c)/(-a)^(1/2)/b^(1/2))^
(1/2)/b/(-a*d+b*c)-1/4*a*arctan(x*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*
```

```
x^4+c)^(1/2))*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/b/(-a*d+b*c)-c^(1/4)*(c
os(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*E
llipticE(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2)
)*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b/d^(3/4)/(d*x^4+c)^(1/2)+1/2*c
^(1/4)*(2*a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(
d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))
*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b/d^(3/4)/
(a*d+b*c)/(d*x^4+c)^(1/2)+1/8*a*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/
cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))
),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(
1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2)
)*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^(3/2)/c^(1/4)/d^(1/4)/((-a)^(
1/2)*b^(1/2)*c^(1/2)-a*d^(1/2))/(d*x^4+c)^(1/2)-1/8*a*(cos(2*arctan(d^(1/4)
*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arc
tan(d^(1/4)*x/c^(1/4))),-1/4*c^(1/2)*(b^(1/2)-(-a)^(1/2)*d^(1/2)/c^(1/2))^2
/(-a)^(1/2)/b^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(
1/2)+(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^(3/2)/
c^(1/4)/d^(1/4)/((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))/(d*x^4+c)^(1/2)
```

Rubi [A] (warning: unable to verify)

Time = 0.84 (sec) , antiderivative size = 982, normalized size of antiderivative = 1.22, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used

$= \{494, 311, 226, 1210, 504, 1231, 1721\}$

$$\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx =$$

$$-\frac{\sqrt{-a}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)}{8b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}$$

$$+\frac{(-a)^{3/4}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{4b^{5/4}\sqrt{bc-ad}}-\frac{(-a)^{3/4}\arctan\left(\frac{\sqrt{ad-bc}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{4b^{5/4}\sqrt{ad-bc}}$$

$$-\frac{\sqrt[4]{c}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{bd^{3/4}\sqrt{dx^4+c}}$$

$$+\frac{\sqrt[4]{c}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2bd^{3/4}\sqrt{dx^4+c}}$$

$$+\frac{a\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4b\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}$$

$$+\frac{a\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4b\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}$$

$$+\frac{\sqrt{-a}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}$$

$$+\frac{x\sqrt{dx^4+c}}{b\sqrt{d}\left(\sqrt{d}x^2+\sqrt{c}\right)}$$

```
[In] Int[x^6/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (x*Sqrt[c + d*x^4])/(b*Sqrt[d]*(Sqrt[c] + Sqrt[d]*x^2)) + ((-a)^(3/4)*ArcTa
n[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*b^(5/4)*Sqr
t[b*c - a*d]) - ((-a)^(3/4)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/4)*b^(1/
4)*Sqrt[c + d*x^4])])/(4*b^(5/4)*Sqrt[-(b*c) + a*d]) - (c^(1/4)*(Sqrt[c] +
Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan
[(d^(1/4)*x)/c^(1/4)], 1/2])/(b*d^(3/4)*Sqrt[c + d*x^4]) + (c^(1/4)*(Sqrt[c
] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*Ar
cTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*b*d^(3/4)*Sqrt[c + d*x^4]) + (a*(Sqrt[c
] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d
*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1
```

```
/2])/(4*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (a*(Sqrt[c] + (Sqrt[-a]*Sq
rt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] +
 Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b*c^(1/4
)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[-a]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt
[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*
EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sq
rt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b^(3/2)*c^(1/4)*d^(
1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - (Sqrt[-a]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*S
qrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^
2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sq
rt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b^(3/2)*c^(1/4)*d^(
1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
```

<u>Rule 226</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

<u>Rule 311</u>

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

<u>Rule 494</u>

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

<u>Rule 504</u>

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

<u>Rule 1210</u>

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
```

```
}, x] && PosQ[c/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{x^2}{\sqrt{c+dx^4}}\,dx}{b} - \frac{a\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{b}\\
&= \frac{a\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{2b^{3/2}} - \frac{a\int \frac{1}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{2b^{3/2}} + \frac{\sqrt{c}\int \frac{1}{\sqrt{c+dx^4}}\,dx}{b\sqrt{d}} - \frac{\sqrt{c}\int \frac{1-\frac{\sqrt{d}x^2}{\sqrt{c}}}{\sqrt{c+dx^4}}\,dx}{b\sqrt{d}}
\end{aligned}$$

$$= \frac{x\sqrt{c+dx^4}}{b\sqrt{d}\left(\sqrt{c}+\sqrt{d}x^2\right)} - \frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{bd^{3/4}\sqrt{c+dx^4}}$$

$$+ \frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2bd^{3/4}\sqrt{c+dx^4}}$$

$$+ \frac{\left(a\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{2b(bc+ad)}$$

$$- \frac{\left(a\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{2b(bc+ad)}$$

$$+ \frac{\left(a\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt{d}\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{2b^{3/2}(bc+ad)}$$

$$+ \frac{\left(a\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt{d}\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{2b^{3/2}(bc+ad)}$$

$$= \frac{x\sqrt{c+dx^4}}{b\sqrt{d}\left(\sqrt{c}+\sqrt{d}x^2\right)} + \frac{(-a)^{3/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4b^{5/4}\sqrt{bc-ad}}$$
$$- \frac{(-a)^{3/4}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4b^{5/4}\sqrt{-bc+ad}}$$
$$- \frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{bd^{3/4}\sqrt{c+dx^4}}$$
$$+ \frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2bd^{3/4}\sqrt{c+dx^4}}$$
$$+ \frac{a\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4b^{3/2}\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}}$$
$$+ \frac{a\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4b^{3/2}\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}}$$
$$+ \frac{\sqrt{-a}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}}$$
$$- \frac{\sqrt{-a}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.08

$$\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{x^7\sqrt{\frac{c+dx^4}{c}}\,\text{AppellF1}\left(\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{7a\sqrt{c+dx^4}}$$

```
[In] Integrate[x^6/((a + b*x^4)*Sqrt[c + d*x^4]),x]
[Out] (x^7*Sqrt[(c + d*x^4)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x^4
)/a)])/(7*a*Sqrt[c + d*x^4])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.76 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.36

method	result
default	$\frac{i\sqrt{c}\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\sqrt{d\,x^4+c}\,\sqrt{d}}-\frac{a\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha}{}}{}\right)}{8b^2}$
elliptic	$\frac{i\sqrt{c}\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\sqrt{d\,x^4+c}\,\sqrt{d}}-\frac{a\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha}{}}{}\right)}{8b^2}$

```
[In] int(x^6/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] I/b*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/
c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)/d^(1/2)*(EllipticF(x*(I/c^(1/2)*
d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(1/2))^(1/2),I))-1/8*a/b^2*sum(1
/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+
b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-
I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2
)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I
/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a)
)
```

Fricas [F]

$$\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx=\int \frac{x^6}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

```
[In] integrate(x^6/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^4 + c)*x^6/(b*d*x^8 + (b*c + a*d)*x^4 + a*c), x)
```

Sympy [F]

$$\int \frac{x^6}{(a+bx^4)\,\sqrt{c+dx^4}}\,dx = \int \frac{x^6}{(a+bx^4)\,\sqrt{c+dx^4}}\,dx$$

```
[In] integrate(x**6/(b*x**4+a)/(d*x**4+c)**(1/2),x)
[Out] Integral(x**6/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{x^6}{(a+bx^4)\,\sqrt{c+dx^4}}\,dx = \int \frac{x^6}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

```
[In] integrate(x^6/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^6/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{x^6}{(a+bx^4)\,\sqrt{c+dx^4}}\,dx = \int \frac{x^6}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

```
[In] integrate(x^6/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^6/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^4)\,\sqrt{c+dx^4}}\,dx = \int \frac{x^6}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}\,dx$$

```
[In] int(x^6/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
[Out] int(x^6/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.820 $\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 656

$$\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4(bc-ad)}+\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4(bc-ad)}$$
$$-\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2(bc+ad)\sqrt{c+dx^4}}$$
$$-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8\sqrt{b}\sqrt[4]{c}\left(\sqrt{-a}\sqrt{b}\sqrt{c}-a\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$
$$+\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8\sqrt{b}\sqrt[4]{c}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$

```
[Out] 1/4*arctan(x*((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*((a*d-b*
c)/(-a)^(1/2)/b^(1/2))^(1/2)/(-a*d+b*c)+1/4*arctan(x*((-a*d+b*c)/(-a)^(1/2)
/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(-a*
d+b*c)-1/2*c^(1/4)*d^(1/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2
*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*
2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/(a
*d+b*c)/(d*x^4+c)^(1/2)-1/8*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(
2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/
4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2)
,1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((
```

```
d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/c^(1/4)/d^(1/4)/b^(1/2)/((-a)^(1/2)
*b^(1/2)*c^(1/2)-a*d^(1/2))/(d*x^4+c)^(1/2)+1/8*(cos(2*arctan(d^(1/4)*x/c^(
1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^
(1/4)*x/c^(1/4))),-1/4*c^(1/2)*(b^(1/2)-(-a)^(1/2)*d^(1/2)/c^(1/2))^2/(-a)^
(1/2)/b^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(
-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/c^(1/4)/d^(1/4
)/b^(1/2)/((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))/(d*x^4+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 756, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {504, 1231, 226, 1721}

$$\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}} - \frac{\arctan\left(\frac{x\sqrt{ad-bc}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4\sqrt[4]{-a}\sqrt[4]{b}\sqrt{ad-bc}}$$
$$- \frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}$$
$$- \frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}+\sqrt{c}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}$$
$$- \frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8\sqrt{-a}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}(ad+bc)}$$
$$+ \frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)^2\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8\sqrt{-a}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}(ad+bc)}$$

```
[In] Int[x^2/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])]/(4*(-a)^(1
/4)*b^(1/4)*Sqrt[b*c - a*d]) - ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/4)*b^
(1/4)*Sqrt[c + d*x^4])]/(4*(-a)^(1/4)*b^(1/4)*Sqrt[-(b*c) + a*d]) - ((Sqrt[
c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c +
d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)],
1/2])/(4*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[c] + (Sqrt[-a]*Sqrt[
d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sq
rt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*c^(1/4)*(b*
c + a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c
```

```
] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/
4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d])
, 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*Sqrt[-a]*Sqrt[b]*c^(1/4)*d^(1/4)*
(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqr
t[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[
(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d])
, 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*Sqrt[-a]*Sqrt[b]*c^(1/4)*d^(1/4)*
(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{2\sqrt{b}}+\frac{\int \frac{1}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{2\sqrt{b}}$$

$$= -\frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{2(bc+ad)} + \frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{2(bc+ad)} - \frac{\left(\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt{d}\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{2(bc+ad)} - \frac{\left(\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt{d}\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{2(bc+ad)}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}} - \frac{\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4\sqrt[4]{-a}\sqrt[4]{b}\sqrt{-bc+ad}} - \frac{\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}} - \frac{\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}} + \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8\sqrt{-a}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}} - \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8\sqrt{-a}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.10

$$\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{x^3\sqrt{\frac{c+dx^4}{c}}\operatorname{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{3a\sqrt{c+dx^4}}$$

```
[In] Integrate[x^2/((a + b*x^4)*Sqrt[c + d*x^4]),x]
[Out] (x^3*Sqrt[(c + d*x^4)/c]*AppellF1[3/4, 1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)
/a)])/(3*a*Sqrt[c + d*x^4])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.80 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.29

method	result	size
default	$\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^4 b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2 d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^3 b\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\Pi\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},\frac{i\sqrt{c}\,\_\alpha^2 b}{\sqrt{d}\,a},\frac{\sqrt{-\frac{i\sqrt{d}}{\sqrt{c}}}}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}}\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,a\sqrt{d\,x^4+c}}}{\_\alpha}}{8b}$	191
elliptic	$\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^4 b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2 d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^3 b\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\Pi\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},\frac{i\sqrt{c}\,\_\alpha^2 b}{\sqrt{d}\,a},\frac{\sqrt{-\frac{i\sqrt{d}}{\sqrt{c}}}}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}}\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,a\sqrt{d\,x^4+c}}}{\_\alpha}}{8b}$	191

```
[In] int(x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/b*sum(1/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2
*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alph
a^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*
x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alph
a^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootO
f(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \text{Timed out}$$

```
[In] integrate(x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

```
[In] integrate(x**2/(b*x**4+a)/(d*x**4+c)**(1/2),x)
[Out] Integral(x**2/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^2}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

```
[In] integrate(x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^2}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

```
[In] integrate(x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^2}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}\,dx$$

```
[In] int(x^2/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
[Out] int(x^2/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.821 $\int \frac{1}{x^2(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 833

$$\begin{aligned}\int \frac{1}{x^2(a+bx^4)\sqrt{c+dx^4}}\,dx &= -\frac{\sqrt{c+dx^4}}{acx}+\frac{\sqrt{d}x\sqrt{c+dx^4}}{ac\left(\sqrt{c}+\sqrt{d}x^2\right)} \\ &\quad-\frac{b\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4a(bc-ad)}-\frac{b\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4a(bc-ad)} \\ &\quad-\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{ac^{3/4}\sqrt{c+dx^4}} \\ &\quad+\frac{\sqrt[4]{d}(2bc+ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2ac^{3/4}(bc+ad)\sqrt{c+dx^4}} \\ &\quad+\frac{\sqrt{b}\left(\frac{\sqrt{b}\sqrt[4]{c}}{\sqrt[4]{d}}-\frac{\sqrt{-a}\sqrt[4]{d}}{\sqrt[4]{c}}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a\left(\sqrt{-a}\sqrt{b}\sqrt{c}-a\sqrt{d}\right)\sqrt{c+dx^4}} \\ &\quad-\frac{\sqrt{b}\left(\frac{\sqrt{b}\sqrt[4]{c}}{\sqrt[4]{d}}+\frac{\sqrt{-a}\sqrt[4]{d}}{\sqrt[4]{c}}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt{c+dx^4}}\end{aligned}$$

```
[Out] -(d*x^4+c)^(1/2)/a/c/x+x*d^(1/2)*(d*x^4+c)^(1/2)/a/c/(c^(1/2)+x^2*d^(1/2))-
1/4*b*arctan(x*((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*((a*d-
b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/a/(-a*d+b*c)-1/4*b*arctan(x*((-a*d+b*c)/(-a)
^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2
```

```
)/a/(-a*d+b*c)-d^(1/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arc
tan(d^(1/4)*x/c^(1/4)))*EllipticE(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1
/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/c^(3
/4)/(d*x^4+c)^(1/2)+1/2*d^(1/4)*(a*d+2*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)
))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)
*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(
1/2))^2)^(1/2)/a/c^(3/4)/(a*d+b*c)/(d*x^4+c)^(1/2)+1/8*(cos(2*arctan(d^(1/4
)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*ar
ctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1
/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*b^(1/2)*(-d^(1/4)*(-a)^(1/2)/c^(1/
4)+c^(1/4)*b^(1/2)/d^(1/4))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d
^(1/2))^2)^(1/2)/a/((-a)^(1/2)*b^(1/2)*c^(1/2)-a*d^(1/2))/(d*x^4+c)^(1/2)-1
/8*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4
)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*c^(1/2)*(b^(1/2)-(-a)^
(1/2)*d^(1/2)/c^(1/2))^2/(-a)^(1/2)/b^(1/2)/d^(1/2),1/2*2^(1/2))*b^(1/2)*(d
^(1/4)*(-a)^(1/2)/c^(1/4)+c^(1/4)*b^(1/2)/d^(1/4))*(c^(1/2)+x^2*d^(1/2))*((
d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(
1/2))/(d*x^4+c)^(1/2)
```

Rubi [A] (warning: unable to verify)

Time = 1.02 (sec) , antiderivative size = 1007, normalized size of antiderivative = 1.21, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{491, 598, 311, 226, 1210, 504, 1231, 1721\}$

$$\int \frac{1}{x^2 (a+bx^4) \sqrt{c+dx^4}} dx =$$

$$-\frac{\sqrt{b}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}}, 2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right), \frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8(-a)^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}$$

$$+\frac{b^{3/4}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{4(-a)^{5/4}\sqrt{bc-ad}} - \frac{b^{3/4}\arctan\left(\frac{\sqrt{ad-bc}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{4(-a)^{5/4}\sqrt{ad-bc}}$$

$$-\frac{\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{ac^{3/4}\sqrt{dx^4+c}}$$

$$+\frac{\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right), \frac{1}{2}\right)}{2ac^{3/4}\sqrt{dx^4+c}}$$

$$+\frac{b\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right), \frac{1}{2}\right)}{4a\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}$$

$$+\frac{b\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right), \frac{1}{2}\right)}{4a\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}$$

$$+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}}, 2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right), \frac{1}{2}\right)}{8(-a)^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}$$

$$-\frac{\sqrt{dx^4+c}}{acx}+\frac{\sqrt{d}x\sqrt{dx^4+c}}{ac\left(\sqrt{d}x^2+\sqrt{c}\right)}$$

```
[In] Int[1/(x^2*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -(Sqrt[c + d*x^4]/(a*c*x)) + (Sqrt[d]*x*Sqrt[c + d*x^4])/(a*c*(Sqrt[c] + Sq
rt[d]*x^2)) + (b^(3/4)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[
c + d*x^4])])/(4*(-a)^(5/4)*Sqrt[b*c - a*d]) - (b^(3/4)*ArcTan[(Sqrt[-(b*c)
 + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(5/4)*Sqrt[-(b*c)
 + a*d]) - (d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqr
t[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(a*c^(3/4)*Sqrt
[c + d*x^4]) + (d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] +
 Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*a*c^(3/4
)*Sqrt[c + d*x^4]) + (b*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqr
t[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2
```

```
*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4
]) + (b*(Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x
^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)
*x)/c^(1/4)], 1/2])/(4*a*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(S
qrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^
4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*S
qrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)]
, 1/2])/(8*(-a)^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - (Sqrt[
b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c +
 d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*S
qrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4
)], 1/2])/(8*(-a)^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
```

```
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{c+dx^4}}{acx}+\frac{\int \frac{x^2\left(-bc+ad+bdx^4\right)}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{ac}\\
&= -\frac{\sqrt{c+dx^4}}{acx}+\frac{\int \left(\frac{dx^2}{\sqrt{c+dx^4}}-\frac{bcx^2}{(a+bx^4)\sqrt{c+dx^4}}\right)dx}{ac}\\
&= -\frac{\sqrt{c+dx^4}}{acx}-\frac{b\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{a}+\frac{d\int \frac{x^2}{\sqrt{c+dx^4}}\,dx}{ac}\\
&= -\frac{\sqrt{c+dx^4}}{acx}+\frac{\sqrt{b}\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{2a}\\
&\quad -\frac{\sqrt{b}\int \frac{1}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{2a}+\frac{\sqrt{d}\int \frac{1}{\sqrt{c+dx^4}}\,dx}{a\sqrt{c}}-\frac{\sqrt{d}\int \frac{1-\frac{\sqrt{d}x^2}{\sqrt{c}}}{\sqrt{c+dx^4}}\,dx}{a\sqrt{c}}
\end{aligned}
$$

$$= -\frac{\sqrt{c+dx^4}}{acx} + \frac{\sqrt{d}x\sqrt{c+dx^4}}{ac\left(\sqrt{c}+\sqrt{d}x^2\right)}$$
$$- \frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{ac^{3/4}\sqrt{c+dx^4}}$$
$$+ \frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2ac^{3/4}\sqrt{c+dx^4}}$$
$$+ \frac{\left(b\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{2a(bc+ad)}$$
$$- \frac{\left(b\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{2a(bc+ad)}$$
$$+ \frac{\left(\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt{d}\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{2a(bc+ad)}$$
$$+ \frac{\left(\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt{d}\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{2a(bc+ad)}$$

$$
\begin{aligned}
&= -\frac{\sqrt{c+dx^4}}{acx} + \frac{\sqrt{d}x\sqrt{c+dx^4}}{ac\left(\sqrt{c}+\sqrt{d}x^2\right)} \\
&+ \frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{5/4}\sqrt{bc-ad}} - \frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{5/4}\sqrt{-bc+ad}} \\
&- \frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{ac^{3/4}\sqrt{c+dx^4}} \\
&+ \frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2ac^{3/4}\sqrt{c+dx^4}} \\
&+ \frac{\sqrt{b}\left(\sqrt{b}\sqrt[4]{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt[4]{c}}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4a(bc+ad)\sqrt{c+dx^4}} \\
&+ \frac{\sqrt{b}\left(\sqrt{b}\sqrt[4]{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt[4]{c}}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4a(bc+ad)\sqrt{c+dx^4}} \\
&+ \frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8(-a)^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}} \\
&- \frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8(-a)^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.17

$$
\int \frac{1}{x^2(a+bx^4)\sqrt{c+dx^4}}\,dx
$$

$$
= \frac{-21a(c+dx^4)+7(-bc+ad)x^4\sqrt{1+\frac{dx^4}{c}}\operatorname{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+3bdx^8\sqrt{1+\frac{dx^4}{c}}\operatorname{AppellF1}\left(\right.}{21a^2cx\sqrt{c+dx^4}}
$$

```
[In] Integrate[1/(x^2*(a + b*x^4)*Sqrt[c + d*x^4]),x]
[Out] (-21*a*(c + d*x^4) + 7*(-(b*c) + a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[3/4,
 1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)] + 3*b*d*x^8*Sqrt[1 + (d*x^4)/c]*A
ppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x^4)/a)])/(21*a^2*c*x*Sqrt[c
+ d*x^4])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.49 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.37

method	result
default	$\frac{-\frac{\sqrt{d\,x^4+c}}{cx}+\frac{i\sqrt{d}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{\sqrt{c}\,\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}}{a}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{-\frac{\mathrm{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_}{}}{}}{8}$
elliptic	$-\frac{\sqrt{d\,x^4+c}}{acx}+\frac{i\sqrt{d}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{a\sqrt{c}\,\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{-\frac{\mathrm{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4}}\right)}{\sqrt{\frac{-ad+bc}{b}}}}{}}{}$
risch	$-\frac{\sqrt{d\,x^4+c}}{acx}+\frac{\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{c\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{-\frac{\mathrm{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}}{}\right.}{}}{ac}$

```
[In] int(1/x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/c*(d*x^4+c)^(1/2)/x+I*d^(1/2)/c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-
I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2
)*(EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(1/2))
^(1/2),I)))-1/8/a*sum(1/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alp
ha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))
^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^
2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d
^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),
_alpha=RootOf(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + bx^4) \sqrt{c + dx^4}} \, dx = \text{Timed out}$$

```
[In] integrate(1/x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^2 (a + bx^4) \sqrt{c + dx^4}} \, dx = \int \frac{1}{x^2 (a + bx^4) \sqrt{c + dx^4}} \, dx$$

```
[In] integrate(1/x**2/(b*x**4+a)/(d*x**4+c)**(1/2),x)
[Out] Integral(1/(x**2*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a + bx^4) \sqrt{c + dx^4}} \, dx = \int \frac{1}{(bx^4 + a)\sqrt{dx^4 + c}x^2} \, dx$$

```
[In] integrate(1/x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a + bx^4) \sqrt{c + dx^4}} \, dx = \int \frac{1}{(bx^4 + a)\sqrt{dx^4 + c}x^2} \, dx$$

```
[In] integrate(1/x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^4) \sqrt{c+dx^4}} \, dx = \int \frac{1}{x^2 \, (b\,x^4+a) \, \sqrt{d\,x^4+c}} \, dx$$

```
[In] int(1/(x^2*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
[Out] int(1/(x^2*(a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.822 $\int \frac{x^{15}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 175

$$\int \frac{x^{15}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{ax^8\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} - \frac{\sqrt{c+dx^4}(4b^2c^2+8abcd-15a^2d^2-bd(2bc-5ad)x^4)}{12b^3d^2(bc-ad)} - \frac{a^2(6bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{7/2}(bc-ad)^{3/2}}$$

```
[Out] -1/4*a^2*(-5*a*d+6*b*c)*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/b
^(7/2)/(-a*d+b*c)^(3/2)+1/4*a*x^8*(d*x^4+c)^(1/2)/b/(-a*d+b*c)/(b*x^4+a)-1/
12*(4*b^2*c^2+8*a*b*c*d-15*a^2*d^2-b*d*(-5*a*d+2*b*c)*x^4)*(d*x^4+c)^(1/2)/
b^3/d^2/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 100, 152, 65, 214}

$$\int \frac{x^{15}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = -\frac{a^2(6bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{7/2}(bc-ad)^{3/2}} - \frac{\sqrt{c+dx^4}(-15a^2d^2-bdx^4(2bc-5ad)+8abcd+4b^2c^2)}{12b^3d^2(bc-ad)} + \frac{ax^8\sqrt{c+dx^4}}{4b(a+bx^4)(bc-ad)}$$

```
[In] Int[x^15/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (a*x^8*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) - (Sqrt[c + d*x^4]*(4
*b^2*c^2 + 8*a*b*c*d - 15*a^2*d^2 - b*d*(2*b*c - 5*a*d)*x^4))/(12*b^3*d^2*(
b*c - a*d)) - (a^2*(6*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b
*c - a*d]])/(4*b^(7/2)*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{x^3}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^4\right)\\
&= \frac{ax^8\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} - \frac{\text{Subst}\left(\int \frac{x\left(2ac+\frac{1}{2}(-2bc+5ad)x\right)}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4b(bc-ad)}\\
&= \frac{ax^8\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt{c+dx^4}\left(4b^2c^2+8abcd-15a^2d^2-bd(2bc-5ad)x^4\right)}{12b^3d^2(bc-ad)}\\
&\quad + \frac{\left(a^2(6bc-5ad)\right)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{8b^3(bc-ad)}\\
&= \frac{ax^8\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt{c+dx^4}\left(4b^2c^2+8abcd-15a^2d^2-bd(2bc-5ad)x^4\right)}{12b^3d^2(bc-ad)}\\
&\quad + \frac{\left(a^2(6bc-5ad)\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{4b^3d(bc-ad)}\\
&= \frac{ax^8\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt{c+dx^4}\left(4b^2c^2+8abcd-15a^2d^2-bd(2bc-5ad)x^4\right)}{12b^3d^2(bc-ad)}\\
&\quad - \frac{a^2(6bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{7/2}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00

$$\begin{aligned}
&\int \frac{x^{15}}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx\\
&= -\frac{\sqrt{c+dx^4}\left(-15a^3d^2+2a^2bd\left(4c-5dx^4\right)+2b^3cx^4\left(2c-dx^4\right)+2ab^2\left(2c^2+3cdx^4+d^2x^8\right)\right)}{12b^3d^2(bc-ad)\left(a+bx^4\right)}\\
&\quad + \frac{a^2(-6bc+5ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{4b^{7/2}(-bc+ad)^{3/2}}
\end{aligned}$$

```
[In] Integrate[x^15/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
[Out] -1/12*(Sqrt[c + d*x^4]*(-15*a^3*d^2 + 2*a^2*b*d*(4*c - 5*d*x^4) + 2*b^3*c*x
^4*(2*c - d*x^4) + 2*a*b^2*(2*c^2 + 3*c*d*x^4 + d^2*x^8)))/(b^3*d^2*(b*c -
a*d)*(a + b*x^4)) + (a^2*(-6*b*c + 5*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^4])/
Sqrt[-(b*c) + a*d]])/(4*b^(7/2)*(-(b*c) + a*d)^(3/2))
```

Maple [A] (verified)

Time = 5.15 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.98

method	result
pseudoelliptic	$-\frac{5\left(-d^2\left(ad-\frac{6bc}{5}\right)(b\,x^4+a)a^2\arctan\left(\frac{b\sqrt{d\,x^4+c}}{\sqrt{(ad-bc)b}}\right)+\left(-\frac{4x^4\left(-\frac{d\,x^4}{2}+c\right)c\,b^3}{15}-\frac{4\left(d\,x^4+c\right)\left(\frac{d\,x^4}{2}+c\right)a\,b^2}{15}-\frac{8\left(-\frac{5d\,x^4}{4}+c\right)d\,a^2b}{15}+\right.}{4\sqrt{(ad-bc)b}\,d^2b^3(ad-bc)(b\,x^4+a)}$
risch	$-\frac{(-bd\,x^4+6ad+2bc)\sqrt{d\,x^4+c}}{6d^2b^3}-\frac{3a^2\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-}}{b}\right.}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right.}{4b^4\sqrt{-\frac{ad-bc}{b}}}$
default	$-\frac{\sqrt{d\,x^4+c}\,(-d\,x^4+2c)}{6b^2d^2}-\frac{a\sqrt{d\,x^4+c}}{b^3d}+\frac{3a^2\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-2d\sqrt{-}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4b\sqrt{-\frac{ad-bc}{b}}}\right.}{}$
elliptic	$\frac{x^4\sqrt{d\,x^4+c}}{6b^2d}-\frac{c\sqrt{d\,x^4+c}}{3b^2d^2}-\frac{a\sqrt{d\,x^4+c}}{b^3d}-\frac{3a^2\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-2d\sqrt{-a}}}{x^2+\frac{\sqrt{-ab}}{b}}\right.}{4b^4\sqrt{-\frac{ad-bc}{b}}}$

```
[In] int(x^15/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -5/4*(-d^2*(a*d-6/5*b*c)*(b*x^4+a)*a^2*arctan(b*(d*x^4+c)^(1/2)/((a*d-b*c)*
b)^(1/2))+(-4/15*x^4*(-1/2*d*x^4+c)*c*b^3-4/15*(d*x^4+c)*(1/2*d*x^4+c)*a*b^
2-8/15*(-5/4*d*x^4+c)*d*a^2*b+a^3*d^2)*((a*d-b*c)*b)^(1/2)*(d*x^4+c)^(1/2))
/((a*d-b*c)*b)^(1/2)/d^2/b^3/(a*d-b*c)/(b*x^4+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.62 (sec) , antiderivative size = 622, normalized size of antiderivative = 3.55

$$\int \frac{x^{15}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$
$$=\left[\frac{3\,(6\,a^3bcd^2-5\,a^4d^3+(6\,a^2b^2cd^2-5\,a^3bd^3)x^4)\sqrt{b^2c-abd}\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)+2\,(2\,(b^5c^2d\right.}{24\,(ab^6c^2d^2-2\,a^2b^5cd^3}\right.$$

```
[In] integrate(x^15/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(6*a^3*b*c*d^2 - 5*a^4*d^3 + (6*a^2*b^2*c*d^2 - 5*a^3*b*d^3)*x^4)*
sqrt(b^2*c - a*b*d)*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*sqrt(b^2
*c - a*b*d))/(b*x^4 + a)) + 2*(2*(b^5*c^2*d - 2*a*b^4*c*d^2 + a^2*b^3*d^3)*
x^8 - 4*a*b^4*c^3 - 4*a^2*b^3*c^2*d + 23*a^3*b^2*c*d^2 - 15*a^4*b*d^3 - 2*(
2*b^5*c^3 + a*b^4*c^2*d - 8*a^2*b^3*c*d^2 + 5*a^3*b^2*d^3)*x^4)*sqrt(d*x^4
+ c))/(a*b^6*c^2*d^2 - 2*a^2*b^5*c*d^3 + a^3*b^4*d^4 + (b^7*c^2*d^2 - 2*a*b
^6*c*d^3 + a^2*b^5*d^4)*x^4), 1/12*(3*(6*a^3*b*c*d^2 - 5*a^4*d^3 + (6*a^2*b
^2*c*d^2 - 5*a^3*b*d^3)*x^4)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^4 + c)*sq
rt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) + (2*(b^5*c^2*d - 2*a*b^4*c*d^2 + a^2*b
^3*d^3)*x^8 - 4*a*b^4*c^3 - 4*a^2*b^3*c^2*d + 23*a^3*b^2*c*d^2 - 15*a^4*b*d
^3 - 2*(2*b^5*c^3 + a*b^4*c^2*d - 8*a^2*b^3*c*d^2 + 5*a^3*b^2*d^3)*x^4)*sqr
t(d*x^4 + c))/(a*b^6*c^2*d^2 - 2*a^2*b^5*c*d^3 + a^3*b^4*d^4 + (b^7*c^2*d^2
 - 2*a*b^6*c*d^3 + a^2*b^5*d^4)*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{15}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Timed out}$$

```
[In] integrate(x**15/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{15}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^15/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.03

$$\int \frac{x^{15}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{\sqrt{dx^4+c}a^3d}{4\,(b^4c-ab^3d)((dx^4+c)b-bc+ad)} + \frac{(6\,a^2bc-5\,a^3d)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{4\,(b^4c-ab^3d)\sqrt{-b^2c+abd}} + \frac{(dx^4+c)^{\frac{3}{2}}b^4d^4-3\,\sqrt{dx^4+c}b^4cd^4-6\,\sqrt{dx^4+c}ab^3d^5}{6\,b^6d^6}$$

```
[In] integrate(x^15/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(d*x^4 + c)*a^3*d/((b^4*c - a*b^3*d)*((d*x^4 + c)*b - b*c + a*d)) +
 1/4*(6*a^2*b*c - 5*a^3*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(
(b^4*c - a*b^3*d)*sqrt(-b^2*c + a*b*d)) + 1/6*((d*x^4 + c)^(3/2)*b^4*d^4 -
3*sqrt(d*x^4 + c)*b^4*c*d^4 - 6*sqrt(d*x^4 + c)*a*b^3*d^5)/(b^6*d^6)
```

Mupad [B] (verification not implemented)

Time = 9.45 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.06

$$\int \frac{x^{15}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{(d\,x^4+c)^{3/2}}{6\,b^2\,d^2} - \left(\frac{3\,c}{2\,b^2\,d^2}+\frac{a\,d-b\,c}{b^3\,d^2}\right)\sqrt{d\,x^4+c} + \frac{a^2\,\mathrm{atan}\left(\frac{a^2\,\sqrt{b}\,\sqrt{d\,x^4+c}\,(5\,a\,d-6\,b\,c)}{\sqrt{a\,d-b\,c}\,(5\,a^3\,d-6\,a^2\,b\,c)}\right)\,(5\,a\,d-6\,b\,c)}{4\,b^{7/2}\,(a\,d-b\,c)^{3/2}} - \frac{a^3\,d\,\sqrt{d\,x^4+c}}{2\,(a\,d-b\,c)\,(2\,b^4\,(d\,x^4+c)-2\,b^4\,c+2\,a\,b^3\,d)}$$

```
[In] int(x^15/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] (c + d*x^4)^(3/2)/(6*b^2*d^2) - ((3*c)/(2*b^2*d^2) + (a*d - b*c)/(b^3*d^2))
*(c + d*x^4)^(1/2) + (a^2*atan((a^2*b^(1/2)*(c + d*x^4)^(1/2)*(5*a*d - 6*b*
c))/((a*d - b*c)^(1/2)*(5*a^3*d - 6*a^2*b*c)))*(5*a*d - 6*b*c))/(4*b^(7/2)*
(a*d - b*c)^(3/2)) - (a^3*d*(c + d*x^4)^(1/2))/(2*(a*d - b*c)*(2*b^4*(c + d
*x^4) - 2*b^4*c + 2*a*b^3*d))
```

3.823 $\int \frac{x^{11}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 123

$$\int \frac{x^{11}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{\sqrt{c+dx^4}}{2b^2d} - \frac{a^2\sqrt{c+dx^4}}{4b^2(bc-ad)(a+bx^4)} + \frac{a(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{5/2}(bc-ad)^{3/2}}$$

```
[Out] 1/4*a*(-3*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(5
/2)/(-a*d+b*c)^(3/2)+1/2*(d*x^4+c)^(1/2)/b^2/d-1/4*a^2*(d*x^4+c)^(1/2)/b^2/
(-a*d+b*c)/(b*x^4+a)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 91, 81, 65, 214}

$$\int \frac{x^{11}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = -\frac{a^2\sqrt{c+dx^4}}{4b^2(a+bx^4)(bc-ad)} + \frac{a(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{5/2}(bc-ad)^{3/2}} + \frac{\sqrt{c+dx^4}}{2b^2d}$$

```
[In] Int[x^11/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] Sqrt[c + d*x^4]/(2*b^2*d) - (a^2*Sqrt[c + d*x^4])/(4*b^2*(b*c - a*d)*(a + b
*x^4)) + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*
d]])/(4*b^(5/2)*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^4\right)$$

$$= -\frac{a^2\sqrt{c+dx^4}}{4b^2(bc-ad)\,(a+bx^4)} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(2bc-ad)+b(bc-ad)x}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4b^2(bc-ad)}$$

$$= \frac{\sqrt{c+dx^4}}{2b^2d} - \frac{a^2\sqrt{c+dx^4}}{4b^2(bc-ad)\left(a+bx^4\right)} - \frac{(a(4bc-3ad))\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{8b^2(bc-ad)}$$

$$= \frac{\sqrt{c+dx^4}}{2b^2d} - \frac{a^2\sqrt{c+dx^4}}{4b^2(bc-ad)\left(a+bx^4\right)} - \frac{(a(4bc-3ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{4b^2d(bc-ad)}$$

$$= \frac{\sqrt{c+dx^4}}{2b^2d} - \frac{a^2\sqrt{c+dx^4}}{4b^2(bc-ad)\left(a+bx^4\right)} + \frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{5/2}(bc-ad)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.06

$$\int \frac{x^{11}}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{\frac{\sqrt{b}\sqrt{c+dx^4}\left(-3a^2d+2b^2cx^4+2ab\left(c-dx^4\right)\right)}{d(bc-ad)(a+bx^4)} + \frac{a(4bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{4b^{5/2}}$$

```
[In] Integrate[x^11/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
[Out] ((Sqrt[b]*Sqrt[c + d*x^4]*(-3*a^2*d + 2*b^2*c*x^4 + 2*a*b*(c - d*x^4)))/(d*
(b*c - a*d)*(a + b*x^4)) + (a*(4*b*c - 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^
4])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/(4*b^(5/2))
```

Maple [A] (verified)

Time = 5.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.08

method	result
pseudoelliptic	$\frac{-\frac{3d\left(b\,x^{4}+a\right)a\left(ad-\frac{4bc}{3}\right)\arctan\left(\frac{b\sqrt{d\,x^{4}+c}}{\sqrt{(ad-bc)b}}\right)}{4}+\frac{3\left(-\frac{2b^{2}c\,x^{4}}{3}-\frac{2a\left(-d\,x^{4}+c\right)b}{3}+a^{2}d\right)\sqrt{d\,x^{4}+c}\,\sqrt{(ad-bc)b}}{4}}{b^{2}(ad-bc)d(b\,x^{4}+a)\sqrt{(ad-bc)b}}$
risch	$\frac{\sqrt{d\,x^{4}+c}}{2b^{2}d}+\frac{a\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^{2}+\frac{\sqrt{-ab}}{b}}\right)}{2b^{3}\sqrt{-\frac{ad-bc}{b}}}+\frac{a\ln\Big(}{}$
elliptic	$\frac{\sqrt{d\,x^{4}+c}}{2b^{2}d}+\frac{a\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^{2}+\frac{\sqrt{-ab}}{b}}\right)}{2b^{3}\sqrt{-\frac{ad-bc}{b}}}+\frac{a\ln\Big(}{}$
default	$\frac{\sqrt{d\,x^{4}+c}}{2b^{2}d}+\frac{a^{2}\left(\frac{\sqrt{-ab}\,\sqrt{d\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{8ab(ad-bc)\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}-\frac{d\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}}{x^{2}+}\right.}{8b(ad-bc)\sqrt{}}\right.}{}$

```
[In] int(x^11/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/4/((a*d-b*c)*b)^(1/2)*(-d*(b*x^4+a)*a*(a*d-4/3*b*c)*arctan(b*(d*x^4+c)^(1
/2)/((a*d-b*c)*b)^(1/2))+(-2/3*b^2*c*x^4-2/3*a*(-d*x^4+c)*b+a^2*d)*(d*x^4+c
)^(1/2)*((a*d-b*c)*b)^(1/2))/d/b^2/(a*d-b*c)/(b*x^4+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(103) = 206$.

Time = 0.64 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.86

$$\int \frac{x^{11}}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$
$$=\left[\frac{(4\,a^2bcd-3\,a^3d^2+(4\,ab^2cd-3\,a^2bd^2)x^4)\sqrt{b^2c-abd}\log\left(\frac{bdx^4+2\,bc-ad+2\,\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)+2\,(2\,ab^3c^2-5\,a}{8\,(ab^5c^2d-2\,a^2b^4cd^2+a^3b^3d^3+(b^6c^2d-2\,ab^5cd^2+a^2b}\right.$$
$$-\frac{(4\,a^2bcd-3\,a^3d^2+(4\,ab^2cd-3\,a^2bd^2)x^4)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^4+c}\sqrt{-b^2c+abd}}{bdx^4+bc}\right)-(2\,ab^3c^2-5\,a^2b^2cd}{4\,(ab^5c^2d-2\,a^2b^4cd^2+a^3b^3d^3+(b^6c^2d-2\,ab^5cd^2+a^2b^4d}$$

```
[In] integrate(x^11/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^4)*sqrt(b^2*
c - a*b*d)*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*sqrt(b^2*c - a*b*
d))/(b*x^4 + a)) + 2*(2*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2 + 2*(b^4*c^
2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^4)*sqrt(d*x^4 + c))/(a*b^5*c^2*d - 2*a^2*b
^4*c*d^2 + a^3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^4), -1
/4*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^4)*sqrt(-b^2*c
 + a*b*d)*arctan(sqrt(d*x^4 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) - (2
*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2 + 2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b
^2*d^2)*x^4)*sqrt(d*x^4 + c))/(a*b^5*c^2*d - 2*a^2*b^4*c*d^2 + a^3*b^3*d^3
+ (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Timed out}$$

```
[In] integrate(x**11/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^11/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.09

$$\int \frac{x^{11}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = -\frac{\sqrt{dx^4+c}a^2d}{4\,(b^3c-ab^2d)((dx^4+c)b-bc+ad)} - \frac{(4\,abc-3\,a^2d)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{4\,(b^3c-ab^2d)\sqrt{-b^2c+abd}} + \frac{\sqrt{dx^4+c}}{2\,b^2d}$$

```
[In] integrate(x^11/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(d*x^4 + c)*a^2*d/((b^3*c - a*b^2*d)*((d*x^4 + c)*b - b*c + a*d))
- 1/4*(4*a*b*c - 3*a^2*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/((
b^3*c - a*b^2*d)*sqrt(-b^2*c + a*b*d)) + 1/2*sqrt(d*x^4 + c)/(b^2*d)
```

Mupad [B] (verification not implemented)

Time = 9.83 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.17

$$\int \frac{x^{11}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\frac{\sqrt{d\,x^4+c}}{2\,b^2\,d}-\frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^4+c}\,(3\,a\,d-4\,b\,c)}{(3\,a^2\,d-4\,a\,b\,c)\,\sqrt{a\,d-b\,c}}\right)\,(3\,a\,d-4\,b\,c)}{4\,b^{5/2}\,(a\,d-b\,c)^{3/2}}+\frac{a^2\,d\,\sqrt{d\,x^4+c}}{2\,(a\,d-b\,c)\,\left(2\,b^3\,\left(d\,x^4+c\right)-2\,b^3\,c+2\,a\,b^2\,d\right)}$$

```
[In] int(x^11/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] (c + d*x^4)^(1/2)/(2*b^2*d) - (a*atan((a*b^(1/2)*(c + d*x^4)^(1/2)*(3*a*d -
 4*b*c))/((3*a^2*d - 4*a*b*c)*(a*d - b*c)^(1/2)))*(3*a*d - 4*b*c))/(4*b^(5/
2)*(a*d - b*c)^(3/2)) + (a^2*d*(c + d*x^4)^(1/2))/(2*(a*d - b*c)*(2*b^3*(c
+ d*x^4) - 2*b^3*c + 2*a*b^2*d))
```

3.824 $\int \frac{x^7}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 99

$$\int \frac{x^7}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{a\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} - \frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}}$$

```
[Out] -1/4*(-a*d+2*b*c)*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)
/(-a*d+b*c)^(3/2)+1/4*a*(d*x^4+c)^(1/2)/b/(-a*d+b*c)/(b*x^4+a)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {457, 79, 65, 214}

$$\int \frac{x^7}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{a\sqrt{c+dx^4}}{4b(a+bx^4)(bc-ad)} - \frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}}$$

```
[In] Int[x^7/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (a*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(4*b^(3/2)*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{x}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^4\right)\\
&= \frac{a\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} + \frac{(2bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{8b(bc-ad)}\\
&= \frac{a\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} + \frac{(2bc-ad)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{4bd(bc-ad)}\\
&= \frac{a\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} - \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

$$\int \frac{x^7}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{\frac{a\sqrt{b}\sqrt{c+dx^4}}{(bc-ad)(a+bx^4)} - \frac{(2bc-ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{4b^{3/2}}$$

```
[In] Integrate[x^7/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((a*Sqrt[b]*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4)) - ((2*b*c - a*d)*Arc
Tan[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/(4
*b^(3/2))
```

Maple [A] (verified)

Time = 5.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{-\frac{a\sqrt{d\,x^4+c}}{b\,x^4+a}+\frac{(ad-2bc)\arctan\left(\frac{b\sqrt{d\,x^4+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}}{4(ad-bc)b}$
elliptic	$-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4b^2\sqrt{-\frac{ad-bc}{b}}} - \frac{\ln\left(\frac{-\frac{2(ad-bc)}{b} \cdots}{\cdots}\right)}{\cdots}$
default	$\frac{-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4b\sqrt{-\frac{ad-bc}{b}}} - \frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{\cdots}}{\cdots}}{\cdots}\right)}{\cdots}}{b}$

```
[In] int(x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/(a*d-b*c)/b*(-a*(d*x^4+c)^(1/2)/(b*x^4+a)+(a*d-2*b*c)/((a*d-b*c)*b)^(1/
2)*arctan(b*(d*x^4+c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.91 (sec) , antiderivative size = 348, normalized size of antiderivative = 3.52

$$\int \frac{x^7}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$
$$= \left[\frac{((2\,b^2c-abd)x^4+2\,abc-a^2d)\sqrt{b^2c-abd}\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)+2\,\sqrt{dx^4+c}(ab^2c-a^2bd)}{8\,(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2+(b^5c^2-2\,ab^4cd+a^2b^3d^2)x^4)},\right.$$

```
[In] integrate(x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(((2*b^2*c - a*b*d)*x^4 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b*
d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) +
 2*sqrt(d*x^4 + c)*(a*b^2*c - a^2*b*d))/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^
2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^4), 1/4*(((2*b^2*c - a*b*d)
*x^4 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^4 + c)*sqrt(-b
^2*c + a*b*d)/(b*d*x^4 + b*c)) + sqrt(d*x^4 + c)*(a*b^2*c - a^2*b*d))/(a*b^
4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)
*x^4)]
```

Sympy [F]

$$\int \frac{x^7}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^7}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

```
[In] integrate(x**7/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**7/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.17

$$\int \frac{x^7}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{\frac{\sqrt{dx^4+c}ad^2}{(b^2c-abd)((dx^4+c)b-bc+ad)}+\frac{(2\,bcd-ad^2)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^2c-abd)\sqrt{-b^2c+abd}}}{4\,d}$$

```
[In] integrate(x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(d*x^4 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^4 + c)*b - b*c + a*d)) +
(2*b*c*d - a*d^2)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c -
a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

Mupad [B] (verification not implemented)

Time = 9.69 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{b}\sqrt{d\,x^4+c}}{\sqrt{a\,d-b\,c}}\right)\,(a\,d-2\,b\,c)}{4\,b^{3/2}\,(a\,d-b\,c)^{3/2}} - \frac{a\,d\sqrt{d\,x^4+c}}{2\,b\,(a\,d-b\,c)\,\left(2\,b\,(d\,x^4+c)+2\,a\,d-2\,b\,c\right)}$$

```
[In] int(x^7/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] (atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c)^(1/2))*(a*d - 2*b*c))/(4*b^(3
/2)*(a*d - b*c)^(3/2)) - (a*d*(c + d*x^4)^(1/2))/(2*b*(a*d - b*c)*(2*b*(c +
 d*x^4) + 2*a*d - 2*b*c))
```

3.825 $\int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = -\frac{\sqrt{c+dx^4}}{4(bc-ad)\,(a+bx^4)}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}(bc-ad)^{3/2}}$$

```
[Out] 1/4*d*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(3/2)/b^
(1/2)-1/4*(d*x^4+c)^(1/2)/(-a*d+b*c)/(b*x^4+a)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {455, 44, 65, 214}

$$\int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^4}}{4\,(a+bx^4)\,(bc-ad)}$$

```
[In] Int[x^3/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/4*Sqrt[c + d*x^4]/((b*c - a*d)*(a + b*x^4)) + (d*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^4])/Sqrt[b*c - a*d]])/(4*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !Int
```

```
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^4\right)\\
&= -\frac{\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)} - \frac{d\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{8(bc-ad)}\\
&= -\frac{\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)} - \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{4(bc-ad)}\\
&= -\frac{\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{1}{4}\left(-\frac{\sqrt{c+dx^4}}{(bc-ad)(a+bx^4)}+\frac{d\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}(-bc+ad)^{3/2}}\right)$$

```
[In] Integrate[x^3/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (-(Sqrt[c + d*x^4]/((b*c - a*d)*(a + b*x^4))) + (d*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^4])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/4
```

Maple [A] (verified)

Time = 4.89 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{d\left(b\,x^{4}+a\right)\arctan\left(\frac{b\sqrt{d\,x^{4}+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{d\,x^{4}+c}\,\sqrt{(ad-bc)b}}{4\sqrt{(ad-bc)b}\,(ad-bc)\left(b\,x^{4}+a\right)}$
default	$\frac{\sqrt{-ab}\,\sqrt{d\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{-ab}\,\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{8ab(ad-bc)\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}-\frac{d\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\,\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{d\left(x^{2}+\frac{\sqrt{-ab}}{b}\cdots\right.}}{x^{2}+\frac{\sqrt{-ab}}{b}}\cdots\right.}{8b(ad-bc)\sqrt{-\frac{ad-bc}{b}}\cdots}$
elliptic	$\frac{\sqrt{-ab}\,\sqrt{d\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{-ab}\,\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{8ab(ad-bc)\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}-\frac{d\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\,\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{d\left(x^{2}+\frac{\sqrt{-ab}}{b}\cdots\right.}}{x^{2}+\frac{\sqrt{-ab}}{b}}\cdots\right.}{8b(ad-bc)\sqrt{-\frac{ad-bc}{b}}\cdots}$

```
[In] int(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/((a*d-b*c)*b)^(1/2)*(d*(b*x^4+a)*arctan(b*(d*x^4+c)^(1/2)/((a*d-b*c)*b)
^(1/2))+(d*x^4+c)^(1/2)*((a*d-b*c)*b)^(1/2))/(a*d-b*c)/(b*x^4+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(71) = 142.

Time = 0.35 (sec) , antiderivative size = 302, normalized size of antiderivative = 3.47

$$\int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \left[-\frac{(bdx^4+ad)\sqrt{b^2c-abd}\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)+2\,\sqrt{dx^4+c}(b^2c-abd)}{8\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^4)}, -\frac{(bdx^4+ad)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^4+c}\sqrt{-b^2c+abd}}{bdx^4+bc}\right)+\sqrt{dx^4+c}(b^2c-abd)}{4\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^4)}\right]$$

```
[In] integrate(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
[Out] [-1/8*((b*d*x^4 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^4 + 2*b*c - a*d - 2*s
qrt(d*x^4 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) + 2*sqrt(d*x^4 + c)*(b^2*c
 - a*b*d))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d
+ a^2*b^2*d^2)*x^4), -1/4*((b*d*x^4 + a*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt
(d*x^4 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) + sqrt(d*x^4 + c)*(b^2*c
- a*b*d))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d +
 a^2*b^2*d^2)*x^4)]
```

Sympy [F]

$$\int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

```
[In] integrate(x**3/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
[Out] Integral(x**3/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.07

$$\int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = -\frac{d\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{4\sqrt{-b^2c+abd}(bc-ad)} - \frac{\sqrt{dx^4+c}d}{4\left((dx^4+c)b-bc+ad\right)(bc-ad)}$$

```
[In] integrate(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*d*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*(b*c - a*d)) - 1/4*sqrt(d*x^4 + c)*d/(((d*x^4 + c)*b - b*c + a*d)*(b*c - a
*d))
```

Mupad [B] (verification not implemented)

Time = 9.54 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.97

$$\int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{d\sqrt{d\,x^4+c}}{2\,(a\,d-b\,c)\,\left(2\,b\,(d\,x^4+c)+2\,a\,d-2\,b\,c\right)} + \frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^4+c}}{\sqrt{a\,d-b\,c}}\right)}{4\,\sqrt{b}\,(a\,d-b\,c)^{3/2}}$$

```
[In] int(x^3/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] (d*(c + d*x^4)^(1/2))/(2*(a*d - b*c)*(2*b*(c + d*x^4) + 2*a*d - 2*b*c)) + (
d*atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c)^(1/2)))/(4*b^(1/2)*(a*d - b*
c)^(3/2))
```

3.826 $\int \frac{1}{x(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 132

$$\int \frac{1}{x(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{b\sqrt{c+dx^4}}{4a(bc-ad)(a+bx^4)} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a^2\sqrt{c}} + \frac{\sqrt{b}(2bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4a^2(bc-ad)^{3/2}}$$

```
[Out] 1/4*(-3*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2
)/a^2/(-a*d+b*c)^(3/2)-1/2*arctanh((d*x^4+c)^(1/2)/c^(1/2))/a^2/c^(1/2)+1/4
*b*(d*x^4+c)^(1/2)/a/(-a*d+b*c)/(b*x^4+a)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 105, 162, 65, 214}

$$\int \frac{1}{x(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{\sqrt{b}(2bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4a^2(bc-ad)^{3/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a^2\sqrt{c}} + \frac{b\sqrt{c+dx^4}}{4a(a+bx^4)(bc-ad)}$$

```
[In] Int[1/(x*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (b*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*(a + b*x^4)) - ArcTanh[Sqrt[c + d*x^4]
/Sqrt[c]]/(2*a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[
c + d*x^4])/Sqrt[b*c - a*d]])/(4*a^2*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{4}\text{Subst}\left(\int \frac{1}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^4\right)$$

$$= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)\,(a+bx^4)} + \frac{\text{Subst}\left(\int \frac{bc-ad+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4a(bc-ad)}$$

$$= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)(a+bx^4)} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^4\right)}{4a^2} - \frac{(b(2bc-3ad))\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{8a^2(bc-ad)}$$

$$= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)(a+bx^4)} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2a^2d} - \frac{(b(2bc-3ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{4a^2d(bc-ad)}$$

$$= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)(a+bx^4)} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a^2\sqrt{c}} + \frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4a^2(bc-ad)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.94

$$\int \frac{1}{x(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{-\frac{ab\sqrt{c+dx^4}}{(-bc+ad)(a+bx^4)} + \frac{\sqrt{b}(2bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}} - \frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{\sqrt{c}}}{4a^2}$$

```
[In] Integrate[1/(x*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (-((a*b*Sqrt[c + d*x^4])/((-(b*c) + a*d)*(a + b*x^4))) + (Sqrt[b]*(2*b*c -
3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)
^(3/2) - (2*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/Sqrt[c])/(4*a^2)
```

Maple [A] (verified)

Time = 5.01 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11

method	result
pseudoelliptic	$\frac{b\sqrt{c}\,(b\,x^4+a)\left(bc-\frac{3ad}{2}\right)\arctan\left(\frac{b\sqrt{d\,x^4+c}}{\sqrt{(ad-bc)b}}\right)-\frac{\left(2(ad-bc)\left(b\,x^4+a\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}}{\sqrt{c}}\right)+\sqrt{d\,x^4+c}\,\sqrt{c}\,ab\right)\sqrt{(ad-bc)b}}{2}}{2\sqrt{c}\,\sqrt{(ad-bc)b}\,a^2(ad-bc)(b\,x^4+a)}$
elliptic	$-\frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)}{2a^2\sqrt{c}}+\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-b}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4a^2\sqrt{-\frac{ad-bc}{b}}}$
default	$-\frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)}{2a^2\sqrt{c}}-\frac{b\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{4b\sqrt{-\frac{ad-bc}{b}}}\right.}{}$

```
[In] int(1/x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(b*c^(1/2)*(b*x^4+a)*(b*c-3/2*a*d)*arctan(b*(d*x^4+c)^(1/2)/((a*d-b*c)*
b)^(1/2))-1/2*(2*(a*d-b*c)*(b*x^4+a)*arctanh((d*x^4+c)^(1/2)/c^(1/2))+(d*x^
4+c)^(1/2)*c^(1/2)*a*b)*((a*d-b*c)*b)^(1/2))/c^(1/2)/((a*d-b*c)*b)^(1/2)/a^
2/(a*d-b*c)/(b*x^4+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.54 (sec) , antiderivative size = 862, normalized size of antiderivative = 6.53

$$\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$
$$=\left[\frac{2\sqrt{dx^4+c}abc+((2b^2c^2-3abcd)x^4+2abc^2-3a^2cd)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^4+2bc-ad+2\sqrt{dx^4+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^4+a}\right)+}{8\left(a^3bc^2-a^4cd+(a^2b^2c^2-a^3bcd)x^4\right)}\right.$$

```
[In] integrate(1/x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(d*x^4 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^4 + 2*a*b*c^2 -
3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 +
c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + 2*((b^2*c - a*b*d)*x^4 +
 a*b*c - a^2*d)*sqrt(c)*log((d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4))
/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^4), 1/4*(sqrt(d*x^4 + c
```

```
)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^4 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b*
c - a*d))*arctan(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4
 + b*c)) + ((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(c)*log((d*x^4 - 2*sqr
t(d*x^4 + c)*sqrt(c) + 2*c)/x^4))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3
*b*c*d)*x^4), 1/8*(2*sqrt(d*x^4 + c)*a*b*c + 4*((b^2*c - a*b*d)*x^4 + a*b*c
 - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) + ((2*b^2*c^2 - 3*a*b
*c*d)*x^4 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c
 - a*d + 2*sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)))/(
a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^4), 1/4*(sqrt(d*x^4 + c)*
a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^4 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b*c
- a*d))*arctan(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 +
 b*c)) + 2*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^4
 + c)*sqrt(-c)/c))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^4)]
```

Sympy [F]

$$\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx=\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

```
[In] integrate(1/x/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
[Out] Integral(1/(x*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx=\int \frac{1}{\left(bx^4+a\right)^2\sqrt{dx^4+c}\,x}\,dx$$

```
[In] integrate(1/x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx=\frac{\sqrt{dx^4+c}bd}{4\,(abc-a^2d)((dx^4+c)b-bc+ad)}-\frac{(2\,b^2c-3\,abd)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{4\,(a^2bc-a^3d)\sqrt{-b^2c+abd}}+\frac{\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{2\,a^2\sqrt{-c}}$$

```
[In] integrate(1/x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(d*x^4 + c)*b*d/((a*b*c - a^2*d)*((d*x^4 + c)*b - b*c + a*d)) - 1/4
*(2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b
*c - a^3*d)*sqrt(-b^2*c + a*b*d)) + 1/2*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a
^2*sqrt(-c))
```

Mupad [B] (verification not implemented)

Time = 10.72 (sec) , antiderivative size = 3017, normalized size of antiderivative = 22.86

$$\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \text{Too large to display}$$

```
[In] int(1/(x*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] (atan((((((c + d*x^4)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^
3))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((2*a^6*b^2*d^5 - 3*a^5*b^
3*c*d^4 + a^4*b^4*c^2*d^3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - ((c + d*
x^4)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(64*a^7*b^2*d^5 - 256*a
^6*b^3*c*d^4 - 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3))/(64*(a^4*d^2 + a
^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*
b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(8*(a^5*d^3 - a^2*b^3*
c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)
^(1/2)*1i)/(8*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) +
((((c + d*x^4)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(8*
(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + (((2*a^6*b^2*d^5 - 3*a^5*b^3*c*d^4
 + a^4*b^4*c^2*d^3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) + ((c + d*x^4)^(1
/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(64*a^7*b^2*d^5 - 256*a^6*b^3*
c*d^4 - 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3))/(64*(a^4*d^2 + a^2*b^2*
c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2
)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(8*(a^5*d^3 - a^2*b^3*c^3 + 3
*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*
1i)/(8*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))/(((3*a*b
^3*d^4)/16 - (b^4*c*d^3)/8)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - ((((c +
 d*x^4)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(8*(a^4*d^
2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((2*a^6*b^2*d^5 - 3*a^5*b^3*c*d^4 + a^4*
b^4*c^2*d^3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - ((c + d*x^4)^(1/2)*(3*
a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(64*a^7*b^2*d^5 - 256*a^6*b^3*c*d^4 -
 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3))/(64*(a^4*d^2 + a^2*b^2*c^2 - 2
*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*
a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(8*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^
2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(8*(a^
5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) + ((((c + d*x^4)^(1
/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(8*(a^4*d^2 + a^2*b^
```

```
2*c^2 - 2*a^3*b*c*d)) + (((2*a^6*b^2*d^5 - 3*a^5*b^3*c*d^4 + a^4*b^4*c^2*d^
3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) + ((c + d*x^4)^(1/2)*(3*a*d - 2*b*
c)*(-b*(a*d - b*c)^3)^(1/2)*(64*a^7*b^2*d^5 - 256*a^6*b^3*c*d^4 - 128*a^4*b
^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3))/(64*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d
)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*
c)*(-b*(a*d - b*c)^3)^(1/2))/(8*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d -
3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(8*(a^5*d^3 - a^
2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2))))*(3*a*d - 2*b*c)*(-b*(a*d -
b*c)^3)^(1/2)*1i)/(4*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d
^2)) - (atan((((((2*a^6*b^2*d^5 - 3*a^5*b^3*c*d^4 + a^4*b^4*c^2*d^3)/(4*(a^
5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) - ((c + d*x^4)^(1/2)*(64*a^7*b^2*d^5 -
256*a^6*b^3*c*d^4 - 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3))/(128*a^2*c^
(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(4*a^2*c^(1/2)) - ((c + d*x^4
)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(32*(a^4*d^2 + a
^2*b^2*c^2 - 2*a^3*b*c*d)))*1i)/(a^2*c^(1/2)) - ((((2*a^6*b^2*d^5 - 3*a^5*b
^3*c*d^4 + a^4*b^4*c^2*d^3)/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) + ((c
 + d*x^4)^(1/2)*(64*a^7*b^2*d^5 - 256*a^6*b^3*c*d^4 - 128*a^4*b^5*c^3*d^2 +
 320*a^5*b^4*c^2*d^3))/(128*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*
d)))/(4*a^2*c^(1/2)) + ((c + d*x^4)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 -
 20*a*b^4*c*d^3))/(32*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))*1i)/(a^2*c^(1
/2)))/(((3*a*b^3*d^4)/16 - (b^4*c*d^3)/8)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*
c*d) + (((2*a^6*b^2*d^5 - 3*a^5*b^3*c*d^4 + a^4*b^4*c^2*d^3)/(4*(a^5*d^2 +
a^3*b^2*c^2 - 2*a^4*b*c*d)) - ((c + d*x^4)^(1/2)*(64*a^7*b^2*d^5 - 256*a^6*
b^3*c*d^4 - 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3))/(128*a^2*c^(1/2)*(a
^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(4*a^2*c^(1/2)) - ((c + d*x^4)^(1/2)*
(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(32*(a^4*d^2 + a^2*b^2*c
^2 - 2*a^3*b*c*d)))/(a^2*c^(1/2)) + (((2*a^6*b^2*d^5 - 3*a^5*b^3*c*d^4 + a^
4*b^4*c^2*d^3)/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) + ((c + d*x^4)^(1/
2)*(64*a^7*b^2*d^5 - 256*a^6*b^3*c*d^4 - 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*
c^2*d^3))/(128*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(4*a^2*c
^(1/2)) + ((c + d*x^4)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d
^3))/(32*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(a^2*c^(1/2))))*1i)/(2*a^2
*c^(1/2)) - (b*d*(c + d*x^4)^(1/2))/(2*(a^2*d - a*b*c)*(2*b*(c + d*x^4) + 2
*a*d - 2*b*c))
```

3.827 $\int \frac{1}{x^5(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 185

$$\int \frac{1}{x^5\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = -\frac{b(2bc-ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt{c+dx^4}}{4acx^4\left(a+bx^4\right)} + \frac{(4bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^3c^{3/2}} - \frac{b^{3/2}(4bc-5ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4a^3(bc-ad)^{3/2}}$$

```
[Out] 1/4*(a*d+4*b*c)*arctanh((d*x^4+c)^(1/2)/c^(1/2))/a^3/c^(3/2)-1/4*b^(3/2)*(-
5*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/a^3/(-a*d+b*
c)^(3/2)-1/4*b*(-a*d+2*b*c)*(d*x^4+c)^(1/2)/a^2/c/(-a*d+b*c)/(b*x^4+a)-1/4*
(d*x^4+c)^(1/2)/a/c/x^4/(b*x^4+a)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {457, 105, 156, 162, 65, 214}

$$\int \frac{1}{x^5\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = -\frac{b^{3/2}(4bc-5ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4a^3(bc-ad)^{3/2}} + \frac{(ad+4bc)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^3c^{3/2}} - \frac{b\sqrt{c+dx^4}(2bc-ad)}{4a^2c\left(a+bx^4\right)(bc-ad)} - \frac{\sqrt{c+dx^4}}{4acx^4\left(a+bx^4\right)}$$

```
[In] Int[1/(x^5*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/4*(b*(2*b*c - a*d)*Sqrt[c + d*x^4])/(a^2*c*(b*c - a*d)*(a + b*x^4)) - Sq
rt[c + d*x^4]/(4*a*c*x^4*(a + b*x^4)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x
^4]/Sqrt[c]])/(4*a^3*c^(3/2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*S
qrt[c + d*x^4])/Sqrt[b*c - a*d]])/(4*a^3*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2\sqrt{c+dx}}\,dx, x, x^4\right)$$

$$= -\frac{\sqrt{c+dx^4}}{4acx^4\left(a+bx^4\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4bc+ad)+\frac{3bdx}{2}}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^4\right)}{4ac}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt{c+dx^4}}{4acx^4\left(a+bx^4\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-ad)(4bc+ad)+\frac{1}{2}bd(2bc-ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4a^2c(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt{c+dx^4}}{4acx^4\left(a+bx^4\right)} + \frac{\left(b^2(4bc-5ad)\right)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{8a^3(bc-ad)} - \frac{(4bc+ad)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^4\right)}{8a^3c}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt{c+dx^4}}{4acx^4\left(a+bx^4\right)} + \frac{\left(b^2(4bc-5ad)\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{4a^3d(bc-ad)} - \frac{(4bc+ad)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{4a^3cd}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt{c+dx^4}}{4acx^4\left(a+bx^4\right)} + \frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^3c^{3/2}} - \frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4a^3(bc-ad)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^5 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \frac{\frac{a\sqrt{c+dx^4}(-a^2d+2b^2cx^4+ab(c-dx^4))}{c(-bc+ad)x^4(a+bx^4)} - \frac{b^{3/2}(4bc-5ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}} + \frac{(4bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{c^{3/2}}}{4a^3}$$

```
[In] Integrate[1/(x^5*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((a*Sqrt[c + d*x^4]*(-(a^2*d) + 2*b^2*c*x^4 + a*b*(c - d*x^4)))/(c*(-(b*c)
+ a*d)*x^4*(a + b*x^4)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^4])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2) + ((4*b*c + a*d)*ArcTanh
[Sqrt[c + d*x^4]/Sqrt[c]])/c^(3/2))/(4*a^3)
```

Maple [A] (verified)

Time = 5.13 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{-4x^4c^{\frac{5}{2}}b^2\left(bc-\frac{5ad}{4}\right)(b\,x^4+a)\arctan\left(\frac{b\sqrt{d\,x^4+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{(ad-bc)b}\left(c\,x^4(b\,x^4+a)(ad+4bc)(ad-bc)\arctanh\left(\frac{\sqrt{d\,x^4+c}}{\sqrt{c}}\right)+c^{\frac{3}{2}}(2b\right.}{4\sqrt{(ad-bc)b}\,a^3(ad-bc)(b\,x^4+a)x^4c^{\frac{5}{2}}}$
risch	$-\frac{\sqrt{d\,x^4+c}}{4c\,a^2x^4}-\frac{-\frac{(ad+4bc)\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)}{2a\sqrt{c}}-2b^2c\left(\frac{\sqrt{-ab}\,\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{8ab(ad-bc)\left(x^2+\frac{\sqrt{-ab}}{b}\right)}-\frac{d\ln\left(\frac{-\frac{2(ad-}{b}}{}\right.}{}\right.}{}$
elliptic	$-\frac{\sqrt{d\,x^4+c}}{4c\,a^2x^4}+\frac{d\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)}{4a^2c^{\frac{3}{2}}}+\frac{b\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)}{a^3\sqrt{c}}-\frac{b\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{}}{x^2+\sqrt{}}\right.}{2a^3\sqrt{-\frac{a}{}}}$
default	$\frac{-\frac{\sqrt{d\,x^4+c}}{4c\,x^4}+\frac{d\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)}{4c^{\frac{3}{2}}}}{a^2}+\frac{b\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)}{a^3\sqrt{c}}+\frac{b^2\left(\frac{\sqrt{-ab}\,\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{8ab(ad-bc)\left(x^2+\frac{\sqrt{-ab}}{b}\right)}\right.}{}$

```
[In] int(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/((a*d-b*c)*b)^(1/2)*(-4*x^4*c^(5/2)*b^2*(b*c-5/4*a*d)*(b*x^4+a)*arctan(
b*(d*x^4+c)^(1/2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(c*x^4*(b*x^4+a)
*(a*d+4*b*c)*(a*d-b*c)*arctanh((d*x^4+c)^(1/2)/c^(1/2))+c^(3/2)*(2*b^2*c*x^
4+a*(-d*x^4+c)*b-a^2*d)*a*(d*x^4+c)^(1/2)))/a^3/(a*d-b*c)/(b*x^4+a)/x^4/c^(
5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.54 (sec) , antiderivative size = 1236, normalized size of antiderivative = 6.68

$$\int \frac{1}{x^5 (a+bx^4)^2 \sqrt{c+dx^4}} dx = \text{Too large to display}$$

```
[In] integrate(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
[Out] [1/8*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^8 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^4)
*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*(b*c -
a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*
d^2)*x^8 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^4)*sqrt(c)*log((d*x^4 +
2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) - 2*(a^2*b*c^2 - a^3*c*d + (2*a*b^2*c
^2 - a^2*b*c*d)*x^4)*sqrt(d*x^4 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^8 + (a
^4*b*c^3 - a^5*c^2*d)*x^4), -1/8*(2*((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^8 + (4*a
*b^2*c^3 - 5*a^2*b*c^2*d)*x^4)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^4 + c)
*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c)) - ((4*b^3*c^2 - 3*a*b^2*
c*d - a^2*b*d^2)*x^8 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^4)*sqrt(c)*l
og((d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) + 2*(a^2*b*c^2 - a^3*c*d
 + (2*a*b^2*c^2 - a^2*b*c*d)*x^4)*sqrt(d*x^4 + c))/((a^3*b^2*c^3 - a^4*b*c^2
*d)*x^8 + (a^4*b*c^3 - a^5*c^2*d)*x^4), -1/8*(2*((4*b^3*c^2 - 3*a*b^2*c*d -
 a^2*b*d^2)*x^8 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^4)*sqrt(-c)*arcta
n(sqrt(d*x^4 + c)*sqrt(-c)/c) - ((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^8 + (4*a*b^2
*c^3 - 5*a^2*b*c^2*d)*x^4)*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d -
 2*sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + 2*(a^2*b
*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^4)*sqrt(d*x^4 + c))/((a^3*b^2*
c^3 - a^4*b*c^2*d)*x^8 + (a^4*b*c^3 - a^5*c^2*d)*x^4), -1/4*(((4*b^3*c^3 -
5*a*b^2*c^2*d)*x^8 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^4)*sqrt(-b/(b*c - a*d)
)*arctan(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c))
 + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^8 + (4*a*b^2*c^2 - 3*a^2*b*c*d
- a^3*d^2)*x^4)*sqrt(-c)*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) + (a^2*b*c^2 -
a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^4)*sqrt(d*x^4 + c))/((a^3*b^2*c^3 - a
^4*b*c^2*d)*x^8 + (a^4*b*c^3 - a^5*c^2*d)*x^4)]
```

Sympy [F]

$$\int \frac{1}{x^5 (a+bx^4)^2 \sqrt{c+dx^4}} dx = \int \frac{1}{x^5 (a+bx^4)^2 \sqrt{c+dx^4}} dx$$

```
[In] integrate(1/x**5/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
[Out] Integral(1/(x**5*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^5 \left(a+bx^4\right)^2 \sqrt{c+dx^4}} \, dx = \int \frac{1}{\left(bx^4+a\right)^2 \sqrt{dx^4+c}x^5} \, dx$$

```
[In] integrate(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^5), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^5 \left(a+bx^4\right)^2 \sqrt{c+dx^4}} \, dx = \frac{(4\,b^3c-5\,ab^2d)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{4\,(a^3bc-a^4d)\sqrt{-b^2c+abd}} - \frac{2\,(dx^4+c)^{\frac{3}{2}}b^2cd-2\,\sqrt{dx^4+c}b^2c^2d-(dx^4+c)^{\frac{3}{2}}abd^2+2\,\sqrt{dx^4+c}abcd^2-\sqrt{dx^4+c}a^2d^3}{4\,(a^2bc^2-a^3cd)\left((dx^4+c)^2b-2\,(dx^4+c)bc+bc^2+(dx^4+c)ad-acd\right)} - \frac{(4\,bc+ad)\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{4\,a^3\sqrt{-c}c}$$

```
[In] integrate(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] 1/4*(4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/((
a^3*b*c - a^4*d)*sqrt(-b^2*c + a*b*d)) - 1/4*(2*(d*x^4 + c)^(3/2)*b^2*c*d -
 2*sqrt(d*x^4 + c)*b^2*c^2*d - (d*x^4 + c)^(3/2)*a*b*d^2 + 2*sqrt(d*x^4 + c
)*a*b*c*d^2 - sqrt(d*x^4 + c)*a^2*d^3)/((a^2*b*c^2 - a^3*c*d)*((d*x^4 + c)^
2*b - 2*(d*x^4 + c)*b*c + b*c^2 + (d*x^4 + c)*a*d - a*c*d)) - 1/4*(4*b*c +
a*d)*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a^3*sqrt(-c)*c)
```

Mupad [B] (verification not implemented)

Time = 11.90 (sec) , antiderivative size = 3822, normalized size of antiderivative = 20.66

$$\int \frac{1}{x^5 \left(a+bx^4\right)^2 \sqrt{c+dx^4}} \, dx = \text{Too large to display}$$

```
[In] int(1/(x^5*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
[Out] (((c + d*x^4)^(1/2)*(a^2*d^3 + 2*b^2*c^2*d - 2*a*b*c*d^2))/(2*a^2*(b*c^2 -
a*c*d)) + (b*(c + d*x^4)^(3/2)*(a*d^2 - 2*b*c*d))/(2*a^2*(b*c^2 - a*c*d)))/
```

((c + d*x^4)*(2*a*d - 4*b*c) + 2*b*(c + d*x^4)^2 + 2*b*c^2 - 2*a*c*d) + (at
an((((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^4)^(1/2)*(a^4*b^
3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^
2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((-b^3*(a*d - b*c
)^3)^(1/2)*(5*a*d - 4*b*c)*((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*
c^3*d^4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((
-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^4)^(1/2)*(5*a*d - 4*b*c)*(128*a^6*b^5*c^
5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2*d^5))/(6
4*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^
4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d
 - 3*a^5*b*c*d^2)))*1i)/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5
*b*c*d^2)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^4)^(1/2
)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a
^2*b^5*c^2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - ((-b^3*(
a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4
*a^7*b^4*c^3*d^4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^
3*d) + ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^4)^(1/2)*(5*a*d - 4*b*c)*(128*a
^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2
d^5))/(64(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c
^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*
b^2*c^2*d - 3*a^5*b*c*d^2)))*1i)/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*
d - 3*a^5*b*c*d^2)))/(((5*a^3*b^4*d^6)/32 + b^7*c^3*d^3 - (3*a*b^6*c^2*d^4)
/2 + (3*a^2*b^5*c*d^5)/16)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((
-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^4)^(1/2)*(a^4*b^3*d^6
 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4)
)/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((-b^3*(a*d - b*c)^3)^(
1/2)*(5*a*d - 4*b*c)*((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*c^3*d^
4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((-b^3*(
a*d - b*c)^3)^(1/2)*(c + d*x^4)^(1/2)*(5*a*d - 4*b*c)*(128*a^6*b^5*c^5*d^2
 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2*d^5))/(64*(a^4
*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*
c^2*d - 3*a^5*b*c*d^2))))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a
^5*b*c*d^2))))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)
) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^4)^(1/2)*(a^4*b^
3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^
2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - ((-b^3*(a*d - b*c
)^3)^(1/2)*(5*a*d - 4*b*c)*((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*
c^3*d^4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((
-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^4)^(1/2)*(5*a*d - 4*b*c)*(128*a^6*b^5*c^
5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2*d^5))/(6
4*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^
4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d
 - 3*a^5*b*c*d^2))))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*
c*d^2))))*(-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*1i)/(4*(a^6*d^3 - a^3*
b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + (atan(((((c + d*x^4)^(1/2)*(

a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + (((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*c^3*d^4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((c + d*x^4)^(1/2)*(a*d + 4*b*c)*(128*a^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2*d^5))/(64*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(8*a^3*(c^3)^(1/2)))*(a*d + 4*b*c)*1i)/(8*a^3*(c^3)^(1/2)) + ((((c + d*x^4)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - (((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*c^3*d^4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((c + d*x^4)^(1/2)*(a*d + 4*b*c)*(128*a^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2*d^5))/(64*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(8*a^3*(c^3)^(1/2)))*(a*d + 4*b*c)*1i)/(8*a^3*(c^3)^(1/2)))/(((5*a^3*b^4*d^6)/32 + b^7*c^3*d^3 - (3*a*b^6*c^2*d^4)/2 + (3*a^2*b^5*c*d^5)/16)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((((c + d*x^4)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + (((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*c^3*d^4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((c + d*x^4)^(1/2)*(a*d + 4*b*c)*(128*a^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2*d^5))/(64*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(8*a^3*(c^3)^(1/2)))*(a*d + 4*b*c))/(8*a^3*(c^3)^(1/2)) + ((((c + d*x^4)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - (((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*c^3*d^4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((c + d*x^4)^(1/2)*(a*d + 4*b*c)*(128*a^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2*d^5))/(64*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(8*a^3*(c^3)^(1/2)))*(a*d + 4*b*c))/(8*a^3*(c^3)^(1/2))))*(a*d + 4*b*c)*1i)/(4*a^3*(c^3)^(1/2))

3.828 $\int \frac{x^{13}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 191

$$\int \frac{x^{13}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{(bc-2ad)x^2\sqrt{c+dx^4}}{4b^2d(bc-ad)} + \frac{ax^6\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} + \frac{a^{3/2}(5bc-4ad)\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4b^3(bc-ad)^{3/2}} - \frac{(bc+4ad)\operatorname{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^3d^{3/2}}$$

```
[Out] 1/4*a^(3/2)*(-4*a*d+5*b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1
/2))/b^3/(-a*d+b*c)^(3/2)-1/4*(4*a*d+b*c)*arctanh(x^2*d^(1/2)/(d*x^4+c)^(1/
2))/b^3/d^(3/2)+1/4*(-2*a*d+b*c)*x^2*(d*x^4+c)^(1/2)/b^2/d/(-a*d+b*c)+1/4*a
*x^6*(d*x^4+c)^(1/2)/b/(-a*d+b*c)/(b*x^4+a)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {476, 481, 596, 537, 223, 212, 385, 211}

$$\int \frac{x^{13}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{a^{3/2}(5bc-4ad)\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4b^3(bc-ad)^{3/2}} - \frac{(4ad+bc)\operatorname{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^3d^{3/2}} + \frac{x^2\sqrt{c+dx^4}(bc-2ad)}{4b^2d(bc-ad)} + \frac{ax^6\sqrt{c+dx^4}}{4b(a+bx^4)(bc-ad)}$$

```
[In] Int[x^13/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((b*c - 2*a*d)*x^2*Sqrt[c + d*x^4])/(4*b^2*d*(b*c - a*d)) + (a*x^6*Sqrt[c +
 d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) + (a^(3/2)*(5*b*c - 4*a*d)*ArcTan[(S
qrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*b^3*(b*c - a*d)^(3/2)) -
 ((b*c + 4*a*d)*ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]])/(4*b^3*d^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x^6}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx, x, x^2\right)$$

$$= \frac{ax^6\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} - \frac{\text{Subst}\left(\int \frac{x^2(3ac-2(bc-2ad)x^2)}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4b(bc-ad)}$$

$$= \frac{(bc-2ad)x^2\sqrt{c+dx^4}}{4b^2d(bc-ad)} + \frac{ax^6\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} + \frac{\text{Subst}\left(\int \frac{-2ac(bc-2ad)-2(bc-ad)(bc+4ad)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{8b^2d(bc-ad)}$$

$$= \frac{(bc-2ad)x^2\sqrt{c+dx^4}}{4b^2d(bc-ad)} + \frac{ax^6\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} + \frac{(a^2(5bc-4ad))\,\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4b^3(bc-ad)} - \frac{(bc+4ad)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4b^3d}$$

$$= \frac{(bc-2ad)x^2\sqrt{c+dx^4}}{4b^2d(bc-ad)} + \frac{ax^6\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} + \frac{(a^2(5bc-4ad))\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{4b^3(bc-ad)} - \frac{(bc+4ad)\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{4b^3d}$$

$$
= \frac{(bc-2ad)x^2\sqrt{c+dx^4}}{4b^2d(bc-ad)} + \frac{ax^6\sqrt{c+dx^4}}{4b(bc-ad)\,(a+bx^4)} + \frac{a^{3/2}(5bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4b^3(bc-ad)^{3/2}} - \frac{(bc+4ad)\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^3d^{3/2}}
$$

Mathematica [A] (verified)

Time = 2.77 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.99

$$
\int \frac{x^{13}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx
$$

$$
= \frac{\frac{bx^2\sqrt{c+dx^4}(-2a^2d+b^2cx^4+ab(c-dx^4))}{d(bc-ad)(a+bx^4)} + \frac{a^{3/2}(5bc-4ad)\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} - \frac{(bc+4ad)\log\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{d^{3/2}}}{4b^3}
$$

```
[In] Integrate[x^13/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((b*x^2*Sqrt[c + d*x^4]*(-2*a^2*d + b^2*c*x^4 + a*b*(c - d*x^4)))/(d*(b*c -
 a*d)*(a + b*x^4)) + (a^(3/2)*(5*b*c - 4*a*d)*ArcTan[(a*Sqrt[d] + b*x^2*(Sq
rt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(b*c - a*d)^(3/2)
 - ((b*c + 4*a*d)*Log[Sqrt[d]*x^2 + Sqrt[c + d*x^4]])/d^(3/2))/(4*b^3)
```

Maple [A] (verified)

Time = 6.40 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$-\frac{\frac{a^2\left(-\frac{b\sqrt{d\,x^4+c}\,x^2}{b\,x^4+a}-\frac{(4ad-5bc)\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}\,a}{x^2\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)}{ad-bc}-\frac{b\sqrt{d\,x^4+c}\,x^2d^{\frac{3}{2}}-4\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}}{x^2\sqrt{d}}\right)a\,d^2-\operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}}{x^2\sqrt{d}}\right)bcd}{d^{\frac{5}{2}}}}{4b^3}$
risch	$\frac{x^2\sqrt{d\,x^4+c}}{4b^2d}-\frac{a\ln\left(x^2\sqrt{d}+\sqrt{d\,x^4+c}\right)}{b^3\sqrt{d}}-\frac{\ln\left(x^2\sqrt{d}+\sqrt{d\,x^4+c}\right)c}{4b^2d^{\frac{3}{2}}}+\frac{a^2\sqrt{d\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{8b^3(ad-bc)\left(x^2-\frac{\sqrt{-ab}}{b}\right)}-$
elliptic	$\frac{x^2\sqrt{d\,x^4+c}}{4b^2d}-\frac{a\ln\left(x^2\sqrt{d}+\sqrt{d\,x^4+c}\right)}{b^3\sqrt{d}}-\frac{\ln\left(x^2\sqrt{d}+\sqrt{d\,x^4+c}\right)c}{4b^2d^{\frac{3}{2}}}+\frac{a^2\sqrt{d\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{8b^3(ad-bc)\left(x^2-\frac{\sqrt{-ab}}{b}\right)}-$
default	Expression too large to display

```
[In] int(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b^3*(a^2/(a*d-b*c)*(-b*(d*x^4+c)^(1/2)*x^2/(b*x^4+a)-(4*a*d-5*b*c)/((a
*d-b*c)*a)^(1/2)*arctanh((d*x^4+c)^(1/2)/x^2*a/((a*d-b*c)*a)^(1/2)))-(b*(d*
x^4+c)^(1/2)*x^2*d^(3/2)-4*arctanh((d*x^4+c)^(1/2)/x^2/d^(1/2))*a*d^2-arcta
nh((d*x^4+c)^(1/2)/x^2/d^(1/2))*b*c*d)/d^(5/2))
```

Fricas [A] (verification not implemented)

none

Time = 1.45 (sec) , antiderivative size = 1386, normalized size of antiderivative = 7.26

$$\int \frac{x^{13}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4*
a^2*b*d^2)*x^4)*sqrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c) +
 (5*a^2*b*c*d^2 - 4*a^3*d^3 + (5*a*b^2*c*d^2 - 4*a^2*b*d^3)*x^4)*sqrt(-a/(b
*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^
2*c*d)*x^4 + a^2*c^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2
- a^2*c*d)*x^2)*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4
+ a^2)) + 4*((b^3*c*d - a*b^2*d^2)*x^6 + (a*b^2*c*d - 2*a^2*b*d^2)*x^2)*sqr
t(d*x^4 + c))/(a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*x^4), 1/
16*(4*(a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4*a^2
*b*d^2)*x^4)*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) + (5*a^2*b*c*d^2
 - 4*a^3*d^3 + (5*a*b^2*c*d^2 - 4*a^2*b*d^3)*x^4)*sqrt(-a/(b*c - a*d))*log(
((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^
2*c^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)
*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 4*((b
^3*c*d - a*b^2*d^2)*x^6 + (a*b^2*c*d - 2*a^2*b*d^2)*x^2)*sqrt(d*x^4 + c))/(
a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*x^4), -1/8*((5*a^2*b*c*
d^2 - 4*a^3*d^3 + (5*a*b^2*c*d^2 - 4*a^2*b*d^3)*x^4)*sqrt(a/(b*c - a*d))*ar
ctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*
d*x^6 + a*c*x^2)) - (a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b
^2*c*d - 4*a^2*b*d^2)*x^4)*sqrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)
*x^2 - c) - 2*((b^3*c*d - a*b^2*d^2)*x^6 + (a*b^2*c*d - 2*a^2*b*d^2)*x^2)*s
qrt(d*x^4 + c))/(a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*x^4),
1/8*(2*(a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4*a^
2*b*d^2)*x^4)*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) - (5*a^2*b*c*d^
2 - 4*a^3*d^3 + (5*a*b^2*c*d^2 - 4*a^2*b*d^3)*x^4)*sqrt(a/(b*c - a*d))*arct
an(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*d*
x^6 + a*c*x^2)) + 2*((b^3*c*d - a*b^2*d^2)*x^6 + (a*b^2*c*d - 2*a^2*b*d^2)*
x^2)*sqrt(d*x^4 + c))/(a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*
x^4)]
```

Sympy [F]

$$\int \frac{x^{13}}{\left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx = \int \frac{x^{13}}{\left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx$$

```
[In] integrate(x**13/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
[Out] Integral(x**13/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{x^{13}}{\left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx = \int \frac{x^{13}}{\left(bx^4+a\right)^2\sqrt{dx^4+c}}\,dx$$

```
[In] integrate(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^13/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(163) = 326$.

Time = 0.39 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.74

$$\int \frac{x^{13}}{\left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx$$
$$= -\frac{\left(5\,a^2bc\sqrt{d}-4\,a^3d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{4\left(b^4c-ab^3d\right)\sqrt{abcd-a^2d^2}} + \frac{\sqrt{dx^4+c}x^2}{4\,b^2d}$$
$$+ \frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2a^2bcd-2\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2a^3d^2-a^2bc^2d}{2\left(b^4c\sqrt{d}-ab^3d^{\frac{3}{2}}\right)\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4b-2\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2bc+4\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2ad+\cdots}$$
$$+ \frac{(bc+4\,ad)\log\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2\right)}{8\,b^3d^{\frac{3}{2}}}$$

```
[In] integrate(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] -1/4*(5*a^2*b*c*sqrt(d) - 4*a^3*d^(3/2))*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*
x^4 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((b^4*c - a*b^3*d)*sq
rt(a*b*c*d - a^2*d^2)) + 1/4*sqrt(d*x^4 + c)*x^2/(b^2*d) + 1/2*((sqrt(d)*x^
```

```
2 - sqrt(d*x^4 + c))^2*a^2*b*c*d - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a^3*
d^2 - a^2*b*c^2*d)/((b^4*c*sqrt(d) - a*b^3*d^(3/2))*((sqrt(d)*x^2 - sqrt(d*
x^4 + c))^4*b - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c + 4*(sqrt(d)*x^2 -
sqrt(d*x^4 + c))^2*a*d + b*c^2)) + 1/8*(b*c + 4*a*d)*log((sqrt(d)*x^2 - sqr
t(d*x^4 + c))^2)/(b^3*d^(3/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{13}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^{13}}{(b\,x^4+a)^2\,\sqrt{d\,x^4+c}}\,dx$$

```
[In] int(x^13/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
[Out] int(x^13/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.829 $\int \frac{x^9}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 141

$$\int \frac{x^9}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{ax^2\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} - \frac{\sqrt{a}(3bc-2ad)\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4b^2(bc-ad)^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{2b^2\sqrt{d}}$$

```
[Out] -1/4*(-2*a*d+3*b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))*a^
(1/2)/b^2/(-a*d+b*c)^(3/2)+1/2*arctanh(x^2*d^(1/2)/(d*x^4+c)^(1/2))/b^2/d^(
1/2)+1/4*a*x^2*(d*x^4+c)^(1/2)/b/(-a*d+b*c)/(b*x^4+a)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {476, 481, 537, 223, 212, 385, 211}

$$\int \frac{x^9}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = -\frac{\sqrt{a}(3bc-2ad)\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4b^2(bc-ad)^{3/2}} + \frac{ax^2\sqrt{c+dx^4}}{4b(a+bx^4)(bc-ad)} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{2b^2\sqrt{d}}$$

```
[In] Int[x^9/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (a*x^2*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) - (Sqrt[a]*(3*b*c - 2
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*b^2*(b*c
- a*d)^(3/2)) + ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]]/(2*b^2*Sqrt[d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx,x,x^2\right)$$
$$= \frac{ax^2\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} - \frac{\text{Subst}\left(\int \frac{ac-2(bc-ad)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{4b(bc-ad)}$$
$$= \frac{ax^2\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,x,x^2\right)}{2b^2} - \frac{(a(3bc-2ad))\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{4b^2(bc-ad)}$$
$$= \frac{ax^2\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} + \frac{\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x^2}{\sqrt{c+dx^4}}\right)}{2b^2} - \frac{(a(3bc-2ad))\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^2}{\sqrt{c+dx^4}}\right)}{4b^2(bc-ad)}$$
$$= \frac{ax^2\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} - \frac{\sqrt{a}(3bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4b^2(bc-ad)^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{2b^2\sqrt{d}}$$

Mathematica [A] (verified)

Time = 1.96 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.09

$$\int \frac{x^9}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{\frac{abx^2\sqrt{c+dx^4}}{(bc-ad)(a+bx^4)} + \frac{\sqrt{a}(-3bc+2ad)\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} + \frac{2\log\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{d}}}{4b^2}$$

```
[In] Integrate[x^9/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((a*b*x^2*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4)) + (Sqrt[a]*(-3*b*c + 2
*a*d)*ArcTan[(a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*S
qrt[b*c - a*d])])/(b*c - a*d)^(3/2) + (2*Log[Sqrt[d]*x^2 + Sqrt[c + d*x^4]]
)/Sqrt[d])/(4*b^2)
```

Maple [A] (verified)

Time = 5.70 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$-\frac{-\frac{a\left(-\frac{b\sqrt{d\,x^{4}+c}\,x^{2}}{b\,x^{4}+a}-\frac{(2ad-3bc)\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{4}+c}\,a}{x^{2}\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)}{ad-bc}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{4}+c}}{x^{2}\sqrt{d}}\right)}{\sqrt{d}}}{4b^{2}}$
elliptic	$\frac{\ln\left(x^{2}\sqrt{d}+\sqrt{d\,x^{4}+c}\right)}{2b^{2}\sqrt{d}}-\frac{a\sqrt{d\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{8b^{2}(ad-bc)\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}-\frac{ad\sqrt{-ab}\,\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-}}{b}\right.}{b}}{}\right.}{}$
default	Expression too large to display

[In] int(x^9/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/4/b^2*(-a/(a*d-b*c)*(-b*(d*x^4+c)^(1/2)*x^2/(b*x^4+a)-(2*a*d-3*b*c)/((a*d-b*c)*a)^(1/2)*arctanh((d*x^4+c)^(1/2)/x^2*a/((a*d-b*c)*a)^(1/2)))-2/d^(1/2)*arctanh((d*x^4+c)^(1/2)/x^2/d^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 1077, normalized size of antiderivative = 7.64

$$\int \frac{x^9}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \left[\frac{4\sqrt{dx^4+c}abdx^2+4\left((b^2c-abd)x^4+abc-a^2d\right)\sqrt{d}\log\left(-2\,dx^4-2\sqrt{dx^4+c}\sqrt{d}x^2-c\right)+\left((3\,b^2cd-\right.}{16\,(a}\right.$$

[In] integrate(x^9/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")

[Out] [1/16*(4*sqrt(d*x^4 + c)*a*b*d*x^2 + 4*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c) + ((3*b^2*c*d - 2*a*b*d^2)*x^4 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^4), 1/16*(4*sqrt(d*x^4 + c)*a*b*d*x^2 - 8*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^4 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(-a

```
/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4
*a^2*c*d)*x^4 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c
^2 - a^2*c*d)*x^2)*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x
^4 + a^2)))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^4), 1/8*(2*s
qrt(d*x^4 + c)*a*b*d*x^2 + ((3*b^2*c*d - 2*a*b*d^2)*x^4 + 3*a*b*c*d - 2*a^2
*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4
+ c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)) + 2*((b^2*c - a*b*d)*x^4 + a*
b*c - a^2*d)*sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c))/(a*
b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^4), 1/8*(2*sqrt(d*x^4 + c)*
a*b*d*x^2 - 4*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(-d)*arctan(sqrt(-d
)*x^2/sqrt(d*x^4 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^4 + 3*a*b*c*d - 2*a^2*d
^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 +
c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)))/(a*b^3*c*d - a^2*b^2*d^2 + (b^
4*c*d - a*b^3*d^2)*x^4)]
```

Sympy [F]

$$\int \frac{x^9}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^9}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

```
[In] integrate(x**9/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
[Out] Integral(x**9/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{x^9}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^9}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

```
[In] integrate(x^9/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^9/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(117) = 234$.

Time = 0.37 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.11

$$\int \frac{x^9}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = -\frac{\left(3\,abc\sqrt{d}-2\,a^2d^{\frac{3}{2}}\right)\arctan\left(-\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{4\,(b^3c-ab^2d)\sqrt{abcd-a^2d^2}} - \frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2abc\sqrt{d}-2\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2a^2d^{\frac{3}{2}}-abc^2\sqrt{d}}{2\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4b-2\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2bc+4\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2ad+bc^2\right)(b^3c-ab^2d)} - \frac{\log\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2\right)}{4\,b^2\sqrt{d}}$$

```
[In] integrate(x^9/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(3*a*b*c*sqrt(d) - 2*a^2*d^(3/2))*arctan(-1/2*((sqrt(d)*x^2 - sqrt(d*x
^4 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((b^3*c - a*b^2*d)*sqr
t(a*b*c*d - a^2*d^2)) - 1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*b*c*sqrt(d
) - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a^2*d^(3/2) - a*b*c^2*sqrt(d))/(((s
qrt(d)*x^2 - sqrt(d*x^4 + c))^4*b - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c
 + 4*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + b*c^2)*(b^3*c - a*b^2*d)) - 1/
4*log((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2)/(b^2*sqrt(d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^9}{\left(b\,x^4+a\right)^2\sqrt{d\,x^4+c}}\,dx$$

```
[In] int(x^9/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(x^9/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.830 $\int \frac{x^5}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{x^5}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=-\frac{x^2\sqrt{c+dx^4}}{4(bc-ad)(a+bx^4)}+\frac{c\arctan\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4\sqrt{a}(bc-ad)^{3/2}}$$

```
[Out] 1/4*c*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))/(-a*d+b*c)^(3/2)
/a^(1/2)-1/4*x^2*(d*x^4+c)^(1/2)/(-a*d+b*c)/(b*x^4+a)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {476, 482, 12, 385, 211}

$$\int \frac{x^5}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\frac{c\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4\sqrt{a}(bc-ad)^{3/2}}-\frac{x^2\sqrt{c+dx^4}}{4(a+bx^4)(bc-ad)}$$

```
[In] Int[x^5/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/4*(x^2*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4)) + (c*ArcTan[(Sqrt[b*c
- a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*Sqrt[a]*(b*c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx,x,x^2\right)\\
&= -\frac{x^2\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}+\frac{\text{Subst}\left(\int \frac{c}{(a+bx^2)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{4(bc-ad)}\\
&= -\frac{x^2\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}+\frac{c\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{4(bc-ad)}\\
&= -\frac{x^2\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}+\frac{c\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^2}{\sqrt{c+dx^4}}\right)}{4(bc-ad)}\\
&= -\frac{x^2\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}+\frac{c\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4\sqrt{a}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.20

$$\int \frac{x^5}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{1}{4}\left(-\frac{x^2\sqrt{c+dx^4}}{(bc-ad)(a+bx^4)}+\frac{c\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}(bc-ad)^{3/2}}\right)$$

```
[In] Integrate[x^5/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (-((x^2*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4))) + (c*ArcTan[(a*Sqrt[d]
+ b*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(Sqrt[
a]*(b*c - a*d)^(3/2)))/4
```

Maple [A] (verified)

Time = 5.56 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$-\frac{c\left(-\frac{\sqrt{d\,x^4+c}\,x^2}{c\left(b\,x^4+a\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}\,a}{x^2\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)}{4(ad-bc)}$
elliptic	$\frac{\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{8b(ad-bc)\left(x^2+\frac{\sqrt{-ab}}{b}\right)}+\frac{d\sqrt{-ab}\,\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right.\cdots}}{x^2+\frac{\sqrt{-ab}}{b}}\cdots\right)}{8b^2(ad-bc)\sqrt{-\frac{ad-bc}{b}}\cdots}$
default	Expression too large to display

```
[In] int(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*c/(a*d-b*c)*(-(d*x^4+c)^(1/2)*x^2/c/(b*x^4+a)+1/((a*d-b*c)*a)^(1/2)*ar
ctanh((d*x^4+c)^(1/2)/x^2*a/((a*d-b*c)*a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(77) = 154$.

Time = 0.45 (sec) , antiderivative size = 426, normalized size of antiderivative = 4.58

$$\int \frac{x^5}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$
$$= \left[-\frac{4\sqrt{dx^4+c}(abc-a^2d)x^2-(bcx^4+ac)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,a^2cd)x^4+a^2c^2+4\,((b}{b^2x^8+2\,abx^4+a^2}\right.}{16\,(a^2b^2c^2-2\,a^3bcd+a^4d^2+(ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^4)}\right.$$
$$\left.-\frac{2\sqrt{dx^4+c}(abc-a^2d)x^2-(bcx^4+ac)\sqrt{abc-a^2d}\arctan\left(\frac{((bc-2\,ad)x^4-ac)\sqrt{dx^4+c}\sqrt{abc-a^2d}}{2\,((abcd-a^2d^2)x^6+(abc^2-a^2cd)x^2)}\right)}{8\,(a^2b^2c^2-2\,a^3bcd+a^4d^2+(ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^4)}\right]$$

```
[In] integrate(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(4*sqrt(d*x^4 + c)*(a*b*c - a^2*d)*x^2 - (b*c*x^4 + a*c)*sqrt(-a*b*c
 + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2
*c*d)*x^4 + a^2*c^2 + 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sqrt(d*x^4 + c)*sqrt(
-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/(a^2*b^2*c^2 - 2*a^3*b*c*d +
 a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^4), -1/8*(2*sqrt(d*x^4
 + c)*(a*b*c - a^2*d)*x^2 - (b*c*x^4 + a*c)*sqrt(a*b*c - a^2*d)*arctan(1/2*
((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a
^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d
^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^4)]
```

Sympy [F]

$$\int \frac{x^5}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^5}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

```
[In] integrate(x**5/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{x^5}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^5}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

[In] integrate(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")

[Out] integrate(x^5/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(77) = 154$.

Time = 0.86 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.62

$$\int \frac{x^5}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\frac{c\sqrt{d}\arctan\left(-\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{4\,\sqrt{abcd-a^2d^2}(bc-ad)}+\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2bc\sqrt{d}-2\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2ad^{\frac{3}{2}}-bc^2\sqrt{d}}{2\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4b-2\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2bc+4\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2ad+bc^2\right)(b^2c-abd)}$$

[In] integrate(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")

[Out] 1/4*c*sqrt(d)*arctan(-1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b*c - a*d)) + 1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c*sqrt(d) - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d^(3/2) - b*c^2*sqrt(d))/(((sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c + 4*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + b*c^2)*(b^2*c - a*b*d))

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^5}{(b\,x^4+a)^2\,\sqrt{d\,x^4+c}}\,dx$$

[In] int(x^5/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)

[Out] int(x^5/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)

3.831 $\int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\frac{bx^2\sqrt{c+dx^4}}{4a(bc-ad)(a+bx^4)}+\frac{(bc-2ad)\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{3/2}(bc-ad)^{3/2}}$$

```
[Out] 1/4*(-2*a*d+b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))/a^(3/
2)/(-a*d+b*c)^(3/2)+1/4*b*x^2*(d*x^4+c)^(1/2)/a/(-a*d+b*c)/(b*x^4+a)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {476, 390, 385, 211}

$$\int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\frac{(bc-2ad)\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{3/2}(bc-ad)^{3/2}}+\frac{bx^2\sqrt{c+dx^4}}{4a(a+bx^4)(bc-ad)}$$

```
[In] Int[x/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (b*x^2*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*(a + b*x^4)) + ((b*c - 2*a*d)*ArcT
an[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*a^(3/2)*(b*c - a*d)
^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx, x, x^2\right)\\
&= \frac{bx^2\sqrt{c+dx^4}}{4a(bc-ad)\,(a+bx^4)} + \frac{(bc-2ad)\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4a(bc-ad)}\\
&= \frac{bx^2\sqrt{c+dx^4}}{4a(bc-ad)\,(a+bx^4)} + \frac{(bc-2ad)\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{4a(bc-ad)}\\
&= \frac{bx^2\sqrt{c+dx^4}}{4a(bc-ad)\,(a+bx^4)} + \frac{(bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{3/2}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.19

$$\int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = -\frac{bx^2\sqrt{c+dx^4}}{4a(-bc+ad)\,(a+bx^4)} + \frac{(bc-2ad)\arctan\left(\frac{a\sqrt{d}+b\sqrt{d}x^4+bx^2\sqrt{c+dx^4}}{\sqrt{a}\sqrt{bc-ad}}\right)}{4a^{3/2}(bc-ad)^{3/2}}$$

```
[In] Integrate[x/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/4*(b*x^2*Sqrt[c + d*x^4])/(a*(-(b*c) + a*d)*(a + b*x^4)) + ((b*c - 2*a*d
)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^4 + b*x^2*Sqrt[c + d*x^4])/(Sqrt[a]*Sqrt[
b*c - a*d])])/(4*a^(3/2)*(b*c - a*d)^(3/2))
```

Maple [A] (verified)

Time = 5.85 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$\frac{-\frac{b\sqrt{d\,x^4+c}\,x^2}{b\,x^4+a}+\frac{(2ad-bc)\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^4+c}\,a}{x^2\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}}{4(ad-bc)a}$
default	$-\frac{\sqrt{d\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{8a(ad-bc)\left(x^2-\frac{\sqrt{-ab}}{b}\right)}+\frac{d\sqrt{-ab}\,\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2-\cdots\right.}}{x^2-\frac{\sqrt{-ab}}{b}}\right.}{8ba(ad-bc)\sqrt{-\frac{ad-bc}{b}}}$
elliptic	$-\frac{\sqrt{d\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{8a(ad-bc)\left(x^2-\frac{\sqrt{-ab}}{b}\right)}+\frac{d\sqrt{-ab}\,\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2-\cdots\right.}}{x^2-\frac{\sqrt{-ab}}{b}}\right.}{8ba(ad-bc)\sqrt{-\frac{ad-bc}{b}}}$

```
[In] int(x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/(a*d-b*c)/a*(-b*(d*x^4+c)^(1/2)*x^2/(b*x^4+a)+(2*a*d-b*c)/((a*d-b*c)*a)
^(1/2)*arctanh((d*x^4+c)^(1/2)/x^2*a/((a*d-b*c)*a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(88) = 176$.

Time = 0.66 (sec) , antiderivative size = 467, normalized size of antiderivative = 4.49

$$\int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \left[\frac{4\sqrt{dx^4+c}(ab^2c-a^2bd)x^2-((b^2c-2\,abd)x^4+abc-2\,a^2d)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc}{}\right.}{16\,(a^3b^2c^2-2\,a^4bcd+a^5d^2+(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2)x}\right.$$

```
[In] integrate(x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*sqrt(d*x^4 + c)*(a*b^2*c - a^2*b*d)*x^2 - ((b^2*c - 2*a*b*d)*x^4 +
 a*b*c - 2*a^2*d)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^
2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x^6 - a
*c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)))
/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*
b*d^2)*x^4), 1/8*(2*sqrt(d*x^4 + c)*(a*b^2*c - a^2*b*d)*x^2 + ((b^2*c - 2*a
*b*d)*x^4 + a*b*c - 2*a^2*d)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*
x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (
a*b*c^2 - a^2*c*d)*x^2)))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c
^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x^4)]
```

Sympy [F]

$$\int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

```
[In] integrate(x/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

```
[In] integrate(x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(88) = 176$.

Time = 0.32 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.28

$$\int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = -\frac{1}{4}d^{\frac{3}{2}}\left(\frac{(bc-2\,ad)\arctan\left(\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2 b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{(abcd-a^2d^2)^{\frac{3}{2}}}+\frac{2\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2 bc \cdots\right.}{\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4 b-2\left(\sqrt{d}x^2-\sqrt{dx^4+\cdots}\right.\right.}\right.$$

```
[In] integrate(x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b
 - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d^2)^(3/2) + 2*((sq
rt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*
d - b*c^2)/(((sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b - 2*(sqrt(d)*x^2 - sqrt(d*
x^4 + c))^2*b*c + 4*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + b*c^2)*(a*b*c*d
 - a^2*d^2)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x}{(b\,x^4+a)^2\sqrt{d\,x^4+c}}\,dx$$

```
[In] int(x/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(x/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.832 $\int \frac{1}{x^3(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 149

$$\int \frac{1}{x^3 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = -\frac{(3bc-2ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^2(a+bx^4)} - \frac{b(3bc-4ad)\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{5/2}(bc-ad)^{3/2}}$$

```
[Out] -1/4*b*(-4*a*d+3*b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))/
a^(5/2)/(-a*d+b*c)^(3/2)-1/4*(-2*a*d+3*b*c)*(d*x^4+c)^(1/2)/a^2/c/(-a*d+b*c
)/x^2+1/4*b*(d*x^4+c)^(1/2)/a/(-a*d+b*c)/x^2/(b*x^4+a)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {476, 483, 597, 12, 385, 211}

$$\int \frac{1}{x^3 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = -\frac{b(3bc-4ad)\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{5/2}(bc-ad)^{3/2}} - \frac{\sqrt{c+dx^4}(3bc-2ad)}{4a^2cx^2(bc-ad)} + \frac{b\sqrt{c+dx^4}}{4ax^2(a+bx^4)(bc-ad)}$$

```
[In] Int[1/(x^3*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/4*((3*b*c - 2*a*d)*Sqrt[c + d*x^4])/(a^2*c*(b*c - a*d)*x^2) + (b*Sqrt[c
+ d*x^4])/(4*a*(b*c - a*d)*x^2*(a + b*x^4)) - (b*(3*b*c - 4*a*d)*ArcTan[(Sq
```

```
rt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*a^(5/2)*(b*c - a*d)^(3/2)
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2 (a+bx^2)^2 \sqrt{c+dx^2}}\,dx, x, x^2\right)$$

$$= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^2 (a+bx^4)} - \frac{\text{Subst}\left(\int \frac{-3bc+2ad-2bdx^2}{x^2(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4a(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^2 (a+bx^4)} - \frac{\text{Subst}\left(\int \frac{bc(3bc-4ad)}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4a^2c(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^2 (a+bx^4)} - \frac{(b(3bc-4ad))\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4a^2(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^2 (a+bx^4)} - \frac{(b(3bc-4ad))\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{4a^2(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^2 (a+bx^4)} - \frac{b(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{5/2}(bc-ad)^{3/2}}$$

Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^3 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \frac{\sqrt{c+dx^4}(2abc-2a^2d+3b^2cx^4-2abdx^4)}{4a^2c(-bc+ad)x^2 (a+bx^4)} - \frac{b(3bc-4ad)\arctan\left(\frac{a\sqrt{d}+b\sqrt{d}x^4+bx^2\sqrt{c+dx^4}}{\sqrt{a}\sqrt{bc-ad}}\right)}{4a^{5/2}(bc-ad)^{3/2}}$$

```
[In] Integrate[1/(x^3*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (Sqrt[c + d*x^4]*(2*a*b*c - 2*a^2*d + 3*b^2*c*x^4 - 2*a*b*d*x^4))/(4*a^2*c*
(-(b*c) + a*d)*x^2*(a + b*x^4)) - (b*(3*b*c - 4*a*d)*ArcTan[(a*Sqrt[d] + b*
Sqrt[d]*x^4 + b*x^2*Sqrt[c + d*x^4])/(Sqrt[a]*Sqrt[b*c - a*d])])/(4*a^(5/2)
*(b*c - a*d)^(3/2))
```

Maple [A] (verified)

Time = 6.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^4+c}}{x^2}+\frac{bc\left(\frac{b\sqrt{d\,x^4+c}\,x^2}{b\,x^4+a}-\frac{(4ad-3bc)\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}\,a}{x^2\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)}{2ad-2bc}}{2a^2c}$
risch	$-\frac{\sqrt{d\,x^4+c}}{2a^2c\,x^2}-\frac{3b\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{8a^2\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}}+$
elliptic	$-\frac{\sqrt{d\,x^4+c}}{2a^2c\,x^2}-\frac{3b\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x^2+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x^2+\frac{\sqrt{-ab}}{b}}\right)}{8a^2\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}}+$
default	Expression too large to display

```
[In] int(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^2*(-(d*x^4+c)^(1/2)/x^2+1/2*b*c/(a*d-b*c)*(b*(d*x^4+c)^(1/2)*x^2/(b*x
^4+a)-(4*a*d-3*b*c)/((a*d-b*c)*a)^(1/2)*arctanh((d*x^4+c)^(1/2)/x^2*a/((a*d
-b*c)*a)^(1/2))))/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(129) = 258$.

Time = 0.72 (sec) , antiderivative size = 612, normalized size of antiderivative = 4.11

$$\int \frac{1}{x^3(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$
$$= \left[-\frac{((3\,b^3c^2-4\,ab^2cd)x^6+(3\,ab^2c^2-4\,a^2bcd)x^2)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,a^2cd)x^4+a^2}{b^2x^8+2\,ab}\right.}{16\,((a^3b^3c^3-2\,a^4b^2c^2d+a^5b}\right.$$
$$-\frac{((3\,b^3c^2-4\,ab^2cd)x^6+(3\,ab^2c^2-4\,a^2bcd)x^2)\sqrt{abc-a^2d}\arctan\left(\frac{((bc-2\,ad)x^4-ac)\sqrt{dx^4+c}\sqrt{abc-a^2d}}{2\,((abcd-a^2d^2)x^6+(abc^2-a^2cd)x^2)}\right)+2\,(2\,a}{8\,((a^3b^3c^3-2\,a^4b^2c^2d+a^5bcd^2)x^6+(a^4b^2c^3-2\,a^5}$$

```
[In] integrate(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^6 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^2)*s
qrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c
```

```
^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sqrt(d*x^4
+ c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 4*(2*a^2*b^2*c^2
- 4*a^3*b*c*d + 2*a^4*d^2 + (3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^4
)*sqrt(d*x^4 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^6 + (a^
4*b^2*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^2), -1/8*(((3*b^3*c^2 - 4*a*b^2*c*
d)*x^6 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^2)*sqrt(a*b*c - a^2*d)*arctan(1/2*((
b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2
*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)) + 2*(2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*
a^4*d^2 + (3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^4)*sqrt(d*x^4 + c))
/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^6 + (a^4*b^2*c^3 - 2*a^5*
b*c^2*d + a^6*c*d^2)*x^2)]
```

Sympy [F]

$$\int \frac{1}{x^3 (a+bx^4)^2 \sqrt{c+dx^4}} \, dx = \int \frac{1}{x^3 (a+bx^4)^2 \sqrt{c+dx^4}} \, dx$$

```
[In] integrate(1/x**3/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
[Out] Integral(1/(x**3*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (a+bx^4)^2 \sqrt{c+dx^4}} \, dx = \int \frac{1}{(bx^4+a)^2 \sqrt{dx^4+c}x^3} \, dx$$

```
[In] integrate(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. 2(129) = 258.

Time = 0.89 (sec) , antiderivative size = 418, normalized size of antiderivative = 2.81

$$\int \frac{1}{x^3 (a+bx^4)^2 \sqrt{c+dx^4}} \, dx$$
$$= \frac{1}{4} d^{\frac{5}{2}} \left(\frac{(3\,b^2c - 4\,abd) \arctan\left(\frac{\left(\sqrt{d}x^2 - \sqrt{dx^4+c}\right)^2 b - bc + 2\,ad}{2\sqrt{abcd - a^2d^2}}\right)}{(a^2bcd^2 - a^3d^3)\sqrt{abcd - a^2d^2}} + \frac{2\left(3\left(\sqrt{d}x^2 - \sqrt{dx^4+c}\right)^4 b^2c - 4\left(\sqrt{d}x^2 - \sqrt{dx\right.\right.}{\left(\left(\left(\sqrt{d}x^2 - \sqrt{dx^4+c}\right)^6 b - 3\left(\sqrt{d}x^2 - \sqrt{dx\right.\right.\right.} \right.$$

```
[In] integrate(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*d^(5/2)*((3*b^2*c - 4*a*b*d)*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c)
)^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^2*b*c*d^2 - a^3*d^3)*sqrt
(a*b*c*d - a^2*d^2)) + 2*(3*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b^2*c - 4*(sq
rt(d)*x^2 - sqrt(d*x^4 + c))^4*a*b*d - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*
b^2*c^2 + 14*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*b*c*d - 8*(sqrt(d)*x^2 - s
qrt(d*x^4 + c))^2*a^2*d^2 + 3*b^2*c^3 - 2*a*b*c^2*d)/((((sqrt(d)*x^2 - sqrt(
d*x^4 + c))^6*b - 3*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b*c + 4*(sqrt(d)*x^2
- sqrt(d*x^4 + c))^4*a*d + 3*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c^2 - 4*(s
qrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*c*d - b*c^3)*(a^2*b*c*d^2 - a^3*d^3)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx=\int \frac{1}{x^3\,{\left(b\,x^4+a\right)}^2\,\sqrt{d\,x^4+c}} \,d x$$

```
[In] int(1/(x^3*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^3*(a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.833 $\int \frac{1}{x^7(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 208

$$\int \frac{1}{x^7(a+bx^4)^2\sqrt{c+dx^4}}\,dx = -\frac{(5bc-2ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^6}+\frac{(15b^2c^2-8abcd-4a^2d^2)\sqrt{c+dx^4}}{12a^3c^2(bc-ad)x^2}+\frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^6(a+bx^4)}+\frac{b^2(5bc-6ad)\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{7/2}(bc-ad)^{3/2}}$$

```
[Out] 1/4*b^2*(-6*a*d+5*b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))
/a^(7/2)/(-a*d+b*c)^(3/2)-1/12*(-2*a*d+5*b*c)*(d*x^4+c)^(1/2)/a^2/c/(-a*d+b
*c)/x^6+1/12*(-4*a^2*d^2-8*a*b*c*d+15*b^2*c^2)*(d*x^4+c)^(1/2)/a^3/c^2/(-a*
d+b*c)/x^2+1/4*b*(d*x^4+c)^(1/2)/a/(-a*d+b*c)/x^6/(b*x^4+a)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {476, 483, 597, 12, 385, 211}

$$\int \frac{1}{x^7(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{b^2(5bc-6ad)\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{7/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^4}(5bc-2ad)}{12a^2cx^6(bc-ad)}+\frac{\sqrt{c+dx^4}(-4a^2d^2-8abcd+15b^2c^2)}{12a^3c^2x^2(bc-ad)}+\frac{b\sqrt{c+dx^4}}{4ax^6(a+bx^4)(bc-ad)}$$

```
[In] Int[1/(x^7*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/12*((5*b*c - 2*a*d)*Sqrt[c + d*x^4])/(a^2*c*(b*c - a*d)*x^6) + ((15*b^2*
c^2 - 8*a*b*c*d - 4*a^2*d^2)*Sqrt[c + d*x^4])/(12*a^3*c^2*(b*c - a*d)*x^2)
+ (b*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*x^6*(a + b*x^4)) + (b^2*(5*b*c - 6*a
*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*a^(7/2)*(b*
c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
```

```
] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4 (a+bx^2)^2 \sqrt{c+dx^2}}\, dx, x, x^2\right)$$

$$= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^6 (a+bx^4)} - \frac{\text{Subst}\left(\int \frac{-5bc+2ad-4bdx^2}{x^4(a+bx^2)\sqrt{c+dx^2}}\, dx, x, x^2\right)}{4a(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^6} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^6 (a+bx^4)} + \frac{\text{Subst}\left(\int \frac{-15b^2c^2+8abcd+4a^2d^2-2bd(5bc-2ad)x^2}{x^2(a+bx^2)\sqrt{c+dx^2}}\, dx, x, x^2\right)}{12a^2c(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^6} + \frac{(15b^2c^2-8abcd-4a^2d^2)\sqrt{c+dx^4}}{12a^3c^2(bc-ad)x^2} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^6 (a+bx^4)} - \frac{\text{Subst}\left(\int -\frac{3b^2c^2(5bc-6ad)}{(a+bx^2)\sqrt{c+dx^2}}\, dx, x, x^2\right)}{12a^3c^2(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^6} + \frac{(15b^2c^2-8abcd-4a^2d^2)\sqrt{c+dx^4}}{12a^3c^2(bc-ad)x^2} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^6 (a+bx^4)} + \frac{(b^2(5bc-6ad))\,\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\, dx, x, x^2\right)}{4a^3(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^6} + \frac{(15b^2c^2-8abcd-4a^2d^2)\sqrt{c+dx^4}}{12a^3c^2(bc-ad)x^2} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^6 (a+bx^4)} + \frac{(b^2(5bc-6ad))\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\, dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{4a^3(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^6} + \frac{(15b^2c^2-8abcd-4a^2d^2)\sqrt{c+dx^4}}{12a^3c^2(bc-ad)x^2} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^6 (a+bx^4)} + \frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{7/2}(bc-ad)^{3/2}}$$

Mathematica [A] (verified)

Time = 2.50 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^7 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = -\frac{\sqrt{c+dx^4}(15b^3c^2x^8+2ab^2cx^4(5c-4dx^4)+2a^3d(c-2dx^4)-2a^2b(c^2+3cdx^4+2d^2x^8))}{12a^3c^2(-bc+ad)x^6(a+bx^4)} + \frac{b^2(5bc-6ad)\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{4a^{7/2}(bc-ad)^{3/2}}$$

```
[In] Integrate[1/(x^7*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/12*(Sqrt[c + d*x^4]*(15*b^3*c^2*x^8 + 2*a*b^2*c*x^4*(5*c - 4*d*x^4) + 2*
a^3*d*(c - 2*d*x^4) - 2*a^2*b*(c^2 + 3*c*d*x^4 + 2*d^2*x^8)))/(a^3*c^2*(-(b
*c) + a*d)*x^6*(a + b*x^4)) + (b^2*(5*b*c - 6*a*d)*ArcTan[(a*Sqrt[d] + b*x^
2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(4*a^(7/2)*(
b*c - a*d)^(3/2))
```

Maple [A] (verified)

Time = 6.75 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{4}+c}\,\left(-2ad\,x^{4}-6bc\,x^{4}+ac\right)}{3x^{6}}-\frac{b^{2}c^{2}\left(\frac{b\sqrt{d\,x^{4}+c}\,x^{2}}{b\,x^{4}+a}-\frac{(6ad-5bc)\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{4}+c}\,a}{x^{2}\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)}{2(ad-bc)}}{2a^{3}c^{2}}$
risch	$-\frac{\sqrt{d\,x^{4}+c}\,\left(-2ad\,x^{4}-6bc\,x^{4}+ac\right)}{6c^{2}a^{3}x^{6}}-\frac{b^{2}\sqrt{d\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)^{2}+\frac{2d\sqrt{-ab}\,\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{8a^{3}(ad-bc)\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}+\frac{bd\sqrt{-ab}\,\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\cdots}{\cdots}}{\cdots}\right.}{\cdots}$
elliptic	$-\frac{\sqrt{d\,x^{4}+c}}{6a^{2}c\,x^{6}}+\frac{d\sqrt{d\,x^{4}+c}}{3a^{2}c^{2}x^{2}}+\frac{b\sqrt{d\,x^{4}+c}}{a^{3}c\,x^{2}}-\frac{b^{2}\sqrt{d\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{-ab}\,\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{8a^{3}(ad-bc)\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)}-\frac{bd\sqrt{-ab}\,\ln\left(\frac{-\frac{2(ad-b\cdots}{b}}{\cdots}\right.}{\cdots}$
default	Expression too large to display

```
[In] int(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^3*(-1/3*(d*x^4+c)^(1/2)*(-2*a*d*x^4-6*b*c*x^4+a*c)/x^6-1/2*b^2*c^2/(a
*d-b*c)*(b*(d*x^4+c)^(1/2)*x^2/(b*x^4+a)-(6*a*d-5*b*c)/((a*d-b*c)*a)^(1/2)*
```

arctanh((d*x^4+c)^(1/2)/x^2*a/((a*d-b*c)*a)^(1/2))))/c^2

Fricas [A] (verification not implemented)

none

Time = 0.74 (sec) , antiderivative size = 760, normalized size of antiderivative = 3.65

$$\int \frac{1}{x^7 (a+bx^4)^2 \sqrt{c+dx^4}} \, dx$$
$$= \left[-\frac{3\left((5\,b^4c^3-6\,ab^3c^2d)x^{10}+(5\,ab^3c^3-6\,a^2b^2c^2d)x^6\right)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,a^2cd)x}{b^2x^8} \right.}{} \right.$$

```
[In] integrate(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^10 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2*
d)*x^6)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2
*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sq
rt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)) - 4*((15*a
*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^8 - 2*a^3*b^
2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2 + 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d +
a^4*b*c*d^2 + 2*a^5*d^3)*x^4)*sqrt(d*x^4 + c))/((a^4*b^3*c^4 - 2*a^5*b^2*c^
3*d + a^6*b*c^2*d^2)*x^10 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7*c^2*d^2)*x^6
), 1/24*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^10 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2
*d)*x^6)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^
4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x
^2)) + 2*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)
*x^8 - 2*a^3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2 + 2*(5*a^2*b^3*c^3 - 8*a
^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^4)*sqrt(d*x^4 + c))/((a^4*b^3*c^4
 - 2*a^5*b^2*c^3*d + a^6*b*c^2*d^2)*x^10 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a
^7*c^2*d^2)*x^6)]
```

Sympy [F]

$$\int \frac{1}{x^7 (a+bx^4)^2 \sqrt{c+dx^4}} \, dx = \int \frac{1}{x^7 (a+bx^4)^2 \sqrt{c+dx^4}} \, dx$$

```
[In] integrate(1/x**7/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/(x**7*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^7 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{(bx^4+a)^2\sqrt{dx^4+c}x^7}\,dx$$

```
[In] integrate(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^7), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(184) = 368.

Time = 0.94 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.90

$$\int \frac{1}{x^7 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \frac{1}{12} d^{\frac{7}{2}} \left(\frac{3\,(5\,b^3c - 6\,ab^2d)\arctan\left(-\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2 b - bc + 2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{(a^3bcd^3 - a^4d^4)\sqrt{abcd-a^2d^2}} - \frac{6\left(\left(\sqrt{d}x^2 - \sqrt{dx^4+c}\right.\right.}{(a^3bcd^3-a^4d^4)\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4 b\right.} \right.$$

```
[In] integrate(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] 1/12*d^(7/2)*(3*(5*b^3*c - 6*a*b^2*d)*arctan(-1/2*((sqrt(d)*x^2 - sqrt(d*x^
4 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^3*b*c*d^3 - a^4*d^4
)*sqrt(a*b*c*d - a^2*d^2)) - 6*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b^3*c - 2
*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*b^2*d - b^3*c^2)/((a^3*b*c*d^3 - a^4*d
^4)*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c)
)^2*b*c + 4*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + b*c^2)) - 8*(3*(sqrt(d)
*x^2 - sqrt(d*x^4 + c))^4*b - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c - 3*(
sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + 3*b*c^2 + a*c*d)/(((sqrt(d)*x^2 - sq
rt(d*x^4 + c))^2 - c)^3*a^3*d^3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^7 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{x^7\,{\left(b\,x^4+a\right)}^2\,\sqrt{d\,x^4+c}}\,dx$$

```
[In] int(1/(x^7*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
[Out] int(1/(x^7*(a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.834 $\int \frac{x^8}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 996

$$\int \frac{x^8}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

$$= \frac{ax\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} - \frac{\sqrt[4]{-a}(5bc-3ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16b^{7/4}(bc-ad)^{3/2}}$$

$$+ \frac{\sqrt[4]{-a}(5bc-3ad)\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16b^{7/4}(-bc+ad)^{3/2}}$$

$$+ \frac{(4bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)\sqrt{c+dx^4}}$$

$$- \frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}(5bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16b^2\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$

$$- \frac{\sqrt{-a}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(5bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16b^2\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$

$$- \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(5bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{32b^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$

$$- \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(5bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{32b^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$

```
[Out] -1/16*(-a)^(1/4)*(-3*a*d+5*b*c)*arctan(x*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4
)/(d*x^4+c)^(1/2))/b^(7/4)/(-a*d+b*c)^(3/2)+1/16*(-a)^(1/4)*(-3*a*d+5*b*c)*
arctan(x*(a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/b^(7/4)/(a*d-b
*c)^(3/2)+1/4*a*x*(d*x^4+c)^(1/2)/b/(-a*d+b*c)/(b*x^4+a)+1/8*(-3*a*d+4*b*c)
*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4))
)*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1
/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^2/c^(1/4)/d^(1/4)/(-a*d+b*
c)/(d*x^4+c)^(1/2)-1/16*a*d^(1/4)*(-3*a*d+5*b*c)*(cos(2*arctan(d^(1/4)*x/c^
(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^
(1/4)*x/c^(1/4))),1/2*2^(1/2))*(b^(1/2)*c^(1/2)/(-a)^(1/2)+d^(1/2))*(c^(1/2
)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^2/c^(1/4)/(-a*d+
b*c)/(a*d+b*c)/(d*x^4+c)^(1/2)-1/16*d^(1/4)*(-3*a*d+5*b*c)*(cos(2*arctan(d^
```

```
(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2
*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(-a)^(1/2)*(c^(1/2)+x^2*d^(1/2))*(
b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/
2)/b^2/c^(1/4)/(-a^2*d^2+b^2*c^2)/(d*x^4+c)^(1/2)-1/32*(-3*a*d+5*b*c)*(cos(
2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*Elli
pticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(
1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2
))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^
2)^(1/2)/b^2/c^(1/4)/d^(1/4)/(-a^2*d^2+b^2*c^2)/(d*x^4+c)^(1/2)-1/32*(-3*a*
d+5*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/
c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)
-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(
1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2
)+x^2*d^(1/2))^2)^(1/2)/b^2/c^(1/4)/d^(1/4)/(-a^2*d^2+b^2*c^2)/(d*x^4+c)^(1
/2)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 996, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used

$= \{481, 537, 226, 418, 1231, 1721\}$

$$\int \frac{x^8}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx =$$

$$-\frac{(5bc-3ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\right.}{32b^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^4+c}}$$
$$-\frac{\sqrt{-a}\sqrt[4]{d}(5bc-3ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)}{16b^2\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{dx^4+c}}$$
$$-\frac{\sqrt[4]{-a}(5bc-3ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{16b^{7/4}(bc-ad)^{3/2}}$$
$$+\frac{\sqrt[4]{-a}(5bc-3ad)\arctan\left(\frac{\sqrt{ad-bc}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{16b^{7/4}(ad-bc)^{3/2}}$$
$$+\frac{(4bc-3ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)\sqrt{dx^4+c}}$$
$$-\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}(5bc-3ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16b^2\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{dx^4+c}}$$
$$-\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(5bc-3ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{32b^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^4+c}}$$
$$+\frac{ax\sqrt{dx^4+c}}{4b(bc-ad)\left(bx^4+a\right)}$$

```
[In] Int[x^8/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (a*x*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) - ((-a)^(1/4)*(5*b*c -
3*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(1
6*b^(7/4)*(b*c - a*d)^(3/2)) + ((-a)^(1/4)*(5*b*c - 3*a*d)*ArcTan[(Sqrt[-(b
*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*b^(7/4)*(-(b*c) +
a*d)^(3/2)) + ((4*b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sq
rt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b
^2*c^(1/4)*d^(1/4)*(b*c - a*d)*Sqrt[c + d*x^4]) - (a*((Sqrt[b]*Sqrt[c])/Sqr
t[-a] + Sqrt[d])*d^(1/4)*(5*b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c +
d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)],
1/2])/(16*b^2*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) - (Sqrt[-a]*
```

```
(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*d^(1/4)*(5*b*c - 3*a*d)*(Sqrt[c] + Sqr
t[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d
^(1/4)*x)/c^(1/4)], 1/2])/(16*b^2*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c +
d*x^4]) - ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(5*b*c - 3*a*d)*(Sqrt[c]
+ Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*
(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]),
2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*b^2*c^(1/4)*d^(1/4)*(b*c - a*d)*(b
*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(5*b*c
 - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^
2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sq
rt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*b^2*c^(1/4)*d^(1/4
)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
```

```
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = \frac{ax\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} - \frac{\int \frac{ac+(-4bc+3ad)x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{4b(bc-ad)}$$

$$= \frac{ax\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} + \frac{(4bc-3ad)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{4b^2(bc-ad)} - \frac{(a(5bc-3ad))\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{4b^2(bc-ad)}$$

$$= \frac{ax\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} + \frac{(4bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8b^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)\sqrt{c+dx^4}} - \frac{(5bc-3ad)\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{8b^2(bc-ad)} - \frac{(5bc-3ad)\int \frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{8b^2(bc-ad)}$$

$$=\frac{ax\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)}+\frac{(4bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8b^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)\sqrt{c+dx^4}}-\frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(5bc-3ad)\right)\int\frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{8b^{3/2}(bc-ad)(bc+ad)}-\frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(5bc-3ad)\right)\int\frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{8b^{3/2}(bc-ad)(bc+ad)}-\frac{\left(a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt{d}(5bc-3ad)\right)\int\frac{1}{\sqrt{c+dx^4}}\,dx}{8b^2(bc-ad)(bc+ad)}-\frac{\left(\sqrt{-a}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt{d}(5bc-3ad)\right)\int\frac{1}{\sqrt{c+dx^4}}\,dx}{8b^2(bc-ad)(bc+ad)}$$

$$=\frac{ax\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)}-\frac{\sqrt[4]{-a}(5bc-3ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16b^{7/4}(bc-ad)^{3/2}}+\frac{\sqrt[4]{-a}(5bc-3ad)\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16b^{7/4}(-bc+ad)^{3/2}}+\frac{(4bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8b^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)\sqrt{c+dx^4}}-\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}(5bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16b^2\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}}-\frac{\sqrt{-a}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(5bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16b^2\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}}-\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(5bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}}{\sqrt[4]{c}}\right.\right.}{32b^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}}-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(5bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)\right.}{32b^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.30 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.25

$$\int \frac{x^8}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$
$$= \frac{x\left(\frac{(4bc-3ad)x^4\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{ab}+\frac{5a\left(c+dx^4+\frac{5ac^2\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx}{c}\right.}{-5ac\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2x^4\left(2bc\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\right.\right.}\right.}{b(a+bx^4)}\right.}{20(bc-ad)\sqrt{c+dx^4}}$$

```
[In] Integrate[x^8/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (x*(((4*b*c - 3*a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((
d*x^4)/c), -((b*x^4)/a)])/(a*b) + (5*a*(c + d*x^4 + (5*a*c^2*AppellF1[1/4,
1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)])/(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4
, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((
d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((
b*x^4)/a)]))))/(b*(a + b*x^4))))/(20*(b*c - a*d)*Sqrt[c + d*x^4])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.31 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.34

method	result
elliptic	$-\frac{ax\sqrt{d\,x^4+c}}{4(ad-bc)b(b\,x^4+a)}+\frac{\left(\frac{1}{b^2}-\frac{ad}{4(ad-bc)b^2}\right)\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{a\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{(3ad-5bc)\left(-\frac{\mathrm{arctanh}}{}\right.}{}\right.}{}$
default	$\frac{\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b^2\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}+\frac{a^2\left(-\frac{bx\sqrt{d\,x^4+c}}{4(ad-bc)a\left(b\,x^4+a\right)}-\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{4(ad-bc)a\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\right.}{}$

[In] int(x^8/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/4*a/(a*d-b*c)/b*x*(d*x^4+c)^(1/2)/(b*x^4+a)+(1/b^2-1/4*a*d/(a*d-b*c)/b^2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-1/32*a/b^3*sum((3*a*d-5*b*c)/(a*d-b*c)/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a))

Fricas [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Timed out}$$

```
[In] integrate(x^8/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^8}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^8}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

```
[In] integrate(x**8/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
[Out] Integral(x**8/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{x^8}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^8}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

```
[In] integrate(x^8/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^8/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{x^8}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^8}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

```
[In] integrate(x^8/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^8/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^8}{(b\,x^4+a)^2\,\sqrt{d\,x^4+c}}\,dx$$

```
[In] int(x^8/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
[Out] int(x^8/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.835 $\int \frac{x^4}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 908

$$\int \frac{x^4}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = -\frac{x\sqrt{c+dx^4}}{4(bc-ad)(a+bx^4)} - \frac{(bc+ad)\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{3/4}b^{3/4}(bc-ad)^{3/2}} + \frac{(bc+ad)\arctan\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{3/4}b^{3/4}(-bc+ad)^{3/2}}$$
$$+ \frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16b\sqrt[4]{c}(bc-ad)\sqrt{c+dx^4}}$$
$$+ \frac{\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16ab\sqrt[4]{c}(bc-ad)\sqrt{c+dx^4}}$$
$$- \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b\sqrt[4]{c}(bc-ad)\sqrt{c+dx^4}}$$
$$+ \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32ab\sqrt[4]{c}\sqrt[4]{d}(bc-ad)\sqrt{c+dx^4}}$$
$$+ \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32ab\sqrt[4]{c}\sqrt[4]{d}(bc-ad)\sqrt{c+dx^4}}$$

```
[Out] -1/16*(a*d+b*c)*arctan(x*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2
))/(-a)^(3/4)/b^(3/4)/(-a*d+b*c)^(3/2)+1/16*(a*d+b*c)*arctan(x*(a*d-b*c)^(1
```

```
/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(3/4)/b^(3/4)/(a*d-b*c)^(3/2)-
1/4*x*(d*x^4+c)^(1/2)/(-a*d+b*c)/(b*x^4+a)-1/8*d^(3/4)*(cos(2*arctan(d^(1/4
)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arc
tan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1
/2)+x^2*d^(1/2))^2)^(1/2)/b/c^(1/4)/(-a*d+b*c)/(d*x^4+c)^(1/2)+1/16*d^(1/4)
*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4))
)*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(b^(1/2)*c^(1/2)/
(-a)^(1/2)+d^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^
2)^(1/2)/b/c^(1/4)/(-a*d+b*c)/(d*x^4+c)^(1/2)+1/16*d^(1/4)*(cos(2*arctan(d^
(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2
*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1
/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/b/c^
(1/4)/(-a*d+b*c)/(d*x^4+c)^(1/2)+1/32*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^
(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^
(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/
2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d
^(1/2))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/b/c^(1/4)/d^(1/4)/(-a
*d+b*c)/(d*x^4+c)^(1/2)+1/32*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos
(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-
1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/
2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^
2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/b/c^(1/4)/d^(1/4)/(-a*d+b*c)/
(d*x^4+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 908, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used

$= \{482, 537, 226, 418, 1231, 1721\}$

$$\int \frac{x^4}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

$$= \frac{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{32ab\sqrt[4]{c}\sqrt[4]{d}(bc-ad)\sqrt{dx^4+c}}$$
$$-\frac{(bc+ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{16(-a)^{3/4}b^{3/4}(bc-ad)^{3/2}}+\frac{(bc+ad)\arctan\left(\frac{\sqrt{ad-bc}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{16(-a)^{3/4}b^{3/4}(ad-bc)^{3/2}}$$
$$-\frac{d^{3/4}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b\sqrt[4]{c}(bc-ad)\sqrt{dx^4+c}}$$
$$+\frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16b\sqrt[4]{c}(bc-ad)\sqrt{dx^4+c}}$$
$$+\frac{\left(\sqrt{d}a+\sqrt{-a}\sqrt{b}\sqrt{c}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16ab\sqrt[4]{c}(bc-ad)\sqrt{dx^4+c}}$$
$$+\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32ab\sqrt[4]{c}\sqrt[4]{d}(bc-ad)\sqrt{dx^4+c}}$$
$$-\frac{x\sqrt{dx^4+c}}{4(bc-ad)\left(bx^4+a\right)}$$

```
[In] Int[x^4/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/4*(x*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4)) - ((b*c + a*d)*ArcTan[(S
qrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(3/4)*b^(
3/4)*(b*c - a*d)^(3/2)) + ((b*c + a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^
(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(3/4)*b^(3/4)*(-(b*c) + a*d)^(3/2
)) + (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2
)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x
)/c^(1/4)], 1/2])/(16*b*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^4]) + ((Sqrt[-a]*S
qrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4
)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])
/(16*a*b*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^4]) - (d^(3/4)*(Sqrt[c] + Sqrt[d]
*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/
4)*x)/c^(1/4)], 1/2])/(8*b*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]
*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sq
rt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d]
```

```
)^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2]
)/(32*a*b*c^(1/4)*d^(1/4)*(b*c - a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c]
- Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + S
qrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a
]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*a*b*c^
(1/4)*d^(1/4)*(b*c - a*d)*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{x\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}+\frac{\int \frac{c-dx^4}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{4(bc-ad)}$$

$$= -\frac{x\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}-\frac{d\int \frac{1}{\sqrt{c+dx^4}}\,dx}{4b(bc-ad)}+\frac{(bc+ad)\int \frac{1}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{4b(bc-ad)}$$

$$= -\frac{x\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8b\sqrt[4]{c}(bc-ad)\sqrt{c+dx^4}}+\frac{(bc+ad)\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{8ab(bc-ad)}+\frac{(bc+ad)\int \frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{8ab(bc-ad)}$$

$$= -\frac{x\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8b\sqrt[4]{c}(bc-ad)\sqrt{c+dx^4}}+\frac{\left(\sqrt{c}\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{8a(bc-ad)}+\frac{\left(\sqrt{c}\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{8a(bc-ad)}+\frac{\left(\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt{d}\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{8b(bc-ad)}+\frac{\left(\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt{d}\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{8ab(bc-ad)}$$

$$= -\frac{x\sqrt{c+dx^4}}{4(bc-ad)(a+bx^4)} - \frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{3/4}b^{3/4}(bc-ad)^{3/2}}$$
$$+ \frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{3/4}b^{3/4}(-bc+ad)^{3/2}}$$
$$+ \frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16b\sqrt[4]{c}(bc-ad)\sqrt{c+dx^4}}$$
$$+ \frac{\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16ab\sqrt[4]{c}(bc-ad)\sqrt{c+dx^4}}$$
$$- \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8b\sqrt[4]{c}(bc-ad)\sqrt{c+dx^4}}$$
$$+ \frac{\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)^2\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{32a\sqrt[4]{c}\sqrt[4]{d}(bc-ad)\sqrt{c+dx^4}}$$
$$+ \frac{\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)^2\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{32a\sqrt[4]{c}\sqrt[4]{d}(bc-ad)\sqrt{c+dx^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.23 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.26

$$\int \frac{x^4}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$
$$= \frac{x\left(\frac{dx^4\sqrt{1+\frac{dx^4}{c}}\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{a}+\frac{5\left(c+dx^4+\frac{5ac^2\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{-5ac\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2x^4\left(2bc\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+\right.}\right.}{a+bx^4}\right)}{20(-bc+ad)\sqrt{c+dx^4}}$$

```
[In] Integrate[x^4/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (x*((d*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c), -((
b*x^4)/a)])/a + (5*(c + d*x^4 + (5*a*c^2*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^
4)/c), -((b*x^4)/a)])/(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b
```

```
*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)
/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]))))/(a +
b*x^4)))/(20*(-(b*c) + a*d)*Sqrt[c + d*x^4])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.20 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.36

method	result
elliptic	$\frac{x\sqrt{d\,x^4+c}}{4(ad-bc)(b\,x^4+a)}+\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{4(ad-bc)b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{(ad+bc)\left(-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}\right.}{}}{32}$
default	$\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^3b\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\Pi\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},\frac{i\sqrt{c}\,\_\alpha^2b}{\sqrt{d}\,a},\frac{\sqrt{-\frac{i\sqrt{d}}{\sqrt{c}}}}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}}\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,a\sqrt{d\,x^4+c}}}{\_\alpha^3}}{8b^2}-\frac{a\left(-\frac{}{4(ad}\right.}{}$

```
[In] int(x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/(a*d-b*c)*x*(d*x^4+c)^(1/2)/(b*x^4+a)+1/4*d/(a*d-b*c)/b/(I/c^(1/2)*d^(1
/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/
(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-1/32/b^2*sum((a*d+
b*c)/(a*d-b*c)/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*
x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*
_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2
)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*
_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=
RootOf(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Timed out}$$

```
[In] integrate(x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^4}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^4}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

```
[In] integrate(x**4/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
[Out] Integral(x**4/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^4}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

```
[In] integrate(x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^4/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^4}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

```
[In] integrate(x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^4}{(b\,x^4+a)^2\,\sqrt{d\,x^4+c}}\,dx$$

```
[In] int(x^4/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
[Out] int(x^4/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.836 $\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 983

$$\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{bx\sqrt{c+dx^4}}{4a(bc-ad)(a+bx^4)}$$
$$+\frac{\sqrt[4]{b}(3bc-5ad)\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{7/4}(bc-ad)^{3/2}}-\frac{\sqrt[4]{b}(3bc-5ad)\arctan\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{7/4}(-bc+ad)^{3/2}}$$
$$+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a\sqrt[4]{c}(bc-ad)\sqrt{c+dx^4}}$$
$$+\frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}(3bc-5ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$
$$+\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(3bc-5ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16(-a)^{3/2}\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$
$$+\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(3bc-5ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{32a^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$
$$+\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(3bc-5ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}}{\sqrt[4]{c}}\right.\right.}{32a^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$

```
[Out] 1/16*b^(1/4)*(-5*a*d+3*b*c)*arctan(x*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d
*x^4+c)^(1/2))/(-a)^(7/4)/(-a*d+b*c)^(3/2)-1/16*b^(1/4)*(-5*a*d+3*b*c)*arct
```

```
an(x*(a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(7/4)/(a*d-b*
c)^(3/2)+1/4*b*x*(d*x^4+c)^(1/2)/a/(-a*d+b*c)/(b*x^4+a)+1/8*d^(3/4)*(cos(2*
arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*Ellipt
icF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d
*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/c^(1/4)/(-a*d+b*c)/(d*x^4+c)^(1/2)
+1/16*d^(1/4)*(-5*a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos
(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/
2*2^(1/2))*(b^(1/2)*c^(1/2)/(-a)^(1/2)+d^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x
^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/c^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^4+
c)^(1/2)+1/16*d^(1/4)*(-5*a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(
1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1
/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2
))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/(-a)^(3/2)/c^(1/4)/(-a*d+b*c)/
(a*d+b*c)/(d*x^4+c)^(1/2)+1/32*(-5*a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/
4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1
/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/
2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)
^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a^2/c^(1/4)/d^(
1/4)/(-a^2*d^2+b^2*c^2)/(d*x^4+c)^(1/2)+1/32*(-5*a*d+3*b*c)*(cos(2*arctan(d
^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin
(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(
-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/
2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/
a^2/c^(1/4)/d^(1/4)/(-a^2*d^2+b^2*c^2)/(d*x^4+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 983, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used

$= \{425, 537, 226, 418, 1231, 1721\}$

$$\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

$$= \frac{(3bc-5ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\right.}{32a^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^4+c}}$$
$$+\frac{\sqrt[4]{d}(3bc-5ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)}{16(-a)^{3/2}\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{dx^4+c}}$$
$$+\frac{\sqrt[4]{b}(3bc-5ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{16(-a)^{7/4}(bc-ad)^{3/2}}-\frac{\sqrt[4]{b}(3bc-5ad)\arctan\left(\frac{\sqrt{ad-bc}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{16(-a)^{7/4}(ad-bc)^{3/2}}$$
$$+\frac{d^{3/4}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a\sqrt[4]{c}(bc-ad)\sqrt{dx^4+c}}$$
$$+\frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}(3bc-5ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{dx^4+c}}$$
$$+\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(3bc-5ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.\right.}{32a^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^4+c}}$$
$$+\frac{bx\sqrt{dx^4+c}}{4a(bc-ad)\left(bx^4+a\right)}$$

```
[In] Int[1/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (b*x*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*(a + b*x^4)) + (b^(1/4)*(3*b*c - 5*a
*d)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(
-a)^(7/4)*(b*c - a*d)^(3/2)) - (b^(1/4)*(3*b*c - 5*a*d)*ArcTan[(Sqrt[-(b*c)
 + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(7/4)*(-(b*c) +
a*d)^(3/2)) + (d^(3/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] +
Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*a*c^(1/4)
*(b*c - a*d)*Sqrt[c + d*x^4]) + (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(
1/4)*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sq
rt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(16*a*c^(1/4)*
(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqr
t[d])*d^(1/4)*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqr
t[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(16*(
-a)^(3/2)*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt
[c] + Sqrt[-a]*Sqrt[d])^2*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c +
```

```
 d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[
-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(
1/4)], 1/2])/(32*a^2*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4
]) + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(3*b*c - 5*a*d)*(Sqrt[c] + Sqr
t[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*S
qrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan
[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*a^2*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d
)*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = \frac{bx\sqrt{c+dx^4}}{4a(bc-ad)\,(a+bx^4)} - \frac{\int \frac{-3bc+4ad-bdx^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{4a(bc-ad)}$$

$$= \frac{bx\sqrt{c+dx^4}}{4a(bc-ad)\,(a+bx^4)} + \frac{d\int \frac{1}{\sqrt{c+dx^4}}\,dx}{4a(bc-ad)} + \frac{(3bc-5ad)\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{4a(bc-ad)}$$

$$= \frac{bx\sqrt{c+dx^4}}{4a(bc-ad)\,(a+bx^4)} + \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a\sqrt[4]{c}(bc-ad)\sqrt{c+dx^4}} + \frac{(3bc-5ad)\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{8a^2(bc-ad)} + \frac{(3bc-5ad)\int \frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{8a^2(bc-ad)}$$

$$= \frac{bx\sqrt{c+dx^4}}{4a(bc-ad)\,(a+bx^4)} + \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a\sqrt[4]{c}(bc-ad)\sqrt{c+dx^4}} + \frac{\left(\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(3bc-5ad)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{8a^2(bc-ad)(bc+ad)} + \frac{\left(\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(3bc-5ad)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{8a^2(bc-ad)(bc+ad)} + \frac{\left(\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt{d}(3bc-5ad)\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{8a(bc-ad)(bc+ad)} + \frac{\left(\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt{d}(3bc-5ad)\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{8(-a)^{3/2}(bc-ad)(bc+ad)}$$

$$= \frac{bx\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} + \frac{\sqrt[4]{b}(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{7/4}(bc-ad)^{3/2}}$$
$$- \frac{\sqrt[4]{b}(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{7/4}(-bc+ad)^{3/2}}$$
$$+ \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a\sqrt[4]{c}(bc-ad)\sqrt{c+dx^4}}$$
$$+ \frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}(3bc-5ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16a\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$
$$+ \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(3bc-5ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16(-a)^{3/2}\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$
$$+ \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(3bc-5ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.\right.}{32a^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$
$$+ \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(3bc-5ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}}{\sqrt[4]{c}}\right.\right.}{32a^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.28 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.40

$$\int \frac{1}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$
$$= \frac{-5acx\,\mathrm{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\left(5a(4bc-4ad+bdx^4)+bdx^4(a+bx^4)\sqrt{1+\frac{dx^4}{c}}\,\mathrm{AppellF1}\left(\frac{5}{4},\right.\right.}{20a^2(bc-ad)\left(a+bx^4\right)\sqrt{c+dx^4}\left(-5ac\right.}$$

```
[In] Integrate[1/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (-5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)]*(5*a*(4*b*
c - 4*a*d + b*d*x^4) + b*d*x^4*(a + b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4
, 1/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]) + 2*b*x^5*(5*a*(c + d*x^4) + d*
x^4*(a + b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c)
, -((b*x^4)/a)])*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/
```

a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]))/(20*a^2*(b*c - a*d)*(a + b*x^4)*Sqrt[c + d*x^4]*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.96 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.34

method	result
default	$-\frac{bx\sqrt{d\,x^4+c}}{4(ad-bc)a(b\,x^4+a)}-\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{4(ad-bc)a\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{(-5ad+3bc)\left(-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4}}\right)}{\sqrt{\frac{-ad+bc}{b}}}\right.}{}$
elliptic	$-\frac{bx\sqrt{d\,x^4+c}}{4(ad-bc)a(b\,x^4+a)}-\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{4(ad-bc)a\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{(-5ad+3bc)\left(-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4}}\right)}{\sqrt{\frac{-ad+bc}{b}}}\right.}{}$

[In] int(1/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/4*b/(a*d-b*c)/a*x*(d*x^4+c)^(1/2)/(b*x^4+a)-1/4*d/(a*d-b*c)/a/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-1/32/b/a*sum((-5*a*d+3*b*c)/(a*d-b*c)/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a))

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

```
[In] integrate(1/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
[Out] Integral(1/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{1}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{1}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{1}{(b\,x^4+a)^2\,\sqrt{d\,x^4+c}}\,dx$$

```
[In] int(1/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
[Out] int(1/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.837 $\int \frac{1}{x^4(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1046

$$\int \frac{1}{x^4(a+bx^4)^2\sqrt{c+dx^4}}\,dx = -\frac{(7bc-4ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^3}+\frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^3(a+bx^4)}$$
$$+\frac{b^{5/4}(7bc-9ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{11/4}(bc-ad)^{3/2}}-\frac{b^{5/4}(7bc-9ad)\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{11/4}(-bc+ad)^{3/2}}$$
$$-\frac{d^{3/4}(7bc-4ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{24a^2c^{5/4}(bc-ad)\sqrt{c+dx^4}}$$
$$+\frac{b\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(7bc-9ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16(-a)^{5/2}\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$
$$-\frac{b\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(7bc-9ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16(-a)^{5/2}\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$
$$-\frac{b\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(7bc-9ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{32a^3\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$
$$-\frac{b\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(7bc-9ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{32a^3\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$

```
[Out] 1/16*b^(5/4)*(-9*a*d+7*b*c)*arctan(x*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d
*x^4+c)^(1/2))/(-a)^(11/4)/(-a*d+b*c)^(3/2)-1/16*b^(5/4)*(-9*a*d+7*b*c)*arc
```

```
tan(x*(a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(11/4)/(a*d-
b*c)^(3/2)-1/12*(-4*a*d+7*b*c)*(d*x^4+c)^(1/2)/a^2/c/(-a*d+b*c)/x^3+1/4*b*(
d*x^4+c)^(1/2)/a/(-a*d+b*c)/x^3/(b*x^4+a)-1/24*d^(3/4)*(-4*a*d+7*b*c)*(cos(
2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*Elli
pticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(
(d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a^2/c^(5/4)/(-a*d+b*c)/(d*x^4+c)^(
1/2)+1/16*b*d^(1/4)*(-9*a*d+7*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/
2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4
))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))
*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/(-a)^(5/2)/c^(1/4)/(-a*d+b*c)/(a
*d+b*c)/(d*x^4+c)^(1/2)-1/32*b*(-9*a*d+7*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/
4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1
/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/
2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)
^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a^3/c^(1/4)/d^(
1/4)/(-a^2*d^2+b^2*c^2)/(d*x^4+c)^(1/2)-1/16*b*d^(1/4)*(-9*a*d+7*b*c)*(cos(
2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*Elli
pticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(
b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/
2)/(-a)^(5/2)/c^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^4+c)^(1/2)-1/32*b*(-9*a*d+7
*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(
1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-
a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2
)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x
^2*d^(1/2))^2)^(1/2)/a^3/c^(1/4)/d^(1/4)/(-a^2*d^2+b^2*c^2)/(d*x^4+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 1046, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used

$= \{483, 597, 537, 226, 418, 1231, 1721\}$

$$\int \frac{1}{x^4 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx =$$

$$-\frac{b(7bc-9ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\right.}{32a^3\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^4+c}}$$

$$+\frac{b\sqrt[4]{d}(7bc-9ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)}{16(-a)^{5/2}\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{dx^4+c}}$$

$$+\frac{b^{5/4}(7bc-9ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{16(-a)^{11/4}(bc-ad)^{3/2}}-\frac{b^{5/4}(7bc-9ad)\arctan\left(\frac{\sqrt{ad-bc}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{16(-a)^{11/4}(ad-bc)^{3/2}}$$

$$-\frac{d^{3/4}(7bc-4ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{24a^2c^{5/4}(bc-ad)\sqrt{dx^4+c}}$$

$$-\frac{b\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(7bc-9ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16(-a)^{5/2}\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{dx^4+c}}$$

$$-\frac{b\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(7bc-9ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{32a^3\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^4+c}}$$

$$+\frac{b\sqrt{dx^4+c}}{4a(bc-ad)x^3(bx^4+a)}-\frac{(7bc-4ad)\sqrt{dx^4+c}}{12a^2c(bc-ad)x^3}$$

```
[In] Int[1/(x^4*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/12*((7*b*c - 4*a*d)*Sqrt[c + d*x^4])/(a^2*c*(b*c - a*d)*x^3) + (b*Sqrt[c
 + d*x^4])/(4*a*(b*c - a*d)*x^3*(a + b*x^4)) + (b^(5/4)*(7*b*c - 9*a*d)*Arc
Tan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(11
/4)*(b*c - a*d)^(3/2)) - (b^(5/4)*(7*b*c - 9*a*d)*ArcTan[(Sqrt[-(b*c) + a*d
]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(11/4)*(-(b*c) + a*d)^
(3/2)) - (d^(3/4)*(7*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/
(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(
24*a^2*c^(5/4)*(b*c - a*d)*Sqrt[c + d*x^4]) + (b*(Sqrt[b]*Sqrt[c] - Sqrt[-a
]*Sqrt[d])*d^(1/4)*(7*b*c - 9*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)
/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/
(16*(-a)^(5/2)*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) - (b*(Sqrt[
b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*d^(1/4)*(7*b*c - 9*a*d)*(Sqrt[c] + Sqrt[d]*x
^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)
*x)/c^(1/4)], 1/2])/(16*(-a)^(5/2)*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c +
```

```
d*x^4]) - (b*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(7*b*c - 9*a*d)*(Sqrt[
c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1
/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]
), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*a^3*c^(1/4)*d^(1/4)*(b*c - a*d)
*(b*c + a*d)*Sqrt[c + d*x^4]) - (b*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(
7*b*c - 9*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*
x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[
b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*a^3*c^(1/4)*d
^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
```

```
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^3\left(a+bx^4\right)} - \frac{\int \frac{-7bc+4ad-5bdx^4}{x^4(a+bx^4)\sqrt{c+dx^4}}\,dx}{4a(bc-ad)} \\
&= -\frac{(7bc-4ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^3\left(a+bx^4\right)} + \frac{\int \frac{-21b^2c^2+20abcd+4a^2d^2-bd(7bc-4ad)x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{12a^2c(bc-ad)} \\
&= -\frac{(7bc-4ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^3\left(a+bx^4\right)} \\
&\quad - \frac{(b(7bc-9ad))\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{4a^2(bc-ad)} - \frac{(d(7bc-4ad))\int \frac{1}{\sqrt{c+dx^4}}\,dx}{12a^2c(bc-ad)} \\
&= -\frac{(7bc-4ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^3\left(a+bx^4\right)} \\
&\quad - \frac{d^{3/4}(7bc-4ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{24a^2c^{5/4}(bc-ad)\sqrt{c+dx^4}} \\
&\quad - \frac{(b(7bc-9ad))\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{8a^3(bc-ad)} - \frac{(b(7bc-9ad))\int \frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{8a^3(bc-ad)}
\end{aligned}
$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^3\,(a+bx^4)}$$

$$- \frac{d^{3/4}(7bc-4ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{24a^2c^{5/4}(bc-ad)\sqrt{c+dx^4}}$$

$$- \frac{\left(b^{3/2}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(7bc-9ad)\right)\int\frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{8a^3(bc-ad)(bc+ad)}$$

$$- \frac{\left(b^{3/2}\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(7bc-9ad)\right)\int\frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{8a^3(bc-ad)(bc+ad)}$$

$$- \frac{\left(b\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt{d}(7bc-9ad)\right)\int\frac{1}{\sqrt{c+dx^4}}\,dx}{8a^2(bc-ad)(bc+ad)}$$

$$+ \frac{\left(b\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt{d}(7bc-9ad)\right)\int\frac{1}{\sqrt{c+dx^4}}\,dx}{8(-a)^{5/2}(bc-ad)(bc+ad)}$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^3\,(a+bx^4)}$$
$$+ \frac{b^{5/4}(7bc-9ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{11/4}(bc-ad)^{3/2}}$$
$$- \frac{b^{5/4}(7bc-9ad)\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{11/4}(-bc+ad)^{3/2}}$$
$$- \frac{d^{3/4}(7bc-4ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{24a^2c^{5/4}(bc-ad)\sqrt{c+dx^4}}$$
$$- \frac{b\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}(7bc-9ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16a^2\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$
$$+ \frac{b\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(7bc-9ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16(-a)^{5/2}\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$
$$- \frac{b\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(7bc-9ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\right.\right.}{32a^3\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$
$$- \frac{b\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(7bc-9ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\right.\right.}{32a^3\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.55 (sec) , antiderivative size = 408, normalized size of antiderivative = 0.39

$$\int \frac{1}{x^4\,(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$
$$= \frac{bd(7bc-4ad)x^8\sqrt{1+\frac{dx^4}{c}}\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+\frac{a\left(25ac\left(-7b^2cx^4(4c+dx^4)+4a^2d(c+2dx^4)+4ab(-c^2+5cd\right.\right.}{(a+bx^4)\left(\right.}}{60a^3}$$

```
[In] Integrate[1/(x^4*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (b*d*(7*b*c - 4*a*d)*x^8*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((
d*x^4)/c), -((b*x^4)/a)] + (a*(25*a*c*(-7*b^2*c*x^4*(4*c + d*x^4) + 4*a^2*d
```

```
*(c + 2*d*x^4) + 4*a*b*(-c^2 + 5*c*d*x^4 + d^2*x^8))*AppellF1[1/4, 1/2, 1,
5/4, -((d*x^4)/c), -((b*x^4)/a)] + 10*x^4*(c + d*x^4)*(-4*a^2*d + 7*b^2*c*x
^4 + 4*a*b*(c - d*x^4))*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((
b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))/
((a + b*x^4)*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)]
 + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*
d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]))))/(60*a^3*c*(-(b
*c) + a*d)*x^3*Sqrt[c + d*x^4])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 8.39 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.35

method	result
elliptic	$\frac{b^2 x\sqrt{d\,x^4+c}}{4a^2(ad-bc)(b\,x^4+a)}-\frac{\sqrt{d\,x^4+c}}{3c\,a^2x^3}+\frac{\left(\frac{bd}{4(ad-bc)a^2}-\frac{d}{3c\,a^2}\right)\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\sqrt{d\,x^4+c}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}(9ad-\ldots}{\ldots}$
default	$\frac{-\frac{\sqrt{d\,x^4+c}}{3c\,x^3}-\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{3c\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\sqrt{d\,x^4+c}}}{a^2}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^3b\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1\ldots}}{\ldots}}{\_\alpha^3}}{8a^2}$ $\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\sqrt{d\,x^4+c}}+\frac{3c\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\_\alpha^3b\sqrt{1-\frac{i\sqrt{d}\,x\ldots}{\sqrt{c}}}\ldots}{\ldots}}{\_\alpha^3}\ldots\right.}{8}$
risch	$-\frac{\sqrt{d\,x^4+c}}{3c\,a^2x^3}-\frac{\ldots}{\ldots}$

[In] int(1/x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/4*b^2/a^2/(a*d-b*c)*x*(d*x^4+c)^(1/2)/(b*x^4+a)-1/3/c/a^2*(d*x^4+c)^(1/2)/x^3+(1/4*b*d/(a*d-b*c)/a^2-1/3*d/c/a^2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-1/32/a^2*sum((9*a*d-7*b*c)/(a*d-b*c)/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a))

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^4 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{x^4 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx$$

```
[In] integrate(1/x**4/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
[Out] Integral(1/(x**4*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{(bx^4+a)^2\sqrt{dx^4+c}x^4}\,dx$$

```
[In] integrate(1/x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{(bx^4+a)^2\sqrt{dx^4+c}x^4}\,dx$$

```
[In] integrate(1/x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{x^4 \left(b\,x^4+a\right)^2 \sqrt{d\,x^4+c}}\,dx$$

```
[In] int(1/(x^4*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
[Out] int(1/(x^4*(a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.838 $\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1146

$$\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{\sqrt{d}x\sqrt{c+dx^4}}{4b(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)} - \frac{x^3\sqrt{c+dx^4}}{4(bc-ad)\,(a+bx^4)}$$
$$+ \frac{(3bc-ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16\sqrt[4]{-a}b^{5/4}(bc-ad)^{3/2}} + \frac{(3bc-ad)\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16\sqrt[4]{-a}b^{5/4}(-bc+ad)^{3/2}}$$
$$- \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4b(bc-ad)\sqrt{c+dx^4}}$$
$$+ \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b(bc-ad)\sqrt{c+dx^4}}$$
$$- \frac{\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(3bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16b\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$
$$- \frac{\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(3bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16b\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$
$$+ \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(3bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{32\sqrt{-a}b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$
$$- \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(3bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{32\sqrt{-a}b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$

```
[Out] 1/16*(-a*d+3*b*c)*arctan(x*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1
/2))/(-a)^(1/4)/b^(5/4)/(-a*d+b*c)^(3/2)+1/16*(-a*d+3*b*c)*arctan(x*(a*d-b*
c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(1/4)/b^(5/4)/(a*d-b*c)^(
3/2)-1/4*x^3*(d*x^4+c)^(1/2)/(-a*d+b*c)/(b*x^4+a)+1/4*x*d^(1/2)*(d*x^4+c)^(
1/2)/b/(-a*d+b*c)/(c^(1/2)+x^2*d^(1/2))-1/4*c^(1/4)*d^(1/4)*(cos(2*arctan(d
^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticE(sin(
2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/
(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b/(-a*d+b*c)/(d*x^4+c)^(1/2)+1/8*c^(1/4)*d^(
1/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1
/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*
d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b/(-a*d+b*c)/(d*x^4+c)^(
```

```
1/2)-1/16*d^(1/4)*(-a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/c
os(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),
1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d
*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^(3/2)/c^(1/4)/(-a^2*d^2+b^2*c^2)/(
d*x^4+c)^(1/2)-1/32*(-a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)
/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4)
)),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^
(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2
))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^(3/2)/c^(1/4)/d^(1/4)/(-a*
d+b*c)/(a*d+b*c)/(-a)^(1/2)/(d*x^4+c)^(1/2)-1/16*d^(1/4)*(-a*d+3*b*c)*(cos(
2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*Elli
pticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(
b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/
2)/b^(3/2)/c^(1/4)/(-a^2*d^2+b^2*c^2)/(d*x^4+c)^(1/2)+1/32*(-a*d+3*b*c)*(co
s(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*El
lipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*
d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(
1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2
))^2)^(1/2)/b^(3/2)/c^(1/4)/d^(1/4)/(-a*d+b*c)/(a*d+b*c)/(-a)^(1/2)/(d*x^4+
c)^(1/2)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 1146, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{482, 598, 311, 226, 1210, 504, 1231, 1721\}$

$$\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = -\frac{\sqrt{dx^4+c}\,x^3}{4(bc-ad)(bx^4+a)} + \frac{\sqrt{d}\sqrt{dx^4+c}\,x}{4b(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)}$$
$$+ \frac{(3bc-ad)\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{16\sqrt[4]{-a}\,b^{5/4}(bc-ad)^{3/2}} + \frac{(3bc-ad)\arctan\left(\frac{\sqrt{ad-bc}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{16\sqrt[4]{-a}\,b^{5/4}(ad-bc)^{3/2}}$$
$$- \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4b(bc-ad)\sqrt{dx^4+c}}$$
$$+ \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b(bc-ad)\sqrt{dx^4+c}}$$
$$- \frac{\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(3bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16b\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{dx^4+c}}$$
$$- \frac{\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(3bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16b\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{dx^4+c}}$$
$$+ \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(3bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{32\sqrt{-a}\,b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^4+c}}$$
$$- \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(3bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{32\sqrt{-a}\,b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^4+c}}$$

```
[In] Int[x^6/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)) - (x^
3*Sqrt[c + d*x^4])/(4*(b*c - a*d)*(a + b*x^4)) + ((3*b*c - a*d)*ArcTan[(Sqr
t[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(1/4)*b^(5/
4)*(b*c - a*d)^(3/2)) + ((3*b*c - a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^
(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(1/4)*b^(5/4)*(-(b*c) + a*d)^(3/2
)) - (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + S
qrt[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b*(b*c - a
*d)*Sqrt[c + d*x^4]) + (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d
*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1
/2])/(8*b*(b*c - a*d)*Sqrt[c + d*x^4]) - ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqr
t[b])*d^(1/4)*(3*b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[
c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(16*b*c
```

```
^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[c] + (Sqrt[-a]*Sqr
t[d])/Sqrt[b])*d^(1/4)*(3*b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^
4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2]
)/(16*b*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c
] + Sqrt[-a]*Sqrt[d])^2*(3*b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x
^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*
Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)
], 1/2])/(32*Sqrt[-a]*b^(3/2)*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[
c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(3*b*c - a*d)*(Sqrt[c
] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sq
rt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2
*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*Sqrt[-a]*b^(3/2)*c^(1/4)*d^(1/4)*(b
*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
```

```
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^3\sqrt{c+dx^4}}{4(bc-ad)\,(a+bx^4)}+\frac{\int \frac{x^2(3c+dx^4)}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{4(bc-ad)}\\
&= -\frac{x^3\sqrt{c+dx^4}}{4(bc-ad)\,(a+bx^4)}+\frac{\int \left(\frac{dx^2}{b\sqrt{c+dx^4}}+\frac{(3bc-ad)x^2}{b(a+bx^4)\sqrt{c+dx^4}}\right)dx}{4(bc-ad)}\\
&= -\frac{x^3\sqrt{c+dx^4}}{4(bc-ad)\,(a+bx^4)}+\frac{d\int \frac{x^2}{\sqrt{c+dx^4}}\,dx}{4b(bc-ad)}+\frac{(3bc-ad)\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{4b(bc-ad)}\\
&= -\frac{x^3\sqrt{c+dx^4}}{4(bc-ad)\,(a+bx^4)}+\frac{\left(\sqrt{c}\sqrt{d}\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{4b(bc-ad)}-\frac{\left(\sqrt{c}\sqrt{d}\right)\int \frac{1-\frac{\sqrt{d}x^2}{\sqrt{c}}}{\sqrt{c+dx^4}}\,dx}{4b(bc-ad)}\\
&\quad -\frac{(3bc-ad)\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{8b^{3/2}(bc-ad)}+\frac{(3bc-ad)\int \frac{1}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{8b^{3/2}(bc-ad)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\sqrt{d}x\sqrt{c+dx^4}}{4b(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)} - \frac{x^3\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)} \\
&\quad - \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4b(bc-ad)\sqrt{c+dx^4}} \\
&\quad + \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8b(bc-ad)\sqrt{c+dx^4}} \\
&\quad - \frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(3bc-ad)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{8b(bc-ad)(bc+ad)} \\
&\quad + \frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(3bc-ad)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{8b(bc-ad)(bc+ad)} \\
&\quad - \frac{\left(\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt{d}(3bc-ad)\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{8b^{3/2}(bc-ad)(bc+ad)} \\
&\quad - \frac{\left(\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt{d}(3bc-ad)\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{8b^{3/2}(bc-ad)(bc+ad)}
\end{aligned}
$$

$$= \frac{\sqrt{d}x\sqrt{c+dx^4}}{4b(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)} - \frac{x^3\sqrt{c+dx^4}}{4(bc-ad)(a+bx^4)}$$
$$+ \frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16\sqrt[4]{-a}b^{5/4}(bc-ad)^{3/2}} + \frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16\sqrt[4]{-a}b^{5/4}(-bc+ad)^{3/2}}$$
$$- \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4b(bc-ad)\sqrt{c+dx^4}}$$
$$+ \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8b(bc-ad)\sqrt{c+dx^4}}$$
$$- \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(3bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16b^{3/2}\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$
$$- \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(3bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16b^{3/2}\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$
$$+ \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(3bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\right.\right.}{32\sqrt{-a}b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$
$$- \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(3bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{32\sqrt{-a}b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.39 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.14

$$\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$
$$= \frac{-7ax^3(c+dx^4)+7cx^3(a+bx^4)\sqrt{1+\frac{dx^4}{c}}\operatorname{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+dx^7(a+bx^4)\sqrt{1+\frac{dx^4}{c}}\operatorname{Ap}}{28a(bc-ad)(a+bx^4)\sqrt{c+dx^4}}$$

```
[In] Integrate[x^6/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (-7*a*x^3*(c + d*x^4) + 7*c*x^3*(a + b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[3/
4, 1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)] + d*x^7*(a + b*x^4)*Sqrt[1 + (d
```

```
*x^4)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x^4)/a)])/(28*a*(b*
c - a*d)*(a + b*x^4)*Sqrt[c + d*x^4])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.02 (sec) , antiderivative size = 353, normalized size of antiderivative = 0.31

method	result
elliptic	$\frac{x^{3}\sqrt{d\,x^{4}+c}}{4(ad-bc)(b\,x^{4}+a)}-\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{4(ad-bc)b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}}-\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^{4}b+a\right)}\frac{(-ad+3bc)\left(-\frac{\mathrm{arct}\ldots}{\ldots}\right.}{\ldots}$
default	$\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^{4}b+a\right)}\frac{-\frac{\mathrm{arctanh}\left(\frac{2d\,x^{2}\,\_\alpha^{2}+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^{4}+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+\frac{2\,\_\alpha^{3}b\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\Pi\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},\frac{i\sqrt{c}\,\_\alpha^{2}b}{\sqrt{d}\,a},\frac{\sqrt{-\frac{i\sqrt{d}}{\sqrt{c}}}}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}}\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,a\sqrt{d\,x^{4}+c}}}{\_\alpha}}{8b^{2}}-\frac{a\left(-\frac{b\,x^{3}\ldots}{4(ad-b\ldots}\right.}{\ldots}$

```
[In] int(x^6/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/(a*d-b*c)*x^3*(d*x^4+c)^(1/2)/(b*x^4+a)-1/4*I*d^(1/2)/(a*d-b*c)/b*c^(1/
2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d
^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*(EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)
-EllipticE(x*(I/c^(1/2)*d^(1/2))^(1/2),I))-1/32/b^2*sum((-a*d+3*b*c)/(a*d-b
*c)/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a
*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*
(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(
1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,
(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b
+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Timed out}$$

```
[In] integrate(x^6/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

```
[In] integrate(x**6/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
[Out] Integral(x**6/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^6}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

```
[In] integrate(x^6/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^6/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^6}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

```
[In] integrate(x^6/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^6/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^6}{(b\,x^4+a)^2\,\sqrt{d\,x^4+c}}\,dx$$

```
[In] int(x^6/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
[Out] int(x^6/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.839 $\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1144

$$\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = -\frac{\sqrt{d}x\sqrt{c+dx^4}}{4a(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)}+\frac{bx^3\sqrt{c+dx^4}}{4a(bc-ad)(a+bx^4)}$$
$$-\frac{(bc-3ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{5/4}\sqrt[4]{b}(bc-ad)^{3/2}}-\frac{(bc-3ad)\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{5/4}\sqrt[4]{b}(-bc+ad)^{3/2}}$$
$$+\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4a(bc-ad)\sqrt{c+dx^4}}$$
$$-\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a(bc-ad)\sqrt{c+dx^4}}$$
$$-\frac{\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$
$$-\frac{\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$
$$-\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{32(-a)^{3/2}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$
$$+\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{32(-a)^{3/2}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$

```
[Out] -1/16*(-3*a*d+b*c)*arctan(x*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(
1/2))/(-a)^(5/4)/b^(1/4)/(-a*d+b*c)^(3/2)-1/16*(-3*a*d+b*c)*arctan(x*(a*d-b
*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(5/4)/b^(1/4)/(a*d-b*c)^
(3/2)+1/4*b*x^3*(d*x^4+c)^(1/2)/a/(-a*d+b*c)/(b*x^4+a)-1/4*x*d^(1/2)*(d*x^4
+c)^(1/2)/a/(-a*d+b*c)/(c^(1/2)+x^2*d^(1/2))+1/4*c^(1/4)*d^(1/4)*(cos(2*arc
tan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticE
(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^
4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/(-a*d+b*c)/(d*x^4+c)^(1/2)-1/8*c^(1/4
)*d^(1/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x
/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)
+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/(-a*d+b*c)/(d*x^4
```

```
+c)^(1/2)+1/32*(-3*a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(
2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/
4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2)
,1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*
((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/(-a)^(3/2)/c^(1/4)/d^(1/4)/(-a*d+
b*c)/(a*d+b*c)/b^(1/2)/(d*x^4+c)^(1/2)-1/32*(-3*a*d+b*c)*(cos(2*arctan(d^(1
/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*
arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)
^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*
c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/(-a
)^(3/2)/c^(1/4)/d^(1/4)/(-a*d+b*c)/(a*d+b*c)/b^(1/2)/(d*x^4+c)^(1/2)-1/16*d
^(1/4)*(-3*a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan
(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2)
)*(c^(1/2)+x^2*d^(1/2))*(c^(1/2)-(-a)^(1/2)*d^(1/2)/b^(1/2))*((d*x^4+c)/(c^
(1/2)+x^2*d^(1/2))^2)^(1/2)/a/c^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^4+c)^(1/2)-
1/16*d^(1/4)*(-3*a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*
arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2
^(1/2))*(c^(1/2)+x^2*d^(1/2))*(c^(1/2)+(-a)^(1/2)*d^(1/2)/b^(1/2))*((d*x^4+
c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/c^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^4+c)^
(1/2)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 1144, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{483, 598, 311, 226, 1210, 504, 1231, 1721\}$

$$\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{b\sqrt{dx^4+c}x^3}{4a(bc-ad)(bx^4+a)} - \frac{\sqrt{d}\sqrt{dx^4+c}x}{4a(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)} - \frac{(bc-3ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{16(-a)^{5/4}\sqrt[4]{b}(bc-ad)^{3/2}} - \frac{(bc-3ad)\arctan\left(\frac{\sqrt{ad-bc}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{16(-a)^{5/4}\sqrt[4]{b}(ad-bc)^{3/2}} + \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4a(bc-ad)\sqrt{dx^4+c}} - \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a(bc-ad)\sqrt{dx^4+c}} - \frac{\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(bc-3ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{dx^4+c}} - \frac{\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(bc-3ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{dx^4+c}} - \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(bc-3ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{c}}{\sqrt[4]{}}\right.\right.}{32(-a)^{3/2}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^4+c}} + \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(bc-3ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{32(-a)^{3/2}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^4+c}}$$

```
[In] Int[x^2/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/4*(Sqrt[d]*x*Sqrt[c + d*x^4])/(a*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)) +
(b*x^3*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*(a + b*x^4)) - ((b*c - 3*a*d)*ArcT
an[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(5/4
)*b^(1/4)*(b*c - a*d)^(3/2)) - ((b*c - 3*a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x)
/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(5/4)*b^(1/4)*(-(b*c) + a*
d)^(3/2)) + (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt
[c] + Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a*(
b*c - a*d)*Sqrt[c + d*x^4]) - (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt
[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1
/4)], 1/2])/(8*a*(b*c - a*d)*Sqrt[c + d*x^4]) - ((Sqrt[c] - (Sqrt[-a]*Sqrt[
d])/Sqrt[b])*d^(1/4)*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)
/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/
```

```
(16*a*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[c] + (Sqrt[
-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c
 + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)
], 1/2])/(16*a*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]
*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(
c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sq
rt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/
c^(1/4)], 1/2])/(32*(-a)^(3/2)*Sqrt[b]*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a
*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c - 3*a*d
)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ellip
ticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sq
rt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*(-a)^(3/2)*Sqrt[b]*c^(1/4)
*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
```

```
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx^3\sqrt{c+dx^4}}{4a(bc-ad)\,(a+bx^4)} - \frac{\int \frac{x^2(-bc+4ad+bdx^4)}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{4a(bc-ad)} \\
&= \frac{bx^3\sqrt{c+dx^4}}{4a(bc-ad)\,(a+bx^4)} - \frac{\int \left(\frac{dx^2}{\sqrt{c+dx^4}} + \frac{(-bc+3ad)x^2}{(a+bx^4)\sqrt{c+dx^4}}\right)dx}{4a(bc-ad)} \\
&= \frac{bx^3\sqrt{c+dx^4}}{4a(bc-ad)\,(a+bx^4)} - \frac{d\int \frac{x^2}{\sqrt{c+dx^4}}\,dx}{4a(bc-ad)} + \frac{(bc-3ad)\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{4a(bc-ad)} \\
&= \frac{bx^3\sqrt{c+dx^4}}{4a(bc-ad)\,(a+bx^4)} - \frac{\left(\sqrt{c}\sqrt{d}\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{4a(bc-ad)} + \frac{\left(\sqrt{c}\sqrt{d}\right)\int \frac{1-\frac{\sqrt{d}x^2}{\sqrt{c}}}{\sqrt{c+dx^4}}\,dx}{4a(bc-ad)} \\
&\quad - \frac{(bc-3ad)\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{8a\sqrt{b}(bc-ad)} + \frac{(bc-3ad)\int \frac{1}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{8a\sqrt{b}(bc-ad)}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\sqrt{d}x\sqrt{c+dx^4}}{4a(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)} + \frac{bx^3\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} \\
&+ \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4a(bc-ad)\sqrt{c+dx^4}} \\
&- \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a(bc-ad)\sqrt{c+dx^4}} \\
&- \frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(bc-3ad)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{8a(bc-ad)(bc+ad)} \\
&+ \frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(bc-3ad)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{8a(bc-ad)(bc+ad)} \\
&- \frac{\left(\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt{d}(bc-3ad)\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{8a\sqrt{b}(bc-ad)(bc+ad)} \\
&- \frac{\left(\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt{d}(bc-3ad)\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{8a\sqrt{b}(bc-ad)(bc+ad)}
\end{aligned}
$$

$$= -\frac{\sqrt{d}x\sqrt{c+dx^4}}{4a(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)} + \frac{bx^3\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)}$$

$$- \frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{5/4}\sqrt[4]{b}(bc-ad)^{3/2}} - \frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{5/4}\sqrt[4]{b}(-bc+ad)^{3/2}}$$

$$+ \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4a(bc-ad)\sqrt{c+dx^4}}$$

$$- \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a(bc-ad)\sqrt{c+dx^4}}$$

$$- \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16a\sqrt{b}\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$

$$- \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16a\sqrt{b}\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$

$$- \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{32(-a)^{3/2}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$

$$+ \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)\right.}{32(-a)^{3/2}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.15

$$\int \frac{x^2}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

$$= \frac{21abx^3(c+dx^4)+7(bc-4ad)x^3(a+bx^4)\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-3bdx^7(a+bx^4)\sqrt{}}{84a^2(bc-ad)\left(a+bx^4\right)\sqrt{c+dx^4}}$$

```
[In] Integrate[x^2/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (21*a*b*x^3*(c + d*x^4) + 7*(b*c - 4*a*d)*x^3*(a + b*x^4)*Sqrt[1 + (d*x^4)/
c]*AppellF1[3/4, 1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)] - 3*b*d*x^7*(a +
```

```
b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x
^4)/a)])/(84*a^2*(b*c - a*d)*(a + b*x^4)*Sqrt[c + d*x^4])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.98 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.31

method	result
default	$-\frac{b\,x^3\sqrt{d\,x^4+c}}{4(ad-bc)a(b\,x^4+a)}+\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{4(ad-bc)a\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{(-3ad+bc)}{}}{}$
elliptic	$-\frac{b\,x^3\sqrt{d\,x^4+c}}{4(ad-bc)a(b\,x^4+a)}+\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{4(ad-bc)a\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{(-3ad+bc)}{}}{}$

```
[In] int(x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*b/(a*d-b*c)/a*x^3*(d*x^4+c)^(1/2)/(b*x^4+a)+1/4*I*d^(1/2)/(a*d-b*c)/a*
c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1
/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*(EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/
2),I)-EllipticE(x*(I/c^(1/2)*d^(1/2))^(1/2),I))-1/32/b/a*sum((-3*a*d+b*c)/(
a*d-b*c)/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)
/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3
*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4
+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2
/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_
Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Timed out}$$

```
[In] integrate(x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

```
[In] integrate(x**2/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
[Out] Integral(x**2/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^2}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

```
[In] integrate(x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^2}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

```
[In] integrate(x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^2}{{\left(b\,x^4+a\right)}^2\,\sqrt{d\,x^4+c}}\,dx$$

```
[In] int(x^2/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
[Out] int(x^2/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.840 $\int \frac{1}{x^2(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1225

$$\int \frac{1}{x^2 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx$$
$$= -\frac{(5bc-4ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x}+\frac{\sqrt{d}(5bc-4ad)x\sqrt{c+dx^4}}{4a^2c(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)}+\frac{b\sqrt{c+dx^4}}{4a(bc-ad)x\,(a+bx^4)}$$
$$-\frac{b^{3/4}(5bc-7ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{9/4}(bc-ad)^{3/2}}-\frac{b^{3/4}(5bc-7ad)\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{9/4}(-bc+ad)^{3/2}}$$
$$-\frac{\sqrt[4]{d}(5bc-4ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4a^2c^{3/4}(bc-ad)\sqrt{c+dx^4}}$$
$$+\frac{\sqrt[4]{d}(5bc-4ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a^2c^{3/4}(bc-ad)\sqrt{c+dx^4}}$$
$$+\frac{b\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(5bc-7ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a^2\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$
$$+\frac{b\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(5bc-7ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a^2\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$
$$-\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(5bc-7ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\right.}{32(-a)^{5/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$
$$+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(5bc-7ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{32(-a)^{5/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$

```
[Out] -1/16*b^(3/4)*(-7*a*d+5*b*c)*arctan(x*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(
d*x^4+c)^(1/2))/(-a)^(9/4)/(-a*d+b*c)^(3/2)-1/16*b^(3/4)*(-7*a*d+5*b*c)*arc
tan(x*(a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(9/4)/(a*d-b
*c)^(3/2)-1/4*(-4*a*d+5*b*c)*(d*x^4+c)^(1/2)/a^2/c/(-a*d+b*c)/x+1/4*b*(d*x^
4+c)^(1/2)/a/(-a*d+b*c)/x/(b*x^4+a)+1/4*(-4*a*d+5*b*c)*x*d^(1/2)*(d*x^4+c)^
(1/2)/a^2/c/(-a*d+b*c)/(c^(1/2)+x^2*d^(1/2))-1/4*d^(1/4)*(-4*a*d+5*b*c)*(co
s(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*El
lipticE(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))
*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a^2/c^(3/4)/(-a*d+b*c)/(d*x^4+c)
```

```
^(1/2)+1/8*d^(1/4)*(-4*a*d+5*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2
)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4)
)),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(
1/2)/a^2/c^(3/4)/(-a*d+b*c)/(d*x^4+c)^(1/2)+1/32*(-7*a*d+5*b*c)*(cos(2*arct
an(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi
(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^
2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*b^(1/2)*(c^(1/2)+x^2*d^(1
/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2)
)^2)^(1/2)/(-a)^(5/2)/c^(1/4)/d^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^4+c)^(1/2)-
1/32*(-7*a*d+5*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan
(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1
/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^
(1/2))*b^(1/2)*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2
*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/(-a)^(5/2)/c^(1/4)/d^(1/4)/(-a*d
+b*c)/(a*d+b*c)/(d*x^4+c)^(1/2)+1/16*b*d^(1/4)*(-7*a*d+5*b*c)*(cos(2*arctan
(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(si
n(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(c^(1/2)-
(-a)^(1/2)*d^(1/2)/b^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a^2/c
^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^4+c)^(1/2)+1/16*b*d^(1/4)*(-7*a*d+5*b*c)*(
cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*
EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2
))*(c^(1/2)+(-a)^(1/2)*d^(1/2)/b^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)
^(1/2)/a^2/c^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^4+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 1225, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used

$= \{483, 597, 598, 311, 226, 1210, 504, 1231, 1721\}$

$$\int \frac{1}{x^2 (a+bx^4)^2 \sqrt{c+dx^4}}\, dx$$

$$= \frac{\sqrt{b}(5bc-7ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}}, 2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right), \frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\right.}{32(-a)^{5/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^4+c}}$$

$$- \frac{b^{3/4}(5bc-7ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{16(-a)^{9/4}(bc-ad)^{3/2}} - \frac{b^{3/4}(5bc-7ad)\arctan\left(\frac{\sqrt{ad-bc}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{16(-a)^{9/4}(ad-bc)^{3/2}}$$

$$- \frac{\sqrt[4]{d}(5bc-4ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4a^2c^{3/4}(bc-ad)\sqrt{dx^4+c}}$$

$$+ \frac{\sqrt[4]{d}(5bc-4ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right), \frac{1}{2}\right)}{8a^2c^{3/4}(bc-ad)\sqrt{dx^4+c}}$$

$$+ \frac{b\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(5bc-7ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right), \frac{1}{2}\right)}{16a^2\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{dx^4+c}}$$

$$+ \frac{b\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(5bc-7ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right), \frac{1}{2}\right)}{16a^2\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{dx^4+c}}$$

$$- \frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(5bc-7ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}}, 2\arctan\right.}{32(-a)^{5/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^4+c}}$$

$$- \frac{(5bc-4ad)\sqrt{dx^4+c}}{4a^2c(bc-ad)x} + \frac{\sqrt{d}(5bc-4ad)x\sqrt{dx^4+c}}{4a^2c(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)} + \frac{b\sqrt{dx^4+c}}{4a(bc-ad)x\,(bx^4+a)}$$

```
[In] Int[1/(x^2*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/4*((5*b*c - 4*a*d)*Sqrt[c + d*x^4])/(a^2*c*(b*c - a*d)*x) + (Sqrt[d]*(5*
b*c - 4*a*d)*x*Sqrt[c + d*x^4])/(4*a^2*c*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2
)) + (b*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*x*(a + b*x^4)) - (b^(3/4)*(5*b*c
- 7*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/
(16*(-a)^(9/4)*(b*c - a*d)^(3/2)) - (b^(3/4)*(5*b*c - 7*a*d)*ArcTan[(Sqrt[-
(b*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(9/4)*(-(b*
c) + a*d)^(3/2)) - (d^(1/4)*(5*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c
 + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)
], 1/2])/(4*a^2*c^(3/4)*(b*c - a*d)*Sqrt[c + d*x^4]) + (d^(1/4)*(5*b*c - 4*
```

```
a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*El
lipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*a^2*c^(3/4)*(b*c - a*d)*Sqr
t[c + d*x^4]) + (b*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(5*b*c -
7*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*
EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(16*a^2*c^(1/4)*(b*c - a*d)*
(b*c + a*d)*Sqrt[c + d*x^4]) + (b*(Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^
(1/4)*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + S
qrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(16*a^2*c^(1/
4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) - (Sqrt[b]*(Sqrt[b]*Sqrt[c] + S
qrt[-a]*Sqrt[d])^2*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)
/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqr
t[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)],
1/2])/(32*(-a)^(5/2)*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4
]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(5*b*c - 7*a*d)*(Sqrt[
c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(S
qrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]),
2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*(-a)^(5/2)*c^(1/4)*d^(1/4)*(b*c -
a*d)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral}=\frac{b\sqrt{c+dx^4}}{4a(bc-ad)x\left(a+bx^4\right)}-\frac{\int\frac{-5bc+4ad-3bdx^4}{x^2(a+bx^4)\sqrt{c+dx^4}}\,dx}{4a(bc-ad)}$$

$$=-\frac{(5bc-4ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x}+\frac{b\sqrt{c+dx^4}}{4a(bc-ad)x\left(a+bx^4\right)}+\frac{\int\frac{x^2\left(-((bc-2ad)(5bc-2ad))+bd(5bc-4ad)x^4\right)}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{4a^2c(bc-ad)}$$

$$=-\frac{(5bc-4ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x}+\frac{b\sqrt{c+dx^4}}{4a(bc-ad)x\left(a+bx^4\right)}+\frac{\int\left(\frac{d(5bc-4ad)x^2}{\sqrt{c+dx^4}}+\frac{(-5b^2c^2+7abcd)x^2}{(a+bx^4)\sqrt{c+dx^4}}\right)dx}{4a^2c(bc-ad)}$$

$$=-\frac{(5bc-4ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x}+\frac{b\sqrt{c+dx^4}}{4a(bc-ad)x\left(a+bx^4\right)}-\frac{(b(5bc-7ad))\int\frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{4a^2(bc-ad)}+\frac{(d(5bc-4ad))\int\frac{x^2}{\sqrt{c+dx^4}}\,dx}{4a^2c(bc-ad)}$$

$$=-\frac{(5bc-4ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x}+\frac{b\sqrt{c+dx^4}}{4a(bc-ad)x\left(a+bx^4\right)}+\frac{\left(\sqrt{b}(5bc-7ad)\right)\int\frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{8a^2(bc-ad)}-\frac{\left(\sqrt{b}(5bc-7ad)\right)\int\frac{1}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{8a^2(bc-ad)}+\frac{\left(\sqrt{d}(5bc-4ad)\right)\int\frac{1}{\sqrt{c+dx^4}}\,dx}{4a^2\sqrt{c}(bc-ad)}-\frac{\left(\sqrt{d}(5bc-4ad)\right)\int\frac{1-\frac{\sqrt{d}x^2}{\sqrt{c}}}{\sqrt{c+dx^4}}\,dx}{4a^2\sqrt{c}(bc-ad)}$$

$$
\begin{aligned}
&= -\frac{(5bc-4ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x} + \frac{\sqrt{d}(5bc-4ad)x\sqrt{c+dx^4}}{4a^2c(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x\,(a+bx^4)} \\
&- \frac{\sqrt[4]{d}(5bc-4ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4a^2c^{3/4}(bc-ad)\sqrt{c+dx^4}} \\
&+ \frac{\sqrt[4]{d}(5bc-4ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a^2c^{3/4}(bc-ad)\sqrt{c+dx^4}} \\
&+ \frac{\left(b\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(5bc-7ad)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{8a^2(bc-ad)(bc+ad)} \\
&- \frac{\left(b\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(5bc-7ad)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{8a^2(bc-ad)(bc+ad)} \\
&+ \frac{\left(\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt{d}(5bc-7ad)\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{8a^2(bc-ad)(bc+ad)} \\
&+ \frac{\left(\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt{d}(5bc-7ad)\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{8a^2(bc-ad)(bc+ad)}
\end{aligned}
$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x} + \frac{\sqrt{d}(5bc-4ad)x\sqrt{c+dx^4}}{4a^2c(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)}$$
$$+ \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x\,(a+bx^4)} - \frac{b^{3/4}(5bc-7ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{9/4}(bc-ad)^{3/2}}$$
$$- \frac{b^{3/4}(5bc-7ad)\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{9/4}(-bc+ad)^{3/2}}$$
$$- \frac{\sqrt[4]{d}(5bc-4ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4a^2c^{3/4}(bc-ad)\sqrt{c+dx^4}}$$
$$+ \frac{\sqrt[4]{d}(5bc-4ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a^2c^{3/4}(bc-ad)\sqrt{c+dx^4}}$$
$$+ \frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(5bc-7ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16a^2\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$
$$+ \frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(5bc-7ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16a^2\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$
$$- \frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(5bc-7ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\right.\right.}{32(-a)^{5/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$
$$+ \frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(5bc-7ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\right.\right.}{32(-a)^{5/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.18

$$\int \frac{1}{x^2(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$
$$= \frac{21a(c+dx^4)(4a^2d-5b^2cx^4-4ab(c-dx^4))-7(5b^2c^2-12abcd+4a^2d^2)x^4(a+bx^4)\sqrt{1+\frac{dx^4}{c}}\text{AppellF1}}{84a^3c(bc-ad)x(a+b}$$

```
[In] Integrate[1/(x^2*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

[Out] (21*a*(c + d*x^4)*(4*a^2*d - 5*b^2*c*x^4 - 4*a*b*(c - d*x^4)) - 7*(5*b^2*c^
2 - 12*a*b*c*d + 4*a^2*d^2)*x^4*(a + b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[3/
4, 1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)] + 3*b*d*(5*b*c - 4*a*d)*x^8*(a
+ b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b
*x^4)/a)])/(84*a^3*c*(b*c - a*d)*x*(a + b*x^4)*Sqrt[c + d*x^4])

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 8.14 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.32

method	result
elliptic	$\frac{b^2x^3\sqrt{dx^4+c}}{4(ad-bc)a^2(bx^4+a)}-\frac{\sqrt{dx^4+c}}{c\,a^2x}+\frac{i\left(-\frac{bd}{4(ad-bc)a^2}+\frac{d}{c\,a^2}\right)\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{dx^4+c}\,\sqrt{d}}-\frac{\_\alpha}{}$
default	$\frac{-\frac{\sqrt{dx^4+c}}{cx}+\frac{i\sqrt{d}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{\sqrt{c}\,\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{dx^4+c}}}{a^2}-\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^4b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\,\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{dx^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}+}{}}{}$ $\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{dx^4+c}}-\frac{c\left(\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^4b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\,\_\alpha^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{dx^4}}\right)}{\sqrt{\frac{-ad+bc}{b}}}}{}\right.}{}$
risch	$-\frac{\sqrt{dx^4+c}}{c\,a^2x}+\frac{}{}$

```
[In] int(1/x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/(a*d-b*c)/a^2*b^2*x^3*(d*x^4+c)^(1/2)/(b*x^4+a)-1/c/a^2*(d*x^4+c)^(1/2)
/x+I*(-1/4*b*d/(a*d-b*c)/a^2+d/c/a^2)*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-
I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2
)/d^(1/2)*(EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*
d^(1/2))^(1/2),I))-1/32/a^2*sum((7*a*d-5*b*c)/(a*d-b*c)/_alpha*(-1/((-a*d+b
*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4
+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^
2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^
(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1
/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^2 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{x^2 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx$$

```
[In] integrate(1/x**2/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{(bx^4+a)^2 \sqrt{dx^4+c}x^2}\,dx$$

```
[In] integrate(1/x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 \left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{\left(bx^4+a\right)^2 \sqrt{dx^4+c}x^2}\,dx$$

```
[In] integrate(1/x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{x^2 \left(b\,x^4+a\right)^2 \sqrt{d\,x^4+c}}\,dx$$

```
[In] int(1/(x^2*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
[Out] int(1/(x^2*(a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.841 $\int \frac{(ex)^m(a+bx^4)^2}{\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 200

$$\int \frac{(ex)^m\left(a+bx^4\right)^2}{\sqrt{c+dx^4}}\,dx = -\frac{b(bc(5+m)-2ad(7+m))(ex)^{1+m}\sqrt{c+dx^4}}{d^2e(3+m)(7+m)}+\frac{b^2(ex)^{5+m}\sqrt{c+dx^4}}{de^5(7+m)} + \frac{\left(a^2d^2(3+m)(7+m)+bc(1+m)(bc(5+m)-2ad(7+m))\right)(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\right.}{d^2e(1+m)(3+m)(7+m)\sqrt{c+dx^4}}$$

```
[Out] -b*(b*c*(5+m)-2*a*d*(7+m))*(e*x)^(1+m)*(d*x^4+c)^(1/2)/d^2/e/(3+m)/(7+m)+b^
2*(e*x)^(5+m)*(d*x^4+c)^(1/2)/d/e^5/(7+m)+(a^2*d^2*(3+m)*(7+m)+b*c*(1+m)*(b
*c*(5+m)-2*a*d*(7+m)))*(e*x)^(1+m)*hypergeom([1/2, 1/4+1/4*m],[5/4+1/4*m],-
d*x^4/c)*(1+d*x^4/c)^(1/2)/d^2/e/(1+m)/(3+m)/(7+m)/(d*x^4+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {475, 470, 372, 371}

$$\int \frac{(ex)^m\left(a+bx^4\right)^2}{\sqrt{c+dx^4}}\,dx = \frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\left(\frac{a^2d^2(m+7)}{m+1}+\frac{bc(bc(m+5)-2ad(m+7))}{m+3}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{4},\frac{m+5}{4},-\frac{dx^4}{c}\right)}{d^2e(m+7)\sqrt{c+dx^4}} - \frac{b\sqrt{c+dx^4}(ex)^{m+1}(bc(m+5)-2ad(m+7))}{d^2e(m+3)(m+7)}+\frac{b^2\sqrt{c+dx^4}(ex)^{m+5}}{de^5(m+7)}$$

```
[In] Int[((e*x)^m*(a + b*x^4)^2)/Sqrt[c + d*x^4],x]
```

```
[Out] -((b*(b*c*(5 + m) - 2*a*d*(7 + m))*(e*x)^(1 + m)*Sqrt[c + d*x^4])/(d^2*e*(3
 + m)*(7 + m))) + (b^2*(e*x)^(5 + m)*Sqrt[c + d*x^4])/(d*e^5*(7 + m)) + (((
a^2*d^2*(7 + m))/(1 + m) + (b*c*(b*c*(5 + m) - 2*a*d*(7 + m)))/(3 + m))*(e*
x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1 + m)/4, (5 + m)/4,
 -((d*x^4)/c)])/(d^2*e*(7 + m)*Sqrt[c + d*x^4])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 475

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[d^2*(e*x)^(m + n + 1)*((a + b*x^n)^(p + 1)/(b*e^(n
+ 1)*(m + n*(p + 2) + 1))), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b^2(ex)^{5+m}\sqrt{c+dx^4}}{de^5(7+m)} + \frac{\int \frac{(ex)^m\left(a^2d(7+m)-b(bc(5+m)-2ad(7+m))x^4\right)}{\sqrt{c+dx^4}}\,dx}{d(7+m)} \\
&= -\frac{b(bc(5+m)-2ad(7+m))(ex)^{1+m}\sqrt{c+dx^4}}{d^2e(3+m)(7+m)} + \frac{b^2(ex)^{5+m}\sqrt{c+dx^4}}{de^5(7+m)} \\
&\quad - \left(-a^2 - \frac{bc(1+m)(bc(5+m)-2ad(7+m))}{d^2(3+m)(7+m)}\right)\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx
\end{aligned}
$$

$$= -\frac{b(bc(5+m)-2ad(7+m))(ex)^{1+m}\sqrt{c+dx^4}}{d^2e(3+m)(7+m)} + \frac{b^2(ex)^{5+m}\sqrt{c+dx^4}}{de^5(7+m)} - \frac{\left(\left(-a^2 - \frac{bc(1+m)(bc(5+m)-2ad(7+m))}{d^2(3+m)(7+m)}\right)\sqrt{1+\frac{dx^4}{c}}\right)\int \frac{(ex)^m}{\sqrt{1+\frac{dx^4}{c}}}\,dx}{\sqrt{c+dx^4}}$$

$$= -\frac{b(bc(5+m)-2ad(7+m))(ex)^{1+m}\sqrt{c+dx^4}}{d^2e(3+m)(7+m)} + \frac{b^2(ex)^{5+m}\sqrt{c+dx^4}}{de^5(7+m)} + \frac{\left(a^2 + \frac{bc(1+m)(bc(5+m)-2ad(7+m))}{d^2(3+m)(7+m)}\right)(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{4};\frac{5+m}{4};-\frac{dx^4}{c}\right)}{e(1+m)\sqrt{c+dx^4}}$$

Mathematica [A] (verified)

Time = 11.13 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.82

$$\int \frac{(ex)^m (a+bx^4)^2}{\sqrt{c+dx^4}}\,dx = \frac{x(ex)^m\sqrt{1+\frac{dx^4}{c}}\left(a^2(45+14m+m^2)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{4},\frac{5+m}{4},-\frac{dx^4}{c}\right)+b(1+m)x^4\left(2a(9+m)\right.\right.}{(1+m)(5+m)(9+}$$

```
[In] Integrate[((e*x)^m*(a + b*x^4)^2)/Sqrt[c + d*x^4],x]
```

```
[Out] (x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*(a^2*(45 + 14*m + m^2)*Hypergeometric2F1[1/2
, (1 + m)/4, (5 + m)/4, -((d*x^4)/c)] + b*(1 + m)*x^4*(2*a*(9 + m)*Hypergeo
metric2F1[1/2, (5 + m)/4, (9 + m)/4, -((d*x^4)/c)] + b*(5 + m)*x^4*Hypergeo
metric2F1[1/2, (9 + m)/4, (13 + m)/4, -((d*x^4)/c)])))/((1 + m)*(5 + m)*(9
+ m)*Sqrt[c + d*x^4])
```

Maple [F]

$$\int \frac{(ex)^m (b\,x^4+a)^2}{\sqrt{d\,x^4+c}}dx$$

```
[In] int((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] int((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(ex)^m (a+bx^4)^2}{\sqrt{c+dx^4}}\,dx = \int \frac{(bx^4+a)^2(ex)^m}{\sqrt{dx^4+c}}\,dx$$

```
[In] integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^8 + 2*a*b*x^4 + a^2)*(e*x)^m/sqrt(d*x^4 + c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.38 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.91

$$\int \frac{(ex)^m (a+bx^4)^2}{\sqrt{c+dx^4}}\,dx = \frac{a^2e^mx^{m+1}\Gamma\left(\frac{m}{4}+\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{4}+\frac{1}{4}\\ \frac{m}{4}+\frac{5}{4}\end{matrix}\middle|\frac{dx^4e^{i\pi}}{c}\right)}{4\sqrt{c}\,\Gamma\left(\frac{m}{4}+\frac{5}{4}\right)} + \frac{abe^mx^{m+5}\Gamma\left(\frac{m}{4}+\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{4}+\frac{5}{4}\\ \frac{m}{4}+\frac{9}{4}\end{matrix}\middle|\frac{dx^4e^{i\pi}}{c}\right)}{2\sqrt{c}\,\Gamma\left(\frac{m}{4}+\frac{9}{4}\right)} + \frac{b^2e^mx^{m+9}\Gamma\left(\frac{m}{4}+\frac{9}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{4}+\frac{9}{4}\\ \frac{m}{4}+\frac{13}{4}\end{matrix}\middle|\frac{dx^4e^{i\pi}}{c}\right)}{4\sqrt{c}\,\Gamma\left(\frac{m}{4}+\frac{13}{4}\right)}$$

```
[In] integrate((e*x)**m*(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] a**2*e**m*x**(m + 1)*gamma(m/4 + 1/4)*hyper((1/2, m/4 + 1/4), (m/4 + 5/4,),
 d*x**4*exp_polar(I*pi)/c)/(4*sqrt(c)*gamma(m/4 + 5/4)) + a*b*e**m*x**(m +
5)*gamma(m/4 + 5/4)*hyper((1/2, m/4 + 5/4), (m/4 + 9/4,), d*x**4*exp_polar(
I*pi)/c)/(2*sqrt(c)*gamma(m/4 + 9/4)) + b**2*e**m*x**(m + 9)*gamma(m/4 + 9/
4)*hyper((1/2, m/4 + 9/4), (m/4 + 13/4,), d*x**4*exp_polar(I*pi)/c)/(4*sqrt
(c)*gamma(m/4 + 13/4))
```

Maxima [F]

$$\int \frac{(ex)^m (a+bx^4)^2}{\sqrt{c+dx^4}}\,dx = \int \frac{(bx^4+a)^2(ex)^m}{\sqrt{dx^4+c}}\,dx$$

```
[In] integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^2*(e*x)^m/sqrt(d*x^4 + c), x)
```

Giac [F]

$$\int \frac{(ex)^m (a+bx^4)^2}{\sqrt{c+dx^4}}\,dx = \int \frac{(bx^4+a)^2(ex)^m}{\sqrt{dx^4+c}}\,dx$$

```
[In] integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^4 + a)^2*(e*x)^m/sqrt(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^4)^2}{\sqrt{c+dx^4}}\,dx = \int \frac{(e\,x)^m\,(b\,x^4+a)^2}{\sqrt{d\,x^4+c}}\,dx$$

```
[In] int(((e*x)^m*(a + b*x^4)^2)/(c + d*x^4)^(1/2),x)
[Out] int(((e*x)^m*(a + b*x^4)^2)/(c + d*x^4)^(1/2), x)
```

3.842 $\int \frac{(ex)^m(a+bx^4)}{\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 123

$$\int \frac{(ex)^m (a+bx^4)}{\sqrt{c+dx^4}}\,dx = \frac{b(ex)^{1+m}\sqrt{c+dx^4}}{de(3+m)} - \frac{(bc(1+m)-ad(3+m))(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{4},\frac{5+m}{4},-\frac{dx^4}{c}\right)}{de(1+m)(3+m)\sqrt{c+dx^4}}$$

```
[Out] b*(e*x)^(1+m)*(d*x^4+c)^(1/2)/d/e/(3+m)-(b*c*(1+m)-a*d*(3+m))*(e*x)^(1+m)*h
ypergeom([1/2, 1/4+1/4*m],[5/4+1/4*m],-d*x^4/c)*(1+d*x^4/c)^(1/2)/d/e/(1+m)
/(3+m)/(d*x^4+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {470, 372, 371}

$$\int \frac{(ex)^m (a+bx^4)}{\sqrt{c+dx^4}}\,dx = \frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\left(\frac{a}{m+1}-\frac{bc}{d(m+3)}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{4},\frac{m+5}{4},-\frac{dx^4}{c}\right)}{e\sqrt{c+dx^4}} + \frac{b\sqrt{c+dx^4}(ex)^{m+1}}{de(m+3)}$$

```
[In] Int[((e*x)^m*(a + b*x^4))/Sqrt[c + d*x^4],x]
```

```
[Out] (b*(e*x)^(1 + m)*Sqrt[c + d*x^4])/(d*e*(3 + m)) + ((a/(1 + m) - (b*c)/(d*(3
 + m)))*(e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1 + m)/4,
 (5 + m)/4, -((d*x^4)/c)])/(e*Sqrt[c + d*x^4])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b(ex)^{1+m}\sqrt{c+dx^4}}{de(3+m)} - \left(-a+\frac{bc(1+m)}{d(3+m)}\right)\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx \\
&= \frac{b(ex)^{1+m}\sqrt{c+dx^4}}{de(3+m)} - \frac{\left(\left(-a+\frac{bc(1+m)}{d(3+m)}\right)\sqrt{1+\frac{dx^4}{c}}\right)\int \frac{(ex)^m}{\sqrt{1+\frac{dx^4}{c}}}\,dx}{\sqrt{c+dx^4}} \\
&= \frac{b(ex)^{1+m}\sqrt{c+dx^4}}{de(3+m)} + \frac{\left(a-\frac{bc(1+m)}{d(3+m)}\right)(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{4};\frac{5+m}{4};-\frac{dx^4}{c}\right)}{e(1+m)\sqrt{c+dx^4}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\int \frac{(ex)^m (a+bx^4)}{\sqrt{c+dx^4}} dx = \frac{x(ex)^m\sqrt{1+\frac{dx^4}{c}}\left(a(5+m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{4},\frac{5+m}{4},-\frac{dx^4}{c}\right)+b(1+m)x^4\,\text{Hypergeometric2F1}\right.}{(1+m)(5+m)\sqrt{c+dx^4}}$$

```
[In] Integrate[((e*x)^m*(a + b*x^4))/Sqrt[c + d*x^4],x]
[Out] (x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*(a*(5 + m)*Hypergeometric2F1[1/2, (1 + m)/4,
 (5 + m)/4, -((d*x^4)/c)] + b*(1 + m)*x^4*Hypergeometric2F1[1/2, (5 + m)/4,
 (9 + m)/4, -((d*x^4)/c)]))/((1 + m)*(5 + m)*Sqrt[c + d*x^4])
```

Maple [F]

$$\int \frac{(ex)^m (b\,x^4+a)}{\sqrt{d\,x^4+c}}dx$$

```
[In] int((e*x)^m*(b*x^4+a)/(d*x^4+c)^(1/2),x)
[Out] int((e*x)^m*(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(ex)^m (a+bx^4)}{\sqrt{c+dx^4}} dx = \int \frac{(bx^4+a)(ex)^m}{\sqrt{dx^4+c}} dx$$

```
[In] integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
[Out] integral((b*x^4 + a)*(e*x)^m/sqrt(d*x^4 + c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.19 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.95

$$\int \frac{(ex)^m (a+bx^4)}{\sqrt{c+dx^4}} dx = \frac{ae^m x^{m+1}\Gamma\left(\frac{m}{4}+\frac{1}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{m}{4}+\frac{1}{4}\\ \frac{m}{4}+\frac{5}{4}\end{matrix}\right|\frac{dx^4e^{i\pi}}{c}\right)}{4\sqrt{c}\Gamma\left(\frac{m}{4}+\frac{5}{4}\right)} + \frac{be^m x^{m+5}\Gamma\left(\frac{m}{4}+\frac{5}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{m}{4}+\frac{5}{4}\\ \frac{m}{4}+\frac{9}{4}\end{matrix}\right|\frac{dx^4e^{i\pi}}{c}\right)}{4\sqrt{c}\Gamma\left(\frac{m}{4}+\frac{9}{4}\right)}$$

```
[In] integrate((e*x)**m*(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] a*e**m*x**(m + 1)*gamma(m/4 + 1/4)*hyper((1/2, m/4 + 1/4), (m/4 + 5/4,), d*
x**4*exp_polar(I*pi)/c)/(4*sqrt(c)*gamma(m/4 + 5/4)) + b*e**m*x**(m + 5)*ga
mma(m/4 + 5/4)*hyper((1/2, m/4 + 5/4), (m/4 + 9/4,), d*x**4*exp_polar(I*pi)
/c)/(4*sqrt(c)*gamma(m/4 + 9/4))
```

Maxima [F]

$$\int \frac{(ex)^m (a + bx^4)}{\sqrt{c + dx^4}}\, dx = \int \frac{(bx^4 + a)(ex)^m}{\sqrt{dx^4 + c}}\, dx$$

```
[In] integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)*(e*x)^m/sqrt(d*x^4 + c), x)
```

Giac [F]

$$\int \frac{(ex)^m (a + bx^4)}{\sqrt{c + dx^4}}\, dx = \int \frac{(bx^4 + a)(ex)^m}{\sqrt{dx^4 + c}}\, dx$$

```
[In] integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)*(e*x)^m/sqrt(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a + bx^4)}{\sqrt{c + dx^4}}\, dx = \int \frac{(e\,x)^m\,(b\,x^4 + a)}{\sqrt{d\,x^4 + c}}\, dx$$

```
[In] int(((e*x)^m*(a + b*x^4))/(c + d*x^4)^(1/2),x)
```

```
[Out] int(((e*x)^m*(a + b*x^4))/(c + d*x^4)^(1/2), x)
```

3.843 $\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 68

$$\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx=\frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{4},\frac{5+m}{4},-\frac{dx^4}{c}\right)}{e(1+m)\sqrt{c+dx^4}}$$

```
[Out] (e*x)^(1+m)*hypergeom([1/2, 1/4+1/4*m],[5/4+1/4*m],-d*x^4/c)*(1+d*x^4/c)^(1
/2)/e/(1+m)/(d*x^4+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {372, 371}

$$\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx=\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{4},\frac{m+5}{4},-\frac{dx^4}{c}\right)}{e(m+1)\sqrt{c+dx^4}}$$

```
[In] Int[(e*x)^m/Sqrt[c + d*x^4],x]
```

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1 + m)/4, (5 + m
)/4, -((d*x^4)/c)])/(e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^4}{c}}\int\frac{(ex)^m}{\sqrt{1+\frac{dx^4}{c}}}\,dx}{\sqrt{c+dx^4}}$$

$$= \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{4};\frac{5+m}{4};-\frac{dx^4}{c}\right)}{e(1+m)\sqrt{c+dx^4}}$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int\frac{(ex)^m}{\sqrt{c+dx^4}}\,dx = \frac{x(ex)^m\sqrt{1+\frac{dx^4}{c}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{4},1+\frac{1+m}{4},-\frac{dx^4}{c}\right)}{(1+m)\sqrt{c+dx^4}}$$

```
[In] Integrate[(e*x)^m/Sqrt[c + d*x^4],x]
[Out] (x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1 + m)/4, 1 + (1 + m
)/4, -((d*x^4)/c)])/((1 + m)*Sqrt[c + d*x^4])
```

Maple [F]

$$\int\frac{(ex)^m}{\sqrt{d\,x^4+c}}dx$$

```
[In] int((e*x)^m/(d*x^4+c)^(1/2),x)
[Out] int((e*x)^m/(d*x^4+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx = \int \frac{(ex)^m}{\sqrt{dx^4+c}}\,dx$$

```
[In] integrate((e*x)^m/(d*x^4+c)^(1/2),x, algorithm="fricas")
[Out] integral((e*x)^m/sqrt(d*x^4 + c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx = \frac{e^m x^{m+1}\Gamma\left(\frac{m}{4}+\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{m}{4}+\frac{1}{4} \\ \frac{m}{4}+\frac{5}{4} \end{matrix}\middle| \frac{dx^4 e^{i\pi}}{c}\right)}{4\sqrt{c}\,\Gamma\left(\frac{m}{4}+\frac{5}{4}\right)}$$

```
[In] integrate((e*x)**m/(d*x**4+c)**(1/2),x)
[Out] e**m*x**(m + 1)*gamma(m/4 + 1/4)*hyper((1/2, m/4 + 1/4), (m/4 + 5/4,), d*x*
*4*exp_polar(I*pi)/c)/(4*sqrt(c)*gamma(m/4 + 5/4))
```

Maxima [F]

$$\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx = \int \frac{(ex)^m}{\sqrt{dx^4+c}}\,dx$$

```
[In] integrate((e*x)^m/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x)^m/sqrt(d*x^4 + c), x)
```

Giac [F]

$$\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx = \int \frac{(ex)^m}{\sqrt{dx^4+c}}\,dx$$

```
[In] integrate((e*x)^m/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] integrate((e*x)^m/sqrt(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx=\int \frac{{\left(e\,x\right)}^m}{\sqrt{d\,x^4+c}} \,d x$$

```
[In] int((e*x)^m/(c + d*x^4)^(1/2),x)
[Out] int((e*x)^m/(c + d*x^4)^(1/2), x)
```

3.844 $\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 81

$$\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\mathrm{AppellF1}\left(\frac{1+m}{4},1,\frac{1}{2},\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae(1+m)\sqrt{c+dx^4}}$$

```
[Out] (e*x)^(1+m)*AppellF1(1/4+1/4*m,1,1/2,5/4+1/4*m,-b*x^4/a,-d*x^4/c)*(1+d*x^4/
c)^(1/2)/a/e/(1+m)/(d*x^4+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {525, 524}

$$\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\,\mathrm{AppellF1}\left(\frac{m+1}{4},1,\frac{1}{2},\frac{m+5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae(m+1)\sqrt{c+dx^4}}$$

```
[In] Int[(e*x)^m/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 1, 1/2, (5 + m)/4, -
((b*x^4)/a), -((d*x^4)/c)])/(a*e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^4}{c}}\int \frac{(ex)^m}{(a+bx^4)\sqrt{1+\frac{dx^4}{c}}}\,dx}{\sqrt{c+dx^4}}$$

$$= \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}F_1\left(\frac{1+m}{4};1,\frac{1}{2};\frac{5+m}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae(1+m)\sqrt{c+dx^4}}$$

Mathematica [A] (verified)

Time = 5.61 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.54

$$\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{x(ex)^m\sqrt{c+dx^4}\left(bc\,\text{AppellF1}\left(\frac{1+m}{4},-\frac{1}{2},1,\frac{5+m}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-ad\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{4},\frac{5+m}{4},-\frac{dx^4}{c}\right.\right.}{ac(bc-ad)(1+m)\sqrt{1+\frac{dx^4}{c}}}$$

```
[In] Integrate[(e*x)^m/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (x*(e*x)^m*Sqrt[c + d*x^4]*(b*c*AppellF1[(1 + m)/4, -1/2, 1, (5 + m)/4, -((
d*x^4)/c), -((b*x^4)/a)] - a*d*Hypergeometric2F1[1/2, (1 + m)/4, (5 + m)/4,
 -((d*x^4)/c)]))/(a*c*(b*c - a*d)*(1 + m)*Sqrt[1 + (d*x^4)/c])
```

Maple [F]

$$\int \frac{(ex)^m}{(b\,x^4+a)\sqrt{d\,x^4+c}}dx$$

```
[In] int((e*x)^m/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] int((e*x)^m/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{(ex)^m}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

```
[In] integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*x^4 + c)*(e*x)^m/(b*d*x^8 + (b*c + a*d)*x^4 + a*c), x)
```

Sympy [F]

$$\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

```
[In] integrate((e*x)**m/(b*x**4+a)/(d*x**4+c)**(1/2),x)
[Out] Integral((e*x)**m/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{(ex)^m}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

```
[In] integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x)^m/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{(ex)^m}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

```
[In] integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] integrate((e*x)^m/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{(e\,x)^m}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}\,dx$$

```
[In] int((e*x)^m/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
[Out] int((e*x)^m/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

3.845 $\int \frac{(ex)^m}{(a+bx^4)^2\sqrt{c+dx^4}} dx$

Optimal result

Integrand size = 26, antiderivative size = 81

$$\int \frac{(ex)^m}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{1+m}{4},2,\frac{1}{2},\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2e(1+m)\sqrt{c+dx^4}}$$

```
[Out] (e*x)^(1+m)*AppellF1(1/4+1/4*m,2,1/2,5/4+1/4*m,-b*x^4/a,-d*x^4/c)*(1+d*x^4/
c)^(1/2)/a^2/e/(1+m)/(d*x^4+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {525, 524}

$$\int \frac{(ex)^m}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\,\text{AppellF1}\left(\frac{m+1}{4},2,\frac{1}{2},\frac{m+5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2e(m+1)\sqrt{c+dx^4}}$$

```
[In] Int[(e*x)^m/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 2, 1/2, (5 + m)/4, -
((b*x^4)/a), -((d*x^4)/c)])/(a^2*e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^4}{c}}\int \frac{(ex)^m}{(a+bx^4)^2\sqrt{1+\frac{dx^4}{c}}}\,dx}{\sqrt{c+dx^4}}$$

$$= \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}F_1\left(\frac{1+m}{4};2,\frac{1}{2};\frac{5+m}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2e(1+m)\sqrt{c+dx^4}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 179 vs. $2(81) = 162$.

Time = 11.18 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.21

$$\int \frac{(ex)^m}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

$$= \frac{x(ex)^m\sqrt{c+dx^4}\left(-abcd\,\text{AppellF1}\left(\frac{1+m}{4},-\frac{1}{2},1,\frac{5+m}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+bc(bc-ad)\,\text{AppellF1}\left(\frac{1+m}{4},2,-\frac{1}{2},\frac{5+m}{4}\right.\right.}{a^2c(bc-ad)^2(1+m)\sqrt{1+\frac{dx^4}{c}}}$$

```
[In] Integrate[(e*x)^m/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (x*(e*x)^m*Sqrt[c + d*x^4]*(-(a*b*c*d*AppellF1[(1 + m)/4, -1/2, 1, (5 + m)/
4, -((d*x^4)/c), -((b*x^4)/a)]) + b*c*(b*c - a*d)*AppellF1[(1 + m)/4, 2, -1
/2, (5 + m)/4, -((b*x^4)/a), -((d*x^4)/c)] + a^2*d^2*Hypergeometric2F1[1/2,
 (1 + m)/4, (5 + m)/4, -((d*x^4)/c)]))/(a^2*c*(b*c - a*d)^2*(1 + m)*Sqrt[1
+ (d*x^4)/c])
```

Maple [F]

$$\int \frac{(ex)^m}{(b\,x^4+a)^2\,\sqrt{d\,x^4+c}}dx$$

```
[In] int((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
[Out] int((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(ex)^m}{(a+bx^4)^2\,\sqrt{c+dx^4}}\,dx = \int \frac{(ex)^m}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

```
[In] integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*x^4 + c)*(e*x)^m/(b^2*d*x^12 + (b^2*c + 2*a*b*d)*x^8 + (2*a
*b*c + a^2*d)*x^4 + a^2*c), x)
```

Sympy [F]

$$\int \frac{(ex)^m}{(a+bx^4)^2\,\sqrt{c+dx^4}}\,dx = \int \frac{(ex)^m}{(a+bx^4)^2\,\sqrt{c+dx^4}}\,dx$$

```
[In] integrate((e*x)**m/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
[Out] Integral((e*x)**m/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{(ex)^m}{(a+bx^4)^2\,\sqrt{c+dx^4}}\,dx = \int \frac{(ex)^m}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

```
[In] integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate((e*x)^m/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{(ex)^m}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{(ex)^m}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

```
[In] integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] integrate((e*x)^m/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{(e\,x)^m}{(b\,x^4+a)^2\sqrt{d\,x^4+c}}\,dx$$

```
[In] int((e*x)^m/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
[Out] int((e*x)^m/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

3.846 $\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 81

$$\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{c+dx^4}}\,dx = \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\mathrm{AppellF1}\left(\frac{1+m}{4},3,\frac{1}{2},\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3e(1+m)\sqrt{c+dx^4}}$$

```
[Out] (e*x)^(1+m)*AppellF1(1/4+1/4*m,3,1/2,5/4+1/4*m,-b*x^4/a,-d*x^4/c)*(1+d*x^4/
c)^(1/2)/a^3/e/(1+m)/(d*x^4+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {525, 524}

$$\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{c+dx^4}}\,dx = \frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\,\mathrm{AppellF1}\left(\frac{m+1}{4},3,\frac{1}{2},\frac{m+5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3e(m+1)\sqrt{c+dx^4}}$$

```
[In] Int[(e*x)^m/((a + b*x^4)^3*Sqrt[c + d*x^4]),x]
```

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 3, 1/2, (5 + m)/4, -
((b*x^4)/a), -((d*x^4)/c)])/(a^3*e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^4}{c}}\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{1+\frac{dx^4}{c}}}\,dx}{\sqrt{c+dx^4}}$$

$$= \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}F_1\left(\frac{1+m}{4};3,\frac{1}{2};\frac{5+m}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3e(1+m)\sqrt{c+dx^4}}$$

Mathematica [A] (verified)

Time = 11.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{c+dx^4}}\,dx = \frac{x(ex)^m\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{1+m}{4},3,\frac{1}{2},\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3(1+m)\sqrt{c+dx^4}}$$

```
[In] Integrate[(e*x)^m/((a + b*x^4)^3*Sqrt[c + d*x^4]),x]
```

```
[Out] (x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 3, 1/2, (5 + m)/4, -((b*
x^4)/a), -((d*x^4)/c)])/(a^3*(1 + m)*Sqrt[c + d*x^4])
```

Maple [F]

$$\int \frac{(ex)^m}{(b\,x^4+a)^3\sqrt{d\,x^4+c}}dx$$

```
[In] int((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(1/2),x)
```

```
[Out] int((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{c+dx^4}}\,dx=\int \frac{(ex)^m}{(bx^4+a)^3\sqrt{dx^4+c}}\,dx$$

```
[In] integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^4 + c)*(e*x)^m/(b^3*d*x^16 + (b^3*c + 3*a*b^2*d)*x^12 + 3
*(a*b^2*c + a^2*b*d)*x^8 + (3*a^2*b*c + a^3*d)*x^4 + a^3*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{c+dx^4}}\,dx=\text{Timed out}$$

```
[In] integrate((e*x)**m/(b*x**4+a)**3/(d*x**4+c)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{c+dx^4}}\,dx=\int \frac{(ex)^m}{(bx^4+a)^3\sqrt{dx^4+c}}\,dx$$

```
[In] integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m/((b*x^4 + a)^3*sqrt(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{c+dx^4}}\,dx=\int \frac{(ex)^m}{(bx^4+a)^3\sqrt{dx^4+c}}\,dx$$

```
[In] integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/((b*x^4 + a)^3*sqrt(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{c+dx^4}}\,dx = \int \frac{(e\,x)^m}{(b\,x^4+a)^3\,\sqrt{d\,x^4+c}}\,dx$$

```
[In] int((e*x)^m/((a + b*x^4)^3*(c + d*x^4)^(1/2)),x)
[Out] int((e*x)^m/((a + b*x^4)^3*(c + d*x^4)^(1/2)), x)
```

3.847 $\int \frac{(ex)^m(a+bx^4)^2}{(c+dx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 198

$$\int \frac{(ex)^m\left(a+bx^4\right)^2}{\left(c+dx^4\right)^{3/2}}\,dx=\frac{(bc-ad)^2(ex)^{1+m}}{2cd^2e\sqrt{c+dx^4}}+\frac{b^2(ex)^{1+m}\sqrt{c+dx^4}}{d^2e(3+m)}-\frac{\left(2b^2c^2(1+m)-(3+m)\left(2a^2d^2-(bc-ad)^2(1+m)\right)\right)(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{4},\frac{5+m}{4}\ldots\right)}{2cd^2e(1+m)(3+m)\sqrt{c+dx^4}}$$

```
[Out] 1/2*(-a*d+b*c)^2*(e*x)^(1+m)/c/d^2/e/(d*x^4+c)^(1/2)+b^2*(e*x)^(1+m)*(d*x^4
+c)^(1/2)/d^2/e/(3+m)-1/2*(2*b^2*c^2*(1+m)-(3+m)*(2*a^2*d^2-(-a*d+b*c)^2*(1
+m)))*(e*x)^(1+m)*hypergeom([1/2, 1/4+1/4*m],[5/4+1/4*m],-d*x^4/c)*(1+d*x^4
/c)^(1/2)/c/d^2/e/(1+m)/(3+m)/(d*x^4+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {474, 470, 372, 371}

$$\int \frac{(ex)^m\left(a+bx^4\right)^2}{\left(c+dx^4\right)^{3/2}}\,dx=-\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\left(2b^2c^2(m+1)-(m+3)\left(2a^2d^2-(m+1)(bc-ad)^2\right)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{4},\frac{m\ldots}{4}\ldots\right)}{2cd^2e(m+1)(m+3)\sqrt{c+dx^4}}+\frac{(ex)^{m+1}(bc-ad)^2}{2cd^2e\sqrt{c+dx^4}}+\frac{b^2\sqrt{c+dx^4}(ex)^{m+1}}{d^2e(m+3)}$$

```
[In] Int[((e*x)^m*(a + b*x^4)^2)/(c + d*x^4)^(3/2),x]
```

```
[Out] ((b*c - a*d)^2*(e*x)^(1 + m))/(2*c*d^2*e*Sqrt[c + d*x^4]) + (b^2*(e*x)^(1 +
 m)*Sqrt[c + d*x^4])/(d^2*e*(3 + m)) - ((2*b^2*c^2*(1 + m) - (3 + m)*(2*a^2
*d^2 - (b*c - a*d)^2*(1 + m)))*(e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeome
tric2F1[1/2, (1 + m)/4, (5 + m)/4, -((d*x^4)/c)])/(2*c*d^2*e*(1 + m)*(3 + m
)*Sqrt[c + d*x^4])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(bc-ad)^2(ex)^{1+m}}{2cd^2e\sqrt{c+dx^4}} - \frac{\int \frac{(ex)^m\left(-2a^2d^2+(bc-ad)^2(1+m)-2b^2cdx^4\right)}{\sqrt{c+dx^4}}\,dx}{2cd^2} \\
&= \frac{(bc-ad)^2(ex)^{1+m}}{2cd^2e\sqrt{c+dx^4}} + \frac{b^2(ex)^{1+m}\sqrt{c+dx^4}}{d^2e(3+m)} \\
&\quad - \frac{\left(-a^2d^2(1-m)-2abcd(1+m)+\frac{b^2c^2(1+m)(5+m)}{3+m}\right)\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx}{2cd^2}
\end{aligned}
$$

$$=\frac{(bc-ad)^2(ex)^{1+m}}{2cd^2e\sqrt{c+dx^4}}+\frac{b^2(ex)^{1+m}\sqrt{c+dx^4}}{d^2e(3+m)}-\frac{\left(\left(-a^2d^2(1-m)-2abcd(1+m)+\frac{b^2c^2(1+m)(5+m)}{3+m}\right)\sqrt{1+\frac{dx^4}{c}}\right)\int\frac{(ex)^m}{\sqrt{1+\frac{dx^4}{c}}}\,dx}{2cd^2\sqrt{c+dx^4}}$$

$$=\frac{(bc-ad)^2(ex)^{1+m}}{2cd^2e\sqrt{c+dx^4}}+\frac{b^2(ex)^{1+m}\sqrt{c+dx^4}}{d^2e(3+m)}+\frac{\left(a^2d^2(1-m)+2abcd(1+m)-\frac{b^2c^2(1+m)(5+m)}{3+m}\right)(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{4};\frac{5+m}{4};-\frac{dx^4}{c}\right)}{2cd^2e(1+m)\sqrt{c+dx^4}}$$

Mathematica [A] (verified)

Time = 11.19 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.84

$$\int\frac{(ex)^m\left(a+bx^4\right)^2}{\left(c+dx^4\right)^{3/2}}\,dx=\frac{x(ex)^m\sqrt{1+\frac{dx^4}{c}}\left(a^2(45+14m+m^2)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{4},\frac{5+m}{4},-\frac{dx^4}{c}\right)+\right.}{}$$

```
[In] Integrate[((e*x)^m*(a + b*x^4)^2)/(c + d*x^4)^(3/2),x]
```

```
[Out] (x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*(a^2*(45 + 14*m + m^2)*Hypergeometric2F1[3/2
, (1 + m)/4, (5 + m)/4, -((d*x^4)/c)] + b*(1 + m)*x^4*(2*a*(9 + m)*Hypergeo
metric2F1[3/2, (5 + m)/4, (9 + m)/4, -((d*x^4)/c)] + b*(5 + m)*x^4*Hypergeo
metric2F1[3/2, (9 + m)/4, (13 + m)/4, -((d*x^4)/c)])))/(c*(1 + m)*(5 + m)*(
9 + m)*Sqrt[c + d*x^4])
```

Maple [F]

$$\int\frac{(ex)^m\left(b\,x^4+a\right)^2}{\left(d\,x^4+c\right)^{\frac{3}{2}}}dx$$

```
[In] int((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(3/2),x)
```

```
[Out] int((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(ex)^m (a+bx^4)^2}{(c+dx^4)^{3/2}} \, dx = \int \frac{(bx^4+a)^2 (ex)^m}{(dx^4+c)^{\frac{3}{2}}} \, dx$$

[In] integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="fricas")

[Out] integral((b^2*x^8 + 2*a*b*x^4 + a^2)*sqrt(d*x^4 + c)*(e*x)^m/(d^2*x^8 + 2*c*d*x^4 + c^2), x)

Sympy [F]

$$\int \frac{(ex)^m (a+bx^4)^2}{(c+dx^4)^{3/2}} \, dx = \int \frac{(ex)^m (a+bx^4)^2}{(c+dx^4)^{\frac{3}{2}}} \, dx$$

[In] integrate((e*x)**m*(b*x**4+a)**2/(d*x**4+c)**(3/2),x)

[Out] Integral((e*x)**m*(a + b*x**4)**2/(c + d*x**4)**(3/2), x)

Maxima [F]

$$\int \frac{(ex)^m (a+bx^4)^2}{(c+dx^4)^{3/2}} \, dx = \int \frac{(bx^4+a)^2 (ex)^m}{(dx^4+c)^{\frac{3}{2}}} \, dx$$

[In] integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="maxima")

[Out] integrate((b*x^4 + a)^2*(e*x)^m/(d*x^4 + c)^(3/2), x)

Giac [F]

$$\int \frac{(ex)^m (a+bx^4)^2}{(c+dx^4)^{3/2}} \, dx = \int \frac{(bx^4+a)^2 (ex)^m}{(dx^4+c)^{\frac{3}{2}}} \, dx$$

[In] integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="giac")

[Out] integrate((b*x^4 + a)^2*(e*x)^m/(d*x^4 + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^4)^2}{(c+dx^4)^{3/2}}\,dx = \int \frac{(e\,x)^m\,(b\,x^4+a)^2}{(d\,x^4+c)^{3/2}}\,dx$$

```
[In] int(((e*x)^m*(a + b*x^4)^2)/(c + d*x^4)^(3/2),x)
[Out] int(((e*x)^m*(a + b*x^4)^2)/(c + d*x^4)^(3/2), x)
```

3.848 $\int \frac{(ex)^m(a+bx^4)}{(c+dx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 132

$$\int \frac{(ex)^m\,(a+bx^4)}{(c+dx^4)^{3/2}}\,dx = -\frac{(bc-ad)(ex)^{1+m}}{2cde\sqrt{c+dx^4}} + \frac{(ad(1-m)+bc(1+m))(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{4},\frac{5+m}{4},-\frac{dx^4}{c}\right)}{2cde(1+m)\sqrt{c+dx^4}}$$

```
[Out] -1/2*(-a*d+b*c)*(e*x)^(1+m)/c/d/e/(d*x^4+c)^(1/2)+1/2*(a*d*(-m+1)+b*c*(1+m)
)*(e*x)^(1+m)*hypergeom([1/2, 1/4+1/4*m],[5/4+1/4*m],-d*x^4/c)*(1+d*x^4/c)^
(1/2)/c/d/e/(1+m)/(d*x^4+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {468, 372, 371}

$$\int \frac{(ex)^m\,(a+bx^4)}{(c+dx^4)^{3/2}}\,dx = \frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}(ad(1-m)+bc(m+1))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{4},\frac{m+5}{4},-\frac{dx^4}{c}\right)}{2cde(m+1)\sqrt{c+dx^4}} - \frac{(ex)^{m+1}(bc-ad)}{2cde\sqrt{c+dx^4}}$$

```
[In] Int[((e*x)^m*(a + b*x^4))/(c + d*x^4)^(3/2),x]
```

```
[Out] -1/2*((b*c - a*d)*(e*x)^(1 + m))/(c*d*e*Sqrt[c + d*x^4]) + ((a*d*(1 - m) +
b*c*(1 + m))*(e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1 +
m)/4, (5 + m)/4, -((d*x^4)/c)])/(2*c*d*e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(bc-ad)(ex)^{1+m}}{2cde\sqrt{c+dx^4}}+\frac{(-ad(-1+m)+bc(1+m))\int\frac{(ex)^m}{\sqrt{c+dx^4}}\,dx}{2cd}\\
&= -\frac{(bc-ad)(ex)^{1+m}}{2cde\sqrt{c+dx^4}}+\frac{\left((-ad(-1+m)+bc(1+m))\sqrt{1+\frac{dx^4}{c}}\right)\int\frac{(ex)^m}{\sqrt{1+\frac{dx^4}{c}}}\,dx}{2cd\sqrt{c+dx^4}}\\
&= -\frac{(bc-ad)(ex)^{1+m}}{2cde\sqrt{c+dx^4}}+\frac{(ad(1-m)+bc(1+m))(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{4};\frac{5+m}{4};-\frac{dx^4}{c}\right)}{2cde(1+m)\sqrt{c+dx^4}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 5.58 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.86

$$\int\frac{(ex)^m\left(a+bx^4\right)}{\left(c+dx^4\right)^{3/2}}\,dx=\frac{x(ex)^m\sqrt{1+\frac{dx^4}{c}}\left(a(5+m)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{4},\frac{5+m}{4},-\frac{dx^4}{c}\right)+b(1+m)x\right.}{c(1+m)(5+m)\sqrt{c+dx^4}}$$

```
[In] Integrate[((e*x)^m*(a + b*x^4))/(c + d*x^4)^(3/2),x]
```

```
[Out] (x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*(a*(5 + m)*Hypergeometric2F1[3/2, (1 + m)/4,
 (5 + m)/4, -((d*x^4)/c)] + b*(1 + m)*x^4*Hypergeometric2F1[3/2, (5 + m)/4,
 (9 + m)/4, -((d*x^4)/c)]))/(c*(1 + m)*(5 + m)*Sqrt[c + d*x^4])
```

Maple [F]

$$\int \frac{(ex)^m (b\,x^4+a)}{(d\,x^4+c)^{\frac{3}{2}}}dx$$

```
[In] int((e*x)^m*(b*x^4+a)/(d*x^4+c)^(3/2),x)
[Out] int((e*x)^m*(b*x^4+a)/(d*x^4+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(ex)^m (a+bx^4)}{(c+dx^4)^{3/2}}\,dx = \int \frac{(bx^4+a)(ex)^m}{(dx^4+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="fricas")
[Out] integral((b*x^4 + a)*sqrt(d*x^4 + c)*(e*x)^m/(d^2*x^8 + 2*c*d*x^4 + c^2), x
)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 17.87 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.89

$$\int \frac{(ex)^m (a+bx^4)}{(c+dx^4)^{3/2}}\,dx = \frac{ae^m x^{m+1}\Gamma\left(\frac{m}{4}+\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{m}{4}+\frac{1}{4}\\ \frac{m}{4}+\frac{5}{4}\end{matrix}\right|\frac{dx^4e^{i\pi}}{c}\right)}{4c^{\frac{3}{2}}\Gamma\left(\frac{m}{4}+\frac{5}{4}\right)} + \frac{be^m x^{m+5}\Gamma\left(\frac{m}{4}+\frac{5}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{m}{4}+\frac{5}{4}\\ \frac{m}{4}+\frac{9}{4}\end{matrix}\right|\frac{dx^4e^{i\pi}}{c}\right)}{4c^{\frac{3}{2}}\Gamma\left(\frac{m}{4}+\frac{9}{4}\right)}$$

```
[In] integrate((e*x)**m*(b*x**4+a)/(d*x**4+c)**(3/2),x)
[Out] a*e**m*x**(m + 1)*gamma(m/4 + 1/4)*hyper((3/2, m/4 + 1/4), (m/4 + 5/4,), d*
x**4*exp_polar(I*pi)/c)/(4*c**(3/2)*gamma(m/4 + 5/4)) + b*e**m*x**(m + 5)*g
amma(m/4 + 5/4)*hyper((3/2, m/4 + 5/4), (m/4 + 9/4,), d*x**4*exp_polar(I*pi
)/c)/(4*c**(3/2)*gamma(m/4 + 9/4))
```

Maxima [F]

$$\int \frac{(ex)^m (a + bx^4)}{(c + dx^4)^{3/2}}\, dx = \int \frac{(bx^4 + a)(ex)^m}{(dx^4 + c)^{\frac{3}{2}}}\, dx$$

```
[In] integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="maxima")
[Out] integrate((b*x^4 + a)*(e*x)^m/(d*x^4 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(ex)^m (a + bx^4)}{(c + dx^4)^{3/2}}\, dx = \int \frac{(bx^4 + a)(ex)^m}{(dx^4 + c)^{\frac{3}{2}}}\, dx$$

```
[In] integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^4 + a)*(e*x)^m/(d*x^4 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a + bx^4)}{(c + dx^4)^{3/2}}\, dx = \int \frac{(e\,x)^m\,(b\,x^4 + a)}{(d\,x^4 + c)^{3/2}}\, dx$$

```
[In] int(((e*x)^m*(a + b*x^4))/(c + d*x^4)^(3/2),x)
[Out] int(((e*x)^m*(a + b*x^4))/(c + d*x^4)^(3/2), x)
```

3.849 $\int \frac{(ex)^m}{(c+dx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 71

$$\int \frac{(ex)^m}{(c+dx^4)^{3/2}}\,dx = \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{4},\frac{5+m}{4},-\frac{dx^4}{c}\right)}{ce(1+m)\sqrt{c+dx^4}}$$

```
[Out] (e*x)^(1+m)*hypergeom([3/2, 1/4+1/4*m],[5/4+1/4*m],-d*x^4/c)*(1+d*x^4/c)^(1
/2)/c/e/(1+m)/(d*x^4+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {372, 371}

$$\int \frac{(ex)^m}{(c+dx^4)^{3/2}}\,dx = \frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{4},\frac{m+5}{4},-\frac{dx^4}{c}\right)}{ce(m+1)\sqrt{c+dx^4}}$$

```
[In] Int[(e*x)^m/(c + d*x^4)^(3/2),x]
```

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[3/2, (1 + m)/4, (5 + m
)/4, -((d*x^4)/c)])/(c*e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^4}{c}}\int \frac{(ex)^m}{\left(1+\frac{dx^4}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^4}}$$

$$= \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,{}_2F_1\left(\frac{3}{2},\frac{1+m}{4};\frac{5+m}{4};-\frac{dx^4}{c}\right)}{ce(1+m)\sqrt{c+dx^4}}$$

Mathematica [A] (verified)

Time = 1.93 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

$$\int \frac{(ex)^m}{(c+dx^4)^{3/2}}\,dx = \frac{x(ex)^m\sqrt{1+\frac{dx^4}{c}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{4},1+\frac{1+m}{4},-\frac{dx^4}{c}\right)}{c(1+m)\sqrt{c+dx^4}}$$

```
[In] Integrate[(e*x)^m/(c + d*x^4)^(3/2),x]
[Out] (x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[3/2, (1 + m)/4, 1 + (1 + m
)/4, -((d*x^4)/c)])/(c*(1 + m)*Sqrt[c + d*x^4])
```

Maple [F]

$$\int \frac{(ex)^m}{(d\,x^4+c)^{\frac{3}{2}}}dx$$

```
[In] int((e*x)^m/(d*x^4+c)^(3/2),x)
[Out] int((e*x)^m/(d*x^4+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(ex)^m}{(c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(dx^4+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x)^m/(d*x^4+c)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(d*x^4 + c)*(e*x)^m/(d^2*x^8 + 2*c*d*x^4 + c^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

$$\int \frac{(ex)^m}{(c+dx^4)^{3/2}}\,dx = \frac{e^m x^{m+1}\Gamma\left(\frac{m}{4}+\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{4}+\frac{1}{4}\\ \frac{m}{4}+\frac{5}{4}\end{matrix}\middle|\frac{dx^4e^{i\pi}}{c}\right)}{4c^{\frac{3}{2}}\Gamma\left(\frac{m}{4}+\frac{5}{4}\right)}$$

```
[In] integrate((e*x)**m/(d*x**4+c)**(3/2),x)
[Out] e**m*x**(m + 1)*gamma(m/4 + 1/4)*hyper((3/2, m/4 + 1/4), (m/4 + 5/4,), d*x*
*4*exp_polar(I*pi)/c)/(4*c**(3/2)*gamma(m/4 + 5/4))
```

Maxima [F]

$$\int \frac{(ex)^m}{(c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(dx^4+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x)^m/(d*x^4+c)^(3/2),x, algorithm="maxima")
[Out] integrate((e*x)^m/(d*x^4 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(ex)^m}{(c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(dx^4+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x)^m/(d*x^4+c)^(3/2),x, algorithm="giac")
[Out] integrate((e*x)^m/(d*x^4 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(c+dx^4)^{3/2}}\,dx = \int \frac{(e\,x)^m}{(d\,x^4+c)^{3/2}}\,dx$$

```
[In] int((e*x)^m/(c + d*x^4)^(3/2),x)
[Out] int((e*x)^m/(c + d*x^4)^(3/2), x)
```

3.850 $\int \frac{(ex)^m}{(a+bx^4)(c+dx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 84

$$\int \frac{(ex)^m}{(a+bx^4)(c+dx^4)^{3/2}}\,dx = \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{1+m}{4},1,\frac{3}{2},\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ace(1+m)\sqrt{c+dx^4}}$$

```
[Out] (e*x)^(1+m)*AppellF1(1/4+1/4*m,1,3/2,5/4+1/4*m,-b*x^4/a,-d*x^4/c)*(1+d*x^4/
c)^(1/2)/a/c/e/(1+m)/(d*x^4+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {525, 524}

$$\int \frac{(ex)^m}{(a+bx^4)(c+dx^4)^{3/2}}\,dx = \frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\,\text{AppellF1}\left(\frac{m+1}{4},1,\frac{3}{2},\frac{m+5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ace(m+1)\sqrt{c+dx^4}}$$

```
[In] Int[(e*x)^m/((a + b*x^4)*(c + d*x^4)^(3/2)),x]
```

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 1, 3/2, (5 + m)/4, -
((b*x^4)/a), -((d*x^4)/c)])/(a*c*e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{1+\frac{dx^4}{c}}\int \frac{(ex)^m}{(a+bx^4)\left(1+\frac{dx^4}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^4}}\\ &= \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}F_1\left(\frac{1+m}{4};1,\frac{3}{2};\frac{5+m}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ace(1+m)\sqrt{c+dx^4}}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 169 vs. 2(84) = 168.

Time = 11.19 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.01

$$\int \frac{(ex)^m}{(a+bx^4)(c+dx^4)^{3/2}}\,dx = \frac{x(ex)^m\sqrt{c+dx^4}\left(b^2c^2\,\text{AppellF1}\left(\frac{1+m}{4},-\frac{1}{2},1,\frac{5+m}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+ad\left(-bc\,\text{Hy}\right.\right.}{ac^2(b}$$

```
[In] Integrate[(e*x)^m/((a + b*x^4)*(c + d*x^4)^(3/2)),x]
```

```
[Out] (x*(e*x)^m*Sqrt[c + d*x^4]*(b^2*c^2*AppellF1[(1 + m)/4, -1/2, 1, (5 + m)/4,
 -((d*x^4)/c), -((b*x^4)/a)] + a*d*(-(b*c*Hypergeometric2F1[1/2, (1 + m)/4,
 (5 + m)/4, -((d*x^4)/c)]) + (-(b*c) + a*d)*Hypergeometric2F1[3/2, (1 + m)/
4, (5 + m)/4, -((d*x^4)/c)])))/(a*c^2*(b*c - a*d)^2*(1 + m)*Sqrt[1 + (d*x^4
)/c])
```

Maple [F]

$$\int \frac{(ex)^m}{(b\,x^4+a)\,(d\,x^4+c)^{\frac{3}{2}}}dx$$

[In] int((e*x)^m/(b*x^4+a)/(d*x^4+c)^(3/2),x)

[Out] int((e*x)^m/(b*x^4+a)/(d*x^4+c)^(3/2),x)

Fricas [F]

$$\int \frac{(ex)^m}{(a+bx^4)\,(c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(bx^4+a)(dx^4+c)^{\frac{3}{2}}}\,dx$$

[In] integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(d*x^4 + c)*(e*x)^m/(b*d^2*x^12 + (2*b*c*d + a*d^2)*x^8 + (b*c ^2 + 2*a*c*d)*x^4 + a*c^2), x)

Sympy [F]

$$\int \frac{(ex)^m}{(a+bx^4)\,(c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(a+bx^4)\,(c+dx^4)^{\frac{3}{2}}}\,dx$$

[In] integrate((e*x)**m/(b*x**4+a)/(d*x**4+c)**(3/2),x)

[Out] Integral((e*x)**m/((a + b*x**4)*(c + d*x**4)**(3/2)), x)

Maxima [F]

$$\int \frac{(ex)^m}{(a+bx^4)\,(c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(bx^4+a)(dx^4+c)^{\frac{3}{2}}}\,dx$$

[In] integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="maxima")

[Out] integrate((e*x)^m/((b*x^4 + a)*(d*x^4 + c)^(3/2)), x)

Giac [F]

$$\int \frac{(ex)^m}{(a+bx^4)(c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(bx^4+a)(dx^4+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="giac")
[Out] integrate((e*x)^m/((b*x^4 + a)*(d*x^4 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^4)(c+dx^4)^{3/2}}\,dx = \int \frac{(e\,x)^m}{(b\,x^4+a)\,(d\,x^4+c)^{3/2}}\,dx$$

```
[In] int((e*x)^m/((a + b*x^4)*(c + d*x^4)^(3/2)),x)
[Out] int((e*x)^m/((a + b*x^4)*(c + d*x^4)^(3/2)), x)
```

3.851 $\int \frac{(ex)^m}{(a+bx^4)^2(c+dx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 84

$$\int \frac{(ex)^m}{(a+bx^4)^2(c+dx^4)^{3/2}}\,dx = \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{1+m}{4},2,\frac{3}{2},\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2ce(1+m)\sqrt{c+dx^4}}$$

```
[Out] (e*x)^(1+m)*AppellF1(1/4+1/4*m,2,3/2,5/4+1/4*m,-b*x^4/a,-d*x^4/c)*(1+d*x^4/
c)^(1/2)/a^2/c/e/(1+m)/(d*x^4+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {525, 524}

$$\int \frac{(ex)^m}{(a+bx^4)^2(c+dx^4)^{3/2}}\,dx = \frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\,\text{AppellF1}\left(\frac{m+1}{4},2,\frac{3}{2},\frac{m+5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2ce(m+1)\sqrt{c+dx^4}}$$

```
[In] Int[(e*x)^m/((a + b*x^4)^2*(c + d*x^4)^(3/2)),x]
```

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 2, 3/2, (5 + m)/4, -
((b*x^4)/a), -((d*x^4)/c)])/(a^2*c*e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^4}{c}}\int \frac{(ex)^m}{(a+bx^4)^2\left(1+\frac{dx^4}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^4}}$$

$$= \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}F_1\left(\frac{1+m}{4};2,\frac{3}{2};\frac{5+m}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2ce(1+m)\sqrt{c+dx^4}}$$

Mathematica [A] (verified)

Time = 11.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

$$\int \frac{(ex)^m}{(a+bx^4)^2\,(c+dx^4)^{3/2}}\,dx = \frac{x(ex)^m\left(1+\frac{dx^4}{c}\right)^{3/2}\text{AppellF1}\left(\frac{1+m}{4},2,\frac{3}{2},\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2(1+m)\,(c+dx^4)^{3/2}}$$

```
[In] Integrate[(e*x)^m/((a + b*x^4)^2*(c + d*x^4)^(3/2)),x]
[Out] (x*(e*x)^m*(1 + (d*x^4)/c)^(3/2)*AppellF1[(1 + m)/4, 2, 3/2, (5 + m)/4, -((
b*x^4)/a), -((d*x^4)/c)])/(a^2*(1 + m)*(c + d*x^4)^(3/2))
```

Maple [F]

$$\int \frac{(ex)^m}{(b\,x^4+a)^2\,(d\,x^4+c)^{\frac{3}{2}}}dx$$

```
[In] int((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(3/2),x)
[Out] int((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(ex)^m}{(a+bx^4)^2(c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(bx^4+a)^2(dx^4+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(d*x^4 + c)*(e*x)^m/(b^2*d^2*x^16 + 2*(b^2*c*d + a*b*d^2)*x^12
 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^8 + 2*(a*b*c^2 + a^2*c*d)*x^4 + a^2*c^
2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^4)^2(c+dx^4)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*x)**m/(b*x**4+a)**2/(d*x**4+c)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ex)^m}{(a+bx^4)^2(c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(bx^4+a)^2(dx^4+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="maxima")
[Out] integrate((e*x)^m/((b*x^4 + a)^2*(d*x^4 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{(ex)^m}{(a+bx^4)^2(c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(bx^4+a)^2(dx^4+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="giac")
[Out] integrate((e*x)^m/((b*x^4 + a)^2*(d*x^4 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^4)^2\,(c+dx^4)^{3/2}}\,dx = \int \frac{(e\,x)^m}{(b\,x^4+a)^2\,(d\,x^4+c)^{3/2}}\,dx$$

```
[In] int((e*x)^m/((a + b*x^4)^2*(c + d*x^4)^(3/2)),x)
[Out] int((e*x)^m/((a + b*x^4)^2*(c + d*x^4)^(3/2)), x)
```

3.852 $\int \frac{(ex)^m}{(a+bx^4)^3 (c+dx^4)^{3/2}} dx$

Optimal result

Integrand size = 26, antiderivative size = 84

$$\int \frac{(ex)^m}{(a+bx^4)^3 (c+dx^4)^{3/2}}\, dx = \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{1+m}{4},3,\frac{3}{2},\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3ce(1+m)\sqrt{c+dx^4}}$$

```
[Out] (e*x)^(1+m)*AppellF1(1/4+1/4*m,3,3/2,5/4+1/4*m,-b*x^4/a,-d*x^4/c)*(1+d*x^4/
c)^(1/2)/a^3/c/e/(1+m)/(d*x^4+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {525, 524}

$$\int \frac{(ex)^m}{(a+bx^4)^3 (c+dx^4)^{3/2}}\, dx = \frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\,\text{AppellF1}\left(\frac{m+1}{4},3,\frac{3}{2},\frac{m+5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3ce(m+1)\sqrt{c+dx^4}}$$

```
[In] Int[(e*x)^m/((a + b*x^4)^3*(c + d*x^4)^(3/2)),x]
```

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 3, 3/2, (5 + m)/4, -
((b*x^4)/a), -((d*x^4)/c)])/(a^3*c*e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^4}{c}}\int \frac{(ex)^m}{(a+bx^4)^3\left(1+\frac{dx^4}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^4}}$$
$$= \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}F_1\left(\frac{1+m}{4};3,\frac{3}{2};\frac{5+m}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3ce(1+m)\sqrt{c+dx^4}}$$

Mathematica [A] (verified)

Time = 11.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

$$\int \frac{(ex)^m}{(a+bx^4)^3\,(c+dx^4)^{3/2}}\,dx = \frac{x(ex)^m\left(1+\frac{dx^4}{c}\right)^{3/2}\text{AppellF1}\left(\frac{1+m}{4},3,\frac{3}{2},\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3(1+m)\,(c+dx^4)^{3/2}}$$

```
[In] Integrate[(e*x)^m/((a + b*x^4)^3*(c + d*x^4)^(3/2)),x]
[Out] (x*(e*x)^m*(1 + (d*x^4)/c)^(3/2)*AppellF1[(1 + m)/4, 3, 3/2, (5 + m)/4, -((
b*x^4)/a), -((d*x^4)/c)])/(a^3*(1 + m)*(c + d*x^4)^(3/2))
```

Maple [F]

$$\int \frac{(ex)^m}{(b\,x^4+a)^3\,(d\,x^4+c)^{\frac{3}{2}}}dx$$

```
[In] int((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(3/2),x)
[Out] int((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(ex)^m}{(a+bx^4)^3\,(c+dx^4)^{3/2}}\,dx=\int \frac{(ex)^m}{(bx^4+a)^3(dx^4+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^4 + c)*(e*x)^m/(b^3*d^2*x^20 + (2*b^3*c*d + 3*a*b^2*d^2)*
x^16 + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^12 + (3*a*b^2*c^2 + 6*a^2*b*
c*d + a^3*d^2)*x^8 + a^3*c^2 + (3*a^2*b*c^2 + 2*a^3*c*d)*x^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^4)^3\,(c+dx^4)^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate((e*x)**m/(b*x**4+a)**3/(d*x**4+c)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ex)^m}{(a+bx^4)^3\,(c+dx^4)^{3/2}}\,dx=\int \frac{(ex)^m}{(bx^4+a)^3(dx^4+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m/((b*x^4 + a)^3*(d*x^4 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{(ex)^m}{(a+bx^4)^3\,(c+dx^4)^{3/2}}\,dx=\int \frac{(ex)^m}{(bx^4+a)^3(dx^4+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/((b*x^4 + a)^3*(d*x^4 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^4)^3\,(c+dx^4)^{3/2}}\,dx = \int \frac{(e\,x)^m}{(b\,x^4+a)^3\,(d\,x^4+c)^{3/2}}\,dx$$

```
[In] int((e*x)^m/((a + b*x^4)^3*(c + d*x^4)^(3/2)),x)
[Out] int((e*x)^m/((a + b*x^4)^3*(c + d*x^4)^(3/2)), x)
```

3.853 $\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = -\frac{(bc+ad)\sqrt{c+dx^6}}{3b^2d^2} + \frac{(c+dx^6)^{3/2}}{9bd^2} - \frac{a^2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}$$

```
[Out] 1/9*(d*x^6+c)^(3/2)/b/d^2-1/3*a^2*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c
)^(1/2))/b^(5/2)/(-a*d+b*c)^(1/2)-1/3*(a*d+b*c)*(d*x^6+c)^(1/2)/b^2/d^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {457, 90, 65, 214}

$$\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = -\frac{a^2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}} - \frac{\sqrt{c+dx^6}(ad+bc)}{3b^2d^2} + \frac{(c+dx^6)^{3/2}}{9bd^2}$$

```
[In] Int[x^17/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/3*((b*c + a*d)*Sqrt[c + d*x^6])/(b^2*d^2) + (c + d*x^6)^(3/2)/(9*b*d^2)
- (a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(3*b^(5/2)*Sqrt[
b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{6}\text{Subst}\left(\int \frac{x^2}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)\\
&= \frac{1}{6}\text{Subst}\left(\int \left(\frac{-bc-ad}{b^2d\sqrt{c+dx}}+\frac{a^2}{b^2(a+bx)\sqrt{c+dx}}+\frac{\sqrt{c+dx}}{bd}\right)\,dx, x, x^6\right)\\
&= -\frac{(bc+ad)\sqrt{c+dx^6}}{3b^2d^2}+\frac{\left(c+dx^6\right)^{3/2}}{9bd^2}+\frac{a^2\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{6b^2}\\
&= -\frac{(bc+ad)\sqrt{c+dx^6}}{3b^2d^2}+\frac{\left(c+dx^6\right)^{3/2}}{9bd^2}+\frac{a^2\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{3b^2d}\\
&= -\frac{(bc+ad)\sqrt{c+dx^6}}{3b^2d^2}+\frac{\left(c+dx^6\right)^{3/2}}{9bd^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{\sqrt{c+dx^6}(-2bc-3ad+bdx^6)}{9b^2d^2} + \frac{a^2\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{3b^{5/2}\sqrt{-bc+ad}}$$

```
[In] Integrate[x^17/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (Sqrt[c + d*x^6]*(-2*b*c - 3*a*d + b*d*x^6))/(9*b^2*d^2) + (a^2*ArcTan[(Sqr
t[b]*Sqrt[c + d*x^6])/Sqrt[-(b*c) + a*d]])/(3*b^(5/2)*Sqrt[-(b*c) + a*d])
```

Maple [A] (verified)

Time = 7.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

method	result	size
pseudoelliptic	$\frac{a^2\arctan\left(\frac{b\sqrt{d\,x^6+c}}{\sqrt{(ad-bc)b}}\right)d^2-\left(\left(-\frac{b\,x^6}{3}+a\right)d+\frac{2bc}{3}\right)\sqrt{d\,x^6+c}\,\sqrt{(ad-bc)b}}{3\sqrt{(ad-bc)b}\,b^2d^2}$	91

```
[In] int(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(a^2*arctan(b*(d*x^6+c)^(1/2)/((a*d-b*c)*b)^(1/2))*d^2-((-1/3*b*x^6+a)*
d+2/3*b*c)*(d*x^6+c)^(1/2)*((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)/b^2/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.53 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.77

$$\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \left[\frac{3\sqrt{b^2c-abd}a^2d^2\log\left(\frac{bdx^6+2\,bc-ad-2\,\sqrt{dx^6+c}\sqrt{b^2c-abd}}{bx^6+a}\right)+2\left((b^3cd-ab^2d^2)x^6-2\,b^3c^2-ab^2cd+3\,a^2bd^2\right)\sqrt{dx}}{18\,(b^4cd^2-ab^3d^3)}\right.$$

```
[In] integrate(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt(d*
x^6 + c)*sqrt(b^2*c - a*b*d))/(b*x^6 + a)) + 2*((b^3*c*d - a*b^2*d^2)*x^6 -
 2*b^3*c^2 - a*b^2*c*d + 3*a^2*b*d^2)*sqrt(d*x^6 + c))/(b^4*c*d^2 - a*b^3*d
^3), 1/9*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(sqrt(d*x^6 + c)*sqrt(-b^2*c
 + a*b*d)/(b*d*x^6 + b*c)) + ((b^3*c*d - a*b^2*d^2)*x^6 - 2*b^3*c^2 - a*b^2
*c*d + 3*a^2*b*d^2)*sqrt(d*x^6 + c))/(b^4*c*d^2 - a*b^3*d^3)]
```

Sympy [F]

$$\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

```
[In] integrate(x**17/(b*x**6+a)/(d*x**6+c)**(1/2),x)
[Out] Integral(x**17/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.02

$$\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{a^2\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}b^2} + \frac{(dx^6+c)^{\frac{3}{2}}b^2d^4 - 3\sqrt{dx^6+c}b^2cd^4 - 3\sqrt{dx^6+c}abd^5}{9\,b^3d^6}$$

```
[In] integrate(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] 1/3*a^2*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*b^2) + 1/9*((d*x^6 + c)^(3/2)*b^2*d^4 - 3*sqrt(d*x^6 + c)*b^2*c*d^4 - 3*s
qrt(d*x^6 + c)*a*b*d^5)/(b^3*d^6)
```

Mupad [B] (verification not implemented)

Time = 9.19 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.99

$$\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{(d\,x^6+c)^{3/2}}{9\,b\,d^2} - \left(\frac{2\,c}{3\,b\,d^2}+\frac{3\,a\,d^3-3\,b\,c\,d^2}{9\,b^2\,d^4}\right)\sqrt{d\,x^6+c}+\frac{a^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^6+c}}{\sqrt{a\,d-b\,c}}\right)}{3\,b^{5/2}\,\sqrt{a\,d-b\,c}}$$

```
[In] int(x^17/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] (c + d*x^6)^(3/2)/(9*b*d^2) - ((2*c)/(3*b*d^2) + (3*a*d^3 - 3*b*c*d^2)/(9*b
^2*d^4))*(c + d*x^6)^(1/2) + (a^2*atan((b^(1/2)*(c + d*x^6)^(1/2))/(a*d - b
*c)^(1/2)))/(3*b^(5/2)*(a*d - b*c)^(1/2))
```

3.854 $\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{\sqrt{c+dx^6}}{3bd} + \frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}$$

```
[Out] 1/3*a*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)^
(1/2)+1/3*(d*x^6+c)^(1/2)/b/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {457, 81, 65, 214}

$$\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}} + \frac{\sqrt{c+dx^6}}{3bd}$$

```
[In] Int[x^11/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] Sqrt[c + d*x^6]/(3*b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a
*d]])/(3*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{6}\text{Subst}\left(\int \frac{x}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right) \\
&= \frac{\sqrt{c+dx^6}}{3bd} - \frac{a\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{6b} \\
&= \frac{\sqrt{c+dx^6}}{3bd} - \frac{a\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{3bd} \\
&= \frac{\sqrt{c+dx^6}}{3bd} + \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{1}{3}\left(\frac{\sqrt{c+dx^6}}{bd} - \frac{a\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{b^{3/2}\sqrt{-bc+ad}}\right)$$

```
[In] Integrate[x^11/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (Sqrt[c + d*x^6]/(b*d) - (a*ArcTan[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[-(b*c) +
a*d]])/(b^(3/2)*Sqrt[-(b*c) + a*d]))/3
```

Maple [A] (verified)

Time = 5.40 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

method	result	size
pseudoelliptic	$\frac{-a\arctan\left(\frac{b\sqrt{d\,x^6+c}}{\sqrt{(ad-bc)b}}\right)d+\sqrt{d\,x^6+c}\,\sqrt{(ad-bc)b}}{3bd\sqrt{(ad-bc)b}}$	72

```
[In] int(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(-a*arctan(b*(d*x^6+c)^(1/2)/((a*d-b*c)*b)^(1/2))*d+(d*x^6+c)^(1/2)*((a
*d-b*c)*b)^(1/2))/b/d/((a*d-b*c)*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.77

$$\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \left[\frac{\sqrt{b^2c-abd}ad\log\left(\frac{bdx^6+2\,bc-ad+2\,\sqrt{dx^6+c}\sqrt{b^2c-abd}}{bx^6+a}\right)+2\,\sqrt{dx^6+c}(b^2c-abd)}{6\,(b^3cd-ab^2d^2)}, -\frac{\sqrt{-b^2c+abd}ad\arctan\left(\frac{\sqrt{dx^6+c}\sqrt{-b^2c+abd}}{bdx^6+bc}\right)-\sqrt{dx^6+c}(b^2c-abd)}{3\,(b^3cd-ab^2d^2)}\right]$$

```
[In] integrate(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(b^2*c - a*b*d)*a*d*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 + c
)*sqrt(b^2*c - a*b*d))/(b*x^6 + a)) + 2*sqrt(d*x^6 + c)*(b^2*c - a*b*d))/(b
```

```
^3*c*d - a*b^2*d^2), -1/3*(sqrt(-b^2*c + a*b*d)*a*d*arctan(sqrt(d*x^6 + c)*
sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c)) - sqrt(d*x^6 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2)]
```

Sympy [F]

$$\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

```
[In] integrate(x**11/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x**11/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = -\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b} - \frac{\sqrt{dx^6+c}}{b}}{3\,d}$$

```
[In] integrate(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*(a*d*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b
*d)*b) - sqrt(d*x^6 + c)/b)/d
```

Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{\sqrt{d\,x^6+c}}{3\,b\,d} - \frac{a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^6+c}}{\sqrt{a\,d-b\,c}}\right)}{3\,b^{3/2}\,\sqrt{a\,d-b\,c}}$$

```
[In] int(x^11/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] (c + d*x^6)^(1/2)/(3*b*d) - (a*atan((b^(1/2)*(c + d*x^6)^(1/2))/(a*d - b*c)
^(1/2)))/(3*b^(3/2)*(a*d - b*c)^(1/2))
```

3.855 $\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

```
[Out] -1/3*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(1/2)/(-a*d+b*c)^(
1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {455, 65, 214}

$$\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

```
[In] Int[x^5/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/3*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]]/(Sqrt[b]*Sqrt[b*c -
 a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{6}\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{3d}\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{3\sqrt{b}\sqrt{-bc+ad}}$$

```
[In] Integrate[x^5/((a + b*x^6)*Sqrt[c + d*x^6]),x]
[Out] ArcTan[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[-(b*c) + a*d]]/(3*Sqrt[b]*Sqrt[-(b*c)
 + a*d])
```

Maple [A] (verified)

Time = 5.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

method	result	size
pseudoelliptic	$\frac{\arctan\left(\frac{b\sqrt{d\,x^{6}+c}}{\sqrt{(ad-bc)b}}\right)}{3\sqrt{(ad-bc)b}}$	39

```
[In] int(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^6+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.55

$$\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \left[\frac{\log\left(\frac{bdx^6+2\,bc-ad-2\,\sqrt{dx^6+c}\sqrt{b^2c-abd}}{bx^6+a}\right)}{6\,\sqrt{b^2c-abd}}, \frac{\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^6+c}\sqrt{-b^2c+abd}}{bdx^6+bc}\right)}{3\,(b^2c-abd)}\right]$$

```
[In] integrate(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*sqrt(b^2*c - a*b*d))/(b
*x^6 + a))/sqrt(b^2*c - a*b*d), 1/3*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^6
+ c)*sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c))/(b^2*c - a*b*d)]
```

Sympy [A] (verification not implemented)

Time = 8.63 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.71

$$\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \begin{cases} \frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^6}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b\sqrt{\frac{ad-bc}{b}}} & \text{for}\ d\neq 0 \\ \begin{cases} \frac{x^6}{6a\sqrt{c}} & \text{for}\ b=0 \\ \tilde{\infty}x^6 & \text{for}\ \sqrt{c}=0 \\ \frac{\log\left(6a\sqrt{c}+6b\sqrt{c}x^6\right)}{6b\sqrt{c}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**5/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Piecewise((atan(sqrt(c + d*x**6)/sqrt((a*d - b*c)/b))/(3*b*sqrt((a*d - b*c)
/b)), Ne(d, 0)), (Piecewise((x**6/(6*a*sqrt(c)), Eq(b, 0)), (zoo*x**6, Eq(s
qrt(c), 0)), (log(6*a*sqrt(c) + 6*b*sqrt(c)*x**6)/(6*b*sqrt(c)), True)), Tr
ue))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}}$$

```
[In] integrate(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{\mathrm{atan}\left(\frac{b\sqrt{d\,x^6+c}}{\sqrt{a\,b\,d-b^2\,c}}\right)}{3\sqrt{a\,b\,d-b^2\,c}}$$

```
[In] int(x^5/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] atan((b*(c + d*x^6)^(1/2))/(a*b*d - b^2*c)^(1/2))/(3*(a*b*d - b^2*c)^(1/2))
```

3.856 $\int \frac{1}{x(a+bx^6)\sqrt{c+dx^6}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 85

$$\int \frac{1}{x\,(a+bx^6)\,\sqrt{c+dx^6}} \, dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a\sqrt{c}} + \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}$$

```
[Out] -1/3*arctanh((d*x^6+c)^(1/2)/c^(1/2))/a/c^(1/2)+1/3*arctanh(b^(1/2)*(d*x^6+
c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/a/(-a*d+b*c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {457, 88, 65, 214}

$$\int \frac{1}{x\,(a+bx^6)\,\sqrt{c+dx^6}} \, dx = \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a\sqrt{c}}$$

```
[In] Int[1/(x*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/3*ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]]/(a*Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[
b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(3*a*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{6}\text{Subst}\left(\int \frac{1}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^6\right)}{6a} - \frac{b\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{6a}\\
&= \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{3ad} - \frac{b\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{3ad}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a\sqrt{c}} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = -\frac{\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a}$$

```
[In] Integrate[1/(x*(a + b*x^6)*Sqrt[c + d*x^6]),x]
[Out] -1/3*((Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[-(b*c) + a*d]])/Sqrt[-
(b*c) + a*d] + ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]]/Sqrt[c])/a
```

Maple [A] (verified)

Time = 5.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

method	result	size
pseudoelliptic	$-\frac{b \arctan\left(\frac{b\sqrt{d\,x^{6}+c}}{\sqrt{(a d-b c) b}}\right)\sqrt{c}+\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{6}+c}}{\sqrt{c}}\right)\sqrt{(a d-b c) b}}{3 a \sqrt{(a d-b c) b}\,\sqrt{c}}$	78

[In] int(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/3*(b*arctan(b*(d*x^6+c)^(1/2)/((a*d-b*c)*b)^(1/2))*c^(1/2)+arctanh((d*x^6+c)^(1/2)/c^(1/2))*((a*d-b*c)*b)^(1/2))/a/((a*d-b*c)*b)^(1/2)/c^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 431, normalized size of antiderivative = 5.07

$$\int \frac{1}{x\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = \left[\frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^6+2\,bc-ad+2\,\sqrt{dx^6+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^6+a}\right)+\sqrt{c}\log\left(\frac{dx^6-2\,\sqrt{dx^6+c}\sqrt{c}+2\,c}{x^6}\right)}{6\,ac}, \frac{2\,c\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx}}{}\right.}{}\right.$$

[In] integrate(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")

[Out] [1/6*(c*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + sqrt(c)*log((d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6))/(a*c), 1/6*(2*c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^6 + b*c)) + sqrt(c)*log((d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6))/(a*c), 1/6*(c*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + 2*sqrt(-c)*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c))/(a*c), 1/3*(c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^6 + b*c)) + sqrt(-c)*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c))/(a*c)]

Sympy [A] (verification not implemented)

Time = 6.99 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{1}{x\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=\begin{cases}\frac{2\left(-\frac{d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^6}}{\sqrt{\frac{ad-bc}{b}}}\right)}{6a\sqrt{\frac{ad-bc}{b}}}+\frac{d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^6}}{\sqrt{-c}}\right)}{6a\sqrt{-c}}\right)}{d} & \text{for } d\neq 0\\ \frac{\mathrm{atan}\left(\frac{2\left(\frac{a}{2b}+x^6\right)}{\sqrt{-\frac{a^2}{b^2}}}\right)}{3b\sqrt{c}\sqrt{-\frac{a^2}{b^2}}} & \text{otherwise}\end{cases}$$

```
[In] integrate(1/x/(b*x**6+a)/(d*x**6+c)**(1/2),x)
[Out] Piecewise((2*(-d*atan(sqrt(c + d*x**6)/sqrt((a*d - b*c)/b))/(6*a*sqrt((a*d
- b*c)/b)) + d*atan(sqrt(c + d*x**6)/sqrt(-c))/(6*a*sqrt(-c)))/d, Ne(d, 0))
, (atan(2*(a/(2*b) + x**6)/sqrt(-a**2/b**2))/(3*b*sqrt(c)*sqrt(-a**2/b**2))
, True))
```

Maxima [F]

$$\int \frac{1}{x\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=\int \frac{1}{\left(bx^6+a\right)\sqrt{dx^6+c}\,x}\,dx$$

```
[In] integrate(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int \frac{1}{x\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=-\frac{b\arctan\left(\frac{\sqrt{dx^6+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\,a}+\frac{\arctan\left(\frac{\sqrt{dx^6+c}}{\sqrt{-c}}\right)}{3\,a\sqrt{-c}}$$

```
[In] integrate(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] -1/3*b*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*a) + 1/3*arctan(sqrt(d*x^6 + c)/sqrt(-c))/(a*sqrt(-c))
```

Mupad [B] (verification not implemented)

Time = 9.30 (sec) , antiderivative size = 652, normalized size of antiderivative = 7.67

$$\int \frac{1}{x\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = -\frac{\operatorname{atanh}\left(\frac{\sqrt{d\,x^6+c}}{\sqrt{c}}\right)}{3\,a\,\sqrt{c}} - \frac{\operatorname{atan}\left(\frac{\frac{\sqrt{b^2\,c-a\,b\,d}\left(\frac{2\,b^3\,d^2\,\sqrt{d\,x^6+c}}{27}-\frac{\sqrt{b^2\,c-a\,b\,d}\left(\frac{2\,a^2\,b^2\,d^3}{9}-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\sqrt{d\,x^6+c}\,\sqrt{b^2\,c-a\,b\,d}}{36\left(a^2\,d-a\,b\,c\right)}\right)}{6\left(a^2\,d-a\,b\,c\right)}\right)1\mathrm{i}}{a^2\,d-a\,b\,c}+\frac{\sqrt{b^2\,c-a\,b\,d}\left(\frac{2\,b^3\,d^2\,\sqrt{d\,x^6+c}}{27}+\cdots\right.}{\cdots}}{\frac{\sqrt{b^2\,c-a\,b\,d}\left(\frac{2\,b^3\,d^2\,\sqrt{d\,x^6+c}}{27}-\frac{\sqrt{b^2\,c-a\,b\,d}\left(\frac{2\,a^2\,b^2\,d^3}{9}-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\sqrt{d\,x^6+c}\,\sqrt{b^2\,c-a\,b\,d}}{36\left(a^2\,d-a\,b\,c\right)}\right)}{6\left(a^2\,d-a\,b\,c\right)}\right)}{a^2\,d-a\,b\,c}-\frac{\sqrt{b^2\,c-a\,b\,d}\left(\frac{2\,b^3\,d^2\,\sqrt{d\,x^6+c}}{27}+\cdots\right.}{\cdots}}\right)\cdots}{3\left(a^2\,d-a\,b\,c\right)}$$

```
[In] int(1/(x*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] - atanh((c + d*x^6)^(1/2)/c^(1/2))/(3*a*c^(1/2)) - (atan((((b^2*c - a*b*d)^
(1/2)*((2*b^3*d^2*(c + d*x^6)^(1/2))/27 - ((b^2*c - a*b*d)^(1/2)*((2*a^2*b^
2*d^3)/9 - ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^6)^(1/2)*(b^2*c - a
*b*d)^(1/2))/(36*(a^2*d - a*b*c))))/(6*(a^2*d - a*b*c))*1i)/(a^2*d - a*b*c
) + ((b^2*c - a*b*d)^(1/2)*((2*b^3*d^2*(c + d*x^6)^(1/2))/27 + ((b^2*c - a*
b*d)^(1/2)*((2*a^2*b^2*d^3)/9 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*
x^6)^(1/2)*(b^2*c - a*b*d)^(1/2))/(36*(a^2*d - a*b*c))))/(6*(a^2*d - a*b*c)
))*1i)/(a^2*d - a*b*c))/(((b^2*c - a*b*d)^(1/2)*((2*b^3*d^2*(c + d*x^6)^(1/
2))/27 - ((b^2*c - a*b*d)^(1/2)*((2*a^2*b^2*d^3)/9 - ((8*a^3*b^2*d^3 - 16*a
^2*b^3*c*d^2)*(c + d*x^6)^(1/2)*(b^2*c - a*b*d)^(1/2))/(36*(a^2*d - a*b*c))
))/(6*(a^2*d - a*b*c))))/(a^2*d - a*b*c) - ((b^2*c - a*b*d)^(1/2)*((2*b^3*d
^2*(c + d*x^6)^(1/2))/27 + ((b^2*c - a*b*d)^(1/2)*((2*a^2*b^2*d^3)/9 + ((8*
a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^6)^(1/2)*(b^2*c - a*b*d)^(1/2))/(3
6*(a^2*d - a*b*c))))/(6*(a^2*d - a*b*c))))/(a^2*d - a*b*c)))*(b^2*c - a*b*d
)^(1/2)*1i)/(3*(a^2*d - a*b*c))
```

3.857 $\int \frac{1}{x^7(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 117

$$\int \frac{1}{x^7(a+bx^6)\sqrt{c+dx^6}}\,dx = -\frac{\sqrt{c+dx^6}}{6acx^6}+\frac{(2bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^2c^{3/2}}-\frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}}$$

```
[Out] 1/6*(a*d+2*b*c)*arctanh((d*x^6+c)^(1/2)/c^(1/2))/a^2/c^(3/2)-1/3*b^(3/2)*ar
ctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(1/2)-1/6*(d
*x^6+c)^(1/2)/a/c/x^6
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 105, 162, 65, 214}

$$\int \frac{1}{x^7(a+bx^6)\sqrt{c+dx^6}}\,dx = -\frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\text{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^2c^{3/2}}-\frac{\sqrt{c+dx^6}}{6acx^6}$$

```
[In] Int[1/(x^7*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/6*Sqrt[c + d*x^6]/(a*c*x^6) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^6]/Sqr
t[c]])/(6*a^2*c^(3/2)) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*
c - a*d]])/(3*a^2*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{6}\text{Subst}\left(\int \frac{1}{x^2(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)\\
&= -\frac{\sqrt{c+dx^6}}{6acx^6} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2bc+ad)+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{6ac}\\
&= -\frac{\sqrt{c+dx^6}}{6acx^6} + \frac{b^2\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{6a^2} - \frac{(2bc+ad)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^6\right)}{12a^2c}
\end{aligned}$$

$$= -\frac{\sqrt{c+dx^6}}{6acx^6} + \frac{b^2\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{3a^2d} - \frac{(2bc+ad)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{6a^2cd}$$

$$= -\frac{\sqrt{c+dx^6}}{6acx^6} + \frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^2c^{3/2}} - \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^7(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{-\frac{a\sqrt{c+dx^6}}{cx^6} + \frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}} + \frac{(2bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{c^{3/2}}}{6a^2}$$

```
[In] Integrate[1/(x^7*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (-((a*Sqrt[c + d*x^6])/(c*x^6)) + (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^6
])/Sqrt[-(b*c) + a*d]])/Sqrt[-(b*c) + a*d] + ((2*b*c + a*d)*ArcTanh[Sqrt[c
+ d*x^6]/Sqrt[c]])/c^(3/2))/(6*a^2)
```

Maple [A] (verified)

Time = 5.39 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.79

method	result	size
pseudoelliptic	$\frac{\frac{2b^2\arctan\left(\frac{b\sqrt{dx^6+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}} - \frac{a\sqrt{dx^6+c}}{cx^6} + \frac{(ad+2bc)\,\text{arctanh}\left(\frac{\sqrt{dx^6+c}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}}{6a^2}$	92

```
[In] int(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/a^2*(2*b^2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^6+c)^(1/2)/((a*d-b*c)*b)^(
1/2))-a/c*(d*x^6+c)^(1/2)/x^6+(a*d+2*b*c)/c^(3/2)*arctanh((d*x^6+c)^(1/2)/c
^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 565, normalized size of antiderivative = 4.83

$$\int \frac{1}{x^7 (a + bx^6) \sqrt{c + dx^6}}\, dx$$

$$= \left[\frac{2\, bc^2 x^6 \sqrt{\frac{b}{bc-ad}} \log\left(\frac{bdx^6+2\, bc-ad-2\, \sqrt{dx^6+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^6+a}\right) + (2\, bc + ad)\sqrt{c}x^6 \log\left(\frac{dx^6+2\, \sqrt{dx^6+c}\sqrt{c}+2\, c}{x^6}\right) - 2\, \sqrt{dx}}{12\, a^2 c^2 x^6} \right.$$

$$- \frac{4\, bc^2 x^6 \sqrt{-\frac{b}{bc-ad}} \arctan\left(-\frac{\sqrt{dx^6+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^6+bc}\right) - (2\, bc + ad)\sqrt{c}x^6 \log\left(\frac{dx^6+2\, \sqrt{dx^6+c}\sqrt{c}+2\, c}{x^6}\right) + 2\, \sqrt{dx^6 +}}{12\, a^2 c^2 x^6}$$

$$- \frac{2\, bc^2 x^6 \sqrt{-\frac{b}{bc-ad}} \arctan\left(-\frac{\sqrt{dx^6+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^6+bc}\right) + (2\, bc + ad)\sqrt{-c}x^6 \arctan\left(\frac{\sqrt{dx^6+c}\sqrt{-c}}{c}\right) + \sqrt{dx^6 + ca}}{6\, a^2 c^2 x^6}$$

```
[In] integrate(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(2*b*c^2*x^6*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt(
d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + (2*b*c + a*d)*sq
rt(c)*x^6*log((d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6) - 2*sqrt(d*x^6
 + c)*a*c)/(a^2*c^2*x^6), -1/12*(4*b*c^2*x^6*sqrt(-b/(b*c - a*d))*arctan(-s
qrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^6 + b*c)) - (2*b*c +
 a*d)*sqrt(c)*x^6*log((d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6) + 2*sq
rt(d*x^6 + c)*a*c)/(a^2*c^2*x^6), 1/6*(b*c^2*x^6*sqrt(b/(b*c - a*d))*log((b
*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(
b*x^6 + a)) - (2*b*c + a*d)*sqrt(-c)*x^6*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c)
 - sqrt(d*x^6 + c)*a*c)/(a^2*c^2*x^6), -1/6*(2*b*c^2*x^6*sqrt(-b/(b*c - a*d
))*arctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^6 + b*c)
) + (2*b*c + a*d)*sqrt(-c)*x^6*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c) + sqrt(d*
x^6 + c)*a*c)/(a^2*c^2*x^6)]
```

Sympy [F]

$$\int \frac{1}{x^7 (a+bx^6) \sqrt{c+dx^6}}\,dx = \int \frac{1}{x^7 (a+bx^6) \sqrt{c+dx^6}}\,dx$$

```
[In] integrate(1/x**7/(b*x**6+a)/(d*x**6+c)**(1/2),x)
[Out] Integral(1/(x**7*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{x^7 (a+bx^6) \sqrt{c+dx^6}}\,dx = \int \frac{1}{(bx^6+a)\sqrt{dx^6+c}x^7}\,dx$$

```
[In] integrate(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^7), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^7 (a+bx^6) \sqrt{c+dx^6}}\,dx = \frac{b^2 \arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}a^2} - \frac{(2\,bc+ad)\arctan\left(\frac{\sqrt{dx^6+c}}{\sqrt{-c}}\right)}{6\,a^2\sqrt{-c}c} - \frac{\sqrt{dx^6+c}}{6\,acx^6}$$

```
[In] integrate(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] 1/3*b^2*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*a^2) - 1/6*(2*b*c + a*d)*arctan(sqrt(d*x^6 + c)/sqrt(-c))/(a^2*sqrt(-c)*c
) - 1/6*sqrt(d*x^6 + c)/(a*c*x^6)
```

Mupad [B] (verification not implemented)

Time = 9.85 (sec) , antiderivative size = 396, normalized size of antiderivative = 3.38

$$\int \frac{1}{x^7\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$
$$=\frac{\ln\left(\sqrt{d\,x^6+c}\,\left(b^4\,c-a\,b^3\,d\right)^{3/2}+b^6\,c^2+a^2\,b^4\,d^2-2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{6\,a^3\,d-6\,a^2\,b\,c}$$
$$-\frac{\ln\left(\sqrt{d\,x^6+c}\,\left(b^4\,c-a\,b^3\,d\right)^{3/2}-b^6\,c^2-a^2\,b^4\,d^2+2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{6\,\left(a^3\,d-a^2\,b\,c\right)}-\frac{\sqrt{d\,x^6+c}}{6\,a\,c\,x^6}$$
$$-\frac{\mathrm{atan}\left(\frac{b^4\,d^4\,\sqrt{d\,x^6+c}\,1\mathrm{i}}{18\,\sqrt{c^3}\,\left(\frac{b^4\,d^4}{18\,c}+\frac{5\,a\,b^3\,d^5}{108\,c^2}+\frac{a^2\,b^2\,d^6}{108\,c^3}\right)}+\frac{b^2\,d^6\,\sqrt{d\,x^6+c}\,1\mathrm{i}}{108\,\sqrt{c^3}\,\left(\frac{5\,b^3\,d^5}{108\,a}+\frac{b^2\,d^6}{108\,c}+\frac{b^4\,c\,d^4}{18\,a^2}\right)}+\frac{b^3\,d^5\,\sqrt{d\,x^6+c}\,5\mathrm{i}}{108\,\sqrt{c^3}\,\left(\frac{b^4\,d^4}{18\,a}+\frac{5\,b^3\,d^5}{108\,c}+\frac{a\,b^2\,d^6}{108\,c^2}\right)}\right)\,\left(a\,d+2\,b\,c\right)\,1}{6\,a^2\,\sqrt{c^3}}$$

```
[In] int(1/(x^7*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] (log((c + d*x^6)^(1/2)*(b^4*c - a*b^3*d)^(3/2) + b^6*c^2 + a^2*b^4*d^2 - 2*
a*b^5*c*d)*(b^4*c - a*b^3*d)^(1/2))/(6*a^3*d - 6*a^2*b*c) - (log((c + d*x^6
)^(1/2)*(b^4*c - a*b^3*d)^(3/2) - b^6*c^2 - a^2*b^4*d^2 + 2*a*b^5*c*d)*(b^4
*c - a*b^3*d)^(1/2))/(6*(a^3*d - a^2*b*c)) - (c + d*x^6)^(1/2)/(6*a*c*x^6)
- (atan((b^4*d^4*(c + d*x^6)^(1/2)*1i)/(18*(c^3)^(1/2)*((b^4*d^4)/(18*c) +
(5*a*b^3*d^5)/(108*c^2) + (a^2*b^2*d^6)/(108*c^3))) + (b^2*d^6*(c + d*x^6)^
(1/2)*1i)/(108*(c^3)^(1/2)*((5*b^3*d^5)/(108*a) + (b^2*d^6)/(108*c) + (b^4*
c*d^4)/(18*a^2))) + (b^3*d^5*(c + d*x^6)^(1/2)*5i)/(108*(c^3)^(1/2)*((b^4*d
^4)/(18*a) + (5*b^3*d^5)/(108*c) + (a*b^2*d^6)/(108*c^2))))*(a*d + 2*b*c)*1
i)/(6*a^2*(c^3)^(1/2))
```

3.858 $\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 123

$$\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{x^3\sqrt{c+dx^6}}{6bd} + \frac{a^{3/2}\arctan\left(\frac{\sqrt{bc-ad}x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3b^2\sqrt{bc-ad}} - \frac{(bc+2ad)\text{arctanh}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{6b^2d^{3/2}}$$

```
[Out] -1/6*(2*a*d+b*c)*arctanh(x^3*d^(1/2)/(d*x^6+c)^(1/2))/b^2/d^(3/2)+1/3*a^(3/
2)*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))/b^2/(-a*d+b*c)^(1/2
)+1/6*x^3*(d*x^6+c)^(1/2)/b/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {476, 490, 537, 223, 212, 385, 211}

$$\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{a^{3/2}\arctan\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3b^2\sqrt{bc-ad}} - \frac{(2ad+bc)\text{arctanh}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{6b^2d^{3/2}} + \frac{x^3\sqrt{c+dx^6}}{6bd}$$

```
[In] Int[x^14/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^3*Sqrt[c + d*x^6])/(6*b*d) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt
[a]*Sqrt[c + d*x^6])])/(3*b^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTanh[(Sq
rt[d]*x^3)/Sqrt[c + d*x^6]])/(6*b^2*d^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)\\
&= \frac{x^3\sqrt{c+dx^6}}{6bd} - \frac{\text{Subst}\left(\int \frac{ac+(bc+2ad)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6bd}\\
&= \frac{x^3\sqrt{c+dx^6}}{6bd} + \frac{a^2\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{3b^2} - \frac{(bc+2ad)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6b^2d}\\
&= \frac{x^3\sqrt{c+dx^6}}{6bd} + \frac{a^2\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{3b^2}\\
&\quad - \frac{(bc+2ad)\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{6b^2d}\\
&= \frac{x^3\sqrt{c+dx^6}}{6bd} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3b^2\sqrt{bc-ad}} - \frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x^3}{\sqrt{c+dx^6}}\right)}{6b^2d^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$
\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{\frac{bx^3\sqrt{c+dx^6}}{d} + \frac{2a^{3/2}\arctan\left(\frac{a\sqrt{d}+bx^3\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}} - \frac{(bc+2ad)\log\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{d^{3/2}}}{6b^2}
$$

```
[In] Integrate[x^14/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] ((b*x^3*Sqrt[c + d*x^6])/d + (2*a^(3/2)*ArcTan[(a*Sqrt[d] + b*x^3*(Sqrt[d]*
x^3 + Sqrt[c + d*x^6]))/(Sqrt[a]*Sqrt[b*c - a*d])])/Sqrt[b*c - a*d] - ((b*c
 + 2*a*d)*Log[Sqrt[d]*x^3 + Sqrt[c + d*x^6]])/d^(3/2))/(6*b^2)
```

Maple [A] (verified)

Time = 9.61 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.24

method	result
pseudoelliptic	$\frac{b\sqrt{(ad-bc)a}\,\sqrt{d\,x^6+c}\,x^3\sqrt{d}+2a^2\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^6+c}\,a}{x^3\sqrt{(ad-bc)a}}\right)d^{\frac{3}{2}}-2\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^6+c}}{x^3\sqrt{d}}\right)ad\sqrt{(ad-bc)a}-\mathrm{arctanh}\left(\frac{\sqrt{d\,x^6+c}}{x^3\sqrt{d}}\right)bc\sqrt{}}{6b^2\sqrt{(ad-bc)a}\,d^{\frac{3}{2}}}$

```
[In] int(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(b*((a*d-b*c)*a)^(1/2)*(d*x^6+c)^(1/2)*x^3*d^(1/2)+2*a^2*arctanh((d*x^6
+c)^(1/2)/x^3*a/((a*d-b*c)*a)^(1/2))*d^(3/2)-2*arctanh((d*x^6+c)^(1/2)/x^3/
d^(1/2))*a*d*((a*d-b*c)*a)^(1/2)-arctanh((d*x^6+c)^(1/2)/x^3/d^(1/2))*b*c*(
(a*d-b*c)*a)^(1/2))/b^2/((a*d-b*c)*a)^(1/2)/d^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.83 (sec) , antiderivative size = 739, normalized size of antiderivative = 6.01

$$\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \left[\frac{2\sqrt{dx^6+c}bdx^3+ad^2\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{12}-2\,(3\,abc^2-4\,a^2cd)x^6+a^2c^2+4\,((b^2c^2-3\,abcd+2\,a^2d^2)x^9-(abc^2-}{b^2x^{12}+2\,abx^6+a^2}\right.}{12\,b^2d^2}\right.$$

```
[In] integrate(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(2*sqrt(d*x^6 + c)*b*d*x^3 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 +
4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*
x^6 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)) + (b*c + 2*a*d
)*sqrt(d)*log(-2*d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c))/(b^2*d^2), 1/1
2*(2*sqrt(d*x^6 + c)*b*d*x^3 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8
*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 + 4*((
b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*x^6
+ c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)) + 2*(b*c + 2*a*d)*
sqrt(-d)*arctan(sqrt(-d)*x^3/sqrt(d*x^6 + c)))/(b^2*d^2), 1/12*(2*sqrt(d*x^
6 + c)*b*d*x^3 - 2*a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^6
 - a*c)*sqrt(d*x^6 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c*x^3)) + (b*c + 2
*a*d)*sqrt(d)*log(-2*d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c))/(b^2*d^2),
 1/6*(sqrt(d*x^6 + c)*b*d*x^3 - a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c
 - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c*x^3
```

```
)) + (b*c + 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x^3/sqrt(d*x^6 + c)))/(b^2*d^2)
]
```

Sympy [F]

$$\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

```
[In] integrate(x**14/(b*x**6+a)/(d*x**6+c)**(1/2),x)
[Out] Integral(x**14/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^{14}}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

```
[In] integrate(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^14/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Giac [B] (verification not implemented)

Error detected during grading. Assigning place holder grade for now.

Time = 0.43 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.93

$$\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = Recursive assumption c \geq \frac{\sqrt{dx^6+c}\,x^3}{6\,bd} - \frac{a^2\arctan\left(\frac{a\sqrt{d+\frac{c}{x^6}}}{\sqrt{abc-a^2d}}\right)}{3\sqrt{abc-a^2d}\,b^2\operatorname{sgn}(x)} + \frac{\left(2\,a^2\sqrt{-d}d\arctan\left(\frac{a\sqrt{d}}{\sqrt{abc-a^2d}}\right) - \sqrt{abc-a^2d}\,bc\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right) - 2\sqrt{abc-a^2d}\,da\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)\right)\operatorname{sgn}(x)}{6\sqrt{abc-a^2d}\,b^2\sqrt{-d}d} + \frac{(bc+2\,ad)\arctan\left(\frac{\sqrt{d+\frac{c}{x^6}}}{\sqrt{-d}}\right)}{6\,b^2\sqrt{-d}d\operatorname{sgn}(x)} - \frac{dignored}{t_{nostep}^6}$$

```
[In] integrate(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] Recursive*assumption*c >= 1/6*sqrt(d*x^6 + c)*x^3/(b*d) - 1/3*a^2*arctan(a*
sqrt(d + c/x^6)/sqrt(a*b*c - a^2*d))/(sqrt(a*b*c - a^2*d)*b^2*sgn(x)) + 1/6
```

```
*(2*a^2*sqrt(-d)*d*arctan(a*sqrt(d)/sqrt(a*b*c - a^2*d)) - sqrt(a*b*c - a^2
*d)*b*c*arctan(sqrt(d)/sqrt(-d)) - 2*sqrt(a*b*c - a^2*d)*a*d*arctan(sqrt(d)
/sqrt(-d)))*sgn(x)/(sqrt(a*b*c - a^2*d)*b^2*sqrt(-d)*d) + 1/6*(b*c + 2*a*d)
*arctan(sqrt(d + c/x^6)/sqrt(-d))/(b^2*sqrt(-d)*d*sgn(x)) - d*ignored/t_nos
tep^6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^{14}}{(b\,x^6+a)\,\sqrt{d\,x^6+c}}\,dx$$

```
[In] int(x^14/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
[Out] int(x^14/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.859 $\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}} dx$

Optimal result

Integrand size = 24, antiderivative size = 91

$$\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}} dx = -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{bc-ad}x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3b\sqrt{bc-ad}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{3b\sqrt{d}}$$

```
[Out] 1/3*arctanh(x^3*d^(1/2)/(d*x^6+c)^(1/2))/b/d^(1/2)-1/3*arctan(x^3*(-a*d+b*c
)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))*a^(1/2)/b/(-a*d+b*c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {476, 494, 223, 212, 385, 211}

$$\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}} dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{3b\sqrt{d}} - \frac{\sqrt{a}\arctan\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3b\sqrt{bc-ad}}$$

```
[In] Int[x^8/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/3*(Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(b*S
qrt[b*c - a*d]) + ArcTanh[(Sqrt[d]*x^3)/Sqrt[c + d*x^6]]/(3*b*Sqrt[d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^3\right)}{3b} - \frac{a\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{3b}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{3b} - \frac{a\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{3b}\\
&= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bc-ad}x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3b\sqrt{bc-ad}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{3b\sqrt{d}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.19

$$\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{-\frac{\sqrt{a}\arctan\left(\frac{a\sqrt{d}+bx^3\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}+\frac{\log\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{\sqrt{d}}}{3b}$$

```
[In] Integrate[x^8/((a + b*x^6)*Sqrt[c + d*x^6]),x]
[Out] (-((Sqrt[a]*ArcTan[(a*Sqrt[d] + b*x^3*(Sqrt[d]*x^3 + Sqrt[c + d*x^6]))/(Sqr
t[a]*Sqrt[b*c - a*d])])/Sqrt[b*c - a*d]) + Log[Sqrt[d]*x^3 + Sqrt[c + d*x^6
]]/Sqrt[d])/(3*b)
```

Maple [A] (verified)

Time = 7.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.93

method	result	size
pseudoelliptic	$\frac{-a\ \mathrm{arctanh}\left(\frac{\sqrt{d\,x^6+c}\,a}{x^3\sqrt{(ad-bc)a}}\right)\sqrt{d}+\mathrm{arctanh}\left(\frac{\sqrt{d\,x^6+c}}{x^3\sqrt{d}}\right)\sqrt{(ad-bc)a}}{3b\sqrt{(ad-bc)a}\,\sqrt{d}}$	85

```
[In] int(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(-a*arctanh((d*x^6+c)^(1/2)/x^3*a/((a*d-b*c)*a)^(1/2))*d^(1/2)+arctanh(
(d*x^6+c)^(1/2)/x^3/d^(1/2))*((a*d-b*c)*a)^(1/2))/b/((a*d-b*c)*a)^(1/2)/d^(
1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.61 (sec) , antiderivative size = 632, normalized size of antiderivative = 6.95

$$\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \left[\frac{d\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{12}-2\,(3\,abc^2-4\,a^2cd)x^6+a^2c^2-4\,((b^2c^2-3\,abcd+2\,a^2d^2)x^9-(abc^2-a^2cd)x^3)\sqrt{dx^6+c}\sqrt{-\frac{a}{bc-a}}}{b^2x^{12}+2\,abx^6+a^2}\right.}{12\,bd}\right.$$

```
[In] integrate(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
[Out] [1/12*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 -
 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*
```

```
d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*x^6 + c)*sqrt(-a/(b*c - a*d)))/(
b^2*x^12 + 2*a*b*x^6 + a^2)) + 2*sqrt(d)*log(-2*d*x^6 - 2*sqrt(d*x^6 + c)*s
qrt(d)*x^3 - c))/(b*d), 1/12*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*
c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b^2*c
^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*x^6 + c)*
sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)) - 4*sqrt(-d)*arctan(sqr
t(-d)*x^3/sqrt(d*x^6 + c)))/(b*d), 1/6*(d*sqrt(a/(b*c - a*d))*arctan(-1/2*(
(b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c
*x^3)) + sqrt(d)*log(-2*d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c))/(b*d),
1/6*(d*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6
 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c*x^3)) - 2*sqrt(-d)*arctan(sqrt(-d)
*x^3/sqrt(d*x^6 + c)))/(b*d)]
```

Sympy [F]

$$\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

```
[In] integrate(x**8/(b*x**6+a)/(d*x**6+c)**(1/2),x)
[Out] Integral(x**8/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^8}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

```
[In] integrate(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^8/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(71) = 142$.

Time = 0.30 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.71

$$\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx = -\frac{\left(a\sqrt{-d}\arctan\left(\frac{a\sqrt{d}}{\sqrt{abc-a^2d}}\right)-\sqrt{abc-a^2d}\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)\right)\mathrm{sgn}(x)}{3\sqrt{abc-a^2d}\,b\sqrt{-d}} + \frac{a\arctan\left(\frac{a\sqrt{d+\frac{c}{x^6}}}{\sqrt{abc-a^2d}}\right)}{3\sqrt{abc-a^2d}\,b\,\mathrm{sgn}(x)} - \frac{\arctan\left(\frac{\sqrt{d+\frac{c}{x^6}}}{\sqrt{-d}}\right)}{3\,b\sqrt{-d}\,\mathrm{sgn}(x)}$$

```
[In] integrate(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*(a*sqrt(-d)*arctan(a*sqrt(d)/sqrt(a*b*c - a^2*d)) - sqrt(a*b*c - a^2*d
)*arctan(sqrt(d)/sqrt(-d)))*sgn(x)/(sqrt(a*b*c - a^2*d)*b*sqrt(-d)) + 1/3*a
*arctan(a*sqrt(d + c/x^6)/sqrt(a*b*c - a^2*d))/(sqrt(a*b*c - a^2*d)*b*sgn(x
)) - 1/3*arctan(sqrt(d + c/x^6)/sqrt(-d))/(b*sqrt(-d)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx=\int \frac{x^8}{(b\,x^6+a)\,\sqrt{d\,x^6+c}}\,dx$$

```
[In] int(x^8/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(x^8/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.860 $\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 54

$$\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx=\frac{\arctan\left(\frac{\sqrt{bc-ad}x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3\sqrt{a}\sqrt{bc-ad}}$$

```
[Out] 1/3*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))/a^(1/2)/(-a*d+b*c)
^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {476, 385, 211}

$$\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx=\frac{\arctan\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3\sqrt{a}\sqrt{bc-ad}}$$

```
[In] Int[x^2/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])]/(3*Sqrt[a]*Sqrt[b*c
 - a*d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3\sqrt{a}\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.37

$$\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{\arctan\left(\frac{a\sqrt{d}+bx^3\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{3\sqrt{a}\sqrt{bc-ad}}$$

```
[In] Integrate[x^2/((a + b*x^6)*Sqrt[c + d*x^6]),x]
[Out] ArcTan[(a*Sqrt[d] + b*x^3*(Sqrt[d]*x^3 + Sqrt[c + d*x^6]))/(Sqrt[a]*Sqrt[b*
c - a*d])]/(3*Sqrt[a]*Sqrt[b*c - a*d])
```

Maple [A] (verified)

Time = 6.88 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{6}+c}\,a}{x^{3}\sqrt{(ad-bc)a}}\right)}{3\sqrt{(ad-bc)a}}$	42

```
[In] int(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/((a*d-b*c)*a)^(1/2)*arctanh((d*x^6+c)^(1/2)/x^3*a/((a*d-b*c)*a)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(42) = 84$.

Time = 0.73 (sec) , antiderivative size = 245, normalized size of antiderivative = 4.54

$$\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \left[-\frac{\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{12}-2\,(3\,abc^2-4\,a^2cd)x^6+a^2c^2-4\,((bc-2\,ad)x^9-acx^3)\sqrt{dx^6+c}\sqrt{-abc+a^2d}}{b^2x^{12}+2\,abx^6+a^2}\right)}{12\,(abc-a^2d)}, \frac{\arctan\left(\right.}{}\right.$$

```
[In] integrate(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2
*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*sq
rt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2))/(a*b*c -
a^2*d), 1/6*arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b*c
 - a^2*d)/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3))/sqrt(a*b*c -
 a^2*d)]
```

Sympy [F]

$$\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

```
[In] integrate(x**2/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x**2/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^2}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

```
[In] integrate(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.33

$$\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx = -\frac{\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x^3-\sqrt{dx^6+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{3\sqrt{abcd-a^2d^2}}$$

```
[In] integrate(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] -1/3*sqrt(d)*arctan(1/2*((sqrt(d)*x^3 - sqrt(d*x^6 + c))^2*b - b*c + 2*a*d)
/sqrt(a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^2}{(b\,x^6+a)\,\sqrt{d\,x^6+c}}\,dx$$

```
[In] int(x^2/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
[Out] int(x^2/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.861 $\int \frac{1}{x^4(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \frac{1}{x^4(a+bx^6)\sqrt{c+dx^6}}\,dx = -\frac{\sqrt{c+dx^6}}{3acx^3} - \frac{b\arctan\left(\frac{\sqrt{bc-ad}x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3a^{3/2}\sqrt{bc-ad}}$$

```
[Out] -1/3*b*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))/a^(3/2)/(-a*d+b
*c)^(1/2)-1/3*(d*x^6+c)^(1/2)/a/c/x^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {476, 491, 12, 385, 211}

$$\int \frac{1}{x^4(a+bx^6)\sqrt{c+dx^6}}\,dx = -\frac{b\arctan\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3a^{3/2}\sqrt{bc-ad}} - \frac{\sqrt{c+dx^6}}{3acx^3}$$

```
[In] Int[1/(x^4*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/3*Sqrt[c + d*x^6]/(a*c*x^3) - (b*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*S
qrt[c + d*x^6])])/(3*a^(3/2)*Sqrt[b*c - a*d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{c+dx^6}}{3acx^3} - \frac{\text{Subst}\left(\int \frac{bc}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{3ac}\\
&= -\frac{\sqrt{c+dx^6}}{3acx^3} - \frac{b\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{3a}\\
&= -\frac{\sqrt{c+dx^6}}{3acx^3} - \frac{b\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{3a}\\
&= -\frac{\sqrt{c+dx^6}}{3acx^3} - \frac{b\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3a^{3/2}\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^4 (a+bx^6)\sqrt{c+dx^6}}\,dx = -\frac{\sqrt{c+dx^6}}{3acx^3} - \frac{b\arctan\left(\frac{a\sqrt{d}+bx^3\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{3a^{3/2}\sqrt{bc-ad}}$$

```
[In] Integrate[1/(x^4*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/3*Sqrt[c + d*x^6]/(a*c*x^3) - (b*ArcTan[(a*Sqrt[d] + b*x^3*(Sqrt[d]*x^3
+ Sqrt[c + d*x^6]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(3*a^(3/2)*Sqrt[b*c - a*d])
```

Maple [A] (verified)

Time = 8.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

method	result	size
pseudoelliptic	$-\frac{cb\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^6+c}\,a}{x^3\sqrt{(ad-bc)a}}\right)x^3+\sqrt{d\,x^6+c}\,\sqrt{(ad-bc)a}}{3a\,x^3\sqrt{(ad-bc)a}\,c}$	80

```
[In] int(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(c*b*arctanh((d*x^6+c)^(1/2)/x^3*a/((a*d-b*c)*a)^(1/2))*x^3+(d*x^6+c)^
(1/2)*((a*d-b*c)*a)^(1/2))/a/x^3/((a*d-b*c)*a)^(1/2)/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(64) = 128.

Time = 0.37 (sec) , antiderivative size = 332, normalized size of antiderivative = 4.15

$$\int \frac{1}{x^4 (a+bx^6)\sqrt{c+dx^6}}\,dx = \left[-\frac{\sqrt{-abc+a^2d}bcx^3\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{12}-2\,(3\,abc^2-4\,a^2cd)x^6+a^2c^2+4\,((bc-2\,ad)x^9-acx^3)\sqrt{dx^6+c}\sqrt{-abc+a^2d}}{b^2x^{12}+2\,abx^6+a^2}\right)+4\,\sqrt{}}{12\,(a^2bc^2-a^3cd)x^3}\right.$$
$$\left.-\frac{\sqrt{abc-a^2d}bcx^3\arctan\left(\frac{((bc-2\,ad)x^6-ac)\sqrt{dx^6+c}\sqrt{abc-a^2d}}{2\,((abcd-a^2d^2)x^9+(abc^2-a^2cd)x^3)}\right)+2\,\sqrt{dx^6+c}(abc-a^2d)}{6\,(a^2bc^2-a^3cd)x^3}\right]$$

```
[In] integrate(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(sqrt(-a*b*c + a^2*d)*b*c*x^3*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)
*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 + 4*((b*c - 2*a*d)*x^9 - a*
```

```
c*x^3)*sqrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2))
+ 4*sqrt(d*x^6 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^3), -1/6*(sqr
t(a*b*c - a^2*d)*b*c*x^3*arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 +
c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3))
 + 2*sqrt(d*x^6 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4 (a + bx^6) \sqrt{c + dx^6}} \, dx = \int \frac{1}{x^4 (a + bx^6) \sqrt{c + dx^6}} \, dx$$

```
[In] integrate(1/x**4/(b*x**6+a)/(d*x**6+c)**(1/2),x)
[Out] Integral(1/(x**4*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (a + bx^6) \sqrt{c + dx^6}} \, dx = \int \frac{1}{(bx^6 + a)\sqrt{dx^6 + c}x^4} \, dx$$

```
[In] integrate(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a + bx^6) \sqrt{c + dx^6}} \, dx = \int \frac{1}{(bx^6 + a)\sqrt{dx^6 + c}x^4} \, dx$$

```
[In] integrate(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^6) \sqrt{c+dx^6}} \, dx = \int \frac{1}{x^4 \, (b\,x^6+a) \, \sqrt{d\,x^6+c}} \, dx$$

```
[In] int(1/(x^4*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
[Out] int(1/(x^4*(a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.862 $\int \frac{1}{x^{10}(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 115

$$\int \frac{1}{x^{10}(a+bx^6)\sqrt{c+dx^6}}\,dx = -\frac{\sqrt{c+dx^6}}{9acx^9}+\frac{(3bc+2ad)\sqrt{c+dx^6}}{9a^2c^2x^3}+\frac{b^2\arctan\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3a^{5/2}\sqrt{bc-ad}}$$

```
[Out] 1/3*b^2*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))/a^(5/2)/(-a*d+
b*c)^(1/2)-1/9*(d*x^6+c)^(1/2)/a/c/x^9+1/9*(2*a*d+3*b*c)*(d*x^6+c)^(1/2)/a^
2/c^2/x^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {476, 491, 597, 12, 385, 211}

$$\int \frac{1}{x^{10}(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{b^2\arctan\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3a^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^6}(2ad+3bc)}{9a^2c^2x^3}-\frac{\sqrt{c+dx^6}}{9acx^9}$$

```
[In] Int[1/(x^10*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/9*Sqrt[c + d*x^6]/(a*c*x^9) + ((3*b*c + 2*a*d)*Sqrt[c + d*x^6])/(9*a^2*c
^2*x^3) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(3*
a^(5/2)*Sqrt[b*c - a*d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^4(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^6}}{9acx^9} + \frac{\text{Subst}\left(\int \frac{-3bc-2ad-2bdx^2}{x^2(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{9ac}$$

$$= -\frac{\sqrt{c+dx^6}}{9acx^9} + \frac{(3bc+2ad)\sqrt{c+dx^6}}{9a^2c^2x^3} - \frac{\text{Subst}\left(\int -\frac{3b^2c^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{9a^2c^2}$$

$$= -\frac{\sqrt{c+dx^6}}{9acx^9} + \frac{(3bc+2ad)\sqrt{c+dx^6}}{9a^2c^2x^3} + \frac{b^2\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{3a^2}$$

$$= -\frac{\sqrt{c+dx^6}}{9acx^9} + \frac{(3bc+2ad)\sqrt{c+dx^6}}{9a^2c^2x^3} + \frac{b^2\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{3a^2}$$

$$= -\frac{\sqrt{c+dx^6}}{9acx^9} + \frac{(3bc+2ad)\sqrt{c+dx^6}}{9a^2c^2x^3} + \frac{b^2\tan^{-1}\left(\frac{\sqrt{bc-ad}x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3a^{5/2}\sqrt{bc-ad}}$$

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^{10}(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{\sqrt{c+dx^6}(-ac+3bcx^6+2adx^6)}{9a^2c^2x^9} + \frac{b^2\arctan\left(\frac{a\sqrt{d}+bx^3\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{3a^{5/2}\sqrt{bc-ad}}$$

```
[In] Integrate[1/(x^10*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (Sqrt[c + d*x^6]*(-(a*c) + 3*b*c*x^6 + 2*a*d*x^6))/(9*a^2*c^2*x^9) + (b^2*A
rcTan[(a*Sqrt[d] + b*x^3*(Sqrt[d]*x^3 + Sqrt[c + d*x^6]))/(Sqrt[a]*Sqrt[b*c
 - a*d])])/(3*a^(5/2)*Sqrt[b*c - a*d])
```

Maple [A] (verified)

Time = 10.95 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

method	result	size
pseudoelliptic	$-\frac{-3b^2c^2\,\text{arctanh}\left(\frac{\sqrt{d\,x^6+c}\,a}{x^3\sqrt{(ad-bc)a}}\right)x^9+\left((-3b\,x^6+a)c-2ad\,x^6\right)\sqrt{d\,x^6+c}\,\sqrt{(ad-bc)a}}{9\sqrt{(ad-bc)a}\,a^2x^9c^2}$	103

```
[In] int(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*(-3*b^2*c^2*arctanh((d*x^6+c)^(1/2)/x^3*a/((a*d-b*c)*a)^(1/2))*x^9+((-
3*b*x^6+a)*c-2*a*d*x^6)*(d*x^6+c)^(1/2)*((a*d-b*c)*a)^(1/2))/((a*d-b*c)*a)^
(1/2)/a^2/x^9/c^2
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 416, normalized size of antiderivative = 3.62

$$\int \frac{1}{x^{10} \left(a+b x^6\right) \sqrt{c+d x^6}} \, dx$$
$$= \left[-\frac{3 \sqrt{-abc+a^2d} b^2 c^2 x^9 \log \left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{12}-2\left(3\,abc^2-4\,a^2cd\right)x^6+a^2c^2-4\left(\left(bc-2\,ad\right)x^9-acx^3\right)\sqrt{dx^6+c}\sqrt{-abc+a^2d}}{b^2x^{12}+2\,abx^6+a^2}\right) -}{36\left(a^3bc^3-a^4c^2d\right)x^9}\right.$$

```
[In] integrate(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/36*(3*sqrt(-a*b*c + a^2*d)*b^2*c^2*x^9*log(((b^2*c^2 - 8*a*b*c*d + 8*a^
2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b*c - 2*a*d)*x^
9 - a*c*x^3)*sqrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 +
a^2)) - 4*((3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^6 - a^2*b*c^2 + a^3*c*d)
*sqrt(d*x^6 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^9), 1/18*(3*sqrt(a*b*c - a^2*d
)*b^2*c^2*x^9*arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b
*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3)) + 2*((3*a*
b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^6 - a^2*b*c^2 + a^3*c*d)*sqrt(d*x^6 + c)
)/((a^3*b*c^3 - a^4*c^2*d)*x^9)]
```

Sympy [F]

$$\int \frac{1}{x^{10} \left(a+b x^6\right) \sqrt{c+d x^6}} \, dx = \int \frac{1}{x^{10} \left(a+b x^6\right) \sqrt{c+d x^6}} \, dx$$

```
[In] integrate(1/x**10/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(1/(x**10*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{x^{10} \left(a+b x^6\right) \sqrt{c+d x^6}} \, dx = \int \frac{1}{\left(b x^6+a\right)\sqrt{d x^6+c}\, x^{10}} \, dx$$

```
[In] integrate(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^10), x)
```

Giac [F]

$$\int \frac{1}{x^{10}\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}x^{10}}\,dx$$

```
[In] integrate(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{10}\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=\int \frac{1}{x^{10}\,\left(b\,x^6+a\right)\,\sqrt{d\,x^6+c}}\,dx$$

```
[In] int(1/(x^10*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
[Out] int(1/(x^10*(a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.863 $\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{x^5\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(\frac{5}{6},1,\frac{1}{2},\frac{11}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{5a\sqrt{c+dx^6}}$$

```
[Out] 1/5*x^5*AppellF1(5/6,1,1/2,11/6,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a/(d*x
^6+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{x^5\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(\frac{5}{6},1,\frac{1}{2},\frac{11}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{5a\sqrt{c+dx^6}}$$

```
[In] Int[x^4/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^5*Sqrt[1 + (d*x^6)/c]*AppellF1[5/6, 1, 1/2, 11/6, -((b*x^6)/a), -((d*x^6
)/c)])/(5*a*Sqrt[c + d*x^6])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^6}{c}}\int \frac{x^4}{(a+bx^6)\sqrt{1+\frac{dx^6}{c}}}\,dx}{\sqrt{c+dx^6}}$$
$$= \frac{x^5\sqrt{1+\frac{dx^6}{c}}F_1\left(\frac{5}{6};1,\frac{1}{2};\frac{11}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{5a\sqrt{c+dx^6}}$$

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{x^5\sqrt{\frac{c+dx^6}{c}}\,\text{AppellF1}\left(\frac{5}{6},\frac{1}{2},1,\frac{11}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)}{5a\sqrt{c+dx^6}}$$

```
[In] Integrate[x^4/((a + b*x^6)*Sqrt[c + d*x^6]),x]
[Out] (x^5*Sqrt[(c + d*x^6)/c]*AppellF1[5/6, 1/2, 1, 11/6, -((d*x^6)/c), -((b*x^6
)/a)])/(5*a*Sqrt[c + d*x^6])
```

Maple [F]

$$\int \frac{x^4}{(b\,x^6+a)\sqrt{d\,x^6+c}}dx$$

```
[In] int(x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x)
[Out] int(x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   Not i
ntegrable (provided residues have no relations)
```

Sympy [F]

$$\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

```
[In] integrate(x**4/(b*x**6+a)/(d*x**6+c)**(1/2),x)
[Out] Integral(x**4/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^4}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

```
[In] integrate(x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^4/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^4}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

```
[In] integrate(x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^4}{(b\,x^6+a)\,\sqrt{d\,x^6+c}}\,dx$$

```
[In] int(x^4/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
[Out] int(x^4/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.864 $\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx=\frac{x^4\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(\frac{2}{3},1,\frac{1}{2},\frac{5}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a\sqrt{c+dx^6}}$$

```
[Out] 1/4*x^4*AppellF1(2/3,1,1/2,5/3,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a/(d*x^
6+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {476, 525, 524}

$$\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx=\frac{x^4\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(\frac{2}{3},1,\frac{1}{2},\frac{5}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a\sqrt{c+dx^6}}$$

```
[In] Int[x^3/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^6)/c]*AppellF1[2/3, 1, 1/2, 5/3, -((b*x^6)/a), -((d*x^6)
/c)])/(4*a*Sqrt[c + d*x^6])
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx, x, x^2\right)\\ &= \frac{\sqrt{1+\frac{dx^6}{c}}\text{Subst}\left(\int \frac{x}{(a+bx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx, x, x^2\right)}{2\sqrt{c+dx^6}}\\ &= \frac{x^4\sqrt{1+\frac{dx^6}{c}}F_1\left(\frac{2}{3};1,\frac{1}{2};\frac{5}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a\sqrt{c+dx^6}}\end{aligned}$$

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{x^4\sqrt{\frac{c+dx^6}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)}{4a\sqrt{c+dx^6}}$$

```
[In] Integrate[x^3/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^4*Sqrt[(c + d*x^6)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^6)/c), -((b*x^6)
/a)])/(4*a*Sqrt[c + d*x^6])
```

Maple [F]

$$\int \frac{x^3}{(b\,x^6+a)\,\sqrt{d\,x^6+c}}dx$$

```
[In] int(x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x)
[Out] int(x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^6)\,\sqrt{c+dx^6}}\,dx = \text{Timed out}$$

```
[In] integrate(x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^3}{(a+bx^6)\,\sqrt{c+dx^6}}\,dx = \int \frac{x^3}{(a+bx^6)\,\sqrt{c+dx^6}}\,dx$$

```
[In] integrate(x**3/(b*x**6+a)/(d*x**6+c)**(1/2),x)
[Out] Integral(x**3/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{x^3}{(a+bx^6)\,\sqrt{c+dx^6}}\,dx = \int \frac{x^3}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

```
[In] integrate(x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Giac [F]

$$\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^3}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

```
[In] integrate(x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^3/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^3}{(b\,x^6+a)\,\sqrt{d\,x^6+c}}\,dx$$

```
[In] int(x^3/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
[Out] int(x^3/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.865 $\int \frac{x}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{x}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{x^2\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(\frac{1}{3},1,\frac{1}{2},\frac{4}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a\sqrt{c+dx^6}}$$

```
[Out] 1/2*x^2*AppellF1(1/3,1,1/2,4/3,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a/(d*x^
6+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {476, 441, 440}

$$\int \frac{x}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{x^2\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},1,\frac{1}{2},\frac{4}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a\sqrt{c+dx^6}}$$

```
[In] Int[x/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^2*Sqrt[1 + (d*x^6)/c]*AppellF1[1/3, 1, 1/2, 4/3, -((b*x^6)/a), -((d*x^6)
/c)])/(2*a*Sqrt[c + d*x^6])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx, x, x^2\right)\\ &= \frac{\sqrt{1+\frac{dx^6}{c}}\text{Subst}\left(\int \frac{1}{(a+bx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx, x, x^2\right)}{2\sqrt{c+dx^6}}\\ &= \frac{x^2\sqrt{1+\frac{dx^6}{c}}F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a\sqrt{c+dx^6}}\end{aligned}$$

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{x^2\sqrt{\frac{c+dx^6}{c}}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)}{2a\sqrt{c+dx^6}}$$

```
[In] Integrate[x/((a + b*x^6)*Sqrt[c + d*x^6]),x]
[Out] (x^2*Sqrt[(c + d*x^6)/c]*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^6)/c), -((b*x^6)
/a)])/(2*a*Sqrt[c + d*x^6])
```

Maple [F]

$$\int \frac{x}{\left(b\,x^{6}+a\right)\sqrt{d\,x^{6}+c}}dx$$

```
[In] int(x/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^6)\,\sqrt{c+dx^6}}\,dx = \text{Timed out}$$

```
[In] integrate(x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x}{(a+bx^6)\,\sqrt{c+dx^6}}\,dx = \int \frac{x}{(a+bx^6)\,\sqrt{c+dx^6}}\,dx$$

```
[In] integrate(x/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{x}{(a+bx^6)\,\sqrt{c+dx^6}}\,dx = \int \frac{x}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

```
[In] integrate(x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Giac [F]

$$\int \frac{x}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

```
[In] integrate(x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] integrate(x/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x}{(b\,x^6+a)\,\sqrt{d\,x^6+c}}\,dx$$

```
[In] int(x/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
[Out] int(x/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.866 $\int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{x\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(\frac{1}{6},1,\frac{1}{2},\frac{7}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a\sqrt{c+dx^6}}$$

```
[Out] x*AppellF1(1/6,1,1/2,7/6,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a/(d*x^6+c)^(
1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {441, 440}

$$\int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{x\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(\frac{1}{6},1,\frac{1}{2},\frac{7}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a\sqrt{c+dx^6}}$$

```
[In] Int[1/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (x*Sqrt[1 + (d*x^6)/c]*AppellF1[1/6, 1, 1/2, 7/6, -((b*x^6)/a), -((d*x^6)/c
)])/(a*Sqrt[c + d*x^6])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^6}{c}}\int\frac{1}{(a+bx^6)\sqrt{1+\frac{dx^6}{c}}}\,dx}{\sqrt{c+dx^6}}$$

$$= \frac{x\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{1}{6};1,\frac{1}{2};\frac{7}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a\sqrt{c+dx^6}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 161 vs. $2(59) = 118$.

Time = 10.19 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.73

$$\int\frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx = -\frac{7acx\,\text{AppellF1}\left(\frac{1}{6},\frac{1}{2},1,\frac{7}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)}{(a+bx^6)\sqrt{c+dx^6}\,\big(-7ac\,\text{AppellF1}\big(\frac{1}{6},\frac{1}{2},1,\frac{7}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\big)+3x^6\,\big(2bc\,\text{AppellF1}\big(\frac{7}{6},\frac{1}{2},2,\frac{13}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}}$$

```
[In] Integrate[1/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (-7*a*c*x*AppellF1[1/6, 1/2, 1, 7/6, -((d*x^6)/c), -((b*x^6)/a)])/((a + b*x
^6)*Sqrt[c + d*x^6]*(-7*a*c*AppellF1[1/6, 1/2, 1, 7/6, -((d*x^6)/c), -((b*x
^6)/a)] + 3*x^6*(2*b*c*AppellF1[7/6, 1/2, 2, 13/6, -((d*x^6)/c), -((b*x^6)/
a)] + a*d*AppellF1[7/6, 3/2, 1, 13/6, -((d*x^6)/c), -((b*x^6)/a)])))
```

Maple [F]

$$\int\frac{1}{(b\,x^6+a)\sqrt{d\,x^6+c}}dx$$

```
[In] int(1/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   Not i
ntegrable (provided residues have no relations)
```

Sympy [F]

$$\int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

```
[In] integrate(1/(b*x**6+a)/(d*x**6+c)**(1/2),x)
[Out] Integral(1/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

```
[In] integrate(1/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

```
[In] integrate(1/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{1}{(b\,x^6+a)\,\sqrt{d\,x^6+c}}\,dx$$

```
[In] int(1/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
[Out] int(1/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.867 $\int \frac{1}{x^2(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{1}{x^2 (a+bx^6) \sqrt{c+dx^6}}\,dx=-\frac{\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(-\frac{1}{6},1,\frac{1}{2},\frac{5}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{ax\sqrt{c+dx^6}}$$

```
[Out] -AppellF1(-1/6,1,1/2,5/6,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a/x/(d*x^6+c)
^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{1}{x^2 (a+bx^6) \sqrt{c+dx^6}}\,dx=-\frac{\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(-\frac{1}{6},1,\frac{1}{2},\frac{5}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{ax\sqrt{c+dx^6}}$$

```
[In] Int[1/(x^2*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -((Sqrt[1 + (d*x^6)/c]*AppellF1[-1/6, 1, 1/2, 5/6, -((b*x^6)/a), -((d*x^6)/
c)])/(a*x*Sqrt[c + d*x^6]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^6}{c}}\int \frac{1}{x^2(a+bx^6)\sqrt{1+\frac{dx^6}{c}}}\,dx}{\sqrt{c+dx^6}}$$
$$= -\frac{\sqrt{1+\frac{dx^6}{c}}F_1\left(-\frac{1}{6};1,\frac{1}{2};\frac{5}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{ax\sqrt{c+dx^6}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 141 vs. $2(62) = 124$.

Time = 10.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.27

$$\int \frac{1}{x^2(a+bx^6)\sqrt{c+dx^6}}\,dx$$
$$= \frac{-55a(c+dx^6) - 11(bc-2ad)x^6\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(\frac{5}{6},\frac{1}{2},1,\frac{11}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right) + 10bdx^{12}\sqrt{1+\frac{dx^6}{c}}\,\text{AppellI}}{55a^2cx\sqrt{c+dx^6}}$$

```
[In] Integrate[1/(x^2*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (-55*a*(c + d*x^6) - 11*(b*c - 2*a*d)*x^6*Sqrt[1 + (d*x^6)/c]*AppellF1[5/6,
 1/2, 1, 11/6, -((d*x^6)/c), -((b*x^6)/a)] + 10*b*d*x^12*Sqrt[1 + (d*x^6)/c
]*AppellF1[11/6, 1/2, 1, 17/6, -((d*x^6)/c), -((b*x^6)/a)])/(55*a^2*c*x*Sqr
t[c + d*x^6])
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b\,x^{6}+a\right) \sqrt{d\,x^{6}+c}}dx$$

```
[In] int(1/x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x)
[Out] int(1/x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{x^2 (a+bx^6) \sqrt{c+dx^6}} \, dx = \int \frac{1}{(bx^6+a)\sqrt{dx^6+c}x^2} \, dx$$

```
[In] integrate(1/x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*x^6 + c)/(b*d*x^14 + (b*c + a*d)*x^8 + a*c*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2 (a+bx^6) \sqrt{c+dx^6}} \, dx = \int \frac{1}{x^{2} \left(a + b x^{6}\right) \sqrt{c + d x^{6}}} \, dx$$

```
[In] integrate(1/x**2/(b*x**6+a)/(d*x**6+c)**(1/2),x)
[Out] Integral(1/(x**2*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a+bx^6) \sqrt{c+dx^6}} \, dx = \int \frac{1}{(bx^6+a)\sqrt{dx^6+c}x^2} \, dx$$

```
[In] integrate(1/x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a + bx^6) \sqrt{c + dx^6}} \, dx = \int \frac{1}{(bx^6 + a)\sqrt{dx^6 + cx^2}} \, dx$$

```
[In] integrate(1/x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + bx^6) \sqrt{c + dx^6}} \, dx = \int \frac{1}{x^2 \, (b \, x^6 + a) \, \sqrt{d \, x^6 + c}} \, dx$$

```
[In] int(1/(x^2*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
[Out] int(1/(x^2*(a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.868 $\int \frac{1}{x^3(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{x^3(a+bx^6)\sqrt{c+dx^6}}\,dx = -\frac{\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(-\frac{1}{3},1,\frac{1}{2},\frac{2}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2ax^2\sqrt{c+dx^6}}$$

```
[Out] -1/2*AppellF1(-1/3,1,1/2,2/3,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a/x^2/(d*
x^6+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {476, 525, 524}

$$\int \frac{1}{x^3(a+bx^6)\sqrt{c+dx^6}}\,dx = -\frac{\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(-\frac{1}{3},1,\frac{1}{2},\frac{2}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2ax^2\sqrt{c+dx^6}}$$

```
[In] Int[1/(x^3*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/2*(Sqrt[1 + (d*x^6)/c]*AppellF1[-1/3, 1, 1/2, 2/3, -((b*x^6)/a), -((d*x^
6)/c)])/(a*x^2*Sqrt[c + d*x^6])
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\,(a+bx^3)\,\sqrt{c+dx^3}}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{1+\frac{dx^6}{c}}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx, x, x^2\right)}{2\sqrt{c+dx^6}}$$

$$= -\frac{\sqrt{1+\frac{dx^6}{c}}\,F_1\left(-\frac{1}{3};1,\frac{1}{2};\frac{2}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2ax^2\sqrt{c+dx^6}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(64) = 128.

Time = 10.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.20

$$\int \frac{1}{x^3\,(a+bx^6)\,\sqrt{c+dx^6}}\,dx$$

$$= \frac{-20a(c+dx^6)+5(-2bc+ad)x^6\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+2bdx^{12}\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}}{40a^2cx^2\sqrt{c+dx^6}}$$

```
[In] Integrate[1/(x^3*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (-20*a*(c + d*x^6) + 5*(-2*b*c + a*d)*x^6*Sqrt[1 + (d*x^6)/c]*AppellF1[2/3,
 1/2, 1, 5/3, -((d*x^6)/c), -((b*x^6)/a)] + 2*b*d*x^12*Sqrt[1 + (d*x^6)/c]*
AppellF1[5/3, 1/2, 1, 8/3, -((d*x^6)/c), -((b*x^6)/a)])/(40*a^2*c*x^2*Sqrt[
c + d*x^6])
```

Maple [F]

$$\int \frac{1}{x^{3} \left(b\,x^{6}+a \right) \sqrt{d\,x^{6}+c}}d x$$

```
[In] int(1/x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x)
[Out] int(1/x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^6) \sqrt{c+dx^6}} \, dx = \text{Timed out}$$

```
[In] integrate(1/x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^3 (a+bx^6) \sqrt{c+dx^6}} \, dx = \int \frac{1}{x^{3} \left(a + b x^{6}\right) \sqrt{c + d x^{6}}}\, dx$$

```
[In] integrate(1/x**3/(b*x**6+a)/(d*x**6+c)**(1/2),x)
[Out] Integral(1/(x**3*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (a+bx^6) \sqrt{c+dx^6}} \, dx = \int { \frac {1}{{\left (b x^{6} + a\right)} \sqrt {d x^{6} + c} x^{3}} \,d x }$$

```
[In] integrate(1/x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a + bx^6) \sqrt{c + dx^6}} dx = \int \frac{1}{(bx^6 + a)\sqrt{dx^6 + c}x^3} dx$$

```
[In] integrate(1/x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a + bx^6) \sqrt{c + dx^6}} dx = \int \frac{1}{x^3 (b x^6 + a) \sqrt{d x^6 + c}} dx$$

```
[In] int(1/(x^3*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
[Out] int(1/(x^3*(a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.869 $\int \frac{1}{x^5(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{x^5(a+bx^6)\sqrt{c+dx^6}}\,dx = -\frac{\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4ax^4\sqrt{c+dx^6}}$$

```
[Out] -1/4*AppellF1(-2/3,1,1/2,1/3,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a/x^4/(d*
x^6+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {476, 525, 524}

$$\int \frac{1}{x^5(a+bx^6)\sqrt{c+dx^6}}\,dx = -\frac{\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4ax^4\sqrt{c+dx^6}}$$

```
[In] Int[1/(x^5*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/4*(Sqrt[1 + (d*x^6)/c]*AppellF1[-2/3, 1, 1/2, 1/3, -((b*x^6)/a), -((d*x^
6)/c)])/(a*x^4*Sqrt[c + d*x^6])
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx, x, x^2\right)\\ &= \frac{\sqrt{1+\frac{dx^6}{c}}\text{Subst}\left(\int \frac{1}{x^3(a+bx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx, x, x^2\right)}{2\sqrt{c+dx^6}}\\ &= -\frac{\sqrt{1+\frac{dx^6}{c}}F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4ax^4\sqrt{c+dx^6}}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 141 vs. $2(64) = 128$.

Time = 10.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.20

$$\int \frac{1}{x^5\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = \frac{-8a(c+dx^6)-4(4bc+ad)x^6\sqrt{1+\frac{dx^6}{c}}\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)-bdx^{12}\sqrt{1+\frac{dx^6}{c}}\text{AppellF1}\left(\frac{4}{3}\right.}{32a^2cx^4\sqrt{c+dx^6}}$$

```
[In] Integrate[1/(x^5*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (-8*a*(c + d*x^6) - 4*(4*b*c + a*d)*x^6*Sqrt[1 + (d*x^6)/c]*AppellF1[1/3, 1
/2, 1, 4/3, -((d*x^6)/c), -((b*x^6)/a)] - b*d*x^12*Sqrt[1 + (d*x^6)/c]*Appe
llF1[4/3, 1/2, 1, 7/3, -((d*x^6)/c), -((b*x^6)/a)])/(32*a^2*c*x^4*Sqrt[c +
d*x^6])
```

Maple [F]

$$\int \frac{1}{x^{5} \left(b\,x^{6}+a \right) \sqrt{d\,x^{6}+c}}dx$$

```
[In] int(1/x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x)
[Out] int(1/x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (a+bx^6) \sqrt{c+dx^6}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^5 (a+bx^6) \sqrt{c+dx^6}}\,dx = \int \frac{1}{x^{5} \left(a + b x^{6}\right) \sqrt{c + d x^{6}}}\, dx$$

```
[In] integrate(1/x**5/(b*x**6+a)/(d*x**6+c)**(1/2),x)
[Out] Integral(1/(x**5*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{x^5 (a+bx^6) \sqrt{c+dx^6}}\,dx = \int { \frac{1}{{\left(b x^{6} + a\right)} \sqrt{d x^{6} + c} x^{5}} \,d x }$$

```
[In] integrate(1/x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5 (a+bx^6) \sqrt{c+dx^6}} \, dx = \int \frac{1}{(bx^6+a)\sqrt{dx^6+c}x^5} \, dx$$

```
[In] integrate(1/x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (a+bx^6) \sqrt{c+dx^6}} \, dx = \int \frac{1}{x^5 \, (b\,x^6+a) \, \sqrt{d\,x^6+c}} \, dx$$

```
[In] int(1/(x^5*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
[Out] int(1/(x^5*(a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

3.870 $\int \frac{x^{17}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 123

$$\int \frac{x^{17}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{\sqrt{c+dx^6}}{3b^2d} - \frac{a^2\sqrt{c+dx^6}}{6b^2(bc-ad)\,(a+bx^6)} + \frac{a(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{5/2}(bc-ad)^{3/2}}$$

```
[Out] 1/6*a*(-3*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(5
/2)/(-a*d+b*c)^(3/2)+1/3*(d*x^6+c)^(1/2)/b^2/d-1/6*a^2*(d*x^6+c)^(1/2)/b^2/
(-a*d+b*c)/(b*x^6+a)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 91, 81, 65, 214}

$$\int \frac{x^{17}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = -\frac{a^2\sqrt{c+dx^6}}{6b^2\,(a+bx^6)\,(bc-ad)} + \frac{a(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{5/2}(bc-ad)^{3/2}} + \frac{\sqrt{c+dx^6}}{3b^2d}$$

```
[In] Int[x^17/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] Sqrt[c + d*x^6]/(3*b^2*d) - (a^2*Sqrt[c + d*x^6])/(6*b^2*(b*c - a*d)*(a + b
*x^6)) + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*
d]])/(6*b^(5/2)*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{6}\text{Subst}\left(\int \frac{x^2}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^6\right)$$

$$= -\frac{a^2\sqrt{c+dx^6}}{6b^2(bc-ad)\,(a+bx^6)} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(2bc-ad)+b(bc-ad)x}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{6b^2(bc-ad)}$$

$$=\frac{\sqrt{c+dx^6}}{3b^2d}-\frac{a^2\sqrt{c+dx^6}}{6b^2(bc-ad)\,(a+bx^6)}-\frac{(a(4bc-3ad))\text{Subst}\left(\int\frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^6\right)}{12b^2(bc-ad)}$$

$$=\frac{\sqrt{c+dx^6}}{3b^2d}-\frac{a^2\sqrt{c+dx^6}}{6b^2(bc-ad)\,(a+bx^6)}-\frac{(a(4bc-3ad))\text{Subst}\left(\int\frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^6}\right)}{6b^2d(bc-ad)}$$

$$=\frac{\sqrt{c+dx^6}}{3b^2d}-\frac{a^2\sqrt{c+dx^6}}{6b^2(bc-ad)\,(a+bx^6)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{5/2}(bc-ad)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.06

$$\int\frac{x^{17}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\frac{\frac{\sqrt{b}\sqrt{c+dx^6}(-3a^2d+2b^2cx^6+2ab(c-dx^6))}{d(bc-ad)(a+bx^6)}+\frac{a(4bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{6b^{5/2}}$$

```
[In] Integrate[x^17/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] ((Sqrt[b]*Sqrt[c + d*x^6]*(-3*a^2*d + 2*b^2*c*x^6 + 2*a*b*(c - d*x^6)))/(d*
(b*c - a*d)*(a + b*x^6)) + (a*(4*b*c - 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^
6])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/(6*b^(5/2))
```

Maple [A] (verified)

Time = 5.36 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.08

method	result	size
pseudoelliptic	$\frac{-(b\,x^6+a)\,d\left(ad-\frac{4bc}{3}\right)a\arctan\left(\frac{b\sqrt{d\,x^6+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{(ad-bc)b}\,\sqrt{d\,x^6+c}\left(-\frac{2b^2c\,x^6}{3}-\frac{2a\left(-d\,x^6+c\right)b}{3}+a^2d\right)}{2\sqrt{(ad-bc)b}\,d\,b^2(ad-bc)(b\,x^6+a)}$	133

```
[In] int(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/((a*d-b*c)*b)^(1/2)*(-(b*x^6+a)*d*(a*d-4/3*b*c)*a*arctan(b*(d*x^6+c)^(1
/2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(d*x^6+c)^(1/2)*(-2/3*b^2*c*x^
6-2/3*a*(-d*x^6+c)*b+a^2*d))/d/b^2/(a*d-b*c)/(b*x^6+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(103) = 206$.

Time = 0.64 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.86

$$\int \frac{x^{17}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \left[\frac{((4\,ab^2cd-3\,a^2bd^2)x^6+4\,a^2bcd-3\,a^3d^2)\sqrt{b^2c-abd}\log\left(\frac{bdx^6+2\,bc-ad+2\,\sqrt{dx^6+c}\sqrt{b^2c-abd}}{bx^6+a}\right)+2\,(2\,(b^4c^2-2\right.}{12\,(ab^5c^2d-2\,a^2b^4cd^2+a^3b^3d^3+(b^6c^2d-2\,ab^5cd^2+a}$$

$$-\frac{((4\,ab^2cd-3\,a^2bd^2)x^6+4\,a^2bcd-3\,a^3d^2)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^6+c}\sqrt{-b^2c+abd}}{bdx^6+bc}\right)-(2\,(b^4c^2-2\,ab^3cd}{6\,(ab^5c^2d-2\,a^2b^4cd^2+a^3b^3d^3+(b^6c^2d-2\,ab^5cd^2+a^2b^4}$$

```
[In] integrate(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(((4*a*b^2*c*d - 3*a^2*b*d^2)*x^6 + 4*a^2*b*c*d - 3*a^3*d^2)*sqrt(b^2
*c - a*b*d)*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 + c)*sqrt(b^2*c - a*b
*d))/(b*x^6 + a)) + 2*(2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^6 + 2*a*b^
3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2)*sqrt(d*x^6 + c))/(a*b^5*c^2*d - 2*a^2*
b^4*c*d^2 + a^3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^6), -
1/6*(((4*a*b^2*c*d - 3*a^2*b*d^2)*x^6 + 4*a^2*b*c*d - 3*a^3*d^2)*sqrt(-b^2*
c + a*b*d)*arctan(sqrt(d*x^6 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c)) - (
2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^6 + 2*a*b^3*c^2 - 5*a^2*b^2*c*d +
 3*a^3*b*d^2)*sqrt(d*x^6 + c))/(a*b^5*c^2*d - 2*a^2*b^4*c*d^2 + a^3*b^3*d^3
 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{17}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \text{Timed out}$$

```
[In] integrate(x**17/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{17}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.09

$$\int \frac{x^{17}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = -\frac{\sqrt{dx^6+c}a^2d}{6\,(b^3c-ab^2d)((dx^6+c)b-bc+ad)} - \frac{(4\,abc-3\,a^2d)\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{6\,(b^3c-ab^2d)\sqrt{-b^2c+abd}} + \frac{\sqrt{dx^6+c}}{3\,b^2d}$$

```
[In] integrate(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(d*x^6 + c)*a^2*d/((b^3*c - a*b^2*d)*((d*x^6 + c)*b - b*c + a*d))
- 1/6*(4*a*b*c - 3*a^2*d)*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/((
b^3*c - a*b^2*d)*sqrt(-b^2*c + a*b*d)) + 1/3*sqrt(d*x^6 + c)/(b^2*d)
```

Mupad [B] (verification not implemented)

Time = 9.47 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.17

$$\int \frac{x^{17}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{\sqrt{d\,x^6+c}}{3\,b^2\,d} - \frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^6+c}\,(3\,a\,d-4\,b\,c)}{(3\,a^2\,d-4\,a\,b\,c)\,\sqrt{a\,d-b\,c}}\right)\,(3\,a\,d-4\,b\,c)}{6\,b^{5/2}\,(a\,d-b\,c)^{3/2}} + \frac{a^2\,d\,\sqrt{d\,x^6+c}}{2\,(a\,d-b\,c)\,\left(3\,b^3\,\left(d\,x^6+c\right)-3\,b^3\,c+3\,a\,b^2\,d\right)}$$

```
[In] int(x^17/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] (c + d*x^6)^(1/2)/(3*b^2*d) - (a*atan((a*b^(1/2)*(c + d*x^6)^(1/2)*(3*a*d -
 4*b*c))/((3*a^2*d - 4*a*b*c)*(a*d - b*c)^(1/2)))*(3*a*d - 4*b*c))/(6*b^(5/
2)*(a*d - b*c)^(3/2)) + (a^2*d*(c + d*x^6)^(1/2))/(2*(a*d - b*c)*(3*b^3*(c
+ d*x^6) - 3*b^3*c + 3*a*b^2*d))
```

3.871 $\int \frac{x^{11}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 99

$$\int \frac{x^{11}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\frac{a\sqrt{c+dx^6}}{6b(bc-ad)(a+bx^6)}-\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{3/2}(bc-ad)^{3/2}}$$

```
[Out] -1/6*(-a*d+2*b*c)*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)
/(-a*d+b*c)^(3/2)+1/6*a*(d*x^6+c)^(1/2)/b/(-a*d+b*c)/(b*x^6+a)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {457, 79, 65, 214}

$$\int \frac{x^{11}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\frac{a\sqrt{c+dx^6}}{6b(a+bx^6)(bc-ad)}-\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{3/2}(bc-ad)^{3/2}}$$

```
[In] Int[x^11/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (a*Sqrt[c + d*x^6])/(6*b*(b*c - a*d)*(a + b*x^6)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(6*b^(3/2)*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{6}\text{Subst}\left(\int \frac{x}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^6\right)\\
&= \frac{a\sqrt{c+dx^6}}{6b(bc-ad)\left(a+bx^6\right)} + \frac{(2bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{12b(bc-ad)}\\
&= \frac{a\sqrt{c+dx^6}}{6b(bc-ad)\left(a+bx^6\right)} + \frac{(2bc-ad)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{6bd(bc-ad)}\\
&= \frac{a\sqrt{c+dx^6}}{6b(bc-ad)\left(a+bx^6\right)} - \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{3/2}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

$$\int \frac{x^{11}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{\frac{a\sqrt{b}\sqrt{c+dx^6}}{(bc-ad)(a+bx^6)} - \frac{(2bc-ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{6b^{3/2}}$$

```
[In] Integrate[x^11/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] ((a*Sqrt[b]*Sqrt[c + d*x^6])/((b*c - a*d)*(a + b*x^6)) - ((2*b*c - a*d)*Arc
Tan[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/(6
*b^(3/2))
```

Maple [A] (verified)

Time = 5.32 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$\frac{-\frac{a\sqrt{d\,x^6+c}}{b\,x^6+a}+\frac{(ad-2bc)\arctan\left(\frac{b\sqrt{d\,x^6+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}}{6(ad-bc)b}$	83

```
[In] int(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/(a*d-b*c)/b*(-a*(d*x^6+c)^(1/2)/(b*x^6+a)+(a*d-2*b*c)/((a*d-b*c)*b)^(1/
2)*arctan(b*(d*x^6+c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 348, normalized size of antiderivative = 3.52

$$\int \frac{x^{11}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \left[\frac{((2\,b^2c-abd)x^6+2\,abc-a^2d)\sqrt{b^2c-abd}\log\left(\frac{bdx^6+2\,bc-ad-2\,\sqrt{dx^6+c}\sqrt{b^2c-abd}}{bx^6+a}\right)+2\,\sqrt{dx^6+c}(ab^2c-a^2bd)}{12\,(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2+(b^5c^2-2\,ab^4cd+a^2b^3d^2)x^6)}\right.$$

```
[In] integrate(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(((2*b^2*c - a*b*d)*x^6 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b
*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*sqrt(b^2*c - a*b*d))/(b*x^6 + a))
+ 2*sqrt(d*x^6 + c)*(a*b^2*c - a^2*b*d))/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b
^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^6), 1/6*(((2*b^2*c - a*b*d
```

```
)*x^6 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^6 + c)*sqrt(-
b^2*c + a*b*d)/(b*d*x^6 + b*c)) + sqrt(d*x^6 + c)*(a*b^2*c - a^2*b*d))/(a*b
^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2
)*x^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\text{Timed out}$$

```
[In] integrate(x**11/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.17

$$\int \frac{x^{11}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\frac{\frac{\sqrt{dx^6+c}ad^2}{(b^2c-abd)((dx^6+c)b-bc+ad)}+\frac{(2\,bcd-ad^2)\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^2c-abd)\sqrt{-b^2c+abd}}}{6\,d}$$

```
[In] integrate(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] 1/6*(sqrt(d*x^6 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^6 + c)*b - b*c + a*d)) +
(2*b*c*d - a*d^2)*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c -
a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

Mupad [B] (verification not implemented)

Time = 9.80 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.96

$$\int \frac{x^{11}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^6+c}}{\sqrt{a\,d-b\,c}}\right)\,(a\,d-2\,b\,c)}{6\,b^{3/2}\,(a\,d-b\,c)^{3/2}} - \frac{a\,d\,\sqrt{d\,x^6+c}}{2\,b\,(a\,d-b\,c)\,(3\,b\,(d\,x^6+c)+3\,a\,d-3\,b\,c)}$$

```
[In] int(x^11/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] (atan((b^(1/2)*(c + d*x^6)^(1/2))/(a*d - b*c)^(1/2))*(a*d - 2*b*c))/(6*b^(3
/2)*(a*d - b*c)^(3/2)) - (a*d*(c + d*x^6)^(1/2))/(2*b*(a*d - b*c)*(3*b*(c +
 d*x^6) + 3*a*d - 3*b*c))
```

3.872 $\int \frac{x^5}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{x^5}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = -\frac{\sqrt{c+dx^6}}{6(bc-ad)\,(a+bx^6)} + \frac{d\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6\sqrt{b}(bc-ad)^{3/2}}$$

```
[Out] 1/6*d*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(3/2)/b^
(1/2)-1/6*(d*x^6+c)^(1/2)/(-a*d+b*c)/(b*x^6+a)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {455, 44, 65, 214}

$$\int \frac{x^5}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{d\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6\sqrt{b}(bc-ad)^{3/2}} - \frac{\sqrt{c+dx^6}}{6\,(a+bx^6)\,(bc-ad)}$$

```
[In] Int[x^5/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/6*Sqrt[c + d*x^6]/((b*c - a*d)*(a + b*x^6)) + (d*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^6])/Sqrt[b*c - a*d]])/(6*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !Int
```

```
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{6}\text{Subst}\left(\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^6\right)\\
&= -\frac{\sqrt{c+dx^6}}{6(bc-ad)\left(a+bx^6\right)} - \frac{d\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{12(bc-ad)}\\
&= -\frac{\sqrt{c+dx^6}}{6(bc-ad)\left(a+bx^6\right)} - \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{6(bc-ad)}\\
&= -\frac{\sqrt{c+dx^6}}{6(bc-ad)\left(a+bx^6\right)} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6\sqrt{b}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \frac{x^5}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\frac{1}{6}\left(-\frac{\sqrt{c+dx^6}}{(bc-ad)(a+bx^6)}+\frac{d\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}(-bc+ad)^{3/2}}\right)$$

```
[In] Integrate[x^5/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
[Out] (-(Sqrt[c + d*x^6]/((b*c - a*d)*(a + b*x^6))) + (d*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^6])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/6
```

Maple [A] (verified)

Time = 5.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

method	result	size
pseudoelliptic	$\frac{d(b\,x^6+a)\arctan\left(\frac{b\sqrt{d\,x^6+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{d\,x^6+c}\,\sqrt{(ad-bc)b}}{6\sqrt{(ad-bc)b}\,(ad-bc)(b\,x^6+a)}$	90

```
[In] int(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/6*(d*(b*x^6+a)*arctan(b*(d*x^6+c)^(1/2)/((a*d-b*c)*b)^(1/2))+(d*x^6+c)^(1
/2)*((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)/(a*d-b*c)/(b*x^6+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(71) = 142.

Time = 0.41 (sec) , antiderivative size = 302, normalized size of antiderivative = 3.47

$$\int \frac{x^5}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \left[-\frac{(bdx^6+ad)\sqrt{b^2c-abd}\log\left(\frac{bdx^6+2\,bc-ad-2\,\sqrt{dx^6+c}\sqrt{b^2c-abd}}{bx^6+a}\right)+2\,\sqrt{dx^6+c}(b^2c-abd)}{12\,((b^4c^2-2\,ab^3cd+a^2b^2d^2)x^6+ab^3c^2-2\,a^2b^2cd+a^3bd^2)}, -\frac{(bdx^6+ad)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^6+c}\sqrt{-b^2c+abd}}{bdx^6+bc}\right)+\sqrt{dx^6+c}(b^2c-abd)}{6\,((b^4c^2-2\,ab^3cd+a^2b^2d^2)x^6+ab^3c^2-2\,a^2b^2cd+a^3bd^2)}\right]$$

```
[In] integrate(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
[Out] [-1/12*((b*d*x^6 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^6 + 2*b*c - a*d - 2*
sqrt(d*x^6 + c)*sqrt(b^2*c - a*b*d))/(b*x^6 + a)) + 2*sqrt(d*x^6 + c)*(b^2*
```

```
c - a*b*d))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^6 + a*b^3*c^2 - 2*a^2*
b^2*c*d + a^3*b*d^2), -1/6*((b*d*x^6 + a*d)*sqrt(-b^2*c + a*b*d)*arctan(sqr
t(d*x^6 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c)) + sqrt(d*x^6 + c)*(b^2*c
 - a*b*d))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^6 + a*b^3*c^2 - 2*a^2*b
^2*c*d + a^3*b*d^2)]
```

Sympy [F]

$$\int \frac{x^5}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x^5}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$

```
[In] integrate(x**5/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
[Out] Integral(x**5/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.07

$$\int \frac{x^5}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = -\frac{d\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{6\sqrt{-b^2c+abd}(bc-ad)} - \frac{\sqrt{dx^6+c}d}{6\left((dx^6+c)b-bc+ad\right)(bc-ad)}$$

```
[In] integrate(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] -1/6*d*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*(b*c - a*d)) - 1/6*sqrt(d*x^6 + c)*d/(((d*x^6 + c)*b - b*c + a*d)*(b*c - a
*d))
```

Mupad [B] (verification not implemented)

Time = 9.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.97

$$\int \frac{x^5}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\frac{d\sqrt{d\,x^6+c}}{2\,\left(a\,d-b\,c\right)\,\left(3\,b\,\left(d\,x^6+c\right)+3\,a\,d-3\,b\,c\right)}+\frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^6+c}}{\sqrt{a\,d-b\,c}}\right)}{6\,\sqrt{b}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

```
[In] int(x^5/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] (d*(c + d*x^6)^(1/2))/(2*(a*d - b*c)*(3*b*(c + d*x^6) + 3*a*d - 3*b*c)) + (
d*atan((b^(1/2)*(c + d*x^6)^(1/2))/(a*d - b*c)^(1/2)))/(6*b^(1/2)*(a*d - b*
c)^(3/2))
```

3.873 $\int \frac{1}{x(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 132

$$\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\frac{b\sqrt{c+dx^6}}{6a(bc-ad)\left(a+bx^6\right)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}+\frac{\sqrt{b}(2bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6a^2(bc-ad)^{3/2}}$$

```
[Out] 1/6*(-3*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2
)/a^2/(-a*d+b*c)^(3/2)-1/3*arctanh((d*x^6+c)^(1/2)/c^(1/2))/a^2/c^(1/2)+1/6
*b*(d*x^6+c)^(1/2)/a/(-a*d+b*c)/(b*x^6+a)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 105, 162, 65, 214}

$$\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\frac{\sqrt{b}(2bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6a^2(bc-ad)^{3/2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}+\frac{b\sqrt{c+dx^6}}{6a\left(a+bx^6\right)(bc-ad)}$$

```
[In] Int[1/(x*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (b*Sqrt[c + d*x^6])/(6*a*(b*c - a*d)*(a + b*x^6)) - ArcTanh[Sqrt[c + d*x^6]
/Sqrt[c]]/(3*a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[
c + d*x^6])/Sqrt[b*c - a*d]])/(6*a^2*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{6}\text{Subst}\left(\int \frac{1}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^6\right)$$

$$= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)\,(a+bx^6)} + \frac{\text{Subst}\left(\int \frac{bc-ad+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{6a(bc-ad)}$$

$$
\begin{aligned}
&= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)\left(a+bx^6\right)} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^6\right)}{6a^2} \\
&\quad - \frac{(b(2bc-3ad))\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^6\right)}{12a^2(bc-ad)} \\
&= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)\left(a+bx^6\right)} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^6}\right)}{3a^2d} \\
&\quad - \frac{(b(2bc-3ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^6}\right)}{6a^2d(bc-ad)} \\
&= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)\left(a+bx^6\right)} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} + \frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6a^2(bc-ad)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.94

$$
\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{-\frac{ab\sqrt{c+dx^6}}{(-bc+ad)(a+bx^6)} + \frac{\sqrt{b}(2bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}} - \frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{\sqrt{c}}}{6a^2}
$$

```
[In] Integrate[1/(x*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (-((a*b*Sqrt[c + d*x^6])/((-(b*c) + a*d)*(a + b*x^6))) + (Sqrt[b]*(2*b*c -
3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)
^(3/2) - (2*ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]])/Sqrt[c])/(6*a^2)
```

Maple [A] (verified)

Time = 5.35 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11

method	result
pseudoelliptic	$-\frac{-2\left(b\,x^6+a\right)\left(bc-\frac{3ad}{2}\right)\sqrt{c}\,b\arctan\left(\frac{b\sqrt{d\,x^6+c}}{\sqrt{(ad-bc)b}}\right)+\left(2(ad-bc)\left(b\,x^6+a\right)\text{arctanh}\left(\frac{\sqrt{d\,x^6+c}}{\sqrt{c}}\right)+\sqrt{d\,x^6+c}\,\sqrt{c}\,ab\right)\sqrt{(ad-bc)b}}{6\sqrt{c}\,\sqrt{(ad-bc)b}\,a^2(ad-bc)\left(b\,x^6+a\right)}$

```
[In] int(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(-2*(b*x^6+a)*(b*c-3/2*a*d)*c^(1/2)*b*arctan(b*(d*x^6+c)^(1/2)/((a*d-b
*c)*b)^(1/2))+(2*(a*d-b*c)*(b*x^6+a)*arctanh((d*x^6+c)^(1/2)/c^(1/2))+(d*x^
```

```
6+c)^(1/2)*c^(1/2)*a*b)*((a*d-b*c)*b)^(1/2))/c^(1/2)/((a*d-b*c)*b)^(1/2)/a^
2/(a*d-b*c)/(b*x^6+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 862, normalized size of antiderivative = 6.53

$$\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \left[\frac{2\sqrt{dx^6+c}abc+((2\,b^2c^2-3\,abcd)x^6+2\,abc^2-3\,a^2cd)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^6+2\,bc-ad+2\,\sqrt{dx^6+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^6+a}\right)+}{12\left((a^2b^2c^2-a^3bcd)x^6+a^3bc^2-a^4cd\right)}\right.$$

```
[In] integrate(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(2*sqrt(d*x^6 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^6 + 2*a*b*c^2 -
 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 +
 c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + 2*((b^2*c - a*b*d)*x^6
+ a*b*c - a^2*d)*sqrt(c)*log((d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6)
)/((a^2*b^2*c^2 - a^3*b*c*d)*x^6 + a^3*b*c^2 - a^4*c*d), 1/6*(sqrt(d*x^6 +
c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^6 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b
*c - a*d))*arctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^
6 + b*c)) + ((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(c)*log((d*x^6 - 2*sq
rt(d*x^6 + c)*sqrt(c) + 2*c)/x^6))/((a^2*b^2*c^2 - a^3*b*c*d)*x^6 + a^3*b*c
^2 - a^4*c*d), 1/12*(2*sqrt(d*x^6 + c)*a*b*c + 4*((b^2*c - a*b*d)*x^6 + a*b
*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c) + ((2*b^2*c^2 - 3*a
*b*c*d)*x^6 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b
*c - a*d + 2*sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)))
/((a^2*b^2*c^2 - a^3*b*c*d)*x^6 + a^3*b*c^2 - a^4*c*d), 1/6*(sqrt(d*x^6 + c
)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^6 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b*
c - a*d))*arctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^6
 + b*c)) + 2*((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x
^6 + c)*sqrt(-c)/c))/((a^2*b^2*c^2 - a^3*b*c*d)*x^6 + a^3*b*c^2 - a^4*c*d)]
```

Sympy [F]

$$\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$

```
[In] integrate(1/x/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
[Out] Integral(1/(x*(a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}x}\,dx$$

```
[In] integrate(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\frac{\sqrt{dx^6+c}bd}{6\,(abc-a^2d)((dx^6+c)b-bc+ad)}-\frac{(2\,b^2c-3\,abd)\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{6\,(a^2bc-a^3d)\sqrt{-b^2c+abd}}+\frac{\arctan\left(\frac{\sqrt{dx^6+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}}$$

```
[In] integrate(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] 1/6*sqrt(d*x^6 + c)*b*d/((a*b*c - a^2*d)*((d*x^6 + c)*b - b*c + a*d)) - 1/6
*(2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b
*c - a^3*d)*sqrt(-b^2*c + a*b*d)) + 1/3*arctan(sqrt(d*x^6 + c)/sqrt(-c))/(a
^2*sqrt(-c))
```

Mupad [B] (verification not implemented)

Time = 10.31 (sec) , antiderivative size = 3025, normalized size of antiderivative = 22.92

$$\int \frac{1}{x\,(a+bx^6)^2\,\sqrt{c+dx^6}}\,dx = \text{Too large to display}$$

```
[In] int(1/(x*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] (atan((((((c + d*x^6)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^
3))/(18*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - ((((4*a^6*b^2*d^5)/3 - 2*a
^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)
 - ((c + d*x^6)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(144*a^7*b^2
*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^3))/(216
*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c
^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(12*(a^5*
d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*
(a*d - b*c)^3)^(1/2)*1i)/(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a
^4*b*c*d^2)) + ((((c + d*x^6)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*
b^4*c*d^3))/(18*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + ((((4*a^6*b^2*d^5)
/3 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^
4*b*c*d) + ((c + d*x^6)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(144
*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^
3))/(216*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a
^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(
12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b
*c)*(-b*(a*d - b*c)^3)^(1/2)*1i)/(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2
*d - 3*a^4*b*c*d^2)))/(((a*b^3*d^4)/18 - (b^4*c*d^3)/27)/(a^5*d^2 + a^3*b^2
*c^2 - 2*a^4*b*c*d) - ((((c + d*x^6)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2
 - 20*a*b^4*c*d^3))/(18*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - ((((4*a^6*b
^2*d^5)/3 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(a^5*d^2 + a^3*b^2*c^2
 - 2*a^4*b*c*d) - ((c + d*x^6)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/
2)*(144*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4
*c^2*d^3))/(216*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^
3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(
1/2))/(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*
d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2
*c^2*d - 3*a^4*b*c*d^2)) + ((((c + d*x^6)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2
*d^2 - 20*a*b^4*c*d^3))/(18*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + ((((4*
a^6*b^2*d^5)/3 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(a^5*d^2 + a^3*b^
2*c^2 - 2*a^4*b*c*d) + ((c + d*x^6)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3
)^(1/2)*(144*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^
5*b^4*c^2*d^3))/(216*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b
^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)
^3)^(1/2))/(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*
```

```
(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^
3*b^2*c^2*d - 3*a^4*b*c*d^2))))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*1i
)/(6*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) - (atan((((
(((4*a^6*b^2*d^5)/3 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(6*(a^5*d^2
+ a^3*b^2*c^2 - 2*a^4*b*c*d)) - ((c + d*x^6)^(1/2)*(144*a^7*b^2*d^5 - 576*a
^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^3))/(648*a^2*c^(1/2)
*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(6*a^2*c^(1/2)) - ((c + d*x^6)^(1/
2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(108*(a^4*d^2 + a^2*b
^2*c^2 - 2*a^3*b*c*d)))*1i)/(a^2*c^(1/2)) - (((((4*a^6*b^2*d^5)/3 - 2*a^5*b
^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(6*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d))
 + ((c + d*x^6)^(1/2)*(144*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^
3*d^2 + 720*a^5*b^4*c^2*d^3))/(648*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a
^3*b*c*d)))/(6*a^2*c^(1/2)) + ((c + d*x^6)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^
2*d^2 - 20*a*b^4*c*d^3))/(108*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))*1i)/(
a^2*c^(1/2)))/(((a*b^3*d^4)/18 - (b^4*c*d^3)/27)/(a^5*d^2 + a^3*b^2*c^2 - 2
*a^4*b*c*d) + ((((4*a^6*b^2*d^5)/3 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/
3)/(6*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) - ((c + d*x^6)^(1/2)*(144*a^7*
b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^3))/(
648*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(6*a^2*c^(1/2)) - (
(c + d*x^6)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(108*(
a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(a^2*c^(1/2)) + ((((4*a^6*b^2*d^5)/3
 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(6*(a^5*d^2 + a^3*b^2*c^2 - 2*a
^4*b*c*d)) + ((c + d*x^6)^(1/2)*(144*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*
a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^3))/(648*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2
*c^2 - 2*a^3*b*c*d)))/(6*a^2*c^(1/2)) + ((c + d*x^6)^(1/2)*(13*a^2*b^3*d^4
+ 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(108*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*
d)))/(a^2*c^(1/2))))*1i)/(3*a^2*c^(1/2)) - (b*d*(c + d*x^6)^(1/2))/(2*(a^2*
d - a*b*c)*(3*b*(c + d*x^6) + 3*a*d - 3*b*c))
```

3.874 $\int \frac{1}{x^7(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 185

$$\int \frac{1}{x^7\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=-\frac{b(2bc-ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)\left(a+bx^6\right)}-\frac{\sqrt{c+dx^6}}{6acx^6\left(a+bx^6\right)}+\frac{(4bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^3c^{3/2}}-\frac{b^{3/2}(4bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6a^3(bc-ad)^{3/2}}$$

```
[Out] 1/6*(a*d+4*b*c)*arctanh((d*x^6+c)^(1/2)/c^(1/2))/a^3/c^(3/2)-1/6*b^(3/2)*(-
5*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/a^3/(-a*d+b*
c)^(3/2)-1/6*b*(-a*d+2*b*c)*(d*x^6+c)^(1/2)/a^2/c/(-a*d+b*c)/(b*x^6+a)-1/6*
(d*x^6+c)^(1/2)/a/c/x^6/(b*x^6+a)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {457, 105, 156, 162, 65, 214}

$$\int \frac{1}{x^7\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=-\frac{b^{3/2}(4bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\text{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^3c^{3/2}}-\frac{b\sqrt{c+dx^6}(2bc-ad)}{6a^2c\left(a+bx^6\right)(bc-ad)}-\frac{\sqrt{c+dx^6}}{6acx^6\left(a+bx^6\right)}$$

```
[In] Int[1/(x^7*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/6*(b*(2*b*c - a*d)*Sqrt[c + d*x^6])/(a^2*c*(b*c - a*d)*(a + b*x^6)) - Sq
rt[c + d*x^6]/(6*a*c*x^6*(a + b*x^6)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x
^6]/Sqrt[c]])/(6*a^3*c^(3/2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*S
qrt[c + d*x^6])/Sqrt[b*c - a*d]])/(6*a^3*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{6}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2\sqrt{c+dx}}\,dx, x, x^6\right)$$

$$= -\frac{\sqrt{c+dx^6}}{6acx^6\left(a+bx^6\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4bc+ad)+\frac{3bdx}{2}}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^6\right)}{6ac}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)\left(a+bx^6\right)} - \frac{\sqrt{c+dx^6}}{6acx^6\left(a+bx^6\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-ad)(4bc+ad)+\frac{1}{2}bd(2bc-ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{6a^2c(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)\left(a+bx^6\right)} - \frac{\sqrt{c+dx^6}}{6acx^6\left(a+bx^6\right)} + \frac{\left(b^2(4bc-5ad)\right)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{12a^3(bc-ad)} - \frac{(4bc+ad)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^6\right)}{12a^3c}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)\left(a+bx^6\right)} - \frac{\sqrt{c+dx^6}}{6acx^6\left(a+bx^6\right)} + \frac{\left(b^2(4bc-5ad)\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{6a^3d(bc-ad)} - \frac{(4bc+ad)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{6a^3cd}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)\left(a+bx^6\right)} - \frac{\sqrt{c+dx^6}}{6acx^6\left(a+bx^6\right)} + \frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^3c^{3/2}} - \frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6a^3(bc-ad)^{3/2}}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^7 (a+bx^6)^2 \sqrt{c+dx^6}} \, dx = \frac{\frac{a\sqrt{c+dx^6}(-a^2d+2b^2cx^6+ab(c-dx^6))}{c(-bc+ad)x^6(a+bx^6)} - \frac{b^{3/2}(4bc-5ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}} + \frac{(4bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{c^{3/2}}}{6a^3}$$

```
[In] Integrate[1/(x^7*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] ((a*Sqrt[c + d*x^6]*(-(a^2*d) + 2*b^2*c*x^6 + a*b*(c - d*x^6)))/(c*(-(b*c)
+ a*d)*x^6*(a + b*x^6)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^6])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2) + ((4*b*c + a*d)*ArcTanh
[Sqrt[c + d*x^6]/Sqrt[c]])/c^(3/2))/(6*a^3)
```

Maple [A] (verified)

Time = 5.45 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$\frac{-4x^6(b\,x^6+a)\left(bc-\frac{5ad}{4}\right)b^2c^{\frac{5}{2}}\arctan\left(\frac{b\sqrt{d\,x^6+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{(ad-bc)b}\left(c\,x^6(b\,x^6+a)(ad+4bc)(ad-bc)\,\text{arctanh}\left(\frac{\sqrt{d\,x^6+c}}{\sqrt{c}}\right)+\left(-\right.\right.}{6\sqrt{(ad-bc)b}\,c^{\frac{5}{2}}a^3(ad-bc)(b\,x^6+a)x^6}$

```
[In] int(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/((a*d-b*c)*b)^(1/2)/c^(5/2)*(-4*x^6*(b*x^6+a)*(b*c-5/4*a*d)*b^2*c^(5/2)
*arctan(b*(d*x^6+c)^(1/2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(c*x^6*(
b*x^6+a)*(a*d+4*b*c)*(a*d-b*c)*arctanh((d*x^6+c)^(1/2)/c^(1/2))+(-a*d*(b*x^
6+a)*c^(3/2)+b*(2*b*x^6+a)*c^(5/2))*(d*x^6+c)^(1/2)*a))/a^3/(a*d-b*c)/(b*x^
6+a)/x^6
```

Fricas [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 1236, normalized size of antiderivative = 6.68

$$\int \frac{1}{x^7 (a+bx^6)^2 \sqrt{c+dx^6}} \, dx = \text{Too large to display}$$

```
[In] integrate(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^12 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^
6)*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*(b*c
```

```
- a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*
b*d^2)*x^12 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^6)*sqrt(c)*log((d*x^6
 + 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6) - 2*((2*a*b^2*c^2 - a^2*b*c*d)*x^6
 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^6 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^12
+ (a^4*b*c^3 - a^5*c^2*d)*x^6), -1/12*(2*((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^12
+ (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^6)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^
6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^6 + b*c)) - ((4*b^3*c^2 - 3*
a*b^2*c*d - a^2*b*d^2)*x^12 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^6)*sq
rt(c)*log((d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6) + 2*((2*a*b^2*c^2
- a^2*b*c*d)*x^6 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^6 + c))/((a^3*b^2*c^3 - a^
4*b*c^2*d)*x^12 + (a^4*b*c^3 - a^5*c^2*d)*x^6), -1/12*(2*((4*b^3*c^2 - 3*a*
b^2*c*d - a^2*b*d^2)*x^12 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^6)*sqrt
(-c)*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c) - ((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^12
 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^6)*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*
b*c - a*d - 2*sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a))
 + 2*((2*a*b^2*c^2 - a^2*b*c*d)*x^6 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^6 + c))
/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^12 + (a^4*b*c^3 - a^5*c^2*d)*x^6), -1/6*(((
4*b^3*c^3 - 5*a*b^2*c^2*d)*x^12 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^6)*sqrt(-
b/(b*c - a*d))*arctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*
d*x^6 + b*c)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^12 + (4*a*b^2*c^2
- 3*a^2*b*c*d - a^3*d^2)*x^6)*sqrt(-c)*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c) +
 ((2*a*b^2*c^2 - a^2*b*c*d)*x^6 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^6 + c))/((a
^3*b^2*c^3 - a^4*b*c^2*d)*x^12 + (a^4*b*c^3 - a^5*c^2*d)*x^6)]
```

Sympy [F]

$$\int \frac{1}{x^7 (a+bx^6)^2 \sqrt{c+dx^6}}\, dx = \int \frac{1}{x^7 (a+bx^6)^2 \sqrt{c+dx^6}}\, dx$$

```
[In] integrate(1/x**7/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
[Out] Integral(1/(x**7*(a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{x^7 (a+bx^6)^2 \sqrt{c+dx^6}}\, dx = \int \frac{1}{(bx^6+a)^2 \sqrt{dx^6+c}x^7}\, dx$$

```
[In] integrate(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^7), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^7 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \frac{(4\,b^3c-5\,ab^2d)\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{6\,(a^3bc-a^4d)\sqrt{-b^2c+abd}} - \frac{2\,(dx^6+c)^{\frac{3}{2}}b^2cd-2\,\sqrt{dx^6+c}b^2c^2d-(dx^6+c)^{\frac{3}{2}}abd^2+2\,\sqrt{dx^6+c}abcd^2-\sqrt{dx^6+c}a^2d^3}{6\,(a^2bc^2-a^3cd)\left((dx^6+c)^2b-2\,(dx^6+c)bc+bc^2+(dx^6+c)ad-acd\right)} - \frac{(4\,bc+ad)\arctan\left(\frac{\sqrt{dx^6+c}}{\sqrt{-c}}\right)}{6\,a^3\sqrt{-c}c}$$

```
[In] integrate(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/((
a^3*b*c - a^4*d)*sqrt(-b^2*c + a*b*d)) - 1/6*(2*(d*x^6 + c)^(3/2)*b^2*c*d -
 2*sqrt(d*x^6 + c)*b^2*c^2*d - (d*x^6 + c)^(3/2)*a*b*d^2 + 2*sqrt(d*x^6 + c
)*a*b*c*d^2 - sqrt(d*x^6 + c)*a^2*d^3)/((a^2*b*c^2 - a^3*c*d)*((d*x^6 + c)^
2*b - 2*(d*x^6 + c)*b*c + b*c^2 + (d*x^6 + c)*a*d - a*c*d)) - 1/6*(4*b*c +
a*d)*arctan(sqrt(d*x^6 + c)/sqrt(-c))/(a^3*sqrt(-c)*c)
```

Mupad [B] (verification not implemented)

Time = 11.63 (sec) , antiderivative size = 3860, normalized size of antiderivative = 20.86

$$\int \frac{1}{x^7 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \text{Too large to display}$$

```
[In] int(1/(x^7*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] (((c + d*x^6)^(1/2)*(a^2*d^3 + 2*b^2*c^2*d - 2*a*b*c*d^2))/(2*a^2*(b*c^2 -
a*c*d)) + (b*(c + d*x^6)^(3/2)*(a*d^2 - 2*b*c*d))/(2*a^2*(b*c^2 - a*c*d)))/
((c + d*x^6)*(3*a*d - 6*b*c) + 3*b*(c + d*x^6)^2 + 3*b*c^2 - 3*a*c*d) + (at
an((((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^6)^(1/2)*(a^4*b^
3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^
2*d^4))/(18*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((-b^3*(a*d - b*
c)^3)^(1/2)*(5*a*d - 4*b*c)*((144*a^9*b^2*c*d^6 + 288*a^6*b^5*c^4*d^3 - 576
*a^7*b^4*c^3*d^4 + 144*a^8*b^3*c^2*d^5)/(216*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2
*a^7*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^6)^(1/2)*(5*a*d - 4*b
*c)*(288*a^6*b^5*c^5*d^2 - 720*a^7*b^4*c^4*d^3 + 576*a^8*b^3*c^3*d^4 - 144*
a^9*b^2*c^2*d^5))/(216*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3
 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(12*(a^6*d^3 - a^3*b^3
*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*1i)/(12*(a^6*d^3 - a^3*b^3*c^3 +
```

3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*
b*c)*(((c + d*x^6)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 +
6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(18*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*
a^5*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*((144*a^9*b^2*c
*d^6 + 288*a^6*b^5*c^4*d^3 - 576*a^7*b^4*c^3*d^4 + 144*a^8*b^3*c^2*d^5)/(21
6*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) + ((-b^3*(a*d - b*c)^3)^(1/2
)*(c + d*x^6)^(1/2)*(5*a*d - 4*b*c)*(288*a^6*b^5*c^5*d^2 - 720*a^7*b^4*c^4*
d^3 + 576*a^8*b^3*c^3*d^4 - 144*a^9*b^2*c^2*d^5))/(216*(a^4*b^2*c^4 + a^6*c
^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*
c*d^2))))/(12*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*1
i)/(12*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))/((5*a^3*
b^4*d^6 + 32*b^7*c^3*d^3 - 48*a*b^6*c^2*d^4 + 6*a^2*b^5*c*d^5)/(108*(a^6*b^
2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d
- 4*b*c)*(((c + d*x^6)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d
^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(18*(a^4*b^2*c^4 + a^6*c^2*d^2
- 2*a^5*b*c^3*d)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*((144*a^9*b
^2*c*d^6 + 288*a^6*b^5*c^4*d^3 - 576*a^7*b^4*c^3*d^4 + 144*a^8*b^3*c^2*d^5)
/(216*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^
(1/2)*(c + d*x^6)^(1/2)*(5*a*d - 4*b*c)*(288*a^6*b^5*c^5*d^2 - 720*a^7*b^4*
c^4*d^3 + 576*a^8*b^3*c^3*d^4 - 144*a^9*b^2*c^2*d^5))/(216*(a^4*b^2*c^4 + a
^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^
5*b*c*d^2))))/(12*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)
)))/(12*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + ((-b^3
*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^6)^(1/2)*(a^4*b^3*d^6 + 32
*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(1
8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2
)*(5*a*d - 4*b*c)*((144*a^9*b^2*c*d^6 + 288*a^6*b^5*c^4*d^3 - 576*a^7*b^4*c
^3*d^4 + 144*a^8*b^3*c^2*d^5)/(216*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3
d)) + ((-b^3(a*d - b*c)^3)^(1/2)*(c + d*x^6)^(1/2)*(5*a*d - 4*b*c)*(288*a
^6*b^5*c^5*d^2 - 720*a^7*b^4*c^4*d^3 + 576*a^8*b^3*c^3*d^4 - 144*a^9*b^2*c^
2*d^5))/(216*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3
*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(12*(a^6*d^3 - a^3*b^3*c^3 + 3*a
^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(12*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2
*d - 3*a^5*b*c*d^2))))*(-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*1i)/(6*(a
^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + (atan((((((c + d
x^6)^(1/2)(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*
d^5 + 26*a^2*b^5*c^2*d^4))/(18*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d))
+ (((144*a^9*b^2*c*d^6 + 288*a^6*b^5*c^4*d^3 - 576*a^7*b^4*c^3*d^4 + 144*a
^8*b^3*c^2*d^5)/(216*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) - ((c + d
x^6)^(1/2)(a*d + 4*b*c)*(288*a^6*b^5*c^5*d^2 - 720*a^7*b^4*c^4*d^3 + 576*
a^8*b^3*c^3*d^4 - 144*a^9*b^2*c^2*d^5))/(216*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 +
a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(12*a^3*(c^3)^(1/2)))*(a*d +
4*b*c)*1i)/(12*a^3*(c^3)^(1/2)) + ((((c + d*x^6)^(1/2)*(a^4*b^3*d^6 + 32*b
^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(18*
(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - (((144*a^9*b^2*c*d^6 + 288*a

^6*b^5*c^4*d^3 - 576*a^7*b^4*c^3*d^4 + 144*a^8*b^3*c^2*d^5)/(216*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) + ((c + d*x^6)^(1/2)*(a*d + 4*b*c)*(288*a^6*b^5*c^5*d^2 - 720*a^7*b^4*c^4*d^3 + 576*a^8*b^3*c^3*d^4 - 144*a^9*b^2*c^2*d^5))/(216*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(12*a^3*(c^3)^(1/2)))*(a*d + 4*b*c)*1i)/(12*a^3*(c^3)^(1/2)))/((5*a^3*b^4*d^6 + 32*b^7*c^3*d^3 - 48*a*b^6*c^2*d^4 + 6*a^2*b^5*c*d^5)/(108*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) - ((((c + d*x^6)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(18*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + (((144*a^9*b^2*c*d^6 + 288*a^6*b^5*c^4*d^3 - 576*a^7*b^4*c^3*d^4 + 144*a^8*b^3*c^2*d^5)/(216*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) - ((c + d*x^6)^(1/2)*(a*d + 4*b*c)*(288*a^6*b^5*c^5*d^2 - 720*a^7*b^4*c^4*d^3 + 576*a^8*b^3*c^3*d^4 - 144*a^9*b^2*c^2*d^5))/(216*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(12*a^3*(c^3)^(1/2)))*(a*d + 4*b*c))/(12*a^3*(c^3)^(1/2)) + ((((c + d*x^6)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(18*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - (((144*a^9*b^2*c*d^6 + 288*a^6*b^5*c^4*d^3 - 576*a^7*b^4*c^3*d^4 + 144*a^8*b^3*c^2*d^5)/(216*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) + ((c + d*x^6)^(1/2)*(a*d + 4*b*c)*(288*a^6*b^5*c^5*d^2 - 720*a^7*b^4*c^4*d^3 + 576*a^8*b^3*c^3*d^4 - 144*a^9*b^2*c^2*d^5))/(216*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(12*a^3*(c^3)^(1/2)))*(a*d + 4*b*c))/(12*a^3*(c^3)^(1/2))))*(a*d + 4*b*c)*1i)/(6*a^3*(c^3)^(1/2))

3.875 $\int \frac{x^{14}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 141

$$\int \frac{x^{14}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{ax^3\sqrt{c+dx^6}}{6b(bc-ad)\,(a+bx^6)} - \frac{\sqrt{a}(3bc-2ad)\arctan\left(\frac{\sqrt{bc-ad}x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6b^2(bc-ad)^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{3b^2\sqrt{d}}$$

```
[Out] -1/6*(-2*a*d+3*b*c)*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))*a^
(1/2)/b^2/(-a*d+b*c)^(3/2)+1/3*arctanh(x^3*d^(1/2)/(d*x^6+c)^(1/2))/b^2/d^(
1/2)+1/6*a*x^3*(d*x^6+c)^(1/2)/b/(-a*d+b*c)/(b*x^6+a)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {476, 481, 537, 223, 212, 385, 211}

$$\int \frac{x^{14}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = -\frac{\sqrt{a}(3bc-2ad)\arctan\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6b^2(bc-ad)^{3/2}} + \frac{ax^3\sqrt{c+dx^6}}{6b\,(a+bx^6)\,(bc-ad)} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{3b^2\sqrt{d}}$$

```
[In] Int[x^14/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (a*x^3*Sqrt[c + d*x^6])/(6*b*(b*c - a*d)*(a + b*x^6)) - (Sqrt[a]*(3*b*c - 2
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*b^2*(b*c
- a*d)^(3/2)) + ArcTanh[(Sqrt[d]*x^3)/Sqrt[c + d*x^6]]/(3*b^2*Sqrt[d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx, x, x^3\right)\\
&= \frac{ax^3\sqrt{c+dx^6}}{6b(bc-ad)(a+bx^6)} - \frac{\text{Subst}\left(\int \frac{ac-2(bc-ad)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6b(bc-ad)}\\
&= \frac{ax^3\sqrt{c+dx^6}}{6b(bc-ad)(a+bx^6)} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^3\right)}{3b^2}\\
&\quad - \frac{(a(3bc-2ad))\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6b^2(bc-ad)}\\
&= \frac{ax^3\sqrt{c+dx^6}}{6b(bc-ad)(a+bx^6)} + \frac{\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{3b^2}\\
&\quad - \frac{(a(3bc-2ad))\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{6b^2(bc-ad)}\\
&= \frac{ax^3\sqrt{c+dx^6}}{6b(bc-ad)(a+bx^6)} - \frac{\sqrt{a}(3bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6b^2(bc-ad)^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{3b^2\sqrt{d}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.99 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.09

$$
\int \frac{x^{14}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{\frac{abx^3\sqrt{c+dx^6}}{(bc-ad)(a+bx^6)} + \frac{\sqrt{a}(-3bc+2ad)\arctan\left(\frac{a\sqrt{d}+bx^3\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} + \frac{2\log\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{\sqrt{d}}}{6b^2}
$$

```
[In] Integrate[x^14/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] ((a*b*x^3*Sqrt[c + d*x^6])/((b*c - a*d)*(a + b*x^6)) + (Sqrt[a]*(-3*b*c + 2
*a*d)*ArcTan[(a*Sqrt[d] + b*x^3*(Sqrt[d]*x^3 + Sqrt[c + d*x^6]))/(Sqrt[a]*S
qrt[b*c - a*d])])/(b*c - a*d)^(3/2) + (2*Log[Sqrt[d]*x^3 + Sqrt[c + d*x^6]]
)/Sqrt[d])/(6*b^2)
```

Maple [A] (verified)

Time = 10.58 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.83

method	result	size
pseudoelliptic	$-\frac{-\frac{a\left(-\frac{\sqrt{d\,x^{6}+c}\,b\,x^{3}}{b\,x^{6}+a}-\frac{(2ad-3bc)\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{6}+c}\,a}{x^{3}\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)}{ad-bc}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{6}+c}}{x^{3}\sqrt{d}}\right)}{\sqrt{d}}}{6b^{2}}$	117

[In] int(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/6/b^2*(-a/(a*d-b*c)*(-(d*x^6+c)^(1/2)*b*x^3/(b*x^6+a)-(2*a*d-3*b*c)/((a*d-b*c)*a)^(1/2)*arctanh((d*x^6+c)^(1/2)/x^3*a/((a*d-b*c)*a)^(1/2)))-2/d^(1/2)*arctanh((d*x^6+c)^(1/2)/x^3/d^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.85 (sec) , antiderivative size = 1077, normalized size of antiderivative = 7.64

$$\int \frac{x^{14}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$
$$= \left[\frac{4\sqrt{dx^6+c}abdx^3+4\left((b^2c-abd)x^6+abc-a^2d\right)\sqrt{d}\log\left(-2\,dx^6-2\sqrt{dx^6+c}\sqrt{d}x^3-c\right)+\left((3\,b^2cd-\right.}{24\left(\left(\right.\right.}\right.$$

[In] integrate(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")

[Out] [1/24*(4*sqrt(d*x^6 + c)*a*b*d*x^3 + 4*((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(d)*log(-2*d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c) + ((3*b^2*c*d - 2*a*b*d^2)*x^6 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*x^6 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)))/((b^4*c*d - a*b^3*d^2)*x^6 + a*b^3*c*d - a^2*b^2*d^2), 1/24*(4*sqrt(d*x^6 + c)*a*b*d*x^3 - 8*((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(-d)*arctan(sqrt(-d)*x^3/sqrt(d*x^6 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^6 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*x^6 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)))/((b^4*c*d - a*b^3*d^2)*x^6 + a*b^3*c*d - a^2*b^2*d^2), 1/12*(2*sqrt(d*x^6 + c)*a*b*d*x^3 + ((3*b^2*c*d - 2*a*b*d^2)*x^6 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d

```
*x^6 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c*x^3)) + 2*((b^2*c - a*b*d)*x^6
 + a*b*c - a^2*d)*sqrt(d)*log(-2*d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c)
)/((b^4*c*d - a*b^3*d^2)*x^6 + a*b^3*c*d - a^2*b^2*d^2), 1/12*(2*sqrt(d*x^6
 + c)*a*b*d*x^3 - 4*((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(-d)*arctan(s
qrt(-d)*x^3/sqrt(d*x^6 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^6 + 3*a*b*c*d - 2
*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*
x^6 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c*x^3)))/((b^4*c*d - a*b^3*d^2)*x
^6 + a*b^3*c*d - a^2*b^2*d^2)]
```

Sympy [F]

$$\int \frac{x^{14}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x^{14}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$

```
[In] integrate(x**14/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
[Out] Integral(x**14/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{x^{14}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x^{14}}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

```
[In] integrate(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^14/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(117) = 234$.

Time = 0.30 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.43

$$\int \frac{x^{14}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = -\frac{\left(3\,abc\sqrt{-d}\arctan\left(\frac{a\sqrt{d}}{\sqrt{abc-a^2d}}\right)-2\,a^2\sqrt{-d}d\arctan\left(\frac{a\sqrt{d}}{\sqrt{abc-a^2d}}\right)-2\,\sqrt{abc-a^2d}bc\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)+2\,\sqrt{abc}\right.}{6\left(\sqrt{abc-a^2d}b^3c\sqrt{-d}-\sqrt{abc-a^2d}ab^2\sqrt{-d}d\right)} + \frac{ac\sqrt{d+\frac{c}{x^6}}}{6\left(b^2c\,\mathrm{sgn}\left(x\right)-abd\,\mathrm{sgn}\left(x\right)\right)\left(bc+a\left(d+\frac{c}{x^6}\right)-ad\right)} + \frac{\left(3\,abc-2\,a^2d\right)\arctan\left(\frac{a\sqrt{d+\frac{c}{x^6}}}{\sqrt{abc-a^2d}}\right)}{6\left(b^3c\,\mathrm{sgn}\left(x\right)-ab^2d\,\mathrm{sgn}\left(x\right)\right)\sqrt{abc-a^2d}} - \frac{\arctan\left(\frac{\sqrt{d+\frac{c}{x^6}}}{\sqrt{-d}}\right)}{3\,b^2\sqrt{-d}\,\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*(3*a*b*c*sqrt(-d)*arctan(a*sqrt(d)/sqrt(a*b*c - a^2*d)) - 2*a^2*sqrt(-
d)*d*arctan(a*sqrt(d)/sqrt(a*b*c - a^2*d)) - 2*sqrt(a*b*c - a^2*d)*b*c*arct
an(sqrt(d)/sqrt(-d)) + 2*sqrt(a*b*c - a^2*d)*a*d*arctan(sqrt(d)/sqrt(-d)) +
 sqrt(a*b*c - a^2*d)*a*sqrt(-d)*sqrt(d))*sgn(x)/(sqrt(a*b*c - a^2*d)*b^3*c*
sqrt(-d) - sqrt(a*b*c - a^2*d)*a*b^2*sqrt(-d)*d) + 1/6*a*c*sqrt(d + c/x^6)/
((b^2*c*sgn(x) - a*b*d*sgn(x))*(b*c + a*(d + c/x^6) - a*d)) + 1/6*(3*a*b*c
- 2*a^2*d)*arctan(a*sqrt(d + c/x^6)/sqrt(a*b*c - a^2*d))/((b^3*c*sgn(x) - a
*b^2*d*sgn(x))*sqrt(a*b*c - a^2*d)) - 1/3*arctan(sqrt(d + c/x^6)/sqrt(-d))/
(b^2*sqrt(-d)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{14}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\int \frac{x^{14}}{(b\,x^6+a)^2\,\sqrt{d\,x^6+c}}\,dx$$

```
[In] int(x^14/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(x^14/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.876 $\int \frac{x^8}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{x^8}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = -\frac{x^3\sqrt{c+dx^6}}{6(bc-ad)\,(a+bx^6)} + \frac{c\arctan\left(\frac{\sqrt{bc-ad}x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6\sqrt{a}(bc-ad)^{3/2}}$$

```
[Out] 1/6*c*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))/(-a*d+b*c)^(3/2)
/a^(1/2)-1/6*x^3*(d*x^6+c)^(1/2)/(-a*d+b*c)/(b*x^6+a)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {476, 482, 12, 385, 211}

$$\int \frac{x^8}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{c\arctan\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6\sqrt{a}(bc-ad)^{3/2}} - \frac{x^3\sqrt{c+dx^6}}{6\,(a+bx^6)\,(bc-ad)}$$

```
[In] Int[x^8/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/6*(x^3*Sqrt[c + d*x^6])/((b*c - a*d)*(a + b*x^6)) + (c*ArcTan[(Sqrt[b*c
- a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*Sqrt[a]*(b*c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx, x, x^3\right)\\
&= -\frac{x^3\sqrt{c+dx^6}}{6(bc-ad)\,(a+bx^6)} + \frac{\text{Subst}\left(\int \frac{c}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6(bc-ad)}\\
&= -\frac{x^3\sqrt{c+dx^6}}{6(bc-ad)\,(a+bx^6)} + \frac{c\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6(bc-ad)}\\
&= -\frac{x^3\sqrt{c+dx^6}}{6(bc-ad)\,(a+bx^6)} + \frac{c\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{6(bc-ad)}\\
&= -\frac{x^3\sqrt{c+dx^6}}{6(bc-ad)\,(a+bx^6)} + \frac{c\tan^{-1}\left(\frac{\sqrt{bc-ad}x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6\sqrt{a}(bc-ad)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.20

$$\int \frac{x^8}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\frac{1}{6}\left(-\frac{x^3\sqrt{c+dx^6}}{(bc-ad)(a+bx^6)}+\frac{c\arctan\left(\frac{a\sqrt{d}+bx^3\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}(bc-ad)^{3/2}}\right)$$

```
[In] Integrate[x^8/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (-((x^3*Sqrt[c + d*x^6])/((b*c - a*d)*(a + b*x^6))) + (c*ArcTan[(a*Sqrt[d]
+ b*x^3*(Sqrt[d]*x^3 + Sqrt[c + d*x^6]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(Sqrt[
a]*(b*c - a*d)^(3/2)))/6
```

Maple [A] (verified)

Time = 9.34 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.87

method	result	size
pseudoelliptic	$-\frac{c\left(-\frac{\sqrt{d\,x^{6}+c}\,x^{3}}{c\left(b\,x^{6}+a\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{6}+c}\,a}{x^{3}\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)}{6(ad-bc)}$	81

```
[In] int(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*c/(a*d-b*c)*(-(d*x^6+c)^(1/2)*x^3/c/(b*x^6+a)+1/((a*d-b*c)*a)^(1/2)*ar
ctanh((d*x^6+c)^(1/2)/x^3*a/((a*d-b*c)*a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(77) = 154.

Time = 0.36 (sec) , antiderivative size = 426, normalized size of antiderivative = 4.58

$$\int \frac{x^8}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$

$$=\left[-\frac{4\sqrt{dx^6+c}(abc-a^2d)x^3-(bcx^6+ac)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{12}-2\left(3\,abc^2-4\,a^2cd\right)x^6+a^2c^2+4\left((bc\right.}{b^2x^{12}+2\,abx^6+a^2}\right.}{24\left((ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^6+a^2b^2c^2-2\,a^3bcd+a^4d^2\right)}\right.$$

$$\left.-\frac{2\sqrt{dx^6+c}(abc-a^2d)x^3-(bcx^6+ac)\sqrt{abc-a^2d}\arctan\left(\frac{((bc-2\,ad)x^6-ac)\sqrt{dx^6+c}\sqrt{abc-a^2d}}{2\left((abcd-a^2d^2)x^9+(abc^2-a^2cd)x^3\right)}\right)}{12\left((ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^6+a^2b^2c^2-2\,a^3bcd+a^4d^2\right)}\right]$$

```
[In] integrate(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(4*sqrt(d*x^6 + c)*(a*b*c - a^2*d)*x^3 - (b*c*x^6 + a*c)*sqrt(-a*b*c
 + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^
2*c*d)*x^6 + a^2*c^2 + 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*sqrt(d*x^6 + c)*sqrt
(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2)))/((a*b^3*c^2 - 2*a^2*b^2*c*
d + a^3*b*d^2)*x^6 + a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2), -1/12*(2*sqrt(d*
x^6 + c)*(a*b*c - a^2*d)*x^3 - (b*c*x^6 + a*c)*sqrt(a*b*c - a^2*d)*arctan(1
/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d
- a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3)))/((a*b^3*c^2 - 2*a^2*b^2*c*d + a
^3*b*d^2)*x^6 + a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)]
```

Sympy [F]

$$\int \frac{x^8}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\int \frac{x^8}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$

```
[In] integrate(x**8/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
[Out] Integral(x**8/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{x^8}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\int \frac{x^8}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

```
[In] integrate(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^8/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Giac [F]

$$\int \frac{x^8}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\int \frac{x^8}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

```
[In] integrate(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\int \frac{x^8}{(b\,x^6+a)^2\,\sqrt{d\,x^6+c}}\,dx$$

```
[In] int(x^8/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
[Out] int(x^8/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.877 $\int \frac{x^2}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \frac{x^2}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\frac{bx^3\sqrt{c+dx^6}}{6a(bc-ad)\,(a+bx^6)}+\frac{(bc-2ad)\arctan\left(\frac{\sqrt{bc-ad}x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{3/2}(bc-ad)^{3/2}}$$

```
[Out] 1/6*(-2*a*d+b*c)*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))/a^(3/
2)/(-a*d+b*c)^(3/2)+1/6*b*x^3*(d*x^6+c)^(1/2)/a/(-a*d+b*c)/(b*x^6+a)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {476, 390, 385, 211}

$$\int \frac{x^2}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\frac{(bc-2ad)\arctan\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{3/2}(bc-ad)^{3/2}}+\frac{bx^3\sqrt{c+dx^6}}{6a\,(a+bx^6)\,(bc-ad)}$$

```
[In] Int[x^2/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (b*x^3*Sqrt[c + d*x^6])/(6*a*(b*c - a*d)*(a + b*x^6)) + ((b*c - 2*a*d)*ArcT
an[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*a^(3/2)*(b*c - a*d)
^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx, x, x^3\right) \\
&= \frac{bx^3\sqrt{c+dx^6}}{6a(bc-ad)\,(a+bx^6)} + \frac{(bc-2ad)\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6a(bc-ad)} \\
&= \frac{bx^3\sqrt{c+dx^6}}{6a(bc-ad)\,(a+bx^6)} + \frac{(bc-2ad)\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{6a(bc-ad)} \\
&= \frac{bx^3\sqrt{c+dx^6}}{6a(bc-ad)\,(a+bx^6)} + \frac{(bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{3/2}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.19

$$\int \frac{x^2}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = -\frac{bx^3\sqrt{c+dx^6}}{6a(-bc+ad)\left(a+bx^6\right)} + \frac{(bc-2ad)\arctan\left(\frac{a\sqrt{d}+b\sqrt{d}x^6+bx^3\sqrt{c+dx^6}}{\sqrt{a}\sqrt{bc-ad}}\right)}{6a^{3/2}(bc-ad)^{3/2}}$$

```
[In] Integrate[x^2/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/6*(b*x^3*Sqrt[c + d*x^6])/(a*(-(b*c) + a*d)*(a + b*x^6)) + ((b*c - 2*a*d
)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^6 + b*x^3*Sqrt[c + d*x^6])/(Sqrt[a]*Sqrt[
b*c - a*d])])/(6*a^(3/2)*(b*c - a*d)^(3/2))
```

Maple [A] (verified)

Time = 9.44 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

method	result	size
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^6+c}\,b\,x^3}{b\,x^6+a}+\frac{(2ad-bc)\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^6+c}\,a}{x^3\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}}{6(ad-bc)a}$	90

```
[In] int(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/(a*d-b*c)/a*(-(d*x^6+c)^(1/2)*b*x^3/(b*x^6+a)+(2*a*d-b*c)/((a*d-b*c)*a)
^(1/2)*arctanh((d*x^6+c)^(1/2)/x^3*a/((a*d-b*c)*a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(88) = 176.

Time = 0.51 (sec) , antiderivative size = 467, normalized size of antiderivative = 4.49

$$\int \frac{x^2}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \left[\frac{4\sqrt{dx^6+c}(ab^2c-a^2bd)x^3-((b^2c-2\,abd)x^6+abc-2\,a^2d)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{12}-2\,(3\,c}{}\right.}{24\,(a^3b^2c^2-2\,a^4bcd+a^5d^2+(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2)}\right.$$

```
[In] integrate(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(4*sqrt(d*x^6 + c)*(a*b^2*c - a^2*b*d)*x^3 - ((b^2*c - 2*a*b*d)*x^6 +
 a*b*c - 2*a^2*d)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^
```

```
2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b*c - 2*a*d)*x^9 -
a*c*x^3)*sqrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2)
))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^
4*b*d^2)*x^6), 1/12*(2*sqrt(d*x^6 + c)*(a*b^2*c - a^2*b*d)*x^3 + ((b^2*c -
2*a*b*d)*x^6 + a*b*c - 2*a^2*d)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*
d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^9
+ (a*b*c^2 - a^2*c*d)*x^3)))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^
3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x^6)]
```

Sympy [F]

$$\int \frac{x^2}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x^2}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$

```
[In] integrate(x**2/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
[Out] Integral(x**2/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x^2}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

```
[In] integrate(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(88) = 176.

Time = 0.28 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.28

$$\int \frac{x^2}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = -\frac{1}{6}\,d^{\frac{3}{2}}\left(\frac{(bc-2\,ad)\arctan\left(\frac{\left(\sqrt{d}x^3-\sqrt{dx^6+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\left(abcd-a^2d^2\right)^{\frac{3}{2}}} + \frac{2\left(\left(\sqrt{d}x^3-\sqrt{dx^6+c}\right)^2bc-\right.}{\left(\left(\sqrt{d}x^3-\sqrt{dx^6+c}\right)^4b-2\left(\sqrt{d}x^3-\sqrt{dx^6+c}\right)\right.}\right.$$

```
[In] integrate(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x^3 - sqrt(d*x^6 + c))^2*b
 - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d^2)^(3/2) + 2*((sq
rt(d)*x^3 - sqrt(d*x^6 + c))^2*b*c - 2*(sqrt(d)*x^3 - sqrt(d*x^6 + c))^2*a*
d - b*c^2)/(((sqrt(d)*x^3 - sqrt(d*x^6 + c))^4*b - 2*(sqrt(d)*x^3 - sqrt(d*
x^6 + c))^2*b*c + 4*(sqrt(d)*x^3 - sqrt(d*x^6 + c))^2*a*d + b*c^2)*(a*b*c*d
 - a^2*d^2)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x^2}{(b\,x^6+a)^2\,\sqrt{d\,x^6+c}}\,dx$$

```
[In] int(x^2/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(x^2/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.878 $\int \frac{1}{x^4(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 149

$$\int \frac{1}{x^4\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=-\frac{(3bc-2ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)x^3}+\frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^3\left(a+bx^6\right)}-\frac{b(3bc-4ad)\arctan\left(\frac{\sqrt{bc-ad}x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{5/2}(bc-ad)^{3/2}}$$

```
[Out] -1/6*b*(-4*a*d+3*b*c)*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))/
a^(5/2)/(-a*d+b*c)^(3/2)-1/6*(-2*a*d+3*b*c)*(d*x^6+c)^(1/2)/a^2/c/(-a*d+b*c
)/x^3+1/6*b*(d*x^6+c)^(1/2)/a/(-a*d+b*c)/x^3/(b*x^6+a)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {476, 483, 597, 12, 385, 211}

$$\int \frac{1}{x^4\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=-\frac{b(3bc-4ad)\arctan\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{5/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^6}(3bc-2ad)}{6a^2cx^3(bc-ad)}+\frac{b\sqrt{c+dx^6}}{6ax^3\left(a+bx^6\right)(bc-ad)}$$

```
[In] Int[1/(x^4*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/6*((3*b*c - 2*a*d)*Sqrt[c + d*x^6])/(a^2*c*(b*c - a*d)*x^3) + (b*Sqrt[c
+ d*x^6])/(6*a*(b*c - a*d)*x^3*(a + b*x^6)) - (b*(3*b*c - 4*a*d)*ArcTan[(Sq
```

```
rt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*a^(5/2)*(b*c - a*d)^(3/2)
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2 (a+bx^2)^2 \sqrt{c+dx^2}}\,dx, x, x^3\right)$$

$$= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^3 (a+bx^6)} - \frac{\text{Subst}\left(\int \frac{-3bc+2ad-2bdx^2}{x^2(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6a(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^3 (a+bx^6)} - \frac{\text{Subst}\left(\int \frac{bc(3bc-4ad)}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6a^2c(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^3 (a+bx^6)} - \frac{(b(3bc-4ad))\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6a^2(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^3 (a+bx^6)} - \frac{(b(3bc-4ad))\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{6a^2(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^3 (a+bx^6)} - \frac{b(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{5/2}(bc-ad)^{3/2}}$$

Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^4 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \frac{\sqrt{c+dx^6}(2abc-2a^2d+3b^2cx^6-2abdx^6)}{6a^2c(-bc+ad)x^3 (a+bx^6)} - \frac{b(3bc-4ad)\arctan\left(\frac{a\sqrt{d}+b\sqrt{d}x^6+bx^3\sqrt{c+dx^6}}{\sqrt{a}\sqrt{bc-ad}}\right)}{6a^{5/2}(bc-ad)^{3/2}}$$

```
[In] Integrate[1/(x^4*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (Sqrt[c + d*x^6]*(2*a*b*c - 2*a^2*d + 3*b^2*c*x^6 - 2*a*b*d*x^6))/(6*a^2*c*
(-(b*c) + a*d)*x^3*(a + b*x^6)) - (b*(3*b*c - 4*a*d)*ArcTan[(a*Sqrt[d] + b*
Sqrt[d]*x^6 + b*x^3*Sqrt[c + d*x^6])/(Sqrt[a]*Sqrt[b*c - a*d])])/(6*a^(5/2)
*(b*c - a*d)^(3/2))
```

Maple [A] (verified)

Time = 12.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.75

method	result	size
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{6}+c}}{x^{3}}+\frac{bc\left(\frac{\sqrt{d\,x^{6}+c}\,b\,x^{3}}{b\,x^{6}+a}-\frac{(4ad-3bc)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{6}+c}\,a}{x^{3}\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)}{2ad-2bc}}{3a^{2}c}$	112

```
[In] int(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a^2*(-(d*x^6+c)^(1/2)/x^3+1/2*b*c/(a*d-b*c)*((d*x^6+c)^(1/2)*b*x^3/(b*x
^6+a)-(4*a*d-3*b*c)/((a*d-b*c)*a)^(1/2)*arctanh((d*x^6+c)^(1/2)/x^3*a/((a*d
-b*c)*a)^(1/2))))/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(129) = 258$.

Time = 0.51 (sec) , antiderivative size = 612, normalized size of antiderivative = 4.11

$$\int \frac{1}{x^4 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx$$

$$= \left[-\frac{((3\,b^3c^2-4\,ab^2cd)x^9+(3\,ab^2c^2-4\,a^2bcd)x^3)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{12}-2\,(3\,abc^2-4\,a^2cd)x^6+a}{b^2x^{12}+2\,a}\right.}{24\,((a^3b^3c^3-2\,a^4b^2c^2d+a^5b}\right.$$

$$-\frac{((3\,b^3c^2-4\,ab^2cd)x^9+(3\,ab^2c^2-4\,a^2bcd)x^3)\sqrt{abc-a^2d}\arctan\left(\frac{((bc-2\,ad)x^6-ac)\sqrt{dx^6+c}\sqrt{abc-a^2d}}{2\,((abcd-a^2d^2)x^9+(abc^2-a^2cd)x^3)}\right)+2\,((3}{12\,((a^3b^3c^3-2\,a^4b^2c^2d+a^5bcd^2)x^9+(a^4b^2c^3-2\,a}$$

```
[In] integrate(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^9 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^3)*s
qrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*
c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 + 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*sqrt(d*x^6
 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2)) + 4*((3*a*b^3*c^2
 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^6 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4*d
^2)*sqrt(d*x^6 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^9 + (
a^4*b^2*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^3), -1/12*(((3*b^3*c^2 - 4*a*b^2
*c*d)*x^9 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^3)*sqrt(a*b*c - a^2*d)*arctan(1/2
*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d -
a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3)) + 2*((3*a*b^3*c^2 - 5*a^2*b^2*c*d
+ 2*a^3*b*d^2)*x^6 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4*d^2)*sqrt(d*x^6 +
c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^9 + (a^4*b^2*c^3 - 2*a
^5*b*c^2*d + a^6*c*d^2)*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \int \frac{1}{x^4 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx$$

```
[In] integrate(1/x**4/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
[Out] Integral(1/(x**4*(a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \int \frac{1}{(bx^6+a)^2 \sqrt{dx^6+c}x^4}\,dx$$

```
[In] integrate(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \int \frac{1}{(bx^6+a)^2 \sqrt{dx^6+c}x^4}\,dx$$

```
[In] integrate(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \int \frac{1}{x^4 (b\,x^6+a)^2 \sqrt{d\,x^6+c}}\,dx$$

```
[In] int(1/(x^4*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
[Out] int(1/(x^4*(a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.879 $\int \frac{1}{x^{10}(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 208

$$\int \frac{1}{x^{10}(a+bx^6)^2\sqrt{c+dx^6}}\,dx = -\frac{(5bc-2ad)\sqrt{c+dx^6}}{18a^2c(bc-ad)x^9} + \frac{(15b^2c^2-8abcd-4a^2d^2)\sqrt{c+dx^6}}{18a^3c^2(bc-ad)x^3} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^9(a+bx^6)} + \frac{b^2(5bc-6ad)\arctan\left(\frac{\sqrt{bc-ad}x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{7/2}(bc-ad)^{3/2}}$$

```
[Out] 1/6*b^2*(-6*a*d+5*b*c)*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))
/a^(7/2)/(-a*d+b*c)^(3/2)-1/18*(-2*a*d+5*b*c)*(d*x^6+c)^(1/2)/a^2/c/(-a*d+b
*c)/x^9+1/18*(-4*a^2*d^2-8*a*b*c*d+15*b^2*c^2)*(d*x^6+c)^(1/2)/a^3/c^2/(-a*
d+b*c)/x^3+1/6*b*(d*x^6+c)^(1/2)/a/(-a*d+b*c)/x^9/(b*x^6+a)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used

$= \{476, 483, 597, 12, 385, 211\}$

$$\int \frac{1}{x^{10} (a+bx^6)^2 \sqrt{c+dx^6}} \, dx = \frac{b^2(5bc-6ad) \arctan\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{7/2}(bc-ad)^{3/2}} - \frac{\sqrt{c+dx^6}(5bc-2ad)}{18a^2cx^9(bc-ad)} + \frac{\sqrt{c+dx^6}(-4a^2d^2-8abcd+15b^2c^2)}{18a^3c^2x^3(bc-ad)} + \frac{b\sqrt{c+dx^6}}{6ax^9(a+bx^6)(bc-ad)}$$

```
[In] Int[1/(x^10*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/18*((5*b*c - 2*a*d)*Sqrt[c + d*x^6])/(a^2*c*(b*c - a*d)*x^9) + ((15*b^2*
c^2 - 8*a*b*c*d - 4*a^2*d^2)*Sqrt[c + d*x^6])/(18*a^3*c^2*(b*c - a*d)*x^3)
+ (b*Sqrt[c + d*x^6])/(6*a*(b*c - a*d)*x^9*(a + b*x^6)) + (b^2*(5*b*c - 6*a
*d)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*a^(7/2)*(b*
c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
```

```
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^4 (a+bx^2)^2 \sqrt{c+dx^2}}\,dx, x, x^3\right)$$

$$= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^9 (a+bx^6)} - \frac{\text{Subst}\left(\int \frac{-5bc+2ad-4bdx^2}{x^4(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6a(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^6}}{18a^2c(bc-ad)x^9} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^9 (a+bx^6)} + \frac{\text{Subst}\left(\int \frac{-15b^2c^2+8abcd+4a^2d^2-2bd(5bc-2ad)x^2}{x^2(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{18a^2c(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^6}}{18a^2c(bc-ad)x^9} + \frac{(15b^2c^2-8abcd-4a^2d^2)\sqrt{c+dx^6}}{18a^3c^2(bc-ad)x^3} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^9 (a+bx^6)} - \frac{\text{Subst}\left(\int -\frac{3b^2c^2(5bc-6ad)}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{18a^3c^2(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^6}}{18a^2c(bc-ad)x^9} + \frac{(15b^2c^2-8abcd-4a^2d^2)\sqrt{c+dx^6}}{18a^3c^2(bc-ad)x^3} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^9 (a+bx^6)} + \frac{(b^2(5bc-6ad))\,\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6a^3(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^6}}{18a^2c(bc-ad)x^9} + \frac{(15b^2c^2-8abcd-4a^2d^2)\sqrt{c+dx^6}}{18a^3c^2(bc-ad)x^3} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^9 (a+bx^6)} + \frac{(b^2(5bc-6ad))\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{6a^3(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^6}}{18a^2c(bc-ad)x^9} + \frac{(15b^2c^2-8abcd-4a^2d^2)\sqrt{c+dx^6}}{18a^3c^2(bc-ad)x^3} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^9(a+bx^6)} + \frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{7/2}(bc-ad)^{3/2}}$$

Mathematica [A] (verified)

Time = 3.18 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^{10}(a+bx^6)^2\sqrt{c+dx^6}}\,dx = -\frac{\sqrt{c+dx^6}(15b^3c^2x^{12}+2ab^2cx^6(5c-4dx^6)+2a^3d(c-2dx^6)-2a^2b(c^2+3cdx^6+2d^2x^{12}))}{18a^3c^2(-bc+ad)x^9(a+bx^6)} + \frac{b^2(5bc-6ad)\arctan\left(\frac{a\sqrt{d}+bx^3\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{6a^{7/2}(bc-ad)^{3/2}}$$

```
[In] Integrate[1/(x^10*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/18*(Sqrt[c + d*x^6]*(15*b^3*c^2*x^12 + 2*a*b^2*c*x^6*(5*c - 4*d*x^6) + 2
*a^3*d*(c - 2*d*x^6) - 2*a^2*b*(c^2 + 3*c*d*x^6 + 2*d^2*x^12)))/(a^3*c^2*(-
(b*c) + a*d)*x^9*(a + b*x^6)) + (b^2*(5*b*c - 6*a*d)*ArcTan[(a*Sqrt[d] + b*
x^3*(Sqrt[d]*x^3 + Sqrt[c + d*x^6]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(6*a^(7/2)
*(b*c - a*d)^(3/2))
```

Maple [A] (verified)

Time = 18.35 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^6+c}\,\left(-2ad\,x^6-6bc\,x^6+ac\right)}{3x^9}-\frac{b^2c^2\left(\frac{\sqrt{d\,x^6+c}\,b\,x^3}{b\,x^6+a}-\frac{(6ad-5bc)\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^6+c}\,a}{x^3\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)}{2(ad-bc)}}{3a^3c^2}$	134

```
[In] int(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a^3*(-1/3*(d*x^6+c)^(1/2)*(-2*a*d*x^6-6*b*c*x^6+a*c)/x^9-1/2*b^2*c^2/(a
*d-b*c)*((d*x^6+c)^(1/2)*b*x^3/(b*x^6+a)-(6*a*d-5*b*c)/((a*d-b*c)*a)^(1/2)*
arctanh((d*x^6+c)^(1/2)/x^3*a/((a*d-b*c)*a)^(1/2))))/c^2
```

Fricas [A] (verification not implemented)

none

Time = 0.69 (sec) , antiderivative size = 760, normalized size of antiderivative = 3.65

$$\int \frac{1}{x^{10} (a + bx^6)^2 \sqrt{c + dx^6}} \, dx$$
$$= \left[-\frac{3 \left((5 b^4 c^3 - 6 ab^3 c^2 d) x^{15} + (5 ab^3 c^3 - 6 a^2 b^2 c^2 d) x^9 \right) \sqrt{-abc + a^2 d} \log \left(\frac{(b^2 c^2 - 8 abcd + 8 a^2 d^2) x^{12} - 2 (3 abc^2 - 4 a^2 cd}{b^2 x} \right.}{} \right.$$

```
[In] integrate(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
[Out] [-1/72*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^15 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2*
d)*x^9)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 -
2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*s
qrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2)) - 4*((15
*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^12 - 2*a^3
*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2 + 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d
 + a^4*b*c*d^2 + 2*a^5*d^3)*x^6)*sqrt(d*x^6 + c))/((a^4*b^3*c^4 - 2*a^5*b^2
*c^3*d + a^6*b*c^2*d^2)*x^15 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7*c^2*d^2)*
x^9), 1/36*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^15 + (5*a*b^3*c^3 - 6*a^2*b^2*
c^2*d)*x^9)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d
*x^6 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d
)*x^3)) + 2*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d
^3)*x^12 - 2*a^3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2 + 2*(5*a^2*b^3*c^3 -
 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^6)*sqrt(d*x^6 + c))/((a^4*b^3
*c^4 - 2*a^5*b^2*c^3*d + a^6*b*c^2*d^2)*x^15 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d
 + a^7*c^2*d^2)*x^9)]
```

Sympy [F]

$$\int \frac{1}{x^{10} (a + bx^6)^2 \sqrt{c + dx^6}} \, dx = \int \frac{1}{x^{10} (a + bx^6)^2 \sqrt{c + dx^6}} \, dx$$

```
[In] integrate(1/x**10/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
[Out] Integral(1/(x**10*(a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{x^{10} (a + bx^6)^2 \sqrt{c + dx^6}} dx = \int \frac{1}{(bx^6 + a)^2 \sqrt{dx^6 + c} x^{10}} dx$$

```
[In] integrate(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^10), x)
```

Giac [F]

$$\int \frac{1}{x^{10} (a + bx^6)^2 \sqrt{c + dx^6}} dx = \int \frac{1}{(bx^6 + a)^2 \sqrt{dx^6 + c} x^{10}} dx$$

```
[In] integrate(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{10} (a + bx^6)^2 \sqrt{c + dx^6}} dx = \int \frac{1}{x^{10} (b x^6 + a)^2 \sqrt{d x^6 + c}} dx$$

```
[In] int(1/(x^10*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
[Out] int(1/(x^10*(a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.880 $\int \frac{x^4}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^4}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\frac{x^5\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(\frac{5}{6},2,\frac{1}{2},\frac{11}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{5a^2\sqrt{c+dx^6}}$$

```
[Out] 1/5*x^5*AppellF1(5/6,2,1/2,11/6,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a^2/(d
*x^6+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{x^4}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\frac{x^5\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(\frac{5}{6},2,\frac{1}{2},\frac{11}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{5a^2\sqrt{c+dx^6}}$$

```
[In] Int[x^4/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^5*Sqrt[1 + (d*x^6)/c]*AppellF1[5/6, 2, 1/2, 11/6, -((b*x^6)/a), -((d*x^6
)/c)])/(5*a^2*Sqrt[c + d*x^6])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^6}{c}}\int \frac{x^4}{(a+bx^6)^2\sqrt{1+\frac{dx^6}{c}}}\,dx}{\sqrt{c+dx^6}}$$
$$= \frac{x^5\sqrt{1+\frac{dx^6}{c}}F_1\left(\frac{5}{6};2,\frac{1}{2};\frac{11}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{5a^2\sqrt{c+dx^6}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 169 vs. $2(64) = 128$.

Time = 10.20 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.64

$$\int \frac{x^4}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$
$$= \frac{x^5\left(55ab(c+dx^6)+11(bc-6ad)\,(a+bx^6)\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(\frac{5}{6},\frac{1}{2},1,\frac{11}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)-10bdx^6(a+bx^6)\right.}{330a^2(bc-ad)\,(a+bx^6)\sqrt{c+dx^6}}$$

```
[In] Integrate[x^4/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^5*(55*a*b*(c + d*x^6) + 11*(b*c - 6*a*d)*(a + b*x^6)*Sqrt[1 + (d*x^6)/c]
*AppellF1[5/6, 1/2, 1, 11/6, -((d*x^6)/c), -((b*x^6)/a)] - 10*b*d*x^6*(a +
b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[11/6, 1/2, 1, 17/6, -((d*x^6)/c), -((b*
x^6)/a)]))/(330*a^2*(b*c - a*d)*(a + b*x^6)*Sqrt[c + d*x^6])
```

Maple [F]

$$\int \frac{x^4}{\left(b\,x^6+a\right)^2\sqrt{d\,x^6+c}}dx$$

[In] int(x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)

[Out] int(x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)

Fricas [F]

$$\int \frac{x^4}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x^4}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

[In] integrate(x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(d*x^6 + c)*x^4/(b^2*d*x^18 + (b^2*c + 2*a*b*d)*x^12 + (2*a*b*c + a^2*d)*x^6 + a^2*c), x)

Sympy [F]

$$\int \frac{x^4}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x^4}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$

[In] integrate(x**4/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)

[Out] Integral(x**4/((a + b*x**6)**2*sqrt(c + d*x**6)), x)

Maxima [F]

$$\int \frac{x^4}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x^4}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

[In] integrate(x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")

[Out] integrate(x^4/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)

Giac [F]

$$\int \frac{x^4}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x^4}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

```
[In] integrate(x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x^4}{(b\,x^6+a)^2\,\sqrt{d\,x^6+c}}\,dx$$

```
[In] int(x^4/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
[Out] int(x^4/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.881 $\int \frac{x^3}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^3}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{x^4\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(\frac{2}{3},2,\frac{1}{2},\frac{5}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a^2\sqrt{c+dx^6}}$$

```
[Out] 1/4*x^4*AppellF1(2/3,2,1/2,5/3,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a^2/(d*
x^6+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {476, 525, 524}

$$\int \frac{x^3}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{x^4\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(\frac{2}{3},2,\frac{1}{2},\frac{5}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a^2\sqrt{c+dx^6}}$$

```
[In] Int[x^3/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^6)/c]*AppellF1[2/3, 2, 1/2, 5/3, -((b*x^6)/a), -((d*x^6)
/c)])/(4*a^2*Sqrt[c + d*x^6])
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{1+\frac{dx^6}{c}}\text{Subst}\left(\int \frac{x}{(a+bx^3)^2\sqrt{1+\frac{dx^3}{c}}}\,dx, x, x^2\right)}{2\sqrt{c+dx^6}}$$

$$= \frac{x^4\sqrt{1+\frac{dx^6}{c}}F_1\left(\frac{2}{3};2,\frac{1}{2};\frac{5}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a^2\sqrt{c+dx^6}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 168 vs. 2(64) = 128.

Time = 10.19 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.62

$$\int \frac{x^3}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = -\frac{x^4\left(-10ab(c+dx^6)-5(bc-3ad)\left(a+bx^6\right)\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+bdx^6(a+bx^6)\right.}{60a^2(bc-ad)\left(a+bx^6\right)\sqrt{c+dx^6}}$$

```
[In] Integrate[x^3/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/60*(x^4*(-10*a*b*(c + d*x^6) - 5*(b*c - 3*a*d)*(a + b*x^6)*Sqrt[1 + (d*x
^6)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^6)/c), -((b*x^6)/a)] + b*d*x^6*(a
+ b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^6)/c), -((b*
x^6)/a)]))/(a^2*(b*c - a*d)*(a + b*x^6)*Sqrt[c + d*x^6])
```

Maple [F]

$$\int \frac{x^{3}}{\left(b\,x^{6}+a\right)^{2} \sqrt{d\,x^{6}+c}}d x$$

```
[In] int(x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
[Out] int(x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\text{Timed out}$$

```
[In] integrate(x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^3}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\int \frac{x^3}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$

```
[In] integrate(x**3/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
[Out] Integral(x**3/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{x^3}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\int \frac{x^3}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

```
[In] integrate(x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Giac [F]

$$\int \frac{x^3}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x^3}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

```
[In] integrate(x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^3/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x^3}{(b\,x^6+a)^2\,\sqrt{d\,x^6+c}}\,dx$$

```
[In] int(x^3/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
[Out] int(x^3/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.882 $\int \frac{x}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{x}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\frac{x^2\sqrt{1+\frac{dx^6}{c}}\operatorname{AppellF1}\left(\frac{1}{3},2,\frac{1}{2},\frac{4}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a^2\sqrt{c+dx^6}}$$

```
[Out] 1/2*x^2*AppellF1(1/3,2,1/2,4/3,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a^2/(d*
x^6+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {476, 441, 440}

$$\int \frac{x}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\frac{x^2\sqrt{\frac{dx^6}{c}+1}\operatorname{AppellF1}\left(\frac{1}{3},2,\frac{1}{2},\frac{4}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a^2\sqrt{c+dx^6}}$$

```
[In] Int[x/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^2*Sqrt[1 + (d*x^6)/c]*AppellF1[1/3, 2, 1/2, 4/3, -((b*x^6)/a), -((d*x^6)
/c)])/(2*a^2*Sqrt[c + d*x^6])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{1+\frac{dx^6}{c}}\,\text{Subst}\left(\int \frac{1}{(a+bx^3)^2\sqrt{1+\frac{dx^3}{c}}}\,dx, x, x^2\right)}{2\sqrt{c+dx^6}}$$

$$= \frac{x^2\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{1}{3};2,\frac{1}{2};\frac{4}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a^2\sqrt{c+dx^6}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 172 vs. $2(64) = 128$.

Time = 10.17 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.69

$$\int \frac{x}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$
$$= \frac{8abx^2(c+dx^6)+8(2bc-3ad)x^2(a+bx^6)\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+bdx^8(a+bx^6)\sqrt{1}}{48a^2(bc-ad)(a+bx^6)\sqrt{c+dx^6}}$$

```
[In] Integrate[x/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (8*a*b*x^2*(c + d*x^6) + 8*(2*b*c - 3*a*d)*x^2*(a + b*x^6)*Sqrt[1 + (d*x^6)
/c]*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^6)/c), -((b*x^6)/a)] + b*d*x^8*(a + b
*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^6)/c), -((b*x^6
)/a)])/(48*a^2*(b*c - a*d)*(a + b*x^6)*Sqrt[c + d*x^6])
```

Maple [F]

$$\int \frac{x}{\left(b\,x^{6}+a\right)^{2}\sqrt{d\,x^{6}+c}}dx$$

[In] int(x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)

[Out] int(x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx = \text{Timed out}$$

[In] integrate(x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{x}{\left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx = \int \frac{x}{\left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx$$

[In] integrate(x/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)

[Out] Integral(x/((a + b*x**6)**2*sqrt(c + d*x**6)), x)

Maxima [F]

$$\int \frac{x}{\left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx = \int \frac{x}{\left(bx^6+a\right)^2\sqrt{dx^6+c}}\,dx$$

[In] integrate(x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")

[Out] integrate(x/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)

Giac [F]

$$\int \frac{x}{(a+bx^6)^2 \sqrt{c+dx^6}} \, dx = \int \frac{x}{(bx^6+a)^2 \sqrt{dx^6+c}} \, dx$$

[In] integrate(x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")

[Out] integrate(x/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^6)^2 \sqrt{c+dx^6}} \, dx = \int \frac{x}{(b\,x^6+a)^2 \sqrt{d\,x^6+c}} \, dx$$

[In] int(x/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)

[Out] int(x/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)

3.883 $\int \frac{1}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{x\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(\frac{1}{6},2,\frac{1}{2},\frac{7}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a^2\sqrt{c+dx^6}}$$

```
[Out] x*AppellF1(1/6,2,1/2,7/6,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a^2/(d*x^6+c)
^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {441, 440}

$$\int \frac{1}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{x\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(\frac{1}{6},2,\frac{1}{2},\frac{7}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a^2\sqrt{c+dx^6}}$$

```
[In] Int[1/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (x*Sqrt[1 + (d*x^6)/c]*AppellF1[1/6, 2, 1/2, 7/6, -((b*x^6)/a), -((d*x^6)/c
)])/(a^2*Sqrt[c + d*x^6])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^6}{c}}\int \frac{1}{(a+bx^6)^2\sqrt{1+\frac{dx^6}{c}}}\,dx}{\sqrt{c+dx^6}}$$

$$= \frac{x\sqrt{1+\frac{dx^6}{c}}F_1\left(\frac{1}{6};2,\frac{1}{2};\frac{7}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a^2\sqrt{c+dx^6}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 329 vs. $2(59) = 118$.

Time = 10.30 (sec) , antiderivative size = 329, normalized size of antiderivative = 5.58

$$\int \frac{1}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$

$$= \frac{x\left(-2bdx^6\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(\frac{7}{6},\frac{1}{2},1,\frac{13}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)-\frac{7a\left(7ac(6ad-b(6c+dx^6))\,\text{AppellF1}\left(\frac{1}{6},\frac{1}{2},1,\frac{7}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+3bx^6(\cdots}{(a+bx^6)\left(-7ac\,\text{AppellF1}\left(\frac{1}{6},\frac{1}{2},1,\frac{7}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+3x^6(2\cdots}\right)}{42a^2(-bc+ad)\sqrt{c+dx^6}}$$

```
[In] Integrate[1/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (x*(-2*b*d*x^6*Sqrt[1 + (d*x^6)/c]*AppellF1[7/6, 1/2, 1, 13/6, -((d*x^6)/c)
, -((b*x^6)/a)] - (7*a*(7*a*c*(6*a*d - b*(6*c + d*x^6))*AppellF1[1/6, 1/2,
1, 7/6, -((d*x^6)/c), -((b*x^6)/a)] + 3*b*x^6*(c + d*x^6)*(2*b*c*AppellF1[7
/6, 1/2, 2, 13/6, -((d*x^6)/c), -((b*x^6)/a)] + a*d*AppellF1[7/6, 3/2, 1, 1
3/6, -((d*x^6)/c), -((b*x^6)/a)])))/((a + b*x^6)*(-7*a*c*AppellF1[1/6, 1/2,
 1, 7/6, -((d*x^6)/c), -((b*x^6)/a)] + 3*x^6*(2*b*c*AppellF1[7/6, 1/2, 2, 1
3/6, -((d*x^6)/c), -((b*x^6)/a)] + a*d*AppellF1[7/6, 3/2, 1, 13/6, -((d*x^6
)/c), -((b*x^6)/a)])))))/(42*a^2*(-(b*c) + a*d)*Sqrt[c + d*x^6])
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{6}+a\right)^{2} \sqrt{d\,x^{6}+c}}dx$$

```
[In] int(1/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
[Out] int(1/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx = \int \frac{1}{\left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx$$

```
[In] integrate(1/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
[Out] Integral(1/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx = \int \frac{1}{{\left(bx^6+a\right)}^2\sqrt{dx^6+c}}\,dx$$

```
[In] integrate(1/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{1}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

```
[In] integrate(1/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{1}{(b\,x^6+a)^2\,\sqrt{d\,x^6+c}}\,dx$$

```
[In] int(1/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
[Out] int(1/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.884 $\int \frac{1}{x^2(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{1}{x^2\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=-\frac{\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(-\frac{1}{6},2,\frac{1}{2},\frac{5}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a^2x\sqrt{c+dx^6}}$$

```
[Out] -AppellF1(-1/6,2,1/2,5/6,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a^2/x/(d*x^6+
c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{1}{x^2\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=-\frac{\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(-\frac{1}{6},2,\frac{1}{2},\frac{5}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a^2x\sqrt{c+dx^6}}$$

```
[In] Int[1/(x^2*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -((Sqrt[1 + (d*x^6)/c]*AppellF1[-1/6, 2, 1/2, 5/6, -((b*x^6)/a), -((d*x^6)/
c)]])/(a^2*x*Sqrt[c + d*x^6]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^6}{c}}\int \frac{1}{x^2(a+bx^6)^2\sqrt{1+\frac{dx^6}{c}}}\,dx}{\sqrt{c+dx^6}}$$
$$= -\frac{\sqrt{1+\frac{dx^6}{c}}F_1\left(-\frac{1}{6};2,\frac{1}{2};\frac{5}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a^2x\sqrt{c+dx^6}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 226 vs. $2(62) = 124$.

Time = 10.30 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.65

$$\int \frac{1}{x^2(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$
$$= \frac{55a(c+dx^6)(6a^2d-7b^2cx^6-6ab(c-dx^6))-11(7b^2c^2-24abcd+12a^2d^2)x^6(a+bx^6)\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF}}{330a^3c(bc-ad)x(a}$$

```
[In] Integrate[1/(x^2*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (55*a*(c + d*x^6)*(6*a^2*d - 7*b^2*c*x^6 - 6*a*b*(c - d*x^6)) - 11*(7*b^2*c
^2 - 24*a*b*c*d + 12*a^2*d^2)*x^6*(a + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[
5/6, 1/2, 1, 11/6, -((d*x^6)/c), -((b*x^6)/a)] + 10*b*d*(7*b*c - 6*a*d)*x^1
2*(a + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[11/6, 1/2, 1, 17/6, -((d*x^6)/c)
, -((b*x^6)/a)])/(330*a^3*c*(b*c - a*d)*x*(a + b*x^6)*Sqrt[c + d*x^6])
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b\,x^{6}+a \right)^{2} \sqrt{d\,x^{6}+c}}d x$$

```
[In] int(1/x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
[Out] int(1/x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{x^2 \left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx = \int \frac{1}{{\left(bx^{6} + a\right)}^{2} \sqrt{dx^{6} + c}x^{2}}\,dx$$

```
[In] integrate(1/x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*x^6 + c)/(b^2*d*x^20 + (b^2*c + 2*a*b*d)*x^14 + (2*a*b*c +
a^2*d)*x^8 + a^2*c*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2 \left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx = \int \frac{1}{x^{2} \left(a + bx^{6}\right)^{2} \sqrt{c + dx^{6}}}\,dx$$

```
[In] integrate(1/x**2/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
[Out] Integral(1/(x**2*(a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 \left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx = \int \frac{1}{{\left(bx^{6} + a\right)}^{2} \sqrt{dx^{6} + c}x^{2}}\,dx$$

```
[In] integrate(1/x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \int \frac{1}{(bx^6+a)^2\sqrt{dx^6+c}x^2}\,dx$$

```
[In] integrate(1/x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \int \frac{1}{x^2 (b\,x^6+a)^2 \sqrt{d\,x^6+c}}\,dx$$

```
[In] int(1/(x^2*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
[Out] int(1/(x^2*(a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.885 $\int \frac{1}{x^3(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{x^3\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=-\frac{\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(-\frac{1}{3},2,\frac{1}{2},\frac{2}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a^2x^2\sqrt{c+dx^6}}$$

```
[Out] -1/2*AppellF1(-1/3,2,1/2,2/3,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a^2/x^2/(
d*x^6+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {476, 525, 524}

$$\int \frac{1}{x^3\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=-\frac{\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(-\frac{1}{3},2,\frac{1}{2},\frac{2}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a^2x^2\sqrt{c+dx^6}}$$

```
[In] Int[1/(x^3*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/2*(Sqrt[1 + (d*x^6)/c]*AppellF1[-1/3, 2, 1/2, 2/3, -((b*x^6)/a), -((d*x^
6)/c)])/(a^2*x^2*Sqrt[c + d*x^6])
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\,(a+bx^3)^2\,\sqrt{c+dx^3}}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{1+\frac{dx^6}{c}}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx^3)^2\sqrt{1+\frac{dx^3}{c}}}\,dx, x, x^2\right)}{2\sqrt{c+dx^6}}$$

$$= -\frac{\sqrt{1+\frac{dx^6}{c}}\,F_1\left(-\frac{1}{3}; 2, \frac{1}{2}; \frac{2}{3}; -\frac{bx^6}{a}, -\frac{dx^6}{c}\right)}{2a^2x^2\sqrt{c+dx^6}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 226 vs. 2(64) = 128.

Time = 10.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.53

$$\int \frac{1}{x^3\,(a+bx^6)^2\,\sqrt{c+dx^6}}\,dx$$
$$= \frac{20a(c+dx^6)\,(3a^2d-4b^2cx^6-3ab(c-dx^6))-5(8b^2c^2-15abcd+3a^2d^2)\,x^6(a+bx^6)\,\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}}{120a^3c(bc-ad)x^2\,(a+}$$

```
[In] Integrate[1/(x^3*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (20*a*(c + d*x^6)*(3*a^2*d - 4*b^2*c*x^6 - 3*a*b*(c - d*x^6)) - 5*(8*b^2*c^
2 - 15*a*b*c*d + 3*a^2*d^2)*x^6*(a + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[2/
3, 1/2, 1, 5/3, -((d*x^6)/c), -((b*x^6)/a)] + 2*b*d*(4*b*c - 3*a*d)*x^12*(a
 + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^6)/c), -((b
*x^6)/a)])/(120*a^3*c*(b*c - a*d)*x^2*(a + b*x^6)*Sqrt[c + d*x^6])
```

Maple [F]

$$\int \frac{1}{x^{3} \left(b\,x^{6}+a \right)^{2} \sqrt{d\,x^{6}+c}}d x$$

```
[In] int(1/x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
[Out] int(1/x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^3 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \int \frac{1}{x^{3} \left(a + b x^{6}\right)^{2} \sqrt{c + d x^{6}}}\, dx$$

```
[In] integrate(1/x**3/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
[Out] Integral(1/(x**3*(a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \int { \frac {1}{{\left (b x^{6} + a\right)}^{2} \sqrt {d x^{6} + c} x^{3}} \,d x }$$

```
[In] integrate(1/x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a + bx^6)^2 \sqrt{c + dx^6}} \, dx = \int \frac{1}{(bx^6 + a)^2 \sqrt{dx^6 + c}x^3} \, dx$$

```
[In] integrate(1/x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a + bx^6)^2 \sqrt{c + dx^6}} \, dx = \int \frac{1}{x^3 (b\,x^6 + a)^2 \sqrt{d\,x^6 + c}} \, dx$$

```
[In] int(1/(x^3*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
[Out] int(1/(x^3*(a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.886 $\int \frac{1}{x^5(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{x^5\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=-\frac{\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(-\frac{2}{3},2,\frac{1}{2},\frac{1}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a^2x^4\sqrt{c+dx^6}}$$

```
[Out] -1/4*AppellF1(-2/3,2,1/2,1/3,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a^2/x^4/(
d*x^6+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {476, 525, 524}

$$\int \frac{1}{x^5\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=-\frac{\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(-\frac{2}{3},2,\frac{1}{2},\frac{1}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a^2x^4\sqrt{c+dx^6}}$$

```
[In] Int[1/(x^5*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/4*(Sqrt[1 + (d*x^6)/c]*AppellF1[-2/3, 2, 1/2, 1/3, -((b*x^6)/a), -((d*x^
6)/c)])/(a^2*x^4*Sqrt[c + d*x^6])
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3 (a+bx^3)^2 \sqrt{c+dx^3}}\,dx, x, x^2\right)\\ &= \frac{\sqrt{1+\frac{dx^6}{c}}\,\text{Subst}\left(\int \frac{1}{x^3(a+bx^3)^2\sqrt{1+\frac{dx^3}{c}}}\,dx, x, x^2\right)}{2\sqrt{c+dx^6}}\\ &= -\frac{\sqrt{1+\frac{dx^6}{c}}\,F_1\left(-\frac{2}{3}; 2, \frac{1}{2}; \frac{1}{3}; -\frac{bx^6}{a}, -\frac{dx^6}{c}\right)}{4a^2x^4\sqrt{c+dx^6}}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 225 vs. $2(64) = 128$.

Time = 10.29 (sec) , antiderivative size = 225, normalized size of antiderivative = 3.52

$$\int \frac{1}{x^5 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \frac{8a(c+dx^6)\left(3a^2d-5b^2cx^6-3ab(c-dx^6)\right)+4\left(-20b^2c^2+21abcd+3a^2d^2\right)x^6(a+bx^6)\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF}}{96a^3c(bc-ad)x^4\,(a+}$$

```
[In] Integrate[1/(x^5*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (8*a*(c + d*x^6)*(3*a^2*d - 5*b^2*c*x^6 - 3*a*b*(c - d*x^6)) + 4*(-20*b^2*c
^2 + 21*a*b*c*d + 3*a^2*d^2)*x^6*(a + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[1
/3, 1/2, 1, 4/3, -((d*x^6)/c), -((b*x^6)/a)] + b*d*(-5*b*c + 3*a*d)*x^12*(a
 + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^6)/c), -((b
*x^6)/a)])/(96*a^3*c*(b*c - a*d)*x^4*(a + b*x^6)*Sqrt[c + d*x^6])
```

Maple [F]

$$\int \frac{1}{x^{5} \left(b\,x^{6}+a\right)^{2} \sqrt{d\,x^{6}+c}}d x$$

```
[In] int(1/x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
[Out] int(1/x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^5 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \int \frac{1}{x^5 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx$$

```
[In] integrate(1/x**5/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
[Out] Integral(1/(x**5*(a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{x^5 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \int \frac{1}{(bx^6+a)^2 \sqrt{dx^6+c}\,x^5}\,dx$$

```
[In] integrate(1/x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \int \frac{1}{(bx^6+a)^2\sqrt{dx^6+c}x^5}\,dx$$

```
[In] integrate(1/x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \int \frac{1}{x^5 (b\,x^6+a)^2 \sqrt{d\,x^6+c}}\,dx$$

```
[In] int(1/(x^5*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
[Out] int(1/(x^5*(a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

3.887 $\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = -\frac{(bc+ad)\sqrt{c+dx^8}}{4b^2d^2}+\frac{(c+dx^8)^{3/2}}{12bd^2}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}$$

```
[Out] 1/12*(d*x^8+c)^(3/2)/b/d^2-1/4*a^2*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*
c)^(1/2))/b^(5/2)/(-a*d+b*c)^(1/2)-1/4*(a*d+b*c)*(d*x^8+c)^(1/2)/b^2/d^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {457, 90, 65, 214}

$$\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = -\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^8}(ad+bc)}{4b^2d^2}+\frac{(c+dx^8)^{3/2}}{12bd^2}$$

```
[In] Int[x^23/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/4*((b*c + a*d)*Sqrt[c + d*x^8])/(b^2*d^2) + (c + d*x^8)^(3/2)/(12*b*d^2)
 - (a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(4*b^(5/2)*Sqrt
[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{8}\text{Subst}\left(\int \frac{x^2}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)\\
&= \frac{1}{8}\text{Subst}\left(\int \left(\frac{-bc-ad}{b^2d\sqrt{c+dx}} + \frac{a^2}{b^2(a+bx)\sqrt{c+dx}} + \frac{\sqrt{c+dx}}{bd}\right)\,dx, x, x^8\right)\\
&= -\frac{(bc+ad)\sqrt{c+dx^8}}{4b^2d^2} + \frac{\left(c+dx^8\right)^{3/2}}{12bd^2} + \frac{a^2\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)}{8b^2}\\
&= -\frac{(bc+ad)\sqrt{c+dx^8}}{4b^2d^2} + \frac{\left(c+dx^8\right)^{3/2}}{12bd^2} + \frac{a^2\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{4b^2d}\\
&= -\frac{(bc+ad)\sqrt{c+dx^8}}{4b^2d^2} + \frac{\left(c+dx^8\right)^{3/2}}{12bd^2} - \frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\sqrt{c+dx^8}(-2bc-3ad+bdx^8)}{12b^2d^2} + \frac{a^2\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{4b^{5/2}\sqrt{-bc+ad}}$$

```
[In] Integrate[x^23/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (Sqrt[c + d*x^8]*(-2*b*c - 3*a*d + b*d*x^8))/(12*b^2*d^2) + (a^2*ArcTan[(Sq
rt[b]*Sqrt[c + d*x^8])/Sqrt[-(b*c) + a*d]])/(4*b^(5/2)*Sqrt[-(b*c) + a*d])
```

Maple [A] (verified)

Time = 8.89 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

method	result	size
pseudoelliptic	$\frac{a^2\arctan\left(\frac{b\sqrt{d\,x^8+c}}{\sqrt{(ad-bc)b}}\right)d^2-\sqrt{d\,x^8+c}\left(\left(-\frac{b\,x^8}{3}+a\right)d+\frac{2bc}{3}\right)\sqrt{(ad-bc)b}}{4\sqrt{(ad-bc)b}\,b^2d^2}$	91

```
[In] int(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/((a*d-b*c)*b)^(1/2)*(a^2*arctan(b*(d*x^8+c)^(1/2)/((a*d-b*c)*b)^(1/2))*
d^2-(d*x^8+c)^(1/2)*((-1/3*b*x^8+a)*d+2/3*b*c)*((a*d-b*c)*b)^(1/2))/b^2/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.77

$$\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \left[\frac{3\sqrt{b^2c-abd}a^2d^2\log\left(\frac{bdx^8+2\,bc-ad-2\,\sqrt{dx^8+c}\sqrt{b^2c-abd}}{bx^8+a}\right)+2\left((b^3cd-ab^2d^2)x^8-2\,b^3c^2-ab^2cd+3\,a^2bd^2\right)\sqrt{c}}{24\left(b^4cd^2-ab^3d^3\right)}\right.$$

```
[In] integrate(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(d*
x^8 + c)*sqrt(b^2*c - a*b*d))/(b*x^8 + a)) + 2*((b^3*c*d - a*b^2*d^2)*x^8 -
 2*b^3*c^2 - a*b^2*c*d + 3*a^2*b*d^2)*sqrt(d*x^8 + c))/(b^4*c*d^2 - a*b^3*d
^3), 1/12*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(sqrt(d*x^8 + c)*sqrt(-b^2*
c + a*b*d)/(b*d*x^8 + b*c)) + ((b^3*c*d - a*b^2*d^2)*x^8 - 2*b^3*c^2 - a*b^
2*c*d + 3*a^2*b*d^2)*sqrt(d*x^8 + c))/(b^4*c*d^2 - a*b^3*d^3)]
```

Sympy [F]

$$\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

```
[In] integrate(x**23/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Integral(x**23/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.02

$$\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{a^2\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{4\sqrt{-b^2c+abd}b^2} + \frac{(dx^8+c)^{\frac{3}{2}}b^2d^4 - 3\sqrt{dx^8+c}b^2cd^4 - 3\sqrt{dx^8+c}abd^5}{12\,b^3d^6}$$

```
[In] integrate(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] 1/4*a^2*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*b^2) + 1/12*((d*x^8 + c)^(3/2)*b^2*d^4 - 3*sqrt(d*x^8 + c)*b^2*c*d^4 - 3*
sqrt(d*x^8 + c)*a*b*d^5)/(b^3*d^6)
```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.99

$$\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\left(d\,x^8+c\right)^{3/2}}{12\,b\,d^2} - \left(\frac{c}{2\,b\,d^2}+\frac{4\,a\,d^3-4\,b\,c\,d^2}{16\,b^2\,d^4}\right)\sqrt{d\,x^8+c}+\frac{a^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^8+c}}{\sqrt{a\,d-b\,c}}\right)}{4\,b^{5/2}\,\sqrt{a\,d-b\,c}}$$

```
[In] int(x^23/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] (c + d*x^8)^(3/2)/(12*b*d^2) - (c/(2*b*d^2) + (4*a*d^3 - 4*b*c*d^2)/(16*b^2
*d^4))*(c + d*x^8)^(1/2) + (a^2*atan((b^(1/2)*(c + d*x^8)^(1/2))/(a*d - b*c
)^(1/2)))/(4*b^(5/2)*(a*d - b*c)^(1/2))
```

3.888 $\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\sqrt{c+dx^8}}{4bd} + \frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{3/2}\sqrt{bc-ad}}$$

```
[Out] 1/4*a*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)^
(1/2)+1/4*(d*x^8+c)^(1/2)/b/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {457, 81, 65, 214}

$$\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{3/2}\sqrt{bc-ad}} + \frac{\sqrt{c+dx^8}}{4bd}$$

```
[In] Int[x^15/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] Sqrt[c + d*x^8]/(4*b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a
*d]])/(4*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{8}\text{Subst}\left(\int \frac{x}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right) \\
&= \frac{\sqrt{c+dx^8}}{4bd} - \frac{a\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)}{8b} \\
&= \frac{\sqrt{c+dx^8}}{4bd} - \frac{a\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{4bd} \\
&= \frac{\sqrt{c+dx^8}}{4bd} + \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{3/2}\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{1}{4}\left(\frac{\sqrt{c+dx^8}}{bd} - \frac{a\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{b^{3/2}\sqrt{-bc+ad}}\right)$$

```
[In] Integrate[x^15/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (Sqrt[c + d*x^8]/(b*d) - (a*ArcTan[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[-(b*c) +
a*d]])/(b^(3/2)*Sqrt[-(b*c) + a*d]))/4
```

Maple [A] (verified)

Time = 5.99 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

method	result	size
pseudoelliptic	$\frac{-a\arctan\left(\frac{b\sqrt{d\,x^8+c}}{\sqrt{(ad-bc)b}}\right)d+\sqrt{d\,x^8+c}\,\sqrt{(ad-bc)b}}{4bd\sqrt{(ad-bc)b}}$	72

```
[In] int(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-a*arctan(b*(d*x^8+c)^(1/2)/((a*d-b*c)*b)^(1/2))*d+(d*x^8+c)^(1/2)*((a
*d-b*c)*b)^(1/2))/b/d/((a*d-b*c)*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.77

$$\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \left[\frac{\sqrt{b^2c-abd}ad\log\left(\frac{bdx^8+2\,bc-ad+2\,\sqrt{dx^8+c}\sqrt{b^2c-abd}}{bx^8+a}\right)+2\,\sqrt{dx^8+c}(b^2c-abd)}{8\,(b^3cd-ab^2d^2)}, -\frac{\sqrt{-b^2c+abd}ad\arctan\left(\frac{\sqrt{dx^8+c}\sqrt{-b^2c+abd}}{bdx^8+bc}\right)-\sqrt{dx^8+c}(b^2c-abd)}{4\,(b^3cd-ab^2d^2)}\right]$$

```
[In] integrate(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(b^2*c - a*b*d)*a*d*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 + c
)*sqrt(b^2*c - a*b*d))/(b*x^8 + a)) + 2*sqrt(d*x^8 + c)*(b^2*c - a*b*d))/(b
```

```
^3*c*d - a*b^2*d^2), -1/4*(sqrt(-b^2*c + a*b*d)*a*d*arctan(sqrt(d*x^8 + c)*
sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c)) - sqrt(d*x^8 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2)]
```

Sympy [F]

$$\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

```
[In] integrate(x**15/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Integral(x**15/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = -\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b} - \frac{\sqrt{dx^8+c}}{b}}{4\,d}$$

```
[In] integrate(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] -1/4*(a*d*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b
*d)*b) - sqrt(d*x^8 + c)/b)/d
```

Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\sqrt{d\,x^8+c}}{4\,b\,d} - \frac{a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^8+c}}{\sqrt{a\,d-b\,c}}\right)}{4\,b^{3/2}\,\sqrt{a\,d-b\,c}}$$

```
[In] int(x^15/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] (c + d*x^8)^(1/2)/(4*b*d) - (a*atan((b^(1/2)*(c + d*x^8)^(1/2))/(a*d - b*c)
^(1/2)))/(4*b^(3/2)*(a*d - b*c)^(1/2))
```

3.889 $\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}} dx$

Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}} dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\sqrt{bc-ad}}$$

```
[Out] -1/4*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(1/2)/(-a*d+b*c)^(
1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {455, 65, 214}

$$\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}} dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\sqrt{bc-ad}}$$

```
[In] Int[x^7/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/4*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]]/(Sqrt[b]*Sqrt[b*c -
 a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{8}\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{4d}\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{4\sqrt{b}\sqrt{-bc+ad}}$$

```
[In] Integrate[x^7/((a + b*x^8)*Sqrt[c + d*x^8]),x]
[Out] ArcTan[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[-(b*c) + a*d]]/(4*Sqrt[b]*Sqrt[-(b*c)
 + a*d])
```

Maple [A] (verified)

Time = 5.90 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

method	result	size
pseudoelliptic	$\frac{\arctan\left(\frac{b\sqrt{d\,x^8+c}}{\sqrt{(ad-bc)b}}\right)}{4\sqrt{(ad-bc)b}}$	39

```
[In] int(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/4/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^8+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.55

$$\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \left[\frac{\log\left(\frac{bdx^8+2\,bc-ad-2\,\sqrt{dx^8+c}\sqrt{b^2c-abd}}{bx^8+a}\right)}{8\,\sqrt{b^2c-abd}},\ \frac{\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^8+c}\sqrt{-b^2c+abd}}{bdx^8+bc}\right)}{4\,(b^2c-abd)}\right]$$

```
[In] integrate(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] [1/8*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*sqrt(b^2*c - a*b*d))/(b
*x^8 + a))/sqrt(b^2*c - a*b*d), 1/4*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^8
+ c)*sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c))/(b^2*c - a*b*d)]
```

Sympy [A] (verification not implemented)

Time = 10.99 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.71

$$\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \begin{cases} \frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^8}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4b\sqrt{\frac{ad-bc}{b}}} & \text{for } d \neq 0 \\ \begin{cases} \frac{x^8}{8a\sqrt{c}} & \text{for } b = 0 \\ \tilde{\infty}x^8 & \text{for } \sqrt{c} = 0 \\ \frac{\log\left(8a\sqrt{c}+8b\sqrt{c}x^8\right)}{8b\sqrt{c}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**7/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Piecewise((atan(sqrt(c + d*x**8)/sqrt((a*d - b*c)/b))/(4*b*sqrt((a*d - b*c)
/b)), Ne(d, 0)), (Piecewise((x**8/(8*a*sqrt(c)), Eq(b, 0)), (zoo*x**8, Eq(s
qrt(c), 0)), (log(8*a*sqrt(c) + 8*b*sqrt(c)*x**8)/(8*b*sqrt(c)), True)), Tr
ue))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{4\sqrt{-b^2c+abd}}$$

```
[In] integrate(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\operatorname{atan}\left(\frac{b\sqrt{d\,x^8+c}}{\sqrt{a\,b\,d-b^2\,c}}\right)}{4\sqrt{a\,b\,d-b^2\,c}}$$

```
[In] int(x^7/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] atan((b*(c + d*x^8)^(1/2))/(a*b*d - b^2*c)^(1/2))/(4*(a*b*d - b^2*c)^(1/2))
```

3.890 $\int \frac{1}{x(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 85

$$\int \frac{1}{x\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a\sqrt{c}}+\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a\sqrt{bc-ad}}$$

```
[Out] -1/4*arctanh((d*x^8+c)^(1/2)/c^(1/2))/a/c^(1/2)+1/4*arctanh(b^(1/2)*(d*x^8+
c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/a/(-a*d+b*c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {457, 88, 65, 214}

$$\int \frac{1}{x\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a\sqrt{bc-ad}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a\sqrt{c}}$$

```
[In] Int[1/(x*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/4*ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]]/(a*Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[
b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(4*a*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{8}\text{Subst}\left(\int \frac{1}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^8\right)}{8a} - \frac{b\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)}{8a}\\
&= \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{4ad} - \frac{b\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{4ad}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a\sqrt{c}} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int \frac{1}{x(a+bx^8)\sqrt{c+dx^8}}\,dx = -\frac{\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{\sqrt{c}}}{4a}$$

```
[In] Integrate[1/(x*(a + b*x^8)*Sqrt[c + d*x^8]),x]
[Out] -1/4*((Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[-(b*c) + a*d]])/Sqrt[-
(b*c) + a*d] + ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]]/Sqrt[c])/a
```

Maple [A] (verified)

Time = 6.18 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

method	result	size
pseudoelliptic	$-\frac{b\arctan\left(\frac{b\sqrt{d\,x^{8}+c}}{\sqrt{(ad-bc)b}}\right)\sqrt{c}+\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{8}+c}}{\sqrt{c}}\right)\sqrt{(ad-bc)b}}{4a\sqrt{(ad-bc)b}\,\sqrt{c}}$	78

```
[In] int(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(b*arctan(b*(d*x^8+c)^(1/2)/((a*d-b*c)*b)^(1/2))*c^(1/2)+arctanh((d*x^
8+c)^(1/2)/c^(1/2))*((a*d-b*c)*b)^(1/2))/a/((a*d-b*c)*b)^(1/2)/c^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 431, normalized size of antiderivative = 5.07

$$\int \frac{1}{x\,(a+bx^8)\,\sqrt{c+dx^8}}\,dx = \left[\frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^8+2\,bc-ad+2\,\sqrt{dx^8+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^8+a}\right)+\sqrt{c}\log\left(\frac{dx^8-2\,\sqrt{dx^8+c}\sqrt{c}+2\,c}{x^8}\right)}{8\,ac}, \frac{2\,c\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{}}{}\right.}{}\right.$$

```
[In] integrate(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(c*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 + c)*
(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + sqrt(c)*log((d*x^8 - 2*sqrt
(d*x^8 + c)*sqrt(c) + 2*c)/x^8))/(a*c), 1/8*(2*c*sqrt(-b/(b*c - a*d))*arcta
n(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^8 + b*c)) + sqrt
(c)*log((d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8))/(a*c), 1/8*(c*sqrt(
b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 + c)*(b*c - a*d)*s
qrt(b/(b*c - a*d)))/(b*x^8 + a)) + 2*sqrt(-c)*arctan(sqrt(d*x^8 + c)*sqrt(-
c)/c))/(a*c), 1/4*(c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^8 + c)*(b*c - a*
d)*sqrt(-b/(b*c - a*d))/(b*d*x^8 + b*c)) + sqrt(-c)*arctan(sqrt(d*x^8 + c)*
sqrt(-c)/c))/(a*c)]
```

Sympy [A] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{1}{x\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=\begin{cases}\frac{2\left(-\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^8}}{\sqrt{\frac{ad-bc}{b}}}\right)}{8a\sqrt{\frac{ad-bc}{b}}}+\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^8}}{\sqrt{-c}}\right)}{8a\sqrt{-c}}\right)}{d} & \text{for } d\neq 0\\ \frac{\operatorname{atan}\left(\frac{2\left(\frac{a}{2b}+x^8\right)}{\sqrt{-\frac{a^2}{b^2}}}\right)}{4b\sqrt{c}\sqrt{-\frac{a^2}{b^2}}} & \text{otherwise}\end{cases}$$

```
[In] integrate(1/x/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Piecewise((2*(-d*atan(sqrt(c + d*x**8)/sqrt((a*d - b*c)/b))/(8*a*sqrt((a*d
- b*c)/b)) + d*atan(sqrt(c + d*x**8)/sqrt(-c))/(8*a*sqrt(-c)))/d, Ne(d, 0))
, (atan(2*(a/(2*b) + x**8)/sqrt(-a**2/b**2))/(4*b*sqrt(c)*sqrt(-a**2/b**2))
, True))
```

Maxima [F]

$$\int \frac{1}{x\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}x}\,dx$$

```
[In] integrate(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int \frac{1}{x\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=-\frac{b\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{4\sqrt{-b^2c+abd}a}+\frac{\arctan\left(\frac{\sqrt{dx^8+c}}{\sqrt{-c}}\right)}{4a\sqrt{-c}}$$

```
[In] integrate(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] -1/4*b*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*a) + 1/4*arctan(sqrt(d*x^8 + c)/sqrt(-c))/(a*sqrt(-c))
```

Mupad [B] (verification not implemented)

Time = 9.36 (sec) , antiderivative size = 652, normalized size of antiderivative = 7.67

$$\int \frac{1}{x\,(a+bx^8)\,\sqrt{c+dx^8}}\,dx = -\frac{\operatorname{atanh}\left(\frac{\sqrt{d\,x^8+c}}{\sqrt{c}}\right)}{4\,a\,\sqrt{c}} - \frac{\operatorname{atan}\left(\frac{\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(\frac{b^3\,d^2\,\sqrt{d\,x^8+c}}{4}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(a^2\,b^2\,d^3-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^8+c}\,\sqrt{b^2\,c-a\,b\,d}}{8\,\left(a^2\,d-a\,b\,c\right)}\right)}{8\,\left(a^2\,d-a\,b\,c\right)}\right)\,1i}{8\,\left(a^2\,d-a\,b\,c\right)}+\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(\frac{b^3\,d^2\,\sqrt{d\,x^8+c}}{4}+\cdots\right.}{\cdots}}{\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(\frac{b^3\,d^2\,\sqrt{d\,x^8+c}}{4}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(a^2\,b^2\,d^3-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^8+c}\,\sqrt{b^2\,c-a\,b\,d}}{8\,\left(a^2\,d-a\,b\,c\right)}\right)}{8\,\left(a^2\,d-a\,b\,c\right)}\right)}{8\,\left(a^2\,d-a\,b\,c\right)}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(\frac{b^3\,d^2\,\sqrt{d\,x^8+c}}{4}+\cdots\right.}{\cdots}}\right.\cdots}{4\,\left(a^2\,d-a\,b\,c\right)}$$

```
[In] int(1/(x*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] - atanh((c + d*x^8)^(1/2)/c^(1/2))/(4*a*c^(1/2)) - (atan((((b^2*c - a*b*d)^
(1/2)*((b^3*d^2*(c + d*x^8)^(1/2))/4 - ((b^2*c - a*b*d)^(1/2)*(a^2*b^2*d^3
- ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^8)^(1/2)*(b^2*c - a*b*d)^(1/
2))/(8*(a^2*d - a*b*c))))/(8*(a^2*d - a*b*c)))*1i)/(8*(a^2*d - a*b*c)) + ((
b^2*c - a*b*d)^(1/2)*((b^3*d^2*(c + d*x^8)^(1/2))/4 + ((b^2*c - a*b*d)^(1/2
)*(a^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^8)^(1/2)*(b^2
*c - a*b*d)^(1/2))/(8*(a^2*d - a*b*c))))/(8*(a^2*d - a*b*c)))*1i)/(8*(a^2*d
 - a*b*c)))/(((b^2*c - a*b*d)^(1/2)*((b^3*d^2*(c + d*x^8)^(1/2))/4 - ((b^2*
c - a*b*d)^(1/2)*(a^2*b^2*d^3 - ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*
x^8)^(1/2)*(b^2*c - a*b*d)^(1/2))/(8*(a^2*d - a*b*c))))/(8*(a^2*d - a*b*c))
))/(8*(a^2*d - a*b*c)) - ((b^2*c - a*b*d)^(1/2)*((b^3*d^2*(c + d*x^8)^(1/2)
)/4 + ((b^2*c - a*b*d)^(1/2)*(a^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*
d^2)*(c + d*x^8)^(1/2)*(b^2*c - a*b*d)^(1/2))/(8*(a^2*d - a*b*c))))/(8*(a^2
*d - a*b*c))))/(8*(a^2*d - a*b*c))))*(b^2*c - a*b*d)^(1/2)*1i)/(4*(a^2*d -
a*b*c))
```

3.891 $\int \frac{1}{x^9(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 117

$$\int \frac{1}{x^9(a+bx^8)\sqrt{c+dx^8}}\,dx = -\frac{\sqrt{c+dx^8}}{8acx^8}+\frac{(2bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^2c^{3/2}} - \frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a^2\sqrt{bc-ad}}$$

```
[Out] 1/8*(a*d+2*b*c)*arctanh((d*x^8+c)^(1/2)/c^(1/2))/a^2/c^(3/2)-1/4*b^(3/2)*ar
ctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(1/2)-1/8*(d
*x^8+c)^(1/2)/a/c/x^8
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 105, 162, 65, 214}

$$\int \frac{1}{x^9(a+bx^8)\sqrt{c+dx^8}}\,dx = -\frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a^2\sqrt{bc-ad}} + \frac{(ad+2bc)\text{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^2c^{3/2}} - \frac{\sqrt{c+dx^8}}{8acx^8}$$

```
[In] Int[1/(x^9*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/8*Sqrt[c + d*x^8]/(a*c*x^8) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^8]/Sqr
t[c]])/(8*a^2*c^(3/2)) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*
c - a*d]])/(4*a^2*Sqrt[b*c - a*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{8}\text{Subst}\left(\int \frac{1}{x^2(a+bx)\sqrt{c+dx}}\,dx,x,x^8\right)\\
&= -\frac{\sqrt{c+dx^8}}{8acx^8}-\frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2bc+ad)+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^8\right)}{8ac}\\
&= -\frac{\sqrt{c+dx^8}}{8acx^8}+\frac{b^2\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^8\right)}{8a^2}-\frac{(2bc+ad)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^8\right)}{16a^2c}
\end{aligned}
$$

$$= -\frac{\sqrt{c+dx^8}}{8acx^8} + \frac{b^2\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{4a^2d} - \frac{(2bc+ad)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{8a^2cd}$$

$$= -\frac{\sqrt{c+dx^8}}{8acx^8} + \frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^2c^{3/2}} - \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a^2\sqrt{bc-ad}}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^9(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{-\frac{a\sqrt{c+dx^8}}{cx^8} + \frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}} + \frac{(2bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{c^{3/2}}}{8a^2}$$

```
[In] Integrate[1/(x^9*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (-((a*Sqrt[c + d*x^8])/(c*x^8)) + (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^8
])/Sqrt[-(b*c) + a*d]])/Sqrt[-(b*c) + a*d] + ((2*b*c + a*d)*ArcTanh[Sqrt[c
+ d*x^8]/Sqrt[c]])/c^(3/2))/(8*a^2)
```

Maple [A] (verified)

Time = 6.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.79

method	result	size
pseudoelliptic	$\frac{\frac{2b^2\arctan\left(\frac{b\sqrt{d\,x^8+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}} - \frac{a\sqrt{d\,x^8+c}}{c\,x^8} + \frac{(ad+2bc)\,\text{arctanh}\left(\frac{\sqrt{d\,x^8+c}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}}{8a^2}$	92

```
[In] int(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/a^2*(2*b^2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^8+c)^(1/2)/((a*d-b*c)*b)^(
1/2))-a/c*(d*x^8+c)^(1/2)/x^8+(a*d+2*b*c)/c^(3/2)*arctanh((d*x^8+c)^(1/2)/c
^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.88 (sec) , antiderivative size = 565, normalized size of antiderivative = 4.83

$$\int \frac{1}{x^9 (a+bx^8) \sqrt{c+dx^8}}\,dx = \left[\frac{2\,bc^2x^8\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^8+2\,bc-ad-2\,\sqrt{dx^8+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^8+a}\right)+(2\,bc+ad)\sqrt{c}x^8\log\left(\frac{dx^8+2\,\sqrt{dx^8+c}\sqrt{c}+2\,c}{x^8}\right)-2\,\sqrt{c}\right.$$

$$-\frac{4\,bc^2x^8\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^8+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^8+bc}\right)-(2\,bc+ad)\sqrt{c}x^8\log\left(\frac{dx^8+2\,\sqrt{dx^8+c}\sqrt{c}+2\,c}{x^8}\right)+2\,\sqrt{dx^8}}{16\,a^2c^2x^8}$$

$$-\frac{2\,bc^2x^8\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^8+c}(bc-ad)\sqrt{-\frac{b}{bc-ad}}}{bdx^8+bc}\right)+(2\,bc+ad)\sqrt{-c}x^8\arctan\left(\frac{\sqrt{dx^8+c}\sqrt{-c}}{c}\right)+\sqrt{dx^8+c}}{8\,a^2c^2x^8}$$

```
[In] integrate(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*b*c^2*x^8*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(
d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + (2*b*c + a*d)*sq
rt(c)*x^8*log((d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8) - 2*sqrt(d*x^8
 + c)*a*c)/(a^2*c^2*x^8), -1/16*(4*b*c^2*x^8*sqrt(-b/(b*c - a*d))*arctan(-s
qrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^8 + b*c)) - (2*b*c +
 a*d)*sqrt(c)*x^8*log((d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8) + 2*sq
rt(d*x^8 + c)*a*c)/(a^2*c^2*x^8), 1/8*(b*c^2*x^8*sqrt(b/(b*c - a*d))*log((b
*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(
b*x^8 + a)) - (2*b*c + a*d)*sqrt(-c)*x^8*arctan(sqrt(d*x^8 + c)*sqrt(-c)/c)
 - sqrt(d*x^8 + c)*a*c)/(a^2*c^2*x^8), -1/8*(2*b*c^2*x^8*sqrt(-b/(b*c - a*d
))*arctan(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^8 + b*c)
) + (2*b*c + a*d)*sqrt(-c)*x^8*arctan(sqrt(d*x^8 + c)*sqrt(-c)/c) + sqrt(d*
x^8 + c)*a*c)/(a^2*c^2*x^8)]
```

Sympy [F]

$$\int \frac{1}{x^9 (a+bx^8) \sqrt{c+dx^8}}\, dx = \int \frac{1}{x^9 (a+bx^8) \sqrt{c+dx^8}}\, dx$$

[In] integrate(1/x**9/(b*x**8+a)/(d*x**8+c)**(1/2),x)

[Out] Integral(1/(x**9*(a + b*x**8)*sqrt(c + d*x**8)), x)

Maxima [F]

$$\int \frac{1}{x^9 (a+bx^8) \sqrt{c+dx^8}}\, dx = \int \frac{1}{(bx^8+a)\sqrt{dx^8+c}x^9}\, dx$$

[In] integrate(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^9), x)

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^9 (a+bx^8) \sqrt{c+dx^8}}\, dx = \frac{b^2 \arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{4\sqrt{-b^2c+abd}a^2} - \frac{(2\,bc+ad)\arctan\left(\frac{\sqrt{dx^8+c}}{\sqrt{-c}}\right)}{8\,a^2\sqrt{-c}c} - \frac{\sqrt{dx^8+c}}{8\,acx^8}$$

[In] integrate(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")

[Out] 1/4*b^2*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*a^2) - 1/8*(2*b*c + a*d)*arctan(sqrt(d*x^8 + c)/sqrt(-c))/(a^2*sqrt(-c)*c
) - 1/8*sqrt(d*x^8 + c)/(a*c*x^8)

Mupad [B] (verification not implemented)

Time = 10.11 (sec) , antiderivative size = 396, normalized size of antiderivative = 3.38

$$\int \frac{1}{x^9 (a+bx^8) \sqrt{c+dx^8}}\,dx = \frac{\ln\left(\sqrt{d\,x^8+c}\,{\left(b^4\,c-a\,b^3\,d\right)}^{3/2}+b^6\,c^2+a^2\,b^4\,d^2-2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{8\,a^3\,d-8\,a^2\,b\,c} - \frac{\ln\left(\sqrt{d\,x^8+c}\,{\left(b^4\,c-a\,b^3\,d\right)}^{3/2}-b^6\,c^2-a^2\,b^4\,d^2+2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{8\,\left(a^3\,d-a^2\,b\,c\right)} - \frac{\sqrt{d\,x^8+c}}{8\,a\,c\,x^8} - \frac{\mathrm{atan}\left(\frac{b^4\,d^4\,\sqrt{d\,x^8+c}\,3{}\mathrm{i}}{128\,\sqrt{c^3}\,\left(\frac{3\,b^4\,d^4}{128\,c}+\frac{5\,a\,b^3\,d^5}{256\,c^2}+\frac{a^2\,b^2\,d^6}{256\,c^3}\right)}+\frac{b^2\,d^6\,\sqrt{d\,x^8+c}\,1{}\mathrm{i}}{256\,\sqrt{c^3}\,\left(\frac{5\,b^3\,d^5}{256\,a}+\frac{b^2\,d^6}{256\,c}+\frac{3\,b^4\,c\,d^4}{128\,a^2}\right)}+\frac{b^3\,d^5\,\sqrt{d\,x^8+c}\,5{}\mathrm{i}}{256\,\sqrt{c^3}\,\left(\frac{3\,b^4\,d^4}{128\,a}+\frac{5\,b^3\,d^5}{256\,c}+\frac{a\,b^2\,d^6}{256\,c^2}\right)}\right)\,\left(a\,d+2\right.}{8\,a^2\,\sqrt{c^3}}$$

```
[In] int(1/(x^9*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] (log((c + d*x^8)^(1/2)*(b^4*c - a*b^3*d)^(3/2) + b^6*c^2 + a^2*b^4*d^2 - 2*
a*b^5*c*d)*(b^4*c - a*b^3*d)^(1/2))/(8*a^3*d - 8*a^2*b*c) - (log((c + d*x^8
)^(1/2)*(b^4*c - a*b^3*d)^(3/2) - b^6*c^2 - a^2*b^4*d^2 + 2*a*b^5*c*d)*(b^4
*c - a*b^3*d)^(1/2))/(8*(a^3*d - a^2*b*c)) - (c + d*x^8)^(1/2)/(8*a*c*x^8)
- (atan((b^4*d^4*(c + d*x^8)^(1/2)*3i)/(128*(c^3)^(1/2)*((3*b^4*d^4)/(128*c
) + (5*a*b^3*d^5)/(256*c^2) + (a^2*b^2*d^6)/(256*c^3))) + (b^2*d^6*(c + d*x
^8)^(1/2)*1i)/(256*(c^3)^(1/2)*((5*b^3*d^5)/(256*a) + (b^2*d^6)/(256*c) + (
3*b^4*c*d^4)/(128*a^2))) + (b^3*d^5*(c + d*x^8)^(1/2)*5i)/(256*(c^3)^(1/2)*
((3*b^4*d^4)/(128*a) + (5*b^3*d^5)/(256*c) + (a*b^2*d^6)/(256*c^2))))*(a*d
+ 2*b*c)*1i)/(8*a^2*(c^3)^(1/2))
```

3.892 $\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 123

$$\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{x^4\sqrt{c+dx^8}}{8bd} + \frac{a^{3/2}\arctan\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4b^2\sqrt{bc-ad}} - \frac{(bc+2ad)\text{arctanh}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{8b^2d^{3/2}}$$

```
[Out] -1/8*(2*a*d+b*c)*arctanh(x^4*d^(1/2)/(d*x^8+c)^(1/2))/b^2/d^(3/2)+1/4*a^(3/
2)*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))/b^2/(-a*d+b*c)^(1/2
)+1/8*x^4*(d*x^8+c)^(1/2)/b/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {476, 490, 537, 223, 212, 385, 211}

$$\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{a^{3/2}\arctan\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4b^2\sqrt{bc-ad}} - \frac{(2ad+bc)\text{arctanh}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{8b^2d^{3/2}} + \frac{x^4\sqrt{c+dx^8}}{8bd}$$

```
[In] Int[x^19/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^4*Sqrt[c + d*x^8])/(8*b*d) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt
[a]*Sqrt[c + d*x^8])])/(4*b^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTanh[(Sq
rt[d]*x^4)/Sqrt[c + d*x^8]])/(8*b^2*d^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\text{integral} = \frac{1}{4}\text{Subst}\left(\int \frac{x^4}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)$$

$$= \frac{x^4\sqrt{c+dx^8}}{8bd} - \frac{\text{Subst}\left(\int \frac{ac+(bc+2ad)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{8bd}$$

$$= \frac{x^4\sqrt{c+dx^8}}{8bd} + \frac{a^2\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{4b^2} - \frac{(bc+2ad)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^4\right)}{8b^2d}$$

$$= \frac{x^4\sqrt{c+dx^8}}{8bd} + \frac{a^2\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)}{4b^2} - \frac{(bc+2ad)\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)}{8b^2d}$$

$$= \frac{x^4\sqrt{c+dx^8}}{8bd} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4b^2\sqrt{bc-ad}} - \frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{8b^2d^{3/2}}$$

Mathematica [A] (verified)

Time = 2.36 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\frac{bx^4\sqrt{c+dx^8}}{d} + \frac{2a^{3/2}\arctan\left(\frac{a\sqrt{d}+bx^4\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}} - \frac{(bc+2ad)\log\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{d^{3/2}}}{8b^2}$$

```
[In] Integrate[x^19/((a + b*x^8)*Sqrt[c + d*x^8]),x]
[Out] ((b*x^4*Sqrt[c + d*x^8])/d + (2*a^(3/2)*ArcTan[(a*Sqrt[d] + b*x^4*(Sqrt[d]*
x^4 + Sqrt[c + d*x^8]))/(Sqrt[a]*Sqrt[b*c - a*d])])/Sqrt[b*c - a*d] - ((b*c
 + 2*a*d)*Log[Sqrt[d]*x^4 + Sqrt[c + d*x^8]])/d^(3/2))/(8*b^2)
```

Maple [A] (verified)

Time = 22.37 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.24

method	result
pseudoelliptic	$\frac{b\sqrt{(ad-bc)a}\,\sqrt{d\,x^8+c}\,\sqrt{d}\,x^4+2a^2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^8+c}\,a}{x^4\sqrt{(ad-bc)a}}\right)d^{\frac{3}{2}}-2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^8+c}}{x^4\sqrt{d}}\right)ad\sqrt{(ad-bc)a}-\operatorname{arctanh}\left(\frac{\sqrt{d\,x^8+c}}{x^4\sqrt{d}}\right)bc}{8b^2\sqrt{(ad-bc)a}\,d^{\frac{3}{2}}}$

```
[In] int(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(b*((a*d-b*c)*a)^(1/2)*(d*x^8+c)^(1/2)*d^(1/2)*x^4+2*a^2*arctanh((d*x^8
+c)^(1/2)/x^4*a/((a*d-b*c)*a)^(1/2))*d^(3/2)-2*arctanh((d*x^8+c)^(1/2)/x^4/
d^(1/2))*a*d*((a*d-b*c)*a)^(1/2)-arctanh((d*x^8+c)^(1/2)/x^4/d^(1/2))*b*c*(
(a*d-b*c)*a)^(1/2))/b^2/((a*d-b*c)*a)^(1/2)/d^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 739, normalized size of antiderivative = 6.01

$$\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \left[\frac{2\sqrt{dx^8+c}bdx^4+ad^2\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{16}-2\,(3\,abc^2-4\,a^2cd)x^8+a^2c^2+4\,((b^2c^2-3\,abcd+2\,a^2d^2)x^{12}-(abc}{b^2x^{16}+2\,abx^8+a^2}\right.}{16\,b^2d^2}\right.$$

```
[In] integrate(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*sqrt(d*x^8 + c)*b*d*x^4 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 +
4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d
*x^8 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2*a*b*x^8 + a^2)) + (b*c + 2*a*
d)*sqrt(d)*log(-2*d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 - c))/(b^2*d^2), 1/
16*(2*sqrt(d*x^8 + c)*b*d*x^4 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 -
8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 + 4*(
(b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d*x^
8 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2*a*b*x^8 + a^2)) + 2*(b*c + 2*a*d
)*sqrt(-d)*arctan(sqrt(-d)*x^4/sqrt(d*x^8 + c)))/(b^2*d^2), 1/16*(2*sqrt(d*
x^8 + c)*b*d*x^4 - 2*a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x
^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 + a*c*x^4)) + (b*c
+ 2*a*d)*sqrt(d)*log(-2*d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 - c))/(b^2*d^
2), 1/8*(sqrt(d*x^8 + c)*b*d*x^4 - a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((
b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 + a*c
```

```
*x^4)) + (b*c + 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x^4/sqrt(d*x^8 + c)))/(b^2*
d^2)]
```

Sympy [F]

$$\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

```
[In] integrate(x**19/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Integral(x**19/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^{19}}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^19/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^{19}}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

```
[In] int(x^19/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] int(x^19/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.893 $\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}} dx$

Optimal result

Integrand size = 24, antiderivative size = 91

$$\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}} dx = -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{bc-ad}x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4b\sqrt{bc-ad}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{4b\sqrt{d}}$$

```
[Out] 1/4*arctanh(x^4*d^(1/2)/(d*x^8+c)^(1/2))/b/d^(1/2)-1/4*arctan(x^4*(-a*d+b*c
)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))*a^(1/2)/b/(-a*d+b*c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {476, 494, 223, 212, 385, 211}

$$\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}} dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{4b\sqrt{d}} - \frac{\sqrt{a}\arctan\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4b\sqrt{bc-ad}}$$

```
[In] Int[x^11/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/4*(Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(b*S
qrt[b*c - a*d]) + ArcTanh[(Sqrt[d]*x^4)/Sqrt[c + d*x^8]]/(4*b*Sqrt[d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^4\right)}{4b} - \frac{a\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{4b}\\ &= \frac{\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)}{4b} - \frac{a\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)}{4b}\\ &= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bc-ad}x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4b\sqrt{bc-ad}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{4b\sqrt{d}}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.19

$$\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{-\frac{\sqrt{a}\arctan\left(\frac{a\sqrt{d}+bx^4\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}+\frac{\log\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{\sqrt{d}}}{4b}$$

```
[In] Integrate[x^11/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (-((Sqrt[a]*ArcTan[(a*Sqrt[d] + b*x^4*(Sqrt[d]*x^4 + Sqrt[c + d*x^8]))/(Sqr
t[a]*Sqrt[b*c - a*d])])/Sqrt[b*c - a*d]) + Log[Sqrt[d]*x^4 + Sqrt[c + d*x^8
]]/Sqrt[d])/(4*b)
```

Maple [A] (verified)

Time = 12.91 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.93

method	result	size
pseudoelliptic	$\frac{-a\ \operatorname{arctanh}\left(\frac{\sqrt{d\,x^8+c}\,a}{x^4\sqrt{(ad-bc)a}}\right)\sqrt{d}+\operatorname{arctanh}\left(\frac{\sqrt{d\,x^8+c}}{x^4\sqrt{d}}\right)\sqrt{(ad-bc)a}}{4b\sqrt{(ad-bc)a}\,\sqrt{d}}$	85

```
[In] int(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-a*arctanh((d*x^8+c)^(1/2)/x^4*a/((a*d-b*c)*a)^(1/2))*d^(1/2)+arctanh(
(d*x^8+c)^(1/2)/x^4/d^(1/2))*((a*d-b*c)*a)^(1/2))/b/((a*d-b*c)*a)^(1/2)/d^(
1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 632, normalized size of antiderivative = 6.95

$$\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$
$$= \left[\frac{d\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{16}-2\,(3\,abc^2-4\,a^2cd)x^8+a^2c^2-4\,((b^2c^2-3\,abcd+2\,a^2d^2)x^{12}-(abc^2-a^2cd)x^4)\sqrt{dx^8+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^{16}+2\,abx^8+a^2}\right.}{16\,bd}\right.$$

```
[In] integrate(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 -
 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*
```

```
d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d*x^8 + c)*sqrt(-a/(b*c - a*d)))/
(b^2*x^16 + 2*a*b*x^8 + a^2)) + 2*sqrt(d)*log(-2*d*x^8 - 2*sqrt(d*x^8 + c)*
sqrt(d)*x^4 - c))/(b*d), 1/16*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b
*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b^2*
c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d*x^8 + c
)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2*a*b*x^8 + a^2)) - 4*sqrt(-d)*arctan(s
qrt(-d)*x^4/sqrt(d*x^8 + c)))/(b*d), 1/8*(d*sqrt(a/(b*c - a*d))*arctan(-1/2
*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 +
a*c*x^4)) + sqrt(d)*log(-2*d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 - c))/(b*d
), 1/8*(d*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*
x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 + a*c*x^4)) - 2*sqrt(-d)*arctan(sqrt
(-d)*x^4/sqrt(d*x^8 + c)))/(b*d)]
```

Sympy [F]

$$\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

```
[In] integrate(x**11/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Integral(x**11/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^{11}}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^11/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^{11}}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

```
[In] int(x^11/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] int(x^11/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.894 $\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 54

$$\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\frac{\arctan\left(\frac{\sqrt{bc-ad}x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4\sqrt{a}\sqrt{bc-ad}}$$

```
[Out] 1/4*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))/a^(1/2)/(-a*d+b*c)
^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {476, 385, 211}

$$\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\frac{\arctan\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4\sqrt{a}\sqrt{bc-ad}}$$

```
[In] Int[x^3/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])]/(4*Sqrt[a]*Sqrt[b*c
 - a*d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4\sqrt{a}\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.37

$$\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\arctan\left(\frac{a\sqrt{d}+bx^4\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{4\sqrt{a}\sqrt{bc-ad}}$$

```
[In] Integrate[x^3/((a + b*x^8)*Sqrt[c + d*x^8]),x]
[Out] ArcTan[(a*Sqrt[d] + b*x^4*(Sqrt[d]*x^4 + Sqrt[c + d*x^8]))/(Sqrt[a]*Sqrt[b*
c - a*d])]/(4*Sqrt[a]*Sqrt[b*c - a*d])
```

Maple [A] (verified)

Time = 10.89 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^8+c}\,a}{x^4\sqrt{(ad-bc)a}}\right)}{4\sqrt{(ad-bc)a}}$	42

```
[In] int(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/((a*d-b*c)*a)^(1/2)*arctanh((d*x^8+c)^(1/2)/x^4*a/((a*d-b*c)*a)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(42) = 84$.

Time = 0.45 (sec) , antiderivative size = 245, normalized size of antiderivative = 4.54

$$\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \left[-\frac{\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{16}-2\,(3\,abc^2-4\,a^2cd)x^8+a^2c^2-4\,((bc-2\,ad)x^{12}-acx^4)\sqrt{dx^8+c}\sqrt{-abc+a^2d}}{b^2x^{16}+2\,abx^8+a^2}\right)}{16\,(abc-a^2d)}, \text{arctan}\right.$$

```
[In] integrate(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2
*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b*c - 2*a*d)*x^12 - a*c*x^4)*s
qrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2))/(a*b*c -
 a^2*d), 1/8*arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a*b*
c - a^2*d)/((a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4))/sqrt(a*b*c
 - a^2*d)]
```

Sympy [F]

$$\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

```
[In] integrate(x**3/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^3}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.33

$$\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx = -\frac{\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{4\,\sqrt{abcd-a^2d^2}}$$

```
[In] integrate(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] -1/4*sqrt(d)*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b - b*c + 2*a*d)
/sqrt(a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^3}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

```
[In] int(x^3/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] int(x^3/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.895 $\int \frac{1}{x^5(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \frac{1}{x^5\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=-\frac{\sqrt{c+dx^8}}{4acx^4}-\frac{b\arctan\left(\frac{\sqrt{bc-ad}x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4a^{3/2}\sqrt{bc-ad}}$$

```
[Out] -1/4*b*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))/a^(3/2)/(-a*d+b
*c)^(1/2)-1/4*(d*x^8+c)^(1/2)/a/c/x^4
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {476, 491, 12, 385, 211}

$$\int \frac{1}{x^5\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=-\frac{b\arctan\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^8}}{4acx^4}$$

```
[In] Int[1/(x^5*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/4*Sqrt[c + d*x^8]/(a*c*x^4) - (b*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*S
qrt[c + d*x^8])])/(4*a^(3/2)*Sqrt[b*c - a*d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^4\right)\\
&= -\frac{\sqrt{c+dx^8}}{4acx^4}-\frac{\text{Subst}\left(\int \frac{bc}{(a+bx^2)\sqrt{c+dx^2}}\,dx,x,x^4\right)}{4ac}\\
&= -\frac{\sqrt{c+dx^8}}{4acx^4}-\frac{b\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx,x,x^4\right)}{4a}\\
&= -\frac{\sqrt{c+dx^8}}{4acx^4}-\frac{b\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^4}{\sqrt{c+dx^8}}\right)}{4a}\\
&= -\frac{\sqrt{c+dx^8}}{4acx^4}-\frac{b\tan^{-1}\left(\frac{\sqrt{bc-ad}x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4a^{3/2}\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^5 (a+bx^8) \sqrt{c+dx^8}}\,dx = -\frac{\sqrt{c+dx^8}}{4acx^4} - \frac{b\arctan\left(\frac{a\sqrt{d}+bx^4\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{4a^{3/2}\sqrt{bc-ad}}$$

```
[In] Integrate[1/(x^5*(a + b*x^8)*Sqrt[c + d*x^8]),x]
[Out] -1/4*Sqrt[c + d*x^8]/(a*c*x^4) - (b*ArcTan[(a*Sqrt[d] + b*x^4*(Sqrt[d]*x^4
+ Sqrt[c + d*x^8]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(4*a^(3/2)*Sqrt[b*c - a*d])
```

Maple [A] (verified)

Time = 14.68 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

method	result	size
pseudoelliptic	$-\frac{cb\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^8+c}\,a}{x^4\sqrt{(ad-bc)a}}\right)x^4+\sqrt{d\,x^8+c}\,\sqrt{(ad-bc)a}}{4a\,x^4\sqrt{(ad-bc)a}\,c}$	80

```
[In] int(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/4*(c*b*arctanh((d*x^8+c)^(1/2)/x^4*a/((a*d-b*c)*a)^(1/2))*x^4+(d*x^8+c)^
(1/2)*((a*d-b*c)*a)^(1/2))/a/x^4/((a*d-b*c)*a)^(1/2)/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(64) = 128$.

Time = 0.50 (sec) , antiderivative size = 332, normalized size of antiderivative = 4.15

$$\int \frac{1}{x^5 (a+bx^8) \sqrt{c+dx^8}}\,dx = \left[-\frac{\sqrt{-abc+a^2d}bcx^4\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{16}-2\,(3\,abc^2-4\,a^2cd)x^8+a^2c^2+4\,((bc-2\,ad)x^{12}-acx^4)\sqrt{dx^8+c}\sqrt{-abc+a^2d}}{b^2x^{16}+2\,abx^8+a^2}\right)+ \cdots}{16\,(a^2bc^2-a^3cd)x^4}\right.$$
$$\left. -\frac{\sqrt{abc-a^2d}bcx^4\arctan\left(\frac{((bc-2\,ad)x^8-ac)\sqrt{dx^8+c}\sqrt{abc-a^2d}}{2\,((abcd-a^2d^2)x^{12}+(abc^2-a^2cd)x^4)}\right)+2\,\sqrt{dx^8+c}(abc-a^2d)}{8\,(a^2bc^2-a^3cd)x^4}\right]$$

```
[In] integrate(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] [-1/16*(sqrt(-a*b*c + a^2*d)*b*c*x^4*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)
*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 + 4*((b*c - 2*a*d)*x^12 - a
```

```
*c*x^4)*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2))
 + 4*sqrt(d*x^8 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^4), -1/8*(sq
rt(a*b*c - a^2*d)*b*c*x^4*arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 +
 c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4
)) + 2*sqrt(d*x^8 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^4)]
```

Sympy [F]

$$\int \frac{1}{x^5 (a + bx^8) \sqrt{c + dx^8}}\, dx = \int \frac{1}{x^5 (a + bx^8) \sqrt{c + dx^8}}\, dx$$

```
[In] integrate(1/x**5/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Integral(1/(x**5*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{1}{x^5 (a + bx^8) \sqrt{c + dx^8}}\, dx = \int \frac{1}{(bx^8 + a)\sqrt{dx^8 + c}x^5}\, dx$$

```
[In] integrate(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^5), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.45

$$\int \frac{1}{x^5 (a + bx^8) \sqrt{c + dx^8}}\, dx = \frac{1}{4} d^{\frac{3}{2}} \left(\frac{b \arctan\left(\frac{\left(\sqrt{d}x^4 - \sqrt{dx^8 + c}\right)^2 b - bc + 2\,ad}{2\sqrt{abcd - a^2d^2}}\right)}{\sqrt{abcd - a^2d^2}ad} + \frac{2}{\left(\left(\sqrt{d}x^4 - \sqrt{dx^8 + c}\right)^2 - c\right)ad} \right)$$

```
[In] integrate(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] 1/4*d^(3/2)*(b*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b - b*c + 2*a*
d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a*d) + 2/(((sqrt(d)*x^
4 - sqrt(d*x^8 + c))^2 - c)*a*d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (a + bx^8) \sqrt{c + dx^8}} \, dx = \int \frac{1}{x^5 \, (b \, x^8 + a) \, \sqrt{d \, x^8 + c}} \, dx$$

```
[In] int(1/(x^5*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] int(1/(x^5*(a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.896 $\int \frac{1}{x^{13}(a+bx^8)\sqrt{c+dx^8}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 115

$$\int \frac{1}{x^{13}(a+bx^8)\sqrt{c+dx^8}} \, dx = -\frac{\sqrt{c+dx^8}}{12acx^{12}} + \frac{(3bc+2ad)\sqrt{c+dx^8}}{12a^2c^2x^4} + \frac{b^2\arctan\left(\frac{\sqrt{bc-ad}x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4a^{5/2}\sqrt{bc-ad}}$$

```
[Out] 1/4*b^2*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))/a^(5/2)/(-a*d+
b*c)^(1/2)-1/12*(d*x^8+c)^(1/2)/a/c/x^12+1/12*(2*a*d+3*b*c)*(d*x^8+c)^(1/2)
/a^2/c^2/x^4
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {476, 491, 597, 12, 385, 211}

$$\int \frac{1}{x^{13}(a+bx^8)\sqrt{c+dx^8}} \, dx = \frac{b^2\arctan\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4a^{5/2}\sqrt{bc-ad}} + \frac{\sqrt{c+dx^8}(2ad+3bc)}{12a^2c^2x^4} - \frac{\sqrt{c+dx^8}}{12acx^{12}}$$

```
[In] Int[1/(x^13*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/12*Sqrt[c + d*x^8]/(a*c*x^12) + ((3*b*c + 2*a*d)*Sqrt[c + d*x^8])/(12*a^
2*c^2*x^4) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/
(4*a^(5/2)*Sqrt[b*c - a*d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^4\,(a+bx^2)\,\sqrt{c+dx^2}}\,dx, x, x^4\right)$$

$$= -\frac{\sqrt{c+dx^8}}{12acx^{12}} + \frac{\text{Subst}\left(\int \frac{-3bc-2ad-2bdx^2}{x^2(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{12ac}$$

$$= -\frac{\sqrt{c+dx^8}}{12acx^{12}} + \frac{(3bc+2ad)\sqrt{c+dx^8}}{12a^2c^2x^4} - \frac{\text{Subst}\left(\int -\frac{3b^2c^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{12a^2c^2}$$

$$= -\frac{\sqrt{c+dx^8}}{12acx^{12}} + \frac{(3bc+2ad)\sqrt{c+dx^8}}{12a^2c^2x^4} + \frac{b^2\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{4a^2}$$

$$= -\frac{\sqrt{c+dx^8}}{12acx^{12}} + \frac{(3bc+2ad)\sqrt{c+dx^8}}{12a^2c^2x^4} + \frac{b^2\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)}{4a^2}$$

$$= -\frac{\sqrt{c+dx^8}}{12acx^{12}} + \frac{(3bc+2ad)\sqrt{c+dx^8}}{12a^2c^2x^4} + \frac{b^2\tan^{-1}\left(\frac{\sqrt{bc-ad}x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4a^{5/2}\sqrt{bc-ad}}$$

Mathematica [A] (verified)

Time = 2.50 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^{13}(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\sqrt{c+dx^8}(-ac+3bcx^8+2adx^8)}{12a^2c^2x^{12}} + \frac{b^2\arctan\left(\frac{a\sqrt{d}+bx^4\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{4a^{5/2}\sqrt{bc-ad}}$$

```
[In] Integrate[1/(x^13*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (Sqrt[c + d*x^8]*(-(a*c) + 3*b*c*x^8 + 2*a*d*x^8))/(12*a^2*c^2*x^12) + (b^2
*ArcTan[(a*Sqrt[d] + b*x^4*(Sqrt[d]*x^4 + Sqrt[c + d*x^8]))/(Sqrt[a]*Sqrt[b
*c - a*d])])/(4*a^(5/2)*Sqrt[b*c - a*d])
```

Maple [A] (verified)

Time = 25.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

method	result	size
pseudoelliptic	$-\frac{-3b^2c^2\,\text{arctanh}\left(\frac{\sqrt{d\,x^8+c}\,a}{x^4\sqrt{(ad-bc)a}}\right)x^{12}+\sqrt{d\,x^8+c}\,\left((-3b\,x^8+a)c-2ad\,x^8\right)\sqrt{(ad-bc)a}}{12\sqrt{(ad-bc)a}\,x^{12}a^2c^2}$	103

```
[In] int(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(-3*b^2*c^2*arctanh((d*x^8+c)^(1/2)/x^4*a/((a*d-b*c)*a)^(1/2))*x^12+(
d*x^8+c)^(1/2)*((-3*b*x^8+a)*c-2*a*d*x^8)*((a*d-b*c)*a)^(1/2))/((a*d-b*c)*a
)^(1/2)/x^12/a^2/c^2
```

Fricas [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 416, normalized size of antiderivative = 3.62

$$\int \frac{1}{x^{13}\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$$
$$= \left[-\frac{3\sqrt{-abc+a^2d}\,b^2c^2x^{12}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{16}-2\,(3\,abc^2-4\,a^2cd)x^8+a^2c^2-4\,((bc-2\,ad)x^{12}-acx^4)\sqrt{dx^8+c}\sqrt{-abc+a^2d}}{b^2x^{16}+2\,abx^8+a^2}\right.}{48\,(a^3bc^3-a^4c^2d)x^{12}}\right.$$

```
[In] integrate(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*sqrt(-a*b*c + a^2*d)*b^2*c^2*x^12*log(((b^2*c^2 - 8*a*b*c*d + 8*a
^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b*c - 2*a*d)*x
^12 - a*c*x^4)*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8
+ a^2)) - 4*((3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^8 - a^2*b*c^2 + a^3*c*
d)*sqrt(d*x^8 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^12), 1/24*(3*sqrt(a*b*c - a^
2*d)*b^2*c^2*x^12*arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt
(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4)) + 2*(
(3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^8 - a^2*b*c^2 + a^3*c*d)*sqrt(d*x^8
 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^12)]
```

Sympy [F]

$$\int \frac{1}{x^{13}\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx = \int \frac{1}{x^{13}\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$$

```
[In] integrate(1/x**13/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(1/(x**13*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{1}{x^{13}\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx = \int \frac{1}{(bx^8+a)\sqrt{dx^8+c}x^{13}}\,dx$$

```
[In] integrate(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^13), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(95) = 190$.

Time = 1.27 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.78

$$\int \frac{1}{x^{13}(a+bx^8)\sqrt{c+dx^8}}\,dx = -\frac{1}{12}d^{\frac{5}{2}}\left(\frac{3b^2\arctan\left(\frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2b-bc+2ad}{2\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}a^2d^2} + \frac{2\left(3\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^4b-6\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2b\right.}{\left(\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right.\right.}\right.$$

```
[In] integrate(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/12*d^(5/2)*(3*b^2*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b - b*c
+ 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2*d^2) + 2*(3*
(sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b - 6*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b
*c - 6*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*d + 3*b*c^2 + 2*a*c*d)/(((sqrt(d
)*x^4 - sqrt(d*x^8 + c))^2 - c)^3*a^2*d^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{13}(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{1}{x^{13}\,\left(b\,x^8+a\right)\,\sqrt{d\,x^8+c}}\,dx$$

```
[In] int(1/(x^13*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(1/(x^13*(a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.897 $\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 851

$$
\begin{aligned}
&\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx \\
&= -\frac{\sqrt[4]{-a}\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8b^{3/4}\sqrt{bc-ad}} - \frac{\sqrt[4]{-a}\arctan\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8b^{3/4}\sqrt{-bc+ad}} \\
&\quad + \frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}} \\
&\quad - \frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}} \\
&\quad - \frac{\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}} \\
&\quad - \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16b\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}} \\
&\quad - \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16b\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}
\end{aligned}
$$

```
[Out] -1/8*(-a)^(1/4)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1
/2))/b^(3/4)/(-a*d+b*c)^(1/2)-1/8*(-a)^(1/4)*arctan(x^2*(a*d-b*c)^(1/2)/(-a
```

```
)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/b^(3/4)/(a*d-b*c)^(1/2)+1/4*(cos(2*arctan(
d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF
(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*
x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b/c^(1/4)/d^(1/4)/(d*x^8+c)^(1/2)-1/8
*a*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4
)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(
b^(1/2)*c^(1/2)/(-a)^(1/2)+d^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/
2)+x^4*d^(1/2))^2)^(1/2)/b/c^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)-1/8*d^(1/4)*(c
os(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)
))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*((-a)^(1/2)*b^
(1/2)*c^(1/2)+a*d^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1
/2))^2)^(1/2)/b/c^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)-1/16*(cos(2*arctan(d^(1/4
)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(
2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(
-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/
2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/
b/c^(1/4)/d^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)-1/16*(cos(2*arctan(d^(1/4)*x^2/
c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arct
an(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(
1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^
(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b/c^(
1/4)/d^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 851, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used

$= \{476, 494, 226, 418, 1231, 1721\}$

$$\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx =$$

$$-\frac{\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{16b\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^8+c}}$$
$$-\frac{\sqrt[4]{-a}\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{8b^{3/4}\sqrt{bc-ad}}-\frac{\sqrt[4]{-a}\arctan\left(\frac{\sqrt{ad-bc}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{8b^{3/4}\sqrt{ad-bc}}$$
$$+\frac{\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4b\sqrt[4]{c}\sqrt[4]{d}\sqrt{dx^8+c}}$$
$$-\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b\sqrt[4]{c}(bc+ad)\sqrt{dx^8+c}}$$
$$-\frac{\left(\sqrt{d}a+\sqrt{-a}\sqrt{b}\sqrt{c}\right)\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b\sqrt[4]{c}(bc+ad)\sqrt{dx^8+c}}$$
$$-\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16b\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^8+c}}$$

```
[In] Int[x^9/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/8*((-a)^(1/4)*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c +
d*x^8])])/(b^(3/4)*Sqrt[b*c - a*d]) - ((-a)^(1/4)*ArcTan[(Sqrt[-(b*c) + a*d
]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*b^(3/4)*Sqrt[-(b*c) + a*d]
) + ((Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*El
lipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*b*c^(1/4)*d^(1/4)*Sqrt[c
+ d*x^8]) - (a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sq
rt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(
d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((
Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[
(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(
1/4)], 1/2])/(8*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[b]*Sqrt[c]
+ Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + S
qrt[d]*x^4)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt
[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*b
*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a
]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^
```

```
4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]
*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*b*c^(1/4)*d^(
1/4)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
```

```
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{2b} - \frac{a\text{Subst}\left(\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{2b}$$

$$= \frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}} - \frac{\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4b} - \frac{\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4b}$$

$$= \frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}} - \frac{\left(\sqrt{c}\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4(bc+ad)} - \frac{\left(\sqrt{c}\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4(bc+ad)} - \frac{\left(a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4b(bc+ad)} - \frac{\left(\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4b(bc+ad)}$$

$$= -\frac{\sqrt[4]{-a}\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8b^{3/4}\sqrt{bc-ad}} - \frac{\sqrt[4]{-a}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8b^{3/4}\sqrt{-bc+ad}}$$
$$+ \frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}}$$
$$- \frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8b\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}}$$
$$- \frac{\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8b\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}}$$
$$- \frac{\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}$$
$$- \frac{\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.08

$$\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{x^{10}\sqrt{\frac{c+dx^8}{c}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{10a\sqrt{c+dx^8}}$$

```
[In] Integrate[x^9/((a + b*x^8)*Sqrt[c + d*x^8]),x]
[Out] (x^10*Sqrt[(c + d*x^8)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c), -((b*x^8
)/a)])/(10*a*Sqrt[c + d*x^8])
```

Maple [F]

$$\int \frac{x^9}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}dx$$

```
[In] int(x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x)
[Out] int(x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^9}{(a+bx^8)\,\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

```
[In] integrate(x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^9}{(a+bx^8)\,\sqrt{c+dx^8}}\,dx = \int \frac{x^9}{(a+bx^8)\,\sqrt{c+dx^8}}\,dx$$

```
[In] integrate(x**9/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Integral(x**9/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x^9}{(a+bx^8)\,\sqrt{c+dx^8}}\,dx = \int \frac{x^9}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^9/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Giac [F]

$$\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^9}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^9/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^9}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

```
[In] int(x^9/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] int(x^9/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.898 $\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 754

$$\begin{aligned}\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx &= -\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{3/4}\sqrt{bc-ad}}-\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{3/4}\sqrt{-bc+ad}}\\
&+\frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}}\\
&+\frac{\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}}\\
&+\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}\\
&+\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}\end{aligned}$$

```
[Out] -1/8*b^(1/4)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2)
)/(-a)^(3/4)/(-a*d+b*c)^(1/2)-1/8*b^(1/4)*arctan(x^2*(a*d-b*c)^(1/2)/(-a)^(
1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(3/4)/(a*d-b*c)^(1/2)+1/8*d^(1/4)*(cos(2
*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*E
llipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(b^(1/2)*c^(1/2)/(
-a)^(1/2)+d^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2
)^(1/2)/c^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)+1/8*d^(1/4)*(cos(2*arctan(d^(1/4)
```

```
*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*
arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(
1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^(
1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)+1/16*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^
(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^
2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c
^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/
2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^(1/4)/d^(1/4)/(
a*d+b*c)/(d*x^8+c)^(1/2)+1/16*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/
cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1
/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2
)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^
(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^(1/4)/d^(1/4)/(a*d+b
*c)/(d*x^8+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 754, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {476, 418, 1231, 226, 1721}

$$\begin{aligned}
\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx = & -\frac{\sqrt[4]{b}\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{3/4}\sqrt{bc-ad}}-\frac{\sqrt[4]{b}\arctan\left(\frac{x^2\sqrt{ad-bc}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{3/4}\sqrt{ad-bc}} \\
& +\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8\sqrt[4]{c}\sqrt{c+dx^8}(ad+bc)} \\
& +\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a\sqrt[4]{c}\sqrt{c+dx^8}(ad+bc)} \\
& +\frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}(ad+bc)} \\
& +\frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)^2\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}(ad+bc)}
\end{aligned}$$

```
[In] Int[x/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/8*(b^(1/4)*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x
^8])])/((-a)^(3/4)*Sqrt[b*c - a*d]) - (b^(1/4)*ArcTan[(Sqrt[-(b*c) + a*d]*x
```

```
^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*(-a)^(3/4)*Sqrt[-(b*c) + a*d]
) + (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)
*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^
2)/c^(1/4)], 1/2])/(8*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + ((Sqrt[-a]*Sqr
t[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/
(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])
/(8*a*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*S
qrt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^
2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]
*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*a*c^(1/4)*d^(
1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2
*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Ellipt
icPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqr
t[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*a*c^(1/4)*d^(1/4)*(b*c +
a*d)*Sqrt[c + d*x^8])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
```

```
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4a} + \frac{\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4a}$$

$$= \frac{\left(\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4a(bc+ad)}$$
$$+ \frac{\left(\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4a(bc+ad)}$$
$$+ \frac{\left(\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4(bc+ad)}$$
$$+ \frac{\left(\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4a(bc+ad)}$$

$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{3/4}\sqrt{bc-ad}} - \frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{3/4}\sqrt{-bc+ad}}$$
$$+ \frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}}$$
$$+ \frac{\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}}$$
$$+ \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16a\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}$$
$$+ \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16a\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.09

$$\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{x^2\sqrt{\frac{c+dx^8}{c}}\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{2a\sqrt{c+dx^8}}$$

```
[In] Integrate[x/((a + b*x^8)*Sqrt[c + d*x^8]),x]
[Out] (x^2*Sqrt[(c + d*x^8)/c]*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^8)/c), -((b*x^8)
/a)])/(2*a*Sqrt[c + d*x^8])
```

Maple [F]

$$\int \frac{x}{(b\,x^8+a)\sqrt{d\,x^8+c}}dx$$

```
[In] int(x/(b*x^8+a)/(d*x^8+c)^(1/2),x)
[Out] int(x/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

```
[In] integrate(x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

```
[In] integrate(x/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Integral(x/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Giac [F]

$$\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(x/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

```
[In] int(x/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] int(x/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.899 $\int \frac{1}{x^7(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 878

$$\begin{aligned}
&\int \frac{1}{x^7\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx\\
&= -\frac{\sqrt{c+dx^8}}{6acx^6} - \frac{b^{5/4}\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{7/4}\sqrt{bc-ad}} - \frac{b^{5/4}\arctan\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{7/4}\sqrt{-bc+ad}}\\
&\quad - \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{12ac^{5/4}\sqrt{c+dx^8}}\\
&\quad - \frac{b\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}}\\
&\quad - \frac{b\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a^2\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}}\\
&\quad - \frac{b\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}\\
&\quad - \frac{b\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}
\end{aligned}$$

```
[Out] -1/8*b^(5/4)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2)
)/(-a)^(7/4)/(-a*d+b*c)^(1/2)-1/8*b^(5/4)*arctan(x^2*(a*d-b*c)^(1/2)/(-a)^(
```

```
1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(7/4)/(a*d-b*c)^(1/2)-1/6*(d*x^8+c)^(1/2
)/a/c/x^6-1/12*d^(3/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*a
rctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/
2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/
a/c^(5/4)/(d*x^8+c)^(1/2)-1/8*b*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))
^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)
*x^2/c^(1/4))),1/2*2^(1/2))*(b^(1/2)*c^(1/2)/(-a)^(1/2)+d^(1/2))*(c^(1/2)+x
^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^(1/4)/(a*d+b*c)/(
d*x^8+c)^(1/2)-1/8*b*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/c
os(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4
))),1/2*2^(1/2))*((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))*(c^(1/2)+x^4*d^(1/2
))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a^2/c^(1/4)/(a*d+b*c)/(d*x^8+c
)^(1/2)-1/16*b*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^
(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1
/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^
(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8
+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a^2/c^(1/4)/d^(1/4)/(a*d+b*c)/(d*x^8+c)^
(1/2)-1/16*b*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1
/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/
2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(
1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^8+
c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a^2/c^(1/4)/d^(1/4)/(a*d+b*c)/(d*x^8+c)^(
1/2)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 878, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used

$= \{476, 491, 537, 226, 418, 1231, 1721\}$

$$\int \frac{1}{x^7 (a+bx^8)\sqrt{c+dx^8}}\,dx = -\frac{b\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}}, 2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right), \frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{16a^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^8+c}}$$
$$-\frac{b^{5/4}\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{8(-a)^{7/4}\sqrt{bc-ad}} - \frac{b^{5/4}\arctan\left(\frac{\sqrt{ad-bc}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{8(-a)^{7/4}\sqrt{ad-bc}}$$
$$-\frac{d^{3/4}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right), \frac{1}{2}\right)}{12ac^{5/4}\sqrt{dx^8+c}}$$
$$-\frac{b\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right), \frac{1}{2}\right)}{8a\sqrt[4]{c}(bc+ad)\sqrt{dx^8+c}}$$
$$-\frac{b\left(\sqrt{d}a+\sqrt{-a}\sqrt{b}\sqrt{c}\right)\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right), \frac{1}{2}\right)}{8a^2\sqrt[4]{c}(bc+ad)\sqrt{dx^8+c}}$$
$$-\frac{b\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}}, 2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right), \frac{1}{2}\right)}{16a^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^8+c}}$$
$$-\frac{\sqrt{dx^8+c}}{6acx^6}$$

```
[In] Int[1/(x^7*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/6*Sqrt[c + d*x^8]/(a*c*x^6) - (b^(5/4)*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a
)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*(-a)^(7/4)*Sqrt[b*c - a*d]) - (b^(5/4
)*ArcTan[(Sqrt[-(b*c) + a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(8
*(-a)^(7/4)*Sqrt[-(b*c) + a*d]) - (d^(3/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c
+ d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4
)], 1/2])/(12*a*c^(5/4)*Sqrt[c + d*x^8]) - (b*((Sqrt[b]*Sqrt[c])/Sqrt[-a] +
 Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[
d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*a*c^(1/4)*(b
*c + a*d)*Sqrt[c + d*x^8]) - (b*(Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1
/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Ell
ipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*a^2*c^(1/4)*(b*c + a*d)*Sq
rt[c + d*x^8]) - (b*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[
d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-1/4*(Sqrt[b
]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTa
```

```
n[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*a^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c
+ d*x^8]) - (b*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^
4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c]
+ Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/
4)*x^2)/c^(1/4)], 1/2])/(16*a^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]
)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
```

```
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{c+dx^8}}{6acx^6}+\frac{\text{Subst}\left(\int \frac{-3bc-ad-bdx^4}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{6ac}\\
&= -\frac{\sqrt{c+dx^8}}{6acx^6}-\frac{b\text{Subst}\left(\int \frac{1}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{2a}-\frac{d\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx,x,x^2\right)}{6ac}\\
&= -\frac{\sqrt{c+dx^8}}{6acx^6}-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{12ac^{5/4}\sqrt{c+dx^8}}\\
&\quad-\frac{b\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{4a^2}-\frac{b\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{4a^2}
\end{aligned}
$$

$$= -\frac{\sqrt{c+dx^8}}{6acx^6} - \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{12ac^{5/4}\sqrt{c+dx^8}}$$
$$-\frac{\left(b^{3/2}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\right)\text{Subst}\left(\int\frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{4a^2(bc+ad)}$$
$$-\frac{\left(b^{3/2}\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\right)\text{Subst}\left(\int\frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{4a^2(bc+ad)}$$
$$-\frac{\left(b\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt{d}\right)\text{Subst}\left(\int\frac{1}{\sqrt{c+dx^4}}\,dx,x,x^2\right)}{4a(bc+ad)}$$
$$-\frac{\left(b\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt{d}\right)\text{Subst}\left(\int\frac{1}{\sqrt{c+dx^4}}\,dx,x,x^2\right)}{4a^2(bc+ad)}$$

$$= -\frac{\sqrt{c+dx^8}}{6acx^6} - \frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{7/4}\sqrt{bc-ad}} - \frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{7/4}\sqrt{-bc+ad}}$$
$$-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{12ac^{5/4}\sqrt{c+dx^8}}$$
$$-\frac{b\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}}$$
$$-\frac{b\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a^2\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}}$$
$$-\frac{b\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16a^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}$$
$$-\frac{b\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16a^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.16

$$\int \frac{1}{x^7 (a+bx^8) \sqrt{c+dx^8}} dx = \frac{-5a(c+dx^8) - 5(3bc+ad)x^8 \sqrt{1+\frac{dx^8}{c}} \text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^8}{c},-\frac{bx^8}{a}\right) - bdx^{16} \sqrt{1+\frac{dx^8}{c}} \text{AppellF1}\left(\frac{5}{4}\right.}{30a^2cx^6\sqrt{c+dx^8}}$$

```
[In] Integrate[1/(x^7*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (-5*a*(c + d*x^8) - 5*(3*b*c + a*d)*x^8*Sqrt[1 + (d*x^8)/c]*AppellF1[1/4, 1
/2, 1, 5/4, -((d*x^8)/c), -((b*x^8)/a)] - b*d*x^16*Sqrt[1 + (d*x^8)/c]*Appe
llF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c), -((b*x^8)/a)])/(30*a^2*c*x^6*Sqrt[c +
d*x^8])
```

Maple [F]

$$\int \frac{1}{x^7 (b\,x^8+a) \sqrt{d\,x^8+c}} dx$$

```
[In] int(1/x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{x^7 (a+bx^8) \sqrt{c+dx^8}} dx = \int \frac{1}{(bx^8+a)\sqrt{dx^8+c}x^7} dx$$

```
[In] integrate(1/x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^8 + c)/(b*d*x^23 + (b*c + a*d)*x^15 + a*c*x^7), x)
```

Sympy [F]

$$\int \frac{1}{x^7 (a + bx^8) \sqrt{c + dx^8}} \, dx = \int \frac{1}{x^7 (a + bx^8) \sqrt{c + dx^8}} \, dx$$

```
[In] integrate(1/x**7/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Integral(1/(x**7*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{1}{x^7 (a + bx^8) \sqrt{c + dx^8}} \, dx = \int \frac{1}{(bx^8 + a)\sqrt{dx^8 + c}x^7} \, dx$$

```
[In] integrate(1/x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^7), x)
```

Giac [F]

$$\int \frac{1}{x^7 (a + bx^8) \sqrt{c + dx^8}} \, dx = \int \frac{1}{(bx^8 + a)\sqrt{dx^8 + c}x^7} \, dx$$

```
[In] integrate(1/x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^7 (a + bx^8) \sqrt{c + dx^8}} \, dx = \int \frac{1}{x^7 \left(b\,x^8 + a\right) \sqrt{d\,x^8 + c}} \, dx$$

```
[In] int(1/(x^7*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] int(1/(x^7*(a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.900 $\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1005

$$\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{x^2\sqrt{c+dx^8}}{2b\sqrt{d}\left(\sqrt{c}+\sqrt{d}x^4\right)} + \frac{(-a)^{3/4}\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8b^{5/4}\sqrt{bc-ad}} - \frac{(-a)^{3/4}\arctan\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8b^{5/4}\sqrt{-bc+ad}} - \frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2bd^{3/4}\sqrt{c+dx^8}} + \frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4bd^{3/4}\sqrt{c+dx^8}} + \frac{a\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}} + \frac{a\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}} + \frac{\sqrt{-a}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\right.}{16b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}} - \frac{\sqrt{-a}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right.}{16b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}$$

```
[Out] 1/8*(-a)^(3/4)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/
2))/b^(5/4)/(-a*d+b*c)^(1/2)-1/8*(-a)^(3/4)*arctan(x^2*(a*d-b*c)^(1/2)/(-a)
^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/b^(5/4)/(a*d-b*c)^(1/2)+1/2*x^2*(d*x^8+c)^(
1/2)/b/d^(1/2)/(c^(1/2)+x^4*d^(1/2))-1/2*c^(1/4)*(cos(2*arctan(d^(1/4)*x^2/
c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticE(sin(2*arcta
n(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1
/2)+x^4*d^(1/2))^2)^(1/2)/b/d^(3/4)/(d*x^8+c)^(1/2)+1/4*c^(1/4)*(cos(2*arct
an(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*Ellipt
icF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(
(d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b/d^(3/4)/(d*x^8+c)^(1/2)-1/16*(co
s(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4))
)*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(
```

```
1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(-a)^(1/2)*
(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^
(1/2)+x^4*d^(1/2))^2)^(1/2)/b^(3/2)/c^(1/4)/d^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/
2)+1/16*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x
^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/2)*c^
(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))
*(-a)^(1/2)*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((
d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b^(3/2)/c^(1/4)/d^(1/4)/(a*d+b*c)/(
d*x^8+c)^(1/2)+1/8*a*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/c
os(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4
))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(c^(1/2)-(-a)^(1/2)*d^(1/2)/b^(1/2))
*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b/c^(1/4)/(a*d+b*c)/(d*x^8+c)^(1
/2)+1/8*a*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan
(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(
1/2))*(c^(1/2)+x^4*d^(1/2))*(c^(1/2)+(-a)^(1/2)*d^(1/2)/b^(1/2))*((d*x^8+c)
/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b/c^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 1005, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{476, 494, 311, 226, 1210, 504, 1231, 1721\}$

$$\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}\,dx =$$

$$-\frac{\sqrt{-a}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)}{16b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^8+c}}$$

$$+\frac{(-a)^{3/4}\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{8b^{5/4}\sqrt{bc-ad}}-\frac{(-a)^{3/4}\arctan\left(\frac{\sqrt{ad-bc}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{8b^{5/4}\sqrt{ad-bc}}$$

$$-\frac{\sqrt[4]{c}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2bd^{3/4}\sqrt{dx^8+c}}$$

$$+\frac{\sqrt[4]{c}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4bd^{3/4}\sqrt{dx^8+c}}$$

$$+\frac{a\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b\sqrt[4]{c}(bc+ad)\sqrt{dx^8+c}}$$

$$+\frac{a\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b\sqrt[4]{c}(bc+ad)\sqrt{dx^8+c}}$$

$$+\frac{\sqrt{-a}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\right.}{16b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^8+c}}$$

$$+\frac{x^2\sqrt{dx^8+c}}{2b\sqrt{d}\left(\sqrt{d}x^4+\sqrt{c}\right)}$$

```
[In] Int[x^13/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^2*Sqrt[c + d*x^8])/(2*b*Sqrt[d]*(Sqrt[c] + Sqrt[d]*x^4)) + ((-a)^(3/4)*A
rcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*b^(5/
4)*Sqrt[b*c - a*d]) - ((-a)^(3/4)*ArcTan[(Sqrt[-(b*c) + a*d]*x^2)/((-a)^(1/
4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*b^(5/4)*Sqrt[-(b*c) + a*d]) - (c^(1/4)*(Sq
rt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticE[
2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*b*d^(3/4)*Sqrt[c + d*x^8]) + (c^(
1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*El
lipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*b*d^(3/4)*Sqrt[c + d*x^8]
) + (a*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^
4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*
```

```
x^2)/c^(1/4)], 1/2])/(8*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + (a*(Sqrt[c
] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d
*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)],
 1/2])/(8*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + (Sqrt[-a]*(Sqrt[b]*Sqrt[
c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c]
+ Sqrt[d]*x^4)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(S
qrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(1
6*b^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) - (Sqrt[-a]*(Sqrt[b]
*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sq
rt[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(
4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])
/(16*b^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{2b} - \frac{a\text{Subst}\left(\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{2b}\\
&= \frac{a\text{Subst}\left(\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4b^{3/2}} - \frac{a\text{Subst}\left(\int \frac{1}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4b^{3/2}}\\
&\quad + \frac{\sqrt{c}\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{2b\sqrt{d}} - \frac{\sqrt{c}\text{Subst}\left(\int \frac{1-\frac{\sqrt{d}x^2}{\sqrt{c}}}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{2b\sqrt{d}}
\end{aligned}
$$

$$= \frac{x^2\sqrt{c+dx^8}}{2b\sqrt{d}\left(\sqrt{c}+\sqrt{d}x^4\right)} - \frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2bd^{3/4}\sqrt{c+dx^8}}$$

$$+ \frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4bd^{3/4}\sqrt{c+dx^8}}$$

$$+ \frac{\left(a\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\right)\text{Subst}\left(\int\frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{4b(bc+ad)}$$

$$- \frac{\left(a\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\right)\text{Subst}\left(\int\frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{4b(bc+ad)}$$

$$+ \frac{\left(a\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt{d}\right)\text{Subst}\left(\int\frac{1}{\sqrt{c+dx^4}}\,dx,x,x^2\right)}{4b^{3/2}(bc+ad)}$$

$$+ \frac{\left(a\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt{d}\right)\text{Subst}\left(\int\frac{1}{\sqrt{c+dx^4}}\,dx,x,x^2\right)}{4b^{3/2}(bc+ad)}$$

$$= \frac{x^2\sqrt{c+dx^8}}{2b\sqrt{d}\left(\sqrt{c}+\sqrt{d}x^4\right)} + \frac{(-a)^{3/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8b^{5/4}\sqrt{bc-ad}}$$
$$- \frac{(-a)^{3/4}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8b^{5/4}\sqrt{-bc+ad}}$$
$$- \frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2bd^{3/4}\sqrt{c+dx^8}}$$
$$+ \frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4bd^{3/4}\sqrt{c+dx^8}}$$
$$+ \frac{a\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8b^{3/2}\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}}$$
$$+ \frac{a\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8b^{3/2}\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}}$$
$$+ \frac{\sqrt{-a}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}$$
$$- \frac{\sqrt{-a}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.06

$$\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{x^{14}\sqrt{\frac{c+dx^8}{c}}\,\text{AppellF1}\left(\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{14a\sqrt{c+dx^8}}$$

```
[In] Integrate[x^13/((a + b*x^8)*Sqrt[c + d*x^8]),x]
[Out] (x^14*Sqrt[(c + d*x^8)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^8)/c), -((b*x^
8)/a)])/(14*a*Sqrt[c + d*x^8])
```

Maple [F]

$$\int \frac{x^{13}}{\left(b\,x^{8}+a\right)\sqrt{d\,x^{8}+c}}dx$$

```
[In] int(x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x)
[Out] int(x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^{13}}{(a+bx^8)\,\sqrt{c+dx^8}}\,dx = \int \frac{x^{13}}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*x^8 + c)*x^13/(b*d*x^16 + (b*c + a*d)*x^8 + a*c), x)
```

Sympy [F]

$$\int \frac{x^{13}}{(a+bx^8)\,\sqrt{c+dx^8}}\,dx = \int \frac{x^{13}}{(a+bx^8)\,\sqrt{c+dx^8}}\,dx$$

```
[In] integrate(x**13/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Integral(x**13/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x^{13}}{(a+bx^8)\,\sqrt{c+dx^8}}\,dx = \int \frac{x^{13}}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^13/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Giac [F]

$$\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^{13}}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^13/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^{13}}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

```
[In] int(x^13/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] int(x^13/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.901 $\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 768

$$\begin{aligned}
\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx &= \frac{\arctan\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}} - \frac{\arctan\left(\frac{\sqrt{-bc+ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8\sqrt[4]{-a}\sqrt[4]{b}\sqrt{-bc+ad}} \\
&- \frac{\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}} \\
&- \frac{\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}} \\
&+ \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16\sqrt{-a}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}} \\
&- \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16\sqrt{-a}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}
\end{aligned}$$

```
[Out] 1/8*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(1
/4)/b^(1/4)/(-a*d+b*c)^(1/2)-1/8*arctan(x^2*(a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1
/4)/(d*x^8+c)^(1/2))/(-a)^(1/4)/b^(1/4)/(a*d-b*c)^(1/2)-1/16*(cos(2*arctan(
d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticP
i(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2
))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*
(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^
```

```
(1/2)/c^(1/4)/d^(1/4)/(a*d+b*c)/(-a)^(1/2)/b^(1/2)/(d*x^8+c)^(1/2)+1/16*(co
s(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4))
)*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^
(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x
^4*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*
d^(1/2))^2)^(1/2)/c^(1/4)/d^(1/4)/(a*d+b*c)/(-a)^(1/2)/b^(1/2)/(d*x^8+c)^(1
/2)-1/8*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d
^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/
2))*(c^(1/2)+x^4*d^(1/2))*(c^(1/2)-(-a)^(1/2)*d^(1/2)/b^(1/2))*((d*x^8+c)/(
c^(1/2)+x^4*d^(1/2))^2)^(1/2)/c^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)-1/8*d^(1/4)
*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1
/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^
4*d^(1/2))*(c^(1/2)+(-a)^(1/2)*d^(1/2)/b^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(
1/2))^2)^(1/2)/c^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 768, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {476, 504, 1231, 226, 1721}

$$\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}} - \frac{\arctan\left(\frac{x^2\sqrt{ad-bc}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8\sqrt[4]{-a}\sqrt[4]{b}\sqrt{ad-bc}}$$
$$- \frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8\sqrt[4]{c}\sqrt{c+dx^8}(ad+bc)}$$
$$- \frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}+\sqrt{c}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8\sqrt[4]{c}\sqrt{c+dx^8}(ad+bc)}$$
$$- \frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16\sqrt{-a}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}(ad+bc)}$$
$$+ \frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)^2\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16\sqrt{-a}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}(ad+bc)}$$

```
[In] Int[x^5/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])]/(8*(-a)^
(1/4)*b^(1/4)*Sqrt[b*c - a*d]) - ArcTan[(Sqrt[-(b*c) + a*d]*x^2)/((-a)^(1/4
```

```
)*b^(1/4)*Sqrt[c + d*x^8])]/(8*(-a)^(1/4)*b^(1/4)*Sqrt[-(b*c) + a*d]) - ((S
qrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(
c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1
/4)], 1/2])/(8*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[c] + (Sqrt[-a]
*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c
] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*c^(
1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2
*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Ellipt
icPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*
Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*Sqrt[-a]*Sqrt[b]*c^(1/
4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt
[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*
EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[
c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*Sqrt[-a]*Sqrt[b]*c^
(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{1}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4\sqrt{b}}$$

$$= -\frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4(bc+ad)}$$
$$+ \frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4(bc+ad)}$$
$$- \frac{\left(\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4(bc+ad)}$$
$$- \frac{\left(\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4(bc+ad)}$$

$$=\frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}}-\frac{\tan^{-1}\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8\sqrt[4]{-a}\sqrt[4]{b}\sqrt{-bc+ad}}$$
$$-\frac{\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}}$$
$$-\frac{\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}}$$
$$+\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16\sqrt{-a}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}$$
$$-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16\sqrt{-a}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.08

$$\int\frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\frac{x^6\sqrt{\frac{c+dx^8}{c}}\operatorname{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{6a\sqrt{c+dx^8}}$$

```
[In] Integrate[x^5/((a + b*x^8)*Sqrt[c + d*x^8]),x]
[Out] (x^6*Sqrt[(c + d*x^8)/c]*AppellF1[3/4, 1/2, 1, 7/4, -((d*x^8)/c), -((b*x^8)
/a)])/(6*a*Sqrt[c + d*x^8])
```

Maple [F]

$$\int\frac{x^5}{(b\,x^8+a)\sqrt{d\,x^8+c}}dx$$

```
[In] int(x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x)
[Out] int(x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

```
[In] integrate(x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

```
[In] integrate(x**5/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Integral(x**5/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^5}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^5/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Giac [F]

$$\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^5}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^5/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^5}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

```
[In] int(x^5/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] int(x^5/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.902 $\int \frac{1}{x^3(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1032

$$\int \frac{1}{x^3(a+bx^8)\sqrt{c+dx^8}}\,dx = -\frac{\sqrt{c+dx^8}}{2acx^2}+\frac{\sqrt{d}x^2\sqrt{c+dx^8}}{2ac\left(\sqrt{c}+\sqrt{d}x^4\right)}$$
$$+\frac{b^{3/4}\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{5/4}\sqrt{bc-ad}}-\frac{b^{3/4}\arctan\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{5/4}\sqrt{-bc+ad}}$$
$$-\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2ac^{3/4}\sqrt{c+dx^8}}$$
$$+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4ac^{3/4}\sqrt{c+dx^8}}$$
$$+\frac{b\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}}$$
$$+\frac{b\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}}$$
$$+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\right.}{16(-a)^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}$$
$$-\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right.}{16(-a)^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}$$

```
[Out] 1/8*b^(3/4)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))
/(-a)^(5/4)/(-a*d+b*c)^(1/2)-1/8*b^(3/4)*arctan(x^2*(a*d-b*c)^(1/2)/(-a)^(1
/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(5/4)/(a*d-b*c)^(1/2)-1/2*(d*x^8+c)^(1/2)
/a/c/x^2+1/2*x^2*d^(1/2)*(d*x^8+c)^(1/2)/a/c/(c^(1/2)+x^4*d^(1/2))-1/2*d^(1
/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c
^(1/4)))*EllipticE(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)
+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^(3/4)/(d*x^8+c)
^(1/2)+1/4*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arcta
n(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^
(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^
(3/4)/(d*x^8+c)^(1/2)-1/16*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos
(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4)
```

```
)),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^
(1/2),1/2*2^(1/2))*b^(1/2)*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2
)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/(-a)^(3/2)/c^(1/4)/d
^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)+1/16*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2
)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*
x^2/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2
)/c^(1/2)/d^(1/2),1/2*2^(1/2))*b^(1/2)*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/
2)+(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/(-a)^(3/
2)/c^(1/4)/d^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)+1/8*b*d^(1/4)*(cos(2*arctan(d^
(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(s
in(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(c^(1/
2)-(-a)^(1/2)*d^(1/2)/b^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/
c^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)+1/8*b*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c
^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan
(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(c^(1/2)+(-a)^(1/
2)*d^(1/2)/b^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^(1/4)/(a*
d+b*c)/(d*x^8+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 1032, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used

$= \{476, 491, 598, 311, 226, 1210, 504, 1231, 1721\}$

$$\int \frac{1}{x^3 (a + bx^8) \sqrt{c + dx^8}} \, dx =$$

$$-\frac{\sqrt{b}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}}, 2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right), \frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)}{16(-a)^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^8+c}}$$
$$+\frac{b^{3/4}\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{8(-a)^{5/4}\sqrt{bc-ad}}-\frac{b^{3/4}\arctan\left(\frac{\sqrt{ad-bc}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{8(-a)^{5/4}\sqrt{ad-bc}}$$
$$-\frac{\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2ac^{3/4}\sqrt{dx^8+c}}$$
$$+\frac{\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right), \frac{1}{2}\right)}{4ac^{3/4}\sqrt{dx^8+c}}$$
$$+\frac{b\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right), \frac{1}{2}\right)}{8a\sqrt[4]{c}(bc+ad)\sqrt{dx^8+c}}$$
$$+\frac{b\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right), \frac{1}{2}\right)}{8a\sqrt[4]{c}(bc+ad)\sqrt{dx^8+c}}$$
$$+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}}, 2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right), \right.}{16(-a)^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^8+c}}$$
$$+\frac{\sqrt{d}x^2\sqrt{dx^8+c}}{2ac\left(\sqrt{d}x^4+\sqrt{c}\right)}-\frac{\sqrt{dx^8+c}}{2acx^2}$$

```
[In] Int[1/(x^3*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/2*Sqrt[c + d*x^8]/(a*c*x^2) + (Sqrt[d]*x^2*Sqrt[c + d*x^8])/(2*a*c*(Sqrt
[c] + Sqrt[d]*x^4)) + (b^(3/4)*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(
1/4)*Sqrt[c + d*x^8])])/(8*(-a)^(5/4)*Sqrt[b*c - a*d]) - (b^(3/4)*ArcTan[(S
qrt[-(b*c) + a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*(-a)^(5/4)
*Sqrt[-(b*c) + a*d]) - (d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(S
qrt[c] + Sqrt[d]*x^4)^2]*EllipticE[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(
2*a*c^(3/4)*Sqrt[c + d*x^8]) + (d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d
*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)],
 1/2])/(4*a*c^(3/4)*Sqrt[c + d*x^8]) + (b*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqr
t[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x
```

```
^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*a*c^(1/4)*(b*c +
 a*d)*Sqrt[c + d*x^8]) + (b*(Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*
(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Ellipti
cF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*a*c^(1/4)*(b*c + a*d)*Sqrt[c +
 d*x^8]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[
d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-1/4*(Sqrt[b
]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTa
n[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*(-a)^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*
Sqrt[c + d*x^8]) - (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c]
 + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqr
t[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*
ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*(-a)^(3/2)*c^(1/4)*d^(1/4)*(b*c +
a*d)*Sqrt[c + d*x^8])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
```

```
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2 (a+bx^4) \sqrt{c+dx^4}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt{c+dx^8}}{2acx^2} + \frac{\text{Subst}\left(\int \frac{x^2(-bc+ad+bdx^4)}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{2ac}$$

$$= -\frac{\sqrt{c+dx^8}}{2acx^2} + \frac{\text{Subst}\left(\int \left(\frac{dx^2}{\sqrt{c+dx^4}} - \frac{bcx^2}{(a+bx^4)\sqrt{c+dx^4}}\right) dx, x, x^2\right)}{2ac}$$

$$= -\frac{\sqrt{c+dx^8}}{2acx^2} - \frac{b\text{Subst}\left(\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}} dx, x, x^2\right)}{2a} + \frac{d\text{Subst}\left(\int \frac{x^2}{\sqrt{c+dx^4}} dx, x, x^2\right)}{2ac}$$

$$= -\frac{\sqrt{c+dx^8}}{2acx^2} + \frac{\sqrt{b}\text{Subst}\left(\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}} dx, x, x^2\right)}{4a} - \frac{\sqrt{b}\text{Subst}\left(\int \frac{1}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}} dx, x, x^2\right)}{4a} + \frac{\sqrt{d}\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}} dx, x, x^2\right)}{2a\sqrt{c}} - \frac{\sqrt{d}\text{Subst}\left(\int \frac{1-\frac{\sqrt{d}x^2}{\sqrt{c}}}{\sqrt{c+dx^4}} dx, x, x^2\right)}{2a\sqrt{c}}$$

$$= -\frac{\sqrt{c+dx^8}}{2acx^2} + \frac{\sqrt{d}x^2\sqrt{c+dx^8}}{2ac\left(\sqrt{c}+\sqrt{d}x^4\right)} - \frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2ac^{3/4}\sqrt{c+dx^8}} + \frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4ac^{3/4}\sqrt{c+dx^8}} + \frac{\left(b\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}} dx, x, x^2\right)}{4a(bc+ad)} - \frac{\left(b\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}} dx, x, x^2\right)}{4a(bc+ad)} + \frac{\left(\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}} dx, x, x^2\right)}{4a(bc+ad)} + \frac{\left(\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}} dx, x, x^2\right)}{4a(bc+ad)}$$

$$
\begin{aligned}
&= -\frac{\sqrt{c+dx^8}}{2acx^2} + \frac{\sqrt{d}x^2\sqrt{c+dx^8}}{2ac\left(\sqrt{c}+\sqrt{d}x^4\right)} \\
&+ \frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{5/4}\sqrt{bc-ad}} - \frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{5/4}\sqrt{-bc+ad}} \\
&- \frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2ac^{3/4}\sqrt{c+dx^8}} \\
&+ \frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4ac^{3/4}\sqrt{c+dx^8}} \\
&+ \frac{\sqrt{b}\left(\sqrt{b}\sqrt[4]{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt[4]{c}}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a(bc+ad)\sqrt{c+dx^8}} \\
&+ \frac{\sqrt{b}\left(\sqrt{b}\sqrt[4]{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt[4]{c}}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a(bc+ad)\sqrt{c+dx^8}} \\
&+ \frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16(-a)^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}} \\
&- \frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16(-a)^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.14

$$
\int \frac{1}{x^3\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx
$$

$$
= \frac{-21a(c+dx^8)+7(-bc+ad)x^8\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+3bdx^{16}\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}}{42a^2cx^2\sqrt{c+dx^8}}
$$

```
[In] Integrate[1/(x^3*(a + b*x^8)*Sqrt[c + d*x^8]),x]
[Out] (-21*a*(c + d*x^8) + 7*(-(b*c) + a*d)*x^8*Sqrt[1 + (d*x^8)/c]*AppellF1[3/4,
 1/2, 1, 7/4, -((d*x^8)/c), -((b*x^8)/a)] + 3*b*d*x^16*Sqrt[1 + (d*x^8)/c]*
AppellF1[7/4, 1/2, 1, 11/4, -((d*x^8)/c), -((b*x^8)/a)])/(42*a^2*c*x^2*Sqrt
[c + d*x^8])
```

Maple [F]

$$\int \frac{1}{x^{3} \left(b\,x^{8}+a \right) \sqrt{d\,x^{8}+c}}d x$$

```
[In] int(1/x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x)
[Out] int(1/x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^8) \sqrt{c+dx^8}} \, dx=\text{Timed out}$$

```
[In] integrate(1/x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^3 (a+bx^8) \sqrt{c+dx^8}} \, dx=\int \frac{1}{x^{3} \left(a + b x^{8}\right) \sqrt{c + d x^{8}}}\, dx$$

```
[In] integrate(1/x**3/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Integral(1/(x**3*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (a+bx^8) \sqrt{c+dx^8}} \, dx=\int { \frac{1}{{\left(b x^{8} + a\right)} \sqrt{d x^{8} + c} x^{3}} \,d x }$$

```
[In] integrate(1/x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a + bx^8) \sqrt{c + dx^8}} \, dx = \int \frac{1}{(bx^8 + a)\sqrt{dx^8 + c}x^3} \, dx$$

```
[In] integrate(1/x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a + bx^8) \sqrt{c + dx^8}} \, dx = \int \frac{1}{x^3 \, (b \, x^8 + a) \, \sqrt{d \, x^8 + c}} \, dx$$

```
[In] int(1/(x^3*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] int(1/(x^3*(a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.903 $\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}} dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\frac{x^5\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{5}{8},1,\frac{1}{2},\frac{13}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{5a\sqrt{c+dx^8}}$$

```
[Out] 1/5*x^5*AppellF1(5/8,1,1/2,13/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a/(d*x
^8+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\frac{x^5\sqrt{\frac{dx^8}{c}+1}\,\text{AppellF1}\left(\frac{5}{8},1,\frac{1}{2},\frac{13}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{5a\sqrt{c+dx^8}}$$

```
[In] Int[x^4/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^5*Sqrt[1 + (d*x^8)/c]*AppellF1[5/8, 1, 1/2, 13/8, -((b*x^8)/a), -((d*x^8
)/c)])/(5*a*Sqrt[c + d*x^8])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{x^4}{(a+bx^8)\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}$$

$$= \frac{x^5\sqrt{1+\frac{dx^8}{c}}F_1\left(\frac{5}{8};1,\frac{1}{2};\frac{13}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{5a\sqrt{c+dx^8}}$$

Mathematica [A] (verified)

Time = 10.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{x^5\sqrt{\frac{c+dx^8}{c}}\,\text{AppellF1}\left(\frac{5}{8},\frac{1}{2},1,\frac{13}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{5a\sqrt{c+dx^8}}$$

```
[In] Integrate[x^4/((a + b*x^8)*Sqrt[c + d*x^8]),x]
[Out] (x^5*Sqrt[(c + d*x^8)/c]*AppellF1[5/8, 1/2, 1, 13/8, -((d*x^8)/c), -((b*x^8
)/a)])/(5*a*Sqrt[c + d*x^8])
```

Maple [F]

$$\int \frac{x^4}{(b\,x^8+a)\sqrt{d\,x^8+c}}dx$$

```
[In] int(x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x)
[Out] int(x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

```
[In] integrate(x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

```
[In] integrate(x**4/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Integral(x**4/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^4}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^4/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^4}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\int \frac{x^4}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

```
[In] int(x^4/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] int(x^4/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.904 $\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{x^3\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{3}{8},1,\frac{1}{2},\frac{11}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a\sqrt{c+dx^8}}$$

```
[Out] 1/3*x^3*AppellF1(3/8,1,1/2,11/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a/(d*x
^8+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{x^3\sqrt{\frac{dx^8}{c}+1}\,\text{AppellF1}\left(\frac{3}{8},1,\frac{1}{2},\frac{11}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a\sqrt{c+dx^8}}$$

```
[In] Int[x^2/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^3*Sqrt[1 + (d*x^8)/c]*AppellF1[3/8, 1, 1/2, 11/8, -((b*x^8)/a), -((d*x^8
)/c)])/(3*a*Sqrt[c + d*x^8])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{x^2}{(a+bx^8)\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}$$

$$= \frac{x^3\sqrt{1+\frac{dx^8}{c}}F_1\left(\frac{3}{8};1,\frac{1}{2};\frac{11}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a\sqrt{c+dx^8}}$$

Mathematica [A] (verified)

Time = 10.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{x^3\sqrt{\frac{c+dx^8}{c}}\,\text{AppellF1}\left(\frac{3}{8},\frac{1}{2},1,\frac{11}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{3a\sqrt{c+dx^8}}$$

```
[In] Integrate[x^2/((a + b*x^8)*Sqrt[c + d*x^8]),x]
[Out] (x^3*Sqrt[(c + d*x^8)/c]*AppellF1[3/8, 1/2, 1, 11/8, -((d*x^8)/c), -((b*x^8
)/a)])/(3*a*Sqrt[c + d*x^8])
```

Maple [F]

$$\int \frac{x^2}{(b\,x^8+a)\sqrt{d\,x^8+c}}dx$$

```
[In] int(x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x)
[Out] int(x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   Not i
ntegrable (provided residues have no relations)
```

Sympy [F]

$$\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

```
[In] integrate(x**2/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Integral(x**2/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^2}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Giac [F]

$$\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^2}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\int \frac{x^2}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

```
[In] int(x^2/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] int(x^2/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.905 $\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{x\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{1}{8},1,\frac{1}{2},\frac{9}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a\sqrt{c+dx^8}}$$

```
[Out] x*AppellF1(1/8,1,1/2,9/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a/(d*x^8+c)^(
1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {441, 440}

$$\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{x\sqrt{\frac{dx^8}{c}+1}\,\text{AppellF1}\left(\frac{1}{8},1,\frac{1}{2},\frac{9}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a\sqrt{c+dx^8}}$$

```
[In] Int[1/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x*Sqrt[1 + (d*x^8)/c]*AppellF1[1/8, 1, 1/2, 9/8, -((b*x^8)/a), -((d*x^8)/c
)])/(a*Sqrt[c + d*x^8])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^8}{c}} \int \frac{1}{(a+bx^8)\sqrt{1+\frac{dx^8}{c}}} \, dx}{\sqrt{c+dx^8}}$$

$$= \frac{x\sqrt{1+\frac{dx^8}{c}} F_1\left(\frac{1}{8};1,\frac{1}{2};\frac{9}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a\sqrt{c+dx^8}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 161 vs. 2(59) = 118.

Time = 10.20 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.73

$$\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}} \, dx =$$

$$-\frac{9acx \operatorname{AppellF1}\left(\frac{1}{8},\frac{1}{2},1,\frac{9}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{(a+bx^8)\sqrt{c+dx^8}\left(-9ac \operatorname{AppellF1}\left(\frac{1}{8},\frac{1}{2},1,\frac{9}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+4x^8\left(2bc \operatorname{AppellF1}\left(\frac{9}{8},\frac{1}{2},2,\frac{17}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right.\right.\right.}$$

```
[In] Integrate[1/((a + b*x^8)*Sqrt[c + d*x^8]),x]

[Out] (-9*a*c*x*AppellF1[1/8, 1/2, 1, 9/8, -((d*x^8)/c), -((b*x^8)/a)])/((a + b*x
^8)*Sqrt[c + d*x^8]*(-9*a*c*AppellF1[1/8, 1/2, 1, 9/8, -((d*x^8)/c), -((b*x
^8)/a)] + 4*x^8*(2*b*c*AppellF1[9/8, 1/2, 2, 17/8, -((d*x^8)/c), -((b*x^8)/
a)] + a*d*AppellF1[9/8, 3/2, 1, 17/8, -((d*x^8)/c), -((b*x^8)/a)])))
```

Maple [F]

$$\int \frac{1}{(b\,x^8+a)\sqrt{d\,x^8+c}}dx$$

```
[In] int(1/(b*x^8+a)/(d*x^8+c)^(1/2),x)

[Out] int(1/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

```
[In] integrate(1/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Integral(1/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(1/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(1/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{1}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

```
[In] int(1/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] int(1/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.906 $\int \frac{1}{x^2(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{1}{x^2(a+bx^8)\sqrt{c+dx^8}}\,dx = -\frac{\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(-\frac{1}{8},1,\frac{1}{2},\frac{7}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{ax\sqrt{c+dx^8}}$$

```
[Out] -AppellF1(-1/8,1,1/2,7/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a/x/(d*x^8+c)
^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{1}{x^2(a+bx^8)\sqrt{c+dx^8}}\,dx = -\frac{\sqrt{\frac{dx^8}{c}+1}\,\text{AppellF1}\left(-\frac{1}{8},1,\frac{1}{2},\frac{7}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{ax\sqrt{c+dx^8}}$$

```
[In] Int[1/(x^2*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -((Sqrt[1 + (d*x^8)/c]*AppellF1[-1/8, 1, 1/2, 7/8, -((b*x^8)/a), -((d*x^8)/
c)])/(a*x*Sqrt[c + d*x^8]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{1}{x^2(a+bx^8)\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}$$
$$= -\frac{\sqrt{1+\frac{dx^8}{c}}F_1\left(-\frac{1}{8};1,\frac{1}{2};\frac{7}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{ax\sqrt{c+dx^8}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(62) = 124.

Time = 10.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.27

$$\int \frac{1}{x^2(a+bx^8)\sqrt{c+dx^8}}\,dx$$
$$= \frac{-35a(c+dx^8)-5(bc-3ad)x^8\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{7}{8},\frac{1}{2},1,\frac{15}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+7bdx^{16}\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}}{35a^2cx\sqrt{c+dx^8}}$$

```
[In] Integrate[1/(x^2*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (-35*a*(c + d*x^8) - 5*(b*c - 3*a*d)*x^8*Sqrt[1 + (d*x^8)/c]*AppellF1[7/8,
1/2, 1, 15/8, -((d*x^8)/c), -((b*x^8)/a)] + 7*b*d*x^16*Sqrt[1 + (d*x^8)/c]*
AppellF1[15/8, 1/2, 1, 23/8, -((d*x^8)/c), -((b*x^8)/a)])/(35*a^2*c*x*Sqrt[
c + d*x^8])
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b\,x^{8}+a \right) \sqrt{d\,x^{8}+c}}d x$$

[In] int(1/x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x)

[Out] int(1/x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x)

Fricas [F]

$$\int \frac{1}{x^2 (a+bx^8) \sqrt{c+dx^8}} \, dx = \int \frac{1}{(bx^8+a)\sqrt{dx^8+c}x^2} \, dx$$

[In] integrate(1/x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(d*x^8 + c)/(b*d*x^18 + (b*c + a*d)*x^10 + a*c*x^2), x)

Sympy [F]

$$\int \frac{1}{x^2 (a+bx^8) \sqrt{c+dx^8}} \, dx = \int \frac{1}{x^{2} \left(a + b x^{8}\right) \sqrt{c + d x^{8}}}\, dx$$

[In] integrate(1/x**2/(b*x**8+a)/(d*x**8+c)**(1/2),x)

[Out] Integral(1/(x**2*(a + b*x**8)*sqrt(c + d*x**8)), x)

Maxima [F]

$$\int \frac{1}{x^2 (a+bx^8) \sqrt{c+dx^8}} \, dx = \int \frac{1}{(bx^8+a)\sqrt{dx^8+c}x^2} \, dx$$

[In] integrate(1/x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^2), x)

Giac [F]

$$\int \frac{1}{x^2 (a + bx^8) \sqrt{c + dx^8}} dx = \int \frac{1}{(bx^8 + a)\sqrt{dx^8 + c}x^2} dx$$

```
[In] integrate(1/x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + bx^8) \sqrt{c + dx^8}} dx = \int \frac{1}{x^2 (b x^8 + a) \sqrt{d x^8 + c}} dx$$

```
[In] int(1/(x^2*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] int(1/(x^2*(a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.907 $\int \frac{1}{x^4(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{x^4\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx = -\frac{\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(-\frac{3}{8},1,\frac{1}{2},\frac{5}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3ax^3\sqrt{c+dx^8}}$$

```
[Out] -1/3*AppellF1(-3/8,1,1/2,5/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a/x^3/(d*
x^8+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{1}{x^4\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx = -\frac{\sqrt{\frac{dx^8}{c}+1}\,\text{AppellF1}\left(-\frac{3}{8},1,\frac{1}{2},\frac{5}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3ax^3\sqrt{c+dx^8}}$$

```
[In] Int[1/(x^4*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/3*(Sqrt[1 + (d*x^8)/c]*AppellF1[-3/8, 1, 1/2, 5/8, -((b*x^8)/a), -((d*x^
8)/c)])/(a*x^3*Sqrt[c + d*x^8])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{1}{x^4(a+bx^8)\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}$$

$$= -\frac{\sqrt{1+\frac{dx^8}{c}}F_1\left(-\frac{3}{8};1,\frac{1}{2};\frac{5}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3ax^3\sqrt{c+dx^8}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(64) = 128.

Time = 10.15 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.20

$$\int \frac{1}{x^4\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$$
$$= \frac{-65a(c+dx^8)+13(-3bc+ad)x^8\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{5}{8},\frac{1}{2},1,\frac{13}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+5bdx^{16}\sqrt{1+\frac{dx^8}{c}}\,\text{Appell}}{195a^2cx^3\sqrt{c+dx^8}}$$

```
[In] Integrate[1/(x^4*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (-65*a*(c + d*x^8) + 13*(-3*b*c + a*d)*x^8*Sqrt[1 + (d*x^8)/c]*AppellF1[5/8
, 1/2, 1, 13/8, -((d*x^8)/c), -((b*x^8)/a)] + 5*b*d*x^16*Sqrt[1 + (d*x^8)/c
]*AppellF1[13/8, 1/2, 1, 21/8, -((d*x^8)/c), -((b*x^8)/a)])/(195*a^2*c*x^3*
Sqrt[c + d*x^8])
```

Maple [F]

$$\int \frac{1}{x^{4} \left(b\,x^{8}+a \right) \sqrt{d\,x^{8}+c}}d x$$

```
[In] int(1/x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x)
[Out] int(1/x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^8) \sqrt{c+dx^8}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^4 (a+bx^8) \sqrt{c+dx^8}}\,dx = \int \frac{1}{x^{4} \left(a + b x^{8}\right) \sqrt{c + d x^{8}}}\, dx$$

```
[In] integrate(1/x**4/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Integral(1/(x**4*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (a+bx^8) \sqrt{c+dx^8}}\,dx = \int { \frac{1}{{\left(b x^{8} + a\right)} \sqrt{d x^{8} + c} x^{4}} \,d x}$$

```
[In] integrate(1/x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a + bx^8) \sqrt{c + dx^8}} \, dx = \int \frac{1}{(bx^8 + a)\sqrt{dx^8 + cx^4}} \, dx$$

```
[In] integrate(1/x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a + bx^8) \sqrt{c + dx^8}} \, dx = \int \frac{1}{x^4 \, (b\, x^8 + a) \, \sqrt{d\, x^8 + c}} \, dx$$

```
[In] int(1/(x^4*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] int(1/(x^4*(a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

3.908 $\int \frac{x^{23}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 123

$$\int \frac{x^{23}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\frac{\sqrt{c+dx^8}}{4b^2d}-\frac{a^2\sqrt{c+dx^8}}{8b^2(bc-ad)\,(a+bx^8)}+\frac{a(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}}$$

```
[Out] 1/8*a*(-3*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(5
/2)/(-a*d+b*c)^(3/2)+1/4*(d*x^8+c)^(1/2)/b^2/d-1/8*a^2*(d*x^8+c)^(1/2)/b^2/
(-a*d+b*c)/(b*x^8+a)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 91, 81, 65, 214}

$$\int \frac{x^{23}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=-\frac{a^2\sqrt{c+dx^8}}{8b^2\,(a+bx^8)\,(bc-ad)}+\frac{a(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^8}}{4b^2d}$$

```
[In] Int[x^23/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] Sqrt[c + d*x^8]/(4*b^2*d) - (a^2*Sqrt[c + d*x^8])/(8*b^2*(b*c - a*d)*(a + b
*x^8)) + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*
d]])/(8*b^(5/2)*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{8}\text{Subst}\left(\int \frac{x^2}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^8\right)$$

$$= -\frac{a^2\sqrt{c+dx^8}}{8b^2(bc-ad)\,(a+bx^8)} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(2bc-ad)+b(bc-ad)x}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)}{8b^2(bc-ad)}$$

$$=\frac{\sqrt{c+dx^8}}{4b^2d}-\frac{a^2\sqrt{c+dx^8}}{8b^2(bc-ad)\left(a+bx^8\right)}-\frac{(a(4bc-3ad))\text{Subst}\left(\int\frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^8\right)}{16b^2(bc-ad)}$$

$$=\frac{\sqrt{c+dx^8}}{4b^2d}-\frac{a^2\sqrt{c+dx^8}}{8b^2(bc-ad)\left(a+bx^8\right)}-\frac{(a(4bc-3ad))\text{Subst}\left(\int\frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^8}\right)}{8b^2d(bc-ad)}$$

$$=\frac{\sqrt{c+dx^8}}{4b^2d}-\frac{a^2\sqrt{c+dx^8}}{8b^2(bc-ad)\left(a+bx^8\right)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.06

$$\int\frac{x^{23}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\frac{\frac{\sqrt{b}\sqrt{c+dx^8}\left(-3a^2d+2b^2cx^8+2ab\left(c-dx^8\right)\right)}{d(bc-ad)\left(a+bx^8\right)}+\frac{a(4bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{8b^{5/2}}$$

```
[In] Integrate[x^23/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] ((Sqrt[b]*Sqrt[c + d*x^8]*(-3*a^2*d + 2*b^2*c*x^8 + 2*a*b*(c - d*x^8)))/(d*
(b*c - a*d)*(a + b*x^8)) + (a*(4*b*c - 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^
8])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/(8*b^(5/2))
```

Maple [A] (verified)

Time = 5.95 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.08

method	result	size
pseudoelliptic	$\frac{-\frac{3d\left(ad-\frac{4bc}{3}\right)a\left(b\,x^8+a\right)\arctan\left(\frac{b\sqrt{d\,x^8+c}}{\sqrt{(ad-bc)b}}\right)}{8}+\frac{3\left(-\frac{2b^2c\,x^8}{3}-\frac{2a\left(-d\,x^8+c\right)b}{3}+a^2d\right)\sqrt{(ad-bc)b}\,\sqrt{d\,x^8+c}}{8}}{b^2(ad-bc)d\left(b\,x^8+a\right)\sqrt{(ad-bc)b}}$	133

```
[In] int(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/8/((a*d-b*c)*b)^(1/2)*(-d*(a*d-4/3*b*c)*a*(b*x^8+a)*arctan(b*(d*x^8+c)^(1
/2)/((a*d-b*c)*b)^(1/2))+(-2/3*b^2*c*x^8-2/3*a*(-d*x^8+c)*b+a^2*d)*((a*d-b*
c)*b)^(1/2)*(d*x^8+c)^(1/2))/d/b^2/(a*d-b*c)/(b*x^8+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(103) = 206.

Time = 0.28 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.86

$$\int \frac{x^{23}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$
$$= \left[\frac{((4\,ab^2cd-3\,a^2bd^2)x^8+4\,a^2bcd-3\,a^3d^2)\sqrt{b^2c-abd}\log\left(\frac{bdx^8+2\,bc-ad+2\,\sqrt{dx^8+c}\sqrt{b^2c-abd}}{bx^8+a}\right)+2\,(2\,(b^4c^2-2}{16\,(ab^5c^2d-2\,a^2b^4cd^2+a^3b^3d^3+(b^6c^2d-2\,ab^5cd^2+a}\right.$$
$$-\frac{((4\,ab^2cd-3\,a^2bd^2)x^8+4\,a^2bcd-3\,a^3d^2)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^8+c}\sqrt{-b^2c+abd}}{bdx^8+bc}\right)-(2\,(b^4c^2-2\,ab^3cd}{8\,(ab^5c^2d-2\,a^2b^4cd^2+a^3b^3d^3+(b^6c^2d-2\,ab^5cd^2+a^2b^4}$$

```
[In] integrate(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((4*a*b^2*c*d - 3*a^2*b*d^2)*x^8 + 4*a^2*b*c*d - 3*a^3*d^2)*sqrt(b^2
*c - a*b*d)*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 + c)*sqrt(b^2*c - a*b
*d))/(b*x^8 + a)) + 2*(2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^8 + 2*a*b^
3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2)*sqrt(d*x^8 + c))/(a*b^5*c^2*d - 2*a^2*
b^4*c*d^2 + a^3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^8), -
1/8*(((4*a*b^2*c*d - 3*a^2*b*d^2)*x^8 + 4*a^2*b*c*d - 3*a^3*d^2)*sqrt(-b^2*
c + a*b*d)*arctan(sqrt(d*x^8 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c)) - (
2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^8 + 2*a*b^3*c^2 - 5*a^2*b^2*c*d +
 3*a^3*b*d^2)*sqrt(d*x^8 + c))/(a*b^5*c^2*d - 2*a^2*b^4*c*d^2 + a^3*b^3*d^3
 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^8)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{23}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

```
[In] integrate(x**23/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{23}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.09

$$\int \frac{x^{23}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{\sqrt{dx^8+c}a^2d}{8\,(b^3c-ab^2d)((dx^8+c)b-bc+ad)} - \frac{(4\,abc-3\,a^2d)\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{8\,(b^3c-ab^2d)\sqrt{-b^2c+abd}} + \frac{\sqrt{dx^8+c}}{4\,b^2d}$$

```
[In] integrate(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(d*x^8 + c)*a^2*d/((b^3*c - a*b^2*d)*((d*x^8 + c)*b - b*c + a*d))
- 1/8*(4*a*b*c - 3*a^2*d)*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/((
b^3*c - a*b^2*d)*sqrt(-b^2*c + a*b*d)) + 1/4*sqrt(d*x^8 + c)/(b^2*d)
```

Mupad [B] (verification not implemented)

Time = 9.48 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.17

$$\int \frac{x^{23}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{\sqrt{d\,x^8+c}}{4\,b^2\,d} - \frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^8+c}\,(3\,a\,d-4\,b\,c)}{(3\,a^2\,d-4\,a\,b\,c)\,\sqrt{a\,d-b\,c}}\right)\,(3\,a\,d-4\,b\,c)}{8\,b^{5/2}\,(a\,d-b\,c)^{3/2}} + \frac{a^2\,d\,\sqrt{d\,x^8+c}}{2\,(a\,d-b\,c)\,\left(4\,b^3\,\left(d\,x^8+c\right)-4\,b^3\,c+4\,a\,b^2\,d\right)}$$

```
[In] int(x^23/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] (c + d*x^8)^(1/2)/(4*b^2*d) - (a*atan((a*b^(1/2)*(c + d*x^8)^(1/2)*(3*a*d -
 4*b*c))/((3*a^2*d - 4*a*b*c)*(a*d - b*c)^(1/2)))*(3*a*d - 4*b*c))/(8*b^(5/
2)*(a*d - b*c)^(3/2)) + (a^2*d*(c + d*x^8)^(1/2))/(2*(a*d - b*c)*(4*b^3*(c
+ d*x^8) - 4*b^3*c + 4*a*b^2*d))
```

3.909 $\int \frac{x^{15}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 99

$$\int \frac{x^{15}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{a\sqrt{c+dx^8}}{8b(bc-ad)(a+bx^8)} - \frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{3/2}}$$

```
[Out] -1/8*(-a*d+2*b*c)*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)
/(-a*d+b*c)^(3/2)+1/8*a*(d*x^8+c)^(1/2)/b/(-a*d+b*c)/(b*x^8+a)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {457, 79, 65, 214}

$$\int \frac{x^{15}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{a\sqrt{c+dx^8}}{8b(a+bx^8)(bc-ad)} - \frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{3/2}}$$

```
[In] Int[x^15/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (a*Sqrt[c + d*x^8])/(8*b*(b*c - a*d)*(a + b*x^8)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(8*b^(3/2)*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{8}\text{Subst}\left(\int \frac{x}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^8\right)\\
&= \frac{a\sqrt{c+dx^8}}{8b(bc-ad)\left(a+bx^8\right)}+\frac{(2bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)}{16b(bc-ad)}\\
&= \frac{a\sqrt{c+dx^8}}{8b(bc-ad)\left(a+bx^8\right)}+\frac{(2bc-ad)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{8bd(bc-ad)}\\
&= \frac{a\sqrt{c+dx^8}}{8b(bc-ad)\left(a+bx^8\right)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

$$\int \frac{x^{15}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{\frac{a\sqrt{b}\sqrt{c+dx^8}}{(bc-ad)(a+bx^8)} - \frac{(2bc-ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{8b^{3/2}}$$

```
[In] Integrate[x^15/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] ((a*Sqrt[b]*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8)) - ((2*b*c - a*d)*Arc
Tan[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/(8
*b^(3/2))
```

Maple [A] (verified)

Time = 5.84 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^8+c}\,a}{b\,x^8+a}+\frac{(ad-2bc)\arctan\left(\frac{b\sqrt{d\,x^8+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}}{8(ad-bc)b}$	83

```
[In] int(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/(a*d-b*c)/b*(-(d*x^8+c)^(1/2)*a/(b*x^8+a)+(a*d-2*b*c)/((a*d-b*c)*b)^(1/
2)*arctan(b*(d*x^8+c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 348, normalized size of antiderivative = 3.52

$$\int \frac{x^{15}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$
$$= \left[\frac{((2\,b^2c-abd)x^8+2\,abc-a^2d)\sqrt{b^2c-abd}\log\left(\frac{bdx^8+2\,bc-ad-2\,\sqrt{dx^8+c}\sqrt{b^2c-abd}}{bx^8+a}\right)+2\,\sqrt{dx^8+c}(ab^2c-a^2bd)}{16\,((b^5c^2-2\,ab^4cd+a^2b^3d^2)x^8+ab^4c^2-2\,a^2b^3cd+a^3b^2d^2)}\right.$$

```
[In] integrate(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((2*b^2*c - a*b*d)*x^8 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b
*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*sqrt(b^2*c - a*b*d))/(b*x^8 + a))
+ 2*sqrt(d*x^8 + c)*(a*b^2*c - a^2*b*d))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*
d^2)*x^8 + a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2), 1/8*(((2*b^2*c - a*b*d
```

```
)*x^8 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^8 + c)*sqrt(-
b^2*c + a*b*d)/(b*d*x^8 + b*c)) + sqrt(d*x^8 + c)*(a*b^2*c - a^2*b*d))/((b^
5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^8 + a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^
2*d^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{15}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

```
[In] integrate(x**15/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{15}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.17

$$\int \frac{x^{15}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{\frac{\sqrt{dx^8+c}ad^2}{(b^2c-abd)((dx^8+c)b-bc+ad)} + \frac{(2\,bcd-ad^2)\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^2c-abd)\sqrt{-b^2c+abd}}}{8\,d}$$

```
[In] integrate(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(d*x^8 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^8 + c)*b - b*c + a*d)) +
(2*b*c*d - a*d^2)*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c -
a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

Mupad [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.96

$$\int \frac{x^{15}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^8+c}}{\sqrt{a\,d-b\,c}}\right)\,(a\,d-2\,b\,c)}{8\,b^{3/2}\,(a\,d-b\,c)^{3/2}} - \frac{a\,d\,\sqrt{d\,x^8+c}}{2\,b\,(a\,d-b\,c)\,(4\,b\,(d\,x^8+c)+4\,a\,d-4\,b\,c)}$$

```
[In] int(x^15/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] (atan((b^(1/2)*(c + d*x^8)^(1/2))/(a*d - b*c)^(1/2))*(a*d - 2*b*c))/(8*b^(3
/2)*(a*d - b*c)^(3/2)) - (a*d*(c + d*x^8)^(1/2))/(2*b*(a*d - b*c)*(4*b*(c +
 d*x^8) + 4*a*d - 4*b*c))
```

3.910 $\int \frac{x^7}{(a+bx^8)^2\sqrt{c+dx^8}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{x^7}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{\sqrt{c+dx^8}}{8(bc-ad)\,(a+bx^8)} + \frac{d\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8\sqrt{b}(bc-ad)^{3/2}}$$

```
[Out] 1/8*d*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(3/2)/b^
(1/2)-1/8*(d*x^8+c)^(1/2)/(-a*d+b*c)/(b*x^8+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {455, 44, 65, 214}

$$\int \frac{x^7}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{d\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8\sqrt{b}(bc-ad)^{3/2}} - \frac{\sqrt{c+dx^8}}{8\,(a+bx^8)\,(bc-ad)}$$

```
[In] Int[x^7/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/8*Sqrt[c + d*x^8]/((b*c - a*d)*(a + b*x^8)) + (d*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^8])/Sqrt[b*c - a*d]])/(8*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !Int
```

```
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{8}\text{Subst}\left(\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^8\right)\\
&= -\frac{\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} - \frac{d\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)}{16(bc-ad)}\\
&= -\frac{\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} - \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{8(bc-ad)}\\
&= -\frac{\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8\sqrt{b}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \frac{x^7}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{1}{8}\left(-\frac{\sqrt{c+dx^8}}{(bc-ad)(a+bx^8)}+\frac{d\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}(-bc+ad)^{3/2}}\right)$$

```
[In] Integrate[x^7/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (-(Sqrt[c + d*x^8]/((b*c - a*d)*(a + b*x^8))) + (d*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^8])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/8
```

Maple [A] (verified)

Time = 5.82 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

method	result	size
pseudoelliptic	$\frac{d(b\,x^8+a)\arctan\left(\frac{b\sqrt{d\,x^8+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{d\,x^8+c}\,\sqrt{(ad-bc)b}}{8\sqrt{(ad-bc)b}\,(ad-bc)(b\,x^8+a)}$	90

```
[In] int(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(d*(b*x^8+a)*arctan(b*(d*x^8+c)^(1/2)/((a*d-b*c)*b)^(1/2))+(d*x^8+c)^(1
/2)*((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)/(a*d-b*c)/(b*x^8+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(71) = 142.

Time = 0.41 (sec) , antiderivative size = 302, normalized size of antiderivative = 3.47

$$\int \frac{x^7}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \left[-\frac{(bdx^8+ad)\sqrt{b^2c-abd}\log\left(\frac{bdx^8+2\,bc-ad-2\,\sqrt{dx^8+c}\sqrt{b^2c-abd}}{bx^8+a}\right)+2\,\sqrt{dx^8+c}(b^2c-abd)}{16\,((b^4c^2-2\,ab^3cd+a^2b^2d^2)x^8+ab^3c^2-2\,a^2b^2cd+a^3bd^2)}, -\frac{(bdx^8+ad)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^8+c}\sqrt{-b^2c+abd}}{bdx^8+bc}\right)+\sqrt{dx^8+c}(b^2c-abd)}{8\,((b^4c^2-2\,ab^3cd+a^2b^2d^2)x^8+ab^3c^2-2\,a^2b^2cd+a^3bd^2)}\right]$$

```
[In] integrate(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((b*d*x^8 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^8 + 2*b*c - a*d - 2*
sqrt(d*x^8 + c)*sqrt(b^2*c - a*b*d))/(b*x^8 + a)) + 2*sqrt(d*x^8 + c)*(b^2*
```

```
c - a*b*d))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^8 + a*b^3*c^2 - 2*a^2*
b^2*c*d + a^3*b*d^2), -1/8*((b*d*x^8 + a*d)*sqrt(-b^2*c + a*b*d)*arctan(sqr
t(d*x^8 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c)) + sqrt(d*x^8 + c)*(b^2*c
 - a*b*d))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^8 + a*b^3*c^2 - 2*a^2*b
^2*c*d + a^3*b*d^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

```
[In] integrate(x**7/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.07

$$\int \frac{x^7}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{d\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{8\sqrt{-b^2c+abd}(bc-ad)} - \frac{\sqrt{dx^8+c}d}{8\left((dx^8+c)b-bc+ad\right)(bc-ad)}$$

```
[In] integrate(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] -1/8*d*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*(b*c - a*d)) - 1/8*sqrt(d*x^8 + c)*d/(((d*x^8 + c)*b - b*c + a*d)*(b*c - a
*d))
```

Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.97

$$\int \frac{x^7}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\frac{d\sqrt{d\,x^8+c}}{2\,\left(a\,d-b\,c\right)\,\left(4\,b\,\left(d\,x^8+c\right)+4\,a\,d-4\,b\,c\right)}+\frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^8+c}}{\sqrt{a\,d-b\,c}}\right)}{8\,\sqrt{b}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

```
[In] int(x^7/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] (d*(c + d*x^8)^(1/2))/(2*(a*d - b*c)*(4*b*(c + d*x^8) + 4*a*d - 4*b*c)) + (
d*atan((b^(1/2)*(c + d*x^8)^(1/2))/(a*d - b*c)^(1/2)))/(8*b^(1/2)*(a*d - b*
c)^(3/2))
```

3.911 $\int \frac{1}{x(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 132

$$\int \frac{1}{x\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{b\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a^2\sqrt{c}} + \frac{\sqrt{b}(2bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8a^2(bc-ad)^{3/2}}$$

```
[Out] 1/8*(-3*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2
)/a^2/(-a*d+b*c)^(3/2)-1/4*arctanh((d*x^8+c)^(1/2)/c^(1/2))/a^2/c^(1/2)+1/8
*b*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/(b*x^8+a)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {457, 105, 162, 65, 214}

$$\int \frac{1}{x\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{\sqrt{b}(2bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8a^2(bc-ad)^{3/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a^2\sqrt{c}} + \frac{b\sqrt{c+dx^8}}{8a\left(a+bx^8\right)(bc-ad)}$$

```
[In] Int[1/(x*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (b*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*(a + b*x^8)) - ArcTanh[Sqrt[c + d*x^8]
/Sqrt[c]]/(4*a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[
c + d*x^8])/Sqrt[b*c - a*d]])/(8*a^2*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{8}\text{Subst}\left(\int \frac{1}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^8\right)\\ &= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)\,(a+bx^8)} + \frac{\text{Subst}\left(\int \frac{bc-ad+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)}{8a(bc-ad)}\end{aligned}$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^8\right)}{8a^2} - \frac{(b(2bc-3ad))\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)}{16a^2(bc-ad)}$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{4a^2d} - \frac{(b(2bc-3ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{8a^2d(bc-ad)}$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a^2\sqrt{c}} + \frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8a^2(bc-ad)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{-\frac{ab\sqrt{c+dx^8}}{(-bc+ad)(a+bx^8)} + \frac{\sqrt{b}(2bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}} - \frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{\sqrt{c}}}{8a^2}$$

```
[In] Integrate[1/(x*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (-((a*b*Sqrt[c + d*x^8])/((-(b*c) + a*d)*(a + b*x^8))) + (Sqrt[b]*(2*b*c -
3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)
^(3/2) - (2*ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]])/Sqrt[c])/(8*a^2)
```

Maple [A] (verified)

Time = 5.92 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11

method	result
pseudoelliptic	$-\frac{-2b\left(bc-\frac{3ad}{2}\right)\sqrt{c}\,(b\,x^8+a)\arctan\left(\frac{b\sqrt{d\,x^8+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{(ad-bc)b}\left(2(ad-bc)(b\,x^8+a)\,\text{arctanh}\left(\frac{\sqrt{d\,x^8+c}}{\sqrt{c}}\right)+\sqrt{c}\,\sqrt{d\,x^8+c}\,ab\right)}{8\sqrt{c}\,\sqrt{(ad-bc)b}\,a^2(ad-bc)(b\,x^8+a)}$

```
[In] int(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-2*b*(b*c-3/2*a*d)*c^(1/2)*(b*x^8+a)*arctan(b*(d*x^8+c)^(1/2)/((a*d-b
*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(2*(a*d-b*c)*(b*x^8+a)*arctanh((d*x^8+c)^
```

```
(1/2)/c^(1/2))+c^(1/2)*(d*x^8+c)^(1/2)*a*b))/c^(1/2)/((a*d-b*c)*b)^(1/2)/a^
2/(a*d-b*c)/(b*x^8+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 862, normalized size of antiderivative = 6.53

$$\int \frac{1}{x\,(a+bx^8)^2\,\sqrt{c+dx^8}}\,dx = \left[\frac{2\,\sqrt{dx^8+c}abc+((2\,b^2c^2-3\,abcd)x^8+2\,abc^2-3\,a^2cd)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^8+2\,bc-ad+2\,\sqrt{dx^8+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^8+a}\right)+}{16\,((a^2b^2c^2-a^3bcd)x^8+a^3bc^2-a^4cd)}\right.$$

```
[In] integrate(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*sqrt(d*x^8 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^8 + 2*a*b*c^2 -
 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 +
 c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + 2*((b^2*c - a*b*d)*x^8
+ a*b*c - a^2*d)*sqrt(c)*log((d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8)
)/((a^2*b^2*c^2 - a^3*b*c*d)*x^8 + a^3*b*c^2 - a^4*c*d), 1/8*(sqrt(d*x^8 +
c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^8 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b
*c - a*d))*arctan(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^
8 + b*c)) + ((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(c)*log((d*x^8 - 2*sq
rt(d*x^8 + c)*sqrt(c) + 2*c)/x^8))/((a^2*b^2*c^2 - a^3*b*c*d)*x^8 + a^3*b*c
^2 - a^4*c*d), 1/16*(2*sqrt(d*x^8 + c)*a*b*c + 4*((b^2*c - a*b*d)*x^8 + a*b
*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^8 + c)*sqrt(-c)/c) + ((2*b^2*c^2 - 3*a
*b*c*d)*x^8 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b
*c - a*d + 2*sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)))
/((a^2*b^2*c^2 - a^3*b*c*d)*x^8 + a^3*b*c^2 - a^4*c*d), 1/8*(sqrt(d*x^8 + c
)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^8 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b*
c - a*d))*arctan(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^8
 + b*c)) + 2*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x
^8 + c)*sqrt(-c)/c))/((a^2*b^2*c^2 - a^3*b*c*d)*x^8 + a^3*b*c^2 - a^4*c*d)]
```

Sympy [F]

$$\int \frac{1}{x\left(a+b x^{8}\right)^{2} \sqrt{c+d x^{8}}} d x=\int \frac{1}{x\left(a+b x^{8}\right)^{2} \sqrt{c+d x^{8}}} d x$$

```
[In] integrate(1/x/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
[Out] Integral(1/(x*(a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(a+b x^{8}\right)^{2} \sqrt{c+d x^{8}}} d x=\int \frac{1}{\left(b x^{8}+a\right)^{2} \sqrt{d x^{8}+c} x} d x$$

```
[In] integrate(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(a+b x^{8}\right)^{2} \sqrt{c+d x^{8}}} d x=\frac{\sqrt{d x^{8}+c} b d}{8\left(a b c-a^{2} d\right)\left(\left(d x^{8}+c\right) b-b c+a d\right)} -\frac{\left(2 b^{2} c-3 a b d\right) \arctan \left(\frac{\sqrt{d x^{8}+c} b}{\sqrt{-b^{2} c+a b d}}\right)}{8\left(a^{2} b c-a^{3} d\right) \sqrt{-b^{2} c+a b d}}+\frac{\arctan \left(\frac{\sqrt{d x^{8}+c}}{\sqrt{-c}}\right)}{4 a^{2} \sqrt{-c}}$$

```
[In] integrate(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] 1/8*sqrt(d*x^8 + c)*b*d/((a*b*c - a^2*d)*((d*x^8 + c)*b - b*c + a*d)) - 1/8
*(2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b
*c - a^3*d)*sqrt(-b^2*c + a*b*d)) + 1/4*arctan(sqrt(d*x^8 + c)/sqrt(-c))/(a
^2*sqrt(-c))
```

Mupad [B] (verification not implemented)

Time = 10.15 (sec) , antiderivative size = 3017, normalized size of antiderivative = 22.86

$$\int \frac{1}{x\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \text{Too large to display}$$

```
[In] int(1/(x*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] (atan((((((c + d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^
3))/(32*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((a^6*b^2*d^5 - (3*a^5*b^
3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - (
(c + d*x^8)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(256*a^7*b^2*d^5
 - 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*c^3*d^2 + 1280*a^5*b^4*c^2*d^3))/(512*(
a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2
*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(16*(a^5*d^
3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a
*d - b*c)^3)^(1/2)*1i)/(16*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4
*b*c*d^2)) + ((((c + d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^
4*c*d^3))/(32*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + (((a^6*b^2*d^5 - (3*
a^5*b^3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*
d) + ((c + d*x^8)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(256*a^7*b
^2*d^5 - 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*c^3*d^2 + 1280*a^5*b^4*c^2*d^3))/
(512*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b
^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(16*(
a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*
(-b*(a*d - b*c)^3)^(1/2)*1i)/(16*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d -
 3*a^4*b*c*d^2)))/(((3*a*b^3*d^4)/128 - (b^4*c*d^3)/64)/(a^5*d^2 + a^3*b^2*
c^2 - 2*a^4*b*c*d) - ((((c + d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 -
 20*a*b^4*c*d^3))/(32*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((a^6*b^2*d
^5 - (3*a^5*b^3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(a^5*d^2 + a^3*b^2*c^2 - 2*
a^4*b*c*d) - ((c + d*x^8)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(2
56*a^7*b^2*d^5 - 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*c^3*d^2 + 1280*a^5*b^4*c^
2*d^3))/(512*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 +
 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2
))/(16*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d -
 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(16*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^
2*d - 3*a^4*b*c*d^2)) + ((((c + d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^
2 - 20*a*b^4*c*d^3))/(32*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + (((a^6*b^
2*d^5 - (3*a^5*b^3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(a^5*d^2 + a^3*b^2*c^2 -
 2*a^4*b*c*d) + ((c + d*x^8)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)
*(256*a^7*b^2*d^5 - 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*c^3*d^2 + 1280*a^5*b^4
*c^2*d^3))/(512*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^
3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(
1/2))/(16*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*
```

d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(16*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2))))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*1i)/(8*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) - (atan((((((a^6*b^2*d^5 - (3*a^5*b^3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(8*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) - ((c + d*x^8)^(1/2)*(256*a^7*b^2*d^5 - 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*c^3*d^2 + 1280*a^5*b^4*c^2*d^3))/(2048*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(8*a^2*c^(1/2)) - ((c + d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(256*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))*1i)/(a^2*c^(1/2)) - ((((a^6*b^2*d^5 - (3*a^5*b^3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(8*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) + ((c + d*x^8)^(1/2)*(256*a^7*b^2*d^5 - 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*c^3*d^2 + 1280*a^5*b^4*c^2*d^3))/(2048*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(8*a^2*c^(1/2)) + ((c + d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(256*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))*1i)/(a^2*c^(1/2)))/(((3*a*b^3*d^4)/128 - (b^4*c*d^3)/64)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) + (((a^6*b^2*d^5 - (3*a^5*b^3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(8*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) - ((c + d*x^8)^(1/2)*(256*a^7*b^2*d^5 - 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*c^3*d^2 + 1280*a^5*b^4*c^2*d^3))/(2048*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(8*a^2*c^(1/2)) - ((c + d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(256*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(a^2*c^(1/2)) + (((a^6*b^2*d^5 - (3*a^5*b^3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(8*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) + ((c + d*x^8)^(1/2)*(256*a^7*b^2*d^5 - 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*c^3*d^2 + 1280*a^5*b^4*c^2*d^3))/(2048*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(8*a^2*c^(1/2)) + ((c + d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(256*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(a^2*c^(1/2))))*1i)/(4*a^2*c^(1/2)) - (b*d*(c + d*x^8)^(1/2))/(2*(a^2*d - a*b*c)*(4*b*(c + d*x^8) + 4*a*d - 4*b*c))

3.912 $\int \frac{1}{x^9(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 185

$$\int \frac{1}{x^9(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{b(2bc-ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)(a+bx^8)} - \frac{\sqrt{c+dx^8}}{8acx^8(a+bx^8)} + \frac{(4bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^3c^{3/2}} - \frac{b^{3/2}(4bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8a^3(bc-ad)^{3/2}}$$

```
[Out] 1/8*(a*d+4*b*c)*arctanh((d*x^8+c)^(1/2)/c^(1/2))/a^3/c^(3/2)-1/8*b^(3/2)*(-
5*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/a^3/(-a*d+b*
c)^(3/2)-1/8*b*(-a*d+2*b*c)*(d*x^8+c)^(1/2)/a^2/c/(-a*d+b*c)/(b*x^8+a)-1/8*
(d*x^8+c)^(1/2)/a/c/x^8/(b*x^8+a)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {457, 105, 156, 162, 65, 214}

$$\int \frac{1}{x^9(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{b^{3/2}(4bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8a^3(bc-ad)^{3/2}} + \frac{(ad+4bc)\text{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^3c^{3/2}} - \frac{b\sqrt{c+dx^8}(2bc-ad)}{8a^2c(a+bx^8)(bc-ad)} - \frac{\sqrt{c+dx^8}}{8acx^8(a+bx^8)}$$

```
[In] Int[1/(x^9*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/8*(b*(2*b*c - a*d)*Sqrt[c + d*x^8])/(a^2*c*(b*c - a*d)*(a + b*x^8)) - Sq
rt[c + d*x^8]/(8*a*c*x^8*(a + b*x^8)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x
^8]/Sqrt[c]])/(8*a^3*c^(3/2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*S
qrt[c + d*x^8])/Sqrt[b*c - a*d]])/(8*a^3*(b*c - a*d)^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{8}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2\sqrt{c+dx}}\,dx,x,x^8\right)\\
&= -\frac{\sqrt{c+dx^8}}{8acx^8\left(a+bx^8\right)}-\frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4bc+ad)+\frac{3bdx}{2}}{x(a+bx)^2\sqrt{c+dx}}\,dx,x,x^8\right)}{8ac}\\
&= -\frac{b(2bc-ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)\left(a+bx^8\right)}-\frac{\sqrt{c+dx^8}}{8acx^8\left(a+bx^8\right)}-\frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-ad)(4bc+ad)+\frac{1}{2}bd(2bc-ad)x}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^8\right)}{8a^2c(bc-ad)}\\
&= -\frac{b(2bc-ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)\left(a+bx^8\right)}-\frac{\sqrt{c+dx^8}}{8acx^8\left(a+bx^8\right)}\\
&\quad+\frac{\left(b^2(4bc-5ad)\right)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^8\right)}{16a^3(bc-ad)}\\
&\quad-\frac{(4bc+ad)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^8\right)}{16a^3c}\\
&= -\frac{b(2bc-ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)\left(a+bx^8\right)}-\frac{\sqrt{c+dx^8}}{8acx^8\left(a+bx^8\right)}\\
&\quad+\frac{\left(b^2(4bc-5ad)\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^8}\right)}{8a^3d(bc-ad)}\\
&\quad-\frac{(4bc+ad)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^8}\right)}{8a^3cd}\\
&= -\frac{b(2bc-ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)\left(a+bx^8\right)}-\frac{\sqrt{c+dx^8}}{8acx^8\left(a+bx^8\right)}\\
&\quad+\frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^3c^{3/2}}-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8a^3(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^9 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx$$
$$= \frac{\frac{a\sqrt{c+dx^8}(-a^2d+2b^2cx^8+ab(c-dx^8))}{c(-bc+ad)x^8(a+bx^8)} - \frac{b^{3/2}(4bc-5ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}} + \frac{(4bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{c^{3/2}}}{8a^3}$$

```
[In] Integrate[1/(x^9*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] ((a*Sqrt[c + d*x^8]*(-(a^2*d) + 2*b^2*c*x^8 + a*b*(c - d*x^8)))/(c*(-(b*c)
+ a*d)*x^8*(a + b*x^8)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^8])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2) + ((4*b*c + a*d)*ArcTanh
[Sqrt[c + d*x^8]/Sqrt[c]])/c^(3/2))/(8*a^3)
```

Maple [A] (verified)

Time = 6.15 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$-\frac{x^{8} b^{2} \left(b\,x^{8}+a \right) \left(bc-\frac{5ad}{4}\right) c^{\frac{5}{2}} \arctan\left(\frac{b\sqrt{d\,x^{8}+c}}{\sqrt{\left(ad-bc \right) b}}\right)-\frac{\left(c\,x^{8} \left(b\,x^{8}+a \right) \left(ad+4bc \right) \left(ad-bc \right) \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{8}+c}}{\sqrt{c}}\right)+\left(2b^{2} c\,x^{8}+a\left(-d\,x^{8}+c \right) \cdots\right.\right.}{4}}{2\sqrt{\left(ad-bc \right) b}\, a^{3} \left(ad-bc \right) \left(b\,x^{8}+a \right) c^{\frac{5}{2}} x^{8}}$

```
[In] int(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/((a*d-b*c)*b)^(1/2)*(x^8*b^2*(b*x^8+a)*(b*c-5/4*a*d)*c^(5/2)*arctan(b*
(d*x^8+c)^(1/2)/((a*d-b*c)*b)^(1/2))-1/4*(c*x^8*(b*x^8+a)*(a*d+4*b*c)*(a*d-
b*c)*arctanh((d*x^8+c)^(1/2)/c^(1/2))+(2*b^2*c*x^8+a*(-d*x^8+c)*b-a^2*d)*a*
c^(3/2)*(d*x^8+c)^(1/2))*((a*d-b*c)*b)^(1/2))/a^3/(a*d-b*c)/(b*x^8+a)/c^(5/
2)/x^8
```

Fricas [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 1236, normalized size of antiderivative = 6.68

$$\int \frac{1}{x^9 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^16 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^
8)*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*(b*c
- a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*
b*d^2)*x^16 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^8)*sqrt(c)*log((d*x^8
 + 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8) - 2*((2*a*b^2*c^2 - a^2*b*c*d)*x^8
 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^8 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^16
+ (a^4*b*c^3 - a^5*c^2*d)*x^8), -1/16*(2*((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^16
+ (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^8)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^
8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^8 + b*c)) - ((4*b^3*c^2 - 3*
a*b^2*c*d - a^2*b*d^2)*x^16 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^8)*sq
rt(c)*log((d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8) + 2*((2*a*b^2*c^2
- a^2*b*c*d)*x^8 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^8 + c))/((a^3*b^2*c^3 - a^
4*b*c^2*d)*x^16 + (a^4*b*c^3 - a^5*c^2*d)*x^8), -1/16*(2*((4*b^3*c^2 - 3*a*
b^2*c*d - a^2*b*d^2)*x^16 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^8)*sqrt
(-c)*arctan(sqrt(d*x^8 + c)*sqrt(-c)/c) - ((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^16
 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^8)*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*
b*c - a*d - 2*sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a))
 + 2*((2*a*b^2*c^2 - a^2*b*c*d)*x^8 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^8 + c))
/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^16 + (a^4*b*c^3 - a^5*c^2*d)*x^8), -1/8*(((
4*b^3*c^3 - 5*a*b^2*c^2*d)*x^16 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^8)*sqrt(-
b/(b*c - a*d))*arctan(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*
d*x^8 + b*c)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^16 + (4*a*b^2*c^2
- 3*a^2*b*c*d - a^3*d^2)*x^8)*sqrt(-c)*arctan(sqrt(d*x^8 + c)*sqrt(-c)/c) +
 ((2*a*b^2*c^2 - a^2*b*c*d)*x^8 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^8 + c))/((a
^3*b^2*c^3 - a^4*b*c^2*d)*x^16 + (a^4*b*c^3 - a^5*c^2*d)*x^8)]
```

Sympy [F]

$$\int \frac{1}{x^9 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}\, dx = \int \frac{1}{x^9 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}\, dx$$

```
[In] integrate(1/x**9/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
[Out] Integral(1/(x**9*(a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{1}{x^9 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{1}{(bx^8+a)^2\sqrt{dx^8+c}x^9}\,dx$$

```
[In] integrate(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^9), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^9 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \frac{(4\,b^3c-5\,ab^2d)\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{8\,(a^3bc-a^4d)\sqrt{-b^2c+abd}} - \frac{2\,(dx^8+c)^{\frac{3}{2}}b^2cd-2\,\sqrt{dx^8+c}b^2c^2d-(dx^8+c)^{\frac{3}{2}}abd^2+2\,\sqrt{dx^8+c}abcd^2-\sqrt{dx^8+c}a^2d^3}{8\,(a^2bc^2-a^3cd)\left((dx^8+c)^2b-2\,(dx^8+c)bc+bc^2+(dx^8+c)ad-acd\right)} - \frac{(4\,bc+ad)\arctan\left(\frac{\sqrt{dx^8+c}}{\sqrt{-c}}\right)}{8\,a^3\sqrt{-c}c}$$

```
[In] integrate(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] 1/8*(4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/((
a^3*b*c - a^4*d)*sqrt(-b^2*c + a*b*d)) - 1/8*(2*(d*x^8 + c)^(3/2)*b^2*c*d -
 2*sqrt(d*x^8 + c)*b^2*c^2*d - (d*x^8 + c)^(3/2)*a*b*d^2 + 2*sqrt(d*x^8 + c
)*a*b*c*d^2 - sqrt(d*x^8 + c)*a^2*d^3)/((a^2*b*c^2 - a^3*c*d)*((d*x^8 + c)^
2*b - 2*(d*x^8 + c)*b*c + b*c^2 + (d*x^8 + c)*a*d - a*c*d)) - 1/8*(4*b*c +
a*d)*arctan(sqrt(d*x^8 + c)/sqrt(-c))/(a^3*sqrt(-c)*c)
```

Mupad [B] (verification not implemented)

Time = 12.25 (sec) , antiderivative size = 3832, normalized size of antiderivative = 20.71

$$\int \frac{1}{x^9 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \text{Too large to display}$$

```
[In] int(1/(x^9*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
[Out] (((c + d*x^8)^(1/2)*(a^2*d^3 + 2*b^2*c^2*d - 2*a*b*c*d^2))/(2*a^2*(b*c^2 -
a*c*d)) + (b*(c + d*x^8)^(3/2)*(a*d^2 - 2*b*c*d))/(2*a^2*(b*c^2 - a*c*d)))/
```

((c + d*x^8)*(4*a*d - 8*b*c) + 4*b*(c + d*x^8)^2 + 4*b*c^2 - 4*a*c*d) + (at
an((((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^8)^(1/2)*(a^4*b^
3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^
2*d^4))/(32*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((-b^3*(a*d - b*
c)^3)^(1/2)*(5*a*d - 4*b*c)*(((a^9*b^2*c*d^6)/2 + a^6*b^5*c^4*d^3 - 2*a^7*b
^4*c^3*d^4 + (a^8*b^3*c^2*d^5)/2)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*
d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^8)^(1/2)*(5*a*d - 4*b*c)*(512*a^6
*b^5*c^5*d^2 - 1280*a^7*b^4*c^4*d^3 + 1024*a^8*b^3*c^3*d^4 - 256*a^9*b^2*c^
2*d^5))/(512*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3
*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(16*(a^6*d^3 - a^3*b^3*c^3 + 3*a
^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*1i)/(16*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*
c^2*d - 3*a^5*b*c*d^2)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c
+ d*x^8)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4
*c*d^5 + 26*a^2*b^5*c^2*d^4))/(32*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*
d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((a^9*b^2*c*d^6)/2 + a^6
*b^5*c^4*d^3 - 2*a^7*b^4*c^3*d^4 + (a^8*b^3*c^2*d^5)/2)/(a^6*b^2*c^4 + a^8*
c^2*d^2 - 2*a^7*b*c^3*d) + ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^8)^(1/2)*(5
*a*d - 4*b*c)*(512*a^6*b^5*c^5*d^2 - 1280*a^7*b^4*c^4*d^3 + 1024*a^8*b^3*c^
3*d^4 - 256*a^9*b^2*c^2*d^5))/(512*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3
d)(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(16*(a^6*d
^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*1i)/(16*(a^6*d^3 - a^
3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))/(((5*a^3*b^4*d^6)/256 + (b^7
*c^3*d^3)/8 - (3*a*b^6*c^2*d^4)/16 + (3*a^2*b^5*c*d^5)/128)/(a^6*b^2*c^4 +
a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*
(((c + d*x^8)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^
3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(32*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b
*c^3*d)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((a^9*b^2*c*d^6)/2
+ a^6*b^5*c^4*d^3 - 2*a^7*b^4*c^3*d^4 + (a^8*b^3*c^2*d^5)/2)/(a^6*b^2*c^4 +
a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^8)^(1/
2)*(5*a*d - 4*b*c)*(512*a^6*b^5*c^5*d^2 - 1280*a^7*b^4*c^4*d^3 + 1024*a^8*b
^3*c^3*d^4 - 256*a^9*b^2*c^2*d^5))/(512*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*
b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(16*(
a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(16*(a^6*d^3 -
a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + ((-b^3*(a*d - b*c)^3)^(1/
2)*(5*a*d - 4*b*c)*(((c + d*x^8)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a
*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(32*(a^4*b^2*c^4 + a^
6*c^2*d^2 - 2*a^5*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(
((a^9*b^2*c*d^6)/2 + a^6*b^5*c^4*d^3 - 2*a^7*b^4*c^3*d^4 + (a^8*b^3*c^2*d^5
)/2)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((-b^3*(a*d - b*c)^3)^(1
/2)*(c + d*x^8)^(1/2)*(5*a*d - 4*b*c)*(512*a^6*b^5*c^5*d^2 - 1280*a^7*b^4*c
^4*d^3 + 1024*a^8*b^3*c^3*d^4 - 256*a^9*b^2*c^2*d^5))/(512*(a^4*b^2*c^4 + a
^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^
5*b*c*d^2))))/(16*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)
)))/(16*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))*(-b^3*
(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*1i)/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*

b^2*c^2*d - 3*a^5*b*c*d^2)) + (atan((((((c + d*x^8)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(32*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((((a^9*b^2*c*d^6)/2 + a^6*b^5*c^4*d^3 - 2*a^7*b^4*c^3*d^4 + (a^8*b^3*c^2*d^5)/2)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((c + d*x^8)^(1/2)*(a*d + 4*b*c)*(512*a^6*b^5*c^5*d^2 - 1280*a^7*b^4*c^4*d^3 + 1024*a^8*b^3*c^3*d^4 - 256*a^9*b^2*c^2*d^5))/(512*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(16*a^3*(c^3)^(1/2)))*(a*d + 4*b*c)*1i)/(16*a^3*(c^3)^(1/2)) + ((((c + d*x^8)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(32*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - ((((a^9*b^2*c*d^6)/2 + a^6*b^5*c^4*d^3 - 2*a^7*b^4*c^3*d^4 + (a^8*b^3*c^2*d^5)/2)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((c + d*x^8)^(1/2)*(a*d + 4*b*c)*(512*a^6*b^5*c^5*d^2 - 1280*a^7*b^4*c^4*d^3 + 1024*a^8*b^3*c^3*d^4 - 256*a^9*b^2*c^2*d^5))/(512*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(16*a^3*(c^3)^(1/2)))*(a*d + 4*b*c)*1i)/(16*a^3*(c^3)^(1/2)))/((((5*a^3*b^4*d^6)/256 + (b^7*c^3*d^3)/8 - (3*a*b^6*c^2*d^4)/16 + (3*a^2*b^5*c*d^5)/128)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((((c + d*x^8)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(32*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((((a^9*b^2*c*d^6)/2 + a^6*b^5*c^4*d^3 - 2*a^7*b^4*c^3*d^4 + (a^8*b^3*c^2*d^5)/2)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((c + d*x^8)^(1/2)*(a*d + 4*b*c)*(512*a^6*b^5*c^5*d^2 - 1280*a^7*b^4*c^4*d^3 + 1024*a^8*b^3*c^3*d^4 - 256*a^9*b^2*c^2*d^5))/(512*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(16*a^3*(c^3)^(1/2)))*(a*d + 4*b*c))/(16*a^3*(c^3)^(1/2)) + ((((c + d*x^8)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(32*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - ((((a^9*b^2*c*d^6)/2 + a^6*b^5*c^4*d^3 - 2*a^7*b^4*c^3*d^4 + (a^8*b^3*c^2*d^5)/2)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((c + d*x^8)^(1/2)*(a*d + 4*b*c)*(512*a^6*b^5*c^5*d^2 - 1280*a^7*b^4*c^4*d^3 + 1024*a^8*b^3*c^3*d^4 - 256*a^9*b^2*c^2*d^5))/(512*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(16*a^3*(c^3)^(1/2)))*(a*d + 4*b*c))/(16*a^3*(c^3)^(1/2))))*(a*d + 4*b*c)*1i)/(8*a^3*(c^3)^(1/2))

3.913 $\int \frac{x^{19}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 141

$$\int \frac{x^{19}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{ax^4\sqrt{c+dx^8}}{8b(bc-ad)(a+bx^8)} - \frac{\sqrt{a}(3bc-2ad)\arctan\left(\frac{\sqrt{bc-ad}x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8b^2(bc-ad)^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{4b^2\sqrt{d}}$$

```
[Out] -1/8*(-2*a*d+3*b*c)*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))*a^
(1/2)/b^2/(-a*d+b*c)^(3/2)+1/4*arctanh(x^4*d^(1/2)/(d*x^8+c)^(1/2))/b^2/d^(
1/2)+1/8*a*x^4*(d*x^8+c)^(1/2)/b/(-a*d+b*c)/(b*x^8+a)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {476, 481, 537, 223, 212, 385, 211}

$$\int \frac{x^{19}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{\sqrt{a}(3bc-2ad)\arctan\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8b^2(bc-ad)^{3/2}} + \frac{ax^4\sqrt{c+dx^8}}{8b(a+bx^8)(bc-ad)} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{4b^2\sqrt{d}}$$

```
[In] Int[x^19/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (a*x^4*Sqrt[c + d*x^8])/(8*b*(b*c - a*d)*(a + b*x^8)) - (Sqrt[a]*(3*b*c - 2
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*b^2*(b*c
- a*d)^(3/2)) + ArcTanh[(Sqrt[d]*x^4)/Sqrt[c + d*x^8]]/(4*b^2*Sqrt[d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{x^4}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx, x, x^4\right)\\
&= \frac{ax^4\sqrt{c+dx^8}}{8b(bc-ad)(a+bx^8)} - \frac{\text{Subst}\left(\int \frac{ac-2(bc-ad)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{8b(bc-ad)}\\
&= \frac{ax^4\sqrt{c+dx^8}}{8b(bc-ad)(a+bx^8)} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^4\right)}{4b^2}\\
&\quad - \frac{(a(3bc-2ad))\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{8b^2(bc-ad)}\\
&= \frac{ax^4\sqrt{c+dx^8}}{8b(bc-ad)(a+bx^8)} + \frac{\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)}{4b^2}\\
&\quad - \frac{(a(3bc-2ad))\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)}{8b^2(bc-ad)}\\
&= \frac{ax^4\sqrt{c+dx^8}}{8b(bc-ad)(a+bx^8)} - \frac{\sqrt{a}(3bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8b^2(bc-ad)^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{4b^2\sqrt{d}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 4.19 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.09

$$
\int \frac{x^{19}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{\frac{abx^4\sqrt{c+dx^8}}{(bc-ad)(a+bx^8)} + \frac{\sqrt{a}(-3bc+2ad)\arctan\left(\frac{a\sqrt{d}+bx^4\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} + \frac{2\log\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{\sqrt{d}}}{8b^2}
$$

```
[In] Integrate[x^19/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] ((a*b*x^4*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8)) + (Sqrt[a]*(-3*b*c + 2
*a*d)*ArcTan[(a*Sqrt[d] + b*x^4*(Sqrt[d]*x^4 + Sqrt[c + d*x^8]))/(Sqrt[a]*S
qrt[b*c - a*d])])/(b*c - a*d)^(3/2) + (2*Log[Sqrt[d]*x^4 + Sqrt[c + d*x^8]]
)/Sqrt[d])/(8*b^2)
```

Maple [A] (verified)

Time = 24.90 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.83

method	result	size
pseudoelliptic	$-\frac{-\frac{a\left(-\frac{b\sqrt{d\,x^{8}+c}\,x^{4}}{b\,x^{8}+a}-\frac{(2ad-3bc)\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{8}+c}\,a}{x^{4}\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)}{ad-bc}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{8}+c}}{x^{4}\sqrt{d}}\right)}{\sqrt{d}}}{8b^{2}}$	117

```
[In] int(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/b^2*(-a/(a*d-b*c)*(-b*(d*x^8+c)^(1/2)*x^4/(b*x^8+a)-(2*a*d-3*b*c)/((a*
d-b*c)*a)^(1/2)*arctanh((d*x^8+c)^(1/2)/x^4*a/((a*d-b*c)*a)^(1/2)))-2/d^(1/
2)*arctanh((d*x^8+c)^(1/2)/x^4/d^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 1.19 (sec) , antiderivative size = 1077, normalized size of antiderivative = 7.64

$$\int \frac{x^{19}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

$$= \left[\frac{4\sqrt{dx^8+c}abdx^4+4\left((b^2c-abd)x^8+abc-a^2d\right)\sqrt{d}\log\left(-2\,dx^8-2\sqrt{dx^8+c}\sqrt{d}x^4-c\right)+((3\,b^2cd-2}{32\,((}\right.$$

```
[In] integrate(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*(4*sqrt(d*x^8 + c)*a*b*d*x^4 + 4*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d
)*sqrt(d)*log(-2*d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 - c) + ((3*b^2*c*d -
 2*a*b*d^2)*x^8 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 -
4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d
*x^8 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2*a*b*x^8 + a^2)))/((b^4*c*d -
a*b^3*d^2)*x^8 + a*b^3*c*d - a^2*b^2*d^2), 1/32*(4*sqrt(d*x^8 + c)*a*b*d*x^
4 - 8*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(-d)*arctan(sqrt(-d)*x^4/sq
rt(d*x^8 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^8 + 3*a*b*c*d - 2*a^2*d^2)*sqrt
(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2
 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (
a*b*c^2 - a^2*c*d)*x^4)*sqrt(d*x^8 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2
*a*b*x^8 + a^2)))/((b^4*c*d - a*b^3*d^2)*x^8 + a*b^3*c*d - a^2*b^2*d^2), 1/
16*(2*sqrt(d*x^8 + c)*a*b*d*x^4 + ((3*b^2*c*d - 2*a*b*d^2)*x^8 + 3*a*b*c*d
- 2*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt
```

```
(d*x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 + a*c*x^4)) + 2*((b^2*c - a*b*d)*
x^8 + a*b*c - a^2*d)*sqrt(d)*log(-2*d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 -
 c))/((b^4*c*d - a*b^3*d^2)*x^8 + a*b^3*c*d - a^2*b^2*d^2), 1/16*(2*sqrt(d*
x^8 + c)*a*b*d*x^4 - 4*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(-d)*arcta
n(sqrt(-d)*x^4/sqrt(d*x^8 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^8 + 3*a*b*c*d
- 2*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt
(d*x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 + a*c*x^4)))/((b^4*c*d - a*b^3*d^
2)*x^8 + a*b^3*c*d - a^2*b^2*d^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{19}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

```
[In] integrate(x**19/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{19}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^{19}}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^19/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(117) = 234$.

Time = 0.37 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.11

$$\int \frac{x^{19}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{\left(3\,abc\sqrt{d}-2\,a^2d^{\frac{3}{2}}\right)\arctan\left(-\frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{8\,(b^3c-ab^2d)\sqrt{abcd-a^2d^2}} - \frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2abc\sqrt{d}-2\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2a^2d^{\frac{3}{2}}-abc^2\sqrt{d}}{4\left(\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^4b-2\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2bc+4\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2ad+bc^2\right)(b^3c-ab^2d)} - \frac{\log\left(\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2\right)}{8\,b^2\sqrt{d}}$$

```
[In] integrate(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(3*a*b*c*sqrt(d) - 2*a^2*d^(3/2))*arctan(-1/2*((sqrt(d)*x^4 - sqrt(d*x
^8 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((b^3*c - a*b^2*d)*sqr
t(a*b*c*d - a^2*d^2)) - 1/4*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*b*c*sqrt(d
) - 2*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a^2*d^(3/2) - a*b*c^2*sqrt(d))/(((s
qrt(d)*x^4 - sqrt(d*x^8 + c))^4*b - 2*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c
 + 4*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*d + b*c^2)*(b^3*c - a*b^2*d)) - 1/
8*log((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2)/(b^2*sqrt(d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{19}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^{19}}{{\left(b\,x^8+a\right)}^2\,\sqrt{d\,x^8+c}} \,d x$$

```
[In] int(x^19/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(x^19/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.914 $\int \frac{x^{11}}{(a+bx^8)^2\sqrt{c+dx^8}} dx$

Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{x^{11}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{x^4\sqrt{c+dx^8}}{8(bc-ad)(a+bx^8)} + \frac{c\arctan\left(\frac{\sqrt{bc-ad}x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8\sqrt{a}(bc-ad)^{3/2}}$$

```
[Out] 1/8*c*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))/(-a*d+b*c)^(3/2)
/a^(1/2)-1/8*x^4*(d*x^8+c)^(1/2)/(-a*d+b*c)/(b*x^8+a)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {476, 482, 12, 385, 211}

$$\int \frac{x^{11}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{c\arctan\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8\sqrt{a}(bc-ad)^{3/2}} - \frac{x^4\sqrt{c+dx^8}}{8(a+bx^8)(bc-ad)}$$

```
[In] Int[x^11/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/8*(x^4*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8)) + (c*ArcTan[(Sqrt[b*c
- a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*Sqrt[a]*(b*c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx, x, x^4\right)\\
&= -\frac{x^4\sqrt{c+dx^8}}{8(bc-ad)(a+bx^8)} + \frac{\text{Subst}\left(\int \frac{c}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{8(bc-ad)}\\
&= -\frac{x^4\sqrt{c+dx^8}}{8(bc-ad)(a+bx^8)} + \frac{c\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{8(bc-ad)}\\
&= -\frac{x^4\sqrt{c+dx^8}}{8(bc-ad)(a+bx^8)} + \frac{c\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)}{8(bc-ad)}\\
&= -\frac{x^4\sqrt{c+dx^8}}{8(bc-ad)(a+bx^8)} + \frac{c\tan^{-1}\left(\frac{\sqrt{bc-ad}x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8\sqrt{a}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.20

$$\int \frac{x^{11}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\frac{1}{8}\left(-\frac{x^4\sqrt{c+dx^8}}{(bc-ad)(a+bx^8)}+\frac{c\arctan\left(\frac{a\sqrt{d}+bx^4\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}(bc-ad)^{3/2}}\right)$$

```
[In] Integrate[x^11/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (-((x^4*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8))) + (c*ArcTan[(a*Sqrt[d]
+ b*x^4*(Sqrt[d]*x^4 + Sqrt[c + d*x^8]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(Sqrt[
a]*(b*c - a*d)^(3/2)))/8
```

Maple [A] (verified)

Time = 20.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.87

method	result	size
pseudoelliptic	$-\frac{c\left(-\frac{\sqrt{d\,x^{8}+c}\,x^{4}}{c\left(b\,x^{8}+a\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{8}+c}\,a}{x^{4}\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)}{8(ad-bc)}$	81

```
[In] int(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*c/(a*d-b*c)*(-(d*x^8+c)^(1/2)*x^4/c/(b*x^8+a)+1/((a*d-b*c)*a)^(1/2)*ar
ctanh((d*x^8+c)^(1/2)/x^4*a/((a*d-b*c)*a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(77) = 154.

Time = 0.35 (sec) , antiderivative size = 426, normalized size of antiderivative = 4.58

$$\int \frac{x^{11}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

$$=\left[-\frac{4\sqrt{dx^8+c}(abc-a^2d)x^4-(bcx^8+ac)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{16}-2\,(3\,abc^2-4\,a^2cd)x^8+a^2c^2+4\,((bc}{b^2x^{16}+2\,abx^8+a^2}\right.}{32\,((ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^8+a^2b^2c^2-2\,a^3bcd+a^4d^2)}\right.$$

$$\left.-\frac{2\sqrt{dx^8+c}(abc-a^2d)x^4-(bcx^8+ac)\sqrt{abc-a^2d}\arctan\left(\frac{((bc-2\,ad)x^8-ac)\sqrt{dx^8+c}\sqrt{abc-a^2d}}{2\,((abcd-a^2d^2)x^{12}+(abc^2-a^2cd)x^4)}\right)}{16\,((ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^8+a^2b^2c^2-2\,a^3bcd+a^4d^2)}\right]$$

```
[In] integrate(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(4*sqrt(d*x^8 + c)*(a*b*c - a^2*d)*x^4 - (b*c*x^8 + a*c)*sqrt(-a*b*c
 + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^
2*c*d)*x^8 + a^2*c^2 + 4*((b*c - 2*a*d)*x^12 - a*c*x^4)*sqrt(d*x^8 + c)*sqr
t(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2)))/((a*b^3*c^2 - 2*a^2*b^2*c
*d + a^3*b*d^2)*x^8 + a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2), -1/16*(2*sqrt(d
*x^8 + c)*(a*b*c - a^2*d)*x^4 - (b*c*x^8 + a*c)*sqrt(a*b*c - a^2*d)*arctan(
1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d
 - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4)))/((a*b^3*c^2 - 2*a^2*b^2*c*d +
 a^3*b*d^2)*x^8 + a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

```
[In] integrate(x**11/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{11}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^{11}}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^11/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(77) = 154$.

Time = 0.99 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.62

$$\int \frac{x^{11}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{c\sqrt{d}\arctan\left(-\frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{8\sqrt{abcd-a^2d^2}(bc-ad)} + \frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2bc\sqrt{d}-2\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2ad^{\frac{3}{2}}-bc^2\sqrt{d}}{4\left(\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^4b-2\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2bc+4\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2ad+bc^2\right)(b^2c-abd)}$$

```
[In] integrate(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*c*sqrt(d)*arctan(-1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b - b*c + 2*a*
d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b*c - a*d)) + 1/4*((s
qrt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c*sqrt(d) - 2*(sqrt(d)*x^4 - sqrt(d*x^8 +
 c))^2*a*d^(3/2) - b*c^2*sqrt(d))/(((sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b - 2
*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c + 4*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^
2*a*d + b*c^2)*(b^2*c - a*b*d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^{11}}{\left(b\,x^8+a\right)^2\,\sqrt{d\,x^8+c}}\,dx$$

```
[In] int(x^11/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(x^11/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.915 $\int \frac{x^3}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \frac{x^3}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{bx^4\sqrt{c+dx^8}}{8a(bc-ad)\,(a+bx^8)} + \frac{(bc-2ad)\arctan\left(\frac{\sqrt{bc-ad}x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{3/2}(bc-ad)^{3/2}}$$

```
[Out] 1/8*(-2*a*d+b*c)*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))/a^(3/
2)/(-a*d+b*c)^(3/2)+1/8*b*x^4*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/(b*x^8+a)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {476, 390, 385, 211}

$$\int \frac{x^3}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{(bc-2ad)\arctan\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{3/2}(bc-ad)^{3/2}} + \frac{bx^4\sqrt{c+dx^8}}{8a\,(a+bx^8)\,(bc-ad)}$$

```
[In] Int[x^3/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (b*x^4*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*(a + b*x^8)) + ((b*c - 2*a*d)*ArcT
an[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*a^(3/2)*(b*c - a*d)
^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx, x, x^4\right)\\
&= \frac{bx^4\sqrt{c+dx^8}}{8a(bc-ad)\,(a+bx^8)} + \frac{(bc-2ad)\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{8a(bc-ad)}\\
&= \frac{bx^4\sqrt{c+dx^8}}{8a(bc-ad)\,(a+bx^8)} + \frac{(bc-2ad)\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)}{8a(bc-ad)}\\
&= \frac{bx^4\sqrt{c+dx^8}}{8a(bc-ad)\,(a+bx^8)} + \frac{(bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{3/2}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.83 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.19

$$\int \frac{x^3}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{bx^4\sqrt{c+dx^8}}{8a(-bc+ad)(a+bx^8)} + \frac{(bc-2ad)\arctan\left(\frac{a\sqrt{d}+b\sqrt{d}x^8+bx^4\sqrt{c+dx^8}}{\sqrt{a}\sqrt{bc-ad}}\right)}{8a^{3/2}(bc-ad)^{3/2}}$$

```
[In] Integrate[x^3/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/8*(b*x^4*Sqrt[c + d*x^8])/(a*(-(b*c) + a*d)*(a + b*x^8)) + ((b*c - 2*a*d
)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^8 + b*x^4*Sqrt[c + d*x^8])/(Sqrt[a]*Sqrt[
b*c - a*d])])/(8*a^(3/2)*(b*c - a*d)^(3/2))
```

Maple [A] (verified)

Time = 20.21 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

method	result	size
pseudoelliptic	$\frac{-\frac{b\sqrt{d\,x^8+c}\,x^4}{b\,x^8+a}+\frac{(2ad-bc)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^8+c}\,a}{x^4\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}}{8a(ad-bc)}$	90

```
[In] int(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/a/(a*d-b*c)*(-b*(d*x^8+c)^(1/2)*x^4/(b*x^8+a)+(2*a*d-b*c)/((a*d-b*c)*a)
^(1/2)*arctanh((d*x^8+c)^(1/2)/x^4*a/((a*d-b*c)*a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(88) = 176.

Time = 0.39 (sec) , antiderivative size = 467, normalized size of antiderivative = 4.49

$$\int \frac{x^3}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \left[\frac{4\sqrt{dx^8+c}(ab^2c-a^2bd)x^4 - ((b^2c-2abd)x^8+abc-2a^2d)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^{16}-2(3c}{}\right.}{32((a^2b^3c^2-2a^3b^2cd+a^4bd^2)x^8+a^3b^2c^2-2a^4bcd+a}\right.$$

```
[In] integrate(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*(4*sqrt(d*x^8 + c)*(a*b^2*c - a^2*b*d)*x^4 - ((b^2*c - 2*a*b*d)*x^8 +
 a*b*c - 2*a^2*d)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^
```

```
2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b*c - 2*a*d)*x^12 -
 a*c*x^4)*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2
)))/((a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x^8 + a^3*b^2*c^2 - 2*a^4*b*
c*d + a^5*d^2), 1/16*(2*sqrt(d*x^8 + c)*(a*b^2*c - a^2*b*d)*x^4 + ((b^2*c -
 2*a*b*d)*x^8 + a*b*c - 2*a^2*d)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a
*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^1
2 + (a*b*c^2 - a^2*c*d)*x^4)))/((a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x
^8 + a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2)]
```

Sympy [F]

$$\int \frac{x^3}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^3}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

```
[In] integrate(x**3/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
[Out] Integral(x**3/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x^3}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^3}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(88) = 176$.

Time = 0.37 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.28

$$\int \frac{x^3}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{1}{8}\,d^{\frac{3}{2}}\left(\frac{(bc-2\,ad)\arctan\left(\frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\left(abcd-a^2d^2\right)^{\frac{3}{2}}} + \frac{2\left(\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2bc-\right.}{\left(\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^4b-2\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right.\right.}\right.$$

```
[In] integrate(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b
 - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d^2)^(3/2) + 2*((sq
rt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c - 2*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*
d - b*c^2)/(((sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b - 2*(sqrt(d)*x^4 - sqrt(d*
x^8 + c))^2*b*c + 4*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*d + b*c^2)*(a*b*c*d
 - a^2*d^2)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^3}{\left(b\,x^8+a\right)^2\,\sqrt{d\,x^8+c}}\,dx$$

```
[In] int(x^3/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(x^3/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.916 $\int \frac{1}{x^5(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 149

$$\int \frac{1}{x^5(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{(3bc-2ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^4}+\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^4(a+bx^8)} - \frac{b(3bc-4ad)\arctan\left(\frac{\sqrt{bc-ad}x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{5/2}(bc-ad)^{3/2}}$$

```
[Out] -1/8*b*(-4*a*d+3*b*c)*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))/
a^(5/2)/(-a*d+b*c)^(3/2)-1/8*(-2*a*d+3*b*c)*(d*x^8+c)^(1/2)/a^2/c/(-a*d+b*c
)/x^4+1/8*b*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/x^4/(b*x^8+a)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {476, 483, 597, 12, 385, 211}

$$\int \frac{1}{x^5(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{b(3bc-4ad)\arctan\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{5/2}(bc-ad)^{3/2}} - \frac{\sqrt{c+dx^8}(3bc-2ad)}{8a^2cx^4(bc-ad)}+\frac{b\sqrt{c+dx^8}}{8ax^4(a+bx^8)(bc-ad)}$$

```
[In] Int[1/(x^5*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/8*((3*b*c - 2*a*d)*Sqrt[c + d*x^8])/(a^2*c*(b*c - a*d)*x^4) + (b*Sqrt[c
+ d*x^8])/(8*a*(b*c - a*d)*x^4*(a + b*x^8)) - (b*(3*b*c - 4*a*d)*ArcTan[(Sq
```

```
rt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*a^(5/2)*(b*c - a*d)^(3/2)
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^2 (a+bx^2)^2 \sqrt{c+dx^2}}\,dx, x, x^4\right)$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^4 (a+bx^8)} - \frac{\text{Subst}\left(\int \frac{-3bc+2ad-2bdx^2}{x^2(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{8a(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^4} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^4 (a+bx^8)} - \frac{\text{Subst}\left(\int \frac{bc(3bc-4ad)}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{8a^2c(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^4} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^4 (a+bx^8)} - \frac{(b(3bc-4ad))\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{8a^2(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^4} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^4 (a+bx^8)} - \frac{(b(3bc-4ad))\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)}{8a^2(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^4} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^4 (a+bx^8)} - \frac{b(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{5/2}(bc-ad)^{3/2}}$$

Mathematica [A] (verified)

Time = 2.13 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^5 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \frac{\sqrt{c+dx^8}(2abc-2a^2d+3b^2cx^8-2abdx^8)}{8a^2c(-bc+ad)x^4 (a+bx^8)} - \frac{b(3bc-4ad)\arctan\left(\frac{a\sqrt{d}+b\sqrt{d}x^8+bx^4\sqrt{c+dx^8}}{\sqrt{a}\sqrt{bc-ad}}\right)}{8a^{5/2}(bc-ad)^{3/2}}$$

```
[In] Integrate[1/(x^5*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (Sqrt[c + d*x^8]*(2*a*b*c - 2*a^2*d + 3*b^2*c*x^8 - 2*a*b*d*x^8))/(8*a^2*c*
(-(b*c) + a*d)*x^4*(a + b*x^8)) - (b*(3*b*c - 4*a*d)*ArcTan[(a*Sqrt[d] + b*
Sqrt[d]*x^8 + b*x^4*Sqrt[c + d*x^8])/(Sqrt[a]*Sqrt[b*c - a*d])])/(8*a^(5/2)
*(b*c - a*d)^(3/2))
```

Maple [A] (verified)

Time = 31.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.75

method	result	size
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^8+c}}{x^4}+\frac{bc\left(\frac{b\sqrt{d\,x^8+c}\,x^4}{b\,x^8+a}-\frac{(4ad-3bc)\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^8+c}\,a}{x^4\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)}{2ad-2bc}}{4a^2c}$	112

```
[In] int(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a^2*(-(d*x^8+c)^(1/2)/x^4+1/2*b*c/(a*d-b*c)*(b*(d*x^8+c)^(1/2)*x^4/(b*x
^8+a)-(4*a*d-3*b*c)/((a*d-b*c)*a)^(1/2)*arctanh((d*x^8+c)^(1/2)/x^4*a/((a*d
-b*c)*a)^(1/2))))/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(129) = 258.

Time = 0.55 (sec) , antiderivative size = 612, normalized size of antiderivative = 4.11

$$\int \frac{1}{x^5\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$$
$$= \left[-\frac{((3\,b^3c^2-4\,ab^2cd)x^{12}+(3\,ab^2c^2-4\,a^2bcd)x^4)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{16}-2\,(3\,abc^2-4\,a^2cd)x^8+\cdots}{b^2x^{16}+2\cdots}\right.}{32\,((a^3b^3c^3-2\,a^4b^2c^2d+a^5\cdots}\right.$$
$$-\frac{((3\,b^3c^2-4\,ab^2cd)x^{12}+(3\,ab^2c^2-4\,a^2bcd)x^4)\sqrt{abc-a^2d}\arctan\left(\frac{((bc-2\,ad)x^8-ac)\sqrt{dx^8+c}\sqrt{abc-a^2d}}{2\,((abcd-a^2d^2)x^{12}+(abc^2-a^2cd)x^4)}\right)+2\,((\cdots}{16\,((a^3b^3c^3-2\,a^4b^2c^2d+a^5bcd^2)x^{12}+(a^4b^2c^3-2\,\cdots}$$

```
[In] integrate(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^12 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^4)*
sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b
*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 + 4*((b*c - 2*a*d)*x^12 - a*c*x^4)*sqrt(d*x
^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2)) + 4*((3*a*b^3*c
^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^8 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4
*d^2)*sqrt(d*x^8 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^12
+ (a^4*b^2*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^4), -1/16*(((3*b^3*c^2 - 4*a*
b^2*c*d)*x^12 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^4)*sqrt(a*b*c - a^2*d)*arctan
(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*
d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4)) + 2*((3*a*b^3*c^2 - 5*a^2*b^2
*c*d + 2*a^3*b*d^2)*x^8 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4*d^2)*sqrt(d*x
^8 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^12 + (a^4*b^2*c^3
 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^4)]
```

Sympy [F]

$$\int \frac{1}{x^5 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}\,dx = \int \frac{1}{x^5 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}\,dx$$

```
[In] integrate(1/x**5/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
[Out] Integral(1/(x**5*(a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{1}{x^5 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}\,dx = \int \frac{1}{\left(bx^8+a\right)^2 \sqrt{dx^8+c}\,x^5}\,dx$$

```
[In] integrate(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. 2(129) = 258.

Time = 0.93 (sec) , antiderivative size = 418, normalized size of antiderivative = 2.81

$$\int \frac{1}{x^5 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}\,dx$$
$$= \frac{1}{8}d^{\frac{5}{2}}\left(\frac{\left(3\,b^2c-4\,abd\right)\arctan\left(\frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\left(a^2bcd^2-a^3d^3\right)\sqrt{abcd-a^2d^2}} + \frac{2\left(3\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^4b^2c-4\left(\sqrt{d}x^4-\sqrt{dx\right.\right.}{\left(\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^6b-3\left(\sqrt{d}x^4-\sqrt{dx\right.\right.}\right.$$

```
[In] integrate(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] 1/8*d^(5/2)*((3*b^2*c - 4*a*b*d)*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c)
)^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^2*b*c*d^2 - a^3*d^3)*sqrt
(a*b*c*d - a^2*d^2)) + 2*(3*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b^2*c - 4*(sq
rt(d)*x^4 - sqrt(d*x^8 + c))^4*a*b*d - 6*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*
b^2*c^2 + 14*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*b*c*d - 8*(sqrt(d)*x^4 - s
qrt(d*x^8 + c))^2*a^2*d^2 + 3*b^2*c^3 - 2*a*b*c^2*d)/(((sqrt(d)*x^4 - sqrt(
d*x^8 + c))^6*b - 3*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b*c + 4*(sqrt(d)*x^4
- sqrt(d*x^8 + c))^4*a*d + 3*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c^2 - 4*(s
qrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*c*d - b*c^3)*(a^2*b*c*d^2 - a^3*d^3)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{1}{x^5 \left(b\,x^8+a\right)^2 \sqrt{d\,x^8+c}}\,dx$$

```
[In] int(1/(x^5*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
[Out] int(1/(x^5*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.917 $\int \frac{1}{x^{13}(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 208

$$\int \frac{1}{x^{13}(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{(5bc-2ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^{12}} + \frac{(15b^2c^2-8abcd-4a^2d^2)\sqrt{c+dx^8}}{24a^3c^2(bc-ad)x^4} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^{12}(a+bx^8)} + \frac{b^2(5bc-6ad)\arctan\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{7/2}(bc-ad)^{3/2}}$$

```
[Out] 1/8*b^2*(-6*a*d+5*b*c)*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))
/a^(7/2)/(-a*d+b*c)^(3/2)-1/24*(-2*a*d+5*b*c)*(d*x^8+c)^(1/2)/a^2/c/(-a*d+b
*c)/x^12+1/24*(-4*a^2*d^2-8*a*b*c*d+15*b^2*c^2)*(d*x^8+c)^(1/2)/a^3/c^2/(-a
*d+b*c)/x^4+1/8*b*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/x^12/(b*x^8+a)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

$= \{476, 483, 597, 12, 385, 211\}$

$$\int \frac{1}{x^{13}\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{b^2(5bc-6ad)\arctan\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{7/2}(bc-ad)^{3/2}} - \frac{\sqrt{c+dx^8}(5bc-2ad)}{24a^2cx^{12}(bc-ad)} + \frac{\sqrt{c+dx^8}(-4a^2d^2-8abcd+15b^2c^2)}{24a^3c^2x^4(bc-ad)} + \frac{b\sqrt{c+dx^8}}{8ax^{12}\left(a+bx^8\right)(bc-ad)}$$

```
[In] Int[1/(x^13*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/24*((5*b*c - 2*a*d)*Sqrt[c + d*x^8])/(a^2*c*(b*c - a*d)*x^12) + ((15*b^2
*c^2 - 8*a*b*c*d - 4*a^2*d^2)*Sqrt[c + d*x^8])/(24*a^3*c^2*(b*c - a*d)*x^4)
 + (b*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*x^12*(a + b*x^8)) + (b^2*(5*b*c - 6
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*a^(7/2)*(
b*c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
```

```
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx,x,x^4\right)$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^{12}\left(a+bx^8\right)} - \frac{\text{Subst}\left(\int \frac{-5bc+2ad-4bdx^2}{x^4(a+bx^2)\sqrt{c+dx^2}}\,dx,x,x^4\right)}{8a(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^{12}} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^{12}\left(a+bx^8\right)} + \frac{\text{Subst}\left(\int \frac{-15b^2c^2+8abcd+4a^2d^2-2bd(5bc-2ad)x^2}{x^2(a+bx^2)\sqrt{c+dx^2}}\,dx,x,x^4\right)}{24a^2c(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^{12}} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^8}}{24a^3c^2(bc-ad)x^4} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^{12}\left(a+bx^8\right)} - \frac{\text{Subst}\left(\int -\frac{3b^2c^2(5bc-6ad)}{(a+bx^2)\sqrt{c+dx^2}}\,dx,x,x^4\right)}{24a^3c^2(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^{12}} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^8}}{24a^3c^2(bc-ad)x^4} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^{12}\left(a+bx^8\right)} + \frac{\left(b^2(5bc-6ad)\right)\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx,x,x^4\right)}{8a^3(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^{12}} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^8}}{24a^3c^2(bc-ad)x^4} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^{12}\left(a+bx^8\right)} + \frac{\left(b^2(5bc-6ad)\right)\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^4}{\sqrt{c+dx^8}}\right)}{8a^3(bc-ad)}$$

$$
= -\frac{(5bc-2ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^{12}} + \frac{(15b^2c^2-8abcd-4a^2d^2)\sqrt{c+dx^8}}{24a^3c^2(bc-ad)x^4} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^{12}(a+bx^8)} + \frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{7/2}(bc-ad)^{3/2}}
$$

Mathematica [A] (verified)

Time = 5.34 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.97

$$
\int \frac{1}{x^{13}(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{\sqrt{c+dx^8}(15b^3c^2x^{16}+2ab^2cx^8(5c-4dx^8)+2a^3d(c-2dx^8)-2a^2b(c^2+3cdx^8+2d^2x^{16}))}{24a^3c^2(-bc+ad)x^{12}(a+bx^8)} + \frac{b^2(5bc-6ad)\arctan\left(\frac{a\sqrt{d}+bx^4\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{8a^{7/2}(bc-ad)^{3/2}}
$$

```
[In] Integrate[1/(x^13*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/24*(Sqrt[c + d*x^8]*(15*b^3*c^2*x^16 + 2*a*b^2*c*x^8*(5*c - 4*d*x^8) + 2
*a^3*d*(c - 2*d*x^8) - 2*a^2*b*(c^2 + 3*c*d*x^8 + 2*d^2*x^16)))/(a^3*c^2*(-
(b*c) + a*d)*x^12*(a + b*x^8)) + (b^2*(5*b*c - 6*a*d)*ArcTan[(a*Sqrt[d] + b
*x^4*(Sqrt[d]*x^4 + Sqrt[c + d*x^8]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(8*a^(7/2
)*(b*c - a*d)^(3/2))
```

Maple [A] (verified)

Time = 47.79 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^8+c}\,\left(-2ad\,x^8-6bc\,x^8+ac\right)}{3x^{12}}-\frac{b^2c^2\left(\frac{b\sqrt{d\,x^8+c}\,x^4}{b\,x^8+a}-\frac{(6ad-5bc)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^8+c}\,a}{x^4\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)}{2(ad-bc)}}{4a^3c^2}$	134

```
[In] int(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a^3*(-1/3*(d*x^8+c)^(1/2)*(-2*a*d*x^8-6*b*c*x^8+a*c)/x^12-1/2*b^2*c^2/(
a*d-b*c)*(b*(d*x^8+c)^(1/2)*x^4/(b*x^8+a)-(6*a*d-5*b*c)/((a*d-b*c)*a)^(1/2)
*arctanh((d*x^8+c)^(1/2)/x^4*a/((a*d-b*c)*a)^(1/2))))/c^2
```

Fricas [A] (verification not implemented)

none

Time = 0.66 (sec) , antiderivative size = 760, normalized size of antiderivative = 3.65

$$\int \frac{1}{x^{13}(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$
$$= \left[-\frac{3\left((5\,b^4c^3-6\,ab^3c^2d)x^{20}+(5\,ab^3c^3-6\,a^2b^2c^2d)x^{12}\right)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{16}-2\,(3\,abc^2-4\,a^2cd)}{b^2x}\right.}{}\right.$$

```
[In] integrate(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^20 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2*
d)*x^12)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 -
 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b*c - 2*a*d)*x^12 - a*c*x^4)
*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2)) - 4*((
15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^16 + 2*(
5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^8 - 2*a^3*b^2*
c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2)*sqrt(d*x^8 + c))/((a^4*b^3*c^4 - 2*a^5*b
^2*c^3*d + a^6*b*c^2*d^2)*x^20 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7*c^2*d^2
)*x^12), 1/48*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^20 + (5*a*b^3*c^3 - 6*a^2*b
^2*c^2*d)*x^12)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sq
rt(d*x^8 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^
2*c*d)*x^4)) + 2*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^
4*b*d^3)*x^16 + 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^
3)*x^8 - 2*a^3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2)*sqrt(d*x^8 + c))/((a^
4*b^3*c^4 - 2*a^5*b^2*c^3*d + a^6*b*c^2*d^2)*x^20 + (a^5*b^2*c^4 - 2*a^6*b*
c^3*d + a^7*c^2*d^2)*x^12)]
```

Sympy [F]

$$\int \frac{1}{x^{13}(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{1}{x^{13}(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

```
[In] integrate(1/x**13/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(1/(x**13*(a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{1}{x^{13}\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{1}{\left(bx^8+a\right)^2\sqrt{dx^8+c}\,x^{13}}\,dx$$

```
[In] integrate(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^13), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(184) = 368.

Time = 1.05 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.90

$$\int \frac{1}{x^{13}\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{1}{24}\,d^{\frac{7}{2}}\left(\frac{3\left(5\,b^3c-6\,ab^2d\right)\arctan\left(-\frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\left(a^3bcd^3-a^4d^4\right)\sqrt{abcd-a^2d^2}}-\frac{6\left(\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right.\right.}{\left(a^3bcd^3-a^4d^4\right)\left(\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^4b\right.}\right.$$

```
[In] integrate(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] 1/24*d^(7/2)*(3*(5*b^3*c - 6*a*b^2*d)*arctan(-1/2*((sqrt(d)*x^4 - sqrt(d*x^
8 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^3*b*c*d^3 - a^4*d^4
)*sqrt(a*b*c*d - a^2*d^2)) - 6*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b^3*c - 2
*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*b^2*d - b^3*c^2)/((a^3*b*c*d^3 - a^4*d
^4)*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b - 2*(sqrt(d)*x^4 - sqrt(d*x^8 + c)
)^2*b*c + 4*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*d + b*c^2)) - 8*(3*(sqrt(d)
*x^4 - sqrt(d*x^8 + c))^4*b - 6*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c - 3*(
sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*d + 3*b*c^2 + a*c*d)/(((sqrt(d)*x^4 - sq
rt(d*x^8 + c))^2 - c)^3*a^3*d^3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{13}\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{1}{x^{13}\,\left(b\,x^8+a\right)^2\,\sqrt{d\,x^8+c}}\,dx$$

```
[In] int(1/(x^13*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
[Out] int(1/(x^13*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.918 $\int \frac{x^9}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 924

$$\int \frac{x^9}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{x^2\sqrt{c+dx^8}}{8(bc-ad)(a+bx^8)} - \frac{(bc+ad)\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{3/4}b^{3/4}(bc-ad)^{3/2}} + \frac{(bc+ad)\arctan\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{3/4}b^{3/4}(-bc+ad)^{3/2}}$$
$$+ \frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32b\sqrt[4]{c}(bc-ad)\sqrt{c+dx^8}}$$
$$+ \frac{\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32ab\sqrt[4]{c}(bc-ad)\sqrt{c+dx^8}}$$
$$- \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16b\sqrt[4]{c}(bc-ad)\sqrt{c+dx^8}}$$
$$+ \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{64ab\sqrt[4]{c}\sqrt[4]{d}(bc-ad)\sqrt{c+dx^8}}$$
$$+ \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{64ab\sqrt[4]{c}\sqrt[4]{d}(bc-ad)\sqrt{c+dx^8}}$$

```
[Out] -1/32*(a*d+b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1
/2))/(-a)^(3/4)/b^(3/4)/(-a*d+b*c)^(3/2)+1/32*(a*d+b*c)*arctan(x^2*(a*d-b*c
```

```
)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(3/4)/b^(3/4)/(a*d-b*c)^(3
/2)-1/8*x^2*(d*x^8+c)^(1/2)/(-a*d+b*c)/(b*x^8+a)-1/16*d^(3/4)*(cos(2*arctan
(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*Elliptic
F(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d
*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b/c^(1/4)/(-a*d+b*c)/(d*x^8+c)^(1/2)
+1/32*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(
1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2)
)*(b^(1/2)*c^(1/2)/(-a)^(1/2)+d^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^
(1/2)+x^4*d^(1/2))^2)^(1/2)/b/c^(1/4)/(-a*d+b*c)/(d*x^8+c)^(1/2)+1/32*d^(1/
4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^
(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*((-a)^(1/
2)*b^(1/2)*c^(1/2)+a*d^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4
*d^(1/2))^2)^(1/2)/a/b/c^(1/4)/(-a*d+b*c)/(d*x^8+c)^(1/2)+1/64*(cos(2*arcta
n(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*Ellipti
cPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1
/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2)
)*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2
)^(1/2)/a/b/c^(1/4)/d^(1/4)/(-a*d+b*c)/(d*x^8+c)^(1/2)+1/64*(cos(2*arctan(d
^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi
(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2
))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*
(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^
(1/2)/a/b/c^(1/4)/d^(1/4)/(-a*d+b*c)/(d*x^8+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 924, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used

$= \{476, 482, 537, 226, 418, 1231, 1721\}$

$$\int \frac{x^9}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

$$= \frac{\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{64ab\sqrt[4]{c}\sqrt[4]{d}(bc-ad)\sqrt{dx^8+c}}$$
$$-\frac{(bc+ad)\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{32(-a)^{3/4}b^{3/4}(bc-ad)^{3/2}}+\frac{(bc+ad)\arctan\left(\frac{\sqrt{ad-bc}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{32(-a)^{3/4}b^{3/4}(ad-bc)^{3/2}}$$
$$-\frac{d^{3/4}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16b\sqrt[4]{c}(bc-ad)\sqrt{dx^8+c}}$$
$$+\frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32b\sqrt[4]{c}(bc-ad)\sqrt{dx^8+c}}$$
$$+\frac{\left(\sqrt{d}a+\sqrt{-a}\sqrt{b}\sqrt{c}\right)\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32ab\sqrt[4]{c}(bc-ad)\sqrt{dx^8+c}}$$
$$+\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{64ab\sqrt[4]{c}\sqrt[4]{d}(bc-ad)\sqrt{dx^8+c}}$$
$$-\frac{x^2\sqrt{dx^8+c}}{8(bc-ad)(bx^8+a)}$$

```
[In] Int[x^9/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/8*(x^2*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8)) - ((b*c + a*d)*ArcTan[
(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(3/4)
*b^(3/4)*(b*c - a*d)^(3/2)) + ((b*c + a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x^2)/
((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(3/4)*b^(3/4)*(-(b*c) + a*d
)^(3/2)) + (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[
d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(
1/4)*x^2)/c^(1/4)], 1/2])/(32*b*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^8]) + ((Sq
rt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c
 + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/
4)], 1/2])/(32*a*b*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^8]) - (d^(3/4)*(Sqrt[c]
 + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*Arc
Tan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*b*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^8]
) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c
 + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqr
```

```
t[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)
/c^(1/4)], 1/2])/(64*a*b*c^(1/4)*d^(1/4)*(b*c - a*d)*Sqrt[c + d*x^8]) + ((S
qrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^
8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d
])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)],
 1/2])/(64*a*b*c^(1/4)*d^(1/4)*(b*c - a*d)*Sqrt[c + d*x^8])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
```

```
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx, x, x^2\right)$$

$$= -\frac{x^2\sqrt{c+dx^8}}{8(bc-ad)(a+bx^8)} + \frac{\text{Subst}\left(\int \frac{c-dx^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8(bc-ad)}$$

$$= -\frac{x^2\sqrt{c+dx^8}}{8(bc-ad)(a+bx^8)} - \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8b(bc-ad)} + \frac{(bc+ad)\text{Subst}\left(\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8b(bc-ad)}$$

$$= -\frac{x^2\sqrt{c+dx^8}}{8(bc-ad)(a+bx^8)} - \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16b\sqrt[4]{c}(bc-ad)\sqrt{c+dx^8}} + \frac{(bc+ad)\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{16ab(bc-ad)} + \frac{(bc+ad)\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{16ab(bc-ad)}$$

$$= -\frac{x^2\sqrt{c+dx^8}}{8(bc-ad)(a+bx^8)} - \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16b\sqrt[4]{c}(bc-ad)\sqrt{c+dx^8}}$$
$$+ \frac{\left(\sqrt{c}\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\right)\text{Subst}\left(\int\frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{16a(bc-ad)}$$
$$+ \frac{\left(\sqrt{c}\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\right)\text{Subst}\left(\int\frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{16a(bc-ad)}$$
$$+ \frac{\left(\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt{d}\right)\text{Subst}\left(\int\frac{1}{\sqrt{c+dx^4}}\,dx,x,x^2\right)}{16b(bc-ad)}$$
$$+ \frac{\left(\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt{d}\right)\text{Subst}\left(\int\frac{1}{\sqrt{c+dx^4}}\,dx,x,x^2\right)}{16ab(bc-ad)}$$

$$= -\frac{x^2\sqrt{c+dx^8}}{8(bc-ad)(a+bx^8)} - \frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{3/4}b^{3/4}(bc-ad)^{3/2}}$$
$$+ \frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{3/4}b^{3/4}(-bc+ad)^{3/2}}$$
$$+ \frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{32b\sqrt[4]{c}(bc-ad)\sqrt{c+dx^8}}$$
$$+ \frac{\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{32ab\sqrt[4]{c}(bc-ad)\sqrt{c+dx^8}}$$
$$- \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16b\sqrt[4]{c}(bc-ad)\sqrt{c+dx^8}}$$
$$+ \frac{\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{64a\sqrt[4]{c}\sqrt[4]{d}(bc-ad)\sqrt{c+dx^8}}$$
$$+ \frac{\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)^2\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{64a\sqrt[4]{c}\sqrt[4]{d}(bc-ad)\sqrt{c+dx^8}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.17

$$\int \frac{x^9}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{x^2\left(5a(c+dx^8)-5c(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+dx^8(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,\text{App}\right.}{40a(bc-ad)\,(a+bx^8)\sqrt{c+dx^8}}$$

```
[In] Integrate[x^9/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/40*(x^2*(5*a*(c + d*x^8) - 5*c*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[
1/4, 1/2, 1, 5/4, -((d*x^8)/c), -((b*x^8)/a)] + d*x^8*(a + b*x^8)*Sqrt[1 +
(d*x^8)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c), -((b*x^8)/a)]))/(a*(b*c
 - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

Maple [F]

$$\int \frac{x^9}{(b\,x^8+a)^2\sqrt{d\,x^8+c}}dx$$

```
[In] int(x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^9}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

```
[In] integrate(x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^9}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

```
[In] integrate(x**9/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^9}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^9}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^9/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Giac [F]

$$\int \frac{x^9}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^9}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^9/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^9}{(b\,x^8+a)^2\,\sqrt{d\,x^8+c}}\,dx$$

```
[In] int(x^9/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
[Out] int(x^9/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.919 $\int \frac{x}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 999

$$\int \frac{x}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{bx^2\sqrt{c+dx^8}}{8a(bc-ad)(a+bx^8)} + \frac{\sqrt[4]{b}(3bc-5ad)\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{7/4}(bc-ad)^{3/2}} - \frac{\sqrt[4]{b}(3bc-5ad)\arctan\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{7/4}(-bc+ad)^{3/2}} + \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a\sqrt[4]{c}(bc-ad)\sqrt{c+dx^8}} + \frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}(3bc-5ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32a\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^8}} + \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(3bc-5ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32(-a)^{3/2}\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^8}} + \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(3bc-5ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{}}{}\right.\right.}{64a^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^8}} + \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(3bc-5ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}}{\sqrt[4]{}}\right.\right.}{64a^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$

```
[Out] 1/32*b^(1/4)*(-5*a*d+3*b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/
(d*x^8+c)^(1/2))/(-a)^(7/4)/(-a*d+b*c)^(3/2)-1/32*b^(1/4)*(-5*a*d+3*b*c)*ar
```

```
ctan(x^2*(a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(7/4)/(a*
d-b*c)^(3/2)+1/8*b*x^2*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/(b*x^8+a)+1/16*d^(3/4)*
(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/
4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4
*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^(1/4)/(-a*d+b*c)/(d
*x^8+c)^(1/2)+1/32*d^(1/4)*(-5*a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)
))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/
4)*x^2/c^(1/4))),1/2*2^(1/2))*(b^(1/2)*c^(1/2)/(-a)^(1/2)+d^(1/2))*(c^(1/2)
+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^(1/4)/(-a*d+b*c
)/(a*d+b*c)/(d*x^8+c)^(1/2)+1/32*d^(1/4)*(-5*a*d+3*b*c)*(cos(2*arctan(d^(1/
4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(
2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*
c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/(-a)^
(3/2)/c^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^8+c)^(1/2)+1/64*(-5*a*d+3*b*c)*(cos
(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))
*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1
/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4
*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^
(1/2))^2)^(1/2)/a^2/c^(1/4)/d^(1/4)/(-a^2*d^2+b^2*c^2)/(d*x^8+c)^(1/2)+1/64
*(-5*a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d
^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),-1/4*(b^
(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*
2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x
^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a^2/c^(1/4)/d^(1/4)/(-a^2*d^2+b^2*c^2)
/(d*x^8+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 999, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used

$= \{476, 425, 537, 226, 418, 1231, 1721\}$

$$\int \frac{x}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

$$= \frac{(3bc-5ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\right.}{64a^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^8+c}}$$

$$+\frac{\sqrt[4]{d}(3bc-5ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)}{32(-a)^{3/2}\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{dx^8+c}}$$

$$+\frac{\sqrt[4]{b}(3bc-5ad)\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{32(-a)^{7/4}(bc-ad)^{3/2}}-\frac{\sqrt[4]{b}(3bc-5ad)\arctan\left(\frac{\sqrt{ad-bc}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{32(-a)^{7/4}(ad-bc)^{3/2}}$$

$$+\frac{d^{3/4}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a\sqrt[4]{c}(bc-ad)\sqrt{dx^8+c}}$$

$$+\frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}(3bc-5ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32a\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{dx^8+c}}$$

$$+\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(3bc-5ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{}}{}\right.\right.}{64a^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^8+c}}$$

$$+\frac{bx^2\sqrt{dx^8+c}}{8a(bc-ad)\left(bx^8+a\right)}$$

```
[In] Int[x/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (b*x^2*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*(a + b*x^8)) + (b^(1/4)*(3*b*c - 5
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(
32*(-a)^(7/4)*(b*c - a*d)^(3/2)) - (b^(1/4)*(3*b*c - 5*a*d)*ArcTan[(Sqrt[-(
b*c) + a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(7/4)*(-(b
*c) + a*d)^(3/2)) + (d^(3/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt
[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*
a*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^8]) + (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqr
t[d])*d^(1/4)*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqr
t[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(32
*a*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] - S
qrt[-a]*Sqrt[d])*d^(1/4)*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c +
d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)]
, 1/2])/(32*(-a)^(3/2)*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) + (
(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x
```

```
^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-1/4*(Sqrt[b]*Sq
rt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d
^(1/4)*x^2)/c^(1/4)], 1/2])/(64*a^2*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)
*Sqrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(3*b*c - 5*a*d)
*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Ellipt
icPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqr
t[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*a^2*c^(1/4)*d^(1/4)*(b*c
- a*d)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx, x, x^2\right)\\
&= \frac{bx^2\sqrt{c+dx^8}}{8a(bc-ad)\,(a+bx^8)} - \frac{\text{Subst}\left(\int \frac{-3bc+4ad-bdx^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8a(bc-ad)}\\
&= \frac{bx^2\sqrt{c+dx^8}}{8a(bc-ad)\,(a+bx^8)} + \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8a(bc-ad)}\\
&\quad + \frac{(3bc-5ad)\text{Subst}\left(\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8a(bc-ad)}\\
&= \frac{bx^2\sqrt{c+dx^8}}{8a(bc-ad)\,(a+bx^8)} + \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16a\sqrt[4]{c}(bc-ad)\sqrt{c+dx^8}}\\
&\quad + \frac{(3bc-5ad)\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{16a^2(bc-ad)}\\
&\quad + \frac{(3bc-5ad)\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{16a^2(bc-ad)}
\end{aligned}$$

$$= \frac{bx^2\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} + \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16a\sqrt[4]{c}(bc-ad)\sqrt{c+dx^8}}$$
$$+ \frac{\left(\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(3bc-5ad)\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{16a^2(bc-ad)(bc+ad)}$$
$$+ \frac{\left(\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(3bc-5ad)\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{16a^2(bc-ad)(bc+ad)}$$
$$+ \frac{\left(\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt{d}(3bc-5ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{16a(bc-ad)(bc+ad)}$$
$$+ \frac{\left(\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt{d}(3bc-5ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{16(-a)^{3/2}(bc-ad)(bc+ad)}$$

$$= \frac{bx^2\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} + \frac{\sqrt[4]{b}(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{7/4}(bc-ad)^{3/2}}$$
$$- \frac{\sqrt[4]{b}(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{7/4}(-bc+ad)^{3/2}}$$
$$+ \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16a\sqrt[4]{c}(bc-ad)\sqrt{c+dx^8}}$$
$$+ \frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}(3bc-5ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{32a\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$
$$+ \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(3bc-5ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{32(-a)^{3/2}\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$
$$+ \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(3bc-5ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\cdots\right.\right.}{64a^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$
$$+ \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(3bc-5ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\cdots\right.\right.}{64a^2\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.17

$$\int \frac{x}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$
$$= \frac{x^2\left(5ab(c+dx^8)+5(3bc-4ad)\,(a+bx^8)\,\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+bdx^8(a+bx^8)\,\sqrt{1\right.}{40a^2(bc-ad)\,(a+bx^8)\,\sqrt{c+dx^8}}$$

```
[In] Integrate[x/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^2*(5*a*b*(c + d*x^8) + 5*(3*b*c - 4*a*d)*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]
*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^8)/c), -((b*x^8)/a)] + b*d*x^8*(a + b*x^
8)*Sqrt[1 + (d*x^8)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c), -((b*x^8)/a
)]))/(40*a^2*(b*c - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

Maple [F]

$$\int \frac{x}{(b\,x^8+a)^2\sqrt{d\,x^8+c}}dx$$

```
[In] int(x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

```
[In] integrate(x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{x}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$$

```
[In] integrate(x/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
[Out] Integral(x/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{x}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Giac [F]

$$\int \frac{x}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{x}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(x/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{x}{\left(b\,x^8+a\right)^2\,\sqrt{d\,x^8+c}}\,dx$$

```
[In] int(x/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
[Out] int(x/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.920 $\int \frac{1}{x^7(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1060

$$\int \frac{1}{x^7(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{(7bc-4ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^6}+\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^6(a+bx^8)}$$
$$+\frac{b^{5/4}(7bc-9ad)\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{11/4}(bc-ad)^{3/2}}-\frac{b^{5/4}(7bc-9ad)\arctan\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{11/4}(-bc+ad)^{3/2}}$$
$$-\frac{d^{3/4}(7bc-4ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{48a^2c^{5/4}(bc-ad)\sqrt{c+dx^8}}$$
$$+\frac{b\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(7bc-9ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32(-a)^{5/2}\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$
$$-\frac{b\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(7bc-9ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32(-a)^{5/2}\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$
$$-\frac{b\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(7bc-9ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{64a^3\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$
$$-\frac{b\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(7bc-9ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{64a^3\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$

```
[Out] 1/32*b^(5/4)*(-9*a*d+7*b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/
(d*x^8+c)^(1/2))/(-a)^(11/4)/(-a*d+b*c)^(3/2)-1/32*b^(5/4)*(-9*a*d+7*b*c)*a
```

```
rctan(x^2*(a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(11/4)/(
a*d-b*c)^(3/2)-1/24*(-4*a*d+7*b*c)*(d*x^8+c)^(1/2)/a^2/c/(-a*d+b*c)/x^6+1/8
*b*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/x^6/(b*x^8+a)-1/48*d^(3/4)*(-4*a*d+7*b*c)*(
cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4
)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*
d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a^2/c^(5/4)/(-a*d+b*c)/(
d*x^8+c)^(1/2)+1/32*b*d^(1/4)*(-9*a*d+7*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1
/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^
(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)-(-a
)^(1/2)*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/(-a)^(5/2)/c^(1/
4)/(-a*d+b*c)/(a*d+b*c)/(d*x^8+c)^(1/2)-1/64*b*(-9*a*d+7*b*c)*(cos(2*arctan
(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*Elliptic
Pi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/
2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))
*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)
^(1/2)/a^3/c^(1/4)/d^(1/4)/(-a^2*d^2+b^2*c^2)/(d*x^8+c)^(1/2)-1/32*b*d^(1/4
)*(-9*a*d+7*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(
d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1
/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^8+c)/
(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/(-a)^(5/2)/c^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x
^8+c)^(1/2)-1/64*b*(-9*a*d+7*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1
/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/
c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^
(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2
)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a^3/c^(1/4)/d^(1/4)/
(-a^2*d^2+b^2*c^2)/(d*x^8+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 1060, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{476, 483, 597, 537, 226, 418, 1231, 1721\}$

$$\int \frac{1}{x^7 (a+bx^8)^2 \sqrt{c+dx^8}} \, dx =$$

$$-\frac{b(7bc-9ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{\right.}{64a^3\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^8+c}}$$
$$+\frac{b\sqrt[4]{d}(7bc-9ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)}{32(-a)^{5/2}\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{dx^8+c}}$$
$$+\frac{b^{5/4}(7bc-9ad)\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{32(-a)^{11/4}(bc-ad)^{3/2}}-\frac{b^{5/4}(7bc-9ad)\arctan\left(\frac{\sqrt{ad-bc}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{32(-a)^{11/4}(ad-bc)^{3/2}}$$
$$-\frac{d^{3/4}(7bc-4ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{48a^2c^{5/4}(bc-ad)\sqrt{dx^8+c}}$$
$$-\frac{b\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(7bc-9ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32(-a)^{5/2}\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{dx^8+c}}$$
$$-\frac{b\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(7bc-9ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{64a^3\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^8+c}}$$
$$+\frac{b\sqrt{dx^8+c}}{8a(bc-ad)x^6\left(bx^8+a\right)}-\frac{(7bc-4ad)\sqrt{dx^8+c}}{24a^2c(bc-ad)x^6}$$

```
[In] Int[1/(x^7*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/24*((7*b*c - 4*a*d)*Sqrt[c + d*x^8])/(a^2*c*(b*c - a*d)*x^6) + (b*Sqrt[c
 + d*x^8])/(8*a*(b*c - a*d)*x^6*(a + b*x^8)) + (b^(5/4)*(7*b*c - 9*a*d)*Arc
Tan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(
11/4)*(b*c - a*d)^(3/2)) - (b^(5/4)*(7*b*c - 9*a*d)*ArcTan[(Sqrt[-(b*c) + a
*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(11/4)*(-(b*c) + a
*d)^(3/2)) - (d^(3/4)*(7*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x
^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1
/2])/(48*a^2*c^(5/4)*(b*c - a*d)*Sqrt[c + d*x^8]) + (b*(Sqrt[b]*Sqrt[c] - S
qrt[-a]*Sqrt[d])*d^(1/4)*(7*b*c - 9*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c +
d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)]
, 1/2])/(32*(-a)^(5/2)*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) - (
b*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*d^(1/4)*(7*b*c - 9*a*d)*(Sqrt[c] + S
qrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[
(d^(1/4)*x^2)/c^(1/4)], 1/2])/(32*(-a)^(5/2)*c^(1/4)*(b*c - a*d)*(b*c + a*d
```

```
)*Sqrt[c + d*x^8]) - (b*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(7*b*c - 9*a
*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Ell
ipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[
c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*a^3*c^(1/4)*d^(1/4)
*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) - (b*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*
Sqrt[d])^2*(7*b*c - 9*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c
] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sq
rt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(64
*a^3*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx,x,x^2\right)\\
&= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^6\left(a+bx^8\right)}-\frac{\text{Subst}\left(\int \frac{-7bc+4ad-5bdx^4}{x^4(a+bx^4)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{8a(bc-ad)}\\
&= -\frac{(7bc-4ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^6}+\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^6\left(a+bx^8\right)}\\
&\quad+\frac{\text{Subst}\left(\int \frac{-21b^2c^2+20abcd+4a^2d^2-bd(7bc-4ad)x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{24a^2c(bc-ad)}
\end{aligned}$$

$$=-\frac{(7bc-4ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^6}+\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^6\left(a+bx^8\right)}-\frac{(b(7bc-9ad))\text{Subst}\left(\int\frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{8a^2(bc-ad)}-\frac{(d(7bc-4ad))\text{Subst}\left(\int\frac{1}{\sqrt{c+dx^4}}\,dx,x,x^2\right)}{24a^2c(bc-ad)}$$

$$=-\frac{(7bc-4ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^6}+\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^6\left(a+bx^8\right)}-\frac{d^{3/4}(7bc-4ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{48a^2c^{5/4}(bc-ad)\sqrt{c+dx^8}}-\frac{(b(7bc-9ad))\text{Subst}\left(\int\frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{16a^3(bc-ad)}-\frac{(b(7bc-9ad))\text{Subst}\left(\int\frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{16a^3(bc-ad)}$$

$$=-\frac{(7bc-4ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^6}+\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^6\left(a+bx^8\right)}-\frac{d^{3/4}(7bc-4ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{48a^2c^{5/4}(bc-ad)\sqrt{c+dx^8}}-\frac{\left(b^{3/2}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(7bc-9ad)\right)\text{Subst}\left(\int\frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{16a^3(bc-ad)(bc+ad)}-\frac{\left(b^{3/2}\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(7bc-9ad)\right)\text{Subst}\left(\int\frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{16a^3(bc-ad)(bc+ad)}-\frac{\left(b\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt{d}(7bc-9ad)\right)\text{Subst}\left(\int\frac{1}{\sqrt{c+dx^4}}\,dx,x,x^2\right)}{16a^2(bc-ad)(bc+ad)}+\frac{\left(b\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt{d}(7bc-9ad)\right)\text{Subst}\left(\int\frac{1}{\sqrt{c+dx^4}}\,dx,x,x^2\right)}{16(-a)^{5/2}(bc-ad)(bc+ad)}$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^6} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^6(a+bx^8)}$$
$$+ \frac{b^{5/4}(7bc-9ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{11/4}(bc-ad)^{3/2}}$$
$$- \frac{b^{5/4}(7bc-9ad)\tan^{-1}\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{11/4}(-bc+ad)^{3/2}}$$
$$- \frac{d^{3/4}(7bc-4ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{48a^2c^{5/4}(bc-ad)\sqrt{c+dx^8}}$$
$$- \frac{b\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}(7bc-9ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{32a^2\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$
$$+ \frac{b\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(7bc-9ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{32(-a)^{5/2}\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$
$$- \frac{b\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(7bc-9ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.\right.}{64a^3\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$
$$- \frac{b\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(7bc-9ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{64a^3\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.32 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.21

$$\int \frac{1}{x^7(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$
$$= \frac{5a(c+dx^8)\left(4a^2d-7b^2cx^8-4ab(c-dx^8)\right)+5(-21b^2c^2+20abcd+4a^2d^2)x^8(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,\mathrm{AppellF}}{120a^3c(bc-ad)x^6(a-}$$

```
[In] Integrate[1/(x^7*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
[Out] (5*a*(c + d*x^8)*(4*a^2*d - 7*b^2*c*x^8 - 4*a*b*(c - d*x^8)) + 5*(-21*b^2*c
^2 + 20*a*b*c*d + 4*a^2*d^2)*x^8*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[1
/4, 1/2, 1, 5/4, -((d*x^8)/c), -((b*x^8)/a)] + b*d*(-7*b*c + 4*a*d)*x^16*(a
 + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c), -((b
*x^8)/a)])/(120*a^3*c*(b*c - a*d)*x^6*(a + b*x^8)*Sqrt[c + d*x^8])
```

Maple [F]

$$\int \frac{1}{x^{7} \left(b \,x^{8}+a \right)^{2} \sqrt{d \,x^{8}+c}}d x$$

```
[In] int(1/x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
[Out] int(1/x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^7 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^7 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{1}{x^{7} \left(a + b x^{8}\right)^{2} \sqrt{c + d x^{8}}}\, dx$$

```
[In] integrate(1/x**7/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
[Out] Integral(1/(x**7*(a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{1}{x^7 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int { \frac {1}{{\left (b x^{8} + a\right)}^{2} \sqrt {d x^{8} + c} x^{7}} \,d x }$$

```
[In] integrate(1/x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^7), x)
```

Giac [F]

$$\int \frac{1}{x^7 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{1}{(bx^8+a)^2 \sqrt{dx^8+c}x^7}\,dx$$

```
[In] integrate(1/x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^7 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{1}{x^7 (b\,x^8+a)^2 \sqrt{d\,x^8+c}}\,dx$$

```
[In] int(1/(x^7*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
[Out] int(1/(x^7*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.921 $\int \frac{x^{13}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1164

$$
\begin{aligned}
\int \frac{x^{13}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx &= \frac{\sqrt{d}x^2\sqrt{c+dx^8}}{8b(bc-ad)\left(\sqrt{c}+\sqrt{d}x^4\right)}-\frac{x^6\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} \\
&+\frac{(3bc-ad)\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32\sqrt[4]{-a}b^{5/4}(bc-ad)^{3/2}}+\frac{(3bc-ad)\arctan\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32\sqrt[4]{-a}b^{5/4}(-bc+ad)^{3/2}} \\
&-\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8b(bc-ad)\sqrt{c+dx^8}} \\
&+\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16b(bc-ad)\sqrt{c+dx^8}} \\
&-\frac{\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(3bc-ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32b\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^8}} \\
&-\frac{\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(3bc-ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32b\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^8}} \\
&+\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(3bc-ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right.\right.}{64\sqrt{-a}b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^8}} \\
&-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(3bc-ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right.\right.}{64\sqrt{-a}b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^8}}
\end{aligned}
$$

```
[Out] 1/32*(-a*d+3*b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^
(1/2))/(-a)^(1/4)/b^(5/4)/(-a*d+b*c)^(3/2)+1/32*(-a*d+3*b*c)*arctan(x^2*(a*
d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(1/4)/b^(5/4)/(a*d-b*
c)^(3/2)-1/8*x^6*(d*x^8+c)^(1/2)/(-a*d+b*c)/(b*x^8+a)+1/8*x^2*d^(1/2)*(d*x^
8+c)^(1/2)/b/(-a*d+b*c)/(c^(1/2)+x^4*d^(1/2))-1/8*c^(1/4)*d^(1/4)*(cos(2*ar
ctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*Elli
pticE(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))
*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b/(-a*d+b*c)/(d*x^8+c)^(1/2)+1/1
6*c^(1/4)*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan
(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(
1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b/(-a
```

```
*d+b*c)/(d*x^8+c)^(1/2)-1/32*d^(1/4)*(-a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x^2
/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arct
an(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2
)-(-a)^(1/2)*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b^(3/2)/c^(
1/4)/(-a^2*d^2+b^2*c^2)/(d*x^8+c)^(1/2)-1/64*(-a*d+3*b*c)*(cos(2*arctan(d^(
1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(s
in(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^
2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^
(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/
2)/b^(3/2)/c^(1/4)/d^(1/4)/(-a*d+b*c)/(a*d+b*c)/(-a)^(1/2)/(d*x^8+c)^(1/2)-
1/32*d^(1/4)*(-a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(
2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4)))
,1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*((
d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b^(3/2)/c^(1/4)/(-a^2*d^2+b^2*c^2)/
(d*x^8+c)^(1/2)+1/64*(-a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1
/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/
c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^
(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2
)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b^(3/2)/c^(1/4)/d^(1
/4)/(-a*d+b*c)/(a*d+b*c)/(-a)^(1/2)/(d*x^8+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 1164, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used

$= \{476, 482, 598, 311, 226, 1210, 504, 1231, 1721\}$

$$\int \frac{x^{13}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{\sqrt{dx^8+c}\,x^6}{8(bc-ad)\,(bx^8+a)} + \frac{\sqrt{d}\sqrt{dx^8+c}\,x^2}{8b(bc-ad)\left(\sqrt{d}x^4+\sqrt{c}\right)}$$
$$+ \frac{(3bc-ad)\arctan\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{32\sqrt[4]{-a}\,b^{5/4}(bc-ad)^{3/2}} + \frac{(3bc-ad)\arctan\left(\frac{\sqrt{ad-bc}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{32\sqrt[4]{-a}\,b^{5/4}(ad-bc)^{3/2}}$$
$$- \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8b(bc-ad)\sqrt{dx^8+c}}$$
$$+ \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16b(bc-ad)\sqrt{dx^8+c}}$$
$$- \frac{\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(3bc-ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32b\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{dx^8+c}}$$
$$- \frac{\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(3bc-ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32b\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{dx^8+c}}$$
$$+ \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(3bc-ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right.\right.}{64\sqrt{-a}\,b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^8+c}}$$
$$- \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(3bc-ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right.\right.}{64\sqrt{-a}\,b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^8+c}}$$

```
[In] Int[x^13/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (Sqrt[d]*x^2*Sqrt[c + d*x^8])/(8*b*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^4)) - (
x^6*Sqrt[c + d*x^8])/(8*(b*c - a*d)*(a + b*x^8)) + ((3*b*c - a*d)*ArcTan[(S
qrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(1/4)*b
^(5/4)*(b*c - a*d)^(3/2)) + ((3*b*c - a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x^2)/
((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(1/4)*b^(5/4)*(-(b*c) + a*d
)^(3/2)) - (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[
c] + Sqrt[d]*x^4)^2]*EllipticE[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*b*
(b*c - a*d)*Sqrt[c + d*x^8]) + (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqr
t[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c
^(1/4)], 1/2])/(16*b*(b*c - a*d)*Sqrt[c + d*x^8]) - ((Sqrt[c] - (Sqrt[-a]*S
qrt[d])/Sqrt[b])*d^(1/4)*(3*b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*
x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)],
```

```
1/2])/(32*b*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[c] +
(Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(3*b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^4)*S
qrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)
/c^(1/4)], 1/2])/(32*b*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) + (
(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(3*b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^4
)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt
[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(
1/4)*x^2)/c^(1/4)], 1/2])/(64*Sqrt[-a]*b^(3/2)*c^(1/4)*d^(1/4)*(b*c - a*d)*
(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(3*b
*c - a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^
2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sq
rt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*Sqrt[-a]*b^(3/2)
*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
```

```
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx, x, x^2\right)$$

$$= -\frac{x^6\sqrt{c+dx^8}}{8(bc-ad)\,(a+bx^8)} + \frac{\text{Subst}\left(\int \frac{x^2(3c+dx^4)}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8(bc-ad)}$$

$$= -\frac{x^6\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)}+\frac{\text{Subst}\left(\int\left(\frac{dx^2}{b\sqrt{c+dx^4}}+\frac{(3bc-ad)x^2}{b(a+bx^4)\sqrt{c+dx^4}}\right)dx,x,x^2\right)}{8(bc-ad)}$$

$$= -\frac{x^6\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)}+\frac{d\text{Subst}\left(\int\frac{x^2}{\sqrt{c+dx^4}}dx,x,x^2\right)}{8b(bc-ad)}+\frac{(3bc-ad)\text{Subst}\left(\int\frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}dx,x,x^2\right)}{8b(bc-ad)}$$

$$= -\frac{x^6\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)}+\frac{\left(\sqrt{c}\sqrt{d}\right)\text{Subst}\left(\int\frac{1}{\sqrt{c+dx^4}}dx,x,x^2\right)}{8b(bc-ad)}-\frac{\left(\sqrt{c}\sqrt{d}\right)\text{Subst}\left(\int\frac{1-\frac{\sqrt{d}x^2}{\sqrt{c}}}{\sqrt{c+dx^4}}dx,x,x^2\right)}{8b(bc-ad)}-\frac{(3bc-ad)\text{Subst}\left(\int\frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}dx,x,x^2\right)}{16b^{3/2}(bc-ad)}+\frac{(3bc-ad)\text{Subst}\left(\int\frac{1}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}dx,x,x^2\right)}{16b^{3/2}(bc-ad)}$$

$$= \frac{\sqrt{d}x^2\sqrt{c+dx^8}}{8b(bc-ad)\left(\sqrt{c}+\sqrt{d}x^4\right)}-\frac{x^6\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)}-\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8b(bc-ad)\sqrt{c+dx^8}}+\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16b(bc-ad)\sqrt{c+dx^8}}-\frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(3bc-ad)\right)\text{Subst}\left(\int\frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}dx,x,x^2\right)}{16b(bc-ad)(bc+ad)}+\frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(3bc-ad)\right)\text{Subst}\left(\int\frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}dx,x,x^2\right)}{16b(bc-ad)(bc+ad)}-\frac{\left(\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt{d}(3bc-ad)\right)\text{Subst}\left(\int\frac{1}{\sqrt{c+dx^4}}dx,x,x^2\right)}{16b^{3/2}(bc-ad)(bc+ad)}-\frac{\left(\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt{d}(3bc-ad)\right)\text{Subst}\left(\int\frac{1}{\sqrt{c+dx^4}}dx,x,x^2\right)}{16b^{3/2}(bc-ad)(bc+ad)}$$

$$= \frac{\sqrt{d}x^2\sqrt{c+dx^8}}{8b(bc-ad)\left(\sqrt{c}+\sqrt{d}x^4\right)} - \frac{x^6\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)}$$
$$+ \frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32\sqrt[4]{-a}b^{5/4}(bc-ad)^{3/2}} + \frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32\sqrt[4]{-a}b^{5/4}(-bc+ad)^{3/2}}$$
$$- \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8b(bc-ad)\sqrt{c+dx^8}}$$
$$+ \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16b(bc-ad)\sqrt{c+dx^8}}$$
$$- \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(3bc-ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{32b^{3/2}\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$
$$- \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(3bc-ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{32b^{3/2}\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$
$$+ \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(3bc-ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right.\right.}{64\sqrt{-a}b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$
$$- \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(3bc-ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right.\right.}{64\sqrt{-a}b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.14

$$\int \frac{x^{13}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$$
$$= \frac{x^6\left(-7a(c+dx^8)+7c(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\operatorname{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+dx^8(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\operatorname{App}\right.}{56a(bc-ad)\left(a+bx^8\right)\sqrt{c+dx^8}}$$

```
[In] Integrate[x^13/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^6*(-7*a*(c + d*x^8) + 7*c*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[3/4,
1/2, 1, 7/4, -((d*x^8)/c), -((b*x^8)/a)] + d*x^8*(a + b*x^8)*Sqrt[1 + (d*x^
```

```
8)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^8)/c), -((b*x^8)/a)]))/(56*a*(b*c
- a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

Maple [F]

$$\int \frac{x^{13}}{\left(b\,x^{8}+a\right)^{2} \sqrt{d\,x^{8}+c}}dx$$

```
[In] int(x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
[Out] int(x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^{13}}{\left(a+bx^8\right)^2 \sqrt{c+dx^8}}\,dx = \text{Timed out}$$

```
[In] integrate(x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{13}}{\left(a+bx^8\right)^2 \sqrt{c+dx^8}}\,dx = \text{Timed out}$$

```
[In] integrate(x**13/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{13}}{\left(a+bx^8\right)^2 \sqrt{c+dx^8}}\,dx = \int \frac{x^{13}}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^13/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Giac [F]

$$\int \frac{x^{13}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^{13}}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^13/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{13}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^{13}}{(b\,x^8+a)^2\,\sqrt{d\,x^8+c}}\,dx$$

```
[In] int(x^13/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
[Out] int(x^13/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.922 $\int \frac{x^5}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1162

$$\int \frac{x^5}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{\sqrt{d}x^2\sqrt{c+dx^8}}{8a(bc-ad)\left(\sqrt{c}+\sqrt{d}x^4\right)}+\frac{bx^6\sqrt{c+dx^8}}{8a(bc-ad)\,(a+bx^8)}$$

$$-\frac{(bc-3ad)\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{5/4}\sqrt[4]{b}(bc-ad)^{3/2}}-\frac{(bc-3ad)\arctan\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{5/4}\sqrt[4]{b}(-bc+ad)^{3/2}}$$

$$+\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a(bc-ad)\sqrt{c+dx^8}}$$

$$-\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a(bc-ad)\sqrt{c+dx^8}}$$

$$-\frac{\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(bc-3ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32a\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$

$$-\frac{\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(bc-3ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32a\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$

$$-\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(bc-3ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}\ldots}{\sqrt[4]{\ldots}}\right.\right.}{64(-a)^{3/2}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$

$$+\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(bc-3ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right.\right.}{64(-a)^{3/2}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$

```
[Out] -1/32*(-3*a*d+b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)
^(1/2))/(-a)^(5/4)/b^(1/4)/(-a*d+b*c)^(3/2)-1/32*(-3*a*d+b*c)*arctan(x^2*(a
*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(5/4)/b^(1/4)/(a*d-b
*c)^(3/2)+1/8*b*x^6*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/(b*x^8+a)-1/8*x^2*d^(1/2)*
(d*x^8+c)^(1/2)/a/(-a*d+b*c)/(c^(1/2)+x^4*d^(1/2))+1/8*c^(1/4)*d^(1/4)*(cos
(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))
*EllipticE(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(
1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/(-a*d+b*c)/(d*x^8+c)^(1/2
)-1/16*c^(1/4)*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*a
rctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/
2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/
```

```
a/(-a*d+b*c)/(d*x^8+c)^(1/2)+1/64*(-3*a*d+b*c)*(cos(2*arctan(d^(1/4)*x^2/c^
(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan
(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2
)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/
2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/(-a)^(3/
2)/c^(1/4)/d^(1/4)/(-a*d+b*c)/(a*d+b*c)/b^(1/2)/(d*x^8+c)^(1/2)-1/64*(-3*a*
d+b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^
2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/2)*c^(
1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*
(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^
(1/2)+x^4*d^(1/2))^2)^(1/2)/(-a)^(3/2)/c^(1/4)/d^(1/4)/(-a*d+b*c)/(a*d+b*c)
/b^(1/2)/(d*x^8+c)^(1/2)-1/32*d^(1/4)*(-3*a*d+b*c)*(cos(2*arctan(d^(1/4)*x^
2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arc
tan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(c^(1/2)-(-a)^
(1/2)*d^(1/2)/b^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^(1/4)/
(-a*d+b*c)/(a*d+b*c)/(d*x^8+c)^(1/2)-1/32*d^(1/4)*(-3*a*d+b*c)*(cos(2*arcta
n(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*Ellipti
cF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(c
^(1/2)+(-a)^(1/2)*d^(1/2)/b^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2
)/a/c^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^8+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 1162, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used

$= \{476, 483, 598, 311, 226, 1210, 504, 1231, 1721\}$

$$\int \frac{x^5}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{b\sqrt{dx^8+c}x^6}{8a(bc-ad)(bx^8+a)} - \frac{\sqrt{d}\sqrt{dx^8+c}x^2}{8a(bc-ad)\left(\sqrt{d}x^4+\sqrt{c}\right)} - \frac{(bc-3ad)\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{32(-a)^{5/4}\sqrt[4]{b}(bc-ad)^{3/2}} - \frac{(bc-3ad)\arctan\left(\frac{\sqrt{ad-bc}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{32(-a)^{5/4}\sqrt[4]{b}(ad-bc)^{3/2}} + \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a(bc-ad)\sqrt{dx^8+c}} - \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a(bc-ad)\sqrt{dx^8+c}} - \frac{\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(bc-3ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32a\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{dx^8+c}} - \frac{\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(bc-3ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32a\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{dx^8+c}} - \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(bc-3ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{c}}{\sqrt[4]{}}\right.\right.}{64(-a)^{3/2}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^8+c}} + \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(bc-3ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{64(-a)^{3/2}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^8+c}}$$

```
[In] Int[x^5/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/8*(Sqrt[d]*x^2*Sqrt[c + d*x^8])/(a*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^4))
+ (b*x^6*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*(a + b*x^8)) - ((b*c - 3*a*d)*Ar
cTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^
(5/4)*b^(1/4)*(b*c - a*d)^(3/2)) - ((b*c - 3*a*d)*ArcTan[(Sqrt[-(b*c) + a*d
]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(5/4)*b^(1/4)*(-(b*c
) + a*d)^(3/2)) + (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)
/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticE[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2]
)/(8*a*(b*c - a*d)*Sqrt[c + d*x^8]) - (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x
^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)
*x^2)/c^(1/4)], 1/2])/(16*a*(b*c - a*d)*Sqrt[c + d*x^8]) - ((Sqrt[c] - (Sqr
t[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[
(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(
```

```
1/4)], 1/2])/(32*a*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sqr
t[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]
*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/
4)*x^2)/c^(1/4)], 1/2])/(32*a*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^
8]) - ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(b*c - 3*a*d)*(Sqrt[c] + Sqrt
[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-1/4*(Sqrt[
b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcT
an[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*(-a)^(3/2)*Sqrt[b]*c^(1/4)*d^(1/4)*(b*
c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d
])^2*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt
[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*S
qrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*(-a)^(3
/2)*Sqrt[b]*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
```

```
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx, x, x^2\right)$$

$$= \frac{bx^6\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(-bc+4ad+bdx^4\right)}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8a(bc-ad)}$$

$$= \frac{bx^6\sqrt{c+dx^8}}{8a(bc-ad)\,(a+bx^8)} - \frac{\text{Subst}\left(\int \left(\frac{dx^2}{\sqrt{c+dx^4}} + \frac{(-bc+3ad)x^2}{(a+bx^4)\sqrt{c+dx^4}}\right)\,dx, x, x^2\right)}{8a(bc-ad)}$$

$$= \frac{bx^6\sqrt{c+dx^8}}{8a(bc-ad)\,(a+bx^8)} - \frac{d\text{Subst}\left(\int \frac{x^2}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8a(bc-ad)} + \frac{(bc-3ad)\text{Subst}\left(\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8a(bc-ad)}$$

$$= \frac{bx^6\sqrt{c+dx^8}}{8a(bc-ad)\,(a+bx^8)} - \frac{\left(\sqrt{c}\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8a(bc-ad)} + \frac{\left(\sqrt{c}\sqrt{d}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{d}x^2}{\sqrt{c}}}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8a(bc-ad)} - \frac{(bc-3ad)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{16a\sqrt{b}(bc-ad)} + \frac{(bc-3ad)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{16a\sqrt{b}(bc-ad)}$$

$$= -\frac{\sqrt{d}x^2\sqrt{c+dx^8}}{8a(bc-ad)\left(\sqrt{c}+\sqrt{d}x^4\right)} + \frac{bx^6\sqrt{c+dx^8}}{8a(bc-ad)\,(a+bx^8)} + \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a(bc-ad)\sqrt{c+dx^8}} - \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16a(bc-ad)\sqrt{c+dx^8}} - \frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(bc-3ad)\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{16a(bc-ad)(bc+ad)} + \frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(bc-3ad)\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{16a(bc-ad)(bc+ad)} - \frac{\left(\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt{d}(bc-3ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{16a\sqrt{b}(bc-ad)(bc+ad)} - \frac{\left(\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt{d}(bc-3ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{16a\sqrt{b}(bc-ad)(bc+ad)}$$

$$= -\frac{\sqrt{d}x^2\sqrt{c+dx^8}}{8a(bc-ad)\left(\sqrt{c}+\sqrt{d}x^4\right)} + \frac{bx^6\sqrt{c+dx^8}}{8a(bc-ad)(a+bx^8)}$$

$$- \frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{5/4}\sqrt[4]{b}(bc-ad)^{3/2}} - \frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{5/4}\sqrt[4]{b}(-bc+ad)^{3/2}}$$

$$+ \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a(bc-ad)\sqrt{c+dx^8}}$$

$$- \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16a(bc-ad)\sqrt{c+dx^8}}$$

$$- \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-3ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{32a\sqrt{b}\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$

$$- \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-3ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{32a\sqrt{b}\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$

$$- \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(bc-3ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{64(-a)^{3/2}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$

$$+ \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(bc-3ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right.\right.}{64(-a)^{3/2}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.20 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.15

$$\int \frac{x^5}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

$$= \frac{x^6\left(21ab(c+dx^8)+7(bc-4ad)(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)-3bdx^8(a+bx^8)\sqrt{\right.}{168a^2(bc-ad)(a+bx^8)\sqrt{c+dx^8}}$$

```
[In] Integrate[x^5/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^6*(21*a*b*(c + d*x^8) + 7*(b*c - 4*a*d)*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*
AppellF1[3/4, 1/2, 1, 7/4, -((d*x^8)/c), -((b*x^8)/a)] - 3*b*d*x^8*(a + b*x
```

```
^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^8)/c), -((b*x^8)
/a)]))/(168*a^2*(b*c - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

Maple [F]

$$\int \frac{x^5}{(b\,x^8+a)^2\,\sqrt{d\,x^8+c}}dx$$

```
[In] int(x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
[Out] int(x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+bx^8)^2\,\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

```
[In] integrate(x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^5}{(a+bx^8)^2\,\sqrt{c+dx^8}}\,dx = \int \frac{x^5}{(a+bx^8)^2\,\sqrt{c+dx^8}}\,dx$$

```
[In] integrate(x**5/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
[Out] Integral(x**5/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x^5}{(a+bx^8)^2\,\sqrt{c+dx^8}}\,dx = \int \frac{x^5}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^5/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Giac [F]

$$\int \frac{x^5}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^5}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^5/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^5}{(b\,x^8+a)^2\,\sqrt{d\,x^8+c}}\,dx$$

```
[In] int(x^5/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
[Out] int(x^5/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.923 $\int \frac{1}{x^3 (a+bx^8)^2 \sqrt{c+dx^8}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 1243

$$\int \frac{1}{x^3 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx$$
$$= -\frac{(5bc-4ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^2}+\frac{\sqrt{d}(5bc-4ad)x^2\sqrt{c+dx^8}}{8a^2c(bc-ad)\left(\sqrt{c}+\sqrt{d}x^4\right)}+\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^2(a+bx^8)}$$
$$-\frac{b^{3/4}(5bc-7ad)\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{9/4}(bc-ad)^{3/2}}-\frac{b^{3/4}(5bc-7ad)\arctan\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{9/4}(-bc+ad)^{3/2}}$$
$$-\frac{\sqrt[4]{d}(5bc-4ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a^2c^{3/4}(bc-ad)\sqrt{c+dx^8}}$$
$$+\frac{\sqrt[4]{d}(5bc-4ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a^2c^{3/4}(bc-ad)\sqrt{c+dx^8}}$$
$$+\frac{b\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(5bc-7ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32a^2\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$
$$+\frac{b\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(5bc-7ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32a^2\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$
$$-\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(5bc-7ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\right.}{64(-a)^{5/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$
$$+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(5bc-7ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{64(-a)^{5/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$

```
[Out] -1/32*b^(3/4)*(-7*a*d+5*b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)
/(d*x^8+c)^(1/2))/(-a)^(9/4)/(-a*d+b*c)^(3/2)-1/32*b^(3/4)*(-7*a*d+5*b*c)*a
rctan(x^2*(a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(9/4)/(a
*d-b*c)^(3/2)-1/8*(-4*a*d+5*b*c)*(d*x^8+c)^(1/2)/a^2/c/(-a*d+b*c)/x^2+1/8*b
*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/x^2/(b*x^8+a)+1/8*(-4*a*d+5*b*c)*x^2*d^(1/2)*
(d*x^8+c)^(1/2)/a^2/c/(-a*d+b*c)/(c^(1/2)+x^4*d^(1/2))-1/8*d^(1/4)*(-4*a*d+
5*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^
2/c^(1/4)))*EllipticE(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1
/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a^2/c^(3/4)/(-a*
```

```
d+b*c)/(d*x^8+c)^(1/2)+1/16*d^(1/4)*(-4*a*d+5*b*c)*(cos(2*arctan(d^(1/4)*x^
2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arc
tan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^
(1/2)+x^4*d^(1/2))^2)^(1/2)/a^2/c^(3/4)/(-a*d+b*c)/(d*x^8+c)^(1/2)+1/64*(-7
*a*d+5*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/
4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)
*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/
2))*b^(1/2)*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((
d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/(-a)^(5/2)/c^(1/4)/d^(1/4)/(-a*d+b*
c)/(a*d+b*c)/(d*x^8+c)^(1/2)-1/64*(-7*a*d+5*b*c)*(cos(2*arctan(d^(1/4)*x^2/
c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arct
an(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(
1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*b^(1/2)*(c^(1/2)+x^4*d^(1/2))*(b^
(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/
2)/(-a)^(5/2)/c^(1/4)/d^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^8+c)^(1/2)+1/32*b*d
^(1/4)*(-7*a*d+5*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*ar
ctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2
*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(c^(1/2)-(-a)^(1/2)*d^(1/2)/b^(1/2))*((d*x^
8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a^2/c^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^8
+c)^(1/2)+1/32*b*d^(1/4)*(-7*a*d+5*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))
^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)
*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(c^(1/2)+(-a)^(1/2)*d^(1/
2)/b^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a^2/c^(1/4)/(-a*d+b*c
)/(a*d+b*c)/(d*x^8+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 1243, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules

used = {476, 483, 597, 598, 311, 226, 1210, 504, 1231, 1721}

$$\int \frac{1}{x^3 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx$$

$$= \frac{\sqrt{b}(5bc-7ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\right.}{64(-a)^{5/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^8+c}}$$

$$-\frac{b^{3/4}(5bc-7ad)\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{32(-a)^{9/4}(bc-ad)^{3/2}}-\frac{b^{3/4}(5bc-7ad)\arctan\left(\frac{\sqrt{ad-bc}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{32(-a)^{9/4}(ad-bc)^{3/2}}$$

$$-\frac{\sqrt[4]{d}(5bc-4ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a^2c^{3/4}(bc-ad)\sqrt{dx^8+c}}$$

$$+\frac{\sqrt[4]{d}(5bc-4ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a^2c^{3/4}(bc-ad)\sqrt{dx^8+c}}$$

$$+\frac{b\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(5bc-7ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32a^2\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{dx^8+c}}$$

$$+\frac{b\left(\sqrt{c}+\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}(5bc-7ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{32a^2\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{dx^8+c}}$$

$$-\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(5bc-7ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\right.}{64(-a)^{5/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{dx^8+c}}$$

$$+\frac{\sqrt{d}(5bc-4ad)x^2\sqrt{dx^8+c}}{8a^2c(bc-ad)\left(\sqrt{d}x^4+\sqrt{c}\right)}+\frac{b\sqrt{dx^8+c}}{8a(bc-ad)x^2(bx^8+a)}-\frac{(5bc-4ad)\sqrt{dx^8+c}}{8a^2c(bc-ad)x^2}$$

```
[In] Int[1/(x^3*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/8*((5*b*c - 4*a*d)*Sqrt[c + d*x^8])/(a^2*c*(b*c - a*d)*x^2) + (Sqrt[d]*(
5*b*c - 4*a*d)*x^2*Sqrt[c + d*x^8])/(8*a^2*c*(b*c - a*d)*(Sqrt[c] + Sqrt[d]
*x^4)) + (b*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*x^2*(a + b*x^8)) - (b^(3/4)*(
5*b*c - 7*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*
x^8])])/(32*(-a)^(9/4)*(b*c - a*d)^(3/2)) - (b^(3/4)*(5*b*c - 7*a*d)*ArcTan
[(Sqrt[-(b*c) + a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(
9/4)*(-(b*c) + a*d)^(3/2)) - (d^(1/4)*(5*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^
4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticE[2*ArcTan[(d^(1/4)*
x^2)/c^(1/4)], 1/2])/(8*a^2*c^(3/4)*(b*c - a*d)*Sqrt[c + d*x^8]) + (d^(1/4)
```

```
*(5*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d
]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*a^2*c^(3/4)*
(b*c - a*d)*Sqrt[c + d*x^8]) + (b*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^
(1/4)*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + S
qrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(32*a^2*c^(
1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) + (b*(Sqrt[c] + (Sqrt[-a]*Sqr
t[d])/Sqrt[b])*d^(1/4)*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*
x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)],
1/2])/(32*a^2*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) - (Sqrt[b]*(
Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^
4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqr
t[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^
(1/4)*x^2)/c^(1/4)], 1/2])/(64*(-a)^(5/2)*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c
+ a*d)*Sqrt[c + d*x^8]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(
5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*
x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[
b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*(-a)^(5/2)*
c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 476

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
```

```
IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
```

```
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx, x, x^2\right)$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^2\left(a+bx^8\right)} - \frac{\text{Subst}\left(\int \frac{-5bc+4ad-3bdx^4}{x^2(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8a(bc-ad)}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^2\left(a+bx^8\right)} + \frac{\text{Subst}\left(\int \frac{x^2\left(-((bc-2ad)(5bc-2ad))+bd(5bc-4ad)x^4\right)}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8a^2c(bc-ad)}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^2\left(a+bx^8\right)} + \frac{\text{Subst}\left(\int \left(\frac{d(5bc-4ad)x^2}{\sqrt{c+dx^4}} + \frac{(-5b^2c^2+7abcd)x^2}{(a+bx^4)\sqrt{c+dx^4}}\right)\,dx, x, x^2\right)}{8a^2c(bc-ad)}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^2\left(a+bx^8\right)} - \frac{(b(5bc-7ad))\text{Subst}\left(\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8a^2(bc-ad)} + \frac{(d(5bc-4ad))\text{Subst}\left(\int \frac{x^2}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8a^2c(bc-ad)}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^2(a+bx^8)} + \frac{\left(\sqrt{b}(5bc-7ad)\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{16a^2(bc-ad)} - \frac{\left(\sqrt{b}(5bc-7ad)\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{16a^2(bc-ad)} + \frac{\left(\sqrt{d}(5bc-4ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8a^2\sqrt{c}(bc-ad)} - \frac{\left(\sqrt{d}(5bc-4ad)\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{d}x^2}{\sqrt{c}}}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8a^2\sqrt{c}(bc-ad)}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^2} + \frac{\sqrt{d}(5bc-4ad)x^2\sqrt{c+dx^8}}{8a^2c(bc-ad)\left(\sqrt{c}+\sqrt{d}x^4\right)} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^2(a+bx^8)} - \frac{\sqrt[4]{d}(5bc-4ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a^2c^{3/4}(bc-ad)\sqrt{c+dx^8}} + \frac{\sqrt[4]{d}(5bc-4ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16a^2c^{3/4}(bc-ad)\sqrt{c+dx^8}} + \frac{\left(b\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(5bc-7ad)\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{16a^2(bc-ad)(bc+ad)} - \frac{\left(b\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(5bc-7ad)\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{16a^2(bc-ad)(bc+ad)} + \frac{\left(\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt{d}(5bc-7ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{16a^2(bc-ad)(bc+ad)} + \frac{\left(\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt{d}(5bc-7ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{16a^2(bc-ad)(bc+ad)}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^2} + \frac{\sqrt{d}(5bc-4ad)x^2\sqrt{c+dx^8}}{8a^2c(bc-ad)\left(\sqrt{c}+\sqrt{d}x^4\right)}$$

$$+ \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^2\left(a+bx^8\right)} - \frac{b^{3/4}(5bc-7ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{9/4}(bc-ad)^{3/2}}$$

$$- \frac{b^{3/4}(5bc-7ad)\tan^{-1}\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{9/4}(-bc+ad)^{3/2}}$$

$$- \frac{\sqrt[4]{d}(5bc-4ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a^2c^{3/4}(bc-ad)\sqrt{c+dx^8}}$$

$$+ \frac{\sqrt[4]{d}(5bc-4ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16a^2c^{3/4}(bc-ad)\sqrt{c+dx^8}}$$

$$+ \frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(5bc-7ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{32a^2\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$

$$+ \frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(5bc-7ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{32a^2\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$

$$- \frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2(5bc-7ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\right.\right.}{64(-a)^{5/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$

$$+ \frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2(5bc-7ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\right.\right.}{64(-a)^{5/2}\sqrt[4]{c}\sqrt[4]{d}(bc-ad)(bc+ad)\sqrt{c+dx^8}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.31 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.18

$$\int \frac{1}{x^3\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$$

$$= \frac{21a(c+dx^8)\left(4a^2d-5b^2cx^8-4ab(c-dx^8)\right)-7\left(5b^2c^2-12abcd+4a^2d^2\right)x^8(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF}}{168a^3c(bc-ad)x^2(a}$$

```
[In] Integrate[1/(x^3*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

[Out] (21*a*(c + d*x^8)*(4*a^2*d - 5*b^2*c*x^8 - 4*a*b*(c - d*x^8)) - 7*(5*b^2*c^
2 - 12*a*b*c*d + 4*a^2*d^2)*x^8*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[3/
4, 1/2, 1, 7/4, -((d*x^8)/c), -((b*x^8)/a)] + 3*b*d*(5*b*c - 4*a*d)*x^16*(a
 + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^8)/c), -((
b*x^8)/a)])/(168*a^3*c*(b*c - a*d)*x^2*(a + b*x^8)*Sqrt[c + d*x^8])

Maple [F]

$$\int \frac{1}{x^{3} \left(b\,x^{8}+a \right)^{2} \sqrt{d\,x^{8}+c}}dx$$

[In] int(1/x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)

[Out] int(1/x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^8)^2 \sqrt{c+dx^8}} \, dx = \text{Timed out}$$

[In] integrate(1/x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{1}{x^3 (a+bx^8)^2 \sqrt{c+dx^8}} \, dx = \int \frac{1}{x^{3} \left(a + b x^{8}\right)^{2} \sqrt{c + d x^{8}}}\, dx$$

[In] integrate(1/x**3/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)

[Out] Integral(1/(x**3*(a + b*x**8)**2*sqrt(c + d*x**8)), x)

Maxima [F]

$$\int \frac{1}{x^3 (a+bx^8)^2 \sqrt{c+dx^8}} \, dx = \int { \frac{1}{{\left(b x^{8} + a\right)}^{2} \sqrt{d x^{8} + c} x^{3}} \,d x }$$

[In] integrate(1/x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^3), x)

Giac [F]

$$\int \frac{1}{x^3 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{1}{(bx^8+a)^2\sqrt{dx^8+c}x^3}\,dx$$

```
[In] integrate(1/x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{1}{x^3\,(b\,x^8+a)^2\,\sqrt{d\,x^8+c}}\,dx$$

```
[In] int(1/(x^3*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
[Out] int(1/(x^3*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.924 $\int \frac{x^4}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^4}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{x^5\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{5}{8},2,\frac{1}{2},\frac{13}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{5a^2\sqrt{c+dx^8}}$$

```
[Out] 1/5*x^5*AppellF1(5/8,2,1/2,13/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a^2/(d
*x^8+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{x^4}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{x^5\sqrt{\frac{dx^8}{c}+1}\,\text{AppellF1}\left(\frac{5}{8},2,\frac{1}{2},\frac{13}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{5a^2\sqrt{c+dx^8}}$$

```
[In] Int[x^4/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^5*Sqrt[1 + (d*x^8)/c]*AppellF1[5/8, 2, 1/2, 13/8, -((b*x^8)/a), -((d*x^8
)/c)])/(5*a^2*Sqrt[c + d*x^8])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{x^4}{(a+bx^8)^2\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}$$

$$= \frac{x^5\sqrt{1+\frac{dx^8}{c}}F_1\left(\frac{5}{8};2,\frac{1}{2};\frac{13}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{5a^2\sqrt{c+dx^8}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 170 vs. 2(64) = 128.

Time = 10.22 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.66

$$\int \frac{x^4}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

$$= \frac{x^5\left(65ab(c+dx^8)+13(3bc-8ad)\left(a+bx^8\right)\sqrt{1+\frac{dx^8}{c}}\text{AppellF1}\left(\frac{5}{8},\frac{1}{2},1,\frac{13}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)-5bdx^8(a+bx\right.}{520a^2(bc-ad)\left(a+bx^8\right)\sqrt{c+dx^8}}$$

```
[In] Integrate[x^4/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^5*(65*a*b*(c + d*x^8) + 13*(3*b*c - 8*a*d)*(a + b*x^8)*Sqrt[1 + (d*x^8)/
c]*AppellF1[5/8, 1/2, 1, 13/8, -((d*x^8)/c), -((b*x^8)/a)] - 5*b*d*x^8*(a +
 b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[13/8, 1/2, 1, 21/8, -((d*x^8)/c), -((b
*x^8)/a)]))/(520*a^2*(b*c - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

Maple [F]

$$\int \frac{x^4}{\left(b\,x^8+a\right)^2 \sqrt{d\,x^8+c}}dx$$

```
[In] int(x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
[Out] int(x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \text{Timed out}$$

```
[In] integrate(x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^4}{(a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{x^4}{\left(a+bx^8\right)^2 \sqrt{c+dx^8}}\,dx$$

```
[In] integrate(x**4/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
[Out] Integral(x**4/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{x^4}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^4/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Giac [F]

$$\int \frac{x^4}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^4}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^4}{\left(b\,x^8+a\right)^2\,\sqrt{d\,x^8+c}}\,dx$$

```
[In] int(x^4/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
[Out] int(x^4/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.925 $\int \frac{x^2}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^2}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{x^3\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{3}{8},2,\frac{1}{2},\frac{11}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a^2\sqrt{c+dx^8}}$$

```
[Out] 1/3*x^3*AppellF1(3/8,2,1/2,11/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a^2/(d
*x^8+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{x^2}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{x^3\sqrt{\frac{dx^8}{c}+1}\,\text{AppellF1}\left(\frac{3}{8},2,\frac{1}{2},\frac{11}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a^2\sqrt{c+dx^8}}$$

```
[In] Int[x^2/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^3*Sqrt[1 + (d*x^8)/c]*AppellF1[3/8, 2, 1/2, 11/8, -((b*x^8)/a), -((d*x^8
)/c)])/(3*a^2*Sqrt[c + d*x^8])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{x^2}{(a+bx^8)^2\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}$$

$$= \frac{x^3\sqrt{1+\frac{dx^8}{c}}F_1\left(\frac{3}{8};2,\frac{1}{2};\frac{11}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a^2\sqrt{c+dx^8}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 170 vs. $2(64) = 128$.

Time = 10.19 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.66

$$\int \frac{x^2}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

$$= \frac{x^3\left(33ab(c+dx^8)+11(5bc-8ad)\,(a+bx^8)\,\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{3}{8},\frac{1}{2},1,\frac{11}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+3bdx^8(a+bx\right.}{264a^2(bc-ad)\,(a+bx^8)\,\sqrt{c+dx^8}}$$

```
[In] Integrate[x^2/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^3*(33*a*b*(c + d*x^8) + 11*(5*b*c - 8*a*d)*(a + b*x^8)*Sqrt[1 + (d*x^8)/
c]*AppellF1[3/8, 1/2, 1, 11/8, -((d*x^8)/c), -((b*x^8)/a)] + 3*b*d*x^8*(a +
 b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[11/8, 1/2, 1, 19/8, -((d*x^8)/c), -((b
*x^8)/a)]))/(264*a^2*(b*c - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

Maple [F]

$$\int \frac{x^2}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}}dx$$

[In] int(x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)

[Out] int(x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

[In] integrate(x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{x^2}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^2}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

[In] integrate(x**2/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)

[Out] Integral(x**2/((a + b*x**8)**2*sqrt(c + d*x**8)), x)

Maxima [F]

$$\int \frac{x^2}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^2}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

[In] integrate(x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")

[Out] integrate(x^2/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)

Giac [F]

$$\int \frac{x^2}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^2}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^2}{(b\,x^8+a)^2\,\sqrt{d\,x^8+c}}\,dx$$

```
[In] int(x^2/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
[Out] int(x^2/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.926 $\int \frac{1}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{x\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{1}{8},2,\frac{1}{2},\frac{9}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a^2\sqrt{c+dx^8}}$$

```
[Out] x*AppellF1(1/8,2,1/2,9/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a^2/(d*x^8+c)
^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {441, 440}

$$\int \frac{1}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{x\sqrt{\frac{dx^8}{c}+1}\,\text{AppellF1}\left(\frac{1}{8},2,\frac{1}{2},\frac{9}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a^2\sqrt{c+dx^8}}$$

```
[In] Int[1/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x*Sqrt[1 + (d*x^8)/c]*AppellF1[1/8, 2, 1/2, 9/8, -((b*x^8)/a), -((d*x^8)/c
)])/(a^2*Sqrt[c + d*x^8])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{1}{(a+bx^8)^2\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}$$

$$= \frac{x\sqrt{1+\frac{dx^8}{c}}F_1\left(\frac{1}{8};2,\frac{1}{2};\frac{9}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a^2\sqrt{c+dx^8}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 328 vs. 2(59) = 118.

Time = 10.30 (sec) , antiderivative size = 328, normalized size of antiderivative = 5.56

$$\int \frac{1}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{x\left(bdx^8\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{9}{8},\frac{1}{2},1,\frac{17}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+\frac{3a\left(9ac\left(8ad-b\left(8c+dx^8\right)\right)\text{AppellF1}\left(\frac{1}{8},\frac{1}{2},1,\frac{9}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+4bx^8\left(c+dx^8\right)\left(2bc\,\text{AppellF1}\left(\frac{9}{8},\frac{1}{2},2,\frac{17}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+ad\,\text{AppellF1}\left(\frac{9}{8},\frac{3}{2},1,\frac{17}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)\right)\right)}{(a+bx^8)\left(-9ac\,\text{AppellF1}\left(\frac{1}{8},\frac{1}{2},1,\frac{9}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+4x^8\left(2bc\,\text{AppellF1}\left(\frac{9}{8},\frac{1}{2},2,\frac{17}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+ad\,\text{AppellF1}\left(\frac{9}{8},\frac{3}{2},1,\frac{17}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)\right)\right)}\right)}{24a^2(-bc+ad)\sqrt{c+dx^8}}$$

```
[In] Integrate[1/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/24*(x*(b*d*x^8*Sqrt[1 + (d*x^8)/c]*AppellF1[9/8, 1/2, 1, 17/8, -((d*x^8)
/c), -((b*x^8)/a)] + (3*a*(9*a*c*(8*a*d - b*(8*c + d*x^8))*AppellF1[1/8, 1/
2, 1, 9/8, -((d*x^8)/c), -((b*x^8)/a)] + 4*b*x^8*(c + d*x^8)*(2*b*c*AppellF
1[9/8, 1/2, 2, 17/8, -((d*x^8)/c), -((b*x^8)/a)] + a*d*AppellF1[9/8, 3/2, 1
, 17/8, -((d*x^8)/c), -((b*x^8)/a)])))/((a + b*x^8)*(-9*a*c*AppellF1[1/8, 1
/2, 1, 9/8, -((d*x^8)/c), -((b*x^8)/a)] + 4*x^8*(2*b*c*AppellF1[9/8, 1/2, 2
, 17/8, -((d*x^8)/c), -((b*x^8)/a)] + a*d*AppellF1[9/8, 3/2, 1, 17/8, -((d*
x^8)/c), -((b*x^8)/a)])))))/(a^2*(-(b*c) + a*d)*Sqrt[c + d*x^8])
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{8}+a\right)^{2} \sqrt{d\,x^{8}+c}}dx$$

```
[In] int(1/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
[Out] int(1/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{1}{(a+bx^8)^2 \sqrt{c+dx^8}}\,dx$$

```
[In] integrate(1/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
[Out] Integral(1/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{1}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(1/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{1}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

```
[In] integrate(1/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{1}{(b\,x^8+a)^2\,\sqrt{d\,x^8+c}}\,dx$$

```
[In] int(1/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
[Out] int(1/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.927 $\int \frac{1}{x^2(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{1}{x^2 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = -\frac{\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(-\frac{1}{8},2,\frac{1}{2},\frac{7}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a^2x\sqrt{c+dx^8}}$$

```
[Out] -AppellF1(-1/8,2,1/2,7/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a^2/x/(d*x^8+
c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{1}{x^2 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = -\frac{\sqrt{\frac{dx^8}{c}+1}\,\text{AppellF1}\left(-\frac{1}{8},2,\frac{1}{2},\frac{7}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a^2x\sqrt{c+dx^8}}$$

```
[In] Int[1/(x^2*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -((Sqrt[1 + (d*x^8)/c]*AppellF1[-1/8, 2, 1/2, 7/8, -((b*x^8)/a), -((d*x^8)/
c)])/(a^2*x*Sqrt[c + d*x^8]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{1}{x^2(a+bx^8)^2\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}$$
$$= -\frac{\sqrt{1+\frac{dx^8}{c}}F_1\left(-\frac{1}{8};2,\frac{1}{2};\frac{7}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a^2x\sqrt{c+dx^8}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 226 vs. $2(62) = 124$.

Time = 10.33 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.65

$$\int \frac{1}{x^2\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$$
$$= \frac{35a(c+dx^8)\left(8a^2d-9b^2cx^8-8ab(c-dx^8)\right)-5(9b^2c^2-40abcd+24a^2d^2)\,x^8(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,\text{Appell}}{280a^3c(bc-ad)x\,(a}$$

```
[In] Integrate[1/(x^2*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (35*a*(c + d*x^8)*(8*a^2*d - 9*b^2*c*x^8 - 8*a*b*(c - d*x^8)) - 5*(9*b^2*c^
2 - 40*a*b*c*d + 24*a^2*d^2)*x^8*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[7
/8, 1/2, 1, 15/8, -((d*x^8)/c), -((b*x^8)/a)] + 7*b*d*(9*b*c - 8*a*d)*x^16*
(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[15/8, 1/2, 1, 23/8, -((d*x^8)/c),
-((b*x^8)/a)])/(280*a^3*c*(b*c - a*d)*x*(a + b*x^8)*Sqrt[c + d*x^8])
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b\,x^{8}+a \right)^{2} \sqrt{d\,x^{8}+c}}d x$$

[In] int(1/x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)

[Out] int(1/x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)

Fricas [F]

$$\int \frac{1}{x^2 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}\, dx = \int \frac{1}{{\left(bx^{8} + a\right)}^{2} \sqrt{dx^{8} + c}x^{2}}\, dx$$

[In] integrate(1/x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(d*x^8 + c)/(b^2*d*x^26 + (b^2*c + 2*a*b*d)*x^18 + (2*a*b*c + a^2*d)*x^10 + a^2*c*x^2), x)

Sympy [F]

$$\int \frac{1}{x^2 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}\, dx = \int \frac{1}{x^{2} \left(a + bx^{8}\right)^{2} \sqrt{c + dx^{8}}}\, dx$$

[In] integrate(1/x**2/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)

[Out] Integral(1/(x**2*(a + b*x**8)**2*sqrt(c + d*x**8)), x)

Maxima [F]

$$\int \frac{1}{x^2 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}\, dx = \int \frac{1}{{\left(bx^{8} + a\right)}^{2} \sqrt{dx^{8} + c}x^{2}}\, dx$$

[In] integrate(1/x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^2), x)

Giac [F]

$$\int \frac{1}{x^2 (a+bx^8)^2 \sqrt{c+dx^8}} \, dx = \int \frac{1}{(bx^8+a)^2 \sqrt{dx^8+c} x^2} \, dx$$

```
[In] integrate(1/x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^8)^2 \sqrt{c+dx^8}} \, dx = \int \frac{1}{x^2 (b\,x^8+a)^2 \sqrt{d\,x^8+c}} \, dx$$

```
[In] int(1/(x^2*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
[Out] int(1/(x^2*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.928 $\int \frac{1}{x^4(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{x^4\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=-\frac{\sqrt{1+\frac{dx^8}{c}}\,\mathrm{AppellF1}\left(-\frac{3}{8},2,\frac{1}{2},\frac{5}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a^2x^3\sqrt{c+dx^8}}$$

```
[Out] -1/3*AppellF1(-3/8,2,1/2,5/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a^2/x^3/(
d*x^8+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {525, 524}

$$\int \frac{1}{x^4\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=-\frac{\sqrt{\frac{dx^8}{c}+1}\,\mathrm{AppellF1}\left(-\frac{3}{8},2,\frac{1}{2},\frac{5}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a^2x^3\sqrt{c+dx^8}}$$

```
[In] Int[1/(x^4*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/3*(Sqrt[1 + (d*x^8)/c]*AppellF1[-3/8, 2, 1/2, 5/8, -((b*x^8)/a), -((d*x^
8)/c)])/(a^2*x^3*Sqrt[c + d*x^8])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^8}{c}}\int\frac{1}{x^4(a+bx^8)^2\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}$$

$$= -\frac{\sqrt{1+\frac{dx^8}{c}}F_1\left(-\frac{3}{8};2,\frac{1}{2};\frac{5}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a^2x^3\sqrt{c+dx^8}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 226 vs. $2(64) = 128$.

Time = 10.32 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.53

$$\int\frac{1}{x^4\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$$
$$= \frac{65a(c+dx^8)\left(8a^2d-11b^2cx^8-8ab(c-dx^8)\right)-13(33b^2c^2-56abcd+8a^2d^2)\,x^8(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,\text{Appe}}{1560a^3c(bc-ad)x^3}$$

```
[In] Integrate[1/(x^4*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (65*a*(c + d*x^8)*(8*a^2*d - 11*b^2*c*x^8 - 8*a*b*(c - d*x^8)) - 13*(33*b^2
*c^2 - 56*a*b*c*d + 8*a^2*d^2)*x^8*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1
[5/8, 1/2, 1, 13/8, -((d*x^8)/c), -((b*x^8)/a)] + 5*b*d*(11*b*c - 8*a*d)*x^
16*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[13/8, 1/2, 1, 21/8, -((d*x^8)/c
), -((b*x^8)/a)])/(1560*a^3*c*(b*c - a*d)*x^3*(a + b*x^8)*Sqrt[c + d*x^8])
```

Maple [F]

$$\int \frac{1}{x^{4} \left(b \,x^{8}+a \right)^{2} \sqrt{d \,x^{8}+c}}d x$$

```
[In] int(1/x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
[Out] int(1/x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^4 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{1}{x^{4} \left(a + b x^{8}\right)^{2} \sqrt{c + d x^{8}}}\, dx$$

```
[In] integrate(1/x**4/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
[Out] Integral(1/(x**4*(a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{1}{{\left(b x^{8} + a\right)}^{2} \sqrt{d x^{8} + c} x^{4}} \,d x$$

```
[In] integrate(1/x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a+bx^8)^2 \sqrt{c+dx^8}} \, dx = \int \frac{1}{(bx^8+a)^2 \sqrt{dx^8+c} x^4} \, dx$$

```
[In] integrate(1/x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^8)^2 \sqrt{c+dx^8}} \, dx = \int \frac{1}{x^4 (b\,x^8+a)^2 \sqrt{d\,x^8+c}} \, dx$$

```
[In] int(1/(x^4*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
[Out] int(1/(x^4*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

3.929 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}x^5 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 123

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}x^5 \, dx=\frac{d(2bc-ad)\sqrt{c+\frac{d}{x^2}}x^2}{16c^2}+\frac{(2bc-ad)\sqrt{c+\frac{d}{x^2}}x^4}{8c}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^6}{6c}-\frac{d^2(2bc-ad)\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{16c^{5/2}}$$

```
[Out] 1/6*a*(c+d/x^2)^(3/2)*x^6/c-1/16*d^2*(-a*d+2*b*c)*arctanh((c+d/x^2)^(1/2)/c
^(1/2))/c^(5/2)+1/16*d*(-a*d+2*b*c)*x^2*(c+d/x^2)^(1/2)/c^2+1/8*(-a*d+2*b*c
)*x^4*(c+d/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {457, 79, 43, 44, 65, 214}

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}x^5 \, dx=-\frac{d^2(2bc-ad)\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{16c^{5/2}}+\frac{dx^2\sqrt{c+\frac{d}{x^2}}(2bc-ad)}{16c^2}+\frac{x^4\sqrt{c+\frac{d}{x^2}}(2bc-ad)}{8c}+\frac{ax^6\left(c+\frac{d}{x^2}\right)^{3/2}}{6c}$$

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^5,x]
```

```
[Out] (d*(2*b*c - a*d)*Sqrt[c + d/x^2]*x^2)/(16*c^2) + ((2*b*c - a*d)*Sqrt[c + d/
x^2]*x^4)/(8*c) + (a*(c + d/x^2)^(3/2)*x^6)/(6*c) - (d^2*(2*b*c - a*d)*ArcT
anh[Sqrt[c + d/x^2]/Sqrt[c]])/(16*c^(5/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)\sqrt{c+dx}}{x^4}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^6}{6c} - \frac{\left(3bc-\frac{3ad}{2}\right)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^3}\,dx, x, \frac{1}{x^2}\right)}{6c}$$

$$= \frac{(2bc-ad)\sqrt{c+\frac{d}{x^2}}x^4}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^6}{6c} - \frac{(d(2bc-ad))\text{Subst}\left(\int \frac{1}{x^2\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)}{16c}$$

$$= \frac{d(2bc-ad)\sqrt{c+\frac{d}{x^2}}x^2}{16c^2} + \frac{(2bc-ad)\sqrt{c+\frac{d}{x^2}}x^4}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^6}{6c} + \frac{(d^2(2bc-ad))\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)}{32c^2}$$

$$= \frac{d(2bc-ad)\sqrt{c+\frac{d}{x^2}}x^2}{16c^2} + \frac{(2bc-ad)\sqrt{c+\frac{d}{x^2}}x^4}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^6}{6c} + \frac{(d(2bc-ad))\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{16c^2}$$

$$= \frac{d(2bc-ad)\sqrt{c+\frac{d}{x^2}}x^2}{16c^2} + \frac{(2bc-ad)\sqrt{c+\frac{d}{x^2}}x^4}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^6}{6c} - \frac{d^2(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{16c^{5/2}}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.96

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^5\,dx = \frac{\sqrt{c+\frac{d}{x^2}}x\left(\sqrt{c}x(6bc(d+2cx^2)+a(-3d^2+2cdx^2+8c^2x^4)) + \frac{6d^2(-2bc+ad)\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{d}+\sqrt{d+cx^2}}\right)}{\sqrt{d+cx^2}}\right)}{48c^{5/2}}$$

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^5,x]
[Out] (Sqrt[c + d/x^2]*x*(Sqrt[c]*x*(6*b*c*(d + 2*c*x^2) + a*(-3*d^2 + 2*c*d*x^2
+ 8*c^2*x^4)) + (6*d^2*(-2*b*c + a*d)*ArcTanh[(Sqrt[c]*x)/(-Sqrt[d] + Sqrt[
d + c*x^2])])/Sqrt[d + c*x^2]))/(48*c^(5/2))
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

method	result
risch	$\frac{x^{2}\left(8a\,x^{4}c^{2}+2acd\,x^{2}+12b\,c^{2}x^{2}-3a\,d^{2}+6bcd\right)\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}{48c^{2}}+\frac{d^{2}\left(ad-2bc\right)\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}{16c^{\frac{5}{2}}\sqrt{c\,x^{2}+d}}$
default	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x\left(8\left(c\,x^{2}+d\right)^{\frac{3}{2}}c^{\frac{3}{2}}a\,x^{3}-6\left(c\,x^{2}+d\right)^{\frac{3}{2}}\sqrt{c}\,adx+12\left(c\,x^{2}+d\right)^{\frac{3}{2}}c^{\frac{3}{2}}bx+3\sqrt{c\,x^{2}+d}\,\sqrt{c}\,a\,d^{2}x-6\sqrt{c\,x^{2}+d}\,c^{\frac{3}{2}}bdx+3\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}\right.}}{48\sqrt{c\,x^{2}+d}\,c^{\frac{5}{2}}}$

[In] int((a+b/x^2)*x^5*(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/48*x^2*(8*a*c^2*x^4+2*a*c*d*x^2+12*b*c^2*x^2-3*a*d^2+6*b*c*d)/c^2*((c*x^2+d)/x^2)^(1/2)+1/16*d^2*(a*d-2*b*c)/c^(5/2)*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*((c*x^2+d)/x^2)^(1/2)*x/(c*x^2+d)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.97

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^5\,dx$$
$$=\left[-\frac{3\left(2\,bcd^2-ad^3\right)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\left(8\,ac^3x^6+2\left(6\,bc^3+ac^2d\right)x^4+3\left(2\,bc^2d-\right.\right.}{96\,c^3}\right.$$

[In] integrate((a+b/x^2)*x^5*(c+d/x^2)^(1/2),x, algorithm="fricas")

[Out] [-1/96*(3*(2*b*c*d^2 - a*d^3)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d) - 2*(8*a*c^3*x^6 + 2*(6*b*c^3 + a*c^2*d)*x^4 + 3*(2*b*c^2*d - a*c*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/c^3, 1/48*(3*(2*b*c*d^2 - a*d^3)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (8*a*c^3*x^6 + 2*(6*b*c^3 + a*c^2*d)*x^4 + 3*(2*b*c^2*d - a*c*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/c^3]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(107) = 214.

Time = 24.90 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.84

$$\int\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^5\,dx=\frac{acx^7}{6\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{5a\sqrt{d}x^5}{24\sqrt{\frac{cx^2}{d}+1}}-\frac{ad^{\frac{3}{2}}x^3}{48c\sqrt{\frac{cx^2}{d}+1}}-\frac{ad^{\frac{5}{2}}x}{16c^2\sqrt{\frac{cx^2}{d}+1}}+\frac{ad^3\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{16c^{\frac{5}{2}}}+\frac{bcx^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{3b\sqrt{d}x^3}{8\sqrt{\frac{cx^2}{d}+1}}+\frac{bd^{\frac{3}{2}}x}{8c\sqrt{\frac{cx^2}{d}+1}}-\frac{bd^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{8c^{\frac{3}{2}}}$$

```
[In] integrate((a+b/x**2)*x**5*(c+d/x**2)**(1/2),x)
```

```
[Out] a*c*x**7/(6*sqrt(d)*sqrt(c*x**2/d + 1)) + 5*a*sqrt(d)*x**5/(24*sqrt(c*x**2/
d + 1)) - a*d**(3/2)*x**3/(48*c*sqrt(c*x**2/d + 1)) - a*d**(5/2)*x/(16*c**2
*sqrt(c*x**2/d + 1)) + a*d**3*asinh(sqrt(c)*x/sqrt(d))/(16*c**(5/2)) + b*c*
x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*b*sqrt(d)*x**3/(8*sqrt(c*x**2/d + 1
)) + b*d**(3/2)*x/(8*c*sqrt(c*x**2/d + 1)) - b*d**2*asinh(sqrt(c)*x/sqrt(d)
)/(8*c**(3/2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(103) = 206.

Time = 0.29 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.98

$$\int\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^5\,dx=-\frac{1}{96}\left(\frac{3\,d^3\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^3-8\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}cd^3-3\sqrt{c+\frac{d}{x^2}}c^2d^3\right)}{\left(c+\frac{d}{x^2}\right)^3c^2-3\left(c+\frac{d}{x^2}\right)^2c^3+3\left(c+\frac{d}{x^2}\right)c^4-c^5}\right)a+\frac{1}{16}\left(\frac{d^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2+\sqrt{c+\frac{d}{x^2}}cd^2\right)}{\left(c+\frac{d}{x^2}\right)^2c-2\left(c+\frac{d}{x^2}\right)c^2+c^3}\right)b$$

```
[In] integrate((a+b/x^2)*x^5*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/96*(3*d^3*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c
^(5/2) + 2*(3*(c + d/x^2)^(5/2)*d^3 - 8*(c + d/x^2)^(3/2)*c*d^3 - 3*sqrt(c
+ d/x^2)*c^2*d^3)/((c + d/x^2)^3*c^2 - 3*(c + d/x^2)^2*c^3 + 3*(c + d/x^2)*
c^4 - c^5))*a + 1/16*(d^2*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2)
+ sqrt(c)))/c^(3/2) + 2*((c + d/x^2)^(3/2)*d^2 + sqrt(c + d/x^2)*c*d^2)/((c
 + d/x^2)^2*c - 2*(c + d/x^2)*c^2 + c^3))*b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.16

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^5\,dx$$
$$=\frac{1}{48}\left(2\left(4\,ax^2\mathrm{sgn}(x)+\frac{6\,bc^4\mathrm{sgn}(x)+ac^3d\mathrm{sgn}(x)}{c^4}\right)x^2+\frac{3\,(2\,bc^3d\mathrm{sgn}(x)-ac^2d^2\mathrm{sgn}(x))}{c^4}\right)\sqrt{cx^2+d}x$$
$$+\frac{(2\,bcd^2\mathrm{sgn}(x)-ad^3\mathrm{sgn}(x))\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+d}\right|\right)}{16\,c^{\frac{5}{2}}}$$
$$-\frac{(2\,bcd^2\log\left(|d|\right)-ad^3\log\left(|d|\right))\mathrm{sgn}(x)}{32\,c^{\frac{5}{2}}}$$

```
[In] integrate((a+b/x^2)*x^5*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*a*x^2*sgn(x) + (6*b*c^4*sgn(x) + a*c^3*d*sgn(x))/c^4)*x^2 + 3*(2
*b*c^3*d*sgn(x) - a*c^2*d^2*sgn(x))/c^4)*sqrt(c*x^2 + d)*x + 1/16*(2*b*c*d^
2*sgn(x) - a*d^3*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)))/c^(5/2) - 1
/32*(2*b*c*d^2*log(abs(d)) - a*d^3*log(abs(d)))*sgn(x)/c^(5/2)
```

Mupad [B] (verification not implemented)

Time = 10.01 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.09

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^5\,dx=\frac{a\,x^6\,\sqrt{c+\frac{d}{x^2}}}{16}+\frac{b\,x^4\,\sqrt{c+\frac{d}{x^2}}}{8}+\frac{a\,x^6\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{6\,c}-\frac{a\,x^6\,{\left(c+\frac{d}{x^2}\right)}^{5/2}}{16\,c^2}+\frac{b\,x^4\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{8\,c}-\frac{b\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,c^{3/2}}-\frac{a\,d^3\,\mathrm{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}\,1{}\mathrm{i}}{\sqrt{c}}\right)\,1{}\mathrm{i}}{16\,c^{5/2}}$$

```
[In] int(x^5*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] (a*x^6*(c + d/x^2)^(1/2))/16 + (b*x^4*(c + d/x^2)^(1/2))/8 + (a*x^6*(c + d/
x^2)^(3/2))/(6*c) - (a*x^6*(c + d/x^2)^(5/2))/(16*c^2) + (b*x^4*(c + d/x^2)
^(3/2))/(8*c) - (a*d^3*atan(((c + d/x^2)^(1/2)*1i)/c^(1/2))*1i)/(16*c^(5/2)
) - (b*d^2*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(8*c^(3/2))
```

3.930 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^3 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^3 \, dx = \frac{(4bc-ad)\sqrt{c+\frac{d}{x^2}}x^2}{8c}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{4c} + \frac{d(4bc-ad)\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{3/2}}$$

```
[Out] 1/4*a*(c+d/x^2)^(3/2)*x^4/c+1/8*d*(-a*d+4*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1
/2))/c^(3/2)+1/8*(-a*d+4*b*c)*x^2*(c+d/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {457, 79, 43, 65, 214}

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^3 \, dx = \frac{d(4bc-ad)\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{3/2}} + \frac{x^2\sqrt{c+\frac{d}{x^2}}(4bc-ad)}{8c}+\frac{ax^4\left(c+\frac{d}{x^2}\right)^{3/2}}{4c}$$

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^3,x]
```

```
[Out] ((4*b*c - a*d)*Sqrt[c + d/x^2]*x^2)/(8*c) + (a*(c + d/x^2)^(3/2)*x^4)/(4*c)
 + (d*(4*b*c - a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(8*c^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)\sqrt{c+dx}}{x^3}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{4c} - \frac{\left(2bc-\frac{ad}{2}\right)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2}\,dx, x, \frac{1}{x^2}\right)}{4c}$$

$$= \frac{(4bc-ad)\sqrt{c+\frac{d}{x^2}}x^2}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{4c} - \frac{(d(4bc-ad))\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)}{16c}$$

$$=\frac{(4bc-ad)\sqrt{c+\frac{d}{x^2}}x^2}{8c}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{4c}-\frac{(4bc-ad)\text{Subst}\left(\int\frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+\frac{d}{x^2}}\right)}{8c}$$

$$=\frac{(4bc-ad)\sqrt{c+\frac{d}{x^2}}x^2}{8c}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{4c}+\frac{d(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{3/2}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08

$$\int\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^3\,dx$$
$$=\frac{\sqrt{c+\frac{d}{x^2}}x\left(\sqrt{c}x\sqrt{d+cx^2}(4bc+a(d+2cx^2))+d(-4bc+ad)\log\left(-\sqrt{c}x+\sqrt{d+cx^2}\right)\right)}{8c^{3/2}\sqrt{d+cx^2}}$$

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^3,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(Sqrt[c]*x*Sqrt[d + c*x^2]*(4*b*c + a*(d + 2*c*x^2)) + d
*(-4*b*c + a*d)*Log[-(Sqrt[c]*x) + Sqrt[d + c*x^2]]))/(8*c^(3/2)*Sqrt[d + c
*x^2])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.01

method	result	size
risch	$\frac{x^2(2acx^2+ad+4bc)\sqrt{\frac{cx^2+d}{x^2}}}{8c}-\frac{d(ad-4bc)\ln\left(\sqrt{c}x+\sqrt{cx^2+d}\right)\sqrt{\frac{cx^2+d}{x^2}}\,x}{8c^{\frac{3}{2}}\sqrt{cx^2+d}}$	91
default	$\frac{\sqrt{\frac{cx^2+d}{x^2}}\,x\left(2(cx^2+d)^{\frac{3}{2}}\sqrt{c}\,ax-\sqrt{cx^2+d}\,\sqrt{c}\,adx+4\sqrt{cx^2+d}\,c^{\frac{3}{2}}bx-\ln\left(\sqrt{c}x+\sqrt{cx^2+d}\right)a\,d^2+4\ln\left(\sqrt{c}x+\sqrt{cx^2+d}\right)bcd\right)}{8\sqrt{cx^2+d}\,c^{\frac{3}{2}}}$	122

```
[In] int((a+b/x^2)*x^3*(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x^2*(2*a*c*x^2+a*d+4*b*c)/c*((c*x^2+d)/x^2)^(1/2)-1/8*d*(a*d-4*b*c)/c^(
3/2)*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*((c*x^2+d)/x^2)^(1/2)*x/(c*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.12

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^3\,dx = \left[-\frac{(4\,bcd-ad^2)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\,(2\,ac^2x^4+(4\,bc^2+acd)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{16\,c^2},\ -\frac{(4\,bcd-ad^2)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)-(2\,ac^2x^4+(4\,bc^2+acd)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{8\,c^2}\right]$$

```
[In] integrate((a+b/x^2)*x^3*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((4*b*c*d - a*d^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c)*x^2*sqrt((c*x^2
+ d)/x^2) - d) - 2*(2*a*c^2*x^4 + (4*b*c^2 + a*c*d)*x^2)*sqrt((c*x^2 + d)/x
^2))/c^2, -1/8*((4*b*c*d - a*d^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2
+ d)/x^2)/(c*x^2 + d)) - (2*a*c^2*x^4 + (4*b*c^2 + a*c*d)*x^2)*sqrt((c*x^2
+ d)/x^2))/c^2]
```

Sympy [A] (verification not implemented)

Time = 17.85 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.60

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^3\,dx = \frac{acx^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{3a\sqrt{d}x^3}{8\sqrt{\frac{cx^2}{d}+1}}+\frac{ad^{\frac{3}{2}}x}{8c\sqrt{\frac{cx^2}{d}+1}}-\frac{ad^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{8c^{\frac{3}{2}}}+\frac{b\sqrt{d}x\sqrt{\frac{cx^2}{d}+1}}{2}+\frac{bd\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2\sqrt{c}}$$

```
[In] integrate((a+b/x**2)*x**3*(c+d/x**2)**(1/2),x)
```

```
[Out] a*c*x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*a*sqrt(d)*x**3/(8*sqrt(c*x**2/d
 + 1)) + a*d**(3/2)*x/(8*c*sqrt(c*x**2/d + 1)) - a*d**2*asinh(sqrt(c)*x/sqr
t(d))/(8*c**(3/2)) + b*sqrt(d)*x*sqrt(c*x**2/d + 1)/2 + b*d*asinh(sqrt(c)*x
/sqrt(d))/(2*sqrt(c))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(74) = 148.

Time = 0.30 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.77

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^3\,dx = \frac{1}{16}\left(\frac{d^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2+\sqrt{c+\frac{d}{x^2}}cd^2\right)}{\left(c+\frac{d}{x^2}\right)^2c-2\left(c+\frac{d}{x^2}\right)c^2+c^3}\right)a + \frac{1}{4}\left(2\sqrt{c+\frac{d}{x^2}}x^2-\frac{d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{\sqrt{c}}\right)b$$

```
[In] integrate((a+b/x^2)*x^3*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(d^2*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(3
/2) + 2*((c + d/x^2)^(3/2)*d^2 + sqrt(c + d/x^2)*c*d^2)/((c + d/x^2)^2*c -
2*(c + d/x^2)*c^2 + c^3))*a + 1/4*(2*sqrt(c + d/x^2)*x^2 - d*log((sqrt(c +
d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/sqrt(c))*b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.17

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^3\,dx = \frac{1}{8}\left(2\,ax^2\mathrm{sgn}(x)+\frac{4\,bc^2\mathrm{sgn}(x)+acd\mathrm{sgn}(x)}{c^2}\right)\sqrt{cx^2+d}x - \frac{(4\,bcd\mathrm{sgn}(x)-ad^2\mathrm{sgn}(x))\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+d}\right|\right)}{8\,c^{\frac{3}{2}}} + \frac{(4\,bcd\log(|d|)-ad^2\log(|d|))\mathrm{sgn}(x)}{16\,c^{\frac{3}{2}}}$$

```
[In] integrate((a+b/x^2)*x^3*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*a*x^2*sgn(x) + (4*b*c^2*sgn(x) + a*c*d*sgn(x))/c^2)*sqrt(c*x^2 + d)*
x - 1/8*(4*b*c*d*sgn(x) - a*d^2*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d
)))/c^(3/2) + 1/16*(4*b*c*d*log(abs(d)) - a*d^2*log(abs(d)))*sgn(x)/c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 9.66 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.03

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^3\,dx=\frac{a\,x^4\,\sqrt{c+\frac{d}{x^2}}}{8}+\frac{b\,x^2\,\sqrt{c+\frac{d}{x^2}}}{2}+\frac{a\,x^4\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{8\,c}+\frac{b\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,\sqrt{c}}-\frac{a\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,c^{3/2}}$$

```
[In] int(x^3*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] (a*x^4*(c + d/x^2)^(1/2))/8 + (b*x^2*(c + d/x^2)^(1/2))/2 + (a*x^4*(c + d/x
^2)^(3/2))/(8*c) + (b*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(2*c^(1/2)) - (a*
d^2*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(8*c^(3/2))
```

3.931 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x \, dx$

Optimal result

Integrand size = 20, antiderivative size = 84

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x \, dx = -\frac{(2bc+ad)\sqrt{c+\frac{d}{x^2}}}{2c}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{2c}+\frac{(2bc+ad)\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\sqrt{c}}$$

```
[Out] 1/2*a*(c+d/x^2)^(3/2)*x^2/c+1/2*(a*d+2*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2)
)/c^(1/2)-1/2*(a*d+2*b*c)*(c+d/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 79, 52, 65, 214}

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x \, dx = \frac{(ad+2bc)\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\sqrt{c}}-\frac{\sqrt{c+\frac{d}{x^2}}(ad+2bc)}{2c}+\frac{ax^2\left(c+\frac{d}{x^2}\right)^{3/2}}{2c}$$

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x,x]
```

```
[Out] -1/2*((2*b*c + a*d)*Sqrt[c + d/x^2])/c + (a*(c + d/x^2)^(3/2)*x^2)/(2*c) +
((2*b*c + a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(2*Sqrt[c])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)\sqrt{c+dx}}{x^2}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{2c} - \frac{(2bc+ad)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x}\,dx, x, \frac{1}{x^2}\right)}{4c}$$

$$= -\frac{(2bc+ad)\sqrt{c+\frac{d}{x^2}}}{2c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{2c} - \frac{1}{4}(2bc+ad)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{(2bc+ad)\sqrt{c+\frac{d}{x^2}}}{2c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{2c} - \frac{(2bc+ad)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{2d}$$

$$= -\frac{(2bc+ad)\sqrt{c+\frac{d}{x^2}}}{2c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{2c} + \frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\sqrt{c}}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.95

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x\,dx = \frac{1}{2}\sqrt{c+\frac{d}{x^2}}\left(-2b+ax^2+\frac{2(2bc+ad)x\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{d}+\sqrt{d+cx^2}}\right)}{\sqrt{c}\sqrt{d+cx^2}}\right)$$

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x,x]
```

```
[Out] (Sqrt[c + d/x^2]*(-2*b + a*x^2 + (2*(2*b*c + a*d)*x*ArcTanh[(Sqrt[c]*x)/(-S
qrt[d] + Sqrt[d + c*x^2])])/(Sqrt[c]*Sqrt[d + c*x^2])))/2
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

method	result
risch	$\frac{(a\,x^2-2b)\sqrt{\frac{c\,x^2+d}{x^2}}}{2}+\frac{\left(\frac{ad}{2}+bc\right)\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)\sqrt{\frac{c\,x^2+d}{x^2}}\,x}{\sqrt{c}\,\sqrt{c\,x^2+d}}$
default	$-\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\left(-\sqrt{c\,x^2+d}\,\sqrt{c}\,ad\,x^2-2\sqrt{c\,x^2+d}\,c^{\frac{3}{2}}\,b\,x^2+2\left(c\,x^2+d\right)^{\frac{3}{2}}\sqrt{c}\,b-\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)a\,d^2x-2\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)bcdx\right)}{2\sqrt{c\,x^2+d}\,d\sqrt{c}}$

```
[In] int((a+b/x^2)*x*(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a*x^2-2*b)*((c*x^2+d)/x^2)^(1/2)+(1/2*a*d+b*c)*ln(c^(1/2)*x+(c*x^2+d)^
(1/2))/c^(1/2)*((c*x^2+d)/x^2)^(1/2)*x/(c*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.85

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x\,dx = \left[\frac{(2\,bc+ad)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)+2\,(acx^2-2\,bc)\sqrt{\frac{cx^2+d}{x^2}}}{4\,c}, -\frac{(2\,bc+ad)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)-(acx^2-2\,bc)\sqrt{\frac{cx^2+d}{x^2}}}{2\,c}\right]$$

```
[In] integrate((a+b/x^2)*x*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((2*b*c + a*d)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x
^2) - d) + 2*(a*c*x^2 - 2*b*c)*sqrt((c*x^2 + d)/x^2))/c, -1/2*((2*b*c + a*d
)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (a*c*x^
2 - 2*b*c)*sqrt((c*x^2 + d)/x^2))/c]
```

Sympy [A] (verification not implemented)

Time = 17.14 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.27

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x\,dx = \frac{a\sqrt{d}x\sqrt{\frac{cx^2}{d}+1}}{2}+\frac{ad\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2\sqrt{c}} + b\sqrt{c}\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)-\frac{bcx}{\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}-\frac{b\sqrt{d}}{x\sqrt{\frac{cx^2}{d}+1}}$$

```
[In] integrate((a+b/x**2)*x*(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(d)*x*sqrt(c*x**2/d + 1)/2 + a*d*asinh(sqrt(c)*x/sqrt(d))/(2*sqrt(c))
 + b*sqrt(c)*asinh(sqrt(c)*x/sqrt(d)) - b*c*x/(sqrt(d)*sqrt(c*x**2/d + 1))
- b*sqrt(d)/(x*sqrt(c*x**2/d + 1))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.29

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x\,dx=\frac{1}{4}\left(2\sqrt{c+\frac{d}{x^2}}x^2-\frac{d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{\sqrt{c}}\right)a-\frac{1}{2}\left(\sqrt{c}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)+2\sqrt{c+\frac{d}{x^2}}\right)b$$

```
[In] integrate((a+b/x^2)*x*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(c + d/x^2)*x^2 - d*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/
x^2) + sqrt(c)))/sqrt(c))*a - 1/2*(sqrt(c)*log((sqrt(c + d/x^2) - sqrt(c))/
(sqrt(c + d/x^2) + sqrt(c))) + 2*sqrt(c + d/x^2))*b
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.04

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x\,dx=\frac{1}{2}\sqrt{cx^2+d}ax\text{sgn}(x)+\frac{2b\sqrt{c}d\text{sgn}(x)}{\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d}-\frac{(2bc\text{sgn}(x)+ad\text{sgn}(x))\log\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2\right)}{4\sqrt{c}}$$

```
[In] integrate((a+b/x^2)*x*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + d)*a*x*sgn(x) + 2*b*sqrt(c)*d*sgn(x)/((sqrt(c)*x - sqrt(c*
x^2 + d))^2 - d) - 1/4*(2*b*c*sgn(x) + a*d*sgn(x))*log((sqrt(c)*x - sqrt(c*
x^2 + d))^2)/sqrt(c)
```

Mupad [B] (verification not implemented)

Time = 9.61 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.81

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x\,dx=\frac{a\,x^2\,\sqrt{c+\frac{d}{x^2}}}{2}-b\,\sqrt{c+\frac{d}{x^2}}+b\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)+\frac{a\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,\sqrt{c}}$$

```
[In] int(x*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] (a*x^2*(c + d/x^2)^(1/2))/2 - b*(c + d/x^2)^(1/2) + b*c^(1/2)*atanh((c + d/
x^2)^(1/2)/c^(1/2)) + (a*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(2*c^(1/2))
```

3.932 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 59

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx=-a\sqrt{c+\frac{d}{x^2}}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d}+a\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)$$

```
[Out] -1/3*b*(c+d/x^2)^(3/2)/d+a*arctanh((c+d/x^2)^(1/2)/c^(1/2))*c^(1/2)-a*(c+d/
x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 81, 52, 65, 214}

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx=a\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)-a\sqrt{c+\frac{d}{x^2}}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d}$$

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x,x]
```

```
[Out] -(a*Sqrt[c + d/x^2]) - (b*(c + d/x^2)^(3/2))/(3*d) + a*Sqrt[c]*ArcTanh[Sqrt
[c + d/x^2]/Sqrt[c]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)\sqrt{c+dx}}{x}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= -\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d} - \frac{1}{2}a\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x}\,dx, x, \frac{1}{x^2}\right)\\
&= -a\sqrt{c+\frac{d}{x^2}} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d} - \frac{1}{2}(ac)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)\\
&= -a\sqrt{c+\frac{d}{x^2}} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d} - \frac{(ac)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{d}
\end{aligned}$$

$$= -a\sqrt{c+\frac{d}{x^2}} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d} + a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.39

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx = -\frac{\sqrt{c+\frac{d}{x^2}}\left(3adx^2+b(d+cx^2)+\frac{3a\sqrt{c}dx^3\log\left(-\sqrt{c}x+\sqrt{d+cx^2}\right)}{\sqrt{d+cx^2}}\right)}{3dx^2}$$

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x,x]
[Out] -1/3*(Sqrt[c + d/x^2]*(3*a*d*x^2 + b*(d + c*x^2) + (3*a*Sqrt[c]*d*x^3*Log[-
(Sqrt[c]*x) + Sqrt[d + c*x^2]])/Sqrt[d + c*x^2]))/(d*x^2)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.42

method	result	size
risch	$-\frac{\left(3ad\,x^2+cb\,x^2+bd\right)\sqrt{\frac{c\,x^2+d}{x^2}}}{3x^2 d}+\frac{a\sqrt{c}\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)\sqrt{\frac{c\,x^2+d}{x^2}}\,x}{\sqrt{c\,x^2+d}}$	84
default	$-\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\left(-3\sqrt{c\,x^2+d}\,c^{\frac{3}{2}}\,a\,x^4+3\left(c\,x^2+d\right)^{\frac{3}{2}}\sqrt{c}\,a\,x^2-3\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)acd\,x^3+\left(c\,x^2+d\right)^{\frac{3}{2}}\sqrt{c}\,b\right)}{3x^2\sqrt{c\,x^2+d}\,d\sqrt{c}}$	109

```
[In] int((a+b/x^2)*(c+d/x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] -1/3*(3*a*d*x^2+b*c*x^2+b*d)/x^2/d*((c*x^2+d)/x^2)^(1/2)+a*c^(1/2)*ln(c^(1/
2)*x+(c*x^2+d)^(1/2))*((c*x^2+d)/x^2)^(1/2)*x/(c*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.81

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx = \left[\frac{3\,a\sqrt{c}dx^2\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\,((bc+3\,ad)x^2+bd)\sqrt{\frac{cx^2+d}{x^2}}}{6\,dx^2},\ -\frac{3\,a\sqrt{-c}dx^2\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+((bc+3\,ad)x^2+bd)\sqrt{\frac{cx^2+d}{x^2}}}{3\,dx^2}\right]$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/6*(3*a*sqrt(c)*d*x^2*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2)
- d) - 2*((b*c + 3*a*d)*x^2 + b*d)*sqrt((c*x^2 + d)/x^2))/(d*x^2), -1/3*(3*
a*sqrt(-c)*d*x^2*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (
(b*c + 3*a*d)*x^2 + b*d)*sqrt((c*x^2 + d)/x^2))/(d*x^2)]
```

Sympy [A] (verification not implemented)

Time = 4.53 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.44

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx = -\frac{a\left(\begin{cases}\frac{2c\,\mathrm{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2\sqrt{c+\frac{d}{x^2}} & \text{for}\ d\neq 0\\ -\sqrt{c}\log\left(x^2\right) & \text{otherwise}\end{cases}\right)}{2} + \frac{b\left(\begin{cases}-\frac{\sqrt{c}}{x^2} & \text{for}\ d=0\\ -\frac{2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)}{2}$$

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x,x)
```

```
[Out] -a*Piecewise((2*c*atan(sqrt(c + d/x**2)/sqrt(-c))/sqrt(-c) + 2*sqrt(c + d/x
**2), Ne(d, 0)), (-sqrt(c)*log(x**2), True))/2 + b*Piecewise((-sqrt(c)/x**2
, Eq(d, 0)), (-2*(c + d/x**2)**(3/2)/(3*d), True))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx = -\frac{1}{2}\left(\sqrt{c}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)+2\sqrt{c+\frac{d}{x^2}}\right)a-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3\,d}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x,x, algorithm="maxima")
[Out] -1/2*(sqrt(c)*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))
+ 2*sqrt(c + d/x^2))*a - 1/3*b*(c + d/x^2)^(3/2)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(47) = 94.

Time = 0.47 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.76

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx = -\frac{1}{2}\,a\sqrt{c}\log\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2\right)\operatorname{sgn}(x) + \frac{2\left(3\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4bc^{\frac{3}{2}}\operatorname{sgn}(x)+3\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4a\sqrt{c}d\operatorname{sgn}(x)-6\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2a\sqrt{c}d^2\operatorname{s}\right.}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)^3}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x,x, algorithm="giac")
[Out] -1/2*a*sqrt(c)*log((sqrt(c)*x - sqrt(c*x^2 + d))^2)*sgn(x) + 2/3*(3*(sqrt(c
)*x - sqrt(c*x^2 + d))^4*b*c^(3/2)*sgn(x) + 3*(sqrt(c)*x - sqrt(c*x^2 + d))
^4*a*sqrt(c)*d*sgn(x) - 6*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*sqrt(c)*d^2*sgn
(x) + b*c^(3/2)*d^2*sgn(x) + 3*a*sqrt(c)*d^3*sgn(x))/((sqrt(c)*x - sqrt(c*x
^2 + d))^2 - d)^3
```

Mupad [B] (verification not implemented)

Time = 9.55 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx = a\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)-a\,\sqrt{c+\frac{d}{x^2}}-\frac{b\,\sqrt{c+\frac{d}{x^2}}\,\left(c\,x^2+d\right)}{3\,d\,x^2}$$

```
[In] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x,x)
[Out] a*c^(1/2)*atanh((c + d/x^2)^(1/2)/c^(1/2)) - a*(c + d/x^2)^(1/2) - (b*(c +
d/x^2)^(1/2)*(d + c*x^2))/(3*d*x^2)
```

3.933 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx = \frac{(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^2} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^2}$$

```
[Out] 1/3*(-a*d+b*c)*(c+d/x^2)^(3/2)/d^2-1/5*b*(c+d/x^2)^(5/2)/d^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {455, 45}

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx = \frac{\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^2} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^2}$$

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^3,x]
```

```
[Out] ((b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^2) - (b*(c + d/x^2)^(5/2))/(5*d^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{2}\text{Subst}\left(\int (a+bx)\sqrt{c+dx}\,dx, x, \frac{1}{x^2}\right)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{(-bc+ad)\sqrt{c+dx}}{d}+\frac{b(c+dx)^{3/2}}{d}\right)\,dx, x, \frac{1}{x^2}\right)\right)\\ &= \frac{(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx = \frac{\sqrt{c+\frac{d}{x^2}}(d+cx^2)\left(-3bd+2bcx^2-5adx^2\right)}{15d^2x^4}$$

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^3,x]
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)*(-3*b*d + 2*b*c*x^2 - 5*a*d*x^2))/(15*d^2*x^4)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

method	result	size
gosper	$-\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,\left(5ad\,x^2-2cb\,x^2+3bd\right)\left(c\,x^2+d\right)}{15d^2x^4}$	48
default	$-\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,\left(5ad\,x^2-2cb\,x^2+3bd\right)\left(c\,x^2+d\right)}{15d^2x^4}$	48
risch	$-\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,\left(5acd\,x^4-2b\,c^2x^4+5a\,d^2x^2+bcd\,x^2+3b\,d^2\right)}{15x^4d^2}$	62
trager	$-\frac{\left(5acd\,x^4-2b\,c^2x^4+5a\,d^2x^2+bcd\,x^2+3b\,d^2\right)\sqrt{-\frac{-c\,x^2-d}{x^2}}}{15x^4d^2}$	66

```
[In] int((a+b/x^2)*(c+d/x^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/15*((c*x^2+d)/x^2)^(1/2)*(5*a*d*x^2-2*b*c*x^2+3*b*d)*(c*x^2+d)/d^2/x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx = \frac{\left((2\,bc^2-5\,acd)x^4-3\,bd^2-(bcd+5\,ad^2)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\,d^2x^4}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/15*((2*b*c^2 - 5*a*c*d)*x^4 - 3*b*d^2 - (b*c*d + 5*a*d^2)*x^2)*sqrt((c*x^
2 + d)/x^2)/(d^2*x^4)
```

Sympy [A] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.59

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx = -\frac{a\left(\begin{cases}\frac{2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3d} & \text{for } d\neq 0\\ \frac{\sqrt{c}}{x^2} & \text{otherwise}\end{cases}\right)}{2} - \frac{b\left(\begin{cases}\frac{2\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2} & \text{for } d\neq 0\\ \frac{\sqrt{c}}{2x^4} & \text{otherwise}\end{cases}\right)}{2}$$

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**3,x)
```

```
[Out] -a*Piecewise((2*(c + d/x**2)**(3/2)/(3*d), Ne(d, 0)), (sqrt(c)/x**2, True))
/2 - b*Piecewise((2*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d**2
, Ne(d, 0)), (sqrt(c)/(2*x**4), True))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx = -\frac{1}{15}\,b\left(\frac{3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^2}-\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^2}\right)-\frac{a\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3\,d}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/15*b*(3*(c + d/x^2)^(5/2)/d^2 - 5*(c + d/x^2)^(3/2)*c/d^2) - 1/3*a*(c +
d/x^2)^(3/2)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(38) = 76$.

Time = 0.59 (sec) , antiderivative size = 250, normalized size of antiderivative = 5.43

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx = \frac{2\left(15\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^8 ac^{\frac{3}{2}}\operatorname{sgn}(x)+30\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^6 bc^{\frac{5}{2}}\operatorname{sgn}(x)-30\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^6 ac^{\frac{3}{2}}d\operatorname{sgn}\right.}{}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 2/15*(15*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(3/2)*sgn(x) + 30*(sqrt(c)*x -
 sqrt(c*x^2 + d))^6*b*c^(5/2)*sgn(x) - 30*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a
*c^(3/2)*d*sgn(x) + 10*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(5/2)*d*sgn(x) +
 20*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(3/2)*d^2*sgn(x) + 10*(sqrt(c)*x -
sqrt(c*x^2 + d))^2*b*c^(5/2)*d^2*sgn(x) - 10*(sqrt(c)*x - sqrt(c*x^2 + d))^
2*a*c^(3/2)*d^3*sgn(x) - 2*b*c^(5/2)*d^3*sgn(x) + 5*a*c^(3/2)*d^4*sgn(x))/(
(sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^5
```

Mupad [B] (verification not implemented)

Time = 9.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.98

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx = \frac{\sqrt{c+\frac{d}{x^2}}\,(b\,c^2+a\,d\,c)}{5\,d^2} - \frac{b\,\sqrt{c+\frac{d}{x^2}}}{5\,x^4} - \frac{\sqrt{c+\frac{d}{x^2}}\,(5\,a\,d^2+b\,c\,d)}{15\,d^2\,x^2} - \frac{c\,\sqrt{c+\frac{d}{x^2}}\,(8\,a\,d+b\,c)}{15\,d^2}$$

```
[In] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^3,x)
```

```
[Out] ((c + d/x^2)^(1/2)*(b*c^2 + a*c*d))/(5*d^2) - (b*(c + d/x^2)^(1/2))/(5*x^4)
 - ((c + d/x^2)^(1/2)*(5*a*d^2 + b*c*d))/(15*d^2*x^2) - (c*(c + d/x^2)^(1/2
)*(8*a*d + b*c))/(15*d^2)
```

3.934 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx = -\frac{c(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}+\frac{(2bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^3}$$

```
[Out] -1/3*c*(-a*d+b*c)*(c+d/x^2)^(3/2)/d^3+1/5*(-a*d+2*b*c)*(c+d/x^2)^(5/2)/d^3-
1/7*b*(c+d/x^2)^(7/2)/d^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 78}

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx = \frac{\left(c+\frac{d}{x^2}\right)^{5/2}(2bc-ad)}{5d^3}-\frac{c\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^3}$$

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^5,x]
```

```
[Out] -1/3*(c*(b*c - a*d)*(c + d/x^2)^(3/2))/d^3 + ((2*b*c - a*d)*(c + d/x^2)^(5/
2))/(5*d^3) - (b*(c + d/x^2)^(7/2))/(7*d^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
```

```
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f])))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\text{Subst}\left(\int x(a+bx)\sqrt{c+dx}\,dx, x, \frac{1}{x^2}\right)\right) \\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{c(bc-ad)\sqrt{c+dx}}{d^2}+\frac{(-2bc+ad)(c+dx)^{3/2}}{d^2}+\frac{b(c+dx)^{5/2}}{d^2}\right)\,dx, x, \frac{1}{x^2}\right)\right) \\
&= -\frac{c(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}+\frac{(2bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.93

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx = \frac{\sqrt{c+\frac{d}{x^2}}(d+cx^2)\left(-15bd^2+12bcdx^2-21ad^2x^2-8bc^2x^4+14acdx^4\right)}{105d^3x^6}$$

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^5,x]
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)*(-15*b*d^2 + 12*b*c*d*x^2 - 21*a*d^2*x^2 - 8*b
*c^2*x^4 + 14*a*c*d*x^4))/(105*d^3*x^6)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(14 a c d\,x^{4}-8 b\,c^{2} x^{4}-21 a\,d^{2} x^{2}+12 b c d\,x^{2}-15 b\,d^{2}\right) \left(c\,x^{2}+d\right)}{105 d^{3} x^{6}}$	70
default	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(14 a c d\,x^{4}-8 b\,c^{2} x^{4}-21 a\,d^{2} x^{2}+12 b c d\,x^{2}-15 b\,d^{2}\right) \left(c\,x^{2}+d\right)}{105 d^{3} x^{6}}$	70
risch	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(14 a\,c^{2} d\,x^{6}-8 b\,c^{3} x^{6}-7 a c\,d^{2} x^{4}+4 b\,c^{2} d\,x^{4}-21 a\,d^{3} x^{2}-3 b c\,d^{2} x^{2}-15 b\,d^{3}\right)}{105 x^{6} d^{3}}$	87
trager	$\frac{\left(14 a\,c^{2} d\,x^{6}-8 b\,c^{3} x^{6}-7 a c\,d^{2} x^{4}+4 b\,c^{2} d\,x^{4}-21 a\,d^{3} x^{2}-3 b c\,d^{2} x^{2}-15 b\,d^{3}\right) \sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{105 x^{6} d^{3}}$	91

```
[In] int((a+b/x^2)*(c+d/x^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/105*((c*x^2+d)/x^2)^(1/2)*(14*a*c*d*x^4-8*b*c^2*x^4-21*a*d^2*x^2+12*b*c*d
*x^2-15*b*d^2)*(c*x^2+d)/d^3/x^6
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx = -\frac{\left(2\,\left(4\,bc^{3}-7\,ac^{2}d\right)x^{6}-\left(4\,bc^{2}d-7\,acd^{2}\right)x^{4}+15\,bd^{3}+3\,\left(bcd^{2}+7\,ad^{3}\right)x^{2}\right)\sqrt{\frac{cx^{2}+d}{x^{2}}}}{105\,d^{3}x^{6}}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/105*(2*(4*b*c^3 - 7*a*c^2*d)*x^6 - (4*b*c^2*d - 7*a*c*d^2)*x^4 + 15*b*d^
3 + 3*(b*c*d^2 + 7*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^3*x^6)
```

Sympy [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.47

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx = -\frac{a\left(\begin{cases}\frac{2\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}}{2x^4} & \text{otherwise}\end{cases}\right)}{2} - \frac{b\left(\begin{cases}\frac{2\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}}{3x^6} & \text{otherwise}\end{cases}\right)}{2}$$

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**5,x)
```

```
[Out] -a*Piecewise((2*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d**2, Ne
(d, 0)), (sqrt(c)/(2*x**4), True))/2 - b*Piecewise((2*(c**2*(c + d/x**2)**(
3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**3, Ne(d, 0))
, (sqrt(c)/(3*x**6), True))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.14

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx = -\frac{1}{105}\,b\left(\frac{15\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^3}-\frac{42\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^3}+\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2}{d^3}\right) - \frac{1}{15}\,a\left(\frac{3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^2}-\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^2}\right)$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/105*b*(15*(c + d/x^2)^(7/2)/d^3 - 42*(c + d/x^2)^(5/2)*c/d^3 + 35*(c + d
/x^2)^(3/2)*c^2/d^3) - 1/15*a*(3*(c + d/x^2)^(5/2)/d^2 - 5*(c + d/x^2)^(3/2
)*c/d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(62) = 124$.

Time = 0.81 (sec) , antiderivative size = 310, normalized size of antiderivative = 4.19

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx$$
$$= \frac{4\left(105\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}ac^{\frac{5}{2}}\mathrm{sgn}(x)+280\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{8}bc^{\frac{7}{2}}\mathrm{sgn}(x)-175\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{8}ac^{\frac{5}{2}}d\ldots}{\ldots}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 4/105*(105*(sqrt(c)*x - sqrt(c*x^2 + d))^10*a*c^(5/2)*sgn(x) + 280*(sqrt(c)
*x - sqrt(c*x^2 + d))^8*b*c^(7/2)*sgn(x) - 175*(sqrt(c)*x - sqrt(c*x^2 + d)
)^8*a*c^(5/2)*d*sgn(x) + 140*(sqrt(c)*x - sqrt(c*x^2 + d))^6*b*c^(7/2)*d*sg
n(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(5/2)*d^2*sgn(x) + 84*(sqrt(c
)*x - sqrt(c*x^2 + d))^4*b*c^(7/2)*d^2*sgn(x) - 42*(sqrt(c)*x - sqrt(c*x^2
+ d))^4*a*c^(5/2)*d^3*sgn(x) - 28*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(7/2)
*d^3*sgn(x) + 49*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(5/2)*d^4*sgn(x) + 4*b
*c^(7/2)*d^4*sgn(x) - 7*a*c^(5/2)*d^5*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d)
)^2 - d)^7
```

Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.70

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx = \frac{2\,a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{15\,d^2}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{7\,x^6}-\frac{a\,\sqrt{c+\frac{d}{x^2}}}{5\,x^4}-\frac{8\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{105\,d^3}-\frac{a\,c\,\sqrt{c+\frac{d}{x^2}}}{15\,d\,x^2}-\frac{b\,c\,\sqrt{c+\frac{d}{x^2}}}{35\,d\,x^4}+\frac{4\,b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d^2\,x^2}$$

```
[In] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^5,x)
```

```
[Out] (2*a*c^2*(c + d/x^2)^(1/2))/(15*d^2) - (b*(c + d/x^2)^(1/2))/(7*x^6) - (a*(
c + d/x^2)^(1/2))/(5*x^4) - (8*b*c^3*(c + d/x^2)^(1/2))/(105*d^3) - (a*c*(c
 + d/x^2)^(1/2))/(15*d*x^2) - (b*c*(c + d/x^2)^(1/2))/(35*d*x^4) + (4*b*c^2
*(c + d/x^2)^(1/2))/(105*d^2*x^2)
```

3.935 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx = \frac{c^2(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^4} - \frac{c(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4} + \frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4} - \frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^4}$$

```
[Out] 1/3*c^2*(-a*d+b*c)*(c+d/x^2)^(3/2)/d^4-1/5*c*(-2*a*d+3*b*c)*(c+d/x^2)^(5/2)
/d^4+1/7*(-a*d+3*b*c)*(c+d/x^2)^(7/2)/d^4-1/9*b*(c+d/x^2)^(9/2)/d^4
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 78}

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx = \frac{c^2\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^4} + \frac{\left(c+\frac{d}{x^2}\right)^{7/2}(3bc-ad)}{7d^4} - \frac{c\left(c+\frac{d}{x^2}\right)^{5/2}(3bc-2ad)}{5d^4} - \frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^4}$$

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^7,x]
```

```
[Out] (c^2*(b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^4) - (c*(3*b*c - 2*a*d)*(c + d/x^2
)^(5/2))/(5*d^4) + ((3*b*c - a*d)*(c + d/x^2)^(7/2))/(7*d^4) - (b*(c + d/x^
2)^(9/2))/(9*d^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{2}\text{Subst}\left(\int x^2(a+bx)\sqrt{c+dx}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-\frac{c^2(bc-ad)\sqrt{c+dx}}{d^3}+\frac{c(3bc-2ad)(c+dx)^{3/2}}{d^3}+\frac{(-3bc+ad)(c+dx)^{5/2}}{d^3}+\frac{b(c+\right.\right.\right.$$

$$= \frac{c^2(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^4}-\frac{c(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}+\frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^4}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx$$

$$= \frac{\sqrt{c+\frac{d}{x^2}}(d+cx^2)\left(-35bd^3+30bcd^2x^2-45ad^3x^2-24bc^2dx^4+36acd^2x^4+16bc^3x^6-24ac^2dx^6\right)}{315d^4x^8}$$

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^7,x]
```

```
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)*(-35*b*d^3 + 30*b*c*d^2*x^2 - 45*a*d^3*x^2 - 2
4*b*c^2*d*x^4 + 36*a*c*d^2*x^4 + 16*b*c^3*x^6 - 24*a*c^2*d*x^6))/(315*d^4*x
^8)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

method	result	size
gosper	$-\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(24a\,c^{2}d\,x^{6}-16b\,c^{3}x^{6}-36ac\,d^{2}x^{4}+24b\,c^{2}d\,x^{4}+45a\,d^{3}x^{2}-30bc\,d^{2}x^{2}+35b\,d^{3}\right)\left(c\,x^{2}+d\right)}{315d^{4}x^{8}}$	94
default	$-\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(24a\,c^{2}d\,x^{6}-16b\,c^{3}x^{6}-36ac\,d^{2}x^{4}+24b\,c^{2}d\,x^{4}+45a\,d^{3}x^{2}-30bc\,d^{2}x^{2}+35b\,d^{3}\right)\left(c\,x^{2}+d\right)}{315d^{4}x^{8}}$	94
risch	$-\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(24a\,c^{3}d\,x^{8}-16b\,c^{4}x^{8}-12a\,c^{2}d^{2}x^{6}+8b\,c^{3}d\,x^{6}+9ac\,d^{3}x^{4}-6b\,c^{2}d^{2}x^{4}+45a\,d^{4}x^{2}+5bc\,d^{3}x^{2}+35b\,d^{4}\right)}{315x^{8}d^{4}}$	111
trager	$-\frac{\left(24a\,c^{3}d\,x^{8}-16b\,c^{4}x^{8}-12a\,c^{2}d^{2}x^{6}+8b\,c^{3}d\,x^{6}+9ac\,d^{3}x^{4}-6b\,c^{2}d^{2}x^{4}+45a\,d^{4}x^{2}+5bc\,d^{3}x^{2}+35b\,d^{4}\right)\sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{315x^{8}d^{4}}$	115

```
[In] int((a+b/x^2)*(c+d/x^2)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/315*((c*x^2+d)/x^2)^(1/2)*(24*a*c^2*d*x^6-16*b*c^3*x^6-36*a*c*d^2*x^4+24
*b*c^2*d*x^4+45*a*d^3*x^2-30*b*c*d^2*x^2+35*b*d^3)*(c*x^2+d)/d^4/x^8
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx$$
$$=\frac{\left(8\,(2\,bc^4-3\,ac^3d)x^8-4\,(2\,bc^3d-3\,ac^2d^2)x^6-35\,bd^4+3\,(2\,bc^2d^2-3\,acd^3)x^4-5\,(bcd^3+9\,ad^4)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{315\,d^4x^8}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/315*(8*(2*b*c^4 - 3*a*c^3*d)*x^8 - 4*(2*b*c^3*d - 3*a*c^2*d^2)*x^6 - 35*b
*d^4 + 3*(2*b*c^2*d^2 - 3*a*c*d^3)*x^4 - 5*(b*c*d^3 + 9*a*d^4)*x^2)*sqrt((c
*x^2 + d)/x^2)/(d^4*x^8)
```

Sympy [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.38

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx = -\frac{a\left(\begin{cases}\frac{2\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3} & \text{for } d\neq 0\\ \frac{\sqrt{c}}{3x^6} & \text{otherwise}\end{cases}\right)}{2} - \frac{b\left(\begin{cases}\frac{2\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4} & \text{for } d\neq 0\\ \frac{\sqrt{c}}{4x^8} & \text{otherwise}\end{cases}\right)}{2}$$

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**7,x)
```

```
[Out] -a*Piecewise((2*(c**2*(c + d/x**2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (
c + d/x**2)**(7/2)/7)/d**3, Ne(d, 0)), (sqrt(c)/(3*x**6), True))/2 - b*Piec
ewise((2*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)/5 - 3*c*
(c + d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**4, Ne(d, 0)), (sqrt(c)/(4
*x**8), True))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.13

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx = -\frac{1}{315}\,b\left(\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{d^4}-\frac{135\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c}{d^4}+\frac{189\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^2}{d^4}-\frac{105\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^3}{d^4}\right) - \frac{1}{105}\,a\left(\frac{15\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^3}-\frac{42\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^3}+\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2}{d^3}\right)$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/315*b*(35*(c + d/x^2)^(9/2)/d^4 - 135*(c + d/x^2)^(7/2)*c/d^4 + 189*(c +
 d/x^2)^(5/2)*c^2/d^4 - 105*(c + d/x^2)^(3/2)*c^3/d^4) - 1/105*a*(15*(c + d
/x^2)^(7/2)/d^3 - 42*(c + d/x^2)^(5/2)*c/d^3 + 35*(c + d/x^2)^(3/2)*c^2/d^3
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(88) = 176$.

Time = 0.99 (sec) , antiderivative size = 370, normalized size of antiderivative = 3.56

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx = \frac{16\left(210\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}ac^{\frac{7}{2}}\operatorname{sgn}(x)+630\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}bc^{\frac{9}{2}}\operatorname{sgn}(x)-315\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}c\right.}{}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 16/315*(210*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(7/2)*sgn(x) + 630*(sqrt(c
)*x - sqrt(c*x^2 + d))^10*b*c^(9/2)*sgn(x) - 315*(sqrt(c)*x - sqrt(c*x^2 +
d))^10*a*c^(7/2)*d*sgn(x) + 378*(sqrt(c)*x - sqrt(c*x^2 + d))^8*b*c^(9/2)*d
*sgn(x) + 63*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(7/2)*d^2*sgn(x) + 168*(sq
rt(c)*x - sqrt(c*x^2 + d))^6*b*c^(9/2)*d^2*sgn(x) - 42*(sqrt(c)*x - sqrt(c*
x^2 + d))^6*a*c^(7/2)*d^3*sgn(x) - 72*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(
9/2)*d^3*sgn(x) + 108*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(7/2)*d^4*sgn(x)
+ 18*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(9/2)*d^4*sgn(x) - 27*(sqrt(c)*x -
 sqrt(c*x^2 + d))^2*a*c^(7/2)*d^5*sgn(x) - 2*b*c^(9/2)*d^5*sgn(x) + 3*a*c^(
7/2)*d^6*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^9
```

Mupad [B] (verification not implemented)

Time = 9.70 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.62

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx = \frac{16\,b\,c^4\,\sqrt{c+\frac{d}{x^2}}}{315\,d^4} - \frac{b\,\sqrt{c+\frac{d}{x^2}}}{9\,x^8} - \frac{8\,a\,c^3\,\sqrt{c+\frac{d}{x^2}}}{105\,d^3} - \frac{a\,\sqrt{c+\frac{d}{x^2}}}{7\,x^6} - \frac{a\,c\,\sqrt{c+\frac{d}{x^2}}}{35\,d\,x^4} - \frac{b\,c\,\sqrt{c+\frac{d}{x^2}}}{63\,d\,x^6} + \frac{4\,a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d^2\,x^2} + \frac{2\,b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d^2\,x^4} - \frac{8\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{315\,d^3\,x^2}$$

```
[In] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^7,x)
```

```
[Out] (16*b*c^4*(c + d/x^2)^(1/2))/(315*d^4) - (b*(c + d/x^2)^(1/2))/(9*x^8) - (8
*a*c^3*(c + d/x^2)^(1/2))/(105*d^3) - (a*(c + d/x^2)^(1/2))/(7*x^6) - (a*c*
(c + d/x^2)^(1/2))/(35*d*x^4) - (b*c*(c + d/x^2)^(1/2))/(63*d*x^6) + (4*a*c
^2*(c + d/x^2)^(1/2))/(105*d^2*x^2) + (2*b*c^2*(c + d/x^2)^(1/2))/(105*d^2*
x^4) - (8*b*c^3*(c + d/x^2)^(1/2))/(315*d^3*x^2)
```

3.936 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 134

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx = -\frac{c^3(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^5}+\frac{c^2(4bc-3ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^5} - \frac{3c(2bc-ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^5} + \frac{(4bc-ad)\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^5}-\frac{b\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^5}$$

```
[Out] -1/3*c^3*(-a*d+b*c)*(c+d/x^2)^(3/2)/d^5+1/5*c^2*(-3*a*d+4*b*c)*(c+d/x^2)^(5
/2)/d^5-3/7*c*(-a*d+2*b*c)*(c+d/x^2)^(7/2)/d^5+1/9*(-a*d+4*b*c)*(c+d/x^2)^(
9/2)/d^5-1/11*b*(c+d/x^2)^(11/2)/d^5
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 78}

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx = -\frac{c^3\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^5} + \frac{c^2\left(c+\frac{d}{x^2}\right)^{5/2}(4bc-3ad)}{5d^5}+\frac{\left(c+\frac{d}{x^2}\right)^{9/2}(4bc-ad)}{9d^5} - \frac{3c\left(c+\frac{d}{x^2}\right)^{7/2}(2bc-ad)}{7d^5}-\frac{b\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^5}$$

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^9,x]
```

```
[Out] -1/3*(c^3*(b*c - a*d)*(c + d/x^2)^(3/2))/d^5 + (c^2*(4*b*c - 3*a*d)*(c + d/
x^2)^(5/2))/(5*d^5) - (3*c*(2*b*c - a*d)*(c + d/x^2)^(7/2))/(7*d^5) + ((4*b
*c - a*d)*(c + d/x^2)^(9/2))/(9*d^5) - (b*(c + d/x^2)^(11/2))/(11*d^5)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{2}\text{Subst}\left(\int x^3(a+bx)\sqrt{c+dx}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{c^3(bc-ad)\sqrt{c+dx}}{d^4} - \frac{c^2(4bc-3ad)(c+dx)^{3/2}}{d^4} + \frac{3c(2bc-ad)(c+dx)^{5/2}}{d^4} + \frac{(-4b\right.\right.\right.$$

$$= -\frac{c^3(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^5} + \frac{c^2(4bc-3ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^5} - \frac{3c(2bc-ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^5} + \frac{(4bc-ad)\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^5} - \frac{b\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^5}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.84

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx = \frac{\sqrt{c+\frac{d}{x^2}}(d+cx^2)\left(11adx^2(-35d^3+30cd^2x^2-24c^2dx^4+16c^3x^6)+b(-315d^4+280cd^3x^2-240c^2d^2x^4+1\right.}{3465d^5x^{10}}$$

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^9,x]
```

[Out] (Sqrt[c + d/x^2]*(d + c*x^2)*(11*a*d*x^2*(-35*d^3 + 30*c*d^2*x^2 - 24*c^2*d
*x^4 + 16*c^3*x^6) + b*(-315*d^4 + 280*c*d^3*x^2 - 240*c^2*d^2*x^4 + 192*c^
3*d*x^6 - 128*c^4*x^8)))/(3465*d^5*x^10)

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.88

method	result
gosper	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(176a\,c^{3}d\,x^{8}-128b\,c^{4}x^{8}-264a\,c^{2}d^{2}x^{6}+192b\,c^{3}d\,x^{6}+330ac\,d^{3}x^{4}-240b\,c^{2}d^{2}x^{4}-385a\,d^{4}x^{2}+280bc\,d^{3}x^{2}-315b\,d^{4}\right)\left(c\,x^{2}+d\right)}{3465d^{5}x^{10}}$
default	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(176a\,c^{3}d\,x^{8}-128b\,c^{4}x^{8}-264a\,c^{2}d^{2}x^{6}+192b\,c^{3}d\,x^{6}+330ac\,d^{3}x^{4}-240b\,c^{2}d^{2}x^{4}-385a\,d^{4}x^{2}+280bc\,d^{3}x^{2}-315b\,d^{4}\right)\left(c\,x^{2}+d\right)}{3465d^{5}x^{10}}$
risch	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(176a\,c^{4}d\,x^{10}-128b\,c^{5}x^{10}-88a\,c^{3}d^{2}x^{8}+64b\,c^{4}d\,x^{8}+66a\,c^{2}d^{3}x^{6}-48b\,c^{3}d^{2}x^{6}-55ac\,d^{4}x^{4}+40b\,c^{2}d^{3}x^{4}-385a\,d^{5}x^{2}-35bc\,d^{4}x\right.}{3465x^{10}d^{5}}$
trager	$\frac{\left(176a\,c^{4}d\,x^{10}-128b\,c^{5}x^{10}-88a\,c^{3}d^{2}x^{8}+64b\,c^{4}d\,x^{8}+66a\,c^{2}d^{3}x^{6}-48b\,c^{3}d^{2}x^{6}-55ac\,d^{4}x^{4}+40b\,c^{2}d^{3}x^{4}-385a\,d^{5}x^{2}-35bc\,d^{4}x^{2}-315b\,d\right.}{3465x^{10}d^{5}}$

[In] int((a+b/x^2)*(c+d/x^2)^(1/2)/x^9,x,method=_RETURNVERBOSE)

[Out] 1/3465*((c*x^2+d)/x^2)^(1/2)*(176*a*c^3*d*x^8-128*b*c^4*x^8-264*a*c^2*d^2*x
^6+192*b*c^3*d*x^6+330*a*c*d^3*x^4-240*b*c^2*d^2*x^4-385*a*d^4*x^2+280*b*c*
d^3*x^2-315*b*d^4)*(c*x^2+d)/d^5/x^10

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.99

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx =$$
$$-\frac{(16\,(8\,bc^5-11\,ac^4d)x^{10}-8\,(8\,bc^4d-11\,ac^3d^2)x^8+6\,(8\,bc^3d^2-11\,ac^2d^3)x^6+315\,bd^5-5\,(8\,bc^2d^3-11}{3465\,d^5x^{10}}$$

[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^9,x, algorithm="fricas")

[Out] -1/3465*(16*(8*b*c^5 - 11*a*c^4*d)*x^10 - 8*(8*b*c^4*d - 11*a*c^3*d^2)*x^8
+ 6*(8*b*c^3*d^2 - 11*a*c^2*d^3)*x^6 + 315*b*d^5 - 5*(8*b*c^2*d^3 - 11*a*c*
d^4)*x^4 + 35*(b*c*d^4 + 11*a*d^5)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^5*x^10)

Sympy [A] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.32

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx = -\frac{a\left(\begin{cases} \frac{2\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4} & \text{for } d\neq 0 \\ \frac{\sqrt{c}}{4x^8} & \text{otherwise} \end{cases}\right)}{2} - \frac{b\left(\begin{cases} \frac{2\left(\frac{c^4\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{4c^3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{6c^2\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}-\frac{4c\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{11}\right)}{d^5} & \text{for } d\neq 0 \\ \frac{\sqrt{c}}{5x^{10}} & \text{otherwise} \end{cases}\right)}{2}$$

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**9,x)
[Out] -a*Piecewise((2*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)/5
 - 3*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**4, Ne(d, 0)), (sqr
t(c)/(4*x**8), True))/2 - b*Piecewise((2*(c**4*(c + d/x**2)**(3/2)/3 - 4*c*
*3*(c + d/x**2)**(5/2)/5 + 6*c**2*(c + d/x**2)**(7/2)/7 - 4*c*(c + d/x**2)*
*(9/2)/9 + (c + d/x**2)**(11/2)/11)/d**5, Ne(d, 0)), (sqrt(c)/(5*x**10), Tr
ue))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.13

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx = -\frac{1}{3465}\,b\left(\frac{315\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{d^5}-\frac{1540\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}c}{d^5}+\frac{2970\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c^2}{d^5}-\frac{2772\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3}{d^5}+\frac{1155\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^4}{d^5}\right) - \frac{1}{315}\,a\left(\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{d^4}-\frac{135\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c}{d^4}+\frac{189\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^2}{d^4}-\frac{105\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^3}{d^4}\right)$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/3465*b*(315*(c + d/x^2)^(11/2)/d^5 - 1540*(c + d/x^2)^(9/2)*c/d^5 + 2970
*(c + d/x^2)^(7/2)*c^2/d^5 - 2772*(c + d/x^2)^(5/2)*c^3/d^5 + 1155*(c + d/x
^2)^(3/2)*c^4/d^5) - 1/315*a*(35*(c + d/x^2)^(9/2)/d^4 - 135*(c + d/x^2)^(7
/2)*c/d^4 + 189*(c + d/x^2)^(5/2)*c^2/d^4 - 105*(c + d/x^2)^(3/2)*c^3/d^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(114) = 228$.

Time = 1.49 (sec) , antiderivative size = 430, normalized size of antiderivative = 3.21

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx = \frac{32\left(3465\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{14}ac^{\frac{9}{2}}\operatorname{sgn}(x)+11088\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}bc^{\frac{11}{2}}\operatorname{sgn}(x)-4851\left(\sqrt{c}x-\sqrt{cx^2+\right.\right.}{}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] 32/3465*(3465*(sqrt(c)*x - sqrt(c*x^2 + d))^14*a*c^(9/2)*sgn(x) + 11088*(sq
rt(c)*x - sqrt(c*x^2 + d))^12*b*c^(11/2)*sgn(x) - 4851*(sqrt(c)*x - sqrt(c*
x^2 + d))^12*a*c^(9/2)*d*sgn(x) + 7392*(sqrt(c)*x - sqrt(c*x^2 + d))^10*b*c
^(11/2)*d*sgn(x) + 231*(sqrt(c)*x - sqrt(c*x^2 + d))^10*a*c^(9/2)*d^2*sgn(x
) + 2640*(sqrt(c)*x - sqrt(c*x^2 + d))^8*b*c^(11/2)*d^2*sgn(x) - 165*(sqrt(
c)*x - sqrt(c*x^2 + d))^8*a*c^(9/2)*d^3*sgn(x) - 1320*(sqrt(c)*x - sqrt(c*x
^2 + d))^6*b*c^(11/2)*d^3*sgn(x) + 1815*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c
^(9/2)*d^4*sgn(x) + 440*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(11/2)*d^4*sgn(
x) - 605*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(9/2)*d^5*sgn(x) - 88*(sqrt(c)
*x - sqrt(c*x^2 + d))^2*b*c^(11/2)*d^5*sgn(x) + 121*(sqrt(c)*x - sqrt(c*x^2
 + d))^2*a*c^(9/2)*d^6*sgn(x) + 8*b*c^(11/2)*d^6*sgn(x) - 11*a*c^(9/2)*d^7*
sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^11
```

Mupad [B] (verification not implemented)

Time = 10.11 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.57

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx = \frac{16\,a\,c^4\,\sqrt{c+\frac{d}{x^2}}}{315\,d^4}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{11\,x^{10}}-\frac{a\,\sqrt{c+\frac{d}{x^2}}}{9\,x^8}-\frac{128\,b\,c^5\,\sqrt{c+\frac{d}{x^2}}}{3465\,d^5}-\frac{a\,c\,\sqrt{c+\frac{d}{x^2}}}{63\,d\,x^6}-\frac{b\,c\,\sqrt{c+\frac{d}{x^2}}}{99\,d\,x^8}+\frac{2\,a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d^2\,x^4}-\frac{8\,a\,c^3\,\sqrt{c+\frac{d}{x^2}}}{315\,d^3\,x^2}+\frac{8\,b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{693\,d^2\,x^6}-\frac{16\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{1155\,d^3\,x^4}+\frac{64\,b\,c^4\,\sqrt{c+\frac{d}{x^2}}}{3465\,d^4\,x^2}$$

```
[In] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^9,x)
```

```
[Out] (16*a*c^4*(c + d/x^2)^(1/2))/(315*d^4) - (b*(c + d/x^2)^(1/2))/(11*x^10) -
(a*(c + d/x^2)^(1/2))/(9*x^8) - (128*b*c^5*(c + d/x^2)^(1/2))/(3465*d^5) -
(a*c*(c + d/x^2)^(1/2))/(63*d*x^6) - (b*c*(c + d/x^2)^(1/2))/(99*d*x^8) + (
2*a*c^2*(c + d/x^2)^(1/2))/(105*d^2*x^4) - (8*a*c^3*(c + d/x^2)^(1/2))/(315
*d^3*x^2) + (8*b*c^2*(c + d/x^2)^(1/2))/(693*d^2*x^6) - (16*b*c^3*(c + d/x^
2)^(1/2))/(1155*d^3*x^4) + (64*b*c^4*(c + d/x^2)^(1/2))/(3465*d^4*x^2)
```

3.937 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^{10} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 150

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^{10} \, dx = -\frac{16d^3(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^3}{3465c^5} + \frac{8d^2(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^5}{1155c^4} - \frac{2d(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^7}{231c^3} + \frac{(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^9}{99c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2} x^{11}}{11c}$$

```
[Out] -16/3465*d^3*(-8*a*d+11*b*c)*(c+d/x^2)^(3/2)*x^3/c^5+8/1155*d^2*(-8*a*d+11*
b*c)*(c+d/x^2)^(3/2)*x^5/c^4-2/231*d*(-8*a*d+11*b*c)*(c+d/x^2)^(3/2)*x^7/c^
3+1/99*(-8*a*d+11*b*c)*(c+d/x^2)^(3/2)*x^9/c^2+1/11*a*(c+d/x^2)^(3/2)*x^11/
c
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used

$= \{464, 277, 270\}$

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^{10}\,dx = -\frac{16d^3x^3\left(c+\frac{d}{x^2}\right)^{3/2}(11bc-8ad)}{3465c^5} + \frac{8d^2x^5\left(c+\frac{d}{x^2}\right)^{3/2}(11bc-8ad)}{1155c^4} - \frac{2dx^7\left(c+\frac{d}{x^2}\right)^{3/2}(11bc-8ad)}{231c^3} + \frac{x^9\left(c+\frac{d}{x^2}\right)^{3/2}(11bc-8ad)}{99c^2} + \frac{ax^{11}\left(c+\frac{d}{x^2}\right)^{3/2}}{11c}$$

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^10,x]
```

```
[Out] (-16*d^3*(11*b*c - 8*a*d)*(c + d/x^2)^(3/2)*x^3)/(3465*c^5) + (8*d^2*(11*b*
c - 8*a*d)*(c + d/x^2)^(3/2)*x^5)/(1155*c^4) - (2*d*(11*b*c - 8*a*d)*(c + d
/x^2)^(3/2)*x^7)/(231*c^3) + ((11*b*c - 8*a*d)*(c + d/x^2)^(3/2)*x^9)/(99*c
^2) + (a*(c + d/x^2)^(3/2)*x^11)/(11*c)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^{11}}{11c} + \frac{(11bc-8ad)\int \sqrt{c+\frac{d}{x^2}}x^8\,dx}{11c}$$

$$
\begin{aligned}
&=\frac{(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^9}{99c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^{11}}{11c}-\frac{(2d(11bc-8ad))\int\sqrt{c+\frac{d}{x^2}}x^6\,dx}{33c^2}\\
&=-\frac{2d(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{231c^3}+\frac{(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^9}{99c^2}\\
&\quad+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^{11}}{11c}+\frac{(8d^2(11bc-8ad))\int\sqrt{c+\frac{d}{x^2}}x^4\,dx}{231c^3}\\
&=\frac{8d^2(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{1155c^4}-\frac{2d(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{231c^3}\\
&\quad+\frac{(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^9}{99c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^{11}}{11c}\\
&\quad-\frac{(16d^3(11bc-8ad))\int\sqrt{c+\frac{d}{x^2}}x^2\,dx}{1155c^4}\\
&=-\frac{16d^3(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3465c^5}+\frac{8d^2(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{1155c^4}\\
&\quad-\frac{2d(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{231c^3}+\frac{(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^9}{99c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^{11}}{11c}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.72

$$
\int\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^{10}\,dx
=\frac{\sqrt{c+\frac{d}{x^2}}x(d+cx^2)\left(11bc(-16d^3+24cd^2x^2-30c^2dx^4+35c^3x^6)+a(128d^4-192cd^3x^2+240c^2d^2x^4-28\right.}{3465c^5}
$$

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^10,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)*(11*b*c*(-16*d^3 + 24*c*d^2*x^2 - 30*c^2*d*x
^4 + 35*c^3*x^6) + a*(128*d^4 - 192*c*d^3*x^2 + 240*c^2*d^2*x^4 - 280*c^3*d
*x^6 + 315*c^4*x^8)))/(3465*c^5)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75

method	result
gosper	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x\left(315a\,x^{8}c^{4}-280a\,c^{3}d\,x^{6}+385b\,c^{4}x^{6}+240a\,c^{2}d^{2}x^{4}-330b\,c^{3}d\,x^{4}-192ac\,d^{3}x^{2}+264b\,c^{2}d^{2}x^{2}+128a\,d^{4}-176bc\,d^{3}\right)\left(c\,x^{2}+d\right)}{3465c^{5}}$
default	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x\left(315a\,x^{8}c^{4}-280a\,c^{3}d\,x^{6}+385b\,c^{4}x^{6}+240a\,c^{2}d^{2}x^{4}-330b\,c^{3}d\,x^{4}-192ac\,d^{3}x^{2}+264b\,c^{2}d^{2}x^{2}+128a\,d^{4}-176bc\,d^{3}\right)\left(c\,x^{2}+d\right)}{3465c^{5}}$
risch	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x\left(315a\,c^{5}x^{10}+35a\,c^{4}d\,x^{8}+385b\,c^{5}x^{8}-40a\,c^{3}d^{2}x^{6}+55b\,c^{4}d\,x^{6}+48a\,c^{2}d^{3}x^{4}-66b\,c^{3}d^{2}x^{4}-64ac\,d^{4}x^{2}+88b\,c^{2}d^{3}x^{2}+128a\,d^{5}-17\right.}{3465c^{5}}$
trager	$\frac{\left(315a\,c^{5}x^{10}+35a\,c^{4}d\,x^{8}+385b\,c^{5}x^{8}-40a\,c^{3}d^{2}x^{6}+55b\,c^{4}d\,x^{6}+48a\,c^{2}d^{3}x^{4}-66b\,c^{3}d^{2}x^{4}-64ac\,d^{4}x^{2}+88b\,c^{2}d^{3}x^{2}+128a\,d^{5}-176bc\,d^{4}\right)x\sqrt{}}{3465c^{5}}$

```
[In] int((a+b/x^2)*x^10*(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3465*((c*x^2+d)/x^2)^(1/2)*x*(315*a*c^4*x^8-280*a*c^3*d*x^6+385*b*c^4*x^6
+240*a*c^2*d^2*x^4-330*b*c^3*d*x^4-192*a*c*d^3*x^2+264*b*c^2*d^2*x^2+128*a*
d^4-176*b*c*d^3)*(c*x^2+d)/c^5
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^{10}\,dx$$
$$=\frac{(315\,ac^5x^{11}+35\,(11\,bc^5+ac^4d)x^9+5\,(11\,bc^4d-8\,ac^3d^2)x^7-6\,(11\,bc^3d^2-8\,ac^2d^3)x^5+8\,(11\,bc^2d^3-8\,ac}{3465\,c^5}$$

```
[In] integrate((a+b/x^2)*x^10*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(315*a*c^5*x^11 + 35*(11*b*c^5 + a*c^4*d)*x^9 + 5*(11*b*c^4*d - 8*a*
c^3*d^2)*x^7 - 6*(11*b*c^3*d^2 - 8*a*c^2*d^3)*x^5 + 8*(11*b*c^2*d^3 - 8*a*c
*d^4)*x^3 - 16*(11*b*c*d^4 - 8*a*d^5)*x)*sqrt((c*x^2 + d)/x^2)/c^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1386 vs. 2(146) = 292.

Time = 2.76 (sec) , antiderivative size = 1386, normalized size of antiderivative = 9.24

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^{10}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b/x**2)*x**10*(c+d/x**2)**(1/2),x)
```

```
[Out] 315*a*c**9*d**(33/2)*x**18*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860
*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**
5*d**20) + 1295*a*c**8*d**(35/2)*x**16*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*
x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**
2 + 3465*c**5*d**20) + 1990*a*c**7*d**(37/2)*x**14*sqrt(c*x**2/d + 1)/(3465
*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c*
*6*d**19*x**2 + 3465*c**5*d**20) + 1358*a*c**6*d**(39/2)*x**12*sqrt(c*x**2/
d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**
4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 343*a*c**5*d**(41/2)*x**10*s
qrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**
7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 35*a*c**4*d**(43/
2)*x**8*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 +
20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 280*a*c*
*3*d**(45/2)*x**6*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**
17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20)
+ 560*a*c**2*d**(47/2)*x**4*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 1386
0*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c*
*5*d**20) + 448*a*c*d**(49/2)*x**2*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8
 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 +
3465*c**5*d**20) + 128*a*d**(51/2)*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8
 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 +
3465*c**5*d**20) + 35*b*c**7*d**(19/2)*x**14*sqrt(c*x**2/d + 1)/(315*c**7*d
**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 11
0*b*c**6*d**(21/2)*x**12*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*
d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 114*b*c**5*d**(23/2)*x
**10*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**
5*d**11*x**2 + 315*c**4*d**12) + 40*b*c**4*d**(25/2)*x**8*sqrt(c*x**2/d + 1
)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**
4*d**12) - 5*b*c**3*d**(27/2)*x**6*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 +
 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) - 30*b*c**2*d*
*(29/2)*x**4*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 +
 945*c**5*d**11*x**2 + 315*c**4*d**12) - 40*b*c*d**(31/2)*x**2*sqrt(c*x**2/
d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 31
5*c**4*d**12) - 16*b*d**(33/2)*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945
*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.05

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^{10}\,dx$$
$$=\frac{\left(35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}x^9-135\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}dx^7+189\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^2x^5-105\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^3x^3\right)b}{315\,c^4}$$
$$+\frac{\left(315\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}x^{11}-1540\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}dx^9+2970\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}d^2x^7-2772\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^3x^5+1155\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d}{3465\,c^5}$$

```
[In] integrate((a+b/x^2)*x^10*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/315*(35*(c + d/x^2)^(9/2)*x^9 - 135*(c + d/x^2)^(7/2)*d*x^7 + 189*(c + d/
x^2)^(5/2)*d^2*x^5 - 105*(c + d/x^2)^(3/2)*d^3*x^3)*b/c^4 + 1/3465*(315*(c
+ d/x^2)^(11/2)*x^11 - 1540*(c + d/x^2)^(9/2)*d*x^9 + 2970*(c + d/x^2)^(7/2
)*d^2*x^7 - 2772*(c + d/x^2)^(5/2)*d^3*x^5 + 1155*(c + d/x^2)^(3/2)*d^4*x^3
)*a/c^5
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.17

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^{10}\,dx=\frac{16\left(11\,bcd^{\frac{9}{2}}-8\,ad^{\frac{11}{2}}\right)\mathrm{sgn}(x)}{3465\,c^5}$$
$$+\frac{315\left(cx^2+d\right)^{\frac{11}{2}}a\mathrm{sgn}(x)+385\left(cx^2+d\right)^{\frac{9}{2}}bc\mathrm{sgn}(x)-1540\left(cx^2+d\right)^{\frac{9}{2}}ad\mathrm{sgn}(x)-1485\left(cx^2+d\right)^{\frac{7}{2}}bcd\mathrm{sgn}(x}{}$$

```
[In] integrate((a+b/x^2)*x^10*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 16/3465*(11*b*c*d^(9/2) - 8*a*d^(11/2))*sgn(x)/c^5 + 1/3465*(315*(c*x^2 + d
)^(11/2)*a*sgn(x) + 385*(c*x^2 + d)^(9/2)*b*c*sgn(x) - 1540*(c*x^2 + d)^(9/
2)*a*d*sgn(x) - 1485*(c*x^2 + d)^(7/2)*b*c*d*sgn(x) + 2970*(c*x^2 + d)^(7/2
)*a*d^2*sgn(x) + 2079*(c*x^2 + d)^(5/2)*b*c*d^2*sgn(x) - 2772*(c*x^2 + d)^(
5/2)*a*d^3*sgn(x) - 1155*(c*x^2 + d)^(3/2)*b*c*d^3*sgn(x) + 1155*(c*x^2 + d
)^(3/2)*a*d^4*sgn(x))/c^5
```

Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.78

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^{10}\,dx=\sqrt{c+\frac{d}{x^2}}\left(\frac{a\,x^{11}}{11}+\frac{x\,(128\,a\,d^5-176\,b\,c\,d^4)}{3465\,c^5}+\frac{x^9\,(385\,b\,c^5+35\,a\,d\,c^4)}{3465\,c^5}-\frac{d\,x^7\,(8\,a\,d-11\,b\,c)}{693\,c^2}+\frac{2\,d^2\,x^5\,(8\,a\,d-11\,b\,c)}{1155\,c^3}-\frac{8\,d^3\,x^3\,(8\,a\,d-11\,b\,c)}{3465\,c^4}\right)$$

```
[In] int(x^10*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] (c + d/x^2)^(1/2)*((a*x^11)/11 + (x*(128*a*d^5 - 176*b*c*d^4))/(3465*c^5) +
 (x^9*(385*b*c^5 + 35*a*c^4*d))/(3465*c^5) - (d*x^7*(8*a*d - 11*b*c))/(693*
c^2) + (2*d^2*x^5*(8*a*d - 11*b*c))/(1155*c^3) - (8*d^3*x^3*(8*a*d - 11*b*c
))/(3465*c^4))
```

3.938 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^8 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 117

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^8 \, dx = \frac{8d^2(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^3}{315c^4} - \frac{4d(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^5}{105c^3} + \frac{(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^7}{21c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2} x^9}{9c}$$

```
[Out] 8/315*d^2*(-2*a*d+3*b*c)*(c+d/x^2)^(3/2)*x^3/c^4-4/105*d*(-2*a*d+3*b*c)*(c+
d/x^2)^(3/2)*x^5/c^3+1/21*(-2*a*d+3*b*c)*(c+d/x^2)^(3/2)*x^7/c^2+1/9*a*(c+d
/x^2)^(3/2)*x^9/c
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {464, 277, 270}

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^8 \, dx = \frac{8d^2x^3\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{315c^4} - \frac{4dx^5\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{105c^3} + \frac{x^7\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{21c^2} + \frac{ax^9\left(c+\frac{d}{x^2}\right)^{3/2}}{9c}$$

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^8,x]
```

```
[Out] (8*d^2*(3*b*c - 2*a*d)*(c + d/x^2)^(3/2)*x^3)/(315*c^4) - (4*d*(3*b*c - 2*a
*d)*(c + d/x^2)^(3/2)*x^5)/(105*c^3) + ((3*b*c - 2*a*d)*(c + d/x^2)^(3/2)*x
^7)/(21*c^2) + (a*(c + d/x^2)^(3/2)*x^9)/(9*c)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2} x^9}{9c} + \frac{(9bc-6ad)\int \sqrt{c+\frac{d}{x^2}}x^6\,dx}{9c} \\
&= \frac{(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^7}{21c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2} x^9}{9c} - \frac{(4d(3bc-2ad))\int \sqrt{c+\frac{d}{x^2}}x^4\,dx}{21c^2} \\
&= -\frac{4d(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^5}{105c^3} + \frac{(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^7}{21c^2} \\
&\quad + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2} x^9}{9c} + \frac{(8d^2(3bc-2ad))\int \sqrt{c+\frac{d}{x^2}}x^2\,dx}{105c^3} \\
&= \frac{8d^2(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^3}{315c^4} - \frac{4d(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^5}{105c^3} \\
&\quad + \frac{(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^7}{21c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2} x^9}{9c}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.75

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^8\,dx$$
$$=\frac{\sqrt{c+\frac{d}{x^2}}x(d+cx^2)\left(24bcd^2-16ad^3-36bc^2dx^2+24acd^2x^2+45bc^3x^4-30ac^2dx^4+35ac^3x^6\right)}{315c^4}$$

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^8,x]
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)*(24*b*c*d^2 - 16*a*d^3 - 36*b*c^2*d*x^2 + 24
*a*c*d^2*x^2 + 45*b*c^3*x^4 - 30*a*c^2*d*x^4 + 35*a*c^3*x^6))/(315*c^4)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.76

method	result	size
gosper	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,x\left(35x^6a\,c^3-30a\,c^2d\,x^4+45b\,c^3x^4+24ac\,d^2x^2-36b\,c^2d\,x^2-16a\,d^3+24bc\,d^2\right)\left(c\,x^2+d\right)}{315c^4}$	89
default	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,x\left(35x^6a\,c^3-30a\,c^2d\,x^4+45b\,c^3x^4+24ac\,d^2x^2-36b\,c^2d\,x^2-16a\,d^3+24bc\,d^2\right)\left(c\,x^2+d\right)}{315c^4}$	89
risch	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,x\left(35a\,x^8c^4+5a\,c^3d\,x^6+45b\,c^4x^6-6a\,c^2d^2x^4+9b\,c^3d\,x^4+8ac\,d^3x^2-12b\,c^2d^2x^2-16a\,d^4+24bc\,d^3\right)}{315c^4}$	106
trager	$\frac{\left(35a\,x^8c^4+5a\,c^3d\,x^6+45b\,c^4x^6-6a\,c^2d^2x^4+9b\,c^3d\,x^4+8ac\,d^3x^2-12b\,c^2d^2x^2-16a\,d^4+24bc\,d^3\right)x\sqrt{-\frac{-c\,x^2-d}{x^2}}}{315c^4}$	110

```
[In] int((a+b/x^2)*x^8*(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/315*((c*x^2+d)/x^2)^(1/2)*x*(35*a*c^3*x^6-30*a*c^2*d*x^4+45*b*c^3*x^4+24*
a*c*d^2*x^2-36*b*c^2*d*x^2-16*a*d^3+24*b*c*d^2)*(c*x^2+d)/c^4
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^8\,dx$$
$$=\frac{\left(35\,ac^4x^9+5\,(9\,bc^4+ac^3d)x^7+3\,(3\,bc^3d-2\,ac^2d^2)x^5-4\,(3\,bc^2d^2-2\,acd^3)x^3+8\,(3\,bcd^3-2\,ad^4)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{315\,c^4}$$

```
[In] integrate((a+b/x^2)*x^8*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*a*c^4*x^9 + 5*(9*b*c^4 + a*c^3*d)*x^7 + 3*(3*b*c^3*d - 2*a*c^2*d^
2)*x^5 - 4*(3*b*c^2*d^2 - 2*a*c*d^3)*x^3 + 8*(3*b*c*d^3 - 2*a*d^4)*x)*sqrt(
(c*x^2 + d)/x^2)/c^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(112) = 224$.

Time = 2.25 (sec) , antiderivative size = 910, normalized size of antiderivative = 7.78

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^8\,dx = \frac{35ac^7d^{\frac{19}{2}}x^{14}\sqrt{\frac{cx^2}{d}+1}}{315c^7d^9x^6+945c^6d^{10}x^4+945c^5d^{11}x^2+315c^4d^{12}} + \frac{110ac^6d^{\frac{21}{2}}x^{12}\sqrt{\frac{cx^2}{d}+1}}{315c^7d^9x^6+945c^6d^{10}x^4+945c^5d^{11}x^2+315c^4d^{12}} + \frac{114ac^5d^{\frac{23}{2}}x^{10}\sqrt{\frac{cx^2}{d}+1}}{315c^7d^9x^6+945c^6d^{10}x^4+945c^5d^{11}x^2+315c^4d^{12}} + \frac{40ac^4d^{\frac{25}{2}}x^8\sqrt{\frac{cx^2}{d}+1}}{315c^7d^9x^6+945c^6d^{10}x^4+945c^5d^{11}x^2+315c^4d^{12}} - \frac{5ac^3d^{\frac{27}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{315c^7d^9x^6+945c^6d^{10}x^4+945c^5d^{11}x^2+315c^4d^{12}} - \frac{30ac^2d^{\frac{29}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{315c^7d^9x^6+945c^6d^{10}x^4+945c^5d^{11}x^2+315c^4d^{12}} - \frac{40acd^{\frac{31}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{315c^7d^9x^6+945c^6d^{10}x^4+945c^5d^{11}x^2+315c^4d^{12}} - \frac{16ad^{\frac{33}{2}}\sqrt{\frac{cx^2}{d}+1}}{315c^7d^9x^6+945c^6d^{10}x^4+945c^5d^{11}x^2+315c^4d^{12}} + \frac{15bc^5d^{\frac{9}{2}}x^{10}\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} + \frac{33bc^4d^{\frac{11}{2}}x^8\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} + \frac{17bc^3d^{\frac{13}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} + \frac{3bc^2d^{\frac{15}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} + \frac{12bcd^{\frac{17}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} + \frac{8bd^{\frac{19}{2}}\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}$$

```
[In] integrate((a+b/x**2)*x**8*(c+d/x**2)**(1/2),x)
```

```
[Out] 35*a*c**7*d**(19/2)*x**14*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6
*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 110*a*c**6*d**(21/2)*
```

```
x**12*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c*
*5*d**11*x**2 + 315*c**4*d**12) + 114*a*c**5*d**(23/2)*x**10*sqrt(c*x**2/d
+ 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*
c**4*d**12) + 40*a*c**4*d**(25/2)*x**8*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x*
*6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) - 5*a*c**3
*d**(27/2)*x**6*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**
4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) - 30*a*c**2*d**(29/2)*x**4*sqrt(c
*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**
2 + 315*c**4*d**12) - 40*a*c*d**(31/2)*x**2*sqrt(c*x**2/d + 1)/(315*c**7*d*
*9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) - 16*
a*d**(33/2)*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 +
945*c**5*d**11*x**2 + 315*c**4*d**12) + 15*b*c**5*d**(9/2)*x**10*sqrt(c*x**
2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 33*b*c
**4*d**(11/2)*x**8*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x
**2 + 105*c**3*d**6) + 17*b*c**3*d**(13/2)*x**6*sqrt(c*x**2/d + 1)/(105*c**
5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 3*b*c**2*d**(15/2)*x**4
*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**
6) + 12*b*c*d**(17/2)*x**2*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**
4*d**5*x**2 + 105*c**3*d**6) + 8*b*d**(19/2)*sqrt(c*x**2/d + 1)/(105*c**5*d
**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^8\,dx = \frac{\left(15\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}x^7-42\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}dx^5+35\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2x^3\right)b}{105\,c^3} + \frac{\left(35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}x^9-135\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}dx^7+189\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^2x^5-105\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^3x^3\right)a}{315\,c^4}$$

```
[In] integrate((a+b/x^2)*x^8*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/105*(15*(c + d/x^2)^(7/2)*x^7 - 42*(c + d/x^2)^(5/2)*d*x^5 + 35*(c + d/x^
2)^(3/2)*d^2*x^3)*b/c^3 + 1/315*(35*(c + d/x^2)^(9/2)*x^9 - 135*(c + d/x^2)
^(7/2)*d*x^7 + 189*(c + d/x^2)^(5/2)*d^2*x^5 - 105*(c + d/x^2)^(3/2)*d^3*x^
3)*a/c^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.20

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^8 \, dx = -\frac{8 \left(3 \, bcd^{\frac{7}{2}} - 2 \, ad^{\frac{9}{2}}\right) \mathrm{sgn}(x)}{315 \, c^4} + \frac{35 \, (cx^2+d)^{\frac{9}{2}} a \mathrm{sgn}(x) + 45 \, (cx^2+d)^{\frac{7}{2}} bc \mathrm{sgn}(x) - 135 \, (cx^2+d)^{\frac{7}{2}} ad \mathrm{sgn}(x) - 126 \, (cx^2+d)^{\frac{5}{2}} bcd \mathrm{sgn}(x) + 1}{315 \, c^4}$$

[In] integrate((a+b/x^2)*x^8*(c+d/x^2)^(1/2),x, algorithm="giac")

[Out] -8/315*(3*b*c*d^(7/2) - 2*a*d^(9/2))*sgn(x)/c^4 + 1/315*(35*(c*x^2 + d)^(9/2)*a*sgn(x) + 45*(c*x^2 + d)^(7/2)*b*c*sgn(x) - 135*(c*x^2 + d)^(7/2)*a*d*sgn(x) - 126*(c*x^2 + d)^(5/2)*b*c*d*sgn(x) + 189*(c*x^2 + d)^(5/2)*a*d^2*sgn(x) + 105*(c*x^2 + d)^(3/2)*b*c*d^2*sgn(x) - 105*(c*x^2 + d)^(3/2)*a*d^3*sgn(x))/c^4

Mupad [B] (verification not implemented)

Time = 8.91 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.83

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^8 \, dx = \sqrt{c+\frac{d}{x^2}} \left(\frac{a\,x^9}{9} - \frac{x\,(16\,a\,d^4 - 24\,b\,c\,d^3)}{315\,c^4} + \frac{x^7\,(45\,b\,c^4 + 5\,a\,d\,c^3)}{315\,c^4} - \frac{d\,x^5\,(2\,a\,d - 3\,b\,c)}{105\,c^2} + \frac{4\,d^2\,x^3\,(2\,a\,d - 3\,b\,c)}{315\,c^3}\right)$$

[In] int(x^8*(a + b/x^2)*(c + d/x^2)^(1/2),x)

[Out] (c + d/x^2)^(1/2)*((a*x^9)/9 - (x*(16*a*d^4 - 24*b*c*d^3))/(315*c^4) + (x^7*(45*b*c^4 + 5*a*c^3*d))/(315*c^4) - (d*x^5*(2*a*d - 3*b*c))/(105*c^2) + (4*d^2*x^3*(2*a*d - 3*b*c))/(315*c^3))

3.939 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^6 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^6 \, dx = -\frac{2d(7bc-4ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^3}{105c^3} + \frac{(7bc-4ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^5}{35c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2} x^7}{7c}$$

```
[Out] -2/105*d*(-4*a*d+7*b*c)*(c+d/x^2)^(3/2)*x^3/c^3+1/35*(-4*a*d+7*b*c)*(c+d/x^
2)^(3/2)*x^5/c^2+1/7*a*(c+d/x^2)^(3/2)*x^7/c
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {464, 277, 270}

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^6 \, dx = -\frac{2dx^3\left(c+\frac{d}{x^2}\right)^{3/2}(7bc-4ad)}{105c^3} + \frac{x^5\left(c+\frac{d}{x^2}\right)^{3/2}(7bc-4ad)}{35c^2} + \frac{ax^7\left(c+\frac{d}{x^2}\right)^{3/2}}{7c}$$

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^6,x]
```

```
[Out] (-2*d*(7*b*c - 4*a*d)*(c + d/x^2)^(3/2)*x^3)/(105*c^3) + ((7*b*c - 4*a*d)*(
c + d/x^2)^(3/2)*x^5)/(35*c^2) + (a*(c + d/x^2)^(3/2)*x^7)/(7*c)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{7c}+\frac{(7bc-4ad)\int\sqrt{c+\frac{d}{x^2}}x^4\,dx}{7c}\\
&= \frac{(7bc-4ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{35c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{7c}-\frac{(2d(7bc-4ad))\int\sqrt{c+\frac{d}{x^2}}x^2\,dx}{35c^2}\\
&= -\frac{2d(7bc-4ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{105c^3}+\frac{(7bc-4ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{35c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{7c}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.76

$$\int\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^6\,dx = \frac{\sqrt{c+\frac{d}{x^2}}x(d+cx^2)\left(-14bcd+8ad^2+21bc^2x^2-12acdx^2+15ac^2x^4\right)}{105c^3}$$

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^6,x]
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)*(-14*b*c*d + 8*a*d^2 + 21*b*c^2*x^2 - 12*a*c
*d*x^2 + 15*a*c^2*x^4))/(105*c^3)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77

method	result	size
gosper	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x\left(15a\,x^{4}c^{2}-12acd\,x^{2}+21b\,c^{2}x^{2}+8a\,d^{2}-14bcd\right)\left(c\,x^{2}+d\right)}{105c^{3}}$	65
default	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x\left(15a\,x^{4}c^{2}-12acd\,x^{2}+21b\,c^{2}x^{2}+8a\,d^{2}-14bcd\right)\left(c\,x^{2}+d\right)}{105c^{3}}$	65
risch	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x\left(15x^{6}a\,c^{3}+3a\,c^{2}d\,x^{4}+21b\,c^{3}x^{4}-4ac\,d^{2}x^{2}+7b\,c^{2}d\,x^{2}+8a\,d^{3}-14bc\,d^{2}\right)}{105c^{3}}$	82
trager	$\frac{\left(15x^{6}a\,c^{3}+3a\,c^{2}d\,x^{4}+21b\,c^{3}x^{4}-4ac\,d^{2}x^{2}+7b\,c^{2}d\,x^{2}+8a\,d^{3}-14bc\,d^{2}\right)x\sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{105c^{3}}$	86

```
[In] int((a+b/x^2)*x^6*(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/105*((c*x^2+d)/x^2)^(1/2)*x*(15*a*c^2*x^4-12*a*c*d*x^2+21*b*c^2*x^2+8*a*d
^2-14*b*c*d)*(c*x^2+d)/c^3
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^6\,dx$$
$$=\frac{(15\,ac^3x^7+3\,(7\,bc^3+ac^2d)x^5+(7\,bc^2d-4\,acd^2)x^3-2\,(7\,bcd^2-4\,ad^3)x)\sqrt{\frac{cx^2+d}{x^2}}}{105\,c^3}$$

```
[In] integrate((a+b/x^2)*x^6*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*a*c^3*x^7 + 3*(7*b*c^3 + a*c^2*d)*x^5 + (7*b*c^2*d - 4*a*c*d^2)*x
^3 - 2*(7*b*c*d^2 - 4*a*d^3)*x)*sqrt((c*x^2 + d)/x^2)/c^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(78) = 156$.

Time = 1.65 (sec) , antiderivative size = 422, normalized size of antiderivative = 5.02

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^6\,dx = \frac{15ac^5d^{\frac{9}{2}}x^{10}\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} + \frac{33ac^4d^{\frac{11}{2}}x^8\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} + \frac{17ac^3d^{\frac{13}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} + \frac{3ac^2d^{\frac{15}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} + \frac{12acd^{\frac{17}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} + \frac{8ad^{\frac{19}{2}}\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} + \frac{b\sqrt{d}x^4\sqrt{\frac{cx^2}{d}+1}}{5} + \frac{bd^{\frac{3}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{15c} - \frac{2bd^{\frac{5}{2}}\sqrt{\frac{cx^2}{d}+1}}{15c^2}$$

```
[In] integrate((a+b/x**2)*x**6*(c+d/x**2)**(1/2),x)
```

```
[Out] 15*a*c**5*d**(9/2)*x**10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*
d**5*x**2 + 105*c**3*d**6) + 33*a*c**4*d**(11/2)*x**8*sqrt(c*x**2/d + 1)/(1
05*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 17*a*c**3*d**(13/
2)*x**6*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c
**3*d**6) + 3*a*c**2*d**(15/2)*x**4*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4
+ 210*c**4*d**5*x**2 + 105*c**3*d**6) + 12*a*c*d**(17/2)*x**2*sqrt(c*x**2/d
 + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 8*a*d**(1
9/2)*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3
*d**6) + b*sqrt(d)*x**4*sqrt(c*x**2/d + 1)/5 + b*d**(3/2)*x**2*sqrt(c*x**2/
d + 1)/(15*c) - 2*b*d**(5/2)*sqrt(c*x**2/d + 1)/(15*c**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^6\,dx = \frac{\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5 - 5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}dx^3\right)b}{15\,c^2} + \frac{\left(15\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}x^7 - 42\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}dx^5 + 35\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2x^3\right)a}{105\,c^3}$$

```
[In] integrate((a+b/x^2)*x^6*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*(c + d/x^2)^(5/2)*x^5 - 5*(c + d/x^2)^(3/2)*d*x^3)*b/c^2 + 1/105*(1
5*(c + d/x^2)^(7/2)*x^7 - 42*(c + d/x^2)^(5/2)*d*x^5 + 35*(c + d/x^2)^(3/2)
*d^2*x^3)*a/c^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.25

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^6\,dx = \frac{2\left(7\,bcd^{\frac{5}{2}}-4\,ad^{\frac{7}{2}}\right)\mathrm{sgn}(x)}{105\,c^3} + \frac{15\,(cx^2+d)^{\frac{7}{2}}a\mathrm{sgn}(x)+21\,(cx^2+d)^{\frac{5}{2}}bc\mathrm{sgn}(x)-42\,(cx^2+d)^{\frac{5}{2}}ad\mathrm{sgn}(x)-35\,(cx^2+d)^{\frac{3}{2}}bcd\mathrm{sgn}(x)+35}{105\,c^3}$$

```
[In] integrate((a+b/x^2)*x^6*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(7*b*c*d^(5/2) - 4*a*d^(7/2))*sgn(x)/c^3 + 1/105*(15*(c*x^2 + d)^(7/2
)*a*sgn(x) + 21*(c*x^2 + d)^(5/2)*b*c*sgn(x) - 42*(c*x^2 + d)^(5/2)*a*d*sgn
(x) - 35*(c*x^2 + d)^(3/2)*b*c*d*sgn(x) + 35*(c*x^2 + d)^(3/2)*a*d^2*sgn(x)
)/c^3
```

Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^6\,dx = \sqrt{c+\frac{d}{x^2}}\left(\frac{a\,x^7}{7}+\frac{x\,(8\,a\,d^3-14\,b\,c\,d^2)}{105\,c^3}+\frac{x^5\,(21\,b\,c^3+3\,a\,d\,c^2)}{105\,c^3} - \frac{d\,x^3\,(4\,a\,d-7\,b\,c)}{105\,c^2}\right)$$

```
[In] int(x^6*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] (c + d/x^2)^(1/2)*((a*x^7)/7 + (x*(8*a*d^3 - 14*b*c*d^2))/(105*c^3) + (x^5*
(21*b*c^3 + 3*a*c^2*d))/(105*c^3) - (d*x^3*(4*a*d - 7*b*c))/(105*c^2))
```

3.940 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^4 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^4 \, dx=\frac{(5bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^3}{15c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2} x^5}{5c}$$

```
[Out] 1/15*(-2*a*d+5*b*c)*(c+d/x^2)^(3/2)*x^3/c^2+1/5*a*(c+d/x^2)^(3/2)*x^5/c
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {464, 270}

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^4 \, dx=\frac{x^3\left(c+\frac{d}{x^2}\right)^{3/2} (5bc-2ad)}{15c^2}+\frac{ax^5\left(c+\frac{d}{x^2}\right)^{3/2}}{5c}$$

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^4,x]
```

```
[Out] ((5*b*c - 2*a*d)*(c + d/x^2)^(3/2)*x^3)/(15*c^2) + (a*(c + d/x^2)^(3/2)*x^5
)/(5*c)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{5c}+\frac{(5bc-2ad)\int\sqrt{c+\frac{d}{x^2}}x^2\,dx}{5c}\\ &= \frac{(5bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{15c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{5c}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^4\,dx=\frac{\sqrt{c+\frac{d}{x^2}}x(d+cx^2)\,(5bc-2ad+3acx^2)}{15c^2}$$

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^4,x]
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)*(5*b*c - 2*a*d + 3*a*c*x^2))/(15*c^2)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,x\left(3ac\,x^2-2ad+5bc\right)\left(c\,x^2+d\right)}{15c^2}$	43
default	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,x\left(3ac\,x^2-2ad+5bc\right)\left(c\,x^2+d\right)}{15c^2}$	43
risch	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,x\left(3a\,x^4c^2+acd\,x^2+5b\,c^2x^2-2a\,d^2+5bcd\right)}{15c^2}$	57
trager	$\frac{\left(3a\,x^4c^2+acd\,x^2+5b\,c^2x^2-2a\,d^2+5bcd\right)x\sqrt{-\frac{-c\,x^2-d}{x^2}}}{15c^2}$	61

```
[In] int((a+b/x^2)*x^4*(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/15*((c*x^2+d)/x^2)^(1/2)*x*(3*a*c*x^2-2*a*d+5*b*c)*(c*x^2+d)/c^2
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^4\,dx=\frac{\left(3\,ac^2x^5+\left(5\,bc^2+acd\right)x^3+\left(5\,bcd-2\,ad^2\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\,c^2}$$

```
[In] integrate((a+b/x^2)*x^4*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*a*c^2*x^5 + (5*b*c^2 + a*c*d)*x^3 + (5*b*c*d - 2*a*d^2)*x)*sqrt((c*
x^2 + d)/x^2)/c^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(46) = 92.

Time = 1.23 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.25

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^4\,dx=\frac{a\sqrt{d}x^4\sqrt{\frac{cx^2}{d}+1}}{5}+\frac{ad^{\frac{3}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{15c}-\frac{2ad^{\frac{5}{2}}\sqrt{\frac{cx^2}{d}+1}}{15c^2}+\frac{b\sqrt{d}x^2\sqrt{\frac{cx^2}{d}+1}}{3}+\frac{bd^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c}$$

```
[In] integrate((a+b/x**2)*x**4*(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(d)*x**4*sqrt(c*x**2/d + 1)/5 + a*d**(3/2)*x**2*sqrt(c*x**2/d + 1)/(1
5*c) - 2*a*d**(5/2)*sqrt(c*x**2/d + 1)/(15*c**2) + b*sqrt(d)*x**2*sqrt(c*x*
*2/d + 1)/3 + b*d**(3/2)*sqrt(c*x**2/d + 1)/(3*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^4\,dx=\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3}{3\,c}+\frac{\left(3\,\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5-5\,\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}dx^3\right)a}{15\,c^2}$$

```
[In] integrate((a+b/x^2)*x^4*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*b*(c + d/x^2)^(3/2)*x^3/c + 1/15*(3*(c + d/x^2)^(5/2)*x^5 - 5*(c + d/x^
2)^(3/2)*d*x^3)*a/c^2
```

Giac [A] (verification not implemented)

none

Time = 0.62 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.36

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^4\,dx$$
$$=-\frac{\left(5\,bcd^{\frac{3}{2}}-2\,ad^{\frac{5}{2}}\right)\operatorname{sgn}(x)}{15\,c^2}$$
$$+\frac{3\,(cx^2+d)^{\frac{5}{2}}a\operatorname{sgn}(x)+5\,(cx^2+d)^{\frac{3}{2}}bc\operatorname{sgn}(x)-5\,(cx^2+d)^{\frac{3}{2}}ad\operatorname{sgn}(x)}{15\,c^2}$$

```
[In] integrate((a+b/x^2)*x^4*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/15*(5*b*c*d^(3/2) - 2*a*d^(5/2))*sgn(x)/c^2 + 1/15*(3*(c*x^2 + d)^(5/2)*
a*sgn(x) + 5*(c*x^2 + d)^(3/2)*b*c*sgn(x) - 5*(c*x^2 + d)^(3/2)*a*d*sgn(x))
/c^2
```

Mupad [B] (verification not implemented)

Time = 8.91 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^4\,dx=\sqrt{c+\frac{d}{x^2}}\left(\frac{a\,x^5}{5}-\frac{x\,(2\,a\,d^2-5\,b\,c\,d)}{15\,c^2}+\frac{x^3\,(5\,b\,c^2+a\,d\,c)}{15\,c^2}\right)$$

```
[In] int(x^4*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] (c + d/x^2)^(1/2)*((a*x^5)/5 - (x*(2*a*d^2 - 5*b*c*d))/(15*c^2) + (x^3*(5*b
*c^2 + a*c*d))/(15*c^2))
```

3.941 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 66

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^2 \, dx = b\sqrt{c+\frac{d}{x^2}}x+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3c}-b\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)$$

```
[Out] 1/3*a*(c+d/x^2)^(3/2)*x^3/c-b*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))*d^(1/2)+b*
x*(c+d/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {462, 248, 283, 223, 212}

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^2 \, dx = \frac{ax^3\left(c+\frac{d}{x^2}\right)^{3/2}}{3c}-b\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)+bx\sqrt{c+\frac{d}{x^2}}$$

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^2,x]
```

```
[Out] b*Sqrt[c + d/x^2]*x + (a*(c + d/x^2)^(3/2)*x^3)/(3*c) - b*Sqrt[d]*ArcTanh[S
qrt[d]/(Sqrt[c + d/x^2]*x)]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3c}+b\int\sqrt{c+\frac{d}{x^2}}\,dx\\
&= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3c}-b\text{Subst}\left(\int\frac{\sqrt{c+dx^2}}{x^2}\,dx,x,\frac{1}{x}\right)\\
&= b\sqrt{c+\frac{d}{x^2}}x+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3c}-(bd)\text{Subst}\left(\int\frac{1}{\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right)\\
&= b\sqrt{c+\frac{d}{x^2}}x+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3c}-(bd)\text{Subst}\left(\int\frac{1}{1-dx^2}\,dx,x,\frac{1}{\sqrt{c+\frac{d}{x^2}}x}\right)\\
&= b\sqrt{c+\frac{d}{x^2}}x+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3c}-b\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^2\,dx=\frac{1}{3}\sqrt{c+\frac{d}{x^2}}x\left(3b+a\left(\frac{d}{c}+x^2\right)-\frac{3b\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{\sqrt{d+cx^2}}\right)$$

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^2,x]
[Out] (Sqrt[c + d/x^2]*x*(3*b + a*(d/c + x^2) - (3*b*Sqrt[d]*ArcTanh[Sqrt[d + c*x
^2]/Sqrt[d]])/Sqrt[d + c*x^2]))/3
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.26

method	result	size
default	$-\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x\left(3\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)bc-a\left(c\,x^{2}+d\right)^{\frac{3}{2}}-3\sqrt{c\,x^{2}+d}\,bc\right)}{3\sqrt{c\,x^{2}+d}\,c}$	83

```
[In] int((a+b/x^2)*x^2*(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*((c*x^2+d)/x^2)^(1/2)*x*(3*d^(1/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)
*b*c-a*(c*x^2+d)^(3/2)-3*(c*x^2+d)^(1/2)*b*c)/(c*x^2+d)^(1/2)/c
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.36

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^2\,dx$$
$$=\left[\frac{3\,bc\sqrt{d}\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\left(acx^3+(3\,bc+ad)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{6\,c},\frac{3\,bc\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+(}{3\,c}\right.$$

```
[In] integrate((a+b/x^2)*x^2*(c+d/x^2)^(1/2),x, algorithm="fricas")
[Out] [1/6*(3*b*c*sqrt(d)*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/
x^2) + 2*(a*c*x^3 + (3*b*c + a*d)*x)*sqrt((c*x^2 + d)/x^2))/c, 1/3*(3*b*c*s
qrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (a*c*x^3 + (
3*b*c + a*d)*x)*sqrt((c*x^2 + d)/x^2))/c]
```

Sympy [A] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.62

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^2\,dx = \frac{a\sqrt{d}x^2\sqrt{\frac{cx^2}{d}+1}}{3}+\frac{ad^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c}+\frac{b\sqrt{c}x}{\sqrt{1+\frac{d}{cx^2}}} - b\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)+\frac{bd}{\sqrt{c}x\sqrt{1+\frac{d}{cx^2}}}$$

```
[In] integrate((a+b/x**2)*x**2*(c+d/x**2)**(1/2),x)
[Out] a*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/3 + a*d**(3/2)*sqrt(c*x**2/d + 1)/(3*c) +
 b*sqrt(c)*x/sqrt(1 + d/(c*x**2)) - b*sqrt(d)*asinh(sqrt(d)/(sqrt(c)*x)) +
b*d/(sqrt(c)*x*sqrt(1 + d/(c*x**2)))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^2\,dx = \frac{a\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3}{3\,c} + \frac{1}{2}\left(2\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)\right)b$$

```
[In] integrate((a+b/x^2)*x^2*(c+d/x^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*a*(c + d/x^2)^(3/2)*x^3/c + 1/2*(2*sqrt(c + d/x^2)*x + sqrt(d)*log((sqr
t(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d))))*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(54) = 108.

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.76

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^2\,dx = \frac{bd\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)\operatorname{sgn}(x)}{\sqrt{-d}} - \frac{\left(3\,bcd\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)+3\,bc\sqrt{-d}\sqrt{d}+a\sqrt{-d}d^{\frac{3}{2}}\right)\operatorname{sgn}(x)}{3\,c\sqrt{-d}} + \frac{(cx^2+d)^{\frac{3}{2}}ac^2\operatorname{sgn}(x)+3\sqrt{cx^2+d}bc^3\operatorname{sgn}(x)}{3\,c^3}$$

```
[In] integrate((a+b/x^2)*x^2*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] b*d*arctan(sqrt(c*x^2 + d)/sqrt(-d))*sgn(x)/sqrt(-d) - 1/3*(3*b*c*d*arctan(
sqrt(d)/sqrt(-d)) + 3*b*c*sqrt(-d)*sqrt(d) + a*sqrt(-d)*d^(3/2))*sgn(x)/(c*
sqrt(-d)) + 1/3*((c*x^2 + d)^(3/2)*a*c^2*sgn(x) + 3*sqrt(c*x^2 + d)*b*c^3*s
gn(x))/c^3
```

Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.21

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^2\,dx=b\,x\,\sqrt{c+\frac{d}{x^2}}+\frac{a\,x\,\sqrt{c+\frac{d}{x^2}}\,\left(c\,x^2+d\right)}{3\,c}+\frac{b\,\sqrt{d}\,\mathrm{asin}\left(\frac{\sqrt{d}\,1\mathrm{i}}{\sqrt{c}\,x}\right)\,\sqrt{c+\frac{d}{x^2}}\,1\mathrm{i}}{\sqrt{c}\,\sqrt{\frac{d}{c\,x^2}+1}}$$

```
[In] int(x^2*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] b*x*(c + d/x^2)^(1/2) + (a*x*(c + d/x^2)^(1/2)*(d + c*x^2))/(3*c) + (b*d^(1
/2)*asin((d^(1/2)*1i)/(c^(1/2)*x))*(c + d/x^2)^(1/2)*1i)/(c^(1/2)*(d/(c*x^2
) + 1)^(1/2))
```

3.942 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 85

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}\,dx=-\frac{(bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x}{c}-\frac{(bc+2ad)\text{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{2\sqrt{d}}$$

```
[Out] a*(c+d/x^2)^(3/2)*x/c-1/2*(2*a*d+b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^
(1/2)-1/2*(2*a*d+b*c)*(c+d/x^2)^(1/2)/c/x
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {382, 464, 201, 223, 212}

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}\,dx=-\frac{(2ad+bc)\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2\sqrt{d}}-\frac{\sqrt{c+\frac{d}{x^2}}(2ad+bc)}{2cx}+\frac{ax\left(c+\frac{d}{x^2}\right)^{3/2}}{c}$$

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2],x]
```

```
[Out] -1/2*((b*c + 2*a*d)*Sqrt[c + d/x^2])/(c*x) + (a*(c + d/x^2)^(3/2)*x)/c - ((
b*c + 2*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(2*Sqrt[d])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{(a+bx^2)\sqrt{c+dx^2}}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x}{c} + \frac{(-bc-2ad)\text{Subst}\left(\int \sqrt{c+dx^2}\,dx, x, \frac{1}{x}\right)}{c}\\
&= -\frac{(bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x}{c} + \frac{1}{2}(-bc-2ad)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)
\end{aligned}
$$

$$=-\frac{(bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x}{c}+\frac{1}{2}(-bc-2ad)\text{Subst}\left(\int\frac{1}{1-dx^2}\,dx,x,\frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right)$$

$$=-\frac{(bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x}{c}-\frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{2\sqrt{d}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\int\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,dx=\frac{\sqrt{c+\frac{d}{x^2}}\left(-b+2ax^2-\frac{(bc+2ad)x^2\text{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{d+cx^2}}\right)}{2x}$$

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2],x]
```

```
[Out] (Sqrt[c + d/x^2]*(-b + 2*a*x^2 - ((b*c + 2*a*d)*x^2*ArcTanh[Sqrt[d + c*x^2]
/Sqrt[d]])/(Sqrt[d]*Sqrt[d + c*x^2])))/(2*x)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.09

method	result	si
risch	$-\frac{b\sqrt{\frac{c\,x^2+d}{x^2}}}{2x}+\frac{\left(a\sqrt{c\,x^2+d}-\frac{(2ad+bc)\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)}{2\sqrt{d}}\right)\sqrt{\frac{c\,x^2+d}{x^2}}\,x}{\sqrt{c\,x^2+d}}$	9
default	$-\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\left(2d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)a\,x^2+\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)b c\,x^2-2\sqrt{c\,x^2+d}\,a d\,x^2-\sqrt{c\,x^2+d}\,b c\,x^2+\left(c\,x^2+d\right)^{\frac{3}{2}}b\right)}{2x\sqrt{c\,x^2+d}\,d}$	1

```
[In] int((a+b/x^2)*(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b/x*((c*x^2+d)/x^2)^(1/2)+(a*(c*x^2+d)^(1/2)-1/2*(2*a*d+b*c)/d^(1/2)*l
n((2*d+2*d^(1/2)*(c*x^2+d)^(1/2))/x))*((c*x^2+d)/x^2)^(1/2)*x/(c*x^2+d)^(1/
2)
```

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.93

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,dx = \left[\frac{(bc+2\,ad)\sqrt{d}x\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,(2\,adx^2-bd)\sqrt{\frac{cx^2+d}{x^2}}}{4\,dx}, \frac{(bc+2\,ad)\sqrt{-d}x\arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+ \cdots}{2\,dx}\right]$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b*c + 2*a*d)*sqrt(d)*x*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^
2) + 2*d)/x^2) + 2*(2*a*d*x^2 - b*d)*sqrt((c*x^2 + d)/x^2))/(d*x), 1/2*((b*
c + 2*a*d)*sqrt(-d)*x*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d))
+ (2*a*d*x^2 - b*d)*sqrt((c*x^2 + d)/x^2))/(d*x)]
```

Sympy [A] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.26

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,dx = \frac{a\sqrt{c}x}{\sqrt{1+\frac{d}{cx^2}}} - a\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right) + \frac{ad}{\sqrt{c}x\sqrt{1+\frac{d}{cx^2}}} - \frac{b\sqrt{c}\sqrt{1+\frac{d}{cx^2}}}{2x} - \frac{bc\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2\sqrt{d}}$$

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(c)*x/sqrt(1 + d/(c*x**2)) - a*sqrt(d)*asinh(sqrt(d)/(sqrt(c)*x)) + a
*d/(sqrt(c)*x*sqrt(1 + d/(c*x**2))) - b*sqrt(c)*sqrt(1 + d/(c*x**2))/(2*x)
- b*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*sqrt(d))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.56

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,dx = \frac{1}{2}\left(2\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)\right)a - \frac{1}{4}\left(\frac{2\sqrt{c+\frac{d}{x^2}}cx}{\left(c+\frac{d}{x^2}\right)x^2-d}-\frac{c\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{\sqrt{d}}\right)b$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sqrt(c + d/x^2)*x + sqrt(d)*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(
c + d/x^2)*x + sqrt(d))))*a - 1/4*(2*sqrt(c + d/x^2)*c*x/((c + d/x^2)*x^2 -
 d) - c*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/sq
rt(d))*b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,dx = \frac{2\sqrt{cx^2+d}ac\,\mathrm{sgn}(x)+\frac{\left(bc^2\mathrm{sgn}(x)+2\,acd\,\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}}-\frac{\sqrt{cx^2+d}bc\,\mathrm{sgn}(x)}{x^2}}{2\,c}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(c*x^2 + d)*a*c*sgn(x) + (b*c^2*sgn(x) + 2*a*c*d*sgn(x))*arctan(
sqrt(c*x^2 + d)/sqrt(-d))/sqrt(-d) - sqrt(c*x^2 + d)*b*c*sgn(x)/x^2)/c
```

Mupad [B] (verification not implemented)

Time = 9.63 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.14

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,dx = a\,x\,\sqrt{c+\frac{d}{x^2}}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{2\,x}-\frac{b\,c\,\ln\left(\sqrt{c+\frac{d}{x^2}}+\frac{\sqrt{d}}{x}\right)}{2\,\sqrt{d}}+\frac{a\,\sqrt{d}\,\mathrm{asin}\left(\frac{\sqrt{d}\,1\mathrm{i}}{\sqrt{c}\,x}\right)\,\sqrt{c+\frac{d}{x^2}}\,1\mathrm{i}}{\sqrt{c}\,\sqrt{\frac{d}{c\,x^2}+1}}$$

```
[In] int((a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] a*x*(c + d/x^2)^(1/2) - (b*(c + d/x^2)^(1/2))/(2*x) - (b*c*log((c + d/x^2)^
(1/2) + d^(1/2)/x))/(2*d^(1/2)) + (a*d^(1/2)*asin((d^(1/2)*1i)/(c^(1/2)*x))
*(c + d/x^2)^(1/2)*1i)/(c^(1/2)*(d/(c*x^2) + 1)^(1/2))
```

3.943 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 91

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx=\frac{(bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}+\frac{c(bc-4ad)\operatorname{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{8d^{3/2}}$$

```
[Out] -1/4*b*(c+d/x^2)^(3/2)/d/x+1/8*c*(-4*a*d+b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(
1/2))/d^(3/2)+1/8*(-4*a*d+b*c)*(c+d/x^2)^(1/2)/d/x
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {470, 342, 201, 223, 212}

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx=\frac{c(bc-4ad)\operatorname{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{8d^{3/2}}+\frac{\sqrt{c+\frac{d}{x^2}}(bc-4ad)}{8dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}$$

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^2,x]
```

```
[Out] ((b*c - 4*a*d)*Sqrt[c + d/x^2])/(8*d*x) - (b*(c + d/x^2)^(3/2))/(4*d*x) + (
c*(b*c - 4*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(8*d^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
```

```
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}+\frac{(-bc+4ad)\int\frac{\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx}{4d}\\
&= -\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}-\frac{(-bc+4ad)\text{Subst}\left(\int\sqrt{c+dx^2}\,dx,x,\frac{1}{x}\right)}{4d}\\
&= \frac{(bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}+\frac{(c(bc-4ad))\text{Subst}\left(\int\frac{1}{\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right)}{8d}\\
&= \frac{(bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}+\frac{(c(bc-4ad))\text{Subst}\left(\int\frac{1}{1-dx^2}\,dx,x,\frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{8d}\\
&= \frac{(bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}+\frac{c(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{8d^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx = \frac{\sqrt{c+\frac{d}{x^2}}\left(-\sqrt{d}(2bd+bcx^2+4adx^2)+\frac{c(bc-4ad)x^4\text{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{\sqrt{d+cx^2}}\right)}{8d^{3/2}x^3}$$

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^2,x]
```

```
[Out] (Sqrt[c + d/x^2]*(-(Sqrt[d]*(2*b*d + b*c*x^2 + 4*a*d*x^2)) + (c*(b*c - 4*a*
d)*x^4*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/Sqrt[d + c*x^2]))/(8*d^(3/2)*x^3)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{\left(4ad\,x^2+cb\,x^2+2bd\right)\sqrt{\frac{c\,x^2+d}{x^2}}}{8x^3d}-\frac{c(4ad-bc)\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)\sqrt{\frac{c\,x^2+d}{x^2}}\,x}{8d^{\frac{3}{2}}\sqrt{c\,x^2+d}}$
default	$-\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\left(4d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)ac\,x^4-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)b\,c^2x^4-4\sqrt{c\,x^2+d}\,acd\,x^4+\sqrt{c\,x^2+d}\,b\,c^2x^4+4\left(c\,x^2+d\right)^{\frac{3}{2}}ad\right.}{8x^3\sqrt{c\,x^2+d}\,d^2}$

```
[In] int((a+b/x^2)*(c+d/x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(4*a*d*x^2+b*c*x^2+2*b*d)/x^3/d*((c*x^2+d)/x^2)^(1/2)-1/8*c*(4*a*d-b*c
)/d^(3/2)*ln((2*d+2*d^(1/2)*(c*x^2+d)^(1/2))/x)*((c*x^2+d)/x^2)^(1/2)*x/(c*
x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.13

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx = \left[-\frac{(bc^2-4\,acd)\sqrt{d}x^3\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,(2\,bd^2+(bcd+4\,ad^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{16\,d^2x^3}, -\frac{(bc^2-4\,acd)\sqrt{-d}x^3\arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+(2\,bd^2+(bcd+4\,ad^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{8\,d^2x^3}\right]$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/16*((b*c^2 - 4*a*c*d)*sqrt(d)*x^3*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2
 + d)/x^2) + 2*d)/x^2) + 2*(2*b*d^2 + (b*c*d + 4*a*d^2)*x^2)*sqrt((c*x^2 +
d)/x^2))/(d^2*x^3), -1/8*((b*c^2 - 4*a*c*d)*sqrt(-d)*x^3*arctan(sqrt(-d)*x*
sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (2*b*d^2 + (b*c*d + 4*a*d^2)*x^2)*sqrt
((c*x^2 + d)/x^2))/(d^2*x^3)]
```

Sympy [A] (verification not implemented)

Time = 3.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.58

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx = -\frac{a\sqrt{c}\sqrt{1+\frac{d}{cx^2}}}{2x} - \frac{ac\,\mathrm{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2\sqrt{d}} - \frac{bc^{\frac{3}{2}}}{8dx\sqrt{1+\frac{d}{cx^2}}} - \frac{3b\sqrt{c}}{8x^3\sqrt{1+\frac{d}{cx^2}}} + \frac{bc^2\,\mathrm{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{8d^{\frac{3}{2}}} - \frac{bd}{4\sqrt{c}x^5\sqrt{1+\frac{d}{cx^2}}}$$

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**2,x)
```

```
[Out] -a*sqrt(c)*sqrt(1 + d/(c*x**2))/(2*x) - a*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*s
qrt(d)) - b*c**(3/2)/(8*d*x*sqrt(1 + d/(c*x**2))) - 3*b*sqrt(c)/(8*x**3*sqr
t(1 + d/(c*x**2))) + b*c**2*asinh(sqrt(d)/(sqrt(c)*x))/(8*d**(3/2)) - b*d/(
4*sqrt(c)*x**5*sqrt(1 + d/(c*x**2)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(75) = 150$.

Time = 0.29 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.12

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx$$
$$= -\frac{1}{4}\left(\frac{2\sqrt{c+\frac{d}{x^2}}cx}{\left(c+\frac{d}{x^2}\right)x^2-d}-\frac{c\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{\sqrt{d}}\right)a$$
$$-\frac{1}{16}\left(\frac{c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2x^3+\sqrt{c+\frac{d}{x^2}}c^2dx\right)}{\left(c+\frac{d}{x^2}\right)^2dx^4-2\left(c+\frac{d}{x^2}\right)d^2x^2+d^3}\right)b$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(c + d/x^2)*c*x/((c + d/x^2)*x^2 - d) - c*log((sqrt(c + d/x^2)*
x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/sqrt(d))*a - 1/16*(c^2*log((sqr
t(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(3/2) + 2*((c +
d/x^2)^(3/2)*c^2*x^3 + sqrt(c + d/x^2)*c^2*d*x)/((c + d/x^2)^2*d*x^4 - 2*(c
 + d/x^2)*d^2*x^2 + d^3))*b
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.43

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx =$$
$$-\frac{\frac{\left(bc^3\mathrm{sgn}(x)-4\,ac^2d\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}d}+\frac{\left(cx^2+d\right)^{\frac{3}{2}}bc^3\mathrm{sgn}(x)+4\left(cx^2+d\right)^{\frac{3}{2}}ac^2d\mathrm{sgn}(x)+\sqrt{cx^2+d}bc^3d\mathrm{sgn}(x)-4\sqrt{cx^2+d}ac^2d^2\mathrm{sgn}(x)}{c^2dx^4}}{8\,c}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/8*((b*c^3*sgn(x) - 4*a*c^2*d*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d))/(s
qrt(-d)*d) + ((c*x^2 + d)^(3/2)*b*c^3*sgn(x) + 4*(c*x^2 + d)^(3/2)*a*c^2*d*
sgn(x) + sqrt(c*x^2 + d)*b*c^3*d*sgn(x) - 4*sqrt(c*x^2 + d)*a*c^2*d^2*sgn(x
))/(c^2*d*x^4))/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)\,\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx$$

```
[In] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^2,x)
[Out] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^2, x)
```

3.944 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 123

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx=\frac{(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{8dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}+\frac{c(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{16d^2x}-\frac{c^2(bc-2ad)\operatorname{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{16d^{5/2}}$$

```
[Out] -1/6*b*(c+d/x^2)^(3/2)/d/x^3-1/16*c^2*(-2*a*d+b*c)*arctanh(d^(1/2)/x/(c+d/x
^2)^(1/2))/d^(5/2)+1/8*(-2*a*d+b*c)*(c+d/x^2)^(1/2)/d/x^3+1/16*c*(-2*a*d+b*
c)*(c+d/x^2)^(1/2)/d^2/x
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {470, 342, 285, 327, 223, 212}

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx=-\frac{c^2(bc-2ad)\operatorname{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{16d^{5/2}}+\frac{c\sqrt{c+\frac{d}{x^2}}(bc-2ad)}{16d^2x}+\frac{\sqrt{c+\frac{d}{x^2}}(bc-2ad)}{8dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}$$

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^4,x]
```

```
[Out] ((b*c - 2*a*d)*Sqrt[c + d/x^2])/(8*d*x^3) - (b*(c + d/x^2)^(3/2))/(6*d*x^3)
 + (c*(b*c - 2*a*d)*Sqrt[c + d/x^2])/(16*d^2*x) - (c^2*(b*c - 2*a*d)*ArcTan
h[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(16*d^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3} + \frac{(-3bc+6ad)\int \frac{\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx}{6d}$$

$$= -\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3} - \frac{(-3bc+6ad)\text{Subst}\left(\int x^2\sqrt{c+dx^2}\,dx, x, \frac{1}{x}\right)}{6d}$$

$$= \frac{(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{8dx^3} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3} + \frac{(c(bc-2ad))\text{Subst}\left(\int \frac{x^2}{\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{8d}$$

$$= \frac{(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{8dx^3} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3} + \frac{c(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{16d^2x} - \frac{(c^2(bc-2ad))\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{16d^2}$$

$$= \frac{(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{8dx^3} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3} + \frac{c(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{16d^2x} - \frac{(c^2(bc-2ad))\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{16d^2}$$

$$= \frac{(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{8dx^3} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3} + \frac{c(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{16d^2x} - \frac{c^2(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{16d^{5/2}}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.01

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx = \frac{\sqrt{c+\frac{d}{x^2}}(-8bd^2-2bcdx^2-12ad^2x^2+3bc^2x^4-6acdx^4)}{48d^2x^5} - \frac{c^2(bc-2ad)\sqrt{c+\frac{d}{x^2}}\,x\,\text{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{16d^{5/2}\sqrt{d+cx^2}}$$

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^4,x]
[Out] (Sqrt[c + d/x^2]*(-8*b*d^2 - 2*b*c*d*x^2 - 12*a*d^2*x^2 + 3*b*c^2*x^4 - 6*a
*c*d*x^4))/(48*d^2*x^5) - (c^2*(b*c - 2*a*d)*Sqrt[c + d/x^2]*x*ArcTanh[Sqrt
[d + c*x^2]/Sqrt[d]])/(16*d^(5/2)*Sqrt[d + c*x^2])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{\left(6acd\,x^{4}-3b\,c^{2}x^{4}+12a\,d^{2}x^{2}+2bcd\,x^{2}+8b\,d^{2}\right)\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}{48x^{5}d^{2}}+\frac{c^{2}\left(2ad-bc\right)\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}{16d^{\frac{5}{2}}\sqrt{c\,x^{2}+d}}$
default	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\left(6d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)a\,c^{2}x^{6}-3\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)b\,c^{3}x^{6}-6\sqrt{c\,x^{2}+d}\,a\,c^{2}d\,x^{6}+3\sqrt{c\,x^{2}+d}\,b\,c^{3}x^{6}+6\left(c\,x^{2}+d\right)^{\frac{3}{2}}ac\right.}{48x^{5}\sqrt{c\,x^{2}+d}\,d^{3}}$

[In] int((a+b/x^2)*(c+d/x^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)

[Out] -1/48*(6*a*c*d*x^4-3*b*c^2*x^4+12*a*d^2*x^2+2*b*c*d*x^2+8*b*d^2)/x^5/d^2*((c*x^2+d)/x^2)^(1/2)+1/16*c^2*(2*a*d-b*c)/d^(5/2)*ln((2*d+2*d^(1/2)*(c*x^2+d)^(1/2))/x)*((c*x^2+d)/x^2)^(1/2)*x/(c*x^2+d)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.98

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx = \left[-\frac{3\,(bc^3-2\,ac^2d)\sqrt{d}x^5\log\left(-\frac{cx^2+2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)-2\,(3\,(bc^2d-2\,acd^2)x^4-8\,bd^3-2\,(bcd^2+6\,ad^3)x^2)}{96\,d^3x^5}\right.$$

[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^4,x, algorithm="fricas")

[Out] [-1/96*(3*(b*c^3 - 2*a*c^2*d)*sqrt(d)*x^5*log(-(c*x^2 + 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2) - 2*(3*(b*c^2*d - 2*a*c*d^2)*x^4 - 8*b*d^3 - 2*(b*c*d^2 + 6*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^3*x^5), 1/48*(3*(b*c^3 - 2*a*c^2*d)*sqrt(-d)*x^5*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (3*(b*c^2*d - 2*a*c*d^2)*x^4 - 8*b*d^3 - 2*(b*c*d^2 + 6*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^3*x^5)]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(107) = 214$.

Time = 7.14 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.84

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx = -\frac{ac^{\frac{3}{2}}}{8dx\sqrt{1+\frac{d}{cx^2}}} - \frac{3a\sqrt{c}}{8x^3\sqrt{1+\frac{d}{cx^2}}} + \frac{ac^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{8d^{\frac{3}{2}}} - \frac{ad}{4\sqrt{c}x^5\sqrt{1+\frac{d}{cx^2}}} + \frac{bc^{\frac{5}{2}}}{16d^2x\sqrt{1+\frac{d}{cx^2}}} + \frac{bc^{\frac{3}{2}}}{48dx^3\sqrt{1+\frac{d}{cx^2}}} - \frac{5b\sqrt{c}}{24x^5\sqrt{1+\frac{d}{cx^2}}} - \frac{bc^3\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{16d^{\frac{5}{2}}} - \frac{bd}{6\sqrt{c}x^7\sqrt{1+\frac{d}{cx^2}}}$$

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**4,x)
```

```
[Out] -a*c**(3/2)/(8*d*x*sqrt(1 + d/(c*x**2))) - 3*a*sqrt(c)/(8*x**3*sqrt(1 + d/(
c*x**2))) + a*c**2*asinh(sqrt(d)/(sqrt(c)*x))/(8*d**(3/2)) - a*d/(4*sqrt(c)
*x**5*sqrt(1 + d/(c*x**2))) + b*c**(5/2)/(16*d**2*x*sqrt(1 + d/(c*x**2))) +
 b*c**(3/2)/(48*d*x**3*sqrt(1 + d/(c*x**2))) - 5*b*sqrt(c)/(24*x**5*sqrt(1
+ d/(c*x**2))) - b*c**3*asinh(sqrt(d)/(sqrt(c)*x))/(16*d**(5/2)) - b*d/(6*s
qrt(c)*x**7*sqrt(1 + d/(c*x**2)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(103) = 206$.

Time = 0.28 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.25

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx = -\frac{1}{16}\left(\frac{c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{3}{2}}} + \frac{2\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2x^3+\sqrt{c+\frac{d}{x^2}}c^2dx\right)}{\left(c+\frac{d}{x^2}\right)^2dx^4-2\left(c+\frac{d}{x^2}\right)d^2x^2+d^3}\right)a + \frac{1}{96}\left(\frac{3\,c^3\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{5}{2}}} + \frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3x^5-8\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^3dx^3-3\sqrt{c+\frac{d}{x^2}}c^3d^2x\right)}{\left(c+\frac{d}{x^2}\right)^3d^2x^6-3\left(c+\frac{d}{x^2}\right)^2d^3x^4+3\left(c+\frac{d}{x^2}\right)d^4x^2-d^5}\right)b$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^4,x, algorithm="maxima")
```

[Out] -1/16*(c^2*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))
/d^(3/2) + 2*((c + d/x^2)^(3/2)*c^2*x^3 + sqrt(c + d/x^2)*c^2*d*x)/((c + d/
x^2)^2*d*x^4 - 2*(c + d/x^2)*d^2*x^2 + d^3))*a + 1/96*(3*c^3*log((sqrt(c +
d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(5/2) + 2*(3*(c + d/x^
2)^(5/2)*c^3*x^5 - 8*(c + d/x^2)^(3/2)*c^3*d*x^3 - 3*sqrt(c + d/x^2)*c^3*d^
2*x)/((c + d/x^2)^3*d^2*x^6 - 3*(c + d/x^2)^2*d^3*x^4 + 3*(c + d/x^2)*d^4*x
^2 - d^5))*b

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.24

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx = \frac{\frac{3\left(bc^4\mathrm{sgn}(x)-2\,ac^3d\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}d^2}+\frac{3\left(cx^2+d\right)^{\frac{5}{2}}bc^4\mathrm{sgn}(x)-6\left(cx^2+d\right)^{\frac{5}{2}}ac^3d\mathrm{sgn}(x)-8\left(cx^2+d\right)^{\frac{3}{2}}bc^4d\mathrm{sgn}(x)-3\sqrt{cx^2+d}bc^4d^2\mathrm{sgn}(x}{c^3d^2x^6}}{48\,c}$$

[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^4,x, algorithm="giac")

[Out] 1/48*(3*(b*c^4*sgn(x) - 2*a*c^3*d*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d))/
(sqrt(-d)*d^2) + (3*(c*x^2 + d)^(5/2)*b*c^4*sgn(x) - 6*(c*x^2 + d)^(5/2)*a*
c^3*d*sgn(x) - 8*(c*x^2 + d)^(3/2)*b*c^4*d*sgn(x) - 3*sqrt(c*x^2 + d)*b*c^4
*d^2*sgn(x) + 6*sqrt(c*x^2 + d)*a*c^3*d^3*sgn(x))/(c^3*d^2*x^6))/c

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)\,\sqrt{c+\frac{d}{x^2}}}{x^4} \,dx$$

[In] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^4,x)

[Out] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^4, x)

3.945 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^5 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 123

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^5 \, dx = \frac{d(6bc-ad)\sqrt{c+\frac{d}{x^2}}x^2}{16c} + \frac{(6bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^4}{24c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^6}{6c} + \frac{d^2(6bc-ad)\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{16c^{3/2}}$$

```
[Out] 1/24*(-a*d+6*b*c)*(c+d/x^2)^(3/2)*x^4/c+1/6*a*(c+d/x^2)^(5/2)*x^6/c+1/16*d^
2*(-a*d+6*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(3/2)+1/16*d*(-a*d+6*b*c)
*x^2*(c+d/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 79, 43, 65, 214}

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^5 \, dx = \frac{d^2(6bc-ad)\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{16c^{3/2}} + \frac{dx^2\sqrt{c+\frac{d}{x^2}}(6bc-ad)}{16c} + \frac{x^4\left(c+\frac{d}{x^2}\right)^{3/2}(6bc-ad)}{24c} + \frac{ax^6\left(c+\frac{d}{x^2}\right)^{5/2}}{6c}$$

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^5,x]
```

```
[Out] (d*(6*b*c - a*d)*Sqrt[c + d/x^2]*x^2)/(16*c) + ((6*b*c - a*d)*(c + d/x^2)^(
3/2)*x^4)/(24*c) + (a*(c + d/x^2)^(5/2)*x^6)/(6*c) + (d^2*(6*b*c - a*d)*Arc
Tanh[Sqrt[c + d/x^2]/Sqrt[c]])/(16*c^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)(c+dx)^{3/2}}{x^4}\,dx,x,\frac{1}{x^2}\right)\right)$$

$$= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^6}{6c} - \frac{\left(3bc-\frac{ad}{2}\right)\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^3}\,dx,x,\frac{1}{x^2}\right)}{6c}$$

$$= \frac{(6bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{24c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^6}{6c} - \frac{(d(6bc-ad))\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2}\,dx,x,\frac{1}{x^2}\right)}{16c}$$

$$= \frac{d(6bc-ad)\sqrt{c+\frac{d}{x^2}}x^2}{16c} + \frac{(6bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{24c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^6}{6c} - \frac{(d^2(6bc-ad))\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)}{32c}$$

$$= \frac{d(6bc-ad)\sqrt{c+\frac{d}{x^2}}x^2}{16c} + \frac{(6bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{24c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^6}{6c} - \frac{(d(6bc-ad))\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{16c}$$

$$= \frac{d(6bc-ad)\sqrt{c+\frac{d}{x^2}}x^2}{16c} + \frac{(6bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{24c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^6}{6c} + \frac{d^2(6bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{16c^{3/2}}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^5\,dx = \frac{\sqrt{c+\frac{d}{x^2}}x\left(\sqrt{c}x\sqrt{d+cx^2}(6bc(5d+2cx^2)+a(3d^2+14cdx^2+8c^2x^4))+3d^2(-6bc+ad)\log\right.}{48c^{3/2}\sqrt{d+cx^2}}$$

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^5,x]
[Out] (Sqrt[c + d/x^2]*x*(Sqrt[c]*x*Sqrt[d + c*x^2]*(6*b*c*(5*d + 2*c*x^2) + a*(3
*d^2 + 14*c*d*x^2 + 8*c^2*x^4)) + 3*d^2*(-6*b*c + a*d)*Log[-(Sqrt[c]*x) + S
qrt[d + c*x^2]]))/(48*c^(3/2)*Sqrt[d + c*x^2])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

method	result
risch	$\frac{x^2(8ax^4c^2+14acdx^2+12bc^2x^2+3ad^2+30bcd)\sqrt{\frac{cx^2+d}{x^2}}}{48c} - \frac{d^2(ad-6bc)\ln\left(\sqrt{c}x+\sqrt{cx^2+d}\right)\sqrt{\frac{cx^2+d}{x^2}}\,x}{16c^{\frac{3}{2}}\sqrt{cx^2+d}}$
default	$\frac{\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}x^3\left(8(cx^2+d)^{\frac{5}{2}}\sqrt{c}ax-2(cx^2+d)^{\frac{3}{2}}\sqrt{c}adx+12(cx^2+d)^{\frac{3}{2}}c^{\frac{3}{2}}bx-3\sqrt{cx^2+d}\sqrt{c}ad^2x+18\sqrt{cx^2+d}c^{\frac{3}{2}}bdx-3\ln\left(\sqrt{c}x+\sqrt{c}\right.\right.}{48(cx^2+d)^{\frac{3}{2}}c^{\frac{3}{2}}}$

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/c*x^2*(8*a*c^2*x^4+14*a*c*d*x^2+12*b*c^2*x^2+3*a*d^2+30*b*c*d)*((c*x^2
+d)/x^2)^(1/2)-1/16*d^2*(a*d-6*b*c)/c^(3/2)*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*(
(c*x^2+d)/x^2)^(1/2)*x/(c*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.69 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.98

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^5\,dx = \left[-\frac{3\,(6\,bcd^2-ad^3)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\,(8\,ac^3x^6+2\,(6\,bc^3+7\,ac^2d)x^4 \ldots}{96\,c^2}\right.$$

$$-\frac{3\,(6\,bcd^2-ad^3)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)-(8\,ac^3x^6+2\,(6\,bc^3+7\,ac^2d)x^4+3\,(10\,bc^2d+acd^2)x^2)\sqrt{\frac{cx^2+d}{x^2}} \ldots}{48\,c^2}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^5,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(6*b*c*d^2 - a*d^3)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c)*x^2*sqrt((c*
x^2 + d)/x^2) - d) - 2*(8*a*c^3*x^6 + 2*(6*b*c^3 + 7*a*c^2*d)*x^4 + 3*(10*b
*c^2*d + a*c*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/c^2, -1/48*(3*(6*b*c*d^2 - a*
d^3)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (8*a
*c^3*x^6 + 2*(6*b*c^3 + 7*a*c^2*d)*x^4 + 3*(10*b*c^2*d + a*c*d^2)*x^2)*sqrt
((c*x^2 + d)/x^2))/c^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(105) = 210.

Time = 33.53 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.06

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^5\,dx = \frac{ac^2x^7}{6\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{11ac\sqrt{d}x^5}{24\sqrt{\frac{cx^2}{d}+1}} + \frac{17ad^{\frac{3}{2}}x^3}{48\sqrt{\frac{cx^2}{d}+1}}+\frac{ad^{\frac{5}{2}}x}{16c\sqrt{\frac{cx^2}{d}+1}}-\frac{ad^3\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{16c^{\frac{3}{2}}}+\frac{bc^2x^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}} + \frac{3bc\sqrt{d}x^3}{8\sqrt{\frac{cx^2}{d}+1}}+\frac{bd^{\frac{3}{2}}x\sqrt{\frac{cx^2}{d}+1}}{2}+\frac{bd^{\frac{3}{2}}x}{8\sqrt{\frac{cx^2}{d}+1}}+\frac{3bd^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{8\sqrt{c}}$$

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**5,x)
```

```
[Out] a*c**2*x**7/(6*sqrt(d)*sqrt(c*x**2/d + 1)) + 11*a*c*sqrt(d)*x**5/(24*sqrt(c
*x**2/d + 1)) + 17*a*d**(3/2)*x**3/(48*sqrt(c*x**2/d + 1)) + a*d**(5/2)*x/(
16*c*sqrt(c*x**2/d + 1)) - a*d**3*asinh(sqrt(c)*x/sqrt(d))/(16*c**(3/2)) +
b*c**2*x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*b*c*sqrt(d)*x**3/(8*sqrt(c*x
**2/d + 1)) + b*d**(3/2)*x*sqrt(c*x**2/d + 1)/2 + b*d**(3/2)*x/(8*sqrt(c*x*
*2/d + 1)) + 3*b*d**2*asinh(sqrt(c)*x/sqrt(d))/(8*sqrt(c))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(103) = 206$.

Time = 0.30 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.95

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^5\,dx = \frac{1}{96}\left(\frac{3\,d^3\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^3+8\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}cd^3-3\sqrt{c+\frac{d}{x^2}}c^2d^3\right)}{\left(c+\frac{d}{x^2}\right)^3c-3\left(c+\frac{d}{x^2}\right)^2c^2+3\left(c+\frac{d}{x^2}\right)c^3-c^4}\right)a - \frac{1}{16}\left(\frac{3\,d^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{\sqrt{c}}-\frac{2\left(5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2-3\sqrt{c+\frac{d}{x^2}}cd^2\right)}{\left(c+\frac{d}{x^2}\right)^2-2\left(c+\frac{d}{x^2}\right)c+c^2}\right)b$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^5,x, algorithm="maxima")
```

```
[Out] 1/96*(3*d^3*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^
(3/2) + 2*(3*(c + d/x^2)^(5/2)*d^3 + 8*(c + d/x^2)^(3/2)*c*d^3 - 3*sqrt(c +
 d/x^2)*c^2*d^3)/((c + d/x^2)^3*c - 3*(c + d/x^2)^2*c^2 + 3*(c + d/x^2)*c^3
 - c^4))*a - 1/16*(3*d^2*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) +
 sqrt(c)))/sqrt(c) - 2*(5*(c + d/x^2)^(3/2)*d^2 - 3*sqrt(c + d/x^2)*c*d^2)/
((c + d/x^2)^2 - 2*(c + d/x^2)*c + c^2))*b
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.17

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^5\,dx = \frac{1}{48}\left(2\left(4\,acx^2\mathrm{sgn}(x)+\frac{6\,bc^5\mathrm{sgn}(x)+7\,ac^4d\mathrm{sgn}(x)}{c^4}\right)x^2+\frac{3\,(10\,bc^4d\mathrm{sgn}(x)+ac^3d^2\mathrm{sgn}(x))}{c^4}\right) - \frac{(6\,bcd^2\mathrm{sgn}(x)-ad^3\mathrm{sgn}(x))\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+d}\right|\right)}{16\,c^{\frac{3}{2}}} + \frac{(6\,bcd^2\log(|d|)-ad^3\log(|d|))\mathrm{sgn}(x)}{32\,c^{\frac{3}{2}}}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^5,x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*a*c*x^2*sgn(x) + (6*b*c^5*sgn(x) + 7*a*c^4*d*sgn(x))/c^4)*x^2 +
3*(10*b*c^4*d*sgn(x) + a*c^3*d^2*sgn(x))/c^4)*sqrt(c*x^2 + d)*x - 1/16*(6*b
*c*d^2*sgn(x) - a*d^3*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)))/c^(3/2
) + 1/32*(6*b*c*d^2*log(abs(d)) - a*d^3*log(abs(d)))*sgn(x)/c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 10.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.06

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^5\,dx = \frac{a\,x^6\,\left(c+\frac{d}{x^2}\right)^{3/2}}{6}+\frac{5\,b\,x^4\,\left(c+\frac{d}{x^2}\right)^{3/2}}{8}+\frac{a\,x^6\,\left(c+\frac{d}{x^2}\right)^{5/2}}{16\,c} + \frac{3\,b\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,\sqrt{c}} - \frac{a\,c\,x^6\,\sqrt{c+\frac{d}{x^2}}}{16} - \frac{3\,b\,c\,x^4\,\sqrt{c+\frac{d}{x^2}}}{8} + \frac{a\,d^3\,\mathrm{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}\,1\mathrm{i}}{\sqrt{c}}\right)\,1\mathrm{i}}{16\,c^{3/2}}$$

```
[In] int(x^5*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] (a*x^6*(c + d/x^2)^(3/2))/6 + (5*b*x^4*(c + d/x^2)^(3/2))/8 + (a*x^6*(c + d
/x^2)^(5/2))/(16*c) + (a*d^3*atan(((c + d/x^2)^(1/2)*1i)/c^(1/2))*1i)/(16*c
^(3/2)) + (3*b*d^2*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(8*c^(1/2)) - (a*c*x^6
*(c + d/x^2)^(1/2))/16 - (3*b*c*x^4*(c + d/x^2)^(1/2))/8
```

3.946 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^3 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 115

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^3 \, dx = -\frac{3d(4bc+ad)\sqrt{c+\frac{d}{x^2}}}{8c} + \frac{(4bc+ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^2}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^4}{4c} + \frac{3d(4bc+ad)\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\sqrt{c}}$$

```
[Out] 1/8*(a*d+4*b*c)*(c+d/x^2)^(3/2)*x^2/c+1/4*a*(c+d/x^2)^(5/2)*x^4/c+3/8*d*(a*
d+4*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(1/2)-3/8*d*(a*d+4*b*c)*(c+d/x^
2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {457, 79, 43, 52, 65, 214}

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^3 \, dx = \frac{3d(ad+4bc)\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\sqrt{c}} + \frac{x^2\left(c+\frac{d}{x^2}\right)^{3/2}(ad+4bc)}{8c} - \frac{3d\sqrt{c+\frac{d}{x^2}}(ad+4bc)}{8c} + \frac{ax^4\left(c+\frac{d}{x^2}\right)^{5/2}}{4c}$$

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^3,x]
```

```
[Out] (-3*d*(4*b*c + a*d)*Sqrt[c + d/x^2])/(8*c) + ((4*b*c + a*d)*(c + d/x^2)^(3/
2)*x^2)/(8*c) + (a*(c + d/x^2)^(5/2)*x^4)/(4*c) + (3*d*(4*b*c + a*d)*ArcTan
h[Sqrt[c + d/x^2]/Sqrt[c]])/(8*Sqrt[c])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)(c+dx)^{3/2}}{x^3}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^4}{4c} - \frac{(4bc+ad)\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2}\,dx, x, \frac{1}{x^2}\right)}{8c}$$

$$= \frac{(4bc+ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^4}{4c} - \frac{(3d(4bc+ad))\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x}\,dx, x, \frac{1}{x^2}\right)}{16c}$$

$$= -\frac{3d(4bc+ad)\sqrt{c+\frac{d}{x^2}}}{8c} + \frac{(4bc+ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^4}{4c} - \frac{1}{16}(3d(4bc+ad))\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{3d(4bc+ad)\sqrt{c+\frac{d}{x^2}}}{8c} + \frac{(4bc+ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^4}{4c} - \frac{1}{8}(3(4bc+ad))\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)$$

$$= -\frac{3d(4bc+ad)\sqrt{c+\frac{d}{x^2}}}{8c} + \frac{(4bc+ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^4}{4c} + \frac{3d(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\sqrt{c}}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^3\,dx = \frac{1}{8}\sqrt{c+\frac{d}{x^2}}\left(-8bd+4bcx^2 + 5adx^2+2acx^4+\frac{6d(4bc+ad)x\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{d}+\sqrt{d+cx^2}}\right)}{\sqrt{c}\sqrt{d+cx^2}}\right)$$

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^3,x]
```

```
[Out] (Sqrt[c + d/x^2]*(-8*b*d + 4*b*c*x^2 + 5*a*d*x^2 + 2*a*c*x^4 + (6*d*(4*b*c
+ a*d)*x*ArcTanh[(Sqrt[c]*x)/(-Sqrt[d] + Sqrt[d + c*x^2])])/(Sqrt[c]*Sqrt[d
 + c*x^2])))/8
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83

method	result
risch	$\frac{\left(2 a \,x^{4} c +5 a d \,x^{2}+4 c b \,x^{2}-8 b d \right) \sqrt{\frac{c \,x^{2}+d}{x^{2}}}}{8}+\frac{3 d \left(a d +4 b c \right) \ln \left(\sqrt{c}\, x +\sqrt{c \,x^{2}+d}\right) \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, x}{8 \sqrt{c}\, \sqrt{c \,x^{2}+d}}$
default	$-\frac{\left(\frac{c \,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{2} \left(-2 \left(c \,x^{2}+d \right)^{\frac{3}{2}} \sqrt{c}\, a d \,x^{2}-8 \left(c \,x^{2}+d \right)^{\frac{3}{2}} c^{\frac{3}{2}} b \,x^{2}+8 \left(c \,x^{2}+d \right)^{\frac{5}{2}} \sqrt{c}\, b -3 \sqrt{c \,x^{2}+d}\, \sqrt{c}\, a \,d^{2} x^{2}-12 \sqrt{c \,x^{2}+d}\, c^{\frac{3}{2}} b d \,x^{2}-3 \ln \left(\sqrt{c}\right.}{8 \left(c \,x^{2}+d \right)^{\frac{3}{2}} d \sqrt{c}}$

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(2*a*c*x^4+5*a*d*x^2+4*b*c*x^2-8*b*d)*((c*x^2+d)/x^2)^(1/2)+3/8*d*(a*d+
4*b*c)*ln(c^(1/2)*x+(c*x^2+d)^(1/2))/c^(1/2)*((c*x^2+d)/x^2)^(1/2)*x/(c*x^2
+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.77

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^3 \, dx = \left[\frac{3 \, {\left(4 \, b c d + a d^{2}\right)} \sqrt{c} \log \left(-2 \, c x^{2} - 2 \, \sqrt{c} x^{2} \sqrt{\frac{c x^{2} + d}{x^{2}}} - d\right) + 2 \, {\left(2 \, a c^{2} x^{4} - 8 \, b c d + {\left(4 \, b c^{2} + 5 \, a c d\right.}}{16 \, c}, -\frac{3 \, {\left(4 \, b c d + a d^{2}\right)} \sqrt{-c} \arctan\left(\frac{\sqrt{-c} x^{2} \sqrt{\frac{c x^{2} + d}{x^{2}}}}{c x^{2} + d}\right) - {\left(2 \, a c^{2} x^{4} - 8 \, b c d + {\left(4 \, b c^{2} + 5 \, a c d\right)} x^{2}\right)} \sqrt{\frac{c x^{2} + d}{x^{2}}}}{8 \, c}\right]$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^3,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(4*b*c*d + a*d^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2
 + d)/x^2) - d) + 2*(2*a*c^2*x^4 - 8*b*c*d + (4*b*c^2 + 5*a*c*d)*x^2)*sqrt(
(c*x^2 + d)/x^2))/c, -1/8*(3*(4*b*c*d + a*d^2)*sqrt(-c)*arctan(sqrt(-c)*x^2
*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (2*a*c^2*x^4 - 8*b*c*d + (4*b*c^2 + 5
*a*c*d)*x^2)*sqrt((c*x^2 + d)/x^2))/c]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(104) = 208$.

Time = 55.22 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.88

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^3\,dx = \frac{ac^2x^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}} + \frac{3ac\sqrt{d}x^3}{8\sqrt{\frac{cx^2}{d}+1}} + \frac{ad^{\frac{3}{2}}x\sqrt{\frac{cx^2}{d}+1}}{2} + \frac{ad^{\frac{3}{2}}x}{8\sqrt{\frac{cx^2}{d}+1}} + \frac{3ad^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{8\sqrt{c}} + \frac{3b\sqrt{c}d\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2} + \frac{bc\sqrt{d}x\sqrt{\frac{cx^2}{d}+1}}{2} - \frac{bc\sqrt{d}x}{\sqrt{\frac{cx^2}{d}+1}} - \frac{bd^{\frac{3}{2}}}{x\sqrt{\frac{cx^2}{d}+1}}$$

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**3,x)
```

```
[Out] a*c**2*x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*a*c*sqrt(d)*x**3/(8*sqrt(c*x
**2/d + 1)) + a*d**(3/2)*x*sqrt(c*x**2/d + 1)/2 + a*d**(3/2)*x/(8*sqrt(c*x*
*2/d + 1)) + 3*a*d**2*asinh(sqrt(c)*x/sqrt(d))/(8*sqrt(c)) + 3*b*sqrt(c)*d*
asinh(sqrt(c)*x/sqrt(d))/2 + b*c*sqrt(d)*x*sqrt(c*x**2/d + 1)/2 - b*c*sqrt(
d)*x/sqrt(c*x**2/d + 1) - b*d**(3/2)/(x*sqrt(c*x**2/d + 1))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.49

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^3\,dx = -\frac{1}{16}\left(\frac{3\,d^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{\sqrt{c}} - \frac{2\left(5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2-3\sqrt{c+\frac{d}{x^2}}cd^2\right)}{\left(c+\frac{d}{x^2}\right)^2-2\left(c+\frac{d}{x^2}\right)c+c^2}\right)a + \frac{1}{4}\left(2\sqrt{c+\frac{d}{x^2}}cx^2-3\sqrt{c}d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)-4\sqrt{c+\frac{d}{x^2}}d\right)b$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^3,x, algorithm="maxima")
```

```
[Out] -1/16*(3*d^2*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/s
qrt(c) - 2*(5*(c + d/x^2)^(3/2)*d^2 - 3*sqrt(c + d/x^2)*c*d^2)/((c + d/x^2)
^2 - 2*(c + d/x^2)*c + c^2))*a + 1/4*(2*sqrt(c + d/x^2)*c*x^2 - 3*sqrt(c)*d
*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c))) - 4*sqrt(c +
d/x^2)*d)*b
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^3\,dx = \frac{2\,b\sqrt{c}d^2\mathrm{sgn}(x)}{\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d} + \frac{1}{8}\left(2\,acx^2\mathrm{sgn}(x)+\frac{4\,bc^3\mathrm{sgn}(x)+5\,ac^2d\mathrm{sgn}(x)}{c^2}\right)\sqrt{cx^2+d}x - \frac{3\left(4\,bcd\mathrm{sgn}(x)+ad^2\mathrm{sgn}(x)\right)\log\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2\right)}{16\,\sqrt{c}}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^3,x, algorithm="giac")
```

```
[Out] 2*b*sqrt(c)*d^2*sgn(x)/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d) + 1/8*(2*a*c*x
^2*sgn(x) + (4*b*c^3*sgn(x) + 5*a*c^2*d*sgn(x))/c^2)*sqrt(c*x^2 + d)*x - 3/
16*(4*b*c*d*sgn(x) + a*d^2*sgn(x))*log((sqrt(c)*x - sqrt(c*x^2 + d))^2)/sqr
t(c)
```

Mupad [B] (verification not implemented)

Time = 10.18 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.91

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^3\,dx = \frac{5\,a\,x^4\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{8} - b\,d\,\sqrt{c+\frac{d}{x^2}} + \frac{3\,b\,\sqrt{c}\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2} + \frac{3\,a\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,\sqrt{c}} - \frac{3\,a\,c\,x^4\,\sqrt{c+\frac{d}{x^2}}}{8} + \frac{b\,c\,x^2\,\sqrt{c+\frac{d}{x^2}}}{2}$$

```
[In] int(x^3*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] (5*a*x^4*(c + d/x^2)^(3/2))/8 - b*d*(c + d/x^2)^(1/2) + (3*b*c^(1/2)*d*atan
h((c + d/x^2)^(1/2)/c^(1/2)))/2 + (3*a*d^2*atanh((c + d/x^2)^(1/2)/c^(1/2))
)/(8*c^(1/2)) - (3*a*c*x^4*(c + d/x^2)^(1/2))/8 + (b*c*x^2*(c + d/x^2)^(1/2
))/2
```

3.947 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x \, dx$

Optimal result

Integrand size = 20, antiderivative size = 110

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x \, dx = -\frac{1}{2}(2bc+3ad)\sqrt{c+\frac{d}{x^2}} - \frac{(2bc+3ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{6c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^2}{2c} + \frac{1}{2}\sqrt{c}(2bc+3ad)\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)$$

```
[Out] -1/6*(3*a*d+2*b*c)*(c+d/x^2)^(3/2)/c+1/2*a*(c+d/x^2)^(5/2)*x^2/c+1/2*(3*a*d
+2*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))*c^(1/2)-1/2*(3*a*d+2*b*c)*(c+d/x^2
)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {457, 79, 52, 65, 214}

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x \, dx = \frac{1}{2}\sqrt{c}(3ad+2bc)\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right) - \frac{\left(c+\frac{d}{x^2}\right)^{3/2}(3ad+2bc)}{6c} - \frac{1}{2}\sqrt{c+\frac{d}{x^2}}(3ad+2bc) + \frac{ax^2\left(c+\frac{d}{x^2}\right)^{5/2}}{2c}$$

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x,x]
```

```
[Out] -1/2*((2*b*c + 3*a*d)*Sqrt[c + d/x^2]) - ((2*b*c + 3*a*d)*(c + d/x^2)^(3/2)
)/(6*c) + (a*(c + d/x^2)^(5/2)*x^2)/(2*c) + (Sqrt[c]*(2*b*c + 3*a*d)*ArcTan
h[Sqrt[c + d/x^2]/Sqrt[c]])/2
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)(c+dx)^{3/2}}{x^2}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^2}{2c} - \frac{\left(bc+\frac{3ad}{2}\right)\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x}\,dx, x, \frac{1}{x^2}\right)}{2c}$$

$$= -\frac{(2bc+3ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{6c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^2}{2c} - \frac{1}{4}(2bc+3ad)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{1}{2}(2bc+3ad)\sqrt{c+\frac{d}{x^2}} - \frac{(2bc+3ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{6c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^2}{2c} - \frac{1}{4}(c(2bc+3ad))\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{1}{2}(2bc+3ad)\sqrt{c+\frac{d}{x^2}} - \frac{(2bc+3ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{6c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^2}{2c} - \frac{(c(2bc+3ad))\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{2d}$$

$$= -\frac{1}{2}(2bc+3ad)\sqrt{c+\frac{d}{x^2}} - \frac{(2bc+3ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{6c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^2}{2c} + \frac{1}{2}\sqrt{c}(2bc+3ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x\,dx = \frac{\sqrt{c+\frac{d}{x^2}}\left(-6adx^2+3acx^4-2b(d+4cx^2)+\frac{6\sqrt{c}(2bc+3ad)x^3\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{d}+\sqrt{d+cx^2}}\right)}{\sqrt{d+cx^2}}\right)}{6x^2}$$

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x,x]
```

```
[Out] (Sqrt[c + d/x^2]*(-6*a*d*x^2 + 3*a*c*x^4 - 2*b*(d + 4*c*x^2) + (6*Sqrt[c]*(
2*b*c + 3*a*d)*x^3*ArcTanh[(Sqrt[c]*x)/(-Sqrt[d] + Sqrt[d + c*x^2])])/Sqrt[
d + c*x^2]))/(6*x^2)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.90

method	result
risch	$\frac{(3a\,x^4c-6ad\,x^2-8cb\,x^2-2bd)\sqrt{\frac{c\,x^2+d}{x^2}}}{6x^2}+\frac{(3ad+2bc)\sqrt{c}\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)\sqrt{\frac{c\,x^2+d}{x^2}}\,x}{2\sqrt{c\,x^2+d}}$
default	$-\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\left(-6(c\,x^2+d)^{\frac{3}{2}}c^{\frac{3}{2}}ad\,x^4-4(c\,x^2+d)^{\frac{3}{2}}c^{\frac{5}{2}}b\,x^4+6(c\,x^2+d)^{\frac{5}{2}}\sqrt{c}\,ad\,x^2+4(c\,x^2+d)^{\frac{5}{2}}c^{\frac{3}{2}}b\,x^2-9\sqrt{c\,x^2+d}\,c^{\frac{3}{2}}a\,d^2x^4-6\sqrt{c\,x^2+}\right.}{6(c\,x^2+d)^{\frac{3}{2}}d^2\sqrt{c}}$

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(3*a*c*x^4-6*a*d*x^2-8*b*c*x^2-2*b*d)/x^2*((c*x^2+d)/x^2)^(1/2)+1/2*(3*
a*d+2*b*c)*c^(1/2)*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*((c*x^2+d)/x^2)^(1/2)*x/(c
*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.77

$$\int\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x\,dx=\left[\frac{3\,(2\,bc+3\,ad)\sqrt{c}x^2\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)+2\,(3\,acx^4-2\,(4\,bc+3\,ad)x^2-2\,b}{12\,x^2}\right.$$
$$\left.-\frac{3\,(2\,bc+3\,ad)\sqrt{-c}x^2\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)-(3\,acx^4-2\,(4\,bc+3\,ad)x^2-2\,bd)\sqrt{\frac{cx^2+d}{x^2}}}{6\,x^2}\right]$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(2*b*c + 3*a*d)*sqrt(c)*x^2*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x
^2 + d)/x^2) - d) + 2*(3*a*c*x^4 - 2*(4*b*c + 3*a*d)*x^2 - 2*b*d)*sqrt((c*x
^2 + d)/x^2))/x^2, -1/6*(3*(2*b*c + 3*a*d)*sqrt(-c)*x^2*arctan(sqrt(-c)*x^2
*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (3*a*c*x^4 - 2*(4*b*c + 3*a*d)*x^2 -
2*b*d)*sqrt((c*x^2 + d)/x^2))/x^2]
```

Sympy [A] (verification not implemented)

Time = 16.51 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.70

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x\,dx=\frac{3a\sqrt{c}d\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2}+\frac{ac\sqrt{d}x\sqrt{\frac{cx^2}{d}+1}}{2}-\frac{ac\sqrt{d}x}{\sqrt{\frac{cx^2}{d}+1}}-\frac{ad^{\frac{3}{2}}}{x\sqrt{\frac{cx^2}{d}+1}}+bc^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)-\frac{bc^2x}{\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}-\frac{bc\sqrt{d}}{x\sqrt{\frac{cx^2}{d}+1}}+bd\left(\begin{cases}-\frac{\sqrt{c}}{2x^2} & \text{for } d=0\\ -\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x,x)
```

```
[Out] 3*a*sqrt(c)*d*asinh(sqrt(c)*x/sqrt(d))/2 + a*c*sqrt(d)*x*sqrt(c*x**2/d + 1)
/2 - a*c*sqrt(d)*x/sqrt(c*x**2/d + 1) - a*d**(3/2)/(x*sqrt(c*x**2/d + 1)) +
 b*c**(3/2)*asinh(sqrt(c)*x/sqrt(d)) - b*c**2*x/(sqrt(d)*sqrt(c*x**2/d + 1)
) - b*c*sqrt(d)/(x*sqrt(c*x**2/d + 1)) + b*d*Piecewise((-sqrt(c)/(2*x**2),
Eq(d, 0)), (-(c + d/x**2)**(3/2)/(3*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.22

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x\,dx=\frac{1}{4}\left(2\sqrt{c+\frac{d}{x^2}}cx^2-3\sqrt{c}d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)-4\sqrt{c+\frac{d}{x^2}}d\right)a-\frac{1}{6}\left(3c^{\frac{3}{2}}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)+2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}+6\sqrt{c+\frac{d}{x^2}}c\right)b$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(c + d/x^2)*c*x^2 - 3*sqrt(c)*d*log((sqrt(c + d/x^2) - sqrt(c))/
(sqrt(c + d/x^2) + sqrt(c))) - 4*sqrt(c + d/x^2)*d)*a - 1/6*(3*c^(3/2)*log(
(sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c))) + 2*(c + d/x^2)^(3
/2) + 6*sqrt(c + d/x^2)*c)*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(90) = 180$.

Time = 0.48 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.05

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x\,dx = \frac{1}{2}\sqrt{cx^2+d}acx\mathrm{sgn}(x) - \frac{1}{4}\left(2\,bc^{\frac{3}{2}}\mathrm{sgn}(x)+3\,a\sqrt{c}d\mathrm{sgn}(x)\right)\log\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2\right) + \frac{2\left(6\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4bc^{\frac{3}{2}}d\mathrm{sgn}(x)+3\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4a\sqrt{c}d^2\mathrm{sgn}(x)-6\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2bc^{\frac{3}{2}}d^2\mathrm{sgn}(\right.}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)^3}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + d)*a*c*x*sgn(x) - 1/4*(2*b*c^(3/2)*sgn(x) + 3*a*sqrt(c)*d*
sgn(x))*log((sqrt(c)*x - sqrt(c*x^2 + d))^2) + 2/3*(6*(sqrt(c)*x - sqrt(c*x
^2 + d))^4*b*c^(3/2)*d*sgn(x) + 3*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*sqrt(c)
*d^2*sgn(x) - 6*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(3/2)*d^2*sgn(x) - 6*(s
qrt(c)*x - sqrt(c*x^2 + d))^2*a*sqrt(c)*d^3*sgn(x) + 4*b*c^(3/2)*d^3*sgn(x)
 + 3*a*sqrt(c)*d^4*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^3
```

Mupad [B] (verification not implemented)

Time = 10.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x\,dx = b\,c^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right) - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3} - a\,d\sqrt{c+\frac{d}{x^2}} - b\,c\sqrt{c+\frac{d}{x^2}} + \frac{3\,a\,\sqrt{c}\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2} + \frac{a\,c\,x^2\,\sqrt{c+\frac{d}{x^2}}}{2}$$

```
[In] int(x*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] b*c^(3/2)*atanh((c + d/x^2)^(1/2)/c^(1/2)) - (b*(c + d/x^2)^(3/2))/3 - a*d*
(c + d/x^2)^(1/2) - b*c*(c + d/x^2)^(1/2) + (3*a*c^(1/2)*d*atanh((c + d/x^2
)^(1/2)/c^(1/2)))/2 + (a*c*x^2*(c + d/x^2)^(1/2))/2
```

3.948 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 76

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx = -ac\sqrt{c+\frac{d}{x^2}} - \frac{1}{3}a\left(c+\frac{d}{x^2}\right)^{3/2} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d} + ac^{3/2}\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)$$

```
[Out] -1/3*a*(c+d/x^2)^(3/2)-1/5*b*(c+d/x^2)^(5/2)/d+a*c^(3/2)*arctanh((c+d/x^2)^
(1/2)/c^(1/2))-a*c*(c+d/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 81, 52, 65, 214}

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx = ac^{3/2}\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right) - \frac{1}{3}a\left(c+\frac{d}{x^2}\right)^{3/2} - ac\sqrt{c+\frac{d}{x^2}} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d}$$

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x,x]
```

```
[Out] -(a*c*Sqrt[c + d/x^2]) - (a*(c + d/x^2)^(3/2))/3 - (b*(c + d/x^2)^(5/2))/(5
*d) + a*c^(3/2)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)(c+dx)^{3/2}}{x}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d} - \frac{1}{2}a\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x}\,dx, x, \frac{1}{x^2}\right)$$

$$= -\frac{1}{3}a\left(c+\frac{d}{x^2}\right)^{3/2} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d} - \frac{1}{2}(ac)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x}\,dx, x, \frac{1}{x^2}\right)$$

$$= -ac\sqrt{c+\frac{d}{x^2}} - \frac{1}{3}a\left(c+\frac{d}{x^2}\right)^{3/2} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d} - \frac{1}{2}\left(ac^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)$$

$$= -ac\sqrt{c+\frac{d}{x^2}} - \frac{1}{3}a\left(c+\frac{d}{x^2}\right)^{3/2} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d} - \frac{\left(ac^2\right)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{d}$$

$$= -ac\sqrt{c+\frac{d}{x^2}} - \frac{1}{3}a\left(c+\frac{d}{x^2}\right)^{3/2} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d} + ac^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.20

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx = \frac{\sqrt{c+\frac{d}{x^2}}\left(-\frac{3b(d+cx^2)^2}{d} - 5ax^2(d+4cx^2) - \frac{15ac^{3/2}x^5\log\left(-\sqrt{c}x+\sqrt{d+cx^2}\right)}{\sqrt{d+cx^2}}\right)}{15x^4}$$

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x,x]
```

```
[Out] (Sqrt[c + d/x^2]*((-3*b*(d + c*x^2)^2)/d - 5*a*x^2*(d + 4*c*x^2) - (15*a*c^
(3/2)*x^5*Log[-(Sqrt[c]*x) + Sqrt[d + c*x^2]])/Sqrt[d + c*x^2]))/(15*x^4)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.42

method	result
risch	$-\frac{\left(20acd\,x^4+3b\,c^2x^4+5a\,d^2x^2+6bcd\,x^2+3b\,d^2\right)\sqrt{\frac{c\,x^2+d}{x^2}}}{15x^4d}+\frac{c^{\frac{3}{2}}a\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)\sqrt{\frac{c\,x^2+d}{x^2}}\,x}{\sqrt{c\,x^2+d}}$
default	$-\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\left(-10\left(c\,x^2+d\right)^{\frac{3}{2}}c^{\frac{5}{2}}a\,x^6+10\left(c\,x^2+d\right)^{\frac{5}{2}}c^{\frac{3}{2}}a\,x^4-15\sqrt{c\,x^2+d}\,c^{\frac{5}{2}}ad\,x^6-15\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)a\,c^2d^2x^5+5\left(c\,x^2+d\right)^{\frac{5}{2}}\sqrt{c}\right.}{15x^2\left(c\,x^2+d\right)^{\frac{3}{2}}d^2\sqrt{c}}$

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(20*a*c*d*x^4+3*b*c^2*x^4+5*a*d^2*x^2+6*b*c*d*x^2+3*b*d^2)/x^4/d*((c*
x^2+d)/x^2)^(1/2)+c^(3/2)*a*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*((c*x^2+d)/x^2)^(
1/2)*x/(c*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.80

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx = \left[\frac{15\,ac^{\frac{3}{2}}dx^4\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\left((3\,bc^2+20\,acd)x^4+3\,bd^2+(6\,bcd+5\,ad^2)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{30\,dx^4}, -\frac{15\,a\sqrt{-c}cdx^4\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+\left((3\,bc^2+20\,acd)x^4+3\,bd^2+(6\,bcd+5\,ad^2)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\,dx^4}\right]$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/30*(15*a*c^(3/2)*d*x^4*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2
) - d) - 2*((3*b*c^2 + 20*a*c*d)*x^4 + 3*b*d^2 + (6*b*c*d + 5*a*d^2)*x^2)*s
qrt((c*x^2 + d)/x^2))/(d*x^4), -1/15*(15*a*sqrt(-c)*c*d*x^4*arctan(sqrt(-c)
*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + ((3*b*c^2 + 20*a*c*d)*x^4 + 3*b*d
^2 + (6*b*c*d + 5*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/(d*x^4)]
```

Sympy [A] (verification not implemented)

Time = 9.84 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.50

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx = \frac{\begin{cases} -\frac{2ac^2\operatorname{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{-c}}\right)}{\sqrt{-c}}-2ac\sqrt{c+\frac{d}{x^2}}-\frac{2a\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2b\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5d} & \text{for}\ d\neq 0 \\ -ac^{\frac{3}{2}}\log\left(-\frac{bc^{\frac{3}{2}}}{x^2}\right)-\frac{bc^{\frac{3}{2}}}{x^2} & \text{otherwise} \end{cases}}{2}$$

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x,x)
```

```
[Out] Piecewise((-2*a*c**2*atan(sqrt(c + d/x**2)/sqrt(-c))/sqrt(-c) - 2*a*c*sqrt(
c + d/x**2) - 2*a*(c + d/x**2)**(3/2)/3 - 2*b*(c + d/x**2)**(5/2)/(5*d), Ne
(d, 0)), (-a*c**(3/2)*log(-b*c**(3/2)/x**2) - b*c**(3/2)/x**2, True))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx = -\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5\,d} - \frac{1}{6}\left(3\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)+2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}+6\sqrt{c+\frac{d}{x^2}}c\right)a$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] -1/5*b*(c + d/x^2)^(5/2)/d - 1/6*(3*c^(3/2)*log((sqrt(c + d/x^2) - sqrt(c))
/(sqrt(c + d/x^2) + sqrt(c))) + 2*(c + d/x^2)^(3/2) + 6*sqrt(c + d/x^2)*c)*
a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(60) = 120.

Time = 0.84 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.34

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx = -\frac{1}{2}\,ac^{\frac{3}{2}}\log\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2\right)\operatorname{sgn}(x) + \frac{2\left(15\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^8bc^{\frac{5}{2}}\operatorname{sgn}(x)+30\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^8ac^{\frac{3}{2}}d\operatorname{sgn}(x)-90\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^6ac^{\frac{3}{2}}d^2\mathrm{sg}}{}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] -1/2*a*c^(3/2)*log((sqrt(c)*x - sqrt(c*x^2 + d))^2)*sgn(x) + 2/15*(15*(sqrt
(c)*x - sqrt(c*x^2 + d))^8*b*c^(5/2)*sgn(x) + 30*(sqrt(c)*x - sqrt(c*x^2 +
d))^8*a*c^(3/2)*d*sgn(x) - 90*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(3/2)*d^2
*sgn(x) + 30*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(5/2)*d^2*sgn(x) + 110*(sq
rt(c)*x - sqrt(c*x^2 + d))^4*a*c^(3/2)*d^3*sgn(x) - 70*(sqrt(c)*x - sqrt(c*
x^2 + d))^2*a*c^(3/2)*d^4*sgn(x) + 3*b*c^(5/2)*d^4*sgn(x) + 20*a*c^(3/2)*d^
5*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^5
```

Mupad [B] (verification not implemented)

Time = 10.37 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx = a\,c^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right) - \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}}{3} - a\,c\,\sqrt{c+\frac{d}{x^2}} - \frac{b\,\sqrt{c+\frac{d}{x^2}}\,\left(c\,x^2+d\right)^2}{5\,d\,x^4}$$

```
[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x,x)
[Out] a*c^(3/2)*atanh((c + d/x^2)^(1/2)/c^(1/2)) - (a*(c + d/x^2)^(3/2))/3 - a*c*
(c + d/x^2)^(1/2) - (b*(c + d/x^2)^(1/2)*(d + c*x^2)^2)/(5*d*x^4)
```

3.949 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3} dx = \frac{(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^2} - \frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^2}$$

```
[Out] 1/5*(-a*d+b*c)*(c+d/x^2)^(5/2)/d^2-1/7*b*(c+d/x^2)^(7/2)/d^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {455, 45}

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3} dx = \frac{\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^2} - \frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^2}$$

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^3,x]
```

```
[Out] ((b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^2) - (b*(c + d/x^2)^(7/2))/(7*d^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{2}\text{Subst}\left(\int (a+bx)(c+dx)^{3/2}\,dx, x, \frac{1}{x^2}\right)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{(-bc+ad)(c+dx)^{3/2}}{d}+\frac{b(c+dx)^{5/2}}{d}\right)\,dx, x, \frac{1}{x^2}\right)\right)\\ &= \frac{(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx = -\frac{\sqrt{c+\frac{d}{x^2}}\,(d+cx^2)^2\,(5bd-2bcx^2+7adx^2)}{35d^2x^6}$$

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^3,x]
[Out] -1/35*(Sqrt[c + d/x^2]*(d + c*x^2)^2*(5*b*d - 2*b*c*x^2 + 7*a*d*x^2))/(d^2*
x^6)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

method	result	size
gosper	$-\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}\left(7ad\,x^{2}-2cb\,x^{2}+5bd\right)\left(c\,x^{2}+d\right)}{35d^{2}x^{4}}$	48
default	$-\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}\left(7ad\,x^{2}-2cb\,x^{2}+5bd\right)\left(c\,x^{2}+d\right)}{35d^{2}x^{4}}$	48
risch	$-\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(7a\,c^{2}d\,x^{6}-2b\,c^{3}x^{6}+14ac\,d^{2}x^{4}+b\,c^{2}d\,x^{4}+7a\,d^{3}x^{2}+8bc\,d^{2}x^{2}+5b\,d^{3}\right)}{35x^{6}d^{2}}$	86
trager	$-\frac{\left(7a\,c^{2}d\,x^{6}-2b\,c^{3}x^{6}+14ac\,d^{2}x^{4}+b\,c^{2}d\,x^{4}+7a\,d^{3}x^{2}+8bc\,d^{2}x^{2}+5b\,d^{3}\right)\sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{35x^{6}d^{2}}$	90

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/35*((c*x^2+d)/x^2)^(3/2)*(7*a*d*x^2-2*b*c*x^2+5*b*d)*(c*x^2+d)/d^2/x^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(38) = 76.

Time = 0.31 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.83

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx = \frac{\left((2\,bc^3-7\,ac^2d)x^6-(bc^2d+14\,acd^2)x^4-5\,bd^3-(8\,bcd^2+7\,ad^3)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{35\,d^2x^6}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/35*((2*b*c^3 - 7*a*c^2*d)*x^6 - (b*c^2*d + 14*a*c*d^2)*x^4 - 5*b*d^3 - (8
*b*c*d^2 + 7*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^2*x^6)
```

Sympy [A] (verification not implemented)

Time = 3.39 (sec) , antiderivative size = 189, normalized size of antiderivative = 4.11

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx = -\frac{ac\left(\begin{cases}\frac{2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3d} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}}{x^2} & \text{otherwise}\end{cases}\right)}{2} - \frac{ad\left(\begin{cases}\frac{2\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}}{2x^4} & \text{otherwise}\end{cases}\right)}{2} - \frac{bc\left(\begin{cases}\frac{2\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}}{2x^4} & \text{otherwise}\end{cases}\right)}{2} - \frac{bd\left(\begin{cases}\frac{2\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}}{3x^6} & \text{otherwise}\end{cases}\right)}{2}$$

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**3,x)
```

```
[Out] -a*c*Piecewise((2*(c + d/x**2)**(3/2)/(3*d), Ne(d, 0)), (sqrt(c)/x**2, True
))/2 - a*d*Piecewise((2*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/
d**2, Ne(d, 0)), (sqrt(c)/(2*x**4), True))/2 - b*c*Piecewise((2*(-c*(c + d/
x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d**2, Ne(d, 0)), (sqrt(c)/(2*x**4),
 True))/2 - b*d*Piecewise((2*(c**2*(c + d/x**2)**(3/2)/3 - 2*c*(c + d/x**2)
**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**3, Ne(d, 0)), (sqrt(c)/(3*x**6), True
))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx=-\frac{a\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5\,d}-\frac{1}{35}\left(\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^2}-\frac{7\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^2}\right)b$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/5*a*(c + d/x^2)^(5/2)/d - 1/35*(5*(c + d/x^2)^(7/2)/d^2 - 7*(c + d/x^2)^
(5/2)*c/d^2)*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(38) = 76.

Time = 0.94 (sec) , antiderivative size = 370, normalized size of antiderivative = 8.04

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx=\frac{2\left(35\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}ac^{\frac{5}{2}}\operatorname{sgn}(x)+70\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}bc^{\frac{7}{2}}\operatorname{sgn}(x)-70\left(\right.\right.}{}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 2/35*(35*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(5/2)*sgn(x) + 70*(sqrt(c)*x
- sqrt(c*x^2 + d))^10*b*c^(7/2)*sgn(x) - 70*(sqrt(c)*x - sqrt(c*x^2 + d))^1
0*a*c^(5/2)*d*sgn(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + d))^8*b*c^(7/2)*d*sgn(x
) + 105*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(5/2)*d^2*sgn(x) + 140*(sqrt(c)
*x - sqrt(c*x^2 + d))^6*b*c^(7/2)*d^2*sgn(x) - 140*(sqrt(c)*x - sqrt(c*x^2
+ d))^6*a*c^(5/2)*d^3*sgn(x) + 28*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(7/2)
*d^3*sgn(x) + 77*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(5/2)*d^4*sgn(x) + 14*
(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(7/2)*d^4*sgn(x) - 14*(sqrt(c)*x - sqrt
(c*x^2 + d))^2*a*c^(5/2)*d^5*sgn(x) - 2*b*c^(7/2)*d^5*sgn(x) + 7*a*c^(5/2)*
d^6*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^7
```

Mupad [B] (verification not implemented)

Time = 10.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.65

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx=\frac{2\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{35\,d^2}-\frac{a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{5\,d}-\frac{2\,a\,c\,\sqrt{c+\frac{d}{x^2}}}{5\,x^2}-\frac{a\,d\,\sqrt{c+\frac{d}{x^2}}}{5\,x^4}-\frac{8\,b\,c\,\sqrt{c+\frac{d}{x^2}}}{35\,x^4}-\frac{b\,d\,\sqrt{c+\frac{d}{x^2}}}{7\,x^6}-\frac{b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{35\,d\,x^2}$$

```
[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^3,x)
```

```
[Out] (2*b*c^3*(c + d/x^2)^(1/2))/(35*d^2) - (a*c^2*(c + d/x^2)^(1/2))/(5*d) - (2
*a*c*(c + d/x^2)^(1/2))/(5*x^2) - (a*d*(c + d/x^2)^(1/2))/(5*x^4) - (8*b*c*
(c + d/x^2)^(1/2))/(35*x^4) - (b*d*(c + d/x^2)^(1/2))/(7*x^6) - (b*c^2*(c +
 d/x^2)^(1/2))/(35*d*x^2)
```

3.950 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5} dx$

Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5} dx = -\frac{c(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}+\frac{(2bc-ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^3}$$

```
[Out] -1/5*c*(-a*d+b*c)*(c+d/x^2)^(5/2)/d^3+1/7*(-a*d+2*b*c)*(c+d/x^2)^(7/2)/d^3-
1/9*b*(c+d/x^2)^(9/2)/d^3
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 78}

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5} dx = \frac{\left(c+\frac{d}{x^2}\right)^{7/2}(2bc-ad)}{7d^3}-\frac{c\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^3}$$

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^5,x]
```

```
[Out] -1/5*(c*(b*c - a*d)*(c + d/x^2)^(5/2))/d^3 + ((2*b*c - a*d)*(c + d/x^2)^(7/
2))/(7*d^3) - (b*(c + d/x^2)^(9/2))/(9*d^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{2}\text{Subst}\left(\int x(a+bx)(c+dx)^{3/2}\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int\left(\frac{c(bc-ad)(c+dx)^{3/2}}{d^2}+\frac{(-2bc+ad)(c+dx)^{5/2}}{d^2}+\frac{b(c+dx)^{7/2}}{d^2}\right)dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\frac{c(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}+\frac{(2bc-ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^3}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int\frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}\,dx = \frac{\sqrt{c+\frac{d}{x^2}}\,(d+cx^2)^2\,(9adx^2(-5d+2cx^2)+b(-35d^2+20cdx^2-8c^2x^4))}{315d^3x^8}$$

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^5,x]
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)^2*(9*a*d*x^2*(-5*d + 2*c*x^2) + b*(-35*d^2 + 2
0*c*d*x^2 - 8*c^2*x^4)))/(315*d^3*x^8)
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}\left(18acd\,x^{4}-8b\,c^{2}x^{4}-45a\,d^{2}x^{2}+20bcd\,x^{2}-35b\,d^{2}\right)\left(c\,x^{2}+d\right)}{315d^{3}x^{6}}$	70
default	$\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}\left(18acd\,x^{4}-8b\,c^{2}x^{4}-45a\,d^{2}x^{2}+20bcd\,x^{2}-35b\,d^{2}\right)\left(c\,x^{2}+d\right)}{315d^{3}x^{6}}$	70
risch	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(18a\,c^{3}d\,x^{8}-8b\,c^{4}x^{8}-9a\,c^{2}d^{2}x^{6}+4b\,c^{3}d\,x^{6}-72ac\,d^{3}x^{4}-3b\,c^{2}d^{2}x^{4}-45a\,d^{4}x^{2}-50bc\,d^{3}x^{2}-35b\,d^{4}\right)}{315x^{8}d^{3}}$	111
trager	$\frac{\left(18a\,c^{3}d\,x^{8}-8b\,c^{4}x^{8}-9a\,c^{2}d^{2}x^{6}+4b\,c^{3}d\,x^{6}-72ac\,d^{3}x^{4}-3b\,c^{2}d^{2}x^{4}-45a\,d^{4}x^{2}-50bc\,d^{3}x^{2}-35b\,d^{4}\right)\sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{315x^{8}d^{3}}$	115

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/315*((c*x^2+d)/x^2)^(3/2)*(18*a*c*d*x^4-8*b*c^2*x^4-45*a*d^2*x^2+20*b*c*d
*x^2-35*b*d^2)*(c*x^2+d)/d^3/x^6
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.47

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}\,dx =$$
$$-\frac{\left(2\,(4\,bc^4-9\,ac^3d)x^8-(4\,bc^3d-9\,ac^2d^2)x^6+35\,bd^4+3\,(bc^2d^2+24\,acd^3)x^4+5\,(10\,bcd^3+9\,ad^4)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{315\,d^3x^8}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/315*(2*(4*b*c^4 - 9*a*c^3*d)*x^8 - (4*b*c^3*d - 9*a*c^2*d^2)*x^6 + 35*b*
d^4 + 3*(b*c^2*d^2 + 24*a*c*d^3)*x^4 + 5*(10*b*c*d^3 + 9*a*d^4)*x^2)*sqrt((
c*x^2 + d)/x^2)/(d^3*x^8)
```

Sympy [A] (verification not implemented)

Time = 3.55 (sec) , antiderivative size = 258, normalized size of antiderivative = 3.49

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}\,dx = -\frac{ac\left(\begin{cases} \frac{2\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2} & \text{for } d\neq 0 \\ \frac{\sqrt{c}}{2x^4} & \text{otherwise} \end{cases}\right)}{2} - \frac{ad\left(\begin{cases} \frac{2\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3} & \text{for } d\neq 0 \\ \frac{\sqrt{c}}{3x^6} & \text{otherwise} \end{cases}\right)}{2} - \frac{bc\left(\begin{cases} \frac{2\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3} & \text{for } d\neq 0 \\ \frac{\sqrt{c}}{3x^6} & \text{otherwise} \end{cases}\right)}{2} - \frac{bd\left(\begin{cases} \frac{2\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4} & \text{for } d\neq 0 \\ \frac{\sqrt{c}}{4x^8} & \text{otherwise} \end{cases}\right)}{2}$$

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**5,x)
```

```
[Out] -a*c*Piecewise((2*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d**2,
Ne(d, 0)), (sqrt(c)/(2*x**4), True))/2 - a*d*Piecewise((2*(c**2*(c + d/x**2
)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**3, Ne(d,
 0)), (sqrt(c)/(3*x**6), True))/2 - b*c*Piecewise((2*(c**2*(c + d/x**2)**(3
/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**3, Ne(d, 0)),
 (sqrt(c)/(3*x**6), True))/2 - b*d*Piecewise((2*(-c**3*(c + d/x**2)**(3/2)/
3 + 3*c**2*(c + d/x**2)**(5/2)/5 - 3*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)
**(9/2)/9)/d**4, Ne(d, 0)), (sqrt(c)/(4*x**8), True))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.14

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}\,dx = -\frac{1}{35}\left(\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^2}-\frac{7\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^2}\right)a - \frac{1}{315}\left(\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{d^3}-\frac{90\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c}{d^3}+\frac{63\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^2}{d^3}\right)b$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/35*(5*(c + d/x^2)^(7/2)/d^2 - 7*(c + d/x^2)^(5/2)*c/d^2)*a - 1/315*(35*(
c + d/x^2)^(9/2)/d^3 - 90*(c + d/x^2)^(7/2)*c/d^3 + 63*(c + d/x^2)^(5/2)*c^
2/d^3)*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(62) = 124$.

Time = 1.31 (sec) , antiderivative size = 430, normalized size of antiderivative = 5.81

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}\,dx = \frac{4\left(315\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{14}ac^{\frac{7}{2}}\mathrm{sgn}(x)+840\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}bc^{\frac{9}{2}}\mathrm{sgn}(x)-3\right.}{}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 4/315*(315*(sqrt(c)*x - sqrt(c*x^2 + d))^14*a*c^(7/2)*sgn(x) + 840*(sqrt(c)
*x - sqrt(c*x^2 + d))^12*b*c^(9/2)*sgn(x) - 315*(sqrt(c)*x - sqrt(c*x^2 + d
))^12*a*c^(7/2)*d*sgn(x) + 1260*(sqrt(c)*x - sqrt(c*x^2 + d))^10*b*c^(9/2)*
d*sgn(x) + 315*(sqrt(c)*x - sqrt(c*x^2 + d))^10*a*c^(7/2)*d^2*sgn(x) + 1764
*(sqrt(c)*x - sqrt(c*x^2 + d))^8*b*c^(9/2)*d^2*sgn(x) - 819*(sqrt(c)*x - sq
rt(c*x^2 + d))^8*a*c^(7/2)*d^3*sgn(x) + 504*(sqrt(c)*x - sqrt(c*x^2 + d))^6
*b*c^(9/2)*d^3*sgn(x) + 441*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(7/2)*d^4*s
gn(x) + 144*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(9/2)*d^4*sgn(x) - 9*(sqrt(
c)*x - sqrt(c*x^2 + d))^4*a*c^(7/2)*d^5*sgn(x) - 36*(sqrt(c)*x - sqrt(c*x^2
 + d))^2*b*c^(9/2)*d^5*sgn(x) + 81*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(7/2
)*d^6*sgn(x) + 4*b*c^(9/2)*d^6*sgn(x) - 9*a*c^(7/2)*d^7*sgn(x))/((sqrt(c)*x
 - sqrt(c*x^2 + d))^2 - d)^9
```

Mupad [B] (verification not implemented)

Time = 10.25 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.22

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}\,dx = \frac{2\,a\,c^3\,\sqrt{c+\frac{d}{x^2}}}{35\,d^2}-\frac{8\,b\,c^4\,\sqrt{c+\frac{d}{x^2}}}{315\,d^3}-\frac{8\,a\,c\,\sqrt{c+\frac{d}{x^2}}}{35\,x^4}-\frac{a\,d\,\sqrt{c+\frac{d}{x^2}}}{7\,x^6}-\frac{10\,b\,c\,\sqrt{c+\frac{d}{x^2}}}{63\,x^6}-\frac{b\,d\,\sqrt{c+\frac{d}{x^2}}}{9\,x^8}-\frac{a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{35\,d\,x^2}-\frac{b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d\,x^4}+\frac{4\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{315\,d^2\,x^2}$$

```
[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^5,x)
```

```
[Out] (2*a*c^3*(c + d/x^2)^(1/2))/(35*d^2) - (8*b*c^4*(c + d/x^2)^(1/2))/(315*d^3
) - (8*a*c*(c + d/x^2)^(1/2))/(35*x^4) - (a*d*(c + d/x^2)^(1/2))/(7*x^6) -
(10*b*c*(c + d/x^2)^(1/2))/(63*x^6) - (b*d*(c + d/x^2)^(1/2))/(9*x^8) - (a*
c^2*(c + d/x^2)^(1/2))/(35*d*x^2) - (b*c^2*(c + d/x^2)^(1/2))/(105*d*x^4) +
 (4*b*c^3*(c + d/x^2)^(1/2))/(315*d^2*x^2)
```

3.951 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7} dx$

Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7} dx = \frac{c^2(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4} - \frac{c(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4} + \frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^4} - \frac{b\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^4}$$

```
[Out] 1/5*c^2*(-a*d+b*c)*(c+d/x^2)^(5/2)/d^4-1/7*c*(-2*a*d+3*b*c)*(c+d/x^2)^(7/2)
/d^4+1/9*(-a*d+3*b*c)*(c+d/x^2)^(9/2)/d^4-1/11*b*(c+d/x^2)^(11/2)/d^4
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 78}

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7} dx = \frac{c^2\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^4} + \frac{\left(c+\frac{d}{x^2}\right)^{9/2}(3bc-ad)}{9d^4} - \frac{c\left(c+\frac{d}{x^2}\right)^{7/2}(3bc-2ad)}{7d^4} - \frac{b\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^4}$$

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^7,x]
```

```
[Out] (c^2*(b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^4) - (c*(3*b*c - 2*a*d)*(c + d/x^2
)^(7/2))/(7*d^4) + ((3*b*c - a*d)*(c + d/x^2)^(9/2))/(9*d^4) - (b*(c + d/x^
2)^(11/2))/(11*d^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{2}\text{Subst}\left(\int x^2(a+bx)(c+dx)^{3/2}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-\frac{c^2(bc-ad)(c+dx)^{3/2}}{d^3}+\frac{c(3bc-2ad)(c+dx)^{5/2}}{d^3}+\frac{(-3bc+ad)(c+dx)^{7/2}}{d^3}+\cdots\right.\right.\right.$$

$$= \frac{c^2(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}-\frac{c(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4}+\frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^4}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}\,dx = \frac{\sqrt{c+\frac{d}{x^2}}(d+cx^2)^2\left(-11adx^2(35d^2-20cdx^2+8c^2x^4)-3b(105d^3-70cd^2x^2+\cdots\right.}{3465d^4x^{10}}$$

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^7,x]
```

```
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)^2*(-11*a*d*x^2*(35*d^2 - 20*c*d*x^2 + 8*c^2*x^
4) - 3*b*(105*d^3 - 70*c*d^2*x^2 + 40*c^2*d*x^4 - 16*c^3*x^6)))/(3465*d^4*x
^10)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

method	result
gosper	$-\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} \left(88a\,c^{2} d\,x^{6}-48b\,c^{3} x^{6}-220ac\,d^{2} x^{4}+120b\,c^{2} d\,x^{4}+385a\,d^{3} x^{2}-210bc\,d^{2} x^{2}+315b\,d^{3}\right) \left(c\,x^{2}+d\right)}{3465 d^{4} x^{8}}$
default	$-\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} \left(88a\,c^{2} d\,x^{6}-48b\,c^{3} x^{6}-220ac\,d^{2} x^{4}+120b\,c^{2} d\,x^{4}+385a\,d^{3} x^{2}-210bc\,d^{2} x^{2}+315b\,d^{3}\right) \left(c\,x^{2}+d\right)}{3465 d^{4} x^{8}}$
risch	$-\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, \left(88a\,c^{4} d\,x^{10}-48b\,c^{5} x^{10}-44a\,c^{3} d^{2} x^{8}+24b\,c^{4} d\,x^{8}+33a\,c^{2} d^{3} x^{6}-18b\,c^{3} d^{2} x^{6}+550ac\,d^{4} x^{4}+15b\,c^{2} d^{3} x^{4}+385a\,d^{5} x^{2}+420bc\,d^{4}\right.}{3465 x^{10} d^{4}}$
trager	$-\frac{\left(88a\,c^{4} d\,x^{10}-48b\,c^{5} x^{10}-44a\,c^{3} d^{2} x^{8}+24b\,c^{4} d\,x^{8}+33a\,c^{2} d^{3} x^{6}-18b\,c^{3} d^{2} x^{6}+550ac\,d^{4} x^{4}+15b\,c^{2} d^{3} x^{4}+385a\,d^{5} x^{2}+420bc\,d^{4} x^{2}+315b\right.}{3465 x^{10} d^{4}}$

[In] int((a+b/x^2)*(c+d/x^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)

[Out] -1/3465*((c*x^2+d)/x^2)^(3/2)*(88*a*c^2*d*x^6-48*b*c^3*x^6-220*a*c*d^2*x^4+
120*b*c^2*d*x^4+385*a*d^3*x^2-210*b*c*d^2*x^2+315*b*d^3)*(c*x^2+d)/d^4/x^8

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.29

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}\,dx = \frac{(8\,(6\,bc^5-11\,ac^4d)x^{10}-4\,(6\,bc^4d-11\,ac^3d^2)x^8+3\,(6\,bc^3d^2-11\,ac^2d^3)x^6-31}{3465\,d^4x^{10}}$$

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^7,x, algorithm="fricas")

[Out] 1/3465*(8*(6*b*c^5 - 11*a*c^4*d)*x^10 - 4*(6*b*c^4*d - 11*a*c^3*d^2)*x^8 +
3*(6*b*c^3*d^2 - 11*a*c^2*d^3)*x^6 - 315*b*d^5 - 5*(3*b*c^2*d^3 + 110*a*c*d
^4)*x^4 - 35*(12*b*c*d^4 + 11*a*d^5)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^4*x^10)

Sympy [A] (verification not implemented)

Time = 3.77 (sec) , antiderivative size = 326, normalized size of antiderivative = 3.13

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}\,dx = -\frac{ac\left(\begin{cases}\frac{2\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}}{3x^6} & \text{otherwise}\end{cases}\right)}{2} - \frac{ad\left(\begin{cases}\frac{2\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}}{4x^8} & \text{otherwise}\end{cases}\right)}{2} - \frac{bc\left(\begin{cases}\frac{2\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}}{4x^8} & \text{otherwise}\end{cases}\right)}{2} - \frac{bd\left(\begin{cases}\frac{2\left(\frac{c^4\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{4c^3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{6c^2\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}-\frac{4c\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{11}\right)}{d^5} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}}{5x^{10}} & \text{otherwise}\end{cases}\right)}{2}$$

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**7,x)
```

```
[Out] -a*c*Piecewise((2*(c**2*(c + d/x**2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 +
 (c + d/x**2)**(7/2)/7)/d**3, Ne(d, 0)), (sqrt(c)/(3*x**6), True))/2 - a*d*
Piecewise((2*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)/5 -
3*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**4, Ne(d, 0)), (sqrt(c
)/(4*x**8), True))/2 - b*c*Piecewise((2*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c*
*2*(c + d/x**2)**(5/2)/5 - 3*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/
9)/d**4, Ne(d, 0)), (sqrt(c)/(4*x**8), True))/2 - b*d*Piecewise((2*(c**4*(c
 + d/x**2)**(3/2)/3 - 4*c**3*(c + d/x**2)**(5/2)/5 + 6*c**2*(c + d/x**2)**(
7/2)/7 - 4*c*(c + d/x**2)**(9/2)/9 + (c + d/x**2)**(11/2)/11)/d**5, Ne(d, 0
)), (sqrt(c)/(5*x**10), True))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.13

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}\,dx = -\frac{1}{315}\left(\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{d^3} - \frac{90\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c}{d^3} + \frac{63\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^2}{d^3}\right)a - \frac{1}{1155}\left(\frac{105\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{d^4} - \frac{385\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}c}{d^4} + \frac{495\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c^2}{d^4} - \frac{231\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3}{d^4}\right)b$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/315*(35*(c + d/x^2)^(9/2)/d^3 - 90*(c + d/x^2)^(7/2)*c/d^3 + 63*(c + d/x
^2)^(5/2)*c^2/d^3)*a - 1/1155*(105*(c + d/x^2)^(11/2)/d^4 - 385*(c + d/x^2)
^(9/2)*c/d^4 + 495*(c + d/x^2)^(7/2)*c^2/d^4 - 231*(c + d/x^2)^(5/2)*c^3/d^
4)*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. 2(88) = 176.

Time = 2.03 (sec) , antiderivative size = 490, normalized size of antiderivative = 4.71

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}\,dx = \frac{16\left(2310\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{16}ac^{\frac{9}{2}}\operatorname{sgn}(x) + 6930\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{14}bc^{\frac{11}{2}}\operatorname{sgn}(x)\right.}{}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 16/3465*(2310*(sqrt(c)*x - sqrt(c*x^2 + d))^16*a*c^(9/2)*sgn(x) + 6930*(sqr
t(c)*x - sqrt(c*x^2 + d))^14*b*c^(11/2)*sgn(x) - 1155*(sqrt(c)*x - sqrt(c*x
^2 + d))^14*a*c^(9/2)*d*sgn(x) + 12474*(sqrt(c)*x - sqrt(c*x^2 + d))^12*b*c
^(11/2)*d*sgn(x) + 231*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(9/2)*d^2*sgn(x
) + 15246*(sqrt(c)*x - sqrt(c*x^2 + d))^10*b*c^(11/2)*d^2*sgn(x) - 4851*(sq
rt(c)*x - sqrt(c*x^2 + d))^10*a*c^(9/2)*d^3*sgn(x) + 4950*(sqrt(c)*x - sqrt
(c*x^2 + d))^8*b*c^(11/2)*d^3*sgn(x) + 2475*(sqrt(c)*x - sqrt(c*x^2 + d))^8
*a*c^(9/2)*d^4*sgn(x) + 990*(sqrt(c)*x - sqrt(c*x^2 + d))^6*b*c^(11/2)*d^4*
sgn(x) + 495*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(9/2)*d^5*sgn(x) - 330*(sq
rt(c)*x - sqrt(c*x^2 + d))^4*b*c^(11/2)*d^5*sgn(x) + 605*(sqrt(c)*x - sqrt(
c*x^2 + d))^4*a*c^(9/2)*d^6*sgn(x) + 66*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c
^(11/2)*d^6*sgn(x) - 121*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(9/2)*d^7*sgn(
x) - 6*b*c^(11/2)*d^7*sgn(x) + 11*a*c^(9/2)*d^8*sgn(x))/((sqrt(c)*x - sqrt(
c*x^2 + d))^2 - d)^11
```

Mupad [B] (verification not implemented)

Time = 10.68 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.98

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}\,dx = \frac{16\,b\,c^5\,\sqrt{c+\frac{d}{x^2}}}{1155\,d^4}-\frac{8\,a\,c^4\,\sqrt{c+\frac{d}{x^2}}}{315\,d^3}-\frac{10\,a\,c\,\sqrt{c+\frac{d}{x^2}}}{63\,x^6} -\frac{a\,d\,\sqrt{c+\frac{d}{x^2}}}{9\,x^8}-\frac{4\,b\,c\,\sqrt{c+\frac{d}{x^2}}}{33\,x^8}-\frac{b\,d\,\sqrt{c+\frac{d}{x^2}}}{11\,x^{10}}-\frac{a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d\,x^4} +\frac{4\,a\,c^3\,\sqrt{c+\frac{d}{x^2}}}{315\,d^2\,x^2}-\frac{b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{231\,d\,x^6}+\frac{2\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{385\,d^2\,x^4}-\frac{8\,b\,c^4\,\sqrt{c+\frac{d}{x^2}}}{1155\,d^3\,x^2}$$

```
[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^7,x)
```

```
[Out] (16*b*c^5*(c + d/x^2)^(1/2))/(1155*d^4) - (8*a*c^4*(c + d/x^2)^(1/2))/(315*
d^3) - (10*a*c*(c + d/x^2)^(1/2))/(63*x^6) - (a*d*(c + d/x^2)^(1/2))/(9*x^8
) - (4*b*c*(c + d/x^2)^(1/2))/(33*x^8) - (b*d*(c + d/x^2)^(1/2))/(11*x^10)
- (a*c^2*(c + d/x^2)^(1/2))/(105*d*x^4) + (4*a*c^3*(c + d/x^2)^(1/2))/(315*
d^2*x^2) - (b*c^2*(c + d/x^2)^(1/2))/(231*d*x^6) + (2*b*c^3*(c + d/x^2)^(1/
2))/(385*d^2*x^4) - (8*b*c^4*(c + d/x^2)^(1/2))/(1155*d^3*x^2)
```

3.952 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 134

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx = -\frac{c^3(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^5}+\frac{c^2(4bc-3ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^5} - \frac{c(2bc-ad)\left(c+\frac{d}{x^2}\right)^{9/2}}{3d^5}+\frac{(4bc-ad)\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^5}-\frac{b\left(c+\frac{d}{x^2}\right)^{13/2}}{13d^5}$$

```
[Out] -1/5*c^3*(-a*d+b*c)*(c+d/x^2)^(5/2)/d^5+1/7*c^2*(-3*a*d+4*b*c)*(c+d/x^2)^(7
/2)/d^5-1/3*c*(-a*d+2*b*c)*(c+d/x^2)^(9/2)/d^5+1/11*(-a*d+4*b*c)*(c+d/x^2)^
(11/2)/d^5-1/13*b*(c+d/x^2)^(13/2)/d^5
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 78}

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx = -\frac{c^3\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^5}+\frac{c^2\left(c+\frac{d}{x^2}\right)^{7/2}(4bc-3ad)}{7d^5} + \frac{\left(c+\frac{d}{x^2}\right)^{11/2}(4bc-ad)}{11d^5}-\frac{c\left(c+\frac{d}{x^2}\right)^{9/2}(2bc-ad)}{3d^5}-\frac{b\left(c+\frac{d}{x^2}\right)^{13/2}}{13d^5}$$

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^9,x]
```

```
[Out] -1/5*(c^3*(b*c - a*d)*(c + d/x^2)^(5/2))/d^5 + (c^2*(4*b*c - 3*a*d)*(c + d/
x^2)^(7/2))/(7*d^5) - (c*(2*b*c - a*d)*(c + d/x^2)^(9/2))/(3*d^5) + ((4*b*c
 - a*d)*(c + d/x^2)^(11/2))/(11*d^5) - (b*(c + d/x^2)^(13/2))/(13*d^5)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{2}\text{Subst}\left(\int x^3(a+bx)(c+dx)^{3/2}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{c^3(bc-ad)(c+dx)^{3/2}}{d^4} - \frac{c^2(4bc-3ad)(c+dx)^{5/2}}{d^4} + \frac{3c(2bc-ad)(c+dx)^{7/2}}{d^4} + \frac{(\right.\right.\right.$$

$$= -\frac{c^3(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^5} + \frac{c^2(4bc-3ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^5} - \frac{c(2bc-ad)\left(c+\frac{d}{x^2}\right)^{9/2}}{3d^5} + \frac{(4bc-ad)\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^5} - \frac{b\left(c+\frac{d}{x^2}\right)^{13/2}}{13d^5}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx = \frac{\sqrt{c+\frac{d}{x^2}}(d+cx^2)^2\left(13adx^2(-105d^3+70cd^2x^2-40c^2dx^4+16c^3x^6)+b(-115\right.}{15015d^5x^{12}}$$

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^9,x]
```

```
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)^2*(13*a*d*x^2*(-105*d^3 + 70*c*d^2*x^2 - 40*c^
2*d*x^4 + 16*c^3*x^6) + b*(-1155*d^4 + 840*c*d^3*x^2 - 560*c^2*d^2*x^4 + 32
0*c^3*d*x^6 - 128*c^4*x^8)))/(15015*d^5*x^12)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.88

method	result
gosper	$\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}\left(208a\,c^{3}d\,x^{8}-128b\,c^{4}x^{8}-520a\,c^{2}d^{2}x^{6}+320b\,c^{3}d\,x^{6}+910ac\,d^{3}x^{4}-560b\,c^{2}d^{2}x^{4}-1365a\,d^{4}x^{2}+840bc\,d^{3}x^{2}-1155b\,d^{4}\right)\left(c\,x^{2}+d\right)}{15015d^{5}x^{10}}$
default	$\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}\left(208a\,c^{3}d\,x^{8}-128b\,c^{4}x^{8}-520a\,c^{2}d^{2}x^{6}+320b\,c^{3}d\,x^{6}+910ac\,d^{3}x^{4}-560b\,c^{2}d^{2}x^{4}-1365a\,d^{4}x^{2}+840bc\,d^{3}x^{2}-1155b\,d^{4}\right)\left(c\,x^{2}+d\right)}{15015d^{5}x^{10}}$
risch	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(208a\,c^{5}d\,x^{12}-128b\,c^{6}x^{12}-104a\,c^{4}d^{2}x^{10}+64b\,c^{5}d\,x^{10}+78a\,c^{3}d^{3}x^{8}-48b\,c^{4}d^{2}x^{8}-65a\,c^{2}d^{4}x^{6}+40b\,c^{3}d^{3}x^{6}-1820ac\,d^{5}x^{4}-35b\right.}{15015x^{12}d^{5}}$
trager	$\frac{\left(208a\,c^{5}d\,x^{12}-128b\,c^{6}x^{12}-104a\,c^{4}d^{2}x^{10}+64b\,c^{5}d\,x^{10}+78a\,c^{3}d^{3}x^{8}-48b\,c^{4}d^{2}x^{8}-65a\,c^{2}d^{4}x^{6}+40b\,c^{3}d^{3}x^{6}-1820ac\,d^{5}x^{4}-35b\,c^{2}d^{4}x^{4}-\right.}{15015x^{12}d^{5}}$

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15015*((c*x^2+d)/x^2)^(3/2)*(208*a*c^3*d*x^8-128*b*c^4*x^8-520*a*c^2*d^2*
x^6+320*b*c^3*d*x^6+910*a*c*d^3*x^4-560*b*c^2*d^2*x^4-1365*a*d^4*x^2+840*b*
c*d^3*x^2-1155*b*d^4)*(c*x^2+d)/d^5/x^10
```

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.17

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx =$$

$$-\frac{(16\,(8\,bc^6-13\,ac^5d)x^{12}-8\,(8\,bc^5d-13\,ac^4d^2)x^{10}+6\,(8\,bc^4d^2-13\,ac^3d^3)x^8+1155\,bd^6-5\,(8\,bc^3d^3-13\,a}{15015\,d^5x^{12}}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/15015*(16*(8*b*c^6 - 13*a*c^5*d)*x^12 - 8*(8*b*c^5*d - 13*a*c^4*d^2)*x^1
0 + 6*(8*b*c^4*d^2 - 13*a*c^3*d^3)*x^8 + 1155*b*d^6 - 5*(8*b*c^3*d^3 - 13*a
*c^2*d^4)*x^6 + 35*(b*c^2*d^4 + 52*a*c*d^5)*x^4 + 105*(14*b*c*d^5 + 13*a*d^
6)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^5*x^12)
```

Sympy [A] (verification not implemented)

Time = 3.93 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.93

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx = -\frac{ac\left(\begin{cases}\frac{2\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4} & \text{for } d\neq 0\\ \frac{\sqrt{c}}{4x^8} & \text{otherwise}\end{cases}\right)}{2} - \frac{ad\left(\begin{cases}\frac{2\left(\frac{c^4\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{4c^3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{6c^2\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}-\frac{4c\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{11}\right)}{d^5} & \text{for } d\neq 0\\ \frac{\sqrt{c}}{5x^{10}} & \text{otherwise}\end{cases}\right)}{2} - \frac{bc\left(\begin{cases}\frac{2\left(\frac{c^4\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{4c^3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{6c^2\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}-\frac{4c\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{11}\right)}{d^5} & \text{for } d\neq 0\\ \frac{\sqrt{c}}{5x^{10}} & \text{otherwise}\end{cases}\right)}{2} - \frac{bd\left(\begin{cases}\frac{2\left(-\frac{c^5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+c^4\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}-\frac{10c^3\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{10c^2\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}-\frac{5c\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{11}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{13}{2}}}{13}\right)}{d^6} & \text{for } d\neq 0\\ \frac{\sqrt{c}}{6x^{12}} & \text{otherwise}\end{cases}\right)}{2}$$

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**9,x)
```

```
[Out] -a*c*Piecewise((2*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)
/5 - 3*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**4, Ne(d, 0)), (s
qrt(c)/(4*x**8), True))/2 - a*d*Piecewise((2*(c**4*(c + d/x**2)**(3/2)/3 -
4*c**3*(c + d/x**2)**(5/2)/5 + 6*c**2*(c + d/x**2)**(7/2)/7 - 4*c*(c + d/x*
*2)**(9/2)/9 + (c + d/x**2)**(11/2)/11)/d**5, Ne(d, 0)), (sqrt(c)/(5*x**10)
, True))/2 - b*c*Piecewise((2*(c**4*(c + d/x**2)**(3/2)/3 - 4*c**3*(c + d/x
**2)**(5/2)/5 + 6*c**2*(c + d/x**2)**(7/2)/7 - 4*c*(c + d/x**2)**(9/2)/9 +
(c + d/x**2)**(11/2)/11)/d**5, Ne(d, 0)), (sqrt(c)/(5*x**10), True))/2 - b*
d*Piecewise((2*(-c**5*(c + d/x**2)**(3/2)/3 + c**4*(c + d/x**2)**(5/2) - 10
*c**3*(c + d/x**2)**(7/2)/7 + 10*c**2*(c + d/x**2)**(9/2)/9 - 5*c*(c + d/x*
*2)**(11/2)/11 + (c + d/x**2)**(13/2)/13)/d**6, Ne(d, 0)), (sqrt(c)/(6*x**1
2), True))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.13

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx = -\frac{1}{1155}\left(\frac{105\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{d^4}-\frac{385\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}c}{d^4}+\frac{495\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c^2}{d^4}-\frac{231\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3}{d^4}\right)a - \frac{1}{15015}\left(\frac{1155\left(c+\frac{d}{x^2}\right)^{\frac{13}{2}}}{d^5}-\frac{5460\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}c}{d^5}+\frac{10010\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}c^2}{d^5}-\frac{8580\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c^3}{d^5}+\frac{3003\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^4}{d^5}\right.$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/1155*(105*(c + d/x^2)^(11/2)/d^4 - 385*(c + d/x^2)^(9/2)*c/d^4 + 495*(c
+ d/x^2)^(7/2)*c^2/d^4 - 231*(c + d/x^2)^(5/2)*c^3/d^4)*a - 1/15015*(1155*(
c + d/x^2)^(13/2)/d^5 - 5460*(c + d/x^2)^(11/2)*c/d^5 + 10010*(c + d/x^2)^(
9/2)*c^2/d^5 - 8580*(c + d/x^2)^(7/2)*c^3/d^5 + 3003*(c + d/x^2)^(5/2)*c^4/
d^5)*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. 2(114) = 228.

Time = 2.05 (sec) , antiderivative size = 550, normalized size of antiderivative = 4.10

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx = \frac{32\left(15015\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{18}ac^{\frac{11}{2}}\operatorname{sgn}(x)+48048\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{16}bc^{\frac{13}{2}}\operatorname{sgn}\right.}{}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] 32/15015*(15015*(sqrt(c)*x - sqrt(c*x^2 + d))^18*a*c^(11/2)*sgn(x) + 48048*
(sqrt(c)*x - sqrt(c*x^2 + d))^16*b*c^(13/2)*sgn(x) - 3003*(sqrt(c)*x - sqrt
(c*x^2 + d))^16*a*c^(11/2)*d*sgn(x) + 96096*(sqrt(c)*x - sqrt(c*x^2 + d))^1
4*b*c^(13/2)*d*sgn(x) - 6006*(sqrt(c)*x - sqrt(c*x^2 + d))^14*a*c^(11/2)*d^
2*sgn(x) + 109824*(sqrt(c)*x - sqrt(c*x^2 + d))^12*b*c^(13/2)*d^2*sgn(x) -
28314*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(11/2)*d^3*sgn(x) + 37752*(sqrt(
c)*x - sqrt(c*x^2 + d))^10*b*c^(13/2)*d^3*sgn(x) + 13728*(sqrt(c)*x - sqrt(
c*x^2 + d))^10*a*c^(11/2)*d^4*sgn(x) + 5720*(sqrt(c)*x - sqrt(c*x^2 + d))^8
*b*c^(13/2)*d^4*sgn(x) + 5720*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(11/2)*d^
5*sgn(x) - 2288*(sqrt(c)*x - sqrt(c*x^2 + d))^6*b*c^(13/2)*d^5*sgn(x) + 371
```

8*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(11/2)*d^6*sgn(x) + 624*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(13/2)*d^6*sgn(x) - 1014*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(11/2)*d^7*sgn(x) - 104*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(13/2)*d^7*sgn(x) + 169*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(11/2)*d^8*sgn(x) + 8*b*c^(13/2)*d^8*sgn(x) - 13*a*c^(11/2)*d^9*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^13

Mupad [B] (verification not implemented)

Time = 11.33 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.85

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx=\frac{16\,a\,c^5\,\sqrt{c+\frac{d}{x^2}}}{1155\,d^4}-\frac{128\,b\,c^6\,\sqrt{c+\frac{d}{x^2}}}{15015\,d^5}-\frac{4\,a\,c\,\sqrt{c+\frac{d}{x^2}}}{33\,x^8}-\frac{a\,d\,\sqrt{c+\frac{d}{x^2}}}{11\,x^{10}}-\frac{14\,b\,c\,\sqrt{c+\frac{d}{x^2}}}{143\,x^{10}}-\frac{b\,d\,\sqrt{c+\frac{d}{x^2}}}{13\,x^{12}}-\frac{a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{231\,d\,x^6}+\frac{2\,a\,c^3\,\sqrt{c+\frac{d}{x^2}}}{385\,d^2\,x^4}-\frac{8\,a\,c^4\,\sqrt{c+\frac{d}{x^2}}}{1155\,d^3\,x^2}-\frac{b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{429\,d\,x^8}+\frac{8\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{3003\,d^2\,x^6}-\frac{16\,b\,c^4\,\sqrt{c+\frac{d}{x^2}}}{5005\,d^3\,x^4}+\frac{64\,b\,c^5\,\sqrt{c+\frac{d}{x^2}}}{15015\,d^4\,x^2}$$

[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^9,x)

[Out] (16*a*c^5*(c + d/x^2)^(1/2))/(1155*d^4) - (128*b*c^6*(c + d/x^2)^(1/2))/(15015*d^5) - (4*a*c*(c + d/x^2)^(1/2))/(33*x^8) - (a*d*(c + d/x^2)^(1/2))/(11*x^10) - (14*b*c*(c + d/x^2)^(1/2))/(143*x^10) - (b*d*(c + d/x^2)^(1/2))/(13*x^12) - (a*c^2*(c + d/x^2)^(1/2))/(231*d*x^6) + (2*a*c^3*(c + d/x^2)^(1/2))/(385*d^2*x^4) - (8*a*c^4*(c + d/x^2)^(1/2))/(1155*d^3*x^2) - (b*c^2*(c + d/x^2)^(1/2))/(429*d*x^8) + (8*b*c^3*(c + d/x^2)^(1/2))/(3003*d^2*x^6) - (16*b*c^4*(c + d/x^2)^(1/2))/(5005*d^3*x^4) + (64*b*c^5*(c + d/x^2)^(1/2))/(15015*d^4*x^2)

3.953 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^{12}\, dx$

Optimal result

Integrand size = 22, antiderivative size = 150

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^{12}\, dx = -\frac{16d^3(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{15015c^5} + \frac{8d^2(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^7}{3003c^4} - \frac{2d(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^9}{429c^3} + \frac{(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^{11}}{143c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^{13}}{13c}$$

```
[Out] -16/15015*d^3*(-8*a*d+13*b*c)*(c+d/x^2)^(5/2)*x^5/c^5+8/3003*d^2*(-8*a*d+13
*b*c)*(c+d/x^2)^(5/2)*x^7/c^4-2/429*d*(-8*a*d+13*b*c)*(c+d/x^2)^(5/2)*x^9/c
^3+1/143*(-8*a*d+13*b*c)*(c+d/x^2)^(5/2)*x^11/c^2+1/13*a*(c+d/x^2)^(5/2)*x^
13/c
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {464, 277, 270}

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^{12}\, dx = -\frac{16d^3x^5\left(c+\frac{d}{x^2}\right)^{5/2}(13bc-8ad)}{15015c^5} + \frac{8d^2x^7\left(c+\frac{d}{x^2}\right)^{5/2}(13bc-8ad)}{3003c^4} - \frac{2dx^9\left(c+\frac{d}{x^2}\right)^{5/2}(13bc-8ad)}{429c^3} + \frac{x^{11}\left(c+\frac{d}{x^2}\right)^{5/2}(13bc-8ad)}{143c^2} + \frac{ax^{13}\left(c+\frac{d}{x^2}\right)^{5/2}}{13c}$$

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^12,x]
```

```
[Out] (-16*d^3*(13*b*c - 8*a*d)*(c + d/x^2)^(5/2)*x^5)/(15015*c^5) + (8*d^2*(13*b
*c - 8*a*d)*(c + d/x^2)^(5/2)*x^7)/(3003*c^4) - (2*d*(13*b*c - 8*a*d)*(c +
d/x^2)^(5/2)*x^9)/(429*c^3) + ((13*b*c - 8*a*d)*(c + d/x^2)^(5/2)*x^11)/(14
3*c^2) + (a*(c + d/x^2)^(5/2)*x^13)/(13*c)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^{13}}{13c}+\frac{(13bc-8ad)\int\left(c+\frac{d}{x^2}\right)^{3/2}x^{10}\,dx}{13c}\\
&= \frac{(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^{11}}{143c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^{13}}{13c}-\frac{(6d(13bc-8ad))\int\left(c+\frac{d}{x^2}\right)^{3/2}x^{8}\,dx}{143c^2}\\
&= -\frac{2d(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^{9}}{429c^3}+\frac{(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^{11}}{143c^2}\\
&\quad+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^{13}}{13c}+\frac{(8d^2(13bc-8ad))\int\left(c+\frac{d}{x^2}\right)^{3/2}x^{6}\,dx}{429c^3}\\
&= \frac{8d^2(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^{7}}{3003c^4}-\frac{2d(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^{9}}{429c^3}\\
&\quad+\frac{(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^{11}}{143c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^{13}}{13c}\\
&\quad-\frac{(16d^3(13bc-8ad))\int\left(c+\frac{d}{x^2}\right)^{3/2}x^{4}\,dx}{3003c^4}
\end{aligned}$$

$$= -\frac{16d^3(13bc - 8ad)\left(c + \frac{d}{x^2}\right)^{5/2} x^5}{15015c^5} + \frac{8d^2(13bc - 8ad)\left(c + \frac{d}{x^2}\right)^{5/2} x^7}{3003c^4} - \frac{2d(13bc - 8ad)\left(c + \frac{d}{x^2}\right)^{5/2} x^9}{429c^3} + \frac{(13bc - 8ad)\left(c + \frac{d}{x^2}\right)^{5/2} x^{11}}{143c^2} + \frac{a\left(c + \frac{d}{x^2}\right)^{5/2} x^{13}}{13c}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.73

$$\int \left(a + \frac{b}{x^2}\right)\left(c + \frac{d}{x^2}\right)^{3/2} x^{12}\,dx = \frac{\sqrt{c + \frac{d}{x^2}}x(d + cx^2)^2\,(13bc(-16d^3 + 40cd^2x^2 - 70c^2dx^4 + 105c^3x^6) + a(128d^4 - 320cd^3x^2 +}{15015c^5}$$

[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^12,x]

[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(13*b*c*(-16*d^3 + 40*c*d^2*x^2 - 70*c^2*d*x^4 + 105*c^3*x^6) + a*(128*d^4 - 320*c*d^3*x^2 + 560*c^2*d^2*x^4 - 840*c^3*d*x^6 + 1155*c^4*x^8)))/(15015*c^5)

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.77

method	result
gosper	$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}} x^3 \left(1155a\,x^8c^4-840a\,c^3d\,x^6+1365b\,c^4x^6+560a\,c^2d^2x^4-910b\,c^3d\,x^4-320ac\,d^3x^2+520b\,c^2d^2x^2+128a\,d^4-208bc\,d^3\right)\left(c\,x^2+d\right)}{15015c^5}$
default	$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}} x^3 \left(1155a\,x^8c^4-840a\,c^3d\,x^6+1365b\,c^4x^6+560a\,c^2d^2x^4-910b\,c^3d\,x^4-320ac\,d^3x^2+520b\,c^2d^2x^2+128a\,d^4-208bc\,d^3\right)\left(c\,x^2+d\right)}{15015c^5}$
risch	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,x\left(1155a\,c^6x^{12}+1470a\,c^5d\,x^{10}+1365b\,c^6x^{10}+35a\,c^4d^2x^8+1820b\,c^5d\,x^8-40a\,c^3d^3x^6+65b\,c^4d^2x^6+48a\,c^2d^4x^4-78b\,c^3d^3x^4-6}{15015c^5}$
trager	$\frac{(1155a\,c^6x^{12}+1470a\,c^5d\,x^{10}+1365b\,c^6x^{10}+35a\,c^4d^2x^8+1820b\,c^5d\,x^8-40a\,c^3d^3x^6+65b\,c^4d^2x^6+48a\,c^2d^4x^4-78b\,c^3d^3x^4-64ac\,d^5x^2+}{15015c^5}$

[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^12,x,method=_RETURNVERBOSE)

[Out] 1/15015*((c*x^2+d)/x^2)^(3/2)*x^3*(1155*a*c^4*x^8-840*a*c^3*d*x^6+1365*b*c^4*x^6+560*a*c^2*d^2*x^4-910*b*c^3*d*x^4-320*a*c*d^3*x^2+520*b*c^2*d^2*x^2+128*a*d^4-208*b*c*d^3)*(c*x^2+d)/c^5

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.03

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^{12}\,dx=\frac{(1155\,ac^6x^{13}+105\,(13\,bc^6+14\,ac^5d)x^{11}+35\,(52\,bc^5d+ac^4d^2)x^9+5\,(13\,bc^4d^2-8\,ac^3d^3)}{15015}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^12,x, algorithm="fricas")
```

```
[Out] 1/15015*(1155*a*c^6*x^13 + 105*(13*b*c^6 + 14*a*c^5*d)*x^11 + 35*(52*b*c^5*
d + a*c^4*d^2)*x^9 + 5*(13*b*c^4*d^2 - 8*a*c^3*d^3)*x^7 - 6*(13*b*c^3*d^3 -
 8*a*c^2*d^4)*x^5 + 8*(13*b*c^2*d^4 - 8*a*c*d^5)*x^3 - 16*(13*b*c*d^5 - 8*a
*d^6)*x)*sqrt((c*x^2 + d)/x^2)/c^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3351 vs. 2(146) = 292.

Time = 5.61 (sec) , antiderivative size = 3351, normalized size of antiderivative = 22.34

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^{12}\,dx=\text{Too large to display}$$

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**12,x)
```

```
[Out] 693*a*c**12*d**(51/2)*x**22*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45
045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 4504
5*c**7*d**29*x**2 + 9009*c**6*d**30) + 3528*a*c**11*d**(53/2)*x**20*sqrt(c*
x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d
**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30
) + 7175*a*c**10*d**(55/2)*x**18*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10
 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 +
 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 7290*a*c**9*d**(57/2)*x**16*sqr
t(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c*
*9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d
**30) + 315*a*c**9*d**(35/2)*x**18*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8
 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 +
3465*c**5*d**20) + 3699*a*c**8*d**(59/2)*x**14*sqrt(c*x**2/d + 1)/(9009*c**
11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**
8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 1295*a*c**8*d**(3
7/2)*x**16*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6
```

+ 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 756*a
*c**7*d**(61/2)*x**12*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c*
*10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7
*d**29*x**2 + 9009*c**6*d**30) + 1990*a*c**7*d**(39/2)*x**14*sqrt(c*x**2/d
+ 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4
+ 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 63*a*c**6*d**(63/2)*x**10*sqrt
(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**
9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d*
*30) + 1358*a*c**6*d**(41/2)*x**12*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8
+ 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 +
3465*c**5*d**20) - 630*a*c**5*d**(65/2)*x**8*sqrt(c*x**2/d + 1)/(9009*c**11
*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*
d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 343*a*c**5*d**(43/2
)*x**10*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 +
20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 1680*a*c
4*d(67/2)*x**6*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10
*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d*
*29*x**2 + 9009*c**6*d**30) + 35*a*c**4*d**(45/2)*x**8*sqrt(c*x**2/d + 1)/(
3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 1386
0*c**6*d**19*x**2 + 3465*c**5*d**20) - 2016*a*c**3*d**(69/2)*x**4*sqrt(c*x*
*2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**
27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30)
+ 280*a*c**3*d**(47/2)*x**6*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 1386
0*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c*
*5*d**20) - 1152*a*c**2*d**(71/2)*x**2*sqrt(c*x**2/d + 1)/(9009*c**11*d**25
*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*
x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 560*a*c**2*d**(49/2)*x**4
*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c
7*d18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 256*a*c*d**(73/
2)*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90
090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*
c**6*d**30) + 448*a*c*d**(51/2)*x**2*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x*
*8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2
+ 3465*c**5*d**20) + 128*a*d**(53/2)*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x*
*8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2
+ 3465*c**5*d**20) + 315*b*c**10*d**(33/2)*x**18*sqrt(c*x**2/d + 1)/(3465*c
9*d16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6
*d**19*x**2 + 3465*c**5*d**20) + 1295*b*c**9*d**(35/2)*x**16*sqrt(c*x**2/d
+ 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4
+ 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 1990*b*c**8*d**(37/2)*x**14*sq
rt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7
*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 1358*b*c**7*d**(39
/2)*x**12*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6
+ 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 35*b*c
7*d(21/2)*x**14*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10

```
*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 343*b*c**6*d**(41/2)*x**10*
sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c*
*7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 110*b*c**6*d**(2
3/2)*x**12*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 9
45*c**5*d**11*x**2 + 315*c**4*d**12) + 35*b*c**5*d**(43/2)*x**8*sqrt(c*x**2
/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x*
*4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 114*b*c**5*d**(25/2)*x**10*
sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**
11*x**2 + 315*c**4*d**12) + 280*b*c**4*d**(45/2)*x**6*sqrt(c*x**2/d + 1)/(3
465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860
*c**6*d**19*x**2 + 3465*c**5*d**20) + 40*b*c**4*d**(27/2)*x**8*sqrt(c*x**2/
d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 31
5*c**4*d**12) + 560*b*c**3*d**(47/2)*x**4*sqrt(c*x**2/d + 1)/(3465*c**9*d**
16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*
x**2 + 3465*c**5*d**20) - 5*b*c**3*d**(29/2)*x**6*sqrt(c*x**2/d + 1)/(315*c
**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12)
 + 448*b*c**2*d**(49/2)*x**2*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 138
60*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c
**5*d**20) - 30*b*c**2*d**(31/2)*x**4*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**
6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 128*b*c*d
**(51/2)*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 +
 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 40*b*c*
d**(33/2)*x**2*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4
 + 945*c**5*d**11*x**2 + 315*c**4*d**12) - 16*b*d**(35/2)*sqrt(c*x**2/d + 1
)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**
4*d**12)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.05

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^{12}\,dx=\frac{\left(105\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}x^{11}-385\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}dx^9+495\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}d^2x^7-231\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^3x^5\right)b}{1155\,c^4}+\frac{\left(1155\left(c+\frac{d}{x^2}\right)^{\frac{13}{2}}x^{13}-5460\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}dx^{11}+10010\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}d^2x^9-8580\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}d^3x^7+3003\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}\right.}{15015\,c^5}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^12,x, algorithm="maxima")
```

```
[Out] 1/1155*(105*(c + d/x^2)^(11/2)*x^11 - 385*(c + d/x^2)^(9/2)*d*x^9 + 495*(c
+ d/x^2)^(7/2)*d^2*x^7 - 231*(c + d/x^2)^(5/2)*d^3*x^5)*b/c^4 + 1/15015*(11
```

```
55*(c + d/x^2)^(13/2)*x^13 - 5460*(c + d/x^2)^(11/2)*d*x^11 + 10010*(c + d/
x^2)^(9/2)*d^2*x^9 - 8580*(c + d/x^2)^(7/2)*d^3*x^7 + 3003*(c + d/x^2)^(5/2
)*d^4*x^5)*a/c^5
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.17

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^{12}\,dx=\frac{16\left(13\,bcd^{\frac{11}{2}}-8\,ad^{\frac{13}{2}}\right)\mathrm{sgn}(x)}{15015\,c^5}$$
$$+\frac{1155\,(cx^2+d)^{\frac{13}{2}}a\mathrm{sgn}(x)+1365\,(cx^2+d)^{\frac{11}{2}}bc\mathrm{sgn}(x)-5460\,(cx^2+d)^{\frac{11}{2}}ad\mathrm{sgn}(x)-5005\,(cx^2+d)^{\frac{9}{2}}bcd\mathrm{sgn}(}{}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^12,x, algorithm="giac")
```

```
[Out] 16/15015*(13*b*c*d^(11/2) - 8*a*d^(13/2))*sgn(x)/c^5 + 1/15015*(1155*(c*x^2
 + d)^(13/2)*a*sgn(x) + 1365*(c*x^2 + d)^(11/2)*b*c*sgn(x) - 5460*(c*x^2 +
d)^(11/2)*a*d*sgn(x) - 5005*(c*x^2 + d)^(9/2)*b*c*d*sgn(x) + 10010*(c*x^2 +
 d)^(9/2)*a*d^2*sgn(x) + 6435*(c*x^2 + d)^(7/2)*b*c*d^2*sgn(x) - 8580*(c*x^
2 + d)^(7/2)*a*d^3*sgn(x) - 3003*(c*x^2 + d)^(5/2)*b*c*d^3*sgn(x) + 3003*(c
*x^2 + d)^(5/2)*a*d^4*sgn(x))/c^5
```

Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.91

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^{12}\,dx=\sqrt{c+\frac{d}{x^2}}\,\left(\frac{x\,(128\,a\,d^6-208\,b\,c\,d^5)}{15015\,c^5}\right.$$
$$+\frac{x^{11}\,(1365\,b\,c^6+1470\,a\,d\,c^5)}{15015\,c^5}+\frac{a\,c\,x^{13}}{13}+\frac{d\,x^9\,(a\,d+52\,b\,c)}{429\,c}$$
$$\left.-\frac{d^2\,x^7\,(8\,a\,d-13\,b\,c)}{3003\,c^2}+\frac{2\,d^3\,x^5\,(8\,a\,d-13\,b\,c)}{5005\,c^3}-\frac{8\,d^4\,x^3\,(8\,a\,d-13\,b\,c)}{15015\,c^4}\right)$$

```
[In] int(x^12*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] (c + d/x^2)^(1/2)*((x*(128*a*d^6 - 208*b*c*d^5))/(15015*c^5) + (x^11*(1365*
b*c^6 + 1470*a*c^5*d))/(15015*c^5) + (a*c*x^13)/13 + (d*x^9*(a*d + 52*b*c))
/(429*c) - (d^2*x^7*(8*a*d - 13*b*c))/(3003*c^2) + (2*d^3*x^5*(8*a*d - 13*b
*c))/(5005*c^3) - (8*d^4*x^3*(8*a*d - 13*b*c))/(15015*c^4))
```

3.954 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^{10} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 117

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^{10} \, dx=\frac{8d^2(11bc-6ad) \left(c+\frac{d}{x^2}\right)^{5/2} x^5}{3465c^4}-\frac{4d(11bc-6ad) \left(c+\frac{d}{x^2}\right)^{5/2} x^7}{693c^3}+\frac{(11bc-6ad) \left(c+\frac{d}{x^2}\right)^{5/2} x^9}{99c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^{11}}{11c}$$

```
[Out] 8/3465*d^2*(-6*a*d+11*b*c)*(c+d/x^2)^(5/2)*x^5/c^4-4/693*d*(-6*a*d+11*b*c)*
(c+d/x^2)^(5/2)*x^7/c^3+1/99*(-6*a*d+11*b*c)*(c+d/x^2)^(5/2)*x^9/c^2+1/11*a
*(c+d/x^2)^(5/2)*x^11/c
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {464, 277, 270}

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^{10} \, dx=\frac{8d^2x^5\left(c+\frac{d}{x^2}\right)^{5/2}(11bc-6ad)}{3465c^4}-\frac{4dx^7\left(c+\frac{d}{x^2}\right)^{5/2}(11bc-6ad)}{693c^3}+\frac{x^9\left(c+\frac{d}{x^2}\right)^{5/2}(11bc-6ad)}{99c^2}+\frac{ax^{11}\left(c+\frac{d}{x^2}\right)^{5/2}}{11c}$$

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^10,x]
```

```
[Out] (8*d^2*(11*b*c - 6*a*d)*(c + d/x^2)^(5/2)*x^5)/(3465*c^4) - (4*d*(11*b*c -
6*a*d)*(c + d/x^2)^(5/2)*x^7)/(693*c^3) + ((11*b*c - 6*a*d)*(c + d/x^2)^(5/
2)*x^9)/(99*c^2) + (a*(c + d/x^2)^(5/2)*x^11)/(11*c)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^{11}}{11c}+\frac{(11bc-6ad)\int\left(c+\frac{d}{x^2}\right)^{3/2}x^8\,dx}{11c}\\
&= \frac{(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^9}{99c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^{11}}{11c}-\frac{(4d(11bc-6ad))\int\left(c+\frac{d}{x^2}\right)^{3/2}x^6\,dx}{99c^2}\\
&= -\frac{4d(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^7}{693c^3}+\frac{(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^9}{99c^2}\\
&\quad+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^{11}}{11c}+\frac{(8d^2(11bc-6ad))\int\left(c+\frac{d}{x^2}\right)^{3/2}x^4\,dx}{693c^3}\\
&= \frac{8d^2(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^5}{3465c^4}-\frac{4d(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^7}{693c^3}\\
&\quad+\frac{(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^9}{99c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^{11}}{11c}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^{10}\,dx = \frac{\sqrt{c+\frac{d}{x^2}}x(d+cx^2)^2 (11bc(8d^2-20cdx^2+35c^2x^4)+3a(-16d^3+40cd^2x^2-70c^2dx^4+10}{3465c^4}$$

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^10,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(11*b*c*(8*d^2 - 20*c*d*x^2 + 35*c^2*x^4)
+ 3*a*(-16*d^3 + 40*c*d^2*x^2 - 70*c^2*d*x^4 + 105*c^3*x^6)))/(3465*c^4)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.78

method	result
gosper	$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}} x^3 \left(315x^6 a\,c^3-210a\,c^2 d\,x^4+385b\,c^3 x^4+120ac\,d^2 x^2-220b\,c^2 d\,x^2-48a\,d^3+88bc\,d^2\right) \left(c\,x^2+d\right)}{3465c^4}$
default	$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}} x^3 \left(315x^6 a\,c^3-210a\,c^2 d\,x^4+385b\,c^3 x^4+120ac\,d^2 x^2-220b\,c^2 d\,x^2-48a\,d^3+88bc\,d^2\right) \left(c\,x^2+d\right)}{3465c^4}$
risch	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\, x (315a\,c^5 x^{10}+420a\,c^4 d\,x^8+385b\,c^5 x^8+15a\,c^3 d^2 x^6+550b\,c^4 d\,x^6-18a\,c^2 d^3 x^4+33b\,c^3 d^2 x^4+24ac\,d^4 x^2-44b\,c^2 d^3 x^2-48a\,d^5}{3465c^4}$
trager	$\frac{(315a\,c^5 x^{10}+420a\,c^4 d\,x^8+385b\,c^5 x^8+15a\,c^3 d^2 x^6+550b\,c^4 d\,x^6-18a\,c^2 d^3 x^4+33b\,c^3 d^2 x^4+24ac\,d^4 x^2-44b\,c^2 d^3 x^2-48a\,d^5+88bc\,d^4)x}{3465c^4}$

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3465*((c*x^2+d)/x^2)^(3/2)*x^3*(315*a*c^3*x^6-210*a*c^2*d*x^4+385*b*c^3*x
^4+120*a*c*d^2*x^2-220*b*c^2*d*x^2-48*a*d^3+88*b*c*d^2)*(c*x^2+d)/c^4
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.13

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^{10}\,dx = \frac{(315\,ac^5x^{11}+35\,(11\,bc^5+12\,ac^4d)x^9+5\,(110\,bc^4d+3\,ac^3d^2)x^7+3\,(11\,bc^3d^2-6\,ac^2d^3)x^5}{3465\,c^4}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^10,x, algorithm="fricas")
```

```
[Out] 1/3465*(315*a*c^5*x^11 + 35*(11*b*c^5 + 12*a*c^4*d)*x^9 + 5*(110*b*c^4*d +
3*a*c^3*d^2)*x^7 + 3*(11*b*c^3*d^2 - 6*a*c^2*d^3)*x^5 - 4*(11*b*c^2*d^3 - 6
*a*c*d^4)*x^3 + 8*(11*b*c*d^4 - 6*a*d^5)*x)*sqrt((c*x^2 + d)/x^2)/c^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2304 vs. 2(112) = 224.

Time = 4.20 (sec) , antiderivative size = 2304, normalized size of antiderivative = 19.69

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^{10}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**10,x)
```

```
[Out] 315*a*c**10*d**(33/2)*x**18*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 1386
0*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c*
*5*d**20) + 1295*a*c**9*d**(35/2)*x**16*sqrt(c*x**2/d + 1)/(3465*c**9*d**16
*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x*
*2 + 3465*c**5*d**20) + 1990*a*c**8*d**(37/2)*x**14*sqrt(c*x**2/d + 1)/(346
5*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c
**6*d**19*x**2 + 3465*c**5*d**20) + 1358*a*c**7*d**(39/2)*x**12*sqrt(c*x**2
/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x*
*4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 35*a*c**7*d**(21/2)*x**14*s
qrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**1
1*x**2 + 315*c**4*d**12) + 343*a*c**6*d**(41/2)*x**10*sqrt(c*x**2/d + 1)/(3
465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860
*c**6*d**19*x**2 + 3465*c**5*d**20) + 110*a*c**6*d**(23/2)*x**12*sqrt(c*x**
2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 +
315*c**4*d**12) + 35*a*c**5*d**(43/2)*x**8*sqrt(c*x**2/d + 1)/(3465*c**9*d*
*16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19
*x**2 + 3465*c**5*d**20) + 114*a*c**5*d**(25/2)*x**10*sqrt(c*x**2/d + 1)/(3
15*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d*
*12) + 280*a*c**4*d**(45/2)*x**6*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 +
 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 34
65*c**5*d**20) + 40*a*c**4*d**(27/2)*x**8*sqrt(c*x**2/d + 1)/(315*c**7*d**9
*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 560*a
*c**3*d**(47/2)*x**4*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*
d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**2
0) - 5*a*c**3*d**(29/2)*x**6*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c
**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 448*a*c**2*d**(49/
2)*x**2*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 +
20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 30*a*c**
2*d**(31/2)*x**4*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x*
```

```
*4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 128*a*c*d**(51/2)*sqrt(c*x**2/
d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**
4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 40*a*c*d**(33/2)*x**2*sqrt(c
*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**
2 + 315*c**4*d**12) - 16*a*d**(35/2)*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6
 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 35*b*c**8*
d**(19/2)*x**14*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**
4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 110*b*c**7*d**(21/2)*x**12*sqrt
(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x
**2 + 315*c**4*d**12) + 114*b*c**6*d**(23/2)*x**10*sqrt(c*x**2/d + 1)/(315*
c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12
) + 40*b*c**5*d**(25/2)*x**8*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c
**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 15*b*c**5*d**(11/2
)*x**10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c
**3*d**6) - 5*b*c**4*d**(27/2)*x**6*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6
+ 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 33*b*c**4*d
**(13/2)*x**8*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 +
 105*c**3*d**6) - 30*b*c**3*d**(29/2)*x**4*sqrt(c*x**2/d + 1)/(315*c**7*d**
9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 17*b
*c**3*d**(15/2)*x**6*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5
*x**2 + 105*c**3*d**6) - 40*b*c**2*d**(31/2)*x**2*sqrt(c*x**2/d + 1)/(315*c
**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12)
 + 3*b*c**2*d**(17/2)*x**4*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**
4*d**5*x**2 + 105*c**3*d**6) - 16*b*c*d**(33/2)*sqrt(c*x**2/d + 1)/(315*c**
7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) +
 12*b*c*d**(19/2)*x**2*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d*
*5*x**2 + 105*c**3*d**6) + 8*b*d**(21/2)*sqrt(c*x**2/d + 1)/(105*c**5*d**4*
x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^{10}\,dx=\frac{\left(35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}x^9-90\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}dx^7+63\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^2x^5\right)b}{315\,c^3}+\frac{\left(105\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}x^{11}-385\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}dx^9+495\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}d^2x^7-231\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^3x^5\right)a}{1155\,c^4}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^10,x, algorithm="maxima")
```

[Out] 1/315*(35*(c + d/x^2)^(9/2)*x^9 - 90*(c + d/x^2)^(7/2)*d*x^7 + 63*(c + d/x^
2)^(5/2)*d^2*x^5)*b/c^3 + 1/1155*(105*(c + d/x^2)^(11/2)*x^11 - 385*(c + d/
x^2)^(9/2)*d*x^9 + 495*(c + d/x^2)^(7/2)*d^2*x^7 - 231*(c + d/x^2)^(5/2)*d^
3*x^5)*a/c^4

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.20

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^{10}\,dx = -\frac{8\left(11\,bcd^{\frac{9}{2}}-6\,ad^{\frac{11}{2}}\right)\mathrm{sgn}(x)}{3465\,c^4} + \frac{315\,(cx^2+d)^{\frac{11}{2}}a\mathrm{sgn}(x)+385\,(cx^2+d)^{\frac{9}{2}}bc\mathrm{sgn}(x)-1155\,(cx^2+d)^{\frac{9}{2}}ad\mathrm{sgn}(x)-990\,(cx^2+d)^{\frac{7}{2}}bcd\mathrm{sgn}(x)+}{3465\,c^4}$$

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^10,x, algorithm="giac")

[Out] -8/3465*(11*b*c*d^(9/2) - 6*a*d^(11/2))*sgn(x)/c^4 + 1/3465*(315*(c*x^2 + d
)^(11/2)*a*sgn(x) + 385*(c*x^2 + d)^(9/2)*b*c*sgn(x) - 1155*(c*x^2 + d)^(9/
2)*a*d*sgn(x) - 990*(c*x^2 + d)^(7/2)*b*c*d*sgn(x) + 1485*(c*x^2 + d)^(7/2)
*a*d^2*sgn(x) + 693*(c*x^2 + d)^(5/2)*b*c*d^2*sgn(x) - 693*(c*x^2 + d)^(5/2
)*a*d^3*sgn(x))/c^4

Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.01

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^{10}\,dx = \sqrt{c+\frac{d}{x^2}}\left(\frac{x^9\,(385\,b\,c^5+420\,a\,d\,c^4)}{3465\,c^4}-\frac{x\,(48\,a\,d^5-88\,b\,c\,d^4)}{3465\,c^4}+\frac{a\,c\,x^{11}}{11}+\frac{d\,x^7\,(3\,a\,d+110\,b\,c)}{693\,c}-\frac{d^2\,x^5\,(6\,a\,d-11\,b\,c)}{1155\,c^2}+\frac{4\,d^3\,x^3\,(6\,a\,d-11\,b\,c)}{3465\,c^3}\right)$$

[In] int(x^10*(a + b/x^2)*(c + d/x^2)^(3/2),x)

[Out] (c + d/x^2)^(1/2)*((x^9*(385*b*c^5 + 420*a*c^4*d))/(3465*c^4) - (x*(48*a*d^
5 - 88*b*c*d^4))/(3465*c^4) + (a*c*x^11)/11 + (d*x^7*(3*a*d + 110*b*c))/(69
3*c) - (d^2*x^5*(6*a*d - 11*b*c))/(1155*c^2) + (4*d^3*x^3*(6*a*d - 11*b*c))
/(3465*c^3))

3.955 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^8 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^8 \, dx = -\frac{2d(9bc-4ad) \left(c+\frac{d}{x^2}\right)^{5/2} x^5}{315c^3} + \frac{(9bc-4ad) \left(c+\frac{d}{x^2}\right)^{5/2} x^7}{63c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^9}{9c}$$

```
[Out] -2/315*d*(-4*a*d+9*b*c)*(c+d/x^2)^(5/2)*x^5/c^3+1/63*(-4*a*d+9*b*c)*(c+d/x^
2)^(5/2)*x^7/c^2+1/9*a*(c+d/x^2)^(5/2)*x^9/c
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {464, 277, 270}

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^8 \, dx = -\frac{2dx^5\left(c+\frac{d}{x^2}\right)^{5/2} (9bc-4ad)}{315c^3} + \frac{x^7\left(c+\frac{d}{x^2}\right)^{5/2} (9bc-4ad)}{63c^2} + \frac{ax^9\left(c+\frac{d}{x^2}\right)^{5/2}}{9c}$$

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^8,x]
```

```
[Out] (-2*d*(9*b*c - 4*a*d)*(c + d/x^2)^(5/2)*x^5)/(315*c^3) + ((9*b*c - 4*a*d)*(
c + d/x^2)^(5/2)*x^7)/(63*c^2) + (a*(c + d/x^2)^(5/2)*x^9)/(9*c)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
```

```
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^9}{9c}+\frac{(9bc-4ad)\int\left(c+\frac{d}{x^2}\right)^{3/2}x^6\,dx}{9c}\\
&= \frac{(9bc-4ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^7}{63c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^9}{9c}-\frac{(2d(9bc-4ad))\int\left(c+\frac{d}{x^2}\right)^{3/2}x^4\,dx}{63c^2}\\
&= -\frac{2d(9bc-4ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^5}{315c^3}+\frac{(9bc-4ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^7}{63c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^9}{9c}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.79

$$\int\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^8\,dx=\frac{\sqrt{c+\frac{d}{x^2}}\,x(d+cx^2)^2\left(9bc(-2d+5cx^2)+a(8d^2-20cdx^2+35c^2x^4)\right)}{315c^3}$$

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^8,x]
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(9*b*c*(-2*d + 5*c*x^2) + a*(8*d^2 - 20*c*
d*x^2 + 35*c^2*x^4)))/(315*c^3)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}} x^3 \left(35a\,x^4c^2-20acd\,x^2+45b\,c^2x^2+8a\,d^2-18bcd\right)\left(c\,x^2+d\right)}{315c^3}$	67
default	$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}} x^3 \left(35a\,x^4c^2-20acd\,x^2+45b\,c^2x^2+8a\,d^2-18bcd\right)\left(c\,x^2+d\right)}{315c^3}$	67
risch	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,x\left(35a\,x^8c^4+50a\,c^3d\,x^6+45b\,c^4x^6+3a\,c^2d^2x^4+72b\,c^3d\,x^4-4ac\,d^3x^2+9b\,c^2d^2x^2+8a\,d^4-18bc\,d^3\right)}{315c^3}$	106
trager	$\frac{\left(35a\,x^8c^4+50a\,c^3d\,x^6+45b\,c^4x^6+3a\,c^2d^2x^4+72b\,c^3d\,x^4-4ac\,d^3x^2+9b\,c^2d^2x^2+8a\,d^4-18bc\,d^3\right)x\sqrt{-\frac{-c\,x^2-d}{x^2}}}{315c^3}$	110

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/315*((c*x^2+d)/x^2)^(3/2)*x^3*(35*a*c^2*x^4-20*a*c*d*x^2+45*b*c^2*x^2+8*a
*d^2-18*b*c*d)*(c*x^2+d)/c^3
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.26

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^8\,dx = \frac{(35\,ac^4x^9+5\,(9\,bc^4+10\,ac^3d)x^7+3\,(24\,bc^3d+ac^2d^2)x^5+(9\,bc^2d^2-4\,acd^3)x^3-2\,(9\,bcd^3}{315\,c^3}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^8,x, algorithm="fricas")
```

```
[Out] 1/315*(35*a*c^4*x^9 + 5*(9*b*c^4 + 10*a*c^3*d)*x^7 + 3*(24*b*c^3*d + a*c^2*
d^2)*x^5 + (9*b*c^2*d^2 - 4*a*c*d^3)*x^3 - 2*(9*b*c*d^3 - 4*a*d^4)*x)*sqrt(
(c*x^2 + d)/x^2)/c^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1340 vs. 2(78) = 156.

Time = 3.15 (sec) , antiderivative size = 1340, normalized size of antiderivative = 15.95

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^8\,dx = \text{Too large to display}$$

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**8,x)
```

```
[Out] 35*a*c**8*d**(19/2)*x**14*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6
*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 110*a*c**7*d**(21/2)*
x**12*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c*
*5*d**11*x**2 + 315*c**4*d**12) + 114*a*c**6*d**(23/2)*x**10*sqrt(c*x**2/d
+ 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*
c**4*d**12) + 40*a*c**5*d**(25/2)*x**8*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x*
*6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 15*a*c**
5*d**(11/2)*x**10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x*
*2 + 105*c**3*d**6) - 5*a*c**4*d**(27/2)*x**6*sqrt(c*x**2/d + 1)/(315*c**7*
d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 3
3*a*c**4*d**(13/2)*x**8*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d
**5*x**2 + 105*c**3*d**6) - 30*a*c**3*d**(29/2)*x**4*sqrt(c*x**2/d + 1)/(31
5*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**
12) + 17*a*c**3*d**(15/2)*x**6*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210
*c**4*d**5*x**2 + 105*c**3*d**6) - 40*a*c**2*d**(31/2)*x**2*sqrt(c*x**2/d +
 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c
**4*d**12) + 3*a*c**2*d**(17/2)*x**4*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4
 + 210*c**4*d**5*x**2 + 105*c**3*d**6) - 16*a*c*d**(33/2)*sqrt(c*x**2/d + 1
)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**
4*d**12) + 12*a*c*d**(19/2)*x**2*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 2
10*c**4*d**5*x**2 + 105*c**3*d**6) + 8*a*d**(21/2)*sqrt(c*x**2/d + 1)/(105*
c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 15*b*c**6*d**(9/2)*x
**10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3
*d**6) + 33*b*c**5*d**(11/2)*x**8*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 +
210*c**4*d**5*x**2 + 105*c**3*d**6) + 17*b*c**4*d**(13/2)*x**6*sqrt(c*x**2/
d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 3*b*c**3
*d**(15/2)*x**4*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2
 + 105*c**3*d**6) + 12*b*c**2*d**(17/2)*x**2*sqrt(c*x**2/d + 1)/(105*c**5*d
**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 8*b*c*d**(19/2)*sqrt(c*x**
2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + b*d**(
3/2)*x**4*sqrt(c*x**2/d + 1)/5 + b*d**(5/2)*x**2*sqrt(c*x**2/d + 1)/(15*c)
- 2*b*d**(7/2)*sqrt(c*x**2/d + 1)/(15*c**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^8\,dx = \frac{\left(5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}x^7-7\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}dx^5\right)b}{35\,c^2} + \frac{\left(35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}x^9-90\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}dx^7+63\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^2x^5\right)a}{315\,c^3}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^8,x, algorithm="maxima")
```

```
[Out] 1/35*(5*(c + d/x^2)^(7/2)*x^7 - 7*(c + d/x^2)^(5/2)*d*x^5)*b/c^2 + 1/315*(3
5*(c + d/x^2)^(9/2)*x^9 - 90*(c + d/x^2)^(7/2)*d*x^7 + 63*(c + d/x^2)^(5/2)
*d^2*x^5)*a/c^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.25

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^8\,dx = \frac{2\left(9\,bcd^{\frac{7}{2}}-4\,ad^{\frac{9}{2}}\right)\mathrm{sgn}(x)}{315\,c^3} + \frac{35\,(cx^2+d)^{\frac{9}{2}}a\mathrm{sgn}(x)+45\,(cx^2+d)^{\frac{7}{2}}bc\mathrm{sgn}(x)-90\,(cx^2+d)^{\frac{7}{2}}ad\mathrm{sgn}(x)-63\,(cx^2+d)^{\frac{5}{2}}bcd\mathrm{sgn}(x)+63\,(c}{315\,c^3}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^8,x, algorithm="giac")
```

```
[Out] 2/315*(9*b*c*d^(7/2) - 4*a*d^(9/2))*sgn(x)/c^3 + 1/315*(35*(c*x^2 + d)^(9/2
)*a*sgn(x) + 45*(c*x^2 + d)^(7/2)*b*c*sgn(x) - 90*(c*x^2 + d)^(7/2)*a*d*sgn
(x) - 63*(c*x^2 + d)^(5/2)*b*c*d*sgn(x) + 63*(c*x^2 + d)^(5/2)*a*d^2*sgn(x)
)/c^3
```

Mupad [B] (verification not implemented)

Time = 8.96 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.15

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^8\,dx = \sqrt{c+\frac{d}{x^2}}\left(\frac{x\,(8\,a\,d^4-18\,b\,c\,d^3)}{315\,c^3} + \frac{x^7\,(45\,b\,c^4+50\,a\,d\,c^3)}{315\,c^3}+\frac{a\,c\,x^9}{9}+\frac{d\,x^5\,(a\,d+24\,b\,c)}{105\,c}-\frac{d^2\,x^3\,(4\,a\,d-9\,b\,c)}{315\,c^2}\right)$$

```
[In] int(x^8*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] (c + d/x^2)^(1/2)*((x*(8*a*d^4 - 18*b*c*d^3))/(315*c^3) + (x^7*(45*b*c^4 +
50*a*c^3*d))/(315*c^3) + (a*c*x^9)/9 + (d*x^5*(a*d + 24*b*c))/(105*c) - (d^
2*x^3*(4*a*d - 9*b*c))/(315*c^2))
```

3.956 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^6 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^6 \, dx=\frac{(7bc-2ad) \left(c+\frac{d}{x^2}\right)^{5/2} x^5}{35c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^7}{7c}$$

```
[Out] 1/35*(-2*a*d+7*b*c)*(c+d/x^2)^(5/2)*x^5/c^2+1/7*a*(c+d/x^2)^(5/2)*x^7/c
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {464, 270}

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^6 \, dx=\frac{x^5\left(c+\frac{d}{x^2}\right)^{5/2} (7bc-2ad)}{35c^2}+\frac{ax^7\left(c+\frac{d}{x^2}\right)^{5/2}}{7c}$$

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^6,x]
```

```
[Out] ((7*b*c - 2*a*d)*(c + d/x^2)^(5/2)*x^5)/(35*c^2) + (a*(c + d/x^2)^(5/2)*x^7
)/(7*c)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^7}{7c}+\frac{(7bc-2ad)\int\left(c+\frac{d}{x^2}\right)^{3/2}x^4\,dx}{7c}\\ &= \frac{(7bc-2ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^5}{35c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^7}{7c}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^6\,dx=\frac{\sqrt{c+\frac{d}{x^2}}\,x(d+cx^2)^2\,(7bc-2ad+5acx^2)}{35c^2}$$

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^6,x]
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(7*b*c - 2*a*d + 5*a*c*x^2))/(35*c^2)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

method	result	size
gosper	$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}x^3\left(5ac\,x^2-2ad+7bc\right)\left(c\,x^2+d\right)}{35c^2}$	45
default	$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}x^3\left(5ac\,x^2-2ad+7bc\right)\left(c\,x^2+d\right)}{35c^2}$	45
risch	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,x\left(5x^6a\,c^3+8a\,c^2d\,x^4+7b\,c^3x^4+ac\,d^2x^2+14b\,c^2d\,x^2-2a\,d^3+7bc\,d^2\right)}{35c^2}$	81
trager	$\frac{\left(5x^6a\,c^3+8a\,c^2d\,x^4+7b\,c^3x^4+ac\,d^2x^2+14b\,c^2d\,x^2-2a\,d^3+7bc\,d^2\right)x\sqrt{-\frac{-c\,x^2-d}{x^2}}}{35c^2}$	85

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^6,x,method=_RETURNVERBOSE)
[Out] 1/35*((c*x^2+d)/x^2)^(3/2)*x^3*(5*a*c*x^2-2*a*d+7*b*c)*(c*x^2+d)/c^2
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.51

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^6\,dx=\frac{\left(5\,ac^3x^7+(7\,bc^3+8\,ac^2d)x^5+(14\,bc^2d+acd^2)x^3+(7\,bcd^2-2\,ad^3)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{35\,c^2}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^6,x, algorithm="fricas")
```

```
[Out] 1/35*(5*a*c^3*x^7 + (7*b*c^3 + 8*a*c^2*d)*x^5 + (14*b*c^2*d + a*c*d^2)*x^3
+ (7*b*c*d^2 - 2*a*d^3)*x)*sqrt((c*x^2 + d)/x^2)/c^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(46) = 92$.

Time = 2.37 (sec) , antiderivative size = 498, normalized size of antiderivative = 9.40

$$\begin{aligned}\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^6\,dx &= \frac{15ac^6d^{\frac{9}{2}}x^{10}\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} \\ &+ \frac{33ac^5d^{\frac{11}{2}}x^8\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{17ac^4d^{\frac{13}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} \\ &+ \frac{3ac^3d^{\frac{15}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{12ac^2d^{\frac{17}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} \\ &+ \frac{8acd^{\frac{19}{2}}\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{ad^{\frac{3}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{5}+\frac{ad^{\frac{5}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{15c} \\ &- \frac{2ad^{\frac{7}{2}}\sqrt{\frac{cx^2}{d}+1}}{15c^2}+\frac{bc\sqrt{d}x^4\sqrt{\frac{cx^2}{d}+1}}{5}+\frac{2bd^{\frac{3}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{5}+\frac{bd^{\frac{5}{2}}\sqrt{\frac{cx^2}{d}+1}}{5c}\end{aligned}$$

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**6,x)
```

```
[Out] 15*a*c**6*d**(9/2)*x**10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*
d**5*x**2 + 105*c**3*d**6) + 33*a*c**5*d**(11/2)*x**8*sqrt(c*x**2/d + 1)/(1
05*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 17*a*c**4*d**(13/
2)*x**6*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c
**3*d**6) + 3*a*c**3*d**(15/2)*x**4*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4
+ 210*c**4*d**5*x**2 + 105*c**3*d**6) + 12*a*c**2*d**(17/2)*x**2*sqrt(c*x**
2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 8*a*c*
```

```
d**(19/2)*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105
*c**3*d**6) + a*d**(3/2)*x**4*sqrt(c*x**2/d + 1)/5 + a*d**(5/2)*x**2*sqrt(c
*x**2/d + 1)/(15*c) - 2*a*d**(7/2)*sqrt(c*x**2/d + 1)/(15*c**2) + b*c*sqrt(
d)*x**4*sqrt(c*x**2/d + 1)/5 + 2*b*d**(3/2)*x**2*sqrt(c*x**2/d + 1)/5 + b*d
**(5/2)*sqrt(c*x**2/d + 1)/(5*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^6\,dx=\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5}{5\,c}+\frac{\left(5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}x^7-7\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}dx^5\right)a}{35\,c^2}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^6,x, algorithm="maxima")
```

```
[Out] 1/5*b*(c + d/x^2)^(5/2)*x^5/c + 1/35*(5*(c + d/x^2)^(7/2)*x^7 - 7*(c + d/x^
2)^(5/2)*d*x^5)*a/c^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.36

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^6\,dx=-\frac{\left(7\,bcd^{\frac{5}{2}}-2\,ad^{\frac{7}{2}}\right)\operatorname{sgn}(x)}{35\,c^2}+\frac{5\,(cx^2+d)^{\frac{7}{2}}a\operatorname{sgn}(x)+7\,(cx^2+d)^{\frac{5}{2}}bc\operatorname{sgn}(x)-7\,(cx^2+d)^{\frac{5}{2}}ad\operatorname{sgn}(x)}{35\,c^2}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^6,x, algorithm="giac")
```

```
[Out] -1/35*(7*b*c*d^(5/2) - 2*a*d^(7/2))*sgn(x)/c^2 + 1/35*(5*(c*x^2 + d)^(7/2)*
a*sgn(x) + 7*(c*x^2 + d)^(5/2)*b*c*sgn(x) - 7*(c*x^2 + d)^(5/2)*a*d*sgn(x))
/c^2
```

Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.45

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^6 \, dx = \sqrt{c+\frac{d}{x^2}} \left(\frac{x^5 \left(7 b c^3+8 a d c^2\right)}{35 c^2} - \frac{x \left(2 a d^3-7 b c d^2\right)}{35 c^2}+\frac{a c x^7}{7}+\frac{d x^3 \left(a d+14 b c\right)}{35 c}\right)$$

```
[In] int(x^6*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] (c + d/x^2)^(1/2)*((x^5*(7*b*c^3 + 8*a*c^2*d))/(35*c^2) - (x*(2*a*d^3 - 7*b
*c*d^2))/(35*c^2) + (a*c*x^7)/7 + (d*x^3*(a*d + 14*b*c))/(35*c))
```

3.957 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^4 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^4 \, dx = bd\sqrt{c+\frac{d}{x^2}}x + \frac{1}{3}b\left(c+\frac{d}{x^2}\right)^{3/2} x^3 + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{5c} - bd^{3/2}\text{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)$$

```
[Out] 1/3*b*(c+d/x^2)^(3/2)*x^3+1/5*a*(c+d/x^2)^(5/2)*x^5/c-b*d^(3/2)*arctanh(d^(
1/2)/x/(c+d/x^2)^(1/2))+b*d*x*(c+d/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {462, 342, 283, 223, 212}

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^4 \, dx = \frac{ax^5\left(c+\frac{d}{x^2}\right)^{5/2}}{5c} - bd^{3/2}\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right) + bdx\sqrt{c+\frac{d}{x^2}} + \frac{1}{3}bx^3\left(c+\frac{d}{x^2}\right)^{3/2}$$

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^4,x]
```

```
[Out] b*d*Sqrt[c + d/x^2]*x + (b*(c + d/x^2)^(3/2)*x^3)/3 + (a*(c + d/x^2)^(5/2)*
x^5)/(5*c) - b*d^(3/2)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^5}{5c} + b\int\left(c+\frac{d}{x^2}\right)^{3/2}x^2\,dx \\
&= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^5}{5c} - b\text{Subst}\left(\int\frac{(c+dx^2)^{3/2}}{x^4}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{3}b\left(c+\frac{d}{x^2}\right)^{3/2}x^3 + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^5}{5c} - (bd)\text{Subst}\left(\int\frac{\sqrt{c+dx^2}}{x^2}\,dx, x, \frac{1}{x}\right) \\
&= bd\sqrt{c+\frac{d}{x^2}}x + \frac{1}{3}b\left(c+\frac{d}{x^2}\right)^{3/2}x^3 + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^5}{5c} - \left(bd^2\right)\text{Subst}\left(\int\frac{1}{\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)
\end{aligned}$$

$$= bd\sqrt{c+\frac{d}{x^2}}x+\frac{1}{3}b\left(c+\frac{d}{x^2}\right)^{3/2}x^3+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^5}{5c} - (bd^2)\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{1}{\sqrt{c+\frac{d}{x^2}}x}\right)$$

$$= bd\sqrt{c+\frac{d}{x^2}}x+\frac{1}{3}b\left(c+\frac{d}{x^2}\right)^{3/2}x^3+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^5}{5c} - bd^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^4\,dx = \frac{1}{15}\sqrt{c+\frac{d}{x^2}}x\left(\frac{3a(d+cx^2)^2}{c} + 5b(4d+cx^2) - \frac{15bd^{3/2}\text{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{\sqrt{d+cx^2}}\right)$$

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^4,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*((3*a*(d + c*x^2)^2)/c + 5*b*(4*d + c*x^2) - (15*b*d^(3/
2)*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/Sqrt[d + c*x^2]))/15
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}} x^3 \left(3a\left(c\,x^2+d\right)^{\frac{5}{2}}+5\left(c\,x^2+d\right)^{\frac{3}{2}} bc-15d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)bc+15\sqrt{c\,x^2+d}\,bcd\right)}{15\left(c\,x^2+d\right)^{\frac{3}{2}} c}$	99

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*((c*x^2+d)/x^2)^(3/2)*x^3*(3*a*(c*x^2+d)^(5/2)+5*(c*x^2+d)^(3/2)*b*c-1
5*d^(3/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*b*c+15*(c*x^2+d)^(1/2)*b*c*d)
/(c*x^2+d)^(3/2)/c
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.36

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^4\,dx = \left[\frac{15\,bcd^{\frac{3}{2}}\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,(3\,ac^2x^5+(5\,bc^2+6\,acd)x^3+(20\,bcd+3\,ad^2)x)\right.}{30\,c}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^4,x, algorithm="fricas")
```

```
[Out] [1/30*(15*b*c*d^(3/2)*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d
)/x^2) + 2*(3*a*c^2*x^5 + (5*b*c^2 + 6*a*c*d)*x^3 + (20*b*c*d + 3*a*d^2)*x)
*sqrt((c*x^2 + d)/x^2))/c, 1/15*(15*b*c*sqrt(-d)*d*arctan(sqrt(-d)*x*sqrt((
c*x^2 + d)/x^2)/(c*x^2 + d)) + (3*a*c^2*x^5 + (5*b*c^2 + 6*a*c*d)*x^3 + (20
*b*c*d + 3*a*d^2)*x)*sqrt((c*x^2 + d)/x^2))/c]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(75) = 150.

Time = 2.18 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.14

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^4\,dx = \frac{ac\sqrt{d}x^4\sqrt{\frac{cx^2}{d}+1}}{5}+\frac{2ad^{\frac{3}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{5}+\frac{ad^{\frac{5}{2}}\sqrt{\frac{cx^2}{d}+1}}{5c} + \frac{b\sqrt{c}dx}{\sqrt{1+\frac{d}{cx^2}}}+\frac{bc\sqrt{d}x^2\sqrt{\frac{cx^2}{d}+1}}{3}+\frac{bd^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3}-bd^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)+\frac{bd^2}{\sqrt{c}x\sqrt{1+\frac{d}{cx^2}}}$$

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**4,x)
```

```
[Out] a*c*sqrt(d)*x**4*sqrt(c*x**2/d + 1)/5 + 2*a*d**(3/2)*x**2*sqrt(c*x**2/d + 1
)/5 + a*d**(5/2)*sqrt(c*x**2/d + 1)/(5*c) + b*sqrt(c)*d*x/sqrt(1 + d/(c*x**
2)) + b*c*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/3 + b*d**(3/2)*sqrt(c*x**2/d + 1)
/3 - b*d**(3/2)*asinh(sqrt(d)/(sqrt(c)*x)) + b*d**2/(sqrt(c)*x*sqrt(1 + d/(
c*x**2)))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.06

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^4\,dx=\frac{a\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5}{5\,c}+\frac{1}{6}\left(2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3+6\sqrt{c+\frac{d}{x^2}}dx+3\,d^{\frac{3}{2}}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)\right)b$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^4,x, algorithm="maxima")
```

```
[Out] 1/5*a*(c + d/x^2)^(5/2)*x^5/c + 1/6*(2*(c + d/x^2)^(3/2)*x^3 + 6*sqrt(c + d
/x^2)*d*x + 3*d^(3/2)*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x
+ sqrt(d))))*b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.63

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^4\,dx=\frac{bd^2\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)\mathrm{sgn}(x)}{\sqrt{-d}}-\frac{\left(15\,bcd^2\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)+20\,bc\sqrt{-d}d^{\frac{3}{2}}+3\,a\sqrt{-d}d^{\frac{5}{2}}\right)\mathrm{sgn}(x)}{15\,c\sqrt{-d}}+\frac{3\,(cx^2+d)^{\frac{5}{2}}ac^4\mathrm{sgn}(x)+5\,(cx^2+d)^{\frac{3}{2}}bc^5\mathrm{sgn}(x)+15\,\sqrt{cx^2+d}bc^5d\mathrm{sgn}(x)}{15\,c^5}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^4,x, algorithm="giac")
```

```
[Out] b*d^2*arctan(sqrt(c*x^2 + d)/sqrt(-d))*sgn(x)/sqrt(-d) - 1/15*(15*b*c*d^2*a
rctan(sqrt(d)/sqrt(-d)) + 20*b*c*sqrt(-d)*d^(3/2) + 3*a*sqrt(-d)*d^(5/2))*s
gn(x)/(c*sqrt(-d)) + 1/15*(3*(c*x^2 + d)^(5/2)*a*c^4*sgn(x) + 5*(c*x^2 + d)
^(3/2)*b*c^5*sgn(x) + 15*sqrt(c*x^2 + d)*b*c^5*d*sgn(x))/c^5
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^4\,dx=\int x^4\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}dx$$

```
[In] int(x^4*(a + b/x^2)*(c + d/x^2)^(3/2),x)
[Out] int(x^4*(a + b/x^2)*(c + d/x^2)^(3/2), x)
```

3.958 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 121

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^2 \, dx = -\frac{d(3bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx}+\frac{(3bc+2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x}{3c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^3}{3c}-\frac{1}{2}\sqrt{d}(3bc+2ad)\text{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)$$

```
[Out] 1/3*(2*a*d+3*b*c)*(c+d/x^2)^(3/2)*x/c+1/3*a*(c+d/x^2)^(5/2)*x^3/c-1/2*(2*a*
d+3*b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))*d^(1/2)-1/2*d*(2*a*d+3*b*c)*(c+
d/x^2)^(1/2)/c/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {464, 248, 283, 201, 223, 212}

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^2 \, dx = -\frac{1}{2}\sqrt{d}(2ad+3bc)\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right) + \frac{x\left(c+\frac{d}{x^2}\right)^{3/2}(2ad+3bc)}{3c}-\frac{d\sqrt{c+\frac{d}{x^2}}(2ad+3bc)}{2cx}+\frac{ax^3\left(c+\frac{d}{x^2}\right)^{5/2}}{3c}$$

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^2,x]
```

```
[Out] -1/2*(d*(3*b*c + 2*a*d)*Sqrt[c + d/x^2])/(c*x) + ((3*b*c + 2*a*d)*(c + d/x^
2)^(3/2)*x)/(3*c) + (a*(c + d/x^2)^(5/2)*x^3)/(3*c) - (Sqrt[d]*(3*b*c + 2*a
*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^3}{3c}+\frac{(3bc+2ad)\int\left(c+\frac{d}{x^2}\right)^{3/2}\,dx}{3c}\\
&= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^3}{3c}-\frac{(3bc+2ad)\text{Subst}\left(\int\frac{(c+dx^2)^{3/2}}{x^2}\,dx,x,\frac{1}{x}\right)}{3c}\\
&= \frac{(3bc+2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x}{3c}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^3}{3c}-\frac{(d(3bc+2ad))\text{Subst}\left(\int\sqrt{c+dx^2}\,dx,x,\frac{1}{x}\right)}{c}\\
&= -\frac{d(3bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx}+\frac{(3bc+2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x}{3c}\\
&\quad+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^3}{3c}-\frac{1}{2}(d(3bc+2ad))\text{Subst}\left(\int\frac{1}{\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right)\\
&= -\frac{d(3bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx}+\frac{(3bc+2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x}{3c}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^3}{3c}\\
&\quad-\frac{1}{2}(d(3bc+2ad))\text{Subst}\left(\int\frac{1}{1-dx^2}\,dx,x,\frac{1}{\sqrt{c+\frac{d}{x^2}}x}\right)\\
&= -\frac{d(3bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx}+\frac{(3bc+2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x}{3c}\\
&\quad+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^3}{3c}-\frac{1}{2}\sqrt{d}(3bc+2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.87

$$
\int\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^2\,dx=\frac{\sqrt{c+\frac{d}{x^2}}\left(\sqrt{d+cx^2}(-3bd+6bcx^2+8adx^2+2acx^4)-3\sqrt{d}(3bc+2ad)x^2\text{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)\right)}{6x\sqrt{d+cx^2}}
$$

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^2,x]
```

```
[Out] (Sqrt[c + d/x^2]*(Sqrt[d + c*x^2]*(-3*b*d + 6*b*c*x^2 + 8*a*d*x^2 + 2*a*c*x
^4) - 3*Sqrt[d]*(3*b*c + 2*a*d)*x^2*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]]))/(6*x
*Sqrt[d + c*x^2])
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.21

method	result
risch	$-\frac{bd\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}{2x}+\frac{\left(c^{2}a\left(\frac{x^{2}\sqrt{c\,x^{2}+d}}{3c}-\frac{2d\sqrt{c\,x^{2}+d}}{3c^{2}}\right)+\sqrt{c\,x^{2}+d}\,bc+2ad\sqrt{c\,x^{2}+d}-\frac{\sqrt{d}\,(2ad+3bc)\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)}{2}\right)\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}{\sqrt{c\,x^{2}+d}}$
default	$-\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}x\left(6d^{\frac{5}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)a\,x^{2}+9d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)bc\,x^{2}-2\left(c\,x^{2}+d\right)^{\frac{3}{2}}ad\,x^{2}-3\left(c\,x^{2}+d\right)^{\frac{3}{2}}bc\,x^{2}+3\left(c\,x^{2}+d\right)^{\frac{5}{2}}\right.}{6\left(c\,x^{2}+d\right)^{\frac{3}{2}}d}$

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b*d/x*((c*x^2+d)/x^2)^(1/2)+(c^2*a*(1/3*x^2/c*(c*x^2+d)^(1/2)-2/3*d/c^
2*(c*x^2+d)^(1/2))+(c*x^2+d)^(1/2)*b*c+2*a*d*(c*x^2+d)^(1/2)-1/2*d^(1/2)*(2
*a*d+3*b*c)*ln((2*d+2*d^(1/2)*(c*x^2+d)^(1/2))/x))*((c*x^2+d)/x^2)^(1/2)*x/
(c*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.57

$$\int\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^2\,dx=\left[\frac{3\,(3\,bc+2\,ad)\sqrt{d}x\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,(2\,acx^4+2\,(3\,bc+4\,ad)x^2-3\,bd)\sqrt{\frac{cx^2+d}{x^2}}}{12\,x}\right.$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(3*b*c + 2*a*d)*sqrt(d)*x*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 +
d)/x^2) + 2*d)/x^2) + 2*(2*a*c*x^4 + 2*(3*b*c + 4*a*d)*x^2 - 3*b*d)*sqrt((c
*x^2 + d)/x^2))/x, 1/6*(3*(3*b*c + 2*a*d)*sqrt(-d)*x*arctan(sqrt(-d)*x*sqrt
((c*x^2 + d)/x^2)/(c*x^2 + d)) + (2*a*c*x^4 + 2*(3*b*c + 4*a*d)*x^2 - 3*b*d
)*sqrt((c*x^2 + d)/x^2))/x]
```

Sympy [A] (verification not implemented)

Time = 3.24 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.67

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^2\,dx = \frac{a\sqrt{c}dx}{\sqrt{1+\frac{d}{cx^2}}} + \frac{ac\sqrt{d}x^2\sqrt{\frac{cx^2}{d}+1}}{3} + \frac{ad^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3} - ad^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right) + \frac{ad^2}{\sqrt{c}x\sqrt{1+\frac{d}{cx^2}}} + \frac{bc^{\frac{3}{2}}x}{\sqrt{1+\frac{d}{cx^2}}} - \frac{b\sqrt{c}d\sqrt{1+\frac{d}{cx^2}}}{2x} + \frac{b\sqrt{c}d}{x\sqrt{1+\frac{d}{cx^2}}} - \frac{3bc\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2}$$

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**2,x)
```

```
[Out] a*sqrt(c)*d*x/sqrt(1 + d/(c*x**2)) + a*c*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/3
+ a*d**(3/2)*sqrt(c*x**2/d + 1)/3 - a*d**(3/2)*asinh(sqrt(d)/(sqrt(c)*x)) +
 a*d**2/(sqrt(c)*x*sqrt(1 + d/(c*x**2))) + b*c**(3/2)*x/sqrt(1 + d/(c*x**2)
) - b*sqrt(c)*d*sqrt(1 + d/(c*x**2))/(2*x) + b*sqrt(c)*d/(x*sqrt(1 + d/(c*x
**2))) - 3*b*c*sqrt(d)*asinh(sqrt(d)/(sqrt(c)*x))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.35

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^2\,dx = \frac{1}{6}\left(2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3 + 6\sqrt{c+\frac{d}{x^2}}dx + 3\,d^{\frac{3}{2}}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)\right)a + \frac{1}{4}\left(4\sqrt{c+\frac{d}{x^2}}cx - \frac{2\sqrt{c+\frac{d}{x^2}}cdx}{\left(c+\frac{d}{x^2}\right)x^2-d} + 3\,c\sqrt{d}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)\right)b$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(c + d/x^2)^(3/2)*x^3 + 6*sqrt(c + d/x^2)*d*x + 3*d^(3/2)*log((sqrt(
c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d))))*a + 1/4*(4*sqrt(c +
 d/x^2)*c*x - 2*sqrt(c + d/x^2)*c*d*x/((c + d/x^2)*x^2 - d) + 3*c*sqrt(d)*l
og((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d))))*b
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.95

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^2\,dx = \frac{2\left(cx^2+d\right)^{\frac{3}{2}}ac\operatorname{sgn}(x)+6\sqrt{cx^2+d}\,bc^2\operatorname{sgn}(x)+6\sqrt{cx^2+d}\,acd\operatorname{sgn}(x)-\frac{3\sqrt{cx^2+d}\,bcd\operatorname{sgn}(x)}{x^2}+\cdots}{6\,c}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] 1/6*(2*(c*x^2 + d)^(3/2)*a*c*sgn(x) + 6*sqrt(c*x^2 + d)*b*c^2*sgn(x) + 6*sq
rt(c*x^2 + d)*a*c*d*sgn(x) - 3*sqrt(c*x^2 + d)*b*c*d*sgn(x)/x^2 + 3*(3*b*c^
2*d*sgn(x) + 2*a*c*d^2*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d))/sqrt(-d))/c
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^2\,dx = \int x^2\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}\,dx$$

```
[In] int(x^2*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] int(x^2*(a + b/x^2)*(c + d/x^2)^(3/2), x)
```

3.959 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 112

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} \, dx=-\frac{3(bc+4ad)\sqrt{c+\frac{d}{x^2}}}{8x}-\frac{(bc+4ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{4cx}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x}{c}-\frac{3c(bc+4ad)\text{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{8\sqrt{d}}$$

```
[Out] -1/4*(4*a*d+b*c)*(c+d/x^2)^(3/2)/c/x+a*(c+d/x^2)^(5/2)*x/c-3/8*c*(4*a*d+b*c
)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(1/2)-3/8*(4*a*d+b*c)*(c+d/x^2)^(1/2
)/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {382, 464, 201, 223, 212}

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} \, dx=-\frac{3c(4ad+bc)\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{8\sqrt{d}}-\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(4ad+bc)}{4cx}-\frac{3\sqrt{c+\frac{d}{x^2}}(4ad+bc)}{8x}+\frac{ax\left(c+\frac{d}{x^2}\right)^{5/2}}{c}$$

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2),x]
```

```
[Out] (-3*(b*c + 4*a*d)*Sqrt[c + d/x^2])/(8*x) - ((b*c + 4*a*d)*(c + d/x^2)^(3/2)
)/(4*c*x) + (a*(c + d/x^2)^(5/2)*x)/c - (3*c*(b*c + 4*a*d)*ArcTanh[Sqrt[d]/
(Sqrt[c + d/x^2]*x)])/(8*Sqrt[d])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{(a+bx^2)\,(c+dx^2)^{3/2}}{x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x}{c} + \frac{(-bc-4ad)\text{Subst}\left(\int (c+dx^2)^{3/2}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{(bc+4ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{4cx} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x}{c} - \frac{1}{4}(3(bc+4ad))\text{Subst}\left(\int \sqrt{c+dx^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{3(bc+4ad)\sqrt{c+\frac{d}{x^2}}}{8x} - \frac{(bc+4ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{4cx} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x}{c} - \frac{1}{8}(3c(bc+4ad))\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{3(bc+4ad)\sqrt{c+\frac{d}{x^2}}}{8x} - \frac{(bc+4ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{4cx} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x}{c} - \frac{1}{8}(3c(bc+4ad))\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right)$$

$$= -\frac{3(bc+4ad)\sqrt{c+\frac{d}{x^2}}}{8x} - \frac{(bc+4ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{4cx} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x}{c} - \frac{3c(bc+4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{8\sqrt{d}}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.82

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} dx = \frac{\sqrt{c+\frac{d}{x^2}}\left(-2bd-5bcx^2-4adx^2+8acx^4-\frac{3c(bc+4ad)x^4\text{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{d+cx^2}}\right)}{8x^3}$$

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2),x]
[Out] (Sqrt[c + d/x^2]*(-2*b*d - 5*b*c*x^2 - 4*a*d*x^2 + 8*a*c*x^4 - (3*c*(b*c +
4*a*d)*x^4*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/(Sqrt[d]*Sqrt[d + c*x^2])))/(8
*x^3)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{(4ad\,x^2+5cb\,x^2+2bd)\sqrt{\frac{c\,x^2+d}{x^2}}}{8x^3}+\frac{c\left(8a\sqrt{c\,x^2+d}-\frac{(12ad+3bc)\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)}{\sqrt{d}}\right)\sqrt{\frac{c\,x^2+d}{x^2}}\,x}{8\sqrt{c\,x^2+d}}$
default	$-\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\left(-4(c\,x^2+d)^{\frac{3}{2}}acd\,x^4-(c\,x^2+d)^{\frac{3}{2}}b\,c^2x^4+12d^{\frac{5}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)ac\,x^4+3d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)b\,c^2x^4+4(c\,x^2\right.}{8x(c\,x^2+d)^{\frac{3}{2}}d^2}$

[In] int((a+b/x^2)*(c+d/x^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/8*(4*a*d*x^2+5*b*c*x^2+2*b*d)/x^3*((c*x^2+d)/x^2)^(1/2)+1/8*c*(8*a*(c*x^2+d)^(1/2)-(12*a*d+3*b*c)/d^(1/2)*ln((2*d+2*d^(1/2)*(c*x^2+d)^(1/2))/x))*((c*x^2+d)/x^2)^(1/2)*x/(c*x^2+d)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.93

$$\int\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}dx=\left[\frac{3\,(bc^2+4\,acd)\sqrt{d}x^3\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,(8\,acdx^4-2\,bd^2-(5\,bcd+4\,ad^2)x^2)\sqrt{}}{16\,dx^3}\right.$$

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2),x, algorithm="fricas")

[Out] [1/16*(3*(b*c^2 + 4*a*c*d)*sqrt(d)*x^3*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2) + 2*(8*a*c*d*x^4 - 2*b*d^2 - (5*b*c*d + 4*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/(d*x^3), 1/8*(3*(b*c^2 + 4*a*c*d)*sqrt(-d)*x^3*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (8*a*c*d*x^4 - 2*b*d^2 - (5*b*c*d + 4*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/(d*x^3)]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(100) = 200.

Time = 5.38 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.93

$$\int\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}dx=\frac{ac^{\frac{3}{2}}x}{\sqrt{1+\frac{d}{cx^2}}}-\frac{a\sqrt{c}d\sqrt{1+\frac{d}{cx^2}}}{2x}+\frac{a\sqrt{c}d}{x\sqrt{1+\frac{d}{cx^2}}}-\frac{3ac\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2}-\frac{bc^{\frac{3}{2}}\sqrt{1+\frac{d}{cx^2}}}{2x}-\frac{bc^{\frac{3}{2}}}{8x\sqrt{1+\frac{d}{cx^2}}}-\frac{3b\sqrt{c}d}{8x^3\sqrt{1+\frac{d}{cx^2}}}-\frac{3bc^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{8\sqrt{d}}-\frac{bd^2}{4\sqrt{c}x^5\sqrt{1+\frac{d}{cx^2}}}$$

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2),x)
[Out] a*c**(3/2)*x/sqrt(1 + d/(c*x**2)) - a*sqrt(c)*d*sqrt(1 + d/(c*x**2))/(2*x)
+ a*sqrt(c)*d/(x*sqrt(1 + d/(c*x**2))) - 3*a*c*sqrt(d)*asinh(sqrt(d)/(sqrt(
c)*x))/2 - b*c**(3/2)*sqrt(1 + d/(c*x**2))/(2*x) - b*c**(3/2)/(8*x*sqrt(1 +
 d/(c*x**2))) - 3*b*sqrt(c)*d/(8*x**3*sqrt(1 + d/(c*x**2))) - 3*b*c**2*asin
h(sqrt(d)/(sqrt(c)*x))/(8*sqrt(d)) - b*d**2/(4*sqrt(c)*x**5*sqrt(1 + d/(c*x
**2)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(94) = 188.

Time = 0.29 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.85

$$\int\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}dx=\frac{1}{4}\left(4\sqrt{c+\frac{d}{x^2}}cx-\frac{2\sqrt{c+\frac{d}{x^2}}cdx}{(c+\frac{d}{x^2})x^2-d}+3c\sqrt{d}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)\right)a+\frac{1}{16}\left(\frac{3c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{\sqrt{d}}-\frac{2\left(5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2x^3-3\sqrt{c+\frac{d}{x^2}}c^2dx\right)}{\left(c+\frac{d}{x^2}\right)^2x^4-2\left(c+\frac{d}{x^2}\right)dx^2+d^2}\right)b$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2),x, algorithm="maxima")
[Out] 1/4*(4*sqrt(c + d/x^2)*c*x - 2*sqrt(c + d/x^2)*c*d*x/((c + d/x^2)*x^2 - d)
+ 3*c*sqrt(d)*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d
```

```
))))*a + 1/16*(3*c^2*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x +
 sqrt(d)))/sqrt(d) - 2*(5*(c + d/x^2)^(3/2)*c^2*x^3 - 3*sqrt(c + d/x^2)*c^2
*d*x)/((c + d/x^2)^2*x^4 - 2*(c + d/x^2)*d*x^2 + d^2))*b
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.29

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}\,dx=\frac{8\sqrt{cx^2+d}ac^2\mathrm{sgn}(x)+\frac{3\,(bc^3\mathrm{sgn}(x)+4\,ac^2d\mathrm{sgn}(x))\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}}-\frac{5\,(cx^2+d)^{\frac{3}{2}}bc^3\mathrm{sgn}(x)+4\,(cx^2+d)^{\frac{3}{2}}ac^2d\mathrm{sgn}}{}}{8\,c}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(8*sqrt(c*x^2 + d)*a*c^2*sgn(x) + 3*(b*c^3*sgn(x) + 4*a*c^2*d*sgn(x))*a
rctan(sqrt(c*x^2 + d)/sqrt(-d))/sqrt(-d) - (5*(c*x^2 + d)^(3/2)*b*c^3*sgn(x
) + 4*(c*x^2 + d)^(3/2)*a*c^2*d*sgn(x) - 3*sqrt(c*x^2 + d)*b*c^3*d*sgn(x) -
 4*sqrt(c*x^2 + d)*a*c^2*d^2*sgn(x))/(c^2*x^4))/c
```

Mupad [B] (verification not implemented)

Time = 10.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.70

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}\,dx=\frac{a\,x\,{\left(c\,x^2+d\right)}^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{d}{c\,x^2}\right)}{{\left(\frac{d}{c}+x^2\right)}^{3/2}}-\frac{b\,{\left(c\,x^2+d\right)}^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{d}{c\,x^2}\right)}{x\,{\left(\frac{d}{c}+x^2\right)}^{3/2}}$$

```
[In] int((a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] (a*x*(d + c*x^2)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -d/(c*x^2)))/(d/c + x^2
)^(3/2) - (b*(d + c*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -d/(c*x^2)))/(x*
(d/c + x^2)^(3/2))
```

3.960 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 123

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2} dx = \frac{c(bc-6ad)\sqrt{c+\frac{d}{x^2}}}{16dx} + \frac{(bc-6ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{24dx} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx} + \frac{c^2(bc-6ad)\text{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)}{16d^{3/2}}$$

```
[Out] 1/24*(-6*a*d+b*c)*(c+d/x^2)^(3/2)/d/x-1/6*b*(c+d/x^2)^(5/2)/d/x+1/16*c^2*(-
6*a*d+b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(3/2)+1/16*c*(-6*a*d+b*c)*(
c+d/x^2)^(1/2)/d/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {470, 342, 201, 223, 212}

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2} dx = \frac{c^2(bc-6ad)\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{16d^{3/2}} + \frac{\left(c+\frac{d}{x^2}\right)^{3/2}(bc-6ad)}{24dx} + \frac{c\sqrt{c+\frac{d}{x^2}}(bc-6ad)}{16dx} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}$$

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^2,x]
```

```
[Out] (c*(b*c - 6*a*d)*Sqrt[c + d/x^2])/(16*d*x) + ((b*c - 6*a*d)*(c + d/x^2)^(3/
2))/(24*d*x) - (b*(c + d/x^2)^(5/2))/(6*d*x) + (c^2*(b*c - 6*a*d)*ArcTanh[S
qrt[d]/(Sqrt[c + d/x^2]*x)])/(16*d^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}+\frac{(-bc+6ad)\int\frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx}{6d}$$

$$=-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}-\frac{(-bc+6ad)\text{Subst}\left(\int\left(c+dx^2\right)^{3/2}\,dx,x,\frac{1}{x}\right)}{6d}$$

$$=\frac{(bc-6ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{24dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}+\frac{(c(bc-6ad))\text{Subst}\left(\int\sqrt{c+dx^2}\,dx,x,\frac{1}{x}\right)}{8d}$$

$$= \frac{c(bc-6ad)\sqrt{c+\frac{d}{x^2}}}{16dx} + \frac{(bc-6ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{24dx} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx} + \frac{(c^2(bc-6ad))\operatorname{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{16d}$$

$$= \frac{c(bc-6ad)\sqrt{c+\frac{d}{x^2}}}{16dx} + \frac{(bc-6ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{24dx} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx} + \frac{(c^2(bc-6ad))\operatorname{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{1}{\sqrt{c+\frac{d}{x^2}}x}\right)}{16d}$$

$$= \frac{c(bc-6ad)\sqrt{c+\frac{d}{x^2}}}{16dx} + \frac{(bc-6ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{24dx} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx} + \frac{c^2(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)}{16d^{3/2}}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx = \frac{\sqrt{c+\frac{d}{x^2}}\left(-\sqrt{d}(6adx^2(2d+5cx^2)+b(8d^2+14cdx^2+3c^2x^4)) + \frac{3c^2(bc-6ad)x^6\text{arc}}{\sqrt{d+}}\right.}{48d^{3/2}x^5}$$

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^2,x]
[Out] (Sqrt[c + d/x^2]*(-(Sqrt[d]*(6*a*d*x^2*(2*d + 5*c*x^2) + b*(8*d^2 + 14*c*d*
x^2 + 3*c^2*x^4))) + (3*c^2*(b*c - 6*a*d)*x^6*ArcTanh[Sqrt[d + c*x^2]/Sqrt[
d]])/Sqrt[d + c*x^2]))/(48*d^(3/2)*x^5)
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{(30acd\,x^4+3b\,c^2x^4+12a\,d^2x^2+14bcd\,x^2+8b\,d^2)\sqrt{\frac{c\,x^2+d}{x^2}}}{48x^5d} - \frac{c^2(6ad-bc)\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)\sqrt{\frac{c\,x^2+d}{x^2}}\,x}{16d^{\frac{3}{2}}\sqrt{c\,x^2+d}}$
default	$-\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\left(18d^{\frac{5}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)a\,c^2x^6-3d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)b\,c^3x^6-6(c\,x^2+d)^{\frac{3}{2}}a\,c^2d\,x^6+(c\,x^2+d)^{\frac{3}{2}}b\,c^3x^6+6(c\,x^2+\right.}{48x^3(c\,x}$

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*(30*a*c*d*x^4+3*b*c^2*x^4+12*a*d^2*x^2+14*b*c*d*x^2+8*b*d^2)/x^5/d*((
c*x^2+d)/x^2)^(1/2)-1/16*c^2*(6*a*d-b*c)/d^(3/2)*ln((2*d+2*d^(1/2)*(c*x^2+d
)^(1/2))/x)*((c*x^2+d)/x^2)^(1/2)*x/(c*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx = \left[-\frac{3\,(bc^3-6\,ac^2d)\sqrt{d}x^5\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,(3\,(bc^2d+10\,acd^2)x^4}{96\,d^2x^5}\right.$$

$$-\frac{3\,(bc^3-6\,ac^2d)\sqrt{-d}x^5\arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+(3\,(bc^2d+10\,acd^2)x^4+8\,bd^3+2\,(7\,bcd^2+6\,ad^3)x^2)\sqrt{\frac{cx^2}{x}}}{48\,d^2x^5}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b*c^3 - 6*a*c^2*d)*sqrt(d)*x^5*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c
*x^2 + d)/x^2) + 2*d)/x^2) + 2*(3*(b*c^2*d + 10*a*c*d^2)*x^4 + 8*b*d^3 + 2*
(7*b*c*d^2 + 6*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^2*x^5), -1/48*(3*(b*c^
3 - 6*a*c^2*d)*sqrt(-d)*x^5*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2
+ d)) + (3*(b*c^2*d + 10*a*c*d^2)*x^4 + 8*b*d^3 + 2*(7*b*c*d^2 + 6*a*d^3)*x
^2)*sqrt((c*x^2 + d)/x^2))/(d^2*x^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(102) = 204$.

Time = 10.45 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.06

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx = -\frac{ac^{\frac{3}{2}}\sqrt{1+\frac{d}{cx^2}}}{2x} - \frac{ac^{\frac{3}{2}}}{8x\sqrt{1+\frac{d}{cx^2}}} - \frac{3a\sqrt{c}d}{8x^3\sqrt{1+\frac{d}{cx^2}}} - \frac{3ac^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{8\sqrt{d}} - \frac{ad^2}{4\sqrt{c}x^5\sqrt{1+\frac{d}{cx^2}}} - \frac{bc^{\frac{5}{2}}}{16dx\sqrt{1+\frac{d}{cx^2}}} - \frac{17bc^{\frac{3}{2}}}{48x^3\sqrt{1+\frac{d}{cx^2}}} - \frac{11b\sqrt{c}d}{24x^5\sqrt{1+\frac{d}{cx^2}}} + \frac{bc^3\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{16d^{\frac{3}{2}}} - \frac{bd^2}{6\sqrt{c}x^7\sqrt{1+\frac{d}{cx^2}}}$$

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**2,x)
```

```
[Out] -a*c**(3/2)*sqrt(1 + d/(c*x**2))/(2*x) - a*c**(3/2)/(8*x*sqrt(1 + d/(c*x**2
))) - 3*a*sqrt(c)*d/(8*x**3*sqrt(1 + d/(c*x**2))) - 3*a*c**2*asinh(sqrt(d)/
(sqrt(c)*x))/(8*sqrt(d)) - a*d**2/(4*sqrt(c)*x**5*sqrt(1 + d/(c*x**2))) - b
*c**(5/2)/(16*d*x*sqrt(1 + d/(c*x**2))) - 17*b*c**(3/2)/(48*x**3*sqrt(1 + d
/(c*x**2))) - 11*b*sqrt(c)*d/(24*x**5*sqrt(1 + d/(c*x**2))) + b*c**3*asinh(
sqrt(d)/(sqrt(c)*x))/(16*d**(3/2)) - b*d**2/(6*sqrt(c)*x**7*sqrt(1 + d/(c*x
**2)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(103) = 206$.

Time = 0.30 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.24

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx = \frac{1}{16}\left(\frac{3\,c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{\sqrt{d}} - \frac{2\left(5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2x^3 - 3\sqrt{c+\frac{d}{x^2}}c^2dx\right)}{\left(c+\frac{d}{x^2}\right)^2x^4 - 2\left(c+\frac{d}{x^2}\right)dx^2 + d^2}\right)a - \frac{1}{96}\left(\frac{3\,c^3\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{3}{2}}} + \frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3x^5 + 8\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^3dx^3 - 3\sqrt{c+\frac{d}{x^2}}c^3d^2x\right)}{\left(c+\frac{d}{x^2}\right)^3dx^6 - 3\left(c+\frac{d}{x^2}\right)^2d^2x^4 + 3\left(c+\frac{d}{x^2}\right)d^3x^2 - d^4}\right)b$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/16*(3*c^2*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d))
)/sqrt(d) - 2*(5*(c + d/x^2)^(3/2)*c^2*x^3 - 3*sqrt(c + d/x^2)*c^2*d*x)/((c
 + d/x^2)^2*x^4 - 2*(c + d/x^2)*d*x^2 + d^2))*a - 1/96*(3*c^3*log((sqrt(c +
 d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(3/2) + 2*(3*(c + d/x
^2)^(5/2)*c^3*x^5 + 8*(c + d/x^2)^(3/2)*c^3*d*x^3 - 3*sqrt(c + d/x^2)*c^3*d
^2*x)/((c + d/x^2)^3*d*x^6 - 3*(c + d/x^2)^2*d^2*x^4 + 3*(c + d/x^2)*d^3*x^
2 - d^4))*b
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.41

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx = -\frac{\frac{3\left(bc^4\mathrm{sgn}(x)-6\,ac^3d\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}d} + \frac{3\left(cx^2+d\right)^{\frac{5}{2}}bc^4\mathrm{sgn}(x)+30\left(cx^2+d\right)^{\frac{5}{2}}ac^3d\mathrm{sgn}(x)+8\left(cx^2+d\right)^{\frac{3}{2}}bc^4d\mathrm{sgn}(x)-48\left(cx^2+d\right)^{\frac{3}{2}}ac^3d^2\mathrm{sg}}{c^3dx^6}}{48\,c}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/48*(3*(b*c^4*sgn(x) - 6*a*c^3*d*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d))
/(sqrt(-d)*d) + (3*(c*x^2 + d)^(5/2)*b*c^4*sgn(x) + 30*(c*x^2 + d)^(5/2)*a*
c^3*d*sgn(x) + 8*(c*x^2 + d)^(3/2)*b*c^4*d*sgn(x) - 48*(c*x^2 + d)^(3/2)*a*
c^3*d^2*sgn(x) - 3*sqrt(c*x^2 + d)*b*c^4*d^2*sgn(x) + 18*sqrt(c*x^2 + d)*a*
c^3*d^3*sgn(x))/(c^3*d*x^6))/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)\,\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx$$

```
[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^2,x)
```

```
[Out] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^2, x)
```

3.961 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 159

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx = \frac{c(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{64dx^3}+\frac{(3bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{48dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}+\frac{c^2(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{128d^2x}-\frac{c^3(3bc-8ad)\text{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{128d^{5/2}}$$

```
[Out] 1/48*(-8*a*d+3*b*c)*(c+d/x^2)^(3/2)/d/x^3-1/8*b*(c+d/x^2)^(5/2)/d/x^3-1/128
*c^3*(-8*a*d+3*b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(5/2)+1/64*c*(-8*a
*d+3*b*c)*(c+d/x^2)^(1/2)/d/x^3+1/128*c^2*(-8*a*d+3*b*c)*(c+d/x^2)^(1/2)/d^
2/x
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {470, 342, 285, 327, 223, 212}

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx = -\frac{c^3(3bc-8ad)\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{128d^{5/2}}+\frac{c^2\sqrt{c+\frac{d}{x^2}}(3bc-8ad)}{128d^2x}+\frac{c\sqrt{c+\frac{d}{x^2}}(3bc-8ad)}{64dx^3}+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-8ad)}{48dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}$$

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^4,x]
```

```
[Out] (c*(3*b*c - 8*a*d)*Sqrt[c + d/x^2])/(64*d*x^3) + ((3*b*c - 8*a*d)*(c + d/x^
2)^(3/2))/(48*d*x^3) - (b*(c + d/x^2)^(5/2))/(8*d*x^3) + (c^2*(3*b*c - 8*a*
d)*Sqrt[c + d/x^2])/(128*d^2*x) - (c^3*(3*b*c - 8*a*d)*ArcTanh[Sqrt[d]/(Sqr
t[c + d/x^2]*x)])/(128*d^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}+\frac{(-3bc+8ad)\int\frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx}{8d}\\
&= -\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}-\frac{(-3bc+8ad)\text{Subst}\left(\int x^2\left(c+dx^2\right)^{3/2}\,dx,x,\frac{1}{x}\right)}{8d}\\
&= \frac{(3bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{48dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}+\frac{(c(3bc-8ad))\text{Subst}\left(\int x^2\sqrt{c+dx^2}\,dx,x,\frac{1}{x}\right)}{16d}\\
&= \frac{c(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{64dx^3}+\frac{(3bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{48dx^3}\\
&\quad -\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}+\frac{\left(c^2(3bc-8ad)\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right)}{64d}\\
&= \frac{c(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{64dx^3}+\frac{(3bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{48dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}\\
&\quad +\frac{c^2(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{128d^2x}-\frac{\left(c^3(3bc-8ad)\right)\text{Subst}\left(\int\frac{1}{\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right)}{128d^2}\\
&= \frac{c(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{64dx^3}+\frac{(3bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{48dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}\\
&\quad +\frac{c^2(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{128d^2x}-\frac{\left(c^3(3bc-8ad)\right)\text{Subst}\left(\int\frac{1}{1-dx^2}\,dx,x,\frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{128d^2}\\
&= \frac{c(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{64dx^3}+\frac{(3bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{48dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}\\
&\quad +\frac{c^2(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{128d^2x}-\frac{c^3(3bc-8ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{128d^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.94

$$\int\frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx=$$

$$-\frac{\sqrt{c+\frac{d}{x^2}}\left(\sqrt{d}\sqrt{d+cx^2}\left(8adx^2\left(8d^2+14cdx^2+3c^2x^4\right)+b\left(48d^3+72cd^2x^2+6c^2dx^4-9c^3x^6\right)\right)+3c^3(3bc-8\right.}{384d^{5/2}x^7\sqrt{d+cx^2}}$$

[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^4,x]

[Out] -1/384*(Sqrt[c + d/x^2]*(Sqrt[d]*Sqrt[d + c*x^2]*(8*a*d*x^2*(8*d^2 + 14*c*d*x^2 + 3*c^2*x^4) + b*(48*d^3 + 72*c*d^2*x^2 + 6*c^2*d*x^4 - 9*c^3*x^6)) + 3*c^3*(3*b*c - 8*a*d)*x^8*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]]))/(d^(5/2)*x^7*Sqrt[d + c*x^2])

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.95

method	result
risch	$-\frac{\left(24 a\,c^{2} d\,x^{6}-9 b\,c^{3} x^{6}+112 a c\,d^{2} x^{4}+6 b\,c^{2} d\,x^{4}+64 a\,d^{3} x^{2}+72 b c\,d^{2} x^{2}+48 b\,d^{3}\right) \sqrt{\frac{c\,x^{2}+d}{x^{2}}}}{384 x^{7} d^{2}}+\frac{c^{3} \left(8 a d -3 b c \right) \ln \left(\frac{2 d +2 \sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right) \sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}{128 d^{\frac{5}{2}} \sqrt{c\,x^{2}+d}}$
default	$\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} \left(-8 \left(c\,x^{2}+d \right)^{\frac{3}{2}} a\,c^{3} d\,x^{8}+3 \left(c\,x^{2}+d \right)^{\frac{3}{2}} b\,c^{4} x^{8}+24 d^{\frac{5}{2}} \ln \left(\frac{2 d +2 \sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right) a\,c^{3} x^{8}-9 d^{\frac{3}{2}} \ln \left(\frac{2 d +2 \sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right) b\,c^{4} x^{8}+8 (c x \cdots\right.}{\cdots}$

[In] int((a+b/x^2)*(c+d/x^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)

[Out] -1/384*(24*a*c^2*d*x^6-9*b*c^3*x^6+112*a*c*d^2*x^4+6*b*c^2*d*x^4+64*a*d^3*x^2+72*b*c*d^2*x^2+48*b*d^3)/x^7/d^2*((c*x^2+d)/x^2)^(1/2)+1/128*c^3*(8*a*d-3*b*c)/d^(5/2)*ln((2*d+2*d^(1/2)*(c*x^2+d)^(1/2))/x)*((c*x^2+d)/x^2)^(1/2)*x/(c*x^2+d)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.87

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx=\left[-\frac{3\left(3\,bc^4-8\,ac^3d\right)\sqrt{d}x^7\log\left(-\frac{cx^2+2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)-2\left(3\left(3\,bc^3d-8\,ac^2d^2\right)\cdots\right.}{768\,d^3x^{7}}\cdots\right.$$

[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^4,x, algorithm="fricas")

[Out] [-1/768*(3*(3*b*c^4 - 8*a*c^3*d)*sqrt(d)*x^7*log(-(c*x^2 + 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2) - 2*(3*(3*b*c^3*d - 8*a*c^2*d^2)*x^6 - 48*b*d^4 - 2*(3*b*c^2*d^2 + 56*a*c*d^3)*x^4 - 8*(9*b*c*d^3 + 8*a*d^4)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^3*x^7), 1/384*(3*(3*b*c^4 - 8*a*c^3*d)*sqrt(-d)*x^7*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (3*(3*b*c^3*d - 8*a*c^2*d^2)*x^6 - 48*b*d^4 - 2*(3*b*c^2*d^2 + 56*a*c*d^3)*x^4 - 8*(9*b*c*d^3 + 8*a*d^4)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^3*x^7)]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(141) = 282.

Time = 31.78 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.81

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx = -\frac{ac^{\frac{5}{2}}}{16dx\sqrt{1+\frac{d}{cx^2}}} - \frac{17ac^{\frac{3}{2}}}{48x^3\sqrt{1+\frac{d}{cx^2}}} - \frac{11a\sqrt{c}d}{24x^5\sqrt{1+\frac{d}{cx^2}}} + \frac{ac^3\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{16d^{\frac{3}{2}}} - \frac{ad^2}{6\sqrt{c}x^7\sqrt{1+\frac{d}{cx^2}}} + \frac{3bc^{\frac{7}{2}}}{128d^2x\sqrt{1+\frac{d}{cx^2}}} + \frac{bc^{\frac{5}{2}}}{128dx^3\sqrt{1+\frac{d}{cx^2}}} - \frac{13bc^{\frac{3}{2}}}{64x^5\sqrt{1+\frac{d}{cx^2}}} - \frac{5b\sqrt{c}d}{16x^7\sqrt{1+\frac{d}{cx^2}}} - \frac{3bc^4\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{128d^{\frac{5}{2}}} - \frac{bd^2}{8\sqrt{c}x^9\sqrt{1+\frac{d}{cx^2}}}$$

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**4,x)
```

```
[Out] -a*c**(5/2)/(16*d*x*sqrt(1 + d/(c*x**2))) - 17*a*c**(3/2)/(48*x**3*sqrt(1 +
 d/(c*x**2))) - 11*a*sqrt(c)*d/(24*x**5*sqrt(1 + d/(c*x**2))) + a*c**3*asin
h(sqrt(d)/(sqrt(c)*x))/(16*d**(3/2)) - a*d**2/(6*sqrt(c)*x**7*sqrt(1 + d/(c
*x**2))) + 3*b*c**(7/2)/(128*d**2*x*sqrt(1 + d/(c*x**2))) + b*c**(5/2)/(128
*d*x**3*sqrt(1 + d/(c*x**2))) - 13*b*c**(3/2)/(64*x**5*sqrt(1 + d/(c*x**2))
) - 5*b*sqrt(c)*d/(16*x**7*sqrt(1 + d/(c*x**2))) - 3*b*c**4*asinh(sqrt(d)/(
sqrt(c)*x))/(128*d**(5/2)) - b*d**2/(8*sqrt(c)*x**9*sqrt(1 + d/(c*x**2)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(135) = 270.

Time = 0.28 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.23

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx = -\frac{1}{96}\left(\frac{3\,c^3\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{3}{2}}} + \frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3x^5 + 8\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^3dx^3 - 3\sqrt{c+\frac{d}{x^2}}c^3d^2x\right)}{\left(c+\frac{d}{x^2}\right)^3dx^6 - 3\left(c+\frac{d}{x^2}\right)^2d^2x^4 + 3\left(c+\frac{d}{x^2}\right)d^3x^2 - d^4}\right)a + \frac{1}{256}\left(\frac{3\,c^4\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{5}{2}}} + \frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c^4x^7 - 11\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^4dx^5 - 11\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^4d^2x^3 + 3\sqrt{c+\frac{d}{x^2}}c\right.}{\left(c+\frac{d}{x^2}\right)^4d^2x^8 - 4\left(c+\frac{d}{x^2}\right)^3d^3x^6 + 6\left(c+\frac{d}{x^2}\right)^2d^4x^4 - 4\left(c+\frac{d}{x^2}\right)d^5x^2 + \right.}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/96*(3*c^3*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)
))/d^(3/2) + 2*(3*(c + d/x^2)^(5/2)*c^3*x^5 + 8*(c + d/x^2)^(3/2)*c^3*d*x^3
 - 3*sqrt(c + d/x^2)*c^3*d^2*x)/((c + d/x^2)^3*d*x^6 - 3*(c + d/x^2)^2*d^2*
x^4 + 3*(c + d/x^2)*d^3*x^2 - d^4))*a + 1/256*(3*c^4*log((sqrt(c + d/x^2)*x
 - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(5/2) + 2*(3*(c + d/x^2)^(7/2)
*c^4*x^7 - 11*(c + d/x^2)^(5/2)*c^4*d*x^5 - 11*(c + d/x^2)^(3/2)*c^4*d^2*x^
3 + 3*sqrt(c + d/x^2)*c^4*d^3*x)/((c + d/x^2)^4*d^2*x^8 - 4*(c + d/x^2)^3*d
^3*x^6 + 6*(c + d/x^2)^2*d^4*x^4 - 4*(c + d/x^2)*d^5*x^2 + d^6))*b
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.35

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx = \frac{\frac{3\left(3\,bc^5\mathrm{sgn}(x)-8\,ac^4d\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}d^2} + \frac{9\left(cx^2+d\right)^{\frac{7}{2}}bc^5\mathrm{sgn}(x)-24\left(cx^2+d\right)^{\frac{7}{2}}ac^4d\mathrm{sgn}(x)-33\,(c}{}}{}$$

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/384*(3*(3*b*c^5*sgn(x) - 8*a*c^4*d*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d
))/(sqrt(-d)*d^2) + (9*(c*x^2 + d)^(7/2)*b*c^5*sgn(x) - 24*(c*x^2 + d)^(7/2
)*a*c^4*d*sgn(x) - 33*(c*x^2 + d)^(5/2)*b*c^5*d*sgn(x) - 40*(c*x^2 + d)^(5/
2)*a*c^4*d^2*sgn(x) - 33*(c*x^2 + d)^(3/2)*b*c^5*d^2*sgn(x) + 88*(c*x^2 + d
)^(3/2)*a*c^4*d^3*sgn(x) + 9*sqrt(c*x^2 + d)*b*c^5*d^3*sgn(x) - 24*sqrt(c*x
^2 + d)*a*c^4*d^4*sgn(x))/(c^4*d^2*x^8))/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)\,\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx$$

```
[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^4,x)
```

```
[Out] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^4, x)
```

3.962 $\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{(4bc-3ad)\sqrt{c+\frac{d}{x^2}}x^2}{8c^2}+\frac{a\sqrt{c+\frac{d}{x^2}}x^4}{4c}-\frac{d(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{5/2}}$$

```
[Out] -1/8*d*(-3*a*d+4*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(5/2)+1/8*(-3*a*d+
4*b*c)*x^2*(c+d/x^2)^(1/2)/c^2+1/4*a*x^4*(c+d/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 79, 44, 65, 214}

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx=-\frac{d(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{5/2}}+\frac{x^2\sqrt{c+\frac{d}{x^2}}(4bc-3ad)}{8c^2}+\frac{ax^4\sqrt{c+\frac{d}{x^2}}}{4c}$$

```
[In] Int[((a + b/x^2)*x^3)/Sqrt[c + d/x^2],x]
```

```
[Out] ((4*b*c - 3*a*d)*Sqrt[c + d/x^2]*x^2)/(8*c^2) + (a*Sqrt[c + d/x^2]*x^4)/(4*
c) - (d*(4*b*c - 3*a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(8*c^(5/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{a+bx}{x^3\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= \frac{a\sqrt{c+\frac{d}{x^2}}x^4}{4c} - \frac{\left(2bc-\frac{3ad}{2}\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)}{4c}\\
&= \frac{(4bc-3ad)\sqrt{c+\frac{d}{x^2}}x^2}{8c^2} + \frac{a\sqrt{c+\frac{d}{x^2}}x^4}{4c} + \frac{(d(4bc-3ad))\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)}{16c^2}
\end{aligned}$$

$$= \frac{(4bc-3ad)\sqrt{c+\frac{d}{x^2}}x^2}{8c^2} + \frac{a\sqrt{c+\frac{d}{x^2}}x^4}{4c} + \frac{(4bc-3ad)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{8c^2}$$

$$= \frac{(4bc-3ad)\sqrt{c+\frac{d}{x^2}}x^2}{8c^2} + \frac{a\sqrt{c+\frac{d}{x^2}}x^4}{4c} - \frac{d(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{5/2}}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.17

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx$$

$$= \frac{\sqrt{c}(d+cx^2)\left(4bc-3ad+2acx^2\right) + \frac{2d(-4bc+3ad)\sqrt{d+cx^2}\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{d}+\sqrt{d+cx^2}}\right)}{x}}{8c^{5/2}\sqrt{c+\frac{d}{x^2}}}$$

```
[In] Integrate[((a + b/x^2)*x^3)/Sqrt[c + d/x^2],x]
```

```
[Out] (Sqrt[c]*(d + c*x^2)*(4*b*c - 3*a*d + 2*a*c*x^2) + (2*d*(-4*b*c + 3*a*d)*Sq
rt[d + c*x^2]*ArcTanh[(Sqrt[c]*x)/(-Sqrt[d] + Sqrt[d + c*x^2])])/x)/(8*c^(5
/2)*Sqrt[c + d/x^2])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.10

method	result	size
risch	$\frac{(2ac\,x^2-3ad+4bc)(c\,x^2+d)}{8c^2\sqrt{\frac{c\,x^2+d}{x^2}}} + \frac{d(3ad-4bc)\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)\sqrt{c\,x^2+d}}{8c^{\frac{5}{2}}\sqrt{\frac{c\,x^2+d}{x^2}}\,x}$	99
default	$\frac{\sqrt{c\,x^2+d}\left(2\sqrt{c\,x^2+d}\,c^{\frac{5}{2}}\,a\,x^3-3\sqrt{c\,x^2+d}\,c^{\frac{3}{2}}\,adx+4\sqrt{c\,x^2+d}\,c^{\frac{5}{2}}\,bx+3\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)ac\,d^2-4\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)b\,c^2d\right)}{8\sqrt{\frac{c\,x^2+d}{x^2}}\,x\,c^{\frac{7}{2}}}$	129

```
[In] int((a+b/x^2)*x^3/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(2*a*c*x^2-3*a*d+4*b*c)*(c*x^2+d)/c^2/((c*x^2+d)/x^2)^(1/2)+1/8*d*(3*a*
d-4*b*c)/c^(5/2)*ln(c^(1/2)*x+(c*x^2+d)^(1/2))/((c*x^2+d)/x^2)^(1/2)/x*(c*x
^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.13

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx = \left[-\frac{(4\,bcd-3\,ad^2)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\,(2\,ac^2x^4+(4\,bc^2-3\,acd)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{16\,c^3}, \frac{(4\,bcd}{}\right.$$

```
[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((4*b*c*d - 3*a*d^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^
2 + d)/x^2) - d) - 2*(2*a*c^2*x^4 + (4*b*c^2 - 3*a*c*d)*x^2)*sqrt((c*x^2 +
d)/x^2))/c^3, 1/8*((4*b*c*d - 3*a*d^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c
*x^2 + d)/x^2)/(c*x^2 + d)) + (2*a*c^2*x^4 + (4*b*c^2 - 3*a*c*d)*x^2)*sqrt(
(c*x^2 + d)/x^2))/c^3]
```

Sympy [A] (verification not implemented)

Time = 14.38 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.67

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{ax^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}} - \frac{a\sqrt{d}x^3}{8c\sqrt{\frac{cx^2}{d}+1}} - \frac{3ad^{\frac{3}{2}}x}{8c^2\sqrt{\frac{cx^2}{d}+1}} + \frac{3ad^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{8c^{\frac{5}{2}}} + \frac{b\sqrt{d}x\sqrt{\frac{cx^2}{d}+1}}{2c} - \frac{bd\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2c^{\frac{3}{2}}}$$

```
[In] integrate((a+b/x**2)*x**3/(c+d/x**2)**(1/2),x)
```

```
[Out] a*x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) - a*sqrt(d)*x**3/(8*c*sqrt(c*x**2/d +
 1)) - 3*a*d**(3/2)*x/(8*c**2*sqrt(c*x**2/d + 1)) + 3*a*d**2*asinh(sqrt(c)*
x/sqrt(d))/(8*c**(5/2)) + b*sqrt(d)*x*sqrt(c*x**2/d + 1)/(2*c) - b*d*asinh(
sqrt(c)*x/sqrt(d))/(2*c**(3/2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(74) = 148$.

Time = 0.28 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.98

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{1}{4}\,b\left(\frac{2\sqrt{c+\frac{d}{x^2}}d}{\left(c+\frac{d}{x^2}\right)c-c^2}+\frac{d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right) - \frac{1}{16}\,a\left(\frac{3\,d^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2-5\sqrt{c+\frac{d}{x^2}}cd^2\right)}{\left(c+\frac{d}{x^2}\right)^2c^2-2\left(c+\frac{d}{x^2}\right)c^3+c^4}\right)$$

```
[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*b*(2*sqrt(c + d/x^2)*d/((c + d/x^2)*c - c^2) + d*log((sqrt(c + d/x^2) -
 sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(3/2)) - 1/16*a*(3*d^2*log((sqrt(c
 + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(5/2) + 2*(3*(c + d/x^2
)^(3/2)*d^2 - 5*sqrt(c + d/x^2)*c*d^2)/((c + d/x^2)^2*c^2 - 2*(c + d/x^2)*c
^3 + c^4))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.23

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{1}{8}\sqrt{cx^2+d}x\left(\frac{2\,ax^2}{c\mathrm{sgn}\,(x)}+\frac{4\,bc^2\mathrm{sgn}(x)-3\,acd\mathrm{sgn}(x)}{c^3}\right) - \frac{(4\,bcd\log\left(|d|\right)-3\,ad^2\log\left(|d|\right))\mathrm{sgn}(x)}{16\,c^{\frac{5}{2}}} + \frac{(4\,bcd-3\,ad^2)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+d}\right|\right)}{8\,c^{\frac{5}{2}}\mathrm{sgn}\,(x)}$$

```
[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^2 + d)*x*(2*a*x^2/(c*sgn(x)) + (4*b*c^2*sgn(x) - 3*a*c*d*sgn(x
))/c^3) - 1/16*(4*b*c*d*log(abs(d)) - 3*a*d^2*log(abs(d)))*sgn(x)/c^(5/2) +
 1/8*(4*b*c*d - 3*a*d^2)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)))/(c^(5/2)*sg
n(x))
```

Mupad [B] (verification not implemented)

Time = 9.85 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{5\,a\,x^4\,\sqrt{c+\frac{d}{x^2}}}{8\,c} - \frac{3\,a\,x^4\,\left(c+\frac{d}{x^2}\right)^{3/2}}{8\,c^2} + \frac{b\,x^2\,\sqrt{c+\frac{d}{x^2}}}{2\,c} - \frac{b\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,c^{3/2}} + \frac{3\,a\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,c^{5/2}}$$

```
[In] int((x^3*(a + b/x^2))/(c + d/x^2)^(1/2),x)
```

```
[Out] (5*a*x^4*(c + d/x^2)^(1/2))/(8*c) - (3*a*x^4*(c + d/x^2)^(3/2))/(8*c^2) + (
b*x^2*(c + d/x^2)^(1/2))/(2*c) - (b*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(2*
c^(3/2)) + (3*a*d^2*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(8*c^(5/2))
```

3.963 $\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 59

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{a\sqrt{c+\frac{d}{x^2}}x^2}{2c}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{3/2}}$$

```
[Out] 1/2*(-a*d+2*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(3/2)+1/2*a*x^2*(c+d/x^
2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {457, 79, 65, 214}

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{3/2}}+\frac{ax^2\sqrt{c+\frac{d}{x^2}}}{2c}$$

```
[In] Int[((a + b/x^2)*x)/Sqrt[c + d/x^2],x]
```

```
[Out] (a*Sqrt[c + d/x^2]*x^2)/(2*c) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt
[c]])/(2*c^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{a+bx}{x^2\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= \frac{a\sqrt{c+\frac{d}{x^2}}x^2}{2c} - \frac{\left(bc-\frac{ad}{2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)}{2c}\\
&= \frac{a\sqrt{c+\frac{d}{x^2}}x^2}{2c} - \frac{\left(bc-\frac{ad}{2}\right)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{cd}\\
&= \frac{a\sqrt{c+\frac{d}{x^2}}x^2}{2c} + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{a\sqrt{c}x(d+cx^2)+(-2bc+ad)\sqrt{d+cx^2}\log\left(-\sqrt{c}x+\sqrt{d+cx^2}\right)}{2c^{3/2}\sqrt{c+\frac{d}{x^2}}x}$$

```
[In] Integrate[((a + b/x^2)*x)/Sqrt[c + d/x^2],x]
```

```
[Out] (a*Sqrt[c]*x*(d + c*x^2) + (-2*b*c + a*d)*Sqrt[d + c*x^2]*Log[-(Sqrt[c]*x)
+ Sqrt[d + c*x^2]])/(2*c^(3/2)*Sqrt[c + d/x^2]*x)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.39

method	result	size
risch	$\frac{a(c\,x^2+d)}{2c\sqrt{\frac{c\,x^2+d}{x^2}}}-\frac{(ad-2bc)\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)\sqrt{c\,x^2+d}}{2c^{\frac{3}{2}}\sqrt{\frac{c\,x^2+d}{x^2}}\,x}$	82
default	$\frac{\sqrt{c\,x^2+d}\left(\sqrt{c\,x^2+d}\,c^{\frac{3}{2}}\,ax+2b\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)c^2-\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)acd\right)}{2\sqrt{\frac{c\,x^2+d}{x^2}}\,x\,c^{\frac{5}{2}}}$	90

```
[In] int((a+b/x^2)*x/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c*a*(c*x^2+d)/((c*x^2+d)/x^2)^(1/2)-1/2*(a*d-2*b*c)/c^(3/2)*ln(c^(1/2)*
x+(c*x^2+d)^(1/2))/((c*x^2+d)/x^2)^(1/2)/x*(c*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.47

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx$$
$$=\left[\frac{2\,acx^2\sqrt{\frac{cx^2+d}{x^2}}-(2\,bc-ad)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)}{4\,c^2},\frac{acx^2\sqrt{\frac{cx^2+d}{x^2}}-(2\,bc-ad)\sqrt{-c}\,\mathrm{arctan}}{2\,c^2}\right.$$

```
[In] integrate((a+b/x^2)*x/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*c*x^2*sqrt((c*x^2 + d)/x^2) - (2*b*c - a*d)*sqrt(c)*log(-2*c*x^2
+ 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d))/c^2, 1/2*(a*c*x^2*sqrt((c*x^2 +
 d)/x^2) - (2*b*c - a*d)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)
/(c*x^2 + d)))/c^2]
```

Sympy [A] (verification not implemented)

Time = 14.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{a\sqrt{d}x\sqrt{\frac{cx^2}{d}+1}}{2c}-\frac{ad\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2c^{\frac{3}{2}}}+\frac{b\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{\sqrt{c}}$$

```
[In] integrate((a+b/x**2)*x/(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(d)*x*sqrt(c*x**2/d + 1)/(2*c) - a*d*asinh(sqrt(c)*x/sqrt(d))/(2*c**(
3/2)) + b*asinh(sqrt(c)*x/sqrt(d))/sqrt(c)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(47) = 94.

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.85

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{1}{4}a\left(\frac{2\sqrt{c+\frac{d}{x^2}}d}{\left(c+\frac{d}{x^2}\right)c-c^2}+\frac{d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)-\frac{b\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{2\sqrt{c}}$$

```
[In] integrate((a+b/x^2)*x/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*a*(2*sqrt(c + d/x^2)*d/((c + d/x^2)*c - c^2) + d*log((sqrt(c + d/x^2) -
 sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(3/2)) - 1/2*b*log((sqrt(c + d/x^2
) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/sqrt(c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.34

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{\sqrt{cx^2+d}ax}{2\,c\operatorname{sgn}(x)}+\frac{(2\,bc\log(|d|)-ad\log(|d|))\operatorname{sgn}(x)}{4\,c^{\frac{3}{2}}}-\frac{(2\,bc-ad)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+d}\right|\right)}{2\,c^{\frac{3}{2}}\operatorname{sgn}(x)}$$

```
[In] integrate((a+b/x^2)*x/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + d)*a*x/(c*sgn(x)) + 1/4*(2*b*c*log(abs(d)) - a*d*log(abs(d
)))*sgn(x)/c^(3/2) - 1/2*(2*b*c - a*d)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)
))/(c^(3/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.56 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{a\,x^2\,\sqrt{c+\frac{d}{x^2}}}{2\,c}-\frac{a\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,c^{3/2}}$$

```
[In] int((x*(a + b/x^2))/(c + d/x^2)^(1/2),x)
```

```
[Out] (b*atanh((c + d/x^2)^(1/2)/c^(1/2)))/c^(1/2) + (a*x^2*(c + d/x^2)^(1/2))/(2
*c) - (a*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(2*c^(3/2))
```

3.964 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 43

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x}\,dx = -\frac{b\sqrt{c+\frac{d}{x^2}}}{d}+\frac{a\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}}$$

```
[Out] a*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(1/2)-b*(c+d/x^2)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {457, 81, 65, 214}

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x}\,dx = \frac{a\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{d}$$

```
[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x),x]
```

```
[Out] -((b*Sqrt[c + d/x^2])/d) + (a*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/Sqrt[c]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{a+bx}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= -\frac{b\sqrt{c+\frac{d}{x^2}}}{d} - \frac{1}{2}a\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)\\
&= -\frac{b\sqrt{c+\frac{d}{x^2}}}{d} - \frac{a\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{d}\\
&= -\frac{b\sqrt{c+\frac{d}{x^2}}}{d} + \frac{a\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.77

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x}\,dx=\frac{-b\sqrt{c}(d+cx^2)-adx\sqrt{d+cx^2}\log\left(-\sqrt{c}x+\sqrt{d+cx^2}\right)}{\sqrt{c}d\sqrt{c+\frac{d}{x^2}}x^2}$$

[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x),x]

[Out] (-(b*Sqrt[c]*(d + c*x^2)) - a*d*x*Sqrt[d + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[d + c*x^2]])/(Sqrt[c]*d*Sqrt[c + d/x^2]*x^2)

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.60

method	result	size
default	$\frac{\sqrt{c\,x^2+d}\,\left(a\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)dx-b\sqrt{c\,x^2+d}\,\sqrt{c}\right)}{\sqrt{\frac{c\,x^2+d}{x^2}}\,x^2\sqrt{c}\,d}$	69
risch	$-\frac{b(c\,x^2+d)}{d\,x^2\sqrt{\frac{c\,x^2+d}{x^2}}}+\frac{a\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)\sqrt{c\,x^2+d}}{\sqrt{c}\,\sqrt{\frac{c\,x^2+d}{x^2}}\,x}$	77

[In] int((a+b/x^2)/x/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] (c*x^2+d)^(1/2)*(a*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*d*x-b*(c*x^2+d)^(1/2)*c^(1/2))/((c*x^2+d)/x^2)^(1/2)/x^2/c^(1/2)/d

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 130, normalized size of antiderivative = 3.02

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x}\,dx=\left[\frac{a\sqrt{c}d\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\,bc\sqrt{\frac{cx^2+d}{x^2}}}{2\,cd},\ -\frac{a\sqrt{-c}d\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+bc\sqrt{\frac{cx^2+d}{x^2}}}{cd}\right]$$

[In] integrate((a+b/x^2)/x/(c+d/x^2)^(1/2),x, algorithm="fricas")

```
[Out] [1/2*(a*sqrt(c)*d*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d) -
 2*b*c*sqrt((c*x^2 + d)/x^2))/(c*d), -(a*sqrt(-c)*d*arctan(sqrt(-c)*x^2*sqr
t((c*x^2 + d)/x^2)/(c*x^2 + d)) + b*c*sqrt((c*x^2 + d)/x^2))/(c*d)]
```

Sympy [A] (verification not implemented)

Time = 2.95 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x}\,dx = -\frac{a\left(\begin{cases} \frac{2\operatorname{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{-c}}\right)}{\sqrt{-c}} & \text{for } d\neq 0 \\ -\frac{\log\left(x^2\right)}{\sqrt{c}} & \text{otherwise} \end{cases}\right)}{2} + \frac{b\left(\begin{cases} -\frac{1}{\sqrt{c}x^2} & \text{for } d=0 \\ -\frac{2\sqrt{c+\frac{d}{x^2}}}{d} & \text{otherwise} \end{cases}\right)}{2}$$

```
[In] integrate((a+b/x**2)/x/(c+d/x**2)**(1/2),x)
```

```
[Out] -a*Piecewise((2*atan(sqrt(c + d/x**2)/sqrt(-c))/sqrt(-c), Ne(d, 0)), (-log(
x**2)/sqrt(c), True))/2 + b*Piecewise((-1/(sqrt(c)*x**2), Eq(d, 0)), (-2*sq
rt(c + d/x**2)/d, True))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x}\,dx = -\frac{a\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{2\sqrt{c}} - \frac{b\sqrt{c+\frac{d}{x^2}}}{d}$$

```
[In] integrate((a+b/x^2)/x/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*a*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/sqrt(c)
 - b*sqrt(c + d/x^2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.53

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x}\,dx = -\frac{a\log\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2\right)}{2\sqrt{c}\mathrm{sgn}\left(x\right)} + \frac{2\,b\sqrt{c}}{\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)\mathrm{sgn}\left(x\right)}$$

```
[In] integrate((a+b/x^2)/x/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*a*log((sqrt(c)*x - sqrt(c*x^2 + d))^2)/(sqrt(c)*sgn(x)) + 2*b*sqrt(c)/
(((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x}\,dx = \frac{a\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{b\,\sqrt{c+\frac{d}{x^2}}}{d}$$

```
[In] int((a + b/x^2)/(x*(c + d/x^2)^(1/2)),x)
```

```
[Out] (a*atanh((c + d/x^2)^(1/2)/c^(1/2)))/c^(1/2) - (b*(c + d/x^2)^(1/2))/d
```

3.965 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 43

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^3} dx = \frac{(bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^2} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^2}$$

```
[Out] -1/3*b*(c+d/x^2)^(3/2)/d^2+(-a*d+b*c)*(c+d/x^2)^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {455, 45}

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^3} dx = \frac{\sqrt{c+\frac{d}{x^2}}(bc-ad)}{d^2} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^2}$$

```
[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^3),x]
```

```
[Out] ((b*c - a*d)*Sqrt[c + d/x^2])/d^2 - (b*(c + d/x^2)^(3/2))/(3*d^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{a+bx}{\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{-bc+ad}{d\sqrt{c+dx}}+\frac{b\sqrt{c+dx}}{d}\right)\,dx, x, \frac{1}{x^2}\right)\right)\\
&= \frac{(bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^3}\,dx = -\frac{\sqrt{c+\frac{d}{x^2}}(3adx^2+b(d-2cx^2))}{3d^2x^2}$$

```
[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^3),x]
[Out] -1/3*(Sqrt[c + d/x^2]*(3*a*d*x^2 + b*(d - 2*c*x^2)))/(d^2*x^2)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

method	result	size
trager	$-\frac{(3ad\,x^2-2cb\,x^2+bd)\sqrt{-\frac{-c\,x^2-d}{x^2}}}{3x^2d^2}$	44
gosper	$-\frac{(3ad\,x^2-2cb\,x^2+bd)(c\,x^2+d)}{3\sqrt{\frac{c\,x^2+d}{x^2}}\,d^2x^4}$	47
default	$-\frac{(3ad\,x^2-2cb\,x^2+bd)(c\,x^2+d)}{3\sqrt{\frac{c\,x^2+d}{x^2}}\,d^2x^4}$	47
risch	$-\frac{(3ad\,x^2-2cb\,x^2+bd)(c\,x^2+d)}{3\sqrt{\frac{c\,x^2+d}{x^2}}\,d^2x^4}$	47

```
[In] int((a+b/x^2)/x^3/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3/x^2*(3*a*d*x^2-2*b*c*x^2+b*d)/d^2*(-(-c*x^2-d)/x^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^3}\,dx = \frac{((2\,bc-3\,ad)x^2-bd)\sqrt{\frac{cx^2+d}{x^2}}}{3\,d^2x^2}$$

```
[In] integrate((a+b/x^2)/x^3/(c+d/x^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*((2*b*c - 3*a*d)*x^2 - b*d)*sqrt((c*x^2 + d)/x^2)/(d^2*x^2)
```

Sympy [A] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.51

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^3}\,dx = \frac{\begin{cases} \frac{-2a\sqrt{c+\frac{d}{x^2}}-\frac{2b\left(-c\sqrt{c+\frac{d}{x^2}}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}\right)}{d}}{d} & \text{for}\ d\neq 0 \\ \frac{-\frac{a}{x^2}-\frac{b}{2x^4}}{\sqrt{c}} & \text{otherwise} \end{cases}}{2}$$

```
[In] integrate((a+b/x**2)/x**3/(c+d/x**2)**(1/2),x)
[Out] Piecewise(((-2*a*sqrt(c + d/x**2) - 2*b*(-c*sqrt(c + d/x**2) + (c + d/x**2)
**(3/2)/3)/d)/d, Ne(d, 0)), ((-a/x**2 - b/(2*x**4))/sqrt(c), True))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^3}\,dx = -\frac{1}{3}\,b\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{d^2}-\frac{3\,\sqrt{c+\frac{d}{x^2}}c}{d^2}\right)-\frac{a\sqrt{c+\frac{d}{x^2}}}{d}$$

```
[In] integrate((a+b/x^2)/x^3/(c+d/x^2)^(1/2),x, algorithm="maxima")
[Out] -1/3*b*((c + d/x^2)^(3/2)/d^2 - 3*sqrt(c + d/x^2)*c/d^2) - a*sqrt(c + d/x^2
)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(37) = 74.

Time = 0.45 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.88

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^3}\,dx = \frac{2\left(3\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4a\sqrt{c}+6\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2bc^{\frac{3}{2}}-6\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2a\sqrt{c}d-2\,bc^{\frac{3}{2}}d+3\,a\sqrt{c}d^2\right)}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)^3\operatorname{sgn}(x)}$$

```
[In] integrate((a+b/x^2)/x^3/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*sqrt(c) + 6*(sqrt(c)*x - sqrt(c*x^
2 + d))^2*b*c^(3/2) - 6*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*sqrt(c)*d - 2*b*c
^(3/2)*d + 3*a*sqrt(c)*d^2)/(((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^3*sgn(x)
)
```

Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^3}\,dx = -\frac{\sqrt{c+\frac{d}{x^2}}\,\left(b\,d+3\,a\,d\,x^2-2\,b\,c\,x^2\right)}{3\,d^2\,x^2}$$

```
[In] int((a + b/x^2)/(x^3*(c + d/x^2)^(1/2)),x)
```

```
[Out] -((c + d/x^2)^(1/2)*(b*d + 3*a*d*x^2 - 2*b*c*x^2))/(3*d^2*x^2)
```

3.966 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^5}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^5}\,dx=-\frac{c(bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^3}+\frac{(2bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}$$

```
[Out] 1/3*(-a*d+2*b*c)*(c+d/x^2)^(3/2)/d^3-1/5*b*(c+d/x^2)^(5/2)/d^3-c*(-a*d+b*c)
*(c+d/x^2)^(1/2)/d^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 78}

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^5}\,dx=\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(2bc-ad)}{3d^3}-\frac{c\sqrt{c+\frac{d}{x^2}}(bc-ad)}{d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}$$

```
[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^5),x]
```

```
[Out] -((c*(b*c - a*d)*Sqrt[c + d/x^2])/d^3) + ((2*b*c - a*d)*(c + d/x^2)^(3/2))/
(3*d^3) - (b*(c + d/x^2)^(5/2))/(5*d^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{x(a+bx)}{\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{c(bc-ad)}{d^2\sqrt{c+dx}} + \frac{(-2bc+ad)\sqrt{c+dx}}{d^2} + \frac{b(c+dx)^{3/2}}{d^2}\right)\,dx, x, \frac{1}{x^2}\right)\right)\\
&= -\frac{c(bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^3} + \frac{(2bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^5}\,dx = \frac{\sqrt{c+\frac{d}{x^2}}(-5adx^2(d-2cx^2)+b(-3d^2+4cdx^2-8c^2x^4))}{15d^3x^4}$$

```
[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^5),x]
[Out] (Sqrt[c + d/x^2]*(-5*a*d*x^2*(d - 2*c*x^2) + b*(-3*d^2 + 4*c*d*x^2 - 8*c^2*
x^4)))/(15*d^3*x^4)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

method	result	size
trager	$\frac{\left(10acd\,x^{4}-8b\,c^{2}x^{4}-5a\,d^{2}x^{2}+4bcd\,x^{2}-3b\,d^{2}\right)\sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{15x^{4}d^{3}}$	67
gosper	$\frac{\left(10acd\,x^{4}-8b\,c^{2}x^{4}-5a\,d^{2}x^{2}+4bcd\,x^{2}-3b\,d^{2}\right)\left(c\,x^{2}+d\right)}{15\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,d^{3}x^{6}}$	70
default	$\frac{\left(10acd\,x^{4}-8b\,c^{2}x^{4}-5a\,d^{2}x^{2}+4bcd\,x^{2}-3b\,d^{2}\right)\left(c\,x^{2}+d\right)}{15\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,d^{3}x^{6}}$	70
risch	$\frac{\left(10acd\,x^{4}-8b\,c^{2}x^{4}-5a\,d^{2}x^{2}+4bcd\,x^{2}-3b\,d^{2}\right)\left(c\,x^{2}+d\right)}{15\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,d^{3}x^{6}}$	70

```
[In] int((a+b/x^2)/x^5/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/15/x^4*(10*a*c*d*x^4-8*b*c^2*x^4-5*a*d^2*x^2+4*b*c*d*x^2-3*b*d^2)/d^3*(-(
-c*x^2-d)/x^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^5}\,dx = -\frac{\left(2\,(4\,bc^2-5\,acd)x^4+3\,bd^2-(4\,bcd-5\,ad^2)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\,d^3x^4}$$

```
[In] integrate((a+b/x^2)/x^5/(c+d/x^2)^(1/2),x, algorithm="fricas")
[Out] -1/15*(2*(4*b*c^2 - 5*a*c*d)*x^4 + 3*b*d^2 - (4*b*c*d - 5*a*d^2)*x^2)*sqrt(
(c*x^2 + d)/x^2)/(d^3*x^4)
```

Sympy [A] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.39

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^5}\,dx = \frac{\begin{cases} \frac{-\frac{2a\left(-c\sqrt{c+\frac{d}{x^2}}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}\right)}{d}-\frac{2b\left(c^2\sqrt{c+\frac{d}{x^2}}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2}}{d} & \text{for } d\neq 0 \\ \frac{-\frac{a}{2x^4}-\frac{b}{3x^6}}{\sqrt{c}} & \text{otherwise} \end{cases}}{2}$$

```
[In] integrate((a+b/x**2)/x**5/(c+d/x**2)**(1/2),x)
```

```
[Out] Piecewise(((-2*a*(-c*sqrt(c + d/x**2) + (c + d/x**2)**(3/2)/3)/d - 2*b*(c**
2*sqrt(c + d/x**2) - 2*c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d**
2)/d, Ne(d, 0)), ((-a/(2*x**4) - b/(3*x**6))/sqrt(c), True))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.15

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^5}\,dx = -\frac{1}{15}\,b\left(\frac{3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^3} - \frac{10\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^3} + \frac{15\sqrt{c+\frac{d}{x^2}}c^2}{d^3}\right) - \frac{1}{3}\,a\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{d^2} - \frac{3\sqrt{c+\frac{d}{x^2}}c}{d^2}\right)$$

```
[In] integrate((a+b/x^2)/x^5/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/15*b*(3*(c + d/x^2)^(5/2)/d^3 - 10*(c + d/x^2)^(3/2)*c/d^3 + 15*sqrt(c +
 d/x^2)*c^2/d^3) - 1/3*a*((c + d/x^2)^(3/2)/d^2 - 3*sqrt(c + d/x^2)*c/d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(62) = 124.

Time = 0.66 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.50

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^5}\,dx = \frac{4\left(15\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^6ac^{\frac{3}{2}} + 40\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4bc^{\frac{5}{2}} - 35\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4ac^{\frac{3}{2}}d - 20\left(\sqrt{c}x-\sqrt{\right.}\right.}{15\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)^5\operatorname{sgn}(x)}$$

```
[In] integrate((a+b/x^2)/x^5/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*(15*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(3/2) + 40*(sqrt(c)*x - sqrt(c
*x^2 + d))^4*b*c^(5/2) - 35*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(3/2)*d - 2
0*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(5/2)*d + 25*(sqrt(c)*x - sqrt(c*x^2
+ d))^2*a*c^(3/2)*d^2 + 4*b*c^(5/2)*d^2 - 5*a*c^(3/2)*d^3)/(((sqrt(c)*x - s
qrt(c*x^2 + d))^2 - d)^5*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^5}\,dx=-\frac{\sqrt{c+\frac{d}{x^2}}\,\left(8\,b\,c^2\,x^4-10\,a\,c\,d\,x^4-4\,b\,c\,d\,x^2+5\,a\,d^2\,x^2+3\,b\,d^2\right)}{15\,d^3\,x^4}$$

```
[In] int((a + b/x^2)/(x^5*(c + d/x^2)^(1/2)),x)
[Out] -((c + d/x^2)^(1/2)*(3*b*d^2 + 5*a*d^2*x^2 + 8*b*c^2*x^4 - 10*a*c*d*x^4 - 4
*b*c*d*x^2))/(15*d^3*x^4)
```

3.967 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^7} dx$

Optimal result

Integrand size = 22, antiderivative size = 101

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^7} dx = \frac{c^2(bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^4} - \frac{c(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^4} + \frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4} - \frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4}$$

```
[Out] -1/3*c*(-2*a*d+3*b*c)*(c+d/x^2)^(3/2)/d^4+1/5*(-a*d+3*b*c)*(c+d/x^2)^(5/2)/
d^4-1/7*b*(c+d/x^2)^(7/2)/d^4+c^2*(-a*d+b*c)*(c+d/x^2)^(1/2)/d^4
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {457, 78}

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^7} dx = \frac{c^2\sqrt{c+\frac{d}{x^2}}(bc-ad)}{d^4} + \frac{\left(c+\frac{d}{x^2}\right)^{5/2}(3bc-ad)}{5d^4} - \frac{c\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{3d^4} - \frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4}$$

```
[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^7),x]
```

```
[Out] (c^2*(b*c - a*d)*Sqrt[c + d/x^2])/d^4 - (c*(3*b*c - 2*a*d)*(c + d/x^2)^(3/2
))/(3*d^4) + ((3*b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^4) - (b*(c + d/x^2)^(7/
2))/(7*d^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{x^2(a+bx)}{\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-\frac{c^2(bc-ad)}{d^3\sqrt{c+dx}}+\frac{c(3bc-2ad)\sqrt{c+dx}}{d^3}+\frac{(-3bc+ad)(c+dx)^{3/2}}{d^3}+\frac{b(c+dx)^{5/2}}{d^3}\right)\right.\right.$$

$$= \frac{c^2(bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^4}-\frac{c(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^4}+\frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^7}\,dx = \frac{(d+cx^2)\left(-15bd^3+18bcd^2x^2-21ad^3x^2-24bc^2dx^4+28acd^2x^4+48bc^3x^6-56ac^2dx^6\right)}{105d^4\sqrt{c+\frac{d}{x^2}}x^8}$$

```
[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^7),x]
[Out] ((d + c*x^2)*(-15*b*d^3 + 18*b*c*d^2*x^2 - 21*a*d^3*x^2 - 24*b*c^2*d*x^4 +
28*a*c*d^2*x^4 + 48*b*c^3*x^6 - 56*a*c^2*d*x^6))/(105*d^4*Sqrt[c + d/x^2]*x
^8)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.90

method	result	size
trager	$-\frac{\left(56 a\,c^{2} d\,x^{6}-48 b\,c^{3} x^{6}-28 a c\,d^{2} x^{4}+24 b\,c^{2} d\,x^{4}+21 a\,d^{3} x^{2}-18 b c\,d^{2} x^{2}+15 b\,d^{3}\right) \sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{105 x^{6} d^{4}}$	91
gosper	$-\frac{\left(56 a\,c^{2} d\,x^{6}-48 b\,c^{3} x^{6}-28 a c\,d^{2} x^{4}+24 b\,c^{2} d\,x^{4}+21 a\,d^{3} x^{2}-18 b c\,d^{2} x^{2}+15 b\,d^{3}\right) \left(c\,x^{2}+d\right)}{105 \sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, d^{4} x^{8}}$	94
default	$-\frac{\left(56 a\,c^{2} d\,x^{6}-48 b\,c^{3} x^{6}-28 a c\,d^{2} x^{4}+24 b\,c^{2} d\,x^{4}+21 a\,d^{3} x^{2}-18 b c\,d^{2} x^{2}+15 b\,d^{3}\right) \left(c\,x^{2}+d\right)}{105 \sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, d^{4} x^{8}}$	94
risch	$-\frac{\left(56 a\,c^{2} d\,x^{6}-48 b\,c^{3} x^{6}-28 a c\,d^{2} x^{4}+24 b\,c^{2} d\,x^{4}+21 a\,d^{3} x^{2}-18 b c\,d^{2} x^{2}+15 b\,d^{3}\right) \left(c\,x^{2}+d\right)}{105 \sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, d^{4} x^{8}}$	94

```
[In] int((a+b/x^2)/x^7/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/105/x^6*(56*a*c^2*d*x^6-48*b*c^3*x^6-28*a*c*d^2*x^4+24*b*c^2*d*x^4+21*a*
d^3*x^2-18*b*c*d^2*x^2+15*b*d^3)/d^4*(-(-c*x^2-d)/x^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.85

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^7}\,dx = \frac{(8\,(6\,bc^3-7\,ac^2d)x^6-4\,(6\,bc^2d-7\,acd^2)x^4-15\,bd^3+3\,(6\,bcd^2-7\,ad^3)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{105\,d^4x^6}$$

```
[In] integrate((a+b/x^2)/x^7/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*(6*b*c^3 - 7*a*c^2*d)*x^6 - 4*(6*b*c^2*d - 7*a*c*d^2)*x^4 - 15*b*d
^3 + 3*(6*b*c*d^2 - 7*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^4*x^6)
```

Sympy [A] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.32

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^7}\,dx = \frac{\begin{cases} \frac{-\frac{2a\left(c^2\sqrt{c+\frac{d}{x^2}}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2}-\frac{2b\left(-c^3\sqrt{c+\frac{d}{x^2}}+c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3}}{d} & \text{for } d\neq 0 \\ \frac{-\frac{a}{3x^6}-\frac{b}{4x^8}}{\sqrt{c}} & \text{otherwise} \end{cases}}{2}$$

```
[In] integrate((a+b/x**2)/x**7/(c+d/x**2)**(1/2),x)
```

```
[Out] Piecewise(((-2*a*(c**2*sqrt(c + d/x**2) - 2*c*(c + d/x**2)**(3/2)/3 + (c +
d/x**2)**(5/2)/5)/d**2 - 2*b*(-c**3*sqrt(c + d/x**2) + c**2*(c + d/x**2)**(
3/2) - 3*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**3)/d, Ne(d, 0)
), ((-a/(3*x**6) - b/(4*x**8))/sqrt(c), True))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.17

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^7}\,dx = -\frac{1}{35}\,b\left(\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^4}-\frac{21\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^4}+\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2}{d^4}-\frac{35\sqrt{c+\frac{d}{x^2}}c^3}{d^4}\right) - \frac{1}{15}\,a\left(\frac{3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^3}-\frac{10\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^3}+\frac{15\sqrt{c+\frac{d}{x^2}}c^2}{d^3}\right)$$

```
[In] integrate((a+b/x^2)/x^7/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/35*b*(5*(c + d/x^2)^(7/2)/d^4 - 21*(c + d/x^2)^(5/2)*c/d^4 + 35*(c + d/x
^2)^(3/2)*c^2/d^4 - 35*sqrt(c + d/x^2)*c^3/d^4) - 1/15*a*(3*(c + d/x^2)^(5/
2)/d^3 - 10*(c + d/x^2)^(3/2)*c/d^3 + 15*sqrt(c + d/x^2)*c^2/d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(87) = 174$.

Time = 0.87 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.34

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^7}\,dx = \frac{16\left(70\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^8ac^{\frac{5}{2}}+210\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^6bc^{\frac{7}{2}}-175\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^6ac^{\frac{5}{2}}d-126\left(\sqrt{c}x\right.\right.}{105}$$

```
[In] integrate((a+b/x^2)/x^7/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 16/105*(70*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(5/2) + 210*(sqrt(c)*x - sqr
t(c*x^2 + d))^6*b*c^(7/2) - 175*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(5/2)*d
 - 126*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(7/2)*d + 147*(sqrt(c)*x - sqrt(
c*x^2 + d))^4*a*c^(5/2)*d^2 + 42*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(7/2)*
d^2 - 49*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(5/2)*d^3 - 6*b*c^(7/2)*d^3 +
7*a*c^(5/2)*d^4)/(((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^7*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^7}\,dx = \frac{\sqrt{c+\frac{d}{x^2}}\,(48\,b\,c^3-56\,a\,c^2\,d)}{105\,d^4} - \frac{b\sqrt{c+\frac{d}{x^2}}}{7\,d\,x^6} - \frac{\sqrt{c+\frac{d}{x^2}}\,(24\,b\,c^2-28\,a\,c\,d)}{105\,d^3\,x^2} - \frac{\sqrt{c+\frac{d}{x^2}}\,(7\,a\,d-6\,b\,c)}{35\,d^2\,x^4}$$

```
[In] int((a + b/x^2)/(x^7*(c + d/x^2)^(1/2)),x)
```

```
[Out] ((c + d/x^2)^(1/2)*(48*b*c^3 - 56*a*c^2*d))/(105*d^4) - (b*(c + d/x^2)^(1/2
))/(7*d*x^6) - ((c + d/x^2)^(1/2)*(24*b*c^2 - 28*a*c*d))/(105*d^3*x^2) - ((
c + d/x^2)^(1/2)*(7*a*d - 6*b*c))/(35*d^2*x^4)
```

3.968 $\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 82

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx=-\frac{2d(5bc-4ad)\sqrt{c+\frac{d}{x^2}}x}{15c^3}+\frac{(5bc-4ad)\sqrt{c+\frac{d}{x^2}}x^3}{15c^2}+\frac{a\sqrt{c+\frac{d}{x^2}}x^5}{5c}$$

```
[Out] -2/15*d*(-4*a*d+5*b*c)*x*(c+d/x^2)^(1/2)/c^3+1/15*(-4*a*d+5*b*c)*x^3*(c+d/x
^2)^(1/2)/c^2+1/5*a*x^5*(c+d/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {464, 277, 197}

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx=-\frac{2dx\sqrt{c+\frac{d}{x^2}}(5bc-4ad)}{15c^3}+\frac{x^3\sqrt{c+\frac{d}{x^2}}(5bc-4ad)}{15c^2}+\frac{ax^5\sqrt{c+\frac{d}{x^2}}}{5c}$$

```
[In] Int[((a + b/x^2)*x^4)/Sqrt[c + d/x^2],x]
```

```
[Out] (-2*d*(5*b*c - 4*a*d)*Sqrt[c + d/x^2]*x)/(15*c^3) + ((5*b*c - 4*a*d)*Sqrt[c
 + d/x^2]*x^3)/(15*c^2) + (a*Sqrt[c + d/x^2]*x^5)/(5*c)
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a\sqrt{c+\frac{d}{x^2}}x^5}{5c}+\frac{(5bc-4ad)\int\frac{x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx}{5c}\\
&= \frac{(5bc-4ad)\sqrt{c+\frac{d}{x^2}}x^3}{15c^2}+\frac{a\sqrt{c+\frac{d}{x^2}}x^5}{5c}-\frac{(2d(5bc-4ad))\int\frac{1}{\sqrt{c+\frac{d}{x^2}}}\,dx}{15c^2}\\
&= -\frac{2d(5bc-4ad)\sqrt{c+\frac{d}{x^2}}x}{15c^3}+\frac{(5bc-4ad)\sqrt{c+\frac{d}{x^2}}x^3}{15c^2}+\frac{a\sqrt{c+\frac{d}{x^2}}x^5}{5c}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.68

$$\int\frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{\sqrt{c+\frac{d}{x^2}}x(5bc(-2d+cx^2)+a(8d^2-4cdx^2+3c^2x^4))}{15c^3}$$

```
[In] Integrate[((a + b/x^2)*x^4)/Sqrt[c + d/x^2],x]
[Out] (Sqrt[c + d/x^2]*x*(5*b*c*(-2*d + c*x^2) + a*(8*d^2 - 4*c*d*x^2 + 3*c^2*x^4
)))/(15*c^3)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.76

method	result	size
trager	$\frac{\left(3 a \,x^{4} c^{2}-4 a c d \,x^{2}+5 b \,c^{2} x^{2}+8 a \,d^{2}-10 b c d \right) x \sqrt{-\frac{-c \,x^{2}-d}{x^{2}}}}{15 c^{3}}$	62
gosper	$\frac{\left(3 a \,x^{4} c^{2}-4 a c d \,x^{2}+5 b \,c^{2} x^{2}+8 a \,d^{2}-10 b c d \right) \left(c \,x^{2}+d \right)}{15 x \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, c^{3}}$	67
default	$\frac{\left(3 a \,x^{4} c^{2}-4 a c d \,x^{2}+5 b \,c^{2} x^{2}+8 a \,d^{2}-10 b c d \right) \left(c \,x^{2}+d \right)}{15 x \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, c^{3}}$	67
risch	$\frac{\left(3 a \,x^{4} c^{2}-4 a c d \,x^{2}+5 b \,c^{2} x^{2}+8 a \,d^{2}-10 b c d \right) \left(c \,x^{2}+d \right)}{15 x \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, c^{3}}$	67

```
[In] int((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/15*(3*a*c^2*x^4-4*a*c*d*x^2+5*b*c^2*x^2+8*a*d^2-10*b*c*d)*x/c^3*(-(-c*x^2
-d)/x^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.72

$$\int \frac{\left(a+\frac{b}{x^2}\right) x^4}{\sqrt{c+\frac{d}{x^2}}}\, dx = \frac{(3\, ac^2x^5 + (5\, bc^2 - 4\, acd)x^3 - 2\,(5\, bcd - 4\, ad^2)x)\sqrt{\frac{cx^2+d}{x^2}}}{15\, c^3}$$

```
[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x, algorithm="fricas")
[Out] 1/15*(3*a*c^2*x^5 + (5*b*c^2 - 4*a*c*d)*x^3 - 2*(5*b*c*d - 4*a*d^2)*x)*sqrt
((c*x^2 + d)/x^2)/c^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(76) = 152.

Time = 1.46 (sec) , antiderivative size = 338, normalized size of antiderivative = 4.12

$$\int \frac{\left(a+\frac{b}{x^2}\right) x^4}{\sqrt{c+\frac{d}{x^2}}}\, dx = \frac{3ac^4d^{\frac{9}{2}}x^8\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^5x^2+15c^3d^6} + \frac{2ac^3d^{\frac{11}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^5x^2+15c^3d^6} + \frac{3ac^2d^{\frac{13}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^5x^2+15c^3d^6} + \frac{12acd^{\frac{15}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^5x^2+15c^3d^6} + \frac{8ad^{\frac{17}{2}}\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^5x^2+15c^3d^6} + \frac{b\sqrt{d}x^2\sqrt{\frac{cx^2}{d}+1}}{3c} - \frac{2bd^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c^2}$$

```
[In] integrate((a+b/x**2)*x**4/(c+d/x**2)**(1/2),x)
```

```
[Out] 3*a*c**4*d**(9/2)*x**8*sqrt(c*x**2/d + 1)/(15*c**5*d**4*x**4 + 30*c**4*d**5
*x**2 + 15*c**3*d**6) + 2*a*c**3*d**(11/2)*x**6*sqrt(c*x**2/d + 1)/(15*c**5
*d**4*x**4 + 30*c**4*d**5*x**2 + 15*c**3*d**6) + 3*a*c**2*d**(13/2)*x**4*sq
rt(c*x**2/d + 1)/(15*c**5*d**4*x**4 + 30*c**4*d**5*x**2 + 15*c**3*d**6) + 1
2*a*c*d**(15/2)*x**2*sqrt(c*x**2/d + 1)/(15*c**5*d**4*x**4 + 30*c**4*d**5*x
**2 + 15*c**3*d**6) + 8*a*d**(17/2)*sqrt(c*x**2/d + 1)/(15*c**5*d**4*x**4 +
 30*c**4*d**5*x**2 + 15*c**3*d**6) + b*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/(3*c
) - 2*b*d**(3/2)*sqrt(c*x**2/d + 1)/(3*c**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.04

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3-3\sqrt{c+\frac{d}{x^2}}dx\right)b}{3\,c^2} + \frac{\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5-10\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}dx^3+15\sqrt{c+\frac{d}{x^2}}d^2x\right)a}{15\,c^3}$$

```
[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*((c + d/x^2)^(3/2)*x^3 - 3*sqrt(c + d/x^2)*d*x)*b/c^2 + 1/15*(3*(c + d/
x^2)^(5/2)*x^5 - 10*(c + d/x^2)^(3/2)*d*x^3 + 15*sqrt(c + d/x^2)*d^2*x)*a/c
^3
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.21

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{2\left(5\,bcd^{\frac{3}{2}}-4\,ad^{\frac{5}{2}}\right)\mathrm{sgn}(x)}{15\,c^3} - \frac{(bcd-ad^2)\sqrt{cx^2+d}}{c^3\mathrm{sgn}\,(x)} + \frac{3\,(cx^2+d)^{\frac{5}{2}}a+5\,(cx^2+d)^{\frac{3}{2}}bc-10\,(cx^2+d)^{\frac{3}{2}}ad}{15\,c^3\mathrm{sgn}\,(x)}$$

```
[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(5*b*c*d^(3/2) - 4*a*d^(5/2))*sgn(x)/c^3 - (b*c*d - a*d^2)*sqrt(c*x^2
+ d)/(c^3*sgn(x)) + 1/15*(3*(c*x^2 + d)^(5/2)*a + 5*(c*x^2 + d)^(3/2)*b*c -
 10*(c*x^2 + d)^(3/2)*a*d)/(c^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.70 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.65

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{x\,\sqrt{c+\frac{d}{x^2}}\,\left(3\,a\,c^2\,x^4+5\,b\,c^2\,x^2-4\,a\,c\,d\,x^2-10\,b\,c\,d+8\,a\,d^2\right)}{15\,c^3}$$

```
[In] int((x^4*(a + b/x^2))/(c + d/x^2)^(1/2),x)
[Out] (x*(c + d/x^2)^(1/2)*(8*a*d^2 + 3*a*c^2*x^4 + 5*b*c^2*x^2 - 10*b*c*d - 4*a*
c*d*x^2))/(15*c^3)
```

3.969 $\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 51

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{(3bc-2ad)\sqrt{c+\frac{d}{x^2}}\,x}{3c^2}+\frac{a\sqrt{c+\frac{d}{x^2}}\,x^3}{3c}$$

```
[Out] 1/3*(-2*a*d+3*b*c)*x*(c+d/x^2)^(1/2)/c^2+1/3*a*x^3*(c+d/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {464, 197}

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{x\sqrt{c+\frac{d}{x^2}}(3bc-2ad)}{3c^2}+\frac{ax^3\sqrt{c+\frac{d}{x^2}}}{3c}$$

```
[In] Int[((a + b/x^2)*x^2)/Sqrt[c + d/x^2],x]
```

```
[Out] ((3*b*c - 2*a*d)*Sqrt[c + d/x^2]*x)/(3*c^2) + (a*Sqrt[c + d/x^2]*x^3)/(3*c)
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{a\sqrt{c+\frac{d}{x^2}}\,x^3}{3c} + \frac{(3bc-2ad)\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}\,dx}{3c}$$
$$= \frac{(3bc-2ad)\sqrt{c+\frac{d}{x^2}}\,x}{3c^2} + \frac{a\sqrt{c+\frac{d}{x^2}}\,x^3}{3c}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{\sqrt{c+\frac{d}{x^2}}\,x(3bc-2ad+acx^2)}{3c^2}$$

```
[In] Integrate[((a + b/x^2)*x^2)/Sqrt[c + d/x^2],x]
[Out] (Sqrt[c + d/x^2]*x*(3*b*c - 2*a*d + a*c*x^2))/(3*c^2)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

method	result	size
trager	$\frac{(a c\,x^2-2ad+3bc)x\sqrt{-\frac{-c\,x^2-d}{x^2}}}{3c^2}$	39
gosper	$\frac{(a c\,x^2-2ad+3bc)\,(c\,x^2+d)}{3x\sqrt{\frac{c\,x^2+d}{x^2}}\,c^2}$	44
default	$\frac{(a c\,x^2-2ad+3bc)\,(c\,x^2+d)}{3x\sqrt{\frac{c\,x^2+d}{x^2}}\,c^2}$	44
risch	$\frac{(a c\,x^2-2ad+3bc)\,(c\,x^2+d)}{3x\sqrt{\frac{c\,x^2+d}{x^2}}\,c^2}$	44

```
[In] int((a+b/x^2)*x^2/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(a*c*x^2-2*a*d+3*b*c)*x/c^2*(-(-c*x^2-d)/x^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{\left(acx^3+(3\,bc-2\,ad)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{3\,c^2}$$

```
[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*(a*c*x^3 + (3*b*c - 2*a*d)*x)*sqrt((c*x^2 + d)/x^2)/c^2
```

Sympy [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.37

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{a\sqrt{d}x^2\sqrt{\frac{cx^2}{d}+1}}{3c} - \frac{2ad^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c^2} + \frac{b\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}{c}$$

```
[In] integrate((a+b/x**2)*x**2/(c+d/x**2)**(1/2),x)
[Out] a*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/(3*c) - 2*a*d**(3/2)*sqrt(c*x**2/d + 1)/(
3*c**2) + b*sqrt(d)*sqrt(c*x**2/d + 1)/c
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{b\sqrt{c+\frac{d}{x^2}}x}{c} + \frac{\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3 - 3\,\sqrt{c+\frac{d}{x^2}}dx\right)a}{3\,c^2}$$

```
[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(1/2),x, algorithm="maxima")
[Out] b*sqrt(c + d/x^2)*x/c + 1/3*((c + d/x^2)^(3/2)*x^3 - 3*sqrt(c + d/x^2)*d*x)
*a/c^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx=-\frac{\left(3\,bc\sqrt{d}-2\,ad^{\frac{3}{2}}\right)\mathrm{sgn}(x)}{3\,c^2}+\frac{(cx^2+d)^{\frac{3}{2}}a}{3\,c^2\mathrm{sgn}\,(x)}+\frac{\sqrt{cx^2+d}(bc-ad)}{c^2\mathrm{sgn}\,(x)}$$

```
[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*b*c*sqrt(d) - 2*a*d^(3/2))*sgn(x)/c^2 + 1/3*(c*x^2 + d)^(3/2)*a/(c^
2*sgn(x)) + sqrt(c*x^2 + d)*(b*c - a*d)/(c^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.42 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.31

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{a\,x^3\,\sqrt{c+\frac{d}{x^2}}\,\left(c-\frac{2\,d}{x^2}\right)}{3\,c^2}+\frac{b\,x\,\sqrt{\frac{c\,x^2}{d}+1}}{\sqrt{c+\frac{d}{x^2}}\,\left(\sqrt{\frac{c\,x^2}{d}+1}+1\right)}$$

```
[In] int((x^2*(a + b/x^2))/(c + d/x^2)^(1/2),x)
```

```
[Out] (a*x^3*(c + d/x^2)^(1/2)*(c - (2*d)/x^2))/(3*c^2) + (b*x*((c*x^2)/d + 1)^(1
/2))/((c + d/x^2)^(1/2)*(((c*x^2)/d + 1)^(1/2) + 1))
```

3.970 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}} dx$

Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}} dx = \frac{a\sqrt{c+\frac{d}{x^2}}x}{c} - \frac{b\text{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)}{\sqrt{d}}$$

```
[Out] -b*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(1/2)+a*x*(c+d/x^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {382, 462, 223, 212}

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}} dx = \frac{ax\sqrt{c+\frac{d}{x^2}}}{c} - \frac{b\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{\sqrt{d}}$$

```
[In] Int[(a + b/x^2)/Sqrt[c + d/x^2],x]
```

```
[Out] (a*Sqrt[c + d/x^2]*x)/c - (b*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/Sqrt[d]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{a+bx^2}{x^2\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{a\sqrt{c+\frac{d}{x^2}}\,x}{c} - b\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{a\sqrt{c+\frac{d}{x^2}}\,x}{c} - b\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right)\\
&= \frac{a\sqrt{c+\frac{d}{x^2}}\,x}{c} - \frac{b\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{\sqrt{d}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.51

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{a\sqrt{d}(d+cx^2) - bc\sqrt{d+cx^2}\,\text{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{c\sqrt{d}\sqrt{c+\frac{d}{x^2}}\,x}$$

```
[In] Integrate[(a + b/x^2)/Sqrt[c + d/x^2],x]
[Out] (a*Sqrt[d]*(d + c*x^2) - b*c*Sqrt[d + c*x^2]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d
]])/(c*Sqrt[d]*Sqrt[c + d/x^2]*x)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.55

method	result	size
default	$\frac{\sqrt{c\,x^{2}+d}\,\left(a\sqrt{c\,x^{2}+d}\,\sqrt{d}-b\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)c\right)}{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,xc\sqrt{d}}$	73

[In] int((a+b/x^2)/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] (c*x^2+d)^(1/2)*(a*(c*x^2+d)^(1/2)*d^(1/2)-b*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*c)/((c*x^2+d)/x^2)^(1/2)/x/c/d^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.79

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}\,dx = \left[\frac{2\,adx\sqrt{\frac{cx^2+d}{x^2}}+bc\sqrt{d}\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)}{2\,cd}, \frac{adx\sqrt{\frac{cx^2+d}{x^2}}+bc\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)}{cd}\right]$$

[In] integrate((a+b/x^2)/(c+d/x^2)^(1/2),x, algorithm="fricas")

[Out] [1/2*(2*a*d*x*sqrt((c*x^2 + d)/x^2) + b*c*sqrt(d)*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2))/(c*d), (a*d*x*sqrt((c*x^2 + d)/x^2) + b*c*sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)))/(c*d)]

Sympy [A] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{a\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}{c} - \frac{b\,\text{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{\sqrt{d}}$$

[In] integrate((a+b/x**2)/(c+d/x**2)**(1/2),x)

[Out] a*sqrt(d)*sqrt(c*x**2/d + 1)/c - b*asinh(sqrt(d)/(sqrt(c)*x))/sqrt(d)

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{a\sqrt{c+\frac{d}{x^2}}\,x}{c}+\frac{b\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{2\sqrt{d}}$$

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(1/2),x, algorithm="maxima")
[Out] a*sqrt(c + d/x^2)*x/c + 1/2*b*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d
/x^2)*x + sqrt(d)))/sqrt(d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(39) = 78$.

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.70

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}\,dx=-\frac{\left(bc\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)+a\sqrt{-d}\sqrt{d}\right)\operatorname{sgn}(x)}{c\sqrt{-d}}+\frac{\frac{b\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}}+\frac{\sqrt{cx^2+d}a}{c}}{\operatorname{sgn}(x)}$$

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(1/2),x, algorithm="giac")
[Out] -(b*c*arctan(sqrt(d)/sqrt(-d)) + a*sqrt(-d)*sqrt(d))*sgn(x)/(c*sqrt(-d)) +
(b*arctan(sqrt(c*x^2 + d)/sqrt(-d))/sqrt(-d) + sqrt(c*x^2 + d)*a/c)/sgn(x)
```

Mupad [B] (verification not implemented)

Time = 9.46 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.38

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{a\,x\,\sqrt{\frac{c\,x^2}{d}+1}}{\sqrt{c+\frac{d}{x^2}}\,\left(\sqrt{\frac{c\,x^2}{d}+1}+1\right)}-\frac{b\,\ln\left(\sqrt{c+\frac{d}{x^2}}+\frac{\sqrt{d}}{x}\right)}{\sqrt{d}}$$

```
[In] int((a + b/x^2)/(c + d/x^2)^(1/2),x)
[Out] (a*x*((c*x^2)/d + 1)^(1/2))/((c + d/x^2)^(1/2)*(((c*x^2)/d + 1)^(1/2) + 1))
 - (b*log((c + d/x^2)^(1/2) + d^(1/2)/x))/d^(1/2)
```

3.971 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 61

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^2} dx = -\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx} + \frac{(bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)}{2d^{3/2}}$$

```
[Out] 1/2*(-2*a*d+b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(3/2)-1/2*b*(c+d/x^2)
^(1/2)/d/x
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {470, 342, 223, 212}

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^2} dx = \frac{(bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2d^{3/2}} - \frac{b\sqrt{c+\frac{d}{x^2}}}{2dx}$$

```
[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^2),x]
```

```
[Out] -1/2*(b*Sqrt[c + d/x^2])/(d*x) + ((b*c - 2*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d
/x^2]*x)])/(2*d^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx}+\frac{(-bc+2ad)\int\frac{1}{\sqrt{c+\frac{d}{x^2}}\,x^2}\,dx}{2d}\\
&= -\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx}-\frac{(-bc+2ad)\text{Subst}\left(\int\frac{1}{\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right)}{2d}\\
&= -\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx}-\frac{(-bc+2ad)\text{Subst}\left(\int\frac{1}{1-dx^2}\,dx,x,\frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{2d}\\
&= -\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{2d^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.31

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^2}\,dx = \frac{-b\sqrt{d}(d+cx^2)+(bc-2ad)x^2\sqrt{d+cx^2}\text{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{c+\frac{d}{x^2}}x^3}$$

```
[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^2),x]
[Out] (-(b*Sqrt[d]*(d + c*x^2)) + (b*c - 2*a*d)*x^2*Sqrt[d + c*x^2]*ArcTanh[Sqrt[
d + c*x^2]/Sqrt[d]])/(2*d^(3/2)*Sqrt[c + d/x^2]*x^3)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.52

method	result	size
risch	$-\frac{b(c\,x^2+d)}{2d\,x^3\sqrt{\frac{c\,x^2+d}{x^2}}}-\frac{(2ad-bc)\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)\sqrt{c\,x^2+d}}{2d^{\frac{3}{2}}\sqrt{\frac{c\,x^2+d}{x^2}}\,x}$	93
default	$-\frac{\sqrt{c\,x^2+d}\left(2a\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)d^2x^2-\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)bcd\,x^2+\sqrt{c\,x^2+d}\,d^{\frac{3}{2}}b\right)}{2\sqrt{\frac{c\,x^2+d}{x^2}}\,x^3d^{\frac{5}{2}}}$	105

```
[In] int((a+b/x^2)/x^2/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2/d*b*(c*x^2+d)/x^3/((c*x^2+d)/x^2)^(1/2)-1/2*(2*a*d-b*c)/d^(3/2)*ln((2*
d+2*d^(1/2)*(c*x^2+d)^(1/2))/x)/((c*x^2+d)/x^2)^(1/2)/x*(c*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.36

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^2}\,dx = \left[-\frac{(bc-2\,ad)\sqrt{d}x\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,bd\sqrt{\frac{cx^2+d}{x^2}}}{4\,d^2x}, -\frac{(bc-2\,ad)\sqrt{-d}x\arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+bd\sqrt{\frac{cx^2+d}{x^2}}}{2\,d^2x}\right]$$

```
[In] integrate((a+b/x^2)/x^2/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*c - 2*a*d)*sqrt(d)*x*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x
^2) + 2*d)/x^2) + 2*b*d*sqrt((c*x^2 + d)/x^2))/(d^2*x), -1/2*((b*c - 2*a*d)
*sqrt(-d)*x*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + b*d*sqrt
((c*x^2 + d)/x^2))/(d^2*x)]
```

Sympy [A] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.08

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^2}\,dx = -\frac{a\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{\sqrt{d}} - \frac{b\sqrt{c}\sqrt{1+\frac{d}{cx^2}}}{2dx} + \frac{bc\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2d^{\frac{3}{2}}}$$

```
[In] integrate((a+b/x**2)/x**2/(c+d/x**2)**(1/2),x)
```

```
[Out] -a*asinh(sqrt(d)/(sqrt(c)*x))/sqrt(d) - b*sqrt(c)*sqrt(1 + d/(c*x**2))/(2*d
*x) + b*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*d**(3/2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(49) = 98.

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.98

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^2}\,dx = -\frac{1}{4}\left(\frac{2\sqrt{c+\frac{d}{x^2}}cx}{\left(c+\frac{d}{x^2}\right)dx^2-d^2} + \frac{c\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}\right)b + \frac{a\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{2\sqrt{d}}$$

```
[In] integrate((a+b/x^2)/x^2/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(c + d/x^2)*c*x/((c + d/x^2)*d*x^2 - d^2) + c*log((sqrt(c + d/x
^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(3/2))*b + 1/2*a*log((sqr
t(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/sqrt(d)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^2}\,dx = -\frac{\frac{(bc^2-2\,acd)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}d}+\frac{\sqrt{cx^2+d}bc}{dx^2}}{2\,c\mathrm{sgn}\left(x\right)}$$

```
[In] integrate((a+b/x^2)/x^2/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*((b*c^2 - 2*a*c*d)*arctan(sqrt(c*x^2 + d)/sqrt(-d))/(sqrt(-d)*d) + sqr
t(c*x^2 + d)*b*c/(d*x^2))/(c*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 10.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.54

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^2}\,dx = \begin{cases} -\frac{3\,a\,x^2+b}{3\,\sqrt{c}\,x^3} & \text{if}\ \ d=0 \\ \frac{b\,c\,\ln\left(2\,\sqrt{c+\frac{d}{x^2}}+\frac{2\,\sqrt{d}}{x}\right)}{2\,d^{3/2}}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{2\,d\,x}-\frac{a\,\ln\left(\sqrt{c+\frac{d}{x^2}}+\frac{\sqrt{d}}{x}\right)}{\sqrt{d}} & \text{if}\ \ d\ne 0 \end{cases}$$

```
[In] int((a + b/x^2)/(x^2*(c + d/x^2)^(1/2)),x)
```

```
[Out] piecewise(d == 0, -(b + 3*a*x^2)/(3*c^(1/2)*x^3), d ~= 0, - (a*log((c + d/x
^2)^(1/2) + d^(1/2)/x))/d^(1/2) - (b*(c + d/x^2)^(1/2))/(2*d*x) + (b*c*log(
2*(c + d/x^2)^(1/2) + (2*d^(1/2))/x))/(2*d^(3/2)))
```

3.972 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^4} dx$

Optimal result

Integrand size = 22, antiderivative size = 93

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^4} dx = -\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}+\frac{(3bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^2x}-\frac{c(3bc-4ad)\operatorname{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)}{8d^{5/2}}$$

```
[Out] -1/8*c*(-4*a*d+3*b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(5/2)-1/4*b*(c+d
/x^2)^(1/2)/d/x^3+1/8*(-4*a*d+3*b*c)*(c+d/x^2)^(1/2)/d^2/x
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {470, 342, 327, 223, 212}

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^4} dx = -\frac{c(3bc-4ad)\operatorname{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{8d^{5/2}}+\frac{\sqrt{c+\frac{d}{x^2}}(3bc-4ad)}{8d^2x}-\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}$$

```
[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^4),x]
```

```
[Out] -1/4*(b*Sqrt[c + d/x^2])/(d*x^3) + ((3*b*c - 4*a*d)*Sqrt[c + d/x^2])/(8*d^2
*x) - (c*(3*b*c - 4*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(8*d^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}+\frac{(-3bc+4ad)\int\frac{1}{\sqrt{c+\frac{d}{x^2}}\,x^4}\,dx}{4d}\\
&= -\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}-\frac{(-3bc+4ad)\mathrm{Subst}\left(\int\frac{x^2}{\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right)}{4d}\\
&= -\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}+\frac{(3bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^2x}-\frac{(c(3bc-4ad))\mathrm{Subst}\left(\int\frac{1}{\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right)}{8d^2}\\
&= -\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}+\frac{(3bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^2x}-\frac{(c(3bc-4ad))\mathrm{Subst}\left(\int\frac{1}{1-dx^2}\,dx,x,\frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{8d^2}\\
&= -\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}+\frac{(3bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^2x}-\frac{c(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{8d^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.09

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^4}\,dx = \frac{-\sqrt{d}(d+cx^2)\left(2bd-3bcx^2+4adx^2\right)-c(3bc-4ad)x^4\sqrt{d+cx^2}\operatorname{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{c+\frac{d}{x^2}}x^5}$$

[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^4),x]

[Out] (-(Sqrt[d]*(d + c*x^2)*(2*b*d - 3*b*c*x^2 + 4*a*d*x^2)) - c*(3*b*c - 4*a*d)*x^4*Sqrt[d + c*x^2]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/(8*d^(5/2)*Sqrt[c + d/x^2]*x^5)

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.20

method	result
risch	$-\frac{(c\,x^2+d)\left(4ad\,x^2-3cb\,x^2+2bd\right)}{8d^2x^5\sqrt{\frac{c\,x^2+d}{x^2}}}+\frac{c(4ad-3bc)\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)\sqrt{c\,x^2+d}}{8d^{\frac{5}{2}}\sqrt{\frac{c\,x^2+d}{x^2}}\,x}$
default	$-\frac{\sqrt{c\,x^2+d}\left(-4\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)ac\,d^2x^4+3\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)b\,c^2d\,x^4+4d^{\frac{5}{2}}\sqrt{c\,x^2+d}\,a\,x^2-3d^{\frac{3}{2}}\sqrt{c\,x^2+d}\,bc\,x^2+2d^{\frac{5}{2}}\sqrt{c\,x^2+d}}{8\sqrt{\frac{c\,x^2+d}{x^2}}\,x^5d^{\frac{7}{2}}}$

[In] int((a+b/x^2)/x^4/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/8*(c*x^2+d)*(4*a*d*x^2-3*b*c*x^2+2*b*d)/d^2/x^5/((c*x^2+d)/x^2)^(1/2)+1/8*c*(4*a*d-3*b*c)/d^(5/2)*ln((2*d+2*d^(1/2)*(c*x^2+d)^(1/2))/x)/((c*x^2+d)/x^2)^(1/2)/x*(c*x^2+d)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.16

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^4}\,dx = \left[-\frac{(3\,bc^2-4\,acd)\sqrt{d}x^3\log\left(-\frac{cx^2+2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,(2\,bd^2-(3\,bcd-4\,ad^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{16\,d^3x^3}, \frac{(3\,bc^2-4\,acd}{}\right.$$

[In] integrate((a+b/x^2)/x^4/(c+d/x^2)^(1/2),x, algorithm="fricas")

[Out] [-1/16*((3*b*c^2 - 4*a*c*d)*sqrt(d)*x^3*log(-(c*x^2 + 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2) + 2*(2*b*d^2 - (3*b*c*d - 4*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^3*x^3), 1/8*((3*b*c^2 - 4*a*c*d)*sqrt(-d)*x^3*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (2*b*d^2 - (3*b*c*d - 4*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^3*x^3)]

Sympy [A] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.61

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^4}\,dx = -\frac{a\sqrt{c}\sqrt{1+\frac{d}{cx^2}}}{2dx}+\frac{ac\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2d^{\frac{3}{2}}}+\frac{3bc^{\frac{3}{2}}}{8d^2x\sqrt{1+\frac{d}{cx^2}}}+\frac{b\sqrt{c}}{8dx^3\sqrt{1+\frac{d}{cx^2}}}-\frac{3bc^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{8d^{\frac{5}{2}}}-\frac{b}{4\sqrt{c}x^5\sqrt{1+\frac{d}{cx^2}}}$$

[In] integrate((a+b/x**2)/x**4/(c+d/x**2)**(1/2),x)

[Out] -a*sqrt(c)*sqrt(1 + d/(c*x**2))/(2*d*x) + a*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*d**(3/2)) + 3*b*c**(3/2)/(8*d**2*x*sqrt(1 + d/(c*x**2))) + b*sqrt(c)/(8*d*x**3*sqrt(1 + d/(c*x**2))) - 3*b*c**2*asinh(sqrt(d)/(sqrt(c)*x))/(8*d**(5/2)) - b/(4*sqrt(c)*x**5*sqrt(1 + d/(c*x**2)))

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(77) = 154$.

Time = 0.28 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.15

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^4}\,dx = -\frac{1}{4}\left(\frac{2\sqrt{c+\frac{d}{x^2}}cx}{\left(c+\frac{d}{x^2}\right)dx^2-d^2}+\frac{c\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}\right)a + \frac{1}{16}b\left(\frac{3\,c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{5}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2x^3-5\sqrt{c+\frac{d}{x^2}}c^2dx\right)}{\left(c+\frac{d}{x^2}\right)^2d^2x^4-2\left(c+\frac{d}{x^2}\right)d^3x^2+d^4}\right)$$

```
[In] integrate((a+b/x^2)/x^4/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(c + d/x^2)*c*x/((c + d/x^2)*d*x^2 - d^2) + c*log((sqrt(c + d/x
^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(3/2))*a + 1/16*b*(3*c^2*
log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(5/2) +
2*(3*(c + d/x^2)^(3/2)*c^2*x^3 - 5*sqrt(c + d/x^2)*c^2*d*x)/((c + d/x^2)^2*
d^2*x^4 - 2*(c + d/x^2)*d^3*x^2 + d^4))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.34

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^4}\,dx = \frac{\frac{(3\,bc^3-4\,ac^2d)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}d^2}+\frac{3\,(cx^2+d)^{\frac{3}{2}}bc^3-4\,(cx^2+d)^{\frac{3}{2}}ac^2d-5\,\sqrt{cx^2+d}bc^3d+4\,\sqrt{cx^2+d}ac^2d^2}{c^2d^2x^4}}{8\,c\mathrm{sgn}\left(x\right)}$$

```
[In] integrate((a+b/x^2)/x^4/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*((3*b*c^3 - 4*a*c^2*d)*arctan(sqrt(c*x^2 + d)/sqrt(-d))/(sqrt(-d)*d^2)
+ (3*(c*x^2 + d)^(3/2)*b*c^3 - 4*(c*x^2 + d)^(3/2)*a*c^2*d - 5*sqrt(c*x^2 +
 d)*b*c^3*d + 4*sqrt(c*x^2 + d)*a*c^2*d^2)/(c^2*d^2*x^4))/(c*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^4}\,dx=\int \frac{a+\frac{b}{x^2}}{x^4\,\sqrt{c+\frac{d}{x^2}}}\,dx$$

```
[In] int((a + b/x^2)/(x^4*(c + d/x^2)^(1/2)),x)
[Out] int((a + b/x^2)/(x^4*(c + d/x^2)^(1/2)), x)
```

3.973 $\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 118

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\frac{3d(4bc-5ad)}{8c^3\sqrt{c+\frac{d}{x^2}}}+\frac{(4bc-5ad)x^2}{8c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}}-\frac{3d(4bc-5ad)\operatorname{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{7/2}}$$

```
[Out] -3/8*d*(-5*a*d+4*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(7/2)+3/8*d*(-5*a*
d+4*b*c)/c^3/(c+d/x^2)^(1/2)+1/8*(-5*a*d+4*b*c)*x^2/c^2/(c+d/x^2)^(1/2)+1/4
*a*x^4/c/(c+d/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {457, 79, 44, 53, 65, 214}

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=-\frac{3d(4bc-5ad)\operatorname{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{7/2}}+\frac{3d(4bc-5ad)}{8c^3\sqrt{c+\frac{d}{x^2}}}+\frac{x^2(4bc-5ad)}{8c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}}$$

```
[In] Int[((a + b/x^2)*x^3)/(c + d/x^2)^(3/2),x]
```

```
[Out] (3*d*(4*b*c - 5*a*d))/(8*c^3*Sqrt[c + d/x^2]) + ((4*b*c - 5*a*d)*x^2)/(8*c^
2*Sqrt[c + d/x^2]) + (a*x^4)/(4*c*Sqrt[c + d/x^2]) - (3*d*(4*b*c - 5*a*d)*A
rcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(8*c^(7/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{a+bx}{x^3(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= \frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}} - \frac{\left(2bc-\frac{5ad}{2}\right)\text{Subst}\left(\int \frac{1}{x^2(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)}{4c}\\
&= \frac{(4bc-5ad)x^2}{8c^2\sqrt{c+\frac{d}{x^2}}} + \frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}} + \frac{(3d(4bc-5ad))\text{Subst}\left(\int \frac{1}{x(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)}{16c^2}\\
&= \frac{3d(4bc-5ad)}{8c^3\sqrt{c+\frac{d}{x^2}}} + \frac{(4bc-5ad)x^2}{8c^2\sqrt{c+\frac{d}{x^2}}} + \frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}} + \frac{(3d(4bc-5ad))\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)}{16c^3}\\
&= \frac{3d(4bc-5ad)}{8c^3\sqrt{c+\frac{d}{x^2}}} + \frac{(4bc-5ad)x^2}{8c^2\sqrt{c+\frac{d}{x^2}}} + \frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}}\\
&\quad + \frac{(3(4bc-5ad))\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{8c^3}\\
&= \frac{3d(4bc-5ad)}{8c^3\sqrt{c+\frac{d}{x^2}}} + \frac{(4bc-5ad)x^2}{8c^2\sqrt{c+\frac{d}{x^2}}} + \frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}} - \frac{3d(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.35

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{\sqrt{c}x(4bc(3d+cx^2)+a(-15d^2-5cdx^2+2c^2x^4))+24bcd\sqrt{d+cx^2}\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{d}-\sqrt{d+cx^2}}\right)}{8c^{7/2}\sqrt{c+\frac{d}{x^2}}x}$$

```
[In] Integrate[((a + b/x^2)*x^3)/(c + d/x^2)^(3/2),x]
[Out] (Sqrt[c]*x*(4*b*c*(3*d + c*x^2) + a*(-15*d^2 - 5*c*d*x^2 + 2*c^2*x^4)) + 24
*b*c*d*Sqrt[d + c*x^2]*ArcTanh[(Sqrt[c]*x)/(Sqrt[d] - Sqrt[d + c*x^2])] + 3
0*a*d^2*Sqrt[d + c*x^2]*ArcTanh[(Sqrt[c]*x)/(-Sqrt[d] + Sqrt[d + c*x^2])])/
(8*c^(7/2)*Sqrt[c + d/x^2]*x)
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.19

method	result
default	$\frac{(c\,x^2+d)\left(2c^{\frac{7}{2}}\,a\,x^5-5c^{\frac{5}{2}}\,ad\,x^3+4c^{\frac{7}{2}}\,b\,x^3-15c^{\frac{3}{2}}\,a\,d^2x+12c^{\frac{5}{2}}\,bdx+15\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)\sqrt{c\,x^2+d}\,ac\,d^2-12\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)\sqrt{c\,x^2}\right.}{8\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}x^3c^{\frac{9}{2}}}$
risch	$\frac{(2ac\,x^2-7ad+4bc)\,(c\,x^2+d)}{8c^3\sqrt{\frac{c\,x^2+d}{x^2}}}+\frac{d\left(\frac{7adx}{\sqrt{c\,x^2+d}}-\frac{4bcx}{\sqrt{c\,x^2+d}}+(15acd-12b\,c^2)\left(-\frac{x}{c\sqrt{c\,x^2+d}}+\frac{\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)}{c^{\frac{3}{2}}}\right)\right)\sqrt{c\,x^2+d}}{8c^3\sqrt{\frac{c\,x^2+d}{x^2}}\,x}$

[In] int((a+b/x^2)*x^3/(c+d/x^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/8*(c*x^2+d)*(2*c^(7/2)*a*x^5-5*c^(5/2)*a*d*x^3+4*c^(7/2)*b*x^3-15*c^(3/2)*a*d^2*x+12*c^(5/2)*b*d*x+15*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*(c*x^2+d)^(1/2)*a*c*d^2-12*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*(c*x^2+d)^(1/2)*b*c^2*d)/((c*x^2+d)/x^2)^(3/2)/x^3/c^(9/2)

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.58

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \left[-\frac{3\,(4\,bcd^2-5\,ad^3+(4\,bc^2d-5\,acd^2)x^2)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\,(}{16\,(c^5x^2+c^4d)}\right.$$

[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(3/2),x, algorithm="fricas")

[Out] [-1/16*(3*(4*b*c*d^2 - 5*a*d^3 + (4*b*c^2*d - 5*a*c*d^2)*x^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d) - 2*(2*a*c^3*x^6 + (4*b*c^3 - 5*a*c^2*d)*x^4 + 3*(4*b*c^2*d - 5*a*c*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/(c^5*x^2 + c^4*d), 1/8*(3*(4*b*c*d^2 - 5*a*d^3 + (4*b*c^2*d - 5*a*c*d^2)*x^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (2*a*c^3*x^6 + (4*b*c^3 - 5*a*c^2*d)*x^4 + 3*(4*b*c^2*d - 5*a*c*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/(c^5*x^2 + c^4*d)]

Sympy [A] (verification not implemented)

Time = 34.41 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.50

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = a\left(\frac{x^5}{4c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}} - \frac{5\sqrt{d}x^3}{8c^2\sqrt{\frac{cx^2}{d}+1}} - \frac{15d^{\frac{3}{2}}x}{8c^3\sqrt{\frac{cx^2}{d}+1}} + \frac{15d^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{8c^{\frac{7}{2}}}\right) + b\left(\frac{x^3}{2c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}} + \frac{3\sqrt{d}x}{2c^2\sqrt{\frac{cx^2}{d}+1}} - \frac{3d\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2c^{\frac{5}{2}}}\right)$$

```
[In] integrate((a+b/x**2)*x**3/(c+d/x**2)**(3/2),x)
```

```
[Out] a*(x**5/(4*c*sqrt(d)*sqrt(c*x**2/d + 1)) - 5*sqrt(d)*x**3/(8*c**2*sqrt(c*x*
*2/d + 1)) - 15*d**(3/2)*x/(8*c**3*sqrt(c*x**2/d + 1)) + 15*d**2*asinh(sqrt
(c)*x/sqrt(d))/(8*c**(7/2))) + b*(x**3/(2*c*sqrt(d)*sqrt(c*x**2/d + 1)) + 3
*sqrt(d)*x/(2*c**2*sqrt(c*x**2/d + 1)) - 3*d*asinh(sqrt(c)*x/sqrt(d))/(2*c*
*(5/2)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(98) = 196.

Time = 0.29 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.82

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = -\frac{1}{16}a\left(\frac{2\left(15\left(c+\frac{d}{x^2}\right)^2d^2 - 25\left(c+\frac{d}{x^2}\right)cd^2 + 8c^2d^2\right)}{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3 - 2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^4 + \sqrt{c+\frac{d}{x^2}}c^5} + \frac{15d^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{7}{2}}}\right) + \frac{1}{4}b\left(\frac{2\left(3\left(c+\frac{d}{x^2}\right)d - 2cd\right)}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2 - \sqrt{c+\frac{d}{x^2}}c^3} + \frac{3d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right)$$

```
[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/16*a*(2*(15*(c + d/x^2)^2*d^2 - 25*(c + d/x^2)*c*d^2 + 8*c^2*d^2)/((c +
d/x^2)^(5/2)*c^3 - 2*(c + d/x^2)^(3/2)*c^4 + sqrt(c + d/x^2)*c^5) + 15*d^2*
log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(7/2)) + 1/4
*b*(2*(3*(c + d/x^2)*d - 2*c*d)/((c + d/x^2)^(3/2)*c^2 - sqrt(c + d/x^2)*c^
3) + 3*d*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(5/
2))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.22

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{\left(x^2\left(\frac{2\,ax^2}{c\mathrm{sgn}(x)}+\frac{4\,bc^4\mathrm{sgn}(x)-5\,ac^3d\mathrm{sgn}(x)}{c^5}\right)+\frac{3\,(4\,bc^3d\mathrm{sgn}(x)-5\,ac^2d^2\mathrm{sgn}(x))}{c^5}\right)x}{8\,\sqrt{cx^2+d}} - \frac{3\,(4\,bcd\log\left(|d|\right)-5\,ad^2\log\left(|d|\right))\mathrm{sgn}(x)}{16\,c^{\frac{7}{2}}} + \frac{3\,(4\,bcd-5\,ad^2)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+d}\right|\right)}{8\,c^{\frac{7}{2}}\mathrm{sgn}\left(x\right)}$$

```
[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(x^2*(2*a*x^2/(c*sgn(x)) + (4*b*c^4*sgn(x) - 5*a*c^3*d*sgn(x))/c^5) + 3
*(4*b*c^3*d*sgn(x) - 5*a*c^2*d^2*sgn(x))/c^5)*x/sqrt(c*x^2 + d) - 3/16*(4*b
*c*d*log(abs(d)) - 5*a*d^2*log(abs(d)))*sgn(x)/c^(7/2) + 3/8*(4*b*c*d - 5*a
*d^2)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)))/(c^(7/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 10.34 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.14

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{a\,x^4}{4\,c\,\sqrt{c+\frac{d}{x^2}}} - \frac{15\,a\,d^2}{8\,c^3\,\sqrt{c+\frac{d}{x^2}}} + \frac{b\,x^2}{2\,c\,\sqrt{c+\frac{d}{x^2}}} - \frac{3\,b\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,c^{5/2}} + \frac{15\,a\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,c^{7/2}} + \frac{3\,b\,d}{2\,c^2\,\sqrt{c+\frac{d}{x^2}}} - \frac{5\,a\,d\,x^2}{8\,c^2\,\sqrt{c+\frac{d}{x^2}}}$$

```
[In] int((x^3*(a + b/x^2))/(c + d/x^2)^(3/2),x)
```

```
[Out] (a*x^4)/(4*c*(c + d/x^2)^(1/2)) - (15*a*d^2)/(8*c^3*(c + d/x^2)^(1/2)) + (b
*x^2)/(2*c*(c + d/x^2)^(1/2)) - (3*b*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(2
*c^(5/2)) + (15*a*d^2*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(8*c^(7/2)) + (3*b*
d)/(2*c^2*(c + d/x^2)^(1/2)) - (5*a*d*x^2)/(8*c^2*(c + d/x^2)^(1/2))
```

3.974 $\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 86

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=-\frac{2bc-3ad}{2c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^2}{2c\sqrt{c+\frac{d}{x^2}}}+\frac{(2bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{5/2}}$$

```
[Out] 1/2*(-3*a*d+2*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(5/2)+1/2*(3*a*d-2*b*
c)/c^2/(c+d/x^2)^(1/2)+1/2*a*x^2/c/(c+d/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {457, 79, 53, 65, 214}

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\frac{(2bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{5/2}}-\frac{2bc-3ad}{2c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^2}{2c\sqrt{c+\frac{d}{x^2}}}$$

```
[In] Int[((a + b/x^2)*x)/(c + d/x^2)^(3/2),x]
```

```
[Out] -1/2*(2*b*c - 3*a*d)/(c^2*Sqrt[c + d/x^2]) + (a*x^2)/(2*c*Sqrt[c + d/x^2])
+ ((2*b*c - 3*a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(2*c^(5/2))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{a+bx}{x^2(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{ax^2}{2c\sqrt{c+\frac{d}{x^2}}} - \frac{\left(bc-\frac{3ad}{2}\right)\text{Subst}\left(\int \frac{1}{x(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)}{2c}$$

$$=-\frac{2bc-3ad}{2c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^2}{2c\sqrt{c+\frac{d}{x^2}}}-\frac{(2bc-3ad)\text{Subst}\left(\int\frac{1}{x\sqrt{c+dx}}\,dx,x,\frac{1}{x^2}\right)}{4c^2}$$

$$=-\frac{2bc-3ad}{2c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^2}{2c\sqrt{c+\frac{d}{x^2}}}-\frac{(2bc-3ad)\text{Subst}\left(\int\frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+\frac{d}{x^2}}\right)}{2c^2d}$$

$$=-\frac{2bc-3ad}{2c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^2}{2c\sqrt{c+\frac{d}{x^2}}}+\frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{5/2}}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.62

$$\int\frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\frac{6ad\sqrt{d+cx^2}\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{d}-\sqrt{d+cx^2}}\right)+\sqrt{c}\left(-2bcx+3adx+acx^3+4b\sqrt{c}\sqrt{d+cx^2}\text{arctanh}\right.}{2c^{5/2}\sqrt{c+\frac{d}{x^2}}x}$$

```
[In] Integrate[((a + b/x^2)*x)/(c + d/x^2)^(3/2),x]
```

```
[Out] (6*a*d*Sqrt[d + c*x^2]*ArcTanh[(Sqrt[c]*x)/(Sqrt[d] - Sqrt[d + c*x^2])] + S
qrt[c]*(-2*b*c*x + 3*a*d*x + a*c*x^3 + 4*b*Sqrt[c]*Sqrt[d + c*x^2]*ArcTanh[
(Sqrt[c]*x)/(-Sqrt[d] + Sqrt[d + c*x^2])]))/(2*c^(5/2)*Sqrt[c + d/x^2]*x)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.34

method	result	size
default	$-\frac{(c\,x^2+d)\left(-c^{\frac{5}{2}}a\,x^3-3c^{\frac{3}{2}}adx+2c^{\frac{5}{2}}bx+3\sqrt{c\,x^2+d}\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)acd-2\sqrt{c\,x^2+d}\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)b\,c^2\right)}{2\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}x^3c^{\frac{7}{2}}}$	115
risch	$\frac{a(c\,x^2+d)}{2c^2\sqrt{\frac{c\,x^2+d}{x^2}}}-\frac{\left(\frac{adx}{\sqrt{c\,x^2+d}}+(3acd-2b\,c^2)\left(-\frac{x}{c\sqrt{c\,x^2+d}}+\frac{\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)}{c^{\frac{3}{2}}}\right)\right)\sqrt{c\,x^2+d}}{2c^2\sqrt{\frac{c\,x^2+d}{x^2}}\,x}$	121

```
[In] int((a+b/x^2)*x/(c+d/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(c*x^2+d)*(-c^(5/2)*a*x^3-3*c^(3/2)*a*d*x+2*c^(5/2)*b*x+3*(c*x^2+d)^(1
/2)*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*a*c*d-2*(c*x^2+d)^(1/2)*ln(c^(1/2)*x+(c*x
^2+d)^(1/2))*b*c^2)/((c*x^2+d)/x^2)^(3/2)/x^3/c^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.90

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \left[-\frac{(2\,bcd-3\,ad^2+(2\,bc^2-3\,acd)x^2)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\,(ac^2x^4 \ldots}{4\,(c^4x^2+c^3d)}, -\frac{(2\,bcd-3\,ad^2+(2\,bc^2-3\,acd)x^2)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)-(ac^2x^4-(2\,bc^2-3\,acd)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{2\,(c^4x^2+c^3d)}\right]$$

```
[In] integrate((a+b/x^2)*x/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*b*c*d - 3*a*d^2 + (2*b*c^2 - 3*a*c*d)*x^2)*sqrt(c)*log(-2*c*x^2 +
 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d) - 2*(a*c^2*x^4 - (2*b*c^2 - 3*a*c
*d)*x^2)*sqrt((c*x^2 + d)/x^2))/(c^4*x^2 + c^3*d), -1/2*((2*b*c*d - 3*a*d^2
 + (2*b*c^2 - 3*a*c*d)*x^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x
^2)/(c*x^2 + d)) - (a*c^2*x^4 - (2*b*c^2 - 3*a*c*d)*x^2)*sqrt((c*x^2 + d)/x
^2))/(c^4*x^2 + c^3*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(73) = 146$.

Time = 16.94 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.07

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = a\left(\frac{x^3}{2c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{3\sqrt{d}x}{2c^2\sqrt{\frac{cx^2}{d}+1}}-\frac{3d\,\mathrm{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2c^{\frac{5}{2}}}\right) + b\left(-\frac{2c^3x^2\sqrt{1+\frac{d}{cx^2}}}{2c^{\frac{9}{2}}x^2+2c^{\frac{7}{2}}d}-\frac{c^3x^2\log\left(\frac{d}{cx^2}\right)}{2c^{\frac{9}{2}}x^2+2c^{\frac{7}{2}}d}+\frac{2c^3x^2\log\left(\sqrt{1+\frac{d}{cx^2}}+1\right)}{2c^{\frac{9}{2}}x^2+2c^{\frac{7}{2}}d} - \frac{c^2d\log\left(\frac{d}{cx^2}\right)}{2c^{\frac{9}{2}}x^2+2c^{\frac{7}{2}}d}+\frac{2c^2d\log\left(\sqrt{1+\frac{d}{cx^2}}+1\right)}{2c^{\frac{9}{2}}x^2+2c^{\frac{7}{2}}d}\right)$$

```
[In] integrate((a+b/x**2)*x/(c+d/x**2)**(3/2),x)
```

```
[Out] a*(x**3/(2*c*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*sqrt(d)*x/(2*c**2*sqrt(c*x**2/
d + 1)) - 3*d*asinh(sqrt(c)*x/sqrt(d))/(2*c**(5/2))) + b*(-2*c**3*x**2*sqrt
```

```
(1 + d/(c*x**2))/(2*c**(9/2)*x**2 + 2*c**(7/2)*d) - c**3*x**2*log(d/(c*x**2
))/(2*c**(9/2)*x**2 + 2*c**(7/2)*d) + 2*c**3*x**2*log(sqrt(1 + d/(c*x**2))
+ 1)/(2*c**(9/2)*x**2 + 2*c**(7/2)*d) - c**2*d*log(d/(c*x**2))/(2*c**(9/2)*
x**2 + 2*c**(7/2)*d) + 2*c**2*d*log(sqrt(1 + d/(c*x**2)) + 1)/(2*c**(9/2)*x
**2 + 2*c**(7/2)*d))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(70) = 140$.

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.67

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{1}{4}\,a\left(\frac{2\left(3\left(c+\frac{d}{x^2}\right)d-2\,cd\right)}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2-\sqrt{c+\frac{d}{x^2}}\,c^3}+\frac{3\,d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right) - \frac{1}{2}\,b\left(\frac{\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2}{\sqrt{c+\frac{d}{x^2}}\,c}\right)$$

```
[In] integrate((a+b/x^2)*x/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*a*(2*(3*(c + d/x^2)*d - 2*c*d)/((c + d/x^2)^(3/2)*c^2 - sqrt(c + d/x^2)
*c^3) + 3*d*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^
(5/2)) - 1/2*b*(log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c))
)/c^(3/2) + 2/(sqrt(c + d/x^2)*c))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.22

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{x\left(\frac{ax^2}{c\text{sgn}(x)}-\frac{2\,bc^2\text{sgn}(x)-3\,acd\text{sgn}(x)}{c^3}\right)}{2\sqrt{cx^2+d}} + \frac{(2\,bc\log\left(|d|\right)-3\,ad\log\left(|d|\right))\text{sgn}(x)}{4\,c^{\frac{5}{2}}} - \frac{(2\,bc-3\,ad)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+d}\right|\right)}{2\,c^{\frac{5}{2}}\text{sgn}\left(x\right)}$$

```
[In] integrate((a+b/x^2)*x/(c+d/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*x*(a*x^2/(c*sgn(x)) - (2*b*c^2*sgn(x) - 3*a*c*d*sgn(x))/c^3)/sqrt(c*x^2
 + d) + 1/4*(2*b*c*log(abs(d)) - 3*a*d*log(abs(d)))*sgn(x)/c^(5/2) - 1/2*(2
*b*c - 3*a*d)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)))/(c^(5/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.82 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}-\frac{b}{c\,\sqrt{c+\frac{d}{x^2}}}+\frac{a\,x^2}{2\,c\,\sqrt{c+\frac{d}{x^2}}}-\frac{3\,a\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,c^{5/2}}+\frac{3\,a\,d}{2\,c^2\,\sqrt{c+\frac{d}{x^2}}}$$

```
[In] int((x*(a + b/x^2))/(c + d/x^2)^(3/2),x)
```

```
[Out] (b*atanh((c + d/x^2)^(1/2)/c^(1/2)))/c^(3/2) - b/(c*(c + d/x^2)^(1/2)) + (a
*x^2)/(2*c*(c + d/x^2)^(1/2)) - (3*a*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(2
*c^(5/2)) + (3*a*d)/(2*c^2*(c + d/x^2)^(1/2))
```

3.975 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 52

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x}\,dx=\frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}}+\frac{a\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}$$

```
[Out] a*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(3/2)+(-a*d+b*c)/c/d/(c+d/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {457, 79, 65, 214}

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x}\,dx=\frac{a\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}+\frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}}$$

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x),x]
```

```
[Out] (b*c - a*d)/(c*d*Sqrt[c + d/x^2]) + (a*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/c^
(3/2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{a+bx}{x(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}} - \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)}{2c}\\
&= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}} - \frac{a\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{cd}\\
&= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}} + \frac{a\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.44

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x}\,dx=\frac{\sqrt{c}(bc-ad)x-ad\sqrt{d+cx^2}\log\left(-\sqrt{c}x+\sqrt{d+cx^2}\right)}{c^{3/2}d\sqrt{c+\frac{d}{x^2}}x}$$

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x),x]
```

```
[Out] (Sqrt[c]*(b*c - a*d)*x - a*d*Sqrt[d + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[d + c*
x^2]])/(c^(3/2)*d*Sqrt[c + d/x^2]*x)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.44

method	result	size
default	$\frac{(c\,x^{2}+d)\left(c^{\frac{5}{2}}\,b x-c^{\frac{3}{2}}\,a d x+\sqrt{c\,x^{2}+d}\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right) a c d\right)}{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{3} d\,c^{\frac{5}{2}}}$	75

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2+d)*(c^(5/2)*b*x-c^(3/2)*a*d*x+(c*x^2+d)^(1/2)*ln(c^(1/2)*x+(c*x^2+d)
^(1/2))*a*c*d)/((c*x^2+d)/x^2)^(3/2)/x^3/d/c^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(44) = 88$.

Time = 0.46 (sec) , antiderivative size = 200, normalized size of antiderivative = 3.85

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x}\,dx=\left[\frac{2\,(bc^2-acd)x^2\sqrt{\frac{cx^2+d}{x^2}}+(acdx^2+ad^2)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)}{2\,(c^3dx^2+c^2d^2)},\ \frac{(bc^2-\right.$$

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b*c^2 - a*c*d)*x^2*sqrt((c*x^2 + d)/x^2) + (a*c*d*x^2 + a*d^2)*sqr
t(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d))/(c^3*d*x^2 +
c^2*d^2), ((b*c^2 - a*c*d)*x^2*sqrt((c*x^2 + d)/x^2) - (a*c*d*x^2 + a*d^2)*
sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)))/(c^3*d*x^2
 + c^2*d^2)]
```

Sympy [A] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.44

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x}\,dx=\begin{cases}\frac{2\left(-\frac{ad\operatorname{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{-c}}\right)}{2c\sqrt{-c}}-\frac{ad-bc}{2c\sqrt{c+\frac{d}{x^2}}}\right)}{d} & \text{for } d\neq 0\\ \frac{-a\log\left(-\frac{b}{x^2}\right)-\frac{b}{x^2}}{2c^{\frac{3}{2}}} & \text{otherwise}\end{cases}$$

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x,x)
```

```
[Out] Piecewise((2*(-a*d*atan(sqrt(c + d/x**2)/sqrt(-c))/(2*c*sqrt(-c)) - (a*d -
b*c)/(2*c*sqrt(c + d/x**2)))/d, Ne(d, 0)), ((-a*log(-b/x**2) - b/x**2)/(2*c
**(3/2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.33

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x}\,dx=-\frac{1}{2}\,a\left(\frac{\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2}{\sqrt{c+\frac{d}{x^2}}c}\right)+\frac{b}{\sqrt{c+\frac{d}{x^2}}d}$$

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*(log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(3/2
) + 2/(sqrt(c + d/x^2)*c)) + b/(sqrt(c + d/x^2)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.33

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x}\,dx=\frac{a\log\left(|d|\right)\operatorname{sgn}(x)}{2\,c^{\frac{3}{2}}}+\frac{(bc\operatorname{sgn}(x)-ad\operatorname{sgn}(x))x}{\sqrt{cx^2+d}cd}-\frac{a\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+d}\right|\right)}{c^{\frac{3}{2}}\operatorname{sgn}(x)}$$

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/2*a*log(abs(d))*sgn(x)/c^(3/2) + (b*c*sgn(x) - a*d*sgn(x))*x/(sqrt(c*x^2
+ d)*c*d) - a*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)))/(c^(3/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.47 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x}\,dx = \frac{a\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}} - \frac{a}{c\,\sqrt{c+\frac{d}{x^2}}} + \frac{b\,\sqrt{x^2}}{d\,\sqrt{c\,x^2+d}}$$

```
[In] int((a + b/x^2)/(x*(c + d/x^2)^(3/2)),x)
[Out] (a*atanh((c + d/x^2)^(1/2)/c^(1/2)))/c^(3/2) - a/(c*(c + d/x^2)^(1/2)) + (b
*(x^2)^(1/2))/(d*(d + c*x^2)^(1/2))
```

3.976 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 42

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^3}\,dx=-\frac{bc-ad}{d^2\sqrt{c+\frac{d}{x^2}}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{d^2}$$

```
[Out] (a*d-b*c)/d^2/(c+d/x^2)^(1/2)-b*(c+d/x^2)^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {455, 45}

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^3}\,dx=-\frac{bc-ad}{d^2\sqrt{c+\frac{d}{x^2}}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{d^2}$$

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^3),x]
```

```
[Out] -((b*c - a*d)/(d^2*Sqrt[c + d/x^2])) - (b*Sqrt[c + d/x^2])/d^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{a+bx}{(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{-bc+ad}{d(c+dx)^{3/2}}+\frac{b}{d\sqrt{c+dx}}\right)dx, x, \frac{1}{x^2}\right)\right)\\ &= -\frac{bc-ad}{d^2\sqrt{c+\frac{d}{x^2}}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{d^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^3}\,dx = \frac{adx^2-b(d+2cx^2)}{d^2\sqrt{c+\frac{d}{x^2}}x^2}$$

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^3),x]
[Out] (a*d*x^2 - b*(d + 2*c*x^2))/(d^2*Sqrt[c + d/x^2]*x^2)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

method	result	size
gosper	$\frac{(ad\,x^2-2cb\,x^2-bd)(c\,x^2+d)}{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}} d^2x^4}$	46
default	$\frac{(ad\,x^2-2cb\,x^2-bd)(c\,x^2+d)}{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}} d^2x^4}$	46
trager	$\frac{(ad\,x^2-2cb\,x^2-bd)\sqrt{-\frac{-c\,x^2-d}{x^2}}}{d^2(c\,x^2+d)}$	49
risch	$-\frac{b(c\,x^2+d)}{d^2x^2\sqrt{\frac{c\,x^2+d}{x^2}}}+\frac{ad-bc}{d^2\sqrt{\frac{c\,x^2+d}{x^2}}}$	56

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
[Out] (a*d*x^2-2*b*c*x^2-b*d)*(c*x^2+d)/((c*x^2+d)/x^2)^(3/2)/d^2/x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^3}\,dx=-\frac{\left((2\,bc-ad)x^2+bd\right)\sqrt{\frac{cx^2+d}{x^2}}}{cd^2x^2+d^3}$$

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^3,x, algorithm="fricas")
[Out] -((2*b*c - a*d)*x^2 + b*d)*sqrt((c*x^2 + d)/x^2)/(c*d^2*x^2 + d^3)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.62

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^3}\,dx=\begin{cases}\frac{a}{d\sqrt{c+\frac{d}{x^2}}}-\frac{2bc}{d^2\sqrt{c+\frac{d}{x^2}}}-\frac{b}{dx^2\sqrt{c+\frac{d}{x^2}}} & \text{for } d\neq 0\\ \frac{-\frac{a}{2x^2}-\frac{b}{4x^4}}{c^{\frac{3}{2}}} & \text{otherwise}\end{cases}$$

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**3,x)
[Out] Piecewise((a/(d*sqrt(c + d/x**2)) - 2*b*c/(d**2*sqrt(c + d/x**2)) - b/(d*x*
*2*sqrt(c + d/x**2)), Ne(d, 0)), ((-a/(2*x**2) - b/(4*x**4))/c**(3/2), True
))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^3}\,dx=-b\left(\frac{\sqrt{c+\frac{d}{x^2}}}{d^2}+\frac{c}{\sqrt{c+\frac{d}{x^2}}d^2}\right)+\frac{a}{\sqrt{c+\frac{d}{x^2}}d}$$

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^3,x, algorithm="maxima")
[Out] -b*(sqrt(c + d/x^2)/d^2 + c/(sqrt(c + d/x^2)*d^2)) + a/(sqrt(c + d/x^2)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.57

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^3}\,dx = \frac{2\,b\sqrt{c}}{\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)d\mathrm{sgn}\,(x)} - \frac{(bc-ad)x}{\sqrt{cx^2+d}d^2\mathrm{sgn}\,(x)}$$

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^3,x, algorithm="giac")
[Out] 2*b*sqrt(c)/(((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)*d*sgn(x)) - (b*c - a*d)*
x/(sqrt(c*x^2 + d)*d^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^3}\,dx = \frac{x\,\sqrt{c+\frac{d}{x^2}}\,\left(x^2\,\left(\frac{a}{d}-\frac{2\,b\,c}{d^2}\right)-\frac{b}{d}\right)}{c\,x^3+d\,x}$$

```
[In] int((a + b/x^2)/(x^3*(c + d/x^2)^(3/2)),x)
[Out] (x*(c + d/x^2)^(1/2)*(x^2*(a/d - (2*b*c)/d^2) - b/d))/(d*x + c*x^3)
```

3.977 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^5}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 68

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^5}\,dx=\frac{c(bc-ad)}{d^3\sqrt{c+\frac{d}{x^2}}}+\frac{(2bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}$$

```
[Out] -1/3*b*(c+d/x^2)^(3/2)/d^3+c*(-a*d+b*c)/d^3/(c+d/x^2)^(1/2)+(-a*d+2*b*c)*(c
+d/x^2)^(1/2)/d^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 78}

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^5}\,dx=\frac{\sqrt{c+\frac{d}{x^2}}(2bc-ad)}{d^3}+\frac{c(bc-ad)}{d^3\sqrt{c+\frac{d}{x^2}}}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}$$

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^5),x]
```

```
[Out] (c*(b*c - a*d))/(d^3*Sqrt[c + d/x^2]) + ((2*b*c - a*d)*Sqrt[c + d/x^2])/d^3
 - (b*(c + d/x^2)^(3/2))/(3*d^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
```

```
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{x(a+bx)}{(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{c(bc-ad)}{d^2(c+dx)^{3/2}} + \frac{-2bc+ad}{d^2\sqrt{c+dx}} + \frac{b\sqrt{c+dx}}{d^2}\right)\,dx, x, \frac{1}{x^2}\right)\right)\\
&= \frac{c(bc-ad)}{d^3\sqrt{c+\frac{d}{x^2}}} + \frac{(2bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^3} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^5}\,dx = \frac{-3adx^2(d+2cx^2)+b(-d^2+4cdx^2+8c^2x^4)}{3d^3\sqrt{c+\frac{d}{x^2}}x^4}$$

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^5),x]
[Out] (-3*a*d*x^2*(d + 2*c*x^2) + b*(-d^2 + 4*c*d*x^2 + 8*c^2*x^4))/(3*d^3*Sqrt[c
 + d/x^2]*x^4)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

method	result	size
gosper	$-\frac{(6acd\,x^{4}-8b\,c^{2}x^{4}+3a\,d^{2}x^{2}-4bcd\,x^{2}+b\,d^{2})(c\,x^{2}+d)}{3\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}d^{3}x^{6}}$	69
default	$-\frac{(6acd\,x^{4}-8b\,c^{2}x^{4}+3a\,d^{2}x^{2}-4bcd\,x^{2}+b\,d^{2})(c\,x^{2}+d)}{3\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}d^{3}x^{6}}$	69
trager	$-\frac{(6acd\,x^{4}-8b\,c^{2}x^{4}+3a\,d^{2}x^{2}-4bcd\,x^{2}+b\,d^{2})\sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{3x^{2}d^{3}(c\,x^{2}+d)}$	75
risch	$-\frac{(c\,x^{2}+d)(3ad\,x^{2}-5cb\,x^{2}+bd)}{3d^{3}x^{4}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}-\frac{(ad-bc)c}{d^{3}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}$	75

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
[Out] -1/3*(6*a*c*d*x^4-8*b*c^2*x^4+3*a*d^2*x^2-4*b*c*d*x^2+b*d^2)*(c*x^2+d)/((c*
x^2+d)/x^2)^(3/2)/d^3/x^6
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^5}\,dx = \frac{(2\,(4\,bc^2-3\,acd)x^4-bd^2+(4\,bcd-3\,ad^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{3\,(cd^3x^4+d^4x^2)}$$

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x, algorithm="fricas")
[Out] 1/3*(2*(4*b*c^2 - 3*a*c*d)*x^4 - b*d^2 + (4*b*c*d - 3*a*d^2)*x^2)*sqrt((c*x
^2 + d)/x^2)/(c*d^3*x^4 + d^4*x^2)
```

Sympy [A] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.25

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^5}\,dx = \begin{cases} \frac{2\left(-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{6d^2}-\frac{c(ad-bc)}{2d^2\sqrt{c+\frac{d}{x^2}}}-\frac{\sqrt{c+\frac{d}{x^2}}\,(ad-2bc)}{2d^2}\right)}{d} & \text{for } d\neq 0 \\ \frac{-\frac{a}{2x^4}-\frac{b}{3x^6}}{2c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**5,x)
[Out] Piecewise((2*(-b*(c + d/x**2)**(3/2)/(6*d**2) - c*(a*d - b*c)/(2*d**2*sqrt(
c + d/x**2)) - sqrt(c + d/x**2)*(a*d - 2*b*c)/(2*d**2))/d, Ne(d, 0)), ((-a/
(2*x**4) - b/(3*x**6))/(2*c**(3/2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.19

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^5}\,dx = -\frac{1}{3}\,b\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{d^3} - \frac{6\,\sqrt{c+\frac{d}{x^2}}\,c}{d^3} - \frac{3\,c^2}{\sqrt{c+\frac{d}{x^2}}\,d^3}\right) - a\left(\frac{\sqrt{c+\frac{d}{x^2}}}{d^2} + \frac{c}{\sqrt{c+\frac{d}{x^2}}\,d^2}\right)$$

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/3*b*((c + d/x^2)^(3/2)/d^3 - 6*sqrt(c + d/x^2)*c/d^3 - 3*c^2/(sqrt(c + d
/x^2)*d^3)) - a*(sqrt(c + d/x^2)/d^2 + c/(sqrt(c + d/x^2)*d^2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(60) = 120$.

Time = 0.53 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.76

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^5}\,dx = \frac{(bc^2-acd)x}{\sqrt{cx^2+d}d^3\mathrm{sgn}(x)} - \frac{2\left(3\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4bc^{\frac{3}{2}} - 3\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4a\sqrt{c}d - 12\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2bc^{\frac{3}{2}}d + 6\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2a\sqrt{c}d^2 + 5bc^{\frac{3}{2}}d^2 - 3a\sqrt{c}d^3\right)}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)^3d^2\mathrm{sgn}(x)}$$

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] (b*c^2 - a*c*d)*x/(sqrt(c*x^2 + d)*d^3*sgn(x)) - 2/3*(3*(sqrt(c)*x - sqrt(c
*x^2 + d))^4*b*c^(3/2) - 3*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*sqrt(c)*d - 12
*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(3/2)*d + 6*(sqrt(c)*x - sqrt(c*x^2 +
d))^2*a*sqrt(c)*d^2 + 5*b*c^(3/2)*d^2 - 3*a*sqrt(c)*d^3)/(((sqrt(c)*x - sqr
t(c*x^2 + d))^2 - d)^3*d^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^5}\,dx=-\frac{\sqrt{c+\frac{d}{x^2}}\,\left(-8\,b\,c^2\,x^4+6\,a\,c\,d\,x^4-4\,b\,c\,d\,x^2+3\,a\,d^2\,x^2+b\,d^2\right)}{3\,d^3\,x^2\,\left(c\,x^2+d\right)}$$

```
[In] int((a + b/x^2)/(x^5*(c + d/x^2)^(3/2)),x)
```

```
[Out] -((c + d/x^2)^(1/2)*(b*d^2 + 3*a*d^2*x^2 - 8*b*c^2*x^4 + 6*a*c*d*x^4 - 4*b*
c*d*x^2))/(3*d^3*x^2*(d + c*x^2))
```

3.978 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^7}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 100

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^7}\,dx = -\frac{c^2(bc-ad)}{d^4\sqrt{c+\frac{d}{x^2}}}-\frac{c(3bc-2ad)\sqrt{c+\frac{d}{x^2}}}{d^4}+\frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}$$

```
[Out] 1/3*(-a*d+3*b*c)*(c+d/x^2)^(3/2)/d^4-1/5*b*(c+d/x^2)^(5/2)/d^4-c^2*(-a*d+b*
c)/d^4/(c+d/x^2)^(1/2)-c*(-2*a*d+3*b*c)*(c+d/x^2)^(1/2)/d^4
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 78}

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^7}\,dx = -\frac{c^2(bc-ad)}{d^4\sqrt{c+\frac{d}{x^2}}}+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-ad)}{3d^4}-\frac{c\sqrt{c+\frac{d}{x^2}}(3bc-2ad)}{d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}$$

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^7),x]
```

```
[Out] -((c^2*(b*c - a*d))/(d^4*Sqrt[c + d/x^2])) - (c*(3*b*c - 2*a*d)*Sqrt[c + d/
x^2])/d^4 + ((3*b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^4) - (b*(c + d/x^2)^(5/2
))/(5*d^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{x^2(a+bx)}{(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-\frac{c^2(bc-ad)}{d^3(c+dx)^{3/2}}+\frac{c(3bc-2ad)}{d^3\sqrt{c+dx}}+\frac{(-3bc+ad)\sqrt{c+dx}}{d^3}+\frac{b(c+dx)^{3/2}}{d^3}\right)\,dx, x, \frac{1}{x^2}\right.\right.$$

$$= -\frac{c^2(bc-ad)}{d^4\sqrt{c+\frac{d}{x^2}}}-\frac{c(3bc-2ad)\sqrt{c+\frac{d}{x^2}}}{d^4}+\frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.81

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^7}\,dx = \frac{-5adx^2(d^2-4cdx^2-8c^2x^4)-3b(d^3-2cd^2x^2+8c^2dx^4+16c^3x^6)}{15d^4\sqrt{c+\frac{d}{x^2}}x^6}$$

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^7),x]
[Out] (-5*a*d*x^2*(d^2 - 4*c*d*x^2 - 8*c^2*x^4) - 3*b*(d^3 - 2*c*d^2*x^2 + 8*c^2*
d*x^4 + 16*c^3*x^6))/(15*d^4*Sqrt[c + d/x^2]*x^6)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{\left(40 a \,c^{2} d \,x^{6}-48 b \,c^{3} x^{6}+20 a c \,d^{2} x^{4}-24 b \,c^{2} d \,x^{4}-5 a \,d^{3} x^{2}+6 b c \,d^{2} x^{2}-3 b \,d^{3}\right) \left(c \,x^{2}+d \right)}{15 \left(\frac{c \,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} d^{4} x^{8}}$	94
default	$\frac{\left(40 a \,c^{2} d \,x^{6}-48 b \,c^{3} x^{6}+20 a c \,d^{2} x^{4}-24 b \,c^{2} d \,x^{4}-5 a \,d^{3} x^{2}+6 b c \,d^{2} x^{2}-3 b \,d^{3}\right) \left(c \,x^{2}+d \right)}{15 \left(\frac{c \,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} d^{4} x^{8}}$	94
risch	$\frac{\left(c \,x^{2}+d \right) \left(25 a c d \,x^{4}-33 b \,c^{2} x^{4}-5 a \,d^{2} x^{2}+9 b c d \,x^{2}-3 b \,d^{2}\right)}{15 d^{4} x^{6} \sqrt{\frac{c \,x^{2}+d}{x^{2}}}}+\frac{\left(a d -b c \right) c^{2}}{d^{4} \sqrt{\frac{c \,x^{2}+d}{x^{2}}}}$	99
trager	$\frac{\left(40 a \,c^{2} d \,x^{6}-48 b \,c^{3} x^{6}+20 a c \,d^{2} x^{4}-24 b \,c^{2} d \,x^{4}-5 a \,d^{3} x^{2}+6 b c \,d^{2} x^{2}-3 b \,d^{3}\right) \sqrt{-\frac{-c \,x^{2}-d}{x^{2}}}}{15 x^{4} d^{4} \left(c \,x^{2}+d \right)}$	100

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(40*a*c^2*d*x^6-48*b*c^3*x^6+20*a*c*d^2*x^4-24*b*c^2*d*x^4-5*a*d^3*x^2
+6*b*c*d^2*x^2-3*b*d^3)*(c*x^2+d)/((c*x^2+d)/x^2)^(3/2)/d^4/x^8
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^7} dx = -\frac{\left(8 \left(6 bc^3-5 ac^2 d\right)x^6+4 \left(6 bc^2 d-5 acd^2\right)x^4+3 bd^3-\left(6 bcd^2-5 ad^3\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{15 \left(cd^4 x^6+d^5 x^4\right)}$$

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/15*(8*(6*b*c^3 - 5*a*c^2*d)*x^6 + 4*(6*b*c^2*d - 5*a*c*d^2)*x^4 + 3*b*d^
3 - (6*b*c*d^2 - 5*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2)/(c*d^4*x^6 + d^5*x^4)
```

Sympy [A] (verification not implemented)

Time = 2.65 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.14

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^7}\,dx=\begin{cases} \frac{2\left(-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{10d^3}+\frac{c^2(ad-bc)}{2d^3\sqrt{c+\frac{d}{x^2}}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}(ad-3bc)}{6d^3}-\frac{\sqrt{c+\frac{d}{x^2}}\left(-2acd+3bc^2\right)}{2d^3}\right)}{d} & \text{for } d\neq 0 \\ \frac{-\frac{a}{3x^6}-\frac{b}{4x^8}}{2c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**7,x)
```

```
[Out] Piecewise((2*(-b*(c + d/x**2)**(5/2)/(10*d**3) + c**2*(a*d - b*c)/(2*d**3*s
qrt(c + d/x**2)) - (c + d/x**2)**(3/2)*(a*d - 3*b*c)/(6*d**3) - sqrt(c + d/
x**2)*(-2*a*c*d + 3*b*c**2)/(2*d**3))/d, Ne(d, 0)), ((-a/(3*x**6) - b/(4*x*
*8))/(2*c**(3/2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.16

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^7}\,dx = -\frac{1}{5}b\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^4}-\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^4}+\frac{15\sqrt{c+\frac{d}{x^2}}c^2}{d^4}+\frac{5c^3}{\sqrt{c+\frac{d}{x^2}}d^4}\right) - \frac{1}{3}a\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{d^3}-\frac{6\sqrt{c+\frac{d}{x^2}}c}{d^3}-\frac{3c^2}{\sqrt{c+\frac{d}{x^2}}d^3}\right)$$

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/5*b*((c + d/x^2)^(5/2)/d^4 - 5*(c + d/x^2)^(3/2)*c/d^4 + 15*sqrt(c + d/x
^2)*c^2/d^4 + 5*c^3/(sqrt(c + d/x^2)*d^4)) - 1/3*a*((c + d/x^2)^(3/2)/d^3 -
 6*sqrt(c + d/x^2)*c/d^3 - 3*c^2/(sqrt(c + d/x^2)*d^3))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(88) = 176.

Time = 0.70 (sec) , antiderivative size = 303, normalized size of antiderivative = 3.03

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^7}\,dx = -\frac{(bc^3-ac^2d)x}{\sqrt{cx^2+d}d^4\mathrm{sgn}\,(x)} + \frac{2\left(15\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^8 bc^{\frac{5}{2}}-15\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^8 ac^{\frac{3}{2}}d-90\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^6 bc^{\frac{5}{2}}d+90\left(\sqrt{c}x-\sqrt{c}\right.}{}$$

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] -(b*c^3 - a*c^2*d)*x/(sqrt(c*x^2 + d)*d^4*sgn(x)) + 2/15*(15*(sqrt(c)*x - s
qrt(c*x^2 + d))^8*b*c^(5/2) - 15*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(3/2)*
d - 90*(sqrt(c)*x - sqrt(c*x^2 + d))^6*b*c^(5/2)*d + 90*(sqrt(c)*x - sqrt(c
*x^2 + d))^6*a*c^(3/2)*d^2 + 240*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(5/2)*
d^2 - 160*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(3/2)*d^3 - 150*(sqrt(c)*x -
sqrt(c*x^2 + d))^2*b*c^(5/2)*d^3 + 110*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^
(3/2)*d^4 + 33*b*c^(5/2)*d^4 - 25*a*c^(3/2)*d^5)/(((sqrt(c)*x - sqrt(c*x^2
+ d))^2 - d)^5*d^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^7}\,dx = -\frac{\sqrt{c+\frac{d}{x^2}}\,\left(48\,b\,c^3\,x^6-40\,a\,c^2\,d\,x^6+24\,b\,c^2\,d\,x^4-20\,a\,c\,d^2\,x^4-6\,b\,c\,d^2\,x^2+5\,a\,d^3\,x^2+3\,b\,d^3\right)}{15\,d^4\,x^4\,\left(c\,x^2+d\right)}$$

```
[In] int((a + b/x^2)/(x^7*(c + d/x^2)^(3/2)),x)
```

```
[Out] -((c + d/x^2)^(1/2)*(3*b*d^3 + 5*a*d^3*x^2 + 48*b*c^3*x^6 - 20*a*c*d^2*x^4
- 40*a*c^2*d*x^6 - 6*b*c*d^2*x^2 + 24*b*c^2*d*x^4))/(15*d^4*x^4*(d + c*x^2)
)
```

3.979 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^9} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 126

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^9} \, dx = \frac{c^3(bc-ad)}{d^5\sqrt{c+\frac{d}{x^2}}}+\frac{c^2(4bc-3ad)\sqrt{c+\frac{d}{x^2}}}{d^5} - \frac{c(2bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{d^5}+\frac{(4bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^5}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^5}$$

```
[Out] -c*(-a*d+2*b*c)*(c+d/x^2)^(3/2)/d^5+1/5*(-a*d+4*b*c)*(c+d/x^2)^(5/2)/d^5-1/
7*b*(c+d/x^2)^(7/2)/d^5+c^3*(-a*d+b*c)/d^5/(c+d/x^2)^(1/2)+c^2*(-3*a*d+4*b*
c)*(c+d/x^2)^(1/2)/d^5
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 78}

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^9} \, dx = \frac{c^3(bc-ad)}{d^5\sqrt{c+\frac{d}{x^2}}}+\frac{c^2\sqrt{c+\frac{d}{x^2}}(4bc-3ad)}{d^5} + \frac{\left(c+\frac{d}{x^2}\right)^{5/2}(4bc-ad)}{5d^5}-\frac{c\left(c+\frac{d}{x^2}\right)^{3/2}(2bc-ad)}{d^5}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^5}$$

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^9),x]
```

```
[Out] (c^3*(b*c - a*d))/(d^5*Sqrt[c + d/x^2]) + (c^2*(4*b*c - 3*a*d)*Sqrt[c + d/x
^2])/d^5 - (c*(2*b*c - a*d)*(c + d/x^2)^(3/2))/d^5 + ((4*b*c - a*d)*(c + d/
x^2)^(5/2))/(5*d^5) - (b*(c + d/x^2)^(7/2))/(7*d^5)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{x^3(a+bx)}{(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{c^3(bc-ad)}{d^4(c+dx)^{3/2}} - \frac{c^2(4bc-3ad)}{d^4\sqrt{c+dx}} + \frac{3c(2bc-ad)\sqrt{c+dx}}{d^4} + \frac{(-4bc+ad)(c+dx)^{3/2}}{d^4} + \right.\right.\right.$$
$$= \frac{c^3(bc-ad)}{d^5\sqrt{c+\frac{d}{x^2}}} + \frac{c^2(4bc-3ad)\sqrt{c+\frac{d}{x^2}}}{d^5} - \frac{c(2bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{d^5} + \frac{(4bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^5} - \frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^5}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.83

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^9}\,dx = \frac{-7adx^2(d^3-2cd^2x^2+8c^2dx^4+16c^3x^6)+b(-5d^4+8cd^3x^2-16c^2d^2x^4+64c^3dx^6+}{35d^5\sqrt{c+\frac{d}{x^2}}x^8}$$

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^9),x]
```

```
[Out] (-7*a*d*x^2*(d^3 - 2*c*d^2*x^2 + 8*c^2*d*x^4 + 16*c^3*x^6) + b*(-5*d^4 + 8*
c*d^3*x^2 - 16*c^2*d^2*x^4 + 64*c^3*d*x^6 + 128*c^4*x^8))/(35*d^5*Sqrt[c +
d/x^2]*x^8)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{\left(112 a\,c^{3} d\,x^{8}-128 b\,c^{4} x^{8}+56 a\,c^{2} d^{2} x^{6}-64 b\,c^{3} d\,x^{6}-14 a c\,d^{3} x^{4}+16 b\,c^{2} d^{2} x^{4}+7 a\,d^{4} x^{2}-8 b c\,d^{3} x^{2}+5 b\,d^{4}\right) \left(c\,x^{2}+d \right)}{35 \left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} d^{5} x^{10}}$	118
default	$-\frac{\left(112 a\,c^{3} d\,x^{8}-128 b\,c^{4} x^{8}+56 a\,c^{2} d^{2} x^{6}-64 b\,c^{3} d\,x^{6}-14 a c\,d^{3} x^{4}+16 b\,c^{2} d^{2} x^{4}+7 a\,d^{4} x^{2}-8 b c\,d^{3} x^{2}+5 b\,d^{4}\right) \left(c\,x^{2}+d \right)}{35 \left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} d^{5} x^{10}}$	118
trager	$-\frac{\left(112 a\,c^{3} d\,x^{8}-128 b\,c^{4} x^{8}+56 a\,c^{2} d^{2} x^{6}-64 b\,c^{3} d\,x^{6}-14 a c\,d^{3} x^{4}+16 b\,c^{2} d^{2} x^{4}+7 a\,d^{4} x^{2}-8 b c\,d^{3} x^{2}+5 b\,d^{4}\right) \sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{35 x^{6} d^{5} \left(c\,x^{2}+d \right)}$	124
risch	$-\frac{\left(c\,x^{2}+d \right) \left(77 a\,c^{2} d\,x^{6}-93 b\,c^{3} x^{6}-21 a c\,d^{2} x^{4}+29 b\,c^{2} d\,x^{4}+7 a\,d^{3} x^{2}-13 b c\,d^{2} x^{2}+5 b\,d^{3}\right)}{35 d^{5} x^{8} \sqrt{\frac{c\,x^{2}+d}{x^{2}}}}-\frac{c^{3} \left(a d -b c \right)}{d^{5} \sqrt{\frac{c\,x^{2}+d}{x^{2}}}}$	124

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/35*(112*a*c^3*d*x^8-128*b*c^4*x^8+56*a*c^2*d^2*x^6-64*b*c^3*d*x^6-14*a*c
*d^3*x^4+16*b*c^2*d^2*x^4+7*a*d^4*x^2-8*b*c*d^3*x^2+5*b*d^4)*(c*x^2+d)/((c*
x^2+d)/x^2)^(3/2)/d^5/x^10
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.96

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^9}\,dx = \frac{(16\,(8\,bc^4-7\,ac^3d)x^8+8\,(8\,bc^3d-7\,ac^2d^2)x^6-5\,bd^4-2\,(8\,bc^2d^2-7\,acd^3)x^4+(8\,bc}{35\,(cd^5x^8+d^6x^6)}$$

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/35*(16*(8*b*c^4 - 7*a*c^3*d)*x^8 + 8*(8*b*c^3*d - 7*a*c^2*d^2)*x^6 - 5*b*
d^4 - 2*(8*b*c^2*d^2 - 7*a*c*d^3)*x^4 + (8*b*c*d^3 - 7*a*d^4)*x^2)*sqrt((c*
x^2 + d)/x^2)/(c*d^5*x^8 + d^6*x^6)
```

Sympy [A] (verification not implemented)

Time = 3.06 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.16

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^9}\,dx = \begin{cases} \frac{2\left(-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{14d^4}-\frac{c^3(ad-bc)}{2d^4\sqrt{c+\frac{d}{x^2}}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}(ad-4bc)}{10d^4}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}\left(-3acd+6bc^2\right)}{6d^4}-\frac{\sqrt{c+\frac{d}{x^2}}\cdot\left(3ac^2d-4bc^3\right)}{2d^4}\right)}{d} & \text{for } d \\ \frac{-\frac{a}{4x^8}-\frac{b}{5x^{10}}}{2c^{\frac{3}{2}}} & \text{other} \end{cases}$$

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**9,x)
```

```
[Out] Piecewise((2*(-b*(c + d/x**2)**(7/2)/(14*d**4) - c**3*(a*d - b*c)/(2*d**4*s
qrt(c + d/x**2)) - (c + d/x**2)**(5/2)*(a*d - 4*b*c)/(10*d**4) - (c + d/x**
2)**(3/2)*(-3*a*c*d + 6*b*c**2)/(6*d**4) - sqrt(c + d/x**2)*(3*a*c**2*d - 4
*b*c**3)/(2*d**4))/d, Ne(d, 0)), ((-a/(4*x**8) - b/(5*x**10))/(2*c**(3/2)),
 True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.20

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^9}\,dx = -\frac{1}{35}\,b\left(\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^5}-\frac{28\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^5}+\frac{70\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2}{d^5}-\frac{140\sqrt{c+\frac{d}{x^2}}c^3}{d^5}-\frac{35\,c^4}{\sqrt{c+\frac{d}{x^2}}d^5}\right)-\frac{1}{5}\,a\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^4}-\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^4}+\frac{15\sqrt{c+\frac{d}{x^2}}c^2}{d^4}+\frac{5\,c^3}{\sqrt{c+\frac{d}{x^2}}d^4}\right)$$

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/35*b*(5*(c + d/x^2)^(7/2)/d^5 - 28*(c + d/x^2)^(5/2)*c/d^5 + 70*(c + d/x
^2)^(3/2)*c^2/d^5 - 140*sqrt(c + d/x^2)*c^3/d^5 - 35*c^4/(sqrt(c + d/x^2)*d
^5)) - 1/5*a*((c + d/x^2)^(5/2)/d^4 - 5*(c + d/x^2)^(3/2)*c/d^4 + 15*sqrt(c
 + d/x^2)*c^2/d^4 + 5*c^3/(sqrt(c + d/x^2)*d^4))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(112) = 224$.

Time = 1.19 (sec) , antiderivative size = 414, normalized size of antiderivative = 3.29

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^9}\,dx = \frac{(bc^4-ac^3d)x}{\sqrt{cx^2+d}d^5\mathrm{sgn}\left(x\right)} - \frac{2\left(35\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}bc^{\frac{7}{2}}-35\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}ac^{\frac{5}{2}}d-280\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}bc^{\frac{7}{2}}d+280\left(\sqrt{c}x\right.\right.}{}$$

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] (b*c^4 - a*c^3*d)*x/(sqrt(c*x^2 + d)*d^5*sgn(x)) - 2/35*(35*(sqrt(c)*x - sq
rt(c*x^2 + d))^12*b*c^(7/2) - 35*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(5/2)
*d - 280*(sqrt(c)*x - sqrt(c*x^2 + d))^10*b*c^(7/2)*d + 280*(sqrt(c)*x - sq
rt(c*x^2 + d))^10*a*c^(5/2)*d^2 + 1015*(sqrt(c)*x - sqrt(c*x^2 + d))^8*b*c^
(7/2)*d^2 - 1015*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(5/2)*d^3 - 2240*(sqrt
(c)*x - sqrt(c*x^2 + d))^6*b*c^(7/2)*d^3 + 1680*(sqrt(c)*x - sqrt(c*x^2 + d
))^6*a*c^(5/2)*d^4 + 1673*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(7/2)*d^4 - 1
337*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(5/2)*d^5 - 616*(sqrt(c)*x - sqrt(c
*x^2 + d))^2*b*c^(7/2)*d^5 + 504*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(5/2)*
d^6 + 93*b*c^(7/2)*d^6 - 77*a*c^(5/2)*d^7)/(((sqrt(c)*x - sqrt(c*x^2 + d))^
2 - d)^7*d^4*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.22

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^9}\,dx = \frac{c\sqrt{c+\frac{d}{x^2}}\,(21\,a\,d-29\,b\,c)}{35\,d^4\,x^2} - \frac{b\sqrt{c+\frac{d}{x^2}}}{7\,d^2\,x^6} - \frac{\sqrt{c+\frac{d}{x^2}}\,(7\,a\,d^2-13\,b\,c\,d)}{35\,d^4\,x^4} - \frac{\sqrt{c+\frac{d}{x^2}}\left(x^2\left(\frac{58\,b\,c^4-42\,a\,c^3\,d}{35\,d^5}+\frac{2\,c^3\,(77\,a\,d-93\,b\,c)}{35\,d^5}\right)+\frac{c^2\,(77\,a\,d-93\,b\,c)}{35\,d^4}\right)}{c\,x^2+d}$$

```
[In] int((a + b/x^2)/(x^9*(c + d/x^2)^(3/2)),x)
```

```
[Out] (c*(c + d/x^2)^(1/2)*(21*a*d - 29*b*c))/(35*d^4*x^2) - (b*(c + d/x^2)^(1/2)
)/(7*d^2*x^6) - ((c + d/x^2)^(1/2)*(7*a*d^2 - 13*b*c*d))/(35*d^4*x^4) - ((c
 + d/x^2)^(1/2)*(x^2*((58*b*c^4 - 42*a*c^3*d)/(35*d^5) + (2*c^3*(77*a*d - 9
3*b*c))/(35*d^5)) + (c^2*(77*a*d - 93*b*c))/(35*d^4)))/(d + c*x^2)
```

3.980 $\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 111

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\frac{4d(5bc-6ad)x}{15c^3\sqrt{c+\frac{d}{x^2}}}-\frac{8d(5bc-6ad)\sqrt{c+\frac{d}{x^2}}\,x}{15c^4}+\frac{(5bc-6ad)x^3}{15c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}$$

```
[Out] 4/15*d*(-6*a*d+5*b*c)*x/c^3/(c+d/x^2)^(1/2)+1/15*(-6*a*d+5*b*c)*x^3/c^2/(c+
d/x^2)^(1/2)+1/5*a*x^5/c/(c+d/x^2)^(1/2)-8/15*d*(-6*a*d+5*b*c)*x*(c+d/x^2)^
(1/2)/c^4
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {464, 277, 198, 197}

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=-\frac{8dx\sqrt{c+\frac{d}{x^2}}(5bc-6ad)}{15c^4}+\frac{4dx(5bc-6ad)}{15c^3\sqrt{c+\frac{d}{x^2}}}+\frac{x^3(5bc-6ad)}{15c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}$$

```
[In] Int[((a + b/x^2)*x^4)/(c + d/x^2)^(3/2),x]
```

```
[Out] (4*d*(5*b*c - 6*a*d)*x)/(15*c^3*Sqrt[c + d/x^2]) - (8*d*(5*b*c - 6*a*d)*Sqr
t[c + d/x^2]*x)/(15*c^4) + ((5*b*c - 6*a*d)*x^3)/(15*c^2*Sqrt[c + d/x^2]) +
 (a*x^5)/(5*c*Sqrt[c + d/x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}+\frac{(5bc-6ad)\int \frac{x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx}{5c}\\
&= \frac{(5bc-6ad)x^3}{15c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}-\frac{(4d(5bc-6ad))\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx}{15c^2}\\
&= \frac{4d(5bc-6ad)x}{15c^3\sqrt{c+\frac{d}{x^2}}}+\frac{(5bc-6ad)x^3}{15c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}-\frac{(8d(5bc-6ad))\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}\,dx}{15c^3}\\
&= \frac{4d(5bc-6ad)x}{15c^3\sqrt{c+\frac{d}{x^2}}}-\frac{8d(5bc-6ad)\sqrt{c+\frac{d}{x^2}}\,x}{15c^4}+\frac{(5bc-6ad)x^3}{15c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.72

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\frac{5bc(-8d^2-4cdx^2+c^2x^4)+3a(16d^3+8cd^2x^2-2c^2dx^4+c^3x^6)}{15c^4\sqrt{c+\frac{d}{x^2}}\,x}$$

```
[In] Integrate[((a + b/x^2)*x^4)/(c + d/x^2)^(3/2),x]
[Out] (5*b*c*(-8*d^2 - 4*c*d*x^2 + c^2*x^4) + 3*a*(16*d^3 + 8*c*d^2*x^2 - 2*c^2*d
*x^4 + c^3*x^6))/(15*c^4*Sqrt[c + d/x^2]*x)
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{\left(3x^6ac^3-6ac^2dx^4+5bc^3x^4+24acd^2x^2-20bc^2dx^2+48ad^3-40bcd^2\right)\left(cx^2+d\right)}{15\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}x^3c^4}$	91
default	$\frac{\left(3x^6ac^3-6ac^2dx^4+5bc^3x^4+24acd^2x^2-20bc^2dx^2+48ad^3-40bcd^2\right)\left(cx^2+d\right)}{15\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}x^3c^4}$	91
trager	$\frac{\left(3x^6ac^3-6ac^2dx^4+5bc^3x^4+24acd^2x^2-20bc^2dx^2+48ad^3-40bcd^2\right)x\sqrt{-\frac{-cx^2-d}{x^2}}}{15(cx^2+d)c^4}$	95
risch	$\frac{\left(3ax^4c^2-9acdx^2+5bc^2x^2+33ad^2-25bcd\right)\left(cx^2+d\right)}{15c^4\sqrt{\frac{cx^2+d}{x^2}}\,x}+\frac{(ad-bc)d^2}{c^4\sqrt{\frac{cx^2+d}{x^2}}\,x}$	99

```
[In] int((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/15*(3*a*c^3*x^6-6*a*c^2*d*x^4+5*b*c^3*x^4+24*a*c*d^2*x^2-20*b*c^2*d*x^2+4
8*a*d^3-40*b*c*d^2)*(c*x^2+d)/((c*x^2+d)/x^2)^(3/2)/x^3/c^4
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\frac{(3\,ac^3x^7+(5\,bc^3-6\,ac^2d)x^5-4\,(5\,bc^2d-6\,acd^2)x^3-8\,(5\,bcd^2-6\,ad^3)x)\sqrt{\frac{cx^2+d}{x^2}}}{15\,(c^5x^2+c^4d)}$$

```
[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x, algorithm="fricas")
[Out] 1/15*(3*a*c^3*x^7 + (5*b*c^3 - 6*a*c^2*d)*x^5 - 4*(5*b*c^2*d - 6*a*c*d^2)*x
^3 - 8*(5*b*c*d^2 - 6*a*d^3)*x)*sqrt((c*x^2 + d)/x^2)/(c^5*x^2 + c^4*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(107) = 214$.

Time = 2.92 (sec) , antiderivative size = 561, normalized size of antiderivative = 5.05

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = a\left(\frac{c^5d^{\frac{19}{2}}x^{10}\sqrt{\frac{cx^2}{d}+1}}{5c^7d^9x^6+15c^6d^{10}x^4+15c^5d^{11}x^2+5c^4d^{12}} + \frac{5c^3d^{\frac{23}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{5c^7d^9x^6+15c^6d^{10}x^4+15c^5d^{11}x^2+5c^4d^{12}} + \frac{30c^2d^{\frac{25}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{5c^7d^9x^6+15c^6d^{10}x^4+15c^5d^{11}x^2+5c^4d^{12}} + \frac{40cd^{\frac{27}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{5c^7d^9x^6+15c^6d^{10}x^4+15c^5d^{11}x^2+5c^4d^{12}} + \frac{16d^{\frac{29}{2}}\sqrt{\frac{cx^2}{d}+1}}{5c^7d^9x^6+15c^6d^{10}x^4+15c^5d^{11}x^2+5c^4d^{12}}\right) + b\left(\frac{c^3d^{\frac{9}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6} - \frac{3c^2d^{\frac{11}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6} - \frac{12cd^{\frac{13}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6} - \frac{8d^{\frac{15}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6}\right)$$

```
[In] integrate((a+b/x**2)*x**4/(c+d/x**2)**(3/2),x)
```

```
[Out] a*(c**5*d**(19/2)*x**10*sqrt(c*x**2/d + 1)/(5*c**7*d**9*x**6 + 15*c**6*d**1
0*x**4 + 15*c**5*d**11*x**2 + 5*c**4*d**12) + 5*c**3*d**(23/2)*x**6*sqrt(c*
x**2/d + 1)/(5*c**7*d**9*x**6 + 15*c**6*d**10*x**4 + 15*c**5*d**11*x**2 + 5
*c**4*d**12) + 30*c**2*d**(25/2)*x**4*sqrt(c*x**2/d + 1)/(5*c**7*d**9*x**6
+ 15*c**6*d**10*x**4 + 15*c**5*d**11*x**2 + 5*c**4*d**12) + 40*c*d**(27/2)*
x**2*sqrt(c*x**2/d + 1)/(5*c**7*d**9*x**6 + 15*c**6*d**10*x**4 + 15*c**5*d*
*11*x**2 + 5*c**4*d**12) + 16*d**(29/2)*sqrt(c*x**2/d + 1)/(5*c**7*d**9*x**
6 + 15*c**6*d**10*x**4 + 15*c**5*d**11*x**2 + 5*c**4*d**12)) + b*(c**3*d**(
9/2)*x**6*sqrt(c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*
d**6) - 3*c**2*d**(11/2)*x**4*sqrt(c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4
*d**5*x**2 + 3*c**3*d**6) - 12*c*d**(13/2)*x**2*sqrt(c*x**2/d + 1)/(3*c**5*
d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6) - 8*d**(15/2)*sqrt(c*x**2/d + 1
)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.15

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{1}{3}\,b\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3-6\,\sqrt{c+\frac{d}{x^2}}dx}{c^3}-\frac{3\,d^2}{\sqrt{c+\frac{d}{x^2}}c^3x}\right) + \frac{1}{5}\,a\left(\frac{5\,d^3}{\sqrt{c+\frac{d}{x^2}}c^4x}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5-5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}dx^3+15\,\sqrt{c+\frac{d}{x^2}}d^2x}{c^4}\right)$$

```
[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*b*(((c + d/x^2)^(3/2)*x^3 - 6*sqrt(c + d/x^2)*d*x)/c^3 - 3*d^2/(sqrt(c
+ d/x^2)*c^3*x)) + 1/5*a*(5*d^3/(sqrt(c + d/x^2)*c^4*x) + ((c + d/x^2)^(5/2
)*x^5 - 5*(c + d/x^2)^(3/2)*d*x^3 + 15*sqrt(c + d/x^2)*d^2*x)/c^4)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.32

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{8\,(5\,bcd^2-6\,ad^3)\mathrm{sgn}(x)}{15\,c^4\sqrt{d}}-\frac{bcd^2-ad^3}{\sqrt{cx^2+d}c^4\mathrm{sgn}\left(x\right)} + \frac{3\left(cx^2+d\right)^{\frac{5}{2}}ac^{16}+5\left(cx^2+d\right)^{\frac{3}{2}}bc^{17}-15\left(cx^2+d\right)^{\frac{3}{2}}ac^{16}d-30\,\sqrt{cx^2+d}bc^{17}d+45\,\sqrt{cx^2+d}ac^{16}d^2}{15\,c^{20}\mathrm{sgn}\left(x\right)}$$

```
[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 8/15*(5*b*c*d^2 - 6*a*d^3)*sgn(x)/(c^4*sqrt(d)) - (b*c*d^2 - a*d^3)/(sqrt(c
*x^2 + d)*c^4*sgn(x)) + 1/15*(3*(c*x^2 + d)^(5/2)*a*c^16 + 5*(c*x^2 + d)^(3
/2)*b*c^17 - 15*(c*x^2 + d)^(3/2)*a*c^16*d - 30*sqrt(c*x^2 + d)*b*c^17*d +
45*sqrt(c*x^2 + d)*a*c^16*d^2)/(c^20*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 10.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.71

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\frac{3\,a\,c^3\,x^6+5\,b\,c^3\,x^4-6\,a\,c^2\,d\,x^4-20\,b\,c^2\,d\,x^2+24\,a\,c\,d^2\,x^2-40\,b\,c\,d^2+48\,a\,d^3}{15\,c^4\,x\,\sqrt{c+\frac{d}{x^2}}}$$

```
[In] int((x^4*(a + b/x^2))/(c + d/x^2)^(3/2),x)
[Out] (48*a*d^3 + 3*a*c^3*x^6 + 5*b*c^3*x^4 - 40*b*c*d^2 + 24*a*c*d^2*x^2 - 6*a*c
^2*d*x^4 - 20*b*c^2*d*x^2)/(15*c^4*x*(c + d/x^2)^(1/2))
```

3.981 $\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 79

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=-\frac{(3bc-4ad)x}{3c^2\sqrt{c+\frac{d}{x^2}}}+\frac{2(3bc-4ad)\sqrt{c+\frac{d}{x^2}}\,x}{3c^3}+\frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}}$$

```
[Out] -1/3*(-4*a*d+3*b*c)*x/c^2/(c+d/x^2)^(1/2)+1/3*a*x^3/c/(c+d/x^2)^(1/2)+2/3*(
-4*a*d+3*b*c)*x*(c+d/x^2)^(1/2)/c^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {464, 198, 197}

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\frac{2x\sqrt{c+\frac{d}{x^2}}(3bc-4ad)}{3c^3}-\frac{x(3bc-4ad)}{3c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}}$$

```
[In] Int[((a + b/x^2)*x^2)/(c + d/x^2)^(3/2),x]
```

```
[Out] -1/3*((3*b*c - 4*a*d)*x)/(c^2*Sqrt[c + d/x^2]) + (2*(3*b*c - 4*a*d)*Sqrt[c
+ d/x^2]*x)/(3*c^3) + (a*x^3)/(3*c*Sqrt[c + d/x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}}+\frac{(3bc-4ad)\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx}{3c} \\
&= -\frac{(3bc-4ad)x}{3c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}}+\frac{(2(3bc-4ad))\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}\,dx}{3c^2} \\
&= -\frac{(3bc-4ad)x}{3c^2\sqrt{c+\frac{d}{x^2}}}+\frac{2(3bc-4ad)\sqrt{c+\frac{d}{x^2}}x}{3c^3}+\frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.72

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\frac{3bc(2d+cx^2)+a(-8d^2-4cdx^2+c^2x^4)}{3c^3\sqrt{c+\frac{d}{x^2}}x}$$

```
[In] Integrate[((a + b/x^2)*x^2)/(c + d/x^2)^(3/2),x]
[Out] (3*b*c*(2*d + c*x^2) + a*(-8*d^2 - 4*c*d*x^2 + c^2*x^4))/(3*c^3*Sqrt[c + d/
x^2]*x)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{\left(a\,x^{4} c^{2}-4 a c d\,x^{2}+3 b\,c^{2} x^{2}-8 a\,d^{2}+6 b c d \right) \left(c\,x^{2}+d \right)}{3 \left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} c^{3} x^{3}}$	66
default	$\frac{\left(a\,x^{4} c^{2}-4 a c d\,x^{2}+3 b\,c^{2} x^{2}-8 a\,d^{2}+6 b c d \right) \left(c\,x^{2}+d \right)}{3 \left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} c^{3} x^{3}}$	66
trager	$\frac{\left(a\,x^{4} c^{2}-4 a c d\,x^{2}+3 b\,c^{2} x^{2}-8 a\,d^{2}+6 b c d \right) x \sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{3 \left(c\,x^{2}+d \right) c^{3}}$	70
risch	$\frac{\left(a c\,x^{2}-5 a d+3 b c \right) \left(c\,x^{2}+d \right)}{3 c^{3} \sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, x}-\frac{\left(a d-b c \right) d}{c^{3} \sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, x}$	75

```
[In] int((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(a*c^2*x^4-4*a*c*d*x^2+3*b*c^2*x^2-8*a*d^2+6*b*c*d)*(c*x^2+d)/((c*x^2+d
)/x^2)^(3/2)/c^3/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$\int \frac{\left(a+\frac{b}{x^2}\right) x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\frac{\left(ac^2x^5+\left(3\,bc^2-4\,acd\right)x^3+2\,\left(3\,bcd-4\,ad^2\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{3\,\left(c^4x^2+c^3d\right)}$$

```
[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(a*c^2*x^5 + (3*b*c^2 - 4*a*c*d)*x^3 + 2*(3*b*c*d - 4*a*d^2)*x)*sqrt((c
*x^2 + d)/x^2)/(c^4*x^2 + c^3*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(70) = 140$.

Time = 2.61 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.38

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = a\left(\frac{c^3d^{\frac{9}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6} - \frac{3c^2d^{\frac{11}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6} - \frac{12cd^{\frac{13}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6} - \frac{8d^{\frac{15}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6}\right) + b\left(\frac{x^2}{c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}} + \frac{2\sqrt{d}}{c^2\sqrt{\frac{cx^2}{d}+1}}\right)$$

```
[In] integrate((a+b/x**2)*x**2/(c+d/x**2)**(3/2),x)
```

```
[Out] a*(c**3*d**(9/2)*x**6*sqrt(c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x*
*2 + 3*c**3*d**6) - 3*c**2*d**(11/2)*x**4*sqrt(c*x**2/d + 1)/(3*c**5*d**4*x
**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6) - 12*c*d**(13/2)*x**2*sqrt(c*x**2/d +
 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6) - 8*d**(15/2)*sqrt(
c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6)) + b*(x**
2/(c*sqrt(d)*sqrt(c*x**2/d + 1)) + 2*sqrt(d)/(c**2*sqrt(c*x**2/d + 1)))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.14

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = b\left(\frac{\sqrt{c+\frac{d}{x^2}}x}{c^2} + \frac{d}{\sqrt{c+\frac{d}{x^2}}c^2x}\right) + \frac{1}{3}a\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3 - 6\sqrt{c+\frac{d}{x^2}}dx}{c^3} - \frac{3d^2}{\sqrt{c+\frac{d}{x^2}}c^3x}\right)$$

```
[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] b*(sqrt(c + d/x^2)*x/c^2 + d/(sqrt(c + d/x^2)*c^2*x)) + 1/3*a*(((c + d/x^2)
^(3/2)*x^3 - 6*sqrt(c + d/x^2)*d*x)/c^3 - 3*d^2/(sqrt(c + d/x^2)*c^3*x))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.34

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = -\frac{2\,(3\,bcd-4\,ad^2)\mathrm{sgn}(x)}{3\,c^3\sqrt{d}}+\frac{bcd-ad^2}{\sqrt{cx^2+d}c^3\mathrm{sgn}\,(x)} + \frac{(cx^2+d)^{\frac{3}{2}}ac^6+3\,\sqrt{cx^2+d}bc^7-6\,\sqrt{cx^2+d}ac^6d}{3\,c^9\mathrm{sgn}\,(x)}$$

```
[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*b*c*d - 4*a*d^2)*sgn(x)/(c^3*sqrt(d)) + (b*c*d - a*d^2)/(sqrt(c*x^2
 + d)*c^3*sgn(x)) + 1/3*((c*x^2 + d)^(3/2)*a*c^6 + 3*sqrt(c*x^2 + d)*b*c^7
- 6*sqrt(c*x^2 + d)*a*c^6*d)/(c^9*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.70 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.03

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{b\,c^2\,x^4+3\,b\,c\,d\,x^2+2\,b\,d^2}{c^2\,x^3\,\left(c+\frac{d}{x^2}\right)^{3/2}}-\frac{-a\,c^2\,x^4+4\,a\,c\,d\,x^2+8\,a\,d^2}{3\,c^3\,x\,\sqrt{c+\frac{d}{x^2}}}$$

```
[In] int((x^2*(a + b/x^2))/(c + d/x^2)^(3/2),x)
```

```
[Out] (2*b*d^2 + b*c^2*x^4 + 3*b*c*d*x^2)/(c^2*x^3*(c + d/x^2)^(3/2)) - (8*a*d^2
- a*c^2*x^4 + 4*a*c*d*x^2)/(3*c^3*x*(c + d/x^2)^(1/2))
```

3.982 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}} dx$

Optimal result

Integrand size = 19, antiderivative size = 45

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}} dx = -\frac{bc-2ad}{c^2\sqrt{c+\frac{d}{x^2}}x}+\frac{ax}{c\sqrt{c+\frac{d}{x^2}}}$$

```
[Out] (2*a*d-b*c)/c^2/x/(c+d/x^2)^(1/2)+a*x/c/(c+d/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {382, 464, 197}

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}} dx = \frac{ax}{c\sqrt{c+\frac{d}{x^2}}}-\frac{bc-2ad}{c^2x\sqrt{c+\frac{d}{x^2}}}$$

```
[In] Int[(a + b/x^2)/(c + d/x^2)^(3/2),x]
```

```
[Out] -((b*c - 2*a*d)/(c^2*Sqrt[c + d/x^2]*x)) + (a*x)/(c*Sqrt[c + d/x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
```

```
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{a+bx^2}{x^2\left(c+dx^2\right)^{3/2}}\,dx, x, \frac{1}{x}\right) \\
&= \frac{ax}{c\sqrt{c+\frac{d}{x^2}}} + \frac{(-bc+2ad)\text{Subst}\left(\int \frac{1}{\left(c+dx^2\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{c} \\
&= -\frac{bc-2ad}{c^2\sqrt{c+\frac{d}{x^2}}\,x} + \frac{ax}{c\sqrt{c+\frac{d}{x^2}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{-bc+2ad+acx^2}{c^2\sqrt{c+\frac{d}{x^2}}\,x}$$

```
[In] Integrate[(a + b/x^2)/(c + d/x^2)^(3/2),x]
[Out] (-(b*c) + 2*a*d + a*c*x^2)/(c^2*Sqrt[c + d/x^2]*x)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{\left(a c \,x^{2}+2 a d -b c \right) \left(c \,x^{2}+d \right)}{\left(\frac{c \,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} c^{2} x^{3}}$	43
default	$\frac{\left(a c \,x^{2}+2 a d -b c \right) \left(c \,x^{2}+d \right)}{\left(\frac{c \,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} c^{2} x^{3}}$	43
trager	$\frac{\left(a c \,x^{2}+2 a d -b c \right) x \sqrt{-\frac{-c \,x^{2}-d}{x^{2}}}}{\left(c \,x^{2}+d \right) c^{2}}$	47
risch	$\frac{a \left(c \,x^{2}+d \right)}{c^{2} \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, x}+\frac{a d -b c}{c^{2} \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, x}$	58

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] (a*c*x^2+2*a*d-b*c)*(c*x^2+d)/((c*x^2+d)/x^2)^(3/2)/c^2/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\frac{(acx^3-(bc-2\,ad)x)\sqrt{\frac{cx^2+d}{x^2}}}{c^3x^2+c^2d}$$

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2),x, algorithm="fricas")
[Out] (a*c*x^3 - (b*c - 2*a*d)*x)*sqrt((c*x^2 + d)/x^2)/(c^3*x^2 + c^2*d)
```

Sympy [A] (verification not implemented)

Time = 2.46 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=a\left(\frac{x^2}{c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{2\sqrt{d}}{c^2\sqrt{\frac{cx^2}{d}+1}}\right)-\frac{b}{c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}$$

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2),x)
[Out] a*(x**2/(c*sqrt(d)*sqrt(c*x**2/d + 1)) + 2*sqrt(d)/(c**2*sqrt(c*x**2/d + 1)
)) - b/(c*sqrt(d)*sqrt(c*x**2/d + 1))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = a\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x}{c^2}+\frac{d}{\sqrt{c+\frac{d}{x^2}}\,c^2x}\right)-\frac{b}{\sqrt{c+\frac{d}{x^2}}\,cx}$$

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] a*(sqrt(c + d/x^2)*x/c^2 + d/(sqrt(c + d/x^2)*c^2*x)) - b/(sqrt(c + d/x^2)*
c*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.38

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{(bc-2\,ad)\mathrm{sgn}(x)}{c^2\sqrt{d}}+\frac{\sqrt{cx^2+d}a}{c^2\mathrm{sgn}\left(x\right)}-\frac{bc-ad}{\sqrt{cx^2+d}c^2\mathrm{sgn}\left(x\right)}$$

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] (b*c - 2*a*d)*sgn(x)/(c^2*sqrt(d)) + sqrt(c*x^2 + d)*a/(c^2*sgn(x)) - (b*c
- a*d)/(sqrt(c*x^2 + d)*c^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{\left(c\,x^2+d\right)\,\left(a\,c\,x^2+2\,a\,d-b\,c\right)}{c^2\,x^3\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}$$

```
[In] int((a + b/x^2)/(c + d/x^2)^(3/2),x)
```

```
[Out] ((d + c*x^2)*(2*a*d - b*c + a*c*x^2))/(c^2*x^3*(c + d/x^2)^(3/2))
```

3.983 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 59

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^2}\,dx=\frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}x}-\frac{b\text{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)}{d^{3/2}}$$

```
[Out] -b*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(3/2)+(-a*d+b*c)/c/d/x/(c+d/x^2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {463, 342, 223, 212}

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^2}\,dx=\frac{bc-ad}{cdx\sqrt{c+\frac{d}{x^2}}}-\frac{b\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{d^{3/2}}$$

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^2),x]
```

```
[Out] (b*c - a*d)/(c*d*Sqrt[c + d/x^2]*x) - (b*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x
)])/d^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}\,x}+\frac{b\int\frac{1}{\sqrt{c+\frac{d}{x^2}}\,x^2}\,dx}{d}\\
&= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}\,x}-\frac{b\text{Subst}\left(\int\frac{1}{\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right)}{d}\\
&= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}\,x}-\frac{b\text{Subst}\left(\int\frac{1}{1-dx^2}\,dx,x,\frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{d}\\
&= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}\,x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{d^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.20

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^2}\,dx=\frac{\sqrt{d}(bc-ad)-bc\sqrt{d+cx^2}\operatorname{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{cd^{3/2}\sqrt{c+\frac{d}{x^2}}x}$$

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^2),x]
[Out] (Sqrt[d]*(b*c - a*d) - b*c*Sqrt[d + c*x^2]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]]
)/(c*d^(3/2)*Sqrt[c + d/x^2]*x)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.34

method	result	size
default	$-\frac{(c\,x^2+d)\left(\sqrt{c\,x^2+d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)bcd+d^{\frac{5}{2}}a-b\,d^{\frac{3}{2}}c\right)}{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}x^3c\,d^{\frac{5}{2}}}$	79

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
[Out] -(c*x^2+d)*((c*x^2+d)^(1/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*b*c*d+d^(5/
2)*a-b*d^(3/2)*c)/((c*x^2+d)/x^2)^(3/2)/x^3/c/d^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 195, normalized size of antiderivative = 3.31

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^2}\,dx=\left[\frac{2\,(bcd-ad^2)x\sqrt{\frac{cx^2+d}{x^2}}+(bc^2x^2+bcd)\sqrt{d}\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)}{2\,(c^2d^2x^2+cd^3)},\ \frac{(bcd-ad^2)x}{}\right.$$

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x, algorithm="fricas")
[Out] [1/2*(2*(b*c*d - a*d^2)*x*sqrt((c*x^2 + d)/x^2) + (b*c^2*x^2 + b*c*d)*sqrt(
d)*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2))/(c^2*d^2*x^
2 + c*d^3), ((b*c*d - a*d^2)*x*sqrt((c*x^2 + d)/x^2) + (b*c^2*x^2 + b*c*d)*
sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)))/(c^2*d^2*x^2
 + c*d^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(46) = 92$.

Time = 3.84 (sec) , antiderivative size = 206, normalized size of antiderivative = 3.49

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^2}\,dx = -\frac{a}{c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}} + b\left(\frac{cd^2x^2\log\left(\frac{cx^2}{d}\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}} - \frac{2cd^2x^2\log\left(\sqrt{\frac{cx^2}{d}+1}+1\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}} + \frac{2d^3\sqrt{\frac{cx^2}{d}+1}}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}} + \frac{d^3\log\left(\frac{cx^2}{d}\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}} - \frac{2d^3\log\left(\sqrt{\frac{cx^2}{d}+1}+1\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}\right)$$

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**2,x)
```

```
[Out] -a/(c*sqrt(d)*sqrt(c*x**2/d + 1)) + b*(c*d**2*x**2*log(c*x**2/d)/(2*c*d**(7
/2)*x**2 + 2*d**(9/2)) - 2*c*d**2*x**2*log(sqrt(c*x**2/d + 1) + 1)/(2*c*d**
(7/2)*x**2 + 2*d**(9/2)) + 2*d**3*sqrt(c*x**2/d + 1)/(2*c*d**(7/2)*x**2 + 2
*d**(9/2)) + d**3*log(c*x**2/d)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) - 2*d**3*l
og(sqrt(c*x**2/d + 1) + 1)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^2}\,dx = \frac{1}{2}\,b\left(\frac{\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{3}{2}}} + \frac{2}{\sqrt{c+\frac{d}{x^2}}dx}\right) - \frac{a}{\sqrt{c+\frac{d}{x^2}}cx}$$

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*(log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(
3/2) + 2/(sqrt(c + d/x^2)*d*x)) - a/(sqrt(c + d/x^2)*c*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(51) = 102$.

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.83

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^2}\,dx = \frac{b\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}d\operatorname{sgn}(x)} - \frac{\left(bc\sqrt{d}\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)+bc\sqrt{-d}-a\sqrt{-d}d\right)\operatorname{sgn}(x)}{c\sqrt{-d}d^{\frac{3}{2}}} + \frac{bc-ad}{\sqrt{cx^2+d}cd\operatorname{sgn}(x)}$$

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] b*arctan(sqrt(c*x^2 + d)/sqrt(-d))/(sqrt(-d)*d*sgn(x)) - (b*c*sqrt(d)*arcta
n(sqrt(d)/sqrt(-d)) + b*c*sqrt(-d) - a*sqrt(-d)*d)*sgn(x)/(c*sqrt(-d)*d^(3/
2)) + (b*c - a*d)/(sqrt(c*x^2 + d)*c*d*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.45 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^2}\,dx = \frac{b}{d\,x\,\sqrt{c+\frac{d}{x^2}}} - \frac{a}{c\,x\,\sqrt{c+\frac{d}{x^2}}} - \frac{b\,\ln\left(\sqrt{c+\frac{d}{x^2}}+\frac{\sqrt{d}}{x}\right)}{d^{3/2}}$$

```
[In] int((a + b/x^2)/(x^2*(c + d/x^2)^(3/2)),x)
```

```
[Out] b/(d*x*(c + d/x^2)^(1/2)) - a/(c*x*(c + d/x^2)^(1/2)) - (b*log((c + d/x^2)^
(1/2) + d^(1/2)/x))/d^(3/2)
```

3.984 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 92

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}\,dx=-\frac{b}{2d\sqrt{c+\frac{d}{x^2}}\,x^3}-\frac{3bc-2ad}{2d^2\sqrt{c+\frac{d}{x^2}}\,x}+\frac{(3bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{2d^{5/2}}$$

```
[Out] 1/2*(-2*a*d+3*b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(5/2)-1/2*b/d/x^3/(
c+d/x^2)^(1/2)+1/2*(2*a*d-3*b*c)/d^2/x/(c+d/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {470, 342, 294, 223, 212}

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}\,dx=\frac{(3bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2d^{5/2}}-\frac{3bc-2ad}{2d^2x\sqrt{c+\frac{d}{x^2}}}-\frac{b}{2dx^3\sqrt{c+\frac{d}{x^2}}}$$

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^4),x]
```

```
[Out] -1/2*b/(d*Sqrt[c + d/x^2]*x^3) - (3*b*c - 2*a*d)/(2*d^2*Sqrt[c + d/x^2]*x)
+ ((3*b*c - 2*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(2*d^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b}{2d\sqrt{c+\frac{d}{x^2}}x^3}+\frac{(-3bc+2ad)\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}\,dx}{2d} \\
&= -\frac{b}{2d\sqrt{c+\frac{d}{x^2}}x^3}-\frac{(-3bc+2ad)\text{Subst}\left(\int \frac{x^2}{(c+dx^2)^{3/2}}\,dx,x,\frac{1}{x}\right)}{2d} \\
&= -\frac{b}{2d\sqrt{c+\frac{d}{x^2}}x^3}-\frac{3bc-2ad}{2d^2\sqrt{c+\frac{d}{x^2}}x}+\frac{(3bc-2ad)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right)}{2d^2} \\
&= -\frac{b}{2d\sqrt{c+\frac{d}{x^2}}x^3}-\frac{3bc-2ad}{2d^2\sqrt{c+\frac{d}{x^2}}x}+\frac{(3bc-2ad)\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{1}{\sqrt{c+\frac{d}{x^2}}x}\right)}{2d^2}
\end{aligned}$$

$$= -\frac{b}{2d\sqrt{c+\frac{d}{x^2}}x^3} - \frac{3bc-2ad}{2d^2\sqrt{c+\frac{d}{x^2}}x} + \frac{(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)}{2d^{5/2}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.99

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}\,dx = \frac{\sqrt{d}(2adx^2-b(d+3cx^2))+(3bc-2ad)x^2\sqrt{d+cx^2}\,\text{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{2d^{5/2}\sqrt{c+\frac{d}{x^2}}x^3}$$

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^4),x]
[Out] (Sqrt[d]*(2*a*d*x^2 - b*(d + 3*c*x^2)) + (3*b*c - 2*a*d)*x^2*Sqrt[d + c*x^2
]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/(2*d^(5/2)*Sqrt[c + d/x^2]*x^3)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.38

method	result	size
risch	$-\frac{b(c\,x^2+d)}{2d^2x^3\sqrt{\frac{c\,x^2+d}{x^2}}}+\frac{\left(\frac{bc}{\sqrt{c\,x^2+d}}+d(2ad-3bc)\left(\frac{1}{d\sqrt{c\,x^2+d}}-\frac{\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)}{d^{\frac{3}{2}}}\right)\right)\sqrt{c\,x^2+d}}{2d^2\sqrt{\frac{c\,x^2+d}{x^2}}\,x}$	127
default	$-\frac{(c\,x^2+d)\left(2\sqrt{c\,x^2+d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)a\,d^2x^2-3\sqrt{c\,x^2+d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)bcd\,x^2-2d^{\frac{5}{2}}\,a\,x^2+3d^{\frac{3}{2}}\,bc\,x^2+d^{\frac{5}{2}}\,b\right)}{2\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}x^5d^{\frac{7}{2}}}$	131

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
[Out] -1/2/d^2*b*(c*x^2+d)/x^3/((c*x^2+d)/x^2)^(1/2)+1/2/d^2*(b*c/(c*x^2+d)^(1/2)
+d*(2*a*d-3*b*c)*(1/d/(c*x^2+d)^(1/2)-1/d^(3/2)*ln((2*d+2*d^(1/2)*(c*x^2+d)
^(1/2))/x)))/((c*x^2+d)/x^2)^(1/2)/x*(c*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.70

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^4}\,dx = \left[-\frac{((3\,bc^2-2\,acd)x^3+(3\,bcd-2\,ad^2)x)\sqrt{d}\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,(bd^2+(3}{4\,(cd^3x^3+d^4x)}\right.$$

$$\left.-\frac{((3\,bc^2-2\,acd)x^3+(3\,bcd-2\,ad^2)x)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+(bd^2+(3\,bcd-2\,ad^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{2\,(cd^3x^3+d^4x)}\right]$$

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/4*(((3*b*c^2 - 2*a*c*d)*x^3 + (3*b*c*d - 2*a*d^2)*x)*sqrt(d)*log(-(c*x^
2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2) + 2*(b*d^2 + (3*b*c*d - 2
*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/(c*d^3*x^3 + d^4*x), -1/2*(((3*b*c^2 -
2*a*c*d)*x^3 + (3*b*c*d - 2*a*d^2)*x)*sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^
2 + d)/x^2)/(c*x^2 + d)) + (b*d^2 + (3*b*c*d - 2*a*d^2)*x^2)*sqrt((c*x^2 +
d)/x^2))/(c*d^3*x^3 + d^4*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(76) = 152.

Time = 6.59 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.85

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^4}\,dx = a\left(\frac{cd^2x^2\log\left(\frac{cx^2}{d}\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}-\frac{2cd^2x^2\log\left(\sqrt{\frac{cx^2}{d}+1}+1\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}+\frac{2d^3\sqrt{\frac{cx^2}{d}+1}}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}+\frac{d^3\log\left(\frac{cx^2}{d}\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}-\frac{2d^3\log\left(\sqrt{\frac{cx^2}{d}+1}+1\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}\right)+b\left(-\frac{3\sqrt{c}}{2d^2x\sqrt{1+\frac{d}{cx^2}}}+\frac{3c\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2d^{\frac{5}{2}}}-\frac{1}{2\sqrt{c}dx^3\sqrt{1+\frac{d}{cx^2}}}\right)$$

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**4,x)
```

```
[Out] a*(c*d**2*x**2*log(c*x**2/d)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) - 2*c*d**2*x**2*log(sqrt(c*x**2/d + 1) + 1)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) + 2*d**3*sqrt(c*x**2/d + 1)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) + d**3*log(c*x**2/d)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) - 2*d**3*log(sqrt(c*x**2/d + 1) + 1)/(2*c*d**(7/2)*x**2 + 2*d**(9/2))) + b*(-3*sqrt(c)/(2*d**2*x*sqrt(1 + d/(c*x**2))) + 3*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*d**(5/2)) - 1/(2*sqrt(c)*d*x**3*sqrt(1 + d/(c*x**2))))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(76) = 152$.

Time = 0.28 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.76

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}\,dx = -\frac{1}{4}\,b\left(\frac{2\,\left(3\,\left(c+\frac{d}{x^2}\right)cx^2-2\,cd\right)}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2x^3-\sqrt{c+\frac{d}{x^2}}d^3x}+\frac{3\,c\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{5}{2}}}\right)+\frac{1}{2}\,a\left(\frac{\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2}{\sqrt{c+\frac{d}{x^2}}dx}\right)$$

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/4*b*(2*(3*(c + d/x^2)*c*x^2 - 2*c*d)/((c + d/x^2)^(3/2)*d^2*x^3 - sqrt(c + d/x^2)*d^3*x) + 3*c*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(5/2)) + 1/2*a*(log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(3/2) + 2/(sqrt(c + d/x^2)*d*x))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.16

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}\,dx = -\frac{(3\,bc-2\,ad)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{2\,\sqrt{-d}d^2\mathrm{sgn}\left(x\right)}-\frac{3\,(cx^2+d)bc-2\,(cx^2+d)ad-2\,bcd+2\,ad^2}{2\left(\left(cx^2+d\right)^{\frac{3}{2}}-\sqrt{cx^2+d}d\right)d^2\mathrm{sgn}\left(x\right)}$$

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/2*(3*b*c - 2*a*d)*arctan(sqrt(c*x^2 + d)/sqrt(-d))/(sqrt(-d)*d^2*sgn(x)) - 1/2*(3*(c*x^2 + d)*b*c - 2*(c*x^2 + d)*a*d - 2*b*c*d + 2*a*d^2)/(((c*x^2 + d)^(3/2) - sqrt(c*x^2 + d)*d)*d^2*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}\,dx=\int \frac{a+\frac{b}{x^2}}{x^4\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$$

```
[In] int((a + b/x^2)/(x^4*(c + d/x^2)^(3/2)),x)
[Out] int((a + b/x^2)/(x^4*(c + d/x^2)^(3/2)), x)
```

3.985 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^6}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 123

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^6}\,dx=-\frac{b}{4d\sqrt{c+\frac{d}{x^2}}x^5}-\frac{5bc-4ad}{4d^2\sqrt{c+\frac{d}{x^2}}x^3}+\frac{3(5bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^3x}-\frac{3c(5bc-4ad)\operatorname{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)}{8d^{7/2}}$$

```
[Out] -3/8*c*(-4*a*d+5*b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(7/2)-1/4*b/d/x^
5/(c+d/x^2)^(1/2)+1/4*(4*a*d-5*b*c)/d^2/x^3/(c+d/x^2)^(1/2)+3/8*(-4*a*d+5*b
*c)*(c+d/x^2)^(1/2)/d^3/x
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {470, 342, 294, 327, 223, 212}

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^6}\,dx=-\frac{3c(5bc-4ad)\operatorname{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{8d^{7/2}}+\frac{3\sqrt{c+\frac{d}{x^2}}(5bc-4ad)}{8d^3x}-\frac{5bc-4ad}{4d^2x^3\sqrt{c+\frac{d}{x^2}}}-\frac{b}{4dx^5\sqrt{c+\frac{d}{x^2}}}$$

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^6),x]
```

```
[Out] -1/4*b/(d*Sqrt[c + d/x^2]*x^5) - (5*b*c - 4*a*d)/(4*d^2*Sqrt[c + d/x^2]*x^3
) + (3*(5*b*c - 4*a*d)*Sqrt[c + d/x^2])/(8*d^3*x) - (3*c*(5*b*c - 4*a*d)*Ar
cTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(8*d^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{b}{4d\sqrt{c+\frac{d}{x^2}}x^5} + \frac{(-5bc+4ad)\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}x^6}\,dx}{4d}$$

$$= -\frac{b}{4d\sqrt{c+\frac{d}{x^2}}x^5} - \frac{(-5bc+4ad)\text{Subst}\left(\int \frac{x^4}{(c+dx^2)^{3/2}}\,dx, x, \frac{1}{x}\right)}{4d}$$

$$= -\frac{b}{4d\sqrt{c+\frac{d}{x^2}}x^5} - \frac{5bc-4ad}{4d^2\sqrt{c+\frac{d}{x^2}}x^3} + \frac{(3(5bc-4ad))\text{Subst}\left(\int \frac{x^2}{\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{4d^2}$$

$$= -\frac{b}{4d\sqrt{c+\frac{d}{x^2}}x^5} - \frac{5bc-4ad}{4d^2\sqrt{c+\frac{d}{x^2}}x^3} + \frac{3(5bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^3x} - \frac{(3c(5bc-4ad))\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{8d^3}$$

$$= -\frac{b}{4d\sqrt{c+\frac{d}{x^2}}x^5} - \frac{5bc-4ad}{4d^2\sqrt{c+\frac{d}{x^2}}x^3} + \frac{3(5bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^3x} - \frac{(3c(5bc-4ad))\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{1}{\sqrt{c+\frac{d}{x^2}}x}\right)}{8d^3}$$

$$= -\frac{b}{4d\sqrt{c+\frac{d}{x^2}}x^5} - \frac{5bc-4ad}{4d^2\sqrt{c+\frac{d}{x^2}}x^3} + \frac{3(5bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^3x} - \frac{3c(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)}{8d^{7/2}}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.92

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^6}\,dx = \frac{\sqrt{d}(-4adx^2(d+3cx^2)+b(-2d^2+5cdx^2+15c^2x^4)) - 3c(5bc-4ad)x^4\sqrt{d+cx^2}\text{arcta}}{8d^{7/2}\sqrt{c+\frac{d}{x^2}}x^5}$$

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^6),x]
[Out] (Sqrt[d]*(-4*a*d*x^2*(d + 3*c*x^2) + b*(-2*d^2 + 5*c*d*x^2 + 15*c^2*x^4)) -
 3*c*(5*b*c - 4*a*d)*x^4*Sqrt[d + c*x^2]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/
(8*d^(7/2)*Sqrt[c + d/x^2]*x^5)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.26

method	result
risch	$-\frac{(c\,x^{2}+d)\left(4ad\,x^{2}-7cb\,x^{2}+2bd\right)}{8d^{3}x^{5}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}-\frac{c\left(-\frac{4ad-7bc}{\sqrt{c\,x^{2}+d}}+3d(4ad-5bc)\left(\frac{1}{d\sqrt{c\,x^{2}+d}}-\frac{\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)}{d^{\frac{3}{2}}}\right)\right)\sqrt{c\,x^{2}+d}}{8d^{3}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}$
default	$-\frac{(c\,x^{2}+d)\left(12d^{\frac{5}{2}}ac\,x^{4}-15d^{\frac{3}{2}}b\,c^{2}x^{4}-12\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)\sqrt{c\,x^{2}+d}\,ac\,d^{2}x^{4}+15\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)\sqrt{c\,x^{2}+d}\,b\,c^{2}d\,x^{4}+4d^{\frac{7}{2}}a}{8\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}x^{7}d^{\frac{9}{2}}}$

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(c*x^2+d)*(4*a*d*x^2-7*b*c*x^2+2*b*d)/d^3/x^5/((c*x^2+d)/x^2)^(1/2)-1/
8*c/d^3*(-(4*a*d-7*b*c)/(c*x^2+d)^(1/2)+3*d*(4*a*d-5*b*c)*(1/d/(c*x^2+d)^(1
/2)-1/d^(3/2)*ln((2*d+2*d^(1/2)*(c*x^2+d)^(1/2))/x)))/((c*x^2+d)/x^2)^(1/2)
/x*(c*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.55

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^6}\,dx=\left[-\frac{3\left((5\,bc^3-4\,ac^2d)x^5+(5\,bc^2d-4\,acd^2)x^3\right)\sqrt{d}\log\left(-\frac{cx^2+2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)-2\,(3}{16\,(cd^4x^5+d^5x^3)}\right.$$

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] [-1/16*(3*((5*b*c^3 - 4*a*c^2*d)*x^5 + (5*b*c^2*d - 4*a*c*d^2)*x^3)*sqrt(d)
*log(-(c*x^2 + 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2) - 2*(3*(5*b*c^
2*d - 4*a*c*d^2)*x^4 - 2*b*d^3 + (5*b*c*d^2 - 4*a*d^3)*x^2)*sqrt((c*x^2 + d
)/x^2))/(c*d^4*x^5 + d^5*x^3), 1/8*(3*((5*b*c^3 - 4*a*c^2*d)*x^5 + (5*b*c^2
*d - 4*a*c*d^2)*x^3)*sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^
2 + d)) + (3*(5*b*c^2*d - 4*a*c*d^2)*x^4 - 2*b*d^3 + (5*b*c*d^2 - 4*a*d^3)*
x^2)*sqrt((c*x^2 + d)/x^2))/(c*d^4*x^5 + d^5*x^3)]
```

Sympy [A] (verification not implemented)

Time = 11.33 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.46

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^6}\,dx = a\left(-\frac{3\sqrt{c}}{2d^2x\sqrt{1+\frac{d}{cx^2}}}+\frac{3c\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2d^{\frac{5}{2}}}-\frac{1}{2\sqrt{c}dx^3\sqrt{1+\frac{d}{cx^2}}}\right) + b\left(\frac{15c^{\frac{3}{2}}}{8d^3x\sqrt{1+\frac{d}{cx^2}}}+\frac{5\sqrt{c}}{8d^2x^3\sqrt{1+\frac{d}{cx^2}}}-\frac{15c^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{8d^{\frac{7}{2}}}-\frac{1}{4\sqrt{c}dx^5\sqrt{1+\frac{d}{cx^2}}}\right)$$

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**6,x)
```

```
[Out] a*(-3*sqrt(c)/(2*d**2*x*sqrt(1 + d/(c*x**2))) + 3*c*asinh(sqrt(d)/(sqrt(c)*
x))/(2*d**(5/2)) - 1/(2*sqrt(c)*d*x**3*sqrt(1 + d/(c*x**2)))) + b*(15*c**(3
/2)/(8*d**3*x*sqrt(1 + d/(c*x**2))) + 5*sqrt(c)/(8*d**2*x**3*sqrt(1 + d/(c*
x**2))) - 15*c**2*asinh(sqrt(d)/(sqrt(c)*x))/(8*d**(7/2)) - 1/(4*sqrt(c)*d*
x**5*sqrt(1 + d/(c*x**2))))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(103) = 206.

Time = 0.27 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.98

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^6}\,dx = \frac{1}{16}\,b\left(\frac{2\left(15\left(c+\frac{d}{x^2}\right)^2c^2x^4-25\left(c+\frac{d}{x^2}\right)c^2dx^2+8\,c^2d^2\right)}{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^3x^5-2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^4x^3+\sqrt{c+\frac{d}{x^2}}d^5x}+\frac{15\,c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{7}{2}}}\right) - \frac{1}{4}\,a\left(\frac{2\left(3\left(c+\frac{d}{x^2}\right)cx^2-2\,cd\right)}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2x^3-\sqrt{c+\frac{d}{x^2}}d^3x}+\frac{3\,c\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{5}{2}}}\right)$$

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] 1/16*b*(2*(15*(c + d/x^2)^2*c^2*x^4 - 25*(c + d/x^2)*c^2*d*x^2 + 8*c^2*d^2)
/((c + d/x^2)^(5/2)*d^3*x^5 - 2*(c + d/x^2)^(3/2)*d^4*x^3 + sqrt(c + d/x^2)
*d^5*x) + 15*c^2*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqr
t(d)))/d^(7/2)) - 1/4*a*(2*(3*(c + d/x^2)*c*x^2 - 2*c*d)/((c + d/x^2)^(3/2)
*d^2*x^3 - sqrt(c + d/x^2)*d^3*x) + 3*c*log((sqrt(c + d/x^2)*x - sqrt(d))/(
sqrt(c + d/x^2)*x + sqrt(d)))/d^(5/2))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.20

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^6}\,dx = \frac{3\,(5\,bc^2-4\,acd)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{8\,\sqrt{-d}d^3\mathrm{sgn}\,(x)} + \frac{bc^2-acd}{\sqrt{cx^2+d}d^3\mathrm{sgn}\,(x)} + \frac{7\,(cx^2+d)^{\frac{3}{2}}bc^2-4\,(cx^2+d)^{\frac{3}{2}}acd-9\,\sqrt{cx^2+d}bc^2d+4\,\sqrt{cx^2+d}acd^2}{8\,c^2d^3x^4\mathrm{sgn}\,(x)}$$

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 3/8*(5*b*c^2 - 4*a*c*d)*arctan(sqrt(c*x^2 + d)/sqrt(-d))/(sqrt(-d)*d^3*sgn(
x)) + (b*c^2 - a*c*d)/(sqrt(c*x^2 + d)*d^3*sgn(x)) + 1/8*(7*(c*x^2 + d)^(3/
2)*b*c^2 - 4*(c*x^2 + d)^(3/2)*a*c*d - 9*sqrt(c*x^2 + d)*b*c^2*d + 4*sqrt(c
*x^2 + d)*a*c*d^2)/(c^2*d^3*x^4*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^6}\,dx = \int \frac{a+\frac{b}{x^2}}{x^6\,\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$$

```
[In] int((a + b/x^2)/(x^6*(c + d/x^2)^(3/2)),x)
```

```
[Out] int((a + b/x^2)/(x^6*(c + d/x^2)^(3/2)), x)
```

3.986 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m \, dx$

Optimal result

Integrand size = 24, antiderivative size = 105

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m \, dx = \frac{\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} (ex)^{1+m} \operatorname{AppellF1}\left(\frac{1}{2}(-1-m),-p,-q,\frac{1-m}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e(1+m)}$$

```
[Out] (a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1+m)*AppellF1(-1/2-1/2*m,-p,-q,-1/2*m+1/2,-b
/a/x^2,-d/c/x^2)/e/(1+m)/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {511, 525, 524}

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m \, dx = \frac{x(ex)^m \left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \operatorname{AppellF1}\left(\frac{1}{2}(-m-1),-p,-q,\frac{1-m}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{m+1}$$

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^m,x]
```

```
[Out] ((a + b/x^2)^p*(c + d/x^2)^q*x*(e*x)^m*AppellF1[(-1 - m)/2, -p, -q, (1 - m)
/2, -(b/(a*x^2)), -(d/(c*x^2))])/((1 + m)*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))
^q)
```

Rule 511

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Dist[(-(e*x)^m)*(x^(-1))^m, Subst[Int[(a + b/x^n)^p*((
c + d/x^n)^q/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m, p, q},
 x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = -\left(\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int x^{-2-m} \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx, x, \frac{1}{x}\right)\right)$$

$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int x^{-2-m} \left(1+\frac{bx^2}{a}\right)^p \left(c+dx^2\right)^q \, dx, x, \frac{1}{x}\right)\right)$$

$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} \left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int x^{-2-m} \left(1+\frac{bx^2}{a}\right)\right.\right.$$

$$= \frac{\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} x(ex)^m F_1\left(\frac{1}{2}(-1-m); -p, -q; \frac{1-m}{2}; -\frac{b}{ax^2}, -\frac{d}{cx^2}\right)}{1+m}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.10

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m\,dx$$
$$=\frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x(ex)^m\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{1}{2}(1+m-2p-2q),-p,-q,\frac{1}{2}(3+m-2p\right.}{1+m-2p-2q}$$

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^m,x]
[Out] ((a + b/x^2)^p*(c + d/x^2)^q*x*(e*x)^m*AppellF1[(1 + m - 2*p - 2*q)/2, -p,
-q, (3 + m - 2*p - 2*q)/2, -((a*x^2)/b), -((c*x^2)/d)])/((1 + m - 2*p - 2*q
)*(1 + (a*x^2)/b)^p*(1 + (c*x^2)/d)^q)
```

Maple [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m\,dx$$

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^m,x)
[Out] int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^m,x)
```

Fricas [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m\,dx=\int (ex)^m\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q\,dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^m,x, algorithm="fricas")
[Out] integral((e*x)^m*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m\,dx=\text{Timed out}$$

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q*(e*x)**m,x)
[Out] Timed out
```

Maxima [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m\,dx = \int (ex)^m \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q\,dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^m,x, algorithm="maxima")
[Out] integrate((e*x)^m*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

Giac [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m\,dx = \int (ex)^m \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q\,dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^m,x, algorithm="giac")
[Out] integrate((e*x)^m*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m\,dx = \int (e\,x)^m \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q\,dx$$

```
[In] int((e*x)^m*(a + b/x^2)^p*(c + d/x^2)^q,x)
[Out] int((e*x)^m*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.987 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx=\frac{1}{5}\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} x^5\,\text{AppellF1}\left(-\frac{5}{2},-p,-q,-\frac{3}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

```
[Out] 1/5*(a+b/x^2)^p*(c+d/x^2)^q*x^5*AppellF1(-5/2,-p,-q,-3/2,-b/a/x^2,-d/c/x^2)
/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {509, 525, 524}

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx=\frac{1}{5}x^5\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q}\text{AppellF1}\left(-\frac{5}{2},-p,-q,-\frac{3}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q*x^4,x]
```

```
[Out] ((a + b/x^2)^p*(c + d/x^2)^q*x^5*AppellF1[-5/2, -p, -q, -3/2, -(b/(a*x^2)),
 -(d/(c*x^2))])/(5*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rule 509

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^(m + 2)), x], x, 1/
```

```
x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] && In
tegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{(a+bx^2)^p\,(c+dx^2)^q}{x^6}\,dx, x, \frac{1}{x}\right)$$

$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p (c+dx^2)^q}{x^6}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p \left(1+\frac{dx^2}{c}\right)^q}{x^6}\,dx, x, \frac{1}{x}\right)\right)$$

$$= \frac{1}{5}\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} x^5 F_1\left(-\frac{5}{2}; -p, -q; -\frac{3}{2}; -\frac{b}{ax^2}, -\frac{d}{cx^2}\right)$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.26

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^5 \left(1+\frac{ax^2}{b}\right)^{-p} \left(1+\frac{cx^2}{d}\right)^{-q} \text{AppellF1}\left(\frac{5}{2}-p-q,-p,-q,\frac{7}{2}-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{-5+2p+2q}$$

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q*x^4,x]
[Out] -(((a + b/x^2)^p*(c + d/x^2)^q*x^5*AppellF1[5/2 - p - q, -p, -q, 7/2 - p -
q, -((a*x^2)/b), -((c*x^2)/d)])/((-5 + 2*p + 2*q)*(1 + (a*x^2)/b)^p*(1 + (c
*x^2)/d)^q))
```

Maple [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4dx$$

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q*x^4,x)
[Out] int((a+b/x^2)^p*(c+d/x^2)^q*x^4,x)
```

Fricas [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^4,x, algorithm="fricas")
[Out] integral(x^4*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx = \text{Timed out}$$

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q*x**4,x)
[Out] Timed out
```

Maxima [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^4,x, algorithm="maxima")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x^4, x)
```

Giac [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^4,x, algorithm="giac")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx = \int x^4 \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q\,dx$$

```
[In] int(x^4*(a + b/x^2)^p*(c + d/x^2)^q,x)
[Out] int(x^4*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.988 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 100

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx = \frac{b^2\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\operatorname{AppellF1}\left(1+p,-q,3,2+p,-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a^3(1+p)}$$

```
[Out] 1/2*b^2*(a+b/x^2)^(p+1)*(c+d/x^2)^q*AppellF1(p+1,3,-q,2+p,(a+b/x^2)/a,-d*(a
+b/x^2)/(-a*d+b*c))/a^3/(p+1)/((b*(c+d/x^2)/(-a*d+b*c))^q)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {457, 142, 141}

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx = \frac{b^2\left(a+\frac{b}{x^2}\right)^{p+1}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\operatorname{AppellF1}\left(p+1,-q,3,p+2,-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a^3(p+1)}$$

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q*x^3,x]
```

```
[Out] (b^2*(a + b/x^2)^(1 + p)*(c + d/x^2)^q*AppellF1[1 + p, -q, 3, 2 + p, -((d*(
a + b/x^2))/(b*c - a*d)), (a + b/x^2)/a])/(2*a^3*(1 + p)*((b*(c + d/x^2))/(
b*c - a*d))^q)
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^p(c+dx)^q}{x^3}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= -\left(\frac{1}{2}\left(\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\right)\text{Subst}\left(\int \frac{(a+bx)^p\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^q}{x^3}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= \frac{b^2\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}F_1\left(1+p;-q,3;2+p;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a^3(1+p)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3 \, dx = -\frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4 \left(1+\frac{ax^2}{b}\right)^{-p} \left(1+\frac{cx^2}{d}\right)^{-q} \operatorname{AppellF1}\left(2-p-q,-p,-q,3-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{2(-2+p+q)}$$

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q*x^3,x]
[Out] -1/2*((a + b/x^2)^p*(c + d/x^2)^q*x^4*AppellF1[2 - p - q, -p, -q, 3 - p - q
, -((a*x^2)/b), -((c*x^2)/d)])/((-2 + p + q)*(1 + (a*x^2)/b)^p*(1 + (c*x^2)
/d)^q)
```

Maple [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3 dx$$

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q*x^3,x)
[Out] int((a+b/x^2)^p*(c+d/x^2)^q*x^3,x)
```

Fricas [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3 \, dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3 \, dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^3,x, algorithm="fricas")
[Out] integral(x^3*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3 \, dx = \text{Timed out}$$

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q*x**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^3,x, algorithm="maxima")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x^3, x)
```

Giac [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^3,x, algorithm="giac")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx = \int x^3 \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q\,dx$$

```
[In] int(x^3*(a + b/x^2)^p*(c + d/x^2)^q,x)
[Out] int(x^3*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.989 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2 \, dx=\frac{1}{3}\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} x^3 \operatorname{AppellF1}\left(-\frac{3}{2},-p,-q,-\frac{1}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

```
[Out] 1/3*(a+b/x^2)^p*(c+d/x^2)^q*x^3*AppellF1(-3/2,-p,-q,-1/2,-b/a/x^2,-d/c/x^2)
/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {509, 525, 524}

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2 \, dx=\frac{1}{3}x^3\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \operatorname{AppellF1}\left(-\frac{3}{2},-p,-q,-\frac{1}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q*x^2,x]
```

```
[Out] ((a + b/x^2)^p*(c + d/x^2)^q*x^3*AppellF1[-3/2, -p, -q, -1/2, -(b/(a*x^2)),
 -(d/(c*x^2))])/(3*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rule 509

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^(m + 2)), x], x, 1/
```

```
x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] && In
tegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{(a+bx^2)^p\,(c+dx^2)^q}{x^4}\,dx, x, \frac{1}{x}\right)$$

$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p (c+dx^2)^q}{x^4}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p\left(1+\frac{dx^2}{c}\right)^q}{x^4}\,dx, x,\right.\right.$$

$$= \frac{1}{3}\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}x^3F_1\left(-\frac{3}{2};-p,-q;-\frac{1}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.26

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3 \left(1+\frac{ax^2}{b}\right)^{-p} \left(1+\frac{cx^2}{d}\right)^{-q} \text{AppellF1}\left(\frac{3}{2}-p-q,-p,-q,\frac{5}{2}-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{-3+2p+2q}$$

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q*x^2,x]
```

```
[Out] -(((a + b/x^2)^p*(c + d/x^2)^q*x^3*AppellF1[3/2 - p - q, -p, -q, 5/2 - p - q, -((a*x^2)/b), -((c*x^2)/d)])/((-3 + 2*p + 2*q)*(1 + (a*x^2)/b)^p*(1 + (c*x^2)/d)^q))
```

Maple [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2 dx$$

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q*x^2,x)
```

```
[Out] int((a+b/x^2)^p*(c+d/x^2)^q*x^2,x)
```

Fricas [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx = \text{Timed out}$$

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q*x**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^2,x, algorithm="maxima")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x^2, x)
```

Giac [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^2,x, algorithm="giac")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx = \int x^2 \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q\,dx$$

```
[In] int(x^2*(a + b/x^2)^p*(c + d/x^2)^q,x)
[Out] int(x^2*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.990 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx$

Optimal result

Integrand size = 20, antiderivative size = 98

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx = -\frac{b\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q \left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q} \text{AppellF1}\left(1+p,-q,2,2+p,-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a^2(1+p)}$$

```
[Out] -1/2*b*(a+b/x^2)^(p+1)*(c+d/x^2)^q*AppellF1(p+1,2,-q,2+p,(a+b/x^2)/a,-d*(a+
b/x^2)/(-a*d+b*c))/a^2/(p+1)/((b*(c+d/x^2)/(-a*d+b*c))^q)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {457, 142, 141}

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx = -\frac{b\left(a+\frac{b}{x^2}\right)^{p+1}\left(c+\frac{d}{x^2}\right)^q \left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q} \text{AppellF1}\left(p+1,-q,2,p+2,-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a^2(p+1)}$$

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q*x,x]
```

```
[Out] -1/2*(b*(a + b/x^2)^(1 + p)*(c + d/x^2)^q*AppellF1[1 + p, -q, 2, 2 + p, -((
d*(a + b/x^2))/(b*c - a*d)), (a + b/x^2)/a])/(a^2*(1 + p)*((b*(c + d/x^2))/
(b*c - a*d))^q)
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^p(c+dx)^q}{x^2}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= -\left(\frac{1}{2}\left(\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\right)\text{Subst}\left(\int \frac{(a+bx)^p\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^q}{x^2}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= -\frac{b\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}F_1\left(1+p;-q,2;2+p;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a^2(1+p)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2 \left(1+\frac{ax^2}{b}\right)^{-p} \left(1+\frac{cx^2}{d}\right)^{-q} \text{AppellF1}\left(1-p-q,-p,-q,2-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{2(-1+p+q)}$$

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q*x,x]
[Out] -1/2*((a + b/x^2)^p*(c + d/x^2)^q*x^2*AppellF1[1 - p - q, -p, -q, 2 - p - q
, -((a*x^2)/b), -((c*x^2)/d)])/((-1 + p + q)*(1 + (a*x^2)/b)^p*(1 + (c*x^2)
/d)^q)
```

Maple [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x dx$$

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q*x,x)
[Out] int((a+b/x^2)^p*(c+d/x^2)^q*x,x)
```

Fricas [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x,x, algorithm="fricas")
[Out] integral(x*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx = \text{Timed out}$$

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q*x,x)
[Out] Timed out
```

Maxima [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x,x, algorithm="maxima")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x, x)
```

Giac [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x,x, algorithm="giac")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx = \int x\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \,dx$$

```
[In] int(x*(a + b/x^2)^p*(c + d/x^2)^q,x)
[Out] int(x*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.991 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \, dx$

Optimal result

Integrand size = 19, antiderivative size = 79

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \, dx = \left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} x \operatorname{AppellF1}\left(-\frac{1}{2},-p,-q,\frac{1}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

```
[Out] (a+b/x^2)^p*(c+d/x^2)^q*x*AppellF1(-1/2,-p,-q,1/2,-b/a/x^2,-d/c/x^2)/((1+b/
a/x^2)^p)/((1+d/c/x^2)^q)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {382, 525, 524}

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \, dx = x\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \operatorname{AppellF1}\left(-\frac{1}{2},-p,-q,\frac{1}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q,x]
```

```
[Out] ((a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[-1/2, -p, -q, 1/2, -(b/(a*x^2)), -(
d/(c*x^2))])/((1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
```

```
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{(a+bx^2)^p\,(c+dx^2)^q}{x^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p (c+dx^2)^q}{x^2}\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p\left(1+\frac{dx^2}{c}\right)^q}{x^2}\,dx, x,\right.\right.$$

$$= \left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q} xF_1\left(-\frac{1}{2}; -p, -q; \frac{1}{2}; -\frac{b}{ax^2}, -\frac{d}{cx^2}\right)$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.32

$$\int \left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q dx = -\frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{1}{2}-p-q, -p, -q, \frac{3}{2}-p-q, -\frac{ax^2}{b}, -\frac{cx^2}{d}\right)}{-1+2p+2q}$$

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q,x]
```

```
[Out] -(((a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[1/2 - p - q, -p, -q, 3/2 - p - q,
 -((a*x^2)/b), -((c*x^2)/d)])/((-1 + 2*p + 2*q)*(1 + (a*x^2)/b)^p*(1 + (c*x
^2)/d)^q))
```

Maple [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q,x)
[Out] int((a+b/x^2)^p*(c+d/x^2)^q,x)
```

Fricas [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="fricas")
[Out] integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = \text{Timed out}$$

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q,x)
[Out] Timed out
```

Maxima [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="maxima")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q, x)
```

Giac [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="giac")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

```
[In] int((a + b/x^2)^p*(c + d/x^2)^q,x)
[Out] int((a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.992 $\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 97

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x}\,dx = \frac{\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q \left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q} \text{AppellF1}\left(1+p,-q,1,2+p,-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a(1+p)}$$

```
[Out] 1/2*(a+b/x^2)^(p+1)*(c+d/x^2)^q*AppellF1(p+1,1,-q,2+p,(a+b/x^2)/a,-d*(a+b/x
^2)/(-a*d+b*c))/a/(p+1)/((b*(c+d/x^2)/(-a*d+b*c))^q)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {457, 142, 141}

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x}\,dx = \frac{\left(a+\frac{b}{x^2}\right)^{p+1}\left(c+\frac{d}{x^2}\right)^q \left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q} \text{AppellF1}\left(p+1,-q,1,p+2,-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a(p+1)}$$

```
[In] Int[((a + b/x^2)^p*(c + d/x^2)^q)/x,x]
```

```
[Out] ((a + b/x^2)^(1 + p)*(c + d/x^2)^q*AppellF1[1 + p, -q, 1, 2 + p, -((d*(a +
b/x^2))/(b*c - a*d)), (a + b/x^2)/a])/(2*a*(1 + p)*((b*(c + d/x^2))/(b*c -
a*d))^q)
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 142

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^p(c+dx)^q}{x}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= -\left(\frac{1}{2}\left(\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\right)\text{Subst}\left(\int \frac{(a+bx)^p\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^q}{x}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= \frac{\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q} F_1\left(1+p;-q,1;2+p;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a(1+p)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x}\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{ax^2}{b}\right)^{-p} \left(1+\frac{cx^2}{d}\right)^{-q} \text{AppellF1}\left(-p-q,-p,-q,1-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{2(p+q)}$$

```
[In] Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/x,x]
[Out] -1/2*((a + b/x^2)^p*(c + d/x^2)^q*AppellF1[-p - q, -p, -q, 1 - p - q, -((a*
x^2)/b), -((c*x^2)/d)])/((p + q)*(1 + (a*x^2)/b)^p*(1 + (c*x^2)/d)^q)
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x}dx$$

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q/x,x)
[Out] int((a+b/x^2)^p*(c+d/x^2)^q/x,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x}\,dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x,x, algorithm="fricas")
[Out] integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x}\,dx = \text{Timed out}$$

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q/x,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x}\,dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x,x, algorithm="maxima")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/x, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x}\,dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x,x, algorithm="giac")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x}\,dx$$

```
[In] int(((a + b/x^2)^p*(c + d/x^2)^q)/x,x)
[Out] int(((a + b/x^2)^p*(c + d/x^2)^q)/x, x)
```

3.993 $\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 82

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{x}$$

```
[Out] -(a+b/x^2)^p*(c+d/x^2)^q*AppellF1(1/2,-p,-q,3/2,-b/a/x^2,-d/c/x^2)/((1+b/a/
x^2)^p)/((1+d/c/x^2)^q)/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {509, 441, 440}

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{x}$$

```
[In] Int[((a + b/x^2)^p*(c + d/x^2)^q)/x^2,x]
```

```
[Out] -(((a + b/x^2)^p*(c + d/x^2)^q*AppellF1[1/2, -p, -q, 3/2, -(b/(a*x^2)), -(d
/(c*x^2))])/((1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q*x))
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
```

```
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 509

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^(m + 2)), x], x, 1/
x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] && In
tegerQ[m]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int (a+bx^2)^p\,(c+dx^2)^q\,dx, x, \frac{1}{x}\right)$$

$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int\left(1+\frac{bx^2}{a}\right)^p(c+dx^2)^q\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int\left(1+\frac{bx^2}{a}\right)^p\left(1+\frac{dx^2}{c}\right)^q\,dx, x,\right.\right.$$

$$= -\frac{\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{x}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.29

$$\int\frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q}\text{AppellF1}\left(-\frac{1}{2}-p-q,-p,-q,\frac{1}{2}-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{(1+2p+2q)x}$$

```
[In] Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/x^2,x]
```

```
[Out] -(((a + b/x^2)^p*(c + d/x^2)^q*AppellF1[-1/2 - p - q, -p, -q, 1/2 - p - q,
-((a*x^2)/b), -((c*x^2)/d)])/((1 + 2*p + 2*q)*x*(1 + (a*x^2)/b)^p*(1 + (c*x
^2)/d)^q))
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^2}dx$$

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q/x^2,x)
[Out] int((a+b/x^2)^p*(c+d/x^2)^q/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^2,x, algorithm="fricas")
[Out] integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q/x**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^2,x, algorithm="maxima")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/x^2, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^2,x, algorithm="giac")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx$$

```
[In] int(((a + b/x^2)^p*(c + d/x^2)^q)/x^2,x)
[Out] int(((a + b/x^2)^p*(c + d/x^2)^q)/x^2, x)
```

3.994 $\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3} \, dx = -\frac{\left(a+\frac{b}{x^2}\right)^{1+p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q} \text{Hypergeometric2F1}\left(1+p,-q,2+p,-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad}\right)}{2b(1+p)}$$

```
[Out] -1/2*(a+b/x^2)^(p+1)*(c+d/x^2)^q*hypergeom([-q, p+1],[2+p],-d*(a+b/x^2)/(-a
*d+b*c))/b/(p+1)/((b*(c+d/x^2)/(-a*d+b*c))^q)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {455, 72, 71}

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3} \, dx = -\frac{\left(a+\frac{b}{x^2}\right)^{p+1} \left(c+\frac{d}{x^2}\right)^q \left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q} \text{Hypergeometric2F1}\left(p+1,-q,p+2,-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad}\right)}{2b(p+1)}$$

```
[In] Int[((a + b/x^2)^p*(c + d/x^2)^q)/x^3,x]
```

```
[Out] -1/2*((a + b/x^2)^(1 + p)*(c + d/x^2)^q*Hypergeometric2F1[1 + p, -q, 2 + p,
 -((d*(a + b/x^2))/(b*c - a*d))])/(b*(1 + p)*((b*(c + d/x^2))/(b*c - a*d))^
q)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{2}\text{Subst}\left(\int (a+bx)^p(c+dx)^q\,dx, x, \frac{1}{x^2}\right)\right)\\ &= -\left(\frac{1}{2}\left(\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\right)\text{Subst}\left(\int (a+bx)^p\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^q dx, x, \frac{1}{x^2}\right)\right)\\ &= -\frac{\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}{}_2F_1\left(1+p,-q;2+p;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad}\right)}{2b(1+p)}\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.29

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{ax^2}{b}\right)^{-p}(d+cx^2)\left(1+\frac{cx^2}{d}\right)^p\text{Hypergeometric2F1}\left(-p,-1-p-q,-p-q,\frac{(bc}{b(}\right.}{2d(1+p+q)x^2}$$

```
[In] Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/x^3,x]
```

```
[Out] -1/2*((a + b/x^2)^p*(c + d/x^2)^q*(d + c*x^2)*(1 + (c*x^2)/d)^p*Hypergeomet
ric2F1[-p, -1 - p - q, -p - q, ((b*c - a*d)*x^2)/(b*(d + c*x^2))])/(d*(1 +
p + q)*x^2*(1 + (a*x^2)/b)^p)
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3} dx$$

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q/x^3,x)
```

```
[Out] int((a+b/x^2)^p*(c+d/x^2)^q/x^3,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3} dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3} dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^3,x, algorithm="fricas")
```

```
[Out] integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3} dx = \text{Timed out}$$

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q/x**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3} dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3} dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/x^3, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^3,x, algorithm="giac")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx$$

```
[In] int(((a + b/x^2)^p*(c + d/x^2)^q)/x^3,x)
[Out] int(((a + b/x^2)^p*(c + d/x^2)^q)/x^3, x)
```

3.995 $\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4} \, dx = -\frac{\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} \text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3x^3}$$

```
[Out] -1/3*(a+b/x^2)^p*(c+d/x^2)^q*AppellF1(3/2,-p,-q,5/2,-b/a/x^2,-d/c/x^2)/((1+
b/a/x^2)^p)/((1+d/c/x^2)^q)/x^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {509, 525, 524}

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4} \, dx = -\frac{\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3x^3}$$

```
[In] Int[((a + b/x^2)^p*(c + d/x^2)^q)/x^4,x]
```

```
[Out] -1/3*((a + b/x^2)^p*(c + d/x^2)^q*AppellF1[3/2, -p, -q, 5/2, -(b/(a*x^2)),
-(d/(c*x^2))])/((1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q*x^3)
```

Rule 509

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^(m + 2)), x], x, 1/
```

```
x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] && In
tegerQ[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int x^2\left(a+bx^2\right)^p\left(c+dx^2\right)^q\,dx, x, \frac{1}{x}\right)$$

$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int x^2\left(1+\frac{bx^2}{a}\right)^p\left(c+dx^2\right)^q\,dx, x, \frac{1}{x}\right)\right)$$

$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int x^2\left(1+\frac{bx^2}{a}\right)^p\left(1+\frac{dx^2}{c}\right)^q\,dx,\right.\right.$$

$$= -\frac{\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}F_1\left(\frac{3}{2};-p,-q;\frac{5}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3x^3}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.26

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q}\text{AppellF1}\left(-\frac{3}{2}-p-q,-p,-q,-\frac{1}{2}-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{(3+2p+2q)x^3}$$

```
[In] Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/x^4,x]
```

[Out] -(((a + b/x^2)^p*(c + d/x^2)^q*AppellF1[-3/2 - p - q, -p, -q, -1/2 - p - q, -((a*x^2)/b), -((c*x^2)/d)])/((3 + 2*p + 2*q)*x^3*(1 + (a*x^2)/b)^p*(1 + (c*x^2)/d)^q))

Maple [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4}dx$$

[In] int((a+b/x^2)^p*(c+d/x^2)^q/x^4,x)

[Out] int((a+b/x^2)^p*(c+d/x^2)^q/x^4,x)

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx$$

[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^4,x, algorithm="fricas")

[Out] integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/x^4, x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx=\text{Timed out}$$

[In] integrate((a+b/x**2)**p*(c+d/x**2)**q/x**4,x)

[Out] Timed out

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx$$

[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^4,x, algorithm="maxima")

[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/x^4, x)

Giac [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^4,x, algorithm="giac")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx$$

```
[In] int(((a + b/x^2)^p*(c + d/x^2)^q)/x^4,x)
[Out] int(((a + b/x^2)^p*(c + d/x^2)^q)/x^4, x)
```

3.996 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{5/2} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 91

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{5/2} \, dx = \frac{2\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} (ex)^{7/2} \text{AppellF1}\left(-\frac{7}{4},-p,-q,-\frac{3}{4},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{7e}$$

```
[Out] 2/7*(a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(7/2)*AppellF1(-7/4,-p,-q,-3/4,-b/a/x^2,-
d/c/x^2)/e/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {510, 525, 524}

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{5/2} \, dx = \frac{2(ex)^{7/2} \left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \text{AppellF1}\left(-\frac{7}{4},-p,-q,-\frac{3}{4},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{7e}$$

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(5/2),x]
```

```
[Out] (2*(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(7/2)*AppellF1[-7/4, -p, -q, -3/4, -(b
/(a*x^2)), -(d/(c*x^2))])/(7*e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rule 510

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, Dist[-g/e, Subst[Int[(a + b
```

```
/(e^n*x^(g*n)))^p*((c + d/(e^n*x^(g*n)))^q/x^(g*(m + 1) + 1)), x], x, 1/(e*
x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fraction
Q[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = -\frac{2\text{Subst}\left(\int \frac{(a+be^2x^4)^p(c+de^2x^4)^q}{x^8}\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{be^2x^4}{a}\right)^p(c+de^2x^4)^q}{x^8}\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int \frac{\left(1+\frac{be^2x^4}{a}\right)^p\left(1+\frac{de^2x^4}{c}\right)^q}{x^8}\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= \frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}(ex)^{7/2}F_1\left(-\frac{7}{4};-p,-q;-\frac{3}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{7e}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.22

$$\int \left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q(ex)^{5/2}\,dx =$$

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x(ex)^{5/2}\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{7}{4}-p-q,-p,-q,\frac{11}{4}-p-q,-\frac{ax^2}{b},-\right.}{-7+4p+4q}$$

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(5/2),x]
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*x*(e*x)^(5/2)*AppellF1[7/4 - p - q, -p, -q,
 11/4 - p - q, -((a*x^2)/b), -((c*x^2)/d)])/((-7 + 4*p + 4*q)*(1 + (a*x^2)/
b)^p*(1 + (c*x^2)/d)^q)
```

Maple [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{\frac{5}{2}}\, dx$$

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(5/2),x)
[Out] int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(5/2),x)
```

Fricas [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{5/2}\, dx = \int (ex)^{\frac{5}{2}} \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x)*e^2*x^2*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{5/2}\, dx = \text{Timed out}$$

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q*(e*x)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{5/2}\, dx = \int (ex)^{\frac{5}{2}} \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(5/2),x, algorithm="maxima")
[Out] integrate((e*x)^(5/2)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{5/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[0,6,1,1,0]%%%} / %%%{1,[0,0,0,0,3]%%%} Error: Bad Argument
 Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{5/2}\,dx = \int (e\,x)^{5/2} \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

```
[In] int((e*x)^(5/2)*(a + b/x^2)^p*(c + d/x^2)^q,x)
```

```
[Out] int((e*x)^(5/2)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.997 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 91

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{3/2}\,dx=\frac{2\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} (ex)^{5/2}\operatorname{AppellF1}\left(-\frac{5}{4},-p,-q,-\frac{1}{4},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{5e}$$

```
[Out] 2/5*(a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(5/2)*AppellF1(-5/4,-p,-q,-1/4,-b/a/x^2,-
d/c/x^2)/e/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {510, 525, 524}

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{3/2}\,dx=\frac{2(ex)^{5/2}\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q}\operatorname{AppellF1}\left(-\frac{5}{4},-p,-q,-\frac{1}{4},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{5e}$$

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(3/2),x]
```

```
[Out] (2*(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(5/2)*AppellF1[-5/4, -p, -q, -1/4, -(b
/(a*x^2)), -(d/(c*x^2))])/(5*e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rule 510

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, Dist[-g/e, Subst[Int[(a + b
```

```
/(e^n*x^(g*n)))^p*((c + d/(e^n*x^(g*n)))^q/x^(g*(m + 1) + 1)), x], x, 1/(e*
x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fraction
Q[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = -\frac{2\text{Subst}\left(\int \frac{(a+be^2x^4)^p(c+de^2x^4)^q}{x^6}\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{be^2x^4}{a}\right)^p(c+de^2x^4)^q}{x^6}\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int \frac{\left(1+\frac{be^2x^4}{a}\right)^p\left(1+\frac{de^2x^4}{c}\right)^q}{x^6}\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= \frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}(ex)^{5/2}F_1\left(-\frac{5}{4};-p,-q;-\frac{1}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{5e}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.22

$$\int \left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q(ex)^{3/2}\,dx =$$

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x(ex)^{3/2}\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{5}{4}-p-q,-p,-q,\frac{9}{4}-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{-5+4p+4q}$$

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(3/2),x]
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*x*(e*x)^(3/2)*AppellF1[5/4 - p - q, -p, -q,
 9/4 - p - q, -((a*x^2)/b), -((c*x^2)/d)])/((-5 + 4*p + 4*q)*(1 + (a*x^2)/b
)^p*(1 + (c*x^2)/d)^q)
```

Maple [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{\frac{3}{2}}\,dx$$

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2),x)
[Out] int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2),x)
```

Fricas [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{3/2}\,dx=\int (ex)^{\frac{3}{2}}\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q\,dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x)*e*x*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{3/2}\,dx=\text{Timed out}$$

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q*(e*x)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{3/2}\,dx=\int (ex)^{\frac{3}{2}}\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q\,dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2),x, algorithm="maxima")
[Out] integrate((e*x)^(3/2)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[0,4,1,1,0]%%%} / %%%{1,[0,0,0,0,2]%%%} Error: Bad Argument
 Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{3/2}\,dx = \int (e\,x)^{3/2}\,\left(a+\frac{b}{x^2}\right)^p\,\left(c+\frac{d}{x^2}\right)^q\,dx$$

```
[In] int((e*x)^(3/2)*(a + b/x^2)^p*(c + d/x^2)^q,x)
```

```
[Out] int((e*x)^(3/2)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.998 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 91

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}\,dx = \frac{2\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} (ex)^{3/2}\,\text{AppellF1}\left(-\frac{3}{4},-p,-q,\frac{1}{4},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3e}$$

```
[Out] 2/3*(a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2)*AppellF1(-3/4,-p,-q,1/4,-b/a/x^2,-d
/c/x^2)/e/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {510, 525, 524}

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}\,dx = \frac{2(ex)^{3/2} \left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \text{AppellF1}\left(-\frac{3}{4},-p,-q,\frac{1}{4},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3e}$$

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q*Sqrt[e*x],x]
```

```
[Out] (2*(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(3/2)*AppellF1[-3/4, -p, -q, 1/4, -(b/
(a*x^2)), -(d/(c*x^2))])/(3*e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rule 510

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, Dist[-g/e, Subst[Int[(a + b
```

```
/(e^n*x^(g*n)))^p*((c + d/(e^n*x^(g*n)))^q/x^(g*(m + 1) + 1)), x], x, 1/(e*
x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fraction
Q[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\text{Subst}\left(\int \frac{(a+be^2x^4)^p(c+de^2x^4)^q}{x^4}\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e} \\
&= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{be^2x^4}{a}\right)^p(c+de^2x^4)^q}{x^4}\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e} \\
&= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int \frac{\left(1+\frac{be^2x^4}{a}\right)^p\left(1+\frac{de^2x^4}{c}\right)^q}{x^4}\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e} \\
&= \frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}(ex)^{3/2}F_1\left(-\frac{3}{4};-p,-q;\frac{1}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3e}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.22

$$
\int \left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q\sqrt{ex}\,dx = -\frac{2\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x\sqrt{ex}\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{3}{4}-p-q,-p,-q,\frac{7}{4}-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{-3+4p+4q}
$$

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q*Sqrt[e*x],x]
```

```
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*x*Sqrt[e*x]*AppellF1[3/4 - p - q, -p, -q, 7
/4 - p - q, -((a*x^2)/b), -((c*x^2)/d)])/((-3 + 4*p + 4*q)*(1 + (a*x^2)/b)^
p*(1 + (c*x^2)/d)^q)
```

Maple [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}dx$$

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1/2),x)
```

```
[Out] int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1/2),x)
```

Fricas [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}\,dx=\int \sqrt{ex}\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x)*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}\,dx=\text{Timed out}$$

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q*(e*x)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}\,dx=\int \sqrt{ex}\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

Giac [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}\,dx = \int \sqrt{ex}\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}\,dx = \int \sqrt{e\,x}\,\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

```
[In] int((e*x)^(1/2)*(a + b/x^2)^p*(c + d/x^2)^q,x)
[Out] int((e*x)^(1/2)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.999 $\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 89

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}} \, dx = \frac{2\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} \sqrt{ex}\,\text{AppellF1}\left(-\frac{1}{4},-p,-q,\frac{3}{4},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e}$$

```
[Out] 2*(a+b/x^2)^p*(c+d/x^2)^q*AppellF1(-1/4,-p,-q,3/4,-b/a/x^2,-d/c/x^2)*(e*x)^
(1/2)/e/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {510, 525, 524}

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}} \, dx = \frac{2\sqrt{ex}\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \text{AppellF1}\left(-\frac{1}{4},-p,-q,\frac{3}{4},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e}$$

```
[In] Int[((a + b/x^2)^p*(c + d/x^2)^q)/Sqrt[e*x],x]
```

```
[Out] (2*(a + b/x^2)^p*(c + d/x^2)^q*Sqrt[e*x]*AppellF1[-1/4, -p, -q, 3/4, -(b/(a
*x^2)), -(d/(c*x^2))])/(e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rule 510

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, Dist[-g/e, Subst[Int[(a + b
/(e^n*x^(g*n)))^p*((c + d/(e^n*x^(g*n)))^q/x^(g*(m + 1) + 1)), x], x, 1/(e*
x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fraction
Q[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = -\frac{2\text{Subst}\left(\int \frac{(a+be^2x^4)^p(c+de^2x^4)^q}{x^2}\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{be^2x^4}{a}\right)^p(c+de^2x^4)^q}{x^2}\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int \frac{\left(1+\frac{be^2x^4}{a}\right)^p\left(1+\frac{de^2x^4}{c}\right)^q}{x^2}\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= \frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\sqrt{ex}F_1\left(-\frac{1}{4};-p,-q;\frac{3}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.25

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx = -\frac{2\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{1}{4}-p-q,-p,-q,\frac{5}{4}-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{(-1+4p+4q)\sqrt{ex}}$$

```
[In] Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/Sqrt[e*x],x]
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[1/4 - p - q, -p, -q, 5/4 - p - q
, -((a*x^2)/b), -((c*x^2)/d)])/((-1 + 4*p + 4*q)*Sqrt[e*x]*(1 + (a*x^2)/b)^
p*(1 + (c*x^2)/d)^q)
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}dx$$

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(1/2),x)
[Out] int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x)*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/(e*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q/(e*x)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx = \int \frac{\left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(1/2),x, algorithm="maxima")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/sqrt(e*x), x)
```

Giac [F]

$$\int \frac{\left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx = \int \frac{\left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(1/2),x, algorithm="giac")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/sqrt(e*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx = \int \frac{\left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q}{\sqrt{e\,x}}\,dx$$

```
[In] int(((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(1/2),x)
[Out] int(((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(1/2), x)
```

3.1000 $\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 89

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}}\,dx = -\frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{4},-p,-q,\frac{5}{4},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e\sqrt{ex}}$$

```
[Out] -2*(a+b/x^2)^p*(c+d/x^2)^q*AppellF1(1/4,-p,-q,5/4,-b/a/x^2,-d/c/x^2)/e/((1+
b/a/x^2)^p)/((1+d/c/x^2)^q)/(e*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {510, 441, 440}

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}}\,dx = -\frac{2\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{4},-p,-q,\frac{5}{4},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e\sqrt{ex}}$$

```
[In] Int[((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(3/2),x]
```

```
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*AppellF1[1/4, -p, -q, 5/4, -(b/(a*x^2)), -(
d/(c*x^2))])/(e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q*Sqrt[e*x])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, Dist[-g/e, Subst[Int[(a + b
/(e^n*x^(g*n)))^p*((c + d/(e^n*x^(g*n)))^q/x^(g*(m + 1) + 1)), x], x, 1/(e*
x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fraction
Q[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\text{Subst}\left(\int (a+be^2x^4)^p\,(c+de^2x^4)^q\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e} \\
&= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int\left(1+\frac{be^2x^4}{a}\right)^p(c+de^2x^4)^q\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e} \\
&= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int\left(1+\frac{be^2x^4}{a}\right)^p\left(1+\frac{de^2x^4}{c}\right)^q dx, x, \frac{1}{\sqrt{ex}}\right)}{e} \\
&= -\frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}F_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e\sqrt{ex}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.25

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}}\,dx = -\frac{2\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q}\text{AppellF1}\left(-\frac{1}{4}-p-q,-p,-q,\frac{3}{4}-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{(1+4p+4q)(ex)^{3/2}}$$

```
[In] Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(3/2),x]
```

```
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[-1/4 - p - q, -p, -q, 3/4 - p -
q, -((a*x^2)/b), -((c*x^2)/d)])/((1 + 4*p + 4*q)*(e*x)^(3/2)*(1 + (a*x^2)/b
)^p*(1 + (c*x^2)/d)^q)
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x^{2}}\right)^{p} \left(c+\frac{d}{x^{2}}\right)^{q}}{\left(e x\right)^{\frac{3}{2}}}d x$$

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(3/2),x)
```

```
[Out] int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(3/2),x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}}\,dx = \int \frac{\left(a+\frac{b}{x^{2}}\right)^{p}\left(c+\frac{d}{x^{2}}\right)^{q}}{\left(e x\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x)*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/(e^2*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q/(e*x)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}}\,dx = \int \frac{\left(a+\frac{b}{x^{2}}\right)^{p}\left(c+\frac{d}{x^{2}}\right)^{q}}{\left(e x\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/(e*x)^(3/2), x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{\frac{3}{2}}}\,dx$$

[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(3/2),x, algorithm="giac")

[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/(e*x)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(e\,x)^{3/2}}\,dx$$

[In] int(((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(3/2),x)

[Out] int(((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(3/2), x)

3.1001 $\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 91

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}}\,dx = -\frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\operatorname{AppellF1}\left(\frac{3}{4},-p,-q,\frac{7}{4},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3e(ex)^{3/2}}$$

```
[Out] -2/3*(a+b/x^2)^p*(c+d/x^2)^q*AppellF1(3/4,-p,-q,7/4,-b/a/x^2,-d/c/x^2)/e/((
1+b/a/x^2)^p)/((1+d/c/x^2)^q)/(e*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {510, 525, 524}

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}}\,dx = -\frac{2\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}\operatorname{AppellF1}\left(\frac{3}{4},-p,-q,\frac{7}{4},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3e(ex)^{3/2}}$$

```
[In] Int[((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(5/2),x]
```

```
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*AppellF1[3/4, -p, -q, 7/4, -(b/(a*x^2)), -(
d/(c*x^2))])/(3*e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q*(e*x)^(3/2))
```

Rule 510

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, Dist[-g/e, Subst[Int[(a + b
/(e^n*x^(g*n)))^p*((c + d/(e^n*x^(g*n)))^q/x^(g*(m + 1) + 1)), x], x, 1/(e*
x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fraction
Q[m]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = -\frac{2\text{Subst}\left(\int x^2(a+be^2x^4)^p\,(c+de^2x^4)^q\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int x^2\left(1+\frac{be^2x^4}{a}\right)^p(c+de^2x^4)^q\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int x^2\left(1+\frac{be^2x^4}{a}\right)^p\left(1+\frac{de^2x^4}{c}\right)^q dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}F_1\left(\frac{3}{4};-p,-q;\frac{7}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3e(ex)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.22

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}} \, dx = -\frac{2\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\left(1+\frac{ax^2}{b}\right)^{-p} \left(1+\frac{cx^2}{d}\right)^{-q} \text{AppellF1}\left(-\frac{3}{4}-p-q,-p,-q,\frac{1}{4}-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{(3+4p+4q)(ex)^{5/2}}$$

```
[In] Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(5/2),x]
```

```
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[-3/4 - p - q, -p, -q, 1/4 - p - q, -((a*x^2)/b), -((c*x^2)/d)])/((3 + 4*p + 4*q)*(e*x)^(5/2)*(1 + (a*x^2)/b)^p*(1 + (c*x^2)/d)^q)
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{\frac{5}{2}}} dx$$

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(5/2),x)
```

```
[Out] int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(5/2),x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}} \, dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{\frac{5}{2}}} \, dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x)*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/(e^3*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}} \, dx = \text{Timed out}$$

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q/(e*x)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(5/2),x, algorithm="maxima")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/(e*x)^(5/2), x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(5/2),x, algorithm="giac")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/(e*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(e\,x)^{5/2}}\,dx$$

```
[In] int(((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(5/2),x)
[Out] int(((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(5/2), x)
```

3.1002 $\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 135

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}\,dx=-\frac{5}{64}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}-\frac{5}{96}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}-\frac{1}{24}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}+\frac{1}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}-\frac{5\operatorname{arccosh}\left(\sqrt{x}\right)}{64}$$

```
[Out] -5/64*arccosh(x^(1/2))-5/96*x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)-1/
24*x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)+1/4*x^(7/2)*(-1+x^(1/2))^(1
/2)*(1+x^(1/2))^(1/2)-5/64*x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {286, 329, 336, 54}

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}\,dx=-\frac{5\operatorname{arccosh}\left(\sqrt{x}\right)}{64}+\frac{1}{4}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{7/2}-\frac{1}{24}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}-\frac{5}{96}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}-\frac{5}{64}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}$$

```
[In] Int[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2),x]
```

```
[Out] (-5*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/64 - (5*Sqrt[-1 + Sqrt[x]
]*Sqrt[1 + Sqrt[x]]*x^(3/2))/96 - (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(
5/2))/24 + (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(7/2))/4 - (5*ArcCosh[Sq
rt[x]])/64
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 286

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^p*((a2 + b2*x^n)
^p/(c*(m + 2*n*p + 1))), x] + Dist[2*a1*a2*n*(p/(m + 2*n*p + 1)), Int[(c*x)
^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a2
, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && Ne
Q[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Dist[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1))), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 336

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}-\frac{1}{8}\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx\\ &= -\frac{1}{24}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}+\frac{1}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}-\frac{5}{48}\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx\\ &= -\frac{5}{96}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}-\frac{1}{24}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}\\ &\quad+\frac{1}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}-\frac{5}{64}\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx\end{aligned}$$

$$
\begin{aligned}
&= -\frac{5}{64}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} - \frac{5}{96}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} \\
&\quad - \frac{1}{24}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} + \frac{1}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2} \\
&\quad - \frac{5}{128}\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx \\
&= -\frac{5}{64}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} - \frac{5}{96}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} \\
&\quad - \frac{1}{24}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} + \frac{1}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2} \\
&\quad - \frac{5}{64}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}}\,dx, x, \sqrt{x}\right) \\
&= -\frac{5}{64}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} - \frac{5}{96}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} \\
&\quad - \frac{1}{24}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} + \frac{1}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2} - \frac{5}{64}\cosh^{-1}\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A] (warning: unable to verify)

Time = 7.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.73

$$
\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}\,dx = \frac{1}{192}\left(\sqrt{\frac{-1+\sqrt{x}}{1+\sqrt{x}}}\sqrt{x}(-15-15\sqrt{x}-10x-10x^{3/2}-8x^2-8x^{5/2}+48x^3+48x^7
$$

[In] Integrate[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2),x]

[Out] (Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]*Sqrt[x]*(-15 - 15*Sqrt[x] - 10*x - 10*x^(3/2) - 8*x^2 - 8*x^(5/2) + 48*x^3 + 48*x^(7/2)) - 30*ArcTanh[Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]])/192

Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.56

method	result	size
derivativedivides	$-\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(-48\sqrt{-1+x}\,x^{\frac{7}{2}}+8x^{\frac{5}{2}}\sqrt{-1+x}+10x^{\frac{3}{2}}\sqrt{-1+x}+15\sqrt{x}\,\sqrt{-1+x}+15\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{192\sqrt{-1+x}}$	75
default	$-\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(-48\sqrt{-1+x}\,x^{\frac{7}{2}}+8x^{\frac{5}{2}}\sqrt{-1+x}+10x^{\frac{3}{2}}\sqrt{-1+x}+15\sqrt{x}\,\sqrt{-1+x}+15\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{192\sqrt{-1+x}}$	75

[In] int(x^(5/2)*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2),x,method=_RETURNVERBOSE)

```
[Out] -1/192*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(-48*(-1+x)^(1/2)*x^(7/2)+8*x^(5
/2)*(-1+x)^(1/2)+10*x^(3/2)*(-1+x)^(1/2)+15*x^(1/2)*(-1+x)^(1/2)+15*ln(x^(1
/2)+(-1+x)^(1/2)))/(-1+x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.46

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}\,dx = \frac{1}{192}\left(48\,x^3-8\,x^2-10\,x-15\right)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} + \frac{5}{128}\,\log\left(2\,\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2\,x+1\right)$$

```
[In] integrate(x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/192*(48*x^3 - 8*x^2 - 10*x - 15)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) -
 1) + 5/128*log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

Sympy [F]

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}\,dx = \int x^{\frac{5}{2}}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\,dx$$

```
[In] integrate(x**(5/2)*(-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(x**(5/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.42

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}\,dx = \frac{1}{4}\,(x-1)^{\frac{3}{2}}x^{\frac{5}{2}} + \frac{5}{24}\,(x-1)^{\frac{3}{2}}x^{\frac{3}{2}} + \frac{5}{32}\,(x-1)^{\frac{3}{2}}\sqrt{x} + \frac{5}{64}\,\sqrt{x-1}\sqrt{x} - \frac{5}{64}\,\log\left(2\,\sqrt{x-1}+2\,\sqrt{x}\right)$$

```
[In] integrate(x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/4*(x - 1)^(3/2)*x^(5/2) + 5/24*(x - 1)^(3/2)*x^(3/2) + 5/32*(x - 1)^(3/2)
*sqrt(x) + 5/64*sqrt(x - 1)*sqrt(x) - 5/64*log(2*sqrt(x - 1) + 2*sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.20

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}\,dx = \frac{1}{6720}\left(\left(2\left(\left(4\left(5\left(6\left(7\sqrt{x}-50\right)\left(\sqrt{x}+1\right)+1219\right)\left(\sqrt{x}+1\right)-12463\right)\left(\sqrt{x}+\right.\right.\right.\right.\right.$$
$$+\frac{1}{840}\left(\left(2\left(\left(4\left(5\left(6\sqrt{x}-37\right)\left(\sqrt{x}+1\right)+661\right)\left(\sqrt{x}+1\right)-4551\right)\left(\sqrt{x}+1\right)+4781\right)\left(\sqrt{x}+1\right)-6335\right)\left(\sqrt{x}+1\right.\right.$$
$$+\frac{5}{32}\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

```
[In] integrate(x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="giac")
[Out] 1/6720*((2*((4*(5*(6*(7*sqrt(x) - 50)*(sqrt(x) + 1) + 1219)*(sqrt(x) + 1) -
 12463)*(sqrt(x) + 1) + 64233)*(sqrt(x) + 1) - 53963)*(sqrt(x) + 1) + 59465
)*(sqrt(x) + 1) - 23205)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/840*((2*((
4*(5*(6*sqrt(x) - 37)*(sqrt(x) + 1) + 661)*(sqrt(x) + 1) - 4551)*(sqrt(x) +
 1) + 4781)*(sqrt(x) + 1) - 6335)*(sqrt(x) + 1) + 2835)*sqrt(sqrt(x) + 1)*s
qrt(sqrt(x) - 1) + 5/32*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 75.08 (sec) , antiderivative size = 831, normalized size of antiderivative = 6.16

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}\,dx = \text{Too large to display}$$

```
[In] int(x^(5/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2),x)
[Out] ((1723*((x^(1/2) - 1)^(1/2) - 1i)^5)/(48*((x^(1/2) + 1)^(1/2) - 1)^5) - (23
5*((x^(1/2) - 1)^(1/2) - 1i)^3)/(48*((x^(1/2) + 1)^(1/2) - 1)^3) + (72283*(
(x^(1/2) - 1)^(1/2) - 1i)^7)/(16*((x^(1/2) + 1)^(1/2) - 1)^7) + (848801*((x
^(1/2) - 1)^(1/2) - 1i)^9)/(16*((x^(1/2) + 1)^(1/2) - 1)^9) + (4181067*((x^
(1/2) - 1)^(1/2) - 1i)^11)/(16*((x^(1/2) + 1)^(1/2) - 1)^11) + (10994181*((
x^(1/2) - 1)^(1/2) - 1i)^13)/(16*((x^(1/2) + 1)^(1/2) - 1)^13) + (17457599*
((x^(1/2) - 1)^(1/2) - 1i)^15)/(16*((x^(1/2) + 1)^(1/2) - 1)^15) + (1745759
9*((x^(1/2) - 1)^(1/2) - 1i)^17)/(16*((x^(1/2) + 1)^(1/2) - 1)^17) + (10994
181*((x^(1/2) - 1)^(1/2) - 1i)^19)/(16*((x^(1/2) + 1)^(1/2) - 1)^19) + (418
1067*((x^(1/2) - 1)^(1/2) - 1i)^21)/(16*((x^(1/2) + 1)^(1/2) - 1)^21) + (84
8801*((x^(1/2) - 1)^(1/2) - 1i)^23)/(16*((x^(1/2) + 1)^(1/2) - 1)^23) + (72
283*((x^(1/2) - 1)^(1/2) - 1i)^25)/(16*((x^(1/2) + 1)^(1/2) - 1)^25) + (172
3*((x^(1/2) - 1)^(1/2) - 1i)^27)/(48*((x^(1/2) + 1)^(1/2) - 1)^27) - (235*(
(x^(1/2) - 1)^(1/2) - 1i)^29)/(48*((x^(1/2) + 1)^(1/2) - 1)^29) + (5*((x^(1
/2) - 1)^(1/2) - 1i)^31)/(16*((x^(1/2) + 1)^(1/2) - 1)^31) + (5*((x^(1/2) -
```

```
1)^(1/2) - 1i))/(16*((x^(1/2) + 1)^(1/2) - 1)))/((120*((x^(1/2) - 1)^(1/2)
 - 1i)^4)/((x^(1/2) + 1)^(1/2) - 1)^4 - (16*((x^(1/2) - 1)^(1/2) - 1i)^2)/(
(x^(1/2) + 1)^(1/2) - 1)^2 - (560*((x^(1/2) - 1)^(1/2) - 1i)^6)/((x^(1/2) +
 1)^(1/2) - 1)^6 + (1820*((x^(1/2) - 1)^(1/2) - 1i)^8)/((x^(1/2) + 1)^(1/2)
 - 1)^8 - (4368*((x^(1/2) - 1)^(1/2) - 1i)^10)/((x^(1/2) + 1)^(1/2) - 1)^10
 + (8008*((x^(1/2) - 1)^(1/2) - 1i)^12)/((x^(1/2) + 1)^(1/2) - 1)^12 - (114
40*((x^(1/2) - 1)^(1/2) - 1i)^14)/((x^(1/2) + 1)^(1/2) - 1)^14 + (12870*((x
^(1/2) - 1)^(1/2) - 1i)^16)/((x^(1/2) + 1)^(1/2) - 1)^16 - (11440*((x^(1/2)
 - 1)^(1/2) - 1i)^18)/((x^(1/2) + 1)^(1/2) - 1)^18 + (8008*((x^(1/2) - 1)^(
1/2) - 1i)^20)/((x^(1/2) + 1)^(1/2) - 1)^20 - (4368*((x^(1/2) - 1)^(1/2) -
1i)^22)/((x^(1/2) + 1)^(1/2) - 1)^22 + (1820*((x^(1/2) - 1)^(1/2) - 1i)^24)
/((x^(1/2) + 1)^(1/2) - 1)^24 - (560*((x^(1/2) - 1)^(1/2) - 1i)^26)/((x^(1/
2) + 1)^(1/2) - 1)^26 + (120*((x^(1/2) - 1)^(1/2) - 1i)^28)/((x^(1/2) + 1)^
(1/2) - 1)^28 - (16*((x^(1/2) - 1)^(1/2) - 1i)^30)/((x^(1/2) + 1)^(1/2) - 1
)^30 + ((x^(1/2) - 1)^(1/2) - 1i)^32/((x^(1/2) + 1)^(1/2) - 1)^32 + 1) - (5
*atanh(((x^(1/2) - 1)^(1/2) - 1i)/((x^(1/2) + 1)^(1/2) - 1)))/16
```

3.1003 $\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 104

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\,dx = -\frac{1}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} - \frac{1}{12}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} + \frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} - \frac{\operatorname{arccosh}\left(\sqrt{x}\right)}{8}$$

```
[Out] -1/8*arccosh(x^(1/2))-1/12*x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)+1/3
*x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)-1/8*x^(1/2)*(-1+x^(1/2))^(1/2
)*(1+x^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {286, 329, 336, 54}

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\,dx = -\frac{\operatorname{arccosh}\left(\sqrt{x}\right)}{8} + \frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2} - \frac{1}{12}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2} - \frac{1}{8}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}$$

```
[In] Int[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2),x]
```

```
[Out] -1/8*(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]) - (Sqrt[-1 + Sqrt[x]]*S
qrt[1 + Sqrt[x]]*x^(3/2))/12 + (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2
))/3 - ArcCosh[Sqrt[x]]/8
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 286

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^p*((a2 + b2*x^n)
^p/(c*(m + 2*n*p + 1))), x] + Dist[2*a1*a2*n*(p/(m + 2*n*p + 1)), Int[(c*x)
^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a2
, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && Ne
Q[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Dist[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1))), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 336

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} - \frac{1}{6}\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx \\
&= -\frac{1}{12}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} + \frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} - \frac{1}{8}\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx \\
&= -\frac{1}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} - \frac{1}{12}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} \\
&\quad + \frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} - \frac{1}{16}\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx
\end{aligned}$$

$$= -\frac{1}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} - \frac{1}{12}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} + \frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} - \frac{1}{8}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{1}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} - \frac{1}{12}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} + \frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} - \frac{1}{8}\cosh^{-1}\left(\sqrt{x}\right)$$

Mathematica [A] (warning: unable to verify)

Time = 1.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\,dx = \frac{1}{24}\left(\sqrt{\frac{-1+\sqrt{x}}{1+\sqrt{x}}}\sqrt{x}\left(-3-3\sqrt{x}-2x-2x^{3/2}+8x^2+8x^{5/2}\right)-6\text{arctanh}\left(\sqrt{\frac{-}{1}}\right.\right.$$

```
[In] Integrate[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2),x]
```

```
[Out] (Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]*Sqrt[x]*(-3 - 3*Sqrt[x] - 2*x - 2*x^(3/
2) + 8*x^2 + 8*x^(5/2)) - 6*ArcTanh[Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]])/24
```

Maple [A] (verified)

Time = 4.60 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.62

method	result	size
derivativedivides	$-\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(-8x^{\frac{5}{2}}\sqrt{-1+x}+2x^{\frac{3}{2}}\sqrt{-1+x}+3\sqrt{x}\,\sqrt{-1+x}+3\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{24\sqrt{-1+x}}$	65
default	$-\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(-8x^{\frac{5}{2}}\sqrt{-1+x}+2x^{\frac{3}{2}}\sqrt{-1+x}+3\sqrt{x}\,\sqrt{-1+x}+3\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{24\sqrt{-1+x}}$	65

```
[In] int(x^(3/2)*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(-8*x^(5/2)*(-1+x)^(1/2)+2*x^(3/2
)*(-1+x)^(1/2)+3*x^(1/2)*(-1+x)^(1/2)+3*ln(x^(1/2)+(-1+x)^(1/2)))/(-1+x)^(1
/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.55

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\,dx = \frac{1}{24}\left(8\,x^2-2\,x-3\right)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} + \frac{1}{16}\log\left(2\,\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2\,x+1\right)$$

```
[In] integrate(x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/24*(8*x^2 - 2*x - 3)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/16*l
og(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

Sympy [F]

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\,dx = \int x^{\frac{3}{2}}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\,dx$$

```
[In] integrate(x**(3/2)*(-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(x**(3/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.45

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\,dx = \frac{1}{3}\,(x-1)^{\frac{3}{2}}x^{\frac{3}{2}} + \frac{1}{4}\,(x-1)^{\frac{3}{2}}\sqrt{x} + \frac{1}{8}\,\sqrt{x-1}\sqrt{x} - \frac{1}{8}\,\log\left(2\,\sqrt{x-1}+2\,\sqrt{x}\right)$$

```
[In] integrate(x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/3*(x - 1)^(3/2)*x^(3/2) + 1/4*(x - 1)^(3/2)*sqrt(x) + 1/8*sqrt(x - 1)*sqr
t(x) - 1/8*log(2*sqrt(x - 1) + 2*sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.22

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\,dx = \frac{1}{120}\left(\left(2\left(\left(4\left(5\sqrt{x}-26\right)\left(\sqrt{x}+1\right)+321\right)\left(\sqrt{x}+1\right)-451\right)\left(\sqrt{x}+1\right)+745\right.\right.$$
$$+\frac{1}{60}\left(\left(2\left(3\left(4\sqrt{x}-17\right)\left(\sqrt{x}+1\right)+133\right)\left(\sqrt{x}+1\right)-295\right)\left(\sqrt{x}+1\right)+195\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}$$
$$+\frac{1}{4}\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

```
[In] integrate(x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/120*((2*((4*(5*sqrt(x) - 26)*(sqrt(x) + 1) + 321)*(sqrt(x) + 1) - 451)*(s
qrt(x) + 1) + 745)*(sqrt(x) + 1) - 405)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1)
 + 1/60*((2*(3*(4*sqrt(x) - 17)*(sqrt(x) + 1) + 133)*(sqrt(x) + 1) - 295)*(
sqrt(x) + 1) + 195)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/4*log(sqrt(sqrt
(x) + 1) - sqrt(sqrt(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 45.13 (sec) , antiderivative size = 632, normalized size of antiderivative = 6.08

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\,dx = -\frac{\operatorname{atanh}\left(\frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{\sqrt{\sqrt{x}+1}-1}\right)}{2}$$
$$-\frac{\frac{35\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^3}{6\left(\sqrt{\sqrt{x}+1}-1\right)^3}+\frac{757\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^5}{2\left(\sqrt{\sqrt{x}+1}-1\right)^5}+\frac{7339\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^7}{2\left(\sqrt{\sqrt{x}+1}-1\right)^7}+\frac{41929\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^9}{3\left(\sqrt{\sqrt{x}+1}-1\right)^9}+\frac{25661\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{11}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{11}}+\frac{25661\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{13}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{13}}}{1+\frac{66\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\sqrt{x}+1}-1\right)^4}-\frac{220\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\sqrt{x}+1}-1\right)^6}+\frac{495\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\sqrt{x}+1}-1\right)^8}-\frac{792\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{10}}+\frac{924\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{12}}-\frac{792\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{1}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{14}}}$$

```
[In] int(x^(3/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2),x)
```

```
[Out] - atanh(((x^(1/2) - 1)^(1/2) - 1i)/((x^(1/2) + 1)^(1/2) - 1))/2 - ((35*((x^
(1/2) - 1)^(1/2) - 1i)^3)/(6*((x^(1/2) + 1)^(1/2) - 1)^3) + (757*((x^(1/2)
- 1)^(1/2) - 1i)^5)/(2*((x^(1/2) + 1)^(1/2) - 1)^5) + (7339*((x^(1/2) - 1)^
(1/2) - 1i)^7)/(2*((x^(1/2) + 1)^(1/2) - 1)^7) + (41929*((x^(1/2) - 1)^(1/2
) - 1i)^9)/(3*((x^(1/2) + 1)^(1/2) - 1)^9) + (25661*((x^(1/2) - 1)^(1/2) -
1i)^11)/((x^(1/2) + 1)^(1/2) - 1)^11 + (25661*((x^(1/2) - 1)^(1/2) - 1i)^13
)/((x^(1/2) + 1)^(1/2) - 1)^13 + (41929*((x^(1/2) - 1)^(1/2) - 1i)^15)/(3*(
(x^(1/2) + 1)^(1/2) - 1)^15) + (7339*((x^(1/2) - 1)^(1/2) - 1i)^17)/(2*((x^
(1/2) + 1)^(1/2) - 1)^17) + (757*((x^(1/2) - 1)^(1/2) - 1i)^19)/(2*((x^(1/2
) + 1)^(1/2) - 1)^19) + (35*((x^(1/2) - 1)^(1/2) - 1i)^21)/(6*((x^(1/2) + 1
```

```
)^(1/2) - 1)^21) - ((x^(1/2) - 1)^(1/2) - 1i)^23/(2*((x^(1/2) + 1)^(1/2) -
1)^23) - ((x^(1/2) - 1)^(1/2) - 1i)/(2*((x^(1/2) + 1)^(1/2) - 1)))/((66*((x
^(1/2) - 1)^(1/2) - 1i)^4)/((x^(1/2) + 1)^(1/2) - 1)^4 - (12*((x^(1/2) - 1)
^(1/2) - 1i)^2)/((x^(1/2) + 1)^(1/2) - 1)^2 - (220*((x^(1/2) - 1)^(1/2) - 1
i)^6)/((x^(1/2) + 1)^(1/2) - 1)^6 + (495*((x^(1/2) - 1)^(1/2) - 1i)^8)/((x^
(1/2) + 1)^(1/2) - 1)^8 - (792*((x^(1/2) - 1)^(1/2) - 1i)^10)/((x^(1/2) + 1
)^(1/2) - 1)^10 + (924*((x^(1/2) - 1)^(1/2) - 1i)^12)/((x^(1/2) + 1)^(1/2)
- 1)^12 - (792*((x^(1/2) - 1)^(1/2) - 1i)^14)/((x^(1/2) + 1)^(1/2) - 1)^14
+ (495*((x^(1/2) - 1)^(1/2) - 1i)^16)/((x^(1/2) + 1)^(1/2) - 1)^16 - (220*(
(x^(1/2) - 1)^(1/2) - 1i)^18)/((x^(1/2) + 1)^(1/2) - 1)^18 + (66*((x^(1/2)
- 1)^(1/2) - 1i)^20)/((x^(1/2) + 1)^(1/2) - 1)^20 - (12*((x^(1/2) - 1)^(1/2
) - 1i)^22)/((x^(1/2) + 1)^(1/2) - 1)^22 + ((x^(1/2) - 1)^(1/2) - 1i)^24/((
x^(1/2) + 1)^(1/2) - 1)^24 + 1)
```

3.1004 $\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 73

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}\,dx = -\frac{1}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} - \frac{\operatorname{arccosh}(\sqrt{x})}{4}$$

```
[Out] -1/4*arccosh(x^(1/2))+1/2*x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)-1/4*
x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {286, 329, 336, 54}

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}\,dx = -\frac{\operatorname{arccosh}(\sqrt{x})}{4} + \frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2} - \frac{1}{4}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}$$

```
[In] Int[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x],x]
```

```
[Out] -1/4*(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]) + (Sqrt[-1 + Sqrt[x]]*S
qrt[1 + Sqrt[x]]*x^(3/2))/2 - ArcCosh[Sqrt[x]]/4
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
```

```
- d, 0] && GtQ[a, 0]
```

Rule 286

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^p*((a2 + b2*x^n)
^p/(c*(m + 2*n*p + 1))), x] + Dist[2*a1*a2*n*(p/(m + 2*n*p + 1)), Int[(c*x)
^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a2
, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && Ne
Q[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Dist[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1))), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 336

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}-\frac{1}{4}\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx \\
&= -\frac{1}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}+\frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}-\frac{1}{8}\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx \\
&= -\frac{1}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}+\frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} \\
&\quad -\frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,\sqrt{x}\right) \\
&= -\frac{1}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}+\frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}-\frac{1}{4}\cosh^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 404 vs. $2(73) = 146$.

Time = 1.93 (sec) , antiderivative size = 404, normalized size of antiderivative = 5.53

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}\,dx = \frac{-4\sqrt{1+\sqrt{x}}\left(-18816+28224\sqrt{x}+55360x+17296x^{3/2}+7240x^2-1096x^{5/2}-4752x^3-1136x^{7/2}\right)-4\sqrt{}}{-12416+}$$

$$+\operatorname{arctanh}\left(\frac{-1+\sqrt{-1+\sqrt{x}}}{\sqrt{3}-\sqrt{1+\sqrt{x}}}\right)$$

```
[In] Integrate[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x],x]
```

```
[Out] (-4*Sqrt[1 + Sqrt[x]]*(-18816 + 28224*Sqrt[x] + 55360*x + 17296*x^(3/2) + 7
240*x^2 - 1096*x^(5/2) - 4752*x^3 - 1136*x^(7/2)) - 4*Sqrt[-1 + Sqrt[x]]*Sq
rt[1 + Sqrt[x]]*(32592 + 74488*Sqrt[x] + 38632*x + 6992*x^(3/2) - 104*x^2 -
 6079*x^(5/2) - 3120*x^3 - 194*x^(7/2)) + Sqrt[3]*(-4*Sqrt[-1 + Sqrt[x]]*(-
18816 - 52416*Sqrt[x] - 41472*x - 10928*x^(3/2) - 1192*x^2 + 3832*x^(5/2) +
 3408*x^3 + 656*x^(7/2)) - 4*(10864 - 10872*Sqrt[x] - 41440*x - 23268*x^(3/
2) - 6678*x^2 - 1148*x^(5/2) + 3416*x^3 + 1800*x^(7/2) + 112*x^4)))/(-12416
 + 13312*Sqrt[x] + 49408*x + 24960*x^(3/2) + 1552*x^2 + Sqrt[3]*Sqrt[1 + Sq
rt[x]]*(7168 - 11264*Sqrt[x] - 22016*x - 5248*x^(3/2)) + Sqrt[-1 + Sqrt[x]]
*(21504 + 60416*Sqrt[x] + 47104*x + 9088*x^(3/2) + Sqrt[3]*Sqrt[1 + Sqrt[x]
]*(-12416 - 28672*Sqrt[x] - 14400*x - 896*x^(3/2)))) + ArcTanh[(-1 + Sqrt[-
1 + Sqrt[x]])/(Sqrt[3] - Sqrt[1 + Sqrt[x]])]
```

Maple [A] (verified)

Time = 4.59 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$-\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(-2x^{\frac{3}{2}}\sqrt{-1+x}+\sqrt{x}\,\sqrt{-1+x}+\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{4\sqrt{-1+x}}$	52
default	$-\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(-2x^{\frac{3}{2}}\sqrt{-1+x}+\sqrt{x}\,\sqrt{-1+x}+\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{4\sqrt{-1+x}}$	52

```
[In] int(x^(1/2)*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(-2*x^(3/2)*(-1+x)^(1/2)+x^(1/2)*(
-1+x)^(1/2)+ln(x^(1/2)+(-1+x)^(1/2)))/(-1+x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}\,dx = \frac{1}{4}\,(2\,x-1)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} + \frac{1}{8}\,\log\left(2\,\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2\,x+1\right)$$

```
[In] integrate(x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/4*(2*x - 1)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/8*log(2*sqrt(
x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

Sympy [F]

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}\,dx = \int \sqrt{x}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\,dx$$

```
[In] integrate(x**(1/2)*(-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.51

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}\,dx = \frac{1}{2}\,(x-1)^{\frac{3}{2}}\sqrt{x} + \frac{1}{4}\,\sqrt{x-1}\sqrt{x} - \frac{1}{4}\,\log\left(2\,\sqrt{x-1}+2\,\sqrt{x}\right)$$

```
[In] integrate(x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/2*(x - 1)^(3/2)*sqrt(x) + 1/4*sqrt(x - 1)*sqrt(x) - 1/4*log(2*sqrt(x - 1)
 + 2*sqrt(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(45) = 90$.

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.26

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}\,dx = \frac{1}{12}\left(\left(2\left(3\sqrt{x}-10\right)\left(\sqrt{x}+1\right)+43\right)\left(\sqrt{x}+1\right)-39\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} + \frac{1}{3}\left(\left(2\sqrt{x}-5\right)\left(\sqrt{x}+1\right)+9\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} + \frac{1}{2}\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

```
[In] integrate(x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="giac")
[Out] 1/12*((2*(3*sqrt(x) - 10)*(sqrt(x) + 1) + 43)*(sqrt(x) + 1) - 39)*sqrt(sqrt
(x) + 1)*sqrt(sqrt(x) - 1) + 1/3*((2*sqrt(x) - 5)*(sqrt(x) + 1) + 9)*sqrt(s
qrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/2*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) -
1))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}\,dx = \int \sqrt{x}\,\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,dx$$

```
[In] int(x^(1/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2),x)
[Out] int(x^(1/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2), x)
```

3.1005 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 37

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx = \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} - \operatorname{arccosh}\left(\sqrt{x}\right)$$

```
[Out] -arccosh(x^(1/2))+x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {286, 336, 54}

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx = \sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x} - \operatorname{arccosh}\left(\sqrt{x}\right)$$

```
[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x],x]
```

```
[Out] Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] - ArcCosh[Sqrt[x]]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 286

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^p*((a2 + b2*x^n)
```

```
^p/(c*(m + 2*n*p + 1))), x] + Dist[2*a1*a2*n*(p/(m + 2*n*p + 1)), Int[(c*x)
^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a2
, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && Ne
Q[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 336

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}-\frac{1}{2}\int\frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx\\ &= \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}-\text{Subst}\left(\int\frac{1}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,\sqrt{x}\right)\\ &= \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}-\cosh^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 264 vs. $2(37) = 74$.

Time = 1.49 (sec) , antiderivative size = 264, normalized size of antiderivative = 7.14

$$\int\frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx$$
$$=4\left(\frac{4\sqrt{1+\sqrt{x}}\left(-12-24\sqrt{x}+x+5x^{3/2}\right)+\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\left(-84-10\sqrt{x}+28x+7x^{3/2}\right)+\sqrt{3}\Big(28}{56-16\sqrt{3}\sqrt{1+\sqrt{x}}\left(2+3\sqrt{x}\right)+\sqrt{-1+\sqrt{x}}\left(96-8\sqrt{3}\sqrt{1+\sqrt{}}\right.}\right.$$
$$\left.+\operatorname{arctanh}\left(\frac{-1+\sqrt{-1+\sqrt{x}}}{\sqrt{3}-\sqrt{1+\sqrt{x}}}\right)\right)$$

```
[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x],x]
```

```
[Out] 4*((4*Sqrt[1 + Sqrt[x]]*(-12 - 24*Sqrt[x] + x + 5*x^(3/2)) + Sqrt[-1 + Sqrt
[x]]*Sqrt[1 + Sqrt[x]]*(-84 - 10*Sqrt[x] + 28*x + 7*x^(3/2)) + Sqrt[3]*(28
+ 70*Sqrt[x] + 18*x - 14*x^(3/2) - 4*x^2 - 4*Sqrt[-1 + Sqrt[x]]*(-12 - 8*Sq
```

```
rt[x] + 5*x + 3*x^(3/2))))/(56 - 16*Sqrt[3]*Sqrt[1 + Sqrt[x]]*(2 + 3*Sqrt[x
]) + Sqrt[-1 + Sqrt[x]]*(96 - 8*Sqrt[3]*Sqrt[1 + Sqrt[x]]*(7 + 2*Sqrt[x]) +
 80*Sqrt[x]) + 112*Sqrt[x] + 28*x) + ArcTanh[(-1 + Sqrt[-1 + Sqrt[x]])/(Sqr
t[3] - Sqrt[1 + Sqrt[x]])])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(25) = 50$.

Time = 4.59 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.95

method	result	size
derivativedivides	$\sqrt{\sqrt{x}-1}\,\left(\sqrt{x}+1\right)^{\frac{3}{2}}-\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}-\frac{\sqrt{\left(\sqrt{x}+1\right)\left(\sqrt{x}-1\right)}\,\ln\left(\sqrt{x}+\sqrt{-1+x}\right)}{\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}}$	72
default	$\sqrt{\sqrt{x}-1}\,\left(\sqrt{x}+1\right)^{\frac{3}{2}}-\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}-\frac{\sqrt{\left(\sqrt{x}+1\right)\left(\sqrt{x}-1\right)}\,\ln\left(\sqrt{x}+\sqrt{-1+x}\right)}{\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}}$	72

```
[In] int((x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(3/2)-(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)-((x
^(1/2)+1)*(x^(1/2)-1))^(1/2)/(x^(1/2)+1)^(1/2)/(x^(1/2)-1)^(1/2)*ln(x^(1/2)
+(-1+x)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx = \sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} + \frac{1}{2}\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2\,x+1\right)$$

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="fricas
")
```

```
[Out] sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/2*log(2*sqrt(x)*sqrt(sqrt(x
) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.65

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx = 4\sqrt{\sqrt{x}-1}\left(\frac{\left(\sqrt{x}+1\right)^{\frac{3}{2}}}{4} - \frac{\sqrt{\sqrt{x}+1}}{4}\right) - 2\log\left(2\sqrt{\sqrt{x}-1}+2\sqrt{\sqrt{x}+1}\right)$$

```
[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(1/2),x)
[Out] 4*sqrt(sqrt(x) - 1)*((sqrt(x) + 1)**(3/2)/4 - sqrt(sqrt(x) + 1)/4) - 2*log(
2*sqrt(sqrt(x) - 1) + 2*sqrt(sqrt(x) + 1))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx = \sqrt{x-1}\sqrt{x} - \log\left(2\sqrt{x-1}+2\sqrt{x}\right)$$

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="maxima
")
[Out] sqrt(x - 1)*sqrt(x) - log(2*sqrt(x - 1) + 2*sqrt(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(25) = 50.

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx = \sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}(\sqrt{x}-2) + 2\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} + 2\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="giac")
[Out] sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1)*(sqrt(x) - 2) + 2*sqrt(sqrt(x) + 1)*sqr
t(sqrt(x) - 1) + 2*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 9.38 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx = \sqrt{x}\,\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1} - \ln\left(\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}+\sqrt{x}\right)$$

```
[In] int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(1/2),x)
```

```
[Out] x^(1/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2) - log((x^(1/2) - 1)^(1/2)*(
x^(1/2) + 1)^(1/2) + x^(1/2))
```

3.1006 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 67

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx = \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{\sqrt{x}} - 2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + 2\text{arccosh}\left(\sqrt{x}\right)$$

```
[Out] 2*arccosh(x^(1/2))+2*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(1/2)-2*x^(1/2)
*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {333, 286, 336, 54}

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx = 2\text{arccosh}\left(\sqrt{x}\right) + \frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{\sqrt{x}} - 2\sqrt{\sqrt{x}-1}\sqrt{x}\sqrt{\sqrt{x}+1}$$

```
[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(3/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/Sqrt[x] - 2*Sqrt[-1 + Sqrt[x]]
*Sqrt[1 + Sqrt[x]]*Sqrt[x] + 2*ArcCosh[Sqrt[x]]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
```

```
- d, 0] && GtQ[a, 0]
```

Rule 286

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^p*((a2 + b2*x^n)
^p/(c*(m + 2*n*p + 1))), x] + Dist[2*a1*a2*n*(p/(m + 2*n*p + 1)), Int[(c*x)
^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a2
, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && Ne
Q[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 333

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b2
*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] - Dist[b1*b2*((m + 2*n*(p + 1) + 1)/(a
1*a2*c^(2*n)*(m + 1))), Int[(c*x)^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p
, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2, 0] && IG
tQ[2*n, 0] && LtQ[m, -1] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 336

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{\sqrt{x}} - 2\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx \\
&= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{\sqrt{x}} - 2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx \\
&= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{\sqrt{x}} - 2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} \\
&\quad + 2\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}}\,dx, x, \sqrt{x}\right) \\
&= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{\sqrt{x}} - 2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + 2\cosh^{-1}\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 184 vs. $2(67) = 134$.

Time = 1.19 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.75

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx = \frac{\left(-1+\sqrt{-1+\sqrt{x}}\right)\left(\sqrt{3}-\sqrt{1+\sqrt{x}}\right)\left(-2+\sqrt{-1+\sqrt{x}}+\sqrt{3}\sqrt{1+\sqrt{x}}-\right.}{\left(-3-2\sqrt{-1+\sqrt{x}}+2\sqrt{3}\sqrt{1+\sqrt{x}}+\sqrt{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}-2\sqrt{x}\right)\sqrt{}} - 8\operatorname{arctanh}\left(\frac{-1+\sqrt{-1+\sqrt{x}}}{\sqrt{3}-\sqrt{1+\sqrt{x}}}\right)$$

```
[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(3/2),x]
```

```
[Out] ((-1 + Sqrt[-1 + Sqrt[x]])*(Sqrt[3] - Sqrt[1 + Sqrt[x]])*(-2 + Sqrt[-1 + Sq
rt[x]] + Sqrt[3]*Sqrt[1 + Sqrt[x]] - Sqrt[x]))/((-3 - 2*Sqrt[-1 + Sqrt[x]]
+ 2*Sqrt[3]*Sqrt[1 + Sqrt[x]] + Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]
] - 2*Sqrt[x])*Sqrt[x]) - 8*ArcTanh[(-1 + Sqrt[-1 + Sqrt[x]])/(Sqrt[3] - Sq
rt[1 + Sqrt[x]])]
```

Maple [A] (verified)

Time = 4.57 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\sqrt{x}-\sqrt{-1+x}\right)}{\sqrt{x}\,\sqrt{-1+x}}$	47
default	$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\sqrt{x}-\sqrt{-1+x}\right)}{\sqrt{x}\,\sqrt{-1+x}}$	47

```
[In] int((x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(ln(x^(1/2)+(-1+x)^(1/2))*x^(1/2)-(-1
+x)^(1/2))/x^(1/2)/(-1+x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx = -\frac{x\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2x+1\right)+2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+2x}{x}$$

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2),x, algorithm="fricas
")
```

```
[Out] -(x*log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1) + 2*sqrt(x
)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 2*x)/x
```

Sympy [F]

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx = \int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(3/2),x)
```

```
[Out] Integral(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)/x**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.40

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx = -\frac{2\sqrt{x-1}}{\sqrt{x}} + 2\log\left(2\sqrt{x-1} + 2\sqrt{x}\right)$$

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2),x, algorithm="maxima
")
```

```
[Out] -2*sqrt(x - 1)/sqrt(x) + 2*log(2*sqrt(x - 1) + 2*sqrt(x))
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 10.55 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.93

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx = 8\,\mathrm{atanh}\left(\frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{\sqrt{\sqrt{x}+1}-1}\right) - \frac{\frac{5\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^2}{2\left(\sqrt{\sqrt{x}+1}-1\right)^2}+\frac{1}{2}}{\frac{\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^3}{\left(\sqrt{\sqrt{x}+1}-1\right)^3}+\frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{\sqrt{\sqrt{x}+1}-1}} - \frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{2\left(\sqrt{\sqrt{x}+1}-1\right)}$$

```
[In] int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(3/2),x)
```

```
[Out] 8*atanh(((x^(1/2) - 1)^(1/2) - 1i)/((x^(1/2) + 1)^(1/2) - 1)) - ((5*((x^(1/
2) - 1)^(1/2) - 1i)^2)/(2*((x^(1/2) + 1)^(1/2) - 1)^2) + 1/2)/((((x^(1/2) -
1)^(1/2) - 1i)^3/((x^(1/2) + 1)^(1/2) - 1)^3 + ((x^(1/2) - 1)^(1/2) - 1i)/(
(x^(1/2) + 1)^(1/2) - 1)) - ((x^(1/2) - 1)^(1/2) - 1i)/(2*((x^(1/2) + 1)^(1
/2) - 1))
```

3.1007 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 31

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx=\frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{3x^{3/2}}$$

```
[Out] 2/3*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(3/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {271}

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx=\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{3x^{3/2}}$$

```
[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(5/2),x]
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(3*x^(3/2))
```

Rule 271

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b
2*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rubi steps

$$\text{integral}=\frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{3x^{3/2}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 421 vs. 2(31) = 62.

Time = 2.51 (sec) , antiderivative size = 421, normalized size of antiderivative = 13.58

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx = \frac{\left(-1+\sqrt{-1+\sqrt{x}}\right)\left(\sqrt{3}-\sqrt{1+\sqrt{x}}\right)\left(-2+\sqrt{-1+\sqrt{x}}+\sqrt{3}\sqrt{1+\sqrt{x}}-\right.}{}$$

```
[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(5/2),x]
```

```
[Out] ((-1 + Sqrt[-1 + Sqrt[x]])*(Sqrt[3] - Sqrt[1 + Sqrt[x]])*(-2 + Sqrt[-1 + Sq
rt[x]] + Sqrt[3]*Sqrt[1 + Sqrt[x]] - Sqrt[x])*(8*(7 + 12*Sqrt[-1 + Sqrt[x]]
 - 4*Sqrt[3]*Sqrt[1 + Sqrt[x]] - 7*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt
[x]]) + 4*(49 + 8*Sqrt[-1 + Sqrt[x]] - 24*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 3*Sqr
t[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*Sqrt[x] + 2*(-23 - 144*Sqrt[-1 +
 Sqrt[x]] + 32*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 77*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sq
rt[1 + Sqrt[x]])*x + 2*(-140 - 106*Sqrt[-1 + Sqrt[x]] + 62*Sqrt[3]*Sqrt[1 +
 Sqrt[x]] + 21*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*x^(3/2) - 73*x
^2))/(12*(-3 - 2*Sqrt[-1 + Sqrt[x]] + 2*Sqrt[3]*Sqrt[1 + Sqrt[x]] + Sqrt[3]
*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]] - 2*Sqrt[x])^3*x^(3/2))
```

Maple [A] (verified)

Time = 4.57 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,(-1+x)}{3x^{\frac{3}{2}}}$	23
default	$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,(-1+x)}{3x^{\frac{3}{2}}}$	23

```
[In] int((x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(-1+x)/x^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx = \frac{2\left((x-1)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+x^2\right)}{3\,x^2}$$

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(5/2),x, algorithm="fricas
")
[Out] 2/3*((x - 1)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + x^2)/x^2
```

Sympy [F]

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx = \int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(5/2),x)
[Out] Integral(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)/x**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.32

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx = \frac{2\,(x-1)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(5/2),x, algorithm="maxima
")
[Out] 2/3*(x - 1)^(3/2)/x^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(19) = 38$.

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx = \frac{16\left(3\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^8+16\right)}{3\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^4+4\right)^3}$$

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(5/2),x, algorithm="giac")
[Out] 16/3*(3*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^8 + 16)/((sqrt(sqrt(x) + 1)
 - sqrt(sqrt(x) - 1))^4 + 4)^3
```

Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx = \frac{\sqrt{\sqrt{x}-1}\left(\frac{2\,x\,\sqrt{\sqrt{x}+1}}{3}-\frac{2\,\sqrt{\sqrt{x}+1}}{3}\right)}{x^{3/2}}$$

```
[In] int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(5/2),x)
[Out] ((x^(1/2) - 1)^(1/2)*((2*x*(x^(1/2) + 1)^(1/2))/3 - (2*(x^(1/2) + 1)^(1/2))
/3))/x^(3/2)
```

3.1008 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 63

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx=\frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{5x^{5/2}}+\frac{4\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{15x^{3/2}}$$

```
[Out] 2/5*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(5/2)+4/15*(-1+x^(1/2))^(3/2)*(1
+x^(1/2))^(3/2)/x^(3/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {278, 271}

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx=\frac{4\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{15x^{3/2}}+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{5x^{5/2}}$$

```
[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(7/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(5*x^(5/2)) + (4*(-1 + Sqrt[x]
)^(3/2)*(1 + Sqrt[x])^(3/2))/(15*x^(3/2))
```

Rule 271

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b
2*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rule 278

```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[x^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b2*x^n)^(p + 1
)/(a1*a2*(m + 1))), x] - Dist[b1*b2*((m + 2*n*(p + 1) + 1)/(a1*a2*(m + 1)))
, Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1,
 a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*
n) + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{5x^{5/2}}+\frac{2}{5}\int\frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx\\ &= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{5x^{5/2}}+\frac{4\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{15x^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 10.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.57

$$\int\frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx=\frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}(3+2x)}{15x^{5/2}}$$

```
[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(7/2),x]
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2)*(3 + 2*x))/(15*x^(5/2))
```

Maple [A] (verified)

Time = 4.82 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.44

method	result	size
derivativedivides	$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,(-1+x)(2x+3)}{15x^{\frac{5}{2}}}$	28
default	$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,(-1+x)(2x+3)}{15x^{\frac{5}{2}}}$	28

```
[In] int((x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(7/2),x,method=_RETURNVERBOSE)
[Out] 2/15*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(-1+x)*(2*x+3)/x^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx = \frac{2\left(2\,x^{3}+\left(2\,x^{2}+x-3\right)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}\right)}{15\,x^{3}}$$

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(7/2),x, algorithm="fricas
")
[Out] 2/15*(2*x^3 + (2*x^2 + x - 3)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1))/
x^3
```

Sympy [F]

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx = \int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{\frac{7}{2}}}\,dx$$

```
[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(7/2),x)
[Out] Integral(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)/x**(7/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx = \frac{4\left(x-1\right)^{\frac{3}{2}}}{15\,x^{\frac{3}{2}}} + \frac{2\left(x-1\right)^{\frac{3}{2}}}{5\,x^{\frac{5}{2}}}$$

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(7/2),x, algorithm="maxima
")
[Out] 4/15*(x - 1)^(3/2)/x^(3/2) + 2/5*(x - 1)^(3/2)/x^(5/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(39) = 78.

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx = \frac{128\left(15\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{12}-20\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{8}+80\left(\sqrt{\cdot}\right.\right.}{15\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+4\right)^{5}}$$

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] 128/15*(15*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^12 - 20*(sqrt(sqrt(x) +
1) - sqrt(sqrt(x) - 1))^8 + 80*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 +
64)/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 4)^5
```

Mupad [B] (verification not implemented)

Time = 9.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx = \frac{\sqrt{\sqrt{x}-1}\left(\frac{2\,x\,\sqrt{\sqrt{x}+1}}{15}-\frac{2\,\sqrt{\sqrt{x}+1}}{5}+\frac{4\,x^2\,\sqrt{\sqrt{x}+1}}{15}\right)}{x^{5/2}}$$

```
[In] int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(7/2),x)
```

```
[Out] ((x^(1/2) - 1)^(1/2)*((2*x*(x^(1/2) + 1)^(1/2))/15 - (2*(x^(1/2) + 1)^(1/2)
)/5 + (4*x^2*(x^(1/2) + 1)^(1/2))/15))/x^(5/2)
```

3.1009 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 94

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx=\frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{7x^{7/2}}+\frac{8\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{35x^{5/2}}+\frac{16\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{105x^{3/2}}$$

```
[Out] 2/7*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(7/2)+8/35*(-1+x^(1/2))^(3/2)*(1
+x^(1/2))^(3/2)/x^(5/2)+16/105*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(3/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {278, 271}

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx=\frac{16\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{105x^{3/2}}+\frac{8\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{35x^{5/2}}+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{7x^{7/2}}$$

```
[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(9/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(7*x^(7/2)) + (8*(-1 + Sqrt[x]
)^(3/2)*(1 + Sqrt[x])^(3/2))/(35*x^(5/2)) + (16*(-1 + Sqrt[x])^(3/2)*(1 + S
qrt[x])^(3/2))/(105*x^(3/2))
```

Rule 271

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b
2*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rule 278

```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[x^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b2*x^n)^(p + 1
)/(a1*a2*(m + 1))), x] - Dist[b1*b2*((m + 2*n*(p + 1) + 1)/(a1*a2*(m + 1)))
, Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1,
 a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*
n) + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{7x^{7/2}}+\frac{4}{7}\int\frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx\\
&= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{7x^{7/2}}+\frac{8\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{35x^{5/2}}+\frac{8}{35}\int\frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx\\
&= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{7x^{7/2}}+\frac{8\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{35x^{5/2}}+\frac{16\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{105x^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.44

$$\int\frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx=\frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}\left(15+12x+8x^2\right)}{105x^{7/2}}$$

```
[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(9/2),x]
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2)*(15 + 12*x + 8*x^2))/(105*x^(7/
2))
```

Maple [A] (verified)

Time = 4.61 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.35

method	result	size
derivativedivides	$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,(-1+x)(8x^2+12x+15)}{105x^{\frac{7}{2}}}$	33
default	$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,(-1+x)(8x^2+12x+15)}{105x^{\frac{7}{2}}}$	33

```
[In] int((x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
[Out] 2/105*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(-1+x)*(8*x^2+12*x+15)/x^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx = \frac{2\left(8\,x^4+(8\,x^3+4\,x^2+3\,x-15)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}\right)}{105\,x^4}$$

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(9/2),x, algorithm="fricas
")
[Out] 2/105*(8*x^4 + (8*x^3 + 4*x^2 + 3*x - 15)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sq
rt(x) - 1))/x^4
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(9/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx = \frac{16\,(x-1)^{\frac{3}{2}}}{105\,x^{\frac{3}{2}}} + \frac{8\,(x-1)^{\frac{3}{2}}}{35\,x^{\frac{5}{2}}} + \frac{2\,(x-1)^{\frac{3}{2}}}{7\,x^{\frac{7}{2}}}$$

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(9/2),x, algorithm="maxima
")
```

```
[Out] 16/105*(x - 1)^(3/2)/x^(3/2) + 8/35*(x - 1)^(3/2)/x^(5/2) + 2/7*(x - 1)^(3/
2)/x^(7/2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx = \frac{4096\left(35\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{16} - 70\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{12} + 168\left(\cdots\right.\right.}{105\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}\cdots}\right.\right.}$$

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] 4096/105*(35*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^16 - 70*(sqrt(sqrt(x)
+ 1) - sqrt(sqrt(x) - 1))^12 + 168*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^
8 + 224*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 128)/((sqrt(sqrt(x) + 1
) - sqrt(sqrt(x) - 1))^4 + 4)^7
```

Mupad [B] (verification not implemented)

Time = 9.42 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx = \frac{\sqrt{\sqrt{x}-1}\left(\frac{2\,x\,\sqrt{\sqrt{x}+1}}{35} - \frac{2\,\sqrt{\sqrt{x}+1}}{7} + \frac{8\,x^2\,\sqrt{\sqrt{x}+1}}{105} + \frac{16\,x^3\,\sqrt{\sqrt{x}+1}}{105}\right)}{x^{7/2}}$$

```
[In] int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(9/2),x)
```

```
[Out] ((x^(1/2) - 1)^(1/2)*((2*x*(x^(1/2) + 1)^(1/2))/35 - (2*(x^(1/2) + 1)^(1/2)
)/7 + (8*x^2*(x^(1/2) + 1)^(1/2))/105 + (16*x^3*(x^(1/2) + 1)^(1/2))/105))/
x^(7/2)
```

3.1010 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 125

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx = \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{9x^{9/2}} + \frac{4\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{21x^{7/2}} + \frac{16\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{105x^{5/2}} + \frac{32\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{315x^{3/2}}$$

```
[Out] 2/9*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(9/2)+4/21*(-1+x^(1/2))^(3/2)*(1
+x^(1/2))^(3/2)/x^(7/2)+16/105*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(5/2)
+32/315*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {278, 271}

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx = \frac{32\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{315x^{3/2}} + \frac{16\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{105x^{5/2}} + \frac{4\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{21x^{7/2}} + \frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{9x^{9/2}}$$

```
[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(11/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(9*x^(9/2)) + (4*(-1 + Sqrt[x]
)^(3/2)*(1 + Sqrt[x])^(3/2))/(21*x^(7/2)) + (16*(-1 + Sqrt[x])^(3/2)*(1 + S
```

```
qrt[x])^(3/2))/(105*x^(5/2)) + (32*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2)
)/(315*x^(3/2))
```

Rule 271

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b
2*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rule 278

```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[x^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b2*x^n)^(p + 1
)/(a1*a2*(m + 1))), x] - Dist[b1*b2*((m + 2*n*(p + 1) + 1)/(a1*a2*(m + 1)))
, Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1,
 a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*
n) + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{9x^{9/2}}+\frac{2}{3}\int\frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx\\
&= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{9x^{9/2}}+\frac{4\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{21x^{7/2}}+\frac{8}{21}\int\frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx\\
&= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{9x^{9/2}}+\frac{4\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{21x^{7/2}}\\
&\quad+\frac{16\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{105x^{5/2}}+\frac{16}{105}\int\frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx\\
&= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{9x^{9/2}}+\frac{4\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{21x^{7/2}}\\
&\quad+\frac{16\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{105x^{5/2}}+\frac{32\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{315x^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 10.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx = \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}\left(35+30x+24x^2+16x^3\right)}{315x^{9/2}}$$

```
[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(11/2),x]
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2)*(35 + 30*x + 24*x^2 + 16*x^3))/
(315*x^(9/2))
```

Maple [A] (verified)

Time = 4.70 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.30

method	result	size
derivativedivides	$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,(-1+x)\left(16x^3+24x^2+30x+35\right)}{315x^{\frac{9}{2}}}$	38
default	$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,(-1+x)\left(16x^3+24x^2+30x+35\right)}{315x^{\frac{9}{2}}}$	38

```
[In] int((x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(11/2),x,method=_RETURNVERBOSE)
[Out] 2/315*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(-1+x)*(16*x^3+24*x^2+30*x+35)/x^
(9/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx = \frac{2\left(16\,x^5+\left(16\,x^4+8\,x^3+6\,x^2+5\,x-35\right)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}\right)}{315\,x^5}$$

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(11/2),x, algorithm="frica
s")
[Out] 2/315*(16*x^5 + (16*x^4 + 8*x^3 + 6*x^2 + 5*x - 35)*sqrt(x)*sqrt(sqrt(x) +
1)*sqrt(sqrt(x) - 1))/x^5
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx = \text{Timed out}$$

```
[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(11/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx = \frac{32\,(x-1)^{\frac{3}{2}}}{315\,x^{\frac{3}{2}}} + \frac{16\,(x-1)^{\frac{3}{2}}}{105\,x^{\frac{5}{2}}} + \frac{4\,(x-1)^{\frac{3}{2}}}{21\,x^{\frac{7}{2}}} + \frac{2\,(x-1)^{\frac{3}{2}}}{9\,x^{\frac{9}{2}}}$$

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(11/2),x, algorithm="maxim
a")
```

```
[Out] 32/315*(x - 1)^(3/2)/x^(3/2) + 16/105*(x - 1)^(3/2)/x^(5/2) + 4/21*(x - 1)^
(3/2)/x^(7/2) + 2/9*(x - 1)^(3/2)/x^(9/2)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx = \frac{16384\left(315\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{20} - 756\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{16} + 13\right.}{315}$$

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(11/2),x, algorithm="giac"
)
```

```
[Out] 16384/315*(315*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^20 - 756*(sqrt(sqrt(
x) + 1) - sqrt(sqrt(x) - 1))^16 + 1344*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) -
1))^12 + 2304*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^8 + 2304*(sqrt(sqrt(x
) + 1) - sqrt(sqrt(x) - 1))^4 + 1024)/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) -
1))^4 + 4)^9
```

Mupad [B] (verification not implemented)

Time = 9.36 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx = \frac{\sqrt{\sqrt{x}-1}\,\left(\frac{2\,x\,\sqrt{\sqrt{x}+1}}{63}-\frac{2\,\sqrt{\sqrt{x}+1}}{9}+\frac{4\,x^2\,\sqrt{\sqrt{x}+1}}{105}+\frac{16\,x^3\,\sqrt{\sqrt{x}+1}}{315}+\frac{32\,x^4\,\sqrt{\sqrt{x}+1}}{315}\right)}{x^{9/2}}$$

```
[In] int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(11/2),x)
```

```
[Out] ((x^(1/2) - 1)^(1/2)*((2*x*(x^(1/2) + 1)^(1/2))/63 - (2*(x^(1/2) + 1)^(1/2)
)/9 + (4*x^2*(x^(1/2) + 1)^(1/2))/105 + (16*x^3*(x^(1/2) + 1)^(1/2))/315 +
(32*x^4*(x^(1/2) + 1)^(1/2))/315))/x^(9/2)
```

3.1011 $\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 104

$$\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{5}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \frac{5}{12}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} + \frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} + \frac{5\text{arccosh}\left(\sqrt{x}\right)}{8}$$

```
[Out] 5/8*arccosh(x^(1/2))+5/12*x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)+1/3*
x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)+5/8*x^(1/2)*(-1+x^(1/2))^(1/2)
*(1+x^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {329, 336, 54}

$$\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{5\text{arccosh}\left(\sqrt{x}\right)}{8} + \frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2} + \frac{5}{12}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2} + \frac{5}{8}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}$$

```
[In] Int[x^(5/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] (5*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/8 + (5*Sqrt[-1 + Sqrt[x]]*
Sqrt[1 + Sqrt[x]]*x^(3/2))/12 + (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/
2))/3 + (5*ArcCosh[Sqrt[x]])/8
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Dist[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1))), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 336

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}+\frac{5}{6}\int\frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx\\
&=\frac{5}{12}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}+\frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}+\frac{5}{8}\int\frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx\\
&=\frac{5}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}+\frac{5}{12}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\\
&\quad+\frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}+\frac{5}{16}\int\frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx\\
&=\frac{5}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}+\frac{5}{12}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\\
&\quad+\frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}+\frac{5}{8}\text{Subst}\left(\int\frac{1}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,\sqrt{x}\right)\\
&=\frac{5}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}+\frac{5}{12}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\\
&\quad+\frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}+\frac{5}{8}\cosh^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 1.36 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{1}{24}\sqrt{\frac{-1+\sqrt{x}}{1+\sqrt{x}}}\sqrt{x}\big(15+15\sqrt{x} + 10x+10x^{3/2}+8x^2+8x^{5/2}\big)+\frac{5}{4}\text{arctanh}\left(\sqrt{\frac{-1+\sqrt{x}}{1+\sqrt{x}}}\right)$$

```
[In] Integrate[x^(5/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] (Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]*Sqrt[x]*(15 + 15*Sqrt[x] + 10*x + 10*x^
(3/2) + 8*x^2 + 8*x^(5/2)))/24 + (5*ArcTanh[Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x
])]])/4
```

Maple [A] (verified)

Time = 4.82 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.62

method	result	size
derivativedivides	$\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(8x^{\frac{5}{2}}\sqrt{-1+x}+10x^{\frac{3}{2}}\sqrt{-1+x}+15\sqrt{x}\,\sqrt{-1+x}+15\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{24\sqrt{-1+x}}$	65
default	$\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(8x^{\frac{5}{2}}\sqrt{-1+x}+10x^{\frac{3}{2}}\sqrt{-1+x}+15\sqrt{x}\,\sqrt{-1+x}+15\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{24\sqrt{-1+x}}$	65

```
[In] int(x^(5/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(8*x^(5/2)*(-1+x)^(1/2)+10*x^(3/2)
*(-1+x)^(1/2)+15*x^(1/2)*(-1+x)^(1/2)+15*ln(x^(1/2)+(-1+x)^(1/2)))/(-1+x)^(
1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.55

$$\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{1}{24}\left(8\,x^2+10\,x+15\right)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} - \frac{5}{16}\,\log\left(2\,\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2\,x+1\right)$$

```
[In] integrate(x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fricas
")
```

[Out] 1/24*(8*x^2 + 10*x + 15)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 5/16
*log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)

Sympy [F]

$$\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \int \frac{x^{\frac{5}{2}}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\,dx$$

[In] integrate(x**(5/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)

[Out] Integral(x**(5/2)/(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.45

$$\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{1}{3}\sqrt{x-1}x^{\frac{5}{2}} + \frac{5}{12}\sqrt{x-1}x^{\frac{3}{2}} + \frac{5}{8}\sqrt{x-1}\sqrt{x} + \frac{5}{8}\log\left(2\sqrt{x-1} + 2\sqrt{x}\right)$$

[In] integrate(x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxima
")

[Out] 1/3*sqrt(x - 1)*x^(5/2) + 5/12*sqrt(x - 1)*x^(3/2) + 5/8*sqrt(x - 1)*sqrt(x
) + 5/8*log(2*sqrt(x - 1) + 2*sqrt(x))

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.73

$$\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{1}{24}\left(\left(2\left(\left(4\left(\sqrt{x}+1\right)\left(\sqrt{x}-4\right)+45\right)\left(\sqrt{x}+1\right)-55\right)\left(\sqrt{x}+1\right)+85\right)\left(\sqrt{x}+1\right. - \frac{5}{4}\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

[In] integrate(x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac")

[Out] 1/24*((2*((4*(sqrt(x) + 1)*(sqrt(x) - 4) + 45)*(sqrt(x) + 1) - 55)*(sqrt(x)
 + 1) + 85)*(sqrt(x) + 1) - 33)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 5/4*l
og(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))

Mupad [B] (verification not implemented)

Time = 38.44 (sec) , antiderivative size = 632, normalized size of antiderivative = 6.08

$$\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{5\,\mathrm{atanh}\left(\frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{\sqrt{\sqrt{x}+1}-1}\right)}{2} - \frac{-\frac{175\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^3}{6\left(\sqrt{\sqrt{x}+1}-1\right)^3} + \frac{311\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^5}{2\left(\sqrt{\sqrt{x}+1}-1\right)^5} + \frac{8361\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^7}{2\left(\sqrt{\sqrt{x}+1}-1\right)^7} + \frac{42259\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^9}{3\left(\sqrt{\sqrt{x}+1}-1\right)^9} + \frac{25295\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{11}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{11}} + \frac{25295\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{13}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{13}} \cdots}{1 + \frac{66\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\sqrt{x}+1}-1\right)^4} - \frac{220\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\sqrt{x}+1}-1\right)^6} + \frac{495\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\sqrt{x}+1}-1\right)^8} - \frac{792\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{10}} + \frac{924\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{12}} - \frac{792\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{14}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{14}} \cdots}$$

```
[In] int(x^(5/2)/((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

```
[Out] (5*atanh(((x^(1/2) - 1)^(1/2) - 1i)/((x^(1/2) + 1)^(1/2) - 1)))/2 - ((311*(
(x^(1/2) - 1)^(1/2) - 1i)^5)/(2*((x^(1/2) + 1)^(1/2) - 1)^5) - (175*((x^(1/
2) - 1)^(1/2) - 1i)^3)/(6*((x^(1/2) + 1)^(1/2) - 1)^3) + (8361*((x^(1/2) -
1)^(1/2) - 1i)^7)/(2*((x^(1/2) + 1)^(1/2) - 1)^7) + (42259*((x^(1/2) - 1)^(
1/2) - 1i)^9)/(3*((x^(1/2) + 1)^(1/2) - 1)^9) + (25295*((x^(1/2) - 1)^(1/2)
 - 1i)^11)/((x^(1/2) + 1)^(1/2) - 1)^11 + (25295*((x^(1/2) - 1)^(1/2) - 1i)
^13)/((x^(1/2) + 1)^(1/2) - 1)^13 + (42259*((x^(1/2) - 1)^(1/2) - 1i)^15)/(
3*((x^(1/2) + 1)^(1/2) - 1)^15) + (8361*((x^(1/2) - 1)^(1/2) - 1i)^17)/(2*(
(x^(1/2) + 1)^(1/2) - 1)^17) + (311*((x^(1/2) - 1)^(1/2) - 1i)^19)/(2*((x^(
1/2) + 1)^(1/2) - 1)^19) - (175*((x^(1/2) - 1)^(1/2) - 1i)^21)/(6*((x^(1/2)
 + 1)^(1/2) - 1)^21) + (5*((x^(1/2) - 1)^(1/2) - 1i)^23)/(2*((x^(1/2) + 1)^
(1/2) - 1)^23) + (5*((x^(1/2) - 1)^(1/2) - 1i))/(2*((x^(1/2) + 1)^(1/2) - 1
)))/((66*((x^(1/2) - 1)^(1/2) - 1i)^4)/((x^(1/2) + 1)^(1/2) - 1)^4 - (12*((
x^(1/2) - 1)^(1/2) - 1i)^2)/((x^(1/2) + 1)^(1/2) - 1)^2 - (220*((x^(1/2) -
1)^(1/2) - 1i)^6)/((x^(1/2) + 1)^(1/2) - 1)^6 + (495*((x^(1/2) - 1)^(1/2) -
 1i)^8)/((x^(1/2) + 1)^(1/2) - 1)^8 - (792*((x^(1/2) - 1)^(1/2) - 1i)^10)/(
(x^(1/2) + 1)^(1/2) - 1)^10 + (924*((x^(1/2) - 1)^(1/2) - 1i)^12)/((x^(1/2)
 + 1)^(1/2) - 1)^12 - (792*((x^(1/2) - 1)^(1/2) - 1i)^14)/((x^(1/2) + 1)^(1
/2) - 1)^14 + (495*((x^(1/2) - 1)^(1/2) - 1i)^16)/((x^(1/2) + 1)^(1/2) - 1)
^16 - (220*((x^(1/2) - 1)^(1/2) - 1i)^18)/((x^(1/2) + 1)^(1/2) - 1)^18 + (6
6*((x^(1/2) - 1)^(1/2) - 1i)^20)/((x^(1/2) + 1)^(1/2) - 1)^20 - (12*((x^(1/
2) - 1)^(1/2) - 1i)^22)/((x^(1/2) + 1)^(1/2) - 1)^22 + ((x^(1/2) - 1)^(1/2)
 - 1i)^24/((x^(1/2) + 1)^(1/2) - 1)^24 + 1)
```

3.1012 $\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 73

$$\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{3}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} + \frac{3\operatorname{arccosh}\left(\sqrt{x}\right)}{4}$$

```
[Out] 3/4*arccosh(x^(1/2))+1/2*x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)+3/4*x
^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {329, 336, 54}

$$\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{3\operatorname{arccosh}\left(\sqrt{x}\right)}{4} + \frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2} + \frac{3}{4}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}$$

```
[In] Int[x^(3/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] (3*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/4 + (Sqrt[-1 + Sqrt[x]]*Sq
rt[1 + Sqrt[x]]*x^(3/2))/2 + (3*ArcCosh[Sqrt[x]])/4
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
```

```
- d, 0] && GtQ[a, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Dist[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1))), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 336

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}+\frac{3}{4}\int\frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx\\
&= \frac{3}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}+\frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}+\frac{3}{8}\int\frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx\\
&= \frac{3}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}+\frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\\
&\quad+\frac{3}{4}\text{Subst}\left(\int\frac{1}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,\sqrt{x}\right)\\
&= \frac{3}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}+\frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}+\frac{3}{4}\cosh^{-1}\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A] (warning: unable to verify)

Time = 1.37 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$
\int\frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx=\frac{1}{4}\left(\sqrt{\frac{-1+\sqrt{x}}{1+\sqrt{x}}}\sqrt{x}\left(3+3\sqrt{x}+2x+2x^{3/2}\right)+6\operatorname{arctanh}\left(\sqrt{\frac{-1+\sqrt{x}}{1+\sqrt{x}}}\right)\right)
$$

```
[In] Integrate[x^(3/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] (Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]*Sqrt[x]*(3 + 3*Sqrt[x] + 2*x + 2*x^(3/2
)) + 6*ArcTanh[Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]])/4
```

Maple [A] (verified)

Time = 4.75 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(2x^{\frac{3}{2}}\sqrt{-1+x}+3\sqrt{x}\,\sqrt{-1+x}+3\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{4\sqrt{-1+x}}$	55
default	$\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(2x^{\frac{3}{2}}\sqrt{-1+x}+3\sqrt{x}\,\sqrt{-1+x}+3\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{4\sqrt{-1+x}}$	55

```
[In] int(x^(3/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/4*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(2*x^(3/2)*(-1+x)^(1/2)+3*x^(1/2)*(
-1+x)^(1/2)+3*ln(x^(1/2)+(-1+x)^(1/2)))/(-1+x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{1}{4}\,(2\,x+3)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} - \frac{3}{8}\,\log\left(2\,\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2\,x+1\right)$$

```
[In] integrate(x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fricas
")
[Out] 1/4*(2*x + 3)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 3/8*log(2*sqrt(
x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

Sympy [F]

$$\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \int \frac{x^{\frac{3}{2}}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\,dx$$

```
[In] integrate(x**(3/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
[Out] Integral(x**(3/2)/(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.51

$$\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{1}{2}\,\sqrt{x-1}x^{\frac{3}{2}} + \frac{3}{4}\,\sqrt{x-1}\sqrt{x} + \frac{3}{4}\,\log\left(2\,\sqrt{x-1} + 2\,\sqrt{x}\right)$$

```
[In] integrate(x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/2*sqrt(x - 1)*x^(3/2) + 3/4*sqrt(x - 1)*sqrt(x) + 3/4*log(2*sqrt(x - 1) +
 2*sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{1}{4}\,\left(\left(2\,\left(\sqrt{x}+1\right)\left(\sqrt{x}-2\right)+9\right)\left(\sqrt{x}+1\right)-5\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} - \frac{3}{2}\,\log\left(\sqrt{\sqrt{x}+1} - \sqrt{\sqrt{x}-1}\right)$$

```
[In] integrate(x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*((2*(sqrt(x) + 1)*(sqrt(x) - 2) + 9)*(sqrt(x) + 1) - 5)*sqrt(sqrt(x) +
1)*sqrt(sqrt(x) - 1) - 3/2*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 26.04 (sec) , antiderivative size = 429, normalized size of antiderivative = 5.88

$$\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = 3\,\mathrm{atanh}\left(\frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{\sqrt{\sqrt{x}+1}-1}\right) + \frac{\frac{23\,\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^3}{\left(\sqrt{\sqrt{x}+1}-1\right)^3} + \frac{333\,\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^5}{\left(\sqrt{\sqrt{x}+1}-1\right)^5} + \frac{671\,\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^7}{\left(\sqrt{\sqrt{x}+1}-1\right)^7} + \frac{671\,\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^9}{\left(\sqrt{\sqrt{x}+1}-1\right)^9} + \frac{333\,\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{11}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{11}} + \frac{23\,\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{13}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{13}} - \frac{3\,(\sqrt{\cdots}}{(\sqrt{\cdots}}}{1 + \frac{28\,\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\sqrt{x}+1}-1\right)^4} - \frac{56\,\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\sqrt{x}+1}-1\right)^6} + \frac{70\,\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\sqrt{x}+1}-1\right)^8} - \frac{56\,\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{10}} + \frac{28\,\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{12}} - \frac{8\,\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{14}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{14}} + \frac{(\sqrt{\cdots}}{(\sqrt{\cdots}}}$$

```
[In] int(x^(3/2)/((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

```
[Out] 3*atanh(((x^(1/2) - 1)^(1/2) - 1i)/((x^(1/2) + 1)^(1/2) - 1)) + ((23*((x^(1
/2) - 1)^(1/2) - 1i)^3)/((x^(1/2) + 1)^(1/2) - 1)^3 + (333*((x^(1/2) - 1)^(
1/2) - 1i)^5)/((x^(1/2) + 1)^(1/2) - 1)^5 + (671*((x^(1/2) - 1)^(1/2) - 1i)
^7)/((x^(1/2) + 1)^(1/2) - 1)^7 + (671*((x^(1/2) - 1)^(1/2) - 1i)^9)/((x^(1
/2) + 1)^(1/2) - 1)^9 + (333*((x^(1/2) - 1)^(1/2) - 1i)^11)/((x^(1/2) + 1)^
(1/2) - 1)^11 + (23*((x^(1/2) - 1)^(1/2) - 1i)^13)/((x^(1/2) + 1)^(1/2) - 1
)^13 - (3*((x^(1/2) - 1)^(1/2) - 1i)^15)/((x^(1/2) + 1)^(1/2) - 1)^15 - (3*
((x^(1/2) - 1)^(1/2) - 1i))/((x^(1/2) + 1)^(1/2) - 1))/((28*((x^(1/2) - 1)^
(1/2) - 1i)^4)/((x^(1/2) + 1)^(1/2) - 1)^4 - (8*((x^(1/2) - 1)^(1/2) - 1i)^
2)/((x^(1/2) + 1)^(1/2) - 1)^2 - (56*((x^(1/2) - 1)^(1/2) - 1i)^6)/((x^(1/2
) + 1)^(1/2) - 1)^6 + (70*((x^(1/2) - 1)^(1/2) - 1i)^8)/((x^(1/2) + 1)^(1/2
) - 1)^8 - (56*((x^(1/2) - 1)^(1/2) - 1i)^10)/((x^(1/2) + 1)^(1/2) - 1)^10
+ (28*((x^(1/2) - 1)^(1/2) - 1i)^12)/((x^(1/2) + 1)^(1/2) - 1)^12 - (8*((x^
(1/2) - 1)^(1/2) - 1i)^14)/((x^(1/2) + 1)^(1/2) - 1)^14 + ((x^(1/2) - 1)^(1
/2) - 1i)^16/((x^(1/2) + 1)^(1/2) - 1)^16 + 1)
```

3.1013 $\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 35

$$\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx=\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}+\operatorname{arccosh}\left(\sqrt{x}\right)$$

```
[Out] arccosh(x^(1/2))+x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {329, 336, 54}

$$\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx=\operatorname{arccosh}\left(\sqrt{x}\right)+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}$$

```
[In] Int[Sqrt[x]/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] + ArcCosh[Sqrt[x]]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
```

```
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Dist[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1))), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 336

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}+\frac{1}{2}\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx\\ &= \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}+\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}}\,dx,x,\sqrt{x}\right)\\ &= \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}+\cosh^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 265 vs. $2(35) = 70$.

Time = 1.52 (sec) , antiderivative size = 265, normalized size of antiderivative = 7.57

$$\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{4\Big(4\sqrt{1+\sqrt{x}}\left(-12-24\sqrt{x}+x+5x^{3/2}\right)+\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\left(-84-10\sqrt{x}+28x+7x^{3/2}\right)+\sqrt{3}\Big(28}{56-16\sqrt{3}\sqrt{1+\sqrt{x}}\left(2+3\sqrt{x}\right)+\sqrt{-1+\sqrt{x}}\left(96-8\sqrt{3}\sqrt{1+\sqrt{}}\right.} - 4\text{arctanh}\left(\frac{-1+\sqrt{-1+\sqrt{x}}}{\sqrt{3}-\sqrt{1+\sqrt{x}}}\right)$$

```
[In] Integrate[Sqrt[x]/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] (4*(4*Sqrt[1 + Sqrt[x]]*(-12 - 24*Sqrt[x] + x + 5*x^(3/2)) + Sqrt[-1 + Sqrt
[x]]*Sqrt[1 + Sqrt[x]]*(-84 - 10*Sqrt[x] + 28*x + 7*x^(3/2)) + Sqrt[3]*(28
+ 70*Sqrt[x] + 18*x - 14*x^(3/2) - 4*x^2 - 4*Sqrt[-1 + Sqrt[x]]*(-12 - 8*Sq
```

```
rt[x] + 5*x + 3*x^(3/2)))))/(56 - 16*Sqrt[3]*Sqrt[1 + Sqrt[x]]*(2 + 3*Sqrt[
x]) + Sqrt[-1 + Sqrt[x]]*(96 - 8*Sqrt[3]*Sqrt[1 + Sqrt[x]]*(7 + 2*Sqrt[x])
+ 80*Sqrt[x]) + 112*Sqrt[x] + 28*x) - 4*ArcTanh[(-1 + Sqrt[-1 + Sqrt[x]])/(
Sqrt[3] - Sqrt[1 + Sqrt[x]])]
```

Maple [A] (verified)

Time = 4.61 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(\sqrt{x}\,\sqrt{-1+x}+\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{\sqrt{-1+x}}$	41
default	$\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(\sqrt{x}\,\sqrt{-1+x}+\ln\left(\sqrt{x}+\sqrt{-1+x}\right)\right)}{\sqrt{-1+x}}$	41

```
[In] int(x^(1/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(x^(1/2)*(-1+x)^(1/2)+ln(x^(1/2)+(-1+x)
^(1/2)))/(-1+x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} - \frac{1}{2}\,\log\left(2\,\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2\,x+1\right)$$

```
[In] integrate(x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 1/2*log(2*sqrt(x)*sqrt(sqrt(x
) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

Sympy [F]

$$\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \int \frac{\sqrt{x}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\,dx$$

```
[In] integrate(x**(1/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
[Out] Integral(sqrt(x)/(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \sqrt{x-1}\sqrt{x} + \log\left(2\sqrt{x-1} + 2\sqrt{x}\right)$$

```
[In] integrate(x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxima
")
[Out] sqrt(x - 1)*sqrt(x) + log(2*sqrt(x - 1) + 2*sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} - 2\log\left(\sqrt{\sqrt{x}+1} - \sqrt{\sqrt{x}-1}\right)$$

```
[In] integrate(x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac")
[Out] sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*log(sqrt(sqrt(x) + 1) - sqr
t(sqrt(x) - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \int \frac{\sqrt{x}}{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}\,dx$$

```
[In] int(x^(1/2)/((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
[Out] int(x^(1/2)/((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)), x)
```

3.1014 $\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 8

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx = 2\text{arccosh}\left(\sqrt{x}\right)$$

```
[Out] 2*arccosh(x^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {336, 54}

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx = 2\text{arccosh}\left(\sqrt{x}\right)$$

```
[In] Int[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]),x]
```

```
[Out] 2*ArcCosh[Sqrt[x]]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 336

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
```

```
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}}\,dx, x, \sqrt{x}\right)\\ &= 2\cosh^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 38 vs. 2(8) = 16.

Time = 1.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 4.75

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx = -8\text{arctanh}\left(\frac{-1+\sqrt{-1+\sqrt{x}}}{\sqrt{3}-\sqrt{1+\sqrt{x}}}\right)$$

```
[In] Integrate[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]),x]
[Out] -8*ArcTanh[(-1 + Sqrt[-1 + Sqrt[x]])/(Sqrt[3] - Sqrt[1 + Sqrt[x]])]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(6) = 12.

Time = 4.59 (sec) , antiderivative size = 40, normalized size of antiderivative = 5.00

method	result	size
derivativedivides	$\frac{2\sqrt{(\sqrt{x}+1)(\sqrt{x}-1)}\,\ln(\sqrt{x}+\sqrt{-1+x})}{\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}}$	40
default	$\frac{2\sqrt{(\sqrt{x}+1)(\sqrt{x}-1)}\,\ln(\sqrt{x}+\sqrt{-1+x})}{\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}}$	40

```
[In] int(1/x^(1/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*((x^(1/2)+1)*(x^(1/2)-1))^(1/2)/(x^(1/2)+1)^(1/2)/(x^(1/2)-1)^(1/2)*ln(x^
(1/2)+(-1+x)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(6) = 12$.

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 3.38

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx = -\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2x+1\right)$$

```
[In] integrate(1/x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fric
as")
[Out] -log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx = \int \frac{1}{\sqrt{x}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\,dx$$

```
[In] integrate(1/x**(1/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
[Out] Integral(1/(sqrt(x)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(6) = 12$.

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx = 2\log\left(2\sqrt{x-1}+2\sqrt{x}\right)$$

```
[In] integrate(1/x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxi
ma")
[Out] 2*log(2*sqrt(x - 1) + 2*sqrt(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(6) = 12$.

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.50

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx = -4\,\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

```
[In] integrate(1/x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac
")
```

```
[Out] -4*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx = 2\,\mathrm{acosh}\left(\sqrt{x}\right)$$

```
[In] int(1/(x^(1/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

```
[Out] 2*acosh(x^(1/2))
```

3.1015 $\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 29

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx = \frac{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}$$

```
[Out] 2*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {271}

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx = \frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{\sqrt{x}}$$

```
[In] Int[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2)),x]
```

```
[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x]
```

Rule 271

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b
2*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rubi steps

$$\text{integral} = \frac{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 146 vs. $2(29) = 58$.

Time = 1.13 (sec) , antiderivative size = 146, normalized size of antiderivative = 5.03

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx = \frac{\left(-1+\sqrt{-1+\sqrt{x}}\right)\left(\sqrt{3}-\sqrt{1+\sqrt{x}}\right)\left(-2+\sqrt{-1+\sqrt{x}}+\sqrt{3}\sqrt{1+\sqrt{\cdots}}\right.}{\left(3+2\sqrt{-1+\sqrt{x}}-2\sqrt{3}\sqrt{1+\sqrt{x}}-\sqrt{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}+2\sqrt{x}\cdots\right.}$$

```
[In] Integrate[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2)),x]
```

```
[Out] ((-1 + Sqrt[-1 + Sqrt[x]])*(Sqrt[3] - Sqrt[1 + Sqrt[x]])*(-2 + Sqrt[-1 + Sq
rt[x]] + Sqrt[3]*Sqrt[1 + Sqrt[x]] - Sqrt[x]))/((3 + 2*Sqrt[-1 + Sqrt[x]] -
 2*Sqrt[3]*Sqrt[1 + Sqrt[x]] - Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]
 + 2*Sqrt[x])*Sqrt[x])
```

Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{\sqrt{x}}$	20
default	$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{\sqrt{x}}$	20

```
[In] int(1/x^(3/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx = \frac{2\left(\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+x\right)}{x}$$

```
[In] integrate(1/x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fric
as")
```

```
[Out] 2*(sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + x)/x
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx = \int \frac{1}{x^{\frac{3}{2}}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\,dx$$

```
[In] integrate(1/x**(3/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
[Out] Integral(1/(x**(3/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.34

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx = \frac{2\sqrt{x-1}}{\sqrt{x}}$$

```
[In] integrate(1/x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxi
ma")
[Out] 2*sqrt(x - 1)/sqrt(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx = \frac{16}{\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^4+4}$$

```
[In] integrate(1/x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac
")
[Out] 16/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 4)
```

Mupad [B] (verification not implemented)

Time = 9.68 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx = \frac{2\,\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{\sqrt{x}}$$

```
[In] int(1/(x^(3/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
[Out] (2*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(1/2)
```

3.1016 $\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 63

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}}\,dx=\frac{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{3x^{3/2}}+\frac{4\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{3\sqrt{x}}$$

```
[Out] 2/3*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2)+4/3*(-1+x^(1/2))^(1/2)*(1+
x^(1/2))^(1/2)/x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {278, 271}

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}}\,dx=\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{3x^{3/2}}+\frac{4\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{3\sqrt{x}}$$

```
[In] Int[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2)),x]
```

```
[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/(3*x^(3/2)) + (4*Sqrt[-1 + Sqrt[x]
]*Sqrt[1 + Sqrt[x]])/(3*Sqrt[x])
```

Rule 271

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b
2*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rule 278

```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[x^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b2*x^n)^(p + 1
)/(a1*a2*(m + 1))), x] - Dist[b1*b2*((m + 2*n*(p + 1) + 1)/(a1*a2*(m + 1)))
, Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1,
 a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*
n) + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{3x^{3/2}}+\frac{2}{3}\int\frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx\\ &= \frac{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{3x^{3/2}}+\frac{4\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{3\sqrt{x}}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 407 vs. $2(63) = 126$.

Time = 2.56 (sec) , antiderivative size = 407, normalized size of antiderivative = 6.46

$$\int\frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}}\,dx=\frac{\left(-1+\sqrt{-1+\sqrt{x}}\right)\left(\sqrt{3}-\sqrt{1+\sqrt{x}}\right)\left(-2+\sqrt{-1+\sqrt{x}}+\sqrt{3}\sqrt{1+\sqrt{\ }}\right.}{}$$

```
[In] Integrate[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2)),x]
```

```
[Out] ((-1 + Sqrt[-1 + Sqrt[x]])*(Sqrt[3] - Sqrt[1 + Sqrt[x]])*(-2 + Sqrt[-1 + Sq
rt[x]] + Sqrt[3]*Sqrt[1 + Sqrt[x]] - Sqrt[x])*(8*(-7 - 12*Sqrt[-1 + Sqrt[x]
] + 4*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 7*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqr
t[x]]) - 4*(49 + 8*Sqrt[-1 + Sqrt[x]] - 24*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 3*Sq
rt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*Sqrt[x] + 2*(-61 + 16*Sqrt[3]*S
qrt[1 + Sqrt[x]] + 7*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*x + (-56
 - 28*Sqrt[-1 + Sqrt[x]] + 20*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 6*Sqrt[3]*Sqrt[-1
 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*x^(3/2) - 11*x^2))/(12*(-3 - 2*Sqrt[-1 + Sqr
t[x]] + 2*Sqrt[3]*Sqrt[1 + Sqrt[x]] + Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + S
qrt[x]] - 2*Sqrt[x])^3*x^(3/2))
```

Maple [A] (verified)

Time = 4.59 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.40

method	result	size
derivativedivides	$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,(1+2x)}{3x^{\frac{3}{2}}}$	25
default	$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,(1+2x)}{3x^{\frac{3}{2}}}$	25

```
[In] int(1/x^(5/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(1+2*x)/x^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.54

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}}\,dx = \frac{2\left((2\,x+1)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+2\,x^2\right)}{3\,x^2}$$

```
[In] integrate(1/x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fric
as")
[Out] 2/3*((2*x + 1)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 2*x^2)/x^2
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}}\,dx = \int \frac{1}{x^{\frac{5}{2}}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\,dx$$

```
[In] integrate(1/x**(5/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
[Out] Integral(1/(x**(5/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.33

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}}\,dx = \frac{4\sqrt{x-1}}{3\sqrt{x}} + \frac{2\sqrt{x-1}}{3\,x^{\frac{3}{2}}}$$

```
[In] integrate(1/x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 4/3*sqrt(x - 1)/sqrt(x) + 2/3*sqrt(x - 1)/x^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}}\,dx = \frac{128\left(3\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^4+4\right)}{3\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^4+4\right)^3}$$

```
[In] integrate(1/x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac
")
```

```
[Out] 128/3*(3*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 4)/((sqrt(sqrt(x) + 1)
 - sqrt(sqrt(x) - 1))^4 + 4)^3
```

Mupad [B] (verification not implemented)

Time = 9.68 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}}\,dx = \frac{\sqrt{\sqrt{x}-1}\left(\frac{4\,x}{3}+\frac{2\,\sqrt{x}}{3}+\frac{4\,x^{3/2}}{3}+\frac{2}{3}\right)}{x^{3/2}\,\sqrt{\sqrt{x}+1}}$$

```
[In] int(1/(x^(5/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

```
[Out] ((x^(1/2) - 1)^(1/2)*((4*x)/3 + (2*x^(1/2))/3 + (4*x^(3/2))/3 + 2/3))/(x^(3
/2)*(x^(1/2) + 1)^(1/2))
```

3.1017 $\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 94

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}}\,dx=\frac{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{5x^{5/2}}+\frac{8\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{15x^{3/2}}+\frac{16\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{15\sqrt{x}}$$

```
[Out] 2/5*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(5/2)+8/15*(-1+x^(1/2))^(1/2)*(1
+x^(1/2))^(1/2)/x^(3/2)+16/15*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {278, 271}

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}}\,dx=\frac{8\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{15x^{3/2}}+\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{5x^{5/2}}+\frac{16\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{15\sqrt{x}}$$

```
[In] Int[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(7/2)),x]
```

```
[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/(5*x^(5/2)) + (8*Sqrt[-1 + Sqrt[x]
]*Sqrt[1 + Sqrt[x]])/(15*x^(3/2)) + (16*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]
])/(15*Sqrt[x])
```

Rule 271

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b
2*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rule 278

```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[x^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b2*x^n)^(p + 1
)/(a1*a2*(m + 1))), x] - Dist[b1*b2*((m + 2*n*(p + 1) + 1)/(a1*a2*(m + 1)))
, Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1,
 a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*
n) + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{5x^{5/2}}+\frac{4}{5}\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}}\,dx\\
&= \frac{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{5x^{5/2}}+\frac{8\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{15x^{3/2}}+\frac{8}{15}\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx\\
&= \frac{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{5x^{5/2}}+\frac{8\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{15x^{3/2}}+\frac{16\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{15\sqrt{x}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}}\,dx=\frac{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}(3+4x+8x^2)}{15x^{5/2}}$$

```
[In] Integrate[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(7/2)),x]
[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*(3 + 4*x + 8*x^2))/(15*x^(5/2))
```

Maple [A] (verified)

Time = 4.59 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.32

method	result	size
derivativedivides	$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,(8x^{2}+4x+3)}{15x^{\frac{5}{2}}}$	30
default	$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,(8x^{2}+4x+3)}{15x^{\frac{5}{2}}}$	30

```
[In] int(1/x^(7/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/15*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(8*x^2+4*x+3)/x^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.41

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}}\,dx = \frac{2\left(8\,x^{3}+(8\,x^{2}+4\,x+3)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}\right)}{15\,x^{3}}$$

```
[In] integrate(1/x^(7/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fric
as")
[Out] 2/15*(8*x^3 + (8*x^2 + 4*x + 3)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1)
)/x^3
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}}\,dx = \int \frac{1}{x^{\frac{7}{2}}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\,dx$$

```
[In] integrate(1/x**(7/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
[Out] Integral(1/(x**(7/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.33

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}}\,dx = \frac{16\sqrt{x-1}}{15\sqrt{x}} + \frac{8\sqrt{x-1}}{15\,x^{\frac{3}{2}}} + \frac{2\sqrt{x-1}}{5\,x^{\frac{5}{2}}}$$

```
[In] integrate(1/x^(7/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 16/15*sqrt(x - 1)/sqrt(x) + 8/15*sqrt(x - 1)/x^(3/2) + 2/5*sqrt(x - 1)/x^(5
/2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}}\,dx = \frac{4096\left(5\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{8}+10\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+8\right)}{15\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+4\right)^{5}}$$

```
[In] integrate(1/x^(7/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac
")
```

```
[Out] 4096/15*(5*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^8 + 10*(sqrt(sqrt(x) + 1
) - sqrt(sqrt(x) - 1))^4 + 8)/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 +
4)^5
```

Mupad [B] (verification not implemented)

Time = 9.71 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.46

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}}\,dx = \frac{\sqrt{\sqrt{x}-1}\left(\frac{8\,x}{15}+\frac{16\,x^2}{15}+\frac{2\sqrt{x}}{5}+\frac{8\,x^{3/2}}{15}+\frac{16\,x^{5/2}}{15}+\frac{2}{5}\right)}{x^{5/2}\sqrt{\sqrt{x}+1}}$$

```
[In] int(1/(x^(7/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

```
[Out] ((x^(1/2) - 1)^(1/2)*((8*x)/15 + (16*x^2)/15 + (2*x^(1/2))/5 + (8*x^(3/2))/
15 + (16*x^(5/2))/15 + 2/5))/(x^(5/2)*(x^(1/2) + 1)^(1/2))
```

3.1018 $\int x^2(-a+bx^n)^p (a+bx^n)^p \, dx$

Optimal result

Integrand size = 24, antiderivative size = 78

$$\int x^2(-a+bx^n)^p (a+bx^n)^p \, dx = \frac{1}{3}x^3(-a+bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{2n},-p,1+\frac{3}{2n},\frac{b^2x^{2n}}{a^2}\right)$$

```
[Out] 1/3*x^3*(-a+b*x^n)^p*(a+b*x^n)^p*hypergeom([-p, 3/2/n],[1+3/2/n],b^2*x^(2*n
)/a^2)/((1-b^2*x^(2*n)/a^2)^p)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {373, 372, 371}

$$\int x^2(-a+bx^n)^p (a+bx^n)^p \, dx = \frac{1}{3}x^3(bx^n-a)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{2n},-p,1+\frac{3}{2n},\frac{b^2x^{2n}}{a^2}\right)$$

```
[In] Int[x^2*(-a + b*x^n)^p*(a + b*x^n)^p,x]
```

```
[Out] (x^3*(-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[3/(2*n), -p, 1 + 3/(2*n
), (b^2*x^(2*n))/a^2])/(3*(1 - (b^2*x^(2*n))/a^2)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 373

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Dist[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^Fra
cPart[p]/(a1*a2 + b1*b2*x^(2*n))^FracPart[p]), Int[(c*x)^m*(a1*a2 + b1*b2*x
^(2*n))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 +
a1*b2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left((-a+bx^n)^p(a+bx^n)^p\left(-a^2+b^2x^{2n}\right)^{-p}\right)\int x^2\left(-a^2+b^2x^{2n}\right)^p\,dx \\
&= \left((-a+bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\right)\int x^2\left(1-\frac{b^2x^{2n}}{a^2}\right)^p\,dx \\
&= \frac{1}{3}x^3(-a+bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{3}{2n},-p;1+\frac{3}{2n};\frac{b^2x^{2n}}{a^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.03

$$\int x^2(-a+bx^n)^p(a+bx^n)^p\,dx = \frac{1}{3}x^3(-a+bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{3}{2n},-p;1+\frac{3}{2n};\frac{b^2x^{2n}}{a^2}\right)$$

```
[In] Integrate[x^2*(-a + b*x^n)^p*(a + b*x^n)^p,x]
```

```
[Out] (x^3*(-a + b*x^n)^p*(a + b*x^n)^p*HypergeometricPFQ[{3/(2*n), -p}, {1 + 3/(
2*n)}, (b^2*x^(2*n))/a^2])/(3*(1 - (b^2*x^(2*n))/a^2)^p)
```

Maple [F]

$$\int x^2 (b\,x^n - a)^p\,(a + b\,x^n)^p\,dx$$

```
[In] int(x^2*(b*x^n-a)^p*(a+b*x^n)^p,x)
[Out] int(x^2*(b*x^n-a)^p*(a+b*x^n)^p,x)
```

Fricas [F]

$$\int x^2 (-a + bx^n)^p\,(a + bx^n)^p\,dx = \int (bx^n + a)^p (bx^n - a)^p x^2\,dx$$

```
[In] integrate(x^2*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="fricas")
[Out] integral((b*x^n + a)^p*(b*x^n - a)^p*x^2, x)
```

Sympy [F]

$$\int x^2 (-a + bx^n)^p\,(a + bx^n)^p\,dx = \int x^2 (-a + bx^n)^p\,(a + bx^n)^p\,dx$$

```
[In] integrate(x**2*(-a+b*x**n)**p*(a+b*x**n)**p,x)
[Out] Integral(x**2*(-a + b*x**n)**p*(a + b*x**n)**p, x)
```

Maxima [F]

$$\int x^2 (-a + bx^n)^p\,(a + bx^n)^p\,dx = \int (bx^n + a)^p (bx^n - a)^p x^2\,dx$$

```
[In] integrate(x^2*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="maxima")
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p*x^2, x)
```

Giac [F]

$$\int x^2(-a+bx^n)^p\,(a+bx^n)^p\;dx=\int (bx^n+a)^p(bx^n-a)^px^2\,dx$$

```
[In] integrate(x^2*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="giac")
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(-a+bx^n)^p\,(a+bx^n)^p\;dx=\int x^2\,(a+b\,x^n)^p\,(b\,x^n-a)^p\,dx$$

```
[In] int(x^2*(a + b*x^n)^p*(b*x^n - a)^p,x)
[Out] int(x^2*(a + b*x^n)^p*(b*x^n - a)^p, x)
```

3.1019 $\int x(-a+bx^n)^p (a+bx^n)^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 70

$$\int x(-a+bx^n)^p (a+bx^n)^p \, dx = \frac{1}{2}x^2(-a+bx^n)^p (a+bx^n)^p \left(1 - \frac{b^2x^{2n}}{a^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},\frac{b^2x^{2n}}{a^2}\right)$$

```
[Out] 1/2*x^2*(-a+b*x^n)^p*(a+b*x^n)^p*hypergeom([1/n, -p],[1+1/n],b^2*x^(2*n)/a^
2)/((1-b^2*x^(2*n)/a^2)^p)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {373, 372, 371}

$$\int x(-a+bx^n)^p (a+bx^n)^p \, dx = \frac{1}{2}x^2(bx^n-a)^p (a+bx^n)^p \left(1 - \frac{b^2x^{2n}}{a^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},\frac{b^2x^{2n}}{a^2}\right)$$

```
[In] Int[x*(-a + b*x^n)^p*(a + b*x^n)^p,x]
```

```
[Out] (x^2*(-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1),
 (b^2*x^(2*n))/a^2])/(2*(1 - (b^2*x^(2*n))/a^2)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 373

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Dist[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^Fra
cPart[p]/(a1*a2 + b1*b2*x^(2*n))^FracPart[p]), Int[(c*x)^m*(a1*a2 + b1*b2*x
^(2*n))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 +
a1*b2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left((-a+bx^n)^p(a+bx^n)^p\left(-a^2+b^2x^{2n}\right)^{-p}\right)\int x\left(-a^2+b^2x^{2n}\right)^p\,dx\\ &= \left((-a+bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\right)\int x\left(1-\frac{b^2x^{2n}}{a^2}\right)^p\,dx\\ &= \frac{1}{2}x^2(-a+bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};\frac{b^2x^{2n}}{a^2}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03

$$\int x(-a+bx^n)^p(a+bx^n)^p\,dx=\frac{1}{2}x^2(-a+bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};\frac{b^2x^{2n}}{a^2}\right)$$

```
[In] Integrate[x*(-a + b*x^n)^p*(a + b*x^n)^p,x]
[Out] (x^2*(-a + b*x^n)^p*(a + b*x^n)^p*HypergeometricPFQ[{n^(-1), -p}, {1 + n^(-
1)}, (b^2*x^(2*n))/a^2])/(2*(1 - (b^2*x^(2*n))/a^2)^p)
```

Maple [F]

$$\int x(b\,x^n-a)^p\,(a+b\,x^n)^p\,dx$$

```
[In] int(x*(b*x^n-a)^p*(a+b*x^n)^p,x)
[Out] int(x*(b*x^n-a)^p*(a+b*x^n)^p,x)
```

Fricas [F]

$$\int x(-a+bx^n)^p\,(a+bx^n)^p\;dx = \int (bx^n+a)^p(bx^n-a)^px\,dx$$

```
[In] integrate(x*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="fricas")
[Out] integral((b*x^n + a)^p*(b*x^n - a)^p*x, x)
```

Sympy [F]

$$\int x(-a+bx^n)^p\,(a+bx^n)^p\;dx = \int x(-a+bx^n)^p\,(a+bx^n)^p\;dx$$

```
[In] integrate(x*(-a+b*x**n)**p*(a+b*x**n)**p,x)
[Out] Integral(x*(-a + b*x**n)**p*(a + b*x**n)**p, x)
```

Maxima [F]

$$\int x(-a+bx^n)^p\,(a+bx^n)^p\;dx = \int (bx^n+a)^p(bx^n-a)^px\,dx$$

```
[In] integrate(x*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="maxima")
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p*x, x)
```

Giac [F]

$$\int x(-a+bx^n)^p\,(a+bx^n)^p\;dx=\int (bx^n+a)^p(bx^n-a)^px\,dx$$

```
[In] integrate(x*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="giac")
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(-a+bx^n)^p\,(a+bx^n)^p\;dx=\int x\,(a+b\,x^n)^p\,(b\,x^n-a)^p\,dx$$

```
[In] int(x*(a + b*x^n)^p*(b*x^n - a)^p,x)
[Out] int(x*(a + b*x^n)^p*(b*x^n - a)^p, x)
```

3.1020 $\int (-a+bx^n)^p (a+bx^n)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 73

$$\int (-a+bx^n)^p (a+bx^n)^p \, dx = x(-a+bx^n)^p (a+bx^n)^p \left(1 - \frac{b^2x^{2n}}{a^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2n}, -p, \frac{1}{2}\left(2+\frac{1}{n}\right), \frac{b^2x^{2n}}{a^2}\right)$$

```
[Out] x*(-a+b*x^n)^p*(a+b*x^n)^p*hypergeom([-p, 1/2/n],[1+1/2/n],b^2*x^(2*n)/a^2)
/((1-b^2*x^(2*n)/a^2)^p)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {259, 252, 251}

$$\int (-a+bx^n)^p (a+bx^n)^p \, dx = x(bx^n-a)^p (a+bx^n)^p \left(1 - \frac{b^2x^{2n}}{a^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2n}, -p, \frac{1}{2}\left(2+\frac{1}{n}\right), \frac{b^2x^{2n}}{a^2}\right)$$

```
[In] Int[(-a + b*x^n)^p*(a + b*x^n)^p,x]
```

```
[Out] (x*(-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p, (2 + n^(-1))
/2, (b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 259

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^FracPart[p]/(a1*a2 +
 b1*b2*x^(2*n))^FracPart[p]), Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left((-a+bx^n)^p\,(a+bx^n)^p\left(-a^2+b^2x^{2n}\right)^{-p}\right)\int\left(-a^2+b^2x^{2n}\right)^p\,dx\\
&= \left((-a+bx^n)^p\,(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\right)\int\left(1-\frac{b^2x^{2n}}{a^2}\right)^p\,dx\\
&= x(-a+bx^n)^p\,(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int(-a+bx^n)^p\,(a+bx^n)^p\,dx = x(-a+bx^n)^p\,(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2n},-p,1+\frac{1}{2n},\frac{b^2x^{2n}}{a^2}\right)$$

```
[In] Integrate[(-a + b*x^n)^p*(a + b*x^n)^p,x]
[Out] (x*(-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p, 1 + 1/(2*n),
 (b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p
```

Maple [F]

$$\int (b\,x^n-a)^p\,(a+b\,x^n)^p\,dx$$

```
[In] int((b*x^n-a)^p*(a+b*x^n)^p,x)
[Out] int((b*x^n-a)^p*(a+b*x^n)^p,x)
```

Fricas [F]

$$\int (-a+bx^n)^p\,(a+bx^n)^p\;dx=\int (bx^n+a)^p(bx^n-a)^p\,dx$$

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="fricas")
[Out] integral((b*x^n + a)^p*(b*x^n - a)^p, x)
```

Sympy [F]

$$\int (-a+bx^n)^p\,(a+bx^n)^p\;dx=\int (-a+bx^n)^p\,(a+bx^n)^p\;dx$$

```
[In] integrate((-a+b*x**n)**p*(a+b*x**n)**p,x)
[Out] Integral((-a + b*x**n)**p*(a + b*x**n)**p, x)
```

Maxima [F]

$$\int (-a+bx^n)^p\,(a+bx^n)^p\;dx=\int (bx^n+a)^p(bx^n-a)^p\,dx$$

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="maxima")
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p, x)
```

Giac [F]

$$\int (-a+bx^n)^p (a+bx^n)^p \, dx = \int (bx^n+a)^p(bx^n-a)^p \, dx$$

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="giac")
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (-a+bx^n)^p (a+bx^n)^p \, dx = \int (a+b\,x^n)^p \,(b\,x^n-a)^p \, dx$$

```
[In] int((a + b*x^n)^p*(b*x^n - a)^p,x)
[Out] int((a + b*x^n)^p*(b*x^n - a)^p, x)
```

3.1021 $\int \frac{(-a+bx^n)^p(a+bx^n)^p}{x} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 72

$$\int \frac{(-a+bx^n)^p (a+bx^n)^p}{x} \, dx = -\frac{(-a+bx^n)^p (a+bx^n)^p (a^2-b^2x^{2n}) \text{Hypergeometric2F1}\left(1,1+p,2+p,1-\frac{b^2x^{2n}}{a^2}\right)}{2a^2n(1+p)}$$

```
[Out] -1/2*(-a+b*x^n)^p*(a+b*x^n)^p*(a^2-b^2*x^(2*n))*hypergeom([1, p+1],[2+p],1-
b^2*x^(2*n)/a^2)/a^2/n/(p+1)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {273, 127, 272, 67}

$$\int \frac{(-a+bx^n)^p (a+bx^n)^p}{x} \, dx = -\frac{(a^2-b^2x^{2n}) (bx^n-a)^p (a+bx^n)^p \text{Hypergeometric2F1}\left(1,p+1,p+2,1-\frac{b^2x^{2n}}{a^2}\right)}{2a^2n(p+1)}$$

```
[In] Int[((-a + b*x^n)^p*(a + b*x^n)^p)/x,x]
```

```
[Out] -1/2*((-a + b*x^n)^p*(a + b*x^n)^p*(a^2 - b^2*x^(2*n))*Hypergeometric2F1[1,
 1 + p, 2 + p, 1 - (b^2*x^(2*n))/a^2])/(a^2*n*(1 + p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
```

```
d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
|| GtQ[-d/(b*c), 0])
```

Rule 127

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*
d*x^2)^FracPart[m]), Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b,
c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 273

```
Int[(x_)^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(
p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a1 + b1*
x)^p*(a2 + b2*x)^p, x], x, x^n], x] /; FreeQ[{a1, b1, a2, b2, m, n, p}, x]
&& EqQ[a2*b1 + a1*b2, 0] && IntegerQ[Simplify[(m + 1)/(2*n)]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-a+bx)^p(a+bx)^p}{x}\,dx, x, x^n\right)}{n}\\
&= \frac{\left((-a+bx^n)^p\,(a+bx^n)^p\,(-a^2+b^2x^{2n})^{-p}\right)\text{Subst}\left(\int \frac{(-a^2+b^2x^2)^p}{x}\,dx, x, x^n\right)}{n}\\
&= \frac{\left((-a+bx^n)^p\,(a+bx^n)^p\,(-a^2+b^2x^{2n})^{-p}\right)\text{Subst}\left(\int \frac{(-a^2+b^2x)^p}{x}\,dx, x, x^{2n}\right)}{2n}\\
&= -\frac{(-a+bx^n)^p\,(a+bx^n)^p\,(a^2-b^2x^{2n})\;{}_2F_1\left(1,1+p;2+p;1-\frac{b^2x^{2n}}{a^2}\right)}{2a^2n(1+p)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

$$\int \frac{(-a+bx^n)^p (a+bx^n)^p}{x} \, dx = \frac{(-a+bx^n)^p (a+bx^n)^p (-a^2+b^2x^{2n}) \text{Hypergeometric2F1}\left(1,1+p,2+p,1-\frac{b^2x^{2n}}{a^2}\right)}{2a^2n(1+p)}$$

```
[In] Integrate[((-a + b*x^n)^p*(a + b*x^n)^p)/x,x]
[Out] ((-a + b*x^n)^p*(a + b*x^n)^p*(-a^2 + b^2*x^(2*n))*Hypergeometric2F1[1, 1 +
 p, 2 + p, 1 - (b^2*x^(2*n))/a^2])/(2*a^2*n*(1 + p))
```

Maple [F]

$$\int \frac{(b\,x^n-a)^p (a+b\,x^n)^p}{x}dx$$

```
[In] int((b*x^n-a)^p*(a+b*x^n)^p/x,x)
[Out] int((b*x^n-a)^p*(a+b*x^n)^p/x,x)
```

Fricas [F]

$$\int \frac{(-a+bx^n)^p (a+bx^n)^p}{x} \, dx = \int \frac{(bx^n+a)^p(bx^n-a)^p}{x} \, dx$$

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p/x,x, algorithm="fricas")
[Out] integral((b*x^n + a)^p*(b*x^n - a)^p/x, x)
```

Sympy [F]

$$\int \frac{(-a+bx^n)^p (a+bx^n)^p}{x} \, dx = \int \frac{(-a+bx^n)^p (a+bx^n)^p}{x} \, dx$$

```
[In] integrate((-a+b*x**n)**p*(a+b*x**n)**p/x,x)
[Out] Integral((-a + b*x**n)**p*(a + b*x**n)**p/x, x)
```

Maxima [F]

$$\int \frac{(-a+bx^n)^p (a+bx^n)^p}{x} \, dx = \int \frac{(bx^n+a)^p (bx^n-a)^p}{x} \, dx$$

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p/x,x, algorithm="maxima")
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p/x, x)
```

Giac [F]

$$\int \frac{(-a+bx^n)^p (a+bx^n)^p}{x} \, dx = \int \frac{(bx^n+a)^p (bx^n-a)^p}{x} \, dx$$

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p/x,x, algorithm="giac")
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(-a+bx^n)^p (a+bx^n)^p}{x} \, dx = \int \frac{(a+b\,x^n)^p\,(b\,x^n-a)^p}{x} \, dx$$

```
[In] int(((a + b*x^n)^p*(b*x^n - a)^p)/x,x)
[Out] int(((a + b*x^n)^p*(b*x^n - a)^p)/x, x)
```

3.1022 $\int \frac{(-a+bx^n)^p(a+bx^n)^p}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 76

$$\int \frac{(-a+bx^n)^p\,(a+bx^n)^p}{x^2}\,dx = -\frac{(-a+bx^n)^p\,(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2n},-p,1-\frac{1}{2n},\frac{b^2x^{2n}}{a^2}\right)}{x}$$

```
[Out] -(-a+b*x^n)^p*(a+b*x^n)^p*hypergeom([-p, -1/2/n],[1-1/2/n],b^2*x^(2*n)/a^2)
/x/((1-b^2*x^(2*n)/a^2)^p)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {373, 372, 371}

$$\int \frac{(-a+bx^n)^p\,(a+bx^n)^p}{x^2}\,dx = -\frac{(bx^n-a)^p\,(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2n},-p,1-\frac{1}{2n},\frac{b^2x^{2n}}{a^2}\right)}{x}$$

```
[In] Int[((-a + b*x^n)^p*(a + b*x^n)^p)/x^2,x]
```

```
[Out] -(((-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[-1/2*1/n, -p, 1 - 1/(2*n)
, (b^2*x^(2*n))/a^2])/(x*(1 - (b^2*x^(2*n))/a^2)^p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 373

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Dist[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^Fra
cPart[p]/(a1*a2 + b1*b2*x^(2*n))^FracPart[p]), Int[(c*x)^m*(a1*a2 + b1*b2*x
^(2*n))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 +
a1*b2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left((-a+bx^n)^p(a+bx^n)^p\left(-a^2+b^2x^{2n}\right)^{-p}\right)\int\frac{\left(-a^2+b^2x^{2n}\right)^p}{x^2}\,dx\\
&= \left((-a+bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\right)\int\frac{\left(1-\frac{b^2x^{2n}}{a^2}\right)^p}{x^2}\,dx\\
&= -\frac{(-a+bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2n},-p;1-\frac{1}{2n};\frac{b^2x^{2n}}{a^2}\right)}{x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int\frac{(-a+bx^n)^p(a+bx^n)^p}{x^2}\,dx = -\frac{(-a+bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(-\frac{1}{2n},-p;1-\frac{1}{2n};\frac{b^2x^{2n}}{a^2}\right)}{x}$$

```
[In] Integrate[((-a + b*x^n)^p*(a + b*x^n)^p)/x^2,x]
```

```
[Out] -(((-a + b*x^n)^p*(a + b*x^n)^p*HypergeometricPFQ[{-1/2*1/n, -p}, {1 - 1/(2
*n)}, (b^2*x^(2*n))/a^2])/(x*(1 - (b^2*x^(2*n))/a^2)^p))
```

Maple [F]

$$\int \frac{(b\,x^n-a)^p\,(a+b\,x^n)^p}{x^2}dx$$

```
[In] int((b*x^n-a)^p*(a+b*x^n)^p/x^2,x)
[Out] int((b*x^n-a)^p*(a+b*x^n)^p/x^2,x)
```

Fricas [F]

$$\int \frac{(-a+bx^n)^p\,(a+bx^n)^p}{x^2}\,dx = \int \frac{(bx^n+a)^p(bx^n-a)^p}{x^2}\,dx$$

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p/x^2,x, algorithm="fricas")
[Out] integral((b*x^n + a)^p*(b*x^n - a)^p/x^2, x)
```

Sympy [F]

$$\int \frac{(-a+bx^n)^p\,(a+bx^n)^p}{x^2}\,dx = \int \frac{(-a+bx^n)^p\,(a+bx^n)^p}{x^2}\,dx$$

```
[In] integrate((-a+b*x**n)**p*(a+b*x**n)**p/x**2,x)
[Out] Integral((-a + b*x**n)**p*(a + b*x**n)**p/x**2, x)
```

Maxima [F]

$$\int \frac{(-a+bx^n)^p\,(a+bx^n)^p}{x^2}\,dx = \int \frac{(bx^n+a)^p(bx^n-a)^p}{x^2}\,dx$$

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p/x^2,x, algorithm="maxima")
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p/x^2, x)
```

Giac [F]

$$\int \frac{(-a+bx^n)^p\,(a+bx^n)^p}{x^2}\,dx = \int \frac{(bx^n+a)^p(bx^n-a)^p}{x^2}\,dx$$

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p/x^2,x, algorithm="giac")
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(-a+bx^n)^p\,(a+bx^n)^p}{x^2}\,dx = \int \frac{(a+b\,x^n)^p\,(b\,x^n-a)^p}{x^2}\,dx$$

```
[In] int(((a + b*x^n)^p*(b*x^n - a)^p)/x^2,x)
[Out] int(((a + b*x^n)^p*(b*x^n - a)^p)/x^2, x)
```

3.1023 $\int \frac{1+x^6}{x(1-x^6)} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \frac{1+x^6}{x(1-x^6)} \, dx = \log(x) - \frac{1}{3}\log\left(1-x^6\right)$$

```
[Out] ln(x)-1/3*ln(-x^6+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {457, 78}

$$\int \frac{1+x^6}{x(1-x^6)} \, dx = \log(x) - \frac{1}{3}\log\left(1-x^6\right)$$

```
[In] Int[(1 + x^6)/(x*(1 - x^6)),x]
```

```
[Out] Log[x] - Log[1 - x^6]/3
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{6}\text{Subst}\left(\int \frac{1+x}{(1-x)x}\,dx, x, x^6\right)\\ &= \frac{1}{6}\text{Subst}\left(\int \left(-\frac{2}{-1+x}+\frac{1}{x}\right)\,dx, x, x^6\right)\\ &= \log(x)-\frac{1}{3}\log\left(1-x^6\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1+x^6}{x\left(1-x^6\right)}\,dx = \log(x)-\frac{1}{3}\log\left(1-x^6\right)$$

```
[In] Integrate[(1 + x^6)/(x*(1 - x^6)),x]
[Out] Log[x] - Log[1 - x^6]/3
```

Maple [A] (verified)

Time = 4.60 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
risch	$\ln\left(x\right)-\frac{\ln\left(x^{6}-1\right)}{3}$	12
meijerg	$-\frac{\ln\left(-x^{6}+1\right)}{3}+\ln\left(x\right)+\frac{i\pi}{6}$	18
default	$\ln\left(x\right)-\frac{\ln\left(-1+x\right)}{3}-\frac{\ln\left(1+x\right)}{3}-\frac{\ln\left(x^{2}+x+1\right)}{3}-\frac{\ln\left(x^{2}-x+1\right)}{3}$	36
norman	$\ln\left(x\right)-\frac{\ln\left(-1+x\right)}{3}-\frac{\ln\left(1+x\right)}{3}-\frac{\ln\left(x^{2}+x+1\right)}{3}-\frac{\ln\left(x^{2}-x+1\right)}{3}$	36
parallelrisch	$\ln\left(x\right)-\frac{\ln\left(-1+x\right)}{3}-\frac{\ln\left(1+x\right)}{3}-\frac{\ln\left(x^{2}+x+1\right)}{3}-\frac{\ln\left(x^{2}-x+1\right)}{3}$	36

```
[In] int((x^6+1)/x/(-x^6+1),x,method=_RETURNVERBOSE)
[Out] ln(x)-1/3*ln(x^6-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{1+x^6}{x(1-x^6)}\,dx = -\frac{1}{3}\log\left(x^6-1\right)+\log\left(x\right)$$

```
[In] integrate((x^6+1)/x/(-x^6+1),x, algorithm="fricas")
[Out] -1/3*log(x^6 - 1) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{1+x^6}{x(1-x^6)}\,dx = \log\left(x\right)-\frac{\log\left(x^6-1\right)}{3}$$

```
[In] integrate((x**6+1)/x/(-x**6+1),x)
[Out] log(x) - log(x**6 - 1)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1+x^6}{x(1-x^6)}\,dx = -\frac{1}{3}\log\left(x^6-1\right)+\frac{1}{6}\log\left(x^6\right)$$

```
[In] integrate((x^6+1)/x/(-x^6+1),x, algorithm="maxima")
[Out] -1/3*log(x^6 - 1) + 1/6*log(x^6)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{1+x^6}{x(1-x^6)}\,dx = \frac{1}{6}\log\left(x^6\right)-\frac{1}{3}\log\left(\left|x^6-1\right|\right)$$

```
[In] integrate((x^6+1)/x/(-x^6+1),x, algorithm="giac")
[Out] 1/6*log(x^6) - 1/3*log(abs(x^6 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{1+x^6}{x\,(1-x^6)}\,dx = \ln(x) - \frac{\ln(x^6-1)}{3}$$

```
[In] int(-(x^6 + 1)/(x*(x^6 - 1)),x)
[Out] log(x) - log(x^6 - 1)/3
```

3.1024 $\int (ex)^m (a+bx^n)^p (a(1+m)+b(1+m+n+np)$

Optimal result

Integrand size = 33, antiderivative size = 22

$$\int (ex)^m (a+bx^n)^p (a(1+m)+b(1+m+n+np)x^n) \, dx = \frac{(ex)^{1+m} (a+bx^n)^{1+p}}{e}$$

```
[Out] (e*x)^(1+m)*(a+b*x^n)^(p+1)/e
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {460}

$$\int (ex)^m (a+bx^n)^p (a(1+m)+b(1+m+n+np)x^n) \, dx = \frac{(ex)^{m+1} (a+bx^n)^{p+1}}{e}$$

```
[In] Int[(e*x)^m*(a + b*x^n)^p*(a*(1 + m) + b*(1 + m + n + n*p)*x^n),x]
```

```
[Out] ((e*x)^(1 + m)*(a + b*x^n)^(1 + p))/e
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{(ex)^{1+m} (a+bx^n)^{1+p}}{e}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 5.00

$$\int (ex)^m (a+bx^n)^p (a(1+m)+b(1+m+n+np)x^n)\,dx$$
$$=x(ex)^m (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \left(a\,\text{Hypergeometric2F1}\left(\frac{1+m}{n},-p,\frac{1+m+n}{n},-\frac{bx^n}{a}\right)\right.$$
$$\left.+\frac{b(1+m+n+np)x^n\,\text{Hypergeometric2F1}\left(\frac{1+m+n}{n},-p,\frac{1+m+2n}{n},-\frac{bx^n}{a}\right)}{1+m+n}\right)$$

```
[In] Integrate[(e*x)^m*(a + b*x^n)^p*(a*(1 + m) + b*(1 + m + n + n*p)*x^n),x]
```

```
[Out] (x*(e*x)^m*(a + b*x^n)^p*(a*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n)/n,
 -((b*x^n)/a)] + (b*(1 + m + n + n*p)*x^n*Hypergeometric2F1[(1 + m + n)/n,
-p, (1 + m + 2*n)/n, -((b*x^n)/a)])/(1 + m + n)))/(1 + (b*x^n)/a)^p
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(22) = 44.

Time = 12.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

method	result	size
parallelrisch	$\frac{x\,x^n (ex)^m (a+b\,x^n)^p b^2+x(ex)^m (a+b\,x^n)^p ab}{b}$	46

```
[In] int((e*x)^m*(a+b*x^n)^p*(a*(1+m)+b*(n*p+m+n+1)*x^n),x,method=_RETURNVERBOSE
)
```

```
[Out] (x*x^n*(e*x)^m*(a+b*x^n)^p*b^2+x*(e*x)^m*(a+b*x^n)^p*a*b)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int (ex)^m (a+bx^n)^p (a(1+m)+b(1+m+n+np)x^n)\,dx$$
$$=\left(bxx^n e^{(m\log(e)+m\log(x))}+axe^{(m\log(e)+m\log(x))}\right)(bx^n+a)^p$$

```
[In] integrate((e*x)^m*(a+b*x^n)^p*(a*(1+m)+b*(n*p+m+n+1)*x^n),x, algorithm="fri
cas")
```

```
[Out] (b*x*x^n*e^(m*log(e) + m*log(x)) + a*x*e^(m*log(e) + m*log(x)))*(b*x^n + a)
^p
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 2.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int (ex)^m (a+bx^n)^p (a(1+m)+b(1+m+n+np)x^n) \, dx = ax(ex)^m (a+bx^n)^p + bxx^n(ex)^m (a+bx^n)^p$$

```
[In] integrate((e*x)**m*(a+b*x**n)**p*(a*(1+m)+b*(n*p+m+n+1)*x**n),x)
```

```
[Out] a*x*(e*x)**m*(a + b*x**n)**p + b*x*x**n*(e*x)**m*(a + b*x**n)**p
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int (ex)^m (a+bx^n)^p (a(1+m)+b(1+m+n+np)x^n) \, dx = \left(ae^m xx^m + be^m xe^{(m\log(x)+n\log(x))}\right)(bx^n+a)^p$$

```
[In] integrate((e*x)^m*(a+b*x^n)^p*(a*(1+m)+b*(n*p+m+n+1)*x^n),x, algorithm="max
ima")
```

```
[Out] (a*e^m*x*x^m + b*e^m*x*e^(m*log(x) + n*log(x)))*(b*x^n + a)^p
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(22) = 44$.

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int (ex)^m (a+bx^n)^p (a(1+m)+b(1+m+n+np)x^n) \, dx = (bx^n+a)^p bxx^n e^{(m\log(e)+m\log(x))} + (bx^n+a)^p axe^{(m\log(e)+m\log(x))}$$

```
[In] integrate((e*x)^m*(a+b*x^n)^p*(a*(1+m)+b*(n*p+m+n+1)*x^n),x, algorithm="gia
c")
```

```
[Out] (b*x^n + a)^p*b*x*x^n*e^(m*log(e) + m*log(x)) + (b*x^n + a)^p*a*x*e^(m*log(
e) + m*log(x))
```

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int (ex)^m (a+bx^n)^p (a(1+m)+b(1+m+n+np)x^n) \, dx$$
$$= \left(a\,x\,{\left(e\,x\right)}^m+b\,x^{n+1}\,{\left(e\,x\right)}^m\right)\,{\left(a+b\,x^n\right)}^p$$

```
[In] int((e*x)^m*(a*(m + 1) + b*x^n*(m + n + n*p + 1))*(a + b*x^n)^p,x)
[Out] (a*x*(e*x)^m + b*x^(n + 1)*(e*x)^m)*(a + b*x^n)^p
```

3.1025 $\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)} dx$

Optimal result

Integrand size = 24, antiderivative size = 114

$$\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)} dx = \frac{b(ex)^{1+m} \text{Hypergeometric2F1}\left(1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{bx^n}{a}\right)}{a(bc-ad)e(1+m)} - \frac{d(ex)^{1+m} \text{Hypergeometric2F1}\left(1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{dx^n}{c}\right)}{c(bc-ad)e(1+m)}$$

```
[Out] b*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a/(-a*d+b*c)/e/(
1+m)-d*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-d*x^n/c)/c/(-a*d+b*c
)/e/(1+m)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {522, 371}

$$\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)} dx = \frac{b(ex)^{m+1} \text{Hypergeometric2F1}\left(1, \frac{m+1}{n}, \frac{m+n+1}{n}, -\frac{bx^n}{a}\right)}{ae(m+1)(bc-ad)} - \frac{d(ex)^{m+1} \text{Hypergeometric2F1}\left(1, \frac{m+1}{n}, \frac{m+n+1}{n}, -\frac{dx^n}{c}\right)}{ce(m+1)(bc-ad)}$$

```
[In] Int[(e*x)^m/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a
)])/(a*(b*c - a*d)*e*(1 + m)) - (d*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 +
m)/n, (1 + m + n)/n, -((d*x^n)/c)])/(c*(b*c - a*d)*e*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 522

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, n, m}
, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\int \frac{(ex)^m}{a+bx^n}\,dx}{bc-ad}-\frac{d\int \frac{(ex)^m}{c+dx^n}\,dx}{bc-ad}\\ &= \frac{b(ex)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)e(1+m)}-\frac{d(ex)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)e(1+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

$$\int \frac{(ex)^m}{(a+bx^n)\,(c+dx^n)}\,dx = \frac{x(ex)^m\left(-bc\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)+ad\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{dx^n}{c}\right)\right)}{ac(-bc+ad)(1+m)}$$

```
[In] Integrate[(e*x)^m/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (x*(e*x)^m*(-(b*c*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/
a)]) + a*d*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)]))/(
a*c*(-(b*c) + a*d)*(1 + m))
```

Maple [F]

$$\int \frac{(ex)^m}{(a+b\,x^n)\,(c+d\,x^n)}dx$$

```
[In] int((e*x)^m/(a+b*x^n)/(c+d*x^n),x)
```

```
[Out] int((e*x)^m/(a+b*x^n)/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{(ex)^m}{(bx^n+a)(dx^n+c)}\,dx$$

```
[In] integrate((e*x)^m/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
[Out] integral((e*x)^m/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

Sympy [F]

$$\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}\,dx$$

```
[In] integrate((e*x)**m/(a+b*x**n)/(c+d*x**n),x)
[Out] Integral((e*x)**m/((a + b*x**n)*(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{(ex)^m}{(bx^n+a)(dx^n+c)}\,dx$$

```
[In] integrate((e*x)^m/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
[Out] integrate((e*x)^m/((b*x^n + a)*(d*x^n + c)), x)
```

Giac [F]

$$\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{(ex)^m}{(bx^n+a)(dx^n+c)}\,dx$$

```
[In] integrate((e*x)^m/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
[Out] integrate((e*x)^m/((b*x^n + a)*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{(e\,x)^m}{(a+b\,x^n)\,(c+d\,x^n)}\,dx$$

```
[In] int((e*x)^m/((a + b*x^n)*(c + d*x^n)),x)
[Out] int((e*x)^m/((a + b*x^n)*(c + d*x^n)), x)
```

3.1026 $\int \frac{x^2}{(a+bx^n)(c+dx^n)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 89

$$\int \frac{x^2}{(a+bx^n)(c+dx^n)}\,dx = \frac{bx^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)}{3a(bc-ad)} - \frac{dx^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{dx^n}{c}\right)}{3c(bc-ad)}$$

```
[Out] 1/3*b*x^3*hypergeom([1, 3/n],[(3+n)/n],-b*x^n/a)/a/(-a*d+b*c)-1/3*d*x^3*hyp
ergeom([1, 3/n],[(3+n)/n],-d*x^n/c)/c/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {522, 371}

$$\int \frac{x^2}{(a+bx^n)(c+dx^n)}\,dx = \frac{bx^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{n+3}{n},-\frac{bx^n}{a}\right)}{3a(bc-ad)} - \frac{dx^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{n+3}{n},-\frac{dx^n}{c}\right)}{3c(bc-ad)}$$

```
[In] Int[x^2/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*x^n)/a)])/(3*a*(b*c - a*d)
) - (d*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((d*x^n)/c)])/(3*c*(b*c -
a*d))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 522

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, n, m}
, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\int \frac{x^2}{a+bx^n}\,dx}{bc-ad} - \frac{d\int \frac{x^2}{c+dx^n}\,dx}{bc-ad}\\ &= \frac{bx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{bx^n}{a}\right)}{3a(bc-ad)} - \frac{dx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{dx^n}{c}\right)}{3c(bc-ad)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(a+bx^n)(c+dx^n)}\,dx = \frac{bcx^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right) - adx^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{dx^n}{c}\right)}{3abc^2-3a^2cd}$$

```
[In] Integrate[x^2/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*c*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*x^n)/a)] - a*d*x^3*Hyper
geometric2F1[1, 3/n, (3 + n)/n, -((d*x^n)/c)])/(3*a*b*c^2 - 3*a^2*c*d)
```

Maple [F]

$$\int \frac{x^2}{(a+b\,x^n)\,(c+d\,x^n)}dx$$

```
[In] int(x^2/(a+b*x^n)/(c+d*x^n),x)
```

```
[Out] int(x^2/(a+b*x^n)/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{x^2}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{x^2}{(bx^n+a)(dx^n+c)}\,dx$$

```
[In] integrate(x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
[Out] integral(x^2/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

Sympy [F]

$$\int \frac{x^2}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{x^2}{(a+bx^n)(c+dx^n)}\,dx$$

```
[In] integrate(x**2/(a+b*x**n)/(c+d*x**n),x)
[Out] Integral(x**2/((a + b*x**n)*(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{x^2}{(bx^n+a)(dx^n+c)}\,dx$$

```
[In] integrate(x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
[Out] integrate(x^2/((b*x^n + a)*(d*x^n + c)), x)
```

Giac [F]

$$\int \frac{x^2}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{x^2}{(bx^n+a)(dx^n+c)}\,dx$$

```
[In] integrate(x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
[Out] integrate(x^2/((b*x^n + a)*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^n)\,(c+dx^n)}\,dx=\int \frac{x^2}{(a+b\,x^n)\,\,(c+d\,x^n)}\,dx$$

```
[In] int(x^2/((a + b*x^n)*(c + d*x^n)),x)
[Out] int(x^2/((a + b*x^n)*(c + d*x^n)), x)
```

3.1027 $\int \frac{x}{(a+bx^n)(c+dx^n)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 89

$$\int \frac{x}{(a+bx^n)(c+dx^n)}\,dx = \frac{bx^2\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)}{2a(bc-ad)} - \frac{dx^2\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{2+n}{n},-\frac{dx^n}{c}\right)}{2c(bc-ad)}$$

```
[Out] 1/2*b*x^2*hypergeom([1, 2/n],[(2+n)/n],-b*x^n/a)/a/(-a*d+b*c)-1/2*d*x^2*hyp
ergeom([1, 2/n],[(2+n)/n],-d*x^n/c)/c/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {522, 371}

$$\int \frac{x}{(a+bx^n)(c+dx^n)}\,dx = \frac{bx^2\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{n+2}{n},-\frac{bx^n}{a}\right)}{2a(bc-ad)} - \frac{dx^2\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{n+2}{n},-\frac{dx^n}{c}\right)}{2c(bc-ad)}$$

```
[In] Int[x/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a*(b*c - a*d)
) - (d*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((d*x^n)/c)])/(2*c*(b*c -
a*d))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 522

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, n, m}
, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\int \frac{x}{a+bx^n}\,dx}{bc-ad} - \frac{d\int \frac{x}{c+dx^n}\,dx}{bc-ad}\\ &= \frac{bx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{bx^n}{a}\right)}{2a(bc-ad)} - \frac{dx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{dx^n}{c}\right)}{2c(bc-ad)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88

$$\int \frac{x}{(a+bx^n)(c+dx^n)}\,dx = \frac{bcx^2\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right) - adx^2\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{2+n}{n},-\frac{dx^n}{c}\right)}{2abc^2-2a^2cd}$$

```
[In] Integrate[x/((a + b*x^n)*(c + d*x^n)),x]
[Out] (b*c*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a)] - a*d*x^2*Hyper
geometric2F1[1, 2/n, (2 + n)/n, -((d*x^n)/c)])/(2*a*b*c^2 - 2*a^2*c*d)
```

Maple [F]

$$\int \frac{x}{(a+b\,x^n)\,(c+d\,x^n)}dx$$

```
[In] int(x/(a+b*x^n)/(c+d*x^n),x)
[Out] int(x/(a+b*x^n)/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{x}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{x}{(bx^n+a)(dx^n+c)}\,dx$$

```
[In] integrate(x/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
[Out] integral(x/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

Sympy [F]

$$\int \frac{x}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{x}{(a+bx^n)(c+dx^n)}\,dx$$

```
[In] integrate(x/(a+b*x**n)/(c+d*x**n),x)
[Out] Integral(x/((a + b*x**n)*(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{x}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{x}{(bx^n+a)(dx^n+c)}\,dx$$

```
[In] integrate(x/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
[Out] integrate(x/((b*x^n + a)*(d*x^n + c)), x)
```

Giac [F]

$$\int \frac{x}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{x}{(bx^n+a)(dx^n+c)}\,dx$$

```
[In] integrate(x/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
[Out] integrate(x/((b*x^n + a)*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^n)(c+dx^n)} \, dx = \int \frac{x}{(a+b\,x^n)\,(c+d\,x^n)} \, dx$$

```
[In] int(x/((a + b*x^n)*(c + d*x^n)),x)
[Out] int(x/((a + b*x^n)*(c + d*x^n)), x)
```

3.1028 $\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx=\frac{bx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{dx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)}$$

```
[Out] b*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/(-a*d+b*c)-d*x*hypergeom([1, 1/n
],[1+1/n],-d*x^n/c)/c/(-a*d+b*c)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {400, 251}

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx=\frac{bx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{dx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)}$$

```
[In] Int[1/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d)
) - (d*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(b*c -
a*d))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\int\frac{1}{a+bx^n}\,dx}{bc-ad}-\frac{d\int\frac{1}{c+dx^n}\,dx}{bc-ad}\\ &= \frac{bx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{dx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$\int\frac{1}{(a+bx^n)(c+dx^n)}\,dx = \frac{x\left(-bc\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)+ad\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)\right)}{ac(-bc+ad)}$$

```
[In] Integrate[1/((a + b*x^n)*(c + d*x^n)),x]
[Out] (x*(-(b*c*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)]) + a*d*Hyp
ergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(a*c*(-(b*c) + a*d))
```

Maple [F]

$$\int\frac{1}{(a+b\,x^n)\,(c+d\,x^n)}dx$$

```
[In] int(1/(a+b*x^n)/(c+d*x^n),x)
[Out] int(1/(a+b*x^n)/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx=\int \frac{1}{(bx^n+a)(dx^n+c)}\,dx$$

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
[Out] integral(1/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx=\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx$$

```
[In] integrate(1/(a+b*x**n)/(c+d*x**n),x)
[Out] Integral(1/((a + b*x**n)*(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx=\int \frac{1}{(bx^n+a)(dx^n+c)}\,dx$$

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx=\int \frac{1}{(bx^n+a)(dx^n+c)}\,dx$$

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{1}{(a+b\,x^n)\,(c+d\,x^n)}\,dx$$

```
[In] int(1/((a + b*x^n)*(c + d*x^n)),x)
[Out] int(1/((a + b*x^n)*(c + d*x^n)), x)
```

3.1029 $\int \frac{1}{x(a+bx^n)(c+dx^n)} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 63

$$\int \frac{1}{x(a+bx^n)(c+dx^n)} \, dx = \frac{\log(x)}{ac} - \frac{b\log(a+bx^n)}{a(bc-ad)n} + \frac{d\log(c+dx^n)}{c(bc-ad)n}$$

```
[Out] ln(x)/a/c-b*ln(a+b*x^n)/a/(-a*d+b*c)/n+d*ln(c+d*x^n)/c/(-a*d+b*c)/n
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 84}

$$\int \frac{1}{x(a+bx^n)(c+dx^n)} \, dx = -\frac{b\log(a+bx^n)}{an(bc-ad)} + \frac{d\log(c+dx^n)}{cn(bc-ad)} + \frac{\log(x)}{ac}$$

```
[In] Int[1/(x*(a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] Log[x]/(a*c) - (b*Log[a + b*x^n])/(a*(b*c - a*d)*n) + (d*Log[c + d*x^n])/(c
*(b*c - a*d)*n)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{acx}+\frac{b^2}{a(-bc+ad)(a+bx)}+\frac{d^2}{c(bc-ad)(c+dx)}\right)dx, x, x^n\right)}{n}\\ &= \frac{\log(x)}{ac}-\frac{b\log(a+bx^n)}{a(bc-ad)n}+\frac{d\log(c+dx^n)}{c(bc-ad)n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{1}{x(a+bx^n)(c+dx^n)}\,dx = \frac{bc\log(x^n)-ad\log(x^n)-bc\log(a+bx^n)+ad\log(c+dx^n)}{abc^2n-a^2cdn}$$

```
[In] Integrate[1/(x*(a + b*x^n)*(c + d*x^n)),x]
[Out] (b*c*Log[x^n] - a*d*Log[x^n] - b*c*Log[a + b*x^n] + a*d*Log[c + d*x^n])/(a*
b*c^2*n - a^2*c*d*n)
```

Maple [A] (verified)

Time = 4.88 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$\frac{\ln(x)adn-\ln(x)bcn+b\ln(a+b\,x^n)c-d\ln(c+d\,x^n)a}{(ad-bc)acn}$	58
derivativedivides	$\frac{\frac{\ln(x^n)}{ac}-\frac{d\ln(c+d\,x^n)}{c(ad-bc)}+\frac{b\ln(a+b\,x^n)}{(ad-bc)a}}{n}$	64
default	$\frac{\frac{\ln(x^n)}{ac}-\frac{d\ln(c+d\,x^n)}{c(ad-bc)}+\frac{b\ln(a+b\,x^n)}{(ad-bc)a}}{n}$	64
norman	$\frac{\ln(x)}{ac}+\frac{b\ln(a+b\,e^{n\ln(x)})}{(ad-bc)an}-\frac{d\ln(c+d\,e^{n\ln(x)})}{cn(ad-bc)}$	68
risch	$-\frac{\ln(x)b}{(ad-bc)a}+\frac{\ln(x)d}{c(ad-bc)}+\frac{b\ln(x^n+\frac{a}{b})}{(ad-bc)an}-\frac{d\ln(x^n+\frac{c}{d})}{cn(ad-bc)}$	94

```
[In] int(1/x/(a+b*x^n)/(c+d*x^n),x,method=_RETURNVERBOSE)
[Out] (ln(x)*a*d*n-ln(x)*b*c*n+b*ln(a+b*x^n)*c-d*ln(c+d*x^n)*a)/(a*d-b*c)/a/c/n
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{1}{x(a+bx^n)(c+dx^n)}\,dx = -\frac{bc\log(bx^n+a)-ad\log(dx^n+c)-(bc-ad)n\log(x)}{(abc^2-a^2cd)n}$$

```
[In] integrate(1/x/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] -(b*c*log(b*x^n + a) - a*d*log(d*x^n + c) - (b*c - a*d)*n*log(x))/((a*b*c^2
 - a^2*c*d)*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(46) = 92.

Time = 1.34 (sec) , antiderivative size = 348, normalized size of antiderivative = 5.52

$$\int \frac{1}{x(a+bx^n)(c+dx^n)}\,dx = \begin{cases} \frac{\frac{\log(x)}{c}-\frac{\log\left(\frac{c}{d}+x^n\right)}{cn}}{a} & \text{for } b=0 \\ \frac{\frac{\log(x)}{a}-\frac{\log\left(\frac{a}{b}+x^n\right)}{an}}{c} & \text{for } d=0 \\ \frac{-\frac{x^{-n}}{cn}+\frac{d\log\left(x^{-n}+\frac{d}{c}\right)}{c^2n}}{b} & \text{for } a=0 \\ \frac{cdn\log(x)}{bc^3n+bc^2dnx^n}-\frac{cd\log\left(\frac{c}{d}+x^n\right)}{bc^3n+bc^2dnx^n}+\frac{cd}{bc^3n+bc^2dnx^n}+\frac{d^2nx^n\log(x)}{bc^3n+bc^2dnx^n}-\frac{d^2x^n\log\left(\frac{c}{d}+x^n\right)}{bc^3n+bc^2dnx^n} & \text{for } a=\frac{bc}{d} \\ \frac{-\frac{x^{-n}}{an}+\frac{b\log\left(x^{-n}+\frac{b}{a}\right)}{a^2n}}{d} & \text{for } c=0 \\ \frac{\log(x)}{(a+b)(c+d)} & \text{for } n=0 \\ \frac{adn\log(x)}{a^2cdn-abc^2n}-\frac{ad\log\left(\frac{c}{d}+x^n\right)}{a^2cdn-abc^2n}-\frac{bcn\log(x)}{a^2cdn-abc^2n}+\frac{bc\log\left(\frac{a}{b}+x^n\right)}{a^2cdn-abc^2n} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/x/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Piecewise(((log(x)/c - log(c/d + x**n)/(c*n))/a, Eq(b, 0)), ((log(x)/a - lo
g(a/b + x**n)/(a*n))/c, Eq(d, 0)), ((-1/(c*n*x**n) + d*log(x**(-n) + d/c)/(
c**2*n))/b, Eq(a, 0)), (c*d*n*log(x)/(b*c**3*n + b*c**2*d*n*x**n) - c*d*log
(c/d + x**n)/(b*c**3*n + b*c**2*d*n*x**n) + c*d/(b*c**3*n + b*c**2*d*n*x**n
) + d**2*n*x**n*log(x)/(b*c**3*n + b*c**2*d*n*x**n) - d**2*x**n*log(c/d + x
**n)/(b*c**3*n + b*c**2*d*n*x**n), Eq(a, b*c/d)), ((-1/(a*n*x**n) + b*log(x
**(-n) + b/a)/(a**2*n))/d, Eq(c, 0)), (log(x)/((a + b)*(c + d)), Eq(n, 0)),
 (a*d*n*log(x)/(a**2*c*d*n - a*b*c**2*n) - a*d*log(c/d + x**n)/(a**2*c*d*n
- a*b*c**2*n) - b*c*n*log(x)/(a**2*c*d*n - a*b*c**2*n) + b*c*log(a/b + x**n
)/(a**2*c*d*n - a*b*c**2*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\left(a+bx^{n}\right)\left(c+dx^{n}\right)}\,dx = -\frac{b\log\left(\frac{bx^{n}+a}{b}\right)}{abcn-a^{2}dn}+\frac{d\log\left(\frac{dx^{n}+c}{d}\right)}{bc^{2}n-acdn}+\frac{\log\left(x\right)}{ac}$$

```
[In] integrate(1/x/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] -b*log((b*x^n + a)/b)/(a*b*c*n - a^2*d*n) + d*log((d*x^n + c)/d)/(b*c^2*n -
 a*c*d*n) + log(x)/(a*c)
```

Giac [F]

$$\int \frac{1}{x\left(a+bx^{n}\right)\left(c+dx^{n}\right)}\,dx = \int \frac{1}{(bx^{n}+a)(dx^{n}+c)x}\,dx$$

```
[In] integrate(1/x/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)*x), x)
```

Mupad [B] (verification not implemented)

Time = 9.98 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.57

$$\int \frac{1}{x\left(a+bx^{n}\right)\left(c+dx^{n}\right)}\,dx = \frac{b\,\ln\left(-\frac{1}{b\,d\,x}-\frac{2\,a\,c\,n+a\,d\,n\,x^{n}+b\,c\,n\,x^{n}}{d\,x\,\left(a^{2}\,d\,n-a\,b\,c\,n\right)}\right)}{a^{2}\,d\,n-a\,b\,c\,n}+\frac{d\,\ln\left(-\frac{1}{b\,d\,x}-\frac{2\,a\,c\,n+a\,d\,n\,x^{n}+b\,c\,n\,x^{n}}{b\,x\,\left(b\,c^{2}\,n-a\,c\,d\,n\right)}\right)}{b\,c^{2}\,n-a\,c\,d\,n}+\frac{\ln\left(x\right)\,\left(n-1\right)}{a\,c\,n}$$

```
[In] int(1/(x*(a + b*x^n)*(c + d*x^n)),x)
```

```
[Out] (b*log(- 1/(b*d*x) - (2*a*c*n + a*d*n*x^n + b*c*n*x^n)/(d*x*(a^2*d*n - a*b*
c*n))))/(a^2*d*n - a*b*c*n) + (d*log(- 1/(b*d*x) - (2*a*c*n + a*d*n*x^n + b
*c*n*x^n)/(b*x*(b*c^2*n - a*c*d*n))))/(b*c^2*n - a*c*d*n) + (log(x)*(n - 1)
)/(a*c*n)
```

3.1030 $\int \frac{1}{x^2(a+bx^n)(c+dx^n)} dx$

Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \frac{1}{x^2 (a+bx^n) (c+dx^n)} dx = -\frac{b \text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)x} + \frac{d \text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)x}$$

```
[Out] -b*hypergeom([1, -1/n],[(-1+n)/n],-b*x^n/a)/a/(-a*d+b*c)/x+d*hypergeom([1,
-1/n],[(-1+n)/n],-d*x^n/c)/c/(-a*d+b*c)/x
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {522, 371}

$$\int \frac{1}{x^2 (a+bx^n) (c+dx^n)} dx = \frac{d \text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{dx^n}{c}\right)}{cx(bc-ad)} - \frac{b \text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{ax(bc-ad)}$$

```
[In] Int[1/(x^2*(a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] -((b*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((b*x^n)/a)])/(a*(b*c - a
*d)*x)) + (d*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((d*x^n)/c)])/(c*
(b*c - a*d)*x)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 522

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, n, m}
, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\int \frac{1}{x^2(a+bx^n)}\,dx}{bc-ad} - \frac{d\int \frac{1}{x^2(c+dx^n)}\,dx}{bc-ad}\\ &= -\frac{b\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)x} + \frac{d\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)x}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2\,(a+bx^n)\,(c+dx^n)}\,dx = \frac{bc\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},\frac{-1+n}{n},-\frac{bx^n}{a}\right) - ad\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},\frac{-1+n}{n},-\frac{dx^n}{c}\right)}{ac(-bc+ad)x}$$

```
[In] Integrate[1/(x^2*(a + b*x^n)*(c + d*x^n)),x]
[Out] (b*c*Hypergeometric2F1[1, -n^(-1), (-1 + n)/n, -((b*x^n)/a)] - a*d*Hypergeo
metric2F1[1, -n^(-1), (-1 + n)/n, -((d*x^n)/c)])/(a*c*(-(b*c) + a*d)*x)
```

Maple [F]

$$\int \frac{1}{x^2\,(a+b\,x^n)\,(c+d\,x^n)}dx$$

```
[In] int(1/x^2/(a+b*x^n)/(c+d*x^n),x)
[Out] int(1/x^2/(a+b*x^n)/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{1}{x^2\,(a+bx^n)\,(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
[Out] integral(1/(b*d*x^2*x^(2*n) + (b*c + a*d)*x^2*x^n + a*c*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2\,(a+bx^n)\,(c+dx^n)}\,dx = \int \frac{1}{x^2\,(a+bx^n)\,(c+dx^n)}\,dx$$

```
[In] integrate(1/x**2/(a+b*x**n)/(c+d*x**n),x)
[Out] Integral(1/(x**2*(a + b*x**n)*(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\,(a+bx^n)\,(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\,(a+bx^n)\,(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+bx^n\right)\left(c+dx^n\right)}\,dx=\int \frac{1}{x^2\,\left(a+b\,x^n\right)\,\left(c+d\,x^n\right)}\,dx$$

```
[In] int(1/(x^2*(a + b*x^n)*(c + d*x^n)),x)
[Out] int(1/(x^2*(a + b*x^n)*(c + d*x^n)), x)
```

3.1031 $\int \frac{1}{x^3(a+bx^n)(c+dx^n)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 95

$$\int \frac{1}{x^3(a+bx^n)(c+dx^n)}\,dx = -\frac{b\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2a(bc-ad)x^2} + \frac{d\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{dx^n}{c}\right)}{2c(bc-ad)x^2}$$

```
[Out] -1/2*b*hypergeom([1, -2/n],[(-2+n)/n],-b*x^n/a)/a/(-a*d+b*c)/x^2+1/2*d*hype
rgeom([1, -2/n],[(-2+n)/n],-d*x^n/c)/c/(-a*d+b*c)/x^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {522, 371}

$$\int \frac{1}{x^3(a+bx^n)(c+dx^n)}\,dx = \frac{d\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{dx^n}{c}\right)}{2cx^2(bc-ad)} - \frac{b\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2ax^2(bc-ad)}$$

```
[In] Int[1/(x^3*(a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] -1/2*(b*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((b*x^n)/a)])/(a*(b*c - a
*d)*x^2) + (d*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((d*x^n)/c)])/(2*c*
(b*c - a*d)*x^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 522

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, n, m}
, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\int\frac{1}{x^3(a+bx^n)}\,dx}{bc-ad}-\frac{d\int\frac{1}{x^3(c+dx^n)}\,dx}{bc-ad}\\ &= -\frac{b\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a(bc-ad)x^2}+\frac{d\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{dx^n}{c}\right)}{2c(bc-ad)x^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.81

$$\int\frac{1}{x^3\left(a+bx^n\right)\left(c+dx^n\right)}\,dx = \frac{bc\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},\frac{-2+n}{n},-\frac{bx^n}{a}\right)-ad\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},\frac{-2+n}{n},-\frac{dx^n}{c}\right)}{2ac(-bc+ad)x^2}$$

```
[In] Integrate[1/(x^3*(a + b*x^n)*(c + d*x^n)),x]
[Out] (b*c*Hypergeometric2F1[1, -2/n, (-2 + n)/n, -((b*x^n)/a)] - a*d*Hypergeomet
ric2F1[1, -2/n, (-2 + n)/n, -((d*x^n)/c)])/(2*a*c*(-(b*c) + a*d)*x^2)
```

Maple [F]

$$\int\frac{1}{x^3\left(a+b\,x^n\right)\left(c+d\,x^n\right)}dx$$

```
[In] int(1/x^3/(a+b*x^n)/(c+d*x^n),x)
[Out] int(1/x^3/(a+b*x^n)/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{1}{x^3 (a + bx^n) (c + dx^n)} dx = \int \frac{1}{(bx^n + a)(dx^n + c)x^3} dx$$

```
[In] integrate(1/x^3/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
[Out] integral(1/(b*d*x^3*x^(2*n) + (b*c + a*d)*x^3*x^n + a*c*x^3), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (a + bx^n) (c + dx^n)} dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(1/x**3/(a+b*x**n)/(c+d*x**n),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{1}{x^3 (a + bx^n) (c + dx^n)} dx = \int \frac{1}{(bx^n + a)(dx^n + c)x^3} dx$$

```
[In] integrate(1/x^3/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a + bx^n) (c + dx^n)} dx = \int \frac{1}{(bx^n + a)(dx^n + c)x^3} dx$$

```
[In] integrate(1/x^3/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a + bx^n) (c + dx^n)} dx = \int \frac{1}{x^3 (a + b x^n) (c + d x^n)} dx$$

```
[In] int(1/(x^3*(a + b*x^n)*(c + d*x^n)),x)
[Out] int(1/(x^3*(a + b*x^n)*(c + d*x^n)), x)
```

3.1032 $\int \frac{(ex)^m}{(a+bx^n)^2(c+dx^n)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 175

$$\int \frac{(ex)^m}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{b(ex)^{1+m}}{a(bc-ad)en(a+bx^n)} + \frac{b(ad(1+m-2n)-bc(1+m-n))(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a^2(bc-ad)^2e(1+m)n} + \frac{d^2(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)^2e(1+m)}$$

```
[Out] b*(e*x)^(1+m)/a/(-a*d+b*c)/e/n/(a+b*x^n)+b*(a*d*(1+m-2*n)-b*c*(1+m-n))*(e*x
)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/e/(1+
m)/n+d^2*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-d*x^n/c)/c/(-a*d+b
*c)^2/e/(1+m)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {518, 611, 371}

$$\int \frac{(ex)^m}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{b(ex)^{m+1}(ad(m-2n+1)-bc(m-n+1))\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{a^2e(m+1)n(bc-ad)^2} + \frac{d^2(ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{dx^n}{c}\right)}{ce(m+1)(bc-ad)^2} + \frac{b(ex)^{m+1}}{aen(bc-ad)(a+bx^n)}$$

```
[In] Int[(e*x)^m/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (b*(e*x)^(1 + m))/(a*(b*c - a*d)*e*n*(a + b*x^n)) + (b*(a*d*(1 + m - 2*n) -
 b*c*(1 + m - n))*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)
/n, -((b*x^n)/a)])/(a^2*(b*c - a*d)^2*e*(1 + m)*n) + (d^2*(e*x)^(1 + m)*Hyp
ergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)])/(c*(b*c - a*d)^2
*e*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 518

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 611

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b(ex)^{1+m}}{a(bc-ad)en\,(a+bx^n)} - \frac{\int \frac{(ex)^m(bc(1+m-n)+adn+bd(1+m-n)x^n)}{(a+bx^n)(c+dx^n)}\,dx}{a(bc-ad)n} \\
&= \frac{b(ex)^{1+m}}{a(bc-ad)en\,(a+bx^n)} - \frac{\int \left(\frac{b(-ad(1+m-2n)+bc(1+m-n))(ex)^m}{(bc-ad)(a+bx^n)} + \frac{ad^2n(ex)^m}{(-bc+ad)(c+dx^n)}\right)dx}{a(bc-ad)n} \\
&= \frac{b(ex)^{1+m}}{a(bc-ad)en\,(a+bx^n)} + \frac{d^2\int \frac{(ex)^m}{c+dx^n}\,dx}{(bc-ad)^2} + \frac{(b(ad(1+m-2n)-bc(1+m-n)))\int \frac{(ex)^m}{a+bx^n}\,dx}{a(bc-ad)^2n}
\end{aligned}
$$

$$
= \frac{b(ex)^{1+m}}{a(bc-ad)en\,(a+bx^n)} + \frac{b(ad(1+m-2n)-bc(1+m-n))(ex)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{a^2(bc-ad)^2e(1+m)n} + \frac{d^2(ex)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)^2e(1+m)}
$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.81

$$
\int \frac{(ex)^m}{(a+bx^n)^2\,(c+dx^n)}\,dx = \frac{x(ex)^m\left(\frac{b^2c-abd}{a^2n+abnx^n} + \frac{b(ad(1+m-2n)-bc(1+m-n))\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a^2(1+m)n} + \frac{d^2\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\ldots\right)}{c+cm}\right.}{(bc-ad)^2}
$$

```
[In] Integrate[(e*x)^m/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (x*(e*x)^m*((b^2*c - a*b*d)/(a^2*n + a*b*n*x^n) + (b*(a*d*(1 + m - 2*n) - b
*c*(1 + m - n))*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)
])/(a^2*(1 + m)*n) + (d^2*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -(
(d*x^n)/c)])/(c + c*m)))/(b*c - a*d)^2
```

Maple [F]

$$
\int \frac{(ex)^m}{(a+b\,x^n)^2\,(c+d\,x^n)}dx
$$

```
[In] int((e*x)^m/(a+b*x^n)^2/(c+d*x^n),x)
```

```
[Out] int((e*x)^m/(a+b*x^n)^2/(c+d*x^n),x)
```

Fricas [F]

$$
\int \frac{(ex)^m}{(a+bx^n)^2\,(c+dx^n)}\,dx = \int \frac{(ex)^m}{(bx^n+a)^2(dx^n+c)}\,dx
$$

```
[In] integrate((e*x)^m/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2*n) + (2*a*
b*c + a^2*d)*x^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(ex)^m}{(a+bx^n)^2 (c+dx^n)} \, dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((e*x)**m/(a+b*x**n)**2/(c+d*x**n),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(ex)^m}{(a+bx^n)^2 (c+dx^n)} \, dx = \int \frac{(ex)^m}{(bx^n+a)^2(dx^n+c)} \, dx$$

```
[In] integrate((e*x)^m/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
[Out] d^2*e^m*integrate(x^m/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a
*b*c*d^2 + a^2*d^3)*x^n), x) + b*e^m*x*x^m/(a^2*b*c*n - a^3*d*n + (a*b^2*c*
n - a^2*b*d*n)*x^n) - (b^2*c*e^m*(m - n + 1) - a*b*d*e^m*(m - 2*n + 1))*int
egrate(x^m/(a^2*b^2*c^2*n - 2*a^3*b*c*d*n + a^4*d^2*n + (a*b^3*c^2*n - 2*a^
2*b^2*c*d*n + a^3*b*d^2*n)*x^n), x)
```

Giac [F]

$$\int \frac{(ex)^m}{(a+bx^n)^2 (c+dx^n)} \, dx = \int \frac{(ex)^m}{(bx^n+a)^2(dx^n+c)} \, dx$$

```
[In] integrate((e*x)^m/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
[Out] integrate((e*x)^m/((b*x^n + a)^2*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^n)^2 (c+dx^n)} \, dx = \int \frac{(e\,x)^m}{(a+b\,x^n)^2\,(c+d\,x^n)} \, dx$$

```
[In] int((e*x)^m/((a + b*x^n)^2*(c + d*x^n)),x)
[Out] int((e*x)^m/((a + b*x^n)^2*(c + d*x^n)), x)
```

3.1033 $\int \frac{x^2}{(a+bx^n)^2(c+dx^n)} dx$

Optimal result

Integrand size = 22, antiderivative size = 142

$$\int \frac{x^2}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{bx^3}{a(bc-ad)n(a+bx^n)} + \frac{b(ad(3-2n)-bc(3-n))x^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)}{3a^2(bc-ad)^2n} + \frac{d^2x^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{dx^n}{c}\right)}{3c(bc-ad)^2}$$

```
[Out] b*x^3/a/(-a*d+b*c)/n/(a+b*x^n)+1/3*b*(a*d*(3-2*n)-b*c*(3-n))*x^3*hypergeom(
[1, 3/n],[(3+n)/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/n+1/3*d^2*x^3*hypergeom([1, 3
/n],[(3+n)/n],-d*x^n/c)/c/(-a*d+b*c)^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {518, 611, 371}

$$\int \frac{x^2}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{bx^3(ad(3-2n)-bc(3-n))\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{n+3}{n},-\frac{bx^n}{a}\right)}{3a^2n(bc-ad)^2} + \frac{d^2x^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{n+3}{n},-\frac{dx^n}{c}\right)}{3c(bc-ad)^2} + \frac{bx^3}{an(bc-ad)(a+bx^n)}$$

```
[In] Int[x^2/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (b*x^3)/(a*(b*c - a*d)*n*(a + b*x^n)) + (b*(a*d*(3 - 2*n) - b*c*(3 - n))*x^
3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*x^n)/a)])/(3*a^2*(b*c - a*d)^2*
n) + (d^2*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((d*x^n)/c)])/(3*c*(b*c
 - a*d)^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 518

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 611

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{bx^3}{a(bc-ad)n\left(a+bx^n\right)} - \frac{\int \frac{x^2(bc(3-n)+adn+bd(3-n)x^n)}{(a+bx^n)(c+dx^n)}\,dx}{a(bc-ad)n} \\
&= \frac{bx^3}{a(bc-ad)n\left(a+bx^n\right)} - \frac{\int \left(\frac{b(-ad(3-2n)+bc(3-n))x^2}{(bc-ad)(a+bx^n)} + \frac{ad^2nx^2}{(-bc+ad)(c+dx^n)}\right)dx}{a(bc-ad)n} \\
&= \frac{bx^3}{a(bc-ad)n\left(a+bx^n\right)} + \frac{d^2\int \frac{x^2}{c+dx^n}\,dx}{(bc-ad)^2} + \frac{(b(ad(3-2n)-bc(3-n)))\int \frac{x^2}{a+bx^n}\,dx}{a(bc-ad)^2n} \\
&= \frac{bx^3}{a(bc-ad)n\left(a+bx^n\right)} + \frac{b(ad(3-2n)-bc(3-n))x^3\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{bx^n}{a}\right)}{3a^2(bc-ad)^2n} \\
&\quad + \frac{d^2x^3\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{dx^n}{c}\right)}{3c(bc-ad)^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{x^3\left(bc(ad(3-2n)+bc(-3+n))(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)+a\left(3bc(bc-ad)+ad^2n\right.\right.}{3a^2c(bc-ad)^2n\,(a+bx^n)}$$

[In] Integrate[x^2/((a + b*x^n)^2*(c + d*x^n)),x]

[Out] (x^3*(b*c*(a*d*(3 - 2*n) + b*c*(-3 + n))*(a + b*x^n)*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*x^n)/a)] + a*(3*b*c*(b*c - a*d) + a*d^2*n*(a + b*x^n)*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((d*x^n)/c)])))/(3*a^2*c*(b*c - a*d)^2*n*(a + b*x^n))

Maple [F]

$$\int \frac{x^2}{(a+b\,x^n)^2\,(c+d\,x^n)}dx$$

[In] int(x^2/(a+b*x^n)^2/(c+d*x^n),x)

[Out] int(x^2/(a+b*x^n)^2/(c+d*x^n),x)

Fricas [F]

$$\int \frac{x^2}{(a+bx^n)^2(c+dx^n)}\,dx = \int \frac{x^2}{(bx^n+a)^2(dx^n+c)}\,dx$$

[In] integrate(x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")

[Out] integral(x^2/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2*n) + (2*a*b*c + a^2*d)*x^n), x)

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx^n)^2(c+dx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

[In] integrate(x**2/(a+b*x**n)**2/(c+d*x**n),x)

[Out] Exception raised: HeuristicGCDFailed >> no luck

Maxima [F]

$$\int \frac{x^2}{(a+bx^n)^2 (c+dx^n)}\,dx = \int \frac{x^2}{(bx^n+a)^2(dx^n+c)}\,dx$$

```
[In] integrate(x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] b*x^3/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*d*n)*x^n) + d^2*integrate(x
^2/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)
*x^n), x) - (a*b*d*(2*n - 3) - b^2*c*(n - 3))*integrate(x^2/(a^2*b^2*c^2*n
- 2*a^3*b*c*d*n + a^4*d^2*n + (a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a^3*b*d^2*n)
*x^n), x)
```

Giac [F]

$$\int \frac{x^2}{(a+bx^n)^2 (c+dx^n)}\,dx = \int \frac{x^2}{(bx^n+a)^2(dx^n+c)}\,dx$$

```
[In] integrate(x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^2/((b*x^n + a)^2*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^n)^2 (c+dx^n)}\,dx = \int \frac{x^2}{(a+b\,x^n)^2\,(c+d\,x^n)}\,dx$$

```
[In] int(x^2/((a + b*x^n)^2*(c + d*x^n)),x)
```

```
[Out] int(x^2/((a + b*x^n)^2*(c + d*x^n)), x)
```

3.1034 $\int \frac{x}{(a+bx^n)^2(c+dx^n)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 143

$$\int \frac{x}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{bx^2}{a(bc-ad)n(a+bx^n)} + \frac{b(2ad(1-n)-bc(2-n))x^2\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)}{2a^2(bc-ad)^2n} + \frac{d^2x^2\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{2+n}{n},-\frac{dx^n}{c}\right)}{2c(bc-ad)^2}$$

```
[Out] b*x^2/a/(-a*d+b*c)/n/(a+b*x^n)+1/2*b*(2*a*d*(1-n)-b*c*(2-n))*x^2*hypergeom(
[1, 2/n],[(2+n)/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/n+1/2*d^2*x^2*hypergeom([1, 2
/n],[(2+n)/n],-d*x^n/c)/c/(-a*d+b*c)^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {518, 611, 371}

$$\int \frac{x}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{bx^2(2ad(1-n)-bc(2-n))\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{n+2}{n},-\frac{bx^n}{a}\right)}{2a^2n(bc-ad)^2} + \frac{d^2x^2\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{n+2}{n},-\frac{dx^n}{c}\right)}{2c(bc-ad)^2} + \frac{bx^2}{an(bc-ad)(a+bx^n)}$$

```
[In] Int[x/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (b*x^2)/(a*(b*c - a*d)*n*(a + b*x^n)) + (b*(2*a*d*(1 - n) - b*c*(2 - n))*x^
2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a^2*(b*c - a*d)^2*
n) + (d^2*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((d*x^n)/c)])/(2*c*(b*c
 - a*d)^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 518

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 611

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{bx^2}{a(bc-ad)n\,(a+bx^n)} - \frac{\int \frac{x(bc(2-n)+adn+bd(2-n)x^n)}{(a+bx^n)(c+dx^n)}\,dx}{a(bc-ad)n} \\
&= \frac{bx^2}{a(bc-ad)n\,(a+bx^n)} - \frac{\int \left(\frac{b(-2ad(1-n)+bc(2-n))x}{(bc-ad)(a+bx^n)} + \frac{ad^2nx}{(-bc+ad)(c+dx^n)}\right)dx}{a(bc-ad)n} \\
&= \frac{bx^2}{a(bc-ad)n\,(a+bx^n)} + \frac{d^2\int \frac{x}{c+dx^n}\,dx}{(bc-ad)^2} + \frac{(b(2ad(1-n)-bc(2-n)))\int \frac{x}{a+bx^n}\,dx}{a(bc-ad)^2n} \\
&= \frac{bx^2}{a(bc-ad)n\,(a+bx^n)} + \frac{b(2ad(1-n)-bc(2-n))x^2\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{bx^n}{a}\right)}{2a^2(bc-ad)^2n} \\
&\quad + \frac{d^2x^2\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{dx^n}{c}\right)}{2c(bc-ad)^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.94

$$\int \frac{x}{(a+bx^n)^2 (c+dx^n)} \, dx$$
$$= \frac{x^2\left(bc(bc(-2+n)-2ad(-1+n))\,(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)+a\left(2bc(bc-ad)+ad\right.}{2a^2c(bc-ad)^2n\,(a+bx^n)}$$

```
[In] Integrate[x/((a + b*x^n)^2*(c + d*x^n)),x]
[Out] (x^2*(b*c*(b*c*(-2 + n) - 2*a*d*(-1 + n))*(a + b*x^n)*Hypergeometric2F1[1,
2/n, (2 + n)/n, -((b*x^n)/a)] + a*(2*b*c*(b*c - a*d) + a*d^2*n*(a + b*x^n)*
Hypergeometric2F1[1, 2/n, (2 + n)/n, -((d*x^n)/c)])))/(2*a^2*c*(b*c - a*d)^
2*n*(a + b*x^n))
```

Maple [F]

$$\int \frac{x}{(a+b\,x^n)^2 (c+d\,x^n)} dx$$

```
[In] int(x/(a+b*x^n)^2/(c+d*x^n),x)
[Out] int(x/(a+b*x^n)^2/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{x}{(a+bx^n)^2 (c+dx^n)} \, dx = \int \frac{x}{(bx^n+a)^2 (dx^n+c)} \, dx$$

```
[In] integrate(x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
[Out] integral(x/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2*n) + (2*a*b*c +
a^2*d)*x^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x}{(a+bx^n)^2 (c+dx^n)} \, dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(x/(a+b*x**n)**2/(c+d*x**n),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{x}{(a+bx^n)^2 (c+dx^n)}\,dx = \int \frac{x}{(bx^n+a)^2(dx^n+c)}\,dx$$

[In] integrate(x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")

[Out] d^2*integrate(x/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^n), x) + b*x^2/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*d*n)*x^n) - (2*a*b*d*(n - 1) - b^2*c*(n - 2))*integrate(x/(a^2*b^2*c^2*n - 2*a^3*b*c*d*n + a^4*d^2*n + (a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a^3*b*d^2*n)*x^n), x)

Giac [F]

$$\int \frac{x}{(a+bx^n)^2 (c+dx^n)}\,dx = \int \frac{x}{(bx^n+a)^2(dx^n+c)}\,dx$$

[In] integrate(x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")

[Out] integrate(x/((b*x^n + a)^2*(d*x^n + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^n)^2 (c+dx^n)}\,dx = \int \frac{x}{(a+b\,x^n)^2\,(c+d\,x^n)}\,dx$$

[In] int(x/((a + b*x^n)^2*(c + d*x^n)),x)

[Out] int(x/((a + b*x^n)^2*(c + d*x^n)), x)

3.1035 $\int \frac{1}{(a+bx^n)^2(c+dx^n)} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 122

$$\int \frac{1}{(a+bx^n)^2 (c+dx^n)} \, dx = \frac{bx}{a(bc-ad)n(a+bx^n)} + \frac{b(ad(1-2n)-bc(1-n))x \text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2(bc-ad)^2n} + \frac{d^2x \text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)^2}$$

```
[Out] b*x/a/(-a*d+b*c)/n/(a+b*x^n)+b*(a*d*(1-2*n)-b*c*(1-n))*x*hypergeom([1, 1/n]
,[1+1/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/n+d^2*x*hypergeom([1, 1/n],[1+1/n],-d*x
^n/c)/c/(-a*d+b*c)^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {425, 536, 251}

$$\int \frac{1}{(a+bx^n)^2 (c+dx^n)} \, dx = \frac{bx(ad(1-2n)-bc(1-n)) \text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2n(bc-ad)^2} + \frac{d^2x \text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)^2} + \frac{bx}{an(bc-ad)(a+bx^n)}$$

```
[In] Int[1/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*n*(a + b*x^n)) + (b*(a*d*(1 - 2*n) - b*c*(1 - n))*x*Hy
pergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*(b*c - a*d)^2*n)
 + (d^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(b*c -
 a*d)^2)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx}{a(bc-ad)n\left(a+bx^n\right)} - \frac{\int \frac{adn+b(c-cn)+bd(1-n)x^n}{(a+bx^n)(c+dx^n)}\,dx}{a(bc-ad)n} \\
&= \frac{bx}{a(bc-ad)n\left(a+bx^n\right)} + \frac{d^2\int \frac{1}{c+dx^n}\,dx}{(bc-ad)^2} + \frac{(b(ad(1-2n)-bc(1-n)))\int \frac{1}{a+bx^n}\,dx}{a(bc-ad)^2n} \\
&= \frac{bx}{a(bc-ad)n\left(a+bx^n\right)} + \frac{b(ad(1-2n)-bc(1-n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2(bc-ad)^2n} \\
&\quad + \frac{d^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{x\left(\frac{b^2c-abd}{a^2n+abnx^n}+\frac{b(ad(1-2n)+bc(-1+n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2n}+\frac{d^2\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c}\right)}{(bc-ad)^2}$$

```
[In] Integrate[1/((a + b*x^n)^2*(c + d*x^n)),x]
[Out] (x*((b^2*c - a*b*d)/(a^2*n + a*b*n*x^n) + (b*(a*d*(1 - 2*n) + b*c*(-1 + n))
*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*n) + (d^2*Hyp
ergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/c))/(b*c - a*d)^2
```

Maple [F]

$$\int \frac{1}{(a+b\,x^n)^2(c+d\,x^n)}dx$$

```
[In] int(1/(a+b*x^n)^2/(c+d*x^n),x)
[Out] int(1/(a+b*x^n)^2/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)}\,dx$$

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
[Out] integral(1/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2*n) + (2*a*b*c +
a^2*d)*x^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(1/(a+b*x**n)**2/(c+d*x**n),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{1}{(a+bx^n)^2 (c+dx^n)} dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)} dx$$

[In] integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")

[Out] d^2*integrate(1/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^n), x) - (a*b*d*(2*n - 1) - b^2*c*(n - 1))*integrate(1/(a^2*b^2*c^2*n - 2*a^3*b*c*d*n + a^4*d^2*n + (a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a^3*b*d^2*n)*x^n), x) + b*x/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*d*n)*x^n)

Giac [F]

$$\int \frac{1}{(a+bx^n)^2 (c+dx^n)} dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)} dx$$

[In] integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")

[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n)^2 (c+dx^n)} dx = \int \frac{1}{(a+b\,x^n)^2\,(c+d\,x^n)} dx$$

[In] int(1/((a + b*x^n)^2*(c + d*x^n)),x)

[Out] int(1/((a + b*x^n)^2*(c + d*x^n)), x)

3.1036 $\int \frac{1}{x(a+bx^n)^2(c+dx^n)} dx$

Optimal result

Integrand size = 22, antiderivative size = 101

$$\int \frac{1}{x(a+bx^n)^2(c+dx^n)} dx = \frac{b}{a(bc-ad)n(a+bx^n)} + \frac{\log(x)}{a^2c} - \frac{b(bc-2ad)\log(a+bx^n)}{a^2(bc-ad)^2n} - \frac{d^2\log(c+dx^n)}{c(bc-ad)^2n}$$

```
[Out] b/a/(-a*d+b*c)/n/(a+b*x^n)+ln(x)/a^2/c-b*(-2*a*d+b*c)*ln(a+b*x^n)/a^2/(-a*d
+b*c)^2/n-d^2*ln(c+d*x^n)/c/(-a*d+b*c)^2/n
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {457, 84}

$$\int \frac{1}{x(a+bx^n)^2(c+dx^n)} dx = -\frac{b(bc-2ad)\log(a+bx^n)}{a^2n(bc-ad)^2} + \frac{\log(x)}{a^2c} - \frac{d^2\log(c+dx^n)}{cn(bc-ad)^2} + \frac{b}{an(bc-ad)(a+bx^n)}$$

```
[In] Int[1/(x*(a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] b/(a*(b*c - a*d)*n*(a + b*x^n)) + Log[x]/(a^2*c) - (b*(b*c - 2*a*d)*Log[a +
 b*x^n])/(a^2*(b*c - a*d)^2*n) - (d^2*Log[c + d*x^n])/(c*(b*c - a*d)^2*n)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
```

```
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^2(c+dx)}\,dx, x, x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2cx} + \frac{b^2}{a(-bc+ad)(a+bx)^2} + \frac{b^2(-bc+2ad)}{a^2(-bc+ad)^2(a+bx)} - \frac{d^3}{c(bc-ad)^2(c+dx)}\right)dx, x, x^n\right)}{n}\\
&= \frac{b}{a(bc-ad)n\,(a+bx^n)} + \frac{\log(x)}{a^2c} - \frac{b(bc-2ad)\log\left(a+bx^n\right)}{a^2(bc-ad)^2n} - \frac{d^2\log\left(c+dx^n\right)}{c(bc-ad)^2n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06

$$
\begin{aligned}
\int \frac{1}{x\left(a+bx^n\right)^2\left(c+dx^n\right)}\,dx = &-\frac{b}{a(-bc+ad)n\,(a+bx^n)} + \frac{\log\left(x^n\right)}{a^2cn}\\
&+ \frac{b(-bc+2ad)\log\left(a+bx^n\right)}{a^2(-bc+ad)^2n} - \frac{d^2\log\left(c+dx^n\right)}{c(bc-ad)^2n}
\end{aligned}
$$

```
[In] Integrate[1/(x*(a + b*x^n)^2*(c + d*x^n)),x]
[Out] -(b/(a*(-(b*c) + a*d)*n*(a + b*x^n))) + Log[x^n]/(a^2*c*n) + (b*(-(b*c) + 2
*a*d)*Log[a + b*x^n])/(a^2*(-(b*c) + a*d)^2*n) - (d^2*Log[c + d*x^n])/(c*(b
*c - a*d)^2*n)
```

Maple [A] (verified)

Time = 5.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{\frac{\ln(x^n)}{a^2c}-\frac{d^2\ln(c+d\,x^n)}{(ad-bc)^2c}-\frac{b}{(ad-bc)a(a+b\,x^n)}+\frac{b(2ad-bc)\ln(a+b\,x^n)}{a^2(ad-bc)^2}}{n}$
default	$\frac{\frac{\ln(x^n)}{a^2c}-\frac{d^2\ln(c+d\,x^n)}{(ad-bc)^2c}-\frac{b}{(ad-bc)a(a+b\,x^n)}+\frac{b(2ad-bc)\ln(a+b\,x^n)}{a^2(ad-bc)^2}}{n}$
norman	$\frac{\frac{b^2e^{n\ln(x)}}{n\,a^2(ad-bc)}+\frac{\ln(x)}{ac}+\frac{b\ln(x)e^{n\ln(x)}}{a^2c}}{a+b\,e^{n\ln(x)}}+\frac{b(2ad-bc)\ln\left(a+b\,e^{n\ln(x)}\right)}{(a^2d^2-2abcd+b^2c^2)a^2n}-\frac{d^2\ln\left(c+d\,e^{n\ln(x)}\right)}{cn(a^2d^2-2abcd+b^2c^2)}$
parallelrisch	$\frac{\ln(x)x^na^2b\,d^2n-2\ln(x)x^na\,b^2cdn+\ln(x)x^nb^3c^2n+\ln(x)a^3d^2n-2\ln(x)a^2bcdn+\ln(x)a\,b^2c^2n+2\ln(a+b\,x^n)x^na\,b^2cd-\mathrm{l}}{(a^2d^2-2abcd+b^2c^2}$
risch	$-\frac{2\ln(x)bd}{(a^2d^2-2abcd+b^2c^2)a}+\frac{\ln(x)b^2c}{(a^2d^2-2abcd+b^2c^2)a^2}+\frac{\ln(x)d^2}{c(a^2d^2-2abcd+b^2c^2)}-\frac{b}{(ad-bc)an(a+b\,x^n)}+\frac{2b\ln(x^n}{(a^2d^2-2abc}$

```
[In] int(1/x/(a+b*x^n)^2/(c+d*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(1/a^2/c*ln(x^n)-d^2/(a*d-b*c)^2/c*ln(c+d*x^n)-b/(a*d-b*c)/a/(a+b*x^n)+
b*(2*a*d-b*c)/a^2/(a*d-b*c)^2*ln(a+b*x^n))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(101) = 202$.

Time = 0.30 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.22

$$\int \frac{1}{x\left(a+bx^n\right)^2\left(c+dx^n\right)}\,dx$$
$$= \frac{ab^2c^2-a^2bcd+(b^3c^2-2\,ab^2cd+a^2bd^2)nx^n\log(x)+(ab^2c^2-2\,a^2bcd+a^3d^2)n\log(x)-(ab^2c^2-2\,a^2bc}{(a^2b^3c^3-2\,a^3b^2c^2d+a^4bcd^2)nx^n+(a^3b^2c^3-2\,a^4}$$

```
[In] integrate(1/x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] (a*b^2*c^2 - a^2*b*c*d + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*n*x^n*log(x) +
 (a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*n*log(x) - (a*b^2*c^2 - 2*a^2*b*c*d +
(b^3*c^2 - 2*a*b^2*c*d)*x^n)*log(b*x^n + a) - (a^2*b*d^2*x^n + a^3*d^2)*log
(d*x^n + c))/((a^2*b^3*c^3 - 2*a^3*b^2*c^2*d + a^4*b*c*d^2)*n*x^n + (a^3*b^
2*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2)*n)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(a+bx^n\right)^2\left(c+dx^n\right)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(1/x/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.50

$$\int \frac{1}{x\left(a+bx^n\right)^2\left(c+dx^n\right)}\,dx = -\frac{d^2\log\left(\frac{dx^n+c}{d}\right)}{b^2c^3n-2\,abc^2dn+a^2cd^2n}-\frac{\left(b^2c-2\,abd\right)\log\left(\frac{bx^n+a}{b}\right)}{a^2b^2c^2n-2\,a^3bcdn+a^4d^2n}+\frac{b}{a^2bcn-a^3dn+\left(ab^2cn-a^2bdn\right)x^n}+\frac{\log\left(x\right)}{a^2c}$$

```
[In] integrate(1/x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
[Out] -d^2*log((d*x^n + c)/d)/(b^2*c^3*n - 2*a*b*c^2*d*n + a^2*c*d^2*n) - (b^2*c
 - 2*a*b*d)*log((b*x^n + a)/b)/(a^2*b^2*c^2*n - 2*a^3*b*c*d*n + a^4*d^2*n) +
 b/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*d*n)*x^n) + log(x)/(a^2*c)
```

Giac [F]

$$\int \frac{1}{x\left(a+bx^n\right)^2\left(c+dx^n\right)}\,dx = \int \frac{1}{\left(bx^n+a\right)^2\left(dx^n+c\right)x}\,dx$$

```
[In] integrate(1/x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+bx^n\right)^2\left(c+dx^n\right)}\,dx = \int \frac{1}{x\,\left(a+b\,x^n\right)^2\,\left(c+d\,x^n\right)}\,dx$$

```
[In] int(1/(x*(a + b*x^n)^2*(c + d*x^n)),x)
[Out] int(1/(x*(a + b*x^n)^2*(c + d*x^n)), x)
```

3.1037 $\int \frac{1}{x^2(a+bx^n)^2(c+dx^n)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 142

$$\int \frac{1}{x^2(a+bx^n)^2(c+dx^n)}\,dx = \frac{b}{a(bc-ad)nx(a+bx^n)} - \frac{b(bc(1+n)-ad(1+2n))\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{a^2(bc-ad)^2nx} - \frac{d^2\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)^2x}$$

```
[Out] b/a/(-a*d+b*c)/n/x/(a+b*x^n)-b*(b*c*(1+n)-a*d*(1+2*n))*hypergeom([1, -1/n],
[(-1+n)/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/n/x-d^2*hypergeom([1, -1/n],[(-1+n)/n
],-d*x^n/c)/c/(-a*d+b*c)^2/x
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {518, 611, 371}

$$\int \frac{1}{x^2(a+bx^n)^2(c+dx^n)}\,dx = -\frac{b(bc(n+1)-ad(2n+1))\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{a^2nx(bc-ad)^2} - \frac{d^2\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{dx^n}{c}\right)}{cx(bc-ad)^2} + \frac{b}{anx(bc-ad)(a+bx^n)}$$

```
[In] Int[1/(x^2*(a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] b/(a*(b*c - a*d)*n*x*(a + b*x^n)) - (b*(b*c*(1 + n) - a*d*(1 + 2*n))*Hyperg
eometric2F1[1, -n^(-1), -((1 - n)/n), -((b*x^n)/a)])/(a^2*(b*c - a*d)^2*n*x
) - (d^2*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((d*x^n)/c)])/(c*(b*c
 - a*d)^2*x)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 518

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 611

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b}{a(bc-ad)nx\left(a+bx^n\right)}-\frac{\int \frac{adn-bc(1+n)-bd(1+n)x^n}{x^2(a+bx^n)(c+dx^n)}\,dx}{a(bc-ad)n}\\
&= \frac{b}{a(bc-ad)nx\left(a+bx^n\right)}-\frac{\int \left(\frac{b(-bc(1+n)+ad(1+2n))}{(bc-ad)x^2(a+bx^n)}+\frac{ad^2n}{(-bc+ad)x^2(c+dx^n)}\right)dx}{a(bc-ad)n}\\
&= \frac{b}{a(bc-ad)nx\left(a+bx^n\right)}+\frac{d^2\int \frac{1}{x^2(c+dx^n)}\,dx}{(bc-ad)^2}+\frac{(b(bc(1+n)-ad(1+2n)))\int \frac{1}{x^2(a+bx^n)}\,dx}{a(bc-ad)^2n}\\
&= \frac{b}{a(bc-ad)nx\left(a+bx^n\right)}-\frac{b(bc(1+n)-ad(1+2n))\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a^2(bc-ad)^2nx}\\
&\quad -\frac{d^2\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)^2x}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2 (a+bx^n)^2 (c+dx^n)}\,dx = \frac{bc(-bc(1+n)+ad(1+2n))(a+bx^n)\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},\frac{-1+n}{n},-\frac{bx^n}{a}\right)-a\big(bc(-bc+ad)+ad^2}{a^2c(bc-ad)^2nx(a+bx^n)}$$

```
[In] Integrate[1/(x^2*(a + b*x^n)^2*(c + d*x^n)),x]
[Out] (b*c*(-(b*c*(1 + n)) + a*d*(1 + 2*n))*(a + b*x^n)*Hypergeometric2F1[1, -n^(
-1), (-1 + n)/n, -((b*x^n)/a)] - a*(b*c*(-(b*c) + a*d) + a*d^2*n*(a + b*x^n
)*Hypergeometric2F1[1, -n^(-1), (-1 + n)/n, -((d*x^n)/c)]))/(a^2*c*(b*c - a
*d)^2*n*x*(a + b*x^n))
```

Maple [F]

$$\int \frac{1}{x^2 (a+b\,x^n)^2 (c+d\,x^n)}dx$$

```
[In] int(1/x^2/(a+b*x^n)^2/(c+d*x^n),x)
[Out] int(1/x^2/(a+b*x^n)^2/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{1}{x^2 (a+bx^n)^2 (c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
[Out] integral(1/(b^2*d*x^2*x^(3*n) + a^2*c*x^2 + (b^2*c + 2*a*b*d)*x^2*x^(2*n) +
 (2*a*b*c + a^2*d)*x^2*x^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (a+bx^n)^2 (c+dx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(1/x**2/(a+b*x**n)**2/(c+d*x**n),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{1}{x^2 (a+bx^n)^2 (c+dx^n)} dx = \int \frac{1}{(bx^n+a)^2 (dx^n+c)x^2} dx$$

```
[In] integrate(1/x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] d^2*integrate(1/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^2*x^n + (b^2*c^3 - 2
*a*b*c^2*d + a^2*c*d^2)*x^2), x) - (a*b*d*(2*n + 1) - b^2*c*(n + 1))*integr
ate(1/((a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a^3*b*d^2*n)*x^2*x^n + (a^2*b^2*c^2
*n - 2*a^3*b*c*d*n + a^4*d^2*n)*x^2), x) + b/((a*b^2*c*n - a^2*b*d*n)*x*x^n
 + (a^2*b*c*n - a^3*d*n)*x)
```

Giac [F]

$$\int \frac{1}{x^2 (a+bx^n)^2 (c+dx^n)} dx = \int \frac{1}{(bx^n+a)^2 (dx^n+c)x^2} dx$$

```
[In] integrate(1/x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^n)^2 (c+dx^n)} dx = \int \frac{1}{x^2 (a+b\,x^n)^2 \,(c+d\,x^n)} dx$$

```
[In] int(1/(x^2*(a + b*x^n)^2*(c + d*x^n)),x)
```

```
[Out] int(1/(x^2*(a + b*x^n)^2*(c + d*x^n)), x)
```

3.1038 $\int \frac{1}{x^3(a+bx^n)^2(c+dx^n)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 145

$$\int \frac{1}{x^3 (a+bx^n)^2 (c+dx^n)}\,dx = \frac{b}{a(bc-ad)nx^2(a+bx^n)} + \frac{b(2ad(1+n)-bc(2+n))\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2a^2(bc-ad)^2nx^2} - \frac{d^2\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{dx^n}{c}\right)}{2c(bc-ad)^2x^2}$$

```
[Out] b/a/(-a*d+b*c)/n/x^2/(a+b*x^n)+1/2*b*(2*a*d*(1+n)-b*c*(2+n))*hypergeom([1,
-2/n],[(-2+n)/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/n/x^2-1/2*d^2*hypergeom([1, -2/
n],[(-2+n)/n],-d*x^n/c)/c/(-a*d+b*c)^2/x^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {518, 611, 371}

$$\int \frac{1}{x^3 (a+bx^n)^2 (c+dx^n)}\,dx = \frac{b(2ad(n+1)-bc(n+2))\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2a^2nx^2(bc-ad)^2} - \frac{d^2\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{dx^n}{c}\right)}{2cx^2(bc-ad)^2} + \frac{b}{anx^2(bc-ad)(a+bx^n)}$$

```
[In] Int[1/(x^3*(a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] b/(a*(b*c - a*d)*n*x^2*(a + b*x^n)) + (b*(2*a*d*(1 + n) - b*c*(2 + n))*Hype
rgeometric2F1[1, -2/n, -((2 - n)/n), -((b*x^n)/a)])/(2*a^2*(b*c - a*d)^2*n*
x^2) - (d^2*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((d*x^n)/c)])/(2*c*(b
*c - a*d)^2*x^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 518

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 611

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b}{a(bc-ad)nx^2\left(a+bx^n\right)}-\frac{\int \frac{adn-bc(2+n)-bd(2+n)x^n}{x^3(a+bx^n)(c+dx^n)}\,dx}{a(bc-ad)n}\\
&= \frac{b}{a(bc-ad)nx^2\left(a+bx^n\right)}-\frac{\int \left(\frac{b(2ad(1+n)-bc(2+n))}{(bc-ad)x^3(a+bx^n)}+\frac{ad^2n}{(-bc+ad)x^3(c+dx^n)}\right)dx}{a(bc-ad)n}\\
&= \frac{b}{a(bc-ad)nx^2\left(a+bx^n\right)}+\frac{d^2\int \frac{1}{x^3(c+dx^n)}\,dx}{(bc-ad)^2}-\frac{(b(2ad(1+n)-bc(2+n)))\int \frac{1}{x^3(a+bx^n)}\,dx}{a(bc-ad)^2n}\\
&= \frac{b}{a(bc-ad)nx^2\left(a+bx^n\right)}+\frac{b(2ad(1+n)-bc(2+n))\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a^2(bc-ad)^2nx^2}\\
&\quad -\frac{d^2\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{dx^n}{c}\right)}{2c(bc-ad)^2x^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3 (a+bx^n)^2 (c+dx^n)}\,dx = \frac{bc(2ad(1+n)-bc(2+n))(a+bx^n)\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},\frac{-2+n}{n},-\frac{bx^n}{a}\right)-a\left(2bc(-bc+ad)+ad^2n\right.}{2a^2c(bc-ad)^2nx^2(a+bx^n)}$$

```
[In] Integrate[1/(x^3*(a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (b*c*(2*a*d*(1 + n) - b*c*(2 + n))*(a + b*x^n)*Hypergeometric2F1[1, -2/n, (
-2 + n)/n, -((b*x^n)/a)] - a*(2*b*c*(-(b*c) + a*d) + a*d^2*n*(a + b*x^n)*Hy
pergeometric2F1[1, -2/n, (-2 + n)/n, -((d*x^n)/c)]))/(2*a^2*c*(b*c - a*d)^2
*n*x^2*(a + b*x^n))
```

Maple [F]

$$\int \frac{1}{x^3 (a+b\,x^n)^2 (c+d\,x^n)}dx$$

```
[In] int(1/x^3/(a+b*x^n)^2/(c+d*x^n),x)
```

```
[Out] int(1/x^3/(a+b*x^n)^2/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{1}{x^3 (a+bx^n)^2 (c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)x^3}\,dx$$

```
[In] integrate(1/x^3/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*d*x^3*x^(3*n) + a^2*c*x^3 + (b^2*c + 2*a*b*d)*x^3*x^(2*n) +
 (2*a*b*c + a^2*d)*x^3*x^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (a+bx^n)^2 (c+dx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(1/x**3/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{1}{x^3 (a+bx^n)^2 (c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)x^3}\,dx$$

[In] integrate(1/x^3/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")

[Out] d^2*integrate(1/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^3*x^n + (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*x^3), x) + (b^2*c*(n + 2) - 2*a*b*d*(n + 1))*integrate(1/((a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a^3*b*d^2*n)*x^3*x^n + (a^2*b^2*c^2*n - 2*a^3*b*c*d*n + a^4*d^2*n)*x^3), x) + b/((a*b^2*c*n - a^2*b*d*n)*x^2*x^n + (a^2*b*c*n - a^3*d*n)*x^2)

Giac [F]

$$\int \frac{1}{x^3 (a+bx^n)^2 (c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)x^3}\,dx$$

[In] integrate(1/x^3/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")

[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)*x^3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^n)^2 (c+dx^n)}\,dx = \int \frac{1}{x^3\,(a+b\,x^n)^2\,(c+d\,x^n)}\,dx$$

[In] int(1/(x^3*(a + b*x^n)^2*(c + d*x^n)),x)

[Out] int(1/(x^3*(a + b*x^n)^2*(c + d*x^n)), x)

3.1039 $\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 130

$$\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n}\,dx = -\frac{(bc-ad)^3x^n}{d^4n}+\frac{b(b^2c^2-3abcd+3a^2d^2)\,x^{2n}}{2d^3n} - \frac{b^2(bc-3ad)x^{3n}}{3d^2n}+\frac{b^3x^{4n}}{4dn}+\frac{c(bc-ad)^3\log\left(c+dx^n\right)}{d^5n}$$

```
[Out] -(-a*d+b*c)^3*x^n/d^4/n+1/2*b*(3*a^2*d^2-3*a*b*c*d+b^2*c^2)*x^(2*n)/d^3/n-1
/3*b^2*(-3*a*d+b*c)*x^(3*n)/d^2/n+1/4*b^3*x^(4*n)/d/n+c*(-a*d+b*c)^3*ln(c+d
*x^n)/d^5/n
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {457, 78}

$$\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n}\,dx = \frac{bx^{2n}(3a^2d^2-3abcd+b^2c^2)}{2d^3n}-\frac{b^2x^{3n}(bc-3ad)}{3d^2n}+\frac{c(bc-ad)^3\log\left(c+dx^n\right)}{d^5n}-\frac{x^n(bc-ad)^3}{d^4n}+\frac{b^3x^{4n}}{4dn}$$

```
[In] Int[(x^(-1 + 2*n)*(a + b*x^n)^3)/(c + d*x^n),x]
```

```
[Out] -(((b*c - a*d)^3*x^n)/(d^4*n)) + (b*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x^(2*
n))/(2*d^3*n) - (b^2*(b*c - 3*a*d)*x^(3*n))/(3*d^2*n) + (b^3*x^(4*n))/(4*d*
n) + (c*(b*c - a*d)^3*Log[c + d*x^n])/(d^5*n)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x(a+bx)^3}{c+dx}\,dx, x, x^n\right)}{n} \\
&= \frac{\text{Subst}\left(\int \left(\frac{(-bc+ad)^3}{d^4} + \frac{b(b^2c^2-3abcd+3a^2d^2)x}{d^3} - \frac{b^2(bc-3ad)x^2}{d^2} + \frac{b^3x^3}{d} + \frac{c(bc-ad)^3}{d^4(c+dx)}\right) dx, x, x^n\right)}{n} \\
&= -\frac{(bc-ad)^3x^n}{d^4n} + \frac{b(b^2c^2-3abcd+3a^2d^2)\,x^{2n}}{2d^3n} \\
&\quad - \frac{b^2(bc-3ad)x^{3n}}{3d^2n} + \frac{b^3x^{4n}}{4dn} + \frac{c(bc-ad)^3\log\left(c+dx^n\right)}{d^5n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.03

$$
\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n}\,dx
= \frac{dx^n(12a^3d^3 + 18a^2bd^2(-2c+dx^n) + 6ab^2d(6c^2-3cdx^n+2d^2x^{2n}) + b^3(-12c^3+6c^2dx^n-4cd^2x^{2n}+3d^3x}{12d^5n}
$$

```
[In] Integrate[(x^(-1 + 2*n)*(a + b*x^n)^3)/(c + d*x^n),x]
```

```
[Out] (d*x^n*(12*a^3*d^3 + 18*a^2*b*d^2*(-2*c + d*x^n) + 6*a*b^2*d*(6*c^2 - 3*c*d
*x^n + 2*d^2*x^(2*n)) + b^3*(-12*c^3 + 6*c^2*d*x^n - 4*c*d^2*x^(2*n) + 3*d^
3*x^(3*n))) + 12*c*(b*c - a*d)^3*Log[c + d*x^n])/(12*d^5*n)
```

Maple [A] (verified)

Time = 5.41 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.45

method	result
norman	$\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\mathrm{e}^{n\ln(x)}}{d^4n}+\frac{b^3\mathrm{e}^{4n\ln(x)}}{4dn}+\frac{b(3a^2d^2-3abcd+b^2c^2)\mathrm{e}^{2n\ln(x)}}{2d^3n}+\frac{b^2(3ad-bc)\mathrm{e}^{3n\ln(x)}}{3d^2n}-\frac{c(a^3d^3-3a}{}$
risch	$\frac{b^3x^{4n}}{4dn}+\frac{b^2x^{3n}a}{dn}-\frac{b^3x^{3n}c}{3d^2n}+\frac{3b\,x^{2n}a^2}{2dn}-\frac{3b^2x^{2n}ac}{2d^2n}+\frac{b^3x^{2n}c^2}{2d^3n}+\frac{x^na^3}{dn}-\frac{3x^na^2bc}{d^2n}+\frac{3x^na\,b^2c^2}{d^3n}-\frac{x^nb^3c^3}{d^4n}-\frac{c\ln(x^n-}{d^2}$

```
[In] int(x^(-1+2*n)*(a+b*x^n)^3/(c+d*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/n*exp(n*ln(x))+1/4*b^3/
d/n*exp(n*ln(x))^4+1/2*b*(3*a^2*d^2-3*a*b*c*d+b^2*c^2)/d^3/n*exp(n*ln(x))^2
+1/3*b^2*(3*a*d-b*c)/d^2/n*exp(n*ln(x))^3-c*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*
c^2*d-b^3*c^3)/d^5/n*ln(c+d*exp(n*ln(x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.36

$$\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n}\,dx$$
$$=\frac{3\,b^3d^4x^{4\,n}-4\,(b^3cd^3-3\,ab^2d^4)x^{3\,n}+6\,(b^3c^2d^2-3\,ab^2cd^3+3\,a^2bd^4)x^{2\,n}-12\,(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd}{12\,d^5n}$$

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^3*d^4*x^(4*n) - 4*(b^3*c*d^3 - 3*a*b^2*d^4)*x^(3*n) + 6*(b^3*c^2*
d^2 - 3*a*b^2*c*d^3 + 3*a^2*b*d^4)*x^(2*n) - 12*(b^3*c^3*d - 3*a*b^2*c^2*d^
2 + 3*a^2*b*c*d^3 - a^3*d^4)*x^n + 12*(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^
2*d^2 - a^3*c*d^3)*log(d*x^n + c))/(d^5*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(114) = 228$.

Time = 3.83 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.67

$$\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n}\,dx$$
$$=\begin{cases} \frac{(a+b)^3\log(x)}{c} \\ \frac{\frac{a^3xx^{2n-1}}{2n}+\frac{a^2bxx^nx^{2n-1}}{n}+\frac{3ab^2xx^{2n}x^{2n-1}}{4n}+\frac{b^3xx^{3n}x^{2n-1}}{5n}}{c} \\ \frac{(a+b)^3\log(x)}{c+d} \\ -\frac{a^3c\log\left(\frac{c}{d}+x^n\right)}{d^2n}+\frac{a^3x^n}{dn}+\frac{3a^2bc^2\log\left(\frac{c}{d}+x^n\right)}{d^3n}-\frac{3a^2bcx^n}{d^2n}+\frac{3a^2bx^{2n}}{2dn}-\frac{3ab^2c^3\log\left(\frac{c}{d}+x^n\right)}{d^4n}+\frac{3ab^2c^2x^n}{d^3n}-\frac{3ab^2cx^{2n}}{2d^2n}+\frac{ab^2x^{3n}}{dn} \end{cases}$$

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)**3/(c+d*x**n),x)
```

```
[Out] Piecewise(((a + b)**3*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a**3*x*x**(2*n - 1)
/(2*n) + a**2*b*x*x**n*x**(2*n - 1)/n + 3*a*b**2*x*x**(2*n)*x**(2*n - 1)/(4
*n) + b**3*x*x**(3*n)*x**(2*n - 1)/(5*n))/c, Eq(d, 0)), ((a + b)**3*log(x)/
(c + d), Eq(n, 0)), (-a**3*c*log(c/d + x**n)/(d**2*n) + a**3*x**n/(d*n) + 3
*a**2*b*c**2*log(c/d + x**n)/(d**3*n) - 3*a**2*b*c*x**n/(d**2*n) + 3*a**2*b
*x**(2*n)/(2*d*n) - 3*a*b**2*c**3*log(c/d + x**n)/(d**4*n) + 3*a*b**2*c**2*
x**n/(d**3*n) - 3*a*b**2*c*x**(2*n)/(2*d**2*n) + a*b**2*x**(3*n)/(d*n) + b*
*3*c**4*log(c/d + x**n)/(d**5*n) - b**3*c**3*x**n/(d**4*n) + b**3*c**2*x**(
2*n)/(2*d**3*n) - b**3*c*x**(3*n)/(3*d**2*n) + b**3*x**(4*n)/(4*d*n), True)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.78

$$\begin{aligned}
&\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n}\,dx \\
&= a^3\left(\frac{x^n}{dn} - \frac{c\log\left(\frac{dx^n+c}{d}\right)}{d^2n}\right) \\
&\quad + \frac{1}{12}\,b^3\left(\frac{12\,c^4\log\left(\frac{dx^n+c}{d}\right)}{d^5n} + \frac{3\,d^3x^{4\,n} - 4\,cd^2x^{3\,n} + 6\,c^2dx^{2\,n} - 12\,c^3x^n}{d^4n}\right) \\
&\quad - \frac{1}{2}\,ab^2\left(\frac{6\,c^3\log\left(\frac{dx^n+c}{d}\right)}{d^4n} - \frac{2\,d^2x^{3\,n} - 3\,cdx^{2\,n} + 6\,c^2x^n}{d^3n}\right) \\
&\quad + \frac{3}{2}\,a^2b\left(\frac{2\,c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n} + \frac{dx^{2\,n} - 2\,cx^n}{d^2n}\right)
\end{aligned}$$

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] a^3*(x^n/(d*n) - c*log((d*x^n + c)/d)/(d^2*n)) + 1/12*b^3*(12*c^4*log((d*x^
n + c)/d)/(d^5*n) + (3*d^3*x^(4*n) - 4*c*d^2*x^(3*n) + 6*c^2*d*x^(2*n) - 12
*c^3*x^n)/(d^4*n)) - 1/2*a*b^2*(6*c^3*log((d*x^n + c)/d)/(d^4*n) - (2*d^2*x
^(3*n) - 3*c*d*x^(2*n) + 6*c^2*x^n)/(d^3*n)) + 3/2*a^2*b*(2*c^2*log((d*x^n
+ c)/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))
```

Giac [F]

$$\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n}\,dx = \int \frac{(bx^n+a)^3x^{2\,n-1}}{dx^n+c}\,dx$$

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")
[Out] integrate((b*x^n + a)^3*x^(2*n - 1)/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n}\,dx = \int \frac{x^{2\,n-1}\,(a+b\,x^n)^3}{c+d\,x^n}\,dx$$

```
[In] int((x^(2*n - 1)*(a + b*x^n)^3)/(c + d*x^n),x)
[Out] int((x^(2*n - 1)*(a + b*x^n)^3)/(c + d*x^n), x)
```

3.1040 $\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n} dx$

Optimal result

Integrand size = 26, antiderivative size = 90

$$\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n} dx = \frac{(bc-ad)^2x^n}{d^3n} - \frac{b(bc-2ad)x^{2n}}{2d^2n} + \frac{b^2x^{3n}}{3dn} - \frac{c(bc-ad)^2\log(c+dx^n)}{d^4n}$$

```
[Out] (-a*d+b*c)^2*x^n/d^3/n-1/2*b*(-2*a*d+b*c)*x^(2*n)/d^2/n+1/3*b^2*x^(3*n)/d/n
-c*(-a*d+b*c)^2*ln(c+d*x^n)/d^4/n
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {457, 78}

$$\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n} dx = -\frac{c(bc-ad)^2\log(c+dx^n)}{d^4n} + \frac{x^n(bc-ad)^2}{d^3n} - \frac{bx^{2n}(bc-2ad)}{2d^2n} + \frac{b^2x^{3n}}{3dn}$$

```
[In] Int[(x^(-1 + 2*n)*(a + b*x^n)^2)/(c + d*x^n),x]
```

```
[Out] ((b*c - a*d)^2*x^n)/(d^3*n) - (b*(b*c - 2*a*d)*x^(2*n))/(2*d^2*n) + (b^2*x^
(3*n))/(3*d*n) - (c*(b*c - a*d)^2*Log[c + d*x^n])/(d^4*n)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{x(a+bx)^2}{c+dx}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{(-bc+ad)^2}{d^3} - \frac{b(bc-2ad)x}{d^2} + \frac{b^2x^2}{d} - \frac{c(bc-ad)^2}{d^3(c+dx)}\right)dx, x, x^n\right)}{n}\\ &= \frac{(bc-ad)^2x^n}{d^3n} - \frac{b(bc-2ad)x^{2n}}{2d^2n} + \frac{b^2x^{3n}}{3dn} - \frac{c(bc-ad)^2\log(c+dx^n)}{d^4n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

$$\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n}\,dx = \frac{dx^n(6a^2d^2+6abd(-2c+dx^n)+b^2(6c^2-3cdx^n+2d^2x^{2n}))-6c(bc-ad)^2\log(c+dx^n)}{6d^4n}$$

```
[In] Integrate[(x^(-1 + 2*n)*(a + b*x^n)^2)/(c + d*x^n),x]
[Out] (d*x^n*(6*a^2*d^2 + 6*a*b*d*(-2*c + d*x^n) + b^2*(6*c^2 - 3*c*d*x^n + 2*d^2
*x^(2*n))) - 6*c*(b*c - a*d)^2*Log[c + d*x^n])/(6*d^4*n)
```

Maple [A] (verified)

Time = 4.93 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.31

method	result	size
norman	$\frac{(a^2d^2-2abcd+b^2c^2)\mathrm{e}^{n\ln(x)}}{d^3n}+\frac{b^2\mathrm{e}^{3n\ln(x)}}{3dn}+\frac{b(2ad-bc)\mathrm{e}^{2n\ln(x)}}{2d^2n}-\frac{c(a^2d^2-2abcd+b^2c^2)\ln(c+d\,\mathrm{e}^{n\ln(x)})}{d^4n}$	118
risch	$\frac{b^2x^{3n}}{3dn}+\frac{b\,x^{2n}a}{dn}-\frac{b^2x^{2n}c}{2d^2n}+\frac{x^na^2}{dn}-\frac{2x^nabc}{d^2n}+\frac{x^nb^2c^2}{d^3n}-\frac{c\ln(x^n+\frac{c}{d})a^2}{d^2n}+\frac{2c^2\ln(x^n+\frac{c}{d})ab}{d^3n}-\frac{c^3\ln(x^n+\frac{c}{d})b^2}{d^4n}$	161

```
[In] int(x^(-1+2*n)*(a+b*x^n)^2/(c+d*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)/n*exp(n*ln(x))+1/3*b^2/d/n*exp(n*ln(x))^3
+1/2*b*(2*a*d-b*c)/d^2/n*exp(n*ln(x))^2-c*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^4/n
*ln(c+d*exp(n*ln(x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.20

$$\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n}\,dx = \frac{2\,b^2d^3x^{3\,n}-3\,(b^2cd^2-2\,abd^3)x^{2\,n}+6\,(b^2c^2d-2\,abcd^2+a^2d^3)x^n-6\,(b^2c^3-2\,abc^2d+a^2cd^2)\log\left(dx^n+c\right)}{6\,d^4n}$$

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^2*d^3*x^(3*n) - 3*(b^2*c*d^2 - 2*a*b*d^3)*x^(2*n) + 6*(b^2*c^2*d -
 2*a*b*c*d^2 + a^2*d^3)*x^n - 6*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*log(d*x
^n + c))/(d^4*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(75) = 150.

Time = 2.13 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.46

$$\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n}\,dx = \begin{cases} \frac{(a+b)^2\log(x)}{c} & \text{for } d=0 \\ \frac{\frac{a^2xx^{2n-1}}{2n}+\frac{2abxx^nx^{2n-1}}{3n}+\frac{b^2xx^{2n}x^{2n-1}}{4n}}{c} & \text{for } d=0 \\ \frac{(a+b)^2\log(x)}{c+d} & \text{for } n=0 \\ -\frac{a^2c\log\left(\frac{c}{d}+x^n\right)}{d^2n}+\frac{a^2x^n}{dn}+\frac{2abc^2\log\left(\frac{c}{d}+x^n\right)}{d^3n}-\frac{2abcx^n}{d^2n}+\frac{abx^{2n}}{dn}-\frac{b^2c^3\log\left(\frac{c}{d}+x^n\right)}{d^4n}+\frac{b^2c^2x^n}{d^3n}-\frac{b^2cx^{2n}}{2d^2n}+\frac{b^2x^{3n}}{3dn} & \text{otherwis} \end{cases}$$

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Piecewise(((a + b)**2*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a**2*x*x**(2*n - 1)
/(2*n) + 2*a*b*x*x**n*x**(2*n - 1)/(3*n) + b**2*x*x**(2*n)*x**(2*n - 1)/(4*
n))/c, Eq(d, 0)), ((a + b)**2*log(x)/(c + d), Eq(n, 0)), (-a**2*c*log(c/d +
 x**n)/(d**2*n) + a**2*x**n/(d*n) + 2*a*b*c**2*log(c/d + x**n)/(d**3*n) - 2
*a*b*c*x**n/(d**2*n) + a*b*x**(2*n)/(d*n) - b**2*c**3*log(c/d + x**n)/(d**4
*n) + b**2*c**2*x**n/(d**3*n) - b**2*c*x**(2*n)/(2*d**2*n) + b**2*x**(3*n)/
(3*d*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.67

$$\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n}\,dx = a^2\left(\frac{x^n}{dn} - \frac{c\log\left(\frac{dx^n+c}{d}\right)}{d^2n}\right) - \frac{1}{6}b^2\left(\frac{6\,c^3\log\left(\frac{dx^n+c}{d}\right)}{d^4n} - \frac{2\,d^2x^{3\,n} - 3\,cdx^{2\,n} + 6\,c^2x^n}{d^3n}\right) + ab\left(\frac{2\,c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n} + \frac{dx^{2\,n} - 2\,cx^n}{d^2n}\right)$$

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] a^2*(x^n/(d*n) - c*log((d*x^n + c)/d)/(d^2*n)) - 1/6*b^2*(6*c^3*log((d*x^n
+ c)/d)/(d^4*n) - (2*d^2*x^(3*n) - 3*c*d*x^(2*n) + 6*c^2*x^n)/(d^3*n)) + a*
b*(2*c^2*log((d*x^n + c)/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))
```

Giac [F]

$$\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n}\,dx = \int \frac{(bx^n+a)^2x^{2\,n-1}}{dx^n+c}\,dx$$

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^2*x^(2*n - 1)/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n}\,dx = \int \frac{x^{2\,n-1}\,(a+b\,x^n)^2}{c+d\,x^n}\,dx$$

```
[In] int((x^(2*n - 1)*(a + b*x^n)^2)/(c + d*x^n),x)
```

```
[Out] int((x^(2*n - 1)*(a + b*x^n)^2)/(c + d*x^n), x)
```

3.1041 $\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n} \, dx = -\frac{(bc-ad)x^n}{d^2n} + \frac{bx^{2n}}{2dn} + \frac{c(bc-ad)\log(c+dx^n)}{d^3n}$$

```
[Out] -(-a*d+b*c)*x^n/d^2/n+1/2*b*x^(2*n)/d/n+c*(-a*d+b*c)*ln(c+d*x^n)/d^3/n
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {457, 78}

$$\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n} \, dx = \frac{c(bc-ad)\log(c+dx^n)}{d^3n} - \frac{x^n(bc-ad)}{d^2n} + \frac{bx^{2n}}{2dn}$$

```
[In] Int[(x^(-1 + 2*n)*(a + b*x^n))/(c + d*x^n),x]
```

```
[Out] -(((b*c - a*d)*x^n)/(d^2*n)) + (b*x^(2*n))/(2*d*n) + (c*(b*c - a*d)*Log[c +
 d*x^n])/(d^3*n)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{x(a+bx)}{c+dx}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{-bc+ad}{d^2}+\frac{bx}{d}+\frac{c(bc-ad)}{d^2(c+dx)}\right)\,dx, x, x^n\right)}{n}\\ &= -\frac{(bc-ad)x^n}{d^2n}+\frac{bx^{2n}}{2dn}+\frac{c(bc-ad)\log\left(c+dx^n\right)}{d^3n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n}\,dx = \frac{dx^n(-2bc+2ad+bdx^n)+2c(bc-ad)\log\left(c+dx^n\right)}{2d^3n}$$

```
[In] Integrate[(x^(-1 + 2*n)*(a + b*x^n))/(c + d*x^n),x]
[Out] (d*x^n*(-2*b*c + 2*a*d + b*d*x^n) + 2*c*(b*c - a*d)*Log[c + d*x^n])/(2*d^3*
n)
```

Maple [A] (verified)

Time = 4.69 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

method	result	size
norman	$\frac{(ad-bc)\mathrm{e}^{n\ln(x)}}{d^2n}+\frac{b\,\mathrm{e}^{2n\ln(x)}}{2dn}-\frac{(ad-bc)c\ln\left(c+d\,\mathrm{e}^{n\ln(x)}\right)}{d^3n}$	65
risch	$\frac{b\,x^{2n}}{2dn}+\frac{x^n a}{dn}-\frac{x^n bc}{d^2n}-\frac{c\ln\left(x^n+\frac{c}{d}\right)a}{d^2n}+\frac{c^2\ln\left(x^n+\frac{c}{d}\right)b}{d^3n}$	81

```
[In] int(x^(-1+2*n)*(a+b*x^n)/(c+d*x^n),x,method=_RETURNVERBOSE)
[Out] 1/d^2*(a*d-b*c)/n*exp(n*ln(x))+1/2*b/d/n*exp(n*ln(x))^2-(a*d-b*c)*c/d^3/n*l
n(c+d*exp(n*ln(x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n}\,dx = \frac{bd^2x^{2\,n}-2\,(bcd-ad^2)x^n+2\,(bc^2-acd)\log\left(dx^n+c\right)}{2\,d^3n}$$

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
[Out] 1/2*(b*d^2*x^(2*n) - 2*(b*c*d - a*d^2)*x^n + 2*(b*c^2 - a*c*d)*log(d*x^n +
c))/(d^3*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(48) = 96.

Time = 1.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.93

$$\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n}\,dx = \begin{cases} \frac{(a+b)\log(x)}{c} & \text{for } d=0\wedge n=0 \\ \frac{\frac{axx^{2n-1}}{2n}+\frac{bxx^nx^{2n-1}}{3n}}{c} & \text{for } d=0 \\ \frac{(a+b)\log(x)}{c+d} & \text{for } n=0 \\ -\frac{ac\log\left(\frac{c}{d}+x^n\right)}{d^2n}+\frac{ax^n}{dn}+\frac{bc^2\log\left(\frac{c}{d}+x^n\right)}{d^3n}-\frac{bcx^n}{d^2n}+\frac{bx^{2n}}{2dn} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)/(c+d*x**n),x)
[Out] Piecewise(((a + b)*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a*x*x**(2*n - 1)/(2*n)
 + b*x*x**n*x**(2*n - 1)/(3*n))/c, Eq(d, 0)), ((a + b)*log(x)/(c + d), Eq(n
, 0)), (-a*c*log(c/d + x**n)/(d**2*n) + a*x**n/(d*n) + b*c**2*log(c/d + x**
n)/(d**3*n) - b*c*x**n/(d**2*n) + b*x**(2*n)/(2*d*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.38

$$\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n}\,dx = a\left(\frac{x^n}{dn}-\frac{c\log\left(\frac{dx^n+c}{d}\right)}{d^2n}\right)+\frac{1}{2}\,b\left(\frac{2\,c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n}+\frac{dx^{2\,n}-2\,cx^n}{d^2n}\right)$$

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
[Out] a*(x^n/(d*n) - c*log((d*x^n + c)/d)/(d^2*n)) + 1/2*b*(2*c^2*log((d*x^n + c)
/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))
```

Giac [F]

$$\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n}\,dx = \int \frac{(bx^n+a)x^{2\,n-1}}{dx^n+c}\,dx$$

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
[Out] integrate((b*x^n + a)*x^(2*n - 1)/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n}\,dx = \int \frac{x^{2\,n-1}\,(a+b\,x^n)}{c+d\,x^n}\,dx$$

```
[In] int((x^(2*n - 1)*(a + b*x^n))/(c + d*x^n),x)
[Out] int((x^(2*n - 1)*(a + b*x^n))/(c + d*x^n), x)
```

3.1042 $\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 54

$$\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)}\,dx = -\frac{a\log(a+bx^n)}{b(bc-ad)n}+\frac{c\log(c+dx^n)}{d(bc-ad)n}$$

```
[Out] -a*ln(a+b*x^n)/b/(-a*d+b*c)/n+c*ln(c+d*x^n)/d/(-a*d+b*c)/n
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {457, 78}

$$\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)}\,dx = \frac{c\log(c+dx^n)}{dn(bc-ad)}-\frac{a\log(a+bx^n)}{bn(bc-ad)}$$

```
[In] Int[x^(-1 + 2*n)/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] -((a*Log[a + b*x^n])/(b*(b*c - a*d)*n)) + (c*Log[c + d*x^n])/(d*(b*c - a*d)
*n)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{x}{(a+bx)(c+dx)}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)}+\frac{c}{(bc-ad)(c+dx)}\right)\,dx, x, x^n\right)}{n}\\ &= -\frac{a\log(a+bx^n)}{b(bc-ad)n}+\frac{c\log(c+dx^n)}{d(bc-ad)n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)}\,dx = -\frac{ad\log(a+bx^n)-bc\log(c+dx^n)}{b^2cdn-abd^2n}$$

```
[In] Integrate[x^(-1 + 2*n)/((a + b*x^n)*(c + d*x^n)),x]
[Out] -((a*d*Log[a + b*x^n] - b*c*Log[c + d*x^n])/(b^2*c*d*n - a*b*d^2*n))
```

Maple [A] (verified)

Time = 5.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

method	result	size
norman	$\frac{a\ln\left(a+b\,\mathrm{e}^{n\ln(x)}\right)}{(ad-bc)bn}-\frac{c\ln\left(c+d\,\mathrm{e}^{n\ln(x)}\right)}{dn(ad-bc)}$	59
risch	$\frac{\ln(x)}{bd}-\frac{\ln(x)a}{(ad-bc)b}+\frac{\ln(x)c}{d(ad-bc)}+\frac{a\ln\left(x^n+\frac{a}{b}\right)}{(ad-bc)bn}-\frac{c\ln\left(x^n+\frac{c}{d}\right)}{dn(ad-bc)}$	103

```
[In] int(x^(-1+2*n)/(a+b*x^n)/(c+d*x^n),x,method=_RETURNVERBOSE)
[Out] a/(a*d-b*c)/b/n*ln(a+b*exp(n*ln(x)))-c/d/n/(a*d-b*c)*ln(c+d*exp(n*ln(x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)}\,dx = -\frac{ad\log(bx^n+a)-bc\log(dx^n+c)}{(b^2cd-abd^2)n}$$

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
[Out] -(a*d*log(b*x^n + a) - b*c*log(d*x^n + c))/((b^2*c*d - a*b*d^2)*n)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)}\,dx = \text{Exception raised: NotImplementedError}$$

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)/(c+d*x**n),x)
[Out] Exception raised: NotImplementedError >> no valid subset found
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)}\,dx = -\frac{a\log\left(\frac{bx^n+a}{b}\right)}{b^2cn-abdn}+\frac{c\log\left(\frac{dx^n+c}{d}\right)}{bcdn-ad^2n}$$

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
[Out] -a*log((b*x^n + a)/b)/(b^2*c*n - a*b*d*n) + c*log((d*x^n + c)/d)/(b*c*d*n -
 a*d^2*n)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{x^{2\,n-1}}{(bx^n+a)(dx^n+c)}\,dx$$

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
[Out] integrate(x^(2*n - 1)/((b*x^n + a)*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{x^{2\,n-1}}{(a+b\,x^n)\,(c+d\,x^n)}\,dx$$

```
[In] int(x^(2*n - 1)/((a + b*x^n)*(c + d*x^n)),x)
[Out] int(x^(2*n - 1)/((a + b*x^n)*(c + d*x^n)), x)
```

3.1043 $\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 75

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{a}{b(bc-ad)n(a+bx^n)} + \frac{c\log(a+bx^n)}{(bc-ad)^2n} - \frac{c\log(c+dx^n)}{(bc-ad)^2n}$$

```
[Out] a/b/(-a*d+b*c)/n/(a+b*x^n)+c*ln(a+b*x^n)/(-a*d+b*c)^2/n-c*ln(c+d*x^n)/(-a*d
+b*c)^2/n
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {457, 78}

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{a}{bn(bc-ad)(a+bx^n)} + \frac{c\log(a+bx^n)}{n(bc-ad)^2} - \frac{c\log(c+dx^n)}{n(bc-ad)^2}$$

```
[In] Int[x^(-1 + 2*n)/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] a/(b*(b*c - a*d)*n*(a + b*x^n)) + (c*Log[a + b*x^n])/((b*c - a*d)^2*n) - (c
*Log[c + d*x^n])/((b*c - a*d)^2*n)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x}{(a+bx)^2(c+dx)}\,dx, x, x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)^2}+\frac{bc}{(bc-ad)^2(a+bx)}-\frac{cd}{(bc-ad)^2(c+dx)}\right)\,dx, x, x^n\right)}{n}\\
&= \frac{a}{b(bc-ad)n\,(a+bx^n)}+\frac{c\log\left(a+bx^n\right)}{(bc-ad)^2n}-\frac{c\log\left(c+dx^n\right)}{(bc-ad)^2n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+2n}}{\left(a+bx^n\right)^2\left(c+dx^n\right)}\,dx=\frac{a}{b(bc-ad)n\,(a+bx^n)}+\frac{c\log\left(a+bx^n\right)}{(bc-ad)^2n}-\frac{c\log\left(c+dx^n\right)}{(bc-ad)^2n}$$

```
[In] Integrate[x^(-1 + 2*n)/((a + b*x^n)^2*(c + d*x^n)),x]
[Out] a/(b*(b*c - a*d)*n*(a + b*x^n)) + (c*Log[a + b*x^n])/((b*c - a*d)^2*n) - (c
*Log[c + d*x^n])/((b*c - a*d)^2*n)
```

Maple [A] (verified)

Time = 5.38 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.43

method	result	size
risch	$-\frac{a}{(ad-bc)bn(a+b\,x^n)}-\frac{c\ln\left(x^n+\frac{c}{d}\right)}{n(a^2d^2-2abcd+b^2c^2)}+\frac{c\ln\left(x^n+\frac{a}{b}\right)}{n(a^2d^2-2abcd+b^2c^2)}$	107
norman	$\frac{e^{n\ln(x)}}{(ad-bc)n\left(a+b\,e^{n\ln(x)}\right)}+\frac{c\ln\left(a+b\,e^{n\ln(x)}\right)}{n(a^2d^2-2abcd+b^2c^2)}-\frac{c\ln\left(c+d\,e^{n\ln(x)}\right)}{n(a^2d^2-2abcd+b^2c^2)}$	109

```
[In] int(x^(-1+2*n)/(a+b*x^n)^2/(c+d*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] -a/(a*d-b*c)/b/n/(a+b*x^n)-c/n/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(x^n+c/d)+c/n/
(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.60

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{abc-a^2d+(b^2cx^n+abc)\log(bx^n+a)-(b^2cx^n+abc)\log(dx^n+c)}{(b^4c^2-2\,ab^3cd+a^2b^2d^2)nx^n+(ab^3c^2-2\,a^2b^2cd+a^3bd^2)n}$$

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] (a*b*c - a^2*d + (b^2*c*x^n + a*b*c)*log(b*x^n + a) - (b^2*c*x^n + a*b*c)*l
og(d*x^n + c))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*n*x^n + (a*b^3*c^2 -
2*a^2*b^2*c*d + a^3*b*d^2)*n)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.61

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{c\log\left(\frac{bx^n+a}{b}\right)}{b^2c^2n-2\,abcdn+a^2d^2n} - \frac{c\log\left(\frac{dx^n+c}{d}\right)}{b^2c^2n-2\,abcdn+a^2d^2n} + \frac{a}{ab^2cn-a^2bdn+(b^3cn-ab^2dn)x^n}$$

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] c*log((b*x^n + a)/b)/(b^2*c^2*n - 2*a*b*c*d*n + a^2*d^2*n) - c*log((d*x^n +
 c)/d)/(b^2*c^2*n - 2*a*b*c*d*n + a^2*d^2*n) + a/(a*b^2*c*n - a^2*b*d*n + (
b^3*c*n - a*b^2*d*n)*x^n)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)}\,dx = \int \frac{x^{2n-1}}{(bx^n+a)^2(dx^n+c)}\,dx$$

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
[Out] integrate(x^(2*n - 1)/((b*x^n + a)^2*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)}\,dx = \int \frac{x^{2n-1}}{(a+b\,x^n)^2\,(c+d\,x^n)}\,dx$$

```
[In] int(x^(2*n - 1)/((a + b*x^n)^2*(c + d*x^n)),x)
[Out] int(x^(2*n - 1)/((a + b*x^n)^2*(c + d*x^n)), x)
```

3.1044 $\int \frac{x^{-1+2n}}{(a+bx^n)^3(c+dx^n)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 105

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3(c+dx^n)}\,dx = \frac{a}{2b(bc-ad)n(a+bx^n)^2} - \frac{c}{(bc-ad)^2n(a+bx^n)} - \frac{cd\log(a+bx^n)}{(bc-ad)^3n} + \frac{cd\log(c+dx^n)}{(bc-ad)^3n}$$

```
[Out] 1/2*a/b/(-a*d+b*c)/n/(a+b*x^n)^2-c/(-a*d+b*c)^2/n/(a+b*x^n)-c*d*ln(a+b*x^n)
/(-a*d+b*c)^3/n+c*d*ln(c+d*x^n)/(-a*d+b*c)^3/n
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {457, 78}

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3(c+dx^n)}\,dx = \frac{a}{2bn(bc-ad)(a+bx^n)^2} - \frac{c}{n(bc-ad)^2(a+bx^n)} - \frac{cd\log(a+bx^n)}{n(bc-ad)^3} + \frac{cd\log(c+dx^n)}{n(bc-ad)^3}$$

```
[In] Int[x^(-1 + 2*n)/((a + b*x^n)^3*(c + d*x^n)),x]
```

```
[Out] a/(2*b*(b*c - a*d)*n*(a + b*x^n)^2) - c/((b*c - a*d)^2*n*(a + b*x^n)) - (c*
d*Log[a + b*x^n])/((b*c - a*d)^3*n) + (c*d*Log[c + d*x^n])/((b*c - a*d)^3*n
)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{x}{(a+bx)^3(c+dx)}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)^3}+\frac{bc}{(bc-ad)^2(a+bx)^2}-\frac{bcd}{(bc-ad)^3(a+bx)}+\frac{cd^2}{(bc-ad)^3(c+dx)}\right)dx, x, x^n\right)}{n}\\ &= \frac{a}{2b(bc-ad)n\left(a+bx^n\right)^2}-\frac{c}{(bc-ad)^2n\left(a+bx^n\right)}-\frac{cd\log\left(a+bx^n\right)}{(bc-ad)^3n}+\frac{cd\log\left(c+dx^n\right)}{(bc-ad)^3n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.95

$$\int \frac{x^{-1+2n}}{\left(a+bx^n\right)^3\left(c+dx^n\right)}\,dx = \frac{-abc-a^2d-2b^2cx^n}{2b(bc-ad)^2n\left(a+bx^n\right)^2}-\frac{cd\log\left(a+bx^n\right)}{(bc-ad)^3n}+\frac{cd\log\left(c+dx^n\right)}{(bc-ad)^3n}$$

```
[In] Integrate[x^(-1 + 2*n)/((a + b*x^n)^3*(c + d*x^n)),x]
```

```
[Out] (-(a*b*c) - a^2*d - 2*b^2*c*x^n)/(2*b*(b*c - a*d)^2*n*(a + b*x^n)^2) - (c*d
*Log[a + b*x^n])/((b*c - a*d)^3*n) + (c*d*Log[c + d*x^n])/((b*c - a*d)^3*n)
```

Maple [A] (verified)

Time = 6.24 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.50

method	result	si
risch	$-\frac{2b^2 c\, x^n+a^2 d+abc}{2n(ad-bc)^2 b(a+b\, x^n)^2}-\frac{cd\ln\left(x^n+\frac{c}{d}\right)}{n(a^3 d^3-3a^2 bc\, d^2+3a\, b^2 c^2 d-b^3 c^3)}+\frac{cd\ln\left(x^n+\frac{a}{b}\right)}{n(a^3 d^3-3a^2 bc\, d^2+3a\, b^2 c^2 d-b^3 c^3)}$	15
norman	$\frac{-\frac{bc\, e^{n\ln(x)}}{\left(a^2 d^2-2abcd+b^2 c^2\right)n}+\frac{a\left(-abd-b^2 c\right)}{2\left(a^2 d^2-2abcd+b^2 c^2\right)b^2 n}}{\left(a+b\, e^{n\ln(x)}\right)^2}+\frac{cd\ln\left(a+b\, e^{n\ln(x)}\right)}{n(a^3 d^3-3a^2 bc\, d^2+3a\, b^2 c^2 d-b^3 c^3)}-\frac{cd\ln\left(c+d\, e^{n\ln(x)}\right)}{n(a^3 d^3-3a^2 bc\, d^2+3a\, b^2 c^2 d-b^3 c^3)}$	20

```
[In] int(x^(-1+2*n)/(a+b*x^n)^3/(c+d*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(2*b^2*c*x^n+a^2*d+a*b*c)/n/(a*d-b*c)^2/b/(a+b*x^n)^2-c*d/n/(a^3*d^3-3
*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(x^n+c/d)+c*d/n/(a^3*d^3-3*a^2*b*c*d^
2+3*a*b^2*c^2*d-b^3*c^3)*ln(x^n+a/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(103) = 206.

Time = 0.31 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.54

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3\,(c+dx^n)}\,dx = -\frac{ab^2c^2-a^3d^2+2\,(b^3c^2-ab^2cd)x^n+2\,(b^3cdx^{2\,n}+2\,ab^2cdx^n+a^2bcd)\log\left(bx^n+a\right)-2\,(b^3cdx^{2\,n}+2}{2\,((b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3)nx^{2\,n}+2\,(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3cd^2-a^4b^2d^3)nx^n+(a^2b^4c^3}$$

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] -1/2*(a*b^2*c^2 - a^3*d^2 + 2*(b^3*c^2 - a*b^2*c*d)*x^n + 2*(b^3*c*d*x^(2*n
) + 2*a*b^2*c*d*x^n + a^2*b*c*d)*log(b*x^n + a) - 2*(b^3*c*d*x^(2*n) + 2*a*
b^2*c*d*x^n + a^2*b*c*d)*log(d*x^n + c))/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*
b^4*c*d^2 - a^3*b^3*d^3)*n*x^(2*n) + 2*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3
*b^3*c*d^2 - a^4*b^2*d^3)*n*x^n + (a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^
2*c*d^2 - a^5*b*d^3)*n)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3\,(c+dx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)**3/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(103) = 206$.

Time = 0.20 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.31

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3(c+dx^n)}\,dx$$
$$= -\frac{cd\log\left(\frac{bx^n+a}{b}\right)}{b^3c^3n - 3\,ab^2c^2dn + 3\,a^2bcd^2n - a^3d^3n} + \frac{cd\log\left(\frac{dx^n+c}{d}\right)}{b^3c^3n - 3\,ab^2c^2dn + 3\,a^2bcd^2n - a^3d^3n}$$
$$- \frac{2\,b^2cx^n + abc + a^2d}{2\,(a^2b^3c^2n - 2\,a^3b^2cdn + a^4bd^2n + (b^5c^2n - 2\,ab^4cdn + a^2b^3d^2n)x^{2\,n} + 2\,(ab^4c^2n - 2\,a^2b^3cdn + a^3b^2d^2n}$$

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] -c*d*log((b*x^n + a)/b)/(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*d^2*n - a^
3*d^3*n) + c*d*log((d*x^n + c)/d)/(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*
d^2*n - a^3*d^3*n) - 1/2*(2*b^2*c*x^n + a*b*c + a^2*d)/(a^2*b^3*c^2*n - 2*a
^3*b^2*c*d*n + a^4*b*d^2*n + (b^5*c^2*n - 2*a*b^4*c*d*n + a^2*b^3*d^2*n)*x^
(2*n) + 2*(a*b^4*c^2*n - 2*a^2*b^3*c*d*n + a^3*b^2*d^2*n)*x^n)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3(c+dx^n)}\,dx = \int \frac{x^{2\,n-1}}{(bx^n+a)^3(dx^n+c)}\,dx$$

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/((b*x^n + a)^3*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3(c+dx^n)}\,dx = \int \frac{x^{2\,n-1}}{(a+b\,x^n)^3\,(c+d\,x^n)}\,dx$$

```
[In] int(x^(2*n - 1)/((a + b*x^n)^3*(c + d*x^n)),x)
```

```
[Out] int(x^(2*n - 1)/((a + b*x^n)^3*(c + d*x^n)), x)
```

3.1045 $\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 158

$$\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n}\,dx = \frac{c(bc-ad)^3x^n}{d^5n} - \frac{(bc-ad)^3x^{2n}}{2d^4n} + \frac{b(b^2c^2-3abcd+3a^2d^2)\,x^{3n}}{3d^3n} - \frac{b^2(bc-3ad)x^{4n}}{4d^2n} + \frac{b^3x^{5n}}{5dn} - \frac{c^2(bc-ad)^3\log(c+dx^n)}{d^6n}$$

```
[Out] c*(-a*d+b*c)^3*x^n/d^5/n-1/2*(-a*d+b*c)^3*x^(2*n)/d^4/n+1/3*b*(3*a^2*d^2-3*
a*b*c*d+b^2*c^2)*x^(3*n)/d^3/n-1/4*b^2*(-3*a*d+b*c)*x^(4*n)/d^2/n+1/5*b^3*x
^(5*n)/d/n-c^2*(-a*d+b*c)^3*ln(c+d*x^n)/d^6/n
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {457, 90}

$$\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n}\,dx = \frac{bx^{3n}(3a^2d^2-3abcd+b^2c^2)}{3d^3n} - \frac{b^2x^{4n}(bc-3ad)}{4d^2n} - \frac{c^2(bc-ad)^3\log(c+dx^n)}{d^6n} + \frac{cx^n(bc-ad)^3}{d^5n} - \frac{x^{2n}(bc-ad)^3}{2d^4n} + \frac{b^3x^{5n}}{5dn}$$

```
[In] Int[(x^(-1 + 3*n)*(a + b*x^n)^3)/(c + d*x^n),x]
```

```
[Out] (c*(b*c - a*d)^3*x^n)/(d^5*n) - ((b*c - a*d)^3*x^(2*n))/(2*d^4*n) + (b*(b^2
*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x^(3*n))/(3*d^3*n) - (b^2*(b*c - 3*a*d)*x^(4*
```

```
n))/(4*d^2*n) + (b^3*x^(5*n))/(5*d*n) - (c^2*(b*c - a*d)^3*Log[c + d*x^n])/
(d^6*n)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2(a+bx)^3}{c+dx}\,dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{c(bc-ad)^3}{d^5} + \frac{(-bc+ad)^3x}{d^4} + \frac{b(b^2c^2-3abcd+3a^2d^2)x^2}{d^3} - \frac{b^2(bc-3ad)x^3}{d^2} + \frac{b^3x^4}{d} - \frac{c^2(bc-ad)^3}{d^5(c+dx)}\right)dx, x, x^n\right)}{n}$$

$$= \frac{c(bc-ad)^3x^n}{d^5n} - \frac{(bc-ad)^3x^{2n}}{2d^4n} + \frac{b(b^2c^2-3abcd+3a^2d^2)\,x^{3n}}{3d^3n} - \frac{b^2(bc-3ad)x^{4n}}{4d^2n} + \frac{b^3x^{5n}}{5dn} - \frac{c^2(bc-ad)^3\log\left(c+dx^n\right)}{d^6n}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.17

$$\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n}\,dx = \frac{dx^n(30a^3d^3(-2c+dx^n) + 30a^2bd^2(6c^2-3cdx^n+2d^2x^{2n}) + 15ab^2d(-12c^3+6c^2dx^n-4cd^2x^{2n}+3d^3x^{3n}}{60d^6n}$$

```
[In] Integrate[(x^(-1 + 3*n)*(a + b*x^n)^3)/(c + d*x^n),x]
```

```
[Out] (d*x^n*(30*a^3*d^3*(-2*c + d*x^n) + 30*a^2*b*d^2*(6*c^2 - 3*c*d*x^n + 2*d^2
*x^(2*n)) + 15*a*b^2*d*(-12*c^3 + 6*c^2*d*x^n - 4*c*d^2*x^(2*n) + 3*d^3*x^(
3*n)) + b^3*(60*c^4 - 30*c^3*d*x^n + 20*c^2*d^2*x^(2*n) - 15*c*d^3*x^(3*n)
+ 12*d^4*x^(4*n))) - 60*c^2*(b*c - a*d)^3*Log[c + d*x^n])/(60*d^6*n)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(150) = 300.

Time = 5.72 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.16

method	result
risch	$\frac{b^3 x^{5n}}{5dn}+\frac{3b^2 x^{4n} a}{4dn}-\frac{b^3 x^{4n} c}{4d^2 n}+\frac{b x^{3n} a^2}{dn}-\frac{b^2 x^{3n} a c}{d^2 n}+\frac{b^3 x^{3n} c^2}{3d^3 n}+\frac{x^{2n} a^3}{2dn}-\frac{3x^{2n} a^2 b c}{2d^2 n}+\frac{3x^{2n} a b^2 c^2}{2d^3 n}-\frac{x^{2n} b^3 c^3}{2d^4 n}-\frac{c x^n a^3}{d^2 n}$

```
[In] int(x^(-1+3*n)*(a+b*x^n)^3/(c+d*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*b^3/d/n*(x^n)^5+3/4*b^2/d/n*(x^n)^4*a-1/4*b^3/d^2/n*(x^n)^4*c+b/d/n*(x^
n)^3*a^2-b^2/d^2/n*(x^n)^3*a*c+1/3*b^3/d^3/n*(x^n)^3*c^2+1/2/d/n*(x^n)^2*a^
3-3/2/d^2/n*(x^n)^2*a^2*b*c+3/2/d^3/n*(x^n)^2*a*b^2*c^2-1/2/d^4/n*(x^n)^2*b
^3*c^3-c/d^2/n*x^n*a^3+3*c^2/d^3/n*x^n*a^2*b-3*c^3/d^4/n*x^n*a*b^2+c^4/d^5/
n*x^n*b^3+c^2/d^3/n*ln(x^n+c/d)*a^3-3*c^3/d^4/n*ln(x^n+c/d)*a^2*b+3*c^4/d^5
/n*ln(x^n+c/d)*a*b^2-c^5/d^6/n*ln(x^n+c/d)*b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.46

$$\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n}\,dx$$
$$=\frac{12\,b^3d^5x^{5\,n}-15\,(b^3cd^4-3\,ab^2d^5)x^{4\,n}+20\,(b^3c^2d^3-3\,ab^2cd^4+3\,a^2bd^5)x^{3\,n}-30\,(b^3c^3d^2-3\,ab^2c^2d^3+3\,a^2b$$

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/60*(12*b^3*d^5*x^(5*n) - 15*(b^3*c*d^4 - 3*a*b^2*d^5)*x^(4*n) + 20*(b^3*c
^2*d^3 - 3*a*b^2*c*d^4 + 3*a^2*b*d^5)*x^(3*n) - 30*(b^3*c^3*d^2 - 3*a*b^2*c
^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*x^(2*n) + 60*(b^3*c^4*d - 3*a*b^2*c^3*d^2
 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*x^n - 60*(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b
*c^3*d^2 - a^3*c^2*d^3)*log(d*x^n + c))/(d^6*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(138) = 276$.

Time = 5.48 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.71

$$\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n}\,dx$$
$$= \begin{cases} \frac{(a+b)^3\log(x)}{c} \\ \frac{\frac{a^3xx^{3n-1}}{3n}+\frac{3a^2bxx^nx^{3n-1}}{4n}+\frac{3ab^2xx^{2n}x^{3n-1}}{5n}+\frac{b^3xx^{3n}x^{3n-1}}{6n}}{c} \\ \frac{(a+b)^3\log(x)}{c+d} \\ \frac{a^3c^2\log\left(\frac{c}{d}+x^n\right)}{d^3n} - \frac{a^3cx^n}{d^2n} + \frac{a^3x^{2n}}{2dn} - \frac{3a^2bc^3\log\left(\frac{c}{d}+x^n\right)}{d^4n} + \frac{3a^2bc^2x^n}{d^3n} - \frac{3a^2bcx^{2n}}{2d^2n} + \frac{a^2bx^{3n}}{dn} + \frac{3ab^2c^4\log\left(\frac{c}{d}+x^n\right)}{d^5n} - \frac{3ab^2c^3x^n}{d^4n} \end{cases}$$

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**3/(c+d*x**n),x)
```

```
[Out] Piecewise(((a + b)**3*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a**3*x*x**(3*n - 1)
/(3*n) + 3*a**2*b*x*x**n*x**(3*n - 1)/(4*n) + 3*a*b**2*x*x**(2*n)*x**(3*n -
 1)/(5*n) + b**3*x*x**(3*n)*x**(3*n - 1)/(6*n))/c, Eq(d, 0)), ((a + b)**3*l
og(x)/(c + d), Eq(n, 0)), (a**3*c**2*log(c/d + x**n)/(d**3*n) - a**3*c*x**n
/(d**2*n) + a**3*x**(2*n)/(2*d*n) - 3*a**2*b*c**3*log(c/d + x**n)/(d**4*n)
+ 3*a**2*b*c**2*x**n/(d**3*n) - 3*a**2*b*c*x**(2*n)/(2*d**2*n) + a**2*b*x**
(3*n)/(d*n) + 3*a*b**2*c**4*log(c/d + x**n)/(d**5*n) - 3*a*b**2*c**3*x**n/(
d**4*n) + 3*a*b**2*c**2*x**(2*n)/(2*d**3*n) - a*b**2*c*x**(3*n)/(d**2*n) +
3*a*b**2*x**(4*n)/(4*d*n) - b**3*c**5*log(c/d + x**n)/(d**6*n) + b**3*c**4*
x**n/(d**5*n) - b**3*c**3*x**(2*n)/(2*d**4*n) + b**3*c**2*x**(3*n)/(3*d**3*
n) - b**3*c*x**(4*n)/(4*d**2*n) + b**3*x**(5*n)/(5*d*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.81

$$\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n}\,dx$$
$$= -\frac{1}{60}b^3\left(\frac{60\,c^5\log\left(\frac{dx^n+c}{d}\right)}{d^6n} - \frac{12\,d^4x^{5\,n} - 15\,cd^3x^{4\,n} + 20\,c^2d^2x^{3\,n} - 30\,c^3dx^{2\,n} + 60\,c^4x^n}{d^5n}\right)$$
$$+ \frac{1}{4}ab^2\left(\frac{12\,c^4\log\left(\frac{dx^n+c}{d}\right)}{d^5n} + \frac{3\,d^3x^{4\,n} - 4\,cd^2x^{3\,n} + 6\,c^2dx^{2\,n} - 12\,c^3x^n}{d^4n}\right)$$
$$- \frac{1}{2}a^2b\left(\frac{6\,c^3\log\left(\frac{dx^n+c}{d}\right)}{d^4n} - \frac{2\,d^2x^{3\,n} - 3\,cdx^{2\,n} + 6\,c^2x^n}{d^3n}\right)$$
$$+ \frac{1}{2}a^3\left(\frac{2\,c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n} + \frac{dx^{2\,n} - 2\,cx^n}{d^2n}\right)$$

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] -1/60*b^3*(60*c^5*log((d*x^n + c)/d)/(d^6*n) - (12*d^4*x^(5*n) - 15*c*d^3*x
^(4*n) + 20*c^2*d^2*x^(3*n) - 30*c^3*d*x^(2*n) + 60*c^4*x^n)/(d^5*n)) + 1/4
*a*b^2*(12*c^4*log((d*x^n + c)/d)/(d^5*n) + (3*d^3*x^(4*n) - 4*c*d^2*x^(3*n
) + 6*c^2*d*x^(2*n) - 12*c^3*x^n)/(d^4*n)) - 1/2*a^2*b*(6*c^3*log((d*x^n +
c)/d)/(d^4*n) - (2*d^2*x^(3*n) - 3*c*d*x^(2*n) + 6*c^2*x^n)/(d^3*n)) + 1/2*
a^3*(2*c^2*log((d*x^n + c)/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))
```

Giac [F]

$$\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n}\,dx = \int \frac{(bx^n+a)^3x^{3\,n-1}}{dx^n+c}\,dx$$

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^3*x^(3*n - 1)/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n}\,dx = \int \frac{x^{3\,n-1}\,(a+b\,x^n)^3}{c+d\,x^n}\,dx$$

```
[In] int((x^(3*n - 1)*(a + b*x^n)^3)/(c + d*x^n),x)
```

```
[Out] int((x^(3*n - 1)*(a + b*x^n)^3)/(c + d*x^n), x)
```

3.1046 $\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 118

$$\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx = -\frac{c(bc-ad)^2x^n}{d^4n}+\frac{(bc-ad)^2x^{2n}}{2d^3n}-\frac{b(bc-2ad)x^{3n}}{3d^2n}+\frac{b^2x^{4n}}{4dn}+\frac{c^2(bc-ad)^2\log(c+dx^n)}{d^5n}$$

```
[Out] -c*(-a*d+b*c)^2*x^n/d^4/n+1/2*(-a*d+b*c)^2*x^(2*n)/d^3/n-1/3*b*(-2*a*d+b*c)
*x^(3*n)/d^2/n+1/4*b^2*x^(4*n)/d/n+c^2*(-a*d+b*c)^2*ln(c+d*x^n)/d^5/n
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {457, 90}

$$\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx = \frac{c^2(bc-ad)^2\log(c+dx^n)}{d^5n}-\frac{cx^n(bc-ad)^2}{d^4n}+\frac{x^{2n}(bc-ad)^2}{2d^3n}-\frac{bx^{3n}(bc-2ad)}{3d^2n}+\frac{b^2x^{4n}}{4dn}$$

```
[In] Int[(x^(-1 + 3*n)*(a + b*x^n)^2)/(c + d*x^n),x]
```

```
[Out] -((c*(b*c - a*d)^2*x^n)/(d^4*n)) + ((b*c - a*d)^2*x^(2*n))/(2*d^3*n) - (b*(
b*c - 2*a*d)*x^(3*n))/(3*d^2*n) + (b^2*x^(4*n))/(4*d*n) + (c^2*(b*c - a*d)^
2*Log[c + d*x^n])/(d^5*n)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2(a+bx)^2}{c+dx}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{c(bc-ad)^2}{d^4}+\frac{(-bc+ad)^2x}{d^3}-\frac{b(bc-2ad)x^2}{d^2}+\frac{b^2x^3}{d}+\frac{c^2(bc-ad)^2}{d^4(c+dx)}\right)dx, x, x^n\right)}{n}\\ &= -\frac{c(bc-ad)^2x^n}{d^4n}+\frac{(bc-ad)^2x^{2n}}{2d^3n}-\frac{b(bc-2ad)x^{3n}}{3d^2n}+\frac{b^2x^{4n}}{4dn}+\frac{c^2(bc-ad)^2\log(c+dx^n)}{d^5n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.06

$$\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx = \frac{dx^n(6a^2d^2(-2c+dx^n)+4abd(6c^2-3cdx^n+2d^2x^{2n})+b^2(-12c^3+6c^2dx^n-4cd^2x^{2n}+3d^3x^{3n}))+12c^2(b}{12d^5n}$$

```
[In] Integrate[(x^(-1 + 3*n)*(a + b*x^n)^2)/(c + d*x^n),x]
[Out] (d*x^n*(6*a^2*d^2*(-2*c + d*x^n) + 4*a*b*d*(6*c^2 - 3*c*d*x^n + 2*d^2*x^(2*
n)) + b^2*(-12*c^3 + 6*c^2*d*x^n - 4*c*d^2*x^(2*n) + 3*d^3*x^(3*n))) + 12*c
^2*(b*c - a*d)^2*Log[c + d*x^n])/(12*d^5*n)
```

Maple [A] (verified)

Time = 4.90 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.33

method	result
norman	$\frac{b^2 e^{4n\ln(x)}}{4dn}+\frac{(a^2d^2-2abcd+b^2c^2)e^{2n\ln(x)}}{2d^3n}+\frac{b(2ad-bc)e^{3n\ln(x)}}{3d^2n}-\frac{c(a^2d^2-2abcd+b^2c^2)e^{n\ln(x)}}{d^4n}+\frac{c^2(a^2d^2-2abcd+b^2c^2)\ln(c+}{d^5n}$
risch	$\frac{b^2x^{4n}}{4dn}+\frac{2b\,x^{3n}a}{3dn}-\frac{b^2x^{3n}c}{3d^2n}+\frac{x^{2n}a^2}{2dn}-\frac{x^{2n}abc}{d^2n}+\frac{x^{2n}b^2c^2}{2d^3n}-\frac{c\,x^na^2}{d^2n}+\frac{2c^2x^nab}{d^3n}-\frac{c^3x^nb^2}{d^4n}+\frac{c^2\ln(x^n+\frac{c}{d})a^2}{d^3n}-\frac{2c^3\ln(x}{d}$

```
[In] int(x^(-1+3*n)*(a+b*x^n)^2/(c+d*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b^2/d/n*exp(n*ln(x))^4+1/2/d^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)/n*exp(n*ln(x
))^2+1/3*b*(2*a*d-b*c)/d^2/n*exp(n*ln(x))^3-c*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d
^4/n*exp(n*ln(x))+c^2/d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/n*ln(c+d*exp(n*ln(x))
)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.24

$$\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx = \frac{3\,b^2d^4x^{4\,n} - 4\,(b^2cd^3 - 2\,abd^4)x^{3\,n} + 6\,(b^2c^2d^2 - 2\,abcd^3 + a^2d^4)x^{2\,n} - 12\,(b^2c^3d - 2\,abc^2d^2 + a^2cd^3)x^n + 1}{12\,d^5n}$$

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*d^4*x^(4*n) - 4*(b^2*c*d^3 - 2*a*b*d^4)*x^(3*n) + 6*(b^2*c^2*d^
2 - 2*a*b*c*d^3 + a^2*d^4)*x^(2*n) - 12*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*
d^3)*x^n + 12*(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*log(d*x^n + c))/(d^5*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. 2(99) = 198.

Time = 3.20 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.35

$$\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx = \begin{cases} \frac{(a+b)^2\log(x)}{c} \\ \frac{\frac{a^2xx^{3n-1}}{3n} + \frac{abxx^nx^{3n-1}}{2n} + \frac{b^2xx^{2n}x^{3n-1}}{5n}}{c} \\ \frac{(a+b)^2\log(x)}{c+d} \\ \frac{a^2c^2\log\left(\frac{c}{d}+x^n\right)}{d^3n} - \frac{a^2cx^n}{d^2n} + \frac{a^2x^{2n}}{2dn} - \frac{2abc^3\log\left(\frac{c}{d}+x^n\right)}{d^4n} + \frac{2abc^2x^n}{d^3n} - \frac{abcx^{2n}}{d^2n} + \frac{2abx^{3n}}{3dn} + \frac{b^2c^4\log\left(\frac{c}{d}+x^n\right)}{d^5n} - \frac{b^2c^3x^n}{d^4n} + \frac{b^2c^2x^{2}}{2d^3n} \end{cases}$$

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Piecewise(((a + b)**2*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a**2*x*x**(3*n - 1)
/(3*n) + a*b*x*x**n*x**(3*n - 1)/(2*n) + b**2*x*x**(2*n)*x**(3*n - 1)/(5*n)
)/c, Eq(d, 0)), ((a + b)**2*log(x)/(c + d), Eq(n, 0)), (a**2*c**2*log(c/d +
 x**n)/(d**3*n) - a**2*c*x**n/(d**2*n) + a**2*x**(2*n)/(2*d*n) - 2*a*b*c**3
*log(c/d + x**n)/(d**4*n) + 2*a*b*c**2*x**n/(d**3*n) - a*b*c*x**(2*n)/(d**2
```

```
*n) + 2*a*b*x**(3*n)/(3*d*n) + b**2*c**4*log(c/d + x**n)/(d**5*n) - b**2*c*
*3*x**n/(d**4*n) + b**2*c**2*x**(2*n)/(2*d**3*n) - b**2*c*x**(3*n)/(3*d**2*
n) + b**2*x**(4*n)/(4*d*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.63

$$\begin{aligned}\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx &= \frac{1}{12}\,b^2\left(\frac{12\,c^4\log\left(\frac{dx^n+c}{d}\right)}{d^5n}+\frac{3\,d^3x^{4\,n}-4\,cd^2x^{3\,n}+6\,c^2dx^{2\,n}-12\,c^3x^n}{d^4n}\right)\\ &\quad-\frac{1}{3}\,ab\left(\frac{6\,c^3\log\left(\frac{dx^n+c}{d}\right)}{d^4n}-\frac{2\,d^2x^{3\,n}-3\,cdx^{2\,n}+6\,c^2x^n}{d^3n}\right)\\ &\quad+\frac{1}{2}\,a^2\left(\frac{2\,c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n}+\frac{dx^{2\,n}-2\,cx^n}{d^2n}\right)\end{aligned}$$

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] 1/12*b^2*(12*c^4*log((d*x^n + c)/d)/(d^5*n) + (3*d^3*x^(4*n) - 4*c*d^2*x^(3
*n) + 6*c^2*d*x^(2*n) - 12*c^3*x^n)/(d^4*n)) - 1/3*a*b*(6*c^3*log((d*x^n +
c)/d)/(d^4*n) - (2*d^2*x^(3*n) - 3*c*d*x^(2*n) + 6*c^2*x^n)/(d^3*n)) + 1/2*
a^2*(2*c^2*log((d*x^n + c)/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))
```

Giac [F]

$$\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx=\int \frac{(bx^n+a)^2x^{3\,n-1}}{dx^n+c}\,dx$$

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^2*x^(3*n - 1)/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx = \int \frac{x^{3\,n-1}\,(a+b\,x^n)^2}{c+d\,x^n}\,dx$$

```
[In] int((x^(3*n - 1)*(a + b*x^n)^2)/(c + d*x^n),x)
[Out] int((x^(3*n - 1)*(a + b*x^n)^2)/(c + d*x^n), x)
```

3.1047 $\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 86

$$\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n}\,dx=\frac{c(bc-ad)x^n}{d^3n}-\frac{(bc-ad)x^{2n}}{2d^2n}+\frac{bx^{3n}}{3dn}-\frac{c^2(bc-ad)\log(c+dx^n)}{d^4n}$$

```
[Out] c*(-a*d+b*c)*x^n/d^3/n-1/2*(-a*d+b*c)*x^(2*n)/d^2/n+1/3*b*x^(3*n)/d/n-c^2*(
-a*d+b*c)*ln(c+d*x^n)/d^4/n
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {457, 78}

$$\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n}\,dx=-\frac{c^2(bc-ad)\log(c+dx^n)}{d^4n}+\frac{cx^n(bc-ad)}{d^3n}-\frac{x^{2n}(bc-ad)}{2d^2n}+\frac{bx^{3n}}{3dn}$$

```
[In] Int[(x^(-1 + 3*n)*(a + b*x^n))/(c + d*x^n),x]
```

```
[Out] (c*(b*c - a*d)*x^n)/(d^3*n) - ((b*c - a*d)*x^(2*n))/(2*d^2*n) + (b*x^(3*n))
/(3*d*n) - (c^2*(b*c - a*d)*Log[c + d*x^n])/(d^4*n)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2(a+bx)}{c+dx}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{c(bc-ad)}{d^3}+\frac{(-bc+ad)x}{d^2}+\frac{bx^2}{d}-\frac{c^2(bc-ad)}{d^3(c+dx)}\right)dx, x, x^n\right)}{n}\\ &= \frac{c(bc-ad)x^n}{d^3n}-\frac{(bc-ad)x^{2n}}{2d^2n}+\frac{bx^{3n}}{3dn}-\frac{c^2(bc-ad)\log\left(c+dx^n\right)}{d^4n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88

$$\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n}\,dx = \frac{dx^n(3ad(-2c+dx^n)+b(6c^2-3cdx^n+2d^2x^{2n}))+6c^2(-bc+ad)\log\left(c+dx^n\right)}{6d^4n}$$

```
[In] Integrate[(x^(-1 + 3*n)*(a + b*x^n))/(c + d*x^n),x]
[Out] (d*x^n*(3*a*d*(-2*c + d*x^n) + b*(6*c^2 - 3*c*d*x^n + 2*d^2*x^(2*n))) + 6*c
^2*(-(b*c) + a*d)*Log[c + d*x^n])/(6*d^4*n)
```

Maple [A] (verified)

Time = 4.82 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.06

method	result	size
norman	$\frac{b\,e^{3n\ln(x)}}{3dn}+\frac{(ad-bc)e^{2n\ln(x)}}{2d^2n}-\frac{(ad-bc)c\,e^{n\ln(x)}}{d^3n}+\frac{c^2(ad-bc)\ln\left(c+d\,e^{n\ln(x)}\right)}{d^4n}$	91
risch	$\frac{b\,x^{3n}}{3dn}+\frac{x^{2n}a}{2dn}-\frac{x^{2n}bc}{2d^2n}-\frac{c\,x^n a}{d^2n}+\frac{c^2x^n b}{d^3n}+\frac{c^2\ln\left(x^n+\frac{c}{d}\right)a}{d^3n}-\frac{c^3\ln\left(x^n+\frac{c}{d}\right)b}{d^4n}$	115

```
[In] int(x^(-1+3*n)*(a+b*x^n)/(c+d*x^n),x,method=_RETURNVERBOSE)
[Out] 1/3*b/d/n*exp(n*ln(x))^3+1/2/d^2*(a*d-b*c)/n*exp(n*ln(x))^2-(a*d-b*c)*c/d^3
/n*exp(n*ln(x))+c^2/d^4*(a*d-b*c)/n*ln(c+d*exp(n*ln(x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95

$$\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n}\,dx = \frac{2\,bd^3x^{3\,n}-3\,(bcd^2-ad^3)x^{2\,n}+6\,(bc^2d-acd^2)x^n-6\,(bc^3-ac^2d)\log\left(dx^n+c\right)}{6\,d^4n}$$

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*d^3*x^(3*n) - 3*(b*c*d^2 - a*d^3)*x^(2*n) + 6*(b*c^2*d - a*c*d^2)*
x^n - 6*(b*c^3 - a*c^2*d)*log(d*x^n + c))/(d^4*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(71) = 142.

Time = 1.86 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.74

$$\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n}\,dx = \begin{cases} \frac{(a+b)\log(x)}{c} & \text{for } d=0\wedge n=0 \\ \frac{\frac{axx^{3n-1}}{3n}+\frac{bxx^nx^{3n-1}}{4n}}{c} & \text{for } d=0 \\ \frac{(a+b)\log(x)}{c+d} & \text{for } n=0 \\ \frac{ac^2\log\left(\frac{c}{d}+x^n\right)}{d^3n}-\frac{acx^n}{d^2n}+\frac{ax^{2n}}{2dn}-\frac{bc^3\log\left(\frac{c}{d}+x^n\right)}{d^4n}+\frac{bc^2x^n}{d^3n}-\frac{bcx^{2n}}{2d^2n}+\frac{bx^{3n}}{3dn} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Piecewise(((a + b)*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a*x*x**(3*n - 1)/(3*n)
 + b*x*x**n*x**(3*n - 1)/(4*n))/c, Eq(d, 0)), ((a + b)*log(x)/(c + d), Eq(n
, 0)), (a*c**2*log(c/d + x**n)/(d**3*n) - a*c*x**n/(d**2*n) + a*x**(2*n)/(2
*d*n) - b*c**3*log(c/d + x**n)/(d**4*n) + b*c**2*x**n/(d**3*n) - b*c*x**(2*
n)/(2*d**2*n) + b*x**(3*n)/(3*d*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.30

$$\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n}\,dx = -\frac{1}{6}\,b\left(\frac{6\,c^3\log\left(\frac{dx^n+c}{d}\right)}{d^4n} - \frac{2\,d^2x^{3\,n} - 3\,cdx^{2\,n} + 6\,c^2x^n}{d^3n}\right) + \frac{1}{2}\,a\left(\frac{2\,c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n} + \frac{dx^{2\,n} - 2\,cx^n}{d^2n}\right)$$

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
[Out] -1/6*b*(6*c^3*log((d*x^n + c)/d)/(d^4*n) - (2*d^2*x^(3*n) - 3*c*d*x^(2*n) +
 6*c^2*x^n)/(d^3*n)) + 1/2*a*(2*c^2*log((d*x^n + c)/d)/(d^3*n) + (d*x^(2*n)
 - 2*c*x^n)/(d^2*n))
```

Giac [F]

$$\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n}\,dx = \int \frac{(bx^n+a)x^{3\,n-1}}{dx^n+c}\,dx$$

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
[Out] integrate((b*x^n + a)*x^(3*n - 1)/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n}\,dx = \int \frac{x^{3\,n-1}\,(a+b\,x^n)}{c+d\,x^n}\,dx$$

```
[In] int((x^(3*n - 1)*(a + b*x^n))/(c + d*x^n),x)
[Out] int((x^(3*n - 1)*(a + b*x^n))/(c + d*x^n), x)
```

3.1048 $\int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 71

$$\int \frac{x^{-1+3n}}{(a+bx^n)\,(c+dx^n)}\,dx=\frac{x^n}{bdn}+\frac{a^2\log(a+bx^n)}{b^2(bc-ad)n}-\frac{c^2\log(c+dx^n)}{d^2(bc-ad)n}$$

```
[Out] x^n/b/d/n+a^2*ln(a+b*x^n)/b^2/(-a*d+b*c)/n-c^2*ln(c+d*x^n)/d^2/(-a*d+b*c)/n
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {457, 84}

$$\int \frac{x^{-1+3n}}{(a+bx^n)\,(c+dx^n)}\,dx=\frac{a^2\log(a+bx^n)}{b^2n(bc-ad)}-\frac{c^2\log(c+dx^n)}{d^2n(bc-ad)}+\frac{x^n}{bdn}$$

```
[In] Int[x^(-1 + 3*n)/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] x^n/(b*d*n) + (a^2*Log[a + b*x^n])/(b^2*(b*c - a*d)*n) - (c^2*Log[c + d*x^n
])/(d^2*(b*c - a*d)*n)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx)(c+dx)}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{bd}+\frac{a^2}{b(bc-ad)(a+bx)}+\frac{c^2}{d(-bc+ad)(c+dx)}\right)\,dx, x, x^n\right)}{n}\\ &= \frac{x^n}{bdn}+\frac{a^2\log(a+bx^n)}{b^2(bc-ad)n}-\frac{c^2\log(c+dx^n)}{d^2(bc-ad)n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)}\,dx = \frac{a^2d^2\log(a+bx^n)+b(d(bc-ad)x^n-bc^2\log(c+dx^n))}{b^2d^2(bc-ad)n}$$

```
[In] Integrate[x^(-1 + 3*n)/((a + b*x^n)*(c + d*x^n)),x]
[Out] (a^2*d^2*Log[a + b*x^n] + b*(d*(b*c - a*d)*x^n - b*c^2*Log[c + d*x^n]))/(b^
2*d^2*(b*c - a*d)*n)
```

Maple [A] (verified)

Time = 5.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.10

method	result	size
norman	$\frac{e^{n\ln(x)}}{bdn}+\frac{c^2\ln\left(c+d\,e^{n\ln(x)}\right)}{d^2n(ad-bc)}-\frac{a^2\ln\left(a+b\,e^{n\ln(x)}\right)}{(ad-bc)b^2n}$	78
risch	$-\frac{\ln(x)a}{b^2d}-\frac{\ln(x)c}{b\,d^2}+\frac{x^n}{bdn}+\frac{\ln(x)a^2}{(ad-bc)b^2}-\frac{\ln(x)c^2}{d^2(ad-bc)}-\frac{a^2\ln\left(x^n+\frac{a}{b}\right)}{(ad-bc)b^2n}+\frac{c^2\ln\left(x^n+\frac{c}{d}\right)}{d^2n(ad-bc)}$	137

```
[In] int(x^(-1+3*n)/(a+b*x^n)/(c+d*x^n),x,method=_RETURNVERBOSE)
[Out] 1/b/d/n*exp(n*ln(x))+c^2/d^2/n/(a*d-b*c)*ln(c+d*exp(n*ln(x)))-a^2/(a*d-b*c)
/b^2/n*ln(a+b*exp(n*ln(x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04

$$\int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)}\,dx = \frac{a^2d^2\log(bx^n+a) - b^2c^2\log(dx^n+c) + (b^2cd - abd^2)x^n}{(b^3cd^2 - ab^2d^3)n}$$

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
[Out] (a^2*d^2*log(b*x^n + a) - b^2*c^2*log(d*x^n + c) + (b^2*c*d - a*b*d^2)*x^n)
/((b^3*c*d^2 - a*b^2*d^3)*n)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)/(c+d*x**n),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.14

$$\int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)}\,dx = \frac{a^2\log\left(\frac{bx^n+a}{b}\right)}{b^3cn - ab^2dn} - \frac{c^2\log\left(\frac{dx^n+c}{d}\right)}{bcd^2n - ad^3n} + \frac{x^n}{bdn}$$

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
[Out] a^2*log((b*x^n + a)/b)/(b^3*c*n - a*b^2*d*n) - c^2*log((d*x^n + c)/d)/(b*c*
d^2*n - a*d^3*n) + x^n/(b*d*n)
```

Giac [F]

$$\int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{x^{3\,n-1}}{(bx^n+a)(dx^n+c)}\,dx$$

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
[Out] integrate(x^(3*n - 1)/((b*x^n + a)*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{x^{3\,n-1}}{(a+b\,x^n)\,(c+d\,x^n)}\,dx$$

```
[In] int(x^(3*n - 1)/((a + b*x^n)*(c + d*x^n)),x)
[Out] int(x^(3*n - 1)/((a + b*x^n)*(c + d*x^n)), x)
```

3.1049 $\int \frac{x^{-1+3n}}{(a+bx^n)^2(c+dx^n)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 95

$$\int \frac{x^{-1+3n}}{(a+bx^n)^2(c+dx^n)}\,dx = -\frac{a^2}{b^2(bc-ad)n(a+bx^n)} - \frac{a(2bc-ad)\log(a+bx^n)}{b^2(bc-ad)^2n} + \frac{c^2\log(c+dx^n)}{d(bc-ad)^2n}$$

```
[Out] -a^2/b^2/(-a*d+b*c)/n/(a+b*x^n)-a*(-a*d+2*b*c)*ln(a+b*x^n)/b^2/(-a*d+b*c)^2
/n+c^2*ln(c+d*x^n)/d/(-a*d+b*c)^2/n
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {457, 90}

$$\int \frac{x^{-1+3n}}{(a+bx^n)^2(c+dx^n)}\,dx = -\frac{a^2}{b^2n(bc-ad)(a+bx^n)} - \frac{a(2bc-ad)\log(a+bx^n)}{b^2n(bc-ad)^2} + \frac{c^2\log(c+dx^n)}{dn(bc-ad)^2}$$

```
[In] Int[x^(-1 + 3*n)/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] -(a^2/(b^2*(b*c - a*d)*n*(a + b*x^n))) - (a*(2*b*c - a*d)*Log[a + b*x^n])/(
b^2*(b*c - a*d)^2*n) + (c^2*Log[c + d*x^n])/(d*(b*c - a*d)^2*n)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx)^2(c+dx)}\,dx, x, x^n\right)}{n} \\
&= \frac{\text{Subst}\left(\int \left(\frac{a^2}{b(bc-ad)(a+bx)^2} + \frac{a(-2bc+ad)}{b(bc-ad)^2(a+bx)} + \frac{c^2}{(bc-ad)^2(c+dx)}\right)dx, x, x^n\right)}{n} \\
&= -\frac{a^2}{b^2(bc-ad)n\,(a+bx^n)} - \frac{a(2bc-ad)\log(a+bx^n)}{b^2(bc-ad)^2n} + \frac{c^2\log(c+dx^n)}{d(bc-ad)^2n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.98

$$
\int \frac{x^{-1+3n}}{(a+bx^n)^2\,(c+dx^n)}\,dx = -\frac{a^2}{b^2(bc-ad)n\,(a+bx^n)} + \frac{a(-2bc+ad)\log(a+bx^n)}{b^2(bc-ad)^2n} + \frac{c^2\log(c+dx^n)}{d(-bc+ad)^2n}
$$

```
[In] Integrate[x^(-1 + 3*n)/((a + b*x^n)^2*(c + d*x^n)),x]
[Out] -(a^2/(b^2*(b*c - a*d)*n*(a + b*x^n))) + (a*(-2*b*c + a*d)*Log[a + b*x^n])/
(b^2*(b*c - a*d)^2*n) + (c^2*Log[c + d*x^n])/(d*(-(b*c) + a*d)^2*n)
```

Maple [A] (verified)

Time = 5.40 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.32

method	result
norman	$\frac{a^2}{(ad-bc)b^2n(a+b\,\mathrm{e}^{n\ln(x)})} + \frac{c^2\ln(c+d\,\mathrm{e}^{n\ln(x)})}{dn(a^2d^2-2abcd+b^2c^2)} + \frac{a(ad-2bc)\ln(a+b\,\mathrm{e}^{n\ln(x)})}{(a^2d^2-2abcd+b^2c^2)b^2n}$
risch	$\frac{\ln(x)}{b^2d} - \frac{\ln(x)a^2d}{(a^2d^2-2abcd+b^2c^2)b^2} + \frac{2\ln(x)ac}{(a^2d^2-2abcd+b^2c^2)b} - \frac{\ln(x)c^2}{d(a^2d^2-2abcd+b^2c^2)} + \frac{a^2}{(ad-bc)b^2n(a+b\,x^n)} + \frac{a^2\ln(x^n+\frac{a}{b}}{(a^2d^2-2abcd+b^2}$

```
[In] int(x^(-1+3*n)/(a+b*x^n)^2/(c+d*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] a^2/(a*d-b*c)/b^2/n/(a+b*exp(n*ln(x)))+c^2/d/n/(a^2*d^2-2*a*b*c*d+b^2*c^2)*
ln(c+d*exp(n*ln(x)))+a*(a*d-2*b*c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/n*ln(a+b
*exp(n*ln(x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.75

$$\int \frac{x^{-1+3n}}{(a+bx^n)^2(c+dx^n)}\,dx = -\frac{a^2bcd-a^3d^2+(2\,a^2bcd-a^3d^2+(2\,ab^2cd-a^2bd^2)x^n)\log(bx^n+a)-(b^3c^2x^n+ab^2c^2)\log(dx^n+c)}{(b^5c^2d-2\,ab^4cd^2+a^2b^3d^3)nx^n+(ab^4c^2d-2\,a^2b^3cd^2+a^3b^2d^3)n}$$

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] -(a^2*b*c*d - a^3*d^2 + (2*a^2*b*c*d - a^3*d^2 + (2*a*b^2*c*d - a^2*b*d^2)*
x^n)*log(b*x^n + a) - (b^3*c^2*x^n + a*b^2*c^2)*log(d*x^n + c))/((b^5*c^2*d
 - 2*a*b^4*c*d^2 + a^2*b^3*d^3)*n*x^n + (a*b^4*c^2*d - 2*a^2*b^3*c*d^2 + a^
3*b^2*d^3)*n)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1+3n}}{(a+bx^n)^2(c+dx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.55

$$\int \frac{x^{-1+3n}}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{c^2\log\left(\frac{dx^n+c}{d}\right)}{b^2c^2dn-2\,abcd^2n+a^2d^3n} - \frac{a^2}{ab^3cn-a^2b^2dn+(b^4cn-ab^3dn)x^n} - \frac{(2\,abc-a^2d)\log\left(\frac{bx^n+a}{b}\right)}{b^4c^2n-2\,ab^3cdn+a^2b^2d^2n}$$

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] c^2*log((d*x^n + c)/d)/(b^2*c^2*d*n - 2*a*b*c*d^2*n + a^2*d^3*n) - a^2/(a*b
^3*c*n - a^2*b^2*d*n + (b^4*c*n - a*b^3*d*n)*x^n) - (2*a*b*c - a^2*d)*log((
b*x^n + a)/b)/(b^4*c^2*n - 2*a*b^3*c*d*n + a^2*b^2*d^2*n)
```

Giac [F]

$$\int \frac{x^{-1+3n}}{(a+bx^n)^2(c+dx^n)}\,dx = \int \frac{x^{3n-1}}{(bx^n+a)^2(dx^n+c)}\,dx$$

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/((b*x^n + a)^2*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{(a+bx^n)^2(c+dx^n)}\,dx = \int \frac{x^{3n-1}}{(a+b\,x^n)^2\,(c+d\,x^n)}\,dx$$

```
[In] int(x^(3*n - 1)/((a + b*x^n)^2*(c + d*x^n)),x)
```

```
[Out] int(x^(3*n - 1)/((a + b*x^n)^2*(c + d*x^n)), x)
```

3.1050 $\int \frac{x^{-1+3n}}{(a+bx^n)^3(c+dx^n)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 120

$$\int \frac{x^{-1+3n}}{(a+bx^n)^3(c+dx^n)}\,dx = -\frac{a^2}{2b^2(bc-ad)n(a+bx^n)^2}+\frac{a(2bc-ad)}{b^2(bc-ad)^2n(a+bx^n)} + \frac{c^2\log(a+bx^n)}{(bc-ad)^3n}-\frac{c^2\log(c+dx^n)}{(bc-ad)^3n}$$

```
[Out] -1/2*a^2/b^2/(-a*d+b*c)/n/(a+b*x^n)^2+a*(-a*d+2*b*c)/b^2/(-a*d+b*c)^2/n/(a+
b*x^n)+c^2*ln(a+b*x^n)/(-a*d+b*c)^3/n-c^2*ln(c+d*x^n)/(-a*d+b*c)^3/n
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {457, 90}

$$\int \frac{x^{-1+3n}}{(a+bx^n)^3(c+dx^n)}\,dx = -\frac{a^2}{2b^2n(bc-ad)(a+bx^n)^2}+\frac{a(2bc-ad)}{b^2n(bc-ad)^2(a+bx^n)} + \frac{c^2\log(a+bx^n)}{n(bc-ad)^3}-\frac{c^2\log(c+dx^n)}{n(bc-ad)^3}$$

```
[In] Int[x^(-1 + 3*n)/((a + b*x^n)^3*(c + d*x^n)),x]
```

```
[Out] -1/2*a^2/(b^2*(b*c - a*d)*n*(a + b*x^n)^2) + (a*(2*b*c - a*d))/(b^2*(b*c -
a*d)^2*n*(a + b*x^n)) + (c^2*Log[a + b*x^n])/((b*c - a*d)^3*n) - (c^2*Log[c
 + d*x^n])/((b*c - a*d)^3*n)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx)^3(c+dx)}\,dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{b(bc-ad)(a+bx)^3} + \frac{a(-2bc+ad)}{b(bc-ad)^2(a+bx)^2} + \frac{bc^2}{(bc-ad)^3(a+bx)} - \frac{c^2d}{(bc-ad)^3(c+dx)}\right)\,dx, x, x^n\right)}{n}$$

$$= -\frac{a^2}{2b^2(bc-ad)n\,(a+bx^n)^2} + \frac{a(2bc-ad)}{b^2(bc-ad)^2n\,(a+bx^n)} + \frac{c^2\log(a+bx^n)}{(bc-ad)^3n} - \frac{c^2\log(c+dx^n)}{(bc-ad)^3n}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.78

$$\int \frac{x^{-1+3n}}{(a+bx^n)^3\,(c+dx^n)}\,dx = \frac{\frac{a(-bc+ad)\left(-3abc+a^2d-4b^2cx^n+2abdx^n\right)}{b^2(a+bx^n)^2} + 2c^2\log(a+bx^n) - 2c^2\log(c+dx^n)}{2(bc-ad)^3n}$$

```
[In] Integrate[x^(-1 + 3*n)/((a + b*x^n)^3*(c + d*x^n)),x]
```

```
[Out] ((a*(-(b*c) + a*d)*(-3*a*b*c + a^2*d - 4*b^2*c*x^n + 2*a*b*d*x^n))/(b^2*(a
+ b*x^n)^2) + 2*c^2*Log[a + b*x^n] - 2*c^2*Log[c + d*x^n])/(2*(b*c - a*d)^3
*n)
```

Maple [A] (verified)

Time = 6.23 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.41

method	result	siz
risch	$-\frac{a\left(2abd\,x^{n}-4b^{2}c\,x^{n}+a^{2}d-3abc\right)}{2n\,b^{2}(ad-bc)^{2}(a+b\,x^{n})^{2}}+\frac{c^{2}\ln\left(x^{n}+\frac{c}{d}\right)}{n(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3})}-\frac{c^{2}\ln\left(x^{n}+\frac{a}{b}\right)}{n(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3})}$	16
norman	$\frac{\frac{(-ad+2bc)a\,\mathrm{e}^{n\ln(x)}}{nb\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}+\frac{a^{2}(-ad+3bc)}{2\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)b^{2}n}}{\left(a+b\,\mathrm{e}^{n\ln(x)}\right)^{2}}+\frac{c^{2}\ln\left(c+d\,\mathrm{e}^{n\ln(x)}\right)}{n(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3})}-\frac{c^{2}\ln\left(a+b\,\mathrm{e}^{n\ln(x)}\right)}{n(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3})}$	21

```
[In] int(x^(-1+3*n)/(a+b*x^n)^3/(c+d*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*(2*a*b*d*x^n-4*b^2*c*x^n+a^2*d-3*a*b*c)/n/b^2/(a*d-b*c)^2/(a+b*x^n)^
2+c^2/n/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(x^n+c/d)-c^2/n/(a^
3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(x^n+a/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(118) = 236.

Time = 0.28 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.51

$$\int \frac{x^{-1+3n}}{(a+bx^n)^3(c+dx^n)}\,dx$$
$$= \frac{3\,a^2b^2c^2 - 4\,a^3bcd + a^4d^2 + 2\,(2\,ab^3c^2 - 3\,a^2b^2cd + a^3bd^2)x^n + 2\,(b^4c^2x^{2\,n} + 2\,ab^3c^2x^n + a^2b^2c^2)\log\,(bx^n + \cdots}{2\,((b^7c^3 - 3\,ab^6c^2d + 3\,a^2b^5cd^2 - a^3b^4d^3)nx^{2\,n} + 2\,(ab^6c^3 - 3\,a^2b^5c^2d + 3\,a^3b^4cd^2 - a^4b^3d^3)nx^n + (\cdots}$$

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*(3*a^2*b^2*c^2 - 4*a^3*b*c*d + a^4*d^2 + 2*(2*a*b^3*c^2 - 3*a^2*b^2*c*d
 + a^3*b*d^2)*x^n + 2*(b^4*c^2*x^(2*n) + 2*a*b^3*c^2*x^n + a^2*b^2*c^2)*log
(b*x^n + a) - 2*(b^4*c^2*x^(2*n) + 2*a*b^3*c^2*x^n + a^2*b^2*c^2)*log(d*x^n
 + c))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*n*x^(2*n)
 + 2*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*n*x^n +
(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*n)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1+3n}}{(a+bx^n)^3(c+dx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)**3/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(118) = 236$.

Time = 0.21 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.18

$$\int \frac{x^{-1+3n}}{(a+bx^n)^3 (c+dx^n)} \, dx = \frac{c^2 \log\left(\frac{bx^n+a}{b}\right)}{b^3c^3n - 3\,ab^2c^2dn + 3\,a^2bcd^2n - a^3d^3n} - \frac{c^2 \log\left(\frac{dx^n+c}{d}\right)}{b^3c^3n - 3\,ab^2c^2dn + 3\,a^2bcd^2n - a^3d^3n} + \frac{3\,a^2bc - a^3d + 2\,(2\,ab^2c - a^2bd)x^n}{2\,(a^2b^4c^2n - 2\,a^3b^3cdn + a^4b^2d^2n + (b^6c^2n - 2\,ab^5cdn + a^2b^4d^2n)x^{2\,n} + 2\,(ab^5c^2n - 2\,a^2b^4cdn + a^3b^3d^2n}$$

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] c^2*log((b*x^n + a)/b)/(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*d^2*n - a^3
*d^3*n) - c^2*log((d*x^n + c)/d)/(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*d
^2*n - a^3*d^3*n) + 1/2*(3*a^2*b*c - a^3*d + 2*(2*a*b^2*c - a^2*b*d)*x^n)/(
a^2*b^4*c^2*n - 2*a^3*b^3*c*d*n + a^4*b^2*d^2*n + (b^6*c^2*n - 2*a*b^5*c*d*
n + a^2*b^4*d^2*n)*x^(2*n) + 2*(a*b^5*c^2*n - 2*a^2*b^4*c*d*n + a^3*b^3*d^2
*n)*x^n)
```

Giac [F]

$$\int \frac{x^{-1+3n}}{(a+bx^n)^3 (c+dx^n)} \, dx = \int \frac{x^{3\,n-1}}{(bx^n+a)^3 (dx^n+c)} \, dx$$

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/((b*x^n + a)^3*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{(a+bx^n)^3 (c+dx^n)} \, dx = \int \frac{x^{3\,n-1}}{(a+b\,x^n)^3\,(c+d\,x^n)} \, dx$$

```
[In] int(x^(3*n - 1)/((a + b*x^n)^3*(c + d*x^n)),x)
```

```
[Out] int(x^(3*n - 1)/((a + b*x^n)^3*(c + d*x^n)), x)
```

3.1051 $\int x^{13}(b+cx)^{13}(b+2cx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 14

$$\int x^{13}(b+cx)^{13}(b+2cx)\,dx=\frac{1}{14}x^{14}(b+cx)^{14}$$

```
[Out] 1/14*x^14*(c*x+b)^14
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {75}

$$\int x^{13}(b+cx)^{13}(b+2cx)\,dx=\frac{1}{14}x^{14}(b+cx)^{14}$$

```
[In] Int[x^13*(b + c*x)^13*(b + 2*c*x),x]
[Out] (x^14*(b + c*x)^14)/14
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\text{integral}=\frac{1}{14}x^{14}(b+cx)^{14}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(14) = 28.

Time = 0.01 (sec) , antiderivative size = 172, normalized size of antiderivative = 12.29

$$\int x^{13}(b+cx)^{13}(b+2cx)\,dx=\frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13}{2}b^{12}c^2x^{16}+26b^{11}c^3x^{17}+\frac{143}{2}b^{10}c^4x^{18}+143b^9c^5x^{19}+\frac{429}{2}b^8c^6x^{20}+\frac{1716}{7}b^7c^7x^{21}+\frac{429}{2}b^6c^8x^{22}+143b^5c^9x^{23}+\frac{143}{2}b^4c^{10}x^{24}+26b^3c^{11}x^{25}+\frac{13}{2}b^2c^{12}x^{26}+bc^{13}x^{27}+\frac{c^{14}x^{28}}{14}$$

```
[In] Integrate[x^13*(b + c*x)^13*(b + 2*c*x),x]
```

```
[Out] (b^14*x^14)/14 + b^13*c*x^15 + (13*b^12*c^2*x^16)/2 + 26*b^11*c^3*x^17 + (1
43*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (429*b^8*c^6*x^20)/2 + (1716*b^7*c
^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*c^9*x^23 + (143*b^4*c^10*x^24)/
2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2 + b*c^13*x^27 + (c^14*x^28)/14
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(12) = 24.

Time = 4.54 (sec) , antiderivative size = 155, normalized size of antiderivative = 11.07

method	result
gosper	$143b^5c^9x^{23}+\frac{143}{2}b^4c^{10}x^{24}+26b^3c^{11}x^{25}+\frac{13}{2}b^2c^{12}x^{26}+b\,c^{13}x^{27}+\frac{1}{14}c^{14}x^{28}+\frac{1}{14}b^{14}x^{14}+b^{13}c\,x$
default	$143b^5c^9x^{23}+\frac{143}{2}b^4c^{10}x^{24}+26b^3c^{11}x^{25}+\frac{13}{2}b^2c^{12}x^{26}+b\,c^{13}x^{27}+\frac{1}{14}c^{14}x^{28}+\frac{1}{14}b^{14}x^{14}+b^{13}c\,x$
norman	$143b^5c^9x^{23}+\frac{143}{2}b^4c^{10}x^{24}+26b^3c^{11}x^{25}+\frac{13}{2}b^2c^{12}x^{26}+b\,c^{13}x^{27}+\frac{1}{14}c^{14}x^{28}+\frac{1}{14}b^{14}x^{14}+b^{13}c\,x$
risch	$143b^5c^9x^{23}+\frac{143}{2}b^4c^{10}x^{24}+26b^3c^{11}x^{25}+\frac{13}{2}b^2c^{12}x^{26}+b\,c^{13}x^{27}+\frac{1}{14}c^{14}x^{28}+\frac{1}{14}b^{14}x^{14}+b^{13}c\,x$
parallelrisch	$143b^5c^9x^{23}+\frac{143}{2}b^4c^{10}x^{24}+26b^3c^{11}x^{25}+\frac{13}{2}b^2c^{12}x^{26}+b\,c^{13}x^{27}+\frac{1}{14}c^{14}x^{28}+\frac{1}{14}b^{14}x^{14}+b^{13}c\,x$

```
[In] int(x^13*(c*x+b)^13*(2*c*x+b),x,method=_RETURNVERBOSE)
```

```
[Out] 143*b^5*c^9*x^23+143/2*b^4*c^10*x^24+26*b^3*c^11*x^25+13/2*b^2*c^12*x^26+b*
c^13*x^27+1/14*c^14*x^28+1/14*b^14*x^14+b^13*c*x^15+13/2*b^12*c^2*x^16+26*b
^11*c^3*x^17+143/2*b^10*c^4*x^18+143*b^9*c^5*x^19+429/2*b^8*c^6*x^20+1716/7
*b^7*c^7*x^21+429/2*b^6*c^8*x^22
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(12) = 24.

Time = 0.27 (sec) , antiderivative size = 154, normalized size of antiderivative = 11.00

$$\int x^{13}(b+cx)^{13}(b+2cx)\,dx = \frac{1}{14}\,c^{14}x^{28}+bc^{13}x^{27}+\frac{13}{2}\,b^{2}c^{12}x^{26}+26\,b^{3}c^{11}x^{25}+\frac{143}{2}\,b^{4}c^{10}x^{24}+143\,b^{5}c^{9}x^{23}+\frac{429}{2}\,b^{6}c^{8}x^{22}+\frac{1716}{7}\,b^{7}c^{7}x^{21}+\frac{429}{2}\,b^{8}c^{6}x^{20}+143\,b^{9}c^{5}x^{19}+\frac{143}{2}\,b^{10}c^{4}x^{18}+26\,b^{11}c^{3}x^{17}+\frac{13}{2}\,b^{12}c^{2}x^{16}+b^{13}cx^{15}+\frac{1}{14}\,b^{14}x^{14}$$

```
[In] integrate(x^13*(c*x+b)^13*(2*c*x+b),x, algorithm="fricas")
```

```
[Out] 1/14*c^14*x^28 + b*c^13*x^27 + 13/2*b^2*c^12*x^26 + 26*b^3*c^11*x^25 + 143/
2*b^4*c^10*x^24 + 143*b^5*c^9*x^23 + 429/2*b^6*c^8*x^22 + 1716/7*b^7*c^7*x^
21 + 429/2*b^8*c^6*x^20 + 143*b^9*c^5*x^19 + 143/2*b^10*c^4*x^18 + 26*b^11*
c^3*x^17 + 13/2*b^12*c^2*x^16 + b^13*c*x^15 + 1/14*b^14*x^14
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(10) = 20.

Time = 0.04 (sec) , antiderivative size = 175, normalized size of antiderivative = 12.50

$$\int x^{13}(b+cx)^{13}(b+2cx)\,dx = \frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13b^{12}c^{2}x^{16}}{2}+26b^{11}c^{3}x^{17}+\frac{143b^{10}c^{4}x^{18}}{2}+143b^{9}c^{5}x^{19}+\frac{429b^{8}c^{6}x^{20}}{2}+\frac{1716b^{7}c^{7}x^{21}}{7}+\frac{429b^{6}c^{8}x^{22}}{2}+143b^{5}c^{9}x^{23}+\frac{143b^{4}c^{10}x^{24}}{2}+26b^{3}c^{11}x^{25}+\frac{13b^{2}c^{12}x^{26}}{2}+bc^{13}x^{27}+\frac{c^{14}x^{28}}{14}$$

```
[In] integrate(x**13*(c*x+b)**13*(2*c*x+b),x)
```

```
[Out] b**14*x**14/14 + b**13*c*x**15 + 13*b**12*c**2*x**16/2 + 26*b**11*c**3*x**1
7 + 143*b**10*c**4*x**18/2 + 143*b**9*c**5*x**19 + 429*b**8*c**6*x**20/2 +
1716*b**7*c**7*x**21/7 + 429*b**6*c**8*x**22/2 + 143*b**5*c**9*x**23 + 143*
b**4*c**10*x**24/2 + 26*b**3*c**11*x**25 + 13*b**2*c**12*x**26/2 + b*c**13*
x**27 + c**14*x**28/14
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(12) = 24$.

Time = 0.19 (sec) , antiderivative size = 154, normalized size of antiderivative = 11.00

$$\int x^{13}(b+cx)^{13}(b+2cx)\,dx = \frac{1}{14}\,c^{14}x^{28} + bc^{13}x^{27} + \frac{13}{2}\,b^2c^{12}x^{26} + 26\,b^3c^{11}x^{25} + \frac{143}{2}\,b^4c^{10}x^{24} + 143\,b^5c^9x^{23} + \frac{429}{2}\,b^6c^8x^{22} + \frac{1716}{7}\,b^7c^7x^{21} + \frac{429}{2}\,b^8c^6x^{20} + 143\,b^9c^5x^{19} + \frac{143}{2}\,b^{10}c^4x^{18} + 26\,b^{11}c^3x^{17} + \frac{13}{2}\,b^{12}c^2x^{16} + b^{13}cx^{15} + \frac{1}{14}\,b^{14}x^{14}$$

```
[In] integrate(x^13*(c*x+b)^13*(2*c*x+b),x, algorithm="maxima")
```

```
[Out] 1/14*c^14*x^28 + b*c^13*x^27 + 13/2*b^2*c^12*x^26 + 26*b^3*c^11*x^25 + 143/
2*b^4*c^10*x^24 + 143*b^5*c^9*x^23 + 429/2*b^6*c^8*x^22 + 1716/7*b^7*c^7*x^
21 + 429/2*b^8*c^6*x^20 + 143*b^9*c^5*x^19 + 143/2*b^10*c^4*x^18 + 26*b^11*
c^3*x^17 + 13/2*b^12*c^2*x^16 + b^13*c*x^15 + 1/14*b^14*x^14
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int x^{13}(b+cx)^{13}(b+2cx)\,dx = \frac{1}{14}\left(cx^2+bx\right)^{14}$$

```
[In] integrate(x^13*(c*x+b)^13*(2*c*x+b),x, algorithm="giac")
```

```
[Out] 1/14*(c*x^2 + b*x)^14
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 154, normalized size of antiderivative = 11.00

$$\int x^{13}(b+cx)^{13}(b+2cx)\,dx = \frac{b^{14}\,x^{14}}{14} + b^{13}\,c\,x^{15} + \frac{13\,b^{12}\,c^2\,x^{16}}{2} + 26\,b^{11}\,c^3\,x^{17} + \frac{143\,b^{10}\,c^4\,x^{18}}{2} + 143\,b^9\,c^5\,x^{19} + \frac{429\,b^8\,c^6\,x^{20}}{2} + \frac{1716\,b^7\,c^7\,x^{21}}{7} + \frac{429\,b^6\,c^8\,x^{22}}{2} + 143\,b^5\,c^9\,x^{23} + \frac{143\,b^4\,c^{10}\,x^{24}}{2} + 26\,b^3\,c^{11}\,x^{25} + \frac{13\,b^2\,c^{12}\,x^{26}}{2} + b\,c^{13}\,x^{27} + \frac{c^{14}\,x^{28}}{14}$$

```
[In] int(x^13*(b + c*x)^13*(b + 2*c*x),x)
```

```
[Out] (b^14*x^14)/14 + (c^14*x^28)/14 + b^13*c*x^15 + b*c^13*x^27 + (13*b^12*c^2*
x^16)/2 + 26*b^11*c^3*x^17 + (143*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (42
9*b^8*c^6*x^20)/2 + (1716*b^7*c^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*
c^9*x^23 + (143*b^4*c^10*x^24)/2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2
```

3.1052 $\int x^{27}(b+cx^2)^{13}(b+2cx^2)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 16

$$\int x^{27}\left(b+cx^2\right)^{13}\left(b+2cx^2\right)\,dx=\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

```
[Out] 1/28*x^28*(c*x^2+b)^14
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {457, 75}

$$\int x^{27}\left(b+cx^2\right)^{13}\left(b+2cx^2\right)\,dx=\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

```
[In] Int[x^27*(b + c*x^2)^13*(b + 2*c*x^2),x]
```

```
[Out] (x^28*(b + c*x^2)^14)/28
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx, x, x^2\right)$$
$$= \frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 182 vs. 2(16) = 32.

Time = 0.01 (sec) , antiderivative size = 182, normalized size of antiderivative = 11.38

$$\int x^{27}\left(b+cx^2\right)^{13}\left(b+2cx^2\right)\,dx = \frac{b^{14}x^{28}}{28} + \frac{1}{2}b^{13}cx^{30} + \frac{13}{4}b^{12}c^2x^{32} + 13b^{11}c^3x^{34} + \frac{143}{4}b^{10}c^4x^{36} + \frac{143}{2}b^9c^5x^{38} + \frac{429}{4}b^8c^6x^{40} + \frac{858}{7}b^7c^7x^{42} + \frac{429}{4}b^6c^8x^{44} + \frac{143}{2}b^5c^9x^{46} + \frac{143}{4}b^4c^{10}x^{48} + 13b^3c^{11}x^{50} + \frac{13}{4}b^2c^{12}x^{52} + \frac{1}{2}bc^{13}x^{54} + \frac{c^{14}x^{56}}{28}$$

```
[In] Integrate[x^27*(b + c*x^2)^13*(b + 2*c*x^2),x]
```

```
[Out] (b^14*x^28)/28 + (b^13*c*x^30)/2 + (13*b^12*c^2*x^32)/4 + 13*b^11*c^3*x^34
+ (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x^38)/2 + (429*b^8*c^6*x^40)/4 + (85
8*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4 + (143*b^5*c^9*x^46)/2 + (143*b^4*
c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^2*c^12*x^52)/4 + (b*c^13*x^54)/2 +
(c^14*x^56)/28
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(14) = 28.

Time = 4.74 (sec) , antiderivative size = 157, normalized size of antiderivative = 9.81

method	result
gosper	$\frac{1}{28}b^{14}x^{28}+\frac{1}{2}b^{13}c\,x^{30}+\frac{13}{4}b^{12}c^2x^{32}+13b^{11}c^3x^{34}+\frac{143}{4}b^{10}c^4x^{36}+\frac{143}{2}b^9c^5x^{38}+\frac{429}{4}b^8c^6x^{40}+\frac{858}{7}b$
default	$\frac{1}{28}b^{14}x^{28}+\frac{1}{2}b^{13}c\,x^{30}+\frac{13}{4}b^{12}c^2x^{32}+13b^{11}c^3x^{34}+\frac{143}{4}b^{10}c^4x^{36}+\frac{143}{2}b^9c^5x^{38}+\frac{429}{4}b^8c^6x^{40}+\frac{858}{7}b$
risch	$\frac{1}{28}b^{14}x^{28}+\frac{1}{2}b^{13}c\,x^{30}+\frac{13}{4}b^{12}c^2x^{32}+13b^{11}c^3x^{34}+\frac{143}{4}b^{10}c^4x^{36}+\frac{143}{2}b^9c^5x^{38}+\frac{429}{4}b^8c^6x^{40}+\frac{858}{7}b$
parallelrisch	$\frac{1}{28}b^{14}x^{28}+\frac{1}{2}b^{13}c\,x^{30}+\frac{13}{4}b^{12}c^2x^{32}+13b^{11}c^3x^{34}+\frac{143}{4}b^{10}c^4x^{36}+\frac{143}{2}b^9c^5x^{38}+\frac{429}{4}b^8c^6x^{40}+\frac{858}{7}b$

```
[In] int(x^27*(c*x^2+b)^13*(2*c*x^2+b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/28*b^14*x^28+1/2*b^13*c*x^30+13/4*b^12*c^2*x^32+13*b^11*c^3*x^34+143/4*b^
10*c^4*x^36+143/2*b^9*c^5*x^38+429/4*b^8*c^6*x^40+858/7*b^7*c^7*x^42+429/4*
b^6*c^8*x^44+143/2*b^5*c^9*x^46+143/4*b^4*c^10*x^48+13*b^3*c^11*x^50+13/4*b
^2*c^12*x^52+1/2*b*c^13*x^54+1/28*c^14*x^56
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.27 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{27}\left(b+cx^2\right)^{13}\left(b+2cx^2\right)\,dx = \frac{1}{28}\,c^{14}x^{56}+\frac{1}{2}\,bc^{13}x^{54}+\frac{13}{4}\,b^2c^{12}x^{52}+13\,b^3c^{11}x^{50}+\frac{143}{4}\,b^4c^{10}x^{48}+\frac{143}{2}\,b^5c^9x^{46}+\frac{429}{4}\,b^6c^8x^{44}+\frac{858}{7}\,b^7c^7x^{42}+\frac{429}{4}\,b^8c^6x^{40}+\frac{143}{2}\,b^9c^5x^{38}+\frac{143}{4}\,b^{10}c^4x^{36}+13\,b^{11}c^3x^{34}+\frac{13}{4}\,b^{12}c^2x^{32}+\frac{1}{2}\,b^{13}cx^{30}+\frac{1}{28}\,b^{14}x^{28}$$

```
[In] integrate(x^27*(c*x^2+b)^13*(2*c*x^2+b),x, algorithm="fricas")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7*c
^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36 + 1
3*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(12) = 24$.

Time = 0.04 (sec) , antiderivative size = 182, normalized size of antiderivative = 11.38

$$\int x^{27}\left(b+cx^2\right)^{13}\left(b+2cx^2\right)\,dx = \frac{b^{14}x^{28}}{28}+\frac{b^{13}cx^{30}}{2}+\frac{13b^{12}c^2x^{32}}{4}+13b^{11}c^3x^{34}+\frac{143b^{10}c^4x^{36}}{4}+\frac{143b^9c^5x^{38}}{2}+\frac{429b^8c^6x^{40}}{4}+\frac{858b^7c^7x^{42}}{7}+\frac{429b^6c^8x^{44}}{4}+\frac{143b^5c^9x^{46}}{2}+\frac{143b^4c^{10}x^{48}}{4}+13b^3c^{11}x^{50}+\frac{13b^2c^{12}x^{52}}{4}+\frac{bc^{13}x^{54}}{2}+\frac{c^{14}x^{56}}{28}$$

```
[In] integrate(x**27*(c*x**2+b)**13*(2*c*x**2+b),x)
```

```
[Out] b**14*x**28/28 + b**13*c*x**30/2 + 13*b**12*c**2*x**32/4 + 13*b**11*c**3*x*
*34 + 143*b**10*c**4*x**36/4 + 143*b**9*c**5*x**38/2 + 429*b**8*c**6*x**40/
```

```
4 + 858*b**7*c**7*x**42/7 + 429*b**6*c**8*x**44/4 + 143*b**5*c**9*x**46/2 +
 143*b**4*c**10*x**48/4 + 13*b**3*c**11*x**50 + 13*b**2*c**12*x**52/4 + b*c
**13*x**54/2 + c**14*x**56/28
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.20 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{27}(b+cx^2)^{13}(b+2cx^2)\,dx = \frac{1}{28}c^{14}x^{56} + \frac{1}{2}bc^{13}x^{54} + \frac{13}{4}b^2c^{12}x^{52} + 13\,b^3c^{11}x^{50} + \frac{143}{4}b^4c^{10}x^{48} + \frac{143}{2}b^5c^9x^{46} + \frac{429}{4}b^6c^8x^{44} + \frac{858}{7}b^7c^7x^{42} + \frac{429}{4}b^8c^6x^{40} + \frac{143}{2}b^9c^5x^{38} + \frac{143}{4}b^{10}c^4x^{36} + 13\,b^{11}c^3x^{34} + \frac{13}{4}b^{12}c^2x^{32} + \frac{1}{2}b^{13}cx^{30} + \frac{1}{28}b^{14}x^{28}$$

```
[In] integrate(x^27*(c*x^2+b)^13*(2*c*x^2+b),x, algorithm="maxima")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7*c
^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36 + 1
3*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{27}(b+cx^2)^{13}(b+2cx^2)\,dx = \frac{1}{28}c^{14}x^{56} + \frac{1}{2}bc^{13}x^{54} + \frac{13}{4}b^2c^{12}x^{52} + 13\,b^3c^{11}x^{50} + \frac{143}{4}b^4c^{10}x^{48} + \frac{143}{2}b^5c^9x^{46} + \frac{429}{4}b^6c^8x^{44} + \frac{858}{7}b^7c^7x^{42} + \frac{429}{4}b^8c^6x^{40} + \frac{143}{2}b^9c^5x^{38} + \frac{143}{4}b^{10}c^4x^{36} + 13\,b^{11}c^3x^{34} + \frac{13}{4}b^{12}c^2x^{32} + \frac{1}{2}b^{13}cx^{30} + \frac{1}{28}b^{14}x^{28}$$

```
[In] integrate(x^27*(c*x^2+b)^13*(2*c*x^2+b),x, algorithm="giac")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7*c
^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36 + 1
3*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{27}(b+cx^2)^{13}\left(b+2cx^2\right)\,dx=\frac{b^{14}\,x^{28}}{28}+\frac{b^{13}\,c\,x^{30}}{2}+\frac{13\,b^{12}\,c^2\,x^{32}}{4}+13\,b^{11}\,c^3\,x^{34}+\frac{143\,b^{10}\,c^4\,x^{36}}{4}+\frac{143\,b^9\,c^5\,x^{38}}{2}+\frac{429\,b^8\,c^6\,x^{40}}{4}+\frac{858\,b^7\,c^7\,x^{42}}{7}+\frac{429\,b^6\,c^8\,x^{44}}{4}+\frac{143\,b^5\,c^9\,x^{46}}{2}+\frac{143\,b^4\,c^{10}\,x^{48}}{4}+13\,b^3\,c^{11}\,x^{50}+\frac{13\,b^2\,c^{12}\,x^{52}}{4}+\frac{b\,c^{13}\,x^{54}}{2}+\frac{c^{14}\,x^{56}}{28}$$

```
[In] int(x^27*(b + c*x^2)^13*(b + 2*c*x^2),x)
```

```
[Out] (b^14*x^28)/28 + (c^14*x^56)/28 + (b^13*c*x^30)/2 + (b*c^13*x^54)/2 + (13*b
^12*c^2*x^32)/4 + 13*b^11*c^3*x^34 + (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x
^38)/2 + (429*b^8*c^6*x^40)/4 + (858*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4
 + (143*b^5*c^9*x^46)/2 + (143*b^4*c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^
2*c^12*x^52)/4
```

3.1053 $\int x^{41}(b+cx^3)^{13}(b+2cx^3)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 16

$$\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx=\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

```
[Out] 1/42*x^42*(c*x^3+b)^14
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {457, 75}

$$\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx=\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

```
[In] Int[x^41*(b + c*x^3)^13*(b + 2*c*x^3),x]
```

```
[Out] (x^42*(b + c*x^3)^14)/42
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx, x, x^3\right)$$
$$= \frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 186 vs. $2(16) = 32$.

Time = 0.01 (sec) , antiderivative size = 186, normalized size of antiderivative = 11.62

$$\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx = \frac{b^{14}x^{42}}{42}+\frac{1}{3}b^{13}cx^{45}+\frac{13}{6}b^{12}c^2x^{48}+\frac{26}{3}b^{11}c^3x^{51}+\frac{143}{6}b^{10}c^4x^{54}+\frac{143}{3}b^9c^5x^{57}+\frac{143}{2}b^8c^6x^{60}+\frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^6c^8x^{66}+\frac{143}{3}b^5c^9x^{69}+\frac{143}{6}b^4c^{10}x^{72}+\frac{26}{3}b^3c^{11}x^{75}+\frac{13}{6}b^2c^{12}x^{78}+\frac{1}{3}bc^{13}x^{81}+\frac{c^{14}x^{84}}{42}$$

```
[In] Integrate[x^41*(b + c*x^3)^13*(b + 2*c*x^3),x]
[Out] (b^14*x^42)/42 + (b^13*c*x^45)/3 + (13*b^12*c^2*x^48)/6 + (26*b^11*c^3*x^51
)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c^5*x^57)/3 + (143*b^8*c^6*x^60)/2 +
 (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^66)/2 + (143*b^5*c^9*x^69)/3 + (143*
b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3 + (13*b^2*c^12*x^78)/6 + (b*c^13*x^
81)/3 + (c^14*x^84)/42
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 4.66 (sec) , antiderivative size = 157, normalized size of antiderivative = 9.81

method	result
gosper	$\frac{1}{42}b^{14}x^{42}+\frac{1}{3}b^{13}c\,x^{45}+\frac{13}{6}b^{12}c^2x^{48}+\frac{26}{3}b^{11}c^3x^{51}+\frac{143}{6}b^{10}c^4x^{54}+\frac{143}{3}b^9c^5x^{57}+\frac{13}{6}b^2c^{12}x^{78}+\frac{1}{3}b$
default	$\frac{1}{42}b^{14}x^{42}+\frac{1}{3}b^{13}c\,x^{45}+\frac{13}{6}b^{12}c^2x^{48}+\frac{26}{3}b^{11}c^3x^{51}+\frac{143}{6}b^{10}c^4x^{54}+\frac{143}{3}b^9c^5x^{57}+\frac{13}{6}b^2c^{12}x^{78}+\frac{1}{3}b$
risch	$\frac{1}{42}b^{14}x^{42}+\frac{1}{3}b^{13}c\,x^{45}+\frac{13}{6}b^{12}c^2x^{48}+\frac{26}{3}b^{11}c^3x^{51}+\frac{143}{6}b^{10}c^4x^{54}+\frac{143}{3}b^9c^5x^{57}+\frac{13}{6}b^2c^{12}x^{78}+\frac{1}{3}b$
parallelrisch	$\frac{1}{42}b^{14}x^{42}+\frac{1}{3}b^{13}c\,x^{45}+\frac{13}{6}b^{12}c^2x^{48}+\frac{26}{3}b^{11}c^3x^{51}+\frac{143}{6}b^{10}c^4x^{54}+\frac{143}{3}b^9c^5x^{57}+\frac{13}{6}b^2c^{12}x^{78}+\frac{1}{3}b$

```
[In] int(x^41*(c*x^3+b)^13*(2*c*x^3+b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/42*b^14*x^42+1/3*b^13*c*x^45+13/6*b^12*c^2*x^48+26/3*b^11*c^3*x^51+143/6*
b^10*c^4*x^54+143/3*b^9*c^5*x^57+13/6*b^2*c^12*x^78+1/3*b*c^13*x^81+1/42*c^
14*x^84+143/2*b^8*c^6*x^60+572/7*b^7*c^7*x^63+143/2*b^6*c^8*x^66+143/3*b^5*
c^9*x^69+143/6*b^4*c^10*x^72+26/3*b^3*c^11*x^75
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx = \frac{1}{42}\,c^{14}x^{84}+\frac{1}{3}\,bc^{13}x^{81}+\frac{13}{6}\,b^2c^{12}x^{78}+\frac{26}{3}\,b^3c^{11}x^{75}+\frac{143}{6}\,b^4c^{10}x^{72}+\frac{143}{3}\,b^5c^9x^{69}+\frac{143}{2}\,b^6c^8x^{66}+\frac{572}{7}\,b^7c^7x^{63}+\frac{143}{2}\,b^8c^6x^{60}+\frac{143}{3}\,b^9c^5x^{57}+\frac{143}{6}\,b^{10}c^4x^{54}+\frac{26}{3}\,b^{11}c^3x^{51}+\frac{13}{6}\,b^{12}c^2x^{48}+\frac{1}{3}\,b^{13}cx^{45}+\frac{1}{42}\,b^{14}x^{42}$$

```
[In] integrate(x^41*(c*x^3+b)^13*(2*c*x^3+b),x, algorithm="fricas")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
+ 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b^7
*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^54 +
 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(12) = 24$.

Time = 0.05 (sec) , antiderivative size = 185, normalized size of antiderivative = 11.56

$$\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx = \frac{b^{14}x^{42}}{42}+\frac{b^{13}cx^{45}}{3}+\frac{13b^{12}c^2x^{48}}{6}+\frac{26b^{11}c^3x^{51}}{3}+\frac{143b^{10}c^4x^{54}}{6}+\frac{143b^9c^5x^{57}}{3}+\frac{143b^8c^6x^{60}}{2}+\frac{572b^7c^7x^{63}}{7}+\frac{143b^6c^8x^{66}}{2}+\frac{143b^5c^9x^{69}}{3}+\frac{143b^4c^{10}x^{72}}{6}+\frac{26b^3c^{11}x^{75}}{3}+\frac{13b^2c^{12}x^{78}}{6}+\frac{bc^{13}x^{81}}{3}+\frac{c^{14}x^{84}}{42}$$

```
[In] integrate(x**41*(c*x**3+b)**13*(2*c*x**3+b),x)
```

```
[Out] b**14*x**42/42 + b**13*c*x**45/3 + 13*b**12*c**2*x**48/6 + 26*b**11*c**3*x*
*51/3 + 143*b**10*c**4*x**54/6 + 143*b**9*c**5*x**57/3 + 143*b**8*c**6*x**6
```

```
0/2 + 572*b**7*c**7*x**63/7 + 143*b**6*c**8*x**66/2 + 143*b**5*c**9*x**69/3
 + 143*b**4*c**10*x**72/6 + 26*b**3*c**11*x**75/3 + 13*b**2*c**12*x**78/6 +
 b*c**13*x**81/3 + c**14*x**84/42
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.20 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{41}(b+cx^3)^{13}(b+2cx^3)\,dx = \frac{1}{42}c^{14}x^{84} + \frac{1}{3}bc^{13}x^{81} + \frac{13}{6}b^2c^{12}x^{78} + \frac{26}{3}b^3c^{11}x^{75} + \frac{143}{6}b^4c^{10}x^{72} + \frac{143}{3}b^5c^9x^{69} + \frac{143}{2}b^6c^8x^{66} + \frac{572}{7}b^7c^7x^{63} + \frac{143}{2}b^8c^6x^{60} + \frac{143}{3}b^9c^5x^{57} + \frac{143}{6}b^{10}c^4x^{54} + \frac{26}{3}b^{11}c^3x^{51} + \frac{13}{6}b^{12}c^2x^{48} + \frac{1}{3}b^{13}cx^{45} + \frac{1}{42}b^{14}x^{42}$$

```
[In] integrate(x^41*(c*x^3+b)^13*(2*c*x^3+b),x, algorithm="maxima")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
+ 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b^7
*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^54 +
 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.29 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{41}(b+cx^3)^{13}(b+2cx^3)\,dx = \frac{1}{42}c^{14}x^{84} + \frac{1}{3}bc^{13}x^{81} + \frac{13}{6}b^2c^{12}x^{78} + \frac{26}{3}b^3c^{11}x^{75} + \frac{143}{6}b^4c^{10}x^{72} + \frac{143}{3}b^5c^9x^{69} + \frac{143}{2}b^6c^8x^{66} + \frac{572}{7}b^7c^7x^{63} + \frac{143}{2}b^8c^6x^{60} + \frac{143}{3}b^9c^5x^{57} + \frac{143}{6}b^{10}c^4x^{54} + \frac{26}{3}b^{11}c^3x^{51} + \frac{13}{6}b^{12}c^2x^{48} + \frac{1}{3}b^{13}cx^{45} + \frac{1}{42}b^{14}x^{42}$$

```
[In] integrate(x^41*(c*x^3+b)^13*(2*c*x^3+b),x, algorithm="giac")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
+ 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b^7
*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^54 +
 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx=\frac{b^{14}\,x^{42}}{42}+\frac{b^{13}\,c\,x^{45}}{3}+\frac{13\,b^{12}\,c^2\,x^{48}}{6}+\frac{26\,b^{11}\,c^3\,x^{51}}{3}+\frac{143\,b^{10}\,c^4\,x^{54}}{6}+\frac{143\,b^9\,c^5\,x^{57}}{3}+\frac{143\,b^8\,c^6\,x^{60}}{2}+\frac{572\,b^7\,c^7\,x^{63}}{7}+\frac{143\,b^6\,c^8\,x^{66}}{2}+\frac{143\,b^5\,c^9\,x^{69}}{3}+\frac{143\,b^4\,c^{10}\,x^{72}}{6}+\frac{26\,b^3\,c^{11}\,x^{75}}{3}+\frac{13\,b^2\,c^{12}\,x^{78}}{6}+\frac{b\,c^{13}\,x^{81}}{3}+\frac{c^{14}\,x^{84}}{42}$$

```
[In] int(x^41*(b + c*x^3)^13*(b + 2*c*x^3),x)
```

```
[Out] (b^14*x^42)/42 + (c^14*x^84)/42 + (b^13*c*x^45)/3 + (b*c^13*x^81)/3 + (13*b
^12*c^2*x^48)/6 + (26*b^11*c^3*x^51)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c
^5*x^57)/3 + (143*b^8*c^6*x^60)/2 + (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^6
6)/2 + (143*b^5*c^9*x^69)/3 + (143*b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3
+ (13*b^2*c^12*x^78)/6
```

3.1054 $\int x^{-1+14n}(b+cx^n)^{13}(b+2cx^n)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 21

$$\int x^{-1+14n}(b+cx^n)^{13}(b+2cx^n)\,dx=\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

```
[Out] 1/14*x^(14*n)*(b+c*x^n)^14/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {457, 75}

$$\int x^{-1+14n}(b+cx^n)^{13}(b+2cx^n)\,dx=\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

```
[In] Int[x^(-1 + 14*n)*(b + c*x^n)^13*(b + 2*c*x^n),x]
```

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx, x, x^n\right)}{n} \\ &= \frac{x^{14n}(b+cx^n)^{14}}{14n} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x^{-1+14n}(b+cx^n)^{13}\left(b+2cx^n\right)\,dx = \frac{x^{14n}(b+cx^n)^{14}}{14n}$$

```
[In] Integrate[x^(-1 + 14*n)*(b + c*x^n)^13*(b + 2*c*x^n),x]
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(19) = 38.

Time = 183.42 (sec) , antiderivative size = 230, normalized size of antiderivative = 10.95

method	result
risch	$\frac{c^{14}x^{28n}}{14n}+\frac{b\,c^{13}x^{27n}}{n}+\frac{13b^{2}c^{12}x^{26n}}{2n}+\frac{26b^{3}c^{11}x^{25n}}{n}+\frac{143b^{4}c^{10}x^{24n}}{2n}+\frac{143b^{5}c^{9}x^{23n}}{n}+\frac{429b^{6}c^{8}x^{22n}}{2n}+\frac{1716b^{7}c^{7}x^{21n}}{7n}$
parallelrisch	$c^{14}x^{-1+14n}x^{14n}x+14b\,c^{13}x^{-1+14n}x^{13n}x+91b^{2}c^{12}x^{-1+14n}x^{12n}x+364b^{3}c^{11}x^{-1+14n}x^{11n}x+1001b^{4}c^{10}x^{-1+14n}x^{10n}x+2002b^{5}$

```
[In] int(x^(-1+14*n)*(b+c*x^n)^13*(b+2*c*x^n),x,method=_RETURNVERBOSE)
[Out] 1/14*c^14/n*(x^n)^28+b*c^13/n*(x^n)^27+13/2*b^2*c^12/n*(x^n)^26+26*b^3*c^11
/n*(x^n)^25+143/2*b^4*c^10/n*(x^n)^24+143*b^5*c^9/n*(x^n)^23+429/2*b^6*c^8/
n*(x^n)^22+1716/7*b^7*c^7/n*(x^n)^21+429/2*b^8*c^6/n*(x^n)^20+143*b^9*c^5/n
*(x^n)^19+143/2*b^10*c^4/n*(x^n)^18+26*b^11*c^3/n*(x^n)^17+13/2*b^12*c^2/n*
(x^n)^16+b^13*c/n*(x^n)^15+1/14*b^14/n*(x^n)^14
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(19) = 38$.

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 9.00

$$\int x^{-1+14n}(b+cx^n)^{13}(b+2cx^n)\,dx$$
$$= \frac{c^{14}x^{28\,n} + 14\,bc^{13}x^{27\,n} + 91\,b^2c^{12}x^{26\,n} + 364\,b^3c^{11}x^{25\,n} + 1001\,b^4c^{10}x^{24\,n} + 2002\,b^5c^9x^{23\,n} + 3003\,b^6c^8x^{22\,n} + \cdots}{\cdots}$$

```
[In] integrate(x^(-1+14*n)*(b+c*x^n)^13*(b+2*c*x^n),x, algorithm="fricas")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*c
^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*c^
8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^5*x
^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*x^(1
6*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(15) = 30$.

Time = 10.80 (sec) , antiderivative size = 360, normalized size of antiderivative = 17.14

$$\int x^{-1+14n}(b+cx^n)^{13}(b+2cx^n)\,dx$$
$$= \begin{cases} \frac{b^{14}xx^{14n-1}}{14n} + \frac{b^{13}cxx^nx^{14n-1}}{n} + \frac{13b^{12}c^2xx^{2n}x^{14n-1}}{2n} + \frac{26b^{11}c^3xx^{3n}x^{14n-1}}{n} + \frac{143b^{10}c^4xx^{4n}x^{14n-1}}{2n} + \frac{143b^9c^5xx^{5n}x^{14n-1}}{n} + \frac{429b\cdots}{} \\ (b+c)^{13}(b+2c)\log(x) \end{cases}$$

```
[In] integrate(x**(-1+14*n)*(b+c*x**n)**13*(b+2*c*x**n),x)
```

```
[Out] Piecewise((b**14*x*x**(14*n - 1)/(14*n) + b**13*c*x*x**n*x**(14*n - 1)/n +
13*b**12*c**2*x*x**(2*n)*x**(14*n - 1)/(2*n) + 26*b**11*c**3*x*x**(3*n)*x**
(14*n - 1)/n + 143*b**10*c**4*x*x**(4*n)*x**(14*n - 1)/(2*n) + 143*b**9*c**
5*x*x**(5*n)*x**(14*n - 1)/n + 429*b**8*c**6*x*x**(6*n)*x**(14*n - 1)/(2*n)
 + 1716*b**7*c**7*x*x**(7*n)*x**(14*n - 1)/(7*n) + 429*b**6*c**8*x*x**(8*n)
*x**(14*n - 1)/(2*n) + 143*b**5*c**9*x*x**(9*n)*x**(14*n - 1)/n + 143*b**4*
c**10*x*x**(10*n)*x**(14*n - 1)/(2*n) + 26*b**3*c**11*x*x**(11*n)*x**(14*n
- 1)/n + 13*b**2*c**12*x*x**(12*n)*x**(14*n - 1)/(2*n) + b*c**13*x*x**(13*n
)*x**(14*n - 1)/n + c**14*x*x**(14*n)*x**(14*n - 1)/(14*n), Ne(n, 0)), ((b
+ c)**13*(b + 2*c)*log(x), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(19) = 38.

Time = 0.21 (sec) , antiderivative size = 229, normalized size of antiderivative = 10.90

$$\int x^{-1+14n}(b+cx^n)^{13}(b+2cx^n)\,dx = \frac{c^{14}x^{28\,n}}{14\,n} + \frac{bc^{13}x^{27\,n}}{n} + \frac{13\,b^2c^{12}x^{26\,n}}{2\,n} + \frac{26\,b^3c^{11}x^{25\,n}}{n} + \frac{143\,b^4c^{10}x^{24\,n}}{2\,n} + \frac{143\,b^5c^9x^{23\,n}}{n} + \frac{429\,b^6c^8x^{22\,n}}{2\,n} + \frac{1716\,b^7c^7x^{21\,n}}{7\,n} + \frac{429\,b^8c^6x^{20\,n}}{2\,n} + \frac{143\,b^9c^5x^{19\,n}}{n} + \frac{143\,b^{10}c^4x^{18\,n}}{2\,n} + \frac{26\,b^{11}c^3x^{17\,n}}{n} + \frac{13\,b^{12}c^2x^{16\,n}}{2\,n} + \frac{b^{13}cx^{15\,n}}{n} + \frac{b^{14}x^{14\,n}}{14\,n}$$

```
[In] integrate(x^(-1+14*n)*(b+c*x^n)^13*(b+2*c*x^n),x, algorithm="maxima")
```

```
[Out] 1/14*c^14*x^(28*n)/n + b*c^13*x^(27*n)/n + 13/2*b^2*c^12*x^(26*n)/n + 26*b^
3*c^11*x^(25*n)/n + 143/2*b^4*c^10*x^(24*n)/n + 143*b^5*c^9*x^(23*n)/n + 42
9/2*b^6*c^8*x^(22*n)/n + 1716/7*b^7*c^7*x^(21*n)/n + 429/2*b^8*c^6*x^(20*n)
/n + 143*b^9*c^5*x^(19*n)/n + 143/2*b^10*c^4*x^(18*n)/n + 26*b^11*c^3*x^(17
*n)/n + 13/2*b^12*c^2*x^(16*n)/n + b^13*c*x^(15*n)/n + 1/14*b^14*x^(14*n)/n
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(19) = 38.

Time = 0.36 (sec) , antiderivative size = 189, normalized size of antiderivative = 9.00

$$\int x^{-1+14n}(b+cx^n)^{13}(b+2cx^n)\,dx = \frac{c^{14}x^{28\,n} + 14\,bc^{13}x^{27\,n} + 91\,b^2c^{12}x^{26\,n} + 364\,b^3c^{11}x^{25\,n} + 1001\,b^4c^{10}x^{24\,n} + 2002\,b^5c^9x^{23\,n} + 3003\,b^6c^8x^{22\,n} + 3}{}$$

```
[In] integrate(x^(-1+14*n)*(b+c*x^n)^13*(b+2*c*x^n),x, algorithm="giac")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*c
^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*c^
8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^5*x
^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*x^(1
6*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n
```

Mupad [B] (verification not implemented)

Time = 9.55 (sec) , antiderivative size = 229, normalized size of antiderivative = 10.90

$$\int x^{-1+14n}(b+cx^n)^{13}\,(b+2cx^n)\;dx=\frac{b^{14}\,x^{14\,n}}{14\,n}+\frac{c^{14}\,x^{28\,n}}{14\,n}+\frac{13\,b^{12}\,c^2\,x^{16\,n}}{2\,n}+\frac{26\,b^{11}\,c^3\,x^{17\,n}}{n}+\frac{143\,b^{10}\,c^4\,x^{18\,n}}{2\,n}+\frac{143\,b^9\,c^5\,x^{19\,n}}{n}+\frac{429\,b^8\,c^6\,x^{20\,n}}{2\,n}+\frac{1716\,b^7\,c^7\,x^{21\,n}}{7\,n}+\frac{429\,b^6\,c^8\,x^{22\,n}}{2\,n}+\frac{143\,b^5\,c^9\,x^{23\,n}}{n}+\frac{143\,b^4\,c^{10}\,x^{24\,n}}{2\,n}+\frac{26\,b^3\,c^{11}\,x^{25\,n}}{n}+\frac{13\,b^2\,c^{12}\,x^{26\,n}}{2\,n}+\frac{b^{13}\,c\,x^{15\,n}}{n}+\frac{b\,c^{13}\,x^{27\,n}}{n}$$

```
[In] int(x^(14*n - 1)*(b + c*x^n)^13*(b + 2*c*x^n),x)
```

```
[Out] (b^14*x^(14*n))/(14*n) + (c^14*x^(28*n))/(14*n) + (13*b^12*c^2*x^(16*n))/(2
*n) + (26*b^11*c^3*x^(17*n))/n + (143*b^10*c^4*x^(18*n))/(2*n) + (143*b^9*c
^5*x^(19*n))/n + (429*b^8*c^6*x^(20*n))/(2*n) + (1716*b^7*c^7*x^(21*n))/(7*
n) + (429*b^6*c^8*x^(22*n))/(2*n) + (143*b^5*c^9*x^(23*n))/n + (143*b^4*c^1
0*x^(24*n))/(2*n) + (26*b^3*c^11*x^(25*n))/n + (13*b^2*c^12*x^(26*n))/(2*n)
 + (b^13*c*x^(15*n))/n + (b*c^13*x^(27*n))/n
```

3.1055 $\int x^{-1+m}(a+bx^n)^{-1+p}(am+b(m+np)x^n)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 13

$$\int x^{-1+m}(a+bx^n)^{-1+p}(am+b(m+np)x^n)\,dx=x^m(a+bx^n)^p$$

```
[Out] x^m*(a+b*x^n)^p
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {460}

$$\int x^{-1+m}(a+bx^n)^{-1+p}(am+b(m+np)x^n)\,dx=x^m(a+bx^n)^p$$

```
[In] Int[x^(-1 + m)*(a + b*x^n)^(-1 + p)*(a*m + b*(m + n*p)*x^n),x]
```

```
[Out] x^m*(a + b*x^n)^p
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=x^m(a+bx^n)^p$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 8.23

$$\int x^{-1+m}(a+bx^n)^{-1+p}\left(am+b(m+np)x^n\right)\,dx$$
$$=\frac{x^m(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(a(m+n)\,\text{Hypergeometric2F1}\left(\frac{m}{n},1-p,\frac{m+n}{n},-\frac{bx^n}{a}\right)+b(m+np)x^n\,\text{Hypergeo}\right.}{a(m+n)}$$

```
[In] Integrate[x^(-1 + m)*(a + b*x^n)^(-1 + p)*(a*m + b*(m + n*p)*x^n),x]
```

```
[Out] (x^m*(a + b*x^n)^p*(a*(m + n)*Hypergeometric2F1[m/n, 1 - p, (m + n)/n, -((b
*x^n)/a)] + b*(m + n*p)*x^n*Hypergeometric2F1[(m + n)/n, 1 - p, 2 + m/n, -(
(b*x^n)/a)]))/(a*(m + n)*(1 + (b*x^n)/a)^p)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(13) = 26.

Time = 12.57 (sec) , antiderivative size = 50, normalized size of antiderivative = 3.85

method	result	size
parallelrisch	$\frac{x\,x^n x^{-1+m}(a+b\,x^n)^{p-1}b^2+x\,x^{-1+m}(a+b\,x^n)^{p-1}ab}{b}$	50

```
[In] int(x^(-1+m)*(a+b*x^n)^(p-1)*(a*m+b*(n*p+m)*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] (x*x^n*x^(-1+m)*(a+b*x^n)^(p-1)*b^2+x*x^(-1+m)*(a+b*x^n)^(p-1)*a*b)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(13) = 26.

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.46

$$\int x^{-1+m}(a+bx^n)^{-1+p}\left(am+b(m+np)x^n\right)\,dx=\left(bxx^{m-1}x^n+axx^{m-1}\right)(bx^n+a)^{p-1}$$

```
[In] integrate(x^(-1+m)*(a+b*x^n)^(-1+p)*(a*m+b*(n*p+m)*x^n),x, algorithm="frica
s")
```

```
[Out] (b*x*x^(m - 1)*x^n + a*x*x^(m - 1))*(b*x^n + a)^(p - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(10) = 20.

Time = 2.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 3.00

$$\int x^{-1+m}(a+bx^n)^{-1+p}\left(am+b(m+np)x^n\right)\,dx = axx^{m-1}(a+bx^n)^{p-1}+bxx^nx^{m-1}(a+bx^n)^{p-1}$$

```
[In] integrate(x**(-1+m)*(a+b*x**n)**(-1+p)*(a*m+b*(n*p+m)*x**n),x)
```

```
[Out] a*x*x**(m - 1)*(a + b*x**n)**(p - 1) + b*x*x**n*x**(m - 1)*(a + b*x**n)**(p
 - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int x^{-1+m}(a+bx^n)^{-1+p}\left(am+b(m+np)x^n\right)\,dx = e^{(p\log(bx^n+a)+m\log(x))}$$

```
[In] integrate(x^(-1+m)*(a+b*x^n)^(-1+p)*(a*m+b*(n*p+m)*x^n),x, algorithm="maxim
a")
```

```
[Out] e^(p*log(b*x^n + a) + m*log(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(13) = 26.

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 5.38

$$\int x^{-1+m}(a+bx^n)^{-1+p}\left(am+b(m+np)x^n\right)\,dx = bxx^ne^{(p\log(bx^n+a)+m\log(x)-\log(bx^n+a)-\log(x))}+axe^{(p\log(bx^n+a)+m\log(x)-\log(bx^n+a)-\log(x))}$$

```
[In] integrate(x^(-1+m)*(a+b*x^n)^(-1+p)*(a*m+b*(n*p+m)*x^n),x, algorithm="giac"
)
```

```
[Out] b*x*x^n*e^(p*log(b*x^n + a) + m*log(x) - log(b*x^n + a) - log(x)) + a*x*e^(
p*log(b*x^n + a) + m*log(x) - log(b*x^n + a) - log(x))
```

Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int x^{-1+m}(a+bx^n)^{-1+p}\left(am+b(m+np)x^n\right)\,dx=\left(a\,x^m+b\,x^{m+n}\right)\,\left(a+b\,x^n\right)^{p-1}$$

```
[In] int(x^(m - 1)*(a*m + b*x^n*(m + n*p))*(a + b*x^n)^(p - 1),x)
[Out] (a*x^m + b*x^(m + n))*(a + b*x^n)^(p - 1)
```

3.1056 $\int \frac{b+2cx}{x(b+cx)} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 8

$$\int \frac{b+2cx}{x(b+cx)} \, dx = \log(x(b+cx))$$

```
[Out] ln(x*(c*x+b))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {78}

$$\int \frac{b+2cx}{x(b+cx)} \, dx = \log(b+cx) + \log(x)$$

```
[In] Int[(b + 2*c*x)/(x*(b + c*x)),x]
[Out] Log[x] + Log[b + c*x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{x}+\frac{c}{b+cx}\right) \, dx \\ &= \log(x)+\log(b+cx) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

$$\int \frac{b+2cx}{x(b+cx)}\,dx = \log(x)+\log(b+cx)$$

```
[In] Integrate[(b + 2*c*x)/(x*(b + c*x)),x]
[Out] Log[x] + Log[b + c*x]
```

Maple [A] (verified)

Time = 4.62 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
default	$\ln(x(cx+b))$	9
norman	$\ln(x)+\ln(cx+b)$	10
parallelrisch	$\ln(x)+\ln(cx+b)$	10
risch	$\ln(c\,x^2+bx)$	11

```
[In] int((2*c*x+b)/x/(c*x+b),x,method=_RETURNVERBOSE)
[Out] ln(x*(c*x+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{b+2cx}{x(b+cx)}\,dx = \log\left(cx^2+bx\right)$$

```
[In] integrate((2*c*x+b)/x/(c*x+b),x, algorithm="fricas")
[Out] log(c*x^2 + b*x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{x(b+cx)}\,dx=\log\left(bx+cx^2\right)$$

```
[In] integrate((2*c*x+b)/x/(c*x+b),x)
[Out] log(b*x + c*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

$$\int \frac{b+2cx}{x(b+cx)}\,dx=\log\left(cx+b\right)+\log\left(x\right)$$

```
[In] integrate((2*c*x+b)/x/(c*x+b),x, algorithm="maxima")
[Out] log(c*x + b) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.38

$$\int \frac{b+2cx}{x(b+cx)}\,dx=\log\left(|cx+b|\right)+\log\left(|x|\right)$$

```
[In] integrate((2*c*x+b)/x/(c*x+b),x, algorithm="giac")
[Out] log(abs(c*x + b)) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{x(b+cx)}\,dx=\ln\left(x\,\left(b+c\,x\right)\right)$$

```
[In] int((b + 2*c*x)/(x*(b + c*x)),x)
[Out] log(x*(b + c*x))
```

3.1057 $\int \frac{b+2cx^2}{x(b+cx^2)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 15

$$\int \frac{b+2cx^2}{x\left(b+cx^2\right)}\,dx=\log(x)+\frac{1}{2}\log\left(b+cx^2\right)$$

```
[Out] ln(x)+1/2*ln(c*x^2+b)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {457, 78}

$$\int \frac{b+2cx^2}{x\left(b+cx^2\right)}\,dx=\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

```
[In] Int[(b + 2*c*x^2)/(x*(b + c*x^2)),x]
```

```
[Out] Log[x] + Log[b + c*x^2]/2
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)\,dx, x, x^2\right)\\ &= \log(x)+\frac{1}{2}\log\left(b+cx^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx^2}{x\left(b+cx^2\right)}\,dx = \log(x)+\frac{1}{2}\log\left(b+cx^2\right)$$

```
[In] Integrate[(b + 2*c*x^2)/(x*(b + c*x^2)),x]
[Out] Log[x] + Log[b + c*x^2]/2
```

Maple [A] (verified)

Time = 4.60 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\ln\left(x\right)+\frac{\ln\left(c\,x^{2}+b\right)}{2}$	14
norman	$\ln\left(x\right)+\frac{\ln\left(c\,x^{2}+b\right)}{2}$	14
risch	$\ln\left(x\right)+\frac{\ln\left(c\,x^{2}+b\right)}{2}$	14
parallelrisch	$\ln\left(x\right)+\frac{\ln\left(c\,x^{2}+b\right)}{2}$	14

```
[In] int((2*c*x^2+b)/x/(c*x^2+b),x,method=_RETURNVERBOSE)
[Out] ln(x)+1/2*ln(c*x^2+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx^2}{x(b+cx^2)}\,dx = \frac{1}{2}\,\log\left(cx^2+b\right)+\log\left(x\right)$$

```
[In] integrate((2*c*x^2+b)/x/(c*x^2+b),x, algorithm="fricas")
[Out] 1/2*log(c*x^2 + b) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{b+2cx^2}{x(b+cx^2)}\,dx = \log\left(x\right)+\frac{\log\left(\frac{b}{c}+x^2\right)}{2}$$

```
[In] integrate((2*c*x**2+b)/x/(c*x**2+b),x)
[Out] log(x) + log(b/c + x**2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{b+2cx^2}{x(b+cx^2)}\,dx = \frac{1}{2}\,\log\left(cx^2+b\right)+\frac{1}{2}\,\log\left(x^2\right)$$

```
[In] integrate((2*c*x^2+b)/x/(c*x^2+b),x, algorithm="maxima")
[Out] 1/2*log(c*x^2 + b) + 1/2*log(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{b+2cx^2}{x(b+cx^2)}\,dx = \frac{1}{2}\,\log\left(x^2\right)+\frac{1}{2}\,\log\left(\left|cx^2+b\right|\right)$$

```
[In] integrate((2*c*x^2+b)/x/(c*x^2+b),x, algorithm="giac")
[Out] 1/2*log(x^2) + 1/2*log(abs(c*x^2 + b))
```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx^2}{x\left(b+cx^2\right)}\,dx=\frac{\ln\left(c\,x^2+b\right)}{2}+\ln\left(x\right)$$

```
[In] int((b + 2*c*x^2)/(x*(b + c*x^2)),x)
[Out] log(b + c*x^2)/2 + log(x)
```

3.1058 $\int \frac{b+2cx^3}{x(b+cx^3)} dx$

Optimal result

Integrand size = 21, antiderivative size = 15

$$\int \frac{b+2cx^3}{x\left(b+cx^3\right)} dx = \log(x) + \frac{1}{3}\log\left(b+cx^3\right)$$

```
[Out] ln(x)+1/3*ln(c*x^3+b)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {457, 78}

$$\int \frac{b+2cx^3}{x\left(b+cx^3\right)} dx = \frac{1}{3}\log\left(b+cx^3\right) + \log(x)$$

```
[In] Int[(b + 2*c*x^3)/(x*(b + c*x^3)),x]
```

```
[Out] Log[x] + Log[b + c*x^3]/3
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)\,dx, x, x^3\right)\\ &= \log(x)+\frac{1}{3}\log\left(b+cx^3\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx^3}{x\left(b+cx^3\right)}\,dx = \log(x)+\frac{1}{3}\log\left(b+cx^3\right)$$

```
[In] Integrate[(b + 2*c*x^3)/(x*(b + c*x^3)),x]
[Out] Log[x] + Log[b + c*x^3]/3
```

Maple [A] (verified)

Time = 4.63 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\ln(x)+\frac{\ln(cx^3+b)}{3}$	14
norman	$\ln(x)+\frac{\ln(cx^3+b)}{3}$	14
risch	$\ln(x)+\frac{\ln(cx^3+b)}{3}$	14
parallelrisch	$\ln(x)+\frac{\ln(cx^3+b)}{3}$	14

```
[In] int((2*c*x^3+b)/x/(c*x^3+b),x,method=_RETURNVERBOSE)
[Out] ln(x)+1/3*ln(c*x^3+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx^3}{x(b+cx^3)}\,dx = \frac{1}{3}\log\left(cx^3+b\right)+\log\left(x\right)$$

```
[In] integrate((2*c*x^3+b)/x/(c*x^3+b),x, algorithm="fricas")
[Out] 1/3*log(c*x^3 + b) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{b+2cx^3}{x(b+cx^3)}\,dx = \log\left(x\right)+\frac{\log\left(\frac{b}{c}+x^3\right)}{3}$$

```
[In] integrate((2*c*x**3+b)/x/(c*x**3+b),x)
[Out] log(x) + log(b/c + x**3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{b+2cx^3}{x(b+cx^3)}\,dx = \frac{1}{3}\log\left(cx^3+b\right)+\frac{1}{3}\log\left(x^3\right)$$

```
[In] integrate((2*c*x^3+b)/x/(c*x^3+b),x, algorithm="maxima")
[Out] 1/3*log(c*x^3 + b) + 1/3*log(x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx^3}{x(b+cx^3)}\,dx = \frac{1}{3}\log\left(\left|cx^3+b\right|\right)+\log\left(\left|x\right|\right)$$

```
[In] integrate((2*c*x^3+b)/x/(c*x^3+b),x, algorithm="giac")
[Out] 1/3*log(abs(c*x^3 + b)) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx^3}{x\,(b+cx^3)}\,dx = \frac{\ln\left(c\,x^3+b\right)}{3}+\ln\left(x\right)$$

```
[In] int((b + 2*c*x^3)/(x*(b + c*x^3)),x)
[Out] log(b + c*x^3)/3 + log(x)
```

3.1059 $\int \frac{b+2cx^n}{x(b+cx^n)} dx$

Optimal result

Integrand size = 21, antiderivative size = 15

$$\int \frac{b+2cx^n}{x(b+cx^n)} dx = \log(x) + \frac{\log(b+cx^n)}{n}$$

```
[Out] ln(x)+ln(b+c*x^n)/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {457, 78}

$$\int \frac{b+2cx^n}{x(b+cx^n)} dx = \frac{\log(b+cx^n)}{n} + \log(x)$$

```
[In] Int[(b + 2*c*x^n)/(x*(b + c*x^n)),x]
```

```
[Out] Log[x] + Log[b + c*x^n]/n
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx, x, x^n\right)}{n} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)\,dx, x, x^n\right)}{n} \\ &= \log(x)+\frac{\log\left(b+cx^n\right)}{n} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{b+2cx^n}{x\left(b+cx^n\right)}\,dx=\frac{\log\left(x^n\right)+\log\left(n(b+cx^n)\right)}{n}$$

```
[In] Integrate[(b + 2*c*x^n)/(x*(b + c*x^n)),x]
[Out] (Log[x^n] + Log[n*(b + c*x^n)])/n
```

Maple [A] (verified)

Time = 4.69 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

method	result	size
derivativedivides	$\frac{\ln(x^n(b+c\,x^n))}{n}$	17
default	$\frac{\ln(x^n(b+c\,x^n))}{n}$	17
norman	$\ln\left(x\right)+\frac{\ln\left(b+c\,{\mathrm e}^{n\ln(x)}\right)}{n}$	18
risch	$\ln\left(x\right)+\frac{\ln\left(x^n+\frac{b}{c}\right)}{n}$	18
parallelrisch	$\frac{n\ln(x)+\ln(b+c\,x^n)}{n}$	18

```
[In] int((b+2*c*x^n)/x/(b+c*x^n),x,method=_RETURNVERBOSE)
[Out] 1/n*ln(x^n*(b+c*x^n))
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{b+2cx^n}{x(b+cx^n)}\,dx=\frac{n\log(x)+\log(cx^n+b)}{n}$$

```
[In] integrate((b+2*c*x^n)/x/(b+c*x^n),x, algorithm="fricas")
[Out] (n*log(x) + log(c*x^n + b))/n
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \frac{b+2cx^n}{x(b+cx^n)}\,dx=\begin{cases}\log(x) & \text{for } c=0 \wedge (c=0 \vee n=0)\\ \frac{(b+2c)\log(x)}{b+c} & \text{for } n=0\\ \log(x)+\frac{\log\left(\frac{b}{c}+x^n\right)}{n} & \text{otherwise}\end{cases}$$

```
[In] integrate((b+2*c*x**n)/x/(b+c*x**n),x)
[Out] Piecewise((log(x), Eq(c, 0) & (Eq(c, 0) | Eq(n, 0))), ((b + 2*c)*log(x)/(b
+ c), Eq(n, 0)), (log(x) + log(b/c + x**n)/n, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(15) = 30.

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 3.13

$$\int \frac{b+2cx^n}{x(b+cx^n)}\,dx=b\left(\frac{\log(x)}{b}-\frac{\log\left(\frac{cx^n+b}{c}\right)}{bn}\right)+\frac{2\log\left(\frac{cx^n+b}{c}\right)}{n}$$

```
[In] integrate((b+2*c*x^n)/x/(b+c*x^n),x, algorithm="maxima")
[Out] b*(log(x)/b - log((c*x^n + b)/c)/(b*n)) + 2*log((c*x^n + b)/c)/n
```

Giac [F]

$$\int \frac{b+2cx^n}{x\left(b+cx^n\right)}\,dx = \int \frac{2\,cx^n+b}{(cx^n+b)x}\,dx$$

```
[In] integrate((b+2*c*x^n)/x/(b+c*x^n),x, algorithm="giac")
[Out] integrate((2*c*x^n + b)/((c*x^n + b)*x), x)
```

Mupad [B] (verification not implemented)

Time = 8.70 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx^n}{x\left(b+cx^n\right)}\,dx = \ln\left(x\right)+\frac{\ln\left(b+c\,x^n\right)}{n}$$

```
[In] int((b + 2*c*x^n)/(x*(b + c*x^n)),x)
[Out] log(x) + log(b + c*x^n)/n
```

3.1060 $\int \frac{b+2cx}{x^8(b+cx)^8} dx$

Optimal result

Integrand size = 17, antiderivative size = 14

$$\int \frac{b+2cx}{x^8(b+cx)^8} dx = -\frac{1}{7x^7(b+cx)^7}$$

```
[Out] -1/7/x^7/(c*x+b)^7
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{75\}$

$$\int \frac{b+2cx}{x^8(b+cx)^8} dx = -\frac{1}{7x^7(b+cx)^7}$$

```
[In] Int[(b + 2*c*x)/(x^8*(b + c*x)^8),x]
```

```
[Out] -1/7*1/(x^7*(b + c*x)^7)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\text{integral} = -\frac{1}{7x^7(b+cx)^7}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{x^8(b+cx)^8}\,dx = -\frac{1}{7x^7(b+cx)^7}$$

```
[In] Integrate[(b + 2*c*x)/(x^8*(b + c*x)^8),x]
[Out] -1/7*1/(x^7*(b + c*x)^7)
```

Maple [A] (verified)

Time = 4.63 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result
gosper	$-\frac{1}{7x^7(cx+b)^7}$
norman	$-\frac{1}{7x^7(cx+b)^7}$
risch	$-\frac{1}{7x^7(cx+b)^7}$
parallelrisch	$-\frac{1}{7x^7(cx+b)^7}$
default	$-\frac{1}{7b^7x^7}-\frac{132c^6}{b^{13}x}+\frac{66c^5}{b^{12}x^2}-\frac{30c^4}{b^{11}x^3}+\frac{12c^3}{b^{10}x^4}-\frac{4c^2}{b^9x^5}+\frac{c}{b^8x^6}+\frac{132c^7}{b^{13}(cx+b)}+\frac{66c^7}{b^{12}(cx+b)^2}+\frac{30c^7}{b^{11}(cx+b)^3}+\frac{12c^7}{b^{10}(cx+$

```
[In] int((2*c*x+b)/x^8/(c*x+b)^8,x,method=_RETURNVERBOSE)
[Out] -1/7/x^7/(c*x+b)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(12) = 24$.

Time = 0.49 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.79

$$\int \frac{b+2cx}{x^8(b+cx)^8}\,dx = -\frac{1}{7\,(c^7x^{14}+7\,bc^6x^{13}+21\,b^2c^5x^{12}+35\,b^3c^4x^{11}+35\,b^4c^3x^{10}+21\,b^5c^2x^9+7\,b^6cx^8+b^7x^7)}$$

```
[In] integrate((2*c*x+b)/x^8/(c*x+b)^8,x, algorithm="fricas")
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11 + 35*b^4*
c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(14) = 28$.

Time = 0.43 (sec) , antiderivative size = 87, normalized size of antiderivative = 6.21

$$\int \frac{b + 2cx}{x^8(b + cx)^8}\,dx = -\frac{1}{7b^7x^7 + 49b^6cx^8 + 147b^5c^2x^9 + 245b^4c^3x^{10} + 245b^3c^4x^{11} + 147b^2c^5x^{12} + 49bc^6x^{13} + 7c^7x^{14}}$$

```
[In] integrate((2*c*x+b)/x**8/(c*x+b)**8,x)
[Out] -1/(7*b**7*x**7 + 49*b**6*c*x**8 + 147*b**5*c**2*x**9 + 245*b**4*c**3*x**10
 + 245*b**3*c**4*x**11 + 147*b**2*c**5*x**12 + 49*b*c**6*x**13 + 7*c**7*x**
14)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(12) = 24$.

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.79

$$\int \frac{b + 2cx}{x^8(b + cx)^8}\,dx = -\frac{1}{7\,(c^7x^{14} + 7\,bc^6x^{13} + 21\,b^2c^5x^{12} + 35\,b^3c^4x^{11} + 35\,b^4c^3x^{10} + 21\,b^5c^2x^9 + 7\,b^6cx^8 + b^7x^7)}$$

```
[In] integrate((2*c*x+b)/x^8/(c*x+b)^8,x, algorithm="maxima")
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11 + 35*b^4*
c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{b + 2cx}{x^8(b + cx)^8}\,dx = -\frac{1}{7\,(cx^2 + bx)^7}$$

```
[In] integrate((2*c*x+b)/x^8/(c*x+b)^8,x, algorithm="giac")
[Out] -1/7/(c*x^2 + b*x)^7
```

Mupad [B] (verification not implemented)

Time = 11.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{b+2cx}{x^8(b+cx)^8}\,dx=-\frac{1}{7\,x^7\,{\left(b+c\,x\right)}^7}$$

```
[In] int((b + 2*c*x)/(x^8*(b + c*x)^8),x)
[Out] -1/(7*x^7*(b + c*x)^7)
```

3.1061 $\int \frac{b+2cx^2}{x^{15}(b+cx^2)^8}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 16

$$\int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8}\,dx=-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

```
[Out] -1/14/x^14/(c*x^2+b)^7
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {457, 75}

$$\int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8}\,dx=-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

```
[In] Int[(b + 2*c*x^2)/(x^15*(b + c*x^2)^8),x]
```

```
[Out] -1/14*1/(x^14*(b + c*x^2)^7)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx, x, x^2\right) \\ &= -\frac{1}{14x^{14}\left(b+cx^2\right)^7} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8}\,dx = -\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

```
[In] Integrate[(b + 2*c*x^2)/(x^15*(b + c*x^2)^8),x]
[Out] -1/14*1/(x^14*(b + c*x^2)^7)
```

Maple [A] (verified)

Time = 4.65 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
gosper	$-\frac{1}{14x^{14}(c\,x^2+b)^7}$
norman	$-\frac{1}{14x^{14}(c\,x^2+b)^7}$
risch	$-\frac{1}{14x^{14}(c\,x^2+b)^7}$
parallelrisch	$-\frac{1}{14x^{14}(c\,x^2+b)^7}$
default	$-\frac{1}{14b^7x^{14}}-\frac{66c^6}{b^{13}x^2}+\frac{33c^5}{b^{12}x^4}-\frac{15c^4}{b^{11}x^6}+\frac{6c^3}{b^{10}x^8}-\frac{2c^2}{b^9x^{10}}+\frac{c}{2b^8x^{12}}-\frac{c^8\left(-\frac{132}{c(c\,x^2+b)}-\frac{b^5}{c(c\,x^2+b)^6}-\frac{b^6}{7c(c\,x^2+b)^7}-\frac{}{c(c\,x}\right.}{2b^1}$

```
[In] int((2*c*x^2+b)/x^15/(c*x^2+b)^8,x,method=_RETURNVERBOSE)
[Out] -1/14/x^14/(c*x^2+b)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.

Time = 0.39 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \frac{b+2cx^2}{x^{15}\,(b+cx^2)^8}\,dx = -\frac{1}{14\,(c^7x^{28}+7\,bc^6x^{26}+21\,b^2c^5x^{24}+35\,b^3c^4x^{22}+35\,b^4c^3x^{20}+21\,b^5c^2x^{18}+7\,b^6cx^{16}+b^7x^{14})}$$

```
[In] integrate((2*c*x^2+b)/x^15/(c*x^2+b)^8,x, algorithm="fricas")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4
*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(15) = 30$.

Time = 0.66 (sec) , antiderivative size = 87, normalized size of antiderivative = 5.44

$$\int \frac{b+2cx^2}{x^{15}\,(b+cx^2)^8}\,dx = -\frac{1}{14b^7x^{14}+98b^6cx^{16}+294b^5c^2x^{18}+490b^4c^3x^{20}+490b^3c^4x^{22}+294b^2c^5x^{24}+98bc^6x^{26}+14c^7x^{28}}$$

```
[In] integrate((2*c*x**2+b)/x**15/(c*x**2+b)**8,x)
```

```
[Out] -1/(14*b**7*x**14 + 98*b**6*c*x**16 + 294*b**5*c**2*x**18 + 490*b**4*c**3*x
**20 + 490*b**3*c**4*x**22 + 294*b**2*c**5*x**24 + 98*b*c**6*x**26 + 14*c**
7*x**28)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \frac{b+2cx^2}{x^{15}\,(b+cx^2)^8}\,dx = -\frac{1}{14\,(c^7x^{28}+7\,bc^6x^{26}+21\,b^2c^5x^{24}+35\,b^3c^4x^{22}+35\,b^4c^3x^{20}+21\,b^5c^2x^{18}+7\,b^6cx^{16}+b^7x^{14})}$$

```
[In] integrate((2*c*x^2+b)/x^15/(c*x^2+b)^8,x, algorithm="maxima")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4
*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8}\,dx=-\frac{1}{14\left(cx^4+bx^2\right)^7}$$

```
[In] integrate((2*c*x^2+b)/x^15/(c*x^2+b)^8,x, algorithm="giac")
[Out] -1/14/(c*x^4 + b*x^2)^7
```

Mupad [B] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8}\,dx=-\frac{1}{14\,x^{14}\left(c\,x^2+b\right)^7}$$

```
[In] int((b + 2*c*x^2)/(x^15*(b + c*x^2)^8),x)
[Out] -1/(14*x^14*(b + c*x^2)^7)
```

3.1062 $\int \frac{b+2cx^3}{x^{22}(b+cx^3)^8}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 16

$$\int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx=-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

```
[Out] -1/21/x^21/(c*x^3+b)^7
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {457, 75}

$$\int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx=-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

```
[In] Int[(b + 2*c*x^3)/(x^22*(b + c*x^3)^8),x]
```

```
[Out] -1/21*1/(x^21*(b + c*x^3)^7)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx, x, x^3\right)$$
$$= -\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx = -\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

```
[In] Integrate[(b + 2*c*x^3)/(x^22*(b + c*x^3)^8),x]
[Out] -1/21*1/(x^21*(b + c*x^3)^7)
```

Maple [A] (verified)

Time = 4.82 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
gosper	$-\frac{1}{21x^{21}\left(c\,x^{3}+b\right)^{7}}$
risch	$-\frac{1}{21x^{21}\left(c\,x^{3}+b\right)^{7}}$
parallelrisch	$-\frac{1}{21x^{21}\left(c\,x^{3}+b\right)^{7}}$
default	$-\frac{1}{21b^{7}x^{21}}-\frac{44c^{6}}{b^{13}x^{3}}+\frac{22c^{5}}{b^{12}x^{6}}-\frac{10c^{4}}{b^{11}x^{9}}+\frac{4c^{3}}{b^{10}x^{12}}-\frac{4c^{2}}{3b^{9}x^{15}}+\frac{c}{3b^{8}x^{18}}-\frac{c^{8}\left(-\frac{132}{c\left(c\,x^{3}+b\right)}-\frac{b^{5}}{c\left(c\,x^{3}+b\right)^{6}}-\frac{b^{6}}{7c\left(c\,x^{3}+b\right)^{7}}-\frac{}{c(}\right.}{3}$

```
[In] int((2*c*x^3+b)/x^22/(c*x^3+b)^8,x,method=_RETURNVERBOSE)
[Out] -1/21/x^21/(c*x^3+b)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.

Time = 0.69 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx = -\frac{1}{21\left(c^7x^{42}+7\,bc^6x^{39}+21\,b^2c^5x^{36}+35\,b^3c^4x^{33}+35\,b^4c^3x^{30}+21\,b^5c^2x^{27}+7\,b^6cx^{24}+b^7x^{21}\right)}$$

```
[In] integrate((2*c*x^3+b)/x^22/(c*x^3+b)^8,x, algorithm="fricas")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4
*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(15) = 30$.

Time = 0.89 (sec) , antiderivative size = 87, normalized size of antiderivative = 5.44

$$\int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx = -\frac{1}{21b^7x^{21}+147b^6cx^{24}+441b^5c^2x^{27}+735b^4c^3x^{30}+735b^3c^4x^{33}+441b^2c^5x^{36}+147bc^6x^{39}+21c^7x^{42}}$$

```
[In] integrate((2*c*x**3+b)/x**22/(c*x**3+b)**8,x)
```

```
[Out] -1/(21*b**7*x**21 + 147*b**6*c*x**24 + 441*b**5*c**2*x**27 + 735*b**4*c**3*
x**30 + 735*b**3*c**4*x**33 + 441*b**2*c**5*x**36 + 147*b*c**6*x**39 + 21*c
**7*x**42)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx = -\frac{1}{21\left(c^7x^{42}+7\,bc^6x^{39}+21\,b^2c^5x^{36}+35\,b^3c^4x^{33}+35\,b^4c^3x^{30}+21\,b^5c^2x^{27}+7\,b^6cx^{24}+b^7x^{21}\right)}$$

```
[In] integrate((2*c*x^3+b)/x^22/(c*x^3+b)^8,x, algorithm="maxima")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4
*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{b+2cx^3}{x^{22}(b+cx^3)^8}\,dx = -\frac{1}{21\,(cx^6+bx^3)^7}$$

```
[In] integrate((2*c*x^3+b)/x^22/(c*x^3+b)^8,x, algorithm="giac")
[Out] -1/21/(c*x^6 + b*x^3)^7
```

Mupad [B] (verification not implemented)

Time = 13.81 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{b+2cx^3}{x^{22}(b+cx^3)^8}\,dx = -\frac{1}{21\,x^{21}\,(c\,x^3+b)^7}$$

```
[In] int((b + 2*c*x^3)/(x^22*(b + c*x^3)^8),x)
[Out] -1/(21*x^21*(b + c*x^3)^7)
```

3.1063 $\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8} dx$

Optimal result

Integrand size = 25, antiderivative size = 21

$$\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8} dx = -\frac{x^{-7n}}{7n(b+cx^n)^7}$$

```
[Out] -1/7/n/(x^(7*n))/(b+c*x^n)^7
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {457, 75}

$$\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8} dx = -\frac{x^{-7n}}{7n(b+cx^n)^7}$$

```
[In] Int[(x^(-1 - 7*n)*(b + 2*c*x^n))/(b + c*x^n)^8,x]
```

```
[Out] -1/7*1/(n*x^(7*n)*(b + c*x^n)^7)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx, x, x^n\right)}{n} \\ &= -\frac{x^{-7n}}{7n\,(b+cx^n)^7} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8}\,dx = -\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

```
[In] Integrate[(x^(-1 - 7*n)*(b + 2*c*x^n))/(b + c*x^n)^8,x]
[Out] -1/7*1/(n*x^(7*n)*(b + c*x^n)^7)
```

Maple [A] (verified)

Time = 14.92 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

method	result
parallelrisch	$-\frac{x\,x^{-1-7n}}{7n(b+c\,x^n)^7}$
risch	$-\frac{132c^6x^{-n}}{b^{13}n}+\frac{66c^5x^{-2n}}{b^{12}n}-\frac{30c^4x^{-3n}}{b^{11}n}+\frac{12c^3x^{-4n}}{b^{10}n}-\frac{4c^2x^{-5n}}{b^9n}+\frac{c\,x^{-6n}}{b^8n}-\frac{x^{-7n}}{7b^7n}+\frac{c^7\left(924x^{6n}c^6+6006b\,c^5x^{5n}+163\right.}{}$

```
[In] int(x^(-1-7*n)*(b+2*c*x^n)/(b+c*x^n)^8,x,method=_RETURNVERBOSE)
[Out] -1/7*x*x^(-1-7*n)/n/(b+c*x^n)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(21) = 42.

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 5.00

$$\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8}\,dx = -\frac{1}{7\,(c^7nx^{14\,n} + 7\,bc^6nx^{13\,n} + 21\,b^2c^5nx^{12\,n} + 35\,b^3c^4nx^{11\,n} + 35\,b^4c^3nx^{10\,n} + 21\,b^5c^2nx^{9\,n} + 7\,b^6cnx^{8\,n} + b^7n}$$

```
[In] integrate(x^(-1-7*n)*(b+2*c*x^n)/(b+c*x^n)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 21*b^2*c^5*n*x^(12*n) + 35*b^3*
c^4*n*x^(11*n) + 35*b^4*c^3*n*x^(10*n) + 21*b^5*c^2*n*x^(9*n) + 7*b^6*c*n*x
^(8*n) + b^7*n*x^(7*n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(19) = 38.

Time = 41.77 (sec) , antiderivative size = 129, normalized size of antiderivative = 6.14

$$\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8}\,dx = \begin{cases} -\frac{xx^{-7n-1}}{7b^7n+49b^6cnx^n+147b^5c^2nx^{2n}+245b^4c^3nx^{3n}+245b^3c^4nx^{4n}+147b^2c^5nx^{5n}+49bc^6nx^{6n}+7c^7nx^{7n}} & \text{for } n\neq 0 \\ \frac{(b+2c)\log{(x)}}{(b+c)^8} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**(-1-7*n)*(b+2*c*x**n)/(b+c*x**n)**8,x)
```

```
[Out] Piecewise((-x*x**(-7*n - 1)/(7*b**7*n + 49*b**6*c*n*x**n + 147*b**5*c**2*n*
x**(2*n) + 245*b**4*c**3*n*x**(3*n) + 245*b**3*c**4*n*x**(4*n) + 147*b**2*c
**5*n*x**(5*n) + 49*b*c**6*n*x**(6*n) + 7*c**7*n*x**(7*n)), Ne(n, 0)), ((b
+ 2*c)*log(x)/(b + c)**8, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 612 vs. 2(21) = 42.

Time = 0.25 (sec) , antiderivative size = 612, normalized size of antiderivative = 29.14

$$\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8}\,dx = -\frac{1}{105}\,b\left(\frac{360360\,c^{13}x^{13\,n} + 2342340\,bc^{12}x^{12\,n} + 6426420\,b^2c^{11}x^{11\,n} + 9579570\,b^3c^{10}x^{10\,n} + 8270262\,b^4c^9x^{9\,n}}{b^{14}c^7nx^{14\,n} + 7\,b^{15}c^6nx^{13\,n} + 21\,b^{16}c^5n}\right.$$
$$+\frac{1}{105}\,c\left(\frac{360360\,c^{12}x^{12\,n} + 2342340\,bc^{11}x^{11\,n} + 6426420\,b^2c^{10}x^{10\,n} + 9579570\,b^3c^9x^{9\,n} + 8270262\,b^4c^8x^{8\,n} +}{b^{13}c^7nx^{13\,n} + 7\,b^{14}c^6nx^{12\,n} + 21\,b^{15}c^5nx^{11\,n} +}\right.$$

[In] integrate(x^(-1-7*n)*(b+2*c*x^n)/(b+c*x^n)^8,x, algorithm="maxima")

[Out] -1/105*b*((360360*c^13*x^(13*n) + 2342340*b*c^12*x^(12*n) + 6426420*b^2*c^11*x^(11*n) + 9579570*b^3*c^10*x^(10*n) + 8270262*b^4*c^9*x^(9*n) + 4018014*b^5*c^8*x^(8*n) + 934362*b^6*c^7*x^(7*n) + 45045*b^7*c^6*x^(6*n) - 5005*b^8*c^5*x^(5*n) + 1001*b^9*c^4*x^(4*n) - 273*b^10*c^3*x^(3*n) + 91*b^11*c^2*x^(2*n) - 35*b^12*c*x^n + 15*b^13)/(b^14*c^7*n*x^(14*n) + 7*b^15*c^6*n*x^(13*n) + 21*b^16*c^5*n*x^(12*n) + 35*b^17*c^4*n*x^(11*n) + 35*b^18*c^3*n*x^(10*n) + 21*b^19*c^2*n*x^(9*n) + 7*b^20*c*n*x^(8*n) + b^21*n*x^(7*n)) + 360360*c^7*log(x)/b^15 - 360360*c^7*log((c*x^n + b)/c)/(b^15*n)) + 1/105*c*((360360*c^12*x^(12*n) + 2342340*b*c^11*x^(11*n) + 6426420*b^2*c^10*x^(10*n) + 9579570*b^3*c^9*x^(9*n) + 8270262*b^4*c^8*x^(8*n) + 4018014*b^5*c^7*x^(7*n) + 934362*b^6*c^6*x^(6*n) + 45045*b^7*c^5*x^(5*n) - 5005*b^8*c^4*x^(4*n) + 1001*b^9*c^3*x^(3*n) - 273*b^10*c^2*x^(2*n) + 91*b^11*c*x^n - 35*b^12)/(b^13*c^7*n*x^(13*n) + 7*b^14*c^6*n*x^(12*n) + 21*b^15*c^5*n*x^(11*n) + 35*b^16*c^4*n*x^(10*n) + 35*b^17*c^3*n*x^(9*n) + 21*b^18*c^2*n*x^(8*n) + 7*b^19*c*n*x^(7*n) + b^20*n*x^(6*n)) + 360360*c^6*log(x)/b^14 - 360360*c^6*log((c*x^n + b)/c)/(b^14*n))

Giac [F]

$$\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8}\,dx = \int \frac{(2\,cx^n+b)x^{-7\,n-1}}{(cx^n+b)^8}\,dx$$

[In] integrate(x^(-1-7*n)*(b+2*c*x^n)/(b+c*x^n)^8,x, algorithm="giac")

[Out] integrate((2*c*x^n + b)*x^(-7*n - 1)/(c*x^n + b)^8, x)

Mupad [B] (verification not implemented)

Time = 9.39 (sec) , antiderivative size = 105, normalized size of antiderivative = 5.00

$$\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8}\,dx =$$

$$-\frac{1}{7\,x^{7\,n}\,\left(b^7\,n+c^7\,n\,x^{7\,n}+7\,b^6\,c\,n\,x^n+7\,b\,c^6\,n\,x^{6\,n}+21\,b^5\,c^2\,n\,x^{2\,n}+35\,b^4\,c^3\,n\,x^{3\,n}+35\,b^3\,c^4\,n\,x^{4\,n}+21\,b^2\,c\right.}$$

[In] int((b + 2*c*x^n)/(x^(7*n + 1)*(b + c*x^n)^8),x)

[Out] -1/(7*x^(7*n)*(b^7*n + c^7*n*x^(7*n) + 7*b^6*c*n*x^n + 7*b*c^6*n*x^(6*n) + 21*b^5*c^2*n*x^(2*n) + 35*b^4*c^3*n*x^(3*n) + 35*b^3*c^4*n*x^(4*n) + 21*b^2*c^5*n*x^(5*n)))

3.1064 $\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}} dx$

Optimal result

Integrand size = 22, antiderivative size = 52

$$\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}} dx = -\frac{1}{8}\sqrt{1+x^{16}} - \frac{1}{24}\left(1+x^{16}\right)^{3/2} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+x^{16}}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] -1/24*(x^16+1)^(3/2)+1/8*arctanh(1/2*(x^16+1)^(1/2)*2^(1/2))*2^(1/2)-1/8*(x
^16+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 81, 52, 65, 212}

$$\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}} dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{x^{16}+1}}{\sqrt{2}}\right)}{4\sqrt{2}} - \frac{1}{24}\left(x^{16}+1\right)^{3/2} - \frac{\sqrt{x^{16}+1}}{8}$$

```
[In] Int[(x^31*Sqrt[1 + x^16])/(1 - x^16),x]
```

```
[Out] -1/8*Sqrt[1 + x^16] - (1 + x^16)^(3/2)/24 + ArcTanh[Sqrt[1 + x^16]/Sqrt[2]]
/(4*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{16}\text{Subst}\left(\int \frac{x\sqrt{1+x}}{1-x}\,dx, x, x^{16}\right) \\
&= -\frac{1}{24}\left(1+x^{16}\right)^{3/2} + \frac{1}{16}\text{Subst}\left(\int \frac{\sqrt{1+x}}{1-x}\,dx, x, x^{16}\right) \\
&= -\frac{1}{8}\sqrt{1+x^{16}} - \frac{1}{24}\left(1+x^{16}\right)^{3/2} + \frac{1}{8}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{1+x}}\,dx, x, x^{16}\right) \\
&= -\frac{1}{8}\sqrt{1+x^{16}} - \frac{1}{24}\left(1+x^{16}\right)^{3/2} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+x^{16}}\right) \\
&= -\frac{1}{8}\sqrt{1+x^{16}} - \frac{1}{24}\left(1+x^{16}\right)^{3/2} + \frac{\tanh^{-1}\left(\frac{\sqrt{1+x^{16}}}{\sqrt{2}}\right)}{4\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}}\,dx = \frac{1}{24}\left(-4-x^{16}\right)\sqrt{1+x^{16}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+x^{16}}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[In] Integrate[(x^31*Sqrt[1 + x^16])/(1 - x^16),x]
[Out] ((-4 - x^16)*Sqrt[1 + x^16])/24 + ArcTanh[Sqrt[1 + x^16]/Sqrt[2]]/(4*Sqrt[2
])
```

Maple [A] (verified)

Time = 5.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.33

method	result	size
pseudoelliptic	$-\frac{x^{16}\sqrt{x^{16}+1}}{24}-\frac{\sqrt{x^{16}+1}}{6}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(x^{8}+1\right)\sqrt{2}}{2\sqrt{x^{16}+1}}\right)}{16}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(x^{8}-1\right)\sqrt{2}}{2\sqrt{x^{16}+1}}\right)}{16}$	69
risch	$-\frac{\left(x^{16}+4\right)\sqrt{x^{16}+1}}{24}-\frac{\operatorname{RootOf}\left(\_Z^{2}-2\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^{2}-2\right)x^{16}+3\operatorname{RootOf}\left(\_Z^{2}-2\right)-4\sqrt{x^{16}+1}}{(-1+x)(1+x)\left(x^{2}+1\right)\left(x^{4}+1\right)\left(x^{8}+1\right)}\right)}{16}$	85
trager	$\left(-\frac{x^{16}}{24}-\frac{1}{6}\right)\sqrt{x^{16}+1}+\frac{\operatorname{RootOf}\left(\_Z^{2}-2\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^{2}-2\right)x^{16}+3\operatorname{RootOf}\left(\_Z^{2}-2\right)+4\sqrt{x^{16}+1}}{(-1+x)(1+x)\left(x^{2}+1\right)\left(x^{4}+1\right)\left(x^{8}+1\right)}\right)}{16}$	87

```
[In] int(x^31*(x^16+1)^(1/2)/(-x^16+1),x,method=_RETURNVERBOSE)
[Out] -1/24*x^16*(x^16+1)^(1/2)-1/6*(x^16+1)^(1/2)+1/16*2^(1/2)*arctanh(1/2*(x^8+
1)*2^(1/2)/(x^16+1)^(1/2))-1/16*2^(1/2)*arctanh(1/2*(x^8-1)*2^(1/2)/(x^16+1
)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}}\,dx = -\frac{1}{24}\left(x^{16}+4\right)\sqrt{x^{16}+1} + \frac{1}{16}\sqrt{2}\log\left(\frac{x^{16}+2\sqrt{2}\sqrt{x^{16}+1}+3}{x^{16}-1}\right)$$

```
[In] integrate(x^31*(x^16+1)^(1/2)/(-x^16+1),x, algorithm="fricas")
[Out] -1/24*(x^16 + 4)*sqrt(x^16 + 1) + 1/16*sqrt(2)*log((x^16 + 2*sqrt(2)*sqrt(x
^16 + 1) + 3)/(x^16 - 1))
```

Sympy [A] (verification not implemented)

Time = 27.37 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12

$$\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}}\,dx = -\frac{\left(x^{16}+1\right)^{\frac{3}{2}}}{24} - \frac{\sqrt{x^{16}+1}}{8} - \frac{\sqrt{2}\left(\log\left(\sqrt{x^{16}+1}-\sqrt{2}\right) - \log\left(\sqrt{x^{16}+1}+\sqrt{2}\right)\right)}{16}$$

```
[In] integrate(x**31*(x**16+1)**(1/2)/(-x**16+1),x)
[Out] -(x**16 + 1)**(3/2)/24 - sqrt(x**16 + 1)/8 - sqrt(2)*(log(sqrt(x**16 + 1) -
 sqrt(2)) - log(sqrt(x**16 + 1) + sqrt(2)))/16
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}}\,dx = -\frac{1}{24}\left(x^{16}+1\right)^{\frac{3}{2}} - \frac{1}{16}\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{x^{16}+1}}{\sqrt{2}+\sqrt{x^{16}+1}}\right) - \frac{1}{8}\sqrt{x^{16}+1}$$

```
[In] integrate(x^31*(x^16+1)^(1/2)/(-x^16+1),x, algorithm="maxima")
[Out] -1/24*(x^16 + 1)^(3/2) - 1/16*sqrt(2)*log(-(sqrt(2) - sqrt(x^16 + 1))/(sqrt
(2) + sqrt(x^16 + 1))) - 1/8*sqrt(x^16 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08

$$\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}}\,dx = -\frac{1}{24}\left(x^{16}+1\right)^{\frac{3}{2}} - \frac{1}{16}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\sqrt{x^{16}+1}\right|}{2\left(\sqrt{2}+\sqrt{x^{16}+1}\right)}\right) - \frac{1}{8}\sqrt{x^{16}+1}$$

```
[In] integrate(x^31*(x^16+1)^(1/2)/(-x^16+1),x, algorithm="giac")
[Out] -1/24*(x^16 + 1)^(3/2) - 1/16*sqrt(2)*log(1/2*abs(-2*sqrt(2) + 2*sqrt(x^16
+ 1))/(sqrt(2) + sqrt(x^16 + 1))) - 1/8*sqrt(x^16 + 1)
```

Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.71

$$\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{x^{16}+1}}{2}\right)}{8} - \frac{\sqrt{x^{16}+1}}{8} - \frac{\left(x^{16}+1\right)^{3/2}}{24}$$

```
[In] int(-(x^31*(x^16 + 1)^(1/2))/(x^16 - 1),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(x^16 + 1)^(1/2))/2))/8 - (x^16 + 1)^(1/2)/8 - (x^1
6 + 1)^(3/2)/24
```

3.1065 $\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x} dx$

Optimal result

Integrand size = 26, antiderivative size = 93

$$\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x} dx = \frac{2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}} - \frac{2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{b}}$$

```
[Out] 2*arctanh(c^(1/2)*(a+b/x)^(1/2)/a^(1/2)/(c+d/x)^(1/2))*c^(1/2)/a^(1/2)-2*ar
ctanh(d^(1/2)*(a+b/x)^(1/2)/b^(1/2)/(c+d/x)^(1/2))*d^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {457, 132, 65, 223, 212, 12, 95, 214}

$$\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x} dx = \frac{2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}} - \frac{2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{b}}$$

```
[In] Int[Sqrt[c + d/x]/(Sqrt[a + b/x]*x),x]
```

```
[Out] (2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a + b/x])/(Sqrt[a]*Sqrt[c + d/x])])/Sqrt[a
] - (2*Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[a + b/x])/(Sqrt[b]*Sqrt[c + d/x])])/Sq
rt[b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\
&= -\left(d\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, \frac{1}{x}\right)\right) - \text{Subst}\left(\int \frac{c}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, \frac{1}{x}\right)\\
&= -\left(c\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, \frac{1}{x}\right)\right) - \frac{(2d)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{b}\\
&= -\left((2c)\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\right)\right) - \frac{(2d)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\right)}{b}\\
&= \frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}} - \frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 10.50 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x}\,dx = -\frac{2\sqrt{d}\sqrt{bc-ad}\sqrt{c+\frac{d}{x}}x\sqrt{\frac{b(d+cx)}{(bc-ad)x}}\text{arcsinh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{bd+bcx} + \frac{2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}}$$

```
[In] Integrate[Sqrt[c + d/x]/(Sqrt[a + b/x]*x),x]
```

```
[Out] (-2*Sqrt[d]*Sqrt[b*c - a*d]*Sqrt[c + d/x]*x*Sqrt[(b*(d + c*x))/((b*c - a*d)
*x)]*ArcSinh[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(b*d + b*c*x) + (2*S
qrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a + b/x])/(Sqrt[a]*Sqrt[c + d/x])])/Sqrt[a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(69) = 138$.

Time = 0.15 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.54

method	result	size
default	$-\frac{\sqrt{\frac{ax+b}{x}}\, x \sqrt{\frac{cx+d}{x}}\left(\sqrt{ac}\, \ln\left(\frac{adx+bcx+2\sqrt{bd}\, \sqrt{(ax+b)(cx+d)}+2bd}{x}\right)d-\ln\left(\frac{2acx+2\sqrt{(ax+b)(cx+d)}\, \sqrt{ac}+ad+bc}{2\sqrt{ac}}\right)\sqrt{bd}\, c\right)}{\sqrt{(ax+b)(cx+d)}\, \sqrt{ac}\, \sqrt{bd}}$	143

```
[In] int((c+d/x)^(1/2)/x/(a+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*((a*c)^(1/2)*ln((a*d*x+b*c*x+2*(b*d)
^(1/2)*((a*x+b)*(c*x+d))^(1/2)+2*b*d)/x)*d-ln(1/2*(2*a*c*x+2*((a*x+b)*(c*x+
d))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*(b*d)^(1/2)*c)/((a*x+b)*(c*x+d)
)^(1/2)/(a*c)^(1/2)/(b*d)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(69) = 138$.

Time = 0.42 (sec) , antiderivative size = 757, normalized size of antiderivative = 8.14

$$\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x}\,dx = \left[\frac{1}{2}\sqrt{\frac{c}{a}}\log\left(-8\,a^2c^2x^2-b^2c^2-6\,abcd-a^2d^2 -4\left(2\,a^2cx^2+\left(abc+a^2d\right)x\right)\sqrt{\frac{c}{a}}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}-8\left(abc^2+a^2cd\right)x\right) +\frac{1}{2}\sqrt{\frac{d}{b}}\log\left(-\frac{8\,b^2d^2+(b^2c^2+6\,abcd+a^2d^2)x^2-4\,(2\,b^2dx+(b^2c+abd)x^2)\sqrt{\frac{d}{b}}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}+8\,(b^2cd\cdots}{x^2}\right. -\sqrt{-\frac{c}{a}}\arctan\left(\frac{2\,ax\sqrt{-\frac{c}{a}}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}}{2\,acx+bc+ad}\right) +\frac{1}{2}\sqrt{\frac{d}{b}}\log\left(-\frac{8\,b^2d^2+(b^2c^2+6\,abcd+a^2d^2)x^2-4\,(2\,b^2dx+(b^2c+abd)x^2)\sqrt{\frac{d}{b}}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}+8\,(b^2cd\cdots}{x^2}\right. +\frac{1}{2}\sqrt{\frac{c}{a}}\log\left(-8\,a^2c^2x^2-b^2c^2-6\,abcd-a^2d^2 -4\left(2\,a^2cx^2+\left(abc+a^2d\right)x\right)\sqrt{\frac{c}{a}}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}-8\left(abc^2+a^2cd\right)x\right), -\sqrt{-\frac{c}{a}}\arctan\left(\frac{2\,ax\sqrt{-\frac{c}{a}}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}}{2\,acx+bc+ad}\right) +\sqrt{-\frac{d}{b}}\arctan\left(\frac{(2\,bdx+(bc+ad)x^2)\sqrt{-\frac{d}{b}}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}}{2\,(acdx^2+bd^2+(bcd+ad^2)x)}\right)\right]$$

```
[In] integrate((c+d/x)^(1/2)/x/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(c/a)*log(-8*a^2*c^2*x^2 - b^2*c^2 - 6*a*b*c*d - a^2*d^2 - 4*(2*a^
2*c*x^2 + (a*b*c + a^2*d)*x)*sqrt(c/a)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)
- 8*(a*b*c^2 + a^2*c*d)*x) + 1/2*sqrt(d/b)*log(-(8*b^2*d^2 + (b^2*c^2 + 6*a
*b*c*d + a^2*d^2)*x^2 - 4*(2*b^2*d*x + (b^2*c + a*b*d)*x^2)*sqrt(d/b)*sqrt(
(a*x + b)/x)*sqrt((c*x + d)/x) + 8*(b^2*c*d + a*b*d^2)*x)/x^2), -sqrt(-c/a)
*arctan(2*a*x*sqrt(-c/a)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*a*c*x + b*c
 + a*d)) + 1/2*sqrt(d/b)*log(-(8*b^2*d^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*
x^2 - 4*(2*b^2*d*x + (b^2*c + a*b*d)*x^2)*sqrt(d/b)*sqrt((a*x + b)/x)*sqrt(
(c*x + d)/x) + 8*(b^2*c*d + a*b*d^2)*x)/x^2), sqrt(-d/b)*arctan(1/2*(2*b*d*
x + (b*c + a*d)*x^2)*sqrt(-d/b)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(a*c*d*
x^2 + b*d^2 + (b*c*d + a*d^2)*x)) + 1/2*sqrt(c/a)*log(-8*a^2*c^2*x^2 - b^2*
c^2 - 6*a*b*c*d - a^2*d^2 - 4*(2*a^2*c*x^2 + (a*b*c + a^2*d)*x)*sqrt(c/a)*s
```

```
qrt((a*x + b)/x)*sqrt((c*x + d)/x) - 8*(a*b*c^2 + a^2*c*d)*x), -sqrt(-c/a)*
arctan(2*a*x*sqrt(-c/a)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*a*c*x + b*c
+ a*d)) + sqrt(-d/b)*arctan(1/2*(2*b*d*x + (b*c + a*d)*x^2)*sqrt(-d/b)*sqrt
((a*x + b)/x)*sqrt((c*x + d)/x)/(a*c*d*x^2 + b*d^2 + (b*c*d + a*d^2)*x))]
```

Sympy [F]

$$\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x}\,dx=\int \frac{\sqrt{c+\frac{d}{x}}}{x\sqrt{a+\frac{b}{x}}}\,dx$$

```
[In] integrate((c+d/x)**(1/2)/x/(a+b/x)**(1/2),x)
[Out] Integral(sqrt(c + d/x)/(x*sqrt(a + b/x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x}\,dx=\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x}\,dx$$

```
[In] integrate((c+d/x)^(1/2)/x/(a+b/x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c + d/x)/(sqrt(a + b/x)*x), x)
```

Giac [F]

$$\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x}\,dx=\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x}\,dx$$

```
[In] integrate((c+d/x)^(1/2)/x/(a+b/x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c + d/x)/(sqrt(a + b/x)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x}\,dx = \int \frac{\sqrt{c+\frac{d}{x}}}{x\,\sqrt{a+\frac{b}{x}}}\,dx$$

```
[In] int((c + d/x)^(1/2)/(x*(a + b/x)^(1/2)),x)
[Out] int((c + d/x)^(1/2)/(x*(a + b/x)^(1/2)), x)
```

3.1066 $\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 252

$$\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64bd^4n} + \frac{5(bc-ad)(7bc+ad)\,(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96bd^3n} - \frac{(7bc+ad)\,(a+bx^n)^{5/2}\sqrt{c+dx^n}}{24bd^2n} + \frac{(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bdn} + \frac{5(bc-ad)^3(7bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{64b^{3/2}d^{9/2}n}$$

```
[Out] 5/64*(-a*d+b*c)^3*(a*d+7*b*c)*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d*
x^n)^(1/2))/b^(3/2)/d^(9/2)/n+5/96*(-a*d+b*c)*(a*d+7*b*c)*(a+b*x^n)^(3/2)*(
c+d*x^n)^(1/2)/b/d^3/n-1/24*(a*d+7*b*c)*(a+b*x^n)^(5/2)*(c+d*x^n)^(1/2)/b/d
^2/n+1/4*(a+b*x^n)^(7/2)*(c+d*x^n)^(1/2)/b/d/n-5/64*(-a*d+b*c)^2*(a*d+7*b*c
)*(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b/d^4/n
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {457, 81, 52, 65, 223, 212}

$$\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}dx = \frac{5(bc-ad)^3(ad+7bc)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{64b^{3/2}d^{9/2}n} - \frac{5(bc-ad)^2(ad+7bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64bd^4n} + \frac{5(bc-ad)(ad+7bc)\,(a+bx^n)^{3/2}\,\sqrt{c+dx^n}}{96bd^3n} - \frac{(ad+7bc)\,(a+bx^n)^{5/2}\,\sqrt{c+dx^n}}{24bd^2n} + \frac{(a+bx^n)^{7/2}\,\sqrt{c+dx^n}}{4bdn}$$

```
[In] Int[(x^(-1 + 2*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n],x]
```

```
[Out] (-5*(b*c - a*d)^2*(7*b*c + a*d)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(64*b*d^4*
n) + (5*(b*c - a*d)*(7*b*c + a*d)*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(96*b*
d^3*n) - ((7*b*c + a*d)*(a + b*x^n)^(5/2)*Sqrt[c + d*x^n])/(24*b*d^2*n) + (
(a + b*x^n)^(7/2)*Sqrt[c + d*x^n])/(4*b*d*n) + (5*(b*c - a*d)^3*(7*b*c + a*
d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(64*b^(3/2
)*d^(9/2)*n)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{n}\\
&= \frac{(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bdn} - \frac{(7bc+ad)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{8bdn}\\
&= -\frac{(7bc+ad)\,(a+bx^n)^{5/2}\sqrt{c+dx^n}}{24bd^2n} + \frac{(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bdn}\\
&\quad + \frac{(5(bc-ad)(7bc+ad))\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{48bd^2n}\\
&= \frac{5(bc-ad)(7bc+ad)\,(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96bd^3n} - \frac{(7bc+ad)\,(a+bx^n)^{5/2}\sqrt{c+dx^n}}{24bd^2n}\\
&\quad + \frac{(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bdn} - \frac{(5(bc-ad)^2(7bc+ad))\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{64bd^3n}\\
&= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64bd^4n}\\
&\quad + \frac{5(bc-ad)(7bc+ad)\,(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96bd^3n} - \frac{(7bc+ad)\,(a+bx^n)^{5/2}\sqrt{c+dx^n}}{24bd^2n}\\
&\quad + \frac{(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bdn} + \frac{(5(bc-ad)^3(7bc+ad))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{128bd^4n}
\end{aligned}$$

$$= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64bd^4n} + \frac{5(bc-ad)(7bc+ad)\left(a+bx^n\right)^{3/2}\sqrt{c+dx^n}}{96bd^3n} - \frac{(7bc+ad)\left(a+bx^n\right)^{5/2}\sqrt{c+dx^n}}{24bd^2n} + \frac{\left(a+bx^n\right)^{7/2}\sqrt{c+dx^n}}{4bdn} + \frac{(5(bc-ad)^3(7bc+ad))\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^n}\right)}{64b^2d^4n}$$

$$= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64bd^4n} + \frac{5(bc-ad)(7bc+ad)\left(a+bx^n\right)^{3/2}\sqrt{c+dx^n}}{96bd^3n} - \frac{(7bc+ad)\left(a+bx^n\right)^{5/2}\sqrt{c+dx^n}}{24bd^2n} + \frac{\left(a+bx^n\right)^{7/2}\sqrt{c+dx^n}}{4bdn} + \frac{(5(bc-ad)^3(7bc+ad))\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\right)}{64b^2d^4n}$$

$$= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64bd^4n} + \frac{5(bc-ad)(7bc+ad)\left(a+bx^n\right)^{3/2}\sqrt{c+dx^n}}{96bd^3n} - \frac{(7bc+ad)\left(a+bx^n\right)^{5/2}\sqrt{c+dx^n}}{24bd^2n} + \frac{\left(a+bx^n\right)^{7/2}\sqrt{c+dx^n}}{4bdn} + \frac{5(bc-ad)^3(7bc+ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{64b^{3/2}d^{9/2}n}$$

Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.88

$$\int \frac{x^{-1+2n}\left(a+bx^n\right)^{5/2}}{\sqrt{c+dx^n}}\,dx = \frac{b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)\left(15a^3d^3+a^2bd^2(-191c+118dx^n)+ab^2d(265c^2-172cd\right.}{}$$

```
[In] Integrate[(x^(-1 + 2*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(15*a^3*d^3 + a^2*b*d^2*(-191*c + 11
8*d*x^n) + a*b^2*d*(265*c^2 - 172*c*d*x^n + 136*d^2*x^(2*n)) + b^3*(-105*c^
3 + 70*c^2*d*x^n - 56*c*d^2*x^(2*n) + 48*d^3*x^(3*n))) + 15*(b*c - a*d)^(7/
2)*(7*b*c + a*d)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a
+ b*x^n])/Sqrt[b*c - a*d]])/(192*b^2*d^(9/2)*n*Sqrt[c + d*x^n])
```

Maple [F]

$$\int \frac{x^{-1+2n}(a+b\,x^n)^{\frac{5}{2}}}{\sqrt{c+d\,x^n}}dx$$

[In] int(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)

[Out] int(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 607, normalized size of antiderivative = 2.41

$$\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \left[-\frac{15\,(7\,b^4c^4-20\,ab^3c^3d+18\,a^2b^2c^2d^2-4\,a^3bcd^3-a^4d^4)\sqrt{bd}\log\left(8\,b^2d^2x^{2\,n}+\right.}{}\right.$$

$$-\frac{15\,(7\,b^4c^4-20\,ab^3c^3d+18\,a^2b^2c^2d^2-4\,a^3bcd^3-a^4d^4)\sqrt{-bd}\arctan\left(\frac{(2\,\sqrt{-bd}bdx^n+(bc+ad)\sqrt{-bd})\sqrt{bx^n+a}\sqrt{dx^n+c}}{2\,(b^2d^2x^{2\,n}+abcd+(b^2cd+abd^2)x^n)}\right)}{}$$

[In] integrate(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")

[Out] [-1/768*(15*(7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) - 4*(48*b^4*d^4*x^(3*n) - 105*b^4*c^3*d + 265*a*b^3*c^2*d^2 - 191*a^2*b^2*c*d^3 + 15*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 17*a*b^3*d^4)*x^(2*n) + 2*(35*b^4*c^2*d^2 - 86*a*b^3*c*d^3 + 59*a^2*b^2*d^4)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^2*d^5*n), -1/384*(15*(7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*d^4)*sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)) - 2*(48*b^4*d^4*x^(3*n) - 105*b^4*c^3*d + 265*a*b^3*c^2*d^2 - 191*a^2*b^2*c*d^3 + 15*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 17*a*b^3*d^4)*x^(2*n) + 2*(35*b^4*c^2*d^2 - 86*a*b^3*c*d^3 + 59*a^2*b^2*d^4)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^2*d^5*n)]

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \text{Timed out}$$

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)**(5/2)/(c+d*x**n)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \int \frac{(bx^n+a)^{\frac{5}{2}}x^{2n-1}}{\sqrt{dx^n+c}}\,dx$$

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^n + a)^(5/2)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

Giac [F]

$$\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \int \frac{(bx^n+a)^{\frac{5}{2}}x^{2n-1}}{\sqrt{dx^n+c}}\,dx$$

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^n + a)^(5/2)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \int \frac{x^{2n-1}\,(a+b\,x^n)^{5/2}}{\sqrt{c+d\,x^n}}\,dx$$

```
[In] int((x^(2*n - 1)*(a + b*x^n)^(5/2))/(c + d*x^n)^(1/2),x)
[Out] int((x^(2*n - 1)*(a + b*x^n)^(5/2))/(c + d*x^n)^(1/2), x)
```

3.1067 $\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 199

$$\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8bd^3n} - \frac{(5bc+ad)\left(a+bx^n\right)^{3/2}\sqrt{c+dx^n}}{12bd^2n} + \frac{\left(a+bx^n\right)^{5/2}\sqrt{c+dx^n}}{3bdn} - \frac{(bc-ad)^2(5bc+ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{8b^{3/2}d^{7/2}n}$$

```
[Out] -1/8*(-a*d+b*c)^2*(a*d+5*b*c)*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d*
x^n)^(1/2))/b^(3/2)/d^(7/2)/n-1/12*(a*d+5*b*c)*(a+b*x^n)^(3/2)*(c+d*x^n)^(1
/2)/b/d^2/n+1/3*(a+b*x^n)^(5/2)*(c+d*x^n)^(1/2)/b/d/n+1/8*(-a*d+b*c)*(a*d+5
*b*c)*(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b/d^3/n
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {457, 81, 52, 65, 223, 212}

$$\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = -\frac{(bc-ad)^2(ad+5bc)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{8b^{3/2}d^{7/2}n} + \frac{(bc-ad)(ad+5bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8bd^3n} - \frac{(ad+5bc)\left(a+bx^n\right)^{3/2}\sqrt{c+dx^n}}{12bd^2n} + \frac{\left(a+bx^n\right)^{5/2}\sqrt{c+dx^n}}{3bdn}$$

```
[In] Int[(x^(-1 + 2*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n],x]
```

```
[Out] ((b*c - a*d)*(5*b*c + a*d)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(8*b*d^3*n) - (
(5*b*c + a*d)*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(12*b*d^2*n) + ((a + b*x^n
)^(5/2)*Sqrt[c + d*x^n])/(3*b*d*n) - ((b*c - a*d)^2*(5*b*c + a*d)*ArcTanh[(
Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(8*b^(3/2)*d^(7/2)*n)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{n}\\
&= \frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bdn} - \frac{(5bc+ad)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{6bdn}\\
&= -\frac{(5bc+ad)\,(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12bd^2n} + \frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bdn}\\
&\quad + \frac{((bc-ad)(5bc+ad))\text{Subst}\left(\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{8bd^2n}\\
&= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8bd^3n} - \frac{(5bc+ad)\,(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12bd^2n}\\
&\quad + \frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bdn} - \frac{\left((bc-ad)^2(5bc+ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{16bd^3n}\\
&= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8bd^3n} - \frac{(5bc+ad)\,(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12bd^2n}\\
&\quad + \frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bdn} - \frac{\left((bc-ad)^2(5bc+ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^n}\right)}{8b^2d^3n}\\
&= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8bd^3n} - \frac{(5bc+ad)\,(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12bd^2n}\\
&\quad + \frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bdn} - \frac{\left((bc-ad)^2(5bc+ad)\right)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\right)}{8b^2d^3n}\\
&= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8bd^3n} - \frac{(5bc+ad)\,(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12bd^2n}\\
&\quad + \frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bdn} - \frac{(bc-ad)^2(5bc+ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{8b^{3/2}d^{7/2}n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.89

$$\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \frac{b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)\left(3a^2d^2+2abd(-11c+7dx^n)+b^2(15c^2-10cdx^n+8d^2x^2\right.}{24b^2d^{7/2}n\sqrt{c+dx^n}}$$

```
[In] Integrate[(x^(-1 + 2*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(3*a^2*d^2 + 2*a*b*d*(-11*c + 7*d*x^
n) + b^2*(15*c^2 - 10*c*d*x^n + 8*d^2*x^(2*n))) - 3*(b*c - a*d)^(5/2)*(5*b*
c + a*d)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n]
)/Sqrt[b*c - a*d]])/(24*b^2*d^(7/2)*n*Sqrt[c + d*x^n])
```

Maple [F]

$$\int \frac{x^{-1+2n}(a+b\,x^n)^{\frac{3}{2}}}{\sqrt{c+d\,x^n}}dx$$

```
[In] int(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

```
[Out] int(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.36

$$\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \left[\frac{3\,(5\,b^3c^3-9\,ab^2c^2d+3\,a^2bcd^2+a^3d^3)\sqrt{bd}\log\left(8\,b^2d^2x^{2\,n}+b^2c^2+6\,abcd+a^2d\right.}{}\right.$$

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*sqrt(b*d)*lo
g(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*x^
n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a
*b*d^2)*x^n) + 4*(8*b^3*d^3*x^(2*n) + 15*b^3*c^2*d - 22*a*b^2*c*d^2 + 3*a^2
*b*d^3 - 2*(5*b^3*c*d^2 - 7*a*b^2*d^3)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c)
)/(b^2*d^4*n), 1/48*(3*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3
)*sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqr
t(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^
2)*x^n)) + 2*(8*b^3*d^3*x^(2*n) + 15*b^3*c^2*d - 22*a*b^2*c*d^2 + 3*a^2*b*d
^3 - 2*(5*b^3*c*d^2 - 7*a*b^2*d^3)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b
^2*d^4*n)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \text{Timed out}$$

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)**(3/2)/(c+d*x**n)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \int \frac{(bx^n+a)^{\frac{3}{2}}x^{2n-1}}{\sqrt{dx^n+c}}\,dx$$

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^n + a)^(3/2)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

Giac [F]

$$\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \int \frac{(bx^n+a)^{\frac{3}{2}}x^{2n-1}}{\sqrt{dx^n+c}}\,dx$$

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^n + a)^(3/2)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \int \frac{x^{2n-1}\,(a+b\,x^n)^{3/2}}{\sqrt{c+d\,x^n}}\,dx$$

```
[In] int((x^(2*n - 1)*(a + b*x^n)^(3/2))/(c + d*x^n)^(1/2),x)
[Out] int((x^(2*n - 1)*(a + b*x^n)^(3/2))/(c + d*x^n)^(1/2), x)
```

3.1068 $\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 146

$$\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx=-\frac{(3bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n}+\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn}+\frac{(bc-ad)(3bc+ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{4b^{3/2}d^{5/2}n}$$

```
[Out] 1/4*(-a*d+b*c)*(a*d+3*b*c)*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d*x^n
)^(1/2))/b^(3/2)/d^(5/2)/n+1/2*(a+b*x^n)^(3/2)*(c+d*x^n)^(1/2)/b/d/n-1/4*(a
*d+3*b*c)*(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b/d^2/n
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {457, 81, 52, 65, 223, 212}

$$\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx=\frac{(bc-ad)(ad+3bc)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{4b^{3/2}d^{5/2}n}-\frac{(ad+3bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n}+\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn}$$

```
[In] Int[(x^(-1 + 2*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n],x]
```

```
[Out] -1/4*((3*b*c + a*d)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(b*d^2*n) + ((a + b*x^
n)^(3/2)*Sqrt[c + d*x^n])/(2*b*d*n) + ((b*c - a*d)*(3*b*c + a*d)*ArcTanh[(S
qrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(4*b^(3/2)*d^(5/2)*n)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$=\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn}-\frac{(3bc+ad)\text{Subst}\left(\int\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx,x,x^n\right)}{4bdn}$$

$$=-\frac{(3bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n}+\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn}+\frac{((bc-ad)(3bc+ad))\text{Subst}\left(\int\frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx,x,x^n\right)}{8bd^2n}$$

$$=-\frac{(3bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n}+\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn}+\frac{((bc-ad)(3bc+ad))\text{Subst}\left(\int\frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx^n}\right)}{4b^2d^2n}$$

$$=-\frac{(3bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n}+\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn}+\frac{((bc-ad)(3bc+ad))\text{Subst}\left(\int\frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\right)}{4b^2d^2n}$$

$$=-\frac{(3bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n}+\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn}+\frac{(bc-ad)(3bc+ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{4b^{3/2}d^{5/2}n}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.97

$$\int\frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx=\frac{b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)\left(-3bc+ad+2bdx^n\right)+(bc-ad)^{3/2}(3bc+ad)\sqrt{\frac{b(c+dx^n)}{bc-ad}}\text{arcsinh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{4b^2d^{5/2}n\sqrt{c+dx^n}}$$

```
[In] Integrate[(x^(-1 + 2*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(-3*b*c + a*d + 2*b*d*x^n) + (b*c -
a*d)^(3/2)*(3*b*c + a*d)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]
*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(4*b^2*d^(5/2)*n*Sqrt[c + d*x^n])
```

Maple [F]

$$\int \frac{x^{-1+2n}\sqrt{a+b\,x^n}}{\sqrt{c+d\,x^n}}dx$$

```
[In] int(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

```
[Out] int(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.46

$$\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx = \left[-\frac{(3\,b^2c^2-2\,abcd-a^2d^2)\sqrt{bd}\log\left(8\,b^2d^2x^{2\,n}+b^2c^2+6\,abcd+a^2d^2-4\left(2\,\sqrt{bd}bdx^n+(bc+ad)\sqrt{bd}\right)\sqrt{\ }\right.}{16\,b^2d^3n} -\frac{(3\,b^2c^2-2\,abcd-a^2d^2)\sqrt{-bd}\arctan\left(\frac{\left(2\,\sqrt{-bd}bdx^n+(bc+ad)\sqrt{-bd}\right)\sqrt{bx^n+a}\sqrt{dx^n+c}}{2\,(b^2d^2x^{2\,n}+abcd+(b^2cd+abd^2)x^n)}\right)-2\,(2\,b^2d^2x^n-3\,b^2cd+a}{8\,b^2d^3n}\right.$$

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) +
 b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(
b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) - 4*(2*b
^2*d^2*x^n - 3*b^2*c*d + a*b*d^2)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^2*d^3
*n), -1/8*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*sqrt(
-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c)/(b^
2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)) - 2*(2*b^2*d^2*x^n - 3*
b^2*c*d + a*b*d^2)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^2*d^3*n)]
```

Sympy [F]

$$\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx = \int \frac{x^{2n-1}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$$

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)**(1/2)/(c+d*x**n)**(1/2),x)
```

```
[Out] Integral(x**(2*n - 1)*sqrt(a + b*x**n)/sqrt(c + d*x**n), x)
```

Maxima [F]

$$\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx = \int \frac{\sqrt{bx^n+a}x^{2\,n-1}}{\sqrt{dx^n+c}}\,dx$$

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^n + a)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

Giac [F]

$$\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx = \int \frac{\sqrt{bx^n+a}x^{2\,n-1}}{\sqrt{dx^n+c}}\,dx$$

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^n + a)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx = \int \frac{x^{2\,n-1}\,\sqrt{a+b\,x^n}}{\sqrt{c+d\,x^n}}\,dx$$

```
[In] int((x^(2*n - 1)*(a + b*x^n)^(1/2))/(c + d*x^n)^(1/2),x)
[Out] int((x^(2*n - 1)*(a + b*x^n)^(1/2))/(c + d*x^n)^(1/2), x)
```

3.1069 $\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 89

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx=\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn}-\frac{(bc+ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}d^{3/2}n}$$

```
[Out] -(a*d+b*c)*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(3/2)
/d^(3/2)/n+(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b/d/n
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {457, 81, 65, 223, 212}

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx=\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn}-\frac{(ad+bc)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}d^{3/2}n}$$

```
[In] Int[x^(-1 + 2*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]),x]
```

```
[Out] (Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(b*d*n) - ((b*c + a*d)*ArcTanh[(Sqrt[d]*S
qrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(3/2)*d^(3/2)*n)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{n} \\
&= \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn} - \frac{(bc+ad)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{2bdn} \\
&= \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn} - \frac{(bc+ad)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^n}\right)}{b^2dn} \\
&= \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn} - \frac{(bc+ad)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\right)}{b^2dn} \\
&= \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn} - \frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}d^{3/2}n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.38

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = \frac{b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)-\sqrt{bc-ad}(bc+ad)\sqrt{\frac{b(c+dx^n)}{bc-ad}}\operatorname{arcsinh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{b^2d^{3/2}n\sqrt{c+dx^n}}$$

```
[In] Integrate[x^(-1 + 2*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]),x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n) - Sqrt[b*c - a*d]*(b*c + a*d)*Sqrt[(
b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*
d]])/(b^2*d^(3/2)*n*Sqrt[c + d*x^n])
```

Maple [F]

$$\int \frac{x^{-1+2n}}{\sqrt{a+b\,x^n}\,\sqrt{c+d\,x^n}}dx$$

```
[In] int(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

```
[Out] int(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 281, normalized size of antiderivative = 3.16

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = \left[\frac{4\sqrt{bx^n+a}\sqrt{dx^n+c}bd+(bc+ad)\sqrt{bd}\log\left(8\,b^2d^2x^{2\,n}+b^2c^2+6\,abcd+a^2d^2-4\left(2\,\sqrt{bd}bdx^n+(bc+a\right.\right.}{4\,b^2d^2n}\right.$$

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(b*x^n + a)*sqrt(d*x^n + c)*b*d + (b*c + a*d)*sqrt(b*d)*log(8*b
^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*x^n + (
b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^
2)*x^n))/(b^2*d^2*n), 1/2*(2*sqrt(b*x^n + a)*sqrt(d*x^n + c)*b*d + (b*c + a
*d)*sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*s
qrt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*
d^2)*x^n)))/(b^2*d^2*n)]
```

Sympy [F]

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = \int \frac{x^{2n-1}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx$$

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)**(1/2)/(c+d*x**n)**(1/2),x)
[Out] Integral(x**(2*n - 1)/(sqrt(a + b*x**n)*sqrt(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = \int \frac{x^{2\,n-1}}{\sqrt{bx^n+a}\sqrt{dx^n+c}}\,dx$$

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(2*n - 1)/(sqrt(b*x^n + a)*sqrt(d*x^n + c)), x)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = \int \frac{x^{2\,n-1}}{\sqrt{bx^n+a}\sqrt{dx^n+c}}\,dx$$

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(x^(2*n - 1)/(sqrt(b*x^n + a)*sqrt(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = \int \frac{x^{2\,n-1}}{\sqrt{a+b\,x^n}\,\sqrt{c+d\,x^n}}\,dx$$

```
[In] int(x^(2*n - 1)/((a + b*x^n)^(1/2)*(c + d*x^n)^(1/2)),x)
[Out] int(x^(2*n - 1)/((a + b*x^n)^(1/2)*(c + d*x^n)^(1/2)), x)
```

3.1070 $\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 91

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \frac{2a\sqrt{c+dx^n}}{b(bc-ad)n\sqrt{a+bx^n}} + \frac{2\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}\sqrt{d}n}$$

```
[Out] 2*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(3/2)/n/d^(1/2
)+2*a*(c+d*x^n)^(1/2)/b/(-a*d+b*c)/n/(a+b*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {457, 79, 65, 223, 212}

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}\sqrt{d}n} + \frac{2a\sqrt{c+dx^n}}{bn(bc-ad)\sqrt{a+bx^n}}$$

```
[In] Int[x^(-1 + 2*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (2*a*Sqrt[c + d*x^n])/(b*(b*c - a*d)*n*Sqrt[a + b*x^n]) + (2*ArcTanh[(Sqrt[
d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(3/2)*Sqrt[d]*n)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx, x, x^n\right)}{n} \\
&= \frac{2a\sqrt{c+dx^n}}{b(bc-ad)n\sqrt{a+bx^n}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{bn} \\
&= \frac{2a\sqrt{c+dx^n}}{b(bc-ad)n\sqrt{a+bx^n}} + \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^n}\right)}{b^2n} \\
&= \frac{2a\sqrt{c+dx^n}}{b(bc-ad)n\sqrt{a+bx^n}} + \frac{2\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\right)}{b^2n}
\end{aligned}$$

$$= \frac{2a\sqrt{c+dx^n}}{b(bc-ad)n\sqrt{a+bx^n}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}\sqrt{d}n}$$

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.34

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \frac{2\left(\frac{ab(c+dx^n)}{(bc-ad)\sqrt{a+bx^n}} + \frac{\sqrt{bc-ad}\sqrt{\frac{b(c+dx^n)}{bc-ad}}\operatorname{arcsinh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{\sqrt{d}}\right)}{b^2n\sqrt{c+dx^n}}$$

```
[In] Integrate[x^(-1 + 2*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]
[Out] (2*((a*b*(c + d*x^n))/((b*c - a*d)*Sqrt[a + b*x^n]) + (Sqrt[b*c - a*d]*Sqrt
[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c -
a*d]])/Sqrt[d]))/(b^2*n*Sqrt[c + d*x^n])
```

Maple [F]

$$\int \frac{x^{-1+2n}}{(a+b\,x^n)^{\frac{3}{2}}\sqrt{c+d\,x^n}}dx$$

```
[In] int(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
[Out] int(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. 2(75) = 150.

Time = 0.35 (sec) , antiderivative size = 408, normalized size of antiderivative = 4.48

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \left[\frac{4\sqrt{bx^n+a}\sqrt{dx^n+c}abd + \left((b^2c-abd)\sqrt{bd}x^n + (abc-a^2d)\sqrt{bd}\right)\log\left(8\,b\right.}{2\,((b^4c}\right.$$

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
[Out] [1/2*(4*sqrt(b*x^n + a)*sqrt(d*x^n + c)*a*b*d + ((b^2*c - a*b*d)*sqrt(b*d)*
x^n + (a*b*c - a^2*d)*sqrt(b*d))*log(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*
d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n +
a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n))/((b^4*c*d - a*b^3*d^2)*n*x
^n + (a*b^3*c*d - a^2*b^2*d^2)*n), (2*sqrt(b*x^n + a)*sqrt(d*x^n + c)*a*b*d
 - ((b^2*c - a*b*d)*sqrt(-b*d)*x^n + (a*b*c - a^2*d)*sqrt(-b*d))*arctan(1/2
*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n
 + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)))/((b^4*c*d - a
*b^3*d^2)*n*x^n + (a*b^3*c*d - a^2*b^2*d^2)*n)]
```

Sympy [F]

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{2n-1}}{(a+bx^n)^{\frac{3}{2}}\sqrt{c+dx^n}}\,dx$$

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)**(3/2)/(c+d*x**n)**(1/2),x)
[Out] Integral(x**(2*n - 1)/((a + b*x**n)**(3/2)*sqrt(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{2n-1}}{(bx^n+a)^{\frac{3}{2}}\sqrt{dx^n+c}}\,dx$$

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(2*n - 1)/((b*x^n + a)^(3/2)*sqrt(d*x^n + c)), x)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{2n-1}}{(bx^n+a)^{\frac{3}{2}}\sqrt{dx^n+c}}\,dx$$

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(x^(2*n - 1)/((b*x^n + a)^(3/2)*sqrt(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{2n-1}}{(a+b\,x^n)^{3/2}\,\sqrt{c+d\,x^n}}\,dx$$

```
[In] int(x^(2*n - 1)/((a + b*x^n)^(3/2)*(c + d*x^n)^(1/2)),x)
[Out] int(x^(2*n - 1)/((a + b*x^n)^(3/2)*(c + d*x^n)^(1/2)), x)
```

3.1071 $\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 95

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \frac{2a\sqrt{c+dx^n}}{3b(bc-ad)n\,(a+bx^n)^{3/2}} - \frac{2(3bc-ad)\sqrt{c+dx^n}}{3b(bc-ad)^2n\sqrt{a+bx^n}}$$

```
[Out] 2/3*a*(c+d*x^n)^(1/2)/b/(-a*d+b*c)/n/(a+b*x^n)^(3/2)-2/3*(-a*d+3*b*c)*(c+d*
x^n)^(1/2)/b/(-a*d+b*c)^2/n/(a+b*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 79, 37}

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \frac{2a\sqrt{c+dx^n}}{3bn(bc-ad)\,(a+bx^n)^{3/2}} - \frac{2(3bc-ad)\sqrt{c+dx^n}}{3bn(bc-ad)^2\sqrt{a+bx^n}}$$

```
[In] Int[x^(-1 + 2*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (2*a*Sqrt[c + d*x^n])/(3*b*(b*c - a*d)*n*(a + b*x^n)^(3/2)) - (2*(3*b*c - a
*d)*Sqrt[c + d*x^n])/(3*b*(b*c - a*d)^2*n*Sqrt[a + b*x^n])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx, x, x^n\right)}{n}\\
&= \frac{2a\sqrt{c+dx^n}}{3b(bc-ad)n\left(a+bx^n\right)^{3/2}} + \frac{(3bc-ad)\text{Subst}\left(\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx, x, x^n\right)}{3b(bc-ad)n}\\
&= \frac{2a\sqrt{c+dx^n}}{3b(bc-ad)n\left(a+bx^n\right)^{3/2}} - \frac{2(3bc-ad)\sqrt{c+dx^n}}{3b(bc-ad)^2n\sqrt{a+bx^n}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.60

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \frac{2\sqrt{c+dx^n}(-2ac-3bcx^n+adx^n)}{3(bc-ad)^2n\left(a+bx^n\right)^{3/2}}$$

```
[In] Integrate[x^(-1 + 2*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),x]
[Out] (2*Sqrt[c + d*x^n]*(-2*a*c - 3*b*c*x^n + a*d*x^n))/(3*(b*c - a*d)^2*n*(a +
b*x^n)^(3/2))
```

Maple [F]

$$\int \frac{x^{-1+2n}}{(a+b\,x^{n})^{\frac{5}{2}}\sqrt{c+d\,x^{n}}}dx$$

[In] int(x^(-1+2*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)

[Out] int(x^(-1+2*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.42

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = -\frac{2\,(2\,ac+(3\,bc-ad)x^n)\sqrt{bx^n+a}\sqrt{dx^n+c}}{3\,((b^4c^2-2\,ab^3cd+a^2b^2d^2)nx^{2\,n}+2\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2)nx^n+(a^2b^2c^2-2\,a^3bcd+a^4d^2)n)}$$

[In] integrate(x^(-1+2*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")

[Out] -2/3*(2*a*c + (3*b*c - a*d)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*n*x^(2*n) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*n*x^n + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*n)

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \text{Timed out}$$

[In] integrate(x**(-1+2*n)/(a+b*x**n)**(5/2)/(c+d*x**n)**(1/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{2\,n-1}}{(bx^n+a)^{\frac{5}{2}}\sqrt{dx^n+c}}\,dx$$

[In] integrate(x^(-1+2*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")

[Out] integrate(x^(2*n - 1)/((b*x^n + a)^(5/2)*sqrt(d*x^n + c)), x)

Giac [F]

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{2\,n-1}}{(bx^n+a)^{\frac{5}{2}}\sqrt{dx^n+c}}\,dx$$

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(x^(2*n - 1)/((b*x^n + a)^(5/2)*sqrt(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{2\,n-1}}{(a+b\,x^n)^{5/2}\,\sqrt{c+d\,x^n}}\,dx$$

```
[In] int(x^(2*n - 1)/((a + b*x^n)^(5/2)*(c + d*x^n)^(1/2)),x)
[Out] int(x^(2*n - 1)/((a + b*x^n)^(5/2)*(c + d*x^n)^(1/2)), x)
```

3.1072 $\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 358

$$\begin{aligned}\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx &= \frac{(bc-ad)^2\left(63b^2c^2+14abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{128b^2d^5n} \\ &- \frac{(bc-ad)\left(63b^2c^2+14abcd+3a^2d^2\right)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{192b^2d^4n} \\ &+ \frac{\left(63b^2c^2+14abcd+3a^2d^2\right)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{240b^2d^3n} \\ &- \frac{3(3bc+ad)(a+bx^n)^{7/2}\sqrt{c+dx^n}}{40b^2d^2n} + \frac{x^n(a+bx^n)^{7/2}\sqrt{c+dx^n}}{5bdn} \\ &- \frac{(bc-ad)^3\left(63b^2c^2+14abcd+3a^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{128b^{5/2}d^{11/2}n}\end{aligned}$$

```
[Out] -1/128*(-a*d+b*c)^3*(3*a^2*d^2+14*a*b*c*d+63*b^2*c^2)*arctanh(d^(1/2)*(a+b*
x^n)^(1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(5/2)/d^(11/2)/n-1/192*(-a*d+b*c)*(3*
a^2*d^2+14*a*b*c*d+63*b^2*c^2)*(a+b*x^n)^(3/2)*(c+d*x^n)^(1/2)/b^2/d^4/n+1/
240*(3*a^2*d^2+14*a*b*c*d+63*b^2*c^2)*(a+b*x^n)^(5/2)*(c+d*x^n)^(1/2)/b^2/d
^3/n-3/40*(a*d+3*b*c)*(a+b*x^n)^(7/2)*(c+d*x^n)^(1/2)/b^2/d^2/n+1/5*x^n*(a+
b*x^n)^(7/2)*(c+d*x^n)^(1/2)/b/d/n+1/128*(-a*d+b*c)^2*(3*a^2*d^2+14*a*b*c*d
+63*b^2*c^2)*(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b^2/d^5/n
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {457, 92, 81, 52, 65, 223, 212}

$$\begin{aligned}
&\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \\
&-\frac{(bc-ad)^3\left(3a^2d^2+14abcd+63b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{128b^{5/2}d^{11/2}n} \\
&+\frac{(bc-ad)^2\left(3a^2d^2+14abcd+63b^2c^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{128b^2d^5n} \\
&-\frac{(bc-ad)\left(3a^2d^2+14abcd+63b^2c^2\right)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{192b^2d^4n} \\
&+\frac{\left(3a^2d^2+14abcd+63b^2c^2\right)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{240b^2d^3n} \\
&-\frac{3(ad+3bc)(a+bx^n)^{7/2}\sqrt{c+dx^n}}{40b^2d^2n}+\frac{x^n(a+bx^n)^{7/2}\sqrt{c+dx^n}}{5bdn}
\end{aligned}$$

```
[In] Int[(x^(-1 + 3*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n],x]
```

```
[Out] ((b*c - a*d)^2*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x^n]*Sqrt[c
 + d*x^n])/(128*b^2*d^5*n) - ((b*c - a*d)*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*
d^2)*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(192*b^2*d^4*n) + ((63*b^2*c^2 + 14
*a*b*c*d + 3*a^2*d^2)*(a + b*x^n)^(5/2)*Sqrt[c + d*x^n])/(240*b^2*d^3*n) -
(3*(3*b*c + a*d)*(a + b*x^n)^(7/2)*Sqrt[c + d*x^n])/(40*b^2*d^2*n) + (x^n*(
a + b*x^n)^(7/2)*Sqrt[c + d*x^n])/(5*b*d*n) - ((b*c - a*d)^3*(63*b^2*c^2 +
14*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c +
 d*x^n])])/(128*b^(5/2)*d^(11/2)*n)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= \frac{x^n(a+bx^n)^{7/2}\sqrt{c+dx^n}}{5bdn} + \frac{\text{Subst}\left(\int \frac{(a+bx)^{5/2}\left(-ac-\frac{3}{2}(3bc+ad)x\right)}{\sqrt{c+dx}}\,dx, x, x^n\right)}{5bdn}$$

$$= -\frac{3(3bc+ad)\left(a+bx^n\right)^{7/2}\sqrt{c+dx^n}}{40b^2d^2n} + \frac{x^n\left(a+bx^n\right)^{7/2}\sqrt{c+dx^n}}{5bdn} + \frac{\left(63b^2c^2+14abcd+3a^2d^2\right)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx,x,x^n\right)}{80b^2d^2n}$$

$$= \frac{\left(63b^2c^2+14abcd+3a^2d^2\right)\left(a+bx^n\right)^{5/2}\sqrt{c+dx^n}}{240b^2d^3n} - \frac{3(3bc+ad)\left(a+bx^n\right)^{7/2}\sqrt{c+dx^n}}{40b^2d^2n} + \frac{x^n\left(a+bx^n\right)^{7/2}\sqrt{c+dx^n}}{5bdn} - \frac{\left((bc-ad)\left(63b^2c^2+14abcd+3a^2d^2\right)\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx,x,x^n\right)}{96b^2d^3n}$$

$$= -\frac{(bc-ad)\left(63b^2c^2+14abcd+3a^2d^2\right)\left(a+bx^n\right)^{3/2}\sqrt{c+dx^n}}{192b^2d^4n} + \frac{\left(63b^2c^2+14abcd+3a^2d^2\right)\left(a+bx^n\right)^{5/2}\sqrt{c+dx^n}}{240b^2d^3n} - \frac{3(3bc+ad)\left(a+bx^n\right)^{7/2}\sqrt{c+dx^n}}{40b^2d^2n} + \frac{x^n\left(a+bx^n\right)^{7/2}\sqrt{c+dx^n}}{5bdn} + \frac{\left((bc-ad)^2\left(63b^2c^2+14abcd+3a^2d^2\right)\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx,x,x^n\right)}{128b^2d^4n}$$

$$= \frac{(bc-ad)^2\left(63b^2c^2+14abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{128b^2d^5n} - \frac{(bc-ad)\left(63b^2c^2+14abcd+3a^2d^2\right)\left(a+bx^n\right)^{3/2}\sqrt{c+dx^n}}{192b^2d^4n} + \frac{\left(63b^2c^2+14abcd+3a^2d^2\right)\left(a+bx^n\right)^{5/2}\sqrt{c+dx^n}}{240b^2d^3n} - \frac{3(3bc+ad)\left(a+bx^n\right)^{7/2}\sqrt{c+dx^n}}{40b^2d^2n} + \frac{x^n\left(a+bx^n\right)^{7/2}\sqrt{c+dx^n}}{5bdn} - \frac{\left((bc-ad)^3\left(63b^2c^2+14abcd+3a^2d^2\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx,x,x^n\right)}{256b^2d^5n}$$

$$= \frac{(bc-ad)^2\left(63b^2c^2+14abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{128b^2d^5n} - \frac{(bc-ad)\left(63b^2c^2+14abcd+3a^2d^2\right)\left(a+bx^n\right)^{3/2}\sqrt{c+dx^n}}{192b^2d^4n} + \frac{\left(63b^2c^2+14abcd+3a^2d^2\right)\left(a+bx^n\right)^{5/2}\sqrt{c+dx^n}}{240b^2d^3n} - \frac{3(3bc+ad)\left(a+bx^n\right)^{7/2}\sqrt{c+dx^n}}{40b^2d^2n} + \frac{x^n\left(a+bx^n\right)^{7/2}\sqrt{c+dx^n}}{5bdn} - \frac{\left((bc-ad)^3\left(63b^2c^2+14abcd+3a^2d^2\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx^n}\right)}{128b^3d^5n}$$

$$= \frac{(bc-ad)^2(63b^2c^2+14abcd+3a^2d^2)\sqrt{a+bx^n}\sqrt{c+dx^n}}{128b^2d^5n} - \frac{(bc-ad)(63b^2c^2+14abcd+3a^2d^2)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{192b^2d^4n} + \frac{(63b^2c^2+14abcd+3a^2d^2)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{240b^2d^3n} - \frac{3(3bc+ad)(a+bx^n)^{7/2}\sqrt{c+dx^n}}{40b^2d^2n} + \frac{x^n(a+bx^n)^{7/2}\sqrt{c+dx^n}}{5bdn} - \frac{((bc-ad)^3(63b^2c^2+14abcd+3a^2d^2))\,\text{Subst}\left(\int\frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\right)}{128b^3d^5n}$$

$$= \frac{(bc-ad)^2(63b^2c^2+14abcd+3a^2d^2)\sqrt{a+bx^n}\sqrt{c+dx^n}}{128b^2d^5n} - \frac{(bc-ad)(63b^2c^2+14abcd+3a^2d^2)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{192b^2d^4n} + \frac{(63b^2c^2+14abcd+3a^2d^2)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{240b^2d^3n} - \frac{3(3bc+ad)(a+bx^n)^{7/2}\sqrt{c+dx^n}}{40b^2d^2n} + \frac{x^n(a+bx^n)^{7/2}\sqrt{c+dx^n}}{5bdn} - \frac{(bc-ad)^3(63b^2c^2+14abcd+3a^2d^2)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{128b^{5/2}d^{11/2}n}$$

Mathematica [A] (verified)

Time = 2.66 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.77

$$\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \frac{\sqrt{c+dx^n}\left(-\frac{24(3bc+ad)(a+bx^n)^4}{bd}+64x^n(a+bx^n)^4+\frac{5(bc-ad)^3(63b^2c^2+14abcd+3a^2d^2)\left(-\right.}{}\right.}{320bdn\sqrt{a+bx^n}}$$

```
[In] Integrate[(x^(-1 + 3*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n],x]
```

```
[Out] (Sqrt[c + d*x^n]*((-24*(3*b*c + a*d)*(a + b*x^n)^4)/(b*d) + 64*x^n*(a + b*x
^n)^4 + (5*(b*c - a*d)^3*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^2)*((-2*d*(a +
b*x^n))/(-(b*c) + a*d) - (4*d^2*(a + b*x^n)^2)/(3*(b*c - a*d)^2) - (16*d^3*
(a + b*x^n)^3)/(15*(-(b*c) + a*d)^3) - (2*Sqrt[d]*Sqrt[a + b*x^n]*ArcSinh[(
Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(Sqrt[b*c - a*d]*Sqrt[(b*(c + d*
x^n))/(b*c - a*d)]))))/(4*b*d^5)))/(320*b*d*n*Sqrt[a + b*x^n])
```

Maple [F]

$$\int \frac{x^{-1+3n}(a+b\,x^{n})^{\frac{5}{2}}}{\sqrt{c+d\,x^{n}}}dx$$

[In] int(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)

[Out] int(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 771, normalized size of antiderivative = 2.15

$$\int \frac{x^{-1+3n}(a+bx^{n})^{5/2}}{\sqrt{c+dx^{n}}}\,dx = \left[-\frac{15\,(63\,b^{5}c^{5}-175\,ab^{4}c^{4}d+150\,a^{2}b^{3}c^{3}d^{2}-30\,a^{3}b^{2}c^{2}d^{3}-5\,a^{4}bcd^{4}-3\,a^{5}d^{5})\sqrt{bd}}{}\right.$$

[In] integrate(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")

```
[Out] [-1/7680*(15*(63*b^5*c^5 - 175*a*b^4*c^4*d + 150*a^2*b^3*c^3*d^2 - 30*a^3*b
^2*c^2*d^3 - 5*a^4*b*c*d^4 - 3*a^5*d^5)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) + b
^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*
d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) - 4*(384*b
^5*d^5*x^(4*n) + 945*b^5*c^4*d - 2310*a*b^4*c^3*d^2 + 1564*a^2*b^3*c^2*d^3
- 90*a^3*b^2*c*d^4 - 45*a^4*b*d^5 - 144*(3*b^5*c*d^4 - 7*a*b^4*d^5)*x^(3*n)
 + 8*(63*b^5*c^2*d^3 - 148*a*b^4*c*d^4 + 93*a^2*b^3*d^5)*x^(2*n) - 2*(315*b
^5*c^3*d^2 - 749*a*b^4*c^2*d^3 + 481*a^2*b^3*c*d^4 - 15*a^3*b^2*d^5)*x^n)*s
qrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^6*n), 1/3840*(15*(63*b^5*c^5 - 175*a
*b^4*c^4*d + 150*a^2*b^3*c^3*d^2 - 30*a^3*b^2*c^2*d^3 - 5*a^4*b*c*d^4 - 3*a
^5*d^5)*sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d
))*sqrt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d +
a*b*d^2)*x^n)) + 2*(384*b^5*d^5*x^(4*n) + 945*b^5*c^4*d - 2310*a*b^4*c^3*d^
2 + 1564*a^2*b^3*c^2*d^3 - 90*a^3*b^2*c*d^4 - 45*a^4*b*d^5 - 144*(3*b^5*c*d
^4 - 7*a*b^4*d^5)*x^(3*n) + 8*(63*b^5*c^2*d^3 - 148*a*b^4*c*d^4 + 93*a^2*b^
3*d^5)*x^(2*n) - 2*(315*b^5*c^3*d^2 - 749*a*b^4*c^2*d^3 + 481*a^2*b^3*c*d^4
 - 15*a^3*b^2*d^5)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^6*n)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \text{Timed out}$$

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**(5/2)/(c+d*x**n)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \int \frac{(bx^n+a)^{\frac{5}{2}}x^{3n-1}}{\sqrt{dx^n+c}}\,dx$$

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^n + a)^(5/2)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

Giac [F]

$$\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \int \frac{(bx^n+a)^{\frac{5}{2}}x^{3n-1}}{\sqrt{dx^n+c}}\,dx$$

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^n + a)^(5/2)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \int \frac{x^{3\,n-1}\,{\left(a+b\,x^n\right)}^{5/2}}{\sqrt{c+d\,x^n}}\,dx$$

```
[In] int((x^(3*n - 1)*(a + b*x^n)^(5/2))/(c + d*x^n)^(1/2),x)
[Out] int((x^(3*n - 1)*(a + b*x^n)^(5/2))/(c + d*x^n)^(1/2), x)
```

3.1073 $\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 291

$$\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = -\frac{(bc-ad)\left(35b^2c^2+10abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64b^2d^4n} + \frac{\left(35b^2c^2+10abcd+3a^2d^2\right)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96b^2d^3n} - \frac{(7bc+3ad)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{24b^2d^2n} + \frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bdn} + \frac{(bc-ad)^2\left(35b^2c^2+10abcd+3a^2d^2\right)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{64b^{5/2}d^{9/2}n}$$

```
[Out] 1/64*(-a*d+b*c)^2*(3*a^2*d^2+10*a*b*c*d+35*b^2*c^2)*arctanh(d^(1/2)*(a+b*x^
n)^(1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(5/2)/d^(9/2)/n+1/96*(3*a^2*d^2+10*a*b*
c*d+35*b^2*c^2)*(a+b*x^n)^(3/2)*(c+d*x^n)^(1/2)/b^2/d^3/n-1/24*(3*a*d+7*b*c
)*(a+b*x^n)^(5/2)*(c+d*x^n)^(1/2)/b^2/d^2/n+1/4*x^n*(a+b*x^n)^(5/2)*(c+d*x^
n)^(1/2)/b/d/n-1/64*(-a*d+b*c)*(3*a^2*d^2+10*a*b*c*d+35*b^2*c^2)*(a+b*x^n)^
(1/2)*(c+d*x^n)^(1/2)/b^2/d^4/n
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used

= {457, 92, 81, 52, 65, 223, 212}

$$\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \frac{(bc-ad)^2\left(3a^2d^2+10abcd+35b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{64b^{5/2}d^{9/2}n} - \frac{(bc-ad)\left(3a^2d^2+10abcd+35b^2c^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64b^2d^4n} + \frac{\left(3a^2d^2+10abcd+35b^2c^2\right)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96b^2d^3n} - \frac{(3ad+7bc)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{24b^2d^2n} + \frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bdn}$$

```
[In] Int[(x^(-1 + 3*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n],x]
```

```
[Out] -1/64*((b*c - a*d)*(35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x^n]*Sq
rt[c + d*x^n])/(b^2*d^4*n) + ((35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*(a + b*
x^n)^(3/2)*Sqrt[c + d*x^n])/(96*b^2*d^3*n) - ((7*b*c + 3*a*d)*(a + b*x^n)^(
5/2)*Sqrt[c + d*x^n])/(24*b^2*d^2*n) + (x^n*(a + b*x^n)^(5/2)*Sqrt[c + d*x^
n])/(4*b*d*n) + ((b*c - a*d)^2*(35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*ArcTan
h[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(64*b^(5/2)*d^(9/2)
*n)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{n} \\
&= \frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bdn} + \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}\left(-ac-\frac{1}{2}(7bc+3ad)x\right)}{\sqrt{c+dx}}\,dx, x, x^n\right)}{4bdn} \\
&= -\frac{(7bc+3ad)\,(a+bx^n)^{5/2}\sqrt{c+dx^n}}{24b^2d^2n} + \frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bdn} \\
&\quad + \frac{(35b^2c^2+10abcd+3a^2d^2)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{48b^2d^2n} \\
&= \frac{(35b^2c^2+10abcd+3a^2d^2)\,(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96b^2d^3n} \\
&\quad - \frac{(7bc+3ad)\,(a+bx^n)^{5/2}\sqrt{c+dx^n}}{24b^2d^2n} + \frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bdn} \\
&\quad - \frac{((bc-ad)\,(35b^2c^2+10abcd+3a^2d^2))\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{64b^2d^3n}
\end{aligned}$$

$$= -\frac{(bc-ad)\left(35b^2c^2+10abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64b^2d^4n} + \frac{\left(35b^2c^2+10abcd+3a^2d^2\right)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96b^2d^3n} - \frac{(7bc+3ad)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{24b^2d^2n} + \frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bdn} + \frac{\left((bc-ad)^2\left(35b^2c^2+10abcd+3a^2d^2\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{128b^2d^4n}$$

$$= -\frac{(bc-ad)\left(35b^2c^2+10abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64b^2d^4n} + \frac{\left(35b^2c^2+10abcd+3a^2d^2\right)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96b^2d^3n} - \frac{(7bc+3ad)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{24b^2d^2n} + \frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bdn} + \frac{\left((bc-ad)^2\left(35b^2c^2+10abcd+3a^2d^2\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^n}\right)}{64b^3d^4n}$$

$$= -\frac{(bc-ad)\left(35b^2c^2+10abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64b^2d^4n} + \frac{\left(35b^2c^2+10abcd+3a^2d^2\right)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96b^2d^3n} - \frac{(7bc+3ad)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{24b^2d^2n} + \frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bdn} + \frac{\left((bc-ad)^2\left(35b^2c^2+10abcd+3a^2d^2\right)\right)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\right)}{64b^3d^4n}$$

$$= -\frac{(bc-ad)\left(35b^2c^2+10abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64b^2d^4n} + \frac{\left(35b^2c^2+10abcd+3a^2d^2\right)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96b^2d^3n} - \frac{(7bc+3ad)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{24b^2d^2n} + \frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bdn} + \frac{(bc-ad)^2\left(35b^2c^2+10abcd+3a^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{64b^{5/2}d^{9/2}n}$$

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.83

$$\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \frac{-b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)\left(9a^3d^3+3a^2bd^2(5c-2dx^n)-ab^2d(145c^2-92cdx^n+7\right.}{}$$

[In] Integrate[(x^(-1 + 3*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n],x]

[Out] (-(b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(9*a^3*d^3 + 3*a^2*b*d^2*(5*c - 2*d*x^n) - a*b^2*d*(145*c^2 - 92*c*d*x^n + 72*d^2*x^(2*n)) + b^3*(105*c^3 - 70*c^2*d*x^n + 56*c*d^2*x^(2*n) - 48*d^3*x^(3*n)))) + 3*(b*c - a*d)^(5/2)*(35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(192*b^3*d^(9/2)*n*Sqrt[c + d*x^n])

Maple [F]

$$\int \frac{x^{-1+3n}(a+b\,x^n)^{\frac{3}{2}}}{\sqrt{c+d\,x^n}}dx$$

[In] int(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)

[Out] int(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 607, normalized size of antiderivative = 2.09

$$\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \left[\frac{3\,(35\,b^4c^4-60\,ab^3c^3d+18\,a^2b^2c^2d^2+4\,a^3bcd^3+3\,a^4d^4)\sqrt{bd}\log\left(8\,b^2d^2x^{2\,n}+b^2\right.}{}\right.$$

$$-\frac{3\,(35\,b^4c^4-60\,ab^3c^3d+18\,a^2b^2c^2d^2+4\,a^3bcd^3+3\,a^4d^4)\sqrt{-bd}\arctan\left(\frac{(2\,\sqrt{-bd}bdx^n+(bc+ad)\sqrt{-bd})\sqrt{bx^n+a}\sqrt{dx^n+c}}{2\,(b^2d^2x^{2\,n}+abcd+(b^2cd+abd^2)x^n)}\right.}{}$$

[In] integrate(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")

[Out] [1/768*(3*(35*b^4*c^4 - 60*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 + 3*a^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) + 4*(48*b^4*d^4*x^(3*n) - 105*b^4*c^3*d + 145*a*b^3*c^2*d^2 - 15*a^2*b^2*c*d^3 - 9*a^3*b*d^4 - 8*(7*b^4*c*d^3 -

```
9*a*b^3*d^4)*x^(2*n) + 2*(35*b^4*c^2*d^2 - 46*a*b^3*c*d^3 + 3*a^2*b^2*d^4)
*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^5*n), -1/384*(3*(35*b^4*c^4 -
 60*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 + 3*a^4*d^4)*sqrt(-b*d
)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a
)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)) -
2*(48*b^4*d^4*x^(3*n) - 105*b^4*c^3*d + 145*a*b^3*c^2*d^2 - 15*a^2*b^2*c*d^
3 - 9*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 9*a*b^3*d^4)*x^(2*n) + 2*(35*b^4*c^2*d^2
 - 46*a*b^3*c*d^3 + 3*a^2*b^2*d^4)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b
^3*d^5*n)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \text{Timed out}$$

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**(3/2)/(c+d*x**n)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \int \frac{(bx^n+a)^{\frac{3}{2}}x^{3\,n-1}}{\sqrt{dx^n+c}}\,dx$$

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^n + a)^(3/2)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

Giac [F]

$$\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \int \frac{(bx^n+a)^{\frac{3}{2}}x^{3\,n-1}}{\sqrt{dx^n+c}}\,dx$$

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^n + a)^(3/2)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \int \frac{x^{3\,n-1}\,(a+b\,x^n)^{3/2}}{\sqrt{c+d\,x^n}}\,dx$$

```
[In] int((x^(3*n - 1)*(a + b*x^n)^(3/2))/(c + d*x^n)^(1/2),x)
[Out] int((x^(3*n - 1)*(a + b*x^n)^(3/2))/(c + d*x^n)^(1/2), x)
```

3.1074 $\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 221

$$\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx = \frac{(5b^2c^2+2abcd+a^2d^2)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8b^2d^3n} - \frac{(5bc+3ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12b^2d^2n} + \frac{x^n(a+bx^n)^{3/2}\sqrt{c+dx^n}}{3bdn} - \frac{(bc-ad)(5b^2c^2+2abcd+a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{8b^{5/2}d^{7/2}n}$$

```
[Out] -1/8*(-a*d+b*c)*(a^2*d^2+2*a*b*c*d+5*b^2*c^2)*arctanh(d^(1/2)*(a+b*x^n)^(1/
2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(5/2)/d^(7/2)/n-1/12*(3*a*d+5*b*c)*(a+b*x^n)^
(3/2)*(c+d*x^n)^(1/2)/b^2/d^2/n+1/3*x^n*(a+b*x^n)^(3/2)*(c+d*x^n)^(1/2)/b/d
/n+1/8*(a^2*d^2+2*a*b*c*d+5*b^2*c^2)*(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b^2/d^
3/n
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {457, 92, 81, 52, 65, 223, 212}

$$\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx = -\frac{(bc-ad)(a^2d^2+2abcd+5b^2c^2)\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{8b^{5/2}d^{7/2}n} + \frac{(a^2d^2+2abcd+5b^2c^2)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8b^2d^3n} - \frac{(3ad+5bc)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12b^2d^2n} + \frac{x^n(a+bx^n)^{3/2}\sqrt{c+dx^n}}{3bdn}$$

```
[In] Int[(x^(-1 + 3*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n],x]
```

```
[Out] ((5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(8*b^2*
d^3*n) - ((5*b*c + 3*a*d)*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(12*b^2*d^2*n)
 + (x^n*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(3*b*d*n) - ((b*c - a*d)*(5*b^2*
c^2 + 2*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[
c + d*x^n])])/(8*b^(5/2)*d^(7/2)*n)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+bx}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{n}\\
&= \frac{x^n(a+bx^n)^{3/2}\sqrt{c+dx^n}}{3bdn} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(-ac-\frac{1}{2}(5bc+3ad)x\right)}{\sqrt{c+dx}}\,dx, x, x^n\right)}{3bdn}\\
&= -\frac{(5bc+3ad)\,(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12b^2d^2n} + \frac{x^n(a+bx^n)^{3/2}\sqrt{c+dx^n}}{3bdn}\\
&\quad + \frac{(5b^2c^2+2abcd+a^2d^2)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{8b^2d^2n}\\
&= \frac{(5b^2c^2+2abcd+a^2d^2)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8b^2d^3n}\\
&\quad - \frac{(5bc+3ad)\,(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12b^2d^2n} + \frac{x^n(a+bx^n)^{3/2}\sqrt{c+dx^n}}{3bdn}\\
&\quad - \frac{((bc-ad)\,(5b^2c^2+2abcd+a^2d^2))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{16b^2d^3n}\\
&= \frac{(5b^2c^2+2abcd+a^2d^2)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8b^2d^3n}\\
&\quad - \frac{(5bc+3ad)\,(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12b^2d^2n} + \frac{x^n(a+bx^n)^{3/2}\sqrt{c+dx^n}}{3bdn}\\
&\quad - \frac{((bc-ad)\,(5b^2c^2+2abcd+a^2d^2))\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^n}\right)}{8b^3d^3n}
\end{aligned}$$

$$= \frac{(5b^2c^2+2abcd+a^2d^2)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8b^2d^3n} - \frac{(5bc+3ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12b^2d^2n} + \frac{x^n(a+bx^n)^{3/2}\sqrt{c+dx^n}}{3bdn} - \frac{((bc-ad)(5b^2c^2+2abcd+a^2d^2))\operatorname{Subst}\left(\int\frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\right)}{8b^3d^3n}$$

$$= \frac{(5b^2c^2+2abcd+a^2d^2)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8b^2d^3n} - \frac{(5bc+3ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12b^2d^2n} + \frac{x^n(a+bx^n)^{3/2}\sqrt{c+dx^n}}{3bdn} - \frac{(bc-ad)(5b^2c^2+2abcd+a^2d^2)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{8b^{5/2}d^{7/2}n}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.86

$$\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$$

$$= \frac{b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)\left(-3a^2d^2+2abd(-2c+dx^n)+b^2(15c^2-10cdx^n+8d^2x^{2n})\right)-3(bc-ad)^{3/2}\left(5b^2c^2\right.}{24b^3d^{7/2}n\sqrt{c+dx^n}}$$

```
[In] Integrate[(x^(-1 + 3*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(-3*a^2*d^2 + 2*a*b*d*(-2*c + d*x^n)
 + b^2*(15*c^2 - 10*c*d*x^n + 8*d^2*x^(2*n))) - 3*(b*c - a*d)^(3/2)*(5*b^2*
c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[
d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(24*b^3*d^(7/2)*n*Sqrt[c + d*x^n])
```

Maple [F]

$$\int \frac{x^{-1+3n}\sqrt{a+b\,x^n}}{\sqrt{c+d\,x^n}}dx$$

```
[In] int(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

```
[Out] int(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.13

$$\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx = \left[-\frac{3\,(5\,b^3c^3-3\,ab^2c^2d-a^2bcd^2-a^3d^3)\sqrt{bd}\log\left(8\,b^2d^2x^{2\,n}+b^2c^2+6\,abcd+a^2d^2+4\left(2\,\sqrt{bd}bdx^n+(bc\right.\right.}{}\right.$$

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*log
(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n
 + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*
b*d^2)*x^n) - 4*(8*b^3*d^3*x^(2*n) + 15*b^3*c^2*d - 4*a*b^2*c*d^2 - 3*a^2*b
*d^3 - 2*(5*b^3*c*d^2 - a*b^2*d^3)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b
^3*d^4*n), 1/48*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*sqrt
(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^
n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n
)) + 2*(8*b^3*d^3*x^(2*n) + 15*b^3*c^2*d - 4*a*b^2*c*d^2 - 3*a^2*b*d^3 - 2*
(5*b^3*c*d^2 - a*b^2*d^3)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^4*n)
]
```

Sympy [F]

$$\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx = \int \frac{x^{3n-1}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$$

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**(1/2)/(c+d*x**n)**(1/2),x)
```

```
[Out] Integral(x**(3*n - 1)*sqrt(a + b*x**n)/sqrt(c + d*x**n), x)
```

Maxima [F]

$$\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx = \int \frac{\sqrt{bx^n+a}x^{3\,n-1}}{\sqrt{dx^n+c}}\,dx$$

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^n + a)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

Giac [F]

$$\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx = \int \frac{\sqrt{bx^n+a}x^{3\,n-1}}{\sqrt{dx^n+c}}\,dx$$

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^n + a)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx = \int \frac{x^{3\,n-1}\,\sqrt{a+b\,x^n}}{\sqrt{c+d\,x^n}}\,dx$$

```
[In] int((x^(3*n - 1)*(a + b*x^n)^(1/2))/(c + d*x^n)^(1/2),x)
[Out] int((x^(3*n - 1)*(a + b*x^n)^(1/2))/(c + d*x^n)^(1/2), x)
```

3.1075 $\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 150

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = -\frac{3(bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n} + \frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn} - \frac{(4abcd-3(bc+ad)^2)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{4b^{5/2}d^{5/2}n}$$

```
[Out] -1/4*(4*a*b*c*d-3*(a*d+b*c)^2)*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d
*x^n)^(1/2))/b^(5/2)/d^(5/2)/n-3/4*(a*d+b*c)*(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2
)/b^2/d^2/n+1/2*x^n*(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b/d/n
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {457, 92, 81, 65, 223, 212}

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = -\frac{(4abcd-3(ad+bc)^2)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{4b^{5/2}d^{5/2}n} - \frac{3(ad+bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n} + \frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn}$$

```
[In] Int[x^(-1 + 3*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]),x]
```

```
[Out] (-3*(b*c + a*d)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(4*b^2*d^2*n) + (x^n*Sqrt[
a + b*x^n]*Sqrt[c + d*x^n])/(2*b*d*n) - ((4*a*b*c*d - 3*(b*c + a*d)^2)*ArcT
anh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(4*b^(5/2)*d^(5/2
)*n)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx\sqrt{c+dx}}}\,dx, x, x^n\right)}{n}$$

$$= \frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn} + \frac{\text{Subst}\left(\int \frac{-ac-\frac{3}{2}(bc+ad)x}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{2bdn}$$

$$= -\frac{3(bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n} + \frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn} - \frac{(4abcd-3(bc+ad)^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{8b^2d^2n}$$

$$= -\frac{3(bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n} + \frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn} - \frac{(4abcd-3(bc+ad)^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^n}\right)}{4b^3d^2n}$$

$$= -\frac{3(bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n} + \frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn} - \frac{(4abcd-3(bc+ad)^2)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\right)}{4b^3d^2n}$$

$$= -\frac{3(bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n} + \frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn} - \frac{(4abcd-3(bc+ad)^2)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{4b^{5/2}d^{5/2}n}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx$$

$$= \frac{b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)\left(-3bc-3ad+2bdx^n\right)+\sqrt{bc-ad}(3b^2c^2+2abcd+3a^2d^2)\sqrt{\frac{b(c+dx^n)}{bc-ad}}\text{arcsinh}\left(\frac{\sqrt{d}\sqrt{}}{\sqrt{b}}\right.}{4b^3d^{5/2}n\sqrt{c+dx^n}}$$

```
[In] Integrate[x^(-1 + 3*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]),x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(-3*b*c - 3*a*d + 2*b*d*x^n) + Sqrt[
b*c - a*d]*(3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*Sqrt[(b*(c + d*x^n))/(b*c -
a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(4*b^3*d^(5/2)*n*
Sqrt[c + d*x^n])
```

Maple [F]

$$\int \frac{x^{-1+3n}}{\sqrt{a+b\,x^{n}}\,\sqrt{c+d\,x^{n}}}dx$$

```
[In] int(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
[Out] int(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.41

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = \left[\frac{(3\,b^2c^2+2\,abcd+3\,a^2d^2)\sqrt{bd}\log\left(8\,b^2d^2x^{2\,n}+b^2c^2+6\,abcd+a^2d^2+4\left(2\,\sqrt{bd}bdx^n+(bc+ad)\sqrt{bd}\right)\sqrt{b\cdots}\right.}{16\,b^3d^3n} - \frac{(3\,b^2c^2+2\,abcd+3\,a^2d^2)\sqrt{-bd}\arctan\left(\frac{(2\,\sqrt{-bd}bdx^n+(bc+ad)\sqrt{-bd})\sqrt{bx^n+a}\sqrt{dx^n+c}}{2\,(b^2d^2x^{2\,n}+abcd+(b^2cd+abd^2)x^n)}\right)-2\,(2\,b^2d^2x^n-3\,b^2cd-3\cdots}{8\,b^3d^3n}\right.$$

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
[Out] [1/16*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n)
+ b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt
(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) + 4*(2*
b^2*d^2*x^n - 3*b^2*c*d - 3*a*b*d^2)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*
d^3*n), -1/8*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*
sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c
)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)) - 2*(2*b^2*d^2*x^n
 - 3*b^2*c*d - 3*a*b*d^2)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^3*n)]
```

Sympy [F]

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = \int \frac{x^{3n-1}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx$$

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)**(1/2)/(c+d*x**n)**(1/2),x)
[Out] Integral(x**(3*n - 1)/(sqrt(a + b*x**n)*sqrt(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx=\int \frac{x^{3\,n-1}}{\sqrt{bx^n+a}\sqrt{dx^n+c}}\,dx$$

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(3*n - 1)/(sqrt(b*x^n + a)*sqrt(d*x^n + c)), x)
```

Giac [F]

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx=\int \frac{x^{3\,n-1}}{\sqrt{bx^n+a}\sqrt{dx^n+c}}\,dx$$

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(x^(3*n - 1)/(sqrt(b*x^n + a)*sqrt(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx=\int \frac{x^{3\,n-1}}{\sqrt{a+b\,x^n}\,\sqrt{c+d\,x^n}}\,dx$$

```
[In] int(x^(3*n - 1)/((a + b*x^n)^(1/2)*(c + d*x^n)^(1/2)),x)
[Out] int(x^(3*n - 1)/((a + b*x^n)^(1/2)*(c + d*x^n)^(1/2)), x)
```

3.1076 $\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 133

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = -\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{b^2dn} - \frac{(bc+3ad)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{5/2}d^{3/2}n}$$

```
[Out] -(3*a*d+b*c)*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(5/
2)/d^(3/2)/n-2*a^2*(c+d*x^n)^(1/2)/b^2/(-a*d+b*c)/n/(a+b*x^n)^(1/2)+(a+b*x^
n)^(1/2)*(c+d*x^n)^(1/2)/b^2/d/n
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {457, 91, 81, 65, 223, 212}

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = -\frac{2a^2\sqrt{c+dx^n}}{b^2n(bc-ad)\sqrt{a+bx^n}} - \frac{(3ad+bc)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{5/2}d^{3/2}n} + \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{b^2dn}$$

```
[In] Int[x^(-1 + 3*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (-2*a^2*Sqrt[c + d*x^n])/(b^2*(b*c - a*d)*n*Sqrt[a + b*x^n]) + (Sqrt[a + b*
x^n]*Sqrt[c + d*x^n])/(b^2*d*n) - ((b*c + 3*a*d)*ArcTanh[(Sqrt[d]*Sqrt[a +
b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(5/2)*d^(3/2)*n)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx, x, x^n\right)}{n}\\
&= -\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{2\text{Subst}\left(\int \frac{-\frac{1}{2}a(bc-ad)+\frac{1}{2}b(bc-ad)x}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{b^2(bc-ad)n}\\
&= -\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{b^2dn} - \frac{(bc+3ad)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{2b^2dn}\\
&= -\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{b^2dn}\\
&\quad - \frac{(bc+3ad)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^n}\right)}{b^3dn}\\
&= -\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{b^2dn} - \frac{(bc+3ad)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\right)}{b^3dn}\\
&= -\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{b^2dn} - \frac{(bc+3ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{5/2}d^{3/2}n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.39

$$
\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \frac{-b\sqrt{d}(c+dx^n)\left(-3a^2d+b^2cx^n+ab(c-dx^n)\right)+\sqrt{bc-ad}(b^2c^2+2abcd-3a}{b^3d^{3/2}(-bc+ad)n\sqrt{a+bx^n}\sqrt{c+d}}
$$

```
[In] Integrate[x^(-1 + 3*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (-(b*Sqrt[d]*(c + d*x^n)*(-3*a^2*d + b^2*c*x^n + a*b*(c - d*x^n))) + Sqrt[b
*c - a*d]*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*Sqrt[a + b*x^n]*Sqrt[(b*(c + d*
x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(b^3
*d^(3/2)*(-(b*c) + a*d)*n*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])
```

Maple [F]

$$\int \frac{x^{-1+3n}}{(a+b\,x^n)^{\frac{3}{2}}\sqrt{c+d\,x^n}}dx$$

[In] `int(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)`

[Out] `int(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(113) = 226.

Time = 0.39 (sec) , antiderivative size = 540, normalized size of antiderivative = 4.06

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \left[\frac{4\,(ab^2cd-3\,a^2bd^2+(b^3cd-ab^2d^2)x^n)\sqrt{bx^n+a}\sqrt{dx^n+c}+\Big((b^3c^2+2\,ab^2}{}\right.$$

[In] `integrate(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")`

[Out]
```
[1/4*(4*(a*b^2*c*d - 3*a^2*b*d^2 + (b^3*c*d - a*b^2*d^2)*x^n)*sqrt(b*x^n +
a)*sqrt(d*x^n + c) + ((b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*sqrt(b*d)*x^n +
 (a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2)*sqrt(b*d))*log(8*b^2*d^2*x^(2*n) + b
^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*
d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n))/((b^5*c*d
^2 - a*b^4*d^3)*n*x^n + (a*b^4*c*d^2 - a^2*b^3*d^3)*n), 1/2*(2*(a*b^2*c*d -
 3*a^2*b*d^2 + (b^3*c*d - a*b^2*d^2)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c) +
 ((b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*sqrt(-b*d)*x^n + (a*b^2*c^2 + 2*a^2
*b*c*d - 3*a^3*d^2)*sqrt(-b*d))*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a
*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d
+ (b^2*c*d + a*b*d^2)*x^n)))/((b^5*c*d^2 - a*b^4*d^3)*n*x^n + (a*b^4*c*d^2
- a^2*b^3*d^3)*n)]
```

Sympy [F]

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{3n-1}}{(a+bx^n)^{\frac{3}{2}}\sqrt{c+dx^n}}\,dx$$

[In] `integrate(x**(-1+3*n)/(a+b*x**n)**(3/2)/(c+d*x**n)**(1/2),x)`

[Out] `Integral(x**(3*n - 1)/((a + b*x**n)**(3/2)*sqrt(c + d*x**n)), x)`

Maxima [F]

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{3n-1}}{(bx^n+a)^{\frac{3}{2}}\sqrt{dx^n+c}}\,dx$$

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(3*n - 1)/((b*x^n + a)^(3/2)*sqrt(d*x^n + c)), x)
```

Giac [F]

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{3n-1}}{(bx^n+a)^{\frac{3}{2}}\sqrt{dx^n+c}}\,dx$$

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(x^(3*n - 1)/((b*x^n + a)^(3/2)*sqrt(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{3n-1}}{(a+b\,x^n)^{3/2}\sqrt{c+d\,x^n}}\,dx$$

```
[In] int(x^(3*n - 1)/((a + b*x^n)^(3/2)*(c + d*x^n)^(1/2)),x)
[Out] int(x^(3*n - 1)/((a + b*x^n)^(3/2)*(c + d*x^n)^(1/2)), x)
```

3.1077 $\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 147

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = -\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)n\,(a+bx^n)^{3/2}} + \frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2(bc-ad)^2n\sqrt{a+bx^n}} + \frac{2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{5/2}\sqrt{d}n}$$

```
[Out] 2*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(5/2)/n/d^(1/2
)-2/3*a^2*(c+d*x^n)^(1/2)/b^2/(-a*d+b*c)/n/(a+b*x^n)^(3/2)+4/3*a*(-2*a*d+3*
b*c)*(c+d*x^n)^(1/2)/b^2/(-a*d+b*c)^2/n/(a+b*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {457, 91, 79, 65, 223, 212}

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = -\frac{2a^2\sqrt{c+dx^n}}{3b^2n(bc-ad)\,(a+bx^n)^{3/2}} + \frac{2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{5/2}\sqrt{d}n} + \frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2n(bc-ad)^2\sqrt{a+bx^n}}$$

```
[In] Int[x^(-1 + 3*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (-2*a^2*Sqrt[c + d*x^n])/(3*b^2*(b*c - a*d)*n*(a + b*x^n)^(3/2)) + (4*a*(3*
b*c - 2*a*d)*Sqrt[c + d*x^n])/(3*b^2*(b*c - a*d)^2*n*Sqrt[a + b*x^n]) + (2*
```

```
ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(5/2)*Sqrt
[d]*n)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)n\,(a+bx^n)^{3/2}} + \frac{2\text{Subst}\left(\int \frac{-\frac{1}{2}a(3bc-ad)+\frac{3}{2}b(bc-ad)x}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx, x, x^n\right)}{3b^2(bc-ad)n}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)n\,(a+bx^n)^{3/2}} + \frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2(bc-ad)^2n\sqrt{a+bx^n}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{b^2n}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)n\,(a+bx^n)^{3/2}} + \frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2(bc-ad)^2n\sqrt{a+bx^n}} + \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^n}\right)}{b^3n}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)n\,(a+bx^n)^{3/2}} + \frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2(bc-ad)^2n\sqrt{a+bx^n}} + \frac{2\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\right)}{b^3n}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)n\,(a+bx^n)^{3/2}} + \frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2(bc-ad)^2n\sqrt{a+bx^n}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{5/2}\sqrt{d}n}$$

Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.48

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \frac{2\sqrt{c+dx^n}\left(\frac{a^2}{-bc+ad} + \frac{(3b^2c^2-a^2d^2)(a+bx^n)}{d(bc-ad)^2} - \frac{3(a+bx^n)\left(\sqrt{bc-ad}\sqrt{\frac{b(c+dx^n)}{bc-ad}}-\sqrt{d}\sqrt{a+bx^n}\mathrm{a}\right.}{d\sqrt{bc-ad}\sqrt{\frac{b(c+dx^n)}{bc-ad}}}\right.}{3b^2n\,(a+bx^n)^{3/2}}$$

```
[In] Integrate[x^(-1 + 3*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (2*Sqrt[c + d*x^n]*(a^2/(-(b*c) + a*d) + ((3*b^2*c^2 - a^2*d^2)*(a + b*x^n)
)/(d*(b*c - a*d)^2) - (3*(a + b*x^n)*(Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^n))/
(b*c - a*d)] - Sqrt[d]*Sqrt[a + b*x^n]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sq
rt[b*c - a*d]]))/(d*Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^n))/(b*c - a*d)])))/(3
*b^2*n*(a + b*x^n)^(3/2))
```

Maple [F]

$$\int \frac{x^{-1+3n}}{\left(a+b\,x^{n}\right)^{\frac{5}{2}}\sqrt{c+d\,x^{n}}}dx$$

```
[In] int(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

```
[Out] int(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(123) = 246$.

Time = 0.49 (sec) , antiderivative size = 769, normalized size of antiderivative = 5.23

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \left[\frac{4\left(5\,a^2b^2cd-3\,a^3bd^2+2\left(3\,ab^3cd-2\,a^2b^2d^2\right)x^n\right)\sqrt{bx^n+a}\sqrt{dx^n+c}+3\left(\left(b^4\right.\right.}{}\right.$$

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(4*(5*a^2*b^2*c*d - 3*a^3*b*d^2 + 2*(3*a*b^3*c*d - 2*a^2*b^2*d^2)*x^n)
*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 3*((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)
*sqrt(b*d)*x^(2*n) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*sqrt(b*d)*x^
n + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(b*d))*log(8*b^2*d^2*x^(2*n)
+ b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt
(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n))/((b^7*
c^2*d - 2*a*b^6*c*d^2 + a^2*b^5*d^3)*n*x^(2*n) + 2*(a*b^6*c^2*d - 2*a^2*b^5
*c*d^2 + a^3*b^4*d^3)*n*x^n + (a^2*b^5*c^2*d - 2*a^3*b^4*c*d^2 + a^4*b^3*d^
3)*n), 1/3*(2*(5*a^2*b^2*c*d - 3*a^3*b*d^2 + 2*(3*a*b^3*c*d - 2*a^2*b^2*d^2
)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c) - 3*((b^4*c^2 - 2*a*b^3*c*d + a^2*b^
2*d^2)*sqrt(-b*d)*x^(2*n) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*sqrt(
-b*d)*x^n + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(-b*d))*arctan(1/2*(2
*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n +
c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)))/((b^7*c^2*d - 2*
a*b^6*c*d^2 + a^2*b^5*d^3)*n*x^(2*n) + 2*(a*b^6*c^2*d - 2*a^2*b^5*c*d^2 + a
^3*b^4*d^3)*n*x^n + (a^2*b^5*c^2*d - 2*a^3*b^4*c*d^2 + a^4*b^3*d^3)*n)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \text{Timed out}$$

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)**(5/2)/(c+d*x**n)**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{3\,n-1}}{(bx^n+a)^{\frac{5}{2}}\sqrt{dx^n+c}}\,dx$$

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(3*n - 1)/((b*x^n + a)^(5/2)*sqrt(d*x^n + c)), x)
```

Giac [F]

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{3\,n-1}}{(bx^n+a)^{\frac{5}{2}}\sqrt{dx^n+c}}\,dx$$

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(x^(3*n - 1)/((b*x^n + a)^(5/2)*sqrt(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{3\,n-1}}{(a+b\,x^n)^{5/2}\,\sqrt{c+d\,x^n}}\,dx$$

```
[In] int(x^(3*n - 1)/((a + b*x^n)^(5/2)*(c + d*x^n)^(1/2)),x)
[Out] int(x^(3*n - 1)/((a + b*x^n)^(5/2)*(c + d*x^n)^(1/2)), x)
```

3.1078 $\int x^p(b+cx)^p(b+2cx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 20

$$\int x^p(b+cx)^p(b+2cx)\,dx=\frac{x^{1+p}(b+cx)^{1+p}}{1+p}$$

```
[Out] x^(p+1)*(c*x+b)^(p+1)/(p+1)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {75}

$$\int x^p(b+cx)^p(b+2cx)\,dx=\frac{x^{p+1}(b+cx)^{p+1}}{p+1}$$

```
[In] Int[x^p*(b + c*x)^p*(b + 2*c*x),x]
[Out] (x^(1 + p)*(b + c*x)^(1 + p))/(1 + p)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\text{integral}=\frac{x^{1+p}(b+cx)^{1+p}}{1+p}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^p(b+cx)^p(b+2cx)\,dx=\frac{x^{1+p}(b+cx)^{1+p}}{1+p}$$

```
[In] Integrate[x^p*(b + c*x)^p*(b + 2*c*x),x]
[Out] (x^(1 + p)*(b + c*x)^(1 + p))/(1 + p)
```

Maple [A] (verified)

Time = 4.81 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
gosper	$\frac{x^{1+p}(cx+b)^{1+p}}{1+p}$	21
risch	$\frac{x(cx+b)x^p(cx+b)^p}{1+p}$	23
parallelrisch	$\frac{x^2x^p(cx+b)^pbc+x\,x^p(cx+b)^pb^2}{b(1+p)}$	42

```
[In] int(x^p*(c*x+b)^p*(2*c*x+b),x,method=_RETURNVERBOSE)
[Out] x^(1+p)*(c*x+b)^(1+p)/(1+p)
```

Fricas [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int x^p(b+cx)^p(b+2cx)\,dx=\frac{(cx^2+bx)(cx+b)^px^p}{p+1}$$

```
[In] integrate(x^p*(c*x+b)^p*(2*c*x+b),x, algorithm="fricas")
[Out] (c*x^2 + b*x)*(c*x + b)^p*x^p/(p + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(15) = 30$.

Time = 0.79 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.30

$$\int x^p(b+cx)^p(b+2cx)\,dx = \begin{cases} \frac{bx x^p (b+cx)^p}{p+1} + \frac{cx^2x^p(b+cx)^p}{p+1} & \text{for } p \neq -1 \\ \log(x) + \log\left(\frac{b}{c}+x\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(x**p*(c*x+b)**p*(2*c*x+b),x)
[Out] Piecewise((b*x*x**p*(b + c*x)**p/(p + 1) + c*x**2*x**p*(b + c*x)**p/(p + 1)
, Ne(p, -1)), (log(x) + log(b/c + x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int x^p(b+cx)^p(b+2cx)\,dx = \frac{(cx^2+bx)e^{(p\log(cx+b)+p\log(x))}}{p+1}$$

```
[In] integrate(x^p*(c*x+b)^p*(2*c*x+b),x, algorithm="maxima")
[Out] (c*x^2 + b*x)*e^(p*log(c*x + b) + p*log(x))/(p + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int x^p(b+cx)^p(b+2cx)\,dx = \frac{(cx+b)^p cx^2x^p + (cx+b)^p bxx^p}{p+1}$$

```
[In] integrate(x^p*(c*x+b)^p*(2*c*x+b),x, algorithm="giac")
[Out] ((c*x + b)^p*c*x^2*x^p + (c*x + b)^p*b*x*x^p)/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^p(b+cx)^p(b+2cx)\,dx=\frac{x\,x^p\,{\left(b+c\,x\right)}^p\,\left(b+c\,x\right)}{p+1}$$

```
[In] int(x^p*(b + c*x)^p*(b + 2*c*x),x)
[Out] (x*x^p*(b + c*x)^p*(b + c*x))/(p + 1)
```

3.1079 $\int x^{-1+2(1+p)}(b+cx^2)^p(b+2cx^2)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int x^{-1+2(1+p)}\left(b+cx^2\right)^p\left(b+2cx^2\right)\,dx=\frac{x^{2(1+p)}\left(b+cx^2\right)^{1+p}}{2(1+p)}$$

```
[Out] 1/2*x^(2+2*p)*(c*x^2+b)^(p+1)/(p+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {460}

$$\int x^{-1+2(1+p)}\left(b+cx^2\right)^p\left(b+2cx^2\right)\,dx=\frac{x^{2(p+1)}\left(b+cx^2\right)^{p+1}}{2(p+1)}$$

```
[In] Int[x^(-1 + 2*(1 + p))*(b + c*x^2)^p*(b + 2*c*x^2),x]
```

```
[Out] (x^(2*(1 + p))*(b + c*x^2)^(1 + p))/(2*(1 + p))
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=\frac{x^{2(1+p)}\left(b+cx^2\right)^{1+p}}{2(1+p)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.59

$$\int x^{-1+2(1+p)}\left(b+cx^2\right)^p\left(b+2cx^2\right)\,dx$$
$$=\frac{x^{2+2p}(b+cx^2)^p\left(1+\frac{cx^2}{b}\right)^{-p}\left(b(2+p)\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,-\frac{cx^2}{b}\right)+2c(1+p)x^2\,\text{Hyper}\right.}{2(1+p)(2+p)}$$

```
[In] Integrate[x^(-1 + 2*(1 + p))*(b + c*x^2)^p*(b + 2*c*x^2),x]
```

```
[Out] (x^(2 + 2*p)*(b + c*x^2)^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, -
((c*x^2)/b)] + 2*c*(1 + p)*x^2*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x^2
)/b)]))/(2*(1 + p)*(2 + p)*(1 + (c*x^2)/b)^p)
```

Maple [A] (verified)

Time = 5.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{x^{2+2p}\left(c\,x^{2}+b\right)^{1+p}}{2+2p}$	26
risch	$\frac{x\left(c\,x^{2}+b\right)x^{1+2p}\left(c\,x^{2}+b\right)^{p}}{2+2p}$	32
parallelrisch	$\frac{x^{3}x^{1+2p}\left(c\,x^{2}+b\right)^{p}bc+x\,x^{1+2p}\left(c\,x^{2}+b\right)^{p}b^{2}}{2b(1+p)}$	55

```
[In] int(x^(1+2*p)*(c*x^2+b)^p*(2*c*x^2+b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^(2+2*p)*(c*x^2+b)^(1+p)/(1+p)
```

Fricas [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int x^{-1+2(1+p)}\left(b+cx^2\right)^p\left(b+2cx^2\right)\,dx=\frac{(cx^3+bx)(cx^2+b)^p x^{2\,p+1}}{2\,(p+1)}$$

```
[In] integrate(x^(1+2*p)*(c*x^2+b)^p*(2*c*x^2+b),x, algorithm="fricas")
```

```
[Out] 1/2*(c*x^3 + b*x)*(c*x^2 + b)^p*x^(2*p + 1)/(p + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(20) = 40$.

Time = 26.65 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.96

$$\int x^{-1+2(1+p)}\left(b+cx^2\right)^p\left(b+2cx^2\right)\,dx = \begin{cases} \frac{bxx^{2p+1}(b+cx^2)^p}{2p+2} + \frac{cx^3x^{2p+1}(b+cx^2)^p}{2p+2} & \text{for } p \neq -1 \\ \log(x) + \frac{\log\left(x-\sqrt{-\frac{b}{c}}\right)}{2} + \frac{\log\left(x+\sqrt{-\frac{b}{c}}\right)}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**(1+2*p)*(c*x**2+b)**p*(2*c*x**2+b),x)
```

```
[Out] Piecewise((b*x*x**(2*p + 1)*(b + c*x**2)**p/(2*p + 2) + c*x**3*x**(2*p + 1)
*(b + c*x**2)**p/(2*p + 2), Ne(p, -1)), (log(x) + log(x - sqrt(-b/c))/2 + l
og(x + sqrt(-b/c))/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int x^{-1+2(1+p)}\left(b+cx^2\right)^p\left(b+2cx^2\right)\,dx = \frac{(cx^4+bx^2)e^{(p\log(cx^2+b)+2\,p\log(x))}}{2\,(p+1)}$$

```
[In] integrate(x^(1+2*p)*(c*x^2+b)^p*(2*c*x^2+b),x, algorithm="maxima")
```

```
[Out] 1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(25) = 50$.

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.93

$$\int x^{-1+2(1+p)}\left(b+cx^2\right)^p\left(b+2cx^2\right)\,dx = \frac{(cx^2+b)^p cx^3 e^{(2\,p\log(x)+\log(x))} + (cx^2+b)^p bxe^{(2\,p\log(x)+\log(x))}}{2\,(p+1)}$$

```
[In] integrate(x^(1+2*p)*(c*x^2+b)^p*(2*c*x^2+b),x, algorithm="giac")
```

```
[Out] 1/2*((c*x^2 + b)^p*c*x^3*e^(2*p*log(x) + log(x)) + (c*x^2 + b)^p*b*x*e^(2*p
*log(x) + log(x)))/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

$$\int x^{-1+2(1+p)}\left(b+cx^2\right)^p\left(b+2cx^2\right)\,dx=\left(c\,x^2+b\right)^p\left(\frac{c\,x^{2\,p+1}\,x^3}{2\,p+2}+\frac{b\,x\,x^{2\,p+1}}{2\,p+2}\right)$$

```
[In] int(x^(2*p + 1)*(b + c*x^2)^p*(b + 2*c*x^2),x)
[Out] (b + c*x^2)^p*((c*x^(2*p + 1)*x^3)/(2*p + 2) + (b*x*x^(2*p + 1))/(2*p + 2))
```

3.1080 $\int x^{-1+3(1+p)}(b+cx^3)^p\left(b+2cx^3\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int x^{-1+3(1+p)}\left(b+cx^3\right)^p\left(b+2cx^3\right)\,dx=\frac{x^{3(1+p)}\left(b+cx^3\right)^{1+p}}{3(1+p)}$$

```
[Out] 1/3*x^(3+3*p)*(c*x^3+b)^(p+1)/(p+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {460}

$$\int x^{-1+3(1+p)}\left(b+cx^3\right)^p\left(b+2cx^3\right)\,dx=\frac{x^{3(p+1)}\left(b+cx^3\right)^{p+1}}{3(p+1)}$$

```
[In] Int[x^(-1 + 3*(1 + p))*(b + c*x^3)^p*(b + 2*c*x^3),x]
```

```
[Out] (x^(3*(1 + p))*(b + c*x^3)^(1 + p))/(3*(1 + p))
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=\frac{x^{3(1+p)}\left(b+cx^3\right)^{1+p}}{3(1+p)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.59

$$\int x^{-1+3(1+p)}\left(b+cx^3\right)^p\left(b+2cx^3\right)\,dx$$
$$=\frac{x^{3+3p}(b+cx^3)^p\left(1+\frac{cx^3}{b}\right)^{-p}\left(b(2+p)\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,-\frac{cx^3}{b}\right)+2c(1+p)x^3\,\text{Hyper}\right.}{3(1+p)(2+p)}$$

```
[In] Integrate[x^(-1 + 3*(1 + p))*(b + c*x^3)^p*(b + 2*c*x^3),x]
[Out] (x^(3 + 3*p)*(b + c*x^3)^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, -
((c*x^3)/b)] + 2*c*(1 + p)*x^3*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x^3
)/b)]))/(3*(1 + p)*(2 + p)*(1 + (c*x^3)/b)^p)
```

Maple [A] (verified)

Time = 6.59 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{x^{3+3p}\left(c\,x^{3}+b\right)^{1+p}}{3+3p}$	26
risch	$\frac{x\left(c\,x^{3}+b\right)x^{2+3p}\left(c\,x^{3}+b\right)^{p}}{3+3p}$	32
parallelrisch	$\frac{x^{4}x^{2+3p}\left(c\,x^{3}+b\right)^{p}c^{2}+x\,x^{2+3p}\left(c\,x^{3}+b\right)^{p}bc}{3c(1+p)}$	55

```
[In] int(x^(2+3*p)*(c*x^3+b)^p*(2*c*x^3+b),x,method=_RETURNVERBOSE)
[Out] 1/3*x^(3+3*p)*(c*x^3+b)^(1+p)/(1+p)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int x^{-1+3(1+p)}\left(b+cx^3\right)^p\left(b+2cx^3\right)\,dx=\frac{(cx^4+bx)(cx^3+b)^p x^{3\,p+2}}{3\,(p+1)}$$

```
[In] integrate(x^(2+3*p)*(c*x^3+b)^p*(2*c*x^3+b),x, algorithm="fricas")
[Out] 1/3*(c*x^4 + b*x)*(c*x^3 + b)^p*x^(3*p + 2)/(p + 1)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1+3(1+p)}\left(b+cx^3\right)^p\left(b+2cx^3\right)\,dx=\text{Timed out}$$

```
[In] integrate(x**(2+3*p)*(c*x**3+b)**p*(2*c*x**3+b),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int x^{-1+3(1+p)}\left(b+cx^3\right)^p\left(b+2cx^3\right)\,dx=\frac{(cx^6+bx^3)e^{(p\log(cx^3+b)+3p\log(x))}}{3(p+1)}$$

```
[In] integrate(x^(2+3*p)*(c*x^3+b)^p*(2*c*x^3+b),x, algorithm="maxima")
[Out] 1/3*(c*x^6 + b*x^3)*e^(p*log(c*x^3 + b) + 3*p*log(x))/(p + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(25) = 50.

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.07

$$\int x^{-1+3(1+p)}\left(b+cx^3\right)^p\left(b+2cx^3\right)\,dx=\frac{(cx^3+b)^pcx^4e^{(3p\log(x)+2\log(x))}+(cx^3+b)^pbxe^{(3p\log(x)+2\log(x))}}{3(p+1)}$$

```
[In] integrate(x^(2+3*p)*(c*x^3+b)^p*(2*c*x^3+b),x, algorithm="giac")
[Out] 1/3*((c*x^3 + b)^p*c*x^4*e^(3*p*log(x) + 2*log(x)) + (c*x^3 + b)^p*b*x*e^(3
*p*log(x) + 2*log(x)))/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 8.92 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

$$\int x^{-1+3(1+p)}\left(b+cx^3\right)^p\left(b+2cx^3\right)\,dx=\left(c\,x^3+b\right)^p\left(\frac{c\,x^{3\,p+2}\,x^4}{3\,p+3}+\frac{b\,x\,x^{3\,p+2}}{3\,p+3}\right)$$

```
[In] int(x^(3*p + 2)*(b + c*x^3)^p*(b + 2*c*x^3),x)
[Out] (b + c*x^3)^p*((c*x^(3*p + 2)*x^4)/(3*p + 3) + (b*x*x^(3*p + 2))/(3*p + 3))
```

3.1081 $\int x^{-1+n(1+p)}(b+cx^n)^p (b+2cx^n) \, dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int x^{-1+n(1+p)}(b+cx^n)^p (b+2cx^n) \, dx = \frac{x^{n(1+p)}(b+cx^n)^{1+p}}{n(1+p)}$$

```
[Out] x^(n*(p+1))*(b+c*x^n)^(p+1)/n/(p+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {460}

$$\int x^{-1+n(1+p)}(b+cx^n)^p (b+2cx^n) \, dx = \frac{x^{n(p+1)}(b+cx^n)^{p+1}}{n(p+1)}$$

```
[In] Int[x^(-1 + n*(1 + p))*(b + c*x^n)^p*(b + 2*c*x^n),x]
```

```
[Out] (x^(n*(1 + p))*(b + c*x^n)^(1 + p))/(n*(1 + p))
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{x^{n(1+p)}(b+cx^n)^{1+p}}{n(1+p)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 3.74

$$\int x^{-1+n(1+p)}(b+cx^n)^p\,(b+2cx^n)\;dx$$
$$=\frac{(b+cx^n)^p\,\left(1+\frac{cx^n}{b}\right)^{-p}\left(b(2+p)x^{n(1+p)}\text{Hypergeometric2F1}\left(-p,1+p,2+p,-\frac{cx^n}{b}\right)+2c(1+p)x^{n(2+p)}\text{H}\right.}{n(1+p)(2+p)}$$

```
[In] Integrate[x^(-1 + n*(1 + p))*(b + c*x^n)^p*(b + 2*c*x^n),x]
```

```
[Out] ((b + c*x^n)^p*(b*(2 + p)*x^(n*(1 + p))*Hypergeometric2F1[-p, 1 + p, 2 + p,
 -((c*x^n)/b)] + 2*c*(1 + p)*x^(n*(2 + p))*Hypergeometric2F1[-p, 2 + p, 3 +
 p, -((c*x^n)/b)]))/(n*(1 + p)*(2 + p)*(1 + (c*x^n)/b)^p)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(27) = 54.

Time = 6.68 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.22

method	result	size
parallelrisch	$\frac{x\,x^{n}x^{np+n-1}(b+c\,x^{n})^{p}c^{2}+x\,x^{np+n-1}(b+c\,x^{n})^{p}bc}{n(1+p)c}$	60

```
[In] int(x^(-1+n*(1+p))*(b+c*x^n)^p*(b+2*c*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] (x*x^n*x^(n*p+n-1)*(b+c*x^n)^p*c^2+x*x^(n*p+n-1)*(b+c*x^n)^p*b*c)/n/(1+p)/c
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int x^{-1+n(1+p)}(b+cx^n)^p\,(b+2cx^n)\;dx=\frac{(cxx^n+bx)(cx^n+b)^px^{np+n-1}}{np+n}$$

```
[In] integrate(x^(-1+n*(1+p))*(b+c*x^n)^p*(b+2*c*x^n),x, algorithm="fricas")
```

```
[Out] (c*x*x^n + b*x)*(c*x^n + b)^p*x^(n*p + n - 1)/(n*p + n)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 17.65 (sec) , antiderivative size = 116, normalized size of antiderivative = 4.30

$$\int x^{-1+n(1+p)}(b+cx^n)^p\,(b+2cx^n)\ dx = \frac{b^{p+1}c^{-p-1}c^{p+1}x^{np+n}\Gamma(p+1)\,{}_2F_1\left(\begin{matrix}-p,p+1\\p+2\end{matrix}\middle|\frac{cx^ne^{i\pi}}{b}\right)}{n\Gamma\left(p+2\right)} + \frac{2b^{p+2}cc^{-p-2}c^{p+2}x^{np+2n}\Gamma(p+2)\,{}_2F_1\left(\begin{matrix}-p,p+2\\p+3\end{matrix}\middle|\frac{cx^ne^{i\pi}}{b}\right)}{b^2n\Gamma\left(p+3\right)}$$

```
[In] integrate(x**(-1+n*(1+p))*(b+c*x**n)**p*(b+2*c*x**n),x)
[Out] b**(p + 1)*c**(-p - 1)*c**(p + 1)*x**(n*p + n)*gamma(p + 1)*hyper((-p, p +
1), (p + 2,), c*x**n*exp_polar(I*pi)/b)/(n*gamma(p + 2)) + 2*b**(p + 2)*c*c
**(-p - 2)*c**(p + 2)*x**(n*p + 2*n)*gamma(p + 2)*hyper((-p, p + 2), (p + 3
,), c*x**n*exp_polar(I*pi)/b)/(b**2*n*gamma(p + 3))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int x^{-1+n(1+p)}(b+cx^n)^p\,(b+2cx^n)\ dx = \frac{(cx^{2\,n}+bx^n)e^{(np\log(x)+p\log(cx^n+b))}}{n(p+1)}$$

```
[In] integrate(x^(-1+n*(1+p))*(b+c*x^n)^p*(b+2*c*x^n),x, algorithm="maxima")
[Out] (c*x^(2*n) + b*x^n)*e^(n*p*log(x) + p*log(c*x^n + b))/(n*(p + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(27) = 54.

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.44

$$\int x^{-1+n(1+p)}(b+cx^n)^p\,(b+2cx^n)\ dx = \frac{(cx^n+b)^pcxx^ne^{(np\log(x)+n\log(x)-\log(x))}+(cx^n+b)^pbxe^{(np\log(x)+n\log(x)-\log(x))}}{np+n}$$

```
[In] integrate(x^(-1+n*(1+p))*(b+c*x^n)^p*(b+2*c*x^n),x, algorithm="giac")
[Out] ((c*x^n + b)^p*c*x*x^n*e^(n*p*log(x) + n*log(x) - log(x)) + (c*x^n + b)^p*b
*x*e^(n*p*log(x) + n*log(x) - log(x)))/(n*p + n)
```

Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.00

$$\int x^{-1+n(1+p)}(b+cx^n)^p\,(b+2cx^n)\ dx=\left(\frac{b\,x\,x^{n\,(p+1)-1}}{n\,(p+1)}+\frac{c\,x\,x^n\,x^{n\,(p+1)-1}}{n\,(p+1)}\right)\,(b+c\,x^n)^p$$

```
[In] int(x^(n*(p + 1) - 1)*(b + c*x^n)^p*(b + 2*c*x^n),x)
[Out] ((b*x*x^(n*(p + 1) - 1))/(n*(p + 1)) + (c*x*x^n*x^(n*(p + 1) - 1))/(n*(p +
1)))*(b + c*x^n)^p
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                      convert(ExpnType_result,string)," vs. order ",
                      convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):



    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```